THE IMPACT OF CONSTRUCTIVIST-BASED TEACHING METHOD ON SECONDARY SCHOOL LEARNERS' ERRORS IN ALGEBRA

by

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DECLARATION

I declare that the project, "*The impact of constructivist-based teaching method on secondary school learners' errors in algebra*" is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

______________ 17 - 07 - 2015

MR J OWUSU DATE

DEDICATION

To my wife, Mrs Linda Konadu Owusu

To my daughters, Jopa Adom Owusu, Hephzibah Nhyira Owusu

and Jessica Kumi Owusu

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ABSTRACT

The aim of this study was to investigate the comparative effects of Constructivist-Based Teaching Method (CBTM) and the Traditional Teaching Method (TTM) on Grade 11 Mathematics learners' errors in algebra. The constructivist learning theory (CLT) was used to frame this study. Mainly, CLT was used to influence the design of CBTI to hone participants' errors in algebra that militate against their performance in Mathematics. The study was conducted in the Mpumalanga Province of South Africa with a four-week intervention programme in each of the two participating secondary schools. Participants consisted of n=78 Grade 11 Mathematics learners and one Grade 11 Mathematics teacher. A non-equivalent control group design consisting of a pre-test and post-test measure was employed. The Grade 11 teacher in the control school employed the TTM while the researcher implemented CBTM in the experimental school.

The main aspects of CBTM entailed participants' construction of their own knowledge from the base of prior knowledge and through group learning approach and exploratory talk in which discussions included argumentation, verbalising explanations, justifications and reflections. Participants in experimental school became familiar with the basic principles of CBTI such as group work, which enhanced the construction of conceptual understanding of algebraic concepts. This reduced most of the errors they commit in algebra and elevated their performance in Mathematics. The principal instruments for data collection consisted of a standardised Algebra Concept Achievement Test and lesson observations.

The pre-test was used to determine participants' initial errors in algebra before the intervention. A post-test was given at the end of intervention to ascertain change in participants' errors in algebra over a four-week intervention period. Using descriptive and inferential statistical techniques, the study found that participants in experimental school significantly reduced their errors in algebra than those in control school. The study showed that CBTM was a more effective pedagogy that improved the errors Grade 11 learners commit in algebra than the TTM.

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LIST OF KEY TERMS

Constructivist-based teaching method

Teaching Strategies or Approaches

Poor performance

Traditional or Conventional instruction

Constructivism

Radical constructivism

Social constructivism

Group learning

Worked-out examples

Exploratory talk

Errors

Scaffolding

Zone of Proximal Development

LIST OF ABBREVIATIONS

ACAT	Algebra Concepts Achievement Test
ANA	Annual National Assessment
C2005	Curriculum 2005
CAPS	Curriculum and Assessment Policy Statement
CBTM	Constructivist-Based Teaching Method
CBTI	Constructivist-Based Teaching Instruction
CSMS	Concepts in Secondary School Mathematics
CLT	Constructivist Learning Theory
DBE	Department of Basic Education
DoE	Department of Education
EER	End-of-year Examiners' Report
EFRM	Examination Feedback Resource Material
FET	Further Education and Training
H_0	Null hypothesis
H_1	Alternative hypothesis
IBM	International Business Machine
KZN	Kwa-Zulu Natal
LTSM	Learning and teaching support materials
MDE	Mpumalanga Department of Education
NCS	National Curriculum Statement
NDR	National Diagnostic Report
NRC	National Research Council
NSC	National Senior Certificate
NSCDR	NSC Diagnostic Report
OBE	Outcomes-Based Education
REC	Research Ethics Committee
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SPSS	Statistical Package for Social Sciences
TIMSS	Trends in International Mathematics and Science Study
TIMSS-R	TIMSS-Repeat

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CHAPTER ONE

A THEORETICAL OVERVIEW OF THE STUDY

1.1 INTRODUCTION

Poor performance in Mathematics is a global problem. In addition, learners'[1] errors in algebra have been associated with poor performance in Mathematics. Luneta and Makonye (2010) established that algebraic errors present epistemological challenges that have a negative impact in learning calculus. One of the most common algebraic errors learners make is writing the expression *3x+2* as *5x* and simplifying the expression *7–5y* as *2y*. Research suggests that some of algebraic errors made by learners are a result of the teachers' instruction (Fleisch, 2008). Hence this study focused on teachers' instructional methods in order to address observed learners' errors in algebra.

The current study investigated the comparative impacts[2] of a constructivist-based teaching method (CBTM) and traditional teaching methods (TTM) on the exposition and treatment of learners' algebraic errors in Grade 11. On the whole, the current study aimed to improve the Mathematics performance of Grade 11 learners in White River Circuit in one of the districts in the Mpumalanga[3] Province in South Africa. The district's name is Ehlanzeni. The investigation, which was conducted as a quasi-experimental design, consisted of n=78 Grade 11 Mathematics learners from two disadvantaged secondary schools (see, Section 1.8.5). One Grade 11 Mathematics teacher employed the TTM in the control group while the researcher employed the CBTM in the experimental group. The experiment lasted for four weeks.

The constructivist learning theory underpinned this study. The tenets of constructivist-based teaching indicate that learners should have the autonomy to actively

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1. In this study the terms *learner* and *student* are treated synonymously. In practice the term *learner* is reserved for one who studies at primary or secondary school and the term *student* is reserved for someone who is older and is studying at a higher education institution.
2. In this study the word *impact* refers to how the instruction, which referred to the method of teaching, led to the exposition and subsequent treatment of learners' errors.
3. Mpumalanga is one of the nine provinces in South Africa.

participate in the lesson to construct their own knowledge through group learning, mathematical discourse, and exploratory talk. Hence a group learning approach was largely incorporated in the CBTM lessons to facilitate the exposition of learners' errors through verbalisation of their mathematical thoughts, which subsequently led to error treatment through guided peer group interactions.

The traditional algebra class for this study was considered to be one that is mainly teacher-centred, textbook-driven, transmission-oriented and with practice algebraic problems done by learners individually in a non-group setting. The teaching and learning environment that is described in the preceding sentence was considered in the current study as the traditional teaching method (TTM), which largely characterised algebraic lessons (conducted by the teacher) in a control group. In this traditional classroom setting, the teacher takes charge of a lot of the intellectual work in that classroom. The teacher plans the scope and sequence, pre-synthesizes and pre-packages most of the learning (Brooks & Brooks, 1999). It is the researcher's view that at time of the study the TTM was the dominant teaching method in South Africa considered to accord opportunity to complete the syllabi within the stipulated time.

1.2 AIM OF THE STUDY

The main aim of this study was to investigate the comparative effects of a constructivist-based teaching method (CBTM) and the traditional teaching methods (TTM) on Grade 11 learners' errors in algebra, in terms of exposing the errors and subsequently providing a treatment for the observed errors. Mainly, the current study sought to highlight the significance of using CBTM as an effective instructional tool to teach certain algebraic topics in selected Grade 11 algebraic topics by focussing on error exposition and treatment.

1.3 THE OBJECTIVES OF THE STUDY

In order to achieve the aim of the study the following objectives were set out:

- To use a group learning approach to facilitate the exposition and treatment of learners' errors when certain algebraic topics are treated in a Grade 11 mathematics lesson;

- To observe the traditional methods of teaching in term of exposing and treating learners' algebraic errors in algebra Grade 11 lesson; and,
- To measure the effect of error treatment when the constructivist-based teaching method is compared with the traditional teaching method.

1.4 CONTEXTUAL BACKGROUND OF THE STUDY

South Africa has experienced several curriculum reforms since becoming a democratic country in 1994. Despite these curriculum reforms the performance of learners in Mathematics still remains a national concern. According to Nkhoma (2002), from 1994 democratic attempts have been made to improve the quality of Mathematics instruction, particularly in black township schools, in order to elevate learners' basic mathematical skills and subsequently reduce their errors. However, only little progress has been made thus far. Statistics and research indicate that learners' performance in Mathematics in South Africa at the National Senior Certificate (NSC) examination and in the Trend in International Mathematics and Science Study (TIMSS) is persistently poor.

For instance, in the 1995 TIMSS study, Grade 8 learners from South Africa participated alongside 41 countries in Mathematics but were ranked in the last position with a mean score of 351 points out of a possible 800 points (Howie, 2001). This mean was significantly lower than the international benchmark of 513 (Howie, 2001; Mji & Makgato, 2006). In the TIMSS-R 1999, South African learners scored a mean of 275 in Mathematics which was far below the international mean of 487. This mean was lower than that of Morocco, Tunisia and other developing countries such as Chile, Indonesia, Malaysia and the Philippines (Howie, 2001; Mji & Makgato, 2006). The TIMSS results in 2003 showed no improvement and even African countries like Egypt, Botswana and Ghana that participated for the first time in 2003 performed better than South Africa, which had participated in the previous TIMSS editions (Reddy, 2006). The results of the Southern African Consortium for Monitoring Education Quality (SACMEQ) show a similar trend, thus highlighting South Africa's poor performance in Mathematics. For instance, in 2000 SACMEQ conducted an evaluation of Grade 6 Mathematics and reading ability in 14 countries and South African learners performed poorly. Also, the SACMEQ (2011) report indicated that

South Africa's mean score for Mathematics was 481.1, which was below the collective SACMEQ average of 500.

In February 2011, more than six million Grade 3 and Grade 6 learners throughout South Africa wrote the Annual National Assessment (ANA[4]) tests in literacy, numeracy, language and Mathematics (Department of Basic Education [DBE], 2011). The national average performance in Mathematics in Grade 6 was 30% (DBE, 2011). According to DBE (2011: 20) in the 2011 ANA "only 12% of Grade 6 learners scored 50% or more in Mathematics". Among Grade 3 learners: "only 17% scored more than 50% in their numeracy assessment; and the national average was 28%" (DBE, 2011: 20). In the wake of these findings, the ANA report concluded that "the challenges for the schooling system in South Africa remain great" (DBE, 2011: 36). These findings show that the traditional methods of teaching Mathematics are not effective in improving learners' performance, thus alleviating learners' errors in mathematical tasks.

The Mathematics results in Grade 12 also raise concern. The National Senior Certificate (NSC) Grade 12 results for the past four years show a marginal improvement in Mathematics (see, Table 1.1). The 2009 National Senior Certificate Diagnostic Report (NSCDR, 2009; 49), states that there was "a need for serious intervention in Mathematics and Physical Science, which performed lower than the other subjects".

Table 1.1: Overall NSC Grade 12 performance trends in mathematics (2010-2013)

Year of examination	Number who wrote	Number who achieved at 30% and above	% achieved at 30% and above	Number who achieved at 40% and above	% achieved at 40% and above
2010	263 034	127 785	47.4	81 473	30.9
2011	224 635	106 327	46.3	61 592	30.1
2012	225 874	121 970	54.0	80 716	35.7
2013	241 509	142 666	59.1	97 790	40.5

Source: NSC Diagnostic Report (NSCDR 2013: 125)

4. The Annual National Assessments (ANA) are grade-specific language and mathematics standardized tests for Grade 1 to Grade 6, and Grade 9 learners that have been arranged by the Department of Basic Education (DBE) in South Africa to monitor and detect learners' problems in specific subjects (DBE, 2012).

In almost all TIMSS and SACMEQ studies, including the ANA and NSC assessments, learners' poor performance in Mathematics is largely characterised with errors they made while trying to respond to the problem solving tasks. A link between learners' errors in Mathematics and teachers' content knowledge which influence their instructional methods has been documented. In addition, a study by Bansilal, Brijlal and Mkhwanazi (2014) revealed that Grade 12 Mathematics teachers in Kwa-Zulu Natal[5] (KZN) Province performed poorly when tested with a past NSC Mathematics paper. Of the 253 Mathematics teachers who participated in the study that explored their mathematical content knowledge most got an average of 59%. A quarter (25%) of the teachers got below 39% (Bansilal *et al*., 2014: 16). The findings of the Bansilal *et al.*' s (2014) study raise concerns about the way Mathematics is taught in South African classrooms when some of the teachers are found to be in possession of poor subject content knowledge.

Swan (2006) highlighted that the approach a teacher takes when teaching a concept in mathematics is influenced by their own conception of those concepts as well as what they want the learners to be able to do with those concepts. Berstein (2011) reported the important and major roles played by teachers in the performance or non-performance of learners. The report by Berstein (2011) also confirmed that the poor performance of teachers is a major reason for the poor performance of learners in the South African schooling system.

Among other things Fleisch (2008) attributed causes of learners' errors in Mathematics to inappropriate teaching strategies. Research also shows that these instructional challenges are more common in disadvantaged schools, which are mostly located in rural and township settings (Dhlamini, 2011; Dhlamini & Mogari, 2011). The term "disadvantaged school" in this study refers to quintile 1 and quintile 2 under-resourced schools. Schools in South Africa are ranked by using a quintile system, which is based on the availability of both human and material resources. Moreover, the quintile rankings take into consideration factors like "income level, unemployment rate, and/ or level of education (literacy rate) of the surrounding community that determine the poverty index of the school" (NSCDR, 2009: 14). On the ranking continuum the

5. Kwa Zulu Natal (KZN) is one of the coastal provinces in South Africa.

most severely under-resourced schools could be placed in quintile 1 and the well-resourced schools are placed in quintile 5. On the basis of this rating system the two schools that participated in the current study were ranked as quintile 2 schools, indicating that they were disadvantaged schools.

Generally, poor performance in Mathematics in quintile 2 has been largely linked to learners' errors, which could be a result of inappropriate and ineffective methods of teaching (Fleisch, 2008). In addition, Gaigher (2006: 2) found that "in 1988, only 13.5% of the black teachers in secondary schools had a degree, and almost 40% had no qualifications to teach in secondary schools". During this time many black teachers depended on the security of a single textbook and notes that had been summarised for them (Gerard, 2011). However, this arrangement would certainly not contribute positively to the teachers' instructional methods, particularly in key subject areas. As a result, for many decades learners from township schools, which are largely placed as quintile 1 and quintile 2 schools, have suffered in the fields of Mathematics, Science and Engineering (Gerard, 2011; Van der Berg, 2007).

It is against this background that the focus of the current study was to highlight an effective teaching approach to expose and subsequently provide error treatment in a Grade 11 Mathematics classroom. To achieve this, the current study advocated the constructivist-based teaching method (CBTM) as an instructional tool to elevate learners' performance in selected Grade 11 algebraic topics. Ross and Wilson (2005: 127) recommended that "studies that examine effects of constructivist teaching approaches on algebraic understanding of different ideas and age groups of students need to be completed". This study therefore investigated the comparative effects of a constructivist-based teaching method (CBTM) and traditional teaching methods (TTM) on the exposition and treatment of Grade 11 learners' algebraic errors. The algebraic concepts that were explored in this investigation are variables, expressions, equations, and word-problems. Mainly, CBTM used group approach to expose and address learners' errors.

The impact of social interaction on learning is very essential for meaningful knowledge construction. Through a group learning approach and exploratory talk learners interacted and exchanged ideas regarding the algebraic errors they experienced. In addition, the relevance of Vygotsky's Zone of Proximal Development (ZPD) in developing learners' ability to restructure and exchange ideas through interaction with other learners was also explored in the CBTM lessons.

The CBTM that was used in this study is based on constructivist learning theories (CLT), which is a theoretical framework advocating that learning occurs when learners are actively involved in the process of meaning making and self-construction of knowledge (see, Section 1.8.3). Constructivist learning theory holds that learning always builds upon knowledge that a learner already knows to facilitate the construction of schema (Noddings, 1990). Tellez (2007: 553) believes that "constructivism provides a platform for learners to take charge of their learning by actively constructing their own knowledge". Several studies have supported the use of constructivist approach in science-related disciplines (Cobb, 1996; Dangel, 2011; Fox, 2001; Phillips, 1995). The influence of CLT in CBTM lessons presented CBTM as an effective instructional methods to meaningfully teach certain algebraic topics in Grade 11 Mathematics.

1.5 THE PROBLEM STATEMENT

As demonstrated in the preceding section, South African learners' performance in Mathematics has been poor for a very long time. As a practicing Mathematics teacher, I believe that in order to address learners' poor performance in this subject, it is necessary to explore learners' errors in algebra. Luneta and Makonye (2010) concluded that algebraic errors have negative impact in learning calculus, which constitutes another algebraic component of Mathematics. The NSCDR (2013), the Examination Feedback and Resource Material [EFRM] (2013) and the End-of-the-year Examiners' Reports [EER] (2012) for Grade 12 Mathematics have all highlighted an important variable contributing to learners' poor performance, which is the tendency by learners to do several errors when attempting to solve Mathematics tasks.

In my teaching experience, I have also observed that some of the basic errors learners consistently commit in a Grade 11 Mathematics lesson are: (1) simplifying the algebraic expression $9m–4m$ as 5 (*It seems in such mathematical phrases learners* *tend to treat numerals and letters separately, hence 9–4=5 and m–m=0, which ultimately produce a difference of 5*); (2) solving the equation *4x=12* as *x=12–4* and eventually arriving at the answer *x=8* (*In such instances learners are more prone to certain components of the equations as like terms*); and, (3) simplifying "*a x a"* as *2a* and *a+a* as a^2 (*In this case learners seem to struggle to distinguish between the operations of multiplication and addition*). The three examples present a few errors learners tend to do when they attempt to solve certain algebraic tasks in Grade 11 Mathematics classrooms. The connection between learners' errors and poor performance in Mathematics has been documented in research (Fajemidagba, 1986; Prakitipong & Nakamura, 2006; Rosnick, 1981).

This study explored teachers' instructional methods, which are believed to contribute to learners' errors (Fleisch, 2008). Studies have consistently highlighted teachers' instruction as an important variable to influence the performance of learners in Mathematics and could also be a contributing factor not only to the learners' poor performance but also the reason for the learners' errors in this subject (Shulman, 1995, 1987). Given the current state of learners' performance in Mathematics (see, Section 1.2) it is reasonable to argue that the traditional teaching methods are not providing meaningful instructional options to address learners' errors in mathematics, particularly in algebra. Traditional teaching methods are known to limit learners' participation in the lesson and to be more teacher-centred in an attempt to chase the syllabus coverage.

In the light of the foregoing background the current study identified a need to search for responsive instructional method to address learners' errors in algebra in terms of exposing the errors and thereby providing a treatment for the observed errors. On this basis this current study investigated the comparative impacts of a constructivist-based instruction (CBTM) and traditional teaching methods (TTM) on learners' errors in algebra in order to search for an alternative instructional approach to improve learners' performance in Grade 11 Mathematics.

1.6 RESEARCH QUESTIONS

The following research questions guided this study:

1.6.1 What characterises the teaching and learning activities in a constructivist-based teaching method and traditional teaching method during a Grade 11 algebraic lesson?

1.6.2 How do the constructivist-based teaching method and the traditional method facilitate the exposition of learners' errors in a Grade 11 algebraic lesson?

1.6.3 What is the comparative effect of constructivist-based teaching method and the traditional teaching method on the treatment of learners' errors in Grade 11 algebraic classrooms?

Given the comparative nature of this study the first research question intended to document the distinguishing aspects of two comparative instructional (teaching) methods, namely, the CBTM and the TTM. The second research question aimed to document the relative potential of each teaching method to expose learners' errors in algebra. The third research question was intended to generate statistical measurement to compare the effect of each teaching method on the treatment of learners' errors before and after the interventions.

1.7 RATIONALE FOR THE STUDY

Mathematics is important for both individuals and the country. According to the National Research Council [NRC] (1989), "Mathematics opens doors to careers and further studies, and it enables informed decisions for nations as it provides knowledge to compete in a technological economy" (p. 1). The NSC Examiner and Internal Moderators report contained in the Mpumalanga Department of Education [MDE] (2013) and the EFRM (2013) revealed that algebra holds the key to improving the performance of learners in Mathematics. In addition, the EFRM (2013) states that, "learners who do very well in algebra tend to do well in other sections (topics) of mathematics or in other subjects" (p. 25). The EFRM (2013) adds that "in most cases the algebraic manipulation is lacking and that cost learners marks" (p. 25).

Some of the factors cited by the EFRM (2013) as the causes of poor performance in Mathematics included: (1) lack of basic skills (to compute the product, doing factorisation, operations with integers and fractions, simplifying algebraic fractions, subject of a formula, etc.); (2) lack of effective teaching and meaningful learning; (3) incorrect use of mathematical language and related notation, which tends to lead to learners making more mathematical errors; and, (4) lack of expressing the same mathematical concept differently. In the same vein, NSCDR (2013: 125) attributed poor performance in the 2013 NSC Mathematics paper 1 and paper 2 examinations to "the errors learners make in algebraic simplification, substitution and solving equations in two unknown". Thus the algebraic errors made by learners seem to influence their performance in Mathematics.

Given this background, the rationale for conducting the current investigation emanated from the fact that: (1) at the time of the current study there was paucity of local studies to explore the influence of learners' algebraic errors on their overall mathematical performance; and, (2) at the time of the current study there was a paucity of local studies to explore the influence of instructional methods on learners' errors in Grade 11 algebraic classrooms.

1.8 SIGNIFICANCE OF THE STUDY

The knowledge of how learners experience difficulties when learning certain specific algebraic concepts such as, algebraic variables, expressions, equations and word-problems is very important in Mathematics education. Without adequate knowledge of how learners undergo the learning of basic algebraic concepts or operations the teacher may underestimate the complexity of the learning process which might lead to learners making errors in algebra. For example, during the learning of *variables, mathematical expressions, equations* and *word-problems* at secondary school level, it is still not clear what errors learners make and how often learners tend to make them. It is also not clear where the errors come from and how the errors could be treated. Not being aware of learners' errors could limit teachers to explore effective teaching strategies to help learners. Given this background, the significance of this current study was to highlight the effectiveness of a Constructivist-Based Teaching Method in exposing and treating learners' errors when they solve certain algebraic tasks in Grade 11. On the whole, the results of this study will introduce new and reformed instructional strategies to improve learners' performance in Mathematics in the White River Circuit in Mpumalanga.

1.9 DEFINITION OF KEY TERMS

The following operational terms are defined for the current study:

1.9.1 A philosophy of constructivism

This is a philosophy or an educational approach that is based on the premise that those who are engaged in a learning process construct their own understanding of the world through their experiences (see, Section 2.3).

1.9.2 Constructivist learning

Constructivist learning theory (CLT) posits that all knowledge is constructed from a base of prior knowledge. According to the CLT, learners are not blank slates and knowledge cannot be imparted effectively without the learner making sense of it according to his or her current conceptions. Brooks and Brooks (1999) stated that in the constructivist classroom the focus tends to shift from the teacher to the learners. In addition, Brooks and Brooks (1999) emphasised that the constructivist classroom is no longer a place where the teacher, who is considered an "expert", pours knowledge into passive learners who wait passively like empty vessels to be filled. In contrast, in the constructivist model, learners are encouraged to be actively involved in their own learning. The teacher functions as a facilitator who coaches, mediates, prompts, and who helps learners develop and assess their understanding and learning. The CBTM, which is espoused in the current study, largely incorporated elements of group and interactive learning that constitute a constructivist learning process. This instructional approach was opted in the current study for its potential to expose and provide a treatment for learners' errors when learners attempt to make ownership of their learning.

1.9.3 Constructivist-based teaching method (CBTM)

Constructivist-based teaching methods (CBTM) are based on constructivist learning theory (see, Section 2.3). In terms of this study, the constructivist-based classroom setting is the one in which learners actively participate in the learning process of selected Grade 11 algebraic topics with the view to encourage them to construct their own mathematical knowledge. As much as the CBTM approach incorporates the transmission-oriented approach, it largely embraces the elements of open-ended problem solving, constructive mathematical discussion, group learning, learner-centred and exploratory talk. In the context of the current study, the researcher, who also posed as a teacher in this investigation, acted more as a facilitator when learners discussed and argued their mathematical views in group learning settings. The learner-centred approach, which was followed in CBTM lessons, facilitated the exposition of learners' algebraic errors through interactive group discussions. Subsequently, the treatment of observed learners' errors was achieved through learners' argumentation and facilitation by the teacher (see, Section 3.8).

1.9.4 Traditional teaching method (TTM)

The traditional algebra class for this study was considered to be the one that was mainly teacher-centred, textbook-driven, transmission-oriented and with practice algebraic problems done by learners individually. The teaching and learning environment that is described in the preceding sentence was considered in the current study as the traditional teaching method (TTM), which largely characterised algebraic lessons (conducted by the teacher) in a control group. The researcher observed that in a TTM learning environment learners did not play an active role in the lesson. Learners were observed to be sitting in their desks and passively receiving the knowledge transmitted by the teacher to them. In this setting, learners were observed to be doing the tasks assigned to them by the teacher individually.

1.9.5 Disadvantaged learner

According to Tsanwani (2009: 12), the term disadvantaged learners refers to a group of learners "from populations with low social status, low educational achievement, tenuous or no employment, limited participation in community or organisation and limited ready potential for upward mobility". Disadvantaged learners have a tendency to commit errors in mathematical tasks because of their low educational background and support, and exposure to classroom instruction by unqualified and under-qualified teachers.

1.9.6 Algebraic errors

The phrase '*algebraic errors'* refers to the mistakes that learners tend to make when they solve certain tasks in algebra. According to the Free Dictionary, an error means a simple lapse of care or concentration, which almost everyone makes at least occasionally. In Mathematics, an error could refer to an observed deviation from an intended correct solution, which could result in wrongly answered problems, which have flaws in the process that generated the answers (Young & O'Shea, 1981). In terms of this study, an error was regarded as a mistake in the process of solving an algebraic problem algorithmically, procedurally or by any other method (see, Section 2.2).

1.9.7 A mathematical variable

A mathematical variable is a general purpose term in Mathematics for an entity that can take various values in any particular context. The domain of the variable may be limited to "a particular set of numbers or algebraic quantities" (Schoenfeld & Arcavi, 1988: 422).

1.9.8 A mathematical equation

An equation is used to model a change or situation. In Mathematics, an equation could be regarded as a statement that asserts the equality of two expressions usually written as linear array of symbols that are separated into left and right sides and joined by an equal sign.

1.9.9 Group approach

In everyday contexts the term 'group work', brings to mind the notion of people working together in order to achieve a certain objective. In the classroom setting the term group work has come to mean that "participants are engaged in a coordinated continuing attempt to solve a problem or in some other way construct common knowledge" (Mercer & Littleton, 2007: 25). In the current study the term "group approach" refers to an arrangement in which two or more participants (learners) worked together in a form of discussing algebraic tasks to achieve shared solutions. Among other things these discussions lead to the exposition of participants' errors, and subsequently the CBTM guided group interactions facilitated the treatment of participants' errors.

1.9.10 An exploratory talk

The term "exploratory talk" refers to a discussion in a group where one person builds critically and constructively on what others have said. Arguments and counterarguments are justified and alternative perspectives are offered. Exploratory talk is considered the most educationally relevant type of talk (Mercer, Wegerif & Dawe, 1999). Central to exploratory talk is the belief that collaborative thinking skills can be taught explicitly in order to enable both teachers and learners to understand talk as “thinking aloud with others” (Monaghan, 2004: 124). This view resonates with the aims of the new South African curriculum that collaborative and constructivist measures are important for meaningful learning to take place (DoE, 2003).

1.9.11 Schema

A schema is a mechanism in human memory that allows for the storage, synthesis, generalisation and retrieval of similar experiences (Marshall, 1995). A schema allows an individual to organise similar experiences in such a way that the individual can easily recognize additional similar experiences. Schemas are triggered when an individual tries to comprehend, understand, organize, or make sense of a new situation (Greeno, Collins & Resnick, 1996). In knowledge construction, there is always a base structure from which to begin construction and this is called a structure of assimilation. The process of continual revision of structures is called accommodation (Noddings, 1990).

1.9.12 Scaffolding

Scaffolding refers to the process of providing learners with instructional support in the initial stages of learning a new subject. A 'scaffold' provides an assurance that learners are not left to understand new knowledge by themselves. The support is removed when the learner is ready like the scaffolding that supports workers who have been constructing or repairing a building, which is removed when construction is complete. As a learner is learning a new concept in an algebra class, the learner might observe it being done step-by-step by a more advanced peer in a small group or by a teacher. This support is the 'scaffold' the learner needs temporarily. Each step is demonstrated and explained, and then the learner tries it alone without the scaffold.

1.9.13 Zone of Proximal Development (ZPD)

The Zone of Proximal Development (ZPD) refers to the limit to which the learner can perform given tasks alone. Beyond that limit his or her success depends on support from other people such as the teacher or a peer in the group. Vygotsky (1978) refers to this limit as ZPD. The ZPD is therefore considered in this current study as the boundary between what a learner can successfully do without support and what he or she will be able to do after the support. Through the ZPD the more knowledgeable learners help the less knowledgeable to gain more knowledge to understand better and this may enable learners to construct new knowledge. It was advocated in the current study that when learners gain better understand the tendency to make algebraic errors could be minimised (see other discussions in Section 2.4).

1.10 ORGANISATION OF THE DISSERTATION

The dissertation consists of five chapters which are arranged to provide deeper insight into the issues raised in Chapter 1 (see, Section 1.1 & Section 1.4), and to provide answers to the research questions that guided the current study (Section 1.3). Chapter 1 provides a theoretical overview of the study. The following issues are addressed in Chapter 1: the introduction and the contextual background of the study, the statement of the problem, the research questions, the aim and objectives of the study, the significance of the study and the rationale for the study. Chapter 2 provides a comprehensive review of related literature to the study, and subsequently a conceptual framework for the study is developed in this chapter.

Chapter 3 provides an outline of a research methodology, which guided the current study. Among other things, Chapter 3 addresses issues relating to the research design that was employed in this study; the study population and sampling techniques that were used in this study. In addition, the following issues are also addressed in Chapter 3: instrumentation, data collection procedures, and data analysis techniques and issues relating to ethical considerations for the current study. Chapter 4 presents the analysis of data for the study. The methods used for data analysis are clearly demonstrated in this chapter. On the whole, the results of the study are presented in Chapter 4. In Chapter 5, the summary of the findings and conclusions are presented. Chapter 5 also presents the recommendations relating to the results of the study; and finally the gaps and limitations of the study are identified in same chapter.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter provides a discussion relating to the following issues: a review of literature pertained to learners' errors in terms of four conceptual areas in algebra; designing instruction to address learners' errors in algebra; the construction of knowledge and issues of teaching and learning in a constructivist learning setting; the comparison between constructivist-based and traditional ideas about teaching; review of literature relating to the constructivist-based instruction, its benefits and critical perspective of constructivism are reviewed in this chapter. In addition, this chapter provides an outline of how to implement a constructivist based teaching method in order to facilitate the reduction of learners' errors. Types of errors learners do when then they solve algebraic tasks also forms part of the discussions in Chapter 2. The Chapter 2 concludes by identifying a suitable theoretical framework for the current study.

2.2 ERRORS RELATING TO FOUR CONCEPTUAL AREAS IN ALGEBRA

Algebra has been described by a number of researchers (Kriegler, 2008) as a field with several aspects including abstract arithmetic, the language, and the tool for the study of functions and mathematical modelling aspects. Kesianye, Durwaarder and Sichinga (2001) reported that the traditional formal approach to teaching algebra looked at algebra as purely a mathematical discipline without linking it to day-to-day circumstances. This created a situation where at the end of algebra course, learners would realise no necessity for it because of its abstract nature thereby resulting in numerous errors. Manly and Ginsburg (2010) stated that algebra teaching is likely to focus on fundamental issues of symbols manipulation, simplifying expressions, and solving equations. This approach of teaching promotes rote learning and does not represent the coherent picture of algebra nor promote conceptual understanding required to avoid making mistakes.

The current study considered learners' errors in terms of four conceptual areas in Grade 11 algebra, namely, the mathematical variables, algebraic expressions, algebraic equations and word problems. This study focussed on these components or areas of algebra because in reviewing the related literature (see, EER, 2012; EFRM, 2013; NSCDR, 2013), it was revealed that, the sections in algebra where learners made most errors that affected their basic mathematical manipulations and their performance in mathematics were in the areas of algebraic variables, expressions, equations, and word-problems. The following sub-sections provide discussion in relation to the four conceptual areas in algebra.

2.2.1 Types of errors

Luneta and Makonye (2010) defined errors as discursive mistakes and challenges learners display in their responses to mathematics tasks. Luneta and Makonye (2010) identified two types of errors, namely, the systematic and the unsystematic errors. According to Lukhele, Murray and Olivier (1999), unsystematic errors are exhibited without the intention of learners and such errors may not be repeated. However, learners can correct unsystematic errors independently (Lukhele *et al*., 1999). In contrast, systematic errors may be repeated, systematically constructed or reconstructed over a period of time due to the grasping of incorrect conceptions when solving a particular problem (Idris, 2011). According to Watanabe (1991), some learners use short cuts to solve mathematical problems, which may result in errors. Erbas (2004) described errors as incorrect application and conclusion of mathematical expressions and ideas.

2.2.2 Learners' difficulties with algebraic variables

Knuth, Alibali, Hattikudor, McNeil and Stephens (2008) emphasised that variables are one of the core algebraic ideas and that the concepts of variable play a very important role in problem solving as well as in thinking and communicating mathematically. The use of variable is important as it forms the basis of generalisations. Understanding the different use of variables is important for learners' success in algebra. Failure to understand this could lead to learners making errors when solving problems. Lodholz (1999) stated that understanding variables, equality, relationship and the technical language of algebra are key requirements for success in the subject.

Letters may be used to represent different meanings in different contexts. The inclusion of letters in algebraic expressions or equations may cause difficulties to the learners because of the variety of meanings that a single letter can take in different contexts. For example, the same letters may carry different meaning in arithmetic and different meaning when it is algebra. In this regard Kieran (1992) explained that in arithmetic, 12m could mean 12 meters, which refers to the number of times a meter is replicated or "*12 times the units of meters appear*". However, in algebra the phrase "12m" could be interpreted as meaning "*12 times some unknown number of meters*" (Kieran, 1990). This means that in algebra, the letter "*m*" could be interpreted as representing the unknown quantity. Therefore, in this context the letter "*m*" may carry two different meanings. Philipp (1999) identified seven categories in which the letters of the alphabet are used to group variables with examples to illustrate their uses. Letters could be used as:

- labels, as is the case with "f" and "y" in *3f=1y*, denoting 3 feet in 1 yard;
- constants π, e, and c;
- unknowns to denote x in $5x - 9 = 11$;
- generalized numbers to denote a and b in $a+b=b+a$;
- varying quantities to denote x and y in $y=9x-2$;
- parameters to denote m and b in $y=mx+b$; and,
- abstract symbols to denote e and x in $e * x = x$.

A detailed classification about children's interpretation of letters is provided by Kuchemann (1981) reporting from the program Concepts in Secondary Mathematics and Science (CSMS). Kuchemann (1981) administered a 51-item paper-and-pencil test to 3000 British secondary school learners. Using a category originally developed by Callis (1975, cited in Kuchemann, 1981) categorised each item in the test in terms of six levels, namely, (1) letter evaluated; (2) letter ignored; (3) letter as an object (4) letter as a specific unknown; (5) letter as a generalised number; and, (6) letter as a variable. Examples of some of Kuchemann (1981) categories are provided in Table 2.1.

Table 2.1: Kuchemann (1981) categories that may lead to learners' algebraic errors

Kuchemann (1981) category	Related category example
1	What can you say about *a* if *a+5 = 8*?
2	If *n −246=762*, then what is *n −247*?
3	Simplify *2a+ 5b+ a*
4	Add *4* onto *3n*
5	What can you say about *c* if *c + d =10*, and *c* is less than *d*?
6	Which is larger *2n* or *n +2*?

The results of Kuchemann's (1981) study indicated that learners' interpretations of letters, such as those given in Table 2.1, were partly depended on the nature and complexity of the question. Philipp (1999) and Kuchemann (1981) classifications provide suitable examples of instances where letters could be used in different situations. Philipp's category is broader in the sense that it includes some of Kuchemann's categories. Another instance of learning difficulty encountered when learners are learning algebraic variables is the variety of meanings that a single letter can take in different contexts. Macgregor and Stacey (1997) found that the majority of learners up to the age of 15 years committed this error as they could not interpret algebraic letters as generalised numbers or even as specific unknowns. Learners simply ignored the letters and "replaced them with numerical values or regarded the letters as standing for shorthand names" (Macgregor & Stacey, 1997: 69). Furthermore, Macgregor and Stacey (1997) claim that the principal explanation given in the literature for this type of error has a general link to levels of cognitive development. However, Macgregor and Stacey (1997) provided alternative explanations for specific origins of misinterpretation that have been overlooked in the literature, which may or may not be associated with cognitive level.

Stacey and MacGregor's argument boils down to the unique language of algebra with its rules, conventions and practices. Mathematical ideas often need to be reformulated before they can be represented as algebraic statement and symbolic notation. One of the difficulties for learners is how to interpret these symbols correctly. The rules for interpreting and manipulating mathematical symbols are not always in accord with the way relationships are conveyed through the English language. Lannin (2005) supported this argument by stating that learners often fail to understand the meaning linked with the formal symbols they use including the operational symbols. The origins of misinterpretation mentioned in the Macgregor and Stacey's (1997) study are:

- intuitive assumptions and pragmatic reasoning about a new notation;
- analogies with familiar symbol systems;
- interference from new learning in mathematics; and;
- the effects of misleading teaching materials.

Macgregor and Stacey (1997: 75) state that the Roman Numeral System is an example for the "analogies with familiar symbol systems" category. In the ancient Roman Numeral System VI means *'1 more than 5'* and IV means *'1 less than 5'*, which indicates that the position and the value of one numeral will change the value of the other numeral. This analogy causes learners to apply their experiences in one number system to a different system where it is inapplicable, thus resulting in an error.

2.2.3 Learners' difficulties with algebraic expressions

Letters are used to build up algebraic expressions. One letter or a combination of letters could be used in an expression. Therefore, there is a close relationship of understanding the meaning of letters in the context of an expression. Mamba (2012) pointed out that the abstract nature of algebraic expressions such as understanding or manipulating them according to accepted rules, procedures, or algorithms posed many problems to learners. Erbas (2004) described errors as incorrect application and conclusion of mathematical expressions and ideas. Blanco and Garrotte (2007), Li (2006), and Erbas (2004) suggested that one of the causes of errors in learning algebra emanates from some obstacles such as lack of closure. That is to say some learners see algebraic expressions as statements that are at times incomplete. Hall (2002) suggested that learners tend to be reluctant to stop before getting to an answer they are comfortable with which is usually a numerical answer. Agnieszka (1997) commented on some misleading instances where learners use objects for symbols or they often refer letters to real life objects. For example, sometimes learners may interpret the algebraic expression *8a* as shorthand for "*8 apples*". Such procedures are efficient in the case of simple tasks such as transforming *2a + 3a* as two apples plus three apples. These interpretations are categorised as lower forms of understanding and they are not sufficient for somewhat more difficult tasks. Agnieszka (1997) provided an example of an expression such as $3a-b+a$, where such low-level procedures cannot be used but both younger and older learners still use the same object such as an apple to represent both a and b.

The duality of mathematical concepts as processes or objects depending on the problem situation and on the learner's conceptualisation provides an explanation for why learners commit the error of conjoining unlike terms such as $4t + 5$ meaninglessly to arrive at $9t$ as the final answer. Conjoining letters in algebra refers to an act of meaninglessly connecting together mathematical letters. This incorrect connection could make learners to commit errors. The researcher was inclined to support this view and considered it to be relevant to the current study because from personal classroom experience most learners persistently make conjoining errors in algebraic expression where they frequently simplify expression such as $5x +3$ to $8x$. Bosse' and Faulconer (2008) affirmed that conjoiners constitute an important component of learners' source of errors in algebra.

One of the most essential steps in learning Mathematics is objectification, which refers to making an object out of a process. This is reflected in the Mathematics curriculum as a way of developing operational thinking, that is, thinking about a process in terms of operations on objects (Dreyfus, Artigue, Eisenberg, Tall & Wheeler, 1990). Due to this dual nature of mathematical notations as processes and objects learners encounter many difficulties. For example, learners may see $3x+2$ as standing for: (1) the process '*add three times x and two*'; and, (2) for an object as $3x+2$. This dual conception may cause learners to be confused between conceptualizing $3x+2$ as a process or as an object. The potentially resulting error is that learners may simplify $3x+2$ to $5x$ as the final answer, when $3x+2$ should actually be conceptualized as an object.

Rule and Hallagan (2006) comments on a teacher model in which learners were asked to visually represent an algebraic expression given in four different forms. The same expression was given in four different forms as follows: 4(s+1); s+s+s+s+4; 2s+2(s+2); and 4(s+2)−4. A square pool with measurements $s \times s$ and a small square with measurements 1×1 were given as manipulative to illustrate the border of a square pool in four different ways related to the above four expressions. There were four main conclusions. Firstly, transition from arithmetic to algebra takes time for learners. Secondly, learners preferred numerical answers and to conjoin algebraic terms. Thirdly, on a positive note, visual representations helped learners to understand the algorithms in algebra. Fourthly, learners could not understand the concept of a variable clearly. Researchers have differences in opinions about reasons for this error.

Given the similar meanings of '*and*' and '*plus*' in natural language learners may consider *ab* to mean the same as *a+b* (Stacey & MacGregor, 1994; Tall & Thomas, 1991). Learners may erroneously draw on previous learning from other subjects that do not differentiate between conjoining and adding. For example, in Chemistry, adding oxygen to carbon produces CO_2. Stephen (2005) explains this tendency as a difficulty in accepting the lack of closure property of algebraic letters. Learners perceive open algebraic expressions as 'incomplete' and try to 'finish' them by oversimplifying. For example, they consider an algebraic expression such as "*m+n*" as incomplete and try to simplify it to "*mn*". A typical explanation for this error is the tendency in many arithmetic problems to have a final single-digit answer or to interpret a symbol such as '+' as an operation to be performed, thus leading to conjoining of terms (see, Tall & Thomas, 1991).

Many common errors in simplifying algebraic expressions seem to be instances of the retrieval of correct but inappropriately applied rules (Matz, 1980, cited in Gunawardena, 2011). For example, learners incorrectly apply $\frac{ax}{bx} = \frac{a}{b}$ into expressions like $\frac{a+x}{b+x} = \frac{a}{b}$. This is an application of a known mathematically correct rule to an inappropriate situation by incorrectly perceiving the similarities of the two situations. These instances result in mathematical errors. In addition, Schoenfeld (1985) argued that an inappropriate use of arithmetical and algebraic procedures is called an algebraic bug. Bugs are procedures that are correct in some situations but may be incorrect if applied to other situations. As an example, Schoenfeld (1985) described that learners sometimes write *x(yz)=xy+xz* by considering the transformation *x(y+z)=xy +xz*. The application of the distributive law is incorrect when the parenthetical values are multiplied. Lack of understanding of the structural features of algebra may cause this type of error.

2.2.4 Learners' difficulties in solving algebraic equations

Research (Falker, Levi & Carpenter, 1999; Ketterlin-Geller, Jungjohann, Chard & Baker, 2007; Knuth *et.al*., 2008) has shown that both younger and older learners alike have serious difficulties understanding the meaning of the equal sign. Learners fail to understand that equality is a relationship expressing the idea that two mathematical expressions hold the same value. Mamba (2012) affirmed that inadequate understanding of the uses of the equal sign and its properties when used in an equation posed a major challenge to learners and hinders them from solving equations correctly.

When two algebraic expressions are combined together with an equal sign they produce an algebraic equation. A definition of a mathematical or an algebraic equation is provided in Section 1.9.8. To solve an algebraic equation correctly one must know the application of rules that are used to simplify algebraic expressions. An equal sign is used to express the equivalence between the two sides of the equation. This is an additional burden to learners. Arithmetic and algebra share many of the same symbols and signs such as the equal sign, the addition and subtraction signs. The interpretation given to the equal sign by learners is sometimes different from its accepted meaning. There are two interpretations attributed to the equal sign, namely, symmetric and transitive relation. The symmetric relation indicates that the quantities on both sides of the equal sign are equal. The transitive relation indicates that the quantity on one side of the equation can be transferred to the other side using rules. Kieran (1992) notes that in elementary school the equal sign is used more to announce a result than to express a symmetric or a transitive relation. An example to explain the Kieran's case is:

> *Daniel went to visit his grandmother, who gave him $1.50. Then he bought a book costing $3.20. If he has $2.30 left, how much money did he have before visiting his grandmother? (Kieran, 1992: 98)*

Sixth graders that attempted to solve the Kieran's task wrote the answer as *2.30+3.20=5.50-1.50=4.00*. It is observed that the symmetric property of the equal sign is violated here; hence participants could be considered to have done errors. Kieran (1992) further claimed that the equal sign is sometimes perceived by learners as implying: "*it gives*", that is, as a left-to-right directional signal rather than a structural property. In other words, learners may perceive the equal sign as a symbol inviting them to do something rather than looking at it as symbolising a relationship (Foster, 2007; Falkner, Levi & Carpenter, 1999; Kieran, 1992; Weinberg, 2007). The type of errors that emanates from this kind of interpretation is extensively elaborated in the literature (for examples, see, Foster, 2007). Weinberg (2007: 170) contends that instead of uniquely denoting sameness the equal sign seems to be a "Swiss army knife" of symbols, representing a ratio, the co-existence of unequal sets, or an undefined relationship between two objects, ideas, or symbols. This variety of meanings may cause problems to learners and cause them to commit more errors. Kieran (1992) further elaborates the sources of errors for the misuse of the equal sign. Furthermore, Kieran (1992) maintains that learners' inclination to interpret the equal sign as a command to compute an answer suggests that aspects of arithmetic instruction could be contributing to the errors they commit in algebra.

When learners use the equal sign as a 'step marker' to indicate the next step of the procedure, they do not properly consider the equivalence property of it. Another explanation for the use of the equal sign in terms of performing a task or doing something could be ascribed to the fact that the equal sign mostly "comes at the end of an equation and only one number comes after it" (Falkner *et. al*., 1999: 3). A possible origin of this error is the '=' button on many calculators, which always returns an answer. Foster (2007) reports that, in the United States, although learners use the equal sign early in their school careers, they often use it to mean that the answer follows. When used in an equation the equal sign indicates that the expressions on the left and right sides have the same value. This serves as an impediment for learners who might have been taught that the equal sign refers to the answer that follows.

The procedures required to solve some equations involve transformations that are different from normal operations that learners are used to employ. The procedure for solving the equation rests on the principle that adding the same number to or subtracting the same number from both sides of an equation conserves the equality (Filloy, Rojano & Puig, 2007; Filloy, Rojano & Solares, 2003). This principle is equally applicable to multiplying or dividing both sides by the same number. Equations that have the variable on one side such as $x+a=b$, $ax=b$ and $ax+b=c$ can be solved by using similar methods. Filloy *et al*., (2007) states that the situation becomes complicated when the equations appear in the form $ax + b = cx + d$. The procedures required to solve equations of this type involve transformations that are different such as subtracting ax or cx from both sides.

Similarly, learners usually have difficulties in solving linear systems of equations with two unknowns (Filloy *et al*., 2007). In the two-unknown linear system such as: $y = 2x + 3$ and $y = 4x + 1$, despite the unknown being represented by a letter y, it has also been represented by an expression that involves another unknown letter x. Therefore, learners will have to operate the unknowns with a second level representation (Filloy *et al.*, 2007). This second level representation of the variables brings additional difficulties to them and as a result they commit more errors.

2.2.5 Learners' difficulties in solving word problems

According to Sönnerhed (2009), algebraic problem solving process require learners to go through three steps: (1) translate problems communicated in daily words into algebraic structure by making use of variables and symbols; (2) formulate algebraic structure with specific rules; and, (3) solving the problem. All these three steps required that the learners are able to handle symbols, variables and concepts, have the requisite skills needed for the operation. However, learners lack these prerequisites and therefore encounter difficulties in this area of algebra.

Bishop, Filloy and Puig (2008) argue that word problems have traditionally been the nemesis of many learners in algebraic classrooms. The primary source of difficulty for learners in solving algebraic word problems is translating the story into appropriate algebraic expressions (Bishop *et al*., 2008). This involves a triple process; assigning variables, noting constants, and representing relationships among variables. Among these processes, relational aspects of the word problem are particularly difficult to translate into symbols. Bishop *et al*. (2008) further claim that difficulties experienced by learners when translating word problems from natural language to algebra and vice versa is one of the three situations that generally arise when learners have just completed elementary education and are beginning secondary education.

According to Bishop *et al*. (2008), the specifics of algebraic translation errors have not been examined as closely as the translation errors that are associated with arithmetic word problems. Bishop *et al.* (2008) further aver that it is reasonable to assume that algebraic translation errors result from the semantic structure and memory demands of the problem. To emphasise learners' difficulties in translating relational statements into algebraic language, some researchers extensively discussed the famous "student-professor" problem (for examples, see, Clement, 1982; Clement, Lochead & Monk, 1981; Kaput, 1985). In these studies, the "student-professor" problem reads as: "There are six times as many students as professors at this university", and students were asked to write an algebraic expression for the student-professor relationship. Many researchers found that there was a translation error such as "*6S=P*", where *S* and *P* represent the number of students and the number of professors respectively (Clement *et al*., 1981; Macgregor & Stacey, 1993; Weinberg, 2007).

Clement (1982) contends that there could be two reasons for the errors identified in the student-professor problem. Firstly, students could have literally translated the syntax of the relational statement into an algebraic expression without considering the magnitude of the relationship. Secondly, students could have used 6S to represent the group of students and P to represent the group of professors. Clement (1982) further claims that for those who committed the error the "=" symbol could have been interpreted as not representing a mathematical relationship but instead as simply separating the two groups, namely, students and professors (Clement, 1982). Resnick and Clement (1982) noted that not only does the reversal error appear in many situations but it has also proven to be difficult to remediate.

MacGregor and Stacey (1993) comment on the reasons for students to write additive totals such as *6s + p* as the answer to the "student professor" problem. Moreover, MacGregor and Stacey (1993) argue in such answers learners do not match the symbols with the words but were expressing features of some underlying cognitive model of an invisible mathematical relationship. In addition, Weinberg (2007) describes this strategy as operative reasoning in that, learners performed hypothetical operations on two quantities to equalise the totals. Not all the errors that occur while solving algebraic word problems result from difficulties in representing and translating problem statements.

Sometimes learners get confused when they try to formulate a solution for an algebraic word problem. Kieran (1992) note that to solve a problem such as "*When 4 is added to 3 times a certain number, the sum is 40*", learners would subtract 4 and divide by 3 using arithmetic. But solving the problem using algebra would require setting up an equation like $3x + 4 = 40$. To set up the equation, learners must think precisely the opposite way they would solve it using arithmetic. Therefore, two different kinds of thinking patterns are involved in these two contexts, which would sometimes confuse learners. In arithmetic, learners think of the operations they use to solve the problem whereas in algebra they must represent the problem situation rather than the solving operations. This dilemma could be interpreted in another way as the interference from previously learned arithmetical procedures hindering the development of subsequent algebraic concepts. Apart from the difficulties encountered by learners when translating word problems into algebraic language, there are other barriers such as interferences from other systems, not understanding the equal sign as a relationship, and other errors in simplifying algebraic expressions.

Fajemidagba (1986) observed that learners perform poorly in word problems solving in Mathematics. In the light of this observation Fajemidagba (1986) investigated factors responsible for learners' poor achievement in mathematics word problem. Factors identified included misconception of mathematical statement, which led to errors. Fajemidagba (1986) identified two types of reversal errors usually committed when solving mathematics word problems. These are static syntactic error and semantic error. According to Fajemidagba (1986), the static syntactic error is committed due to direct translation of the given problem or word matching. The semantic error could be committed as a result of inadequate understanding of the language embedded in the problem. In view of the identified areas of learners' difficulties with mathematical statements, such aspects of Mathematics are poorly responded to in both qualifying and terminal mathematics examinations. Fajemidagba (1986) further affirms that learners excel more in numerical problems than word problem at the secondary school or university level as they have great difficulties in solving word problems in Mathematics. As a result, learners may commit more errors, which may cause poor performances in Mathematics.

2.3 CONSTRUCTIVISM

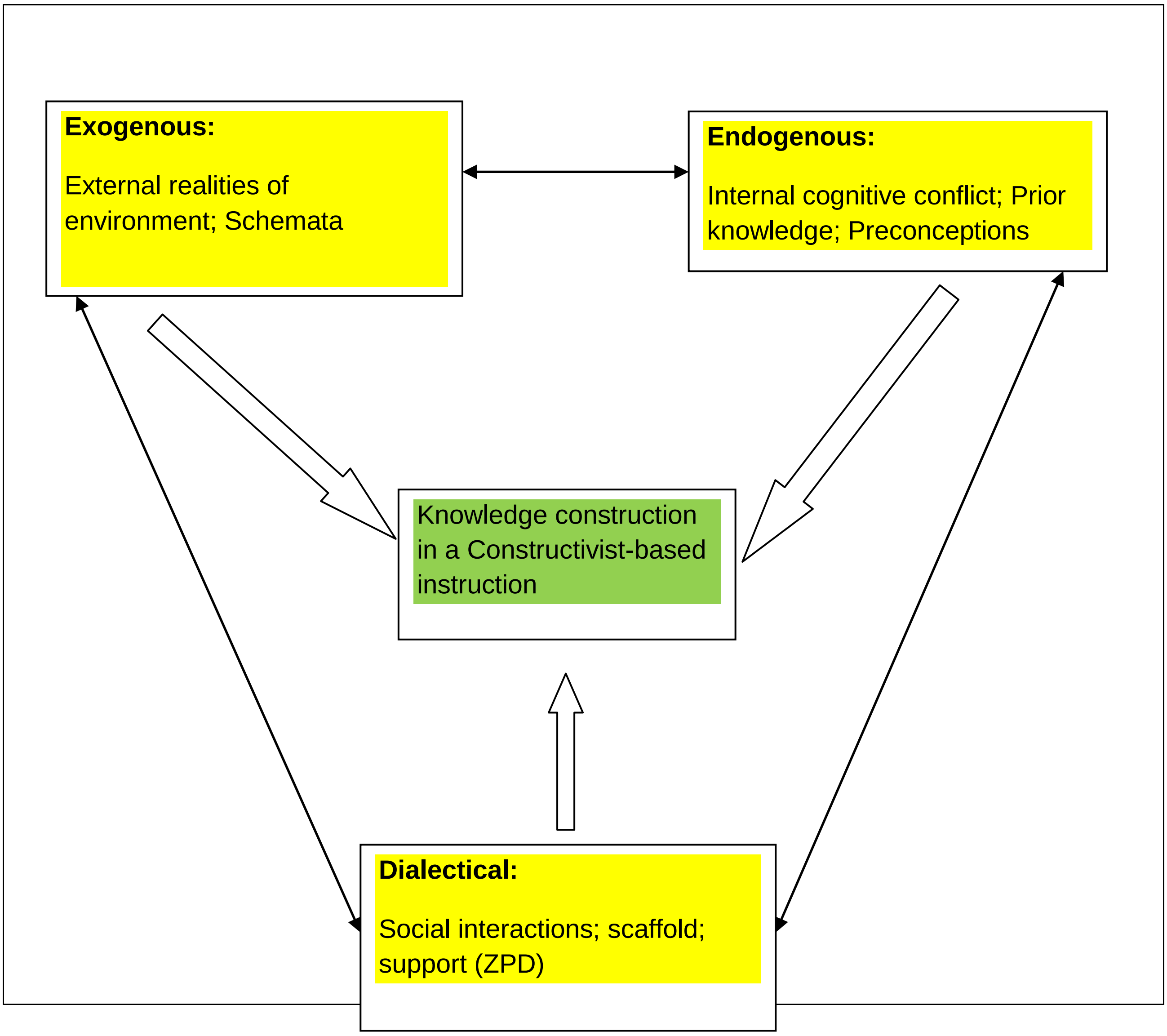
The field of education has undergone a significant shift in thinking about the nature of human learning and the conditions that best promote the varied dimensions of human learning. As in psychology, there has been a paradigm shift in designed instruction from behaviourism to cognitivism and now to constructivism (Cooper, 1993). Whenever a novel conception is introduced it always elicits great resistance and the recent paradigm shift to constructivism is no exception. Constructivism perspective is one of the most influential views of learning during the last two decades of the 20th century. Johri (2005) confirms that many modern pedagogical theory and practice around the world prefer Vygotsky's social constructivist and Piaget's radical constructivist approaches to teaching and learning because of the numerous benefits (see, Section 2.3.5) accrued from this learning theory. For example, the constructivist perspective has contributed to shaping Mathematics reform efforts of National Council of Teachers of Mathematics [NCTM], (2007) (see also, NCTM, 2000).

Constructivism is an epistemological view of knowledge acquisition that emphasises knowledge construction rather than knowledge transmission and the recording of information conveyed by others (see, Section 1.9.2). It is aligned with active learning and promotes comparison of new ideas with prior knowledge (Goldin, 1990; Piaget, 1973; Steffe, 1991; Von Glasersfeld, 1997; Vygotsky, 1978). Constructivism involves learners' interpretation of knowledge and understanding from the experiences encountered as active learners (Slavin, 2000). In addition, Von Glasersfeld (1996: 19) states that "for whatever things we know, we know only insofar as we have constructed them as relatively viable permanent entities in our conceptual world". Communication and justification of ideas are important in helping learners develop problem-solving skills (Piaget, 1973). There is much importance in facilitating correct mathematical language, justifying and sharing ideas with others (Ball & Bass, 2000). Learners can construct meaning in Mathematics from others or from use of individual objects (Von Glasersfeld, 1997). The act of solving one's own problems (Wood, Cobb & Yackel, 2000) as well as the process of question-asking concerning various strategies applicable to the Mathematics topics can increase learners' mathematical abilities and thereby reduce the number of errors they commit (Carpenter, Fennema, Fuson, Hiebert, Human, Murray & Wearne, 1994).

2.3.1 Constructivist notion of how knowledge is constructed

Constructivism views the role of the learner as one of building and transforming knowledge. The question is: "What does constructing knowledge mean?" There are different notions in constructivism about the nature of knowledge and the knowledge construction process. Moshman (1982) identified three types of constructivism as exogenous; endogenous; and dialectical constructivism (see, Figure 2.1).

Figure 2.1: Conceptual framework representing knowledge construction



In exogenous constructivism, there is an external reality that is reconstructed as new knowledge is formulated. In other words, one's mental structures develop to reflect the organisation of the world. This view of constructivism calls attention to how we construct and elaborate schemata and networks of information on the basis of the external realities of the environments we experience. Endogenous constructivism, which is also referred to as cognitive constructivism (Cobb, 1994; Moshman, 1982) views individual constructions of knowledge as internal. Most importantly, this endogenous perspective emphasises individual knowledge construction as stimulated by internal cognitive conflict as learners strive to resolve mental disequilibrium. This is derived from Piaget's theory of equilibration in which he used two main concepts of assimilation and accommodation to explain knowledge construction (Piaget, 1977, 1970). In this process of knowledge construction, children as well as older learners negotiate the meaning of experiences and phenomena that are different from their existing schema. Learners construct their own knowledge through individual or socially mediated discovery-oriented learning activities to advance their cognitive structures by revising and creating new understandings out of existing ones.

Dialectical constructivism, which is also known as social constructivism (Brown, Collins & Duguid, 1989; Rogoff, 1990), views the origin of knowledge construction as being the social intersection of people, engagements that involve sharing, comparing and debating among learners and mentors. This explains and justifies why the current study employed group learning approach to foster these interactions. Through the interactive process, learners are able to expose their errors through their participation, remarks and contributions during group discussion and the subsequent argumentation that ensued assist them to correct their own errors and help others to correct theirs. These engagements improve learners' conceptual understanding and in this way knowledge is considered to have been mutually constructed. This view is derived from Vygotsky's (1978) concept of Zone of Proximal Development (ZPD) in his socio-cultural theory of learning, which accentuates the supportive guidance of mentors as they enable the learner to successively handle difficult tasks that involve more complex skills and understanding. This eventually leads to the development of individual independent competence.

During the social interactions the fundamental nature of group learning through the cognitive exchange enables learners to construct personal knowledge. In addition, the context in which learning occurs is very important. This view is known as contextualism in psychology and has becomes one of the key tenets of constructivism when expressed as situated cognition. Moreover, Prawat (1992) states that there are several interpretations of what constructivist theory means, but most of them agree that constructivism involves a paradigm shift in the focus of teaching by putting the students' own efforts to understand at the centre of the educational enterprise. Despite the differences in opinions about what constructivism is most constructivists agree that the four central characteristics believed to influence all learning are:

- learners construct their own learning;
- new learning depends on learners' existing understanding (prior knowledge);
- the critical role of social interaction; and,
- the necessity of authentic learning tasks (context) for meaningful learning (Bruning, Schraw & Ronning, 1995; Pressley, Harris & Marks, 1992).

For the learner to construct meaning, the learner must actively strive to make sense of new experiences in relation to his or her prior knowledge on a topic. Students develop knowledge through an active construction process, not through the passive reception of information (Brophy, 1992). The manner in which information is presented and how learners are supported in the process of constructing knowledge are very important. Emphasis is placed on the pre-existing knowledge that learners bring to each learning task. Learners' current understanding provides the immediate context for interpreting any new learning and incoming knowledge. A learner's existing schema has a significant influence on what is learned and how conceptual change occurs if needs be. In the same vein, learners interpret mathematical tasks and instructional activities involving new concepts in terms of their prior knowledge. Errors may characterise learners' initial phases of learning because their existing knowledge might be inadequate and could only support partial understanding (Smith, DiSessa & Roschelle, 1993). Given the learners' inadequacy of their existing knowledge, they may not be able to explain mathematical phenomena and solve algebraic problems. Hence at this level, learners learn by transforming and refining their prior knowledge into more sophisticated forms, which may result in errors.

The key element which is largely enhanced in a group learning setting is dialogue. Dialogue is the catalyst for knowledge acquisition (Applefield, Huber & Moallen, 2001). Learners' understanding is facilitated by exchanges that occur through social interaction, questioning and explaining, challenging and offering timely support and feedback. The concept of learning communities has been offered as an ideal learning culture for group instruction (Brown, 1994; Brown & Campione, 1994). These communities focus on helping group members to learn by supporting one another through respectful listening and encouragement. In terms of the current study this process could construed as the ZPD (see, Section 1.9.13). The goal is to engender a spirit and culture of openness, exploration and a shared commitment towards learning.

The concept of situated learning advocated in social constructivist approaches attach importance to the context in which learning occurs. Authentic tasks are embedded in real world experience and context. Knowledge is conceived as being embedded in and connected to the situation where the learning occurs. As a consequence, thinking and knowledge that is constructed are inextricably tied to the immediate social and physical context of the learning experience. What is learned tends to be context-bound or tied to the situation in which the learning process is taking place (Lave & Wenger, 1991). An example of the nature of situational learning can be seen in cases where learners' school learning fails to transfer readily relevant tasks to an out of school context. Brown, Collins and Duguid (1989) documented how people can acquire complex mathematical operations in one setting and yet be unable to apply those same operations in another setting.

The influence of how teachers' and peers' support contribute to learning is clarified by the concepts of scaffolding, cognitive apprenticeship, tutoring and cooperative learning and learning communities (Brown, 1994; Rogoff, 1998). The social nature of learning views cognition as a collaborative process and modern constructivist thought provides the theoretical basis for group learning, project or problem based learning and other discovery-oriented instructional approaches. As learners interact with their peers, conceptual understanding is enhanced to foster knowledge construction.

Therefore, constructivists-based teaching method espoused in the current study makes extensive use of group learning tasks, which involved peer tutoring. Hence the current study believed that learners learn more readily from having a dialogue with each other about significant problems.

The ZPD in Vygotsky's social learning theory considered in the current study focuses on the impact of social factors to the learning process. Learners receive support from the teacher and other capable learners to enable them identify and correct the errors they commit during the knowledge construction process in the classroom. Most significantly, Vygotsky (1978) emphasises that parents, teachers, peers or other adults who interact or live with the learner play an important role in his or her learning process. Furthermore, Vygotsky (1978) argues that there is a limit to which the individual can independently perform given tasks. When it seems that the task demands the learner to go beyond the limit of his or her capabilities, then the success in the task depends on the support from other people. Hence, the ZPD is perceived as the boundary between what a learner can successfully do without support and what he or she will be able to do in the future as new skills are acquired (Shrum & Glisan, 2000).

In the absence of the ZPD learners will be more likely to do errors when they attempt mathematical tasks independently. In this regard, a group learning setting could open up the ZPD opportunities for struggling learners to receive the necessary support. The implication of Vygotsky's theory to instruction is that learners' ability to restructure ideas could be enhanced in a group learning environment that encourages interaction and exchange of ideas with other people. In the context of the current study, it is reasonable to argue that the Vygotsky's theory recognises learners as being able to modify their errors and preconceptions through social negotiation and interaction in a group work environment. In this context, negotiation of meaning refers to an exchange or sharing of ideas between learners, and weighing alternative conceptions from multiple perspectives. Through negotiating meanings, learners could identify the gaps in their preconceptions, which may give rise to their errors, and may modify them. In group learning, learners generally encounter a peer who possesses a slightly higher cognitive level, one within the learner's ZPD. An important aspect of teacher guidance relates to the constructivist notion of generative learning. Since constructivists believe that the learner must transform or appropriate whatever is learned, one can say that all learning is discovered.

The types of tasks that are selected for learners to engage in a complex, problem-based, and real-life context reveal the emphasis of constructivists' top-down view of instruction. Learners are deliberately confronted with complex tasks that can only be performed with a teacher's guidance and that create an immediate need to develop relevant skills. In this context, learners can learn what they need to know in order to figure out how to accomplish authentic but difficult tasks at the upper range of their ZDP. Finally, Von Glasersfeld (1996) and Steffe (1991) perceived constructivism as the acquisition of knowledge with understanding. If learners acquire knowledge with understanding it will be virtually difficult for them to commit numerous systematic errors they presently commit in their algebraic task in the classroom.

2.3.2 Learning in a constructivist-based teaching environment

2.3.2.1 Learning as an active social process

Social constructivism, which is strongly influenced by Vygotsky's (1978) work, suggests that knowledge is first constructed in a social context and is then appropriated by individuals (Bruning, Schraw, & Ronning, 1999; Cole 1991; Eggen & Kauchak, 2004). The social constructivists refer to the process of sharing individual perspectives as *collaborative elaboration* (Van Meter & Stevens, 2000). According to Greeno, Collins and Resnick (1996), collaborative elaboration may produce learners who can construct shared understanding that will not be possible to produce if they had done it individually. Social constructivists view learning as an active process where learners learn to discover principles, concepts and facts for themselves and hence encourage guesswork and intuitive thinking in learners. Kukla (2000) argues that reality is constructed by our own activities and that people, together as members of a society, invent the properties of the world. Other constructivist scholars share Kukla's view and emphasise that individuals make meanings through interactions with each other and with the environment they live in. Knowledge is thus a product of humans and it is socially and culturally constructed (Ernest, 1991; Prawat & Floden, 1994).

2.3.2.2 Learning as dynamic interactions between task, teacher and learner

A further characteristic of the role of the facilitator in the social constructivist viewpoint is that the teacher and the learners equally learn from each other during instruction (Holt & Willard-Holt, 2000). This means that the learning experience is both subjective and objective and requires that the teacher's culture, values and background become an essential part of the interplay between learners and tasks in the shaping of meaning. Learners compare their version of the truth with that of the teacher and fellow learners to arrive at a new, socially tested version of truth (Kukla, 2000). The task becomes the interface between the instructor and the learner (McMahon, 1997). This creates a dynamic interaction between task, teacher and learner. This entails that learners and teachers develop an awareness of each other's viewpoints and then look to their own beliefs, standards and values, thus being both subjective and objective at the same time (Savery & Duffy, 1994).

The social constructivist model thus emphasises the importance of mentoring and the relationship between the learner and the teacher in the learning process (Archee & Duin, 1995; Brown, Collins & Duguid, 1989). Some learning approaches that could harbour this interactive learning include reciprocal teaching, peer collaboration, cognitive apprenticeship, problem-based instruction, web quests, anchored instruction and other approaches that involve learning with others.

2.3.2.3 Group learning

Learners with different skills and backgrounds should collaborate in tasks and discussions to arrive at a shared understanding of the truth in a specific field (Duffy & Jonassen, 1992). In direct contradiction to the traditional teaching approaches, most social constructivist models stress the need for collaboration among learners (Duffy & Jonassen, 1992). The notion that has got significant implications for peer collaboration is that of the zone of proximal development (see, Section 2.3.1 & Section 1.9.13). Through a process of 'scaffolding', learning can be extended beyond the limitations of physical maturation to the extent that the development process lags behind the learning process (Vygotsky 1978; see, also, Section 1.9.12).

Dhlamini and Mogari (2013) argue that group approach to learning has the potential of helping individuals to accomplish more work than they can achieve in solitary pursuits. In essence, when people work in groups, they can share responsibilities and ideas. Consequently, they may be more successful in finding a solution to a problem. In various spheres of inquiry, people are beginning to appreciate the beneficial influence of utilising group learning initiatives to foster productive interdisciplinary approaches. Within the research paradigm, group approach is seen as a useful tool to become familiar with many resources available in the facilitation of research processes, practice and partnership (Erichsen, Goldenstein & Kaiser, 2011).

The term “group work” refers to an arrangement in which two or more people work together to achieve a common goal (Dhlamini & Mogari, 2013). In this arrangement, strategies are integrated in an attempt to address the problem of learners’ errors in algebra, or issues of a complex nature (Erichsen *et al*., 2011). In the current study, “group work” refers to a classroom arrangement in which learners sit together to discuss and solve mathematics tasks. Classroom arrangement that incorporates group learning activities provides learners with “effective tools to reinforce their problem solving system” (Dhlamini, 2012: 241). This is possible because the processes that occur during group discussion include verbalising explanations, justifications and reflections (Beers, Boshuizen & Kirchner, 2007; Kirchner, Beers, Boshuizen & Gijselaers, 2008), giving mutual support (Van Boxtel, Van der Linden & Kanselaar, 2000) and developing arguments about complex problems (Munneke, Andriessen, Kanselaar & Kirchner, 2007). In the same vein, Dhlamini (2012) emphasises three elements of group learning activities, namely, discussion, argumentation and reflection. According to Van Boxtel *et al*. (2000), group learning activities can allow learners to provide explanations of their understanding, which can help them to elaborate and reorganise their knowledge. Lai (2011) also notes that group learning activities such as, providing elaborated explanations to group members improves learners understanding of conceptual knowledge.

Given this background, Dhlamini (2012: 241) proposes that “schools should also see the need to train learners to become effective in collaborative learning settings” and further suggested that instruction that promotes collaborative skills of learners ought to be designed. In the current study, the constructivist-based teaching employed group approach learning environments in the experimental school (see, Section 1.9.3 & Section 1.9.9). During the group discussions, learners through exploratory talk were able to identify and correct the errors they made through the scaffolds in the form of support provided by the other capable learners in the group. The control school used the traditional teaching method in which teaching was associated with transmission of knowledge by the teacher, and learning associated with passive receiving of knowledge (see, Section 1.9.4). The traditional teaching approach was primarily presented in a non-group teaching mode and the lesson was mainly teacher-centred (Section 1.9.4). Consequently, learning in the TTM environment limited learner participation and reflection (Johnes, 2006).

The current study therefore investigated the comparative effects of the two teaching approaches on the performance of Grade 11 Mathematics learners in terms of exposing learners' algebraic errors and subsequently providing a treatment for the observed errors (Section 1.2). In the light of the national search for teaching approaches that can improve Mathematics performance of learners in South Africa, the researcher believed that the outcome variable of the current study, which explored the impact of constructivist-based teaching method on Grade 11 learners' errors in algebra, is timely. The results of this study may be of importance for those interested in empowering teachers to meet the challenges of the new curriculum.

2.3.3 Worked-out examples in constructivist-based instruction

One of the key instructional devices used by the CBTM to reduce learners' errors is the worked-out example strategy. This instructional tool was deemed appropriate because: (1) it serves as a scaffold (Section 1.9.12); and, (2) the steps are meant to provide a model to show how other similar problems might be solved. A worked-out example is an instructional device that provides a model for solving a particular type of problem and typically includes a problem statement and a procedure for solving a problem (see, Atkinson, Derry, Renkl & Wortham, 2000; Van Gog, Ericsson, Rikers & Paas, 2005). A worked-out example provides a learner with an expert's model of solving a typical problem that the learner can learn from and emulate. A worked-out example typically presents a solution in a step-by-step fashion (see, Appendix F). In a worked-out example approach, the idea is to provide the learner with support and resources needed to solve that problem, and then encourage the learner to solve novel problems.

Most studies have demonstrated that learning from worked-out examples leads to superior learning outcomes in algebra problem solving. One of the studies that have reported the beneficial effect of the worked-out examples approach in the domain algebra was conducted by Anthony (2008). In Anthony's (2008) study, a conventional cognitive tutor was compared with an instructional version that included example-problem pairs, consisting of annotated worked-out examples presented with problem solving tasks. Even though no significant differences were observed in the immediate retention, participants in the example-enriched condition attained significantly better long-term retention scores. Furthermore, Anthony's (2008) study added another component of empirical evidence in support of the beneficial influence of worked-out examples approach. In this current study, the learners in the experimental group were given worked-out example problems as scaffold. The step-by-step procedure involved in the worked-out examples provided scaffold that enabled learners to identify the errors committed and why learners were committing such errors. The steps-by-step approach was meant to show how other similar problems might be solved. This process enabled learners to avoid making similar errors.

Constructivist-based teaching approaches play an integral role in developing learners' conceptual understanding and ability to communicate learned ideas. These approaches include teacher promotion of learner independent thinking, creation of problem-centred lessons, and facilitation of shared meanings. Problem-centred lessons can also increase learners' learning in Mathematics by promoting understanding of the relatedness of various topics. Problem-centred lessons include realistic situations and the posing of problems. Teachers need to present cumulative problems to increase learners' abilities to build upon other topics and make much-needed connections (Cunningham, 2004; Wood & Sellers, 1997). Moreover, problem-centred lessons can also significantly increase developmental Mathematics of students' problem-solving skills (Verhovsek & Striplin, 2003). Similarly, Sharp and Adams (2002) reported the same results concerning the manner in which realistic situations can help learners develop a foundational understanding of division of fractions. Shared meanings allow learners to verbalise their mathematical ideas and thus increase their learning. The constructivist approach of facilitation of shared meanings involves the use of negotiated meanings and creation of small group activities. Wood, Cobb & Yackel (2000) advocate the importance of negotiated meanings and small group collaboration in socially communicative situations due to the opportunity for explanations, debates, and discussion.

Yager (1991) provides what can be described as one of the comprehensive guidelines for constructivist teaching. The 18 guidelines of constructivist instruction stressed by Yager (1991) are:

- Seek out and use learner questions and ideas to guide lessons and whole instructional units;
- Accept and encourage learner initiation of ideas;
- Promote learner leadership, collaboration, location of information and taking actions as a result of the learning process;
- Use learners' thinking, experiences, and interests to drive lessons;
- Encourage the use of alternative sources for information both from written materials and experts;
- Encourage learners to suggest causes for events and situations and encourage them to predict consequences;
- Seek out learners' ideas before presenting teacher ideas or before studying ideas from textbooks or other sources;
- Encourage learners to challenge each other's conceptualisations and ideas;
- Encourage adequate time for reflection and analysis;
- Respect and use all ideas that learners generate;
- Encourage self-analysis, collection of real evidence to support ideas and reformulation of ideas in light of new knowledge;
- Use learners' identification of problems with local interest and impact as organisers for the course;
- Use local resources (human and material) as original sources of information that can be used in problem resolution;
- Involve learners in seeking information that can be applied in solving real-life problems;
- Extend learning beyond the class period, classroom and the school;
- Focus on the impact of science on each individual learner;

- Refrain from viewing science content as something that merely exists for learners to master on tests; and,
- Emphasise career awareness especially as related to science and technology.

The highlight of Yager's (1991) guidelines for constructivist teaching is the conception that a shift in the culture of learning is necessary if learners are to become meaning makers. A shift in the culture of learning denotes giving the learners greater responsibility over their own learning, thinking for themselves, reflecting over their own actions and thoughts, evaluating their knowledge, and applying new ideas to solve problems in multiple contexts. The constructivist-based pedagogy advocates that the learner should take the lead in the learning process while the teacher plays the role of a coach or a facilitator. Fundamentally, it is this change in the role of the learner from one who absorbs knowledge transmitted by the teacher to one who constructs new knowledge that distinguishes the constructivist approach from the traditional approach.

This current study focused on teaching the experimental group basic concepts in algebra where learners commit many fundamental errors through constructivist-based instructional approach. These basic algebraic concepts are variables, expressions, equations, and word-problem (see, Section 2.2). To achieve this purpose, two existing in-tact Grade 11 classrooms of Mathematics in a public school in the White River circuit were selected as control and experimental groups (see, Section 3.4.1). Both groups wrote pre-test and post-test before and after experimentation respectively (see, Section 3.3 & Section 3.5). The treatment applied to the experimental group involved Vygotsky's three constructivist processes for each algebraic concept. The three constructivist processes are:

- **Modelling**- In this process, the researcher prepared worked-out-examples on each of the four algebraic concepts (Section 2.3.3). Learners in their respective groups were given worksheets to identify mistakes or errors in the worksheets on the basis of the worked-out-examples. Learners were encouraged to follow the steps in the worked-out-examples on how to approach and look at the problem, how to convert word problem into appropriate algebraic equation using letters (such as *a*, *b*, *c*, *x*, *y*, etc.) as variables, how to put value, and how to simplify or solve equation by doing mathematical operations. This approach did not only serve as modelling but also scaffolding (Section 1.9.12).

- **Scaffolding**- The researcher through step-by-step approach taught learners how to solve algebraic equation, how to simplify algebraic expressions, how to form an equation from word-problem, put value and solve the problem. The presentation was a two-way questioning and discussion using visual aids, that is, blackboard and chalks and charts that served as scaffolding. There is element of collaboration in this approach. Collaboration also overlapped with scaffolding (see, also, Section 1.9.12).

- **Collaboration**- The researcher initially selected bright learners of the class, and divided the class into five groups of at least six learners in the group with one bright learner in each group for appropriate and immediate scaffolding (see, Section 1.9.12; Section 1.9.9 & Section 3.6.2.2.). The groups initially took some time to brainstorm, discuss and share the problem in hand, then worked out steps and solved the problem through collaborative efforts and sharing. The researcher acting as a facilitator was also available to help out any groups if needed (Section 3.6.2.2).

2.3.4 Research supporting constructivist-based instruction

Tellez (2007: 553) reviewed major reform efforts in curriculum and pedagogy to establish that "the importance of constructivism in educational theory and research cannot be underestimated". Several studies support constructivist approach in science-related disciplines (for examples, see, Cobb, 1996; Dangel, 2011; Fox, 2001; Phillips, 1995). Also, Chin, Duncan and Hmelo-Silver (2007) cited several studies supporting the success of the constructivist problem-based and inquiry learning methods. For example, Chin *et al*. (2007) described a project called GenScope, which was an inquiry-based science software application.

Students, who were in the experimental group using the GenScope software, showed significant gains over the control groups. The largest gain was shown by the students who were enrolled in the basic courses. Chin *et al*. (2007) cited a study by Geier on the effectiveness of inquiry-based science for middle school students as demonstrated by their performance on high-stakes standardised tests. The improvement was 14% for the first cohort of students and 13% for the second cohort. Chin *et al*. (2007) also found that inquiry-based teaching methods greatly reduced the achievement gap for African-American students.

Guthrie**,** Taboada, and Humenick (2004) compared three instructional methods for third-grade reading: a traditional approach, a strategies instruction only approach, and an approach with strategies instruction and constructivist motivation techniques including student choices, collaboration, and hands-on activities. The constructivist approach, called Concept-Oriented Reading Instruction (CORI), resulted in better student reading comprehension, cognitive strategies and motivation. Kim (2005) found that using constructivist teaching methods for the 6^{th} Graders resulted in better learner achievement than traditional teaching methods. The Kim's (2005) study also found that learners preferred constructivist methods over traditional ones. However, Kim (2005) did not find any difference in student self-concept or learning strategies between those taught by constructivist and those taught in traditional methods.

Doğru and Kalender (2007) compared science classrooms using traditional teacher-centred approaches to those using student-centred constructivist methods. In the initial test of learner performance, which was administered immediately after the intervention, Doğru and Kalender (2007) found no significant difference between traditional and constructivist methods. However, in the follow-up assessment, which occurred 15 days later, learners who learned through constructivist methods showed better retention of knowledge than those who learned through the comparative traditional methods. Bhutto (2013) researched on the effect of teaching of algebra through social constructivist approach on 7^{th} Graders' learning outcomes in Sindh in Pakistan and found that the experimental group that was taught through social constructivist approach excelled in achieving statistically significant learning outcomes than the control group that was taught through traditional one-way teaching. The studies mentioned in the preceding discussion provided some motivation and justification for the current study to be conducted. Hence the researcher had a belief that the constructivist-based teaching method that was mainly advocated in the current study would succeed in reducing learners' algebraic errors in Grade 11.

2.3.5 A critical perspective on constructivism

Some of the benefits of constructivism have been documented, and these are:

- Children learn more and enjoy learning more when they are actively involved rather than being passive listeners;
- Education works best when it concentrates on thinking and understanding, rather than when it is focussed on rote memorization. Hence constructivism concentrates on learning how to think and understand;
- Constructivist learning is transferable. In constructivist classrooms learners create organising principles that they can readily transfer to other learning settings;
- Constructivism gives learners ownership of what they learn since learning is based on learners' questions and explorations and often learners have a hand in designing the assessments as well. Constructivist assessment engages the learners' initiatives and personal investments in their journals, research reports, physical models, and artistic representations. Engaging the creative instincts develops learners' abilities to express knowledge through a variety of ways. Learners are also more likely to retain and transfer the new knowledge to real life contexts;
- By grounding learning activities in an authentic and real-world context, constructivism may stimulate and provide meaningful engagement for learners. Learners in constructivist classrooms learn to question things and to apply their natural curiosity to the world; and,
- Constructivism promotes social and communication skills by creating a classroom environment that facilitates collaborative interaction and exchange of ideas. Learners learn how to articulate their ideas clearly as well as to

collaborate on tasks effectively by contributing in group projects. Therefore, learners exchange ideas and so learn to "negotiate" with others and to evaluate their contributions in a socially acceptable manner. This is essential to success in the real world since they are always exposed to a variety of experiences in which they have to cooperate and navigate among the ideas of others.

Nevertheless, constructivism has been criticised on various grounds. Some of the charges that critics such as Fox (2001), Phillips (1995) and Terhart (2003) have levelled against constructivism are:

- Constructivism is said to be elitist. Critics say that constructivism and other "progressive" educational theories have been most successful with children from privileged backgrounds who are fortunate in having outstanding teachers, committed parents and prosperous home environments. Conversely, critics argue that disadvantaged children who are lacking such resources may benefit more from explicit instruction;
- Social constructivism leads to group thinking. Critics contend that the collaborative aspects of constructivist classrooms tend to produce a "tyranny of the majority" in which a few learners' voices or interpretations dominate the group's conclusions and dissenting learners are forced to conform to the emerging consensus; and,
- There is little hard evidence that constructivist methods work. Critics argue that constructivists, by rejecting evaluation through testing and other external criteria, have made themselves unaccountable for their learners' progress. Critics also maintain that studies of various kinds of instructions, for example the '*Project Follow Through'* (a long-term government initiative) found that learners in constructivist classrooms lag behind those in more traditional classrooms in basic skills (Brooks & Brooks, 1999).

The relevance of these critical views of constructivism cannot be underestimated. These critical opinions of constructivism serve to: guide constructivist educators' perspectives when planning constructivist lessons; enable constructivist educators to exercise cautions and discretions when implementing constructivist-based instruction; and lastly, provide insight for constructivist-based educators not to see constructivist teaching approaches as the only pedagogical panacea for all the mathematical odds.

Contrary to criticisms by Fox (2001), Phillips (1995), Terhart (2003), and Jin and Cortazzi (1998) and many other conservative or traditional educators, constructivism does not dismiss the active role of the teacher or the value of expert knowledge and disadvantaged learners can also benefit from it. Constructivism modifies that role, so that teachers help learners to construct knowledge rather than to reproduce a series of facts. The constructivist teacher provides tools such as problem-solving and inquiry-based learning activities with which learners formulate and test their ideas, draw conclusions and inferences, and pool and convey their knowledge in a collaborative learning environment. Constructivism transforms the learner from a passive recipient of information to an active participant in the learning process. Always guided by the teacher, learners construct their knowledge actively rather than just mechanically ingesting knowledge from the teacher or the textbook. Such criticisms provide insight for constructivist-based educators not to see constructivist teaching approaches as the only effective approach for all the mathematical odds.

2.3.6 Constructivist and traditional ideas about teaching and learning

In the constructivist classroom, the focus tends to shift from the teacher to the learners (Brooks & Brooks, 1999). One of the teacher's biggest responsibilities becomes that of ‘asking good questions’. Again, in the constructivist classroom both teacher and learners think of knowledge not as inert factoids to be memorised but as a dynamic and ever-changing view of the world we live in and the ability to successfully stretch and explore that view (Brooks & Brooks, 1999).

When comparing the traditional teaching methods to the constructivist-based teaching method, Applefield *et al*. (2001) stated that in the traditional approach a bottom-up strategy, which involves isolating the basic skills, teaching occurs by separating and building these incrementally before tackling higher order tasks. This is an essentially objectivist and behavioural approach to instruction (teaching method) although cognitive information processing views often lead to similar instructional practices. However, constructivist-based teaching method turns this highly sequential approach on its head. Instead of carefully structuring the elements of topics to be learned, learning proceeds from the natural need to develop understanding and skills required for completion of significant tasks. The distinctions between the traditional teaching methods and the constructivist-based teaching method are reflected in Table 2.2.

Table 2.2: A comparison between traditional and constructivist-based classrooms

The traditional classroom	The constructivist classroom
Begins with parts of the whole by emphasising basic skills	Begins with the whole and expand to parts
Strict adherence to fixed curriculum	Focus is on pursuit of learner questions and interests
Textbooks and workbooks-oriented	The use of primary sources and manipulative materials
Teacher is a provider and learners are passive recipients	Learning is interactive and builds on what learners already know
Teacher assumes a directive and authoritative role	Teacher interacts and negotiates with learners
Assessment is via testing and emphasis on correct answers	Assessment is via learner works observations, points of view and tests.
Knowledge is inert	Knowledge is dynamic and changes with experiences. Process is as important as product
Learners work individually and independently	Learners work in groups to facilitate self-construction of knowledge

Table 2.2 shows that there are significant differences in basic assumptions about knowledge, learners, and learning between traditional and constructivist approaches. It is important to stress that constructivists do acknowledge that learners in the traditional classroom are also constructing knowledge but it is just a matter of the emphasis being on the learner and not on the teacher. In terms of the current study, learners' errors, which were observed during a constructivist-based teaching method (CBTM), were meaningfully exposed because learners were given opportunities to be the constructors of their knowledge. This is in line with the last point in Table 2.2 on the constructivist section of the table. As learners verbalise their knowledge during active participation in the group discussion their errors are manifested.

However, in the traditional teaching method (TTM) and learning environment, learners' errors could be observed after instruction through post-lesson activities because the teacher is the main player during instruction (see, 4th point in Table 2.2 on the traditional section of the table).

2.3.7 Designing instruction to address learners' errors

Errors that are commonly done by learners, which could be as a result of misconceptions, must be deconstructed. It must be noted that this study did not explore the causes of the observed learners' errors, hence the fact that these errors could be emanating from learners' misconceptions remains subjective and hypothetical in terms of this study (see, Section 1.1). However, it is the researcher's view that teachers could help learners to reconstruct learners' misconceptions in favour of correct conceptions. Lochead and Mestre (1988) describe an effective inductive technique for the purposes of designing instruction to address learners' errors in algebraic tasks. The following instructional recommendations are provided:

- *Probe for and determine qualitative understanding*
    - Given this perspective, the following question could be raised: *Does the learner understand qualitatively the ideas and topics at hand*? Without qualitative understanding, learning is effectively blocked. For example, *does the learner understand the nature of a variable, or the meaning of fraction*?

- *Probe for and determine quantitative understanding*
    - Quantitative understanding means, for example, that learners have a working understanding of mathematical concepts at hand. In algebra, it means that learners can work comfortably with variables and expressions with variables in them; and,

- *Probe for and determine conceptual reasoning*
    - The following question come to mind: *Can the learner analyse a problem without the use of computational algorithms*? Dependence on an algorithm as the problem-solving method is an indicator that

learners have been taught only algorithmic procedures and/ or the learner fails to understand the conception involved.

In addition, it is helpful to confront learners with counterexamples in an attempt to address their errors. A self-discovered counterexample will have a far stronger and lasting instructional effect. Incorrect beliefs can be loosened somewhat when so confronted.

2.4 THEORETICAL ORIENTATION OF THE STUDY

A constructivist learning theory (CLT) underpinned the current study. Constructivist-based teaching method (CBTM) as used in this study refers to instruction that incorporates elements of Vygotsky's social constructivist learning and Piaget's radical constructivism (see, Section 1.9.3 & Section 1.9.13). The main gist of the constructivist theoretical perspective in this current study was to investigate how learners can be given initial support during the group discussion to help them to identify and correct the errors they commit in their algebra class in the knowledge construction process (see, Section 1.9.9 & Section 1.9.12). The initial support learners require could be provided by the teacher or a more capable learner. This process enables them to be in position to reconstruct their conception.

Vygotsky (1978) stated that learning is basically the travel from Zone of Approximal Development (ZAD) to Zone of Proximal Development (ZPD). Within the ZAD, a learner knows or is able to do certain things on the basis of previously acquired knowledge and experience while within the ZPD a learner can do certain things through assistance or scaffolding provided by a mature or more experienced adult or even peer. Vygotsky (1978) strongly believes that learning could only take place within a cultural and social setting but not in isolation. In the current study, the researcher provided the needed support to assist learners to identify the errors they made through the use of worked-out examples (see, Section 3.6.2.2). In addition, the constructivist-based classroom was characterised by group activity and interactions where the more capable and intelligent learners assisted the other learners through discussion to enable them identify the errors they were making and corrected them by themselves through the assistance received from their peers and the teacher.

Formal teaching has always been a complex process involving diverse knowledge and instructional decisions. Teachers' pedagogical content knowledge has been shown to affect teachers' instructional practice as well as learners learning in the domain of Mathematics (Ball, Goffney & Bass 2005; Baumert, Kunter, Blum, Brunner, Voss & Jordan, 2010; see, also, Section 1.1 & Section 1.4). Teaching approaches have evolved and shown clear digression from teacher-centred to the learner-centred tendency. The importance of algebra taught as integral part of Mathematics in most national curricula cannot be underestimated. For example, the National Council of Teachers of Mathematics (NCTM, 2000) recommended introducing algebra and algebraic reasoning in elementary and middle grades throughout the courses of Mathematics. It is also felt that Mathematics community is concerned about the knowledge required for effective teaching of algebra (Ball & Thames, 2010).

Warren (2008) reports that, learners in developing countries and some developed countries experience difficulties with proper understanding of algebraic concepts, especially algebraic variables. This is because of the mechanical way teachers teach it without explaining real meaning in social context. This obviously leads to poor learning and open opportunities for learners to do errors. The South African Curriculum Assessment Policy Statement (CAPS) acknowledges the danger of inappropriate teaching on learners' conceptual understanding when it stated that "learning procedures and proofs without a good understanding of why they are important will leave learners ill-equipped to use their knowledge in later life" (CAPS, 2012: 8). A number of factors could be linked to effective teaching and some of these include: (1) teacher's subject knowledge (Hill, Ball & Schilling, 2004); (2) teacher's pedagogical knowledge (Baumert *et al*., 2010); and, (3) teacher's knowledge of learners' thinking (Franke & Kazemi, 2001). However, Bhutto (2013) argues that educational psychology and pedagogy do not support the idea of teachers using differentiated methods of instruction, resources and tasks to teach learners differently on the basis of their needs. Therefore, he recommended the use effective and appropriate teaching method suitable for all learners at all times.

Given this background, the researcher believes that CBTM could be an effective teaching method to address the errors learners do when performing algebraic tasks in

Grade 11 classroom (see, Section 1.4 & Section 1.5). Learners come to the algebra class with different preconceptions. Some of the challenges confronting teachers are to understand the nature of learners' preconceptions in order to design and implement appropriate instructional interventions to reconcile their conceptions of Mathematics. Some perspectives have been advocating how learners' preconceptions can be modified through instructions. While traditional theorists subscribe to substitution of inaccurate conceptions with accurate ones, the constructivists' views are identified with giving the learners autonomy to inquire and re-evaluate their own ideas. The former has been confronted with widespread criticism and is becoming less tenable.

The current study is identified with the constructivists' views (see, Table 2.2). Hence the main aim of the current study was to investigate the comparative effects of a constructivist-based teaching method (CBTM) and the traditional teaching methods (TTM) on Grade 11 learners' errors in algebra, in terms of exposing the errors and subsequently providing a treatment for the observed errors (Section 1.2). This aim was explored in terms of four conceptual areas of algebra, namely, the mathematical variables, mathematical expressions, mathematical equations and solving word-problem (see, Section 2.2). The comparison is aimed at determining the instruction that is more effective in terms of highlighting and treating learners' errors algebraic tasks to achieve the ultimate desirable outcome of improving learners' performance in Mathematics (Section 3.3). This study focussed on errors made frequently by many learners and their effects on learners' basic algebraic skills and performance in Mathematics (Section 3.2; see, also, Section, 2.2).

2.5 SUMMARY OF THE CHAPTER

Mainly, this chapter provided a discussion on the following pertinent issues: (1) a review of literature in relation to learners' errors in the four conceptual areas in algebra, namely, the mathematical variables, algebraic expressions, algebraic equations and word-problems; (2) the notion of constructivism and constructivist-based instruction and related emerging issues such as: the notion of knowledge construction, learning in a constructivist-based teaching environment, the role of worked-out examples in constructivist-based instruction, review of literature on studies supporting constructivist-based instruction, and a comparison between constructivist and traditional ideas about teaching and learning of Mathematics.

In addition, Chapter 2 reviewed studies on constructivist-based learning environment, and instruction in terms of the benefits and criticism of constructivism. The last section looked at how to implement reformed instructional strategies to help learners to curtail, if not eliminate, the numerous errors they make in their algebra class. The concluding part of Chapter 2 looked at the discussion on the theoretical orientation of the study in terms of constructivist learning and the two broad groups of the theories, namely, the cognitive and social learning theories.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 INTRODUCTION

The purpose of a study has an influence on the type of research design the researcher chooses to follow (Welman, Kruger & Mitchell, 2005). Subsequently, the design selected influences the data collection methods as well as the techniques and instruments used to collect the data (Welman *et al*., 2005). In this chapter, the research paradigm and methodology that was followed in conducting this research is outlined. In addition, the research design and sampling procedures are presented in this chapter. The following issues are also discussed: instrumentation, reliability and validity issues, data collection and data analysis and how the researcher addressed the ethical issues embedded in the current study.

3.2 THE POSITIVIST RESEARCH PARADIGM

The current study was conducted from a positivist paradigm. The positivist paradigm relies on knowledge obtained through articulated observations and controlled experiment. The assumption of this paradigm is that "truth is established by looking at the hard facts" (Higgs & Smith, 2006: 1). This implies that all obtained results must be substantiated with evidence. Within this paradigm the researcher is able to manipulate the independent variable, which in the current study represented the teaching methods, namely, the constructivist-based teaching method and traditional teaching methods. The effect of the independent variable on the dependent variable, which in the current study represented learners' errors, could also be observed and measured when using a positivist approach. The positivist paradigm is concerned with objectivity, which is, what is or how things are; and also not how things should be.

Given this research perspective, the researcher thought that the actual observation and measurement of the magnitude of learners' error treatment of CBTM should be compared with that of TTM. The differences in learners' error reduction of the two groups would provide objective or quantitative evidence to judge if the constructivist-based teaching method was more effective in reducing learners' errors than traditional teaching method, and if it was more effective, then the magnitude of the effectiveness would also be determined. This would allow for analysis of data by means of mathematical tools and allow for generalising the findings beyond the location or circumstance where the study was conducted (see, Black, 1999; Blaxter, Hughes & Tight, 2005; Burns, 2000; Crotty, 1998; Descombes, 2003; Morrison, 2003). Data gathered from current study were used to determine if a difference in learners' error reduction existed between the participants in the control group, who were taught in TTM, and those in the experiment group that received the CBMT instruction.

3.3 RESEARCH HYPOTHESIS FOR THE STUDY

The aim of this study was to investigate the comparative effects of a constructivist-based teaching method (CBTM) and the traditional teaching methods (TTM) on Grade 11 learners' errors in algebra, in terms of exposing the errors and subsequently providing a treatment for the observed errors (see, Section 1.2). Guided by literature the following hypothesis and null hypothesis were formulated prior to the commencement of the current study:

> **Hypothesis (H_1):** The constructivist-based teaching method is more effective than traditional teaching methods in reducing[6] learners' errors when Grade 11 algebraic tasks are treated.

$$H_1: \mu_{\text{constructivist-based teaching method}} \neq \mu_{\text{traditional teaching method.}}$$

> **Null Hypothesis (H_0):** The constructivist-based teaching method is not more effective than traditional teaching methods in reducing learners' errors when Grade 11 algebraic tasks are treated.

$$H_0: \mu_{\text{ constructivist-based teaching method}} = \mu_{\text{ traditional teaching method.}}$$

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6. In terms of the hypothesis that is stated in Section 3.2 the word *reduction* should be construed in terms of error treatment as reflected in the comparative post-test scores involving learners in the CBTM and TTM instructions (experimental and control groups).

3.4 RESEARCH DESIGN

Mainly this study followed a quantitative research methodology. Within this methodology a quasi-experimental research approach with a non-equivalent control group design was opted. Quasi-experiments are investigations that lack random selection of participants to the study, and also lack of such quality for the assignment of participants to groups (Gall, Gall & Borg, 2007). Hence the schools that participated in the study were conveniently selected (convenience sampling), which is a non-random sampling method. This design was opted to investigate the comparative effects of a constructivist-based teaching method and a traditional teaching method on Grade 11 learners' errors in algebra.

A non-equivalent control group design was opted because it would provide practical options to work with intact classrooms in both the experimental group and the control group. However, it was not possible to randomly assign study participants to these groups as this would interfere with the existing teaching schedules of participating schools. In the experimental school one intact group, consisting of Grade 11 Mathematics learners, participated in the study. Also, a similar arrangement was opted in the control school. Hence there was no need for random selection and assignment of participants within a single classroom setting into the experimental school and the control school.

3.4.1 Non-equivalent control group design

According to Cook (2002: 42), "random assignment is rare in research on the effectiveness of strategies to improve student performance". Arzi and White (2005) observed that "random selection is rarely convenient or even possible in educational research" (p. 141). A number of researchers rate the non-equivalent control group design as worth using in many instances in which true experiments[7] are not possible (for examples, see, Blessing & Florister, 2012; Cohen, Manion & Morrison, 2007; Delamont, 2012; Dhlamini, 2012; Hancock & Mueller, 2010; Jackson, 2012; Johnson & Christenson, 2012). True experiments are probably most common in a pre-test post-test group design with random assignment (Lee & Whalen, 2007).

7. True experiments are experimental designs in which there is random selection of participants to the study and random assignment of participants to different groups.

It was observed in this current study that lack of randomness would pose a threat to internal validity because it was only feasible to assign intact groups to experimental (n=36) and control (n=42) groups, rather than individuals being assigned randomly to these groups. In order to minimise the effect of this challenge, threat schools that shared the following characteristics were selected for participation in the study: (1) schools that shared similar quintile rankings (see, Section 1.4); (2) schools with learners who shared a similar socio-economic profile; and, (3) schools with almost similar academic profiles in terms of learner performance, particularly in mathematics (see, also, Section 3.4.1). In order to verify the equivalence of learners in the two participating schools, which were 12km apart, the pre-test results of the schools were compared prior to the experiment (see, Section 4.3).

A non-equivalent control group design has been used in several studies. Dhlamini and Mogari (2013) used a quasi-experimental study (classroom-based) with a non-equivalent control group design to determine the effect of a group approach on the performance of high school Mathematics learners. Dhlamini and Mogari (2013) opted for the non-equivalent control group design because practical constraints did not permit the possibility of random allocation of participants to either the experimental group or the control group. Gaigher, Rogan and Brown (2006) employed a similar design to investigate the effect of a structured problem solving strategy on 189Grade 12 learners' problem solving skills and their conceptual understanding of Physics.

The rationale for non-randomised assignment of learners into groups was an "attempt to exclude diffusion, contamination and rivalry" (Gaigher *et al*., 2006: 9). In addition, Claire and Michael (2003: 241) used the non-equivalent control group design in a study in which "the effectiveness of a Social Skills Training (SST) programme on 28 learners from four secondary schools was evaluated". They opted for this design due to practical constraints of time and resources. In Claire and Michael's (2003) study, one school was used as experimental group and the other the control group.

Furthermore, Turner and Lapan (2005) employed the non-equivalent control group design because "it was only feasible to randomly assign intact groups with similar characteristics (that is, students from the same grades, from the same type of school, and from a similar socio-economic status) to experimental (n=107) and control (n=53) groups, rather than individuals being assigned to these groups" (p. 518). However, Turner and Lapan (2005) acknowledged that the random assignment was not feasible because "intact classes are already formed before the research is begun" (p. 518). The non-equivalent control group design has been used widely in educational research in recent years (for examples, see, Baker & White, 2003; Chih-Ming & Yi-Lun, 2009; Liu, 2005; Ozmen, 2008).

Similarly, the current study opted for the non-equivalent control group design due to the non-feasibility to randomly select participants for participation in the study, and subsequently not being able to randomly assign participants to the control and experimental groups. Hence intact classrooms were used as experimental and control groups. Random assignment and reorganisation of learners into experimental and control groups would have altered and disrupted the systematic arrangement and normal running of the participating schools.

3.5 SAMPLING

Sampling refers to the process and techniques used to select the study participants. Sampling reduces the cost of collecting data by working with a manageable and accessible group that is representative of the population (Welman *et al*., 2005). As indicated earlier, the participants in this research consisted of one Grade 11 Mathematics teacher and n=78 Mathematics learners in Grade 11.

3.5.1 The population of the study

The targeted population in this study was Grade 11 Mathematics learners from quintile 2 schools, which are historically disadvantaged and are from a rural background in the White River Circuit of Ehlanzeni District in Mpumalanga Province of South Africa. According to the entries and records available at the White River Circuit at the time of conducting the current study, the population of Grade 11 mathematics learners for 2014 in the circuit was n=550. Table 3.1 presents a profile of schools in the White River Circuit for the academic year 2013.

Table 3.1 shows that at the time of conducting the current study there were n=11 secondary schools in the White River circuit. The schools in Table 3.1 constituted the population of the current study. Of the n=11 schools in Table 3.1, n=7 secondary schools are public and n=4 secondary schools are independent schools. With exception of one secondary school in the White River circuit, which is in quintile 5, the remaining public secondary schools in Table 3.1 had been ranked in quintile 2

Table 3.1: White River circuit 2013 NSC mathematics and school performances

Name of school[8]	Quintile ranking	% achieved in mathematics	% achieved as a school
A	Independent[9]	61.1	85.6
B	Independent	23.5	55.8
C	2	100.0	100.0
D	5	95.3	99.4
E	2	66.7	86.5
*F	2	29.8	53.5
G	2	64.7	82.6
H	Independent	51.0	84.0
I	2	90.0	91.7
J	2	65.0	79.2
K	2	64.7	91.5
*L	2	58.3	83.3
M[10]	Independent	N/A	N/A

Source: NSC 2013 Subject Report (p. 300-305) *No longer in White River Circuit effective January 2014

On the basis of these criteria, schools E and G in Table 3.1 were selected as the two participating schools for this study. In Table 3.1 school E was chosen as the experimental school and school G was used as the control school.

3.5.2 The study sample

The sample of the study consisted of n=78 Grade 11 Mathematics learners drawn from two rural secondary schools in Table 3.1. Schools in Table 3.1 consisted of n=550 learners who represented the population of the study. Of the n=78 learners from two participating schools, n=36 were in the experimental group and

8. Actual names of schools are not used for ethical reasons.
9. Schools that are considered to be independent in terms of the quintile rankings are those that are privately-owned, and also privately funded and managed.
10. The school had learners only from Grade 1 to Grade 11 as of 2013 and did not register Grade 12 learners for NSC examination in 2013.

n=42 in the control group. Learners in the control group were taught by their incumbent Grade 11 Mathematics teacher.

Only one teacher participated in the study. The teacher was an existing teacher who was teaching learners who had been allocated in the control group. Hence the teacher played the role of preserving traditional learning conditions in the control group by providing a traditional teaching method (see, Section 1.9.4). Comparatively, learners in the experimental school were taught by the researcher who had embraced the basic aspects of constructivist-based teaching approach (CBTM) prior to the commencement of the experiment (see, Section 1.9.3). This experimental arrangement and subsequent research design (Section 3.3) were thought to provide feasible and realistic options to meaningfully conduct the experiment that is summarily described in Section 1.1. The experiment lasted for four weeks (Section 1.1).

The researcher realised that requesting or training another teacher to administer the CBTM in the experimental group would have demanded an extensive training for the teacher, which could have prolonged the study. Also, the teacher training arrangement would not certainly guarantee the envisaged effectiveness and implementation of the CBTM intervention by the teacher who would have just been trained for a few days. Moreover, the researcher focused on Grade 11 Mathematics learners because: (1) Grade 11 is considered to be a critical stage where any implemented change (intervention) has the potential to make an impact on future performance in Grade12 and beyond; and, (2) most of the errors in algebra acquired from the previous grade levels are commonly exhibited at Grade 11.

3.5.3 Sampling techniques

The two participating schools were selected from a population of n=550 learners in Table 3.1. From this population, the two participating secondary schools had a total of n=123 Grade 11 Mathematics learners. However, out of the n=123 a sample size of n=78 was purposively selected for participation in this study. Johnson and Christenson (2012: 481) note that “larger samples result in smaller a sampling error, which means that your sample values (the statistics), will be closer to the true population values (the parameters)”. Most importantly, in purposive sampling, the goal is to select a sample that is likely to be “information-rich” with respect to the anticipated outcomes of the study (Gall *et al*., 2007: 178). The n=78 learners were selected to ensure that the mean of the sample ($\bar{x}$) would be representative of the population mean (μ). Purposive sampling helped the researcher in discovering, gaining insight and understanding the problem of learners' errors in algebra as a variable of poor performance of secondary school learners in Mathematics in the two participating secondary schools. The two selected schools presented a convenient sample for the study.

Given the profiles of schools in Table 3.1, the researcher developed the criteria to select the two schools (E & G in Table 3.1) for participation in the study. This criterion provided convenient sampling procedures for the selection of the study sample, and it also incorporated the shared characteristics of schools that are presented in Section 3.3.1. Hence the two participating schools had to:

- be a public[11] school in the White River Circuit;
- be in the same quintile ranking category (see, Section 1.4);
- be at least 10 km apart to avoid contamination and diffusion;
- be a rural disadvantaged school located in the same geographical area;
- have performed between 50–69%[12] in Mathematics in the 2013 NSC Examination;
- have performed between 70–90% in the 2013 NSC Exam as a school; and,
- be managed and governed by the same educational policies, rules and regulations (Department of Basic Education [DBE], 2010).

Averagely, schools in disadvantaged township and rural areas tend to experience similar educational challenges such as inadequate allocation of resources such as, inadequacy of learning and teaching support materials (LTSM); being adversely affected by an inadequacy of qualified teachers, particularly in the fields of Mathematics and Science; and, challenges in dealing with domain-specific teaching and learning facilities (Khuzwayo, 2005; Van der Berg, 2007).

11. Public schools are schools that are government-funded.
12. Mpumalanga Department of Education classify any NSC performance in the category of 50-69% as poor performance.

The convenience sampling techniques have been used in various studies. For instance, Dhlamini (2012: 100) used “convenience sampling to select n=783 Grade 10 learners from nine township high schools from Ekurhuleni and Tshwane regions” for participation in a study in which the researcher investigated the comparative effects of context-based problem solving instruction and conventional instructions on learners’ performance. The convenience sampling technique is largely opted in studies that are conducted in naturalistic education settings (for examples, see, Lombard & Grosser, 2008; Mji & Makgato, 2006; Mogari, 2004; Ozsoy & Ataman, 2010).

3.6 INSTRUMENTATION

The primary data collection instruments for the study were Algebra Concept Achievement Test (ACAT) and lesson observation schedules (see, Appendix A; see also, Appendix B & Appendix E).

3.6.1 Purpose of instruments

The ACAT was administered to determine if there would be a significant reduction in the frequency of learners’ errors after the intervention. The lesson observation was conducted to ascertain and determine the comparative effects of instructions on the exposition and treatment of learners’ errors in both groups during instructions.

3.6.1.1 Achievement test

The achievement test (ACAT) was developed by the researcher to principally determine and evaluate learners’ errors in four conceptual areas in algebra, namely, variables; expressions; equations; and word-problems before and after the intervention (see, Section 2.2). The ACAT was administered as a pre-test and a post-test. The pre-test was used to determine participants’ initial errors in algebra before intervention. Mainly, the results of the pre-test helped the researcher to ascertain the level of equivalence prior to the commencement of the study between the two comparative groups, namely, the experimental group and the control group (see, Section 3.3.1 & Section 4.3). A post-test was given at the end of intervention to ascertain any change in participants’ errors in algebra over a four-week period. The same test was administered to both the experimental group and the control group.

3.6.1.2 Lesson observations

According to Mulhall (2003), a lesson observation is an excellent instrument with which to gain a rich picture of any social phenomenon such as, the behaviour of learners in a classroom. Gay, Mills and Airasian (2006) supported this view when they stated that classroom behaviour, which constitutes the behaviour of the teacher, the behaviour of the student, and the interactions between teacher and student, can best be studied through naturalistic observation. In accordance with this assertion, the purpose of the lesson observations in the current study was to determine what could transpire in the classroom during Grade 11 lessons on algebraic concepts in both the control group and the experimental group. In terms of the current study, the lesson observations provided useful data to inform the implementation of a constructivist-based teaching method (CBTM) in Grade 11 Mathematics classrooms with the intent to reduce learners' errors in algebra. Also, these observations provided useful insights in documenting certain aspects of the traditional teaching method (TTM) in terms of dealing with learners' errors in a Grade 11 algebra lesson (see, Section 4.5).

3.6.2 Development of instruments

3.6.2.1 Achievement tests

In order to preserve the content validity, the instruments were developed by the researcher using and comparing a variety of literature. The instruments were validated by high school subject advisors for Mathematics and the university lecturer in Mathematics Education. To develop the ACAT, the researcher was mainly guided by the DBE assessment standards as reflected in the 2012 CAPS document. To strengthen the test objectivity, fairness and consistency, the learners' scripts in the achievement tests (pre-test and post-test) were moderated by colleagues after marking. These colleagues were the practicing teachers who taught Grade 11 Mathematics in the White River Circuit at the time of the study. These teachers had sufficient experience to evaluate qualities of objectivity, fairness and consistency in Grade 11 Mathematics assessment tools.

3.6.2.2 Lesson observation schedule

A lesson observation schedule was constructed to observe lessons in both groups (see, Appendix E). The development of lesson observation schedule for the control school and the experimental school was largely influenced by the definition of traditional teaching method (TTM) and constructivist-based teaching method (CBTM) (see, Section 1.9.3 & Section 1.9.4). The lesson observation was used in order to observe the aspects of TTM and CBTM in relation to the exposition of learners' errors, and also, how each instruction tends to provide treatment for the observed learners' errors.

The researcher observed the lessons in the control school more than once so as to trace the treatment activities. Lesson observations in the experimental group were on-going during the course of the experiment. The main areas of focus during the lesson observations were:

- the format of instruction;
- how the teacher used teaching and learning resources;
- the arrangement of the learning setting to facilitate the exposition and subsequent treatment of learners' errors;
- how the teachers discovered learners' errors; and
- how the teacher provided treatment for the observed learners' errors.

With this observational focus in mind, the lesson observation schedule was developed to address the aim and objectives of the study (see, Section 1.2 & Section 1.3).

3.6.3 Validation of instruments

In the context of this study, validity refers to the extent to which the instruments were able to provide data that related to learners' errors in algebra through intervention.

3.6.3.1 Achievement test

According to Martyn (2009) there are three main types of validity: content, criterion (concurrent and predictive), and construct. Content validity addresses how well the content of the test samples the subject matter. Content validity, including forms of face validity, was established for the achievement test (ACAT). In addition, face validity was pursued in order to judge whether measurement of learners' errors in algebra through the test was worth pursuing or not (Cohen, Manion & Morrison, 2007; Johnson & Christenson, 2012; Rubin & Babbie, 2010). Content validity, which is the degree to which a measure covers the range of meanings included within the concept, was established when Mathematics subjects advisors and the university lecturer confirmed that the content of the test adhered to the requirements of the Grade 11 Mathematics curriculum of the South African Curriculum and Assessment Policy Statement (CAPS) for Mathematics curriculum: Grades10- 12 (DoE, 2012) as a basis. The content of the test was discussed with the Mathematics subject advisors and two Mathematics teachers and their suggestions and insights were incorporated into the test content prior to the administration of the pre-test. Also, similar test construction procedures that are documented in the literature helped the researcher to gain insights in terms of conducting this process (for examples, see, Demircioglu, Demircioglu & Calik, 2009; Donkor, 2010; Hattingh & Killen, 2003; Kasanda, Lubben, Gaoseb, Kandjeo-Marenga, Kapenda, & Campbell, 2005).

In the process of content validation for the achievement test, the current study attempted to locate learners' errors in algebra within the context of learners' basic skills in algebra. Efforts were therefore made to construct an achievement test that met this objective. In order to achieve this, context-rich algebra topics were selected from the Grade 11 Mathematics syllabus. The selected themes from this topic covered themes in algebraic variables, expressions, equations, and word-problem. After the construction, the test was given to credible Mathematics practitioners who constituted school Mathematics curriculum advisors, two Heads of Department for Mathematics at school level and two Mathematics teachers teaching Mathematics at Grade 10-12 levels for validation. These experts worked independently to validate the test.

3.6.3.2 The lesson observation schedule

A pilot study was conducted prior to the commencement of the main study (see, Section 3.6.1). The purpose of the pilot study was to examine the level of bias in the research process, and also to trial the observation process (Johnson &

Christenson, 2012; MacMillan & Schumacher, 2006). The final lesson observation schedule was influenced by the findings of observation schedule that was conducted during the pilot study.

3.6.4 Reliability of the instrument

3.6.4.1 Achievement test

There are several forms of reliability measures described in the literature. MacMillan and Schumacher (2010: 181) suggested that "five methods of reliability exist: stability (test-retest), equivalence, equivalence and stability, internal consistency (split-half, Kuder-Richardson, Cronbach's alpha), and agreement". In the current study, the researcher used the split-half method to obtain the reliability measure. In this method, the test scores were divided into two halves: scores for odd-numbered items and scores for even-numbered items. Then, the correlation between the two halves was determined. The split-half reliability coefficient for the preliminary trial was compared with the reliability coefficient for the whole test using the Spearman correlation coefficient formula. This process reflected an adequate level of reliability and these observations showed that the test was reliable. The Spearman Brown formula was used to measure the "linear relationship between two sets of ranked data" (Charter, 2001: 693) that reflected learners' scores obtained in the pre-test and post-test. The results confirmed that the test was reliable to measure learners' errors in algebra. With a sample of n=70 the value of r=0.82 was computed for reliability of the test.

3.6.4.2 Lesson observation

Reliability of the lesson observations was determined through a process of repeated usage of the observation schedule. It was used during the pilot study as well as in the main study.

3.6.5 Validity of the study

Validity refers to the extent to which the outcomes of a research accurately describe the phenomenon or issues it is supposed to measure (Bush, 2003; Burns, 2000). In this study, the validity of the constructivist-based teaching model was judged from the perspectives of its internal validity and external validity.

3.6.5.1 Internal validity

According to Burns (2000), internal validity is concerned with the question: Do the experimental treatments make a difference in the specific experiment under scrutiny or can the difference be ascribed to other factors? For this study, internal validity referred to the extent to which the researcher would be certain that the findings of the research were solely due to the comparative effects of constructivist-based teaching method, which characterised instruction in experimental group, and also due to the traditional teaching method employed in the control group. Possible threats to internal validity such as diffusion of intervention or contamination, experimenter or researcher effects were controlled.

To minimise the threat of internal contact within the two participating groups, the researcher ensured that the participating groups were 12 kilometres apart. According to Gaigher (2006: 37), such separation effectively "prevents diffusion, contamination, rivalry and demoralisation". Contamination can occur when learners in different group talk to each other or borrow each other's study tools (Shea, Arnold & Mann, 2004). In addition, contamination could have threatened the internal validity in the current study when the control group interacted with the experimental group that was exposed to the CBTM intervention instruction. Results of a study conducted by Howe, Keogh-Brown, Miles and Bachmann (2007: 16-17) to establish expert consensus on contamination in a naturalistic education setting suggested that "geographical overlaps are at the highest risk of contamination". According to Howe *et al.* (2007: 197), contamination can reduce the "statistical significance and precision of effect estimate" needed to make a statistical conclusion that the observed difference between two groups is due only to intervention.

The experimenter or researcher effect which refers to how the deliberate or unintended effect of the researcher can influence the learners' responses in the post-test was controlled. To do this, the lessons were carefully planned in advance and the researcher ensured that the instructions were strictly limited to the lesson plans and activities in the worksheets and the worked-out examples designed for the lesson (see, Appendices F&G). This was to ensure that the researcher was not tempted to teach any aspects of the questions in the achievement tests. Again the question papers were collected from the learners immediately after writing the pre-test. This was done to prevent learners from discussing the questions in the pre-test and possibly ask the researcher for explanation of certain questions in the test during the lesson.

3.6.5.2 External validity

External validity refers to the extent to which the results of a research can be generalised to other settings beyond where the study was conducted (Gay, Mills & Airasian, 2011). The external validity of this study was determined from two perspectives, namely, the population validity and the ecological validity. Whereas population validity refers to the extent to which the results from a research can be generalised to other groups or people, ecological validity refers to the extent to which the results of a study can be generalised to situations outside the research setting (Cardwell, Clark, & Meldrum, 2004; Fraenkel & Wallen, 1996).

At the time of this study there were seven public and four independent secondary schools in the White River Circuit of Ehlanzeni District of Mpumalanga Department of Education. However, only two secondary schools were selected for participation in this study. This represented 18.2% of the secondary schools in the circuit. There were about n=550 Grade 11 Mathematics learners from all the secondary schools in the circuit. From this population two secondary schools, which consisted of n=78 Grade 11 Mathematics learners representing 14.2% were selected to participate in the study. Although the number of learners selected was low compared to the entire population of learners in the schools (n=78 out of n=550), the number of schools chosen was representative of the population of schools in the circuit. The population of learners involved in this study represented only 14.2% of the entire population of Grade 11 Mathematics learners in the circuit. However, since the learners from the participating schools constituted a fair representation of the population of the 11 secondary schools it was inferred that this study has a high population validity and as such its findings could be generalised to the other nine schools in the circuit that were not selected.

Ecological validity as stated earlier is a measure of the extent to which the findings of a study can be interpreted to be true in settings different from the one in which it was conducted (Cardwell *et al.*, 2004). The current study was conducted under normal classroom conditions. All lessons were conducted during normal school lesson periods. In addition, all measurements were conducted during normal lesson times in line with the schedules of the participating schools. All learners in the two participating schools that were used in evaluating the comparative effects of the two teaching methods were exposed to the same ecological conditions irrespective of whether they were in the experimental group or in the control group. Given this background, it was reasonable to expect that the rest of the schools that did not participate in the study had similar ecological conditions that characterised the two participating schools. It was therefore thought that it would be feasible to generalise the findings of the current study to all the eleven secondary schools in the circuit, and most of the secondary schools in the Ehlanzeni District since they all had similar setting and conditions as those in participating schools.

3.7 DATA COLLECTION

3.7.1 The pilot study

The study was piloted in one public secondary school located in the same White River circuit. The pilot sample consisted of n=31 Grade 11 mathematics learners. The pilot school shared similar characteristics with schools in the two schools in the main study in terms of poor learner performance in mathematics, being a public school governed by the same educational policies, similarity in the quintile ranking and also reflecting similar socio-economic factors.

3.7.2 The main study

The same content was taught in both the control and experimental groups. Both groups had equal number of instructional periods. During instruction (see, Section 3.7.2.2) questions were asked to test learners’ understanding and learners asked questions for clarity. Each learner was assessed and scored on each item on the pre-test and post-test. The scores obtained by learners in the pre-test were recorded by the researcher. The post-test scores obtained by learners were recorded by the researcher again. The scores from both the pre-test and post-test were analysed using descriptive and inferential statistics (see, Section 4.3 & Section 4.4).

3.7.2.1 The achievement tests

The study began with the administration of a pre-test which was an achievement test to both groups (experimental and control). The learners were assigned with index numbers (code names) for use in the achievement test in order to conceal their identity. They were given codes such as PRE01C, with the "PRE" prefix denoting the pre-test session, and "01" representing learner 1 and the letter "C" referring to the group in which a learner was allocated, which is the control group in this instance. Following this line of identification, a second learner in the class list who wrote a pre-test in the experimental group would be coded as PRE02E. So, PRE39C referred to learner 39 who wrote the pre-test in the control group. The post-test coding system used the same number as in the pre-test but with the prefix 'POS'. For instance, the 24th learner with a pre-test code of PRE24E used the code POS24E for the post-test. Therefore, the letters E and C served to distinguish between the experimental group and control group.

The test lasted for one-and-half hours and a double period of Mathematics was used for this purpose. The researcher administered the test in the experimental school, while incumbent teacher administered the test in the control school. In order to ensure that conditions remained similar for both groups, which were situated 12km apart, the researcher met with teacher prior to the test. In addition, the teacher was requested to start and end the test on time and to encourage learners to be on time for the test. The teacher was asked to invigilate honestly and credibly, and to remain at the invigilation station during the test. The teacher was also reminded not to provide any assistance to learners while they were writing the test. These precautions ensured that test conditions were fairly similar in the two schools.

3.7.2.2 Instruction

Instruction in the experimental group was guided by the aim of this study, which was to investigate the comparative effects of a constructivist-based teaching method (CBTM) and the traditional teaching methods (TTM) on Grade 11 learners' errors in algebra, in terms of exposing the errors and subsequently providing a treatment for the observed errors (see, Section 1.2). The researcher administered the CBTM instruction in the experimental school while the incumbent teacher in the control school administered the traditional teaching method (TTM).

The researcher planned and designed the CBTM instruction and arranged for the requisite resources such as provision of scaffolds in the form of worked-out examples, and problem solving tools. Both experimental group and the control group used the same worksheets. The four phases of CBTM lesson are: *design*; *instruction*; *learning*; and, *performance*. The design of the lesson emphasised a learning environment that focus on knowledge construction instead of reproduction. The learning environment was regulated to promote knowledge construction task. Designing the instruction for the experimental school was quite a challenging task because of the complexity in knowledge construction process.

The researcher was engaged in a complex planning process that was different from what is prescribed in typical instructional theories. In the process of designing the instruction, the researcher took three factors into consideration. These factors are:

- the definition of the learning focus (that is, a set of instructional goals and objectives that specify what the learner must know to meet the task);
- conceptualisation of anticipated challenges; and,
- the activity which must ideally be authentic in nature.

The learning strategies and the tools that can be used to better understand the task were identified as well. These design decisions were negotiated and refined through a collaborative process between the teacher and learners. At the instruction phase, there were three stages during instructional delivery. The other two phases of CBTM lesson were the learning and the performance phases. Detail discussion of the instruction, learning and performance phases of the CBTM lesson is provided in Section 4.6.

3.7.2.3 Lesson observations

The researcher conducted classroom observations of what transpired during instruction in both the control and the experimental schools. It must be emphasised that the scope of classroom observations covered observation of the teacher, the learner and the instruction. The researcher used a notebook to record the feedback from lesson observations. Areas of focus during classroom observation had been established by the researcher in line with the study research questions (Section 1.6). In the control school, the teacher was observed during the traditional teaching of algebra lessons. The teacher was encouraged to continue using the usual style of teaching, and was only given an activity sheet with algebraic concepts that had been constructed by the researcher with inputs from the teacher (see, Appendix F). The sequence of observation visits to the control group was labelled as CG1, CG2 and CG3. The number following "CG" corresponded with the sequence of visits to the control group. For instance, CG3 referred to the third visit in the sequence of observation visits.

The researcher visited the control school three times, and these visits covered observation of the classroom during instruction in terms of (format of instruction, the arrangement of learning setting, how the teacher discovered and treated learners' errors). The subsequent follow-up visits to the control school were made in order to track down the treatment activities of learners' errors. The observations were limited to only three visits in order to allow lessons to run naturally thereby minimising possible disruption during lessons and avoid over-burdening the teacher and learners with the presence of the researcher. The visits were scheduled for days that were convenient for the researcher who was also implementing the CBTM intervention at the experimental schools. The development of this tool was meant to address the aim and objectives of the study, which was largely influenced by the definition of traditional teaching method (see, Section 1.9.4). This instrument was used in order to observe the aspects of TTM in relation to how learners' errors were exposed and treated in this classroom setting.

3.8 DATA ANALYSIS

The quantitative data collected from the achievement tests were analysed using quantitative methods and the data from lesson observations were analysed using qualitative methods.

3.8.1 Quantitative data analysis

The two statistical techniques used in this analysis were descriptive statistics and inferential statistics. In using the inferential statistics to analyse the quantitative data the independent t-test and the dependent or paired sample t-test were used to analyse participants' scores related to performance on the dependant variable (Gay *et al*., 2011). The dependent variable was learners' mathematics achievement post-test scores; the covariate was learners' pre-test scores. Before performing the t-tests, the researcher evaluated the assumptions underlying study (see, Section 3.7.1.3), namely, the assumption of normality and linearity of data distribution. Various statistical techniques were also employed to analyse certain aspects of quantitative data (see, Section 4.1). An alpha level of 0.05 was used for all statistical data.

3.8.1.1 Descriptive statistics

Descriptive statistics was used to obtain the difference between means and standard deviation for each group on each dependent variable.

3.8.1.1.1 Mean

The means for the control group and the experimental group were used to measure the magnitude of error treatment when the experimental group who received constructivist-based instruction was compared with their counterparts in the control group who received traditional instruction.

3.8.1.1.2 Standard deviation

The standard deviation provided an indication of the degree of variability of the scores in the control and the experimental groups. This current study assumed that the standard deviations of the groups are equal or near equal. For this study, the equality of the variances of Constructivist-Based Teaching Instruction (CBTI) and Traditional Teaching Instruction (TTI) scores was verified using the Levene's test (see, Gastwirth, Gel & Miao, 2006; Lim & Loh, 1996). Levene's test statistic was significant at 0.05 alpha levels, so this research rejected the assumption that the variances of CBTI and TTI groups were not equal. On the other hand, a calculated p-value exceeding 0.05 suggested that the variances for CBTI and TTI groups are equal, and this implied that the assumption of equality or homogeneity of the variances was tenable.

3.8.1.2 The inferential statistics

The inferential statistics used for testing the research hypotheses was the Independent Samples t-test and the Paired-Samples t-test. To determine the statistical significance of the mean difference, in order to affirm the effectiveness of the CBTM, the pre-test and post-test scores were compared using a paired sample t-test at the significance level of 0.05. The paired sample t-test was used because "two mean scores of one sample were compared" (Gall *et al*., 2007: 317). Conclusions were drawn at significance level of 0.05. Notably, the motive behind using these tests was to determine if there was statistically significant difference in the dependent variable between the two groups. The paired sample t-test was used because it is suitable for Pretest-posttest studies in which same groups of subjects are tested twice and the groups are paired or matched. Macmillan and Schumacher (2010) argue that whether the same or different subjects are in each group, as long as there is a systematic relationship between the two groups, "it is necessary to use the paired t-test to calculate the probability of rejecting the null hypothesis" (p. 303). As a result, it is desirable in situations in which there is one group with two measures.

The paired samples t-test was used to test whether there was significant difference between the pre-test and post-test scores of both the experimental and the control groups, whereas the independent samples t-test was used to determine whether there was significant difference between the two groups. The main threat to internal validity of a non-equivalent control group experiment is "the possibility that group differences on the post-test may be due to pre-existing group differences rather than to the treatment effect" (Gall *et al*., 2007: 417). Thus, independent t-test statistic was used to deal with this problem because the tests "statistically reduce the effects of initial group differences by making compensating adjustments to the post-test means of the two groups" (*ibid*).

Given this background, the quantitative data from the achievement tests was analysed using the t-test analysis. The Statistical Package for Social Sciences (SPSS): Output from IBM SPSS Version 16 computer program for windows was used to perform the statistical analysis. Again, Kolmogorov-Smirnov and Shairo-Wilk Levene's test for equality of variance was performed to test the null hypothesis of equal error variance amongst the two groups.

3.8.1.3 Assumption of the study

This study was conducted with two main assumptions in mind (see, also, Section 3.2). This was to ensure that the resultant change in post-test scores in the groups would be attributed to the intervention programme. The first assumption was that the scores in the pre-test and post-test were expected to be normally distributed in all four algebraic conceptual areas in an instance that the CBTM and TTM learners were homogeneous and that assessments would be done as honest as it had been planned.

The second assumption was that if the groups were homogenous and assessments were carried out as honest as it were planned the variances of TTM and CBTM groups would be expected to be equal or near equal in all the sub variables under investigation (equality of variances). A formal normality test was performed by using Kolmogorov-Smirnov and Shapiro-Wilk. The Normal Q-Q plot was used to determine the normality of the pre-test and post-test scores. The p-value for the Kolmogorov-Smirnov and Shapiro-Wilk was used to determine the normality of the pre-test and post-test scores. The null hypothesis for normality is that there is no significant difference between the frequency of learners' errors in the pre-test and post-test in the experimental group, while the alternate hypothesis suggested that there would be significant difference between the frequency of learners' errors in the pre-test and post-test in the experimental group.

If the *p*-value for both Kolmogorov-Smirnov and Shairo-Wilk was more than 5%, the null hypothesis is accepted meaning the post-test and pre-test scores are normally distributed. Paired sample t-test can be applied only when the variables for the study are normally distributed. In Chapter 4, a report is provided on how the researcher conducted this test in this study. The normal Q-Q plot of posttest-pretest scores indicated that the experiment and control groups are normally distributed. The researcher used SPSS to perform a formal normality test by using Kolmogorov-Smirnov and Shapiro-Wilk to determine the linear relationship of data distribution graphically, using a scatter plot. The slope of the regression lines was roughly parallel and it was assumed that there was a linear relationship between the covariate.

3.8.1.4 Testing research hypothesis

The study investigated whether or not learners who were taught with constructivist-based teaching method demonstrated a greater improvement in the errors they commit in algebra than learners taught with traditional method of instruction. The null hypotheses (H_0) and hypotheses (H_1) tested in this study are as stated in Section 3.3. In order to test the null hypothesis, an independent t-test and paired samples t-test were performed. Furthermore, the post-test scores were entered as the dependent variable and the experimental group was entered as the facto variable on SPSS. The pre-test scores were entered as covariates to control for differences among learners before the treatment. The alpha level was established at 0.05 (see more detail in Section 4.4).

3.8.2 Qualitative analysis of lesson observations

Data collected through lesson observations were analysed qualitatively. The researcher used a notebook to record feedback from the lesson observations (see, Section 4.7). The focus areas during classroom observation were indicated in the observation schedule (see, Appendix E) which was guided by the aim and objectives of the study.

3.9 ASPECTS OF CBTM THAT ACCOUNTED FOR ITS EFFECTIVENESS

In this section, the researcher provides a discussion to document aspects of CBTM that accounted for its effectiveness when it was compared with TTM. This discussion is provided in terms of two notions, namely exposition of learners' errors as well as the subsequent treatment of these errors using the idea of group learning setting.

3.9.1 Exposition of learners' errors

During instruction in the group learning setting as learners talk, reflect on personal knowledge, make contributions, interact and participate actively in the group activity, they verbalised their prior knowledge which is likely to have gaps (errors). This process of verbalisation of learners' existing knowledge helped to expose their errors.

3.9.2 Treatment of learners' errors

In providing effective treatment for learners' errors group learning helped to generate a potential error treatment learning environment. Argumentation and exploratory talk by learners in the constructivist-based classroom demonstrated learners' varied level of the ZPD. During the group work, the more knowledgeable learners helped the less knowledgeable to gain more knowledge to understand better and this enabled them to construct new knowledge. Better understanding meant that there are fewer tendencies on the part of the learners to commit errors. It should be noted that the idea of exposition and treatment of learners' errors during instruction did not characterise instruction in TTM.

3.10 ETHICAL CONSIDERATIONS

The researcher received ethical clearance certificate from the University of South Africa (UNISA) (see, Appendix L) and permission letter from the Mpumalanga Department of Education (MDE) (Appendix K) to conduct research in the two participating secondary schools in the White River Circuit Ehlanzeni District of Mpumalanga Province. With the permission letter from MDE, the researcher went to the two schools to request permissions from the principals to use the schools as research sites [see, Appendix J (b)]. Principals were given permission-requesting letters together with consent return-slips to document their responses [see, Appendices J (a - h)].

Permission was secured at school level and a meeting was set up with all prospective participants (Grade 11 learners and the teacher), in which the researcher explained the purpose of this study. In addition, clarity-seeking questions were asked and adequately addressed during that meeting. The Grade 11 teacher and the learners were given participation-requesting letters, together with response-slip to register their feedback [see, Appendices J(a), J(b), J(c), & J(d)]. All the participants agreed to take part in the study and duly signed and returned response-slip.

The letters to all participants were written in a simple language. Before giving letters to participants, all letters were first given to the researcher's supervisor and other colleagues for editing in terms of the language use and the appropriateness of the message contained in them. In each letter, the researcher addressed the following issues: introduction of the researcher; background of the study and its purpose; request of participation; voluntary nature of participation and the fact that withdrawal was permissible; issues of confidentiality was also addressed, such as not revealing the actual identities of learners and those of their schools, and the fact that the study results would be aggregated; the fact that the results of the study be made available to all schools that participated in the study. Nevertheless, all research activities of this study did not interfere with teaching and learning programmes in each of the two participating schools. As mentioned earlier, an ethical clearance was sought from the university's (UNISA) Research Ethical Committee (REC) before commencing the main study and prior to the main study, the instruments of the study were piloted in one secondary school.

This research involved minors (learners under 18 years) who were vulnerable to emotional, verbal abuse, and psychological traumas. To protect them from harm, the normal existing teaching and learning condition or environment was maintained. No corporal punishment, verbal abuse, stigmatisation, intimidation, prejudice or bullying were allowed. The normal security measures during school time were observed; school rules and regulations, and disciplinary codes were enforced.

3.11 SUMMARY OF THE CHAPTER

Chapter 3 provided a discussion that covered methodological issues of the study. The research design and the sampling techniques for this study had been explained in this chapter. Data collection and data analysis techniques were also discussed. The chapter also provided details on intervention procedures for the experiment and control groups. The chapter concluded by providing details on how ethical issues were addressed in the study. In the next chapter, data from the achievement tests and lesson observations are presented and analysed.

CHAPTER FOUR

DATA ANALYSIS

4.1 INTRODUCTION

This chapter presents the results of a quasi-experimental study, which employed a non-equivalent control group design with pre- and post-test measurements (Section 3.4). The study was conducted in a secondary school setting and involved one Grade 11 Mathematics teacher and Grade 11 Mathematics learners from a disadvantaged schooling background (see, Section 1.1; Section 1.4; Section 1.8.5; Section 3.3; Section 3.4). The study compared the relative effects of a constructivist-based teaching method (CBTM) and traditional teaching methods (TTM) on the exposition and treatment of learners' algebraic errors in Grade 11 (Section 1.1). Hence the aim of the current study was to investigate the comparative effects of CBTM and TTM on Grade 11 learners' errors in algebra in terms of exposing the errors and subsequently providing a treatment for the observed errors (Section 1.2). In addition, the following objectives were set out for the study (see, Section 1.3):

- To use a group learning approach to facilitate the exposition and treatment of learners' errors when certain algebraic topics are treated in a Grade 11 mathematics lesson;
- To observe the traditional methods of teaching in term of exposing and treating learners' algebraic errors in algebra Grade 11 lesson; and,
- To measure the effect of error treatment when the constructivist-based teaching method is compared with the traditional teaching method.

Data for the study were collected using both the quantitative and qualitative methods (Section 3.7). However, the study was mainly quantitative (see, Section 3.3). Chapter 4 presents the quantitative analysis of data obtained from the achievement test (Section 3.6.1.1) using the descriptive and inferential statistics. The use of independent t-test and paired sampled t-test as inferential statistics procedures to analyse the quantitative data in the study are also presented in this chapter. The qualitative methods of analysis are used to analyse data obtained from the lesson observations (see, Section 3.6.1.2). The results are also presented in terms of the research questions (Section 1.6) and related hypothesis statements (Section 3.3) that guided the study.

4.2 DESCRIPTION OF PARTICIPATION IN THE STUDY

Table 4.1 provides a summary of the details of learners who participated in the study, and also how they were distributed into the experimental group (EG) and control group (CG). Both groups (EG and CG) wrote a pre-test and a post-test. However, some of the participants missed an opportunity to participate in both test sessions and this scenario is depicted in Table 4.1.

Table 4.1: Information describing learner participation in the achievement test

Participating group	Number of learners	Number of learners who wrote the pre-test	Number of learners who did not write both tests	Number of learners who wrote the post-test
EG	36	35	1	35
CG	42	40	7	35
Total	78(100.0%)	75(96.2%)	8(10.3%)	70(89.7%)

Of the n=78 participants who agreed to take part in the study, n=70(89.7%) of them were considered to have participated fully in the study, that is, they wrote both the pre-test and the post-test. In fact, full participation in the study meant that the participant (learner): (1) was able to attend all teaching lessons; (2) was able to participate in teaching tasks; (3) was able to participate in the writing of both achievement tests at pre- and post-stages in both control and experimental schools; and, (4) was subjected to the lesson observations that also characterised this research. Almost all n=70(89.7%) learners in the last column of Table 4.1 met the four requirements; hence they were designated full participants in the current study.

To monitor the attendance of participants (learners) in both the EG and CG, the researcher (in the experimental school) and the teacher (in the control school) kept records of participants' daily attendance. Using these monitoring tools, it was possible to track down participants who did not participate in all research activities of the study (see, Table 4.1). For instance, Table 4.1 shows that one participant in the experimental school was absent and did not participate fully in the study. In the control school, seven participants did not participate fully. Out of these seven participants who did not write the post-test, five of them wrote the pre-test. Data from participants who did not participate fully in the study from both the CG and the EG were discarded and not analysed. The data analysis that is presented in this report covered only that of the n=70 learners: EG (n=35); and CG (n=35) who participated fully in the study. In total, n=8(10.3%) participants did not participate fully in the study.

Finally, the researcher and the one teacher who participated in the study conducted all research activities in the experimental group and control group, respectively (see, Section 3.5.2). Meaning, the researcher and the teacher participated in all activities that are described in Section 4.2 and those depicted in Table 4.1.

4.3 DESCRIPTIVE STATISTICS

Table 4.2 presents the descriptive statistical analysis of the achievement tests (pre-test and post-test) scores for the experimental group and the control group.

Table 4.2: Descriptive statistics

	Experimental group		Control group	
	Pre-test	Post-test	Pre-test	Post-test
Mean	25.8	44.63	24.14	30.46
Maximum	44	70	40	42
Minimum	15	27	14	17
Standard deviation	7.31	9.68	6.94	6.83
n	35	35	35	35

Table 4.2 shows that the mean scores of the experimental group was 25.8 for the pre-test, and was 44.63 for the post-test. These results suggest that there was a gain score of (44.63-25.8=18.83) in the experimental group as a result of the instruction that was implemented in this group (see, Section 3.7.2.2). The comparable gain of

(30.46-24.14=6.32), which is less than that of the experimental group, is also observed in the control group. In terms of the aim of this study, the observed gains in learner scores (in the experimental group) was interpreted as suggesting a significant reduction in learners' errors in algebra (Section 1.2). Hence these observations showed that meaningful knowledge construction in algebra occurred by using a constructivist-based teaching method in the experimental group.

The standard deviations of the experimental group in Table 4.2 show that the pre-test standard deviation scores are less than the post-test scores. This is an indication that the pre-test scores were more spread around the mean than those of the post-test. The minimum and the maximum marks of the pre-test and the post-test scores in the experimental group were 15 and 44 respectively, and 27 and 70 respectively (see, Table 4.2). These results show that the minimum and maximum marks of post-test scores were higher than that of the pre-test scores in the experimental group. The range of the pre-test and the range of the post-test scores for the experimental group are (44-15=29) and (70-27=43) respectively.

Table 4.2 shows that for the control group, the mean of the pre-test scores was 24.14 and the mean of the post-test score was 30.46, indicating a gain score of (30.46-24.14=6.32). These observations also suggest that there was reduction in learners' errors in the control group, which was taught by the teacher.

However, the observed reduction in the control group was comparatively less than the gain score that was observed in the experiment group. Hence the mean gain of 18.83 by the experiment group is greater than that of the control group with 6.32. The minimum and maximum of the pre-test and post-test scores in the control group were 14 and 40 respectively, and 17 and 42 respectively (see, Table 4.2). Table 4.2 also shows that the range of the pre-test and post-test scores in the control group are (40-14=26) and (42-17=25) respectively.

4.4 INFERENTIAL STATISTICS

In order to test the research hypothesis (Section 3.2), the researcher used the Levene's test for equality of variance, the independent t-test, the paired samples t-test, the equality of variance and the normality of test scores. The tests were performed in order to establish that the two groups (experimental and control) were homogenous before the interventions were administered.

4.4.1 The assumption of homogeneity (equality) of variance

The following hypothesis was formulated to test the assumption of the study (see, Section 3.2).

H_1: The constructivist-based teaching method is more effective than traditional teaching methods in reducing learners' errors when Grade 11 algebraic tasks are treated.

$$H_1: \mu_{\text{constructivist-based teaching method}} \neq \mu_{\text{traditional teaching method.}}$$

H_0: The constructivist-based teaching method is not more effective than traditional teaching methods in reducing learners' errors when Grade 11 Algebraic tasks are treated.

$$H_0: \mu_{\text{constructivist-based teaching method}} = \mu_{\text{traditional teaching method.}}$$

A series of analysis that follow is in line with the analysis of the hypothesis that was set up for the study (Section 3.2). Table 4.3 presents the frequency analysis of pre-test scores in terms of designated mark groups, for both the experimental group and the control group.

Table 4.3 shows that in both the experimental group and the control group, no learner obtained marks that fell between 1 and 10. However, n=10 learners from the experimental group and n=13 learners from the control group, which represented 28.57% and 37.14% respectively, scored marks between 11 and 20. Also, 17(48.57%) learners from the experimental group and 14(40%) learners from the control group obtained marks from 21 to 30 in the pre-test. Furthermore, 7(20%) of the learners from the experimental group and 8(22.86%) from the control group obtained marks from 31 to 40. Only one learner (2.86%) in the experimental group obtained marks between 41 to 50 and none of the test takers in the control group obtained a mark that fell within this designated mark group.

Table 4.3: Analysis of pre-test marks for experiment and control groups

Designated mark group	Experiment group		Control group	
	Frequency	Percentage	Frequency	Percentage
1-10	0.00	0.00	0.00	0.00
11-20	10.00	28.57	13.00	37.14
21-30	17.00	48.57	14.00	40.00
31-40	7.00	20.00	8.00	22.86
41-50	1.00	2.86	0.00	0.00
51-60	0.00	0.00	0.00	0.00
61-70	0.00	0.00	0.00	0.00
71-75	0.00	0.00	0.00	0.00
Total	**35.00**	**100.00**	**35.00**	**100.00**

The results in Table 4.3 show that out of the n=35 learners who participated in the pre-test, n=8 learners each from both the experimental and control groups performed poorly. The latter represented a 22.86% of the study participants. It is also observed from Table 4.3 that out of n=35 learners who participated in the pre-test, n=27 of them from the experiment group and n=27 from the control group, representing 77.14% for each group, failed to obtain half of the total marks. In terms of aim and the context of this study, the observations in Table 4.3 suggest that most of the Grade 11 learners, particularly those who participated in this study, turn to do many errors in algebra and seem to lack basic skills in algebra. It is therefore reasonable to conclude that the poor performance that is depicted by the participants' scores in Table 4.3 could be as a result of these observations.

The independent samples t-test in Table 4.4 was conducted as a formal test for the pre-test of the two groups since they are unrelated. The homogeneity of variance as assessed by Levene's test for equality of variances provided a *p*-value of 0.693, which is more than 5% and hence was interpreted to be not significant. This result suggests that the assumption of homogeneity of variance is not violated.

Table 4.4: Independent t-test for pre-test scores of experimental and control groups

Group	Mean	SD	t	Df	*p*-value
Experiment	25.80	7.31	0.973	68	0.334
Control	24.14	6.94			
Equal variance assumed	(0.693)				

Furthermore, Table 4.4 shows that the pre-test scores for the experimental group, which is observed to be 25.8 ± 7.31, was not significantly higher than the control group (24.14 ± 6.94) t(0.973), *p*=0.334 with a mean difference of 1.66. Since the *p*-value is greater than 5% it is therefore reasonable to accept the null hypothesis and reject the alternate hypothesis. This means that there was no significant difference between the means of the pre-test scores of both the experimental group and control group. Hence the two groups (experimental and control) were considered to be equivalently positioned prior to the commencement of the experiment. In this context, the two groups were considered to be equivalent in terms of the tendency of participants to do errors when they attempted to solve Grade 11 algebraic tasks in the classroom (see the design of the study in Section 3.4).

Table 4.5: Analysis post-test scores for the experiment and control groups

Designated mark group	Experiment group		Control group	
	Frequency	Percentage	Frequency	Percentage
1-10	0.00	0.00	0.00	0.00
11-20	0.00	0.00	6.00	17.14
21-30	2.00	5.71	16.00	45.72
31-40	10.00	28.57	11.00	31.43
41-50	12.00	34.29	2.00	5.71
51-60	10.00	28.57	0.00	0.00
61-70	1.00	2.86	0.00	0.00
71-75	0.00	0.00	0.00	0.00
Total	35.00	100.00	35.00	100.00

Table 4.5 shows that n=6(17.14%) learners from the control group scored marks from 11 to 20 while none of the learners from the experimental group scored marks falling from 1 to 20. Table 4.5 also shows that n=2(5.71%) learners from the experimental group scored marks that fell within the 21 to 30 mark category, while n=16(45.72%) learners in the control group got the scores that fell within this mark

category. In the 31 to 40 mark category, there were n=10(28.57%) learners from the experimental group and n=11(31.43) learners from the control group. The performance differences between learners in the experimental group and control group could further be observed in other designated mark categories. In fact, the differences are persistently observable in all successive designated mark categories. For instance, Table 4.5 shows that in the 41 to 50 mark category, there were n=12 (34.29%) scores from the experimental group and only n=2(5.71%) scorers from the control group. In the 51 to 60 mark category there were n=10(28.57%) scorers from the experimental group and n=0 scorers from the control group. In fact, none of the test takers from the control scored between 51 and 75 score line.

Furthermore, Table 4.5 shows that of n=35 learners who wrote the test in the experimental group only n=2(5.71%) obtained marks less than half of the total marks, and this category of test scores is represented by n=22(62.86%) in the control group. As many as n=33(94.29%) of the learners from the experiment group and n=13(37.14%) learners from the control group obtained a half or more than a half of the total marks. It is a fact that Table 4.5 shows a substantial performance improvement in learners' scores in both groups when the scores are compared with those in Table 4.3. However, the improvements are seemingly more substantial in the experimental group (see, Table 4.3 & Table 4.5). The observed substantial improvements in learners' performance in the experimental group can be attributed to the effect of the use of constructivist-based teaching method (CBTM), which largely incorporated aspects of collaborative or group learning approach when algebra was taught during a Grade 11 Mathematics lesson. In addition, the independent t-test was used to verify the frequency distribution analysis (see, Table 4.4).

The homogeneity of variance as assessed by Levene's test for equality of variances in Table 4.6 yielded a *p*-value of 0.592, which is greater than 5%. However, the obtained *p*-value is not significant implying that the assumption of homogeneity of variance is not violated. Table 4.6 shows that the post-test scores for the experiment group 44.63 ± 9.68 was significantly slightly higher than the control group (30.46 ± 6.83) t(7.767) and a *p*=0.000 with a mean difference of 14.17. Since the *p*-value is less than 5%, therefore, the null hypothesis is rejected and the alternate hypothesis is accepted. This means that there was a significant difference between the mean score of the post-test in both the experimental and the control group.

Table 4.6: Independent t-test of the post-test scores of experiment and control groups

Group	Mean	SD	t	Df	*p*-value
Experiment	44.63	9.68	7.77	68	0.000
Control	30.46	6.83			
Equal variance assumed	(0.592)				

4.4.2 The assumption of normality tests scores

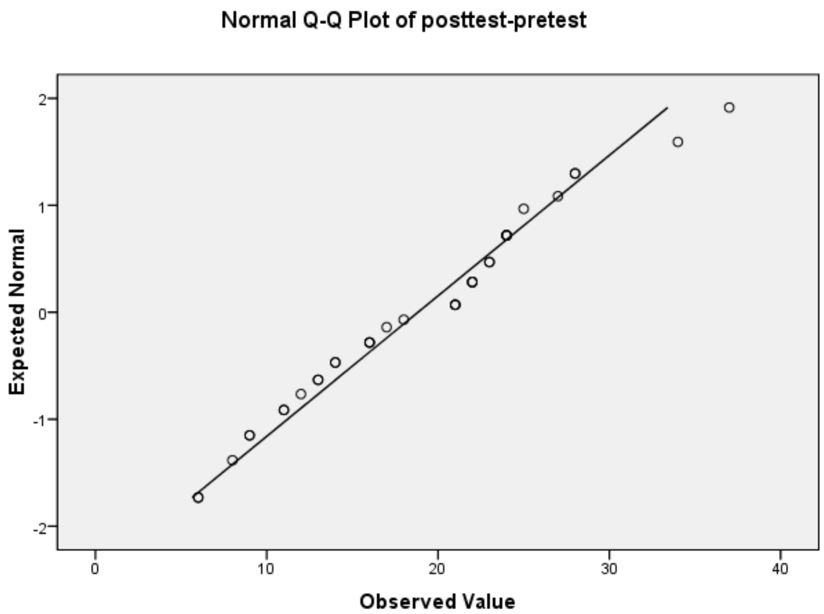
To determine the applicability of the paired sample t-test in order to test the research hypothesis, the assumption of normality of the tests (pre-test and post-test) scores was performed by formulating the following hypothesis.

H_0: The difference between the post-test and pre-test scores in the experimental group is not normally distributed.

H_1: The difference between the post-test and pre-test scores in the experimental group is normally distributed.

It must be noted that the paired sample t-test can be applied only when the variables for the study are normally distributed.

Figure 4.1: The normal Q-Q plot of post-test-pre-test



The normal Q-Q plot for post-test-pre-test scores was used. The paired sample t-test was used to test whether there was significant difference between the test scores in pre-test and post-test of learners who had been taught with a constructivist-based teaching method (in the experimental group). The result obtained for the experimental group is shown in Figure 4.1. The normal Q-Q plot of post-test-pre-test scores indicate that the experimental and control group are normally distributed. In addition, a formal normality test was performed by using Kolmogorov-Smirnov and Shapiro-Wilk. The *p*-values for the Kolmogorov-Smirnov and Shapiro-Wilk yielded 0.171 and 0.424 respectively.

The null hypothesis for normality test was that the difference of the post-test and pre-test scores would not be normally distributed while the alternate asserted that the difference of the post-test and pre-test scores would be normally distributed. Since the *p*-values for both Kolmogorov-Smirnov and Shairo-Wilk were more than 5% the null hypothesis is therefore rejected, meaning the difference of the post-test and pre-test scores are normally distributed. These *p*-values are more than 5%, meaning that the residuals are normally distributed. These results are as shown in the Table 4.7.

Table 4.7: The normality test- experiment group

Normality test	Kolmogorov-Smirnov			Shapiro-Wilk		
	Statistics	df	Sig.	Statistics	df	Sig.
Post-test-pre-test	0.127	35	0.171	0.969	35	0.424

Since the difference in the pre-test and post-test scores were normally distributed, the paired sample t-test can be applied to test the significant difference between the performance of learners in pre-test and post-test scores using constructivist-based teaching method.

4.4.3 Research hypothesis

The null and alternative research hypothesis formulated for this study is (see also, Section 3.3):

H_1: The constructivist-based teaching method is more effective than traditional teaching methods in reducing learners' errors when Grade 11 algebraic tasks are treated.

H_1: $\mu_{\text{constructivist-based teaching method}} \neq \mu_{\text{traditional teaching method.}}$

H_0: The constructivist-based teaching method is not more effective than traditional teaching methods in reducing learners' errors when Grade 11 algebraic tasks are treated.

H_0: $\mu_{\text{constructivist-based teaching method}} = \mu_{\text{traditional teaching method.}}$

The paired sample t-test was performed in order to test the hypothesis in Section 4.5.3. The result of the paired sample t-test for the experimental group is shown in Table 4.8.

Table 4.8: Paired samples statistics for the experimental group

Type of test	Mean	n	SD	SEM	Correlation	*p*-value
Pre-test experiment	25.8	35	7.31	1.24	0.63	0.000
Post-test experiment	44.63	35	9.68	1.64		

Table 4.8 shows that the pre-test mean was 25.8 with a standard deviation of 7.31. The post-test mean was 44.63 and the standard deviation was 9.68. A paired sample performed indicated that n=35 learners took the pre-test and post-test. The correlation between the pre-test and post-test scores was 0.63 with associated probability of 0.000. This result suggests that the correlation was significant. Therefore, it is reasonable to conclude that there was a moderate linear relationship between the pre-test and post-test scores in Table 4.8.

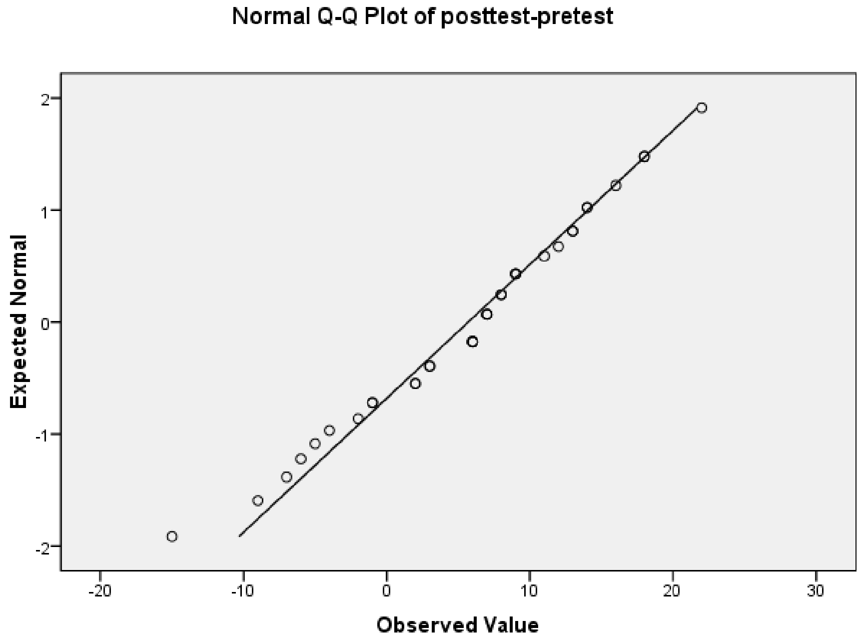
Table 4.9: Paired sample t-test for pre-test and post-test scores of experiment group

Test	N	Mean	SD	SEM	t	df	*p*-value
Pre-test-post-test	35	-18.8	7.61	1.29	-14.6	34	0.000

The result of the paired sample test in Table 4.9 indicated that the pair differences between the pre-test and post-test scores was 18.8.This means that the use of the constructivist-based teaching method potentially reduced learners' error in algebra, hence the improvement in learners' performance. Table 4.9 shows that there was a gain of 18.8 points in the mean scores as a result of using constructivist-based teaching method in the experimental group.

Given these observations, it is therefore reasonable to conclude that there was a statistically significant reduction in learners' algebraic errors, from 25.8 ± 7.31 to 44.63 ± 9.68 (p<0.05), following the implementation of a constructivist-based teaching method in the experimental group. However, the improvement of learners' tendency not to do errors amounted to 18 ± 7.61. Given that p<0.05 it was reasonable to reject $\mathbf{H_0}$ of no effect in favour of $\mathbf{H_1}$. Therefore, it can be concluded that the constructivist-based teaching method is effective in reducing learners' errors.

Figure 4.2: The normal Q-Q plot of post-test-pre-test



In comparison to the observed results in relation to the experimental group, the study further performed the paired sample t-test for the control group. The result of the paired sample t-test for the control group is analysed in Figure 4.2 and the formal test for normality is indicated in Table 4.10.

The normal Q-Q in Figure 4.2 also indicated that the pre-test and post-test scores of the control group were normally distributed. The null hypothesis for normality test is that the difference of the post-test and pre-test scores is not normally distributed while the alternate is that the difference of the post-test and pre-test scores is normally distributed.

Table 4.10: The normality test- control group

Normality test	Kolmogorov-Smirnov			Shapiro-Wilk		
Post-test-pre-test	Statistics	df	Sig.	Statistics	df	Sig.
	0.118	35	0.200	0.962	35	0.261

Since the *p*-values for both Kolmogorov-Smirnov (*p*-value=0.20) and Shairo-Wilk (*p*-value=0.261) were more than 5%, the null hypothesis was rejected in favour of the alternate hypothesis meaning that the difference of the post-test and pre-test scores were normally distributed. Since the difference of the pre-test and post-test scores were normally distributed, the paired sample t-test could then be applied to test the significant difference between the performance of learner's pre-test and post-test scores using traditional teaching method (learners in the control group learning environment). The result of the paired sample t-test for the control group is shown in Table 4.11.

Table 4.11: Paired Samples Statistics for the Control group

Type of test	Mean	n	SD	SEM	Correlation	*p*-value
Pre-test control	24.14	35	6.94	1.17	- 0.54	0.376
Post-test control	30.46	35	6.83	1.40		

Table 4.11 shows that the pre-test mean was 24.14 with *SD*=6.94. The post-test mean was 30.46 and *SD*=6.83. A paired sample conducted showed that n=35 learners took the pre-test and post-test. The correlation between the pre-test and post-test scores was -0.154 with an associated *p*-value of 0.376. This result indicated that the correlation was not significant. Given these observations, the study therefore conclude that even though there was a weak linear relationship between the pre-test and post-test scores for the control group, it was not statistically significant at 10% level.

Table 4.12: The paired sample t-test for pre-test and post-test scores for control group

Test	N	Mean	SD	SEM	t	Df	*p*-value
Pre-test-post-test	35	-3.23	11.61	1.96	-1.65	34	0.109

The findings of the paired sample t-test in Table 4.12 indicate that the pair difference between the pre-test and post-test mean scores was 6.32. This result suggests little improvement in the marks obtained by the Grade 11 algebraic learners who were taught using the traditional teaching method. With reference to the mean scores of the pre-test and post-test and the t-value [from 24.14 ± 6.94 to 30.46 ± 6.83 ($p<0.109$)], the study concluded that there was no statistically significant reduction in learners' algebraic errors in the control group that was taught with the traditional teaching method. Since the probability value of 0.109 is more than 5% then the null hypothesis was accepted, meaning that the traditional teaching method did not significantly reduce learners' errors in algebra.

A comparison was made between the mean gains of 18.83 with an associated *p*-value of 0.000 of the experimental group (see, Table 4.9) to 6.32 with *p*-value of 0.109 of the control group. Therefore, it is reasonable to conclude that the constructivist-based teaching method was more effective in reducing learners' errors than the traditional teaching method and this was statistically significant at 1% level.

4.4.4 Details of other tables used to analyse the inferential statistics

The tables below (see, Table 4.13 to Table 4.25) are the other tables used to analyse the inferential statistics and showed the results of: (1) the independent t-test statistical analysis between the experimental group and the control group; (2) the paired sample t-test within the group; (3) the correlations of scores for the two comparative groups; and, (4) the test of normality from the data used to draw the conclusions for the foregoing analysis.

Table 4.13: Group statistics pre-test scores

	Group*	n	Mean	SD	Std. Error Mean
Pre-test	EG	35	25.8000	7.30753	1.23520
	CG	35	24.1429	6.93747	1.34150

***EG=Experimental group; CG=Control group**

Table 4.14: Independent samples test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
Pre-test	Equal variances assumed	0.000	0.987	2.256	68	0.027	4.11429	1.82355	.47545	7.75312
	Equal variances not assumed			2.256	67.542	0.027	4.11429	1.82355	.47500	7.75357

Table 4.15: Group statistics post-test scores

	Group	n	Mean	Std. Deviation	Std. Error Mean
Post-test	EG	35	44.6286	9.68044	1.63629
	CG	35	30.4571	6.8345	1.40293

Table 4.16: The independent samples test

		Levene's test for equality of variances		t-test for equality of means						
		F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence interval of the difference Lower	Upper
Post-test	Equal variances assumed	0.843	0.362	8.007	68	0.000	17.25714	2.15538	12.95615	21.55814

Table 4.16: The independent samples test

		Levene's test for equality of variances		t-test for equality of means						
		F	Sig.	t	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence interval of the difference: Lower	95% Confidence interval of the difference: Upper
Post-test	Equal variances assumed	0.843	0.362	8.007	68	0.000	17.25714	2.15538	12.95615	21.55814
	Equal variances not assumed			8.007	66.451	0.000	17.25714	2.15538	12.95433	21.55995

Table 4.17: Paired samples statistics

		Mean	n	Std. Deviation	Std. Error Mean
Pair 1	Pretest Experiment	25.8000	35	7.30753	1.23520
	Post-test Experiment	44.6286	35	9.68044	1.63629

Table 4.18: Paired samples correlations

		n	Correlation	Sig.
Pair 1	Pretest Experiment & Post-test Experiment	35	0.630	0.000

Table 4.19: Paired samples test

		Paired differences					T	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper			
Pair 1	Pre-test Experiment Post-test Experiment	-18.82857	7.61379	1.28696	-21.44400	-16.21314	-14.630	34	.000

Table 4.20: Paired samples statistics

		Mean	n	Std. Deviation	Std. Error Mean
Pair 1	Pre-test control	24.1429	35	6.93641	1.34150
	Post-test control	30.4571	35	6.83454	1.40293

Table 4.21 Paired samples correlations

		n	Correlation	Sig.
Pair 1	Pre-test control & post-test control	35	0.470	0.004

Table 4.22: Paired samples test

		Paired differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Pre-test control – Post-test control	-5.68571	8.36228	1.41348	-8.55826	-2.81317	-4.022	34	0.000

Table 4.23: Correlations

		Pre-test Experiment	**Post-test Experiment**
Pre-test Experiment	Pearson Correlation	1	0.630**
	Sig. (2-tailed)		0.000
	N	35	35
Post-test Experiment	Pearson Correlation	0.630**	1
	Sig. (2-tailed)	0.000	
	N	35	35

**** Correlation is significant at the 0.01 level (2- tailed)**

Table 4.24: Test of normality

	Kolmogorov-Smirnov[a]			**Shapiro-Wilk**		
	Statistic	Df	Sig.	Statistic	df	Sig.
Posttest-pretest	0.127	35	0.171	0.969	35	0.424

a. Lilliefors Significance Correction

Table 4.25: Test of normality

	Kolmogorov-Smirnov[a]			**Shapiro-Wilk**		
	Statistic	Df	Sig.	Statistic	Df	Sig.
Post-test-pre-test	0.118	35	0.200	0.962	35	0.261

a. Lilliefors Significance Correction

4.5 DISCUSSION OF LEARNERS' ERRORS IN THE ACHIEVEMENT TESTS

In this section, the errors learners committed in the achievement tests are discussed and analysed. The discussions and analysis start with learners' errors in Part A and follow by learners' errors in Part B and Part C.

4.5.1 Discussion and analysis of learners' errors in PART A of tests

The PART A of both pre-test and post-test was multiple-choice questions. Appendix C provides the rubric of the achievement tests and Appendix H shows learners' responses to Part A of the tests. The summary the number of learners' responses to the various options in the tests in both Experimental Group and Control Group is indicated in Table 4.26 and Table 4.27.

Table 4.26: Number of learners who responded to the various options in the tests -EG

Test	Pre-test				Post-test			
Option/ Question	A	B	C	D	A	B	C	D
1.1	16	2	0	**17**[13]	3	0	2	**30**[14]
1.2	9	**23**	1	2	3	**31**	0	1
1.3	9	0	**26**	0	0	0	**35**	0
1.4	11	10	**12**	2	2	7	**26**	0
1.5	7	**23**	2	3	2	**33**	0	0
1.6	10	**13**	3	9	13	**19**	1	2
1.7	9	11	6	**9**	1	3	2	**29**
1.8	5	12	**11**	7	1	0	**33**	1
1.9	9	**22**	3	1	4	**31**	0	0
1.10	9	**21**	2	3	5	**29**	1	0

Table 4.27: Number of learners who responded to the various options in the tests - CG

Test	Pre-test				Post-test			
Option/ Question	A	B	C	D	A	B	C	D
1.1	12	0	2	**21**	10	2	5	**18**
1.2	5	**30**	0	0	8	**27**	0	0
1.3	3	0	**32**	0	5	0	**30**	0
1.4	15	9	**10**	1	22	4	**4**	5
1.5	2	**29**	0	4	1	**30**	0	4
1.6	5	**23**	1	6	7	**20**	1	7
1.7	4	22	6	**3**	7	16	4	**8**
1.8	5	10	**16**	4	5	9	**16**	5
1.9	8	**23**	0	4	7	**28**	0	0
1.10	13	**19**	2	1	14	**20**	1	0

13. Red colour indicates learners who answered correctly in the pre-test.
14. Green colour indicates number of learners who answered correctly in post-test.

Table 4.28: Summary (in %) of learners' wrong responses in tests in PART A

Question	Experimental group		Control group	
	Pre-test (%)	Post-test (%)	Pre-test (%)	Post-test (%)
1.1	51.43	14.29	40.00	48.57
1.2	34.29	11.43	14.29	22.86
1.3	25.71	0.00	8.57	14.29
1.4	65.71	25.71	71.43	88.57
1.5	34.29	5.71	17.14	14.29
1.6	62.86	45.71	34.29	42.86
1.7	74.29	17.14	91.43	77.14
1.8	68.57	5.71	54.29	54.29
1.9	37.14	11.43	34.29	20.00
1.10	40.00	17.14	45.71	42.86

In Question 1.1 learners were requested to: *expand the bracket and simplify* $(a + b)^2$. The correct option was D. In the pre-test n=16 learners from the EG, representing 45.71%, that is, 16(45.71%) of the participants erroneously chose option A. This implies they expanded and simplified $(a + b)^2$ to $a^2 + b^2$. However, after the intervention only 3(8.57%) learners committed this error in the post-test. Conversely, in the CG 12(34.29%) learners and 10(28.57%) learners in the pre-test and post-test respectively committed this error (see, Table 4.27). It could be seen from Table 4.26 and Table 4.27 that 17(48.57%) learners from EG and 21(60%) learners from CG answered this question correctly. These learners selected the option D in the pre-test and 30(85.71%) of them from EG and 18(51.43%) from the CG answered correctly in the post-test. It is evident from Table 4.28 that 51.43% of learners in the EG answered this question wrongly in the pre-test. However, after the CBTM intervention only 14.29% of the learners answered it wrongly. Given these observations it is reasonable to argue that CBTM significantly improved learners' error in this question. On the contrary, 40% of the learners in the CG answered it wrongly in the pre-test, and after the TTM intervention 48.57% of them answered it wrongly. This implies that the TTM intervention exacerbated the learners' errors in this question.

In Question 1.2 learners were supposed to simplify the expression $3x + 3x$. The correct option was B: $6x$. In simplifying this expression 34.29% of the learners in the

EG and 14.29% of the CG answered incorrectly in the pre-test. However, in the post-test only 11.43% of learners in the EG group and 22.86% of the CG got the answer wrong. This shows an improvement in the error committed by the EG group after the intervention in this question whereas the result from CG after the intervention aggravated the situation. To simplify $3(x + y)$ in Question 1.3 the correct option was C: $3x + 3y$. At this instance 25.71% of the learners from the EG and 8.57% from the CG got it wrong. After the intervention none of the learners from the EG got it wrong in the post-test whereas the percentage of learners who got it wrong from the CG in the post-test worsened to 14.29%. This is an indication that the CBTM intervention completely eliminated this error in the EG.

Question 1.4 was one of the questions that were poorly answered. The correct option was C: $x + 3x = 36$, but 65.71% of the learners from the EG and 71.43% from the CG got it wrong. After the intervention only 25.71% of the learners from EG got it wrong. This is an indication of improvement. However, the scenario worsened in the CG as 88.57% of learners got it wrong. In Question 1.5 learners were required to solve a simple linear equation: $5 + 3x = 11$. It is quite surprising to find Grade 11 learners getting this wrong. It was found that 35.29% of the learners from the EG and 17.14% from the CG solved it wrongly in the pre-test. After the interventions only 5.71% of the learners from the EG and 14.86% from the CG got it wrong. This showed a significant improvement in the EG and a marginal improvement in the CG.

Question 1.6 requested learners to solve a quadratic equation: $(x – 5)(x + 1) = 7$. The correct option was B: $x = -2$ or $x = 6$. This was equally poorly answered as 62.86% of the EG learners and 37.14% from the CG answered it wrongly. After the interventions 45.71% of the learners from the EG and 42.86% of the learners from the CG got it wrong. While EG showed an improvement the CG showed deterioration in performance after the TTM intervention. One of the most poorly answered questions by both groups was Question 1.7. As many as 74.29% of the learners from the EG and a whopping 91.43% of the learners from the CG gave a wrong answer in the pre-test. There was improvement by both groups in the post-test. However, the improvement by the EG was very significant as only 17.14% of the learners got this question wrong while 77.14 in the CG got it wrong.

Learners were required to simplify: $\frac{a}{b}+\frac{c}{d}$ in Question 1.8. It was established that 65.57% of the learners from the EG and 54.29% of learners from the CG answered it wrongly in the pre-test. However, in the post-test, only 5.71% of the learners from EG answered it wrongly and 68.57% from the CG got it wrong. This showed a great improvement by the EG. One of the common errors in algebra learners commit is simplifying exponential expressions like the one in Question 1.9, that is, $(x^2).(x^3)$. Thirteen learners representing 37.14% of the EG and 12 learners representing 34.29% of the CG gave a wrong answer in the pre-test. However, in the post-test, only 4 learners representing 11.43% of the EG, and n=7 learners from the CG representing 20% of the learners got it wrong. This showed in improvement in both groups but EG improved much better.

Question 1.10 requested learners to simplify the phrase $3x–(x–5)$. The purpose of this question was to ascertain one of the errors learners often commit when expressions involve such brackets. Pre-test results indicated that 40% of the learners from the EG, and 45.71% of the learners from the CG got the answer wrong. The correct option was B: $2x + 5$, but as many as 25.71% of learners from the EG and 37.14% of learners from CG chose option A: $2x – 5$. Post-test results showed that only 17.14% of learners from the EG, and 42.86% of learners from the CG got it wrong. This showed a 57.15% improvement in the EG as against 6.23% by the CG.

4.5.2 Discussion and analysis of learners' errors in PART B and PART C of tests

Different types of errors in algebraic variables, expressions, equations, and word-problem characterised learners responses in Part B and Part C of the achievement tests. In both groups, most errors occurred in the pre-test. In this section, the errors committed by learners in the four conceptual areas of algebra by both groups during pre- and post-stage are categorised into four and analysed (see, Table 29).

Table 4.29: Types of errors identified in the four conceptual areas in algebra

Conceptual area	Errors identified
Algebraic Variable Errors	• Misinterpretation of product of two variables • Wrong assignment of arbitrary values for variables • Misjudgement of the magnitude of variables
Algebraic Expression Errors	• Invalid conversion of expression to equations • Reversal errors
Algebraic Equation Errors	• Inability to identify the type of equation • Manipulation and transposition errors
Word-problem Errors	• Translation error • The use of arithmetic instead of algebraic method

Table 4.30: Summary of frequency of learners' errors at pre- and post-stages

Group (n=70)	Test	Number of errors in algebraic variables	Number of errors in algebraic expressions	Number of errors in algebraic equations	Number of errors in word-problems
Experimental group	Pre-test	82	79	68	85
	Post-test	23	18	15	26
% Error reduction		**71.95**	**77.22**	**77.94**	**69.41**
Control group	Pre-test	78	81	65	87
	Post-test	45	34	42	56
% Error reduction		**42.31**	**58.02**	**35.38**	**36.63**

Source: Results of an achievement tests results of the current study

4.5.2.1 Algebraic variable errors

One of the errors learners committed under variables was assigning labels, arbitrary values, or verbs for variables and constants. Some learners misinterpreted a variable as a 'label', as a 'thing', or even as a verb such as 'buying'. Nevertheless, they really did not perceive the correct interpretation of the variable as the 'number of a thing'. It was difficult for them to distinguish between variables and non-variables in terms of the varying and non-varying quantities in the question. Often, they were confused with viewing variables as constants or vice versa. This error type was observed in other questions too.

It was noticed that when learners were asked to name something in the problem that is not a variable (Question 2 of Part C), answers such as 'Thandi', 'cents', 'donuts' were given. In a general sense, these answers may be considered as correct.

Sometimes, the words ‘donuts’ and ‘cents’ could be considered as symbols representing variables in some contexts. However, these answers were considered as incorrect in the context of the given problem since there was a variable or a number attached to these words. Therefore, these words have meanings in the given context when they were taken together with those variables or numbers.

Another type of error committed under variables was misinterpreting the product of two variables. Learners who made the above error had difficulties to perceive the product of two variables as two separate variables combined together by a sign. They viewed the product as one variable of two variables as the second variable is to change the value of the first variable. This indicates that these learners could perceive the product as two separate variables, but they incorrectly perceived an interaction between the two variables. This is a typical property of some numeral systems such as the ancient Roman numeral system but it is not a property of algebraic variables.

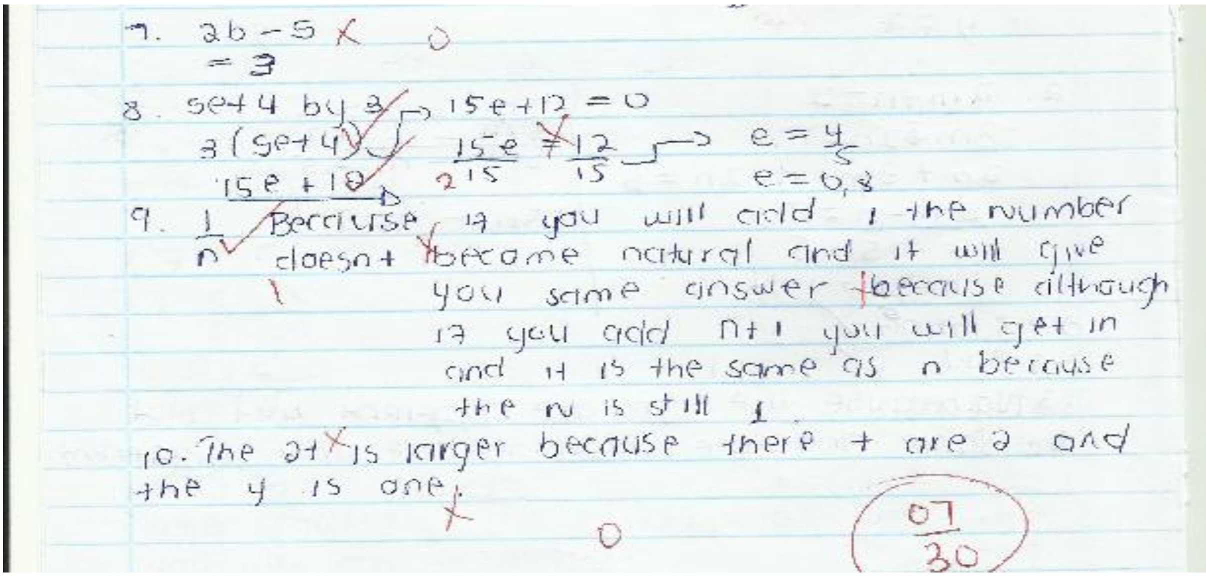
Misjudging the magnitudes of variables and lack of understanding of variables as generalised numbers was another type of error committed under variables. Some learners judged the magnitude of two variables by examining their coefficients when they are in an equation such as question 10 of Part B: $y = 2t + 3$. Since t has a larger value beside it, they thought that 2t is larger than y in the equation. This comparison is correct when comparing two like terms such as 2t and t but it is incorrect when comparing unlike terms and also when they are related to each other in an equation with different coefficients. Not realizing that variables take many values in some contexts was another problem for some learners. In an equation such as $y = 2t + 3$, these learners recognised that both y and t are variables. However, they did not realise that these variables can take more than one value (see vignette 1a below for a learner’s response to Question 10 in Part B).

4.5.2.2 Algebraic expression errors

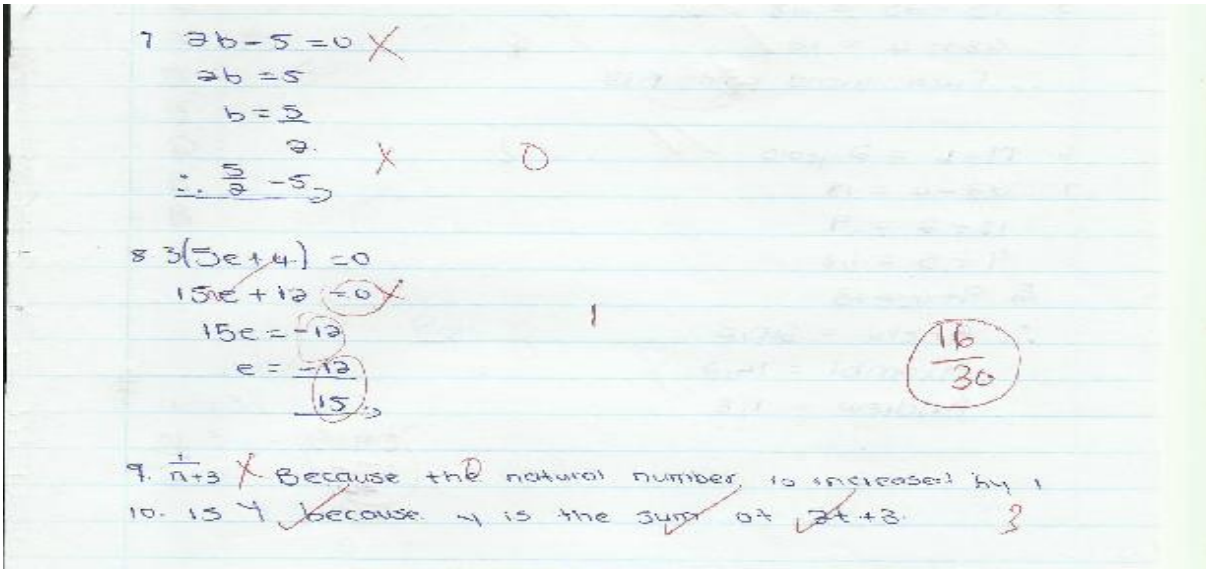
One of the errors learners committed is reversal error. Two different forms of reversal errors were observed in the answers to questions 3 in Part C and question 1.7 in Part A. In question 3, learners were asked to write an algebraic expression for the number of rows in the parade and the correct answer was $\frac{n}{7}$. The answer was considered as a reversal error when it was written as $\frac{7}{n}$. If learners could not understand and use 'n' as representing 'the number of girl scouts', it is difficult for them to write a correct algebraic expression representing the 'the number of rows'. Furthermore, the problem could be difficult for them because the dividend is a variable, not a number. Another possible gap in learners' schemas is that learners are more familiar with multiplying a variable with a given number but dividing may not be that easy for them. In other words, it could be relatively easy for them to calculate the total number of girls when the number of rows is given as a variable and the number of girls in each row is given as a number.

The next category of errors committed was converting algebraic expressions in answers into equations. In this category, some learners formed invalid equations from the answers in the form of algebraic expressions. These learners proceeded further to solve these equations. There were two varieties to this error. Firstly, when simplifying algebraic expressions, learners connected the variables in the problem in a meaningless way to form an equation. Secondly, they were reluctant to accept an algebraic expression as the final answer and came up with a solution by solving the invalid algebraic equation they formed. For example, some of the learners answered questions 7 and 8 in Part B as *5 – 2b = 3b* and others went further to solve *5 – 2b* as: *3 – b* and got the answer b = 3 for question 7. For question 8 some of the learners' solutions were: *3(5e + 4)=15e + 12=27e* and others as 3*(5e + 4)=5e + 12=0* (see, Vignettes 1a).

Vignette 1a: Converting expression into equation and comparing two variables

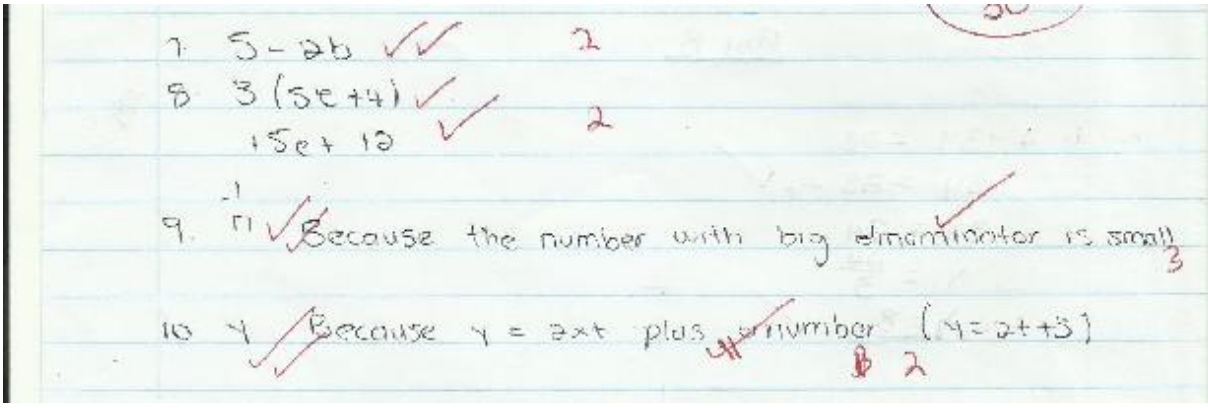


Another learner's response:



After the CBTM intervention, the same learners who committed such errors in the pre-test improved and solved it correctly (see, Vignette 1b).

Vignette 1b: Learner solution after CBTM intervention

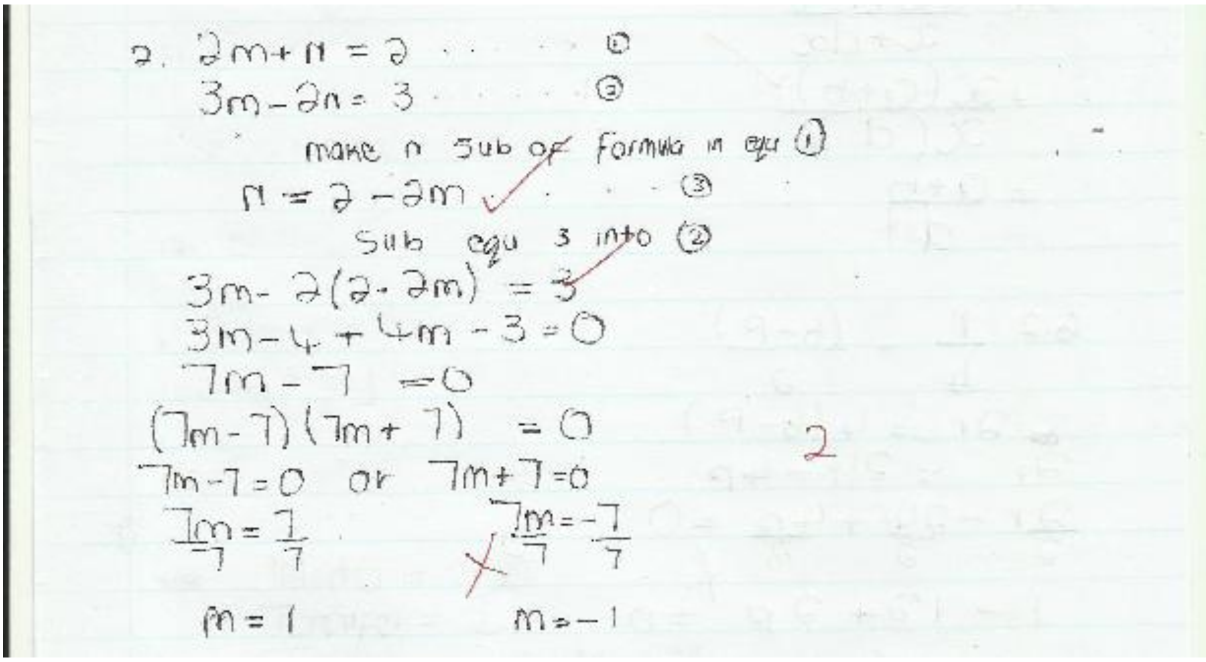


4.5.2.3 Algebraic equation errors

There were some questions in the test for algebraic equations that involved building up and/or solving equations such as Question 1 and Question 2 of Part B, and Questions 4, 5 and 7 in Part C. The problems were in three different formats: algebraic format, word format without a real-life context, and a word format with a real-life context. It is important to mention that some error types appeared more than once in the same question and in different questions. For example, errors associated with manipulation and transposition “Add when the equations have to be subtracted or vice versa” as in Question 1 and Question 2 in Part B, and Questions 4, 5 and 7 in Part C. For instance, some learners solved the simple linear equation in question 1 of Part B, that is, *4+3y=28* as *7y=28* and arrived at the answer *y=4* and others solved it as: *3y=28/4* to get *3y=7* and then again proceeded to the next step as *y=7–3* to arrive at *y=4* as their final answer (see, Vignette 2a). Another error observed in learners’ solution was solving *7m–7=0* as difference of two squares, that is *(7m–7)(7m+7)=0* as part of the solution to the systems of simultaneous linear equations (see, Vignette 2b).

Vignette 2a: Multiple errors in one linear equation

Vignette 2b: Solving linear equation as difference of two squares

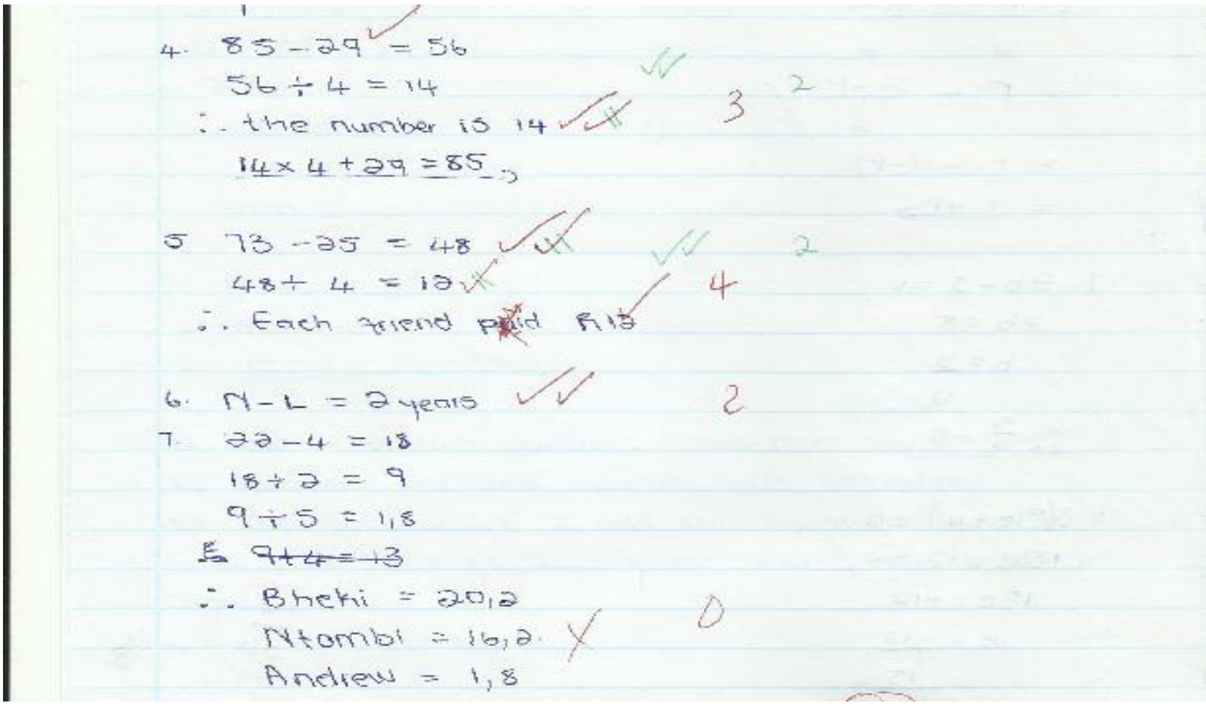


4.5.2.4 Word problem errors

In the past, many empirical studies indicated that learners face difficulties in translating algebra word problems that state relationships between two or more variables into a symbolic form. In this current study, there were seven word problems which consisted mainly of word sentences. Learners had to read the problems, convert them into algebraic forms and solve them. Some of these problems contained relational proportions (Question 7). In some questions, learners had to provide reasons for their answers. Among others, there are two main processes involved in solving a word problem. One is the translation process, which is to read and translate the words of the problem into an algebraic representation. The solution process is to apply standard rules of algebra to arrive at a solution.

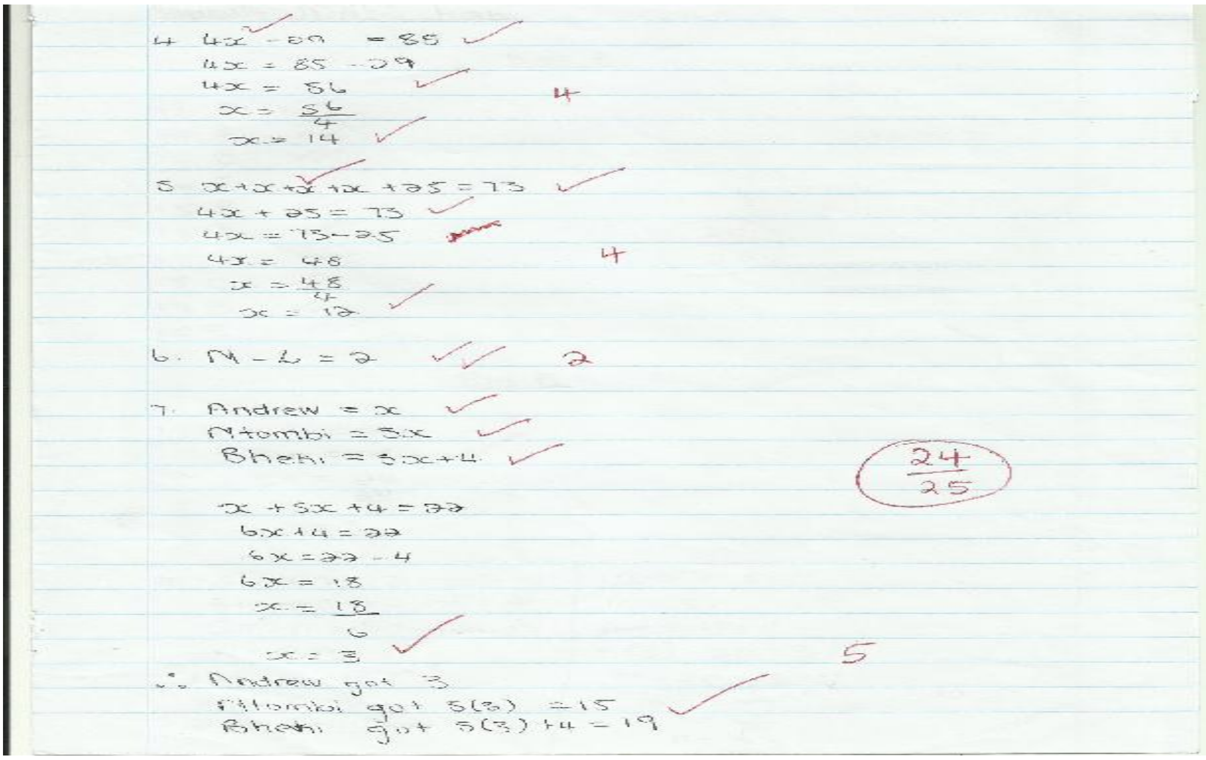
Several types of these errors were seen from the careful analysis of learners' responses. One observation was that a considerable number of learners used arithmetic methods rather than algebraic methods to solve the word problems. For example, 'working backward' and 'trial and error' methods were common. Most of the learners in both groups made this mistake in Question 4 and Question 5 in Part C in the pre-test. Vignette 3 below shows sample of learners who committed this error.

Vignette 3a: Learners using arithmetic method instead of algebraic method



Many learners responded in similar manner to this question (see, Table 29 & Table 30).

Vignette 3b: Learner solution after CBTM intervention



4 4x − 29 = 85
4x = 85 − 29
4x = 56
x = 56/4
x = 14
4

5 x + x + x + x + 25 = 73
4x + 25 = 73
4x = 73 − 25
4x = 48
x = 48/4
x = 12
4

6. M − L = 2   2

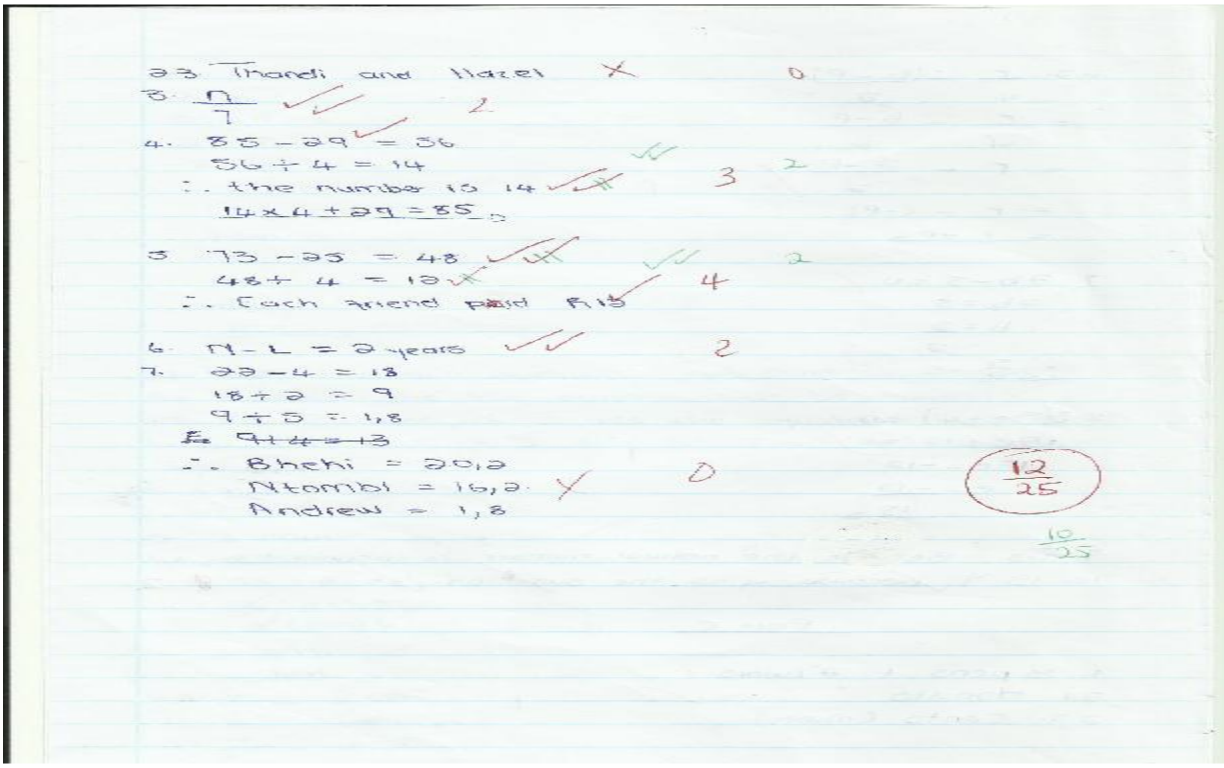
7. Andrew = x
Ntombi = 5x
Bheki = 5x + 4

x + 5x + 4 = 22
6x + 4 = 22
6x = 22 − 4
6x = 18
x = 18/6
x = 3
5
∴ Andrew got 3
Ntombi got 5(3) = 15
Bheki got 5(3) + 4 = 19

24/25

Vignette 4: A typical learner's performance before and after CBTM

4 (a): The learner's script before CBTM intervention



2.3 Thandi and Hazel   0
3. n/7   2
4. 85 − 29 = 56
56 ÷ 4 = 14
∴ the number is 14   3   2
14 × 4 + 29 = 85

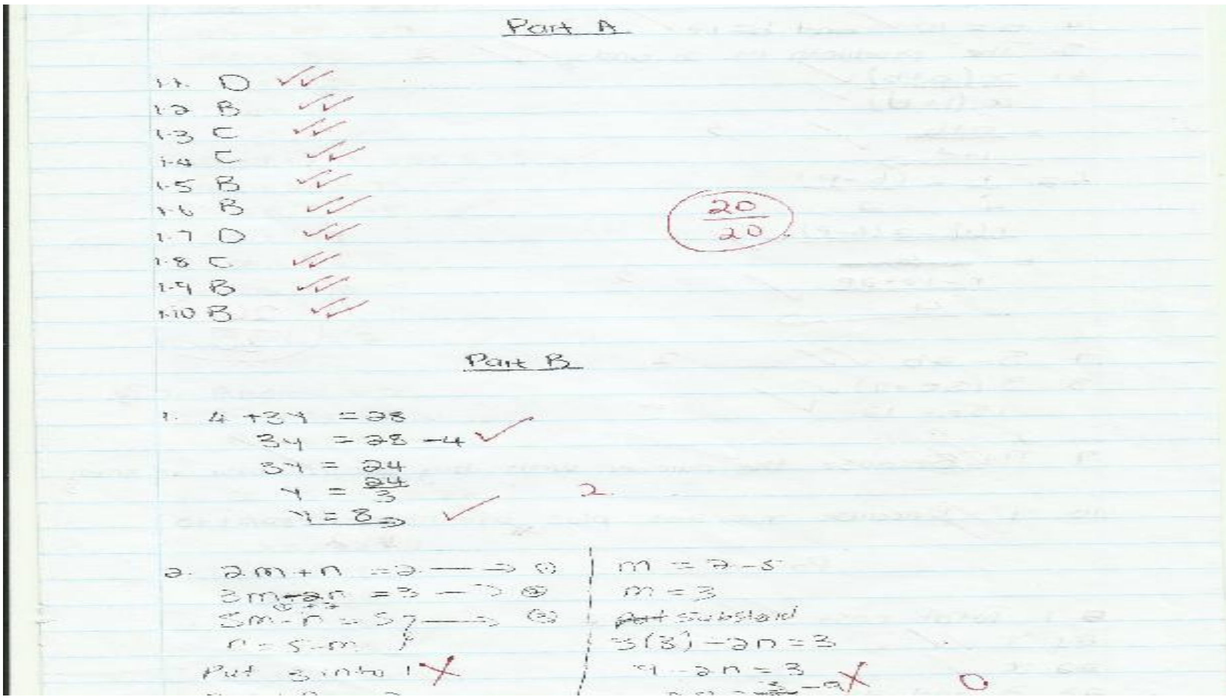
5 73 − 25 = 48
48 ÷ 4 = 12   4   2
∴ Each friend paid R12

6. M − L = 2 years   2
7. 22 − 4 = 18
18 ÷ 2 = 9
9 ÷ 5 = 1,8
~~9 + 4 = 13~~
∴ Bheki = 20,2
Ntombi = 16,2   0
Andrew = 1,8

12/25

10/25

Vignette 4 (b): The learner's script after CBTM intervention

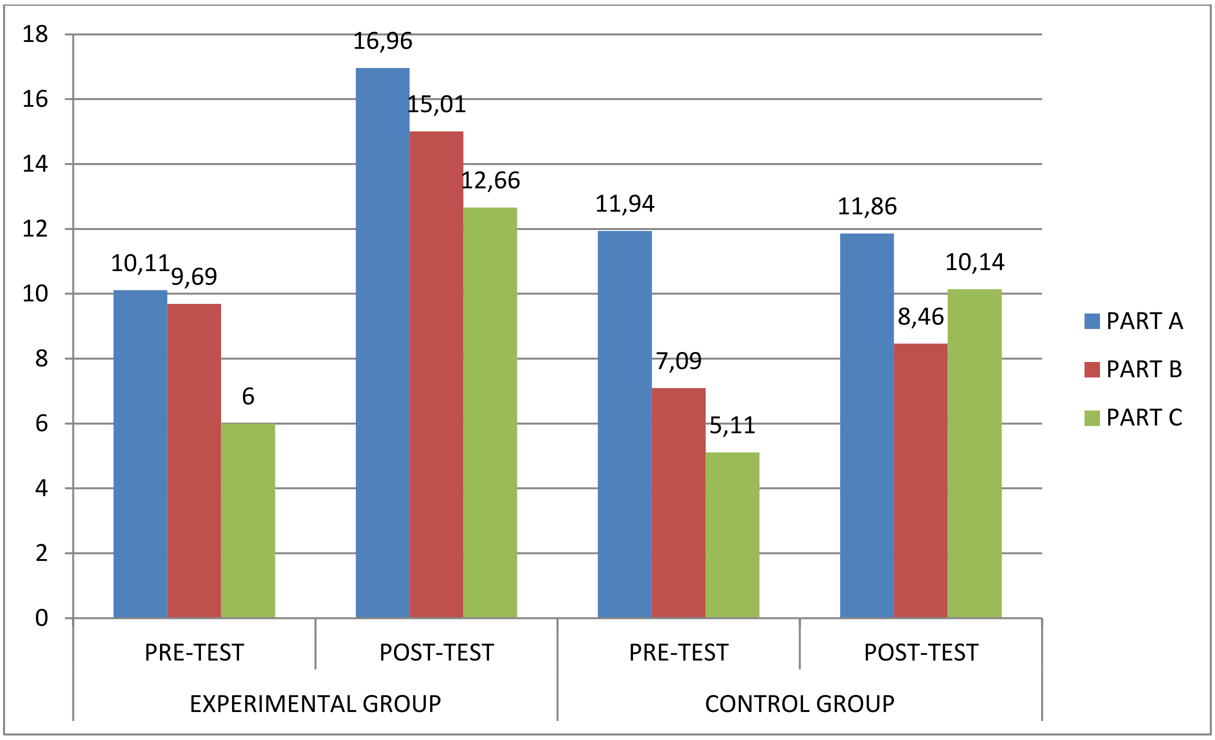


4.5.3 Aggregating the scores and discussions of the achievement test

Figure 4.3 depicts the average scores of learners in the ACATS. It shows the details of the scores of learners in all the three parts of the achievement tests (see also learners' performance indicators in Appendix I). It must be emphasised that an improvement in performance scores is an indication of reduction in learners' errors.

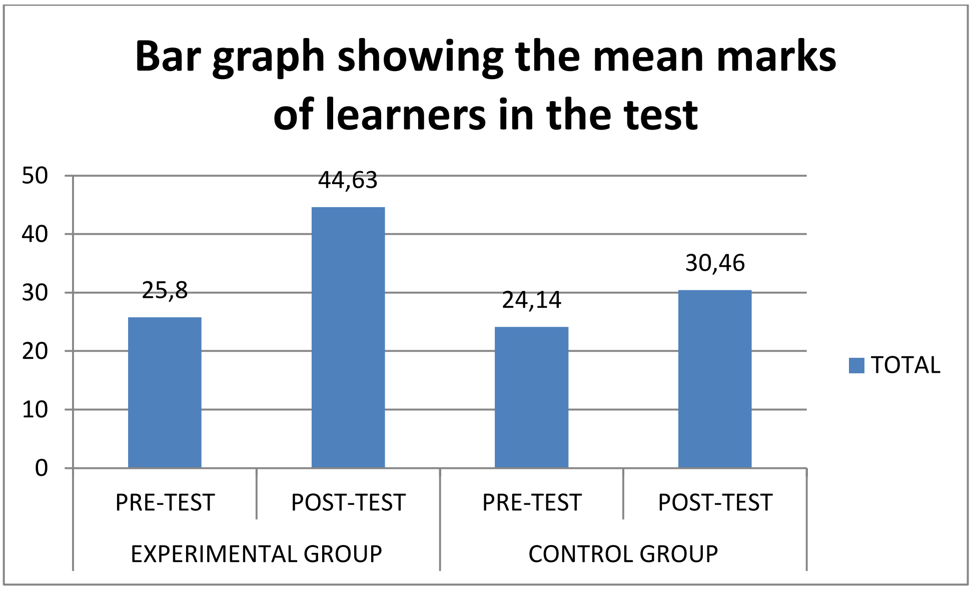
It is evident from Figure 4.3 that the average scores of learners in Part A (multiple choice) of the pre-test was 10.11 out of 20 marks for the EG, and 11.94 out of 20 marks for the CG. Learners in the CG performed slightly better than those in the EG in the pre-test. Part B was poorly performed by learners in the pre-test by both groups. Out of a maximum mark of 30 for this part, the mean mark for the EG was 9.69 and that for the CG was 7.09. Part C of the achievement test which was on word-problem was equally challenging to learners in both groups. Out of a total mark of 25, the mean mark of learners in the EG in the pre-test was 6 and that of CG was 5.11.

Figure 4.3: Grouped bar graph showing learners' average score in the various parts of the ACATS



After the CBTM intervention, the average post-test mark of the EG in Part A increased significantly to 16.96 whereas that of CG who received the TTM intervention decreased marginally to 11.86. Also in Part B, the CBTM intervention yielded an improved average mark of 15.01 for the EG in the post-test whereas that of the CG was marginally increased to 8.46. Finally, in Part C, the average post-test mark of the EG also increased significantly from 6 in the pre-test to 12.66 in the post-test (indicating an improvement of 111%) and that of the CG also improved from 5.11 to 10.14 (indicating an improvement of 98.4%). The aggregate of the learners' mean scores is shown in Figure 4.4. It can be inferred and deduced from the graph that there is no significant difference in the overall mean scores of learners in the pre-test. This is an indication of homogeneity of the two groups before the intervention.

Figure 4.4: Bar graph showing the overall mean marks of learners in the tests



However, the intervention resulted in significant difference in the mean scores of the EG and CG in the post-test. The EG performed significantly better than the CG. The mean mark of the EG increased from 25.8 in the pre-test to 44.63 in the post-test indicating an improvement of 72.98% whereas that of the CG increased from 24.14 to 30.46 indicating an improvement of 26.18%. This huge and significant improvement in the mean mark of the EG is attributable to the CBTM intervention.

It can be concluded from the foregoing discussion and the evidence in Table 4.30, Figure 4.3 and Figure 4.4 that constructivist-based teaching method implemented in the experimental group was more effective in reducing learners' errors in the four conceptual areas in algebra than the traditional teaching method implemented in the control group.

4.6 DISCUSSION OF THE CBTM LESSON

The participating schools followed departmental guidelines on the construction of their school timetables, allowing 4.5 hours of teaching time for mathematics per week. The intervention lasted for four weeks. Figure 4.1 shows the instructional timetable the researcher used to administer the intervention in the experimental school. It must be noted that this was not a researcher developed instructional timetable but it was a school generated schedule.

Figure 4.5: Instructional timetable used by the researcher at the experimental school

Time	**7:30 8:30**	**8:30 9:30**	**9:30 10:20**	**10:20 11:20**	**11:20 12:20**	**12:20 12:30**	**12:30 13:30**	**13:30 14:30**
Periods	1	2		3	4		5	6
Monday			**B**	MAT		**B**		
Tuesday	MAT		**R**			**R**		
Wednesday			**E**		MAT	**E**		
Thursday		MAT	**A**			**A**		
Friday			**K**		MAT	**K**		

Both the control group and the experimental group wrote the pre-test on the first Monday and the post-test on the last Friday of the intervention (see, Figure 4.1). The post-test had exactly the same questions as the pre-test. The challenge was that there was only one period each available for Grade 11 to write the pre-test and the post-test with duration of one-and-half hours each. However, arrangements were made with the school to make use of periods 3 and 4 for this purpose. The researcher invigilated both tests in the experimental school.

The constructivist-based classroom was characterised by group learning approach (see, Section 1.9.3 & Section 1.9.9). Learners were put in groups of at least six members in each group. In addition, the researcher appointed group leaders for each group. The researcher only provided explanations when required to do so. Most importantly, the potential of more robust engagement was exploited with worked-out examples[15] in algebraic variables, expressions, equations and word-problems that were given to groups as worksheets (see, Appendix F & Appendix G). The experimental school and the control school were exposed to identical worksheet tasks. However, the

15. Worked-out examples are a set of activity-related examples used as scaffolds that presented an instructional step-by-step guideline on how to solve algebraic problems.

mode of presenting the worksheet tasks varied between the two groups. At the instruction phase of the lesson, the researcher divided the lesson into three stages: introduction; the body of the lesson; and conclusion. At the *introduction stage*, the researcher introduced the topic to class, explained the key terms and concepts, asked questions to assess learners' prior knowledge of the topic, and established the basic errors and algebraic skills of learners.

At the *body stage* of the lesson, learners in their groups were given the example sheets to discuss the solution steps whiles the researcher monitored group discussion. During this stage self-explanation activity and probing took place. At this stage of instruction, the researcher carefully monitored the group work and whole-class discussion. This was necessary so as to intervene and redirect the learners to correct their errors and misunderstandings. To do this, the teacher occasionally asked the learners probing question such as: *"Why did you do it that way? Will it work if you did it the way your friend suggested? "What makes the answer given by a peer to be wrong?" "What is the correct way to do it?"*

The researcher then asked them to talk about it further in their group as the researcher would get back to them shortly. There were no specific rules that informed the researcher when to intervene or how extensive the intervention should be. Most significantly, the researcher was at liberty to make these decisions and these were made on the basis of the researcher's knowledge of the subject matter and learners' past experiences. The role of the researcher as a teacher was limited to guiding and facilitating rather than telling the learner. The researcher created a purposeful, intentional and collaborative learning environment that enabled learners to actively strive to achieve the cognitive objective.

An example of Worked-out Example used during instruction

A father is three times as old as his son. In eight years' time, the father will be twice as old as the son. Determine the present ages of the father and the son.

Solution:

Step 1: Use a letter to represent the son's age, let y = the son's present age,

Step 2: determine the father's present age using the son's age, thus 3y

Step 3: determine their ages in eight years from now:

The son will be (y + 8) years

The father will be (3y + 8) years

Step 4: generate equation from the statement, thus

In eight years' time, father's age = two times the son's age

$3y + 8 = 2(y + 8)$

Step 5: Solve the equation

$3y + 8 = 2y + 16$

$3y – 2y = 16 – 8$

$y = 8$

Thus the son is 8 years now and the father is 24 years now.

An example of Worksheet activity given during instruction

It was given that x = 1 and Bafana made the following argument:

If x = 1 then,

Step 1: $x^2 = x$ ....multiply both sides by x

Step 2: $x^2 – 1 = x – 1$...subtract 1 from both sides

Step 3: $(x – 1)(x + 1) = x – 1$ ....factorise

Step 4: $\frac{(x-1)(x+1)}{(x-1)} = \frac{(x-1)}{(x-1\ )}$ divide both sides by x – 1

Step 5: $x + 1 = 1$

Step 6: $x = 0$

It was given that x = 1 but Bafana ended his argument by getting x = 0.

1.1 Identify the step in which he made a mistake with his argument?

1.2 Describe how this mistake can be avoided.

The *concluding stage* of the lesson was meant for reflection of the lesson where group discussion of activity took place, and success rate of the lesson was evaluated. The lesson concluded with more tasks given as homework.

Table 31: A constructivist-based lesson plan used in the experimental school

Lesson stages	Planned activities (in a CBTM lesson)
INTRODUCTION (15 min)	• Researcher introduces topic to class; • Explanation of key terms and concepts; • Questions asked to assess learners' prior knowledge of the topic; and, • Researcher establishes the basic errors and algebraic skills of learners.
BODY (20 min)	• Learners arranged in groups; • Example sheets given to groups; • Learners discuss solution steps; • Researcher monitors group discussion; and, • Self-explanation activity and probing takes place.
CONCLUSION (25 min)	• Reflection; • Class work/ group discussion of activity; • Evaluation of success rate; • Reflection on the lesson with more problems/tasks; and, • Homework is given.

At learning and performance phases one could hear different voices and sounds from the various groups like "*I got it*" and sometimes learners exhibited signs of frustration when they encountered challenges in their knowledge construction process with utterances such as "*I don't understand*" and "*your answer is wrong*". Learners' gestures like nodding the head in agreement with the explanation given by peers in the group or the teacher, and their utterances such as "*okay*", "*now I know*" indicated that knowledge constructions was taking place. Table 31 shows the lesson stages delivered by the researcher in the experimental group.

Furthermore, one unique feature of the CBTM lesson was that, it was difficult to identify who the teacher (researcher) was, as the researcher was moving around from group to group in order to monitor, assist and direct learners' discussion. The researcher sometimes sat down with the learners in the group and watched as learners discussed the task assigned to them. Most critically, the learners sometimes did not seem to notice the presence of the teacher in their group and kept on discussing and talking with each other. If someone with traditional preconceived notions that classrooms of learning should be ordered, systematic and quiet had entered the classroom he or she would miss the dynamic learning that was occurring in that classroom and many other classrooms structured for cooperative learning and from constructivist philosophical perspective.

The classroom arrangement was such that one would not even determine which part of the classroom was the back and which one was the front part. In this CBTM generated teaching environment, a teacher's desk was not even seen. All learning activities in this constructivist lesson were centred on the learners. Using the principles of cooperative learning and constructivist learning theory, the researcher carefully built a learning community in which teacher-learner and learner-learner interaction, which was subtly arranged, promoted knowledge construction and deep enduring learning that enabled learners' errors to be exposed and treated during the lesson. The teacher realised that in order to empower learners to verbalise their prior knowledge so as to expose the errors they inhabit and treat them, they must interact with one another as a community of learners frequently and easily.

The unique features of CBTM which distinguished it from the TTM were: the group learning approach; the nature of learner involvement and participation; the guiding and facilitating role of the researcher; the learner-centred lesson; the social interactions that existed in the classroom; availability of scaffolds and problem solving tools; the manner in which learners' errors were exposed and treated during instruction; the prevalence of interactive learning environment and learners critical responses of other learners contributions through verbalisation, argumentation, and exploratory talk.

4.7 ANALYSIS OF LESSON OBSERVATION

One of the objectives of the current study was to observe instruction in the control group in terms of how learners' errors are exposed and treated. In order to achieve this objective, the researcher visited the control school on three occasions to observe the lesson. The main areas of focus during the lesson observation were:

- The format of instruction;
- How the teacher uses teaching and learning resources;
- The arrangement of the learning setting;
- How the teacher discover learners' errors; and,
- How the teacher rectifies or treats learners' errors.

It was observed at the control school that the format of instruction was largely teacher-centred. The teacher was the main role player. The teacher relied on extensive use of text-book as a teaching and learning resources. The teacher constantly directed or referred learners to certain pages of the textbook. The learning setting was arranged to the extent that all the desks were orderly and neatly arranged in rows and the teacher always stood in front of the class while learners sat individually on their well-arranged desks and attentively listened to the teacher as he transmitted knowledge to learners who passively received the transmitted knowledge. Learners attended exclusively to what the teacher said and did, stayed on their seats, worked by themselves and avoided talking to one another as they performed the tasks assigned to them by the teacher individually in a non-group setting.

However, learners did not interact with each other when tasks were given. Learners' participation in the lessons was limited to asking the teacher questions for clarity. The learners were passive listeners during instruction in the classroom except for the relatively few answers that were given by learners who knew the right answers. The learners' thinking was narrowed to what the teacher asked and considered to be a correct response. Instead of being encouraged to ask questions, the role of the learners was to answer questions. This led learners who were not confident that they knew the right answers to minimise their participation in class.

The physical and social environment of the classroom implicitly communicated to learners the idea that the teacher was the centre of all activities in the classroom. Social interaction happened primarily between the teacher and learners and that the teacher was the sole source of information. In addition, the orderly and neatly desk arrangement conveyed the message that the most important activities were those of the teacher that occurred at the front desk. As stated above, the teacher was the main player during instruction. Learners did not play active role in the lesson and as a result, the teacher was only able to discover learners' errors during the post-lesson activities. In the follow-up visit, it was observed severally that the teacher referred to learners' errors committed in the previous lessons. Thus, in the control school it was observed that learners' errors were exposed after the instruction. In terms of error treatment, it was noted in the follow-up visit that learners' error-fixing (corrections) were done by the teacher on the chalkboard while learners copied the rectified errors in their books.

The last observation was about the way the teacher treated the algebra content. Although the teacher provided explanation of the concepts and used examples to demonstrate, it was observed that many of the learners learned to recall only the procedures for doing the activities in the worksheet and to complete their homework assignment. Learners practised remembering the procedures and were able to correct some of the errors. However, it was uncertain whether meaningful knowledge that would enable learners to apply the concepts and rules in novel problem situations had been constructed. Observations in the experimental group were on-going throughout the intervention. This group was the focus group in testing the impact of constructivist-based teaching instruction in terms of exposition and treatment of learners' errors in algebra. The learners were engaged in the same algebra concepts worksheet activities as learners in the control group, but were exposed to the new (CBTM) instruction that was implemented by the researcher. The following were observed:

- learners in groups of six were engaged in group learning activities during instruction;
- learners verbalised their prior knowledge during participation in the group work;
- the intervention instruction influenced learners to expose their errors during participation;
- learners through exploratory talk, argumentation and support provided by their peers and the teacher treated the errors they inhabit during instruction; and,
- the role of the teacher during instruction was limited to guiding and facilitating the lesson.

4.8 SUMMARY OF STATISTICAL FINDINGS

This section summarises the statistical findings from the statistical analyses.

Participation of the study

The data analysis that was presented in this report covered only the n=70 learners: EG (n=35); and CG (n=35) who participated fully in the study. In total, n=70 representing 89.7% participated fully in the study whereas n=8 representing 10.3% participants did not participate fully.

Descriptive statistics

The mean and standard deviation were used as the descriptive statistics to analyse the result obtained from the study. On percentage improvement in the achievement test it was found that the mean performance of learners in the EG improved from 25.8 in the pre-test to 44.63 in the post-test representing 72.98% whiles that of the CG improved from 24.14 to 30.46 representing 26.18%. The CBTM intervention was found to be better than the TTM intervention in improving the performance of learners in the achievement tests.

Again, on the average percentage of error reduction, it was found that the EG improved by 74.13% whereas that of the CG improved by 43.09%. The descriptive statistics found that the CBTM intervention reduced learners' errors better than the TTM intervention in the CG.

Inferential statistics

Inferential statistics used to test the two assumptions of the study were: Levene's independent t-test for homogeneity (equality) of variances; and Kolmogorov-Smirnov and Shapiro-Wilk test for the assumption of normality of test scores. Levene's independent t-test for homogeneity (equality) of variances assessed at 5% with $t = 7.77$ yielded a *p-value of 0.592* which was not significant. This showed that the assumption of homogeneity (equality) of variance was not violated. The formal normality test by Kolmogorov-Smirnov and Shapiro-Wilk performed yielded a *p-value of 0.171 and 0.424* respectively. The paired samples t-test performed corroborated Kolmogorov-Smirnov and Shapiro-Wilk's result. The result showed that the assumption of normality of test scores was not violated and hence the post-test-pre-test scores in the experimental and control group were normally distributed.

On hypothesis testing, the paired samples t-test was used to test the hypothesis of the study. It was found that there was statistically significant reduction in learners' algebraic errors, from 25.8 ± 7.31 to 44.63 ± 9.68 ($p<0.05$), following the implementation of a constructivist-based teaching method in the experimental group. However, the improvement of learners' tendency not to do errors amounted to 18 ± 7.61. It was found that $p<0.05$ so it was reasonable to reject $\mathbf{H_0}$ of no effect in favour of $\mathbf{H_1}$. It was therefore concluded that the constructivist-based teaching method was effective in reducing learners' errors.

4.9 CHAPTER SUMMARY

This chapter presented, analysed and discussed the data collected from the achievement tests and lesson observations. The descriptive statistics (mean and standard deviation) and inferential statistics (independent samples t-test and paired samples t-test) were employed as statistical techniques with the help of the use of IBM SPSS software to analyse the data. As a result of the analysis, the study found that learners who received the CBTM intervention significantly reduced the errors they commit in the four algebraic concepts than learners who received the TTM intervention.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

This study was motivated by the desire to search for pedagogical solution to the perennial trend of poor performance of learners in Mathematics in South Africa. Poor performance in this study was linked to the algebraic errors that learners do in a Grade 11 Mathematics classroom. This study created an opportunity for the researcher to investigate the effectiveness of a teaching method in improving learners' performance in Mathematics by reducing the errors they commit in algebra (see, Section 1.1). The study sought to find out the comparative impact of teaching that is based on constructivist learning theory, which was referred to as CBTM in this study (see, Section 1.9.3), and the traditional teaching method (TTM) (Section 1.9.4) on secondary school learners' errors in algebra (see also, Section 1.2). The study of learners' errors in a Grade 11 algebra lesson was conducted in terms of exposing and providing a suitable treatment for the observed learners' errors (Section 1.2).

In Chapter 1, the research problem, the aim and objectives of the study, the subsequent research questions, the rationale and significance of the study were all summarised and properly stated. The review of literature in Chapter 2 revealed that there is more research in favour of the constructivist-based teaching method (CBTM), in terms of reducing learners' errors in algebra, and subsequently improving learners' performance in Mathematics (see, Section 2.2 & Section 2.3). This chapter summarises the results and findings of the study discussed in Chapter 4. The discussion in this chapter is presented in terms of the aims and objectives, as well as the research questions of the study. The implications of the findings in Chapter 4 suggest that further research is needed to study learners' algebraic errors with an objective to elevate their performance in Grade 11 Mathematics. The limitations of the study are also discussed in this chapter. The chapter concludes with the researcher's views and thoughts regarding the findings of the current study.

5.2 REVISITING THE AIM, THE OBJECTIVES AND RESEARCH QUESTIONS OF THE STUDY

The main aim of this study was to investigate the comparative effects of a constructivist-based teaching method (CBTM) and the traditional teaching methods (TTM) on Grade 11 learners' errors in algebra, in terms of exposing the errors and subsequently providing a treatment for the observed errors (Section 1.2). The participants of the study were drawn from quintile 2 disadvantaged rural schools, which were considered to be poorly performing in Grade 11 Mathematics as a result of the numerous errors learners did in algebraic tasks (see, Section 1.4). The objectives of the current study were (Section 1.3):

- To use a group learning approach to facilitate the exposition and treatment of learners' errors when certain algebraic topics are treated in a Grade 11 mathematics lesson;
- To observe the traditional methods of teaching in term of exposing and treating learners' algebraic errors in algebra Grade 11 lesson; and,
- To measure the effect of error treatment when the constructivist-based teaching method is compared with the traditional teaching method.

It is possible to conclude the aim of the current study and its associated objectives have all been achieved (see discussions in Section 5.3 & Section 5.4). It is the researcher's view that this research has substantially provided evidence to support the view that: (1) the constructivist-based teaching method (CBTM) has a greater potential to enhance the reduction of learners' errors in algebra and also improve the performance of learners in Mathematics when it is compared with the traditional teaching method (TTM); and, (2) the study can serve as a useful point of reference for those who are attempting to improve the teaching and learning of Mathematics in secondary schools, particularly in South Africa. The current study explored the following research questions:

- What characterises the teaching and learning activities in a constructivist-based teaching method and traditional teaching method during a Grade 11 algebraic lesson?
- How do the constructivist-based teaching method and the traditional method facilitate the exposition of learners in a Grade 11 algebraic lesson?
- What is the comparative effect of constructivist-based teaching method and the traditional teaching method on the treatment of learners' errors in Grade 11 algebraic classrooms?

The discussion that follows provides evidence to support the notion that the research questions of the current study have been answered, and that the objectives of the study have been achieved.

5.3 ANSWERING THE RESEARCH QUESTIONS OF THE STUDY

5.3.1 Research question 1

This research question is re-stated in Section 5.2.4 of Section 5.2. In Section 1.9.3 and Section 1.9.4, the researcher provided suitable definitions for what constituted the constructivist-based teaching method (CBTM) and the traditional teaching method (TTM). Hence these definitions provided a suitable context to highlight the key features of differences between the two comparative teaching methods. For instance, in Section 1.9.3 the researcher used Brooks and Brooks (1999) to emphasise the fact that CBTM is more learner-oriented than the TTM, which was explained as largely emphasising the role of the teacher. In TTM, the teacher is considered to be the pourer of knowledge while in the CBTM the self-construction of knowledge by learners is foregrounded. In addition, Section 1.9.2 was used to emphasise a useful link between CBTM and a group learning approach (Section 1.9.9). It was made clear that CBTM is better positioned to embrace elements of group learning approach than TTM, which seems to give a teacher a bigger role during a lesson.

Furthermore, in Table 2.2 of Section 2.3.6, the researcher drew from the existing literature to highlight the distinguishing features between CBTM and TTM (for examples, see, Applefield *et al*., 2001; Brooks & Brooks, 1999). Finally, a description of instructions in the experimental group and control group during the course of this study is documented in Section 3.7.2.2 and Section 4.6. Section 4.6 in particular provided a detail discussion on how CBTM teaching was implemented in the experimental group. This discussion is useful in providing a context of contrasting pedagogical approaches that were meant to expose and provide a suitable treatment for the emerging learners' errors during a sequence of experimental lessons (investigation). Subsequently, the superiority of CBTM over TTM is confirmed by the results of this study, which are presented in Chapter 4. Given this background it is reasonable to conclude that the first research question was addressed through the literature and observations of instructions in both groups.

5.3.2 Research question 2

This research question is re-stated in Section 5.2.5 of Section 5.2. In simple terms, the second research question of the current study was meant to observe the teacher moves, which were primarily meant to encourage learners to expose or reveal their tendencies to do errors when they solve Grade 11 algebraic tasks. It must be noted that the teachers in the experimental group and the control group used contrasting pedagogical strategies to achieve this. In the experimental group, where CBTM was prominent, the researcher opted to incorporate elements of group learning approach (Section 1.9.9) in which participants continuously engaged in constructive learning dialogues. In Section 1.9.10 these dialogues are fittingly described as *exploratory talks* (see, page 13 of this dissertation) because during these robust verbal interactions learners argued, critiqued and probed their group members' points of views.

The process that is described in the preceding paragraph tended to encourage group members (in the CBTM lesson) to be more keen to verbalise their pre-existing algebraic knowledge, which also tended to reveal (expose) their conceptual errors in algebra (see, Section 3.9; see also, Section 4.6). In Section 4.5 several examples of some of the learners' errors that were identified (exposed) during a CBTM lesson were elaborated. In an attempt to observe the teacher's moves that facilitated the exposition of learners' errors in the control group where a TTM was implemented, the researcher conducted a series of lesson observations (see, Section 3.7.2.3). Section 4.7 discussed and described how the lesson observation enabled the researcher to ascertain how learners' errors were exposed and how the teacher treated the errors during instruction.

In comparison with the experimental group, error exposition and treatment were minimal in the control group (see, Section 4.7). Moreover, Table 4.29 and Table 4.30 provided evidence about learners' errors that were exposed and the effects of both interventions on the frequency of learners' errors before and after the intervention. It was evident from Table 4.29 that CBTM intervention significantly reduced learners' errors in all the four conceptual areas in algebra in the experimental group than the TTM intervention in the control group. On the basis of the evidence alluded to, it can reasonably be concluded that the second research question of the current study was adequately answered.

5.3.3 Research question 3

This research question is re-stated in Section 5.2.6 of Section 5.2. Research question 3 focused on the possible treatment of learners' errors that were anticipated in both comparative learning environments (the CBTM and TTM). In this context, the treatment of learners' errors would be measured in terms of learners' successive pre-test and post-test performances during the course of an experiment. Given that this study had initially drawn a link between learners' errors in algebra and performance in Mathematics (see, Section 1.5 & Section 2.2), the current study was therefore premised on the notion that more errors would result in learners' poor performance in Mathematics, and vice versa. In an attempt to answer research question 3, the researcher monitored learners' performance in both learning environments (CBTM and TTM). Hence the performance of learners (participants) in the achievement test at pre- and post-stages of the experiment was considered to be a suitable yardstick to achieve the measurement, namely, the treatment of learners' errors during each type of instruction.

Descriptive statistics presented in Table 4.2 of Section 4.3 shows a relatively better post-test performance of learners in the experimental group when compared with learners in the control group (see comparative mean scores in Table 4.2). In Section 4.3 the researcher concluded that the observed post-test performance, which was observed to show significant gains from the pre-test performance in both groups, suggested "*a significant reduction in learners' errors in algebra*" (see, Table 4.30). Most notably, the comparative post-test performances of the experimental group and control group suggested that CBTM is more effective than TTM in terms of producing a treatment of learners' errors in Grade 11 algebraic classrooms (research question 3).

In fact, Chapter 4 presents several study results that persistently confirm the fact that CBTM is more effective than TTM (for examples, see, Table 4.3; Table 4.4; Table 4.5; Table 4.6; Table 4.8; Table 4.11; Table 4.30). Almost all the information that is contained in the tables that are given as examples in the preceding sentence confirms that CBTM is more effective than TTM. Hence the performance of learners in the CBTM learning environment was superior to that of the learners in the TTM learning environment. Given these observations, it is therefore possible to conclude that the third research question of this study was adequately answered.

5.4 ACHIEVING THE OBJECTIVES OF THE STUDY

The three research objectives of the current study are recast in Section 5.2.

5.4.1 Study objective 1

This research objective is re-stated in Section 5.2.1 of Section 5.2. The study objective 1 focused on the use of group learning approach to expose learners' errors, and subsequently provide a treatment for the observed learners' errors. It must be noted that group approach was specifically incorporated in CBTM lessons. TTM was mainly teacher dominated and eventually overlooked the significance of using the group learning approach. In Section 3.9 of this report, the researcher documented certain aspects of CBTM that accounted for its observed effectiveness. A group learning approach is highlighted as a prominent feature in this section. In Section 3.9.1, the researcher highlights the usefulness of group approach in the exposition of learners' errors during a CBTM lesson. Also, in Section 3.9.2, a discussion is provided on how the group learning approach facilitated the treatment of learners' errors during group learning interactions.

In these sections, it is documented that in the experimental group learners' errors were exposed during instruction through verbalisation as they actively participated in the lesson during instruction in a group learning setting. Most importantly, the lesson was learner-centred and in the process of participating, making remarks and contributing to the group discussion of the algebraic conceptual tasks learners articulated their prior knowledge, which revealed the gaps in their conception. The errors highlighted were treated through learners' exploratory talk, argumentation, and support received from the group members. Finally, the study results that are presented in Chapter 4 further demonstrated that a group learning approach, which was largely embedded in CBTM lessons, was more influential in improving learners' performance in the post-test scores, thus confirming that the CBTM instruction is effective in reducing learners' errors in Grade 11 algebra. Therefore, the study objective 1 for the study was achieved.

5.4.2 Study objective 2

This research objective is re-stated in Section 5.2.2 of Section 5.2. The second research objective for the current study focussed on observing instruction in the TTM learning environment with an intention to see how this form of instruction responded to learners' errors in terms of exposing them and thereby proving a treatment. Therefore, the study objective 2 was meant to observe instruction in the control group in terms of exposing and treating learners' errors during an algebra lesson.

To achieve this, the researcher constructed a lesson observation schedule to monitor the TTM instruction in the control group (see, Section 3.6.1.2 & Section 3.7.2.3). The researcher observed instruction in the control school more than once in order to trace the treatment activities (see, Section 4.7). The discussions in Section 4.7 revealed that in the control group the lesson was mainly teacher-centred and hence learners' participation was limited. As a result, learners' errors were only exposed minimally in post-lesson activities and error-fixing (corrections) was done by only the teacher on the chalkboard for learners to copy.

5.4.3 Study objective 3

This research objective is re-stated in Section 5.2.3 of Section 5.2. The study objective 3 was to measure the magnitude of the effectiveness of error treatment when the constructivist-based teaching method (CBTM) is compared with the traditional teaching method (TTM). The statistical analysis of the post-test results of both the experimental group and the control group suggested that CBTM was more effective in reducing learners' errors than TTM (see, Section 4.4). The independent t-test yielded $p<0.05$, suggesting that CBTM is more effective. The inferential statistical results in Section 4.4 were further corroborated by the descriptive statistics (Section 4.3) that confirmed the superiority and effectiveness of CBTM over TTM.

The inferential statistical tests confirmed the following two comparative results: (1) that learners taught in CBTM performed better than learners who were taught in TTM in the three sections of the achievement tests (see, Table 29 & Section 4.5). This was substantiated by the pre-test-post-test correlation of 0.63 with a p-value of 0.000 (i.e., $p<0.05$) for the CBTM group and -0.54 with p-value of 0.376 (i.e., $p>0.05$) for the TTM group; and, (2) that learners taught in CBTM performed better than learners taught in TTM when measured on the error reduction as a result of meaningful knowledge construction in both groups ($p<0.05$). Learners taught in CBTM constructed knowledge meaningfully and were less likely to commit errors than those taught in TTM group (see, Section 3.9 & Section 4.4). It can therefore be reasonably concluded on the basis of the foregoing discussion that the third objective of the study was met.

5.5 ACHIEVING THE AIM OF THE STUDY

The aim of the current study is re-stated in Section 5.2 (see also, Section 1.2; Section 4.1). The design of the current study made it possible to conduct the investigation in which two types of instructional approaches were investigated in terms of exposing learners' errors during a Grade 11 algebra lesson, and thereby providing a treatment for the observed learners' errors (see, Section 3.4). The design of the study made it possible to create two comparative learning environments to conduct the investigation that would enable the achievement of the study aim. The results of the study, which have been discussed in the preceding sections, suggest that the aim of this has been achieved.

5.6 LINKING THE STUDY RESULTS TO THE THEORETICAL FRAMEWORK OF THE STUDY

The theoretical framework of the current study is presented in Section 2.4. Evidence from constructivist research studies indicate that instructions based on constructivist learning theory are preferred (Johri, 2005). Tellez (2007: 553) found that "the importance of constructivism in educational theory and research cannot be underestimated". Studies by Phillips (1995), Cobb (1996), Fox (2001) and Dangel (2011) support constructivist approach in science-related disciplines. Traditional teaching methods are becoming less tenable to stimulate conceptual understanding as they have ignored the fact that the knowledge, which the learners discover by themselves, is more enduring than the knowledge transmitted to them by the teacher or someone else. Constructivism recognises that learning is a cognitive process involving construction and reconstruction of ideas.

As a learning theory, constructivism recognises the learner as a meaning maker rather than a passive recipient of factual knowledge and conceived learning as process where meaning is modified on the grounds of evidence. Fundamentally, the constructivist approach to teaching recognises the social interaction in the teaching and learning process. Empirical studies conducted by Tellez (2007), Phillips (1995), Cobb (1996), Fox (2001), Dangel (2011), Guthrie *et al*. (2004), Kim (2005), Doğru and Kalender (2007), and Bhutto (2013), which are reviewed in Section 2.3.4 indicate that constructivist teaching methods have more positive effect on learners' performance in Mathematics and Science than traditional teaching methods. Looking closely at the findings of previous empirical studies side by side with the findings of the current study, there is credible evidence that learners' errors in algebraic concepts can be modified by using CBTM as effective methods of teaching.

Although each of the empirical studies reviewed in Section 2.3.4 implemented a different method of constructivist teaching in comparison with traditional method, their results indicated that the learners who received constructivist instruction showed significant gain on the academic achievements than those who received traditional instruction. It was also found that in situations where no significant difference was found between the achievement of the constructivist group and traditional group, it was discerned from qualitative evidence that the learners and teachers who applied the constructivist methods showed preference to the constructivist approach over the traditional approach. The results of the statistical tests indicated a significant difference in error reduction between the learners who received constructivist-based instruction and the learners who received traditional instruction in the four conceptual areas in algebra in this current study.

5.7 CONCLUSIONS

The current study was guided by two assumptions, namely, (1) the assumption of normality; and, (2) the assumption of equality of variances. The results of the test of these assumptions presented in Table 4.4 and in Figure 4.1 show that the assumption of normality in the distribution of scores for CBTM and TTM is fulfilled and the assumption of equality of the variances of CBTM and TTM groups was not violated. This means implicit extraneous variables did not have impact on the outcome of the results, which is an indication that the two groups were homogenous and hence does not raise any doubts about attributing the observed significant reduction in learners' errors in the experimental group to the CBTM intervention.

As mentioned in the rationale that motivated and justified the current study, it is noted that previous studies that evaluated the effect of the constructivist approach on the teaching of science-related disciplines used samples of participants from other parts of the world. However, none of such studies has been conducted using learners in White River circuit of Ehlanzeni District of Mpumalanga Province in South Africa. Conducting the current study using learners from secondary schools in the White River Circuit of Ehlanzeni District in Mpumalanga Province, South Africa has bridged some of the empirical gaps. Although the learners used in this current study were selected from two quintile 2 secondary schools from White River Circuit, the conclusions drawn can be extended to learners in other quintile 2 secondary schools in Mpumalanga Province and other provinces in South Africa.

In addition to bridging empirical gap, the current study highlighted constructivist-based teaching method as an effective pedagogical strategy that inspires a paradigm shift from the dominant traditional teaching approach to the constructivist-based teaching approach in mathematics instruction. This method of teaching can also serve as a resource to practicing teachers, teacher trainers and trainees who aspire to improve their methods of teaching and those who intend to undertake further research on improving the teaching and learning of Mathematics.

5.8 RECOMMENDATIONS

Even though the results of the current study corroborates the findings of previous studies in that they also advocate the use constructivist-based teaching methods over traditional teaching methods in reducing learners' errors in algebra thereby facilitating learners' performance in Mathematics, it must be cautioned that Mathematics educationists in South Africa should not fully cling to the assumption that the constructivist-based approach is a panacea for all the Mathematics learning difficulties.

To guarantee the efficacy of constructivist-based teaching model, teaching should necessitate commitment on the part of the learners, teachers, educational managers and administrators. Effective learning is inspired by good pedagogy. Good pedagogy demands that teachers play the role of facilitators while learners take autonomy of their own learning. Teaching entails giving learners the opportunity to identify the gaps and limitations of their own construction of mathematical knowledge to evaluate their own ideas in applying the new knowledge to find solutions to problems in everyday life.

It is against this backdrop that the researcher would like to make the following inferences and recommendations:

- Mathematics teachers should provide to the learners ample opportunities to discover and construct their own knowledge rather than the learners absorbing the teachers' own ideas. It is important for teachers to note that all knowledge emanates as a hypothetical construction. No individual constructs knowledge for another. The knowledge that the learner constructs by himself is more meaningful than that the one that is transmitted to him by the teacher or someone else;

- Mathematics teaching should aim at encouraging a group learning approach, constructive mathematical discourse in their classroom instruction;
- Mathematics teaching should recognise that learners come to the new lesson with prior knowledge, which may have gaps that may be based on incorrect conceptions that are resistant to change and could result in learners' systematic errors. Consequently, teaching should aim at providing learners with opportunities to identify their errors and modify them in the light of new evidence and support from a capable peer or the teacher during instruction;
- The value of knowledge lies on how it is used. As such, instructions should aim at enhancing learners' ability to apply the mathematical concepts and principles that they have learned to solve given problems;
- Knowledge construction involves giving learners the autonomy to be in charge of their own learning. Teachers must act as facilitators and provide scaffolding to the learner in their knowledge construction; and,
- Teacher educators for Mathematics should organise and create awareness among other teachers that the traditional instruction is becoming less and less relevant to achieving the goals of Mathematics education in this modern dispensation. Mathematics teachers should be encouraged to implement the constructivist methods in their classroom instruction.

5.9 LIMITATIONS OF THE CURRENT STUDY

Firstly, it is acknowledged that the research design that was followed in the current study could pose some challenges to the external validity of the study (Roberts, 2003; see, also, Section 3.4 & Section 3.6.5.2). Participants in the current study were selected from quintile 2 secondary schools and were selected by the qualifying characteristics of their disadvantaged socio-economic background and their poor performance in Mathematics (see, Section 1.4).

Secondly, the design of this study lacked random assignment of participants to the experimental group and control group and as such intact classes were used (Section 3.4.1). While the sample in the study approximated the target population (Section 3.5), caution should be exercised when generalising beyond the study participants. Conclusions may therefore not be extended to schools that are beyond the quintile 2 rural setting with disadvantaged socioeconomic environment in which the experiment was conducted.

Thirdly, the current study was undertaken with the aim to investigate how constructivist-based teaching method could help to reduce learners' errors and thereby improve their performance in only one section of the Grade 11 Mathematics syllabus, namely, algebra (see, Section 1.2). Therefore, the findings of this study may not necessarily be extrapolated to other topics of Grade 11 Mathematics, and also to other grade levels Mathematics syllabi.

Finally, the duration of the intervention was not long enough to warrant a complete reduction and elimination of learners' firmly held systematic errors in algebra (for more discussion see, Section 2.2)

5.10 POSSIBLE GAPS IN THE CURRENT STUDY

One of the possible gaps identified in the current study was of the one relating to the scope of the research methodology. The evidence and findings of the study were based on only extensive quantitative data collection and analysis methods, with a limited qualitative data component that constituted three sessions of lesson observations. It must be acknowledged that participants' semi-structured interviews, which were not considered in the current study, could solicit the views and perceptions of participants about the CBTM as compared to the traditional method to corroborate the lesson observations. Hence sufficient qualitative data were not collected and analysed to support and account for the quantitative evidence obtained from the study.

Secondly, the number of participating secondary schools selected from the quintile 2 strata in the White River Circuit for the study was seemingly not adequate. Likewise, the number of teachers that participated in the study was not enough and hence the conclusions of the study were limited to the traditional teaching method of only one teacher.

The evidence of the educational background information of teacher implementing the traditional teaching method was taken for granted. The qualifications and subject content knowledge of the teacher was assumed but not verified. At no stage was the teacher asked to complete a questionnaire or information collected about him. In addition, the teacher’s personal mathematics knowledge was only inferred from conversations and articulated experiences. No evidence was collected to corroborate the teacher’s verbal claims about his qualifications and teaching experiences. This, however, does not imply or raise any doubt about the teacher’s qualifications and mathematical knowledge as there was no evidence to suggest that any such discrepancy existed, and besides, due processes are usually followed by the schools in the appointment of the teacher in a public school.

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APPENDICES

APPENDIX A: PRE-TEST

Learner Code: Duration: 1h30min

This is a non-evaluative assessment. Your performance in this assessment will have no bearing on your CASS marks. The assessment is designed to help you with algebra by helping your teacher understand the mistakes you make and why you make them.

Instructions:

1. Answer all questions.
2. Use algebraic methods to solve all the problems.

PART A: MULTIPLE CHOICE [20 MARKS]

Instruction: Select the letter of the correct answer. Each question is worth 2 marks.

1.1 Expand the bracket and simplify $(a + b)^2$:

A. $a^2 + b^2$
B. $2a + 2b$
C. $a^2 + ab + b^2$
D. $a^2 + 2ab + b^2$

1.2 What is $3x + 3x =$

A. $6x^2$
B. $6x$
C. $9x^2$
D. $9x$

1.3 Simplify: $3(x + y)$

A. $3xy$
B. $3x + y$
C. $3x + 3y$
D. $x + 3y$

1.4 Jennifer has some trading cards. Lerato has 3 times as many trading cards as Jennifer. They have 36 trading cards in all. Which of these equations represent their trading cards collection?

A. $3x = 36$
B. $x + 3 = 36$
C. $x + 3x = 36$
D. $3x + 36 = x$

1.5 Solve for x if: $5 + 3x = 11$

A. $\frac{11}{8}$

B. 2

C. $\frac{16}{3}$

D. 3

1.6 Solve for x if: $(x - 5)(x + 1) = 7$

A. $x = 12$ or $x = 6$

B. $x = -2$ or $x = 6$

C. $x = -2$ or $x = -6$

D. $x = 2$ or $x = -6$

1.7 There are *n* girls in a girl scouts marching in a parade. There are 6 girls in each row. Which of the expressions below can you use to find out how many rows of girl scouts are marching in the parade?

A. $n - 6$

B. $6n$

C. $\frac{6}{n}$

D. $\frac{n}{6}$

1.8 Simplify: $\frac{a}{b} + \frac{c}{d}$

A. $\frac{a+c}{bd}$

B. $\frac{a+c}{b+d}$

C. $\frac{ad+bc}{bd}$

D. $\frac{ad+bc}{b+d}$

1.9 Simplify: $(x^2).(x^3)$

A. x^6

B. x^5

C. x

D. x^{-1}

1.10 Simplify: $3x - (x - 5)$

A. $2x - 5$

B. $2x + 5$

C. -2

D. 8

PART B SHORT ANSWERS [30 MARKS]

1 Solve for *y if:* $4 + 3y = 28$ (2)

2 Solve the following linear system of equations.

$2m + n = 2$

$3m - 2n = 3$ (4)

3 Consider solving the linear system: $a + b = 5$

$a - b = 7$

3.1 To eliminate ***a*** from both equations, do you add or subtract the two equations (1)

3.2 To eliminate ***b*** from both equations, do you add or subtract the two equations (1)

3.3 Will you obtain the same solution if you add or subtract the two equations? Explain. (3)

4 The statement $a = b - 2$ is true when $a = 5$ and $b = 7$. Find different pair of values of *a* and *b* that also make the statement true. *a=……. and b=…….* (2)

5 What does ***xy*** mean? Write your answer in words. (2)

6 Simplify the following expressions:

6.1 $\frac{ax+bx}{x+dx}$ (3)

6.2 $\frac{r}{4} - \frac{(6-p)}{2}$ (3)

7 Subtract 2b from 5. (2)

8 Multiply $5e + 4$ by 3. (2)

9 The letter n represents a natural number. What is more: $\frac{1}{n}$ or $\frac{1}{n+1}$. Give reason for your answer. (3)

10 In the equation $y = 2t + 3$, which is larger y or t. Explain (2)

PART C WORD PROBLEM [25 MARKS]

1 Pens cost ***p*** rands each and rulers cost ***s*** rands each. If you buy 3 pens and 2 rulers, explain what $3p + 2s$ represents? (2)

2 Thandi sells y donuts. Hazel sells three times as many donuts as Thandi. A donut costs 25 cents.

2.1 Name a variable in this problem. (2)

2.2 Name another variable in the problem. (2)

2.3 Name something in the problem that is not a variable. (2)

3 There are n girl scouts in a parade. There are 7 girls in each row. Write an algebraic expression to find out how many rows of girl scouts are marching in the parade. (2)

4 The sum of four times a certain number and 29 is 85. What is this number? (4)

5 Fakude decided to buy a football with his four friends. Each friend agreed to pay the same amount and Fakude paid the balance of R25. The total cost of the football was R73. How much did each friend pay? (4)

6 Nompilo is exactly two years older than Londeka. Let N stand for Nompilo's age and L stand for Londeka's age. Write an equation to compare Nompilo's age to Londeka's age. (2)

7 Mr Mashaba shared his stamp collection with his two sons and the daughter: Andrew, Bheki and Ntombi. Ntombi received 5 times the number of stamps than Andrew did, and 4 less stamps than those received by Bheki. The whole quantity received by Andrew and Bheki is 22 stamps. How many stamps did Mr Mashaba give to each child? (5)

TOTAL MARKS: 75

APPENDIX B: POST-TEST

Learner Code: Duration: 1h30min

This is a non-evaluative assessment. Your performance in this assessment will have no bearing on your CASS marks. The assessment is designed to help you with algebra by helping your teacher understand the mistakes you make and why you make them.

Instructions:

1. Answer all questions.
2. Use algebraic methods to solve all the problems.

PART A: MULTIPLE CHOICE [20 MARKS]

Instruction: Select the letter of the correct answer. Each question is worth 2 marks.

1.1 Expand the bracket and simplify $(a + b)^2$:

A. $a^2 + b^2$
B. $2a + 2b$
C. $a^2 + ab + b^2$
D. $a^2 + 2ab + b^2$

1.2 What is $3x + 3x =$

A. $6x^2$
B. $6x$
C. $9x^2$
D. $9x$

1.3 Simplify: $3(x + y)$

A. $3xy$
B. $3x + y$
C. $3x + 3y$
D. $x + 3y$

1.4 Jennifer has some trading cards. Lerato has 3 times as many trading cards as Jennifer. They have 36 trading cards in all. Which of these equations represent their trading cards collection?

A. $3x = 36$
B. $x + 3 = 36$
C. $x + 3x = 36$
D. $3x + 36 = x$

1.5 Solve for x if: $5 + 3x = 11$

A. $\frac{11}{8}$
B. 2
C. $\frac{16}{3}$
D. 3

1.6 Solve for x if: $(x - 5)(x + 1) = 7$
A. $x = 12$ or $x = 6$
B. $x = -2$ or $x = 6$
C. $x = -2$ or $x = -6$
D. $x = 2$ or $x = -6$

1.7 There are *n* girls in a girl scouts marching in a parade. There are 6 girls in each row. Which of the expressions below can you use to find out how many rows of girl scouts are marching in the parade?
A. $n - 6$
B. $6n$
C. $\frac{6}{n}$
D. $\frac{n}{6}$

1.8 Simplify: $\frac{a}{b} + \frac{c}{d}$
A. $\frac{a+c}{bd}$
B. $\frac{a+c}{b+d}$
C. $\frac{ad+bc}{bd}$
D. $\frac{ad+bc}{b+d}$

1.9 Simplify: $(x^2).(x^3)$
A. x^6
B. x^5
C. x
D. x^{-1}

1.10 Simplify: $3x - (x - 5)$
A. $2x - 5$
B. $2x + 5$
C. -2
D. 8

PART B SHORT ANSWERS [30 MARKS]

1 Solve for *y if:* $4 + 3y = 28$ (2)

2 Solve the following linear system of equations.

$2m + n = 2$

$3m - 2n = 3$ (4)

3 Consider solving the linear system: $a + b = 5$

$a - b = 7$

3.1 To eliminate ***a*** from both equations, do you add or subtract the two equations (1)

3.2 To eliminate ***b*** from both equations, do you add or subtract the two equations (1)

3.3 Will you obtain the same solution if you add or subtract the two equations? Explain. (3)

4 The statement $a = b - 2$ is true when $a = 5$ and $b = 7$. Find different pair of values of *a* and *b* that also make the statement true. *a=……. and b=…….* (2)

5 What does ***xy*** mean? Write your answer in words. (2)

6 Simplify the following expressions:

6.1 $\frac{ax+bx}{x+dx}$ (3)

6.2 $\frac{r}{4} - \frac{(6-p)}{2}$ (3)

7 Subtract 2b from 5. (2)

8 Multiply $5e + 4$ by 3. (2)

9 The letter n represents a natural number. What is more: $\frac{1}{n}$ or $\frac{1}{n+1}$. Give reason for your answer. (3)

10 In the equation $y = 2t + 3$, which is larger y or t. Explain (2)

PART C WORD PROBLEM [25 MARKS]

1 Pens cost ***p*** rands each and rulers cost ***s*** rands each. If you buy 3 pens and 2 rulers, explain what $3p + 2s$ represents? (2)

2 Thandi sells y donuts. Hazel sells three times as many donuts as Thandi. A donut costs 25 cents.

2.1 Name a variable in this problem. (2)

2.2 Name another variable in the problem. (2)

2.3 Name something in the problem that is not a variable. (2)

3 There are n girl scouts in a parade. There are 7 girls in each row. Write an algebraic expression to find out how many rows of girl scouts are marching in the parade. (2)

4 The sum of four times a certain number and 29 is 85. What is this number? (4)

5 Fakude decided to buy a football with his four friends. Each friend agreed to pay the same amount and Fakude paid the balance of R25. The total cost of the football was R73. How much did each friend pay? (4)

6 Nompilo is exactly two years older than Londeka. Let N stand for Nompilo's age and L stand for Londeka's age. Write an equation to compare Nompilo's age to Londeka's age. (2)

7 Mr Mashaba shared his stamp collection with his two sons and the daughter: Andrew, Bheki and Ntombi. Ntombi received 5 times the number of stamps than Andrew did, and 4 less stamps than those received by Bheki. The whole quantity received by Andrew and Bheki is 22 stamps. How many stamps did Mr Mashaba give to each child? (5)

TOTAL MARKS: 75

APPENDIX C: MARKING RUBRIC FOR THE ACHIEVEMENT TESTS

PART A – 20 Marks

2 marks per question

Question	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.10
Answer	D	B	C	C	B	B	D	C	B	B

PART B – 30 Marks

Question	Answer	Mark Allocation
1	4 + 3y = 28 3y = 28 – 4 3y = 24 y = 24/3 y = 8	✓ 3y = 28 – 4 ✓ answer (2)
2	2m + n = 2................i 3m – 2n = 3...............ii i x 2: 4m + 2n = 4 ....iii ii + iii 7m = 7 m = 1 from i 2(1) + n = 2 n = 2 – 2 n = 0 m = 1 and n = 0 Or 2m + n = 2................i 3m – 2n = 3...............ii From i n = 2 – 2m ....iii Substitute iii into ii 3m – 2(2 – 2m) = 3 3m – 4 + 4m = 3 7m = 7 m = 1 from iii n = 2 – 2(1) n = 0	✓ iii ✓ m = 1 ✓ substitution ✓ n = 0 (4)

3	3.1 Subtract 3.2 Add 3.3 Yes. Addition will eliminate b in order to find a, subtraction will eliminate a in order to find b	✓ ✓ ✓ Yes ✓✓ explanation (5)
4	Any pair in which b – a = 2	✓ b ✓ a (2)
5	*xy* stands for the product of x and y Or *xy* stands for x multiplied by y	✓✓ (2)
6 6.1 6.2	$\frac{ax+bx}{x+dx} = \frac{x(a+b)}{x(1+d)}$ $= \frac{a+b}{1+d}$ $\frac{r}{4} - \frac{(6-p)}{2} = \frac{r-2(6-p)}{4}$ $= \frac{r-12+2p}{4}$	✓ x(a + b) ✓x(1 + d) ✓ answer (3) ✓✓simplification ✓ answer (3)
7	5 – 2b Award only one mark if learner proceeds further	✓✓ (2)
8	3(5e + 4) 15e + 12 (If a learner writes only 3(5e + 4) award maximum mark)	✓ ✓ (2)
9	$\frac{1}{n}$ is more. The numerator is the same so the fraction with the bigger denominator is smaller in value. n + 1> n	✓ right choice ✓✓explanation (3)
10	y is more than t. y is the sum of twice or double of t and 3	✓ y ✓ explanation(2)
Total		**[30]**

PART C – 25 Marks

Question	Answer	Mark allocation
1	3p + 2s represent the total cost of buying three pens and two rulers: Or Total expenditure for three pens and two rulers.	✓✓ (2)

2	Let x = donuts sold by Hazel x = 3y A donut cost 25 cents 2.1 y is a variable 2.2 x is another variable 2.3 25 cents or 3 is not a variable	 ✓✓ (2) ✓✓ (2) ✓✓ (2)
3	$\frac{n}{7}$	✓✓ (2)
4	Let x = the number 4x + 29 = 85 4x = 85 – 29 x = 14	 ✓✓ for equation ✓ solving ✓ x = 14 (4)
5	Let x = the amount contributed by each friend 4x + 25 = 73 4x = 73 – 25 4x = 48 x = 12 (Each friend contributed R12)	✓✓ equation ✓ solving ✓ answer (4)
6	N – L = 2 Or N = L + 2	✓✓ (2)
7	Let the stamps received by Andrew = x Ntombi will receive 5x Bheki will receive 5x + 4 Total stamps received by Andrew and Bheki is 22 x + (5x + 4) = 22 6x + 4 =22 6x = 22 – 4 6x = 18 x = 3 Andrew received 3 stamps Ntombi received 15 stamps Bheki received 19 stamps	✓✓ equation ✓ Andrew ✓ Ntombi ✓Bheki (5)
Total		**[25]**

APPENDIX D: TEST INSTRUMENT FOR PILOT STUDY

Learner Code: Time: 30mins

This is a non-evaluative assessment. Your performance in this assessment will have no bearing on your CASS marks. The assessment is designed to help you with algebra by helping your teacher understand the mistakes you make and why you make them.

Instructions:

1. Answer all questions.

2. Use algebraic methods to solve all the problems.

PART A: MULTIPLE CHOICE

Instruction: Circle the letter of the correct answer

1.1 What does $(-3)^2$=?

A. -9
B. 9
C. -6
D. 6

1.2 Simplify: $5 + 3y$

A. $8y$
B. $2y$
C. $3y = -5$
D. $5 + 3y$

1.3 $\sqrt{9+16}$ =?

A. 3 + 4
B. 19
C. 5
D. 13

1.4 Simplify: $(x^3)^2$

A. x^5
B. x^6
C. x^{-1}
D. x

1.5 Solve for x if: $4 + 3x = 28$

A. $x = 4$
B. $x = -8$
C. $x = 21$
D. $x = 8$

1.6 Solve for x if: $x^2 - 4x = 0$

A. $x = -2$ or $x = 2$
B. $x = 4$

C. $x = 0$ *or* $x = 4$

D. $x = 0$

1.7 Bafana is two times as old as Ntombi. If Ntombi is *n years old*, how old is Bafana?

A. $n - 2$

B. $2n$

C. $n + 2$

D. $\frac{n}{2}$

PART B SHORT ANSWERS

1 Solve for *y if:* $4 + 3y = 28$

2 Solve the following linear system of equations.

$$2x + y = 2$$

$$3x - 2y = 3$$

3 Consider solving the linear system: $m + n = 8$

$$m - n = 4$$

3.1 To eliminate ***m*** from both equations, do you add or subtract the two equations?

3.2 To eliminate ***n*** from both equations, do you add or subtract the two equations?

3.3 Will you obtain the same solution if you add or subtract the two equations? Explain.

4 The statement $a = b - 4$ is true when $a = 5$ and $b = 9$. Find different pair of values of *a* and *b* that also make the statement true. *a*=……. *and b*=…….

5 What does ***xy*** mean? Write your answer in words.

6 Subtract *2b* from *5a*.

7 Multiply $4e + 3$ by 2.

8 Starting with some number, if you multiply it by 3 and then add 27, you get 45. What number did you start with?

APPENDIX E: OBSERVATION SCHEDULE

OBSERVATION FOCUS	FOCUS VARIABLE
TEACHER	1. The format of instruction Mode of instruction 2. The use of learning resources 3. Arrangement of learning setting 4. How does the teacher discover learners' errors 5. How are learners 'errors rectified 6. Learning activities
CONTROL GROUP	1. The teaching strategy used 1.1 Teacher-centred 1.2 Learner-centred 2. Learner involvement and role during instruction 3. Classroom arrangements 3.1 How are the desks arranged 3.2 Seating style or arrangement 4. Interactions in the classroom 5. What constitute learning 6. How are errors identified and rectified 7. Level of learners' dependence on the teacher
EXPERIMENTAL GROUP	1. Format of learning approach Group or non-group learning approach 2. Nature of instruction 3. Role of teacher during instruction 4. Learners participation and involvement 5. What constitute learning 6. How do learners construct knowledge 7. How are learners' errors exposed and treated

APPENDIX F: SAMPLE OF ALGEBRA CONCEPTS WORKSHEET

Use algebraic method to solve the following:

QUESTION 1

Simplify the following:

1.1 $(3x - 2)^2$

1.2 $17y^3 - 3y^2 + 3 - 2(7y^3 - 4y^2 + 1)$

1.3 $10 - 5(a - 2)$

1.4 $3m^4n^2 \times 4m^{-2}n$

QUESTION 2

Simplify the following:

2.1 $\frac{5}{(x-1)} - \frac{3}{(x+1)}$

2.2 $\frac{1}{n+1} + \frac{3}{n}$

QUESTION 3

Solve for x in the following equations:

3.1 $5 - 2x = 21$

3.2 $(3x + 2)(4 - x) = 0$

3.3 $x^2 - 5x - 24 = 0$

3.4 $(3x - 2)(x - 4) = 2 - 3x$

QUESTION 4

Solve for x and y simultaneously in the equations below:

4.1 $3x + 2y = 19$ and $2x - y = 8$

4.2 $x^2 + 2y = 29$ and $2y - 3x = 1$

QUESTION 5

It was given that x = 1, and Bafana made the following argument: If x = 1 then,

Step 1: $x^2 = x$ ....multiply both sides by x

Step 2: $x^2 - 1 = x - 1$...subtract 1 from both sides

Step 3: $(x - 1)(x + 1) = x - 1$ ....factorise

Step 4: $\frac{(x-1)(x+1)}{(x-1)} = \frac{(x-1)}{(x-1)}$ divide both sides by x – 1

Step 5: $x + 1 = 1$

Step 6: $x = 0$

It was given that x = 1 but Bafana ended his argument by getting x = 0.

5.1 Identify the step in which he made a mistake with his argument?

5.2 Describe how this mistake can be avoided.

QUESTION 6

Zanele was given this homework in her trigonometry lesson and this is how she solved it:

2 – 4SinA = 0

-2 SinA = 0

SinA = 0 + 2

SinA = 2

No solution

Her teacher said Zanele's answer is wrong.

6.1 Identify the mistakes she made

6.2 Show her how to solve it correctly

QUESTION 7

The sum of three consecutive natural numbers is 72. Using algebraic method find these numbers.

QUESTION 8

A mother shared R2400 among her three daughters Sharon, Natacia and Patricia. Patricia received three times of what Sharon received, and Natacia received R300 more than Patricia. Calculate how much each of them received.

QUESTION 9

Simplify the following:

9.1 Subtract 12 from 30

9.2 Subtract 8b from 15

APPENDIX G: WORKED-OUT EXAMPLES

EXPRESSIONS:

Grouping of Like and Unlike terms

Example 1

Simplify: $3x^2+5x - 3 - 2x^2 + 6 -3x$

Step 1: group like terms

$3x^2 - 2x^2 +5x - 3x - 3 + 6$

Step 2: simplify

$x^2+2x + 3$

Example 2

Simplify 7x – 3(2x + 4)

Step 1: Expand the brackets

7x – 6x – 12

Step 2: simplify like and unlike terms

x – 12

Example 3

Simplify $4y^5$ x $2y^{-3}$

Step 1: Multiply like terms

(4 x 2) x y^5 x y^{-3}

Step 2: simplify and apply laws of exponents

$8\ y^{5 + (-3)}$

$8y^2$

Example 4

Simplify $\frac{3}{x} + \frac{2}{x+1}$

Step 1: Find the LCM/LCD x(x + 1)

$$\frac{3(x+1)+\ 2x}{x(x+1)}$$

$$\frac{3x+3+\ 2x}{x(x+1)}$$

$$\frac{5x+\ 3}{x(x+1)}$$

EQUATIONS:

Example 1

Solve for x if 8x – 5 = 19

Step 1: Add 5 (additive inverse) to both sides of the equation

8x – 5 + 5 = 19 + 5

8x = 24

Step 2: multiply both sides by the multiplicative inverse of 8

8x(1/8) = 24 (1/8)

x = 3

Example 2

Solve for x if (2x – 3)(x + 1) = 0

Step1: Basic multiplication principle, if axb = 0 then either a = 0 or b = 0

2x – 3 = 0 or x + 1 = 0

Step 2: solve the two linear equations

x = 3/2 or x = -1

Example 3

Solve for x if (x + 1)(x – 3) = 12

Step 1: Expand the brackets since the product of the two brackets is not zero

x^2 -2x -3 = 12

Step 2: Write it in standard form

x^2 -2x – 15 = 0

Step 3: solve the equation by factorisation or any appropriate method

(x + 3)(x – 5) = 0

Step 4: use the basic multiplication principle if since the product is zero

x + 3 = 0 or x – 5 = 0

x = -3 or x = 5

Example 4

Solve for x and y simultaneously if:

4.1 2x + y = 11 and 3x – y = 4

4.2 x + y = 4 and x^2 + xy = 12

Solution

4.1 2x + y = 11...........i

$3x - y = 4$.............ii

Step 1: Co-efficient of y is equal and opposite in sign so y can be eliminated

Step 2: Add equations i and ii

$5x = 15$

$x = 15/5$

$x = 3$

Step 3: substitute the x value in i

$2(3) + y = 11$

$6 + y = 11$

$y = 11 - 6$

$y = 5$

$x = 3$ and $y = 5$

4.2 $x + y = 4$ and $x^2 + xy = 12$

$2x + y = 7$.........................i

$x^2 + xy = 12$...................ii

Step 1: from i make one of the variables the subject

$y = 7 - 2x$ iii

Step 2: substitute iii into ii

$x^2 + x(7 - 2x) = 12$

Step 3: simplify and solve for x

$x^2 + 7x - 2x^2 - 12 = 0$

$-x^2 + 7x - 12 = 0$

$x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x - 3 = 0$ or $x - 4 = 0$

$x = 3$ or $x = 4$

Step 4: substitute the values of x into iii

$y = 7 - 2x$

when $x = 3$, $y = 7 - 2(3) = 1$

when $x = 4$, $y = 7 - 2(4) = -1$

Example 5

Ntombi factorise $10 + 15k - 6mk - 4m$ as

Step 1: $10 + 15k - 6mk - 4m$

Step 2: $5(2 + 3k) - 2m(3k + 2)$

Step 3: $5(2 + 3k) + 2m(2 + 3k)$

Step 4: $(2 + 3k)(5 + 2m)$

Identify the step Ntombi made a mistake and factorise it correctly.

Solution

She made a mistake in step 3

$-2m(3k + 2) \neq 2m(2 + 3k)$ because both terms in the bracket are positive.

The correct factorisation will be

$10 + 15k - 6mk - 4m$

$5(2 + 3k) - 2m(3k + 2)$

$(2 + 3k)(5 - 2m)$

VARIABLE AND WORD-PROBLEM

Example 1

The difference between five times a number and 33 is 52. Find this number using algebraic method.

Solution

Step 1: Let the letter n = the number

Step 2: generate equation for the problem , thus

$5n - 33 = 52$

Step 3: Solve the equation

$5n = 52 + 33$

$5n = 85$

$n = 85/5$

$n = 17$

the number is 17

Example 2

A father is three times as old as his son. In eight years time, the father will be twice as old as the son. Determine the present ages of the father and the son.

Solution

Step 1: Use a letter to represent the son's age, let y = the son's present age,

Step 2: determine the father's present age using the son's age, thus 3y

Step 3: determine their ages in eight years from now:

The son will be (y + 8) years

The father will be (3y + 8) years

Step 4: generate equation from the statement, thus

In eight years' time,

father's age = two times the son's age

$3y + 8 = 2(y + 8)$

Step 5: Solve the equation

$3y + 8 = 2y + 16$

$3y – 2y = 16 – 8$

$y = 8$

The son is 8 years now and the father is 24 years now.

Example 3

The volume of a box with rectangular base is 3072 cm^3. The lengths of the sides are in the ratio 1:2:3. Calculate the length of the shortest side.

Solution

Step 1: let a = the length of shortest side

Step 2: determine the other two sides as 2a and 3a respectively.

Step 3: write down the formula for the volume of the box with rectangular base

V = *l.b.h*

Step 3: substitute the data into the formula and solve for a as

$3072 = a \times 2a \times 3a$

$3072 = 6a^3$

$3072/6 = a^3$

$512 = a^3$

$a^3 = 8^3$

a = 8 cm

Example 4

4.1 There are 84 learners in a classroom and 12 desks in each row. How many rows of desk are in the classroom?

4.2 Now if there are n learners in the classroom, how many rows of desk will be in the classroom?

Solution

4.1 Number of rows = 84/12

= 7

4.2 Number of rows = n/12

Example 5

Simplify your as far as possible if

5.1 35 is subtracted from 63

5.2 12m is subtracted from 15

Solution

5.1 63 – 35

= 28

5.2 15 – 12m

= 15 – 12m

APPENDIX H: LEARNERS RESPONSES IN PART A OF THE ACHIEVEMENT TEST

H (a): Experimental Group learners' responses in Pre-test

Learner Code	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.10
PRE01E	A	A	C	B	B	B	D	B	B	A
PRE02E	A	A	C	D	B	A	C	B	A	B
PRE03E	A	A	C	C	A	D	D	B	A	D
PRE04E	D	B	A	A	C	B	C	A	B	C
PRE05E	D	B	C	C	B	A	C	C	B	B
PRE06E	D	B	C	C	B	D	B	C	B	A
PRE07E	D	B	C	A	D	A	B	D	B	B
PRE08E	A	B	C	B	D	D	B	B	B	B
PRE09E	D	B	C	A	B	B	B	B	A	B
PRE10E	A	B	C	C	B	D	B	C	A	B
PRE11E	A	B	C	B	C	B	A	C	B	B
PRE12E	B	B	C	D	B	A	D	B	D	A
PRE13E	D	B	C	B	B	A	A	C	A	D
PRE14E	A	A	C	A	A	D	A	C	B	B
PRE15E	A	B	A	A	A	A	A	A	A	A
PRE16E	A	B	C	B	B	D	C	B	C	B
PRE17E	A	A	C	B	B	B	D	B	B	A
PRE18E	D	B	C	C	B	B	A	C	A	B
PRE19E	A	C	C	B	A	A	D	B	B	B
PRE20E	A	B	A	C	B	D	A	C	A	B
PRE21E	D	D	C	A	B	B	B	A	B	C
PRE22E	D	B	A	B	D	C	D	C	B	D
PRE23E	D	A	C	A	A	C	B	D	B	A
PRE24E	A	D	A	A	B	A	D	C	C	B
PRE25E	D	A	C	C	B	C	B	C	A	B
PRE26E	D	B	A	A	B	B	D	A	B	B
PRE27E	D	B	C	C	B	B	B	D	B	B
PRE28E	A	B	C	A	B	A	D	B	B	A
PRE29E	A	A	A	A	A	D	B	B	C	B
PRE30E	D	A	A	C	A	A	A	B	B	B
PRE31E	D	B	C	B	B	B	C	D	B	B
PRE32E	D	B	A	C	B	D	B	D	B	B
PRE33E	D	B	C	B	B	B	C	D	B	A
PRE34E	A	B	C	C	B	B	A	A	B	B
PRE35E	B	B	C	C	B	B	A	D	B	A
Wrong response	**18**	**12**	**9**	**24**	**12**	**22**	**26**	**24**	**13**	**14**
% Wrong response	**51.43**	**34.29**	**25.71**	**68.57**	**34.29**	**62.86**	**74.29**	**68.57**	**37.14**	**20.0**

H (b): Control Group learners' responses in Pre-test

Learner Code	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.10
PRE01C	C	B	C	C	D	B	A	C	B	A
PRE02C	A	A	C	A	B	D	B	B	B	B
PRE03C	D	B	C	C	B	B	C	B	A	B
PRE04C	A	B	C	A	B	B	B	C	B	B
PRE05C	A	B	C	C	B	B	B	D	B	D
PRE06C	C	A	C	A	D	D	D	C	B	A
PRE07C	A	B	C	B	B	B	C	A	B	A
PRE09C	A	B	C	A	B	B	A	C	A	A
PRE11C	A	B	C	A	B	A	B	D	B	B
PRE12C	D	B	C	B	B	D	C	C	B	A
PRE13C	D	B	C	B	D	B	B	C	B	A
PRE14C	D	B	C	A	B	B	B	B	B	B
PRE16C	D	A	C	B	D	D	D	B	B	A
PRE17C	A	B	C	A	B	C	A	A	A	C
PRE18C	D	B	C	B	B	B	B	A	D	A
PRE19C	D	B	C	B	B	B	B	B	B	A
PRE20C	D	B	A	C	B	A	C	B	B	A
PRE21C	D	B	C	C	B	B	B	C	B	B
PRE22C	D	B	C	A	B	B	B	C	D	B
PRE23C	D	A	C	C	B	B	B	C	B	B
PRE24C	A	B	C	A	A	D	B	A	B	B
PRE26C	D	B	C	C	B	B	B	A	A	B
PRE27C	D	B	A	C	B	A	B	B	A	B
PRE28C	D	A	A	A	B	A	B	D	A	A
PRE29C	D	B	C	D	B	B	C	B	B	B
PRE30C	A	B	C	B	B	B	B	D	B	B
PRE31C	D	B	C	A	B	B	B	C	B	A
PRE32C	D	B	C	A	B	A	B	C	A	C
PRE33C	D	B	C	A	B	D	B	C	D	B
PRE34C	D	B	C	B	B	B	B	C	B	B
PRE36C	A	B	C	C	B	B	D	C	B	B
PRE37C	D	B	C	B	B	B	B	B	A	B
PRE38C	D	B	C	C	A	B	C	C	D	B
PRE39C	A	B	C	A	B	B	A	C	B	A
PRE40C	A	B	C	A	B	B	B	B	B	B
Wrong response	**12**	**5**	**3**	**25**	**6**	**12**	**32**	**19**	**12**	**16**
% Wrong response	**34.26**	**14.29**	**8.57**	**71.43**	**17.14**	**34.29**	**91.43**	**54.29**	**34.29**	**45.71**

H (c): Experimental Group learners' responses in Post-test

Learner Code	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.10
POS01E	D	B	C	C	B	A	D	C	B	A
POS02E	D	B	C	B	B	A	D	C	B	B
POS03E	A	B	C	C	A	B	D	C	B	B
POS04E	D	B	C	C	B	B	D	C	B	B
POS05E	D	B	C	C	B	A	D	C	B	B
POS06E	D	B	C	C	B	D	B	C	B	B
POS07E	D	B	C	C	B	A	C	C	B	B
POS08E	D	B	C	C	B	A	D	C	B	B
POS09E	D	A	C	C	B	B	D	C	B	B
POS10E	D	B	C	C	B	B	B	C	A	B
POS11E	D	B	C	C	B	A	D	C	B	B
POS12E	D	B	C	C	B	B	D	C	B	B
POS13E	D	B	C	C	B	B	D	C	B	B
POS14E	D	B	C	B	B	A	D	C	B	B
POS15E	C	A	C	A	B	A	B	A	B	B
POS16E	A	D	C	C	B	A	D	C	A	B
POS17E	D	B	C	C	B	B	D	C	B	B
POS18E	D	B	C	C	B	B	D	C	A	B
POS19E	D	B	C	B	B	B	D	C	B	C
POS20E	D	B	C	C	B	A	D	C	B	B
POS21E	D	A	C	B	B	B	D	C	B	A
POS22E	D	B	C	C	B	B	D	C	A	B
POS23E	D	B	C	C	B	B	D	C	B	B
POS24E	D	B	C	C	B	A	D	C	B	B
POS25E	D	B	C	C	B	B	C	C	B	A
POS26E	C	B	C	B	B	B	A	D	B	B
POS27E	D	B	C	A	B	B	D	C	B	B
POS28E	D	B	C	C	B	B	D	C	B	B
POS29E	D	B	C	B	A	C	D	C	B	B
POS30E	D	B	C	C	B	B	D	C	B	B
POS31E	D	B	C	C	B	A	D	C	B	A
POS32E	D	B	C	B	B	D	D	C	B	B
POS33E	D	B	C	C	B	B	D	C	B	A
POS34E	D	B	C	C	B	A	D	C	B	B
POS35E	A	B	C	C	B	B	D	C	B	B
Wrong response	**5**	**4**	**0**	**9**	**2**	**16**	**6**	**2**	**4**	**6**
% Wrong response	**14.29**	**11.43**	**0**	**25.71**	**5.71**	**45.71**	**17.14**	**5.71**	**11.43**	**17.14**

H (d): Control Group learners' responses in Post-test

Learner Code	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.10
POS01C	C	B	C	A	B	D	A	C	B	C
POS02C	D	B	C	A	B	B	A	B	B	B
POS03C	D	B	C	A	B	D	B	C	B	B
POS04C	D	B	C	C	B	D	B	C	B	B
POS05C	C	B	C	A	B	A	D	D	B	A
POS06C	C	B	C	C	B	B	D	A	B	A
POS07C	D	B	C	D	B	A	A	C	B	A
POS09C	A	A	C	A	B	B	B	D	A	A
POS11C	D	B	C	A	B	A	B	C	B	B
POS12C	D	B	C	B	B	D	C	B	B	B
POS13C	D	B	A	B	B	D	D	C	B	B
POS14C	D	B	C	A	B	B	D	B	B	B
POS16C	D	A	C	A	B	B	C	B	B	A
POS17C	B	A	A	B	B	A	B	C	A	A
POS18C	A	A	C	D	B	B	B	C	B	A
POS19C	A	A	A	A	B	B	B	C	B	A
POS20C	A	A	C	A	D	A	C	B	B	A
POS21C	A	B	C	A	B	B	B	A	B	A
POS22C	A	B	C	A	B	C	D	B	B	B
POS23C	D	B	C	C	B	B	A	C	B	B
POS24C	A	B	A	A	B	A	B	A	A	A
POS26C	D	B	C	A	B	B	D	C	B	B
POS27C	A	B	C	A	A	B	A	B	A	A
POS28C	C	A	C	D	B	B	B	C	A	B
POS29C	D	B	C	A	D	B	D	B	B	B
POS30C	D	B	C	A	B	B	A	D	B	B
POS31C	D	A	C	A	B	D	D	C	B	B
POS32C	D	B	C	A	B	D	B	C	B	B
POS33C	B	B	A	D	D	A	C	C	A	A
POS34C	D	B	C	A	B	B	B	A	B	B
POS36C	A	B	C	B	D	B	B	B	B	A
POS37C	D	B	C	A	B	B	B	C	A	B
POS38C	A	B	C	A	B	B	B	A	B	B
POS39C	C	B	C	C	B	B	A	D	B	B
POS40C	D	B	C	D	B	B	B	D	B	B
Wrong response	**17**	**7**	**4**	**31**	**4**	**15**	**27**	**19**	**7**	**15**
% Wrong response	**48.57**	**20.0**	**11.43**	**88.57**	**11.43**	**42.88**	**77.14**	**54.29**	**20.0**	**42.88**

APPENDIX I: LEARNERS' PERFORMANCE INDICATORS IN THE ACHIEVEMENT TESTS

I (a): Mark list - Experimental Group

	Pre-test				Post-test			
Learner No	Part A 20	Part B 30	Part C 25	Total 75	Part A 20	Part B 30	Part C 25	Total 75
1	10	14	13	37	16	19	13	48
2	06	09	03	18	16	06	05	27
3	06	05	05	16	16	06	10	32
4	08	07	05	20	20	13	10	43
5	16	19	03	38	18	26	16	60
6	14	10	03	27	16	18	10	44
7	10	11	03	24	16	19	17	52
8	08	10	08	26	18	21	15	54
9	12	15	02	29	18	20	05	43
10	12	11	09	32	16	21	19	56
11	12	08	03	23	18	08	08	34
12	08	04	05	17	20	12	09	41
13	10	14	04	28	20	10	22	52
14	08	09	05	22	16	07	07	30
15	02	05	10	17	08	14	20	42
16	08	11	03	22	14	14	10	38
17	10	14	06	30	20	11	22	53
18	16	11	05	32	18	14	06	38
19	08	09	12	29	16	16	18	50
20	10	01	05	16	18	16	19	53
21	10	08	05	23	14	11	12	37
22	10	07	06	23	18	17	12	47
23	06	08	05	19	20	13	07	40
24	08	05	06	19	18	13	10	41
25	12	05	06	23	16	10	06	32
26	14	07	07	28	12	12	10	34
27	16	19	09	44	18	20	18	56
28	10	16	10	36	20	26	24	70
29	02	06	07	15	14	15	13	42
30	08	07	05	20	20	09	12	41
31	14	13	09	36	16	24	18	58
32	12	05	04	21	16	14	04	34
33	12	13	04	29	18	16	11	45
34	14	13	08	35	18	21	14	53
35	12	10	07	29	18	15	09	42
36								

I (b): Mark list - Control Group

	Pre-test				Post-test			
Learner No	Part A 20	Part B 30	Part C 25	Total 75	Part A 20	Part B 30	Part C 25	Total 75
1	12	03	07	22	10	04	07	21
2	08	07	03	18	14	11	11	36
3	14	03	04	21	14	07	09	30
4	14	16	09	39	14	16	00	30
5	12	10	06	28	08	09	13	30
6	08	00	06	14	14	00	13	27
7	10	05	04	19	12	11	10	33
8	A	A	A	A	A	A	A	A
9	12	08	04	24	06	06	07	19
10	A	--	--	A	A	A	A	A
11	10	04	03	17	12	06	05	23
12	12	09	04	25	12	15	14	41
13	14	11	06	31	14	11	08	33
14	14	08	04	26	14	12	13	39
15	A	A	A	A	A	A	A	A
16	08	06	04	18	10	06	16	32
17	06	07	06	19	06	05	06	17
18	12	07	00	19	10	08	12	30
19	12	04	04	20	08	08	16	32
20	10	03	06	19	04	02	16	22
21	18	10	12	40	10	15	11	36
22	14	06	03	23	14	07	05	26
23	16	14	04	34	18	08	14	40
24	08	09	00	17	04	09	11	24
25	A	A	A	A	A	A	A	A
26	14	11	08	33	18	15	11	42
27	10	01	04	15	04	04	16	24
28	04	08	07	19	10	10	15	35
29	14	07	10	31	14	07	04	25
30	12	07	02	21	14	03	01	18
31	14	06	05	25	14	10	07	31
32	10	05	07	22	14	07	08	29
33	12	11	06	29	14	07	11	32
34	16	04	01	21	14	05	10	29
35	A	A	A	A	A	A	A	A
36	18	13	05	36	16	13	08	37
37	12	05	06	23	14	06	11	31
38	14	10	10	34	12	08	11	31
39	12	04	02	18	14	12	16	42
40	12	09	07	28	14	13	11	38
41								

A – Absent in one or both tests and therefore not considered in the data analysis

APPENDIX J: CONSENT LETTERS

J (a): Letter of informed consent to Mpumalanga Department of Education requesting for permission to use two secondary schools in White River Circuit as research sites

Research topic: The impact of constructivist-based teaching method on secondary school learners' errors in algebra

Researcher: Mr James Owusu

Dear Sir or Madam,

PERMISSION TO USE TWO SECONDARY SCHOOLS IN WHITE RIVER CIRCUIT AS RESEARCH SITES

The above subject refers.

My name is James Owusu. I am a student at the University of South Africa and am presently enrolled for research master's degree in education with specialization in Mathematics Education. My dissertation supervisor is Dr Joseph J Dhlamini. I am also a Mathematics educator at one of the secondary schools in the White River Circuit. For the final dissertation in my MEd Mathematics Education programme, I am hoping to conduct a research study which examines the impact of constructivist-based teaching method on grade 11 learners' errors in algebra. I have selected two secondary schools in the White River Circuit as research sites to collect data for this study.

The purpose of this study is to identify which teaching method is suitable to improve learners' comprehension in algebra thereby reducing the errors they make in algebra in order to improve their performance in Mathematics. In order to do this I wish to conduct four-week lessons using two different teaching methods with two different teachers. One teacher will use the traditional method of teaching whiles the other use the constructivist-based method of teaching. At the commencement of the study I would administer a pre-test to all the 78 learners in the two grade 11 mathematics classrooms, and at the end of the study a post-test. These tests will take approximately one and half hours. The tests contain about 30 short answer items. The results of these tests will not form part of the continuous assessment (CASS) marks of the learner. These scheduled of activities will take place during the second term of 2014 academic year. I believe the results of this study would help to provide pedagogical way to improve the performance of learners in mathematics in South Africa.

I would like to request for permission from your outfit in order to access these research sites. Please find a copy of a copy of my Research Proposal approved by the university, and proof of registration with UNISA. Should you require further information, you could please contact me by phone at 0780338863 or by e-mail at jambaks@hotmail.com.

Thank you for your consideration.

Yours sincerely

James Owusu

Signature: ___________________

J (b): Letter of informed consent and requesting principals for the participation of the school in the study

Research topic: The impact of constructivist-based teaching method on secondary school learners' errors in algebra

Researcher: Mr James Owusu

Dear Principal

My name is James Owusu. I am a student at the University of South Africa and am presently enrolled for research master's degree in education with specialization in Mathematics Education. My dissertation supervisor is Dr Joseph J Dhlamini. I am also a Mathematics educator at Jacob Mdluli Secondary School. For the final dissertation in my MEd Mathematics Education programme, I am hoping to conduct a research study which examines the impact of constructivist-based teaching method on Grade 11 learners' errors in algebra. I have selected your school as one of the two schools to collect data for this study.

The purpose of this study is to establish the teaching method that is suitable to improve learners' understanding in algebra thereby reducing the errors they make in algebra in order to improve their performance in mathematics. In order to do this I wish to conduct a four-week lesson using two different teaching methods with two different teachers. In the course of the research I will administer a test instrument to 78 learners in two Grade 11 Mathematics classrooms during the second term of 2014 academic year. This test will take approximately one and half hours. The test contains about 30 short answer items. The results of this study may help to find pedagogical way to improve the performance of learners in mathematics in our district. The results of these tests will not form part of the continuous assessment (CASS) marks of the learner.

I would like to request the participation of your school in this study by allowing me to conduct the study in your school. I would also like to request the services of your Grade 11 Mathematics learners and teacher in this study. The teacher will be given a summary of the schedule for study later. You will also be given an opportunity to receive a summary of the findings. I will not use teacher's or learners' names or anything else that might identify them in the written work, oral presentations, or publications. The information remains confidential. They are free to change their minds at any time, and to withdraw even after they have consented to participate. They may decline to answer any specific questions. I will destroy any recording after the research has been presented and/or published which may take up to five years after the data has been collected. There are no known risks to you for assisting in this study.

This study has been approved by the Mpumalanga Department of Basic Education. Please find a copy of the letter of approval from the MDBE. If you would like more information, please contact me by phone at 078 033 8863 or by e-mail at jambaks@hotmail.com. Please contact me at your earliest convenience to discuss the work or to provide your consent to participate.

Thank you for your consideration.

Yours sincerely

James Owusu

Signature: __________________

J (c): Informed response from the principals

Dear Mr Owusu

I, ____, the principal of, ____ high/ secondary school, acknowledge that I have received, read and understood the content of the request letter that you sent me to explain your intentions to conduct research in my school. The title of your research is: **"The impact of constructivist-based teaching method on secondary school learners' errors in algebra,"** and its purpose is explained in your letter.

I therefore **give consent/ do not give consent** that my school (a teacher and specified group of learners) will take part in your research.

Principal signature: _________________________________

Date: _________________________________

Researcher signature: _________________________________

Date: _________________________________

J (d): A consent letter of request to the Mathematics teacher

Research topic: The impact of constructivist-based teaching method on secondary school learners' errors in algebra

Researcher: Mr James Owusu

Dear Grade 11 mathematics teacher

My name is James Owusu. I am a student at the University of South Africa and am presently enrolled for research master's degree in education with specialization in Mathematics Education. My dissertation supervisor is Dr Joseph J Dhlamini. I am also a Mathematics educator at Jacob Mdluli Secondary School. For the final dissertation in my MEd Mathematics Education programme, I am hoping to conduct a school-based research, which examines the impact of constructivist-based teaching method on Grade 11 learners' errors in algebra. I have selected your school, as well as your Grade 11 Mathematics class, to participate in my research. The research will involve two schools.

The main purpose of this study is to establish a teaching method that is suitable to improve learners' understanding in algebra thereby reducing their errors in algebra to improve their performance in mathematics. In order to do this I wish to conduct a four-week lesson using two different teaching methods, one of which could be offered by you. The two schools will be divided into experimental and control groups. I will teach my proposed new instruction in the experimental group, and you could use your own traditional (usual) method in the control group. The aim is to compare the two teaching methods to determine the one which is more effective in reducing learners' errors in algebra. I therefore request you to be part of this research.

In case you agree to participate, you will be expected to administer a performance test to your Grade 11 Mathematics class, at the start and end of the research. The same test will be administered by me in the experimental group at both intervals. We will teach the same content but use different methods to present it. The test scores will be used to measure the influence of each teaching method on the performance of learners. Your name and those of your learners will not be revealed. Pseudonyms will be used instead, and in most cases data will be aggregated. You will be allowed to change your mind at any time, and to withdraw during the course of research if you feel so. There are no known risks to you and to your learners for assisting me in this research.

In case you agree, I will contact the parents of the learners in your class to request their approval and permission for their children to participate in the study. In addition, each child will receive a consent letter from me to explain their involvement in my research. They will also be allowed to choose if they want to participate in the research or not.

This research has been approved by the Mpumalanga Department of Basic Education. Please find a copy of the letter of approval from the MDBE. If you would like more information, please contact me by phone at 078 033 8863 or by e-mail at jambaks@hotmail.com. Please contact me at your earliest convenience to discuss the work or to provide your consent to participate.

Thank you for your consideration.

Yours sincerely

James Owusu

Signature: ________________________

J (e): The informed consent form for the Grade 11 Mathematics teacher

Dear Mr Owusu

I, ____, the teacher of Grade 11 mathematics in ____________________________________ high/ secondary school, acknowledge that I have received, read and understood the content of the request letter that you sent me to explain your intentions to conduct research in my classroom. The title of your research is: *The impact of constructivist-based teaching method on secondary school learners' errors in algebra,* and its purpose, and the purpose of the research is explained in your letter.

I therefore **give consent/ do not give consent** to participate in your research.

Teacher signature: ______________________________

Date: ______________________________

Researcher signature: ______________________________

Date: ___________________________

J (f): Letter of informed consent and requesting parent/guardian for the participation of their children in the research

Research topic: The impact of constructivist-based teaching method on secondary school learners' errors in algebra

Researcher: Mr James Owusu

Dear Parent or Guardian

My name is James Owusu. I am a student at the University of South Africa and am presently enrolled research master's degree in education with specialization in Mathematics Education. My dissertation supervisor is Dr Joseph J Dhlamini. I am also a Mathematics educator at Jacob Mdluli Secondary School. For the final dissertation in my MEd Mathematics Education programme, I am hoping to conduct a research study which examines the impact of constructivist-based teaching method on Grade 11 learners' errors in algebra. I have selected your child as one of the learners from the two schools to collect data for this study.

The purpose of this study is to identify which teaching method is suitable to improve learners' comprehension in algebra thereby reducing the errors learners make in algebra in order to improve their performance in mathematics. In order to examine learner's errors, I wish to conduct four-week lessons using two different teaching methods with two different teachers. In the course of the study I would administer a test instrument to 78 learners in two Grade 11 Mathematics classrooms. Your child will be asked to participate in a written test during the second term of 2014 academic year. This test will take approximately one and half hours. The test contains about 30 short answer items. The results of this study would help to provide a way to improve the performance of learners in mathematics in South Africa. The results of these tests will not form part of the continuous assessment (CASS) marks of the learner. I would like to request the participation of your child in this study. Participation in this study is voluntary and will not affect your child's attendance in class or his/her evaluation by the school. All information collected will be anonymous. In a way, the results of this study may help the school as well to identify students' difficulties in algebra and propose remedial work.

Please indicate on the attached form whether you permit your child to take part in this study. Your cooperation will be very much appreciated. If you have any questions or would like more information, please contact me on phone at 0780338863 or by e-mail at *jambaks@hotmail.com*

Thank you

Yours sincerely

James Owusu

Signature: ___________________

J (g): Informed consent form for the parent/ guardian

Dear Mr Owusu

I, ____, the parent/ guardian of, ____, acknowledge that I have received, read and understood the content of the request letter that you sent me to explain your intentions to conduct research in the school of my child. The title of your research is: **"The impact of constructivist-based teaching method on secondary school learners' errors in algebra,"** and the purpose of the research is explained in the letter.

I therefore **give consent/ do not give consent** for my child participate in your research.

Parent signature: _________________________________

Date: _________________________________

Researcher signature: _________________________________

Date: _________________________________

J (h): Letters of informed assent and requesting Grade 11 learners' participation in the study

Research topic: The impact of constructivist-based teaching method on secondary school learners' errors in algebra

Researcher: Mr James Owusu

Dear Learner,

My name is James Owusu. I am a student at the University of South Africa and am presently enrolled for research master's degree with specialization in Mathematics Education. In order to complete the requirements for the degree, I have to become acquainted with aspects of doing research that will involve Grade 11 Mathematics learners in your school. My research will focus on investigating the appropriate teaching method that will help learners to overcome the difficulties they encounter in learning mathematics as a result of the errors they hold in algebra. The title of my research is: **"The impact of constructivist-based teaching method on secondary school learners' errors in algebra."** My research supervisor is Dr Joseph J Dhlamini who is a Mathematics Education lecturer at the University of South Africa.

The purpose of this research is to assist in trying to find the suitable teaching method to improve learners' performance in mathematics in secondary schools in Mpumalanga province in particular and South Africa in general. I wish to invite you to participate in this research. If you agree to participate in this research you will be requested to attend lessons for a period of four weeks and during this period you will be requested to write two tests; one at the beginning of the research (pre-test) and the other at the end the research (post-test). The results of these tests will not form part of your continuous assessment (CASS) in the school. The assessment is designed to help you with algebra, by helping your teacher understand the mistakes you make, as well as why you make them.

Your identity, and that of your school, will not be revealed. In reporting about the findings from this research pseudonyms will be used. In the end, the results of the study will be made available to you and to your school. All activities related to this research will be conducted between 14h00 and 15h00 in order not to interfere with teaching time. You will be given a timeframe of all the activities involved. Prior to the commencement of the research the researcher will convene a meeting with all participants to explain the objectives of the study and clarify other related issues. Should you decide to participate in the study, you are free to withdraw your participation at any stage of the research without a penalty. After reading this letter, please complete the attached consent form and return to the researcher. I thank you in advance for reading this letter and I hope to hear from you soon. If you have any questions about this research you are free to contact me at 078 033 88633 or jambaks@hotmail.com

Thank you

Yours truly

James Owusu

Signature: _________________

J (i): Informed assent form from learners

Dear Mr Owusu

After reading and understanding the content of the request letter that was given to me by Mr James Owusu, I ……………………………………, the learner of the Grade 11 Mathematics class, **agree/ do not agree** to participate in the research in which the researcher will investigate the impact of the constructivist-based teaching method on secondary school learners' errors in algebra in the two secondary schools in the White River Circuit of Mpumalanga Department of Education.

My decision on the following research activities is as follows:

- To write both the Pretest and the Posttest that will be given to me for data collection.
  Yes ☐ or No ☐ [Use a tick (√) to indicate your choice]

- To participate fully in lessons that would be conducted during the instruction.
  Yes ☐ or No ☐ [Use a tick (√) to indicate your choice]

Student signature : ……………………………… Date: ………………………….

Researcher signature: ………………………………. Date: …………………………..

APPENDIX K: APPROVAL LETTER FROM MPUMALANGA DEPARTMENT EDUCATION

APPROVAL TO CONDUCT RESEARCH FOR MR. JAMES OWUSU: MATHEMATICS EDUCATION: (M.ED DEGREE)

education
DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

Private Bag X 11341
Nelspruit 1200
Government Boulevard
Riverside Park
Building 5
Mpumalanga Province
Republic of South Africa

Litiko leTemfundvo Umnyango weFundo Departement van Onderwys Umnyango wezeMfundo
Enquiries: H.A. Baloyi (013) 766 5476

Mr. J. Owusu
P.O. BOX 1632
Kabokweni
1245

RE: APPLICATION TO CONDUCT RESEARCH: MR. J. OWUSU

Your application to conduct research was received on the on the 09 April 2014. The title of your study is: "*The impact of the constructivist teaching method on secondary school learner's misconceptions and errors in algebra.*" The aims, objectives, the questions and the overall design of your study give an impression that the outcomes of the study especially your findings and recommendations will improve the teaching and learning of the subject. Your request is approved subject to you observing the content of the departmental research manual which is attached. You are required to discuss with the principals of the sampled schools regarding the approach to your observation and data collection as no disruption of tuition will be allowed. You are also requested to adhere to your University's research ethics as spelt out in your research ethics document.

In terms of the attached manual (2.2. bullet number 4 & 6) data or any research activity can only be conducted after school hours as per appointment. You are also requested to share your findings with the department so that we may consider implementing your findings if that will be in the best interest of department.

For more information kindly liaise with the department's research unit @ 013 766 5476 or a.baloyi@education.mpu.gov.za. The department wishes you well in this important project and pledges to give you the necessary support you may need.

APPROVED/NOTAPPROVED:

MRS MOC MHLABANE
HEAD OF DEPARTMENT
25, 4, 14
DATE

APPENDIX L: ETHICAL CLEARANCE CERTIFICATE FROM UNISA

UNISA | college of education

Research Ethics Clearance Certificate

This is to certify that the application for ethical clearance submitted by

J Owusu [45040753]

for a M Ed study entitled

The Impact of constructivist teaching method on secondary school learners' misconceptions and errors in algebra

has met the ethical requirements as specified by the University of South Africa College of Education Research Ethics Committee. This certificate is valid for two years from the date of issue.

Prof KP Dzvimbo
Executive Dean : CEDU

Dr M Claassens
CEDU REC (Chairperson)
mcdtc@netactive.co.za

Reference number: 2014 JUNE /45040753/MC

19 JUNE 2014