THE EFFECT OF INTEGRATION OF GEOGEBRA SOFTWARE IN THE TEACHING OF CIRCLE GEOMETRY ON GRADE 11 STUDENTS' ACHIEVEMENT

By

ALFRED CHIMUKA

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Dr U.I. Ogbonnaya

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DEDICATION

To God be the Glory
To my late parents who believed in industriousness and perseverance.

DECLARATION

Name: CHIMUKA ALFRED

Student number: 46465545

Degree: MSC. MATHEMATICS, SCIENCE AND TECHNOLOGY EDUCATION

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I declare that the above dissertation/thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

________________	9 MAY 2017
SIGNATURE	DATE

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ABSTRACT

This study investigated the effect of integration of GeoGebra into the teaching of circle geometry on Grade 11 students' achievement. The study used a quasi-experimental, non-equivalent control group design to compare achievement, Van Hiele levels, and motivation of students receiving instruction using GeoGebra and those instructed with the traditional 'talk-and-chalk' method.

Two samples of sizes n = 22 (experimental) and n = 25 (control) drawn from two secondary schools in one circuit of the Vhembe district, Limpopo Province in South Africa were used. A pilot study sample of size n = 15, was carried out at different schools in the same circuit, in order to check the reliability and validity of the research instruments, and statistical viability. The results of the pilot study were shown to be reliable, valid and statistically viable. The study was informed by the action, process, object, schema (APOS) and Van Hiele theories, as the joint theoretical framework, and the literature search concentrated on technology integration, especially GeoGebra, in the teaching and learning of mathematics.

The literature was also reviewed on the integration of computer technology (ICT) into mathematics teaching and learning, ICT and mathematical achievement, and ICT and motivation. The study sought to answer three research questions which were hypothetically tested for significance. The findings of this study revealed that there was a significant difference in the achievement of students instructed with GeoGebra compared to those instructed with the traditional teaching method (teacher 'talk-and-chalk'). The average achievement of the experimental group was higher than that of the control group. Significant differences were also established on the Van Hiele levels of students instructed with GeoGebra and those instructed without this software at Levels 1 and 2, while there were no significant differences at Levels 3, 4 and 5. The experimental group achieved a higher group average at the visualisation and analysis Van Hiele levels. It was also statistically inferred from questionnaires through chi-square testing, that students instructed with GeoGebra were more motivated to learn circle geometry than those instructed without the software.

Keywords: Integration of GeoGebra software; APOS theory; Van Hiele theory; achievement; motivation

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ACE	Activities, Classroom discussion and Exercises
APOS	Action, Process, Object Schema
BECTA	British Educational Communications and Technology Agency
CAPS	Curriculum Assessment Policy Statement
CAS	Computer Algebraic System
CVR	Content Validity Ratio
DGS	Dynamic Geometry Systems
DoBE	Department of Basic Education
FET	Further Education and Training
ICT	Information and Computer Technology
ISTE	International Society for Technology
IT	Information Technology
KISS	Keep It Short and Simple
KR20	Kuder-Richardson 20
NCS	National Curriculum Statement
SERA	Scottish Educational Research Association
SPSS	Statistical Package for Social Sciences
Unisa	University of South Africa

CHAPTER 1
BACKGROUND AND OVERVIEW

1.1 Background to the study

Poor achievement in mathematics and other sciences has been a major problem confronting the South African basic education system for quite some time (since the dawn of democracy in 1994). Authorities have from time to time revised the curriculum in general and the mathematics curriculum in particular for various reasons, but mathematical achievement in South Africa by students is yet to compare favourably to international standards.

The South African school curriculum, in particular the secondary school mathematics curriculum, has undergone extensive transformation since 1994. The democratic government of South Africa has issued several curriculum-related reforms intended to democratise education and eliminate inequalities established by the apartheid education system. Chisholm (2003) reported that, since 1994, the curriculum reform process in South Africa has passed through three main stages. She argued that the first stage involved the 'cleansing' of the curriculum of its racist and sexist elements in the immediate aftermath of the democratic elections. The second stage, according to Chisholm (2003), was the implementation of outcomes-based education (OBE) through Curriculum 2005 (C2005) while the third stage involved the review of C2005, culminating in the creation of the Revised Curriculum Statement (Chisholm, 2003). To date, the education curriculum in South Africa has undergone a fourth transformation. The National Curriculum Statement (NCS) of 2002, revised in 2009, has been phased out to make way for the Curriculum and Assessment Policy Statement (CAPS) of 2012. These changes, while they are to all intents and purposes desirable, have inevitably brought with them a number of pedagogical and instructional challenges.

In the NCS, the mathematics Grade 12 examination consisted of three papers (Papers 1, 2 and 3) of which Papers 1 and 2 were compulsory for all learners while Paper 3 was optional (Department of Basic Education, 2012). The topics in Paper 3 were optional topics, hence, they were not taught in many schools because learners had not registered for Paper 3. In the CAPS mathematics curriculum, some of the

NCS optional topics were integrated into either paper 1 or paper 2 so that two Papers, compulsory for all Grade 12 learners were set while Paper 3 was cancelled.

Some of the topics that have been integrated into papers one and two of the new CAPS mathematics curriculum for the Further Education and Training band (FET) (Grades 10–12) are circle geometry, descriptive statistics and interpretation, probability, and bivariate data. The inclusion of Paper three's content into papers one and two was a curriculum change of great magnitude for both teachers and learners. The majority of mathematics teachers in the FET band, at least at the time of its introduction, were ill-equipped pedagogically to teach what appeared to be new topics. Learners, on the other hand, had to brace themselves for an increased workload and more challenging mathematical content.

This study sought to intervene, and investigate how technologically oriented teaching methods could improve student achievement. According to Suan (2014), students' performance and achievement in mathematics is affected by three factors, namely teacher factor, student factor and environmental factor. Suan (2014) argues that the teacher factor is comprised of subject mastery, instructional techniques and strategies, classroom management, communication skills, and personality. The student factor includes study habits, time management, attitude and interest in mathematics; the environmental factor includes issues such as parents' values and attitudes, classroom settings, and peer group (Suan, 2014). This study sought to explore one aspect of the teacher factor (teaching aids), namely, the extent to which technology-inspired instructional techniques and strategies impact on student achievement in mathematics.

This study sought to investigate the effect of integrating GeoGebra into the teaching of circle geometry on students' achievement. Two key variables affecting student achievement focused on in this study were students' Van Hiele levels and student motivation. The researcher believes there are wide knowledge gaps in the effective teaching and learning (instructional techniques and strategies) of circle geometry caused by some of the factors that Suan (2014) alluded to, coupled with the change in the national mathematics curriculum, specifically in Nzhelele East circuit of Vhembe district of Limpopo in South Africa. The researcher believes that the major cause of poor achievement in geometry topics is instructional and knowledge deficiencies among teachers of geometry. In the circuit where this study was carried out, no secondary school was offering mathematics paper three in the NCS curriculum, and most teachers confessed at mathematics workshops to have inadequate knowledge of the geometry topics dating back to their days as students themselves. Moreover, institutional support to schools on presumed difficult topics is either absent or weak. For these reasons, this study explored the effect of technology-inspired teaching.

The traditional talk-and-chalk teacher-centred type of teaching, which assumes that students are passive recipients of knowledge, has lost its lustre among technologically savvy youths. It is one's belief that the integration of information technology (IT), GeoGebra in particular, into the teaching and learning of secondary school mathematics can serve as a scaffold on which changes and developments in curriculum can be better managed.

In this study, the researcher looked specifically at the mathematical content of circle geometry which was formerly in the Grade 11 optional paper three, and quasi-experimentally explored the effect of integrating GeoGebra in teaching the topic on Grade 11 students' achievement, Van Hiele levels, and motivation. The intention was to investigate whether it is worthwhile to integrate GeoGebra into the teaching and learning process in order to narrow the instructional and knowledge gap, seemingly created by the teacher factor and curriculum change.

1.2 Problem statement

Perennial poor performance in mathematics and related science subjects has impacted negatively on the South African tertiary education system. The majority of secondary school graduates fail to meet minimum performance standards in mathematics, required for successful completion of a tertiary qualification in mathematics and science-oriented fields.

According to the Centre for Development and Enterprise (2014), 90% of high school graduates in South Africa fail to meet the minimum performance levels required by tertiary institutions. This state of affairs impacts on the country's human capital and consequently hampers its economic development. From time immemorial, geometry sections in the mathematics examinations in South Africa have contributed significantly to poor performance in mathematics. Examination reports by the Department of Basic Education (DoBE) for the past two decades or so, indicate that geometry in general and circle geometry in particular were major contributors to students' underachievement in mathematics. Geometry is an indispensable pre-requisite for applied mathematics and applied sciences at tertiary level.

Periodical changes in the mathematics curriculum, in which inclusions and/or exclusions of geometric topics, and sometimes making geometric topics optional or compulsory, has exacerbated the problem of poor performance in geometry. These inconsistencies result in some teachers failing to teach effectively those topics which are sometimes removed or made optional. The change of the South African curriculum from NCS to CAPS seems to have created instructional and/or pedagogical challenges for both teachers and students alike. Some of the content of paper three in the NCS mathematics curriculum has been integrated into paper one and paper two of the new CAPS curriculum. The CAPS mathematics curriculum's examination assessment now consists of only two compulsory examination papers (papers one and two), as compared to two compulsory papers (papers one and two) and one optional paper (paper three) in the NCS mathematics curriculum. The implication of this curriculum change is that all students, regardless of ability, have to answer all questions previously assessed in optional paper three, while some teachers have little or no experience in teaching the newly introduced topics. Teaching and learning under these circumstances is no mean task, as evidenced by examination reports from 2008 to 2012, which indicate that students enrolled for paper three did not achieve high marks in the paper (DoBE examination reports, 2008 to 2012).

The major problem that this study sought to solve is poor achievement in circle geometry, that I believe has its origins in an inadequate background in geometry and poor motivation to learn it. The study investigates the effect of the integration of GeoGebra into the teaching of circle geometry on student achievement of Grade 11 students. The emphasis is to discover whether the method of instruction (computer-assisted instruction using GeoGebra) motivates students, enhances their problem-solving techniques and ultimately improves their achievement in geometry.

1.3 Research questions

This study sought to address the following three research questions:

1. Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra in circle geometry?
2. Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at Van Hiele levels of geometric thinking?

To answer research question 2, the following five sub-questions, each focusing on a particular Van Hiele level, were raised:

- 2.1 Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at the visualisation level of geometric thinking?
- 2.2 Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at the analysis level of geometric thinking?
- 2.3 Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at the abstraction level of geometric thinking?
- 2.4 Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at the deduction level of geometric thinking?
- 2.5 Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at the rigour level of geometric thinking?

3. Does GeoGebra aid students' motivation to learn circle geometry?

To answer this research question, a seven-item motivation attributes questionnaire was used and seven hypotheses were chi-square tested for significance of differences.

1.4 Research hypotheses

The following null hypotheses were tested at α = 0.05 (95% confidence interval) level of significance.

1. There is no significant difference in the achievement of students exposed to GeoGebra compared to those not exposed to the software.
2. There is no significant difference in the achievement of students exposed to GeoGebra compared to those not exposed to the software at Van Hiele Level one.
3. There is no significant difference in the achievement of students exposed to GeoGebra compared to those not exposed to the software at Van Hiele Level two.
4. There is no significant difference in the achievement of students exposed to GeoGebra compared to those not exposed to the software at Van Hiele Level three.
5. There is no significant difference in the achievement of students exposed to GeoGebra compared to those not exposed to the software at Van Hiele Level four.
6. There is no significant difference in the achievement of students exposed to GeoGebra compared to those not exposed to the software at Van Hiele Level five.
7. There is no significant difference in the extent to which students are motivated to learn circle geometry between students taught with GeoGebra and those taught without the software.

1.5 Rationale for the study

Many South African students at Grade 12 perform very poorly in Euclidean geometry. Examination reports for different years from the Department of Basic Education in South Africa paint a very gloomy picture about student performance in specific topics, especially Euclidean geometry (DoBE Examination Reports 2012 and 2013; Paper three reports 2014; Paper two reports 2015). Many teachers in the South African school system are not effective in the teaching of geometry, for various reasons, including the fact that some never learnt Euclidian geometry during their school or college education. For these reasons, intervention mechanisms and strategies are needed. There is an urgent need for an improvement of teaching strategies in order to improve poor performance in mathematics. To contribute to the solution of the problem of poor performance, this study sought to employ mathematical technology (GeoGebra) in the teaching and learning process of Euclidean geometry.

It is very evident that the traditional methods of teaching geometry are not achieving the desired results. Consequently, there is a dire need to find alternative teaching and learning methods that incorporate technology. Studies have indicated that technology, if correctly used, could improve student performance (Murray, 2016).

Integrating technology software, such as GeoGebra, into the teaching and learning of mathematics is supported by the theory of experiential learning and learner-centred education (Kolb, 2015). Many studies (e.g. Bester & Brand, 2013; Ogbonnaya, 2010) have advocated the use of technology in mathematics teaching and learning to enhance student learning of mathematical concepts. Willougby and Wood (2008) noted that learning takes place on computer software without the learners realising the amount of attention they are paying to the material.

The use of technological software, such as GeoGebra, in the teaching of circle geometry in mathematics can act as a positive stimulus to students' learning of the concepts.

1.6 Significance of the study

Improving mathematics results in secondary schools in South Africa is a contemporary problem to which practical solutions are yet to be found. This study has sought to contribute in this regard by exploring alternative teaching and learning methods, especially for topics traditionally regarded as problematic to both teachers and students, such as circle geometry.

Geometry is a key mathematics component that is required in all applied physical sciences; its teaching and learning through the use of technology, helps students to develop insights into an understanding of today's technological industry (Ritz, 2009). Teaching and learning methods that incorporate technology are important to developing countries such as South Africa, in order for these countries to catch up with the rest of the developed world. The long-term goal for this study is to inspire mathematics teachers and students to use mathematical software to simplify the teaching and learning processes. Researchers recommend teaching methods that help students to discover and develop their talents in technical fields (Ritz, 2009). By using mathematical software, such as GeoGebra, students begin the process of technical problem-solving in the classroom, which they will be able to transfer to industrial technical problem-solving (Ritz, 2009). Snyder and Hales (1981) argued that using technology in the teaching and learning of geometry can result in positive effect in today's world, in areas such as technological systems of communication, construction, manufacturing and transportation.

Hohenwarter and Preiner (2007) argued that there is a growing belief among international mathematics teachers that GeoGebra has the potential to transform mathematics education. Consequently, this study aimed to add to the list of literature on the use of technology in teaching mathematics, especially in the South African context.

The study may serve as a guide to mathematics educators in finding alternative and/or supplementary ways of teaching circle geometry instead of the teacher talk and-chalk method of teaching. Because many students are not motivated to learn mathematics, this study could help the mathematics education community and other stakeholders in mathematics education in South Africa to determine whether the use of technology in mathematics teaching could motivate students to learn the subject.

1.7 Scope and delimitation of the study

This quasi-experimental research study was of limited scope. The population of the study was derived from Grade 11 mathematics students in Limpopo Province of South Africa. The sample for the study was taken from two Grade 11 mathematics classes at two secondary schools in Nzhelele East circuit in the Vhembe district of Limpopo Province. The topic is limited to circle geometry taught in Grade 11.

1.8 Key concepts

This study compared two methods of teaching and learning, namely Chalk and talk method of instruction and instruction using GeoGebra.

1.8.1 Chalk and talk method of instruction

The chalk-and-talk method of instruction, or direct instruction, is a traditional teaching method in which the students' focus is on what the teacher says and what s/he writes on the chalkboard. In most cases, the teacher speaks from the front of the class, explaining, guiding, controlling and deciding what students must do, and occasionally writes notes, diagrams, questions on the chalkboard. Students are often seated in rows and are expected to pay attention and follow instructions. The chalk-and-talk method of instruction is largely teacher-centred (teacher conveys what s/he knows to students).

1.8.2 Instruction with GeoGebra

GeoGebra is interactive and visual software for teaching and learning geometry, algebra, statistics, calculus, and other sciences. It was developed by Markus Hohenwarter, and can be used for active and problem-oriented teaching and learning. Instruction with GeoGebra enables mathematical experiments and discoveries. GeoGebra has several views (algebraic view; geometric view; spreadsheet view; computer algebra system view; protocol design view; and command line) all linked together. Instruction with GeoGebra is largely student centred.

CHAPTER 2
THEORETICAL FRAMEWORK AND LITERATURE REVIEW

This study is grounded on two learning theories, Van Hiele (1957) and action, process, objects and schema (APOS) (Dubinsky, 1984a) theories. These two theories were deliberately selected as the joint theoretical framework because of their relevance to the teaching and learning process of geometry; the Van Hiele theory was used as a framework to analyse the learners' levels and/or stages that they go through when engaged in circle geometry problem-solving and APOS was used as the general guideline to the research process. In the literature review, the background and context for the study that was briefly explained in chapter one are elaborated. The literature review was intended to discuss findings on the integration of ICT in the teaching and learning process and to identify gaps in current knowledge on the integration of technology with mathematics education at secondary school level.

This chapter also reviews studies that have used Van Hiele and APOS theories. Also reviewed are studies on the integration of information and computer technology (ICT) into mathematics teaching and learning with emphasis on GeoGebra, ICT and learners' achievement, and ICT and learners' motivation to learn. The chapter concludes by affirming the current study. The literature review strategy was the investigation of published databases, journals, research papers, reports and publications in the field of educational technology of the South African Department of Basic Education and other countries.

2.1 Theoretical framework

This study was framed on two learning theories of how students learn (in particular mathematics), namely Van Hiele theory of geometry thinking (Van Hiele, 1957) and APOS theory (Dubinsky, 1984a).

2.1.1 Van Hiele Theory of Geometry Thinking

The Van Hiele theory of geometric thinking is a framework that describes the development of geometric reasoning. Two mathematics educators in the Netherlands, Pierre Van Hiele and Dina Van-Geldorf, developed a pedagogical theory to describe the geometric understanding levels of children by focusing on problems faced by students when they learn geometry (Olkun & Toluk, 2003). According to Van Hiele theory, a student progresses through five stages/levels of development when learning geometry, namely visualisation, analysis, abstraction, deduction and rigour as illustrated in Figure 2.1.

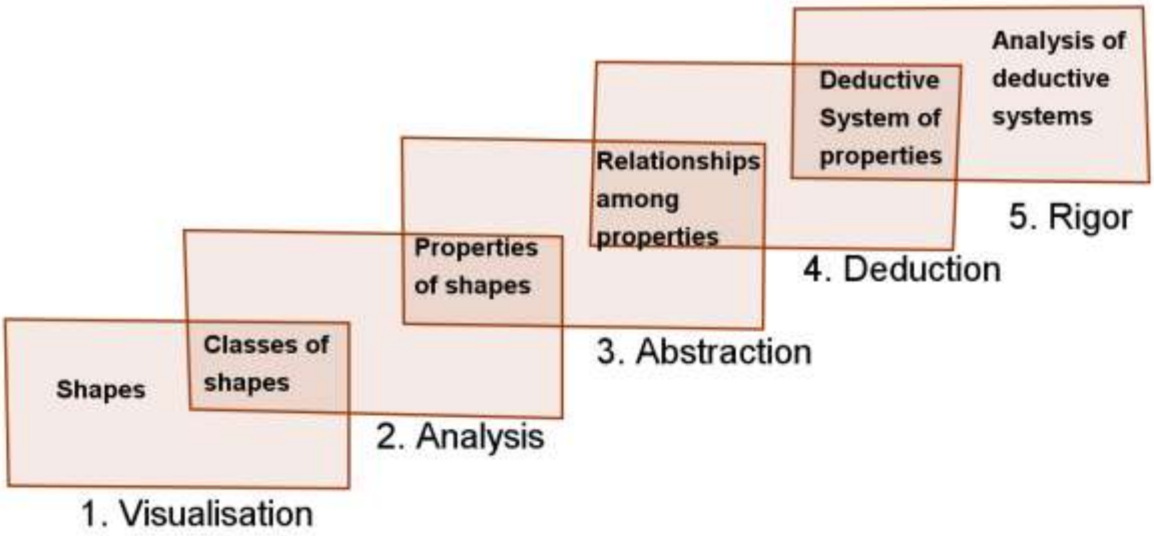

Figure 2.1: Van Hiele Theory (adapted from Van de Walle, 2004, p. 347)

Level 1: Visualisation

The Van Hieles postulated that this stage/level is when students recognize the figures by appearance alone, often by comparing them to a known prototype. The properties of a figure are perceived. Students recognize triangles, squares, circles, parallelograms, and other shapes, but do not identify correctly the properties of these figures. At visualisation level, students make uninformed decisions because they base their arguments on perception, rather than on reasoning, for example, the figure is a square, cube or rectangle because it looks like one (Ball, 1990).

According to Pegg and Davey (1989), there are at least three basic categories of visualization, as follows:

1. Student can focus only on a single feature such as the number of sides.
2. Student can identify certain features of a figure, such as points, sharpness, corners and flatness, but is not able to link these features so as to have an overview of the shape.

3. Student is able to associate a geometric shape with a known shape, for example a cube is like a chalk box or a dice, a rectangle is a long square, parallel lines are like racing tracks (Pegg & Davey, 1989, p. 25).

In essence, successful visualization, according to Pegg and Davey (1989), can be achieved by focusing on one aspect or feature of the geometric figure, or focusing on multiple aspects or features on the figure. Association of a geometric figure with real-life objects also assists visualization.

Level 2: Analysis

A student operating at the analysis level is able to identify each element of a geometric object in terms of its properties in isolation. At this level, students see figures as collections of properties. Although they can recognize and name properties of figures, they do not see relationships between the properties. When describing an object, a student operating at this level might list all the properties the student knows, but does not make connections between figures (Clements & Basttista, 1992). The properties are seen as discrete entities independent of one another. For example, an equilateral triangle has the following properties: three equal sides, three equal angles, and three axes of symmetry but students might not at this level realise that these three properties imply one another. In circle geometry, the following two theorems can be viewed in isolation, yet they imply one another: (i) the theorem that states that, the angle subtended at the centre is twice that subtended at the circumference, and (ii) the theorem that states that, the angle subtended by the diameter is a right angle.

Level 3: Abstraction/Ordering

According to the online Harrap dictionary (2016), an abstraction is ‘an idea or principle considered or discussed in a purely theoretical way without reference to actual examples and instances’. Students at this level are able to perceive relationships between properties and figures. They can define and give informal arguments to justify their reasoning. For example, in circle theorems, students are able to identify that the two theorems, the angle subtended at the centre is twice that subtended at the circumference, and the angle subtended by the diameter is a right angle, imply one another.

The key mental or cognitive activity at this stage is ordering (sequencing). Logical implications and class conclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood (Oliver, 2000). Mayberry (1983) indicates that at this level, logical implications and class inclusions are understood. For example, in an equilateral triangle, all sides are equal, implies that all angles are equal.

Level 4: Deduction

Deduction is the reasoning process by which one concludes something from known facts or circumstances. At this level students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. In addition, at this level, students should be able to construct proofs such as those typically encountered in a secondary school geometry class (Mason, 1998; Van Hiele, 1959; Van Hiele-Geldof, 1984). Most of the higher-order circle geometry theorems or proofs in South African secondary school mathematics can be tackled by students operating at the deductive level, for example, proof of theorems such as: The angle between the tangent to a circle and a chord drawn from point of contact is equal to an angle in the alternate segment.

Level 5: Rigour

The last level, is rigour**.** Students at this level understand the formal aspects of deductive reasoning, such as establishing the similarities and differences between mathematical concepts. For example, in proving circle theorems, students require the whole set of skills, such as statement of what is to be proved, construction of additional lines (abstraction) and statement of implied or given theorems. They can understand the use of indirect proof and proof by contra-positive methods as well as non-Euclidean systems (Simon, 2006).

Some scholars such as Simon (2006) and Mason (1998) concur with the original Van Hiele theory, they argue that the rigour level is the level of college mathematics, where students major in geometry, (Van Hiele, 1986). Their argument is that, students at rigour level understand the relationships between various systems of geometry, through geometrical maturity, thus they are able to describe the effect of adding or deleting an axiom on a given geometric system. This study, however argues that the levels of geometrical understanding by students, as propounded by the Van Hieles can be applied as they are to any group of students regardless of age and/or gender, this assertion is supported by studies such as those carried out by Alex & Mammen, (2012), Abdullah & Zakaria, (2012), and Abidin, (2013). Van Hiele theory is not a developmental theory where students must reach a certain age to progress through the levels; rather, it is dependent on the experiences and activities in which students are engaged. For this reason, the learning environment created by the teacher can provide experiences that helps students to progress from visualisation level to rigour level, hence the use of GeoGebra in this study.

The theory as propounded by the Van Hieles was meant to be hierarchical, meaning that a learner cannot operate with understanding on one level without having been through the previous levels. Although this has been confirmed by some findings from research studies, such as those conducted De Villiers & Njisane (1987), Fuys, Geddes, Lovett & Tischler (1988), Burger & Shaughnessy (1986) suggest, however, that the levels may not be discrete as suggested by the Van Hieles. Rather, it is possible that students learning a particular geometric concept can be in transition between levels, implying that they can oscillate between the two levels. There is also evidence that the student's level of geometrical thinking might vary across topics depending on how recent a topic was studied (Fuys et al., 1988).

The original Van Hiele theory of geometrical thinking regarded each level as having its own vocabulary and students at different geometrical levels were assumed not able to understand one another. For example, a rectangle might have a different meaning for students at different levels. Students at the ordering level might regard rectangles as a special kinds of parallelograms, while students at the lower geometrical level may not understand the relationship between rectangles and parallelograms. Fuys et al., (1988), argued that problems can be encountered when a teacher uses higher level language to that of students. The Van Hiele theory in its original state placed emphasis on pedagogy and the importance of the teacher structuring the students' experiences to facilitate transition through the levels.

Fuys et al. (1988) argued that according to Van Hiele theory, progress from one level to the next involves five phases. Each phase involves a higher level of thinking. These phases are useful in designing activities, as follows:

1. **Information**: The student becomes familiar with the working domain/field of exploration by using the teaching and learning material presented to him/her. The use of GeoGebra (the working domain for this study) in the teaching and learning of circle geometry makes it easy for students to acquire basic, but very important, information that underpins the understanding of the whole topic. Visualisation of angles subtended by various lines, arcs and/or chords, can readily be achieved by students if they are allowed to explore using the computer.
2. **Guided orientation**: The students explore concepts using the teaching and learning materials presented to them, for example, by folding and measuring paper, the will be able to identify symmetrical shapes.
3. **Explanation**: A student becomes conscious of the network of relations, tries to express them in words and learns the required technical language for the subject matter, for example, expresses ideas about the properties of figures. In circle geometry, the learner understands the meaning of key words such as subtending an angle, alternate segment, interior and exterior angles.
4. **Free orientation**: Investigation of relationships between objects is still largely unknown at this stage, but learners are given more complex tasks to find their way round this field, for example, a learner might know about the properties of one kind of shape but is required to investigate the properties for a new shape, for example, a kite. The tasks should be designed so that they can be carried out in different ways. In circle geometry, free orientation of a learner is achieved when the learner is able to identify angle properties on parallel lines and other geometric shapes that may be present in any given diagram.
5. **Integration**: Students are able to summarise all that they have learned about the subject, reflect on their actions and thus obtain an overview of the whole network/field that has been explored, for example, are enabled to summarise.

Van Hiele theory has been adopted by many researchers to explain how students understand basic geometry (Howse & Howse, 2014). For example, Khembo (2011) carried out an investigation into Grade 6 teachers' understanding of geometry, according to the Van Hiele levels of geometric thought. Van der Sandt (2003) carried out a two-year study in South Africa, to investigate the state of the knowledge of prospective teachers of Grade 7 geometry using Van Hiele theory.

There is a need for studies to evaluate the applicability of Van Hiele theory to more advanced geometric concepts, such as Euclidian geometry. In this study, the Van Hiele theory of geometric understanding was extensively used to design the post-test items (questions) for this study.

2.1.2 APOS theory

The APOS theory proposes that in order for an individual to make sense of a given mathematical concept he/she must have appropriate mental structures, (Maharaj, 2010). Dubinsky (1984a) proposed the APOS theory to describe how **actions** become interiorized into **processes** and then encapsulated as mental **objects**, which take place in more sophisticated cognitive **schemas** (Tall, 1999). This study sought to use GeoGebra to teach a topic (circle geometry) perceived as difficult by students, following the four phases of APOS theory: action, process, objects, and schema.

An **action** in APOS theory context is a repeatable physical or mental manipulation that transforms objects (Dubinsky, 1984a). In the teaching and learning of circle geometry, **identifying** equal angles, parallel lines, various line parts and regions of a circle are actions. By using GeoGebra, with its interactive property, the **actions** can be repeated several times until the student interiorizes the properties of angles, line parts or regions of the circle. GeoGebra assists the transformation of a physical action (dragging) into mental action when a student is able to make instant identification of any properties, with or without the diagram.

A **process** in APOS theory context is the cognitive action that takes place entirely in the mind (Dubinsky, 1984b). As an individual repeats and reflects on an action, it may be interiorised into a mental process. The action of identifying various characteristics of angles and lines of a circle can be interiorised in such a way that learners are able to formulate their own mental process without visual diagrams. A process is a mental structure that performs the same operation as the action, but wholly in the mind of the individual (Maharaj, 2010).

Every process will result in an outcome or **object** (Dubinsky, 1984b). If a student becomes aware of the process involved in attempting to solve a problem, then the student has **encapsulated** the process into a cognitive **object** (Dubinsky, 1984b). In circle geometry, students, after having encapsulated or interiorized the various angle properties in circles, parallel lines, triangles and quadrilaterals, will be able to state the various **objects** (circle theorems). The difference between a process and an object is that a process becomes an **object** when it is perceived as an entity upon which actions and processes can be made (Dubinsky, 1984b).

A **schema** in APOS theory context is a more or less coherent collection of cognitive objects and internal processes for manipulating these objects (Dubinsky, 1984b). A schema could aid students to 'understand, deal with, organise, or make sense out of a perceived problem situation' (Dubinsky, 1991b, p. 102). In the circle geometry example, the objects (circle theorems) can be grouped and applied in formal proof of another theorem. For example, to prove the theorem that states: The angle between a tangent and a chord is equal to the angle subtended by the chord on the circumference; a collection of four objects can be used.

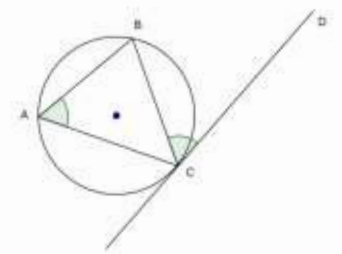

Figure 2.2 Tangent and chord theorem

The formal proof of the theorem illustrated in Figure 2.2, which according to APOS theory is a schema, may require the use of four objects, which are also geometric theories themselves:

1. Angles opposite equal sides are equal.
2. The sum of angles in a triangle is 180^0.

3. The angle subtended at the centre is double that at the circumference.
4. The angle between the tangent and radius/diameter is 90^0.

The above four theorems combined together form a coherent collection of cognitive objects and internal processes that aid the student to understand and deal with the formal proof of the theorem (schema). Table 2.1 illustrates the four phases of APOS theory when applied to circle theorems (geometry).

Table 2.1 APOS theory phases and their descriptors

APOS theory phase	**Descriptors of the phase in circle geometry/theorems**	**Mental/physical process**
1. Actions	Identification of shapes, angles, parallel lines	Interiorisation
2. Processes	Properties of angles, lines	Coordination
3. Objects	Theorems	Encapsulation
4. Schema	Formal proofs of theorems	Generalisation and reversal

According to APOS analysis, the initial step in learning is an **action** of *interiorisation*, followed by a **process** of *coordination* that leads to *encapsulation* of **objects**, whose *generalisation* and *reversal* result in the attainment of **schemas.**

According to Brijlall and Maharaj (2013), interiorisation is the ability of the student to perform various tasks such as applying symbols and short cuts, use of correct language, identify pictures and images to construct internal processes as a way of making sense out of perceived phenomena. Actions on objects are interiorised into a system of operations. The process of interiorisation may in itself involve two or more processes coordinated to form a new higher order process, referred to as **encapsulation**. When encapsulating students demonstrate the ability to apply/conceive previous processes as one object. Encapulation involves generalisation and reversal (Dubinsky, 1991a). Generalisation is the ability to apply schema to a wider range of contexts, and reversal is the ability to reverse the thought process of previously interiorized processes (Dubinsky, 1991a).

Sfard (1991) argued that abstract mathematical notions (concepts) can be conceived in two fundamentally different ways: as processes (operationally) or objects (structurally). In APOS theory, **action** and **process** can be regarded as operational conceptions, while **object** and **schema** are structural (Maharaj, 2010). The development of mathematical concepts often proceeds by taking processes as operators and then turning them into objects.

The four phases, action, process, object, and schema, have been presented by Dubinsky in a hierarchical, ordered list. Each concept in the list must be constructed before the next step is possible. In reality, however, when an individual is developing his or her understanding of a concept, the constructions are not actually made in such a linear manner. With an action conception of circle geometry problems, a student may be limited to think about relevant circle theorem only, without making reference to the process, object and schema relevant to the problem.

In this study, the APOS theory was used directly in the teaching and learning process as illustrated in Table 1 and in the analysis of data by the researcher. The researcher was able to compare the success or failure of students on a mathematical task, (circle theorem-related tasks) with the specific mental constructions they may, or may not, have made. The theory makes testable predictions that if a particular collection of actions, processes, objects and schemas is constructed in a certain manner by a student, then the student will, in all likelihood, be successful in using certain mathematical concepts in certain problem situations (Maharaj, 2010).

Many studies conducted between 1990 and 2000 used the APOS theory in order to analyse how students learn mathematical concepts. For example, studies by Arnon (1992) and those by (1997) and colleagues. Arnon (1992, 1995, 1996, 1998, 1998b) conducted various studies using APOS theory as the theoretical framework, some of which include the following: teaching fractions in elementary school using the software ‘Fractions as equivalence classes’; teaching decimal numbers using concrete objects; refining the use of concrete objects for teaching mathematics to children at the age of concrete operations; and, describing how children develop mathematical concepts.

Following a similar research approach, Asiala led different research teams in using APOS theory to investigate how students learn various concepts. For example, Asiala, Cottril, Dubinsky & Schwindedorf (1997) investigated student understanding of co-sets, normality and quotient groups, and Asiala, Dubinsky, Matthews, Morics and Oktac (1998) examined how students of abstract algebra came to understand permutations of finite sets and symmetries of regular polygons.

The proponent of APOS theory, Dubinsky (1984), has published various scholarly articles in journals and research papers in which the key focus was reflective abstraction. All his works are a testimony of how he (Dubinsky) strongly believed that an APOS-oriented instructional approach was the best method to teach mathematical concepts.

APOS theory has been used extensively in research studies concerning almost every mathematical concept, except geometry. In my literature search of current mathematics journals from 1990 to 2015, I have not found a research study that used the APOS theory to study Euclidian geometry.

The period from 2001 to date has seen very few studies based on the APOS theory and there has also been a shift from strictly APOS-inspired investigations to APOS-fused theoretical frameworks (combination of the APOS theory and one or more learning theories), for example, that of Tall (2004).

This study jointly used Van Hiele and APOS theories to investigate the effect of integration of GeoGebra software on the achievement of Grade 11 students, Van Hiele levels and motivation. Although the theories were propounded at distinctively different times (Van Hiele theory, published in 1957 and APOS theory, published in 1984), they have since made similar contributions to the field of educational instruction and hence form the bases of this study.

Figure 2.3 illustrates the fusion of Van Hiele and APOS theories for this study.

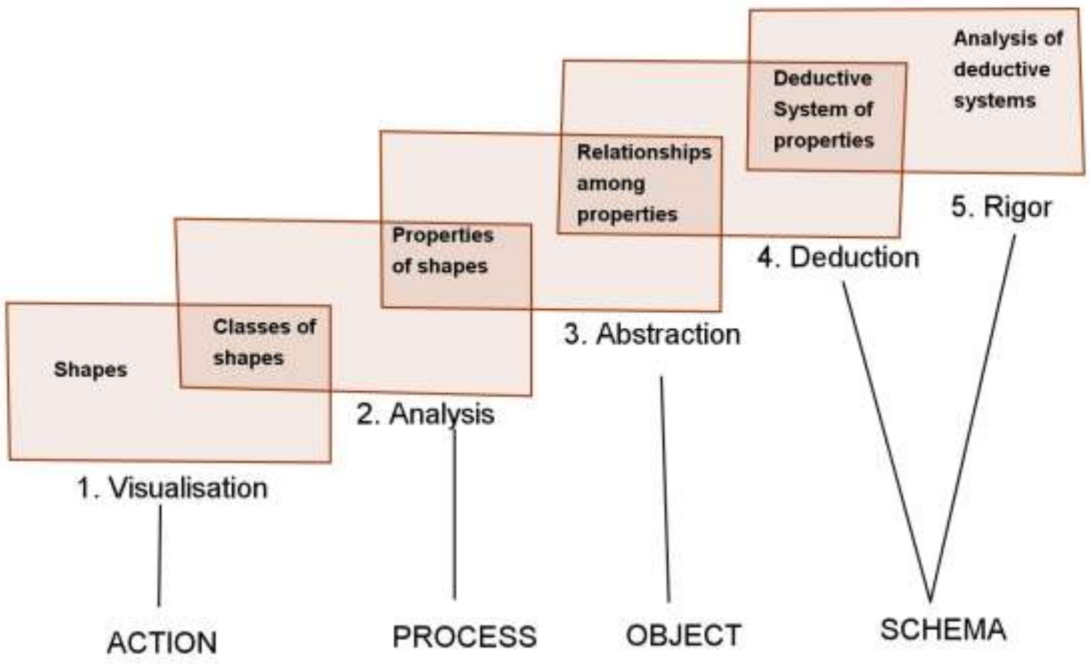

Figure 2.3: Theoretical framework: Fusion of Van Hiele and APOS theories

As illustrated in Figure 2.3, the five Van Hiele levels and the four APOS levels overlap, for example, although actions are directly related to visualisation, they are also related to the analysis level, and this overlapping applies to all the other levels. In some cases, at each Van Hiele level it is possible to achieve all four APOS theory levels, but for the purposes of this study, the relationship between Van Hiele theory and APOS theory is as shown in Figure 2.3.

Both APOS and Van Hiele theories are rooted in the learning theory of constructivism, in which learning is viewed as an active, contextualised process of constructing knowledge rather than the acquisition of knowledge (Devries & Zan, 2003). In this study, the major research question focuses on whether GeoGebra can enhance student achievement, while also assessing the impact of the Van Hiele levels attained by learners on their achievement and whether they were motivated by the teaching and learning method used in their class.

2.2 Literature review

The literature review sought to explore Grade 11 circle geometry content and common errors and/or misconceptions that result in circle geometry being a difficult topic for mathematics students. A few examples of how GeoGebra can be used to correct errors and misconceptions are also presented. The literature review also includes how other research studies employed the Van Hiele and APOS theories and the impact of integrating ICT in teaching and learning in learners' mathematical achievement and motivation.

2.2.1 Circle geometry

Circle geometry is a branch of mathematics that deals with the properties of angles and lines within, on, and outside circles. Circle geometry is a sub-section of Euclidean geometry that incorporates the use of theorems, theorem converses, corollaries and axioms. The South African CAPS (2012) curriculum for Grades 10–12 has circle geometry sections to be covered by students at each grade level. Circle geometry is assessed in paper 2, but South African students continue to perform poorly in this topic. Hence this study seeks to investigate the effect of teaching circle geometry to Grade 11 students by using GeoGebra as a way to improve their poor performance. There could be several reasons responsible for the poor performance in circle geometry, for example, poor teaching and learning methods employed by teachers and lack of student confidence and motivation. Another reason that could contribute to poor performance in circle geometry questions is that circle geometry problems are not intuitively obvious to students. The process of proving theorems often requires students to use results from different sections of geometry and algebra (Stols, 2012).

DoBE (2010, 2011, 2012 & 2013) examination reports on mathematics paper 3 indicate that many students had difficulties in answering circle geometry questions based on Grade 11 content. For example, students may fail to identify that the four diagrams in Figure 2.5 actually depict the same concept.

The focus of this study is circle geometry at Grade 11, hence the presentation of the content of Grade 11 circle geometry and the common errors and misconceptions made by students is important, in order to provide an overview of why students underperform in circle geometry.

At Grade 11 students are expected to achieve three main objectives in circle geometry (DoBE, 2012), namely:

1. Investigate and prove theorems of the geometry of circles assuming results from earlier grades, together with results concerning tangents and radii of circles.
2. Solve circle geometry problems, providing reasons for statements when required.
3. Prove riders.

Presented below are some of the circle geometry theorems and some diagrammatical representations that Grade 11 students are expected to master, as stipulated in the Department of Basic Education Examination Guidelines (DoBE, 2014). The theorems are numbered for the purpose of identifying them only.

1. The line drawn from the centre of a circle perpendicular to a chord bisects the chord.

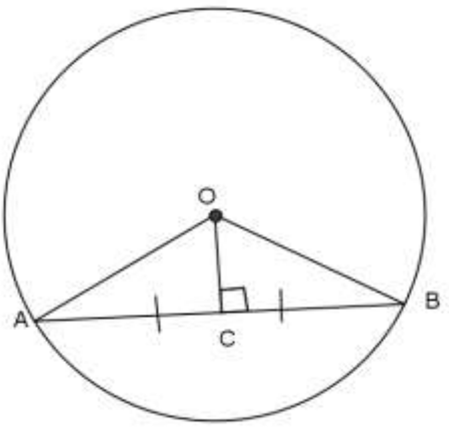

Figure 2.4: Theorem 1

Students are expected to prove that the distance AC = CB.

2. The perpendicular bisector of a chord passes through the centre of the circle.

This theorem is the converse of the theorem stated in 1 and uses the same diagram, but students are expected to prove that angle OCA or OCB is 90^o.

3. The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference (on the same side of the chord as the centre).

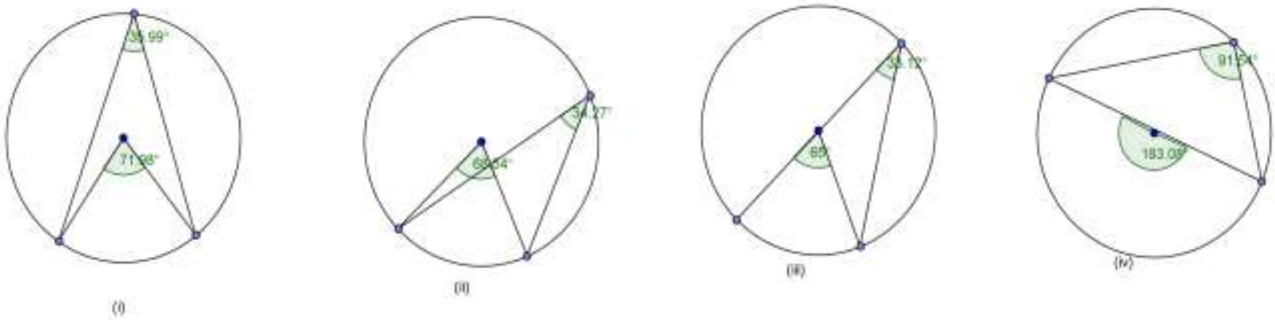

Figure 2.5: Theorem 2

Students are expected to use any of the above four diagrams to prove this theorem. Additional constructions are necessary in order to prove this theorem.

4. Angles subtended by a chord of a circle on the same side of the chord are equal.

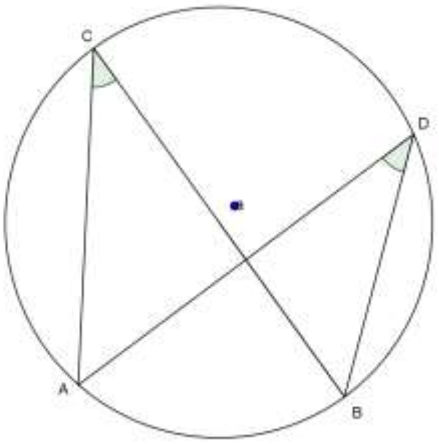

Figure 2.6: Theorem 3

Students are expected to prove that the angles at C and D are equal, with the help of additional constructions.

5. The opposite angles of a cyclic quadrilateral are supplementary.

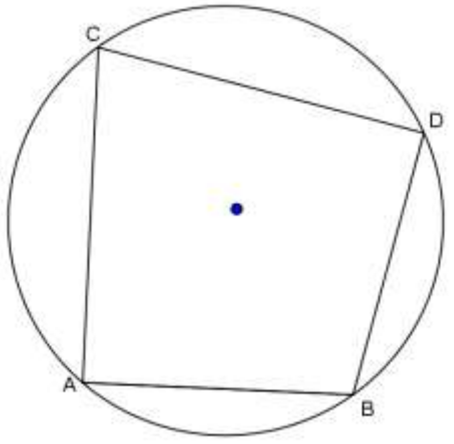

Figure 2.7: Theorem 4

To prove this theorem, students have to pick a pair of opposite angles and then prove that the sum of those angles is 180^o; additional constructions are also necessary in order to prove this theorem.

6. Two tangents drawn to a circle from the same point outside the circle are equal in length.

The examination guidelines stipulate that students must be able to apply the theorem but not to prove this theorem.

7. The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.FF

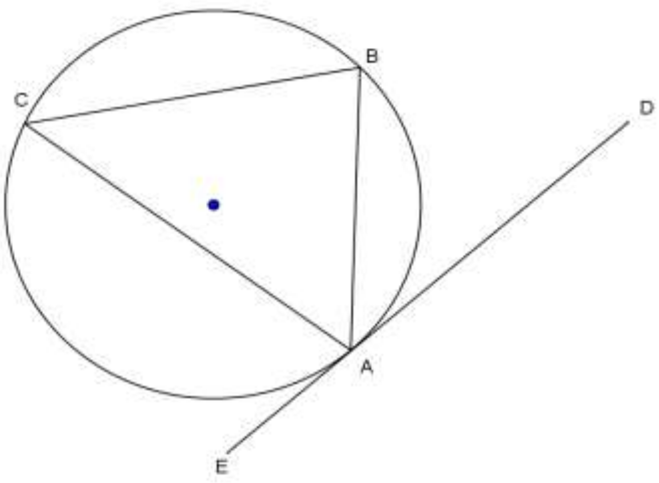

Figure 2.8: Theorem 5

This theorem is commonly known as the ‘Tangent and chord theorem’. Students are expected to prove that the angle DAB is equal to angle ACB.

8. Exterior angle of a cyclic quadrilateral equals the opposite interior angle.

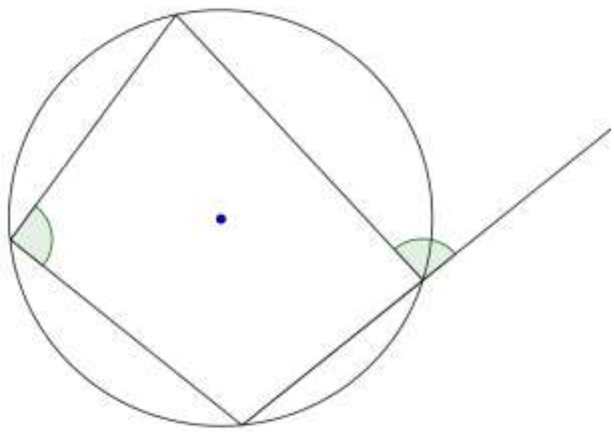

Figure 2.9: Theorem 6

Students are expected to prove that the marked angles are equal.

2.2.2 ICT in mathematics Teaching and learning

The literature review in this section is focused on the role and/or impact of mathematical software, especially GeoGebra in mathematics teaching and learning. I reviewed several research publications that have integrated computer technology, including GeoGebra, in the teaching and learning of mathematics at various levels, ranging from primary education to university education. The aims for this were, among others, to determine whether technology integration had a positive or negative impact on the teaching and learning process, and to identify knowledge gaps in the integration process that need further exploration.

Providing a rich learning environment for students should be every teacher’s priority for successful teaching. Prodromou (2015) discussed the special opportunities for teaching introductory statistics that technology (GeoGebra) offers teachers who aim to provide rich learning experiences to college students. In this study, GeoGebra was integrated into the teaching and learning of introductory statistics, and results indicate that college students were able to perform key statistical investigative tasks, for example, (i) managing data (ii) understanding specific statistical concepts (iii) performing data analysis and inference, and (iv) exploring probability models. Prodromou (2015) strongly recommended the integration of GeoGebra with exploring statistics at college level.

Improvement of student performance in mathematics is largely determined by how effective their teachers are in delivering content to them. Martinovic and Manizade (2013) believe that technology should be used as a partner in the geometry classroom. To support their claim, these authors presented technology-based geometric activities to pre-service teachers, with the aim of promoting pre-service teachers' mathematical reasoning. They found positive results in which these teachers were able to explore carefully structured activities and engage technological skills that enabled them to develop and evaluate geometric conjectures. The study recognized technology as an important part of developing pre-service teachers' professional integrity.

Many studies have also focused on the use or integration of dynamic geometry systems (DGS) in the teaching of specific mathematical topics. Strausova and Hasek (2013) investigated dynamic visual proofs using DGS. Their view is that pictures and diagrams play an important role in the process of understanding various mathematical features, and that an appropriate picture or diagram can be used as visual proof of a geometric property or theorem. They argue that non-verbal proofs (proofs without words) are more attractive and acceptable to students than classical proofs. They presented examples of dynamic visual proofs created by dynamic geometry software to secondary school mathematics teachers and students. However, they do acknowledge the weakness of dynamic visual proofs using DGS because in general they do not capture the chain of thought leading to the proof, but focus only on the result.

Karaibryamov, Tsareva and Zlanatov (2012) carried out a study on optimization of courses in geometry by using DGS known as 'Sam' (mathematical software). In this study, a new approach to the teaching of synthetic geometry in schools and universities with the help of DGS was used. Their aim was to optimize the teaching process. They reported that their new approach increased the benefits of DGS in the teaching and learning of geometry, especially optimising the education process by saving time involved in drawing, generalizing large groups of problems, and stimulating and helping investigations.

Foster and Shah (2015) explored the process of game-based learning in the classroom through the use of the play, curricular activity, reflection, discussion (PCaRD) model. They carried out a mixed-methods study at high school with control and experimental groups, where they implemented three games with the PCaRD model for a period of one year. Pre- and post-assessments were administered in order to measure achievement gains and motivational changes. Their results indicate that PCaRD aided student learning, and motivated them to learn. They also claimed that PCaRD provided teachers with an adaptive structure of integrating games with their teaching process.

Among many scholars who have investigated the effectiveness of mathematical software is Ertekin (2014). His study sought to investigate the effects of teaching analytical geometry using the software Cabri 3D on teacher trainees' ability to write the equation of a given special plane, identify the normal vector of a plane and draw the graph of the plane. The software was used with the intention to improve the trainee teachers' geometric and algebraic competences. The results of this study indicated that students instructed with the software were significantly more successful than those who did not use it, in terms of identifying the equations of special planes and their normal vectors and drawing their graphs.

Swallows (2015), in a case study report titled: 'The year-two decline: Exploring the incremental experiences of a 1:1 technology initiative', reported that 1:1 (one-to-one) technology initiatives result in favourable results in the first year, but in subsequent years results decline. This case study's finding suggests that enthusiasm for the use of technology declines over time, resulting in diminishing favourable outcomes with its continued use.

Design-based research carried out by Donevska-Todorova (2015) on the conceptual understanding of the dot product of vectors in a dynamic geometry environment (DGE), revealed that DGE offers students multiple representations rather than single static representations. This researcher argued that multiple and appropriate combinations of representations are important for students to acquire deeper knowledge about a specific mathematical concept, for example the dot product of vectors.

Perjesi-Hamori (2015) based on her experiences in teaching numerical methods to university students and, using the computer algebraic system (CAS) called Maple, argued that the use of CAS Maple enabled students with limited mathematical skills to understand more complex tasks, such as solutions of multivariate interpolations and regressions, or those of partial differential equations. This study is another success story of technology integration with the teaching of mathematics. A similar study by McAndrew (2015) used CAS calculators to teach and explore numerical methods to third-year pre-service teachers. It was shown from the study that CAS calculators, although very low-powered compared to standard computer-based numerical systems, are also quite capable of handling textbook problems, and as such provide a very accessible learning environment (McAndrew, 2015).

Vajda (2015) used computer algebra software to introduce the classical Chebyshev polynomials as extremal polynomials. The use of computer algebra in this study is reported to have made the exploration of extremal polynomials easy and enjoyable for students. Yet another study that used computer technology to introduce a topic was that conducted by Soon (2015) introducing queuing theory through simulations. In the study, the researchers discussed the role that simulations can play in a classroom to create real-world learning experiences for students. Mathematical principles governing queues are very challenging to students, especially at the introductory stage. Real data was collected from queues at automated teller machines (ATMs) and at cinema ticketing counters, and was used to model activities involving queues. The study found that students were able to understand basic probability theory and statistical concepts, such as the Poisson process and exponential distribution, without the need to know all about classical queuing theory.

The majority of literature available on the integration of computer technology in general and GeoGebra software in particular, indicates that the appropriate use of technology in mathematics teaching and learning supports visualization by learners and assists them to construct geometric and algebraic concepts. However, a research report titled ‘Learning Technology Effectiveness’ developed under the guidance of Culatta and Adams (2014), of the US Department of Education, Office of

Educational Technology, summed up the effectiveness of educational technologies as:

> Technology is not a silver bullet and cannot – by itself – produce the benefits we seek in learning, but without technology, schools have little chance of rising to 21st century expectations. Synthesis of best available evidence consistently indicates the potential for positive effects when technology is a key ingredient in well-designed learning systems (Cullata & Adams, 2014, p. 15).

Hattie (2009), an internationally acclaimed researcher on 'Visible Learning', has conducted extensive research in which he demonstrated, through quasi-experiments, the variables that have the greatest influence on student achievement. His studies have revealed that computer assisted instruction (CAI) is not necessarily among the highest ranging variables in influencing achievement among students. According to Daggett (2011), Hattie carried out a synthesis of over 800 meta-analyses relating to achievement (Dagget, p. 2), in which he analysed and compared about 200 000 'effect size' (the relative impact of one factor compared to other factors). In Hattie's comprehensive study of 'effect size', CAI is not even among the top thirty 'effect size' variables; in fact, CAI is in position thirty-five in Hattie's 'effect size' ranking. Hattie's work demonstrates that CAI is not a panacea to student underachievement, but contributes a significant 0.37 relative impact (effect size), in 'effect size' rankings that ranges from a minimum of 0 to a maximum of 1.44.

Although CAI according to Hattie is not a major panacea to students' underachievement in mathematics, researchers are currently advocating for increased use of computers in the teaching and learning process.

In Slovakia, Guncaga, Majherova and Jancek (2012) found that GeoGebra can be a motivational tool for teaching and learning. Stolls (2014) used the Van Hiele theory to investigate the geometric cognitive development of students in a technology-enriched environment (dynamic geometry software) compared with students in a learning environment without any technology enhancement. The results suggest that the technology-enriched environment helped to improve the conceptual geometric growth of students on Van Hiele Levels 1, 2 and 4. The study suggests that technology can help to create active learning environments in which students can discover, explore, conjecture and visualise.

In Malaysia, Noorbaizura and Leong (2013) studied the effect of using GeoGebra to teach students' learning of fractions. The study showed that the use of GeoGebra to teach fractions is very effective. This was shown through the improved scores of the students in the experimental group. The findings highlighted that students in the experimental group performed better than those in the control group that were taught using the traditional learning method. The software also enhanced visualization and understanding of the fractions concept of both the teacher and students.

Venkataraman (2012) in Singapore carried out a study on innovative activities to develop the geometric reasoning skill in secondary mathematics with the help of open resource software, namely GeoGebra, and found that students taught with the software made progress towards mathematical explanations, which provides a foundation for further deductive reasoning in mathematics. He concluded that the dynamic nature (drag feature) of the software influences the form of explanation and that students are able to generalize the solution and respond with a proper statement. Venkataraman (2012) also concluded that GeoGebra makes learning abstract concepts far more meaningful and helps students to visualize related concepts.

The use of computer technology in the teaching and learning process is believed by some researchers to enhance problem-solving in mathematics, while others are of the view that although users of technology can solve mathematical problems easily, technology in mathematics transforms the learning process from being largely mental to being largely mechanical. The traditional school of thought believes that mathematical problem-solving should minimise the use of technology in the teaching and learning process, while modern-day scholars believe mathematical problem-solving is the ability of students to perform mathematical tasks successfully, with or without computer technologies. Hiebert and Weane (1993) held the view that using computer technology in mathematics teaching and learning provides intellectual challenges that enhance students' mathematical understanding and development. According to Dendane (2009), the goal of mathematical problem-solving is to develop a generic ability to solve real-life problems and apply mathematics in authentic, real-life situations. Problem-solving can also be used as a teaching method for a deeper understanding of concepts. It is an important component of teaching mathematics. It is also a way to present mathematics and a skill which enhances reasoning (Maricic, 2010).

The use of computer technology can easily afford students the opportunity to look back and re-examine the whole problem over and over again. Schoenfeld (1992), in his study on learning to think mathematically, highlighted that one of the difficulties in problem-solving is the fact that several steps are needed to solve any given problem. In addition, at each step, students need to use several skills (Schoenfeld, 1992). Once again the use of computer technologies enables students to perform multiple steps.

With the use of computer technology in the teaching and learning of mathematics, the above factors highlighted by Dendane (2009), can easily be achieved by students. The above points show that there are many skills and factors involved when genuine mathematical problems are being solved. Teachers need to understand and be familiar with these factors and skills. They should also design activities and guide students to develop and use these skills (Burton, 1999). Stanic and Kilpatrick (1989) in their study titled 'Historical perspectives in problem-solving in the mathematics curriculum' argued that students develop these skills only if genuine mathematical problem-solving is taking place.

According to Stacey (2005), problem-solving skills involve a range of processes including analysing, interpreting, reasoning, evaluating and reflecting. Students need deep mathematical knowledge and general reasoning ability as well as heuristic strategies for solving non-routine problems. It is also necessary to have helpful beliefs and personal attributes for organizing and directing one's own work, coupled with good communication skills and the ability to work in cooperative groups.

Problem-solving skills in circle geometry can be aided by the use of CAI, for example GeoGebra. CAI has different modes of teaching and learning, such as drill and practice, tutorial and simulations. GeoGebra, being a dynamic geometry software, presents fascinating classroom experiences which motivate students to actively participate in the learning process. Researchers concur that the key to successful learning is intrinsic motivation. Any teaching and learning method that breeds intrinsic motivation within learners, such as GeoGebra, can be advocated for. Kutluca (2013) argued that the use of CAI increases the students' Van Hiele geometric thinking levels better than traditional methods of instruction.

Serin (2011) carried out a quasi-experimental research study to establish 'The effects of computer-based instruction on the achievement and problem-solving skills of science and technology', and discovered that students who were instructed using CAI had a greater statistically significant increase in their achievements and problem-solving skills than those instructed without computers. Serin (2011) further argues as follows:

> The presentation of teaching materials by means of computer technology helps students to process and develop information, to find alternative solutions, to take an active part in the learning process and to develop their problem solving skills...The computer-based instruction makes teaching techniques far more effective than those of the traditional teaching methods as it is used for presenting information, testing and evaluation and providing feedback. It makes a contribution to the individualization of education. It motivates students and gets them to take an active part in the learning process. It helps develop creativity and problem-solving skills, identity and self-reliance in students (Serin, 2011, p. 184).

Studies conducted by Lappan and Phillip (1998) on problem-solving in mathematics using computer technology proposed the criteria for the selection of topics and problems to be taught using computer technology, and also the guidelines for how to make problem-solving a central aspect of instruction. Kirkley (2003), informed by the study conducted by Lappan and Phillip (1998), outlined the principles to be applied by teachers when teaching in classrooms with computer-based settings (technology set-up).

2.2.3 ICT and students' achievement

That study also reviewed literature on the effect of ICT on students' achievement in general, and achievement in mathematics in particular. This section summarises some of the findings on the impact of educational technology on student achievement and learning and goes on to identify what needs to be studied further. Most studies reviewed in this study on ICT and student academic achievement were done during the period stretching from the 1990s to 2014.

Schlacter (1999) reported on the impact of computer technology on student achievement. He outlined five large-scale studies on computer technology and summarized its positive and negative impact on student achievement. Kulik (1994), as cited in Schlacter (1999), aggregated the findings from more than 500 research studies on computer-based instruction. Kulik's positive findings were that, on average, students instructed using computers scored at the 64th percentile compared to students instructed without computers who scored at the 50th percentile. He also noted that students instructed with computers learnt more in less time and enjoyed their classes more than those instructed without computers. However, Kulik noted that computers did not have positive effects in every area in which they were used.

Sivin-Kachala (1998) reviewed 219 research studies to assess the effect of computer technology on learning and achievement across all learning domains and all ages of learners and found that students in computer technology-rich classrooms experienced positive effects on achievement in all their major subject areas, and their attitudes toward learning and self-concept improved consistently when computers were used for instruction. However, Sivin-Kachala (1998) also noted that the level of effectiveness of computer technology is influenced by the specific student population, software design, the educator's role and level of student access to computer technology.

Baker, Gearhart and Herman (1994) evaluated the impact of interactive technologies of the classrooms of tomorrow (ACOT) across five school sites in the US. Their findings revealed that ACOT experience appeared to result in new learning experiences requiring higher-level reasoning and problem-solving and students developed positive attitudes towards school work. However, students who used computers did not perform any better in vocabulary, reading and comprehension, and mathematical concepts when compared to those who did not have computers.

In a case study conducted in West Virginia on a basic skills/computer education (BS/CE) state-wide initiative, Mann, Shakeshaft, Berker and Kottkamp (1999) reported that students who participated in the programme improved their test scores and their attitude towards technology (both teachers and students) changed from negative to positive. The BS/CE programme was more effective in improving student achievement than class size reduction, increase in instructional time and cross-age tutoring programmes.

Wenglinsky (1998) assessed the effects of simulation and higher-order thinking technologies on a national sample of 6 227 fourth and 7 146 eighth graders with reference to their mathematics achievement in the National Assessment of Educational Progress. He found that eighth-grade students who used simulation and higher-order thinking software showed gains in mathematics scores of up to 15 weeks above grade level, and eighth-grade students whose teachers received professional development on computers, had shown similar gains of up to 13 weeks above grade level. He also noted that higher-order use of computers and the professional development of teachers were positively related to students' academic achievement in mathematics for both fourth-grade and eighth-grade students.

Most companies that manufacture technological gadgets (computers, calculators, and educational software) for educational purposes publish their research findings on the impact of their technologies on student achievement. One such company is Apple.

Apple believes that effective integration of its technology into classroom instruction can result in higher levels of student achievement. Apple (2002) reported four benefits that accrue to students that use their various technologies:

1. Technology integrated into the classroom helps students to master fundamental skills such as reading, writing, and mathematical skills.
2. Proficiency with technology affects the students' ability to write better, express themselves more clearly, and understand presented material faster and with greater recall.
3. Technology prepares students for the demands of the 21st century.
4. Technology can reduce absenteeism, lowers dropout rates, and motivates more students to further their education.

Apple, in all its research studies on the effect of their technologies on students, has established that their technologies result in improved students' academic achievement.

The International Society for Technology in Education (ISTE), an association for educators and education leaders engaged in improving teaching by advancing the effective use of technology, based in the US, has published its policy briefs on the role of technology since 2008. Their first publication in 2008 was on technology and student achievement, titled 'The indelible link'. It reported the positive impact of technology on students' academic achievement. ISTE (2008) documented the fact that the effective integration of technology into teaching and learning was having a positive impact on increasing student achievement through test scores and the acquisition of 21st century skills (skills that are required for students to succeed beyond high school).

Carrillo, Onofa and Ponze (2010) studied the effects of ICT in the school environment on educational achievement, through the use of evidence gathered from a randomized experiment in Ecuador. They evaluated a municipality-sponsored computer-aided instruction project in primary schools, and found that the program had a positive impact on mathematics test scores (about 0.3 of a standard deviation) and a negative but statistically insignificant effect on language test scores.

In Nigeria, Anyamene, Nwokolo and Ifeanyi (2012) investigated the effect of computer-assisted packages on the performance of senior secondary students in mathematics in Awka, Anambra State, and found that students taught using computer-assisted packages performed significantly better than their counterparts taught using the conventional method of instruction.

Although researchers concur that the integration of technology with the learning environment impacts positively on students' achievement, this study views the impact as a difficult aspect to measure quantitatively, because appreciating the usefulness of technology depends on how it has been accepted by the users. All the studies reviewed on the impact of technology on student achievement suggest that the impact is more positive when linked to pedagogies. The impact is dependent upon the type of pedagogies used, for example, whether the transmission mode is teacher-centred or student-centred. Balanskat, Blamaire and Kafal (2006) argued that the factors that impede the successful implementation of ICT in teaching and learning include teachers' poor ICT competence, low motivation and lack of confidence in using new technologies in teaching. Competence and motivation are the key determinants of the levels of engagement in ICT (Balanskat et al., 2006).

2.2.4 ICT and student motivation

The online Business Dictionary (http://www.businessdictionary.com) defines motivation as

> internal and external factors that stimulate desire and energy in people to be continually interested and committed to a job, role or subject, or to make an effort to attain a goal. Motivation results from the interaction of both conscious and unconscious factors such as the intensity of the desire or need, incentive or reward value of the goal, and expectations of the individual and of his or her peers.
>
> (http://www.businessdictionary.com/definition/motivation.html)

There are many studies that testify to the positive impact of ICT on student motivation (BECTA, 2013). Studies have shown that ICT can help to motivate students in many ways, for example, ICT can result in increased commitment to learn, enhance students' sense of achievement, support self-directed study, greater self-esteem, and improved behaviour.

Becker (2000) reported on increased commitment by students to learn if teachers provide their students with technology-enhanced lessons. Becker (2000) argued that students are motivated to continue using computers at other times of the school day and outside school. Harris and Kington (2002) also observed that students who used internet based resources were keen to work in their own time, before and after school, as well as during school hours. Students also developed independence and an autonomous style of learning, a valuable behaviour for life (Harris & Kington, 2002).

Moseley and Higgins (1999) proposed that using ICT could lead to an enhanced sense of achievement among students who previously were underachieving. They found improved achievements and increases in motivation in subjects such as mathematics, geography and English.

The British Educational Communications and Technology Agency (BECTA) (2013) compiled a report on researchers' views on ICT and motivation. This report contains the bulk of what researchers currently believe are the effects of technology on student motivation. According to BECTA:

> The research evidence shows that ICT can stimulate, motivate and spark students' appetites for learning and helps to create a culture of success. This can be demonstrated in their increased commitment to the learning task, increased independence and motivation for self-directed study, their enhanced enjoyment, interest and sense of achievement in learning when using ICT, and their enhanced self-esteem (BECTA, 2013, p. 27).

Mohanty, (2011), in his publication on ICT advantages and disadvantages, had this to say about the motivational effects of ICT in the learning process:

> ICTs such as videos, television and multimedia computer software that combine text, sound, and colourful, moving images can be used to provide challenging and authentic content that will engage the students in the learning process. Interactive radio, likewise, makes use of sound effects, songs, dramatizations, comic skills, and other performance conventions to compel the students to listen and become involved in the lessons being delivered. More so, networked computers with internet connectivity can increase learner motivation as it combines the media richness and interactivity of other ICTs with the opportunity to connect with real people and to participate in real-world events (Mohanty, 2011, p.2).

Baker, Cooley and Trigueros (2011), in their comparative study on exploring secondary school students' motivation using technologies (V-Transformation courseware and GeoGebra) in teaching and learning of transformations, proposed that there was a significant difference between the motivation of students using V-transformation courseware ($M = 3.78$; $SD = 0.403$) compared to that of GeoGebra ($M = 3.50$; $SD = 0.458$; $t(69) = 2.704$; $p = 0.009$). Their findings showed that students who used V-transformation courseware were more motivated when using it in learning transformations than those who used GeoGebra. This study, however, confirms that both technologies can be used to motivate students in the teaching and learning of mathematics.

Furner and Marinas (2014) argued that GeoGebra can motivate students to learn mathematics and that it minimizes anxiety. Real-life photographs that are inserted into GeoGebra provide the basis to observe relationships with different and similar shapes.

This study sought to investigate the effect of ICT (GeoGebra) on students' motivation by analysing data collected through a questionnaire based on indicators (attributes) of motivated students. There are many attributes or manifestations of motivated students, but this study selected only a few (six) in order to answer the additional question: Does GeoGebra aid students' motivation to learn circle geometry? Based on literature reviewed, the six motivational attributes included in the questionnaire are: (i) participation during lesson delivery, (ii) concentration during class activities, (iii) enjoyment of class activities, (iv) Self-confidence, (v) content mastery, and (vi) recommendation/preference of the teaching and learning method.

2.3 Conclusion

This study adopted Van Hiele and APOS theories as the joint theoretical framework. Van Hiele theory was used as the framework to detect students' levels of geometric thinking, hence providing a platform for analysing the data collected from the study, while APOS theory gave this study the general framework for the whole research process. Grade 11 circle-geometry content is the focus for this study. Literature on studies that used Van Hiele and APOS theories, ICT in mathematics teaching and learning, ICT and students' achievement, ICT and students' motivation have been reviewed in this chapter.

CHAPTER 3
RESEARCH METHODOLOGY

In this chapter, we justify the research approach (philosophy) and outline how this study was carried out. This includes a presentation of the research design and its justification, research population and sample, data collection instruments and techniques, validity and reliability issues, and lastly, ethical considerations.

3.1 Research Philosophy

This study adopted the research philosophy of positivism. According to Gordon and Scott (1991), a research philosophy is a belief about the way in which data about a phenomenon should be gathered, analysed and used. Positivism subscribes to the view that only factual knowledge gained through observation or through the senses, including measurement, is genuine knowledge. Collins (2010) argued that positivism depends on quantifiable observations that lend themselves to statistical analysis. Collins noted that

> as a philosophy, positivism is in accordance with the empiricist view that knowledge stems from human experience. It has an atomistic, ontological view of the world as comprising discrete, observable elements and events that interact in an observable, determined and regular manner (Collins, 2010, p. 38).

In this study, positivism was the guiding research philosophy because of its scientific nature. Positivism relies on four aspects of science, namely, that science is deterministic, mechanistic, methodical, and empiricist. The main principles of positivism philosophy adapted from Collins, 2010) that informed this study are:

1. In scientific studies, there are no differences in logic of inquiry.
2. Positivistic studies are aimed at explaining and predicting phenomena.
3. Positivistic researches are empirically observable via human senses. Inductive reasoning is used to develop hypotheses that will be tested during the research process.
4. Positivistic studies do not allow common sense because this may result in biased conclusions.

There are several scientific or positivistic research methodologies, such as laboratory experiments, field experiments, surveys, case studies, theorem proof, forecasting, and simulation. In this study, we chose to carry out a field experiment (quasi-experiment), in order to achieve greater realism and to diminish the extent to which the outcomes could be criticised and contrived (Crowther & Lancaster, 2008). The research design was influenced by the positivistic philosophy.

3.2 Research design

The study adopted the quasi-experimental research design because ‘it provided the best approach to investigating cause and effect relationships’ (McMillan, 2000, p. 207). According to Dinardo (2008), a quasi-experiment is an empirical study used to estimate the causal impact of an intervention on its target population. This view is also supported by Fraenkel and Wallen (2010), who argued that quasi-experimental research is a way to establish cause-and-effect relationships. Gribbons and Herman (1997) concur that quasi-experimental research shares similarities with the traditional experimental design or randomized controlled trial, but quasi-experiment lacks the element of random assignment to treatment or control. This study was a quasi-experimental of non-equivalent comparison group design. The reason for this decision was that practically, it was not possible to assign the students randomly into groups because of the different timetables that the classes followed. The design is depicted in Table 3.1.

Table 3.1 Non-equivalent comparison group design

Group		**Treatment**	
Experimental group (GeoGebra Instructed group)	**Pre-test**	GeoGebra Instructed (Computer assisted)	**Post-test**
Control Group		Traditional instruction (talk and chalk)	

In this study, two groups of students at different schools were involved, namely, the experimental group, which was instructed using GeoGebra software in the school’s computer laboratory, and the control group which was instructed using the conventional teaching method (chalk-and-talk). The same tests were administered to both groups (a pre-test and a post-test). The pre-test was given to both groups in order to check the students' academic level on the topic before intervention and establish the comparability of both groups. The post-test was given in order to assess the students' performance after the intervention and thus ascertain the impact of the intervention.

In support of quasi-experimental designs, Shadish, Cook and Campbell (2002) argued that randomization is impractical and/or unethical in a formal school system. Quasi-experiments are easier to set up than true experiments which require random assignment of subjects. According to Trochim (2001), utilizing quasi-experimental designs minimizes threats to external validity as natural environments do not suffer the same problems of artificiality as compared to a well-controlled laboratory setting. Quasi-experiments, such as this study, are natural experiments whose findings can be applied to other subjects and settings, allowing for some generalizations to be made about population.

In this study, the researcher was, however, aware of the possible disadvantages of quasi-experimentation, such as the fact that study groups may provide weaker evidence because of a lack of randomness. Randomness brings much useful information to a study because it broadens results and therefore gives a better representation of the population as a whole. Cook (2002) highlighted that because randomization is absent in quasi-experiments, some knowledge about the data can be approximated, resulting in difficulties in concluding causal relationships. Regardless of this disadvantage, the results of this study could contribute immensely to the field of instructional pedagogy.

This study was purely a quasi-experimental research study that used the pre-test/post-test quasi-experimental design (pre-test/post-test control group design) to establish the effect of integration of GeoGebra with the teaching of circle geometry on Grade 11 students' problem-solving skills, achievement and motivation. The variables investigated are GeoGebra/teacher talk-and-chalk (independent variable) and Grade 11 students' achievement, Van Hiele levels and motivation (dependent variables).
Before the actual study was carried out, a small-scale pilot study was conducted in order to test the research instruments and feasibility of the study.

Population

The population for this study was all Grade 11 students in Limpopo Province of South Africa. Limpopo Province has been underperforming for several consecutive years when compared to other provinces in the country. It is my belief that the teaching and learning environments in South African schools are not identical, especially across the nine different provinces. Hence, the Limpopo Province was chosen for this study.

Sample and sampling techniques

The sample consisted of two Grade 11 mathematics mixed-ability classes at two different schools. The classes had 22 and 25 students respectively. The sample size for the experimental group (treatment group) was pre-determined by the number of laptops (computers) in the school's computer laboratory. Twenty-two laptops at the experimental school had GeoGebra software installed, and were used by the students (treatment group). The class of 25 at the other school were taught in their usual base room. Coincidentally, in statistical analysis a sample size of 22 or more is classified as a large sample, as a result, any relevant discrete statistical and inferential statistical analysis can be done (Castillo, 2010). Convenience sampling was used because it is inexpensive and participants are readily available (Castillo, 2010). In addition, Ferrance (2000) argued that research studies conducted by educators themselves, in a familiar school setting, with their own learners, would help solve real problems experienced in schools and thus contribute towards improving teaching and learner achievement.

3.3 Treatments

The control group

The control group was taught by their own teacher using the traditional talk-and-chalk teaching method. Four content development worksheets, similar in content to the experimental groups' worksheets, were used. All the questions and tasks were exactly the same for the two groups. The difference was the manner in which students carried out their tasks. In the control group the talk-and-chalk teaching method was used; students learn primarily by listening to the teacher and reading whatever the teacher writes on the chalkboard (auditory and visually). In the experimental group students learn in three different ways: visually, auditory and kinaesthetically. Each lesson was one-hour long, and teaching was done for seven days.

The Experimental group

The experimental group were taught using GeoGebra. Each of the students had a laptop with GeoGebra software installed on it. The teacher instructed and demonstrated with a laptop connected to an overhead projector. After two days of GeoGebra and computer introductory lessons and one day of topic introduction (2-hour lesson per day) content development worksheets were used during lesson delivery. A total of ten lessons were delivered to each group (control and experimental). Each lesson was one-hour long. The worksheets had 'open-ended' questions to allow students to explore different solution strategies and/or skills of answering circle geometry questions. The content development worksheets had the same content for both the control group and the experimental group, although the teaching and learning approaches were different. Each worksheet covered one or two circle theorems depending on the length of the procedures required to prove the theorem(s).

3.4 GeoGebra

GeoGebra, the software used to teach the experimental group, is dynamic mathematics software designed for teaching and learning mathematics (geomentry and algebra) at secondary school and college level, (Hohenwarter & Preiner, 2007).

GeoGebra can be used to perform various mathematical tasks such as to visualise mathematical concepts and create instructional materials. The software if correctly used can foster active student- centred problem-solving and allows for mathematical experiments, interactive explorations, as well as discovery learning (Brunner, 1961).

GeoGebra is based on Java, hence it is truly platform independent and runs on every operating system. GeoGebra is multilingual in its menu, but also in its commands, and has been translated by volunteers from all over the world into more than 35 languages (GeoGebra 3.0). Since GeoGebra joins dynamic geometry with computer algebra, its user interface contains additional components that cannot be found in pure dynamic geometry software. Apart from providing two windows containing the algebraic and graphical representation of objects, components that enable the user to input objects in both representations as well as a menu bar are part of the user interface (see Figure 3.1) below.

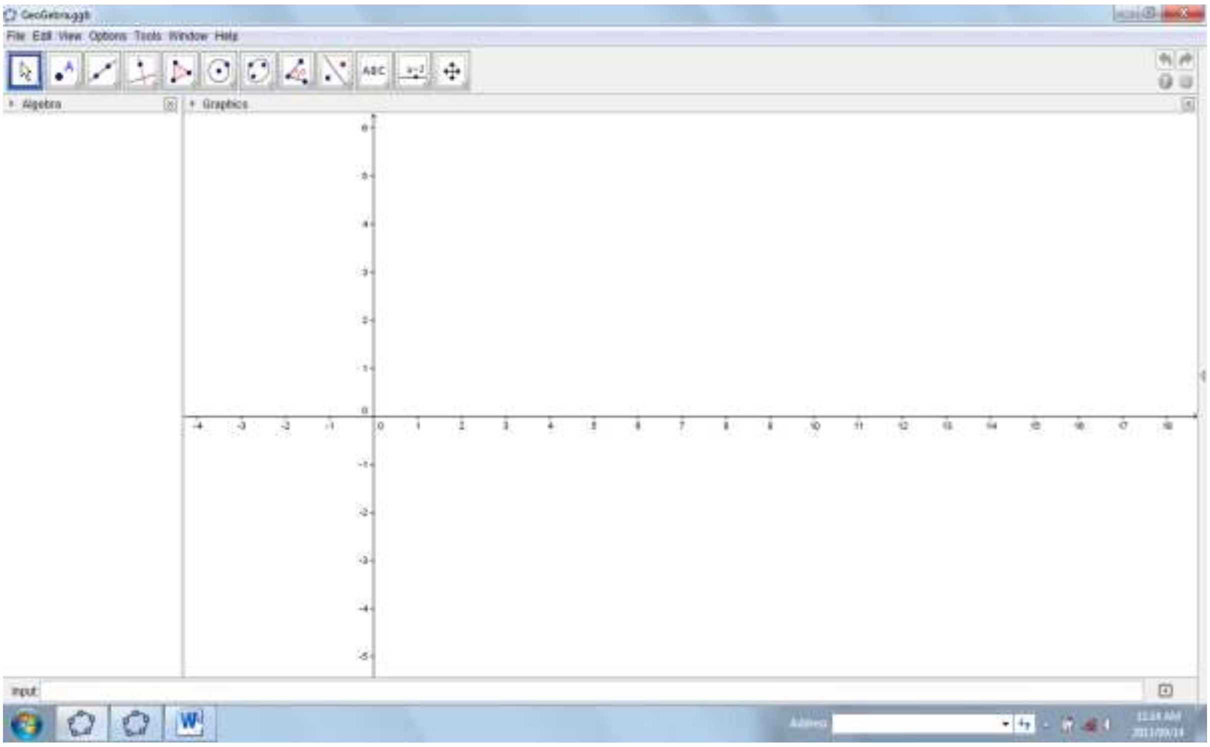

Figure 3.1 GeoGebra's user interface

Graphics window: The graphics window is placed on the right-hand side of the GeoGebra window. It contains a drawing pad on which the geometric representations of objects are displayed. The coordinate axes can be hidden and a coordinate grid can be displayed by the user. In the graphics window, existing objects can be modified directly by dragging them with the mouse, while new objects can be created using the dynamic geometry tools provided in the toolbar.

Toolbar: The toolbar consists of a set of toolboxes in which GeoGebra's dynamic geometry tools are organized. Tools can be activated and applied by using the mouse in a very intuitive way. Both the name of the activated tool as well as the toolbar help, which is placed right next to the toolbar, give useful information on how to operate the corresponding tool and, therefore, how to create new objects. In the right corner of the toolbar the Undo and Redo buttons can be found, which enable the user to undo mistakes step-by-step.

Algebra window: The algebra window is placed on the left-hand side of the GeoGebra window. It contains the numeric and algebraic representations of objects which are organized into two groups:

- **Free objects** can be modified directly by the user and do not depend on any other objects.
- **Dependent objects** are the result of construction processes and depend on 'parent objects'. Although they cannot be modified directly, changing their parent objects influences the dependent objects.

Input field: The input field is placed at the bottom of the GeoGebra window. It permits the input of algebraic expressions by directly using the keyboard. By this means a wide range of pre-defined commands are available which can be applied to already-existing objects in order to create new ones.

Menu bar: The menu bar is placed above the toolbar. It provides a wide range of menu items allowing the user to save, print and export constructions, as well as to change default settings of the program, create custom tools and customize the toolbar.

Construction protocol and navigation bar: Using the view menu, a dynamic construction protocol can be displayed in an additional window. It allows the user to redo a construction step-by-step by using the buttons of a navigation bar. This feature is very useful in terms of finding out how a construction was done or finding and fixing errors within a construction. The order of construction steps can be changed as long as this does not violate the relations between dependant objects. Furthermore, additional objects can be inserted at any position in order to change, extend, or enhance an already existing construction. Additionally, the navigation bar for construction steps can be displayed at the bottom of the graphics window, allowing repetition of a construction without giving away the construction steps ahead of time.

Although GeoGebra's user interface consists of several components, which can be hidden on demand, its design is based on the so-called KISS principle, known from computer engineering. This principle expresses the goal of a programmer to ‘keep it short and simple’, in order to maintain the usability of the software (Hohenwarter & Jones, 2006). In the case of GeoGebra, the developer tried to design the user interface of the software in a straightforward and clear way, which supports the model of the cognitive process of learning with multimedia and reduces the cognitive load for the benefit of more successful learning (Clark & Mayer, 2003).

Since GeoGebra combines ease of use as well as the construction features of dynamic geometry software with the power and functionality of a computer algebra system, it opens up a wide range of application possibilities for teaching mathematics. Its versatility enables teachers to use the software at all grade levels from secondary school to college and for a wide range of different mathematical topics. Accordingly, GeoGebra can be used as a presentation tool as well as for the creation of instructional materials, such as notes or interactive worksheets (Fuchs & Hohenwarter, 2005). Since the software was developed initially for use by students, it fosters active and discovery learning (Bruner, 1961), and can be used by students to carry out mathematical experiments.

GeoGebra has many possibilities to help students experience intuitive feelings and to visualize adequate mathematical processes. The use of this software’s tools allows students to explore a wider range of function types, and provides students with the opportunity to make connections between symbolic and visual representations (Dikovic, 2009).

In Serbia, Maricic (2010) undertook a study on problem-solving in geometry using GeoGebra for mechanical engineering with high school students and found out that students taught using GeoGebra were able to identify initial facts about the task at hand and were also able to offer alternative solutions to the same problem. GeoGebra can be used together with other traditional methods of teaching geometry (e.g. teacher demonstrations).

3.5 Data collection instruments

To answer the research questions and establish the effect of the independent variable (learning with GeoGebra/traditional) on the dependent variable (achievement, Van Hiele levels and motivation), two instruments were used, namely post-test and questionnaire. All the items were based on the topic (Euclidean geometry), aspects and depth of knowledge specified in the NCS, mathematics Grades 10–12 (DoE, 2012).

3.5.1 Pre-Test

The first instrument was a pre-test (Appendix A1) administered to both the control group and the experimental group. The pre-test was based on basic concepts on circles and geometry in general. It was assumed that all learners would use their past experience to answer the pre-test (Grade 8 to 9 mathematics content on geometry). In order to compensate for the non-random assignment of students to the control and experimental classes, the pre-test was used to determine if the classes were comparable at the outset by determining the baseline knowledge or preparedness for learning the topic of circle geometry. The pre-test comprised 25 multiple choice questions covering basic geometry from the Grade 8–10 mathematics syllabus.

3.5.2 Post-Test

The second data collection instrument was the post-test (Appendix A2). The post-test was a comprehensive summative test of 30 questions based on the principles of Van Hiele's theory on levels of geometric understanding. The allocation of marks for this test was dependent on the level to which the questions belong, according to Van Hiele's theory of geometric understanding, as well as the strategy used according to Van Hiele's problem-solving skills strategies indicated in Table 3.2. In order to successfully solve geometry problems, students must be able perform the various skills propounded by the Van Hieles.

Table 3.2 Van Hiele levels question distribution and mark allocation

Van Hiele's levels of geometric understanding	**Question number**	**Total marks**
Visualization	1-7	7
Analysis	8-14	14
Abstraction	15-21	21
Deduction	22-28	28
Rigour	29-30	10

To mark the test an assessment rubric was developed (see Table 3.3).

Table 3.3 Assessment rubric according to Van Hiele levels

Level	Question number	Mark allocation
1. Visual	1-7	0 Incorrect answer 1 Correct answer
2. Analysis	8-14	0 Incorrect analysis of question 1 Partly correct analysis of the problem 2 Correct answer from correct analysis
3. Abstraction	15-21	0 Incorrect abstraction 1 Analysis and/or abstraction partly correct 2 Analysis correct but abstraction incorrect 3 Analysis and abstraction correct (solution correct)
4. Deduction	22-28	0 No understanding of axioms, theorems and definitions 1. Vague or partial understanding of axioms, Theorems and definitions. 2. Partly meaningful definitions and formal arguments 3. Clear logical deductions/correct answer
5. Rigour	29-30	0- No clear visualization 1- Clear visualization and analysis 2- Clear visualisation, analysis and abstraction 3- Clear visualization, analysis, abstraction, and deduction, 4- Clear visualization, analysis, abstraction, deduction, and rigor 5- Excellent presentation of a proof

Each question paper had the rubric attached, and students were expected to return their answer scripts together with the rubric.

3.6 Development of tests

The development of the tests (pre-test and post-test) for this study involved four stages, namely, item generation, content adequacy assessment, factor analysis and internal consistence assessment.

Pre-test

The pre-test items were elementary multiple choice questions on general geometry taken by the researcher from Grade 8 and 9 past examination papers. The past examination papers were district papers set by a team of mathematics subject advisors. Initially, fifty questions were selected and given to experienced mathematics teachers in order to check content adequacy (validity) and then the 25 most appropriate questions were selected. The pre-test questions were of a general nature, in order to measure the baseline knowledge of the two groups and also to determine whether the groups were of comparable ability before the treatment began. The number of questions was decided upon by the experienced teachers after taking into consideration the necessary assumed knowledge required for students to be taught circle geometry successfully.

Post-test

The post-test questions were generated from Grade 11 final national past examination papers. A large pool of circle geometry questions were initially selected by the researcher and were given to experienced mathematics teachers to evaluate the appropriateness of the questions and then to select 30 questions. The experienced teachers used the Department of Basic Education assessment guidelines and taxonomy principles.

3.7 Development of questionnaire

Student motivation can come from many sources, such as students themselves, parents, peers, teachers, and the environment or resources. Each source of motivation has its own benefits although they are interlinked. This study focused on motivation from the teaching and learning methods (Instruction with GeoGebra and traditional talk-sand-chalk). In order to find out which teaching and learning method motivated students better, a motivation questionnaire was designed focusing on only six motivation attributes relevant to teaching and learning process: (i) participation during lesson delivery, (ii) concentration during lesson delivery, (iii) enjoyment of class activities, (iv) self-confidence, (v) content mastery, and (vi) recommendation or preference of teaching/learning method.

In developing the motivation attributes questionnaire this study used a step-by-step approach to developing effective questionnaires and survey procedures for program evaluation and research (Keith & Diem, 1995). The procedure involves the following: determining the purpose of the questionnaire; deciding what is going to be measured (attitude, knowledge or skills); who should be asked; how the questionnaire is delivered to recipients; measurement scale and scoring; and ethical issues such as anonymity and confidentiality. The researcher set the questions and asked experienced researchers to check and validate the questionnaire. Students in both groups were given a questionnaire whose objective was to elucidate their feelings towards the method of instruction that was used in their class (the effect of the instruction method used on motivation). The questionnaire was administered like a test in order to avoid students influencing one another's opinions.

The questions of the questionnaire had two alternative answers, Yes or No. For every Yes response to a question, a student would score one mark, and for every No response, a student would score a zero mark. The student's total score was intended to indicate whether the teaching method used motivated him/her to learn circle geometry. Presented in Appendix A, are two tables of the questionnaire results for the two groups of the study.

3.8 Reliability and validity of instruments

The reliability and validity of the three research instruments explained above was tested. The research instruments were pilot-tested in two schools that did not take part in the main research. The results from the pilot study were used to test the reliability and validity of the instruments and also ascertain the feasibility of the main study. The process for establishing the reliability and validity of the instruments is explained in Sections 3.8.1 and 3.8.2 respectively.

3.8.1 Reliability of test instruments

According to Phelan and Wren (2005), reliability is the degree to which an assessment tool produces stable and consistent results. In this study the reliability of the pre-test and post-test was established using data from the pilot study involving 15 Grade 11 students from another school. Two reliability tests were calculated, the

Kuder-Richardson 20 (KR20) for the pre-test and Cronbach's Spearman-Brown formula for the post-test.

The KR20 was used to measure reliability of the pre-test only because the test involved dichotomous questions (multiple choice items). The KR20 is a mathematical expression of the classical measurement definition of reliability (Grinnel & Unrau, 2005). The classical definition states that reliability is the ratio of true score variance to observed score and is usually expressed symbolically as the following:

$$\rho_{XX/} = \frac{\sigma_T^2}{\sigma_X^2} = \frac{\sigma_T^2}{\sigma_T^2 + \sigma_E^2}$$

Where

The observed score variance is defined as the combination of true score variance (σ_T^2) *and error variance* (σ_E^2). *As error variance is reduced, reliability increases (i.e. a student's observed score is more reflective of the student's true score).*

The internal consistency estimates of this reliability can be mathematically defined as:

$$KR20 = \left[\frac{k}{k-1}\right]\left[\frac{\sigma_X^2 - \sum_{i=1}^{k} p_{i(1-p_i)}}{\sigma_X^2}\right],$$

Where

$KR20$ *is a lower-bound estimate of the true reliability,* k *is the number of items in the test,* σ_X^2 *is the observed score variance, and* p_i *is the proportion of examinees getting item* i *correct, i.e. the item p-value.*

The KR reliability value of the pre-test for this study was calculated as shown in Appendices B4 and B5, and explained in chapter 4 under reliability of the research instruments.

The post-test's reliability was tested using Spearman-Brown because the test items comprised multiple choice questions, short answer questions and long mathematical proofs (Webb, Shavelson & Haetel, 2006). The Spearman-Brown prediction formula is a mathematical formula relating reliability of an assessment test to test length.

According to Stanley (1971), the formula is commonly used to predict the reliability of a test after changing the test length. Drost (2012) argued that the formula is helpful in understanding the non-linear relationship between test reliability and test length. In order to predict reliability, $\rho^{*}_{xx^{/}}$ is estimated as

$$\rho^{*}_{xx^{/}} = \frac{N\rho_{XX^{/}}}{1+(N-1)\rho_{XX^{/}}}$$

Where

N *is the number of tests and* $\rho_{xx^{/}}$ *is the reliability of the first test.*

According to Gay, Mill and Airsian (2011), a test is acceptable for use if its reliability coefficient exceeds 0.60. This study conveniently used a reliability score of 0.70 for both the pre-test (KR20) and post-test (Spearman-Brown formula).

3.8.2 Reliability of questionnaire

A questionnaire is said to be reliable if we can get the same or similar answers repeatedly (Venkitachalam, 2014). In order to assess the reliability and validity of the questionnaire, the questionnaire items are analysed (Nunnally & Bernstein, 1994). Item analysis involves statistical analysis of the questionnaire to identify which items can be retained and which need to be discarded. Though it cannot be calculated exactly, it can be measured by estimating correlation coefficients of various attributes such as questionnaire stability, internal consistency, readability, feasibility, layout, style, and clarity of wording. The current study used one questionnaire, and all the items were checked for validity and reliability.

3.8.3 Validity of instruments

The validity of the test instruments is the extent to which the test accurately measures what it purports to measure. There are several ways to estimate the validity of a test; among them are content validity, concurrent validity, and predictive validity. In this study, only content validity for the pre-test and post-test were determined.

According to Lawshe (1975), content validity is essentially a method for gauging agreement among experts or qualified judges regarding how important a particular test item is. Inspired by Lawshe, the pre-test and post-test for this study were given to experienced mathematics teachers (five in number) and mathematics subject advisors (two in number) within the district, who critiqued, recommended adjustments and rated the validity of these tests using Lawshe's content validity ratio (CVR).

According to Lawshe (1975), if more than half of the panellists (judges) indicate that an item is important, then that item has some content validity. Greater levels of content validity exist when greater numbers of panellists agree that a particular item is essential. The CVR is thus calculated as:

$$CVR_i = \frac{\left[n_{e-\left(\frac{N}{2}\right)}\right]}{\left(\frac{N}{2}\right)}$$

Where

CVR_i *is the CVR for the* i^{th}*item in the test,* n_e *is the number of panellists (judges) rating the item as essential to the domain and* N *is the total number of panellists (judges)* (Lawshe, 1975).

This formula yields values which range from -1 to +1. Positive values indicate that at least half the judges rated the item as essential. As explained above, the mean CVR across all items of the test gives the overall content validity of the test. The mean CVR for both the pre-test and post-test for this study was +1, an indication that the panellists agree that these tests are valid (Wynd, Schmidt & Schaefer, 2003).

3.9 Data analysis and interpretation

This study generated mainly quantitative data from tests (pre-test and pot-test) and questionnaire. Data was jointly analysed using Van Hiele's model of levels of geometric understanding and traditional descriptive statistical methods and inferential statistical methods. Van Hiele's levels of geometric understanding were analysed for both the control group and the treatment group after the treatment (teaching) in order to show whether there was a difference in the achievements of the students in the two groups. Descriptive statistics such as frequency distributions and measures of central tendencies were used to describe and compare sets of data from the study.

The statistical package for social sciences (SPSS) was used for the inferential analysis of the data. Inferential statistics are concerned with making predictions or inferences about a population from observations and analyses of a sample. The results of the analysis of the sample can be used to generalize information about the population that the sample represents.

Descriptive and inferential statistics were calculated and analysed, such as measures of central tendency and significance testing, (t-test and chi-square test). The objective of this study was to reveal whether there is a significant relationship between the independent variable (learning with GeoGebra) and the three dependent variables of this study (problem-solving, achievement and motivation, and/or motivation).

3.10 Ethical considerations

According to the Scottish Educational Research Association (SERA):

> Since education has the fundamental ethical purpose of improving the lives of individuals, communities and society, ethical considerations must lie at the core of educational research (SERA, 2005, p. 3).

Some of the ethical considerations looked at in this study are: ethical clearance, obtaining informed consent from participants, minimising the risk of harm to participants, anonymity and confidentiality of participants.

3.10.1 Ethical clearance by the University

This research study was ethically cleared by the university board responsible for ethical clearances and an ethical clearance certificate was issued.

3.10.2 Informed consent

This study sought for informed consent from participants before they took part in the study. This means that they have to know exactly what they were being asked to do, and what the risks are, before they agree to take part (Laerd, 2010). To obtain informed consent, the researcher designed a consent form for participants. The consent form included, among others, information on the aims of the study, the processes involved as well as the associated demands and inconvenience participants might face. Apart from seeking consent from participants, this study also sought for consent from responsible authorities such as parents of the participants and department of education authorities. Consent and permission letters are attached in the appendices on ethical issues in appendices C1, C2, C3, C4, and C5 respectively.

3.10.3 Minimising the risk of harm

In this study, normal educational research procedures were adhered to, hence the study was deemed unlikely to cause harm or distress to participants. The study did not interfere with normal teaching and learning.

3.10.4 Anonymity and confidentiality

In this study, anonymity and confidentiality was maintained for all the data collected for the study. Data was collected and stored safely and treated confidentially at all stages of the study. Participants' name, title, age and gender are not indicated. In this study the following ethical principles were followed: Researchers should consider the likely consequences of collecting and disseminating various types of data and should guard against predictable misinterpretations or misuse. The researcher

(i) Did not exaggerate the accuracy or explanatory power of the data;
(ii) Alerted potential users of their data to the limits of their reliability and applicability;
(iii) Presented findings and interpretations honestly and objectively;
(iv) Avoided untrue, deceptive, or undocumented statements;
(v) Collected only data needed for the purpose of this inquiry;
(vi) Documented data sources used in this inquiry; highlighted known inaccuracies in the data; took steps to correct or refine the data; applied statistical procedures to the data.

This study was guided by the principles of research outlined above.

3.11 Conclusion

The study sought to investigate the effect of integrating GeoGebra in the teaching of circle geometry on the achievement of Grade 11 students. To address the topic and provide answers to the research questions, a quasi-experimental research, using non-equivalent comparison group design was adopted. Two samples of 22 and 25 students were used as experimental and control groups respectively. A pre-test was administered in order to check whether the two groups were of comparable geometric ability before treatment. Two instruments (post-test and questionnaire) were used to collect data. Both instruments were tested for reliability and validity before they were administered to the two groups. Data collected was quantitatively analysed in order to establish whether there any statistically significant differences between the two groups after instruction by the two methods (instruction by GeoGebra and talk-and-chalk). In addition, the study conformed to ethical requirements such as ethical clearance by the University, informed consent, minimising the risk of harm, anonymity and confidentiality.

CHAPTER 4

FINDINGS

The aim of this study was to investigate the effect of the integration of GeoGebra into the teaching of circle geometry in Grade 11 students' achievement in geometry. Three main research questions were raised namely: (i) Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without the software? (ii) What is the effect of teaching circle geometry to Grade 11 students' Van Hiele levels of geometry learning? (iii) Does GeoGebra aid students' motivation to learn geometry? This chapter presents the results of the data analysed in the study and the interpretation of results. The results are organized and presented using tables, figures, descriptive and inferential statistics. The findings of the pre-test are presented first followed by the findings from the post-test. The post-test results were used to answer the three research questions of this study.

4.1 Pre-test results

A pre-test was administered to both groups (control and experimental groups) two weeks before the interventions in order to check whether the two groups were of comparable geometric abilities before the interventions. Table 4.1 shows the descriptive statistics for the pre-test results for the two groups.

Table 4.1: Group statistics for pre-test

	Groups	N	Mean	Std. deviation	Std. error mean
Marks	Experimental	22	51.2857	24.12270	9.11752
	Control	25	51.7500	22.07617	7.80510

For the experimental group, the average mark (M = 51. 3; SD = 9.1) was slightly lower than the control group average mark (M = 51.8; SD = 7.8).

To check if the difference between the achievements of the groups were statistically significant, independent samples t-test was computed. The following null hypothesis was tested at 95% confidence interval:

Null hypothesis (H_0): There is no significant difference in geometric understanding between the experimental group and control group.

Alternative hypothesis (H_1): There is significant difference in geometric understanding between the experimental group and control group. The results are shown in Table 4.2.

Table 4.2: Independent samples t-test for pre-test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Marks	Equal variances assumed	.000	.985	-.039	45	.970	-.46429	11.92605	-26.22895	25.30038
	Equal variances not assumed			-.039	12.337	.970	-.46429	12.00204	-26.53544	25.60686

Table 4.2 shows that there was no statistically significant difference in the marks for the experimental group (M = 51. 3; SD = 9.1) and control group (M = 51.8; SD = 7.8); t(45) = -0.039; p = 0.97). These results of the pre-test confirmed that the two groups (experimental and control) were of comparable/similar geometric ability before treatment; as such, any differences in geometric ability after treatment could be attributed to the treatment.

4.2 Post-test results

In this section the findings of the post-intervention test (post-test), which was administered after the intervention, are presented and used to address the research questions of the study.

4.2.1 Overall students' achievements in the post-test

Using DoBE performance levels (DoBE, 2012), Table 4.3 shows the mark distribution of students' post-test according to performance. The frequency distribution in Table 4.3 shows a reverse trend between the control and experimental groups. In the control group, more students achieved at the lower levels than at the higher levels, while in the experimental group more students achieved at the higher levels than at the lower levels.

Table 4.3 Frequency distribution of post-test results

Level	Mark range (DoE performance level)	Control group post-test frequency	Experimental group post-test frequency
1	0-29	7	1
2	30-39	3	3
3	40-49	4	2
4	50-59	3	6
5	60-69	5	3
6	70-79	2	1
7	80-100	1	6
TOTAL		25	22

Table 4.4 shows the ranges and standard deviations of the post-test of the control and experimental groups.

Table 4.4 Measures of dispersion and central tendency

Test and group	Minimum	Maximum	Range	Mean	Standard deviation
Post-test Control Group	10	80	68	44.76	21.21
Post-test Experimental Group	26	98	63	61	19.65

From Table 4.4, post-test results for the experimental group have the smallest range and standard deviation, an indication that could imply that the group acquired more or less the same content when compared to the control group's post-test range and standard deviation.

Table 4.5 shows the descriptive statistics for the post-test results for the two groups.

Table 4.5: Group statistics for post-test

	Group	**N**	**Mean**	**Std. deviation**	**Std. error mean**
Post-test	Control Group	25	44.76	21.21179	4.24236
	Experimental Group	22	61.00	19.65415	4.19028

The experimental group post-test average (M = 61; SD = 19.65), is higher than that of the control group post-test average (M = 44.76; SD = 21.21)

Independent samples t-test for the post-test was also carried out. The t-test was calculated in order to check whether there was a significant difference between the two groups' achievements. The following null hypothesis was tested at 95% confidence interval:

Null hypothesis (H_0): There is no significant difference in achievements of the experimental group compared to control group.

Alternative hypothesis (H_1): There is a significant difference in achievements of the experimental group compared to control group. The results are shown in Table 4.6.

Table 4.6 Independent samples t-test for post-test

		Levene's Test for Equality of Variances		**t-test for Equality of Means**						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Post-test	Equal variances assumes	.315	.578	2.710	45	.009	16.24000	5.99258	4.17033	28.30967
	Equal variances not assumes			2.724	44.867	.009	16.24000	5.96289	4.22914	28.25086

Table 4.6 shows that there is a statistically significant difference in achievement (post-test marks) of experimental group (M = 61; SD = 19.65) and control group (M =44.76; SD = 21.21); t(45) = 2.71; p = 0.009).

4.2.2 Students' achievements at van Hiele levels

This section reports the results of student achievements at the five Van Hiele levels.

Students' achievements at Van Hiele Level 1 (visualisation)

The results of the post-test for the two groups for Van Hiele Level 1 are presented in Tables 4.7 and 4.8. Table 4.7 shows the descriptive statistics at Van Hiele Level 1 (visualisation) of the post-test.

Table 4.7 Groups statistics at Van Hiele Level 1 (visualisation)

	Groups	N	Mean	Std. Deviation	Std. Error Mean
Marks	Experimental	22	7.00	.00	.00
	Control	25	5.28	1.95	.39

The experimental group post-test average (M = 7; SD = 0), is higher than that of the control group post-test average (M = 5.25; SD = 1.95).

Independent samples t-test for the post-test was also carried out under the following null hypothesis at 95% confidence interval:

Null hypothesis (H_0): There is no significant difference in achievements of experimental compared to control group at the Van Hiele visualisation level.

Alternative hypothesis (H_1): There is a significant difference in achievements of experimental compared to control group at the Van Hiele visualisation level. The results are shown in Table 4.8.

Table 4.8 Independent samples t-test at Van Hiele level 1 (visualisation)

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Marks	Equal variances assumed	58.4	.00	4.71	45	.00	1.96	.42	1.12	2.80
	Equal variances not assumed			5.03	24.00	.00	1.96	.39	1.16	2.76

Table 4.8 shows that at Van Hiele Level 1 there is a statistically significant difference in post-test marks of the experimental group (M = 7; SD = 0) and control group (M = 5.28; SD = 1.95); t(45) = 4.71; p = 0) in favour of the experimental group. On the basis of this finding, the null hypothesis was rejected and the alternative hypothesis (H1), that there is a significant difference in achievements of students exposed to GeoGebra compared to those not exposed to the software at the Van Hiele visualisation level, was accepted.

Students' achievements at Van Hiele Level 2 (analysis)

The results of the post-test for the two groups for Van Hiele Level 2 are presented in Tables 4.9 and 4.10. Table 4.9 shows the descriptive statistics at Van Hiele Level 2 (analysis) of the post-test.

Table 4.9 Group statistics at Van Hiele Level 2 (analysis)

	Group	N	Mean	Std. deviation	Std. error mean
Marks	Experimental	22	13.45	1.18	.25
	Control	25	9.32	3.73	.75

The experimental group had higher marks (M = 13.35; SD = 1.18) than those in the control group (M = 9.32; SD = 3.73).

Independent samples t-test for the post-test was also carried out, as follows:

Null hypothesis (H_0): There is no significant difference in achievements of the experimental group compared to the control group at the Van Hiele analysis level.

Alternative hypothesis (H_1): There is a significant difference in the achievements of the experimental group compared to the control group at the Van Hiele analysis level.

The results are shown in Table 4.10.

Table 4.10 Independent samples t-test at Van Hiele Level 2 (analysis)

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Marks	Equal variances assumed	20.740	.000	4.981	45	.000	4.13455	.83014	2.46256	5.80653
	Equal variances not assumed			5.253	29.381	.000	4.13455	.78708	2.52570	5.74339

Table 4.10 shows that there is a statistically significant difference in average post-test marks of the experimental group (M = 13.35; SD = 1.18) and control group (M = 9.32; SD = 3.73); t(45) = 4.98; p = 0).

Students' achievements at Van Hiele Level 3 (abstraction/ordering)

The results of the post-test for the two groups for Van Hiele Level 3 are presented in Tables 4.11 and 4.12. Table 4.11 shows the descriptive statistics at Van Hiele Level 2 (abstraction/ordering) of the post-test.

Table 4.11 Groups' statistics at Van Hiele Level 3 (abstraction)

	Groups	N	Mean	Std. deviation	Std. error mean
Marks	Experimental	22	10.36	5.18	1.10
	Control	25	8.28	4.66	.93

As shown in Table 4.11 students in the experimental group had higher marks (M = 10.36; SD = 5.18) than those in the control group (M = 8.28; SD = 4.66).

Independent samples t-test for the post-test was also carried out at 95% confidence interval under the following null hypothesis:

Null hypothesis (H_0): There is no significant difference in achievements of the experimental group compared to the control group at Van Hiele abstraction level.

Alternative hypothesis (H_1): There is a significant difference in achievements of the experimental group compared to the control group at Van Hiele abstraction level. The results are shown in Table 4.12.

Table 4.12 Independent samples t-test for groups at Van Hiele Level 3 (abstraction)

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Marks	Equal variances assumed	.053	.819	1.452	45	.153	2.08364	1.43484	-.80627	4.97354
	Equal variances not assumed			1.442	42.644	.157	2.08364	1.44472	-.83062	4.99789

Table 4.12 shows that there was no statistically significant difference in the average marks for the experimental group (M = 10.36; SD = 5.18) and control group (M = 8.28; SD = 4.66); t(45) = 1.45; p = 0.15).

Students' achievements at Van Hiele Level 4 (deduction)

The results of the post-test for the two groups for Van Hiele Level 4 are presented in Tables 4.13 and 4.14. Table 4.13 shows the descriptive statistics at Van Hiele Level 4 (deduction) of the post-test.

Table 4.13 Groups' statistics at Van Hiele Level 4 (deduction)

	Groups	N	Mean	Std. deviation	Std. error mean
Marks	Experimental	22	12.23	7.55	1.61
	Control	25	8.88	5.37	1.07

As shown in Table 4.13, students in the experimental group had higher marks (M = 12.23; SD = 7.55) than those in the control group (M = 8.88; SD = 5.37).

Independent samples t-test for the post-test was also carried out at 95% confidence interval under the following null hypothesis:

Null hypothesis (H_0): There is no significant difference in achievements of the experimental group compared to the control group at the Van Hiele deduction level.

Alternative hypothesis (H_1): There is a significant difference in achievements of the experimental group compared to the control group at Van Hiele deduction level. The results are shown in Table 4.14.

Table 4.14 Independent samples t-test for groups at Van Hiele Level 4 (deduction)

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Marks	Equal variances assumed	2.489	.122	1.768	45	.084	3.34727	1.89374	-.46691	7.16146
	Equal variances not assumed			1.730	37.399	.092	3.34727	1.93464	-.57128	7.26582

Table 4.14, shows that there is no statistically significant difference in average marks of the experimental group (M = 12.23; SD = 7.55) and control group (M = 8.88; SD = 5.37); t(45) = 1.77; p = 0.084).

Students' achievements at Van Hiele Level 5 (rigour)

The results of the post-test for the two groups for Van Hiele Level 5 (rigour) are presented in Tables 4.15 and 4.16. Table 4.15 shows the descriptive statistics at Van Hiele Level 5 (rigour) of the post-test.

Table 4.15 Groups' statistics at Van Hiele Level 5 (rigor)

	Groups	N	Mean	Std. deviation	Std. error mean
Marks	Experimental	22	5.64	3.49	.74
	Control	25	4.28	3.32	.66

As shown in Table 4.15, the students in the experimental group had higher (M = 5.64; SD = 3.49) than the control group (4.28; SD =3.32).

Independent samples t-test for the post-test was also carried out at 95% confidence interval under the following null hypothesis (H_0):

Null hypothesis (H_0): There is no significant difference in achievements of the experimental group compared to the control group at the Van Hiele rigour level.

Alternative hypothesis (H_1): There is a significant difference in achievements of the experimental group compared to the control group at the Van Hiele rigour level. The results are shown in Table 4.16.

Table 4.16 Independent samples t-test for groups at Van Hiele Level 5 (rigour)

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Marks	Equal variances assumed	.000	.996	1.365	45	.179	1.35636	.99384	-.64534	3.35807
	Not assumed			1.361	43.616	.181	1.35636	.99693	-.6533	3.36605

Table 4.16 shows that there is no statistically significant difference in the average marks for the experimental group (M = 5.64; SD = 3.49) and control group (M = 4.28; SD = 3.32); t(45) = 1.36; p = 0.18).

4.3 Questionnaire results

Although the major research question for this study focused on the achievement of students, this study also sought to investigate whether the use of GeoGebra in the teaching and learning process had any motivational effect on students. The third research question is: Does GeoGebra aid students' motivation to learn circle geometry? Six motivation attributes (indicators of motivation) were used to investigate whether the students were motivated by the teaching and learning method used in their class: (i) participation during lesson delivery, (ii) concentration during lesson delivery, (iii) enjoyment of class activities, (iv) self-confidence, (v) content mastery, and (vi) recommendation/preference of teaching and learning method. Descriptive and inferential statistics were calculated for each motivation attribute; statistical significance was tested through chi-square testing of each of the seven motivation attributes at 0.05 alpha level (95% confidence interval).

The summary of the Yes and No percentages for all the motivation attributes is presented in Appendix D, Figures 4.1 and 4.2. The descriptive results are as follows: Of the 22 experimental group students, 86% answered 'Yes' to the question: Were you motivated to learn circle geometry? Of the 25 of the control group students, 64% answered 'Yes' to the same question. It is evident that the experimental group felt more motivated to learn circle geometry than the control group. In the experimental group, 100% of the students reported that the teaching method (GeoGebra instruction) encouraged them to participate in the learning process, compared to 24% of the control group who responded to the same question. Of the experimental group 95% confirmed that GeoGebra-instructed lessons enabled them to concentrate during lesson delivery, while only 32% of the control group responded that the traditional teacher talk-and-chalk method enabled them to concentrate during lesson delivery.

Regarding enjoyment of lessons, 100% of the respondents of the experimental group confirmed that learning circle geometry using GeoGebra was very enjoyable, while only 36% of the control group felt that the traditional teaching and learning method was enjoyable. Concerning whether the method of instruction enabled students to answer all questions, 82% of the experimental group responded Yes, while 52% of the control group said the traditional teaching and learning method enabled them to answer all questions. For the experimental group 68% responded that the teaching method used led them to master the content, while only 40% of the control group felt the teaching method helped them to master content adequately. All respondents (100%) in the experimental group recommended the use of GeoGebra in the teaching of geometry, compared to 64% of the control group who recommended the traditional talk-and-chalk teaching method.

Having analysed the questionnaire results descriptively, the study also sought to make inferences from these results by performing a chi-square t-test on the total motivation scores obtained by the students. The results of the chi-square tests of each of the six motivation attribute/questions are presented below.

4.3.1 Participation attribute result

The first question that the students were asked was: Did the teaching method encourage you to participate in class activities? The chi-square test for motivation attribute: Encouragement to participate in class activities was carried out at 95% confidence interval, as follows:

Null hypothesis (H_0): There is no difference in encouragement to participate in class activities between students' in the experimental group and students in the control group.

Alternative hypothesis (H_1): There is a difference in encouragement to participate in class activities between students' in the experimental group and students in the control group.

Table 4.17 shows the cross-tabulation of encouragement to participation in class activities attribute and Table 4.18 shows the chi-square test results of encouragement to participation in class activities attribute.

Table 4.17 Cross-tabulation of participation attribute

			Participation	
			Not Encouraged to Participate	Encouraged to Participate
Group	Experimental (GeoGebra Instructed)	Count	0	22
		% within Participation	0.0%	78.6%
	Control (Traditionally Instructed)	Count	19	6
		% within Participation	100.0%	21.4%
Total		Count	19	28
		% within Participation	100.0%	100.0%

Table 4.18 shows chi-square test of encouragement to participation attribute and Figure 4.1 shows the percentage of students from the two groups who were encouraged, or not encouraged, to participate in class activities by the teaching method used in their classes.

Table 4.18 Chi-square test of participation attribute

Chi-Square Tests					
	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	28.066[a]	1	.000		
Continuity Correction[b]	24.999	1	.000		
Likelihood Ratio	35.868	1	.000		
Fisher's Exact Test				.000	.000
Linear-by-Linear Association	27.469	1	.000		
N of Valid Cases	47				

Table 4.18, shows that there was a statistically significant difference between students' participation possibly arising from the teaching and learning method used in their class, $\chi^2(1, N = 47) = 28.07$; $p = 0$.

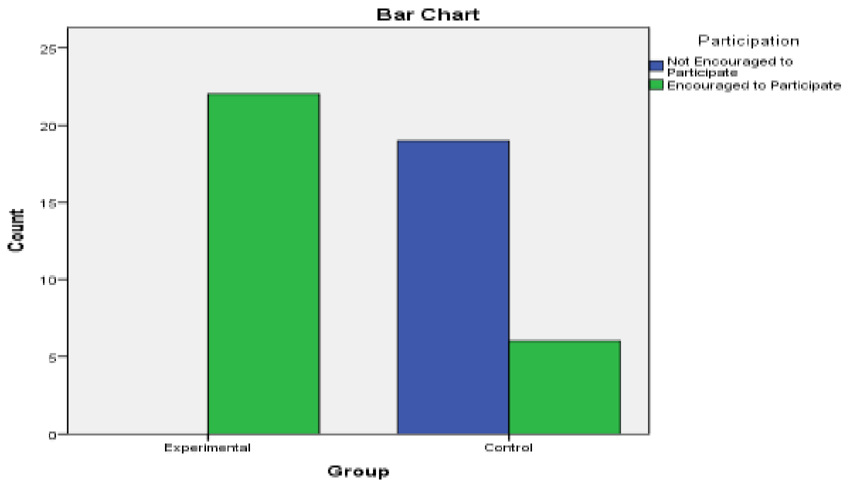

Figure 4.1 Compound bar chart on participation attribute

The compound bar chart shows that all students (100%) in the experimental group said that the teaching method used in their class encouraged them to participate in class activities, while 76% of the control group said the teaching method used in their class did not encourage them to participate in class activities.

4.3.2 Concentration attribute result

The second motivation attribute question that the students were asked was: Did the teaching method enable you to concentrate during and after lesson delivery? The chi-square test for motivation attribute: Concentration during lesson delivery in class activities was carried out at 95% confidence interval, as follows:

Null hypothesis (H_0): There is no difference in concentration of students in experimental compared to students in control group.

Alternative hypothesis (H_1): There is a difference in concentration of students in experimental group compared to students in control group. Table 4.19 shows the cross-tabulation of the concentration attribute and Table 4.20 shows the chi-square test results of the concentration attribute.

Table 4.19 Cross-tabulation of concentration attribute

			Concentration	
			Does not Promote Concentration	Promote Concentration
Group	Experimental (GeoGebra Instructed)	Count	1	21
		% within Concentration	5.6%	72.4%
	Control (Traditionally Instructed)	Count	17	8
		% within Concentration	94.4%	27.6%
Total		Count	18	29
		% within Concentration	100.0%	100.0%

Figure 4.2 show the percentage of students from the two groups who concentrated, or did not concentrate, due to the teaching method used in their class.

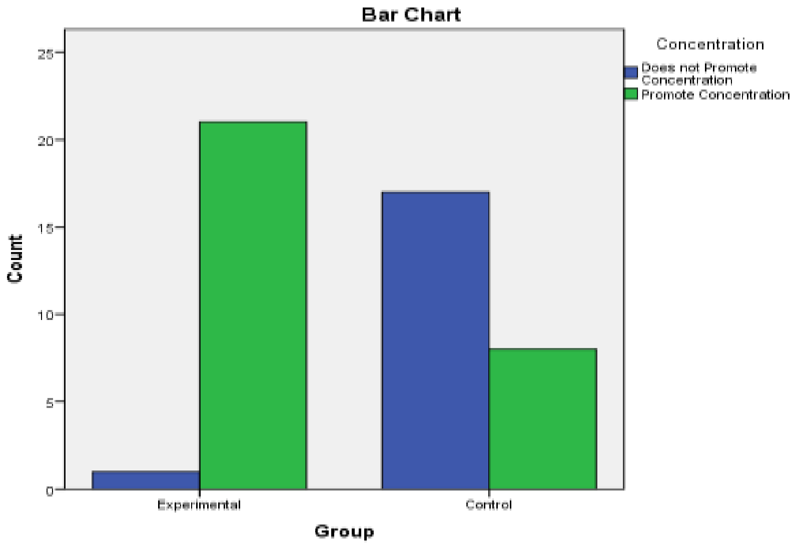

Figure 4.3 Compound bar chart on concentration attribute

The compound bar chart shows that 72.4% of students in the experimental group said that the teaching method used in their class encouraged them to concentrate in class activities, compared to only 27.6% of the students in the control group.

In Table 4.20, the relationship between students' concentration in class activities and the teaching and learning method used in their class was significant, $\chi^2(1, N = 47) = 19.94$; $p = 0$.

Table 4.20 Chi-square tests of concentration attribute

Chi-Square Tests					
	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	19.940[a]	1	.000		
Continuity Correction[b]	17.345	1	.000		
Likelihood Ratio	23.078	1	.000		
Fisher's Exact Test				.000	.000
Linear-by-Linear Association	19.515	1	.000		
N of Valid Cases	47				

4.3.3 Enjoyment attribute result

The third question that the students were asked was: Did you enjoy the lesson(s)? The chi-square test for motivation attribute: Enjoyment of class activities was carried out at 95% confidence interval, as follows:

Null hypothesis (H_0): There is no difference in the enjoyment of students in the experimental group students compared to control group students.

Alternative hypothesis (H_1): There is a difference in the enjoyment of students in the experimental group students compared to control group students.

Table 4.21 shows cross-tabulation of enjoyment attribute and Figure 4.4 shows the percentage of students from the two groups who enjoyed, or did not enjoy, the teaching method used in their class.

Table 4.21 Cross-tabulation of enjoyment attribute

			Enjoyment	
			Does not Enjoy Lessons	Enjoy Lessons
Group	Experimental (GeoGebra Instructed)	Count	0	22
		% within Enjoyment	0.0%	71.0%
	Control (Traditionally Instructed)	Count	16	9
		% within Enjoyment	100.0%	29.0%
Total		Count	16	31
		% within Enjoyment	100.0%	100.0%

The compound bar chart in Figure 4.4 shows that 100% of students in the experimental group enjoyed the lessons compared to only 29.0% of the students in the control group.

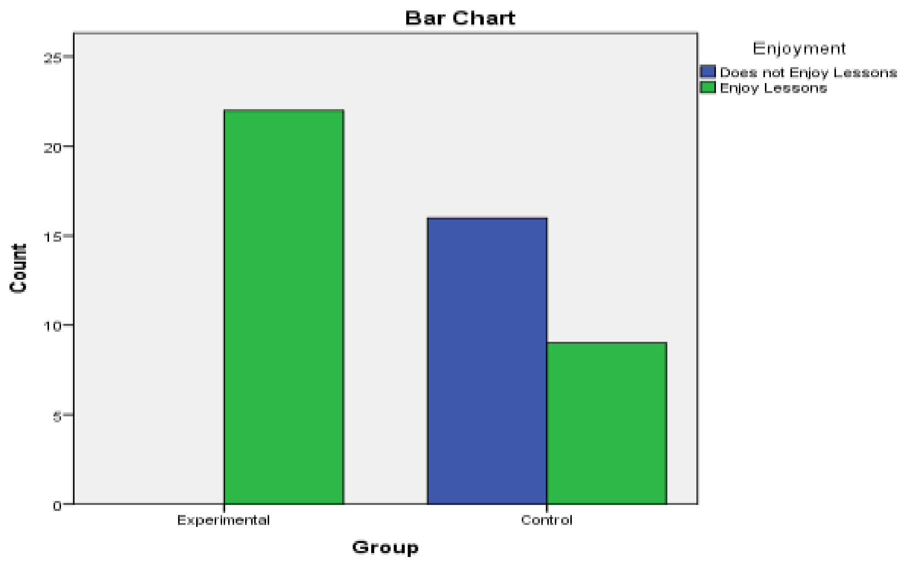

Figure 4.4 Compound bar chart on enjoyment attribute

Table 4.22 shows the results of the chi-square test of independence performed to examine the relation between students' enjoyment during class activities and the teaching and learning method used in their class.

Table 4.22 Chi-square tests of enjoyment attribute

Chi-Square Tests					
	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	21.347[a]	1	.000		
Continuity Correction[b]	18.592	1	.000		
Likelihood Ratio	27.613	1	.000		
Fisher's Exact Test				.000	.000
Linear-by-Linear Association	20.893	1	.000		
N of Valid Cases	47				

The relation between these variables was significant, $\chi^2(1, N = 47) = 21.35$; $p = 0$.

4.3.4 Self-confidence attribute result

The fourth question that the students were asked was: Did the teaching/learning method instil self-confidence in you? The chi-square test for motivation attribute, self-confidence was carried out at 95% confidence interval, as follows:

Null hypothesis (H_0): There is no difference in self-confidence of students in the experimental group compared to students in control group.

Alternative hypothesis (H_1): There is a difference in self-confidence of students in the experimental group compared to students in control group.

Table 4.23 shows cross-tabulation on the self-confidence attribute and Figure 4.5, shows the percentage of students from the two groups who claimed to have been instilled with self-confidence by the teaching and learning method used in their class.

Table 4.23 Cross-tabulation of self-confidence attribute

			All Questions	
			Cannot Answer all Questions	Can Answer all Questions
Group	Experimental (GeoGebra Instructed)	Count	4	18
		% within All Questions	25.0%	58.1%
	Control (Traditionally Instructed)	Count	12	13
		% within All Questions	75.0%	41.9%
Total		Count	16	31
		% within All Questions	100.0%	100.0%

The compound bar chart in Figure 4.5 shows that 75.0% of students in the experimental group enjoyed the lessons compared to 58.1% of the students in the control group.

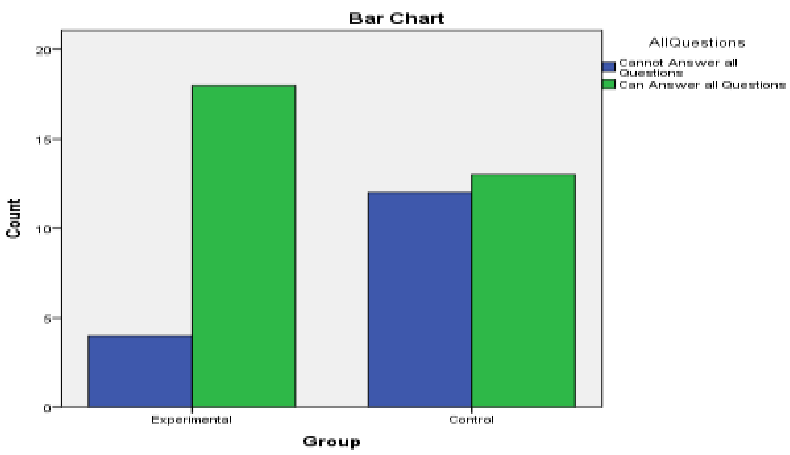

Figure 4.5 Compound bar chart on self-confidence attribute

In Table 4.24 the chi-square test of independence was performed to compare students' self-confidence for the two groups after the teaching and learning method used in their class.

Table 4.24 Chi-square tests of self-confidence attribute

Chi-Square Tests					
	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	4.634[a]	1	.031		
Continuity Correction[b]	3.401	1	.065		
Likelihood Ratio	4.804	1	.028		
Fisher's Exact Test				.063	.031
Linear-by-Linear Association	4.535	1	.033		
N of Valid Cases	47				

The result was significant, $\chi^2(1, N = 47) = 4.63; p = 0.03$.

4.3.5 Content mastery attribute result

The fifth question that students were asked was: Were you able to master the content after the teaching/learning in your class? The chi-square test for motivation attribute, content mastery, was carried out at 95% confidence interval, as follows:

Null hypothesis (H_0): There is no difference in content mastery by students in the experimental group compared to students in the control group.

The alternative hypothesis (H_1): There is a difference in content mastery by students in the experimental group compared to students in the control group.

Table 4.25 shows cross-tabulation of content mastery attribute and Figure 4.6 shows the percentage of students from the two groups who claimed to have mastered the content after treatment.

Table 4.25 Cross-tabulation of content mastery attribute

			ExpectedResults	
			Does not get Expected Results	Gets expected Results
Group	Experimental (GeoGebra Instructed)	Count	6	16
		% within Expected Results	28.6%	61.5%
	Control (Traditionally Instructed)	Count	15	10
		% within Expected Results	71.4%	38.5%
Total		Count	21	26
		% within Expected Results	100.0%	100.0%

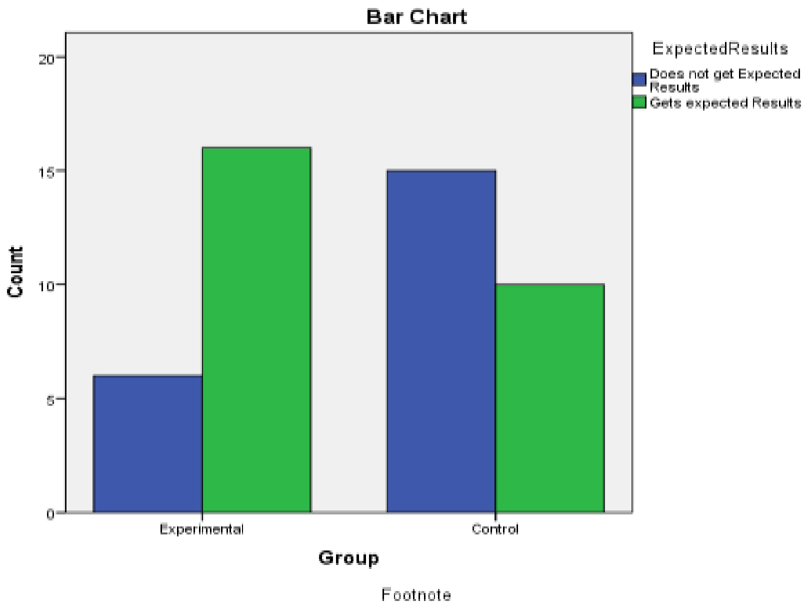

Figure 4.6 Compound bar chart on content mastery attribute

The compound bar chart in Figure 4.6 shows that 71.4% of students in the experimental group mastered their content to 38.5% of the students in the control group.

Table 4.26 shows the chi-square test of independence that was performed to compare students' content mastery after the teaching and learning method used in their class.

Table 4.26 Chi-square tests of content mastery attribute

Chi-Square Tests					
	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	5.071[a]	1	.024		
Continuity Correction[b]	3.833	1	.050		
Likelihood Ratio	5.190	1	.023		
Fisher's Exact Test				.039	.024
Linear-by-Linear Association	4.963	1	.026		
N of Valid Cases	47				

The results were significant, $\chi^2(1, N = 47) = 5.07; p = 0.024$.

4.3 6 Recommendation attribute result

The sixth question asked was: Would you recommend the use of the teaching and learning method used in your class for future geometry lessons?

The chi-square test for motivation attribute: Recommendation of teaching and learning method was carried out at 95% confidence interval under the following null hypothesis:

Null hypothesis (H_0): There is no difference in the recommendations/preference of the teaching/learning method between students in experimental group compared to students in control group.
Alternative hypothesis (H_1): There is no difference in the recommendations or preference of the teaching/learning method between students in the experimental group compared to students in the control group.

Table 4.27 shows the cross-tabulation of the recommendation attribute and Figure 4.7 shows the percentage of students from the two groups who recommended, or did not recommend, the teaching and learning method used by the teacher in their class.

Table 4.27 Cross-tabulation of recommendation/preference attribute

			Recommendation	
			Does not recommend the teaching method	Recommend the teaching method
Group	Experimental (Geogebra-instructed)	Count	0	22
		% within recommendation	0.0%	57.9%
	Control (traditionally Instructed)	Count	9	16
		% within recommendation	100.0%	42.1%

The compound bar chart in Figure 4.7 shows that 100.0% of students in the experimental group enjoyed the lessons compared to 42.1% of the students in the control group.

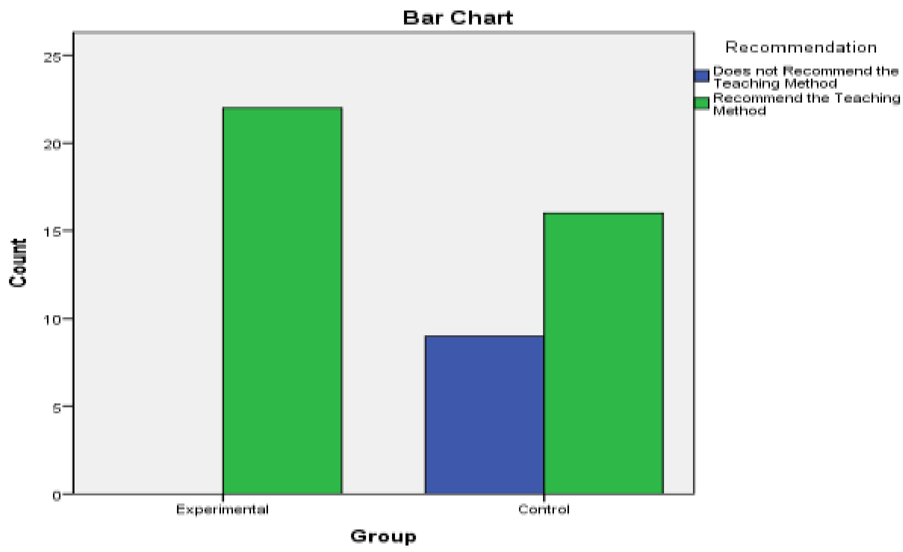

Figure 4.7 Compound bar chart on recommendation/preference attribute

In Table 4.28 the chi-square test of independence was performed to compare the recommendations/preferences of the teaching and learning method used in their class.

Table 4.28 Chi-square tests of recommendations/preferences attribute

Chi-Square Tests					
	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	9.796[a]	1	.002		
Continuity Correction[b]	7.609	1	.006		
Likelihood Ratio	13.236	1	.000		
Fisher's Exact Test				.002	.001
Linear-by-Linear Association	9.587	1	.002		
N of Valid Cases	47				

The relation between these variables was significant, $\chi^2(1, N = 47) = 9.80; p = 0.002$.

The seven motivation attributes presented above were used to answer the research question number three in section 4.4.

4.4 Answering the research questions

The results of the data analyses in section 4.2 were used to answer the research questions advanced in this study. The questions were answered in chronological order.

4.4.1 Research question one

The first research question was: Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra in circle geometry? In testing the hypothesis corresponding to this research question, the data was analysed using independent samples t-test, while statistical inference was taken at 0.05 alpha levels (95% confidence interval). The results are displayed in Tables 4.5 and 4.6. The results show that there is a statistically significant difference in post-test marks of experimental group (M = 61; SD = 19.65) and control group (M = 44.76; SD = 21.21); t(45) = 2.71, p = 0.009). On the basis of these findings, it was concluded that there was is statistically significant difference between the achievements of students exposed to GeoGebra compared to students taught without GeoGebra, hence the null hypothesis (There is no significant difference in achievement of experimental and control groups), was rejected in favour of the alternative hypothesis (There is significant difference in achievement at of experimental and control groups).

4.4.2 Research question two

The second research question states: What is the effect of teaching circle geometry with GeoGebra on Grade 11 students' achievements at Van Hiele levels of geometric thinking? This research question was sub-divided into five questions, each corresponding to a Van Hiele level:

Research question on Van Hiele Level 1 (visualisation)

The first question on Van Hiele levels asks: Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at visualisation level of geometric thinking? In testing the hypothesis corresponding to this research question, the data was analysed using independent samples t-test, while statistical inference was taken at 0.05 alpha level

(95% confidence interval). The results are displayed in Tables 4.7 and 4.8. The results shows that there is a statistically significant difference in post-test marks at Van Hiele Level one of the experimental group ($M = 7$; $SD = 0$) and the control group ($M = 5.28$; $SD = 1.95$); $t(45) = 4.71$; $p = 0$). On the basis of these findings, it was concluded that there was a statistically significant difference between achievement at Van Hiele Level 1 of students exposed to GeoGebra compared to students taught without GeoGebra, hence the null hypothesis (There is no significant difference in achievement at Van Hiele Level 1 of experimental and control groups), was rejected in favour of the alternative hypothesis (There is significant difference in achievement at Van Hiele Level 1 of experimental and control groups).

Research question on Van Hiele Level 2 (analysis)

The second question on Van Hiele levels asks: Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at the analysis level of geometric thinking? In testing the hypothesis corresponding to this research question, the data was analysed using independent samples t-test, while statistical inference was taken at 0.05 alpha level (95% confidence interval). The results are displayed in Tables 4.9 and 4.10. The results show that there is a statistically significant difference in post-test marks at Van Hiele Level 2 of experimental group ($M = 13.35$; $SD = 1.18$) and control group ($M = 9.32$; $SD = 3.73$); $t(45) = 4.98$; $p = 0$). On the basis of these findings, it was concluded that there was a statistically significant difference between achievements at Van Hiele Level 2 of students exposed to GeoGebra compared to students taught without GeoGebra, hence the null hypothesis (There is no significant difference in achievement at Van Hiele Level 2 of the experimental and control groups), was rejected in favour of the alternative hypothesis (There is significant difference in achievement at Van Hiele Level 2 of the experimental and control groups).

Research question on Van Hiele Level 3 (abstraction/ordering)

The third question on Van Hiele level asks: Is there any difference in the achievements of students exposed to GeoGbra compared to students taught without GeoGebra at the abstraction level of geometric thinking? In testing the hypothesis corresponding to this research question, the data was analysed using independent samples t-test, while statistical inference was taken at 0.05 alpha level (95%

confidence interval). The results are displayed in Tables 4.11 and 4.12. The results show that there is no statistically significant difference in post-test marks at Van Hiele Level 3 of the experimental group (M = 10.36; SD = 5.18) and control group (M = 8.2;, SD = 4.66); t(45) = 1.45; p = 0.15). On the basis of these findings, it was concluded that there is no statistically significant difference between achievements at Van Hiele Level 3 of students exposed to GeoGebra compared to students taught without GeoGebra, hence the null hypothesis (There is no significant difference in achievement at Van Hiele Level 3 of experimental and control groups), was accepted, while the alternative hypothesis (There is significant difference in achievement at Van Hiele Level 3 of experimental and control groups) was rejected.

Research question on Van Hiele level 4 (deduction)

The fourth question on Van Hiele levels asks: What is the effect of teaching circle geometry with GeoGebra on Grade 11 students' achievement at Van Hiele deduction level? In testing the hypothesis corresponding to this research question, the data was analysed using independent samples t-test, while statistical inference was taken at 0.05 alpha level (95% confidence interval). The results are displayed in Tables 4.13 and 4.14. The results show that there was no statistically significant difference in post-test marks at Van Hiele Level 4 of experimental group (M = 12.23; SD = 7.15) and control group (M = 8.88; SD = 5.37); t(45) = 1.77; p = 0.08). On the basis of these findings, it was concluded that there was no statistically significant difference between achievements at Van Hiele Level 4 of students exposed to GeoGebra compared to students taught without GeoGebra, hence the null hypothesis (There is no significant difference in achievement at Van Hiele Level 4 of experimental and control groups), was accepted while the alternative hypothesis (There is significant difference in achievement at Van Hiele Level 4 of experimental and control groups) was rejected.

Research question on Van Hiele Level 5 (rigour)

The fifth question on Van Hiele levels asks: Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at the deduction level of geometric thinking? In testing the hypothesis corresponding to this research question, the data was analysed using independent samples t-test, while statistical inference was taken at 0.05 alpha level

(95% confidence interval). The results are displayed in Tables 4.15 and 4.16. The results show that there was no statistically significant difference in post-test marks at Van Hiele Level 5 of the experimental group (M = 12.23; SD = 7.15) and control group (M = 8.88; SD = 5.37); t(45) = 1.77; p = 0.08). On the basis of these findings, it was concluded that there was no statistically significant difference between achievements at Van Hiele level 5 of students exposed to GeoGebra compared to students taught without GeoGebra, hence the null hypothesis (There is no significant difference in achievement at Van Hiele Level 5 of experimental and control groups), was accepted while the alternative hypothesis (There is significant difference in achievement at Van Hiele Level 5 of experimental and control groups) was rejected.

4.4.3 Research question three

The third research question asks: Does GeoGebra aid students' motivation to learn circle geometry? This research question was addressed using the six attributes of motivation. The results of the chi-square analyses (Tables 4.17–4.28), summarised in Table 4.29, show that there were statistically significant differences between the experimental group and the control group in all the motivation attributes, in favour of the experimental group. Hence, it can be concluded that GeoGebra aided students' motivation to learn circle geometry.

Table 4.29 Summary of findings

	Research Question	Answer
1.	Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra?	There is a statistically significant difference between the achievements of students exposed to GeoGebra (experimental group) compared to those not exposed to the software (control group) in favour of experimental group.
2.	Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at visualisation level of geometric thinking?	There is significant difference in achievement of students exposed to GeoGebra compared to those not exposed to the software at visualisation of geometric thinking in favour of the students exposed to GeoGebra.
	Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at analysis level of geometric thinking?	There is significant difference in achievement of students exposed to GeoGebra compared to those not exposed to the software at analysis level of geometric thinking in favour of the students exposed to GeoGebra.
	Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at abstraction level of geometric thinking?	There is no significant difference in achievement of students exposed to GeoGebra compared to those not exposed to the software at abstraction level of geometric thinking.
	Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at deduction level of geometric thinking?	There is no significant difference in achievement of students exposed to GeoGebra compared to those not exposed to the software at at deduction level of geometric thinking.
	Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at rigor level of geometric thinking?	There is no significant difference in achievement of students exposed to GeoGebra compared to those not exposed to the software at rigor level of geometric thinking
3	Does GeoGebra aid students' motivation to learn circle geometry?	GeoGebra aided students' motivation to learn circle geometry

4.5 Conclusion

The study set out to investigate the impact of integration of GeoGebra on students' achievement, Van Hiele levels and motivation to learn circle geometry. The findings have shown that GeoGebra can result in significant differences in student achievements in circle geometry and students' achievement in Van Hiele Levels 1 and 2 (visualisation and analysis respectively), but no significant differences in Van Hiele Levels 3, 4 and 5 (abstraction, deduction and rigour respectively). The questionnaire results for all six motivation attributes show conclusively that GeoGebra motivates students to learn circle geometry.

CHAPTER 5
SUMMARY OF THE STUDY, DISCUSSION, RECOMMENDATIONS AND CONCLUSIONS

This chapter summaries and discusses the findings of the study, its implications and limitations, and makes recommendations for practice and policy. The chapter concludes with suggestions for future research on the integration of mathematical software (GeoGebra) into the teaching and learning of mathematics in South Africa.

5.1 Summary of study

Periodic changes in the South African mathematics curriculum (e.g. the change from NCS of 2002, to CAPS of 2012), pedagogical challenges confronting teachers of mathematics, such as insufficient knowledge of subject matter by teachers due to inadequate training or shear incompetence, addition of new topics to mathematics syllabi, and poor performance in mathematics by students in general and geometry in particular, prompted this study, in order to try and improve the situation. This study has focussed on circle geometry, one of the topics that is highly problematic in secondary school mathematics and that was previously optional.

The theoretical framework of this study was inspired by the positivism paradigm and grounded in two learning philosophies, behaviourism and constructivism. Behaviourist philosophy looks at learning as a system of rewards and targets. The use of GeoGebra in this study was felt to be an appropriate external stimulus. Constructivists view learning as an active contextualised process of constructing knowledge rather than acquiring it. This study concurs with these ideas.

The study adopted the APOS theory as its general theoretical framework and the Van Hiele theory as the theoretical framework for teaching and learning geometry.

5.1.1 Aim of study

The aim of the study was to investigate the effect of integrating GeoGebra with the teaching of circle geometry on Grade 11 students' achievement, Van Hiele levels, and motivation. The study focused on answering three main research questions, namely:

1. Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra in circle geometry?
2. Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at Van Hiele levels of geometric thinking?

To answer this research question, the following five sub-questions of question 2, each focusing on a particular Van Hiele level, were raised:

2.1 Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at visualisation level of geometric thinking?

2.2 Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at analysis level of geometric thinking?

2.3 Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at abstraction level of geometric thinking?

2.4 Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at deduction level of geometric thinking?

2.5 Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without GeoGebra at rigor level of geometric thinking?

3. Does GeoGebra aid students' motivation to learn circle geometry?

Research question three was split into six questions/hypotheses, each focusing on a particular motivation attribute. Research questions one and two were t-tested for statistical significance, while the third research question was chi-square tested for statistical significance.

5.1.2 Methodology

This study was a quasi-experimental research study of non-equivalent group design, in which a pre-test and post-test were used. According to Dinardo (2008), a quasi-experimental study is an empirical study used to estimate the causal impact of an intervention on a targeted population, as was the case in this study.

In this study, a pre-test and post-test were administered to the participants, before and after instruction respectively. The population for the study was all Grade 11 students in Vhembe district of Limpopo Province, South Africa. Two samples of size $n_1 = 22$ (experimental group) and $n_2 = 25$ (control group) were used. A pilot study of sample size n =15 was carried out at another school, in order to check the reliability and validity of the research instruments.

The reliability of the pre-test was tested by calculating the KR20 score, and the reliability of the post-test was established using the Spearman-Brown formula. Validity for both tests was verified using Lawshe's CVR. Data analysis and interpretation was done using Microsoft excel ® and SPSS ®.

5.2 Findings

The results of this study were analysed according to the research question concerned.

5.2.1 Research question one

Is there any difference in the achievements of students exposed to GeoGebra compared to students taught without the software? The independent samples t-test (Table 4.16) shows that there was a statistically significant difference in the average post-test mark of the experimental group (M = 61; SD = 19.65) and control group (M = 44.76; SD = 21.21); $t(45) = 2.71$; $p = 0.009$.

5.2.2 Research question two

What is the effect of teaching circle geometry to Grade 11 students with GeoGebra software on students' Van Hiele levels of geometry learning? This research question was split into five sub-questions, each focusing on a particular Van Hiele level. To answer the five questions, data from the post-test results was analysed using a researcher-developed geometry learning assessment instrument called Van Hiele levels matrix of geometric thinking. The marks of the post-test for every student in both the experimental and control group were analysed to determine the student's

Van Hiele level. Group averages were also calculated and were tested for statistical significance. The results were as follows.

Van Hiele Level 1: Visualisation

The results in Tables 4.7 and 4.8 show that there is no statistically significant difference in the average mark for the experimental group, at Van Hiele Level 1 (M = 7; SD = 0), and the control group (M = 5.28; SD = 1.99); t(45) = 45.03. p = 0.

Van Hiele Level 2: Analysis

The independent samples t-test results in Tables 4.9 and 4.10 show that there was no statistically significant difference in average marks of the experimental group at Van Hiele Level 2 (M = 13.45; SD = 1.18), and control group (M = 9.32; SD = 3.75); t(45) = 5.25; p = 0.

Van Hiele Level 3: Abstraction

The independent samples t-test results in Tables 4.11 and 4.12 show that there was a statistically significant difference in the average mark of the experimental group at Van Hiele level 3 (M = 10.36, SD = 5.18) and control group (M = 8.28, SD = 4.66); t(45) = 1.45, p = 0.15.

Van Hiele Level 4: Deduction

The independent samples t-test results in Tables 4.13 and 4.14 show that there was a statistically significant difference in the average mark of the experimental group at Van Hiele Level 4 (M = 12.23; SD = 7.55) and control group (M = 8.88; SD = 5.37), t(45) = 1.77; p = 0.08).

Van Hiele Level 5: Rigour

The independent samples t-test results in table 4.16 shows that there was statistically significant difference in the average mark of the experimental group at Van Hiele Level 5 (M = 5.64; SD = 3.49) and control group (M = 4.28; SD = 3.32); t(45) = 1.36; p = 0.18.

5.2.3 Research question three

Does GeoGebra software aid students' motivation to learn circle geometry? Research question 3 was investigated using a questionnaire comprising six attributes of scholarly motivation, namely participation, concentration, enjoyment, self-confidence, content mastery and recommendation of the teaching and learning method used. All six motivation attributes were statistically significant, confirming that the use of GeoGebra in the experimental group motivated students more than the traditional teaching and learning method did to the control group.

5.3 Discussion

This section discusses the three main findings of this study, namely the impact of GeoGebra on Grade 11 students' achievement, Van Hiele levels, and motivation.

The results of the analysis of the t-test on the performance of students taught using GeoGebra and those taught using the conventional method of instruction (talk-and-chalk) indicated a significant difference in achievement in favour of the students taught with GeoGebra. The students exposed to GeoGebra achieved a higher average score compared to the control group of students. The possible reasons for this finding could be that GeoGebra enabled students in the experimental group to check the correctness of their methods and the accuracy of their work. Being able to check one's own work goes a long way in determining achievement levels. Because GeoGebra is dynamic, students in the experimental group had opportunities of re-examining their work, while those in the control group could not do the same. In the control group, teaching was limited to a few examples, because drawing many diagrams on the chalkboard consumed both time and space.

In addition, the production of good-quality sketches requires competence in technical drawing skills, which not all teachers possess. GeoGebra-generated sketches are neat and accurate. GeoGebra allowed students in the experimental group real-time exploration opportunities. Consequently, this improved the learning process in terms of speed and quality (Ljajko & Ibro, 2013). When students learn using GeoGebra they spend less time drawing diagrams (sketches) and making calculations; this allows them more time to explore the characteristics of different circle theorems. All these factors could have contributed to the superior achievement of the experimental group.

It is virtually impossible to have passive students when computer technology, such as GeoGebra, is used in the teaching and learning process. GeoGebra changes passive students to independent explorers and the role of the teacher is to direct and monitor students' work. Mathematical concepts and procedures learnt using GeoGebra are long-lasting and better incorporated into students' cognitive structure, which makes them easier to apply (Ljajko & Ibro, 2013).

The findings of this study agree with those of Okoro and Etukudo (2001), Paul and Babaworo (2006), Egunjobi (2002) and Karper, Robinson and Casado-Kehoe (2013) that students taught with CAI packages in chemistry, mathematics and education in general, perform better than those taught with normal classroom instruction.

This study also concluded that the use of GeoGebra in the teaching and learning of circle geometry compared to conventional talk-and-chalk teaching and learning methods results in a significant difference in Level 1 visual and Level 2 analysis but not in the other three levels, namely, Level 3 abstraction; Level 4 deduction; and Level 5 rigour. Contrary to many studies on technology integration that have tended to portray the notion that technology integration yields significant positive differences when compared to traditional teaching methods, this study has shown that significant change depends on the Van Hiele levels.

The results for the Van Hiele analysis level (Level 2) show that the use of GeoGebra in the teaching and learning of circle geometry resulted in significant improvement of achievement of students' geometric thinking at Van Hiele Level 2. GeoGebra could have enabled students to recognise and name different circle theorems and also state the angle properties of those theorems because it is highly interactive and offers countless opportunities to repeat tasks and view them several times, hence students were able to internalise concepts at only these two levels (1and 2).

This finding is similar to that found by Donevska-Todorova (2015), who argued that analysis of mathematical concepts can be made easier by instructing students using technological devices and software because technology offers multiple representations rather than single and static representations that the teacher talk-and-chalk method offers. The results of Levels 1 and 2 are also similar to those obtained by Venkataraman (2012), who found that students taught with GeoGebra made progress towards mathematical explanations which provide a foundation for further deductive reasoning in mathematics (Levels 1 and 2). He concluded that the dynamic nature (drag feature) of the software influences the form of explanation and that students are able to generalize the solution and respond with a proper statement, especially at analysis level. Complex mathematical concepts can be simplified through repetition. Students using GeoGebra have the advantage of repeating difficult mathematical concepts, with countless chances to approach the same problem, and finally become rigorous students.

This study also argues that achievement at the analysis level can be possible if students are well drilled in certain activities and that GeoGebra is a very efficient teaching aid for this purpose.

The results of the other three levels (abstraction, deduction, and rigour) did not show any statistically significant difference of achievement between the experimental group and control group. This is possibly because at these levels students need to independently carry out sequences of logical analysis and presentation of a specific theorem, which may lead to a breakdown of the thought process if a student had not mastered the art of using GeoGebra. In this case, GeoGebra might not help much in attaining correct results for the abstraction, deduction and rigour levels.

Another reason why GeoGebra might not have produced significant effects could be that students had not had enough practice in the use of computers in the first place, and secondly of GeoGebra itself. Very few studies have reported no statistical significance when investigating Van Hiele levels using computer technologies. For example, Johnson (2002) found no significant statistical difference in achievement between students taught with dynamic geometry software and those taught with the traditional method. Johnson argued that teachers teaching with technological software may convert a CAI environment into a traditional one because of insufficient knowledge about DGS.

This study has revealed that using GeoGebra in teaching and learning not only increases students' achievement in general, but also increases achievement at specific Van Hiele levels such as 1 and 2, but also motivates students. All six motivation attributes affirm that GeoGebra enhances student motivation to learn circle geometry. Research provides extensive evidence of GeoGebra having positive motivational effects on geometry students (BECTA, 2013) in the form of increased participation in class activities, improved concentration in class, enjoyment during learning times, self-confidence, content mastery and ultimately recommendation of this teaching and learning method.

5.4 Implications of results

The results of this study have wide implications for mathematics teaching and learning. Improved achievements of students taught using GeoGebra confirm that the use of GeoGebra can reduce the effort devoted to tedious computations and increase students' focus on more important mathematical concepts. Equally importantly, GeoGebra could represent mathematics in ways that help students to understand concepts. In combination, these features would enable teachers to improve both how, and what, students learn. Researchers such as Ellington (2003) have argued that using Interactive geometry software in teaching mathematics, results in students being able to focus on various issues such as:

- More realistic or important problems;
- Exploration and sense-making with multiple representations;
- Development of flexible strategies;
- Mathematical meaning and concepts.

Researchers (e.g. Bransford, Brown & Cocking, 1999; Di-Sessa, 2001; Roschelle, Pear, Goding, Hoadley & Means, 2001) have found that when technology makes abstract ideas tangible, teachers can more easily

- Build upon students' prior knowledge and skills;
- Emphasize connections among mathematical concepts;
- Connect abstractions to real-world settings;
- Address common misunderstandings;
- Introduce more advanced ideas.

Another direct implication of this study is that there may be a need to make computers accessible to all students in schools and consequently the need to train teachers on the use of technology like GeoGebra in the teaching and learning process.

Another implication of using technology such as GeoGebra is a change in the roles of both teachers and students in the teaching and learning process. When teachers use GeoGebra to teach, they are no longer the centre of attention and dispenser of information, as they are with traditional teaching methods. Teachers assume new roles as facilitators, goal setters, and guides who support students. When students use GeoGebra to learn geometry, they assume active roles rather than the passive roles of receiving information from teacher or textbook. They actively make independent choices about how to move forward, and are in a position to define their own goals, make own decisions and evaluate their own progress.

5.5 Limitations of the study

This quasi-experimental research study was of limited scope. The population of the study was derived from Grade 11 mathematics students in Limpopo Province of South Africa. The sample for the study was two Grade 11 mathematics classes at the same school. Consequently, the scale and scope of this study is limited; the validity of this study's results should be tested further by conducting large-scale experimentation using larger samples than this one.

5.7 Recommendations for future research

Future studies on the effect of integration of GeoGebra on students' achievement, Van Hiele levels and motivation would demand comprehensive studies for longer periods, using far larger randomised sample sizes, at different schools of different ethnic composition and socio-economic status that reflect the entire South African economy. As the outcomes from this study show, a mathematics curriculum enriched by GeoGebra can significantly improve not only achievement on some Van Hiele levels, but also can increase motivation among students to learn geometry. This study recommends that mathematics teachers be encouraged to use this software in mathematics classes. Teachers should be introduced to the software in order to experience its effects on themselves and on their students.

This study's results also confirm the need for teachers to employ blended teaching and learning methods, in which computer technologies (software such as GeoGebra) are used simultaneously with the teacher talk-and-chalk teaching strategy. The blended teaching and learning process is a system that combines face-to-face instruction with computer mediated instruction.

The results of Van Hiele Levels 1 and 2 (visual and analysis) for this study showed significant differences after treatment, while Van Hiele Levels 3, 4 and 5 (abstraction, deduction and rigour) did not show significant differences after treatment. Students were able to use GeoGebra to visualise and analyse circle geometry tasks given to them, but were not able to abstract, deduct and carry out rigorous circle geometry tasks using the same GeoGebra. For this reason, blended teaching is highly recommended, where, for visualisation and analysis, students should be instructed with GeoGebra, and for abstraction, deduction and rigour, students should be instructed with the talk-and-chalk method of instruction.

Lloyd-Smith (2010) argued that blended instruction offers more choices for content delivery and is more effective than teaching either fully online or fully classroom-based. In their study Garnham and Kaleta (2002) reported that students learn more in blended learning environments than they do in comparable traditional classes. Blended teaching offers advantages to both the school and the students. The method of instruction is not over-reliant on the physical presence in one room of both the teacher and the student, and it offers greater flexibility for students to carry out their work independently (Lloyd-Smith, 2010).

This study further recommends qualitative research to investigate in depth the root and causes of the effects described in this study. Anderson and Arsenaut (1998, p. 119) argue that the

> Fundamental assumption of the qualitative research paradigm is that an insightful understanding of the world can be gained through observation and conversation in natural settings rather than through experimental manipulation under fabricated conditions.

While quantitative researches seek causal determination, prediction, and generalization of findings, qualitative researchers seek instead illumination, understanding, and extrapolation to similar situations (Strauss & Corbin, 1990, p. 17). As a result, a far more comprehensive study should incorporate qualitative methods of data analysis.

In addition, future research should extend this study to other mathematics topics and grades to see if similar results are obtainable. The findings from such studies might help to improve the quality of mathematics teaching and learning in South Africa.

5.8 Concluding remarks

The study has determined that the use of GeoGebra improves students' achievement, improves students' geometric thinking at some Van Hiele levels (Levels 1 and 2) and motivates students to learn circle geometry. Based on the findings of the study, the researcher recommends GeoGebra assisted instruction in the teaching and learning of geometry. Motivation is the key determinant of student achievement; hence any teaching and learning method that motivates learners to learn will go a long way in solving the South African problem of poor achievement in geometry in particular and poor achievement in mathematics in general.

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APPENDIX A
DATA COLLECTION INSTRUMENTS

A1- Pre-test Question Paper

ANNEXURE A: PRE-TEST QUESTION PAPER

INSTRUCTIONS

1. Answer all questions
2. Choose the best answer and write the letter corresponding to the correct answers in the answer sheet provided.

MARKS: 15 TIME: 30 minutes

Question 1

$\Delta ABC \equiv \Delta A^{I}B^{I}C^{I}$, ∡$C = 3x - 40$ and $C^{I} = 2x - 10$.

Find C.

A. 15

B. 30

C. 50

D. 90

Question 2

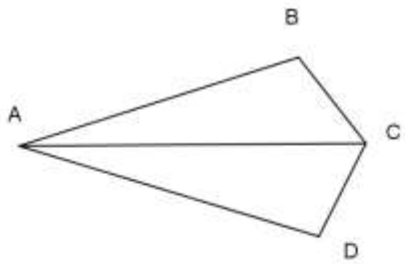
Given: AC bisects ∡∡BAD and ∡BCD. Which of the following methods can be used to prove $\Delta ABC \equiv \Delta ADC$?

A. Side-Angle-Side (SAS)

B. Angle-Side-Angle (ASA)

C. Angle-Angle-Side (AAS)

D. There is insufficient information to determine if the triangles are congruent.



Question 3

In triangle ABC, ∡A = 48⁰, and ∡C = 24⁰.

What type of triangle is triangle ABC?

A. acute

B. right

C. obtuse

D. isosceles

Question 4

In the diagram at the right, AC ≡ DC ≡ DB.

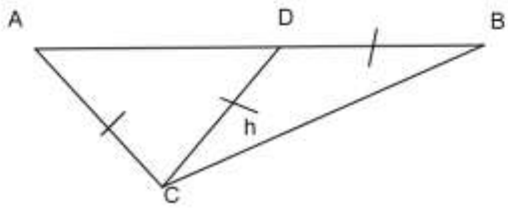
If the ∡ACD = 48⁰, find the ∡B

A. 24⁰

B. 33⁰

C. 48⁰

D. 66⁰⁰



Question 5

Two sides of an isosceles triangle measure 3 and 7.

Which of the following could be the measure of the third side?

A. 9

B. 7

C. 5

D. 3

Question 6

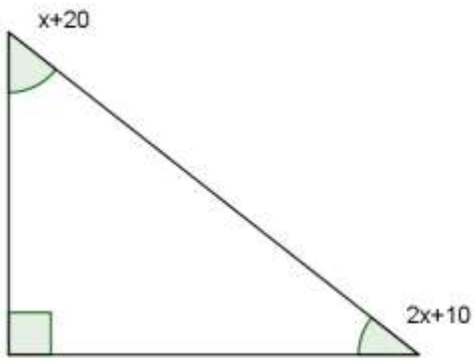
The diagram shows a right triangle. What is the value of x?

A. 15°

B. 20

C. 24°

D. 30°



Question 7

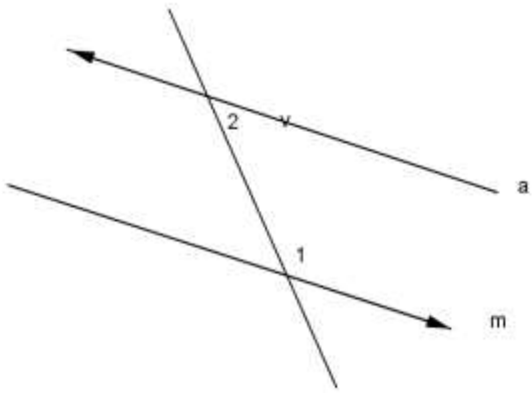
In the diagram, parallel lines a and m are cut by transversal t, angle maqrked 1 is 4x+16 and angle marked 2 is 2x-22. Find the numerical value of 1.

A. 31°

B. 40°

C. 140°

D. 148°



Question 8

A quadrilateral must be a parallelogram if one pair of opposite sides is

A. congruent, only.

B. parallel, only

C. congruent and parallel

D. parallel and the other pair of opposite sides is congruent

Question 9

The perimeter of a rhombus is 60. If the length of its longer diagonal measures 24, the length of the shorter diagonal is

A. 20

B. 18

C. 15

D. 9

Question 10

A parallelogram must be a rectangle if its diagonals

A. bisect each other

B. bisect the angles to which they are drawn

C. are perpendicular to each other

D. are congruent

Question 11

Which statement is true about all parallelograms

A. The diagonals are congruent

B. The are is the product of two adjacent sides.

C. The opposite angles are congruent.

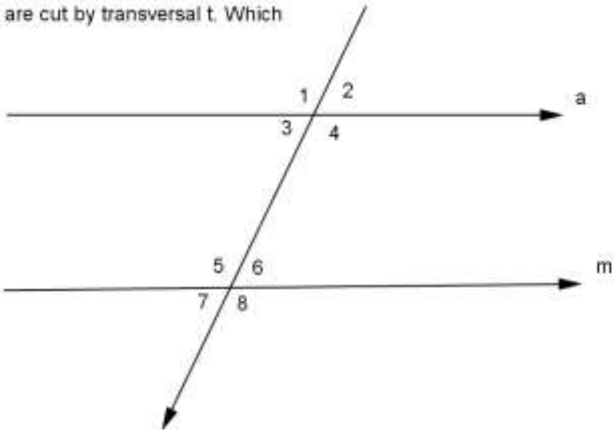
D. The diagonals are perpendicular to eac other.

Question 12

In the diagram, the lines a and m are parallel and are cut by transversal t. Which two angles are not always congruent?

A. 4 and 6B. 1 and 8

C. 4 and 5D. 2 and 3



Question 13

In the diagram, line a is parallel to line b, and line t

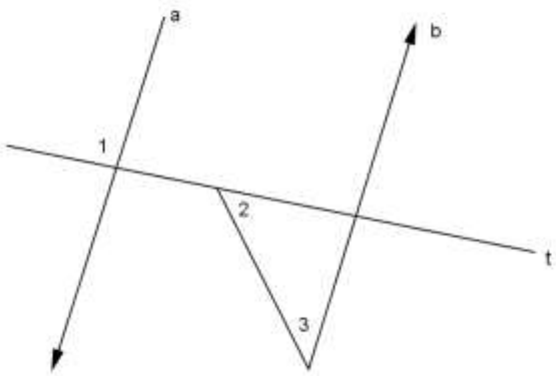
t is a transversal. If angle 1 is 97° and angle 2 is 44°, find angle 3.

A. 44°

B. 53°

C. 83°

D. 97°



Question 14

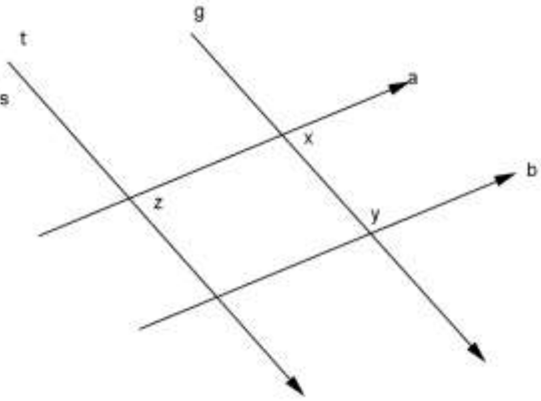
In the diagram, lies f and g are parallel, and lines a and b are parallel. The angle x= 75°. What is the value of angles y+z?

A. 75°

B. 105°

C. 150

D. 180°



Question 15

Which property is true for all trapezoids?

A. Only two opposite sides are parallel.

B. Consecutive angles are supplementary.

C. The base angles are congruent.

D. All angles are equal.

A2-MARKING GUIDELINE FOR PRE-TEST

1. C
2. B
3. A
4. D
5. B
6. B
7. A
8. C
9. B
10. D
11. C
12. A
13. B
14. D

A3- Post-Test Question Paper

APPENDIX: POST-TEST

INSTRUCTIONS

1. This test has three sections:
   Section A: Multiple choice questions (1-14)
   Section B: Short answer questions (15-28)
   Section C: Formal Proofs questions (29-30)
2. Answer ALL questions.
3. Write your answers on the answer sheet(s) provided
4. For questions 15-30, show all necesary working.

MARKS: 80 TIME: 2 HOURS

SECTION A: MULTIPLE CHOICE QUESTIONS

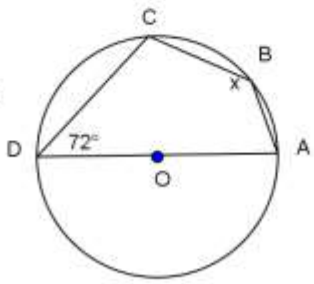
1. In the diagram, AD is a diameter, and ∡CDA=72° , find x

A. 48°

B. 108°

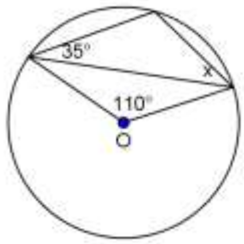
C. 45°

D. 36°

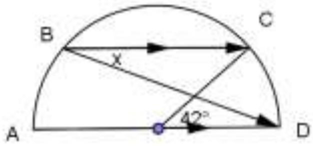


2. In the diagram, O is the centre of the circle. Find x

A. 20°
B. 45°
C. 48°
D. 49°

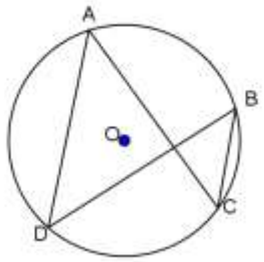


3. In the diagram.ABCD is a semi-circle and BC//AD.
IF ∡COD= 42°, then x= ---
A. 48°
B. 63°
C. 84°
D. 90°



4. In the diagram, which pair of angles confirms the circle theorem that: Angles subtended by the same segiment are equal?

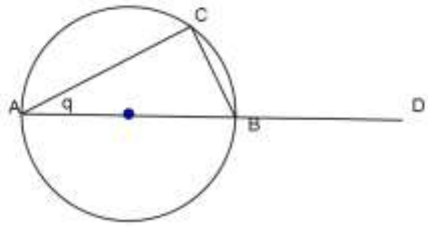
A. (∡A and ∡B) and (∡C and ∡D)
B. (∡C and ∡B) and (∡C and ∡D)
C. (∡A and ∡B) and (∡A and ∡D)
D. (∡A and ∡D) and (∡C and ∡D)



15.

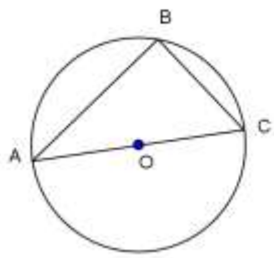
5. In the diagram, AB is a diameter of the circle and ABD is a straight line. Angle ∡CAD is q°. Then ∡CBD=

A. 2q
B. 4q
C. 90° + q
D. 180° - q



USE THE DIAGRAM BELOW TO ANSWER QUESTIONS 6 &7

A, B and C are points on the circuference of a circle, centre O. AC is a diameter of the circle.

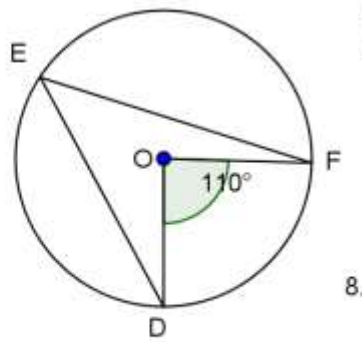


6. What is the size of angle B?

A. 45° B. 60° C. 30° D. 90°

7. Give a reason for your answer in question 6.
A. Angles subtended by the same arc are equal
B. Sum of angles in a right angled triangle is 180°
C. The angle subtended by the same chord are equal
D. The diameter subtends an angle of 90° at the cricumference

USE THE GIVEN DIAGRAM FOR QUESTIONS 8 & 9



D, E and F are points on the circumference of a circle, cenre O. Angle DOF = 110°.

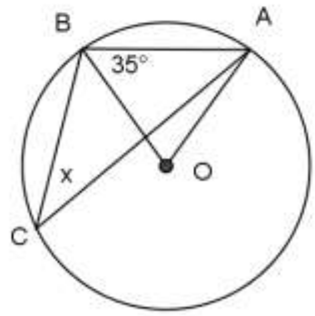
8. What is the size of angle DEF?

A. 220° B. 55° C. 50° D. 230°

9. Give a reason for your answer in question 8.

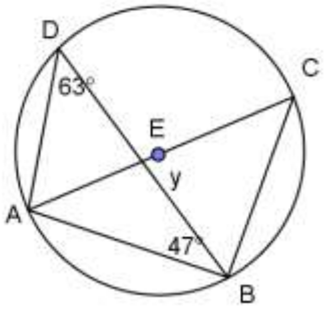
A. Opposite interior angles of a cyclic quadrilateral are supplementayr.
B. sum of angles at a point add up to 180°
C. The angle subtended at the centre is twice that at the circumference
D. Alternate angles are supplementary

10. In the diagram below, find x



A. 35°
B. 70°
C. 110°
D. 55°

11. In the diagram, AEC is a diameter and DB is a straight line.
Find y.



A. 74°
B. 126°
C. 63°
D. 47°

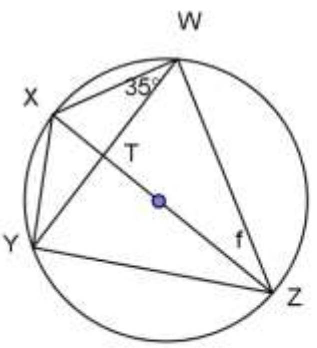
12. Which property does not describe the parallelogram?

A. Both pairs of opposite sides are parallel.
B. Adjacent angles are equal.
C. Both pairs of opposite angles are equal.
D. Both diagonals bisect each other.

13. Complete the theorem: Perpendicular line drawn from the centre of a circle----------------

A. perpendicular to a chord, bisects the chord.
B. is parallel to the chord.
C. is proportional to the chord.
D. forms Pythagoras' theorem.

14. Given circle with centre O, WT = TY and XWT = 35°
Determine f.

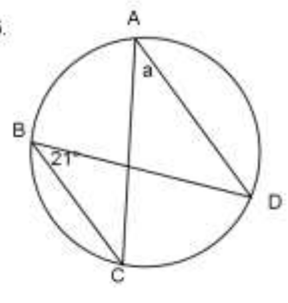


A. 35°
B. 70°°
C. 55°
D. 11o°

SECTION B (Questions 15-28)

Find the the values of the unknown angles, show your working and state reasons for your answers in this section

15.



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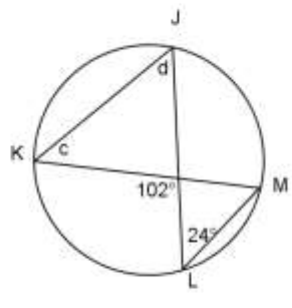

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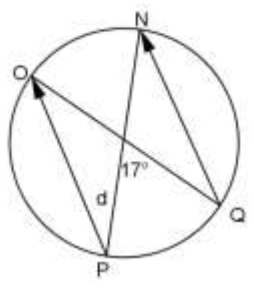

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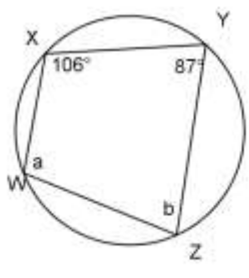

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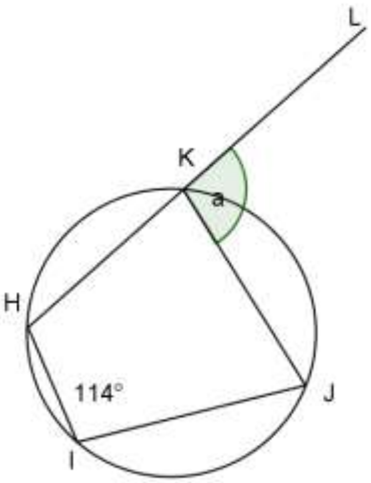

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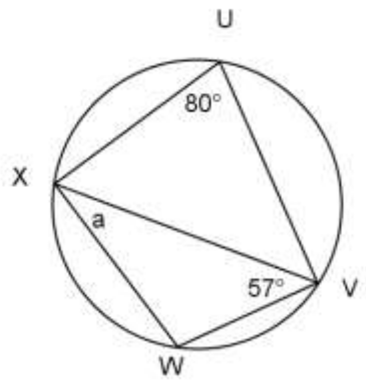

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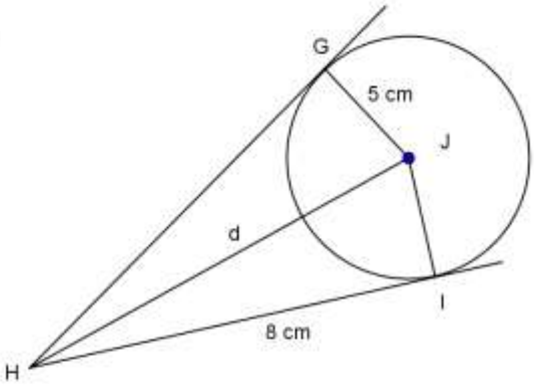

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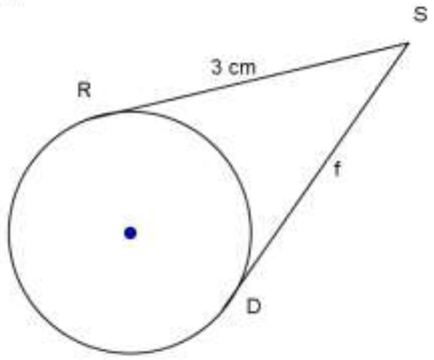

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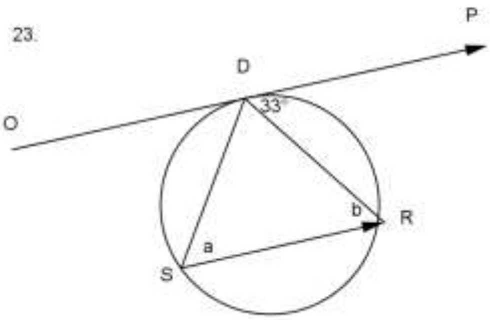

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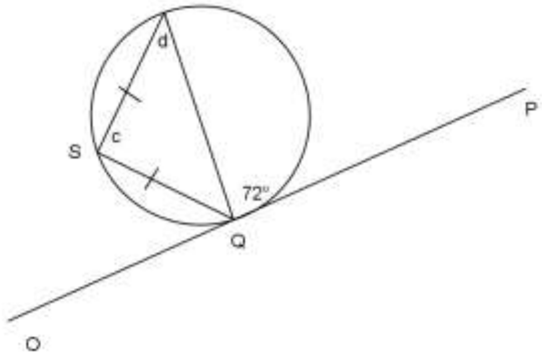
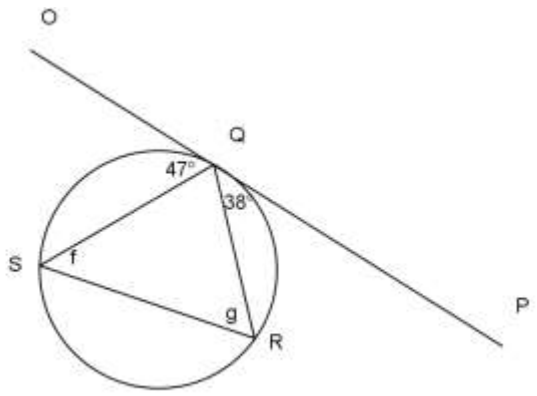
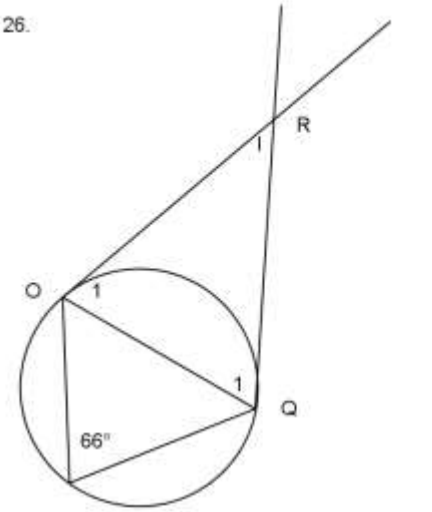


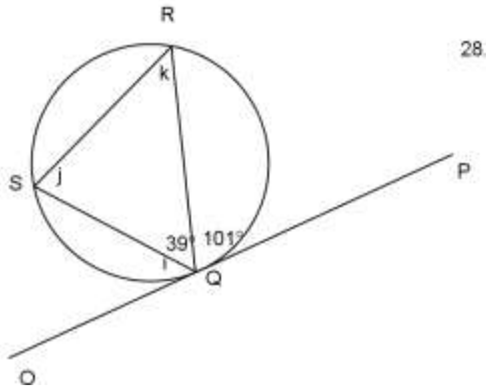
25.



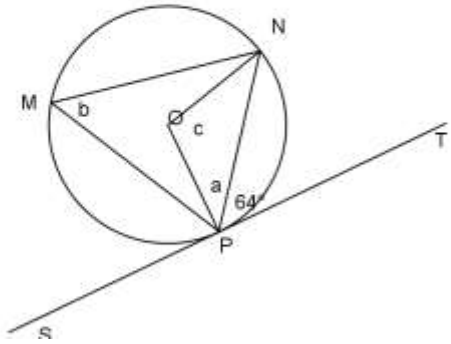
26.



27.



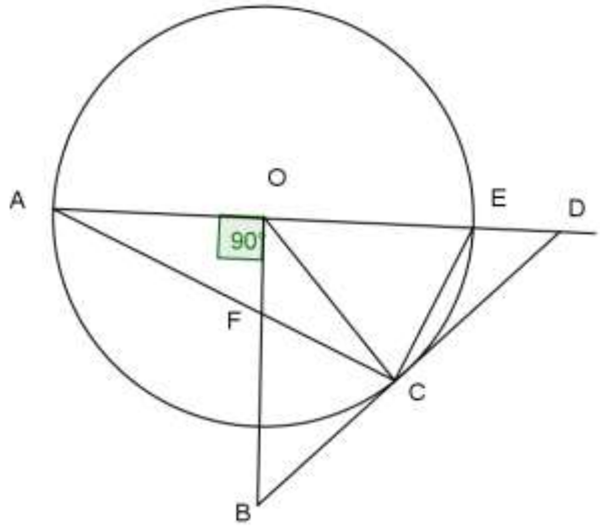
28.



SECTION C - FORMAL PROOFS

29. BD is a tangent to the circle with centre O, BO is perpendicular to AD.

Prove that: CFOE is a cyclic quadrilateral



30. Prove the theorem: A line drawn from the centre of a circle perpendicula to a chord, bisects the chord.

A4- MARKING GUIDELINE FOR POST TEST

1. B
2. A
3. B
4. A
5. C
6. D
7. D
8. B
9. C
10. D
11. A
12. B
13. A
14. A

15. a = 21° (Angles subtended by the same arc equal)

16. c = 24° (Angles subtended by the same arc are equal)
d = 102°-24° = 78° (Exterior angle of a triangle is equal to two opposite interior angles)

17. 17°/2 = 8.5° (OPQN is a parallelogram)
(Exterior angle of a triangle is equal to two opposite interior angles)

18. a = 180°-87°= 93° (Opposite angles of a cyclic quadrilateral are supplementary)
b = 180°-106° = 74°

19. a = 114°° (Exterior angle of a cyclic quadrilateral is equal to opposite interior angle)

20. W = 180°-80°= 100° (Opposite angles are supplementary)
a = 180°- (100°+57°) = 23° (sum of angles in atriangle)

21. $d^2 = 8^2 + 5^2$ (Pythagoras' theorem)
= 89
= $\sqrt{89}$

22. r = 3 (Two tangents from the same point are equal)

23. a = 33°
b = 33° (Angle between tangent and chord is equal to the angle
subtended by the chord in the alternate segiment)

24. c = 72° (Angle between tangent and chord is equal to the angle
subtended by the chord in the alternate segiment)
d = (180°-72°)/2 = 54° (Angles opposite equal sides)

25. f = 38° (Angle between tangent and chord)
g = 47°

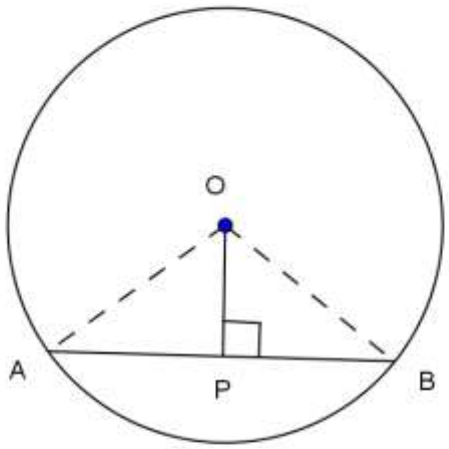
26. i = 180° - (66° + 66°) = 48° (Tan chord theoren)
(Sum of angles in an isosceles triangle)

27. j = 101°
k = 180° - (102° + 39°) = 40°
i = 40°

28. b = 64° (Tan chord theorem)
c = 128° (centre and circumference)
a = 26° (radius and tangent)

29. BO ⊥ OD given
∴ FOE = 90°
∴ FCE = 90° (∠ in semi circle)
∴ CFOE is a cyclic quadrilateral (opp ∠s of a cyclic quadrilateral are supplementary)

30.



PROOF

Draw OA and OB

In ΔOPA and in ΔOPB

$OA^2 = OP^2 + AP^2$ (Pythagoras)

$OB^2 = OP^2 + BP^2$ (Pythagoras)

and OA = OB (radii)

$\therefore AP^2 = BP^2$

$\therefore AP = BP$

ALTERNATIVE PROOF

In ΔOPA and in ΔOPB,

$\measuredangle OPA = \measuredangle OPB$ (given)

OA = OB (equal radii)

OP = OP (common side)

$\therefore \Delta OPA \cong \Delta OPB$ (RHS)

$\therefore AP = PB$

$\therefore$ OP bisects AB.

A6- Questionnaire

POST-TEACHING AND POST-TEST QUESTIONNAIRE

Personal Particulars:

Name: ________________________ Class___________ Gender___________

Instruction

The following questions relate to the teaching method used by your teacher to teach Circle Theorems (Circle Geometry). Answer each question with YES or NO.

	Answer	
Question	YES	NO
1. Did the teaching method encourage you to participate in class activities?		
2. Did the teaching method enable you to concentrate during and after lesson delivery?		
3. Did you enjoy the lesson(s)?		
4. Did the teaching/learning method instil self-confidence in you?		
5. Were you able to master the content after teaching/learning in your class?		
6. Would you recommend the use of the teaching/learning method used in your class for future geometry lessons?		

APPENDIX B: RESULTS AND CALCULATIONS

B1-Van Hiele Levels Results for Control Group Post-test

student	Level 1: Visual							Level 2: Analysis							Level 3: Abstraction							Level 4: Deduction							Level 5: Rigor		Total	%
	Question Number							Question Number							Question Number							Question Number							Question Number			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30		
1	1	0	1	0	1	1	0	2	0	0	0	0	0	0	2	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	12	15
2	1	1	1	0	1	1	1	0	1	0	1	0	1	0	1	0	0	1	2	0	0	0	0	1	1	1	0	0	0	0	16	20
3	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	2	2	0	3	3	3	3	4	4	0	3	0	4	4	5	64	80
4	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	1	3	1	3	4	0	4	0	4	4	0	0	0	54	67
5	1	1	0	1	1	1	1	2	0	2	2	2	2	2	0	2	1	0	2	0	2	1	0	3	0	3	4	0	1	0	37	46
6	1	1	1	0	1	1	1	2	2	2	1	1	2	1	0	2	2	2	2	3	2	1	0	1	2	1	0	1	2	4	42	52
7	1	0	1	1	1	1	1	2	2	0	2	2	1	0	0	0	0	1	1	0	3	0	0	3	3	1	1	0	2	2	32	40
8	1	1	1	1	1	0	1	0	1	2	2	1	1	2	2	1	3	0	2	2	0	1	3	0	2	1	2	0	3	4	41	51
9	1	1	1	1	1	1	1	1	1	2	2	2	0	2	1	0	3	2	2	0	0	3	0	2	0	1	0	2	3	3	39	49
10	1	1	1	1	1	1	0	2	2	2	2	2	2	2	3	3	0	1	3	0	3	0	3	2	3	4	3	4	5	5	62	78
11	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	2	2	0	2	0	3	0	2	3	0	4	2	3	5	4	56	70
12	1	1	1	0	1	1	1	1	2	2	1	2	2	2	2	0	3	0	2	0	3	1	2	1	2	1	1	4	4	4	48	60
13	1	1	0	1	0	1	0	2	2	2	2	1	1	0	1	2	0	0	0	1	2	2	0	0	2	0	2	0	2	2	30	37
14	1	0	0	0	1	1	1	0	0	1	2	2	0	2	0	1	0	1	0	1	0	0	2	1	0	1	0	2	1	1	22	27
15	1	1	1	1	0	1	0	2	1	2	2	1	0	2	0	0	3	0	2	0	1	0	3	2	1	0	2	0	2	3	34	43
16	1	0	1	0	0	0	0	2	1	0	2	1	1	1	0	0	1	1	0	0	0	0	0	0	0	1	0	1	1	1	16	20
17	1	1	0	1	1	1	1	1	1	0	2	0	1	0	0	0	0	1	0	1	0	0	2	1	0	0	1	0	1	2	20	25
18	1	1	1	1	1	1	1	2	1	2	2	0	0	1	1	2	0	3	2	2	1	1	2	3	4	0	1	0	1	3	41	51
19	0	1	1	1	0	0	0	2	0	0	2	1	2	0	0	1	2	0	1	1	0	0	1	2	2	1	0	1	1	1	24	30
20	0	1	0	0	0	0	0	0	1	0	2	0	0	0	1	0	0	0	0	2	0	1	0	0	0	0	0	0	0	0	8	10
21	0	0	1	0	1	1	1	1	2	1	2	1	2	1	3	0	2	0	0	0	2	1	0	1	0	0	0	0	1	1	25	31
22	1	1	1	0	1	1	0	2	2	2	2	2	2	2	3	3	2	0	3	0	1	0	1	3	4	2	4	0	5	3	53	66
23	0	0	1	0	1	0	1	0	0	2	1	0	1	0	0	0	3	0	0	0	0	0	0	1	1	0	0	0	0	0	12	15
24	1	1	1	1	1	1	1	2	2	2	2	0	2	2	2	3	3	0	3	0	3	0	4	3	1	3	2	0	5	3	54	67
25	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	2	3	1	0	3	2	0	2	3	3	4	0	1	4	3	55	69
Tota	21	19	20	15	20	20	17	36	32	34	44	29	31	30	34	30	38	16	35	20	34	20	32	44	31	36	29	23	53	54		

B2- Van Hiele Levels Results for Experimental Group Post-test

Student	Level 1 Visual							Level 2 Analysis							Level 3 Abstraction							Level 4 Deduction							Level 5 Rigor		Total	%
	Question Number							Question Number							Question Number							Question Number							QN			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30		%
1	1	1	1	1	1	1	1	2	1	1	2	0	2	2	1	2	3	0	1	2	1	1	2	2	2	0	1	1	1	0	37	46
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	2	2	3	3	3	4	4	4	4	4	4	4	5	5	78	98
3	1	1	1	1	1	1	1	2	2	2	2	2	2	2	1	2	1	0	2	2	3	0	2	0	0	1	2	3	3	3	46	58
4	1	1	1	1	1	1	1	0	2	2	2	0	2	2	1	0	1	1	0	1	0	2	0	2	1	0	2	0	1	1	30	37
5	1	1	1	1	1	1	1	2	2	2	2	2	2	1	1	2	2	2	0	2	0	1	3	0	1	3	0	2	5	0	44	55
6	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	2	0	3	3	3	1	3	1	2	3	3	2	4	4	60	75
7	1	1	1	1	1	1	1	2	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	21	26
8	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	1	2	3	1	2	2	0	0	1	1	1	0	0	2	3	43	54
9	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	2	1	3	1	0	3	2	0	2	3	2	0	3	2	50	62
10	1	1	1	1	1	1	1	2	2	2	2	1	2	2	1	2	3	0	3	0	0	3	0	3	0	0	0	1	3	1	40	50
11	1	1	1	1	1	1	1	2	2	2	2	2	2	2	1	0	0	0	0	0	0	0	3	2	2	0	0	0	0	0	29	36
12	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	3	2	2	2	2	4	3	3	4	2	1	3	4	5	65	81
13	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	2	3	2	2	2	2	4	3	3	4	2	1	3	4	5	66	83
14	1	1	1	1	1	1	1	2	1	2	2	2	2	2	1	3	3	2	3	0	3	4	3	4	4	4	3	4	5	5	71	89
15	1	1	1	1	1	1	1	2	2	2	2	2	2	2	0	1	2	0	3	1	0	3	1	2	1	0	1	2	2	3	43	54
16	1	1	1	1	1	1	1	2	2	2	2	2	2	2	0	2	2	0	3	2	3	0	2	3	3	0	3	3	2	3	52	65
17	1	1	1	1	1	1	1	2	2	2	2	1	2	2	2	1	0	2	2	1	0	0	1	0	1	0	1	1	1	3	36	45
18	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	2	1	2	3	1	2	1	4	0	3	2	0	3	3	4	55	69
19	1	1	1	1	1	1	1	2	2	2	2	2	2	2	1	1	2	0	2	0	0	0	0	3	2	1	2	2	5	4	46	57
20	1	1	1	1	1	1	1	2	2	2	2	2	2	2	1	3	0	3	3	0	3	4	4	4	0	4	0	4	5	5	64	80
21	1	1	1	1	1	1	1	2	2	2	2	2	2	2	0	3	0	3	3	2	3	4	4	3	4	3	3	4	4	5	69	86
22	1	1	1	1	1	1	1	2	2	2	2	2	2	2	0	0	0	0	1	2	0	0	1	0	0	2	0	2	0	0	29	36
Total	22	22	22	22	22	22	22	42	42	43	44	38	44	43	29	38	34	25	43	29	30	39	45	40	41	35	29	44	62	61		

B3- Motivation Questionnaire Results

Experimental Group Motivation Scores

Student	Question 1	Question 2	Question 3	Question 4	Question5	Question 6	Question7	Total	%
1	1	1	1	1	1	0	1	6	86
2	1	1	1	1	1	1	1	7	100
3	1	1	1	1	1	1	1	7	100
4	1	1	1	1	1	1	1	7	86
5	1	1	1	1	1	1	1	7	100
6	1	1	1	1	1	1	1	7	100
7	1	1	1	1	1	0	1	6	86
8	1	1	1	1	1	1	1	7	100
9	1	1	1	1	1	1	1	7	100
10	1	1	1	1	1	1	1	7	100
11	0	1	0	1	0	0	1	3	43
12	1	1	1	1	1	1	1	7	100
13	1	1	1	1	1	1	1	7	100
14	0	1	1	1	0	0	1	4	57
15	1	1	1	1	1	1	1	7	100
16	1	1	1	1	1	1	1	7	100
17	0	1	1	1	0	0	1	4	57
18	1	1	1	1	1	1	1	7	100
19	1	1	1	1	1	1	1	7	100
20	1	1	1	1	0	0	1	5	71
21	1	1	1	1	1	1	1	7	100
22	1	1	1	1	1	1	1	7	100

Control Group Motivation Scores

Student	Question 1	Question 2	Question 3	Question 4	Question5	Question 6	Question7	Total	%
1	1	1	1	1	1	1	1	7	100
2	1	0	0	0	0	0	0	1	14
3	0	0	0	1	1	1	1	4	57
4	1	1	1	1	1	0	1	6	86
5	1	0	0	0	0	0	0	1	14
6	1	0	0	0	1	1	1	4	57
7	0	0	0	0	0	0	1	1	14
8	1	1	1	1	1	1	1	7	100
9	1	0	0	0	0	0	1	2	29
10	1	1	1	1	1	1	1	7	100
11	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0
13	1	0	0	1	1	0	1	4	57
14	1	1	1	1	1	1	1	7	100
15	0	0	0	0	0	0	0	0	0
16	1	0	0	0	0	0	0	1	14
17	1	0	0	0	1	1	1	4	57
18	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0
20	1	0	1	1	1	1	1	6	86
21	0	0	0	0	0	0	1	1	14
22	0	0	0	0	0	0	0	0	0
23	1	0	0	0	1	1	1	4	57
24	1	1	1	1	1	1	1	7	100
25	1	0	1	0	1	0	1	4	57

B4- Split-half Reliability – Spearman Brown

Spearman-Brown First Half
Question Number

Student	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	1st Half Total
1	1	1	0	1	1	1	1	2	2	1	2	2	0	1	3	**19**
2	1	1	1	1	1	1	1	2	2	2	2	2	0	2	5	**24**
3	0	0	0	1	0	0	1	2	1	2	0	1	1	1	3	**13**
4	1	1	1	1	1	1	0	1	0	0	1	1	0	2	5	**16**
5	1	0	1	0	1	0	0	2	2	0	1	1	1	1	0	**11**
6	1	1	1	1	0	1	1	1	1	1	0	0	0	1	4	**14**
7	1	1	0	1	1	1	1	2	2	2	1	2	1	2	3	**21**
8	1	0	1	1	1	1	1	0	1	2	1	1	0	1	4	**16**
9	1	1	1	1	1	0	0	0	1	0	1	2	0	0	0	**9**
10	1	0	1	1	1	0	1	1	2	2	2	2	1	2	5	**22**
11	1	0	1	0	1	0	1	1	1	0	1	0	1	0	2	**10**
12	1	0	1	1	1	0	0	1	0	2	2	2	1	1	4	**17**
13	1	1	0	0	1	1	0	2	2	2	1	2	1	1	3	**18**
14	1	1	0	0	1	1	1	2	2	2	2	1	1	1	5	**21**
15	1	1	0	1	1	1	1	2	1	2	1	1	1	0	4	**18**

Spearman-Brown Second Half
Question Number

Student	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	2nd Half Total
1	1	0	0	0	0	0	0	1	2	2	2	1	2	2	2	**15**
2	1	1	1	1	1	1	1	2	2	2	2	1	1	2	5	**24**
3	0	0	1	0	1	0	0	0	0	0	0	0	0	2	0	**4**
4	1	0	1	0	1	0	1	1	2	2	2	1	2	2	2	**18**
5	1	1	1	1	1	1	1	2	2	2	2	0	1	1	5	**22**
6	1	1	0	1	1	0	0	0	2	0	0	1	0	2	5	**14**
7	1	1	1	1	0	1	1	2	0	2	2	0	2	1	1	**16**
8	1	1	1	0	1	0	1	2	1	0	0	0	0	2	4	**14**
9	0	0	0	0	1	1	1	2	0	0	0	0	2	2	0	**9**
10	1	0	1	0	1	1	0	2	2	2	2	1	1	2	5	**21**
11	1	0	0	0	1	0	0	0	0	2	2	0	1	2	3	**12**
12	0	0	0	1	0	0	1	2	2	2	2	1	2	2	5	**20**
13	1	0	1	0	0	0	1	0	1	2	2	1	0	1	4	**14**
14	1	1	1	1	1	1	1	2	2	2	2	0	2	2	5	**24**

B5-KUDER-RICHARDSON FORMULA 20

Question Number

Student Number	Q1	Q1	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Total
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	14
2	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	13
3	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	11
4	1	1	1	1	1	1	1	0	1	0	1	0	1	0	0	10
5	1	1	1	1	1	0	1	0	0	1	0	1	0	0	0	8
6	1	1	1	1	1	1	0	0	1	0	1	0	0	0	0	8
7	1	1	1	1	1	1	0	0	0	0	0	1	0	0	0	7
8	1	1	1	0	0	0	0	1	1	0	0	1	0	0	0	6
9	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	6
10	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	5
11	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	5
12	1	1	0	1	0	1	0	0	0	0	0	0	0	0	0	4
13	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	3
14	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	4
15	1	0	1	0	0	1	0	1	0	0	0	0	0	1	0	5
Total	**15**	**13**	**14**	**11**	**8**	**9**	**7**	**5**	**6**	**4**	**5**	**5**	**4**	**2**	**1**	

P	1.00	0.87	0.93	0.73	0.53	0.60	0.47	0.33	0.40	0.27	0.33	0.33	0.27	0.13	0.07	
Q	0.00	0.13	0.07	0.27	0.47	0.40	0.53	0.67	0.60	0.73	0.67	0.67	0.73	0.87	0.93	
Pq	0.00	0.12	0.06	0.20	0.25	0.24	0.25	0.22	0.24	0.20	0.22	0.22	0.20	0.12	0.06	2.59

K	15
Spq	2.59
Var	10.6
P	0.81

APPENDIX B: MOTIVATION ATTRIBUTES FIGURES

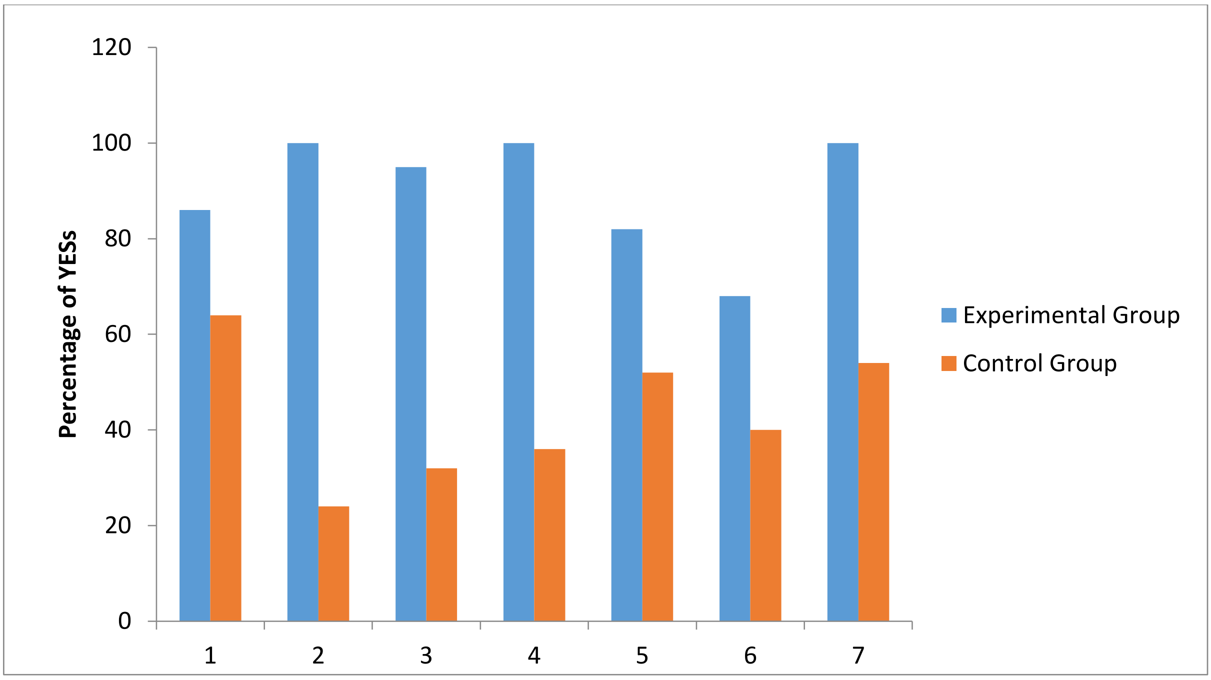


Figure 4.1 Comparison of percentages of Yes responses in motivation questionnaire

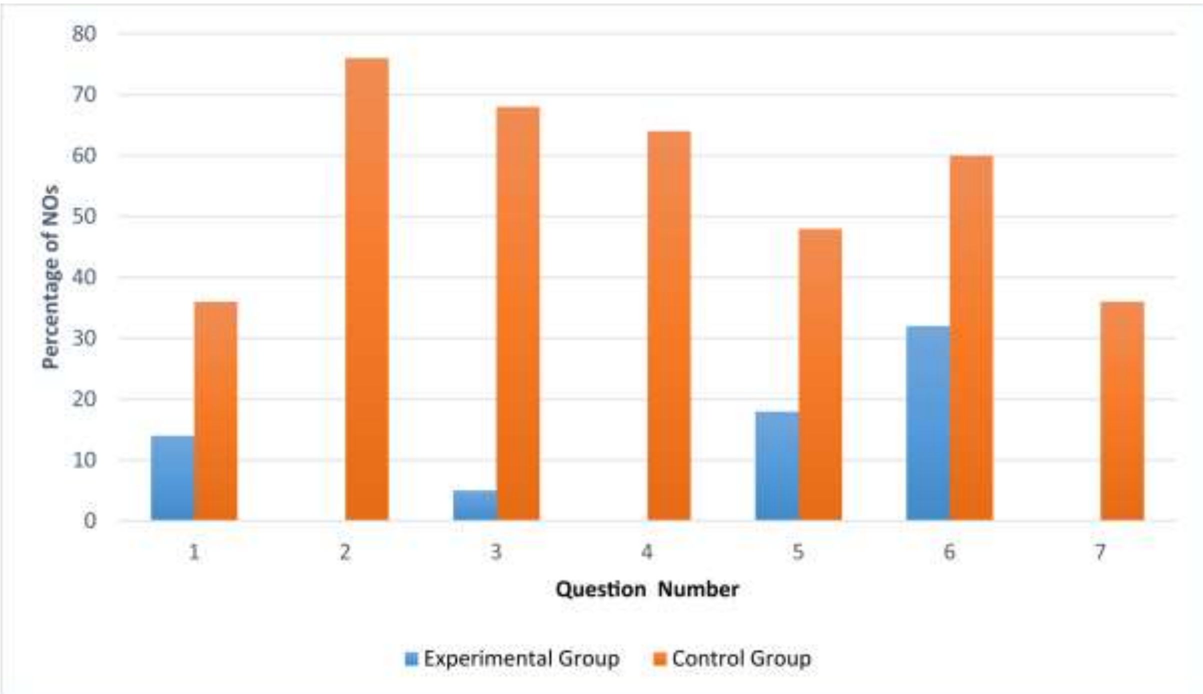


Figure 4.2 Comparison of percentages of No responses in motivation questionnaire

APPENDIX C: ETHICAL ISSUES

This study complies with the UNISA Ethics Policy that applies to all UNISA affiliates who are involved in research on or off campuses of UNISA. To achieve compliance, the UNISA Ethical Clearance Application Form has been completed and will be submitted to the researcher's supervisor for submission to the research Ethics forum (College Research Committee) for approval. The attached consent forms are given below:

C1

Miriyavhavha Technical Secondary School
P.O Box 520
Khakhu
0974
01 October 2013

THE CIRCUIT MANAGER NZHELELE EAST CIRCUIT
P. Bag X 717 Nzhelele
0993
Tel. 015 973 0677 Fax: 015 973 0519

Dear Sir

REF: REQUEST FOR PERMISSION TO CARRY OUT AN EDUCATIONAL RESEARCH (MATHEMATICS) WITH SECONDARY SCHOOLS (GRADE 11) STUDENTS IN YOUR CIRCUIT

I am a mathematics educator at the above mentioned school, and I am currently studying for a masters' degree in Mathematics, Science and Technology education. To complete my studies, I have to submit a dissertation (research project) on my area of study.

I hereby request for permission to carry out an educational research within the schools in your circuit.

Attached is the abstract of my dissertation, just to give you a brief overview of the nature of my study. Also the consent letter for my subjects (students to be involved in the study is also attached).

The actual period for the study is projected to run from January 2014 to March 2014. Only afternoons (after formal schooling time) will be used for this study.

I am looking forward to a favourable response.

Yours faithfully

CHIMUKA ALFRED

C2

Miriyavhavha Technical Secondary School
P.O Box 520
Khakhu
0974
01 October 2013

THE PRINCIPAL MIRIYAVHAVHA TECHNICAL SECONDARY SCHOOL

Dear Sir

REF: REQUEST TO USE THE SCHOOL'S COMPUTER LABORATOR FOR AN EDUCATIONAL RESEARCH

I hereby make a formal request to use the school's computer laboratory for purposes of conducting an educational research in Mathematics education. Six groups of fifteen grade 11 students each (randomly selected from Nzhelele East Circuit) will be taught Mathematics using computers during afternoons on school days for a period of about two weeks.

The research team, myself as team leader, will be responsible for the safe use of the computers in the laboratory and will be liable for any damages during the research period.

If you wish to get more information, please fill free to ask me any questions regarding this intended study.

I am looking forward to your permission.

Yours faithfully

Chimuka A

C3- INFORMED CONSENT/ PERMISSION FORM FOR TRANSPORTATION OF STUDENTS

(For all Students)

This form should be completed together with the INFORMED CONSENT AND ASSENT FORM BY parents and students.

In order for students participating in this research study to arrive and dismiss at designated schools for afternoon lesson delivery, the research team has arranged for transportation of students to and from lesson delivery venue. This situation presents an element of risk to all students that will be transported.

The research team seeks your consent/permission to transport your child using a recommended passenger transport. By consenting to this request, you are accepting the risk that may befall your child during the transportation or any activity done during the research period

____________________________ Date:_______________________________

Student Please print your name

____________________________ Date:_______________________________

Parent Please print your name

Date:_______________________ Date:_______________________________

C4- INFORMED CONSENT AND ASSENT FORM-PARENT/GUARDIAN

(Students under 18 years)

NAME OF STUDY: THE EFFECT OF INTEGRATION OF GEOGEBRA SOFTWARE IN THE TEACHING OF CIRCLE GEOMETRY ON GRADE 11 STUDENTS' ACHIEVEMENT.

NAME OF RESEARCHER: CHIMUKA ALFRED (alfredchimuka7@gmail.com)

SUPERVISOR: Dr. U. I. OGBONNAYA

DEPARTMENT: UNISA- Institute for Science and Technology Education.

My signature below certifies that the research project in which my son/daughter is about to participate has been explained to him/her and that all question regarding this study have been answered satisfactorily. I voluntarily allow my son/daughter to participate in this study and understand that I may withdraw my permission, or my son/daughter can refuse to answer to any question(s), at any time without penalty.

Although my name and that of my son/daughter appears on this form, I understand that this form will not be associated with either my responses or my son/daughter's responses.

I realize that both I and my son/daughter have the right to inquire about the results of this study by conducting the above-named researcher and that neither I nor my son/daughter will be personally identified if the results of this study are published. I do agree that my son/daughter be interviewed by the researchers for educational purposes.

____________________________ Date:________________________________

Student Please print your name

____________________________ Date:________________________________

Parent Please print your name

Date:________________________ Date:________________________________

C5- INFORMED CONSENTFORM-STUDENT

(Students over 18 years)

NAME OF RESEARCHER: CHIMUKA ALFRED (alfredchimuka7@gmail.com)

SUPERVISOR: Dr. U.I. OGBONNAYA

DEPARTMENT: UNISA- Institute for Science and Technology Education.

Informed consent is a requirement of being part of the educational research on:

THE EFFECT OF INTEGRATION OF GEOGEBRA SOFTWARE IN THE TEACHING OF CIRCLE GEOMETRY ON GRADE 11 STUDENTS' ACHIEEMENT.

Please print the requested information in the blank spaces below, sign this form, and return this form to the above mentioned researcher.

NAME________________________________PHONE__________________________

ADDRESS____

I understand and confirm that I will take responsibility for choosing to participate in this research study. My participation is voluntary. I confirm by way of signing below:

____

SIgnature

Date______________________________

UNISA college of science, engineering and technology

Dear Alfred Chimuka (46465545)

Date: 2015-07-26

Application number:
2015_CGS/ISTE_003

REQUEST FOR ETHICAL CLEARANCE: (THE EFFECT OF INTEGRATION OF GEOGEBRA SOFTWARE IN THE TEACHING OF CIRCLE GEOMETRY ON GRADE 11 STUDENTS' PROBLEM SOLVING SKILLS.)

The College of Science, Engineering and Technology's (CSET) Research and Ethics Committee has considered the relevant parts of the studies relating to the abovementioned research project and research methodology and is pleased to inform you that ethical clearance is granted for your research study as set out in your proposal and application for ethical clearance.

Therefore, involved parties may also consider ethics approval as granted. However, the permission granted must not be misconstrued as constituting an instruction from the CSET Executive or the CSET CRIC that sampled interviewees (if applicable) are compelled to take part in the research project. All interviewees retain their individual right to decide whether to participate or not.

We trust that the research will be undertaken in a manner that is respectful of the rights and integrity of those who volunteer to participate, as stipulated in the UNISA Research Ethics policy. The policy can be found at the following URL:
http://cm.unisa.ac.za/contents/departments/res_policies/docs/ResearchEthicsPolicy_apprvCounc_21Sept07.pdf

Please note that the ethical clearance is granted for the duration of this project and if you subsequently do a follow-up study that requires the use of a different research instrument, you will have to submit an addendum to this application, explaining the purpose of the follow-up study and attach the new instrument along with a comprehensive information document and consent form.

Yours sincerely

Prof Ernest Mnkandla
Chair: College of Science, Engineering and Technology Ethics Sub-Committee

Prof IOG Moche
Executive Dean: College of Science, Engineering and Technology

RECEIVED
2015 -07- 2 8
OFFICE OF THE EXECUTIVE DEAN
College of Science, Engineering and Technology

University of South Africa
College of Science, Engineering and Technology
The Science Campus
C/o Christiaan de Wet Road and Pioneer Avenue, Florida Park, Roodepoort
Private Bag X6, Florida, 1710
www.unisa.ac.za/cset

UNISA college of science, engineering and technology