ANALYSIS OF ERRORS MADE BY LEARNERS IN SIMPLIFYING ALGEBRAIC EXPRESSIONS AT GRADE 9 LEVEL

by

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Supervisor: Prof W.W. J. Mwakapenda

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DECLARATION

Name: MILDRET NCUBE

Student number: 48809969

Degree: MASTER OF EDUCATION WITH SPECIALISATION IN MATHEMATICS EDUCATION

Title: ANALYSIS OF ERRORS MADE BY LEARNERS IN SIMPLIFYING ALGEBRAIC EXPRESSIONS AT GRADE 9 LEVEL.

I declare that the above dissertation/thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

_____   08/06/2016

SIGNATURE   DATE

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ABSTRACT

The study investigated errors made by Grade 9 learners when simplifying algebraic expressions. Eighty-two (82) Grade 9 learners from a rural secondary school in Limpopo Province, South Africa participated in the study. The sequential explanatory design method which uses both quantitative and qualitative approaches was used to analyse errors in basic algebra. In the quantitative phase, a 20-item test was administered to the 82 participants. Learners' common errors were identified and grouped according to error type. The qualitative phase involved interviews with selected participants. The interviews focused on each identified common error in order to establish the reasons why learners made the identified errors.

The study identified six (6) common errors in relation to simplifying algebraic expressions. The causes of these errors were attributed to poor arithmetic background; interference from new learning; failure to deal with direction and operation signs; problems with algebraic notation and misapplication of rules.

Key terms

Algebra, simplifying an algebraic expression, error, systematic error, error analysis, misapplication of rules, conjoining terms, distributive property, misinterpretation of algebraic notation, conceptual understanding.

Author Signature: _MaDube_   Date: _08/ 06/ 2016_

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CHAPTER 1: BACKGROUND AND RATIONALE

1.1 Background of the study

Algebra is an influential Mathematics topic in a school curriculum. It is applied in all current branches of Mathematics as well as in Science. In support of this observation, MacIntyre (2005) states that the success in Mathematics largely depends on algebraic concepts. Mensah (2006) views algebra as forming a large proportion of the final matric examination in Mathematics. In the same line, Christmas and Fay (1990) observed that all Mathematics branches use the fundamental ideas of algebra to reason about and model various phenomena. All these views portray the pivotal role played by algebra in the teaching and learning of Mathematics.

Mamba (2012) analysed South Africa's Grade 12 November 2008 Mathematics Paper 1 solutions for one of the classes and discovered that algebraic expressions posed many problems to learners. The algebraic skills of learners are very poor as reported by Barry (2014) in the diagnostic reports. The report also stressed the fact that learners struggle with basic Mathematics of Grades 8-10. This results in learners facing challenges in Grade 11 and 12. The Department of Basic Education (DBE, 2014) also indicates that poor performance in higher grades is linked to poor performance in algebra.

However, algebra, having such a pivotal role in the learning and development of basic Mathematics aspects, gives learners a challenge at school. Algebra is viewed by Booth (1988) as a source of confusion to learners. Bell (1995) regards algebra as a common problem area for learners. Many learners experience difficulties in understanding algebraic concepts. Learners fail to manipulate algebraic concepts according to accepted rules, procedures or algorithms. This in turn affects their performance in Mathematics as success in this subject is largely affected by understanding of concepts in algebra (Mamba, 2012). Learners find it extremely difficult to the extent that some of them drop out from school or if they do not drop out, they struggle to continue with their education (Wellmann, 2008). According to Kinney and Purdy (1952: 59), "Algebra has acquired a reputation among teachers, pupils and parents alike, as one of the most difficult and

troublesome courses in the secondary curriculum." Kilpatrick and Izsac (2008), in the same line, also regarded algebra as an evil force wreaking havoc across the land and also as a source of difficulty and failure.

> Reeve (1936) in the America's National Council of Teachers of Mathematics (NCTM, 1936) eleventh yearbook went to an extent of saying,

> If there is heaven for subjects, then algebra will not go there. It is one subject in the school that has kept children from finishing school, from developing their special interests and from enjoying much of their home studying work. It has caused more family rows, more tears, more heartaches and more sleepless nights than any other subject (p. 2).

This is a serious concern since the South African Mathematics curriculum attaches great importance to algebra as in other countries (Moodley, 2014). Despite the fact that algebra plays such a pivotal role in the development of most Mathematics concepts, learners face a plethora of problems in dealing with it. There is clear evidence that enormous efforts have failed to address the issue of improving students' performance in algebra (National Mathematics Advisory Panel (NMAP), 2008). Therefore, there is a need to identify the causes of these problems faced by learners in algebra. The results would potentially provide information of some of the interminable errors committed by learners in algebra.

Poor performance in Mathematics in South Africa seems not to be declining. According to Moodley (2014) there is no secrecy in the fact that South Africa trails behind the rest of the world in terms of mathematical achievement. This brings more worries to Mathematics teachers, subject advisors, district senior managers and the whole nation at large. Focusing on the Grade 9 learners, it can be seen that they are not performing well in both the national and the international tests. Reddy (2012) sees the curriculum of Grade 9 in South Africa as being on par with the international standards. In the Annual National Assessment (ANA) tests given to Grade 9, the results show a 13%, 14% and 10% pass rates for 2012, 2013 and 2014 respectively. The Trends in International

Mathematics and Science Studies (TIMSS) administers its tests to Grade 8 learners. But South Africa in 2007 and 2011 fielded Grades 9 learners for these tests. The reason, according to Spaull (2013), was that the tests were too difficult for its Grade 8 learners. However, this did not improve the situation as seen in the TIMSS 2011 results. South Africa after having fielded the Grade 9 learners was still at the bottom together with Honduras and Botswana (Reddy, 2012).

Having cited all these problems faced in the Department of Mathematics and also the whole nation, the researcher decided to identify, classify and analyse errors made by learners in simplifying algebraic expressions. The aspect of simplifying algebraic expressions was chosen after having discovered the pivotal role played by algebra in the development of most Mathematics aspects. The researcher hoped that learners and the education system in general could have a chance to improve from the research findings by using the identified errors and their causes to design better ways of addressing the problems in algebra.

1.2 Statement of the problem

Learners' errors give rise to poor performance in any subject. In Mathematics most of these errors are attributed to poor algebraic skills. To make matters worse, almost all topics in Mathematics are developed using algebraic concepts. Therefore, there is a need to recognise common errors made by learners in algebra as well as the causes of those errors. The researcher is of the opinion that analysing errors encountered by learners in simplifying algebra is one way of achieving this. The identification of these errors will help teachers to come up with better ways of minimizing prominent errors in algebra as well as in many branches of Mathematics.

Hence, the intention of this study was to identify and analyse the common errors among 82 Grade 9 learners from one secondary school and find out the root causes of these errors. The objective, after identifying the errors, was to come up with better mathematical instructional practices for learners.

1.3 Aim of the study

- To analyse the common errors learners make in simplifying algebraic expressions.

1.4 Specific objectives

- To determine learners' errors in simplifying algebraic expressions at Grade 9 level; and
- To find out the reasons why learners at Grade 9 level make errors in simplifying algebraic expressions.

1.5 Research questions

- What errors do students make in simplifying algebraic expressions?
- What are the possible reasons that lead learners to make errors in simplifying algebraic expressions?

1.6 Significance of the study

South Africa's performance in the benchmark tests is a serious issue among educators and policy makers, casting doubt about the effectiveness of the curriculum reform efforts of the democratic era (Ndlovu and Mji, 2012). It is a fact that we are aware of the fact that most of the learners have many misconceptions regarding Mathematics. These misconceptions lead them to make errors and hence find the subject very difficult and in particular the branch algebra. Kilpatrick and Izsak (2008) went to an extent of saying that if there was heaven for subjects then algebra would never go there. Therefore, it is necessary to identify means of helping these learners. It is only possible to help them if reasons why they make these errors have been identified first.

There is a plethora of research on errors in algebra. For example, Mamba (2012) investigated on errors exhibited by Grade 12 learners in solving quadratic equations, quadratic inequalities and simultaneous equations. Mensah (2006) also carried a study on college students on their knowledge and understanding of algebra. Another study on college students was carried by McIntyre (2005) who investigated learners' misunderstanding of variables. Bush (2011) analysed Grade 6 and 8 learners' common

algebra-related misconceptions. Mashazi (2014) explored the thinking underlying Grade 9 learner errors in algebra. However, few of these seem to address the Grade 9 level yet it is a crucial level which allows transition from the General Education Training (GET) band to the Further Education Training (FET) band. This study focused attention on the Grade 9 learners. It tried to address the problems faced by learners in algebra. It aimed to improve learners' understanding in algebra through identifying learners' errors in simplifying algebraic expressions. It checked on the frequency of errors in algebra and the causes of these errors.

Learners need to be helped to change their attitudes towards Mathematics through helping them to understand algebra, the topic that has a greater influence in developing most of the mathematical concepts. Hence, the findings of this study targeted to inform teachers, curriculum planners and developers, textbook writers and other stakeholders of the prevalent errors in algebra with the hope that they would be able to identify ways of either reducing or eliminating these algebraic errors. This would in turn improve the performance of learners in Mathematics.

1.7 Chapter outline
This study has been divided into six chapters. The first chapter gives the background of the study, the statement of the problem, the aim and objectives as well as the research questions. It also gives the significance of the study.

Chapter Two gives the basis of the study by examining the previous works done by other researchers in the field of algebra. Pertinent literature review of errors in algebra was carried out. The chapter also gives the research gap. There is also a discussion on the nature of errors in algebra within the constructivist framework.

The devotion of Chapter Three is to discuss the methodological constructs of the research. The chapter explains in detail the research design employed, the sample, sampling methods, the participants of the study, the research instruments, reliability, validity, ethical considerations and limitations of the study.

Chapter Four has data analysis and interpretation of the research test data. The identified common errors are described. The findings are tabled and graphically represented. An analysis of errors from interviews is also presented in this chapter.

Chapter Five is a discussion of the findings of the study. Chapter Six then concludes the study by elucidating the key findings of the study. It also gives the conclusion and recommendations.

1.8 Conclusion

This chapter has given the background, the statement of the problem, the aim of the study and its accompanying objectives together with the research questions. The chapter has also given the significance of the study as well as the chapter outline. The next chapter discusses a review of the related literature. The intention of the literature study is to gain more understanding of the problem.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

Many studies have been carried out on the teaching and learning of Mathematics and in particular, algebra. Numerous studies bring many conceptions about algebra. They bring out the fact that many learners have naive theories, preconceptions or misconceptions that interfere with their learning of Mathematics which result in them making errors when solving problems. This chapter presents a review of related literature in an attempt to provide foundation and ground theory for an organized study of errors encountered by learners in algebra.

2.2 Review on errors in algebra

According to the Oxford dictionary (2003), algebra is part of Mathematics that uses letters and other symbols to represent quantities and situations. Learners generally lack sense of algebra. As a result, they fail to appreciate the power of algebra. They do not know when to use algebra or manipulate it in a range of situations. Booth (1988) regards algebra as a source of confusion and is regarded as a common problem area for learners (Bell, 1995).

The challenge in algebra is that most of the learners fail to understand the main concepts of algebra. Once learners fail to understand the key aspects of algebra, they have difficulties in Mathematics. One of these key aspects is simplifying algebraic expressions. Algebra is a generalized form of arithmetic where letters and both operation and direction signs are used. The use of letters and signs, according to Foster (2007), makes it abstract and difficult. This is because algebraic ideas are based on general ideas instead of real facts or events.

Learners possess a serious arithmetic-algebra gap which, as observed by Seng (2010), remains a fundamental cause of learning difficulties. If learners possess a good arithmetic background, they are not likely to face challenges in algebra. This is because algebra knowledge is built upon the foundation of already acquired arithmetical knowledge.

Learners have many misunderstandings in algebra (Seng, 2010; Mbewe, 2013; MacGregor and Stacey, 1997). These misunderstandings have their own impacts on learners. Li (2006) observed that learners' errors are the symptoms of misunderstanding. According to the free dictionary (2014), an error is, "an act, assertion or belief that unintentionally deviates from what is correct, right or true." Muzangwa and Chifamba (2012) and Donald (2007) view an error as a mistake, blunder, and miscalculation or misjudge. Errors perhaps result from forgetting, confusion or lack of understanding of key concepts. The idea of learners making errors and/or mistakes is a natural part of learning (Lopez-Valero Fernandez & Clarkson 2008). These errors produced by learners as suggested by Mbewe (2013), play an important role in indicating to teachers the stages at which their learners are at as well as showing where there is need for further teaching or study. They assist teachers to advise learners for improvement.

According to Radatz (1979), Melis (2004) and Riccomini (2005), there are two main types of errors, namely; systematic and unsystematic errors. Systematic errors are the common errors made by learners over a long period. "They are recurring erroneous responses methodically constructed and produced beyond space and time", (Mamba, 2012: 19). Following Nesher (1987) and Riccomini (2005)'s views, these errors are symptomatic of a defective cause of thinking. There is misapplication of rules caused by learners' failure to grasp concepts or rules. Drews (2005) observed that these systematic errors are not only produced by children needing assistance but also able students make incorrect generalisations. Unsystematic errors, as suggested by Riccomini (2005), are non-persistent incorrect responses which learners can easily correct themselves without much intervention from the teacher. They are just random and have no evidence of recurring. Kousathana and Tsaparlis (2002) are of the opinion that these errors could be a result of overloading the working memory, hastiness or recklessness. In their view, learners should be able to correct these errors if given another chance.

Given the above definitions, this study concludes that most of the errors in algebra are systematic and therefore can be addressed because of their consistency. It is extremely important to identify learners' errors and their causes. Discovering the errors made by

learners and the reasons for making such errors and identifying the most suitable methods of dealing with them is what Luneta (2008) refers to as error analysis or error diagnosis. Similarly, Ketterlin-Geller and Yovanoff (2009) describe error analysis as focusing on the weaknesses of learners and this is meant to help teachers classify mistakes. Looking at the nature of systematic errors, it is possible to do error analysis in order to identify the reasons for such errors and find ways of helping learners to do away with them.

However, students do not come to class with blank minds (Resnick, 1983). Instead, they come with ideas and facts constructed from their everyday experiences. These ideas and facts having been actively constructed provide everyday functionality to make sense of the world (Mestre, 1987). These conjectures to some extent are the causes of misconceptions which lead learners to make errors in solving Mathematics tasks. Learners try to link what they already know to new information and at times they link unrelated things resulting in them making errors. The way these misconceptions affect learners in learning situations is also evident in algebra.

According to Greens and Rubenstain (2008), most students in Grade 8 and 9 struggle to grasp concepts and skills in algebra. This is the reason why most of the learners discontinue with Mathematics at higher levels. If learners do not discontinue and the misconceptions are not remediated, they go up even to colleges making the same mistakes (Gunawardena, 2011). This researcher investigated errors and misconceptions in algebra with the hope to identify their origins. In the investigation, errors and misconceptions were examined on the four main areas of algebra: variable, algebraic expressions, equations and word problems. From the findings, it was discovered that learners had common misconceptions mostly occurring in algebraic expressions.

Kuchmann (1981) also carried out a study on the 13 to 14 year olds on their errors and misconceptions in algebra. Kuchmann's (1981) study deduced that learners had difficulties in coping with algebraic letters as unknowns or generalized numbers. The study also identified conjoining of terms as one of the most prevalent errors in algebra.

According to Kuchmann (1981), learners seem to have difficulties in accepting lack of closure. When learners are given an expression like *3y+4* and they think the expression is incomplete; so, they tend to write *7y* as their final answer.

Macgregor and Stacey (1997) also conducted a series of studies to investigate the origins of students' misinterpretations of letter usage in algebra. They tried to get explanations for making the errors and also identified the causes of those errors. They deduced that learners can ignore letters while some of them associate them with numerical values. This observation sees learners simplifying m + 4m to 4m as to the appearance of m with no number means there is nothing. The learners who associate the position of a letter in the alphabet with counting number think *a* stands for 1, *b* for 2, *c* for 3 and so on.

Errors are caused by misconceptions and the latter are attributed to lack of conceptualisation and understanding. According to Mbewe (2013), misconceptions are habitual and cannot be solved easily. This was evident from the interviews that Mbewe (2013) conducted with Grade 11 learners after they had written a test on algebra. Mbewe (2013) also discovered that learners' errors occur frequently and repeatedly. In concluding his study, Mbewe (2013) then recommended that teachers and learners need to talk about misconceptions during teaching and learning process so that ways of doing away with them could be identified.

Another study on middle school students was done by Bush (2011). She analysed Grade 6 and 8 learners' common algebra-related errors and misconceptions. In her research, it was discovered that errors and misconceptions in algebra were just the same as those reviewed in the other literatures. However, she confirmed the need for strong and knowledgeable teachers of Mathematics in elementary and middle grades.

There was also a study on college students that was done on student teachers' knowledge and understanding of algebra. It was carried out by Mensah (2006) among final year college of education students in Eastern Cape. The discovery was that even teachers on training also had misconceptions which they carried from their learning

experiences and as they went up there was little change happening. The researcher's worry was that they would go out of college without well-developed algebraic concepts and therefore would not be good enough to assist learners. Therefore, there would be a cascading effect on teaching and learning in schools resulting in a cycle of errors from their misconceptions in algebra.

The issue of effects of misconceptions from early stages was also discovered by McIntyre (2005), who also investigated college students' misunderstanding of variables. In that research a pre-test and a post-test was administered to 731 University of Maine students. In the findings, it was deduced that misconceptions are formed as early as pre-algebra when variables are first introduced to learners. They are then carried on if there is no remediation done. These misconceptions are the causes of errors which are always made by learners.

Wellman (2008) also carried a study on 270 freshman of the school of business at a university by administering a 42 item test on them. The findings were that most of the students performed badly because of their arithmetic and algebraic skills brought from earlier studies. One of the serious learning difficulties in Mathematics is that of misconceptions learners may have from previous or inadequate teaching, informal thinking or poor remembrance (Donald, 2007). These are the causes of learners' errors in solving problems. There is need to reduce if not do away with the misconceptions at early stages before they accumulate and become part of the learners' incorrect conceptions. If these misconceptions are not eradicated, then learners will continuously make errors when solving problems. It is the role of the teacher to let these misconceptions disappear with the framework changes. If misconceptions disappear then errors will also be minimised.

It could be seen that much had been done but still the problem of errors in algebra persisted and it was now the duty of the researcher to give a contribution to what had been done and what had not been done. This brought about the literature gap.

2.3 Literature gap

The review provided the researcher with a better understanding of the problems caused by inadequate knowledge of algebra in Mathematics. The importance of algebra was not only stressed in the South African education system but in the whole world. Both positive and negative impacts of the reviews were identified.

Literature reviewed that there are difficulties in transforming from arithmetic to algebra, understanding the procedural and structural aspects of algebra and use of mini-theories. However, some problems are due to carelessness, overconfidence and also teachers' content gaps. There was evidence from the literature reviews that in the studies, emphasis had been put on many algebraic aspects of algebra at one time for example analysing errors and misconceptions in simplifying algebra, solving linear equations, quadratic equations and linear inequalities all in one research. In that case, there were several issues analysed at the same time. It is a bit difficult to come up with real problems with so many aspects analysed at the same time. Therefore, the researcher decided to concentrate on errors encountered in simplifying algebraic expressions only.

There was evidence that a plethora of researches had been carried out at later stages of learning like at Grade 11 or 12, colleges and even at universities. At those stages, the researcher felt that it was too late for learners to benefit as the introduction of algebra normally starts at Grade 8. It is better to identify problems in early stages than concentrating on later stages when it is no longer going to benefit the learner much. Against this background, this study focuses on Grade 9 level which is a crucial learning stage of High School. Grade 9 is a crucial learning stage because it affects a learner's transition from the GET phase to the Further EET phase. If learners are to grasp this aspect on algebraic expressions, then they will move to higher grades without serious problems. It will be easy for them to grasp concepts in other branches of Mathematics and also in other subjects like Science. Muzangwa and Chifamba (2012) are of the opinion that algebra affects almost all topics in Mathematics. That is the reason why there is need for proper handling of the early secondary education stages so that learners can go up with their Mathematics without encountering serious challenges.

It was from that background that considering the gaps that had been mentioned, the researcher felt that a research on errors in simplifying algebraic expressions at Grade 9 level was worth carrying out.

2.4 Theoretical Framework

The study used the theory of constructivism to view errors made by learners in simplifying algebraic expressions. Constructivism, as described by Fosnot (2005), refers to the process whereby learners actively construct their understanding and knowledge of the world through experiencing things and reflecting on those experiences. Constructivism is a theory that originates from Jean Piaget. According to Piaget (1970), learning is not a simple passive process of receiving from the surrounding environment. Constructivism is a dynamic process of an individual involving interaction between the individual's existing knowledge and new ideas. According to Makonye and Nhlanhla (2014), the constructivist theory implies that learners do not come to a new grade as empty vessels but bring pre-knowledge from previous grades. Resnick (1983) view learners as holding theories constructed from their everyday experiences which is their existing knowledge. The existing knowledge as described by Olivier (1989) is structured in a learner's mind into interrelated concepts called schema. These schemas are retrieved and put into use when a learner encounters a familiar situation. Therefore, the schemas are important tools of each individual learner.

The learning of Mathematics is purely a constructive process because it is cumulative. Mathematical concepts are interrelated with one another. Related knowledge which is possessed by a learner is used to construct new knowledge. Grasping new concepts as described by Smith, DiSessa & Roschelle (1993) is difficult if basic concepts and skills learnt at early stages were not well understood.

During the process of learning, individual thoughts consist of two basic alternative mechanisms, namely; assimilation and accommodation (Piaget, 1970). These two processes describe how an individual adjust the mind to new experiences and be in a position to take new data. Moodley (2014: 11), states, "Assimilation occurs when a

new idea is interpreted in terms of an existing schema." An existing mental structure that is available is used to assimilate a new situation. On the other hand, Moodley (2014) describes accommodation as occurring when there is incorporation of new ideas which are not related to the existing schemas. The existing schema may not be enough to assimilate new ideas. Therefore, existing schemas need to be modified or else new schemas have to be created so that the new experience can be taken care of.

The process of assimilation can be demonstrated using a situation whereby a learner knows that a^2 - $b^2 = (a - b)(a + b)$. When she is asked to evaluate $101^2 - 99^2$ without using a calculator, the learner will be able to express it in the same way used for that a^2-b^2. The learner will have $101^2 - 99^2$ = (101 -99) (101 + 99). The learner can then simplify what is inside brackets to $2 \times 200 = 400$. The demonstration shows that the learner has interpreted the new situation in terms of the already known aspect of factorisation difference of two squares.

The picture of accommodation process can be explained by looking at a situation where learners have to find products of algebraic terms. If a learner has the knowledge that $a \times b = ab$ or ba, then when a problem requires the same learner to simplify $a \times 5$ then s/he might have $a5$ as the answer as the learner will be thinking that $a5$ is the same as $5a$. This means, there is a need for restructuring so that the learner sees when one part of the algebraic term is a number then the number has to be written first. This shows that it is not always possible to connect new ideas to schemas.

The failure to link existing schemas to new situations may result in the creation of a new box in the mind of the individual. The learner may find it difficult to link the knowledge in the box to existing schemas which may force him or her to memorise the ideas or rules to learn. In the process of recalling, some of the rules are partially remembered resulting in the learner being confused and making errors. These errors, as suggested by Olivier (1989), are the natural results of learners' effort to construct knowledge. Labinowicz (1985) also regarded learners' errors as actually natural steps to understanding. According to Brodie (2014), understanding learner errors is a way of

understanding learner thinking. Therefore, errors must be expected and appreciated in the teaching/learning situation.

2.5 Conclusion

The review of literature has deduced that most of the research studies that have been done focus attention on algebra in general and in most cases carried at higher stages of learning like at training colleges where remediation would not be that fruitful. Enough damage would have been left unattended for a long time. It is against this background that the researcher felt that the study on simplifying algebraic expressions at Grade 9 level was worth carrying out.

In the next chapter, the researcher outlines the design and methodology used in this study.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter gives the description of the design and methodology used in the study. There is also a discussion on the targeted population, sample and sampling techniques, research instruments, validity, reliability and data analysis procedures. The chapter also presents time frame and ethical considerations.

3.2 Research design

The purpose of this study was to analyse errors in simplifying algebraic expressions by Grade 9 learners. According to McMillan and Schumacher (2010: 20), "The purpose of a research design is to specify a plan for generating empirical evidence that will be used to answer the research questions. The intent is to use a design that will result in drawing the most valid, credible conclusions from the answers to the research questions." In this study the researcher needed to quantify errors made by learners in simplifying algebraic expressions and also to gain an understanding of the reasons why learners came up with such errors. There was therefore need to bring in both quantitative and qualitative methods into the study. For that purpose, a sequential explanatory design which is a mixed methods approach was employed.

The sequential explanatory design uses both the quantitative and qualitative techniques to collect and analyse data (Cresswell, 1998). The quantitative part helped to understand learners' errors numerically while the qualitative part helped to deepen focus and explain more about the errors through learners' responses to the interview questions. Mbewe (2013) described the main purpose of the sequential explanatory design as to use qualitative results to assist in explaining and interpreting the findings of the quantitative design. In addition, Johnson and Christensen (2008) are of the belief that social phenomena are complex and if one needs to understand them better, s/he must employ mixed methods. Most importantly, mixing quantitative and qualitative methods brought richer data. Merriam (1992) is of the opinion that achieving a deep

understanding of specific phenomena and probing beneath the surface of a situation to come out with strong information for understanding the phenomena under study is the reason of qualitative approach. Hence this study brought in qualitative methods after quantitative methods to provide an understanding of the reasons why learners make errors when simplifying algebraic expressions.

3.3 Research methodology

The learners' thinking processes and procedures used in simplifying algebraic expressions had been obtained. Their underlying concepts were revealed through execution of these procedures. In the quantitative phase, a test instrument was used to identify and classify errors. The findings of the quantitative study were used to determine the type of data that were gathered in the qualitative phase. The study used qualitative data to explain and explore quantitative data and it provided the researcher with information on how learners came up with their answers.

Interviews came in the qualitative phase when learners were asked to answer some questions to justify the procedures they had used in obtaining their solutions. Interviews helped to expose learners' thinking processes that were not clear in their working.

3.4 Target population

Chiromo (2006: 26) defines a population as, "…..all individuals, units, objects or events that will be considered in a research project." In this study, the target population was all Grade 9 learners from one rural Secondary School in Ga-Sekgopo Village, Mopani District in Limpopo Province.

3.5 Sample and sampling techniques

The researcher identified the population on which data collection methods were to be applied to gather information. In the study, the population was Grade 9 learners in Mopani District of Limpopo Province, South Africa. As this was definitely a very large population to handle, the researcher decided to work with a sample of the population.

McMillan and Schumacher (2010) define a sample as a group of individuals from whom data is collected. Therefore, a sample is part of the entire population which usually

represents the whole group under study. Sampling is necessary because it is not always possible or practical to study the whole population.

According to Brink (1991), sampling refers to a process of selecting the sample from a population to obtain information regarding phenomena. Once the general problem has been identified, the task becomes to select the unit of analysis (Merriam, 1992). As described by McMillan and Schumacher (2010), the unit of analysis is the object which is to be studied in terms of research variable that constitute the constructs of interest. In this study, learners' errors and the reasons why they made these errors were the units of analysis.

The participants in this study were randomly selected. A sample of 82 learners was randomly selected from a population of 300 Grade 9 learners from one secondary school. Random numbers were used to select the participants. The researcher assigned each Grade 9 learner a number from numbers 1 to 300. A total of 82 numbers were then selected from the tabled random numbers taking into consideration the last 3 digits whose value was less or equal to 300.

Random selection gives every member of the population equal chances of being selected (McMillan and Schumacher, 2010). The justification of selecting students from only one secondary school was as follows: convenient to the researcher because of easy access and the researcher could relate well with the sample resulting in quality and credibility of research data.

A test was administered to the sampled 82 participants. For the explanation of the reasons why learners made errors, the interviewees were purposively selected from the sampled 82 participants. Tashakkori and Teddie (2003) define purposive sampling as involving selecting certain units or cases based on a specific purpose rather than randomly. The selection of these individuals was based on the specific purpose associated with answering the research study questions. Purposive sampling provided greater depth of the information from a smaller number of units. One learner was selected to represent each type of identified errors. From the tests results, the researcher

grouped the 82 participants according to where they had the most errors. Each group had one representative for the interviews. According to Maxwell (1996), this purposeful sampling technique is a deliberate selection in order to provide important information that cannot be obtained from other choices.

3.6 Research instruments

In this study, the researcher administered a 20 item test on simplifying algebraic expressions to 82 sampled Grade 9 learners. The test was written in one hour. The test involved the computations: addition, subtraction, multiplication and division. Question 1 had seven items, Question 2 had nine items, Question 3 had two items, Questions 4 and 5 had one item each. The first five items in Question 1 tested learners' ability to collect like terms. The last two items of Question 1 tested the ability to use acquired arithmetic knowledge of fractions to simplify algebra and allow the identification of arithmetic-algebra gap. Question 2 items tested the ability of learners to multiply algebraic terms and collection of like terms. Questions 3 and 5 were designed by the researcher in a way to test learners' understanding of letters representing numbers. Question 4 measured the learners' ability to read and understand questions and also collection of like terms.

The solutions provided by learners gave the researcher a chance to identify their errors. The errors made by learners in the test were identified by the researcher according to type. They were then quantified according to occurrence frequencies. From the identified errors, a representative for each type of error was deliberately selected for the interviews. The selected learners were asked to explain the procedures they had followed in coming up with their solutions. The interviews provided an understanding of the reasons that lead learners to make errors. The interviews were held two days after the test so that learners could remember how they had come up with their solutions.

3.7 Reliability

Prior to the main study, a pilot test had been conducted to ascertain validity and reliability of used instruments. Pilot testing helps the researcher to determine ways to identify learners for the interviews. Reliability deals with the ability to come up with the

same review of a given phenomenon if and when the review is conducted under same conditions (Gertz, 1973). On the same note, Opie (2004) views reliability as being synonymous with worthy of trust and unambiguousness of the representation of the total population under study. Therefore, reliability is just an indication of the extent to which results can provide formal assurance in qualitative research because of bias being rooted in all individuals. Data can be interpreted differently by different individuals. Therefore, the researcher needed to try her best to have dependable data for the consistency of the study.

It is the duty of the researcher to ensure reliability. In this study, the reliability of the study was ascertained by a pilot study. According to Cohen and Morrison (2002), the purpose of a pilot study is to impart knowledge of the main study about the quality of the questions in the research task. The pilot study gave an idea of the relevance of the task; it helped to identify the quality of question or instruction and also the context of the question. The pilot study, as suggested by Opie (2004), helped the researcher to get rid of inapplicable data to the study.

3.8 Validity

Test validity as suggested by McMillan and Schumacher (2010) refers to the extent to which inferences based on instrument are reasonable. Validity is not an article that can be purchased with a skill; instead, validity is like nobility, character and quality to be assessed relative to purpose and circumstances (Brinberg and McGrath, 1985). It is a measure of the degree to which explanations of an event match reality. Validity can be viewed as not depending on the data but the interpretation of the data. A test can be viewed as valid if it serves its intended purpose well (Mbewe, 2013). In this study, the test questions were constructed in line with Grade 9 past exam papers as well as relying on common errors encountered by the researcher in the teaching/learning situation.

The researcher constantly discussed with the supervisor concerning the findings and finalised the questions for the follow up interviews. The researcher was very much

aware of the existence of bias. So, she monitored beliefs, insights and preconceptions about learners' misconceptions solely based on practice.

3.9 Ethical issues

Participating in the test and interviews was a voluntary informed consent. Informed consent from parents and permission from the school principal were obtained using documents. The documents included informed invitation letters to learners for their participation and consent forms for parents/guardian for their children to participate. Learners were free to withdraw if they so wished.

Anonymity of participants' information was greatly ensured. The results from the participants were confidential. The researcher kept copies of all participants' responses in a secure place so that they could be accessed at any time for recall and analysis whenever the need arose. Notes on the interviews were kept safe as well.

3.10 Data analysis procedures

Firstly, 82 participants answered a 20 item test in one hour on simplifying algebraic expressions designed by the researcher. The researcher analysed the solutions presented by the participants and grouped the participants according to where they had the most errors.

From the identified errors, 12 learners were selected for the interviews. The interview participants were interviewed by the researcher one by one.

3.11 Time frame

The study was carried out in the second quarter of the year before the mid-year examinations. At that time, learners had covered most of the algebraic aspects. The writing of the test and the interviews were completed within two weeks.

3.12 Conclusion

The researcher has given explanations of the research design and methodology, target population, sampling and sampling techniques and the research instruments used. Having also presented how issues regarding validity, reliability and ethical issues were

dealt with as well as briefly outlining time frame and data analysis procedures, the researcher in the next chapter is going to analyse the key findings of the study.

CHAPTER 4: DATA ANALYSIS AND INTERPRETATION

4.1 Introduction

This chapter provides the findings of the research study in terms of data collected from Grade 9 learners from one secondary school in Limpopo Province, South Africa. The data for the study were collected using two methods: test and interviews. In this chapter, data are analysed and displayed in tables and illustrated graphically.

4.2 Analysis of the research test data

The test responses were analysed and quantified to identify the common errors made by participants in simplifying algebraic expressions. The identified errors were categorised into groups. In addition, frequencies for each type of error per item were recorded. From the results of the test, the researcher identified the common errors and also recorded the number of learners who committed those errors.

Interviews were carried out with 12 learners. These learners were sampled from the group that committed the most prevalent errors. The interviews were held in order to get a clear understanding of how the learners had arrived at particular solutions. The interviews were audio-taped in order to capture as much and accurate information as possible.

4.2.1 Test analysis

The first phase of the study involved the administering of a 20-item test to eighty-two (82) participants of the study. In the analysis of learners' responses to the test items, the researcher identified six (6) common types of errors displayed by the participants. The main errors observed were: conjoining, misapplication of rules, misinterpretation of symbolic notation, misusing the distributive property, sign error and the error of substituting letters by numeric values.

4.2.1.1 Description of identified common errors
Conjoin error

When learners were simplifying expressions, they combined unlike terms. This was mainly seen in item 1.2 in which learners were asked to simplify $3c + 4d$. The most

common solution provided by 83% of the learners was *7cd*. In item 2.1 some learners removed brackets appropriately and simplified *2(3a+4)* to *6a + 8*. However, they proceeded to inappropriately simplify *6a + 8* and obtained *14a* as the answer. Learners lacked understanding of the concept of like and unlike terms.

Misapplication of rules

This error prevailed in most of the questions in the test. Learners misapplied rules in many items. In the first item of the test, learners were asked to simplify $4m + m$ and surprisingly most of them could not get the correct answer. The most common answer was $4m^2$. Instead of adding terms learners multiplied the terms. This error was committed by 37% of the learners. In item 1.6 linked to adding algebraic fractions learners just cross multiplied and ended up without a denominator. In that item, learners reduced the expression $\frac{x}{y} + \frac{w}{z}$ to *xz + wy*. Learners' thinking might have been influenced by previous learnt concepts of exponents and solving of equations with fractions respectively.

Misinterpretation of symbolic notation

This error was committed by 6% of the learners. Learners made misinterpretations of terms with invisible coefficients. Learners assumed that 0 was the coefficient of terms with invisible coefficients as there were no numbers before the letters. As a result, the coefficients of terms like m were taken as 0. That is why some of the learners when asked to simplify $4m + m$ gave their answer as 4m. When simplifying $\frac{ma+mb}{m+md}$ some learners divided each term by m. The learners then cancelled m in each term and their final answer was $\frac{a+b}{d}$. To the learners where there was m only, they cancelled the *m* and nothing was left. As a result, they ended up with only d in the denominator. There was evidence that learners had partial understanding of the factorization procedure. According to Makonye and Nhlanhla (2014), these learners relied on unrefined schema which means the learners possessed disorganised information as their basic structures. Learners presumably had the correct methods in their long-term memory but could not

retrieve the information well. The learners had ideas of rules that were supposed to be used but incorrectly adapted the rule.

Invalid distribution of brackets

The distributive error in bracket expansion was also a very common error especially in Question 2. This error was committed by 23% of the learners in the study. Some of the learners expanded one part of the bracket and left the other part. This was evidenced in the expansion of *2(3a+4)* where the learners gave 6*a*+4 as their solution. The other learners did not know the limits of the pre multiplier. Therefore, when simplifying the expression *3(2m + 3) + (2 + 5m)* they multiplied contents of both the first and second sets of brackets. They came up with *6m + 9 + 6 + 15m*. The learners overgeneralised the distributive law. According to Martz (1980), learners use known rules in appropriate situations but incorrectly adapt the known rules. When expanding the expression $(x + y)^2$ some learners just gave their answer as $x^2 + y^2$. In this case, learners failed to retrieve the correct expansion of a binomial as was found by Mbewe (2013) in his study on misconceptions and errors in algebra at Grade 11 level. Learners relatively executed the distributive rule. The literature attributes invalid distribution of brackets to learners' failure to master basic facts and concepts.

Sign errors

Learners had problems working with integers and operation signs. The percentage of errors due to inappropriate use of signs was 9%. Sign errors were mainly due to failure to combine operation and direction signs. When simplifying the expression *3a-(5c+4),* learners gave *3a -5c + 4* as the answer. It can be deduced that the minus sign was not considered as affecting the second term inside brackets. In the question where learners were asked to subtract the second expression from the first expression, learners understood what they were supposed to do. They correctly wrote $(8x^2 + 3x + 4) - (5x^2 - 7x + 2)$ but failed to work with the signs properly and had $3x^2 - 4x + 2$ as their answer. Learners failed to simplify the middle terms. The other minus sign was ignored as the term to be subtracted already possessed a negative sign. Learners showed too many misunderstandings in working with signs.

Substituting letters by numeric values

Overall, this error contributed 10% to the total errors committed in this study. The substitution of letters by numbers was mostly observed in item 3.2 where 61% of the learners gave 9 as the answer when they were given that $b + d = 6$ and were then asked to determine $b + d + e$. It seems learners assigned 3 to each of the 3 letters in the question.

4.2.1.2 Presentations of results

The researcher then coded these six common errors and also added two more codes. The other two codes were for "correct answer and for question not answered." The researcher used the coding system for easy capturing of the learners' results. The explanation of the coding system used by the researcher is given in the table below.

Table 1: Answer codes descriptions

Description	Correct answer	Conjoin error	Misapplication of rules	Misinterpretation of symbolic notation	Misuse of distributive property	Substitute letter with value	Wrong use of signs	No answer
Code	1	2	3	4	5	6	7	8

As the table displays, there were 8 answer codes used. The first code represented correct answer. The second code was for the error due to conjoin of terms. Code 3 represented the error due to misapplication of rules. Code 4 stood for error due to misinterpretation of algebraic notation. The error due to wrong application of the distributive property was represented by code 5. Substituting a letter by a numeric value was represented by code 6 followed by wrong use of signs as code 7. Lastly, code 8 represented situations where the learner did not attempt to answer the question.

Using the above-mentioned codes, the researcher recorded learners' results and tabled them. The table below is an extract of the tabulated results.

Table 2: An extract of learners' response codes

Item→ Learner ↓	1.1	1.2	1.3	1.4	1.5	1.6	1.7	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.1	3.2	4	5
L1	3	2	3	3	8	8	3	2	5	5	5	5	1	1	4	7	3	6	7	4
L2	3	2	2	2	2	3	3	2	5	5	5	5	1	1	6	6	1	3	7	8
L3	3	2	3	3	3	8	3	1	5	5	5	8	1	3	8	1	1	3	8	0
L4	4	2	3	1	2	3	3	2	8	5	5	8	5	1	4	5	1	6	8	0
L5	4	2	3	2	2	3	3	1	5	7	5	5	1	3	3	5	1	3	7	6
L6	3	2	3	2	2	3	3	2	4	5	5	5	3	1	4	7	6	6	7	8
L7	3	2	3	2	2	3	4	2	5	5	5	5	1	1	5	3	1	6	7	8
L8	3	2	2	2	7	3	3	2	5	5	5	5	3	1	7	7	3	6	8	6
L9	4	3	3	2	7	3	8	2	3	5	2	5	5	5	3	5	1	6	7	8
L10	4	2	3	7	7	3	3	1	3	3	3	1	1	1	3	1	1	8	8	6
L11	1	2	3	7	7	3	4	1	4	7	7	3	1	1	3	7	1	6	7	6

Table 2 is an extract of the results of all learners with all their answers indicated in the appropriate answer codes.

The total number of learners for each answer code was calculated and summarized in a table. The next table illustrates the total number of learners for each of the six common errors per item together with the total number of correct and no answers.

Table 3: Number of learners per answer code per item

ANSWER CODE → ITEM NUMBER ↓	I	2	3	4	5	6	7	8
1.1	31	0	45	6	0	0	0	0
1.2	7	68	7	0	0	0	0	0
1.3	15	10	51	5	0	0	1	0
1.4	15	27	21	1	0	1	14	3
1.5	17	22	26	0	0	0	14	3
1.6	3	0	66	0	0	0	0	13
1.7	2	1	48	19	0	0	1	11
2.1	36	30	0	0	16	0	0	0
2.2	5	1	11	10	51	0	1	3
2.3	6	12	2	5	45	0	10	2
2.4	16	8	4	4	49	0	1	0
2.5	4	0	33	7	32	1	0	5
2.6	56	0	15	0	11	0	0	0
2.7	55	0	14	0	13	0	0	0
2.8	19	0	23	9	24	1	5	1
2.9	30	0	8	0	20	2	21	1
3.1	54	0	7	1	0	15	1	4
3.2	5	0	21	0	0	50	1	5
4	3	4	26	0	0	3	34	12
5	3	0	3	2	0	45	0	29

CODE TOTAL	382	183	431	69	261	118	104	92
% OF CODE	23%	11%	27%	4%	16%	7%	6%	6%

Overall, learners gave 23% of correct answers, 6% of unattended items of questions and 71% of the answers had errors. Also, 37% of answers with errors were in the category of "misapplication of rules" while 22% of the answer codes were associated with "invalid distribution of brackets". Conjoin error had 16% of the distribution, substituting letter by number 10%, sign error 9% and misinterpretation of symbolic notation with 6%.

The table above indicates the distribution of errors among the six error codes together with the other two non-error codes, one for correct answers and the other for missing answer. It can be seen that Question number 1 was dominated by code 3 which represented misapplication of rules

in simplifying algebraic expressions. However, item 1.2 of simplifying *3c + 4d,* 68 out of 82 learners conjoined terms. This means 83% of the learners conjoined terms in that item. Their answer was given as *7cd*. Otherwise, the rest of question 1 was dominated by misapplication of rules.

Question 2 required learners to remove brackets and then simplify the expression. The most prevalent error in this question was due to misusing the distributive rules. The distributive property was wrongly applied in almost all items of this question. Question 3 had most learners substituting the unknown by numeric values. Learners had problems with direction and operation signs in Question 4. Learners failed to deal with integers as a result missed the correct answer for Question 4. Question 5 also found learners mostly substituting letters by numbers as in Question 3.

The overall contribution of each error to this study was also calculated, tabled and illustrated graphically as shown below.

Table 4: Most common errors and their frequencies

Description	Conjoin error	Misapplication of rules	Misinterpretation of symbolic notation	Misuse of distributive property	Substitute letter with value	wrong use of signs
Code	2	3	4	5	6	7
Frequency	183	431	69	261	118	104
Percentage	16	37	6	22	10	9

The contribution of each error to the study was illustrated graphically. The following graph represents the frequencies of each error in the study.
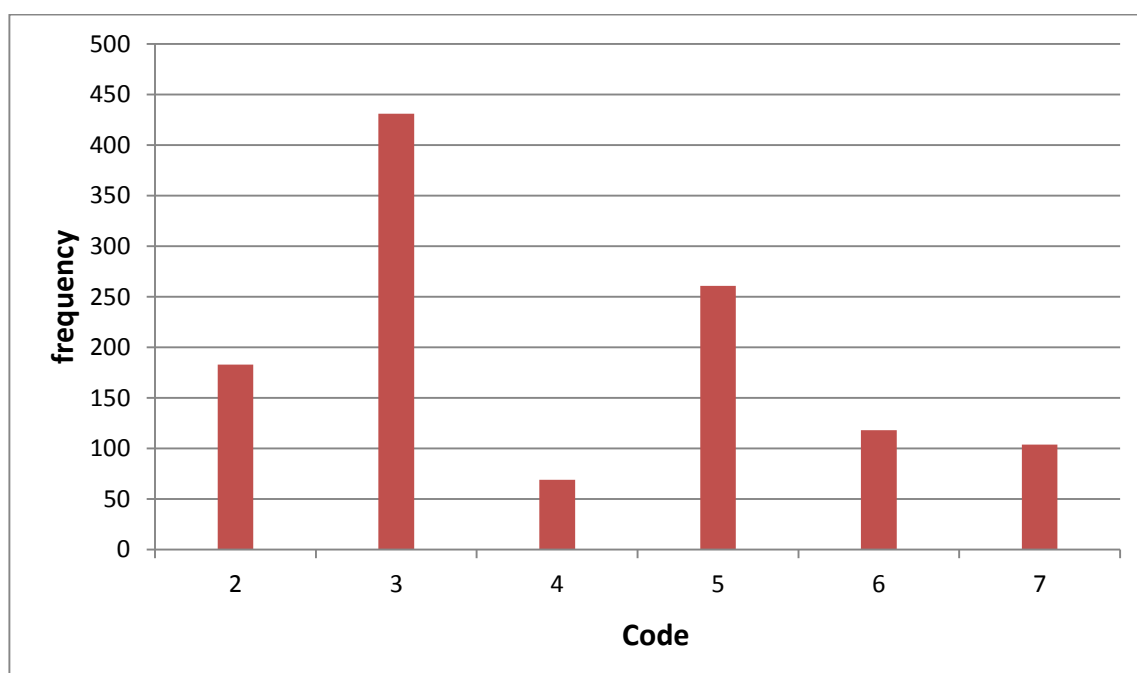
Figure 1: Frequencies of the most common learner errors

It can be clearly seen that the most dominant error was misapplication of rules followed by misuse of the distributive property, conjoin error, substituting letter by number, wrong use of signs and lastly, misinterpretation of symbolic notation.

4.2.2 Interviews

Learners made many errors in simplifying algebraic expressions given in the test. The researcher came up with six common errors with the following percentage distributions: conjoin error, 16%; misapplication of rules, 37%; misinterpretation of symbolic notation, 6%; misuse of the distributive property, 22%; substituting letters by numbers, 10% and sign error, 9%. Learners mostly misapplied rules when they were simplifying algebraic expressions.

The researcher decided to find out the reasons that led learners to commit the six identified errors. Interviewing learners was seen as the best way of getting answers to why learners committed those errors. This became the second phase of the study which involved interviewing 12 learners representing each error code. The number of learners representing each code depended on the prevalence of that error in the test. The table below gives the distribution of learners, the type of error and the item number they represented.

Table 5: Number of learners selected for interviews according to error type
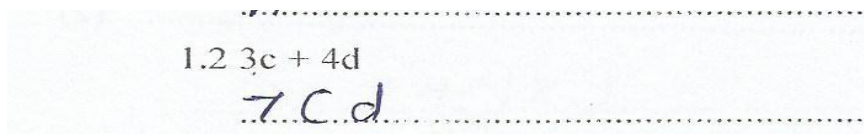
Type of error	% of learners	Number of learners selected	Items selected
Conjoin error	16	2	1.2 2.1
Misapplication of rules	37	4	1.1 1.3 1.6 2.8
Misinterpretation of symbolic notation	6	1	1.7
Misuse of distributive property	22	3	2.2 2.3 2.4
Substituting letters by numbers	10	1	3.2
Sign error	9	1	4

Analysis of errors from interview data

The conjoin error
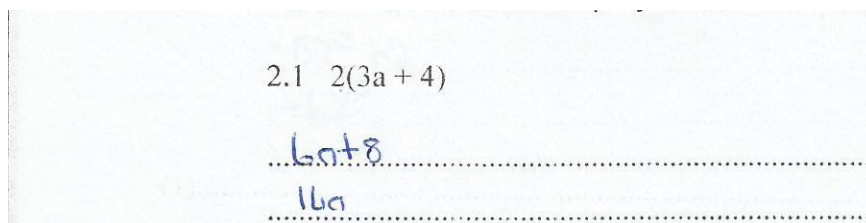
According to Tall and Thomas (1991), learners lack information of the precept of algebraic expressions. Learners were adding coefficients and constants and also coefficients of unlike terms. The expression *3c + 4d* was given as 7cd by 83% of the learners. The other expression where learners added unlike terms was when they were asked to expand and simplify *2(3a + 4)* and getting *6a + 8* was not a problem but went on to give 14a as their final answer. The following extracts demonstrate how conjoin of error terms by learner 1 and learner 12.

Learner 12's response

1.2 3c + 4d

7 C d

Learner 1's response

2.1   2(3a + 4)

6a+8

14a

The two learners had similar explanations as to why they came up with the demonstrated responses. The excerpts below indicate the above two learners' explanations as to why they came up with their answers.

In the excerpts R = researcher and the L = learner.

Excerpt 1: Learner 12

32

R: Look at question 1.2. Go through the question and the solution that you provided.

L: I am done.

R: How did you come up with your answer?

L: The question told me to add *3c* and *4d* and I did.

R: How did you do it?

L: I added 3 and 4 and got 7 then I put *cd*.

R: If you look at *3c* and *4d*, are they like terms?

L: No they are not.

R: So why did you add them?

L: The question said simplify that's why I added to get 1 number.

The learner seemed to have taken the addition sign as an action verb. This was due to the fact that there was misinterpretation of the question. The word 'simplify' had a different meaning to the learner. The question seemed to suggest that the terms were to be *reduced to a single term.* The learner in this case might have followed Mamba's (2012) view that learners may anticipate the *behaviour* of algebraic expressions to be similar to that of arithmetic expressions.

Excerpt 2: learner 1

R: Go to question 2.1 and read the question and the solution that you provided. What is your solution?

L: My solution is *14a.*

R: How did you come up with that answer?

L: I removed brackets first and got *6a + 8* and since the question said I must remove brackets and simplify, I went on to simplify to *14a.*

R: How did you simplify?

L: I added 6 and 8 to get and got 14 then I put and my result was *14a.*

R: Is that correct?

L: Yes it is.

R: If you add *a* and *b* what do you get?

L: I cannot add them.

R: Why?

L: They are not like terms.

R: So are *6a* and 8 like terms?

L: No, I think I made a mistake. I was supposed to leave it as *6a + 8.*

The way the learner responded to the researcher's probing questions shows again that the word simplify had a different meaning to the learner. The learner did not think of like and unlike terms but just considered the term simplify to mean *reduce to a single term*. The learner according to Booth (1998), and Davis (1995) might have had difficulties in accepting lack of closure after expansion.

From the above two excerpts, it can be seen that some learners have a deficiency of prerequisite facts and concepts of algebra. The concept of like terms confuses learners. When learners saw the addition sign, they took it as an instruction to put whatever was given together. To some learners, the word simplify means they must reduce to a single term. They could not think *3c + 4d* could be a solution because they thought having an operation sign meant they had not simplified. That is the reason why most of them gave the answer *7cd* when asked to simplify the expression *3c + 4d*.

Misapplication of rules

Learners used rules inappropriately in simplifying algebraic expressions. Considering $4m + m$ where the most common answer was given as $4m^2$, learners showed confusion between addition and multiplication. Instead of adding the like terms learners tended to multiply $4m$ by m. Cross multiplication was misapplied on $\frac{x}{y} + \frac{w}{z}$ to give the solution $xz + wy$. On $3a-b+a$ most common answer was $3a^2b$. Also on $(x + y)^2$ some learners gave $x^2 + y^2$ as their answer.

Four learners were interviewed on the above briefly described items. The following are extracts of the four learners' responses.

Learner 34's response

1.1 m+4m

$= 4m^2$

1.2 3c + 4d

Learner 50' response

1.3 3a-b+a

$3a^2b$

Learner 36's response

1.6 $\frac{x}{y} + \frac{w}{z}$

$xz + yw$

Learner 41's response

$$2.8(x+y)^2$$
$$(x+y)^2$$
$$(x+y)(x+y)$$
$$x^2+y^2$$

The four extracts demonstrate how learners misapplied algebraic rules. The excerpts below give the explanations provided by four learners whose responses are shown above.

Excerpt 3: Learner 34

R: Go to the first item in your question paper and read it as well as the solution to it.

   Now explain to me how you got that solution.

L: The question asked me to add 4m to m and I did.

R: What did you get when you added?

L: I got $4m^2$

R: How did you add?

L: There was no number on m so I wrote 4 and then added m and m to get m^2.

R: What do you mean when you say on m there was no number?

L: Our teacher said we must add like terms so on m I was looking for a number and I could not see it that's why I wrote 4.

R: If you multiply m by m what do you get?

L: I think it is m^2.

R: So is it the same as $m + m$?

L: Let me think properly.

This excerpt indicates lack of algebraic basics. The learner was not aware of the invisible coefficient and also confused addition and multiplication. The learner thought $m + m$ results in m^2. There was inappropriate application of prior schema. Prior knowledge of exponential expressions was misapplied on that question.

Excerpt 4: Learner 50

R: Can you go to question 1.3. Read it again and the solution that you provided. Explain to me how you got that solution.

L: I subtracted *b* from *3a* to get *2ab* then I added *a* to get $3a^2b$.

R: When you subtract *b* from *3a* do you get *2ab*.

L: Yes.

R: Have you ever heard of like terms?

L: Yes I remember.

R: So are *3a* and *b* like terms which can be simplified to a single term?

L: I see I think I made a mistake.

The above excerpt again shows that the learner lacked prerequisite facts of algebra. There was conjoining of terms together with misapplication of rules. The learner took the operation signs as instructions to add and subtract whether terms were like or not. In

the process of adding and subtracting, learner 50 committed another error linked to misapplication of rules.

Excerpt 5: Learner 36

R: Can you go through question 1.6 and the solution that you gave and then explain how you came up with it.

L: Oh, I did cross multiplication.

R: Was the expression not a fraction?

L: Yes it was.

R: So where is the denominator?

L: When we cross multiply the answer has no denominator.

R: Even if there is no equal you cross multiply like that?

L: Yes.

The learner in this case applied a wrong concept to the question. Cross multiplication is done in situations where there are two fractional terms, one on each side of the equal sign. The learner just saw two fractional terms and related them to algebraic equations with fractions. The absence of the equal sign was not taken into consideration; so, the learner cross multiplied the terms.

Excerpt 6: Learner 41

R: Please go to question 2.8 and read it again. Read your solution as well. What was your solution?

L: It was $x^2 + y^2$.

R. Good. How did you get it?

L: I multiplied x by x to get x^2 and then y by y to get y^2.

R: Do you know the meaning of $(x + y)^2$.

L: yes it means $x + y$ multiplied by $x + y$.

R: Can you use the procedure of removing brackets and write the first stage for me?

L: I will have $x(x + y) + y(x+ y)$, then $x^2 + xy + yx + y^2$

R: Is this going to simplify to the answer that you gave.

L: No it will give me $x^2 + 2xy + y^2$.

R: So what can you say about the answer that you gave in the test?

L: I have seen where I went wrong.

The above excerpt indicates that learners rely on remembered rules and the kind of rules according to Watson (2007) are often misapplied, misremembered or learners do not think about the meaning of situations in which the rules are applicable. The learner might have employed this idea from $(x.y)^2$ which gives $x^2.y^2$ then thought if there is a plus then will just replace multiplication by an addition sign.

Misinterpretation of symbolic notation

This error was seen in situations where the coefficient of one of the unknowns was invisible. Learners misinterpreted the coefficient of such a term to be 0. To learners when there was "nothing" in front of a letter, it meant the coefficient was 0. In question 1.7, learners divided each term of the algebraic fraction by m and were just cancelling the *m* not writing the quotient. There was a term with *m* only. To the learners, when they cancelled *m* nothing was left and they assumed that it had been reduced to 0. The extract below demonstrates how one of the learners cancelled the terms.

Learner 53's response

An explanation by learner 53 of how the *m* disappeared is provided below.

Excerpt 7: Learner 53

R: Go through question 1.7 and your solution to that problem.

R: Here is a paper; I want you to do it again explaining each stage.

L: *Writing the solution again.*

L: I first divided each term by m and cancelled m in each term.

R: What were you left with?

L: I was left with $\frac{a+b}{d}$.

R: Your denominator is now just one term?

L: Yes.

R: Where is the other term?

L: When I cancelled nothing was left on the first term of my denominator.

R: If you divide 2 by 2 what do you get?

L: I will get 1.

R: How about *m* by *m*?
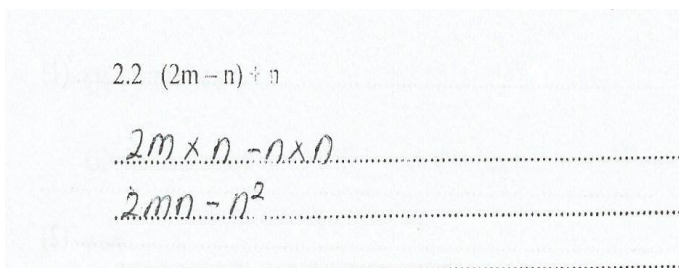
L: I must get 1.

The response given by the learner on the above excerpt indicates that the learner misinterpreted cancelling. To the learner the cancelling meant something had disappeared and nothing was left. The learner lacked basic understanding of algebraic expressions.

Misusing the distributive property

Most of the learners could not operate brackets well. They did not know exactly the terms to be multiplied by the pre or post multiplier. Some learners used the pre-multiplier to multiply terms even outside the brackets which were supposed to be either added or subtracted. The other learners after multiplying with what was in the first set of brackets jumped to the next set of brackets and multiplied with the same pre-multiplier. The following extracts for three learners give a picture how learners misused the distributive property.

 Learner 7's response

2.2 $(2m - n) \div n$

$2m \times n - n \times n$

$2mn - n^2$

Learner 82's response

2.3 $3a - (5c + 4)$

$-15ac + 12a$

Learner 68's response

2.4 $3(2m + 3) + (2 + 5m)$

$6m + 9 + 6 + 15m$

$6m + 15m + 9 + 6$

$21m + 15$

The excerpts below for three learners illustrate how learners failed to apply the distributive property.

Excerpt 8: Learner 7

R: Go to question 2.2 and go through it together with your solution to it. Now explain to me how you got *2mn - n^2*.

L: I got it by multiplying.

R: What did you multiply?

L: First I multiplied *2m* by *n* and wrote *2mn* and put minus then went on to multiply *n* by *n* and wrote n^2 .

R: Is this n supposed to multiply what is inside brackets?

L: Yes thus why they put brackets?

R: So once you see brackets you just multiply by anything?

L: No not by anything here there was nothing to multiply with except *n*.

This excerpt shows lack of understanding of the distributive rules. The meaning of the invisible multiplier was not understood. The learner could not identify the limits of the bracket multiplication.

Excerpt 9: Learner 82

R: I want you to check question 2.3 and your solution.

L: I am done.

R: How did you get *-15ac + 12a*?

L: I multiplied *3a* by *-5c* to get -15ac and then *3a* by 4 to get *12a*.

R: Why were you multiplying by *3a*?

L: Because the question wanted me to remove brackets.

R: So you decided to multiply by *3a*?

L: What must I have multiplied with here?

R: By 1.

L: Why?

R: If I write *a,* what is the coefficient?

L: It is 1.

R: So here there is also no invisible number like for *a* it means 1 is the pre-multiplier.

The learner's response indicates lack of understanding of the distributive laws. The learner again in the process of multiplying by a wrong pre-multiplier went on to make sign errors.

Excerpt 10: Learner 68

R: In your answer script read question 2.4 and the solution that you provided.

L: I have finished.

R: Explain each stage of your solution.

L: At first I multiplied 3 by *2m* then 3 by 3 and also 3 by 2 and 3 by *5m*.

R: What did you get?

L: I got *6m + 9 + 6 + 15m* and collected like terms then got *21m + 15*

R: Look at the question again. Was 3 supposed to multiply the second set of brackets?

L: Yes because it was the only number outside brackets.

R: So if I just write *(a + c)* what do you multiply by?

L: No mam I must have multiplied by 1. I see where I went wrong.

The issue of the invisible multiplier kept on giving learners serious problems. To them brackets means multiplication must be done to bring some changes in the terms. The main problem was that the learner did not know the terms to be multiplied.

Substituting a letter by a numeric value

Learners lack knowledge of variables. They do not understand the meaning of a variable. This was clearly indicated by Question 3.2 which required learners to

determine $b + d + e$ given $b + d = 6$. It was observed that 61% of the learners gave 9 as their answer. An extract of one learner's solution to item 3.2 is provided below.

Learner 68's response

3.2 If b + d = 6,

then b + d + e = ...9
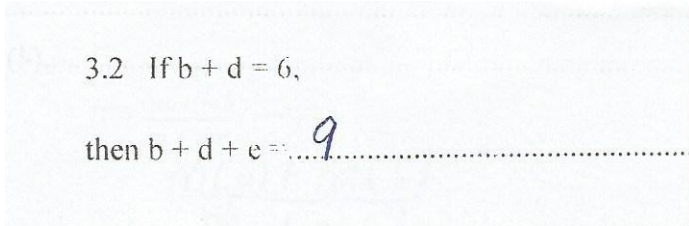
The explanation as to why the learner thought 9 was the answer is demonstrated by the excerpt below.

Excerpt 11: Learner 76

R: Can you please go to question 3. 2. Read the question and the solution that you provided for that question.

L: I am through.

R: Explain to me how you came up with the answer 9.

L: It is because $b + d = 6$.

R: So what?

L: So I added 3 to 6 to get 9.

R: Why did you add 3 and not any other number?

L: It is because to get 6 it is $3 + 3$ so I added another 3.

R: To get 6 do you only add 3 and 3?

L: No.

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R: Which other numbers can you add to get 6?

L: 1 and 5 and also 2 and 4.

R: So here why did you not think they could be other numbers other than 3.

L: Because they told me nothing I thought the numbers were equal.

In the analysis of this response given by learner 76, there is clear evidence that the variable concept was not well understood. Learners seemed to possess in built misunderstandings of the variable. That was the reason why they decided to substitute each of the 3 letters by 3.

Sign error

This error was mainly seen when adding or subtracting a negative term. Learners' answers were affected by wrong application of signs. Learners had problems with both operation and direction signs. This error was mainly seen when adding or subtracting a negative term.
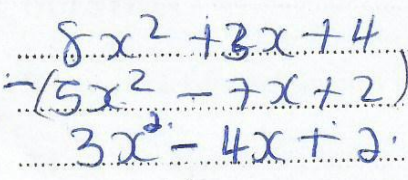
Taking into consideration the question which instructed learners to subtract $5x^2 - 7x + 2$ from $8x^2 + 3x + 4$, some learners had their first stages as $8x^2 + 3x + 4 - 5x^2 - 7x + 2$. The first stage already was wrong because of missing brackets. It meant that the last 2 terms in the expression to be subtracted from the other were automatically going to operate with wrong signs.

There were some learners who had knowledge of using brackets when it came to situations with more than one term. However, some of them could not subtract properly even if their first stage had been put down properly as $(8x^2 + 3x + 4) - (5x^2 - 7x + 2)$. They had problems emanating from failing to deal with the combination of signs. For this question, 41.5% of the learners produced answers with errors which emanated from sign problems. The researcher realized that learners had problems with combining operation and direction signs. An extract of learner 77's response is provided below to

see how learners came up with wrong answers because of failure to deal with both operation and direction signs.

Learner 77's response

The excerpt below brings out the reasons why the learner came up with such sign error.

Excerpt 12: Learner 77

R: Go to question 4 and read the question together with the solution that you provided.

L: I am done.

R: can you please explain how you came up with your answer.

L: I did the subtraction as wanted by the question.

R: Explain how you subtracted.

L: I said $8x^2$ - $5x^2$ and got $3x^2$, $3x$ - $7x$ and got $-4x$ then $4 - 2$ and got 2. So my answer

$3x^2 - 4x + 2.$

R: Before brackets what was your sign?

L: It was a minus.

R: So with the brackets did you use the minus properly?

L: Yes I subtracted $5x^2$ which had no sign before it.

R: How about the other two terms?

L: The next term already had a minus so there was no problem I subtracted.

R: How about the last one?

L: It had a plus so I changed to minus because they said subtract in the question.

R: So you were putting minus on terms which had no minus?

L: Yes because the question wanted me to subtract.

The excerpt shows learners' incompetence in dealing with integers. The error in the learner's response emanated from the learner's failure to deal with signs. It was difficult for the learner to combine operation and direction signs

4.3 Conclusion

The two phases of the investigation have allowed the researcher to discover the most common errors made by learners in simplifying algebraic expressions. It also enabled the researcher to discover the possible causes of those errors. The researcher came up with six types of errors as the common errors made by learners in simplifying algebraic expressions. These errors include: misapplication of rules, conjoin error, misinterpretation of symbolic notation, misusing the distributive property, sign errors and substituting letters by numeric values.

The error due to misapplication of rules was the most common error in this study. Learners misapplied rules mostly because of the fact that algebra has many rules which confuse learners. Learners in many situations learn manipulation of rules without reference to their meaning. Mbewe (2013) suggests that this lack of understanding of the structural features in the conceptual area leads learners to use many rules

inappropriately. Learners then incorrectly apply known rules in appropriate situations. Taking into consideration the learner's solution $xz + wy$ when asked to simplify $\frac{x}{y} + \frac{w}{z}$, the learner had the correct idea of finding the numerator but did not know it was the numerator. As a result, the learner did not put the denominator.

Posamentier (1988) is of the opinion that learners hold naïve theories preconceptions and misconceptions about mathematical ideas which interfere with their learning. Learners possess acquired system of concepts and beliefs which according to Nesher (1987) are wrongly applied to an extended domain. Mbewe (2013) also discovered that learners at times apply inadequate solution schema because of superficial similarities of concepts in disregard of formal similarities. Fischbein and Barash (1993) also viewed learners as possessing solution schema that are deeply rooted in their minds which they mistakenly apply despite intuitive understanding. Therefore, learners possess correct methods in their long term memory. However, it might be difficult for them to recall them resulting in misapplying rules.

The second commonest error involved invalid distribution of brackets. This error was due to misusing the distributive property. Literature attributes this error mainly to learners' failure to master prerequisite facts and concepts. The third commonest error involved conjoining of terms. This error indicated learners' lack of understanding of like and unlike terms. The error due to substitution of letters by numbers was the fourth commonest in terms of frequency. This error related to learners' lack of understanding of variable. The error due to wrong use of signs closely followed that of substituting letters by numbers. There was evidence that learners had difficulties in using direction and operation signs. The error involving misinterpretation of symbolic notation had the least frequency. This error was not frequent probably because most teachers emphasise the use of notations.

This chapter has focused on analysis of the research data. The next chapter discusses key issues that were identified from the analysis of test data and interview transcripts.

The discussion of the key issues will be linked to research studies on errors in algebra as well as to the teaching and learning of Mathematics in general.

CHAPTER 5: DISCUSSION OF THE RESEARCH FINDINGS

5.1 Introduction

The main goal of this research study was to identify the most common errors made by learners in simplifying algebraic expressions and also to find out the possible causes of those errors. Learners committed several errors in simplifying algebraic expressions. This chapter presents a discussion of the findings of the study and relates the findings to the theoretical background and literature review connected to the study.

5.2 Discussion of the findings

A sample of 82 learners participated in the study. The participants were given a 20 item test. In the test, learners were expected to write answers and to show their working. Learners' solutions were analysed to address the first research question of the study which intended to identify the common errors made by Grade 9 learners in simplifying algebraic expressions.

The researcher analysed and interpreted the solutions provided by learners. An item analysis was done and errors were coded as shown in Table 1. The researcher identified six errors as the most common errors from learners' test responses. The six main errors that were identified according to frequency from highest to lowest were: use of inappropriate rules to simplify algebraic expressions; inappropriate use of the distributive property; conjoin of terms; substituting letters by numbers; sign errors and misinterpretation of algebraic notation.

After analysing the test scripts, 12 learners were selected for interviews in relation to the identified common errors. The interviews were meant to address the second research

question which sought to find out the causes of errors which had been displayed by learners in simplifying algebraic expressions in the test.

5.2.1 Misapplication of rules

The findings of the study suggest misapplication of rules to be the main cause of errors in simplifying algebraic expressions at Grade 9 level. The analysis done in this study shows that Grade 9 learners had misapplied rules frequently in simplifying algebraic expressions. In this study, 37% of the errors were due to misapplication of rules. Learners seemed to have got confused and misapplied rules. This was in line with Watson (2007)'s finding that learners get confused and misapply or misremember rules for transforming expressions. Demby (1997) and Kieran (2007) also suggest that the terminology and rules of algebra offer little meaning to many learners resulting in learners memorising algebraic rules with little or no conceptual understanding. Usman (2012) is of the opinion that most of the learners only learn manipulation of rules without reference to meaning of the expression being manipulated. Therefore, learners find it difficult to keep the rules or apply them appropriately. In other situations as suggested by Erlwanger (1975), learners create their own rules which work for themselves only.

According to Mbewe (2013), learners misuse previously learnt procedures and rules in situations where they are not appropriate. When asked to simplify the expression *4m + m*, instead of giving *5m* as the answer most learners gave $4m^2$ as their solution. Learners multiplied the terms instead of adding. In this case, it shows learners failed to differentiate *m + m* from *m × m.* This confusion comes about as learners try to construct knowledge.

Some researchers believe that errors emanate from misconceptions from prior knowledge as learners try to construct mathematical knowledge meanings (Luneta and Makonye, 2010). In the process of constructing mathematical meanings, learners get confused. The confusion arises from too much interference coming from learners themselves, other learners, teachers and also the surrounding environment. According to

Tall and Vinner (1981), errors also result from naive concept images that do not measure up to concept definitions.

The idea of errors emanating from misconceptions from prior knowledge goes in line with the constructivists. According to Smith et al. (1993), constructivists view learning as attained through transforming and refining prior knowledge into more sophisticated concepts. The rise of constructivists theories of learning are viewed by Brodie (2014) as positioning errors as performance of misconceptions, conceptual structures constructed by learners that make sense to learners in relation to their current knowledge. Learners come to a new grade not as empty vessels but they come with pre-knowledge acquired in previous grades (Hatano, 1996). Learners, as suggested by Olivier (1989) then use that knowledge to assimilate and adapt new mathematical concepts. The problem is that at times prior knowledge conflicts with new knowledge making it difficult for learners to judge what is correct or not. Learners then commit errors because they fail to link new knowledge to prior knowledge.

Taking into consideration the explanations provided by learners in the interview, the researcher deduced that this error was mainly due to interference with previously learnt concepts. Looking at the expression $\frac{x}{y} + \frac{w}{z}$ where the most common solution was given as $xz + wy$ and the interviewed learner said that cross multiplication is what had been applied. Cross multiplication must have been done on equations involving fractions and learners thought it was applicable after just seeing two fractions. The learners tried to assimilate new mathematical ideas into a poor inappropriate schema.

On the other hand, learners seemed to lack good arithmetic skills. If fractions were known well from arithmetic, learners could have applied ideas from addition of fractions to simplify $\frac{x}{y} + \frac{w}{z}$. Learners committed errors because of poor arithmetic background. This is supported by Norton and Irvin (2007), MacGregor and Stacey (1997) who also indicated that poor arithmetic skills contribute to algebraic errors.

In this study, it has been discovered that learners at Grade 9 level misapply algebraic rules due to interference from other concepts and also due to the fact that there are many rules in Mathematics. Learners do not make sense of some of the rules; therefore it becomes difficult to keep them in their minds.

5.2.2 Misuse of the distributive property

Misapplication of the distributive property was the second most common error contributing 22% of the errors committed by the learners. Learners made errors in trying to remove brackets. Some of the errors were due to lack of prerequisite facts and concepts as observed Kieran (1992). Learners displayed instrumental knowledge of the distributive property. The learners then got confused and could not even identify the limits of the brackets. Seng (2010), in his study discovered that the distributive property was misapplied in many different ways. This was the same case with this study. Errors emanated from invalid or incomplete distribution in line with Barcellos' (2005) ideas in his suggestions for possible causes of errors linked to expansion of brackets. Moodley (2014) sees learners as not knowing the meaning of brackets. According to Moodley (2014), brackets signify multiplication as soon as learners encounter them.

In this study some learners only multiplied the first number in brackets by the pre or post multiplier. For example, when asked to simplify *2(3a+4)* they got *6a+4*. Other learners, because they saw brackets went on to multiply by a term which was to be added after expansion. This was because they had not seen a visible pre or post multiplier. This means that these learners took brackets to have a different meaning altogether. This is evidenced by items where learners were required to remove brackets. For example when asked to simplify *(2m-n) + n,* most of them gave *2mn - n^2* as their solution. Also, for *3a-(5c+4)* the common solution was *-15ac+12a*. Moodley's (2014) study also reported similar findings. Moodley (2014) found that learners multiply brackets even in the presence of a plus or a minus sign. Learners displayed partial understanding of bracket expansion procedure. The learners were relying on unrefined schema.

In situations where there were two sets of brackets and only the first set having a visible pre-multiplier, learners used the visible pre multiplier for both sets of brackets. This was evidenced on the item which had *3(2m+3) + (2+5m)* where some learners gave *6m+9+6+15* as the first part of their answer. Learners simply multiplied without appreciating the limit of the pre multiplier.

The researcher discovered that learners had difficulties in bracket expansion. Learners inappropriately used the distributive property in a variety of ways as shown in the analysis of learners' solutions. Interview responses also showed that learners had many misconceptions about bracket expansion. These misconceptions led them to produce errors.

5.2.3 Conjoin error

This error contributed the third highest number of errors in the test. The frequency of this error was 16%. Learners made this type of error due to a lack of understanding of the concept of algebraic expression. Learners ignored letters and concentrated on numeric values. They then just added letters in their answers. They also added coefficients and constants and put a letter at the end.

For learners, addition is considered as an 'an action symbol' (Booth, 1999; Davis, 1995). The plus sign as suggested by Mamba (2012) might have been considered as a signal to conjoin terms. This could be the reason why learners conjoined terms. They thought '+' meant put terms together whether like or unlike. Drawing from a constructivist perspective, Brodie (2014) considered errors as arising from conceptual structures constructed by learners. According to Brodie (2014), the conceptual structures make sense in relation to learners' current knowledge but not aligned to convectional mathematical knowledge. In arithmetic, answers are single termed digits but in algebra this does not apply. Tall and Thomas (1991) also support the fact that learners link the idea of single termed answers in arithmetic to algebra. Some learners thought the word simplify meant, reduce to a single term.

Most learners displayed simplified *3c+4d* as being equal to *7cd* and *8a+ 6* as equal *14a* in item 1.2 and 2.1 respectively. They could not believe that *3c+4d* or *8a+6* could be final answers as they possessed more than one term. Learners thought *3c* and *4d* were supposed to be put together to make one term because of the plus sign. The expression *8a+6* was considered to be incomplete. Therefore, learners decided to complete it by reducing it to single term.

The main cause of this error was failure to recognize like terms. Most of the learners just added unlike terms. They were misled by the plus sign which they took as an instruction to conjoin terms. For learners, the word simplify meant reduce to a single term. This suggests that learners have a problem of failing to accept lack of closure and therefore complete or finish expressions. This completing of expressions is what was evidenced in the learners' responses.

5.2.4 Substituting letters by numbers

This error contributed 10% of the errors which were identified in the test. It was mostly identified in item 3.2 where most of the learners thought that if $b + d = 6$ then $b + d + e = 9$. The learners replaced e by 3. The interviewed learner confidently said that he had replaced e by 3 since $b + d = 6$ meant *3+3 = 6.* This meant the learners thought all the three letters had the same value. This implies that learners did not take letters to represent unknown numbers.

Substituting letters by numbers is an error which is produced when learners' responses suggest that the letter has been given numerical value. According to Christou, Vosniadou & Vamvakoussi (2007), this is because learners tend to use their prior experience with numbers in the context of arithmetic. Learners assign numerical values to variables (Kuchemann, 1978). This was also discovered by MacGregor and Stacey (1997) in the study they carried on learners who were around the age of 15. The cause of this replacement of letter by a number as suggested by McIntyre (2005) is that learners have a weak understanding of the variable.

Learners displayed lack of meaning of a variable. Learners possess little knowledge of a variable because the meaning of a variable is often neglected in the teaching and learning of algebra (Usman, 2012). This results in the learners only knowing algebraic manipulation.

5.2.5 Sign errors

This error had a weighting of 9% in the test. Learners failed to subtract integers causing them to make errors in simplifying algebraic expressions. In his study on error analysis of Form 2 learners, Seng (2010) discovered that learners made more errors when negative integers appeared as coefficients in algebraic expressions. In this study, the negative integer also created problems to learners leading them to get wrong answers.

The problem of signs is an indication of poor arithmetic background or a failure to link arithmetic to algebra. Norton and Cooper (1999) are of the opinion that that some learners ignore orders of convention and the operational laws of directed numbers. As a result, these learners disregard some of the signs. At times learners have the first stage of their solutions correctly stated then followed by many errors in simplifying terms (Mamba, 2012). This is evidenced by learners in this study on the question where they were asked to subtract the second expression from the first expression. The learners correctly had $(8x^2+3x+4) - (5x^2-7x+2)$. From that stage most learners had problems with the middle items where they ended up with $3x-7x$. This resulted in the learners giving their final answer as $3x^2-4x+2$. According to the interviewed learner's explanation, there was already a minus on 7; so, he used that minus to subtract as required by the question.

Errors resulting from subtraction of integers prevail because learners have difficulties in operating with negative integers (Seng, 2010). The different uses of the negative sign as suggested by Villasis (2004) are counterintuitive and an obstacle for learners. Learners need to overcome numerous obstacles for them to have few problems in algebra including interpretation of operations (Lee and Messner, 2000). In this study,

the researcher discovered that learners had poor background of operating with directed numbers.

5.2.6 Misinterpretation of symbolic notation

The error due to misinterpretation of symbolic notation contributed 6% of the errors made in the test. Learners misinterpret the symbolic notation when the 'invisible' coefficient of 1 appears in an expression (Seng, 2010). In the test, learners assumed terms like *x* and *y* had no coefficients since they saw no number before the letter. Some learners when they divided a term by another term equal to it thought nothing was left. Barcellos (2005) supports the idea that when learners cancelled equal terms, nothing would be left according to the learners' assumptions. The learner who was interviewed on simplifying algebraic the expression $\frac{ma+mb}{m+md}$ divided each term of the numerator by *m* and got *m + b* and on the denominator divided again by *m* and got *d* only. The learner said that after dividing *m* by *m,* nothing was left.

Barcellos (2005) is of the opinion that learners erroneously cancel terms in simplifying algebraic expressions due to failure to generalize arithmetic rules learned for rational numbers to irrational or complex numbers. Taking the interviewed learner's explanation into consideration, it can be seen that the learner failed to link algebra to arithmetic. When the learner was asked the result of dividing 2 by 2, a correct answer was easily given. The learner then realized that *m* divided by *m* should have given a 1. According to Watson (2007), this is a failure to apply arithmetical meaning. Chamundeswari (2014) is also of the opinion that some of the errors are due to lack of fundamental knowledge in mathematical operations.

5.3 Conclusion

The chapter presented a discussion of the **six** common errors identified in the study. Related literature has also been used to support the findings of the study. The explanations of the origins of the errors have been related to existing literature in a way linking them to broader theoretical views. Having discussed the findings of the study,

the next chapter provides conclusions and recommendations based on the study's findings.

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

6.1 Introduction

This chapter presents conclusions of the study based on the analysis and research findings. The chapter starts by stating the aim and objectives of the research, and research questions. The researcher answers the research questions and provides recommendations for teaching practice and further research.

The aim of this research was:

- To analyse the common errors learners make in simplifying algebraic expressions.

The objectives of the research were:

- To determine learners' errors in simplifying algebraic expressions at Grade 9 level; and
- To find out the reasons why learners at Grade 9 level make errors in simplifying algebraic expressions.

The corresponding research questions were:

- What errors do students make in simplifying algebraic expressions?
- What are the possible reasons that lead learners to make errors in simplifying algebraic expressions?

6.2 Answers to the research questions.
Research question 1

From the test, the researcher analysed and interpreted the solutions provided by the learners. An item analysis was done and the errors were coded as shown in Table 1. The researcher identified six errors as the most common ones in the test. The six errors that were identified according to frequency, from highest to lowest were: misapplication of rules; misuse of the distributive property; conjoin of terms; substituting letters by numbers; wrong use of signs and misinterpretation of symbolic notation.

The most frequent error in this study was due to misapplication of algebraic rules. Luneta and Makonye (2010) have pointed out that misapplication of rules is caused by the fact that learners are taught to manipulate rules. As a result, they lack awareness and understanding of the meaning of expressions. This is because at times teachers mainly put emphasis on procedural rules at the expense of conceptual ideas. According to Luneta and Makonye (2010), this leads learners to depend on procedural knowledge with no conceptual knowledge. This lack of conceptual knowledge leads learners to commit errors. In this study, learners lacked conceptual knowledge and made errors when they were simplifying algebraic expressions. Learners made errors because they did not have appropriate reasons for what they were doing.

Other learners, as discovered by Macgregor and Stacey (1997), overgeneralized correct rules to misapply them in another situation as a result of explicit declarative knowledge gained from the curriculum. Mbewe (2013) also discovered that learners misuse previously learned procedures and rules in situations where they are not applicable. Algebra involves a plethora of terminology and rules which offer little meaning to many learners (Kieran, 2007; Demby1997). Learners overgeneralized and misapplied the algebraic rules because Mathematics has many rules. This makes it difficult for the learners to remember them well and also to apply them appropriately. Watson (2007) also supports the fact that learners find it difficult to remember the rules.

Research question 2

In this study, learners committed a variety of errors due to a number of reasons. The main reason why learners made errors when they were simplifying algebraic expressions was that learners misused algebraic rules. Learners lacked the prerequisite concepts of algebra, making it difficult for them to manipulate algebraic expressions appropriately. Learners also lacked conceptual understanding resulting in them making errors in simplifying algebraic expressions. There was also an indication that learners had poor arithmetic background. This is because learners failed to apply arithmetic

knowledge in algebra. Lack of arithmetic skills or failure to link algebra to arithmetic also led learners to make errors.

The study based its argument on the constructivist perspective which puts strong emphasis on prior knowledge in learning. In this study, there was evidence that learners knew some of the mathematical rules but failed to apply them according to the new situation that they were having. According to Makonye and Luneta (2010), learners' prior knowledge interferes with their new knowledge. Learners try to equilibrate what they already know with the learning. During the equilibration process, learners commit errors as at times they will be in possession of incorrect information or they misremember some of the information.

Overall, the study deduced that learners found it difficult to recall and apply algebraic rules appropriately. As a result learners committed errors when they were simplifying algebraic expressions.

6.3 Recommendations

The results have indicated that learners' errors when simplifying algebraic expressions have their root causes. The researcher has learnt more from identifying learners' errors and their causes. Any study is fruitless if the findings of the study are not useful for future (Mamba, 2012). The researcher is of the opinion that the findings of this study are going to benefit Mathematics Department at this Grade 9 level by making teachers aware of the common errors made by learners when simplifying algebraic expressions.

6.3.1 Recommendations for teaching/learning

The findings from this study mainly showed that learners lacked the basics of algebra, and therefore teachers should assist learners to grasp the basics of algebra like: collecting like and unlike terms; bracket expansion, addition and subtraction of directed algebraic terms. Knowing the basics of algebra will go a long way in understanding the procedural and conceptual aspects of algebra. Teachers should take the constructivist perspective into consideration and be in a position to create a strong arithmetic background for learners so that the arithmetic background could be applied to algebra.

Teachers are encouraged to use teaching methods that enable learners to gain both procedural and conceptual knowledge. The teaching methods should allow learners to give explanations for their answers. Teachers should listen carefully to learners' explanations and be able to identify learners' misconceptions and find ways of helping learners to understand algebraic concepts.

There is need for teachers to create a classroom environment that allows learners to come up with their own conceptions from the procedural and conceptual knowledge taught by the teacher. Learners should also be encouraged to share their successes and problems in algebra in a way trying to clear misconceptions. At times, learners should receive individual attention in order to address the issue of individual differences.

Learners should be given a variety of algebraic expressions to simplify. According to Falle (2007), giving learners a variety of algebraic expressions, makes learners experience the different ways in which algebraic expressions are supposed to be simplified. Learners will get used to algebraic manipulation and algebraic representation.

6.3.2 Recommendations for further research

The findings of this study showed that the methods and approaches used by the teacher to teach algebraic concepts have an effect on the way learners grasp the concepts; therefore, the researcher recommends that there is need to identify the role of the teacher in the errors produced by learners in simplifying algebraic expressions. It is good to understand the way the teacher delivers the concepts to the learners. This will enable identification of the teacher's contribution to the commitment of the errors by learners.

The study also suggests broadening the research by not only concentrating on simplifying algebraic expressions but on algebra as a whole. This may improve the relevance of the research on the teaching/learning situation. The researcher also suggests the use of a bigger sample including participants from several schools.

6.4 Limitations of the study

A relatively small sample was used in this study and all participants were from one school. This is hard to claim to have captured the entire performance in algebra. However, the mixed methods design approach was used in analyzing data in an attempt to promote richness and triangulation.

The second limitation was due to test items of the study. Some of the questions did not bring much information to the study. This was because learners wrote ambiguous answers if they did not leave blank spaces.

6.5 Reflections of the study

Carrying out this study has given the researcher insight into learners' errors in algebra and also the root causes of errors. The study made the researcher realise the importance of listening carefully to learners' explanations to how they come up with their answers. The researcher discovered that learners grasp concepts differently and as a result possess different misconceptions but may come up with the same error.

6.6 Conclusion

This chapter has given a summary of the conclusions made from the study. The chapter also gave recommendations for both teaching and learning as well as for future research. The researcher is of the opinion that the research is going to benefit everyone interested in the improving of the teaching and learning of Mathematics not only in South Africa but the world at large. The findings of this research may be limited to where the research was conducted. Therefore, the generalizability of the findings may be a bit biased. However, there are many implications to almost all Mathematics teachers and learners.

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APPENDIX 1: LETTER TO THE DEPARTMENT OF EDUCATION

P. O. Box 16

Sekgopo

0802

1 December 2014

The Circuit Manager

Department of Education

Private Bag X738

Ga-Kgapane

0838

Dear Sir,

 RE: PERMISSION TO CONDUCT A STUDY AT MAMERIRI HIGH SCHOOL

I am a Masters of Education student specialising in Mathematics Education with UNISA. My dissertation supervisor is Professor Kaino L. M I am requesting to conduct a research at Mameriri High school on Grade 9 learners.

The title of my study is: ANALYSIS OF ERRORS MADE BY LEARNERS IN SIMPLIFYING ALGEBRAIC EXPRESSIONS AT GRADE 9 LEVEL. The study aims to identify learners' errors in algebra and their sources. The intention is to come up with remedial measures which will either do away with or reduce these errors. This will improve the teaching and learning of Mathematics not only to Grade 9 learners but also to the other levels.

I intend to administer a test to 40 sampled Grade 9 learners at the above mentioned school and then interview some of them. The participants will not be disadvantaged in any way. The right of participants to privacy, anonymity, confidentiality and respect for human dignity will be honoured during the research. Participation by learners is voluntary and anyone willing to withdraw can do so without penalty. The participation of learners has no foreseeable risks.

For more information concerning this request you can contact me at 073 713 4810 or at mildretncube@ymail.com or contact my supervisor Prof Kaino L M at 012 429 4314 or at kainolm@unisa.ac.za

Yours faithfully,

Mildret Ncube

APPENDIX 2: LETTER TO THE PRINCIPAL

P. O. Box 16

Sekgopo

0802

1 December 2014

The Principal

Mameririri High School

P. O. Box 16

Sekgopo

0802

Dear Sir,

 RE: PERMISSION TO CONDUCT A STUDY AT YOUR SCHOOL

I am a Masters of Education student specialising in Mathematics Education with UNISA. My dissertation supervisor is Professor Kaino L M. I am requesting to conduct a research at your school on Grade 9 learners.

The title of my study is: ANALYSIS OF ERRORS MADE BY LEARNERS IN SIMPLIFYING ALGEBRAIC EXPRESSIONS AT GRADE 9 LEVEL. The study aims to identify learners' errors in algebra and their sources. The intention is to come up with remedial measures which will either do away with or reduce these errors. This will improve the teaching and learning of Mathematics not only to Grade 9 learners but also to the other levels.

I intend to administer a test to 40 sampled Grade 9 learners at your school and then interview some of them. The participants will not be disadvantaged in any way. The right of participants to privacy, anonymity, confidentiality and respect for human dignity will be honoured during the research. Participation by learners is voluntary and anyone willing to withdraw can do so without penalty. The participation of learners has no foreseeable risks.

For more information concerning this request you can contact me at 073 713 4810 or at mildretncube@ymail.com or contact my supervisor Prof Kaino L. M at 012 429 4314 or at kainolm@unisa.ac.za

Yours faithfully,

Mildret Ncube

APPENDIX 3: LETTER TO THE PARENT/GUARDIAN

P. O. Box 16

Sekgopo

0802

8 December 2014

Dear parent/guardian,

RE: <u>A REQUEST FOR YOUR CHILD'S PARTICIPATION IN RESEARCH STUDY</u>

I am a Masters of Education student specialising in Mathematics Education with UNISA. My dissertation supervisor is Professor Kaino L M. The title of my study is: ANALYSING ERRORS MADE BY LEARNERS IN SIMPLIFYING ALGEBRAIC EXPRESSIONS AT GRADE 9 LEVEL. The intention is to come up with remedial measures which will either do away with or reduce these errors. This will improve the teaching and learning of Mathematics not only to Grade 9 learners but also to the other levels.

I intend to administer a test to 40 Grade 9 learners and then interview some of them. Furthermore, I will observe their teacher revising the administered test with them. I am therefore asking for your permission to allow your child to be one of the participants in this study. The participants will not be disadvantaged in any way. The right of participants to privacy, anonymity, confidentiality and respect for human dignity will be honoured during the research. Participation by learners is voluntary and anyone willing to withdraw can do so without penalty. The participation of learners has no foreseeable risks.

There is attached form at the back of this letter for you to indicate your decision to allow your child to take part in the study. May you please complete it and return it to me at your earliest convenience.

For more information concerning this request you can call me at 0737134810 or email me at <u>mildretncube@ymail.com</u> or contact my supervisor Prof Kaino L. M at 012 429 4314 or at <u>kainolm@unisa.ac.za</u>

Your cooperation will be greatly appreciated.

Yours faithfully,

Mildret Ncube

APPENDIX 4: PARENT/GUARDIAN CONSENT FORM

Please fill in the reply slip on granting permission to your child to participate in the study.

I...have read and understood the conditions of the study.

My child .... can/cannot take part in the study. (Delete the inapplicable).

Parent/guardian's   signature..

Date…………………........

APPENDIX 5: LETTER TO THE LEARNER

P. O. Box 16

Sekgopo

0802

8 December 2014

Dear Learner,

I am a Masters of Education student specialising in Mathematics Education with UNISA. My dissertation supervisor is Professor Kaino L M. The title of my study is: Analysis of errors made in simplifying algebraic expressions at Grade 9 level. The intention is to help Grade 9 learners with algebra by assisting teachers to identify errors in algebra and their causes.

I invite you to participate in my research study. You will write a short test in 1 hour under supervision then, depending on your responses can be interviewed on how you would have come up with your answers. Be assured that you your participation in the study will have no bearing on your grades or evaluation in the subject. The right to privacy, anonymity, confidentiality and respect for human dignity will be honoured during the research. Participation is voluntary and if you decide to withdraw, you can do so at any time without penalty. Participation in the study has no foreseeable risks.

There is an attached form for you to indicate your decision to take part in the study. Discuss your involvement in the study with your parents then complete the form and return it to me at your earliest convenience. A letter has also been sent to your parents to indicate their decision concerning your participation.

For more information concerning this request you can call me at 073 713 4810 or email me at mildretncube@ymail.com or contact my supervisor Prof Kaino L. M at 012 429 4314 or at kainolm@unisa.ac.za

Your participation will be greatly appreciated.

Yours faithfully,

Mildret Ncube

APPENDIX 6: LEARNER'S ASSENT FORM

Please fill in this form to indicate your decision to participate in the mentioned study.

I..hav

e read and understood the conditions for the study. I accept/do not accept to participate

in the study. (Delete the inapplicable).

Learner's   signature..

Date………………………………….

UNISA | college of education

Research Ethics Clearance Certificate
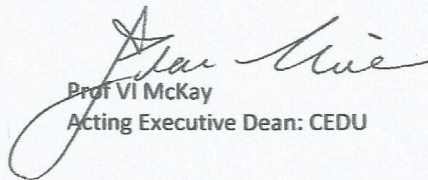
This is to certify that the application for ethical clearance submitted by
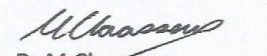
M Ncube [48809969]

For a M Ed study entitled

Analysis of errors made by learners in simplifying Algebraic expressions at Grade 9 level

has met the ethical requirements as specified by the University of South Africa College of Education Research Ethics Committee. This certificate is valid for two years from the date of issue.

Prof VI McKay
Acting Executive Dean: CEDU

Dr M Claassens
CEDU REC (Chairperson)
mcdtc@netactive.co.za

Reference number: 2015 February /48809969 /MC   18 February 2015

UNISA
COLLEGE OF EDUCATION
2015 -02- 2 4
Office of the Deputy Executive Dean
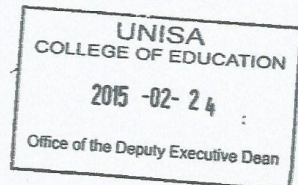
LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF EDUCATION
RAKWADU CIRCUIT

Enquiries: Mashakeni L.A
Persal: 83254781
Date: 01-12-2014

REQUEST TO CONDUCT RESEARCH: NCUBE M. PERSAL NO 83254781.

The above matter refers:

The above named educator is requesting to conduct research in
this circuit under department of Education.

The request is recommended and approved.

Yours faithfully

SETHABA M.D.B
CIRCUIT MANAGER
/rml

Department of Education
Private Bag x 738 Ga-Kgapane 0838

2014 -12- 0 2

Rakwadu Circuit
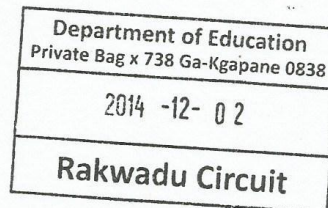
APPENDIX 9: ALGEBRAIC EXPRESSIONS TEST

LEARNER NUMBER: ....

DATE: ....

INSTRUCTIONS

1. Time: 1 hour.

2. Answer all questions.

3. Show all working where necessary in the spaces provided.

4. The use of an electronic calculator is allowed.

QUESTION 1

Simplify where possible.

1.1 m+4m

....

   (1)

1.2 3c + 4d

....

   (1)

1.3 3a-b+a

....

....

   ....(2)

1.4   3pc- 4 + 5cp +8

....

....

   ....(2)

1.5 8k-3y-4k+5y-4x

....

....

....

.......................(3)

1.6 $\frac{x}{y} + \frac{w}{z}$

....

....

....

.......................(3)

1.7 $\frac{ma+mb}{m+md}$

....

....

....

....

......................................(4)

QUESTION 2

Remove brackets and simplify.

2.1 $2(3a + 4)$

....

....

....(2)

2.2 $(2m - n) + n$

....

....

........(2)

2.3 $3a - (5c + 4)$

....

....

.........(2)

2.4   3(2m + 3) + (2 + 5m)

....

....

....

....

....   (4)

2.5 $y\left(\frac{a}{b}\right)$

....

....

.......(2)

2.6   (2a) (3b)

....

....

.......(2)

2.7   (mn) (2m)

....

....

..........(2)

2.8 $(x + y)^2$

....

....

....

....

...................................(4)

2.9   (x + 3) (x - 3)

....

....

....
.......................(3)

QUESTION 3

3.1 If m +n = 6,

then m + n + 4 =..(2)

3.2 If b + d = 6,

then b + d + e =...(2)

QUESTION 4

Subtract first expression from second expression.

$5x^2 - 7x + 2$; $8x^2 + 3x + 4$

....

....

....

....

.....................................(4)

QUESTION 5

Given the straight line MNR is such that NR is 3 times MN and MN is y cm long.

y cm

M   N   R
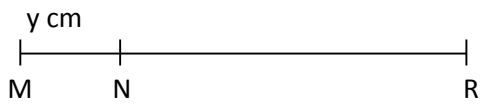
Find MR in terms of y.

....

....

....
....................(3)

TOTAL MARKS: 50

RESEARCHER NAME: MILDRET NCUBE

CONTACT DETAILS: 073 713 4810 or mildretncube@ymail.com

APPENDIX 10: MEMORANDUM FOR ALGEBRAIC EXPRESSIONS TEST

QUESTION 1

1.1 $5m\checkmark$ (1)

1.2 $3c + d\checkmark$ (1)

1.3 $3a + a - b\checkmark$

$= 4a - b \checkmark$ (2)

1.4 $3pc + 5pc + 8 - 4\checkmark$

$= 8pc + 4\checkmark$ (2)

1.5 $8k - 4k + 5y - 3y - 4c\checkmark\checkmark$

$= 4k + 2y - 4c\checkmark$ (3)

1.6 $\frac{x(z) + w(y)}{yz}\checkmark\checkmark$

$= \frac{xz + wy}{yz}\checkmark$ (3)

1.7 $\frac{m(a+b)}{m(1+d)}\checkmark\checkmark$

$= \frac{a+b}{1+d}\checkmark\checkmark$ (4)

QUESTION 2

2.1 $2 \times 3a + 2 \times 4\checkmark$

$= 6a + 8\checkmark$ (2)

2.2 $2m - n + n \checkmark$

$= 2m\checkmark$ (2)

2.3 $3a - 1 \times 5c - 1 \times 4\checkmark$

$= 3a - 5c - 4\checkmark$ (2)

2.4 $3 \times 2m + 3 \times 3 + 1 \times 2 + 1 \times 5m\checkmark$

$= 6m + 9 + 2 + 5m\checkmark$

$= 6m + 5m + 9 + 2\checkmark$

$= 11m + 9\checkmark$ (4)

2.5 $\frac{y \times a}{1 \times b}$√

    $= \frac{ya}{b}$√   (2)

2.6   2 × 3 × a × b √

    = 6ab √   (2)

2.7   2 × m × m × n √

    $= 2m^2$n √   (2)

2.8   (x + y) (x + y) √

    = x (x + y) + y (x + y) √

    $= x^2$ + x y + y x $+ y^2$√

    $= x^2$ + 2x y $+ y^2$√   (4)

2.9   x (x - 3) + 3 (x - 3) √

    $= x^2$ − 3x + 3x − 9√

    $= x^2$ - 9√   (3)

QUESTION 3

3.1   6 + 4 √

    = 10 √   (2)

3.2   6 + e √√   (2)

QUESTION 4

$8x^2$ + 3x + 4 − $(5x^2$ − 7x +2) √

$= 8x^2$ + 3x + 4 - $5x^2$ + 7x − 2√

$= 8x^2$ - $5x^2$ + 3x + 7x + 4 − 2√

$= 3x^2$ + 10x + 2√ **OR**

  $8x^2$ + 3x + 4

- $(5x^2$ − 7x + 2) √

$= 3x^2$ + 10x + 2√√√   (4)

QUESTION 5

MN = 3 y√

∴ MR = y + 3y √

    =4y √   (3)

TOTAL: 50 MARKS

APPENDIX 11: INTERVIEW GUIDE

Process	Interview question
1. Read	Please read the question.
2. Comprehension	What does the question mean?
3. Strategy selection	How do you solve the question?
4. Process test.	Now look at how you solved the question in the
5. Explanation	Explain to me how you came up with this solution.
6. Consolidation	What does the answer mean?
7. Verification	Can you check the correctness of your answer?
8. Conflict	Ask conflicting questions if there is conflict in Solving.

APPENDIX 12: EDITING AND PROOFREADING CERTIFICATE

EDITING AND PROOFREADING CERTIFICATE

7542 Galangal Street

Lotus Gardens

Pretoria

0008

31 May 2016

TO WHOM IT MAY CONCERN

This letter serves to confirm that I have edited and proofread Mrs. M. Ncube's dissertation entitled: **"ANALYSIS OF ERRORS MADE BY LEARNERS IN SIMPLIFYING ALGEBRAIC EXPRESSIONS AT GRADE 9 LEVEL"**

I found the work easy and enjoyable to read. Much of my editing basically dealt with obstructionist technical aspects of language which could have otherwise compromised smooth reading as well as the sense of the information being conveyed. I hope that the work will be found to be of an acceptable standard. I am a member of Professional Editors Group and also a Language Editor at Bureau of Market Research at the University of South Africa.

Hereunder are my particulars:

_____

Jack Chokwe (Mr)

Bureau of Market Research (Unisa)

Contact numbers: 072 214 5489 / 012 429 3327

jmb@executivemail.co.za

Professional
EDITORS
Guild