A FRAMEWORK TO ENABLE ROTORCRAFT MAINTENANCE FREE OPERATING PERIODS

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Andrew T. Bellocchio

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A FRAMEWORK TO ENABLE ROTORCRAFT MAINTENANCE FREE OPERATING PERIODS

Approved By:

Dr. Daniel Schrage, Advisor
School of Aerospace Engineering
Georgia Institute of Technology

Colonel Richard V. Melnyk, Ph.D.
Department of Civil & Mechanical Engineering
United States Military Academy

Dr. Dimitri Mavris
School of Aerospace Engineering
Georgia Institute of Technology

Dr. Sylvester Ashok
School of Aerospace Engineering
Georgia Institute of Technology

Dr. Vitali Volovoi
Engineering Services Lead
Mitek Analytics

Date Approved: April 3, 2018

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NOMENCLATURE

A(t)	Failure area
A_A	Achieved Availability
ADD	Aviation Development Directorate
ADD-AATD	Applied Aviation Technology Directorate
ADS	Aeronautical Design Standard
ADT	Administrative Downtime
AFCS	Automatic Flight Control System
AFLCMC	Air Force Life Cycle Management Center
A_m	Materiel Availability
AMRDEC	Aviation and Missile Research Development and Engineering Center
A_O	Operational Availability
ARH	Armed Reconnaissance Helicopter
ASSIST	A State-based System Integrated Sustainment Tool
BDD	Binary Decision Diagrams
CBM	Condition Based Maintenance
CCA	Cost Capability Analysis
cdf	cumulative distribution function
COST-A	Capability-Based Operations and Sustainment-Aviation
cv	Coefficient of Variance
D	Downtime per Cycle
D_A	Achieved downtime
DES	Discrete Event Simulation
DoD	Department of Defense
DOE	Design of Experiments
DOTMLPF-P	Doctrine, Organization, Training, Materiel, Leadership and education, Personnel, Facilities, and Policy
EMA	Essential Maintenance Action
f(t)	Unreliability or probability failure pdf

F(t)	Unreliability or probability failure cdf
FAMC	Future of Aviation Maintenance
FC	Failure Cause
FDSC	Failure Definition and Scoring Criteria
FMC	Fully Mission Capable
FoS	Family of Systems
FTA	Fault Tree Analysis
FVL	Future Vertical Lift
$g_{(t)}$	Repair time density
$G_{(t)}$	Cumulative failure function
H	Expected number of failures
HOGE	Hover Out of Ground Effect
ICT	Interdisciplinary Concept Team
JCIDS	Joint Capabilities Integration and Development System
JMR	Joint Multi-Role
JTCG/AS	Joint Technical Coordinating Group on Aircraft Survivability
KPP	Key Performance Parameter
KSA	Key System Attribute
LCC	Life Cycle Costs
LDT	Logistical Downtime
MA	Mission Abort
MAC	Maintenance Allocation Chart
MADM	Multi-Attribute Decision Model
MAF	Mission Affecting Failure
MCI	Mission Capability Index
MDT	Mean Downtime
MEF	Mission Essential Function
MFE	Model Fit Error
MFOP	Maintenance Free Operating Period
MFOPS	MFOP Success
$MFOPS_{req}$	MFOPS goal

MLDT	Mean Logistics Delay Time
MMH	Maintenance Man Hours
MMT	Mean Maintenance Time
MPR	Maintenance Policy Reliability
MPS	Minimum Policy Success
MR	Mission Reliability
MRE	Model Response Error
MRP	Maintenance Recovery Period
MRPS	MRP Success
MSE	Mean Square Error
MTBF	Mean Time Between Failures
MTBM	Mean Time Between Maintenance
MTTR	Mean Time To Repair
NPV	Net Present Value
O&S	Operations and Support
OEC	Overall Evaluation Criteria
OEM	Original Equipment Manufacturer
OPTEMPO	Operational Tempo
OSD	Office of the Secretary of Defense
OSD AT&L	Office of the Under Secretary of Defense Acquisition Technology and Logistics
P	Probability
pdf	probability density function
PEO	Program Executive Office
q(t)	Unreliability
QFD	Quality Function Deployment
R&D	Research & Development
R(t)	Reliability function
RAM-C	Reliability, Availability, Maintainability - Cost
RC	Repair Cost
R_m	Materiel Reliability

RUH	Representative Utility Helicopter
t	Time
T_f	Time to Repair Failures
TLD	Time Limited Dispatch
t_{mf}	Maintenance free operating period duration
T_p	Time of preventative replacements
t_p	Replacement interval
t_{phase}	Mission duration
TTF	Time to Failure
TTF	Time to Failure
TTR	Time to Repair
UMA	Unscheduled Maintenance Action
URA	Ultra-Reliable Aircraft
URD	Ultra-Reliable Design
VBA	Value Based Acquisition
V_{Block}	Block Speed
VDD	Value Driven Design
VLC	Vertical Lift Consortium
ZMA	Zero Maintenance Aircraft

SUMMARY

For the past 50 years, the paradigm of on-condition rotorcraft maintenance has suffered from random failures and intrusive scheduled maintenance that regularly disrupted flight operations. The British Ultra-Reliable Aircraft Pilot Program of the late 1990s introduced the paradigm of Maintenance Free Operating Period (MFOP) as a solution. A MFOP aircraft is a fault tolerant, highly reliable system that minimizes disruptive failures and maintenance for an extended period of operations. After the MFOP, a single Maintenance Recovery Period (MRP) consolidates repair of accrued faults and inspections to restore an aircraft's reliability for the next MFOP cycle. A MFOP strategy provides assurance to the user that flight operations will continue without disruption for the duration of the MFOP at a given success rate.

The U.S. Department of Defense recently adopted MFOP as a maintenance strategy for the next generation of rotorcraft named the Future Vertical Lift (FVL) Family of Systems. The U.S. military desires uninterrupted flight operations to enable a more expeditionary force that operates from remote, austere bases. It is thought that a 100-flight hour MFOP at 90% availability will support such deployments; yet, today's fleet has the system reliability to fly less than ten hours without significant repair at 75% availability. The challenge presented is to achieve an order of magnitude improvement to meet the FVL target and set the conditions for near-zero maintenance.

The thesis posits that statistical based metrics using the mean are insufficient in a MFOP strategy and that metrics such as the MFOP, which include the time history of failure, are as important as the rate of failure. It utilizes a Discrete Event Simulation to model the MFOP, MRP, and their success rates as operational metrics. The work identifies

which subsystem(s) limit the MFOP of an aircraft and which components drive MRP higher. It explores the relationship between MFOP and availability where preventive component renewals occur at discrete multiples of the MRP. The thesis provides a framework to a maintenance policy that balances availability, dependability, and maintainability of a MFOP rotorcraft. Finally, it tests the hypothesis that an operational commander has some control over the MFOP by varying the MRP through an aggressive lifing policy.

1 INTRODUCTION

1.1 Progression Towards Ultra-Reliable Design

The paradigm of rotorcraft maintenance is shifting after almost 40 years of time-based, preventive scheduled maintenance. Figure 1 shows the progression of maintenance approaches starting from the 1970s to the desired future state in the mid-21st century. The introduction of new technologies enabling Condition Based Maintenance (CBM) has opened new options for sustainment. The industry is now pushing towards greater reliability, lower costs, and lower maintenance burdens. The promise of steadily developing technologies has inspired aspirations of a near-zero maintenance environment where rotorcraft have the dependability and maintainability of modern automobiles.

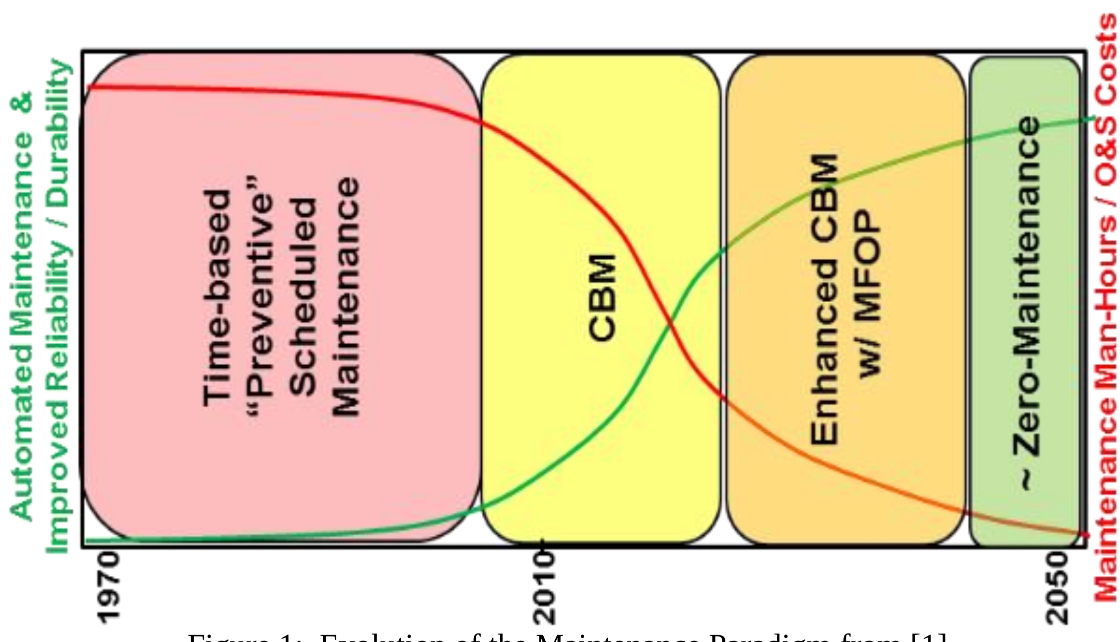


Figure 1: Evolution of the Maintenance Paradigm from [1]

Since the 1970s, rotorcraft maintenance has been a time-based, on-condition maintenance paradigm to ensure readiness [2]. Maintainers considered maintenance activities as unscheduled or scheduled. Unscheduled maintenance is the repair of an aircraft after random component failure. Random failures are very disruptive to operations. They cancel missions and cause aircraft accidents. Despite a slow evolution in part reliability, system reliability has not improved significantly. The increasing complexity of aircraft systems often offset gains in component reliability. Even airplanes are not immune to decreasing returns in overall availability (see Figure 2). Unscheduled maintenance is costly in terms of dollars and lost operating time. As a result, the maintenance burden and operating costs remain high (see Figure 1). Scheduled maintenance has occurred at fixed time intervals, typically flight hours or number of days. It has taken a preventive approach, involving intrusive inspections and replacement of parts with useful life remaining [3].

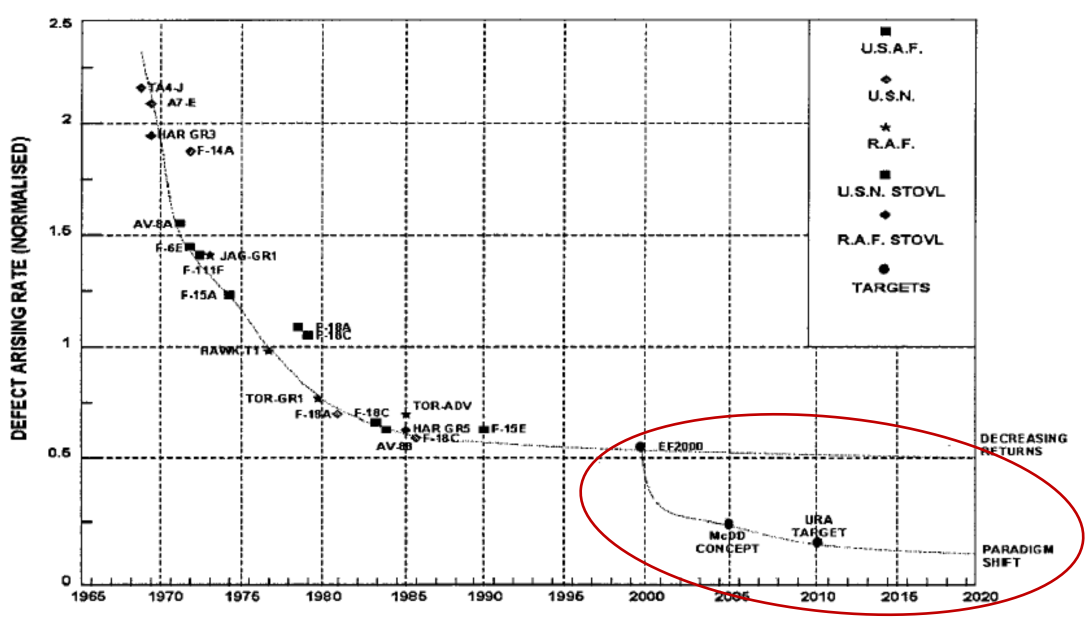


Figure 2: Historical Defect Rate [4]

Operators are demanding improvement and new approaches. Condition Based Maintenance (CBM) attempts to alleviate some of the maintenance burden associated with scheduled inspections by replacing components as needed based upon evidence. The U.S. Army Aeronautical Design Standard (ADS) 79-D Handbook describes CBM as "a set of maintenance processes and capabilities derived primarily from the real-time assessment of system condition which are obtained from embedded sensors and/or external test and measurements using portable equipment" [5]. Over the past ten years, the success of CBM has grown with capability of sensors and health management systems, giving CBM an advantage that was not available to the on-condition maintenance strategy. CBM acknowledges the inefficiencies of the past's on-condition maintenance and eliminates unnecessary removal of a working part; thereby lowering maintenance burden. Minimizing unscheduled maintenance with early detecting of imminent failure further reduces Operation and Support (O&S) cost. CBM is showing itself as a steady evolution with sensor technology and data management maturation. The latest upgrades for Black Hawk and Apache helicopters have integrated CBM technologies.

A program of Maintenance Free Operating Period (MFOP) is the next maintenance paradigm. The British Royal Air Force's Ultra Reliable Aircraft Pilot study introduced the concept of aircraft MFOP in the late 1990s [6]. The Pilot study became a Project with the research goal to find affordable Life Cycle Costs (LCC) and "enable substantial increases in aircraft operational availability and reliability" [4]. A MFOP maintenance program was the first objective of the program. A MFOP maintenance program seeks to eliminate disruptive random failures for over an extended period and consolidate any scheduled maintenance into a succinct, repair period called Maintenance Recovery Period (MRP). Hockley [7] provides the most accepted definition for the terms:

a. Maintenance Free Operating Period (MFOP). A period of operation during which the equipment must be able to carry out all its assigned missions without any maintenance action and without the operator being restricted in any way due to system faults or limitations.

b. Maintenance Recovery Period (MRP). The downtime during which appropriate scheduled or corrective maintenance is done to recover the system to its fully serviceable state so that it can achieve the next MFOP.

MFOP and MRP form a cycle as shown in Figure 3. MFOP is measured in flight hours and MRP is measured in Maintenance Man Hours (MMH) or the total hours the aircraft is unavailable for repair.

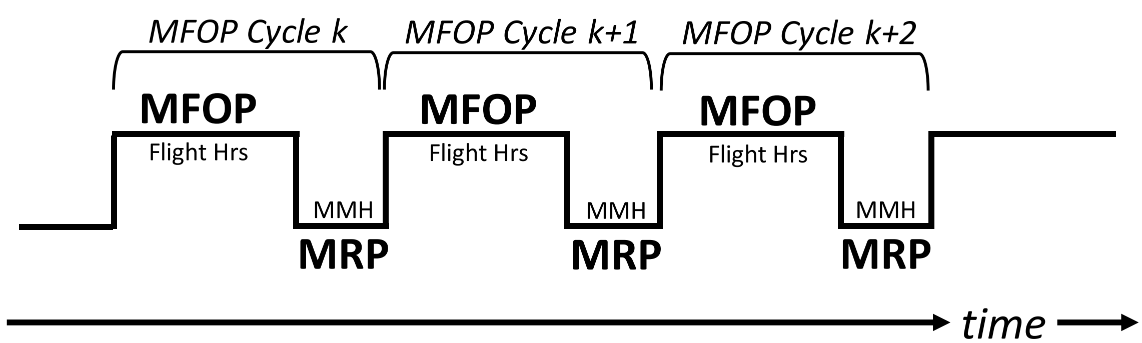


Figure 3: MFOP Cycle

CBM enhances MFOP duration by identifying failures with sufficient lead time to schedule repair at the next MRP. Maintainers preventively replace parts with an impending failure at the next MRP. A MFOP aircraft provides assurance of Fully Mission

Capable (FMC) aircraft to the operator for a specified number of flight hours. MFOP does benefit from improved inherent reliability; however, the focus is on providing a dependable aircraft that operators can use for a long period of time.

The United States Department of Defense (DoD) has adopted MFOP as the maintenance paradigm for its next generation fleet of military aircraft named Future Vertical Lift (FVL) Family of Systems (FoS). DoD expects to begin fielding these aircraft in the next 10-20 years. Unlike CBMs steady inclusion into the current fleet, FVL's FoS design for MFOP capability from the beginning. The work of this thesis occurs within the MFOP maintenance paradigm. For a full discussion on MFOP, please see Section 3.2.

Zero Maintenance Aircraft (ZMA) represents a true revolution in aircraft maintenance. ZMA seeks an order of magnitude change in the MFOP for helicopters beyond 2050 [8]. The U.S. Army Aviation Development Directorate (ADD) envisions FVL setting the conditions for a near zero maintenance program under Ultra Reliable Design (URD) [8]. It requires no scheduled maintenance for extended MFOPs and to have a low maintenance burden and small logistical footprint. Reduced life cycle costs are then a consequence of ultra-reliability.

The progression of the rotorcraft maintenance paradigms is not unlike the story of improvement in automobile reliability. Cars and trucks integrate CBM in the form of system diagnosis such as the check engine light and tire pressure sensor. A MFOP-MRP cycle is not unlike a regular automobile inspection every 10,000 to 20,000 miles. ZMA is the state of today's cars where system availability is near 100% and almost no maintenance, short of oil changes and tire rotation, are due between inspection intervals. For rotorcraft, the future seeks a similar progression, with the goal of ultra-reliable aircraft arriving in the ZMA revolution.

1.2 Relevance

The U.S. Department of Defense recently started a MFOP effort with goal of an ultra-reliable aircraft. The next generation of U.S. military rotorcraft, termed Future Vertical Lift (FVL) Family of Systems (FoS), is “intended to provide the joint force a leap-ahead improvement in vertical lift capabilities over today's rotorcraft” [9]. The U.S. Joint Force is seeking self-deployable, agile aircraft to support a more expeditionary future force. Minimizing the logistical footprint and maintenance burden will be key enablers of a more agile force. FVL FoS will deploy and operate 30 days with minimal support. A conceptual goal of a 100-flight hour Maintenance Free Operating Period (MFOP) at 90% availability [9] will be necessary to support such deployments (see Table 1); yet, today’s fleet has the system reliability to fly less than 10 hours without significant repair at 75% availability [10]. Historical data of MFOP for DoD aircraft is not available because the DoD does not track MFOP as a metric. This research estimates a representative UH-60M Black Hawk model’s MFOP at 5.1 hours (see section 4.3.2.2 for the analysis). Another work found the OV-22 Osprey’s MFOP to be around 2 hours [11]. The 100-flight hour MFOP for FVL represents a single order of magnitude increase in MFOP.

Table 1: Maintenance Metrics

Metric	Current Fleet Benchmark (today)	Future Vertical Lift Target (years 2030s)	Zero Maintenance Threshold (years 2040-50)	Zero Maintenance Objective (years 2040-50)
R_m		TBD	TBD	TBD
MFOP	< 10 hours	100-FH one week	480 FH	720 FH
A_o	75%	90%	90%	95%
MRP			3 days 144 MMH	1.5 days 108 MMH
MTTR (unscheduled)			3 MMH	1.5 MMH

Beyond FVL, Zero Maintenance Aircraft (ZMA) will push the boundaries of reliability and maintainability even further. ZMA will grow MFOP towards a threshold of 480-hour and an objective of 720-hour. This represents two orders of magnitude increase from today's rotorcraft. The challenge is to achieve an order of magnitude changes to meet the FVL target and set the conditions for ZMA.

1.3 Motivation

1.3.1 Driven by FVL Opportunity

The paradigm shift to a MFOP program is driven by the needs of the U.S. future Joint Force. The DoD fleet consists mostly of the UH-1 Iroquois, AH-1 Cobra, UH-60 Black Hawk, AH-64 Apache, CH-47 Chinook, CH-53 Sea Stallion, and the OV-22 Osprey. Of those aircraft, the DoD only fielded one first generation aircraft, the Osprey, in this century.

The current fleet is aging and likely to reach end of life by the mid-2030s. In 2008, the Congressional Rotorcraft Caucus stated its concern "about the lack of a strategic plan for improving the state of vertical lift aircraft" [12].

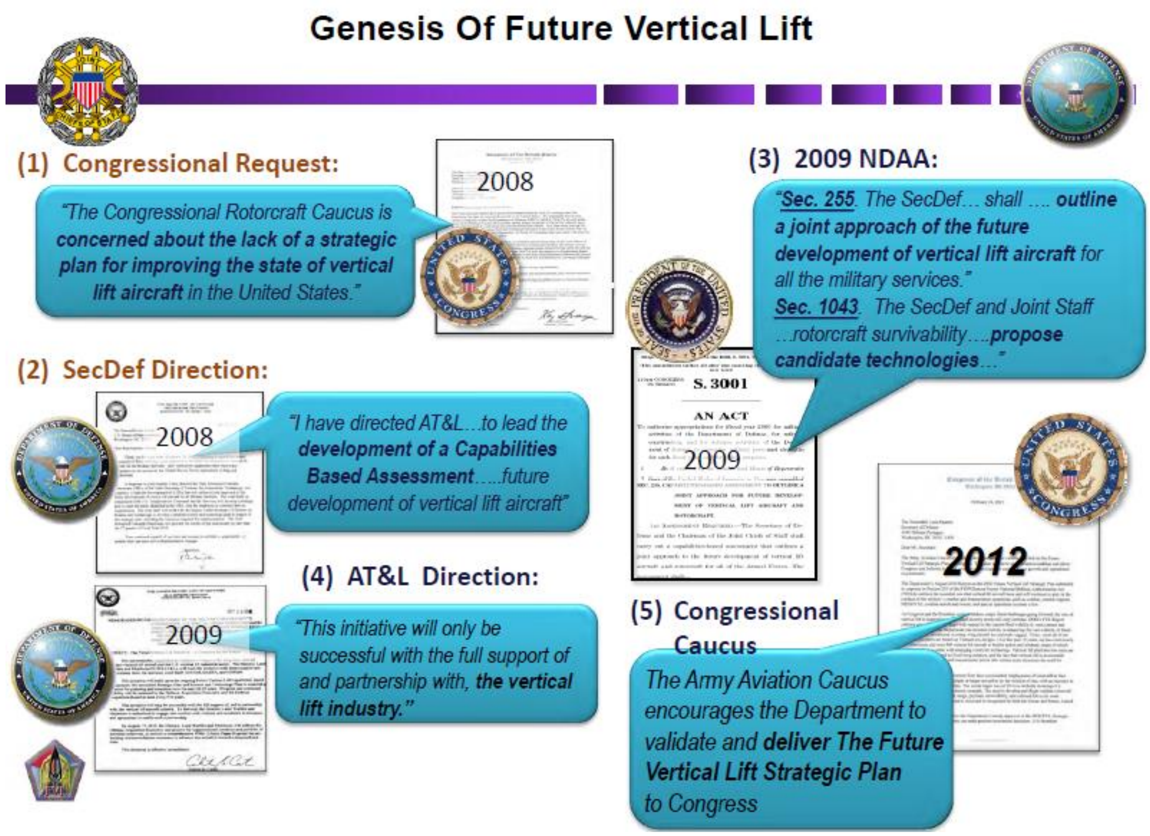


Figure 4: FVL Genesis as Presented by FVL Science and Technology IPT in 2015 [12]

FVL FoS is the subsequent response to the need for a next generation fleet. The current fleet is mature and unable to make gross improvement in availability and dependability due to the diminishing returns previously drawn in Figure 2. FVL provides the impetus and opportunity to make the paradigm shift towards near zero maintenance. By designing for MFOP from the beginning, FVL aircraft are the best opportunity to make the leap forward and achieve the magnitude change in extended MFOP.

1.3.2 Mandated to Balance with Affordability

This section outlines how the valuing of FVL FoS designs and technologies must include affordability as a metric. Improving reliability and MFOP will drive down O&S cost and reduce life cycle cost. This helps achieve affordability requirements. Finally, the evaluation of a MFOP maintenance strategy must include value and cost.

Over the past 30 years, the Army's acquisition effort has not successfully developed a new vertical lift aircraft. In an address to members of the NATO's Future Rotorcraft Requirements in 2015, Daniel Schrage recounts that the technology push did not fit DoD needs or the technology was not affordable. He summarized that "this lack of consensus between the user and materiel developer has resulted in a lack of top Army commitment and; therefore, no new development of Army vertical lift systems over the past thirty years" [13]. The failed acquisition of the RAH-66 Comanche and the Armed Reconnaissance Helicopter (ARH) reflect an era of increasing complexity with long development timelines that neglected affordability. Even the Marine and Air Force V-22 Osprey, which looks to be a success, took 24 years from program start (1983) to first fielding (2007) [14]. Paul Collopy and Peter Hollingsworth, in their formative paper on *Value Driven Design* [15], extrapolated acquisition program cost to completion. They showed that the total loss to delay, overruns, and reductions in materiel (generally caused by overruns) is $55 billion per year, or $150 million each day [15]. Similarly, a RAND Corporation study in 2008 concluded that DoD acquisitions cost growth varies between 38-60% from program start to finish [16]. FVL is attempting to produce a new fleet of helicopters by the 2030s and avoid the financial mistakes of the past. As such, the U.S. Government has issued a mandate for affordability in design and operation of FVL.

The correlation between affordability and system reliability is well documented. The 2015 Future Rotorcraft Requirements Technical Evaluation by NATO concluded that O&S cost accounts for 50% to 70% of Life Cycle Costs over a 40-year life of a modern rotorcraft [17]. Table 2 shows that repairable account for 78% and 88% of total cost per flight hour for the Black Hawk and Apache, respectively. These facts support the conclusion that O&S is most of life cycle cost and that repair of failed components accounts for the greatest portion of O&S costs. An identified path to affordability is with high reliability.

Table 2: Cost per Flight Hour as Reported by GlobalSecurity.org

		Cost per Flight Hour	
System	Total	Consumables	Repairables
UH-60 Black Hawk	$1,602.70	$351.54 (22%)	$1,251.16 (78%)
AH-64D Longbow Apache	$3,851.18	$444.20 (12%)	$3,406.98 (88%)

Data taken from GlobalSecurity.org [18]

The DoD Reliability, Availability, Maintainability, and Cost (RAM-C) Rationale Report Manual summarizes the relationship between cost and reliability. Low reliable systems result in high life cycle costs due to increased O&S cost. Overly high reliable systems drive exorbitant Research and Development (R&D) cost. The RAM-C manual urges to achieve a balance between reliability and cost [19]. Although Figure 5 shows the balance in the middle, historical data demonstrates that the optimal point leans in favor of higher reliability and lower O&S cost (shifting the balance point left). This is due to the mentioned dominance of O&S cost in total life cycle

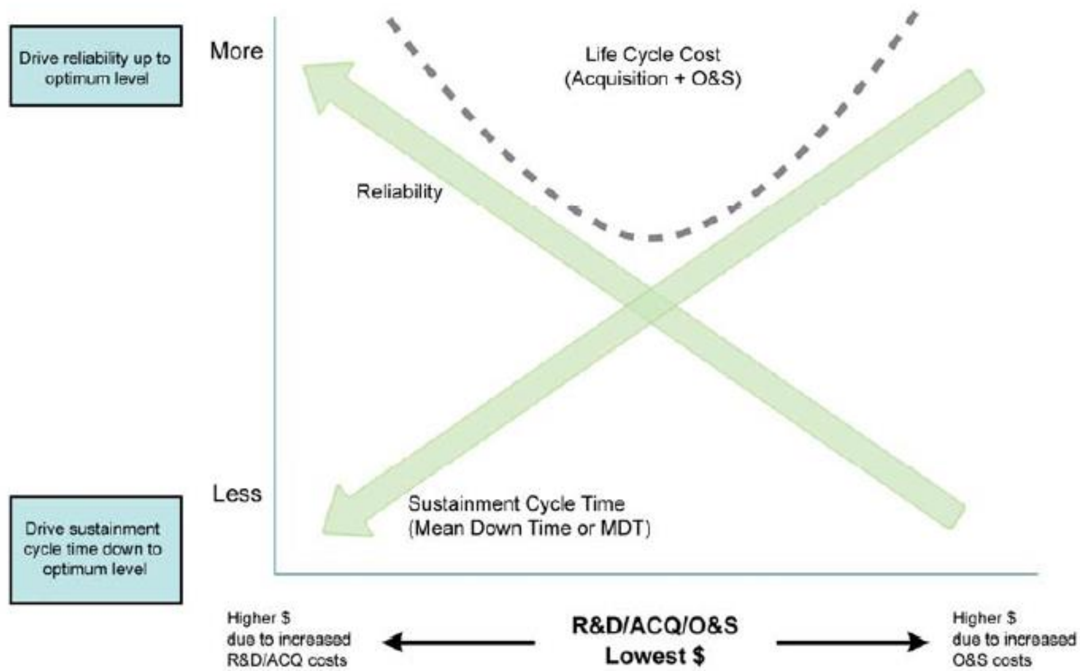


Figure 5: Optimum Life Cycle Cost and Reliability Curve from RAM-C Manual [19]

The Office of the Under Secretary of Defense Acquisition Technology and Logistics (OSD AT&L) initiated the Better Buying Power 1.0 initiative in 2010 that reshaped the DoD acquisition for efficiency. The Better Buying Power framework mandated affordability as a requirement in all new programs so that “cost considerations must shape requirements and design” [20]. The 2.0 initiative added cost trades as a requirement to the process.

Reviewing the Better Buying Power cost trade mandate, DOD RAM-C’s search for balancing reliability and cost, and FVL’s affordability requirements reveals the strong link between sustainability and affordability.

1.3.3 Invested in Science and Technology

The Army is the lead amongst the military services for the FVL Science and Technology development and Acquisition. The Army's Aviation and Missile Research and Development Engineering Center (AMRDEC) heads the Science and Technology IPT. Over the next few years, AMRDEC will lead the discovery and investment into the science and technology. A study to investigate the impacts of a MFOP strategy on the balance between affordability, dependability, and capability is well timed to inform FVL decisions on sustainment. It is the desire of the author to add to the rotorcraft community's efforts in transforming the future sustainment strategy to one of a MFOP paradigm.

Within AMRDEC, the Aviation Development Directorate (ADD) issued a call for proposal in 2015 for Ultra-Reliable Design (URD) [8] in support of FVL and the development of MFOP and ZMA. More recently, ADD stated the need for a path to transform from today's time-based maintenance to a MFOP program in FVL and finally to near zero maintenance. ADD identified several gaps to accomplishing the transformation. These gaps are summarized in Table 3.

Table 3: MFOP Knowledge Gaps

1. Identify ***metrics*** that measure desired sustainment and readiness outcomes 2. *Find* ***tools and methodologies*** needed to support the selected sustainment approach 3. *Create* ***sustainment approaches*** for FVL and near-future to mid-future to far-future (ZMA) technologies 4. ***Account for varying OPTEMPO*** in a future sustainment strategy 5. Realize ***savings in O&S*** and other life cycle cost components

The MFOP knowledge gaps in Table 3 motivate the proposed research to address the gaps from an aircraft system and MFOP maintenance perspective.

1.4 Scope of Work

The scope of the academic effort is to stay within the aircraft materiel solution and associated MFOP-MRP strategy. Considering all potential repercussions of a MFOP strategy on a large bureaucratic organization like the Department of Defense is beyond the scope of a single thesis. As such, research starts from the materiel solution vantage point. It leaves the broader domains of Doctrine, Organization, Training, materiel analysis, Leadership and education, Personnel, Facilities, and Policy (DOTMLPF-P) to future work.

Although a MFOP strategy may equally apply to fixed wing aircraft, the focus of this work is on a vertical lift family of systems. A goal is to demonstrate how to balance benefits and penalties of an applied a MFOP strategy. The thesis applies a MFOP strategy in the context of FVL to achieve relevance. FVL presents the current and largest effort towards near zero maintenance. The framework shown should equally apply to aircraft other than FVL FoS; however, this work does not directly address airplane MFOP.

1.5 Dissertation Outline

This chapter reviewed the history of modern maintenance strategies and addressed the approaching of paradigm change towards MFOP. It identified the relevance of the problem as a need for an order of magnitude change in aircraft dependability to meet future operational requirements. The opportunity that FVL presents to transform rotorcraft maintenance motivates the research. Simultaneously, the mandate to balance any MFOP strategy with affordability constrains the work. MFOP and MRP influence in rotorcraft design is a relevant topic. In 2013, the Army began considering which technologies to invest in to achieve FVL and transition to near zero maintenance. This discovery will continue over the next five years.

The following chapter of the dissertation will begin with defining the problem, its structure, identifying appropriate stakeholders, and identify current challenges to solving the problem. Chapter 3 presents background research to inform the formation of research questions. It presents a literature review on the relevant topics of reliability definitions, MFOP options, the acquisition process, reliability modeling, and value driven design. It concludes with a brief discussion on appropriate performance and cost modeling to the problem and an overview of zero maintenance technologies. Chapter 4 develops the tools to measure MFOP using operational metrics. Chapter 5 introduces a framework to construct a maintenance policy that maximizes availability in a MFOP context. Chapter 6 presents a method to provide some control over MFOPS of a given system. It tests whether the provided framework can improve MFOPS and provides an adaptable policy that maintains MFOPS after an extension to the MFOP duration. Chapter 7 provides concluding statements on the framework and its results. Finally, the dissertation ends with a discussion on future work with a focus on balancing system effectiveness with affordability. To assist the reader, the document's references to page numbers, table, figure, and sections are hyperlinked.

2 PROBLEM DEFINITION

The previous Chapter (see 1.2 Relevance) provided evidence of an order of magnitude gap between the MFOP of today's rotorcraft fleet and the desired MFOP of the FVL FoS. It also established the need to balance system effectiveness against affordability. The larger problem presenting the rotorcraft community is *FVL FoS require a leap ahead in capability and in dependability to meet the future Joint Force's need for agile, dependable aircraft while remaining affordable.*

The larger problem statement spans the entire life cycle of the FoS. This problem is too broad for the scope of a single thesis. A full analysis of the DOTMLPF-P domains as required USD AT&L's Defense Acquisition process is best suited for the larger problem. The larger problem needs scaling to a more manageable problem. To formulate an appropriate problem statement, the next section investigates the stakeholder needs.

2.1 Stakeholders

There are numerous stakeholders in the development of a FoS of MFOP rotorcraft. A sample listing includes Research and Development (R&D) organizations, acquisition organizations, Original Equipment Manufacturers (OEMs), industry vendors, the Vertical Lift Consortium (VLC) consisting of academic and industry rotorcraft experts, academic institutions, Congress, senior leaders (Department of Defense, Joint Staff and Service), operational commanders, training organizations, logistic commands, safety regulators, and more. Stakeholders are grouped by the major role they take in the life cycle of a rotorcraft system. Major stakeholder groups are developers, operational commanders, and senior leaders or decision makers. Each stakeholder has an additional role in bringing ultra-reliable aircraft to the flight line as summarized in Table 4 and presented below.

Table 4: Stakeholder Roles in a MFOP Strategy

Stakeholder Group	Additional Roles in a MFOP strategy
Developers	Calculate reliability statistics Predict operational metrics
Operational Commanders	Manage MFOP-MRP to meet objectives
Decision Makers	Balance system effectiveness against affordability and risk

Developers include research and development organizations, acquisition managers, and industry. R&D organizations encourage science and technology growth. They discover prospective technologies, fund promising technologies, and develop the best to maturity. Acquisition organizations evaluate, integrate, and manage systems developed by industry. Industry, heavily represented by the OEM and vendors, design and produces the aircraft systems and components. Developers have a need to estimate the traditional reliability statistics such as materiel reliability, Mean Time Between Failure (MTBF), and Mean Time to Repair (MTTR). Under a MFOP maintenance strategy, developers also need to predict the MFOP, MRP, and their probability of success.

Operational commanders support strategic goals by employing forces and capabilities. At the tactical level, they are responsible for the daily execution of the mission. They make use of the provided aircraft systems and personnel to meet operational needs. This effort includes integrating maintenance and logistics to achieve the required operational tempo

(OPTEMPO). OPTEMPO defines the type of missions flown, rate of flying, and necessary aircraft availability. Commanders have the need to adapt current systems and procedures to meet a changing operational environment. They need to understand the relationship between MFOP and MRP and their probability of success to assess risk, create supply lines, assign and train personnel, and apply limited resources.

Decision makers are senior civilian and military leadership. They enact policy for the joint force to accomplish strategic goals. Department of Defense senior leaders ensure the strategy is mutually supporting throughout each DOTMLPF-P domain within each service. They liaison with Congress to obtain funding for acquisition programs. Decision makers balance the capability given finite resources. Decision makers set requirements to balance system effectiveness against affordability, schedule, and risk.

2.2 Overall Problem and Problem Structure

Decomposing the problem from the stakeholders' perspectives helps identify the principal issues. The conceptual diagram of Figure 6 shows the linkages between stakeholders and the knowledge gaps found in Table 3.

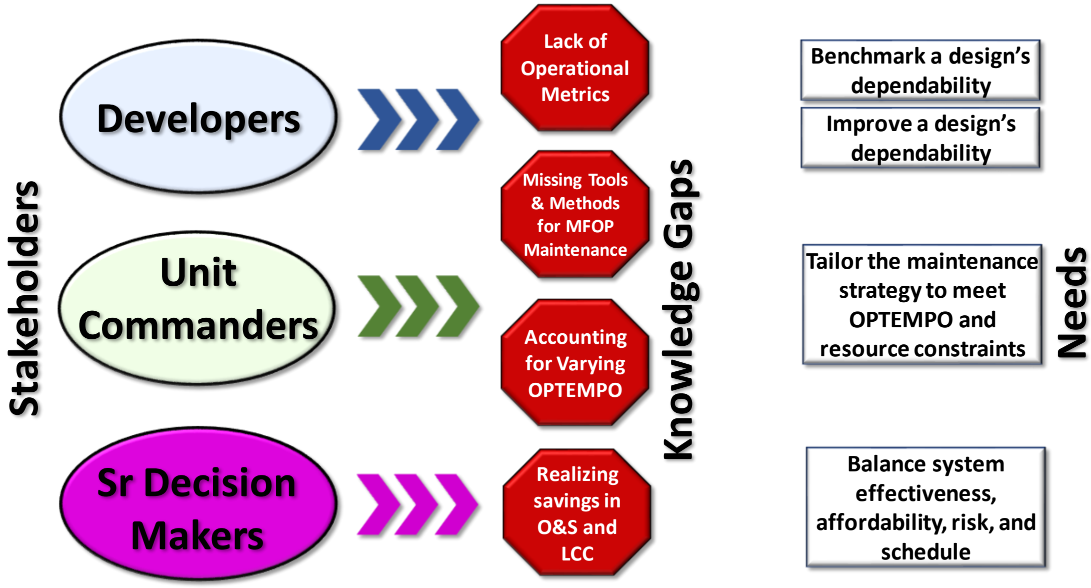


Figure 6: Stakeholder and Knowledge Gaps

Connecting stakeholders to the knowledge gaps reveals that the knowledge gaps inhibit stakeholders from effectively creating a MFOP strategy.

> Overall Problem Statement
>
> Stakeholders' current approaches to aircraft dependability are not suitable for the development of MFOP aircraft.

Decision makers need the ability to measure what a MFOP option or technology does to the value. Such an investigation requires modeling an organizations performance across the DOTMLPF-P spectrum. Any methodology must consider the influence of bureaucratic policies and funding limitations on the acquisition cycle of a MFOP family of systems. Defining value and communicating the balance with affordability is a third problem for decision makers but remained beyond the scope of the thesis. The work provided in this dissertation supports, but does not solve, the larger problem of modeling an organization's performance over the DOTMLPF-P spectrum. Section 3.5 Valuing a MFOP Aircraft discussed how Problem 1 and Problem 2 support the comprehensive need to balance dependability and maintainability with system value.

The proposed research intends to remove the major hindrances that block each stakeholder from creating a balanced MFOP strategy. To achieve a MFOP strategy with an order of magnitude improvement in dependability, stakeholders need to overcome the two remaining problems of Figure 7. The presented solutions to Problem 1 found in Chapter 4 provide stakeholders' with the new tools necessary to measure the dependability of a MFOP aircraft in operational metrics. Chapters 5 and 6 address Problem 2 by providing a framework capable of tailoring maintenance policies to meet low, high, or a changing operational tempo.

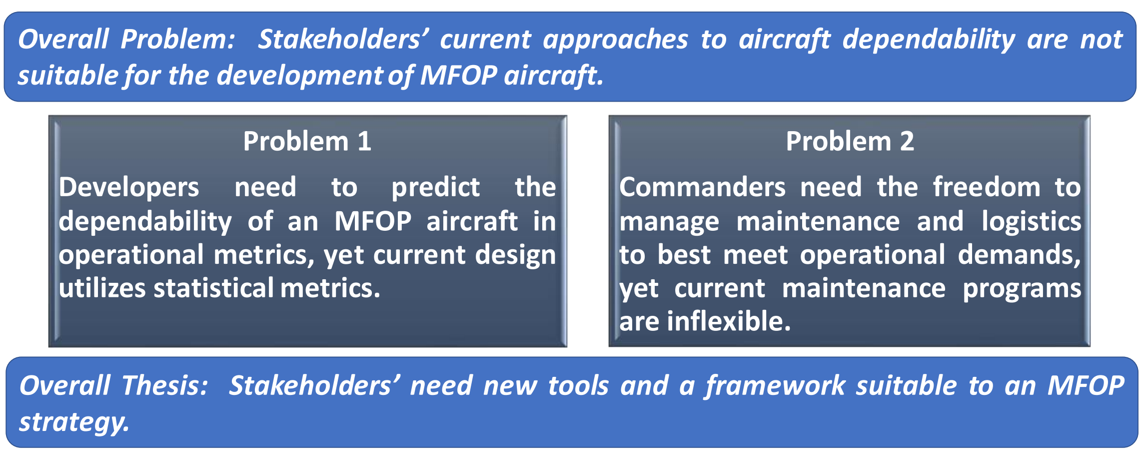


Figure 7: Problem Structure and Overall Thesis

2.3 Problem 1: Measuring MFOP

Problem 1 provides new tools necessary for stakeholders to measure the MFOP of an aircraft using new operational metrics. A challenge to implementing a MFOP strategy is the change in paradigm away from what has always worked (on-condition maintenance). Traditional metrics such as materiel reliability (R_m), MTBF, and MTTR are useful for manufacturing design and safety analysis; however, these statistical metrics using the mean do not tell an operational commander about the dependability of their system in a MFOP strategy.

2.3.1 MTBF: The Wrong Metric

Al Shaalane and Vlok summarized why a paradigm shift in maintenance is necessary to make the transition to MFOP. They stated, "MTBF presumes that failure is inevitable, and thus creates the general assumption that there is no point in striving for the ultimate goal of reliability excellence" [21]. A thought experiment of Figure 8 highlights the fallacy of MTBF as a metric.

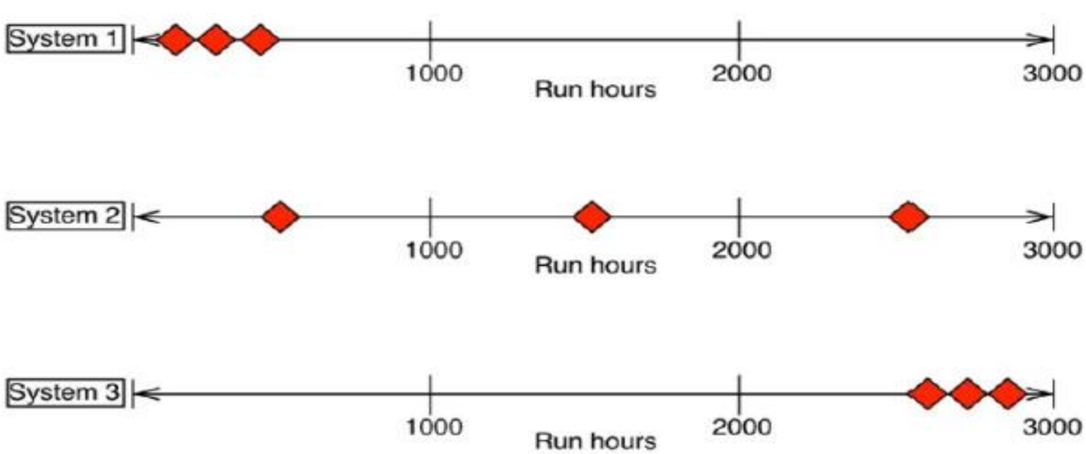


Figure 8: Misleading Nature of MTBF. Figure reprinted with permissions from [21] and from original source [22]

All three systems have three failures over 3,000 hours for the same MTBF of 1000 hours per failure. A designer considering only MTBF has no preference between the systems. This is because MTBF is hiding information about the distribution of failures. System 2 is undesirable to a MFOP designer. System 3, with failures later in run time, is preferable because it provides over 2,500 hours of MFOP before mission disruption. System 1 does provide a similar 2,500 hours if the infant mortality failures are avoided through burn-in. Unlike MTBF, MFOP does not hide the relevant information and provides a better understanding of the impact on operations. MFOPS is a more appropriate metric because it accounts for the random nature of failures, while traditional metrics, like MTBF, falsely assume a deterministic nature. A major consequence of the use of the incorrect metric is a complication in logistics planning (see vignette below).

Vignette on Incorrect Dependability Metrics

As reported in "Air strategy that can't fail" by John Dunn in Professional Engineering, August 1997. [69]

"Assume the maintainer is waiting for an aircraft to return. He knows it has an MTBF of 10 hours and has just flown a five-hour sortie, so he will have half a fault to fix. No one knows is where [sic] that 'fault' might be. To be safe, a crane, a tug, a full set of jacks, a full tool kit and a spare for every replaceable unit on the aircraft, is ordered up on standby. This is real life. The traditional system for defining reliability is a nightmare to the logistics manager and the accountant, who are asked to fund and provision spares that may not be needed for years."

Wg/Cdr Trevor Turner, RAF

2.3.2 Resolving the MTBF Fallacy

According to DI Knowles as reported by Kumar et al. [23], the drawback of MTBF is that it is "almost impossible" to determine the failure rate if the distribution is not exponential. Exponentials are most attractive due to their mathematical ease; however, this becomes a fallacy with aging parts over multiple MFOP cycles. At the Annual Reliability and Maintainability Symposium in 1997, Hockley and Appleton recommended transitioning from MTBF as the reliably metric towards a probability of failure to meet a specified MFOP. They [6] and Relf [24] cautioned MTBF sets an ill-fated acceptance of failure and an inevitability to unscheduled maintenance. The quest for improved dependability with fault free parts becomes ever increasingly expensive in terms of time, weight, and cost. Instead, Hockley and Appleton recommended creating fault tolerant systems [6]. MFOP and MRP are examples of operational metrics that measure the ability of a system to remain failure free (fault tolerant), not fault free. By doing so, the designer adds fault tolerance to inherent reliability as options to improve dependability of the systems.

Problem 1

Designers need to predict the dependability of an MFOP aircraft in operational metrics, yet current design utilizes statistical metrics.

Dr. Michael Hammer, a noted expert in process engineering, created four principles for measurement. (1) "measure what matters, rather than what is convenient or traditional; (2) measure only what matters most; rather than everything; (3) measure what can be controlled, rather than what cannot be controlled; and (4) measure what has impact on desired business goals, rather than ends in themselves" [25]. In short, MTBF is the convenient and MFOP is what matters. This led to the statement of the first problem.

2.3.3 Research Question 1

The shift to what matters, MFOP, has only just started. As late as 2009, the DoD RAM-C Report Manual maintained A_O (operational availability), R_m, and several modes of mean time to repairs as key metrics [19]. A review by Kumar et al. [23] found the Air Force, by the year 2000, began emphasizing reliability metrics based on operational requirements over mean time statistical metrics. The Army's ADD recently acknowledged the need for new metrics in a MFOP strategy gap [9] when it marked it as the first MFOP Knowledge Gap (see Table 3). This knowledge gap and the need for methods to estimate MFOP metrics present a challenge for establishment of a MFOP strategy. With the right metrics identified as MFOP and its probability of success, the next step was to estimate the MFOP by modeling a system. This gave rise to the first research question.

Research Question 1

What method(s) are suited to model MFOP?

Section 3.3 reviews the current literature on predicting MFOP. The majority of analytical and modeling efforts [24], [26], and [27] focused on estimating the MFOP. Kumar et. al [23] and Price et. al [28] add and then use the concept of MFOP probability success to the measurement. Chapter 4 begins with an evaluation of the state of the art modeling methods and their suitability to model an MFOP rotorcraft. The remainder of the chapter develops a customized Discrete Event Simulation, followed by a series of experiments to verify the model's accuracy.

2.3.4 Research Questions 2a and 2b

Once the model estimates a system's MFOP, the designer may wish to improve its performance. To do so, developers need to understand where a component or subsystem is limiting a given MFOP. By locating the limiting component, designers may redesign the architecture or improve component inherent reliability to better achieve targets. Research Question 2a asks to identify components limiting the MFOP duration. Answering this research question attends to *MFOP Knowledge Gap 2: Find tools and methodologies needed to support the selected sustainment approach*. The ability to quantify the limiting component provides the developer with a tool to measure and improve a MFOP design.

Research Question 2a

Which components/subsystems limit an MFOP?

Research Question 2b

Which components/subsystems are the greatest contributor(s) to MRP duration?

Maintainers and logisticians are also keenly interested in repairs that increase the maintenance burden inside the MRP. Identifying what repairs are driving MRP higher is equally as important as what limits a MFOP. The author could not find significant work published on the estimation of the MRP outside of Price et al. [28]. Most of scheduled maintenance modeling in the reliability field follows the on-condition repair paradigm with the goal to minimize total cost. Acknowledging the importance of affordability, the maintenance burden measured in Maintenance Man Hours (MMH) or Maintenance Downtime (hours) is also critical to operators. Measuring the maintenance burden is the topic of Research Question 2b. Answering the research question addresses *MFOP Knowledge Gap 2* by providing developers with an approach to estimating a system's MRP burden.

2.4 Problem 2: Adaptive Maintenance for Agile Aircraft

Problem 2's objective is to provide a framework that leads to maintenance policies suitable to a MFOP strategy. A suitable maintenance policy must enable FVL family of systems to be adaptable and interoperable. The white paper, *Future Aviation Maintenance Concept: Bridging the Gap toward Mobility and FVL* stated, "maintenance doctrine must change to meet the challenges of future enemy" [29]. The Future of Aviation Maintenance Concept (FAMC) Interdisciplinary Concept Team (ICT), led by Lieutenant Colonel J. Peter Velesky, continued to state, "Army Aviation sustainment must become more agile and responsive" [29]. The FMAC ICT is using agile to mean a quick response to changing operational demands. Its context is not as the measure of an aircraft's handling or maneuverability. Today, the Army has begun to limit inspections using CBM to support an agile aircraft. The on-condition paradigm of frequent, scheduled maintenance of the past 40 years was anything but agile. For example, prior to CBM the UH-60 underwent routine "daily, 10-hour/14-day, 40-hour and other required inspections" [9]. Required inspections include 120-hour inspections and intensive phase maintenance every 360 hours. Maintainers conducted 30-day, 90-day, 120-day, 6-month, and yearly special scheduled inspections. CBM has begun to eliminate some of these inspections but its progress is not complete. There still exists a disparity between today's maintenance and the need for adaptive maintenance. The U.S. Army's Aviation Development Directorate denoted the disparity in *MFOP Knowledge Gap 4: Account for varying OPTEMPO in future sustainment strategy* as introduced in Table 3. This disparity or gap constitutes the second problem. MFOP with CBM+ is planned to eliminate the gap.

Problem 2

Commanders need the freedom to manage maintenance and logistics to best meet operational demands, yet current maintenance programs are inflexible.

An adaptive maintenance program should respond to the current operational needs of a commander. A commander has few options on the maintenance of a fielded aircraft under the paradigm of on-condition maintenance. Generally, aircraft architecture remains fixed and vendors ship components with fixed reliability. The versatility of rotorcraft has meant a wide variety of missions in every environment in the world. A fleet-wide scheduled maintenance plan assumes a "one-size fits all" approach to maintenance regardless of mission or environment. Creating a MFOP-MRP cycle fixed to one design tempo is a lost opportunity to maximize dependability.

2.4.1 Research Question 3

A commander that understands the relationship between MFOP and MRP has more flexibility to balance requirements. The war time operational tempo by FVL is thought to be about 100 flight hours in 7 days [9]. Not all aircraft will be at such a high tempo as the FVL war-time target. A unit in training may lower the maintenance burden or save money by trading for a shorter MFOP or seek maximum operational availability. Conversely, a unit with a very high operational tempo may "buy" a longer MFOP with a longer MRP. ADD recognizes the trade between MFOP, MRP, maintainability, and affordability in the third and fourth knowledge gaps of Table 3. The third research question seeks to uncover the maintenance policy that maximizes availability at the potential reduction of a MFOP duration. This simulates a garrison environment where an extended MFOP is not an operational necessity.

Research Question 3
What is the maintenance policy that minimizes downtime?

The development of a policy to minimize downtime is similar to classical renewal theory as presented in its literature review (section 3.4.2). Chapter 5 begins to answer the research question by examining classical renewal theory in a MFOP strategy. Unfortunately, the Chapter 5 shows the theory is unable to protect the MFOP from disruptive maintenance. Section 5.1 revises the theory to satisfy a MFOP strategy. Section 5.2 demonstrates the need for a framework to achieve a suitable policy that minimizes downtime while obtaining a sufficient reliability. The reminder of the chapter constructs a framework (section 5.3) and tests a hypothesis in an experiment on a simple system (sections 5.4 and 5.5). Results, a sensitivity study, and a discussion conclude the chapter.

2.4.2 Research Question 4

The final research question looks at the war-time tempo and FVL's call to minimize the forward footprint by operating aircraft for extended periods without disruptive maintenance. The objective is to maintain the MFOP probability of success above a minimum level for a pre-determined number of MFOP cycles. The minimum level is a quantification of the commander's risk tolerance. The pre-determined number of MFOP cycles accounts for the planned duration at the extended MFOP or high operational tempo.

A policy that supports an extended MFOP is exercising some control over the risk of a failed MFOP. Controlling a MFOP inherently requires management of the MFOP duration and its probability of success. Extending the MFOP naturally results in a decreased MFOPS given the same MRP. Similarly, avoiding risk by raising the MFOPS requirement will shorten the MFOP. This thinking relies upon a common assumption to MFOP modeling in literature: the assumption that only failed parts are replaced. This section questions the general assumption. What if maintainers pursue an aggressive lifing policy that replaces parts before failure? Would this help control MFOPS?

Research Question 4
What is a maintenance policy that controls MFOPS?

The problem is best thought of as managing the MRP to maintain the desired success over time or after an increased MFOP. To increase the MFOPS, one would expect to replace the oldest parts first with the greatest likelihood of mission failure. Additional repairs, consequently, increase the MRP burden. Intuitively, one would assume pushing a system of fixed architecture to a longer MFOP while maintaining the MFOPS results in a longer MRP. The aircraft would fly longer hours and accumulate more repairs for the MRP. The thought process gives rise to Research Question 4.

2.5 Current MFOP Methodologies

In 1999, Mark Relf introduced the broader public to the MFOP paradigm with his formative journal article, titled "Maintenance-Free Operating Periods—The Designer's Challenge" in Quality and Reliability Engineering International [24]. Relf had worked on the British Ultra-Reliable Aircraft project in the late 1990s with British Aerospace and his article served as catalyst for academic exploration of MFOP. He proposed an iterative design methodology drawn in the figure below.

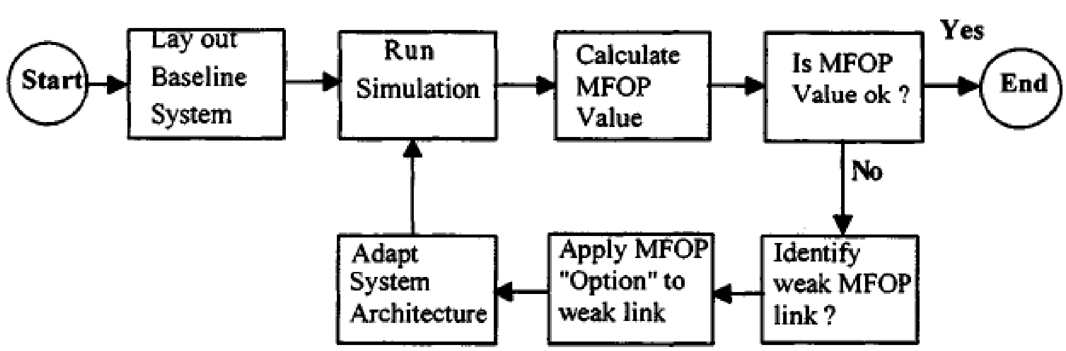


Figure 9: MFOP Design Methodology from Relf [24] Copyright © 1999 John Wiley & Sons, Ltd.

Relf's methodology provided a template to assess where and how to improve a system's MFOP. He recommended a Monte Carlo simulation to approximate the MFOP value of a given system [24]. If the MFOP proved insufficient, the methodology sought the "weak link" that limited the MFOP. MFOP Options (section 3.2.3) provided means to improve a system's MFOP. Calculation of the MFOP value and identification of the weak link constitutes Problem 1 found in Chapter 4. Research Question 1 addresses the selection, development, and validation of a modeling technique to estimate the MFOP. Research Question 2a queries how to find the weak link (section 4.2) and Research Question 3 asks for a technique to quantify the MRP's maintenance burden (section 4.3).

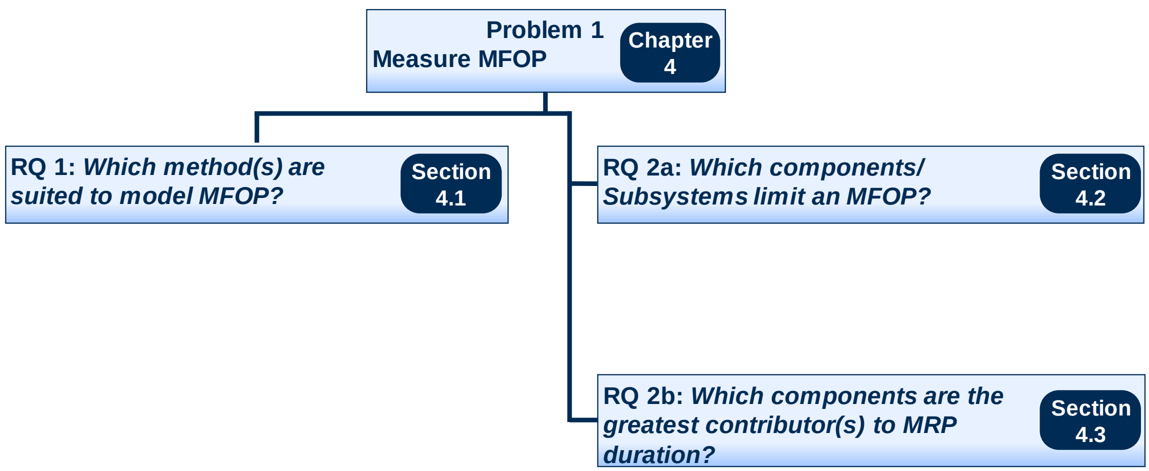


Figure 10: Summary of Problem 1: Measure MFOP

Relf's methodology focused on the improvement of the system to achieve a given MFOP goal. When introducing potential MFOP Options, Relf stated a "policy of hard lifing would be the most viable method to realize a MFOP" outside new technologies of CBM or the use of redundancy. This introduces the notion that a system's MFOP performance is a function of its design and the applied maintenance policy. The original methodology of Figure 9, however, does not directly provide a means to design a maintenance policy in conjunction with the design of the cyber-physical system.

The long-term reliability of a system is a function of its inherent reliability and the supporting maintenance policy. In a survey of reliability modeling techniques published in the Journal of Mechanical Engineering Science, Andrews [26] stated,

> *Traditionally the system design process and the specification of the maintenance programme have been carried out separately. There are advantages to be gained by considering the two aspects simultaneously where the system is designed to enable an efficient and effective maintenance strategy to be employed.*

Like Relf's methodology, none of the recent MFOP modeling methodologies found in a literature search [28], [30], [31] and summarized in section 4.1.2.1 provide a means to construct a MFOP maintenance policy. In summary, there exists the need to link system design and maintenance policy development for a MFOP strategy.

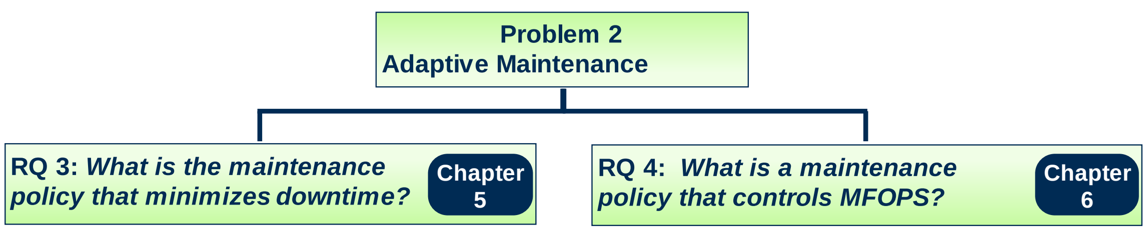


Figure 11: Summary of Problem 2: Adaptive Maintenance

To fill this knowledge gap, Research Question 2b and its conjecture research how to measure the maintenance burden generated by a component or subsystem. Problem 2, consisting of the entirety of Chapters 5 and 6, develops a framework to design a MFOP maintenance policy. Chapter 5 applied a revised maintenance theory in the framework to construct a policy to best meet a MFOP while maximizing availability. Chapter 6 follows the framework to construct an adaptable policy that meets a desired MFOP probability of success over a changing operational tempo.

2.6 Framework Introduction

This section introduces a generic framework that constructs policies to meet operational requirements of a MFOP strategy. The framework establishes an approach to design a system and its maintenance policy to meet availability and dependability requirements. This section provides the reader a roadmap to the research and context to the need, development, and evaluation of the framework. Chapter 3 provides definitions, background, and literature summary on modeling and maintenance methods. Problem 1 of Chapter 4 establishes the tools to model the MFOP of a system, diagnose the weak link, and construct subsystem failure and repair distributions. Chapter 5 establishes the need for the framework, presents the fully developed framework, and shows how the framework satisfies a MFOP strategy. The chapter ends with an experiment that develops a policy to maximize availability for a MFOP system. Chapter 6 elaborates on the framework's use to improve a system's MFOP probability of success and provides an application of an adaptable policy to meet the needs of changing operational tempo.

2.6.1 Framework Overview

The framework to design a MFOP maintenance policy (Figure 12) complements Relf's methodology through three major actions: define, build, and evaluate. It builds upon Relf's methodology by adding the design of a maintenance policy. The framework begins by defining the current system and MFOP setting goals. Principles specific to a MFOP strategy, namely the need to protect the MFOP from disruptive maintenance and ensure sufficient reliability, guide the construction of a MFOP policy. Finally, the framework calls for evaluation of the policy for sufficient MFOP, its probability of success, and resultant downtime. The framework guides the designer in the iterative design of a system and policy until it satisfies the goals for MFOP, MFOP success, and availability.

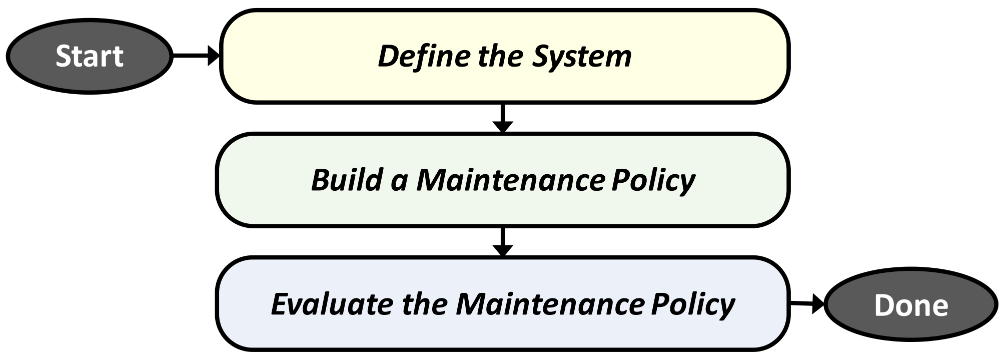


Figure 12: Overview of Designing a Maintenance Policy

Figure 13 presents the conceptual framework on page 34 in full detail.

2.6.2 Define the System

The define action establishes inputs and key metric goals that drive the development of a supporting maintenance policy. Figure 13 (page 34) shows the conceptual framework. The figure lists major sections that introduce or derive key concepts for a process inside rounded, rectangles. Bowlegs (#) denote the input, process, or decision number. Define action events are:

(1) Start. The processes to construct a MFOP design and supporting maintenance policy begins here.

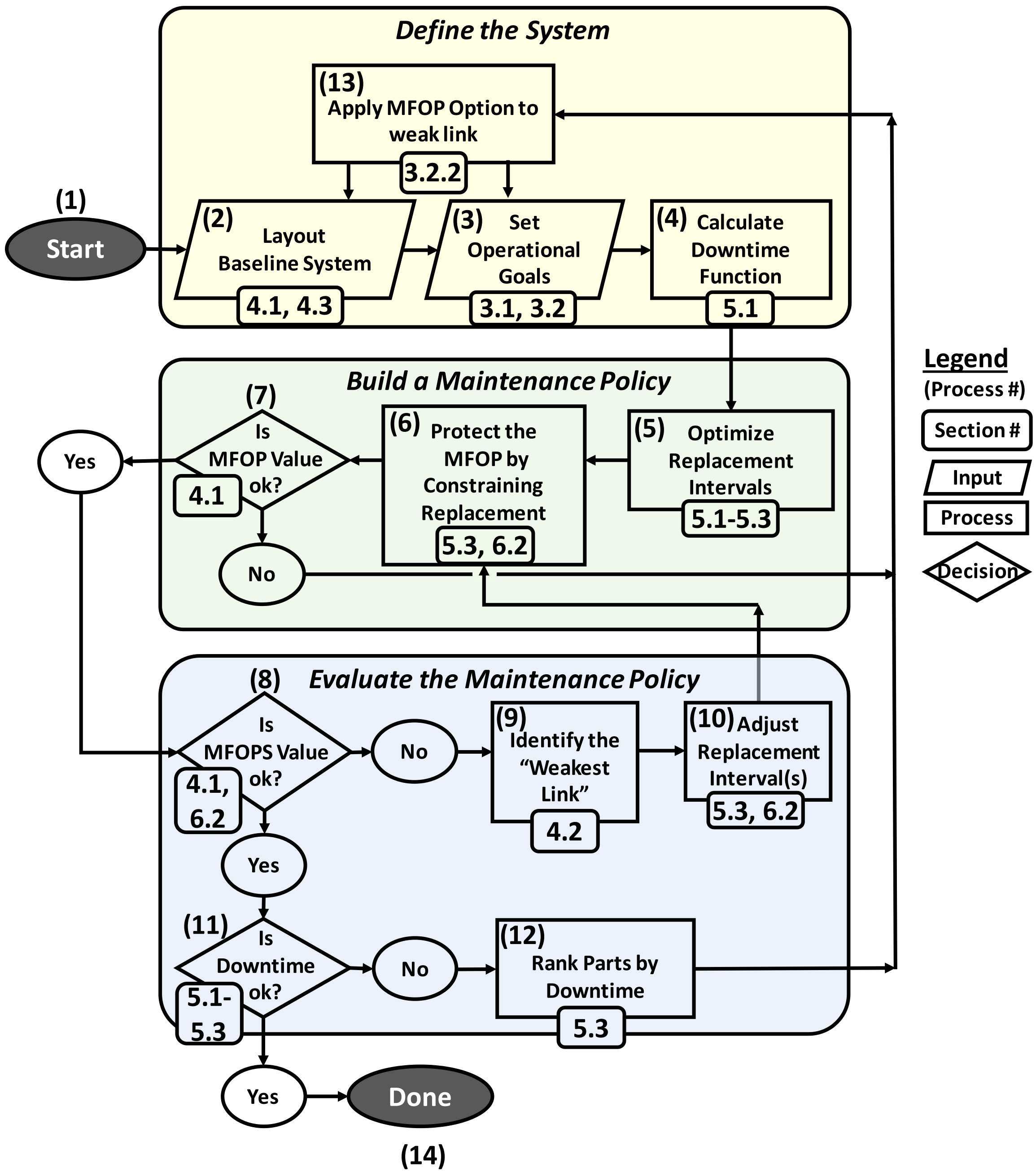


Figure 13: Conceptual Framework to a MFOP Design

(2) Layout Baseline System. Much like Relf's Design Methodology, an initial step is defining the current system. The framework permits the design of the system architecture with performance modeled through a series of phased, fault trees (section 4.1.2.2). Key inputs include component failure distributions (section 4.3.2.1) and associated repair times (sections 4.3.2.3 and 5.1.1).

(3) Set Operational Goals. This process defines the phased or segmented mission based upon model requirements set in section 3.3.1 and methods evaluated in section 4.1.1.1. The MFOP duration requirement and its MFOP Success rate (section 3.2.5) goals are determined based upon operational requirements. The operator must also select the desired availability (section 3.1.2.3).

(4) Calculate Downtime Function. The contribution of component and subsystem downtimes are modeled in accordance with a revised renewal theory developed in section 5.1.1.

2.6.3 Build a Maintenance Policy

The build action constructs the draft policy by constraining preventive maintenance replacement intervals to MRPs. It consists of the following processes:

(5) Optimize Replacement Intervals. Initial renewal intervals are drafted for components assigned to preventive maintenance with tasks of repair, replace, or service. Draft intervals begin by following a policy that maximizes availability.

(6) Protect the MFOP by Constraining Replacement Intervals. This process manages intervals such that all preventive maintenance to occurs in the MRPs. The applied policy determines the way renewal intervals are constrained. Problem 2.1 of

Chapter 5 provides a policy to maximize availability. Chapter 6 provides a policy to control MFOPS over a given number of cycles.

(7) Is MFOP Value Okay? The constraining of replacement intervals provides a limit on the MFOP duration. The decision considers if the policy limited MFOP meets the MFOP duration goal. If the policy's MFOP duration is sufficient, the designer moves to Decision (8). If the policy's MFOP duration is insufficient, the designer must make a change to the system's design by applying one or more of Relf's MFOP Options (section 3.2.3). This feedback to Process (13) is where the design of the system and policy interact.

2.6.4 Evaluate the Maintenance Policy

With the MFOP protected by the build action, the evaluate action checks for sufficient reliability and availability of the system. Checking MFOP and downtime drives the designer to iteratively find the balance between the two.

(8) Is MFOPS Value Okay? The decision checks if the policy's MFOP Success (MFOPS), or probability of success, meets the risk level set in Input (2). Research Question 1's investigates how to model a system's MFOP. The developed Discrete Event Simulation (section 4.1) provides the tool to measure the success rate. If the MFOPS is insufficient to the goal set in Process (2), the designer moves to Process (9). If the MFOPS is sufficient, the designer moves to Process (11).

(9) Identify the Weakest Link. Relf identified the weakest link as a necessary step to improve the system's MFOP. Research Question 2a seeks a means to identify the weakest link(s) that limit a system's MFOP. Conjecture A answers with the

concept of Failure Cause Identification. Section 3.2.1 introduces the concept and section 4.2 provides further development as a quantifiable technique. Problem 2 makes extensive use of Failure Cause Identification in the application of the framework.

(10) Adjust Replacement Intervals. The framework requires the designer to shorten intervals of components identified as weak links. The policy's goals determine the approach to shortening. Sections 5.3 and 6.2 provide guidance for policies that maximize availability and improve MFOPS, respectively. An adjustment to the interval makes a change in the policy that returns the designer to Process (6). The iterative, inner loop between (6) to (10) is the MFOPS control loop supporting Problem 2.2 and is fully discussed in section 6.2.

(11) Is Downtime Okay? The final check of downtime occurs after the policy satisfies both the MFOP and MFOPS targets. Decision (11) ensure the policy balances MFOPS against the downtime calculated using the revised renewal theory created in Chapter 5. If the policy's downtime is unacceptable, the designer moves to Process (12). If the policy's downtime is acceptable, the policy is complete with Process (14).

(12) Rank Parts by Downtime. This process orders components by their individual, expected downtime contribution. The components with the greatest downtime are targets for improvement via a MFOP Option in Process (13) or an improvement to its maintainability in Process (1).

2.6.5 Concluding the Policy

Should the policy become unable to meet the MFOP or downtime goals, the designer must seek a redesign to the cyber-physical system itself. This feedback loop from Decision (7) and Process (12) to the Define action connects the cyber-physical system to the policy development.

(13) Apply MFOP Option to Weak Link. This process originates from Relf's Design Methodology. Possible MFOP Options include inherent reliability, prognostic/diagnostics, redundancy, and reconfigurability [24]. Please see section 3.2.3 for more detail on Relf's MFOP Options.

(14) Done. Policy construction may end once the designed system and policy yield sufficient MFOP duration, sufficient MFOPS, and an acceptable downtime.

The building of a policy begins after the designer defines the system's architecture and component reliability. The framework is intended to guide the operational commander through a series of processes and decisions to create a policy best suited to meet the MFOP need while balancing policy risk measured by MFOPS and the desired availability. The feedback loops to Define the System provide an opportunity for interaction between the system designer and the policy author. In this way, the framework supports conceptual and preliminary design of a new system.

3 LITERATURE REVIEW AND BACKGROUND

3.1 Reliability, Availability, Dependability, and Other -Abilities

In everyday language, the terms reliable and dependable are often synonyms. In the context of aircraft readiness their meanings differ and are significant. Engineers, regrettably, are often ambiguous with their language when using the terms reliability, availability, and dependability. This section provides definitions to avoid confusion. The DoD believes all three metrics are important when predicting readiness and discussing a maintenance strategy. The Joint Capabilities Integration and Development System (JCIDS) Manual requires reliability as a Key System Attribute (KSA) and availability as a Key Performance Parameter (KPP) [19]. In a MFOP strategy, dependability as measured by MFOP and MRP should be a KPPs.

3.1.1 Reliability

3.1.1.1 <u>Basic and Materiel Reliability</u>

The DoD RAM-C Rationale Report Manual [19] defines reliability as "the probability that the system will perform without failure over a specified interval under specified conditions." In [32], Smith provides a similar definition. He summarizes reliability as "the probability of non-failure in a given period." Probability analysis is clearly fundamental to the calculation of reliability. In this thesis, the random variable of interest is the Time to Failure (*TTF*). Let $f(t)$ be the probability density function of the TTF distribution (also called the failure distribution) and is the probability density function, $f(t)$. The failure function, $F(t)$, is the probability that the Time To Failure (*TTF*) occurs before time t. F(t) is the integral of the failure distribution from time zero to t and is known as the cumulative distribution function (cdf).

$$F(t) = \int_0^t f(t)dt = P(TTF \leq t) \tag{1}$$

The reliability function, or survival function, is complementary to the cdf and is defined as

$$R(t) = \int_t^\infty f(t)dt = 1 - F(t) \tag{2}$$

Basic reliability is the probability of the system to operate without faults requiring repair, *R(t)*. Materiel Reliability, R_M, is the basic reliability of a specified system based on materiel condition. DoD RAM-C requires an appropriate materiel reliability to meet the capability needed in the operating environment [19]. Part of fulfilling this requirement in FVL FoS is calculating the R_m necessary to achieve the target MFOPs listed in Table 1.

3.1.1.2 Mission reliability

Mission reliability (MR) is “the probability that the system aged t_b is able to complete mission duration of t_m successfully” [33]. There are two major considerations that distinguish between basic reliability and mission reliability. According to DoD RAM-C, mission reliability considers “failures that cause mission aborts” while basic reliability considers “all failures requiring maintenance” [19]. Basic reliability must be less than or equal to mission reliability because basic reliability includes other failures that do not disrupt mission accomplishment. The second distinction, is that mission reliability is for a system with a given age of each component. As noted by Kumar, in mission reliability ‘we recognize the age of the system before the mission” [33]. Following conditional probability, mission reliability is

$$MR(t_b, t_m) = \frac{R(t_b + t_m)}{R(t_b)} \tag{3}$$

Mission and basic reliability are distinct and have separate uses. DoD RAM-C states that mission reliability is for operational availability calculations and basic reliability supports materiel availability calculations [19].

3.1.1.3 Unreliability

Unreliability, Q, is probability of failure over a specified interval. It is the complement of reliability

$$Q = 1 - R \tag{4}$$

Unreliability is useful in theoretical calculations when it is more convenient to calculate probabilities of failure than probability of success. The use of an inclusion-expansion expression of prime implicants as outlined in Chew et al. [27] provides an analytical solution for system unreliability and, consequently, reliability. Unreliability may be expressed to any level (MFOP cycle, mission, phase, etc.). It is important to denote whether the unreliability is from MFOP start through a level (i.e. start to phase four) or the unreliability is at a level using conditional probability (i.e. phase four). The former decreases with the progression of levels while the latter requires the use of conditional probability (i.e. unreliability of phase 4 given that phases one to three are successful).

3.1.2 Availability

Whereas reliability involves a duration of time, availability is a measure in an instant of time. The DoD divides availability into Materiel Availability (A_M) and Operational Availability (A_O).

3.1.2.1 Materiel Availability

Materiel availability measures the number of in service aircraft against the total fleet inventory. Operational availability is a measure for of a set of aircraft typically under a unit. The RAM-C Rationale Report [19] defines the term as the percentage of systems in operational use.

3.1.2.2 Operational Availability

Operational Availability (A_O) is "the percentage of time that a system or group of systems within a unit are operationally capable of performing an assigned mission" as defined by the RAM-C Rationale Report [19]. It is important to note that A_O, not A_M, is an operational metric. This is because A_O measures downtime beyond materiel condition. Smith [32], expresses A_O as

$$A_O = \frac{Uptime}{Uptime + Down\ time} = \frac{MTBF}{MTBF + MDT} \quad (5)$$

where MTBF is the Mean Time Between Failure and MDT is the Mean Downtime. MTBF is the number of operating hours divided by the total number of failures. MDT is the average downtime of a system. It is the sum of Mean Time to Repair (MTTR), Logistics Downtime (LDT), and Administrative Downtime (ADT). Downtime measured as MTTR only yields inherent (or materiel) reliability [34]. For further definitions, see the DoD RAM-C Rationale Report [19].

3.1.2.3 Achieved Availability

Achieved Availability (A_A) removes the operational aspects of downtime to consider the materiel's performance only. It accounts for downtime due to both corrective and preventive maintenance using Mean Time Between Maintenance (MTBM). The Defense Acquisition Glossary [35] defines Achieved Availability as

> *Availability of a system with respect to operating time and both corrective and preventive maintenance. It ignores Mean Logistics Delay Time (MLDT) and may be calculated as Mean Time Between Maintenance (MTBM) divided by the sum of MTBM and Mean Maintenance Time (MMT) that is*

$$A_A = \frac{MTBM}{MTBM+MMT} \tag{6}$$

A_A allows analysis of a system without accounting for the logistic and administrative delays in maintenance units. A_O is the preferred measure of availability; however, A_A is a metric obtainable when maintenance delays are unknown or unaccounted.

3.1.3 Dependability

Availability and dependability are both strongly a function of reliability but have different meanings. A_O is the probability the system is ready to perform a mission at any given time. The former Reliability Analysis Center, now under the Defense Systems Information Analysis Center (www.dsiac.org), described operational dependability, D_O, as the probability the system remains up during a mission given it started a mission operational [34]. The key difference is that dependability is conditional upon the aircraft beginning a mission up. Figure 14 shows the connection between reliability, materiel availability, and materiel dependability.

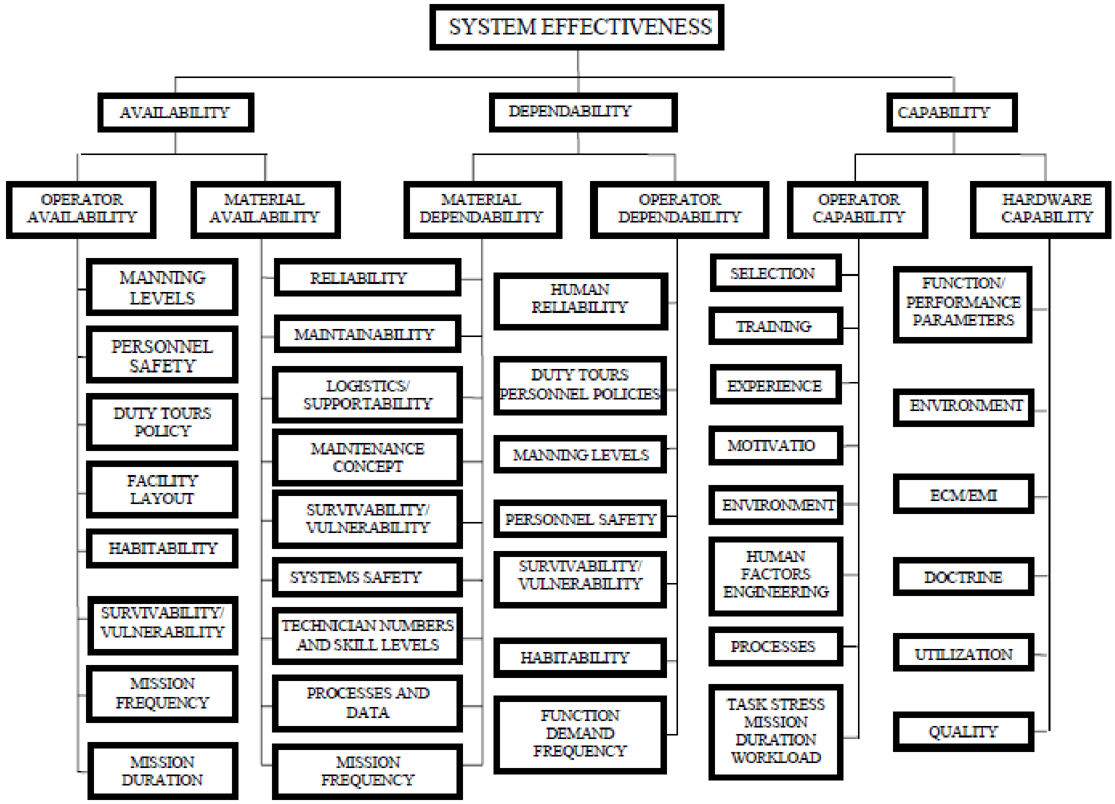


Figure 14: System Effectiveness [34]

A dependable system must stay functional throughout the mission but makes no statement about the system before flight. A dependable system must work when called to work. In a MFOP strategy, a dependable system has little unscheduled maintenance but says nothing about scheduled maintenance. An aircraft may remain dependable and have a large amount of scheduled maintenance (MRP). Dependability is necessary and sufficient for a high MFOP. A reliable system has little unscheduled maintenance and little scheduled maintenance. A reliable aircraft keeps O&S costs low and will support better availability and dependability. MFOP makes a stronger statement about an aircraft's dependability than availability.

3.1.4 Other -Abilities

3.1.4.1 Affordability

Affordability is the measure of financial means and willingness to support a system's life cycle costs. Greater means (and the willingness to spend) or lower costs will increase affordability. This study focusses on the system itself and shall not consider factors external to the system such as an organization's means or willingness to spend. The Cost Capability Analysis curve (see section 3.5.2) informs decision makers of the system effectiveness and life cycle costs. Decision makers are free to consider costs themselves. Consequently, life cycle costs measures affordability absent consideration of an organization.

3.1.4.2 Capability

The purpose of a system is to provide a needed capability to the user. The Joint Capabilities Integration and Development System (JCIDS) compiles capability needs for military systems. JCIDS capability documents cite the vehicle performance, RAM, cost, sustainment, and other requirements. The title system effectiveness addresses the performance capability, availability, and dependability of a system. When using system effectiveness, capability refers to the vehicle's performance to meet operational capability requirements. Availability and dependability is different than operational capability. Equation (22) in section 3.5.1 presents a Mission Capability Index (MCI) as a measurement of operational capability.

3.1.4.3 Maintainability

Like reliability, availability and dependability share the concept of maintainability. The DoD RAM-C does not provide an explicit definition; however, maintainability describes the ease of effort necessary to preserve and restore a system's functionality. Maintainability metrics are measured in time (i.e., Maintenance Man Hours) or cost. Example metrics include MTTR, MDT, LDT, and ADT. In the model of renewal theory introduced in section 3.4.2, the time to repair failures (T_f) and the time to conduct preventive replacements (T_p) measure a components maintainability. Maintainability supports a MFOP strategy with minimal inspections and repairs during a MFOP cycle and limiting the MDT associated with each recovery period.

3.1.4.4 Survivability

Survivability commonly refers to the survival of a crew and system. DoD's Joint Technical Coordinating Group on Aircraft Survivability (JTCG/AS) defined survivability as

> *The capability of a system and crew to avoid or withstand a man-made hostile environment without suffering an abortive impairment of its ability to accomplish its designated mission. Survivability consists of susceptibility, vulnerability, and recoverability. JTCG/AS [36]*

JTCG/AS' definition is more comprehensive than considered in this thesis. Kumar et al. coined the term MFOP Survivability as the confidence level that an item successfully completes the MFOP [23]. This thesis uses the term MFOP Success (MFOPS) to reference Kumar's confidence level to avoid confusion with the JTCG/AS definition. MFOP Success relates to reliability and does not address the vulnerability to enemy action. Section 3.2.5 provides an expanded review of MFOP Success.

3.2 MFOP Concepts

The idea of a MFOP is a different way of approaching aircraft reliability. Traditional reliability seeks fault free operations, yet this section shows why the interest in a MFOP strategy is the duration of failure free operations. A MFOP is as a renewable assurance against system failure. This section discusses how mean based metrics such as MTBF fail to convey the information necessary to predict a MFOP and its probability of success. Finally, it reviews various approaches to improve system MFOP and a few examples of the different non-aviation related industries employing MFOP strategies.

3.2.1 Failure Cause Identification

Statistical metrics using the mean capture the frequency of failure, but not the history of failure (section 2.3.1). Measuring failure using the median is an interesting notion. The median is a step in the right direction with a rough accounting of failure history; however, it is inadequate because the median makes no statement on the frequency of failure. Two components may have the same median time before failure yet failure at different rates. To account for both the frequency of failure and time history, one must look to Failure Cause Identification.

Trindade and Nathan in [22] and [37] demonstrate another useful concept in reliability theory that has a new application in MFOP. The traditional way of showing the cause of failure is through a Failure Cause Pareto chart. Looking at Figure 15, one would assume that Cause A is the biggest limiting factor to the systems MFOP. With this information, Cause E looks like the most reliable and least likely limiting factor to the MFOP.

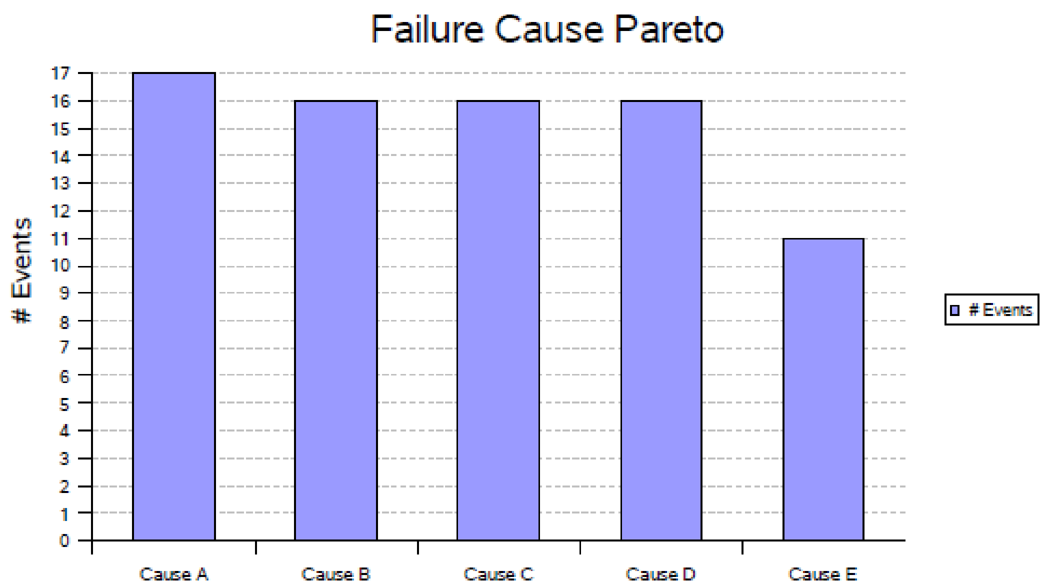


Figure 15: Failure Cause Pareto Chart. Figure reprinted with permissions from [22]

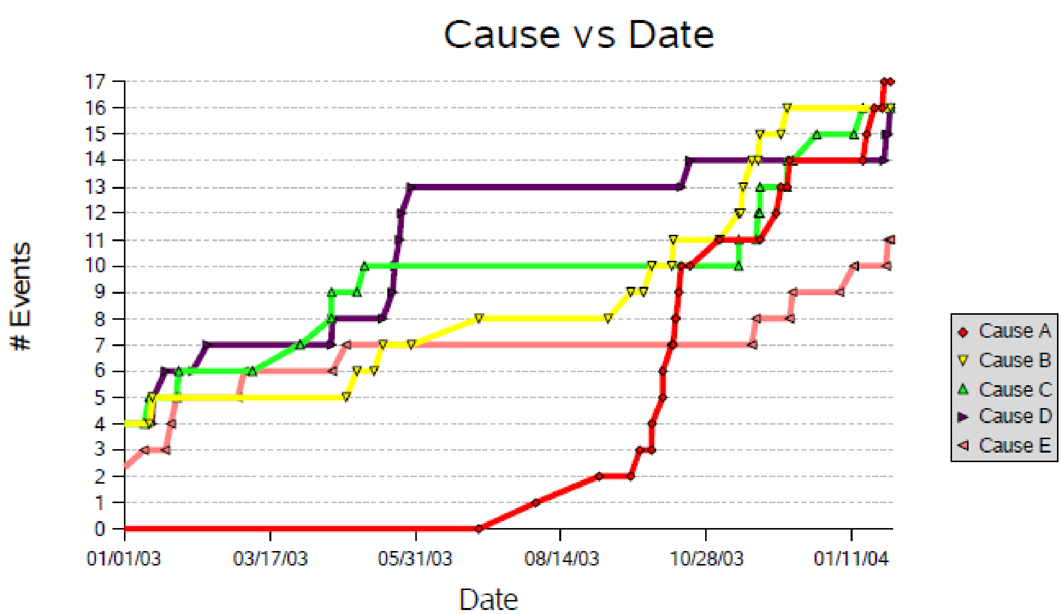


Figure 16: Failure Cause Versus Calendar Date. Figure reprinted with permissions from [22]

Examine the same system proposed by Trindade and Nathan in Figure 16. This figure shows the time history of failures. Although this example uses calendar time, a MFOP designer could easily use flight hours to measure time. Looking at the figure, one sees that Cause E, thought to be the best performer by a Pareto chart, has failures occurring early in the year. Cause A has the most total failures in the year but does not experience failures and limit MFOP until mid-year. To get any significant system MFOP, the designer needs to rework causes B, C, D, and E. Failure Cause history provides the designer the right information to identify which components to improve to better the system's MFOP.

The issues with MTBF and traditional failure bar charts have the same root cause. Reporting the mean destroys the time history information. Under a MFOP strategy, the "when" is as important as the "how often." An effective MFOP strategy may accept more total failures over time in favor of those failures occurring later in usage. This example is an important concept in understanding how to improve MFOP and shapes the approach to solving Problem 1.

3.2.2 The Incompleteness of Availability

In the strictest interpretation of a MFOP cycle, a system does not experience any unscheduled maintenance during the MFOP period and defers all scheduled maintenance to the MRP. The expected downtime due to logistical and administrative delay is minimal because the designer has perfect knowledge of the repairs occurring in the next MRP. Under this interpretation, the ratio of MFOP to MFOP cycle duration may be expressed as the achieved availability, A_A.

$$A_A = \frac{Uptime}{Uptime+Downtime} = \frac{MTBM}{MTBM+MMT} \cong \frac{t_{mf}}{t_{mf}+T_{mr}} \qquad (7)$$

where t_{mf} and T_{mr} denote the duration of the maintenance free operating and maintenance recovery periods, respectively. The equivalency goes even further if the system operates continuously with no standby time.

Equation (7) is valid when two assumptions are true. The MFOP duration becomes an MTBM with no deviation under the strict interpretation which allows system failure only after achieving the MFOP. Likewise, the MRP duration becomes the MMT only if all maintenance is deferred successfully to the MRP.

Availability is an important metric to measure the system's efficiency of uptime to downtime; however, it is an incomplete metric in a MFOP context as it suffers from loss of time information like the MTBF fallacy. Consider two different systems performing the same functions. The systems run continuously with uptime and downtimes as illustrated in Figure 17. The first system has an operating period of 9 hours (t_{mf}), a repair period of 1 hour (T_{mr}), and a total MFOP cycle of 10 hours. From (7), the availability is 90%. A second system has an operating period of 3 hours, a repair period of 20 minutes, and a total MFOP cycle of 3.33 hours. This too has an achieved availability of 90%, yet at more frequent repair intervals. The operational needs will dictate an operator's preference in systems. To an operator seeking greater MFOP, the second system is more disruptive, yet the availability metric alone is incomplete in measuring this deficiency.

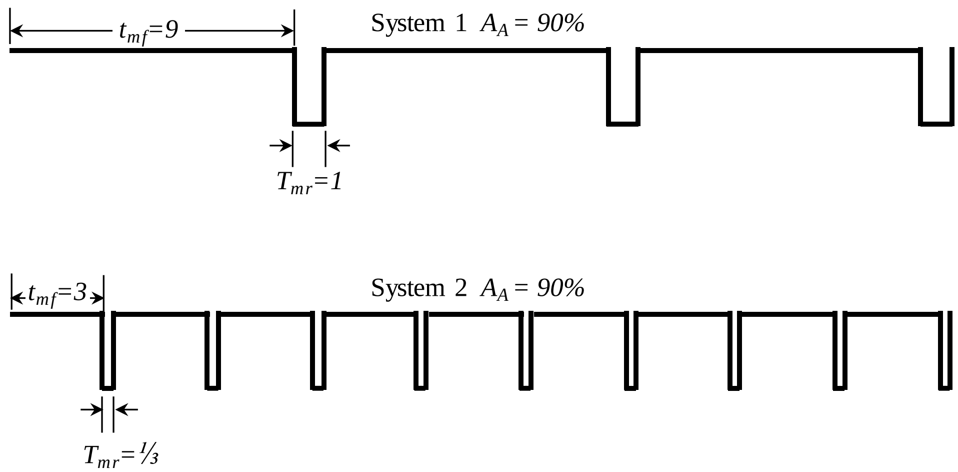


Figure 17: Incompleteness of Operational Availability

The ratio of MFOP to MRP should accompany availability to completely measure a system to satisfy operational needs. In the above example, the systems have MFOP to MRP ratios of 9:1 hours and 3:0.33 hours. Viewing the ratio communicates the time information necessary for a MFOP design.

3.2.3 MFOP Options

Hockley and Appleton first suggested designing from a "bottom-up" approach to achieving a MFOP target [6]. The bottom-up approach involved understanding why, how, and when items failed. They listed the following approaches to improve MFOP:

- Condition Monitoring
- Useful Life
- Fault Tolerance
- Acceptable Degradation
- New Technology

Relf identified a more refined strategy to obtain the capacity for MFOP. He began with the premise that an effective MFOP strategy should include multiple approaches [24]. He introduced six MFOP Options to extend the MFOP as shown in Figure 18. Relf's general MFOP methodology, as previous introduced by Figure 9 of section 2.5, applies the MFOP Options.

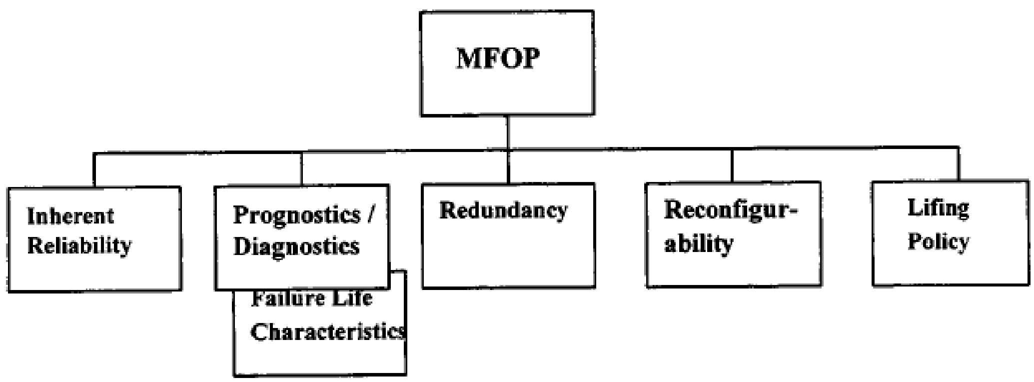


Figure 18: Hierarchy of MFOP Options [24] Copyright © 1999 John Wiley & Sons, Ltd.

Component inherent reliability makes a direct improvement to the systems reliability but has limits based upon manufacturing and material capabilities. Relf warns of runaway costs if a strategy relies only on inherent reliability [24]. Prognostics forecast failure by detecting signs of impending failure with sufficient time to take corrective action. Prognostics are normally on line by working during aircraft operation. Diagnostics identify the source of the fault and are normally off line. Failure life characteristics involves understanding of how parts wear out and fail. Redundancy adds an additional like component or software portioning that performs the task of a failed component at the penalty of additional weight and complexity. Both redundancy and reconfigurability are MFOP options because they are ways for a system to be fault tolerant and continue to function despite a component failure [21]. Designers make the choice to use the first five MFOP options early in the design process; thereby constraining the MFOP before the aircraft reaches a unit.

A lifing policy is the one option to improve MFOP that is under the influence of maintainers in a unit. Some components tend to wear with use and follow a distribution with an increasing failure rate over time or cycles. Lifing policy is the preventive replacement of those aging parts upon reaching the safe life or exceeding a damage tolerance threshold. The goal of a lifing policy is to improve the success of the MFOP. An effective MFOP maintenance plan appropriately times the replacement of worn parts to the MRP. A lifing policy is costly in that it replaces items that are functioning and may be perfectly fine. It also creates the need for item refurbishment before reinstall. Despite these drawbacks, it is the one option most easily changed to by operators. Chapter 6 explores the concept of aggressive lifing to adapt the MFOP to increasing or decreasing operational demands.

3.2.4 Failure Free Versus Fault Free

Hockley and Appleton [6] first introduced the notions of failure free and fault free when outlining the Ultra-Reliable Aircraft Project in 1997. Hockley [7] then elaborated on the terms in 1998. The articles [6], [7] define

1. Failure free to mean "that the equipment is able to operate to its full mission requirement for the period required or specified."
2. Fault free means that "there are no faults and the system is also able to operate to its full mission requirement for the period required or specified."

Failure free means that faults may exist but that they did not prevent the system from accomplishing its task. Fault free is a stricter requirement, requiring no faults to exist in the system. A system that is fault free is always failure free (if designed correctly). A system that is failure free may not necessarily be fault free. The design may be fault or damage tolerant. MFOP options that improve fault tolerance are redundancy and reconfigurability. Inherent reliability, prognostics/diagnostics, and lifing policies are ways to achieve fault free design.

An aircraft may achieve a desired MFOP by reducing failures or being more fault tolerant or both. The important concept is that faults may occur if the system can cope with the fault and continue to fully function. The requirements to be completely fault free or even completely failure free is very stringent; however, the good news is that neither is necessary to obtain a MFOP. A MFOP system just needs to be failure free long enough to achieve the desired period.

3.2.5 MFOP Success

Kumar et al. in [23] introduced the concept of MFOP Survivability. The authors defined MFOPS as "the probability that the item will survive for the duration of the MFOP" [23]. Although Kumar et al. follows the field of reliability engineering's use of the term survivability, it creates a confusion with the military term of survivability (section 3.1.4.4). MFOPS is renamed as MFOP Success to avoid confusion. MFOPS is the probability that a system remains functional after the *ith* MFOP cycle given that it was functional in previous cycles. Conditional probability states that the probability of event B occurring given that event A has already occurred is

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \tag{8}$$

In this case, event B is the probability that the next MFOP cycle is successful given that the previous MFOP cycle (event A) was successful. The MFOPS after a series of i cycles can be calculated then as

$$MFOPS(t_{mf}, i) = \prod_{k=1}^{n} \frac{R_k(i \times t_{mf})}{R_k([i-1] \times t_{mf})} \tag{9}$$

where R_k is the mission reliability of the kth component in the ith cycle and t_{mf} is the period of the cycle (or MFOP) [23]. Equation (9) is constructed for a system with parts arranged in serial. In more complex fault trees or phased missions, a generic equation that represents MFOP is

$$MFOPS(t_{mf}, i) = \frac{R_{sys}(i \times t_{mf})}{R_{sys}([i-1] \times t_{mf})} \tag{10}$$

where R_{sys} is the mission reliability of the system. MFOPS is a named term for the confidence level that a system survives the next cycle. It is like a system hazard rate in that both are conditional probabilities; however, MFOPS measures success over time, while the hazard rate measures failure at an instant of time.

Two sources, [23] and [21], recommend renewal theory to solve the MFOPS of a repairable system. This models the MFOPS after a series of operating periods followed by recovery periods. Kumar et al. [23], again provides a concise application of renewal theory for MFOPS. Assuming a MFOP system has two states, 1 (operating) or 0 (failed), the probability, P_1, that the system survives on the Tth cycle of t_{mf} is found in (11). The probability it does not, P_0, is (12).

$$P_1(T) = R(t_{mf}) + \int_0^T f(u|t_{mf}) P_0(T-u)\, du \tag{11}$$

$$P_0(T) = \int_0^T g(v) P_1(T-v) dv \tag{12}$$

where *f(t)* is the failure density function for the system, *g(t)* is the repair time density function, and *f(u|tmf)* is the probability that a system survives to time *u* given it has survived t_{mf}. Using numerical methods will solve (11) and (12) [23], [26]. P_1 is of the primary concern to find the MFOPS. Finally, [26] gives the probability of failure or unreliability, *q(t)*, of the system as

$$q(T) = \int_0^T [P_1(z) - P_0(z)] dz \tag{13}$$

3.2.6 Examples of MFOP

3.2.6.1 South African Mining Industry

Al Shaalane and Vlok [21] applied the MFOP concept as outlined by Relf and Hockley to the mining industry. The authors modeled three rock crushers working in parallel. They tested each rock crusher's data with the Laplace trend test and then fitted with them to Weibull distributions.

Analysis by the authors showed the MTBF of Crusher 1 and Crusher 2 were similar at 49 and 50 hours, respectively. Figure 19 plots the MFOPS versus MFOP. The MFOPS of each crusher at 50 hours was significantly different despite like MTBFs. Crusher 1 had a 50% chance of reaching the 50-hour mark. Crusher 2 had a 40% chance. This is an example of correctly applied failure cause identification (see section 3.2.1) and shows the advantage in a MFOP analysis.

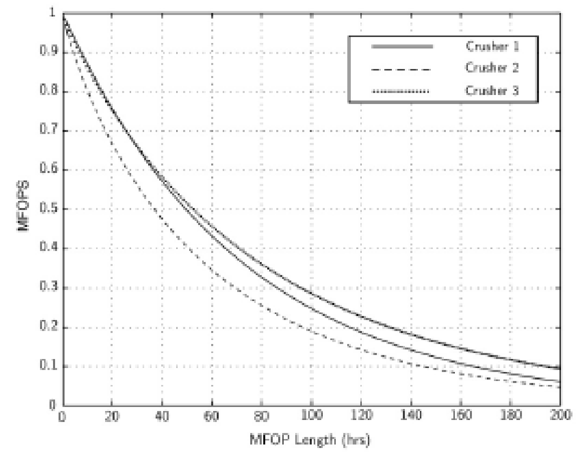


Figure 19: Probability of Crushers Surviving a MFOP Length. Figure reprinted with permissions from [21]

3.2.6.2 U.S. Navy CBM+MFOP Demonstration

In the spring of 2011, the U.S. Navy Program Executive Office (PEO) Ships briefed the results of a CBM + MFOP philosophy. The concept was to take advantage of condition based maintenance to utilize a commercial off the shelf automated monitoring system for self-checking, self-healing capabilities, and remote monitoring and control [38]. The USS Iwo Jima successfully completed a six-month deployment with an A_O of 99.7%. The MFOP program demonstrated improved availability through the MFOP options of redundancy and prognostics with diagnostics. It projected a 99% availability after one year compared to the non-redundant systems value of 83%. At four years, the program estimated an 89% availability compared to the non-redundant systems value of 48%. PEO Ships carried forward a recommendation for further expansion of the MFOP program to the fleet.

3.2.6.3 Airline Industry

The airline industry's maintenance rhythm is loosely a short cycling of MFOP and MRP [39]. Except for "red-eye" and international flights, maintenance resets the airplanes overnight in a short MRP. Airlines dispatch planes for duty throughout the day under a Time Limited Dispatch (TLD) approach. SAE International's Aerospace Recommended Practice (ARP) 5107B [40] states,

> *The TLD concept is one wherein a redundant system is allowed to operate for a predetermined length of time with faults present in the redundant elements of the system, before repairs are required.*

Extending the dispatch to a predetermined length of time creates a MFOP with a given probability of success. Both TLD and a MFOP strategy share the goal to manage risk. They sequence maintenance to the recovery period such that the probability of success meets a desired threshold. In conjunction with the British Ultra-Reliable Aircraft Project, Airbus BAe undertook an experiment with an A320. A benchmarking showed a MFOP of ten days (150 flight hours) with a MFOPS of 0.8 to 0.9 [41]. Albeit rotorcraft maintenance is historically more challenging, this experiment showed the potential for significant MFOP durations by combining options outlined by Relf [24].

3.2.7 MFOP Summary

Improving the duration of failure free operations is the key to unlocking MFOPs. Equally as important, an acceptable risk level of MFOPS must accompany any MFOP projection. Plotting a time history provides the necessary information to determine the MFOP at a given success. Finally, both the U.S. Navy and the Airbus BAe demonstrated the power of multiple MFOP options to great success.

3.3 Reliability System Modeling

The definition of availability, dependability, and MFOPS rely upon the calculation of a system's materiel reliability. It is also worth taking note that reliability is a key metric for safety and risk analysis. Meeting Target Levels of Safety drives much of the certification or air worthiness processes [42]. While not discounting the need for a safe aircraft, this review of modeling techniques focusses on an aircraft's dependability to best address MFOP problems. A hazard, in a MFOP context, is the occurrence of any event that causes mission failure. Unlike a safety hazard, a MFOP hazard may or may not place human lives or equipment at risk. Andrews, in an article in the Journal of Mechanical Engineering Science, stated that the "methods used to quantify the frequency or probability of the system failure resulting in the materialization of the hazard are generally applicable and used across all industrial sectors" [26]. Much like a safety analysis, but with a different purpose, modeling of a MFOP strategy begins with the modeling of a system's materiel reliability.

In that same article, Andrews surveyed state-of-the-art reliability modeling that is applicable to all industries. His primary guidance to the reader was to consider the assumptions of each method. He recommended the selection of a method to be based on the applicability of the model's assumptions [26]. The discussion of the proceeding methods in this chapter will address each's assumptions. Chapter 4 will then consider the assumptions in selecting the best method to solve the thesis' problems.

3.3.1 Model Criteria

A MFOP model for rotorcraft requires certain characteristics and functionality. Like the call for operational metrics, there is a growing expectation that models support operational conditions [26]; therefore, an acceptable MFOP method shall

1. support phased missions
2. be repairable
3. monitor part wear and aging
4. be flexible

A MFOP model must be compatible with a phased mission approach. A MFOP is the compilation of repeated missions. Each mission consists of different phases such as warm-up, hover, take-off, climb, cruise, descent, and landing. Each phase likely has varying duration, flight conditions (e.g. altitude, temperature, airspeed), and component usage. For example, the landing gear only experiences cycles and wear during take-off and landing. An accurate model shall include these differences. Mission unreliability is the probability of at least one failed phase [27]. The three methods, FTA, Markov, and simulation, consider phased missions but with varying complexity.

Integral to a MFOP strategy is the maintenance recovery period at the end of the cycle. Any method must be able to account for a repairable system that adjusts component age and failure distributions.

A great number of parts on a helicopter experience wear and vibration that decreases performance over time or increases unreliability over time. Some components experience burn-in and some components experience wear-out, and others such as electronics tend to have a constant rate of failure. The model must handle a variety of failure distributions.

Flexibility is also important to the model's usability. Methods that are easily to employ have an advantage over those that are difficult to modify or take an extensive effort to calculate. A MFOP model should also provide a time history across the phased mission and throughout multiple missions to predict the MFOPS over flight time.

3.3.2 Analytical Fault Tree Analysis

The origins of Fault Tree Analysis (FTA) date to the 1960s. Andrews called FTA and reliability block diagrams "the cornerstone" of most risk and safety analysis [26]. FTAs are combinatorial methods that work off a minimal cut set to build a systems likelihood of failure [26]. Each cut set represents a failure mode of the system. The MFOPS shown in section 3.2.5 is an example of a combinatorial method.

FTAs prevent several problems in a MFOP strategy. A primary assumption of FTA is that component failure events occur independently [26], [27]. This eliminates FTA from modeling a sequence of events such as cascading failures. If component failure is not independent, then other methods like Markov or simulation are more appropriate [27]. Non-coherent fault trees can handle dependency through the use of prime implicants (combinations of states that cause failure) [27], but Andrews says that they are "highly complex and infrequently used" [26]. Chew et al. [27] demonstrated how to adapt non-coherent FTAs to analytically solve a phased mission. The approach created a new fault tree for each phase and then summed the unreliability for each phase to yield the mission unreliability. It showed the probability of a failure at phase p to be

$$P(phase\ p\ failure) = 1 - P(phase\ p\ success | phase\ p-1\ success)$$

$$= 1 - \frac{P(success\ up\ to\ end\ of\ phase\ p)}{P(success\ up\ to\ end\ of\ phase\ p-1)} \quad (14)$$

For all but the simplest systems, the approach authored by Chew becomes computationally intensive [26]. Finally, FTAs typically model a non-repairable system using the minimal cut sets elimination technique. They become very large and complex with repairable systems [26], [27].

Binary Decision Diagrams (BDDs) are another option. Their structure is like FTAs but each entry takes a system state. BDDs are faster and their calculations are a more efficient depiction than an FTA; however, building their structure is somewhat of an art form and it is difficult to extract the fault structure [26].

3.3.3 Markov Chains

Markov models belong to a family of state space models. The Markov method considers all possible states a system may take [32] and may be continuous or discrete. The general strategy is to solve the probability of failure for all states and then sum them together for the system unreliability. A Markov system is both exhaustive (every state is accounted for) and mutually exclusive (system may only occupy one state at a time) [26], [27], [32]. State outcomes are calculated as

$$\boldsymbol{s}^{(m)} = \boldsymbol{S}.\boldsymbol{A}^{(\boldsymbol{m})} \tag{15}$$

where $\boldsymbol{s}^{(m)}$ is a vector of state probabilities after the *mth* step and $\boldsymbol{A}$ is the transition matrix [26]. Markov analysis model repairs systems by transitioning back from a failed to operating state. This makes it appropriate for MFOP-MRP cycles.

Classical Markov methods have two assumptions: (1) the system lacks a memory; and (2) the system is homogeneous [26], [32]. A homogeneous system has a constant failure rate; therefore, they do not work for non-constant failure or repair rates. Chew et al. in [27] noted that non-homogeneous models support aging parts but they become complex. Another disadvantage of Markov methods is that the model's size grows exponentially as more components join the system [26]. They suffer from state space explosion [43].

3.3.4 Event Simulation

Simulation's greatest advantage is that it is unrestricted by any set of assumptions. It does not suffer from the complexities found in phased mission FTAs or non-homogeneous Markov analysis [27]. Simulation may use different failure distributions including non-constant failure distributions like Weibull. They support repair queuing, component interdependence, and repairable systems [27]. Simulation has become attractive because computing permits multiple iterations that quickly approximates solutions to problems that are mathematically complex or impossible [26]. Most importantly, simulation can easily depict failures in a time history to permit the identification of MFOP to a given success rate. A correctly drawn simulation meets the four criteria (phased mission support, repairable, aging parts, and flexibility) to model cycles of MFOP-MRP in the problem.

3.3.4.1 Discrete Event Simulation

A Discrete Event Simulation (DES) is a type of event simulation. DES models the behavior of a dynamic system by approximating the system as a sequence of instantaneous occurrences [44]. A DES has the following characteristics:

- **Discrete** because the system only occupies one state at a time
- **Event**s cause instantaneous transition from one state to another.
- a **Simulation** of a real-world system that progresses through a series of missions.

The system is in a fixed state until the next event occurs. Events may occur at regular or random intervals. After an event, the event and the new state of the system are recorded. Unlike a time-based simulation, the simulation does not record information at a uniform time step; time is merely an artifact of the simulation [44].

DES provides two advantages over time-based simulations in a MFOP analysis. Depending on the application, recording the state only after event transition may save significant computational time. Secondly, DES is valuable when mixing deterministic and non-deterministic (stochastic) aspects [44]. When considering the MFOP problem, the proposed DES has both aspects as shown in the table below.

Table 5: Discrete Event Simulation Aspects

Deterministic	Stochastic
Phase duration and mission progression	Part failure
System architecture	Part repair time
Repair decisions	Repair cost
Detection of failures	Technology impacts
Performance modeling	

Compiling multiple iterations of the DES approximates the answer to a complex problem. Relf [24] suggested such an effort in 1999 when he first introduced MFOP options. [43], [26], [27] also recommended Monte Carlo simulation to quickly approximate solutions. Chew et al. [27] noted that applying analytical methods like FTA and Markov to a more complicated system requires "extensive calculations," yet simulation like Monte Carlo "does not require a much greater increase in the length of time to find phase reliability."

3.3.4.2 Petri Nets

Several papers [26], [27], [39], [43] endorse the use of state space based Petri nets to run within a Monte Carlo simulation to model MFOP. Petri nets are a flexible, graphical modeling technique that describes a system with places and transitions (see Figure 20). Transitions connect places. Tokens occupy places and move to places via transitions. Transitions may be instantaneous, a fixed time, or follow any random distribution. Total token position marks the system's state. In modeling a system's failure, tokens may assume the role of components as in [39] or work to define component operation or failed states as in [27]. References [27] and [43] demonstrated the use of Petri nets in a systems reliability with comparable results to analytical models. Phase modeling occurs by transitioning tokens from one phase state to another. Volovoi in [43] proves how tokens may age to support component wear and can change color to model Relf's MFOP options. Event simulation is similar to a Petri net in that both are state-based and event driven, but event simulation may not have a graphical depiction of the states.

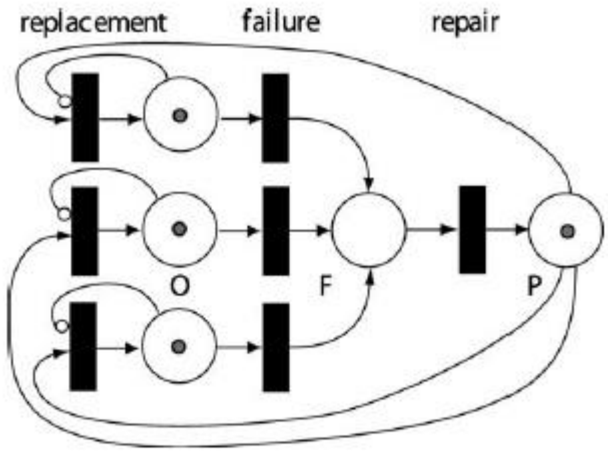


Figure 20: Simple Petri Net with Repairing and Shared Pool of Spares. Figure reprinted with permission[1]

3.4 Maintenance Modeling

A coupling exists between the MFOPS and MRP. A well-designed policy of scheduled maintenance is "the most cost effective way" [26] to maintain the MFOPS for aging systems. Historically, design and maintenance planning are separate [26] . There exists great potential in marrying the system's reliability to a flexible MRP policy. The notion of Time Limited Dispatch (TLD) supports a MRP policy. TLD permits the dispatching of aircraft with known faults but no mission critical failures. TLD operates as failure free but not fault free. Andrews [26] notes that, in TLD, dispatch is halted when the risk of further failure exceeds a risk or safety threshold. The threshold for dispatch in a MFOP strategy is the MFOPS. Generating a maintenance policy that maintains a MFOPS and minimizes downtime is of relevance in military aircraft operations.

[1] Reprinted from *Reliability Engineering & System Safety*, vol. 84, V. Volovoi, "Modeling of system reliability Petri nets with aging tokens," pp. 149-161, Copyright (2004), with permission from Elsevier.

3.4.1 Maintenance Recovery Period

The Maintenance Recovery Period (MRP) enables a MFOP by repairing worn components and replacing failed components. This goal of the MRP is to reset the system reliability to the point where it can achieve the next MFOP at the required success rate (see Figure 21). A secondary goal of a MRP is to trade for less unscheduled maintenance with more scheduled maintenance. The airline industry almost always desires this trade because scheduled maintenance affords the opportunity to reduce administrative and logistical downtimes. The predictability of a MRP limits mission disruption and reduces total downtime. An outcome for a MRP is the reduction in cost. The major "hurt" of airlines was that of unscheduled maintenance; costing on the order £1M per aircraft [6]. The British URA Project predicted a 15-20% savings in O&S cost with the application of a MRP [7].

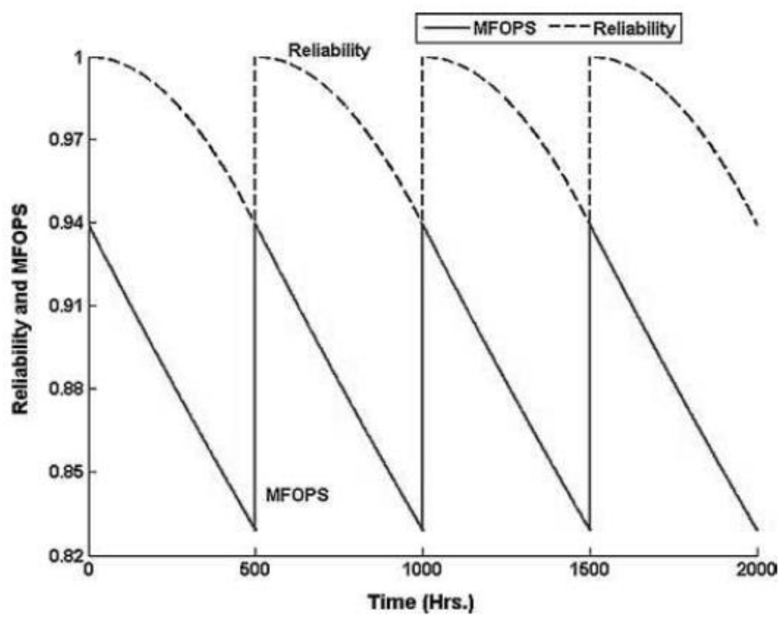


Figure 21: Achieving Reliability and MFOPS Based on Replacement Strategy. Figure reprinted with permissions from [45]

While literature has studied MFOP in recent years, it has neglected MRP. The majority of published work on scheduled maintenance policies to include [3], [46], and [45] are motivated by industry's desire to minimize cost only. In military operations, reduction of cost is not always the primary objective. There is the need for an adaptive maintenance policy that is flexible enough to meet the next MFOP. Each MRP should not look the same because components' usage varies in a phased mission and parts wear at different rates. [21], [41] reinforce the need for such a policy. In a survey of maintenance policies, Galante and Passannanti [46] identified adaptive maintenance policies as a problem that "has been poorly examined." A major effort of the dissertation is to adopt renewal theory as described in the next section to create an adaptive maintenance policy.

3.4.2 Renewal Theory

A previous section, 3.2.1, discussed how to use failure cause identification to discern which components or subsystems are limiting a MFOP. Section 3.2.2 reviewed options to improve the limiting components or subsystems. This section outlines the theory to determine the best time to replace the limiting components. Jardine and Tsang [3] demonstrated the use of renewal theory on a single part, repairable system to minimize downtime. Minimizing downtime is of primary interest because the RAM-C Rationale Report labeled downtime as "a main driver of system life cycle costs partly due to the necessity for additional system acquisitions to meet operational needs" [19]. Figure 22 shows the interval model from [3]. One cycle is the sum of the time to preventive maintenance, t_p, and the time of replacement T_p. This equates to one MFOP cycle where MFOP is t_p and the MRP is T_p. The mean time to replace a failure during unscheduled maintenance is T_f.

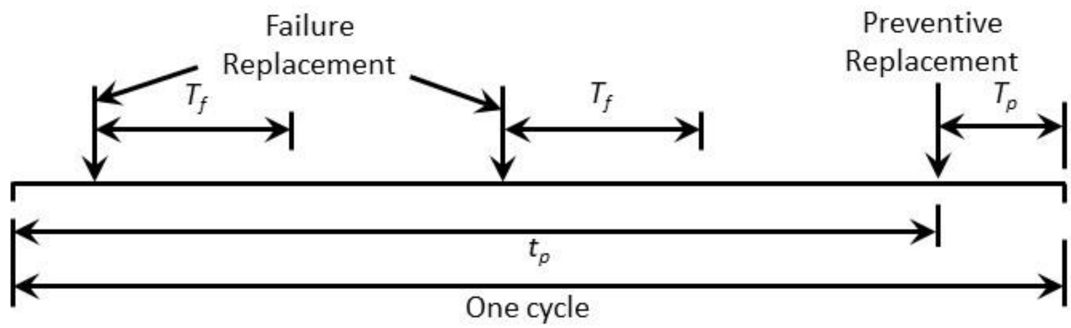


Figure reproduced for clarity

Figure 22: Preventive Replacement Interval Model. Republished with permission of Taylor and Francis from Maintenance, Replacement, and Reliability Theory and Applications, A. K. Jardine and A. H. C. Tsang, 2006; permission conveyed through Copyright Clearance Center, Inc.

The total downtime is represented as a dimensionless quantity in

$$D(t_p) = \frac{\begin{array}{c}\textit{Expected down time due to failures}+\\ \textit{Expected down time due to preventive replacement}\end{array}}{\textit{Cycle Length}} \qquad (16)$$

The expected downtime due to failures is the product of the number of failures in the interval from 0 to t_p, $H(t_p)$, times the mean time to make the replacement, T_f. The expected downtime due to preventive replacement is T_p. Cycle length is $t_p + t_f$. The downtime may be expressed as

$$D(t_p) = \frac{H(t_p)T_f + T_p}{t_p + T_p} \qquad (17)$$

Under renewal theory, $H(t_p)$ has an analytical form of

$$H(t) = \sum_{n=1}^{\infty} F_n(t) \qquad (18)$$

where $F_n(t)$ is the cumulative distribution function of the time up to the *nth* failure. A solution is found by solving for the Laplace transformation of (18). See [3] for a full derivation using renewal theory. Solving with Laplace transformations on continuous functions can only be found on simple distributions like the exponential distribution.

A discrete approach uses all failure distributions including the Weibull and normal distributions. Again, [3] provides the full derivation of *H(T)*. *H(T)* takes the discrete form of

$$H(T) = \sum_{i=0}^{T-1} [1 + H(T - i - 1)] \int_{i}^{i+1} f(t)dt, \;\; T \geq 1 \tag{19}$$

where *f(t)* is failure probability density function. Equation (19) is recursive where the starting value of the recursive equation is *H(0)*. The expected number of failures at the start of the cycle, *H(0)*, is zero because the item is functional at the start of the cycle. Calculation of *H(1)* continues the recursion, followed by *H(2)*, and so on. The intervals of *T* must be discrete increments and can take on any time duration. A 1-hour interval of *T* is probably sufficient for a MFOP maintenance policy. An assumption to the theory is not more than one failure occurs in one interval [3]. A precise estimate using 1-minute intervals reduces the chance of multiple failures in an interval. Cataloging the downtime at each *T* yields a plot like Figure 23 where the optimal point t_p is the minimal downtime.

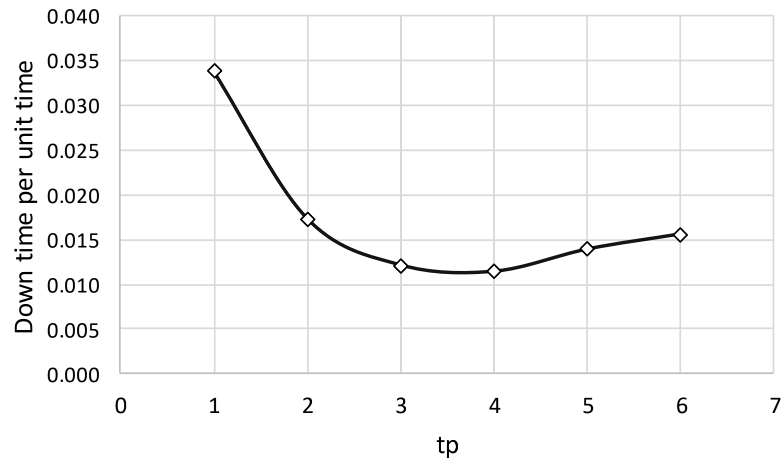


Figure 23: Plot Downtime per Unit Time versus Preventive Maintenance Interval t_p

3.5 Valuing a MFOP Aircraft

Relf states the need for “some form of trade-off” to assess the impact of a MFOP option on system penalties like weight, cost, and logistics [24]. A frequently used technique to assess the impact of a value function is benefit over penalty. Often, Net Present Value (NPS) measures penalty in terms of cost. Value functions are advantageous in trade studies, design optimization as objective functions, an understanding of design spaces, and technology evaluation [47].

$$Value\ Function = \frac{Benefit}{Penalty} \qquad (20)$$

Under pure Value Driven Design, there is no explicit requirements on extensive attributes such as weight, reliability, and cost [15], [48]. A correctly designed value function has no need for extensive requirements. The alternatives value will dip as it nears an undesirable attribute, effectively steering an optimizer away. Design is unrestricted with the only obligation to maximize the value. Collopy and Hollingsworth advise against extensive attributes in [15]. They argue that extensive attributes result in less efficient designs and, instead, objective functions should flow down to each component. A MFOP strategy, by nature, is imposing a minimal duration of failure free operations as an extensive requirement. This suggests the need for another approach to valuing a MFOP aircraft.

Net Present Value is an attractive measure of value because a direct trade may be made between cost and effectiveness. It eases the flowing of objective functions down to the component level and reduces the problem to a consistent unit of measurement. Under NPV, alternatives have a clear, ordered preference. Net Present Value is highly applicable to business but may not be as attractive in military applications. Certain military applications have an effectiveness metric that cannot easily be measured by a dollar value. According to the DoD RAM-C Rationale Report, minimizing LCC is desired but not the goal. In discussing the link between reliability and cost, it cautions, "Note that the optimal reliability value must be sufficient to meet the most strenuous warfighter requirements, which may result in the system having higher than the minimum possible LCC" [19]. In the case of a MFOP strategy, the operationally required MFOP may exist at a less than optimal LCC.

Value Based Acquisition (VBA) provides an approach that accommodates a MFOP requirement and permits quantification of military utility. In VBA, the benefit to cost ratio assumes the form of system effectiveness over LCC. Capability, availability, and dependability comprise a weapons system's effectiveness [49]. The next two sections outline such a single value function for a MFOP aircraft and its balancing with Cost Capability Analysis.

3.5.1 Single Objective Function

An Overall Evaluation Criteria (OEC) from Schrage [49] is an example of an aggregated value function that follows the form of benefit over cost. The function captures the systems benefit through its weighted capability, availability, and dependability. The system penalty is the Life Cycle Costs (LCC).

$$OEC = \frac{\alpha\left(\frac{Capability}{Basline\ Capability}\right)+\beta\left(\frac{Availabliity}{Baseline\ Availablity}\right)+\gamma\left(\frac{Dependability}{Baseline\ Dependability}\right)}{\left(\frac{Baseline\ LCC}{LCC}\right)} \quad (21)$$

The weights (α, β, γ) are typically an output of a Quality Function Deployment's (QFD's) technical weighting. The OEC equation must normalize values of capability, availability, dependability, and life cycle costs to baseline values to remain non-dimensional. In this way, the OEC reflects a design's value with quantities greater than one improving on the baseline's value. Figure 24 from Al Shalaane and Vlok [21] draws the complementary benefits between availability and dependability with cost in a MFOP context. The motivating concept parallels the OEC equation.

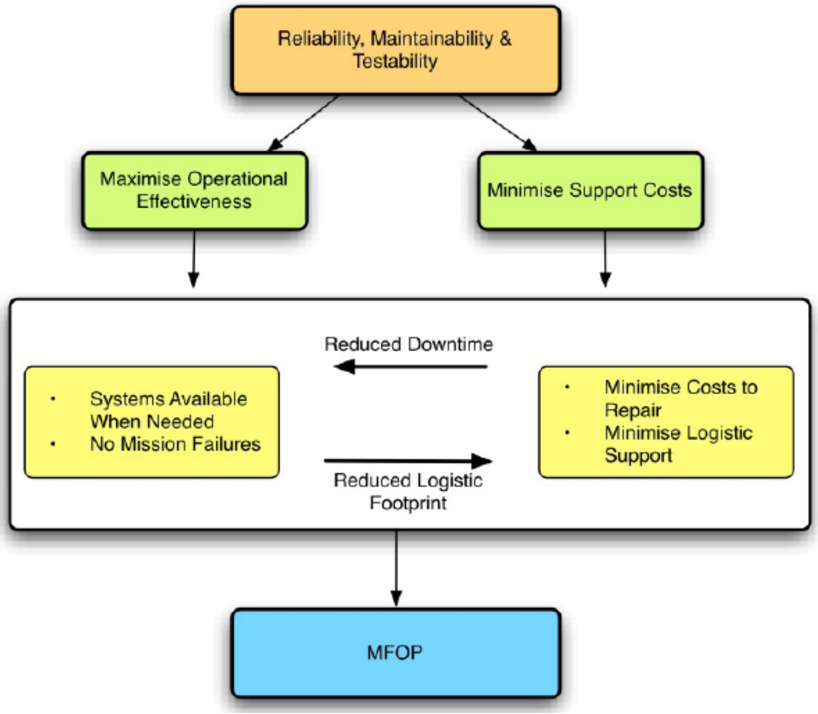


Figure 24: Motivators for MFOP. Figure reprinted with permissions from [21]

The final piece of the OEC is the operational capability. Capability typically captures vehicle performance (i.e. endurance, range, payload) as well as any other metrics desired by the customer. Delsing [50] encourages the use of utility functions for each component of the greater value function. This work proposes the use of a simple utility function called the Mission Capability Index (MCI) from [49]. The MCI measures a helicopter's maximum payload while Hovering Out of Ground Effect (HOGE) and block speed (V_{Block}) against the aircraft's empty weight and fuel weight. Block speed is the total distance travelled divided by the mission time. It accounts for an increasing cruise speed as the engines burn fuel throughout the mission. Conceptual design models are appropriate for the required level of analysis in the thesis, because the intent is to show relative changes, not provide a detailed performance estimate.

$$MCI = \frac{Payload_{HOGE} \times V_{Block}}{Empty\ Weight + Fuel\ Weight} \quad (22)$$

The use of the OEC (21) and a normalized MCI (22) are a simple means to measure and communicate the value associated with a rotorcraft in a MFOP context.

3.5.2 Cost Capability Analysis

The 2016 Defense Acquisition Guidebook described Cost Capability Analysis as an analytical tool to explore affordability and military utility [51]. It stated CCA's purpose is "to support delivery of cost-effective solutions through deliberate trade-off analysis between operational capability and affordability" [51]. Figure 25 is from the Guidebook showing a typical CCA plot of alternatives where effectiveness is on the y-axis and cost is on the x-axis. The term CCA has since been removed from the Guidebook and subsumed by a broader concept describing cost-effectiveness analysis; however, the Defense Acquisition University's website describes the Air Force Life Cycle Management Center's (AFLCMC) Standard Process for Cost Capability Analysis as "a "framework and a high level summary of the steps necessary to properly conduct CCA at various decision points in a program life cycle" [52]. It appears the DoD has delegated the term CCA to the AFLCMC for execution and standardization.

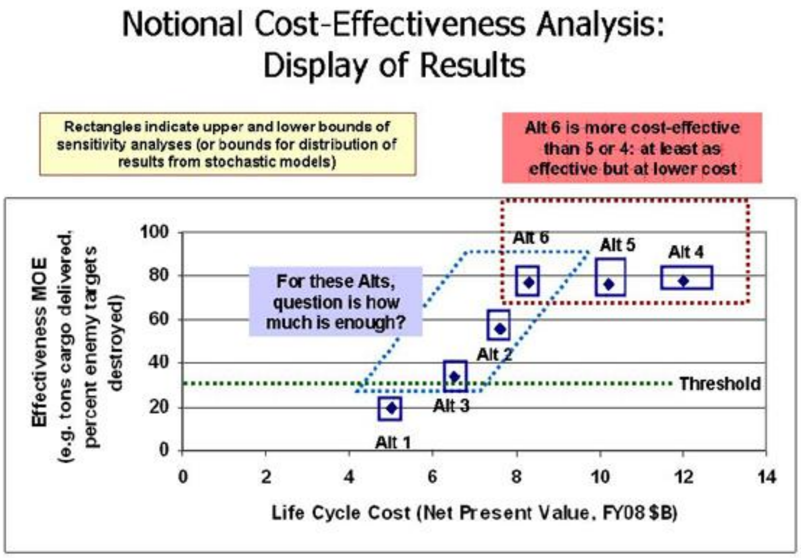


Figure 25: Scatter Plot of Effectiveness versus Cost from RAM-C Rationale Report [19]

CCA is the Air Force's solution to balancing effectiveness and cost throughout the acquisition process as mandated in various DoD directives [19], [52], [53], [54], [55], and recommended in [9], [20], and [38]. Cost Capability Analysis was mandated for all acquisition and materiel solutions per Air Force Instruction 10-601, dated June 2012 [56] and is now required in Air Force Policy Directive 10-6 dated November 2013 [57]. AFLCMC provides the definition of CCA.

> *CCA is an analysis process that uses warfighter involvement, subject matter expertise, and a rigorous multi-attribute, multi-objective decision analysis methodology to define tradespace between cost and warfighting capabilities. AFLCMC [58]*

CCA informs on the trade-offs between effectiveness and affordability. It seeks to identify the right place on the Pareto frontier or Cost Capability Curve. Figure 26 is a relatively simple plot for a multi-objective decision analysis. AFLCMC recommends weighting and then aggregating objectives into a single value function like the OEC (21) as a means of understanding and communicating multi-objective analysis [58]. Another name for the a priori optimization used by OSD AT&L is Multi-Attribute Decision Model (MADM) [50]. AFLCMC cites the reason for supporting the single curve by stating, "Without aggregation, it is often too difficult to simultaneously and objectively consider alternatives with multiple decision criteria" [58]. The curve is an important means to communicate complex design trade-offs to decision makers. Understanding where the curve bends informs decision makers on the point of diminishing returns where further investment provides little additional value. The FVL program mentions this point.

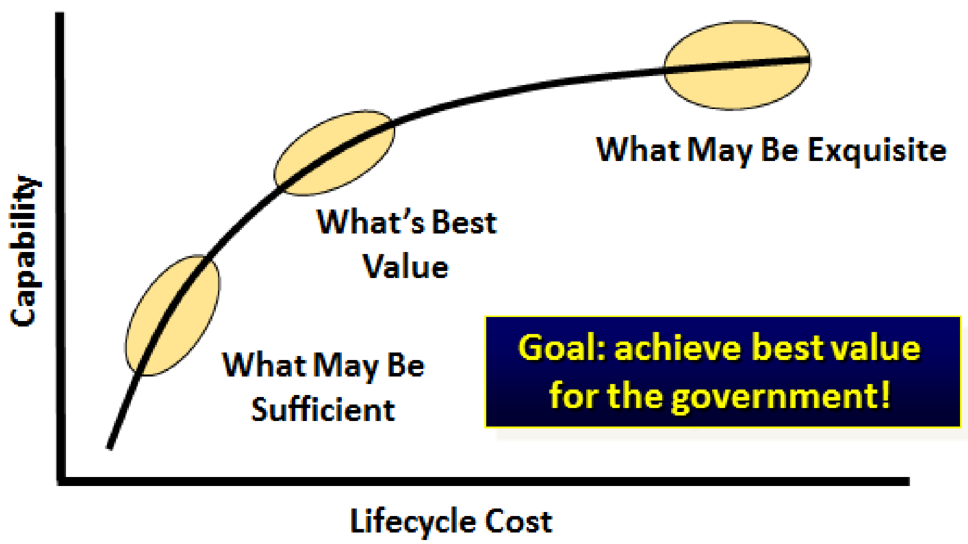


Figure 26: Cost Capability Curve from AFLCMC [58]

3.6 Zero Maintenance Technologies

Technologies either reduce the volume of failures through an increased reliability, improve fault tolerance, or extend component service life are a benefit to a MFOP strategy. Zero maintenance technologies improve inherent reliability or fault tolerance (prognostics/diagnostics, redundancy, and reconfigurability). These four technologies plus lifing policy comprise Relf's MFOP options. Table 6 lists some of the zero maintenance technologies planned for FVL FoS.

Table 6: FVL Technologies

Technology	Benefit
Improved Turbine Engine Program (ITEP)	More Power and better fuel consumption Greater sustainability/reliability
Integrated Health Management	Improved availability
Open System Architecture	Reconfigurability, sustainability
Advanced Control Laws	Reconfigurability

The U.S. Army's Aviation Applied Technology Directorate (ADD-AATD) and Sikorsky Aircraft Corporation joined together for Capability-Based Operations and Sustainment-Aviation (COST-A) to enable the transition to Condition Based Maintenance (CBM) and serve as an initial step towards a near-zero maintenance paradigm. COST-A is a program intended to mature diagnostics and prognostics to reduce O&S costs. COST-A development matured prognostic/diagnostic technologies in propulsion, drive, structural, rotor, electrical, and vehicle management systems to TRL 6 [2]. The team installed the technologies on a prototype UH-60 Black Hawk and assessed the impacts. Table 7 lists the successful COST-A technologies tested in 2015.

Table 7: COST-A Final TRL High-Level Summary from [2]

IPT	Technology Groups	Final TRL
Propulsion	Improved Power Assurance	6
	Main Engine Prognostics	6
	LRU Diagnostics for: AISBV, IPS Blower, Accumulator, Torque Split, Cross-Bleed Valve	6
Drives	Drive Train Loads Monitoring	6-7
	Improved CIs and sensor quality	6
	Tail Drive Shaft Diagnostics	5
	Bearing and Gear Prognostics	4-5
	Maintenance Reasoner	6
Structures	Fatigue and Impacts SHM	6
	Usage/Loads/Damage Prognostics	6
	Corrosion Monitoring	5
	Airframe Health & Prognostics	5
Rotors	Smart LRUs: Rod End, Bearing, Damper, Actuated Push Rod	6
	Wireless LRU and Gateway Communications	6
	Blade Impact & Damage Detection	5-6
Electrical	LRU Distributed Signal Processing	6
	LRU Fault Classifier	5-6
	SSTDR Wire Fault Sensing	5-6
	Wiring Constraint-Based Reasoner	6
	Wiring Interactive Troubleshooting	6
VMS	Pump Reservoir Diagnostic and Leak Trending	6
	TR Servo Diagnostics	6
	Pump Diagnostics	5-6

3.7 Literature Gaps

Andrews [26] and Chew et al. [27] provide a survey of methods to calculate a systems reliability and MFOP. Both promoted the advantages of a Petri-net method in a Monte Carlo simulation. Price et al. [28] has provided the most developed state-space Monte Carlo simulation that approximated MFOP, MFOPS, and added MRP. Renewal theory provides some assistance for a single component. Most of published work such as [3] and [45] minimizes repair cost instead of maximizing a MFOP. This work modifies renewal theory to model a multi-part system's downtime under a MFOP constraint. Failure Cause Identification by Trindade and Nathan [22] provided a helpful guide; but a quantifiable method is missing. To the author's knowledge, literature has not addressed the maintenance policies that support a MFOP strategy; therefore, a desired, but unaddressed topic is adaptive maintenance policies. Adaptive policies, which tailor each MRP to meet the availability or MFOPS requirements, is an immature topic.

Table 8: MFOP Literature Gaps

Gap	State-of-the-Art/Current	Proposed Solution
MFOP metrics	Statistical metrics Rm, MTBF, MTTR	MFOP, MFOPS, MPS
Identifying weakest links in system	Cost based, Graphical Failure Cause Analysis	MFOP based, Quantifiable Failure Cause Analysis
Availability Maintenance Policy	Cost based with renewal theory	Time based in a MFOP context through framework
Adaptable Maintenance Policies to Maintain MFOPS	None	Aggressive Lifing Policy through framework
Cost-effective balance	Value Driven Design, Value Based Acquisition (USAF CCA)	Not fully addressed but CCA in a MFOP strategy is promising

4 PROBLEM 1: MEASURE MFOP

A necessary step to developing a framework for rotorcraft MFOP is the modeling of a system's dependability and its measurement using operational metrics. Section 2.3 formulated three questions to find the necessary tools and models that enable MFOP measurement. They are reshown below for the reader's convenience with the associated hypothesis and conjectures that are developed later in this chapter.

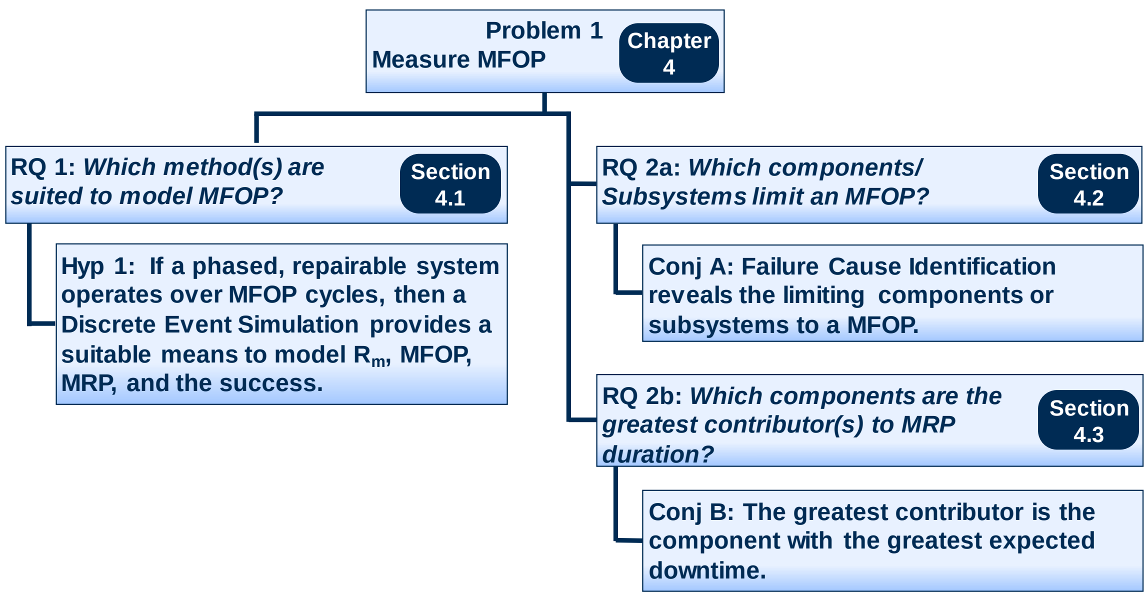


Figure 27: Problem 1 Summary with Hypothesis and Conjectures

This chapter has major subsections dedicated to answer each research question. The first subsection answers Research Question 1 with a validated hypothesis. The next two subsections address the two remaining questions with conjectures. Each of the major subsections give a research question, propose a hypothesis or conjecture, test the proposal by validation or thought experiment, and conclude with a discussion on the results.

4.1 Research Question 1: What Methods Are Suited to Model MFOP

The first research question seeks the calculation of operational metrics like MFOP and MRP as well as the supporting metric of R_m. It asks what method(s) are suited to model MFOP. The research question asks which analytical methods or modeling techniques may predict the MFOP, MRP, and reliability of a system.

4.1.1 Hypothesis 1: Modeling MFOPS with Discrete Event Simulation

4.1.1.1 Selection of a Modeling Technique

Table 9 summarizes the methods and screening criteria discussed in section 3.3. The table compares the discussed modeling methods against the screening criteria (phased mission support, repairable, aging parts, and flexibility). Only non-homogeneous Markov methods and simulation meet three of the screening criteria (phased mission support, repairable, and aging parts) for a supportable MFOP model. Simulation is preferable over non-homogeneous methods because it avoids state space explosion and is more flexible. BDDs and homogeneous Markov models may be the fastest or computationally efficient of the models, but simulation is not far behind using today's computing power. The repairable system screening criteria eliminates both FTAs and BDDs. Homogeneous Markov models are unacceptable because they require unchanging failure rates.

Table 9: Comparison of Modeling Methods

	Fault Tree Analysis	Binary Decision Diagrams	Markov Chain Models		Petri Nets	Customized Discrete Event Sim
			Homog	Non-homog		
Phased missions	✓	✓	✓	✓	✓	✓
Repairable			✓	✓	✓	✓
Aging parts				✓	✓	✓
Flexibility /Ease of Use					?	✓
Dependency	(dynamic only)	✓	✓	✓	✓	✓
Avoids state space explosion	✓	✓			✓	✓
Speed	✓	✓	✓	✓		

Petri nets and Discrete Event Simulations (DES) meet the screening criteria and have the advantages of handling component dependency, modeling flexibility under a variety of architectures and maintenance policies, and avoiding state space explosion. Token locations define the system's state in the Petri net. This graphical depiction is a nice feature for simple models but can become overwhelming when displaying complex systems. The recommended DES runs a state space model like a Petri net, but without the graphical interface. DES requires a considerable bookkeeping effort to record event histories. Both the Petri net and DES approximate the solution of a complex system using a Monte Carlo simulation over many iterations. Speed varies for both simulation models, where system complexity or desire for high precision increase computation time.

Given the availability of increasing computational power, a customized DES was an attractive model technique. This assessment leads to the hypothesis that a discrete event simulation is an appropriate approach to solving for MFOP and MRP.

Research Question 1

What method(s) are suited to model MFOP?

Hypothesis 1: If a phased, repairable system operates over MFOP cycles, then a Discrete Event Simulation provides a suitable means to model Rm, MFOP, MRP, and the success.

The first hypothesis offers that event simulation provides the tools to conduct basic MFOP analysis. Computer simulation easily supports the processing of discrete events necessary to solve the problem. A review of current simulation models informed the modeling technique selected.

4.1.2 Development of a Discrete Event Simulation

4.1.2.1 Review Existing Event Simulation Models

Simulation models have steadily progressed in sophistication over the years. Relf in [24] introduced a MFOP design methodology that was an iterative process to apply MFOP options to reach a desired target (see Figure 9). The Georgia Institute of Technology added to Relf's template in "A State-based System Integrated Sustainment Tool" (ASSIST) [30]. ASSIST accounted for a phased mission, ran a Petri net simulation, and optimized a generic Value Based Acquisition (VBA) function. In a separate project [31], a team improved upon ASSIST to model the UH-60M helicopter. This work created separate FTAs for each aircraft system failure distributions before applying to phase and safety critical events trees (see Figure 28). The most recent work from Price et al [28], added a maintenance manager module that models the downtime in unscheduled repair and scheduled repair (see Figure 29). The figures of the improved ASSIST and Price models are on the next page.

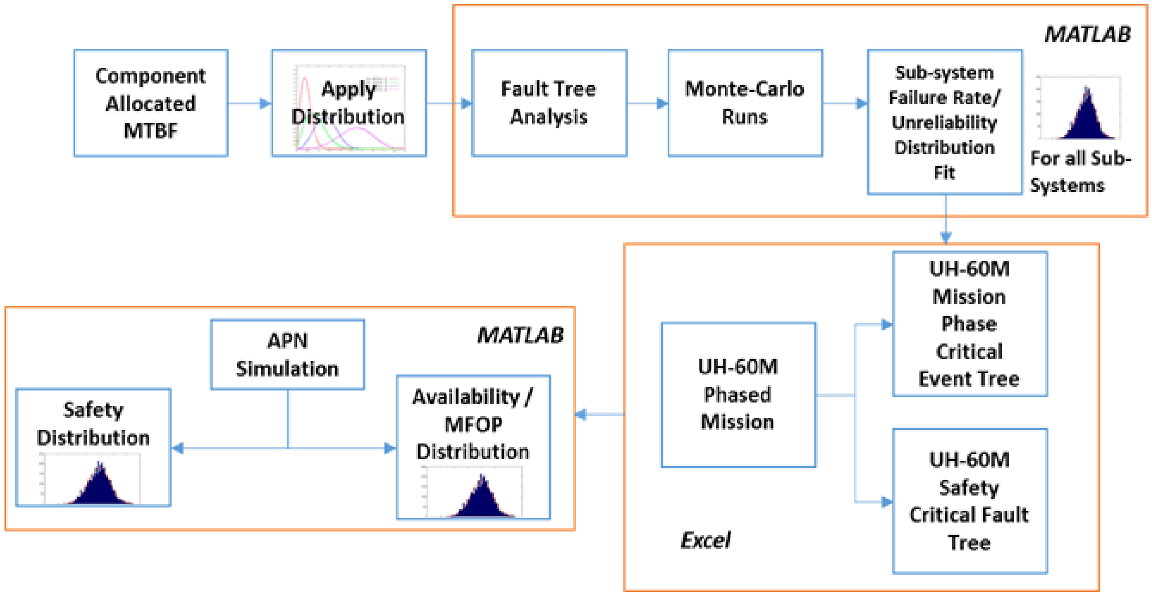


Figure 28: Integrated Petri-Net for Reliability and Safety Analysis [31]

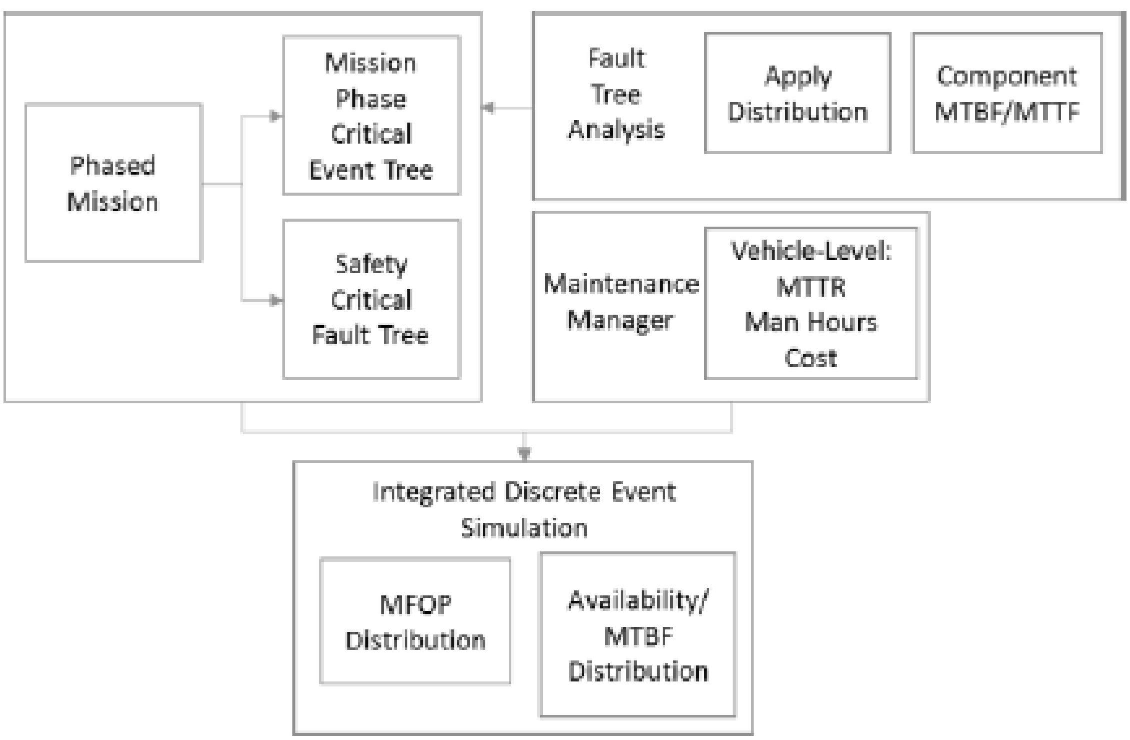


Figure 29: Integrated DES from Price et al. Figure reprinted with permission [28]

4.1.2.2 Shaping a Discrete Event Simulation

A pattern emerges after reviewing the four presented models. Each begins with describing the mission by phase and laying out aircraft systems and component architecture. The models align event trees with the architecture to generate phase specific fault trees. Event trees connect the initiating event failure to possible outcomes [32]. Component failure distributions feed the phased fault trees. A single simulation run progresses through multiple missions and records failure times. The model in Figure 28 runs the Petri net-like simulation on subsystem distributions while Figure 29 deals with components directly.

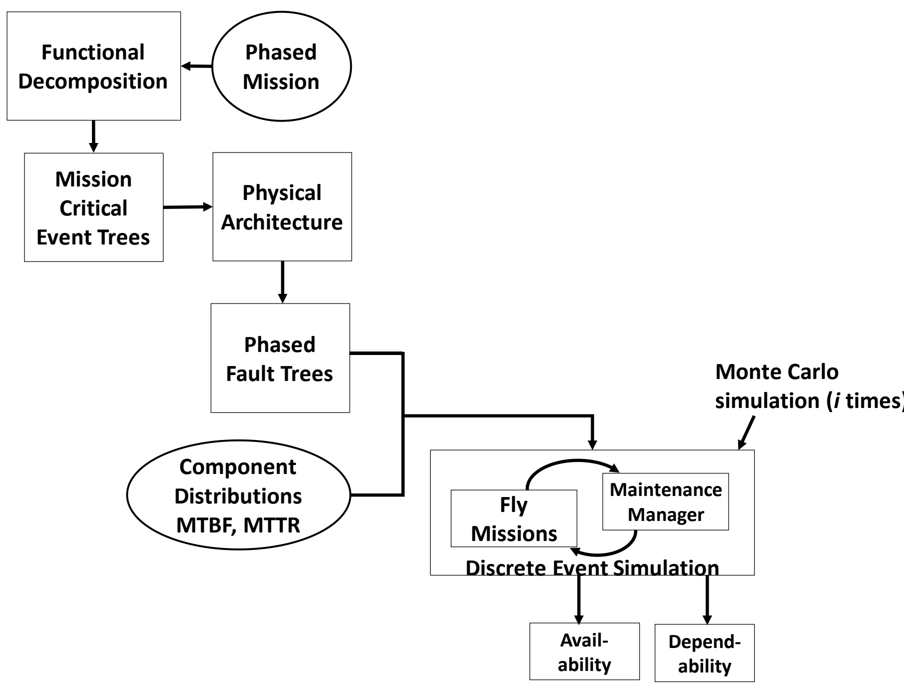
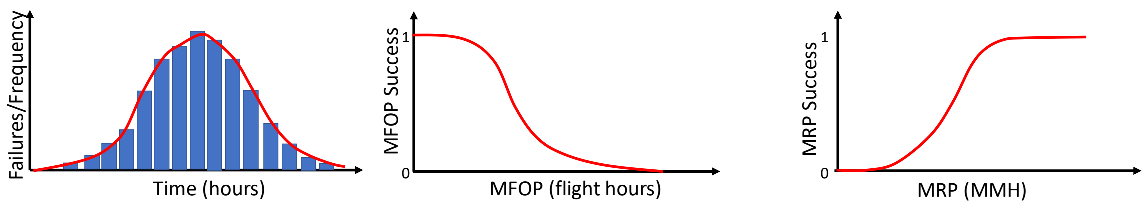


Figure 30: Shape of DES Model

A Monte Carlo simulation repeats each run thousands of times to generate a histogram of failure over time (Figure 31a). With sufficient runs the MFOP distribution takes the shape of a gaussian or normal distribution in accordance with the Central Limit Theorem.

The complement of the cumulative distribution function is the probability the system functions to at least a mission time of t_m and is termed the MFOP Success. A plot over time is in Figure 31b below.



(a) Histogram (b) complementary (inverse) cdf (c) cdf

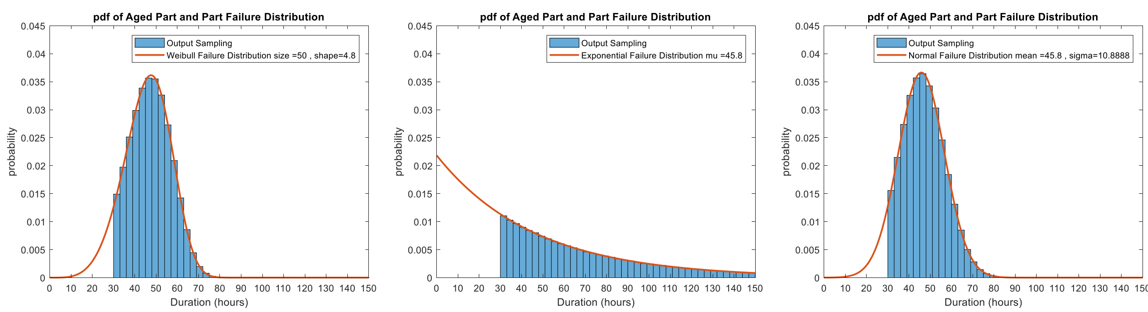
Figure 31: Generating MFOP Success and MRP Success

An important part of this conjecture is the MRP. Just as MFOP has a success rate, MRP should have a success rate. The MRP Success (MRPS) is the probability that the recovery period ends by a given time. MFOP is measured in flight hours while MRP is measured in maintenance man hours (MMH). Both MFOP and MRP are a compilation of component individual failure or repair distributions, respectively. The general structure is in Figure 30. The author conjectures that the MRPS may be found in the same process as the MFOPS by using the cdf of total repair times in Figure 31c. The concept of the Maintenance Manager came from Price et al [28]. A Maintenance Manager follows a policy that determines when to perform maintenance. In the case of a MFOP aircraft, the intent is to perform all maintenance in the MRP.

4.1.2.3 Sampling Part Failure Age

The DES calculates the failure age of each part a priori by sampling from the part's known failure distribution. It then compares the current part age to the known failure age during each phase of flight. If the current part age is greater than or equal to the failure age, than part failure occurs. If the current part age is less than the failure age, then the part is functional throughout the phase. This necessitates the ability to calculate the *mission reliability* (defined in section 3.1.1.2) where parts begin the simulation with a known age.

At the start of the simulation, it is assumed that each part begins in an "up" or fully operational state and that each part has a starting age of $t_{age,0}$. We are seeking to sample from a known failure distribution where the lowest possible outcome of the random variable, T, is the starting age, $t_{age,0}$. Possible outcomes of T must be between $t_{age,0}$ and infinity as shown in Figure 32. The figure shows a $t_{age,0}$ of 30 hours as an example.



(a) Weibull Example (b) Exponential Example (c) Normal Example

Figure 32: pdf of Aged Parts Sampling and Known Failure Distribution

Conditional probability of successful operation through time t+ $t_{age,0}$ given successful operation through $t_{age,0}$ still holds valid as

$$P(TTF \geq t_{age,0} + t \mid TTF \geq t_{age,0}) = \frac{P(TTF \geq t_{age,0} + t)}{P(TFT \geq t_{age,0})} \quad (23)$$

where $t_{age,0}$ is the starting age and *TTF* is the random variable of age to failure. The probability that the part successfully functions in the interval [0, $t_{age,0}$] is known as one because this has already occurred; therefore,

$$P\left(\mathrm{TTF} \geq t_{age,0} + t \mid TTF \geq t_{age,0}\right) = \frac{P\left(TTF \geq t_{age,0} + t\right)}{1}$$

$$= P\left(TTF \geq t_{age,0} + t\right) \qquad (24)$$

Equation (24) is the complement of the failure cumulative distribution function (cdf)

$$P\left(TTF \geq t_{age,0} + t \mid TTF \geq t_{age,0}\right) = 1 - F\left(t_{age,0} + t\right) \qquad (25)$$

The above relationship permits the sampling using an inverse cdf as outlined in the example below.

Consider an example with a part that has a Weibull failure distribution where we wish to sample the random variable *T* from the failure distribution *Weibull(t; η; β)* with a range of [$t_{age,0}$, $+\infty$). The first step is to find the probability that *TTF* is less than $t_{age,0}$ using the Weibull cumulative distribution function of

$$F\left(t_{age,0}; \eta; \beta\right) = P\left(TTF \leq t_{age,0}\right) = 1 - e^{-\left(\frac{t_{age,0}}{\eta}\right)^{\beta}} \qquad (26)$$

where *TTF* is the random variable, η is the distribution's size, and β is the distribution's shape. We need to sample from all possible *t*'s such that

$$F\left(\mathrm{t}_{age,0}\right) < F\left(\mathrm{t}_{age,0} + \mathrm{t}\right) \leq 1 \qquad (27)$$

The above equation is a statement on acceptable percentiles where the minimum is found with (26) and the maximum is 100%. There is no preference for *t* if it is positive; therefore, the percentile follows a uniform distribution *U(0,1)* in

$$pr = F(t_{age,0}) + [1 - F(t_{age,0})] * U(0,1) \qquad (28)$$

where *pr* is the percentile. The quantile function (inverse cdf) may be solved for the random variable *t* using (26) where *F* is equal to *pr*. In the case of Weibull distribution, solving for $t_{age,0}$ from (26) yields the quantile function as

$$t_{age,sample} = \eta[-\ln(1 - pr)]^{1/\beta} \qquad (29)$$

where *pr* is from (28).

Equivalent equations for the exponential and normal (Gaussian) distribution are in Table 10. Example results of the samplings are in Figure 32 above. The figure shows a histogram of samplings from: (a) Weibull distribution; (b) exponential distribution; and (c) normal distribution. The histogram is normalized into a pdf. The mean of each distribution is 45.8 hours and the starting age, $t_{age,0}$, is 30 hours. The histograms show consistency to the known distribution plot (shown as a solid, red line).

Table 10: Finding A Randomly Distributed Failure Age Given a Starting Age

Failure Distribution Type	Equations	
Weibull η is size β is shape	$F = 1 - e^{-\left(\frac{t_{age,0}}{\eta}\right)^{\beta}}$ $pr = F + (1 - F) * U(0,1)$ $t_{age,sample} = \eta[-\ln(1 - pr)]^{1/\beta}$	(26) (28) (29)
Exponential μ is mean or 1/λ	$F = 1 - e^{-\left(\frac{t_{age,0}}{\mu}\right)}$ $pr = F + (1 - F) * U(0,1)$ $t_{age,sample} = \mu[-\ln(1 - pr)]$	(30) (28) (31)

Table 10 (continued)

Failure Distribution Type	Equations	
Normal μ is mean σ is std deviation	$F = \frac{1}{2}\left[1 + erf\left(\frac{t_{age,0} - \mu}{\sigma\sqrt{2}}\right)\right]$	(32)
	$pr = F + (1 - F) * U(0,1)$	(28)
	$t_{age,sample} = \mu + \sigma\sqrt{2}erf^{-1}(2pr - 1)$	(33)

4.1.3 Experiment Plan

Table 9 summarized a qualitative appraisal of state-of-the-art methods. Verifying the DES against known solutions shall support substantiation of the hypothesis. The simulation's MFOP and unreliability results shall be compared to solutions provided from Chew et al. [27] for a phased mission with a simple non-repairable system and a repairable system. Chew et. al [27] calculated an analytical solution from unreliability cut-sets of the simple non-repairable system. The article then calculated the solution to the repairable system using an approximate Markov chain and a Petri net in a Monte Carlo simulation. Results from this work's customized DES was compared against the solutions from Chew et. al for the non-repairable and repairable systems. Finally, DES results to the repairable system were verified against this author's own Markov model that provided an exact solution.

A set of four tests verified the DES before experiments in Problem 2 began. The verification process progressively builds from a simple, single-part test case to a more complicated, multi-phased, multi-part, repairable system. The build-up verifies that the DES successfully handles:

a. varying failure distributions

b. multiple parts

c. a mix of serial and parallel fault tree logic

d. multiple phases with different fault trees

e. non-repairable and repairable systems

f. multiple missions in a MFOP cycle

g. single or multiple MFOPs

Table 11 outlines the progression of test cases for verification.

Table 11: Verification Test Cases

Test Case	Parts	Repairable	Phases	MFOPs	Verification Source
Test Case #1	1	Non-repairable	1	1	Known distributions
Test Case #2a (serial)	11	Non-repairable	1	19	Engine by Kumar [67]
Test Case #2b (parallel)	11	Non-repairable	1	19	Author's analytical solution
Test Case #3	4	Non-repairable	4	1	Chew et al. [26], Author's Markov model
Test Case #4	4	Repairable	4	12	Chew et al. [26], Author's Markov model

The verification sources listed above provide either an analytical answer or different modeling method to compare the simulation's results. Increasing the number of Monte Carlo iterations of the simulation will improve convergence to the true solution and reduce the error; however, the user must balance the desired precision with limiting factors such as computing power and time. For the purposes of model verification, less than one percent error is a sufficient verification each test case. Each test case also examines issues related to convergence.

4.1.4 Verification of the DES

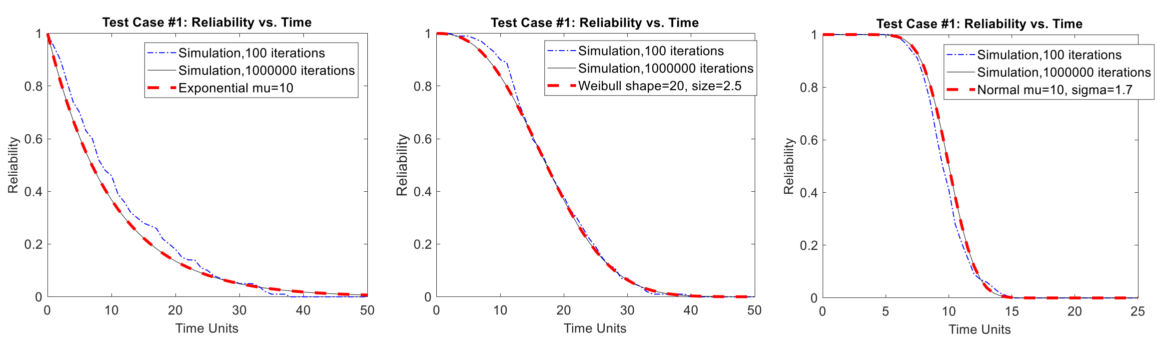
4.1.4.1 Test Case #1: Single Part Distributions

The first test case examines a single-part, single-phase, non-repairable system. This is a relatively simple test where the reliability of three different types of failure distributions are checked against the known analytical answer. Knowing that the system reliability is the complement of the failure distribution's cdf, $R(t) = 1 - F(t)$, we can anticipate the shape of the simulation's output if it is correct. The parameters of the failure distributions are in Table 12. For this test case, the starting part age was zero. The simulation operated the part until failure or until reaching 50 missions. Phase duration was set such that the 50 missions brought the ending system reliability close to zero.

Table 12: Text Case #1 Summary Data

	Exponential	Weibull	Normal
Parameters	λ = 0.1	η = 20 β = 2.5	μ = 10 σ = 1.7
Phase Duration	1 hour	1 hour	0.5 hour
DES Reliability (Known Reliability) @ midpoint	0.081974 (0.082085) t = 25	0.17384 (0.17431) t = 25	0.071266 (0.070701) t = 12.5
Reliability Mean Square Error @ 1e6 iterations	0.0011 from t = 0 to 50	0.0034 from t = 0 to 50	0.0020 from t = 0 to 25

For each distribution, data was collected for a smaller and larger number of iterations (100 and one million iterations, respectively). System reliability of the data sets is in Figure 33 from start through 50 missions. Even at a small number of iterations, the data follows the known reliability shape. At one million iterations, the simulation's curve is indistinguishable from known reliability function. The Mean Square Error (MSE) of system reliability for the 50 missions as shown above and the confirms the plots and success of Test Case #1.



(a) Exponential (b) Weibull (c) Normal

Figure 33: Test Case #1 Reliability Plots

4.1.4.2 Test Case #2: Engine Example

4.1.4.2.1 Test Case #2a: Series System

The second test case is a reliability problem written by U. Dinesh Kumar [33]. The problem assembles eleven parts in series to model a turbine engine's mission reliability (see Figure 34). Weibull failure distributions model the engine components with parameters listed in Table 13. The use of the Weibull distribution accounts for an increasing hazard rate as the part ages.

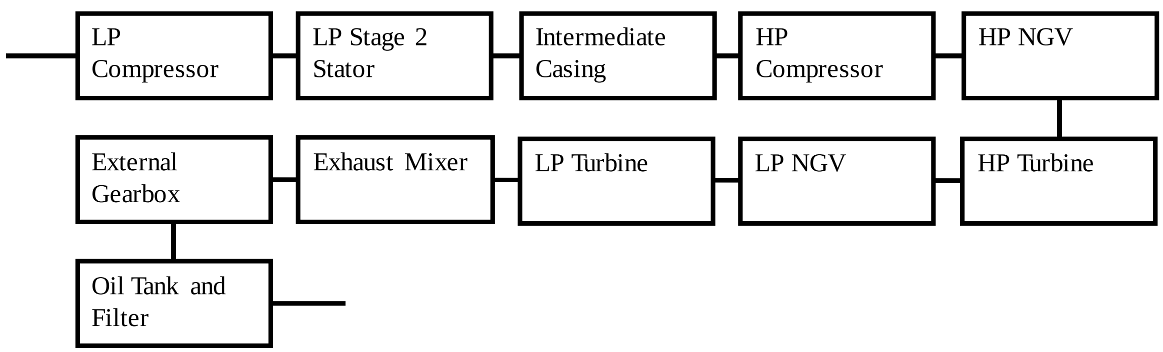


Figure 34: Reliability Block Diagram from Kumar Engine Example [33]

Table 13: Component Failure Distributions from Kumar Engine Example [33]

Item No.	Item	Distribution	Parameter Values
01	LP Compressor	Weibull	η = 15,000, β = 3
02	LP Stage 2 Stator	Weibull	η = 5,000, β = 2.8
03	Intermediate Casing	Weibull	η = 11,000, β = 3
04	HP Compressor	Weibull	η = 12,000, β = 3.5
05	HP NGV	Weibull	η = 8,000, β = 3

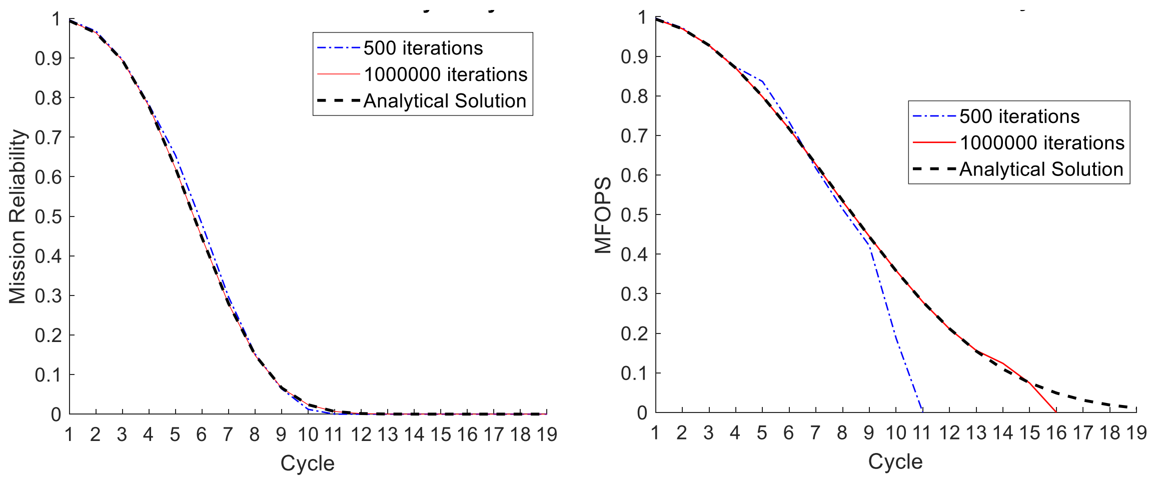
Table 13 (continued)

Item No.	Item	Distribution	Parameter Values
06	HP Turbine	Weibull	η = 25,000, β = 4
07	LP NGV	Weibull	η = 7,000, β = 2.2
08	LP Turbine	Weibull	η = 20,000, β = 2.8
09	Exhaust Mixer	Weibull	η = 7,000, β = 3
10	External Gearbox	Weibull	η = 6,500, β = 3
11	Oil Tank and Filler	Weibull	η = 5,000, β = 3.8

The reliability through time t of components in series is the product of each component's reliability through time t. Time t can be measured as discrete MFOP cycles where t is the product of the *ith* cycle and MFOP duration, t_{mf}. The expression is

$$R(i) = \prod_{k=1}^{n} R_k\left(i \times t_{mf}\right) \tag{34}$$

It is little effort to analytically calculate the system reliability using (34) at each MFOP cycle where the t_{mf} is 500 hours. The system reliability is equivalent to mission reliability for the single phased mission. Plots of Mission reliability for each cycle are in Figure 35(a) for simulations with 500 and one million iterations. Even at a relatively small number of iterations, mission reliability achieves a representative shape of the analytical solution. The Mean Square Error is 0.0527 and 0.0019 for the 500 and one million iterations, respectively.



(a) Mission Reliability (b) MFOPS

Figure 35: Test Case #2a Mission Reliability and MFOPS

Figure 35(b) compares the DES MFOPS results for 500 and one million simulations against the analytical solution. Recall MFOPS is calculated from (9) as a conditional probability and is repeated for the reader's convenience as

$$MFOPS(t_{mf}, i) = \prod_{r=1}^{n} \frac{R_r(i \times t_{mf})}{R_r([i-1] \times t_{mf})} \quad (35)$$

where *r* is the component in a serial system. The simulation with 500 iterations suffers from low surviving iterations and departs from the analytical solution at 11 cycles. One million iterations delay the departure from the analytical solution until the 16th cycle. The Mean Square Error for the MFOPS is 0.3316 through 11 cycles for 500-iterations and 0.0511 through 16 cycles for one million-iterations.

The successful approximation of mission reliability and MFOPS (with sufficient iterations) verifies the DES correctly handles multiple parts and serial logic in a phase, over multiple MFOP cycles. Finally, it is interesting to note that the Mean Square Error for mission reliability with 500 iterations (0.0527) is close to the Mean Square Error for

MFOPS at one million iterations (0.0511). This introduces a recurring phenomenon that MFOPS requires greater iterations than mission reliability due to the nature of conditional probability.

4.1.4.2.2 Test Case#2b: Parallel System

Test Case #2b ordered the same eleven components from Test Case #2a into a parallel configuration (Figure 36) to verify the DES' handling of parallel logic.

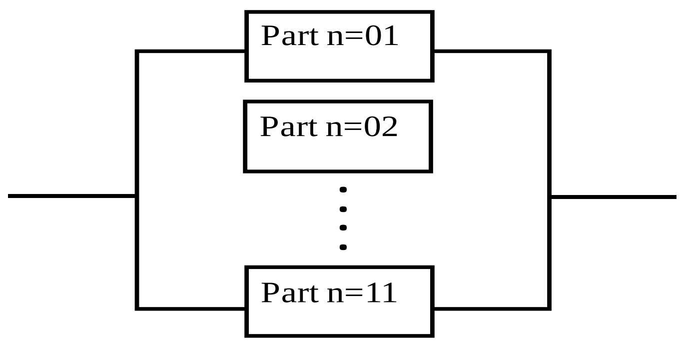
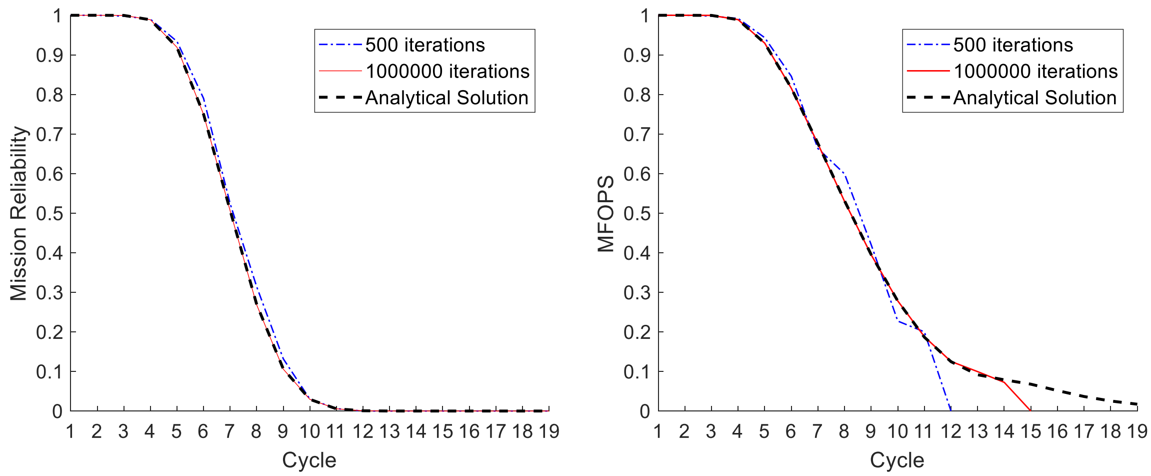


Figure 36: Reliability Block Diagram for Test Case of a Parallel System [33]

The reliability of a parallel system that measures time *t* in *i* discrete intervals of the MFOP, t_{mf}, is written as

$$R(i) = 1 - \prod_{k=1}^{n}\left[1 - R_k\left(i \times t_{mf}\right)\right] \qquad (36)$$

Mission reliability, *MR(i)*, is from (36) by substituting MR_k for R_k. DES output is compared against the analytical solution for system reliability using (36) at each MFOP cycle of 3,500 hours. The MFOP duration as extended from 500 to 3,500 hours to induce sufficient failures in the more reliable parallel system to show the degeneration of reliability over 19 cycles. Figure 37 plots mission reliability at each cycle for 500 and one million iterations. Even at a relatively small number of iterations, mission reliability achieves a representative shape of the analytical solution. The test has similar convergence as in Teste Case #2a. The Mean Square Error of reliability is 0.0689 and 0.0018 for the 500 and one million iterations, respectively.



(a) Mission Reliability (b) MFOPS

Figure 37: Test Case #2b Mission Reliability and MFOPS

MFOPS was calculated from (35) where the MSE was found to be 0.1575 for 500 iterations through twelve cycles. After eleven cycles, only 66 iterations survived for the eleventh cycle and only 15 iterations survived for the twelfth cycle. Calculating MFOPS beyond this point becomes ineffective. This illustrates the weakness of simulation in dealing with low probability events. Increasing the number of starting iterations to ensure a larger surviving set overcomes the weakness. Using one million iterations, the MSE was lowered to 0.0686 and results were useable through 15 cycles. These results lead to the conclusion that the output of the DES adheres to the analytical solution under the stipulation that the measured cycle has sufficient surviving iterations. Test Case #2b successfully verifies the simulation of parallel configurations.

4.1.4.3 Test Case #3: Non-repairable System

Test Case #3 increases the system complexity with a four-phase, four-part system as introduced by Chew et al. [27]. The Test Case is a notional system designed such that each phase has a different fault tree (see Figure 38) and duration (see Table 14). Component failures are all exponential with failures occurring at the rates shown below. Four phases comprise a single mission. Mission essential components vary according to a phase as illustrated in the fault trees. The system undergoes a single MFOP cycle comprised of three missions in the cycle. All components age during each phase and may fail despite not appearing in a phase's fault tree. The system is non-repairable.

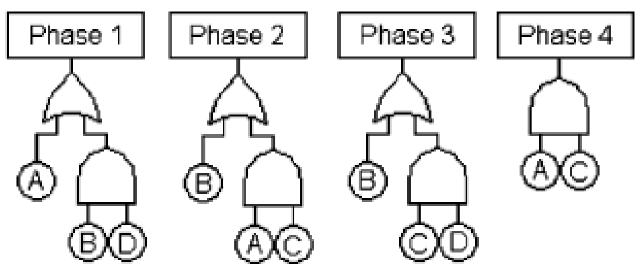


Figure 38: Test Case #3 System Phase Fault Trees. Figure reprinted with permission[2]

Table 14: Test Case #3 Phase Durations and Component Failure Rates

Phase	Phase Duration (h)	Component	Failure Rate λ (h^{-1})
1	0.5	A	0.0045
2	2.5	B	0.0130
3	4.0	C	0.0081
4	1.25	D	0.0011

[2] Reprinted from *Reliability Engineering & System Safety*, vol. 93, S. P. Chew, S.J. Dunnett and J. D. Andrews, "Phased mission modelling of systems with maintenance-free operating periods using simulated Petri nets," pp. 980-994, Copyright (2008), with permission from Elsevier

4.1.4.3.1 Theoretical Solution

The theoretical solution combines the 4-phase, 3-mission cycle into a single, 12-phase mission (i.e. phases 1, 5, and 9 are the same). The combination creates a simple framework for the use of non-coherent fault trees to calculate phase unreliability. The inclusion of NOT gates in a fault tree makes the tree non-coherent. Chew et al. [27] show that the use of non-coherent fault trees is necessary to calculate the unreliability of phase *p* because it requires that phases 1 to 1-*p* to not have failed. They transform any non-coherent fault trees into coherent fault trees using De Morgan's laws shown as

$$(\bar{A} + \bar{B}) = \bar{A} \cdot \bar{B}, \quad (\overline{A \cdot B}) = \bar{A} + \bar{B} \qquad (37)$$

This creates a set of prime implicants (or cut sets) that are all the set of possible causes of phase failure [59]. The inclusion-exclusion principal from Inagaki and Henley [60] using notation from Chew et al. [27] provides the expression for unreliability as

$$Q_i = \sum_{j=1}^{N_i} Pr(C_j) - \sum_{j=1}^{N_i} \sum_{k=1}^{j-1} Pr(C_j \cap C_k) + \cdots + (-1)^{N_i - 1} \times Pr(C_1 \cap C_1 \cap \cdots \cap C_{N_i}) \qquad (38)$$

where *Pr* is probability, C_j is the *jth* prime implicant, and N_i is the number of prime implicants in phase *i*. Chew et al. [27] takes the reader through a sample calculation and provides the theoretical solution used in the Test Case (see Table 15).

Table 15: Test Case #3 Theoretical Solution

Phase i	Theoretical Phase Unreliability, Q_i	Theoretical System Reliability, $R_{1,i}$	Phase i	Theoretical Phase Unreliability, Q_i	Theoretical System Reliability, $R_{1,i}$
1	0.00225	0.9977	7	0.05210	0.7862
2	0.03850	0.9593	8	0.00327	0.7836
3	0.05107	0.9103	9	0.03271	0.7580
4	0.00194	0.9086	10	0.05493	0.7164
5	0.03452	0.8772	11	0.05302	0.6784
6	0.05448	0.8294	12	0.00451	0.6753

Chew et al. remarked, "It is not possible to find the mission reliability by simply multiplying the phase reliabilities, due to the statistically dependent nature of the phases" [27]. The dependency of the current phase on the previous phases manifests in the first fault tree found in Phases 1, 5, and 9. Dependency's influence occurs in Phases 1, 5, and 9 when the phase unreliability rises from 0.00225 to 0.03452 then falls slightly to 0.03271. System reliability shows a consistent decline expected from a system with constant failure rates. Phase pairs 3-4, 7-8, and 11-12 show a very small decline due to the low unreliability of Phase 4 fault tree.

4.1.4.3.2 Markov Chain Model

A Markov Chain model verifies the lengthy theoretical solution for Test Case #3 and provides the means to calculate the solution for the next test case. The presented system supports a Markov analysis because the system is homogeneous due to the four components possessing an exponential failure distribution (see section 3.3.3 Markov Chains). The system is binary in operation where a component is either working (*W*) or failed (*F*). With n binary components, there exist 2^n possible states. The four components ($n = 4$) in the test case's system have a possible 2^4, or sixteen possible states. Figure 39 defines the sixteen possible states in terms of components *A*, *B*, *C*, and *D* as working or failed.

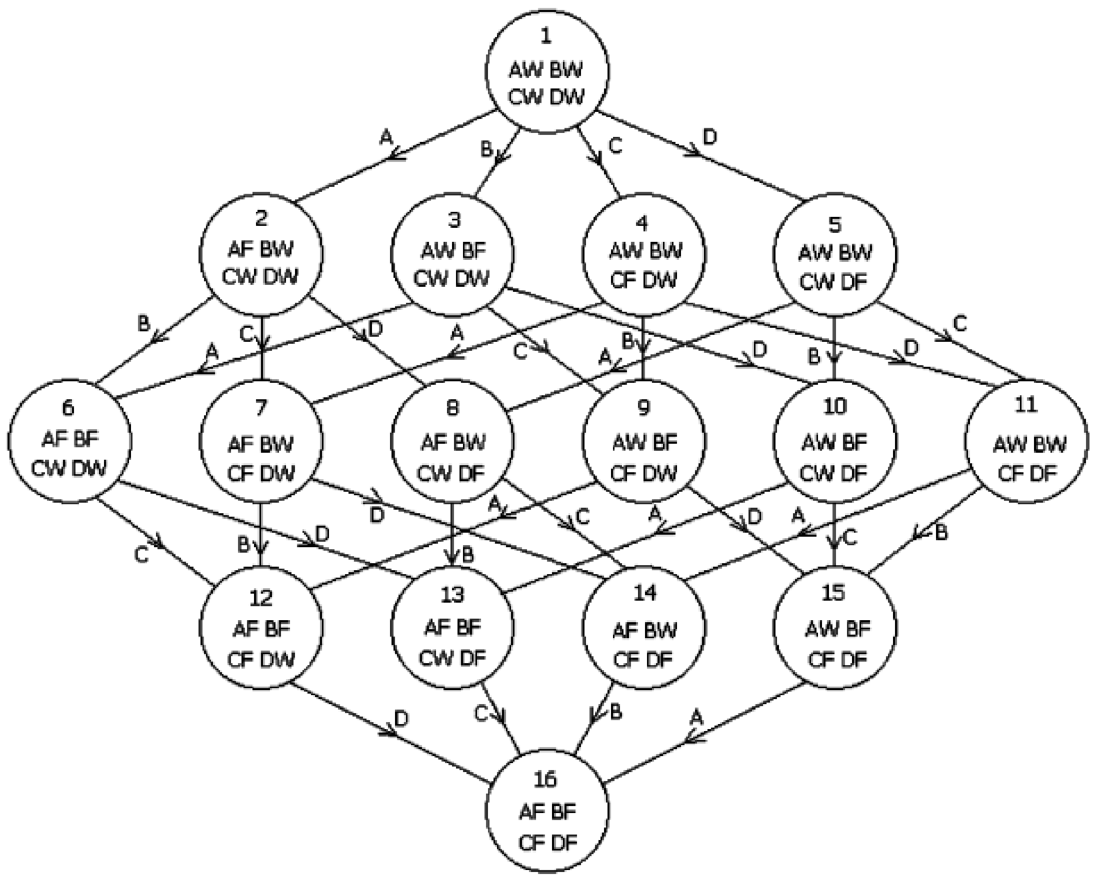


Figure 39: Markov Model of 4-Phase Test Case Figure reprinted with permission[3]

Equation (15) models the progression from one state to the next. Equation (39) defines the steps as phase changes from the current phase, p, to the next phase, $p+1$ as

[3] Reprinted from *Reliability Engineering & System Safety*, vol. 93, S. P. Chew, S.J. Dunnett and J. D. Andrews, "Phased mission modelling of systems with maintenance-free operating periods using simulated Petri nets," pp. 980-994, Copyright (2008), with permission from Elsevier.

$$\boldsymbol{s}^{(p+1)} = \boldsymbol{s}^{(p)}.\boldsymbol{A} \qquad (39)$$

where $\boldsymbol{s}$ is the state space vector and $\boldsymbol{A}$ is the transition matrix. The system starts in a fully working state, $\boldsymbol{s}^{(0)}$, with the first element of $\boldsymbol{s}$ to have a probability of one and all other elements to be zero. The element A_{ij} is the probability of transition from the current state i to the new state j. The 16 x 16 transition matrix, $\boldsymbol{A}$, changes by phase because the probability of component failure is dependent upon the different phase durations. Step (or phase change) must be calculated one step at a time because of the changing transition matrix. For the non-repairable system, the transition matrix is written in a more compact notation as

New State, *j*

Current State, *i*

$$A = \begin{array}{c} \\ 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15\\16 \end{array} \begin{array}{c} \begin{array}{cccccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \end{array} \\ \begin{bmatrix}
\overline{ABCD} & A\overline{BCD} & \bar{A}B\overline{CD} & \overline{AB}C\bar{D} & \overline{ABC}D & AB\overline{CD} & A\bar{B}C\bar{D} & A\overline{BC}D & \bar{A}BC\bar{D} & \bar{A}BC\bar{D} & \bar{A}B\bar{C}D & ABC\bar{D} & AB\bar{C}D & A\bar{B}CD & \bar{A}BCD & ABCD \\
0 & \overline{BCD} & 0 & 0 & 0 & B\overline{CD} & \bar{B}C\bar{D} & \overline{BC}D & 0 & 0 & 0 & BC\bar{D} & B\bar{C}D & \bar{B}CD & 0 & BCD \\
0 & 0 & \overline{ACD} & 0 & 0 & A\overline{CD} & 0 & 0 & \bar{A}C\bar{D} & \overline{AC}D & 0 & AC\bar{D} & A\bar{C}D & 0 & \bar{A}CD & ACD \\
0 & 0 & 0 & \overline{ABD} & 0 & 0 & A\overline{BD} & 0 & \bar{A}B\bar{D} & 0 & \overline{AB}D & AB\bar{D} & 0 & A\bar{B}D & \bar{A}BD & ABD \\
0 & 0 & 0 & 0 & \overline{ABC} & 0 & 0 & A\overline{BC} & 0 & \bar{A}B\bar{C} & \overline{AB}C & 0 & AB\bar{C} & A\bar{B}C & \bar{A}BC & ABC \\
0 & 0 & 0 & 0 & 0 & \overline{CD} & 0 & 0 & 0 & 0 & 0 & C\bar{D} & \bar{C}D & 0 & 0 & CD \\
0 & 0 & 0 & 0 & 0 & 0 & \overline{BD} & 0 & 0 & 0 & 0 & B\bar{D} & 0 & \bar{B}D & 0 & BD \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{BC} & 0 & 0 & 0 & 0 & B\bar{C} & \bar{B}C & 0 & BC \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{AD} & 0 & 0 & A\bar{D} & 0 & 0 & \bar{A}D & AD \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{AC} & 0 & 0 & A\bar{C} & 0 & \bar{A}C & AC \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{AB} & 0 & 0 & A\bar{B} & \bar{A}B & AB \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{D} & 0 & 0 & 0 & D \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{C} & 0 & 0 & C \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{B} & 0 & B \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{A} & A \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \end{array} \qquad (40)$$

where Component B has a probability of failure of B and component non-failure is $\bar{B}$. The transition matrix is an upper triangular and $A_{16,16}$ is always one because the system is non-repairable.

Table 16 shows the Markov Chain results for the non-repairable system next to Chew et al.'s theoretical solution. Listed are the unreliability, Q, in each phase and the system reliability, *R* from start to phase *p*. System reliability is used instead of mission reliability for the test case because components have no age at the start of system operation. The Markov results match Chew et al.'s theoretical solution with the only difference being the number of digits expressed. This validates the Markov model and confirms the solution.

Table 16: Markov Chain Results and Chew et al. Theoretical Solution

Phase	Q_Markov	Q_Chew	R_Markov	R_Chew
1	0.002251	0.00225	0.99775	0.9977
2	0.038504	0.0385	0.95933	0.9593
3	0.051071	0.05107	0.91034	0.9103
4	0.0019358	0.00194	0.90858	0.9086
5	0.034517	0.03452	0.87721	0.8772
6	0.054475	0.05448	0.82943	0.8294
7	0.052096	0.0521	0.78622	0.7862
8	0.0032729	0.00327	0.78365	0.7836
9	0.032713	0.03271	0.75801	0.758
10	0.054927	0.05493	0.71637	0.7164
11	0.053024	0.05302	0.67839	0.6784
12	0.0045089	0.00451	0.67533	0.6753

4.1.4.3.3 Simulation Verification

The DES simulated the non-repairable Chew system of Test Case #3 with 10 million iterations to gain sufficient precision of the known solution presented above. Figure 40 and Figure 41 on the next page plot the simulation's results for phase unreliability and system reliability, respectfully.

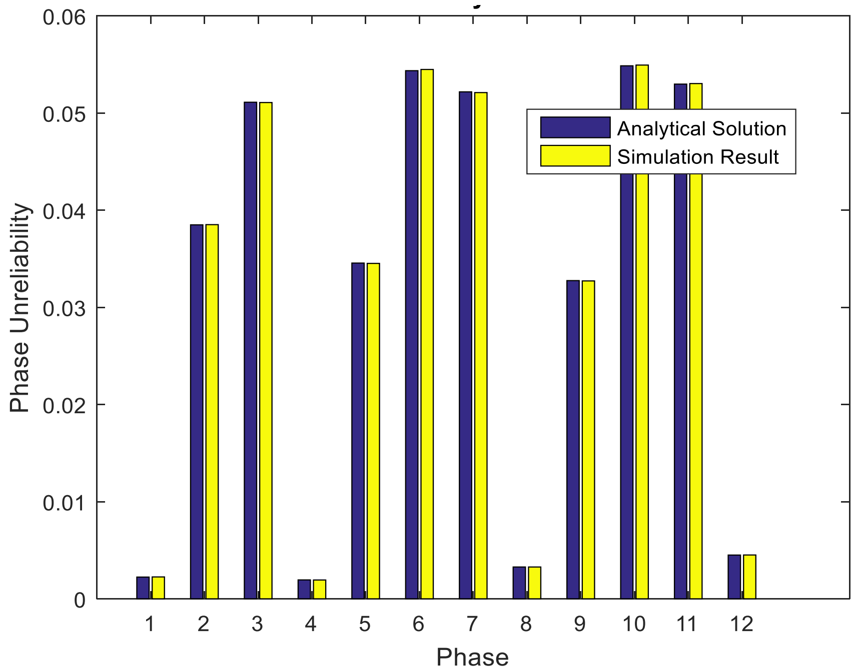


Figure 40: Test Case #3 Phase Unreliability Results

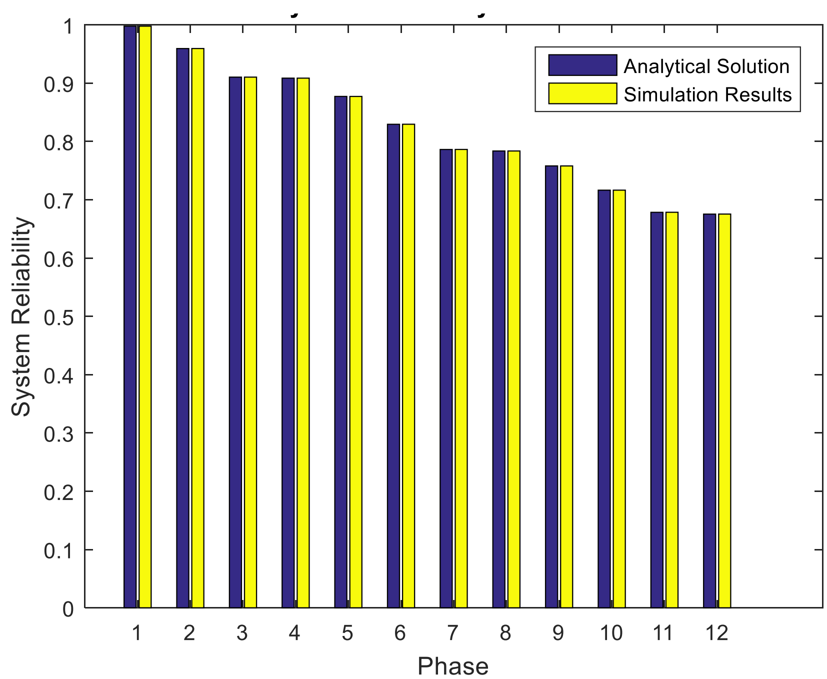


Figure 41: Test Case #3 System Reliability Results

The simulation performed well with a phase unreliability Mean Square Error (MSE) of 0.00020016 and a system reliability MSE of 0.00014546. Percent error for each phase was below the desired 1% as shown in Table 17. Multiple simulations showed no systemic trend in the results. Error in phase unreliability was randomly positive or negative.

Table 17: Test Case #3 Comparison of Analytical and Simulation Unreliability

Phase	1	2	3	4	5	6
Markov	0.002251	0.038504	0.051071	0.001936	0.034517	0.054475
Simulation	0.002270	0.038475	0.051049	0.001947	0.034529	0.054549
% Error	0.843%	-0.075%	-0.043	0.583%	0.034%	0.136%

Phase	7	8	9	10	11	12
Markov	0.052096	0.003273	0.032713	0.054927	0.053024	0.004509
Simulation	0.051940	0.003294	0.032784	0.054970	0.053046	0.004535
% Error	-0.299%	0.655%	0.216%	0.078%	0.040%	0.580%

The system converged on the known phase unreliability with increasing iterations. The next page shows the convergence of each phase in increments of 40,000 iterations to 10 million iterations. All phases showed convergence toward the known solution with Phase 7 (Phase 3, Mission 2) taking the longest. Phase 7's slow convergence is attributed to randomness because: (1) Phase 7 percent error of -0.299% was well under 1% target and; (2) the repetition of the same fault tree in Phase 3 and Phase 11 showed good convergence with low percent errors (-0.043% and 0.040%, respectively).

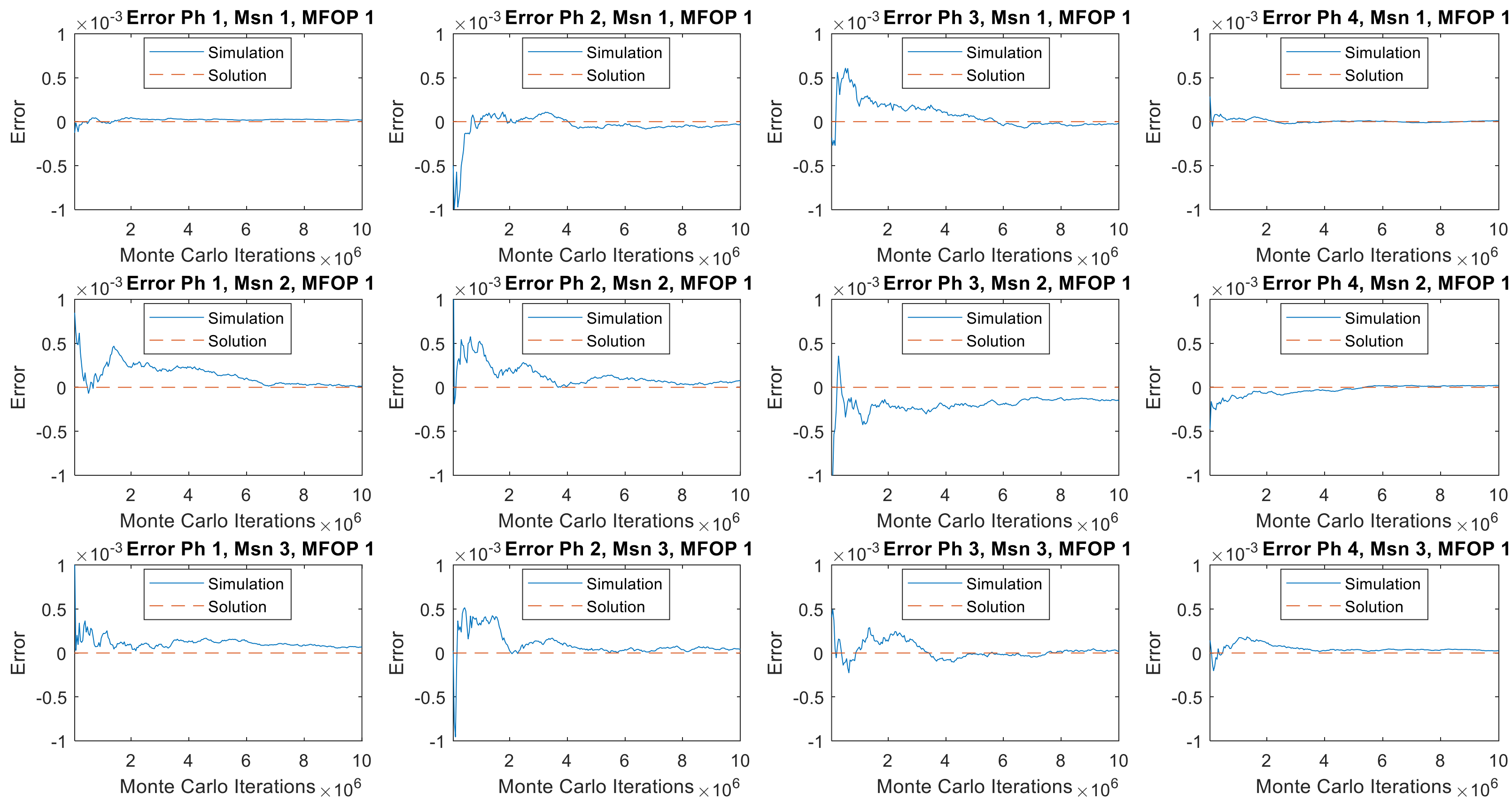


Figure 42: Test Case #3 Phase Unreliability Error

As expected, lower probability events found in the fault trees of Phases 1-5-9 and Phases 4-8-12 had greater percent errors than the higher probability of the other phases. More iterations are needed to gain precision in lower probability events. In summary, the DES successfully handled the phased-mission, non-repairable system with the approximation of phase unreliability and system reliability converging to the known solution.

4.1.4.4 Test Case #4: Repairable System

Test Case #4 is same system of the Test Case #3 with repairs following a maintenance policy. A policy makes repairs to Components A and B at the end of each MFOP cycle. The policy makes repairs to Components C and D every third MFOP cycle. Repairs renew the part to its starting age of zero with no wear. After completing MFOP cycles 3, 6, 9, and 12 the system is reset to its starting condition. MFOP cycles 1, 4, 7, and 10 begin in the starting condition. Mission Reliability is appropriate because unrepaired parts are accumulating age and the system always begins MFOP cycles 2, 3, 5, 6, 8, 9, 11, and 12 with aged parts.

4.1.4.4.1 Repairable Markov Chain Model

Use of a Markov chain model is a means to verify the simulation because it produces an analytical solution to the repairable system. The memoryless property of the system is essential to the Markov model and allows component failure to be dependent upon phase duration, but independent of part age or history. Should any of the components have a failure distribution other than exponential, then homogeneous Markov modeling cannot be used.

Repairs made every cycle partially restore the system. Repairing only Components A and B (as done after cycles 1, 2, 4, 5, 7, 8, 10, 11) may leave the system in states 1, 4, 5, or 11. Repairs made every third MFOP cycle bring the system back to its original state, state 1. Repairs may be approximated by the insertion of zero duration repair phase after the completion of the cycle and before the start of the next cycle. The repair transition matrix, ***A***, is lower triangular. Repairs are perfect with a probability of one. The transition matrix for full repairs every third cycle are shown Figure 43(a) and the matrix for partial repair is shown in Figure 43(b).

$A =$ (rows: Current State, *i*; columns: New State, *j*)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(a) Full Repair

$A =$ (rows: Current State, *i*; columns: New State, *j*)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
12	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

(b) Partial Repair

Figure 43: Repair Transition Matrixes

A mathematically equivalent approach to a repair phase is by altering the state vectors, $\mathbf{s}^{(p)}$ where p is the final state (after phase 4, mission 3) of a cycle. In either approach, a repaired system will have probabilities in states 1, 4, 5, and 11. Table 18 compares results with the solution presented in Chew et al. [27]. The Mean Square Error of Reliability is 0.00025 and MFOPS is 0.00038.

Table 18: Repairable Markov Chain Results and Chew et al. Solution

MFOP_Cycle	Markov_R	Chew_R	Markov_MFOPS	Chew_MFOPS
0	1	1	NaN	NaN
1	0.67533	0.67527	0.67533	0.67527
2	0.45035	0.4502	0.66686	0.6667
3	0.2975	0.2974	0.66059	0.6606
4	0.20091	0.2008	0.67533	0.67518
5	0.13398	0.1339	0.66686	0.66683
6	0.088504	0.08843	0.66059	0.66042
7	0.059769	0.05971	0.67533	0.67522
8	0.039857	0.03982	0.66686	0.66689
9	0.026329	0.0263	0.66059	0.66047
10	0.017781	0.01776	0.67533	0.67529
11	0.011857	0.01184	0.66686	0.66667
12	0.0078329	0.007821	0.66059	0.66056

There exists a slight decay in MFOPS from the first MFOPS cycle (0.67533) to the second MFOPS cycle (0.66686) to the third MFOP cycle (0.66059). The decay is due to the demand to operate Components C and D longer without failure (24.75 to 49.5 to 74.25 hours, respectively). The system is fully restored with full repairs every third cycle and the decay begins again (see Figure 44). The results presented in [27], show small variation between the cycles 1, 4, 7, 10 and so on. The variation may be due to the numerical integration or the precision used by the authors. The analytical solution from this work's Markov model shows no variation with exact repetition of MFOPS.

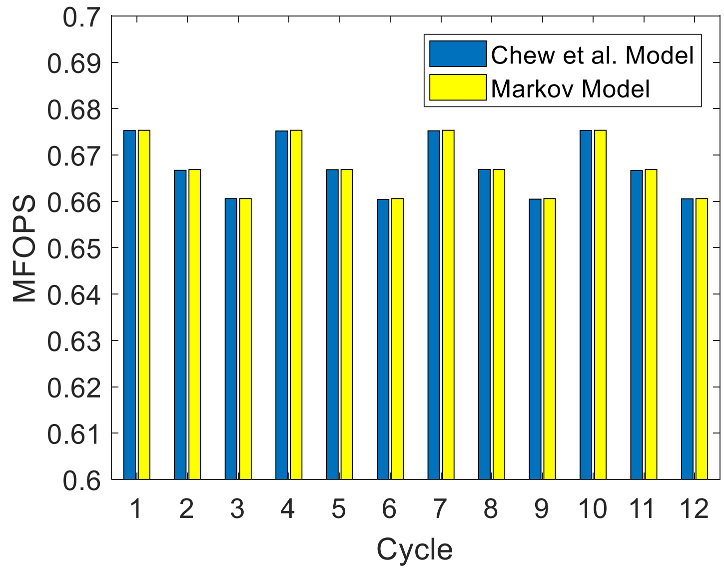


Figure 44: Markov Model Comparison

4.1.4.4.2 Simulation Verification

The simulation of twelve cycles was run one million times with Reliability and MFOPS shown in Figure 45and Figure 46 on page 114. The shape of the curves found in the figures is expected from an exponential system. The Reliability curve follows the complement of an exponential cdf. The MFOPS curve shown in Figure 46 is the complement of the system's hazard curve. The hazard curve of an exponential distribution has zero slope due to the memoryless property; therefore, the MFOPS, which is merely the complement of the hazard curve, is nearly flat. The repair policy manifests the same three-cycle decay as predicted in the analytical solution. MFOP Cycle Reliability Mean Square Error (MSE) was 0.00103 and Cycle Unreliability was 0.00062. Multiple simulations showed no systemic error or trends in the results.

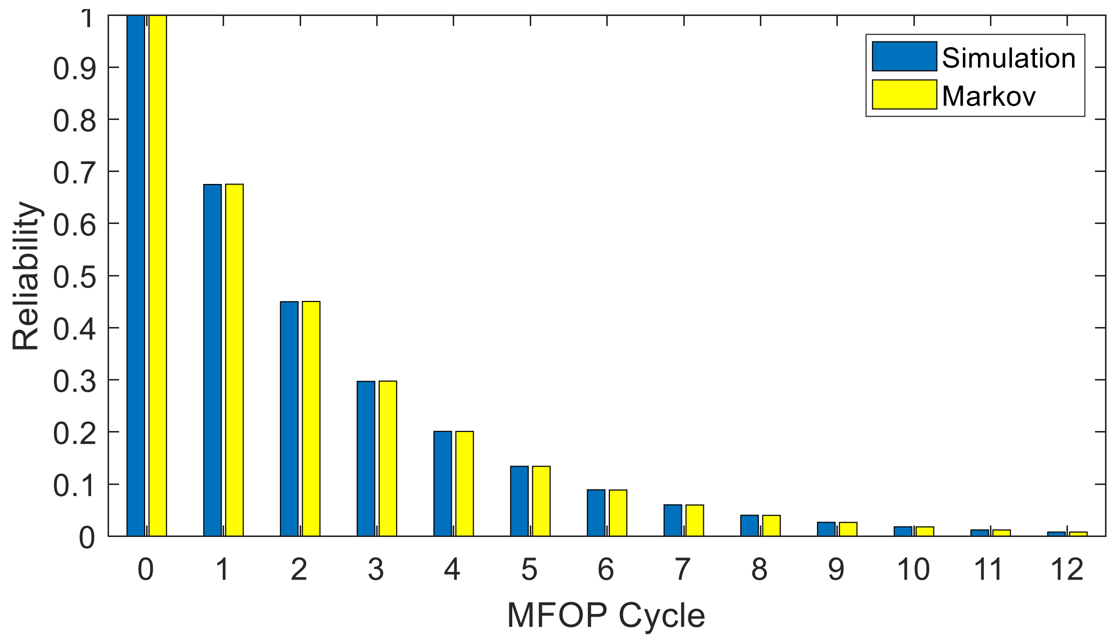


Figure 45: Reliability vs. MFOP Cycle

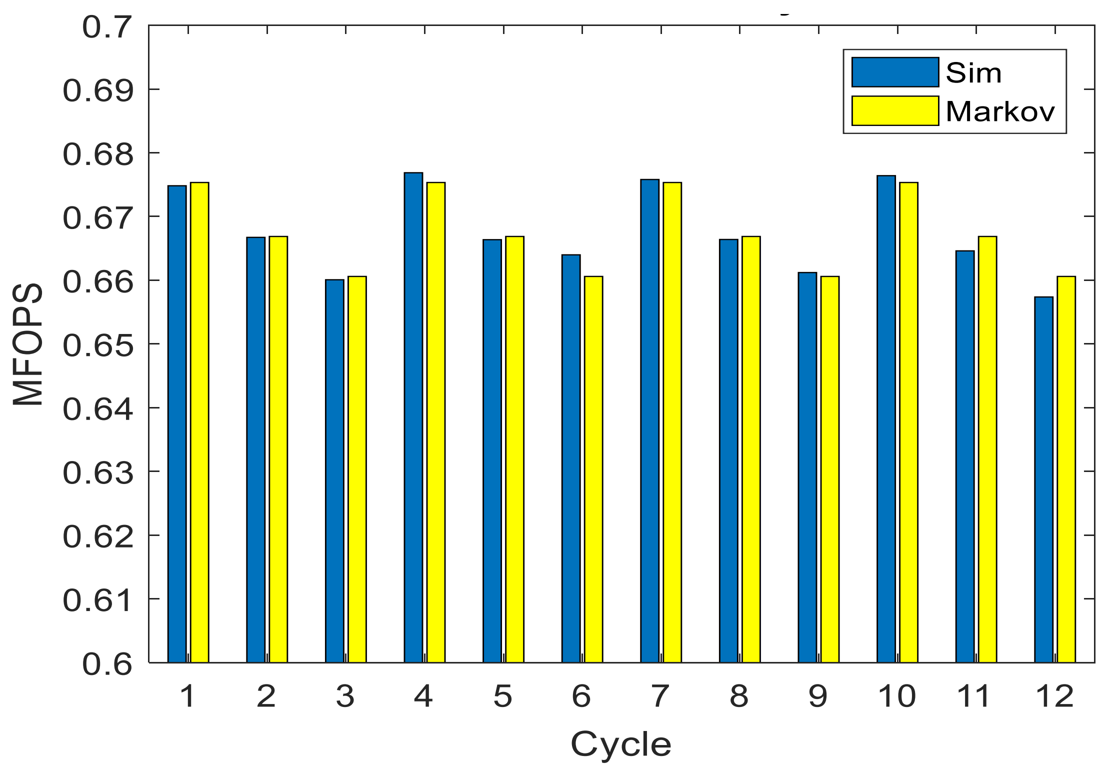


Figure 46: MFOPS vs. MFOP

The simulation approximates the analytical solution at one million iterations to a MFOPS MSE of 0.00853. Percent error for each cycle was below the desired 1% as shown in Table 19.

Table 19: Test Case #4 Comparison of Markov and Simulation MFOPS

Cycle	1	2	3	4	5	6
Markov	0.67533	0.66686	0.66059	0.67533	0.66686	0.66059
Simulation	0.67482	0.66672	0.66007	0.67686	0.66636	0.66398
% Error	-0.076%	-0.020%	-0.079%	0.227%	-0.074%	0.513%

Cycle	7	8	9	10	11	12
Markov	0.67533	0.66686	0.66059	0.67533	0.66686	0.66059
Simulation	0.67580	0.66639	0.66120	0.67640	0.66460	0.65737
% Error	0.070%	-0.069%	0.092%	0.158%	-0.338%	-0.487%

Like Test Case #3, the Test Case #4's convergence of MFOPS (Figure 47) takes more iterations than the reliability (Figure 48) due to the conditional nature of MFOPS. Arrows on the figure highlight the bands of the repeating three cycle decay. The convergence of these cycle results to the analytical Markov solution verify the DES for use with repair policies in complex systems comprised of multiple parts flying phased missions over several MFOP cycles. The progressive verification of each test case provides confidence in the DES' accuracy and for use in experiments that test the hypotheses.

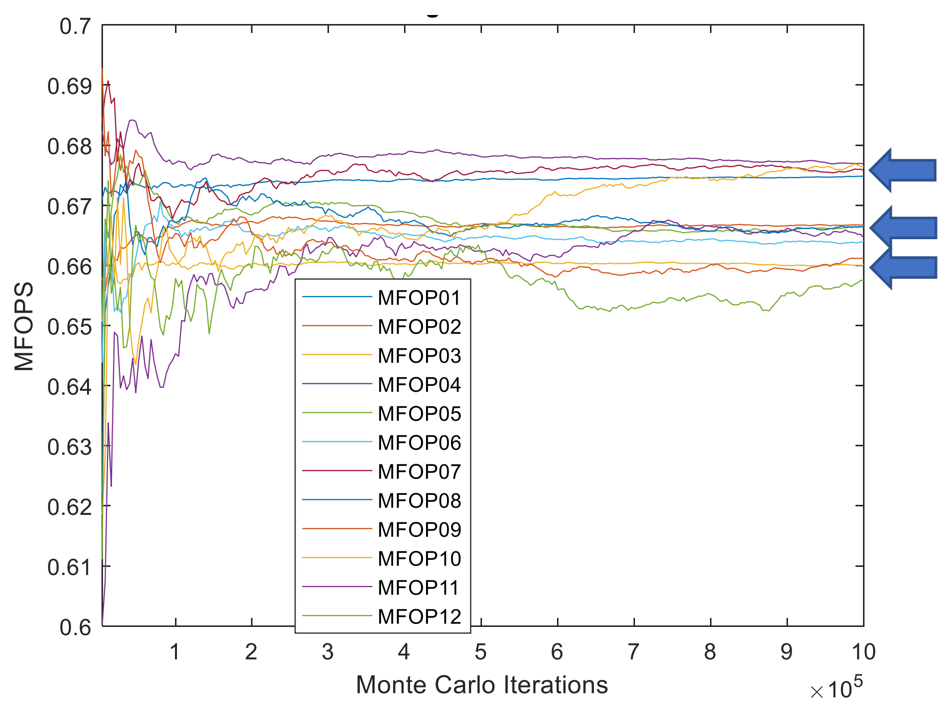


Figure 47: Convergence of MFOPS

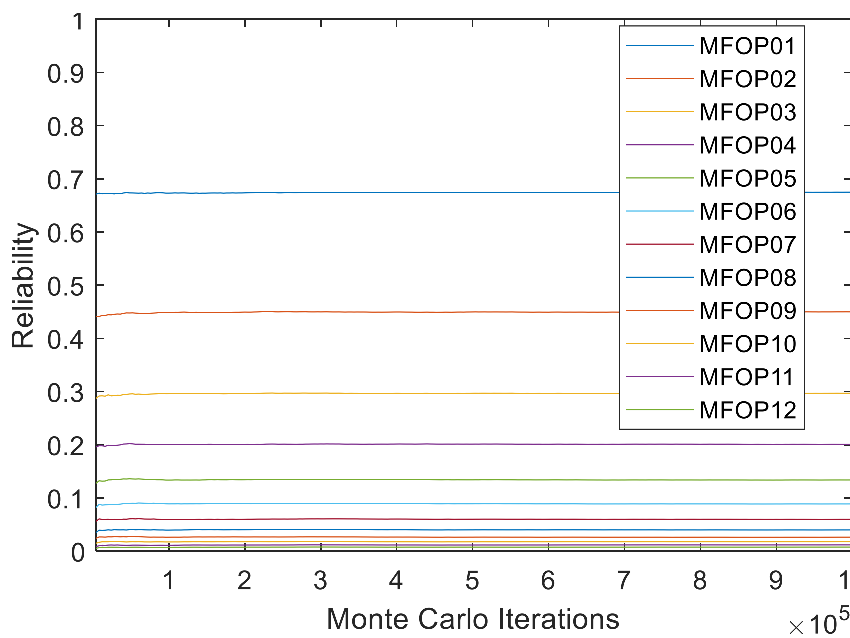


Figure 48: Convergence of MFOP Cycle Reliability

4.2 Research Question 2a: Which components limit a MFOP

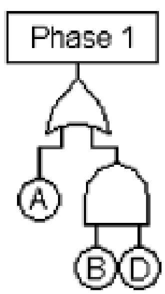
The second research question sought the identification of components or subsystems that limit a given system's dependability. Using MFOP as the metric for dependability and MRP as the metric for maintainability yields the Research Question 2a and 2b, respectively. The identification of limiting component or "weakest link" is a key step in helping the designer improve the MFOP.

> **Research Question 2a**
>
> *Which components/subsystems limit an MFOP?*
>
> **Conjecture A:** Failure Cause Identification reveals the limiting components or subsystems to a MFOP.

The weakest link step found in Relf's methodology (Figure 9) and in the ASSIST methodology tells the designer where to apply a MFOP option. None of the major references [23], [26], [27], [28], [45], directly address how to identify the weak link; however, Trindade and Nathan in [22] offer Failure Cause Identification theory as introduced in section 3.2.1. Finding the part with the greatest mean from component failure distributions is a good indication of a weak link, but that may not necessarily provide the correct prediction. Two concepts hide the answer. First, a phased mission means some components will receive greater usage (and wear) than others. Second, an aircraft and its parts each have their own history after several MFOP cycles. A simulation shows is helpful because it does not have to solve the two problems directly. It merely needs to record the time history of causes. Failure Cause Identification then identifies the weak links to answer Research Question 2.

4.2.1 Development of Failure Cause Identification

Consider the system of Test Case #4 with the first phase's fault tree shown below. There are 2^r states in a binary system where r parts are either operating (0) or failed (1). In the four-part system ($r = 4$), there are sixteen possible states. A binary number of r digits from one to 2^r describes each state. Phase 1 has six operating and ten failed states as shown in Figure 49.



#	A	B	C	D	Status
1	0	0	0	0	Operating
2	0	0	0	1	Operating
3	0	0	1	0	Operating
4	0	0	1	1	Operating
5	0	1	0	0	Operating
6	0	1	0	1	Failed
7	0	1	1	0	Operating
8	0	1	1	1	Failed

#	A	B	C	D	Status
9	1	0	0	0	Failed
10	1	0	0	1	Failed
11	1	0	1	0	Failed
12	1	0	1	1	Failed
13	1	1	0	0	Failed
14	1	1	0	1	Failed
15	1	1	1	0	Failed
16	1	1	1	1	Failed

Figure 49: Failure Causes of Phase 1, Test Case #4 Left most figure reprinted with permission[4]

A detection algorithm denotes when a failed phase occurred by matching the current state's binary sequence to the failed conditions shown above. The algorithm discovers a part's contribution to the failure cause by investigating the current state's binary sequence. It creates a temporary state by changing one failed component to operating. If the temporary repaired state is in an operating condition, the temporary repaired part is a

[4] Reprinted from *Reliability Engineering & System Safety*, vol. 93, S. P. Chew, S.J. Dunnett and J. D. Andrews, "Phased mission modelling of systems with maintenance-free operating periods using simulated Petri nets," pp. 980-994, Copyright (2008), with permission from Elsevier.

member of the failure cause. If the temporary repaired state remains failed, then the temporary repaired part is not a member of the failure cause. The temporary part repairs are conducted one at a time to discover all members of the failure cause.

Phase 1 has two failure causes: (1) Component A failed; and (2) the combination of Components B and D failed. For example, suppose the system enters Phase 1 in an operating condition with Components B and C failed (state 7). Component D then fails during the phase bringing the system into a failed condition (state 8). A failed state is noted and the failure cause algorithm begins. The algorithm temporarily repairs Component B, moving the system into an operating condition (state 4). Component B is denoted as a failure cause and the algorithm returns the system to its original failed state (state 8).

The algorithm next makes a temporarily repair to Component C which moves the system into state 6. Since state 6 remains a failure condition, Component C is not a failure cause. The algorithm removes the temporary repair to Component C, return the system back to State 8. The process checks the last failed part, Component D, and denotes C as a failure cause. Finally, both B and D are recorded as members of the failure cause in Phase 1 for this mission and MFOP's iteration. It is possible that multiple failure causes may occur in the same phase (such as Component A failing after D in the example); however, the first cause is the significant cause because mission abort would have already occurred.

4.2.2 Verification of Failure Cause Algorithm

Verification of the failure cause algorithm begins with a simple test using a single part system. The tested part has an exponential failure distribution with a failure rate of 0.0045 failures/hour. The system is simulated 100,000 times with the same phase durations of

Test #3-4 over twelve MFOPs of three missions each. Simulation results adhere to the known distribution's cdf in Figure 50. The discrete nature of the event simulation, in which no information is collected between events, yields a stair effect to the plot.

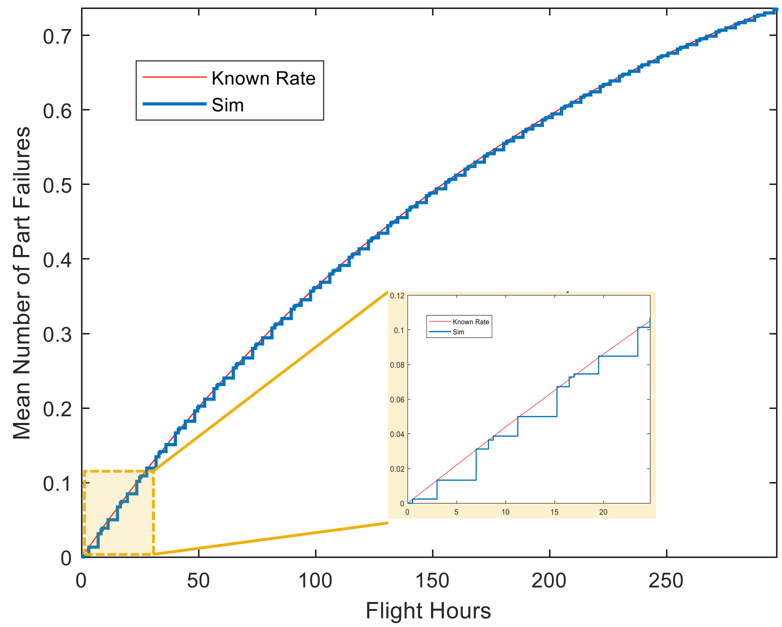


Figure 50: Verification of Failure Cause Identification

4.2.3 Results and Discussion of Failure Cause Identification

In a MFOP strategy, the "when" a system fails is as important as the "how often" or frequency of failure. Metrics using the mean like MTTF and MTBF do not report the "when" that is essential to a MFOP strategy. The problem is further complicated by a phased mission with different durations and fault trees. As outlined in section 3.2.1, Failure Cause Identification provides the necessary time history of the "when" and reveals primary members that fail a system before completing an operating period.

4.2.3.1 The Need to Measure Failure Cause

The traditional ranking failure cause based upon component failure rate does not account for the complexity introduced by an aging system following a phased mission with different fault trees. Even with the constant failure rate system of Test Case #4 where the memoryless property negates aging effects, ranking parts based upon only the part failure rate is misleading. The failure rate predicted order (best to worse) is D (0.0011 fails/hour), A (0.0045 fails/hour), C (0.0081 fails/hour), B (0.0130 fails/hour). Simulating the system through twelve MFOP cycles, reveals the order (best to worse) to be D, C, A, B. Figure 51 plots the failure history of the system (shown as solid lines). Parts A' and C' represent a single part system (shown as dashed lines below). The expected failure history of shown in A' and C' neglect the impacts of architecture and a phased mission.

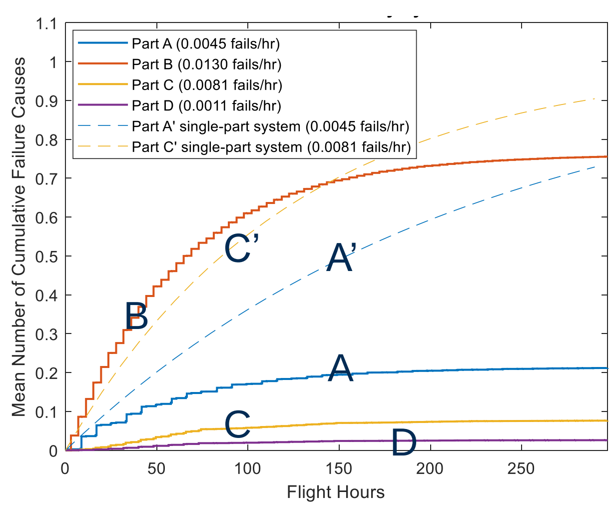


Figure 51: Failure Cause History of Test Case #4

4.2.3.2 Establishing a Metric for Failure Cause Identification

In the previous example, it was easy to see the contribution of each component to system failure by examining the plot of Failure Cause History. Each part was exponential. The system was memoryless and hazard rates were constant. With aging effects, the hazard rate as a function of time, thereby making visual inspection difficult.

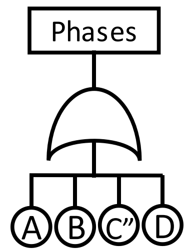
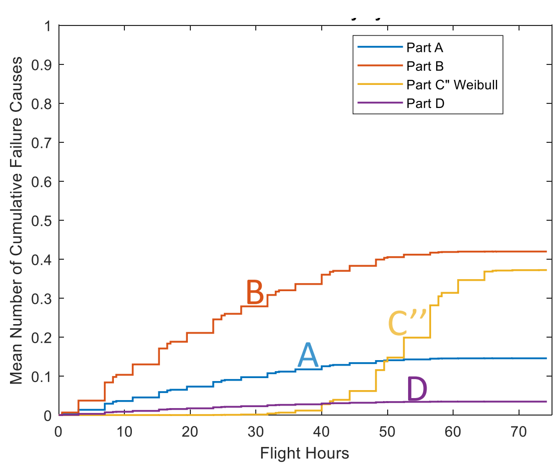
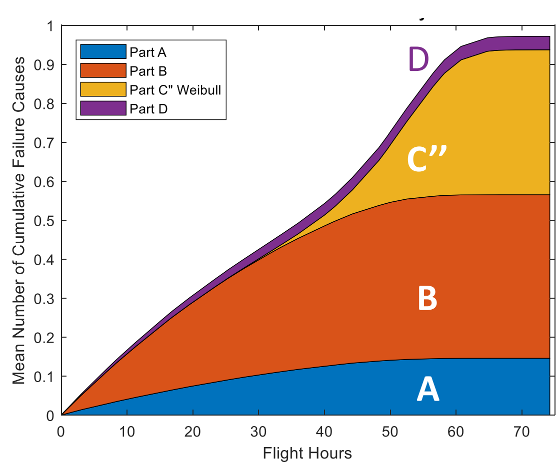


Figure 52: Phase with Components in Series

Consider a four-part system in serial with the fault tree shown in Figure 52 above. The new system replaces Component C's exponential failure distribution with C". C'' has a Weibull failure distribution ($\eta = 55, \ \beta = 9$). The use of a Weibull distribution introduces the effects of aging through a changing hazard rate. The system is simulated over 200,000 iterations through three MFOPs for a total duration of 74.25 hours. The Failure Cause history parts and system are shown in Figure 53.



(a) Failure Cause Part History



(b) Failure Cause System History

Figure 53: Failure Cause History of a Serial System

Part C" begins the simulation with a low hazard rate and does not experience frequent failures until 30 hours. Part C" ends the period with the second most failures; however, it is the best performer through the first half of the mission. Later failures are preferred over early failures in a MFOP strategy. The ideal part has few failures with those failures occurring late in the period. A poor performing part has many failures with those failures occurring early in the period. What is the performance of a part with few failures with those failures occurring early or a part with many failures with those failures occurring later? It is this conflict between the total failures (how often) and the time of failure (when) that complicates identifying the weakest link. This establishes the need to evaluate the weakest link with a new metric.

The development of a metric begins with the failure cause frequency. In the continuous form, let *g(t)* be the failure cause frequency as a function of time *t*. The cumulative failure cause history, *G(t)*, is the integral from start of the system to the end of *i* operating periods t_{mf}.

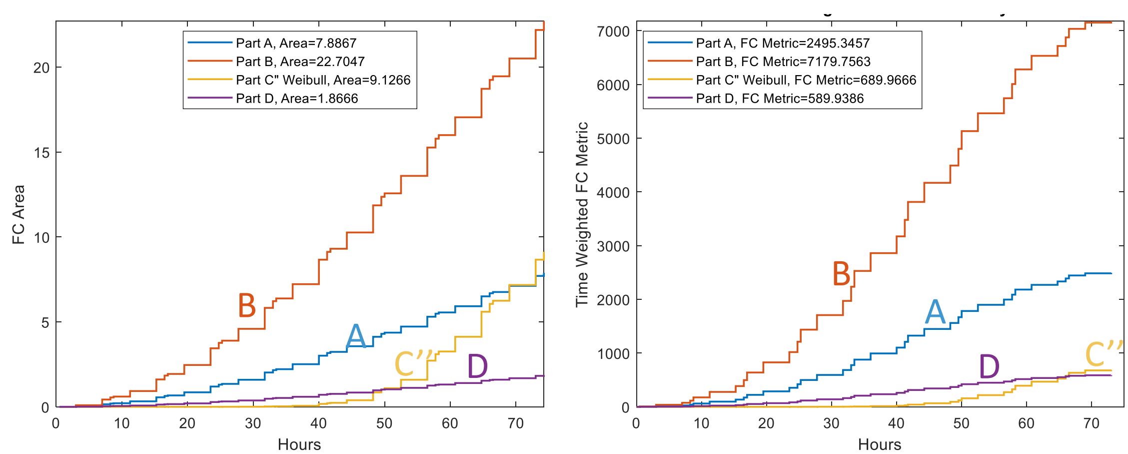
$$G(t) = \int_{t=0}^{(i \times t_{mf})} g(t)dt \tag{41}$$

Note, *g(t)* differs from a failure distributions pdf, *f(t)*. *f(t)* measures the probability of failure whose integral over time is one, while *g(t)* is a frequency of occurrence that does not sum to one. It is necessary to use *g(t)* when comparing different failure causes to capture the "how often" or magnitude of part failures. Both the cdf *F(t)* and *G(t)* are cumulative; however; *F(t)* is integrated over [0,∞] and sums to one, while *G(t)* is integrated over the operating period and does not sum to one.

The area under the failure cause history curve is a metric of cumulative failure that considers both time and frequency. The total area is the sum of the differential areas, where the differential areas are the product of the differential time step and cumulative count. The failure area, *A(t)*, up to time *t* is the integral of *G(t)* of

$$A(t) = \int_{t=0}^{(i \times t_{mf})} G(t)dt \tag{42}$$

The growth of *A(t)* is in Figure 54(a) and values at the end of the period are listed in Table 20.



(a) Failure Cause Area Growth (b) Failure Cause Metric

Figure 54: Measuring Failure Cause in Serial System

The plot of *A(t)* provides information when part performance changes relative to other parts. Higher values of *A(t)* are worse than lower values as they represent greater cumulative failures. The area of Part C" grows past Part D at $t = 7$ hours and exceeds Part A at $t = 69$ hours. The area growth helps answer the question: *What is the part contribution to failure from start to time t?* For example, if the MFOP period is set at 33 hours, then the greatest contributors to failure are B, A, D, and C'' from (greatest to least). If the MFOP period is 66 hours, the greatest contributors to failure are B, A, C'', D.

Table 20: Metrics for Failure Cause Identification of Series System

Component	Mean per Iteration (FC)	Mean per Iteration Normalized	Area (FC-h)	Area Normalized	Metric $(FC\text{-}h)^2$	Metric Normalized
A	0.146	4.22	7.89	4.23	2,495	4.23
B	0.420	12.15	22.71	12.16	7,180	12.17
C" Weibull	0.372	10.78	9.13	4.89	690	1.17
D	0.035	1	1.87	1	590	1

4.2.3.3 Distinguishing Between Early and Late Failures

A(t) is useful to identify the weakest links up to time *t*; however, it cannot differentiate between early and late failures. This is because *A(t)* sums all differential elements equally. For example, the area for Part C" Weibull (9.13) is slightly larger than Part A (7.89); yet, Figure 53(a) shows that Part C" has much later failures than Part A. The area metric is insufficient because a MFOP strategy prefers later failures over earlier failures.

A time weighted metric is needed to understand which component most limits the chance to achieve the longest MFOP period. Such a metric is the Failure Cause Metric (FCM). It weighs early failures worse than later failures and is defined as

$$FCM(t) = \int_{t=0}^{(i\times t_{mf})} \left[\left(i \times t_{mf}\right) - t\right] A(t) dt \qquad (43)$$

Plots of *FCM(t)* are shown in Figure 54(b). The metric is most useful as a quantitative number shown in Table 21. The metric ranks Part C" Weibull with its later failures as better than Part A. FMC provides a quantitative means to identify which component(s) most (and least) limit a desired MFOP based upon when failure occurs.

Table 21: Metrics for Failure Cause Identification of a Sample System

Component	Mean per Iteration (FC)	Mean per Iteration Rank	Area (FC-h)	Area Rank	Metric $(FC\text{-}h)^2$	Metric Rank
A	0.146	2	7.89	2	2,495	*3*
B	0.420	4	22.71	4	7,180	4
C” Weibull	0.372	3	9.13	3	690	*2*
D	0.035	1	1.87	1	590	1

The testing of maintenance policies presented by Hypothesis 2 and 3 utilized Failure Cause Identification extensively to diagnose system behavior.

4.3 Research Question 2b: What is the Greatest Contributor to MRP

The companion to a MFOP’s weakest link is a MRP’s greatest contributor. The greatest contributor is the component or subsystem that maintainers spend the most time repairing in a MRP. If a unit wanted to reduce the length of a MRP, they would ask designers to improve the greatest contributor’s reliability or maintainability (ease of repair). The MRP measures a system’s maintainability. A thought experiment and a practical exercise follows to investigate the greatest contributor to the MRP.

4.3.1 Thought Experiment

A brief thought experiment demonstrates a key concept of the MRP. The repair with the greatest Mean Time To Repair (MTTR) is not necessarily the greatest contributor to the MRP. Similarly, the most failed part is not necessarily the greatest contributor. A brief thought experiment demonstrates this concept. Consider a simple system of three parts in series. Parts A, B, and C have mean repair times shown in Table 22. Suppose a MFOP consists of 100 missions and a counter records the number of failures over the MFOP. Neither the part with greatest MTTR (Part C) nor the part with the highest failure rate (Part A) has the largest total repair time (Part B).

Table 22: MRP Thought Experiment

Part	MTTR	Failures per 100 missions	Expected Cumulative Repair Time
A	1 hour	12	12 hours
B	4 hours	5	20 hours
C	8 hours	2	16 hours

The discussion is looking for the repair with the greatest expectation of contribution to MRP. The greatest contributor should not be as difficult to find as the weakest link because repair times, although stochastic, follow a distribution that does not change over time. It is worth noting that component failure rates calculate the expectation a priori as the part aging is derived from a percentage of flight hours.

Research Question 2b

Which components/subsystems are the greatest contributor(s) to MRP duration?

Conjecture B: The greatest contributor is the component/subsystem with the greatest expected downtime.

The dissertation accepts the conjecture that each component's mean downtime predicts the greatest contributors. The expected downtime is

$$E(H \cdot TTR) = H[t]E(TTR) + Cov(H, TTR) \qquad (44)$$

where H is the expected number of failures to time t, TTR is the Time to Repair distribution, and $(H \cdot TTR)$ is the total repair time over a MFOP cycle. The expected number of failures is a function of the components Time to Failure (TTF) distribution. The expectation of repair time is the MTTR. The expectation of total repair time is the MDT for the part. Assuming H and repair times (TTR) are independent, random variables, the covariance will be zero. A system's total MRP is then

$$MRP = \sum_{r=1}^{n} H_r \cdot E(TTR_r) \qquad (45)$$

where there are n parts in the system.

The use of (45) assumes part unreliability is independent of its time to repair. There is likely a small dependency between frequency of failure and repair times in real operations. A maintainer that sees the same failure frequently is likely to become faster in diagnosing and repairing the item. A unit is also likely to keep spares of commonly failed items, reducing logistical downtime. The repair times are assumed to follow a distribution that is independent of unreliability. Measuring MRP in MMH eliminates the need to model a unit's repair capacity and available manpower to answer the question. The next section conducts a practical exercise to determine the TTF distribution and expected downtime of a utility helicopter.

4.3.2 Construction of a Utility Helicopter Model

FVL develop includes the technology demonstrators of the Joint Multi-Role (JMR) program. The initial effort of JMR focused on fulfilling Capability Set 3, consisting of medium utility and attack configurations. The Black Hawk and AH-64 Apache currently perform these roles in the U.S. Army. The aircraft serve as baselines for the medium class demonstrators and the medium configuration. The Black Hawk is attractive due to its proliferation, long service history, and availability of maintenance data.

Figure 55: Sikorsky Black Hawk Helicopter from [61]

The constructed utility helicopter is representative of the U.S. Army's UH-60M Black Hawk. The Black Hawk is a medium lift, utility helicopter from Sikorsky Aircraft of Lockheed Martin Corporation. There are over 4,000 Black Hawk variants in service with the 2,135 operated by the U.S. Army [61]. The commercial designation of the Black Hawk is the S-70. The UH-60 first saw service in 1979 and has several major model updates. The Army expects an upgraded UH-60M and a future model, the UH-60V, to remain in service through 2045 and the fielding of FVL [62].

Modeling a Representative Utility Helicopter (RUH) model requires the construction of three random variable distributions: (1) Time to Failure (TTF); (2) Time to Repair (TTR); and (3) Repair Cost (if capturing affordability). TTF captures the frequency and nature of failures necessary to calculate MFOPS and understand the weakest link using Failure Cause Identification. TTR establishes the duration of the MRP and the likelihood of meeting a desired duration, MRPS. It also provides downtime, measured in Maintenance Main Hours, for prediction of Achieved Availability by renewal theory.

4.3.2.1 Time to Failure Distributions

The U.S. Army scores aircraft reliability and maintainability events with the Failure Definition and Scoring Criteria (FDSC). Resulting maintenance actions are either scheduled or unscheduled maintenance. The FDSC classifies unscheduled maintenance as Unscheduled Maintenance Action (UMA). The FDSC further distinguishes UMAs as an Essential Maintenance Action (EMA), Mission Affecting Failure (MAF), or Mission Abort (MA) based upon its impact to the mission and time of discovery. FDSC definitions from [63] for the UH-60M event categories are in Table 23. The categories are hierarchical with some terms nested in others as depicted in Figure 56.

Table 23: FDSC Failure Definitions

Term	Name	FDSC Definition [63]
UMA	Unscheduled Maintenance Action	Any maintenance action that is not identified in the technical documentation of the system as scheduled maintenance.
MEF	Mission Essential Function	An MEF is the operational capabilities that the system must perform to complete its missions successfully.
EMA	Essential Maintenance Action	Results from any incident or malfunction which causes the inability to perform, or degrades, one or more MEFs regardless of time of discovery, plus any additional unscheduled maintenance required prior to initiating the next mission, to include restoring mission essential equipment redundancy.

Table 23 (continued)

Term	Name	FDSC Definition [63]
MAF	Mission Affecting Failure	An incident or malfunction which causes the inability to perform, or severely degrades, one or more MEFs and was discovered during mission time, regardless of the operational mission in progress.
MA	Mission Abort	An incident or malfunction that causes the loss of a mission essential function specifically required for the operational mission in progress.

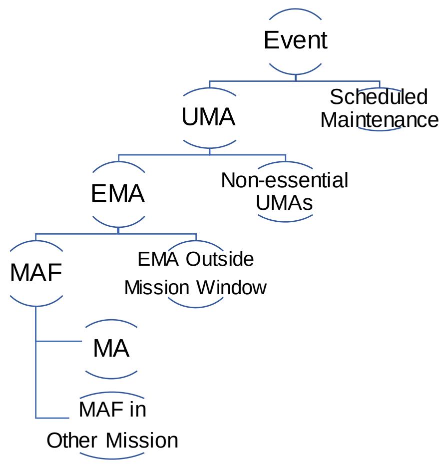


Figure 56: Hierarchical Structure of Failure Classification

EMAs are the classification of interest when studying a MFOP. An "essential" incident prevents the aircraft from performing its Mission Essential Functions (MEFs). MEFs for the Black hawk are fly, communicate, navigate, survive, transport, and provide patient care (HH-60M only) [63]. EMAs are incidents that result in unscheduled maintenance and, therefore, disrupt the MFOP. Even if an EMA incident does not result in a Mission Abort (MA) on a given flight, it still requires maintenance upon returning base. This is an important concept of a MFOP strategy. A successful MFOP system must have a high mission reliability (measured by MA) and minimal unscheduled maintenance (measured by EMA). Given that all MAs are an EMAs, the number of EMAs is equal to or greater than the number of MAs.

EMAs that occur during the mission time (pre-flight to shutdown) are MAFs. EMA's discovered outside the mission time (after post flight inspection and before the next mission time) do not have a specified label. Whether the operator discovers the fault during or after the mission time, it is still an EMA. An incident that does not interfere with an MEF has a non-essential maintenance action. The maintainer may defer the non-essential UMA to the next MRP, to any subsequent MRP, or to the next reset. Although an operator may choose to repair a non-essential UMA, it does not necessarily disrupt a MFOP and should not be used to evaluate an aircraft's MFOP. The ideal near-zero maintenance aircraft has no UMAs because MFOP Options such as prognostics and diagnostics provide warning of a failure with sufficient time to synchronize preventive action at the next MRP; however, the state of the art is far from this capability. Until aircraft achieve the near-zero maintenance paradigm, EMAs represent the proper way to capture both failure and unscheduled maintenance that could disrupt a MFOP.

4.3.2.2 Results of Constructed Time to Failure Distributions

The Reliability, Availability, and Maintainability Engineering and System Assessment Division of the U.S. Army's AMRDEC provided EMA data to study the UH-60M. To ensure public release-ability, data shown in the thesis has been skewed randomly by ±20% from the actual data. This permits the presentation of methodology and results for a Representative Utility Helicopter (RUH) that is similar in class and performance to the UH-60M Black Hawk. Table 24 consolidates EMA data from 55,634 flight hours recorded in a recent two-year period. The data is a compilation of entries manually verified by a RAM expert and represents approximately 24% of all UH-60M flight hours over the two-year period.

Table 24: Representative Utility Helicopter EMAs

Functional Group Code	Subsystem	Essential Maintenance Actions	EMAs per 1,000 h
02	Airframe	2,173	39.06
03	Landing Gear	750	13.48
04	Power Plant	1,423	25.58
05	Rotor	1,800	32.35
06	Drive	804	14.45
07	Pneumatics and Hydraulics	653	11.74
08	Instruments	294	5.28
09	Electrical	395	7.10
10	Fuel	174	3.13
11	Flight Control	1,001	17.99
12	Utility	434	7.80
13	Environmental Control	21	0.38
15	Auxiliary Power Unit	198	3.56
16	Mission Equipment	237	4.26
17	Emergency Equipment	17	0.31
18	Ground Support Equipment	71	1.28
19	Avionics	285	5.12
52	Stabilization (AFCS)	160	2.88
Total		10,890	195.75

The system EMA rate is 195.75 failures per 1,000 flight hours. Recognizing that each subsystem is essential to mission accomplishment leads to a reliability block diagram in series (Figure 57).

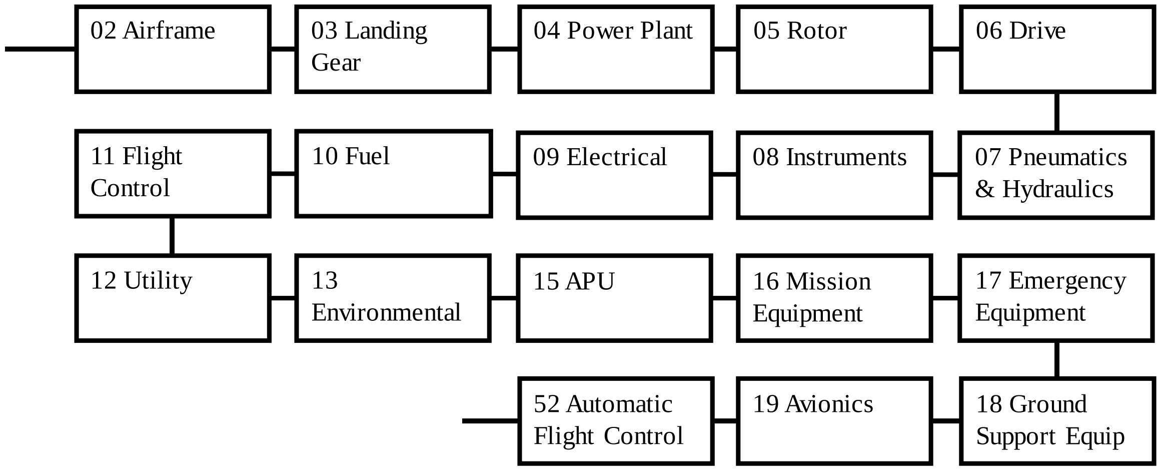


Figure 57: RUH Reliability Block Diagram

The reliability for the serial system is

$$R_{sys} = \prod_{r=1}^{18} R_r(t) \tag{46}$$

where the eighteen subsystems of Figure 57 comprise the RUH. The memoryless property of a system comprised of only exponential TTF distributions system provides that

$$R_{sys} = \prod_{r=1}^{18} e^{-\lambda_r t} = e^{-t\sum_{r=1}^{18}\lambda_r} = e^{-\lambda_{sys}t} \tag{47}$$

and, thus, the system failure rate is

$$\lambda_{sys} = \sum_{r=1}^{18} \lambda_r = 0.19575 \text{ failures/h} \tag{48}$$

Although the RUH is a repairable system, the MTTF provides a useful notion. MTTF is the expected duration the system can achieve without repair. Given that a system must remain without failure and maintenance free during the MFOP duration, the MTTF is a metric of interest. The MTTF of a system is

$$MTTF_{sys} = \int_0^\infty R_{sys}\, dt \tag{49}$$

Substituting (48) into (49) gives

$$MTTF_{sys} = \int_0^\infty e^{-\lambda_{sys} t}\, dt \tag{50}$$

Evaluating the integral at the EMA rates found in Table 30 yields

$$MTTF_{sys} = \frac{1}{\lambda_{sys}} = \frac{1}{0.19575\ EMAs/h} = 5.11\ hours\ per\ EMA \tag{51}$$

The RUH has a mean duration of 5.11 hours before an EMA occurs. This number is of the same order of magnitude to the 5-hour MFOP suggested by [31]. A plot of MFOPS verse MFOP duration is in Figure 58. In this circumstance, the MFOPS equals the probability that the system does not incur an EMA through in the first cycle through time t_{mf} given the system started fully functional.

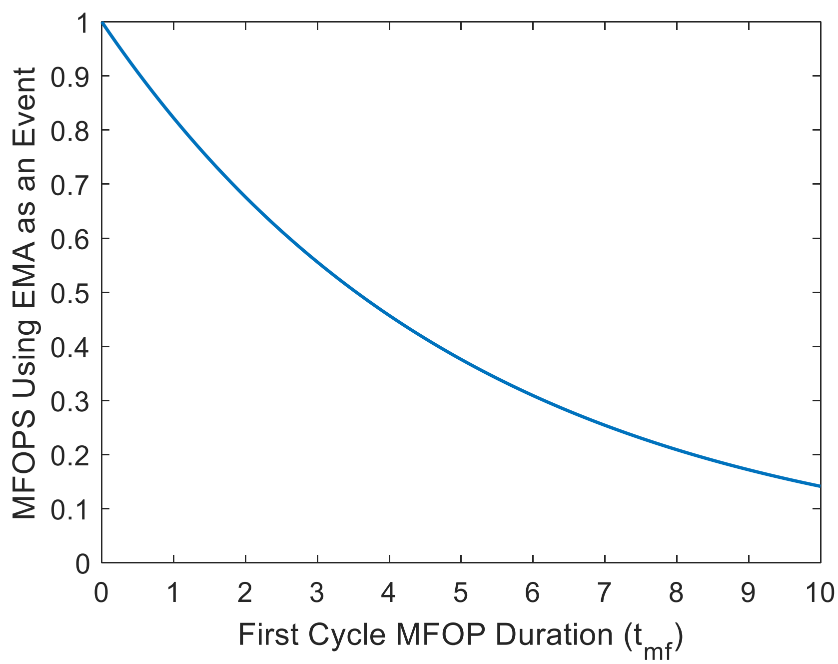


Figure 58: Exponential RUH Model Plot of MFOPS vs MFOP

The memoryless property of an exponential system, specifically expressed as Equation (51), permits a simple analytical solution to MFOP through the quantile (or inverse cdf) function. In a MFOP context, the quantile function of the exponential system is

$$MFOP = -\frac{\ln(MFOPS)}{\lambda_{sys}} \tag{52}$$

Systems with non-exponential TTF distributions or complex reliability block diagrams systems may not have an attainable closed form expression. The table below lists MFOP values at specific, given MFOPS. The RUH has a 50% chance to achieve 3.54 flight hours without incurring any essential maintenance. It has a 36.8% chance to achieve the mean of 5.11 flight hours and only a 14% chance to reach 10 flight hours without essential maintenance.

Table 25: Exponential RUH Model MFOPS

MFOPS	MFOP	Remarks
0.00000039%	100.00 h	
14%	10.00 h	
36.8%	5.11 h	Mean
50%	3.54 h	Median
90%	0.54 h	
95%	0.26 h	

Table 25 shows that a MFOP of 100 hours, as sought by FVL, is a radical improvement in system reliability from today's aircraft.

4.3.2.3 Construction of Time to Repair Distributions

A Time to Repair (TTR) distribution measures the probability that a specific maintenance action takes a duration of *t*. TTR accounts for any resulting maintenance action of an EMA. Ideally, actual repair times build the distribution; however, this data is not always available. New designs may not have a large historical database to generate sufficient data or the information may not be recorded (as is the case of the RUH). Relevant repair times for an EMA event were difficult to identify in the current Army maintenance database. Fortunately, the use of a Maintenance Allocation Chart (MAC) can overcome the lack of comprehensive repair time data.

The MAC accounts for the expected time to complete a maintenance action on a component. Maintenance actions include inspect, service, adjust, repair, replace, and test. The MAC assigns expected action time based upon the experience, skills, and tools at each level of maintenance. Time is tracked as MMH to the tenth of an hour. The author manually compared the 774 working unit codes with an EMA to the 900 component action times of the MAC. Overall, 8,982 of the possible 10,890 EMAs (82%) had a matching TTR. Figure 59 shows the number of matched and unmatched EMAs by subsystem. The distribution fitting excluded unmatched EMAs.

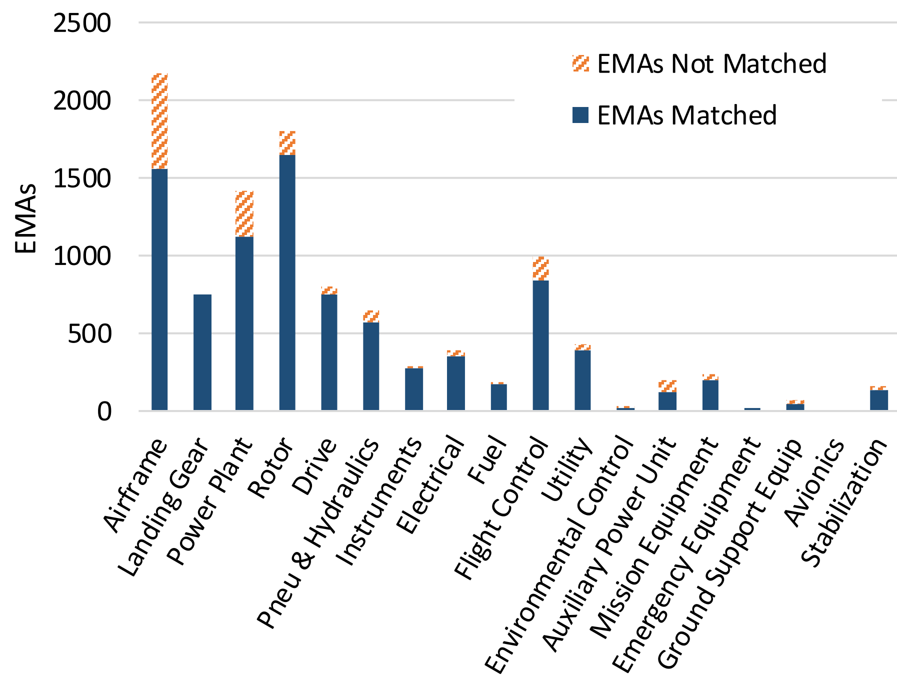


Figure 59: RUH Subsystem EMA to TTR Matching

An excerpt of the Airframe Subsystem is in Table 26 with data organized by working unit code. The table lists EMAs and the matched TTR in third and fourth columns, respectively. Unmatched working unit code EMAs, such as the 24 EMAs of the '02A Forward Fuselage Section, are excluded from building the TTR distribution. 72% (1,557 out of a possible 2,173) of Airframe EMAs matched a TTR. Working Unit Codes' fractions of the 1,557 events are in the fifth column. Column six multiplies the fraction by its respective TTR. The mean TTR for the subsystem is the sum of working unit codes' contributions. The Airframe Subsystem has a mean of almost 2.9 hours per EMA event.

Table 26: Extract of '02 Airframe TTR

(1) Working Unit Code	(2) Nomenclature	(3) EMA	(4) TTR	(5) Fraction	(6) Contribution
'02	Airframe	4	2.3	0.0026	0.0059
'02A	Forward Fuselage Section	(24)	---	0.0000	0.0000
'02A01	Windshield Installation	8	1.4	0.0051	0.0072
'02A01A	Pilot's Windshield	35	1.9	0.0225	0.0427
'02A01B	Co-Pilot's Windshield	29	1.9	0.0186	0.0354
'02A01C	Upper Overhead Window	45	1.4	0.0289	0.0405
'02A01D	Lower Nose Window	49	1.4	0.0315	0.0441
'02A01E	Center Panel Windshield	32	1.4	0.0206	0.0288
⋮	⋮	⋮	⋮	⋮	⋮
'02C15J	Tail Pylon Fitting	1	0.2	0.0006	0.0001
Total		1,557 (2,173)		1	2.8963

The product of a working unit code's EMA and its matched TTR builds a time weighted histogram. The MATLAB's distribution fit application turned the histogram into a density plot with a distribution fit as demonstrated for the Airframe subsystem in Figure 60. Most of the fits are not ideal. The TTR histograms tend to be heavily weighted with TTRs below 3 hours creating a left leaning distribution. Weibull distribution with a shape value between 1 and about 3.4 model the left leaning distributions. The histograms also show a small number of high time repairs. High time repairs represent major maintenance actions like main transmission or rotor installation. The high time maintenance action of the Airframe subsystem is the 29.7 MMH replacement of the tail pylon. This characteristic makes fitting a continuous distribution difficult.

The assumption of a fixed TTR worsens the fit. The assumption removes some of the variability existing in actual maintenance times. Capturing the true variability would reduce the peaks, "smooth" the frequency plot, and provide data easier to fit. For larger subsystems with many components, the Central Limit Theorem predicts the distribution should become normal. The Weibull distribution may approximate the normal (β≈3.44) but with thicker tails. The advantage of the Weibull distribution is that it takes a probability of zero when the random variable is less than zero. This property prevents negative TTRs, unlike the normal which has a range of [-∞, +∞]. Despite the less than ideal fit, the use of a Weibull does provide a better tool than making a generic assumption of a constant TTR rate.

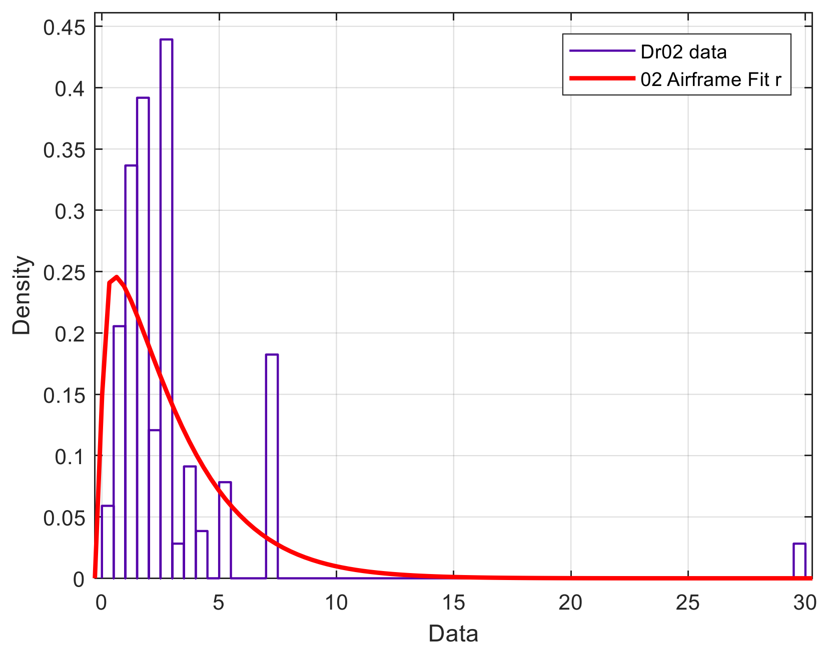


Figure 60: Airframe Subsystem Density Plot and Fit

Table 27 lists the fitted Weibull distribution for the Airframe and the fitted distributions for the remaining subsystems. Eleven of the eighteen distributions are left leaning with the characteristic high-time assembly installation. Every '017 Emergency Equipment essential maintenance action was 0.2 hours. A normal distribution with a very small standard deviation models the short replacement of emergency equipment.

Table 27: RUH Time to Repair Distributions

Subsystem		MTTR (h)	Distribution Type	Parameter	Parameter
02	Airframe	2.9	Weibull	η=3.137	β=1.154
03	Landing Gear	6.8	Weibull	η=7.521	β=1.468
04	Power Plant	1.6	Weibull	η=1.396	β=0.851
05	Rotor	11.6	Weibull	η=11.595	β=1.006
06	Drive	5.5	Weibull	η=4.321	β=0.707
07	Pneu & Hydraulics	4.7	Exponential	μ=4.704	
08	Instruments	0.7	Weibull	η=0.804	β=1.406
09	Electrical	2.3	Weibull	η=2.578	β=1.474
10	Fuel	7.1	Exponential	μ=7.079	
11	Flight Control	5.1	Weibull	η=5.352	β=1.119
12	Utility	1.3	Weibull	η=1.444	β=1.341
13	Enviro Control	3.0	Weibull	η=3.320	β=3.553
15	Auxiliary Power Unit	5.9	Weibull	η=6.675	β=1.753
16	Mission Equipment	1.6	Weibull	η=1.795	β=1.451
17	Emergency Equipment	0.2	Normal	μ=0.2	σ=3e-17
18	Ground Support Equip	0.6	Weibull	η=0.683	β=3.214
19	Avionics	0.7	Weibull	η=0.729	β=3.552
52	Stabilization (AFCS)	0.8	Weibull	η=0.936	β=2.444
00	RUH System	5.5	Lognormal	μ=0.830	σ=1.278

Compiling system data into a single distribution yielded best fit with a lognormal distribution (μ=0.830, σ=1.278). Figure 61 compares the TTR density against the fitted lognormal distribution. Actual mean of matched TTR data was 5.45 hours per EMA. Predicted mean of matched TTR data was 5.19 hours per EMA.

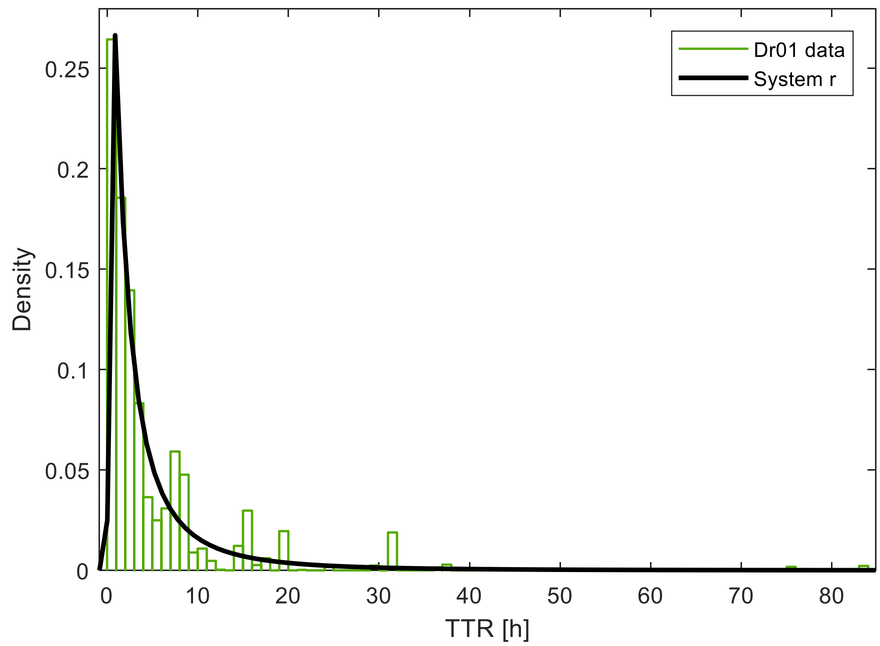


Figure 61: RUH System Density Plot and Fit

5 PROBLEM 2.1: MAXIMIZE AVAILABILITY IN A MFOP POLICY

Problem 1 worked to provide the tools to assess a maintenance strategy and its limiting factors to MFOP and MRP. Both MFOP and availability are important operational metrics to an aircraft's value. Understanding how they influence each other will help develop the right maintenance strategy to meet operational requirements. Problem 2 decomposed this interaction into two research questions. They are reshown below for the reader's convenience with the hypotheses introduced in this chapter.

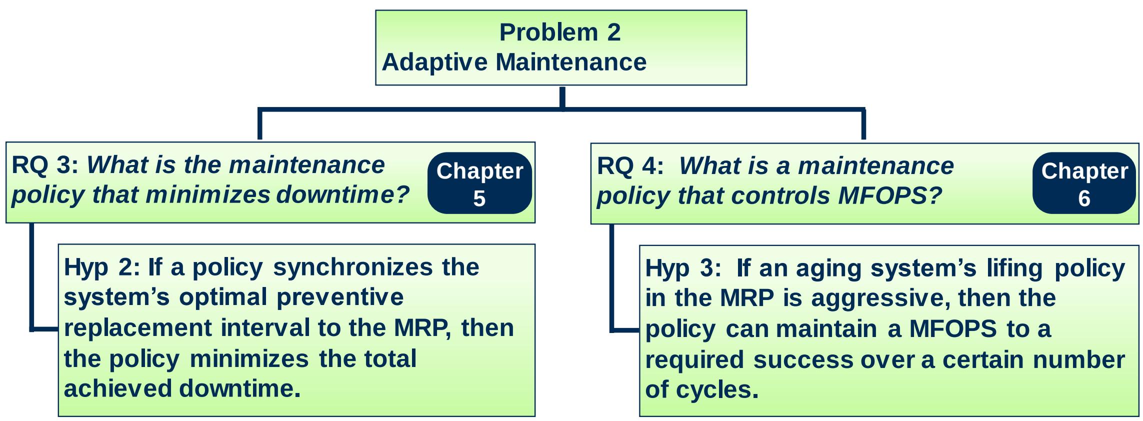


Figure 62: Problem 2 Summary with Hypotheses

Problem 2 addresses MFOP Knowledge Gap 4 (see Table 3): *Account for varying operational tempo in future sustainment strategy.* The problem, consisting of two research questions, explores how to develop adjustable maintenance policies to best meet changing operational demands. This chapter examines Research Question 3 by testing a second hypothesis. The section develops a framework to the creation of a maintenance policy that maximizes availability in a MFOP context. Such a policy of maximum availability is best suited for a garrison or training environment where the operational tempo is low and a shorter MFOP duration is tolerable.

5.1 Research Question 3: What is the Policy to Minimize Downtime

The third research question probes maintenance policies to minimize the downtime and thereby maximize availability. The renewal theory work by Jardine and Tsang in [3] provides a useful model that minimizes the downtime of a single part system. Section 3.4.2 of the background chapter has a review of the Optimal Preventive Replacement Interval model. The time to preventive maintenance in this model may be viewed as a MFOP duration. The downtime is quantified by the dimensionless quantity, D, and is reshown as

$$D(t_p) = \frac{\substack{Expected\ down\ time\ due\ to\ falures + \\ Expected\ down\ time\ due\ to\ preventative\ replacement}}{Cycle\ Length} \quad (16)$$

Total system downtime is the sum of downtime from each part

$$D_{system} = \sum_{i=1}^{N} D_i(t_p) \quad (53)$$

where D_i is the dimensionless downtime caused by the *ith* part and t_p is the MFOP length. Equation (17) shows the calculation for D_i. The optimization statement is the minimization of the system's downtime. This in turn, causes the maximization of materiel availability [3].

$$\min_{t_p,\, m_i} \quad D_{system}$$

$$\text{subject to} \quad t_p - MFOP_{min} \geq 0$$

A discrete approach avoids the difficulties of finding the Laplace transformation for numerous continuous distributions. Considering each part's t_p to be α_r multiples of MFOP will synchronize the maintenance. The t_p must be at least the minimally acceptable MFOP duration. Since the calculation of D is recursive, a computer makes quick work of the calculations.

> **Research Question 3**
>
> *What is the maintenance policy that minimizes downtime?*
>
> **Hypothesis 2:** If a policy synchronizes the system's optimal preventive replacement interval to the MRP, than the policy minimizes the total achieved downtime.

This effort tested a maintenance policy that conducts preventive maintenance repairs during the MRP only. This differs from the modeling conducted in the first two research questions because the earlier work assumed an unoptimized preventive maintenance plan. The hypothesis postulates that a correctly optimized maintenance policy of preventive maintenance will improve the materiel availability with a penalty of more MRP actions. The hypothesis is tested on a simple system with an accompany sensitivity study on the interaction of key variables.

5.1.1 Revised Renewal Theory Model for MFOPs

A MFOP strategy needs to limit corrective action of failures and cluster scheduled maintenance into a MRP. This problem looks at the clustering of scheduled maintenance using renewal theory's Optimal Preventive Replacement Interval model to quantify a system's downtime. The Optimal Preventive Replacement Interval model is appropriate for a MFOP strategy when the policy makes preventive replacements at multiples of the replacement interval (t_p). Synchronizing the preventive replacement interval of all items in the system to designated recovery periods establishes the MRP and creates an assurance that scheduled maintenance will not disrupt operations through the MFOPs.

This section provides an adaptation of the classical renewal theory model to support multiple parts with replacement intervals synchronized as multiples of the MFOP. The work begins with a review of the Optimal Preventive Replacement Interval model, then highlights current limitations of the theory, and finishes with an adaptation to a MFOP strategy. As presented later, the maintenance planner must take care not to minimize downtime through optimal replacement intervals at the expense of system reliability. An effective MFOP maintenance policy balances the operator's need for sufficiently long preventive replacement intervals against the risk of disruptive unscheduled failures occurring in the MFOP.

5.1.1.1 Understanding the Optimal Replacement Interval Model

Section 3.4.2 introduced renewal theory as applied in the Optimal Replacement Interval Model. The reader is encouraged to review this section before continuing with the application below. The model predicts downtime per cycle (*D*) for a single part over a replacement interval (t_p) and is

$$D(t_p) = \frac{H(t_p)T_f + T_p}{t_p + T_p} \qquad (17)$$

where $H(t_p)$ is the expected number of failures over the interval t_p, and where T_f and T_p are the time to make corrective actions and preventative replacements, respectively. The model assumes that corrective repairs times are inclusive of t_p. The assumption is valid when the failure repair times (T_f) are small compared t_p. The assumption keeps t_p*'s* clock running even when the system is down during corrective replacement. The model works well for long running systems that operate for weeks or months and have corrective repairs made in hours or days.

An example of a single item system shown in Figure 63 demonstrates the behavior of the model. The figure plots data points taken from Jardine and Tsang [3] over the model's downtime ratio predicted by Equation (17). It also includes a revised model developed later. The item has a failure distribution that is normal ($\mu=5,\ \sigma=1$), a failure replacement time of 0.07 units, and a preventive replacement time of 0.035 units. Table 28 shows the results for the first 6 intervals. The downtime function begins with a ratio of 1 at a t_p of zero. This represents a state of 100%, continuous, preventive repair with zero operating time ($t_p = 0$). The function decreases rapidly with a series of local minima (3.65, 8.62, and 13.96) and maxima that dampen over time. The global minimum is at a t_p* of 3.65 units of time with a downtime of ratio of 0.0112 or 1.12%.

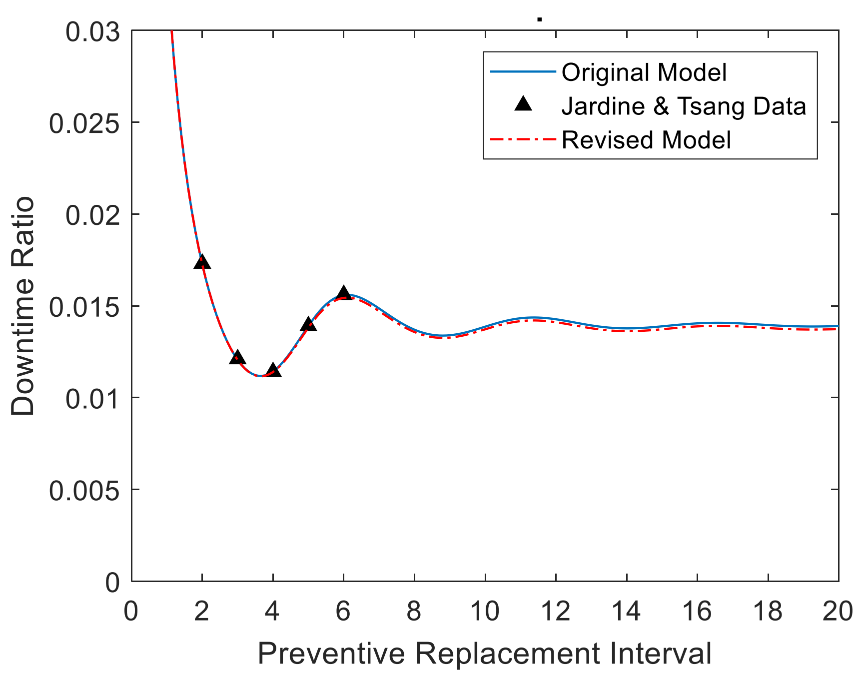


Figure 63: Downtime Ratio versus Preventative Replacement Interval

Table 28: Downtime of Optimal Preventative Replacement Interval

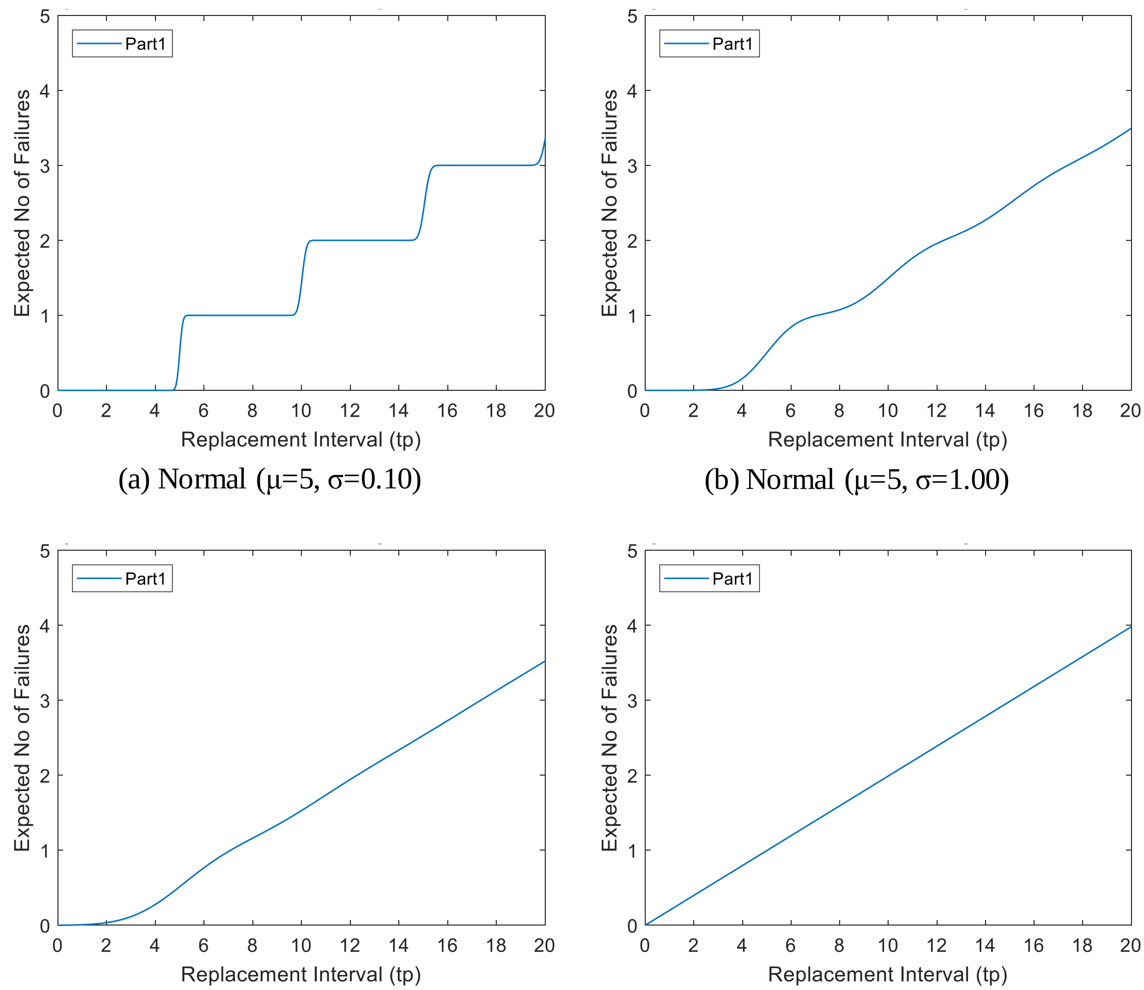
t_p	0	1	2	3	4	5	6
Jardine & Tsang [3] $D(t_p)$		0.0338	0.0173	0.0121	0.0114	0.0139	0.0156
Original Model $D(t_p)$	1	0.03382	0.01725	0.01206	0.01143	0.01391	0.01558
Revised Model $D_A(t_p)$	1	0.03382	0.01724	0.01205	0.01140	0.01381	0.01543

The periodic, non-linear nature of the function is due to the expected number of failures, $H(t_p)$. In the above example, the mean time to failure of the part is five units of time. On average, the first failure occurs at 5 units of time. The model assumes a corrective action after failure at $t=5$. The next failure is then expected to occur five units of time later at $t=10$. In this way, the mean approximates the period of the $H(t_p)$ with the variance of the distribution influencing the amplitude. A failure distribution with a low variance (a pdf that has a pronounced peak and short tails) more closely is periodic at the mean. A helpful non-dimensional measure is the coefficient of variation. The coefficient of variation normalizes the standard deviation with the mean as

$$cv = \frac{\sigma}{\mu} \tag{54}$$

Figure 64(a) shows a low variance distribution with a *cv* of 0.02. In this chart, the failure distribution is normal with a standard deviation of 0.1 units of time. Low variance distributions take a more pronounced step-like shape due to the steepness of the distribution's cdf. Increasing the standard deviation to 1.0 units of time gives a *cv* of 0.2.

Figure 64(b) shows a smoother, less step-like curve as *cv* increases. Increasing the variance to a *cv* of 0.33 creates Figure 64(c). Here, failures occur throughout the interval yielding a flatter curve. The exponential distribution shown in (d) is straight due to its high variance ($Var=\lambda^{-2}=25$) and a *cv* of 1.



(a) Normal (μ=5, σ=0.10)

(b) Normal (μ=5, σ=1.00)

(c) Normal (μ=5, σ=1.67)

(d) Exponential (λ=0.2)

Figure 64: Expected Number of Failures, $H(t_p)$

Since the mean represents the average unit time per failure, the inverse of the mean is failures per unit time, or slope of the $H(t_p)$. The slope of the curve at $t=t_p$ is the instantaneous rate of failure per replacement interval. The mean and the influence of the variance effect the slope. Figure 64(c) and (d) show a normal distribution and exponential distribution with the same mean ($\mu=5$) and same general slope. This holds true if the mean is sufficiently larger than the standard deviation. A best practice provided by Kumar [33], is for the mean to be at least three times the standard deviation ($\mu > 3\sigma$) to ensure the part functions at its starting age. An exponential distribution's slope is the rate of failure, $\lambda=0.2$ failures per unit of time. Both distributions have the same y-intercept by starting at $H(0)=0$. The normal distribution in Figure 64(c) is offset to the right as compared to the exponential distribution in Figure 64(d). The offset indicates that the normal experiences delayed failures early on ($t_p<\mu$) while the exponential distribution has a constant rate of failure.

Understanding the relationship between distribution parameters and $H(t_p)$ provides insight into how failure distributions influence the downtime ratio of a system. Low variance failure distributions have more pronounced dips or lower relative local optima in $D(t_p)$ than higher variance distributions (see Figure 65). It will also have greater peaks or higher relative D. This is an important consideration for the maintenance planner. A low variance distribution will take on greater significance when choosing an optimal replacement interval. A lower variance item introduces greater amplitudes in the system downtime curve making potential selections of t_p either much better or much worse. Figure 65 shows that the selection of t_p of 4.5 units of time yields the global optimum with $D(4.5)=0.0555$. A slight increase to t_p of 5.2 units of time yields local maximum with $D(5.2)=0.0690$. This suggests that, in a highly complex design with many parts and a large dimensionality, the low variance items dominate the sensitivity of t_p. The designer, consequentially, may exclude items with a high variance in time to failure with the purpose of reducing the dimensionality of the problem.

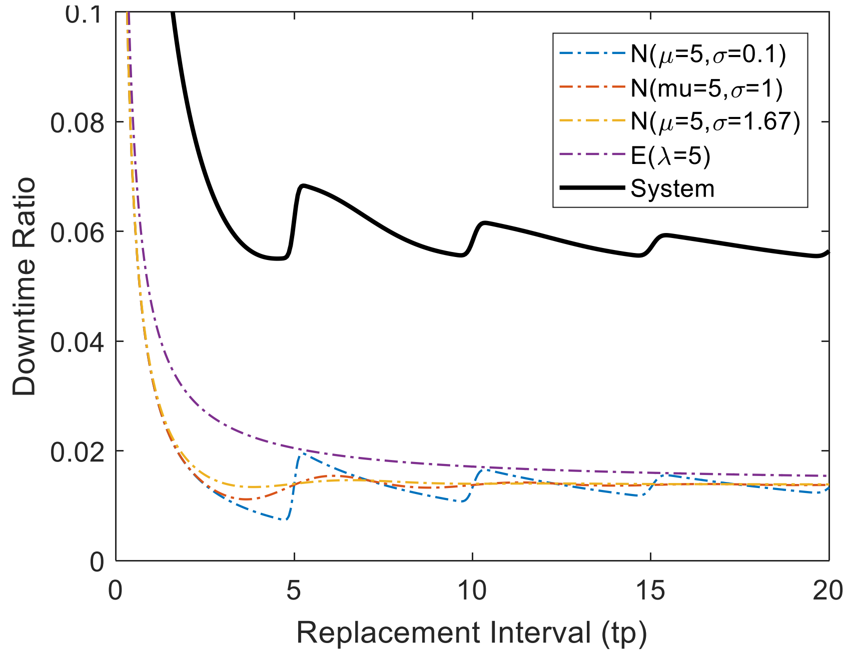


Figure 65: System Downtime Ratio versus Replacement Interval

The limit of the downtime ratio as the replacement interval grows large is

$$\lim_{t_p \to \infty} D(t_p) = \lim_{t_p \to \infty} \frac{H(t_p)T_f + T_p}{t_p + T_p} \tag{55}$$

This limit is dependent upon the rate at which $H(t_p)$ increases over time. With the exponential distribution, we may take advantage of the memoryless property to express the expected number of failures as the integral of the hazard function, *h(t)*, as

$$H(t_p) = \int_0^{t_p} h(t)\,dt = \lambda t_p \tag{56}$$

where *h(t)* is equal to the constant failure rate λ. Jardine and Tsang [3] provide an approximation for the normal distribution as

$$H(t) \approx \frac{t}{\mu} + \frac{\sigma^2 - \mu^2}{2\mu^2} \tag{57}$$

if t_p is sufficiently larger. The second term in (57) creates the right offset discussed above. Other distributions require the use of the recursive formula found in (19) to estimate *H*.

The downtime ratio dampens over time due to the compounding effect of variance in many replaced items. The downtime approaches the limit from above and has a horizontal asymptote. Substituting equations (56) or (57) into (55), yields an approximation of the limit to be

$$\lim_{t_p \to \infty} D(t_p) \approx \frac{1}{\mu} \ T_f \tag{58}$$

where μ is the mean or expectation of the failure distribution. Figure 66 shows the convergence of items towards the limit. In this example, each distribution has a mean of 5 time units per failure with a T_f of 0.07 units of time. Item's individual downtimes converge to

$$\lim_{t_p \to \infty} D(t_p) \approx \frac{1}{5}(0.07) \cong 0.014 \tag{59}$$

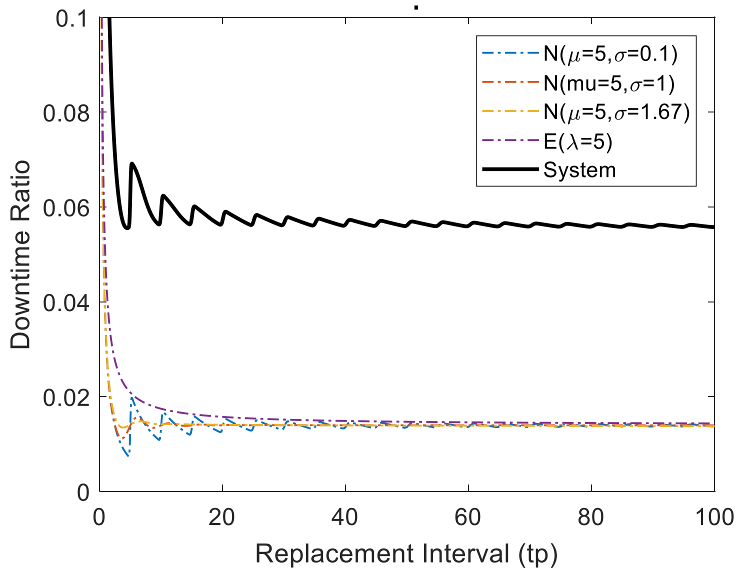


Figure 66: Example Limit of Downtime Ratio

Taking the limit of D as t_p is large, represents an item's downtime ratio without preventive replacement or an item with almost no preventive replacement time, T_p. A policy should preventively replace an item if it has an increasing hazard rate. The intent is to replace the item before it breaks down due to wear. A policy should not replace an item with a constant failure rate (exponential distribution) when attempting to minimize downtime, because the downtime ratio improves towards the limit (see Figure 66). Similarly, a policy should never preventively replace an item with a decreasing hazard rate when attempting to minimize downtime, because the downtime improves over time. Distributions with a decreasing hazard rate include hyper-exponential distributions and Weibull distributions with a shape (β) less than one.

The limit stands as a useful benchmark to ensure that the chosen replacement interval does not worsen the downtime. As shown in Figure 65, it is possible to have a t_p that exacerbates downtime. A maintainer should reject a policy that yields a higher downtime ratio than the limit. A good maintenance policy replaces items near a local minimum if not at its optimal minimum. This results in the best overall system downtime.

5.1.1.2 Limitations of the Current Preventive Replacement Model

Classical renewal theory provides for a method to minimize downtime as a function of a component's preventive replacement interval (t_p); however, the current theory is unsuitable for a MFOP strategy. First, the renewal theory assumes that replacement times due to failure (T_f) are much greater than the replacement interval. This assumption is not necessarily valid for today's helicopters. Section 4.3.2 Construction of a Utility Helicopter Model for further discussion provided an exercise that showed the MMT to be 5.5 hours compared to a MTBM of 5.1 hours. Should FVL make a significant gain in maintainability, the assumption becomes more attractive. Second, current renewal theory considers either single parts or like parts to minimize downtime. A MFOP strategy needs a maintenance policy that handles different components replaced at different intervals. Third, renewal theory permits any range of component intervals that will disruption operations in a complex system of unlike components. Finally, classical renewal theory minimizes downtime only and makes no statement about the reliability performance of the system. Optimization of downtime alone may lead to an unreliable design. The limitations of classical renewal theory establish the need for a modified approach. The following three sections and a later sensitivity study with reliability address the limitations with the development of a new framework.

Table 29: Limitations of Classical Renewal Theory in a MFOP Strategy

MFOP Strategy Need	Classical Renewal Theory Limitations
Include T_f as part of cycle time	Assumes $T_f \ll t_p$. May not be valid (RUH)
Multiple parts	Single part
Synchronize replacements to MRPs	Preventive replacements may disrupt MFOP
Manage the balance between downtime and reliability	Makes no guarantee on sufficient reliability

5.1.1.2.1 Removal of the T_f and T_p Assumption

The original model's assumption that $T_f \ll t_p$ is inappropriate for a helicopter. Rotorcraft typically have an operating period (t_p) measured in hours and repairs (T_f and T_p) that can take hours to days. For example, the rigging of flight controls often takes several days of work after major repair of the system. The construction of the RUH model shows a mean time between EMA of 5.11 hours with a MTTR of 5.5 hours (section 4.3.2). The benchmark suggests this assumption is one to two orders of magnitude from being valid; therefore, the revised model must remove the assumption. A truer assessment of the downtime per cycle should include the repair time lost to unscheduled failures as well as the uptime and downtime due to preventive replacements.

$$D_A(t_p) = \frac{\begin{matrix} Downtime\ due\ to\ failures + \\ Downtime\ due\ to\ preventive\ replacements \end{matrix}}{MFOP\ Cycle\ Time} \qquad (60)$$

where cycle time is

$$MFOP\ Cycle\ Time = MFOP + Downtime\ due\ to\ failures + MRP \tag{61}$$

In this way, the ratio D_A is the Achieved Downtime and represents the percent of cycle time the system is unavailable where cycle time is the sum of the MFOP, MRP, and any downtime due to failures. Achieved downtime is the same as achieved non-availability. Achieved downtime is the compliment of Achieved Availability (A_A) as defined in section 3.1.2.

$$D_A = 1 - A_A \tag{62}$$

A key phrase in A_A's definition is "operating time." In Achieved Availability, an unused aircraft that sits in a hangar or on a ramp does not accumulate operating time. The MFOP duration is like a MTBM where time is operating flight hours. MMT is the average downtime on corrective and preventive repairs. Estimation of MMT is presented in section 4.3.2.3.

Downtime due to failures is the product of the expected number of failures, $H(t_p)$, and the time to replace an unscheduled failure (T_f). Equation (63) adds the term $H(t_p)T_f$ to the denominator in

$$D_A(t_p) = \frac{H(t_p)T_f + T_p}{t_p + H(t_p)T_f + T_p} \tag{63}$$

where H is a function of the operating period. and is found recursively using (19) described in section 3.4.2.

The impact of the additional term changes the definition of the t_p. Unlike the original model, the revised model's operating period clock stops during a repair. The total downtime per unit time (D) of the original model becomes achieved downtime (D_A) in a revised model. In this way, the operating period measures accumulated flight hours and is more reflective of a helicopter's downtime. D_A acknowledges that a system with a MFOPS less than 100% will experience unscheduled failures. This accounting drives the change from downtime per cycle of Equation (17) to achieved downtime of (63). Inclusion of failure repairs creates a slightly larger denominator; hence, the revised model is slightly smaller than the renewal theory's original Optimal Preventive Replacement Model. Table 28 above (page 149) compared the revised model to the original model.

5.1.1.2.2 Expansion to a Multiple Part System

Basic renewal theory and the optimal replacement interval model account for a single item in a system. The proposed optimal MFOP model needs expansion to include multiple items comprising a system. The formulation of equation (63) is advantageous because it permits the system's achieved downtime to be the sum of n component's achieved downtime as

$$D_A(t_p) = \sum_{r=1}^{n} D_{A,r} \qquad (64)$$

Each part's contribution of achieved downtime is

$$D_{A,r}(t_p) = \frac{H_r(t_p)T_{f,r} + T_{p,r}}{t_p + \sum_{r=1}^{n}[H_r(t_p)T_{f,r} + T_{p,r}]} \qquad (65)$$

where each part has its own expected number of failures (H_r), time to repair failures ($T_{f,r}$), and time to make preventive replacements ($T_{p,r}$).

Equation (65) assumes a uniform preventive replacement interval (t_p) for all n parts. A uniform t_p works well when the system is comprised of n identical parts or with parts of similar mean as shown in the example of Figure 65 and Figure 66. A simple way to optimize D_A is to find the common interval, t_p, that minimizes the system downtime. Table 30 shows the part's downtime at specified replacement intervals. Again, the exponential part should never have a replacement to lower downtime; therefore, the exponential part shall have a downtime contribution equal to the limit, 0.014. The last row named System $D_{Asys}(t_p)$ shows the sum of the part downtimes (with the last fixed at 0.014). The first three parts have ideal replacement intervals at t_p's of 4.75, 3.65, and 3.85 units of time, respectively. Let * denote an optimal state. The optimal system D_A* with a uniform preventive replacement interval is 0.0476 at a t_p* of 4.1 units of time. Finally, there is a benefit to preventive replacements. Achieved downtime of the system without preventive replacements ($t_p \to \infty$) increases to 0.0560.

Table 30: D_A of Uniform and Non-Uniform Replacement Interval of Like-System

	Uniform t_p					
	t_p=3.65	t_p=3.85	t_p=4.10	t_p=4.75	$t_p \to \infty$	Ideal Non-Uniform t_p
Norm(μ=5,σ=0.1) $D_{A,1}(t_p)$	0.0095	0.0090	0.0085	0.0074	0.0140	0.0074 (t_p=4.75)
Norm(μ=5,σ=1) $D_{A,2}(t_p)$	0.0112	0.0113	0.0116	0.0132	0.0140	0.0112 (t_p=3.65)
Norm(μ=5,σ=1.67) $D_{A,3}(t_p)$	0.0135	0.0135	0.0135	0.0139	0.0140	0.0135 (t_p=3.85)
Exp(μ=5) $D_{A,4}(t_p)$					0.0140	0.0140 ($t_p \to \infty$)
System $D_{Asys}(t_p)$	0.0482	0.0478	0.0476*	0.0485	0.0560	0.0461**

**Optimal D_A with uniform t_p*
***Optimal D_A with non-uniform t_p*

A uniform replacement interval forces components into an unoptimized condition and results in a higher than necessary D_A. The item replacement interval need not be uniform. Even with the example system of Table 30, a non-uniform preventive replacement interval policy may further improve D_A. Replacing each part at its individual optimal interval as shown in the last column of Table 30 yields a further improved D_A of 0.0461. This represents the unconstrained, global optimum of the system.

Consider a system comprised of Part 1, Part 2, and Part 3 with distributions shown in Table 31 and part D_A plotted in Figure 67. T_f is 0.035 units of time and T_p is 0.07 units of time. The optimal replacement intervals ($t_{p,r}$*) for the parts are 2, 3, and 4 units of time. Like the previous example, the non-uniform replacement interval policy has an improved $D_{A,sys}$ of 0.0455 compared to the uniform replacement interval policy's 0.0502.

Table 31: Three Part System with Non-Uniform Optimal t_p

Part	Failure Distribution	Parameters	Optimal $t_{p,r}$	*Optimal* D_A,*r*($t_{p,r}$)
Part 1	Weibull	η=3.0 β=3.9	2	0.0234
Part 2	Normal	μ=3.6 σ=0.3	3	0.0121
Part 3	Weibull	η=5.5 β=8.0	4	0.0100
$D_{A,sys}(t_{p,r})$				0.0455

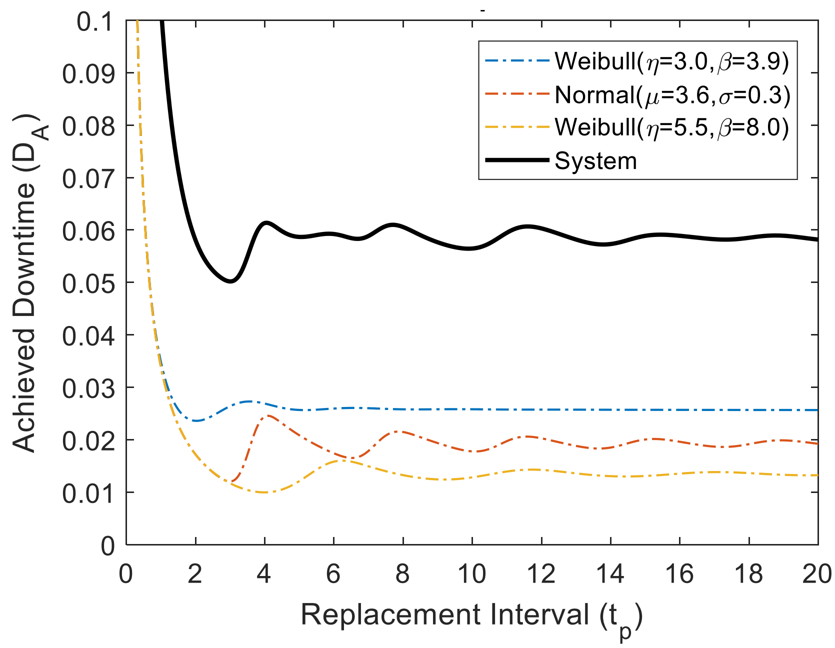


Figure 67: Achieved Downtime of Components with Non-Uniform Optimal t_p

The non-uniform replacement interval policy provides the unconstrained, optimal; however, it may lead to frequent, disruptive scheduled maintenance. Figure 68 draws the operating and downtime of the system and its components. Replacing each component of the example at its ideal interval takes the system offline at t_p values of 2, 3, 4, 6, 8, 9, 10, and 12 units of time as depicted in Figure 68. This prevents a MFOP of no more than two units of time and often one unit of time. The disruptions will be more frequent in a complex system with a variety of part failure distributions. The policy of non-uniform, component optimal replacements represents today's paradigm of preventive maintenance. Although it yields the ideal achieved downtime, it is not supportive of a MFOP strategy.

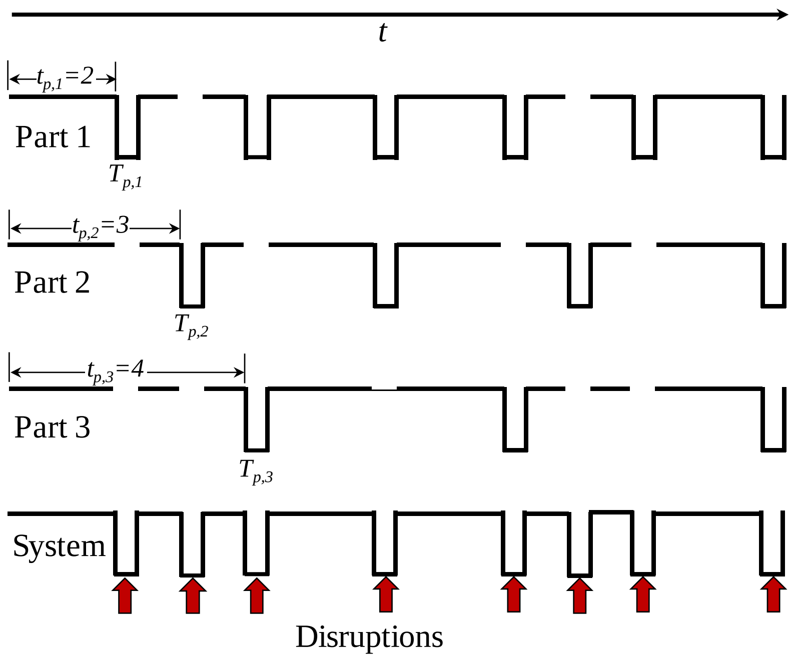


Figure 68: Disruptive Intervals in a Non-Uniform Replacement

5.1.1.2.3 Synchronizing Replacement Intervals to Create a MFOP

A MFOP policy should synchronize item replacement intervals at MRPs to protect the MFOP from disruption. A policy does not have to replace the part at each MRP but it must replace the part in a MRP. Clustering preventive repairs involves the extension or shortening of items' replacement intervals such that the interval $t_{p,r}$ is a multiple of the MFOP duration (t_{mf}) as

$$t_{p,r} = \alpha_r t_{mf} \tag{66}$$

where α_r is a whole number multiple of t_{mf} for the rth part. α_r must be a whole number multiple to synchronize preventive maintenance into MRPs. Figure 69 shows an example synchronization of the three-part system shown earlier. $t_{p,1}$ is extended from two to three units of time and $t_{p,3}$ is extended from four to six units of time. Multipliers of α_1 = 1, α_2 =

1, and α_3 = 2 create a MFOP of three hours. Figure 69 shows a maintenance policy where the sequence repeats every two MFOP cycles and six units of time. The duration of the maintenance policy sequence is the least common multiple of the system's set of α_r and is

$$t_{mp} = \alpha_{lcm} t_{mf} \tag{67}$$

where t_{mp} is the duration of the maintenance policy sequence and α_{lcm} is the least common multiple of all α_r*'s*. In the below example, t_{mp} is 6 units of time.

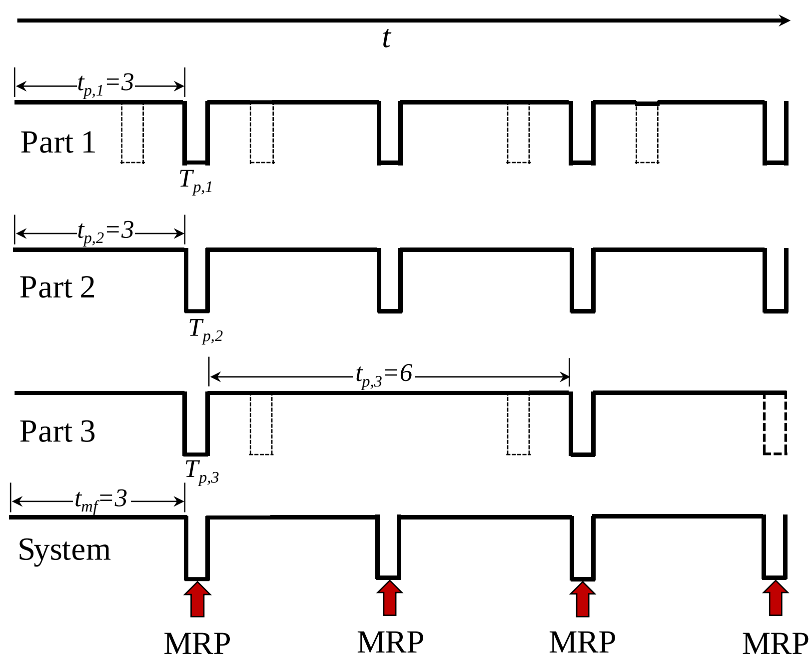


Figure 69: Synchronized Intervals of Replacement

Substituting equations (66) and (67) into equation (65) yields the achieved downtime for a synchronized policy. Summation of n item's D_A is

$$D_A(t_{mf}, \alpha) = \frac{\sum_{r=1}^{n} \left(\frac{\alpha_{lcm}}{\alpha_r}\right) [H_r(\alpha_r t_{mf}) T_{f,r} + T_{p,r}]}{\alpha_{lcm} t_{mf} + \sum_{r=1}^{n} \left(\frac{\alpha_{lcm}}{\alpha_r}\right) [H_r(\alpha_r t_{mf}) T_{f,r} + T_{p,r}]} \tag{68}$$

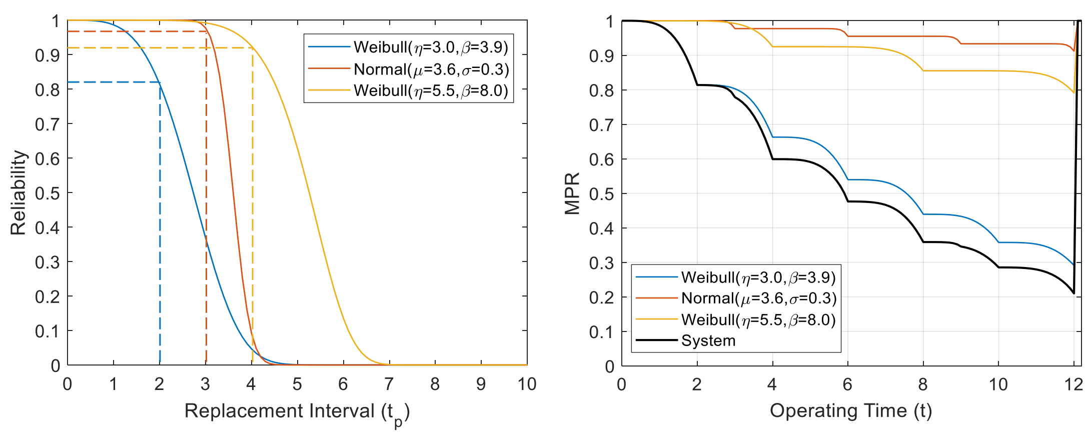
Although α_{lcm} may be divided out, (68) leaves the term to communicate the notion that achieved downtime equals downtime divided by the sum of operating time and downtime. The model considers downtime to be the sum of repair time of all components and all preventive replacements occur in a MRP. A maintainer may defer the scheduled replacement of a still operating item or a non-mission critical failed item from the current MRP to the next. This model does not account for any deferred maintenance. Equation (68) provides D_A given the policy replaces all items in the MRP as dictated by α.

In the discussion above, the renewal theory model is for optimal replacement intervals. Replacements are made with new items, thereby renewing the system. Part renewal may originate from either replacement or repair. The term "replacement interval" refers to a full renewal whether replacement or repair. The theory only requires complete renewal of the component. Partial repairs or installment of partially worn parts means an adjustment to the calculation of expected number of failures (H) and is not addressed in this work.

5.2 Establishing the Need for a Framework

Recall that an acceptable MFOP policy of the revised renewal theory must: (1) protect the MFOP; and (2) balance the desire for low downtime with the requirement for high MFOPS. Neither the uniform or non-uniform models provide a sufficient policy for a MFOP strategy. The uniform replacement interval model protects the MFOP by forcing components outside their optimal setting. Replacing unlike components of a complex system at the same interval may drive reliability low or create an unrealistic maintenance burden with excessive O&S. The non-uniform replacement interval model provides the system's optimal downtime, but it may lead to disruptive scheduled maintenance that destroys the MFOP.

The inclusion of component replacement multipliers, α_r, protect the MFOP; however, it provides no assurance of a reliable system. Figure 70(a) below shows the reliability of each component in the three-part system. Figure 70(b) draws the reliability of the serial system over the life of the policy. In this case, the reliability of the system is low due to the unreliability of the first part. We can conclude that the desire for sufficient reliability as expressed as mission reliability or MFOPS adds a constraint to the optimization of downtime. The models by themselves, therefore, are incomplete in meeting the needs of a MFOP strategy.



(a) Component Reliability (b) Maintenance Policy Reliability

Figure 70: 3-Part System

With both models shown as insufficient, there exists a need for a new way to build policies that meet the final two MFOP needs. This work proposes to use α_r multipliers to protect the MFOP and a framework to balance downtime and MFOPS. The revision of renewal theory permitted the discover of the necessary steps that shaped the framework developed in the next section.

5.3 Framework to Designing a MFOP Maintenance Policy

The framework to design a maintenance policy has three major steps (Figure 71) The framework begins with defining the current system and MFOP setting goals. Principles specific to a MFOP strategy guide the construction of a MFOP policy. Finally, the framework calls for evaluation of the policy for sufficient reliability and acceptable achieved downtime (D_A).

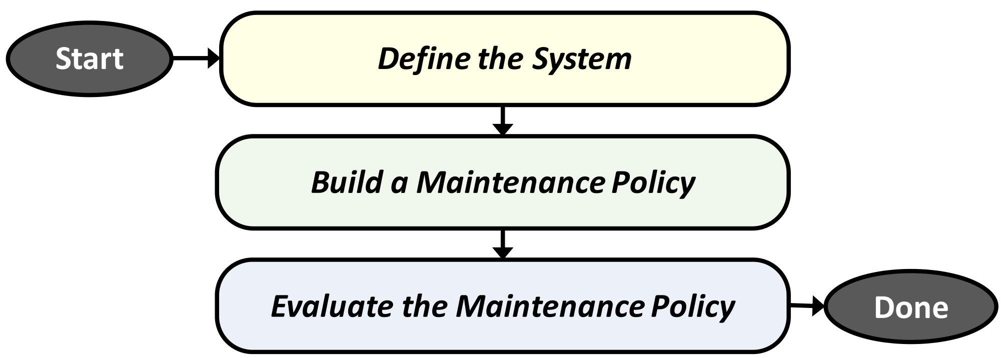


Figure 71: Overview of Designing a Maintenance Policy

5.3.1 Define the System

The first action in developing a MFOP maintenance policy is the definition of the system. Component TTF distributions assembled in a system architecture determine the system's reliability. T_f and T_p are the time to renewal a part, by repair or replacement, under corrective or preventive maintenance. T_f and T_p are fixed values in the revised renewal theory model. Use of the mean time to repair is acceptable for the times. Use of time to repair distributions is possible in a simulation environment but is unnecessary because repair times are additive and the solution will tend towards the mean. Given that unscheduled maintenance is disruptive and unexpected, most components will have a T_f equal to or greater than T_p. This is especially true in a MFOP strategy where logistic and administrative delays are small with the predictability of the MRP.

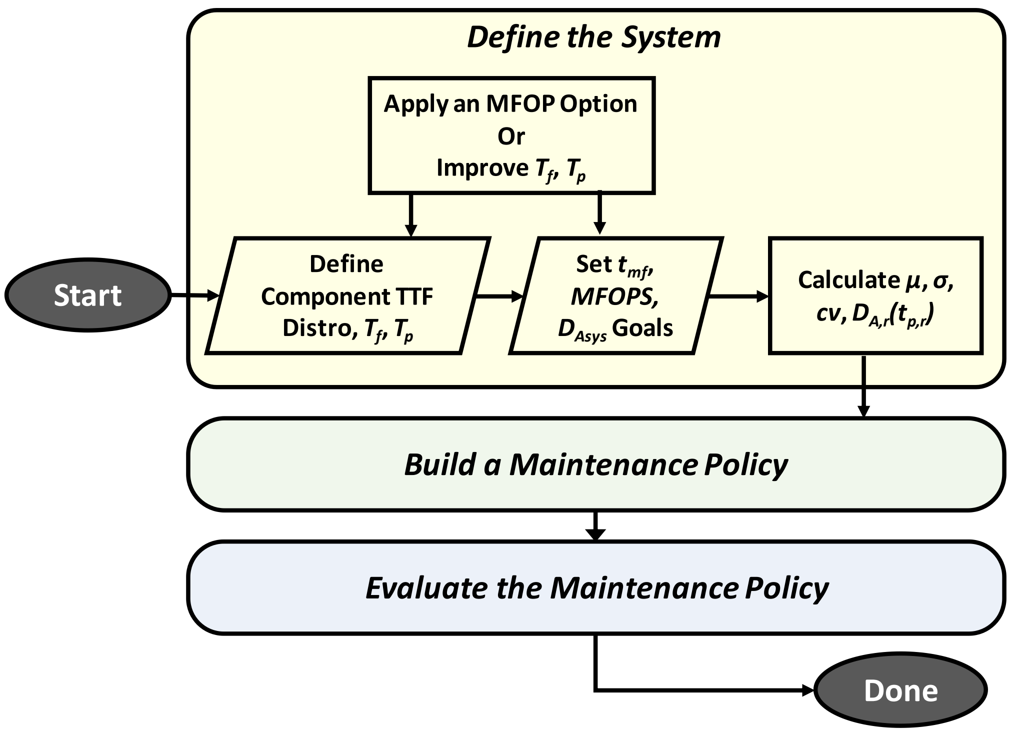


Figure 72: Define the System

The next input to the define action is establishment of the MFOP duration (t_{mf}) and the minimally sufficient MFOPS. The two goals establish the performance needed from the policy. The last process in the define action is to calculate the mean (μ), standard deviation (σ), and coefficient of variance (cv) of each component TTF distribution. From here, each component contribution to achieved downtime ($D_{A,r}$) may be created as a function of the component's replacement interval ($t_{p,r}$).

No decisions are made within the first action related to the maintenance policy. A designer may apply a MFOP option (see section 3.2.3) within the iterative design loop. The framework assumes the operational unit has a fielded aircraft already built and designed. In this way, the framework provides an adaptability to a maintenance strategy that can accommodate different policies to meet changing operational needs. It is possible and may be necessary to conduct a redesign iteration of a subsystem or aircraft if MFOP and availability goals cannot be met with an affordable maintenance policy.

5.3.2 Build a Maintenance Policy

Once the designer defines the system, the policy designer may start to build the policy following Figure 73. Equation (63) provides the first calculation of component's optimal interval, $t_{p,r}$*. Each interval should then be checked with Equation (65). Fixed point iteration using (65) provides a converged solution. The policy is a function of selecting the multipliers α_r and t_{mf} as discussed below.

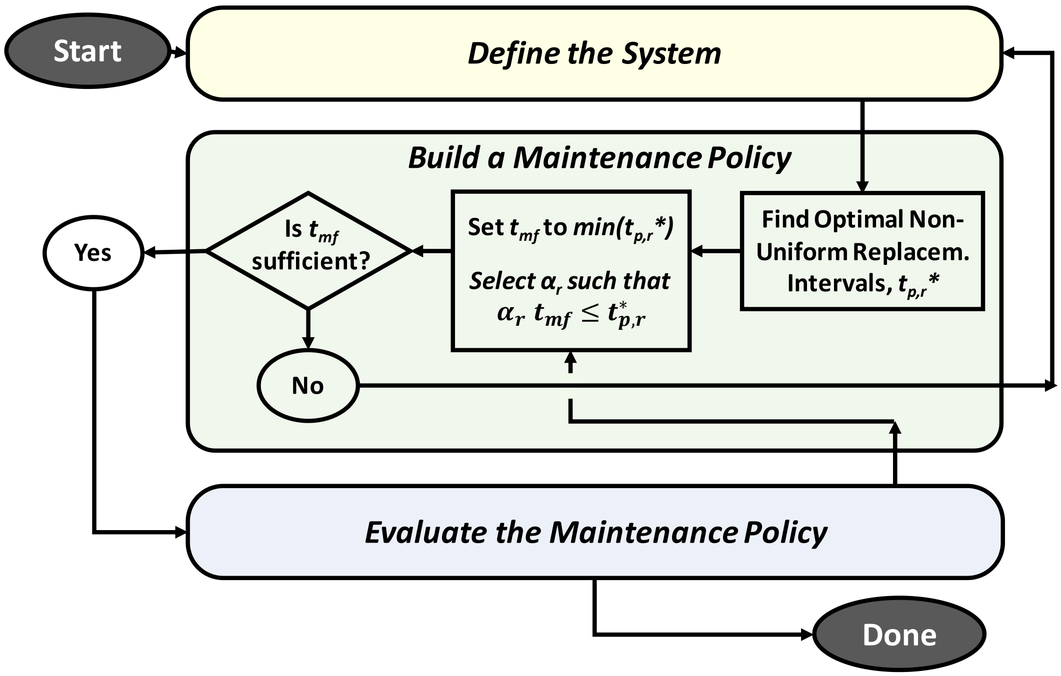


Figure 73: Build a Maintenance Policy

Following the best practice for a reliable design (see section 5.6.1), each component's $t_{p,r}$ should not be extended beyond $t_{p,r}$*. Following this logic, the component with the least $t_{p,r}$* establishes the upper limit for the design's MFOP duration, t_{mf}. The t_{mf} should be set to the minimum $t_{p,r}$* of all components. The multiplier for the component with the minimum $t_{p,r}$* has a multiplier of one. The designer should then select multipliers for the remaining components such that

$$\alpha_r\ t_{mf} \le t^*_{p,r} \qquad (69)$$

to ensure the component is not extended to the point of unreliability.

If the minimum $t_{p,r}$* is less than the MFOP duration goal, the policy designer will have to adjust expectations for the MFOP duration or apply a MFOP Option to improve system reliability. For component redesign, increasing the MTTF provides the greatest gains. Shifts in the variance can alleviate smaller gaps with a carefully chosen $t_{p,r}$. If the minimum

$t_{p,r}$* is equal to or greater than the MFOP goal, the policy designer may move to the evaluation action.

A recurring theme is that the mean and coefficient of variation (*cv*) largely measure the downtime of a component. A lower a component's *cv*, the more crucial the quantify $(\alpha_r t_{mf})$ should approach $t_{p,r}$*. A *cv* close to zero causes a greater amplitude centered about the limit while a *cv* close to one has small amplitudes. Figure 74 is reproduction of Figure 65 where the mean of each component is 5 units of time. Figure 74 shows the *cv* of each component.

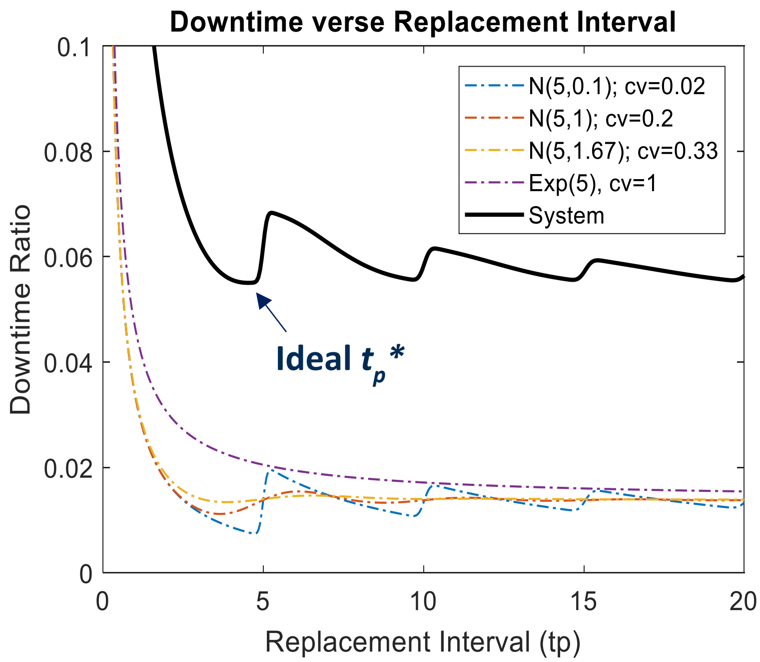


Figure 74: Selecting the Ideal t_p*

A low *cv* describes a distribution where the failure is likely only close to the mean. As the *cv* approaches zero, the expected number of failures just prior to t_p^- (left hand limit) is zero and the expected number of failures just after t_p^+ (right hand limit) is one. The amplitude is one half the difference of the downtime just prior to and just after t_p.

$$\lim_{cv \to 0} a = \frac{1}{2}\left[\frac{T_f \lim_{t \to t_p^+} H(t)}{t_p + T_p} - \frac{T_f \lim_{t \to t_p^-} H(t)}{t_p + T_p}\right]$$

$$\lim_{cv \to 0} a = \frac{1}{2}\left[\frac{T_f(1)}{t_p + T_p} - \frac{T_f(0)}{t_p + T_p}\right]$$

$$\lim_{cv \to 0} a = \frac{{}^{1}/_{2} T_f}{t_p + T_p} \quad (70)$$

The amplitude a captures the extremes that the systems takes about the mean. The amplitude provides an informative measure to evaluate which parts are most important to optimize close to its $t_{p,r}$*. The ideal design point to select is on the left side of the t_p*.

5.3.3 Evaluate the Maintenance Policy

The final action is an evaluation of the maintenance policy against MFOPS and D_A.as shown in Figure 75. The Minimum Policy Success (MPS) is the worst MFOPS through k cycles

$$MPS = \min(MFOPS\ from\ i = 1\ to\ k\ cycles) \quad (71)$$

The policy must maintain the system's MFOPS above the required MFOPS throughout k cycles where

$$MFOPS_{req} \leq MPS\left(0 \leq t \leq k\ t_{mf}\right) \quad (72)$$

If the MPS is less than the MFOPS goal, than the policy is insufficient and needs a redesign. The failure in MFOPS is a function of the system's mission reliability. A technique to identify limiting components in a serial system is to calculate the mission reliability of each component for the duration of $\alpha_r\ t_{mf}$. Failure Cause Identification (see section 4.2) is a more robust method that rank orders components by *A(t)* or *FCM(t)* for any system architecture. The policy designer reduces the weakest component's multiplier. If the weakest component is also the component with the minimum $t_{p,r}*$, then the MFOP duration (t_{mf}) must be reduced. Either change creates a new policy. If the new t_{mf} is insufficient or the new multiplier is undesirable, then the weakest component requires improvement through a MFOP Option. If the MPS is acceptable, then the evaluation continues.

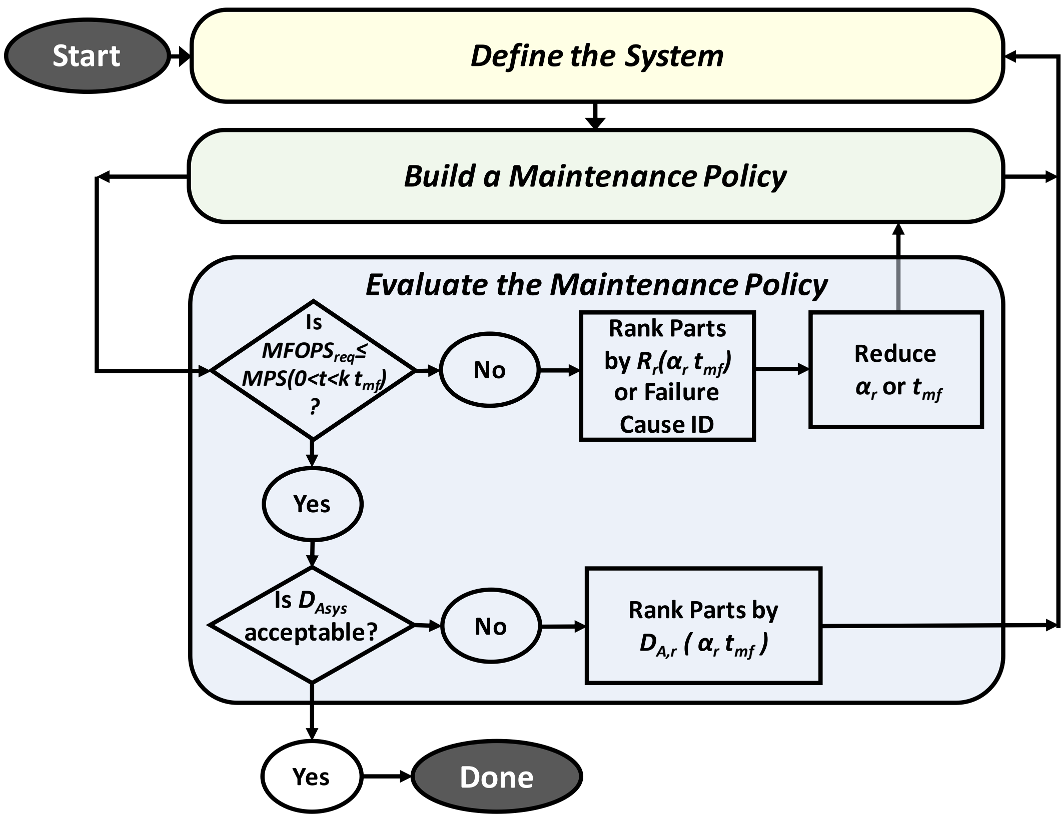


Figure 75: Evaluate the Maintenance Policy

The last check is against the achieved downtime, $D_{Asystem}$. If the downtime is unacceptable, then a redesign of the system is necessary using a MFOP Option or improving component maintainability (T_f or T_p). A redesign of the policy itself will only worsen achieved downtime, because the system was set at its optimal achieved downtime at the first iteration using Equation (69). A MFOP Option or maintainability improvement is necessary. Candidate components for redesign should by their downtime contribution ($D_{A,r}$) using Equation (65). Once t_{mf} and *MPS* are sufficient and D_{Asys} is acceptable, the policy meets the operational demands and the process is complete.

5.3.4 Assembling the Framework to Designing a Maintenance Policy

Figure 76 on the next page assembles the detailed framework from the outcome of the above sections. The framework has three feedback loops that trigger actions in an iterative manner:

1. The first loop occurs if the policy's t_{mf} cannot meet the target goal. The build action block provides the framework to create a policy with the highest t_{mf} where all preventive maintenance occurs in the MRP. A redesign of the system using one or more MFOP Options is necessary to achieve a higher t_{mf} without disrupting the MFOP.
2. The second loop occurs if the MPS cannot meet the MFOPS goal for its expected duration of *k* cycles. The resolution is a lowering of the multiplier(s) or the t_{mf}. If the new t_{mf} is below the MFOP goal, this triggers the first feedback loop.
3. The third loop occurs if a system has an unacceptable achieved downtime. In this case, improving the maintainability of the system by lowering the replacement times (T_f and T_p) is appropriate. A second choice is to select a MFOP Option that improves component reliability.

The framework serves as a guide to building an acceptable maintenance policy that meets the operational requirements for MFOP and MFOPS while maximizing availability.

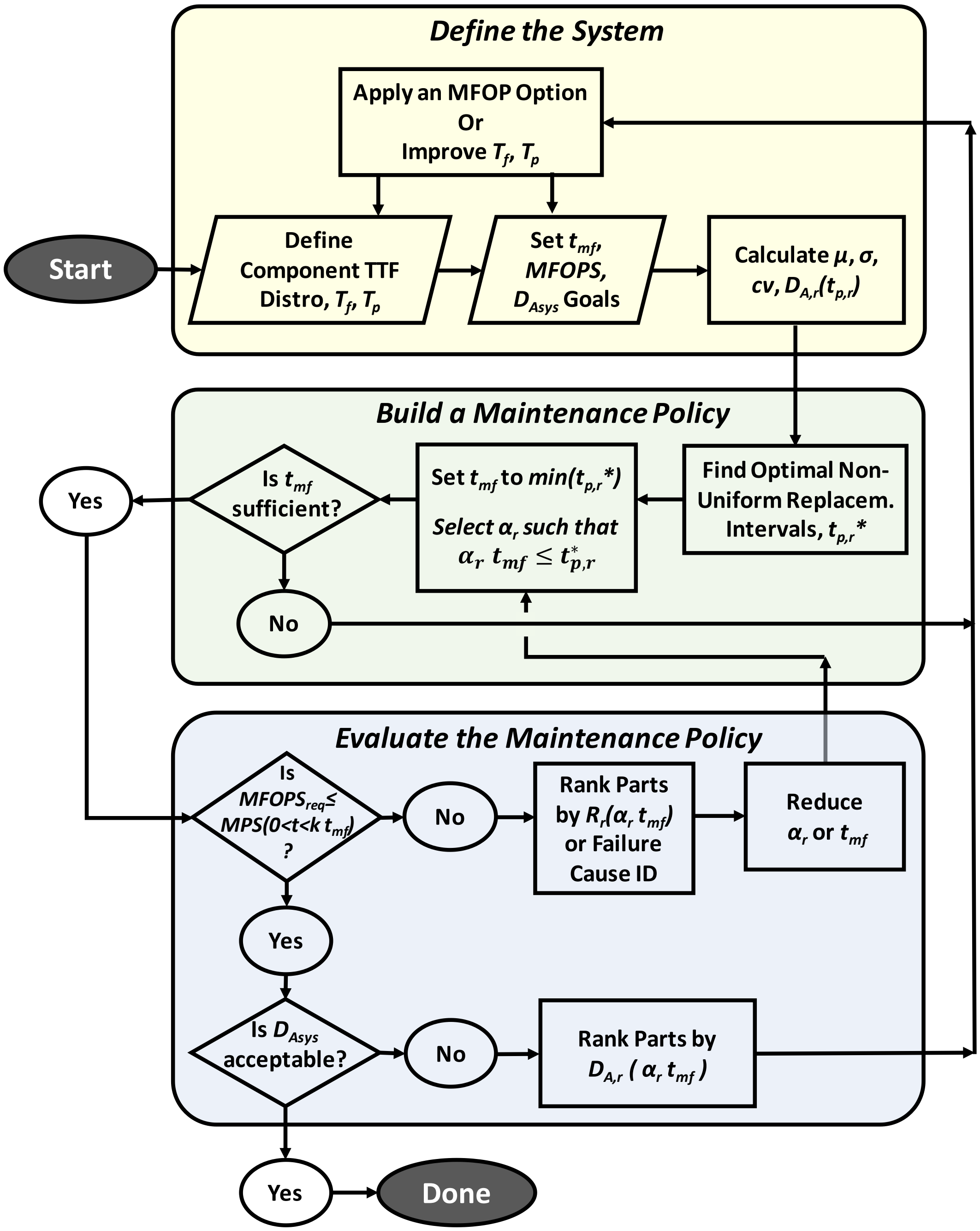


Figure 76: Framework to Design a Maintenance Policy

5.4 Using the Framework to Satisfy a MFOP Strategy

Table 32 summarizes the four approaches introduced in the chapter. Classical renewal theory's optimal replacement interval model provides a means to minimize downtime for a single part only and is unsuitable for a MFOP strategy. The uniform replacement interval model resolves several of classical renewal theory's limitations. It protects the MFOP and minimizes downtime only by forcing each components replacement at the same MRP. The non-uniform replacement interval model provides the global downtime solution but permits disruptive scheduled maintenance that does not protect the MFOP. Neither the uniform or non-uniform models provide assurance of a sufficient reliability or MFOPS.

Table 32: Review of Models to Minimize Downtime in a MFOP Strategy

MFOP Strategy Need	Classical Renewal Theory	Author's Revised Models		
		Uniform Replacement Interval	Non-Uniform Replacement Interval	α_r Multipliers with Framework
Minimizes Downtime	✓	✗	✓	constrained
Include T_f as part of cycle time	✗	✓	✓	✓
Multiple parts	✗	✓	✓	✓
Synchronize replacements to MRPs (protects the MFOP)	✗	✓	✗	✓
Manage the balance between downtime and reliability	✗	✗	✗	✓

The Framework to Design a Maintenance Policy resolves the shortcoming. The policy starts at the unconstrained downtime minimum and changes replacement multipliers iteratively until the policy meets the MFOPS constraint. In this way, the policy built by the framework seeks the constrained downtime solution by exploring designs away from the global optimum.

5.5 Experiment Plan

The first hypothesis utilized a discrete event simulation that assumed a rudimentary maintenance plan without optimization to maximize availability. The second hypothesis postulates that an unconstrained maintenance policy using renewal theory's Optimal Replacement Interval model will maximize availability. The experiment continues by examining the implications of the above model on reliability and MFOP. It will test the framework to build a constrained maintenance policy that synchronizes preventive maintenance to the MRPs and protects the MFOP from disruption. This is done using the revised renewal theory developed in the thesis.

> **Research Question 3**
>
> *What is the maintenance policy that minimizes downtime?*
>
> **Hypothesis 2:** An aggressive lifing policy in the MRP can maintain an MFOPS to a required success over a certain number of cycles.

The framework and revised renewal theory model will be tested on a simple three-part system. A baseline materiel availability for the non-preventive repairs will be compared to the materiel availability of an Optimal Preventive Replacement Interval model as well as revised model.

The experiment has three major assumptions. First, a necessary condition of the renewal theory modeled is that only one failure occurs in a discrete interval. A one-hour interval is assumed but may be shortened to 1-minute to mitigate the chance of multiple failures in one interval. Second, no logistical or administrative downtimes will be included. This is a conservative assumption because preventive maintenance provides predictability to part demand. Units may pre-order parts and store them for use at the next MRP thereby reducing the logistical downtime. A final assumption is that the system has a significant portion of the components experiencing aging, which causes unreliability to increase with usage. This precludes a system comprised of all exponential distributions and constant failure rates. Success is the improvement in downtime while still meeting a MFOPS goal.

5.6 Results and Discussion

5.6.1 Using Coefficient of Variation for Diagnosis

In regards to components qualifying for preventive replacement (those with an increasing hazard rate), the earliest local downtime minimum is always that component's individual, global downtime minimum. This occurs because the earliest minimum does not contend with the compounding replacement time of previous failures. Reliability decreases when extending a component beyond its optimal replacement interval. The drop becomes precipitous in components with a low coefficient of variation.

Figure 77 shows the decrease in reliability after exceeding the optimal replacement interval. Coefficient of variation for the presented system are Part 1 *cv* of 0.287, Part 2 *cv* of 0.083, Part 3 *cv* of 0.148. Low coefficient of variation components, like Part 2's normal distribution, are less likely candidates for extension beyond its optimal replacement interval. Extending a component with a low coefficient of variation beyond the optimal interval results in rapid decrease of reliability. A designer may offset the effects with redundancy or redesign of the component's inherent reliability. Components with higher coefficient of variation are candidates for a MFOP options of prognostics or diagnostics to reduce the uncertainty of failure.

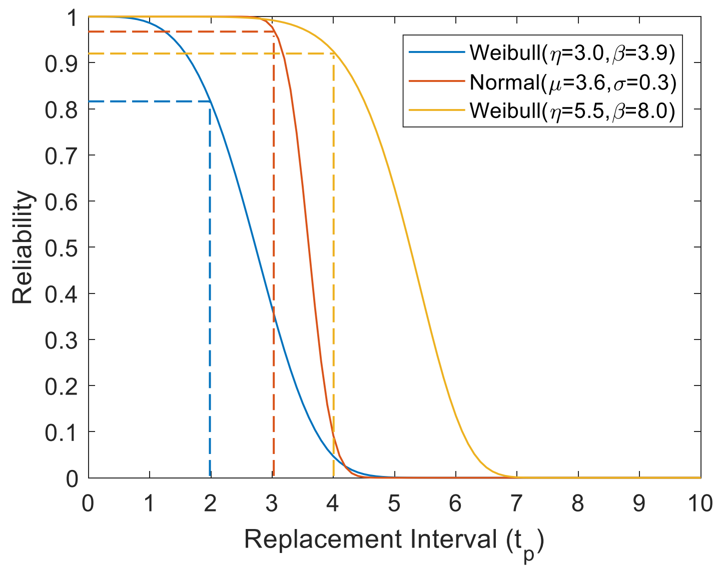


Figure 77: Component Reliability verse Replacement Interval

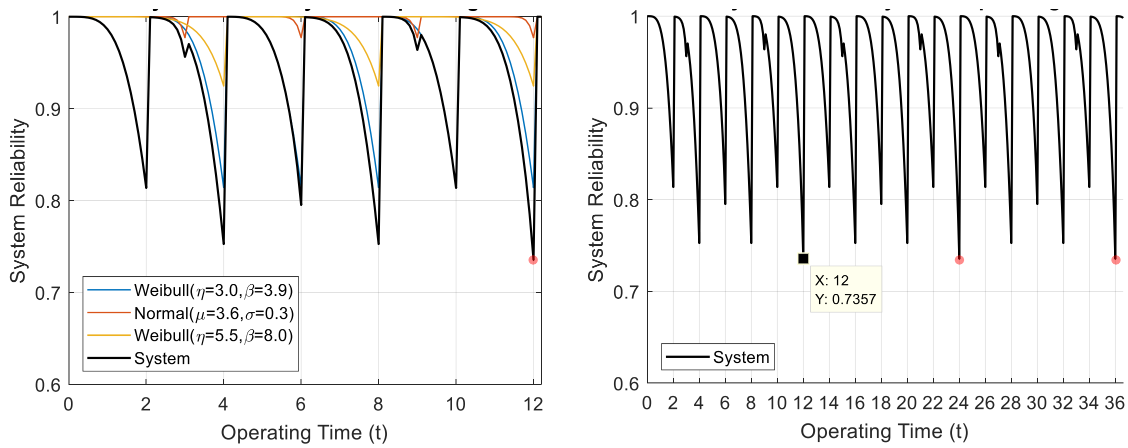
5.6.2 Examining Success of a Maintenance Policy

Selecting part replacement close to its optimal interval yields the least downtime and is the recommended strategy to ensuring high reliability. Extending the component to later local minimums increases the probability of component failure. This places a practical limitation on the extension of the replacement interval and MFOP. The least reliable component is the limiting component. Such a component becomes a candidate for improved inherent reliability, redundancy, or other MFOP options.

5.6.2.1 Creating the Maintenance Policy

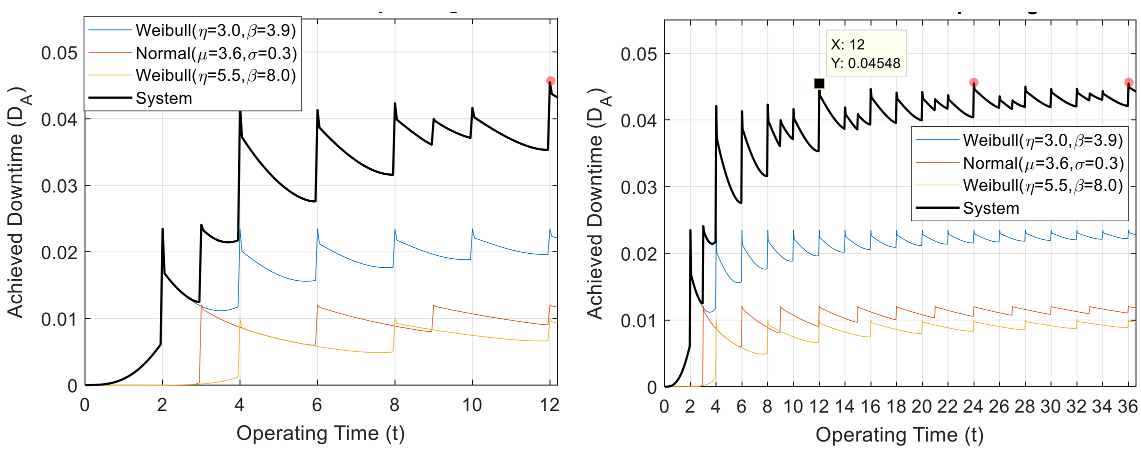
Recall the 3-Part System with a non-uniform replacement interval policy as introduced in Figure 68 (page 163). The maintenance policy duration, t_{mp}, is 12 units of time. The system has components arranged in serial; therefore, the system reliability is the product of component reliability. Component reliability is a function of its replacement interval ($t_{p,r}$). If a policy replaces the component every MRP, then the multiple (α_r) is one and $t_{p,r}$ equals the MFOP duration (t_{mf}). Figure 78 and Figure 79 show the achieved downtime and system reliability. Subfigure (a) is for a single policy and subfigure (b) is over three policy sequences.

The step-like improvements of D_A are the result of component replacement at discrete intervals. The policy duration (t_{mp}) is 12 units of time. Figure 79(a) shows a single policy and (b) shows the policy repeats every 12 units of time. Part 1 is the greatest contributor to downtime of the system. A component's downtime is due to the cost of failure replacement (T_f), expected number of failures (H), and cost of preventive replacement (T_p), and frequency of preventive replacements (α_r). In the example system, all parts have like T_f and T_p. Part's 1 high downtime is due to the lower reliability of Part 1, which raises H and drives a lower multiple α.



(a) Single Sequence (b) Three Sequences

Figure 78: Reliability of a Non-Uniform Optimal Interval Policy



(a) Single Sequence (b) Three Sequences

Figure 79: Achieved Downtime of a Non-Uniform Optimal Interval Policy

The beginning D_A of follow on sequences have a peak value of 0.0455. The peak remains the same at the start of each policy, signifying a full renewal. Cumulative effects of failure variability shrink D_A's amplitude over time (see Figure 80). With enough time, the system converges to the theoretical solution predicted by the revised renewal theory model and Equation (73). The converged values of 0.0234, 0.0121, and 0.010 match those of the theoretical prediction found in Table 31. This verifies the revised model's accuracy as a predicting means of achieved downtime. The benefit of a plot like Figure 80 is that the maintainer can see the early effects of a new policy before it reaches steady state. With short term deployments or large α_{lcm}, the aircraft may never reach the end of the policy and steady state. This has application to FVL where an aircraft deploys for weeks or months under a higher operational tempo and then redeploys to a lower tempo.

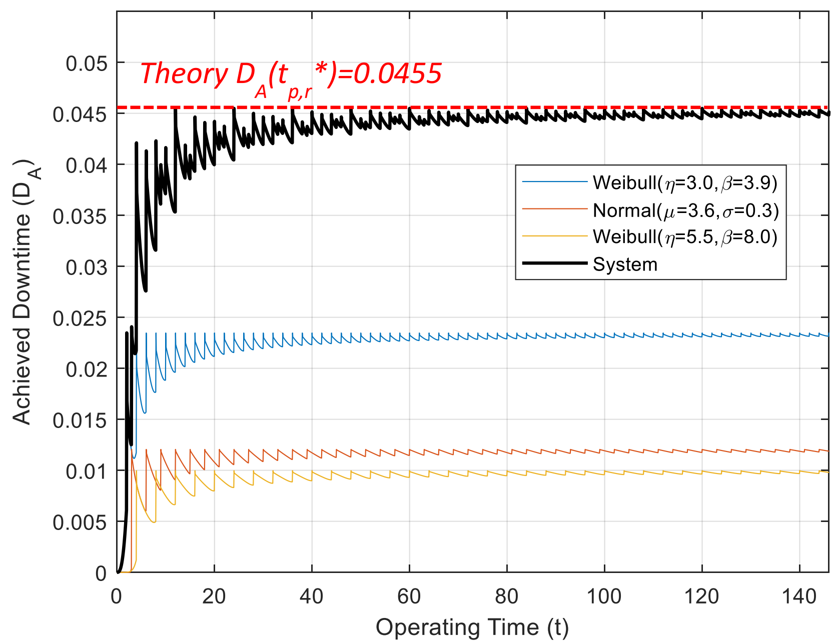


Figure 80: Steady State of Non-Uniform Optimal Interval Policy

5.6.2.2 <u>Measuring Maintenance Policy Reliability and Success</u>

The reliability of a component at time *t* that has survived *k* replacement intervals is

$$R_r(t) = R(t_{p,r})^k R(t - k\, t_{p,r}) \qquad (73)$$

where $t_{p,r}$ is the replacement interval and $k\ t_{p,r} < t < (k+1)\ t_{p,r}$ [64]. To define the rth component's reliability in terms of the MFOP duration, t_{mf}, substitute (66) into (73) to yield

$$R_r(t) = R\left(\alpha_r\ t_{mf}\right)^k R\left(t - k\ \alpha_r\ t_{mf}\right) \tag{74}$$

The equation represents the probability that component r survives to time t. Note k equals α_{lcm}/α_r if the component r survives one sequence of the maintenance policy that is a duration t_{mp}. To assembling component reliabilities of (74) into reliability of a serial system reliability use

$$R_{sys}(t) = \prod_{r=1}^{n} R_r(t) \tag{75}$$

Evaluating the R_{sys} at t_{mp} represents the probability the system survives to the end of the maintenance policy without failure

$$\mathrm{P}\left(TTF \geq t_{mp}\right) = R\left(t_{mp}\right) \tag{76}$$

This is the Maintenance Policy's Reliability (MPR). Substituting (74) into (76) and evaluating at t equal to t_{mp} forms the expression

$$\mathrm{MPR}\left(t_{mp}\right) = \mathrm{P}\left(TTF \geq t_{mp}\right) = R_r\left(\alpha_r\ t_{mf}\right)^{(\alpha_{lcm}/\alpha_r)} \tag{77}$$

Equation (75) or an appropriate reliability block diagram estimates the value. A serial system is

$$\mathrm{MPR}\left(t_{mp}\right) = \prod_{r=1}^{n} R_r\left(\alpha_r\ t_{mf}\right)^{(\alpha_{lcm}/\alpha_r)} \tag{78}$$

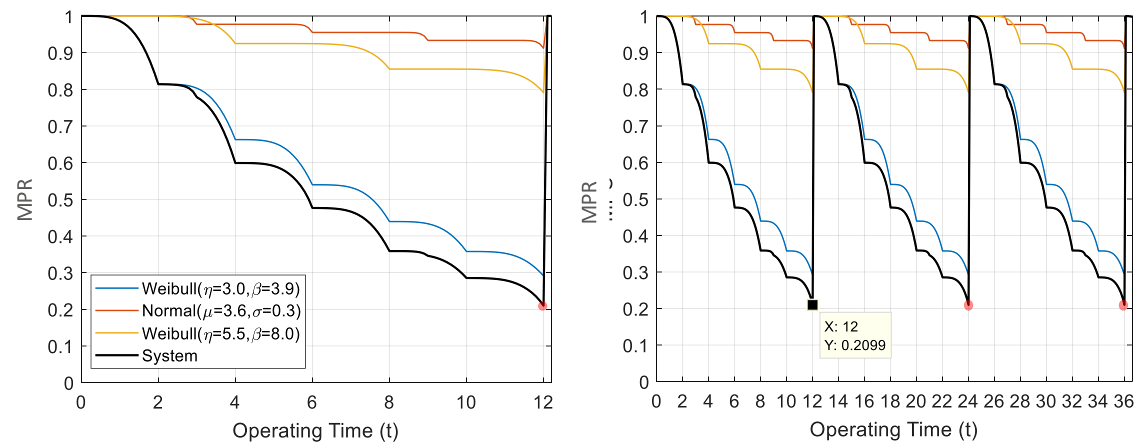
System reliability and MPR are both reliability measurements. System reliability is aircraft's reliability at any instantaneous point of time. MPR measures the cumulative probability the system survives from the start of the policy through the entire life of the policy, t_{mp}.

In the 3-Part Non-Uniform Optimal Interval system, the t_{mf} is one unit of time. The policy renews parts at multiples of 2, 3, and 4. The least common multiple is 12. Table 31 lists the part reliabilities and the probability. There is a 0.2099 probability that the system completes the maintenance policy without a failure.

Table 33: Three Part System with Non-Uniform Optimal t_p

Part	Failure Distribution	$t_{p,r}$	α_r	α_{lcm} / α_r	$R_r(\alpha_r\, t_{mf})$	*MPR*
Part 1	Weibull η=3.0, β=3.9	2	2	6	0.8141	0.2911
Part 2	Normal μ=3.6, σ=0.3	3	3	4	0.9772	0.9119
Part 3	Weibull η=5.5, β=8.0	4	4	3	0.9247	0.7909
System						0.2099

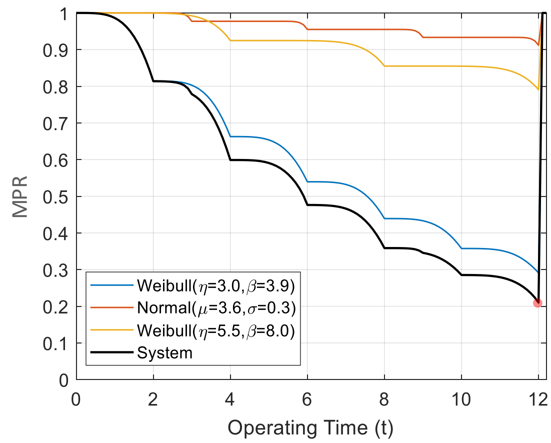
Figure 81 plots MPR of the three-part system at each MFOP cycle. It illustrates of the maintenance policy's performance over time. The plot clearly shows that Part 1 is limiting the system's performance. This makes the component a candidate for improvement through a MFOP Option.



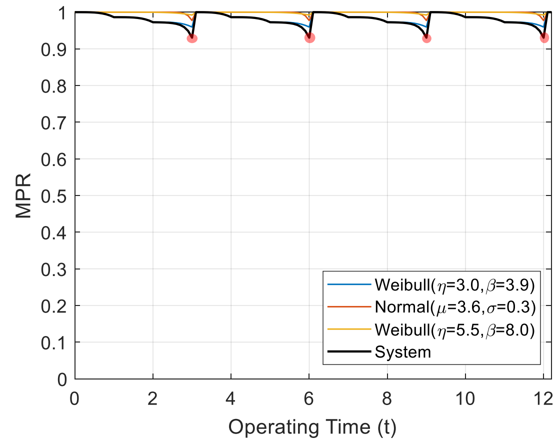
(a) Single Sequence (b) Three Sequences

Figure 81: Maintenance Policy Reliability of a Non-Uniform Optimal Interval Policy

The MPR allows the policy designer to view the policy's effects on component and system reliability. From the figure, Part 1 is driving the system reliability downward. A policy may improve MPR by increasing the frequency of replacement of Part 1. Adjusting α_1 from two to one yields a better performing system; however, the penalty is the reduction of the MFOP from two units of time to one unit of time. Figure 82 shows the policy reliability for an improved policy with α_1=1, α_2=3, and α_3=3. The price paid for better success is the reduction of MFOP to one unit of time along with a doubling of the number of Part 1 replacements and a 33% increase in Part 3 replacements.



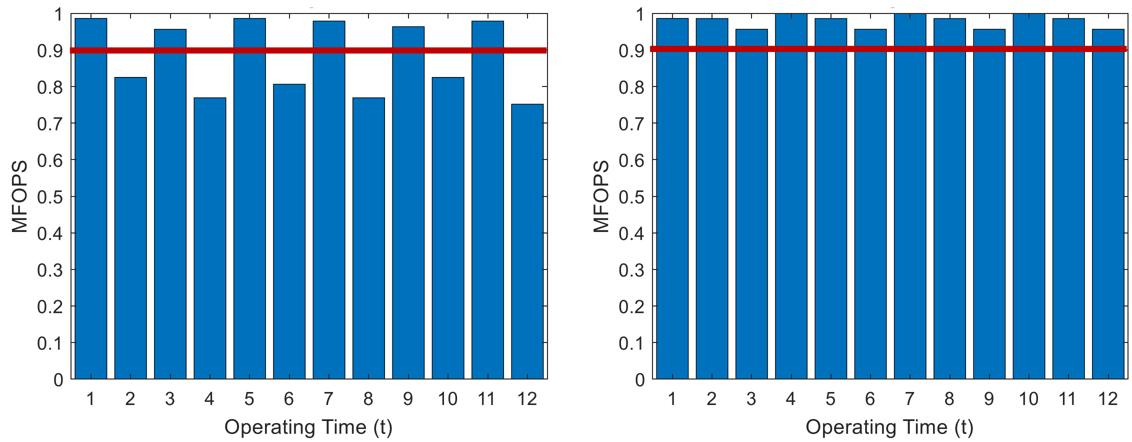
(a) 2-3-4 Design



(b) 1-3-3 Design

Figure 82: Maintenance Policy Reliability Comparison

An insufficient probability indicates the policy is not making enough replacements of one or more components or the t_{mf} is too long. The performance of the system under a policy is denoted as the Minimum Policy Success (MPS). MPS is the minimum MFOPS over k cycles. MFOPS and the Minimum Policy Success communicate different information. MFOPS looks at the probability of completing the next cycle. MPS probability measures the long-term system performance over the life of a policy. The MPS also differs from the policy's reliability(MPR). A policy's ending probability (MPR) states nothing about the intermediate cycles' MFOPS; whereas, MPS does check intermediate cycles. Figure 83 plots the MFOPS of both policies over 12 operating hours (which is also 12 hours with a MFOP duration of 1 hour).



(a) 2-3-4 Design

(b) 1-3-3 Design

Figure 83: Minimum Policy Success

In the 3-part system, the first cycle's MFOPS is one (per definition that the system states in a fully operational state). The second MFOPS is 0.986 and third MFOPS is 0.957. This meets a minimum MFOPS requirement of 90%. The original design with multiplies of 2, 3, and 4 has MFOPS that fall below a 90% requirement as shown in Table 34. In this, we can say that the 2-3-4 design is an insufficient policy and that the 1-3-3 policy is acceptable.

Table 34: Evaluation of Two Maintenance Policies

MFOP Cycle	Operating Time, t	1-3-3 Design MPR	1-3-3 Design MFOPS	2-3-4 Design MPR	2-3-4 Design MFOPS
Start	0	1		1	
1	1	0.9863	0.9863	0.9863	0.9863
2	2	0.9725	0.9860	0.8138	0.8267
3	3	0.9304	0.9567	0.7785	0.9566
4	4			0.5989	0.7693
.				.	.
.				.	.
.				.	.
12	12			0.2099	0.7518
Cycle End			0.9567		0.7518
		MPS	0.9567	MPS	0.7518

5.6.3 Sensitivity Study

The discrete nature of D_{Asys} arises from a maintenance policy where replacements occur at defined intervals. This characteristic results in sharp changes of the gradient in Figure 79 (page 180). Consequently, gradient based optimization techniques are ill-suited to optimize the problem. Instead, a sensitivity study was conducted to explore the relationships between failure replacement time versus preventive replacement time and the response of D_{Asys} to disturbances.

The sensitivity study consisted of creating a Design of Experiments (DoE), fitting a surrogate model, and understanding the behavior of the response. Figure 84 provides a schematic sketch of the study's analysis that yielded conclusions on the policy. The DoE began with defining the model's variables (section 5.6.3.1) and ended with inputs and responses in a completed data table (section 5.6.3.2) . The model fit began with screening out variables with low impact on the responses (section 5.6.3.3). A selection of surrogate model type and fit followed (section 5.6.3.4). The fitting ended with an evaluation of the goodness of fit and acceptance of the surrogate model (section 5.6.3.5). Understanding the downtime and reliability responses began with an observation of trends in a scatter plot (section 5.6.3.6). Use of the software JMP and its built-in prediction profiler enabled observations of downtime and reliability sensitivities to design variables (section 5.6.3.6). The study ends with drawing conclusions from the observed trends (section 5.7).

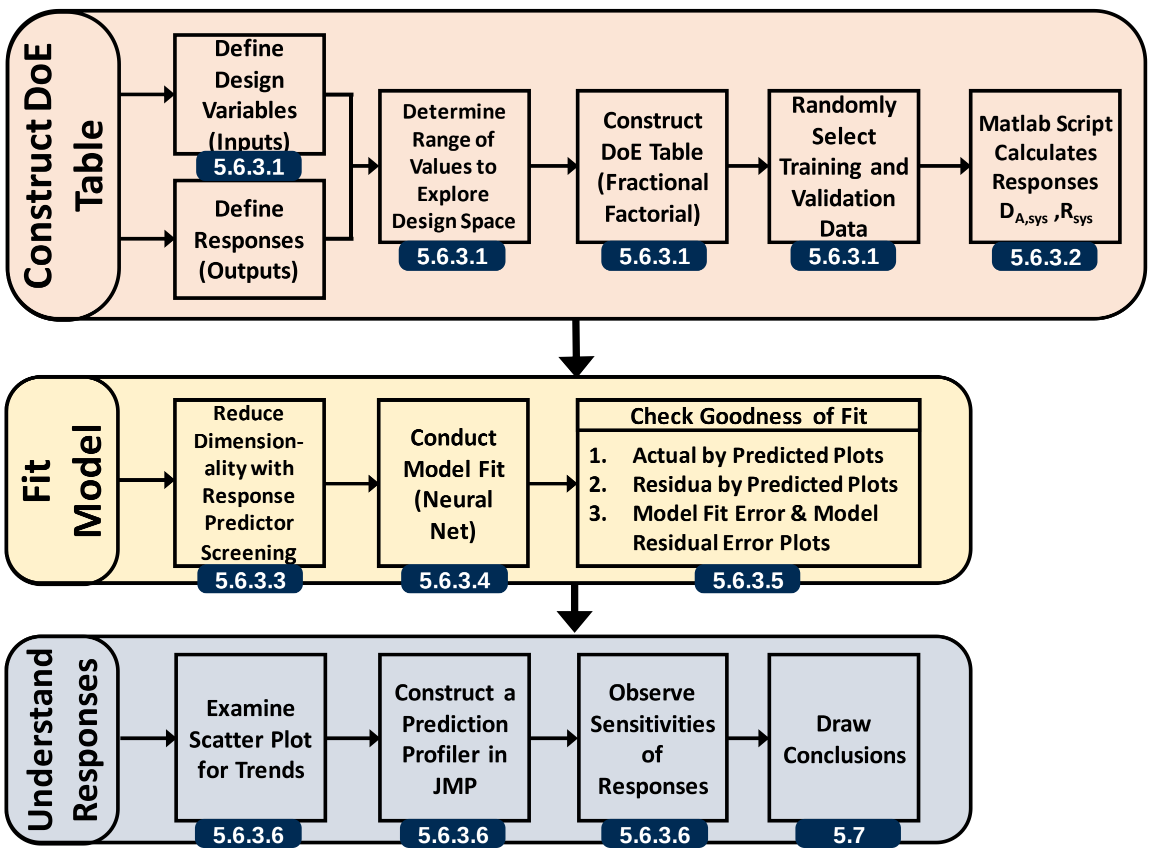


Figure 84: Block Diagram of Sensitivity Study

5.6.3.1 Modeled System Responses

System achieved downtime and system reliability was collected on the 3-part system with non-uniform replacement intervals using a Design Of Experiments (DOE). The independent design variables building the design space were MFOP duration (t_{mf}), component multipliers (α_r), failure replacement times ($T_{f,r}$), and preventive replacement times ($T_{p,r}$).

Table 35 shows the variables and levels that constructed the DOE. The inclusion of intermediate levels ensured exploration of the space to better capture non-linear behavior. T_p and T_f range of 0.01 to 0.20 units of time capture the same behavior of the example's fixed values of 0.070 and 0.035 units of time, respectively. T_p and T_f represent repair times in the achieved downtime analysis; however, they weight the policy more towards failure or preventive replacements.

Table 35: 3-Part DOE Independent Variables

Variable	Units	Levels				
t_{mf}	time	1	2	3	4	5
α_1	---	1	2	3	4	
α_2	---	1	2	3	4	
α_3	---	1	2	3	4	
$T_{f,3}$	time	0.01	0.1	0.2		
$T_{f,3}$	time	0.01	0.1	0.2		
$T_{f,3}$	time	0.01	0.1	0.2		
$T_{p,1}$	time	0.01	0.1	0.2		
$T_{p,2}$	time	0.01	0.1	0.2		
$T_{p,3}$	time	0.01	0.1	0.2		

A MFOP strategy seeks to minimize disruptive failures in favor of preventive replacements at the MRP. Consequently, a MFOP strategy drives failure replacement (T_f) to be costlier than the preventive replacement (T_p). The explored design space considers designs with T_f up to twenty times costlier than T_p. The MFOP duration (t_{mf}) was explored between one to five to cause failures when α_r equaled one. Policy multipliers (α_r) remain whole numbers and drive explored component replacement intervals ($t_{p,r}$) from one to five times the MFOP duration. This causes some failures in all components when t_{mf} was its minimum of one. The potential combinations of $t_{p,r}$ explore the full range of component reliability from near one to near zero.

The DOE had $5 \cdot 4^3 \cdot 3^6$ or 233,280 possible combinations. A fractional factorial reduced the number to 10,000 cases to keep computational time reasonable. The DOE and statistical analysis were conducted with the software JMP Pro v16.1 by SAS Institute Inc [65]. Response of the system were achieved downtime and system reliability at the end of the maintenance policy (ending MPS). Responses were calculated using the same MATLAB script written for the work in section 5.6.2.2 above.

The DoE data table presented in Table 36 shows the design variables, actual and predicted system downtime, and actual system reliability. The table shows the first 15 of 10,000 data samples in the experiment.

Table 36: DoE Data Table with Responses

Design Variables (Inputs)										Downtime Response				
t_{mf}	$T_{f,1}$	$T_{f,2}$	$T_{f,3}$	$T_{p,1}$	$T_{p,2}$	$T_{p,3}$	α_1	α_2	α_3	$D_{A,sys}$ Actual	$D_{A,sys}$ Pre-dicted	$D_{A,sys}$ Percent Error	Rsys	
5	0.2	0.2	0.2	0.1	0.01	0.2	4	4	3	0.1434	0.1435	0.0004	0.0000	T
5	0.1	0.01	0.2	0.1	0.2	0.1	3	3	1	0.0831	0.0821	-0.0116	0.0000	V
5	0.01	0.1	0.01	0.1	0.01	0.2	2	1	2	0.0535	0.0593	0.1083	0.0000	T
4	0.2	0.2	0.01	0.2	0.1	0.2	3	4	1	0.1582	0.1578	-0.0025	0.0000	V
5	0.1	0.01	0.2	0.2	0.1	0.01	1	2	4	0.1008	0.0989	-0.0189	0.0000	T
2	0.1	0.01	0.2	0.01	0.01	0.1	2	2	4	0.0667	0.0681	0.0214	0.0000	T
2	0.1	0.1	0.2	0.1	0.1	0.2	2	3	1	0.1558	0.1605	0.0302	0.0000	T
2	0.1	0.2	0.1	0.2	0.1	0.1	3	4	1	0.1480	0.1436	-0.0295	0.0000	V
5	0.01	0.01	0.2	0.2	0.1	0.2	3	4	1	0.0733	0.0750	0.0234	0.0000	T
2	0.01	0.2	0.01	0.1	0.2	0.2	1	3	2	0.1437	0.1447	0.0072	0.0000	T
5	0.01	0.01	0.2	0.2	0.1	0.2	3	4	1	0.0733	0.0750	0.0234	0.0000	T
2	0.01	0.2	0.01	0.1	0.2	0.2	1	3	2	0.1437	0.1447	0.0072	0.0000	T
3	0.01	0.2	0.01	0.1	0.01	0.2	4	1	2	0.0487	0.0498	0.0231	0.0000	T
2	0.1	0.2	0.01	0.01	0.1	0.01	2	1	1	0.0772	0.0747	-0.0324	0.0464	T
1	0.1	0.1	0.01	0.2	0.1	0.01	3	2	4	0.1235	0.1154	-0.0651	0.0145	T

T = Model Training Data *V = Model Validation Data*

5.6.3.2 Calculation of Responses

Component *r*'s achieved downtime derives from revised renewal theory's Equation (68) and maybe expressed as

$$D_{A,r}\left(t_{mf},\alpha_r\right) = \frac{\left(\frac{\alpha_{lcm}}{\alpha_r}\right)\left[H_r(\alpha_r t_{mf})T_{f,r}+T_{p,r}\right]}{\alpha_{lcm}t_{mf}+\sum_{r=1}^{n}\left(\frac{\alpha_{lcm}}{\alpha_r}\right)\left[H_r(\alpha_r t_{mf})T_{f,r}+T_{p,r}\right]} \quad (79)$$

Sample 15, the last row of Table 36 has a least common multiplier of 12 and yields component achieved downtimes of

$$D_{A,1}(1,3) = \frac{\left(\frac{12}{3}\right)[(0.64459)0.1+0.2]}{12\cdot 1+[1.0578+0.6+0.0323]} = \frac{1.0578}{12+1.6901} = 0.07727 \quad (80a)$$

$$D_{A,2}(1,2) = \frac{\binom{12}{2}[(4.8213E{-}8)0.1+0.1]}{12\cdot 1+[1.0578+0.6+0.0323]} = \frac{0.6000}{12+1.6901} = 0.04383 \quad (80b)$$

$$D_{A,3}(1,4) = \frac{\binom{12}{4}[(0.075283)0.01+0.01]}{12\cdot 1+[1.0578+0.6+0.0323]} = \frac{0.0323}{12+1.6901} = 0.00236 \quad (80c)$$

This leads to a total achieved downtime of

$$D_{A,sys}(1,\boldsymbol{\alpha}) = 0.07727 + 0.04383 + 0.00236 = 0.1235 \quad (81)$$

The surrogate model predicted Sample 15's achieved downtime to be 0.1154 with a percent error of -6.56%.

Calculation of the policies ending reliability followed the calculations of section 5.6.2.2. Unlike downtime, a surrogate model was not necessary for reliability because Equation (77) yielded component reliability as a function of t_{mf}, α_r, and the TTF distributions of Table 31.

$$R_1(\alpha_1\, t_{mf})^{(\alpha_{lcm}/\alpha_1)} = \left[e^{-(3/3)^{3.9}}\right]^{(12/3)} = 0.3679^4 = 0.0184 \quad (82a)$$

$$R_2(\alpha_2\, t_{mf})^{(\alpha_{lcm}/\alpha_2)} = \left[\frac{1}{2}\left(1 + erf\left(\frac{2-3.6}{0.3\sqrt{2}}\right)\right)\right]^{(12/2)} = 1.0000^6 \quad (82b)$$

$$R_3(\alpha_3\, t_{mf})^{(\alpha_{lcm}/\alpha_3)} = \left[e^{-(4/5.5)^{8}}\right]^{(12/4)} = 0.9247^3 = 0.7907 \quad (82c)$$

Likewise, Equation (78) yielded system reliability as the product of each component's reliability.

$$\mathrm{MPR}(t_{mp}) = (0.0184)(1.0000)(0.7907) = 0.0145 \quad (83)$$

5.6.3.3 A First Look at System Sensitivity

A response screening test was conducted on the responses of system achieved downtime and system reliability. The screening test measured the contribution of each design variable (labeled predictor in Figure 85) to the responses D_{Asys} and R_{sys}. Variable contribution measures the degree of change in the response with respect to variable change. The portion column normalizes the contribution value such that all variable portion sums to one. The horizontal bar graph plots portions by variable. The screening plots provide a visualization of variable contribution to responses in a similar manner as a Pareto plot. The benefit of predictor screen is that it "can identify predictors that might be weak alone but strong when used in combination with other predictors" [66]. This attribute is especially useful to analyze the achieved downtime model where the expected number of failures is a function of the product of t_{mf} and α_r.

Predictor Screening

	Dsys			Rsys		
Predictor	Contribution	Portion	Rank	Contribution	Portion	Rank
tmf	1.55586	0.3045	1	77.6793	0.6772	1
Tf1	0.87943	0.1721	2	0.1059	0.0009	9
Tf2	0.30031	0.0588	6	0.1334	0.0012	6
Tf3	0.06379	0.0125	10	0.1514	0.0013	5
Tp1	0.68888	0.1348	3	0.1142	0.0010	8
Tp2	0.51548	0.1009	5	0.1181	0.0010	7
Tp3	0.63112	0.1235	4	0.0976	0.0009	10
alpha1	0.16469	0.0322	8	24.8882	0.2170	2
alpha2	0.11776	0.0230	9	10.0203	0.0874	3
alpha3	0.19289	0.0377	7	1.3976	0.0122	4

Figure 85: Contribution of a 3-Part System's Design Variables on Responses

The most significant factor is the MFOP duration, t_{mf}, because it impacts all components in the system. The top replacement costs belong to Part 1 because it is the weakest link and has its greatest number of expected failures at given replacement interval, $t_{p,1}$. The system is relatively insensitive to Part 3 within the design space because the component has the greatest inherent reliability of all components. Like achieved downtime, system reliability is most sensitive to the MFOP duration. Multipliers for Part

1 and 2 account for almost all the remain contribution and are a function of component inherent reliability. As expected, system reliability is independent of replacement times.

The number of variables is a manageable ten for this system; however, a complex system may quickly grow large. For example, an 18-component system has a single t_{mf}, 36 replacement times, and 18 multipliers for a total of 55 factors. Screened variables remove their small contribution to the responses but provide the benefit of a more manageable model.

5.6.3.4 Fit of the Model

System reliability for serial architecture was readily calculated as a function of design variables; however, achieved downtime was more difficult to measure. The downtime has both a discrete nature and a non-linear shape. The non-linear shape is due to the expected number of failures, $H(t_p)$, first introduced in section 5.1.1.1 (page 147). Models with linear coefficients such as a least squares fit do not perform well in capturing the behavior at smaller t_p's. A neural network with at least ten nodes in the first and second layer performed well in capturing the non-linear behavior of D_{Asys}. The hidden layered approach of a neural net makes it difficult to describe the "functional form of the response surface" [66]. Instead, prediction profilers addressed response relationships later in the section. The neural network used 75% of the 10,000 data points to train the model. The remaining 25% of data points supported the validation of the fit.

5.6.3.5 Goodness of Fit

The fitted neural net had a training data R-square value of 0.9934, which indicated the model was accounting for most of the variation. The validation R-square value of 0.9928 suggested the fit was a candidate for continued consideration. Figure 86 provides the actual by predicted plots for the training and validation data. Both data sets show good adherence of predicted to actual through the entire response range, especially at the troublesome early t_p values that have greater non-linear effects.

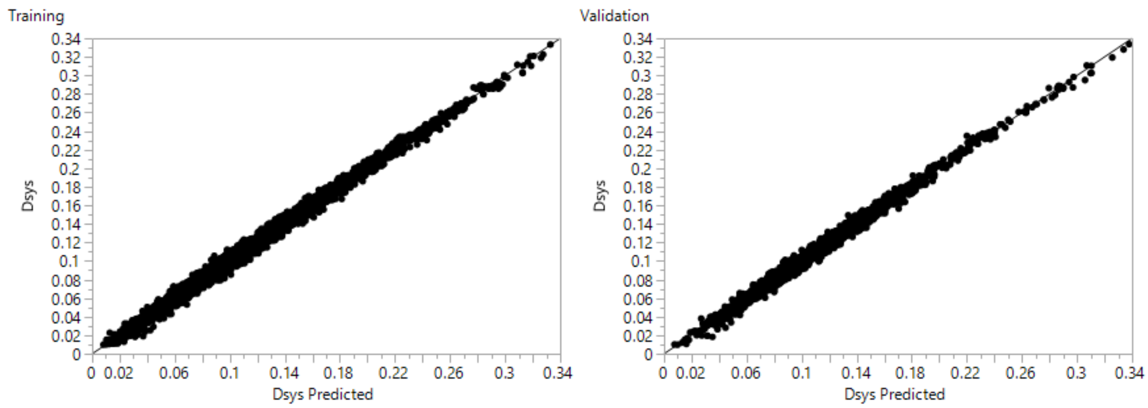


Figure 86: D_{Asys} Actual by Predicted Plots of a 3-Part System Fit

Figure 87 plots of the residual error by predicted D_{Asys}. The sparsity of points above a predicted D_{Asys} of 0.22 are an artifact of the selected design space. The explored space focused more on candidate designs with lower downtimes. There is no discernable clustering and the fit has good symmetry. The validation plot has similar attributes as the training plot. The fit has a larger than residual span to minimum predicted than desired. The fit is good but not great.

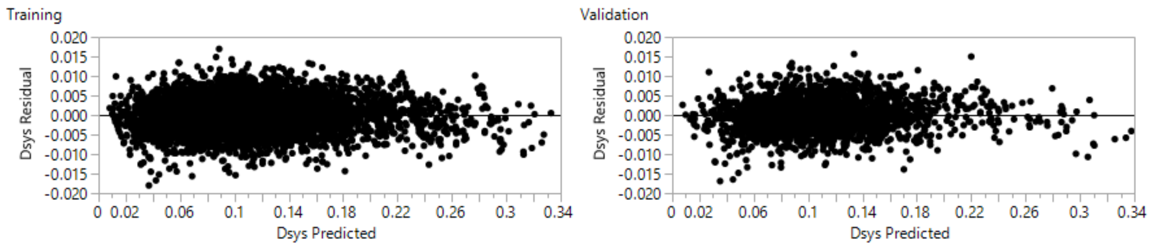
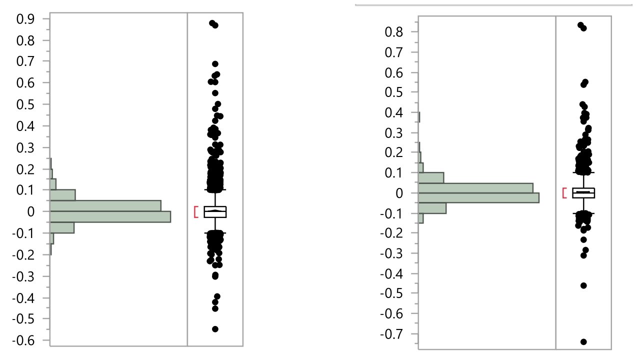


Figure 87: D_{Asys} Residual by Predicted Plots of a 3-Part System Fit

The Model Fit Error (MFE) and Model Response Error (MRE) measure the distribution of the error with respect to actual values. The histograms take the desired bell curve shape with 5.4% of the data greater than ±0.1 normalized error. There was no discernable pattern of conditions to the error. Both training and validation data meet the best practice of means close to zero and standard deviations less than one. This indicates the model has large error for a small minority (5.4%) of the data. The remaining 94% of data points had a small error. Overall, the fit is sufficient to determine trends and the sensitivity of the response D_{Asys} to the design variables.



Quantiles		
100.0%	maximum	0.8789308002
99.5%		0.2721035602
97.5%		0.1155821697
90.0%		0.0566034623
75.0%	quartile	0.0255614128
50.0%	median	-0.000607579
25.0%	quartile	-0.024258977
10.0%		-0.049223124
2.5%		-0.090159262
0.5%		-0.130416616
0.0%	minimum	-0.547377987

Summary Statistics	
Mean	0.0036495
Std Dev	0.0574778
Std Err Mean	0.0006632
Upper 95% Mean	0.0049496
Lower 95% Mean	0.0023495
N	7511

Quantiles		
100.0%	maximum	0.8342028506
99.5%		0.3192506271
97.5%		0.1319438132
90.0%		0.0587985046
75.0%	quartile	0.0250461614
50.0%	median	-0.000607402
25.0%	quartile	-0.025516092
10.0%		-0.053467038
2.5%		-0.091130109
0.5%		-0.140352241
0.0%	minimum	-0.741091995

Summary Statistics	
Mean	0.003562
Std Dev	0.0643706
Std Err Mean	0.0012903
Upper 95% Mean	0.0060921
Lower 95% Mean	0.0010319
N	2489

Figure 88: D_{Asys} MFE and MRE of a 3-Part System Fit

5.6.3.6 Observed Trends

A tool to identify trends in responses is a scatterplot matrix. Figure 89 is a scatterplot of the nine design variables boxed by the system achieved downtime and reliability responses. Linear trend lines are in red. The trend lines assist in reading the scatterplot; however, they can be misleading if the true trend is not linear. System reliability trends, shown in the bottom row of Figure 89, are straightforward. Increasing the t_{mf} or α_r multipliers increases the component's replacement interval. Longer replacement intervals operate components longer before renewal, thus the component accumulates a greater chance of failure and results in lower reliability and more downtime. Horizontal trend lines reflect that replacement times have no impact on system reliability.

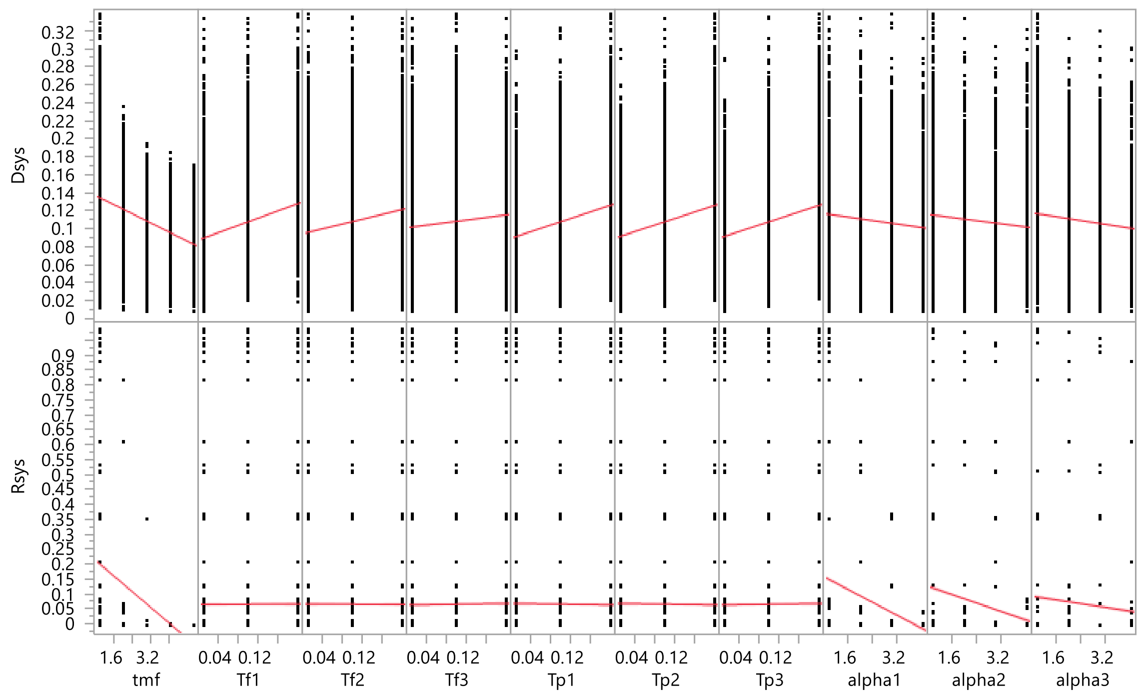


Figure 89: Response Scatter Plot of a 3-Part System Fit

The system's achieved downtime responds as expected with replacement costs. The span of downtime increases as replacement times grow. Greater replacement times, T_f and T_p, magnify the effect of the replacement interval and expand the range of achieved downtime. Since the system cannot occupy a downtime less than zero but can increase toward one, the higher downtimes take an upward trend. The trend line for α_r multipliers is misleading; the top ends of the scatterplots have a concave shape with moderate multipliers yielding lower peaks. This suggest there may be multipliers that are better than others. Finally, the MFOP duration shows very interesting results. The clear downward trend suggests that as t_{mf} increases, the range of downtime shrinks. This speaks to the convergence of the system towards its limit. It does not mean that increasing the MFOP duration will necessarily lower downtime. In fact, the prediction profiler illustrates the effect of longer MFOP durations depends on the weighting dictated by replacement times.

The prediction profiler of Figure 90 draws the systems responses, achieved downtime and reliability, for a given setting of design variables. The black lines indicate a one-on response of the system. It measures the response sensitivity to one variable while fixing all others. The case shown in the profiler has a t_{mf} of 1 and multiplies of 1-3-3. Preventive replacements are favored over failure replacements with repair time costs of 0.01 and 0.1 respectively. This design is similar to the baseline setting used in section 5.4. The model predicts an achieved downtime of 0.0221 and a reliability of 0.9304. Actual achieved downtime is 0.0213 and reliability is 0.9304.

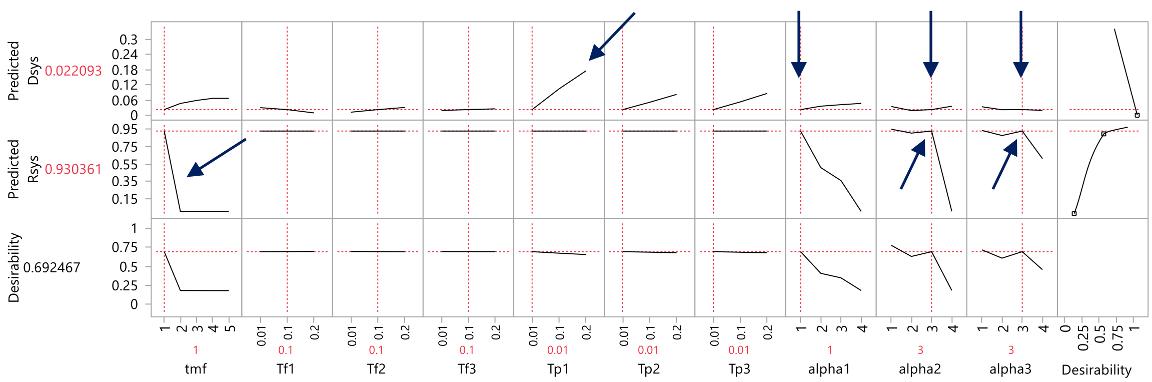


Figure 90: Prediction Profiler

The profiler conveys increasing the MFOP duration (t_{mf}) results in a dramatic decrease in reliability and a doubling of downtime. The model's downtime is most sensitive to an increase in the preventive repair time ($T_{p,1}$) of Part 1 because Part 1 has the lowest inherent reliability and multiplier. The lowest multiplier creates more preventive replacements than the other components leading to more downtime. The multiplier α_2 has a concave curve with 3 being the optimal multiplier. This creates a replacement interval of 3 units of time which coincides with the theoretical prediction. Setting α_3 to a multiplier of 4 puts the component at its optimal interval and yields a system downtime of 0.0221; however, this multiplier pushes reliability to 0.61. This is down from the reliability of 0.93 achieved at α_3 = 3. The behavior of the α_r sensitivity curves matches the behavior of downtime versus replacement interval plots. Part 1 is on the upslope of the global minimum. Part 2 is close to its minimum. Part 3 is on the downslope of its minimum.

Reliability decreases as components operate over longer preventive replacement intervals, t_p. Recall Equation (66) reshown below.

$$t_{p,r} = \alpha_r t_{mf} \qquad (66)$$

An increase in either α_r or t_{mf} raises $t_{p,r}$, reduces component reliability, and reduces system reliability. t_{mf} has a more powerful impact on the system than any one multiplier because t_{mf} raises $t_{p,r}$ for all components in the system. Consequently, extending the t_{mf} is more challenging of a design feat than increasing a single component's multiplier. Figure 90 also assists the policy author to set the right multipliers. All of Part 1's multipliers lie on the system reliability's negative slope. This is indicative of a part sacrificing reliability for downtime or t_{mf}. It also points to the fact that the range of t_{mf} was set too high to fully capture the design space where Part 1's reliability is high. Multipliers α_2 and α_3 are fully explored and showed that they lie on the precipitous of the negative slope. This is an ideal multiplier to minimize downtime while retaining high reliability.

Figure 91 measures the sensitivity of the system to the MFOP duration, t_{mf}. The first, second, and third row have t_{mf}'s of one, three, and five, respectively. As t_{mf} increases, the importance of T_f grows while T_p diminishes. This is a natural consequence of a higher resulting t_p. The longer a component operates without renewal, the greater the chance of failure and the lower the number of preventive replacements. The consequence also occurs with α_r. Downtime's sensitivity to multipliers lessens as the resulting t_p approaches the limit of D_{Asys}.

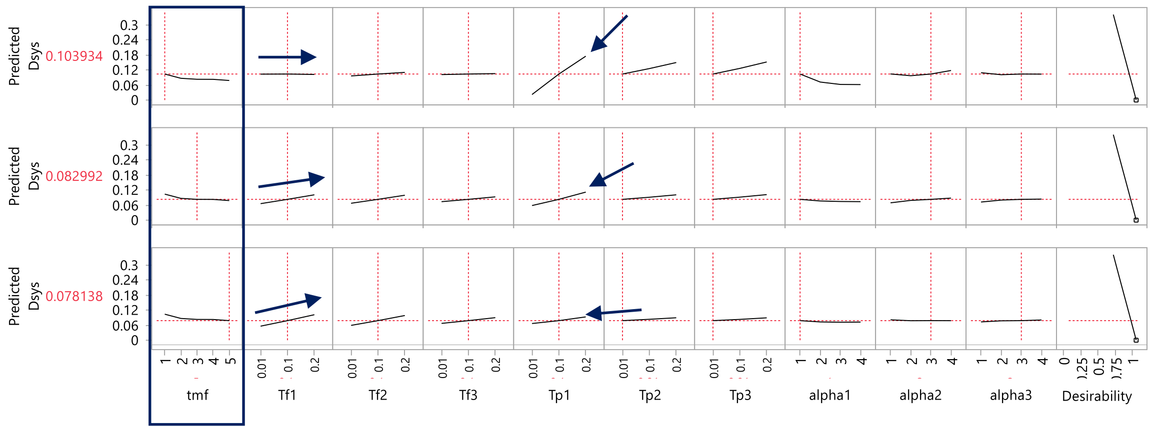


Figure 91: Effect of MFOP Duration

A rise in a replacement time always increases the downtime of the system. Increasing the ratio of T_f to T_p drives the optimal downtime solution towards a shorter t_{mf}., because the penalty for failure replacements is more. Increasing the failure replacement time (T_f) of component *r* has a compounding effect on the system's reaction to other design variables. Greater T_f affects downtime's sensitivity to the multiplier α_r. In Figure 92 below, $T_{f,2}$ varies from 0.01 to 0.1 to 0.2 units of time. The optimal Part 2's multiplier, α_2, shifts to smaller values with more $T_{f,2}$. This is the optimization attempting to avoid the failure penalty by favoring shorter replacement intervals.

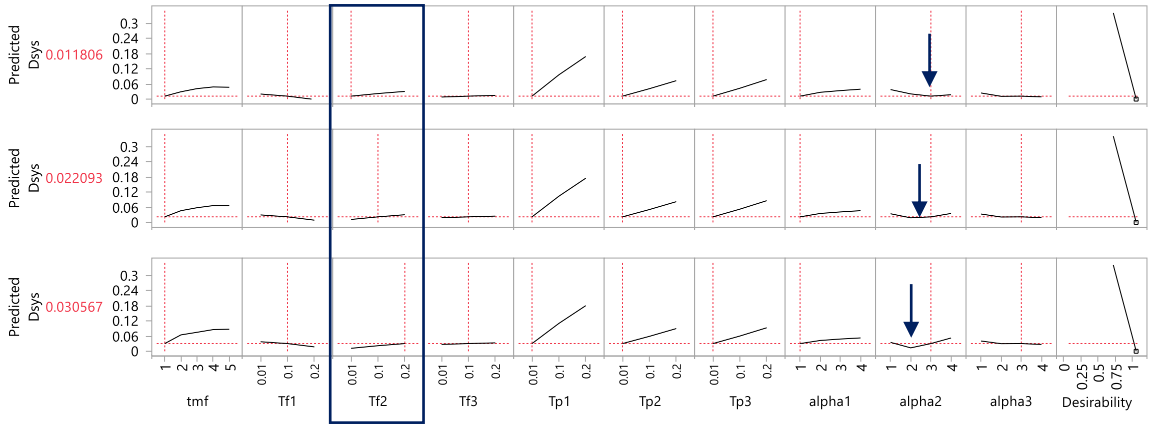


Figure 92: Sensitivity of Failure Replacement Times, T_f

Decreasing the preventive replacement time (T_p) of component r also has a compounding effect on the system's reaction to other design variables. Part 3's T_p changes from 0.2 to 0.1 to 0.01 units of time. Lowering $T_{p,3}$ diminishes downtime's sensitivity to α_3. The other multipliers remain unaffected. The trending direction and concavity of the α_r is a function of how close $t_{p,r}$ is to the component's optimal t_p.

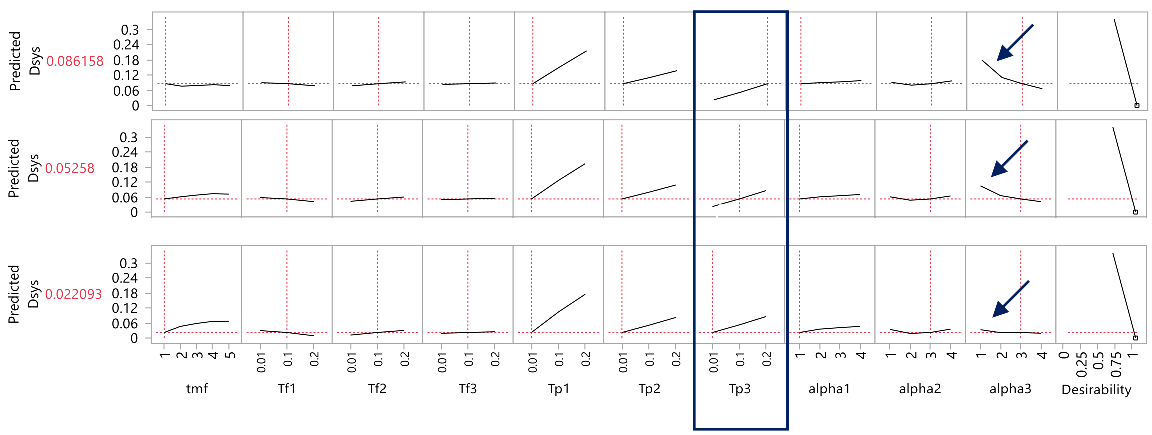


Figure 93: Sensitivity of Preventive Replacement Times, T_p

5.7 Conclusions on a Policy that Minimizes Downtime

A MFOP maintenance policy synchronizes preventive replacement intervals, t_p, to MRPs. Synchronization of different components means an extension or shortening of their optimal replacement interval; doing so takes the components and system out of its optimal state. This has design consequences. An extension sacrifices part reliability and worsens MFOPS. A shortening increases reliability and improves MFOPS but raises the maintenance burden due to more frequent preventive replacements.

The classical Optimal Preventive Replacement model has several limitations that prevent its immediate application in a MFOP strategy. The model accounts for a single part and does not provide an optimum solution for a multi-part system that meets minimum requirements for a MFOP duration and MFOPS. This thesis presented a revised model that establishes the framework to:

1. Account for failure replacements (T_f) and preventive replacements (T_p) that are not significantly greater than the replacement interval (t_p)
2. Find the downtime of a multi-part system
3. Synchronize multiple parts replacement intervals (t_p) without disrupting the MFOP (t_{mf})
4. Manages a system constrained by minimum reliability or MFOPS requirements

A set of recommendations and best practices create a framework to build a maintenance policy that provides a MFOP while balancing downtime and reliability. The primary conclusions from the work are:

a. Availability achieved, A_A, is equal to one minus achieved downtime, D_A; therefore, A_A, is maximized by minimizing D_A. The revised model predicts achieved downtime under MFOP constraints.

b. A time to failure distribution with a low coefficient of variation ($cv \ll 1$) has greater amplitude in the downtime model. A low *cv* component is a component whose D_A contribution is more sensitive to its replacement interval, t_p. Setting t_p at the optimal point creates lower downtimes. A low *cv* component with a slight change (earlier or later) from the optimal results in high downtimes. A distribution with *cv* closer to one is a smoother curve with steadier behavior. It is less sensitive to non-optimal t_p's. A distribution with a *cv* greater than one does has a D_A limit of zero and does not benefit from preventive replacement.

c. A MFOP maintenance policy demands synchronized preventive replacement intervals that are whole number intervals of the MFOP duration.

d. The smaller t_{mf} relative to the components mean, the more choices the policy designer must select a multiplier that improves D_{Asys}.

e. An optimized system with a shorter T_f or longer T_p prefers a longer MFOP duration.

f. Preventively replacing components at the optimal uniform replacement interval is not the global downtime minimum but does create a MFOP. There is no guarantee this MFOP is sufficient for operational needs.

g. The global minimum downtime occurs when components are replaced at their non-uniform preventive replacement intervals. This answers Hypothesis 2. In a complex system, the non-uniform policy will result in disruptive preventive maintenance that opposes a MFOP strategy.

h. A larger replacement interval opposes reliability.

i. There is an optimal replacement interval to minimize downtime, but it may result in an undesirable reliability.

j. Low coefficient of variation components are less likely candidates for extension beyond its optimal replacement interval; however, these components have a more predicable time to failure that makes designing for reliability easier.

k. High reliability designs are systems with components preventively replaced before its first minimal replacement intervals. This places a practical limit on a MFOP duration.

l. Minimum Policy Success (MPS) measures the performance of a maintenance policy over many MFOP cycles. The policy's ending MPS is not necessarily the best judge of a policy, because the ending provides no information about the probability of success of intermediate MFOP cycles. It would not be unusual for a complex system like a helicopter to have the policy duration, tmf, exceed the expected use of the policy. An acceptable policy maintains its MPS throughout the expected number of MFOP cycles that use the policy.

m. Aligning preventive replacements at whole number multiples of the MFOP duration makes the response discrete. The discrete nature makes finding of a constrained, minimum downtime difficult with gradient based optimization techniques. A sensitivity study is an alternate and appropriate means to understanding the problem.

n. Neural networks can adequately model the non-linear characteristics of a system with components that have time to failure distributions with low coefficients of variation.

o. Increasing t_{mf} is more challenging than increasing an α_r, because t_{mf} influences every components behavior.

6 PROBLEM 2.2: CONTROL MFOPS WITH A MFOP POLICY

This chapter utilizes the new framework of the previous chapter to give a maintainer some control over MFOPS given a MFOP duration. A policy providing MFOPS control is attractive for FVL's vision where aircraft conduct maintenance free operating periods from remote, forward operating bases. Availability is less than 100% in a MFOP strategy; however the MFOP is protected from Mission Affecting Failures (see section 4.3.2 Construction of a Utility Helicopter Model) and essential maintenance actions. An established MFOP provides 100% system availability during the operating period. A cycle-ending MRP then completes all deferred and necessary preventive replacements to meet the next cycle.

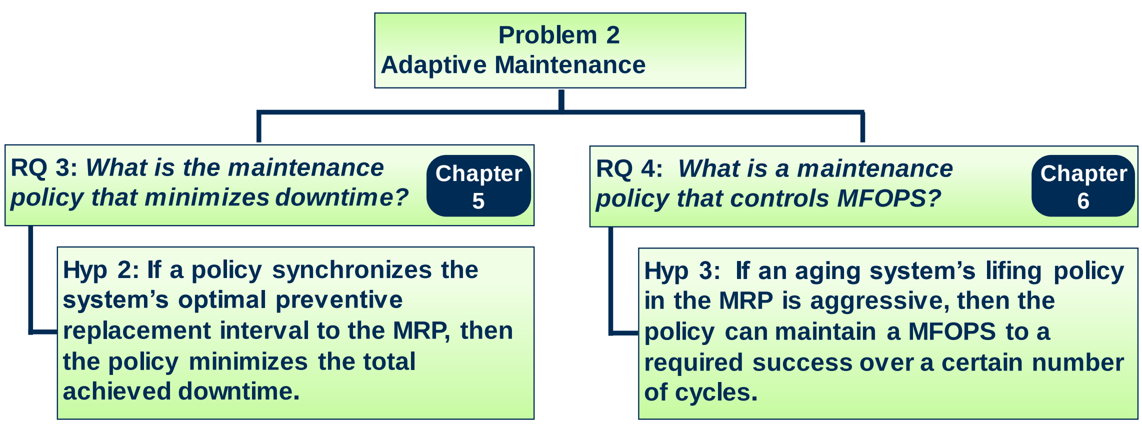


Figure 62: Problem 2 Summary with Hypotheses (reshown)

6.1 Research Question 4: What is a maintenance policy that controls MFOPS

A MRP has a minimum number of repairs needed to restore faults that did not cause mission failure. A system with aging parts, such as a helicopter, will also have a safe lifing policy to meet safety requirements. A maintenance plan must conduct these minimum repairs to replace non-critical faults and required lifing policies. Repairs beyond the minimum number are deemed "aggressive" and are hypothesized to improve the MFOPS of the next MFOP cycle. Two factors upset the stability of a MFOPS. First, an extension to a MFOP will yield a lower MFOPS. Second, the failure rate for wearing parts increases for aging aircraft. Cinci and Griffith [41] in Journal of Quality in Maintenance Engineering, commented

> *"However, there is a view within the reliability engineering community that if a non 'constant failure rate' approach to equipment reliability is used, systems are likely to contribute towards longer MRPs as the aircraft gets older." [41]*

A well-designed policy should minimize the effects of aging and help an aircraft achieve the desired MFOPS in the next MFOP cycle. It is hypothesized that an aggressive lifing policy restores the life of aging parts, improves system reliability, and manages the MFOPS to remain at its goal.

Research Question 4

What is a maintenance policy that controls MFOPS?

Hypothesis 3: If an aging system's lifing policy in the MRP is aggressive, then the policy can maintain a MFOPS to a required success over a certain number of cycles.

The aggressive lifing policy must identify which components to replace preventively to most improve the MFOPS. An algorithm shall rank orders each component and apply the necessary changes until the policy obtains the MFOPS goal. Failure Cause Identification, introduced in section 3.2.1, provides the means to rank order candidates that most limit the MFOPS. This ordering creates a maintenance policy that distinguishes which components to replace first in a MFOP. The methodology to achieve such a policy follows an inner loop of the previously introduced framework.

The development and testing of the hypothesis followed the scientific method. Section 6.2 developed the framework using a simple, 4-part system as a thought experiment. Section 6.4 repeats the experiments on larger, 10-part systems in random, complex architectures. Experiments tested a total of fifteen samples to demonstrate repeatability. Appendix A has full experiment inputs and results to provide transparency.

6.2 Framework to Control MFOPS

The methodology to control MFOPS follows the same framework introduced in the previous chapter. Much of the framework remains unchanged. The processes to manage the MFOPS occurs with an inner design loop between building and evaluating a policy. Figure 94 on page 211 highlights the inner loop within the framework.

Inputs used by the MFOPS control loop arise from the system definition. The system designer determines the system's architecture and component TTF distributions. Aligning system components to the architecture creates the system fault tree. The designer and operator work together to assign the fault trees by mission phase and duration. The operational commander sets the MFOP duration (t_{mf}) and the MFOPS goal ($MFOPS_{req}$) based upon operational demands. The MFOP duration is a function of mission duration (t_{phase}) and the number of missions in the cycle and is

$$t_{mf} = (Missions\ per\ MFOP)\ t_{phase} \tag{84}$$

After establishing the inputs, the MFOP control loop may begin. Figure 94 of page 211 marks the five steps of the MFOPS control loop.

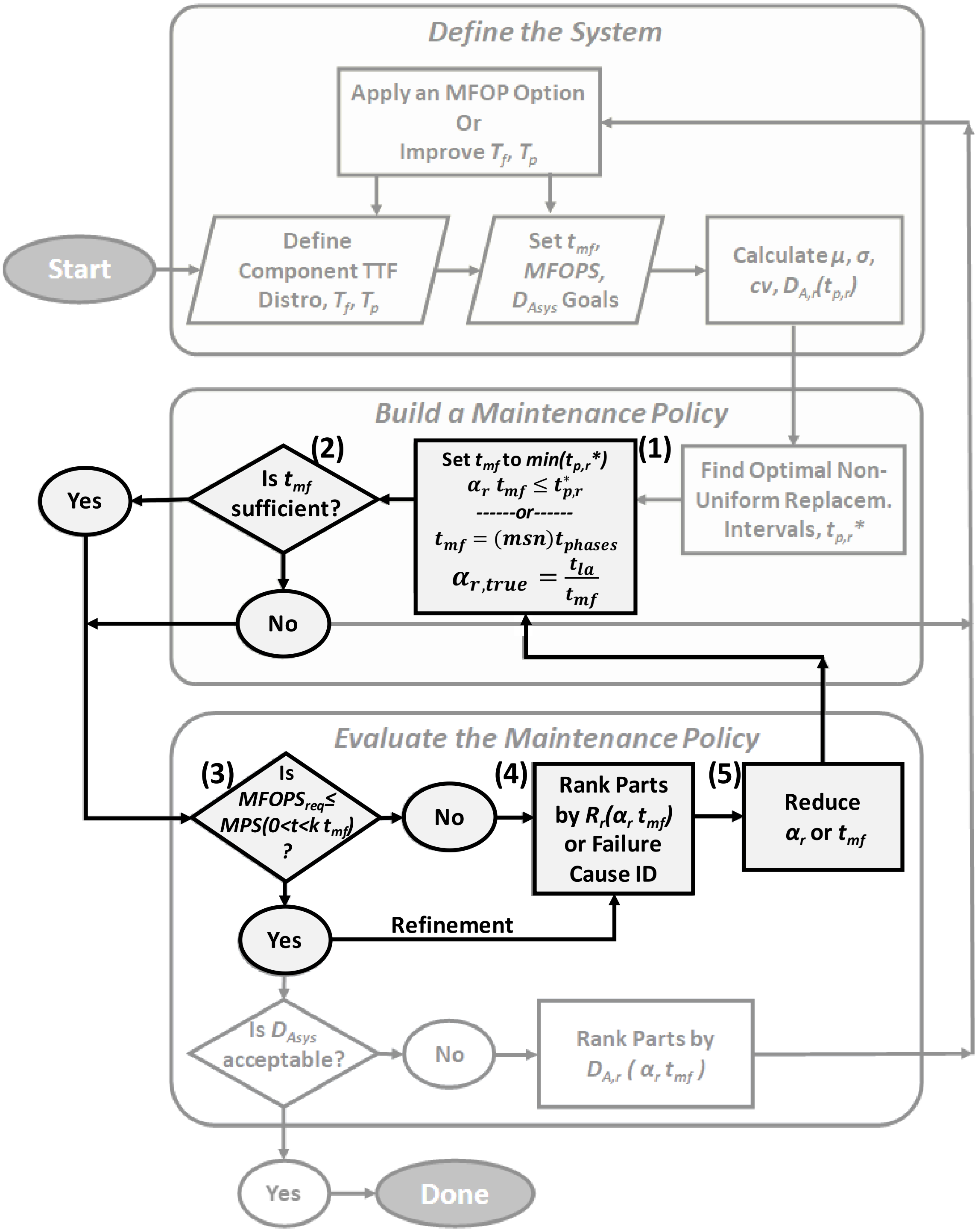


Figure 94: MFOPS Control Loop of the Framework

6.2.1 Step 1: Set α_r

The strategy of controlling MFOPS is one of incremental improvement from a conservative starting point. The policy projects the conservative starting point from the serial arrangement of components. A serial arrangement of components represents the worst possible mission reliability (MR) of *n* components over a given duration (t)

$$MR_{sys}(t) = \prod_{r=1}^{n} MR_r(t) \tag{85}$$

The starting point assumes each of the components has the same mission reliability over *t*. Unless the system is *n*-like components, the assumption is merely a rough order approximation of the system's behavior. The closer a system is to a serial arrangement of like components, the more accurate the approximation. This assumption, however rough, provides a starting point to develop the draft policy of replacement intervals.

Let *mR* denote the mission reliability of each component, then

$$MR_{sys} = mR^n \tag{86}$$

with *n* components. The components do not have to have the same TTF distribution. The assumption only requires each component has a uniform reliability *mR*. Solving for *mR* yields

$$mR = {MR_{sys}}^{1/n} \tag{87}$$

Let *pr* be the probability each component failures before time t. Since *mR* is the probability each component survives through time t, then *pr* is

$$pr = 1 - mR = 1 - {MR_{sys}}^{1/n} \tag{88}$$

If each component has a mission reliability of mR, then the quantile function provides the endurance of the part to that mission reliability. The quantile function, also known as the inverse cdf, is defined as

$$F^{-1}(pr) = t \tag{89}$$

The function returns the time t that yields the probability (pr) for a given TTF cumulative distribution function. Table 10 of section 4.1.2.3 summarizes the quantile function for the exponential, Weibull, and normal distributions. Applying the quantile function to Equation (89) to Equation (88) yields

$$F_r^{-1}(pr) = F^{-1}(1 - mR) = t_{la} \tag{90}$$

where t_{la} is the minimum lifing age of the component r. The minimum lifing age represents the age at which the policy preventively replaces the component to yield a mission reliability of mR.

The true replacement multiplier, α_r, is the number of MFOP durations that occur before the part receives a preventive replacement. Component r has a true replacement multiplier of

$$\alpha_{r,true} = \frac{t_{la}}{t_{mf}} \tag{91}$$

The replacement multiplier, α_r, must be a whole number to ensure replacements occur in a MRP and do not disrupt a MFOP. Rounding down $\alpha_{r,true}$ to the nearest whole number defines α_r as

$$\alpha_r = \begin{cases} 1, & if\ 0 < \alpha_{r,true} \leq 1 \\ floor(\alpha_{r,true}), & if\ \alpha_{r,true} > 1 \end{cases} \tag{92}$$

An $\alpha_{r,true}$ less than one indicates the MFOP duration (t_{mf}) may be too large. The draft set of multipliers provides a minimum estimation to start policy refinement. The first draft is conservative; the true t_{mf} at a MFOPS may be higher, but never lower than the shortest t_{la}. The process of establishing a standard policy for a MFOP duration is driven the setting of t_{mf}'s Equation (84) and α_r's Equation (91). Step 1 adds the two fundamental equations to the framework drawn by Figure 94.

The set of component α_r's creates a conservative draft maintenance policy that guarantees a MFOPS of at least a worst-case condition. The worst-case condition ($MFOPS_{wc}$), is the probability that each component survives through its replacement interval given that it survived up to the last MFOP cycle:

$$MFOPS_{wc} = \prod_{r=1}^{n} \mathrm{P}_r\left(TTF_r > \alpha_r\, t_{mf} \middle| TTF_r > \alpha_r\, t_{mf} - t_{mf}\right) \tag{93}$$

$$MFOPS_{wc} = \prod_{r=1}^{n} \frac{R_r(\alpha_r\, t_{mf})}{R_r(\alpha_r\, t_{mf} - t_{mf})} \tag{94}$$

$MFOPS_{wc}$ is the MFOPS of the ending cycle in a maintenance policy sequence of duration t_{mp}. It represents the most conservative condition of the MFOPS from a serial system. Each component is it its worst reliably in the cycle before replacement. MFOPS_{wc} is the worst case, because, at the end of the policy, all components of the system are replaced in a large MRP or reset. The effects of the final cycle cause the lowest possible MFOPS for the system. The implications of frequency alignments are discussed in the conclusions of the chapter. A system architecture other than serial arrangement will yield a MFOPS higher than MFOPS. $MFOPS_{wc}$ is most useful when the operator plans for the system to use the entire life of t_{mp}.

6.2.2 Step 2: Check t_{mf}

As the first draft policy, the t_{mf} is set to the MFOP duration goal and is sufficient. Step 2 progresses without fail on the first iteration. During later iterations, the policy designer may choose to lower the t_{mf} when faced with an insufficient MFOPS. Step 2 checks if t_{mf} is low enough to drive a system redesign. The lowest t_{la} of all components provides the minimal MFOP duration (t_{mf}) the system will survive. The MFOPS of the system may be higher if any portion of the system has components arranged in a manner other than serial (i.e., parallel or n choose k).

The duration of the maintenance policy sequence is the operational time before the policy renews the entire system in the last MRP. Maintenance policy duration, t_{mp}, is

$$t_{mp} = \alpha_{lcm}\, t_{mf} \tag{95}$$

where α_{lcm} is the least common multiple of the set of α_r. t_{mp} is the amount of flight hours before the system goes into a reset of phase. In a complex system such as a helicopter, the number of varied parts is high and α_{lcm} may be a large number. The effective t_{mp} will be enormous and beyond the practical use of the policy or even the life of the aircraft. If t_{mp} is large, then the operator should define the number of cycles, k, the system must endure above MFOPS.

6.2.3 Step 3: Check MFOPS

Step 3 compares the MFOPS of the system to the goal MFOPS at the given t_{mf}. The Minimum Policy Success (MPS), defined by Equation (72) of section 5.3.3, must last for k cycles. The restated equation is

$$MFOPS_{req} \leq MPS\left(0 < t < k\, t_{mf}\right) \tag{72}$$

Step 3 checks to see if the MPS over *k* cycles remains above the MFOPS goal.

Running the DES over *k* cycles estimates the MPS and behavior of the system. The policy repairs failed components not causing a mission abort at the next MRP. The policy makes preventive replacements of component *r* with cycle frequency of α_r. A plot of MFOPS by cycle provides a graphical depiction of the system behavior under the policy. Figure 95 is an example plot of a policy with a sequence of 30 cycles and a MFOPS goal of 90%. The example has the same fault trees as Test Case #4 with each phase as a duration of 0.5 hours. The policy is sufficient through the eleventh cycle. The system has an α_{lcm} of 30 and fully resets after the 30th cycle. The 31st cycle and 1st cycle have the same MFOPS.

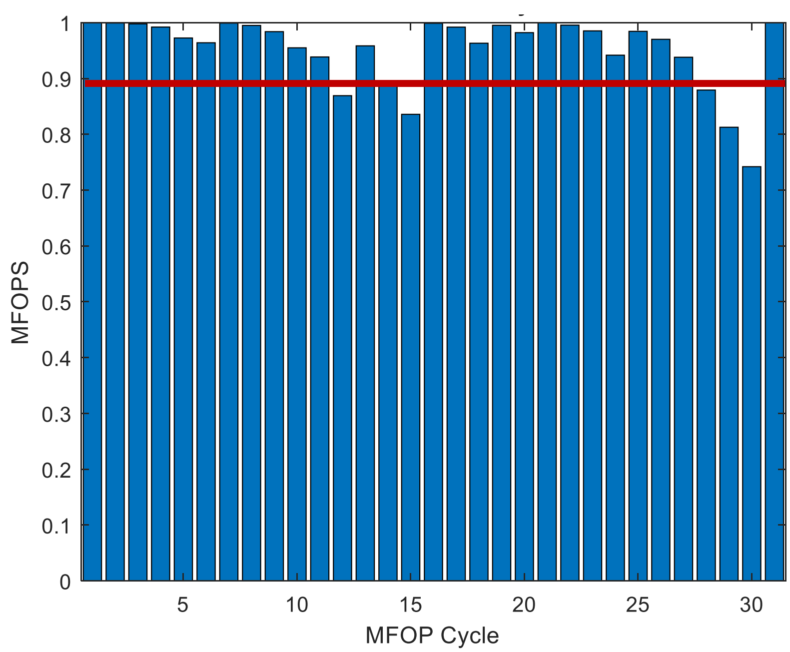


Figure 95: MPS by Cycle

Convergence of the system through *k* cycles is necessary for quality MFOPS estimates. An acceptable MFOPS has convergence to a steady state estimate on the *k*th cycle.

Although convergence of all cycles is necessary, the last cycle is the most critical. The last cycle will always have the least number of iterations surviving and suffers the most from precision error. Figure 96 shows the convergence of 31 cycles as an example. By 10,000 iterations, the system has stabilized and convergence is good by 30,000 iterations. The simulation is more susceptible to poor convergence with a system returning with lower MFOPS.

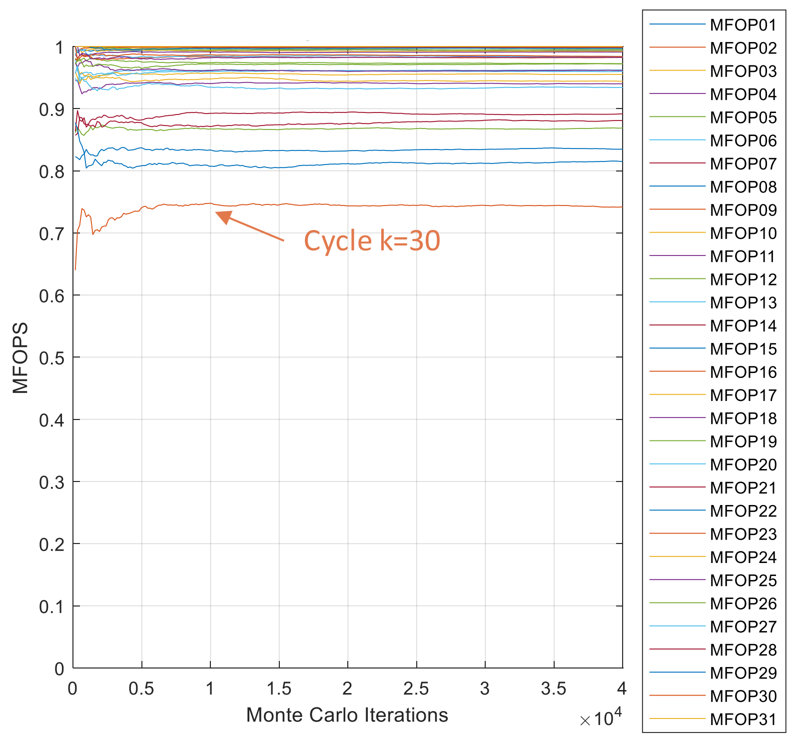


Figure 96: MFOPS Convergence

The cycle performance is the key interest in policy design. A speed advantage is available to the simulation by running fewer iterations at longer mission durations. There is more than one way to simulate a cycle with the same t_{mf} duration. For example, consider a 20-hour MFOP. The first way is to simulate the mission duration of 2 hours over a 10-

mission MFOP. A faster way is to simulate a mission duration of 20 hours in a 1-mission MFOP. Both methods simulate the same 20-hour t_{mf}; however, the first way collects information on each phase, mission, and cycle. The second method runs faster on a computer, because it does not collect information by phase and mission. The second method compiles data into cycle information. The disadvantage of this technique is the inability to assign an event to a phase and mission.

The policy designer may leave the loop if the MPS is above the MFOPS goal and the policy is satisfactory. The designer must continue to Step 4 if the MPS is insufficient or if the operational commander desires to refine the maintenance policy. Reasons to refine the policy may be a desire to increase t_{mf}, improve MFOPS, reduce MRP, or manage O&S resulting from the policy.

6.2.4 Step 4: Rank Parts

The framework assigns draft replacement intervals conservatively. Should one or more component's $\alpha_{r,true}$ be less than one, the system may not support the goal MFOPS. A redesign of the component(s) may be necessary, taking the policy designer back to the define action. Otherwise, the system is likely better than the goal MFOPS. The design can tolerate greater replacement intervals which extend the MFOP duration, reduce the MRP, and lower O&S costs. In either condition, it is necessary to identify which parts are limiting a MFOP. Components performing well are candidates for extension with less penalty to MFOPS.

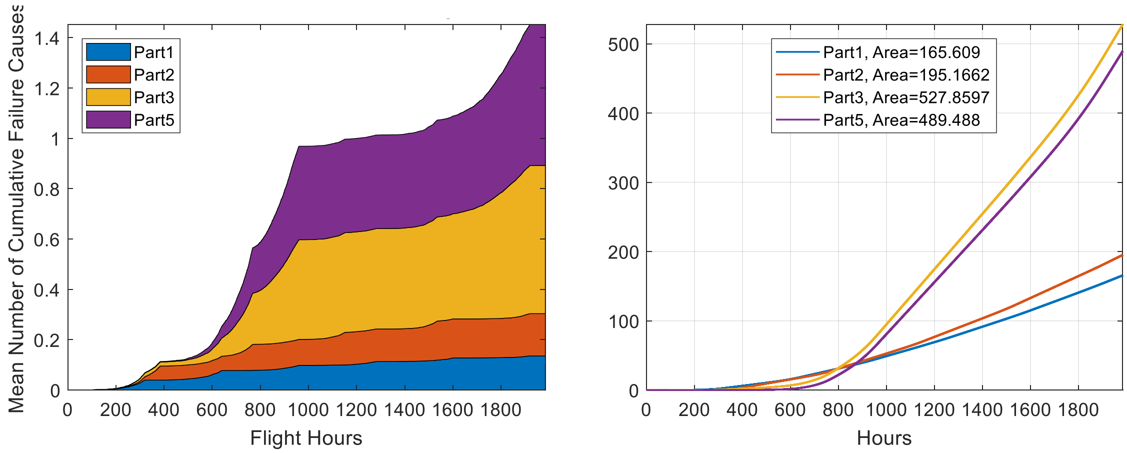
An approximate technique is to rank components by their reliability at the end of the policy. The policy resets the system at the end of the maintenance policy; hence, each component completes its replacement interval of $(\alpha_r\ t_{mf})$ when time equals t_{mp}. The equation below represents component r's reliability when time t is equal to t_{mp}

$$R_r(t_{mp}) = 1 - F_r(\alpha_r t_{mf}) \tag{96}$$

where $F_r(t)$ is the component's TTF cumulative distribution function. The contribution of a component's reliability R_r to the system reliability depends upon the system architecture. The worst-case MFOPS is the final cycle of a serial arrangement before the policy replaces all components in a single MRP and was found in Equation (94). $MFOPS_{wc}$ also makes a weak statement on the MFOPS at the end of the policy. The conditional nature of MFOPS and Equation (10) is such that MFOPS at cycle k is dependent upon the reliability at cycle $k-1$. The reliability at cycle $k-1$ is less than or equal to one; therefore, the MFOPS at cycle k is equal to or better than the reliability at cycle k. Given each component ends its replacement interval at its worst reliability at t_{mp}, Equation (94) serves as the lower limit for a possible MFOPS.

Unlike the above technique, Failure Cause Identification, established in section 3.2.1, provides a direct, quantifiable measure of a component's contribution to failure. Figure 97 provides the cumulative plot and area growth of failure cause for the system with a MFOP duration of 64 hours. Failure Cause Identification linked to a DES tells the full time-history. Figure 97(b) indicates that between the twelfth and fourteenth cycle, the failure causes change. Parts 1 and 2 are the greater contributors until 800 and 870 hours, respectively. Between the twelfth and thirteenth cycles, Parts 3 and 5 overtake Parts 1 and 2. After the 14^{th} cycle (896 hours), Parts 3 and 5 are the greatest causes of system failure through the life of the maintenance policy. A great advantage of Failure Cause

Identification is the process is robust enough to measure non-repairable, repairable systems, and a change in maintenance policy without any adjustment to the process.



(a) Stacked Failure Cause History (b) Failure Cause Area Growth

Figure 97: Failure Cause Identification in the Framework

6.2.5 Step 5: Reduce α_r or t_{mf}

If Step 3 brings an insufficient MFOPS, the policy shall reduce the MFOP duration (t_{mf}) or lower one or more component multiplier (α_r). Lowering t_{mf} lowers the replacement interval for all components. It places less demand on all components and yields greater component reliability. The penalty is a reduced MFOP duration. If the operational commander cannot tolerate a lower MFOP duration, then changing a component's multiplier provides a target means to better MFOPS. Lowering a component's multiplier lowers its replacement interval. The component reliability improves at the penalty of more preventive replacements.

By changing the multiplier, the commander is accepting a strategy of reduced risk (higher MFOPS) through a longer MRP and greater O&S costs. The recommended strategy is to begin the policy close to the true multiplier. The designer should incrementally lower the multipliers of the greatest Failure Cause component(s) until the policy meets the MFOPS goal for k cycles at the designated t_{mf}. If the MFOPS is not sufficient with multiples of one or the maintenance burden is too high, then the operator must lower the MFOP duration.

If the penalty of frequent replacements is intolerable, then the MFOP duration must decrease to meet MFOPS. A method to find the correct MFOPS at a given set of α_r multiples is to start with the MFOP duration of

$$t_{mf} \leq \min(t_{la}) \quad (97)$$

where t_{la} is the minimum lifing age of all components. The number of missions to define a MFOP must be such that

$$Number\ of\ Missions\ per\ MFOP \leq \frac{t_{mf}}{t_{phase}} \quad (98)$$

Setting the MFOP duration as a multiple of mission duration allows easier MFOP management from an operational perspective. If t_{mf} is insufficient but MFOPS is sufficient, then the policy may accommodate more missions in the cycle. Adding more missions will decrease the true multiplier $\alpha_{r,true}$ and close the gap with α_r. The smaller the gap between the actual and true multiplier, the more optimized t_{mf} is at the established MFOPS goal. A MFOP duration with an $\alpha_{r,true}$ less than one will result in a decrease to MFOPS.

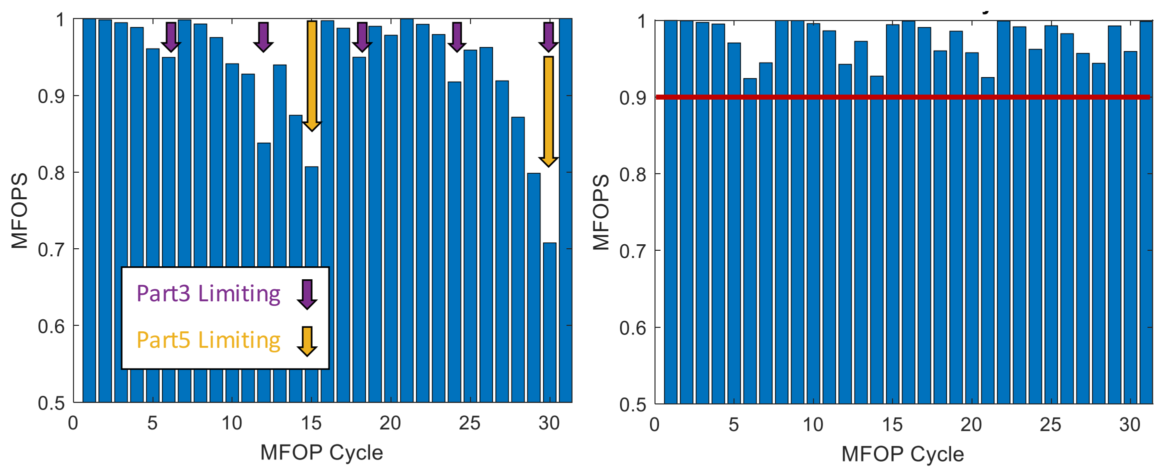
6.2.6 Revaluating the New Policy: A Thought Experiment

Returning to the example problem of this section where the MFOPS fell below the 90% goal after the twelfth cycle. The applied policy had multipliers shown in Table 37. The disparity between multipliers α_r and $\alpha_{r,true}$ in Part 3 and Part 5 are the primary reason the system experiences a drop in MFOPS over time.

Table 37: Example Policy

Component	Distribution	Mean [h]	Multiplier, α_r	True Multiplier, $\alpha_{r,true}$	Lifing Age, t_{la} [h]
Part1	Normal μ=593.42 σ=155.76	593.42	5	4.543	290.75
Part2	Weibull η=523.18 β=8.79	494.9	6	5.405	345.94
Part3	Exponential λ=0.00231	433.35	6	0.178	11.41
Part5	Weibull η=862.27 β=7.08	807.19	15	8.072	516.6

Figure 98(a) below identifies the cycles where MFOPS is low due to the extension of parts beyond their lifing age. The clue to Part 3 is the restoration of MFOPS in cycles 7, 13, 19, 25, and 31. The clue to Part 5 is the restoration of MFOPS in cycles 16 and 30. The restorations occur following a repair of Parts 3 every six cycles and Part 5 every 15 cycles. Figure 98(b) shows that tuning the multipliers of the components improved the MPS of the system. Part 3's replacement frequency doubled using an α_3 of three. Part 4's multiplier dropped to five. Part 2 remained at six and Part 1 increased to seven without major impact. The tuned policy has a minimum MFOPS of 94.2% and is successful.



(a) MPS Limiting Components (b) Improved MPS

Figure 98: Comparison of Maintenance Policies

Figure 98 captures a sensitivity study of MFOP duration for the improved policy above. The plot shows the decrease in the system's MPS (measured as the minimum MFOPS in cycles 1 to 30). Closely following this curve is the final MFOPS at cycle 30. The final curve shows the system reliability at cycle 30. This is the probability the system will successfully complete all 30 cycles without an unscheduled failure. The figure shows the trade-offs available to the operational commander in terms of operational capability (MFOP duration), operational risk (MFOPS), and policy risk (reliability).

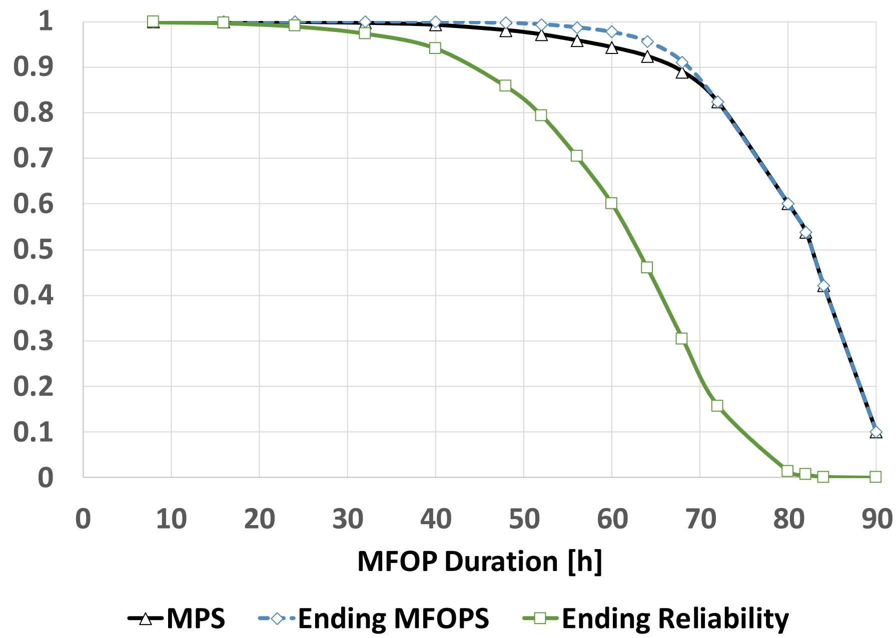


Figure 99: Policy Sensitivity to MFOP Duration

6.3 Design of Experiments

The phenomena of a MRP fully restoring the MFOPS for the next cycle as shown in Figure 21 (page 67) only occurs if the MRP fully renews the life of each component. It is not practical to renew a rotorcraft with tens of thousands of parts at each MRP. A real aircraft ages and the MFOPS will degenerate over many cycles without intervention. The work hypothesized to maintain a MFOPS, an aggressive lifing policy must selectively renew the aged parts that most lower the MFOPS. Following the provided framework created a policy that maintained MFOPS over k cycles by manipulating the replacement multipliers, α_r.

Research Question 4

What is a maintenance policy that controls MFOPS?

Hypothesis 3: If an aging system's lifing policy in the MRP is aggressive, then the policy can maintain a MFOPS to a required success over a certain number of cycles.

A paired t-test examined the significance of an aggressive lifing policy to maintain MFOPS over *k* cycles. The population for the test was rotorcraft system architectures; however, detailed modeling of thirty or more rotorcraft system architecture is an extraordinarily complicated and time-consuming effort. An alternate solution was to randomly generate a master system architecture based upon a set of common constraints. The constraints made the experiment feasible while providing sufficient variety in system architecture and components. Constraints include:

1. A three-level hierarchy
2. A reasonable number of components (no more than 10 components)
3. No more than a sixth of the components followed an exponential failure distribution to ensure sufficient inclusion of parts that experience wear and an increasing hazard rate

The architecture had AND/OR gates randomly selected at each level as well as the number of components inside each branch of the tree. Component TTF distributions were randomly generated from the range of parameters listed in Table 38. The range of MTBF provided sufficient exploration of the FVL-target MFOP duration between ten and one-hundred hours.

Table 38: Ranges of Randomly Generated Component TTF Distributions

Distribution Type	Min MTBF [h per failure]	Max MTBF [h per failure]	Second Parameter
Exponential	1,500	18,200	
Weibull	75	910	$1.5 \leq \beta \leq 10$
Normal	75	910	$0.05 \leq cv \leq 0.286$

The blank fault tree had three levels with 27 slots in the arrangement of Figure 100. The ten components were randomly selected to fill a portion of slots A to AA. Slots not filled with a component were removed from the fault tree and failure logic. Components without a sibling in its branch moved to the higher level.

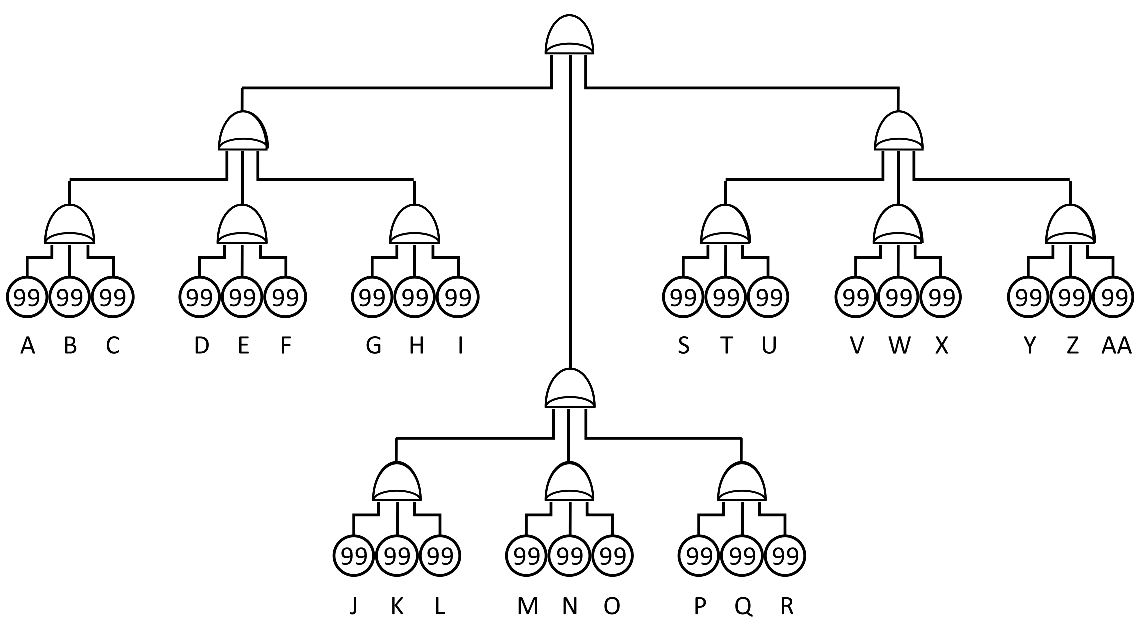


Figure 100: Fault Tree Template

Determination of the MFOP duration was a challenge. Each sample's MFOP varied due to the randomness of the architecture, component selection, and starting age. A possible solution was to generate a pool of components that, on average, yield a similar MFOP; however, the method would introduce bias to the experiment and narrow the exploration of the design space. A better option was to calibrate the MFOP to each sample. Simulating missions without repair until failure provided calibration. Repeating these multiple times via a Monte Carlo simulation built a reliability profile. The MFOP was set as the duration to experience a reliability of 10%. This ensured the MFOP duration induced a sufficient amount of failures to necessitate a preventive replacement policy.

The experiments paired samplings from two different maintenance policies created by following the framework's MFOPS control loop (Figure 94). Three randomly generated fault trees with five different sets of components constructed a total of fifteen, random samples. The DES flew each sample through *k* cycles of the calibrated MFOP and recorded the mean MFOPS and minimum MFOPS (MPS). The experiment collected a paired sample by virtually flying the system for *k* MFOP cycles under two maintenance policies: (1) a maintenance policy that only replaced failed components at the lifing age; and (2) an aggressive lifing policy.

The experiments approached the hypothesis from two perspectives. The first set experiment attempted to improve MFOPS with an aggressive lifing policy. The second experiment checked if a policy could maintain or improve a MFOPS after the extension of the MFOP duration. Each experiment examined MFOPS and MPS for a total of four t-tests on the paired samples.

The t-tests had a null hypothesis of $H_0\colon\ \mu_d = 0$, where the mean difference, μ_d, between the policies was zero. The alternate hypothesis was $H_a\colon\ \mu_d > 0$ where the mean difference was greater than zero. A significance level, α, of 0.05 was applied in the test. Rejection of the null hypothesis in the t-test would conclude the two policies had a statistically different MFOPS and the aggressive policy improved the success rate. The relative preventive maintenance burden was recorded for discussion of results.

6.4 Results and Discussion

6.4.1 Experiment 1: Improving MFOPS

The first experiment investigated if an aggressive lifing policy can improve the MFOPS of a system over *k* cycles. The standard lifing policy follows the control loop of the framework reviewed in the methodology. Two architectures ran five samples a piece for a total of ten paired samples. Figure 101 shows the component slotting for the first sample of the first fault tree. The second fault tree is in Appendix A.

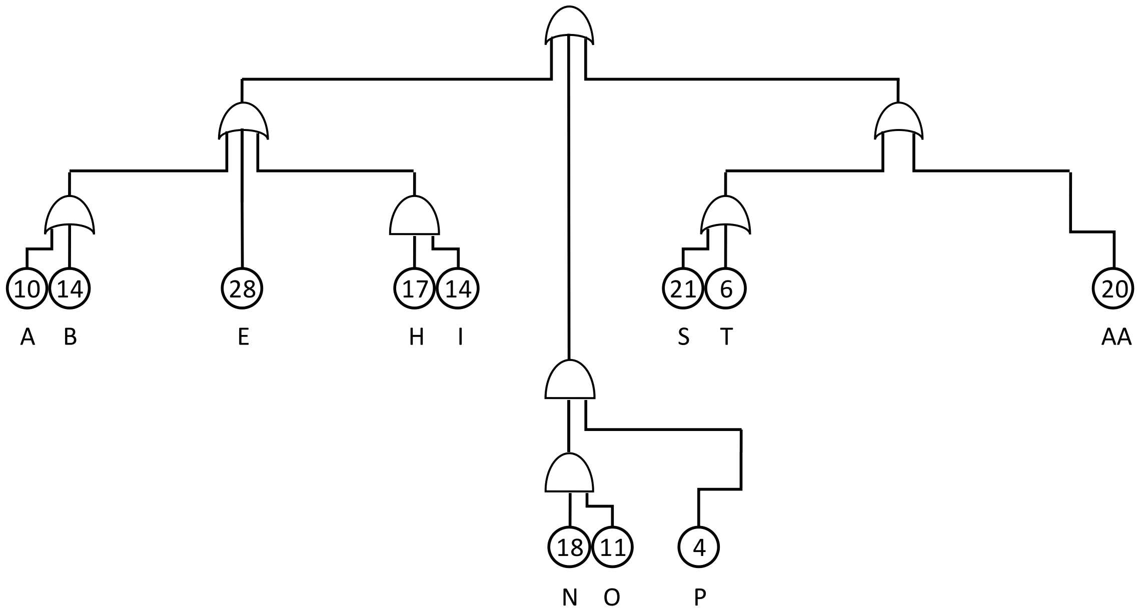
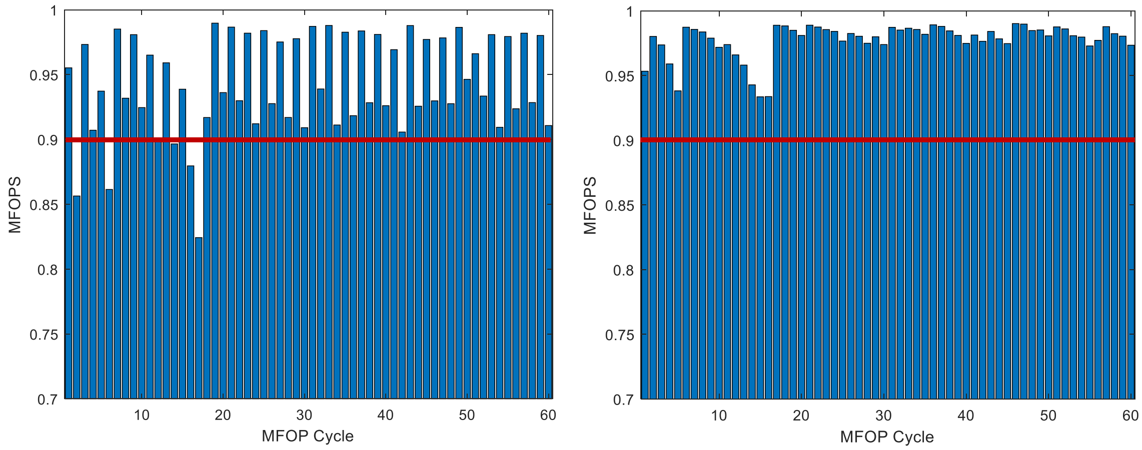


Figure 101: Fault Tree of Architecture 1, Sample 1

6.4.1.1 <u>Following the Control Loop</u>

Step 1 calibrated the system using a MFOP duration that produced a reliability of 10%. Equation (88) provided the target quantile for each component using a system Mission Reliability, MR_{sys}, of 0.80. Next, the framework calculated the component minimum lifing age and the corresponding replacement interval multiplier. The standard lifing policy rounded up all component multipliers to establish a MRP with room for improvement.

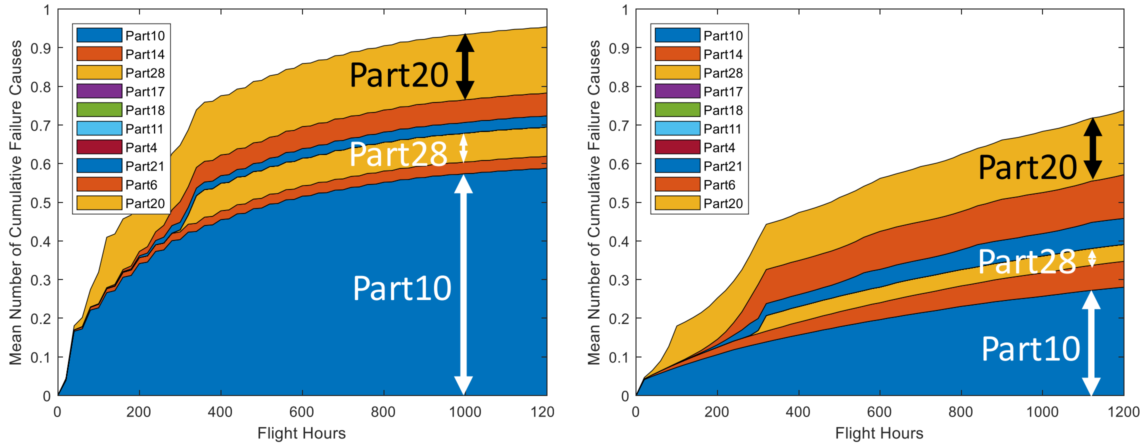
Step 2's t_{mf} remained fixed from the calibration of the previous step. The simulation ran in accordance with Step 3. Convergence typically occurred by 30,000 iterations. Table 40 logged the MFOPS for the standard policy. Figure 102(a) is the plot of MFOPS for Sample 3. This sample had a mean MFOPS of 0.9429 over the 30 cycles. The sample's minimum MFOPS (MPS) of 0.8245 occurred in cycle 6.



(a) Standard Policy (b) Aggressive Policy

Figure 102: Policy Comparison of Sample 1's MFOPS

Step 4 identified Parts 10, 20, and 28 as the primary failure causes for the system. Figure 103(a) shows the stacked failures causes of Sample 1. Part 10 ranked worst with a normalized failure cause area of 24.7. Part 20 was second with an area of 8.04 and Part 28 was 2.8. In Step 5, the aggressive policy improved the MFOPS by reducing Part 10's multiplier from 2 to 1, Part 20's multiplier from 6 to 5, and Part 28's multiplier from 17 to 16. Figure 102(b) and Figure 103(b) provided the aggressive policy's new MFOPS plot and stacked failure cause.



(a) Standard Policy (b) Aggressive Policy

Figure 103: Comparison of Sample 1's Failure Causes

6.4.1.2 Evaluating the Results and the t-Test

The aggressive policy yielded an improved MFOPS of 0.9775 (a 0.0339 improvement from the standard policy) and an MPS of 0.9329 (an improvement of 0.1121). The experiment followed the same control loop for the other nine samples. Appendix A contains the full data set from the experiment. Table 39 and Table 40 summarize the test results.

Table 39: Hypothesis 3, Experiment 1 Paired Test of MFOPS

Sample	Aggressive Policy MFOPS	Standard Policy MFOPS	Difference d
1	0.9775	0.9436	0.0339
2	0.9834	0.9694	0.0140
3	0.9650	0.9411	0.0239
4	0.9710	0.9622	0.0088
5	0.9755	0.9645	0.0110
6	0.9771	0.9250	0.0521
7	0.9918	0.9831	0.0087
8	0.9857	0.9716	0.0141
9	0.9897	0.9222	0.0675
10	0.9955	0.9724	0.0231

Mean difference	$\bar{d} = 0.02571$
Sample std. deviation	$s_d = 0.01995$
Degrees of freedom	$n - 1 = 9$
t-statistic	$t_{n-1} = 4.07441$
Significance level	$\alpha = 0.05$

$$Critical\ t\ (one\ tail) < t - statistic$$
$$1.83311 < 4.07441$$

Reject Ho. The aggressive lifing policy is statistically different.

The tables also show the t-tests of the paired samples for means. The null hypothesis stated assumed a mean difference between the policies of zero. The test t-statistic was far greater than the critical t-statistic for both the mean MFOPS and the MPS tests; leading to the rejection of the null hypothesis. This gave credence to the alternate hypothesis $H_a > \mu_d$ where the aggressive policy provided a statistical improvement to the system's MFOPS.

Table 40: Hypothesis 3, Experiment 1 Paired Test of MPS

Sample	Aggressive Policy MPS	Standard Policy MPS	Difference d
1	0.9329	0.8200	0.1129
2	0.8903	0.6126	0.2777
3	0.7990	0.5313	0.2677
4	0.8562	0.7412	0.1150
5	0.8951	0.8065	0.0886
6	0.9334	0.7806	0.1528
7	0.9571	0.8986	0.0585
8	0.8685	0.7122	0.1563
9	0.9230	0.4987	0.4243
10	0.9621	0.6336	0.3285

Mean difference	$\bar{d} = 0.19823$
Sample std. deviation	$s_d = 0.11963$
Degrees of freedom	$n - 1 = 9$
t-statistic	$t_{n-1} = 5.24016$
Significance level	$\alpha = 0.05$

$$Critical\ t\ (one\ tail) < t - statistic$$
$$1.83311 < 5.24016$$

Reject Ho. The aggressive lifing policy is statistically different.

6.4.2 Experiment 2: Extending the MFOP Duration

The first experiment considered how an aggressive policy may improve a system's MFOPS over k cycles. The next experiment investigates if an aggressive policy may help an operational commander minimize risk when extending a MFOP duration. Both are equally valid questions that an operational commander may ask when planning a deployment.

6.4.2.1 Following the Control Loop

The experiment compares the standard and aggressive policies after an extension to the MFOP duration using the third architecture (see Appendix A for its fault tree). The baseline duration was set as the flight time at which system reliability reached 10% or less. The standard policy started with an MR_{sys} of 0.8. It followed Step 1 of the control loop to establish the replacement multipliers. The process then fixed the standard policy's α_r multipliers while the MFOP duration was incrementally increased. The extension of t_{mf} stopped when the mean MFOPS dropped to approximately 90%. This captured the effects of unreliability after extending the MFOP duration. The extension of the MFOP duration was a dependent on the fault tree and components TTF distribution. On average, the MFOP was successfully extend by 40% more than the baseline value.

The experiment recorded the standard policy's MFOPS and the MPS over 60 cycles in Table 41 and Table 42. The duration of the policy sequences tended to exceed a thousand cycles for even the simplest architectures. Such long sequences were impractical in both operational application and computational time. As a counter, the cycle duration was limited to a reasonable number expected in a deployment but made sufficiently high to encompass at least one replacement interval for each component.

Like Experiment 1, Failure Cause Identification discovered the worst performing components which limited the MFOPS and MPS. The aggressive policy selectively lowered the replacement interval of no more than three components by a single interval. The simulation ran under the aggressive policy over the 60 cycles.

6.4.2.2 Evaluating the Results and t-Test

The experiment conducted paired t-tests on the MFOPS and MPS data to the policies' control of MFOPS and MPS. In both MFOPS and MPS the test statistic was greater than the critical statistic. The tables below summarize the tests and the calculation of the t-statistic.

Table 41: Hypothesis 3, Experiment 2 Paired Test of MFOPS

Sample	Aggressive Policy MFOPS	Standard Policy MFOPS	Difference d
11	0.9675	0.8942	0.0733
12	0.9572	0.9000	0.0572
13	0.9360	0.8994	0.0366
14	0.9620	0.9063	0.0557
15	0.9800	0.9412	0.0388

Mean difference	$\bar{d} = 0.05232$
Sample std. deviation	$s_d = 0.01504$
Degrees of freedom	$n - 1 = 4$
t-statistic	$t_{n-1} = 7.77651$
Significance level	$\alpha = 0.05$

$$Critical\ t\ (one\ tail) < t - statistic$$
$$2.13185 < 7.77651$$

Reject Ho. The aggressive lifing policy is statistically different.

The rejection of the null hypothesis suggests the policies have different impacts on the system mean MFOPS and MPS. The alternate hypothesis proposed the mean difference was greater than zero, $H_a > \mu_d$. This leads to the conclusion that the aggressive policy can restore a system's MFOPS despite a longer MFOP duration.

Table 42: Hypothesis 3, Experiment 2 Paired Test of MPS

Sample	Aggressive Policy MPS	Standard Policy MPS	Difference *d*
11	0.8975	0.5160	0.3815
12	0.8833	0.4445	0.4388
13	0.8489	0.7156	0.1333
14	0.7449	0.6080	0.1369
15	0.9182	0.7269	0.1913

Mean difference	$\bar{d} = 0.25636$
Sample std. deviation	$s_d = 0.14369$
Degrees of freedom	$n - 1 = 4$
t-statistic	$t_{n-1} = 5.45313$
Significance level	$\alpha = 0.05$

$$Critical\ t\ (one\ tail) < t - statistic$$

$$2.13185 < 3.98931$$

Reject Ho. The aggressive lifing policy is statistically different.

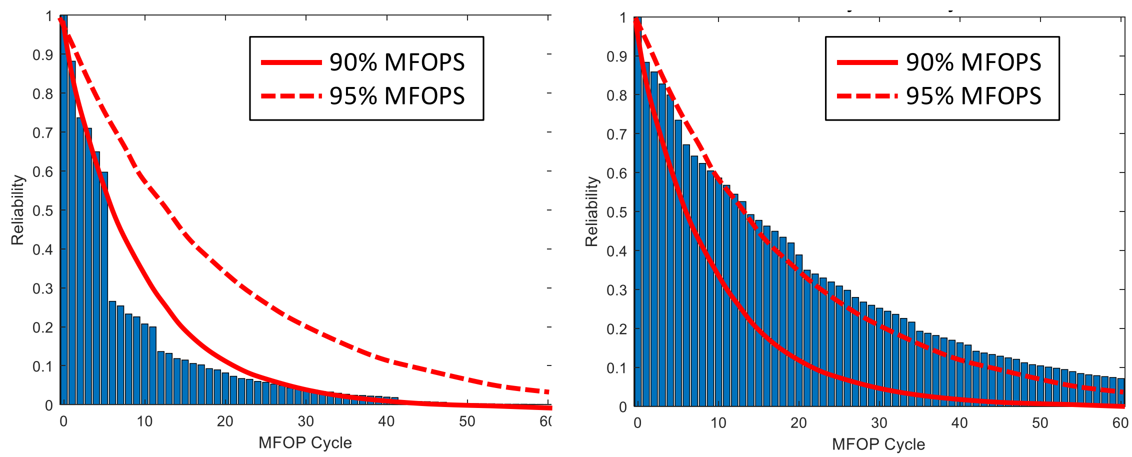
6.5 Conclusions on Controlling MFOPS

6.5.1 Connecting Reliability with MFOPS

A desirable state for a policy is a constant MFOPS above the MFOPS goal. A constant MFOPS provides dependability and a steady maintenance burden. The reliability curve of a constant MFOPS policy is a geometric decay from one to zero over *k* cycles. Equation (99) shows the theoretical, geometric relationship between reliability and MFOPS as

$$MFOPS(t_{mf}, i) = 0.9000 = \frac{R_{sys}(i \times t_{mf})}{R_{sys}([i-1] \times t_{mf})} \qquad (99)$$

where Sample 12 has a mean MFOPS of 0.9000 under the standard policy. The *i*th cycle reliability was 90% of the i-1 cycle. The *i*th cycle under the aggressive policy was 95.72% of the *i-1* cycle. The ideal, constant MFOPS curve for 90% and 95% are drawn over reliability plots of Figure 104. The curves follow the geometric decay of reliability. The standard policy's actual MFOPS had a less ideal shape than the aggressive policy's shape. The ideal curve provides a qualitative measurement of a policy's consistency and ability to meet a MFOPS goal.



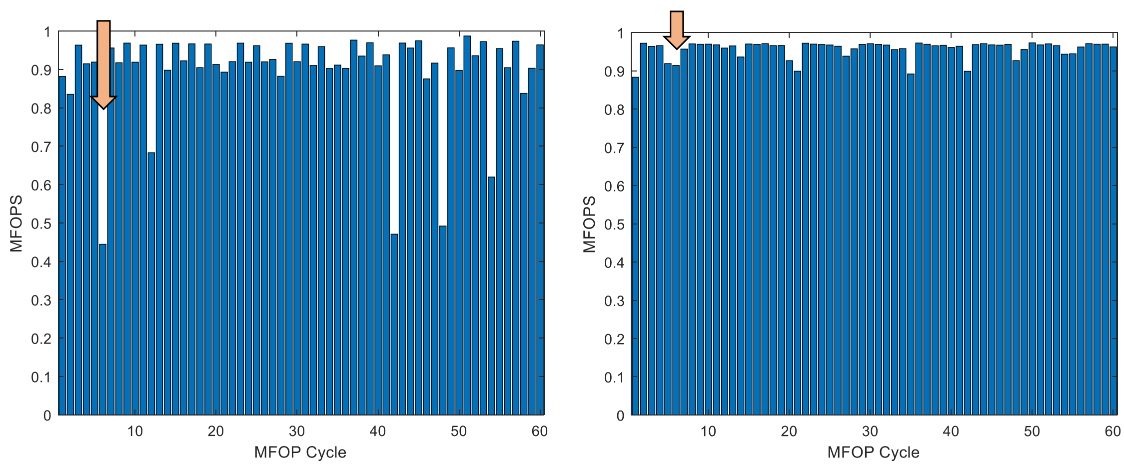
(a) Standard Policy (b) Aggressive Policy

Figure 104: Relationship Between Reliability and MFOPS

6.5.2 Diagnosing MFOPS Hunting

MFOPS hunting frequency measures the alignment of multiple components replacements to a MRP that results in a low MFOPS prior to the MRP. It is like the meshing of teeth of two gears. Gear hunting occurs when tooth *a* of the gear meshes with tooth *b* of the pinion. Based upon the number of teeth in the gear and pinion and their common factors, the teeth *a* and *b* will contact each other with a known frequency. If the gear and pinion each have a defective tooth, a bad mesh occurs at a frequency. A high contact frequency can lead to early tooth failure.

For a maintenance policy, component replacements align at the cycle of a common multiple of α_r. At the cycle of alignment, both components are reaching the end of their useful life. Their unreliability compounds and MFOPS drops sharply at the cycle of alignment. The signature tell of alignment is the return of the MFOPS on the next cycle after both components are renewed. Sample 12's sharp drop and return of MFOPS between cycles 6 and 7 as shown in Figure 105(a) is the result of MFOPS hunting. The Minimum Policy Success (MPS) is especially susceptible to hunting even if it has a high mean MFOPS. The standard policy of sample 12 had an otherwise successful mean MFOPS but suffered from hunting in cycles 6, 12, 18, 36, 42, 48, and 54.



(a) Standard Policy (b) Aggressive Policy

Figure 105: MFOPS Hunting Frequency in Sample 12

Sample 12's standard policy was an exceptionally example of hunting. Table 43 lists the α_r multiples. Part 26 had a multiple of 2. Parts 2, 20, and 15 had multipliers of 3. Parts 11 and 21 had multipliers of 6. At MRP's 6, 12, 18 and so on, the policy renewed six of the ten components. This resulted in a high burden MRP. More importantly, the hunting added a compounding risk of failure as the aligned components reach or exceed their lifing age at cycles 6, 12, 18 and so on. Replacement of Parts 28 and 16 at cycle 42 magnified the effect.

Table 43: Multipliers of Sample 12's Standard Policy

Component	Policy Multiplier, α_r	True Multiplier, $\alpha_{r,true}$	Lifing Age, t_{la} [h]	Failure Cause Area Normalized [fail-h]
Part2	3	1.32	105.4	34.1
Part28	7	4.24	338.8	34.1
Part20	3	1.39	110.8	34.1
Part11	6	3.47	277.9	34.1
Part27	8	4.69	375.5	4.4
Part26	2	0.93	74.6	33.7
Part21	6	3.28	262.2	7.8
Part16	7	4.28	342.4	14.1
Part15	3	1.75	140.4	3.64
Part27	8	4.69	375.5	1

An effective policy resolves hunting by shifting α_r multiples such that they do not have a high frequency of alignment. The best candidate for shifting is one that adds greatest to system failure. For this example, the area of the failure cause curve of Figure 106 provided the means to diagnose which component, Part 11 or 21, to shift. The MFOP duration was 80 hours. At the 480 hours and Cycle 6, Part 11 had a much greater contribution to failure than Part 11; therefore, the policy lowered Part 11 before Part 21.

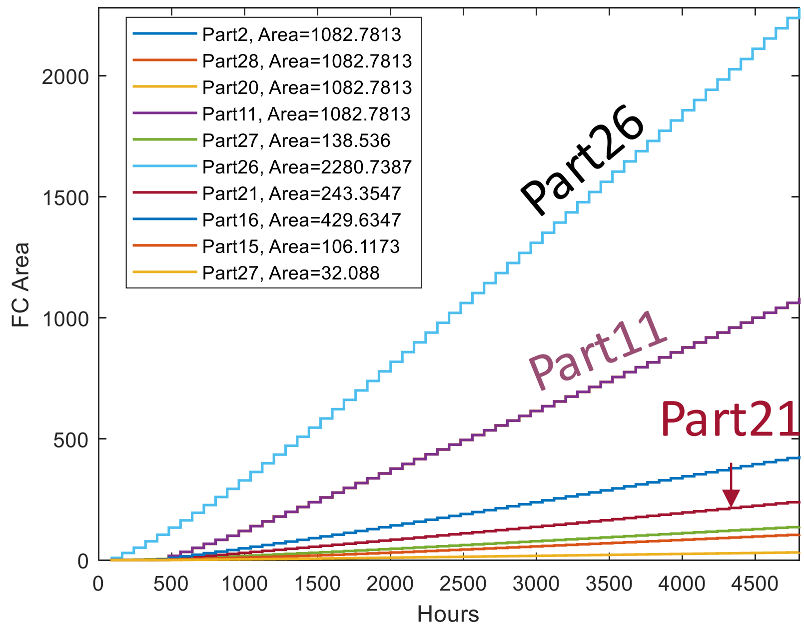
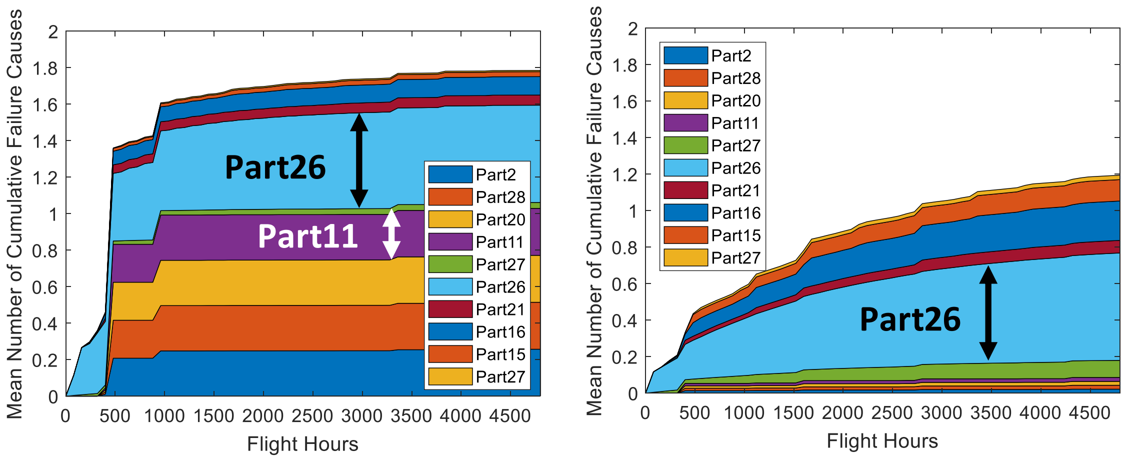


Figure 106: Sample 12's Failure Cause Identification

The aggressive policy lowered the multiplier for parts 26, 11, and 21. It changed Part 26 first, followed by Parts 11 and 21. The change yielded higher reliability and more evenly distributed their alignment with other components as shown in Figure 105(b). The new, aggressive lifing policy improved MPS from 0.4445 to 0.8833 while raising the mean MFOPS by more than 5%.

6.5.3 Reaching the Limit of MFOP Extension

Extending the MFOP duration will ultimately push the limits of system reliability and the maintenance policy will not be sufficient to maintain the MFOPS. This discussion continues the analysis of Sample 12. The experiment extended the sample's MFOP duration from 50 hours to 80 hours. The weakest link of the system is Part 26 as shown in Figure 107(a) and in the Failure Cause Identification of Figure 106 above.



(a) Standard Policy (b) Aggressive Policy

Figure 107: Sample 12 Stacked Failure Causes

Part 26 began the standard policy with a multiplier of 2 and a true multiplier of 0.93. The discrepancy between the multipliers signifies the extension of the component. The true multiplier requested replacement almost every MRP, yet the policy only provided replacement every other MRP. The aggressive policy reduced Part 26's multiplier to 1, which was a slight extension. The slight changes made by the aggressive policy saved just over 0.5 failure causes per iteration. The cost of the reduction in failure was a higher maintenance burden and cost from an additional 0.5 replacements per cycle. Even with the reduction to 1, Part 26 remains the greatest cause of failure in the system. Any further extension of the MFOP duration will exacerbate the worsening condition. The consequence of the further extension will be a precipitous drop in system reliability. At this point, the policy is unable to sustain the MFOPS. It cannot replace the component any more frequently than every MRP ($\alpha_{26} = 1$); thus, there exists a practical limit to a MFOP extension where no MFOP policy will be sufficient.

A policy with fixed multipliers trades MFOPS for a longer duration. At lower relative MFOP durations, the sacrifice of MFOPS may be small. Figure 108 plots MFOPS, MPS, and the number of preventive replacements per cycle as a function of the MFOP duration. Data was collected from the simulation using the α_r multipliers of the aggressive policy found in Table 47 of Appendix A. The figure shows extending the MFOP duration from 40 hours to 75 hours resulted in a MFOPS decrease from 0.991 to 0.966. The extension traded less than 3% of MFOPS for a 75% increase in the MFOP duration. After 75 hours, the MFOPS decayed at a higher rate. Still, the policy achieved a MFOP duration of 95 hours (a 138% increase) at a MFOPS of 0.9.

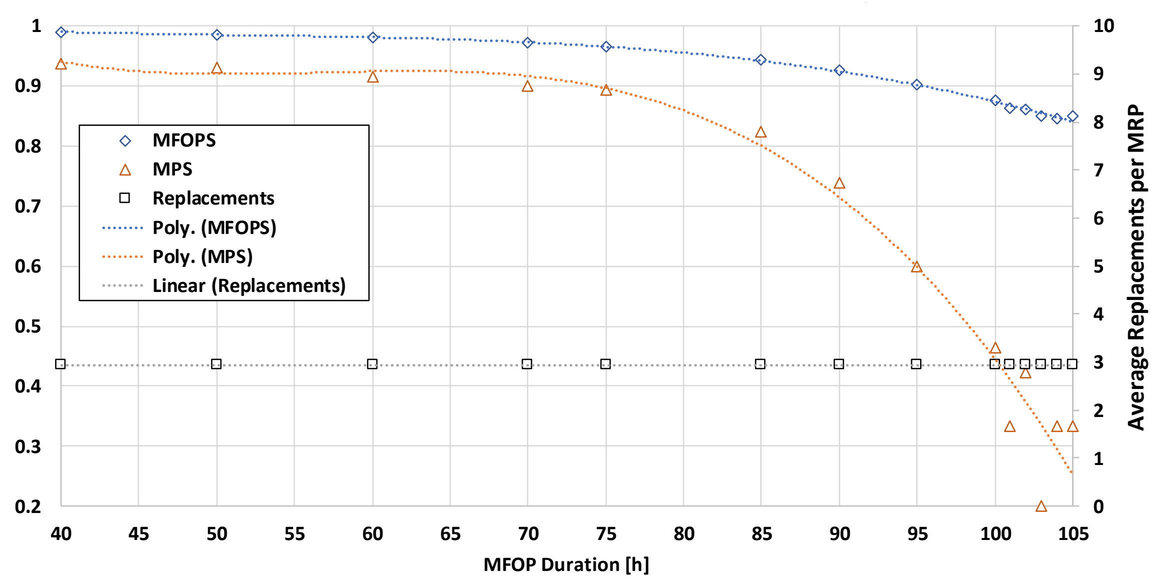


Figure 108: Sample 12, Fixed Policy's Sensitivity to MFOP Duration

The above figure supports several major conclusions. First, the MFOP of a system does have a practical limit. After 105 hours, Sample 12's mean MFOPS dropped to zero. The decay is rapid and the result of the loss of MPS. The drop originated from an interaction of several factors; the most influential factor being Part 2 reached its lifing age of 105.4 hours. Part 2 of Sample 12 occupied slot A of the fault tree (Figure 116). This slot had only OR gates in its path to the top of the fault tree. A failure of Part 2 automatically resulted in a system failure. This was the system's critical part that limits the MFOP duration. It was an excellent candidate for an additional MFOP Option like prognostics/diagnostics in the form of health monitoring to better predict an impending failure.

A second major conclusion is that the drop in MFOPS is sudden at the MFOP duration limit. The system's critical part has a cumulative probability of failure close to one at the limit. Another conclusion is in regards to MPS. The minimum MFOPS (MPS) is naturally less than or equal to the MFOPS value. MPS had a more dramatic break in its drop than MFOPS. The figure shows the break beginning at a MFOP duration of 75 hours and an MPS of 90%. Although the mean MFOPS remain above 90% until 95 hours, the operational commander should expect a select number of cycles to have poor performance with a probability of success of only 60%.

Finally, a commander should weigh the trade in MFOPS for an extended MFOP duration and understand more cycles will perform poorly even if the mean cycle MFOPS remains tolerable. Fortunately, the drop in cycles is, on average, predictable. A MFOP strategy should provide this information as part of the planning process. Of course, the operating unit may always "buy" its way out of the loss of MFOPS and MPS through more frequent preventive replacements.

6.5.4 Trade-Offs of an Adaptive Maintenance Policy

Figure 109 shows the MFOPS, MPS, and replacements per cycle of an unconstrained policy. Unlike the above fixed policy, this policy adapted to the MFOP extension through the successive lowering of its α_r multipliers. The approach rewrote the policy at each extension interval by going through one iteration of the loop. This approach produced positive results in its ability to maintain the MFOPS for 60 cycles. At 100 hours, the MFOPS was above 95%. At 150 hours, the MFOPS crossed below 90%. The adaptable policy achieved a MFOP of 200 hours before experiencing the limit.

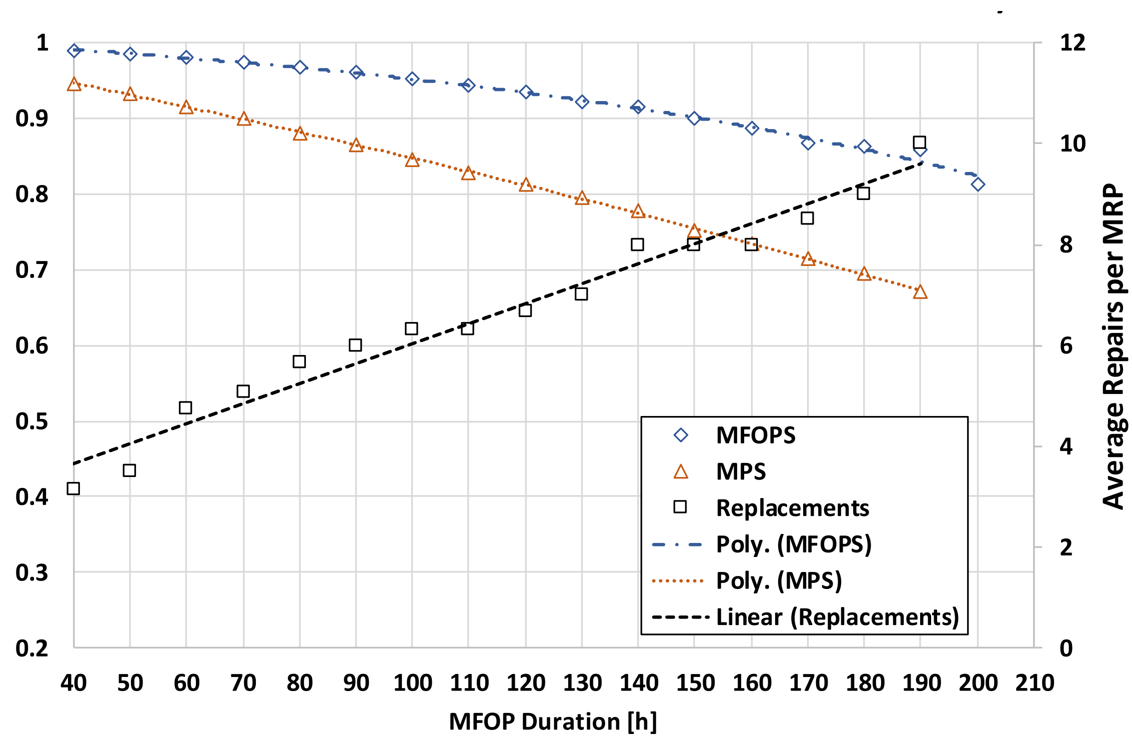


Figure 109: Sample 12, Adaptable Policy's Sensitivity to MFOP Duration

The MPS had a milder decay under the adaptable policy than the fixed policy. It remained below the mean, by definition, but declined in an almost linear fashion. Like a fixed policy, the adaptable approach experienced a sudden loss of MFOPS at the MFOP limit. The loss was sudden and occurred between hours 200 and 201. It may be the true drop was not a step function; however, this was not testable within computational time limits. A large number of iterations at smaller time steps should show a rapidly-decaying, smooth curve. The difference is hypothetical. The practical application of designing MFOP by a discrete number of missions or flight hours creates a finite limit that cannot be exceeded.

Figure 109 includes a plot of the preventive replacements made by the policy at each MFOP duration. The climb in replacement frequency is steady with a linear trend. The theoretical number of preventive replacements in a cycle is

$$Replacments\ per\ Cycle = \sum_r^n \frac{1}{\alpha_r} \tag{100}$$

for a system with n components. The cost per MFOP cycle is estimated as

$$Cost\ of\ Preventive\ Replacments\ Cycle = \sum_r^n \frac{\mathrm{C}_{p,r}}{\alpha_r} \tag{101}$$

where $C_{p,r}$ is the cost of a preventive replacement of component *r*.

The trade-off for a better MFOPS performance policy is an additional maintenance burden. Figure 110 shows the number of preventive replacements per cycle per flight hour of the fixed policy and the adaptable policy. The fixed policy keeps constant the α_r multipliers which holds the maintenance burden fixed. The adaptable policy increases the preventive replacements as needed to maintain MFOPS. The replacements per cycle per hour is an informative plot for the maintenance planners and commanders for their selection of a policy and a MFOP duration. The MRP burden is proportional to the curves of Figure 110, but the MRP varies by cycle depending on the part scheduled for replacement and its time to repair distribution.

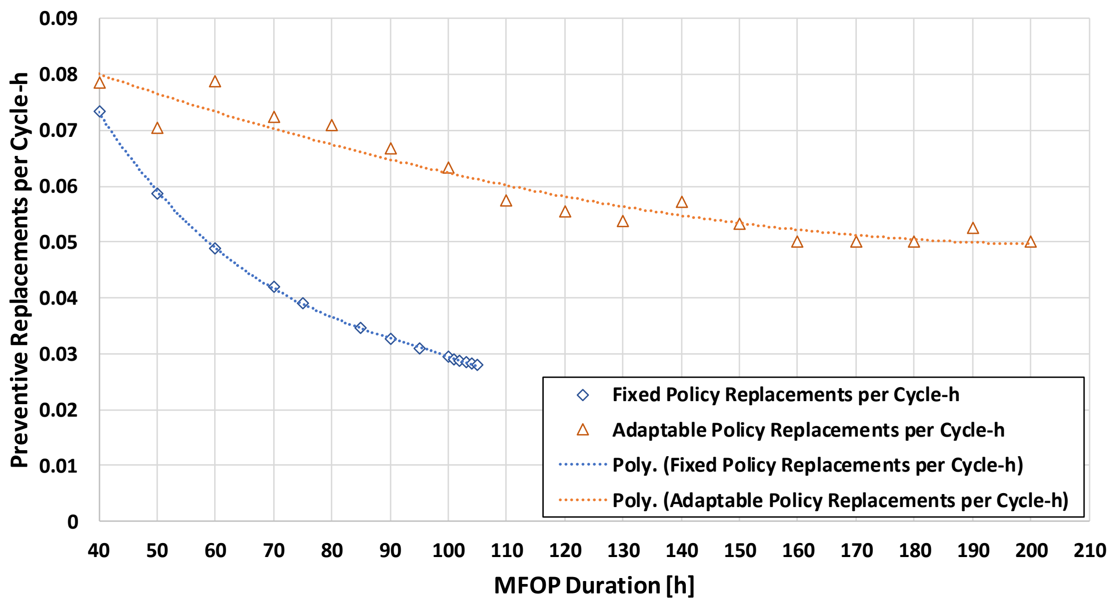


Figure 110: Comparison of Policy Maintenance Burden

6.5.5 Rounding the Multiplier

The ability of an adaptable policy to maintain a MFOPS without degradation over *k* cycles is a testament to the effectiveness to the lifing age approach established as

$$\alpha_{r,true} = \frac{t_{la}}{t_{mf}} \qquad (91)$$

Some loss of MFOPS occurs as the system may only replace components in whole number multiples of t_{mf}. The components with a greater difference between α_r and $\alpha_{r,true}$ contribute more to the failure, because they are further away from their optimal. The policy may round to the nearest whole number of $\alpha_{r,true}$, round up, or round down. Rounding down is more conservative in that it reduces risk (and MFOPS) through slightly more preventive replacements. Rounding up reduces the maintenance burden but may cause a significant decrease in MPS if the $\alpha_{r,true}$ is closer to the lower whole number (i.e., 2.2 is worse than 2.8 if rounding up to 3).

6.5.6 Potential Problems Solved Using the Framework

6.5.6.1 MSG-3

This chapter and the previous chapter presented policies to improve MFOPS and minimize downtime. Additionally, the chapter examined how the framework enables adaptable policies to keep pace with changing operational tempo. Another potential policy for use with the framework is the application of Maintenance Steering Group (MSG-3). MSG-3 is "a rigorous, structured process that identifies optimal scheduled inspection tasks and intervals for aircraft maintenance" [67]. The purpose of MSG-3 is to control maintenance costs and maximize efficiency [67].

The sustainment policy utilizes a top-down approach to optimize maintenance tasks and scheduled inspections. Much like the policy to control MFOPS, MSG-3 permits the use of an aggressive lifing policy to gain sufficient reliability and safety. Unlike the MFOPS policies, MSG-3 has been focused on lowering cost. MSG-3 currently does not protect the MFOP, but it does provide for combing inspections to scheduled intervals that reduce downtime. The interval nature makes MSG-3 a candidate for use in a MFOP strategy with the correct synchronization established by the framework's multiplier approach.

One of the attractive features of MSG-3 is that it seeks the minimization of preventive replacement times (T_p) and maintenance burden through the proper sequencing of replacements by zones. Examples include items that are co-located behind the same access panels or require the same removal of parts to service. An everyday example of zonal analysis occurs in a car's timing belt replacement. Mechanics often recommended replacing the vehicle's water pump during the replacement of the timing belt. The removal of the timing belt is a time-consuming maintenance task and must be done to prior to removing the water pump. Since the labor costs to remove the belt is significantly higher than the pump's replacement, many mechanics suggest replacing both items together. This saves labor and reduces overall costs. Reduction in the overall replacement time reduces downtime and O&S costs.

The U.S. Air Force is employing MSG-3 to maintain its aging C-5 fleet. Major depot renewals and minor scheduled inspections are done over an eight-year policy life [68]. The use of MSG-3 and zonal analysis reduced the Westover Minor ISO inspection from an average of 40 days to 18 days [68].

Applying MSG-3 to a MFOP strategy takes T_p from the deterministic value used by renewal theory to a stochastic Time to Repair (TTR) distribution. Zonal analysis adds a dependency between component TTR distributions. The added complexity of dependency eliminates analytical methods for all but the simplest systems. For complex systems like rotorcraft, Petri nets or DES provide a suitable means to handle dependency. Application MSG-3 to protect the MFOP and reduce costs is potential new research using the framework's multipliers.

6.5.6.2 Heterogenous Fleet

The concept of an adaptable maintenance strategy has tremendous potential for a fleet comprised of a mix of models, variants, and age. Since each variant has different upgrades and components, the maintenance needs differ greatly. A maintainer may force one maintenance policy across all aircraft at the risk of unnecessary or inappropriate procedures. A maintainer must manage each variant differently to avoid risk; therefore, maintenance management of heterogeneous fleet is a time-intensive, expensive effort.

The U.S. Marines V-22 Osprey fleet is an example of a heterogeneous fleet. The MV-22 experienced frequent upgrades since its fielding began in 1999. Seventeen years of war in Afghanistan and Iraq disrupted the complete upgrading of the fleet. Upgrades have occurred at such frequent, disparate intervals, that almost no two MV-22's are alike today [11]. The MV-22 is an excellent candidate for an adaptable policy tailored to each aircraft.

An adaptable policy tailoring maintenance by airframe promises to limit the risk and maximize MFOPS or availability. The notion of a "digital twin" is a fully representative model managed electronically that replicates reliability performance of the aircraft. Automated management of the "digital twin" results in optimization of the aircraft's actual performance. With an adaptable policy tailored to the digital twin, each aircraft achieves its best MFOPS. Younger aircraft no longer undergo excessive scheduled maintenance and aging aircraft experience preventive renewals to maintain reliability. The framework has the demonstrated ability to produce adaptable policies to optimize policy goals by considering upgrade levels, component age, and operational tempo.

7 CONCLUSIONS

After summarizing his formative MFOP methodology, Relf remarked, “There are no broadbrush design rules available at this stage” [24]. The research of this dissertation presented some of the tools and approaches necessary to the initiation of a MFOP strategy. After Relf’s seminal paper, Long et al. suggested a MFOP methodology drives change in “design, operation, and maintenance planning” [45]. This dissertation argued how to measure a MFOP design. It offered a framework to construct maintenance policies that favor availability or MFOP success. Finally, it demonstrated the value of a policy that adapts to a changing operational need.

7.1 Measuring MFOP

The first problem addressed the designers need to measure a MFOP rotorcraft in an effective manner. Figure 111 summarizes Problem 1. The work reasoned the “when” is as important as “how often” of failure analysis in a MFOP strategy. Historical metrics, however, prove ineffective in accounting for the time-history of failure. A discrete event simulation estimated the MFOP and reliability of a repairable aircraft flying phased missions. Its verification was successful and the approximation of a complex system was accurate with sufficient iterations. Failure Cause Identification provided the history of failure that is essential to quantifying which components most limit a MFOP. It was very useful in diagnosing a system’s limitations during the development of policies constructed with the framework. Finally, the research of Problem 1 postulated the greatest contributor to a MRP is the component with the greatest expected downtime. A practical exercise constructed a representative utility helicopter model that supported calculation of the expected downtime. The exercise partially answered the research question because of the author's inability to dedicate sufficient manpower to the data analysis beyond the top, hierarchical level of function groups. More manpower would be necessary to achieve a sufficiently detailed model down to the second or third working code level.

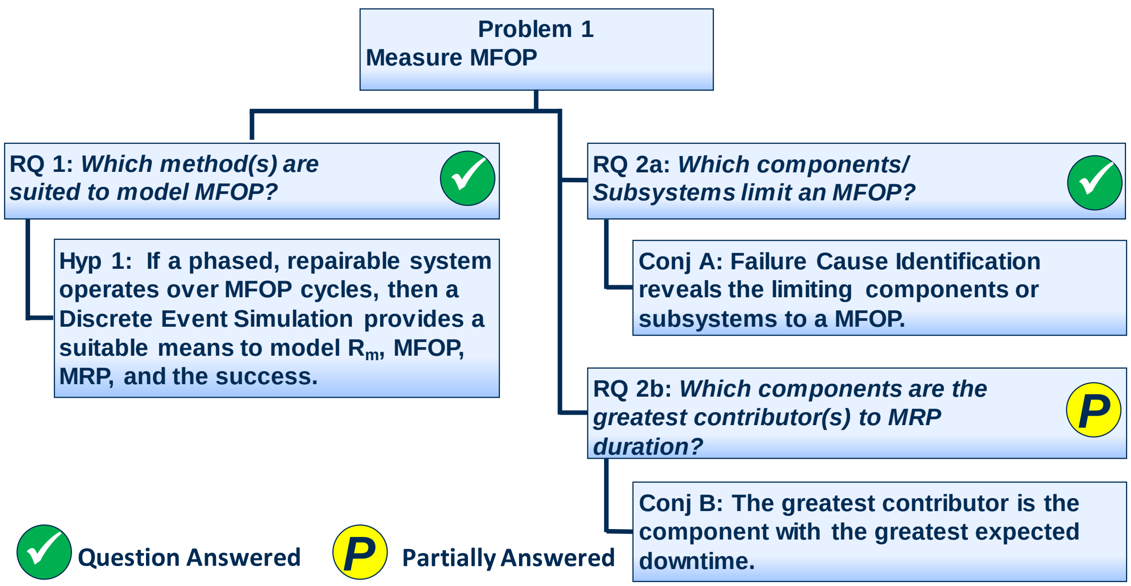


Figure 111: Summary of Problem 1 with Final Status

A primary conclusion of the thesis is tracking the history of failure for a system reveals which components limit the system's MFOP. The discrete event simulation provided an event history that fed Failure Cause Identification's quantification of the weakest links. The area metric captured a components contribution to system failure along its history. The failure cause metric weighed early failures worse than later failures—a trait that is necessary to quantify the "when" of a proper MFOP approach.

The greatest assumption (and challenge) to the accurate modeling of MFOP is component failure distributions. Relf [24] named this as “failure life characteristics” and stressed it as a necessary supporting action to a MFOP strategy. Andrews reinforced the need when he wrote, “lack of good quality data remains a serious challenge and at times an obstacle to credible quantitative analysis” [26]. All the surveyed modeling techniques and models of renewal theory are dependent upon the three distributions: time to failure, time to repair, and cost of repair. The environmental effects on component reliability were absent in this work for simplification yet remains a necessary part of failure life characteristics. The importance and challenges serve to highlight how defining failure life characteristics must be a concerted effort in FVL if the DoD wishes to achieve a MFOP strategy.

A MFOP strategy is an exercise in knowledge—the operator and maintainer may only avoid failure if the failure is known to be imminent. Preventing disruptions through sophisticated health monitoring systems, well sequenced preventive maintenance policies, and on-time logistics require knowledge of the impending failure. It is essential that designers, OEMs and vendors, and acquisition organizations understand the criticality of collecting and distributing accurate failure and repair data to support a fully developed MFOP methodology.

7.2 Maximize Availability

The second problem addressed the need for operators to manage a maintenance policy that best met the operational demands of availability and dependability. A key desire of operational commanders is to maximize availability and to minimize downtime. Solving

Problem 2 informs on how to minimize the downtime while balancing the desire for a long MFOP against the desire for a short MRP. A MFOP maintenance policy synchronized the preventive replacement intervals to the MRP. This applied a constraint on the optimization of downtime that necessitated an adaptation of classical renewal theory.

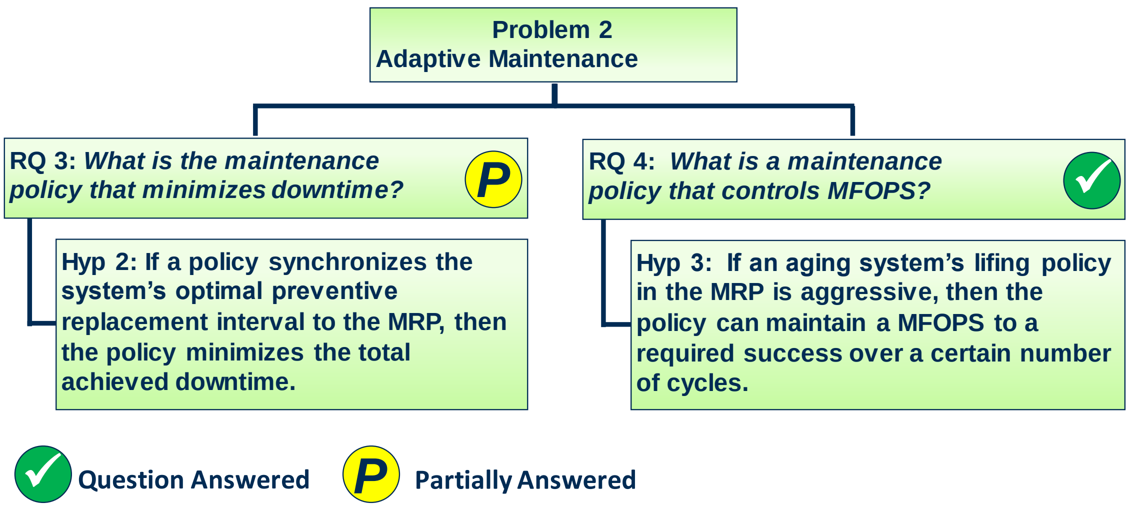


Figure 112: Summary of Problem 2 with Final Status

Renewal theory's Optimal Preventive Replacement Interval model is appropriate for a MFOP strategy when the policy makes preventive replacements at α_r multiples of the replacement interval. Unfortunately, the least replacement interval of all components places a limit on the MFOP duration. The constraint takes components and the system away from their unconstrained optimum. A revised renewal theory accommodated this constraint. The thesis presented a revised model for use in a framework that: (1) accounted for failure and preventive replacement times that are of the same order of magnitude as the replacement interval; (2) support a multi-part system of unlike components; (3)

synchronized component replacement intervals without disrupting the MFOP; and (4) manage a system constrained by reliability or MFOPS requirements.

The framework to design a maintenance policy provided the means to apply the revised renewal theory on a MFOP system. The framework defined the system, built a maintenance policy, and evaluated the maintenance policy as shown on the next page. The framework assumed the operational unit has a fielded aircraft. In this way, the framework provided an adaptability to a maintenance strategy that can accommodate different policies to meet changing operational needs.

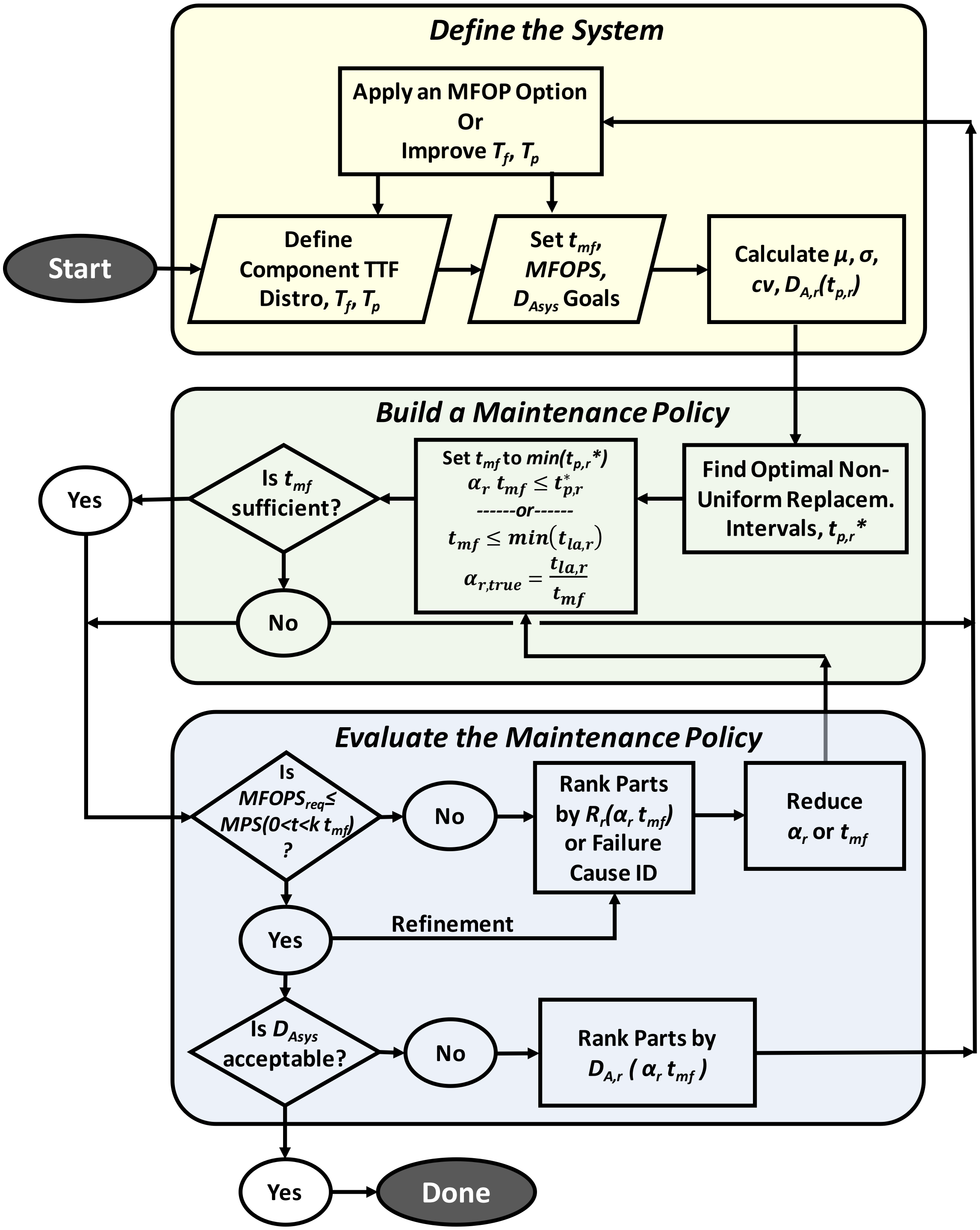


Figure 113: Framework to Enable MFOP Policy

Application of the modified renewal theory through the framework demonstrated preventively replacing components at the optimal uniform replacement interval is not the global downtime minimum. It was shown the global minimum exists using unconstrained, non-uniform replacement intervals. This directly answered Hypothesis 2. Unfortunately, non-uniform replacement intervals lead to disruptive maintenance during the MFOP. This was the state of on-condition maintenance for many years and is a condition in which the MFOP paradigm intends to eliminate. A test on a simple three-part system showed there exists a multiplier-constrained solution that minimizes downtime and protects the MFOP; however, it may result in an undesirable reliability.

Aligning replacement intervals at whole number multiples of the MFOP duration makes the downtime response discrete. The discrete nature made finding the constrained, preventive replacements difficult with traditional gradient based optimization methods. A sensitivity study served as an alternate means to understanding the problem and achieve partial success. The application of a constrained, optimization technique to fit the discrete nature of the problem is remains as the final element to achieve full success.

The sensitivity study uncovered the most influential factors in a policy that minimizes downtime:

a. A component's earliest downtime minimum is always its global minimum. Preventively replacing a component at the earliest minimum yields the least downtime contribution and higher reliability.

b. Components with a time to failure distribution with a low coefficient of variation are less likely candidates for extension beyond their optimal replacement interval. These components, however, have a more distinct amplitude near their optimal replacement time. The larger amplitude creates more distinct local minima and maxima. Consequently, they provide a greater opportunity to minimize downtime than high variance distributions. This suggests that, in a highly complex design with many parts and a large dimensionality, the low variance items dominate the sensitivity of downtime.

c. Increasing the MFOP duration is more challenging than increasing a multiplier because the MFOP duration affects the replacement interval of all components.

d. A larger replacement interval opposes reliability.

e. Failure and preventive replacement times weight the influence of a component on downtime.

7.3 Control MFOPS

A MFOP strategy seeks a dependable aircraft that operates its mission without failure and an assurance of disruption free operations for extended period. MFOPS measures that assurance against failure. Correct synchronization of component renewal (either repair or replacement) must occur outside the MFOP duration and at the next MRP. An effective maintenance policy must manage the need for MFOPS within the constraints of discrete replacement intervals. The fourth research question sought a suitable method to maintain the MFOPS for a given duration or over *k* cycles. The third hypothesis answered that an aggressive lifing policy may successful manage the MFOPS above a given requirement.

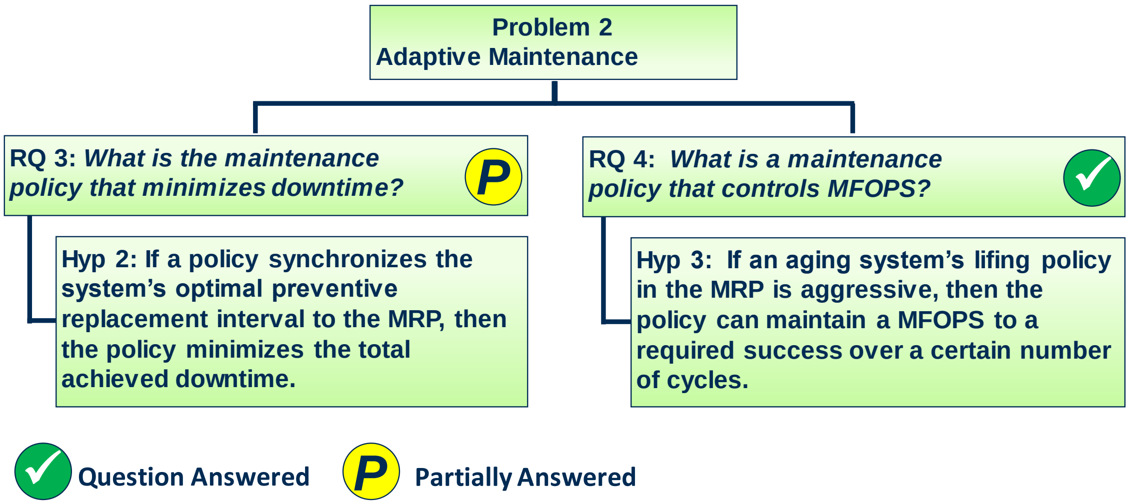


Figure 112: Summary of Problem 2 with Final Status (reshown)

The approach to answering the research question began with a detailed, step by step set of instructions to managing MFOPS within a control loop of the proposed framework. The inner control loop had five steps that set the replacement interval multipliers based upon a conservative approach to the lifing age of components. The starting point for the policy assumed a worst case, serial system. From there, the quantile function provided an estimate for the lifing age of each component. The ideal multiplier was then the quotient of the lifing age and the MFOP duration, rounded to the nearest whole number. After checking the proposed MFOP duration against the operational commander's goal, the framework estimated MFOPS using a discrete event simulation. The framework employed the Failure Cause Identification developed in Problem 1 when the MFOP duration or MFOPS was insufficient. Failure Cause Identification proved to be a robust method that works well with any policy or system architecture. The final step of the loop reduced risk by shortening replacement periods through lower multipliers until the MFOPS was sufficient.

Two experiments tested if the framework provided a sufficient maintenance policy. The first experiment considered how an aggressive policy may improve a system's MFOPS over *k* cycles. The next experiment investigated if an aggressive policy may help an operational commander minimize risk when extending a MFOP duration. Both are equally valid questions an operational commander may ask when planning a deployment.

The first experiment tested the framework's attempt at a more aggressive lifing policy to improve MFOPS over k cycles. Random architectures and part distributions created ten samples. The aggressive lifing policy selectively lowered the multiplier of the worst three Failure Cause components. A t-test rejected the null hypothesis that the standard and aggressive policy have the same mean MFOPS and minimum MFOPS (termed Minimum Policy Success). The test was in favor of the alternate hypothesis—the aggressive policy had a higher mean MFOPS and a higher MPS in the sample.

The second experiment investigated if an aggressive lifing policy would recover the lost MFOPS after an extension of the MFOP duration. The simulation measured the MFOPS and MPS of a standard policy at a baseline MFOP. The experiment then extended the MFOP duration by an average of 40%. The simulation collected the MFOPS and MPS with the aggressive lifing policy. A second set of t-tests rejected the same null hypothesis in favor of an aggressive policy that improved MFOPS.

The phenomenon of sharp drops in MFOPS followed by an equally sharp recovery, named MFOPS hunting, may occur during the design of a policy. The hunting occurred at regular intervals between multiples of failure cause contributors. The experiments discovered hunting was the major cause of a low MPS early in the maintenance policy's sequence. The experiment also showed the mean MFOPS may remain high despite select cycles having a significantly low MFOPS. Understanding the effect of hunting is crucial in designing a maintenance policy that requires consistent, dependable performance through *k* cycles. A plot of MFOPS by cycle or a review of the α_r multipliers diagnosed the hunting between components. Shifting of replacement intervals and lowering of multiplies increased the common factors and reduced the effect of hunting.

The experiments observed a finite limit on the MFOP duration in a MFOP strategy. A policy may not replace components more than every MRP without disrupting the MFOP. Should the ideal multiplier, $\alpha_{r,true}$, be less than one, the MFOP duration extended the component beyond its lifing age. Low variance components with a high Failure Cause metric will affect a precipitous drop in MPS when the duration is beyond the lifing age. Such a component is the system's critical part. Critical parts are ideal candidates for one of Relf's MFOP Options.

An investigation demonstrated a policy with fixed α_r during a MFOP extension will incur a decay in MFOPS and MPS. There does exist trade-space between the MFOP duration and its success; however, when the system reaches the MFOP limit, the MFOPS drops sharply. An adaptable policy that manages the α_r in accordance with the proposed framework corroborated the notion that an operational commander has some control over the MFOPS after changing the MFOP duration. A sample case showed the framework leads to policies robust enough to maintain MFOPS despite a 100% increase in the MFOP duration. Maintaining MFOPS at a greater MFOP duration created a trade space between MFOP duration and the number of preventive replacements per cycle per hour. The adaptable policy had both a better MFOPS and a greater maintenance burden than a fixed policy.

The success of the experiments and sample studies substantiated the claim that a maintainer does have some control over the MFOPS of a system through a lifing policy. This provides one means to fulfill the MFOP Knowledge Gap 4: *Account for varying operational tempo in a future sustainment strategy*. The policy, however aggressive, has a limit to its control. As the MFOP duration approaches its limit, the real number of replacements per MFOP cycle increases proportionally. A policy may have an earlier practical limit that depends on the operator's tolerance for the maintenance burden in the MRP and associated O&S costs.

7.4 Research Contributions

This thesis addressed three of the five knowledge gaps identified by the U.S. Army's Aviation Development Directorate.

Table 3: MFOP Knowledge Gaps (reshown)

1. Identify ***metrics*** that measure desired sustainment and readiness outcomes
2. *Find* ***tools and methodologies*** needed to support the selected sustainment approach
3. *Create* ***sustainment approaches*** for FVL and near-future to mid-future to far-future (ZMA) technologies
4. ***Account for varying operational tempo*** in a future sustainment strategy
5. Realize ***savings in O&S*** and other life cycle cost components

The work contended operational ***metrics*** such as MFOP, MFOPS, MRP, and MPS measure sustainment and readiness in a MFOP strategy. It offered ***tools*** to support a MFOP strategy. The validation of a discrete event simulation substantiated the first hypothesis's objective to model a MFOP aircraft flying phased missions. Failure Cause Identification was refined to support a MFOP context and proved robust enough to handle a variety of maintenance policies. A framework to define, build, and evaluate MFOP maintenance policies enabled the construction of policies that maximized availability and controlled MFOPS. The framework provided an adaptable policy that managed MFOPS over a varying ***operational tempo***. It is the author's desire that the thesis provides some measure of reduction in the knowledge gaps.

In summary, the thesis' major contributions to academia and the sustainment field consist of:

1. The reinforcement of discrete event simulation as a suitable means to model a MFOP aircraft using operational metrics.
2. The maturation of Failure Cause Identification as a primary tool in diagnosing MFOP system failure. It also provided an area-based metric and time weighted metric to quantify which components most limit a MFOP.
3. The introduction of a modified, optimal replacement model that minimized downtime while preserving the MFOP. The research framed the optimization problem's as a synchronization of preventive replacements to the maintenance recovery periods.
4. The presentation of a testable framework that constructed policies to minimize downtime or control MFOP probability of success. The experiments proved the

ability of an aggressive lifing policy to improve a system's MFOPS over k cycles and that of an adaptable policy to maintain MFOPS after an extension to MFOP.

5. The identification of the information requirements necessary for the modeling of a MFOP system as time to failure, time to repair, and cost of repair.

8 FUTURE WORK

Application of an optimization technique to minimize downtime using the modified optimal replacement interval model remains a challenge. Classical gradient based methods are ill-equipped to cope with the discrete nature of synchronized replacements at MRPs. The work of Hypothesis 2 framed the problem and investigated the interaction of the response to MFOP duration, replacement multipliers, and time of replacement. Successfully applying a technique to find the MFOPS-constrained, optimal solution would be a significant benefit. Secondly, there is potential to further revise the model's downtime. Not all renewals will be full in a MFOP strategy. A model should accommodate partial repairs that renew a component enough to reach maintain success in the next MFOP. A robust model that adds a renewal factor would provide an expected number of failures based upon partial repairs or installment of partially worn parts. Both advancements add techniques needed to fill the second knowledge gap and support MFOP-ready, sustainment approaches.

This dissertation examined dependability at the individual aircraft level and sets the conditions for examination of a broader fleetwide problem. Under the statistical metrics and older paradigm of maintenance, maintenance plans considered all periods of time equal; however, to a MFOP planner, all periods of time are not equal. A MFOP strategy prefers later failures over earlier failures. There is future work to study the application of a MFOP strategy where periods of time are not equal across all domains of DOTMLPF-P.

The concept of an adaptable maintenance policy provides the operational commander with a tremendous advantage when coping with change in operational tempo. The presented framework builds policies to manage the change. There exists room for development of a policy that adapts to events occurring inside the MFOP cycle. The ideal policy plans the next MRP based upon the current wear and usage in the current MFOP. A smart aircraft continuously updates its plan for the next MRP using an awareness of its health and expected remaining life of components. It taps into the MFOP Options of prognostics/diagnostics and failure life characteristics to update the maintainer after each flight of the real time MFOPS. The vision of an agile FVL family of systems becomes a reality with the advent of a smart aircraft and a fully adaptable maintenance policy.

The fifth MFOP knowledge gap: *Realize savings in O&S and other life cycle cost components* requires a methodology to realize the MFOP paradigm. DoD has documented the need for cost-effective studies in various directives ([53], [54], [20]) and handbooks ([69] [19]). The Air Force offers Cost Capability Analysis as a solution to show the balance between system effectiveness and affordability. Published work has not fully explored the application of Cost Capability Analysis in a MFOP context.

The goal of acquisition process is to maximize the value within given constraints. Cost Capability Analysis captures a technology impact with a plot like Figure 114. Providing the Cost Capability Curve graphically communicates the value of a design and its balance between effectiveness against life cycle cost.

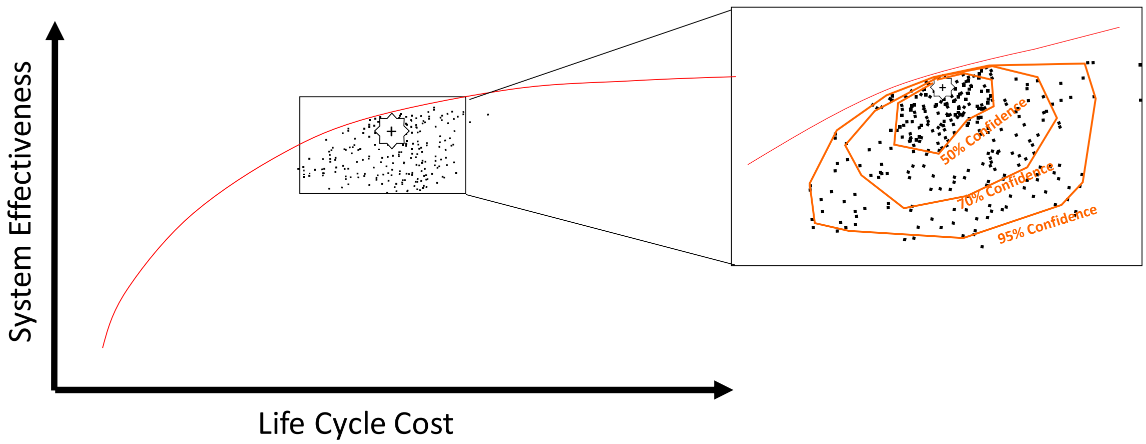


Figure 114: Example of a Cost Capability Plot

The key to communicating the benefit of a technology is to show the movement from a baseline design to a point closer to the Pareto Frontier. Quantification of uncertainty for risk analysis is also important. Bands on the plot depict the probability of achieving an outcome. They advise the decision maker on the risk associated with variability in system effectiveness and life cycle cost modeling.

The thesis provided designers and operators tools and a framework to understand a MFOP design's dependability (MFOP) and maintainability (MRP). The first problem argued for a discrete event simulation and Failure Cause Identification to measure operational metrics like MFOP and its probability of success. The second problem offered a testable framework to generate maintenance policies that adapt to change demands in availability and MFOP duration. Future work should expand the measurement of system effectiveness by incorporating performance and cost modeling. A methodology to communicate the balance between capability, availability, dependability, and affordability meets the final MFOP knowledge gap.

APPENDIX A: EXPERIMENT DATA

Table 44: Experiment Parts Pool

No.	Distribution Type	Parameter 1	Parameter 2	Mean [h]	Starting Age, $t_{age,0}$ [h]
1	Weibull	676.1	8.20	637.5	206.3
2	Weibull	212.2	5.44	195.7	24.8
3	Normal	356.2	57.21	356.2	108.6
4	Weibull	806.3	6.32	750.1	25.8
5	Normal	322.3	88.58	322.3	12.9
6	Weibull	534.2	6.47	497.7	92.1
7	Normal	830.1	115.76	830.1	273.5
8	Exponential	1.407e-4		7,109.5	42.9
9	Normal	150.1	36.97	150.1	41.9
10	Weibull	93.4	3.10	83.6	14.3
11	Weibull	432.6	8.60	408.8	122.6
12	Weibull	88.1	9.72	83.7	15.1
13	Weibull	746.5	8.38	704.6	33.1
14	Exponential	1.160e-4		8,621.6	772.8
15	Normal	306.6	82.61	306.6	82.2
16	Weibull	642.8	6.04	596.5	61.8
17	Weibull	147.0	7.49	138.0	21.6
18	Normal	909.1	85.60	909.1	144.4
19	Weibull	322.9	8.72	305.3	59.5
20	Normal	238.9	63.62	238.9	59.8
21	Weibull	528.5	5.42	487.5	18.1
22	Normal	270.9	17.66	270.9	17.7
23	Weibull	792.4	4.44	722.5	151.4
24	Normal	165.9	37.91	165.9	8.0
25	Normal	134.2	29.85	134.2	7.1
26	Weibull	471.0	2.06	417.2	131.0
27	Exponential	5.943e-5		16,825.3	4,977.2
28	Normal	395.5	28.15	395.5	33.7
29	Normal	750.3	159.50	750.3	170.3
30	Weibull	917.0	3.93	830.3	19.1

The exponential parameter is λ [failures/h]. The Weibull parameters are η [h] and β. The normal parameters are μ [h] and σ [h].

The part assignment, MFOP duration, and multipliers for each architecture is shown in Table 45, Table 46, and Table 47. Parts are slotted into A to AA in sequence. The standard and aggressive policy α_r multipliers are also in sequence. For example, the standard policy multiplier for Parts 10 and 14 of the first sample are 2 and 10. The MFOP duration of Table 47 shows the original and extended for the second experiment of Hypothesis 3. As part of the aggressive policy, up to three components had their multiplier lowered by one. The selected improvement is listed in the last column.

Table 45: Architecture 1 Test Data

Sample	Parts	MFOP t_{mf}	Standard Policy Multipliers	Aggressive Policy Multipliers	Selected Improve-ment
1	[10;14;28;17;18;11;4;21;6;20]	20	[2;10;17;5;37;14;23;14;15;6]	[1;10;16;5;37;14;23;14;15;5]	10,28,20
2	[9;5;25;8;13;24;4;4;12;12]	9	[9;17;9;18;53;10;50;50;7;7]	[9;17;9;18;53;10;50;50;6;5]	11,22,19
3	[14;7;22;12;26;5;4;19;13;11]	25	[8;24;10;3;3;6;18;9;19;12]	[8;24;9;3;3;6;18;8;19;11]	11,23,19
4	[3;13;29;17;1;27;6;15;24;4]	20	[13;24;22;5;22;19;15;8;5;23]	[11;24;22;5;22;19;15;8;5;23]	3,3
5	[17;9;15;17;19;1;14;14;15;24]	16	[6;5;9;6;14;27;13;13;9;6]	[5;4;9;6;14;27;13;13;9;6]	9,17

Table 46: Architecture 2 Test Data

Sample	Parts	MFOP t_{mf}	Standard Policy Multipliers	Aggressive Policy Multipliers	Selected Improvement
6	[20;11;13;23;29;10;27;9;19;17]	25	[5;12;19;14;18;2;16;4;9;4]	[5;12;19;14;18;1;16;4;9;4]	10
7	[40;19;6;10;30;23;30;29;13;20]	78	[8;4;1;2;6;5;6;6;3;4]	[8;4;1;2;6;3;6;6;3;4]	23,23
8	[30;24;10;21;30;21;17;7;5;21]	65	[6;2;1;5;6;5;2;10;3;5]	[6;2;1;5;6;4;2;8;3;5]	7,7,21
9	[5;5;4;7;17;27;10;9;20;7]	40	[4;4;12;15;3;10;1;2;3;15]	[4;4;12;15;2;10;1;2;3;15]	17
10	[13;4;28;16;21;19;15;28;16;18]	41	[12;11;9;5;7;6;4;8;4;18]	[12;11;9;5;6;4;4;8;4;18]	19,19,21

Table 47: Architecture 3 Test Data

Sample	Parts	MFOP t_{mf}	Standard Policy Multipliers	Aggressive Policy Multipliers	Selected Improvement
11	[16;20;9;25;14;30;24;6;23;7]	70→100	[5;2;2;2;3;5;2;5;5;9]	[5;2;2;2;3;4;2;3;5;9]	6,6,30
12	[2;28;20;11;27;26;21;16;15;27]	50→80	[3;7;3;6;8;2;6;7;3;8]	[3;7;3;5;8;1;5;7;3;8]	26,11,21
13	[15;9;6;29;10;2;19;8;23;30]	30→38	[5;3;10;15;1;4;7;6;12;12]	[5;3;10;15;1;2;7;6;12;12]	2,2
14	[16;22;7;6;11;20;17;3;8;30]	30→44	[12;8;20;10;6;4;3;9;6;12]	[12;8;20;10;5;3;3;8;6;12]	3,20,11
15	[23;25;8;22;14;23;16;2;11;7]	41→57	[9;2;4;6;5;9;9;3;7;15]	[9;2;4;6;5;7;9;2;7;15]	2,23,23

The fault trees for the architectures are in the three figures below. These are examples showing the slotted components for a sample of each fault tree. The part slotting of the other samples follows the listing in the tables above Table.

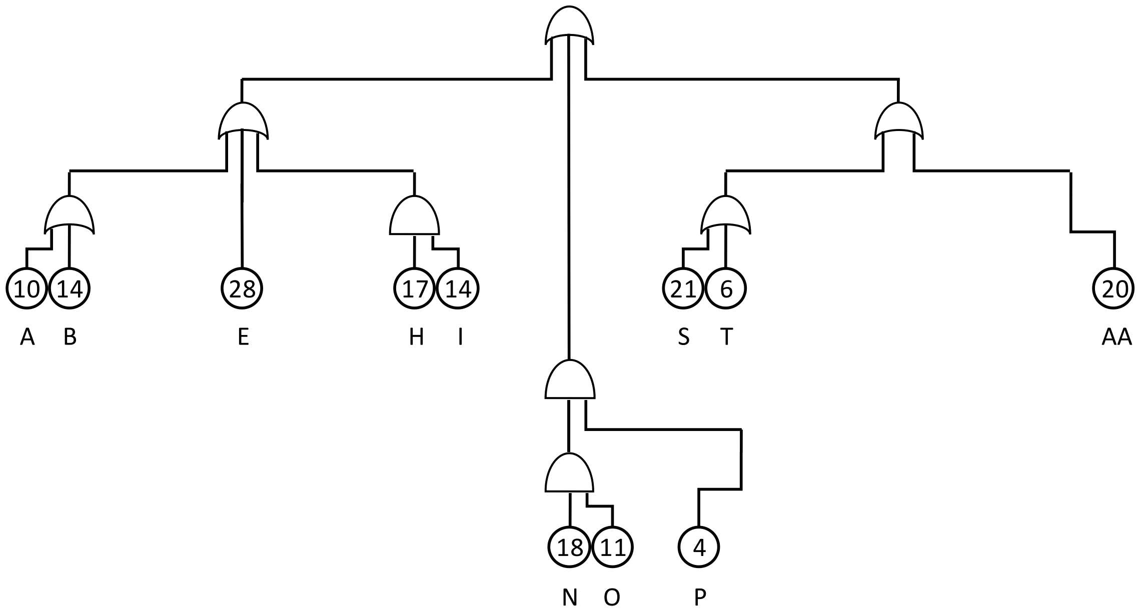


Figure 101: Fault Tree of Architecture 1, Sample

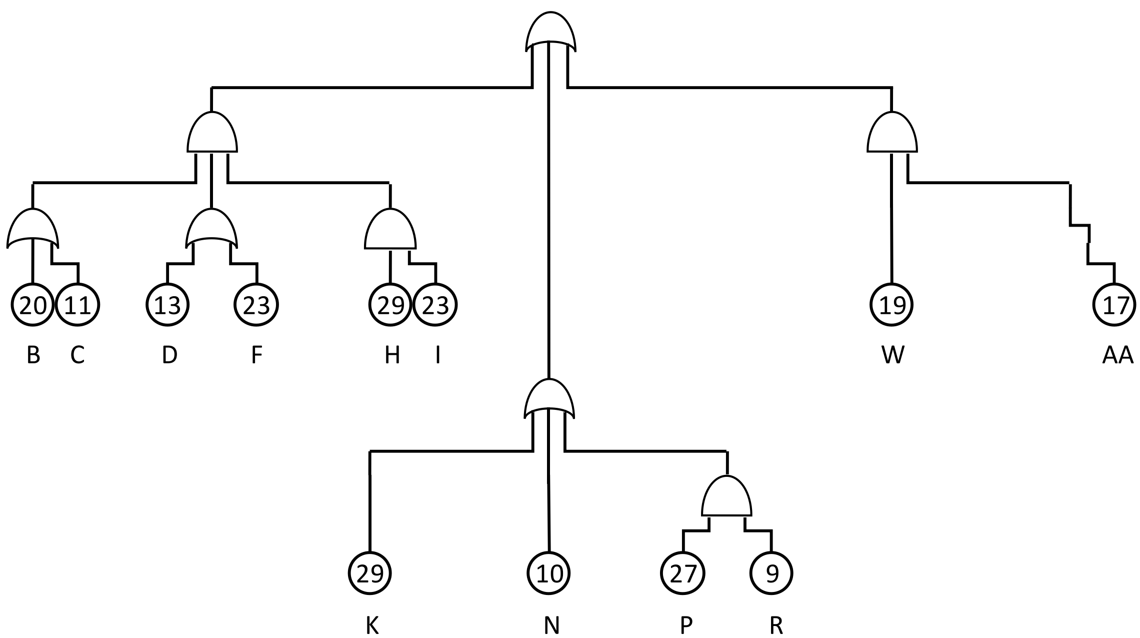


Figure 115: Fault Tree of Architecture 2, Sample 6

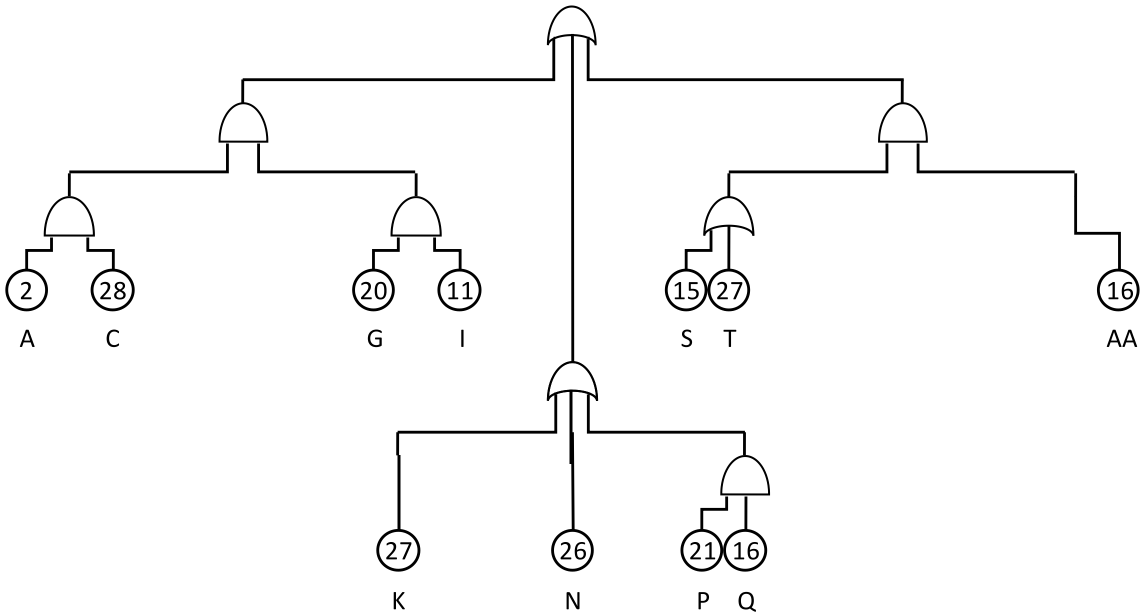


Figure 116: Fault Tree of Architecture 3, Sample 12

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