PERMANENT RACIAL INEQUALITY

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Abstract

Why are the earnings of black households so low compared to those of white households? This question is addressed here within a parsimonious equilibrium model of residential location and intergenerational transmission of human capital. This model features two residential locations, which can be interpreted as neighborhoods. In order to obtain a seamless connection of model and data, Chapter 1 applies off-the-shelf clustering methods to summarize the complexity of US neighborhood-level data into a two-neighborhood characterization. Census tracts from large US Metropolitan Statistical Areas are clustered based jointly on racial configuration (R), average human capital (H) and price of housing services (P). The chapter establishes that metropolitan US can be suitably characterized as a single city with a small number of *representative neighborhoods*. Two representative neighborhoods summarize around one third of the joint variation in (H, R, P) data while three representative neighborhoods summarize around half of the joint variation. Furthermore, census tracts clustered into each abstract representative neighborhood tend to be located close to each other, forming large areas of relatively homogeneous (H, R, P)

within an MSA. The two-neighborhood characterization is employed in Chapter 2 as an empirical counterpart of the theoretical model. In Chapter 2 the model is internally calibrated so that model generated data matches neighborhood segregation by color and earnings, neighborhood population sizes, intergenerational mobility (in earnings and consumption) and the ratio of human capital investment to output in the US. The Chapter's main finding is that, under this calibration, the proposed model produces nearly three quarters of the observed black-white percent difference in household earnings. Permanent racial inequality arises from residential racial segregation coupled with neighborhood human capital externalities. It is established that households' preferences over the racial composition of their neighborhood are at the base of the result. An interesting additional result is that, in contrast with a common line of thought, strong neighborhood color preferences are required to match US data, and halving the importance of racial preferences is enough to eliminate racial segregation and racial inequality from the benchmark equilibrium.

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Introduction

The dissertation attempts to understand the large and persistent differences in average earnings observed across black and white US households by proposing and calibrating a model that links segregation by earnings and race with permanent racial inequality.[1] In the model, white and black households have equal returns in the labor market, equal technologies for improving their human capital, equal access to housing markets, and equal innate abilities. These assumptions describe this work's definition of "equal opportunity". Equal opportunity is admittedly an extreme assumption, since it is clear that racial discrimination may still play a role in generating racial earnings disparities in the US. The purpose behind assuming equal opportunity is twofold. From a positive perspective, the absence of strong evidence of racial discrimination suggests that alternative factors should be considered. From a normative perspective, it is desirable to impose these assumptions in order to address, from a quantitative point of view, whether "equal opportunity is

[1] In some contexts the term "segregation" is interpreted as the result of some kind of discriminatory policy (as in "segregation" vs. "separation" discussions). Here it just refers to a difference in the relative concentration of each color (or earnings level) across two residential areas.

enough" to guarantee the convergence of black and white outcomes over time. The notion that equal opportunity may not be enough is the primary motivation for affirmative action policies (for an early discussion see Loury 1976).

This model is designed to capture four key facts. The first fact says that differences in average earnings across black and white households can be largely traced back to differences in observable skills put in place before individuals go into the labor market (see Neal and Johnson 1996 and Johnson and Neal 1998). In the proposed model all differences in earnings across households will be generated by differences in human capital.

The second fact says that the gap in average observable skills across black and white Americans has remained roughly constant for around 20 years (see Neal 2006). In other words, the skills of black and white Americans show no sign of secular convergence to a common level. The quantitative analysis in Chapter 2 focuses on a stationary equilibrium of the model economy in which any difference across races in skills and earnings that may arise, will be constant over time.

The third fact says that US data reveals substantial intergenerational income mobility and substantial intergenerational neighborhood-income mobility (see Solon 1999 and Sharkey 2008).[2] This critical feature will be captured by the model's mechanics, and its consequences explored in detail in Chapter 2.

[2]Intergenerational neighborhood-income mobility refers, for example, to the extent to which households move from low-income neighborhoods to high-income neighborhoods in one generation.

The fourth fact is purely observational. If one looks at *measurements* of average income and racial composition of neighborhoods (represented by census tracts) within maps of US urban areas, these areas have a striking similarity: many of them display an island-like pattern. A central "island" is composed of low-earnings, predominantly-black neighborhoods, while the surrounding "sea" is composed of high-earnings, predominantly-white neighborhoods. Many economists have viewed racial income inequality as the main cause behind this pattern of residential segregation (as was first suggested by Schelling).[3] The present work encompasses that view, but also allows for the reverse direction of causality: the pattern of residential segregation may generate racial earnings inequality in the future.

The model embeds key elements from standard Beckerian theories of intergenerational transmission of human capital in a Bewley-Huggett-Aiyagari economy (featuring a continuum of heterogeneous households and incomplete asset markets). The economy is populated by dynastic households, each period composed of one parent and one child. Parents rent their human capital in a competitive market and use some of the proceeds to make investment decisions in children's future human capital. The amount of future human capital effectively produced by these investments depends, among other things, on the innate ability of the child. Innate ability is stochastic and exogenous, but can be correlated across generations. Parents cannot

[3] According to this view, black households are highly concentrated in low-priced neighborhoods exclusively because of an exogenous correlation between race and income.

borrow against the future earnings of the child, so that investments are constrained by intragenerational liquidity. There are decreasing returns to scale in human capital production, so that human capital is mean-reverting (i.e. two households of the same color will display identical long-run behavior even if starting off with very different stocks of human capital).

Neighborhoods are incorporated into the Bewley-Huggett-Aiyagari framework. Each generation chooses where to live from a menu containing two neighborhoods. Within a neighborhood, a household chooses the amount of housing services it wishes to consume. Neighborhoods differ in average human capital, racial composition, local price of housing services and in the exogenous local supply of housing services. Introducing neighborhoods allows for three non-standard elements that will be the main sources of permanent racial inequality in the model economy.

The first non-standard element is a difference in the price of housing services across neighborhoods. This price difference will allow for the emergence of segregation by earnings in equilibrium. Holding other household characteristics constant, low earnings households will move into low priced neighborhoods, and high earnings households into high priced ones.

The second non-standard element is a preference by households over the racial composition of neighborhoods. Other neighborhood characteristics held constant, a household decides where to live based on the fraction of households of its color living in each neighborhood. In general, households will prefer to live in a neighborhood that has a large proportion of their own color. However, the formulation allows a taste for diversity whereby a household's ideal neighborhood may contain a certain fraction of households of the other color. The racial preference will allow for the emergence of residential segregation by race in equilibrium.

The third non-standard element is a neighborhood-level externality in the accumulation of human capital. Given the child's innate ability, producing a unit of future human capital requires less investment in a high human capital neighborhood than in a low human capital neighborhood. Therefore, holding the child's innate ability and parental earnings constant, residents of a high human capital neighborhood will accumulate more human capital than residents of a low human capital neighborhood. This fact has a key implication: in this model a combination of segregation by race with segregation by earnings can be a source of future racial inequality.

Each of these three non-standard elements is essential in generating permanent racial inequality. Chapter 2 discusses the role of each element in detail.

Combining externalities and segregation in a theory of persistent racial inequality is not a new idea, researchers in sociology have *theorized* along roughly similar lines before. That literature points to residential segregation as a cause of racial inequality and persistent inner-city poverty.[4] Formal economic theory has explored how exogenous segregation of two groups can lead to group inequality in the presence of group externalities in human capital accumulation.[5] Including the three non-standard elements above allows this study to be the first to endogenize the pattern of segregation. This is an important step in building a deep theory of racial inequality and also in constructing a model that can be used for policy evaluation.

Chapter 1 explores the empirical characteristics of US neighborhoods. The chapter employs 2000 US Census data at the census tract level in order to explore the existence of a suitable representative neighborhood characterization of metropolitan US. First, one chooses a small set of relevant neighborhood variables to describe a neighborhood. Then one attempts to characterize the large number of neighborhoods found in the US (according to the chosen variables) using a small number of typical or representative abstract neighborhoods. The analysis focuses on three neighborhood characteristics: H, R and P. Variable H measures the average human capital of all households in the neighborhood, R measures the fraction of households of a particular color (say white) within the neighborhood, and P measures the price of one unit of housing services.

The purpose of such characterization is the simplification of complex neighborhood level data. A simple characterization permits a transparent interpretation of

[4]See Massey and Denton (1993) ch. 1 for a survey of the sociology literature.

[5]See Loury (1976) for a substantive example in economics.

data through models featuring a small number of neighborhoods.[6] Models with a large number of neighborhoods are impractical given the complexity arising from the mutual interaction between neighborhood characteristics and household location decisions.

The main conclusion of Chapter 1 is that a suitable representative neighborhood characterization employing two or three representative neighborhoods exists. This conclusion is implied by three features of the characterizations. The first feature is good explanatory power. The two and three neighborhood characterizations summarize one third and one half of the joint variation in (H, R, P), respectively. In addition, higher dimensional characterizations provide small increments in explanatory power.[7] The second feature is the contiguity of tracts belonging to each representative neighborhood. Within each MSA, each representative neighborhood is composed by large connected areas of reasonably homogeneous (H, R, P) census tracts. The third feature is robustness. It does not matter whether the analysis focuses on a single city or a large collection of cities. The characterization is (a) remarkably stable across MSA in the sense that the proportions of households in each representative neighborhood are similar across MSA and (b) remarkably invariant to whether the methodology is applied to the full sample or city-by-city. Furthermore,

[6] An example of an interesting theory that can be tested using the two neighborhood characterization is the segregation model by Sethi and Somanathan (2004). As pointed by Durlauf (2003), an evaluation of the empirical implications of this model would improve the limited current understanding of the empirical relevance of the Schelling model of segregation.

[7] A six neighborhood characterization explains around 60% of the joint variation.

the resulting characterizations are robust across several variations in the measurement of (H, R, P), two variations of a key feature of the clustering methodology, and several variations of the sample selection criteria.

Chapter 2 presents the model and the definition of equilibrium and then finds model parameter values such that the resulting computed model equilibrium *quanti–tatively* matches neighborhood and aggregate facts from the US. The empirical neighborhood facts to be matched are obtained from the two neighborhood characterization from Chapter 1.

The neighborhood facts to be matched consist of the ratio of average earnings across neighborhoods, which is .54, the relative price of housing services across neighborhoods, which is .72, the neighborhood population sizes (27% of households live in the first neighborhood and 73% live in the other) and the racial composition of each neighborhood (the fraction of white households is .37 in the first neighborhood and .93 in the second). The aggregate facts matched are the intergenerational correlation of log earnings and log consumption found in US data, .4 and .49, respectively, the cross sectional dispersion of log lifetime earnings, .36, and the fraction of GDP devoted to education, .072, as a measure of the size of the human capital sector.

The first main result, out of two in Chapter 2, says that it is possible to match all the aggregate and neighborhood facts mentioned within the parsimonious model proposed (the model can be formally described in one page). The fact that the model is suitable to this realistic calibration allows for a quantitative assessment of the theory.

The assessment consists on asking how much racial inequality is generated by the model's mechanisms under the matching calibration. The answer is that the model produces three quarters of the measured black-white earnings differential (the model produces 28 out of a total of 39 percentage points). This is the second main result of Chapter 2.

The analysis builds on at least three important strands of the literature: persistent black-white inequality, neighborhood effects and intergenerational mobility. Its place in each of those strands is now discussed.

The first strand attempts to explain persistent black-white inequality both at the theoretical and empirical levels. On the theoretical side, the only two existing views about the sources of black-white inequality (that are known by the author) are "statistical discrimination" theory (see, for example, Coate and Loury 1993) and the intergenerational transmission theory with exogenous segregation of Loury (1976). The current work can be classified under the Loury (1976) view because, conceptually, the forces that lead to persistence of racial inequality here are the same as in that paper: intergenerational transmission and segregation by race and income.[8] Loury (1976) considers a theory with binary investment in skills where the costs of investment vary with innate ability and socio-economic background. In Loury's analysis, the equilibrium distribution of innate ability is identical within every socio-economic background and race category. [9] For tractability, Loury also assumes that each generation only cares about itself, so the benefits from a good family and social environment are transmitted costlessly across generations through "intergenerational externalities".[10] The present work goes beyond Loury's conceptual framework by endogenizing the segregation decision, allowing a continuum of investment possibilities (as opposed to a binary decision), allowing serially correlated innate ability shocks and imposing the discipline of the forward-looking dynastic household with perfect altruism abstraction (there are no intergenerational externalities in this model).[11] Bowles, Loury and Sethi (2008) considers a roughly similar environment to Loury (1976) with binary investment decision and i.i.d. shocks, but

[8]Loury (1976) does not focus exclusively on residential segregation but on any form of segregations into separate social groups.

[9]This is the consequence of two assumptions which are relaxed in this work. The first assumption is that innate abilities are uncorrelated from one generation to the next (i.i.d. shocks), the second assumption is that racial segregation into different neighborhoods, social groups or networks is exogenous. In the present work, descendants of well-off dynasties tend to have higher innate ability, while households endogenously sort into neighborhoods according to neighborhood and household characteristics. Furthermore, the characteristics of neighborhoods are determined in equilibrium by agents' decisions.

[10]Loury (1981), a later paper, considers borrowing constraints, parental investments, and forward looking agents, as a theory of intergenerational correlations beyond intergenerational externalities, but is not explicitly concerned with black-white inequality, or any kind of group inequality.

[11]The assumption of binary investment allows tractability, reducing the distribution of human capital by race from an infinite dimensional object, to two numbers. The side effects of the assumption are unclear, but could be worked out using the computational algorithms and the model employed here.

featuring forward-looking agents and a frictionless intergenerational debt market. Their analysis focuses on transition paths, and their goal is to establish conditions under which two initially equal groups can become unequal over time. Given these two precedents, this work complements the theoretical literature by computing approximations to equilibria with permanent racial inequality under endogenous segregation. ***The present work is the first to provide evidence that even in a setup where households can choose their social environment, and social environment is priced by a competitive market, there can exist equilibria with permanent racial (or group) inequality.*** An interesting extension of this work that would complement the particular findings in Bowles, Loury and Sethi (2008) is the analysis of transition paths starting from initial distributions featuring racial equality. The author intends to approach this task in future work.

The model economy proposed has a reduced number of exogenous elements, and therefore opens the way for the most important contribution of this work with respect to the literature on black-white inequality. ***The present work is the first to provide a quantitative account of persistent black-white inequality within a dynamic equilibrium theory, and to the author's knowledge, the second one to attempt to do so within any formal theory.***

The only paper, to the author's knowledge, that attempts an empirical account of persistent racial inequality in the context of an explicit model is Antonovics (2002).

That paper features a model of statistical discrimination with three potential sources of racial inequality: coordination failures, exogenous differences in the ability of white and black workers to signal their actual skills to employers, and previous racial inequality. Coordination failures and exogenous differences in signal quality impact racial inequality because "workers are only willing to invest up to the expectations of employers". Therefore, racially biased expectations can be self-confirming. Previous racial inequality impacts a binary investment decision in skills because the cost of investment in skills is assumed to be decreasing in household income.[12] The model is estimated using cross sectional census data from 1970 and 2000. The model is strongly rejected by data along a crucial dimension: the estimates imply an intergenerational correlation of wages of only .1.[13]

Antonovics (2002) constitutes an important step in the empirical evaluation of statistical discrimination theories. From the point of view of the black-white inequality literature, the present work complements the findings there by exploring the empirical relevance of neighborhood effects for persistent black-white inequality and perhaps by finding a better agreement with some facts. Both approaches share the methodological discipline of an explicit model and place some emphasis

[12]This feature can be seen as a reduced form counterpart of an intergenerational borrowing constraint.

[13]Besides, under Antonovics's estimates, most racial inequality is attributed to differences in signal quality across races. This seems inconsistent with the observation that AFQT scores can explain racial wage inequality to a large extent (AFQT scores are an example of a racially unbiased signal, perfectly observable by employers) reported by Neal and Johnson (1996).

on intergenerational transmission. The present work imposes a more self-contained structure to the model, introduces forward looking agents and several equilibrium features, but sacrifices on the empirical side by giving up on formal statistical estimation and resorting to (internal) calibration.

The second relevant strand of the literature studies neighborhood effects.[14] The current work bridges two branches of the theoretical neighborhood-effects literature. The first branch contains models of neighborhood formation in which neighborhood effects impact income inequality. The second branch contains neighborhood formation models that deal with racial segregation.[15]

Existing neighborhood formation models of inequality do not deal with racial segregation, while the segregation models, with the exception of Sethi and Somanathan (2004), do not take income inequality into account. Sethi and Somanathan (2004) is the first to allow households to sort by race and income into two neighborhoods. In contrast with the current work, Sethi and Somanathan (2004) views income distributions as exogenous objects, and analyzes a static economy.

Viewed as a neighborhood-formation model of inequality, this work introduces three "sorting mechanisms" (human capital externalities, racial composition, and

[14] The following discussion is mostly based on the survey by Durlauf (2003).

[15] The neighborhood formation and inequality branch is composed by Benabou (1993, 1996), Durlauf (1996a,b) and Fernandez and Rogerson (1996, 1997). The segregation branch contains Schelling (1969, 1971), Panes and Vriend (2002), Bruch and Mare (2003) and Sethi and Somanathan (2004).

price of housing services) as compared to only one, or at most two, in previous work.[16] The "sorting mechanisms" are roughly defined as the characteristics of neighborhoods that attract different types of households into different neighborhoods. In the benchmark equilibrium of this model, households of each race sort according to their combination of income and child innate ability. In comparison with all the neighborhood formation inequality papers cited, this work is unique in exhibiting "partial income stratification". Since households sort according to income but also according to child innate ability, the income distributions corresponding to each neighborhood and race will have considerable overlap.[17] The basic empirical fact that income distributions of different neighborhoods show significant overlap is missed by all the papers mentioned above.[18]

Viewed as a neighborhood-formation model of segregation, the current model is unique in considering forward-looking agents, introducing several factors in the residential location decision besides racial composition, and deriving the price of housing services from market clearing conditions. Durlauf (2003) provides reasons why each of these characteristics is desirable in a neighborhood formation model of segregation.

[16]For example, Benabou (1993, 1996) considers rental price differences across the two neighborhoods, Fernandez and Rogerson (1996) considers differences in income taxes and public education provision, Durlauf (1996a) considers a minimum income entry barrier.

[17]Sethi and Somanathan (2004) refers to outcomes without overlap for each race as "intraracially stratified allocations".

[18]Epple and Platt (1998), a neighborhood-formation paper in public finance, is an exception to this rule where households sort both by income and an idiosyncratic preference shock.

Most importantly, this work is unique with respect to all the neighborhood-formation papers cited, and a valuable methodological contribution to this literature because it takes a quantitative view. Chapter 2 connects model and data seamlessly. The mentioned papers do not attempt a connection to empirical evidence (some neighborhood-formation public finance papers do so, but using different approaches).

As opposed to a common interpretation of Schelling's model of segregation, the benchmark calibration reveals that matching US data requires racial preferences to be sizeable.[19] Within the benchmark equilibrium, in any given period, the aggregate period utilities of black households would be left unchanged by a 3.9% cut of their consumption of housing and non-housing goods if they could enjoy their ideal neighborhood racial configuration instead of the observed one. The aggregate utilities of white households would be unchanged by a 1% decrease in consumption if they could enjoy their ideal neighborhood racial configuration instead of the observed one. Further, it is found that all racial inequality and racial segregation disappear from the benchmark equilibrium when the weight of racial preferences in the utility function is halved. ***From the viewpoint of the segregation literature this finding challenges the idea that overwhelming racial segregation invariably results from many different realistic assumptions about the strength of racial preferences.*** In particular, it shows that this idea collapses when one considers a

[19] A common interpretation of Schelling's work on segregation says that even a very small degree of racial neighborhood preference will result in strong segregation by race.

realistic setting with several “sorting mechanisms” besides race.

There is a large literature that attempts to measure neighborhood effects on economic outcomes. I comment on this literature within the context of intergenerational correlations, discussed below.

The third strand of the literature deals with theories of intergenerational earnings correlations. Becker and Tomes (1979) and Loury (1981) propose parental human capital investments as the main force behind intergenerational correlations. Becker and Tomes (1979) considers that a child’s innate ability, which impacts these investments, can be correlated with parental innate ability. These ideas are applied in this work, as mentioned in the introduction.

The empirical literature on intergenerational correlations of income is surveyed by Solon (1999). This literature has mainly proceeded by using OLS or Instrumental Variables to estimate a regression of log lifetime earnings of sons on log lifetime earnings of parents and other explanatory factors. A popular explanatory factor has been neighborhood background. Solon (1999) concludes that, against intuition, existing work has had a hard time finding large and significant coefficients on the neighborhood background variables. The present work helps explain that puzzle. In the benchmark equilibrium of the model herein, parents of low ability children (all else constant) will move into high earnings neighborhoods in order to mitigate the shortcomings of their children through a better neighborhood environment. Thus, when children's earnings are regressed against parental earnings and neighborhood average human capital using model simulated data, neighborhood effects turn out negative (i.e. higher neighborhood average earnings reduce future household earnings). Once a control for the innate ability of the child is introduced in the regression model, neighborhood effects estimated via OLS reflect what lies in the deep structure of the model: strong positive neighborhood effects. Since innate ability has been hard to measure, this paper should be of aid in interpreting some of the results in the literature.

One related example of particular interest is the assessment of neighborhood effects in the Moving to Opportunity experiment conducted by Kling, Liebman and Katz (2007). Basically, Moving to Opportunity conducts an experiment where a group of low income households in low income neighborhoods are offered housing vouchers to subsidize their moving to better neighborhoods. The study provides results suggesting that schooling outcomes of the children in households of the treated groups are not statistically different from those in the control group.

One should keep in mind that out of all households offered housing vouchers, only around *half* actually used them. A possible explanation for the lack of significant treatment effects on the children of treated households is suggested by the benchmark equilibrium of the model economy in this paper: households with low ability children have stronger incentives to move to a better neighborhood. If the fraction of households that actually used the voucher had lower average ability than the set of households offered a voucher, the fact that educational outcomes for these children is at par with those of children in the control group implies a positive neighborhood effect.

The main message of the paper, which echoes the message of Loury (1976), is that equal opportunity may not be enough to guarantee black-white earnings and skill convergence in the long run. In comparison with Loury's, this work benefits from hindsight in that it is motivated by 20 years of stagnation in the black white skill gap and an even longer tradition of segregation. It also benefits from recent advances in the computation of equilibria in heterogeneous-agent economies and from the theoretical work on neighborhood effects. These allowed a quantitative approach to an already challenging problem. The current work, therefore, goes beyond its predecessor to establish that the theory by which the message is supported is quantitatively consistent with real world facts. Finding adequate policies to foster convergence will be the motivation of future extensions of the analysis herein.

Chapter 1

Representative Neighborhoods of Metropolitan US: A Characterization of Race, Human Capital and Housing Prices

Racial segregation within metropolitan areas and significant differences in average earnings and housing prices across neighborhoods are striking characteristics of metropolitan US.

Residential segregation of black and white households has been a salient trait of US cities for decades. A large literature in sociology deals with the measurement of racial residential segregation using census data (see Massey and Denton, 1998 and Massey, White and Phua 1996). The best known of these measures, the index of dissimilarity, reports that 64% of black population would need to change residence in order for US neighborhoods to become fully integrated (Iceland et al. 2003, pg. 60).

Income differences have also been well documented. A literature decomposes the variation of household incomes into between neighborhood and within neighborhood components. In 2000 Census data, between neighborhood inequality represents around 20% of overall household income inequality (see Wheeler and La Jeuneusse, 2007, and references therein).

Cross sectional variation in housing prices across neighborhoods has been exploited in estimating hedonic-price models of location choice.[1] Important efforts in this literature have identified major drivers of housing prices across communities and also have revealed the complexity involved in the identification of preferences in the presence of several sources of endogeneity, observed and unobserved.

The interaction between household location choices and the characteristics of neighborhood residents has been the focus of abundant work. Several channels through which neighborhood characteristics matter have been proposed. In Sethi and Somanathan (2003) households exhibit preferences over the racial configuration and the average income of neighborhoods. In Benabou (1996) production exhibits local returns to scale in learning. In Fernandez and Rogerson (1996) the quality of schooling is affected by endogenous public expenditures. In de Bartolome (1999) local peer effects affect the cost of public services. In Calabrese, Epple, Romer and Sieg (2006) local peer effects affect the quality of locally provided public goods.

[1] See Calabrese et. al (2006), footnote 4, for a comprehensive list.

Another strand of the literature has resorted to numerical calibrations of equilibrium models. However, even in a numerical environment, the number of communities must be limited because of the computational burden involved. Possibly due to this technical barrier, the empirical literature has not discussed one of the most interesting aspects of neighborhoods, namely, their role in human capital accumulation dynamics. Neighborhoods have been theorized to matter for inequality across groups (Bowles, Loury and Sethi, 2008) growth (Benabou, 1996), inequality (Durlauf , 1992) and the intergenerational persistence of earnings.

The study of neighborhood characteristics and household location decisions clearly requires an equilibrium setting due to the complications arising from the interdependence of neighborhood characteristics and household location decisions. In general, it is hard to deal with equilibrium models of many communities where neighborhood composition matters.[2] Considering that the existing models are static, and take overall income distributions as exogenous objects, discussing human capital accumulation issues within models featuring a large numbers of neighborhoods seems currently implausible. Models with a small number of locations and richer structures constitute a practical alternative.

Chapter 2 proposes and calibrates a dynamic model of human capital accumulation with heterogeneous agents that can replicate the two neighborhood characteri-

[2] For a list of papers discussing this issue see Fernandez and Rogerson, 1996, footnote 11.

zation of data provided in this chapter. Within that model it is possible to evaluate a positive explanation for permanent racial inequality, discuss the role of neighborhoods in the intergenerational transmission of economic status and study the determinants of household location decisions. That work highlights the usefulness of the simple characterizations provided here.

The main attributes of the representative neighborhood characterization are simplicity and clarity, yet there is another appealing feature that applies to clustering-generated characterizations. The $K - means$ clustering algorithm, as applied here, provides a partition of neighborhoods that minimizes a sum of squares criterion which equally weights deviations in (H, R, P) from cluster means. Under this criterion, the empirical model implicit in the representative neighborhood approach achieves the best fit to data that one can hope for with a model featuring a small number of neighborhoods. This feature provides a rationale to fitting more complex models to match aspects of the characterization developed in this chapter.

The rest of the chapter investigates several aspects of the representative neighborhood characterization. Section 1.1 describes the data set, section 1.2 discusses the estimation of the price of housing services from data on housing expenditures, section 1.3 discusses the sample selection criteria, section 1.4 introduces relevant concepts from the clustering literature, section 1.5 describes the representative neighborhoods of the US that are the main output of the chapter, section 1.6 establishes several empirical properties of these representative neighborhoods, and is followed by the concluding section.

1.1 Data

Data for this study comes from the US Census Bureau, 2000 Census of Population and Housing, Summary File 3 (SF3 henceforth). The SF3 contains geographically coded summary statistics at various levels of spatial aggregation. The SF3 contains a total of 813 unique tables, 484 contain population information and 329 contain housing information. Some selected tables are repeated by Race and Hispanic Origin. Each table comprising SF3 is available at the Census Tract aggregation level.[3]

Ideally, one would employ micro level data coded by geographic location. This data would provide maximum flexibility in sample selection, controlling for measurement error, measurement of human capital and measurement of the price of housing services in neighborhoods. However, data aggregated at the census tract level is employed. The reason is twofold. First, the exact geographic location of households and individuals is excluded from Census public access data to guarantee the anonymity of respondents. Geographically coded data can only be obtained

[3]Census tracts are small geographical subdivisions of the US designed by the Census Bureau. The primary purpose of tracts is to provide a unit for presentation of decennial census data. Census tracts generally contain between 1,500 and 8,000 people. The design of tracts aims at generating areas with homogeneous populations in demographic and economic terms, containing around 4,000 people. In contrast with population size, the spatial size of census tracts varies widely depending on local population density.

from Census Data Centers, which have restricted access. Second, SF3 provides a fine level of spatial partitioning (census tract level) and the loss coming from the use of aggregate data is small. In particular, the only departure from the ideal set of variables occurs when comparing earnings of non-Hispanic white, black and "other race" households *within* a Census Tract.[4]

1.1.1 Variables

This subsection obtains measures of (H, R, P) for each neighborhood. The operational definition of a neighborhood will be the census tract. This is a standard choice used in many studies of neighborhoods. However, there are no clear cut reasons for this choice, besides the availability of data. The census tract definition's concern with demographic homogeneity and population homogeneity are desirable properties of census tracts as practical counterparts of neighborhoods.

Human Capital (H)**.** Under standard competitive labor market assumptions, measures of household earnings, and to some extent, measures of household income, capture differences in human capital across households. Four different measures of neighborhood average human capital H are employed. The first two measures are simply the mean of household earnings and the mean of household income in a census tract. The second set of measures seeks to control for observable factors

[4] Unfortunately, the Census 2000 SF3 does not contain information on average earnings by race in a census tract. Summary File 1, an alternative source of tract level data, does not contain income or earnings variables.

that influence human capital. These measures are obtained as regression residuals from a regression of log mean earnings and log mean income against a set of tract characteristics. These characteristics include the age distribution of household heads (7 categories), the distribution of individual educational attainment in the tract (7 categories) and a set of MSA dummies, aimed at controlling for regional differences in returns to human capital. Regression residuals are additively rescaled and then transformed back from logs to levels so that the mean of the original variable in levels is matched. These residuals are referred to as "clean" measures. In the subsequent analysis results using "raw" and "clean" measures are compared.

Racial Composition (R). The ratio of number of non-Hispanic white households to total number of households in a census tract is the measure ofa tract's racial composition R employed for clustering purposes.[5] The fraction of black households and "other race" households are obtained similarly for descriptive purposes.

Price of Housing Services (P). The data contains three sources of information regarding *expenditures* in housing services, which can be used to construct measures of the *price* of housing services P in census tracts. The first source is

[5]Note that this choice is not innocuous. The sum of black and white fractions in a tract is, in general, not equal to 1 because of the presence of other races. Therefore, using the fraction of white households or the fraction of black households as a measure of R could lead to different results. However, no additional results are provided employing the fraction of black households because the number of variable configurations is already large, and the sample is designed to minimize the impact of "other race" households.

the *median house value* variable. This measure is computed by the Census Bureau using market values of housing units reported by home owning households. The housing literature employs a procedure by Poterba (1992) to transform these values into an expenditure measure or Implicit Rental Value (IRV). This procedure consists simply of applying an annual user-cost factor κ_p to the house market value (i.e. $IRV = \kappa_p \times Value$). A factor $\kappa_p = .0893$ is employed here.[6]

The second source is the *median gross rent* variable. This is the median rent paid by renter households in a tract. The measure is designed to include the cost of utilities and fees, such as condo fees, when applicable, in addition to rent. The third source is the *median selected owner costs* variable. This measure is constructed by the Census Bureau in order to estimate the monthly cost of housing for homeowners.[7]

Since expenditures are the product of quantity and price, log expenditures equal the sum of a log price component and the log number of units consumed. In order

[6]Calabrese et. al (2006) employs this approach. The annual user-cost factor is given by

$$\kappa_p = (1 + t_y)(i + t_v) + \psi,$$

where t_y is the income tax rate, i is the nominal interest rate, t_v is the property tax rate, and ψ contains the risk premium for housing investments, maintenance and depreciation costs, and the inflation rate. Calabrese et. al (2006) uses $\kappa_p = \{8.93\%, 13.93\%\}$. This procedure's main criticism comes from evidence suggesting that risk premia vary significantly across locations (see Campbell et al., 2007). This concern is mitigated here by employing alternative measures of housing expenditures.

[7]The "selected owner costs" variable includes reported payments of mortgages, deeds of trust, contracts to purchase, or similar debts on the property (including payments for the first mortgage, second mortgage, home equity loans, and other junior mortgages); real estate taxes; fire, hazard, and flood insurance on the property; utilities (electricity, gas, and water and sewer); and fuels (oil, coal, kerosene, wood, etc.). It also includes monthly condominium fees or mobile home costs (installment loan payments, personal property taxes, site rent, registration fees, and license fees).

to extract the price component, the log of each of the expenditure measures (including median IRV) is regressed against a set of housing characteristics.[8] Regression residuals are then additively rescaled and transformed back from logs to levels so that the mean of the original expenditure variable in levels is matched. The resulting measures are referred to as prices.[9] A detailed exposition of this procedure is provided in the next section.

Definitions of all measures used in this study in terms of Census SF3 variable codes can be found in Table 1.

1.2 Measuring the Price of Housing Services

A standard "cleaning regression" can be applied in order to extract the price component (as opposed to the quantity component) from a measure of expenditures in housing. The rest of the section provides conditions so that a "cleaning regression" estimated by OLS provides consistent estimates of P. The IRV (first measure of housing expenditures) is used for exposition, but the conditions also apply to the other two measures of housing expenditures described in the previous section.

Housing expenditures of household i, living in census tract j are determined

[8] The set of housing characteristics included in the regression contains the median number of rooms in the unit, a distribution of the number of units in the housing structure (10 categories), a distribution of the number of bedrooms (6 categories), fraction of units with telephone service, fraction of units with complete plumbing facilities, fraction of units with complete kitchen facilities and distribution of travel time to work in each tract (12 categories).

[9] The regression coefficients are available from the author on request.

by the local unit price of housing services P_j, and the number of units of housing services consumed by household i. The number of log units of housing services consumed by the household is measured as a quality-quantity index $\log g_i = \varphi w_i^j + MSA_i + \varepsilon_i + v_i$.[10] This index is a linear function of observed characteristics of each house w_i^j, a metropolitan area fixed effect MSA_i, unobserved house characteristics of the house ε_i, and measurement error v_i.

$$P_j g_i = P_j \exp(\varphi w_i^j + MSA_j + \varepsilon_i + v_i)$$

Following Poterba (1992), the left hand side can be measured as the IRV obtained from census data. In logarithms, this yields

$$\log IRV_{i,j} = \log P_j + \varphi w_j^i + MSA_j + \varepsilon_i + v_i.$$

Each term in the above equation can be replaced with its census tract median (the census tract median is simply denoted by dropping the i subscript).

$$\log IRV_j = \log P_j + \varphi w_j + MSA_j + \varepsilon_j + v_j$$

Now two strong assumptions are made which guarantee consistent OLS estimation of relative prices using a linear "cleaning regression" of log median IRV against median house characteristics and an MSA dummy.

A1. $\varepsilon_j = v_j = 0$

[10]The census variables included in the index w_i^j are enumerated in the previous section.

Unobserved characteristics and measurement error are symmetrically distributed around zero within each census tract, so that medians are zero. Under A1 the following equation is obtained. This equation can be used to estimate φ via OLS. Constant φ_0 denotes the intercept and $(\log P_j - \varphi_0)$ is the random disturbance.

$$\log IRV_j = \varphi_0 + \varphi w_j + MSA_j + (\log P_j - \varphi_0)$$

This is the regression equation used.

In order to guarantee that OLS estimates of φ are consistent, the following assumption is made.

A2. $E\left[(\varphi w_j + MSA_j)\log P_j\right] = 0$

Across census tracts, the price of a unit of housing services is uncorrelated with the quantity-quality index of the median home.

Under A1 and A2, OLS estimates of φ are consistent, and the OLS residual is a consistent estimator of P_j up to a constant. This allows estimation of relative prices across two census tracts, say j and j'. Letting $\hat{e}_1 = (\log \hat{P}_j - \hat{\varphi}_0)$ denote the OLS residual, notice that

$$\frac{\hat{P}_j}{\hat{P}_{j'}} = \frac{\exp(\hat{e}_j)}{\exp(\hat{e}_{j'})}.$$

Furthermore, one can compute the relative *average* prices across two *sets* of census tracts. Let C_1 and C_2 be two sets of census tracts. Then the ratio of weighted average prices in C_1 to weighted average prices in C_2 can be obtained as

$$\left(\frac{\sum_{j\in C_2}\omega_j}{\sum_{j\in C_1}\omega_j}\right)\frac{\sum_{j\in C_1}\omega_j\exp(\hat{e}_j)}{\sum_{j\in C_2}\omega_j\exp(\hat{e}_j)}.$$

Where the ω_j are weights. The expression simply displays the ratio of two weighted averages of the exponentials of OLS residuals. This formula will be used to compute the relative price of housing across two clusters of tracts in sections 1.5 and 1.6.

1.3 Sample Selection

The baseline sample aims at providing a comprehensive picture of the distribution (H, R, P) in large US cities. The operational counterpart of a city is the Metropolitan Statistical Area (MSA). Only MSAs with at least 1 million inhabitants are considered. Since the focus is on racial inequality, the sample is further restricted to MSA where at least 10% of the population is black.

Within each selected MSA, the sample is restricted to census tracts with less than 50% of "other race" households. This restriction focuses attention on black-white inequality. To guarantee the exclusion of rural areas, only those census tracts that contain at least 100 people per square kilometer are kept.[11] In order to avoid atypical observations due to small samples, attention is restricted to tracts with 200 households or more. Some census tracts contain a large number of individuals living

[11] This is a standard threshold in the housing literature above which an area is considered urban.

in group quarters.[12] The sample is restricted to census tracts with less than 25% population in group quarters.

Application of these sample selection criteria results in a set of 28 MSA in 25 states, containing 80.7 million people and 17,815 census tracts. The largest MSA in the sample is New York-Northern New Jersey-Long Island with 3,850 tracts, the smallest is Raleigh-Durham-Chapel Hill, with 157 tracts (see the last column of Table 10). Table 2 shows the number of observations deleted by each criterion. Table 3 contains summary statistics from the final sample. Section 1.6.5 compares the results obtained under the baseline sample to those obtained under 4 variations of the sample selection criteria.

1.4 Cluster Analysis

Cluster analysis is concerned with partitioning a set of objects into a small number of classes in such a way that objects within a class are similar to each other and dissimilar with respect to objects in other classes. The main goal of clustering is data simplification. Provided that a limited number of groups of similar objects arises in the data, the properties of these groups can be comprehensively observed, analyzed and modeled by a researcher.

However, the existence of a valid classification is not an assumption. Cluster

[12] Correctional institutions, nursing homes, juvenile institutions, college dormitories, military quarters, and group homes are considered group quarters.

analysis should sometimes conclude that the data in question does not exhibit any discernible pattern.

A commonly used partitional clustering method relies on minimizing a square error (SE) criterion.[13] This method is applied to a set J of objects which are described by continuous variables. Then, each element $j \in J$ can be described as a point x_j in $d-$dimensional Euclidean space. The objective is to obtain a $K-$partition $\{C_1, C_2...C_k...C_K\}$ of J that minimizes within-group variation or SE around each group's centroid c_k. This centroid c_k is usually taken to be the mean or the median of x_j over all elements j belonging to the cluster C_k.

$$SE = \sum_k \sum_{j \in C_k} \omega_j \left(x_j - c_k\right) \cdot \left(x_j - c_k\right)$$

Where ω_j is a weighting factor controlling the importance of element j in the objective function. When centroids are means, this criterion provides a connection between clustering analysis and formal statistical estimation. Under the assumption that deviations from cluster centroids are normally distributed, minimization of SE by choice of partition is equivalent to maximum likelihood estimation of a Gaussian mixture model.[14] Conceptually, the optimal partition can be easily found

[13]Cluster analysis is divided into partitional and hierachical methods. Partitional methods create a single partition of a set J of objects containing N_J objects into K mutually exclusive subsets. In hierarchical methods the data is sequentially split into mutually exclusive groups. One starts with a single group of N_J objects and finishes with N_J groups, each containing one object. New partitions are always nested in previously defined ones. In this sense, hierarchical methods provide a "taxonomy" of the data.

[14]Under the Gaussian mixture model, one assumes there is a "true" partition of the data $\{C_1^0, ..., C_k^0, ..., C_K^0\}$ and that x_j such that $j \in C_k^0$ are normal random variables with mean c_k^0

by computing SE for each of the possible partitions of set J into K disjoint subsets and then choosing the one that minimizes SE. In applications, searching over all possible partitions is infeasible since the number of candidates grows quickly with N_J and K. The search has to be conducted by a heuristic algorithm.[15] A clustering resulting from iterative relocation has two desirable properties. First, each cluster has a centroid which is the mean (or median, alternatively) of the objects in that cluster. Second, each object belongs to the group possessing the nearest centroid. On the downside, this type of algorithm is not guaranteed to find the optimal partition, and its outcome depends on the initial partition chosen.

In the exercises that follow, the clustering procedure is applied 10 times using random starting values and the result that minimizes SE is reported. It's worth noting that the results are nearly identical across the 10 repetitions.

The $K-Means$ algorithm is employed in this study. This algorithm implements the partitional method described above. Centroids c_k are defined as the means of

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and covariance matrix Ω_k^0. The parameters to be estimated are the within-cluster means and covariance matrix, the number of members, and the identity of the members of each group. It is easily shown that when Ω_k^0 is a diagonal matrix, the maximum likelihood estimator of this model minimizes SE.

[15] The most common family of heuristic algorithms employs some variation on a set of steps known as iterative relocation. Iterative relocation proceeds as follows:

1. Assign elements arbitrarily into an initial partition comprised of K clusters and calculate the centroid of each cluster.
2. Generate a new partition by reassigning each element to the nearest cluster centroid. If no objects were reassigned, terminate.
3. Compute new centroids using the partition obtained in Step 2. Return to Step 2.

the elements of each cluster and weights ω_j are set proportional to the number of households in each census tract j.

1.4.1 Normalization of Data

The clustering procedure is sensitive to the relative scaling of variables describing each neighborhood. A common solution to this problem is to normalize each component of x_j to have zero sample mean and sample variance 1. This method is referred to as Z-score normalization in what follows. This procedure is simple and intuitive. However, one should be aware that this kind of normalization does not take into account the correlations across components of x_j.[16] An alternative normalization method, based on the Mahalanobis transformation, takes these correlations into account. This method normalizes the data by an estimate of the inverse covariance matrix $\widehat{\Omega}^{-1}$ of the data. In this case SE becomes

$$SEM = \sum_k \sum_{j \in C_k} \omega_j \left(x_j - c_k\right) \widehat{\Omega}^{-1} \left(x_j - c_k\right).$$

Where $\widehat{\Omega}$ is a sample estimate of the covariance matrix of x_j. The neighborhood characterizations of Section 5 are constructed employing Z-score normalization while sections 1.6.1 and 1.6.2 establish some properties of these characterizations comparing results under Z-score normalization and the Mahalanobis transformation.

[16] Jain and Dubes (1988, pg. 26) contains an example where this neglect leads to the destruction of the underlying cluster structure in a particular dataset.

1.4.2 Cluster Validity

Our main goal is to investigate the extent to which variation in the data can be adequately summarized by a small set of representative neighborhoods. Jain and Dubes (1988) contains an extensive discussion of cluster validity concepts and indicators. In this study "compactness" of the obtained clusters is measured employing a simple indicator.

The indicator employed compares the SE from the optimal cluster with the overall variability of x_j with respect to the sample mean c.[17] In what follows, this measure is referred to as R^2 due to its mechanical similarity with the familiar concept from standard econometric analysis.

$$R^2 = 1 - \frac{SE}{\sum_k \sum_{j \in C_k} \omega_j \left(x_j - c\right) \cdot \left(x_j - c\right)}$$

A value of $R^2 = 1$ means that the data is comprised by K types of identical elements. Section 1.6.1 obtains the R^2 for several clusterings corresponding to combinations of K, variable normalization method and measures of (H, R, P).

1.4.3 Spatial Contiguity

A branch of classification analysis deals with the clustering of objects that are described by a vector of variables (x_j) and also by their position on a plane. In some cases, it may be desirable that objects in the same class are also spatially con-

[17]Since x_j and c are vectors, the "overall variability" is defined as the sum of each component's variation (see the denominator in the expression for R^2).

tiguous.[18] In the extreme, one could restrict all elements in a given class to be contiguous. This constrained clustering problem is known as regionalization (see, for example, Duque et al. 2006). A milder approach is to include the spatial coordinates of each object in the vector of characteristics x_j and apply an unconstrained clustering algorithm. The algorithm will tend towards generating clusters that are "close" in the plane.

In this chapter spatial contiguity is not imposed in any way. However, spatial contiguity is used as an auxiliary measure of cluster adequacy.

Spatial contiguity implies a more adequate clustering of neighborhoods in the following sense. Spatial theories of human capital emphasize spillovers that occur across geographical points. A common view states the strength of these interactions declines with geographical distance. Therefore, the degree to which the representative neighborhoods found by unconstrained clustering are composed of spatially contiguous neighborhoods suggests that the classification is potentially consistent with theories featuring spatial spillovers. Conversely, a low degree of neighborhood contiguity would imply that geographical areas of each type scattered in space, so that spatial spillovers would not be possible.

Two strategies are followed in order to assess spatial contiguity in Section 1.6.4. The first computes a simple indicator that measures the fraction of neighborhoods

[18] In the problem of digital image segmentation, it is usually desirable that adjacent pixels belong to the same class. See, for example, Theiler and Gisler (1997).

of a class C_k to which the average neighborhood in C_k is "connected", in a sense to be made clear. The second presents maps indicating the location of neighborhoods of each class in selected MSAs.

Measuring Spatial Contiguity First, define two neighborhoods to be *adjacent* if their boundaries have a common point.[19] Second, define a *connected* set of neighborhoods as a set of neighborhoods which cannot be separated into two subsets without separating at least one pair of adjacent neighborhoods. The maximum degree of connectedness of a class C_k in a given MSA is obtained when the neighborhoods in C_k form a single connected set of neighborhoods.

If a neighborhood, say j, is picked at random from class C_k, the expected fraction of type C_k tracts in j's MSA ($M(j)$) that are connected to j is given by

$$E[\theta_j|k] = \sum_{j \in C_k} \frac{\theta_j}{N_k}$$
$$\theta_j = \frac{N_{M(j),k,j}}{N_{M(j),k}}.$$

Where $N_{M(j),k,j}$ is the number of class k neighborhoods connected to j in MSA $M(j)$, $N_{M(j),k}$ is the number of tracts of in cluster C_k in MSA $M(j)$ and N_k is the total number of tracts in C_k.

[19]For simplicity, the practical definition of adjacent neighborhoods used differs slightly from the one provided for exposition. Take a pair of neighborhoods A and B. The Census Bureau provides the geographic coordinates at one internal point of each neighborhood, denote these as p_A and p_B. Define a neighborhood's radius as the radius (r_A, r_B) of a circle having the same geographic area as the corresponding neighborhood. Then, say that neighborhoods A and B are connected if $distance(p_A, p_B) \leq \kappa \max(r_A, r_B)$. Where $\kappa \geq 1$ is an arbitrary constant.

1.4.4 Clustering Similarity

In order to evaluate the robustness of a clustering with respect to a change in methodology (e.g. changing the measure of housing prices used, or using SEM versus SE) one needs a way to compare two clusterings and measure the degree to which they have classified the data in the same way. This measure is known as the "similarity" between two clusterings.

There is a natural measure of similarity found in the literature which works well when the number of clusters K is small. The measure takes two different clusterings, say $C^1 = \{C^1_1, C^1_1, ..., C^1_K\}$ and $C^2 = \{C^2_1, C^2_2, ..., C^2_K\}$ and counts the fraction of objects that are classified into the same group in both clusterings. This task is complicated by the fact that sub indices labeling each cluster can be assigned arbitrarily (i.e. there is no way to decide which cluster in C^1 corresponds to any particular cluster in C^2). Therefore, one should examine all possible permutations of the cluster sub indices and choose the one yielding the maximum fraction of coincidences. If Q is the set of all possible permutations $q(k)$ of the indices $(1, 2, 3, ..., k, ..., K)$ then one can express the measure as

$$S1 = \max_{q \in Q} \frac{\sum_{k=1}^{K} |C^1_k \cap C^2_{q(k)}|}{N}$$

Where $|A|$ denotes the number of elements in a set A.

1.5 The Nature of Representative Neighborhoods

This section describes the characteristics of representative neighborhoods obtained by applying the clustering method described in Section 1.4 to the baseline sample of US census tracts obtained in Section 1.3. Results for the $K = 2$ and $K = 3$ cases are presented. The Z-score variable normalization method, described in Section 1.4.1, is employed. A benchmark measure of (H, R, P) is used to describe each tract. In terms of Section 1.3, the vector of object attributes is defined as $x_j = (\log H_j, R_j, \log P_j)$.[20]

The benchmark set of variables measuring (H, R, P) is defined as follows.[21] The "raw" mean tract earnings measure is used for H_j. The fraction of white households in census tract j is the measure of R_j. The price of housing services is obtained by applying the procedure in Section 1.2 to $IRVs$ and setting $P_j = exp(\hat{e}_j)$ for all j.[22]

The characterizations obtained under $K = 2$ and $K = 3$ are referred to as the two neighborhood and three neighborhood characterizations. The two clusters of tracts or representative neighborhoods produced by the two neighborhood characterization are referred to as Neighborhood I and Neighborhood II while the three clusters produced by the three neighborhood characterization are referred to as Neighborhoods

[20] The log transformation is not applied to R because its range is $[0, 1]$.

[21] Section 2.1 discusses the derivation of 1 measure of racial composition R, 4 measures of human capital H, and 3 measures of house prices P.

[22] Note any additive rescaling of $\hat{e}_j$ would leave the clustering unchanged. From section 1.3 it is clear that SE, SEM and R^2 are invariant to additive rescaling of components of x_j, such rescaling is only reflected on the centroids c_k.

X, Y and Z.

1.5.1 Two Representative Neighborhoods ($K = 2$)

The two neighborhood clustering provides the following characterization. Tracts in the first cluster (Neighborhood I hereafter) contain 27% of households. Neighborhood I has 4800 residents per square kilometer and covers around 4600 square kilometers (see Table 4). Neighborhood I's population density is around twice the density in neighborhoods in the second cluster (Neighborhood II hereafter), while I's land area is around one fourth of the area covered by II.

The characterization reflects strong segregation by race. Of the households residing in I, 32% are white, while 84% of households in II are white (see Table 5). If one looks exclusively at black and white households, these fractions become 37% and 93%, respectively.

The characterization exhibits strong segregation by human capital as well. Household earnings averages \$33,591 in I, representing .54 of average earnings in II (\$61,889). Household income averages \$41,747, and represents .55 of average income in II (\$74,577).

Among black households, the ratio of average earnings in I to average earnings in II is .70. This ratio is .58 for white households and .62 for households in other racial categories.

Within I, average income of black households is .90 of the average income of white households. This number decreases to .74 in II.

Finally, the price of a unit of housing services is \$10,405 in I, representing 0.73 of the price in II (\$14,268).

1.5.2 Three Representative Neighborhoods ($K = 3$)

The three neighborhood clustering generates the following characterization. Neighborhood X covers $3,190$ square kilometers while neighborhood Y covers $17,000$ and neighborhood Z covers $10,000$. X has a population density of around $5,300$ residents per square kilometer, while density is much lower in the other two neighborhoods, $2,000$ in Y and $2,700$ in Z (see Table 4).

Average earnings are \$33,142, \$47,106 and \$76,303 in X, Y and Z, respectively (see Table 6). Percentage differences in human capital between X and Y and between Y and Z are similar, generating three approximately equally spaced strata. The ratio of average earnings of X with respect to Y is .70, while the ratio of Y to Z is .61. The picture is similar for average income. Incomes in X, Y and Z average \$40,828, \$57,696 and \$93,407.

In terms of racial configuration, there is strong concentration of black households in Neighborhood X, while neighborhoods Y and Z contain similarly large fractions of white households. Only 20% are white in X, while 80% and 84% of households are white in Y and Z, respectively. Considering exclusively white and black households, the fractions of white households are 24%, 90% and 95% in X, Y and Z, respectively.

As in the two neighborhood characterization, the fraction of white households is proportional to average income in a tract.

For black households, the ratio of average income in X with respect to Y is .81. This ratio is .74 for white households and .75 for households in other racial categories.

Within neighborhood racial inequality is, as in the two neighborhood characterization, smaller than overall inequality for every neighborhood. The black white ratio of average income is .91, .84, .69 in X, Y and Z, respectively, while the overall ratio is .61.

The relative price of housing services in X with respect to Y is .95, while the relative price in Y with respect to Z is .65. As in the two neighborhood case, housing prices are proportional to income across representative neighborhoods.

1.6 Empirical Properties of Representative Neighborhoods

Some properties of the clustering of census tracts obtained by employing the benchmark set of variables (described in Section 1.5) are established in this section. The first property is the validity of the clustering measured by the R^2 statistic of Section 1.4.2. The second property is the robustness of the clustering to two key choices. The first is the configuration of variables used to measure (H, R, P). The second choice is the variable normalization method, discussed in Section 1.4.1. These properties are examined by comparing the benchmark clustering to alternative clusterings which vary some aspect of the methodology. The cluster similarity measure of Section 1.4.4 is used as a comparison device. The third property established is called regional stability. This property requires two things. First, the relative size of neighborhood clusters should not vary too much across MSAs in the benchmark characterization. Second, for any particular MSA the clustering obtained under the benchmark characterization should not be too different from the clustering that would be obtained if the clustering procedure was applied separately to that single MSA. The fourth property is spatial contiguity. This property requires that tracts classified into a particular representative neighborhood are not too far away geographically. This property is established using the indicator developed in Section 1.4.3 and also through map evidence. The fifth property is robustness to sample selection criteria. This property is established by comparing the characteristics of representative neighborhoods obtained in Section 1.5 with those obtained using several variations of sample selection criteria.

1.6.1 Cluster Validity (Results)

The main measure of cluster validity is the R^2 compactness measure, discussed above. Table 8 presents the values obtained for $K = 1, 2, ..6$ and each of the selected variable configurations using Z-score and Mahalanobis normalization.

As expected, compactness increases with K. For $K = 2$, R^2 averages 0.3 across all variable configurations and normalizations. This number increases to 0.5 when $K = 3$ and increases further to 0.6 as K increases from 3 to 6. Thus, most of the gains in explanatory power occur at $K = 2$ and $K = 3$. These clusterings provide a reasonable degree of compactness, while maintaining an acceptable level of complexity. With $K \geq 4$, the complexity becomes substantially greater without a significant increase in explanatory power.

Figures 1 and 2 report a variety of statistics regarding the distribution of (H, R, P) within and between neighborhood classes for $K = 2, 3$. The plots reflect a large degree of similarity across different variable configurations measuring (H, R, P) and large differences in the distributions of each variable across clusters.

These results suggest that using an off-the-shelf clustering procedure, one can capture one third of the variation in racial composition, human capital and housing service prices across US census tracts employing only 2 representative neighborhoods and around one half employing 3 representative neighborhoods.

1.6.2 Robustness to Variable Configuration and Variable Normalization

This subsection determines the degree to which census tracts in the sample are classified in the same way under several variable configurations and variable normalization strategies. First, the clustering procedure is applied under each possible $(variable\ configuration,\ normalization\ strategy)$ combination. Then, all pairwise comparisons of the resulting clusterings are performed, obtaining the fraction of neighborhoods classified in the same group by each pair of clusterings.

The benchmark variable configuration is defined in Section 1.5. Alternative configurations can be constructed by employing all measures of H, R and P described in Section 1.1.1. The alternative configurations considered are constructed by replacing one of the components of (H, R, P) with an alternative measure. This changing-one-at-a-time strategy results in five additional sets of variables. Each of these configurations is denoted by the name of the variable that changes with respect to the benchmark configuration. Table 7 lists variables contained in each variable configuration.

For $K = 2$ the results are striking. In the worst case, 90% of tracts are classified in the same group by the two combinations. On average 94% of tracts are classified in the same group. In many cases the classification is identical.

For $K = 3$ results are less robust (see Table 9), but still remarkably so. In the worst case 48% of tracts are classified in the same group by both combinations, while on average 73% of tracts are classified in the same group, with a standard deviation of 13% across pairs. It is interesting to note that if one excludes the combination employing log average income and Z-score normalization from the pairwise comparisons, the minimum rises to 66% while the average becomes 78%. This particular combination generates a clustering that classifies not more than 53% of tracts in the same way as any other clustering.

This result suggests that racial configuration, human capital and price of housing services provide a meaningful characterization of neighborhoods. Regardless of the diverse measures and normalization employed, the tracts are similarly classified, possibly reflecting a common, deeper, determinant of the classification.

1.6.3 Regional Stability

This subsection investigates whether the partitioning of tracts is coherent across all MSA's for $K = 2, 3$. Two conditions are evaluated.

First, the fraction of tracts classified into each representative neighborhood should be similar across all MSAs. Second, the classification of tracts should not vary too much when centroids are allowed to vary across MSAs.

Table 10 presents the fraction of tracts classified into each representative neighborhoods for each MSA for $K = 2$ and $K = 3$. There is a positive fraction of tracts classified into each group in every city. Furthermore, the fractions are roughly similar across cities. The standard of these fractions across cities are close to 9% for $K = 2$ and 14% for $K = 3$.

In order to allow for different centroids across MSA, tracts are clustered independently for each MSA using the benchmark variable configuration and Z-score normalization for $K = 2, 3$. Then, the obtained clustering is compared with the clustering obtained by pooling all MSAs together. The comparison is performed using the cluster similarity measure.

Table 11 contains the fraction of tracts that were classified in the same representative neighborhood for each MSA under city-by-city versus pooled clustering. For $K = 2$ the two classifications are virtually identical.

I interpret these results as suggestive that the representative neighborhoods obtained reflect general economic and social forces that are common to all the selected MSA and not particular to specific regions or MSAs.

1.6.4 Spatial Contiguity (Results)

This subsection presents results regarding the contiguity of tracts conforming each representative neighborhood. Contiguity is explored in two ways. First, I look at the contiguity indicator described in section 5.3. Second, I comment on maps of selected geographical areas.

Contiguity Indicator The adjacency parameter of the contiguity indicator κ is set to 2.5. This means that two neighborhoods in the same representative neighborhood are considered adjacent if the distance between their Census-assigned internal points is less than 2.5 times the larger of their neighborhood radiuses ("neighborhood radius" is defined in section 1.4.3). The average neighborhood radius in the sample is 1.13 kilometers.

Representative neighborhoods defined by the clustering procedure describe large geographical units of homogeneous characteristics. The contiguitiy indicator in Table 12 shows that in the two neighborhood characterization ($K = 2$), each tract is connected to at least 68% of tracts in its own representative neighborhood within the MSA where it is located. This lower bound is 32% for the three representative neighborhood characterization ($K = 3$). The adjacency indicator in Table 12 shows that the expected number of same-cluster tracts that are adjacent to a randomly selected tract is at least 5.6 for both $K = 2$ and $K = 3$.

For $K = 2$, tracts classified into Neighborhood I tend to be substantially less connected those classified into Neighborhood II. This obeys the fact that tracts in Neighborhood I are less numerous, forming "islands" in a "sea" of tracts classified into Neighborhood II (see maps subsection below). For $K = 3$, tracts classified into Neighborhood B tend to be less connected than those in A and B.

Maps Figures 3 to 18 contain maps corresponding to selected areas of the eight MSA in the sample with largest populations. For each MSA, the $K = 2$ and $K = 3$ characterizations are separately mapped. Each representative neighborhood is represented in a different shade of grey.

The two neighborhood characterization exhibits a striking degree of contiguity. In the selected MSA, tracts in Neighborhood I form a small number of large areas, which are surrounded by tracts in Neighborhood II. This is remarkable given that no geographical location information was used in the clustering procedure, and also given that the number of tracts within each "island" is so large. For example the Washington-Baltimore-Arlington MSA contains $1,435$ tracts of which 378 are in Neighborhood I. These tracts are grouped in virtually 3 "islands".

Many areas in the US are highly diverse, containing a large fraction of households in the "other race" category. This explains that many areas (that, interestingly, lie close to Neighborhood I regions) are blank. This reflects the sample selection criteria that avoids tracts with more than 50% of "other race" households. The Houston-Galveston-Brazoria MSA provides an example.

Finally, the three neighborhood characterization is highly consistent with the two neighborhood characterization. Neighborhood X is basically equal to Neighborhood I.[23] Consequently, Neighborhoods Y and Z turn out to be, approximately, a split of Neighborhood II into two parts. In the three neighborhood characterization, Neighborhood Y exhibits the lowest degree of contiguity. Neighborhood Y appears to the eye as composed of transition areas between clearly defined "islands" composing Neighborhood X and the "sea" of tracts composing Neighborhood Z.

[23]Therefore, the degree of contiguity of tracts in Neighborhood X is also remarkable in the three neighborhood characterization.

1.6.5 Robustness to Variations in Sample Selection

Sample selection criteria are varied in order to examine the robustness of the representative neighborhood characterizations presented in the previous two subsections. I consider the following four variations of the baseline sample selection criteria:

1. Include MSA with population above 250,000 (vs. 1 million in baseline sample).
2. Include MSA with 5% black population or more (vs. 10% in baseline sample).
3. Include neighborhoods with 90% or less of "other race" households (vs. 50% or less included in baseline sample).
4. Exclude neighborhoods with average earnings above $150,000 (vs. no upper limit in baseline sample).

The clustering procedure is applied to each sample variation. Table 14 compares the characteristics of Neighborhood I and Neighborhood II obtained under the baseline sample and sample variations 1 through 4.

Sample variations 1 and 2 result in a dataset containing neighborhoods from 56 and 41 MSA, respectively (compared to 28 MSA in the baseline sample). Table 14 shows that sample variation 1 leaves the two neighborhood characterization virtually unchanged with respect to the baseline sample.[24] Sample variation 2 implies changes

[24]The results shown in Table 14 compare only the means of census tract characteristics. However, inter-quartile ranges and standard deviations are remarkably stable across the considered sample variations as well. Details are available from the author upon request.

in the racial composition of the sample. The overall fraction of black households in the sample falls from .20 under the baseline characterization to .16. This change is reflected in the neighborhood characterization. The fraction of white households in neighborhood I moves from .32 to .4. Remarkably, this is the only appreciable change in the neighborhood characterizations imposed by sample variation 2. Sample variation 3 implies the addition of $1,098$ tracts to the sample (the number of MSA remains 28). This change leaves the neighborhood characterization virtually unchanged (see Table 14). Finally, sample variation 4 implies the deletion of 212 observations, with no appreciable effects on the two neighborhood characterization. Therefore, the results obtained in the baseline sample for the high-earnings neighborhood (Neighborhood II) are not affected by the presence of a small fraction of neighborhoods with very large average earnings.

The results for the three neighborhood characterization are very similar to those described above, and are available from the author upon request.

Conclusions

This chapter has explored the existence of a suitable representative neighborhood characterization of metropolitan US data. The purpose of such a characterization is the simplification of complex neighborhood-level data. A simple characterization permits a transparent interpretation of data through models featuring a small number of neighborhoods. Models with a large number of neighborhoods are impractical given the complexity arising from the mutual interaction between neighborhood characteristics and household location decisions.

The chapter establishes the existence of suitable characterizations employing two or three representative neighborhoods. This finding is based on three features of the two and three representative neighborhood characterizations. First, the two and three neighborhood characterizations summarize one third and one half of the joint variation in (H, R, P), the relevant neighborhood characteristics. In addition, higher dimensional characterizations provide small increments in explanatory power.[25] Second, the census tracts conforming each representative neighborhood turn out to be geographically contiguous, so that, within each MSA, each representative neighborhood is composed of large areas of homogeneous characteristics. Third, it does not matter whether the analysis focuses on a single city or a large collection of cities. The characterization is (a) remarkably stable across MSA in the sense that the proportions of households in each representative neighborhood are similar acros MSA (b) remarkably invariant to whether the methodology is applied to the full sample or city-by-city.

Furthermore, the resulting characterizations are robust across several variations in the measurement of (H, R, P), the variance normalization method used to ensure

[25] A six neighborhood characterization explains around 60% of the joint variation.

the comparability of each component of that vector, and several aspects of the sample selection criteria.

Chapter 2

Understanding Permanent Black-White Inequality: Neighborhood Human Capital Externalities and Residential Segregation

The idea that residential segregation is intimately linked to black-white (BW) disparities in standards of living dates back at least to Myrdal (1944).[1] In departure from the thinking and realities of those days, this chapter studies BW inequality within a model where white and black households have equal opportunities in the labor market, equal technologies for improving their human capital, equal access to housing markets, and equal innate abilities. These assumptions, which constitute the chapter's definition of "equal opportunity" are admittedly imperfect, since it is clear that racial discrimination may still play a role in generating earnings dispar-

[1]See Massey and Denton (1993, Ch.1) for a critical review of the evolution of these ideas in sociology since Myrdal (1944). See Loury (1976) for another early example in economics.

ities in the US. However, as there is not much empirical evidence of labor market discrimination, it is crucial to focus the analysis on the explanatory potential of alternative factors.

The explanatory factors explored in this chapter consist of residential segregation, by color and earnings, combined with neighborhood externalities in human capital accumulation.[2] Racial segregation has been a striking trait of US cities for decades, notwithstanding the Fair Housing Act of 1968, while significant residential segregation by income has also been documented.[3]

Racial inequality appears to be stagnant. Black population has experienced steady economic progress since the abolition of slavery and later with the civil rights laws of the 1960's measured in terms of earnings, wealth and educational attainment, relative to white population.[4] However, recent evidence suggests this progress drastically slowed down since the beginning of the 1990's, as measured by educational attainment and test score indicators (Neal, 2006).

Substantial racial inequality remains. Current measurements suggest that BW family income ratios are in the vicinity of .5 to .63. This magnitude is similar for

[2]The term "color" is used interchangeably with "race" in the rest of the chapter because epidermic color is the only defining characteristic of races that the chapter distinguishes.

[3]The index of dissimilarity reports that 64% of black population would need to change residence in order for US neighborhoods to become fully integrated (Iceland et al. 2003, pg. 60). In 2000 Census data, between neighborhood inequality represents around 20% of overall household income inequality (see Wheeler and La Jeuneusse, 2007, and references therein).

[4]There is a large literature investigating the drivers of this convergence. For a survey of the literature studying the 1960-1990 period see Heckman, Lyons and Todd (2000). White (2007) provides a quantitative theory of the long run path of white and black wealth and earnings based on human capital differentials.

annual incomes and for proxies of lifetime income.[5] For working males, census data from 2000 reports a BW ratio of mean earnings between .61 and .66 (depending on experience) for individuals with 9 to 11 years of education.[6]

Most racial earnings inequality can be traced back to differences in skills. Analysis of hourly earnings data of young adults from the 1990's suggests the currently remaining BW earnings differential is attributable mostly to differences in skills acquired before age 20, rather than to labor market discrimination (Neal and Johnson, 1996).[7] These findings are consistent with empirical tests of statistical discrimination theories in suggesting a secondary role for market discrimination, interpreted as a BW difference in the return to skills (see Antonovics, 2002).

Explaining the sizable BW differential in economic outcomes together with its stability since the beginning of the 1990's is an open challenge for economists. Furthermore,

> A satisfactory explanation of the recent stagnation of BW skill gaps must

[5] Badel (2008a) finds a BW family income ratio of .61 in 2000 census data. Smith and Welch (1986, Table 45) obtains a BW family income ratio of .61 and .63 using Census data from the 1970 and 1980 decennial census. Wolff (1992) reports family income ratios of .63, .62 and .63 from CPS data of 1967, 1983 and 1989, respectively. Charles and Hurst (2008, pg. 9) finds a BW household income ratio of .51 using March CPS data 1990-2002 and a ratio of average total household expenditures (a proxy of permanent income) of .5 in CEX 1986-2002 data.

[6] See Neal (2005), Table 10. For males with 12 years of education the ratio is .73 to .67, depending on experience, and .88 to .69 for those with 16 years of education.

[7] Johnson and Neal (1998) finds that observable skills also go a long way in explaining BW *annual* earnings disparities but finds a comparable role for educational attainment that is not explained by AFQT scores, especially among low-skilled males. Jencks and Phillips (1998) finds a BW ratio of annual earnings of .96 for employed young adult males with AFQT scores above the median and .85 for those with scores between the 30th and 50th percentiles.

> begin on the supply side by describing the factors that raise the cost of investing in skills within the black community. -Neal, 2008

The factors proposed by this chapter are now described.

The mechanics of the model posed here build on heterogeneous agent equilibrium models in the tradition of Bewley (1986), Huggett (1993) and Aiyagari (1994). The interpretation given to agents and shocks builds on the intergenerational mobility literature. See, for example, Becker and Tomes (1986), Loury (1981), and for an empirical application, Restuccia and Urrutia (2004). The introduction of racial preferences borrows from the literature on segregation. See Schelling (1971, 1972) and Sethi and Somanathan (2004). The introduction of neighborhood externalities builds on the theoretical literature on inequality. See Durlauf (1996), Benabou (1993) and Bowles, Loury and Sethi (2008).

The model economy is populated by a continuum of dynastic households. Each period, households are composed of one parent and one child. The parent is replaced by its child at the end of every period and a new child is born. Resources spent by households come from renting the parent's human capital. A fixed rental rate is obtained per unit of parental human capital. Importantly *human capital is observable and its rental rate is identical for black and white households.* Household period resources are exclusively devoted to consumption of housing services, consumption of nondurable non-housing goods, and investments in improving the child's future human capital. There are no intergenerational asset markets.

Households choose the neighborhood in which they live. Relocation is costless and performed at the beginning of each period. Each neighborhood supplies a fixed amount of housing services per period. Housing services are perfectly divisible. The neighborhood's price of a unit of housing services P, average parental human capital H and racial configuration R are endogenously determined attributes. These neighborhood attributes are taken as given by each household.

Average human capital H determines neighborhood-specific human capital externalities. These externalities, in combination with a child's innate ability, determine the effectiveness of parental investments in generating future human capital for the child. The innate ability of the child is random but potentially correlated with its parent's ability. The child's ability is observed by the parent prior to making location, expenditure and investment decisions.

The model is parsimonious. All main model features are necessary in order to generate racial inequality. Two analytical claims (Propositions 1 and 2) show that multiple neighborhoods and racial preferences are necessary in generating racial inequality. Two numerical exercises (see sections 2.4.5 and 2.4.6) show the necessity of human capital externalities and housing markets.

The main mechanisms behind the benchmark equilibrium of the model are easily illustrated by considering the residential decisions of households.

Suppose there are two neighborhoods denoted by I and II. Further suppose that, in equilibrium, neighborhood I has lower average human capital, a lower fraction of white households and lower housing prices compared to II. Finally, suppose that higher neighborhood average human capital generates local externalities that reduce the cost of improving the next generation's human capital.

The search for low housing prices leads black and white households with low human capital to locate in I, effectively generating a lower average human capital there. For these households, the high externalities of II are not as attractive as the low housing prices in I because their marginal utility of current consumption is very high. This leads them to locate in I.

The search for a majority of own race leads black households, in general, to disproportionately locate in neighborhood I and white households to disproportionately locate in II. This effectively generates segregation by race. Since neighborhood I has lower human capital, neighborhood externalities make human capital accumulation costlier for black households. The resulting pattern of segregation and human capital externalities gives rise to BW skill differences that persist permanently across generations.

So far, the story suggests that black and white households could swap roles, generating an equilibrium where most black households reside in the large, high human capital neighborhood. Indeed, this is the case. One can also obtain other interesting equilibria, which are listed in an appendix. Equilibrium multiplicity implies that the story presented here relies on initial conditions. However, as a first step, the chapter focuses on the properties of a steady-state equilibrium that can replicate facts of the US economy, putting only limited effort in exploring the robustness of this equilibrium to varying starting conditions. The chapter only establishes that an arbitrary, counterfactual, variation of households' perceptions about the racial configuration of neighborhoods is not sufficient to drive the economy away from the benchmark equilibrium.

The empirical strategy considers a computationally tractable two neighborhood model. The two neighborhood characterization of the US developed in Chapter 1 is used as the empirical counterpart of the theoretical neighborhoods.

The model's parameters are calibrated to exactly match target neighborhood-level and aggregate-level facts without imposing any degree of racial inequality. Given these parameters, the model's ability to reproduce the observed extent of racial inequality, among other additional facts, is evaluated. The neighborhood facts to be matched consist of the ratio of average earnings across neighborhoods, which is .54, the relative price of housing services across neighborhoods, which is .72, the neighborhood population sizes (27% of households live in the first neighborhood and 73% live in the other) and the racial composition of each neighborhood (the fraction of white households is .37 in the first neighborhood and .93 in the second). The aggregate facts matched are the intergenerational correlation of log earnings and log consumption found in US data, .4 and .49, respectively, the cross sectional dispersion of log lifetime earnings, .36, and the fraction of GDP devoted to education, .072, as a measure of the size of the human capital sector.

In principle, a model could generate a BW earnings ratio between 0 and infinity while respecting the calibration targets enumerated above. To see this, suppose one fixes the ratio of average earnings across neighborhoods and the population of each race in each neighborhood at their empirical values (as in the calibration strategy outlined above). Then simple arithmetic implies that the overall BW earnings ratio (the main variable of interest) will be completely determined by the ratios of black to white earnings that hold *within* each neighborhood. The claim then follows from the fact that these within-neighborhood BW earnings ratios are not imposed by the calibration and may take any non-negative value. Furthermore, one can easily establish numerically that when the calibration targets are imposed, the overall BW earnings ratio is quite sensitive to small movements of the within-neighborhood BW earnings ratios around their empirical values. Section 2.1 provides the details of this analysis, which illustrates the validity and spirit of the calibration strategy.

The main result of the paper states that when the model matches all calibration targets exactly the implied BW earnings ratio is .72. In other words, the model produces a BW earnings differential of 28 percentage points. This is equal to

72% (nearly three quarters) of the observed differential (which equals 39 percentage points, given an empirical BW earnings ratio of .61).

In the benchmark equilibrium, racial inequality is not due to racial differences in parental investments within each neighborhood, but due to differences in the residential location of each race. The location decisions of households account for 86% of the difference in average human capital investments and 97% of the difference in average earnings of households. According to the model, the arguable existence of "cultural differences" that lead to racial differences in investment is only an artifact of residential location.

The benchmark calibration reveals that racial preferences are sizeable. Within the benchmark equilibrium, in any given period, the aggregate period utilities of black households would be left unchanged by a 3.9% cut of their consumption of housing and non-housing goods if they could enjoy their ideal neighborhood racial configuration instead of the observed one. The aggregate utilities of white households would be unchanged by a 1% decrease in consumption if they could enjoy their ideal neighborhood racial configuration instead of the observed one. An unexpected result is that all racial inequality and racial segregation disappear from the benchmark equilibrium when the weight of racial preferences in the utility function is halved.

Households' preferences over the color configuration of their neighborhood are the first principle generating BW asymmetries in economic outcomes in this chapter. For convenience, the chapter interprets these preferences as reflecting well being experienced by each household when living in areas with a large proportion of households having its own color. However, given a two-color world, this component could equivalently be thought of as reflecting an aversion to live in areas with a large proportion of households of the opposite color. These two views are encompassed by the analysis provided here, yielding many possible interpretations.[8] The nature of interpersonal interactions, beliefs or habits that could "endogenize" these racial preferences is not explored here. Instead, the chapter follows a tradition that analyzes the *consequences* of such preferences.[9]

The remainder of the chapter proceeds as follows. Section 2.1 comments on the arithmetics of the two neighborhood characterization and their relationship with the calibration strategy . Section 2.2 describes the model and presents some illustrative theoretical assertions. Section 2.3 finds model parameter values that generate a steady-state equilibrium that exactly replicates residential racial and earnings segregation, housing prices, intergenerational earnings persistence, cross sectional lifetime earnings dispersion, and aggregate investment in human capital facts from US data. Given the estimated parameters, the section then evaluates the model's

[8] For example, the reader could interpret black households' aversion to live in a predominantly white neighborhood as reflecting some form of hostility or indifference exerted by white households there.

[9] See the introduction of Sethi and Somanathan (2004) for a careful exposition of empirical and conceptual arguments in favor of this approach.

ability to predict BW earnings inequality, among other features of the data. Section 2.4 analyzes the estimation results and uses the estimated model to compute counterfactual equilibria where features of the model are arbitrarily modified. The last section concludes and discusses the agenda for future work.

2.1 Implications of the arithmetic of Representative Neighborhoods and Calibration Strategy

Chapter 1 applies clustering analysis to a sample of census tracts from large US MSA. Tracts are clustered based on measures of average human capital (H), racial configuration (R), and price of housing services (P). Table 5 describes the resulting representative neighborhoods when the number of clusters is set to two.

The calibration strategy for model parameters is to set parameter values such that model generated data matches neighborhood level and aggregate level facts. The ratio of average earnings across neighborhoods, which is .54, is among the neighborhood level facts the model should match. Denoting neighborhood average earnings by H_I and H_{II} the condition is $H_I/H_{II} = .54$. The model should also match the fraction of each race living in each neighborhood. Letting B_I and W_I denote the fractions of the black and white populations that live in Neighborhood I this condition reads $B_I = .77$ and $W_I = .13$. This condition directly pins down what the fraction of each race will be within each neighborhood. For example, the fraction Neighborhood I households which are black, is given by

$$R_I(B) = \frac{\chi_B B_I}{\chi_B B_I + \chi_W W_I},$$

where χ_B and χ_W denote the overall fractions of black and white households in the economy ($\chi_B + \chi_W = 1$). These overall fractions are exogenous in the model and are obtained from Table 5.[10] Denote the BW earnings ratio by θ, and the within-neighborhood BW earnings ratios by θ_I and θ_{II}. Then we can express the overall BW earnings ratio as follows

$$\theta = \frac{B_I\theta_I + (1 - B_I)\theta_{II}\nu}{W_I + (1 - W_I)\nu},$$

where

$$\nu = \left(\frac{H_I}{H_{II}}\right) \frac{R_I(B)\theta_I + (1 - R_I(B))}{R_{II}(B)\theta_{II} + (1 - R_{II}(B))}$$

is the ratio of white household earnings in Neighborhood I to white household earnings in Neighborhood II expressed as a function of the neighborhood average earnings ratio, the fraction of black households in each neighborhood, and the within-neighborhood earnings ratios.

From these equations it is easy to see that for given positive values of B_I, W_I and H_I/H_{II}, it is possible to attain any θ between 0 and infinity by choosing θ_I and

[10] The condition also pins down the relative population size of Neighborhood I with respect to Neighborhood II, which can be expressed as

$$\frac{\chi_B B_I + \chi_W W_I}{\chi_B B_{II} + \chi_W W_{II}}.$$

θ_{II} appropriately.

Figure 19 plots θ as a function of θ_I and θ_{II} when $B_I = .77$, $W_I = .13$ and $H_I/H_{II} = .54$. The Figure reveals that θ varies strongly with θ_I and θ_{II} in the $[0.5, 2]$ interval. Also the level curves in the (θ_I, θ_{II}) space are convex. This finding implies that the family of models that can generate a given degree of racial inequality θ must generate within neighborhood BW earnings ratios (θ_I, θ_{II}) that lie in the corresponding convex level curve. Small deviations from the level curve will cause the model to fail in generating the required level of racial inequality. The conclusion is that the calibration strategy asks whether a realistic model parameterized according to the calibration strategy produces (θ_I, θ_{II}) near the level curve corresponding to $\theta = .61$.

2.2 Model

The model economy is populated by a unit continuum of dynastic households. Each period, a household is composed by an adult and a child. Households are described by their epidermic color $r = \{B, W\}$, the innate ability of the child z and the parent's stock of human capital h. An exogenous fraction χ_B of households is black (B) and a fraction χ_W is white (W), with $\chi_B = 1 - \chi_W$.

Each household chooses its residential location from within a finite set of neighborhoods. Adults work full time and obtain earnings wh at a rental rate w per unit of human capital h. Black and white households receive an equal market return w per unit of human capital. Households choose consumption of non-housing goods c, housing services g, and the flow i of private investment in the child's future human capital h'.

Exiting every period the adult dies and the child becomes adult. Entering next period, a new child is born from each adult. The new child's innate ability z' is drawn from a probability distribution $\pi(z'|z)$ which depends on parental innate ability z but is independent of the household's skin color.[11] The new child's color r is equal to its parent's color. Each household is altruistic towards its child's future household, discounting its utility at rate $0 < \beta < 1$. Households cannot borrow against the child's future labor income and, for simplicity, they can only transfer resources to the next generation by investing in the child's future human capital.

2.2.1 Neighborhoods

Each neighborhood n is characterized by an exogenous aggregate local supply of housing services G_n and a vector of endogenous neighborhood characteristics. Endogenous characteristics include the local price of housing services P_n, the fraction of the households living in the neighborhood which are of color r, denoted by $R_n(r)$,

[11] It is assumed that the child's ability is observed before the household makes decisions. Ex-ante observation of ability is consistent with model periods representing several years of investment decisions and observation of the child's characteristics. The literature contains examples of ex-ante and ex-post observation of ability. For the former, see Becker and Tomes (1986), for the latter see Loury (1981).

and the average human capital H_n of adult residents of the neighborhood. For simplicity, I assume that all housing is owned by absentee landlords, who are not part of the model.

2.2.2 Preferences

Households of color r derive utility from the consumption of non-housing goods c, housing services g, and from the fraction R of households living in their neighborhood that have color r.

$$u\left(c, g, R\left(r\right)\right)$$

Function u is jointly concave, twice continuously differentiable, strictly increasing in c and g, and satisfies $u_R\left(c, g, 0\right) > 0$ and $u\left(c, g, 0\right) < u\left(c, g, 1\right)$.

Note that monotonicity in R is not imposed. The last two conditions, which follow those in Sethi and Somanathan (2004), allow an interior satiation point in R. For this reason, households may prefer a neighborhood with some degree of integration over one with an overwhelming majority of their own color. The last condition says, however, that households prefer a neighborhood populated exclusively by their own color to a neighborhood that is exclusively populated by the other color.

2.2.3 Technology

Consumption Good The production technology of non-housing goods is linear in aggregate human capital, and human capital productivity is equal to w.

Human Capital The child's future stock of human capital h' depends on innate, parental and neighborhood factors. A child's future human capital is determined by its innate ability z, parental human capital h, household investment in human capital i and neighborhood average human capital H_n.

$$h' = (1 - \delta) h + zF(i, H_n)$$

Function F is jointly concave, twice continuously differentiable, strictly increasing in each argument and exhibits decreasing returns to scale. Clearly, technology is independent of the household's color.

The specification is broadly consistent with standard theories of intergenerational transmission.[12]

Private investments i are measured in units of human capital. This allows a dual interpretation of wi as the foregone earnings from parental time devoted to the child or as "monetary" investments.

The direct transmission term $(1 - \delta) h$ can be thought of as the part of parental human capital that is passed costlessly across generations. Attitude, personality

[12] In Becker and Tomes (1986, equation 4), for example, production of human capital depends on public and private investments together with an "endowment". The endowment is given by social environment, genetic, family-culture and luck factors. The H_n argument here can be interpreted as a social environment factor, z as the counterpart of genetic and luck factors, and $(1 - \delta) h$ can be interpreted as the family-culture endowment. Like in that chapter, marginal productivity of parental investments rises with environment, genetic and luck endowments here. However, this chapter's counterpart to the family culture component does not affect the returns to private investment. Also, social environment is allowed to vary by neighborhood and public investments are abstracted from.

and social connections are examples of this component.[13]

Innate ability z represents the genetic component of learning ability together with "luck". Finally, the impact of neighborhood average human capital H_n on human capital accumulation captures two main aspects of neighborhoods. First, what is known as neighborhood effects: social connections, positive role models, reduced exposure to violence, and more community resources. Second, differences in local provision of human-capital-enhancing public goods across neighborhoods.[14]

2.2.4 Household's Problem

The problem is most efficiently described in recursive language. The household's state vector is given by $x = (h, r, z)$. The vector of neighborhood characteristics $\{H_n, R_n, P_n\}_{n=1}^{N}$ is taken as given. The decision problem of a household in state x

[13]A growing literature (see, for example, Heckman and Rubinstein, 2001) finds an important role for noncognitive skills in the determination of labor market outcomes. Another literature finds that various personality traits are highly heritable (see Loehlin 1985, and references therein). Finally, Groves (2005) finds that intergenerational persistence in the personality trait denominated "fatalism" can explain 4 percentage points of the intergenerational earnings correlation.

[14]Admittedly, this last component is captured here in a rough way. This is not a critical caveat, and even a desirable feature of this analysis given that focus here is on racial inequality. There is evidence suggesting that, on average, schooling expenditures are similar for black and white youth in the US (see Neal 2006, Section 4.2). Fernandez and Rogerson (1996) focuses specifically on differences in schooling expenditures across neighborhoods.

is given by

$$
\begin{aligned}
V(x) &= \max\left\{V_1(x), ..., V_N(x)\right\} \\
V_n(x) &= \max_{c,g,i} u\left(c, g, R_n(r)\right) + \beta E\left[V(x')\,|z\right] \\
&\quad \textit{subject to} \qquad (2.1) \\
c + P_n g + wi &\leq wh \\
h' &= (1-\delta)\, h + zF\left(i, H_n\right) \\
x' &= \left(h', r, z'\right).
\end{aligned}
$$

The problem formalizes the verbal description in the preceding parts of Section 2.2. Function $V(x)$ denotes the optimal value for a household in state x. This value reflects the maximum utility among those provided by neighborhoods 1 to N. These neighborhood specific utilities $V_n(x)$ for $n = 1, ..., N$ are generated by the household's choice over consumption, housing services, and investment in child's human capital, given each neighborhood's racial configuration R_n, price of housing services P_n and average human capital H_n. The next definition summarizes the objects that constitute a solution to the household problem.

Definition 1 *A solution of the household decision problem consists of neighborhood conditional decision rules* $(c(x,n),\ g(x,n),\ i(x,n))$, *neighborhood value functions* $\{V_n(x)\}_{n=1}^{N}$ *and a value function* $V(x)$ *satisfying (1).*

Due to the discrete neighborhood choice involved in this household problem, only some of the standard properties of value functions and decision rules hold. Appendix A-1 reviews these properties. In summary, it possible to prove the Bellman equation above defines a contraction mapping in the space of continuous bounded functions. However, the fixed point $V(x)$ of such mapping will, in general, not be differentiable or concave.

2.2.5 Location Decision Rules

It was not necessary to specify where households actually choose to live in order to define the solution to the household's problem above. Knowing the utility provided by the best neighborhood was enough. However, specifying a location decision rule is vital for the definition and computation of equilibrium. Location is completely pinned down by the solution of the decision problem when the maximum in the right hand side of the first equation in (1) is unique. However, in states x where the household is indifferent between two or more best neighborhoods, there is no way to determine where the household would actually choose to live. It is assumed that households randomize when indifferent. Therefore, the neighborhood decision rule is defined as a state-conditional probability distribution over neighborhoods.

Definition 2 *A location decision rule $\eta(n|x)$ is consistent with a solution to the household's problem if it satisfies the following conditions:*

1. $\eta(n|x) = 0$ *if* $V_n(x) < V(x)$
2. $0 \leq \eta(n|x) \leq 1 \; \forall \; n$

3. $\sum_n \eta(n|x) = 1$.

When there are no ties, $\eta(n|x) = 1$ if $V_n(x) = V(x)$ and $\eta(n|x) = 0$ if $V_n(x) < V(x)$. Therefore, $\eta(n|x)$ is completely specified by the decision problem. When there are ties, some probabilities are inevitably left unspecified.

With the solution to the household problem and the location decision rule in hand equilibrium can be defined.

2.2.6 Equilibrium

Before proceeding it is important to note that the mentioned indeterminacy of some probabilities in the location decision rule $\eta(n|x)$ does not create any problems in the definition of equilibrium, besides a minor modification to the usual definition of the transition function.[15]

Definition 3 (Equilibrium) *A stationary spatial equilibrium is a probability measure of agents over individual states μ, a vector of neighborhood characteristics $\{(H_n, R_n, P_n)\}_{n=1}^N$, a solution of the household decision problem, and a location decision rule $\eta(n|x)$ that satisfy the following conditions:*

[15] Define the transition function as

$$\begin{aligned}
\mathcal{P}(x,(A_h,r',z')) &= \sum_n \widehat{\mathcal{P}}_n(x,(A_h,r',z')) \\
\widehat{\mathcal{P}}_n(x,(A_h,r',z')) &= \begin{cases} \pi(z'|z)\,\eta(n|x) \text{ if } h(x,n) \in A_h \text{ and } r = r' \\ \qquad 0 \text{ otherwise.} \end{cases} \\
h(x,n) &= (1-\delta)\,h + zF(i(x,n),H_n)
\end{aligned}$$

1. *The solution of the decision problem takes* $\{(H_n, R_n, P_n)\}_{n=1}^{N}$ *as given.*

2. *The location decision rule* $\eta(n|x)$ *is consistent with the solution to the household problem.*

3. *Neighborhood characteristics and aggregate demographics implied by* μ *together with decision rules are consistent with aggregate demographics* χ_r *and neighborhood characteristics* $\{(H_n, R_n, P_n)\}_{n=1}^{N}$.

   $\chi_r = \mu(\{x : r(x) = r\})$ *for* $r = B, W$

   $H_n = \int h\eta(n|x)\, d\mu$ *for all* n

   $R_n(r) = \frac{\int_{x:r(x)=r} \eta(n|x)d\mu}{\int \eta(n|x)d\mu}$ *for* $r = B, W$ *and all* n.

4. *Housing markets clear* $G_n = \int g(x, n)\, \eta(n|x)\, d\mu$ *for all* n.

5. *The probability measure is stationary and consistent with the solution of the household maximization problem*

   $\mu(A) = \int \mathcal{P}(x, A)\, d\mu$ *for any Borel set* A.

I now discuss three propositions that illustrate the mechanics of the model and also highlight important points. First, including more than one neighborhood in the model is essential in generating racial inequality. Second, including racial preferences in the model is essential in generating racial segregation and racial inequality. Third, any equilibrium of the model obtained in the absence of racial preferences can be replicated in the presence of racial preferences.

One Neighborhood When $N = 1$ the decision problem of households collapses to a version of the standard consumption-savings problem with uninsurable idiosyncratic risk. In this case the value function is routinely shown to be concave and differentiable, and the following illustrative first order conditions follow

$$\begin{aligned} u_c P_1 &= u_l \\ u_c &\geq \beta F_i E\left[u_{c'}|z\right] \ (= \text{if } i > 0). \end{aligned}$$

The first condition simply equates the marginal utilities of consumption and housing each period.[16] The second condition is the intertemporal Euler equation, which closely resembles the condition in the incomplete markets economies of Huggett (1993) and Aiyagari (1994). In these economies, households trade a non contingent asset in a centralized market to insure against idiosyncratic employment shocks. Here, households self-insure against low innate ability shocks by investing in human capital, which has deterministic marginal returns. However, claims to future consumption cannot be traded here due to the assumption of no intergen-

[16] In the one neighborhood model the price of housing serves only a utility scaling role when preferences are homothetic in (c, l). In the multiple neighborhood case with idiosyncratic risk, housing prices have a much richer role, see Section 2.4.6

erational asset markets, so the interest rate disappears from the model. To make this point clear, note the marginal productivity term F_i is a function of each household's choices. While households share the productivity level of their neighborhood, the rate of return to private investments in human capital is decreasing in each household's private investment i.

The following proposition shows that equilibria in the one neighborhood economy cannot display racial inequality. The logic behind this result is simple: when there is only one neighborhood and preferences are additively separable in the racial component, decision rules must be identical for black and white households. Therefore, in well behaved cases, the stationary distribution of human capital and innate ability, conditional on color, should be identical for black and white households.

Assumption $A1$, below, says that the utility function is additively separable in the racial component. Assumption $A2$ is rather technical, and guarantees that for any given solution to the household decision problem, a unique stationary distribution is generated by the decision rules. In particular, $A2$ rules out cases where the Markov process implied by the transition function $\mathcal{P}(x,(A,r',z'))$ has multiple ergodic sets. This is a nontrivial concern here, given that decision rules typically exhibit discontinuities.[17]

$A1$. $u(c,g,R(r)) = u(c,g) + v(R(r))$

[17] All equilibria computed in the chapter satisfy the uniqueness of the stationary distribution. See Appendix A-2 for details of the computation.

A2. The sequence of probability measures, $\{\mu_j\}_{j=1}^{\infty}$ *generated from any initial probability measure* μ_0 *such that* $\mu_0(\{x : r = B\}) = \chi^B$ *by the transition function* $\mathcal{P}(x, (A, r', z'))$*, converges weakly to* μ.

Proposition 1 (One Neighborhood) *Suppose A1 holds. Then in any equilibrium* E_0*, the solution of the household decision problem is identical for* $r = W, B$*. If in addition, A2 holds in* E_0*, then* $\mu((A_h, B, z))/\chi_B = \mu((A_h, W, z))/\chi_w$ *for all intervals* A_h *and all* z*.*[18] Proof: See Appendix A-1.

The proposition formalizes a crucial point. In this model, neighborhoods are essential in generating racial inequality.

No Color Preference If $u(c, g, R(r)) = u(c, g)$ households do not care about the color composition of their neighborhood when making residential location decisions. In this case it will also be hard to obtain equilibria that display racial disparities or residential racial segregation when there are *two* or more neighborhoods ($N > 1$).

Assumption $B1$, below, implies that households do not care about the color composition of their neighborhood. Assumption $B2$ guarantees that the location decisions of households are completely specified by the solution to the household's problem. Assumption $B3$ plays the same role of assumption $A2$ above.

[18] The equality of probability measures here and in the other propositions is defined in the weak sense. See Stokey and Lucas (1986), pag. 337.

$B1.$ $u\left(c,g,R(r)\right)=u\left(c,g\right)$

$B2.$ *The measure assigned by μ to the set of households that are indifferent between two or more neighborhoods equals zero.*

$B3.$ *The sequence of probability measures, $\{\mu_j\}_{j=1}^{\infty}$ generated from any initial probability measure μ_0 such that $\mu_0(\{x:r=B\})=\chi^B$ by the transition function $\mathcal{P}\left(x,(A,r',z')\right)$ converges weakly to μ.*

Proposition 2 (No Color Preference) *Suppose B1-B2 hold. Then in any equilibrium E_N the solution of the household decision problem is identical for $r=W,B$. If in addition, B3 holds in E_N, then $\mu\left((A_h,B,z)\right)/\chi_B=\mu\left((A_h,W,z)\right)/\chi_w$ for all intervals A_h and all z.* Proof: See Appendix A-1.

The proposition states that preferences over neighborhoods' color composition are a necessary ingredient in generating racial inequality and racial segregation within this model. On one hand, the result provides motivation for this feature of the model. On the other hand, it raises the question of whether one can obtain substantial inequality and segregation with moderate levels of racial preference. This type of consideration has occupied the literature on segregation since Schelling (1971, 1972), and it is explored numerically here (see Section 2.4.3).

Symmetric R Equilibrium The next proposition shows how equilibria without segregation or racial inequality can arise in the presence of any degree of racial preference. In these equilibria, H and P might or might not vary across neighborhoods. However, R is identical across neighborhoods. As in the no color preference case, symmetric R equilibria do not display any racial inequality.

Proposition 3 (Symmetric R Equilibrium) *Suppose B1-B3 hold. Then for any equilibrium with no color preference E_N^* and any additively separable racial component of the utility function $v(R(r))$, there exists an equilibrium E_N under color preference $v(R(r))$ which only differs from E_N in that value functions V and $\{V_n\}$ are additively rescaled.*

The existence of symmetric R equilibria is viewed here as a particular case of the problem of equilibrium multiplicity. However it is interesting to highlight that there exist equilibria for which racial preferences of any magnitude can be unobservable.

2.3 Fitting the Model to Data

This section first describes the steps taken to find model parameters such that a numerical approximation to an equilibrium of the model produces facts that mimic a set of facts from data. The section then tests the model's performance in its ability to match additional facts.

2.3.1 Choosing Parametric Forms

The initial step is to choose parametric forms for the process of innate abilities $\pi(z'|z)$, the utility function and the human capital production function in order to allow computation of equilibria of the model economy.

Innate Ability Following a standard practice, the process for innate ability is parameterized as a Markov chain taking a small number s of values $(z_1,\ z_2, ..., z_s)$. These values are equally spaced on the log scale. The range of log values and the transition matrix are chosen to approximate a continuous state Markov process with normally distributed innovations

$$\log(z') = \rho \log(z) + \varepsilon \qquad \varepsilon \sim N(0, \sigma_\varepsilon^2).$$

The method in Tauchen (1986) is employed for this purpose.[19]

Utility Function The parametric specification includes two additively separable terms. The first term captures the period utility from consumption of non-housing and housing goods. This term is parameterized as the composition of a Cobb Douglas and a CRRA utility function.[20] The second term captures utility from neighborhood racial configuration. This term is parameterized as a quadratic loss function where deviations of $R(r)$ from parameter R^* generate disutility.

[19]The values of $\log(z)$ used for computation are chosen to lie in $[-2\sigma_z, 2\sigma_z]$ where $\sigma_z^2 = \frac{\sigma_\varepsilon^2}{1-\rho^2}$. The number of values s is set to 9.

[20]The Cobb Douglas specification follows Davis and Ortalo-Magne (2006) who provide evidence in favor of unit elasticity of substitution between housing and non-housing consumption.

$$
\begin{aligned}
u(c,g,R) &= \frac{\left(c^{\alpha}g^{1-\alpha}\right)^{1-\sigma}}{1-\sigma} - \kappa\left(R-R^{*}\right)^{2} \\
\sigma &\geq 0 \\
0 &\leq \alpha \leq 1 \\
\kappa &\geq 0 \\
0 &\leq R^{*}
\end{aligned}
$$

Parameter R^* can be interpreted as a bliss point in neighborhood's racial configuration when $R^* \leq 1$. Parameter κ scales the racial component. Additive separability of the racial component was an assumption in Propositions 1-3. There, separability implied that the neighborhood's racial configuration R does not impact household investment decisions when their residential location is taken as given. This is a desirable discipline in the model.

Production of Human Capital The production of human capital is given by a direct transmission term $(1-\delta)h$ and the production of new human capital. The latter is modeled as a CES production function with technological parameter zA, elasticity of substitution $1/(1-\gamma)$, returns to scale χ and share parameter λ.

$$
h' = (1-\delta)h + zA\left[\lambda i^{\gamma} + (1-\lambda)H^{\gamma}\right]^{\frac{\chi}{\gamma}}
$$

This function provides a reasonable degree of flexibility. In particular, it allows for flexible degree of substitutability between neighborhood human capital externalities H and private investments i. The marginal rate of substitution between i and H is given by $\frac{1}{1-\gamma}$.

2.3.2 Parameter Values and Model Fit

Three steps are followed in order to set the model parameters and assess whether the model is correctly specified. First, a subset of 7 pre-specified parameters is set to standard values in the literature or values directly suggested by data. Then, a numerical search algorithm is used to estimate the remaining 9 parameters to exactly match a set of 9 target facts from data. Third the model's performance is analyzed using a set of 4 additional facts.

Estimation Approach The target moments for estimation are chosen in the following spirit: Choose parameters to exactly match segregation by earnings and racial segregation, along with other facts, without imposing any degree of racial inequality. Then ask what degree of racial inequality is generated by such parameter values.

Pre-Specified Parameters The total fraction of black households is set to $\chi_B = 0.21$, which is the value from Census data, derived from Table 5. This choice implies a fraction of white households $\chi_W = .79$. The rental rate of human capital

w and the price of housing services in neighborhood I, P_I, play only a scaling role in the model. Their values are set to 1 without loss of generality. The subjective discount factor β is set to match an annual factor of 0.96 while the duration of model periods is implicitly set to 25 years. This yields $\beta = 0.36$. Finally, the direct transmission term of human capital $(1 - \delta)h$ is set to zero by setting $\delta = 1$, this decision follows preliminary experimentation with the model, which showed that this term can be dropped without affecting the model's ability to match the target facts.

Parameter σ, which determines relative risk aversion and the elasticity of intertemporal substitution, is set to $\sigma = 2.5$. This is a standard value according to micro-econometric estimates (see Browning, Hansen and Heckman, 1999).[21]

Finally, parameter α which controls the share of housing services in current period household expenditures is set to $\alpha = .75$. This value generates the housing share of expenditures found by Charles et al. (2008) in CEX data (.25). The value is also consistent with Census data evidence presented by Davis and Ortalo-Magne (2008). Table 15(a) summarizes pre-specified parameters and their values.

Estimation Targets The remaining 9 parameters of the model are chosen to exactly match a set of 9 target facts. These parameters and their values are displayed

[21] Some preliminary experimentation showed this value allows a better fit of the model compared to other alternatives considered (namely $\sigma = 1.5$ and $\sigma = 2$).

in Table 15(b). The search is conducted using the simplex routine of Press et. al (1992) in conjunction with the equilibrium search procedure (see Appendix A-2 for further description). The targets and their source are now reviewed.

1. Average Earnings: The average level of earnings, \$54,200, derived from Table 5, is matched in order to keep the numerical scaling of the problem approximately constant across different parameterizations. However, it has no further effect on the behavior of the model economy.

2. Neighborhood Average Earnings Ratio ($I : II$): The ratio of average earnings in I and II from Table 5 is matched in order to reflect the degree of segregation by earnings in the data. The value of this ratio is 0.54.

3. Fraction of total white households living in Neighborhood I: This target and the next, together with the value of χ_B, completely specify the total number of households in each neighborhood and the racial configuration R of each neighborhood.

4. Fraction of total black households living in Neighborhood I: See the previous numeral.

5. Relative Price of Housing ($I : II$): The relative price of housing services across neighborhoods is taken from Table 5. This price ratio could be imposed directly as a pre-specified parameter. It is included as an estimated parameter

because preliminary experimentation showed that the search procedure works better when it can at first deviate from the actual price ratio and then gradually move closer to it.

6. Variance of log lifetime Earnings: This measure is taken from Restuccia and Urrutia (2004), which in turn obtains it from the PSID dataset analyzed by Mulligan (1996). The value is set at 0.36.

7. Intergenerational log lifetime Earnings Correlation: The target value, 0.4, is taken from Solon (1992). This estimate is in line with the Mulligan (1997), whose benchmark value is 0.48. The value 0.4 is used here to maintain comparability with Restuccia and Urrutia (2004).

8. Intergenerational log Consumption Correlation: The target value, .48, comes from Mulligan (1997) Table 7.2.[22]

9. Ratio of Human Capital Investment to Average Earnings: This target is set at the value of combined public and private expenditures in primary, secondary and college education divided by GDP. Data comes originally from the Statistical Abstract of the US (1999), Table 208. Following Restuccia and Urrutia (2004), the value is set at 0.072.

[22] The consumption measure employed by Mulligan (1997) is imputed to PSID data using the coefficients from a regression computed with data from the Consumer Expenditure Survey. Household expenditures in nondurable goods are regressed against expenditures in food at home, expenditures in food away from home, rent expenditures and house value.

Table 16(a) compares data and model generated facts. Despite its parsimonious structure, the model does remarkably well in matching the targets. Most targets are reached at two or more decimals of precision.

A few comments on some of the estimated parameter values are in order.

The parameter $\gamma = -.786$ in the human capital accumulation function implies an elasticity of substitution of $\frac{1}{(1-\gamma)} = .56$. Thus, under the benchmark calibration's technology, private investment i is a poor substitute of neighborhood externality H_n relative to what a standard Cobb Douglas technology would imply (recall that under Cobb Douglas technology, the elasticity of substitution is 1).

The "share" parameter $\lambda = .027$, which seems to attribute almost no importance to private investment, has to be interpreted with caution. The reason why a quick interpretation can be misleading is that i varies widely across households (i approximately ranges from 180 to 13.000), while H_n only displays two values which are in the same order of magnitude (33.591 and 61.889), one for each neighborhood. Examination of model generated data reveals that a unit of investment i produces approximately 30% less output (h') in the low human capital neighborhood. Therefore, the impact of human capital externalities H_n is much more limited than what the value of λ initially suggests.

The racial preference parameter $R^* = .81$ implies a strong preference for diversity where the ideal racial neighborhood configuration (satiation point in R) contains

.19 households of the other race. The "econometric" identification of κ and R^* comes from four points in the $(-v, R)$ space (recall that $-v$ is the value of the racial component of period utility). The four points correspond to the utility loss experienced by each color in each neighborhood under the target racial neighborhood racial configurations.

Additional Facts The ability of the model to match additional facts under the benchmark parameterization is now assessed.

1. Black-White Household Earnings Ratio: The model produces a BW household earnings ratio of .72. This value is consistent with the ratios reviewed in the introduction. The ratio from Census data used to construct Neighborhood facts is .61. In this sense, the model produces 28 out of the 39 percentage points, or 72% of the racial difference in earnings considered in this study. This is one of the main results of the chapter.

2. Black-White Household Earnings Ratio within I: The model produces a BW household earnings ratio of 1.72. This reflects, as will be discussed below, that white households deciding to live in neighborhood I are very poor. In the data, this ratio is .9. This is the only dimension in which the benchmark parameterization fails to qualitatively replicate the data. Given the success of the model in many other dimensions, it is left for future research to investigate

whether this is a generic feature of the model and to what extent it is an important feature of the data. However, it is borne to the reader's attention as a non negligible feature of the benchmark equilibrium.

3. Black-White Household Earnings Ratio within II: The model produces a BW household earnings ratio of .74. This value exactly matches the data from Table 5.

4. Black-White Expenditure Ratio: The model produces a BW ratio of current period expenditures of .72. This value is in the vicinity of its data counterpart, which is .66. The latter is calculated from the CEX statistics reported in Charles et al. (2008), Table A.2.

Table 16(b) summarizes additional facts and their model counterparts. In conclusion, the model is outstanding in reproducing the target facts. The model generates segregation by income and earnings similar to those found in the data while abiding to cross sectional earnings variation, earnings correlations and the size of the education sector of the US economy. The model also does a good job replicating additional facts, remarkably generating a substantial degree of racial inequality.

2.4 Results

2.4.1 Economics of the Benchmark Equilibrium

The economics behind the benchmark equilibrium can be summarized in four basic principles. First, black and white households with very low human capital need to go to I in search of low housing prices. This is caused by the high marginal utility of current expenditures experienced by these households.

Second, black and white low ability households with medium or high human capital need to go to II in search of neighborhood externalities. Households with lower ability are less productive in human capital accumulation and need to increase the inputs into the human capital production function. However, own investment is not a good substitute of neighborhood externalities, requiring these households to move to II (the elasticity of substitution between i and H_n implied by the benchmark parameter values equals 0.56).

Third, high ability black households with medium or high human capital decide to go to neighborhood I due to racial preferences. These households do not care much about housing prices or human capital externalities, due to their stock of human capital and high ability. Since the racial component of preferences is independent of wealth or ability, it becomes more important for these households, leading them to opt for neighborhood I.

Fourth, all white households with medium-low human capital have additional incentives, relative to comparable black households, to go to neighborhood II despite the attractiveness of low house prices in neighborhood I. This is caused by racial preferences.

The first and second principles generate equilibrium differences in human capital and housing prices across neighborhoods. The third and fourth principles generate racial segregation and racial inequality.

These four principles are clearly observed in Figure 20. This figure decomposes the "total change in utility" $V_{II}(x) - V_I(x)$ experimented by a household moving from I to II into three components.

Component (i) contains the change in utility coming from racial preferences $v(R_{II}(r)) - v(R_I(r))$. This component is positive for white households and negative for black households. Importantly, the component does not vary with the ability of the household or with its human capital level.

Component (ii) contains the change in utility coming from current period expenditures $u(c(x, n = II), l(x, n = II)) - u(c(x, n = I), l(x, n = I))$. This component reflects the effect of higher housing prices in II, therefore it is always negative. The key aspect of this component is that it becomes infinitely large as parental human capital goes to zero. From the graph it is clear that Component (i) dominates all other components for low human capital households.

Component (iii) contains the change in utility coming from differences in future human capital $E[V(h(x, n = II), r, z')|z] - E[V(h(x, n = I), r, z')|z]$ with $h(x, n) = zAF(i(x, n), H_n)$. This component is large for low ability households and small for high ability households.

For black and white households with low human capital (panels a, b, c and d), current expenditure component (ii) dominates the other two components as required by principle 1, leading these households to locate in I. For black and white low-ability households with medium or high human capital (panels a and b), investment component (iii) dominates the other two components as required by principle 2, leading them to locate in II. For high ability black households with medium to high human capital (panel c) current expenditure and investment components (ii) and (iii) are small compared to racial component (i) as required by principle 3, leading them to locate in I. For all white households of medium low ability (panels b and d), the "threshold human capital level" at which they decide to move from I to II is lower than it is for comparable black households (panels a and c) as required by principle 4, pushing lower human capital white households towards neighborhood II.

The Role of Neighborhoods Differences in human capital across race are not due to differences in parental investments across races within each neighborhood, but to differences in the residential location of each race. Figure 21 displays the law of motion for human capital that maps parental human capital h, color r and innate ability z into the child's future human capital h'. The horizontal axis contains parental human capital as a fraction of average human capital in the economy. The vertical axis does the same for child's future human capital. Each curve represents the law of motion for one value of the ability shock z. Dotted lines indicate the household decides to live in I while solid lines indicate the household chooses to live in II. In this figure it is clear that the child's future human capital is roughly the same for white and black households of similar ability and parental human capital when they live in the same neighborhood. However, human capital accumulation is very different for similar households located in different neighborhoods. In the benchmark equilibrium, investments in human capital average $\$2,800$ for black households and $\$4,200$ for white households while earnings average $\$41,700$ for black households and $\$57,900$ for white households. If black households kept their within neighborhood investment decision rules but adopted the white households' location decision rules, investments of black households would average $\$4,000$ and their average earnings would average $\$57,400$ in partial equilibrium. We can thus say that, within the benchmark equilibrium, the location decisions of households account for 86% of the difference in investment and 97% of the difference in average earnings.

2.4.2 Equilibrium Multiplicity

One can compute a variety of different equilibria of this model under the benchmark parameterization. Appendix A-3 contains a list of equilibria that have been computed and discusses their possible interest for this and other applications of the model.[23] Equilibrium multiplicity is an inevitable feature of this type of economy. In this sense, it is clear that the story in this chapter is ultimately based on initial conditions. In subsequent work it would be interesting to assess which type of initial conditions give rise to the benchmark equilibrium and which don't.

The rest of the section asks if the benchmark equilibrium is robust to an arbitrary variation in households' perceptions about the racial configuration of neighborhoods. To address this question, Myopic-Household Equilibria are defined and computed. In these equilibria households' perceptions about the racial configuration of neighborhoods are allowed to diverge from those implied by the choices of households.

Definition 4 *A Myopic-Household Equilibrium is a stationary spatial equilibrium where condition* $R_n(r) = \frac{\int_{x:r(x)=r} \eta(n|x)d\mu}{\int \eta(n|x)d\mu}$ *for* $r = B, W$ *and all* n *might not hold.*

Figure 22 displays an array of Myopic-Household Equilibria. The horizontal axis displays the fraction of total black households living in I, denoted by B_I, while

[23] The list includes equilibria where black and white households swap roles respect to the benchmark equilibrium and equilibria with full segregation where human capital is the same in both neighborhoods.

the vertical axis displays the fraction white households living in I, denoted by W_I. The perceptions of households are set by fixing (B_I, W_I) over an equally spaced grid with increments of 10%. Each of these points implies a neighborhood racial configuration perceived by households through the formula $R_I(r) = r_I/(B_I + W_I)$. At each gridpoint, a Myopic-household Equilibrium is computed.[24] The fractions of each group (B_I', W_I') in neighborhood I implied by household choices in equilibrium are then depicted as a small dot, and connected to the perceived fraction by a dark grey line.

The results are reassuring. Households' actions lead to the benchmark equilibrium (B_I, W_I) for all perceptions above the 45 degree line, where the racial configuration in neighborhood I displays a concentration of black households above the full integration level.[25] For all perceptions below the 45 degree line, households' actions lead to an equilibrium where the roles of black and white households are reversed.

2.4.3 The Magnitude of Racial Preferences

In order to grasp the relative magnitude of this component of preferences a simple, conservative, indicator is computed. Consider the stationary distribution of households μ. In any given period, aggregate period utility is given by $\int u(c(x,n), l(x,n))+$

[24]The starting point for the numerical equilibrium search procedure is set at the benchmark equilibrium values (for details on the numerical procedure see Appendix A-2).

[25]All points above the 45 degree line exhibit a higher concentration of black households in I than full integration $(R_I(B) > \chi_B)$. All points below exhibit more concentration of white households than in full integration $(R_I(W) > \chi_W)$. The 45 degree line contains all points of full integration $R_I(B) = \chi_B$.

$v(R_n(r(x))d\mu$. Then one can ask what reduction a of current housing and non-housing consumption leaves households indifferent when the racial composition of neighborhoods is set at the ideal fraction of own race, given by R^*. This reduction satisfies

$$\int u(c(x,n), l(x,n)) + v(R_n(r(x)))d\mu = \int u((1-a)c(x,n), (1-a)l(x,n))d\mu.$$

The racial component drops from the right hand side when $R_n(r(x)) = R^*$ under the functional form assumption made in Section 2.3.1. This indicator is conservative since future utility is not accounted for, and households are not allowed to re-optimize on the right hand side of the previous equation.

Using the benchmark equilibrium's stationary distribution the value $a = .015$ is obtained. Aggregate period utility would be unchanged by a reduction of 1.5% in period consumption accompanied by a change in neighborhood's racial configuration that allowed all households to enjoy their ideal neighborhood racial configuration.

When the proposed indicator is computed only for black households $a = .03$ is obtained, while $a = .01$ is obtained for white households . The larger value of a for black households corresponds to the fact that neighborhoods I and II are both farther from the ideal configuration for black households than for white households.

When the indicator is computed for households with earnings above the median $a = .03$ is obtained. The indicator yields $a = .10$ for black households above the median and $a = .01$ for white households above the median. All high earnings households are willing to pay more than low earnings households to enjoy their ideal neighborhood configuration, however, most high earnings black households live in I which has only 63% black households, while high earnings white households live in II, where the 93% of households is white. This explains the difference across race.

2.4.4 The Role of Color Preferences

Can a lot of segregation and BW inequality be obtained from a small degree of neighborhood color preference? Proposition 2 shows that without any color preference, the model predicts zero BW inequality and no residential segregation. However, a discontinuity could exist whereby adding a small degree of color preferences, substantial segregation and inequality could arise. In terms of the study of racial segregation, this has been a common line from Schelling (1971) to Sethi and Somanathan (2004).

This section first analyzes the effect of decreasing the extent of racial preference on the benchmark equilibrium. For this purpose a sequence of stationary equilibria with gradually lower values of κ is computed. The main finding is that a 46% reduction in κ is enough to eliminate all racial segregation and all racial inequality from the benchmark equilibrium.[26]

The second part of the section asks if these non-segregated equilibria are robust

[26] A similar result holds for other equilibria of the model. In particular, no equilibrium with racial inequality has been found when κ is reduced 46% with respect to its benchmark value. See Appendix A-3.

to arbitrary variations in households' perceptions of color segregation. The main finding is that even if all households perceive neighborhoods to be highly segregated, housing costs and human capital considerations vastly prevail when κ is small, so household decisions still imply little or no color segregation and racial inequality.

A Sequence of Equilibria with Decreasing κ. A sequence of stationary equilibria is computed. At each step of the sequence, parameter κ is decreased by a fixed amount. Then, an equilibrium is computed. Equilibrium $\{H, R, P\}$ values from the preceding step are used as starting values for the numerical equilibrium search procedure (see Appendix A-2). The first equilibrium in the sequence corresponds to the benchmark equilibrium.

Figure 23 displays the results. Panel (a) shows how the equilibrium BW earnings ratio increases monotonically as κ is reduced. The ratio reaches 1 at $\kappa = 1.75 \times 10^{-3}$, which is .54 of the benchmark value. Panel (b) shows how this result follows from increased neighborhood integration. Neighborhoods become monotonically more integrated as κ is reduced, reaching virtually full integration around $\kappa = 1.75 \times 10^{-3}$. The other panels show relatively little variation in other dimensions along the sequence.[27]

Figure 20 provides the key intuition behind this result. For a given ability level

[27] Panel (c) exhibits an interesting non-monotone behavior in relative housing prices, which is not central to the discussion.

z the threshold value of human capital at which households decide to migrate from I to II is lower for white than for black households (the threshold is indicated by the intersection of "Total change in utility" and the horizontal axis). Thus, white residents of I have such low expenditures that the utility cost of paying higher house prices in II is close to infinity (see magnitude of component (ii) in the graph). Therefore, changes in racial preferences barely impact this threshold for whites (in the graph, κ displaces "Total Change in Utility" vertically without any effect on its intersection with the horizontal axis, because its slope is too high at the intersection).

The threshold for black households is higher. Thus, the utility cost of higher house prices in II is lower for them, and a change in racial preference does impact the threshold. However, after κ is low enough, the situation for black households becomes identical to that of white households. The residents of I are so poor that changes in racial preferences play no further role (if "Total change in Utility" is close to vertical at the intersection with horizontal axis, it can be shifted up without any change in the threshold).

Along the described sequence of stationary equilibria, households perceive the neighborhoods as being increasingly integrated. Given this perception, their residential decisions become very similar, effectively implying integration. This is precisely the kind of equilibria discussed in Proposition 3. The following question naturally arises: What would happen if households counter-factually perceived neighborhoods to be partially or completely racially segregated? Would this perception be "self-fulfilling"? The next paragraph addresses this question by looking at a set of Myopic-Household Equilibria, where households' perceptions are arbitrarily determined.

Myopic-Household Equilibria with low κ. An array of equilibria with myopic households is computed.[28] Figure 24 depicts households' reactions to different perceptions about the racial configuration of neighborhoods (see description of Figure 22 in Section 2.4.2). The figure shows that regardless of the perception of households, their actions always imply neighborhood demographics tending towards integration when κ is low. A comparison between panel (a) and (b) shows that as κ is lowered, the implied demographics come closer to the 45 degree line (i.e. full integration). Thus, perceptions of high segregation are not "self-fulfilling" at low values of κ.

2.4.5 The Role of Externalities

In order to investigate the role played by neighborhood externalities, a sequence of stationary equilibria is computed for gradually decreasing values of the share of externalities in the production of human capital (1-λ). Panel (a) of Figure 25 shows that with a small reduction of the importance of externalities in the production of

[28] See Section 2.4.2 for a description of this concept.

human capital, all racial inequality disappears from the equilibrium, while segregation by earnings increases. Panel (b) shows that the reduction in racial inequality responds to the full integration of neighborhoods. Under a small reduction of the importance of externalities households can rely more on own investments to generate human capital for the next generation. This reduces racial inequality. For this reason, more white households are attracted by the low housing prices in I. As white households migrate from II to I, the fraction of black households in II becomes larger, attracting high-earnings black households. As one decreases $(1-\lambda)$ this process continues until all racial segregation has been eliminated from the benchmark equilibrium. This result is very interesting for two reasons. First it shows that human capital externalities are necessary in generating BW inequality in the model. Second, it shows that the observed degree of segregation in the model, requires the interplay of housing prices, racial preferences *and* human capital externalities.

2.4.6 The Role of Housing

In order to investigate the role played by housing, a sequence of stationary equilibria is computed for gradually decreasing values of the share of current expenditures in housing services (1-α). This reduction lowers the importance of housing price differences across neighborhoods in the behavior of households.

Figure 26 contains implications for equilibrium. When housing markets become relatively unimportant, low human capital white households no longer have a reason to live in neighborhood I. The low housing prices of this neighborhood are no longer a large source of current utility. Therefore, the fraction of white households in I decreases (panel b) and total population in I falls (panel b) until full segregation is obtained. With full segregation, average human capital becomes identical in each neighborhood, eliminating racial earnings inequality.

Neighborhood I supplies .27 of total housing services, while the total fraction of black households is .21. Thus, each color group fits nicely into a neighborhood, with only a small difference in per-household consumption of housing services across neighborhoods (panel c). This difference is compensated by a small differential in housing prices across neighborhoods.

This result is analogous to that in Proposition 2. The proposition states that without racial preferences, segregation by race and racial inequality cannot be obtained in this model. This section has established that under the benchmark parameterization, the model is unable to predict any racial inequality, while producing full segregation by race, when housing markets are unimportant. This suggests that housing is an indispensable element of the model.

2.4.7 An Explanation for Results on Empirical Neighborhood Effects?

The empirical literature on intergenerational correlations of income surveyed by Solon (1999) has mainly proceeded by using OLS or Instrumental Variables to estimate a regression of log lifetime earnings of sons on log lifetime earnings of parents and other explanatory factors. The neighborhood effects literature concentrates on neighborhood background. The conclusion in Solon (1999) is that existing work has seldom found large and significant coefficients on the neighborhood background variables. Table 17 presents a recreation of that type of exercise applied to model simulated data. The first two columns contain regressions of children's future log earnings on the earnings of their parents and a Neighborhood II dummy. According to the model mechanics the coefficient on Neighborhood II dummy should be large and positive. However, the coefficient turns out negative for both black and white households. This result is explained by the fact that in the benchmark equilibrium, parents of low ability children (all else constant) will move into high earnings neighborhoods in order to mitigate the low productivity of their children in human capital acquisition. The last two columns of Table 17 show the results of a similar regression adding the log innate ability of the child as a regressor. These two columns show that neighborhood what lies in the deep structure of the model: strong positive neighborhood effects. Since innate ability is hard to measure in data, this paper could be of aid in interpreting some of the results in the literature. The explanation is limited in by the fact that negative and significant (as opposed to small and insignificant) coefficients have been obtained on model regression data. A full claim to explaining the results of the empirical literature should match their results exactly. However, the analysis provided should motivate further study of this particular explanation. It is interesting to note that this explanation applies broadly, even to some experimental evidence. One related example of particular interest is the assessment of neighborhood effects in the Moving to Opportunity experiment conducted by Kling, Liebman and Katz (2007). This example was described in the dissertation's introduction.

Conclusions and Agenda for Future Work

This chapter has crystallized common intuition about the relationship between residential segregation and racial inequality in a parsimonious model economy. In this endeavor it has extended the scope of a popular class of heterogeneous agents models by introducing original modifications to the standard framework.

The chapter has achieved success in taking the posed mechanics to the data and replicating important facts of the US economy. The calibrated model exactly replicates US residential segregation by race and earnings, intergenerational correlations of earnings, and the size of the US education sector with respect to GDP, within a stationary equilibrium. The mapping of a parsimonious two neighborhood model to real life data is allowed by the application of clustering techniques to characterize US neighborhood data through two representative neighborhoods. Clustering is a standard method in itself, but its application to quantitative economic modeling is novel, and evidently useful.

The calibrated model produces 72% of the observed BW family income gap. This important result is obtained while abiding to a set of current stylized facts from the US economy. Prominently, the model reflects the fact that black and white workers seem to receive similar compensation for observable skills. In other words, the model generates large black white differences without appealing to labor market discrimination or informational frictions, which are not strong features of current US data.

It is important to note that no asymmetries by race, beyond a preference over neighborhood color composition, have been assumed. Furthermore, in equilibrium, the decisions of black and white households of similar human capital and ability are similar within each neighborhood. Therefore, differences in human capital across races are generated by differences in the residential location of races and not by BW differences in human capital investment behavior within each location. Within the benchmark equilibrium, the location decisions of households account for 86% of the BW difference in average human capital investments and 97% of the BW difference in average earnings across races.

In summary, the chapter has made progress towards a compelling answer to the crucial question it addresses: Why are the earnings of black households so low compared to those of white households? When the pattern of residential segregation by earnings and color found in data is viewed as generated by the mechanisms in the benchmark equilibrium of the model proposed here, a significant portion of the observed extent of inequality can be explained.

Additionally, the chapter has interesting implications for the study of racial segregation. A common interpretation of Schelling (1971, 1972) is that the extent of segregation observed in the US could be caused by a "small degree" of racial preference. In contrast to this ingrained line of thought, this chapter finds that strong racial preferences are required to match US facts. Furthermore, the chapter measures the sensibility of the equilibrium of interest to a change in the magnitude of racial preferences, finding that segregation and inequality become negligible when one halves the importance of race in utility.

The results of the model rely on initial conditions. Each of the results above is conditional on "things starting out that way". This suggests the main avenue of future extensions of this work. Namely, the exploration of transition paths from different starting points to the equilibrium under study. This type of analysis is computationally challenging, and more so given the specific features of this model (in this model a vector of 6 equilibrium variables must be chosen along the transition path, compared to 1 variable in the standard one-sector growth model with heterogeneous agents). However, analyzing transition paths will yield important lessons both about the validity of the model and hopefully about the mechanisms that have been at play in the US economy. A second avenue for extensions of this work increases the number of neighborhoods in the model economy. This is also computationally challenging, however, it would allow a more direct mapping of model to data.

From a normative point of view, the present chapter shows that "equal opportunity" may not be enough for convergence of black and white earnings over time. This conclusion restates the conclusion of Loury (1976) within a quantitative framework, where the adequacy of the model used to make this prediction can be assessed. The two main results of the paper suggest that the model's mechanics can replicate many features of the data, yielding a stronger case for affirmative action policies.

The approach followed here connects and makes contributions to several branches of the literature. A full analysis of these connection is provided in the introduction to the dissertation.

A-1 Theoretical Appendix

Proposition A-1 *The household problem defines a contraction mapping in the space of continuous bounded functions. There exists a unique bounded continuous function* $V(x)$ *satisfying (1) and the optimal policy correspondence is nonempty and u.h.c.*

Proof *This proof follows directly from the standard proof for the one-sector growth model in Stokey and Lucas (1989, exercise 5.1).*

Proof (Proposition 1) *(i) The household's problem defines a contraction mapping. Therefore, the solution to the household's problem in* E^0 *is unique. If* $A1$ *holds, the period utility function differs by a constant for black and white households. Direct comparison shows the value functions for households in states* (h, B, z) *and* (h, W, z) *differ by* $\frac{v(R(W))-v(R(B))}{1-\beta}$*. Thus, by uniqueness, human capital investment decision rules will be identical for black and white households.*

(ii) Since decision rules are identical for white and black households, the definition of the transition function implies that $\mathcal{P}((h, B, z), (A_h, B, z')) = \mathcal{P}((h, W, z), (A_h, W, z'))$ *for all* h, z, z'*, and all intervals* $A_h \subset [0, \bar{h}]$*.*[29] *Assumption* $A2$ *implies* μ *(the limit of a sequence probability measures* $\{\mu_j\}_{j=1}^{\infty}$ *generated from any initial measure* μ_0 *by the transition function* $\mathcal{P}(x, (A_h, r', z'))$*) is unique.*[30]*. Since households never change*

[29]Decreasing returns to scale in the production of human capital h guarantee that $h \in [0, \bar{h}]$ for some finite $\bar{h}$.

[30]See Stokey and Lucas 1989, pg. 353

color, the limit of a sequence of color conditional probability measures $\{\mu_j(.|r)\}$ *generated from any initial probability measure* $\mu_0(.|r)$ *by the transition function* $\mathcal{P}((h,r,z),(A_h,r,z'))$ *converges weakly to* $\mu(.|r)/\chi^r$*, the equilibrium measure conditional on color. Finally, suppose by way of contradiction that* $\mu(.|r=B)/\chi^B \neq \mu(.|r=W)/\chi^W$*. Then one can set the initial probability measure for black households to equal the initial measure used for white households* $\mu_0(.|r=B)=\mu_0(.|r=W)$ *and apply* $\mathcal{P}((h,B,z),(A_h,B,z'))$ *recursively, generating a sequence* $\{\mu_j(.|r=B)^*\}$*. Since* $\mathcal{P}((h,B,z),(A_h,B,z')) = \mathcal{P}((h,W,z),(A_h,W,z'))$*, then* $\mu_j(.|r=B)^* \to \mu(.|r=W)/\chi^W$*. This contradicts the uniqueness of* $\mu(.|r=B)/\chi^B$*.*

Proof (Proposition 2) *(i) By assumptions B1-B2 and the contraction mapping property of the household problem, the location decision rules* $\eta(n|x)$ *and neighborhood conditional decision rules* $(c(x,n),\ g(x,n),\ i(x,n))$ *are identical for black and white households. Therefore, by definition, the transition function* $\mathcal{P}(\cdot)$ *is also independent of color. Assumption B3 guarantees uniqueness of the stationary distribution* μ *which together with (i) implies the color-conditional stationary distributions are identical (see part* (ii) *of previous proof).*

Proof (Proposition 3) *(i) By Proposition 2, without color preferences* $\mu(.|r=B)/\chi^B=\mu(|r=W)/\chi^W$*. By assumption B2,* $\eta(n|x)$ *is pinned down by the solution* *to the household problem almost everywhere. This implies residential decisions of white and black households are identical almost everywhere. Identical residential decisions imply that in equilibrium E_N, $R_n(r) = \chi^r \ \forall \ n$, so all neighborhoods are fully integrated. Under full integration any racial preference $\nu(R(r))$ just adds a constant to period utility. Thus, the solution to the household problem from E_N also solves the decision problem with racial preference $\nu(R(r))$ given $\{H_n, R_n, P_n\}_{n=1}^N$ from E_N when the value functions are additively rescaled.*

A-2 Computational Appendix

This appendix describes the main equilibrium search algorithm used in the chapter and then discusses its application to model estimation and computation of counterfactual equilibria. The algorithm calculates a Pseudo-Equilibrium of the model.[31] Pseudo-Equilibrium differs from the Stationary Spatial Equilibrium of Definition 3 in allowing demand for housing services in each neighborhood to take any (positive) value, eliminating the market clearing condition for housing services.

This concept is useful to estimate the model. Neighborhood supply of housing services $\{G_n\}_{n=1}^N$ is not observed in the data, so it has to be treated as a free parameter. Fortunately, each neighborhood's population is known. The model can therefore be estimated by including neighborhood *populations* in the set of

[31] Define a Pseudo-Equilibrium in the same way as in Definition 3, but suppress the housing market clearing condition (Condition 4).

estimation targets and then setting model parameters to match these targets under Pseudo-Equilibrium (without considering the housing market clearing condition). Once the benchmark parameters of the model have been estimated one can take the Pseudo-Equilibrium housing *demands* at each neighborhood D_n^{bench} and interpret them as an estimate of housing supply, setting $G_n^{bench} \equiv D_n^{bench}$ for each neighborhood n. Supply, estimated in this way, can then be used to check the housing market clearing conditions in computations of counterfactual equilibria.

Algorithm 1 (Pseudo-Equilibrium)

1 *Set* $\{(H_n, R_n)\}_{n=1}^{N}$ *at starting values* $\{(H_n^0, R_n^0)\}_{n=1}^{N}$. *Fix the parameter vector at some starting value* $\Theta^{pseudo} = \Theta^0$.[32]

2 *Taking* $\{(H_n, R_n)\}_{n=1}^{N}$ *from the previous step and* $\{P_n\}_{n=1}^{N}$ *from the parameter vector* Θ^{pseudo}, *solve the household's decision problem. Use the solution to the household's problem to simulate the path of the individual state vector* x *for one* W *and one* B *households over 500,000 model periods. Assuming stationarity, calculate implied average human capital and racial configuration of each neighborhood,* $H_n^{'}$ *and* $R_n^{'}$, *from the simulated time series by employing the location decision rules and appropriate time series averages.*

3 *If* $\max_n[(H_n - H_n^{'})^2] < \varepsilon_H$ *and* $\max_n[(R_n - R_n^{'})^2] < \varepsilon_R$ *go to step 4. Otherwise,*

[32]The parameter vector Θ contains the parameters listed in Table 16(b). These parameters include the housing price vector (P_n).

for each neighborhood n update

$$H_n = \nu H_n + (1-\nu) H_n^{'}$$
$$R_n = \nu R_n + (1-\nu) R_n^{'}$$

and go back to step 2.[33]

4 *Check for multiple stationary distributions.*[34] *Terminate.*

If the algorithm terminates successfully (i.e. reaches step 4 and there is a unique stationary distribution) one has found a Pseudo-Equilibrium of the model under parameter vector Θ^{pseudo}.

Estimating Model Parameters Parameter estimation proceeds by searching over values of the parameter vector Θ^{pseudo} in order to find a Pseudo-Equilibrium that matches target moments from data. The AMOEBA minimization routine of Press et. al (1992) is used to minimize the squared percentage deviation of model implied moments to estimation targets. These targets are described in Section 2.3.2 and summarized in Table 15(b). Since the number of targets equals the number of parameters one should expect the terminal value of the procedure to be close to zero, implying an exact match. The starting value for the vector Θ^{pseudo} was obtained

[33]Constants ε_H and ε_R control the error tolerance, these are set to the value 0.0001. Constant ν is a "relaxation parameter" that facilitates convergence. This parameter is set to 0.8.

[34]Markov processes sometimes have multiple stationary distributions. A sufficient condition for the uniqueness of the stationary distribution is that the long run averages from the simulations in (2) do not depend on starting values.

by picking the vector providing the best fit from a large set of randomly generated vectors.[35]

Computing Counterfactual Equilibria The computation of counterfactual equilibria proceeds by imposing changes to the benchmark value of Θ^{pseudo} in order to obtain $\Theta^{counter}$. Supply of housing is taken from the benchmark equilibrium by setting $G_n^{bench} \equiv D_n^{bench}$. With these elements in hand, a full equilibrium of the model is computed by adjusting all components of $\{(H_n, R_n, P_n)\}_{n=1}^N$. Mechanically, this is achieved by applying the AMOEBA minimization routine in order to search over the subset of parameters in $\Theta^{counter}$ that determine housing prices $\{P_n\}_{n=1}^N$ to minimize the percentage squared deviation of housing demand $D_n^{counter}$ from housing supply G_n^{bench} in each neighborhood n under Pseudo-Equilibrium.

A-3 Multiple Equilibria

The search for additional equilibria fixes parameter values at the benchmark values estimated in Section 2.3.2. Given these parameter values, the procedure for computing counterfactual equilibria described in Appendix A-3 is applied. This procedure results in the benchmark equilibrium if starting values for $\{(H_n, R_n)\}_{n=1}^N$ are set to benchmark values in step 1 of Numerical Procedure 1 of Appendix A-3. Additional equilibria are obtained by varying these starting values. Equilibria are computed for

[35]Parallel computation using Georgetown's Zappa computer cluster was crucial in computing equilibrium for a large number of randomly generated parameter vectors in reasonable time.

a total of 30 starting values at the benchmark parameter values. Four new equilibria, described below, arise from this exercise.

After examining multiple equilibria at the benchmark parameter values, κ is reduced 46% with respect to its benchmark value, and the same 30 starting values are used to compute multiple equilibria. This exercise results in a total of three equilibria, none of which exhibits racial inequality, and two which exhibit no racial segregation. One of the equilibria exhibits complete racial segregation. When κ is reduced to 25% of the benchmark, only two equilibria are obtained, and both exhibit no racial segregation or racial inequality.

Five starting values for (H_I, H_{II}) are considered. In case 1 (H_I, H_{II}) is set to the benchmark value. In case 2 H_I is set to the benchmark value of H_{II} and H_{II} is set to the benchmark value of H_I. In the third case H_I and H_{II} are both set to the overall average earnings of the benchmark case. In the fourth case H_I is set to 1/3 of its benchmark value while H_{II} is set to 3 times its benchmark value. In the fifth case H_I is set to 3 times the benchmark value of H_{II} while H_{II} is set to 1/3 of the benchmark value of H_I.

Six starting values are considered for (R_I, R_{II}). The first three cases vary (R_I, R_{II}) to maintain neighborhood populations constant in each neighborhood with respect to the benchmark equilibrium. The first case starts with all households of I white. The second case starts with all households in II white. The third case sets the fraction of white households in I to the benchmark fraction of white households in II, which is .93.

The second set of three cases explores starting values that alter total populations in each neighborhood with respect to the benchmark equilibrium. In the first of these cases, all black households live in I while all white households live in II. The second case starts with all white households living in I while all black households live in II. In the third case the fraction of white households in I equals the benchmark fraction of white households in II, and the fraction of white households in II equals the benchmark fraction of white households in I.

The five starting values for (H_I, H_{II}) coupled with the six starting values for (R_I, R_{II}) generate the set of 30 starting values. While many of these starting values result in the benchmark equilibrium, four additional equilibria of the model arise from the configurations of starting values described above.

Reversed-Roles Equilibrium Five of the 30 starting value configurations resulted in an equilibrium where the role played by black and white households in the benchmark equilibrium is reversed. In this equilibrium, white households represent 85% and 75% of the populations of neighborhoods I and II, respectively. Therefore, most black households (82%) reside in II. Average earnings in neighborhood I represent .52 of those in neighborhood II while the relative price of housing services in I, relative to II is .68. These two ratios are roughly in line with data. Of white households .28 is located in neighborhood I, whereas only .17 of black households is. This implies that black households enjoy lower costs of human capital accumulation. The ratio of *white to black* earnings is .93. The white-black gap in this reversed-roles equilibrium is just .07, compared to BW gap of .39 in the benchmark equilibrium. This asymmetry is a consequence of the different black and white population sizes, combined with the housing supplies in each neighborhood. Housing supply in I relative to II is roughly one third, while black population is 21% of total. Therefore, when black households are concentrated in I the ratio of black to white households in I tends to be closer to 1 than the ratio in II when black population is concentrated in II. Thus, the correlation between the fraction of black households with human capital across neighborhoods is lower, leading to milder racial inequality in the reversed-roles equilibrium.

Equilibrium with Full Segregation Two of the 30 configurations result in an equilibrium where all black households live in I, all white households live in II and average human capital is identical in each neighborhood. In this equilibrium there is no racial inequality and the relative price of housing in neighborhood I with respect to neighborhood II is .97, reflecting a slight difference in housing services per household across neighborhoods.

Privileged-Minority Equilibrium One of the 30 configurations results in an interesting equilibrium where the smallest neighborhood (I) becomes the wealthiest one and the one with high housing prices. The ratio of average earnings in I with respect to II is 1.89, and relative price of housing in I with respect to II is 2.7. In this equilibrium, .60 of all black households live in I while .2 of all white households live in I. This implies that the racial minority (black households in this case) faces lower costs of investing in human capital, leading to racial inequality with a "privileged" minority. In this equilibrium, the BW earnings ratio is 1.26, this corresponds to a white-black ratio of .79. This equilibrium immediately suggests the relevance of the model for developing countries, where cities are characterized by small, exclusive residential areas, inhabited by an elite and large surrounding "poverty belts".

Equilibrium with Full Racial Integration Two of the 30 starting values considered result in an equilibrium where neighborhoods are fully racially integrated, but where there is segregation by earnings. In this equilibrium, the ratio of average earnings in I with respect to II is 1.83, while the relative price of housing services in neighborhood I, relative to II is 2.44. This equilibrium presumably corresponds to the *Symmetric R* equilibrium devised in Proposition 3.

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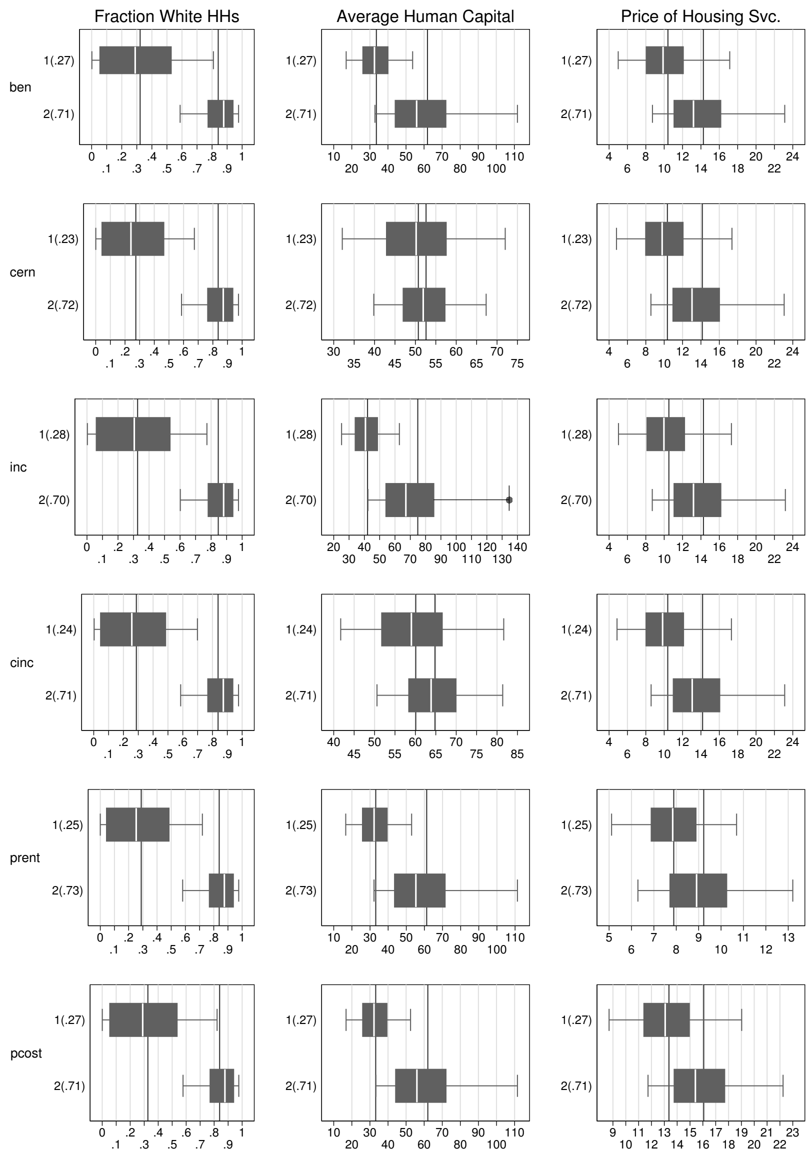
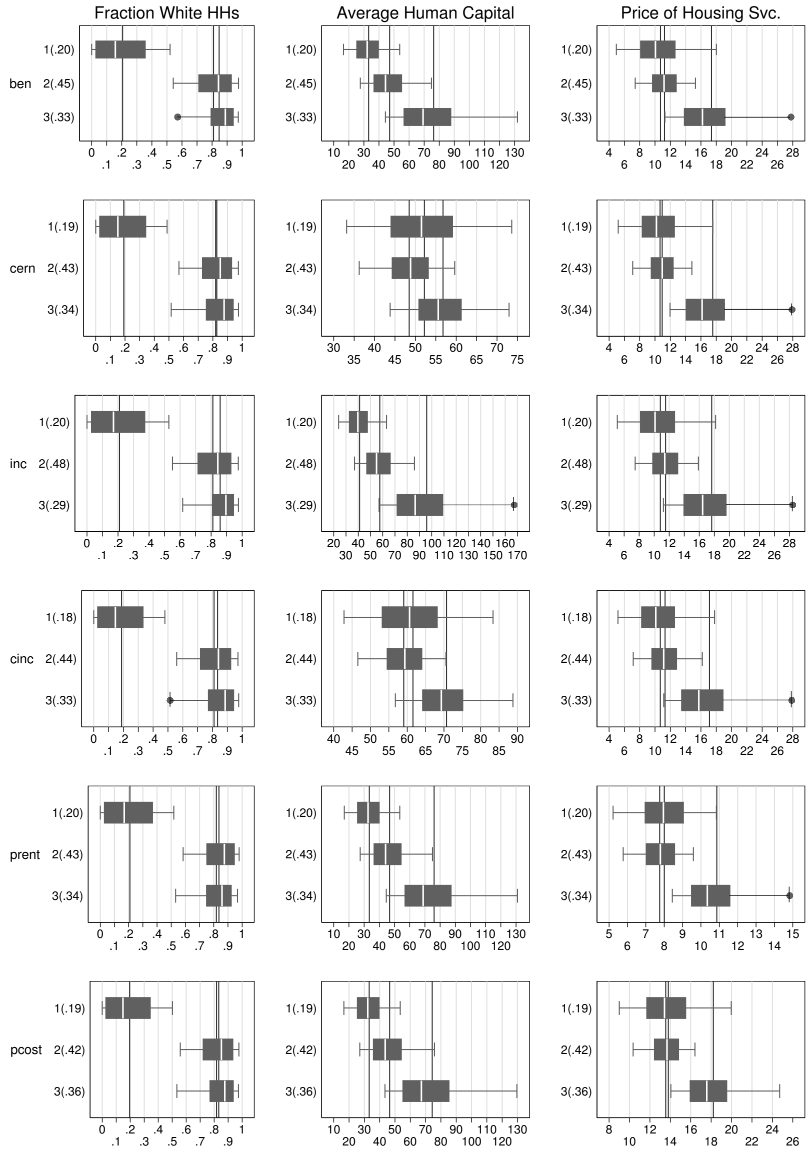
Figure 1: Within-Class Distribution of Neighborhood Characteristics by Variable Configuration, Z-Score Normalization, K=2



Columns display plots of (i) racial configuration, (ii) human capital and (iii) price of housing services measures. Rows correspond to each variable configuration. Within each plot, neighborhood classes are listed in the vertical axis (fraction of HHs in parentheses). Vertical lines indicate neighborhood means (or centroid c_k). Boxes indicate the range between 25th and 75th percentiles. Lines within box indicate medians. Brackets indicate range between 5th and 95th percentile. All statistics are weighted by tract number of households.

Figure 2: Within-Class Distribution of Neighborhood Characteristics by Variable Configuration, Z-Score Normalization, K=3



Columns display plots of (i) racial configuration, (ii) human capital and (iii) price of housing services measures. Rows correspond to each variable configuration. Within each plot, neighborhood classes are listed in the vertical axis (fraction of HHs in parentheses). Vertical lines indicate neighborhood means. Boxes indicate the range between 25th and 75th percentiles. Lines within box indicate medians (or centroid c_k). Brackets indicate range between 5th and 95th percentile. All statistics are weighted by number of households in each tract.

Figure 3: Chicago-Gary-Kenosha MSA, $K = 2$

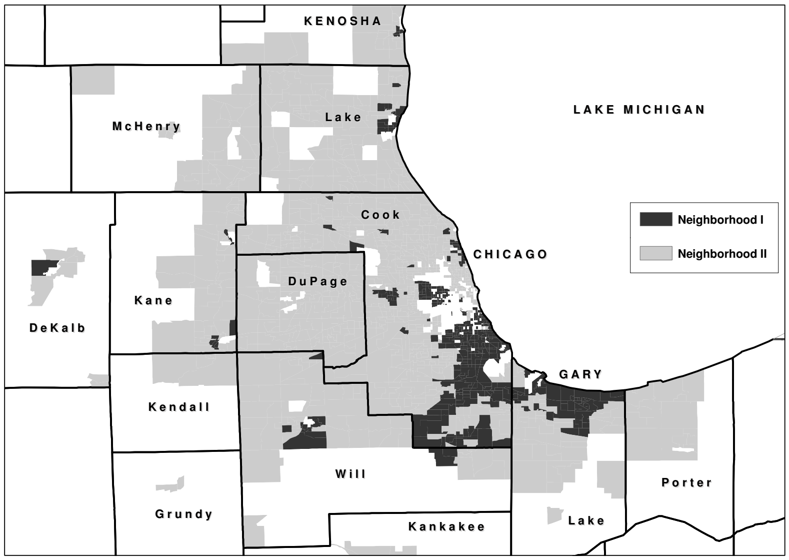
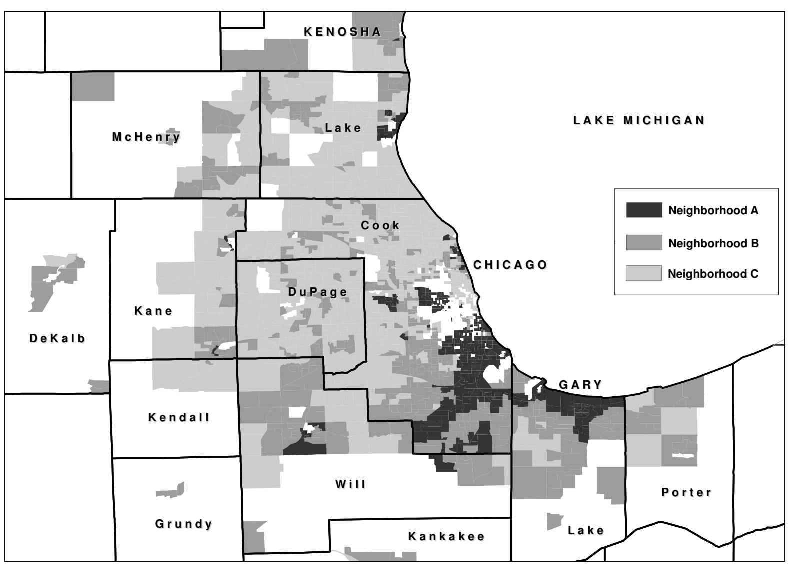


Figure 4: Chicago-Gary-Kenosha MSA, $K = 3$



Figures contain selected neighborhoods of the corresponding MSA.

Figure 5: Dallas-Fort Worth MSA, $K = 2$

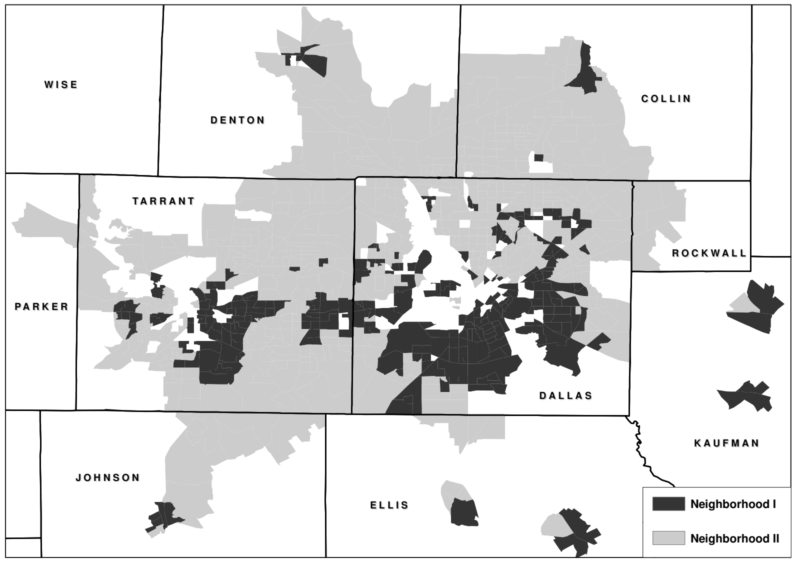
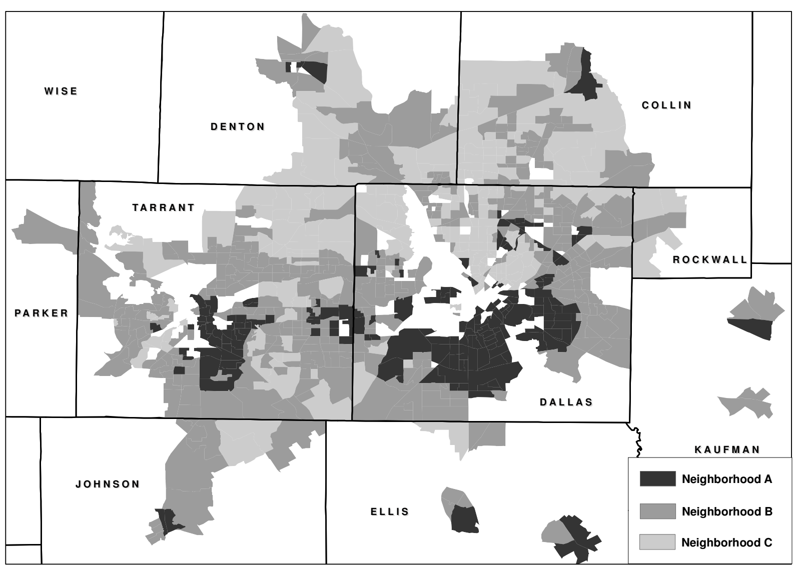


Figure 6: Dallas-Fort Worth MSA, $K = 3$



Figures contain selected neighborhoods of the corresponding MSA.

Figure 7: Detroit-Ann Arbor MSA, $K = 2$

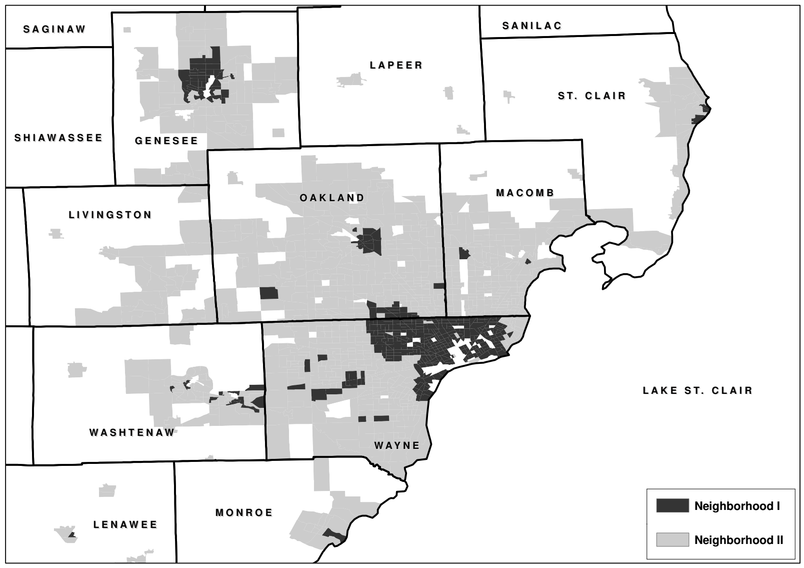
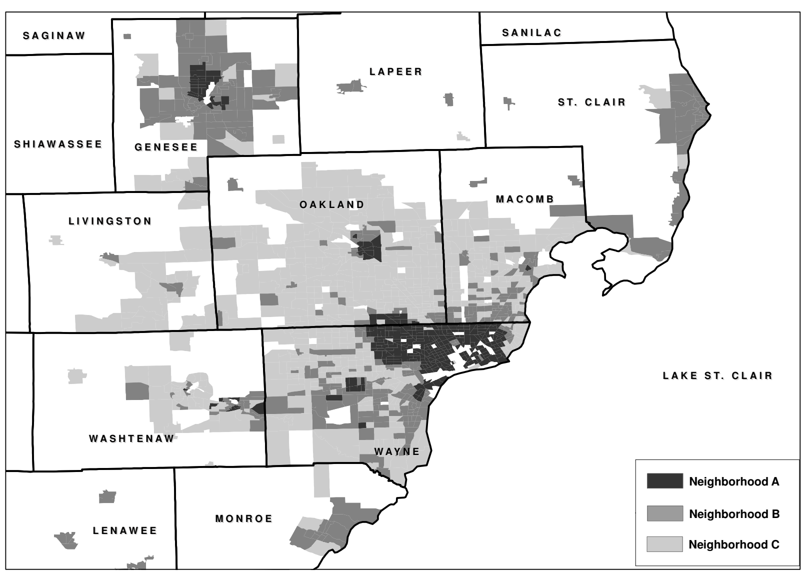


Figure 8: Detroit-Ann Arbor MSA, $K = 3$



Figures contain selected neighborhoods of the corresponding MSA.

Figure 9: Houston-Galveston-Brazoria MSA, $K = 2$

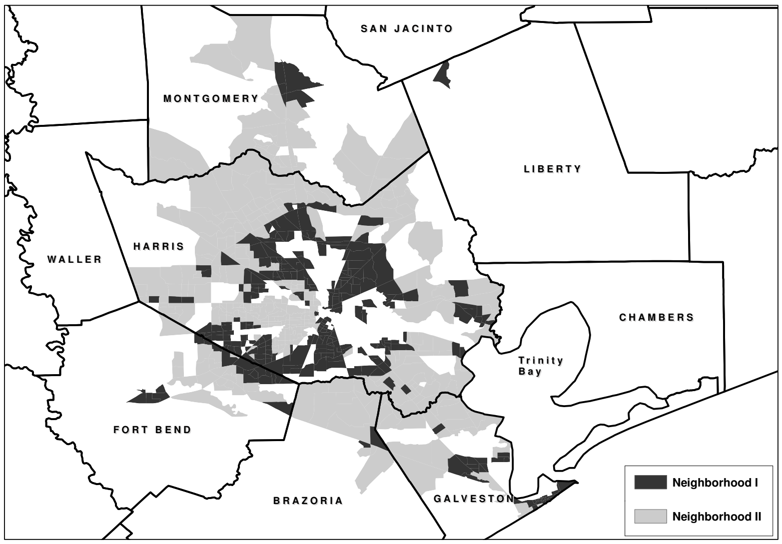
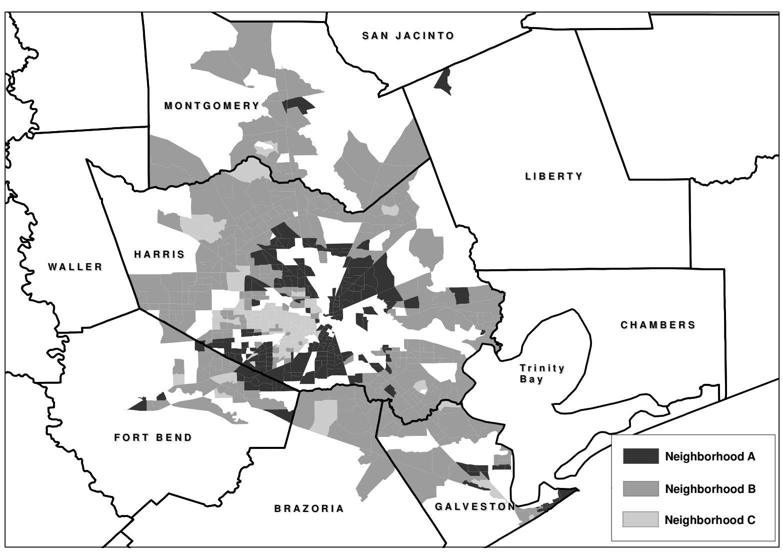


Figure 10: Houston-Galveston-Brazoria MSA, $K = 3$



Figures contain selected neighborhoods of the corresponding MSA.

Figure 11: New York-Northern New Jersey-Long Island MSA, $K = 2$

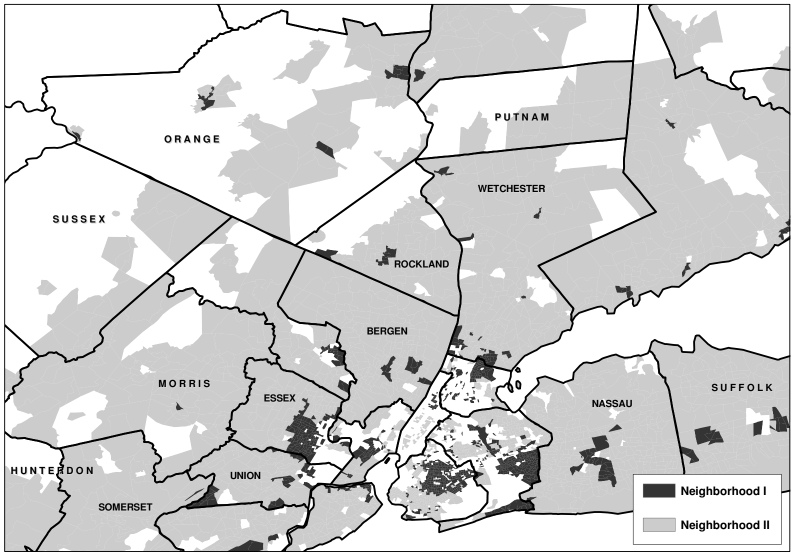
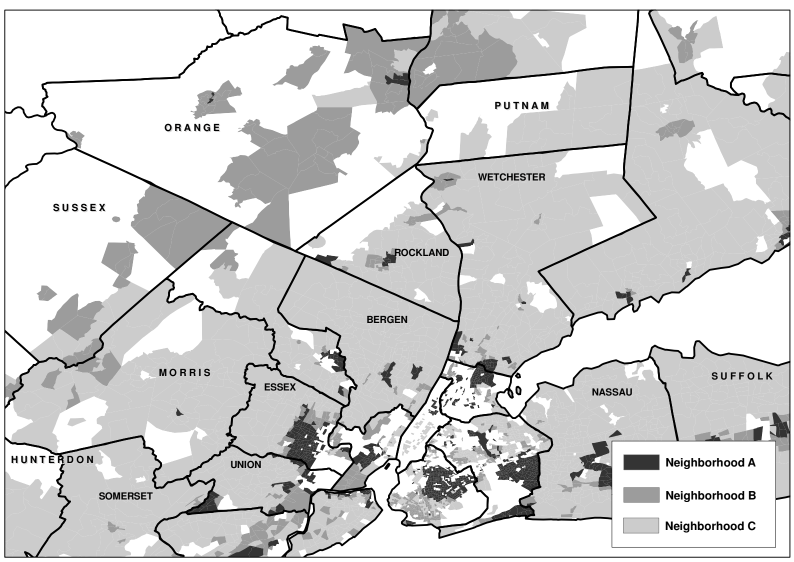


Figure 12: New York-Northern New Jersey-Long Island MSA, $K = 3$



Figures contain selected neighborhoods of the corresponding MSA.

Figure 13: Cleveland-Akron MSA, $K = 2$

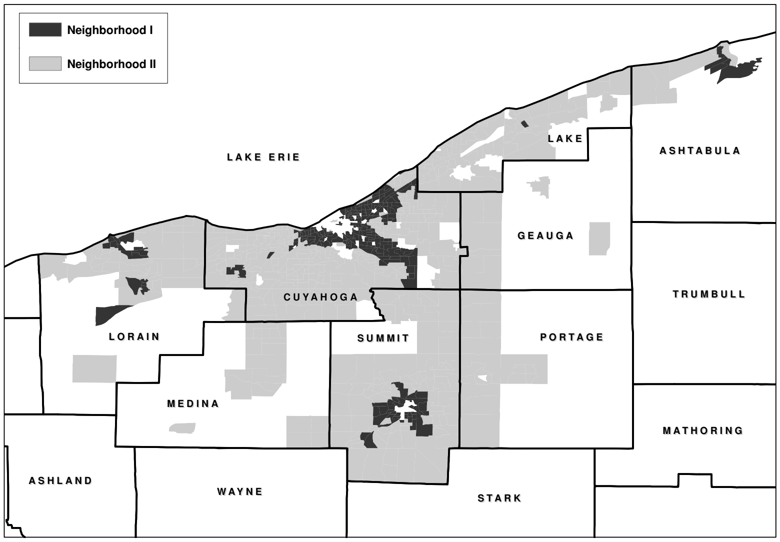
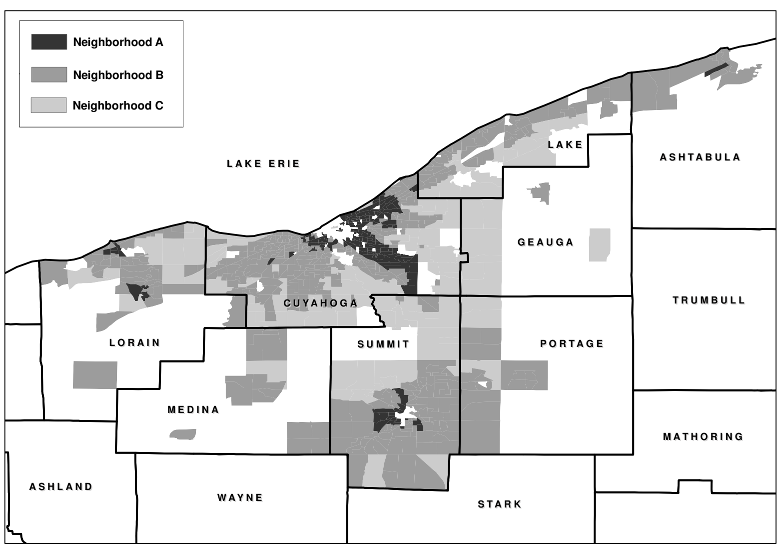


Figure 14: Cleveland-Akron MSA, $K = 3$



Figures contain selected neighborhoods of the corresponding MSA.

Figure 15: Philadelphia MSA, $K = 2$

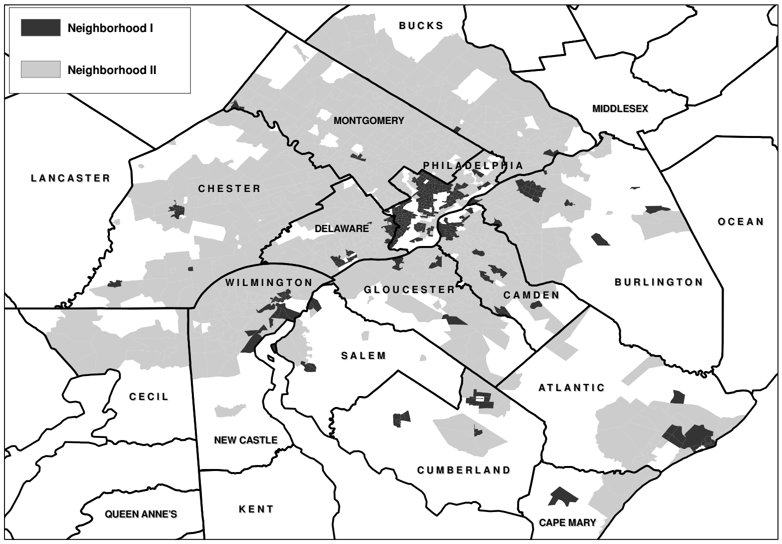
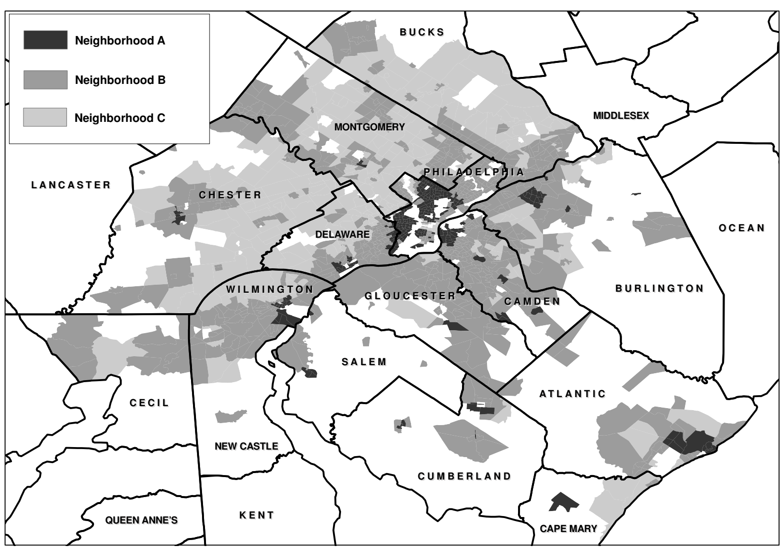


Figure 16: Philadelphia MSA, $K = 3$



Figures contain selected neighborhoods of the corresponding MSA.

Figure 17: Washington-Baltimore-Arlington MSA, $K = 2$

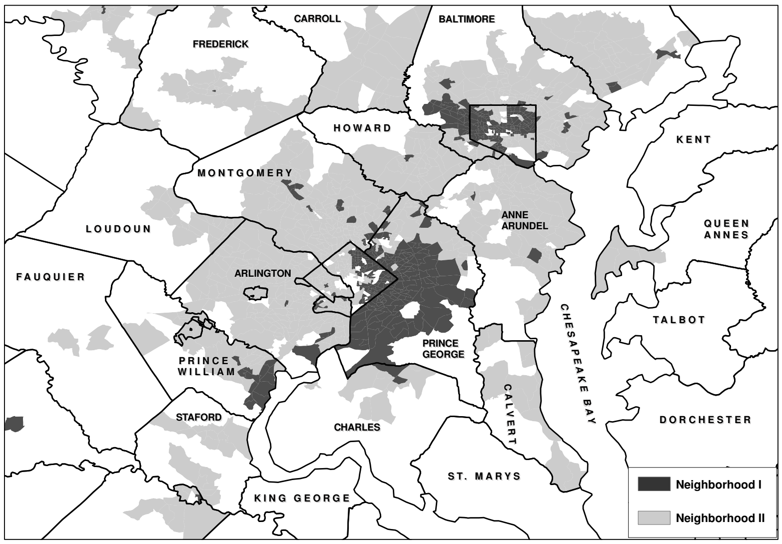
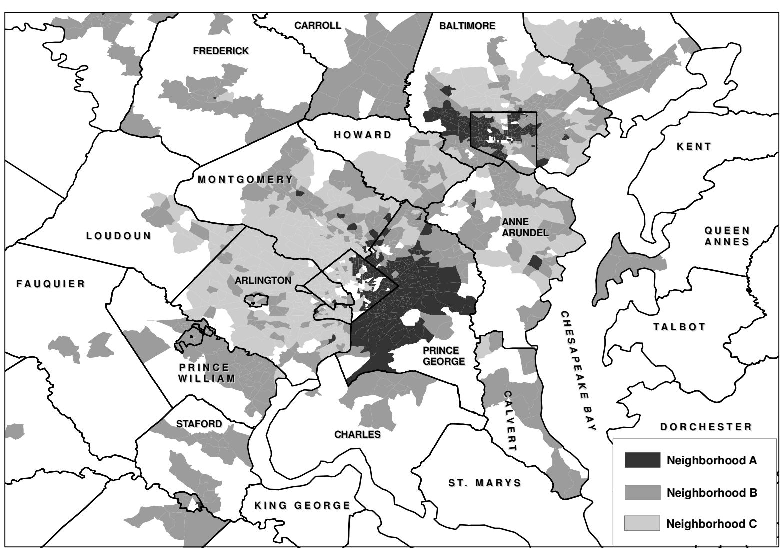
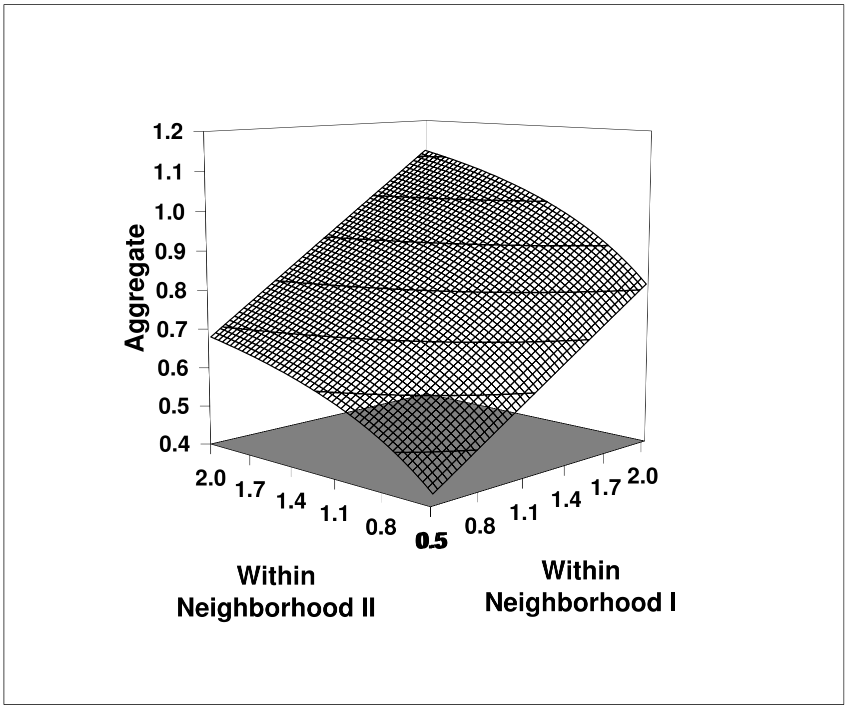


Figure 18: Washington-Baltimore-Arlington MSA, $K = 3$



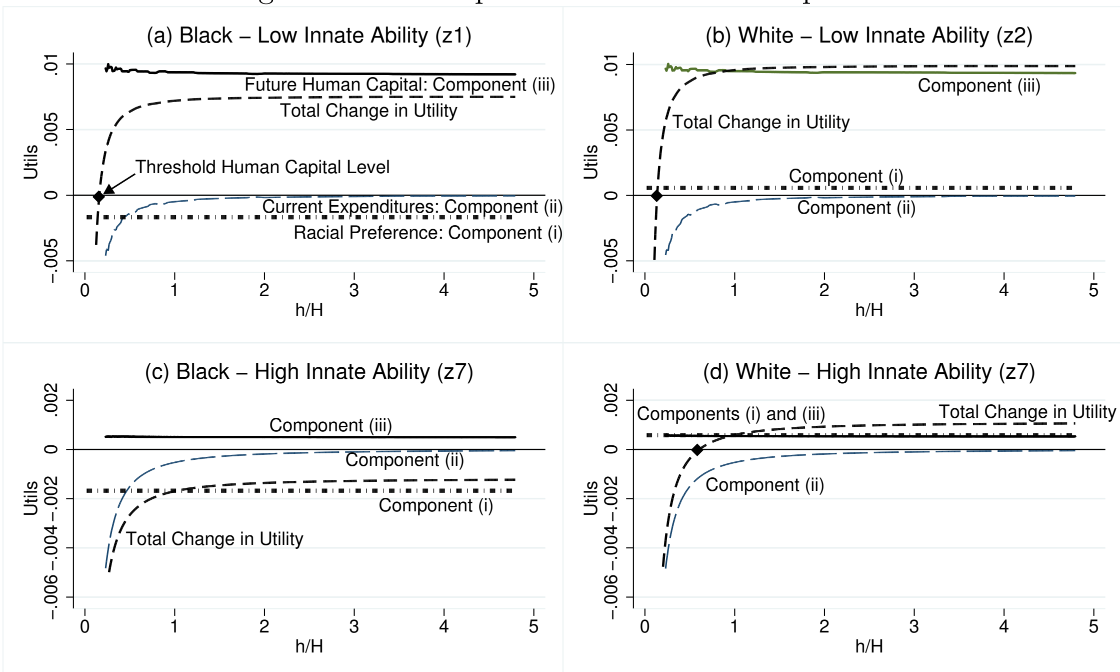
Figures contain selected neighborhoods of the corresponding MSA.

Figure 19: Restrictions on Black White Earnings Ratios



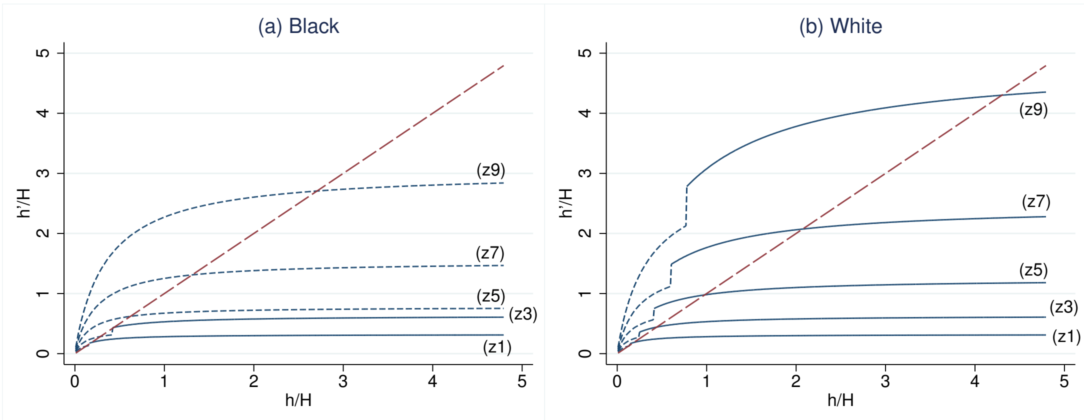
Note: The surface plots the aggregate BW earnings ratio θ as a function of the BW earnings ratios within Neighborhood I (θ_I) and Neighborhood II (θ_{II}). The function assumes that the populations of each color within each neighborhood are given by the empirical values in Table 4.

Figure 20: Decomposition of Bellman's Equation



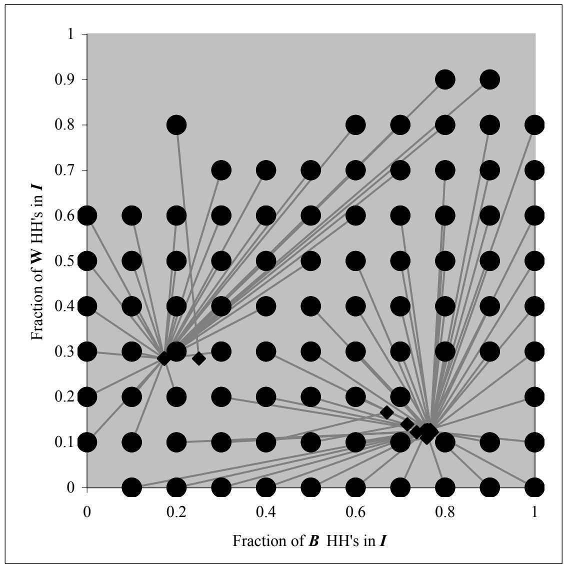
Note: This figure decomposes the total change in utility $V_{II}(x) - V_I(x)$ experienced by a household hypothetically moving from I to II into three components. Component (i) measures the change in period utility from racial preferences $v(R_{II}(r)) - v(R_I(r))$. Component (ii) measures the change in period utility from current expenditures $u(c(x, n = II), l(x, n = II)) - u(c(x, n = I), l(x, n = I))$. Component (iii) measures the change in future utility $E[V(h(x, n = II), r, z')|z] - E[V(h(x, n = I), r, z')|z]$.

Figure 21: Human Capital Law of Motion by Color (selected z shock values).



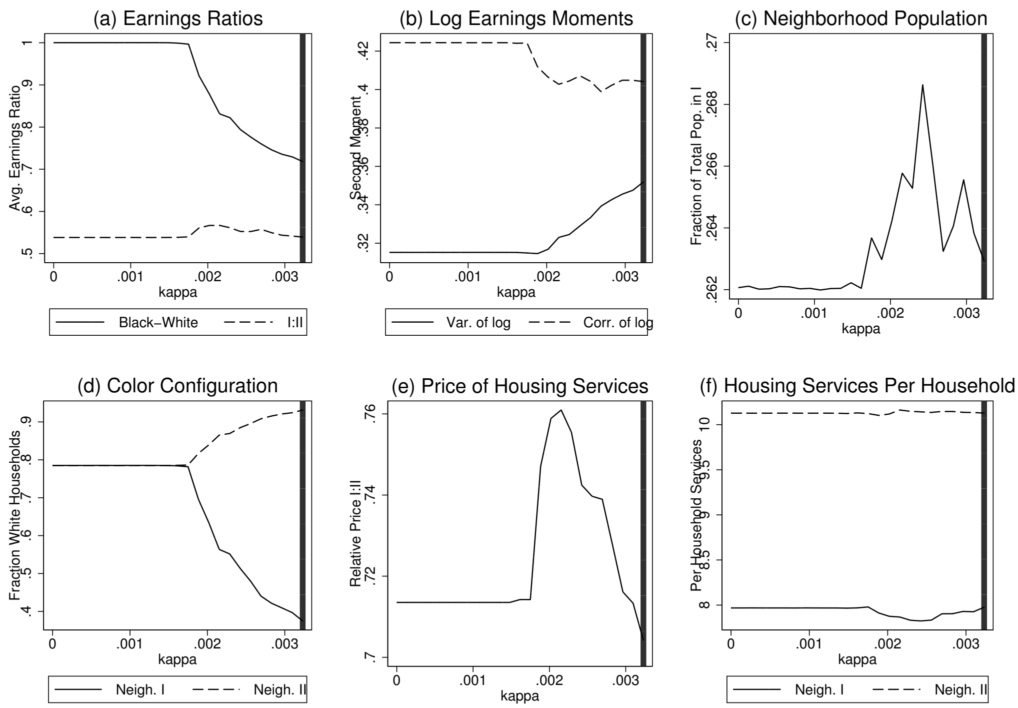
Note: Each curve corresponds to a value of the innate ability shock z. The horizontal axis measures parental human capital while the vertical axis measures the child's future human capital. Parental and child's future human capital are related by the law of motion $h^{'} = zAF(i(x,n), H_n)$ where n is the neighborhood chosen by the household ($\eta(n|x) = 1$). A dotted line is assigned to states x where the household decides to live in neighborhood I and a continuous line represents states where the household decides to live in neighborhood II. Axes are normalized by average human capital H.

Figure 22: Myopic Equilibria Fixing $R(r)$
(Benchmark Parameters)



Note: Each point on the plane represents the distribution of black and white households over neighborhoods I and II, each of these distributions implies a racial configuration through the formula $R_I(r) = r_I/(B_I + W_I)$ where r_I is the fraction of total households of color $r = B, W$ that locate in I. Each large dot represents an arbitrary *perception* of racial configuration imposed to households and is connected to a small dot, representing the racial configuration *implied* by households' choices given the perceived racial configuration. No market clearing prices have been found for missing gridpoints.

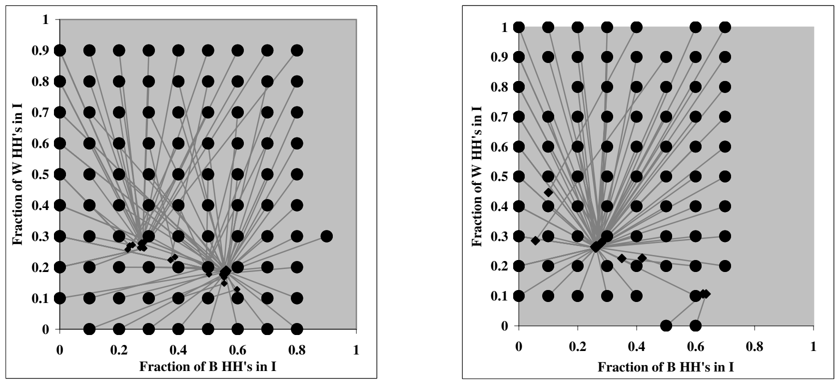
Figure 23: Sequence of Counterfactual Stationary Equilibria
(varying κ)



Note: The benchmark value of the parameter is depicted by a vertical bar. A stationary equilibrium is computed for each of 25 equally spaced values of the parameter in the range of the horizontal axis. The vertical axis measures equilibrium values of model outcomes.

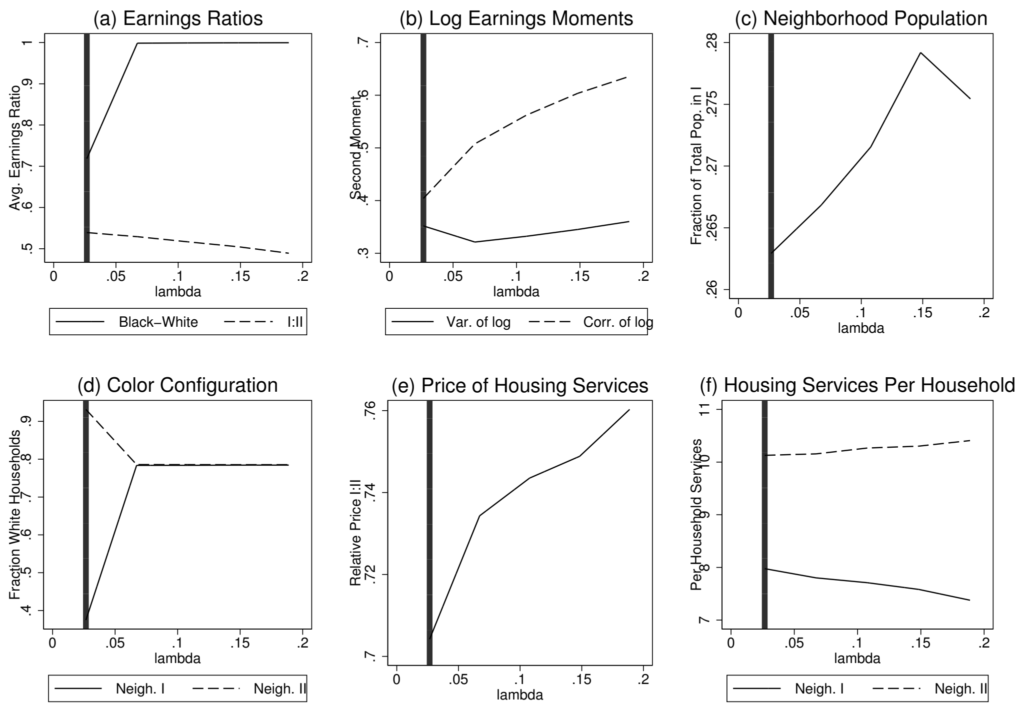
Figure 24: Myopic Equilibria Fixing $R(r)$

κ reduced to 2/3 of benchmark κ reduced to 1/2 of benchmark



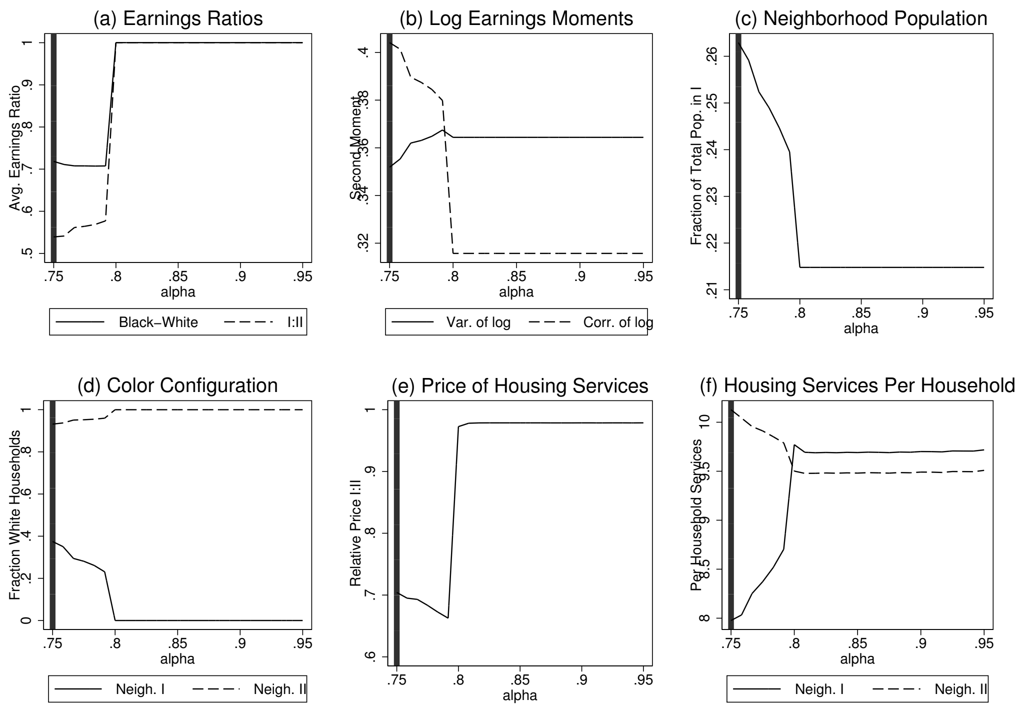
Note: Each point on the plane represents the distribution of black and white households over neighborhoods I and II, each of these distributions implies a racial configuration through the formula $R_I(r) = r_I/(B_I + W_I)$ where r_I is the fraction of total households of color $r = B, W$ that locate in I. Each large dot represents an arbitrary *perception* of racial configuration imposed to households and is connected to a small dot, representing the racial configuration *implied* by households' choices given the perceived racial configuration. No market clearing prices have been found for missing gridpoints.

Figure 25: Sequence of Counterfactual Stationary Equilibria
(varying λ)



Note: The benchmark value of the parameter is depicted by a vertical bar. A stationary equilibrium is computed for each of 25 equally spaced values of the parameter in the range of the horizontal axis. The vertical axis measures equilibrium values of model outcomes.

Figure 26: Sequence of Counterfactual Stationary Equilibria
(Varying α)



Note: The benchmark value of the parameter is depicted by a vertical bar. A stationary equilibrium is computed for each of 25 equally spaced values of the parameter in the range of the horizontal axis. The vertical axis measures equilibrium values of model outcomes.

Table 1: Variable Definitions

Variable	Definition (Census Code)
Racial Configuration	
Fraction black HH	p151b001/(p151a001+..p151g001)
Fraction non-Hispanic white	p151i001/(p151a001+..p151g001)
Earnings and Income	
Average tract HH earnings	p067001/p058001
Average tract HH income	p054001/p052001
Average white HH income	p153i001/p151i001
Average black HH income	p153b001/p151b001
Average oth. race HH income	(P153a001+...P153g001-p153i001-p153b001) /(P151a001+...P151g001-p151i001-p151b001)
Housing	
Median gross rent	H063001
Median value (owner occupied)	H085001
Median selected owner costs	H091001 (owner occ. w/ mortgage)
Median # of rooms in unit	H027002 (owner), H027003 (renter)
Distribution of # units in structure	H032003-012/H032002 (owner), H032014-023/H032013 (renter)
Median yr. structure built	H037002 (owner),H037003 (renter)
Distribution of # bedrooms	H042003-008/H042002 (owner), H042010-015/H042009 (renter)
Fraction w/ tel. service	H043003/H043002 (owner), H043020/H043019 (renter)
Fraction w/ plumbing facilities	H048003/H048002 (owner), H048006/H048005 (renter)
Fraction w/ kitchen facilities	H051003/H051002 (owner), H051006/H051005 (renter)
Distribution of heating fuel	HCT010003-011 (owner), HCT0010013-021 (renter)
Other	
Distribution of time to work	P031003-014/P031002
Fraction of Population in Group Quarters	P009025/P0009001

Source: SF3, 2000 Census of Population and Housing Technical Documentation, released September 2002.

Table 2: Sample Selection Criteria

Criterion	Obs. Dropped	Total Obs.
Initial w/o missing values		50,167
Less than 1 million MSA pop.	14,397	35,770
Less than 10% black HH in MSA	14,244	21,526
Pop. Density less than 100 per sq. km.	1,785	19,741
More than 50% other race	1,421	18,320
Less than 200 HH in tract	226	18,094
More than 25% institutionalized pop.	279	17,815

Each observation corresponds to a census tract.

Table 3: Descriptive Statistics, Main Variables

Variable	Mean	Std. Dev.	5th Pctile.	95th Pctile.
Fraction black HH	.23	.32	0	.95
Fraction white HH	.66	.32	.01	.97
Fraction other races	.1	.11	.01	.35
Av. Tract Income	63,921	33,981	28,178	125,278
Av. Tract HH Income (Blacks)	55,927	43,712	20,508	117,500
Av. Tract HH income (Whites)	66,413	36,393	26,702	130,605
Av. Tract HH Income (oth. Races)	61,015	37,956	21,550	124,834
Median IRV*	13,372	5,593	6,748	22,571
Median gross Rent*	8,806	2,413	5,782	12,743
Median selected owner costs*	15,415	3,795	10,323	21,772
Tract Population	4,427	2,265	1,536	8,403
# HH in tract	1,701	890	573	3,300
Pop. Density (pop. Per Sq. km.)	3,391	6,150	192	13,605
Fraction of pop. in group quarters	.01	.03	0	.08

*Statistics reported after "cleaning regression". Only the mean is invariant with respect to raw variable.

Table 4: Population density and area by variable configuration
Z-Score Normalization

Variable	K = 2		K = 3		
Configuration	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 3$
Population Density					
ben	4837	2,251	5,333	2,070	2,674
cern	5,018	2,298	5,181	2,005	2,907
inc	4,795	2,229	5,295	2,092	2,674
cinc	5,071	2,257	5,310	2,186	2,647
prent	5,112	2,222	5,306	2,251	2,404
pcost	4,959	2,209	5,346	2,255	2,444
Area (1000 sq. km)					
ben	4.59	25.32	3.19	17.07	10.0
cern	3.95	25.09	3.15	16.83	9.42
inc	4.81	25.19	3.31	18.15	8.85
cinc	4.00	25.34	3.01	15.60	10.3
prent	4.04	26.35	3.28	15.13	11.5
pcost	4.45	25.86	3.12	14.61	12.11

Table 5: Characteristics of Representative Neighborhoods, $K = 2$

	Neighborhood		
	I	*II*	
Number of Households (thousands)			
			I/(*I*+*II*)
Black	4,451	1,359	.77
White	2,662	18,577	.13
Other	1,152	2,150	.35
Total	8,265	22,085	.27
W/(B+W)	.37	.93	
Average Income ($)			
			I/*II*
Black	40,076	57,124	.70
White	44,727	76,711	.58
Other	41,320	67,166	.62
Total (B and W only)	41,816	75,376	.55
Total	41,747	74,577	.56
Other ($)			
Average Earnings	33,591	61,889	.54
Average Earn. (B and W only)	41,816	75,376	.61
Price of Housing Services*	10,405	14,268	.73

*Units are normalized to match the value of original IRV measure (see section 2.1).

Table 6: Characteristics of Representative Neighborhoods, $K = 3$

	Neighborhood				
	X	Y	Z		
Number of Households (thousands)					
				$\frac{X}{X+Y+Z}$	$\frac{Y}{X+Y+Z}$
Black	4,074	1,244	492	.70	.21
White	1,268	11,244	8,727	.06	.53
Other	821	1,396	1,084	.25	.42
Total	6,163	13,884	10,303	.20	.46
W/(B+W)	.24	.90	.95		
Average Income ($)					
				X/Z	Y/Z
Black	39,949	49,059	65,481	.61	.75
White	43,955	58,651	94,982	.46	.62
Other	40,363	53,483	77,620	.52	.69
Total	40,899	57,696	93,407	.44	.62
Total (B and W only)	40,828	57,272	91,746	.45	.62
Other					
Average Earnings ($)	33,142	47,106	76,303	.44	.62
Price of Housing Services*	10,715	11,238	17,377	.45	.62

*Units are normalized to match the value of original IRV measure (see section 2.1).

Table 7: Variable Configurations

Name	Racial Composition	Human Capital	Price of Housing Services
ben	%Non Hispanic Whites	log HH earnings	clean IRV* (owners)
cern	%Non Hispanic Whites	clean log HH earnings	clean IRV* (owners)
inc	%Non Hispanic Whites	log HH income	clean home value (owners)
cinc	%Non Hispanic Whites	clean log HH income	clean home value (owners)
prent	%Non Hispanic Whites	log HH earnings	clean rent (renters)
pcost	%Non Hispanic Whites	log HH earnings	clean owner's cost (owners)

*IRV (Implicit Rental Value) is defined as a percentage of a home's market value. See section "Measuring the Price of Housing Services".

Table 8: Cluster compactness (%)

Variable Configuration	$K = 2$	$K = 3$	$K = 4$	$K = 5$	$K = 6$
zben	37	50	57	62	66
zcern	28	41	49	54	58
zinc	37	52	58	63	68
zcinc	28	41	49	55	59
zprent	34	52	58	63	67
zpcost	36	50	57	62	66
mben	26	43	53	58	62
mcern	26	40	49	54	59
minc	26	44	54	58	63
mcinc	26	40	50	55	60
mprent	26	43	54	59	62
mpcost	26	43	54	58	62
Average	30	45	54	58	63

Reported statistic corresponds to the fraction of (H, R, P) sum of variances explained by between cluster variation.

Table 9: Cluster Similarity (%)
$K = 3$. All Variable Configurations and Normalizations

	zben	zcern	zinc	zcinc	zprent	zpcost	mben	mcern	minc	mcinc	mprent	mpcost
zben	-	82	94	79	80	89	76	78	77	72	78	77
zcern	.	-	80	93	70	78	66	77	67	70	70	67
zinc	.	.	-	79	81	87	76	77	78	71	78	77
zcinc	.	.	.	-	70	76	66	73	67	65	69	67
zprent	.	.	.	.	-	81	80	69	80	66	90	80
zpcost	.	.	.	.	.	-	77	74	77	70	80	77
mben	.	.	.	.	.	.	-	62	92	60	88	98
mcern	.	.	.	.	.	.	.	-	64	91	68	64
minc	.	.	.	.	.	.	.	.	-	61	86	92
mcinc	.	.	.	.	.	.	.	.	.	-	65	61
mprent	.	.	.	.	.	.	.	.	.	.	-	88
mpcost	.	.	.	.	.	.	.	.	.	.	.	-

The reported statistic corresponds to the percentage of neighborhoods classified in the same group under two alternative variable configurations. The first letter in each label corresponds to the normalization method (z for Z-Score, m for Mahalanobis). The remainder letters in the label correspond to the name of the variable configuration, described in Table 4.

Table 10: Percentage of Households by Neighborhood Class within each MSA
Benchmark variable configuration, Z-score normalization.

	$K=2$		$K=3$			
Metropolitan Statistical Area	$k=1$	$k=2$	$k=1$	$k=2$	$k=3$	Tracts
Atlanta	33	67	28	55	17	568
Buffalo-Niagara Falls	33	67	16	78	6	250
Charlotte-Gastonia-Rock Hill	24	76	16	46	38	246
Chicago-Gary-Kenosha	24	76	20	34	46	1,658
Cincinnati-Hamilton	18	82	11	67	22	391
Cleveland-Akron	26	74	18	62	19	738
Columbus OH	20	80	13	63	24	310
Dallas-Fort Worth	30	70	19	54	27	833
Detroit-Ann Arbor-Flint	25	75	22	28	50	1,335
Greensboro-Wn Salem-H Point	30	70	21	56	23	196
Houston-Galveston-Brazoria	43	57	31	57	12	630
Indianapolis	22	78	13	65	22	278
Jacksonville FL	32	68	15	64	20	162
Kansas City	25	75	16	65	19	400
Louisville	19	81	13	72	15	207
Memphis	51	49	48	31	21	203
Miami-Ft Lauderdale	51	49	38	38	25	409
Milwaukee-Racine	24	76	20	46	35	392
NY-N New Jersey-Long Island	24	76	21	23	57	3,850
Nashville	18	82	13	53	33	186
New Orleans	46	54	37	44	19	339
Norfolk-V Beach-Newport News	35	65	26	66	8	309
Orlando	33	67	19	63	17	287
Pdelphia-Wilmington-Atl City	26	74	20	57	24	1,356
Raleigh-Durham-Ch Hill	20	80	14	33	53	157
Saint Louis	26	74	18	63	19	429
W Palm Beach-Boca Raton	32	68	16	55	29	243
Washington-Baltimore	29	71	23	46	31	1,453
Total	28	72	22	45	34	17,815
Std. Dev.	9.0	9.0	8.5	13.9	12.7	
Tracts	5,649	12,166	4,458	7,456	5,901	

Table 11: Cluster similarity
Pooled versus MSA by MSA clustering
Benchmark variable configuration, Z-score normalization.

Metropolitan Statistical Area	$K = 2$	$K = 3$	$K = 4$
Atlanta	96	89	80
Buffalo-Niagara Falls	91	74	79
Charlotte-Gastonia-Rock Hill	88	87	65
Chicago-Gary-Kenosha	98	79	84
Cincinnati-Hamilton	98	76	71
Cleveland-Akron	97	85	72
Columbus OH	76	78	67
Dallas-Fort Worth	85	65	87
Detroit-Ann Arbor-Flint	99	91	65
Greensboro-Wn Salem-H Point	98	75	61
Houston-Galveston-Brazoria	96	92	86
Indianapolis	82	71	62
Jacksonville FL	98	78	70
Kansas City	99	82	64
Louisville	98	74	58
Memphis	97	77	76
Miami-Ft Lauderdale	93	63	72
Milwaukee-Racine	98	73	60
NY-N New Jersey-Long Island	83	91	54
Nashville	92	86	63
New Orleans	95	69	59
Norfolk-V Beach-Newport News	93	61	63
Orlando	80	70	55
Pdelphia-Wilmington-Atl City	95	83	69
Raleigh-Durham-Ch Hill	96	70	67
Saint Louis	97	70	67
W Palm Beach-Boca Raton	95	86	93
Washington-Baltimore	86	66	55
Average	93	77	69

The reported statistic corresponds to the fraction of neighborhoods classified in the same group by applying the clustering algorithm to the pooled dataset (all MSA) versus applying it separately to each MSA.

Table 12: Cluster Contiguity. $\kappa = 2.5$
Benchmark variable configuration

	$K = 2$		$K = 3$		
	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 3$
Z-Score Normalization					
Contiguity (%)	40	64	43	32	50
Adjacency	5.6	7.2	5.6	5.7	5.9
Mahalanobis Normalization					
Contiguity (%)	41	68	41	28	50
Adjacency	5.6	7.4	5.6	5.7	6.1

For a randomly chosen neighborhood i of class C_k, contiguity equals the expected fraction of neighborhoods of class j that are connected to i. Adjacency equals the expected number of class C_k neighborhoods that are directly adjacent to i.

Table 13: Mean Income Ratios (%)
Z-Score Normalization

Variable	$K = 2$		$K = 3$		
Configuration	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 3$
Black Households					
ben	89	74	90	83	68
cern	81	70	83	76	62
inc	89	75	89	84	69
cinc	81	70	83	76	64
prent	90	74	89	78	70
cost	90	75	90	81	70
Other Race Households					
ben	92	87	91	91	81
cern	86	86	87	88	80
inc	92	88	91	91	82
cinc	86	86	87	88	81
prent	92	87	90	88	79
cost	92	87	90	89	81

Note: All ratios taken with respect to the corresponding average income of white households.

Table 14: Varying Sample Selection Criteria
Benchmark variable configuration, Z-score normalization, $K = 2$.

	Sample Variation				
Statistic	Baseline	1	2	3	4
Neighborhood I					
Average Earnings (H)	33,591	32,656	33,606	33,795	33,402
Fraction HHs white (R)	.32	.33	.40	.31	.32
Price of Housing Services (P)	10,406	9,976	10,577	10,716	10,063
Neighborhood II					
Average Earnings (H)	61,889	60,222	62,911	61,930	60,311
Fraction HHs white (R)	.84	.84	.83	.84	.84
Price of Housing Services (P)	14,269	13,604	16,562	14,228	13,768
Aggregate					
Fraction of HHs living in I	.71	.71	.69	.67	.71
Overall Fraction White HHs	.70	.70	.70	.67	.69
Overall Fraction Black HHs	.19	.20	.16	.19	.20
Number of MSA	28	56	41	28	28
Number of Tracts	17,815	20,148	24,054	18,913	17,603

Sample variations 1 through 4 correspond to: (1) Include MSA with population above 250.000 (vs. 1 million in baseline sample). (2) Include MSA with 5% black population or more (vs. 10% in baseline sample). (3) Include neighborhoods with fraction of other races less than 90% of other races (vs. 50% or less included in baseline sample). (4) Exclude neighborhoods with average earnings above 150.000 (vs. no upper limit in baseline sample).

Table 15: Model Parameters
(a) Pre-Specified Parameters

Definition	Parameterization	Calibrated Value
Fraction of Black Households	χ_B	$\chi_B = .21$
Human Capital Rental Rate	w	$w = 1$
Housing Service Price Level in I	P_I	$P_I = 1$
Discount Factor	β	$\beta = .36$
Utility Function ((c, l) component)	$u(c, l) = \frac{(c^{\alpha} l^{1-\alpha})^{1-\sigma}}{1-\sigma}$	$\alpha = .75$ $\sigma = 2.5$
Direct transmission of Human Capital	$(1-\delta)h$	$\delta = 1$

(b) Estimated Parameters

Definition	Parameterization	Estimated Value
Innate Ability Process	$ln(z') = \rho ln(z) + \epsilon$ $\epsilon \sim N(0, \sigma_{\varepsilon}^2)$	$\rho = .082$ $\sigma_{\varepsilon} = .678$
Human Capital Production Function	$F(z, i, H_n) =$ $zA(\lambda i^{\gamma} + (1-\lambda)H_n^{\gamma})^{\frac{\chi}{\gamma}}$	$A = 2.761$ $\gamma = -.786$ $\lambda = .027$ $\chi = .769$
Utility Function (racial component)	$v(R(r)) = \kappa(R(r) - R^*)^2$	$\kappa = 3.23 \times 10^{-03}$ $R^* = .812$
Relative Supply of Housing Services	L_I/L_{II}	.28

Table 16: Model Fit

(a) Estimation Targets

Definition	Data	Model
Avg. Earnings in I	$33,591$	$33,543$
Avg. Earnings Ratio ($I : II$)	.54	.54
Log Earnings Variance	.36	.35
Intergenerational log Earnings Correl.	.40	.40
Intergenerational log Consumption Correl.	.49	.45
Fraction W in I	.13	.13
Fraction B in I	.77	.76
Investment-Income Ratio	.072	.072
Housing Price Ratio $I : II$	.73	.71

(b) Additional Facts

Definition	Data	Model
BW Avg. HH Earnings Ratio		
Overall	.61	.72
In Neighborhood I	.90	1.74
In Neighborhood II	.74	.74
BW Expenditure Ratio	.66	.72

Table 17: Neighborhood Effects Regressions

Dependent Variable: child's future log earnings	(1) Black	(2) White	(3) Black	(4) White
Parental log earnings	0.414	0.512	0.221	0.206
"Good" neighborhood dummy	-0.652	-0.437	0.331	0.385
Child's (log) innate ability			0.864	0.890
Constant	2.260	2.276	1.847	1.840
R^2	0.409	0.194	0.996	0.997

All regressions are computed on 300,000 observations of model generated data.