

SEARCH AND MATCHING MODELS OF LABOR MARKETS IN DEVELOPING
ECONOMIES

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ABSTRACT

This dissertation consists of two essays studying the macroeconomics of labor markets with search frictions in developing economies.

The first essay evaluates the impact of market-oriented structural reforms, in particular labor market policies, social assistance programs, and trade liberalization upon long run unemployment, wage inequality and the distribution of employment across sectors in a small open economy with search frictions and idiosyncratic productivity shocks. A search and matching model of a labor market with a large informal sector is built and estimated by Simulated Method of Moments using Colombian household-level data. Results show that changes in labor taxes may have sizable aggregate, compositional and distributional effects if workers associate high payroll taxes with more valuable and efficient social security services. The higher the valuation of SS services, the more progressive these labor market policies become. An expansion of public health insurance to informal sector workers has minor aggregate and distributional effects. Changes in relative prices that negatively affect the relative profitability of the formal sector have quite sizable aggregate effects, producing more long run unemployment and informality, and increasing unemployment duration by 9 weeks. Distributional effects are modest.

The second essay (co-authored with James Albrecht and Susan Vroman) studies the interactions between private and public-sector labor markets by extending the standard Diamond-Mortensen-Pissarides model of equilibrium unemployment to

incorporate public-sector employment. The model is calibrated using Colombian household-level data to investigate the effects of public-sector wage and employment policy on the employment and unemployment rates, as well as the distributions of wages, productivities and human capital levels across sectors. The model matches well selected moments of skill-specific labor market variables. Results indicate that doubling the size of the public sector may crowd out private employment and increase the unemployment rate by 1.3 percentage points, but may also narrow the log-wage gap and reduce log-wage inequality (standard deviation is decreased by 3.4 percentage points). Changes in the public-sector wage rule have minor impact on the employment and unemployment rates but have more sizeable distributional effects. While eliminating the "pure" public-sector premium leads to a small reduction in the log-wage gap of 0.062 log-wage points, placing more weight on formal qualifications when setting public-sector wages reduces the public-sector gap significantly by 0.309 log-wage points.

INDEX WORDS: Developing Economy, Unemployment, Informality, Public Sector, Wage Inequality

DEDICATION

To my family, who has provided tremendous support during this stage of my life,
especially to my husband Juan and my two daughters Emma and Mia

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CHAPTER 1

INTRODUCTION

The research presented in this document is separated into two chapters studying the macroeconomics of labor markets with search frictions in developing economies. Even though each chapter studies distinct features of aggregate labor market outcomes, both use search theory as a structural framework to analyze the long term impact of government policies on the labor markets.

Search frictions are especially important in the labor market, given the existence of heterogeneities and imperfect information. Workers may have different skill levels and jobs may require different qualifications, making the arrival of suitable jobs to job seekers and suitable workers to firms uncertain. Even workers with a known skill level may have uncertain productivities at a particular match, adding an idiosyncratic component to the value of prospective matches. Firms may also not know where the good workers are and may not know all the workers who are in the market. Given this uncertainty and lack of homogeneity, unemployed workers and firms must spend time and effort before job creation takes place. Therefore, trading labor services is a decentralized, time-consuming and costly process for agents in the market.

A key element of this environment is that unemployment persists in equilibrium. This is not a feature of Walrasian labor markets, where there is no voluntary unemployment. Another key element is that we can explain the existence of equilibrium wage dispersion, that is, why workers with the same productivity do not necessarily earn the same wages.

A first class of search models in the literature - partial equilibrium search models - assume that the wage offer distribution is exogenous, so there is no explicit modeling of the demand side of the market, in particular, how firms affect the wage offered distribution. The rate at which offers arrive to workers is also assumed exogenous.

A second class of search models in the literature - equilibrium search models - endogenize the wage offer distribution. One branch of this literature ¹ focuses on wage determination and de-emphasizes the firms' job creation decisions. These models assume that firms are posting (instead of negotiating) wages in the market and explain why there may be a distribution of wages in equilibrium despite assuming ex-ante worker heterogeneity - in other words, why firms may pay workers with equal productivity differently. Another branch of this literature ² focuses on the problem of matching workers to vacancies and assumes that firms are negotiating wages using a Nash bargaining rule that specifies a mapping from match specific productivity to wages. The offer arrival rate is endogenized by introducing a matching function that captures how vacant jobs and unemployed workers are matched. The canonical model of equilibrium unemployment in this branch of the literature is the Diamond-Mortensen-Pissarides (DMP) model. In this type of model, some sort of heterogeneity is required to produce wage dispersion.

The two models presented here follow the DMP tradition with heterogeneity from the worker's side and ex-post idiosyncratic match-specific productivity. Both models are estimated using household-level data to reproduce key empirical features of the Colombian labor markets.

The first essay evaluates the impact of market-oriented reforms on the size of the

¹These are known as wage posting models. See Burdett and Mortensen(1998) and Postel-Vinay and Robin (2002).

²These are known as matching and Nash bargaining models. See Mortensen and Pissarides (1994) and Pissarides (2000).

informal sector, unemployment and wage inequality. After the 1990's, Colombia as well as other Latin American economies implemented a sequence of market-oriented structural reforms, including changes in labor market policies, social assistance programs and trade liberalization. There is no consensus in the literature regarding the extent to which these reforms contributed to deteriorating labor market conditions. Informality has been a persistent phenomenon in the last three decades: regardless of the definition used, approximately one of every two workers is considered informal. Long term unemployment has been rising, and it is quite high for Latin American standards. The formal-sector log-wage premium, although diminishing, is still persistently high, and it is very high compared with the other countries in the region: on average, a formal-sector worker earns about two times what an informal-sector worker earns.

I formulate and estimate an equilibrium search and matching model with formal and informal-sector employment. I also allow *ex-ante* worker heterogeneity in human capital and *ex-post* idiosyncratic match-specific productivity. I endogenize the job separation rate by introducing idiosyncratic shocks to job productivities, and allow direct flows of workers from formal to informal employment (without intervening unemployment) to better match the data. The model can approximately match the aggregate unemployment rate, formal- and informal-sector employment rates, mean employment duration in the formal sector and the relative dispersion in log-wages, but underestimates relative mean-log-wages. Counterfactual experiments consistent with the reforms show that changes in labor taxes may have sizable aggregate, compositional and distributional effects if workers associate high payroll taxes with more valuable and efficient social security services. The higher the valuation of the services, the higher the reduction in the formal-sector log-wage premium. An expansion of public health insurance to informal sector workers has minor aggregate and dis-

tributional effects. Changes in relative prices that negatively affect the relative profitability of the formal sector have quite sizable aggregate effects, producing more long run unemployment and informality, and increasing unemployment duration.

The second essay seeks to understand the interactions between public and private sector labor markets in a model of equilibrium unemployment. In general, we want to understand what types of workers tend to work in each of the sectors, and how the size of the public sector and the way wages are set (i.e. less weight to productivity than to formal qualifications) affect the unemployment rate and the distribution of human capital, productivity and wages in each sector. The public-sector in Colombia is relatively small (about 8.34% of total employment in 2013) when compared with other Latin American countries, and there is a large public-sector log-wage premium, even when controlling for standard observables. In contrast to typical developed and middle-income countries, in which wages tend to be more compressed in the public sector, the degree of wage dispersion is similar in the two sectors.

We build an equilibrium search and matching model that incorporates public-sector employment into a DMP model of equilibrium unemployment. The model is extended in several ways. We allow for both private and public-sector employment, taking into account that the rules that govern public-sector employment and wages are not necessarily the same as those used in the private sector. As in the informality model, workers are heterogeneous in terms of human capital, and stochastic job matching is allowed. However, job destruction is exogenous and there are no direct flows between sectors. The model can match pretty well the aggregate unemployment rate, private and public-sector employment rates, mean employment duration in both sectors, the relative mean log-wages and the overall log-wage variance. The model slightly underestimates dispersion in log-wages within the public-sector relative to the private. The model also matches well the moments of skill-specific labor market

variables, as well as the human capital and wage distributions. Counterfactual simulations of the model indicate that different distributions of education between the two sectors is the most important mechanism contributing to the observed public-sector wage premium. Numerical experiments indicate that doubling the size of the public sector crowds out private-sector employment and increases the unemployment rate by 1.3 percentage points, while narrowing the log-wage gap and reducing log-wage dispersion (standard deviation) by 3.4 percentage points. Changes in the public-sector wage rule have minor impact on the employment and unemployment rates, but have more sizable distributional effects. Placing more weight on formal qualifications when setting public-sector wages reduces the log-wage gap significantly by 0.309 log-wage points, while eliminating the "pure" public-sector premium leads to a minor reduction in the public-sector gap of 0.062 log-wage points.

CHAPTER 2

TAX TRANSFERS SCHEMES, INFORMALITY AND SEARCH FRICTIONS IN A SMALL OPEN ECONOMY

2.1 INTRODUCTION

This paper seeks to understand the impact of market-oriented structural reforms on steady state unemployment, wage dispersion and the distribution of employment across sectors in a small open economy with a persistently large informal sector. To this end, I formulate and estimate an equilibrium search and matching model of a labor market with search frictions and idiosyncratic productivity shocks.

The central idea behind modeling the labor market with a search model (instead of a neoclassical one) is that, given the existence of heterogeneity, frictions or imperfect information, search is a costly activity for both firms and workers who must spend resources before job creation and job destruction occurs. In these types of models equilibrium unemployment in the steady state emerges naturally as a result of the transitions in and out of unemployment, since some existing jobs break up before new matches are made. None of these properties characterize Walrasian labor markets.

The model is particularly relevant for developing economies, in particular Latin American (LA) economies, where wage inequality, high unemployment, job instability and large informal sectors are longstanding concerns, particularly in the last

two decades (Inter-American Development Bank, 2004)¹.

After the 1990's, many LA countries followed a sequence of market-oriented structural reforms, including changes in labor market legislation, social security programs, and changes in the degree of trade and financial openness. The extent to which these countries' labor market, trade and social security reforms contribute to deteriorating labor market conditions is still a debatable question in the literature. Some other factors including aggregate and idiosyncratic market shocks, demographic changes affecting the size of the labor force, or skill biased-technological change may also play an important role².

The model is simulated for the Colombian economy, a country that in the 1990's and 2000's implemented substantial market-oriented reforms whose main goal was to deregulate labor and financial markets.

There are some previous empirical studies focusing on the effects of Colombian structural reforms on labor markets: Cardenas and Gutierrez (1996); Cardenas, Kugler and Bernal (1998); Kugler (1999); Eslava, Haltiwanger, Kugler and Kugler (2010); Camacho, Conover and Hoyos (2013). Most of these studies employ reduced form estimation and none of them build a search model as an analytical framework to understand incentives behind flows between the formal and informal sector. I provide a new lens to study the impact of the reforms by building and estimating a structural model.

¹The informal sector is particularly large in Latin American economies (from 30 to 70 percent of GDP) according to some studies (Maloney, 2004). Other studies (Schneider, 2005) suggest also that the size of the informal sector in other economies is not negligible. They estimate that, for the period 1999-2000, the average size of the informal economy (as percent of GDP) is 41.2 percent in African countries, 26.3 percent in Asian countries, 37.9 percent in transitional economies and 16.7 percent in OECD countries.

²For a survey of the impact of labor market regulations in Latin America see Heckman and Pages (2004), on the impact of trade liberalization and job turnover, see Haltiwanger et al (2004). For a survey of the impact of trade on wage inequality and informality in Latin America and other developing economies see Attanazio, Goldberg and Pavcnik (2004).

Some recent studies have looked at informality in Latin American economies using a flow approach to unemployment: Bosch and Maloney (2007, 2008, 2010); Bosch, Goni and Maloney(2007); Fiess, Fugazza and Maloney (2008).

This model contributes to a growing recent literature that combines informality with labor market search frictions in emerging economies: Albrecht, Navarro and Vroman (2009); Bosch and Esteban-Pretel (2012); Meghir, Narita and Robin (2012); Cosar, Guner and Tybout (2013).

I develop a substantially modified version of the search and matching model with endogenous job destruction by Mortensen and Pissarides (1994)(MP1994 hereafter). The main differences between this model and MP1994 are the following: a) An informal sector is added; b) There is a continuum of worker types; c) The idiosyncratic productivity shock process is modeled differently. While in MP1994 matches start at the “state of the art” or, at the fixed maximum productivity level, in this model they start with a draw from a distribution (stochastic job matching).

The model is similar in spirit to the one developed by Albrecht, Navarro, Vroman (2009) (ANV2009 hereafter) with an informal sector, but the modeling of the informal sector is different, capturing the idea that the informal sector is a ‘disadvantaged’ sector of a dualistic or segmented labor market,³ instead of an unregulated self-employment sector. Even though the informal sector has close linkages to the formal sector, it is still a ‘disadvantaged’ sector characterized by low entry barriers in terms of skills, and therefore, populated by workers with low productivity levels who are excluded or segregated from the formal economy.

The main differences with ANV2009 are the following: 1) In this model there

³There is some supporting empirical evidence of a segmented labor market for the Colombian case. Most of the arguments are supported by the fact that the informal sector is countercyclical, absorbing labor during downturns. See Fiess, Fugazza, Maloney (2008) and Mondragon, Pena, Wills (2010).

are direct flows from formal to informal, while in their model there are no flows between sectors;⁴ 2) In this model there is ex-post match-specific heterogeneity, so a worker's type (individual specific characteristics relevant to the labor market such as human capital level) is not the same as match-specific productivity; workers of a given type are not certain of whether they will be 'good' or 'bad' at a specific job, so I assume initial productivity in a match is a draw from a distribution and not a fixed value (stochastic job matching); 3) Workers in both sectors have productivity shocks that may be 'good' or 'bad' with respect to the match productivity, while in their model, only workers in the formal sector are subject to shocks, and a worker's current productivity can never exceed their type; 4) To understand the impact of social assistance programs and adapt the model to the Colombian case, workers in this model contribute to social security in the formal sector and have access to subsidized health in the informal sector, features that are not present in ANV2009.

Counterfactual experiments show that changes in labor taxes may have sizable aggregate, compositional and distributional effects if workers associate high payroll taxes with more valuable and efficient social security services. The higher the valuation of the SS services, the more progressive these labor market policies become. An expansion of subsidized health to informal sector workers has minor aggregate and distributional effects. Changes in relative prices that affect the profitability of the formal sector negatively have quite sizable aggregate effects, producing more long run unemployment and informality, and increasing unemployment duration.

This essay paper proceeds as follows. Section 2.2 introduces some stylized facts about the Colombian labor markets and a brief summary of the policy reforms.

⁴I estimate that, among the informal-sector workers who switch jobs in a 12-month-period (June 2002 to June 2003), 59.1 percent flow from the formal-sector.

Sections 2.3 details the theoretical model. Section 2.4 describes the data and details the estimation procedure and results. Section 2.5 concludes.

2.2 STYLIZED FACTS

Some stylized facts about the Colombian urban labor markets in the last two decades include: increasing long-run unemployment and unemployment duration, a decreasing but persistently high informality rate, a modest rise in relative earnings favoring informal sector workers, and a decrease in overall wage inequality⁵ driven by a reduction in within-sector inequality.

Figure 2.1 presents the evolution of the unemployment rate from the first quarter of 1984 to the third quarter of 2013. At the beginning of 1984, approximately 13.7 percent of the labor force was unemployed. A period of steadily declining unemployment started. In the fourth quarter of 1993 the unemployment rate had dropped to 7.8 percent. A long period of rising unemployment followed, reaching a peak of 20.5 percent in the third quarter of 2000. The decline of unemployment in the following quarters was slow, reaching a rate of 10.4 percent in the second quarter of 2013.

Notably, short-term unemployment did not fall near its average levels: 11.8 percent over the period 1984-1999 versus 13.9 percent over the period 2000-2013. Mean unemployment duration also increased from 31 weeks to 35 weeks, well above pre-recession norms (see Figure 2.2).

Figure 2.3 shows the evolution of the employment rate. The long run employment

⁵Labor market statistics are constructed using data from Colombian household surveys. These surveys are repeated cross-sections carried out by the Colombian Statistics Department (DANE) on employed and unemployed individuals, and include: Encuesta Nacional de Hogares (ENH) for the period 1984:Q1 to 2000:Q2, Encuesta Continua de Hogares (ECH) for the period 2000:M1 to 2006:M12, and Gran Encuesta Integrada de Hogares (GEIH) for the period 2007:M1-2013:M6. Informality statistics are constructed using the Informality module in the surveys, available every two years before 1996, and yearly afterwards. The statistics presented in this section are for all urban workers.

rate followed an upward trend, increasing from 52 percent in the period 1984-1999 to 55.8 percent in the period 2000-2013.

Informality has been a persistent phenomenon over the last two decades: regardless of the definition of informality used,⁶ approximately one of every two workers are considered informal. Figure 2.4 shows the evolution of the informality rate based on alternative criteria. The average informality rate based on the health criterion diminished slightly from 49.7 percent in 1984-1999 to 48.7 percent in 2000-2013; informality based on pension decreased from 60.2 percent to 57.5 percent in 2000-2013; informality based on health and pension decreased from 60.9 percent to 58.7 percent.

When looking at the sectoral log-wage distributions, two facts are worth emphasizing.

First, there are substantial differences in mean relative earnings, favoring formal-

⁶The following definitions will be used in the paper:

“Social Protection” (SP) Informality Definition: A worker is considered informal if either of the following two conditions hold:

- *Health Affiliation:* Is not affiliated to a health plan, or if affiliated does not make any contributions to the system (either because is part of subsidized regime or beneficiary of the contributory regime)
- *Pension Affiliation:* Not affiliated to a pension fund

This variable is primarily a proxy for non-compliance to labor regulations in Colombia.

“Firm Size and Occupation” (FSO) Informality Definition: A worker is considered informal if the following two criteria hold:

- *Firm Size:* Works in firms with five or fewer employees
- *Occupation:* Works as domestic employee, self-employed, employer or unpaid family worker.

This definition is the one used by the Colombian Statistics Department, DANE, and is consistent with the one used by the ILO, but it does not include any criteria related to non-compliance with regulations.

sector workers. This is evidenced by the size of the log wage premium, which varies between 0.71 and 1.43 over the whole period 1984-2013 (see Figure 2.5). The formal-informal sector mean log wage gap, although diminishing, is still persistently high: the gap decreased only by 0.04 log points in the period under consideration.⁷

Second, overall wage inequality improved even though there were no significant improvements in between-sector inequality. Figure 2.6 shows the variance of the earning distribution in the overall economy. This change was mostly driven by lower wage dispersion within each sector. While between-sector variance decreased only by 9.4 percent, within-sector variance decreased by 32.3 percent, leading to an overall fall in log-wage variance of 29.3 percent. Figure 2.7 shows this variance broken down into the between and within-sector components.

Finally, mean real wages rose in both sectors, suggesting an improvement in living standards. Mean hourly wages increased by 0.15 and 0.13 log-wage points in the informal and formal sector respectively (See Figure 2.8).

In the period under consideration, a sequence of market-oriented structural reforms was implemented: the labor reforms of 1990, 2002 and 2012, the social security reforms of 1993, 2003 and 2007, and the trade liberalization reform that started in 1991.

Prior to the labor reforms, employment protection in Colombia was promoted by labor regulations that imposed high severance payments, early retirement and restrictions on temporary work, affecting labor markets' flexibility, with potential adverse effects on employment.

In this context, the recent labor and social security reforms were intended to con-

⁷Here I use the social security definition of informality, constructed using only health contributions since there is no information for pensions before 1996. The measure of earnings in the survey not only include monetary wages for workers (including tips, commissions) but also remuneration for self-employed, so there is some measurement error in wages.

tribute to the flexibility and efficiency of the labor markets, while still maintaining some level of worker protection.

The labor reform of 1990 was primarily aimed at stimulating job creation by instituting a more flexible system of hiring and layoffs. This was achieved by allowing short-term contracts, promoting a more flexible wage regime, and more importantly, decreasing severance payments. The reform required that formal sector firms make an annual contribution to a private severance fund (including interest payments) instead of paying severance at the time employment terminated.⁸

The social security reform of 1993, implemented in 1994, introduced major changes to the health and pension systems, monopolized by the government until then. The reform increased pension and health contributions and thereby non-wage labor (hereafter NWL) costs with the main goal of expanding social security coverage. Aiming to reach universal health coverage, the reform also created two coexisting health regimes: a contributive regime (CR) and a subsidized regime (SR).

In the CR, employers must provide health insurance by law, regardless of occupation,⁹ and the cost is shared between employers and employees. In the SR, ‘poor’ individuals who meet certain poverty criteria have access to subsidized health, where ‘poor’ is determined by a poverty index score based on the Census to the Poor (SISBEN).¹⁰ This system is financed with transfers from the contributive regime.

The labor reform of 2002 created a system of social protection, aimed to protect unemployed workers and to promote employment in recessionary periods but

⁸However, the law applies only to workers hired after 1991, which means that the reduction in severance payments depends on the turnover rate, and the number of workers who voluntarily switch to the new regime. Hence the reduction in severance payments was gradual rather than immediate.

⁹According to the law, self-employees must also contribute to the system.

¹⁰Individuals may qualify for fully subsidized health if they meet the following criteria: being part of a SISBEN level 1 or 2 household, not being affiliated to the CR, not having an employment relationship, not being a retiree and not being a beneficiary of the CR.

had limited impact on job creation, since most of the measures applied to a very limited group of workers and NWL costs were unaffected. The pension reform of 2003 increased the age requirements for retirement, the length of service required for pension, and the pension contribution rate (gradually until 2008), thus increasing employer and employee NWL costs. The health reform of 2007 increased employer health contributions, raising employer NWL costs.

After three decades of rising NWL costs,¹¹ the more recent labor reform of 2012 reduced employer NWL costs substantially by 8.96 percent through the elimination¹² of para-statal contributions,¹³ with the main goal of stimulating employment.

Changes in social security contributions caused sizable changes in employer and employee NWL costs, potentially affecting job creation and job destruction in the formal and the informal sectors.

Figures 2.9 and 2.10 show the evolution of employer and employee NWL costs in the period 1984-2013.¹⁴ In the period 1985-1990, employer NWL costs were 47.08 percent of the wage. The reduction in severance payments introduced by the reform of 1990 caused a reduction in the average NWL cost paid by the employer from 1991 to 1993, distributing this payment linearly over time. The implementation of this change has been gradual, as discussed before. As a result, NWL costs were reduced to 45.98 percent in 1993. In the period 1994-1996, these costs began to rise to reach a level of 53.41 percent in 1996, since the increasing pension and health contributions implemented by the social security reform of 1993 more than compensate for the gradual reduction of severance payments. In the period 1996-2003, they remained

¹¹The reduction in severance payments introduced by the Reform of 1990 is the exception.

¹²This reduction only applies to workers earning less than 10 times the minimum wage.

¹³In addition to social security, paid vacation and mandatory bonuses, employers must pay taxes to finance social programs. These para-statal contributions include training programs, family allowances and in-kind transfers for low income households.

¹⁴For a detailed disaggregation of these costs see Tables 2.12 and 2.13.

constant. Finally, in the period 2004-2012, these costs continued to climb primarily because of increasing social security contributions, until they reached a peak of 55.78 percent in 2012. The reform of 2012 counteracted the trend by reducing them significantly to 50.78 percent.¹⁵

The employee NWL costs were constant at 4.5 percent during the period 1990-1993, since they were not affected by the labor reform of 1990. From 1994 to 1996 they started to increase, driven primarily by increasing pension contributions introduced by the social security reform. During the period 1996-2003 they were stable, and after 2004 they began to climb until reaching a level of 9 percent in 2013.

Colombia also started a dramatic process of trade liberalization and financial openness in 1991.¹⁶ This may have caused important changes in the real exchange rate, defined as the price of tradables relative to non-tradables. Since the informal sector is mainly comprised of non-tradables,¹⁷ the real exchange rate is a key relative price in this small open economy, and can be quite relevant when it comes to explaining movement of labor across sectors. A sharp increase is observed before 2003, followed by a real appreciation afterwards (see Figure 2.11).

In recent years, investment in social assistance programs (cash transfers, in-kind transfers and subsidized health) has been growing in Colombia. In particular, since 1993, there has been an expansion of subsidized health (Subsidized Regime) to workers who are not part of the contributive regime (informal under the SP defini-

¹⁵These payroll taxes are high when compared to the US, where contributions range between 15 and 20 percent of the gross wage, and similar to the levels observed in some EU countries (i.e. Sweden, Belgium, France), where contributions are close to 50 percent.

¹⁶Interest rate ceilings, exchange rate controls and restrictions on foreign direct investment were eliminated, as well as requirements to invest in government securities. International trade was largely liberalized, due to significant reductions in import tariffs.

¹⁷According to a recent Informality survey by Batini, Kim, Levine and Lotti (2010), 87 percent of self-employed and informal workers in Colombia are concentrated in non-tradables (i.e. commerce and services).

tion). The government is expanding access to health with the main goal of achieving universal healthcare in the near future. Figure 2.12 shows the growing percentage of informal-sector workers who are affiliated to the subsidized regime. While in the second quarter of 2001 approximately 19.42 percent of informal-sector workers were affiliated to the SR, in the second quarter of 2013 this was 47.65 percent.¹⁸

Higher payroll taxes, changes in relative prices affecting the relative profitability of the two sectors, and expanding social assistance programs to informal workers¹⁹ may explain some of the stylized facts observed in the labor markets in recent years.

2.3 MODEL

2.3.1 WORKERS

In this economy agents live forever, discount the future at a constant rate r , and live in a stationary environment where there are no dynamic changes to the structural parameters. The labor force, L , is assumed to be constant and normalized to unity.

In equilibrium, each agent can be in one of three states: unemployed, employed in the formal sector, or employed in the informal sector. I want to allow flows from and to each possible state (except from informal to formal-employment), so I have a total of five transitions in the model.

There is worker heterogeneity ex-ante and ex-post. Ex-ante, workers differ in individual-specific characteristics relevant to the labor market, such as their human capital level, which are called worker types. Ex-post, workers differ in their labor

¹⁸Author's calculations based on Colombian Household Survey. No data are available before 2001.

¹⁹An inflexible wage structure in the formal sector may also be a factor since previous literature (Maloney, Nunez, 2001) suggests that minimum wages in Colombia are high and binding. Here I don't consider any source of wage rigidity.

market status and their initial productivity at the job (ex-post match-specific heterogeneity), as well as their future productivity since they are also subject to idiosyncratic productivity shocks.

Let y be the worker's type, where y is an iid draw from a distribution $F(y)$ with support on the range $[0 \leq y \leq \infty]$. Let y' be the initial productivity of the job (match-specific), where y' is a draw from a distribution $H_i(y' | y)$, for $i = F, I$.²⁰

Let u be the measure of unemployed workers, v_i be the measure of vacancies in sector i , and n_i be the measure of workers in sector i .²¹ Let θ and ϑ be the parameters that measure labor market tightness in the formal and informal sectors respectively, where²²

$$\theta = \frac{v_F}{u}; \vartheta = \frac{v_I}{u + n_F}$$

Let $U(y)$ and $N_i(y', y)$ be the present-discounted value of the expected income stream of an unemployed worker of type y and of an employed worker of type y with match-specific productivity y' in sector i , respectively.

While unemployed, workers enjoy returns b ²³ and receive offers from both sectors, regardless of their type. Let α be an exogenous Poisson rate at which informal sector

²⁰First-order stochastic dominance is assumed, i.e. if $y_1 > y_2$, then $H_i(y' | y_1) < H_i(y' | y_2)$. In the calibration, these distributions are assumed to be log-normal with a scale parameter varying linearly with $\log(y)$, so the first-order stochastic dominance assumption is satisfied.

²¹Notice that v_i is not the total measure of vacancies in the economy, but only the vacancies in sector i . Also, since the labor force is normalized to 1, u is also equal to the aggregate unemployment rate.

²²The tightness measures the number of vacancies relative to potential job seekers in each sector. While job seekers in the formal sector are only unemployed workers, job seekers in the informal are unemployed workers and formal sector workers affected by a 'bad' shock that causes them to look into informal sector opportunities.

²³This is also usually interpreted as an unemployment insurance benefit but there was no such insurance in Colombia before 2003, so b is interpreted as the opportunity cost of leisure. Unemployed workers in Colombia can also receive subsidized health, but there is no data available to measure the size of the subsidy, or the fraction of unemployed who receive it.

offers arrive to the unemployed,²⁴ and $m(\theta)$ be the endogenous rate at which formal sector offers arrive.²⁵ Once contact is made between a worker of type y and a potential employer in the formal sector, a productivity for the prospective match, y' , is drawn.

The details of the job creation process are as follows. Because of the existence of a productivity distribution for new matches, not all meetings create a match. Let $R_{UF}(y)$ and $R_{UI}(y)$ be the minimum productivities in the formal and informal sectors below which neither the firm nor the type- y worker want to start a match (endogenous reservation productivities). If the realization of the productivity draw for a worker of type y is sufficiently ‘high’,²⁶ the worker and the firm in sector i decide to match, and the worker gets a capital gain of $N_i(y', y) - U(y)$; otherwise, the worker returns to the pool of unemployed, and the job remains vacant.²⁷

The flow value of unemployment for a worker of type y is:

$$rU(y) = b + \alpha Emax[N_I(y', y) - U(y), 0] + m(\theta) Emax[N_F(y', y) - U(y), 0]$$

Given the assumptions on the match-specific productivity this gives:

$$rU(y) = b + \alpha \int_{R_{UI}(y)}^{\infty} [N_I(y', y) - U(y)] dH_I(y' | y) + m(\theta) \int_{R_{UF}(y)}^{\infty} [N_F(y', y) - U(y)] dH_F(y' | y) \quad (2.1)$$

While employed in sector i , a worker of type y and current productivity y' enjoys flow utility $u_i(y', y)$. Then, the match that started at productivity y' may continue

²⁴I am assuming no congestion effects in the informal sector (α is not a function of ϑ), so the measure of job seekers does not make it harder for an individual to find an informal sector opportunity. It may be the case that while job seekers are eager to find a formal sector job, they are not eager to find an informal sector job.

²⁵The matching function has standard properties, so $m(\theta)$ is increasing and concave in θ . In the calibration I assume a Cobb Douglas matching function given by $m(\theta) = A\theta^{1-\alpha_m}$.

²⁶Sufficiently ‘high’ means $y' \geq R_{UF}(y)$ for the formal sector and $y' \geq R_{UI}(y)$ for the informal sector.

²⁷Another way of modeling this choice is by assuming that workers choose whether or not to accept jobs based on a reservation wage. This is analogous to the reservation productivity concept.

or be destroyed. The job destruction rate is endogenized by introducing idiosyncratic shocks to job productivities:²⁸ productivity shocks arrive to jobs in sector i at Poisson rate λ , changing the productivity to a new level x . These new productivities are *iid* draws from the conditional distribution $H_i(x | y)$.²⁹

The same mechanism that governs the job creation process applies to job destruction: a match ends when it is in the mutual interest of the worker and the firm to do so, i.e., when a sufficiently bad draw of x is realized.³⁰ The threshold productivities for match dissolution in the informal and the formal sector are $R_{IU}(y)$ and $R_{FU}(y)$, respectively.

I introduce a feature in the model that captures the view of the informal sector as a ‘disadvantaged’ sector of a segmented labor market that expands during downturns to absorb displaced workers from the formal sector. When affected by a ‘bad’ shock, the formal-sector worker has to give up his job before learning his productivity on a potential informal-sector opportunity already available to him. Then, when the productivity is realized, he may choose whether to take the prospective informal-sector job or to become unemployed. On the other hand, the informal-sector worker does not have the option to move directly to the formal sector; he must become unemployed. Also, while the formal-sector worker must make contributions to the social security system, the informal-sector worker receives some subsidized health without incurring

²⁸There are two reasons why the productivity of a job may fall below the reservation value: idiosyncratic or aggregate shocks. Previous evidence for Colombia using plant-level data from the Annual Manufacturing survey estimates that the actual impact of reforms through factor adjustment on aggregate productivity was modest (Eslava, Haltiwanger, Kugler and Kugler, 2010), so introducing idiosyncratic shocks instead of aggregate shocks in the model seems more reasonable.

²⁹Notice that I assume that productivity shocks affect both sectors symmetrically: workers receive shocks at the same rate, regardless of the sector in which they are in. Also, I assume that the idiosyncratic productivity that is drawn after the shock arrives is independent of the initial productivity y' and is irreversible (the firm must produce at the new productivity or shut down), where $x \in [0, \infty]$.

³⁰This means $x < R_{IU}(y)$ for the informal sector, and $x < R_{FU}(y)$ for the formal sector.

any cost.

The flow value of taking a formal-sector job for a worker of type y and current productivity y' (new hire)³¹ is:

$$rN_F(y', y) = u_F(y', y) + \lambda H_F(R_{FU}(y) | y) \text{Emax}[N_I(x, y) - N_F(y', y), U(y) - N_F(y', y)] \\ + \lambda \int_{R_{FU}(y)}^{\infty} [N_F(x, y) - N_F(y', y)] dH_F(x | y) \quad (2.2)$$

where

$$\text{Emax}[\cdot] = \int_0^{R_{FI}(y)} [U(y) - N_F(y', y)] dH_I(x | y) + \int_{R_{FI}(y)}^{\infty} [N_I(x, y) - N_F(y', y)] dH_I(x | y)$$

When affected by a ‘good’ shock, the formal-sector worker stays in his job and gets the capital gain $N_F(x, y) - N_F(y', y)$. When affected by a ‘bad’ shock, he may decide to transition to the informal-sector to get a capital gain of $N_I(x, y) - N_F(y', y)$, or to become unemployed and get $U(y) - N_F(y', y)$.

I assume that $u_F(y', y)$ depends on effective current labor income, net after paying SS contributions, and is adjusted by a subjective valuation of the total (employer and employee) contributions to the system.³² It can be expressed as:

$$u_F(y', y) = w_F(y', y)[(1 - \delta_2) + \tau(\delta_1 + \delta_2)]$$

where $w_F(y', y)$ is the hourly wage in the formal sector, δ_2 and δ_1 are the employee and employer NWL costs as a percentage of the wage,³³ τ is a parameter that measures worker’s valuation of total social security contributions (including employer and employee contributions), $0 \leq \tau \leq 1$.³⁴

³¹This is the flow value for a worker not affected yet by a shock.

³²I assume the utility function is linear in income and total SS benefits constitute a linear function of the total (employer and employee) contributions.

³³These costs consist of social security contributions (health and pension) and non-SS contributions (para-statal contributions and others). See Tables 2.12 and 2.13 for details.

³⁴The valuation of these contributions reflects the value of these contributions and the efficiency of the services provided.

Let $\hat{\delta}_2$ be the non-wage labor costs as a percentage of the wage adjusted by the worker's valuation of the benefits that he/she receives as a result of the total contributions to the social security system,³⁵ so:

$$\hat{\delta}_2 = \delta_2(1 - \tau) - \tau\delta_1$$

The instantaneous utility can be rewritten more compactly as:

$$u_F(y', y) = w_F(y', y)(1 - \hat{\delta}_2) \quad (2.3)$$

An incumbent worker in the formal sector has a different value function than a new hire since, as will be explained later, the wage (and therefore utility) determined in a Nash bargaining negotiation is different. When the firm negotiates with an old hire it must pay severance if they mutually decide to discontinue the match. Therefore, severance weakens the firm's bargaining position.

Let $w_F^s(x, y)$ and $u_F^s(x, y)$ be the wage and utility of an incumbent worker of type y and current productivity x , respectively.³⁶ The flow value of continuing in a formal-sector job for this worker is:

$$\begin{aligned} rN_F(x, y) &= u_F^s(x, y) + \lambda H_F(R_{FU}(y) | y) \text{E}max[N_I(x', y) - N_F(x, y), U(y) - N_F(x, y)] \\ &\quad + \lambda \int_{R_{FU}(y)}^{\infty} [N_F(x', y) - N_F(x, y)] dH_F(x' | y) \end{aligned} \quad (2.4)$$

where x' is a another draw from the distribution $H_F(\cdot | y)$ and

$$u_F^s(x, y) = w_F^s(x, y)(1 - \hat{\delta}_2) \quad (2.5)$$

³⁵Notice that $\hat{\delta}_2 < 0$ if $\tau(\delta_1 + \delta_2) > \delta_2$. Therefore, if the value of the SS services received is higher than the cost of contributing to the system, $\hat{\delta}_2$ works as a transfer and not as a tax.

³⁶The superscripts indicate that the firm would have to make a severance payment if its match with this worker ended.

The flow value of taking an informal-sector job for a worker of type y and match specific productivity y' is:

$$\begin{aligned} rN_I(y', y) &= u_I(y', y) + \lambda H_I(R_{IU}(y) | y)[U(y) - N_I(y', y)] \\ &\quad + \lambda \int_{R_{IU}(y)}^{\infty} [N_I(x, y) - N_I(y', y)] dH_I(x | y) \end{aligned} \quad (2.6)$$

The flow utility for a worker of type y and current productivity y' in the informal sector can be expressed as:

$$u_I(y', y) = w_I(y', y)[1 + \hat{\delta}_3], \text{ with} \quad (2.7)$$

$$\hat{\delta}_3 = \mu \delta_3$$

where $w_I(y', y)$ is the hourly wage³⁷ in the informal sector for a worker of type y and match-specific productivity y' , δ_3 is the amount of social assistance (subsidized health) that workers receive from the government as percentage of their wage in the informal sector,³⁸ and μ is a parameter that measures the worker's valuation of the social assistance benefits received, where $0 \leq \mu \leq 1$.³⁹

So, informal workers are a "vulnerable" population in the sense that, even if they may have access to partial insurance against health shocks due to a government

³⁷Since there are no severance payments in the informal sector, there is no distinction between wages for a new hire versus an incumbent worker.

³⁸I assume the benefit from social assistance is proportional to the informal-sector wage. For the case of Colombia, δ_3 corresponds to allocated health expenditures in the Subsidized Regime program as percentage of nominal wage. However, not all workers receive the subsidy: only those workers whose families are categorized as "poor" as determined by a Poverty Index Score, using the Census of the Poor (SISBEN).

³⁹I am assuming that employee valuations of these services may be lower, equal or higher than in the SS system ($\mu < \tau$, $\mu = \tau$, $\mu > \tau$), depending on the perception of the efficiency of the services provided by the public sector. In the case where $\mu = 0$, informal workers do not value the services offered by the government. In this case, any change in subsidized health expenditures, δ_3 , will not change the flow income in the informal sector. If $\mu > 0$, an expansion in subsidized health expenditures will cause a higher flow income in the informal sector with a consequent behavioral impact.

subsidized health program, they do not have any insurance against unemployment shocks (i.e., severance payments) or aging (i.e., mandatory pension or retirement accounts).

2.3.2 FIRMS

The small economy produces two composite goods: tradables and non-tradables. There are two productive sectors in this economy: formal and informal. The formal and informal sectors are assumed to produce tradables and non-tradables, respectively.

Each sector has a continuum of small firms in the unit interval, which are identical in all respects within each sector. Each firm has one job and maximizes the present discounted value of profits and chooses whether to open a job vacancy and hire a worker or not, so the number of jobs/firms is endogenous. Since the profit maximization condition requires that the marginal value of a vacancy must be zero, this is exactly equivalent to a zero-profit condition for firm entry.

Firms can only adjust to meet demand through changes in the extensive margin (number of jobs offered/employed people), but not through the intensive margin.

The main differences between firms across sectors is that informal-sector firms are not affected by labor market regulations such as social security contributions and severance payments. Distributions of productivity and wages are also different.

Let V_i be the present-discounted value of expected profit from a vacant job in sector i , and $J_i(y', y)$ be the present-discounted value of expected profit from a filled job in sector i with a worker of type y and match-specific productivity y' .

Firms in sector i open vacancies and search among the pool of job seekers, which

involves a hiring cost c , assumed to be constant.⁴⁰ They also face some uncertainty when meeting a job seeker, since they don't know with certainty the type of worker they will meet, and conditional on type, how productive that worker will be at the job.

The flow value of having a vacancy in the formal sector is as follows:

$$rV_F = -c + \frac{m(\theta)}{\theta} Emax[J_F(y', y) - V_F, 0]$$

Formal sector vacancies meet searching workers (only unemployed workers) at the rate $\frac{m(\theta)}{\theta}$. If the job is filled, the firm gets the expected capital gains, $J_F(y', y) - V_F$.

Given the assumption on the match-specific productivity this is equivalent to:⁴¹

$$rV_F = -c + \frac{m(\theta)}{\theta} \int_0^\infty \int_{R_{UF}(y)}^\infty [J_F(y', y) - V_F] dH_F(y' | y) dF_U^*(y) \quad (2.8)$$

where $F_U^*(y)$ is the distribution of y among the unemployed. Using Bayes rule:

$$dF_U^*(y) = \frac{u(y)f(y)}{\int_0^\infty u(y)f(y)dy} dy,$$

where $u(y)$ is the unemployment rate conditional on y .

On the other hand, informal-sector firms meet not only unemployed but also formal-sector workers affected by a 'bad' shock. The rate at which informal-sector vacancies meet unemployed workers is $\frac{\alpha}{\vartheta}$, while the rate at which informal-sector vacancies meet formal-sector workers of type y affected by a 'bad' shock is $\frac{\lambda H_F(R_{FU}(y)|y)}{\vartheta}$.⁴²

The flow value of having a vacancy in the informal sector is:

$$rV_I = -c + \frac{\alpha}{\vartheta} Emax[J_I(y', y) - V_I, 0] + \frac{\lambda H_F(R_{FU}(y)|y)}{\vartheta} Emax[J_I(x, y) - V_I, 0]$$

⁴⁰The hiring cost can also depend on productivity or wages over the business cycle, but it is reasonable to assume is constant in the steady state.

⁴¹Notice also that the value of the vacancy does not depend on y .

⁴²Informal-sector vacancies meet formal-sector workers at rate $\frac{\lambda H_F(R_{FU}(y)|y)}{\vartheta}$ because every time a formal-sector worker experiences a productivity shock, he meets an informal-sector opportunity only after experiencing a 'bad' shock.

Given the assumption on the productivity shock, this is equivalent to:

$$\begin{aligned}
rV_I &= -c + \frac{\alpha}{\vartheta} \left[\frac{u}{u + n_F} \right] \int_0^\infty \int_{R_{UI}(y)}^\infty [J_I(y', y) - V_I] dH_I(y' | y) dF_U^*(y) \\
&\quad + \frac{\lambda}{\vartheta} \left[\frac{n_F}{u + n_F} \right] \int_0^\infty \int_{R_{FI}(y)}^\infty H_F(R_{FU}(y) | y) [J_I(x, y) - V_I] dH_I(x | y) dF_F^*(y)
\end{aligned} \tag{2.9}$$

where $F_F^*(y)$ is the distribution of y among the job seekers in the formal sector that can be expressed as:

$$dF_F^*(y) = \frac{n_F(y)f(y)}{\int_0^\infty n_F(y)f(y)dy} dy.$$

A formal-sector firm matched with a worker of type y and match-specific productivity y' receives some net return for a job, $\pi_F(y', y)$, given by the market value of output minus the net cost of labor (after paying SS contributions). A positive or negative productivity shock arrives at rate λ , and two possible cases arise: if the shock is ‘good’, the match continues with the capital gain (or loss) $J_F(x, y) - J_F(y', y)$; if the shock is ‘bad’, the match ends, the firm pays firing costs, s , and posts a new vacancy, so the capital loss suffered is $V_F(y) - J_F(y', y) - s$.

The flow value of a filled job in the formal sector with a worker of type y and match-specific productivity y' (new hire) is:

$$\begin{aligned}
rJ_F(y', y) &= \pi_F(y', y) + \lambda H_F(R_{FU}(y) | y) [V_F - J_F(y', y) - s] \\
&\quad + \lambda \int_{R_{FU}(y)}^\infty [J_F(x, y) - J_F(y', y)] dH_F(x | y)
\end{aligned} \tag{2.10}$$

where $\pi_F(y', y)$ is the nominal value of a job’s output in the formal sector, which can be defined as:

$$\pi_F(y', y) = p_F y' - w_F(y', y)(1 + \delta_1) \tag{2.11}$$

and p_F is the price of a formal-sector good (price of tradable good).

The flow value of a filled job in the formal sector with a worker of type y and

productivity x (incumbent) is:

$$\begin{aligned} rJ_F(x, y) &= \pi_F^s(x, y) + \lambda H_F(R_{FU}(y) | y)[V_F - J_F(x, y) - s] \\ &\quad + \lambda \int_{R_{FU}(y)}^{\infty} [J_F(x', y) - J_F(x, y)] dH_F(x' | y) \end{aligned} \quad (2.12)$$

where

$$\pi_F^s(x, y) = p_F x - w_F^s(x, y)(1 + \delta_1) \quad (2.13)$$

Notice that $rJ_F(x, y)$ differs from $rJ_F(y', y)$ because the wage associated with each function is different.⁴³

The flow value of a filled job in the informal sector with a worker of type y and current productivity y' is as follows:

$$\begin{aligned} rJ_I(y', y) &= \pi_I(y', y) + \lambda H_I(R_{IU}(y) | y)[V_I - J_I(y', y)] \\ &\quad + \lambda \int_{R_{IU}(y)}^{\infty} [J_I(x, y) - J_I(y', y)] dH_I(x | y) \end{aligned} \quad (2.14)$$

where the nominal value of a job's output in the informal sector, $\pi_I(y', y)$ is defined as:

$$\pi_I(y', y) = p_I y' - w_I(y', y) \quad (2.15)$$

and p_I is the price of the informal sector good (non-tradable good).

The informal-sector job yields net return for the firm $\pi_I(y', y)$. The match may break (without any firing cost involved) if a 'bad' productivity shock arrives (x below some productivity threshold $R_{IU}(y)$), and the firm posts a new vacancy, so the capital loss suffered is $[V_I - J_I(y', y)]$. Otherwise, the match continues and the firm gets the corresponding capital gain or loss, $J_I(x, y) - J_I(y', y)$.

⁴³I will later show that $w_F^s(x, y) \geq w_F(x, y)$.

2.3.3 WAGE DETERMINATION

FORMAL SECTOR

Formal-sector wages in the steady state are determined by workers and firms using Nash bargaining, given the exogenous worker bargaining parameter, β .

An unemployed worker of type y with match-specific productivity y' and a formal sector firm decide to form a match if it is worth it for both, that is if the joint surplus is positive, or equivalently, if $R_{UF}(y) \leq y' \leq \infty$. If they match, they decide how to split the surplus and negotiate a wage contract using Nash bargaining, given the exogenous worker bargaining parameter β .

The initial wage⁴⁴ is given by:

$$\max_{w_F(y', y)} [N_F(y', y) - U(y)]^\beta [J_F(y', y) - V_F]^{1-\beta}$$

The standard sharing rule using the free entry condition ($V_F = 0$) implies

$$(1 - \beta)(1 + \delta_1)[N_F(y', y) - U(y)] = \beta(1 - \hat{\delta}_2)J_F(y', y)$$

After doing some algebra, the corresponding formal-sector wage equation for a new hire of type y and current productivity y' is:

$$\begin{aligned} w_F(y', y) = & \beta \left[\frac{p_F y'}{1 + \delta_1} - \frac{\lambda s}{(1 + \delta_1)} \right] \\ & + (1 - \beta) \left[\frac{rU(y) - \lambda H_F(R_{FU}(y) | y) \int_{R_{FI}(y)}^\infty [N_I(x, y) - U(y)] dH_I(x | y)}{(1 - \hat{\delta}_2)} \right] \end{aligned} \quad (2.16)$$

The wage negotiated in the formal sector is a weighted average between the productivity of the worker (adjusted by the expected severance cost) and the worker's continuation value (adjusted by a term that captures the flows from formal to informal).

⁴⁴This is the wage for new hires in the formal sector that haven't been affected by a shock.

The expected future severance cost reduces the benefits the firm gets if it accepts the bargain with the new hire, improving its bargaining power in the negotiation, and therefore reducing the negotiated wage.

The continuation value reflects not only the value for the worker if he doesn't accept the bargain (the flow value of unemployment), but also the benefits for the worker if he accepts the job, including the possibility of later moving to the informal sector. After being employed in the formal sector, the worker may be affected by a 'bad' productivity shock, and either flow to unemployment or to the informal sector. The expected gains of these two possible states worsen the worker's bargaining position. If the worker expects to get larger gains after being affected by a shock while working in the formal sector, the continuation value is reduced and the worker is willing to accept a lower formal-sector wage.

In general, if the worker has low productivity, a low continuation value, low bargaining power, and has to pay low non-wage labor costs as a formal employee, the worker is willing to accept a lower wage in the negotiation.

Since it is assumed that the wage is renegotiated every time a productivity shock arrives, wages for employed workers of type y and current productivity x ⁴⁵ are determined by workers and firms using Nash bargaining, given the exogenous worker bargaining parameter β , as follows:

$$\max_{w_F^*(x,y)} [N_F(x, y) - U(y)]^\beta [J_F(x, y) - (V_F - s)]^{1-\beta}$$

The corresponding standard sharing rule using the free entry condition ($V_F = 0$) is:

$$(1 - \beta)(1 + \delta_1)[N_F(x, y) - U(y)] = \beta(1 - \hat{\delta}_2)[J_F(x, y) + s]$$

⁴⁵This is the wage for incumbent workers who have been affected by a shock, so $R_{FU}(y) \leq x \leq \infty$.

The formal-sector wage equation for an incumbent worker of type y and current productivity x is:

$$w_F^s(x, y) = \beta \left[\frac{p_F x}{1 + \delta_1} + \frac{rs}{(1 + \delta_1)} \right] + (1 - \beta) \left[\frac{rU(y) - \lambda H_F(R_{FU}(y) | y) \int_{R_{FI}(y)}^{\infty} [N_I(x', y) - U(y)] dH_I(x' | y)}{(1 - \hat{\delta}_2)} \right] \quad (2.17)$$

Notice that $w_F^s(x, y) \geq w_F(x, y)$ since the severance tax now worsens the firm's bargaining position and reduces the benefits the firm gets if it does not accept the bargain with the old hire, thereby reducing its bargaining strength in the negotiation and increasing the negotiated wage.

INFORMAL SECTOR

I assume there is bargaining over wages in the informal sector.⁴⁶

The wage for a worker type y solves:

$$\max_{w_I(y', y)} [N_I(y', y) - U(y)]^\beta [J_I(y', y) - V_I]^{1-\beta}$$

The F.O.C (assuming interior solution), gives us the following standard sharing rule:

$$(1 - \beta)[N_I(y', y) - U(y)] = \beta(1 + \hat{\delta}_3)[J_I(y', y) - V_I]$$

The wage equation for the informal sector is:

$$w_I(y', y) = \beta [p_I y'] + (1 - \beta) \left[\frac{rU(y)}{(1 + \hat{\delta}_3)} \right] \quad (2.18)$$

In this case, neither the productivity nor the continuation value term need to be adjusted since there is no severance tax or flows from informal to formal. However,

⁴⁶I conceive the informal sector as a 'disadvantaged' countercyclical sector in which low-skilled workers are negotiating wages with small firms, rather than as unregulated pro-cyclical self-employment. In Colombia, lack of compliance with social security is also a small firm phenomenon: approximately 87.2 percent of those informal under the SP definition are in small firms (≤ 5 employees).

high benefits in the form of better access to subsidized health (high $\hat{\delta}_3$) worsen the worker's bargaining position, which leads to a lower wage in the bargaining process.

2.3.4 OPTIMAL DECISION RULES AND RESERVATION PRODUCTIVITIES

Optimal decision rules are characterized by a reservation value property.

Reservation productivities are obtained by:

$$R_{FU}(y) : N_F(R_{FU}(y), y) = U(y) \iff J_F(R_{FU}(y), y) = V_F - s = -s$$

$$R_{UF}(y) : N_F(R_{UF}(y), y) = U(y) \iff J_F(R_{UF}(y), y) = V_F = 0$$

$$R_{IU}(y) : N_I(R_{IU}(y), y) = U(y) \iff J_I(R_{IU}(y), y) = V_I = 0$$

$$R_{UI}(y) : N_I(R_{UI}(y), y) = U(y) \iff J_I(R_{UI}(y), y) = V_I = 0$$

$$R_{FI}(y) : N_I(R_{FI}(y), y) = U(y) \iff J_I(R_{FI}(y), y) = V_I = 0$$

The Nash surplus sharing rule guarantees mutual agreement between the worker and the firm. Notice that $R_{IU}(y) = R_{UI}(y) = R_{FI}(y)$. On the other hand $R_{UF}(y) \neq R_{FU}(y)$ since when the match breaks, the formal sector firm has to pay a severance cost.

The corresponding reservation wages can be obtained through the Nash bargaining wage rule which maps productivities into wages, conditioning on type.

RESERVATION PRODUCTIVITY, $R_{FU}(y)$

A match in the formal sector is continued if and only if it is in the mutual interest of the worker and the firm to do so, so a necessary condition for match continuation is that the joint surplus must be non-negative. In equilibrium, the surplus is an increasing function of x ,⁴⁷ and by definition $R_{FU}(y)$ is the threshold productivity

⁴⁷See Appendix 6.1 for the surplus analytical expression.

above which the joint surplus is never negative. Therefore, $R_{FU}(y)$ is defined by the zero surplus condition:

$$N_F(R_{FU}(y), y) - U(y) + J_F(R_{FU}(y), y) + s = 0$$

Using the sharing rule combined with the free entry condition, I get:

$$J_F(R_{FU}(y), y) = -s$$

Substitution and integration by parts gives:

$$R_{FU}(y) = \frac{A_{FU}(y)}{(1 - \hat{\delta}_2)P_F} - \frac{[1 - \beta(1 - \hat{\delta}_2)]rs}{(1 - \beta)P_F} - \frac{\lambda}{r + \lambda} \int_{R_{FU}(y)}^{\infty} [1 - H_F(x' | y)] dx' \quad (2.19)$$

The term $A_{FU}(y)$ captures the worker's outside option adjusted by a term that captures flows from formal to informal as follows:

$$A_{FU}(y) = (1 + \delta_1) \left[rU(y) - \lambda H_F(R_{FU}(y) | y) \int_{R_{FI}(y)}^{\infty} [N_I(x', y) - U(y)] dH_I(x' | y) \right] \quad (2.20)$$

This corresponds to a modified version of the standard upward-sloping job destruction curve, in which, for a given y , higher θ implies better worker's outside opportunities (higher $U(y)$), and therefore more marginal jobs are destroyed (higher $R_{FU}(y)$). There is a secondary effect in play caused by the movement from formality to informality, since higher $R_{FU}(y)$ means higher probability of discontinuing the formal sector match, which negatively affects $A_{FU}(y)$. This equilibrium effect mitigates the impact of θ on $R_{FU}(y)$.

Given $U(y)$, the reservation productivity when transitioning from formal to unemployment is an increasing function of δ_1 and $\hat{\delta}_2$, and a decreasing function of $\hat{\delta}_3$, s and P_F , conditional on y .

The higher the firm's non-wage labor costs and the lower the price of the formal sector good or the severance cost, the higher the reservation productivity firms require

to maintain the match after a productivity shock hits. Firms become pickier about the workers they are willing to ‘retain’ or ‘continue employing’.

The lower the expected utility of the worker in the current match, either because of higher non-wage labor costs (adjusted by valuations), or lower $\hat{\delta}_3$ (receiving less benefits when moving to the IS), the higher the minimum productivity workers require to maintain a match (workers become pickier as well).

Also, $R_{FU}(y)$ is increasing in y if the adjusted outside option term $A_{FU}(y)$ dominates the integral term (‘labor hoarding effect’) in equation (19), since both terms are increasing in y .⁴⁸

RESERVATION PRODUCTIVITY, $R_{UF}(y)$

Analogously, the reservation productivity $R_{UF}(y)$ is:

$$R_{UF}(y) = \frac{A_{UF}(y)}{(1 - \hat{\delta}_2)P_F} + \frac{[1 - \beta(1 - \hat{\delta}_2)]\lambda s}{(1 - \beta)P_F} - \frac{\lambda}{r + \lambda} \int_{R_{FU}(y)}^{\infty} [1 - H_F(x' | y)] dx' \quad (2.21)$$

where

$$A_{UF}(y) = (1 + \delta_1) \left[rU(y) - \lambda H_F(R_{FU}(y) | y) \int_{R_{FI}(y)}^{\infty} [N_I(x, y) - U(y)] dH_I(x | y) \right] \quad (2.22)$$

So:

$$R_{UF}(y) = R_{FU}(y) + \frac{[1 - \beta(1 - \hat{\delta}_2)](\lambda + r)s}{(1 - \beta)P_F} \quad (2.23)$$

Notice that $R_{UF}(y) \geq R_{FU}(y)$ if the second term is weakly positive. Formal sector firms are pickier when hiring a worker than when laying him or her off because in the first case they don’t have to pay a firing cost.

⁴⁸This is the case for the particular set of parameter values chosen in the estimation

RESERVATION PRODUCTIVITY $R_{IU}(y)$

Informal sector matches are destroyed when idiosyncratic productivity $x < R_{IU}(y)$, so the reservation productivity $R_{IU}(y)$ is defined by the condition:

$$J_I(R_{IU}(y), y) = 0$$

Substitution gives:

$$R_{IU}(y) = \frac{A_{IU}(y)}{(1 + \hat{\delta}_3)P_I} - \frac{\lambda}{r + \lambda} \int_{R_{IU}(y)}^{\infty} [1 - H_I(x | y)] dx \quad (2.24)$$

where $A_{IU}(y) = rU(y)$.

Given $U(y)$, the reservation productivity when transitioning from the informal sector to unemployment is a decreasing function in $\hat{\delta}_3$ and P_I . The higher the worker's social assistance benefits (adjusted by valuations), the lower the minimum productivity workers require to maintain a match with an informal-sector firm. Firms are also less picky when profitability is high (high P_I).

2.3.5 JOB CREATION CONDITIONS

In this section, I derive the job creation conditions in the formal and the informal sectors that allow us to pin down equilibrium labor market tightness, θ and ϑ respectively.

In equilibrium, formal sector firms open vacancies until rents are exhausted, so free entry implies $V_F = 0$. The free entry condition and equation (2.8) imply:

$$\frac{c\theta}{m(\theta)} = \int_0^{\infty} \int_{R_{UF}(y)}^{\infty} J_F(y', y) dH_F(y' | y) dF_U^*(y)$$

Equilibrium θ is such that the expected cost of hiring a worker in the formal sector is equal to the expected benefit of hiring an unemployed worker.

Expressing $J_F(y', y)$ as a function of reservation productivity $R_{UF}(y)$ in the latter equation gives:

$$c = \frac{m(\theta)}{\theta} \int_0^\infty \int_{R_{UF}(y)}^\infty \frac{(1-\beta)p_F[y' - R_{UF}(y)]}{r + \lambda} dH_F(y' | y) dF_U^*(y) \quad (2.25)$$

In the informal sector, free entry and equation (2.9) gives:

$$\begin{aligned} c &= \frac{\alpha}{\vartheta} \left[\frac{u}{u + n_F} \right] \int_0^\infty \int_{R_{UI}(y)}^\infty J_I(y', y) dH_I(y' | y) dF_U^*(y) \\ &\quad + \frac{\lambda}{\vartheta} \left[\frac{n_F}{u + n_F} \right] \int_0^\infty \int_{R_{FI}(y)}^\infty H_F(R_{FU}(y) | y) J_I(x, y) dH_I(x | y) dF_F^*(y) \end{aligned} \quad (2.26)$$

Expressing the firm's value function in terms of reservation productivities:

$$\begin{aligned} c &= \frac{\alpha}{\vartheta} \left[\frac{u}{u + n_F} \right] \int_0^\infty \int_{R_{UI}(y)}^\infty \frac{(1-\beta)p_I(y' - R_{IU}(y))}{r + \lambda} dH_I(y' | y) dF_U^*(y) \\ &\quad + \frac{\lambda}{\vartheta} \left[\frac{n_F}{u + n_F} \right] \int_0^\infty \int_{R_{FI}(y)}^\infty H_F(R_{FU}(y) | y) \left[\frac{(1-\beta)p_I(x - R_{IU}(y))}{r + \lambda} \right] dH_I(x | y) dF_F^*(y) \end{aligned} \quad (2.27)$$

2.3.6 STEADY STATE CONDITIONS

In this section I derive the equilibrium rate of unemployment, given the other equilibrium objects. Let $u(y)$ be the fraction of type- y workers in unemployment, $n_I(y)$ be the fraction in informal-sector employment and $n_F(y)$ be the fraction in formal-sector employment, so that $u(y) + n_I(y) + n_F(y) = 1$. In the steady state, the mean rate of unemployment conditional on y is constant, so two steady state conditions apply.

First, the sum of the flows into unemployment must equal the sum of the flows out of unemployment:

$$\begin{aligned} &\lambda H_I(R_{IU}(y) | y) n_I(y) + \lambda H_F(R_{FU}(y)) H_I(R_{UI}(y) | y) n_F(y) = \\ &= \alpha [1 - H_I(R_{UI}(y) | y)] u(y) + m(\theta) [1 - H_F(R_{UF}(y) | y)] u(y) \end{aligned}$$

Second, the analogous condition for the formal sector:

$$\lambda H_F(R_{FU}(y))n_F(y) = m(\theta)[1 - H_F(R_{UF}(y) | y)]u(y)$$

Solving the above equations gives:

$$u(y) = \frac{\lambda H_F(R_{FU}(y) | y)H_I(R_{IU}(y) | y)}{\alpha[1 - H_I(R_{UI}(y) | y)]L(y) + m(\theta)[1 - H_F(R_{UF}(y) | y)]K(y) + \lambda H_F(R_{FU}(y))H_I(R_{IU}(y) | y)} \quad (2.28)$$

$$n_F(y) = \frac{m(\theta)[1 - H_F(R_{UF}(y) | y)]K(y)}{\alpha[1 - H_I(R_{UI}(y) | y)]L(y) + m(\theta)[1 - H_F(R_{UF}(y) | y)]K(y) + \lambda H_F(R_{FU}(y))H_I(R_{IU}(y) | y)} \quad (2.29)$$

$$n_I(y) = \frac{\alpha[1 - H_I(R_{UI}(y) | y)]L(y)}{\alpha[1 - H_I(R_{UI}(y) | y)]L(y) + m(\theta)[1 - H_F(R_{UF}(y) | y)]K(y) + \lambda H_F(R_{FU}(y))H_I(R_{IU}(y) | y)} \quad (2.30)$$

where $L(y)$ and $K(y)$ are

$$L(y) \equiv H_F(R_{FU}(y) | y)$$

$$K(y) \equiv H_F(R_{FU}(y) | y) + H_I(R_{IU}(y) | y) [1 - H_F(R_{FU}(y) | y)]$$

Equation (27) corresponds to a modified version of the Beveridge curve, a negative relation between labor market tightness and unemployment, or alternatively, between vacancies and unemployment. When θ increases, $m(\theta)$ increases as well, encouraging job creation and reducing $u(y)$ ('job creation' effect). Stochastic job matching and endogenous job destruction give us an additional counteracting effect ('reservation productivity' effect): an increase in θ also increases $rU(y)$ (more outside opportunities), which increases $R_{UF}(y)$, $R_{UI}(y)$, and $R_{FU}(y)$. The first two discourage job creation and the latter encourages job destruction, both of which increase unemployment.⁴⁹

⁴⁹I assume that the first effect dominates the second effect (since empirical evidence supports a downward-sloping Beveridge curve). Stochastic job matching means that changes

Aggregate unemployment can be obtained by aggregating across types:

$$u = \int_0^{\infty} u(y)f(y)dy$$

Intuitively, in equilibrium there will be some imperfect sorting of workers among sectors based on their types, y . ‘High’ type workers are more likely to take formal-sector jobs, while ‘low type’ workers are more likely to take informal-sector jobs, and medium type workers will take both.⁵⁰

In addition to compositional effects, changes in policy parameters affect the steady state distributions of productivities among sectors, affecting also the distributions of wages across formal and informal employment.

2.3.7 DETERMINATION OF THE VALUE OF UNEMPLOYMENT

The only thing needed to solve the model is to determine $U(y)$ for all values of y , as a function of reservation productivities and labor market tightness parameters.

Equation (2.1), together with equations (2.26) and (2.32), imply:

$$\begin{aligned} rU(y) = & b + \alpha\beta(1 + \hat{\delta}_3) \int_{R_{UI}(y)}^{\infty} \frac{p_I(y' - R_{IU}(y))}{r + \lambda} dH_I(y' | y) \\ & + m(\theta)\beta(1 - \hat{\delta}_2) \int_{R_{UF}(y)}^{\infty} \frac{p_F(y' - R_{UF}(y))}{(1 + \delta_1)(r + \lambda)} dH_F(x | y) \end{aligned} \quad (2.31)$$

in labor taxes, subsidized health or prices may shift the relationship between vacancies and unemployment, and that the effect of a change in the relevant policy parameter on u via job offers is mitigated, since there are additional effects that offset this demand-side effect.

⁵⁰This is more or less consistent with the empirical fact that the formal sector is skilled labor intensive while the informal sector is unskilled labor intensive in Colombia: average years of schooling for a FS worker is 12.14 , while for an IS worker is 9.12 years (Source: Author’s calculations based on ECH, second quarter of 2003).

To solve the model, some following functional form assumptions are made. Specifically:

$$\begin{aligned}
 F(y) &= \Phi\left(\frac{\ln y - \mu_y}{\sigma_y}\right) \text{ for } y > 0 \\
 H_i(x | y) &= \Phi\left(\frac{\ln x - \mu_i(y)}{\sigma_i}\right) \text{ for } x > 0, i = F, I \\
 H_i(y' | y) &= \Phi\left(\frac{\ln y' - \mu_i(y)}{\sigma_i}\right) \text{ for } y' > 0, i = F, I \\
 \mu_i(y) &= B_i \log(y) \\
 m(\theta) &= A\theta^{1-\alpha_m}
 \end{aligned}$$

Once the model is solved numerically,⁵¹ the empirical strategy consists of estimating the model using micro data, and then simulating how labor market policies, subsidized health programs and relative prices affect the division of the labor force into unemployment, informal and formal sectors (aggregate effects), the mix of worker types in the two sectors (compositional effects) and the distribution of wages (distributional effects).⁵²

2.4 EMPIRICAL STRATEGY

2.4.1 DESCRIPTIVES

To estimate the model, I use data from the Encuesta Continua de Hogares for the second quarter of 2003, representative for thirteen metropolitan areas.⁵³ I choose this period to approximate the pre-reform's steady state equilibrium because: 1) An

⁵¹For details on the computational algorithm to solve the model see Appendix, Section 6.5.

⁵²I simulate the steady state distributions of equilibrium productivity and wages in both sectors, instead of deriving them them analytically. See Appendix, Sections 6.2 and 6.3 for the analytical derivations.

⁵³These areas include the following cities and their metropolitan areas: Bogota, Cali, Medellin, Barranquilla, Bucaramanga, Manizales, Pasto, Pereira, Cucuta, Villavicencio, Ibagu e, Monteria and Cartagena.

informality module is available;⁵⁴ 2) Information on health access disaggregated by contributive or subsidized regime is also available;⁵⁵ 3) The first three quarters of 2003 have a roughly constant unemployment rate, and the model is derived under a steady state assumption.

In addition to standard demographic socio-economic variables (age, gender, marital status, educational attainment, etc), the sample is described by the following labor market variables:

$$(\{W_s\}_{s \in E_i}; \{t_{es}\}_{s \in E_i}; \{t_{nes}\}_{s \in E_i}; \{t_{us}\}_{s \in U}; \{h_s\}_{s \in E_i})_{i=F,I}$$

W_s : Accepted monthly wages for individual s , where $s \in E_i$, so each individual can be employed in sector i (formal or informal)

t_{es} : Employment duration⁵⁶ for individual s , where $s \in E_i$ (right -censored)

t_{nes} : Non-Employment duration⁵⁷ of previous employment for individual s , where $s \in E_i$.

t_{us} : Unemployment duration for individual s , where $s \in U$ (incomplete spells for unemployed, right -censored)

h_s : Weekly hours of work for individual s , where $s \in E_i$.

Even though the data is not longitudinal, retrospective questions about previous unemployment and employment status for both employed and unemployed individuals are available, allowing for the construction of transition flows across the different

⁵⁴An informality module is available for the second quarter of every year after 2001, and every two years before 2001.

⁵⁵Before 2001, there is no information on health access by contributive or subsidized regimes.

⁵⁶This variable measures months in the current job.

⁵⁷This variable measures months without employment between the current job and the previous job (retrospective question), so I cannot identify whether it refers to unemployment or inactivity duration. I also have unemployment duration for the unemployed population; however, this variable is right-censored.

states. There is no retrospective information on social security contributions, so to determine whether a worker was considered formal or informal before being unemployed or employed in the current job, the definition of informality based on firm size and occupation is used.⁵⁸

I choose a subsample of full-time, male workers with completed primary education, and at most 20 years of schooling.⁵⁹ I also drop observations that correspond to the lowest 10% and the highest 2% of wages in each human capital level.⁶⁰ This trimming is performed to reduce measurement error and other sources of observed heterogeneity, since according to the model, agents are heterogeneous ex-ante only because of different human capital levels.⁶¹

The size of the subsample is 20,011 observations, representing 9.7 million people.

Table 2.1 shows the descriptive statistics for the employed population by sector.

When using the SP definition, the estimated informality rate for 2003 is 60.19 percent (s.e. of .0038), or the informal sector is 1.5x the size of the formal sector.

In comparison with the formal sector, the informal sector is characterized by:

- *Lower and Less Dispersed Wages*: Log hourly wages are on average lower and less dispersed than in the formal sector. The log-wage gap in the raw data without any controls is 0.63 (s.e. 0.044)⁶²

⁵⁸In this period, approximately 85.92 percent of informal sector-workers based on the SP definition are also informal based on the FSO definition.

⁵⁹I drop workers from the bottom of the distribution because of measurement error in the wage data for these type of workers; I also drop workers from the top due to the small sample sizes.

⁶⁰The reason for the asymmetry is that I want to avoid measurement error in high and low wages, but I also want to avoid wages on temporary or other jobs that are correctly measured, but fall outside the model scope.

⁶¹Female and part-time workers are also more likely to select themselves out of the labor force.

⁶²This measure of wages includes tips and commissions, but excludes non-monetary payments.

- *More job stability*: Average job duration in the informal sector is 92.3 months (7.69 years) while in the formal sector it is 82.7 months (6.89 years)
- *Higher non-employment duration*: If unemployed/inactive in the previous year, informal sector workers face higher unemployment/inactivity duration on average than their formal sector counterparts (7.34 vs. 6.33 months)
- *Lower human capital levels*: Mean years of schooling among informal-sector workers is 9.13 vs. 12.15 among formal-sector workers.

Table 2.2 shows the variance of the earning distribution in the overall economy, broken down by two components: a) Variability within sectors; b) Variability between sectors. Despite the substantial differences in relative earnings, 97.5 percent of variance in log wages is explained by within-sector variance.⁶³

Table 2.3 shows the descriptive statistics for the unemployed population. The estimated unemployment rate for the year 2003 is quite high: 18.5 percent (s.e. of .00274). Mean unemployment duration is quite high (51.5 weeks),⁶⁴ and higher than the mean non-employment duration for the employed population (6.9 months).

2.4.2 LABOR MARKET DYNAMICS

Standard static analysis of labor markets that analyze stocks of workers in different states does not tell us anything about where those workers arrived from, how long they will stay, or where they will go next. I construct a set of statistics to analyze

⁶³It would be interesting if workers were assigned randomly to sectors, to see whether selection reduces or increases inequality, or what the aggregate wage variability would be if sectoral labor force quality were held constant (see Heckman & Sedlacek, 1985).

⁶⁴This estimate can be upward biased because: 1) Unemployment duration is only observed for the currently unemployed; 2) It is more likely to have workers with higher lengths of unemployment in a sample of unemployed workers at a particular point in time (a 'stock' sample).

labor market dynamics and derive some stylized facts about movement across sectors in the Colombian labor markets, using the retrospective questions from the cross-section database.

ANNUAL DISCRETE TRANSITION MATRIX

Individuals in the sample fall into four labor market states: unemployed (U), employed in the formal sector (F), employed in the informal sector (I), and other (O).⁶⁵

I estimate an annual discrete transition matrix by estimating gross worker flows across states.⁶⁶

Let m_{ij} be the number of individuals in the 2003 sample who were in state i in 2002 and state j in 2003. I observe some, but not all the transitions between states from 2012 to 2013.⁶⁷

When considering transitions into unemployment, I observe all transitions m_{UU}, m_{FU}, m_{IU} and m_{OU} . When considering transitions into the formal sector, I observe m_{FF}, m_{IF} and $m_{UF} + m_{OF}$. When considering transitions into the informal sector, I observe m_{FI}, m_{II} and $m_{UI} + m_{OI}$. I also observed the following transitions into the ‘other’ state: m_{OO} and m_{UO} .

To recover m_{UI}, m_{OI}, m_{UF} and m_{OF} the following assumptions are made:

$$\frac{m_{OF}}{m_{UF} + m_{OF}} = \frac{m_{OI}}{m_{UI} + m_{OI}}$$

⁶⁵Other includes the inactive population.

⁶⁶Another approach may be to think about an underlying continuous-time Markov process that generates the discrete time mobility process, and therefore, estimate the transition probabilities accordingly. See Maloney and Bosch, 2007.

⁶⁷In the survey, I have the following questions for employed: 1) How long (months) have you been employed continuously in the current job?; 2) How long (months) were you without employment between your current job and the previous job? With these questions I can identify job-to-job transitions, and non-employment (unemployment or inactive) to job transitions.

$$m_{FU} + m_{IU} + m_{OU} = m_{UF} + m_{UI} + m_{UO}$$

The transitions from the ‘other’ employment state as a fraction of the total transitions from non-employment to employment are the same across sectors, and the sum of the flows into unemployment are equivalent to the sum of the flows out of unemployment (steady state assumption).

Table 2.4 contains the estimated transitions m_{ij} , for all i and j .

Once all transitions are recovered, the probability of transitioning from state i in 2012 to state j in 2013 can be estimated using:

$$\pi_{ij} = \frac{\hat{m}_{ij}}{\hat{n}_i},$$

where \hat{n}_i is the estimated number of individuals in the 2003 sample who were in state i in 2002.⁶⁸

Let N_{2003} be the number of observations in the 2003 sample and p_i the fraction of the 2002 sample in state i . Then:

$$\hat{n}_i = p_i N_{2003}$$

The estimated transition probabilities can be used to estimate the hazard (separation probabilities) out of sector i , conditional on being in state i up to period t , $h_{ii}(t)$, as follows:

$$\widehat{h_{ii}(t)} = 1 - \widehat{\pi_{ii}(t)}$$

Table 2.5 shows the estimated annual transition probabilities. The probability that an individual who was unemployed in the second quarter of 2002 remains unemployed in 2003 is 72 percent. Ignoring time aggregation bias,⁶⁹ this statistic suggests a hazard

⁶⁸In longitudinal data I observe n_i , but that is not the case in cross-section data since I have a different sample of individuals.

⁶⁹Time aggregation bias may arise because the yearly measurement may combine multiple transitions into a single \hat{O} aggregate \hat{O} transition, so transitions that occur at a higher frequency are not captured here.

rate out of unemployment conditional on being unemployed of 0.28.

Mobility between sectors is dominated by mobility from the formal to the informal sector. When conditioning on working in the formal sector, 81 percent of formal sector workers stay formal over the course of a year. Even if persistence is very high, 6 percent transit to informal sector jobs and only 3 percent to unemployment. These figures imply a hazard out of the formal sector of 0.19.

Transitions from informality to formality are less important, quantitatively. Only approximately 4 percent of informal workers transit to formal sector jobs. Given that persistence is very high (93 percent of informal sector workers stay informal over the course of a year), the estimated hazard out of the informal sector is 0.07, lower than in the formal sector.

Table 2.6 shows the estimated hazards.

This empirical evidence supports the assumption in the model of direct transitions from formal to informal, without intervening unemployment. Transitions from informal to formal are less important.

It is worth noticing that these probabilities are not instantaneous transition probabilities, since there are many workers who change status within a year whose transitions are not captured here.

MEAN EMPLOYMENT AND UNEMPLOYMENT DURATION

Let t_i be the duration spell for state i . Using a partial model of expected unemployment duration where the hazard h_{ii} is constant, it can be deduced that:

$$G(t_i) = 1 - \exp\{-h_{ii}t\}$$

so mean duration in state i can be estimated using the estimated separation rate as follows:

$$\widehat{E}(t_i) = \frac{1}{\widehat{h}_{ii}}$$

In the data there are three sources of bias in unemployment duration since: 1) Duration is only observed for the currently unemployed (incomplete spells); 2) The total length of the duration is not observed (right censoring); 3) Workers with longer unemployment spells are oversampled at a particular point in time. The first and third effect produce an upward bias in the estimate, while the second produces a downward bias in the estimate. Assuming exponential durations, the first two biases cancel out since under-sampling of short durations "cancels" with the underestimate of long durations.

Data on non-employment duration (complete spells for the employed population) and employment duration for workers in the formal and informal sector (incomplete spells) are also available.

Table 2.7 shows the estimates for employment and unemployment duration, using duration data and estimated hazards. The estimated mean unemployment duration is 52 weeks using duration data, much lower than the estimates produced using hazards (183 weeks). This suggests that time aggregation bias can be quite significant.⁷⁰

When using the FSO definition, a formal sector job lasts 69 months on average, an estimate close to the one using hazards rates. Mean length of informal employment (FSO definition) varies more when using the two methodologies: 100 vs. 167 months, suggesting that time aggregation in the informal-sector may also be quantitatively important.

⁷⁰There are many workers who make transitions from unemployment to other states that are not included in the hazard estimate, leading to a downward bias in the hazard rate of unemployment, and an upward bias in unemployment duration.

2.4.3 ESTIMATION

In the estimation, I am particularly interested in matching aggregate unemployment and employment rate figures, getting reasonable labor market tightness parameters for international standards,⁷¹ and matching selected moments of the wage distribution in both sectors.

The parameter space is defined by the following 21 parameters:

$$\Omega = \{r, \lambda, b, \beta, A, \alpha_m, \mu_y, \sigma_y^2, B_F, B_I, \sigma_F, \sigma_I, P_F, P_I, \tau, \mu, \delta_1, \delta_2, \delta_3, c, \alpha\}$$

I partition the parameter space of the benchmark model in two groups: Ω_1 and Ω_2 .

In the first group, parameters are fixed based on previous results from micro studies or data. In the second group, parameters are estimated to match selected empirical moments, including the division of the labor force among unemployment, informal and formal-sector employment, employment and unemployment duration, and selected moments of the wage distribution.

Let Ω_1 be defined as:

$$\Omega_1 = \{r, P_F, P_I, \mu_y, \sigma_y^2, \lambda, b, \beta, A, \alpha_m, \delta_1, \delta_2, \delta_3, \tau, \mu\}$$

The parameter values are chosen with a quarter as the implicit unit of time.

Table 2.8 summarizes the value of the fixed parameters based on data or micro studies.

The real interest rate in Colombia in 2003 was approximately 7.8 percent,⁷² so r is chosen to match the corresponding quarterly rate. Price indexes of tradable and non-tradable goods are used as proxies for the sectoral prices P_F and P_I , respectively.

⁷¹There are no reliable estimates for the Colombian case since there is no data on job vacancies.

⁷²Source: IMF, International Financial Statistics.

The parameters μ_y, σ_y^2 are chosen to coincide with the corresponding empirical moments.⁷³ I choose the rate of arrival of the productivity shock λ equal to 0.04 times per quarter. I use our micro data to test if the estimate falls within an admissible range. Using the separation rates of the formal and informal sector, h_{FF} and h_{II} , I can express λ as follows:

$$\lambda = \frac{h_{FF}}{H_F(R_{FU}(y) | y)}$$

$$\lambda = \frac{h_{II}}{H_I(R_{IU}(y) | y)}$$

Using duration data, I estimate that a formal-sector job lasts on average 6.89 years (27.5 quarters) while an informal-sector job lasts 7.69 years (30.77 quarters). Assuming that employment duration follows an exponential distribution, I estimate formal and informal-sector mean separation rates of 0.036 and 0.032, respectively. Given that $0 \leq H_F(R_{FU}(y) | y) \leq 1$ and $0 \leq H_I(R_{IU}(y) | y) \leq 1$, I have that $0.032 \leq \lambda < \infty$ and $0.036 \leq \lambda < \infty$, so assuming 0.04 shocks per quarter is within an admissible range in the model.⁷⁴

The opportunity cost of leisure, b , is set equal to 1. In Colombia there were no unemployment benefits in this period, so one must think about what the monetary compensation for the unemployed should be, relative to the average wage, or the average ‘replacement rate’. According to HKV2005, such a rate is 0.7 for European countries where benefits are relatively high, and 0.2 for the U.S (at most). In Shimer (2005) the rate is about 0.4, since $b = 0.4$ and the average wage is close to 1. For the Colombian case, it may be reasonable to assume that this ratio would be close to 0.2. Considering that the average wage in the model is close to 6.75, the chosen b gives us a replacement rate of 0.15.

⁷³I use educational attainment as a proxy for worker types.

⁷⁴In ANV2009 λ is set to 0.5 using one year as a unit of time, while in Hornstein, Krusell and Violante (2005) (hereafter HKV2005) λ is set to 0.1 using one quarter as a unit of time.

Following standard results in the literature, β is set equal to 0.5, so shares are split equally between workers and firms.⁷⁵

The parameters of the matching function are set in the following way. I assume a standard Cobb-Douglas function given by $m(\theta) = A\theta^{1-\alpha_m}$, with $A = 2.0$ and $\alpha_m = 0.5$. Since there is no data on vacancies for Colombia, I follow standard results for the U.S. from Mortensen and Nagypal (2007) and Brügemann (2008). The first paper estimates this elasticity at 0.45 and the latter between 0.54 and 0.63.⁷⁶ Also, job-finding rates in the U.S. are estimated to be 0.45 per month, or 1.35 per quarter. I assume job-finding rates in Colombia in the formal sector to be closer to 1.5 per quarter.⁷⁷ Assuming a reasonable labor market tightness, θ , ranging between 0.5 and 1, gives us a technological parameter A ranging between 1.5 and 2.12.

The policy parameters δ_1 , δ_2 and s chosen correspond to the estimated 2003 values: $s = 0$, $\delta_1 = 0.534$, $\delta_2 = 0.0837$, and $\delta_3 = 0.118$.⁷⁸

With regards to workers' preferences, in the benchmark case it is assumed that workers do not value social security services ($\tau = 0$) as in a model without valuations,

⁷⁵Here, the Hosios condition that requires $\beta = \alpha_m$ is satisfied, but in this environment this condition does not imply efficiency search because there is a 'composition' externality that is not internalized. When a firm hires a type- y worker, it changes the mix of types in the pool from which other firms are hiring.

⁷⁶Alternatively I could have set the value of the elasticity closer to the labor share, but there are not reliable estimates for Colombia. Shimer (2005) estimates this elasticity at 0.72.

⁷⁷Notice that job-finding rates in the FS cannot be identified directly using data on unemployment duration because the hazard rate out of unemployment depends not only on $m(\theta)$ but also on the reservation productivity rules.

⁷⁸To get s , severance in the formal sector is one monthly wage per year plus 1 percent interest (or 8.33 percent plus 1 percent interest=9.33 percent of average wage), but since it's paid every year and not when employment terminates, it's included in the total figure of δ_1 . To get δ_3 , I need to calculate the expected value of the services received in the subsidized regime as a percent of the wage. I multiplied the value of the services offered in the contributive regime (I assume their values are the same) by the corresponding probability of getting those services, so $\delta_3 = P_i(\delta_1 + \delta_2)$, where P_i is the proportion of IS workers who receive subsidized health in the data. See detailed employee and employer NWL costs in Tables 2.12 and 2.13.

but they value social assistance benefits partially ($\mu = 0.5$), so a change in δ_3 affects workers' behavior.⁷⁹

Let Ω_2 be defined as follows:

$$\Omega_2 = \{c, \alpha, B_F, B_I, \sigma_F, \sigma_I\}$$

I want to match seven empirical targets:⁸⁰ the unemployment rate, formal and informal-sector employment rates, formal and informal-sector employment durations, the differences in sectoral mean log-wages, and the ratio between the log-wage variance in the informal sector relative to the formal sector.

I simulate the model at the given parameter vector and calculate the moments of interest from the simulated data. Then, I compare these moments to the empirical moments estimated using the real data.⁸¹

Then, for the construction of our simulation-based estimator, I implement the following two-step algorithm:⁸²

1. Set guesses for the parameters of the productivity distributions $\{B_F, B_I, \sigma_F, \sigma_I\}$.
Conditional on these guesses, estimate $\{c, \alpha\}$ to match u , n_f and n_I .
2. Conditional on $\{u, n_f$ and $n_I\}$ obtained in the first step, estimate $\{B_F, B_I, \sigma_F, \sigma_I\}$ to match $\mu_{\ln(W_F)} - \mu_{\ln(W_I)}$, $\sigma_{\ln(W_F)}/\sigma_{\ln(W_I)}$, t_{e_F} and t_{e_I} . Iterate until convergence.

⁷⁹I will analyze in the policy experiments how results change with changes in valuations.

⁸⁰For the model to be identified, at least five empirical moments are needed.

⁸¹To simulate from the distributions $f_F(y)$ and $f_I(y)$, samples are drawn using quantile transform since the analytical expressions of the densities are known, but not the underlying population parameters. After sampling worker types in each sector, one can simulate productivities (conditional on types) given the population parameters B_F and B_I , and then use the wage functions to draw wages. Finally, Kernel densities can be estimated using the simulated data and the simulation-based statistics of interest can be computed.

⁸²This two-step algorithm is similar to the one used in Bradley, Postel-Vinay and Turon (2012).

Table 2.9 shows the estimated parameters.

I estimate \widehat{B}_F equal to 0.62 and \widehat{B}_I equal to 0.41, so the mean productivity varies linearly with education in both sectors, but the conditional mean of the match-specific productivity is less sensitive to y in the informal than in the formal sector.

I estimate \hat{c} equal to 2.50. This implies that the cost of posting a vacancy as a proportion of the model mean wage (vacancy ratio) is 37.04 percent, close to previous results in the literature.⁸³

I estimate $\hat{\alpha}$ equal to 2.91 jobs per quarter in the informal sector, higher than the estimated rate of arrival in the formal sector, $\widehat{m(\theta)}$ equal to 1.67. These estimates seem reasonable given other results in the literature.⁸⁴

The implied replacement ratio in the model is 0.148, reasonable considering there were no unemployment benefits in Colombia in the second quarter of 2003, and the estimated wage share is 0.63, within a reasonable range in the literature.⁸⁵

The model can approximately match the aggregate unemployment rate, formal and informal- sector employment rates, mean employment duration in the formal sector, and the relative dispersion in log-wages within the informal sector relative to the formal. The model underestimates relative mean log-wages. This can be explained by the fact that in the model there is no minimum wage. Thus, if the moments of the wage distribution that the model produces were compared to the moments derived from a counterfactual distribution of wages in the data, absent minimum wages, the size of the bias would be smaller. The model also overestimates mean unemployment duration and produces shorter mean employment duration in the informal sector.

⁸³In Shimer (2005), the vacancy ratio is around 20 percent. In HKV(2005), the vacancy ratio is 32.4 percent.

⁸⁴In Shimer(2005), job-finding rates are 1.35 per quarter.

⁸⁵Wage share is the ratio of mean wages to mean productivities. In Shimer (2005), the wage share is 0.97.

Table 2.10 presents a comparison between the simulated statistics and the data-based statistics.

Figures 2.13- 2.22 show the equilibrium objects and simulated distributions for the benchmark case.

Figure 2.13 shows the analytical density of worker types, y , assumed to be log-normal, compared with the non-parametric estimate of $F(y)$ using sample fractions at each educational level. The figure shows that the lognormal-assumption fits the data relatively well.

Figure 2.14 shows the steady state distribution of types across sectors, where the formal sector distribution is shifted to the right compared to the one in the informal sector, consistent with the empirical distribution. These densities are ‘contaminated’, i.e. only incorporate a restricted pool of workers in each sector.

Figures 2.15 and 2.16 show monotonic unemployment and hazard rates out of unemployment by type. The average unemployment rate and unemployment duration for workers with ‘high’ human capital levels are much lower than for those who are uneducated, given that these workers tend to accept both formal and informal-sector jobs.

Figure 2.17 presents the reservation productivity schedules. The figure shows that $R_{UF}(y) > R_{UI}(y)$ for all y , and both are strictly increasing in y . Conditional on type, workers in the formal sector are pickier and less likely to start a match (and more likely to discontinue a match⁸⁶) than workers in the informal sector. ‘High’ type workers are pickier than ‘low’ type workers since they have more outside opportunities, an effect that for this parametrization is stronger than the ‘labor hoarding’ effect.

Figure 2.18 shows monotone formal and informal-sector employment rates. ‘Low’ type workers are more likely to be informal while ‘high’ type workers are more likely

⁸⁶Here, $R_{FU}(y) = R_{UF}(y)$ since it is assumed in the simulations that $s = 0$.

to be formal. I also observe this monotonicity empirically.

Figure 2.19 shows mean job durations, disaggregated by sector. Figures 2.20, 2.21 and 2.22 show simulated steady state distributions of types, productivities and wages shifted to the right in the formal sector.⁸⁷

2.4.4 RESULTS

LABOR TAX EXPERIMENTS

In these set of experiments I simulate an increase of 4.46 percent and 7.53 percent in employer and employee NWL costs, δ_1 and δ_2 , respectively.⁸⁸ Three scenarios are analyzed under different workers' valuation of social security services.

Scenario 1: No valuation of SS services ($\tau = 0$)

If workers do not value SS contributions ($\tau = 0$), an increase in δ_2 is perceived as a pure net cost from the worker's perspective, while an increase in δ_1 will not directly affect the worker's value function.

A higher value of δ_1 makes vacancy creation less appealing for the formal-sector firm, decreasing θ from 0.67 to 0.62. An increase in δ_2 makes the formal-sector match less attractive from the worker's perspective. Increasing these NWLC shifts the reservation productivity $R_{FU}(y)$ up, so there is a higher probability of discontinuing the formal-sector match, reducing employment duration in the formal sector slightly (less than a month).

Because of the flows in the model from the formal to the informal sector, there is also a positive impact on vacancy creation in the informal sector. Formal-sector workers are more likely to end up in unemployment, so the expected benefits of an

⁸⁷Notice how the simulated distribution of types in Figure 2.20 is a very good approximation to the analytical density shown in Figure 2.14.

⁸⁸This is the actual increase from 2003 to 2012.

informal-sector firm hiring a formal-sector worker affected by a ‘bad’ shock are larger. Vacancy creation in the informal sector is therefore more attractive: ϑ goes up from 0.30 to 0.32. In other words, the informal-sector acts as a ‘buffer’ sector, absorbing some displaced workers from the other sector.

Given that there is more job destruction and less job creation in the formal sector, the formal-sector aggregate employment rate decreases from 29.7 to 27.5 percent, but the level of absorption in the informal sector is quite high, since the informal-sector aggregate employment rate increases from 52.2 to 53.9 percent. As a result, the overall unemployment rate increases slightly from 17.8 to 18.2 percent.⁸⁹

This policy increases unemployment duration slightly (less than a month), and decreases the hazard out of unemployment by shifting up the reservation productivity schedule, $R_{UF}(y)$. Figures 2.23, 2.24 and 2.25 summarize the impact of this policy on the distributions of worker types, productivities and wages. The distribution of types in both sector shifts to the right and is more dispersed with this policy (due to the higher incidence of unemployment among the unskilled workers), but the quantitative impact is very small. Given the compositional effects of this policy and the upward shifts of the reservation productivity schedule, the density of productivities in the formal sector also shifts slightly to the right, causing an average formal-sector productivity rise. Also, when δ_1 increases, the benefits of the formal-sector firm negotiating with the worker are reduced, improving the firm’s bargaining power in the negotiation, and causing a reduction in the formal-sector wage. From the worker’s perspective, a higher δ_2 implies less benefits from the negotiation agreement with the firm, improved bargaining power, and therefore a higher formal sector wage.

There are also some additional effects that need to be considered, since a higher

⁸⁹These statistics cannot be compared one-to-one with the statistics presented in Section 2 since they are valid for a restricted subsample, not for all urban workers.

reservation productivity $R_{UF}(y)$ implies a lower continuation value in the wage negotiation, and therefore a lower formal-sector wage. The flow value of unemployment $U(y)$ is also affected in equilibrium, affecting the continuation value of the negotiation in both sectors.

Finally, the average formal-sector wage increases very little when considering all equilibrium effects, so the log wage gap is barely affected (See Figure 2.25). Total log-wage variance is scarcely affected under this scenario.

In conclusion, the distributional impact of this policy when workers do not value SS contributions is minimal.

Scenario 2: Medium valuation of SS services ($\tau = 0.5$)

When workers value social security services somewhat, $\tau > 0$, qualitative results change since workers may perceive this policy either as a policy that reduces employee NWL costs (or a reduction in the payroll tax) or as a net subsidy from the government.

In this case, formal-sector firms are affected by higher NWL costs (higher δ_1), and therefore, they are more willing to end the match. Under this parametrization, the parameter $\hat{\delta}_2$ is actually reduced (from 8.3 to -23.3 percent), which means that from the workers' perspective, this policy is perceived as a subsidy that increases formal-sector wages by 23 percent. Workers are more willing to continue in a formal-sector match, counteracting the initial upward impact on the reservation productivity. The latter effect dominates, so the probability of discontinuing the formal-sector match goes down, increasing job duration in the formal sector by about 2 months.

From the job creation side, workers are also more willing to start a formal-sector match (a lower $R_{UF}(y)$), affecting job creation positively in the formal sector (θ goes up from 0.67 to 1.69). There is less vacancy creation in the informal sector (ϑ goes down from 0.30 to 0.10).

Overall, this policy will significantly shift employment from the informal to the formal: the informal-sector aggregate employment rate decreases from 52.2 to 25.7 percent, and the formal-sector aggregate employment rate increases from 29.6 to 59.9 percent. The higher labor market tightness and expected formal sector job duration imply a significant reduction in the overall unemployment rate from 17.8 to 14.1 percentage points. The hazard out of unemployment increases substantially, decreasing average unemployment duration from 69 to 53 weeks.

The distributions of worker types in both sectors shift to the left slightly (due to a drop in the unemployment rate among the unskilled workers), and become more compressed in the informal sector (See Figure 2.26). Given these compositional changes and the downward shift in the formal-sector reservation productivity, average formal-sector productivity decreases significantly relative to informal-sector productivity (See Figure 2.27). The strong negative impact on productivity drives formal-sector wages down, producing a significant fall in the log-wage gap (0.26 log wage points), and a significant reduction in between-sector log-wage variance. Even if this policy makes the distribution of wages within each sector more dispersed, the impact of the log-wage gap dominates, and as a result, total variance of log-wages falls substantially by 33.7 percent (see Figure 2.28).

Scenario 3: High valuation of SS services ($\tau = 1.0$)

If workers value these contributions highly ($\tau = 1$), an increase in δ_2 , will not affect the worker's flow utility. Nonetheless, an increase in δ_1 will increase the flow income in the formal sector, since workers associate this increase with more valuable and efficient services.

In this case, aggregate and compositional effects are of of higher magnitude. The informality rate is reduced to 11.7 percent, the formal-sector employment rate rises to 75.3 percent, and the distributions of worker types in both sectors shift to the

left. The distributional impact is also quite sizable. This ‘subsidy’ to formal-sector workers shifts the density of formal-sector productivity to the left and compress it, via the downward shift in reservation productivity. The opposite happens in the informal-sector productivity density. Therefore, the log-wage gap is narrowed and even becomes negative (see Figures 2.29, 2.30 and 2.31).

Overall log-wage inequality is substantially reduced by 28 percent, driven by the important decrease in between-sector variance.

SUBSIDIZED HEALTH EXPERIMENTS

I simulate an expansion in publicly-funded health insurance programs to informal sector workers. While in the benchmark case only 19.2 percent of informal workers have access to subsidized health, here I simulate an increase in coverage up to 50 percent of informal workers, that is an increase in δ_3 by 159.7 percent.⁹⁰ Two scenarios are made for medium and full valuation of these services.

Scenario 1: Medium valuation of SA services ($\mu = 0.5$)

An expansion of subsidized health (increase in δ_3) has a relatively small positive impact on the size of the informal sector (from 52 to 54 percent),⁹¹ even if the simulated percent change in δ_3 is big (159.75 percent). This is explained by the fact that the initial size of δ_3 is quite small, given that a very small percentage of informal-sector workers were covered by the subsidy in the second quarter of 2003.

The job destruction curve in the informal sector shifts down (a lower reservation

⁹⁰There is no good proxy for δ_3 in the data. Therefore, to approximate δ_3 in the benchmark case I assume that $\delta_3 = p_s(\delta_1 + \delta_2)$, where p_s is the proportion of IS workers who receive subsidized health. The value of the SA services is the same as the value of SS services (0.617 in benchmark case), adjusted by the probability of receiving those services. This generates a change in δ_3 from 0.1185 ($0.192 * 0.617$) to 0.3085 ($0.5 * 0.617$).

⁹¹This is more or less consistent with previous results found in Camacho, Conover, Hoyos (2013).

productivity $R_{IU}(y)$ for each labor market tightness ϑ), since workers have more benefits when continuing employment in the informal sector, and therefore are less likely to discontinue the match. This suggests an initial expansion of vacancies in this sector.

Nonetheless, the assumption that formal-sector workers may move to the informal sector in case of a 'bad' shock introduces additional equilibrium effects which may counteract the initial positive effect on informal-sector vacancies. If formal-sector workers have a 'bad' productivity shock and have the option to move to the informal sector, they will receive more continuation value (benefits) from the current formal-sector match when δ_3 rises. This shifts $R_{FU}(y)$ downward, so workers in the formal sector are less likely to end the match and more likely to start a formal sector match (lower $R_{UF}(y)$ as well). If workers in the formal sector are less likely to end the match, the expected benefits of the informal-sector hiring formal-sector workers affected by a 'bad' shock are lower, negatively affecting vacancy creation in the informal sector and counteracting the initial positive effect (ϑ decreases slightly constant).

The health subsidy increases job duration in the informal-sector slightly by shifting down the reservation productivity schedule, and increases the informal-sector employment rate from 52.2 to 54.1 percent. By making formal-sector vacancy creation less attractive, θ is reduced from 0.67 to 0.51.

Given that labor market tightness is reduced significantly, the aggregate unemployment rate increases from 17.8 to 18.1 percent, and the employment rate in the formal sector declines from 29.6 to 27.5 percent.

Both distributions of worker types are shifted to the right (due to a higher incidence of unemployment among the unskilled workers), but these compositional effects are small. Mean productivity in the informal sector falls slightly, driven by the downward shift in the reservation productivity (See Figure 20). Consequently,

the average informal-sector wage falls relative to the formal, slightly increasing the log-wage gap.

The size of these distributional impacts is also quite small (See Figures 2.32, 2.33 and 2.34). Therefore, the effect of this policy on overall log-wage inequality when workers have partial valuation of these services is minimal.

Scenario 2: Full valuation of SA services ($\mu = 1.0$)

When workers view the health subsidies as more valuable, the qualitative impacts are similar to the previous case. Equilibrium effects are of smaller magnitude, so formal-sector reservation productivity rises, leading to a stronger positive-effect on formal-sector productivity. As a result, the log-wage gap increases more than in the previous case.

This policy increases unemployment duration significantly by 7 weeks, and decreases the hazard rate of unemployment by shifting up the reservation productivity schedules, $R_{UF}(y)$. Distributional effects do not vary significantly relative to the previous case (See Figures 2.35, 2.36 and 2.37).

RELATIVE PRICE EXPERIMENTS

I simulate a drop in the price of tradables relative to non-tradables by 5 and 10 percent.⁹²

When the formal sector is less profitable due to trade policies or other factors that reduce relative prices (formal relative to informal), there is more destruction and less creation in the formal sector, increasing reservation productivity $R_{UF}(y)$ and making vacancy creation in the formal sector less attractive.

In the informal sector the impact is quite the opposite, since a lower $R_{IU}(y)$ induces more job creation. A fraction of the workers displaced in the formal sector will be

⁹²Real appreciations of this magnitude occurred after 2003.

absorbed in the informal sector, and the rest will join the pool of unemployed. The rise in the number of job seekers in the informal sector (unemployed and formal-sector workers affected by bad shocks) is more than offset by an increase in vacancy creation in the informal sector, so ϑ rises. The reduction in formal-sector vacancies as well as the higher number of unemployed job seekers make θ fall.

A reduction in relative prices has similar qualitative aggregate effects as the labor taxes policy with $\tau = 0$: there is a shift of resources from formality to informality, but in this case, many of the displaced workers will not be absorbed by the informal sector, inducing a larger rise in overall unemployment. A lower hazard out of unemployment implies an increase in mean unemployment duration, while job duration in the formal-sector decreases and in the informal sector increases. When prices drop significantly by 10%, mean unemployment duration can rise up to 9 weeks.

These aggregate impacts are more sizable than in the case of labor taxes with $\tau = 0$ since there are two effects that come into play: the direct effect of the change in prices on worker and firm surpluses, as well as the equilibrium impact due to changes in reservation productivities and market tightness.

With regard to compositional effects, both distributions of worker types shift slightly to the right (as in the previous experiment, the policy leads to a higher incidence of unemployment among the unskilled workers). With regard to distributional effects, the density of formal-sector productivity shifts to the right, reflecting the upward shift in the reservation productivity, while in the informal-sector, it shifts to the left.

Two forces may affect log wage inequality in opposite ways: a lower relative price of formal-sector goods entails a lower wage rate in the bargaining negotiation, but the rightward changes in the distribution of types and productivities push the average formal-sector wage up. The first effect dominates so mean log wages in the formal

sector decrease.

Despite the higher informal-sector good price, the mean wage in the informal sector drops as well as a result of the downward shift in productivities, so the overall impact on the log-wage gap is quite small. Both wage distributions shift to the left slightly (See Figures 2.38-2.43). Overall wage inequality is not affected significantly.

Table 2.11 summarizes the results of the simulated experiments.

2.5 CONCLUDING REMARKS

In this paper I build a search and matching model to understand the impact of labor market policies and social assistance programs on steady state unemployment, informal sector size and wage distribution in a small open economy with search frictions and idiosyncratic productivity shocks. I solve the model numerically and estimate the structural parameters using Colombian household-level data. The model accounts for the division of the labor force among unemployment, informal and formal sector employment, selected moments of the earning distribution, and formal-sector employment duration. I simulate the model and perform a set of policy experiments consistent with the policy reforms implemented in this developing economy in the last two decades.

My contributions to the literature are on two fronts: within the literature of reforms, I provide a new perspective on the impact of the reforms using a structural model; within the search and matching frictions literature with an informal sector, I generalize previous settings by building the informal sector as a ‘disadvantaged’ sector of a dualistic labor market (consistent with empirical evidence), and add an extra dimension of heterogeneity to fit better the wage distribution.

An expansion of public health insurance to informal sector workers, high pay-

roll taxation and changes in relative prices (tradables vs non-tradables) may affect the division of the labor force into unemployment, employment in the formal and informal sectors (aggregate effects), the mix of workers in the two sectors (compositional effects), and the distribution of productivities and wages (distributional effects). From a policy perspective, it is critical to quantify these effects.

The counterfactual experiments performed suggest that the valuation of social security services and subsidized health is crucial.

Changes in labor taxes have small effects if workers do not value social security services, but may have quite sizable aggregate, compositional and distributional effects if workers associate high payroll taxes with more valuable and efficient services. This perception of labor taxes as net transfers may actually induce a shift of resources from the informal to the formal sector and reduce unemployment, as well as improve overall income inequality by introducing important reductions in the log-wage gap among sectors. The higher the valuation of SS services, the more progressive these labor market policies become.

Social assistance programs that expand subsidized health to informal sector workers may only have mild effects if workers only partially value these services, even if the simulated increase in coverage to workers unregistered in this subsidized regime is quite significant. On the other hand, if workers value these services significantly (i.e. they perceive that the quality is comparable to the benefits offered in the contributive regime), expanding subsidized health may induce an increase in the unemployment rate by 1.5 percentage points, and an increase in unemployment duration of approximately 6 weeks.

Finally, changes in relative prices that negatively affect the relative profitability of the formal sector have quite sizable aggregate effects, producing more long run unemployment and informality. They can also increase the spell of unemployment by

9 weeks. The compositional and distributional effects are modest.

TABLES

Table 2.1: Descriptive Statistics for Employed

	All Employed	Informal	Formal	I/F Ratio
Sample Size	16,405	10,446	5,959	1.75
Population	7,974,924	4,799,786	3,175,138	1.51
$E(\ln W_s)$	-0.73	-1.11	-0.48	2.30
$SD(\ln W_s)$	2.01	1.85	2.06	0.90
$E(t_{es})$	88.51	92.33	82.77	1.12
$SD(t_{es})$	106.22	113.70	93.60	1.21
$E(t_{nes})$	6.90	7.34	6.33	1.16
$SD(t_{nes})$	13.13	13.63	12.45	1.09
$E(Y S_s)$	10.31	9.13	12.15	0.75
$SD(Y S_s)$	4.17	3.82	4.20	0.91

Author's calculations based on ECH, Second Quarter 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. $Y S_s$: years of schooling. Employment and non-employment duration in months. Informality using SP(health and pension) definition. Log of Hourly Wages in current dollars of 2003. I/F ratio is different in the sample than in the population because of sample weights.

Table 2.2: Decomposition of Variance in Log Hourly Wages

	Informal Sector
Mean of Log Wages (M_1)	-1.11
Variance of Log Wages (σ_1)	3.42
Proportion of Employed in Sector (P_1)	0.39
<hr/>	
	Formal Sector
Mean of Log Wages (M_2)	-0.48
Variance of Log Wages (σ_2)	4.27
Proportion of Employed in Sector (P_2)	0.61
<hr/>	
	Economywide
Mean of Log Wages ($P_1M_1 + P_2M_2$)	-0.73
Sum of Within-Sector Variance ($P_1\sigma_1 + P_2\sigma_2$)	3.94
Between-Sector-Variance $P_1P_2(M_1 - M_2)^2$	0.09
Total Variance	4.04

Author's calculations based on ENH, Second quarter of 2003, 13 Metropolitan areas. All statistics weighted using expansion factors. Informality using SP (health and pension) definition. Log of nominal hourly wages in current 2003 dollars (Exchange Rate =2877.65 Pesos per dollar - Source: World Bank)

Table 2.3: Descriptive Statistics for Unemployed

	All Unemployed
Sample Size	3,606
Population	1,810,468
$P(s \in U)$	0.185
$E(t_{us})$	51.53
$SD(t_{us})$	53.15

Author's calculations based on ECH, Second Quarter 2003, 13 Metropolitan Areas. Unemployment duration in weeks

Table 2.4: Transitions across Labor Market States (Population Values)

From \ To	Unemployment	Formal Sector	Informal Sector
Unemployment	1,435,620	106,308	141,453
Inactive	124,238	196,693	261,717
Formal Sector	101,282	2,590,524	192,138
Informal Sector	149,328	191,894	4,294,197

Author's calculations based on ECH, Second Quarter 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Informality using Firm size and Occupation definition. By adding the number of rows in each column I recover the total number of individuals in each labor market state in the second quarter of 2013.

Table 2.5: Annual Discrete Transition Matrix

From \ To	Unemployment	Formal Sector	Informal Sector	Other	Sum
Unemployment	0.72 (0.004)	0.05 (0.0026)	0.07 (0.0020)	0.16 (0.0030)	1.0
Other	0.04 (0.002)	0.06 (0.0042)	0.08 (0.0051)	0.82 (0.0010)	1.0
Formal Sector	0.03 (0.0021)	0.81 (0.0030)	0.06 (0.0010)	0.10 (0.005)	1.0
Informal Sector	0.03 (0.0024)	0.04 (0.0018)	0.93 (0.0022)	0.00 (0.004)	1.0

Author's calculations based on ECH, Second Quarter 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Informality using Firm size and Occupation definition. Standard errors in parentheses

Table 2.6: Estimated Hazards

	Unemployment	Formal Sector	Informal Sector
Estimated Hazards	0.28 (0.0041)	0.19 (0.0030)	0.07 (0.0022)

Author's calculations based on ECH, Second quarter 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Informality using Firm size and Occupation definition. Standard errors in parentheses

Table 2.7: Estimates of Employment and Unemployment Duration, by Sector

	Unemployment (Weeks)	Formal Sector (Months)	Informal Sector (Months)
Estimate using hazards	183 (0.0041)		
FSO Definition		62 (0.0030)	167 (0.0022)
Estimate from Duration Data	52 (0.903)		
FSO Definition		69 (1.16)	100 (1.11)
SP Definition		82 (1.21)	92 (92.33)

Author's calculations based on ECH, Second Quarter 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Standard errors in parentheses

Table 2.8: Fixed Parameters -Based on data or previous micro studies

	Description	Value
PARAMETERS		
r	interest rate	0.02
P_F	price formal-sector good	0.84
P_I	price informal-sector good	0.76
μ_y	mean, worker types	10.31
σ_y^2	variance, worker types	17.38
λ	rate of arrival productivity shock	0.04
b	opportunity cost of leisure	1.0
β	worker's Nash bargaining power	0.5
A	technological parameter, matching function	2.0
α_m	elasticity matching function	0.5
PUBLIC POLICY PARAMETERS		
δ_1	employer NWL cost as percent of FS wage	0.534
δ_2	employee NWL cost as percent of FS wage	0.083
δ_3	subsidized health as percent of IS wage	0.118
s	severance cost	0
VALUATION PARAMETERS		
τ	FS worker, valuation of social security services	0
μ	IS worker, valuation of social assistance services	0.5

Table 2.9: Estimated Parameters (Moment Simulation)

	Description	Value
PARAMETERS		
c	cost of posting a vacancy	2.50
α	rate of arrival IS opportunities	2.91
B_F	FS idiosyncratic productivity, log mean parameter	0.62
B_I	IS idiosyncratic productivity, log mean parameter	0.41
σ_F	FS idiosyncratic and match-specific productivity, log standard deviation	0.65
σ_I	IS idiosyncratic and match-specific productivity, log standard deviation	0.68

Table 2.10: Estimation: Data-based vs. Simulated Statistics

Variable	Model	Data
AGGREGATE UNEMPLOYMENT AND EMPLOYMENT RATES		
u	17.86 (0.0022)	18.50 (0.0027)
n_F	29.66 (0.0152)	32.44 (0.0033)
n_I	52.23 (0.0174)	49.05 (0.0035)
LABOR MARKET TIGHTNESS		
θ	0.67 (0.1048)	n.a.
ϑ	0.30 (0.0043)	n.a.
MEAN EMPLOYMENT AND UNEMPLOYMENT DURATIONS		
t_U	68.6 (1.1401)	51.5 (0.9032)
t_{e_F}	6.52 (0.0151)	6.89 (0.1011)
t_{e_I}	6.52 (0.0125)	7.69 (0.0929)
MEASURES OF WAGE DISPERSION		
$\mu_{\ln(W_F)} - \mu_{\ln(W_I)}$	0.40 (0.0045)	0.63 (0.0430)
$\sigma_{\ln(W_I)} / \sigma_{\ln(W_F)}$	0.88 (0.0167)	0.89 (.0126)

^a Unemployment and employment rates in percentage points. Unemployment duration in weeks. Mean employment and unemployment durations in years. Durations from data are estimated using observed durations, not estimated hazards. Standard errors in parenthesis. The standard errors of the simulated statistics are constructed using bootstrapping (thirty bootstrap replications).

Table 2.11: Compositional and Distributional Effects of Labor Market Policies, Social Assistance and Relative Prices

	Valuations	Aggregate Effects			LM Tightness		Distributional Effects				Mean Durations ^b		
	τ, μ	u	n_F	n_I	θ	ϑ	$\mu_{\ln W_F} - \mu_{\ln W_I}$	σ_{WS}	σ_{BS}	σ	t_u	t_{n_F}	t_{n_I}
Benchmark ^a	$\tau = 0, \mu = 0.5$	17.86	29.66	52.23	0.67	0.30	0.40	0.052	0.043	0.089	69	6.52	6.52
Δ Labor Taxes ^c													
$\delta_1 \uparrow 4.46\%, \delta_2 \uparrow 7.53\%$													
	$\tau = 0$	18.28	27.51	53.94	0.62	0.32	0.41	0.053	0.037	0.090	70	6.50	6.52
	$\tau = 0.5$	14.11	59.90	25.73	1.69	0.10	0.14	0.057	0.004	0.059	53	6.67	6.42
	$\tau = 1.0$	12.68	75.35	11.71	2.46	0.04	-0.06	0.064	0.000	0.064	47	6.73	6.34
Δ SA ^d													
$\delta_3 \uparrow$ by 159.74%													
	$\mu = 0.5$	18.12	27.51	54.11	0.51	0.29	0.42	0.052	0.039	0.091	70	6.53	6.53
	$\mu = 1$	19.49	29.74	50.51	0.717	0.31	0.44	0.054	0.044	0.098	76	6.48	6.49
Δ Relative Prices ^e													
	$P_F/P_I \downarrow$ by 5%	18.69	22.68	58.37	0.43	0.37	0.40	0.053	0.031	0.085	73	6.49	6.54
	$P_F/P_I \downarrow$ by 10%	19.76	18.70	61.28	0.39	0.45	0.40	0.056	0.028	0.085	78	6.45	6.53

^a Unemployment and employment rates in percentage points. Simulation results are robust to changes in initial conditions (guessed initial vectors for approximating steady state equilibrium) and multiple repetitions of the sampling process.

^b Unemployment duration in weeks, employment duration in years.

^c In these set of experiments $\mu = 0.5$.

^d In these set of experiments $\tau = 0$.

^e In these set of experiments $\tau = 0, \mu = 0.5$

Table 2.12: Detailed Employee Non-Wage Labor Costs (as percent of Wage)-Statutory Values (1984-2013)

	1984	1985-1990	Post90 Reform			Post93 Reform			Post2003 Reform			Post2007 Reform		Post2012 Reform
	1984	1985 – 1990	1991	1992	1993	1994	1995	1996-2003	2004	2005	2006	2007	2008-2012	2013
SOCIAL SECURITY														
Pension	1.50	2.17	2.17	2.17	2.17	2.875	3.125	3.375	3.625	3.75	3.875	3.875	4.0	4.0
Health	2.33	2.33	2.33	2.33	2.33	4.00	4.00	4.00	4.00	4.00	4.00	4.0	4.0	4.0
Subtotal Social Security	3.83	4.50	4.50	4.50	4.50	6.875	7.125	7.375	7.625	7.75	7.875	7.875	8.00	8.00
OTHERS														
Solidarity Fund		0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
TOTAL														
EMPLOYEE NWLC	3.83	4.50	4.50	5.50	5.50	7.875	8.125	8.375	8.625	8.75	8.875	8.875	9.00	9.00
(as % of wage)														

Author's Calculations based on statutory values established in the Labor Codes. See Law 100 of 1993, Law 797 of 2003, Law 1122 of 2007 and Law 1607 of 2012.

Table 2.13: Detailed Employer Non-Wage Labor Costs (as percent of Wage)-Statutory Values (1984-2013)

	1984	1985-1990	Post90 Reform			Post93 Reform			Post2003 Reform			Post2007 Reform		Post2012Reform
	1984	1985 – 1990	1991	1992	1993	1994	1995	1996-2003	2004	2005	2006	2007	2008-2012	2013
SOCIAL SECURITY														
Pension	3.00	4.33	4.33	4.33	4.33	8.63	9.38	10.12	10.87	11.25	11.62	11.63	12.00	12.00
Health	4.67	4.67	4.67	4.67	4.67	8.00	8.00	8.00	8.00	8.00	8.00	8.50	8.50	8.50
Work Injury	2.0	2.00	2.00	2.00	2.00	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.4
Subtotal Social Security	9.67	11.00	11.00	11.00	11.00	19.03	19.78	20.53	21.28	21.65	22.03	22.53	22.90	22.90
PARA-STATAL														
Training	2.0	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	0.00
In-Kind Tranfers	2.0	2.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	0.00
Family Allowances	4.0	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Subtotal Para-statal	8.0	8.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	4.0
OTHERS														
Mandatory Bonuses	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Paid Vacations	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
Severance Pay	13.5	13.5	12.8	12.1	11.4	10.70	10.0	9.30	9.30	9.30	9.30	9.30	9.30	9.30
Subtotal Others	28.08	28.08	27.38	26.68	25.98	25.28	24.58	23.88	23.88	23.88	23.88	23.88	23.88	23.88
TOTAL														
EMPLOYER NWLC	45.75	47.08	47.38	46.68	45.98	53.31	53.36	53.41	54.16	54.53	54.91	55.41	55.78	50.78
(as % of wage)														

Author's Calculations based on statutory values established in the Labor Codes. See Law 50 of 1990, Law 100 of 1993, Law 797 of 2003, Law 1122 of 2007, and Law 1607 of 2012. Work injury compensations vary with the degree of risk in the occupation, ranging from degree 1 (lowest risk) to degree 5 (highest risk). Here, I show the values of a level 3 risk. Severance pay was paid upon employment termination prior to the 1990 Labor Reform, but it was turned into a payroll tax afterwards since firm have to deposit payments on a monthly basis in a financial account. Training refers to SENA, In-kind Transfers to ICBF, and Family Allowances to Cajas de Compensacion Familiar. The reduction in parastatal contributions in 2012 only applied to workers earning less than 10x the minimum wage.

FIGURES

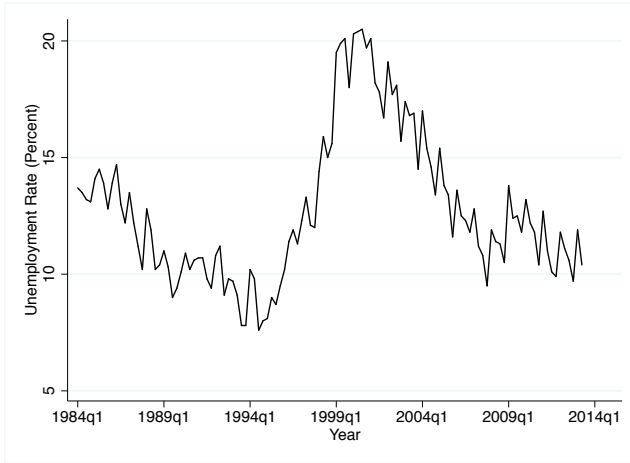


Figure 2.1: Evolution of Unemployment Rate
 All urban workers. Seven Metropolitan areas: Bucaramanga, Barranquilla, Bogota, Cali, Medellin, Manizales and Pasto.
 Source: *Colombian Central Bank*

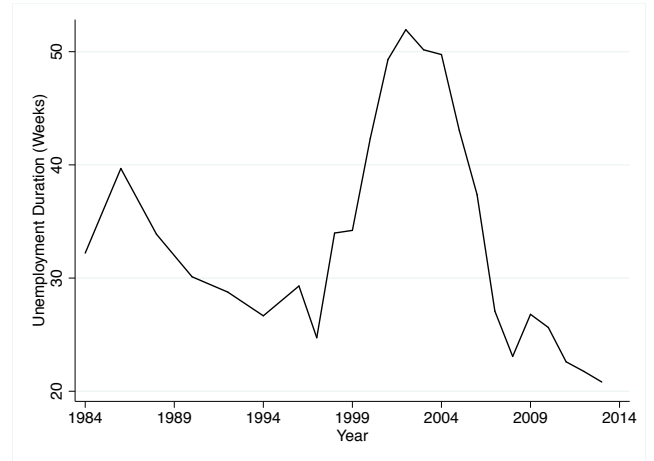


Figure 2.2: Evolution of Mean Unemployment Duration
 Author's calculation based on *Colombian Household Surveys*

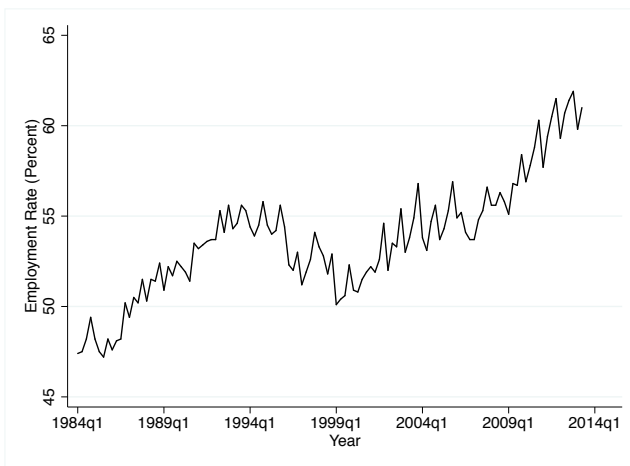


Figure 2.3: Evolution of Employment Rate
 All urban workers. Seven Metropolitan areas: Bucaramanga, Barranquilla, Bogota, Cali, Medellin, Manizales and Pasto.
 Source: *Colombian Central Bank*

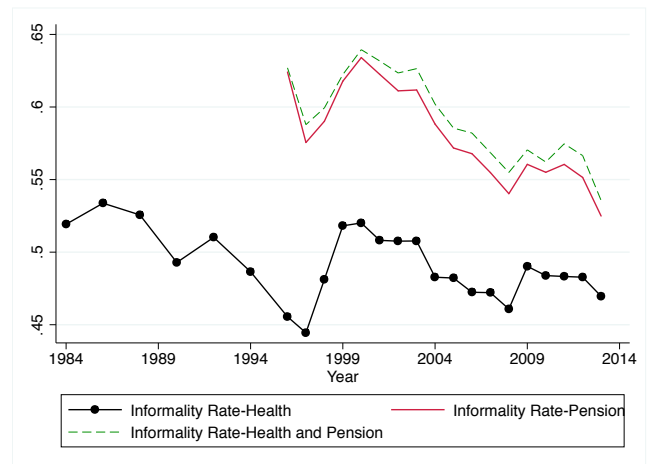


Figure 2.4: Evolution of Informality Rates, by definition of Informality
 Author's calculation based on *Colombian Household Surveys*. There is no information on pension contributions before 1996.

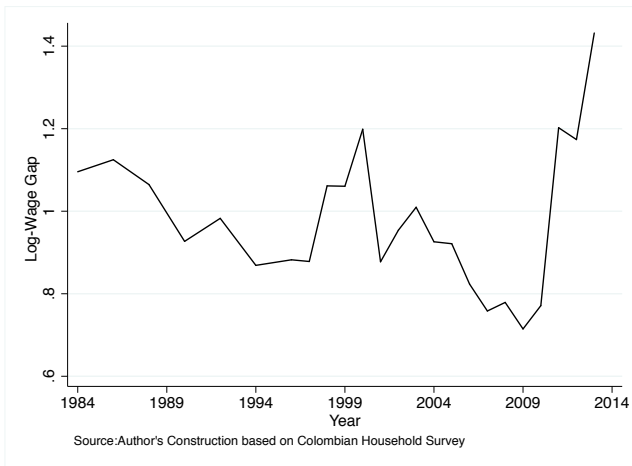


Figure 2.5: Evolution of Log Wage Gap
 Log Wage Gap defined as $\overline{\ln w_F} - \overline{\ln w_I}$. Informality based on health criterion. Log of hourly wages in constant dollars of 2008. Source: Author's calculation based on Colombian Household Surveys

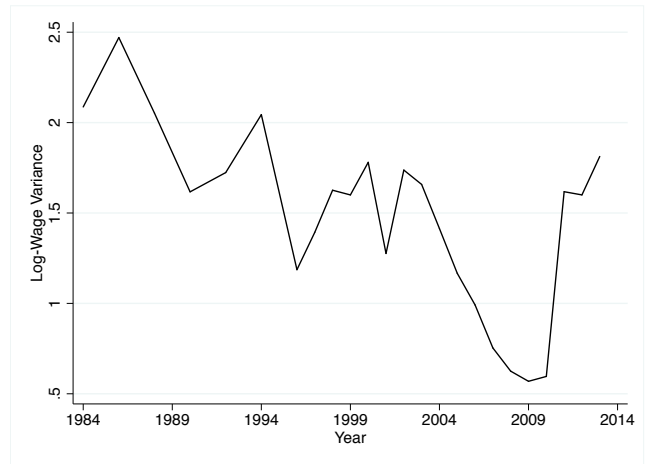


Figure 2.6: Evolution of Variance of Log Hourly Wages
 Log of hourly wages in constant dollars of 2008. Source: Author's calculation based on Colombian Household Surveys

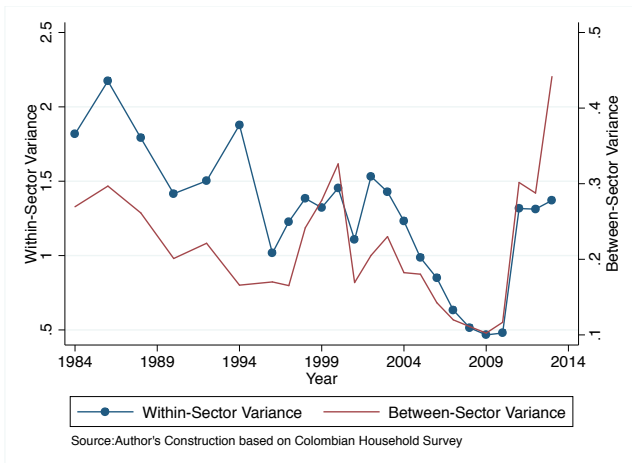


Figure 2.7: Decomposition of Variance of Log Hourly Wages
 Log of hourly wages in constant dollars of 2008. Source: Author's calculation based on Colombian Household Surveys

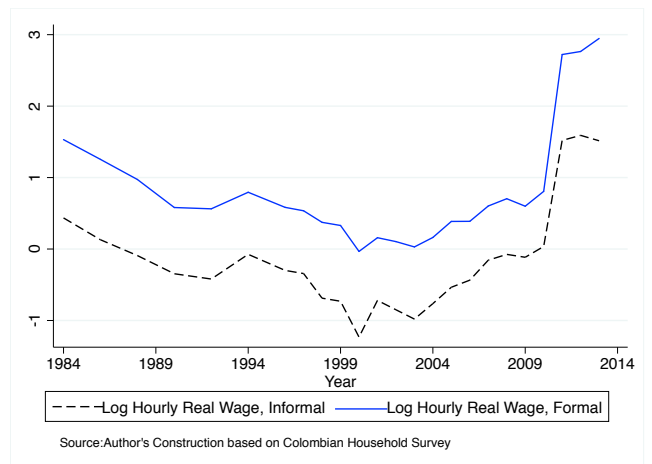


Figure 2.8: Evolution of Mean of Log Hourly Wages
 Log of hourly wages in constant dollars of 2008. Source: Author's calculation based on Colombian Household Surveys

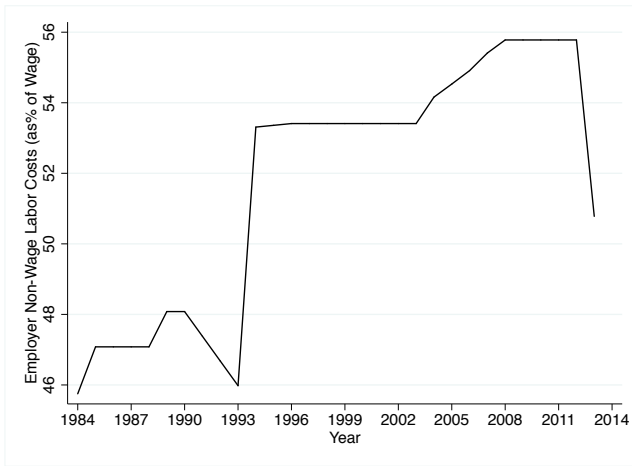


Figure 2.9: Evolution of Employer Non-Wage Labor Costs (as % of Wage)

See Table 13 for a detailed construction of these costs. *Author's Calculations based on Statutory Values*

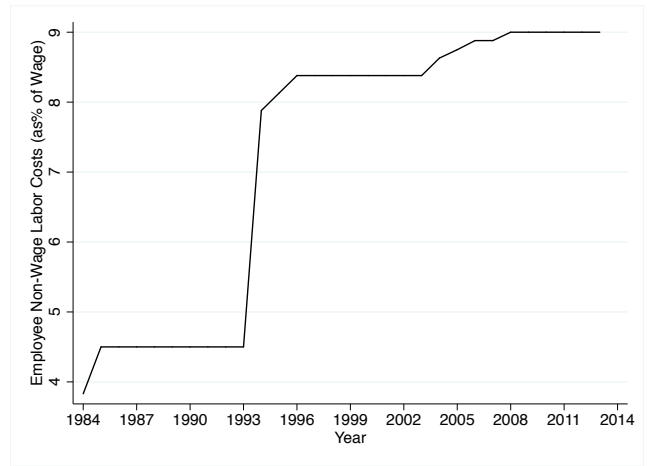


Figure 2.10: Evolution of Employee Non-Wage Labor Costs (as % of Wage)

See Table 12 for a detailed construction of these costs. *Author's Calculations based on Statutory Values*

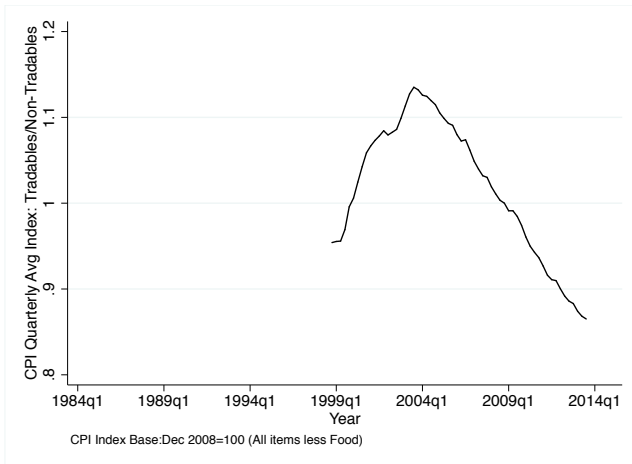


Figure 2.11: Evolution of Relative Prices

CPI Quarterly Average Index- Tradables relative to Non-Tradables. No data availability before 1998 *Source: Colombian Central Bank*

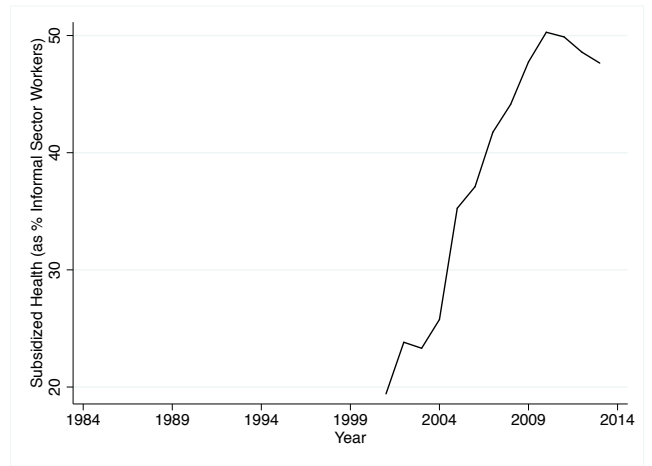


Figure 2.12: Evolution of Subsidized Health

Percentage of Informal-Sector Workers who have access to the Subsidized Regime- No data availability in the surveys before 2001. *Author's Calculations based on Colombian Household Surveys*

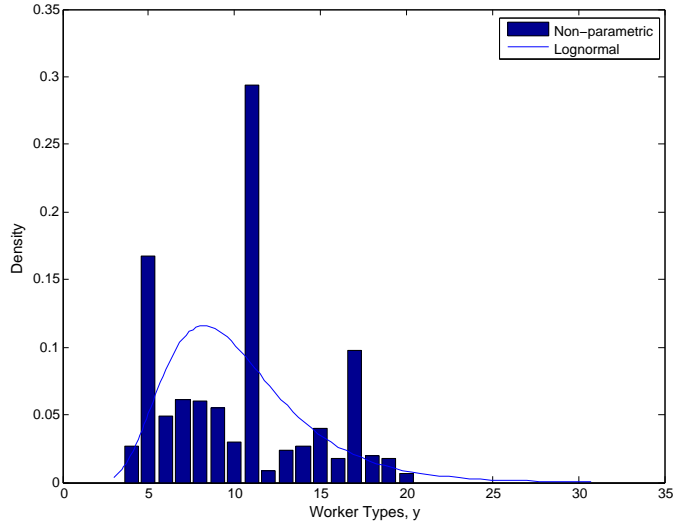


Figure 2.13: Analytical Density of Worker Types: Data vs Lognormal

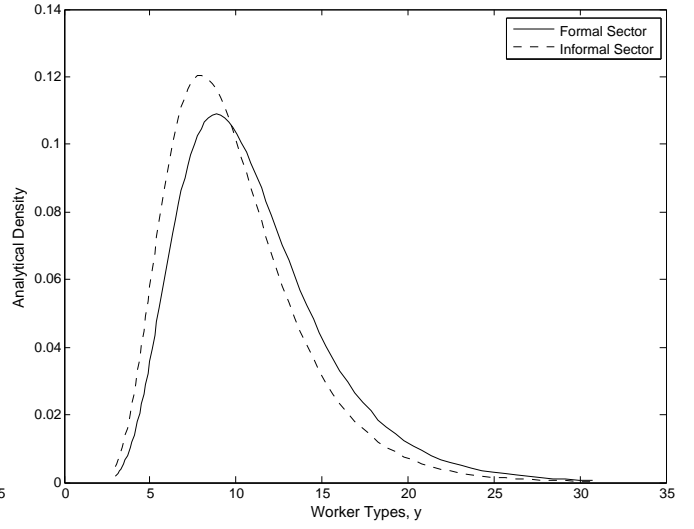


Figure 2.14: Analytical Density of Worker Types, Formal and Informal Sector

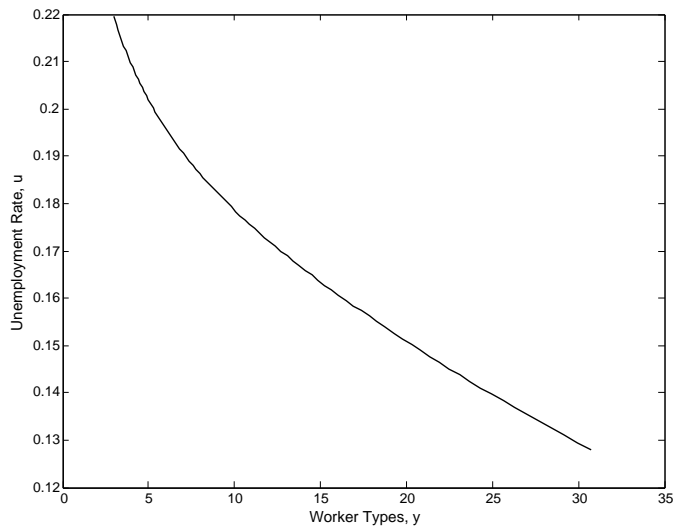


Figure 2.15: Unemployment Rate

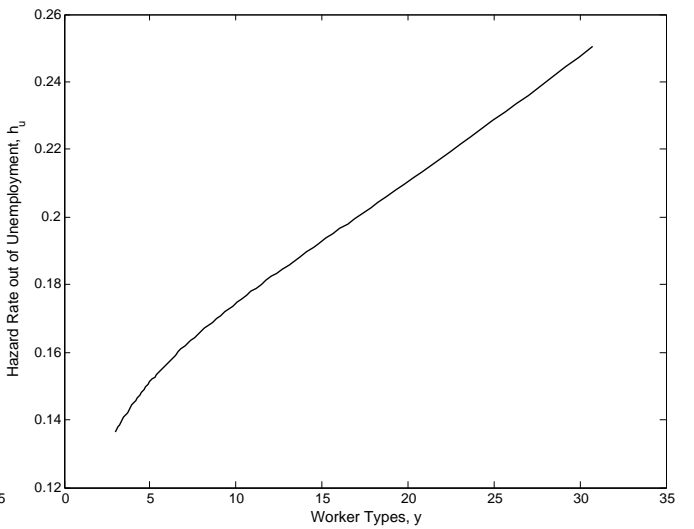


Figure 2.16: Hazard Rate out of Unemployment

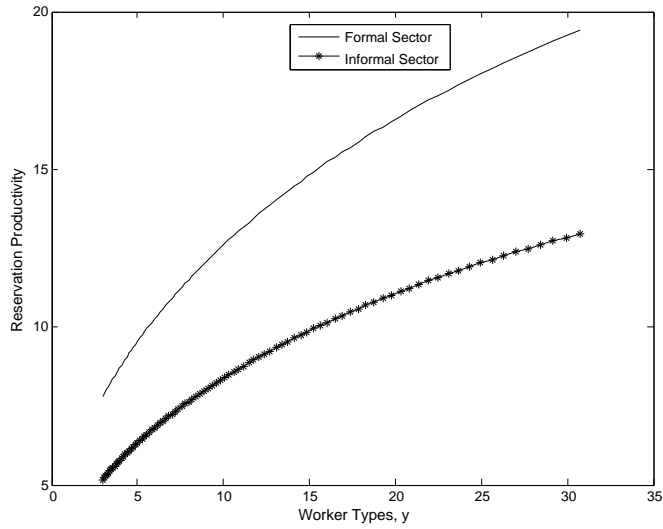


Figure 2.17: Reservation Productivity, Formal and Informal Sector

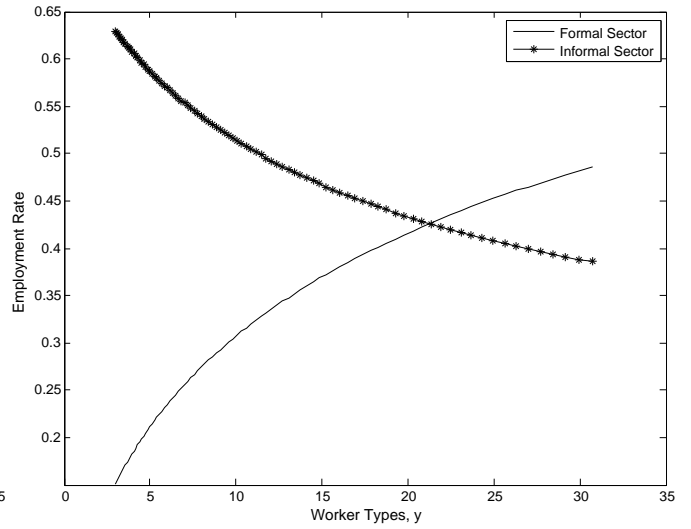


Figure 2.18: Employment Rate, Formal and Informal Sector

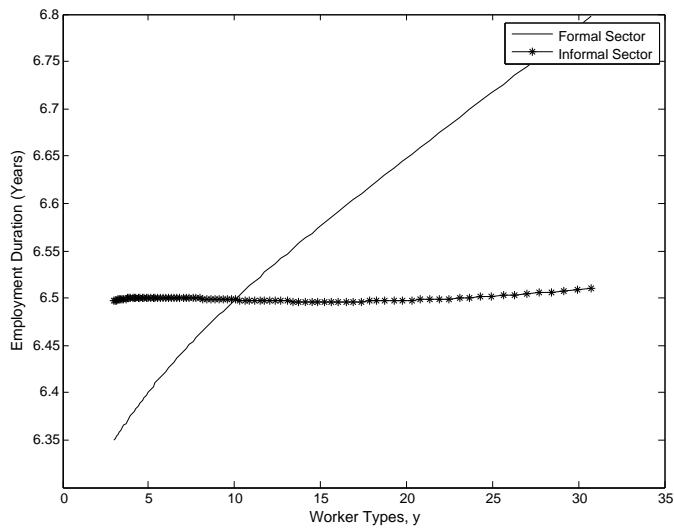


Figure 2.19: Job duration (Years), by Sector

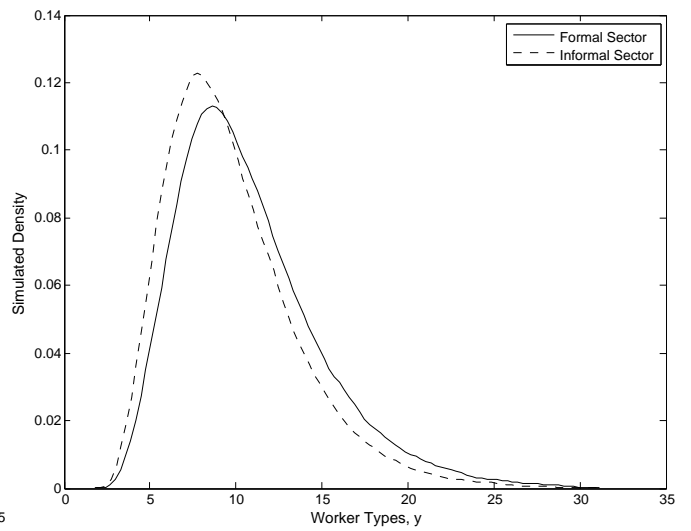


Figure 2.20: Kernel Density Simulated Distribution of Types, by Sector

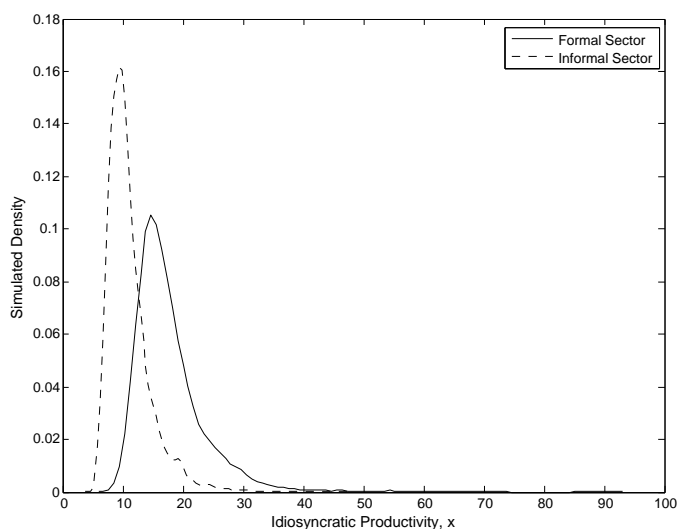


Figure 2.21: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector

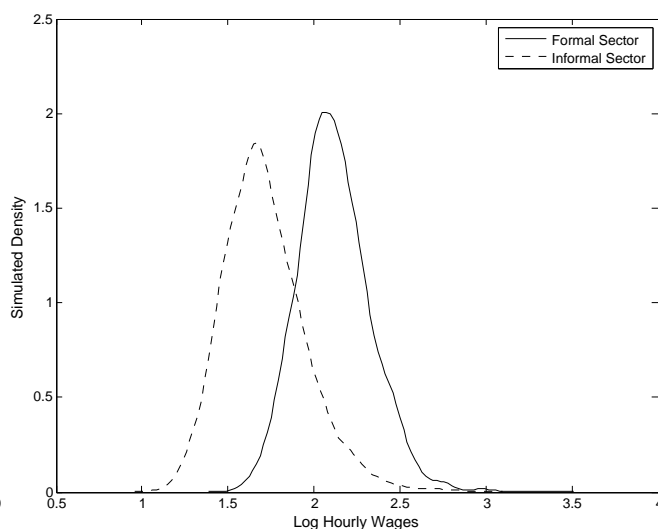


Figure 2.22: Kernel Density Simulated Distribution of Log Wages, by Sector

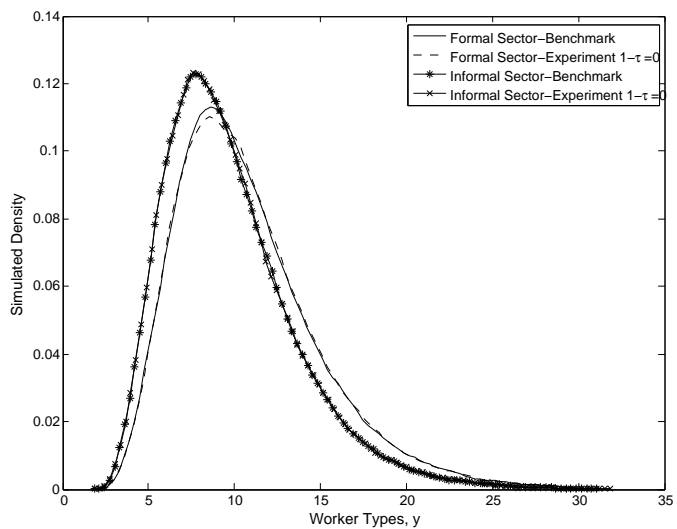


Figure 2.23: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 1, ($\tau = 0$)

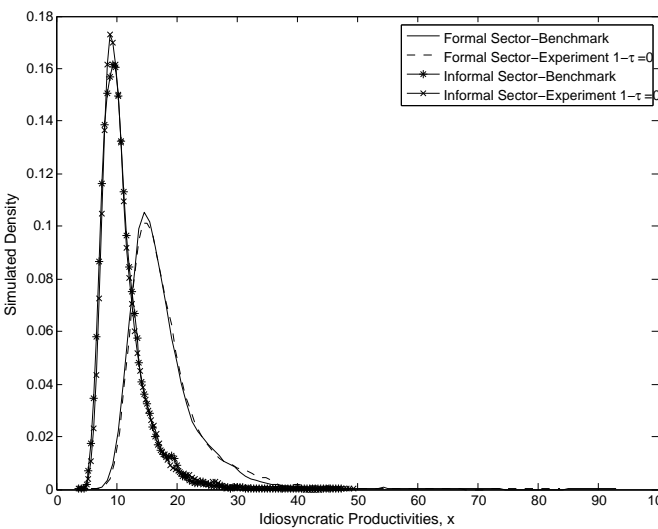


Figure 2.24: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 1 ($\tau = 0$)

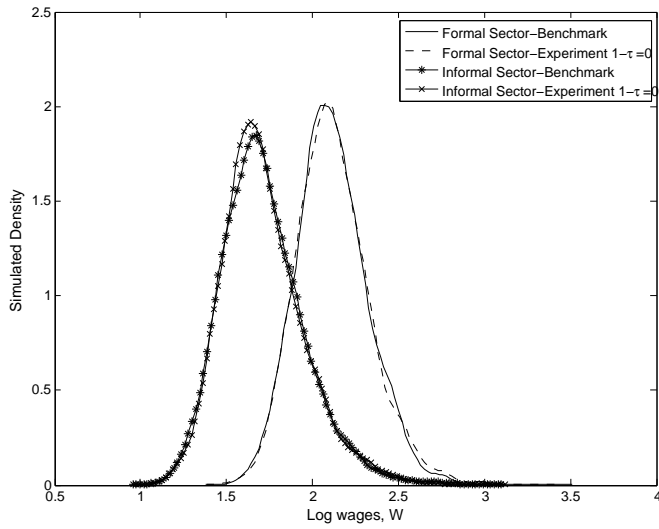


Figure 2.25: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 1 ($\tau = 0$)

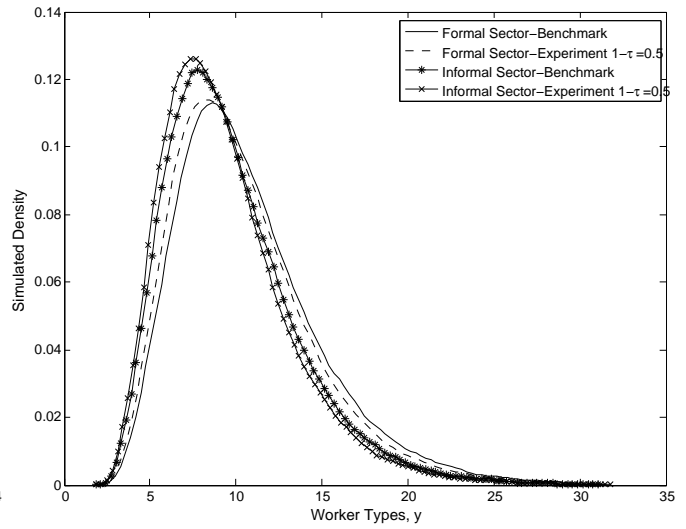


Figure 2.26: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 1, ($\tau = 0.5$)

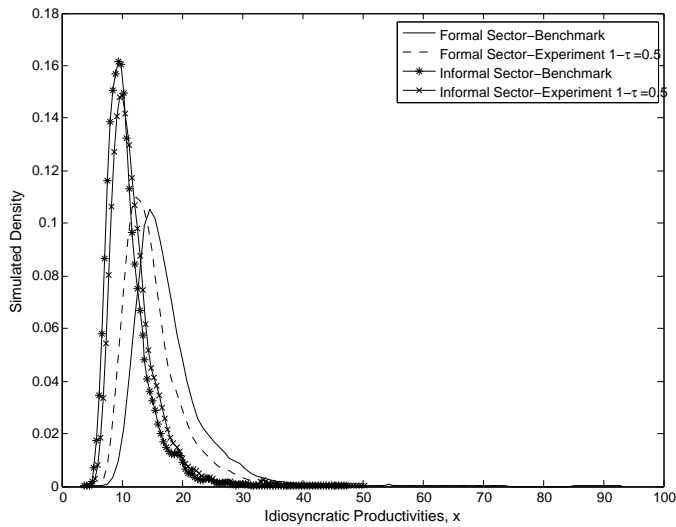


Figure 2.27: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 1 ($\tau = 0.5$)

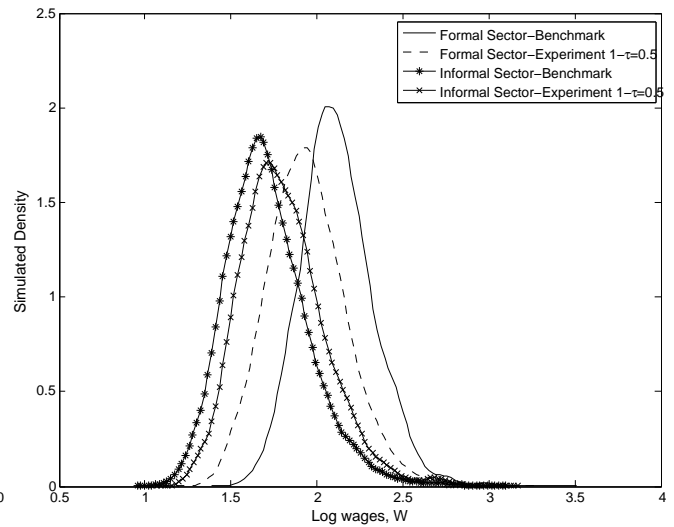


Figure 2.28: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 1 ($\tau = 0.5$)

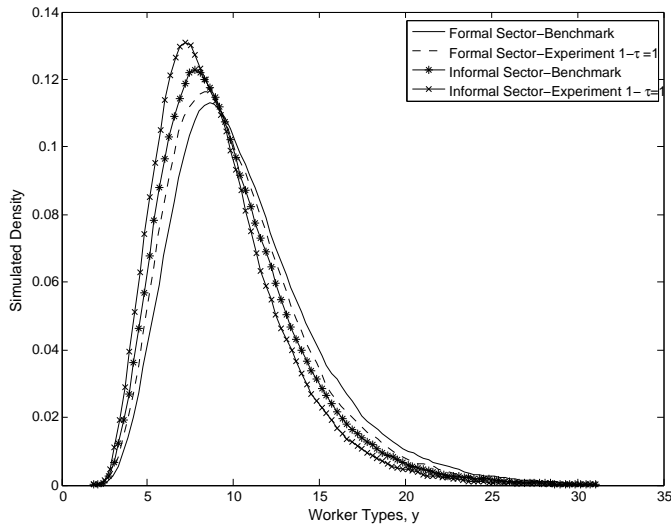


Figure 2.29: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 1, ($\tau = 1$)

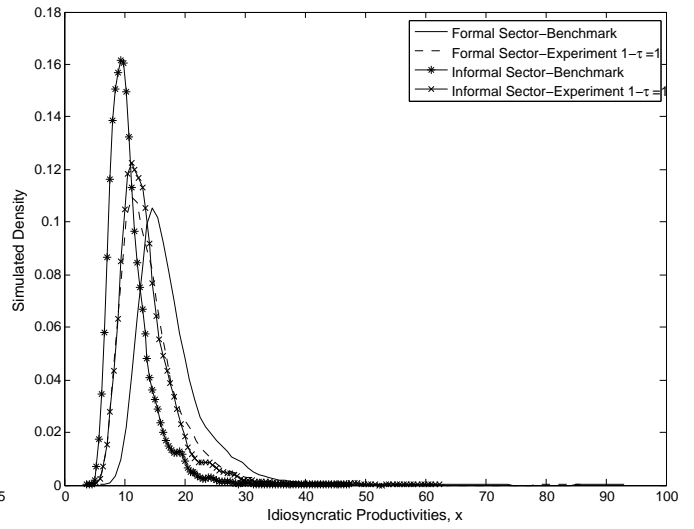


Figure 2.30: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 1 ($\tau = 1$)

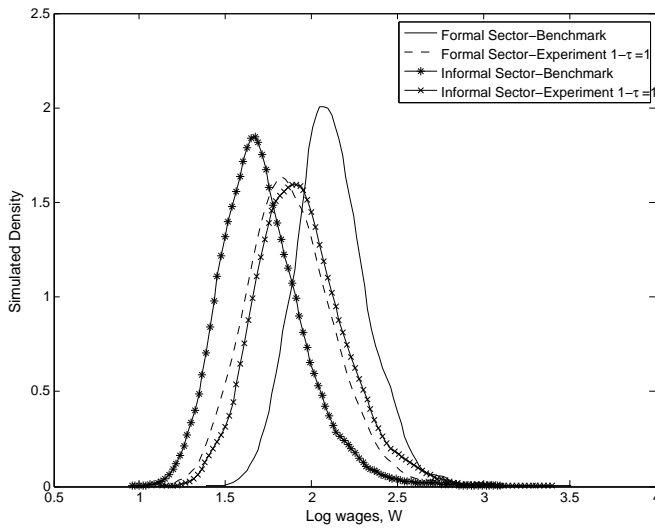


Figure 2.31: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 1 ($\tau = 1$)

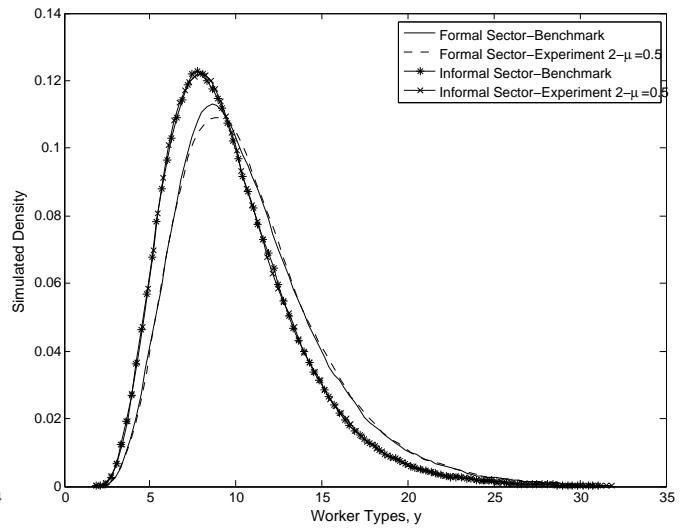


Figure 2.32: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 2 ($\mu = 0.5$)

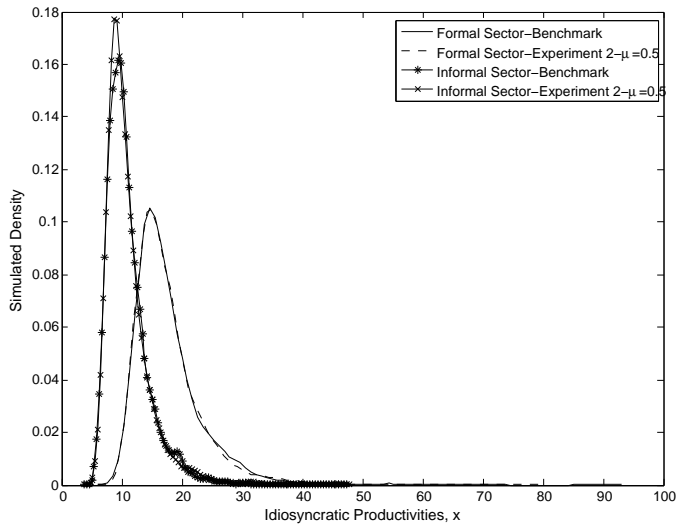


Figure 2.33: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 2 ($\mu = 0.5$)

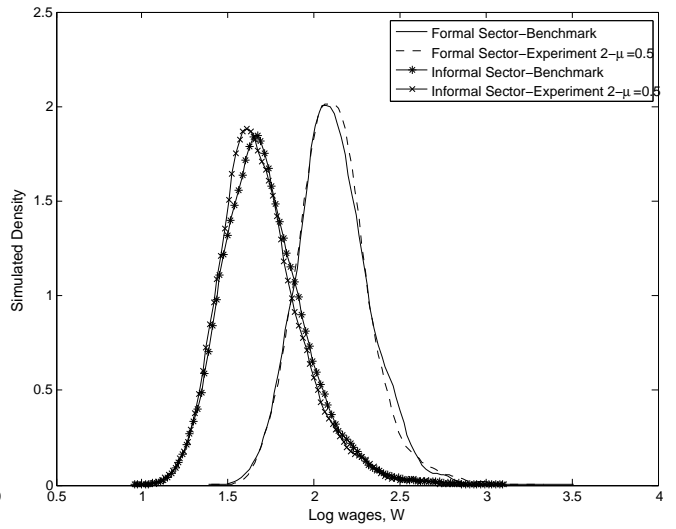


Figure 2.34: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 2 ($\mu = 0.5$)

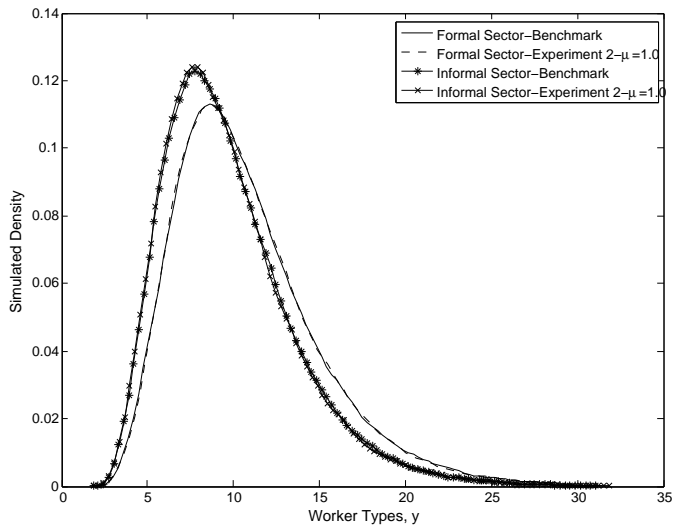


Figure 2.35: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 2 ($\mu = 1$)

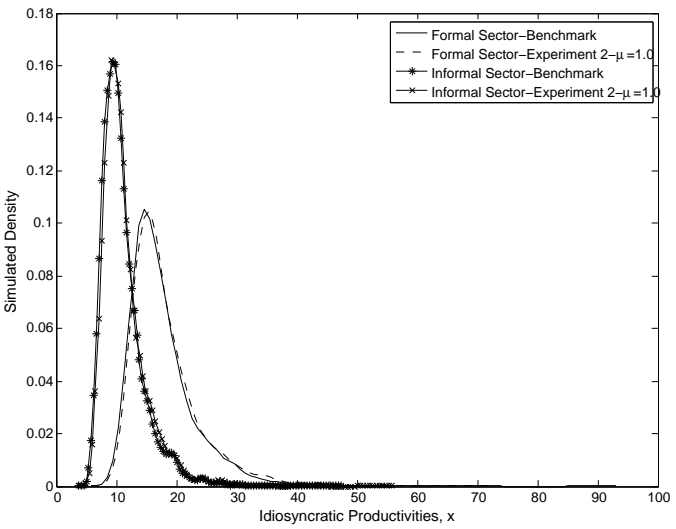


Figure 2.36: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 2 ($\mu = 1$)

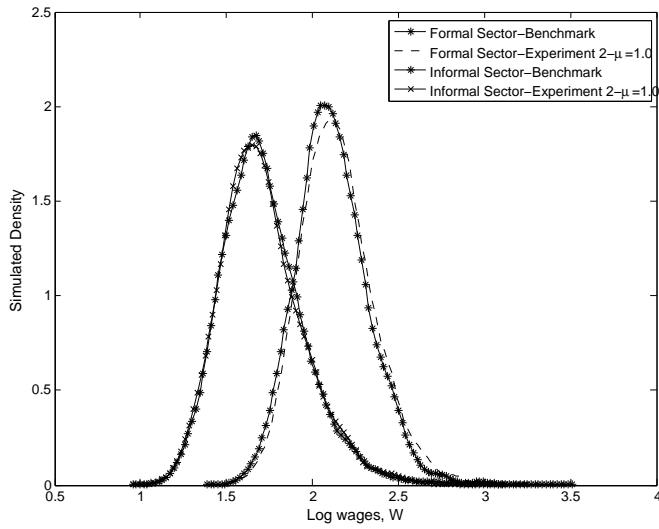


Figure 2.37: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 2 ($\mu = 1$)

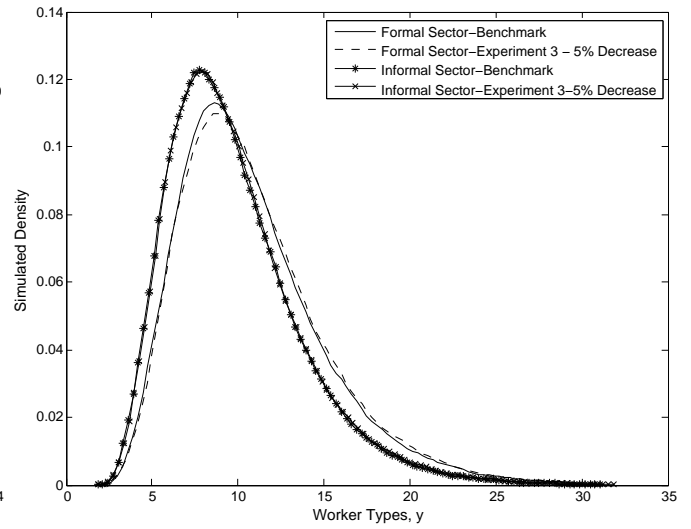


Figure 2.38: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 3 (5% ↓)

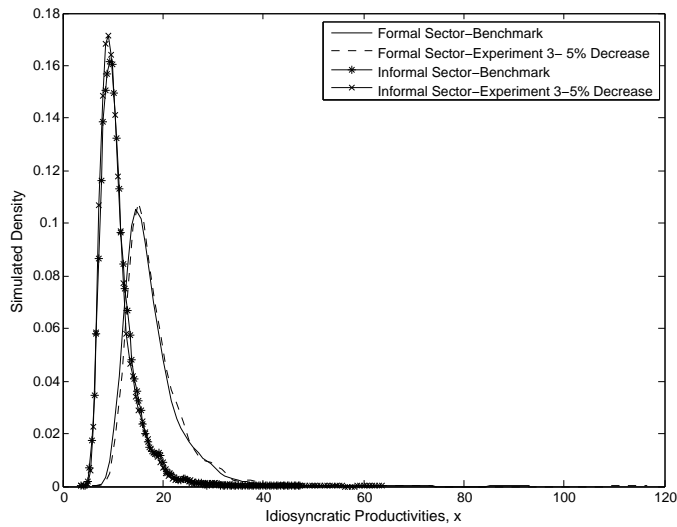


Figure 2.39: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 3 (5% ↓)

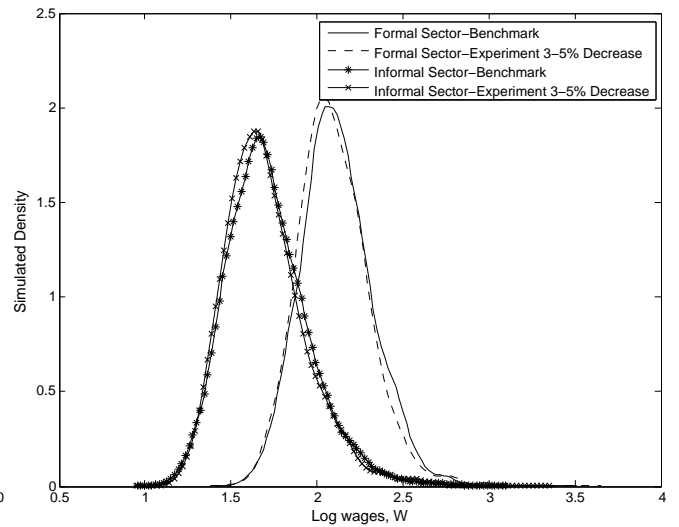


Figure 2.40: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 3 (5% ↓)

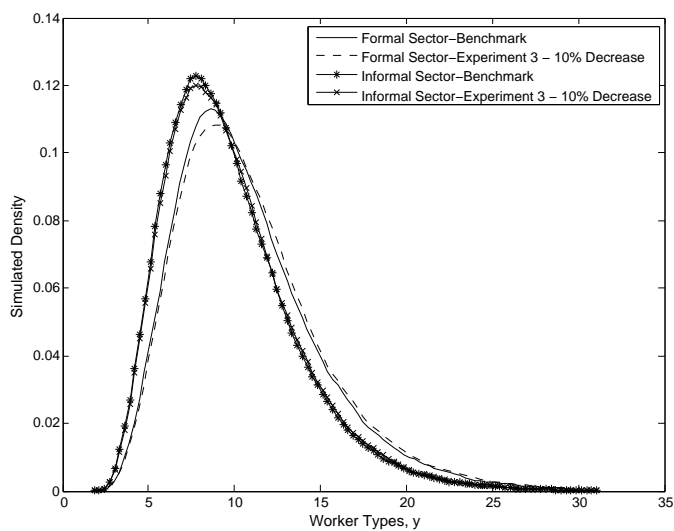


Figure 2.41: Kernel Density Simulated Distribution of Types, by Sector: Benchmark vs. Experiment 3 (10% ↓)

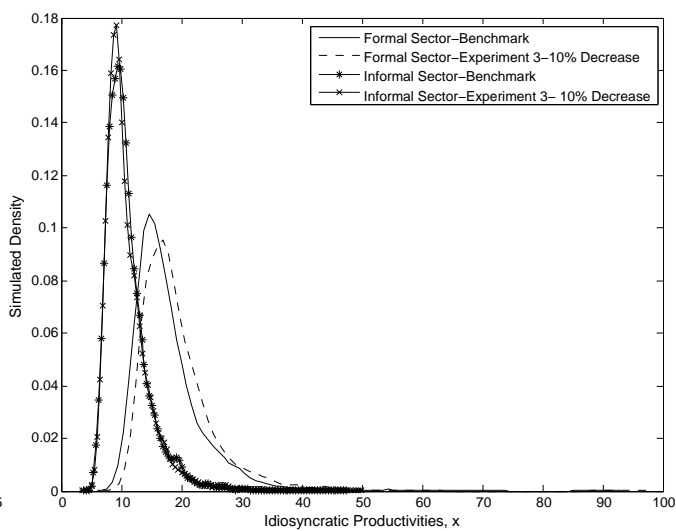


Figure 2.42: Kernel Density Simulated Distribution of Idiosyncratic Productivities, by Sector: Benchmark vs. Experiment 3 (10% ↓)

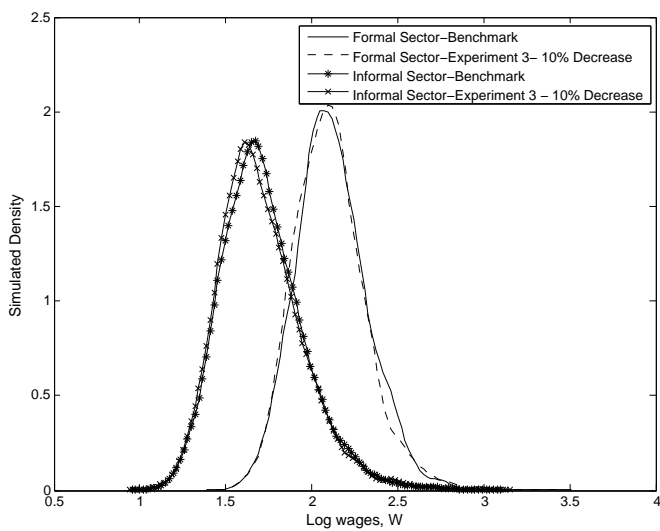


Figure 2.43: Kernel Density Simulated Distribution of Log Wages, by Sector: Benchmark vs. Experiment 3 (10% ↓)

CHAPTER 3

PUBLIC SECTOR EMPLOYMENT IN AN EQUILIBRIUM SEARCH AND MATCHING MODEL

3.1 INTRODUCTION

The public sector accounts for a substantial fraction of employment in both developed and developing economies. Algan et al. (2002) estimates that the public sector accounted for slightly less than 19% of total employment across 17 OECD countries in 2000, and Mizala et al. (2011) estimates that 13% of total urban employment over the period 1996-2007 across eleven Latin American countries was in the public sector.

In this paper, we incorporate a public-sector labor market into an extended version of the Diamond-Mortensen-Pissarides (DMP) search and matching model of equilibrium unemployment (Pissarides 2000). Our model is designed to address distributional questions. What types of workers tend to work in the public sector? What types tend to sort into the private sector? How do the size of the public sector and the hiring and wage-setting rules used in that sector affect the overall unemployment rate and the distributions of workers, productivities and wages across the two sectors?

Our extension to the basic DMP model has three key ingredients. First, we assume an exogenous distribution, $Y \sim F(y)$, $\underline{y} \leq y \leq \bar{y}$, of human capital across workers. This makes it possible to address questions about which types of workers tend to work

in the two sectors.¹ Second, we allow for *ex post* idiosyncratic match productivity. When a worker of type y meets a prospective employer with a vacancy, the worker draws a match-specific productivity, $X \sim G_s(x|y)$, $\underline{x} \leq x \leq \bar{x}$, where the subscript $s \in \{p, g\}$ indicates whether the job in question is in the private or public (government) sector. To give content to our notion of human capital, we assume first-order stochastic dominance, i.e., $y' > y \Rightarrow G_s(x|y') < G_s(x|y)$. The higher is a worker's level of human capital, the more favorable is that worker's distribution of match-specific productivity, and this is the case in both sectors.² Finally, we take into account that the rules governing public-sector employment and wage determination are in general not the same as those used in the private sector. We assume that the public sector posts an exogenous measure of vacancies, v_g , and that a worker of type y who meets a public-sector vacancy and draws match-specific productivity x is offered the job if and only if that productivity is no less than an exogenous threshold that varies with worker type, that is, if and only if $x \geq R_g(y)$. We also assume that a worker's wage in a public-sector job is determined by an exogenous rule, $w_g(x, y)$, and without loss of generality, we set $w_g(x, y) = 0$ for $x < R_g(y)$. Our combination of these three elements – *ex ante* worker heterogeneity, match-specific productivity with a first-order stochastic dominance assumption, and both private- and public-sector employment – is unique in the search and matching literature.

We calibrate our model using Colombian data and then use the calibrated model to simulate the effects of varying (i) the level of public-sector employment and (ii)

¹This feature of our model is taken from Albrecht, Navarro and Vroman (2009). That paper focused on the distribution of worker types across formal employment, informal employment and unemployment.

²This feature of our model is related to Dolado, Jansen and Jimeno (2009), who assume first-order stochastic dominance in conjunction with a two-point distribution for y – “low-skill” and “high-skill” workers. Other papers achieve a similar effect by making a specific functional form assumption, typically that productivity is the product of worker type and an independent match-specific component.

the public-sector hiring and wage-setting rules.

Colombia is an interesting case study because its public-sector wage premium is very large by international standards. Our baseline calibration indicates that most of this premium is attributable to different distributions of education in the two sectors. While more educated workers are more productive in either sector, we find that more highly educated workers sort into the government sector and this largely accounts for the wage premium. In general, our calibration and our numerical experiments suggest that to understand the differences between public- and private-sector wages and productivity, and, more generally, to understand how the two sectors interact requires explicitly considering worker heterogeneity. This allows us to look not only at the differences in means, but also at the differences in wage and productivity distributions and their effects on the aggregate economy.

In terms of related literature, we know of four other papers that incorporate public-sector employment into an equilibrium search and matching framework. Two papers, namely, Burdett (2012) and Bradley, Postel-Vinay and Turon (2014), incorporate a public sector into the Burdett and Mortensen (1998) model of on-the-job search, and two papers, namely, Quadrini and Trigari (2007) and Gomes (2015), use a DMP framework to analyze the effect of public-sector employment and wage policy on the private-sector labor market over the business cycle.³ None of these papers allow for

³One reason that we use a DMP, rather than a Burdett-Mortensen, approach is that direct transitions from the private to the public sector and vice versa are relatively rare in Colombia. Using the method described in Robayo-Abril (2015), we estimate that on an annual basis, the probability of a direct (i.e., without an intervening spell of unemployment) transition from the public to the private sector is less than .04. Our estimate of the transition probability in the opposite direction is less than one quarter of one percent. Because of time aggregation, these estimates are upper bounds.

worker heterogeneity and thus none can address the distributional questions that are the core of our paper.⁴

The rest of our paper is organized as follows. In the next section, we lay out our model and establish the existence of equilibrium. In Section 3, we discuss our calibration. Section 4 presents the results of counterfactual experiments, and Section 5 concludes.

3.2 MODEL

We consider a model with search and matching frictions. Only the unemployed search, and their prospects depend on overall labor market tightness, $\theta = (v_p + v_g)/u$, where v_p and v_g are the measures of private- and public-sector vacancies posted at any instant, and u is the fraction of the workforce that is unemployed. Search is random, so conditional on meeting a prospective employer, the probability that the job is in the private sector is $\phi = v_p/(v_p + v_g)$.⁵ Specifically, job seekers meet prospective employers at Poisson rate $m(\theta)$, and employers meet job seekers at rate $m(\theta)/\theta$. Not all meetings lead to matches. In the private sector, a match forms if and only if the realized value of x is high enough so that the match is jointly worthwhile for the worker and firm. The threshold value of x depends in general on the worker's type. That is, a private-sector match forms if and only if $x \geq R_p(y)$, where $R_p(y)$ is a type-specific reservation productivity. In the public sector, a match forms if and only if $x \geq R_g(y)$. The key equilibrium objects are the reservation productivity schedule, $R_p(y)$, overall labor

⁴In addition to these four completed papers, Navarro and Tejada (work in progress) are applying our approach to analyze the interaction between private- and public-sector labor markets in Chile.

⁵As will be seen below, an assumption of sector-specific search, as in Gomes (2015) and Quadrini and Trigari (2007), would give the unrealistic prediction of perfect sorting. That is, all workers above some type y^* would search exclusively in one sector while all workers of type below y^* would search in the other sector.

market tightness, θ , and the fraction, ϕ , of vacancy postings that are accounted for by the private sector. These objects are determined in equilibrium by (i) the condition that private-sector matches form if and only if doing so is in the joint interest of the worker and firm, (ii) a free-entry condition for private-sector vacancies, and (iii) steady-state conditions for worker flows into and out of unemployment, private-sector employment and public-sector employment.

3.2.1 VALUE FUNCTIONS, WAGES, RESERVATION VALUES

We start with the optimization problem for a worker of type y . Let $U(y)$, $N_p(x, y)$, and $N_g(x, y)$ be the values (expected discounted lifetime incomes) associated with unemployment and employment in, respectively, a private-sector job and a public-sector job with match-specific productivity x . The value of unemployment for a worker of type y is defined by

$$rU(y) = z(y) + \phi m(\theta) E \max[N_p(x, y) - U(y), 0] + (1 - \phi)m(\theta) E \max[N_g(x, y) - U(y), 0] \quad (3.1)$$

This expression reflects the following assumptions. Time is continuous, and the worker lives forever, discounting the future at rate r . The worker of type y receives a flow value $z(y)$ while unemployed. Private-sector vacancies are met at rate $\phi m(\theta)$, and public-sector vacancies are met at rate $(1 - \phi)m(\theta)$. When the worker meets a vacancy, a match-specific productivity is realized, and the worker realizes a capital gain, either $N_p(x, y) - U(y)$ or $N_g(x, y) - U(y)$, if the relevant difference is positive; zero otherwise.⁶

The two employment values are defined by

$$rN_p(x, y) = w_p(x, y) + \delta_p(y)(U(y) - N_p(x, y)) \quad (3.2)$$

⁶Note that we allow the flow value of unemployment, z , to vary with worker type. The contact rates, $\phi m(\theta)$ and $(1 - \phi)m(\theta)$ are assumed to be independent of y ; however, the job accession rates in the two sectors do vary with y since not all contacts lead to a match.

$$rN_g(x, y) = w_g(x, y) + \delta_g(y)(U(y) - N_g(x, y)). \quad (3.3)$$

The private-sector wage is determined by Nash bargaining with an exogenous worker share parameter, as described below, while the public-sector wage schedule is exogenous. Job destruction is assumed to occur at exogenous Poisson rate $\delta_s(y)$, and we allow for the possibility that $\delta_p(y) \neq \delta_g(y)$.

On the private-sector firm side, let $J(x, y)$ be the value (expected discounted profit) associated with a job filled by a worker of type y whose match-specific productivity is x , and let V be the value associated with posting a private-sector vacancy. These values are defined by

$$rJ(x, y) = x - w_p(x, y) + \delta_p(y)(V - J(x, y)) \quad (3.4)$$

$$rV = -c + \frac{m(\theta)}{\theta} E \max[J(x, y) - V, 0]. \quad (3.5)$$

The expectation in equation (3.5) is taken with respect to the joint distribution of (x, y) across the population of unemployed job seekers. A private-sector firm with a vacancy doesn't know what worker type it will meet next nor does it know what match-specific productivity this worker will draw. The firm does know, however, the distribution of worker types among the unemployed and the conditional distribution function $G_p(x|y)$.

We assume that the private-sector wage for a worker of type y with match-specific productivity x is determined via Nash bargaining with exogenous worker share parameter β . Imposing the free-entry condition for private-sector vacancy creation in advance, i.e., $V = 0$, the Nash bargaining solution implies

$$w_p(x, y) = \beta x + (1 - \beta)rU(y); \quad (3.6)$$

that is, the private-sector wage is a weighted average of the flow productivity of the match, x , and the flow value of the worker's outside option, $rU(y)$.

Substituting equation (3.6) into equation (3.2) and assuming that $w_g(x, y)$ is increasing in x for $x \geq R_g(y)$, it is clear that $N_p(x, y)$ and $N_g(x, y)$ are nondecreasing in x for any value of y . Accordingly, reservation productivities can be defined for the type- y worker. The private-sector reservation productivity for a type- y worker, $R_p(y)$, is defined by $N_p(R_p(y), y) = U(y)$. Using equations (3.2) and (3.6), $N_p(R_p(y), y) = U(y)$ implies $R_p(y) = rU(y)$. That is, at $x = R_p(y)$ the net surplus associated with the match equals zero. The public-sector reservation productivity for a type- y worker is simply $R_g(y)$. This is equivalent to assuming that, given the public-sector wage schedule, $N_g(R_g(y), y) \geq U(y)$. If $N_g(R_g(y), y) > U(y)$, there is rationing of public-sector jobs for type- y workers. If $N_g(R_g(y), y) = U(y)$, then $R_g(y) = rU(y) = R_p(y)$; that is, the public- and private-sector reservation productivities are equal for the type- y worker. Finally, we could in principle consider the case of $N_g(R_g(y), y) < U(y)$. In this case, however, matches would not form for $x \in [R_g(y), R_p(y))$ because workers would reject them. In this sense, it is without loss of generality to assume $N_g(R_g(y), y) \geq U(y)$.

To further characterize the private-sector reservation productivity, it is useful to rewrite our expression for $rU(y)$. Using equations (3.2) and (3.6) and integrating by parts gives

$$E \max[N_p(x, y) - U(y), 0] = \frac{\beta}{r + \delta_p(y)} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx.$$

Similarly, using equation (3.3) together with $rU(y) = R_p(y)$ gives

$$E \max[N_g(x, y) - U(y), 0] = \frac{1}{r + \delta_g(y)} \int_{R_g(y)}^{\bar{x}} (w_g(x, y) - R_p(y)) dG_g(x|y).$$

Substituting into equation (3.1) then gives

$$\begin{aligned}
R_p(y) &= z(y) + \phi m(\theta) \frac{\beta}{r + \delta_p(y)} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx \\
&\quad + (1 - \phi) m(\theta) \frac{1}{r + \delta_g(y)} \int_{R_g(y)}^{\bar{x}} (w_g(x, y) - R_p(y)) dG_g(x|y).
\end{aligned} \tag{3.7}$$

Given overall labor market conditions, i.e., θ and ϕ , and the government's employment and wage-setting policy, equation (3.7) gives a unique solution for $R_p(y)$ since the RHS of equation (3.7) is positive at $R_p(y) = 0$, goes to z as $R_p(y) \rightarrow \infty$, and the derivative of the RHS with respect to $R_p(y)$ is negative.

3.2.2 FREE-ENTRY AND STEADY STATE CONDITIONS

The next step is to characterize optimal entry by private-sector firms. Imposing $V = 0$ in advance and using equation (3.4), we have

$$J(x, y) = \frac{x - w_p(x, y)}{r + \delta_p(y)} = (1 - \beta) \frac{x - R_p(y)}{r + \delta_p(y)}.$$

Letting $F_u(y)$ denote the distribution function of Y among the unemployed, the free-entry condition, i.e., equation (3.5) with $V = 0$, can be written as

$$\begin{aligned}
c &= \frac{m(\theta)}{\theta} \left(\frac{1 - \beta}{r + \delta_p} \right) \int_{\underline{y}}^{\bar{y}} \int_{R_p(y)}^{\bar{x}} (x - R_p(y)) dG_p(x|y) dF_u(y) \\
&= \frac{m(\theta)}{\theta} \left(\frac{1 - \beta}{r + \delta_p(y)} \right) \int_{\underline{y}}^{\bar{y}} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx dF_u(y),
\end{aligned} \tag{3.8}$$

where the final equality uses integration by parts.

The only unknown in equation (3.8) is the contaminated distribution function, $F_u(y)$. Using Bayes Law, we can write⁷

$$F_u(y) = \frac{u(y)F(y)}{u};$$

⁷A similar derivation is given in Albrecht, Navarro and Vroman (2009).

that is, the distribution of types among the unemployed, $F_u(y)$, can be written as the type-specific unemployment rate, $u(y)$, times the population distribution function, $F(y)$, normalized by the overall unemployment rate,

$$u = \int_{\underline{y}}^{\bar{y}} u(y) dF(y).$$

To derive the type-specific unemployment rates, $u(y)$, let $n_p(y)$ and $n_g(y)$ be the fractions of time that a type- y worker spends in private-sector and public-sector employment, respectively. In steady state, the following two equations must hold:

$$\delta_p(y)n_p(y) = \phi m(\theta)(1 - G_p(R_p(y)|y))u(y) \quad (3.9)$$

$$\delta_g(y)n_g(y) = (1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))u(y). \quad (3.10)$$

The first condition equates the flow from private-sector employment to unemployment and vice versa, and the second condition equates the flow from public-sector employment to unemployment and vice versa. Using

$$u(y) + n_p(y) + n_g(y) = 1,$$

equations (3.9) and (3.10) imply

$$\begin{aligned} u(y) &= \frac{\delta_g(y)\delta_p(y)}{\delta_g(y)\delta_p(y) + \delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))} \\ n_p(y) &= \frac{\delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y))}{\delta_g(y)\delta_p(y) + \delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))} \\ n_g(y) &= \frac{\delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))}{\delta_g(y)\delta_p(y) + \delta_g(y)\phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p(y)(1 - \phi)m(\theta)(1 - G_g(R_g(y)|y))} \end{aligned} \quad (3.11)$$

Substituting the expression for $u(y)$ into equation (3.8) completes the characterization of the private-sector free-entry condition.

The final unknown that needs to be characterized is ϕ , the fraction of vacancies that are posted by private-sector firms. To do this, note that since

$$v_p + v_g = \theta u,$$

$$\phi = v_p / (v_p + v_g)$$

implies

$$\phi = \frac{\theta u - v_g}{\theta u}. \tag{3.12}$$

This closes the model.

3.2.3 EQUILIBRIUM

Definition: A steady-state equilibrium is a function, $R_p(y)$, that satisfies equation (3.7) for all $y \in [\underline{y}, \bar{y}]$ together with scalars θ and ϕ that satisfy equations (3.8), (3.11) and (3.12).

An equilibrium always exists. First, as noted above, for given values of θ and ϕ , the reservation productivity, $R_p(y)$, is uniquely determined. Second, given any value of ϕ , equation (3.8) has at least one solution for θ . The argument is standard. The RHS of equation (3.8) is continuous in θ , it converges to infinity as $\theta \rightarrow 0$, and it goes to zero as $\theta \rightarrow \infty$. Finally, once $R_p(y)$ and θ are determined as functions of ϕ , equation (3.12) has at least one solution in ϕ . (The complication, of course, is that u depends on ϕ .) Note that we do not claim uniqueness. In equation (3.8), $f_u(y)$ need not be monotonically decreasing in θ nor is it obvious that equation (3.12) has a unique solution. Uniqueness depends on the form of $F(y)$, $G_p(x|y)$, $G_g(x|y)$ and public-sector employment policy and needs to be investigated numerically.⁸

⁸The possibility of non-uniqueness of equilibrium is a common feature of DMP models with worker heterogeneity. See, e.g., Albrecht, Navarro and Vroman (2009) and Chéron, Hairault and Langot (2011).

Given any parameter configuration and given any assumed public-sector wage and employment policy, once we know $\{R_p(y), \theta, \phi\}$, the model can be solved for the equilibrium distributions of wages, productivities and human capital across the two sectors. This can be done analytically. The model gives us the distribution of Y across the unemployed, namely, $F_u(y)$, and the conditional distributions, $G_p(x|y)$ and $G_g(x|y)$, are given exogenously. Then, using the reservation productivity rules, $R_p(y)$ and $R_g(y)$, together with the contact rates, $\phi m(\theta)$ and $(1 - \phi)m(\theta)$, and the job destruction rates, $\delta_p(y)$ and $\delta_g(y)$, we can derive the joint distributions of (X, Y) across the two sectors. Finally, using the Nash bargaining rule for the private sector and the exogenous wage-setting rule, $w_g(x, y)$, for the public sector, we can derive the distributions of wages across the two sectors. Another approach is to find the equilibrium distributions by simulating the model. That is, we feed the assumed distribution of worker types into the model and use the distributions of wages, productivities and human capital across the two sectors that are generated by simulation. This latter approach is the one we use below.

3.3 CALIBRATION

3.3.1 DATA

To calibrate the model, we use data from the Colombian Household Survey (GEIH) from the second quarter of 2013. These surveys are repeated cross sections that are carried out by the Colombian Statistics Department (DANE) and are administered to a sample of employed and unemployed individuals in thirteen metropolitan areas.⁹ We restrict our sample to male salaried full-time workers, and we exclude workers with less

⁹These areas include the following cities and their metropolitan areas: Bogotá, Cali, Medellín, Barranquilla, Bucaramanga, Manizales, Pasto, Pereira, Cucuta, Villavicencio, Ibaguè, Monteria and Cartagena.

than a completed primary school level of education. We also exclude the self-employed, domestic employees and unpaid family workers. The objective of these exclusions is to construct a sample that is primarily comprised of formal-sector workers.¹⁰ Our sample consists of 8,508 individuals, who represent 2.2 million people.

Our calibration strategy is informed by the quality of data available from the household surveys. The data we work with are as follows. First, we know the number of years of education completed by each individual in the sample. Completed primary corresponds to five years of education, and we index workers by years of education completed, $j = 5, \dots, 20$.¹¹ Second, we know whether each respondent is unemployed, employed in the private sector, or employed in the public sector, that is, we know the distribution of workers across the three labor market states of the model. Third, we observe wages for private- and public-sector employees. More precisely, we observe monthly earnings and weekly hours and use these to construct an hourly wage for each employed worker.¹² Wages include tips and commissions. Since we are concerned about measurement error, we trim the top and bottom 1% of wages in each educational class. That is, we trim the top and bottom 1% of wages among workers who have completed 5 years of education, we do the same for workers who have completed 6 years of education, etc. In addition, because we use minimum observed wages in our calibration strategy to estimate reservation productivities (see below), we trim another 9% of wages from the bottom of the distribution within each educational

¹⁰For an equilibrium search and matching model of the interplay between the informal and formal labor markets in Colombia, see Robayo-Abril (2014).

¹¹We drop workers with more than 20 years of education because there are too few individuals in this category.

¹²We express wages in 2008 terms because that is the base year for the Colombian consumer price index.

class. In short, we trim the top 1% and the bottom 10% of observed wages within each educational class.¹³

Educational attainment, labor market state and wage all refer to the respondent's situation as of the survey date, so we are reasonably confident in these data. In addition, retrospective data are available on each respondent's labor market state in the previous year and on his elapsed duration in his current labor market state, but we view these data as less reliable. Regarding previous labor market state, the data suffer from the standard time aggregation problem. For example, a respondent who reports himself as unemployed as of the survey date and also reports that he was unemployed one year prior may have had an employment spell (or spells) in the intervening period. The duration data are also problematic. In particular, an individual who is currently employed reports how many months elapsed between the end of his previous job and the start of his current job, but we cannot tell whether he was unemployed or out of the labor force (a state not included in our model) in the intervening period. Accordingly, we primarily rely on the education, labor market state and wage data in our calibration. We do, however, use data on average durations in private- and public-sector employment in one step in our calibration procedure.

3.3.2 STYLIZED FACTS

We emphasize the following broad facts about the Colombian labor market. First, the level of public sector employment is quite low in Colombia, and the unemployment

¹³We have done a partial robustness check with respect to the trimming rule. Specifically, we have looked at some of the implications of trimming the bottom 5% and the bottom 1% of wages in each educational class, and these alternative trimming rules do not change our results qualitatively. We prefer 10% because with this trimming rule less than one percent of public-sector wages are below the legal minimum (\$1.20 per hour in Colombia in 2008 dollars). In principle, no public-sector wages should be less than this value. With a 5% rule, almost 2% of public-sector wages are below the legal minimum; with a 1% rule, almost 4% are below the minimum wage.

rate is quite high. As can be seen in Table 3.1, the public sector accounts for 8.3% of total employment, which is quite low by developed and middle-income country standards and is approximately half the level of most Latin American countries.¹⁴ Second, wages in the public sector are considerably higher than in the private sector. Figure 3.1 shows the kernel density of log wages by sector. As shown in Table 3.2, the mean log hourly wage in the public sector is \$1.33 as compared to \$0.85 in the private sector, a difference of 0.48 log wage points.¹⁵ This is a large public-sector premium.¹⁶ The degree of wage dispersion is similar in the two sectors – the standard deviation of log hourly wages is \$0.57 in the public versus \$0.60 in the private sector. This is also in contrast to the typical developed and middle-income country pattern, which exhibits a tendency towards wage compression in the public sector. Finally, even though the duration data are less than perfect, it is clear from Table 3.2 that employment tends to last much longer in the public sector than in the private sector.

The primary observation that motivates our calibration strategy is the fact that wages in the public sector are so much higher than those in the private sector. This fact has several possible explanations. First, there may be a pure public-sector premium; that is, the public-sector wage may simply add a bonus to what an equally qualified and equally productive worker would earn in the private sector. Second, public-sector workers are more highly educated on average than are their private-sector counterparts. Specifically, on average the public-sector workers in our sample have completed 13.9 years of education while the corresponding figure for private-sector workers is 11.6 years of education. Third, the weights on the wage determination rule on pro-

¹⁴See Table 1 in Mizala et al. (2011). Note that the figures presented there include all urban workers.

¹⁵The corresponding figures in levels are as follows. The mean hourly wage is \$4.50 in the public sector and \$3.03 in the private sector, which corresponds to a 48.5% difference.

¹⁶See Table 2 in Mizala et al. (2011) for Latin American wage gaps. See Borland and Gregory (1999) for a survey of results on the public-sector premium in developed countries.

ductivity versus qualifications may differ between the two sectors. For example, the public-sector wage may give a higher reward to credentials (years of education) with a corresponding lower reward to productivity than is the case in the private sector. Finally, there may be inherent productivity differences between the two sectors; that is, the distributions of productivity conditional on qualifications may not be the same in the private and public sectors. Conditional on our identifying assumptions, we are able to make some progress in our calibration towards distinguishing among these four explanations of the public-sector wage premium.

3.3.3 CALIBRATION STRATEGY

Our calibration strategy consists of the following steps.

Step 1: We begin by specifying public-sector wage and employment rules. We assume that wages in the public sector are determined by a surplus splitting ruler and, in addition, we allow for the possibility of a pure public-sector premium. Specifically, we assume

$$w_g(x, y) = \psi + \gamma x + (1 - \gamma)R_p(y).$$

Here ψ represents the pure public-sector premium, γ represents the weight placed on match-specific productivity in public-sector jobs, and $1 - \gamma$ represents the weight placed on “qualifications.” In general, we allow for the possibility that $\gamma \neq \beta$ – for example, the public sector may place a relatively strong emphasis on credentials in its wage setting – that is, $\gamma < \beta$ –, but we set $\gamma = \beta$ in our baseline calibration.

We also need to specify which workers the public sector is willing to hire. We do this by assuming that the public sector hires if and only if $x \geq w_g(x, y)$. That is, when an unemployed worker makes contact with a public-sector vacancy, that contact generates a match if and only if the worker’s productivity is at least as great

as the wage he would be paid in the match. This is in the spirit of a basic assumption of the DMP model in the private sector, namely, that a match forms if and only if it is in the joint interest of the worker and employer to do so. Note that setting $R_g(y) = w_g(R_g(y), y)$ implies

$$R_g(y) = \frac{\psi}{1 - \gamma} + R_p(y).$$

The term $\frac{\psi}{1 - \gamma}$ reflects the extent to which public-sector jobs are rationed.¹⁷

Finally, public-sector employment policy is characterized by v_g , the measure of vacancies posted in the public sector. Rather than specifying the level of public-sector vacancy creation exogenously, we estimate v_g as a part of our calibration, as described below.

Step 2: A basic exogenous object in our model is the distribution of human capital, Y , across workers. To estimate this distribution, we equate human capital with completed years of education. We discretize the distribution of Y with p^j estimated as the fraction of the labor force with j years of education for $j = 5, \dots, 20$. We also observe the empirical counterparts of $\{p_p^j\}_{j=5}^{20}$ and $\{p_g^j\}_{j=5}^{20}$, that is, the distributions of educational attainment in private- and public-sector employment. When we assess the goodness of fit of our model, we compare these empirical distributions with the corresponding distributions predicted by our calibrated model.

Step 3: We estimate reservation productivities for each worker type for private- and public-sector employment. Given that private-sector wages are determined by Nash bargaining with exogenous share parameter β , we have

$$w_p^j(x) = \beta x + (1 - \beta)R_p^j; \tag{3.13}$$

¹⁷This discussion implicitly assumes that $\psi \geq 0$, which is what our calibration will indicate.

that is, a worker with j years of education with realized productivity x on a private-sector job receives wage $w_p^j(x)$. A private-sector match with a worker of type j forms if and only if $x \geq R_p^j$, and a worker of this type with match-specific productivity R_p^j receives a wage of $w_p^j(R_p^j) = R_p^j$. Accordingly, we use the minimum observed private-sector wage (after trimming) among workers with j years of education to estimate R_p^j . Similarly, we use the minimum observed public-sector wage among workers with j years of education to estimate R_g^j . This procedure gives us estimates \widehat{R}_p^j and \widehat{R}_g^j for $j = 5, \dots, 20$. We use these estimated reservation productivities to estimate the public-sector rationing factor, namely, $\psi/(1-\gamma)$, using $\sum_{j=5}^{20} \widehat{p}_j \left(\widehat{R}_g^j - \widehat{R}_p^j \right)$. Note that we are imposing the restriction that the public-sector rationing factor is the same for all worker types.

Step 4: Since the private-sector wage for a worker of type j is linear in x and R_p^j , once we have an estimate for R_p^j , we can make assumptions that allow us to use the observed distribution of private-sector wages across workers with j years of education to estimate $G_p^j(x)$, that is, the distribution of private-sector productivity across workers with j years of education. To do this, we assume that $G_p^j(x)$ is a log-normal distribution function with parameters μ_p^j and σ_p^j ; that is, we assume that the log of productivity in potential private-sector jobs across workers having j years of education is normally distributed with mean μ_p^j and standard deviation σ_p^j . Using equation (3.13), we have

$$\ln x = \ln \left(\frac{w_p^j - (1 - \beta)R_p^j}{\beta} \right).$$

As is typical in models in which the wage is determined by Nash bargaining, β is unidentified. Accordingly, we follow the literature and assume a standard value for the share parameter, namely, $\beta = 0.5$.¹⁸ Given this assumed value for β , our estimate

¹⁸Employer-side data on productivity are typically required to identify β . Note that we are assuming the same value of β for all worker types, that is, for all j . In order for equation

for R_p^j , and observed wages, we have a set of estimated values for the log productivity of workers of type j who are employed in private-sector jobs. We then use expressions for the mean and variance of a truncated ($\ln x \geq \ln R_p^j$) log-normal distribution to back out estimates of μ_p^j and σ_p^j .

We use an analogous procedure to estimate $G_g^j(x)$, that is, the conditional distribution of public-sector productivity across workers with j years of education. Again, we assume log normality, this time with parameters μ_g^j and σ_g^j . Using

$$w_g^j(x) = \psi + \gamma x + (1 - \gamma)R_p^j$$

or, equivalently,

$$w_g^j(x) = \gamma x + (1 - \gamma)R_g^j$$

gives

$$\ln x = \ln \left(\frac{w_g^j - (1 - \gamma)R_g^j}{\gamma} \right).$$

Since the public-sector share parameter is unidentified, we assume $\gamma = \beta = 0.5$ in our baseline calibration. We then have a set of estimated values for the log productivity for workers with j years education across public sector employees, and, exactly as we did with the private-sector parameters, we use these values to back out estimates of μ_g^j and σ_g^j .

Step 5: Next, we estimate the parameters governing transitions from unemployment to private- and public-sector employment and vice versa. Since the duration information in our dataset is retrospective and subject to a variety of biases (time aggregation, etc.) we want to minimize the extent to which we use duration data in this estimation procedure. Our assumption that workers contact private-sector vacancies at the same rate independent of type, that is, the assumption that $m(\theta)\phi$ does not vary with j , (3.7) to hold for each worker type, either β or z needs to vary with j . Our choice is to keep β fixed while allowing z to adjust across worker types.

and similarly for the rate at which workers contact public-sector vacancies, helps us achieve this objective.

We proceed as follows. Workers with j years of education move from unemployment to employment at rate $m(\theta)\phi(1-G_p^j(R_p^j))$, and they flow in the opposite direction at rate δ_p^j ; thus, in steady-state,

$$m(\theta)\phi(1-G_p^j(R_p^j))u^j = \delta_p^j n_p^j. \quad (3.14)$$

Similarly, the flow of type j workers from unemployment to public-sector employment and vice versa satisfies

$$m(\theta)(1-\phi)(1-G_g^j(R_g^j))u^j = \delta_g^j n_g^j. \quad (3.15)$$

These steady-state equations hold for each worker type. Once we estimate $m(\theta)$ and ϕ , these equations give us estimates of the job destruction rates, $\{\delta_p^j\}_{j=5}^{20}$ and $\{\delta_g^j\}_{j=5}^{20}$.

To estimate $m(\theta)$ and ϕ , we use expressions for the average durations of private- and public-sector employment. The model assumes exponential durations; thus, for example, the expected duration of private-sector employment for a worker with j years of education is $1/\delta_p^j$. The expected duration of private-sector employment averaged across all worker types can therefore be written as

$$E[T_p] = \sum_{j=5}^{20} p_p^j \left(\frac{1}{\delta_p^j} \right).$$

Using equation (3.14),

$$E[T_p] = \sum_{j=5}^{20} p_p^j \left(\frac{n_p^j}{m(\theta)\phi(1-G_p^j(R_p^j))u^j} \right). \quad (3.16)$$

Similarly, the expected duration of public-sector employment across all worker types is

$$E[T_g] = \sum_{j=5}^{20} p_g^j \left(\frac{n_g^j}{m(\theta)(1-\phi)(1-G_g^j(R_g^j))u^j} \right). \quad (3.17)$$

The only “unknowns” on the right-hand sides of equations (3.16) and (3.17) are $m(\theta)$ and ϕ . Plugging in the sample counterparts for $E[T_p]$ and $E[T_g]$ together with our already-computed estimates of the various objects on the right-hand sides of equations (3.16) and (3.17) gives us estimates of $m(\theta)$ and ϕ .

Step 6: The final step in our calibration procedure assembles a number of loose ends. First, we back out an estimate for θ . To do this, we assume Cobb-Douglas matching, namely,

$$m(\theta) = A\theta^\alpha.$$

Since reliable vacancy data are not available in Colombia, we set values for A and α . Specifically, we choose $\alpha = 0.5$, so the Hosios condition is satisfied, and then set $A = 0.25$. The latter choice is made to be consistent with the literature (e.g., Pissarides and Petrongolo 2001) and to produce a “reasonable” value of θ in the calibration. Given an estimate of $m(\theta)$ from the previous step, we then have an estimate for θ .

Next, we use our estimates of θ and ϕ together with equation (3.12) to set a value for v_g . We also use our estimates of θ and ϕ to back out an estimate of c , using the free-entry condition for private-sector vacancy creation. To do this, we need to fix a value for the discount rate, and we set $r = 0.0217$.¹⁹

Finally, the last set of parameters that we estimate are the type-specific flow values of leisure, that is, the $\{z^j\}_{j=5}^{20}$. These estimates are backed out from a discretized version of equation (3.7). As in many other studies, we find negative values.²⁰

¹⁹This is consistent with an annual real interest rate of 8.96%.

²⁰As discussed in Hornstein, Krusell and Violante (2011), see Table 7 in Eckstein and Wolpin (1995) or Table II in the survey paper by Bunzel et al. (2001).

3.3.4 CALIBRATION RESULTS

We use a quarter as the unit of time in our calibration. Tables 4 and 5 show the estimated parameters. Table 4 presents the estimates of parameters that are assumed to be independent of worker type. First, we estimate that the flow cost of posting a private-sector vacancy is $\widehat{c} = \$3.66$ per hour, about 16% more than the average wage. Second, we estimate the probability that an unemployed worker makes contact with a job opening within a quarter to be $\widehat{m(\theta)} = 0.214$. Since we observe long average durations of employment, a low contact probability is required to fit the observed unemployment rate of $u = 0.230$. Given our assumptions that $A = 0.25$ and $\alpha = 0.5$, our estimate of $m(\theta)$ implies an estimated labor market tightness of $\widehat{\theta} = 0.736$, which is in the usual range of estimates for this variable. Third, we estimate $\widehat{\phi} = 0.923$; that is, about 92% of all vacancy postings are made in the private sector.

Table 4 also gives our estimates of the parameters that describe the public-sector wage and employment policy rules. First, we estimate $\psi = \$0.17$; that is, we estimate a pure-public sector premium of \$0.17 per hour, which is a bit more than 5% of the average wage observed in the data. Given our assumption of $\gamma = 0.5$, this implies a public-sector rationing factor of \$0.34 per hour; that is the reservation productivity in the public sector is higher than the corresponding private-sector cutoff value by 34 cents per hour's worth of output. Finally, we use equation (12) to back out $\widehat{v}_g = 0.013$. Since the size of the labor force is normalized to one, the interpretation is that at any point in time, slightly more than one public-sector vacancy is posted for every 100 workers in the labor force.

Table 5 presents estimates of the parameters that we allow to vary with worker type, that is, with j . In the second column of the table, note the relatively high weights associated with $j = 5$ (completed primary education), $j = 11$ (completed secondary

education), $j = 13$ (2 years of post-secondary education), $j = 14$ (3 years of post-secondary education), and $j = 16$ (university education). Column 3 presents our estimates of the $\{z^j\}_{j=5}^{20}$. These estimates are (i) uniformly negative and (ii) strongly decreasing in j . These estimates lack a natural interpretation. Our general approach in calibrating the model has been to give ourselves sufficient degrees of freedom – in this case, allowing z to vary with j – to allow the model to fit the data well. Starting from a good fit makes it easier to interpret the results of the counterfactual experiments that we present in the next section. As noted above, other studies also find negative z . Columns 4 and 5 present our estimates of the parameters characterizing the conditional distributions $G_p^j(x)$, and columns 6 and 7 present the corresponding estimates of the parameters describing the public-sector distributions. In general, expected productivity increases with education in both sectors, although not perfectly monotonically. Similarly, productivity dispersion increases with j . Comparing the two sectors, our estimates suggest that $\mu_p^j > \mu_g^j$ for very low and very high levels of j . For intermediate levels of j (at least some education beyond basic primary but no more than a university degree), the data suggest the opposite. Comparing columns 5 and 7, there is some tendency for $\sigma_p^j > \sigma_g^j$. The same patterns obtain for the expected values and standard deviations of productivity conditional on $X_s^j \geq R_s^j$ for $s = p, g$. Columns 8 and 9 show the estimated values of the job destruction parameters in the two sectors. These two columns show a very strong tendency for expected duration of employment to increase with years of education in both sectors. In addition, we observe a tendency towards $\delta_g^j > \delta_p^j$ for lower values of j but the opposite (and strongly so) for all levels of education beyond completion of secondary schooling. Finally, the last column of Table 5 shows our estimates of reservation productivities for private-sector employment for each worker type. As expected, these estimated reservation productivities are (almost) monotonically increasing in worker type. Since

$R_g^j = 0.34 + R_p^j$, estimates of reservation productivities for public-sector employment by worker type can be found by adding the public-sector rationing factor (0.34) to the corresponding private-sector reservation productivities given in the last column of Table 5.²¹

Table 6 provides a basis for assessing goodness of fit. In this table, using a set of unconditional moments, we compare the predictions of our calibrated model with the corresponding sample moments. The predicted moments are generated by simulating the model with all parameters set to their calibrated values. The model does a perfect job of matching the aggregate distribution of workers across unemployment, private-sector employment and public-sector employment. In terms of wages, our primary focus is on (i) the mean gap in log wages and (ii) the relative dispersion of log wages between the two sectors and (iii) overall log wage dispersion. Here our model provides a close but not perfect fit. We slightly overestimate the mean public-private log wage gap (0.490 versus 0.476) and slightly underestimate the ratio of the standard deviation of public-sector log wages to corresponding figure for private-sector log wages (a ratio of 0.907 versus 0.950). Our model-generated estimate of the overall standard deviation of log wages is essentially equal to what we see in the data. Table 6 also compares the estimated mean and standard deviation of log wages to the corresponding sample moments for each of the sectors. As can be seen, the estimated moments for the private-sector match what we see in the data almost perfectly, while the estimated public-sector moments match the data well but not perfectly. The main factor lying behind our lack of a perfect fit in the public sector is our assumption that the public-

²¹Note that we can estimate the $\{R_g^j\}$ in two different ways. In our counterfactual simulations we use $R_g^j = \psi + R_p^j$ with R_p^j estimated as the minimum observed private-sector wage among type j workers. Alternatively, we can estimate R_g^j as the minimum observed public-sector wage among type j workers. For low values of j , the minimum observed public-sector wage tends to be greater than $R_g^j = 0.34 + R_p^j$; for higher values of j , the inequality is reversed. In general, however, the two estimators coincide reasonably well.

sector rationing factor, namely, $\psi/(1 - \gamma)$, is the same for all worker types. We make this assumption – which entails some sacrifice of goodness of fit – because we want our counterfactual experiments which examine the effects of varying the public-sector parameters one by one to be easily interpretable.

Table 6 also shows the mean and standard deviation of educational attainment in the public and private sectors. The key observation is that the mean number of years of education among public-sector employees exceeds the corresponding private-sector mean by more than two years. This is what we observe in the data, and the model captures this sorting almost perfectly. Finally, Table 6 also compares the mean employment duration in the two sectors that we see in the data with the corresponding model prediction. As in the data, the mean duration of employment in the public-sector predicted by the model is approximately 2.5 times the corresponding figure for the private sector.

We also compare the distributions of worker types across the three labor market states that are generated by the model with the corresponding distributions observed in the data. These distributions (not shown) match up almost perfectly. The distribution of worker types between the private and public sectors together with our estimates of the sector-specific distributions of productivity conditional on worker type allow us to simulate the distributions of productivity in the two sectors. These are shown in Figure 2. The distribution of productivity across public-sector jobs is clearly to the right of the corresponding private-sector distribution. As can be seen in Tables 5 and 6, the reason is not that public-sector jobs are inherently more productive. Rather, the productivity difference between the two sectors has primarily to do with the fact that more highly educated workers – who are in general more productive – are more likely to be found in the public sector than in the private sector. Finally, we compare the private- and public-sector wage densities generated

by model simulations with the corresponding data-based kernel densities. This is done in Figure 3, and, as can be seen, the model-generated and estimated densities match reasonably well. We interpret the approximate correspondence between the observed and simulated kernels as an indication that years of education is a good approximation for worker human capital and that our log normality assumption is also a good approximation.

What can we learn from our calibration results? In particular, what accounts for the large difference between public- and private-sector wages in Colombia? Our baseline calibration suggests that most of the wage differential can be ascribed to the different distributions of education between the two sectors. More highly educated workers are more productive, both in the private sector and in the public sector. The sorting of the more highly educated workers into the public sector is not an indication that public-sector jobs are inherently more productive. Indeed, our estimates suggest that the means of the private-sector productivity distributions for highly educated workers are higher than the corresponding public-sector means. Instead, the pattern of educational sorting that we see is likely driven by (i) the (relatively small) pure public-sector premium, ψ , and (ii) the type-specific pattern across private- versus public-sector job destruction rates. The public-sector premium affects the distribution of worker types across the two sectors because it leads to job rationing in the public sector, and this rationing is more binding for the less educated part of the workforce. Regarding job destruction rates, the data indicate that $\delta_g^j > \delta_p^j$ for low values of j , but $\delta_g^j < \delta_p^j$ for high values of j . That is, less educated workers tend to retain their private-sector jobs for a relatively long time, but among more highly educated workers, the pattern is reversed. The bottom line is that public-sector workers are paid more because they are more highly educated (because job rationing tends to keep the less educated out of the public sector and because once highly educated

workers find public-sector employment, they tend to keep their jobs for a long time) and because they are more productive (primarily because more educated workers are more productive, irrespective of where they work).

3.4 COUNTERFACTUAL EXPERIMENTS

We now turn to our counterfactual experiments. In these experiments, we change the parameters that characterize the public-sector wage and employment rules and evaluate the impact of these changes on the unemployment rate, the division of employment between the two sectors, average duration of employment in the two sectors, and on the distributions of education, productivities, and wages across the two sectors. Starting from the baseline case, we change each of the public-sector employment policy parameters, namely, v_g , ψ and γ , in turn while holding all other parameters constant. The objective of these experiments is to shed further light on the source of the large public-sector wage premium in Colombia and, more generally, to get a better understanding of how the private- and public-sector labor markets interact in a search and matching framework. The results of our experiments are presented in Tables 7 and 8 and in Figures 4-6.²²

In the first experiment, we increase the size of the public sector by doubling the measure of government vacancies from $v_g = 0.013$ to $v_g = 0.026$. This policy has a straightforward compositional effect; namely, employment is shifted from the private to the public sector. In addition, the policy change generates a one percentage point increase in the unemployment rate. The reason is that, as higher-paying public-sector

²²In Table 8, note that since $R_g^j = \psi + R_p^j$ for all j , the reservation productivity for public-sector jobs averaged across all worker types is the corresponding mean reservation productivity for private-sector jobs plus the public-sector rationing factor (equal to \$0.34 in columns 1, 2 and 4 and equal to zero in column 3). Thus, the standard deviations of reservation productivities for jobs in the two sectors are the same in each column.

jobs become easier to find, private-sector reservation productivities increase, as can be seen in Table 8. Workers are pickier, and, as a result, private-sector firms are less willing to post vacancies; that is, θ falls. The policy change also has distributional effects. As shown in Table 8, the average level of education falls slightly in both the public and in the private sector. This reflects a shift in the incidence of unemployment relative to the baseline case towards more highly educated workers. Table 8 and Figure 4 show how this change in the educational composition of the public- and private-sector workforces spills over into productivities and wages across the two sectors. Mean productivity falls in both sectors, especially in the public sector. This decrease in average productivity translates to a decrease in the mean wage paid in the public sector. In the private-sector, however, the decrease in average productivity is more than offset by the increase in private-sector reservation productivities, and the mean wage rises. This is why the increase in v_g decreases the mean log wage gap between the two sectors. The policy change also reduces productivity dispersion in both sectors. The shift in employment towards somewhat less educated workers – again, caused by increased pickiness among the more highly educated – coupled with our result that the standard deviation of productivity tends to increase with education is what drives this result. This decrease in productivity dispersion is more pronounced among public-sector workers, and this translates into a reduction in relative log wage dispersion between the two sectors and in overall wage inequality; that is, both $\frac{\sigma(\ln w)_g}{\sigma(\ln w)_p}$ and $\sigma(\ln w)$ fall. Finally, since job destruction rates tend to decrease with education, the policy change leads to a slight decrease in average duration of employment in both sectors.

In the second experiment, we eliminate the pure public-sector wage premium by setting $\psi = 0$, while keeping γ and v_g at their baseline values. This policy change reduces the difference between the mean log wages in the two sectors from 0.490 to

0.428, which corresponds to a decrease of 20 cents in the difference in wages between the two sectors in levels.²³ The interesting result of this experiment is that the spillover effects of this policy change are quite small. With $\psi = 0$, the value of search falls; that is, workers become less picky about which private-sector jobs are acceptable. However, since jobs are scarce – and public-sector jobs are particularly difficult to find – this effect is small; the average value of R_p falls by only 2 cents. Since $R_g^j = \psi + R_p^j$ for all j and since ψ is being reduced from 0.34 to zero, the average value of R_g correspondingly falls by 36 cents. The direct effect of setting $\psi = 0$ thus accounts for the lion’s share of the decrease in the public-sector wage. Almost all of the rest of the effect comes from the relative decrease in average productivities between the two sectors. Since private-sector reservation productivities decrease only slightly, setting $\psi = 0$ has only a small effect on private-sector productivity (about 2 cents per hour on average). The decrease in public-sector reservation productivities is larger and has a correspondingly larger effect on average productivity in government jobs (about 8 cents per hour). This difference in productivity effects between the two sectors accounts for almost all of the remaining change in mean wages between the two sectors. See Figure 5 for the full effect of this policy on the wage distributions in the two sectors. Another interesting aspect of setting $\psi = 0$ is that it has almost no effect on the steady-state distribution of worker types across the two sectors. That is, even though the policy change eliminates public-sector job rationing, which is more of a constraint for less educated workers, we do not see a significant decrease in average years of education in the public sector.

In our third and final experiment, we decrease γ from 0.5 to 0.25. With $\gamma = 0.25$, the public sector puts less weight on productivity (and correspondingly more on

²³In the baseline case, the average public-sector wage is \$4.50 per hour, and the corresponding figure for private-sector wages is \$3.03. Setting $\psi = 0$ decreases the mean public-sector wage by 22 cents per hour and the mean private-sector wage by 2 cents per hour.

formal qualifications) in wage setting than the private sector does. This policy change has a substantial impact on relative wages; in particular, the public-sector premium decreases by 0.31 log-wage points. That is, the reweighting in the public-sector wage-setting rule eliminates almost 2/3 of the public-sector log wage premium. The reason for this effect is that there is more dispersion in match-specific productivity than there is in educational attainment. Since there is a concentration of more highly educated workers in the public sector (both before and after the policy change) and since match-specific productivity tends to increase with education, the lower value of γ decreases the reward that public-sector workers receive for their higher levels of productivity.²⁴ In addition, because a lower weight is being placed on the relatively high variation component of wage determination, the standard deviation of wages in the public sector falls substantially. The decrease in γ also has indirect effects. First, the public-sector rationing constraint becomes less binding, that is, $\psi/(1 - \gamma)$ falls. Public-sector reservation productivities fall accordingly. Second, private-sector reservation productivities fall as a result of the decrease in public-sector wages. This leads in turn to a decrease in private-sector wages, although this decrease is much less pronounced than in the public sector. Figure 6 shows the overall effects of the policy change on the distributions of log wages in the two sectors. The decrease in γ also has some (relatively small) compositional effects. The fall in private-sector reservation productivities and the corresponding decrease in wages makes private-sector vacancy creation more attractive. As a result, θ increases slightly with an attendant small shift of workers from unemployment to private-sector employment.

²⁴In many countries, the public-sector wage distribution is more compressed than the corresponding private-sector distribution. See, e.g., Gregory and Borland (1999). A higher weight on formal qualifications (and a correspondingly lower weight on productivity) in the public sector may explain this observation.

The results of our counterfactual experiments refine some of our conclusions about the factors underlying the large gap between public- and private-sector wages that we observe in the data. Starting with the basic conclusion that the public-sector wage premium is primarily driven by the sorting of more educated workers into government jobs, we can see that this pattern is reinforced in Colombia by the weak overall state of the labor market. On average, it takes an unemployed worker over a year to locate a job opportunity ($m(\theta) = 0.214$), and it takes a private-sector firm almost a year to locate a job candidate ($m(\theta)/\theta = 0.291$) for its vacancy. This means that neither workers nor firms can be very selective about which matches they consummate, and, our estimates indeed suggest that a very high percentage of contacts lead to matches. The importance of a weak labor market for our conclusions is illustrated by our first counterfactual experiment. When there are more public-sector vacancies in the market, workers become more selective about which jobs they will accept, and this effect is strongest for the most highly educated. This leads to a decrease in average educational attainment among both private- and public-sector workers, and this in turn reduces the public-private wage differential. Our second experiment helps us better understand why public-sector workers tend to be more highly educated than their private-sector counterparts. The sorting of the more highly educated into the public sector may mean that it is more difficult for the less educated to get jobs in this sector (public-sector job rationing); it may mean that less educated workers find it more difficult to retain their public-sector jobs. The results of our second experiment suggest that the latter factor is the more important one. When we set $\psi = 0$, there is a direct, almost one-for-one, effect on public-sector wages, but the corresponding elimination of public-sector job rationing has virtually no effect on the distribution of education across the two sectors. Finally, our third experiment reinforces our conclusion that the productivity difference between the two sectors is the main driver of the

wage differential. When we reduce the weight on productivity (by setting $\gamma = 0.25$) in the public-sector wage-setting rule, almost 2/3 of the difference in mean log wages between the two sectors is eliminated.

More generally, the results of our calibration and of our counterfactual experiments are consistent with an approach that focuses on worker heterogeneity as a key to understanding the interaction between private- and public-sector labor markets. Wage differences between the public and private sector in Colombia are primarily driven by productivity differences between the two sectors, and these productivity differences are in turn primarily driven by the different distributions of educational attainment across the workers in the two sectors. Although there is a (relatively small) pure public-sector wage premium in Colombia ($\psi = 0.17$) and while such pure premia may well exist in other countries, our approach suggests that it is of first-order importance to understand what lies behind the sorting of different worker types into the public versus the private sector. We focused on two potential explanations. First, more highly educated workers may reject private-sector jobs to wait for more attractive public-sector positions. This is more likely to happen when there is a pure public-sector premium; that is, $\psi > 0$ may, in addition to the direct effect of adding a top-up to public-sector wages, have a strong indirect effect by attracting more qualified workers to public-sector employment. This indirect effect is mostly absent in Colombia because job opportunities arrive too infrequently to allow workers to reject many private-sector jobs, but it is likely important in other countries. Second, there may be differences in retention patterns for different worker types between the private and public sector. Jobs in the public sector are typically viewed as “more secure” than private-sector jobs. To the extent that this is particularly true for highly educated workers (as in Colombia), this is a factor that increases the tendency for more highly educated workers to be concentrated in the public sector. Finally, our calibration suggests that

the inherent productivity associated with public-sector jobs (in our model, this is captured by the moments of the type-specific conditional productivity distributions) does not differ substantially from the inherent productivity of private-sector jobs. That is, productivity differences between the public and private sectors is likely to be primarily a matter of how worker types are sorted across the two sectors. This seems clearly to be the case in Colombia; we suspect the same is true elsewhere. To the extent that the underlying patterns that we observe in the Colombian data generalize to other countries, a better understanding of public-private wage differences and, more generally, how the public- and private-sector labor markets interact requires explicitly taking worker heterogeneity into account.

3.5 CONCLUSION

In this paper, we have developed a search-and-matching model to analyze the interaction between labor markets in the private and public sectors. The focus of our model is on distributional questions. What types of workers sort into the two sectors? How do the size of the public sector and the public sector's wage and employment policies affect the distribution of wages in the private sector and in the public sector? Given this focus, worker heterogeneity is a key element of our model. We calibrate our model using Colombian data. Colombia is an interesting case study because the wage differential between the public and private sectors there is very large. Our calibration and counterfactual experiments are motivated by a desire to differentiate among various potential explanations of this wage gap. Although there is a pure public-sector premium in Colombia, it is small relative to the differential that needs to be explained. Instead, the primary cause of the public-private wage differential in Colombia is that more highly educated workers, who tend to be more productive regardless of whether

they are employed in the private or public sector, get differentially sorted into public-sector employment. A relatively minor aspect of this sorting is that there is rationing of public-sector jobs and that this rationing tends to be more binding for less-educated workers. More importantly, public-sector employment is extremely stable for highly educated workers. Much more so than in the private sector, when a highly educated worker gets a public-sector job, he tends to keep that job for a very long time.

Public sector employment accounts for a significant fraction of employment in most economies, and the effect of public-sector labor market policy on overall labor market performance deserves more attention. The model and the calibration strategy developed in our paper can be applied more generally, and our focus on worker heterogeneity and the sorting of different worker types into the two sectors offers a useful complement to the existing literature.

TABLES

Table 3.1: Labor Market States

Unemployed (u)	0.230 (.0045)
Employed Private Sector (n_p)	0.705 (.0049)
Employed Public Sector (n_g)	0.064 (.0026)

Author's calculations based on GEIH, Second Quarter 2013, 13 Metropolitan Areas.
Standard Errors in parenthesis

Table 3.2: Descriptive Statistics for Employed Population

	Employed	Public	Private
Sample Size	6,265	675	5,590
Population	1,685,135	140,656	1,544,479
$E(\ln W_s)$	0.89	1.33	0.85
$SD(\ln W)$	0.62	0.57	0.60
$E(T_{es})$	4.95	11.01	4.40
$SD(T_{es})$	6.62	8.35	6.15

Author's calculations based on GEIH, Second Quarter of 2013, 13 Metropolitan Areas. Sample includes salaried full-time male workers with completed primary education. Self-employed and unpaid family workers are excluded. All statistics weighted using sampling weights. Log earnings in hourly rates, employment duration in months.

Table 3.3: Fixed Parameters -Based on data or previous micro studies

	Description	Value
PARAMETERS		
r	interest rate	0.02
β	worker's Nash bargaining power	0.50
A	technological parameter, matching function	0.25
α	elasticity matching function	0.50
γ	weight on productivity, public-sector wage rule	0.50

Table 3.4: Estimated Parameters (1)

	Description	Value
c	cost of posting a vacancy	3.658
$m(\theta)$	offer arrival rate	0.219
ϕ	fraction of private-sector vacancies	0.923
v_g	vacancies, public sector	0.013
ψ	public sector fixed factor, wage setting	0.170

Table 3.5: Estimated Parameters (2)

j	p^j	z^j	μ_p^j	σ_p^j	μ_g^j	σ_g^j	δ_p^j	δ_g^j	R_p^j
5	0.08	-0.02	0.99	0.38	0.74	0.40	0.10	0.59	0.75
6	0.03	-0.35	1.09	0.50	0.73	0.05	0.12	0.91	0.67
7	0.03	-0.15	1.02	0.36	1.18	0.22	0.10	1.91	0.64
8	0.03	-0.18	0.98	0.49	1.44	0.25	0.09	1.19	0.82
9	0.04	-0.46	1.08	0.41	1.42	0.42	0.09	0.24	0.75
10	0.02	-0.06	0.85	0.49	1.55	0.06	0.11	0.18	0.69
11	0.36	-0.40	0.92	0.46	1.33	0.48	0.06	0.07	1.00
12	0.05	-0.87	1.01	0.51	1.38	0.54	0.06	0.04	1.05
13	0.10	-1.06	0.99	0.59	1.36	0.42	0.05	0.04	1.25
14	0.08	-1.58	1.12	0.62	1.52	0.57	0.05	0.04	1.44
15	0.03	-1.79	1.25	0.67	1.66	0.41	0.06	0.05	1.51
16	0.09	-9.97	2.08	0.82	2.20	0.54	0.05	0.03	1.89
17	0.03	-18.08	2.42	0.60	2.26	0.39	0.03	0.02	2.24
18	0.02	-25.34	2.56	0.83	2.24	0.59	0.02	0.01	2.89
19	0.01	-24.72	2.72	0.70	2.39	0.47	0.03	0.01	2.82
20	0.00	-8.27	2.89	0.82	2.56	0.51	0.09	0.01	5.12

Table 3.6: Calibration: Model vs. Data

Variable	Model	Data
AGGREGATE UNEMPLOYMENT AND EMPLOYMENT RATES ^a		
u	0.230	0.230
n_P	0.705	0.705
n_G	0.064	0.064
MEASURES OF WAGE DISPERSION		
$\mu(\ln w)_g - \mu(\ln w)_p$	0.490	0.476
$\frac{\sigma(\ln w)_g}{\sigma(\ln w)_p}$	0.907	0.950
$\sigma(\ln w)$	0.618	0.616
LOG WAGE DISTRIBUTION		
$\mu(\ln w)_g$	1.341	1.326
$\mu(\ln w)_p$	0.851	0.850
$\sigma(\ln w)_g$	0.550	0.574
$\sigma(\ln w)_p$	0.607	0.604
HUMAN CAPITAL DISTRIBUTION		
$\mu(y)_g$	13.895	13.897
$\mu(y)_p$	11.637	11.635
$\sigma(y)_g$	3.045	3.019
$\sigma(y)_p$	3.291	3.292
EMPLOYMENT DURATION ^b		
$\mu(t)_p$	4.405	4.405
$\mu(t)_g$	11.027	11.010

^a This unemployment rate represents the number of unemployed as a proportion of the adjusted labor force excluding self-employment, domestic employment and unpaid family workers.

^b Employment durations in years.

Table 3.7: Aggregate and Distributional Effects of Changes in Public-Sector Size and Wage Rules

Variable	Benchmark	Experiment 1 ↑ PS Size $v_g = 0.0261$	Experiment 2 Δ Wage Rule $\psi = 0$	Experiment 3 $\gamma = 0.25$
AGGREGATE EFFECTS				
u	0.230	0.243	0.230	0.228
n_P	0.705	0.630	0.705	0.708
n_G	0.064	0.127	0.064	0.064
LM TIGHTNESS & PUBLIC SECTOR SIZE				
θ	0.736	0.674	0.736	0.738
ϕ	0.923	0.840	0.923	0.922
WAGE DISPERSION				
$\mu(\ln w)_g - \mu(\ln w)_p$	0.490	0.472	0.428	0.181
$\frac{\sigma(\ln w)_g}{\sigma(\ln w)_p}$	0.907	0.901	0.962	0.769
$\sigma(\ln w)$	0.618	0.597	0.618	0.603
JOB DURATION ^a				
$\mu(t)_p$	4.405	4.351	4.405	4.406
$\mu(t)_g$	11.027	10.235	11.005	11.058

^a Employment durations in years.

Table 3.8: Compositional and Distributional Effects of Changes in Public-Sector Size and Wage Rules

Variable	Benchmarck	Experiment 1	Experiment 2	Experiment 3
		\uparrow PS Size $v_g = 0.0261$	Δ Wage Rule $\psi = 0$	$\gamma = 0.25$
HUMAN CAPITAL				
$\mu(y)_g$	13.895	13.705	13.880	13.894
$\mu(y)_p$	11.635	11.496	11.629	11.642
$\sigma(y)_g$	3.045	2.983	3.045	3.054
$\sigma(y)_p$	3.291	3.273	3.290	3.288
PRODUCTIVITY				
$\mu(x)_g$	7.033	6.861	6.955	6.99
$\mu(x)_p$	4.847	4.744	4.828	4.789
$\sigma(x)_g$	5.367	5.201	5.368	5.369
$\sigma(x)_p$	6.721	6.426	6.760	6.677
RES PRODUCTIVITY				
$\mu(x)_g$	1.5450	1.6063	1.1830	1.3948
$\mu(x)_p$	1.2035	1.2649	1.1830	1.0534
$\sigma(x)_g = \sigma(x)_p$	0.7920	0.8091	0.7867	0.6905
LOG WAGE				
$\mu(\ln w)_g$	1.341	1.334	1.270	0.989
$\mu(\ln w)_p$	0.851	0.862	0.842	0.808
$\sigma(\ln w)_g$	0.550	0.522	0.585	0.470
$\sigma(\ln w)_p$	0.607	0.579	0.608	0.611

FIGURES

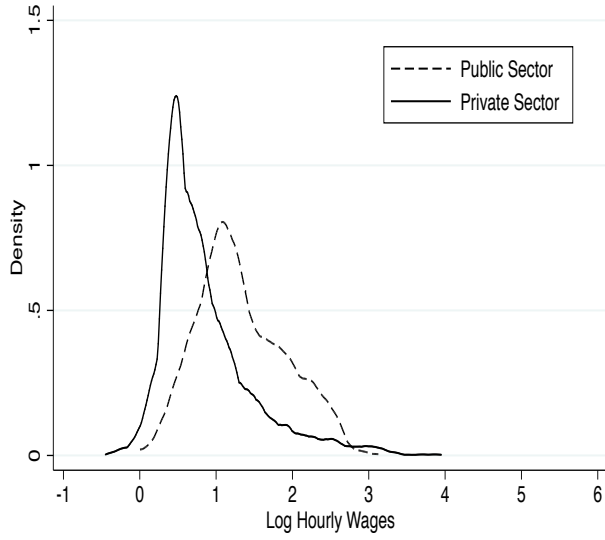


Figure 3.1: Kernel Densities - Log Wages

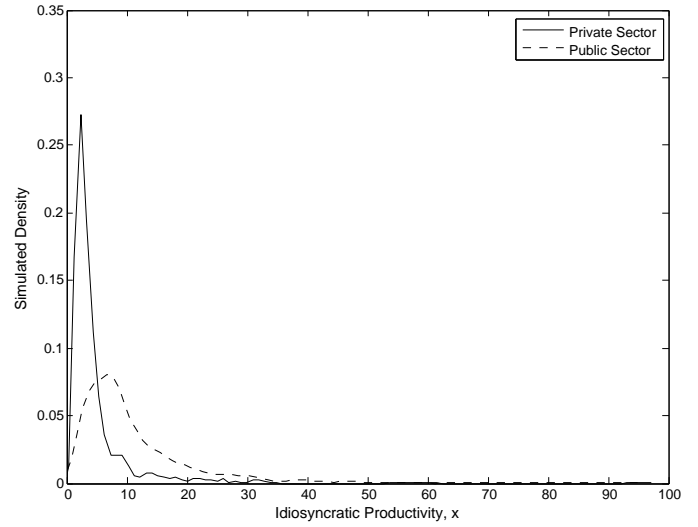


Figure 3.2: Kernel Densities - Simulated Productivities

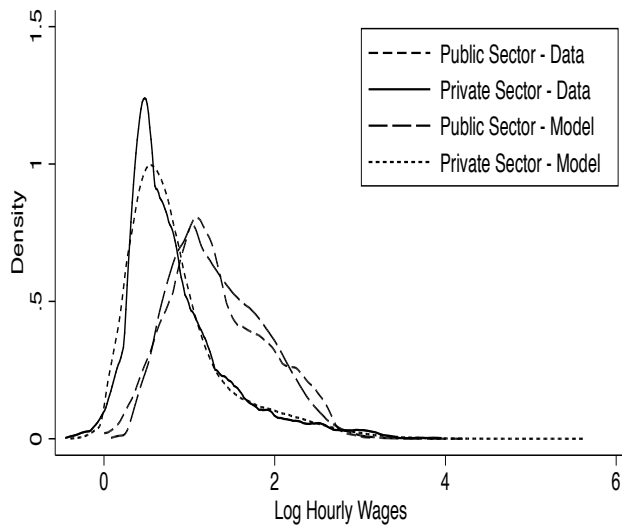


Figure 3.3: Kernel Densities - Log Wages, by Sector: Model vs. Data

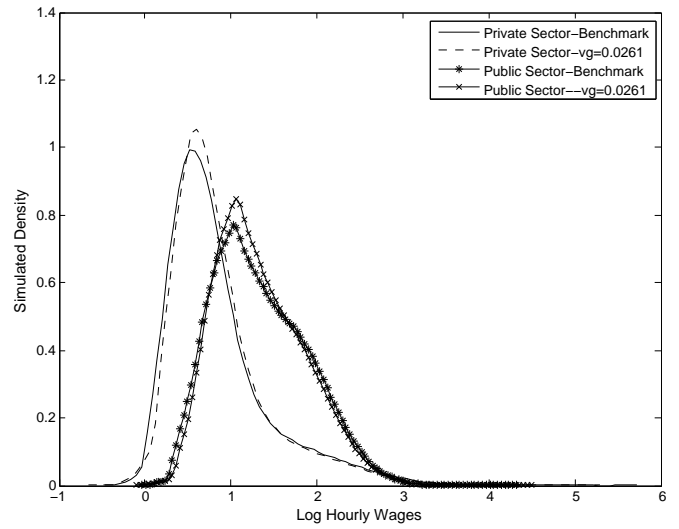


Figure 3.4: Kernel Densities - Log Wages, by Sector: Benchmark vs. Experiment 1

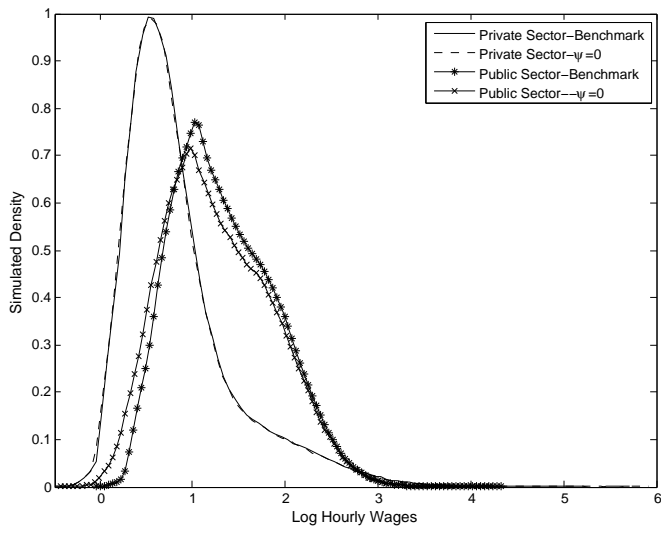


Figure 3.5: Kernel Densities - Log Wages, by Sector: Benchmark vs. Experiment 2

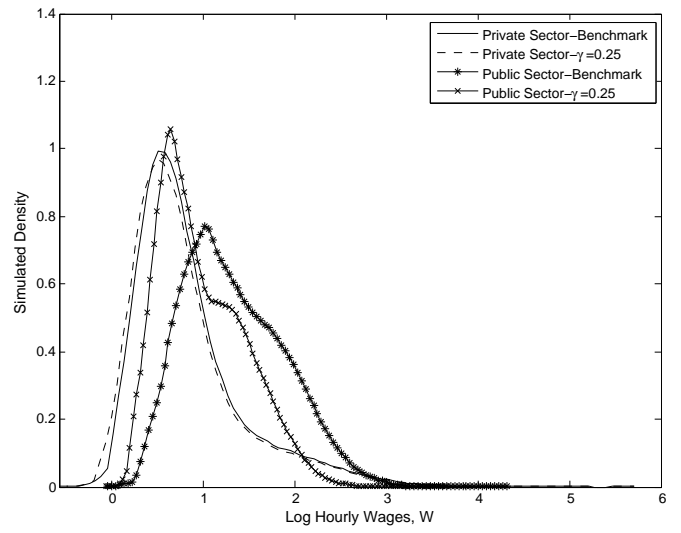


Figure 3.6: Kernel Densities - Log Wages, by Sector: Benchmark vs. Experiment 3

APPENDIX A

APPENDIX

A.0.1 TOTAL SURPLUS, WORKER AND FIRM SURPLUS

Since most equilibrium objects are expressed as functions of workers or firm's surpluses, it is useful to derive expressions for these surpluses as functions of the reservation productivities.

FORMAL SECTOR

In order to get the firm surplus in the formal sector as a function of the reservation productivity $R_{UF}(y)$, I use the value function $J_F(y', y)$ and the wage equation $w_F(y', y)$. Then, I use the Nash sharing rule to obtain total surplus and worker surplus.

The total surplus, worker and firm surplus for a worker of type y and productivity y' in the formal sector are, respectively, given by¹

$$S_F(y', y) = \frac{p_F (1 - \hat{\delta}_2) (y' - R_{UF}(y))}{(1 + \delta_1)(r + \lambda)}$$

$$N_F(y', y) - U(y) = \frac{\beta p_F (1 - \hat{\delta}_2) (y' - R_{UF}(y))}{(1 + \delta_1)(r + \lambda)}$$

$$J_F(y', y) - V_F = \frac{(1 - \beta)p_F (y' - R_{UF}(y))}{(r + \lambda)}$$

¹Notice that workers always get a fixed proportion of the total surplus, so $N_F(y', y) - U(y) = \beta S_F(y', y)$. Substituting this expression in the Nash sharing rule I get $J_F(y', y) - V_F = \frac{(1-\beta)(1+\delta_1)}{(1-\hat{\delta}_2)} S_F(y', y)$. So the total surplus $S_F(y', y) = N_F(y', y) - U(y) + \frac{(1-\hat{\delta}_2)}{(1+\delta_1)} J_F(y', y)$.

The total surplus, workers' and firms' surplus for a worker of type y and productivity x in the formal sector are, respectively, given by:

$$S_F(x, y) = \frac{p_F (1 - \hat{\delta}_2) (x - R_{FU}(y))}{(1 + \delta_1)(r + \lambda)}$$

$$N_F(x, y) - U(y) = \frac{\beta p_F (1 - \hat{\delta}_2) (x - R_{FU}(y))}{(1 + \delta_1)(r + \lambda)}$$

$$J_F(x, y) - V_F = \frac{(1 - \beta)p_F (x - R_{FU}(y))}{(r + \lambda)} - s$$

INFORMAL SECTOR

The corresponding surpluses for a worker of type y and current productivity y' in the informal sector are given by:

$$S_I(y', y) = \frac{p_I (1 + \hat{\delta}_3) (y' - R_{IU}(y))}{r + \lambda}$$

$$N_I(y', y) - U(y) = \frac{\beta p_I (1 + \hat{\delta}_3) (y' - R_{IU}(y))}{r + \lambda}$$

$$J_I(y', y) - V_I = \frac{(1 - \beta)p_I (y' - R_{IU}(y))}{r + \lambda}$$

Therefore, in both sectors, the surpluses depend positively on the market value of the gap between current and minimum productivity at a particular match, and negatively on the rate of arrival of the productivity shock and the interest rate. Higher λ implies a higher turnover rate, and higher r implies that future returns are discounted at a higher rate.

A.0.2 DERIVATION OF STEADY STATE PRODUCTIVITY DISTRIBUTIONS

I want to know how policies affect the marginal distributions of types and productivities in each sector, which also affect the sectoral wage distributions. This section follows closely ANV(2009).

FORMAL SECTOR

First, I want to compute the joint steady state distribution of current productivity and worker types across workers in the formal sector, $f_F(x, y)$.

$$f_F(x, y) = f_F(x | y)f_F(y)$$

I use Bayes rule to compute $f_F(y)$ as follows:

$$f_F(y) = \frac{n_F(y)f(y)}{\int_0^\infty n_F(y)f(y)dy}$$

Now I need $f_F(x | y)$.

Let N be the number of shocks that the worker has experienced to date (in the current spell of unemployment in the FS). If $N = 0$ current productivity x equals match-specific productivity y' with prob 1. If $N > 0$, x is a draw from a truncated density $h_F(x | y)/(1 - H_F(R_{FU}(y) | y))$, for $R_{FU}(y) \leq x \leq \infty$.

So I have:

$$P(N = 0) = P(x = y' | y) \quad \text{for } x=y'.$$

$$f_F(x | y) = \frac{h_F(x | y)}{(1 - H_F(R_{FU}(y) | y))} (1 - P(N = 0)) \quad \text{for } R_{FU}(y) \leq x \leq \infty \text{ and } x \neq y'$$

Let t be elapsed duration of employment in the current formal-sector job. Let N_t be the number of shocks that the worker has experienced to date t . Let's assume that:

$$t \sim \exp(\lambda H_F(R_{FU}(y) | y))$$

$$N_t \sim \text{Poisson}(\lambda(1 - H_F(R_{FU}(y) | y))t)$$

So I have:

$$P(N_t = 0) = \exp(-\lambda(1 - H_F(R_{FU}(y) | y))t)$$

$$P[N = 0] = \int_0^\infty \exp(-\lambda(1 - H_F(R_{FU}(y) | y))t) \lambda(1 - H_F(R_{FU}(y) | y)) \exp(-\lambda H_F(R_{FU}(y) | y)t) dt$$

So I get:

$$P[N = 0] = H_F(R_{FU}(y) | y)$$

The density of current productivity x given worker type y in the formal sector is given by:

$$P(x = y' | y) = H_F(R_{FU}(y) | y) \quad \text{for } x=y'$$

$$f_F(x | y) = h_F(x | y) \quad \text{for } R_{FU}(y) \leq x \leq \infty \text{ and } x \neq y'$$

I can now compute the steady state joint distribution of types and productivity in the formal sector, and the marginal distribution of current productivity in the formal sector as follows:

$$f_F(x, y) = f_F(x | y)f_F(y)$$

$$f_F(x) = \int f_F(x, y)dy$$

INFORMAL SECTOR

Doing the analogous exercise for the informal sector I get:

$$f_I(x) = \int f_I(x, y)dy$$

$$f_I(x, y) = f_I(x | y)f_I(y)$$

where $f_I(y)$ and $f_I(x | y)$ are as follows:

$$f_I(y) = \frac{n_I(y)f(y)}{\int_0^\infty n_I(y)f(y)dy}$$

$$P(x = y' | y) = H_I(R_{IU}(y) | y) \quad \text{for } x=y'.$$

$$f_I(x | y) = h_I(x | y) \quad \text{for } R_{IU}(y) \leq x \leq \infty \text{ and } x \neq y'$$

A.0.3 DERIVATION OF STEADY STATE WAGE DISTRIBUTIONS

To compute the impact of policies on wage inequality in both sectors, I need to derive the distribution of wages across formal and informal sector employment, $m_F(w)$ and $m_I(w)$.

FORMAL SECTOR

There are two types of workers currently employed in the formal sector: the workers that have not received any shock whose current productivity is match-specific ($x = y'$ with probability 1), and those who received a shock and continue in the match ($x \geq R_{FU}(y)$).

This suggests that the distribution of wages in formal-sector employment (conditional on y) consists of a smooth density for $w_F \in [w_F^s[R_{FU}(y), y), w_F^s(\infty, y)]$, and a mass point at $w_F = w_F(y', y)$.

So with $P(N=0)$ the worker of type y receives:

$$w_F(y', y) = \beta \left[\frac{p_F y'}{1 + \delta_1} - \frac{\lambda s}{(1 + \delta_1)} \right] + (1 - \beta) \left[\frac{rU(y) - \lambda H_F(R_{FU}(y) | y) \int_{R_{FI}(y)}^\infty N_I(x, y) - U(y) dH_I(x | y)}{(1 - \hat{\delta}_2)} \right]$$

and with $[1 - P(N = 0)]$ the worker of type y receives:

$$w_F^s(x, y) = \beta \left[\frac{p_F x}{1 + \delta_1} + \frac{rs}{(1 + \delta_1)} \right] + (1 - \beta) \left[\frac{rU(y) - \lambda H_F(R_{FU}(y) | y) \int_{R_{FI}(y)}^\infty N_I(x', y) - U(y) dH_I(x' | y)}{(1 - \hat{\delta}_2)} \right]$$

Let's first calculate calculate $m_F(w/y)$.

To compute the conditional density of a transformed variable (productivity as a

function of wages conditional on y), I know that: $m_F(w/y) = h_F[x = S(w_F^s, y) | y)] \frac{dS(w, y)}{dw}$.

Inverting (81) I get $x = S(w_F^s, y)$ as follows:

$$x \equiv S(w_F^s, y) = \left[\frac{1 + \delta_1}{\beta P_F} \right] w_F^s - \frac{rs}{P_F}$$

$$- \frac{(1 - \beta)(1 + \delta_1)}{\beta P_F} \left[\frac{rU(y) - \lambda H_F(R_{FU}(y) | y) \int_{R_{FI}(y)}^{\infty} N_I(x', y) - U(y) dH_I(x' | y)}{(1 - \hat{\delta}_2)} \right]$$

$$\frac{dx}{dw} = \frac{dS(w_F^s, y)}{dw} = \frac{1 + \delta_1}{\beta P_F}$$

The conditional distribution of wages in formal-sector employment is:

$$P[w_F = w_F(y', y)] = H_F(R_{FU}(y) | y) \quad \text{for } w_F = w_F(y', y).$$

$$m_F(w/y) = \left[\frac{1 + \delta_1}{\beta P_F} \right] h_F(x \equiv S(w_F^s, y) | y) \quad \text{for } w_F \in [w_F^s[R_{FU}(y), y), w_F^s(\infty, y)] \text{ and } w_F \neq w_F(y', y)$$

I finally compute can compute $m_F(w)$ by using:

$$m_F(w) = \int m_F(w/y) f_F(y) dy$$

INFORMAL SECTOR

Doing the analogous exercise for the informal sector I can get $m_I(w)$ as follows:

$$m_I(w) = \int m_I(w/y) f_I(y) dy$$

where $m_I(w/y)$ is the steady state conditional distribution of wages in IS employment and $f_I(y)$ is steady state density of types among IS employment.

The mapping from wages to productivity, conditional on y is:

$$x \equiv S(w_I, y) = \left[\frac{w_I}{\beta P_I} \right] - \left[\frac{(1 - \beta)}{\beta P_I} \right] \left[\frac{rU(y) - m(\theta) \left[\int_{R_{IF}(y', y)}^{\infty} N_F(x, y, y') - U(y) dH_F(x | y) \right]}{(1 + \hat{\delta}_3)} \right]$$

$$\frac{dx}{dw} = \frac{dS(w_I, y)}{dw} = \frac{1}{\beta P_I}$$

After receiving a shock, the worker continues in the match and receives a salary only if $x \geq R_{IU}(y)$. The conditional distribution of wages in formal-sector employment is:

$$P[w_I = w_I(y', y)] = H_I(R_{IU}(y) | y) \quad \text{for } w_I = w_I(y', y).$$

$$m_I(w/y) = \left[\frac{1}{\beta P_I} \right] h_I(x \equiv S(w_I, y) | y) \quad \text{for } w_I \in [w_I^s[R_{IU}(y), y), w_I^s(\infty, y)] \text{ and } w_I \neq w_I(y', y)$$

A.0.4 STEADY STATE EQUILIBRIUM

Definition 1 *Given a vector of parameters $\{b, \alpha, \beta, b, c, \lambda\}$, a vector of prices $\{p_F, p_I, r\}$, a vector of taxes and subsidies $\{\delta_1, \delta_2, \delta_3, s\}$, a vector of valuation of social security and social assistance services $\{\tau, \mu\}$, matching function $m(\cdot)$, and cumulative density functions $F(y)$, $H_i(\cdot | y)$ (for $i=I, F$), a **Steady State Equilibrium with an Informal sector** is a vector formed by the unemployment rate $u(y)$, sector- i employment rates $n_i(y)$, the value of unemployment $U(y)$, the reservation productivities $R_{UF}(y)$, $R_{FU}(y)$, $R_{UI}(y)$, $R_{IU}(y)$ and $R_{FI}(y)$, sector- i wages $w_i(y', y)$ (for $i=I, F$) and $w_i^s(x, y)$ (for $i=F$), and labor market tightness in the formal and informal sectors θ and ϑ , such that:*

1. *The flow value of unemployment $U(y)$ that satisfies (39).*
2. *The reservation productivity schedule $R_{FU}(y)$ that satisfies (19).*
3. *The reservation productivity schedule $R_{UF}(y)$ that satisfies (21).*
4. *The reservation productivity schedule $R_{IU}(y)$ that satisfies (24).*
5. *The unemployment rate and the sector- i employment rates that satisfy (36),*

(37) and (38).

6. The labor market tightness parameter θ that satisfies (34).
7. The labor market tightness parameter ϑ that satisfies (35).
8. The formal-sector wages $w_F(y', y)$ and $w_F^s(x, y)$ that satisfy (16) and (17).
9. The informal-sector wages $w_I(y', y)$ that satisfies (18).

***Proof.*Existence of Equilibrium**

The Steady State Equilibrium with an Informal sector exists if there is a θ that satisfies the job creation condition in the formal sector, equation (34), since all the other equilibrium objects are uniquely determined by θ .

Given that the right-hand side of (34) is continuous in theta, a solution to (34) exists.

To establish uniqueness, I would need to show that the the right-hand side of (34) is strictly monotone. First, $m(\theta)/\theta$ is monotonically decreasing by assumption. Second, $R_{UF}(y)$ is monotonically increasing in θ (since higher θ means higher $U(y)$), so $J_F(y', y)$ is monotonically decreasing in θ . Finally, $u(y)$ should be decreasing in θ due to the dominant negative impact of formal-sector job creation on unemployment (as explained before), and the aggregate unemployment rate u should also be decreasing in θ . However, further assumptions on $H_i(y' | y)$ are required to prove that the ratio $u(y)/u$ is monotonically decreasing as well, so uniqueness is not guaranteed.

A.0.5 COMPUTATIONAL ALGORITHM: APPROXIMATION OF STEADY STATE EQUILIBRIUM

Following the definition of Steady State Equilibrium, I write the following computational algorithm to approximate the steady state of the model numerically:

1. Guess values for θ^0 . Start an outer loop. Guess values for $R_{UF}(y)^0$, $R_{UI}(y)^0$ and $R_{FU}(y)^0$. Start an inner loop, for fixed values of θ^0 . Substitute these values in equation (30) to calculate $rU(y)$.

2. Given $rU(y)$ and $R_{FI}(y)^0$, iterate the Bellman equation (19) to find the fixed point on $R_{FU}(y)$. Call the solution $R_{FU}^{\hat{}}(y)$.

3. Given $R_{FU}^{\hat{}}(y)$, use equation (21) to calculate $R_{UF}^{\hat{}}(y)$.

4. Given $U(y)$, iterate the Bellman equation (24) to find the fixed point on $R_{IU}(y)$.

Call the solution $R_{IU}^{\hat{}}(y)$. Notice that $R_{IU}^{\hat{}}(y) = R_{UI}^{\hat{}}(y) = R_{FI}^{\hat{}}(y)$.

If the following conditions are met:

$$\| R_{UF}(y)^0 - R_{UF}^{\hat{}}(y) \| < \epsilon_{UF} , \text{ and}$$

$$\| R_{UI}(y)^0 - R_{UI}^{\hat{}}(y) \| < \epsilon_{UI} , \text{ and}$$

$$\| R_{FU}(y)^0 - R_{FU}^{\hat{}}(y) \| < \epsilon_{FU} , \text{ and}$$

Then stop inner loop. Otherwise update as follows:

$$R_{UF}(y)^{new} = R_{UF}(y)^0 + \nu_{UF}(R_{UF}(y)^0 - R_{UF}^{\hat{}}(y))$$

$$R_{UI}(y)^{new} = R_{UI}(y)^0 + \nu_{UI}(R_{UI}(y)^0 - R_{UI}^{\hat{}}(y))$$

$$R_{FU}(y)^{new} = R_{FU}(y)^0 + \nu_{FU}(R_{FU}(y)^0 - R_{FU}^{\hat{}}(y))$$

where ϵ_{ij} and ν_{ij} are the tolerance levels and step sizes respectively².

5. Once convergence is reached in the inner loop, use $R_{UI}^{\hat{}}(y)$, $R_{UF}^{\hat{}}(y)$, and $R_{FU}^{\hat{}}(y)$ and θ^0 in equations (27) - (29) to calculate $u(y)$, $n_F(y)$ and $n_I(y)$. Aggregate over y to get u , n_F and n_I .

²The smaller the steps and the tolerance, the more accurate the results but the longer the computational time. I choose $\epsilon_{ij} = 10^{-3}$, $\nu_{ij} = 0.05$.

6. Given $R_{UF}^{\hat{\theta}}(y)$ and $u(y)$, solve equation (25) to get the equilibrium labor market tightness parameter $\hat{\theta}$. If the following condition is met, $|\theta^0 - \hat{\theta}| < \epsilon_{\theta}$, then stop outer loop. Otherwise update as follows: $\theta^{new} = \theta^0 + \nu_{\theta}(\theta^0 - \hat{\theta})$.

7. Once convergence is reached in the outer loop, use equilibrium reservation productivities, $R_{FU}(y)$ and $R_{IU}(y)$, $u(y)$ and $n_F(y)$, to get the labor market tightness parameter ϑ that satisfies equation (26).

8. Given equilibrium $U(y)$, equilibrium reservation productivities $R_{FU}(y)$, $R_{FI}(y)$, $R_{IU}(y)$, get formal sector wages $w_F(y', y)$ and $w_F(x, y)$ that satisfy equations (16) and (17).

9. Given equilibrium $U(y)$, get informal-sector wage $w_I(y', y)$ that satisfies equation (18).

BIBLIOGRAPHY

Albrecht, James, Navarro, Lucas and Vroman, Susan, "The Effects of Labor Market Policies in an Economy with an Informal Sector," *Economic Journal*, July 2009, 119(539), 1105-1129.

Algan, Yann, Cahuc, Pierre and Zylberberg, André "Public Employment and Labour Market Performances," *Economic Policy*, April 2002, 17(34), 9-64.

Andolfatto, David, "Business Cycles and Labor Market Search," *American Economic Review*, March 1996, 86(1), 112-132.

Attanasio, Orazio, Goldberg, Pinelopi K. and Pavcnik, Nina, "Trade Reforms and Wage Inequality in Colombia," *Journal of Development Economics*, August 2004, 74(2), 331-366.

Batini, Nicoletta, Kim, Young-Bae, Levine, Paul, and Lotti, Emanuela, "Informal Labour and Credit Markets: A Survey," *IMF Working Paper No. 10-42*, 2010.

Bosch, Mariano, Goni, Edwin and Maloney, William F., "The Determinants of Rising Informality in Brazil: Evidence from Gross Worker Flows," *World Bank Discussion Paper No. 4375*, 2007.

Bosch, Mariano and Maloney, William F. , "Gross Worker Flows in the Presence of Informal Labor Markets: the Mexican Experience 1987-2002," *IZA Discussion Paper No. 2864*, 2007.

Bosch, Mariano and Maloney, William F. "Cyclical Movements in Unemployment and Informality in Developing Countries," *IZA Discussion Paper No. 3514*, 2008.

Bosch, Mariano and Maloney, William F., "Comparative Analysis of Labor Market Dynamics Using Markov Processes: An Application to Informality," *Labor Economics*, August 2010, 17(4), 621-631.

Bosch, Mariano, and Esteban-Pretel, Julen, "Job Creation and Job Destruction in the Presence of Informal Markets," *Journal of Development Economics*, July 2012, 98(2), 270-286.

Bradley, Jake, Postel-Vinay, Fabien, and Turon, Helene, "Public Sector Wage Policy and Labor Market Equilibrium: A Structural Model," *University of Bristol Mimeo*, 2012.

Bunzel, H., B. J. Christensen, P. Jensen, N. Kiefer, L. Korsholm, L. Muus, G. Neumann and M. Rosholm, "Specification and Estimation of Equilibrium Search Models," *Review of Economic Dynamics*, January 2001, 4(1), 90-126.

Burdett, Kenneth "Towards a Theory of the Labor Market with a Public Sector," *Labour Economics*, January 2012, 19(1), 68-75.

Burdett, Kenneth, and Judd, Kenneth L. "Equilibrium price dispersion," *Econometrica: Journal of the Econometric Society*, July 1983, 51(4), 955-969.

Burdett, Kenneth and Mortensen, Dale, "Wage Differentials, Employer Size and Unemployment," *International Economic Review*, May 1998, 39(2), 257-273.

Cahuc, Pierre, Postel-Vinay, Fabien and Robin, Jean Marc, "Wage Bargaining with On-The Job Search: Theory and Evidence," *Econometrica*, March 2006, 74(2), 323-364.

Cardenas, Mauricio and Gutierrez, Catalina, "Efficiency and Effects of Structural Reform: the Case of Colombia," *Fedesarrollo, Paper Prepared for the Conference in Honor of Albert Fishlow*, 1996.

Cardenas, Mauricio, Kugler, Adriana and Bernal, Raquel, "Labor Market Regulation and Employment in Colombia: 1976-1996," *Fedesarrollo, Regional Research Project Sponsored by the IADB*, 1998.

Camacho, Adriana, Conover Emily, and Hoyos Alejandro, "Effects of Colombia's Social Protection System on Workers' Choice between Formal and Informal Employment," *World Bank Research Working Paper No. 6524*, 2013.

Chéron, Arnaud, Hairault, Jean-Olivier and Langot, François, "Age-Dependent Employment Protection Policy," *Economic Journal.*, December 2011, 121, 1444-1504.

Coşar, Kerem "Adjusting to Trade Liberalization: Reallocation and Labor Market Policies," 2010, mimeo, University of Chicago Booth School of Business.

Cosar, A. Kerem, Guner, Nezih and Tybout, James, "Firm Dynamics, Job Turnover and Wage Distributions in an Open economy," *IZA Discussion Paper No. 7718*, 2013.

Dey, Matthew S., and Flinn, Christopher, "An Equilibrium Model of Health Insurance Provision and Wage Determination," *Econometrica*, March 2005, 73(2), 571-627.

Dolado, Juan, Marcel Jansen, and Juan Jimeno "A Positive Analysis of Targeted Employment Protection Legislation," *B.E. Journal of Macroeconomics*, July 2007, 7(1), 1935-90.

Eckstein, Zvi and Kenneth Wolpin “Duration to First Job and the Return to Schooling: Estimates from a Search-Matching Model,” *Review of Economic Studies*, April 1995, 62(2), 263-86.

Elsby, Michael, Michaels, Ryan and Solon, Gary, “The Ins and Outs of Cyclical Unemployment," *American Economic Journal: Macroeconomics*, January 2009, 1(1), 84-110.

Eslava, Marcela, Haltiwanger, John, Kugler, Adriana and Kugler, Maurice, “Factor Adjustments after Deregulation: Panel Evidence from Colombian Plants," *The Review of Economics and Statistics*, May 2010, 92(2), 378-391.

Fliess, Norbert, Fugazza, Marco and Maloney, William F. ,“Informality and Macroeconomic Fluctuations," *IZA Discussion Paper No. 3519* , 2008.

Fugazza, Marco and Jacques, Jean Francois “Labor Market Institutions, Taxation and the Underground Economy," *Journal of Public Economics*, January 2004, 88(1-2), 395-418.

Goldberg, Pinelopi K. and Pavcnik., Nina,“The Response of the Informal Sector to Trade Liberalization," *Journal of Development Economics*, December 2003, 72(2), 463-496.

Goldberg, Pinelopi K. and Pavcnik., Nina.,“ Distributional Effects of Globalization in Developing Countries," *Journal of Economic Literature*, March 2007, 45(1), 39-82.

Gomes, Pedro “Fiscal Policy and the Labour Market: The Effects of Public Sector Employment and Wages”, February 2011, mimeo, Universidad Carlos III de Madrid.

Gregory, Robert and Jeff Borland (1999) "Recent Developments in Public Sector Labor Markets," in O. Ashenfelter and D. Card (eds), *Handbook of Labor Economics*, vol 3C, North Holland, 3573–3630.

Hornstein, Andreas, Krusell, Per and Violante, Giovanni L., "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism," *Economic Quarterly, Federal Reserve Bank of Richmond*, 91(3), 19-50, 2005.

Hornstein, Andreas, Per Krusell and Giovanni Violante "Frictional Wage Dispersion in Search Models: A Quantitative Assessment," *American Economic Review*, December 2011,101, 2873-98.

Haltiwanger, John, Adriana Kugler, Maurice Kugler, Alejandro Mico, and Carmen Pages, "Effects of Tariffs and Real Exchange Rates on Job Reallocation: Evidence from Latin America," *Journal of Policy Reform*, December 2004, 7(4), 191-208.

Heckman, James J. , and Sedlacek, Guilherme, "Heterogeneity, Aggregation, and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market," *The Journal of Political Economy*, December 1985, 93(6), 1077-1125.

Heckman, James and Carmen Pages, "Law and Enforcement: Lessons from Latin America and the Caribbean: An Introduction," in *Law and Employment: Lessons from Latin America and the Caribbean*, edited by James Heckman and Carmen Pages, University of Chicago Press, Chicago, 2004.

Ihrig, Jane and Moe, Karine, "Lurking in the Shadows: the Informal Sector and Government Policies," *Journal of Development Economics*, April 2004, 73(2), 541-557.

Inter-American Development Bank, "Good Job Wanted: Labor Markets in Latin America. Economic and Social Progress Report," *Published by the Inter-American Development Bank*, Washington, D.C, 2004.

Kugler, Adriana, "The Impact of Firing Costs on Turnover and Unemployment: Evidence from the Colombian Labor Market Reform," *International Tax and Public Finance*, August 1999, 6(3).

Kugler, Adriana and Kugler, Maurice, "Labor Market Effects of Payroll Taxes in Developing Countries: Evidence from Colombia," *Economic Development and Cultural Change*, January 2009, 57(2), 335-358.

Lucifora, Claudio and Dominique Meurs, "The Public Sector Pay Gap in France, Great Britain and Italy," *Review of Income and Wealth*, March 2006, 52(1), 43-59.

Maloney, William and Nunez, Jairo, "Measuring the Impact of Minimum Wages: Evidence from Latin America," *World Bank policy Research Working Paper No 2597*, 2001.

Maloney, William F., "Informality Revisited," *World Development*, July 2004, 32(7), 1159-1178.

Meghir, Costas, Narita, Renata, Robin, Jean-Marc, "Wages and Informality in Developing Countries," *NBER Working Paper No. 18347*, 2012.

Melly, Blaise, "Public-Private Sector Wage Differentials in Germany: Evidence from Quantile Regression," *Empirical Economics*, September 2005, 30(2), 505-520.

Mizala, Alejandra, Pilar Romaguera and Sebastián Gallegos "Private-Public Wage Gap in Latin America (1999-2007): A Matching Approach," Working paper #268, Centro de Economía Aplicada, University of Chile, 2010.

Mondragon, Camilo, Pena, Ximena and Wills, Daniel, "Labor Market Rigidities and Informality in Colombia," *Serie Documentos Cede*, 2010.

Mortensen, Dale, "Equilibrium Unemployment with Wage Posting: Burdett-Mortensen Meets Pissarides" in H. Bunzel, B.J. Christiansen, P. Jensen, N.M. Kiefer and D.T. Mortensen, eds., *Panel Data and Structural Labor Market Models*, Amsterdam: Elsevier, 2000.

Mortensen, Dale, and Pissarides, Christopher, "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, July 1994, 61(3), 397-415.

Mortensen, Dale and Nagypal, Eva, "More on Unemployment and Vacancy Fluctuations," *Review of Economic Dynamics*, July 2007, 10(3), 327-347.

Pissarides, Christopher., *Equilibrium Unemployment Theory*, Cambridge, MA: MIT Press, 2000.

Postel-Vinay, Fabien, "Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, November 2002, 70(6), 2295-2350.

Quadrini, Vincenzo and Antonella Trigari "Public Employment and the Business Cycle," *Scandinavian Journal of Economics*, December 2007, 109(4), 723-42.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment, Vacancies, and Wages: Evidence and Theory," *American Economic Review*, March 2005, 95(1), 25-49.

Shimer, Robert, "Reassessing the Ins and Outs of Unemployment," *NBER Working Paper No.13421*, 2007.

Schneider, Friedrich., "Shadow Economies around the World: What Do We Really Know?," *European Journal of Political Economy*, September 2005, 21(3), 598-642.

Trigari, Antonella, "Equilibrium Unemployment, Job Flows and Inflation Dynamics," *Journal of Money Credit and Banking*, February 2009, 41(1), 1-33.

Walsh, Carl, "Labor Market Search, Sticky Prices, and Interest Rate Policies," *Review of Economic Dynamics*, June 2005, 8(4), 829-849.