Growth Dynamics of Neoclassical Open Economies

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ABSTRACT

This thesis examines how well neoclassical open economy growth models can replicate the qualitative features of the time paths of an emerging economy during its development stages. In particular we look at the trends in saving and investment rates and the current account balance, which have motivated previous research, as well as trends in import and export shares of GDP and the terms of trade, which pertain to an open economy.

We build three open economy models - one with portfolio adjustment cost, one with endogenous discounting and one with 2 sectors. We first look at their stochastic impulse responses. Then we perform the standard exercise of transitional dynamics with no exogenous changes. Finally, we enrich our study by motivating and implementing exogenous changes in technology and foreign demand over a "catch-up" stage, and study the perfect foresight dynamics. With each dynamic experiment, we examine the associated trends, and discuss the intuitions. Each model has its strengths and weaknesses, and each computational trial produces mixed results that match real life observations with varying degrees of success. We compare the results and discuss the implications of each modelling technique.

Keywords: neoclassical, open economy, growth, saving, investment, export, import, terms of trade, real exchange rate, transitional dynamics, endogenous discounting, Uzawa preference, multi-sector, industrial, agricultural.

DEDICATION

This dissertation is dedicated to my parents, who have given me unwavering support and endured agony and joy throughout my pursuit of this degree. I would also like to dedicate it to Ding Liang, who has for the last 20 months been giving me spiritual and technical support for this dissertation.

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Chapter 1

Introduction

In neoclassical macroeconomics, dynamic models have long been used to study long-run economic growth. Previous research has traditionally looked at the transitional dynamics of factors such as consumption, productivity and capital accumulation over time. Impacts of exogenous changes such as technological progress and population growth have also been studied.

The transitional dynamics of neoclassical growth models strongly reflect the law of diminishing marginal product. The standard transitional dynamics exercise assumes constant growth rate of technology and constant parameters, as the growth process involves capital accumulation. Several earlier contributions highlighted counterfactual implications of theory in this context. There has also been a body of literature proposing modelling innovations in an attempt to generate results that provide a better match for the observed trends in economic development.

1.1 Empirical Regularities

We will be looking at growth dynamics of developing economies over time. For real life evidence, our presented data focus on the trends of past and current developing countries in Asia. Some Asian countries were previously developing but are now considered developed, including Japan and the four so-called "Asian tigers" - South Korea, Singapore, Taiwan and Hong Kong. The current ones are China, India and the Southeast Asian countries.

Within the scope of this thesis, when we talk about developing countries, we refer to the above listed group of Asian countries. As illustrated below, we observe quite consistent trends in some variables that interest us. However, we do not claim these trends are also true to developing countries in other parts of the world. In later chapters, we build open economy models, which is motivated by the fact that the Asian economies all gained substantial growth by trading with other countries during their takeoff. This is also a missing feature to many other developing countries.

For analyzing empirical evidence on the variables we are interested in, we posit that a developing economy goes through a catch-up stage, or takeoff, and define its time frame for the Asian countries we look at. As seen in later chapters, the GDP per capita growth rates in developing countries start out high, and slow down over time and converge to that of the US. Specifically, we choose the end of the catch-up to be the year when the difference between the GDP per capita growth rates of a developing nation and the US falls below 3%.

To find real life patterns of growth dynamics we look at data from 4 countries - China, India, Singapore and South Korea. China and India are still very much in their takeoff stages. Following the rule above, Singapore ended its takeoff in 1988, while South Korea's catch-up lasted until 1998. The following table gives descriptive statistics of the variables of our interest, including changes in saving and investment rates, import and export as share of GDP, as well as net foreign assets to GDP ratio for South Korea and Singapore during their takeoff.

Country	Singapore	South Korea
End of Takeoff	1988	1998
Average Growth Rate of Per Capita GDP	6.80%	6.19%
Change in Saving Rate	14.77%	12.10%
Change in Investment Rate	23.13%	13.57%
Change in Import Share	41.27%	20.69%
Change in Export Share	61.92%	43.01%
Change in Net Foreign Assets to GDP	5.84%	9.84%

1.2 The Saving/Investment Puzzle

In much of the neoclassical growth literature, the feature of diminishing marginal product directly translates into the prediction that when an economy starts out poor in resources, it would have high saving and investment to capture the high marginal product; as it becomes richer, it would decrease saving and investment and devote more resources to consumption. An important implication of these models is the monotonic decrease in saving and investment rates over time.

This particular implication is decidedly at odds with what we have seen from several major developing nations over the last several decades. Quite on the contrary to theoretical predictions, the majority of developing economies in Asia have had increasing saving and investment rates over time. This is especially the case during a country's initial stages of development. This saving/investment rate puzzle is exemplified by countries that in the past experienced dramatic growth, such as Singapore and South Korea, as well as currently developing nations like China and India.

Figures 1 and 2 show the saving and investment rates of these prominent former and current examples of developing economies. Clearly, these economies depict a trend of growing saving and investment as percentages of GDP over time, especially during their early stages of super-charged growth. South Korea experienced dramatic growth in the 1980s and 1990s, and its saving and investment rates steadily increased during that period, tailing off somewhat since the late 1990s. Similarly, Singapore has both rates rising during its economic takeoff, with investment and saving rates each starting to decrease in the mid 1980s and late 1990s. As today's beacons of developing nations, China and India have both experienced a fairly persistent increase in saving and investment rates, both seeing a significant spike in the 2000's and no sign of decline.

The existing literature offers a number of ways to extend or modify the basic neoclassical model in order to replicate the observed trends in saving and investment rates. This is often done in the context of closed economy models. Farzin and Wendner (2013)[10] uses hyperbolic discounting to correct for this puzzle. Antras (2001)[3] introduces a consumption subsistence that generates a hump shaped savings profile. Other related works include Goméz (2008)[12] and Smetters (2003)[30] which use a more flexible CES technology, and Litina and Palivos (2010)[17] which introduces endogenous technical progress.

Using a different approach, Chang and Hornstein (2012)[5] is a typical example of a perfect foresight transitional dynamics study, similar to the exercises in later chapters. It addresses the puzzling low saving rates of the South Korean economy early on in its developing stage by imposing structural changes in price of capital and labor distribution between sectors over time. It takes on the view that labor migration happens on its own, instead of endogenously as a thriving industrial sector lures people away from farming, which in is contrast to Gollin et all (2007)[11].

1.3 The Lucas Paradox

Another key implication of diminishing marginal product in neoclassical models is that savings in rich countries should be attracted to poor countries for higher returns, resulting in capital inflow and current account deficit for developing countries. Lucas (1990)[19] made the renowned observation that despite the lower levels of capital per worker and therefore higher marginal product in poor countries, capital does not flow from rich to poor countries, even often going the opposite way. This is commonly known as the "Lucas paradox". He gave some potential explanations for that including human capital and institutional differences.

This paradox is also on full display in some of the Asian countries. While theory predicts current account deficits, two well-known counter-examples are Japan and China, both having experienced persistent current account surpluses. For decades Japan has been holding massive amounts of US assets. It recently overtook China as the largest foreign holder of US securities, with China remaining as the largest US government creditor after years of accumulating Treasuries.

There has also been literature tackling the Lucas paradox. Alfaro et al (2008)[2] empirically finds poor institutional quality to be the leading reason for the lack of capital flow to developing countries. Montiel (2006)[25] provides a list of explanations for Africa's inability to attract investments. Daude and Stein (2007)[8] finds that the unpredictability of laws, regulations and policies, excessive regulatory burden, government instability and lack of commitment play a major role in deterring foreign direct investment. Schularick and Steger (2008)[29] extends the original Lucas (1990)[19] framework to argue that improvements in institutional quality are a key pre-condition for larger capital flows to developing countries.

1.4 Methodologies and More Literature Review

Motivated by the fact that the Asian economies illustrated above all experienced tremendous growth with trade, our work relies heavily on open economy models, with the home country being a developing nation and the foreign country implied as a developed economy. We aim to match our results with real world observations in time trends, in particular making attempts to reconcile the saving/investment rate puzzle and Lucas paradox. Along the way we will examine various types of transitional dynamics, and see how they fit the real world observations.

In a way, our work is complementary to existing literature. An open economy model is in contrast to the closed economy models used in most related work, which allows us to distinguish between investment and saving.[1] Using an open economy model also opens the door to studying import, export and terms of trade, bearing significant relevance to Asian economies' trade-driven growth. The GDP shares of import and export of the Asian developing economies also seem to display an increasing trend over time, as shown in Figure 3 and 4.

For our research, we adopt from Schmitt-Grohé and Uribe (2003)[28], which articulates several methods to mechanically close small open economy models with incomplete markets. These methods remove the unit root in the dynamics of consumption and pins down the steady state. One mechanically practical way to induce stationarity is to include a cost of holding bonds, or portfolio adjustment cost, which we use in chapter 3. As a technical trick, it is straightforward to implement and generates little undesirable complication in economic interpretations. As indicated in Schmitt-Grohé and Uribe (2003)[28], using this trick did not add any numerical side effect to

[1] Saving and investment coincide in closed economy models.

the economics of the model. The advantage of this method is that it allows directly setting the value of steady state level of bonds.

An alternative which we explore in chapter 4 is to introduce an endogenous discount factor, first developed by Uzawa (1968)[32] and applied in Wang (2007)[33]. By letting the home country discount factor be internally determined by consumption and labor, Uzawa preferences imply that agents are more patient when they are poor early on, and become increasingly impatient as they become wealthier. By endogenizing the rate of time preference, not only are we able to close the model mechanically, we are also making the home agents' patience level fully endogenous, which could have a profound intertemporal impact by feeding back into investment and saving decisions of developing countries. Intuitively, this approach could potentially make it harder to match the observed trends in saving and investment rates, as the diminishing patience could lead to decreasing saving and investment rates over time without any changes to exogenous factors. On the other hand, since foreign asset purchase is a form of saving, higher saving induced by higher patience early on could lead to more foreign asset accumulations, which is observed on countries like China and Japan. The impact of endogenous discounting on dynamics turns out to be interestingly subtle, and chapter 4 will make attempts to explore the nature of endogenous discounting and produce results of a decent match with reality.

Several previous papers have used this endogenous discounting technique, which was first developed in Uzawa (1968)[32]. Obstfeld (1990)[26] found that the optimal consumption responses to transitory and anticipated changes in income and interest rates are similar to those implied by models with a constant discount rate, which is similar to Schmitt-Grohé and Uribe's (2003)[28] conclusion that all techniques to close small open economies deliver highly similar dynamics at business cycle frequencies. Mendoza (1991)[23] also studies the saving and investment dynamics of a model in which the rate of time preference increases with past consumption levels. Other works that make use of this technique include Lucas and Stokey (1984)[20], Schmitt-Grohé (1998)[27], Uribe (1997)[31], and Wang (2007)[33].

A final modelling attempt is to shift from a 1-sector to a 2-sector model. Herrendorf et al (2013)[14] talks about the necessity of using multi-sector models. The 1-sector model has long been a workhorse for macroeconomic research. It takes a minimalist approach in describing economic growth and does a fine job of it for the most part. As a result, it abstracts from certain key features of real life economic development. One such feature is the shift of economic activity and transfer of resources between economic sectors. It has been argued that sectoral shifts are in some cases a major force driving growth, whether it arises as a result of policy intervention or equilibrium outcome. Herrendorf et al (2013)[14] surveys the literature using multi-sector models and concludes that allowing sectoral transformation would deliver better insights on important issues concerning economic development, regional income convergence, aggregate productivity trends, hours worked, business cycles, and wage inequality.

It becomes interesting, therefore, to see whether incorporating multiple sectors in modelling efforts would improve our understanding of economic issues from using 1-sector models. In chapter 5, we study growth dynamics by using a 2-sector model. We compare the results with chapters 3 and 4 using 1-sector models, and see how using a 2-sector model may present any potential advantage due to its fundamental nature.

Some previous work on development issues have made use of multi-sector models. Gollin et al (2007)[11] compares the wildly different growth paths between countries by imposing different exogenous factors such as total factor productivity. They include an agricultural sector, suggesting a subsistence level for food consumption (similar to our model) is a major reason for the currently observed income level disparity between nations. Dekle and Vandenbroucke (2012)[9] studies the structural transformation during China's development with a multi-sector framework, attributing growth to TFP increase in the private sector and labor migration. Laitner (2000)[16] also applies a two-sector model and suggests that the historical rise in saving rates is a consequence of agricultural capital gains in land not being counted as part of saving.

While not deviating from the neoclassical framework or drastically tweaking highly standard models, we take on different thought experiments in dynamic analyses. Besides looking at the stochastic impulse responses and simple transitional dynamics without changes to exogenous variables, we also study perfect foresight dynamics by implementing evolving paths of exogenous factors.

We first consider a developing economy that starts out not only with poor resources but also lagging well behind the world technological frontier, and let the occurrence of technological advancement be a significant part of the growth story alongside factor accumulation. This is motivated by the documentation of a large variation in TFP across nations in Hall and Jones (1999)[13], which computes contribution from physical capital, human capital and technology to output. This is in the same spirit as in the numerous papers discussed in Herrendorf et al (2013)[14]. It turns out this in many cases could potentially account for the increase in saving and investment rates.

Acemoglu (2008)[1] has a chapter on diffusion of technology that extensively discusses the adoption of technology by individual countries and its interactions with the world technology level. The rate of world technological growth and developing countries' integration into the world technology frontier are taken as exogenous in our model, but may also be made endogenous in other models. Institutions and policies also affect the developing countries' adoption of new technology.

There has also been a rich body of literature on disentangling the effect of factor accumulation and productivity growth in the Asian developing economies. Lu

(2012)[18] uses business cycle accounting "wedge" method[2] to identify the driving forces of growth for the 4 Asian tigers at different stages, and argues that factor accumulation was the main driver for growth early on, while more growth is attributed to TFP increases in later stages. Young (1994)[34] argues that rapid factor accumulation in capital and labor as well as sectoral reallocation of resources account for the lion's share of the East Asian growth miracle. This is echoed by Young (1995)[35], which claims that the productivity growth in the Asian economies has not been nearly as high as it was believed to be.

A second experiment we consider is to let foreign demand for the developing country's export increase over time. We follow Corsetti et al (2007)[7] in motivating an increase in the quasi-share parameter in the foreign preference as a proxy for improvement in the quality of (or an increase in consumers' knowledge and reduction in purchasing cost of) the home exports. A natural final experiment is to impose the 2 exogenous changes - increase in home technology and in foreign demand simultaneously.

Along the way, when possible, we explore a slightly modified version of our models that would allow for balanced growth paths (BGP). Models with variables that grow on balanced growth paths in the steady state generally depict a more realistic picture of economic growth, as all economies, developed or developing, rarely stop growing at any moment in time.

This dissertation is a first step in trying to match real life observations. The numerical exercises performed in this study do not follow a rigorous calibration methodology, as we hope to first qualitatively capture some of the key patterns of the Asian developing countries. We motivate our numerical exercises by the economic understanding

[2] The "wedge" approach, also followed in Chang and Hornstein (2012)[5] is adapted from Chari et al (2007)[6] business-cycle accounting framework.

of the various computational experiments on the models. With each definition of dynamics experiment, we examine how well the results can match observed time trends and explore the reasons behind the derived dynamics. As we will see, our models just fall short of matching all the trends we are interested in simultaneously. For future efforts, and equipped with a potentially superior model, we would like to perform calibrations with data, although the potential lack of readily available data may present another challenge.

The rest of the thesis is organized as follows. Chapter 2 computes a simple closed economy model that gives a sense of growth dynamics and the evolution of saving rates. Chapter 3 studies an open economy model with portfolio adjustment cost. Chapter 4 explores the mechanism of endogenous discounting, both in closed and open economy models. Chapter 5 builds a 2-sector open economy model and studies its dynamics. Chapter 6 summarizes results of all attempted models, provides ideas for future extension and concludes.

Figure 1.1: Investment Rate of Asian Developing Countries

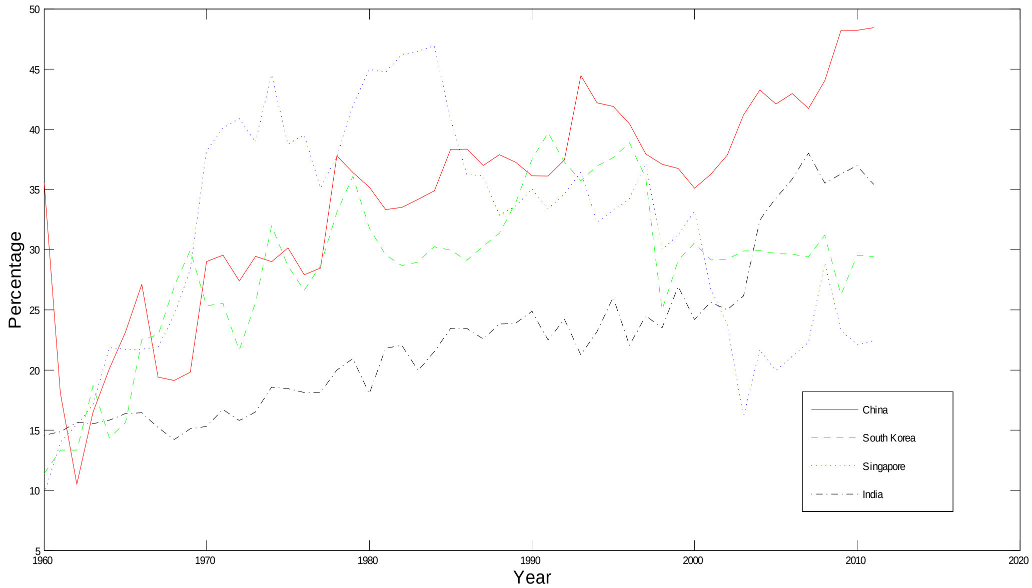


Figure 1.2: Saving Rate of Asian Developing Countries

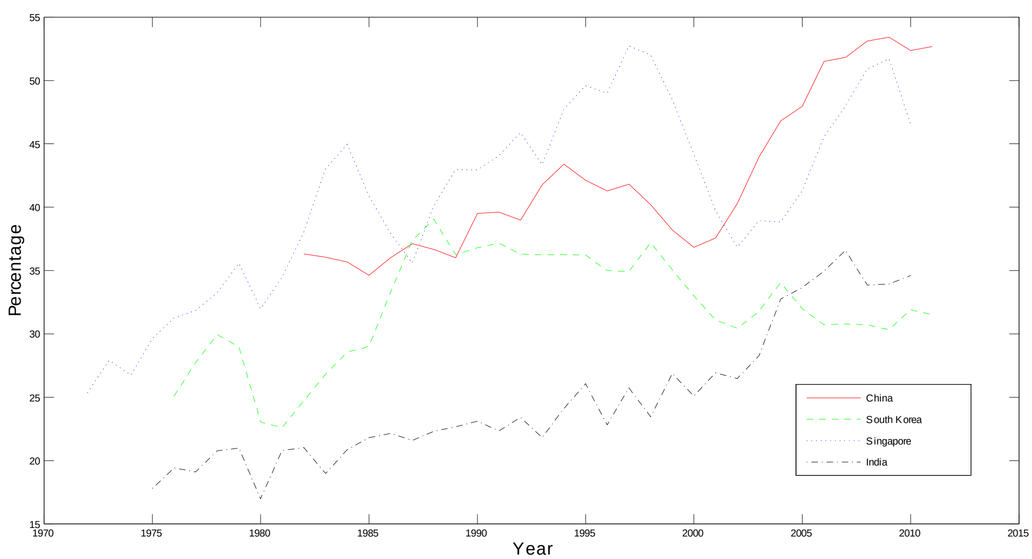


Figure 1.3: Import Share of GDP

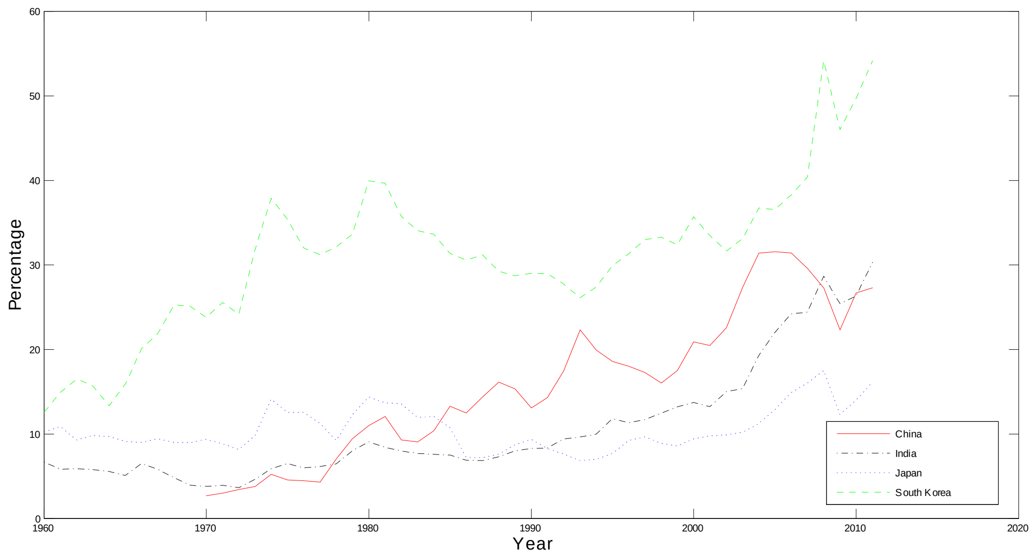
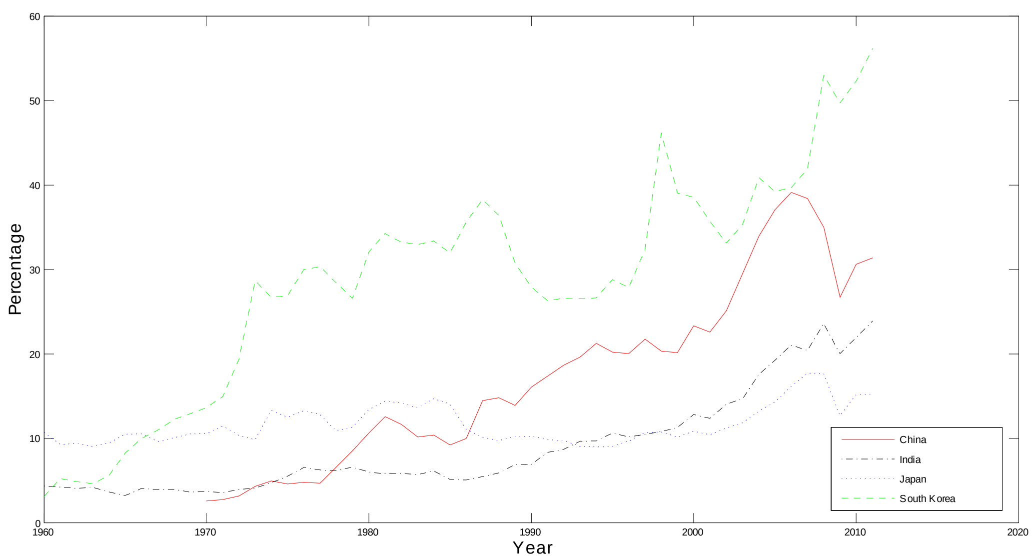


Figure 1.4: Export Share of GDP



Chapter 2

Growth Dynamics of a Baseline Model

Before going into more serious and sophisticated modelling attempts to resolve the saving/investment rate puzzle and the Lucas paradox, to give an example of growth dynamics of a developing economy, we first build a simple dynamic model, and see how a plain closed economy model may fail in explaining our topics of interest.

2.1 Model

This is a basic closed economy model with no modifications that affect the saving rate. It allows for growth on balanced growth paths (BGP) in the steady state. The home country has homogeneous agents that produce, consume and save. They get utility from consumption and disutility from working, and solve

$$\max E \sum_{t=0}^{\infty} \beta^t \frac{(c_t - \tau_t \zeta^{-1} l_t^{\zeta})^{1-\sigma} - 1}{1-\sigma}$$

subject to a flow budget constraint and capital evolution with a capital adjustment cost

$$c_t + i_t = A_t k_t^{\alpha} (z_t l_t)^{1-\alpha} \tag{2.1}$$

$$(1-\delta)k_t + i_t - \frac{\varphi}{2}(\frac{i_t}{k_t} - \delta)^2 k_t = k_{t+1} \tag{2.2}$$

where z_t is the labor-augmenting technology.

Suppose π is the asymptotic growth rate of TFP. The technology grows at a rapid rate in the beginning which slows down to π in the long run. So consumption, capital, investment and output to all grow at the rate of π in the steady state.[1] To accommodate for BGP, we make the coefficient on labor disutility time variant

$$\tau_t = \tau\pi^t$$

We scale all relevant variables by the rate of technological growth π^t, and turn them into units of "per effective worker", including the Lagrange multipliers λ_t on constraint (1) and η_t on (2)

$$\begin{aligned}
\widetilde{c}_t &= c_t/\pi^t \\
\widetilde{i}_t &= i_t/\pi^t \\
\widetilde{k}_t &= k_t/\pi^t \\
\widetilde{z}_t &= z_t/\pi^t \\
\widetilde{\lambda}_t &= \lambda_t\pi^t \\
\widetilde{\eta}_t &= \eta_t\pi^t
\end{aligned}$$

and output per effective worker is expressed with capital and technology per effective worker

$$\widetilde{y}_t = y_t/\pi^t = \widetilde{k}_t^\alpha(\widetilde{z}_t l_t)^{1-\alpha} \tag{2.3}$$

Set $\sigma = 1$, and first order conditions in units of "effective labor" except for labor and percentages are

$$\frac{1}{\widetilde{c}_t - \tau\zeta^{-1}l_t^\zeta} - \widetilde{\lambda}_t = 0 \tag{2.4}$$

$$-\widetilde{\lambda}_t + \widetilde{\eta}_t[1 - \varphi(\frac{\widetilde{i}_t}{\widetilde{k}_t} - \delta)] = 0 \tag{2.5}$$

[1] π corresponds to the growth rate of foreign country (or world) TFP in the open economy models. Chapter 3 provides more details on the rationale of balanced growth path.

$$-\widetilde{\eta}_t + \beta\frac{1}{\pi}E_t\{\widetilde{\lambda}_{t+1}\alpha\widetilde{z}_{t+1}^{1-\alpha}\widetilde{k}_{t+1}^{\alpha-1}{l_{t+1}}^{1-\alpha} + \widetilde{\eta}_{t+1}[1-\delta-\frac{\varphi}{2}(\delta+\frac{\widetilde{i}_{t+1}}{\widetilde{k}_{t+1}})(\delta-\frac{\widetilde{i}_{t+1}}{\widetilde{k}_{t+1}})]\} = 0 \quad (2.6)$$

$$-\frac{\tau l_t^{\zeta-1}}{\widetilde{c}_t - \tau\zeta^{-1}l_t^{\zeta}} + \widetilde{\lambda}_t(1-\alpha)\widetilde{k}_t^{\alpha}\widetilde{z}_t^{1-\alpha}l_t^{-\alpha} = 0 \quad (2.7)$$

and also rewrite budget constraint

$$\widetilde{c}_t + \widetilde{i}_t = \widetilde{k}_t^{\alpha}(\widetilde{z}_t l_t)^{1-\alpha} \quad (2.8)$$

$$(1-\delta)\widetilde{k}_t + \widetilde{i}_t - \frac{\varphi}{2}(\frac{\widetilde{i}_t}{\widetilde{k}_t} - \delta)^2\widetilde{k}_t = \pi\widetilde{k}_{t+1} \quad (2.9)$$

Among all the variables of our interest mentioned in the introduction, only the investment rate applies in this model. In a closed economy, current account and trade do not exist and investment is identical to saving. We define the investment/saving rate as

$$SAV\%_t = \frac{i_t}{y_t} == \frac{\widetilde{i}_t}{\widetilde{y}_t} \quad (2.10)$$

The equilibrium is $\{\widetilde{c}_t, l_t, \widetilde{i}_t, \widetilde{k}_{t+1}, \widetilde{y}_t, \widetilde{\lambda}_t, \widetilde{\eta}_t, SAV_t\}_{t=0}^{\infty}$ and equations (3)-(10), and it delivers a balanced growth path in the steady state, where individual levels of consumption, output, capital, investment and technology all grow at the rate of π.

2.2 Parametric Choices

As explained in chapter 1, we shy away from performing parametric calibrations based on data, and take values from existing literature. We make each period in our model one year, so β is taken to be 0.96. Capital adjustment cost coefficient φ and depreciation rate δ are taken from Canzoneri et al (2012)[4], calibrated to the Canadian economy and adjusted from quarterly to annual.[2] τ and ζ on labor disutility are taken from Mendoza (1991)[23] and Schmitt-Grohé and Uribe (2003)[28], also

[2] δ is quadrupled from the quarterly value of 0.025 to 0.1 per year.

calibrated to the Canadian economy. σ is set to 1 to algebraically allow balanced growth paths. π is set to be 1.02, as the US GDP per capita on average grows at about 2 percent per year.

A list of parameter values are

A	α	φ	δ	σ	τ	ζ	β	π
1	1/3	4.5	0.1	1	1	1.455	0.96	1.02

2.3 Stochastic Impulse Responses

Before going into transitional dynamics, we take a quick look at the impulse responses under the stochastic case. We first turn labor-augmenting technology scaled by the growth rate $\widetilde{z}_t$ into the traditional total factor of productivity (TFP) A_t in the following way

$$\widetilde{z}_t^{1-\alpha} = A_t$$

Then we impose a stochastic shock to the TFP

$$\ln A_t - \ln A = 0.95(\ln A_{t-1} - \ln A) + \epsilon_t, \quad \epsilon_t \sim iid\ N(0, \sigma^2)$$

Size of the shocks are set as $\sigma = 0.01$. Such a shock represents a temporary increase in technology when the system is in the steady state. The impulse response to the 3 shocks are plotted for 30 periods in Figure 1.

Not surprisingly, when technology gets a boost, consumers work more to capitalize on it. They produce more, consume more and invest more. Saving/investment rate increases while the share of consumption decreases. This is because the agents want to take advantage of the increase in productivity, and they do so by allocating more resources to stocking up on capital which contributes to production.

2.4 Simple Transitional Dynamics

We now go into deterministic dynamics to study the time trends of various variables predicted by the model. We start by looking at the transitional dynamics to the steady state. We term this the "simple" dynamics as it does not involve any changes to exogenous factors such as TFP. We start the system with initial levels of capital lower than the steady state value, and trace out the optimal paths of all variables to their steady state values.

For all the deterministic transitional dynamics in this thesis, when choosing initial values of capital (and bonds in later chapters), we make them as small as computationally permissible, usually below 1/3 of their respective steady state values.

Figure 2 shows a case of simple dynamics starting with a relatively low initial capital, just above 20% of its steady state value. Consumption, labor, investment, capital and output all increase. Investment rate rises sharply for the first 30 periods, and then tails off slowly to the steady state. When an economy starts out poor, agents choose to devote more resources to consumption early on, and would rather build up their capital stock slowly, as their future consumption is discounted. After accumulating a decent amount of capital and increasing consumption to a certain level, agents can afford to divert more resources to saving and investment.

We also see in Figure 3 a similar case with higher initial capital, about 60% of its steady state value. Consumption, labor, investment, capital and output still increase in monotone. Saving/investment rate, however, strictly decreases over time. When the economy starts out in a wealthier state and has limited growth potential before reaching the steady state, agents know their capital stock is close to being the maximum value it can possibly be, and simply choose to reduce the amount of resources they devote to investment over time for more consumption in the near future that's less discounted.

In a way we can think of the case in Figure 3 as the second half of the story in Figure 2. If an economy starts out poor, it would increase its saving/investment rate first to ensure a decent amount of consumption while slowly accumulating capital for future production; after reaching a certain level of wealth in capital and consumption, the consumers can afford to decrease investment and devote more resources to consumption as the capital stock approaches the steady state. Therefore, the initial level of wealth critically determines the trend of saving/investment rate in the simple dynamics.

2.5 Transitional Dynamics with Exogenous Increase in TFP

Having looked at the simple dynamics without changes in exogenous factors, we now consider cases of dynamics while implementing exogenous changes to TFP, which is widely believed to be happening to the developing countries especially during their takeoff. The motivation and interpretation for this is provided in greater detail in the following chapters.

We design the path of TFP so that it increases at a diminishing rate for the first T periods. We call this the "catch-up" stage, which is discussed in more details in later chapters. At the end of catch-up, A_t stays at a high value A permanently.

$$A_t = A_{t-1} + \theta(A - A_{t-1}),\ 0 < \theta < 1,\ \forall t < T$$

$$A_t = A,\ \forall t \geq T$$

This states that the gap between the current TFP and its final value decreases at a decreasing speed.[3] If we think of the increase in A_t as a process of catch-up between developing and developed countries in terms of technology, the catch-up starts out very fast, and gradually slows down until the developing country is as technologically advanced as the developed country. The value of θ is taken such that the last period of the catch-up does not see an abrupt increase of A_t. Such a path for TFP with smoother increases ensures TFP is very close to its final value during the last part of the catch-up, and the resulting dynamics would exhibit less of a "kink" point at the end of catch-up. Agents fully anticipate the exact path of A_t in their optimization, hence this is also called perfect foresight transitional dynamics.

Figure 4 shows such a case with low initial capital endowment equal to the case in Figure 2. Not surprisingly with increase in technology, consumption, labor, investment, capital and output would all increase over time. Interestingly, the movement of saving/investment rate is far from monotonic. It experiences a sharp increase followed by decrease before the 20th period. It then rises again before falling slowly toward the steady state. The initial increase is for similar reasons as in the simple dynamics - when the economy is poor, consumers have to devote more resources to consumption first before increasing investment. The increase did not last very long in this case, however, because the rapid increase in TFP makes up for the low capital stock, and allows the consumers to temporarily decrease saving/investment for more consumption. The saving/investment rate would increase again before the catch-up ends, because the last stretch of the non-linear catch-up has limited impact due to the very little increase in TFP.

[3] In later chapters, we adopt a linear increase in exogenous variables to highlight a distinctive catch-up stage.

If we increase the initial capital stock and keep everything else unchanged, as shown in Figure 5, we would get a less sophisticated trend. Now the saving/investment rate first sharply increases but only for a very short time, followed by slight decrease over the long haul. This should not be difficult to comprehend, as consumers who start with a decent amount of wealth would start not in a hurry to invest a lot. Their increase in investment is driven by the sharp increase in the early part of the catch-up, but that incentive quickly draws to an end, as the increase in TFP quickly diminishes and the marginal product of capital also decreases with capital build-up.

2.6 Summary of Results

Using a closed economy model is a good starting exercise for studying growth dynamics. Its plainness allows us to see clearly the interactions between and impacts of all economic factors.

In this simple model, as it turns out, neither simple dynamics nor implementation of exogenous increase in technology can produce even a close-to-realistic trend in the saving/investment rate. The initial endowment of capital plays a critical role in shaping the trend. We are generally unable to produce a persistently increasing saving/investment rate as suggested by historical data on developing countries. In the cases that do generate an initial increase in the rate, the increase in usually short-lived and often followed by reverse trends. Implementing exogenous increases in technology seems to hardly improve the results.

It's clear that the baseline closed economy model does a meager job of matching the observed trend of even just one of the variables we are interested in. In light of this, the following chapters make more sophisticated modelling attempts with an open economy.

Figure 2.1: Stochastic Impulse Responses to Increase in A_t

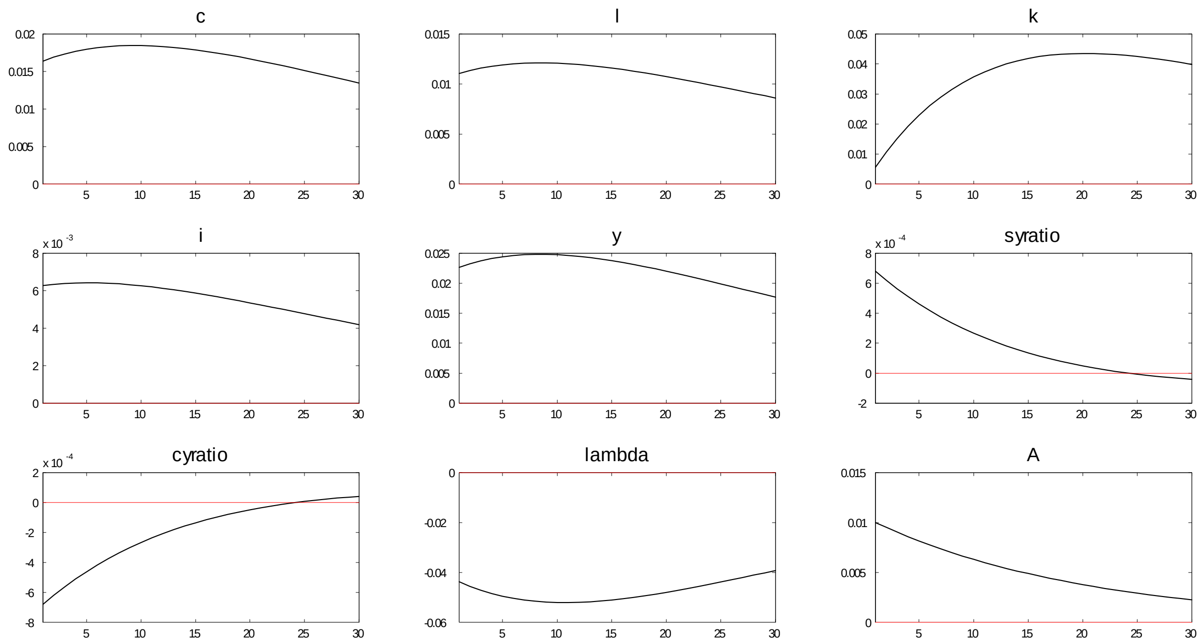


Figure 2.2: Simple Dynamics to Steady State, Low Initial Capital

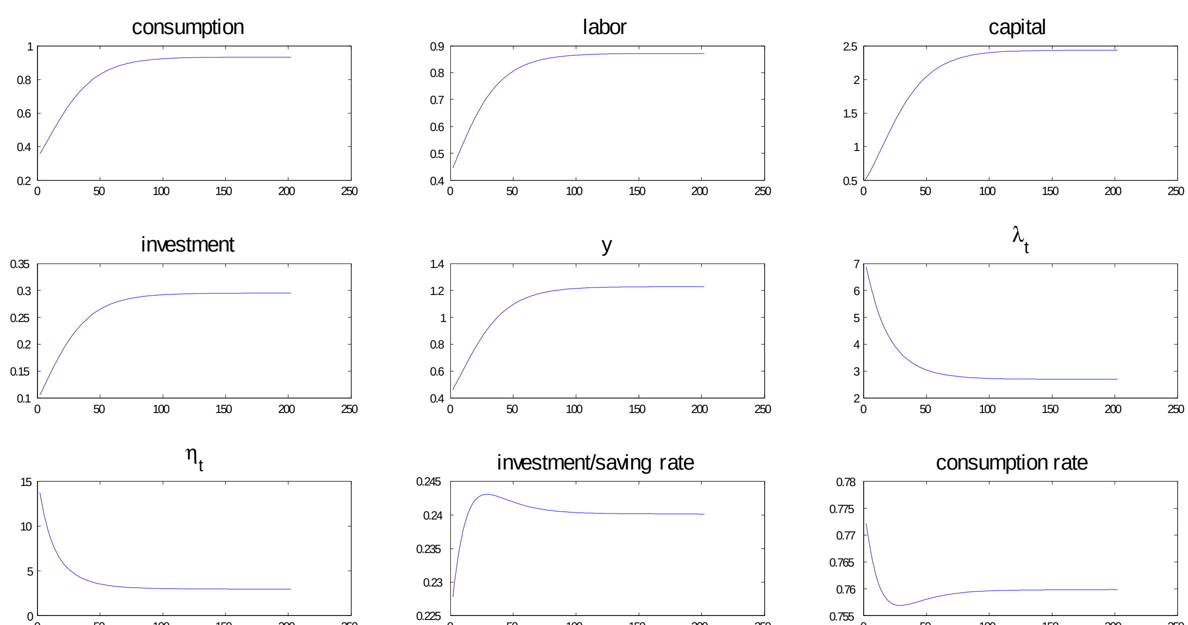


Figure 2.3: Simple Dynamics to Steady State, High Initial Capital

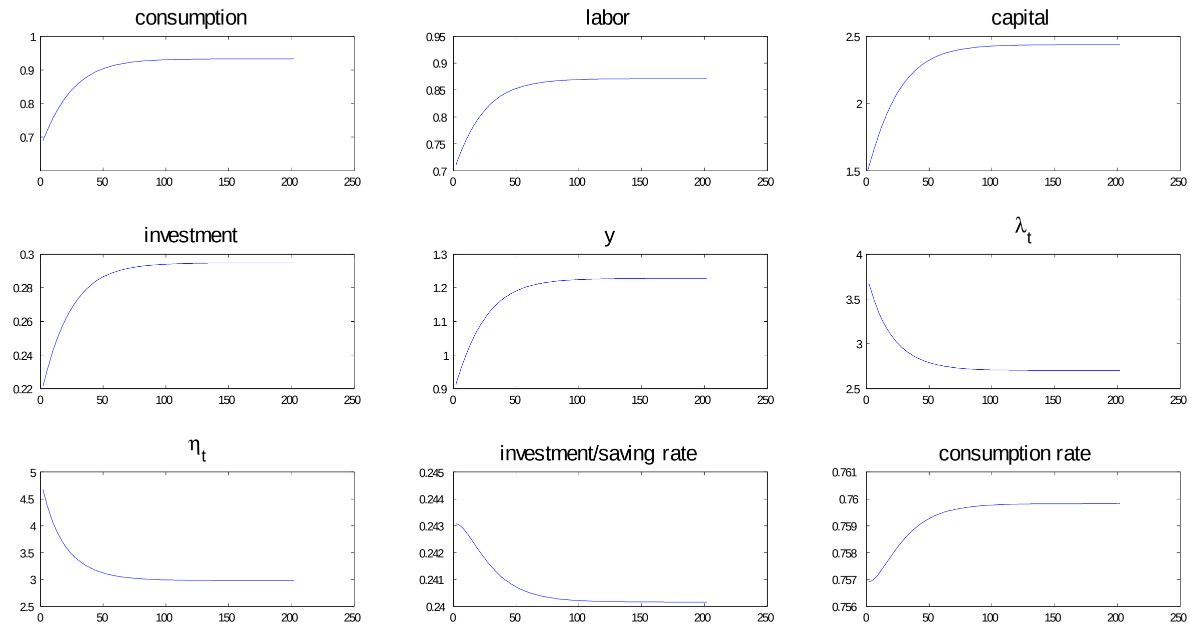


Figure 2.4: Dynamics with Increase in A_t and Low Initial Capital, $\theta = 0.15$

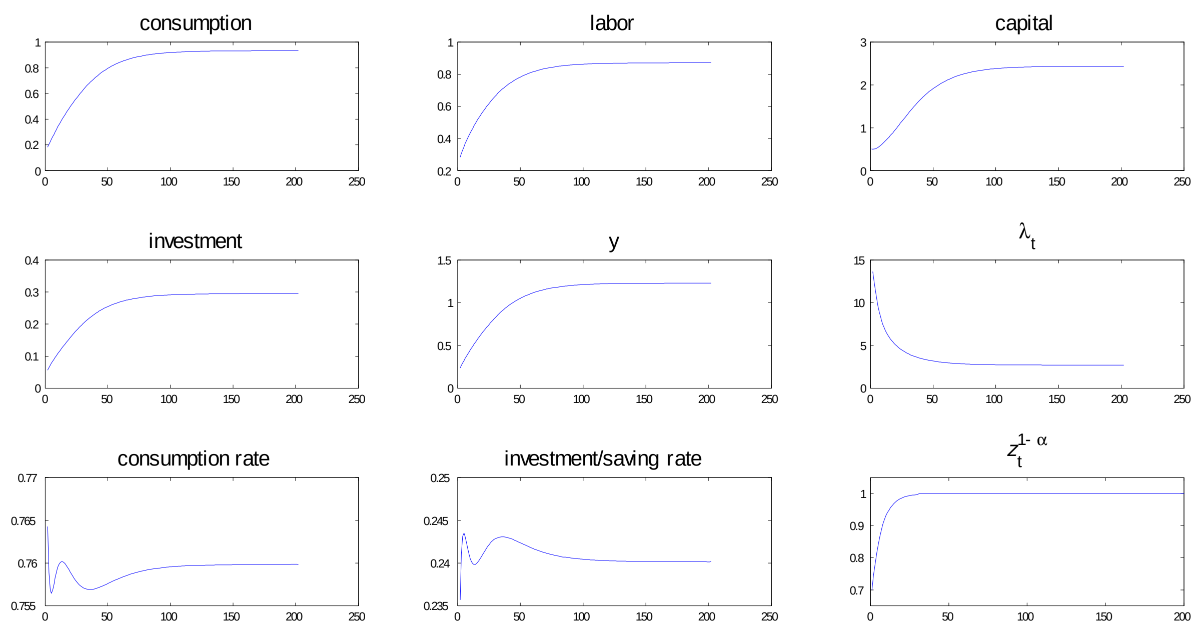
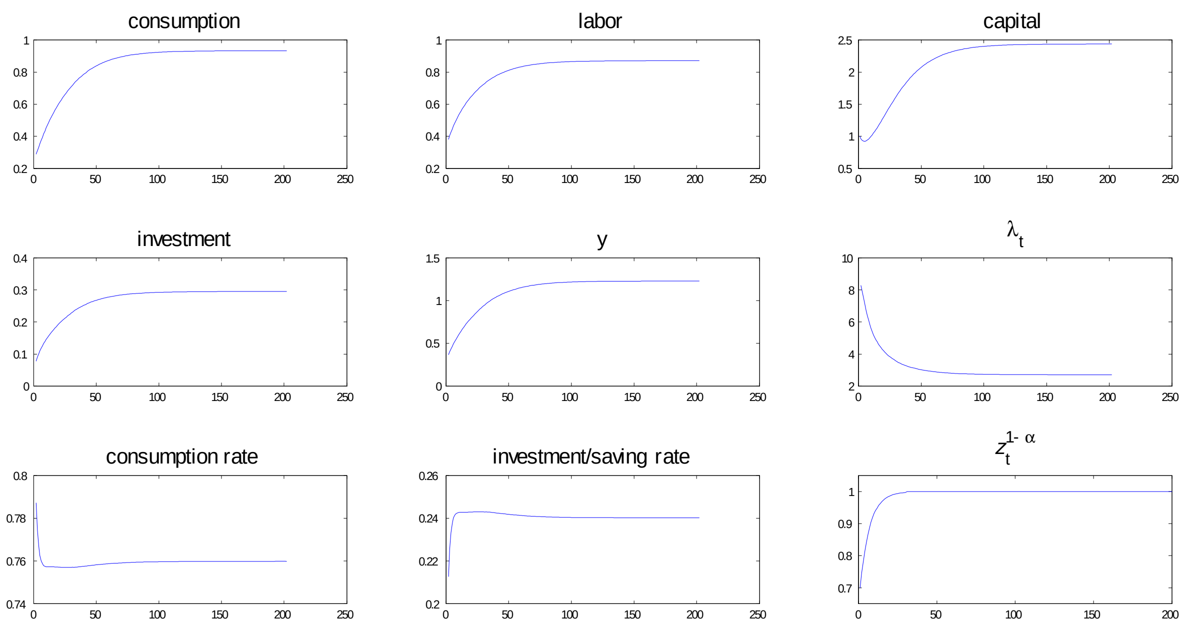


Figure 2.5: Dynamics with Increase in A_t and High Initial Capital, $\theta = 0.15$



Chapter 3

Growth Dynamics of an Open Economy Model

3.1 Introduction

As seen in the previous chapter, using a plain closed economy model can do very little in explaining the growth dynamics we are interested in. Moreover, by design a closed economy model does not distinguish between saving and investment, and also do not allow trade and foreign asset transactions. A natural step to extend the baseline model would be to make it a 2-country open economy model. This is also motivated by the fact that the Asian developing economies attribute a substantial amount of their growth to international trade. In this chapter we build such a model with portfolio adjustment cost, and study its dynamics of not only saving and investment rates, but also import and export shares of output and current account balance.

The rest of the chapter is organized as follows. Section 2 lays out an open economy model and discusses parametric choices. Section 3 takes a quick look at the stochastic impulse responses. Section 4 looks at the simple transitional dynamics and introduces a balanced growth version of the model. Section 5 motivates 2 exogenous changes and implement them to study the perfect foresight dynamics. Section 6 further discusses the economic reasoning of the results. Section 7 summarizes the results.

3.2 Model

This is a relatively straightforward neoclassical open economy general equilibrium model. The home country consists of homogeneous Yeoman farmer type of consumer/firm agents. The foreign country, or the rest of the world (ROW), has an endowment economy. The two countries each have a type of consumption good and engage in trade and asset transactions, with the home good being the numeraire. For notation, variables denoted with a $*$ superscript refers to the subject taking place in the foreign country, and a h or f subscript refers to the type of goods being home or foreign.

3.2.1 Home Country

The representative home agent produces with capital and labor, and trades with the foreign country for consumption variety. She maximizes lifetime utility discounted by β_h by making consumption and saving decisions, subject to an intertemporal budget constraint. She gets disutility from working. She can save by investing in capital stock and buying riskless foreign assets in an incomplete market.

The home consumer's problem is

$$\max E \sum_{t=0}^{\infty} \beta_h^t \frac{(c_t - \tau\zeta^{-1} l_t^{\zeta})^{1-\sigma} - 1}{1-\sigma}$$

subject to

$$c_{h,t} + p_{f,t}c_{f,t} + i_t + b_t p_{f,t} + \frac{\Psi}{2}(b_t - \bar{b})^2 p_{f,t} = y_t + R^*_{t-1} b_{t-1} p_{f,t} \tag{3.1}$$

$$(1-\delta)k_t + i_t - \frac{\varphi}{2}(\frac{i_t}{k_t} - \delta)^2 k_t = k_{t+1} \tag{3.2}$$

where every term including investment and capital is in units of home good. Agents produce with a Cobb-Douglas technology

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha} \tag{3.3}$$

and consumption of home and foreign good is aggregated in the Dixit-Stiglitz form

$$c_t = [\gamma^{1-\rho} c_{h,t}^{\rho} + (1-\gamma)^{1-\rho} c_{f,t}^{\rho}]^{\frac{1}{\rho}}$$

In capital evolution (2) we add in a capital adjustment cost term $\frac{\varphi}{2}(\frac{i_t}{k_t} - \delta)^2 k_t$ that's standard in the literature. To close the model, we also add a cost of holding bonds, or portfolio adjustment cost $\frac{\Psi}{2}(b_t - \bar{b})^2 p_{f,t}$ into (1), as demonstrated in Schmitt-Grohé and Uribe (2003)[28]. R^* is the endogenously determined gross interest rate on bonds, which is in terms of the foreign good. p_f is the relative price of foreign goods in terms of home goods, an increase of which means a real depreciation for the home country. Constraint (1) states that in each period the agent gets production of home goods and bonds payment with interest from the previous period, and spends all that resource on consumption, investment and new bonds purchase, besides paying the portfolio adjustment cost.

Let λ and η be the Lagrange multipliers for constraints in (1) and (2). We get the following first order conditions

$$(c_t - \tau\zeta^{-1} l_t^{\zeta})^{-\sigma} - \lambda_t p_t = 0 \tag{3.4}$$

$$-\eta_t + \beta_h E_t\{\lambda_{t+1} \alpha A_{t+1} k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} + \eta_{t+1}[1 - \delta - \frac{\varphi}{2}(\delta + \frac{i_{t+1}}{k_{t+1}})(\delta - \frac{i_{t+1}}{k_{t+1}})]\} = 0 \tag{3.5}$$

$$-\lambda_t + \eta_t[1 - \varphi(\frac{i_t}{k_t} - \delta)] = 0 \tag{3.6}$$

$$\lambda_t[-p_{f,t} - \Psi(b_t - \bar{b})p_{f,t}] + \beta_h E_t(\lambda_{t+1} p_{f,t+1} R_t^*) = 0 \tag{3.7}$$

$$-(c_t - \tau\zeta^{-1} l_t^{\zeta})^{-\sigma} \tau l_t^{\zeta-1} + \lambda_t (1-\alpha) A_t k_t^{\alpha} l_t^{-\alpha} = 0 \tag{3.8}$$

where p_t is the aggregate price of the consumption bundle, resulting from the static cost minimization problem solutions

$$p_t = [\gamma + (1-\gamma)p_{f,t}^{\frac{\rho}{\rho-1}}]^{\frac{\rho-1}{\rho}} \tag{3.9}$$

$$c_{h,t} = c_t \gamma p_t^{\frac{1}{1-\rho}} \tag{3.10}$$

$$c_{f,t} = c_t(1-\gamma)(\frac{p_{f,t}}{p_t})^{\frac{1}{\rho-1}} \tag{3.11}$$

and

$$p_t c_t = c_{h,t} + p_{f,t} c_{f,t}$$

3.2.2 Foreign Country

The foreign country (or rest of the world) has an endowment of foreign good in each period. There is no production or labor decision. They also desire home good in their consumption via trade. They issue real bonds denominated in their own good. A representative foreign agent maximizes discounted utility by making consumption and saving decisions. Their patience level β_f could potentially be different from home consumers' β_h. A representative foreign agent solves

$$\max E \sum_{t=0}^{\infty} \beta_f^t \ln c_t^*$$

subject to

$$c_{f,t}^* + \frac{1}{p_{f,t}} c_{h,t}^* + b_t^* \leq y_t^* + R_{t-1}^* b_{t-1}^* \tag{3.12}$$

where all terms are in units of foreign good. Consumption bundle is similar to that of home, with different weight between home and foreign good

$$c_t^* = [\gamma^{*1-\rho} c_{f,t}^{*\rho} + (1-\gamma^*)^{1-\rho} c_{h,t}^{*\rho}]^{\frac{1}{\rho}}$$

Let ν be the Lagrange multiplier for the constraint in (12), we get first orders conditions

$$\frac{1}{c_t^*} - \nu_t p_t^* = 0 \tag{3.13}$$

$$-\nu_t + \beta_f R_t^* E_t(\nu_{t+1}) = 0 \tag{3.14}$$

and similarly the static cost minimization solution

$$p_t^* = [\gamma^* + (1-\gamma^*) p_{f,t}^{\frac{\rho}{1-\rho}}]^{\frac{\rho-1}{\rho}} \tag{3.15}$$

$$c_{f,t}^* = c_t^* \gamma^* p_t^{*\frac{1}{1-\rho}} \tag{3.16}$$

$$c_{h,t}^* = c_t^* (1-\gamma^*)(p_{f,t} p_t^*)^{\frac{1}{1-\rho}} \tag{3.17}$$

in which

$$p_t^* c_t^* = \frac{1}{p_{f,t}} c_{h,t}^* + c_{f,t}^*$$

3.2.3 Equilibrium Conditions

Assume the home population is of size 1, and foreign population has size N. We have bonds market clearing condition

$$b_t + N b_t^* = 0 \tag{3.18}$$

and assuming both investment and capital adjustment cost are taken from home output, home and foreign goods market clearing conditions are

$$c_{h,t} + N c_{h,t}^* + i_t + \frac{\Psi}{2}(b_t - \overline{b})^2 p_{f,t} = A_t k_t^\alpha l_t^{1-\alpha} \tag{3.19}$$

$$c_{f,t} + N c_{f,t}^* = N y_t^* \tag{3.20}$$

An equilibrium is defined to be the path of $\{c_t, l_t, i_t, k_{t+1}, y_t, b_t, c_{h,t}, c_{f,t}, c_t^*, b_t^*, c_{f,t}^*, c_{h,t}^*, p_{f,t}, p_t, p_t^*, R_t^*, \lambda_t, \eta_t, \nu_t\}_{t=0}^\infty$ and equations (1)-(18) and (19) or (20). Having both (19) and (20) together is redundant due to Walras' Law.

3.2.4 Parametric Choices

As stated in the introduction, we shy away from doing calibrations based on data of an actual economy. We take some of the commonly accepted parametric values, and for the less obvious parameters, we cite from the literature for calibrated values or give some reasoning.

Capital adjustment cost coefficient φ and depreciation rate δ are taken from Canzoneri et al (2012)[4], calibrated to the Canadian economy. τ and ζ on labor disutility are taken from Mendoza (1991)[23] and Schmitt-Grohé and Uribe (2003)[28], also calibrated to the Canadian economy. β_h and β_f are both 0.99 as we treat each period in the model as a quarter. Setting them equal enables us to directly force the steady state value of bonds to be equal to $\bar{b}$.

For the value of $\bar{b}$, we simply take the fact that China's foreign asset to GDP ratio has recently been approaching one half, and set it such that the steady state bonds to output ratio is close to 0.5. We measure N with the fact that China accounts for about 20% of the world's population. We make Ψ as small as computationally permissible, as the cost of holding bonds is introduced merely to close the model, and we don't want it to interfere with the fundamental dynamics.

γ and γ^* each represent the expenditure share of total consumption spent on the domestic good. By design we conveniently assume all import goes into consumption, and take the values of γ and γ^* to be the ratio between total import and consumption. All imports being consumed is admittedly a troubling assumption, as it is well known that a significant portion of developing countries' imports are investment goods such as machineries. With data on how much of total import is consumed versus invested, γ and γ^* would be calibrated to be lower than their values here, and it would be interesting to distinguish in the model between consumption and investment in total imports. The last chapter talks more about this as a possibility of extension.

The choice for $\sigma = 1$ will be explained when motivating balanced growth path later. For the computational exercises in the following sections, unless otherwise specified as evolving paths in the structural transitional dynamics, the list of used parameters are

A	α	β_h	β_f	φ	δ	σ	ζ	τ	ρ	γ	γ^*	Ψ	$\bar{b}$	N	y^*
1	1/3	0.99	0.99	4.5	0.025	1	1.455	1	$1-1/0.9$	0.85	0.95	0.001	4.1	4.2	1

3.3 Stochastic Impulse Response

Before going into transitional dynamics, we take a quick look at the impulse responses under the stochastic case. We impose 3 stochastic shocks respectively to the system: home total factor productivity (TFP) A_t, foreign endowment y_t^*, and foreign good's weight in the foreign consumption bundle γ_t^*. Studying these shocks also gives a taste of the later section on perfect foresight dynamics, as the exogenous changes implemented there is essentially a sequence of predicted shocks. We apply the following shock mechanism

$$\begin{aligned}
\ln A_t - \ln A &= 0.95(\ln A_{t-1} - \ln A) + \epsilon_t^A, \quad \epsilon_t^A \sim iid\ N(0, \sigma_A^2) \\
\ln y_t^* - \ln y^* &= 0.95(\ln y_{t-1}^* - \ln y^*) + \epsilon_t^y, \quad \epsilon_t^y \sim iid\ N(0, \sigma_y^2) \\
\ln \gamma_t^* - \ln \gamma^* &= 0.95(\ln \gamma_{t-1}^* - \ln \gamma^*) - \epsilon_t^\gamma, \quad \epsilon_t^\gamma \sim iid\ N(0, \sigma_\gamma^2)
\end{aligned}$$

Also for variables of interest, we define savings and current account in home goods as following

$$\begin{aligned}
SAV_t &= y_t - p_t c_t \\
CA_t &= p_{f,t}(b_t - b_{t-1})
\end{aligned}$$

Current account can also be written as the sum of trade surplus and interest payment on asset

$$CA_t = Nc^*_{h,t} - c_{f,t}p_{f,t} + (R^*_{t-1} - 1)b_{t-1}p_{f,t}$$

In relation to the Lucas paradox, a current account surplus means capital outflow for a country, as is the case observed in countries like China and Japan.

Under the stochastic case, we want to see how each variable of interest reacts upon a one time increase in home TFP, foreign endowment or decrease of weight of foreign goods in foreign consumption bundle, and returns back to the steady state. Size of the shocks are set as $\sigma_A = \sigma_y = \sigma_\gamma = 0.01$. The negative sign in front the error term in the evolution of γ^*_t indicates it's a negative shock to demand for foreign goods, effectively an increase in foreign demand for home exports. The impulse responses to the 3 shocks are plotted for 30 periods.

In Figure 1, with a positive shock to A_t, home productivity gets a boost. Investment and labor both increase to capitalize on the boost in productivity. As a result, capital, output and consumption go up. Relative price of foreign good becomes higher as home good becomes more abundant. Agents also dump some foreign assets to be able to afford more investment and consumption at the same time.

In Figure 2, when a positive shock happens to foreign endowment y^*_t, it benefits the home country as well through a falling relative price due to temporary abundance of foreign goods. This, plus the dumping of some foreign asset, allow the home consumers to be able to consume more while also investing more, therefore also increasing labor and output. The welfare impact of an increase in foreign endowment is highly similar to that of an increase in home TFP. Good fortune happening to one country gets transmitted into the other through changes in terms of trade.

In Figure 3, a decrease in γ^*_t means foreigners temporarily have a slightly higher preference for home exports. Home good as a result experiences real appreciation.

Home consumers work and produce more to meet higher foreign demand. Facing a higher import price, foreigners borrow by selling more asset to consume. With foreign good depreciation, home consumers increase foreign asset holding and consumption. Because the increase in foreign demand is temporary, home country wants to sell more now when the price is high, while investment only increases future output. The opportunity cost of investing becomes so high that and the increases in export, consumption and bonds force investment to go down.

A quick check of stochastic impulse responses yields intuitively sensible results and allows us to proceed with analyzing the deterministic dynamics.

3.4 Simple Transitional Dynamics

In this section, we perform the common exercise of transitional dynamics and see how the system converges to the steady state. We term this as "simple" because it does not involve any exogenous changes to parameters. The setting is fully deterministic and there is no uncertainty. For more variables of interest, we define investment and saving rates as

$$
\begin{aligned}
INV\%_t &= \frac{i_t}{y_t} \\
SAV\%_t &= \frac{y_t - p_t c_t}{y_t}
\end{aligned}
$$

Also define import and export as shares of home output, and trade surplus

$$
\begin{aligned}
IMPSH_t &= p_{f,t} c_{f,t} / y_t \\
EXPSH_t &= N c_{h,t}^* / y_t \\
SURPLUS_t &= N c_{h,t}^* - p_{f,t} c_{f,t}
\end{aligned}
$$

3.4.1 Steady State Dynamics

The system starts in a poor state, with low levels of initial endowment in capital and bonds, and the paths of some key variables to the steady state are plotted in Figure 4. We set the initial capital and bonds to be around a quarter of their steady state values, hoping to capture the growth paths of an originally severely underdeveloped economy.

Home consumption, labor, capital and home output all increase monotonically. Home country experiences real depreciation as it becomes more productive, making home goods more abundant and also increasing foreign consumption for approximately the first 100 periods.

Current account is always weakly positive as a result of constant bonds accumulation, which does not violate the Lucas paradox. It must be pointed out that this is easily a direct consequence of starting with an initial bond holding below its steady state level. The dynamics mechanically dictates that bonds must overall increase to eventually reach a high level. Starting with a low initial bond holding is a convenient way to generate current account surplus, yet it's not inconsistent with the reality that many developing countries do start from low or even negative foreign asset holdings.[1]

In the stochastic impulse responses with an increase in foreign demand, we had a decrease in investment because the increase in foreign demand was temporary and agents expect it to go back down, therefore prioritizing consumption, export and bonds purchase. In the steady state dynamics, agents expect continuously increasing foreign demand over time, and increase their investment so they can produce and export more in the future when the price is higher.

[1]Starting with below steady state bonds does not guarantee perpetual current account surplus. It turns out in later chapters that in some cases current account experiences a period of deficit in the begining.

Import share increases, but export share, contrary to data, decreases over time. Investment and savings both go up. However, both the investment and saving rate go down over time. As explained in the introduction, this is a result of the diminishing marginal product. When the economy is poor in resources, marginal product of saving and investing in capital is high; as the economy grows and stocks up on capital, the marginal product of capital decreases. Hence we see a decreasing trend in saving and investment rates. This case perfectly exemplifies the saving/investment rate puzzle.

It also turns out increasing the initial endowment of capital and bonds do not qualitatively change the trends in import and export shares as well as investment and saving rates.

3.4.2 Balanced Growth Path (BGP) Dynamics

So far we have assumed that the model reaches a steady state in which everything is stationary. Alternatively, and for more realism, we can make the model behave with a balanced growth path (BGP) in the steady state, where everything is either constant or growing at a constant rate. As a preview of the detailed discussion in section 6, this would allow for a more credible modelling adaptation of the real world. The developed foreign country grows at a relatively slow but stable "world rate" that's dictated by the latest technological progress, and the developing home country initially plays catch-up by growing at a faster rate and closing the gap with the developed world, and once it catches up with the foreign country in technological build-up, everyone grows at the same world rate. McGrattan (2012)[21] applies a constant known rate of technological growth in their transitional dynamics that's similar to our rate of world technological growth.

To do this, we make a few modifications to the model. We let the foreign endowment increase at a constant rate of π, the world rate of technological progress

$$y_t^* = y^* \pi^t$$

and transform the home production into one with labor-augmenting technology z_t

$$y_t = k_t^\alpha (z_t l_t)^{1-\alpha}$$

so the relation between TFP and labor-augmenting technology is

$$A_t = z_t^{1-\alpha}$$

Also by scaling the cost of holding bonds by the growth rate of foreign technology, we can rewrite the home budget constraint

$$c_{h,t} + p_{f,t} c_{f,t} + i_t + b_t p_{f,t} + \frac{\Psi}{2}(\frac{b_t}{\pi^t} - \bar{b})^2 \pi^t p_{f,t} = y_t + R_{t-1}^* b_{t-1} p_{f,t}$$

To make home consumption in the non-separated utility divisible by π^t without affecting labor, we apply a trick that's similar to those used in Mertens and Ravn (2011)[24] and Jaimovich and Rebelo (2009)[15]. Set $\sigma = 1$, and let τ increase at the same rate as foreign endowment to so labor falls out cleanly when consumption is divided by π^t.

$$\tau_t = \tau \pi^t$$

The new equilibrium with BGP would be the same equations (1)-(19), but scaling all variables of interest (except labor) by the world rate of technological progress π^t. By doing this we turn variables into units of "per effective worker". For example,

$$\begin{aligned} \widetilde{y}_t &= \frac{y_t}{\pi^t} \\ &= (\frac{k_t}{\pi^t})^\alpha (\frac{z_t}{\pi^t} l_t)^{1-\alpha} \\ &= \widetilde{k}_t^\alpha (\widetilde{z}_t l_t)^{1-\alpha} \end{aligned}$$

where

$$\widetilde{k}_t = \frac{k_t}{\pi^t}$$
$$\widetilde{z}_t = \frac{z_t}{\pi^t}$$

$\widetilde{k}_t$ is the capital per unit of effective labor, and $\widetilde{z}_t$ represents the level of home country technology relative to the world technology. In other words the growth rate of $\widetilde{z}_t$ represents the pace at which the home country catches up with the foreign country in productive efficiency. We get a steady state of labor and all other variables in units of effective labor. Labor is not scaled by technological growth, as the hours of working cannot grow to infinity like the other variables measured in goods.

Since we are treating every period as a quarter, we set π to be 1.005, as the US GDP per capita on average grows at about 2 percent per year. Also, in the BGP case we change the discount factor slightly and set $\beta_h = \beta_f = 0.99\pi$, so the optimization conditions exactly match those in the steady state case, making their results more comparable. Note that the two equilibria are not identical, as the budget constraints in the BGP case contains π too. In both cases, $\sigma = 1$.

From here on we will refer to the previous case of a "still" steady state where everything stays constant as the "steady state case", to distinguish from the "balanced growth path (BGP) case", where all applicable terms (capital, consumption etc.) are in units of effective labor. Figure 5 plots the simple dynamics of the BGP case, with variables in units of effective labor except labor and variables expressed in percentages. Not surprisingly, the trends are highly similar to the steady state case, and mismatch on export share, investment and saving rate remains an issue, and for the same reasons explained above.

For robustness check, we tried different starting points of initial capital and bonds. The only possible difference of any significance is that export share goes up over time when starting from a relatively richer state. Figure 6 shows such a case under balanced growth path, where initial bond holding is set at just below three quarters of $\bar{b}$. This is because when starting with a decent amount of resources, the home country does not have to immediately rely on export for wealth accumulation in the early stages, and retains a large portion of its output for domestic consumption and investment. However, it is less conceivable that bond holding can be very high to begin with. For example, China experienced negative net foreign asset holding in the late 1970s when economic reforms began, and now holds a large amount of foreign assets and is still accumulating more. Also, if we further increase the initial capital to be more than half of its steady state value, export share would show a decreasing trend again (not shown here).

To conclude this section, the simple transitional dynamics of this model can capture some of the trends observed in data, such as growth in output, capital and consumption, increasing import share and a current account surplus. It is also clear this alone can't give increasing export share or resolve the saving and investment rates puzzle, where our true interests are. Without changing the model, we need to look beyond the simple dynamics to match the trends in data better.

3.5 Transitional Dynamics with Structural Changes

In the previous section we looked at the dynamics when there is no change to exogenous factors over time and growth takes place solely as a result of endogenous factor accumulation. Noting that the results of this approach have considerably failed in key areas, we now want to consider a scenario where exogenous changes happening in the world also drive the growth of developing countries. Specifically, we provide motivations for two exogenous changes: increase in technology and increase in foreign demand for home goods, and study their impact on the dynamics outcome.

3.5.1 Motivation

Exogenous increases in productive technology are common and widely studied in the literature. The chapter "diffusion of technology" of Acemoglu (2008)[1] shows that an important element of most models of technological diffusion is the built-in advantage for countries that are relatively behind. In an increasingly globalized world with instant information sharing and fast knowledge spillover, developing countries can make up for their technological disadvantage by learning the know-how from advanced countries. As a result, their productivity increases faster than developed countries already equipped with superior technology. Acemoglu (2008)[1] notes that the rate of technology diffusion is higher when the gap between the world technology frontier and the technology level of a particular country is greater. Since there is a large gap for developing countries to close, diffusion of technology takes place wide and fast.

To think of this in the context of the steady state case, TFP increases until it reaches the high constant level of the foreign country. In the context of balanced growth path, the whole world is experiencing technological progress that every nation has knowledge to, and on top of that the developing countries are also picking up existing technology that has been enjoyed by the rest of the world but still new to them, essentially doing catch-up to make up for the gap between itself and the foreign country. Once the gap is closed, productivity in both countries grows at the same rate, and both have an equal amount of perpetually growing technology including the latest.

To think of this as in the case of China, up until the late 1970s China's economy remained severely unproductive due to decades of troubled policies, and that's when China began its economic reforms highlighting opening up to the outside world. Essentially, China's economy was born again at that point with low productivity, scarce capital and even negative foreign asset holding, all of which have experienced drastic growth ever since. Productivity growth results from exchanges with the outside world and learning from other countries' technology and know-how, for example attracting foreign firms to set up plants locally and share its technology with the Chinese engineers in exchange for market access. Capital, output and consumption grew as much as they did in large part because of that.

The exogenous increase in foreign demand for home goods may take some unconventional thinking. While it is more natural to think of the increase in exports as an endogenous outcome alongside productivity growth and real depreciation, it could also have a lot to do with the intrinsic preference of the foreign consumers. We posit that in the beginning of trade consumers know more about their domestic good and very little about imported goods, and they tend to stay safe and consume more of their own good. As trade deepens and information becomes more widely spread (in the same spirit as knowledge spillover in the technological catch-up argument), consumers get to learn more about the traits and qualities of a previously unfamiliar good, and come to like it better and more willingly accept it as a substitute for the domestic good in their consumption. Happening alongside the information spreading is the improvements in home goods' quality, to some degree a result of technological progress. One such example is the Japanese automotive industry. American consumers were largely untrusting of Japanese cars when they first entered the market, but as the Japanese companies made their vehicles increasingly better and American car buyers learned more about them, Japan was able to gain a strong foothold in the US auto market.

Another key factor at play here is the liberalization of trade in recent decades. Although factors like tariffs are not explicitly shown in the model, lifting trade barriers and loosening protectionism has dramatically reduced the purchasing cost of imports to consumers, effectively increasing their demand for imported goods that were desirable but less affordable before. It's hard to deny that the increase in consumers' taste for imported goods partly comes from learning about its quality by using it, which is an endogenous effect; but equally important is the role played by the increasing dissemination of the imports' information, the improvements in quality and the decrease in the effective cost of buying imports.

We reflect these structural changes as specified paths of exogenous variables in the model, and study the dynamics both during and after the occurrence of these changes. Next we will do deterministic perfect foresight computations, in which agents are born with initial endowment of capital and bonds, have full information on how home productivity and foreign preference will be like in each period, and decide the optimal paths to take on. For exogenous structural changes, we implement a path of increasing home TFP and decreasing weight on the foreign good in foreigners' utility, respectively. All other exogenous parameters stay the same all the time.

3.5.2 Increase in Technology

To get a sense of the magnitude of TFP and its increase, Hall and Jones (1999)[13] use a production function with labor-augmenting technology similar to our BGP case, and decomposes output per worker in each country into contributions from three factors - physical capital, human capital, and productivity. They scale everything by the US level, similar to our TFP scaled by the growth rate of world technology in the BGP case. Our exercise is also similar to McGrattan and Prescott (2009)[22], which artificially implements a path of growing TFP in their deterministic transitional dynamics, where the rate of TFP increase is related to the level of economic openness.

From here on and in later chapters, we switch from using non-linear sequences in chapter 2 to arithmetic sequences for exogenous changes. Adopting an arithmetic sequence of increase can result in a clearly defined catch-up stage in growth dynamics, as the catch-up process ends in a distinct break instead of a smooth transition.

Under the steady state context, Figure 7 shows the dynamics of the system through a sequence of technological progress in A_t increasing from 0.8 to 1 in the first 120 periods in equal successions, and staying constant at 1 after that.[2] This path of increase is in contrast to the non-linear path implemented in the previous chapter. The initial capital and bonds are each about one quarter and one half of their respective steady state values.

An immediate observation is that there are kink points at the end of the "catch-up" phase, which is a result of our artificially constructed linear catch-up stage. These bends arise because the system behaves during the catch-up phase expecting further continuous increases in TFP of equal sizes in each future period, instead of shooting directly towards the new steady state. Only after the exogenous changes end and TFP stays constant does the system converge to the steady state corresponding to the high TFP, this part similar to the simple dynamics in the previous section.

The results are similar to simple dynamics in the basic variables. The steady state and BGP results are also highly similar. Consumption and output grow alongside bonds and capital accumulation. Price of foreign good goes up by more now because of

[2] That translates to the home country using the first 30 years to catch up. History suggests that it took Japan and South Korea about this long from takeoff to becoming developed countries. For countries like China, it seems to be taking longer.

the added impact of TFP increase, and export share still goes down. We do manage to get the investment rate to go up, and the saving rate decreases at a slower rate before reaching the kink point. This leads us to suspect that changing initial endowment or path of TFP could lead to an increase in saving rate up to the kink point.

Indeed, Figure 8 confirms that suspicion. Under the BGP context, we have the home labor-augmenting technology relative to the world technology ${\widetilde{z}_t}^{1-\alpha}$ going from 0.8 to 1 in 120 periods. If we increase the initial capital and bonds each to about half and three quarters of their steady state value, we get both saving and investment rates increasing for the entire catch-up phase, and then falling slowly to reach the steady state. It's also noteworthy that import share in this case exhibits non-monotonicity before the kink point, decreasing for a short while before increasing. The home country also goes through post-development current account deficit.

A positive sign is that implementing TFP increases seems to bring about increases to the saving and investment rates at least during the catch-up, contrary to the cases of simple dynamics. This is because when consumers expect future increases in productivity, there is little use in having high saving and investment early on, as it's not very rewarding compared to being consumed. Instead, they slowly increase saving and investment as TFP increases to capitalize on the increase in productivity. Once they reach the end of TFP increase, things go back to the case of simple dynamics. No more increase in TFP is anticipated, and diminishing marginal product mandates that consumers decrease their saving and investment rates.

3.5.3 Increase in Foreign Demand

Following Corsetti et al (2007)[7] we take the quasi-share parameter as a proxy for the quality of home goods. For increase in foreign demand, we make the weight of foreign goods in the consumption bundle γ_t^* decrease from 0.99 to 0.95, also in an arithmetic sequence of 120 periods.

Figure 9 shows the steady state case of this scenario, with low initial endowment. Relative price of foreign good rises shortly before falling in the long run, as increasing foreign demand brings real appreciation for the home country. Investment rate decreases during catch-up and then flattens out. Saving rate rises during catch-up and then falls in the long run. Import share displays a "V" pattern, going down and then up, and export share moves in the exact opposite fashion.

There are two interesting findings. The first is that even with initial bond holding below its steady state value, we now carry a current account deficit deep into the developing stage, which is a case of the Lucas paradox. All previous cases have current account surplus at least during development. So with increasing foreign demand, home country would first attract capital inflow from the outside world. This is because in the beginning the home consumers want to increase investment and capital to meet the increasing foreign demand, and at the same time increase their consumption. These urgent needs combine to force them to borrow from the foreign country. This also explains the investment and saving rate moving in opposite directions. We can write saving by rearranging (1)

$$\begin{aligned} SAV_t &= y_t - p_t c_t \\ &= i_t + p_{f,t}(b_t - R_{t-1}^* b_{t-1}) + \frac{\Psi}{2}(b_t - \bar{b})^2 p_{f,t} \end{aligned}$$

It becomes obvious that even as saving increases, investment could still decrease because the portfolio adjustment cost and changes in bond holding position are increasing. Because this is an open economy, bonds as an extra channel of saving/borrowing essentially balances between saving and investment and allows them to move in opposite directions.

Secondly, the relative price exhibits non-monotonicity during the development stage, increasing shortly before decreasing. This is because in the beginning, the increase in home productivity was dominating, causing real appreciation for the foreign good; in later stages, the increase in foreign demand took over and causes home good appreciation.

It turns out that the level of initial endowment and doing this in the BGP setup do not make much difference. Figure 10 shows the BGP dynamics with initial capital and bonds at three quarters of their steady state levels. It should not come as a surprise that imposing increase in foreign demand alone does a poor job of matching data trends. In reality, increase in foreign demand is not only secondary to, but also to a large degree a consequence of home country's increase in productivity and export. Changing foreign demand alone without addressing its root cause would not produce a decent fit for reality.

3.5.4 Simultaneous Structural Changes

A quick review of all the results so far has none of the cases studied above generating results that perfectly match the observed patterns of developing countries on the following 4 variables: saving and investment rates, import and export shares of output. Data suggests all 4 should increase during the countries' developing stages alongside a current account surplus. The simple transitional dynamics can only get rising import share. Perfect foresight dynamics with only increasing technology has saving rate and export share decreasing, while increasing foreign demand for home exports alone leads to declining investment rate and import share, and also a current account deficit. To generate increasing trends in all 4 variables, a natural next attempt is to combine the 2 structural changes and let them take place at the same time. If the occurrence of either of these 2 exogenous changes is conceivable, then both of them happening simultaneously would also be realistically convincing.

Figure 11 shows a steady state case of such dynamics, with A_t increasing from 0.7 to 1 and γ_t^* decreasing from 0.99 to 0.95 over the first 120 periods. Figure 12 shows a BGP case with the same paths of exogenous changes and starting with a higher level of initial endowment. Both cases generate highly similar trends. Without triggering any abnormality in other variables, now the saving and investment rates, as well as export share are going up, with import share going down during the catch-up phase. Current account has a deficit for the first half of the developing stage, then turning into a surplus for bonds accumulation. This is similar to the trend in the case of increasing foreign demand alone.

It is worth pointing out a potential flaw of studying simultaneous exogenous changes qualitatively. As is clear now, imposing each exogenous change alone gives basically opposite trends in the variables we are interested in, hinting that these two factors may be pulling the system away in two directions. Judging by the results of the simultaneous change case, it seems that the trend in investment rate is dominated by the increase in technology, and the trends in saving rate, import and export share and current account are dominated by increase in foreign demand. This brings up the questions of exactly how much of each change takes place and for how long in reality, both of which require rigorous calibrations with data. Only by doing that can we get more informative results and assess the model's ability to match the real world. For now, though not perfect in terms of results and numerical accuracy, simultaneous changes seems to be the best case scenario that gives the closest match to the trends observed in reality.

3.6 Further Discussions

Having done all the computational exercises, we can now look back at the real world observations that motivate this work, and try to assemble our results into stories that make realistic economic sense. Simple transitional dynamics considers an economy that starts out poor, and grows via saving through investing in capital and foreign asset accumulation. The exogenous situation never changes throughout, and the paths of all variables are smooth and behave in almost unanimous monotonicity.

The story of dynamics with structural changes is more sophisticated. At the beginning of time, part of the world is underdeveloped with growth potential, and the rest maturely developed. Our model shies away from discussing what may have lead to the heterogeneous starting point between the home and foreign country. Nonetheless, we do propose a few possibilities of how the home country starts the growth process exactly at a certain point in time. The examples of the Asian developing countries suggest this could be due to fundamental shifts in the political scene or economic policies, or a combination of both.

As stated before, since the late 1970s, generations of China's leadership have continuously undertaken marketizing economic reforms to liberalize a previously centrally-planned closed economy. The 4 "Asian tigers" had similar episodes of economic boosts from policy shifts. South Korea in the early 1960s adopted outward-looking economic policies that gave strong support to the labor-intensive export industries where they had a comparative advantage at the time. At around the same time, the government of Singapore also adopted pro-business and export-oriented economic policy that boosted foreign investment. In these countries, there was a fairly clear point in time before which the economy was persistently underdeveloped and after which things took off following fundamental shifts in policy.

In our model, after experiencing a period of sustained growth, the home country enters an advanced steady state status and joins the developed world, or "finishes" the development process so to speak. This is also reflected by some of the aforementioned countries, namely Japan, South Korea and Singapore, all of which are now considered well developed countries enjoying high productivity and at the technological forefront of many industrial areas. In this regard of a well defined development stage with clear starting and end points, we believe our model holds significant realism that pertains to the developing-turned–into-developed countries.

Adopting balanced growth alongside the developing countries' catch-up adds another dimension of realism to the model. We deliberately strip the foreign country of capital and production. Besides for simplicity, this is done to establish the foreign country always as an advanced economy that the home country is catching up to and gaining growth from. This is plausible as the Asian economies all experienced tremendous growth in large part due to exporting to developed countries, in many cases taking advantage of their comparative advantage in labor-intensive industries.

At the beginning of development, the home country was not only poor in resources, but also lags behind the foreign country in technology and productivity. During the development stage in which the labor-augmenting technology increases until it reaches a permanent high level, the home country not only learns the existing know-how via information spreading from the rest of the world, but also picks up and even actively contributes to the research and development of new technology. For example, China is a leading player in the field of renewable energy technology as a major producer of wind turbines and electric vehicles; at the same time, it is making up for its disadvantages in fields of electronics and information technology by learning expertise from other countries. That being said, although developing countries do engage in R&D for new technologies, for them adoption of technology from developed countries is much more significant than creating new technologies, as noted in Acemoglu (2008)[1].

The balanced growth rendition of the model would predict that a developing country's technological growth would slow down over time, and eventually stay at a stable rate after the catch-up phase ends. Given enough time, a country would have all the latest technology there is for production, growing only with new inventions and at the world rate like all other developed countries. This is shown by Figure 13 comparing the averaged-filtered GDP per capita growth rate between former Asian developing economies and the United States. The growth rate differences shrink over time, with the Asian countries enjoying fast growth in the beginning and loosely converging to the US in recent decades.

Finally, the kink points in the perfect foresight dynamics, while a mechanic feature of our artificially set paths, interestingly resemble a feature observed in data. The kink points in the model signify the end of the exogenous changes, and the beginning of convergence to the permanent steady state. In most cases studied above, the trends of the things we are interested in simply reverses and become flatter once they hit the kink.[3] Looking back at the currently developed Asian countries (Japan, Singapore and South Korea) in Figures 1-4 in chapter 1, their saving and investment rates also roughly exhibit such an increasing then decreasing toward a constant level pattern, although import and export share show sustained increases, which is rarely the case in our model dynamics. This phenomenon could serve as further proof of a clearly defined developing period with starting and ending points as the premise of studying dynamics with structural changes.

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[3]Things like capital, consumption and output mostly increase in monotone, although still bearing kink points.

3.7 Summary of Results

We build an open economy model that describes the growth of a developing country in the presence of a developed country, and close it with portfolio adjustment cost.

Using simple dynamic analysis without any exogenous changes, it's hard to generate a rising pattern over time for saving and investment rates and export share. We then motivate and implement 2 structural changes that have conceivably occurred to developing countries: technological growth and increase in foreign demand. Having technological growth alone can get investment rate and import share to increase, and enforcing foreign demand increase alone can make saving rate and export share go up, but neither can do it all. Our final experiment was to combine these two changes together, and we were able to get increases in everything but import share, and also a short-lived current account deficit in the beginning. In all the dynamics that we looked at, unless increase in foreign demand is involved, current account always has a surplus throughout, which is consistent with the data.

We present the balanced growth rendition of this model and show its results along the way. We provide evidence to argue that using BGP may be a more realistic depiction of how the developing country draws relative growth from developed countries and eventually gets integrated into the them. We also discuss the concept of a catch-up stage for developing countries and its realistic value. These arguments apply to later chapters as well.

Figure 3.1: Impulse Responses to an Increase in A_t

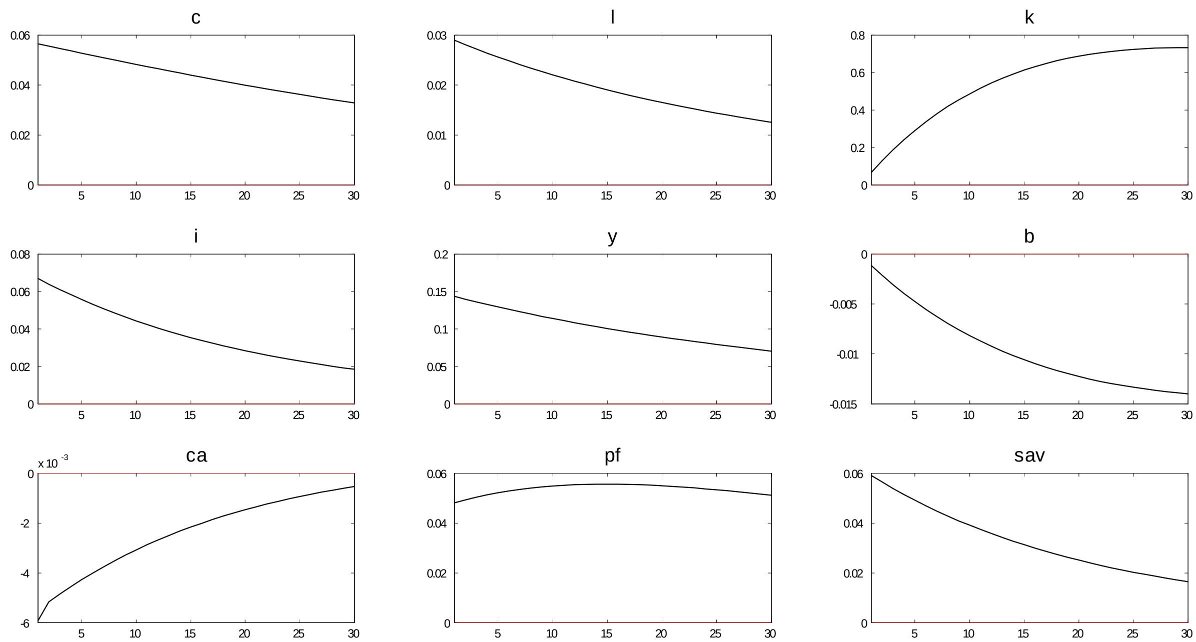


Figure 3.2: Impulse Responses to an Increase in y_t^*

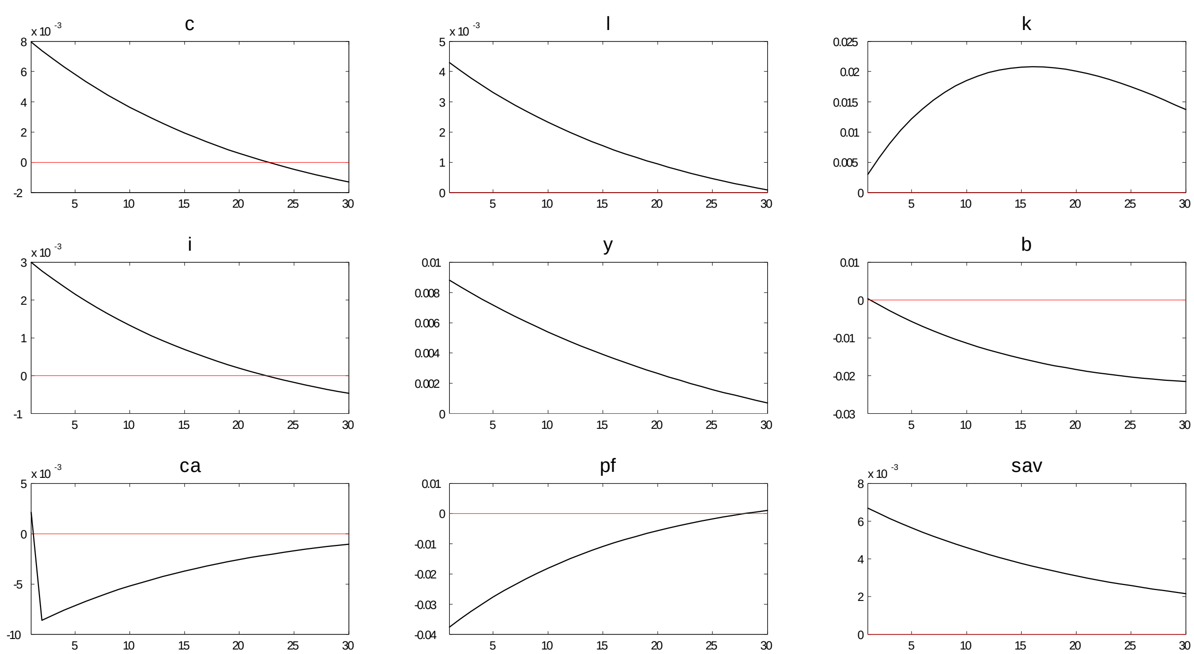


Figure 3.3: Impulse Responses to a Decrease in γ_t^*

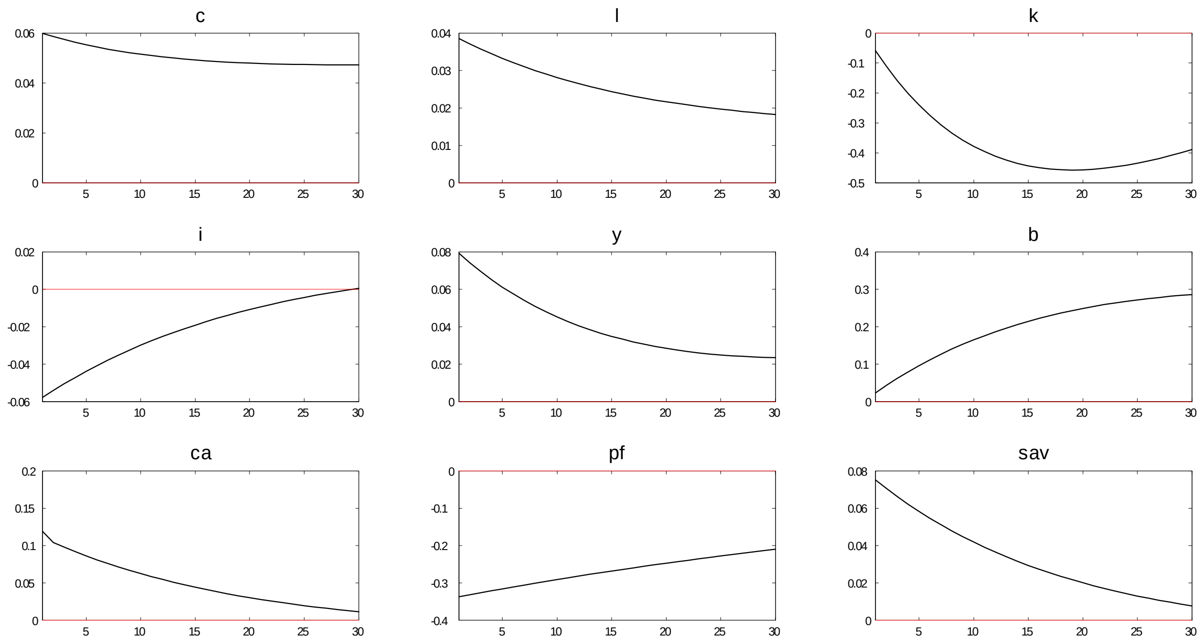


Figure 3.4: Simple Transitional Dynamics, Steady State Case, $k_0 = 20, b_0 = 1$

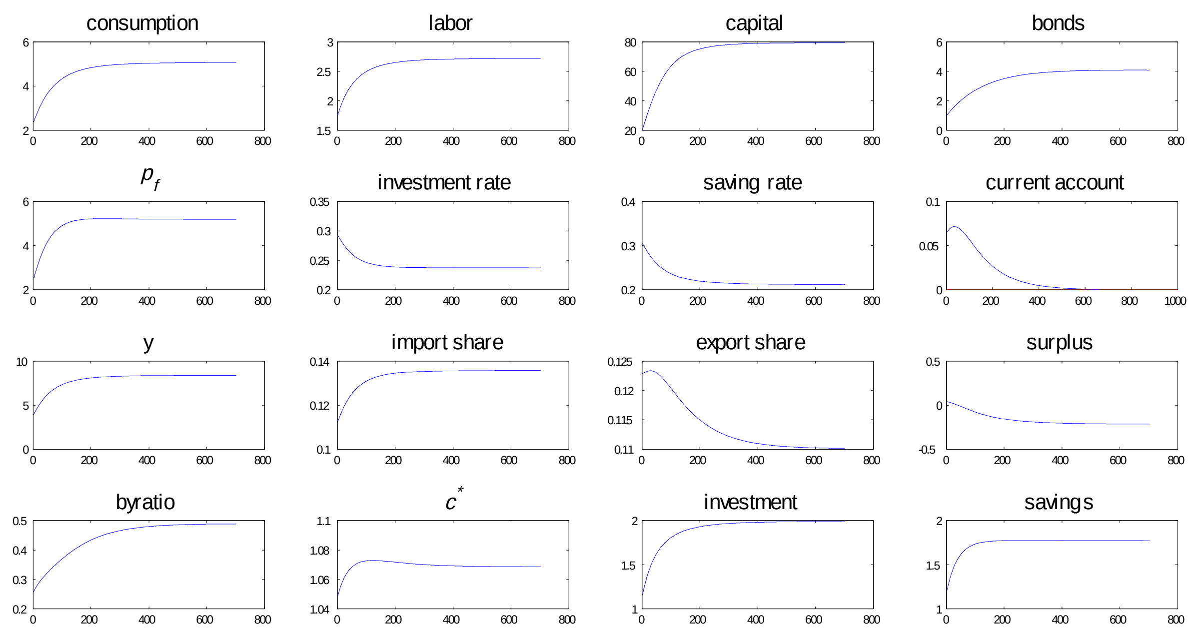


Figure 3.5: Simple Dynamics, BGP case, $k_0 = 20, b_0 = 1$

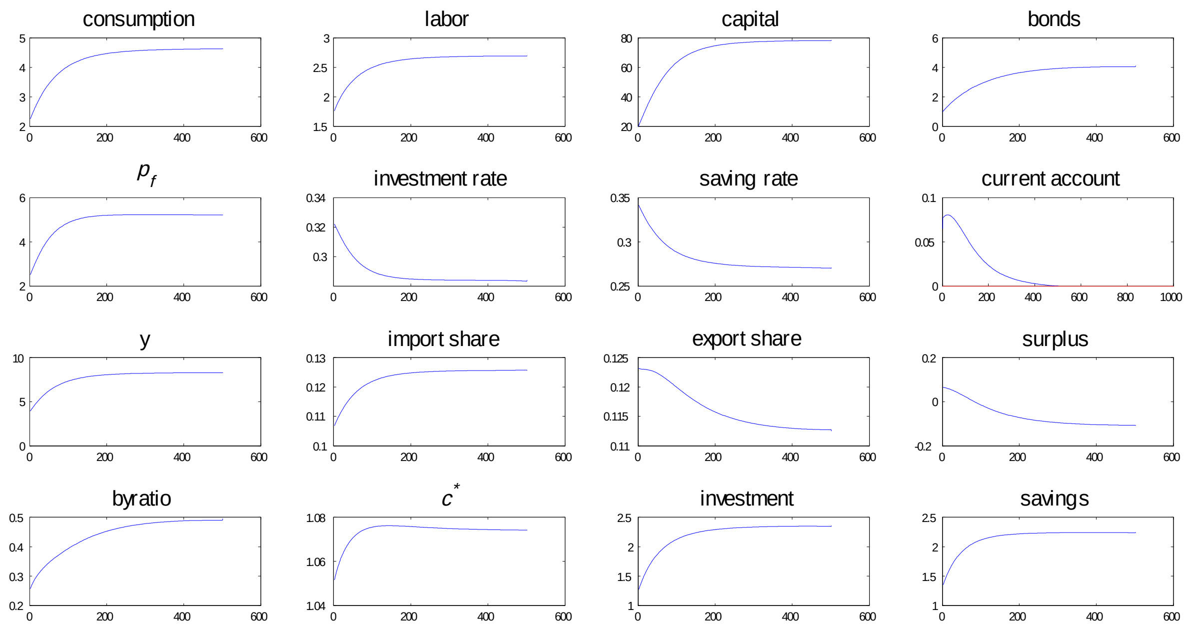


Figure 3.6: Simple Dynamics, BGP case, $k_0 = 40, b_0 = 3$

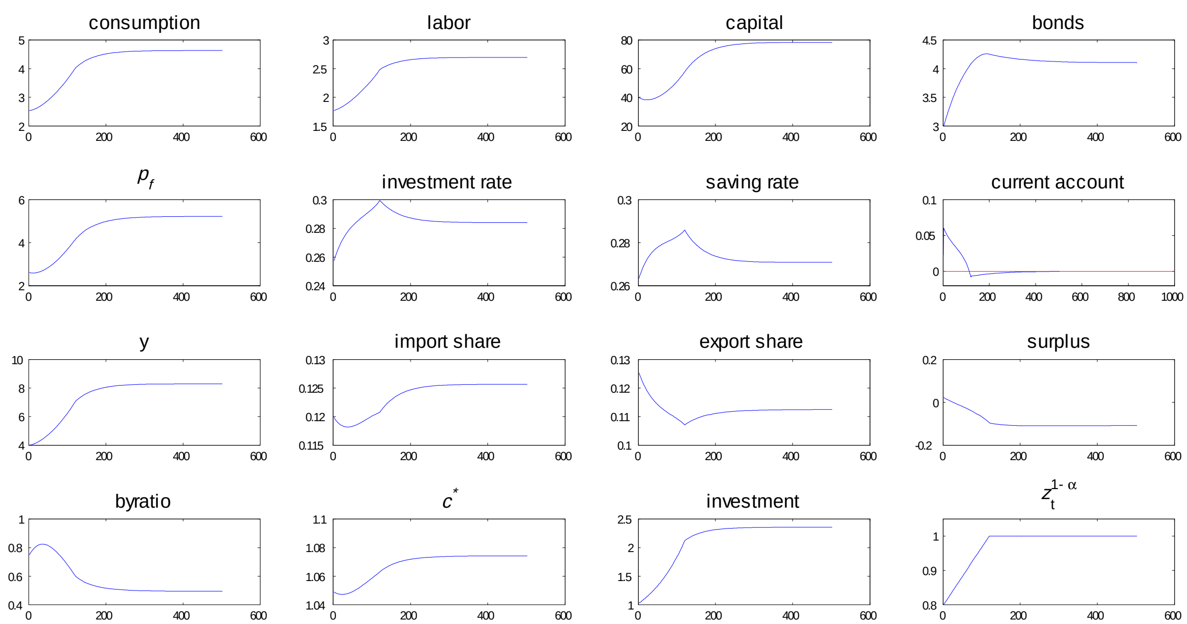


Figure 3.7: Dynamics with Increase in A_t, steady state case, $k_0 = 20, b_0 = 2$

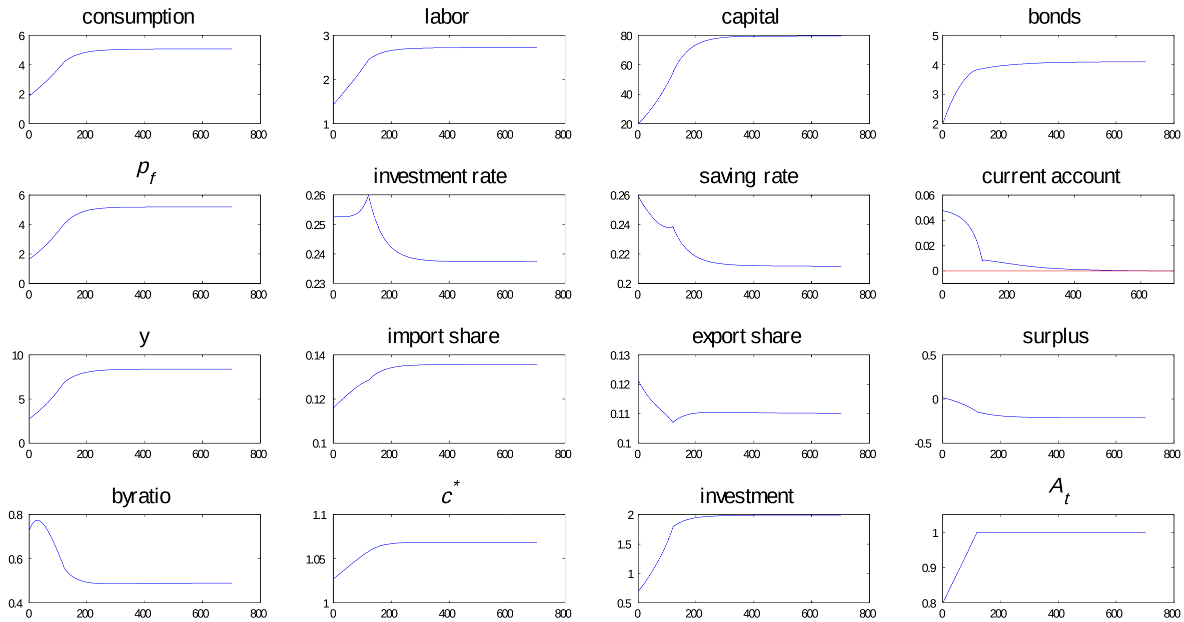


Figure 3.8: Dynamics with increase in $\widetilde{z}_t$, BGP case, $k_0 = 40, b_0 = 3$

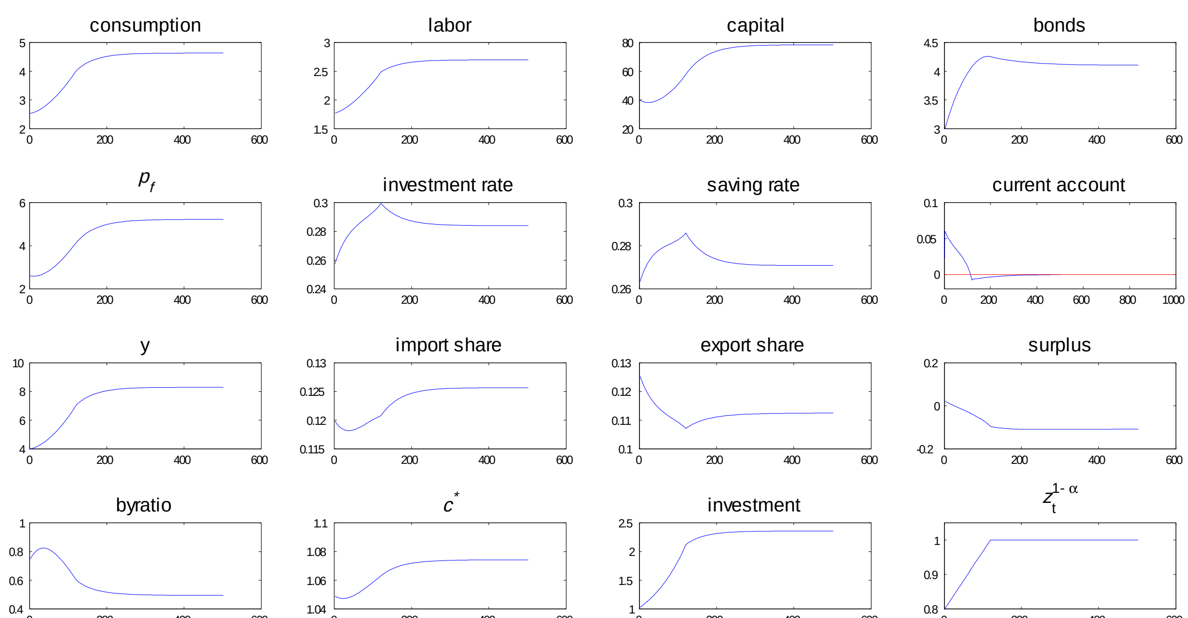


Figure 3.9: Dynamics with decrease in γ_t^*, steady state case, $k_0 = 25, b_0 = 2$

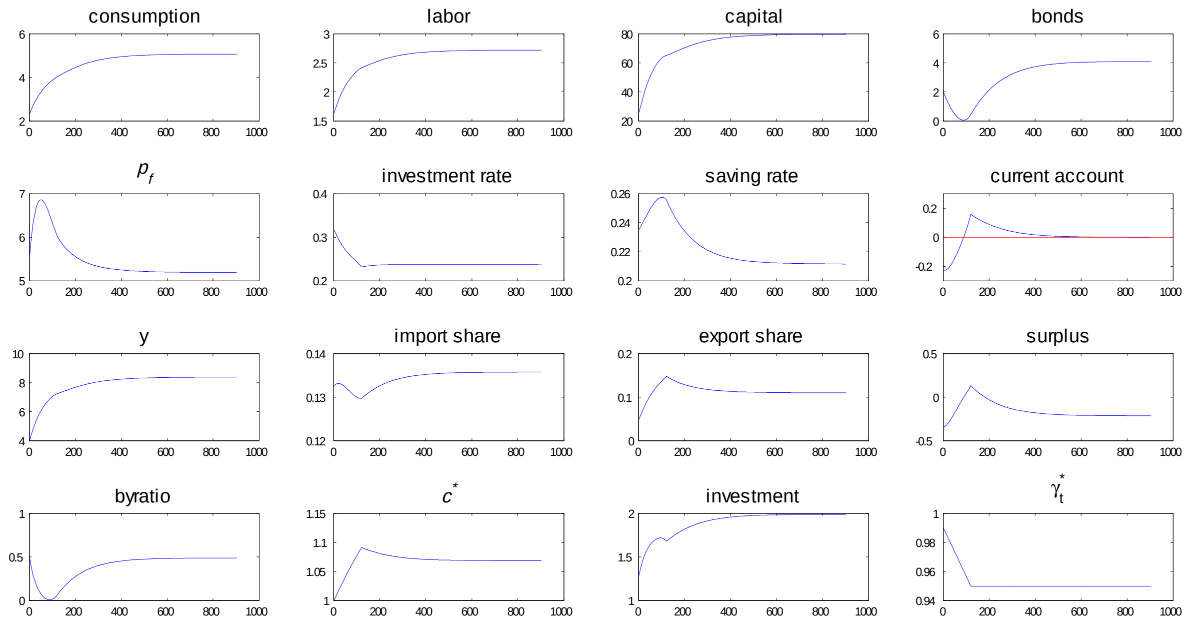


Figure 3.10: Dynamics with decrease in γ_t^*, BGP case, $k_0 = 60, b_0 = 3$

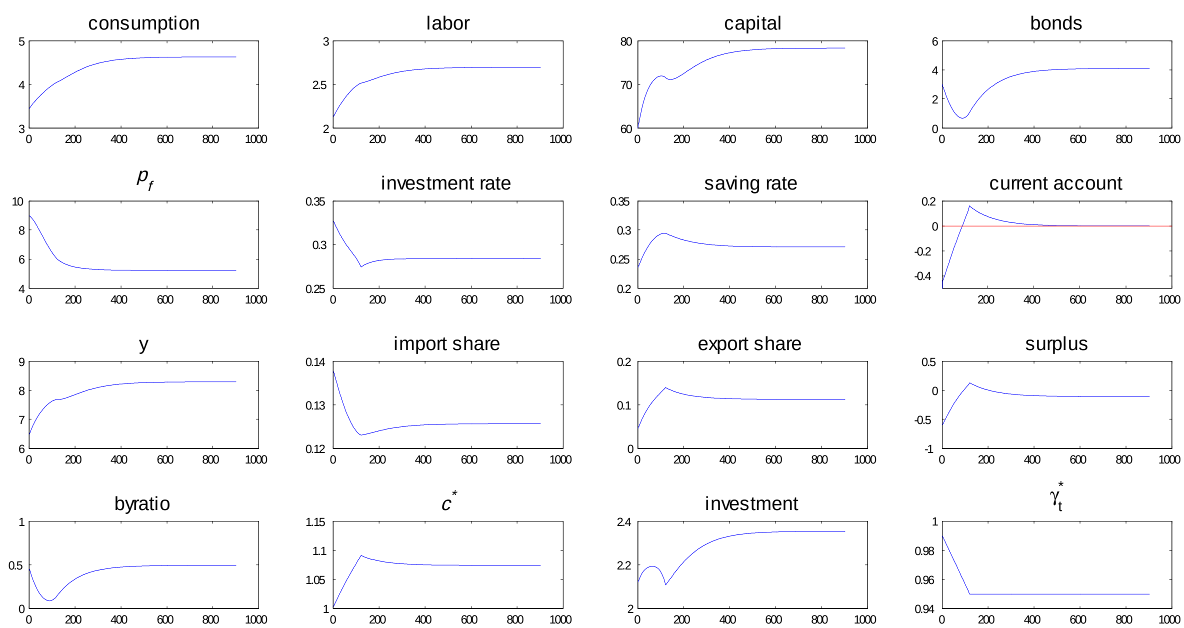


Figure 3.11: Dynamics with increase in A_t and decrease in γ_t^*, steady state case, $k_0 = 30, b_0 = 2$

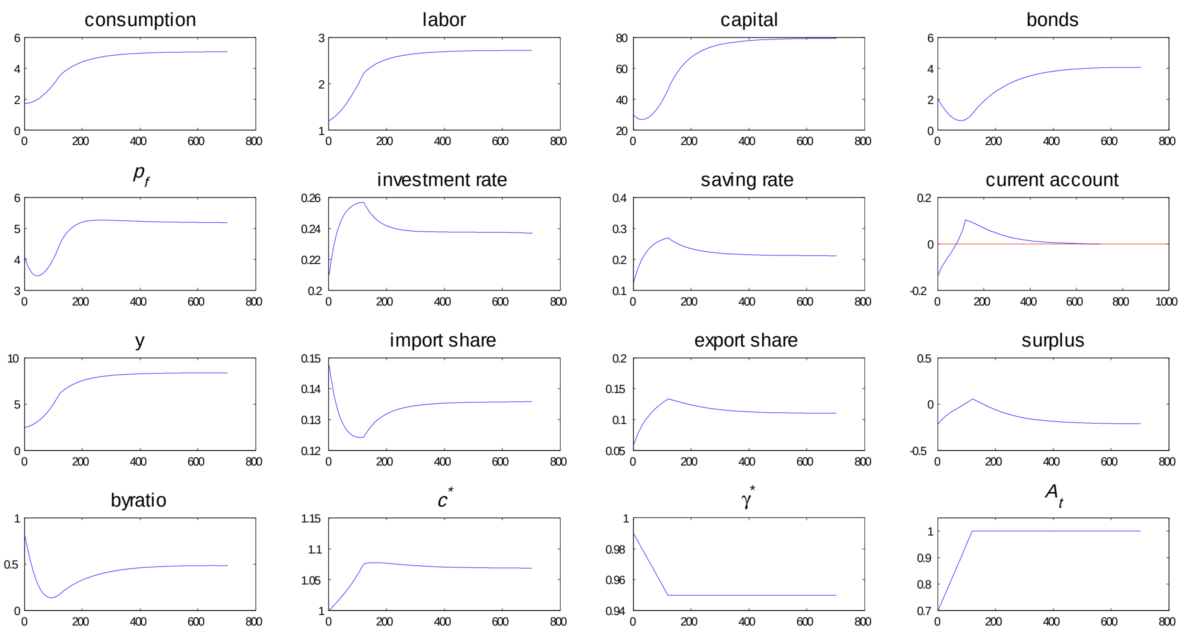


Figure 3.12: Dynamics with increase in $\widetilde{z}_t$ and decrease in γ_t^*, BGP case, $k_0 = 60, b_0 = 3$

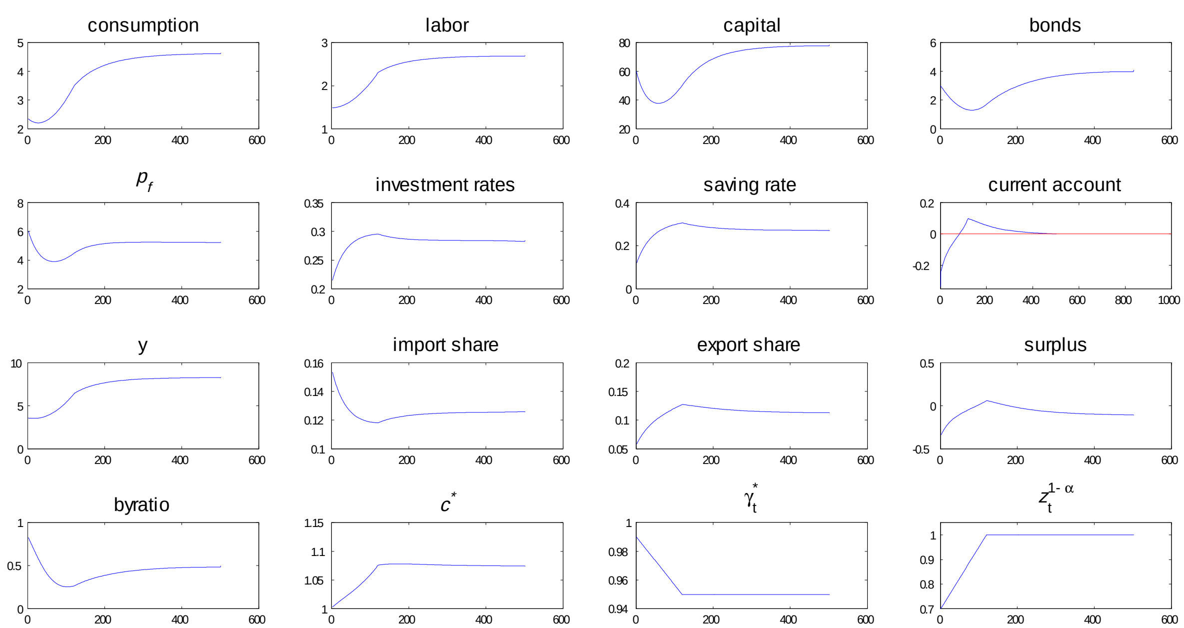
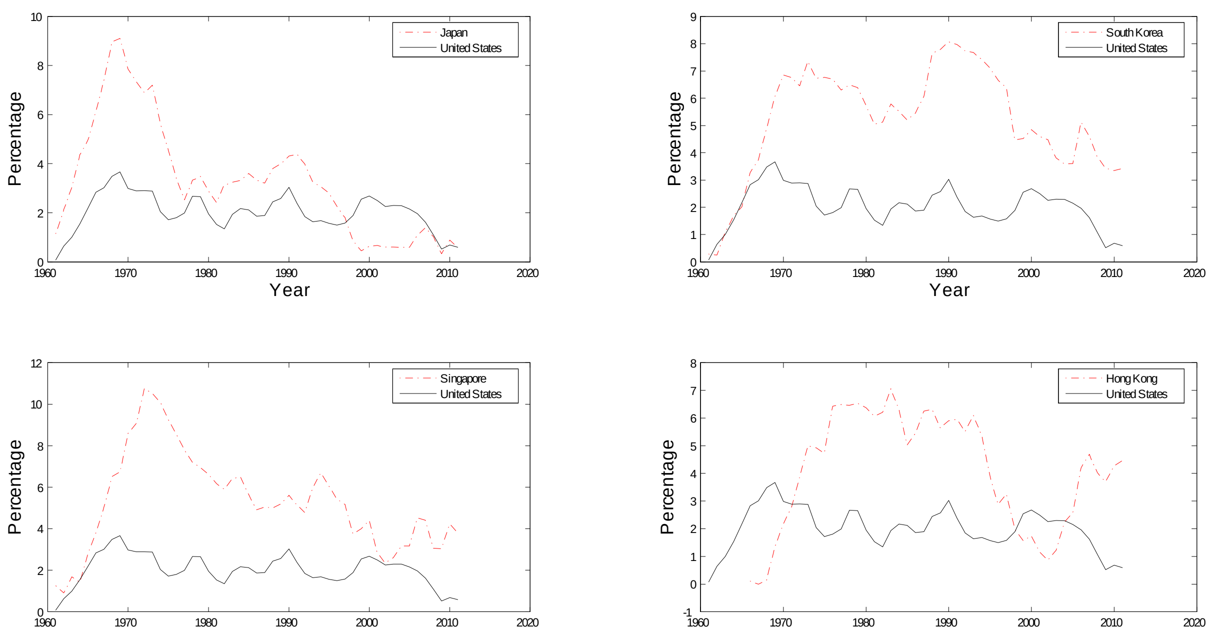


Figure 3.13: Filtered Growth Rates of GDP per capita



Chapter 4

Open Economy Model with Endogenous Discounting

4.1 Introduction

In this chapter we stretch our open economy model built in the previous chapter to include the mechanism of endogenous discounting, and study the various dynamics of our variables of interest produced by it. On the technical front, we deploy the structure of endogenous discounting from Schmitt-Grohé and Uribe (2003)[28], which was first developed in Uzawa (1968)[32], and also used in Mendoza (1991)[23]. To explore such dynamics in the context of balanced growth path, we also study a closed economy model with endogenous discounting.

The rest of the paper is organized as follows. Section 2 lays out the 2 models to be analyzed and discusses parametric choices. Section 3 takes a quick look at the stochastic impulse responses. Section 4 looks at the simple transitional dynamics. Section 5 studies the transitional dynamics with exogenous changes in home productivity and foreign demand. Section 6 provides intuitive discussions on the effects of endogenous discounting on closed and open economy models. Section 7 gives a summary of results.

4.2 Models

We deploy two models and show their results in parallel, both bearing an endogenous discount factor. The first model describes an open economy with bonds, import and export, and the endogenous discount factor depends on the individual level of consumption and labor. The second model is a simple closed economy model, where the discount factor depends on the average level of consumption and labor. In this model saving and investment coincide and there is no bonds or trade, but its algebraic simplicity allows for a balanced growth path (BGP) in the steady state, which enables us to give more realistic interpretations of long run economic growth as explained in chapter 3.

The reason for the difficulty of constructing an open economy model with both endogenous discount factor and balanced growth paths is primarily technical. In an open economy model with the discount factor dependent on average consumption and labor, the paths of bonds turn out to be numerically unstable. That leaves us with the only option of using individual consumption and labor in the discount factor in an open economy. The algebra, however, becomes too tangled up for the discount factor to be properly scaled by the rate of technological growth. Therefore, to demonstrate an economy with endogenous discount factor and also BGP, we resort to a closed economy.

4.2.1 2-Country Model

Except for using endogenous discounting to replace portfolio adjustment cost for the home country, the model is similar to the one built in chapter 3. Home country has Yoeman farmer type of homogeneous agents, who produces with capital and labor, consumes both home and foreign goods, and saves by investing in capital and purchasing foreign bonds. The foreign country is an endowment economy, with no production and labor decision. Homogeneous foreign agents also consume both foreign and home goods, and can borrow by issuing debt denominated in their own good.

Home Country

Home consumer/firm agents solve the following problem

$$\max E\sum_{t=0}^{\infty}\theta_t\frac{(c_t-\tau\zeta^{-1}l_t^{\zeta})^{1-\sigma}-1}{1-\sigma}$$

subject to budget and capital evolution constraints

$$\begin{aligned} c_{h,t}+p_{f,t}c_{f,t}+i_t+b_tp_{f,t} &= y_t+R^*_{t-1}b_{t-1}p_{f,t} \quad (4.1)\\ (1-\delta)k_t+i_t-\frac{\varphi}{2}(\frac{i_t}{k_t}-\delta)^2k_t &= k_{t+1} \quad (4.2)\end{aligned}$$

where home good is the numeraire, and relative price $p_{f,t}$ is the real exchange rate or terms of trade. Production follows a Cobb-Douglas format

$$y_t=A_tk_t^{\alpha}l_t^{1-\alpha} \quad (4.3)$$

and the consumption good is a bundle of home and foreign goods

$$c_t=[\gamma^{1-\rho}c_{h,t}^{\rho}+(1-\gamma)^{1-\rho}c_{f,t}^{\rho}]^{\frac{1}{\rho}}$$

For endogenous discounting, we follow the specification in Schmitt-Grohé and Uribe (2003)[28]. θ_t is equivalent to the additive time preference in models with constant exogenous discount factor, except now the periodical discount factor is endogenous instead of a known constant

$$\begin{aligned} \theta_{t+1} &= \theta_t\beta(c_t,l_t), \quad \theta_0=1\\ \beta(c_t,l_t) &= \mu(\xi+c_t-\omega^{-1}l_t^{\omega})^{-\Psi}, \quad \Psi>0 \quad (4.4)\end{aligned}$$

This says that the discount factor at a certain point in time is the cumulated outcome of previous history of consumption and labor. The more one has consumed and less she has worked in the past, the more she will discount future utility. In other words, the better off one has been in the past, the more impatient she will be looking toward the future.

Let λ_t and η_t each be the Lagrange multipliers for constraints in (1) and (2), and write the Lagrangian at time t as

$$\begin{aligned} L_t &= \frac{(c_t - \tau\zeta^{-1}l_t^{\zeta})^{1-\sigma} - 1}{1-\sigma} \\ &\quad +\lambda_t[A_t k_t^{\alpha} l_t^{1-\alpha} + R_{t-1}^* b_{t-1} p_{f,t} - p_t c_t - i_t - b_t p_{f,t}] \\ &\quad +\eta_t[(1-\delta)k_t + i_t - \frac{\varphi}{2}(\frac{i_t}{k_t} - \delta)^2 k_t - k_{t+1}] \end{aligned}$$

We get the following first order conditions

$$(c_t - \tau\zeta^{-1}l_t^{\zeta})^{-\sigma} - \lambda_t p_t - \Psi\mu(\xi + c_t - \omega^{-1}l_t^{\omega})^{-\Psi-1}L_{t+1} = 0 \tag{4.5}$$

$$-\eta_t + \beta(c_t, l_t)E_t\{\lambda_{t+1}\alpha A_{t+1}k_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha} + \eta_{t+1}[1-\delta-\frac{\varphi}{2}(\delta+\frac{i_{t+1}}{k_{t+1}})(\delta-\frac{i_{t+1}}{k_{t+1}})]\} = 0 \tag{4.6}$$

$$-\lambda_t + \eta_t[1 - \varphi(\frac{i_t}{k_t} - \delta)] = 0 \tag{4.7}$$

$$-\lambda_t p_{f,t} + \beta(c_t, l_t)E_t(\lambda_{t+1}p_{f,t+1}R_t^*) = 0 \tag{4.8}$$

$$-(c_t-\tau\zeta^{-1}l_t^{\zeta})^{-\sigma}\tau l_t^{\zeta-1} + \lambda_t(1-\alpha)A_t k_t^{\alpha} l_t^{-\alpha} + \Psi\mu(\xi+c_t-\omega^{-1}l_t^{\omega})^{-\Psi-1}l_t^{\omega-1}L_{t+1} = 0 \tag{4.9}$$

where p_t is the aggregate price of the consumption bundle, resulting from the static cost minimization problem solutions

$$p_t = [\gamma + (1-\gamma)p_{f,t}^{\frac{\rho}{\rho-1}}]^{\frac{\rho-1}{\rho}} \tag{4.10}$$

$$c_{h,t} = c_t \gamma p_t^{\frac{1}{1-\rho}} \tag{4.11}$$

$$c_{f,t} = c_t(1-\gamma)(\frac{p_{f,t}}{p_t})^{\frac{1}{\rho-1}} \tag{4.12}$$

Also, define saving and current account in home goods as

$$\begin{aligned} SAV_t &= y_t - p_t c_t \\ CA_t &= p_{f,t}(b_t - b_{t-1}) \end{aligned}$$

Current account in home good can also be written as the sum of trade surplus and interest payment on foreign asset holding[1]

$$CA_t = Nc^*_{h,t} - c_{f,t}p_{f,t} + (R^*_{t-1} - 1)b_{t-1}p_{f,t}$$

Foreign Country

A representative foreign agent solves

$$\max E\sum_{t=0}^{\infty}\beta^t \ln c^*_t$$

subject to

$$c^*_{f,t} + \frac{1}{p_{f,t}}c^*_{h,t} + b^*_t \leq y^*_t + R^*_{t-1}b^*_{t-1} \tag{4.13}$$

where y^*_t is the endowment, and consumption bundle has the same format but different shares

$$c^*_t = [\gamma^{*1-\rho}c^{*\rho}_{f,t} + (1-\gamma^*)^{1-\rho}c^{*\rho}_{h,t}]^{\frac{1}{\rho}}$$

We get foreign first order conditions

$$\frac{1}{c^*_t} - \nu_t p^*_t = 0 \tag{4.14}$$

$$-\nu_t + \beta R^*_t E_t(\nu_{t+1}) = 0 \tag{4.15}$$

and similar cost minimization solutions

$$p^*_t = [\gamma^* + (1-\gamma^*)p_{f,t}^{\frac{\rho}{1-\rho}}]^{\frac{\rho-1}{\rho}} \tag{4.16}$$

$$c^*_{f,t} = c^*_t\gamma^* p_t^{*\frac{1}{1-\rho}} \tag{4.17}$$

$$c^*_{h,t} = c^*_t(1-\gamma^*)(p_{f,t}p^*_t)^{\frac{1}{1-\rho}} \tag{4.18}$$

[1] N is the size of foreign population.

Equilibrium

Assuming the home country has a population of 1 and the foreign country has population N, we have bonds and goods market clearing conditions

$$b_t + Nb_t^* = 0 \tag{4.19}$$

$$c_{h,t} + Nc_{h,t}^* + i_t = A_t k_t^{\alpha} l_t^{1-\alpha} \tag{4.20}$$

The equilibrium is defined to be the path of $\{c_t, l_t, i_t, k_{t+1}, y_t, b_t, \beta(c_t, l_t), c_{h,t}, c_{f,t}, c_t^*, b_t^*, c_{f,t}^*, c_{h,t}^*, p_{f,t}, p_t, p_t^*, R_t^*, \lambda_t, \eta_t, \nu_t\}_{t=0}^{\infty}$ and equations (1)-(20).

4.2.2 Closed Economy Model with Balanced Growth Path

We present a simpler closed economy model, where saving is the same as investment. We would also like to make it compatible with balanced growth paths (BGP). The explanation would be that every real term (consumption, output, investment, capital etc.) inherently grows at some world rate of technological growth, and on top of that developing countries in the beginning are catching up with the developed countries (not explicitly modelled here), making up for the gap by learning existing technology from them. Developing nations grows at a pace faster than the world rate during the "takeoff" stage and growth would eventually slow down to that constant rate like the rest of the world in the steady state.

The closed economy model follows closely from the 2 country model, eliminating the foreign country, consumption variety, bonds and trade. Home consumers face a similar maximization problem, capital evolution (2) and internal discounting as in the 2 country model, except some small but crucial tweaks. Now consumers' problem is

$$\max E\sum_{t=0}^{\infty}\theta_t \frac{(c_t - \tau_t \zeta^{-1} l_t^{\zeta})^{1-\sigma} - 1}{1-\sigma}$$

subject to

$$c_t + i_t = y_t$$

Now the coefficient on the labor term is time variant

$$\tau_t = \tau \pi^t$$

and π is the aforementioned fixed world rate of technological growth, similar to the one in the BGP case in chapter 3. Now the Uzawa discounting factor becomes a function of average instead of individual level of consumption and labor, and consumption is scaled by π^t

$$\beta(\bar{c}_t, \bar{l}_t) = \mu(\xi + \frac{\bar{c}_t}{\pi^t} - \omega^{-1}\bar{l}_t^{\omega})^{-\Psi}, \quad \Psi > 0 \tag{4.21}$$

Now taking first order condition does not involve differentiating on $\beta(\bar{c}_t, \bar{l}_t)$, as it has nothing to do with individual choice of consumption and labor.

The logic behind having the discount factor determined by population average level of consumption and labor is actually more intuitive than it seems. In real life people can be prone to making consumption and saving decisions based on observing what others do. This is especially the case in developing countries. For example, an individual is more likely to get a home appliance such as a fridge or cell phone when all her neighbors and friends are buying one. The wild increase in car sales in countries like China is believed to be a result of many people following those who purchased cars earlier.

Rewrite production as

$$y_t = k_t^{\alpha}(z_t l_t)^{1-\alpha} \tag{4.22}$$

To make the equilibrium conform to a balanced growth path, we scale applicable variables by world technological growth rate, and turn them into units of "per effective labor"

$$\begin{aligned}
\widetilde{c}_t &= c_t/\pi^t \\
\widetilde{i}_t &= i_t/\pi^t \\
\widetilde{k}_t &= k_t/\pi^t \\
\widetilde{z}_t &= z_t/\pi^t \\
\widetilde{\lambda}_t &= \lambda_t \pi^t \\
\widetilde{\eta}_t &= \eta_t \pi^t
\end{aligned}$$

and

$$\widetilde{y}_t = y_t/\pi^t = \widetilde{k}_t^{\alpha}(\widetilde{z}_t l_t)^{1-\alpha}$$

Set $\sigma = 1$, and write first order conditions in units of "effective labor" except for labor and percentages

$$\frac{1}{\widetilde{c}_t - \tau\zeta^{-1} l_t^{\zeta}} - \widetilde{\lambda}_t = 0 \tag{4.23}$$

$$-\widetilde{\lambda}_t + \widetilde{\eta}_t[1 - \varphi(\frac{\widetilde{i}_t}{\widetilde{k}_t} - \delta)] = 0 \tag{4.24}$$

$$-\widetilde{\eta}_t + \beta(\bar{c}_t, \bar{l}_t)\frac{1}{\pi}E_t\{\widetilde{\lambda}_{t+1}\alpha\widetilde{z}_{t+1}^{1-\alpha}\widetilde{k}_{t+1}^{\alpha-1}{l_{t+1}}^{1-\alpha} + \widetilde{\eta}_{t+1}[1 - \delta - \frac{\varphi}{2}(\delta + \frac{\widetilde{i}_{t+1}}{\widetilde{k}_{t+1}})(\delta - \frac{\widetilde{i}_{t+1}}{\widetilde{k}_{t+1}})]\} = 0 \tag{4.25}$$

$$-\frac{\tau l_t^{\zeta-1}}{\widetilde{c}_t - \tau\zeta^{-1} l_t^{\zeta}} + \widetilde{\lambda}_t(1-\alpha)\widetilde{k}_t^{\alpha}\widetilde{z}_t^{1-\alpha} l_t^{-\alpha} = 0 \tag{4.26}$$

and also rewrite budget constraint

$$\widetilde{c}_t + \widetilde{i}_t = \widetilde{k}_t^{\alpha}(\widetilde{z}_t l_t)^{1-\alpha} \tag{4.27}$$

$$(1-\delta)\widetilde{k}_t + \widetilde{i}_t - \frac{\varphi}{2}(\frac{\widetilde{i}_t}{\widetilde{k}_t} - \delta)^2\widetilde{k}_t = \pi\widetilde{k}_{t+1} \tag{4.28}$$

In equilibrium we have population average equal to individual level of consumption and labor

$$\widetilde{c}_t = \frac{\overline{c}_t}{\pi^t} \tag{4.29}$$

$$l_t = \bar{l}_t \tag{4.30}$$

The equilibrium is $\{\widetilde{c}_t, l_t, \frac{\overline{c}_t}{\pi^t}, \bar{l}_t, \widetilde{i}_t, \widetilde{k}_{t+1}, \widetilde{y}_t, \beta(\overline{c}_t, \bar{l}_t), \widetilde{\lambda}_t, \widetilde{\eta}_t\}_{t=0}^{\infty}$ and equations (21)-(30), and it delivers a balanced growth path in the steady state, where consumption, output, capital, investment and technology all grow at the rate of π.

4.2.3 Parametric Choices

As explained in chapter 2 and 3, we shy away from performing parametric calibrations based on data, instead taking values from existing literature and applying common economic knowledge. The majority of parameters used here follow from chapter 2 and 3. Additionally, except for μ, the parameters in the endogenous discounting function (ξ, ω, Ψ) all follow from Schmitt-Grohé and Uribe (2003)[28] and Mendoza (1991)[23]. Also, to lessen the chance of $\beta(c_t, l_t) \succeq 1$ at any time[2], we make the timing of the model quarterly, so we make β to be 0.96 and δ to be 0.1, exactly quadrupled the depreciation rate under the quarterly structure used in Canzoneri et al (2012)[4]. Similar to how we dealt with the cost of holding bonds in chapter 3, here we make μ such that the bond to output ratio in the steady state is close to 0.5, reflecting the fact that in recent years China's bond to GDP ratio has been approaching one half.

For the computational exercises in the following sections, unless otherwise specified as evolving paths in the perfect foresight transitional dynamics, the list of used

[2] As the endogenous discount factor, $\beta(c_t, l_t)$ should always be under 1 to ensure the agent is indeed discounting all future utilities.

parameters for the 2 country model are

A	α	β	μ	φ	δ	σ	$\zeta=\omega$	τ	ρ	γ	γ^*	Ψ	ξ	N	y^*
1	1/3	0.96	1.01	4.5	0.1	2	1.455	1	$1-1/0.9$	0.85	0.95	0.11	1	4.2	1

The closed economy model shares most parameter values with the 2-country model with some notable exceptions. In the closed economy model σ is set to 1 to make it algebraically possible for a balanced growth path in the steady state. μ is trivially taken to be 1, as in Schmitt-Grohé and Uribe (2003)[28] and Mendoza (1991)[23], and this gives reasonable steady state values of discount factor and bonds to output ratio. We make π to be 1.02, as in recent decades the US GDP per capita has been growing at around 2% annually on average. A list of additional parameters pertaining to the closed economy model is

σ	μ	π
1	1	1.02

4.3 Stochastic Impulse Response

Before going into deterministic dynamics, we take a look at the stochastic impulse responses. Stochastic shocks are applied to home total factor of productivity (TFP) A_t, foreign endowment y_t^*, and foreign good's weight in the foreign consumption bundle γ_t^*[3]. The stochasticity is defined to follow[4]

$$\begin{aligned}
\ln A_t - \ln A &= 0.95(\ln A_{t-1} - \ln A) + \epsilon_t^A, \quad \epsilon_t^A \sim iid\ N(0, \sigma_A^2) \\
\ln y_t^* - \ln y^* &= 0.95(\ln y_{t-1}^* - \ln y^*) + \epsilon_t^y, \quad \epsilon_t^y \sim iid\ N(0, \sigma_y^2) \\
\ln \gamma_t^* - \ln \gamma^* &= 0.95(\ln \gamma_{t-1}^* - \ln \gamma^*) - \epsilon_t^\gamma, \quad \epsilon_t^\gamma \sim iid\ N(0, \sigma_\gamma^2)
\end{aligned}$$

[3] γ_t^* is a proxy of foreign demand for home export, as explained in chapter 3.

[4] Shocks to y_t^* and γ_t^* only apply to the 2 country model, as they describe the foreign country.

Size of the shocks are set as $\sigma_A = \sigma_y = \sigma_\gamma = 0.01$. The negative sign in front of the error term in the evolution of γ_t^* indicates it's a negative shock. The impulse response to the 3 shocks are plotted for 30 periods.

Figures 1-3 show the impulse responses under the 2 country setup. As expected, the dynamics is partly driven by how shocks transmit through the mechanism of endogenous discount factor. The occurrence of a shock alters the agent's periodical discount factor looking forward, and that further impacts the responses in other variables.

Figure 1 shows that in the 2 country model, when productivity gets a boost, home consumers take advantage of it by investing in more capital. As a result of consuming more also due to higher productivity, they become more impatient, and sell bonds to satisfy the need for more investment and consumption at the same time. Home country experiences real depreciation because of the abundance of home goods.

Figure 2 shows that an increase in foreign endowment easily spreads into the home country. Similar to the responses to an increase in home productivity, home consumers increase investment, capital, output and consumption. As a result, they become less patient and need to sell some of their foreign assets to satisfy the need for more investment and consumption. Home goods sees a real appreciation because of the abundance of foreign good.

Figure 3 shows the impulse responses to an increase in foreign demand for home good. A real appreciation of home goods enables home consumers to consume more. As a result of becoming less patient, they invest less, as more of their own goods goes into export. The home real appreciation is strong enough that they manage to accumulate more foreign assets, even as they become more impatient.

Figure 4 shows the same case as Figure 1 for the closed economy model with very similar responses. As shown by the impulse responses, all 3 types of shocks are considered to be positive shocks to the home consumers.

4.4 Simple Transitional Dynamics

Having made sure that all the impulse responses seem to be intuitively sound, we turn to studying the time paths of our variables of interest. In this section we look at how the system transitions to the steady state, starting with low levels of initial wealth. We call it "simple" dynamics because it does not involve any changes to exogenous factors. It's also fully deterministic. We do this for both models, and compare the computed paths of the variables of interest to the historical trends observed in developing countries.

For our variables of interest, we define investment and saving rates as

$$
\begin{aligned}
INV\%_t &= \frac{i_t}{y_t} \\
SAV\%_t &= \frac{y_t - p_t c_t}{y_t}
\end{aligned}
$$

which differ in the 2-country model and coincide in the closed economy model. Also define import and export as shares of home output, and trade surplus

$$
\begin{aligned}
IMPSH_t &= p_{f,t} c_{f,t} / y_t \\
EXPSH_t &= N c^*_{h,t} / y_t \\
SURPLUS_t &= N c^*_{h,t} - p_{f,t} c_{f,t}
\end{aligned}
$$

4.4.1 Closed Economy Model with Balanced Growth Path

Figure 5 shows a case of simple dynamics of a closed economy. Here the economy starts with well under half of its steady state value of capital per effective worker. Consumption, output, investment and capital all increase. As a result of becoming richer and consuming more, consumers become less patient over time. The investment/saving rate, the only variable among those of interest applicable to the closed economy model, falls over time in the simple dynamics, which contradicts the increases in both rates observed in the Asian developing countries. This is not surprising given the existence of endogenous discounting, as the higher patience level early on due to low consumption makes consumers want to save more, and as they become richer and consequently less patient, they save less and consume more.

It turns out this trend also depends on the starting point to some extent. Starting with an even lower initial capital can generate an initial increase in the saving/investment rate. Figure 6 shows such a case, resulting in increasing saving/investment rate during approximately the first 20 periods, even though the patience level is still strictly decreasing as consumers get wealthier. This is because consumers are so poor in the beginning they have to devote a decent amount of available resources to consumption, as they are aware that their future utilities will be discounted increasingly heavily. Only as they get wealthier can they increase the allocation of resources to saving/investment. Soon, however, when they reach a certain level of wealth, the endogenous discounting kicks in and forces the consumers to decrease saving/investment rate because decreasing discount factor makes them increasingly impatient.

4.4.2 2-Country Model

Figure 7 shows the simple dynamics for the 2 country model, starting with less than half of the steady state values of bonds and capital. Home consumption, labor, capital, output, bonds, saving and investment all increase monotonically. Home country experiences real depreciation as it becomes more productive, making home goods more abundant. Home consumers become more impatient as they get richer over time.

Current account is always positive with monotonic bonds accumulation, which is consistent with observed trends. Import's share of output sees monotonic increase, while export share sees a short-lived increase followed by stronger long-term decrease. In reality, however, we generally observe persistent increase in both shares over time for the Asian developing countries.

Saving and investment rates experience the same trends as export share, increasing slightly for the first 10 years or so and then persistently declining towards steady state. While in reality we do see somewhat of an increasing-then-decreasing trend in saving and investment rates in some developing–turned-developed countries, the initial increase in our model simply lasts too short to provide a good match for reality. The investment and saving rates see brief initial increases because the extreme shortage of resources in the beginning prevents consumers from devoting much to investment and saving. The ensuing decrease is because of the decreasing patience level induced by the endogenous discounting.

The initial increases would disappear if we let the system start from a richer state, as shown in Figure 8. When we start only slightly poorer than the steady state, both saving and investment rates continuously decrease throughout. This is because when starting with a decent amount of resources, the consumers can start by devoting a smaller portion of their wealth to consumption and gradually increase that portion, as they become increasingly more impatient. The impact of endogenous discounting dominates all the way.

To sum it up, simple dynamics gives satisfactory trends in current account and import share, but falls short in explaining the observed trends in export share, as well as saving and investment rates.

4.5 Transitional Dynamics with Structural Changes

The previous section shows that letting development take place on its own without external forces is insufficient for matching reality. Without any exogenous changes, the models' evolution paths don't produce the observed trends in export share, saving rate and investment rate. Like we did in chapter 3, we now perform perfect foresight dynamics by implementing exogenously increasing paths for home technology and foreign demand for home good. Similar to chapter 3, we deploy linear increases to highlight a catch-up phase.

4.5.1 Increase in Technology

We start with a low level of capital (and also bonds in the 2 country model) and low level of TFP, and increase TFP in an arithmetic sequence for the first 30 periods until 1. We call this the "takeoff" or "catch-up" stage, during which the developing country catches up with world technology, and this path is perfectly anticipated by the consumers. At the end of the catch-up, the consumers expect no more exogenous changes, and the economy begins to converge to the steady state corresponding to the high world technology. As seen in chapter 3, the time paths often bends sharply at the end of catch-up, creating kink points.

Closed Economy Model

Figure 9 shows such dynamics for the closed economy model. The investment/saving rate sees a very brief increase before decreasing in monotone over time. This is also driven by the decreasing endogenous discount factor, and the initial increase is because of the low initial wealth as explained before. This result is robust to different starting point of $\widetilde{z}_t$ and initial level of capital. In general applying only TFP increase to the closed economy model does not improve the results.

2 Country Model

Figure 10 shows a case of increasing home TFP for the 2 country model. Real depreciation happens to the home good because of increase in productivity and factor accumulation. A positive difference from the closed economy case is the increase in investment rate during takeoff, although saving rate and export share both see an interesting "V" pattern during takeoff, which is the first time we see non-monotonicity during takeoff so far. Investment rate increases during takeoff, because consumers expect future increase in productivity would more than make up for the loss in early investment as they devote more resources to consumption in the beginning when resources are scarce. Initially the saving rate decreases briefly because of the decreasing marginal product of saving in the face of diminishing patience, but this impact is quickly overpowered by the exogenous increase in productivity, and the trends reverses until the end of catch-up.

Current account is always weakly positive with monotonic bonds accumulation. Only when we have initial endowment very close to the steady state value can we get strictly increasing saving rate during catch-up, but at the same time this generates strictly decreasing export share, as shown in Figure 11.

Applying only increase in TFP to the 2 country model brings about some improvements in reversing the "incorrect" investment rate trend in the simple dynamics case, at least during the takeoff.

4.5.2 Increase in Foreign Demand for Home Good

This scenario only applies to the 2-country model. Similar to chapter 3, we make the weight on foreign good in the foreign preference γ^* to be a proxy for foreigners' demand for home good. We start with low levels of capital and bonds and a high γ^*, and decrease it in an arithmetic sequence for the first 30 periods until 0.95. The only exogenous change during this "catch-up" stage is the increase in foreign demand for home exports, and this is perfectly foreseen by the consumers. Like the in case of technological increase, the dynamics observe kink points.

Figure 12 shows the dynamics with only increase in foreign demand. Real appreciation occurs to the home good because of higher foreign demand. During the catch-up stage, investment rate and import share fall, while saving rate and export share increase, all of them reversing trends after catch-up. A rising export share during catch-up is intuitive as foreign demand increases. The investment rate starts out high, as consumers prepare to meet the future increases in foreign demand.

A curiously troubling sign is that even when bonds holding starts out low, it still drops during catch-up and only recovers afterwards. This results in a current account deficit during catch-up, which contradicts data. Facing increasing foreign demand, even with reduced investment, home consumers decide to dump some foreign asset holding to increase their consumption.

Changing initial level of γ^* or capital and bonds turns out not to affect the resulted trends. Implementing increase in foreign demand alone can "cure" saving rate and export share, but also upset the "correct" trend in investment rate and import share we had in the case of TFP increase.

4.5.3 Simultaneous Increase in Technology and Foreign Demand

It's clear from the above analysis that implementing either exogenous change alone succeeds only in doing part of the matching job. A natural next step is to implement both changes together in hope of getting as many trends right as possible. We start with a low A_t and a high γ_t^*, and let them each increase and decrease in equal successions until their parametric values specified in section 2. This exercise also only applies to the 2 country model.

Figure 13 shows the case that combines precisely the sequences of exogenous changes previously implemented in the single-change cases. The results are very similar to the case of implementing increase in foreign demand alone. Current account bears a deficit during catch-up. Saving rate and export share increase, and import share decreases during catch-up, while investment rate declines throughout. This suggests that the impact of increase in foreign demand may be dominating the increase in technological progress. Reducing the magnitude of foreign demand increase and/or widening the magnitude of home TFP increase could potentially generate more balanced results.

Figure 14 shows such a case with smaller increase in foreign demand and larger increase in home TFP. For the first time, we have export and import shares, investment and saving rates all increasing during almost the entire catch-up. Only the import share briefly decreases initially before rising the rest of the way. However, bonds still declines during catch-up, causing a current account deficit, suggesting the increase in foreign demand is still forcing home consumers to reduce foreign debt in order to increase consumption and also stock up on capital to meet future foreign demand.

Implementing increase in both home technology and foreign demand for home export simultaneously appears to be the "best" scenario we have seen in terms of fitting observed data trends. It serves as proof that taking into consideration changes in all the relevant exogenous factors is instrumental in attempts to reproduce realistic results of growth dynamics.

In the simple dynamics, we get current account surplus because endogenous discounting leads to higher saving by bonds accumulation in the beginning. In the perfect foresight dynamics with increase in foreign demand, home consumers can borrow from the foreign country first by selling assets, as they expect to be able to pay off the debt in the future via home goods appreciation as a result of increase in foreign demand.

4.6 Further Discussions on Endogenous Discounting

With results of closed and open economy models utilizing endogenous discounting, we can now gauge how this mechanism manifests itself in a given model. To do this, we compare models with and without endogenous discounting, and see what effects it may have on the dynamics outcome.

For comparison we revisit the baseline model in chapter 2, which abstracts from using an endogenous discount factor, but is otherwise identical to the closed economy model with BGP described in section 2.2. They share all the common parameter values, except that the exogenous discount factor is always equal to 0.96 in the baseline model.

Figure 15 shows the simple transitional dynamics of this case, analogous to in Figure 5, which depicts the case with endogenous discounting. The two cases bear high resemblance, with just about everything increasing over time except saving/investment rate. Under a closed economy setting without any exogenous changes, the addition of endogenous discounting does not seem to affect the trends of saving/investment rate.

Figure 16 shows the transitional dynamics with increase in TFP for the model without endogenous discounting. It corresponds to the case in Figure 9 with endogenous discounting which experiences the same path of technological increase. A stark difference is that in the case without endogenous discounting, the investment/saving rate observes an increasing trend during the early stages of development, while in the case with endogenous discounting, it sees nearly monotonic decrease. Under a closed economy with exogenous increase in technology, not using endogenous discounting actually depicts a more realistic trend of investment/saving rate. What's behind all this?

Looking back at all the dynamics of models with endogenous discounting, it has always been the case that as growth occurs, the internal discount factor decreases, meaning consumers get more impatient as they become wealthier. As a result of endogenous discounting, consumers should in theory want to consume more and save less over time. This is indeed the case in our closed economy model, as output is allocated between only consumption and investment/saving. As the discount factor falls, they devote more available resources to today's consumption over saving for the future. In the absence of endogenous discounting, the baseline model apparently generates opposite and "correct" trends when technological increase is implemented, as shown in Figure 16.

In our open economy models, however, the presence of endogenous discounting does not seem to do as much harm. In all cases with exogenous changes, at least one of saving and investment rates sees increases. This is because in an open economy with foreign asset transactions, saving is not the same as investment. In an open economy, agents can save by not only investing, but also adjusting foreign asset holding. We can see this by rearranging (1) and decomposing saving

$$\begin{aligned} SAV_t &= y_t - p_t c_t \\ &= i_t + p_{f,t}(b_t - R^*_{t-1} b_{t-1}) \end{aligned}$$

In an open economy saving is the sum of investment and the change in bond position, allowing saving and investment rates to move independently depending on the change in foreign asset holding. Even when endogenous discounting is taking effect and consumers want to increase consumption over time as they become impatient, they don't necessarily have to decrease investment as they can cut back on their bond position. As we saw in the impulse responses in Figure 1, when there is a good shock to productivity, consumers want to capture the full benefit of it by investing in more capital for production, but at the same time want to consume more as a result of becoming more impatient. They achieve this by dumping some foreign asset holding.

The additional channel of saving - foreign asset purchase or sale, which is at the core of an open economy, plays a critical role in making possible the coexistence of endogenous discounting and increasing trends of saving and investment rates, which would be nearly impossible in a closed economy, where by definition an increasing consumption rate as a result of lowering patience must lead to decrease in saving/investment rates. The mechanism of endogenous discounting therefore works better with an open economy framework than a closed economy for the purpose of replicating observed trends in saving and investment rates.

4.7 Summary of Results

In this chapter we make use of the endogenous discounting mechanism, and study the dynamics of both closed and open economy models. The closed economy model with equal saving and investment allows for balanced growth paths, while the open economy model allows us to look at trade and current account.

The closed economy model does poorly in matching the observed trends, with or without exogenous changes in technology. For the open economy model, the simple transitional dynamics without any exogenous change does an inadequate job of matching reality, producing only current account surplus and increasing import share. We then introduce exogenous changes in technology and foreign demand, and look at perfect foresight dynamics. Implementing increase in technology only slightly improves the results, generating increasing investment rate during takeoff. Implementing increase in foreign demand produces increasing saving rate and export share, but also current account deficit and decreasing import share. Finally, combining these two changes together seems to give the best fit possible, with the only flaw being a current account deficit during takeoff.

It is noted that in an open economy, foreign asset holding as an additional channel of saving plays a significant role of adjusting and balancing between saving and investment. As a result, endogenous discounting applied to an open economy framework can generate more realistic trends in saving and investment rates than used in a closed economy model.

Figure 4.1: Impulses Responses to an Increase in A_t, 2-country model

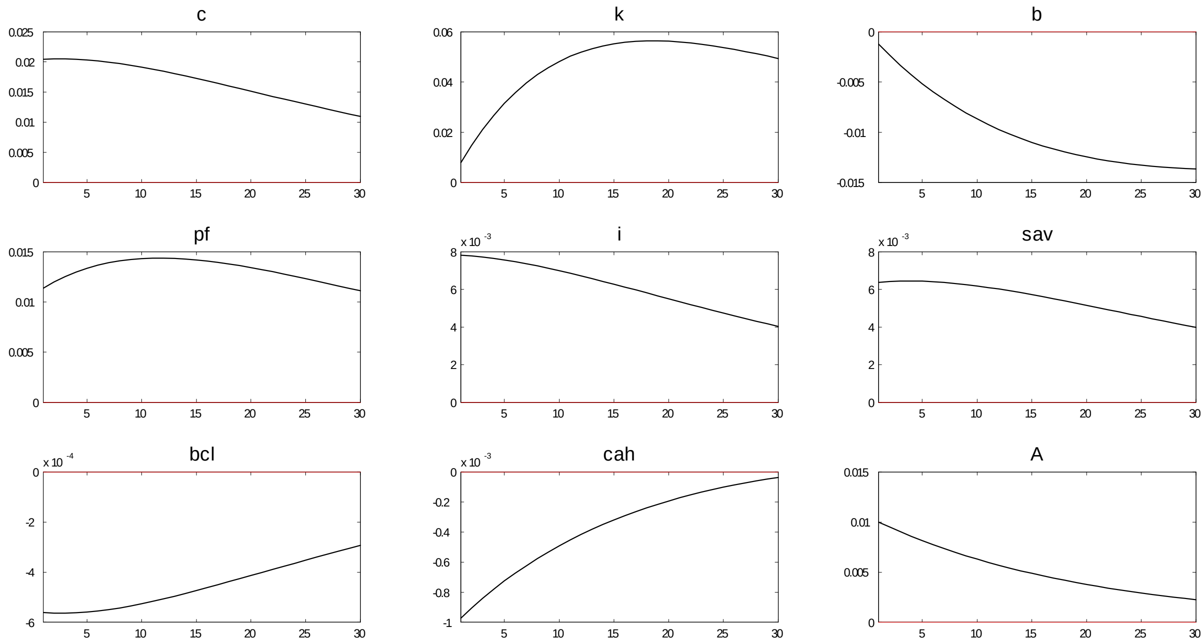


Figure 4.2: Impulses Responses to an Increase in y_t^*, 2-country model

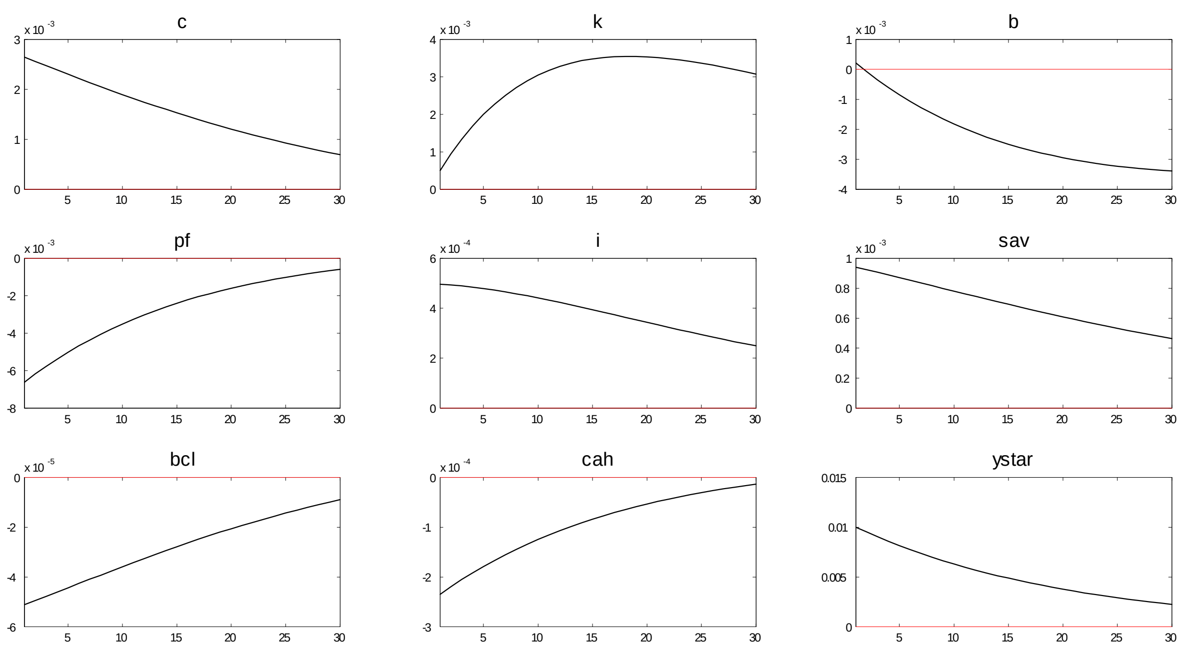


Figure 4.3: Impulses Responses to a Decrease in γ_t^*, 2-country model

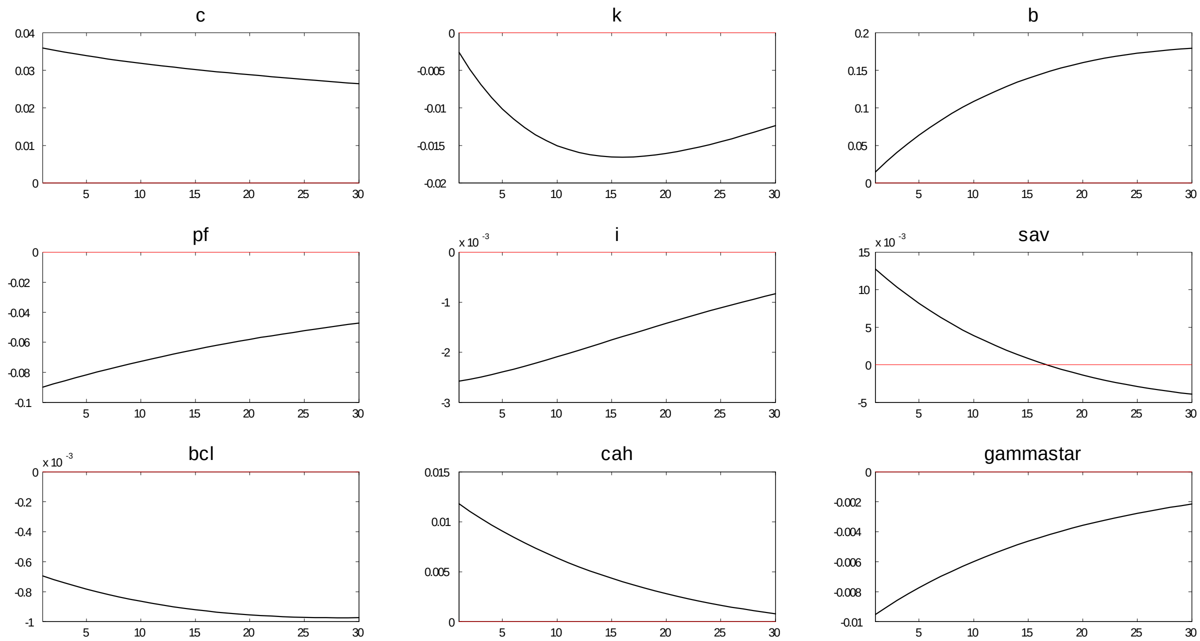


Figure 4.4: Impulses Responses to an Increase in A_t, Closed Economy

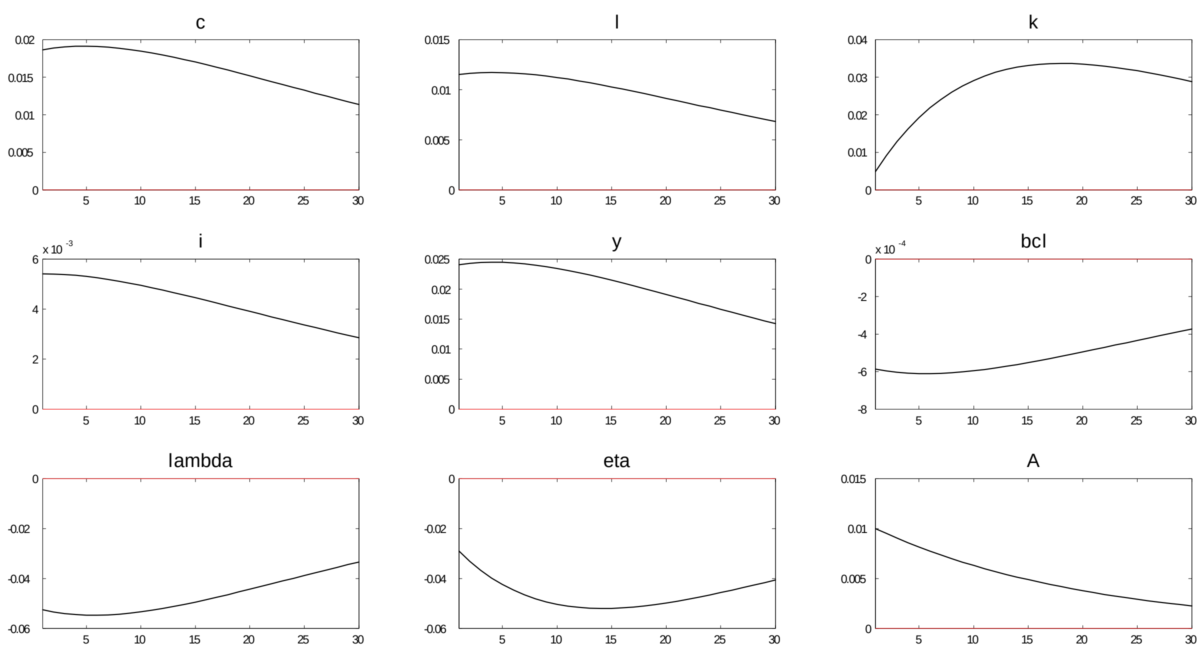


Figure 4.5: Simple Dynamics of Closed Economy, $k_0 = 1$

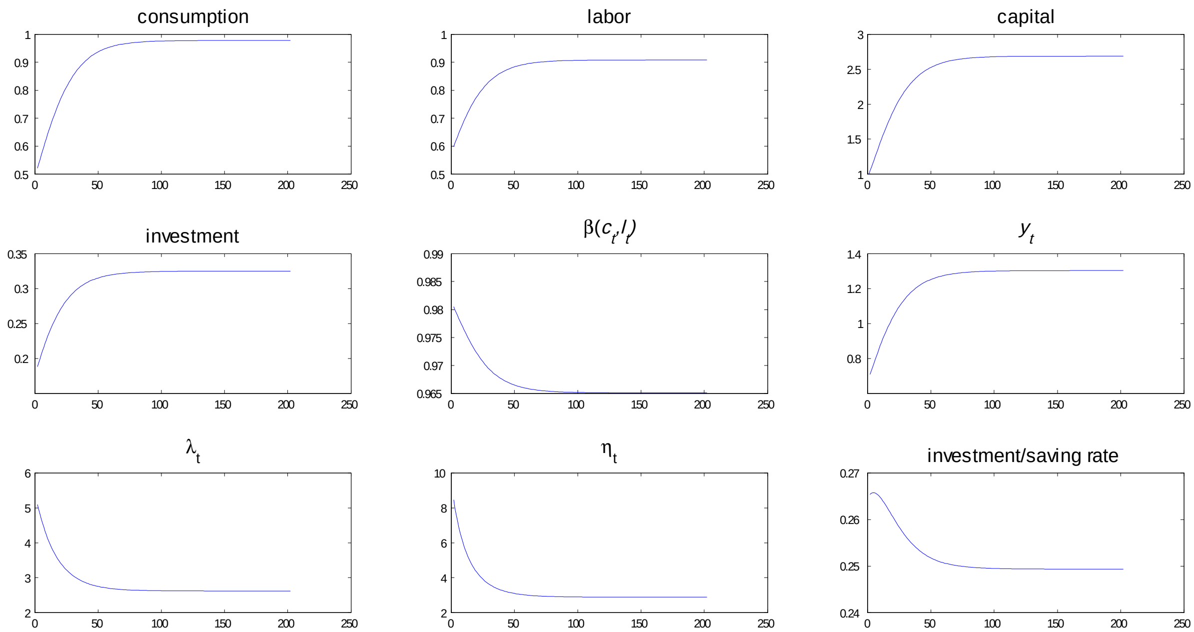


Figure 4.6: Simple Dynamics of Closed Economy, $k_0 = 0.6$

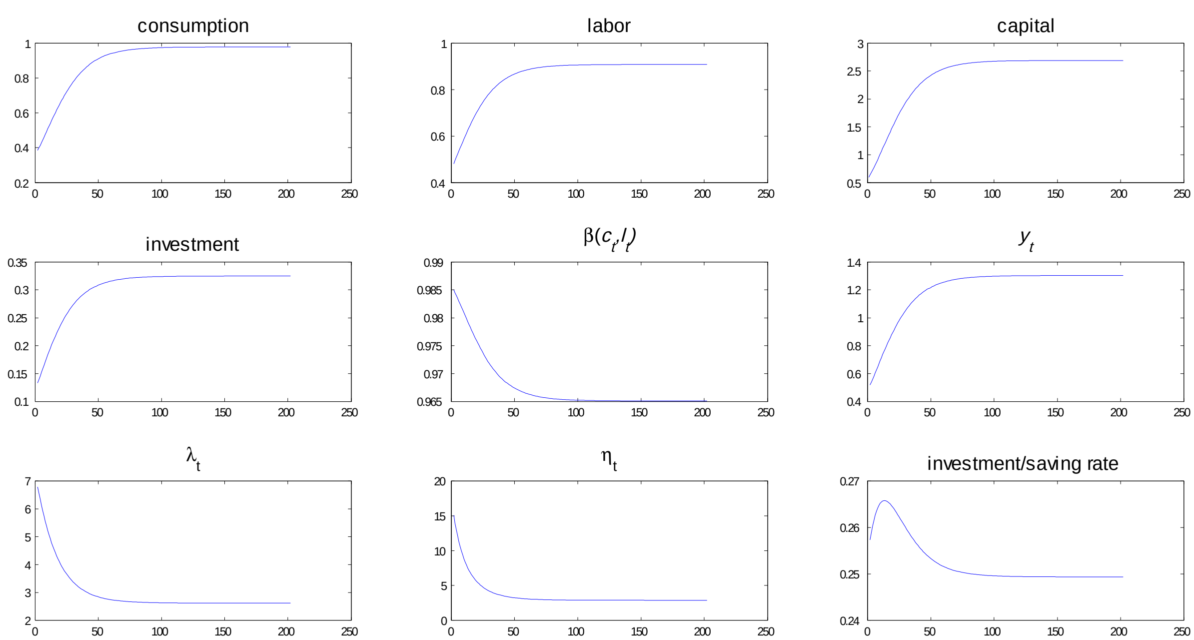


Figure 4.7: Simple Dynamics of 2 Country Model, $k_0 = 1.5, b_0 = 0.4$

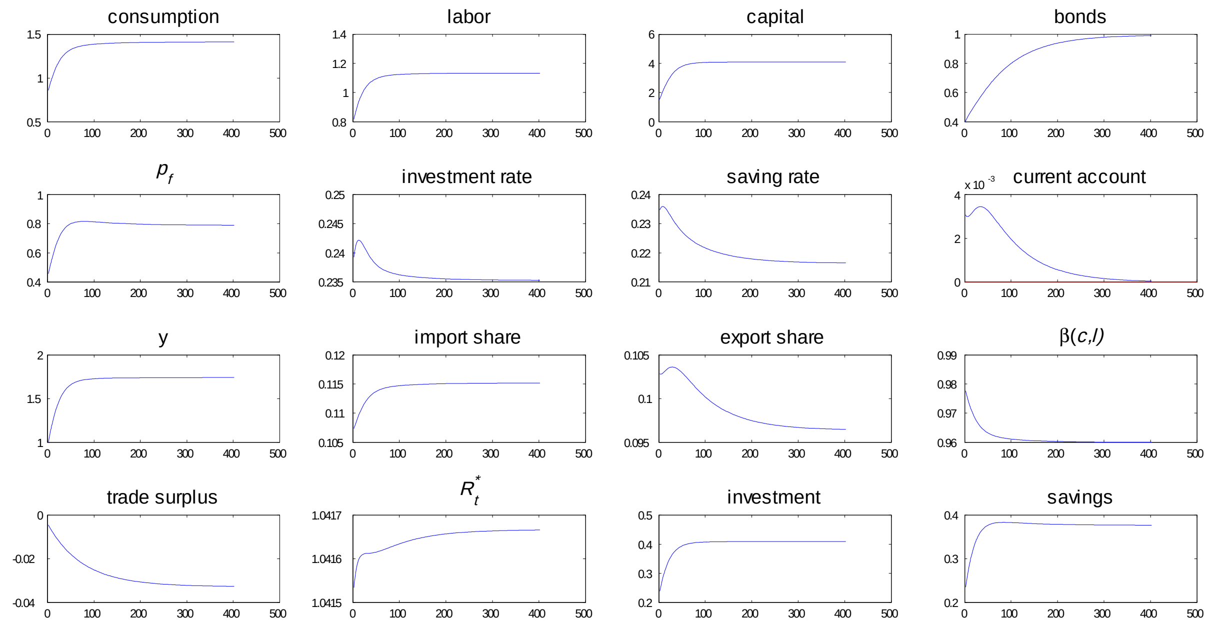


Figure 4.8: Simple Dynamics of 2 Country Model, $k_0 = 3, b_0 = 0.8$

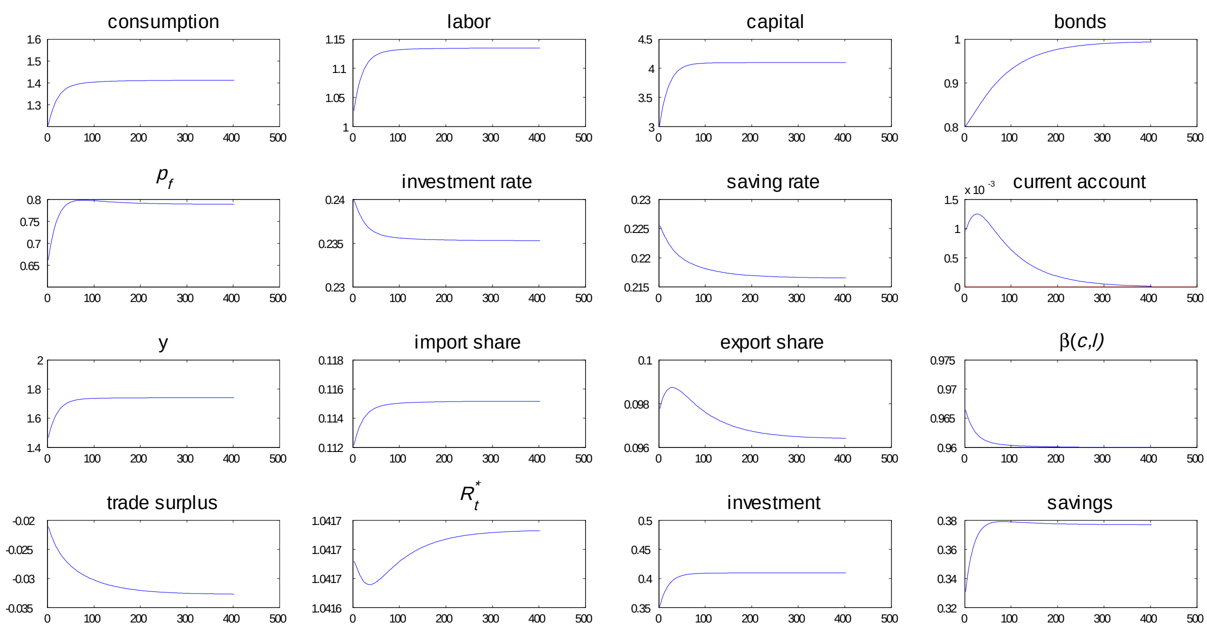


Figure 4.9: Dynamics with Increase in $\widetilde{z}_t$, Closed Economy, $\widetilde{z}_0^{1-\alpha} = 0.7, k_0 = 1$

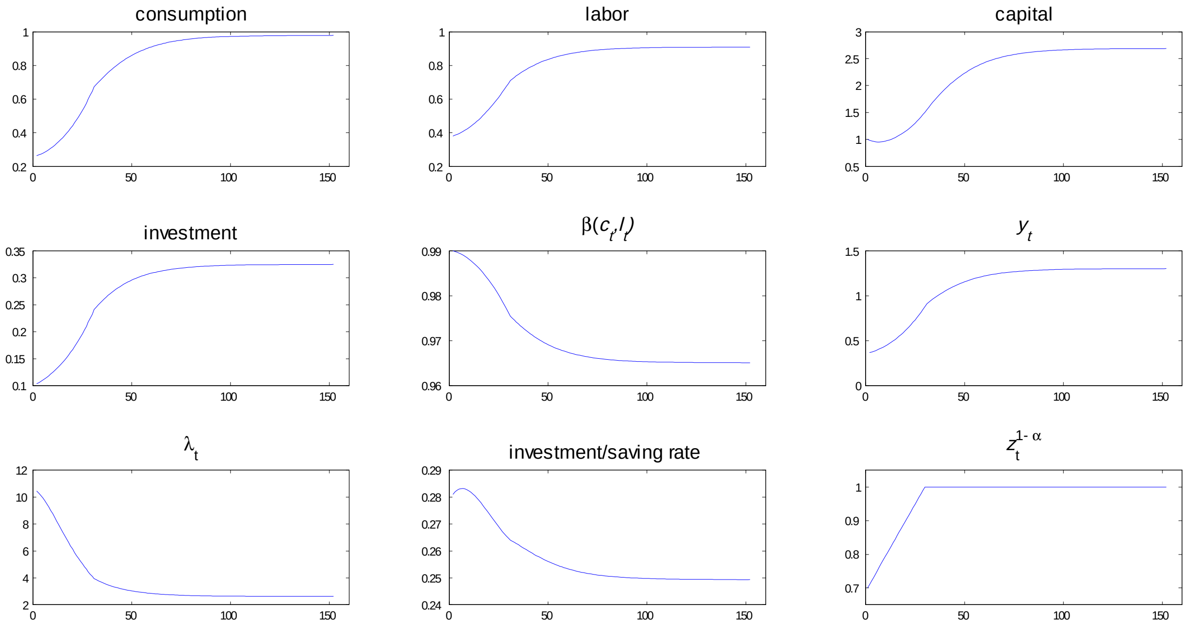


Figure 4.10: Dynamics with Increase in A_t, 2 Country, $A_0 = 0.75, k_0 = 2, b_0 = 0.4$

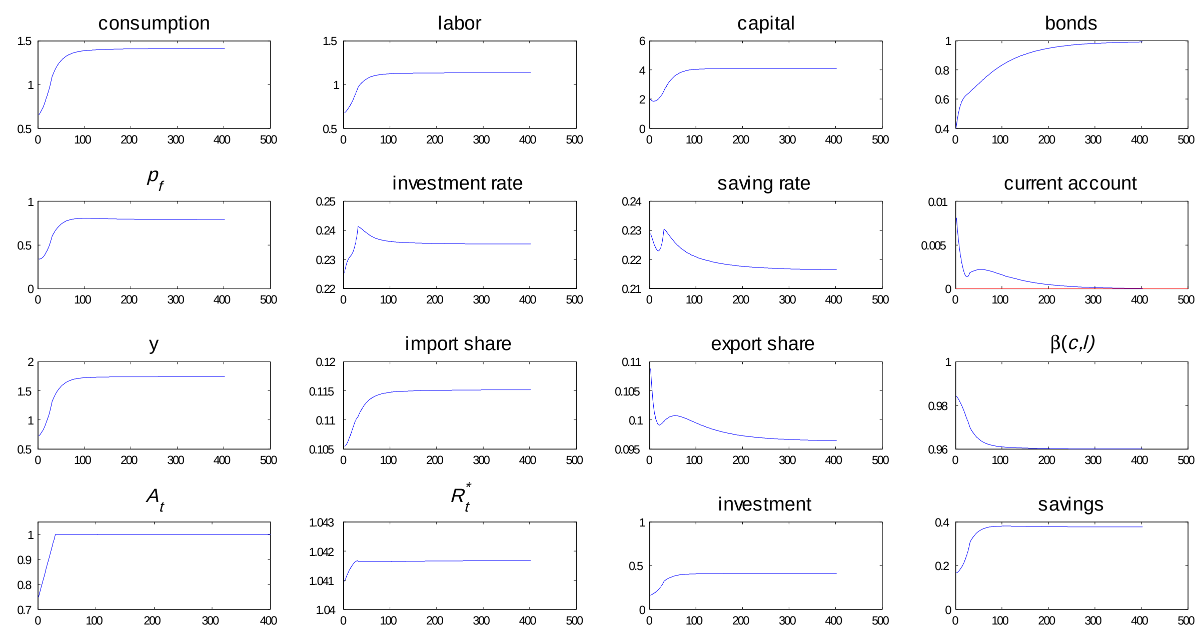


Figure 4.11: Dynamics with Increase in A_t, 2 Country, $A_0 = 0.8, k_0 = 3.7, b_0 = 0.7$

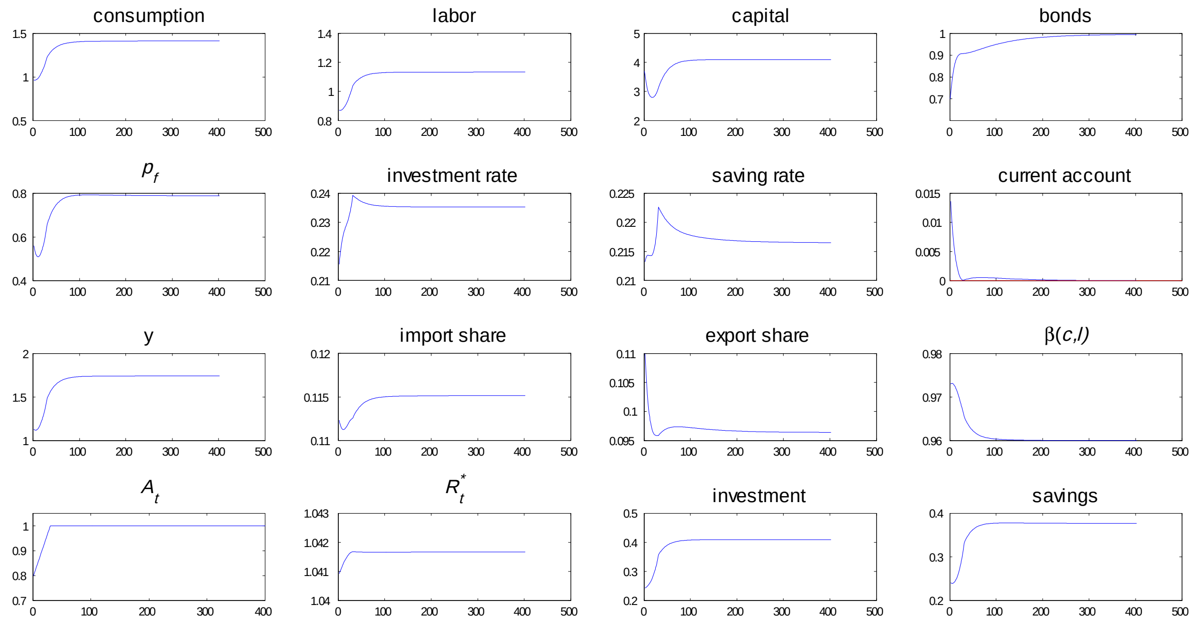


Figure 4.12: Dynamics with Decrease in γ_t^*, 2 Country, $\gamma_0^* = 0.99, k_0 = 2, b_0 = 0.4$

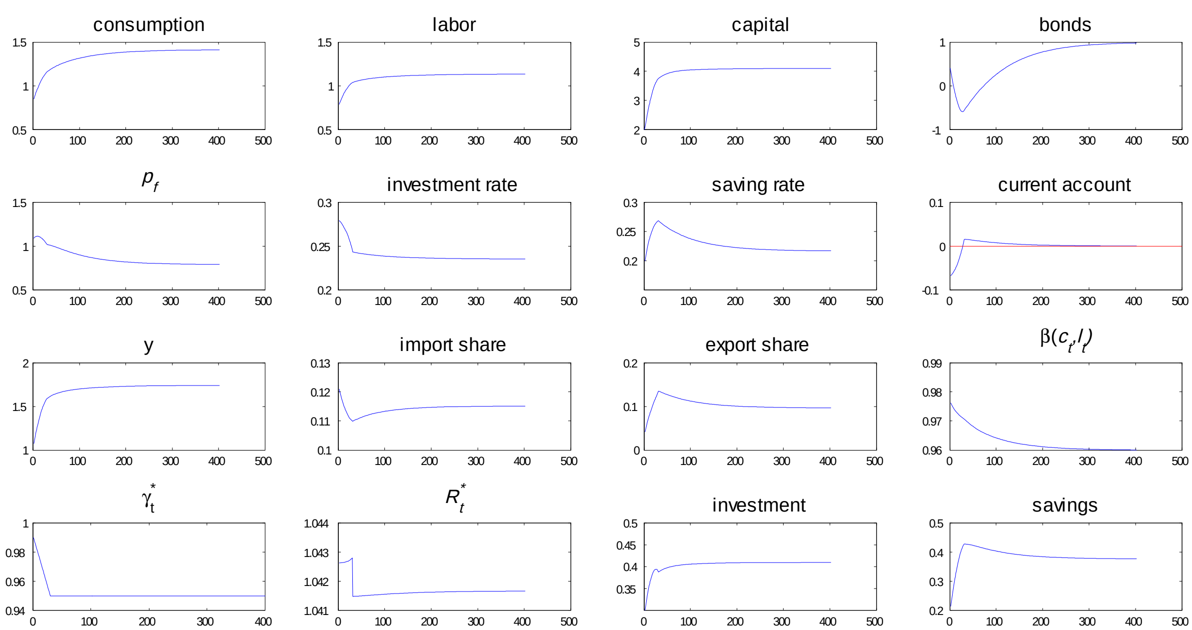


Figure 4.13: Dynamics with Both Changes $A_0 = 0.75, \gamma_0^* = 0.99, k_0 = 2, b_0 = 0.4$

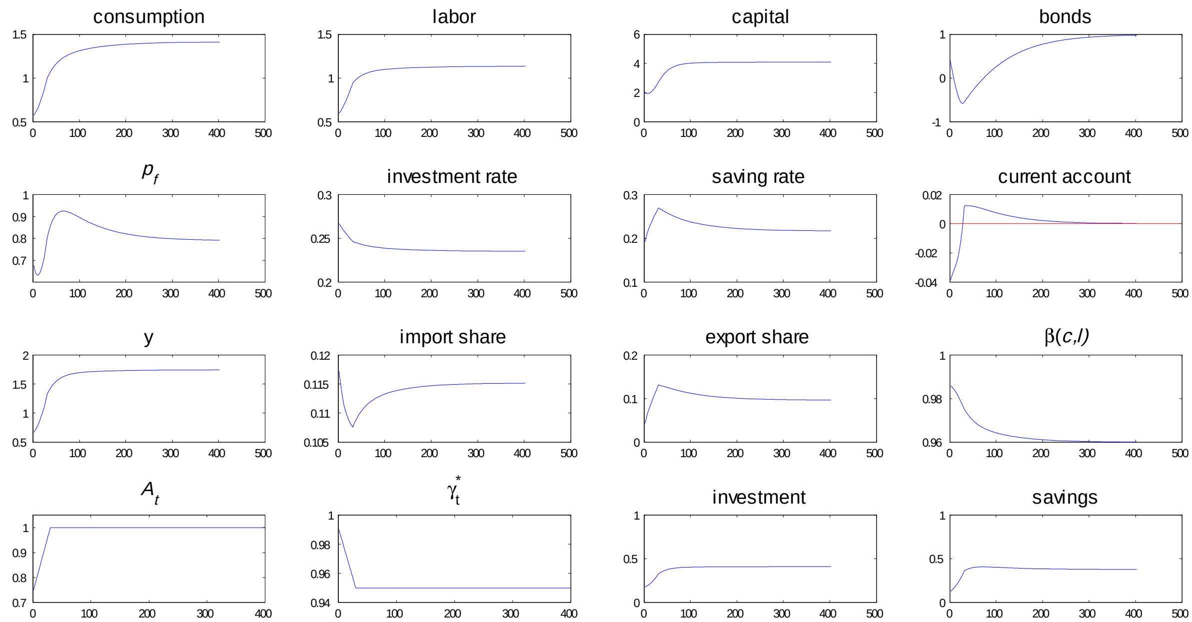


Figure 4.14: Dynamics with Both Changes $A_0 = 0.73, \gamma_0^* = 0.97, k_0 = 2, b_0 = 0.4$

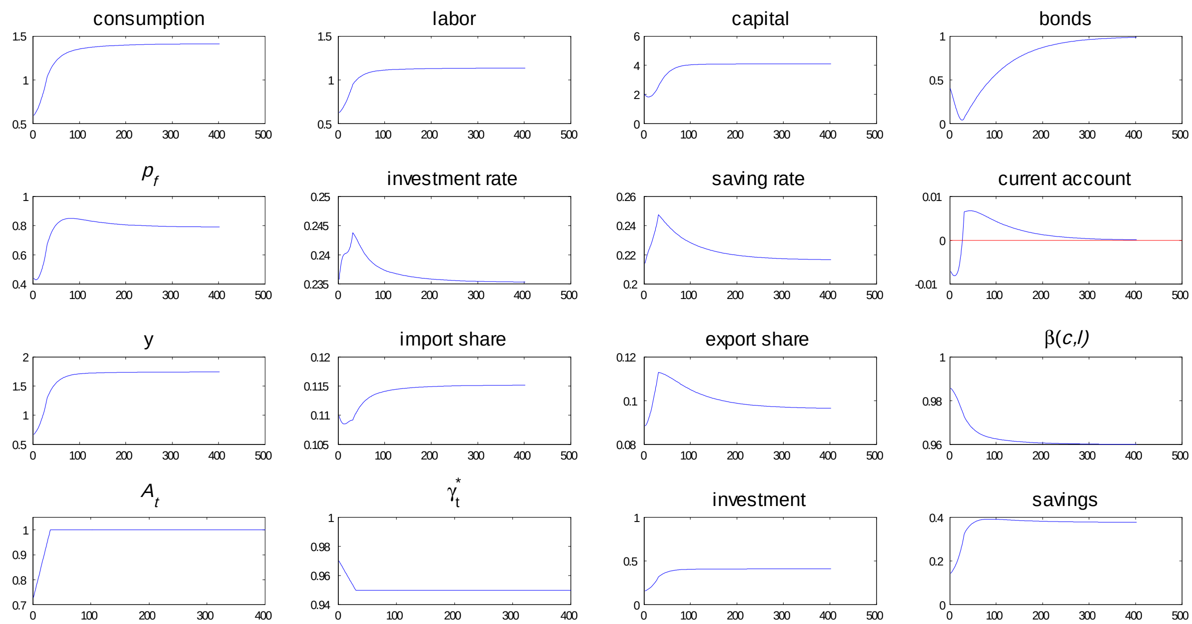


Figure 4.15: Closed Economy without Endogenous Discounting, $k_0 = 1.5$

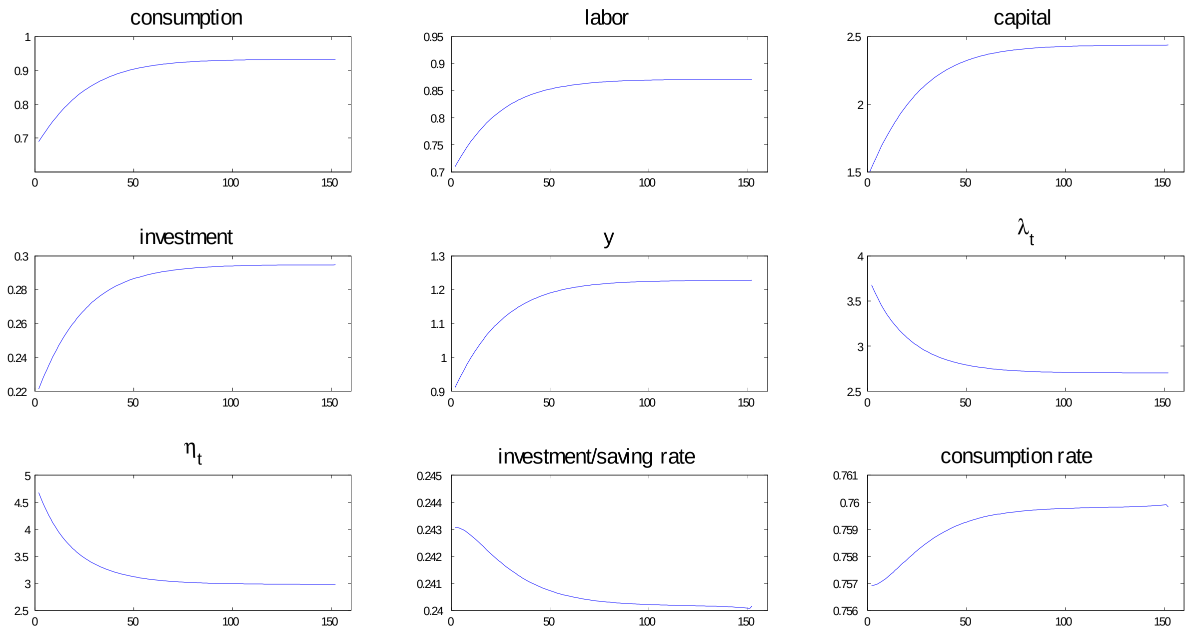
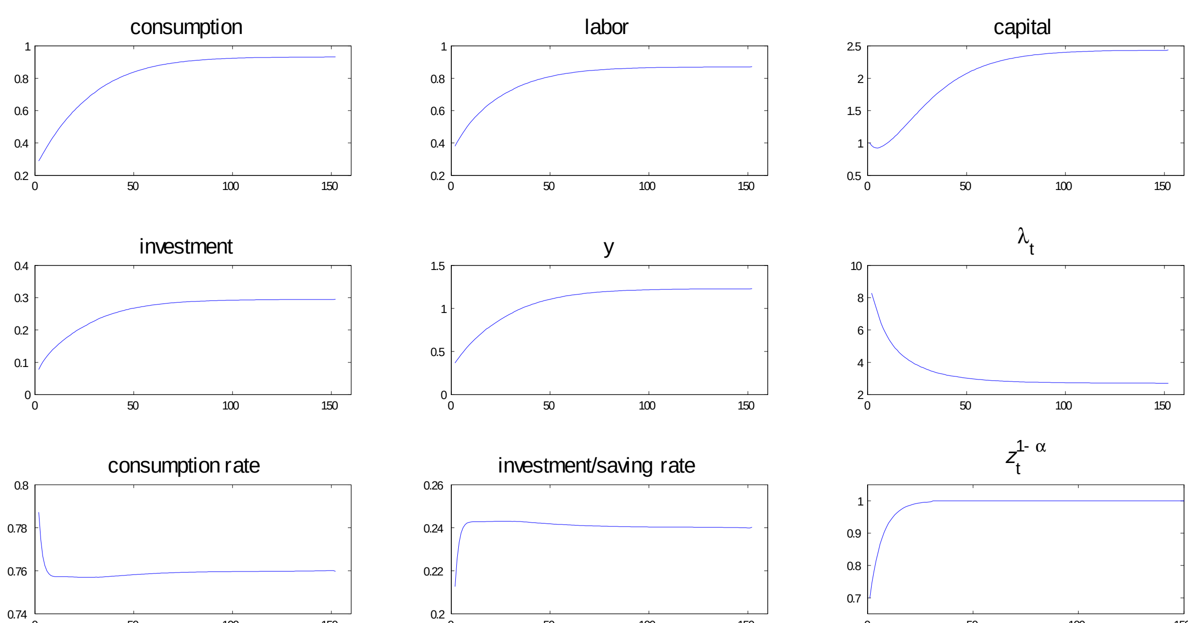


Figure 4.16: $\widetilde{z}_t$ Increasing in Closed Economy w/o Endogenous Discounting, $\widetilde{z}_0^{1-\alpha} = 0.7, k_0 = 1$



Chapter 5

Growth Dynamics of a Multi-Sector Open Economy Model

5.1 Introduction

In this chapter, we further stretch our stylized two-country model to include 2 sectors in the home country. We use portfolio adjustment cost to close the model. This would not only allow us to look at growth dynamics, but also examine the transfer of resources and labor migration between sectors, as well as the Balassa-Samuelson effect during the process of industrialization. We also compare the results with previous chapters. As noted in Herrendorf et al (2013)[14], the conditions under which one can simultaneously generate balanced growth and structural transformation are rather strict, and under these conditions the multi–sector model is not able to account for the broad set of empirical regularities that characterize structural transformation. For these reasons, in this chapter we don't attempt to make our 2-sector open economy model compatible with balanced growth paths.

The rest of the chapter is organized as follows. Section 2 lays out the 2-sector open economy model and discusses parametric choices. Section 3 takes a quick look at the stochastic impulse responses. Section 4 looks at the simple transitional dynamics. Section 5 studies the perfect foresight dynamics with one or more exogenous changes taking place. Section 6 compares results with previous chapters and discusses the reasons for the differences. Section 7 provides ideas of extension and concludes.

5.2 Model

This is a real open economy model of 2 countries, with the home country being a developing nation of our interest. The home country consists of homogenous Yoeman farmer type of agents. They allocate resources between an industrial and an agricultural sector, each producing consumption good and food with capital and labor. The foreign country has an endowment economy. Both home and foreign consumers like variety in their consumption via trade. Food is only produced and consumed by home consumers domestically. Home consumers can save by investing in capital or purchasing foreign assets denominated in foreign goods.

5.2.1 Home Country

Home agents divide 1 unit of labor between working in the industrial and agricultural sectors, and don't gain utility from leisure. They consume an aggregated bundle of home and foreign consumption goods as well as food. Food consumption a_t shows up in utility with a subsistence level $\overline{a}$, below which agents would starve to death.

The home representative consumer's problem is

$$\max E\sum_{t=0}^{\infty}\beta_h^t\left[\frac{c_t^{1-\sigma}}{1-\sigma}+\kappa\ln(a_t-\overline{a})\right]$$

subject to a flow budget constraint, 2 sectoral capital evolutions and labor constraint

$$c_{h,t}+p_{f,t}c_{f,t}+i_{a,t}+i_{m,t}+b_tp_{f,t}+\frac{\Psi}{2}(b_t-\overline{b})^2p_{f,t} = y_t+R^*_{t-1}b_{t-1}p_{f,t} \quad (5.1)$$

$$(1-\delta^a)k_{a,t}+i_{a,t}-\frac{\varphi^a}{2}(\frac{i_{a,t}}{k_{a,t}}-\delta^a)^2k_{a,t} = k_{a,t+1} \quad (5.2)$$

$$(1-\delta^m)k_{m,t}+i_{m,t}-\frac{\varphi^m}{2}(\frac{i_{m,t}}{k_{m,t}}-\delta^m)^2k_{m,t} = k_{m,t+1} \quad (5.3)$$

$$l_{a,t}+l_{m,t} = 1 \quad (5.4)$$

where everything is in terms of home industrial good, and subscript a and m each stands for agricultural and industrial (manufacturing) sector.

The inclusion of food consumption with a subsistence level in the utility follows the Stone-Geary form, which has been extensively used by previous literature with multi-sector models. It states that the marginal utility of food consumption is high when it's close to the subsistence level, and rapidly falls as it rises beyond the subsistence. Food is taken as an inferior good.

$p_{f,t}$ is the relative price between foreign and home industrial good. The capital evolutions contain an adjustment cost term. All food produced is immediately consumed in its entirety as it's perishable. Since food is not traded or saved and its production equals consumption, it does not explicitly enter the flow budget constraint or clearing conditions. Investments use only home industrial good. The budget constraint contains a portfolio adjustment cost to mechanically close the model, as demonstrated in Schmitt-Grohé and Uribe (2003)[28] and also used in chapter 3. The consumption bundle is aggregated between home and foreign consumer good in Dixit-Stiglitz style with unequal weights

$$c_t = [\gamma^{1-\rho} c_{h,t}^{\rho} + (1-\gamma)^{1-\rho} c_{f,t}^{\rho}]^{\frac{1}{\rho}}$$

Cobb-Douglas production takes place in both sectors. Besides capital and labor, the agricultural sector also uses land S_t (for soil) as an input.

$$a_t = A_{a,t} k_{a,t}^{\alpha^a} l_{a,t}^{1-\alpha^a-\omega} S_t^{\omega} \tag{5.5}$$

$$y_t = A_{m,t} k_{m,t}^{\alpha^m} l_{m,t}^{1-\alpha^m} \tag{5.6}$$

The first order conditions to the home maximization problem are

$$c_t^{-\sigma} - \lambda_t p_t = 0 \tag{5.7}$$

$$\begin{aligned} -\eta_{m,t} + \beta_h E_t\{\lambda_{t+1} \alpha^m A_{m,t+1} k_{m,t+1}^{\alpha^m-1} l_{m,t+1}^{1-\alpha^m} \\ +\eta_{m,t+1}[1 - \delta^m - \tfrac{\varphi^m}{2}(\delta^m + \tfrac{i_{m,t+1}}{k_{m,t+1}})(\delta^m - \tfrac{i_{m,t+1}}{k_{m,t+1}})]\} = 0 \end{aligned} \tag{5.8}$$

$$-\eta_{a,t} + \beta_h E_t\{\tfrac{\kappa}{a_{t+1}-\overline{a}}\alpha^a A_{a,t+1}k_{a,t+1}^{\alpha^a-1}l_{a,t+1}^{1-\alpha^a-\omega}s^\omega$$
$$+\eta_{a,t+1}[1-\delta^a-\tfrac{\varphi^a}{2}(\delta^a+\tfrac{i_{a,t+1}}{k_{a,t+1}})(\delta^a-\tfrac{i_{a,t+1}}{k_{a,t+1}})]\} = 0 \tag{5.9}$$

$$-\lambda_t + \eta_{m,t}[1-\varphi^m(\frac{i_{m,t}}{k_{m,t}}-\delta^m)] = 0 \tag{5.10}$$

$$-\lambda_t + \eta_{a,t}[1-\varphi^a(\frac{i_{a,t}}{k_{a,t}}-\delta^a)] = 0 \tag{5.11}$$

$$-\frac{\kappa}{a_t-\overline{a}}(1-\alpha^a-\omega)A_{a,t}k_{a,t}^{\alpha^a}l_{a,t}{}^{-\alpha^a-\omega}s^\omega + \lambda_t(1-\alpha^m)A_{m,t}k_{m,t}^{\alpha^m}l_{m,t}^{-\alpha^m} = 0 \tag{5.12}$$

$$\lambda_t[-p_{f,t}-\Psi(b_t-\overline{b})p_{f,t}] + \beta_h E_t(\lambda_{t+1}p_{f,t+1}R_t^*) = 0 \tag{5.13}$$

The static cost minimization solutions are

$$p_t = [\gamma + (1-\gamma)p_{f,t}^{\frac{\rho}{\rho-1}}]^{\frac{\rho-1}{\rho}} \tag{5.14}$$

$$c_{h,t} = c_t\gamma p_t^{\frac{1}{1-\rho}} \tag{5.15}$$

$$c_{f,t} = c_t(1-\gamma)(\frac{p_{f,t}}{p_t})^{\frac{1}{\rho-1}} \tag{5.16}$$

where p_t is the aggregated price index for the consumption bundle of home and foreign good.

5.2.2 Foreign Country

The representative consumer of the foreign country is endowed with y_t^* units of foreign good in each period. They trade with the home country for home industrial good, and also issues debt denominated in their own good. With a potentially different discount factor, they solve

$$\max E\sum_{t=0}^{\infty}\beta_f^t \ln c_t^*$$

subject to

$$c_{f,t}^* + \frac{1}{p_{f,t}}c_{h,t}^* + b_t^* \le y_t^* + R_{t-1}^*b_{t-1}^* \tag{5.17}$$

where everything is in the foreign good. The consumption bundle is similar to that of home, with potentially different weight distribution between home and foreign good

$$c_t^* = [\gamma^{*1-\rho} c_{f,t}^{*\rho} + (1-\gamma^*)^{1-\rho} c_{h,t}^{*\rho}]^{\frac{1}{\rho}}$$

The first order conditions to the foreign consumer's problem are

$$\frac{1}{c_t^*} - \nu_t p_t^* = 0 \quad (5.18)$$

$$-\nu_t + \beta_f R_t^* E_t(\nu_{t+1}) = 0 \quad (5.19)$$

Similarly we have analogous solutions for the static cost minimization problem

$$p_t^* = [\gamma^* + (1-\gamma^*) p_{f,t}^{\frac{\rho}{1-\rho}}]^{\frac{\rho-1}{\rho}} \quad (5.20)$$

$$c_{f,t}^* = c_t^* \gamma^* p_t^{*\frac{1}{1-\rho}} \quad (5.21)$$

$$c_{h,t}^* = c_t^* (1-\gamma^*)(p_{f,t} p_t^*)^{\frac{1}{1-\rho}} \quad (5.22)$$

where p_t^* represents the amount of foreign good that one unit of aggregated foreign consumption good is worth.

5.2.3 Equilibrium

Assuming the home country has a population normalized to size 1 and the foreign country has a population of size N, we have the following bonds and goods market clearing conditions

$$b_t + N B_t^* = 0 \quad (5.23)$$

$$c_{h,t} + N c_{h,t}^* + i_{a,t} + i_{m,t} + \frac{\Psi}{2}(b_t - \bar{b})^2 p_{f,t} = A_{m,t} k_{m,t}^{\alpha^m} l_{m,t}^{1-\alpha^m} {}^{1} \quad (5.24)$$

[1] All home country investments and the portfolio adjustmet cost come from domestically produced industrial good.

Also for the home country cross-sector relative price, by equating marginal utility of consumption good and food, and adjusting for price

$$\frac{MU(c_t)}{p_t} = \frac{MU(a_t)}{p_{a,t}}$$

The relative price of food in terms of home industrial good can be expressed as

$$p_{a,t} = \frac{\kappa}{a_t - \overline{a}} \frac{p_t}{c_t^{-\sigma}} \tag{5.25}$$

This is the terms of trade formulated in the famed Balassa–Samuelson hypothesis. To see this, imagine instead of a Yoeman farmer economy, we have 2 firms, one industrial and the other agricultural, each hiring workers in a competitive labor market. Because of wage equality between sectors in the labor market, we would have equal marginal product of labor adjusted for sectoral prices

$$MPL_{a,t} * p_{a,t} = MPL_{m,t} * 1 \tag{5.26}$$

which gives

$$p_{a,t} = \frac{MPL_{m,t}}{MPL_{a,t}} = \frac{(1-\alpha^m)A_{m,t}k_{m,t}^{\alpha^m} l_{m,t}^{-\alpha^m}}{(1-\alpha^a-\omega)A_{a,t}k_{a,t}^{\alpha^a} {l_{a,t}}^{-\alpha^a-\omega} s^{\omega}} \tag{5.27}$$

which results in the same expression as in (25) because of the optimal labor allocation condition (12).

We also conveniently define the sum of industrial good and agricultural good as the total output of home country in terms of home industrial good

$$GDP_t = y_t + a_t p_{a,t}$$

The equilibrium is the optimal paths of $\{c_t, l_{m,t}, l_{a,t}, i_{m,t}, i_{a,t}, k_{m,t+1}, k_{a,t+1}, y_t, a_t, b_t, c_{h,t}, c_{f,t}, c_t^*, b_t^*, c_{f,t}^*, c_{h,t}^*, p_{f,t}, p_t, p_t^*, R_t^*, \lambda_t, \eta_{m,t}, \eta_{a,t}, \nu_t, p_{a,t}\}_{t=0}^{\infty}$ and equations (1) to (25).

5.2.4 Parametric Choices

As explained in previous chapters, we shy away from performing parametric calibrations based on data, instead taking values from existing literature and applying common economic knowledge. The common parameters this chapter shares with previous chapters retain the same values. Unlike chapter 4 which treats every period as a year, we make the timing here quarterly, and take depreciation rates and coefficient on the capital adjustment term from Canzoneri et al (2012)[4].

As noted in Herrendorf et al (2013)[14], it's difficult to make multi-sector models compatible with balanced growth path (BGP), so we follow Schmitt-Grohé and Uribe (2003)[28] and make σ to be 2.[2] The value of Ψ is also taken from Schmitt-Grohé and Uribe (2003)[28]. $\bar{b}$ is set such that the bond to output ratio in the steady state is close to 0.5, reflecting the fact that in recent years China's bond to GDP ratio has been approaching one half.[3]. The weight of land in agricultural production ω is set to be one third, with land S normalized to being always 1. κ is given the lowest computationally viable value that does not disrupt the desirable results of the model.

We want to incorporate food into utility such that any extra food consumption beyond subsistence generates very little marginal utility, so we make κ relatively small.

The list of parameter values are

A_m, A_a	α^m, α^a	β_h, β_f	$\bar{a}, S$	φ^m, φ^a	δ^m, δ^a	σ	ω	$\bar{b}$
1	1/3	0.99	1	4.5	0.025	2	1/3	1.8

ρ	γ	γ^*	Ψ	κ	N	y^*
$1-1/0.9$	0.85	0.95	0.00074	0.11	4.2	1

[2] Allowing BGP usually requires $\sigma = 1$.

[3] This refers to China's holding of foreign asset, not the net foreign asset, although these two coincide in our model.

5.3 Stochastic Impulse Response

Before going into deterministic dynamics, we take a look at the stochastic impulse responses. Stochastic shocks are applied to home total factor of productivity (TFP) in both sectors $A_{m,t}$ and $A_{a,t}$, foreign endowment y_t^*, and foreign good's weight in the foreign consumption bundle γ_t^*. The stochasticity is defined to follow

$$\begin{aligned}
\ln A_{m,t} - \ln A_m &= 0.95(\ln A_{m,t-1} - \ln A_m) + \epsilon_t^{A_m}, \quad \epsilon_t^{A_m} \sim iid\ N(0, \sigma_{A_m}^2) \\
\ln A_{a,t} - \ln A_a &= 0.95(\ln A_{a,t-1} - \ln A_a) + \epsilon_t^{A_a}, \quad \epsilon_t^{A_a} \sim iid\ N(0, \sigma_{A_a}^2) \\
\ln y_t^* - \ln y^* &= 0.95(\ln y_{t-1}^* - \ln y^*) + \epsilon_t^{y}, \quad \epsilon_t^{y} \sim iid\ N(0, \sigma_{y}^2) \\
\ln \gamma_t^* - \ln \gamma^* &= 0.95(\ln \gamma_{t-1}^* - \ln \gamma^*) - \epsilon_t^{\gamma}, \quad \epsilon_t^{\gamma} \sim iid\ N(0, \sigma_{\gamma}^2)
\end{aligned}$$

Size of the shocks are set as $\sigma_{Am} = \sigma_{Aa} = \sigma_y = \sigma_\gamma = 0.01$. The negative sign in front of the error term in the evolution of γ_t^* indicates it's a negative shock[4]. The impulse response to the 4 shocks are plotted for 30 periods.

Figure 1 shows the case when the industrial sector TFP gets a boost. To capture this sectoral increase in productivity consumers shift more labor into the industrial production, producing more industrial good and therefore more resources for consumption, bond holding and investment, which in turn boosts the capital stock in both sectors. Because of the abundance of home industrial good, relative prices of both food and foreign good see an appreciation.

Figure 2 shows the case of an increase in the agricultural sector TFP. Because extra food consumption provides very little marginal utility beyond the subsistence level, increase in agricultural TFP actually frees labor from the agricultural sector and moves them into the industrial sector. For the same reason, investment and capital stock in the agricultural sector also decreases, as long as the subsistence level of food

[4] $1 - \gamma_t^*$ is a proxy for foreign demand for home good, as motivated in previous chapters.

production is still met. All this gives a boost to the industrial sector and increases consumption, industrial capital and bond holding, which is interestingly similar to the effect of an increase in industrial sector TFP. Food sees a depreciation against industrial good because of the increase in TFP, while the boost to the industrial sector causes term of trade depreciation for the home good.

Figure 3 shows the case of an increase to foreign endowment. As a result of temporary abundance of foreign good, the foreign country is able to pay back some of their debt, and the real appreciation of home good gives a boost to both sectors. Home industrial sector production grows, increasing consumption and capital in both sectors through investment. Meanwhile, the option to rely on more foreign good for consumption decreases demand for home industrial good, and the resulted food appreciation briefly shifts labor from the agricultural to the industrial sector.

Figure 4 shows the case of an increase to foreign demand for home good. The resulted home good appreciation allows home consumers to consume more and forces foreign consumers to borrow for consumption, hence more bonds sold to home consumers. The increase in foreign demand triggers labor to move from the agricultural to the industrial sector. Agents can even slack off on investments in both sectors and still keep up with the demand. Higher foreign demand also makes home industrial good more coveted domestically, causing depreciation to the relative price of food[5].

5.4 Simple Transitional Dynamics

Starting from this point on we look at deterministic dynamics. As stated in the introduction, our ultimate goal is to trace out development paths from our model

[5] To see this, in (25) the aggregate price for home consumption good p_t is in the expression for $p_{a,t}$, which itself is increasing in $p_{f,t}$.

that could resemble real life observations. In this section we study how the system transitions to the steady state, starting with low levels of initial wealth. We focus on looking at the time paths of our variables of interest, and compare their trends to the historical trends observed from developing countries. We call it "simple' dynamics because it does not involve any changes to exogenous factors.

For our variables of interest, we define investment and saving rates as

$$\begin{aligned} INV\%_t &= \frac{i_t}{GDP_t} \\ SAV\%_t &= \frac{y_t - p_t c_t}{GDP_t} \end{aligned}$$

Also define import and export as shares of home output and trade surplus as

$$\begin{aligned} IMPSH_t &= p_{f,t} c_{f,t} / GDP_t \\ EXPSH_t &= N c^*_{h,t} / GDP_t \\ SURPLUS_t &= N c^*_{h,t} - p_{f,t} c_{f,t} \end{aligned}$$

and by using budget constraint (1) and clearing condition (24), write current account in home good as the sum of trade surplus and interest payment on bonds holding

$$\begin{aligned} CA_t &= p_{f,t} b_t - p_{f,t} b_{t-1} \\ &= N c^*_{h,t} - c_{f,t} p_{f,t} + (R^*_{t-1} - 1) b_{t-1} p_{f,t} \end{aligned}$$

The weight of the industrial sector in the whole economy is the percentage of industrial output in total output

$$WEIGHT_{m,t} = y_t / GDP_t$$

Figure 5 shows a case of such simple dynamics, starting with just above one third of the steady state values of capital in each sector and zero bond holding. As the capital stock builds up, we get steady growth of production in both sectors and in goods consumption through factor accumulation. Home industrial good naturally sees a terms of trade depreciation as productivity grows. The agricultural sector expands by very little beyond subsistence, while food price decreases monotonically.

Import share increases before flattening out, while export share slightly tapers off after steady initial increase. Investment and saving rates exhibit near identical pattern, increasing sharply but only for a very short period of time before decreasing considerably over the long haul. This is due to the urgent needs for consumption and food in the beginning. Agents have to start with low saving and investment rates early on as they allocate more resources to consumption and food subsistence for survival. They increase their investment and saving rates only as they become richer. Soon, however, decreasing marginal product kicks in when they have a considerable amount of wealth and the investment and saving rates drop. Although the initial increases in saving and investment rates don't last as long as data suggests, this is already a considerable "improvement" from the simple dynamics in chapters 3 and 4, in which we got at least half of these 4 variables decreasing from the beginning. In reality, as shown in chapter 1, all 4 variables exhibit steadily increasing trends during the development stages.

For the investment rate in particular, we can divide total investment into 2 sectoral investments and plot the investment rates of each sector. The industrial sector investment rate increases sharply before slightly falling to the steady state, while the agricultural investment rate strictly decreases over time. This is also because of the way utility is modeled with a food subsistence. Given that food production only needs to be just above the subsistence level and consumers want more consumption as long as they are not starving, investing into the agricultural sector becomes less rewarding over time relative to investing into the industrial sector. As capital replaces labor in the agricultural sector, the investment becomes less important and its rate decreases.

In the meantime, as consumers accumulate wealth and get out of poverty, they can afford to commit more resources to investing into the industrial sector, which allows them to consume and export more and further increase the capital stock in both sectors.

The current account, however, experiences initially a deficit before seeing a surplus, which is contrary to real life observations of persistent current account surpluses in developing countries. The models in previous chapters do not have such drawbacks when applying simple dynamics. An easily suspected reason for the cause of initial current account deficit is the need for food subsistence and investment in the beginning. When the poor home country needs enough food for survival and at the same time a decent consumption and investment for capital accumulation in both sectors, it has to borrow from the foreign country to meet the large need for resources otherwise beyond its own means. In the previous 1-sector models, there is no need for food subsistence that must be met at all times and would otherwise cause death (utility of negative infinity), and there was only 1 sector that needs investment, so borrowing from the foreign country was not a necessity.

Another interesting observation, as is the case with any multi-sector model with labor, is the transfer of resources across sectors, particularly the migration of labor. The inclusion of food consumption in the utility, an inherent feature of this model, dictates that extra food beyond the subsistence level presents limited value. This should come conceivable, as humans only need sufficient amounts of food and nutrition, but much more than that does not bring extra benefit and even causes health problems. As a result of this, even though capital in both sectors grow in monotonicity, agents would over time shift labor, a finite resource which does not grow like capital, out of agriculture and into the industrial sector.

The plots in Figure 5 tells a story of industrialization. When the economy starts out poor, food is scarce and critical to survival, and therefore expensive, as agents devote the majority of their resources into agricultural production to feed themselves. As industrial production boosts the capital stock through investment over time, agents have the freedom to essentially replace manual labor with machines in the crops, and move into the factories to produce non-agricultural goods. The subsistence level of food effectively puts a "cap" on the need for food. As labor in the industrial sector increases, the marginal product of labor decreases in the industrial sector and causes food depreciation, as indicated by the Balassa-Samuelson effect expressed in (26). This characterizes the evolution of an industrializing economy, signaled by the long term increase in the weight of industrial sector in the whole economy.

5.5 Transitional Dynamics with Structural Changes

Although successfully characterizing the transitional process of industrialization, the evolution paths produced by the 2-sector model without any exogenous change don't exactly match the observed trends in saving and investment rates and current account balance. As we did in previous chapters, we now implement two exogenous changes to perform perfect foresight dynamics and study the results. Similar to chapters 3 and 4, we deploy linear paths for evolution of exogenous variables in order to highlight a "catch-up" stage during development.

5.5.1 Increase in Technology

Since we have two sectors with distinct TFPs, we face a choice of which sectoral TFP to exogenously increase or both. For the purpose of studying the industrialization process, we want to highlight the increase in technology in the industrial sector in an economy. Therefore, for our experiment we only increase the TFP in the industrial sector and keep the TFP in the agricultural sector constant throughout.

The choice to increase only industrial technology but not agricultural technology is a tricky but strategic one. In reality, it's conceivable that the industrial sector growth benefits substantially more from technological advancement than agriculture. For our experiment, we conveniently assume the agricultural TFP has not improved over time,[6] and the increase in the food production comes purely as a result of more capital stock utilized. Meanwhile, the technology in the industrial sector improves through learning the know-how from developed countries.[7] We are not dismissing the possibility of agricultural technological growth, but focusing on the increase in industrial sector technology allows us to better analyze the process of industrialization. It also turns out in the experiments not shown here that growth paths generally behave similarly with or without implementing increases in agricultural technology.

For the exercise of perfect foresight dynamics, we start with low levels of bonds and capital in both sectors, as well as a low home TFP in the industrial sector $A_{m,t}$. Right from the beginning, we impose exogenous increases in $A_{m,t}$ in an arithmetic sequence for the first 120 periods or 30 years, which we call the catch-up stage, and keep it constant at the ending high value thereafter. The consumers fully anticipate the exact path of TFP, and optimize accordingly. As is the case with previous chapters, there is a kink point at the end of the catch-up stage, after which variables converge to the steady state corresponding to the high $A_{m,t}$ at the end of the catch-up stage.

[6]If thinking in terms of balanced growth path, there is no gap in agricultural technology between developing and developed countries.

[7]If thinking in terms of balanced growth path, the developing country closes the gap in industrial technology between itself and developed countries during the catch-up phase.

Figure 6 shows an example of dynamics with increase in industrial sector TFP, with identical initial resources as in the case in Figure 5. The implementation of the catch-up seems to have a significant impact on the trends of several variables, as compared with results of simple dynamics. Now investment and saving rates both increase in monotone for the entire catch-up stage, which is consistent with real life observations. Labor persistently flows from the agricultural to the industrial sector, although at a much faster pace during catch-up than after. A similar pattern is seen on the weight of the industrial sector, which implies the boom to the industrial sector takes place primarily during the catch-up as a result of TFP increase.

The price of food displays an interestingly different trend from the simple dynamics. It decreases sharply for a short period of time before rising back up by even more until the end of catch-up. A conceivable explanation for this has to do with the relative importance and scarcity of food. When consumers start out poor, they have more desperate demand for food than consumption, therefore making food expensive in the beginning. As the agricultural sector stocks up on capital, human labor is gradually replaced by machines for farming. Labor moves out of the agricultural into the industrial sector as it becomes more attractive to work in the industrial sector, thus increasing the marginal product of labor in the agricultural sector and keeping wages equalized between sectors. This is again because of the rapidly decreasing marginal utility of food consumption beyond subsistence. However, at some point, the food production which has virtually stopped growing, coupled with still rapidly increasing industrial production, makes the food relatively scare and its price go up again. This episode of food price appreciation is mainly due to the boom of the industrial sector powered by the increase in the industrial sector TFP and factor accumulation. In the labor market, the Balassa-Samuelson effects takes place in that the marginal product of labor in the industrial sector continues growing alongside capital accumulation and TFP increase and causes food price to increase as well, as shown in (26).

Other variables behave similarly to the case of simple dynamics. Industrial investment rate increases while agricultural investment rate decreases, for the same aforementioned reasons, exacerbated by the increase in industrial sector TFP. Home good sees terms of trade depreciation as its industrial sector expands. Current account still first sees a deficit for a short time before turning into a surplus, for the same reason explained in the previous section, namely the severe lack of resources and heavy need for investment and food subsistence in the early stages.

5.5.2 Increase in Foreign Demand for Home Good

We now look at the case of exogenously increasing foreign demand for home consumption goods. As explained in previous chapters, we take the weight of foreign good in the foreign utility γ_t^* to be a proxy for foreign demand.[8] Similar to the case of exogenously increasing home TFP, we start with low levels of capital and bonds, and a high value of γ_0^*, and decrease it in equal successions for the first 120 periods until reaching the parametric value specified in section 2, and permanently staying there. The consumers fully anticipate the increase in foreign demand and optimize accordingly.

Figure 7 shows an example of dynamics with increase in foreign demand, with identical initial resources as in the cases in Figure 5 and 6. As is the case in previous cases, consumption and capital in both sectors increase. Labor migrates from the agricultural to the industrial sector over time, the weight of which also increases as

[8]Acutally, foreign demand is proxied by the weight of home good in the foreign utility, $1 - \gamma_t^*$,

industrialization deepens. Current account, like before, bears deficits before experiencing sustained surpluses.

Import share sees steady increase throughout. Saving rate and export share both increase during catch-up and drops back down afterwards. Investment rates follows an interestingly sophisticated path. The total investment rate first sees a drastic spike, immediately ensued by an equally rapid fall until the end of catch-up, and then flattens toward the steady state. Upon closer examination of each sector, the agricultural investment rate still follows a decreasing trend at least during catch-up, but the industrial investment rate sees a sharp increase first before a decrease of lesser magnitude, and finally flattens post catch-up. Looking at all three investment rates, it's clear that during the first half of the catch-up, the investment picture was dominated by the heavy need for industrial sector investment to meet increasing foreign demand, as the decreasing agricultural investment rate was more than negated by the increasing industrial investment rate. For the remainder of the catch-up, both sectoral investment rates decrease, and so does the total investment rate. During this stretch, unlike in the case of productivity increase, increase in foreign demand does not make industrial sector investment more rewarding, thus the effect of diminishing marginal product makes investment rates decrease.

Home terms of trade depreciates for just under 100 periods before appreciating for the long term. This indicates that in the beginning, the effect of home industrial sector expansion dominates the effect of increase in foreign demand, causing home industrial good to become cheaper. In the long run, however, as the home industrial sector expansion slows down, as can be seen after approximately 100 periods in the industrial production plot, the heightened level of foreign demand finally kicks in and raises the price of home industrial good and therefore terms of trade.

The price of the agricultural good first drops shortly before becoming flat. It starts out high for the same reason of food subsistence as before. It depreciates alongside the boom to the industrial sector and the transfer of labor from the agricultural to the industrial sector, like in the case of simple dynamics. The increase of foreign demand as an extra incentive for labor migration causes the marginal product of labor to further decrease in the industrial sector and increase in the agricultural sector, making the agricultural good less valuable against the industrial good, as suggested by the Balassa-Samuelson effect in (27). The increase in foreign demand makes the home industrial good so coveted that we don't see the agricultural good appreciate back up even in later stages when it's relative scarce relative to the industrial good, unlike in the case of increasing industrial sector TFP.

5.5.3 Simultaneous Increase in Technology and Foreign Demand

Compared to the previous chapters, by using a 2-sector open economy model, we have so far been producing considerably better results with simple dynamics and by implementing one single exogenous change at a time. As done in the previous chapters, the final block of our computational exercises is to implement simultaneously the two exogenous changes studied above.

Figure 8 shows such a case with precisely the paths taken on in the above subsections where each change was implemented alone. It looks highly similar to the case of implementing the increase in home industrial sector TFP alone, with smoothly increasing industrial sector investment rate and a "V" shaped pattern for food price during catch-up, suggesting the effect of increase in foreign demand is largely dominated by the effect of increase in home industrial TFP. In previous chapters when implementing a single exogenous change produced seemingly "tilted" results, we tried to find a good mix of magnitude of changes such that the outcome is the "right" blend of the results of the single-change cases, and had some success. Even though most of the things we are interested in seem to display a reasonably good fit already, we try decreasing the size of home industrial TFP increase to see if we get different results.

Figure 9 shows a case similar to Figure 8 except that the starting home industrial TFP $A_{m,0}$ is 0.85 instead of 0.7, so the effect of TFP increase on dynamics is hopefully toned down to some degree. The results seem to combine the trends of the dynamics when implementing only a single change. The investment rate first increases sharply for a short time before decreasing only moderately during catch-up. Food price slightly falls first before climbing back up a little and flattening off. Both of these two trends are qualitatively an average of the trends produced in the single-change cases. In both plots, the general trends remain from the single-change cases but the patterns are much less pronounced.

As discussed in previous chapters, it must be pointed out that the "true" paths can only be pinpointed by calibrating from data. In our study, we implement arbitrary paths of exogenous variables to try to produce trends that match reality. While not claiming that the best match we can find must be associated with the paths of exogenous variables closest to their real life evolutions, we do present the prospect that when equipped with data and calibrated paths, it is possible to produce growth paths of the variables we are interested in that are a decent match for reality.

5.6 Further Discussions

In this chapter we expanded our open economy model into one with 2 sectors in the home country. By doing so not only are we still able to study the time trends of our usual variables of interest, we also manage to explore the process of industrialization, and examine how the Balassa–Samuelson effect takes place through the evolution of food prices. It's worth comparing the results of different dynamics experiments to those produced in previous chapters and discussing what's behind the differences.

This model turns out to bear heavily the symptom of the Lucas paradox. As stated in chapter 3, we mechanically choose to start with an initial bond level below the steady state, and this conveniently gives us current account surpluses in most cases studied in the previous chapters. In all the cases here, however, even when starting with zero bond holding, the home country would run current account deficit for a period of time in the beginning before turning it into a surplus by accumulating foreign assets. Why is this?

In this 2-sector model with a subsistence level of food consumption, when the economy starts out poor, the need for resources by various economic activities is especially desperate. The consumers are forced to devote the majority of their capital and labor to the agricultural sector in order to satisfy the food subsistence. The resulted scant industrial good production is not nearly adequate to be divided across home consumption, export and investments. In an open economy, however, the home country can resort to borrowing from the outside when it's in such a poor state. In our model the foreign good can be consumed by home consumers, so by borrowing from the foreign country, the home country can keep up with consumption without sacrificing on investment, which would boost the growth in the industrial sector, and in turn increase investment into both sectors and speed up the labor migration hence the industrialization process. In previous 1-sector models without a food subsistence in utility, there was no economic activity that must always occupy a certain amount of resources to produce no less than a specific amount of output, and resources were not divided between multiple sectors. Therefore the need for borrowing was less severe, and we rarely get a current account deficit when starting with low initial bond holding.

As noted in the above sections, in all the transitional dynamics with or without exogenous changes, we always have import and export shares increase for a number of periods first. The saving rate also increases during the catch-up stage when implementing one or more exogenous changes. The investment rate would increase during the entire catch-up stage only when implementing increases in home industrial sector TFP, otherwise increasing only very briefly before decreasing. All these are significant improvements over the results in previous chapters, where it's often difficult to get all 4 variables increasing in the beginning without implementing both exogenous changes.

The multi-sector structure of the model is also a major reason for the initial increase in saving and investment rates. Because of the severe lack of resources in the beginning, the majority of which consumers have to utilize for food production to survive, the need for export and consumption dictates that the consumers can't afford to allocate much of their limited industrial production to saving and investment early on. Only after some time of development with stocking up on capital in both sectors and moving labor from the agricultural to the industrial sector, can consumers have plenty of wealth, such that beyond food subsistence and a decent amount of consumption they can still allocate a decent amount of resources into saving and investment. The increases in saving and investment rates are for the same fundamental reason as the initial current account deficit, namely the severe need for resources in the beginning, whose effect overwhelms the effect of diminishing marginal product that causes decreases in saving and investment rates in most 1-sector models.

To sum things up, the two key features produced by this model - increasing saving and investment rates and initial current account deficits, are both deeply rooted in the multi-sector nature of the model.

5.7 Summary of Results

In this chapter, we build an open economy model with 2 sectors, one agricultural and one industrial, to study the growth dynamics of a developing economy and also the process of industrialization. In all the dynamics we explored with or without exogenous changes, we always have the current account bearing a deficit first before turning into a surplus. In all cases we also have all of investment and saving rates, import and export shares all increasing at least for a period of time at first. We attribute these results to the multi-sector feature of the model. The desperate need for a minimum amount of food, coupled with the various needs for resources make home consumers devote less of their wealth to saving and investment and also borrow from the outside when they are poor in the beginning. Along the way, we also discuss the evolution of food price with regard to the Balassa-Samuelson hypothesis.

Compared to 1-sector models in previous chapters, we see only limited improvements in results when moving from simple dynamics to implementing one or more exogenous changes. In fact, the basic dynamics is already doing a decent job in producing trends in our variables of interest except current account balance. Perfect foresight dynamics by implementing exogenous changes does produce more prolonged increases in investment and saving rates.

Figure 5.1: Impulses Responses to an Increase in $A_{m,t}$

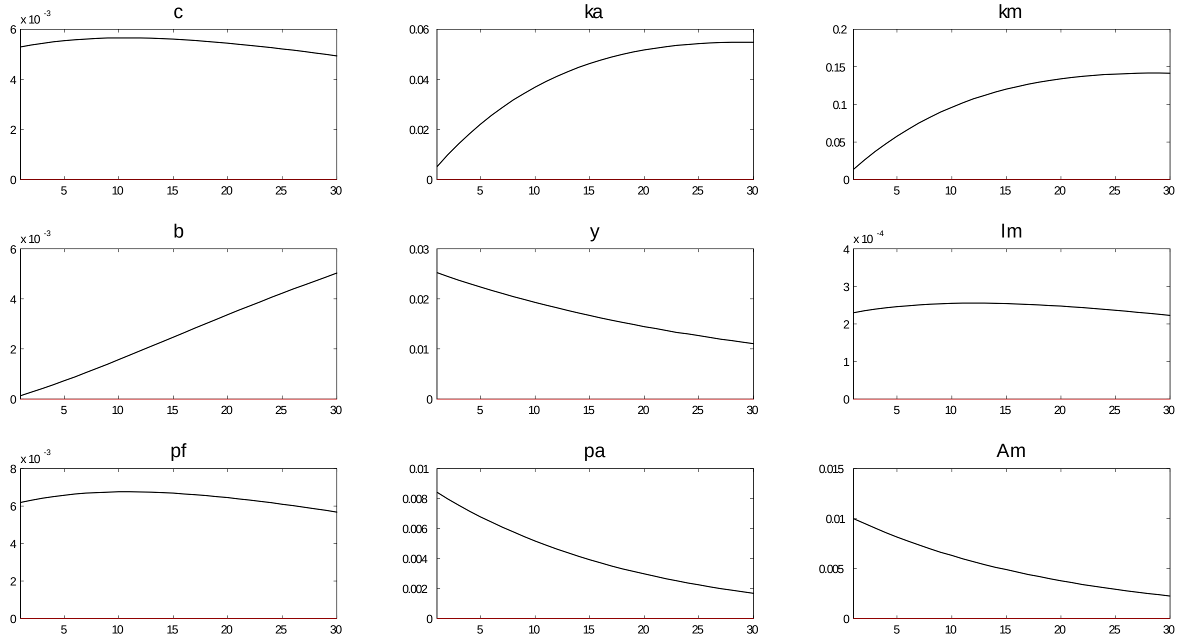


Figure 5.2: Impulses Responses to an Increase in $A_{a,t}$

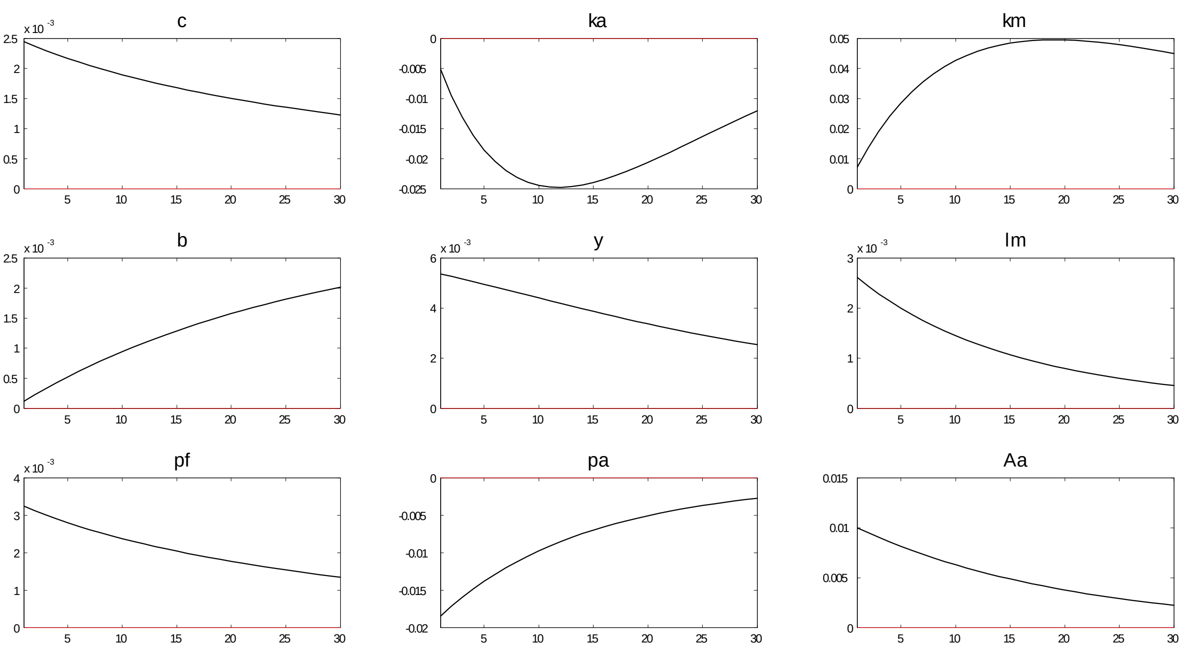


Figure 5.3: Impulses Responses to an Increase in y_t^*

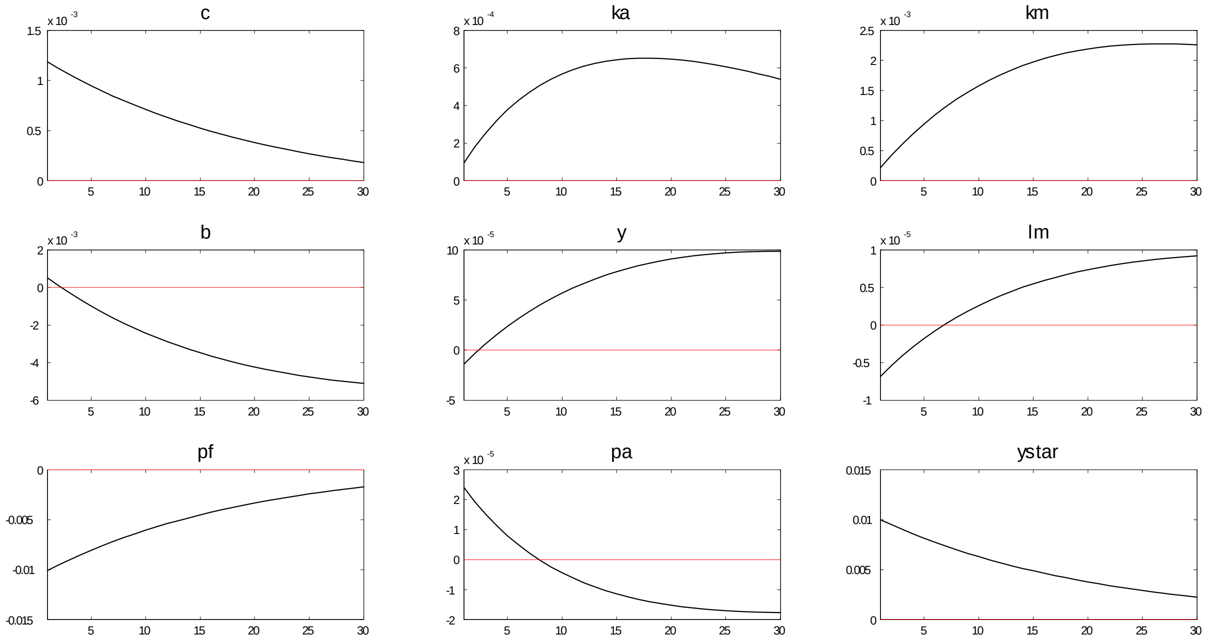


Figure 5.4: Impulses Responses to a Decrease in γ_t^*

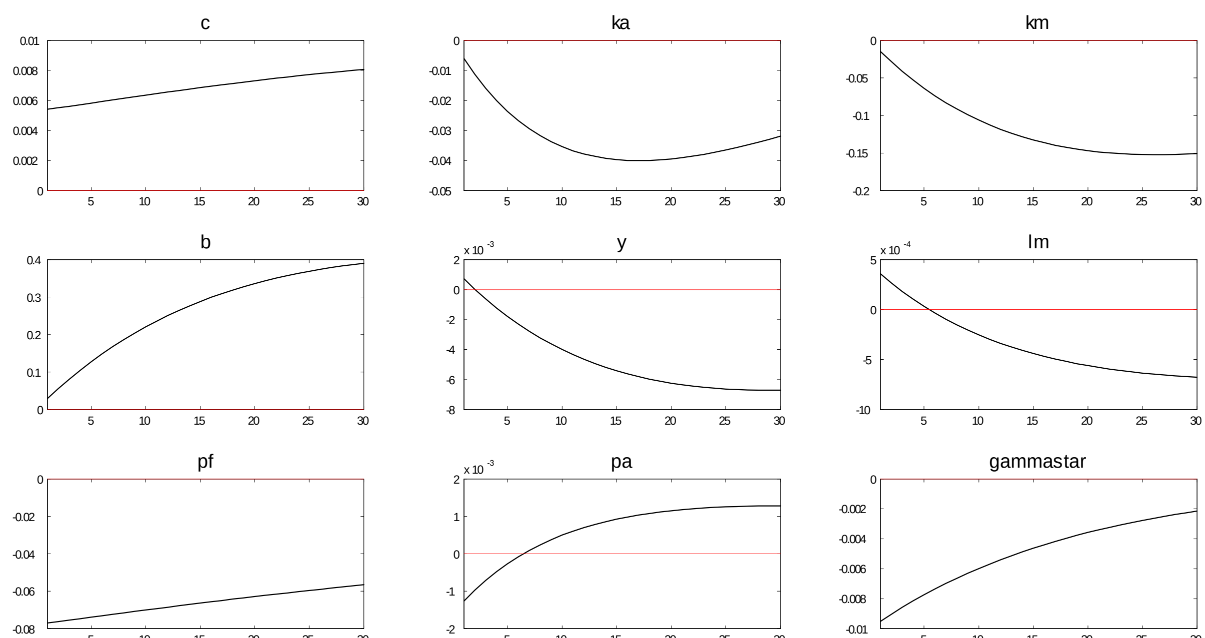


Figure 5.5: Simple Transitional Dynamics, $k_{a,0} = 4, k_{m,0} = 8, b_0 = 0$

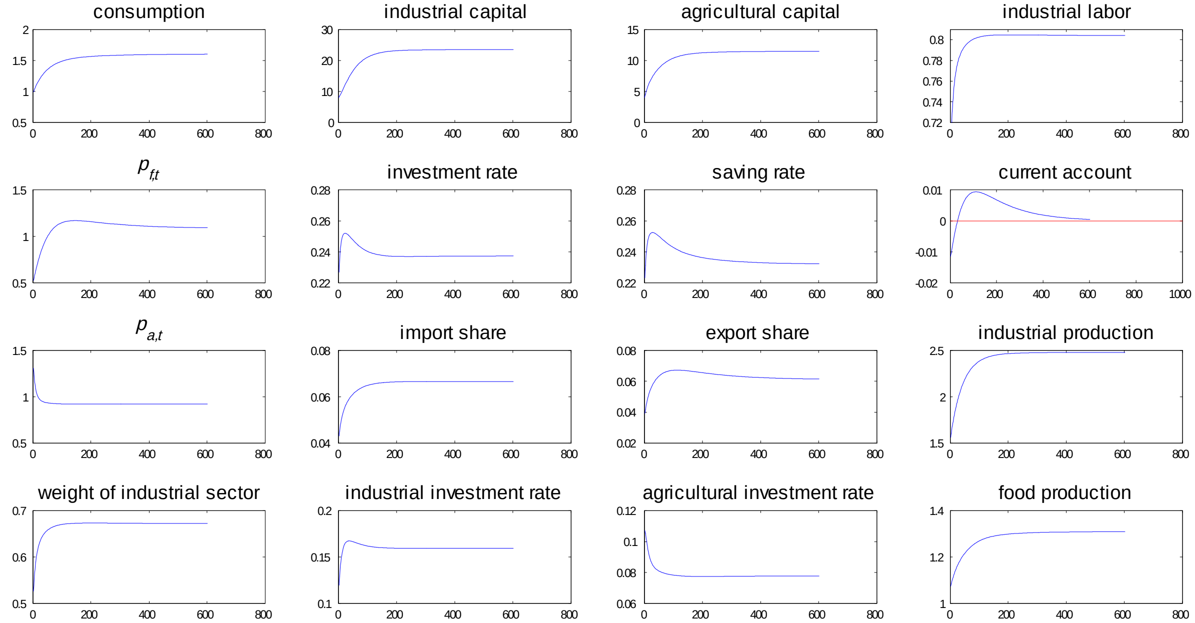


Figure 5.6: Dynamics with Increase in $A_{m,t}$, $k_{a,0} = 4, k_{m,0} = 8, b_0 = 0$

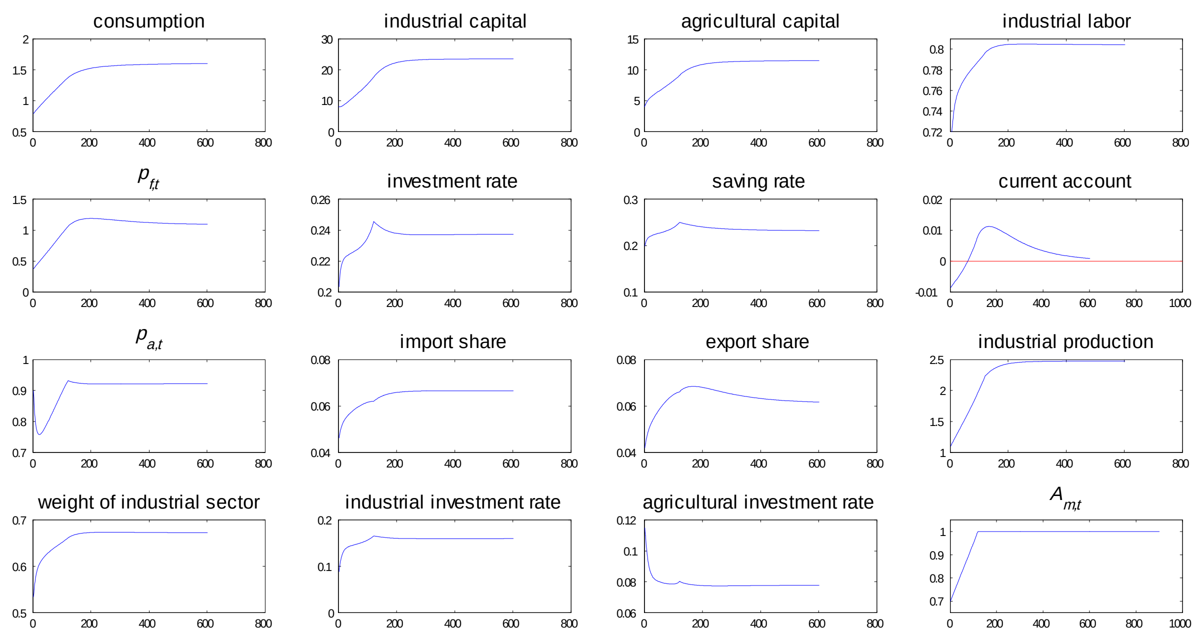


Figure 5.7: Dynamics with Decrease in γ_t^*, $k_{a,0} = 4, k_{m,0} = 8, b_0 = 0$

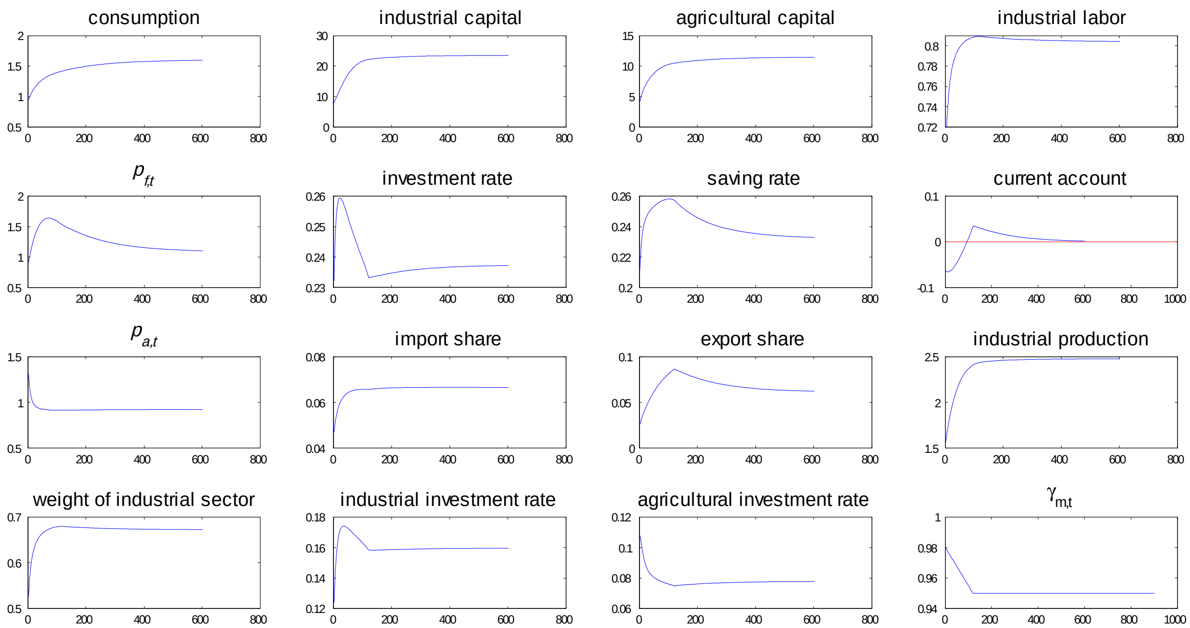


Figure 5.8: Dynamics with Both Changes, $A_{m,0} = 0.7, \gamma_0^* = 0.98$

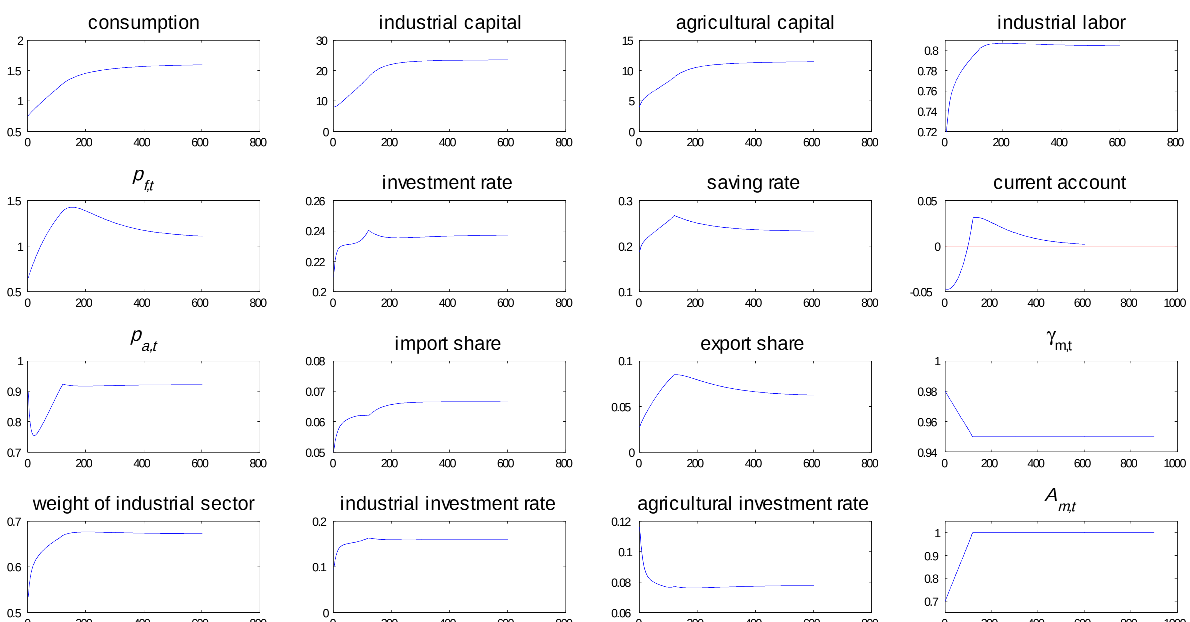
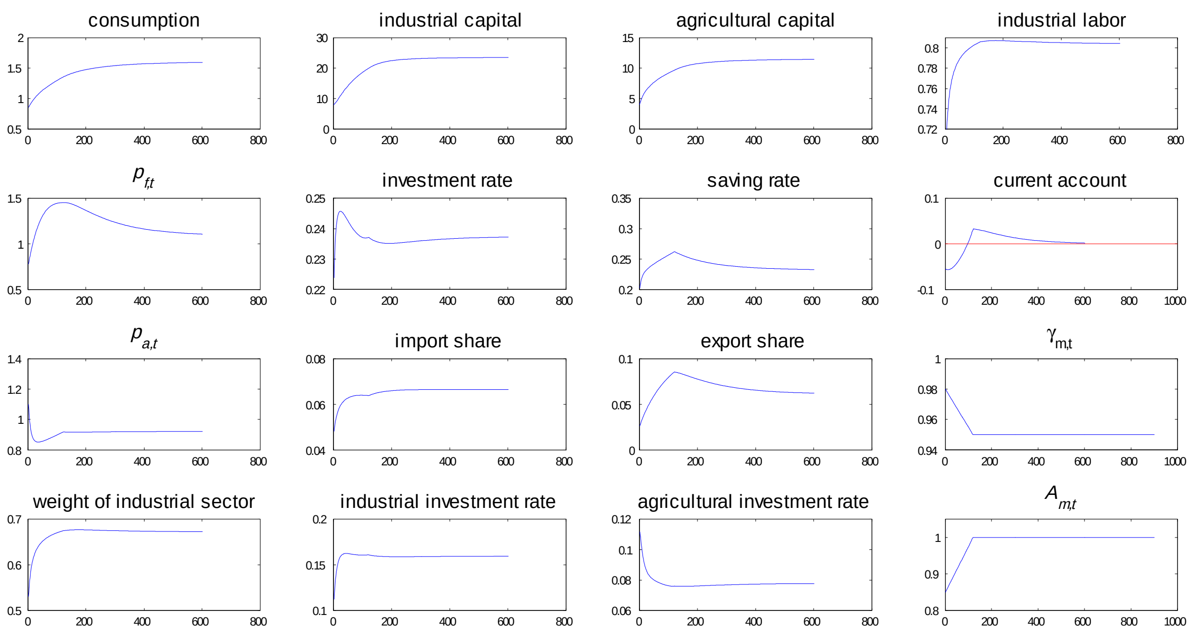


Figure 5.9: Dynamics with Both Changes, $A_{m,0} = 0.85, \gamma_0^* = 0.98$



Chapter 6

Conclusions and Extensions

Traditional neoclassical theory has not always succeeded in explaining all the observed trends of developing countries, which is to a large extent a result of diminishing marginal product embedded in most models. Previous research often had difficulty generating increasing saving and investment rates that are widely documented on developing countries. Also among the theoretical failures is the counterfactual prediction that capital should flow from rich to poor countries in the form of foreign debt, know as the Lucas paradox. We present historical evidence on saving and investment rates, as well as import and export shares from former and current Asian developing economies, hoping to create models that could capture the rising patterns in them as well as producing a current account surplus during a defined "development phase".

We build and study a closed economy model and 3 open economy models. For all of them, we perform a consistent series of exercises. We first check the stochastic impulse responses by imposing shocks to the exogenous variables that we believe may have changed for developing economies in real life, and provide intuitions for them. Then we turn to deterministic dynamics, starting by examining the "simple" dynamics, where the system starts with poor initial resources and converges to the steady state while no change to exogenous factors takes place. Lastly, we perform perfect foresight dynamics by imposing exogenous increases to productivity and/or foreign demand, where agents make optimizing decisions with full knowledge of the paths of exogenous variables. When possible, we make adjustments to make our models compatible with balanced growth paths, which is more realistic as it allows applicable per capita variables to grow at a constant rate of world technology progress in the steady state.

6.1 Summary of Results

To see how a plain model with no gimmicks may fail our purpose, we start with a simple closed economy model. In this baseline model there is no trade or foreign asset and saving is equal to investment. Besides these limitations, the baseline model fails to produce sustained increase in saving/investment rate. Also, imposing increase to technology seems to only complicate rather than improve the results.

Noting that the Asian economies all benefit substantially from trade during development, we build open economy models that describes the growth of a developing country in the presence of a developed country. For our first open economy model, we use portfolio adjustment cost to close the model. This model generally gives current account surpluses. Implementing exogenous changes does a much better job than simple dynamics. When imposing both exogenous changes, we were able to get increases in everything but import share, and also a short-lived current account deficit in the beginning.

For the next modelling attempt, we introduce the mechanism of endogenous discounting into both closed and open economy models. Intuitively, having the discount factor inversely related to consumption and leisure is conducive to generating decreasing trends in saving and investment rates, which is obtained in the closed economy set-up. In the open economy case, similar to the previous model, using perfect foresight dynamics with both changes gives the best fit for data, generating increases in saving and investment rates as well as import and export shares, with the only flaw being an initial current account deficit.

Finally, we further stretch the model to include 2 sectors in the home country, one non-traded agricultural sector and one traded industrial sector. This model allows us to also study the process of industrialization, as labor and other resources shift from the agricultural to the industrial sector over time. This model can hardly generate initial current account surplus, because the severe needs for consumption, food and investment in the beginning force home country to borrow. Otherwise this model does a considerably finer job in other categories compared to 1-sector models. Import and export shares, investment and saving rates always increase at least for a short period of time early on, with or without exogenous changes. We posit that this is also due to the multi-sector nature and food subsistence of the model, as the initial need to avoid starvation and maintain a decent level of consumption prevents consumers from devoting much of their resources to saving and investment early on. Unlike in 1-sector models, the implementation of exogenous changes does not substantially improve the results.

Our models have varying degree of success in resolving the Lucas paradox. In the 1-sector models, we first impose the steady state level of bonds and mechanically generate bonds accumulation, then used the mechanism of endogenous discounting to induce current account surpluses. The 2-sector model always results in current account deficit and offers no remedy to the Lucas paradox.

Comparing all our attempted models, open economy models allows us to distinguish between saving and investment, and also look at more variables of a trade-driven developing economy. Among all open economy models, the 2-sector model proves to be more robust and generates more realistic dynamics than the 1-sector models even without exogenous changes.

6.2 Extensions

This work so far has been largely qualitative, as we only look at the trends but not the numerical changes in the variables. Should data by available, a natural next step is to study a country's specific case by doing rigorous calibrations on not only the parameters but also the paths of exogenous variables, and compare the resulted paths to the observed data not only in trends by also numerically.

Various computational trials of our models did not fit the data trends perfectly, suggesting we may be missing some key features of developing economies. There may also be changes to exogenous factors other than technology and foreign demand that are shaping growth dynamics we are ignoring.

An interesting modification on the multi-sector model would be to allow the foreign good to not only be consumed by home consumers but also enter the home investment. It's a known fact that a substantial portion of developing countries' imports are investment goods such as machineries. Having this feature would not only make more realistic sense for any modelling purpose, but also contribute to the in-depth exploration of growth dynamics. If imports can go into both consumption and investment, the model will potentially reveal more on the interaction between import and investment, the composition of home and foreign goods in the total investment, and the breakdown between investment and consumption in total imports, all of which are so far little known.

In chapters 3-5, we performed perfect foresight transitional dynamics by implementing a "catch-up" stage of linear increases in technology and foreign demand. The advantage of using equal sizes of increase in each period is that the results exhibit pronounced breaks at the end of catch-up with kink points, allowing us to tell growth stories centered on the concept of a catch-up process. Barring the possibility of calibrating the exact evolution paths of exogenous variables, an alternative is to use non-linear increases, such as the one applied in chapter 2, where increases occur at a decreasing speed, producing no kink points. A projected consequence of this is that the resulted dynamics will not display a clear catch-up period, and the shifts in the trends will be smooth and gradual instead of abrupt. As a result, the interpretations will also be different.

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