THREE ESSAYS ON FISCAL POLICY AND GOVERNMENT PRODUCTION

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Abstract

This research investigates government production function, explores its dynamic properties and studies the effects of fiscal policy implemented through government production channels. The first chapter estimates the production function of the U.S. government. The results indicate that we should be cautious when using a Cobb-Douglas function to represent the government production whenever intermediate goods are included. The analysis also shows the CES function has more superior properties to represent the U.S. government production function based on two new measures derived from exact index theory. The second chapter explores the cyclical movement of government spending components as a result of endogenous responses to exogenous private sector and government sector productivity shocks. This chapter also quantifies the relative contributions of exogenous shocks to the volatility of government spending components. The third chapter argues that fiscal policies that can allocate government inputs and determine the categories of government outputs are able to span the whole range to theoretical results on the responses of private consumption, private output, real wage, and private labor to a government spending shock.

INDEX WORDS: Government, Production Function, Fiscal Policy, Business Cycle

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CHAPTER 1

THE PRODUCTION FUNCTION OF U.S. GOVERNMENT: AN EMPIRICAL

INVESTIGATION

The nature of the government production function is an important question with significant theoretical implications. Given an estimate of government outputs and an assumption on the functional form of the government production function, the parameters of the government production function can be estimated. Since the CES production function is widely used in Macroeconomics, I first estimate the U.S. government production function with three-factor and two-factor CES functional forms for the period 1929-2013. The estimated elasticities of substitution of the three-input CES functions are not significantly different from one. However, the estimates of the two-factor CES function show that the elasticity of substitution between intermediate goods and labor is significantly bigger than one, while the elasticity between intermediate goods and capital is significantly smaller than one. The results indicate that we should be cautious when applying a Cobb-Douglas production function to represent the government production function with intermediate goods. In the process, I deal with issues related to the heteroskedasticity problem, the endogeneity problem and the nonstationarity of the series involved in the estimation. Finally, I use exact index theory to construct two measures to assess the self-consistency of a given specification. I use these measures to compare the CES production function against other linearly homogeneous production functions. I find that representing the U.S. government production function with a CES function results in a balanced, if

not better than other functions, self-consistency performance.

1.1 Introduction

The production of government outputs is important for the U.S. economy and it interacts with the private economy through a wide range of markets, including the labor market and the goods market. Using a functional form that is able to represent the government production function can facilitate our understanding of government production behavior as well as the interactions between the government and private economies.

However, estimating the government production function is difficult, in part because the market value of government outputs is unobservable. Solow [1957] first introduced a concept of using an aggregate input quantity index Q_t to replace the output. It has been employed by Jorgenson and Grilliches [1972], Christensen et al. [1973] and Berndt [1976]. This paper adopts this concept and generates an estimate of government outputs, which is based on the quantity index formula corresponding to the assumed government production function. Given the estimate of government output and an assumption on the functional form of the government production function, the parameters of the government production function can be estimated.

Since the constant elasticity of substitution (CES) production function is widely used in the literature, including by Lucas [1969], Klump and Grandville [2000], Acemoglu [2002], Klump et al. [2007] and others, this paper initially assumes that the U.S. government production function follows a three-factor CES functional form for the period 1929–2013. The estimation results show that most estimates of the elasticity of substitution of the three-factor CES function are not significantly different from

one, which indicate that the U.S. government production function with three inputs can be represented by a Cobb-Douglas function. However, some deviating estimates imply that the same elasticity of substitution among three inputs may be a strong assumption. So I relax the three-factor production function assumption and estimate two-input government CES production functions. Results reveal that we should be cautious when applying a Cobb-Douglas function to represent the government production function with intermediate goods as an input. In fact, the estimate of the elasticity of substitution between intermediate goods and government employment is greater than one, while the estimate of the elasticity between intermediate goods and fixed capital is less than one. In the estimation process, I deal with issues related to the heteroskedasticity problem, the endogeneity problem and the nonstationarity of the series problem.

There is concern that the CES function may mis-specify the U.S. government production function. To address this, I expand the specification of the government production function into a wider family of linear homogeneous functions. As discussed by Fare and Mitchell [1989], most of the prevailing production functions in Macroeconomics can be nested within this family. Among them, the Translog function, the CES function and the Square Root Quadratic function are typical. I therefore choose these to represent the family of functions. I estimate these three functional forms and construct consistency measures based on exact index theory to determine which functional form is superior in specifying the government production function.

Following Berndt [1976], aggregate input quantities are constructed to represent government outputs. However, the index formula used to construct aggregate input quantities is not random. A specific index formula corresponds to a specific family of functional forms. Buscheguennce [1925] and Conus and Buscheguennce [1926] first discussed the equality between an index formula and the corresponding production

functions. Later on, this concept was developed as exact index theory in a seminal article by Diewert [1976]. Exact index theory can be interpreted as follows: in a cost-efficient environment, if there is a production function which belongs to a certain family of functional forms, the output of the production function could be calculated without knowing the parameter values of that production function by a corresponding index formula based on the information of the inputs. The corresponding index formula is called an exact index for that family of functional forms.

If the government production function is not mis-specified and the aggregate input quantity is constructed with its corresponding exact index formula, then the estimation of the underlying government production function should have two properties. First, the estimates identified by the first order conditions with the price and inputs information should be the same as the estimates identified by the government production function with the inputs and outputs information. Second, given the estimated production function, the optimal inputs demand should be equal to the actual input purchases in data set . Two measures of self-consistency are constructed accordingly. The assumed government functional form and input data set are called self-consistent if they satisfy these properties. The self-consistency with the government input data set is compared across the CES function, translog function and quadratic function. The results indicate that using CES functions to represent the government production function has a balanced, if not better , self-consistency performance compared to using Translog or Quadratic functions.

The remainder of the paper is organized as follows. Section 2 discusses the data used in the estimates. Section 3 presents the estimates of the CES production function for the government production function with three and two inputs. Section 4 discusses the candidate functional forms from a family of linearly homogeneous func-

tions. Section 5 discusses exact index theory and self-consistency measures. Section 6 concludes.

1.2 Data Construction and Sources

The data set used in this paper is from the National Income and Product Accounts (NIPA) in the Bureau of Economics Analysis (BEA). The quantity of government labor service L_t is defined as the quantity of full-time equivalent government employees. The wage of the government labor service W_t is defined as the wages and salaries for full-time equivalent employees.[1] The price of intermediate goods PM_t is defined to be the same as the chained price index[2] of intermediate goods purchased in the government sector. The quantity of intermediate goods M_t is calculated by dividing the value of intermediate goods and services by the price index of intermediate goods. The rental rate of government fixed capital is assumed to be proportional to the chained price index of fixed capital in the government sector and the quantity of government fixed capital is constructed by dividing the value of the consumption of fixed capital to the price of government fixed capital[3].

For the unobservable value of government output, this paper follows the methods from Berndt [1976] and Antras [2004]. Aggregate input quantity Q_t is calculated with the Sato-Vartia index formula, Tornqvist index formula and Fisher index formula for

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[1]An alternative approach is to directly use the chained price index of employment compensation in the U.S. government sector. Following this method, the quantity of government labor services is calculated by dividing the value of the government employment compensation by the price index. In fact, the data generated through these two methods are extremely similar, except for the scaling difference.

[2]Chained price index in NIPA, which is also referred to as price deflator in NIPA, has been constructed using the Fisher index since 1995.

[3]There is an alternative way to define the rental rate of government fixed capital by dividing the nominal value of government consumption of fixed capital to the real quantity of government fixed capital. In fact, these two methods generate quite similar results.

the CES functional form, Translog functional form and Quadratic functional form separately. I set year 2000 as the base year.[4] Next, the aggregate input price index P_t is constructed by dividing the government consumption expenditure in NIPA by aggregate input quantity Q_t.

1.3 THREE-FACTOR CES PRODUCTION FUNCTION SPECIFICATION

The U.S. government has three inputs for its productions: government employment, government fixed capital and intermediate goods. Since the CES production function is widely used in the field of Macroeconomics, I first assume that the U.S. government production function follows a three-factor CES functional form. Arrow et al. [1961] showed that the assumption of a constant elasticity of substitution implied the following functional form:

$$Y_t = A_t[\alpha_1 K_t^{\frac{\sigma-1}{\sigma}} + \alpha_2 L_t^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_1 - \alpha_2)M_t^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

where Y_t is real output, K_t is the flow of services from real capital stock, L_t is the flow of services from employees, M_t is intermediate goods consumed by the U.S. government, A_t is a Hicks-neutral technology, α_i where $i = 1, 2$ are distribution parameters, and the constant σ is the elasticity of substitution between any two of the inputs. Berndt [1976] points out that we can use an aggregate input quantity index to represent the output based on the information of inputs. Following Berndt [1976]'s method, I construct the aggregate input $Q_t = \frac{Y_t}{A_t}$ with a Sato-Vartia index[5].

$$Q_t = [\alpha_1 K_t^{\frac{\sigma-1}{\sigma}} + \alpha_2 L_t^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_1 - \alpha_2)M_t^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \qquad (A)$$

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[4]In fact, all three index formula generate very similar aggregate input quantities.

[5]Sato-Vartia quantity index Q_t is defined as following:
$\ln Q_t - ln Q_{t-1} = [\sum_i V(w_{t,i}, w_{t-1,i})]^{-1} \cdot [\sum_i V(w_{t,i}, w_{t-1,i})(\ln q_{t,i} - \ln q_{t-1,i})]$
where $w_{t,i} = \frac{p_{t,i}}{q_{t,i}}$; p_i and q_i are the price and quantity of the inputs $i = k, l, m$;

$$V(w_{t,i}, w_{t-1,i}) = \begin{cases} \frac{w_{t,i} - w_{t-1,i}}{\ln w_{t,i} - \ln w_{t-1,i}}, & w_{t,i} \neq w_{t-1,i}. \\ w_{t,i}, & w_{t,i} = w_{t-1,i}. \end{cases}$$

The cost efficient assumption implies three first-order conditions for government production function, equating real factor prices to the real value of their marginal products. These conditions can be rewritten and expanded with an error term to obtain:

$$\ln(Q_t/K_t) = a_1 + \sigma log(R_t/P_t) + \epsilon_{1,t} \tag{1.1}$$

$$\ln(Q_t/L_t) = a_2 + \sigma log(W_t/P_t) + \epsilon_{2,t} \tag{1.2}$$

$$\ln(Q_t/M_t) = a_3 + \sigma log(PM_t/P_t) + \epsilon_{3,t} \tag{1.3}$$

where R_t, W_t, PM_t and P_t are the prices of capital services, labor services, intermediate goods and aggregate input Q_t, respectively. If equation (1.2) is subtracted from equation (1.1), equation (1.3) from equation (1.2) and equation (1.1) from equation (1.3) we should get equations (1.4) through (1.6). Equation (1.4) to (1.6) give us estimates which are independent of the aggregate inputs Q_t.

$$\ln(K_t/L_t) = a_4 + \sigma log(W_t/R_t) + \epsilon_{4,t} \tag{1.4}$$

$$\ln(M_t/L_t) = a_5 + \sigma log(W_t/PM_t) + \epsilon_{5,t} \tag{1.5}$$

$$\ln(M_t/K_t) = a_6 + \sigma log(R_t/PM_t) + \epsilon_{6,t} \tag{1.6}$$

Here a_i, $i = 1, ..., 6$ are constants which depend on α_1 and α_2.

1.3.1 ESTIMATION RESULTS OF THE THREE-FACTOR CES PRODUCTION FUNCTION

I start by presenting the estimates of the elasticity of substitution based on simple Ordinary Least Squares estimates of equations (1.1) to (1.6). Then I adjust the estimates by addressing the issues related to autocorrelation of disturbances, endogeneity of the regressors, as well as the nonstationarity and heteroskedasticity of the variable

series. Finally, I report the full estimates of the three factor-CES production function of the U.S. government.

Ordinary Least Squares Estimation

The top panel of Table 1.1 presents OLS estimates of equation (1.1) to (1.6). The estimates of the elasticity are all close to one for equation (1.1) to (1.3), however, the estimates of equation (1.4) to (1.6) are significantly different from one. The R^2 of the OLS estimations on equations (1.1) through (1.5) are relatively large, ranging 0.64 to 0.94. Table 1.1 also reports the Durbin-Watson statistics for each estimation. The highest Durbin-Watson statistic in the OLS regression is only 0.365. High value of the R^2 and low value of the Durbin-Watson statistics show signs of possible serial autocorrelation in the residuals.

Feasible Generalized Least Squares Estimation

To account for the autocorrelation of the residuals indicated by the OLS Durbin-Watson statistics, I assume the OLS residuals[6] evolve in a standard AR(1) process, i.e., $\mu_t = \rho\mu_{t-1} + \epsilon_t$, where ϵ_t is the white noise. Similar to the estimation of the private sector by Antras [2004], Ljung-Box tests were performed for each of the three specifications at up to three lags with no rejections that estimated $\widehat{\epsilon}_t$ being white noise. This finding supports the use of AR(1) process to model the structure of the disturbances in equations (1.1) to (1.6).

The FGLS column in Table 1.1 presents the estimates of the elasticity obtained by applying two-step Prais-Winsten procedure. The FGLS estimates show different estimates from the OLS regression, which are around 1.7 and 0.46 separately. The

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[6]The residuals could be interpreted as the shock of the combination methods.

Table 1.1: Estimates for Three-Factor CES Production Function

OLS	Eq.1	Eq.2	Eq.3	Eq.4	Eq.5	Eq.6
σ	1.173	1.071	0.972	0.858	1.688	0.458
S.E	(0.096)	(0.030)	(0.075)	(0.056)	(0.065)	(0.079)
R-sqr	0.643	0.941	0.660	0.743	0.927	0.283
D-W	0.127	0.315	0.365	0.100	0.247	0.348
FGLS	Eq.1	Eq.2	Eq.3	Eq.4	Eq.5	Eq.6
σ	1.721	0.994	0.617	1.154	1.232	0.419
S.E	(0.227)	(0.074)	(0.181)	(0.134)	(0.654)	(0.518)
R-sqr	0.684	0.602	0.533	0.710	0.828	0.057
D-W	1.244	1.563	1.372	1.116	1.233	1.859
GIV	Eq.1	Eq.2	Eq.3	Eq.4	Eq.5	Eq.6
σ	0.874	0.985	1.004	0.888	1.598	0.585
S.E	(0.265)	(0.579)	(0.783)	(0.375)	(0.361)	(0.517)
R-sqr	0.422	0.303	0.072	0.113	0.113	0.113
D-W	1.997	1.913	2.032	2.052	2.052	2.052
GMM	Eq.1	Eq.2	Eq.3	Eq.4	Eq.5	Eq.6
σ	1.155	1.074	1.038	0.997	1.727	0.702
S.E	(0.280)	(0.069)	(0.135)	(0.147)	(0.051)	(0.225)
p-v of 'j' test	0.122	0.453	0.166	0.129	0.269	0.282
No.Obs.	85	85	85	85	85	85

Notes: The annual data set is from 1929–2011 of NIPA.

standard errors of FGLS estimations are substantially higher than the OLS estimations. Although the estimates of equation (1.2), equation (1.4), equation (1.5) and equation (1.6) are not significantly from one, the deviations of the estimations leaves doubt on the strong assumption that any two of the inputs share the same elasticity of substitution.

GENERALIZED IV ESTIMATION

Government input prices and quantities are determined by supply and demand sides simultaneously in equilibrium. This simultaneous determination exposes the previous

OLS and FGLS estimation to an endogeneity problem. From the simultaneous endogeneity theory, these demand side equations (1.1) to (1.6), can not be identified unless using a set of exogenous variables which could shift the supply side of the inputs. Therefore, the estimates from OLS and FGLS regressions are likely to be biased.

In addressing the endogeneity problem when estimating the private production function, Berndt [1976] used a two-stage least squares (2SLS) method. Berndt [1976] proposed a large number of instrumental variables from the supply side. However, if some of these variables are only weakly correlated with the independent variables then small sample biases will occur. Therefore, this paper only focuses on a small set of instruments. Specifically, I choose the following three variables as my instruments for the estimation: (1) U.S. population; (2) oil prices; (3) Manufacturing Multifactor Productivity (MMP). These variables can be described as shifters for input supplies. U.S. population clearly determines the supply of labor and capital to the market. The oil price also affects the labor and intermediate goods supply directly. MMP affects the share of the inputs purchased by the private sector.

To tackle the endogeneity problem as well as the autocorrelation problem discussed above, I use generalized instrumental variable (GIV) procedure to estimate equation (1.1) to (1.6). The GIV method is developed by Fair [1970] and a simplified version is introduced by Antras [2004]. The third panel of Table 1.1 presents the results of GIV estimations. The standard deviations of GIV estimates are much higher than the OLS estimations, while the estimates of the elasticities can not be rejected as being equal to one. However, the standard deviations of the GIV methods are so high that estimates of equation (1.2), equation (1.3) and equation (1.6) are not statistically significantly different from zero. However, if heteroskedasticity exists in the estimation then even if the estimates are unbiased, the estimated standard deviation could be wrong. Therefore the generalized method of moments(GMM) method

is used to address the endogeneity problem, the hetroskedasticity problem and the autocorrelation issues together.

GMM Estimation

The GMM estimation in the bottom panel of Table 1.1 requests a heteroskedasticity and autocorrelation-consistent weight martix. I use the Bartlett (Newey-West) kernel and select the lag order using Newey and West [1994] optimal lag-selection algorithm. The estimates of the elasticity of substitution are in fact close to the GIV regression. All of the estimates are significant. They are all not significantly different from one, except the estimate of equation (1.5). All of the P-values of the Hansen 'j' test are bigger than 10%, which imply that we can not reject the null hypothesis that the over-identifying restrictions are valid.

Time Series Estimation

In previous estimations, I focus on addressing the problems of potential endogeneity, autocorrelation and the heteroskedasticity. I now move to discuss the non-stationary problem of the series involved in the estimations.

Six variable series used in equation (1.1) to (1.3) are presented in Figure 1.1. Other variable series used in equation (1.4) through (1.6) are presented in Figure 1.2 to Figure 1.4 separately. These variable series all have apparent trends. It is natural to cast doubt on previous regressions that they may only represent the correlations between the trends for the relevant variable series. This problem is known as the spurious regression problem in Econometric theory. The relatively high R-squares and the low Durbin-Watson statistics obtained in the OLS estimation also point towards this conclusion.

Figure 1.1: Nonstationarity of Fixed Capital and Employment in Government
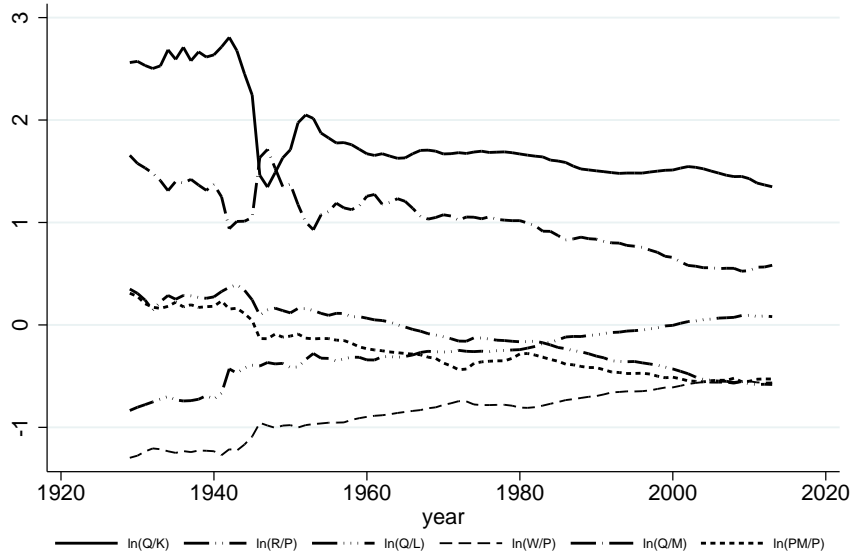
Note: Data is from NIPA 1929-2013 annually

Table 1.2 reports the unit root tests on each of the variable series for the estimation in the government sector. The first row of the top panel of Table 1.2a and Table 1.2b presents the results of a simple Dickey-Fuller test of a unit root in the series against the alternative hypothesis that the variable is generated by a stationary process. It is clear that only $\ln(\frac{Q}{M})$, $\ln(\frac{M}{L})$ and $\ln(\frac{M}{K})$ clearly rejects the hypothesis of a unit root, however, for other dependent variables $\ln(\frac{Q}{K})$, $\ln(\frac{Q}{L})$ and $\ln(\frac{K}{L})$ can not reject the unit root hypothesis. The next two rows extend this simple test to allow for serial correlation by adding higher-order auto-regressive terms to the test. An Augmented Dickey-Fuller test is performed with one and two lags, the rejection results for the dependent variables are the same. In the bottom panel of Table 1.2(a) and Table 1.2(b), I report the results of the same tests performed on each of the twelve series expressed in first differences. In this case, the results indicate a rejection of the null

hypothesis of the series being integrated of order two. Therefore, the OLS estimations of equation (1.3), equation (1.5) and equation (1.6) still provide consistent results.

Table 1.2: Unit Root Test For Three-Factor CES Function Estimation

(a) Unit Root Test of equation (1.1) to equation (1.3)

	$\ln(\frac{Q}{K})$	$\ln(\frac{Q}{L})$	$\ln(\frac{Q}{M})$	$\ln(\frac{R}{P})$	$\ln(\frac{W}{P})$	$\ln(\frac{PM}{P})$	5% Crital Value
ADF 0	-2.040	-2.626	-3.825	-2.905	-1.761	-1.823	-3.466
ADF 1	-3.036	-2.545	-5.088	-4.105	-2.214	-2.050	-3.467
ADF 2	-3.285	-2.385	-4.537	-3.772	-2.128	-1.809	-3.468
	$\triangle\log(\frac{Q}{K})$	$\triangle\log(\frac{Q}{L})$	$\triangle\log(\frac{Q}{M})$	$\triangle\log(\frac{R}{P})$	$\triangle\log(\frac{W}{P})$	$\triangle\log(\frac{PM}{P})$	5% Critical Value
ADF 1	-4.921	-7.153	-7.061	-7.355	-5.684	-5.887	-3.470

(b) Unit Root Test of equation (1.4) to equation (1.6)

	$\ln(\frac{K}{L})$	$\ln(\frac{M}{L})$	$\ln(\frac{M}{K})$	$\ln(\frac{W}{R})$	$\ln(\frac{W}{PM})$	$\ln(\frac{R}{PM})$	5% Crital Value
ADF 0	-1.786	-3.977	-3.713	-2.820	-1.761	-1.675	-3.466
ADF 1	-2.858	-4.142	-4.576	-4.418	-2.214	-2.232	-3.467
ADF 2	-2.588	-3.868	-4.669	-3.666	-2.128	-2.230	-3.468
	$\triangle\ln(\frac{K}{L})$	$\triangle\ln(\frac{M}{L})$	$\triangle\ln(\frac{M}{K})$	$\triangle\ln(\frac{W}{R})$	$\triangle\ln(\frac{W}{PM})$	$\triangle\ln(\frac{R}{PM})$	5% Critical Value
ADF 1	-4.673	-7.635	-6.390	-6.736	-5.329	-8.366	-3.470

Notes: The data set is from 1929–2013 of NIPA.

As indicated by Table 1.2, the OLS and FGLS estimates computed from equation (1.1), equation (1.2) and equation (4) are potentially subject to a spurious regression bias. In fact, as shown by Phillips [1986], in this situation, OLS estimates will not be consistent unless a linear combination of the dependent and independent variables is stationary, that is, only if the two variables entering each regression are co-integrated.

Table 1.3: Global caption

(a) Conintegration Tests For Three-Factor CES Regression

	A. Residual-Based Augmented Dickey-Fuller Tests			
	Residuals of eq.(1)	Residuals of eq.(2)	Residuals of eq.(3)	5% Critical Value
ADF 1	-2.569	-3.358	-4.087	-3.467
ADF 2	-2.863	-3.102	-3.480	-3.468
ADF 3	-2.763	-3.131	-3.790	-3.469

	B. Johansen Cointegration Tests							
	Max				Trace			
Test	r=0 vs r=1		r=1 vs r=2		r=0 vs r=1		r=1 vs r=2	
Num. of lags	1	2	1	2	1	2	1	2
$\ln(\frac{Q}{K})$ & $\ln(\frac{R}{P})$	4.95	12.28	0.12	0.28	5.07	12.56	0.12	0.28
$\ln(\frac{Q}{L})$ & $\ln(\frac{W}{P})$	18.22	19.35	3.71	3.64	21.94	22.99	3.71	3.64
$\ln(\frac{Q}{M})$ & $\ln(\frac{PM}{P})$	10.41	14.03	4.95	5.44	15.36	19.47	4.95	5.44
5% Critical Values	14.07		3.76		15.41		3.76	

(b) Conintegration Tests For Three-Factor CES Regression

	A. Residual-Based Augmented Dickey-Fuller Tests			
	Residuals of eq.(4)	Residuals of eq.(5)	Residuals of eq.(6)	5% Critical Value
ADF 1	-1.651	-4.051	-4.087	-4.537
ADF 2	-2.574	-4.674	-3.480	-4.210
ADF 3	-2.501	-4.550	-3.790	-4.311

	B. Johansen Cointegration Tests							
	Max				Trace			
Test	r=0 vs r=1		r=1 vs r=2		r=0 vs r=1		r=1 vs r=2	
Num. of lags	1	2	1	2	1	2	1	2
$\ln(\frac{K}{L})$ & $\ln(\frac{W}{R})$	3.80	11.70	0.71	0.87	4.51	12.57	0.71	0.87
$\ln(\frac{M}{L})$ & $\ln(\frac{W}{PM})$	18.35	21.36	4.18	4.19	22.54	25.54	4.18	4.19
$\ln(\frac{M}{K})$ & $\ln(\frac{R}{PM})$	36.86	21.36	8.50	8.01	22.54	25.54	4.18	4.19
5% Critical Values	14.07		3.76		15.41		3.76	

Notes: The data set is from 1929-2013 of NIPA.

Table 1.3 presents the results from two co-integration tests. The top panel of Tabel 1.3(a) and Table 1.3(b) consider Engle and Granger [1987] a residual-based Augmented Dickey-Fuller test, which tests the stationary of the residuals from the OLS regressions (1.1) through (1.6). The residuals of equation (1.1), equation (1.2) and equation (1.3) do not show stationary features. But as pointed out by Engle and Granger [1987], the critical values of standard unit root tests are not appropriate when applied to the OLS residuals because they lead to too many rejections of the null hypothesis of no co-integration. MacKinnon [2010] has linked the appropriated critical values to the sample size and to a set of parameters that only vary with the specification of the co-integration equation, the number of variables and the significance level. The appropriated critical values are displayed in the last column.

In the bottom panel of Table 1.3(a) and Table 1.3(b), I implement the maximum likelihood co-integration test suggested by Johansen and Juselius [1990], which tests the null hypothesis of the existence of r co-integrating vectors against the alternative of the existence of $r + 1$ co-integrating vectors. Implementing the test requires specifying a particular model for the co-integration equation as well as choosing the number of lags of the first difference of the variables to be included in the estimation. In light of equations (1.1) to (1.6), I choose a model with a constant and a trend[7] and compute the statistics with one and two lagged first differences of the data. The results in the bottom panel of Table 1.3(a) indicate that in both the estimation including one lag and two lags, the null hypothesis of no co-integration is rejected by $\ln(\frac{Q}{L})$ and $\ln(\frac{W}{P})$. Consequently the OLS estimate for equation (1.2) is consistent.

These tests support the OLS estimates in equation (1.2), equation (1.3), equation (1.5) and equation (1.6). However, the Table 1.1 should be interpreted with caution because there is still a potential spurious regression bias in equation (1.1) and equation

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[7]The trend term is to eliminate the potential time trend in the residuals.

(1.4). As discussed by Hamilton (1994), the spurious regressions would be corrected by differencing the data before estimating the equations. The disadvantage of this approach is that important long-run information would be lost. Interestingly, the existence of a unit root in the OLS residuals still means that the FGLS, GIV and GMM estimates should be consistent with the differenced data. Also, the different time series features and different than one estimates of equation (1.5) and equation (1.6) in Table 1.1 raise the question of whether the assumption of the same elasticity of substitution among three government inputs is too strong.

FULL PARAMETERS ESTIMATION

In the previous discussion, I focused on the estimation of the parameter σ. In this section, I report the estimates of all parameters of three-factor CES function of equation (A). I use feasible generalized nonlinear least squares (FGNLS) to estimate equation (A), discussed in Chapter 6 by Davidson and MacKinnon [2004] and GMM with the instruments world population, oil price and MMF to estimate the equations (1.1) through (1.6) together. The estimates are listed in Table 1.4.

Table 1.4: Full parameter Estimations For the Three-Factor CES function

	FGNLS	GMM
σ	1.090	1.104
	(0.276)	(0.031)
α_1	0.174	0.181
	(0.028)	(0.025)
α_2	0.546	0.547
	(0.014)	(0.008)
No.Obs.	85	85

Notes: The data set is from 1929–2013 of NIPA.

Overall, the full-parameter estimated σ is close to one. So if we assume the production of U.S. government following a three-factor CES function, then these estimates

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indicate that a Cobb-Douglas function can represent the U.S. government production function. However, we have to use it with caution because the assumption of the same elasticity of substitution between any two inputs may be too strong.

1.4 Two-Factor CES production

The assumption that any two inputs of the U.S. government production function have the same elasticity of substitution is strong. So I relax this assumption in this section and assume that any two of the three inputs have their own elasticity of substitution. But I still assume that this two-input production function can be characterized by a two-factor CES function, shown as following:

$$Y_t = A_t[\alpha X_{1t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)X_{2t}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

where Y_t is real output, X_{1t} and X_{2t} are the flows of services from any two inputs of the government sector, A_t is still a Hicks-neutral technological shifter, α is a distribution parameter, and the constant σ is the elasticity of substitution between inputs. Similar to the estimations with three-factor CES function, I construct an aggregate input $Q_t = \frac{Y_t}{A_t}$ with a Sato-Vartia index to represent the output of government sector.

$$Q_t = [\alpha X_{1t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha)X_{2t}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{B}$$

The cost efficient assumption of government production implies two first-order conditions, equating real factor prices to the real value of their marginal products. These conditions can be rewritten and expanded with an error term to obtain:

$$\ln(Q_t/X_{1t}) = \alpha_1 + \sigma\ln(PX_{1t}/P_t) + \epsilon_{1,t} \tag{1.7}$$

$$\ln(Q_t/X_{2t}) = \alpha_2 + \sigma\ln(PX_{2t}/P_t) + \epsilon_{2,t} \tag{1.8}$$

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where PX_{1t} and PX_{2t}, and P_t are the prices of two inputs, and aggregate input Q_t, respectively, and α_1 and α_2 are constants that depend on α. A third alternative specification can be obtained by subtracting equation (1.7) from equation (1.8):

$$\ln(X_{1t}/X_{2t}) = \alpha_3 + \sigma\ln(PX_{2t}/PX_{1t}) + \epsilon_{3,t} \tag{1.9}$$

The bundle of X_1 and X_2 represents three two-inputs combinations of the government production function which are government fixed capital and government employment, government intermediate goods and government employment, government intermediate goods and government capital. I estimate the CES production function based on each of three combinations of the inputs in a similar way as for the estimation of the three-factor CES function. For each combination, I start by reporting simple OLS and FGLS estimates of equation (1.7) to equation (1.9). I then refine these estimates by dealing with the issues related to auto-correlation of the disturbances, the endogeneity problem and nonstationarity of the series. The properties of these estimates are presented in the next three subsections.

1.4.1 Government Capital and Employment

In this section, X_{1t} represents the service flow from government capital, X_{2t} represents the service flow from the government employment. Therefore in this subsection, PX_1 is the cost of using the capital R, and PX_2 is the wage of government employment W. Table 1.5 displays the OLS, FGLS, GIV and GMM estimates of equation (1.7) through equation (1.9) for the elasticity of substitution between government capital and employment.

The Durbin-Watson statistics are reported in the OLS regressions. The highest Durbin-Watson statistics in equation (1.7) through equation (1.9) are much smaller

Table 1.5: Estimates of Fixed Capital and Employment in Government

OLS	Eq.1.7	Eq.1.8	Eq.1.9
σ	0.858	0.857	0.858
S.E	(0.055)	(0.063)	(0.056)
R-sqr	0.753	0.696	0.743
D-W	0.091	0.145	0.100
FGLS	Eq.1.7	Eq.1.8	Eq.1.9
σ	1.117	1.245	1.154
S.E	(0.130)	(0.150)	(0.134)
R-sqr	0.764	0.430	0.710
D-W	1.153	1.038	1.116
GIV	Eq.1.7	Eq.1.8	Eq.1.9
σ	0.888	0.889	0.888
S.E	(0.409)	(0.282)	(0.375)
R-sqr	0.115	0.144	0.113
D-W	2.091	1.894	2.052
GMM	Eq.1.7	Eq.1.8	Eq.1.9
σ	1.009	0.952	0.997
S.E	(0.153)	(0.115)	(0.147)
p-v of 'j' test	0.152	0.140	0.129
No.Obs.	85	85	85

Notes: The data set is from 1929–2011 of NIPA.

than unity, indicating a clear rejection of the null hypothesis of no serial auto-correlation in the residuals. I therefore replicate the estimation process in the three-factor section. I implement FGLS to deal with the auto-correlation problem of the disturbances in the OLS regression, with the assumption that the disturbance follows an AR(1) process. Ljung-Box tests were performed again to back up this process pattern. The FGLS regression results cannot reject the alternative hypothesis that the elasticity of substitution between government capital and government employment is close to one.

Instrumental variables of U.S. population, raw oil prices and MMF are used to perform the GIV and GMM estimations. The GIV address the endogeneity and the auto-correlation issues. The GMM estimation deals with the endogeneity, auo-correlation and hetroskedasticity issues using a Bartlett (Newey-West) kernel and the lag order is selected by the Newey and West(1994) optimal lag-selection algorithm. The estimates of elasticity and substitution between government capital and government employment are all close to one.

Figure 1.2: Nonstationarity of Fixed Capital and Employment in Government
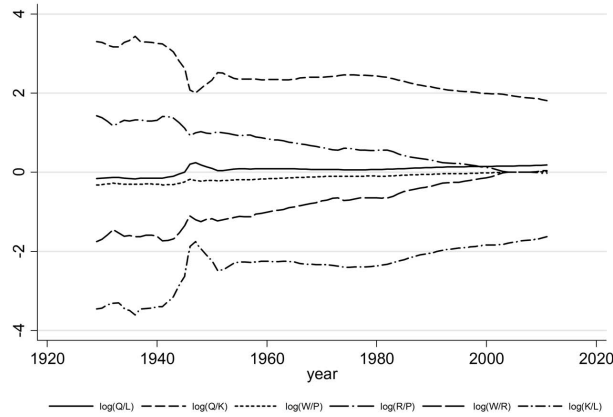
Figure 1.2 displays the variable series used in the estimations for the capital-labor CES production function. All six series show clear upward or downward trends. Table 1.6 performs the unit root test for all of these six variable series. The top panel presents the results of Dickey-Fuller test of a unit root in the series against the alternative hypothesis of trend-stationarity. It is clear from Table 1.6 that in none of the six series does the test reject the hypothesis of a unit root. The next two rows extend this simple test to allow for serial correlation by adding higher-order auto-regressive terms to the test. An Augmented Dickey-Fuller test is performed with one

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and two lags, and the null hypothesis of a unit root is rejected only for $\ln(R)$ and $\ln(W/R)$. Therefore, the estimates computed from equation (1.4) to equation (1.6) for government capital and employment are still potentially subject to a spurious regression bias. In fact, as shown by Phillips [1986], in this situation, OLS estimates will not be consistent unless a linear combination of the dependent and independent variables is stationary, that is, only if the two variables entering each regression are co-integrated.

Table 1.6: Unit Root Test of Fixed Capital and Employment in Government Sector

	$\ln(Q/K)$	$\ln(R/P)$	$\ln(Q/L)$	$\ln(W/P)$	$\ln(K/L)$	$\ln(W/R)$	5% Crital Value
ADF 0	-1.732	-2.836	-1.992	-2.406	-1.786	-2.820	-3.468
ADF 1	-2.760	-4.620	-3.194	-3.149	-2.858	-4.418	-3.469
ADF 2	-2.820	-3.981	-2.536	-2.430	-2.588	-3.666	-3.039
	$\triangle\ln(Q/K)$	$\triangle\ln(R/P)$	$\triangle\ln(Q/L)$	$\triangle\ln(W/P)$	$\triangle\ln(K/L)$	$\triangle\ln(W/R)$	5% Critical Value
ADF 1	-4.462	-6.702	-5.417	-7.143	-4.673	-6.736	-3.470

Notes: The data set is from 1929–2011 of NIPA.

Table 1.7 shows the co-integration tests for the CES function with government capital and employment. The residual-based augmented Dickey-Fuller tests all reject the null hypothesis that there is co-integration. The Johansen cointegration tests indicate that it is hard to determine whether the OLS regressions have the co-integration feature, except $\ln(Q/L)$ on $\ln(W)$ when with more than one lag in this vector error-correction model.

Overall, the results of these cointegration tests imply that the OLS estimates in Table 1.5 should be interpreted with caution because of spurious regression bias. However, as mentioned earlier, the existence of a unit root in the OLS residuals indicates

that the FGLS, GIV and GMM estimates should be consistent with estimating the differenced data.

Table 1.7: Cointegration Tests of Fixed Capital and Employment in Government Sector

	A. Residual-Based Augmented Dickey-Fuller Tests			
	Residuals of eq.(1.7)	Residuals of eq.(1.8)	Residuals of eq.(1.9)	5% Critical Value
ADF 1	-1.586	-1.933	-1.651	-3.468
ADF 2	-2.418	-3.181	-2.574	-3.469
ADF 3	-2.493	-2.579	-2.501	-3.470

	B. Johansen Cointegration Tests							
	Max				Trace			
Test	r=0 vs r=1		r=1 vs r=2		r=0 vs r=1		r=1 vs r=2	
Num. of lags	1	2	1	2	1	2	1	2
$\ln(\frac{Q}{K})$& $\ln(\frac{R}{P})$	3.79	10.09	0.68	0.70	4.47	10.80	0.68	0.70
$\ln(\frac{Q}{L})$& $\ln(\frac{W}{P})$	4.19	19.55	0.90	1.60	5.10	21.36	0.90	1.61
$\ln(\frac{Q}{M})$& $\ln(\frac{PM}{P})$	3.80	11.70	0.71	0.87	4.51	12.57	0.71	0.87
5% Critical Values	14.07		3.76		15.41		3.76	

Notes: The data set is from 1929-2013 of NIPA.

Table 1.8 shows the results of the FGNLS and GMM estimation of the parameters for the CES production function between government capital and government employment. The FGNLS and GMM estimates provide a consistent estimation with the differenced data to address the spurious regression bias. The results indicate a unity elasticity of substitution between government employment and capital. So the CES function with these two inputs is equivalent to a Cobb-Douglass function to represent the U.S. government production function.

1.4.2 GOVERNMENT INTERMEDIATE GOODS AND EMPLOYMENT

In this subsection, I assume that the production function with the inputs of government intermediate goods and employment is weakly separable from other inputs.

Table 1.8: Full parameter Estimations For the Three-Factor CES function

	FGNLS	GMM
σ	1.015	0.952
	(0.048)	(0.045)
α_1	0.210	0.223
	(0.016)	(0.010)
No.Obs.	85	85

Notes: The data set is from 1929–2013 of NIPA.

I still assume that the production function of government intermediate goods and employment follows a CES functional form. Now in equation (B), X_{1t} indicates government intermediate goods and X_{2t} indicates government employment. PM is the price for the service flow of government intermediate goods and W is the price for the service flow of government employment. Table 1.9 shows the estimates with the OLS, FGLS, GIV and GMM methods. The instruments used in the relevant estimations are U.S population and raw oil price. The most significant feature of Table 1.9 is that the values of the estimates of σ are greater than one, which means the government intermediate goods are more like substitutes to government employment.

Figure 1.3 displays the base series of the variables used in the estimations of this section. They seem are all evolve following certain trends. I therefore carry out the unit root test for all these variables. Table 1.10 shows the results. It is interesting to see that $\ln(Q/K)$ and $\ln(K/L)$ pass the unit root test. Table 1.11 shows the Augmented Dicky-Fuller tests for the residuals of all three OLS regressions. The residual series all reject the non-stationarity hypothesis. This suggests that these results are not likely have spurious regression problems.

Table 1.9: Estimates of Intermediate Goods and Employment in Government

| OLS	Eq.1.7	Eq.1.8	Eq.1.9
σ	1.690	1.68	1.688
S.E	(0.069)	(0.058)	(0.065)
R-sqr	0.919	0.940	0.927
D-W	0.249	0.242	0.247
FGLS	Eq.1.7	Eq.1.8	Eq.1.9
σ	1.204	1.280	1.232
S.E	(0.069)	(0.058)	(0.654)
R-sqr	0.895	0.469	0.828
D-W	1.237	1.232	1.233
GIV	Eq.1.7	Eq.1.8	Eq.1.9
σ	1.623	1.530	1.598
S.E	(1.092)	(0.993)	(0.361)
R-sqr	0.83	0.78	0.113
D-W	2.091	1.894	2.052
GMM	Eq.1.7	Eq.1.8	Eq.1.9
σ	1.737	1.711	1.727
S.E	(0.050)	(0.053)	(0.051)
p-v of 'j' test	0.287	0.237	0.269
No.Obs.	85	85	85

Notes: The data set is from 1929–2011 of NIPA.

Table 1.12 shows the full estimation of the two-factor CES production function with the inputs of intermediate goods and government capital. The estimations from FGNLS and GMM are similar and all indicate that the estimate of the elasticity of substitution between government intermediate goods and government employment is greater than one.

1.4.3 Government Capital and intermediate goods

In this section, I assume government capital and government intermediate goods are weakly separable from other inputs of the government production function. The pro-

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Figure 1.3: Nonstationarity of Intermediate Goods and Employment in Government
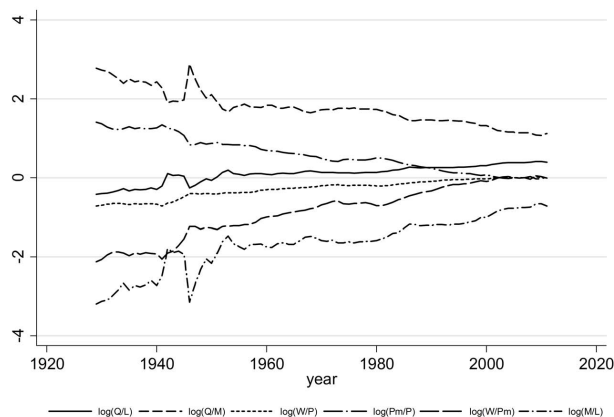
duction function with government capital and intermediate goods can be represented by a two-factor CES function. Now, X_{1t} indicates government intermediate goods and X_{2t} indicates government capital. Table 1.13 presents the related OLS, FGLS, GIV and GMM regressions for equation (1.7) through (1.9). It is striking to see that the OLS and FGLS estimates of equation (1.7) are negative, although not significantly different from zero. One of the possible reasons for this is that there are more demand shocks to the U.S. government purchases of intermediate goods. Alternatively, it could be that the assumption that intermediate goods and capital goods are weakly separable with employment is not valid.

I use U.S. population and Oil prices as instruments to correct the endogeneity bias. The results are shown in the GIV and GMM estimations. The results provide positive estimates of the elasticity of the substitution in equation (1.7), but the standard deviations are big and none of the estimates are significantly different from zero.

Table 1.10: Unit Root Test of Intermediate Goods and Employment in Government

	$\ln(Q/M)$	$\ln(PM/P)$	$\ln(Q/L)$	$\ln(W/P)$	$\ln(M/L)$	$\ln(W/PM)$	5% Crital Value
ADF 0	-4.324	-2.050	-3.332	-1.162	-3.977	-1.675	-3.468
ADF 1	-4.385	-2.614	-3.654	-1.746	-4.142	-2.232	-3.469
ADF 2	-4.090	-2.488	-3.431	-1.966	-3.868	-2.230	-3.470

	$\triangle\ln(Q/M)$	$\triangle\ln(PM/P)$	$\triangle\ln(Q/L)$	$\triangle\ln(W/P)$	$\triangle\ln(M/L)$	$\triangle\ln(W/PM)$	5% Critical Value
ADF 1	-7.803	-5.760	-7.248	-4.605	-7.635	-5.329	-3.470

Notes: The data set is from 1929–2011 of NIPA.

Table 1.11: Cointegration Tests of Fixed for Government Intermediate Goods and Employment

A. Residual-Based Augmented Dickey-Fuller Tests			
	Residuals of eq.(1.7)	Residuals of eq.(1.8)	Residuals of eq.(1.9)	5% Critical Value
---	---	---	---	---
ADF 1	-4.296	-3.600	-4.051	-3.468
ADF 2	-4.708	-4.665	-4.674	-3.469
ADF 3	-4.534	-4.548	-4.550	-3.470

B. Johansen Cointegration Tests							
	Max				Trace		
Test	r=0 vs r=1		r=1 vs r=2		r=0 vs r=1		r=1 vs r=2	
Num. of lags	1	2	1	2	1	2	1	2
$\ln(\frac{Q}{M})$ & $\ln(\frac{PM}{P})$	17.35	21.70	3.97	3.46	21.32	25.16	3.97	3.46
$\ln(\frac{Q}{L})$ & $\ln(\frac{W}{P})$	38.78	24.49	3.73	5.03	32.51	29.52	3.73	5.03
$\ln(\frac{M}{L})$ & $\ln(\frac{W}{PM})$	18.35	21.36	4.18	4.19	22.54	25.54	4.18	4.19
5% Critical Values	14.07		3.76		15.41		3.76	

Notes: The data set is from 1929-2013 of NIPA.

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Table 1.12: Full parameter Estimations CES function with intermediate goods and employment

	FGNLS	GMM
σ	1.577	1.728
	(0.038)	(0.046)
α_1	0.502	0.548
	(0.026)	(0.017)
No.Obs.	85	85

Notes: The data set is from 1929–2013 of NIPA.

Although the estimates of equation (1.8) and equation (1.9) are all positive, some of these estimates are not statistically significant.

Figure 1.4: Nonstationarity of Intermediate Goods and Fixed Capital in Government Sector
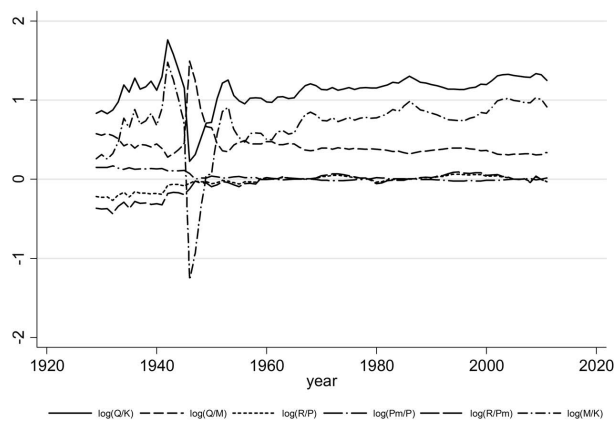

Figure 1.4 shows the series trends of variables used to estimate equation (1.7) to (1.9). Table 1.14 rules out the problem of spurious regression, because $\ln \frac{Q}{M}$, $\ln \frac{Q}{K}$ and $\ln \frac{M}{K}$ all reject the unit root hypothesis. Therefore, OLS should give us consistent esti-

Table 1.13: Estimates of Intermediate Goods and capital in Government

OLS	Eq.1.7	Eq.1.8	Eq.1.9
σ	-0.286	0.579	0.458
S.E	(0.196)	(0.058)	(0.079)
R-sqr	0.025	0.543	0.283
D-W	0.397	0.325	0.348
FGLS	Eq.1.7	Eq.1.8	Eq.1.9
σ	-0.290	0.720	0.419
S.E	(0.196)	(0.058)	(0.518)
R-sqr	0.731	0.830	0.057
D-W	1.309	1.349	1.859
GIV	Eq.1.7	Eq.1.8	Eq.1.9
σ	0.536	0.501	0.585
S.E	(3.618)	(0.233)	(0.517)
R-sqr	0.001	0.062	0.113
D-W	1.83	1.883	2.052
GMM	Eq.1.7	Eq.1.8	Eq.1.9
σ	0.293	0.746	0.702
S.E	(0.586)	(0.168)	(0.225)
p-v of 'j' test	0.337	0.337	0.282
No.Obs.	83	83	83

Notes: The data set is from 1929–2011 of NIPA.

mates if there is no endogeneity problem. GIV and GMM correct the auto-correlation problem and spurious regression problem. The results show that the estimates are smaller than 1, although some of them have relatively big standard deviations which make them not significantly different from zero or one.

Table 1.15 shows the full estimation of the two-factor CES function. The estimates of σ are both significantly smaller than one.

In sum, the estimates of the elasticity of substitution between government inter-mediate goods and government capital are not perfectly conclusive but they are likely to be smaller than one.

Table 1.14: Unit Root Test for Intermediate Goods and Fixed Capital in Government

	ln(Q/M)	ln(PM/P)	ln(Q/K)	ln(R/P)	ln(M/K)	ln(R/PM)	5% Crital Value
ADF 0	-4.096	-1.497	-3.425	-1.922	-3.713	-1.671	-3.468
ADF 1	-4.666	-1.543	-4.477	-1.622	-4.576	-1.476	-3.469
ADF 2	-4.359	-1.360	-4.793	-1.502	-4.669	-1.332	-3.470
	$\triangle$ln(Q/M)	$\triangle$ln(PM/P)	$\triangle$ln(Q/K)	$\triangle$ln(R/P)	$\triangle$ln(M/K)	$\triangle$ln(R/PM)	5% Critical Value
ADF 1	-7.225	-7.594	-5.965	-8.374	-6.390	-8.366	-3.470

Notes: The data set is from 1929–2011 of NIPA.

Table 1.15: Full parameter Estimations CES function with intermediate goods and Capital

	FGNLS	GMM
σ	0.760	0.618
	(0.035)	(0.055)
α_1	0.279	0.126
	(0.13)	(0.19)
No.Obs.	85	85

Notes: The data set is from 1929–2013 of NIPA.

1.5 EXTENSIONS OF PRODUCTION FUNCTION

Because the CES production function is widely used in Macroeconomics, the previous sections use CES functional forms to identify U.S. government production function. In this section, I expand the specifications of the U.S. government production function into a wider family of functional forms. I estimate three representative specifications

of the linear homogeneous function and construct two self-consistency measures based on exact index theory to compare the consistency of each specification with the data set in the real world.

1.5.1 A Family Tree of Production Functions

It is widely believed that Philip Wicksteed(1894) first described the relationship between output and inputs in a parametric way. The most popular functional forms for the production function started to emerge from the enunciation of the Cobb-Douglas function in 1928. Starting in the early 1950's until the late 1970's, production functions attracted much attention in the Macroeconomic world. Since then, different kinds of production functions emerged to become important tools of economic analysis in macroeconomics studies. These production functions include the Translog function, the constant elasticity of substitution (CES) function, the Leontief function, the Cobb-Douglas function, the generalized square-root quadratic function and others. Although these functions look different to each other, they can be nested into one family form known as modified McCarthy function[8] by Rolf and Thomas(1989). Figure 1.5 shows the relationship of different prevailing functional forms, which is cited from Rolf and Thomas's paper.

Figure 1.5 shows the relationships across most of the prevailing production functions in current Macroeconomics world. As we know, the Leontief equation is a special case of the CES function when the limit of the elasticity of substitution approaches zero. I therefore choose the Translog function, the CES function and the Generalized

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[8]The Mc Marthy function is shown as

$$\phi(x) = A \left[\sum_i a_i x_i^\alpha + \frac{\alpha}{2} \sum_i \sum_{j \neq i} \gamma_{ij} x_i^{\delta_{ij}} x_j^{\alpha - \delta_{ij}} \right]^{1/\alpha}$$

where $\alpha \neq 0$, $a_i > 0$ for all i, $\sum_i a_i = 1$, and $\gamma_{ij} = \gamma_{ji}$ and $\delta_{ij} = \alpha - \delta_{ij}$ for all i,j.

Figure 1.5: A family tree of the production function
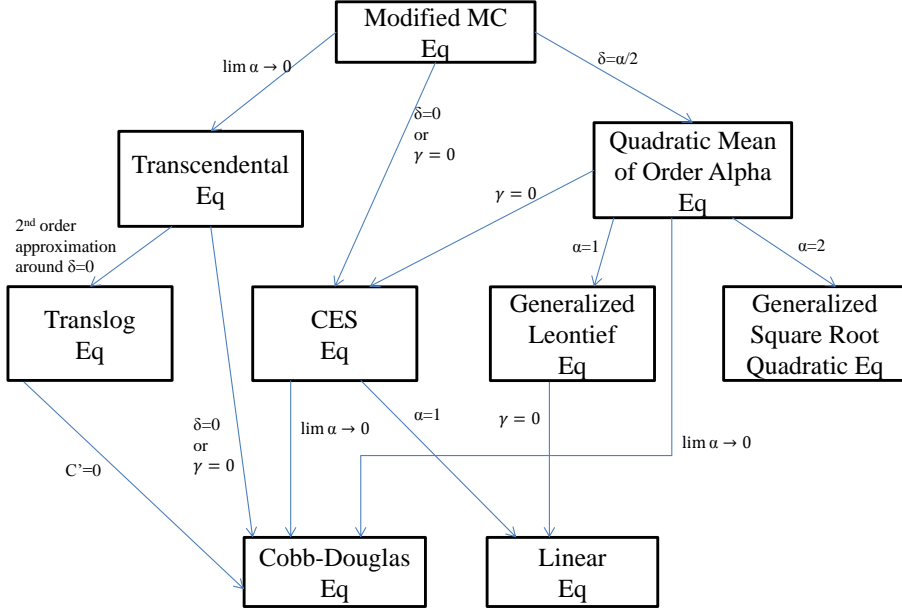
Square Root Quadratic function to represent the whole family of linear homogeneous production functions.

Meanwhile, Milana (2005) argues that a transformed quadratic function can provide a second-order differential approximation to any arbitary function, for example the Box-Cox function. The quadratic mean-of-order-r aggregator function used by Diewert (1976, pp. 129-130) is a special case encompassed by the quadratic Box-Cox function. The quadratic mean-of-order-r aggregator function is listed in equation (1.10).

$$Q^r(q) = \left[\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_{ij}q_i^{\frac{r}{2}}q_j^{\frac{r}{2}}\right]^{\frac{1}{r}} \tag{1.10}$$

where q_i is the inputs and $i = 1, ..., N$. Picking particular values of r, equation (1.10) reduces to some well-known functions. In an unpublished memorandum, Lau [1973] showed that, at the limit as r tends to be zero, it reduces to the homogeneous translog aggregator function. Diewert [1976] noted that, if $r = 2$, then it reduces to a quadratic functional form. Furthermore, if all $\alpha_{ij} = 0$ for $i \neq j$, then it reduces to a CES functional form. The second-order different approximation property provides a foundation to the expanded specification of the government production function.

Because of both representativeness and approximation properties, the CES function, Translog function and Square Root Quadratic function are picked as proper candidate functional forms for the U.S. government production function.

1.5.2   Exact Index Theory and self-Consistency

Although three functional forms are chosen as the candidates to represent the production function of the U.S. government, it is difficult to directly determine which functional form is best fitted to represent the government production function. Fortunately, the method of constructing the aggregate input quantity index leaves us a channel to check if the assumption of a specific functional form is self-consistent with the data set. The idea of self-consistency is based on exact index theory fromDiewert [1976]. In this section, I first provide two preliminary measures of self-consistency, then I compare different functional forms to see which form is more self-consistent.

One of the problems of estimating the government production function is that we can not observe the market value of government outputs. To address this problem, this paper follows Berndt [1976] and Antras [2004] to construct an aggregate input

quantity Q_t to represent the output value of the government sector. This index aggregation process provides us with a way of checking whether our functional form is self-consistent. The self-Consistency comparison is based on Exact Index Theory. Given components, the Exact index theory indicates that an index used to generate an aggregate variable with components is the same as assuming there is a specific production function which combines the components into the aggregate variable.

Suppose there is a producer. p is the normalized prices (nominal prices divided by the price of output) to its inputs q. An index number formula for the output $Q(q)$ is a function of the normalized prices and the quantities of the inputs at the two points 1 and 2, say $I(p_1, p_2, q_1, q_2)$, which gives a measure of the ratio of the outputs at the two points, that is,

$$\frac{Q(q_1)}{Q(q_2)} = I(p_1, p_2, q_1, q_2) \tag{1.11}$$

Under the assumption of a competitive market, and profit-maximization (or cost-minimization), p_i may be identified as the gradient of the production function $Q(q)$ at q_i, so that equation (1.11) should be rewritten as

$$\frac{Q(q_1)}{Q(q_2)} = I(\bigtriangledown Q(q_1), \bigtriangledown Q(q_2), q_1, q_2) \tag{1.12}$$

For any given index number formula $I(p_1, p_2, q_1, q_2)$, one can find the class of functions $Q(q)$ such that equation (1.12) holds exactly for all q_1 and q_2. Diewert [1976] has shown that certain index number formula are exact for certain classes of functional forms. In this paper I use three index number formula: the Törnqvist [1936] index number formula, the Sato [1976] index number formula, and the Fisher [1922] ideal index number formula to be the exact indexes for the translog function, the CES function and the quadratic function.

As discussed by Lau [1979], the relations between exact numbers and their corresponding functions are shown through Theorem 1 to Theorem 3.

Theorem 1 *Fisher index: A once continuously differentiable and homogeneous of degree one function $Q(q)$ is exactly indexed by the Fisher ideal index number formula*

$$I_{fisher,t} = \left[\frac{q^t p^t}{q^{t-1} p^t} \frac{q^t p^{t-1}}{q^{t-1} p^{t-1}} \right]^{1/2}$$

if and only if it can be written in the form

$$Q_t(q) = (q_t' \Lambda q_t)^{1/2}$$

where Λ is a positive definite matrix.

Theorem 2 *Tornqvist index: A once continuously differentiable and homogeneous of degree one function $Q(q)$ is exactly indexed by the Tornqvist index number formula*

$$\ln(Q(q)) = \alpha_0 + \sum_i \alpha_i ln q_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} ln q_i ln q_j$$

where $\sum_i \alpha_i = 1$; $\sum_i \beta_{ij} = 0$, if and only if it is a member of the class of homogeneous of degree one transcendental logarithmic functions, that is,

$$\ln(I_{Tornqvist,t}) = ln Q_t - ln Q_{t-1}$$

$$= \frac{1}{2} \sum_{i=1}^{n} \left[\left(\frac{p_{t,i} q_{t,i}}{\sum_j p_{t,j} q_{t,j}} + \frac{p_{t-1,i} q_{t-1,i}}{\sum_j p_{t-1,j} q_{t-1,j}} \right) \right]$$

Theorem 3 *Sato-Vartia index: A once continuously differentiable and homogeneous of degree one function $Q(q)$ is exactly indexed by the Sato-Vartia index number formula*

$$\ln(I_{sato-vartia,t}) = ln Q_t / ln Q_{t-1}$$

$$= \left[\sum_{k=1}^{n} V(w_{t,k}, w_{t-1,k}) \right]^{-1} \sum_{i=1}^{n} V(w_{t,k}, w_{t-1,k})(ln q_{t,i} - ln q_{t-1,i}]$$

where $w_{j,i} = \frac{p_{j,i}q_{j,i}}{\sum_k p_{j,k}q_{j,k}}$ and $i = 1, ..., 3; j = t, t-1$. $V(x, y) = \begin{cases} \frac{x-y}{lnx-lny} & x \neq y \\ x & x = y \end{cases}$ if and only if it is a member of the class of homogeneous of degree one constant elasticity of substitution functions, that is,

$$Q_t = [\alpha_1 q_{1,t}^{\frac{\sigma-1}{\sigma}} + \alpha_2 q_{2,t}^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_1 - \alpha_2)q_{3,t}^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$$

With the exact index formulas for three representative functions we want to discuss, we now move to the discussion of the methodology of using exact index theory.

Suppose all of the regularity[9] conditions are satisfied, then the estimation processes should have two properties. First, there are two ways to estimate the government production function and they should generate equivalent estimates. One way is to identify the production function based on the optimality conditions of the production function. The other way is to generate the aggregate input quantities first (based on exact index) and then regress the aggregate input quantities on the functional from of government production function directly. According to exact index theory, if the underlying functional form can represent the production function of government sector then these two ways should generate equivalent estimates. Meanwhile, given the estimated production function of government sector and total input cost, we can calculate the optimal demand for each input. This optimal demand should be equal to the input in the data set. I call these two equality properties self-consistency for a production specification and data set. If there are severe deviations from these two equality properties, it is more likely that the assumed production specification is not consistent with our assumptions about government production function and the actual data set.

In order to check the level of self-consistency, I construct two measures: Measure a

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[9]such as the cost efficiency of the government production process, etc.

and Measure b.

Measure a. Self-Consistency measure based on equality between optimal Inputs Demand and actual input demand

If the assumptions of the exact index are all true, then the quantities of the inputs should be consistent with the cost minimization criteria. Also under this condition, given the estimated production function we can calculate the optimal inputs for each budget. The estimated optimal inputs should be the same as the real one. I define a new variable $D_{measure}$, which describes the distance between the calculated optimal inputs and the real inputs.

$$D_{measure} = \frac{1}{T} \sum_{t=1}^{T} \sum^{i} \frac{|\widehat{q}_{i,t} - q_{i,t}|p_{i,t}}{V_t} \tag{1.13}$$

Where $i = 1, 2, ...$ indicates the number of the inputs, $q_{i,t}$ is the quantity of input i at period t, $p_{i,t}$ is the price of $q_{i,t}$ and V_t is the total value (budget) of the inputs at period t. If the outputs of the production can be exactly described by the relevant exact index, then $D_{measure} = 0$. In the real world, if the production is cost-efficient, then $D_{measure}$ should be relative small.

Measure b. Self-Consistency measure based on the equality of Estimates between two identification methods

As discussed earlier, there are two ways to estimate the parameters of government production function. First, we can directly regress the aggregate input quantities on the production function. Second, we can identify the parameters with optimality conditions of the government production function. If all regular conditions are satisfied

according to the exact index theory, these two methods should generate similar estimates. In order to distinguish the estimation process, I call the second estimation process the Robust Estimate Check.

If the estimates from these two methods are more close to each other then the production specification is more consistent with the data set.

1.5.3 Self-Consistency and Three-Factor Functional Forms

In this section, I discuss the three-factor functional forms for the government production function. The tree inputs are the same as before: labor, capital and intermediate goods. The functional forms used to represent the government production function include: the CES function, the Quadratic function and the Translog function. The functional forms and corresponding optimality conditions are listed below separately.

$$Q_t = (\alpha_1 K_t^{\frac{\sigma-1}{\sigma}} + \alpha_2 M_t^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_1 - \alpha_2)L_t^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{1.14}$$

Where the Q_t is aggregate input, K_t, M_t, L_t are the quantity of fixed capital, intermediate goods and the labor separately. P_t, R_t, PM_t and W_t are the corresponding prices.

The first order conditions with respect to the CES function are:

$$\frac{R_t}{PM_t} = \frac{\alpha_1}{\alpha_2} \left(\frac{K_t}{M_t}\right)^{\frac{-1}{\sigma}} + \eta_{1,t} \tag{1.15}$$

$$\frac{R_t}{W_t} = \frac{\alpha_1}{1 - \alpha_1 - \alpha_2} \left(\frac{K_t}{L_t}\right)^{\frac{-1}{\sigma}} + \eta_{2,t} \tag{1.16}$$

Three-Factor Quadratic function:

The three-factor quadratic functional form is displayed in equation (1.17)

$$Q_t = (\alpha_1 K_t^2 + \alpha_2 M_t^2 + \alpha_3 L_t^2 + \alpha_4 K_t M_t + \alpha_5 K_t L_t + \alpha_6 L_t M_t)^{1/2} \tag{1.17}$$

Where α_i are the parameters of the production function. $i = 1, ..., 6$.

The first order conditions with respect to the three-factor quadratic function are:

$$\frac{R_t}{PM_t} = \frac{2\alpha_1 K_t + \alpha_4 M_t + \alpha_5 L_t}{2\alpha_2 M_t + \alpha_4 K_t + \alpha_6 L_t} + \epsilon_{1,t} \tag{1.18}$$

$$\frac{PM_t}{W_t} = \frac{2\alpha_2 M_t + \alpha_4 K_t + \alpha_6 L_t}{2\alpha_3 L_t + \alpha_5 K_t + \alpha_6 M_t} + \epsilon_{2,t} \tag{1.19}$$

Three-Factor Translog function:

The three-factor transcendental logarithmic functional form is displayed in the equation (1.20)

$$\ln Q_t = \alpha_0 + \alpha_1 \ln K_t + \alpha_2 \ln M_t + (1 - \alpha_1 - \alpha_2) \ln L_t$$

$$+ \alpha_3 (\ln K_t)^2 + \alpha_4 \ln M_t{}^2 + \alpha_5 \ln L_t{}^2 + (\alpha_5 - \alpha_3 - \alpha_4) \ln K_t \ln M_t$$

$$+ (\alpha_4 - \alpha_3 - \alpha_5) \ln K_t \ln L_t + (\alpha_3 - \alpha_4 - \alpha_5) \ln M_t \ln L_t \tag{1.20}$$

The first order conditions with respect to the three-factor Transcendental logarithmic function are:

$$\frac{K_t R_t}{Q_t P_t} = \alpha_1 + \alpha_3 \ln \frac{K_t}{M_t} + \alpha_3 \ln \frac{K_t}{L_t} + (\alpha_5 - \alpha_4) \ln \frac{M_t}{L_t} \epsilon_{tg1,t} \tag{1.21}$$

$$\frac{M_t PM_t}{Q_t P_t} = \alpha_2 + \alpha_4 \ln \frac{L_t}{M_t} + \alpha_4 \ln \frac{L_t}{K_t} + (\alpha_5 - \alpha_3) \ln \frac{K_t}{L_t} + \epsilon_{tg2,t} \tag{1.22}$$

Three-Factor Estimation and Self-Consistency Comparison

The top panel of Table 1.16 shows the estimates of nonlinear least square(NLS) estimation on the CES equation (1.14), the Quadratic equation (1.17) and the Translog equation (1.20) separately. The bottom panel of Table 1.16 presents the results of feasible generalized nonlinear least square (FGNLS) estimation on the optimality conditions: equation (1.15) to (1.16) for the CES function; equation (1.18) to (1.19) for the quadratic function; equation (1.21) to (1.22) for the Translog function. It is quite interesting to see that the estimates generated by these two methods in fact do not deviate too much from each other. Although the estimated α_1 for the

Quadratic function show different signs, the value in the robust check estimation is actually insignificant. Regarding the equality of the estimates, the CES specification and the Translog specification do a better job than the Quadratic function. But for the $D_{measure}$ value, the Quadratic production function performs better.

Table 1.16: Estimation of the Government Sector with Intermediate Goods and Capital

		CES		Quadratic		Translog
	a1	0.198	a1	-0.826	a1	0.241
		(0.012)		(0.149)		(0.007)
	a2	0.324	a2	0.223	a2	0.239
		(0.010)		(0.037)		(0.013)
	σ	1.182	a3	0.105	a3	0.057
NLS		(0.058)		(0.006)		(0.001)
			a4	-0.319	a4	-0.115
				(0.118)		(0.003)
			a5	2.483	a5	0.074
				(0.028)		(0.002)
			a6	1.677		
				(0.026)		
$D_{measure}$		0.753		0.338		0.620
	a1	0.191	a1	0.623	a1	0.577
		(0.104)		(0.986)		(0.023)
	a2	0.307	a2	0.657	a2	0.229
		(0.004)		(0.172	(0.020)	
Robust	σ	1.101	a3	0.085	a3	0.046
Check		(0.046)		(0.009)		(0.009)
			a4	-3.911	a4	-0.119
				(0.785)		(0.009)
			a5	5.356	a5	0.071
				(0.107)		(0.005)
			a6	3.846		
				(0.062)		
No.Obs		85		85		85

Notes: The data set is from 1929–2013 of NIPA. The value in the parenthesis is the standard deviation.

1.5.4 SELF-CONSISTENCY AND TWO-FACTOR FUNCTIONAL FORMS

Now I move to discuss the specifications of two-input production functional forms of U.S. government. Similarly to the discussion of three-factor production functions, I first list the functional forms of each specification and derive the optimality conditions accordingly.

The constant elasticity of substitution (CES):

$$Q_t = (\alpha X_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)Y_t^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{1.23}$$

Where Q_t is the aggregate input function. X and Y can be capital K and labor L or intermediate goods M and labor L or capital K and intermediate goods M. $\frac{1}{1-\sigma}$ represents the elasticity of substitution between the inputs in the CES functions. α is the distribution factor.

The corresponding first order condition with respect to the two-factor CES function can be expanded with an error term as:

$$\frac{P_{x,t}}{P_{y,t}} = \frac{\alpha}{1-\alpha}\left(\frac{X_t}{Y_t}\right)^{\frac{-1}{\sigma}} + \epsilon_{ces,t} \tag{1.24}$$

Where p_x and p_x are the price of the inputs of X and Y.

Two-Factor Translog Function:

$$\ln Q_t = \alpha_0 + \alpha_1 \ln X_t + (1-\alpha_1)\ln Y_t + \alpha_2(\ln X_t)^2 + \alpha_3(\ln Y_t)^2 - (\alpha_2+\alpha_3)\ln X_t \ln Y_t \tag{1.25}$$

Where α_i is the parameter. $i = 0, 1, 2, 3$. X,Y are the inputs.

The corresponding first order condition with respect to the Translog function can be expanded with an error term as:

$$\frac{P_{x,t}X_t}{P_{y,t}Y_t} = \frac{\alpha_1 + 2\alpha_2 \ln X_t - (\alpha_2+\alpha_3)\ln Y_t}{(1-\alpha_1) + 2\alpha_3 \ln Y_t - (\alpha_2+\alpha_3)\ln X_t} + \epsilon_{tlog,t} \tag{1.26}$$

Two-Factor Quadratic Function:

$$Q_t = (\alpha_1 X_t^2 + \alpha_2 Y_t^2 + \alpha_3 X_t Y_t)^{\frac{1}{2}} \tag{1.27}$$

The corresponding first order condition with respect to the Quadratic function can be expanded with an error term as:

$$\frac{P_{x,t}}{p_{y,t}} = \frac{2\alpha_1 X_t + \alpha_3 Y_t}{2\alpha_2 Y_t + \alpha_3 X_t} + \epsilon_{qua,t} \tag{1.28}$$

Two-Factor Functional Form Estimation Comparison

Table 1.17 shows the estimation of the government production function with capital and employment. The Quadratic specification has the least value of $D_{measure}$. However, similarity between the estimates from the top panel and the estimates from the bottom panel is not as close as in the CES specification. I general, the CES and Quadratic specifications are more self-consistent than the Translog specification.

Table 1.17: Estimation of the Government Sector with Capital and Labor inputs

		CES		Quadratic		Translog
	α	0.296	a_1		a_1	0.840
		(0.025)				(0.073)
NLS	σ	1.067	a_2	1.551	a_2	0.061
		(0.069)		(0.089)		(0.008)
			a_3	19.248	a_3	0.216
				(0.791)		(0.024)
$D_{measure}$		0.108		0.041		0.075
	α	0.223	a_1	-6.617	a_1	0.001
		(0.007)		(0.107)		(0.016)
Robust	σ	0.999	a_2	0.345	a_2	-0.036
Check		(0.018)		(0.0103)		(0.003)
			a_3	5.097	a_3	-0.044
				(0.035)		(0.004)
No.Obs		85		85		85

Notes: The data set is from 1929–2011 of NIPA. The value in the parenthesis is the standard deviation.

Table 1.18 shows the estimation results of the production functions with the inputs of intermediate goods and employment. The CES specification has the least value of $D_{measure}$. As for the equality between the top panel estimation and the Robust Check, the CES specification and the Translog specification both perform well. So in sum, the CES specification is more self-consistent than the other two specifications.

Table 1.18: Estimation of the Government Sector with Intermediate Goods and Labor inputs

		CES		Quadratic		Translog
	α	0.384	a_1		a_1	0.491
		(0.021)				(0.121)
NLS	σ	1.177	a_2	0.615	a_2	0.029
		(0.067)		(0.129)		(0.009)
			a_3	9.964	a_3	0.059
				(1.499)		(0.033)
$D_{measure}$		0.072		0.091		0.147
	α	0.369	a_1	-0.852	a_1	0.451
		(0.009)		(0.108)		(0.004)
Robust	σ	1.139	a_2	0.312	a_2	0.037
Check		(0.049)		(0.0163)		(0.006)
			a_3	3.290	a_3	0.056
				(0.089)		(0.009)
No.Obs		85		85		85

Notes: The data set is from 1929–2011 of NIPA. The value in the parenthesis is the standard deviation.

Table 1.19 shows the estimation of the production function which includes the inputs of intermediate goods and fixed capital. It is very interesting to see that the values of $D_{measure}$ in this table are all much bigger than in Table 1.17 and in Table 1.18. There are two possible reasons for this: one is that the government is likely to change its consumption of intermediate goods more arbitrarily than other inputs, and the other is that the production function with the inputs of capital and intermediate goods cannot be assumed to be weakly separable with other inputs. However, based

on the self-consistency comparison, the CES and Translog specifications are superior to the Translog specification.

Table 1.19: Estimation of the Government Sector with Intermediate Goods and Capital

		CES		Quadratic		Translog
NLS	α	0.428	a_1		a_1	0.496
		(0.008)				(0.002)
	σ	0.929	a_2	0.784	a_2	0.134
		(0.039)		(0.061)		(0.002)
			a_3	2.043	a_3	0.011
				(0.115)		(0.001)
$D_{measure}$		0.273		0.255		0.762
Robust Check	α	0.337	a_1	0.999	a_1	0.513
		(0.027)		(0.037)		(0.008)
	σ	0.708	a_2	0.933	a_2	0.128
		(0.112)		(0.103)		(0.006)
			a_3	2.046	a_3	0.144
				(0.125)		(0.007)
No.Obs		85		83		85

Notes: The data set is from 1929–2011 of NIPA. The value in the parenthesis is the standard deviation.

1.6 CONCLUSION

In order to estimate the production function of the U.S. government, this paper revisits the method of using an aggregate input quantity index to represent unobservable government outputs. The inputs of U.S. government outputs include three parts: labor, capital and intermediate goods. Following the methods used in [Berndt, 1976] and Antras [2004], I first estimate the U.S. government production function as a CES functional form. During the estimation process I deal with issues related to the heteroskedasticity problem of the disturbances, the endogeneity problem and the nonstationarity of the series. The estimates of the three-factor CES functional form cannot reject the hypothesis that the elasticity of substitution among the three inputs is equal to one. However, the estimates in the two-factor CES functions indicate that we should be cautious when applying a Cobb-Douglas function to represent the government production function with intermediate goods. In fact, government intermediate goods and government employment behave like substitute goods in the context of the government production function. On the other hand, government intermediate goods and government capital behave like complementary goods.

I also expand our discussion of the CES function into a wider family of functional forms. I pick the CES function, the Translog function and the Quadratic function to present the family of linear homogeneous functional forms. Based on exact index theory, I construct two measures to test the consistency between the functional form specification and the actual data set. I find that representing U.S. government production function with a CES production function results in a balanced, if not better, self-consistency performance.

CHAPTER 2

THE COMPONENTS OF GOVERNMENT SPENDING OVER THE BUSINESS CYCLE

The business cycle properties of government spending differ across its components. This paper explores the cyclical movement of government spending components as a result of endogenous responses to exogenous private sector and government sector productivity shocks, and quantifies the relative contributions of these shocks to component volatility. A framework is developed based on a two-sector neoclassical model where a public consumption good is provided through a government production process. Implementation lags and adjustment costs for government spending or its components are included to better capture the business cycle features of the U.S. government. General government and state and local government are analyzed separately to verify the robustness of the model. Simulating the model shows that the model does a good job of accounting for the business cycle dynamics of government spending components. It also reveals that different government spending components respond differently to private and government productivity shocks. In particular, state and local government spending components are affected more by private productivity shocks.

2.1 INTRODUCTION

When the United States economy entered into the zone of the "zero lower bound", research on the business cycle properties of government spending started to pick

up, following a relatively quiet period compared to monetary policy analysis. As a critical part of US economy, government purchases account for around 20% of US GDP. Hence, understanding government spending behaviors and its interactions with the private economy is crucial for policy makers, especially during a period when the monetary policy has extra constraints.

In the literature, there are two complementary approaches in understanding the business cycle properties of government spending. The first approach treats government spending as an exogenous stochastic process and studies, both theoretically and empirically, its consequences to aggregate fluctuations. This approach is featured in the theoretical analysis in Barro and King [1984], Rotemberg and Woodford [1992] and Perotti [2008], and in empirical research on the government spending multiplier, including Mark and Knetter [1997], Price and Kachanovskaya [2010], Lauren et al. [2011] and Gabriel et al. [2010]. The second approach in the literature reverses the perspective of the first approach and instead understands government spending as an endogenous response to other economic shocks. For example, Ambler and Paquet [1996], Azzimonti et al. [2010], Debortoli and Nunes [2010] and Bachmann and Bai [2013a,b] study the amplification mechanisms of government spending in response to total factor productivity (TFP) and woo [2005] and Azzimonti and Talbert [2011] focus on political uncertainty shocks.

In the research mentioned above, government spending is usually assumed to be one homogeneous expenditure on goods. However, this assumption does not fully capture reality, at least in two aspects. First, the cyclical properties of the components of government spending look different to each other. Figure 2.1 shows the cyclical features of U.S. general government spending components. Following the methodology in National Income and Product Accounts (NIPA), government spending can be divided into four components: consumption of fixed capital, government invest-

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ment, compensation to employment and consumption of intermediate goods and services.[1] Figure 2.1.a shows the shares of these four components as a percentage of total government spending for the last four decades. Employment compensation accounts for around 45% of general government spending and consumption to fixed capital accounts for 15%. Government investment and consumption of intermediate goods and services have different trends but they account for 40% in total. Figure 2.1.b shows the fluctuations of these four components over the business cycle. The consumption of fixed capital has the lowest volatility compared with other components. Government investment and consumption of intermediate goods and services are more volatile than government employment. Second, this homogeneity assumption overlooks the different, possibly opposite, impacts on the activities of private economy caused by shocks in different government spending components. In the RBC model for example, an increase in government spending on intermediate goods from the private sector is a positive shock to the private economy, which increases output through the negative wealth effect channel. On the other hand, a rise in government employment that increases compensation to the private sector is more like a transfer from the government sector to the private sector, which not only crowds out private employment but also dampens the negative wealth effect.

Because of these cyclical property and shock impact differences, it is necessary to distinguish government spending components when examine the interactions between government sector and private sector. Finn [1998] and Cavallo [2005] represent the literature which identify exogenous shocks from government spending components and study the effects of these shocks on the private economy. The perspective they use is consistent with the first approach of dealing with the cyclical properties of government

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[1]Intermediate goods used in this paper are a net value, defined as the value of the goods and services purchased from the private sector minus the value of the goods and services sold to other sectors by the U.S. government.

Figure 2.1: Components of General Government Spending Features
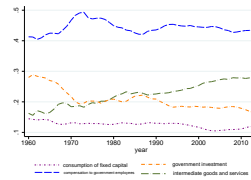
(a) Share of each component
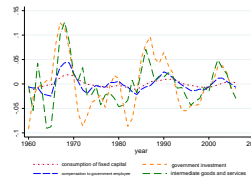
(b) Fluctuations of each component

spending. However, there is few research that studies the endogenous responses of government spending components to economic shocks. This paper's purpose is to fill this gap in the literature.

I first document the business cycle properties of general government as well as state and local government spending and their relevant components. Government spending and its components in this paper are defined to be mainly consistent with "government expenditure on consumption and investment goods", as defined in NIPA. Distinguishing the components of government spending allows for explorations of the transmission channels of the aggregate economy through the interactions between the government production process and the private production process.

I then extend the standard neoclassical model by introducing a benign and cost-efficient government producer. The government produces the public outputs with three inputs - capital, intermediate goods and labor - purchased from competitive markets. The government budget is financed by non-distortionary government taxes

and the public outputs are valued by households. To enable the model to match the dynamic co-movement pattern of government spending components in the data, I assume the productivity shocks of the private sector and the government sector are correlated. In addition, two policy implementation frictions are added to the model economy to fit the empirical patterns of cyclicality. First I assume that today's government decides tomorrow's government's total budget, as well as government employment and investment. Second, an adjustment cost of government investment is added into the government sector, to get a closer approximation to the volatility of the real world. The business cycles in this model economy are generated by productivity shocks from the private sector and the government sector.

I discuss the general government and the state and local government separately. As mentioned in Mark and Knetter [1997], Price and Kachanovskaya [2010] and Lauren et al. [2011], federal defense spending is believed to be more exogenous than state and local government spending. Consequently, we would expect that general government spending is less correlated with private sector shocks compared to state and local government spending. The distinction between the general government and state and local government not only helps to check the robustness of the model; it also provides us an example to illustrate the subtle effects of different productivity correlation levels between government sector and private sector.

In the end, I decompose the fluctuations of the main variables from the private and government sector according to their sources of shocks. The simulation result shows that the model economy can generate a stochastic process of government spending components that is comparable with that of the real world. Meanwhile, the fluctuations of government spending components respond differently to private productivity shocks and government productivity shocks. Among them, government employment is less affected by private productivity shocks. However, for both general government

and state and local government, more than 50% of the fluctuations of government investment, fixed capital consumption and intermediate goods are driven by the private productivity shocks. In particular, when compared with general government, the cyclical properties of state and local government fit better within this endogenous model.

The remainder of this paper is arranged as follows. Section 2 provides a statistical description of the data. Section 3 outlines the model economy. Section 4 describes the methodology of the calibration and simulation. Section 5 and Section 6 present the simulation analysis for the general government and the state and local government separately. Section 7 concludes.

2.2 DATA AND FACTS

The annual data set (1960–2006) used in this paper is from NIPA of the U.S. Bureau of Economic Analysis. The data on the private sector is mainly from Table 1.1.5, Table 2.1 and Table 6.5 of NIPA. The data on the general government and the state and local government is mainly from Table 3.9.4, Table 3.9.5 and Table 7.2 of NIPA. The real values of the variables are either calculated by dividing their current-cost values with the relevant chain-type price indexes or are taken from the real value tables in the NIPA data set directly.

Table 2.1 presents the business fluctuations of the main macro-variables of the U.S. economy from 1960 to 2006. The variables have been Hodrick-Prescott-filtered with the multiplier λ equal to 100. These variables include aggregate output GDP, household consumption c, private investment i_p, total employment in the economy n, private fixed capital k_p, private employment n_p, government fixed capital k_g, govern-

Table 2.1: Business Cyclical features of the U.S. government

j	SD_j	$\frac{SD_j}{SD_{GDP}}$	$\varphi_{j,j-1}$	$\varphi_{j,GDP}$	$\varphi_{j,GDP_{-1}}$
Private Economy					
GDP_t	1.90	1.00	0.55	1.00	0.55
c_t	1.68	0.88	0.61	0.88	0.41
k_{pt}	0.65	0.34	0.75	0.60	0.76
n_{pt}	2.18	1.14	0.57	0.84	0.74
i_{pt}	7.11	3.74	0.41	0.85	0.22
General Government					
G_t	2.47	1.44	0.78	0.36	0.49
k_{gt}	0.78	0.41	0.91	0.24	0.42
n_{gt}	1.58	1.28	0.80	0.24	0.56
i_{gt}	5.32	2.80	0.79	0.50	0.56
m	4.32	2.26	0.75	0.24	0.31
State & Local Government					
G_t	2.14	1.07	0.77	0.40	0.58
k_{gt}	0.60	0.32	0.88	0.11	0.40
n_{gt}	1.41	0.73	0.80	0.20	0.54
i_{gt}	5.16	2.71	0.72	0.63	0.73
m	3.45	1.81	0.77	0.22	0.16

Notes: Sample period of annual data set is 1960–2006; SD denotes percentage standard deviation; $\varphi_{j,j-1}$ is the auto-correlation of the left-side variable; $\varphi_{j,GDP}$ is the correlation between the left-side variable and GDP; $\varphi_{j,GDP_{-1}}$ is the correlation between the left-side variable and one-year lagged GDP.

ment spending G, government employment n_g, public investment i_g and government spending on intermediate goods and services m.

Table 2.1 contains three panels. The top panel shows several well known U.S. cyclical features, such as employment n, which is as volatile as GDP, while private investment i_p is more volatile than GDP, the variables of c, i_p, n, k_p and n_p are strongly pro-cyclical and their contemporaneous correlation with aggregate GDP is higher than the correlation with aggregate one-year lagged GDP.

The second panel of Table 2.1 shows the cyclical features of the variables from the U.S. general government. Two facts need to be emphasized here. First, government spending and its components, except government employment n_g, are more volatile than GDP. Second, in contrast with most of the variables in the private sector of Table 2.1, general government spending and its components have lower contemporaneous correlation with GDP than the correlations with the one-year lagged GDP.

The last panel is at the bottom of Table 2.1, which shows the cyclical fluctuations of the variables from the state and local government. The components of state and local government spending have similar cyclical features as general government spending. However the volatility of the state and local government spending components are smaller than the counterparts in the general government. Also the correlation between government spending components and GDP are relatively higher in state and local government than in general government.

It is important to notice that, in Table 2.1, different components of government spending have different volatility, which reinforces the idea that government spending is not homogeneous.

The main questions left in this paper are: 1) How closely can the model economy replicate these volatility and 2) how much does the cyclical fluctuation of U.S. government spending components depend on private sector shocks and how much does

it depend on those from the government sector. Answering these questions is pursued through a theoretical analysis of the model economy outlined below.

2.3 THE ECONOMIC ENVIRONMENT

This paper is based on a standard neo-classical model. It is expanded to embody a government producer where the production of the government sector is social-welfare-optimized and carried out in a cost-efficient fashion. The policy implementation restrictions of the government sector are added to the economy to be consistent with the real economy. Stochastic exogenous shocks to the productivity of the private sector and of the government sector are the sources of all fluctuations in this model economy. A more exact description of the economy's structure follows.

2.3.1 AGENTS

The model economy has three agents which are the households, the firms and the government. The representative households are the owners of labor and of private capital. The representative firms produce private outputs and the government produces public outputs. Given the production functions and budgets, the private sector and the government sector optimize the productions separately. These agents interact within a perfectly competitive market framework.

HOUSEHOLDS

The economy is populated by a unit mass continuum of infinitely lived identical households. In each period, the household is endowed with one unit of time. It values private consumption c_t, leisure $1 - n_t$, and government public outputs y_{gt}, according

to the utility function

$$\mu(c_t, n_t, y_{gt}) = \frac{1}{1-\gamma} \ln \left[\eta(\theta c_t^{1-\gamma} + (1-\theta)y_{gt}^{1-\gamma}) + (1-\eta)(1-n_t)^{1-\gamma} \right] \qquad (2.1)$$

The series of $\{y_{gt}\}_{t=0}^{\infty}$ are provided to the households by the government sector. The household owns the private capital, k_{pt}. Households rent the private capital out in a perfectly competitive market to the private sector. r_{pt} is the rental rate of the private capital.[2] i_{pt} is the current period investment to private capital. The household also provides labor n_t in a competitive labor market. w_t is the real wage. n_{pt} is the labor hired by the private sector and n_{gt} is the labor hired in the government sector. Total labor n_t supply equals the labor supply in the private sector n_{pt} plus the labor supply in the government sector n_{gt}.

$$n_t = n_{pt} + n_{gt} \qquad (2.2)$$

In the model economy, the household's optimization problem can be expressed as:

$$\max_{c_t, n_{pt}, i_{pt}} E_0 \sum_{t=0}^{\infty} \beta^t \mu(c_t, n_t, y_{gt}) \qquad (2.3)$$

where β is the preference discount factor, with the respect to the budget constraint

$$w_t n_t + r_{pt} k_{pt} = c_t + T_t + i_{pt} \qquad (2.4)$$

where T_t is the lump-sum tax collected by the government at period t. The law of motion of the private capital stock can be written as

$$k_{pt+1} = k_{pt}(1 - \delta_p) + i_{pt} \qquad (2.5)$$

where δ_1 is the depreciation rate of private capital.

---

[2]To be consistent with the government budget restrictions in the real economy, this paper assumes the households do not own the public capital.

Private Firm

The representative firm produces private goods in accordance with a two-factor Cobb-Douglas production function. The firm is profit-maximizing, and hires labor, n_{pt}, and borrows the private capital in competitive markets from the households. The rental rate of the capital is r_{pt}. The price of the private firm's output, y_{pt}, is normalized to be one and the price of labor is w_t. The outputs of the private sector are consumed in four ways, including consumption c_t, private investment i_{pt}, government intermediate goods m_t and government investment i_{gt}.

The production function of the private sector can be expressed as

$$y_{pt} = z_{pt} k_{pt}^b n_{pt}^{1-b} \tag{2.6}$$

where z_{pt} is productivity for the private sector and b is the capital share of the production in the private sector.

I assume the technology of private production z_{pt} evolves following an AR(1) process, as shown in equation (2.7)

$$\ln(z_{pt}) = (1 - \rho_p) \ln(\overline{z}_p) + \rho_p \ln(z_{pt-1}) + \epsilon_{pt} \tag{2.7}$$

where ρ_p is the auto-correlation coefficient of $\ln(z_{pt})$, $\ln(\overline{z}_p)$ is the productivity at steady state, ϵ_{pt} is a serially uncorrelated independent and identically distributed process with mean 0 and standard error σ_{pt}.

Government Producer

The government produces public goods, y_{gt}, in accordance with a three-factor Cobb-Douglas production function, as shown in equation (2.8). The goal of the government sector is to maximize the household's utility. So given the production function, the

government will choose the optimal budget GB_t in equation (2.12)and the optimal inputs y_{gt} that are produced according to

$$y_{gt} = z_{gt} m_t^{d_1} k_{gt}^{d_2} n_{gt}^{1-d_1-d_2} \tag{2.8}$$

The inputs for the production of government sector are government employment n_{gt}, government fixed capital k_{gt} and government intermediate goods and services m_t. Labor services, government investment and government intermediate goods and services are bought from the competitive markets. z_{gt} is the productivity of the government sector. In the model economy, the innovation shocks to z_{gt} and z_{pt} are correlated.For the sake of simplicity, as shown in equation (2.9), I assume the productivity of the government sector is similar to the private sector and follows AR(1) process. The innovation shocks of the government technology, ϵ_{gt}, contains two separable parts $\alpha \epsilon_{pt}$ and $\epsilon_{z_{gt}}$. The value of α and the variances of both shocks determine the level of the correlation between the productivity shock and the public productivity shock. Given the variances of ϵ_{pt} and $\epsilon_{z_{gt}}$, A higher value of α means higher correlation between these two productivity shocks. z_{gt} evolves as shown in equation (2.9) and equation (2.10). In equation (2.9), ρ_g is the auto-correlation coefficient of $ln(z_{gt})$. ϵ_{pt} is the productivity innovations to the private sector. $\epsilon_{z_{gt}}$ is a serially independent and identically distributed shocks with mean 0 and standard error $\sigma_{z_{gt}}$. $\{\epsilon_{z_{gt}}\}_{t=0}^{\infty}$ is independent to $\{\epsilon_{pt}\}_{t=0}^{\infty}$.

$$\ln(z_{gt}) = (1 - \rho_g) \ln(\bar{z}_g) + \rho_g \ln(z_{gt-1}) + \epsilon_{gt} \tag{2.9}$$

$$\epsilon_{gt} = \alpha \epsilon_{pt} + \epsilon_{zgt} \tag{2.10}$$

In this model economy, the government is assumed to be the owner of the public capital. There are two reasons for this assumption. The first one is to make the model economy to be as close as possible to the real economy. In the real world,

the government does not have to pay for the rent of the public capital. Second, this assumption justifies that the government budget includes the purchases of government investment as well as government intermediate goods and government employment. Compared with the private sector, most people believe policy implementation lags are longer in the government sector. If the private sector decided government investment then it would have no such lags. So it is logical to assume that it is the government who makes the decision for the public investment in this model economy, to replicate the cyclical properties of government spending in the real world.

Equation (2.11) is the law of motion of government capital. Government capital has a depreciation rate σ_g. i_{gt} is public investment. To make the cyclical property of the variables as close as possible to the real economy, the model economy assumes an adjustment cost for government investment. This adjustment cost is determined by the parameter Ω. Higher Ω means higher cost of adjusting the public capital.[3] At the steady state, the government investment is constant so the last term of equation (2.11)becomes null.

$$k_{gt+1} = k_{gt}(1 - \sigma_g) + i_{gt} + \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 \tag{2.11}$$

In this model economy, the government budget could be described as in equation (2.12). There is a minor difference between the government output in NIPA and the government spending in the model economy. In NIPA, the consumption of public capital is part of the government consumption expenditure. However, in the real world the consumption of fixed capital is not part of the government outlay, because the government owns the government capital. I define the government budget to be consistent with the real world, as defined below:

---

[3]The main purpose of adding Ω is to bring the standard deviation of government investment to a similar level as the real data.

$$GB_t = m_t + w_t n_{gt} + i_{gt} + \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 \tag{2.12}$$

The government budget, GB_t, is financed by the lum-sum tax, T_t, levied on the current income of the households.

$$GB_t = T_t \tag{2.13}$$

To be consistent with the definition of "government consumption expenditure and investment", government spending G_t in the model economy is defined as:

$$G_t = m_t + w_t n_{gt} + \delta_g k_{gt-1} + i_{gt} + \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 \tag{2.14}$$

where the value of fixed capital consumption $\delta_g k_{gt-1}$ is imputed based on the depreciation rate of government fixed capital.

The goal of the government producer is to maximize the welfare of the household. Because of the budget planning process of the U.S. government, it is quite standard to assume an implementation lag for government spending and its components. Here I assume tomorrow's government's total budget, labor and investment are chosen to maximize the expected welfare function of the representative household today.

2.3.2 Optimality Conditions of the Model Economy

In the model economy, given the prices of the inputs, the government has one-period implementation lag for the government budget GB_t. I assume at period t, the cost-efficient government makes the optimal decision for n_{gt+1}, GB_{t+1} and i_{gt+1} for next period to maximize the welfare of the household. Meanwhile, the representative household chooses c_t, n_t and i_{pt} in period t to maximize its own expected utility. The representative competitive firm takes the prices in the competitive markets as given and maximizes its own profit by choosing the optimal private inputs and outputs.

The equilibrium occurs when the firm, the government and households solve their optimization problems and there exists a price set that makes all competitive markets clear. The price set (w_t, r_{pt}) in the equilibrium is determined in free markets. So there is no market distortion in the model economy at steady state. The equilibrium is implicitly determined by the laws of motion for k_{pt} and k_{gt}, market clearing conditions, the stochastic exogenous process and optimality equations.

Intuitively, this Stackelberg problem is equivalent to a social planner's problem. Let's think about two scenarios. First, the government picks the optimal values of the GB_{t+1}, n_{gt+1} and i_{gt+1} at period t. The second scenario is that there exists a social planner who picks the optimal values of control variables for households and government. Since the best the government can do is incorporated with the second scenario, the best solution in the first scenario should be no better than that of the second scenario. Furthermore, since there is no price or tax distortion in this economy, given the optimal choices of government control variables in the second scenario, the household could pick its own control variables c_t, n_{pt} and i_{pt} to replicate the second scenario economy in the first scenario economy. Therefore, the economy in the second scenario and the first scenario should be the same. Appendix A also provides a strict proof.

In accordance with this logic, the original economic environment could be represented by a social planner and a private producer economy. The private producer is still the same as in the original economy, while the social planner makes optimal decisions for both government and household sector. The optimization problem of the social planner could be described as follows: at period t, given the price of labor w_t and rental rate of the private capital r_{tp}, the household picks GB_{t+1}, k_{pt+1}, n_{pt}, n_{gt+1} and k_{gt+2} to optimize its utility function.

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PLANNER FOR THE GOVERNMENT AND HOUSEHOLD

In this planner and private producer economy, the planner's problem can be displayed as:

$$
\max_{\substack{y_{gt}, n_{gt+1}, k_{gt+2}, i_{gt+1} \\ GB_{t+1}, c_t, n_{pt}, k_{pt+1}}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \ln[\eta \theta c_t^{1-\gamma} + \eta(1-\theta)y_{gt}^{1-\gamma} + (1-\eta)(1-n_{gt}-n_{pt})^{1-\gamma}]
$$

$$(2.15)$$

s.t.

$$
y_{gt} = z_{gt} m_t^{d_1} k_{gt}^{d_2} n_{gt}^{1-d_1-d_2} \tag{2.16}
$$

$$
k_{gt+1} = k_{gt}(1-\delta_g) + i_{gt} + \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 \tag{2.17}
$$

$$
GB_t = m_t + w_t n_{gt} + i_{gt} + \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 \tag{2.18}
$$

$$
w_t(n_{pt} + n_{gt}) + r_{pt} k_{pt} = c_t + k_{pt+1} - (1-\delta_P)k_{pt} + GB_t \tag{2.19}
$$

Assuming $A_t = \eta(\theta c_t^{1-\gamma} + (1-\theta)y_{gt}^{1-\gamma} + (1-\eta)(1-n_t)^{1-\gamma}$, then the optimality conditions for this planner are shown below:

$$\eta(1-\theta)y_{gt}^{-\gamma} + \lambda_{1t}A_t = 0 \quad (2.20)$$

$$-(1-\eta)(1-n_{gt+1}-n_{pt+1})^{-\gamma}$$

$$-A_{t+1}\lambda_{1t+1}z_{gt+1}(1-d_1-d_2)m_{t+1}^{d_1}k_{gt+1}^{d_2}n_{gt+1}^{-d_1-d_2}$$

$$+A_{t+1}(\lambda_{4t+1}-\lambda_{3t+1})(w_{t+1}) = 0 \quad (2.21)$$

$$-\beta\lambda_{1t+2}z_{gt+2}d_2m_{t+2}^{d_1}k_{gt+2}^{d_2-1}n_{gt+2}^{1-d_1-d_2} + \lambda_{2t+1} - \beta\lambda_{2t+2}(1-\delta_g) = 0 \quad (2.22)$$

$$(\lambda_{2t+1}+\lambda_{3t+1})(-1-\Omega(i_{gt+1}-i_{gt})) + \beta(\lambda_{2t+2}+\lambda_{3t+2})\Omega(i_{gt+2}-i_{gt+1}) = 0 \quad (2.23)$$

$$\lambda_{3t} - \lambda_{4t} = 0 \quad (2.24)$$

$$\eta\theta c_t^{-\gamma} - A_t\lambda_{4t} = 0 \quad (2.25)$$

$$-(1-\eta)(1-n_{gt}-n_{pt})^{-\gamma} + A_t\lambda_{4t}w_t = 0 \quad (2.26)$$

$$-\lambda_{4t} + \beta\lambda_{4t+1}r_{pt+1} = 0 \quad (2.27)$$

where λ_{it}, $i=1,2,3,4$ are the Lagrange multipliers for equation (2.16) to(2.19) separately.

Private Firm

The optimization problem for the private producer can be displayed as:

$$\min_{n_{pt},k_{pt}} w_t n_{pt} + r_t k_{pt}$$

$$s.t.$$

$$y_{pt} \le z_{pt}k_{pt}^b n_{pt}^{1-b}$$

The optimality conditions for the private sector are:

$$n_{pt} = z_{pt}bk_{pt}^{b-1}n_{pt}^{1-b} \quad (2.28)$$

$$r_{pt} = z_{pt}(1-b)k_{pt}^b n_{pt}^{1-b} \quad (2.29)$$

EQUILIBRIUM CONDITIONS FOR THE ECONOMY

The market clearing conditions for the goods market and labor market are shown in equation (2.30) and equation (2.31):

$$c_t + GB_t + i_{pt} - w_t n_{gt} = y_{pt} \tag{2.30}$$

$$n_{pt} + n_{gt} = n_t \tag{2.31}$$

where n_t is the total labor supply from the household.

The law of motion of the private capital is defined as:

$$k_{pt+1} = k_{pt}(1 - \delta_p) + i_{pt} \tag{2.32}$$

To be consistent with the accounting method in the NIPA, G_t and GDP_t of this model economy is defined as:

$$G_t = GB_t + \delta_g k_{gt} \tag{2.33}$$

$$GDP_t = c_t + G_t + i_{pt} \tag{2.34}$$

And this model economy can be represented by equation (2.16)–equation (2.34).

2.4   CALIBRATION

The calibration procedure advanced by Kydland and Prescott (1982) is adopted in this paper. In this procedure, values are assigned to the model's parameters to simulate the cyclical properties of the real economy. The data of the real economy are based on an annual data set from 1960–2006 NIPA. The model's time period is defined as one year and the calibration recognizes this definition.

There are 15 parameters in the model. Take the calibration of the general government as an example. As shown in Table 2.2, γ, β, θ and η are related to the

household's utility function. γ is the inverse elasticity of substitution in consumption of the household's utility function. I set it at 0.5 which is typical of the macro business-cycle literature. It means c and y_g are Edgeworth substitutes. β is the Preference discount factor for the utility function. It is calibrated by the ratio of fixed capital to the outputs in the private sector. The calculated value of β is 0.94. θ is the weight of private sector goods in the utility function, which determines the marginal utility ratio of the private goods and public goods consumed by the household. θ in this model economy is calibrated by the ratio of labor inputs between the private and government sector. In the utility function, $1 - \eta$ determines the weight of leisure, compared with the consumption goods. Higher η means higher desire of the household to provide labor. η is calibrated by the fraction of total hours in the year devoted to work which is around 2000/8760=0.23.

The private production sector has four parameters, b, δ_p, ρ_{zp} and σ_{ep}. b is the share of fixed capital in private outputs. The key to calibrate b is to calculate the capital income in the private production sector. Following the standard process, I divide the capital income of the private sector from the NIPA data set into three categories: unambiguous capital income, ambiguous capital income and corporate cash flow. The definition of these two capital income are listed below.

$$Unambiguous\ Capital\ Income = Rental\ Income + Coporate\ Profits + Net\ Interest$$

$$Ambiguous\ Capital\ Income = Proprietors\ Income + GDP - National\ Income$$

And the corporate cash flow is defined as :

$$Cash\ Flow = Undistributed\ Profits + Consumpion\ of\ fixed\ capital - Capital\ Transfer$$

So the capital income in the private sector could be determined by:

$$y_{kp} = Unambiguous\ Capital\ Income + b \times Ambiguous\ Capital\ Income + Cash\ Flow$$

$$= b \times y_p$$

Therefore, the capital share b could be estimated by the following formula:

$$b = \frac{Unambiguous\ Capital\ Income + Cash\ Flow}{y_p - Ambiguous\ Capital\ Income}$$

The depreciation rate of private capital δ_p is pinned down by the ratio of private investment to the private capital stock. δ_p should be the same in both of the models for the general government and state and local government. Next, I estimate the AR(1) process of the private productivity shocks $\ln(z_{pt})$. The auto-regressive parameter ρ_p and the standard deviation of the innovations of private productivity σ_{ep} are used to describe the evolution of the private productivity. Once we calibrate the value of b, the perpetual inventory method is used to estimate both ρ_{zp} and σ_{ep}. This involves inputting the $\{i_t\}_{t=1}^T$ and k_0 into the law of motion of capital to obtain $\{k_t\}_{t=1}^T$.

$$k_{pt+1} = i_t + (1 - \delta)k_{pt}$$

The measure of $\{k_t\}_{t=1}^T$, $\ln(z_{pt})$ is derived following Solow and Swan as the unexplained component of $\ln(y_{pt})$ given the inputs of $\ln(k_{pt})$ and $\ln(n_{pt})$ in the likelihood function:

$$\ln(z_{pt}) = \ln(y_{pt}) - \alpha \ln(k_{pt}) - (1 - \alpha) \ln(n_{pt})$$

I apply the annual Hodrick-Prescott filter to $\ln(z_{pt})$, and estimate ρ_{pz} and δ_{pz} using the HP-filtered version of $\ln(z_{pt})$.

The government production process contains seven parameters. d_1 is the share of the intermediate goods in government consumption expenditure. d_2 is the share of consumption of fixed capital in total government spending. They are calibrated by the corresponding part of NIPA data. δ_2 is the depreciation rate of the capital of the government sector. It is determined by the ratio of the public investment to public capital stock. ω determines the cost of the public investment which is calibrated by standard deviation of government investment. Given e_{zpt}, the parameters

of ρ_{zg}, α and σ_{zgt} describe the cyclical process of government productivity z_{gt}. ρ_{zg} is the auto-regressive parameter of z_{gt}, which is calibrated by the auto-regression from total government purchases. α and σ_{zg} are calibrated by the standard deviation of government purchases and the correlation between government purchases and GDP.

2.5 GENERAL GOVERNMENT CALIBRATION AND SIMULATION RESULTS

Table 2.2 shows the calibration results of the model economy, which uses the general government data set. The first column of Table 2.2 lists all of the necessary parameters. These include $\gamma = 0.5$, which is the inverse of the elasticity of substitution in consumption. It is taken from the literature (eg.Bachmann and Bai [2013b]). ρ_{zp} and σ_{zp} are from estimations for private production. The estimation processes are discussed in the previous section. The remaining parameters are calibrated with the targets listed in the last column. $\alpha = 0.42$ indicates that the correlation between the shock of the private sector to the government sector is positive. That is quite intuitive, as the government sector and the private sector are not independent sectors of the society. The new technology should affect both sectors, so they are positively correlated. The values of all of the parameters are presented in the third column.

Table 2.3 compares key macro-variables of the model economy at the steady state with the average values of the real economy. Steady state variables of the model economy are denoted using the same notation as before except that time subscripts are omitted. The first four variable ratios are used as targets in the calibration. So they are the same as the real economy. The last two ratios are a robustness check to see if the model economy has the same properties as the real economy in the steady state. I find that the characteristics of the model economy at steady state are close to those of the real economy. The ratio of government employment to private

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Table 2.2: Calibration of the Economy with the U.S. General Government

Parameter	Description	Value	
Exogenously Given			
γ	Inverse of elasticity of substitution in consumption	0.5	
Calibration			Target
β	discount factor	0.94	$\frac{k_p}{GDP} = 2.13$
θ	weight of private goods consumption	0.5	$\frac{n_g}{n_p} = 0.22$
η	weight of total goods consumption	0.46	$n = 0.23$
b	capital share of private outputs	0.39	$\frac{y_{kp}}{y_p} = 0.39$
d_1	intermediate goods share of government spending	0.27	$\frac{m}{GB} = 0.27$
d_2	government capital consumption share	0.16	$\frac{CK}{GB} = 0.16$
δ_1	depreciation rate of the capital in private sector	0.1	$\frac{i_p}{k_p} = 0.1$
δ_2	depreciation rate of the capital in government sector	0.08	$\frac{i_g}{k_g} = 0.08$
Ω	government investment implementation cost	13	$std.i_g = 5.32$
ρ_{zg}	auto-regressive parameter of $ln(z_{gt})$	0.9	$corr(g, g_{-1}) = 0.78$
α	weight of ϵ_{pt} in ε_{gt}	0.42	$corr(g, gdp) = 0.36$
σ_{ezg}	standard deviation of ϵ_{zgt}	0.028	$std.g = 2.47$
Estimation			
ρ_{zp}	auto-regressive coefficient of $ln(z_{pt})$ process	0.9	
σ_{zp}	standard deviation of the innovation of $ln(z_{pt})$	0.0123	

Notes: Sample period of data is from 1960–2006 ; The data set for the government is from the entry of the Government Consumption Expenditure and Investment in the NIPA.

employment is smaller than the real data(0.16 to 0.22). The ratio of the consumption to the GDP is 0.57 which is close but smaller than in the real value 0.63.

Table 2.3: General Gov: Simulation Steady States

Parameter	Description	Model	Data
$\frac{k_p}{y_p}$	the capital of the private sector to the GDP	2.10	2.13*
$n_p + n_g$	total labor supply	0.23	0.23*
$\frac{g}{GDP}$	thegovernment spending to the GDP	0.21	0.21*
$\frac{i_p}{k_p}$	the ratio of investment to capital	0.1	0.1*
$\frac{n_g}{n_p}$	the labor in government sector to the private sector	0.16	0.22
$\frac{c}{GDP}$	the private consumption to the GDP	0.57	0.63

Notes: The star mark * indicates that the parameter value is used as a calibration target

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Table 2.4 presents the simulation results over the business cycle of the private sector with those of the general government sector. Column 2 of Table 2.4 reports the standard deviations of the simulated variables. Column 3 reports the ratios of the simulated standard deviations to the real standard deviations. These ratios do not diverge much from one. In fact, the standard deviations of the macro-variables in the private sector are quite typical, such as reported by Hodrick and Prescott [1997]. As for the government sector, the rational government model economy generates sizable fluctuations, which is also comparable with the real world. The only exception is the consumption of the intermediate goods m. The model economy only generates 30% of the variations of m. The less volatility of m in the model economy is possibly because adjusting intermediate goods is easier compared to the employment adjustment and the investment adjustment. Column 4–6 of Table 2.4 presents the auto-correlations of the variables and the correlations of these variables with GDP. The model economy displays the same features as the real economy. For example, the government variables are all mildly pro-cyclical and the contemporaneous correlation between these variables and GDP is smaller than the correlation between them and the one-year lagged GDP. These characteristics back up the assumption about the policy implementation lags.

In the model economy, there are two exogenous shocks, ϵ_{pt} and ϵ_{zgt}, which together generate cyclical volatility. In the model, private productivity is only affected by e_{zp} while the productivity of the government sector is affected by e_{zp} and e_{zgt}. The main question is how much fluctuations of government spending and of relevant components are caused by the productivity shocks from the private sector. As shown in Table 2.5, in the model economy, the shocks to private productivity generate over 90% of the fluctuations in aggregate GDP, private consumption, total employment, private investment and private capital. Although in the model economy only around 30% of

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Table 2.4: General Gov: Moments Comparison

j	SD_j	$\frac{SD_j}{SD_{real}}$	$\varphi_{j,j-1}$	$\varphi_{j,GDP}$	$\varphi_{j,GDP_{-1}}$
			Private Sector		
GDP_t	1.96	1.03	0.53	1.00	0.53
c_t	1.19	0.71	0.75	0.85	0.77
n_t	1.36	0.70	0.46	0.97	0.41
i_{pt}	6.43	0.91	0.34	0.90	0.22
k_{pt}	1.08	1.66	0.81	0.25	0.76
n_{pt}	1.56	0.72	0.43	0.96	0.38
			General Government		
G_t	2.45	0.99	0.42	0.41	0.63
k_{gt}	0.44	0.56	0.82	0.03	0.43
n_{gt}	1.83	1.16	0.48	0.18	0.21
i_{gt}	5.26	0.99	0.18	0.24	0.46
m	2.35	0.54	0.40	0.12	0.16

Notes: SD_j denotes percentage standard deviation of variable J; SD_{real} denotes the standard deviation of the parameter in the real world; $\varphi_{j,j-1}$ is the auto-correlation of the left-side variable; $\varphi_{j,gdp}$ is the correlation between the left-side variable and GDP; $\varphi_{j,gdp-1}$ is the correlation between the left-side variable and one-year lagged GDP

the fluctuations in total government spending are caused by private shocks, nearly 50% of the government investment and intermediate goods fluctuations are provoked by private productivity shocks. But if we take into account the result that the model economy only generates 30% of the fluctuation of the intermediate goods component, then for the general government economy, about 15% of the fluctuations could be explained by the private sector.

Another interesting result in Table 2.5 is that private shocks explain little about the fluctuations in government employment. This implies that government employment has the potential to be used as an instrumental variable to estimate the reactions of the private sector to changes in government spending. As shown in Table 2.5, the

Table 2.5: General Gov: Variance Decomposition

	ϵ_{zp}	ϵ_{zg}
GDP_t	98.39	1.61
c_t	98.55	1.45
n_t	93.05	6.95
i_{pt}	97.57	2.43
k_{pt}	99.25	0.75
n_{pt}	97.42	2.58
G_t	30.94	69.06
k_{gt}	61.13	48.87
n_{gt}	1.08	98.92
i_{gt}	49.50	50.50
m	45.71	54.29

Notes: For correlated shocks, the variance decomposition goes through a Cholesky decomposition of the covariance matrix of the exogenous vairablese_{zp} and the e_{zg}. This table shows the decomposition in the general government environment

shock e_{zgt} of government productivity, which the independent from the private productivity shocks, only barely affect the volatility of the private sector. Such a result is consistent with the results from Finn [1998] within the neo-classical model, which finds that government spending is not a significant driving source of the U.S. business cycle.

To check the validity of alternative assumptions about government productivity and private productivity, two additional experiments are implemented. The first experiment assumes government productivity is constant. This situation happens when we ignore the productivity of the government sector. The cyclical properties of this case are shown in Table 2.6. Without the productivity shock from the government sector, government spending and its components are much less volatile. Meanwhile,

Table 2.6: Constant General Gov Productivity

j	SD_j	$\frac{SD_j}{SD_{real}}$	$\varphi_{j,j-1}$	$\varphi_{j,GDP}$	$\varphi_{j,GDP_{-1}}$
			Private Sector		
GDP_t	1.92	1.01	0.53	1.00	0.53
c_t	1.19	0.71	0.73	0.88	0.77
n_t	1.28	0.66	0.43	0.98	0.38
i_{pt}	6.28	0.88	0.37	0.92	0.24
k_{pt}	1.07	1.65	0.81	0.23	0.76
n_{pt}	1.51	0.69	0.44	0.97	0.39
			General Government		
G_t	1.15	0.47	0.55	0.48	0.98
k_{gt}	0.32	0.41	0.86	-0.39	-0.07
n_{gt}	0.15	0.09	0.71	-0.47	-0.29
i_{gt}	3.31	0.62	0.31	0.24	0.53
m	1.45	0.34	0.22	-0.04	0.81

Notes: This table reports the results of the conterfactual experiment that the productivity of the general government is fixed while other parameters are still used the calibrated ones; SD_j denotes percentage standard deviation of variable J; SD_{real} denotes the standard deviation of the parameter in the real world; $\varphi_{j,j-1}$ is the autocorrelation of the left-side variable; $\varphi_{j,gdp}$ is the correlation between the left-side variable and GDP; $\varphi_{j,gdp-1}$ is the correlation between the left-side variable and one-year lagged GDP

the results indicate that without positive correlation between the private productivity shock and the government productivity shock, k_g and n_g are counter-cyclical, which are contradictory to the real data. The second counter-factual experiment presumes that the shocks to the government productivity are the same as the private sector. Table 2.7 shows the cyclical results in this situation. Now the government employment has much lower fluctuations, and the correlations between the government-related variables and one-year lagged GDP are much higher than in the real world. But the results are closer to the real world than the first counter-factual experiment, which justify the assumption that government productivity is affected by the private sector.

Table 2.7: The Government Sector and the Private Sector Share the Same Shocks

j	SD_j	$\frac{SD_j}{SD_{real}}$	$\varphi_{j,j-1}$	$\varphi_{j,GDP}$	$\varphi_{j,GDP_{-1}}$
		Private Sector			
GDP_t	2.02	1.06	0.54	1.00	0.54
c_t	1.15	0.68	0.74	0.87	0.78
n_t	1.41	0.73	0.47	0.98	0.43
i_{pt}	6.58	0.93	0.32	0.91	0.20
k_{pt}	1.09	1.68	0.80	0.26	0.78
n_{pt}	1.61	0.74	0.42	0.97	0.62
		General Government			
G_t	1.98	0.80	0.46	0.57	0.99
k_{gt}	0.43	0.55	0.83	0.03	0.56
n_{gt}	0.72	0.46	0.52	0.56	1.00
i_{gt}	4.82	0.91	0.22	0.26	0.56
m	2.03	0.47	0.29	0.23	0.94

Notes: This table reports the results of the conterfactual experiment that the productivity of the general government is the same as that of private sector; SD_j denotes percentage standard deviation of variable J; SD_{real} denotes the standard deviation of the parameter in the real world; $\varphi_{j,j-1}$ is the autocorrelation of the left-side variable; $\varphi_{j,gdp}$ is the correlation between the left-side variable and GDP; $\varphi_{j,gdp-1}$ is the correlation between the left-side variable and one-year lagged GDP

In sum, the assumptions of a partial, positively correlated relationship between government productivity and private productivity, as well as a cost efficient endoge-

nous government production process with implementation frictions can capture the main cyclical features of the U.S. general government and private sector. Also in the model economy, over half of the fluctuations of intermediate goods and investment are caused by shocks from the private sector. However, general government employment is less likely affected by shocks from the private sector.

2.6   State and Local Government Calibration and Simulation

Although the definitions of the spending components are the same, the purposes of federal government spending and state and local government spending are notably different. Federal government spending includes large expenses such as public infrastructure construction, federal defense and national education, while the spending of the state and local government is more connected to the local economy. So conventional wisdom suggests that federal government spending should be more exogenous to the private economy than state and local government.

In this section, the government sector of the model economy is re-calibrated to match the state and local government of the U.S. The method used here is the same as the one used in the previous section. The simulation differences of the model calibrated to match the general government, which includes both the federal and state and local government, and the one calibrated to match only the state and local government will help us better understand the origin of the simultaneous endogeneity between the government and private sector, shedding light on the subtle structure of government spending.

Table 2.8 shows the calibration results for the model economy which only takes into account state and local government. There are several differences between the general government and state and local government calibrations. First, the share of

the consumption of capital in general government production is higher than for the state and local government(0.16 compared to 0.08). Also the depreciation rate for the general government is 0.08, which is higher than the state and local government at 0.05. The most important difference is the correlation between shocks to government production and to private production as a result of technological change. The calibration for local government, $\alpha = 0.35$, is smaller than 0.42 for the general government sector. However, the productivity correlation with private sector is higher in state and local government. The reason is because the productivity of general government is more volatile. Another interesting finding is that in order to get the consistent cyclical properties, the adjustment cost should be higher in the state and local government than in the general government.

Table 2.8: Calibration of the Economy with the U.S. State and Local Government

Parameter	Description	Value	
Exogenously Given			
γ	Inverse of elasticity of substitution in consumption	0.5	
Calibration			Target
β	discount factor	0.94	$\frac{k_p}{GDP} = 2.13$
θ	weight of private goods consumption	0.5	$\frac{n_g}{n_p} = 0.11$
η	weight of total goods consumption	0.46	$n = 0.23$
b	capital share of private outputs	0.39	$\frac{y_{kp}}{y_p} = 0.39$
d_1	intermediate goods share of government spending	0.27	$\frac{m}{GB} = 0.27$
d_2	government capital consumption share	0.08	$\frac{CK}{GB} = 0.08$
δ_1	depreciation rate of capital in private sector	0.1	$\frac{i_p}{k_p} = 0.1$
δ_2	depreciation rate of capital in government sector	0.06	$\frac{i_g}{k_g} = 0.05$
Ω	government investment implementation cost	30	$std.i_g = 5.16$
ρ_{zg}	autoregressive parameter of $ln(z_{gt})$	0.9	$corr(g, g_{-1}) = 0.77$
α	weight of ϵ_{pt} in ε_{gt}	0.35	$corr(g, gdp) = 0.40$
σ_{ezg}	standard deviation of ϵ_{zgt}	0.017	$std.g = 2.14$
Estimation			
ρ_{zp}	auto-regressive coefficient of $ln(z_{pt})$ process	0.9	
σ_{zp}	standard deviation of the innovation of $ln(z_{pt})$	0.0123	

Notes: Sample period of data is from 1960–2006 ; The data set for the government is from the entry of the Government Consumption Expenditure and Investment from the Local and State government in the NIPA.

Table 2.9 shows the steady state results in this model economy. Compared with Table 2.3, the steady state ratios in the model economy are closer to the real data. In the two robustness checks, for example, the simulation produces a very close government labor to private labor ratio (0.14 compared with the real ratio 0.15) and consumption to GDP ratio (0.66 compared with the real ratio 0.63). This suggests that the altruistic and cost-efficient government production assumption should be a better fit for the state and local government than for the general government in the U.S.

Table 2.9: State and Local Gov: Simulation Steady States

Parameter	Description	Model	Data
$\frac{k_p}{y_p}$	the capital of the private sector to the GDP	2.18	2.13*
$n_p + n_g$	total labor supply	0.23	0.23*
$\frac{g}{GDP}$	thegovernment spending to the GDP	0.11	0.11*
$\frac{i_p}{k_p}$	the ratio of investment to capital	0.1	0.1*
$\frac{n_g}{n_p}$	the labor in government sector to the private sector	0.14	0.15
$\frac{c}{GDP}$	the private consumption to the GDP	0.66	0.63

Notes: The star mark * indicates that the parameter value is used as a calibration target

Table 2.10 shows the business cycle features of the model economy which only takes into account the state and local government data set. The results suggest even further that the state and local model economy better fits the real economy. The private sector in Table 2.10 is still very standard in the neo-classical framework. The second column of the government sector indicates that the model generates similar fluctuations of the variables. Even for the intermediate good, this model can explain 60% percent of the fluctuations, which is much higher than in the general government sector case, which is around 30%. The cyclical properties in the last three columns of the Table 2.10 are also consistent with the observations in the real data. For example,

the correlation with lagged GDP is higher than with the current period GDP and i_g has the lowest auto-correlation. These similarities indicate that the endogenous and cost-efficient government assumptions are more suitable to the state and local government, and that the conventional wisdom that state and local government is more correlated with the private economy would be correct.

Table 2.10: Moment Properties of State and Local Government

j	SD_j	$\frac{SD_j}{SD_{real}}$	$\varphi_{j,j-1}$	$\varphi_{j,GDP}$	$\varphi_{j,GDP_{-1}}$
		Private Sector			
GDP_t	1.96	1.03	0.53	1.00	0.56
c_t	1.12	0.67	0.73	0.88	0.75
n_t	1.32	0.68	0.45	0.98	0.43
i_{pt}	6.01	0.85	0.38	0.93	0.29
k_{pt}	1.03	1.58	0.82	0.23	0.75
n_{pt}	1.47	0.67	0.40	0.97	0.60
		State and Local Government			
G_t	2.11	0.99	0.50	0.40	0.77
k_{gt}	0.50	0.83	0.80	0.06	0.48
n_{gt}	1.49	1.05	0.49	0.32	0.49
i_{gt}	5.14	0.99	0.14	0.24	0.44
m	2.74	0.79	0.25	0.07	0.80

Notes: SD_j denotes percentage standard deviation of variable J; SD_{real} denotes the standard deviation of the parameter in the real world; $\varphi_{j,j-1}$ is the auto-correlation of the left-side variable; $\varphi_{j,gdp}$ is the correlation between the left-side variable and GDP; $\varphi_{j,gdp-1}$ is the correlation between the left-side variable and one-year lagged GDP

If the conventional wisdom is right, then we should expect higher percentages of government spending fluctuations are driven by shocks in the private sector. Table 2.11 illustrates this point. As shown in Table 2.11, shocks from the private sector cause in the model economy, 56% of the fluctuation of government spending; 57% of the fluctuation of government investment; over 78% of the fluctuations of intermediate goods and 21% of government employment fluctuations. Even taking the second

column of Table 2.10 into account, this model shows that at least 48% of the fluctuation of intermediate goods is driven by the private productivity shocks in the real world.

Table 2.11: State and Local Gov: Variance Decomposition

	ϵ_{zp}	ϵ_{zg}
GDP_t	99.67	0.33
c_t	99.85	0.15
n_t	98.51	1.49
i_{pt}	99.71	0.29
k_{pt}	99.91	0.09
n_{pt}	99.63	0.37
G_t	56.13	43.87
k_{gt}	62.29	37.71
n_{gt}	20.80	79.20
i_{gt}	57.32	42.68
m	78.15	21.85

Notes: For correlated shocks, the variance decomposition goes through a Cholesky decomposition of the covariance matrix of the exogenous vairables e_{zp} and the e_{zg}. This table shows the decomposition in the general government environment

Table 2.12 and Table 2.13 present the simulation results of the two experimental checks of the relationship between the government sector and private sector. The first experiment assumes constant government productivity and the second one assumes the same productivity shocks to the government and the private sectors. Both of these results are similar to the simulation in the general government environment. For the first experiment, the simulation could not generate enough volatility to match the real world, and the simulated counter-cyclical government purchases are contradictory to the real world. For the second case, the main contradiction lies in the higher correlations between the government components spending and GDP. Similar to the

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experiments with the general government environment, the results indicate the necessity of the assumption that the productivity shocks to the government and private sector are partially and positively correlated.

Table 2.12: State and Local Gov has Constant Productivity

j	SD_j	$\frac{SD_j}{SD_{real}}$	$\varphi_{j,j-1}$	$\varphi_{j,GDP}$	$\varphi_{j,GDP_{-1}}$
Private Sector					
GDP_t	1.91	1.01	0.51	1.00	0.51
c_t	1.15	0.68	0.73	0.88	0.77
n_t	1.24	0.64	0.41	0.98	0.37
i_{pt}	5.87	0.82	0.40	0.94	0.29
k_{pt}	1.02	1.56	0.82	0.20	0.73
n_{pt}	1.43	0.66	0.42	0.98	0.59
General Government					
G_t	0.80	0.37	0.64	0.43	0.95
k_{gt}	0.23	0.38	0.85	-0.05	0.47
n_{gt}	0.14	0.10	0.51	-0.58	-0.65
i_{gt}	2.06	0.40	0.27	0.22	0.52
m	1.84	0.53	0.07	-0.18	-0.26

Notes: This table reports the results of the conterfactual experiment that the productivity of the general government is fixed while other parameters are still used the calibrated ones; SD_j denotes percentage standard deviation of variable J; SD_{real} denotes the standard deviation of the parameter in the real world; $\varphi_{j,j-1}$ is the autocorrelation of the left-side variable; $\varphi_{j,gdp}$ is the correlation between the left-side variable and GDP; $\varphi_{j,gdp-1}$ is the correlation between the left-side variable and one-year lagged GDP

Table 2.13: State Local Gov and Private Sector have Same Shocks

j	SD_j	$\frac{SD_j}{SD_{real}}$	$\varphi_{j,j-1}$	$\varphi_{j,GDP}$	$\varphi_{j,GDP_{-1}}$
			Private Sector		
GDP_t	1.91	1.01	0.54	1.00	0.54
c_t	1.11	1.10	0.72	0.85	0.76
n_t	1.27	0.66	0.47	0.98	0.43
i_{pt}	5.66	0.80	0.38	0.93	0.26
k_{pt}	0.99	1.52	0.82	0.23	0.75
n_{pt}	1.40	0.64	0.41	0.95	0.43
			General Government		
G_t	1.82	0.85	0.52	0.57	1.00
k_{gt}	0.53	0.88	0.81	0.14	0.67
n_{gt}	0.86	0.61	0.51	0.59	1.00
i_{gt}	5.32	1.03	0.29	0.29	0.55
m	2.63	0.76	0.15	0.15	0.91

Notes: This table reports the results of the conterfactual experiment that the productivity of the general government is the same as the private sector; SD_j denotes percentage standard deviation of variable J; SD_{real} denotes the standard deviation of the parameter in the real world; $\varphi_{j,j-1}$ is the autocorrelation of the left-side variable; $\varphi_{j,gdp}$ is the correlation between the left-side variable and GDP; $\varphi_{j,gdp-1}$ is the correlation between the left-side variable and one-year lagged GDP

2.7 CONCLUSION

I document the business cycle behavior of government spending components for the general government and the state and local government separately. Different volatility and different cyclical properties indicate that the components of government spending are not homogeneous. To explain the volatilities of government spending components, I provide a tractable workhorse model so as to generate a reasonable fit to the business cycle features of government spending. Implementation lags for government spending, an adjustment cost for government investment and a mildly positive correlation of the production processes between the private sector and government sector are added into the workhorse model. These assumptions are shown to be essential to replicating the cyclical properties of the real world.

The model economy in this paper replicates the fluctuations of main macro-variables in a comparable manner to other standard literature within the framework of the neo-classical model. Although the model economy shows less capability to explain the high volatility of intermediate goods in the general government sector, it generates more acceptable volatility of intermediate goods in state and local government. This result indicates that the general government may have more freedom to adjust its purchases of intermediate goods arbitrarily compared to other components, while the state and local government has a more consistent production process or less policy freedom to adjust its spending arbitrarily.

This paper breaks down the fluctuations of the relevant variables in the model economy according to the exogenous productivity shocks. The variance decomposition indicates that the productivity shocks from the government sector have very small effects on the fluctuations of the private economy. However, the fluctuations of government components can be strongly influenced by the productivity shocks from

the private sector. The level of the influence depends on the correlation between productivity shocks in the government sector and in the private sector. Since the general government has smaller productivity correlation with the private sector, the general government is more affected by its own productivity shocks than is the state and local government.

For both levels of the government, the simulation results indicate that government employment is less affected by the shocks from private sector, whereas government investment and intermediate goods are more affected by private sector shocks. For the general government, 1% of fluctuations in government employment, 50% of fluctuations in government investment and 45% of fluctuations in government intermediate goods are caused by private productivity shocks. For the state and local government, 20% of fluctuations in government employment, 57% of fluctuations in government investment and 78% of fluctuations in government intermediate goods are driven by shocks from the private sector.

CHAPTER 3

GOVERNMENT PRODUCTION, COMPLEMENTARITY AND THE EFFECTS OF
GOVERNMENT SPENDING SHOCKS

This paper reconsiders the effect of government spending on the private sector economy through input and output channels of government production. I introduce a long-run benign government producer to an otherwise standard two-sector business cycle model with price rigidity. I model government production, distinguishing between different categories of inputs, including employment, intermediate goods and capital goods and different categories of outputs. Specific inputs and outputs are classified according to their elasticity of substitution with private consumption. Given the average level of complementarity/substitutability between private consumption, labor supply, and government outputs, I explore the effects of shocks to the different components of government production. The results indicate that a fiscal policy which can fine-tune the allocation of government inputs and the categories of government outputs is able to span the whole range of theoretical results on the responses of private consumption, private output, real wage and private labor to a government spending shock.

3.1 INTRODUCTION

The responses of the private economy to government spending shocks are important for their obvious policy implications. This paper reconsiders the effect of fiscal policy

using a two-sector neoclassical model with price rigidity. I focus on the relationship between private variables (e.g. private consumption, private labor employment, real wage and private output) and different categories of government production to investigate the responses of the private economy to government spending shocks. Particularly, I allow consumer utility to depend on private consumption, government outputs and labor supply in order to reveal the importance of the complementarity/substitutability between them. Government is introduced in this paper as a producer of government outputs. Government spending is defined as the expenditure on government inputs. Distinguishing between different government inputs, intermediate goods, labor and capital, can capture the subtle interaction channels between government spending and private production. Distinguishing the degree of complementarity between different categories of government outputs and private consumption provides additional channels for government spending shocks to affect the private economy. These channels can play a central role in understanding the relationship between the private economy and government spending. Despite their potential, they have rarely been studied in a neoclassical model with a government producer and price rigidity.

Historically, there are two strands of theories which have been advanced in research related to fiscal spending responses. One is Keynesian IS-LM analysis, which claims that an increase in government spending directly boosts aggregate demand and leads to an accommodating expansion in employment and output. The other one is the real business cycle (RBC) model, which argues that an increase in government spending works through a negative wealth effect on households that creates expansions in employment and private output.

Recently, there have been some attempts to build models incorporating private economy responses to government spending shocks. These models have been related

to the complimentarity/substitutability between government spending and private labor supply or private consumption. Linnemann [2006] builds a neoclassical model in which leisure and consumption enter into the utility function. Increases in government spending crowds out the private purchasing power and creates a negative wealth effect. As leisure falls because the negative wealth effect, the substitutability between private consumption and leisure indicates the marginal utility of consumption must increase, making the household want to consume more. Later, Monacelli and Perotti [2009] study the role of substitution between leisure and government spending in a business cycle model with price rigidity. They show that substitution between leisure and government spending can generate large responses of private consumption and real wages to changes in government spending. Linnemann and Schabert [2003] formulate a New-Keynesian model in which they find that government spending causes increased private consumption for sufficiently low values of the elasticity of substitution between private consumption and government spending. Ercolani [2007], however, shows that substitution between private consumption and government spending emerges on the average level. Such substitutability, together with the negative wealth effect, makes private consumption fall after a government spending shock.

Previous literature does not fully capture two important aspects of government spending in reality. First, it assumes that government spending is on homogeneous goods. In fact, as discussed by Finn [1998] and Cavallo [2005], distinguishing between government expenditure on labor and goods can change the conclusions drawn from models that assume an aggregate value of government spending. Baxter and King [1993] reach the same conclusion using an RBC model. Compensation to government employees functions as a government transfer, which dampens the negative wealth effect of government spending.

Second, the previous literature usually ignores the distinctions between government inputs and outputs. In reality, government inputs and government outputs are different. For example, government inputs include labor, intermediate goods and capital while government outputs include education, social security system, national defense and public services. Government outputs can produce certain externalities for private consumption. For example, holding national conferences, carnivals or sporting events can attract travelers to visit the host city and spend money on relevant products. Similarly, making information technology knowledge universal can boost the consumption of high tech-equipment in general. Government output can also produce negative externalities. Providing more public health services can crowd out the need for private hospitals, or providing more public transportation services can reduce the need for private vehicles. In aggregate, government spending will be either a substitute or complement for private consumption. Distinguishing different categories of government outputs, however, is helpful to explain the channels through which government production can affect the private economy.

Since different categories of government inputs and outputs have different interaction properties with the private economy, the shocks to different parts of government production provide extra channels for fiscal policies to affect the economy. This perspective is consistent with the observation that there is no general consensus on the empirical relationship between the private economy and government spending. Blanchard and Perotti [2002], Gali et al. [2007] and Ravn et al. [2012], have found that government spending shocks generate positive responses of private consumption, real wages and real output. Ramey and Shapiro [1998], Edelberg et al. [1999], Burnside et al. [2004], and Ramey [2011] argue instead that the data support the opposite conclusion. According to Ramey [2012], the estimates of the aggregate output multiplier of the government spending vary from 0.5 to 2.

The model I introduce here accommodates these diverse findings by disaggregating the components of government spending and government production. I assume a nested CES-GHH preference to embody the complementarities/substitutabilities between private consumption, labor supply, and government production. The GHH preference structure was introduced by Greenwood et al. [1988], and is used extensively in the business cycle literature as a framework to match a series of empirical regularities. Government outputs and private consumption are combined in a constant elasticity of substitution (CES) form in the preference. I assume in the long run government determines the purchases of inputs and the production of outputs to optimize the household's utility. In the short run, there are shocks to the components of government inputs and outputs which are determined by exogenous fiscal policies. This paper discusses the effects of different fiscal shocks on the private economy through different channels of government production. The results indicate that if fiscal policies can fine-tune input purchases and output production then they can generate a wide range of responses from private consumption, real wages, private employment and private production.

The paper proceeds as follows: In section 2, I discuss the intuition behind the model. In section 3, I set up and calibrate the model economy in the long run with flexible prices. In section 4, I outline the model economy in the short run with government input and output shocks and Calvo staggered price and wage. Section 5 presents the simulation results. Section 6 concludes.

3.2 INTUITION

In the standard neoclassical model, government spending is assumed to be homogeneous and wasteful. An increase in government spending crowds out the purchasing

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power of households. Due to the negative wealth effect, households choose to consume less and work more hours. The labor supply curve shifts out to the right, while labor demand curve remains unchanged. Consequently, real wages decrease, private employment increases and private outputs increase. The assumptions of wasteful and homogenous government spending, however, fail to take into account the fact that government spending is used to purchase the inputs for government production. Taking the U.S. government for example, 60% of government expenditure is for compensation to government employees, 30% is for consumption of intermediate goods and the remainder is for capital consumption. More importantly, government spending also produces productive outputs, including education, national defense, social security, and the legislation system.

Distinguishing between different categories of government inputs and outputs will change the predictions from standard neoclassical models. Increasing the compensation to government employees, for example, transfers wealth from the government sector to households, which dampens the negative wealth effect. It also crowds out employment in the private sector. A necessary condition for private output to increase is that the labor supply shifts out. This paper argues that two mechanisms can make that happen. The first mechanism works through the complementarity channel between labor supply and private consumption. The second mechanism works through the complementarity channel between private consumption and certain categories of government outputs. Both mechanisms require an environment with certain level of price rigidity.

With these two mechanisms, increasing government employment or certain categories of government output can increase the marginal utility of private consumption, as well as the demand of consumption goods. Therefore, in a price staggered environment, private firms encounter an outward shift of the demand curve. Firms that

cannot change their prices meet this extra demand by increasing production, hence shifting out the derived demand for labor. In the short run, as labor increases, private output increases and the marginal dis-utility of labor increases, therefore real wages will rise and the cost of consuming private goods increases. A fine-tuned government spending, then, can produce a new temporary equilibrium with more private output, more private consumption, a higher wage and more private employment.

The degree of complementarity between private consumption and different components of government production determine the effects of fiscal policies. For example, federal defense expenditures probably have a small level of complementarity with private consumption. In the empirical research, therefore, they have little effect on the private economy. On the contrary, if fiscal policy promotes the production of government outputs which have a higher level of complementarity with the private economy, such as holding national sporting events, legalizing marijuana, and controlling environmental pollution near tourist destinations, then the fiscal stimulus effect on the private economy will be stronger.

3.3   MODEL ECONOMY WITH FLEXIBLE PRICES

This section describes and calibrates the model economy without any frictions of price setting. Households, private firms and government are the agents in this economy. Households consume private and government outputs. Monopolistic firms produce private outputs with labor and capital. The government produces government outputs with labor, capital and intermediate goods purchased from private sector. I assume that the government in this flexible price model economy optimizes its production to maximize the utility of households. Because there is no tax distortion in this economy, so this model economy with a benign government is equivalent to a economy with

a competitive government producer and the households determine the quantity of government output according to the competitive price of government outputs.

3.3.1 Households

There is a continuum of households indicated by the index τ. Each household has monopoly power over the supply of its labor. Each households τ maximizes an inter-temporal utility function given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^\tau$$

where β is the discount factor and the instantaneous utility function is separable in consumption c_t^τ, government output $y_{g,t}^\tau$, labor n_t^τ and real cash balances $\frac{m_t^\tau}{p_t}$:

$$U_t^\tau = \frac{1}{1-\sigma_n} \left[\left((c_t^\tau)^{1-\sigma_g} + \psi_g (y_{g,t}^\tau)^{1-\sigma_g} \right)^{\frac{1}{1-\sigma_g}} - \psi_n \frac{(n_t^\tau)^{1+\theta}}{1+\theta} \right]^{1-\sigma_n} + \frac{1}{1-\sigma_m} \left(\frac{m_t^\tau}{p_t} \right)^{1-\sigma_m}$$

(3.1)

The household maximizes his utility function subject to an intertemporal budget constraint which is given by:

$$IBC_t \equiv \frac{m_{t-1}^\tau}{p_t} + \frac{B_{t-1}^\tau}{p_t} + IN_t^\tau - c_t^\tau - \frac{p_{g,t}}{p_t} y_{g,t}^\tau - i_t^\tau - \frac{m_t^\tau}{p_t} - b_t \frac{B_t^\tau}{p_t} - \frac{T_t}{P_t}$$

(3.2)

where $y_{g,t}$ is government output. $p_{g,t}$ is the price of government output. The household's total income is given by:

$$IN_t^\tau = (w_t^\tau n_t^\tau + A_t^\tau) + r_{p,t}^k k_{p,t-1}^\tau + Div_t^\tau + r_{g,t}^k k_{g,t-1}^\tau$$

(3.3)

where $n_t^\tau = n_{g,t}^\tau + n_{p,t}^\tau$ and $i_t^\tau = i_{g,t}^\tau + i_{p,t}^\tau$.

It is assumed, as in Christiano et al. [2005], that there exist state-contingent securities that insure households against variations in household specific labor income. As a result, the first component in the household's income will be equal to aggregate labor income. Furthermore, the marginal utility of wealth will be identical across different types of households.

First, households maximize the objective utility function with respect to consumption c_t, the holding of bonds B_t, the cash balances m_t and the government output $y_{g,t}$ over an infinite life horizon:

$$\max_{c_t, B_t, m_t, y_{g,t}} W \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[U_t^\tau + \lambda_t IBC_t \right] \tag{3.4}$$

This optimization problem gives us the following first order conditions with respect to consumption, labor, cash balance and government outputs.

$$u_1 = \left[\left((c_t^\tau)^{1-\sigma_g} + \psi_g (y_{g,t}^\tau)^{1-\sigma_g} \right)^{\frac{1}{1-\sigma_g}} - \psi_n \frac{(n_t^\tau)^{1+\theta}}{1+\theta} \right]^{-\sigma_n} \left((c_t^\tau)^{1-\sigma_g} + \psi_g (y_{g,t}^\tau)^{1-\sigma_g} \right)^{\frac{\sigma_g}{1-\sigma_g}}$$

$$\lambda_t = u_1 (c_t^\tau)^{-\sigma_g} \tag{3.5}$$

$$\lambda_t = \beta (1 + r_t) \lambda_{t+1} \tag{3.6}$$

$$\left(\frac{m_t}{p_t} \right)^{-\sigma_m} = \lambda_t \frac{nr_t}{1 + nr_t} \tag{3.7}$$

$$\frac{p_{g,t}}{p_t} \lambda_t = u_1 \psi_g (y_{g,t}^\tau)^{-\sigma_g} \tag{3.8}$$

where $1 + nr_t = (1 + r_t)(1 + \pi_t)$ (nr_t is the nominal interest rate, r_t is the real interest rate and π_t is the inflation rate).

LABOR SUPPLY AND WAGE SETTING EQUATION

The household also maximizes function W with respect to the nominal wage w_t^τ:

$$\max_{w_t^\tau} W \equiv \sum_{t=0}^{\infty} \beta^t [U_t^\tau + \lambda_t IBC_t] \tag{3.9}$$

Here I assume the labor supply has a bundler. The bundler hires labor from each household and combines it into final labor. The demand for the labor of a particular

household τ is determined by:

$$n_t^\tau = \left(\frac{w_t^\tau}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} n_t \tag{3.10}$$

This maximization problem of function W results in the following labor supply equation:

$$n_t^\theta = \frac{1}{1+\lambda_w} \lambda_t w_t \frac{1}{\psi_n} \left[\left(c_t^{1-\sigma_g} + \psi_g y_{g,t}^{1-\sigma_g}\right)^{\frac{1}{1-\sigma_g}} - \psi_n \frac{n_t^{1+\theta}}{1+\theta}\right]^{\sigma_n} \tag{3.11}$$

In equilibrium, all households have the same wage w_t^τ. Therefore, w_t^τ is equal to w_t in equation (3.11). As the aggregate function shows

$$w_t = \left[\int_0^1 (w_t^\tau)^{-1/\lambda_w} \, d\tau\right]^{-\lambda_w} \tag{3.12}$$

INVESTMENT AND CAPITAL ACCUMULATION

Finally, households own both private capital stock and government capital stock. They rent out the private capital stock to firm-producers at a given rental rate $r_{p,t}^k$. They also rent out the government capital stock to government-producers at a given rental rate $r_{g,t}^k$. They can increase the supply of capital stock by investing in additional private capital, $i_{p,t}$, or government capital, $i_{g,t}$. Both investments have the same unit cost in terms of consumption.

The law of motion of capital accumulation is given by:

$$k_{p,t} = k_{p,t-1}(1 - \delta_p) + i_{p,t} \tag{3.13}$$

$$k_{g,t} = k_{g,t-1}(1 - \delta_g) + i_{g,t} \tag{3.14}$$

Household chooses next period capital stock $k_{x,t}$ and investment $i_{x,t}$ in order to maximize their intertemporal utility function subject to the intertemporal budget constraint and the capital accumulation equation. letting $x = g \ or \ p$.:

$$\max_{k_{x,t}, i_{x,t}} H \equiv \sum_{t=0}^\infty \beta^t [U_t^\tau + \lambda_t IBC_t + \lambda_t q_{x,t}(k_{x,t-1}(1 - \delta_x) + i_{x,t} - k_{x,t})] \tag{3.15}$$

where δ_x is the depreciation rate and $q_{x,t}$ is the Tobin's q, i.e. the price of one unit of capital stock.

This optimization problem results in the following first order conditions:

$$q_{x,t} = \beta \frac{\lambda_{t+1}}{\lambda_t} [q_{x,t+1}(1 - \delta_x) + r^k_{x,t+1}] \tag{3.16}$$

$$q_{x,t} = 1 \tag{3.17}$$

3.3.2 FIRMS

The country produces a single final good and a continuum of intermediate goods indexed by j, where j is distributed over the unit interval ($j \in [0, 1]$).

FINAL-GOOD SECTOR

The final good is produced using intermediate goods according to the following technology which is similar to the aggregate labor supply in equation (11):

$$y_{p,t} = [\int_0^1 (y^j_{p,t})^{1/1+\lambda_p} \, \mathrm{dj}]^{1+\lambda_p} \tag{3.18}$$

where $y^j_{p,t}$ denotes the quantity of intermediate good of type j that is used in final goods production at date t, and $\lambda_{p,t}$ determines the mark-up in the goods market.

The cost minimization conditions in the final good sector can be written as:

$$y^j_{p,t} = \left[\frac{p^j_t}{p}\right]^{-\frac{1+\lambda_p}{\lambda_p}} y_{p,t}$$

Perfect competition in the final goods market implies the price of the final goods could be also be written as

$$p_t = [\int_0^1 (p^j_t)^{-1/\lambda_p} \, \mathrm{dj}]^{-\lambda_p} \tag{3.19}$$

Intermediate Good Producers

Each intermediate good, j, is produced by firm, j, using the following technology:

$$y_{p,t}^j = Z_{p,t} k_{p,j,t-1}^\alpha n_{p,j,t}^{1-\alpha} \tag{3.20}$$

where A_t is the productivity factor and $k_{p,j,t-1}, n_{p,j,t}$ are the indexed quantity of capital stock and labor used by the intermediate firm. The total costs of the intermediate firm are given by the sum of the wages and the rent. The firm minimizes this total costs subject to its production function:

$$\min_{n_{j,t}, k_{p,j,t-1}} TC_t \equiv w_t n_{p,j,t} + r_{p,t}^k k_{p,j,t-1} + mc_t[y_{p,t}^j - (Z_{p,t} k_{p,j,t-1}^\alpha n_{p,j,t}^{1-\alpha})] \tag{3.21}$$

The results of this minimization problem are the following:

$$w_t = mc_t(1-\alpha)\frac{y_{p,t}}{n_{p,t}}$$

$$r_{p,t}^k = mc_t \alpha \frac{y_{p,t}}{k_{p,t-1}}$$

And then:

$$\frac{w_t n_{p,j,t}}{r_t^k kp, j, t-1} = \frac{1-\alpha}{\alpha} \tag{3.22}$$

$$mc_t = \frac{1}{Z_{p,t}}(w_t)^{1-\alpha}(r_t^k)^\alpha \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} \tag{3.23}$$

Nominal profits of firm j are given by:

$$\Pi_t^j = (p_t^j - mc_t)[\frac{p_t^j}{p}]^{-\frac{1+\lambda_p}{\lambda_p}} y_{p,t} \tag{3.24}$$

Each firm j has market power for its own good and maximizes its profits with respect to the price it sets:

$$\max_{p_t^j} \Pi_t^j \tag{3.25}$$

subject to

$$y_{p,t}^j = [\frac{p_t^j}{p_t}]^{-\frac{1+\lambda_p}{\lambda_p}} y_{p,t}$$

The solution of this maximization problem gives a price, p_t^j, which is a mark-up of the marginal cost:

$$p_t^j = (1 + \lambda_p)mc_t \tag{3.26}$$

3.3.3 GOVERNMENT

The government producer is cost-efficient. Its purpose in the model economy is to maximize the utilities of the households. The production of government output is financed by lump-sum tax. I also assume government is cost-efficient. Therefore, it is equivalent to a model with a competitive government producer instead, in which the profit of government sector is zero. I assume the government prouduction function take a three-factor CES functional form:

$$y_{g,t} = Z_{g,t}im_t^{d_1}k_{g,t}^{d_2}n_{g,t}^{1-d_1-d_2} \tag{3.27}$$

where im are the intermediate goods and services from the private sector. k_g is the capital stock of the government sector. n_g is the labor hired from the competitive labor market. Government spending is given by

$$T_t = p_t w_t n_{g,t} + p_t(im_t + i_{g,t})$$

Non-distortion and zero-profit of the government sector imply the price of the public outputs can be written as

$$p_{g,t} = \frac{p_t(r_{g,t}^k)^{d_2}(w_t)^{1-d_1-d_2}}{Z_{g,t}d_1^{d_1}d_2^{d_2}(1-d_1-d_2)^{1-d_1-d_2}} \tag{3.28}$$

3.3.4 MARKET EQUILIBRIUM

The final goods market is in equilibrium if the supply of final goods equals the demand by households and the purchases of intermediate goods for the government:

$$y_{p,t} = c_t + i_{p,t} + im_t + i_{g,t} \tag{3.29}$$

At the macro level, I consider that all the intermediate firms are symmetric. Moreover, because the capital-labor ratio, will be identical across intermediate goods producers and equal to the aggregate capital-labor ratio and because the marginal cost is independent on the intermediate goods produced, the same technology will be used to characterize the production function of the final good:

$$y_{p,t} = Z_{p,t} k_{t-1}^{\alpha} n_{p,t}^{1-\alpha} \tag{3.30}$$

$$\frac{w_t n_{p,t}}{r_{p,t}^k k_{p,t-1}} = \frac{1-\alpha}{\alpha} \tag{3.31}$$

and the price of the final good will be a mark-up of the marginal cost:

$$p_t^j = (1+\lambda_p) mc_t \tag{3.32}$$

The inflation rate is defined as:

$$\pi_t = \frac{p_{t+1} - p_t}{p_t} \tag{3.33}$$

The money market must also be in equilibrium. Here, I choose to model the process of money as an AR(1) autoregressive process with a constant:

$$m_t = (1-\rho)\eta + \rho m_{t-1} + v_t \tag{3.34}$$

where η is the constant, and ρ is the intertemporal correlation coefficient. η will fix the money and the price since the steady state money demand is equal to:

$$m^* = \frac{\eta}{1-\rho}$$

3.3.5 Calibration for the Economy with Flexible Price

The parameters for this model are drawn from three different sources. The first category is drawn from previous literature. The second category is derived from estimation, and the last category is derived from calibration. To make the model economy

consistent with the U.S. economy, the calibration part uses quarterly data set from 1960:I–2006:IV in NIPA.

Table 3.1 shows the parameters used in the model and their values λ_w and λ_p determine the monopolistic levels of labor supply or production for households and firms separately. Following Canzoneri et al. [2006], I set the value of λ_w and λ_p both equal to 0.2. σ_g (or σ_n) determines the elasticity of substitution between private consumption and government outputs (or labor). Higher σ_g (or σ_n) means government outputs (or labor) and private consumption goods have a higher degree of complementarity. Monacelli and Perotti [2009] assumes σ_n in a range from 1.25 to 3.0. In this paper, I take the average of these values and assume $\sigma_n = 2.0$. In fact, Basu and Kimball [2002] estimate the elasticity of substitution between labor and private consumption and find the value is around 0.35. This value is consistent with $\sigma_n = 2.0$ which indicates that labor and private consumption are complements. Regarding σ_g, Ercolani [2007] finds that government outputs and private consumption are substitutes to each other. Therefore, I make σ_g equal to 0.7. θ is the parameter which governs Frisch elasticity of labor supply, I take its value from Monacelli and Perotti [2009] and make it equal to 0.8. σ_m is the parameter governing elasticity of cash demand, I take its value from Pierre et al. [2003] and set it equal to 1.2.

I assume that the private sector and the government have Cobb-Douglas production functions. All of the estimations are based on the data set from the US government 1960:I–2006:IV in NIPA.

In the model economy, $\beta = 0.99$, $\eta = 18$, $\psi_g = 0.12$ and $\psi_n = 1.3$. These parameters are calibrated according to the target ratios shown in the last column of Table 3.1.

Table 3.2 shows the comparison between the steady state of the model economy and the U.S. economy. .

Table 3.1: Calibration Results

parameter	Value	Description	target
From Literature			
λ_w	0.2	wage mark up	
λ_p	0.2	price mark up	
θ	0.8	Frisch elasticity of labor supply	
σ_n	2.0	elasticity of substitution in labor	
σ_m	1.2	elasticity of cash demand	
σ_g	0.7	inverse of elasticity of substitution in government outputs	
From Estimation			
α	0.3	share of the capital of the private production	
d_1	0.3	intermediate goods share of public production	
d_2	0.1	capital share of public production	
δ_p	0.025	depreciation rate of private capital	
δ_g	0.02	depreciation rate of public capital	
From Calibration			
β	0.99	discount factor	$\frac{k_p}{y_p} = 8.4$
η	18	cash supply in the economy	p=1 at steady state
ψ_g	0.56	weight of public goods in utility function	$\frac{n_g}{n_p} = 0.22$
ψ_n	5.0	weight of labor in utility function	$n = 0.33$

Notes: The data set is from 1960:I-2006:IV in NIPA.

3.4 The Model Economy with Calvo Price

In this section, I introduce Calvo price and wage setting to represent the staggered prices in the model economy. I first build the Calvo price and wage settings in a standard way. Then, I discuss several fiscal policy strategies which can fine-tune the production of government outputs.

3.4.1 Calvo Price Setting For the Private Firms

Using the same assumptions as discussed by Calvo [1983], firms are not allowed to change their prices unless they receive a random price-change signal. The probability that a given price can be re-optimized in any particular period is constant and equal to $(1 - \xi_p)$. The profit optimization problem of the producers that are allowed to

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Table 3.2: Steady States of the Model and Real Economy

parameter	Description	Model Econ	Real Econ
$\frac{k}{y_p}$	The ratio of private capital to the output	7.8	8.4*
n	Total Labor Supply	0.33	0.33*
$\frac{n_g}{n_p}$	ratio of private labor to public employment	0.22	0.22*
$\frac{g}{GDP}$	government expenditure share in GDP	0.20	0.21
$\frac{i_p}{GDP}$	share of investment to GDP	0.17	0.22
$\frac{c}{GDP}$	share of consumption to GDP	0.63	0.60

*Notes:*1.* indicates the average real world ratio is used as the targets to calibrate the model.
2.The data set is from 1960:I-2006:IV.

re-optimize at time t is the following:

$$\max_{\tilde{p}_t^j} D = \sum_{i=0}^{\infty} \beta^i \xi_p^i \lambda_{t+i} y_{t+i}^j \left[\frac{\tilde{p}_t^j}{p_{t+i}} \left(\frac{p_{t+i-1}}{p_{t-1}} \right)^{\gamma_p} - mc_{t+i} \right] \qquad (3.35)$$

where the cost minimization condition in the final good sector is

$$y_{t+i}^j = \left[\frac{\tilde{p}_t^j}{p_{t+i}} \right]^{-\frac{1+\lambda_p}{\lambda_p}} y_{t+i}$$

This results in the following first order condition:

$$\frac{\tilde{p}_t^j}{(1+\lambda_p)} \sum_{i=0}^{\infty} \beta^i \xi_p^i \lambda_{t+i} y_{t+i} p_{t+i}^{1/\lambda_p} (\frac{p_{t+i-1}}{p_{t-1}})^{\gamma_p} = \sum_{i=0}^{\infty} \beta^i \xi_p^i \lambda_{t+i} p_{t+i}^{\frac{1+\lambda_p}{\lambda_p}} y_{t+i} mc_{t+i} \qquad (3.36)$$

Equation (3.36) shows that the price set by firm j is a markup of the future marginal costs. If prices are perfectly flexible ($\xi_p = 0$), the mark-up in period t is equal to $(1 + \lambda_{p,t})$ as in equation (3.26).

Given equation (3.36), the law of motion of the aggregate price index is:

$$p_t^{-1/\lambda_p} = \xi_p \left[p_{t-1} \left(\frac{p_{t-1}}{p_{t-2}} \right)^{\gamma_p} \right]^{-1/\lambda_p} + (1 - \xi_p) \tilde{p}_t^{-1/\lambda_p} \qquad (3.37)$$

I consider the two sums of the equation (3.36) separately and I make each sum equal to a new variable at time t, then solve the resulting equation recursively. For

the first sum, I will follow these steps. I consider the first sum of equation (3.36) as the variable $SUM1_t$:

$$SUM1_t = \sum_{i=0}^{\infty} \beta^i \xi_p^i \lambda_{t+i} y_{t+i} p_{t+i}^{1/\lambda_p} p_{t+i-1}^{\gamma_p}$$

This equation will be the following in a recursive way:

$$SUM1_t = \beta \xi_p SUM1_{t+1} + \lambda_t y_t p_t^{1/\lambda_p} p_{t-1}^{\gamma_p} \qquad (3.38)$$

Then we do the same for the second sum:

$$SUM2_t = \sum_{i=0}^{\infty} \beta^i \xi_p^i \lambda_{t+i} p_{t+i}^{\frac{1+\lambda_p}{\lambda_p}} y_{t+i} mc_{t+i}$$

giving another recursive equation:

$$SUM2_t = \beta \xi_p SUM2_{t+1} + \lambda_t y_t p_t^{\frac{1+\lambda_p}{\lambda_p}} mc_t \qquad (3.39)$$

Given equations (3.38) and (3.39), equation (3.36) becomes:

$$\frac{\tilde{p}_t^j p_{t-1}^{-\gamma_p}}{(1+\lambda_p)} SUM1_t = SUM2_t \qquad (3.40)$$

THE DYNAMICS OF THE CALVO STAGGERED PRICE

In simulating this economy, I will have to consider four endogenous variables in the price setting equation: p_t, $\tilde{p}_t$, $SUM1_t$ and $SUM2_t$. I will replace equation (3.19) by the equations (3.37), (3.38), (3.39) and (3.40):

$$p_t^{-1/\lambda_p} = \xi_p \left[p_{t-1} \left(\frac{p_{t-1}}{p_{t-2}} \right)^{\gamma_p} \right]^{-1/\lambda_p} + (1 - \xi_p) \tilde{p}_t^{-1/\lambda_p}$$

$$SUM1_t = \beta \xi_p SUM1_{t+1} + \lambda_t y_t p_t^{1/\lambda_p} p_{t-1}^{\gamma_p}$$

$$SUM2_t = \beta \xi_p SUM2_{t+1} + \lambda_t y_t p_t^{\frac{1+\lambda_p}{\lambda_p}} mc_t$$

$$\frac{\tilde{p}_t^j p_{t-1}^{-\gamma_p}}{(1+\lambda_p)} SUM3_t = SUM4_t$$

where β, γ_p, ξ_p, λ_p are parameters.

3.4.2 The Model with Calvo-Wage Setting

Here I will consider the model with the sticky wage assumptions. Households act as price-setters in the labor market. Following Erceg et al. [2000] and Canzoneri et al. [2006], I assume that wages can only be optimally adjusted after some random wage-change signal is received. The probability that a particular household can change its nominal wage in period t is constant and equat to $(1 - \xi_w)$. A household τ which receives such a signal in period t will thus set a new nominal wage $\tilde{w}_t^\tau$, taking into account the probability that it will not be re-optimized in the near future. For the households who can not re-optimize, their wages adjust according to:

$$w_t^\tau = \left(\frac{p_{t-1}}{p_{t-2}} \right)^{\gamma_w} w_{t-1}^\tau$$

where γ_w is the degree of wage indexation. When γ_w is equal to 0, there is no indexation and the wages that can not be re-optimized remain constant. When γ_w is equal to 1, there is perfect indexation to past inflation.

Here, the maximization problem of the households becomes:

$$\max_{\tilde{w}_t^\tau} L = \sum_{i=0}^{\infty} \beta^i \xi_w^i \left[U_t^\tau (n_{t+i}^\tau) \right] \tag{3.41}$$

where the particular demand for labor is determined by

$$n_{t+i}^\tau = \left[\frac{\tilde{w}_t^\tau \left(\frac{p_{t+i-1}}{p_{t-1}} \right)^{\gamma_w}}{w_{t+i}} \right]^{-\frac{1+\lambda_w}{\lambda_w}} n_{t+i} \tag{3.42}$$

Household τ chooses $\tilde{w}_t$ to maximize the utility function, subject to

$$IBC_t \equiv \frac{m_{t-1}^\tau}{p_t} + \frac{B_{t-1}^\tau}{p_t} + IN_t^\tau - c_t^\tau - \frac{p_{g,t}}{p_t} y_{g,t} - i_t^\tau - \frac{m_t^\tau}{p_t} - b_t \frac{B_t^\tau}{p_t} - \frac{T_t}{P_t}$$

$$IN_{t+i}^\tau = \left(\frac{\tilde{w}_t^\tau \left(\frac{p_{t+i-1}}{p_{t-1}} \right)^{\gamma_w}}{p_{t+i}} n_{t+i}^\tau + A_{t+i}^\tau \right) + r_{p,t+i}^k k_{p,t+i-1}^\tau + Div_{t+i}^\tau + r_{g,t+i}^k k_{g,t+i-1}^\tau$$

This maximization with staggered wage problem is the following:

$$\max_{\tilde{w}_t^\tau} L = \sum_{i=0}^{\infty} \beta^i \xi_w^i \left[U_t^\tau (n_{t+i}^\tau) + \lambda_{t+i} \tilde{w}_t^\tau p_{t+i}^{-1} (\frac{p_{t+i-1}}{p_{t-1}})^{\gamma_w} n_{t+i}^\tau \right] \tag{3.43}$$

Let $\left(\frac{p_{t+i-1}}{p_{t-1}}\right)^{\gamma_w} = \phi_1$ and $\frac{1+\lambda_w}{\lambda_w} = \phi_2$, So equation (3.42) becomes

$$n_{t+i}^\tau = \left[\frac{\tilde{w}_t^\tau \phi_1}{w_{t+i}}\right]^{-\phi_2} n_{t+i}$$

Plug equation (3.42) into equation (3.46)

$$\max_{\tilde{w}_t^\tau} L = \sum_{i=0}^{\infty} \beta^i \xi_w^i \left[\frac{1}{1-\sigma_n}\left[v(c_{t+i}^\tau, y_{g,t+i}^\tau) - \psi_n \frac{(n_{t+i}^\tau)^{1+\theta}}{1+\theta}\right]^{1-\sigma_n} + \lambda_{t+i}\frac{\tilde{w}_t^\tau}{p_{t+i}}\phi_1\left(\frac{\tilde{w}_t^\tau \phi_1}{w_{t+i}}\right)^{-\phi_2} n_{t+i} + \cdots\right]$$

(3.44)

where $v(c_{t+i}^\tau, y_{g,t+i}^\tau) = \left(\left(c_{t+i}^\tau\right)^{1-\sigma_g} + \psi_g\left(y_{g,t+i}^\tau\right)^{1-\sigma_g}\right)^{\frac{1}{1-\sigma_g}}$.

This optimization problem gives us the following first order conditions.

$$(\tilde{w}_t^\tau)^{1+\phi_2\theta} \sum_{i=0}^{\infty} \beta^i \xi_w^i \frac{\lambda_{t+i}}{p_{t+i}} w_{t+i}^{\phi_2} \phi_1^{1-\phi_2} n_{t+i}$$

$$= \frac{\phi_2}{\phi_2-1}\psi_n \sum_{i=0}^{\infty} \beta^i \xi_w^i \left[v(c_{t+i}^\tau, y_{g,t+i}^\tau) - \psi_n \frac{(n_{t+i}^\tau)^{1+\theta}}{1+\theta}\right]^{-\sigma_n} \phi_1^{-\phi_2(1+\theta)} w_{t+i}^{\phi_2(1+\theta)} n_{t+i}^{1+\theta}$$

(3.45)

Equation (3.45) implies that when wages are perfectly flexible ($\xi_w = 0$), the real wage will be a mark-up (equal to $1 + \lambda_{w,t}$) of the ratio of the marginal disutility of labor over the marginal utility of one unit more consumption.

Given the equation (3.45), the law of motion of the aggregate wage index is given by:

$$w_t^{-1/\lambda_w} = \xi_w \left[w_{t-1}\left(\frac{p_{t-1}}{p_{t-2}}\right)^{\gamma_w}\right]^{-1/\lambda_w} + (1-\xi_w)\tilde{w}_t^{-1/\lambda_w}$$

(3.46)

I consider the two sums of the equation (3.45) separately and make them equal to a new variable at time t. This allows equation (3.45) to be written as a recursive formula. The first sum is simplified as follows:

$$SUM3_t = \sum_{i=0}^{\infty} \beta^i \xi_w^i \frac{\lambda_{t+i}}{p_{t+i}} w_{t+i}^{\phi_2}\left(\frac{P_{t+i-1}}{p_{t-1}}\right)^{\gamma_w(1-\phi_2)} n_{t+i}$$

Then

$$SUM3_{t+1} = \sum_{i=0}^{\infty} \beta^i \xi_w^i \frac{\lambda_{t+1+i}}{p_{t+1+i}} w_{t+1+i}^{\phi_2} (\frac{p_{t+i}}{p_t})^{\gamma_w(1-\phi_2)} n_{t+1+i}$$

$$= \frac{1}{\beta \xi_w} \sum_{j=1}^{\infty} \beta^j \xi_w^j \frac{\lambda_{t+j}}{p_{t+j}} w_{t+j}^{\phi_2} (\frac{p_{t+j-1}}{p_t})^{\gamma_w(1-\phi_2)} n_{t+j}$$

$$= \frac{1}{\beta \xi_w} (\frac{p_{t-1}}{p_t})^{\gamma_w(1-\phi_2)} \sum_{j=1}^{\infty} \beta^j \xi_w^j \frac{\lambda_{t+j}}{p_{t+j}} w_{t+j}^{\phi_2} (\frac{p_{t+j-1}}{p_{t-1}})^{\gamma_w(1-\phi_2)} n_{t+j}$$

$$= \frac{1}{\beta \xi_w} (\frac{p_{t-1}}{p_t})^{\gamma_w(1-\phi_2)} \left[SUM3_t - \frac{\lambda_t}{p_t} w_t^{\phi_2} n_t \right]$$

Such that:

$$SUM3_t = \beta \xi_w (\frac{p_t}{p_{t-1}})^{\gamma_w(1-\phi_2)} SUM3_{t+1} + \frac{\lambda_t}{p_t} w_t^{\phi_2} n_t \qquad (3.47)$$

The second sum cannot be written as recursive formula. Instead, I use the first k terms to represent the infinite sum.[1]

$$SUM4_t = \sum_{i=0}^{k} \beta^i \xi_w^i \left[v(c_{t+i}^\tau, y_{g,t+i}^\tau) - \psi_n \frac{(n_{t+i}^\tau)^{1+\theta}}{1+\theta} \right]^{-\sigma_n} \phi_1^{-\phi_2(1+\theta)} w_{t+i}^{\phi_2(1+\theta)} n_{t+i}^{1+\theta} \qquad (3.48)$$

Given equations (3.47)and (3.48), equation (3.45) becomes:

$$(\tilde{w}_t)^{1+\phi_2\theta} \frac{1}{1+\lambda_w} \frac{1}{\psi_n} SUM3_t = SUM4_t \qquad (3.49)$$

THE DYNAMICS OF CALVO WAGE

I replace the labor supply equation of the model with flexible prices given by equations (3.46), (3.47), (3.48) and (3.49):

$$w_t^{-1/\lambda_w} = \xi_w \left[w_{t-1} \left(\frac{p_{t-1}}{p_{t-2}} \right)^{\gamma_w} \right]^{-1/\lambda_w} + (1 - \xi_w) \tilde{w}_t^{-1/\lambda_w}$$

$$SUM3_t = \beta \xi_w (\frac{p_t}{p_{t-1}})^{\gamma_w(1-\phi_2)} SUM3_{t+1} + \frac{\lambda_t}{p_t} w_t^{\phi_2} n_t$$

---

[1]In the simulation, I use k=20. The residuals become sufficient small.

$$SUM4_t = \beta \xi_w (\frac{p_t}{p_{t-1}})^{\gamma_w(1-\phi_2)} SUM4_{t+1} + w_t^{\phi_2(1+\sigma_n)} n_t^{1+\sigma_n}$$

$$(\tilde{w}_t)^{1+\phi_2\sigma_n} \frac{\phi_2 - 1}{\phi_2} SUM3_t = SUM4_t$$

where β, γ_w, λ_w and ξ_w are parameters.

3.4.3 Market Clearing Conditions

Similar to the model economy with flexible price setting, the model economy with staggered price and wage setting has to satisfy the clearing conditions in both labor, goods, capital and cash markets.

$$y_{p,t} = c_t + i_{p,t} + im_t + i_{g,t}$$

$$n_t = n_{p,t} + n_{g,t}$$

3.5 Shocks to Government Production

Different types of fiscal shocks may have differential impacts on government production. In this section, I explore the reaction of government production to four hypothetical fiscal shocks. I consider shocks to the following aspects of government production: the government production budget, government employment, government intermediate goods and different categories of government outputs.

3.5.1 Shocks to the Budget of Government Production

In this scenario, I assume the budget for government production follows an AR(1) process, as shown in equation (3.50). Government spending is financed by a lump-sum tax levied on households. Furthermore, the government continuously optimizes its production subject to its budget constraint.

$$ln(GB_t) = (1 - \rho_g) \cdot ln(\overline{GB}) + \rho_g \cdot ln(GB_{t-1}) + \epsilon_{gb} \tag{3.50}$$

where GB is the budget of government production. In equilibrium, $GB_t = T_t$. ρ_g is the intertemporal coefficient. ϵ_{gb} is the fiscal shock to $ln(GB)$.

Given the evolution process of the government budget and the CES functional form of government production, purchases of government inputs im_t, $k_{g,t}$ and $n_{g,t}$ can be described as follows:

$$im_t = GB_t \cdot d_1/p_t$$

$$k_{g,t} = GB_t \cdot d_2/(r_{kg,t}p_t)$$

$$n_{g,t} = GB_t \cdot (1 - d_1 - d_2)/(w_t p_t)$$

3.5.2 SHOCKS TO GOVERNMENT EMPLOYMENT

In this scenario, I maintain the assumption that the government production follows an AR(1) process, as in equation (3.50). However, the deviation of the budget from steady state can only affect the compensation to government employees. Therefore, the purchases of government inputs im_t, $k_{g,t}$ and $n_{g,t}$ can be shown as following:

$$im_t = \overline{im}$$

$$k_{g,t} = \overline{k_g}$$

$$n_{g,t} = (GB_t - \overline{im} \cdot p_t - \overline{k_g} \cdot r_{kg,t}p_t)/(w_t p_t)$$

where $\overline{im}$ and $\overline{k_g}$ are the steady state values of government intermediate goods and government capital.

3.5.3 Shocks to Government Intermediate Goods Consumption

In this scenario, government spending evolves according to equation (3.50). But the deviation of government spending from steady state only affect the purchases of government intermediate goods. Therefore, the allocations of government inputs im_t, $k_{g,t}$ and $n_{g,t}$ can be shown as following:

$$im_t = (GB_t - \overline{n_g} \cdot w_t p_t - \overline{k_g} \cdot r_{kg,t} p_t)/p_t$$

$$k_{g,t} = \overline{k_g}$$

$$n_{g,t} = \overline{n_g}$$

3.5.4 Shocks to Specific Categories of Government Outputs

In this scenario, I assume that the government can produce a range of outputs with different degrees of complementarity with private consumption. I assume that the government can adjust its production in response to a positive shock to government spending. Therefore, a government spending shock can also be modeled as a shock to the average value of the elasticity of substitution between government outputs and private consumption.

The features of this scenario can be described as follows:

$$ln(GB_t) = (1 - \rho_g) \cdot ln(\overline{GB}) + \rho_g \cdot ln(GB_{t-1}) + \epsilon_{gb}$$

$$im_t = GB_t \cdot d_1/p_t$$

$$k_{g,t} = GB_t \cdot d_2/(r_{kg,t} p_t)$$

$$n_{g,t} = GB_t \cdot (1 - d_1 - d_2)/(w_t p_t)$$

$$sim_t = \rho_g \cdot sim_{t-1} + \epsilon_{gb}$$

The average level of the inverse of the elasticity of substitution between government outputs and private consumption is defined as:

$$\sigma_g \cdot (1 + mul \cdot sim_t) \tag{3.51}$$

where mul is the weight of the budget shock to the average level of σ_g. In this case, I assume labor and private consumption are separable and re-calibrate the new model economy.

3.6 Simulation of The Economy with Different Fiscal Shocks

This section shows the simulation results of the model economy with the fiscal shocks described in previous section. For each scenario, I will display the results with flexible prices and sticky prices separately. In order to perform the simulation, I first calibrate the AR(1) process of government spending in the U.S. economy. The counterpart of government spending is the value of the government budget in my model economy.

I assume the government expenditure follows an AR(1) process, as shown in equation (3.50). I use the data set of U.S. general government spending from 1960:I–2006:IV in NIPA to estimate this process. Following Hodrick and Prescott [1997], I use $\lambda = 1600$ to detrend the series of the data.

The estimations results of equation (3.50) are shown in Table 3.3.

Table 3.3: Government Expenditure Process Estimation

parameter	Description	Mstimation
ρ_g	Coefficient of AR(1) process	0.79
σ_g	Standard Deviation of ϵ_g	0.00821

Notes: The data set is from 1960:I-2006:IV, NIPA.

3.6.1 Shocks to the Budget of Government Production

The settings of private production and household labor supply in this paper are the same as in Canzoneri et al. [2006]. Households are competitive monopolistic producers for the intermediate goods that are used to produce private final outputs.

I implement the simulation in a flexible price environment. The private firms and households are competitive monopolistic players. They have certain levels of monopolistic power over their production and labor supplies. In this simulation, government production follows a cost-efficient pattern. Figure 3.1 shows the impulse responses of a standard deviation shock on the government budget.

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Figure 3.1: Shocks to Government Spending with Flexible Prices
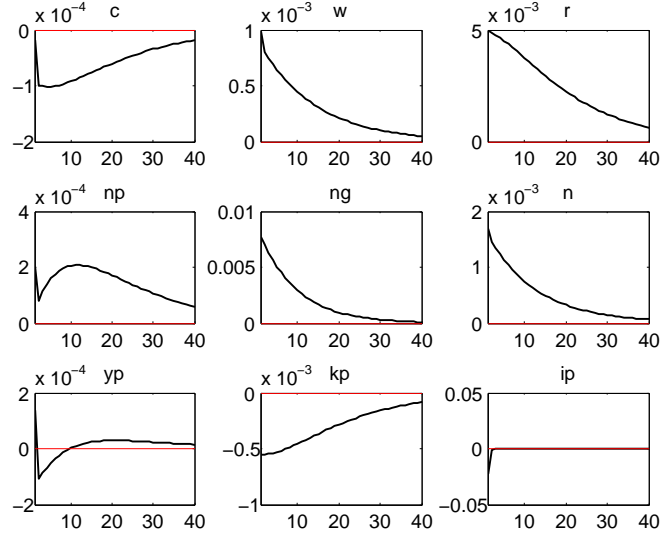
Since the government in this scenario allocates its inputs cost-efficiently, a positive shock to government spending increases government employment and other inputs accordingly. As the labor demand increases, the real wage goes up. Households choose to supply more labor to the market. Households decrease capital supply to the market in order to compensate for the decrease in consumption. The increase of the rental rate makes private firms increase labor demand even though the real wage is slightly higher than the steady state level.

Figure 3.2 shows the responses of a positive shock to government spending in a staggered price environment. Following Pierre et al. [2003], I set $\xi_p = 0.85$ and $\gamma_p = 0.408$. Since a portion of private firms can not adjust their output price immediately, they produce more output to fulfill the positive demand shocks. As shown in Figure

Figure 3.2: Shocks to Government Spending with Sticky Price
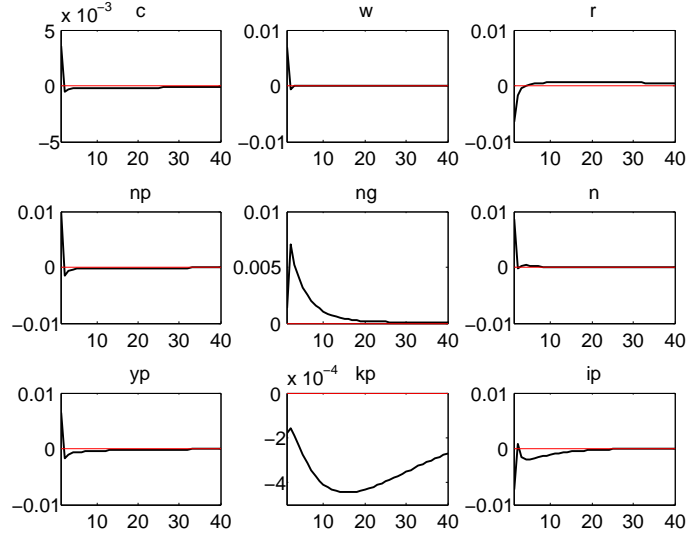
Notes: $\sigma_g = 0.75$, $\sigma_n = 1.5$, $\xi_p = 0.85$, $\gamma_p = 0.408$

3.2, we have a positive response of consumption, labor supply and output to a positive demand shock, at least in the short-run.

Figure 3.3 shows the simulation results in an environment of both sticky price and sticky wages. The results intensify the responses in Figure 3.2 except for the responses of real wage. In figure 3.3, with a similar positive shock to the government budget, the wage does not deviate much from steady state. The reason is that households cannot adjust their wage freely. Furthermore, they must supply more labor if there is a positive demand shock.

3.6.2 SHOCKS TO GOVERNMENT EMPLOYMENT.

Now, I assume that the government can adjust the purchase of certain government inputs but keep the purchase of other inputs fixed at the long-run optimal level.

Figure 3.3: Shocks to Government Spending with Sticky Price and Wage

Notes: $\sigma_g = 0.75$, $\sigma_n = 1.5$, $\xi_p = 0.85$, $\gamma_p = 0.408$, $\xi_w = 0.65$, $\gamma_w = 0.656$
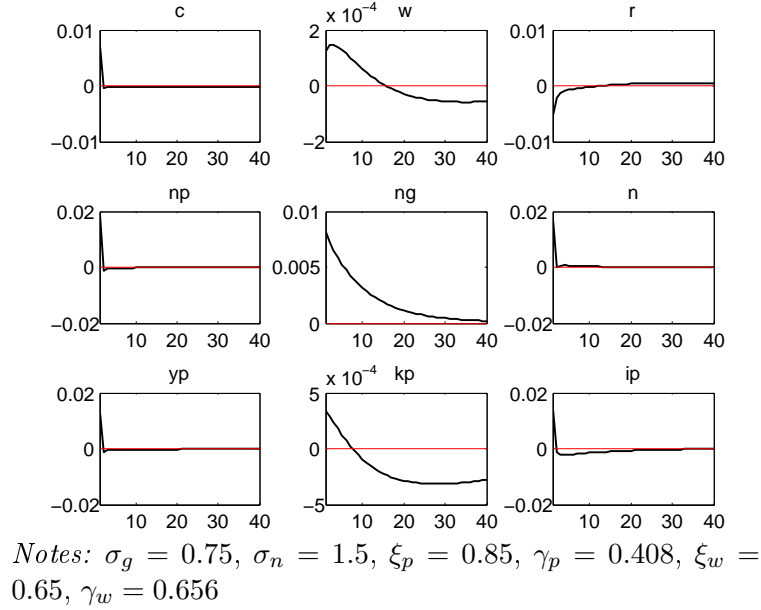
Figure 3.4 displays the results of a simulation where the fiscal budget shock only affects the compensation to government employees in a flexible price environment. With a positive shock to government spending, government hires more labor and crowds out hiring in the private sector. Therefore, total private output decreases and private labor decreases.

Figure 3.5 displays the simulation in a sticky price setting. The main difference from Figure 4 is that a positive shock to government employment can increase private consumption because of the positive complementarity between labor and private consumption. Since prices are sticky in this model, private firms hire more labor and private output increases.

Figure 3.6 displays the simulation of a spending shock on government employment in a sticky price and wage setting. Since wage can not adjust immediately, same

Figure 3.4: Shocks to Government Employment with Flexible Price
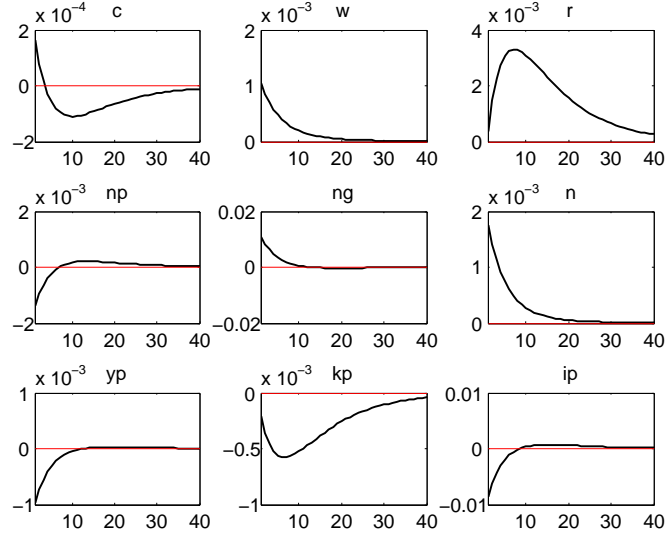
Notes: $\sigma_g = 0.75$, $\sigma_n = 1.5$

positive shock to the compensation of government employee will generate stronger response of government hiring. However, the responses of private consumption, private employment and private output are quite similar to the responses in Figure 3.5. Therefore, the staggered wage setting does not intensify the response of private economy related to the scenario with staggered price.

3.6.3 SHOCKS TO GOVERNMENT INTERMEDIATE GOODS

This subsection shows the results of the simulation where the shocks of government spending only work on the channel of intermediate goods purchasing. Figure 3.8 displays the simulation results of a positive shock to the purchases of government intermediate goods. The responses of the main variables are quite similar to the predictions of a standard neoclassical model. However, the real wage increases as the

Figure 3.5: Shocks to Government Employment with Sticky Price
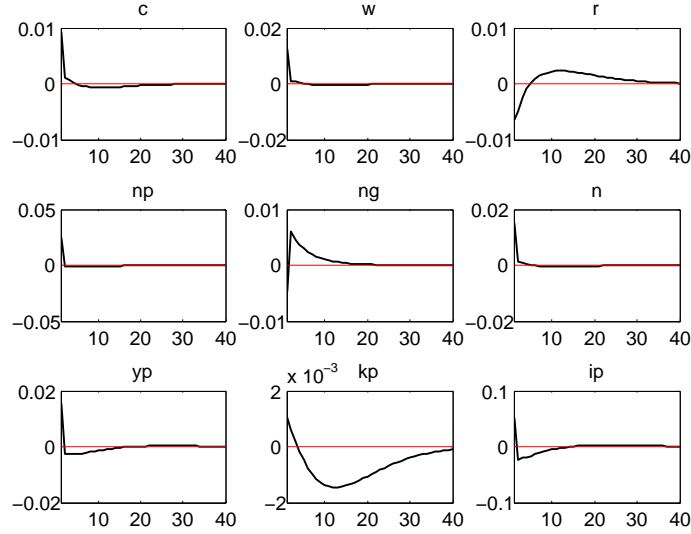
Notes: $\sigma_g = 0.75$, $\sigma_n = 1.5$, $\xi_p = 0.85$, $\gamma_p = 0.408$

government increase the purchases of intermediate goods. One of the possible reasons is that these competitive monopolistic producers shift their labor demand curve when facing extra demand shocks.

Figure 8 shows the results when we introduce a sticky price environment in the model economy. The increase of private consumption is due to the assumption that private consumption and labor supply are complements. Private outputs increase because of the positive demand shock of government intermediate goods.

Figure 9 shows the results when I introduce sticky price and sticky wage into the model economy. The results are similar to Figure 8. However the responses of main macro-variables are generally intensified because of the stickiness of wage. It is not a surprise that the deviation of wage becomes smaller than in Figure 8.

Figure 3.6: Shocks to Government Employment with Sticky Price and Wage

Notes: $\sigma_g = 0.75$, $\sigma_n = 1.5$, $\xi_p = 0.85$, $\gamma_p = 0.408$, $\xi_w = 0.65$, $\gamma_w = 0.656$
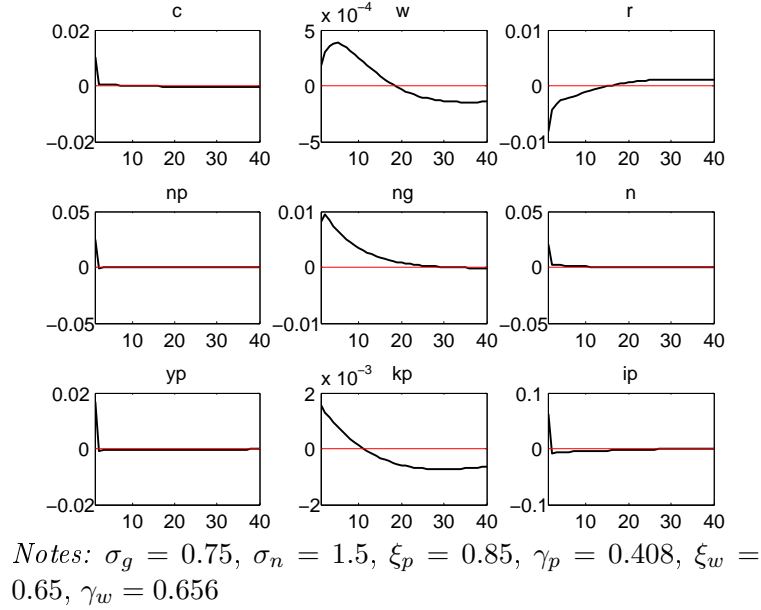
3.6.4   Shocks to Specific Categories of Government Outputs

This subsection focus on the assumption that the government can adjust its production of outputs, altering the level of complementarity with private consumption. From this perspective, government has two ways to stimulate private consumption. First, government can adjust the composition of its production, raising the level of complementarity with private consumption. Second, government increase the quantity of goods produced, which are complements to private consumption. I combine these two strategies and simply assume that the government budget shock works on $\sigma_g{}^2$, the average level of the inverse elasticity of substitution between government outputs

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[2]In order to alleviate the relevant shocks on the elasticity of substitution between labor and private consumption, for simplicity, I assume utility of labor and consumption are separable. I set $\sigma_n = 0$ and re-calibrate the parameters of model economy.

Figure 3.7: Shocks to Government Intermediate Goods with Flexible Price
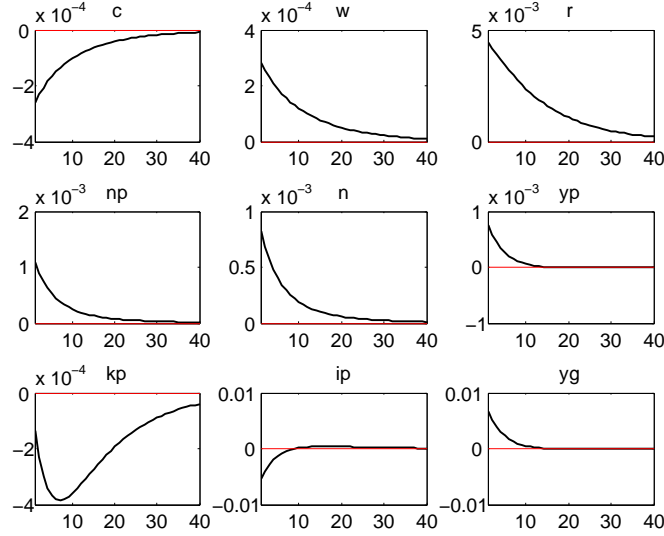
Notes: $\sigma_g = 0.75$, $\sigma_n = 1.5$

and private consumption. The level of the shocks on σ_g is governed by *mul*. In the simulation, I assume $mul = 0.1$. This assumption implies that a standard deviation shock of government spending can only shift σ_g by less than 0.1%.

Figure 3.10 shows the simulation when the government can adjust its outputs according to the complementarity level with private consumption. Private consumption increases because of the positive change of σ_g. Households will save less, so the capital stock will decrease, along with labor demand in private firms. Private sector output will decrease.

Figure 3.11 shows the simulation results in a Calvo staggered price environment. Since private firms can not fully adjust their prices, they instead choose to hire more labor. Therefore, both wages and private output increase.

Figure 3.8: Shocks to Government Intermediate Goods with Sticky Price
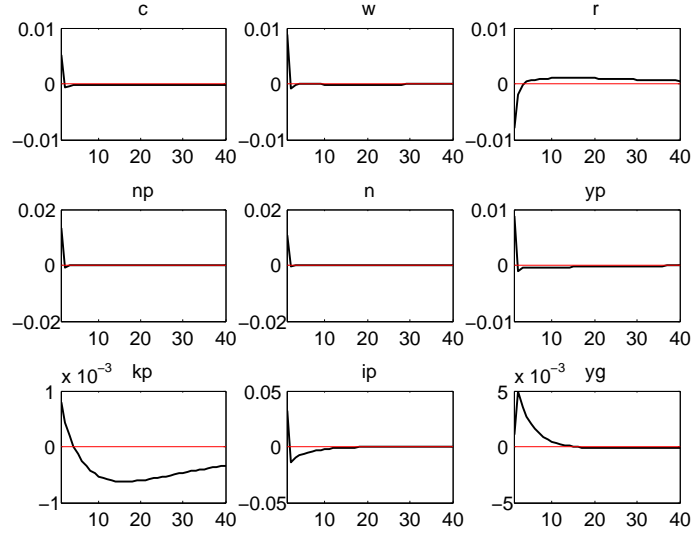
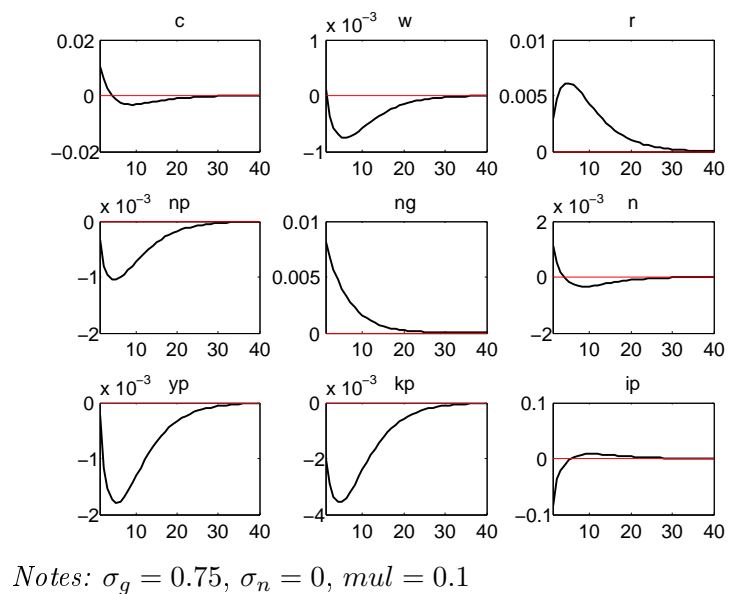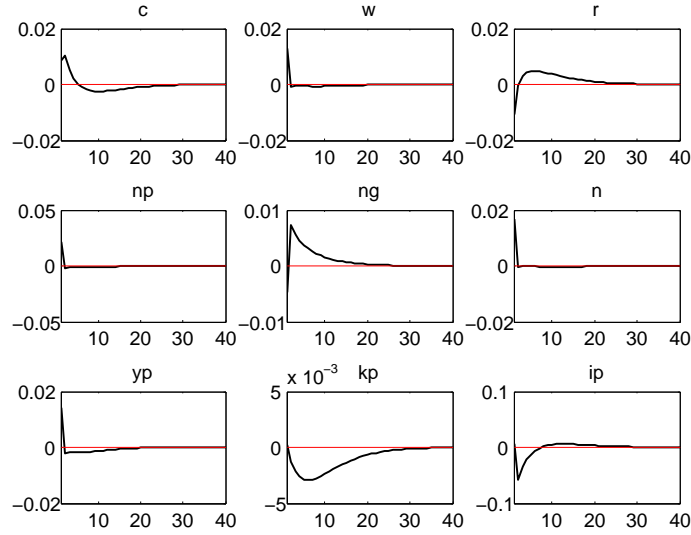*Notes:* $\sigma_g = 0.75$, $\sigma_n = 1.5$, $\xi_p = 0.85$, $\gamma_p = 0.408$

Figure 3.12 shows the simulation results in a staggered wage setting. Except for real wage, other macro-variables all show stronger responses from the shock to government spending comparing with Figure 3.11.

The simulation results of these four fiscal spending strategies imply that government can, in fact, intervene in the private economy through the different channels of government production. The effects and mechanisms of these strategies are not exactly the same. Combined with Calvo price and wage settings, we can span the range of theoretical results on the responses of private consumption, private output, real wage and private labor to a government spending shock.

Figure 3.9: Shocks to Government Intermediate Goods with Sticky Price and Wage

Notes: $\sigma_g = 0.75$, $\sigma_n = 1.5$, $\xi_p = 0.85$, $\gamma_p = 0.408$, $\xi_w = 0.65$, $\gamma_w = 0.656$
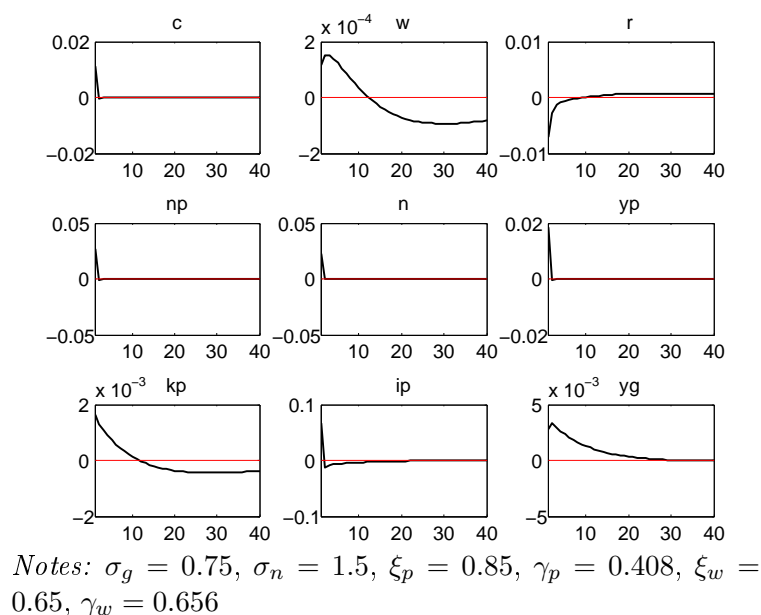
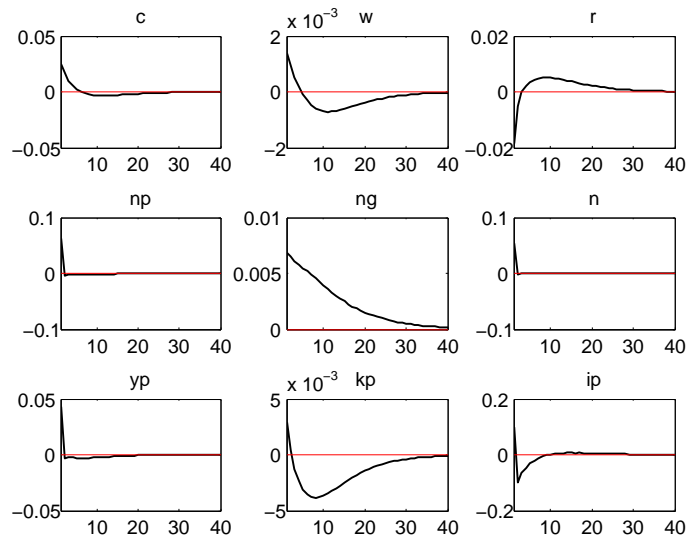Figure 3.10: Shocks to Specific Government output with Flexible Price

Notes: $\sigma_g = 0.75$, $\sigma_n = 0$, $mul = 0.1$

Figure 3.11: Shocks to Government Output with Sticky Price

Notes: $\sigma_g = 0.75$, $\sigma_n = 0$, $\xi_p = 0.85$, $\gamma_p = 0.408$, $mul = 0.1$

Figure 3.12: Shocks to Specific Government Outputs with Sticky Price and Wage

Notes: $\sigma_g = 0.75$, $\sigma_n = 0$, $\xi_p = 0.85$, $\gamma_p = 0.408$, $\xi_w = 0.65$, $\gamma_w = 0.656$

3.7 CONCLUSION

The purchase of government inputs and the production of government outputs plays a crucial role in the operation of the U.S. economy. Distinguishing different categories of government inputs and outputs, according to their natural properties or complimentarity levels with private consumption, provides additional channels for the government to intervene in private economy. This paper studies these channels in a neoclassical model with a benevolent government producer in a Calvo staggered price setting.

I build a standard two-sector business cycle model with price rigidity and calibrate the model economy with U.S. quarterly data from 1960-2006 in NIPA. Then I simulate the model economy. The simulation results indicate that the shocks to government employment and intermediate goods can generate different effects on the private economy.

In sum, in the Calvo staggered price and wage environment, if a government can adjust its fiscal policies according to different channels of government production, as well as the level of complementarity/substitutatbility between private consumption and government production, it is able to span the whole set of theoretical results on the responses of private consumption, private output real wage and private employment to positive government spending shocks.

A. Stackelberg Problem and Social Planner Equivalence

A.1 Stackelberg Problem

Given the choices from the government sector, the household makes their best responses.

Based on the choices from the private sector, the government picks the optimal choices.

Households' Best Responses

$$\max_{c_t, n_{pt}, k_{pt+1}} E_0 \sum_{t=0}^{\infty} \beta^t \mu(c_t, n_{pt}, n_{gt}, y_{gt}) \tag{A.1}$$

$$s.t. \quad w_t n_{pt} + w_t n_{gt} + r_{pt} k_{pt} = c_t + T_t + k_{pt+1} - (1 - \delta_p) k_{pt}$$

where

$$\mu(c_t, n_{pt}, n_{gt}, y_{gt}) = \frac{1}{1-\gamma} ln \left[\eta(\theta c_t{}^{1-\gamma} + (1-\theta) y_{gt}^{1-\gamma}) + (1-\eta)(1 - n_{pt} - n_{gt})^{1-\gamma} \right] \tag{A.2}$$

So the Lagrange equation for household is

$$(c_t, n_{pt}, k_{pt+1}) = E_0 \sum_{t=0}^{\infty} \beta^t [\mu(c_t, n_{pt}, n_{gt}, y_{gt}) \tag{A.3}$$

$$- \lambda_t (c_t + T_t + k_{pt+1} - (1 - \delta_p) k_{pt} - w_t n_{pt} - w_t n_{gt} - r_{pt} k_{pt})] \tag{A.4}$$

The first order conditions are:

$$\frac{\partial}{\partial c_t} = \mu'_{ct} - \lambda_t = 0$$

$$\frac{\partial}{\partial n_{pt}} = \mu'_{npt} + \lambda_t w_t = 0$$

$$\frac{\partial}{\partial k_{pt+1}} = -\lambda_t + \beta \lambda_{t+1} r_{pt+1} = 0$$

Then the household's behavior could be described by the

$$\mu'_{npt} + \mu'_{ct} w_t = 0 \tag{A.5}$$

$$-u'_{ct} + \beta \mu'_{ct+1} r_{pt+1} = 0 \tag{A.6}$$

$$w_t n_{pt} + w_t n_{gt} + r_{pt} k_{pt} = c_t + T_t + k_{pt+1} - (1 - \delta_p) k_{pt} \tag{A.7}$$

Government's Optimization Problem

$$\max_{G_{t+1}, n_{gt+1}, k_{gt+2}, i_{gt+1}} E_0 \sum_{t=0}^{\infty} \beta^t \mu(c_t, n_{pt}, n_{gt}, y_{gt}) \tag{A.8}$$

s.t

$$y_{gt} - z_{gt} m_t^{d_1} k_{gt}^{d_2} n_{gt}^{1-d_1-d_2} = 0$$

$$k_{gt+1} - k_{gt}(1 - \delta_g) - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 = 0$$

$$G_t - m_t - w_t n_{gt} - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 = 0$$

$$\mu'_{npt} + \mu'_{ct} w_t = 0$$

$$-u'_{ct} + \beta \mu'_{ct+1} r_{pt+1} = 0$$

$$w_t n_{pt} + w_t n_{gt} + r_{pt} k_{pt} - c_t - T_t - k_{pt+1} + (1 - \delta_p) k_{pt} = 0$$

The Lagrange problem of the government could be explained as:

$$(y_{gt}, n_{gt+1}, k_{gt+2}, i_{gt+1}, G_{t+1}) = E_0 \sum_{t=0}^{\infty} \beta^t [\mu(c_t, n_{pt}, n_{gt}, y_{gt})$$

$$+ \lambda_{1t}(y_{gt} - z_{gt} m_t^{d_1} k_{gt}^{d_2} n_{gt}^{1-d_1-d_2})$$

$$+ \lambda_{2t}(k_{gt+1} - k_{gt}(1 - \delta_g) - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2)$$

$$+ \lambda_{3t}(G_t - m_t - w_t n_{gt} - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2)$$

$$+ \lambda_{4t}(w_t n_{pt} + w_t n_{gt} + r_{pt} k_{pt} - c_t - G_t - k_{pt+1} + (1 - \delta_p) k_{pt})$$

$$+ \lambda_{5t}(\mu'_{npt} + \mu'_{ct} w_t)$$

$$+ \lambda_{6t}(\beta \mu'_{ct+1} r_{pt+1} - u'_{ct})]$$

The Kuhn Tacker conditions for this problem is:

$$\mu'_{y_{gt}} + \lambda_{1t} + \lambda_{5t} \nabla_{y_{gt}} (\mu'_{npt} + \mu'_{ct} w_t) + \lambda_{6t} \nabla_{y_{gt}} (\beta \mu'_{ct+1} r_{pt+1} - u'_{ct}) = 0 \qquad (A.9)$$

$$\mu'_{n_{gt+1}} + \lambda_{1t+1}(-z_{gt+1}(1 - d_1 - d_2)m_{t+1}^{d_1}k_{gt+1}^{d_2}n_{gt+1}^{-d_1-d_2}) + (\lambda_{4t+1} - \lambda_{3t+1})(-w_{t+1})$$

$$+ \lambda_{5t+1} \nabla_{n_{gt+1}} (\mu'_{npt+1} + \mu'_{ct+1} w_{t+1}) + \lambda_{6t} \nabla_{n_{gt+1}} (\beta \mu'_{ct+2} r_{pt+2} - u'_{ct+1}) = 0 \quad (A.10)$$

$$-\beta \lambda_{1t+2}z_{gt+2}d_2 m_{t+2}^{d_1}k_{gt+2}^{d_2-1}n_{gt+2}^{1-d_1-d_2} + \lambda_{2t+1} - \beta \lambda_{2t+2}(1 - \delta_g) = 0 \quad (A.11)$$

$$(\lambda_{2t+1} + \lambda_{3t+1})(-1 - \Omega(i_{gt+1} - i_{gt})) + \beta(\lambda_{2t+2} + \lambda_{3t+2})\Omega(i_{gt+2} - i_{gt+1}) = 0 \quad (A.12)$$

$$\lambda_{3t} - \lambda_{4t} = 0 \quad (A.13)$$

$$y_{gt} - z_{gt}m_t^{d_1}k_{gt}^{d_2}n_{gt}^{1-d_1-d_2} = 0 \quad (A.14)$$

$$k_{gt+1} - k_{gt}(1 - \delta_g) - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 = 0 \quad (A.15)$$

$$G_t - m_t - w_t n_{gt} - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 = 0 \quad (A.16)$$

$$\mu'_{npt} + \mu'_{ct}w_t = 0 \quad (A.17)$$

$$-u'_{ct} + \beta \mu'_{ct+1}r_{pt+1} = 0 \quad (A.18)$$

$$w_t n_{pt} + w_t n_{gt} + r_{pt}k_{pt} - c_t - G_t - k_{pt+1} + (1 - \delta_p)k_{pt} = 0 \quad (A.19)$$

A.2 Equivalence

The planner's optimization problem

$$\max_{\substack{c_t,n_{pt},k_{pt+1},y_{gt},n_{gt+1} \\ k_{gt+2},i_{gt+1},G_{gt+1}}} E_0 \sum_{t=0}^{\infty} \beta^t \mu(c_t, n_{pt}, n_{gt}, y_{gt}) \qquad (A.20)$$

$$s.t.$$

$$y_{gt} - z_{gt}m_t^{d_1}k_{gt}^{d_2}n_{gt}^{1-d_1-d_2} = 0 \qquad (A.21)$$

$$k_{gt+1} - k_{gt}(1 - \delta_g) - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 = 0 \qquad (A.22)$$

$$G_t - m_t - w_t n_{gt} - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 = 0 \qquad (A.23)$$

$$w_t n_{pt} + w_t n_{gt} + r_{pt}k_{pt} - c_t - G_t - k_{pt+1} + (1 - \delta_p)k_{pt} = 0 \qquad (A.24)$$

The Lagrange equation for the second scenario is

$$(y_{gt}, n_{gt+1}, k_{gt+2}, i_{gt+1}, G_{g+1}, c_t, n_{pt}, k_{pt+1}) \tag{A.25}$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t [\mu(c_t, n_{pt}, n_{gt}, y_{gt})$$

$$+ \lambda_{1t}(y_{gt} - z_{gt} m_t^{d_1} k_{gt}^{d_2} n_{gt}^{1-d_1-d_2})$$

$$+ \lambda_{2t}(k_{gt+1} - k_{gt}(1 - \delta_g) - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2)$$

$$+ \lambda_{3t}(G_t - m_t - w_t n_{gt} - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2)$$

$$+ \lambda_{4t}(w_t n_{pt} + w_t n_{gt} + r_{pt} k_{pt} - c_t - G_t - k_{pt+1} + (1 - \delta_p)k_{pt})]$$

The Kuhn Tacker conditions are:

$$\mu'_{y_{gt}} + \lambda_{1t} = 0 \tag{A.26}$$

$$\mu'_{n_{gt+1}} - \lambda_{1t+1} z_{gt+1}(1 - d_1 - d_2)m_{t+1}^{d_1} k_{gt+1}^{d_2} n_{gt+1}^{-d_1-d_2} + (\lambda_{4t+1} - \lambda_{3t+1})(w_{t+1}) = 0 \tag{A.27}$$

$$-\beta \lambda_{1t+2} z_{gt+2} d_2 m_{t+2}^{d_1} k_{gt+2}^{d_2-1} n_{gt+2}^{1-d_1-d_2} + \lambda_{2t+1} - \beta \lambda_{2t+2}(1 - \delta_g) = 0 \tag{A.28}$$

$$(\lambda_{2t+1} + \lambda_{3t+1})(-1 - \Omega(i_{gt+1} - i_{gt})) + \beta(\lambda_{2t+2} + \lambda_{3t+2})\Omega(i_{gt+2} - i_{gt+1}) = 0 \tag{A.29}$$

$$\lambda_{3t} - \lambda_{4t} = 0 \tag{A.30}$$

$$y_{gt} - z_{gt} m_t^{d_1} k_{gt}^{d_2} n_{gt}^{1-d_1-d_2} = 0 \tag{A.31}$$

$$k_{gt+1} - k_{gt}(1 - \delta_g) - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 = 0 \tag{A.32}$$

$$G_t - m_t - w_t n_{gt} - i_{gt} - \frac{\Omega}{2}(i_{gt} - i_{gt-1})^2 = 0 \tag{A.33}$$

$$\mu'_{npt} + \mu'_{ct} w_t = 0 \tag{A.34}$$

$$-u'_{ct} + \beta \mu'_{ct+1} r_{pt+1} = 0 \tag{A.35}$$

$$w_t n_{pt} + w_t n_{gt} + r_{pt} k_{pt} - c_t - G_t - k_{pt+1} + (1 - \delta_p)k_{pt} = 0 \tag{A.36}$$

First, I assume there exist a vector $A^* = (y_{gt}^*, n_{gt+1}^*, k_{gt}^*, i_{gt}^*, G_{gt}^*, c_t^*, n_{pt}^*, k_{pt}^*)$ and constants λ_i^*, $i = 1, ..., 4$ satisfying the Kuhn Tacker conditions (A.26) through (A.36). Since the utility function μ is concave, it is sufficient to say A^* is the optimal solution of the planner's optimization problem in the second scenario.

Now, let us prove the optimal solution of the second's scenario is also the optimal solution of the first scenario. Comparing the conditions (A.9) to (A.19) with the conditions (A.26) to (A.36), if $\lambda_{5t} = 0$ and $\lambda_{6t} = 0$, then the Kuhn Tacker conditions in the first scenario λ_i becomes the same as in the planner's economy of the second scenario. Therefore, λ_i^*, $i = 1, ..., 4$ with additional constants $\lambda_{5t} = 0$, $\lambda_{6t} = 0$,and A^* should also satisfy the Kuhn Tacker sufficient conditions from (A.9) through (A.19). In this case, according to the Kuhn Tacker sufficient condition, A^* should also be the optimal solution to the economy in the first scenario, which means the optimal solution of the second scenario is also the optimal solution of the first scenario. Q.E.D

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