Three Essays on Microeconomic Dynamics

A Thesis
submitted to the Faculty of the
Graduate School of Arts and Sciences
of Georgetown University
in partial fulfillment of the requirements for the
degree of
Doctor of Philosophy
in Economics

By

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Washington, DC
April 12, 2012

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Abstract

The following three essays discuss dynamics under uncertainty. The first two chapters discuss the question of how one agent can learn about another in an environment with repeated interaction. The third chapter centers on the optimal time to implement a policy in a scenario with stochastic features.

In the first chapter I develop a model of learning in which the focus is on the process of gradual learning. In particular, I examine the question of how the dynamic learning process can improve the long run acquisition of information. The model is based on a repeated interaction between two agents, an Investor and an Associate, where cooperation by both is needed in order to develop a project. The Investor can learn about the Associate by offering different levels of investment in order to infer something about the Associate's type through his response. If the Associate defaults, then he has a short term gain but the interaction is over. The model looks to find the long term level of investment in this interaction. The model can be viewed in a variety of ways, for example as trust development. It can explain why in some cases, even after a long interaction, the level of trust and hence the associated investment will not increase above a certain threshold.

One concept that is relevant in this chapter is the notion of "monotonicity". Monotonicity means roughly that the interaction in every period should be informative. If this happens, then the investment realized by both agents increases gradually, and the amount of information necessary for determining the next period investment is reduced.

The second chapter develops an application of the theory proposed in the first chapter. In this chapter the idea further elaborates upon the theory by using a particular functional form, a Cobb Douglas function. Additionally, this chapter discusses how the requirements of the theorems of the first chapter are related to the fundamentals of the model and develops new results. For example, one result with empirical implications is: a more patient Investor will not necessarily learn more in the long term than a less patient one.

The third chapter analyzes a different problem. Here, an authority has the chance to implement a costly policy, facing the risk that with some probability a similar policy is imposed externally. The question is: what is the best way to implement this policy? Should one just wait until the change is imposed or should one commit to some policy in the future? The difference between both cases is described and we analyze how the parameters of the model affect the optimal solution of the policy maker. Finally, I analysis the effects of time length between the announcement and the implementation. I then characterize an "optimal announcement time" for this model.

INDEX WORDS: Dynamic games, asymmetric information, belief updating

Acknowledgments

I would like to acknowledge everyone who helped me to finish this thesis. I would like to thank my advisor Professor Lagunoff, and the other two committee members, professors Albrecht and Anderson, for their help in the dissertation. I would like to acknowledge the suggestions and comments by Professor Genicot, and the help of Professor Schwartz. I would also like to acknowledge that the third chapter in this dissertation "When Should Developing Countries Announce Their Climate Policy?" was coauthored with Sebastian Miller.

I would also like to thank to the people who helped me during these years, thanks to Aga, Alberto, Christiaan, Daniel, Javier, Maggie, Paula, Sebastian and Yevgeniya. Finally I would thanks to the people in the econ department who made my life easier in the program.

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Chapter 1

Gradualism and Commitment: When to Stop Learning?

1.1 Introduction

This paper studies the way in which asymmetric information between agents can be reduced when they interact in an ongoing project. It focuses on the extent to which this asymmetry is reduced over time and on how much an uninformed agent will learn in that time.

Consider a scenario where an Investor (she) forms a joint venture with an Associate (he). The project they jointly undertake is ongoing as long as they both contribute. The Associate can quit at any moment, and his willingness to participate is private information. If he quits, the Investor incurs a cost and the interaction ends. The joint revenue and the cost derived from the Associate quitting are both increasing with the investment level. The Investor can learn about the Associate's type by offering different levels of investment and observing whether or not he cooperates. The main question here is: how much will the Investor learn about the Associate's type when the cost of learning is related to the investment level?

The scenario outlined above corresponds to several types of relationships. Consider, for instance, a bank credit line, where the investment is the loan amount. The credit line increases with time if the client (Associate) pays his obligations. Alternatively, consider a company that invests in worker training. While the productivity of a worker increases with training, the value of his outside option typically goes up as well. If the worker's willingness to use the outside option is private information, then choosing the optimal level of training is a problem that can be analyzed according to the framework suggested in this paper. A final example is that of a company that wants to invest in a country where the expropriation risk is unknown. In this case the company is the Investor who needs to learn about the country's authority's willingness to expropriate.

The trade-off in all of these cases is between a static effect and a dynamic one. The static effect involves a risk-return trade off. If the investment level is high and the Associate cooperates, then the Investor's return is high too. However, the potential cost is also high if the Associate leaves or defaults.

The dynamic effect involves learning. This learning process has a cost related to the possibility of the Associate quitting and has two characteristics. First, the Investor does not generally learn in just one period. When the learning cost per period is large, he may choose to continue learning over subsequent periods. Secondly, the Associate's decision involves an expected payment in future periods, hence the Investor must account for this while solving for her optimal rate of learning. These features restrict the ways in which the Investor can learn about the Associate. If the Investor expects some types of Associate to leave, then the future path of investment levels will typically be less profitable than the current investment level. There is an implicit upper bound for future decisions.

To some extent, this type of problem has been studied in the literature.[1] In these models there is typically full learning or no learning at all, since the models usually assume two type of Associate. The present paper introduces a model with a continuum of these types. This allows one to examine the question: How much learning takes place in an ongoing relationship? How much does the Investor want to learn about the Associate? I characterize equilibria of the model and find closed form solutions for the steady state investment level.

In equilibrium, the investment path either increases until it approaches a steady state asymptotically, or it remains fixed at a certain amount. Given the path, the Associate will choose the optimal quitting time, which could be "never". The steady state investment level reveals how much the Investor learns overall about the market. If this level is low enough, then the Investor will continue to have the cooperation of almost all the types of Associate, but at the cost of receiving a low long run payoff per each Associate. If this level is high, then the Investor knows that only the better types stay in the market, while the rest of them had left at some moment.

If one restricts the Investor to use Markov strategies, then the equilibrium path will gradually increase, without "jumps". On the other hand, if any equilibrium without restrictions on the strategies exhibits this gradual increase, then this particular equilibrium can be replicated using only Markov strategies. That is, there exits an equivalence between the use of Markov strategies and monotonicity in equilibrium. This property can be restated in a different way, if the probability of an Associate leaving at any time is not zero, then the restriction to Markov strategies does not bind. This result can be extended to the required length of the commitment. Under

[1] See for example Watson (1999, 2002), and Ghosh and Ray (1996, 2001).

a gradual increase, a full commitment solution can be replicated with a short period commitment. The steady state level analysis does not require the Markov property, but the evolution to this level is easy to understand using this result.

Of particular importance in any partnership is the value of commitment to the Investor. I study equilibria with and without commitment by the Investor. Under certain conditions, the Investor can learn more in the steady state with commitment than without commitment. However, even in these cases, the steady state levels with and without commitment are shown to converge as the Investor becomes infinitely patient.

The intuition behind this result is the following: The possibility of committing to future investments allows the Investor to reduce the learning cost, that is, the cost from the Associate decision to quit. Because the Investor can credibly commit to a gradual increase in the investment level, an Associate with a lower outside option decides to quit at lower levels of investment. This ability reduces the learning cost, which is the flow cost during the stage before the steady state is reached. However, if the Investor becomes more patient, this flow cost is less relevant. Moreover, in the limit, just the steady state level matters. When this happens, the cost at which one Associate leaves does not affect the Investor decision, and the reduction that comes from the commitment vanishes.

The rest of the paper is organized as follows: Section 2 presents the literature review. Section 3 introduces the model and describes the players, payoffs, information structure and dynamics of the model. The components of the agents' strategies and their beliefs are studied in Section 4. In Section 5 the equilibrium of the model is described. Section 6 describe the equilibrium in the case in which the Investor can commit. In Section 7 the non-commitment case is analyzed and compared with the commitment scenario. Finally, Section 8 presents the conclusions.

1.2 Related Literature

The key feature of this model is the gradual increase in investment. There are several papers that analyzes gradualism in productive relationships. In these papers "gradualism" occurs in equilibrium even without asymmetric information. Lockwood and Thomas (2002) find that when the investment is not reversible, then in the efficient equilibrium the investment level increases gradually. With incomplete information there are a variety of models which exhibit gradualism. Sobel (1985) presents an example of a lender-borrower framework in which the amount of loan increases over time as the borrower fulfills the payments. Andreoni and Samuelson (2006) provide an experiment where a two period prisoners' dilemma game is played, and collaboration is related to the distribution of the payments between the two stages. If the payment exhibits a gradual increase, then the players are more willing to cooperate than when payments are decreasing or stable.

The present paper is closer to a line of research presented by Watson (1999, 2002). Watson (1999) shows that gradualism is an equilibrium outcome in the context of a dynamic prisoners' dilemma problem with two-sided uncertainty. The key point of his paper is that in a framework with two types of agents who have different payoffs, when the interaction begins with lower levels of investment, equilibrium implies full revelation. That is, only the agent who is more likely to cooperate does so in the end. The level of long term investment, assumed exogenous, is not a central issue. In line with that model, Blonski and Probst (2004) analyze a two-sided uncertainty game with linear payoff functions. Furusawa and Kawakami (2008) study gradualism with outside options that are stochastically available. Their work features two-sided uncertainty but the probability of exercising the option of gradualism is constant while in my paper the probability is endogenous and changes with the path of investments.

Other related works are papers by Ghosh and Ray (1996, 2001) that look at cooperation with limited information. The authors study an equilibrium in a market where agents need to cooperate but face a prisoners' dilemma problem with incomplete information about the other player's impatience level. In the equilibrium they study there is a period of learning, and if the learning phase is successful, there is a steady state stage of cooperation. Ghosh and Ray (2001) show that in an equilibrium with two types and two-sided asymmetric information, uncertainty is resolved after one period. In the present paper the uncertainty is not necessarily eliminated after one period, allowing gradual learning to take place over many periods. Moreover, in some cases it is not optimal to remove all uncertainty; that is, not all the types are revealed even after the steady state is reached.

There are some papers that have addressed related applied problems, like the ones described in the introduction, in a similar way to this work. Sannikov (2008) proposes a model in continuous time in which an agent needs a credit line to finance a project. In the presence of moral hazard and adverse selection the optimal contract involves an increasing credit line. One difference with my model is that in Sannikov's work, the steady state level of investment is exogenous, pinned down by the parameters, whereas here it is endogenous. Hermalin (2002) proposes a model that explains the underprovision of training given adverse selection. The model combines the length of the contract with the heterogeneity of the workers in order to explain why firms do not train workers at the optimal level. Halac (2010) analyzes a Principal-Agent model when the Principal has a outside option that is private information. She shows that in this model the information can be revealed depending on the bargaining power allocation. Rauch and Watson (2002) use a model where the uncertainty about the ability of a external supplier can be reduced by starting with a small level of interaction. Finally, Thomas and Worrall (1994) analyze the case of a company that faces the risk of expropriation, and study the investment path.

Ultimately, the present paper adds to the literature on learning in a productive relationship by introducing structural uncertainty about the outside options involving more than two types. The structure of uncertainty modeled introduces the question of how many of the Associate's types will cooperate with the Investor in the equilibrium. The answer is not that all types cooperate or only the highest one, but rather that there may be an interior solution. This feature allows me to analyze whether a change of parameters affects the steady state investment level through a change in the number of types or the investment levels at which these types cooperate. The advantage of a model with this structure of uncertainty is the possibility of determining the steady state level of investment as a result of the model, not necessarily as a corner solution, there can be partial learning. A richer structure of uncertainty allows the possibility of gradual learning over time, where questions about commitment can be analyzed.

1.3 The Model

There are two players who pursue a joint venture in each of an infinite number of periods. The basic characteristics of the model are:

- There are two agents. An uninformed one, the Investor, and an informed one, the Associate.
- One-sided uncertainty, involving a continuum of Associate types.
- Each period the Investor proposes the level of joint investment.
- The Associate can then choose either to "cooperate" or "quit" after observing the Investor's decision. If he quits then the game ends.
- The payoff from cooperation is increasing in the investment level for both agents, but the ratio between the Associate's payoff from cooperation and his payoff from quitting is decreasing in the investment level.

Thus the interaction can be modeled as an infinite horizon dynamic game. It is common knowledge that the Investor has a discount factor β. The Associate is privately informed of his discount factor, δ. All of above is common knowledge. Time is discrete.

1.3.1 Stage game payoffs and actions

At each stage, the Investor offers the Associate some level of investment x. After observing this level, the Associate chooses whether or not to cooperate. The level of investment can represent any scalar that positively impacts the return to investment, such as assets, money or time. For now assume that this asset fully depreciates after one period. Both the Investor and the Associate are risk neutral. The stage game payoffs are summarized in the following table, where the Investor chooses x and the Associate chooses between the columns, Cooperate or Quit:

$$\begin{array}{c|cc} & C & Q \\ \hline x & \pi(x), \pi(x) & -x, x \end{array} \tag{1.1}$$

Thus $\pi(\cdot)$ represents a profit that is continuously differentiable, strictly concave, and satisfies $\pi(0) = 0$, and $\pi'(0) > 0$.[2] In case of quitting, payoffs correspond with the stylized fact that the Associate can simply keep the asset or represents some outside option that is increasing in the investment[3]. This structure represents a case in which the Associate has some incentive to quit and quitting is costly to the Investor.

Within each period, the stage game has the following two steps:

- The Investor chooses a level of investment, x, which is observed by the Associate.
- The Associate, after observing the level of investment, chooses whether to cooperate or to quit. If he cooperates, both players get $\pi(x)$. If he quits, the Associate takes the investment and the Investor loses it.

1.3.2 Dynamics

The dynamic structure of the game is the following: if the Associate chooses to cooperate, then the payoffs are realized and the game continues. If he chooses to

[2] Throughout the paper results will be illustrated using a Cobb Douglas production function. In this case $\pi(x) = \gamma x^{\alpha} - x$, where $\gamma > 0$ and $\alpha \in (0, 1)$.

[3] Without the assumption of full depreciation the Investor's payoff from quitting would change because one would need to specify how much she can recover from the remaining investment

quit, both receive their "quitting" payoff and the game ends. The investment level can change from one period to another without any physical restriction.

As a reference point, it is useful to analyze the complete information case. If the Associate's discount factor is common knowledge, then the equilibrium of the dynamic game depends on this value, δ, and the profit function π. The equilibrium of this game is a level of investment that maximizes the Investor's profit subject to the Associate's cooperation. Unrestricted maximization, when the Associate cannot quit, gives a unique optimal level of investment. When the option of quitting is included, then the optimal investment level cannot be higher than an investment level that makes the Associate indifferent between cooperating and quitting. Formally the equilibrium of the game is summarized in Lemma 1.1. A formal definition of the two previous investment levels is needed.

Definition: Complete Information upper bounds

- Let $\hat{x}$ denote the unrestricted optimal investment level, that is:

$$\pi'(\hat{x}) = 0 \tag{1.2}$$

- Let $\tilde{x}_\delta$ be the investment level at which an Associate with discount factor δ is indifferent between quitting today or cooperating forever. That is:

$$\tfrac{\pi(\tilde{x}_\delta)}{1-\delta} = \tilde{x}_\delta \tag{1.3}$$

Lemma 1.1 (Complete Information Equilibrium) *Consider a complete information dynamic game with stage payoffs represented in table* (1.1) *and with discount factors β and δ for the Investor and the Associate, respectively. The only subgame perfect equilibrium in pure strategies is one where the Investor offers* $\min\{\hat{x}, \tilde{x}_\delta\}$ *at any time, and the Associate cooperates.*

Proof. The Investor maximizes her expected payoff subject to the Associate's cooperation. The solution is the lower value between the unrestricted optimal, $\hat{x}$, and the upper bound of the feasible set, $\tilde{x}_\delta$.[4]. ∎

[4] In the equilibrium section more details about a generalization of these limits and their use in the asymmetric information case will be given.

1.3.3 Information structure

The general case assumes that the Investor only knows that the Associate's discount factor is uniformly distributed in the interval $[\delta_l, \delta_h]$, with $\delta_l < \delta_h$. The Investor discount factor is public knowledge. All of this is common knowledge.

1.4 Strategies

A strategy for the Investor specifies the level of proposed investment in each period given the previous levels offered. Since the game ends if the Associate quits, it is not necessary to include his previous decisions in the relevant history. A strategy for the Associate specifies his decision to cooperate or to quit given the investment history, the current investment, and potentially future investment levels offered with commitment.

In some scenarios, the Investor can commit in advance to a sequence of investment levels, each of which is conditional on the Associate having remained in the game up to that point. I analyze the model both with and without commitment.[5]

The Investor's strategy involves a possible infinite stream of investments conditional on the Associate's cooperation. Her optimization takes into account the probability that the Associate quits at time t, which is incorporated through her uncertainty about the Associate's type[6]. The Associate's strategy involves choosing at any time to cooperate or to quit given his type and the Investor's proposed

[5] The commitment is not necessarily an explicit one, for example in the case of the bank is just the fact that the client can anticipate the future increments of his credit line given the experience of other clients.

[6] Mixed strategies are not considered, so the probability distribution is just over types

investment level. Start with two different cases; the first one is the *Full Commitment* case, denoted by FC, where the Investor commits to the investment path in the first period and cannot change this path. The second case is the *Non Commitment* one, denoted by NC, where the Investor only offers the current investment level and it is not possible to commit to a future one.

The Investor's strategy in any period involves the choice of an investment level given the public information, that is, the previous investments and the already committed investment levels. Let $\boldsymbol{H}^t$ be the set of all the investment levels until t with a typical element $h^t \in \boldsymbol{H}^t$, that is $h^t = \{x_1, \ldots, x_t\}$. Let $\boldsymbol{H}$ be the set of all the possible histories, that is $\boldsymbol{H} \equiv \bigcup_{t=0}^{\infty} \boldsymbol{H}^t$, where $\boldsymbol{H}^0 \equiv \emptyset$.

The Investor's strategy is defined by $\sigma_t : \boldsymbol{H}^{t-1} \rightarrow \boldsymbol{R}_+$, where $\sigma_t\left(h^{t-1}\right) = x_t$. Implicitly the strategy depends on the Investor's discount factor and the distribution of the Associate's types, but for simplicity this dependence is not include in the specification. Use $\sigma \equiv \bigcup_{t=1}^{\infty} \sigma_t$ as the set off all the Investor's strategies. On the equilibrium path, $h^t = \left(h^{t-1}, \sigma_t\left(h^{t-1}\right)\right)$ for all t. Let $\sigma_{h^t} \equiv \left\{\sigma_{t+1}\left(h^t\right), \sigma_{t+2}\left(h^{t+1}\right), \ldots\right\}$, that is, the expected stream of investment given a starting history h^t and using the decision rule σ. In the Non Commitment case this notation is straightforward, the Investor's decision at any time depends on the previous (accepted) levels of investment. In the Commitment case σ is chosen once and for all at $t = 0$ and no subsequent revision by the Investor can occur. Let σ^{NC} and σ^{FC} denote the Non Commitment and Full Commitment strategies respectively[7], that is $\sigma_{h^1}^{FC}$ is committed at $t = 1$.

[7]The superscript FC or NC will be used when the Full Commitment or Non Commitment case are analyzed, respectively. When the superscript is omitted it is because the statement can be used in both cases

The Associate's decision at time t is denoted by φ_t, where the output is a binary variable. Use 0 as the no cooperation (quitting) decision, while 1 is the decision to cooperate. In the Full Commitment case $\varphi_t^{FC} : \boldsymbol{H} \times (0,1) \to \{0,1\}$. In the Non Commitment case the strategy is $\varphi_t^{NC} : \boldsymbol{H}^t \times (0,1) \to \{0,1\}$, that is, it can only include as argument the past and current investment levels and the Associate's type. Again, let $\varphi \equiv \bigcup_{t=1}^{\infty} \varphi_t$. Let $\varphi_t(h,\delta)$ be the Associate's decision at time t given history h and type δ.

There is a third case that will be used in this chapter, the *One Period Commitment* case, denoted by C. In this case the Investor can commit to the next period investment level, but only to this one. This reflects a short time commitment. In this case the Investor's strategy is given by $\sigma_t^C : \boldsymbol{H}^t \to \boldsymbol{R}_+$, that is the Investor includes the actual investment level as an input, because it was committed to in the previous period. So $\sigma_t^C(h^t) = x_{t+1}$. The Associate's strategy is given by $\varphi_t^C : \boldsymbol{H}^{t+1} \to \{0,1\}$, that is, the Associate knows the next period investment level before deciding whether or not to cooperate today.

1.5 Equilibrium

1.5.1 Characterization of equilibrium of the game

A Perfect Bayesian Equilibrium (PBE) requires sequentially optimal strategies for the Associate and (in the Non Commitment case) the Investor, and a belief updating process that satisfies Bayes' rule whenever possible. The Associate's problem at time t requires a choice between cooperating or quitting given the relevant history and the

Investor's decision rule[8].

I start by working with the Non Commitment case. The Associate's payoff function at time t is:

$$W^{NC}\left(\sigma^{NC},\varphi^{NC}\mid h^{t},\delta\right)=\underbrace{\varphi_{t}^{NC}\left(h^{t}\right)\left[\pi\left(x_{t}\right)+\delta W^{NC}\left(\sigma^{NC},\varphi^{NC}\mid h^{t+1},\delta\right)\right]}_{Cooperate}+$$

$$\underbrace{\left[1-\varphi_{t}^{NC}\left(h^{t}\right)\right]x_{t}}_{Quit} \tag{1.4}$$

s.t.

$h^{t+1}=\left(h^{t},\sigma_{t+1}^{NC}\left(h^{t}\right)\right)$

Note that we look for pure strategies, that is, $\varphi=0$, or $\varphi=1$. The Associate needs to choose between cooperating at t or quitting. Cooperation today and quitting in the future is represented by the first element of the equation (1.4). Before quitting the Associate receives the cooperation payoff. The payoff associated with quitting now is the second term.

The Investor's expected revenue, given her strategy, σ^{NC}, the Associate's one, $\varphi^{NC}(\delta)$,[9] and the belief, are given by:

[8] The problem is the definition of the relevant history in each one of the three cases detailed before.

[9] For simplicity I omit the dependence on h^{t}.

$$V^{NC}\left(\sigma^{NC},\varphi^{NC}\mid h^{t-1}\right)\equiv E_{\delta}\left\{\varphi^{NC}\left(h^{t-1},\sigma_t^{NC}\left(h^{t-1}\right);\delta\right)\left[\pi\left(\sigma_t^{NC}\left(h^{t-1}\right)\right)+\beta V\left(\sigma^{NC},\varphi^{NC}\left(\delta\right)|h^{t}\right)\right]\right.$$

$$\left.-\left[1-\varphi^{NC}\left(h^{t-1},\sigma_t^{NC}\left(h^{t-1}\right);\delta\right)\right]\sigma_t^{NC}\left(h^{t-1}\right)\mid h^{t}\right\} \tag{1.5}$$

s.t.

$h^{t}=\left(h^{t-1},\sigma_t^{NC}\left(h^{t-1}\right)\right)$

E_δ is the expectation operator over δ conditional on history h^t. Let Δ denote the system of conditional beliefs associated with $E_\delta\left\{\cdot\mid h_t\right\}$. This system will be described in detail later and will be one of the key features of the chapter.

A PBE of this game is a triple (σ,φ,Δ) such that:

- Given σ^{NC}, the Associate follows the strategy $\varphi^{NC}\left(h^t,\delta\right)$ that satisfies the following inequality for all h^t,

$$W^{NC}\left(\sigma^{NC},\varphi^{NC}\mid h^{t}\right)\geq W^{NC}\left(\sigma^{NC},\varphi^{\tilde{N}C}\mid h^{t},\delta\right)\quad\forall t,\forall h^{t},\forall\varphi^{\tilde{N}C},\forall\sigma^{NC} \tag{1.6}$$

- Given φ^{NC}, the Investor follows the strategy σ^{NC} that satisfies the following inequality using the belief updating process Δ.

$$V\left(\sigma^{NC},\varphi^{NC}\mid h^{t-1}\right)\geq V\left(\sigma^{\tilde{N}C},\varphi^{NC}\mid h^{t-1}\right)\quad\forall\sigma^{\tilde{N}C},h^{t-1} \tag{1.7}$$

In the Full Commitment case the notation for the Associate's problem is identical to inequality (1.6). The Investor's problem is simpler, she only needs to satisfies the inequality (1.7) given h^0. In the One Period Commitment case the condition is one period ahead.

The belief is given by the lowest Associate's discount factor such that he decides to cooperate in the previous period. Given that the investment levels are weakly positive[10], this variable is enough to characterize the beliefs. In other words, if an Associate's type cooperates at t, then every Associate's type higher will cooperate too. Formally:

$$\hat{\delta}_t = \min_{\delta} \left\{ \varphi_t^{NC*} \left(\sigma^{NC}, h^t, \delta \right) = 1 \right\} \tag{1.8}$$

s.t.

$\hat{\delta}_0 = \delta_l$

That is, the Associate with a discount factor $\hat{\delta}_t$ is the most impatient one that cooperates at t.

1.5.2 Belief monotonicity

One way to analyze this problem is to first restrict the Investor to strategies that depend only on a fixed number of previous investment levels, not all the history, and then show that this restriction is not binding. We will use the term "Markov" strategies in order to refer to this restriction. A Markov Perfect Bayesian Equilibrium (MPBE) will be an PBE where the agents are restricted to use Markov strategies,

[10]This is straightforward from the Associate payoff function. Ceteris paribus, $W^{NC} \left(\sigma^{NC}, \varphi^{NC} \mid h^t, \delta \right)$ is weakly increasing in δ if $x_t \geq 0$ for all t.

that is, strategies that depend only on a fixed number of payoff relevant state variables.

Another question is if under certain conditions the Full Commitment and One Period Commitment are payoff equivalent. I show that, in fact the conditions under which this is true are the same as the conditions where we can use Markov strategies without loss of generality. Roughly speaking, these conditions require that the optimal investment path exhibits no "jumps", that is, the investment level increases gradually.

To establish this result we begin by defining the *equivalent discount factor at time* t, $\tilde{\delta}_t$, by:

$$\tilde{\delta}_t \equiv \tilde{\delta}\left(x_t, x_{t+1}\right) \equiv \frac{x_t - \pi\left(x_t\right)}{x_{t+1}} \tag{1.9}$$

It is easy to see that if the Associate decides to quit at time t his discount factor is below $\tilde{\delta}_t$, because this expression just reflects that is better to quit today than tomorrow. So $\delta < \tilde{\delta}_t$ is a necessary condition for quitting at time t. This condition can be also sufficient under an extra condition. That is stated in the next theorem.

Theorem 1.1 (Equivalent discount factors and Markov strategies) *Consider the One Period Commitment case. Let* $\left(\sigma^C, \varphi^C, \Delta\right)$ *be a PBE. Let* $\{x_t^*\}_{t=1}^{\infty}$ *be the equilibrium investment path. Then the following statements are equivalent:*

- *The sequence* $\left\{\tilde{\delta}\left(x_t^*, x_{t+1}^*\right)\right\}_{t=1}^{\infty}$ *is increasing in* t.
- *There exists a MPBE* $\left(\hat{\sigma}, \hat{\varphi}, \hat{\Delta}\right)$ *with state variables defined as* (x_{t-1}, x_t) *for any arbitrary pair, such that the equilibrium investment path is the same as in the original PBE, that is:*

  i. $\hat{\sigma}_t\left(x_{t-1}^*, x_t^*\right) = \sigma_t^C\left(h^{t*}\right)$

  ii. $\hat{\varphi}_t\left(x_t^*, x_{t+1}^*\right) = \varphi_t^C\left(h^{t+1*}\right)$

The proof is contained in Appendix A. Theorem 1.1 is important because one can now justify the use of Markov strategies based on the characteristics of the equilibrium. Thus it is possible to differentiate between two types of equilibrium: one where the sequence of equivalent discount factors is increasing - call this property *belief monotonicity* - and one where it is not. For an equilibrium of the first type we can use Markov strategies to determine the Investor's optimal strategy without loss of generality. For an equilibrium of the second type, we need a more complex framework in order to determinate the equilibrium.

One key characteristic of a belief monotone equilibrium is that the Investor learns in every period about the Associate's discount factor. An always increasing equivalent discount factor implies that the belief about the most impatient Associate's type is also always increasing[11]. Consequently, under belief monotonicity it is not

[11]Given that strict increase is not required one should include the case when the equivalent discount factor is equal between two periods. In this case is easy to show that the only path

necessary to include more than the last two investment level in order to characterize the equilibrium.

Using this last property, belief monotonicity has one further implication. This result is Theorem 1.2.

Theorem 1.2 (Equivalent discount factors and commitment) *Let* $\left(\sigma^{FC}, \varphi^{FC}, \Delta^{FC}\right)$ *be a PBE under Full Commitment. Let* $\{x_t^*\}_{t=1}^{\infty}$ *be the equilibrium investment path. The sequence* $\left\{\tilde{\delta}\left(x_t^*, x_{t+1}^*\right)\right\}_{t=1}^{\infty}$ *is increasing if and only if there exists a PBE* $\left(\sigma^{C}, \varphi^{C}, \Delta^{C}\right)$ *under One Period Commitment such that both equilibrium paths are equal. That is*

- $\sigma_t^{FC} = \sigma_t^{C}\left(h^{t-1*}\right) = x_t^*$
- $\varphi_t^{FC}\left(h^*\right) = \varphi_t^{C}\left(h^{t*}\right)$

The proof is contained in Appendix A. Theorem 1.2 states that, under belief monotonicity, a one period commitment is enough to replicate the full commitment output. Consequently, for the rest of the chapter the one period commitment case will be used to represent full commitment, maintaining the assumption of belief monotonicity. We now analyze belief monotone equilibria, and use Markov strategies without loss of generality.

1.5.3 Equilibrium bounds

This section characterizes properties of the MPBE steady state. In any MPBE the equilibrium investment path can be shown to be bounded. The investment path

that keeps belief monotonicity is when the investment is constant, and this is a particular case in the set of equilibrium that this chapter will focus on. That is why in the general case belief monotonicity can be understood as a strict increase of the equivalent discount factor.

will be shown to be weakly increasing, as well as bounded above and below.

Lemma 1.2 (Belief monotonicity and investment level increase) *In any MPBE the equilibrium investment level is weakly increasing over time.*

The proof is contained in Appendix B. By Lemma 1.2, belief monotonicity is a sufficient condition for an increasing investment path, but not a necessary condition.

The intuition for the lower bound is the following: if there exists an investment level low enough such that every Associate's type surely cooperates, then maintaining this level will result in no updating of beliefs. Therefore, the Investor is not learning and potentially can increase her revenue by increasing the investment level offered. It is optimal to increase this level until at least the most impatient Associate is indifferent between cooperating and quitting. This limit is given by the Associate's discount factor lower bound, δ_l and is related to the strategic limit concept defined before, in equation (1.3).

Then, from the Investor's point of view, offering an investment level lower than $\tilde{x}_{\delta_l}$ for more than one period is not optimal, as is stated in Lemma 1.3.

Lemma 1.3 (Equilibrium investment level lower limit) *Given any MPBE, let* $\{x_t\}_{t=1}^{\infty}$ *be the equilibrium investment levels. Then* $x_t \geq \tilde{x}_{\delta_l}\ \forall t > 1$.

The argument in the proof show a contradiction if the Investor repeats an investment level below $\tilde{x}_{\delta_l}$. This argument cannot be used for bounding x_1.

Proof. Let $\bar{x} < \tilde{x}_{\delta_l}$. Suppose that the Investor offers $x_t = x_{t+1} = \bar{x}$, for some t. Then this cannot be optimal, because in MPBE the Associate's best response implies that he strictly prefers to cooperate, regardless of his type. So the Investor can increase her payoff in $t+1$ by raising investment until $\tilde{x}_{\delta_l}$. The case when $x_t < x_{t+1} = \bar{x}$ follow the same argument. The case $x_t > x_{t+1}$ is ruled out by Lemma 2. ∎

It is important to note that Lemma 1.3 implies another result. In equilibrium the Associate's payoff from quitting will be always greater than his payoff from cooperating in the stage game. It is easy to see this, using the lowest possible lower bound, 0, and applying Lemma 1.3.

Based on the learning associated with the Investor's offers, one can calculate an upper bound. Once again, this bound is given by the strategic limit, now the one associated with the Associate's upper bound δ_h.

Moreover, there exists another restriction for the upper bound, when Investor does not want to increase the investment even if the Associate will cooperate with certainty. This value is the technological limit, $\hat{x}$, defined in equation (1.2), in the complete information case.

Lemma 1.4 (Equilibrium investment level upper limit) *Given any MPBE, let $\{x_t\}_{t=1}^{\infty}$ be the equilibrium investment levels. Then $x_t \leq min\{\tilde{x}_{\delta_h}, \hat{x}\}$ $\forall t$.*

Proof. First, given the definition of the technological limit, $x_t < \hat{x}$ for all t, regardless of any consideration of the possibility that the Associate quits. If at some

t, $x_t > \tilde{x}_{\delta_h}$, then the only way for the Associate to cooperate is by having higher investments in the future, because if the offer keeps constant then he chooses to quit. Moreover, this future increase in the investment level is unbounded given the concavity of the cooperation payoff. Thus, there exists τ such that $x_\tau > \hat{t}$, and this is suboptimal. ∎

Define the *feasible investment set* (FIS) by the set:

$$X \equiv [\tilde{x}_{\delta_l}, \overline{x}]$$

Where $\overline{x} \equiv \min\{\tilde{x}_{\delta_h}, \hat{x}\}$.

X could be an empty set. The technological limit is not always above the lower bound of X. Lemmata 2 and 4 imply that any MPBE has a least upper bound. In the next sections we will show that this limit is a steady state.

1.6 Equilibrium under commitment

1.6.1 Strategies and equilibrium under belief monotonicity

The Associate's best response can be summarized in an easy way if the equilibrium is belief monotone. The best response at each time depends on current and next period investment:

$$\varphi_t(x_t, x_{t+1}; \delta) = \begin{cases} 1 \text{ (Cooperate)} & \text{if } \pi(x_t) + \delta x_{t+1} \geq x_t \\ 0 \text{ (Quit)} & \text{if } \pi(x_t) + \delta x_{t+1} \leq x_t \end{cases} \tag{1.10}$$

From (1.10) is easy to see that, under belief monotonicity, the Associate's best response is to cooperate at t if his discount factor is greater than or equal to the equivalent discount factor at t. Under belief monotonicity, the lower bound of the Investor's belief about the Associate's discount factor is the equivalent discount factor, defined in equation (1.9), because only Associates with type more patient that this threshold cooperate. This condition provides a simple expression for the probability that Associate quits.

Fix a MPBE in the one period commitment case $\left(\hat{\sigma}, \hat{\varphi}, \hat{\Delta}\right)$. Denote the probability that the Associate quits at t given state (x_{t-1}, x_t), and given the Investor's one period ahead choice x_{t+1}, by:

$$p_t \equiv p_t\left(x_{t-1}, x_t, x_{t+1}\right) \equiv E_\delta\left\{\hat{\varphi}\left(x_t, x_{t+1}, \delta\right) \mid x_{t-1}, x_t\right\}$$

So, the probability can be expressed as:

$$p\left(x_{t-1}, x_t, x_{t+1}\right) = \frac{\tilde{\delta}_t\left(x_t, x_{t+1}\right) - \tilde{\delta}_{t-1}\left(x_{t-1}, x_t\right)}{\delta_h - \tilde{\delta}_{t-1}\left(x_{t-1}, x_t\right)} \tag{1.11}$$

where $\tilde{\delta}_t$ is the equivalent discount factor from equation (1.9).

So under the belief monotone property the Associate's probability of quitting depends only on three variables, x_{t-1}, x_t and x_{t+1}. If the Investor can commit to the next period investment level then the problem can be written as a Bellman equation. Previous and current investment levels are the state variables. The next period investment level is the control variable. Now let us focus on the commitment case and in section 7 I will analyze the non-commitment case.

The commitment problem at time t is:

$$W\left(x_{t-1},x_{t}\right)=\max_{x_{t+1}}\{\left[1-p_{t}\left(x_{t-1},x_{t},x_{t+1}\right)\right]\left[\pi\left(x_{t}\right)+\beta W\left(x_{t},x_{t+1}\right)\right]-p_{t}\left(x_{t-1},x_{t},x_{t+1}\right)x_{t}\} \tag{1.12}$$

$$\text{s.t. } p_t \in [0,1]$$

Where p_t is defined in equation (1.11).

In this problem the only control variable is the future level of investment because, under the assumption of belief monotonicity, this is the only relevant investment level for the Associate's decision to quit or to cooperate at t. Note that the restriction over the value of the function p_t is added explicitly in order to keep this function as general as possible, as it is defined in equation (1.11).

Then a MPBE path is a investment level stream that satisfies equation (1.10) and equation (1.12) at every time t.

1.6.2 Steady state

One way to find the solution to the Investor's Bellman equation is to look for a possible steady state and then analyze its stability. First we will look for a locally stable steady state. Thus, it is necessary to analyze if there is another global solution that rules out this local approach. In the next section this global issue will be addressed.

For simplicity, assume that the upper bound of the Associate's discount factor, δ_h, is the same as the Investor's discount factor, β.[12]

We study the first order condition of equation (1.12) in order to characterize the steady state of the problem. The first order condition assuming an interior solution is:

$$p^{(3)}\left(x_{t-1}, x_t, x_{t+1}\right)\left[\pi\left(x_t^*\right) + \beta W\left(x_t^*, x_{t+1}^*\right) + x_t^*\right] = \beta\left[1 - p\left(x_{t-1}, x_t, x_{t+1}\right)\right] W^{(2)}\left(x_t^*, x_{t+1}^*\right) \tag{1.13}$$

where $p^{(3)}$ is the partial derivative of the probability of quitting with respect to the third argument, and $W^{(2)}\left(x_t^*, x_{t+1}^*\right)$ is the derivative of the function with respect to its second argument. The probability that the Associate quits today is decreasing if tomorrow's investment is increased, because more types of Associate will prefer to wait. Consequently, $p^{(3)}\left(x_{t-1}, x_t, x_{t+1}\right) < 0$. The expression inside the first brackets is the Investor's gain if the Associate does not quit today: the sum of cooperation payoffs, the future payoff, and the avoided quitting cost. The second part of the expression represents the loss to the investor when there is an increase in the investment level tomorrow. This cost is determined by the probability that the Associate cooperates, $1 - p_t$, times the loss in the future value. This second term is negative at the optimal level because an increase in tomorrow's investment level implies that the expected cost of quitting in the future increases. If the investor increases tomorrow's investment level and keeps all the other levels constant, then some types

[12]There are some solutions when those values are different and the results are shown in the next section.

of Associate will cooperate today but at a higher level than before will quit tomorrow.

Denote by x_{LC} the *locally stable steady state level of investment under commitment*[13]. That is, this level is the threshold between a profitable or not profitable marginal increases from the Investor's point of view. The following Lemma establishes conditions that this value must satisfy.

The Euler equation corresponding to the Bellman equation (1.12) evaluated at the steady state gives the following polynomial.

$$2\beta\pi'(x)\,x\pi(x) - [\pi(x) + x[1-\beta]]\,[\pi(x) - x[1-\beta]] = 0$$

So define:

$$F(x) \equiv 2\beta\pi'(x)\,x\pi(x) - [\pi(x) + x[1-\beta]]\,[\pi(x) - x[1-\beta]]$$

Lemma 1.5 (Locally stable steady state under commitment) *In the commitment case, x_{LC} should satisfy:*

i. $F(x_{LC}) = 0$,

ii. $F'(x_{LC}) < 0$,

iii. $x_{LC} \in$ *int X.*

The proof is contained in Appendix B. By the Lemma, if a critical point of the polynomial F satisfies the required properties, then it is a locally stable steady state

[13]Formally, under the belief monotonicity assumption $\exists\epsilon > 0$ s.t. $\forall x_{t-1}, x_t \in B(x_{LC}, \epsilon)$, if $x_{t-1} < x_t < x_{LC} \Rightarrow h_W(x_{t-1}, x_t) > x_t$, or if $x_{t-1} < x_t \geq x_{LC} \Rightarrow h_W(x_{t-1}, x_t) < x_t$, where $h_W(\cdot,\cdot)$ is the policy function of the equation $W(\cdot,\cdot)$.

of the commitment problem. If one rearranges the equation $F(x) = 0$ is easy to see the intuition behind this stable steady state.

$$F(x) = 0 \iff \underbrace{\frac{\pi(x)}{1-\beta} + x}_{Loss\ if\ quits} = \underbrace{\frac{\pi'(x)}{1-\beta}}_{Gain\ if\ cooperates} \left[\frac{2\beta x \pi(x)}{\pi(x) - x[1-\beta]}\right]$$

The left hand side of the last equality is the Investor's loss if the Associate quits at an investment level of x. The first term of the right hand side is the Investor's marginal gain from an increase in the investment level. If the two terms are equal then the investment level x is a steady state. If the right hand side is higher, then the Investor's expected cost is lower than the expected gain and the Investor wants to raise the investment level.

Now the idea is to analyze the possibility of a solution that is not included in the local analysis. The idea here is analyze if x_{LC} is stable when one considers a profitable major change in the investment level.

The key variable in this analysis is the investment level threshold representing a lower bound for a steady state. Call this level the *stable level using jumps* and denote it by x_{SJ}. Any investment level below this value cannot be a steady state, because the Investor wants to increase the investment at some time. The logic behind this condition is the following: given any steady state, as t increases not only does the the investment level converge, but the beliefs too. At that point the question is if the Investor has any incentive to increase the investment level and "jump" to a new level, given the belief. One can prove that this happens if the belief is below some threshold, or, equivalently, if the investment level is below some threshold. This threshold is x_{SJ}. The formal derivation of x_{SJ} is in Appendix B. Here it is enough to use the following polynomial:

$$J(x) \equiv 2\pi'(x)\,x - [\pi(x) + x[1-\beta]]$$

x_{SJ} is a solution of $J(x) = 0$, where $J'(x) < 0$.

With this definition one can state sufficient condition for the steady state under commitment.

Theorem 1.3 (Sufficient conditions for an interior steady state under commitment) *If:*

i. *There exists a unique $x_{LC} \in intX$, and this is the only critical point of F on the FIS*

ii. *There exists $x_{SJ} \in intX$, or $J(x) < 0$ for all $x \in X$*

then in any equilibrium the steady state investment level under commitment will be given by x_{LC}. This steady state is reached by a gradual increase.

Proof. If the conditions above hold then $x_{SJ} < x_{LC}$. See Appendix B for details. ∎

If the conditions of Theorem 3 hold, then in any equilibrium the investment levels will increase gradually to a level in the interior of the feasible investment set.

1.7 Equilibrium without commitment

We now analyze the case of non-commitment. In this case one can follow a strategy similar to the commitment case, looking for a locally stable solution and comparing it to the possibility of jumps. The problem here is the use of a Bellman equation in order to find the equilibrium path. One can think of an artificial problem where the following Bellman equation will give the Investor's optimal decision:

$$V\left(x_{t-1}\right) = \max_{x_t}\{\left[1-\tilde{p}_t\right]\left[\pi\left(x_t\right)+\beta V\left(x_t\right)\right]-\tilde{p}_t x_t\}$$

$$\text{s.t. } \tilde{p}_t = p\left(x_{t-1}, x_t, h_V\left(x_t\right)\right)$$

$$p_t \in [0,1]$$

where $h_V(x)$ is the policy function of the Bellman equation. Given that the Investor cannot commit, the Associate needs to use this equation in order to forecast the next period investment level. This is not the real non-commitment problem for two reasons. First, the Bellman equation only can be evaluated on path; it does not work off path. Second, this equation does not necessarily have a solution. The second problem can be solved by showing that in a local analysis one can find a locally stable steady state. The first problem can be solved using x_{SJ}, because one can show that under some conditions this value is the steady state of the problem. The conditions for this are in the next Lemma.

Lemma 1.6 (Stable steady state under non-commitment) *In the non commitment case, if $x_{SJ} \in int\ X$, then in any equilibrium x_{SJ} is the steady state investment level.*

The proof is contained in Appendix B. If one combines the result of Lemma 1.6 with Theorem 1.3, one can state the theorem that summarizes one main result of this chapter.

Theorem 1.4 (Steady state levels of investment) *If:*

i. There exists a unique $x_{LC} \in intX$*, and this is the only critical point of* F *on the FIS*

ii. There exists $x_{SJ} \in intX$*, or* $J(x) < 0$ *for all* $x \in X$

then in any equilibrium the steady state investment level under commitment will be higher than without it.

The theorem implies the existence of a gap between the commitment and the non-commitment steady state.

Proof. If the conditions above hold then $x_{SJ} < x_{LC}$. See Appendix B for details. ∎

One interesting and unexpected result is that this gap vanishes if the Investor becomes perfectly patient.

Theorem 1.5 (Asymptotic convergence) *Assume that conditions of Theorem 3 hold. When* $\beta \to 1$ *then the steady state without commitment approaches the steady state with commitment.*

The proof is contained in Appendix B. The economic intuition behind this result is the following: As the Investor becomes more patient, he values less the payoff flow of the stage previous to the steady state, because this stage is finite[14]. Given that, in the limit, the only relevant trade-off for the Investor is the one at the steady state investment level. If this level is reached in one step or gradually, then the difference is

[14]Formally the steady state could not be reached in finite time, but it is sufficient to have the investment level close enough in a finite time in order to follow the logic of this theorem.

only at the stage previous to the steady state. Hence, in the limit, this difference goes to zero. For that reason, a gradual increase (commitment case) has no advantage in comparison to a "jump" (non-commitment) when the Investor is perfectly patient.

1.8 Conclusion

This chapter analyzes the case when an Investor needs to learn about an Associate but this learning is costly. Questions about how much the Investor wants to learn and how this differs in cases with or without commitment are the ones that the chapter seeks to answer.

It is possible to find general conditions to characterize the steady state level of investment in the proposed game. This steady state level is related to how much the Investor learns about the Associate in the long run. There are four relevant results.

First, the learning process does not always induce full revelation. In most of the cases the Investor can learn only until some limit.

Second, if the Investor is patient enough and the technological process has low diminishing return then the solution exhibits a gradual increase asymptotically to the steady state. This result looks familiar in the type of examples stated in the introduction, like bank credit lines and workers training.

Third, the steady state depends only on the Associate's upper bound discount factor, that is, the most patient Associate. The only reason why the lower bound matters in equilibrium is because if it is too high, then the steady state level of investment might be too low for the most impatient Associate. In this case the Investor's offer is an amount that makes the most impatient Associate indifferent between cooperating and quitting. As a result, every Associate's type cooperates. There is no learning.

Fourth, if the Investor can commit to future investment levels then she will learn strictly more than if she cannot commit. This result looks quite natural. Unexpectedly however if the Investor becomes more patient, then the difference between both steady states, with and without commitment, converges to zero.

The basic model can be modified in several ways. The first one is to analyze the effect of competition between different Investors. The actual model can be interpreted as a monopoly case. Models that incorporate competition, like perfect competition or an oligopoly case, could be analyzed to study the effect of competition on the steady state. In these models one important variable is the public information, that is, how much the Investors know about the history of the Associate. Models like that can help to analyze the effect of instruments like credit scores in a financial market.

A second extension is to analyze the effect of giving more options to the Associate, for example, the possibility of cooperating at a lower level than the offered one. The question here is how the more patient Associate can use this option in order to reach a higher steady state level. The limiting case is when the Associate has a continuum of possible actions. Finally, a model that incorporates two sided uncertainty can be studied. This case is not an extension of the actual model but the one sided case can give a reference point. Depending on the structure of the game, in particular how the offers are made, one can use the one sided uncertainty case to study the paths that a particular agent will try to follow.

Chapter 2

Gradualism and Commitment: how patient one should be in order to learn?

2.1 Introduction

This chapter is focused in a dynamic learning process using a particular functional form. Starting from the general result in this scenario, showed in the first chapter, here I look for more specific results by using a particular form for the production function.

As we saw in the first chapter, when one Investor wants to learn about one Associate by using a dynamic interaction, different variables affect the outcome. Between these variables one can highlight the ability of the Investor to commit to future investment levels, how patience the Investor is, and the characteristics of the production function. The effect of the two first variables was stated in the previous chapter. In order to analyze the effect of the third one it is necessary to assume a functional form. In this chapter a Cobb Douglas form will be used.

Using this particular functional form one can obtain results than relate the three variables mentioned before. For example, one can analyze the set of parameters where there exists a gradual convergence to the steady state, or if the solution is just one fixed level of investment forever. Other issue to be analyzed is the relationship between the level of patience of the Investor and the diminishing returns to scale of the production function.

2.2 General setup

In this chapter the terminology of chapter one is used, a short explanation of the concepts will be provided. If it is necessary to introduce a new notation it will be mentioned explicitly.

Using the Cobb Douglas example, that is $\pi(x) = \gamma x^{\alpha} - x$, it is possible to get a better understanding of some of the general results of the companion chapter, and obtain extra result given the particular functional form. Using a Cobb Douglas production function makes ir possible to obtain closed expression for the steady state level of investment in the case with commitment (x_{LC}), and non commitment (x_{SJ}), as well as expressions for the bounds of the *feasible investment set* (FIS).[1]

As a starting point, find when the FIS is a non-empty set. After a little bit of algebra the bounds in the Cobb Douglas case are:

$$\tilde{x}_{\delta_l} = \left(\frac{\gamma}{2-\delta_l}\right)^{\frac{1}{1-\alpha}} \qquad \tilde{x}_{\delta_h} = \left(\frac{\gamma}{2-\delta_h}\right)^{\frac{1}{1-\alpha}}$$
$$\hat{x} = (\alpha\gamma)^{\frac{1}{1-\alpha}}$$

[1]The FIS is the set that includes all the investment levels proposed in equilibrium by the Investor. The lower bound of the set is the investment level where the most impatient Associate will be indifferent between cooperate or quit. The upper bound is the minimum between the investment level where the most patient Associate will be indifferent between cooperate or quit, and the unrestricted optimal investment level. Using the notation of chapter 1, $FIS \equiv [\tilde{x}_{\delta_l}, \min\{\tilde{x}_{\delta_h}, \hat{x}\}]$

So if the parameter α in the Cobb Douglas case is low enough, $\alpha < 0.5$, then the FIS is an empty set even when $\delta_l = 0$. Following this argument, one can find a relationship between δ_l and α such that the FIS is not empty.

2.3 Commitment case

Now let go to the local analysis in the commitment case. Depending on the values of parameters α and β, the Euler equation F could have zero, one or two solutions. When there are two real roots of F, the relation between these roots and the upper bound of the FIS is given by the next lemma. Remember that the upper bound of the FIS, $\overline{x}$, is the minimum between the the investment level that leaves the most patient Associate indifferent between cooperates or not, $\tilde{x}_{\delta_h}$, and the unrestricted optimal investment level, $\hat{x}$.

Lemma 2.1 (Relationship between root of F and FIS upper bound) *Suppose that there exist two real positive solutions of $F(x) = 0$. Denote these solutions by x_1 and x_2, where $x_1 < x_2$. Then, depending on the relation between $\tilde{x}_\beta$ and $\hat{x}$*[2] *($\overline{x} \equiv min\{\tilde{x}_\beta, \hat{x}\}$):*

i. $x_1 < \overline{x}$

ii. *If* $\tilde{x}_\beta > \hat{x}$ *then* $x_2 > \tilde{x}_\beta$

iii. *If* $\tilde{x}_\beta < \hat{x}$ *then* $x_2 < \tilde{x}_\beta$

iv. *If* $\tilde{x}_\beta = \hat{x}$ *then* $x_2 = \overline{x}$

The proof is in the Appendix. The lemma states that depending on the discount factor of the most patient Associate and the concavity of the production function there could be one or two solution below the upper bound of the FIS. When the diminishing returns to scale is lower, that is the second derivative of the production function is lower in absolute value, then $F(x) = 0$ is more likely to have both solutions in the FIS. Moreover, the stable condition, $F'(x) < 0$, holds only at the first root, x_1. The economic intuition of these results is that in order to get an interior solution one needs some level of diminishing marginal returns to scale, that is, a production function "concave enough". If this is not the case then there is not a global stable steady state, x_1, but an unstable one, x_2, which rules out the first possible solution.

In the particular case of a Cobb Douglas the equation $F(x) = 0$ can be reduced to a quadratic equation if one focus on solutions other than zero. The smallest strictly positive solution, x_1, is:

[2] We are still assuming that the Investor's discount factor is equal to the most patient Associate one, that is $\delta_h = \beta$.

$$x_1(\alpha,\beta,\gamma) = \left[\frac{\gamma\left[2\alpha\beta - 1\right]}{\beta\left[\alpha+1\right] - 1 + \sqrt{\left[1-\alpha\beta\right]\left[1-\alpha\beta-2\beta+2\beta^2\right]}}\right]^{\frac{1}{1-\alpha}}$$

Then one can differentiate 5 relevant areas in the graph α - β relative to the solutions of the commitment case. The areas differ in the number of solutions of the equation $F(x) = 0$ and the relation between $\tilde{x}_\beta$ and $\hat{x}$. In order to simplify, use W to denote the area where there is no real solution of F.

- Area I: there are two real strictly positive solutions of the equation $F(x) = 0$, and $\tilde{x}_\beta > \hat{x}$, that is $\alpha\beta > 0.5$ and $1/\alpha > 2 - \beta$ respectively.

- Area II: there are two real strictly positive solutions of the equation $F(x) = 0$, and $\tilde{x}_\beta < \hat{x}$, that is $\alpha\beta > 0.5$, and $1/\alpha < 2 - \beta$ respectively.

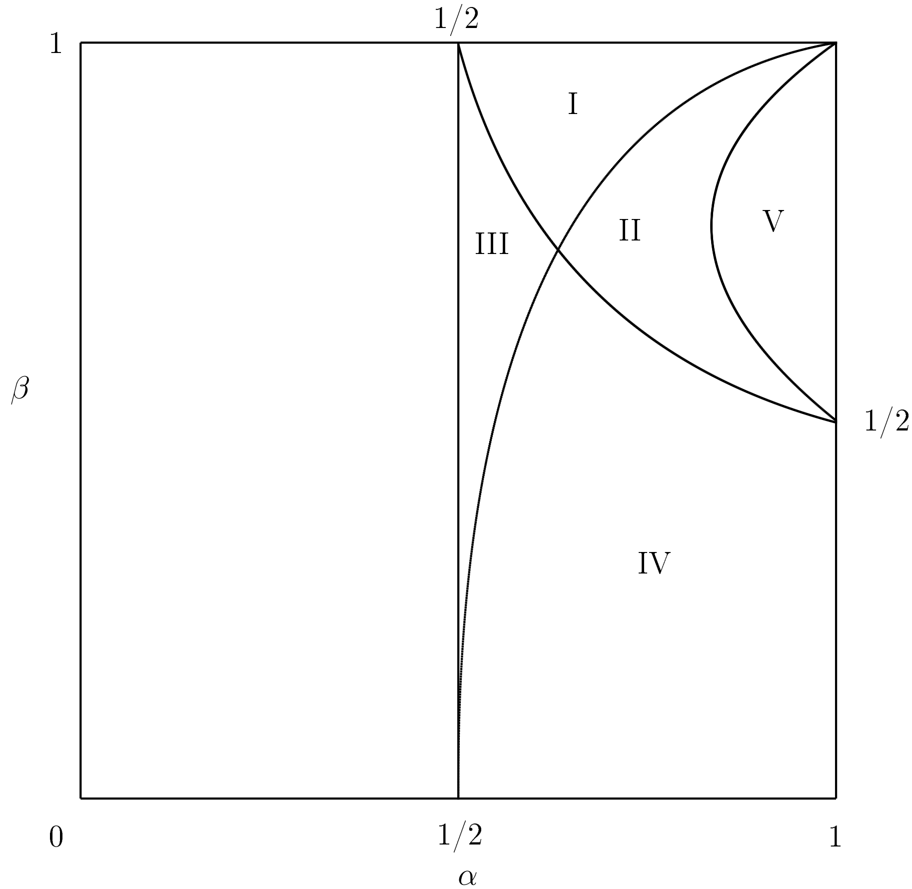
- Area III: there is one real strictly positive solutions of the equation $F(x) = 0$, and $\tilde{x}_\beta > \hat{x}$, that is $\alpha\beta < 0.5$ and $1/\alpha > 2 - \beta$ respectively.

- Area IV: there is one real strictly positive solutions of the equation $F(x) = 0$, and $\tilde{x}_\beta < \hat{x}$, that is $\alpha\beta < 0.5$ and $1/\alpha < 2 - \beta$ respectively.

- Area V: there is no real strictly positive solutions of the equation $F(x) = 0$

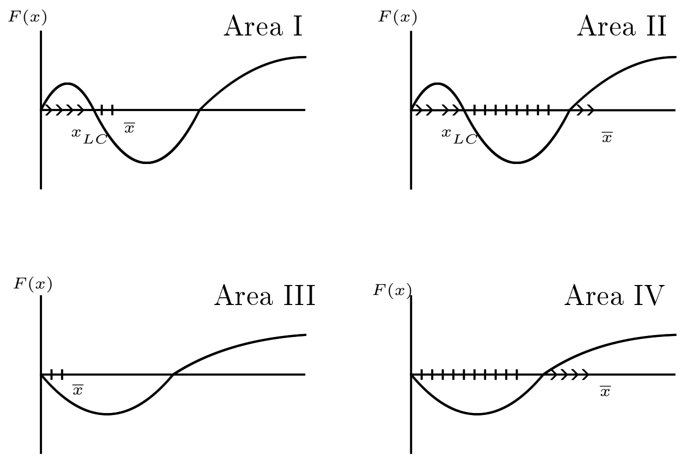
The next graph illustrates the restrictions on parameters that characterize the different cases. On the horizontal axis the value of the parameter α is represented, while the vertical axis is used for the value of the Investor's discount factor, β.

Graph 1: Relevant areas in Cobb Douglas case



Remember that FIS is empty if α is lower than 0.5, which is why this area is not included in the analysis. The next graphs show the relationship between the critical points of F and the FIS upper bound in areas I to IV.

Graph 2: Euler equation in each different area



In area I only the first critical point of F could belong to FIS, and this point satisfies conditions of Lemma 5, so in this area there could be a locally stable steady state. In area II both critical points could belong to FIS, but only the first one is a locally stable steady state. In area III the only critical point is outside FIS, while in area IV the only critical point could be inside the FIS but does not satisfy the properties required for being a locally stable steady state.

The arrows and vertical lines on the axis reflect the marginal incentives when the investment level is in some range. For example, in area I if the investment level is below x_{LC} then the Investor has an incentive to marginally increase investment in the next period. On the other hand, if the level is between x_{LC} and $\overline{x}$ the incentive for the Investor is to keep the level constant in the future. The existence of gradual increases in the equilibrium level of investment is related to the starting point of the investment path.

For clarity of exposition FIS lower bound is not included in the graphs. This lower bound is related to δ_l. If this value is high enough the stable steady state could be below the strategic limit associated with this discount factor and, therefore, outside the FIS. This is the reason why in the previous analysis the word "could" is used. If the lower bound is high enough that the only critical point in area I is outside the FIS, then the solution does not involve a gradual increase, at least not in the local analysis.

Now let us focus on the lower bound of the FIS. Again, it is important to remark that this steady state level of investment does not depend on the Associate's most impatient type, only on the most patient one and on the technological parameters. The only way that the most impatient type affects this steady state is through the lower bound of the FIS. That is, if this lower bound is high enough, above x_{LC} then there is no learning.

Is easy to check that, if $\beta = 4 - 4\alpha$, then $x_1 = \tilde{x}_0$, that is, the first root is equal to the investment level at which an Associate who does not value the future[3] is indifferent between cooperating today versus quitting. This value is the lowest possible lower bound of any FIS.[4] Numerical simulations[5] show that in the Area I, when $\beta < 4 - 4\alpha$, the steady state level of investment under commitment is below the lower bound of the FIS, son the only equilibrium is an offer equal to this lower bound that stays at this level. There is no learning. In the Area I where $\beta > 4 - 4\alpha$ there is gradual learning, because in this area the assumptions of Lemma 1.5 hold.

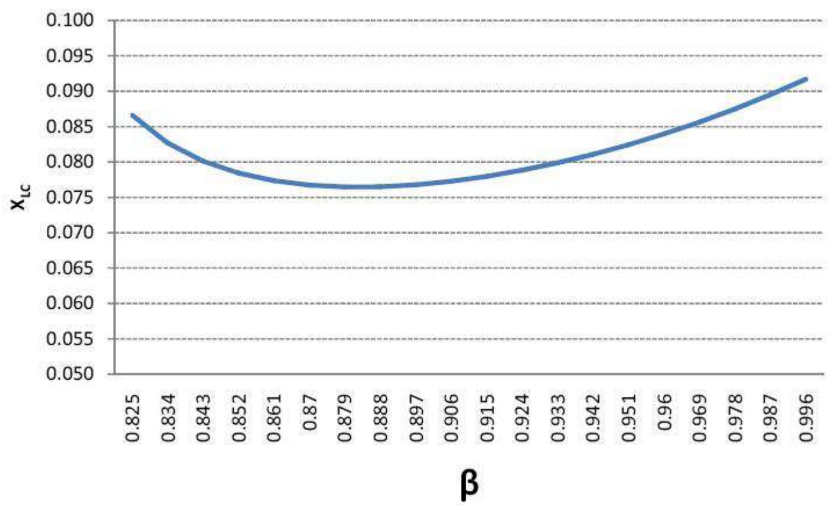
[3] $\delta = 0$.
[4] This is because $\tilde{x}_\delta$ is increasing in δ.
[5] The formal proof is pending.

Now focus on area I, that is, where only one zero of F belongs to the FIS, considering a low enough value of δ_l. In the rest of the section all the statements will be focused on this area. An interesting result is stated in Lemma 2.2.

Lemma 2.2 (Steady state level and Investor's patience) *There exists a set of parameters with non empty interior where the steady state level of investment is not monotonic in the Investor's discount factor* β.

The proof is contained in Appendix C. In area I the steady state is given by the locally stable steady state if the Associate's lower bound is low enough. If the Investor is just patient enough to be in this area, then the steady state level will decrease with β. On the other hand, if the Investor is almost perfectly patient then the steady state is increasing in β. The following graph shows how this value change with β for a particular case.

Figure 1: Steady state level of investment under commitment. Case with $\alpha = 0.85$.



It is important to remember that an increase in β has two effects: One is the direct effect, the Investor is more patient; the other is based on the previous simplification that the Investor's discount factor is equal to the Associate's upper bound. The implication of this is that an increase in β implies an increase in the average Associate's patience. It is useful to separate the two effects and study if one of these is responsible for such belief non-monotonicity. In the case when the Investor's discount factor and the upper bound of the Associate's discount factor can be different, a new polynomial can be obtained for the Euler equation in the steady state.

$$G\left(x\right) \equiv 2\beta\pi'\left(x\right) x\pi\left(x\right) + \beta\pi'\left(x\right) x^{2}\left[\delta_{h} - \beta\right] - \left[\pi\left(x\right) + x\left[1-\beta\right]\right]\left[\pi\left(x\right) - x\left[1-\beta\right]\right]$$

The new polynomial is a generalization of the one when the discount factors are the same, the polynomial F. It is easy to see that when $\beta = \delta_h$ this formula is equal to the polynomial F. Analyzing the case stated in Lemma 2.2, if the investment level belongs to FIS then an increase in δ_h implies that the critical point in the area I, x_{LC}, will increase. That is, if we modify the value of the upper bound of the Associate's discount factor then the steady state investment level will increase. This result is not unexpected. If the probability than the Associate is more patient is higher, because the upper level is higher, then we can expect a higher steady state investment level. However, that means that the result of lemma 2.2 is based on the effect of the Investor's discount factor.

Theorem 2.1 (Steady state level of investment and Investor's patience) *Fix the upper bound of the Associate's discount factor. Then for some range of parameters the steady state level of investment is decreasing in the Investor's discount factor β.*

Proof. Use lemma 2.2 and the polynomial G. ∎

This result is not intuitive. If the Investor is more patient then she will offer a lower steady state investment level. Therefore, the Investor will learn less about the Associate's type in steady state.

2.4 Non commitment case

In the non commitment case the steady state is related to x_{SJ}. In the case of a Cobb Douglas production function the value that is candidate for this limit is:

$$x_{SJ}(\alpha, \beta, \gamma) = \left[\frac{\gamma [2\alpha - 1]}{2 - \beta}\right]^{\frac{1}{1-\alpha}}$$

First, is easy to show that when $\beta = 4 - 4\alpha$, then $x_{SJ} = \tilde{x}_0$. So, if x_1 satisfies the conditions for be a stable steady state under commitment, then $x_{SJ} = x_{LC}$. So one can state the following Lemma for the general case and the Corollary for the particular one:

Lemma 2.3 (Comparison between the steady state with and without commitment) *$x_{LC} = \tilde{x}_0$ if and only if $x_{SJ} = \tilde{x}_0$.*

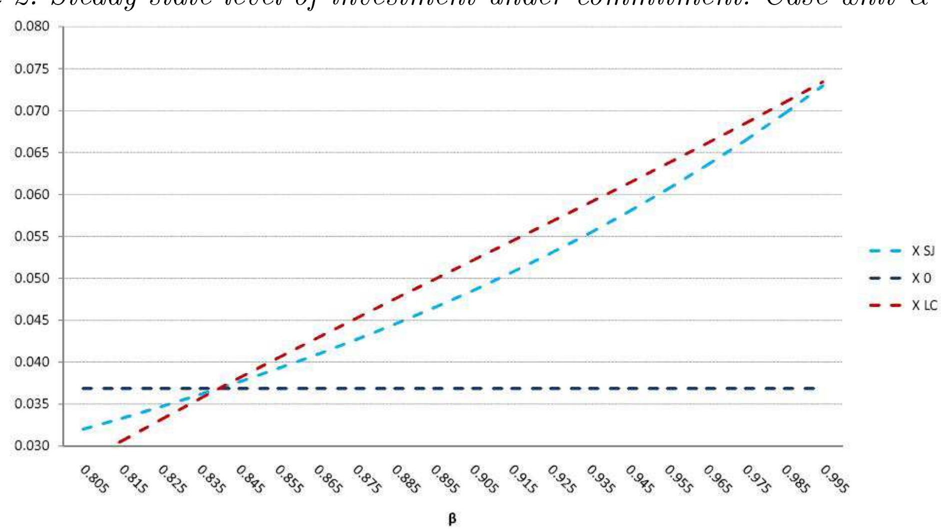
The proof is in the Appendix. The economic intuition behind this result is the following. There is a point where the ability to commit generates no advantage. This happens when in the commitment case there is no learning because investment level for a totally impatient Associate is too high, for the relevant parameters, so there is no incentive to learn even using gradualism. If this happens then in the non commitment case there is the same level. And the implication holds in the other direction too.

Corollary

Suppose that the parameters α and β are such that we are in the Area I described before.[6] *If $\beta = 4 - 4\alpha$ then $x_{SJ} = \tilde{x}_0 = x_{LC}$*

So, in Area I the possible steady state level of investment with and without commitment coincides in two lines. First, in the line defined by $\beta = 4 - 4\alpha$, and second in the (asymptotic) line defined by $\beta = 1$. The following graph shows an example of this case.

Figure 2: Steady state level of investment under commitment. Case whit $\alpha = 0.79$.



[6] $\alpha\beta > 0.5$ and $1/\alpha > 2 - \beta$.

The graph reflects the general result in Area I.

- For $\beta < 4 - 4\alpha$, the steady state investment level is equal to the lower bound of the FIS in the cases with or without commitment.
- For $\beta > 4 - 4\alpha$ and $\delta_h = 0$, the steady state investment level is given by x_{LC} and x_{SJ} in the cases with and without commitment respectively.
- For $\beta > 4 - 4\alpha$ and $\delta_h > 0$, the steady state investment level is given by x_{LC} and x_{SJ} in the cases with and without commitment respectively if these values are higher than $\tilde{x}_{\delta_h}$, or this last value in other case.

2.5 Conclusion

This chapter analyzes game between a Investor and an Associate where the first needs to learn about the second in order to develop an ongoing project. The analysis uses a particular functional form in order to obtain more specific results than in the first chapter.

The chapter shows the conditions on the parameters of the problem required for a gradual learning process. First one needs to have a non-empty set of possible steady states. In this case the diminishing returns to scale is the relevant variable. If these returns decrease at a high rate, the production function is "more concave", then it is possible than there is not a set where the Investor can learn, so there is no learning. Second, one needs that the Investor patience is high enough in order to generate a payoff from learning. When all these conditions are met, then there is a gradual learning process.

Finally the chapter provides a comparative static result on the steady state level of investment under commitment. It is shown that the relationship between this level and the Investor's patience is not monotonic, that is, a more patient Investor will not necessarily learn more in the steady state.

Chapter 3

When Should Developing Countries Announce Their Climate Policy? (co-authored with Sebastian Miller)

3.1 Introduction

According to the latest projections by climate models, in order to achieve the targets of stabilizing concentrations of Greehouse Gases (GHGs) by 2050 at levels that do not lead to potential catastrophic outcomes it is necessary that both developed and developing countries achieve a reduction in emissions[1]. This is, by themselves developed countries even cutting their emissions to zero would not stabilize global concentrations of GHGs. Thus it is very likely that developing countries will have to participate sooner rather than later in terms of reducing their emissions, or at least moderate the rate of growth of their emissions. Developing countries therefore face the challenge of deciding when to participate in climate change mitigation actions.

We argue that even if developing countries do not plan to impose any restrictions on GHGs emissions in the short run, they could benefit from announcing today when they plan to commit to real reductions. The idea relies on the fact that countries will eventually have to reduce emissions (or limit their growth), and given that many infrastructure investments have a long lifespan, eliminating the uncertainty of when the policies will come into effect will eliminate wasteful infrastructure (i.e. in old

[1]See for example Edmonds et. al. (2007) and Calvin et. al. (2009) for two examples of these.

dirty technologies) spending.

To this end we present a simple framework in which a private investor chooses between dirty and clean technologies under uncertainty about future climate policy. We find conditions under which the investor will choose each type. We then compare with the case of announcing the future climate policy. We show conditions under which the investor would choose the latter regime. We also show how the timing of the announcement and other characteristics of the investor affects the choice of technology. One key finding is that announcing future climate policy will induce technological change faster than the announced date, reducing investment in dirty technologies.

This chapter departs from the traditional analysis in several ways. First our focus of analysis is the firm. We claim that under plausible assumptions, even a profit maximizing risk neutral firm would prefer enacting climate policy sooner than later. Our second departure is that since we focus on developing countries we are not interested in the development and deployment of new technologies. Most papers reviewed below do. We assume that these countries are just adopters of technology and therefore the timing of policy would not affect the development of new technology. Our third main difference is that in our analysis we focus on the transition path to a new equilibrium and not on the equilibrium per se. This is we look at the effects over the transition period towards a climate policy that we "know" for sure will be adopted in the future. Therefore our focus is not on the steady state but rather on the path to the new steady state from the private investor's point of view.

The rest of the chapter is organized as follows. The next section summarizes the existing literature on the topic. Section 3 presents our simple framework, while sections 4 and 5 discuss our main results. In section 6 we present an alternative approach that relaxes some assumptions while section 7 discusses some extensions. Section 8 concludes.

3.2 Related Literature

This chapter belongs to the literature on delayed implementation of policies as well as climate policy. When to enact climate policy has been a topic of considerable debate. Thus the literature on the timing of climate policy has received many new contributions in the past few years. Several authors have argued that delaying climate policy adoption would be beneficial since there would be extra time to reduce the uncertainty and verify if the climate is in fact changing[2]. There would also be enough time to develop low carbon technologies if the effects of climate change are as bad as some models predict. In the case of developing countries this would appear to be a dominating strategy as well. As Chisari and Galiani (2010) show, the optimal path for developing countries is to probably grow now and "clean up" later. The only reason for developing countries to adopt climate policy would be to follow some sort of international sanctions (such as carbon border tax adjustments) or given certain

[2]However Weitzman (2009) argues that if as a consequence of climate change the world faces so called catastrophic climate change and if the statistical process behind this event exhibits "fat-tails", then even if these probabilities are low, the expected value of the negative outcome should overtake any possible cost of avoidance and thus delay of implementation would be welfare reducing. Nordhaus (2009) however has challenged this analysis mainly on the grounds that Weitzman (2009) assumes that the cost of climate catastrophe is infinity, while under very high but bounded estimates, the expected costs of catastrophe are significantly reduced.

financial aid to adopt cleaner technologies.

Thus Goulder and Mattai (2000) find a lower time profile of optimal carbon taxes with induced technological change (ITC). When knowledge is gained through R&D investments, the presence of ITC justifies shifting some abatement from the present to the future. However, when knowledge is accumulated via learning-by-doing the impact on the timing of abatement is analytically ambiguous. In addition, Webster (2000) shows though that given the interaction between periods a delay in policy might not be useful even if uncertainty is reduced due to the long term effects of carbon emissions. Requate (2005) looks at the issue of timing and commitment and comparing different second best instruments find that committing to a menu of taxes dominates the other types. Golombek et. al. (2010) look at the time consistency issue and find that subsidizing R&D technologies today would not create a time inconsistency problem with respect to future climate policy.

Finally Gerlagh et. al. (2009) look at the same issue but find that carbon taxes if anything should be higher initially to promote the development of new technologies[3]. Jaccard and Rivers (2007) look of delaying climate policy in the presence of heterogeneous capital stocks. They find that given that significant economic activity uses long lived capital stocks, then the optimal climate policy for these sectors would involve early targeting of CO2 emissions. Shalizi and Lecocq (2009) look at this effect of inertia in long lived capital stocks and show that carbon market on their own are unlikely to be able to generate enough emission reductions required to stabilize emissions, justifying the fund based approach that has taken over carbon market in

[3] It must be noted though that climate-specific R&D subsidies is still preferred as a means to promote development and deployment of new (low carbon) technologies.

developing countries. Finally Strand et. al. (2011) find that if the capital stock can be retrofitted to reduce emissions (for example using carbon capture and sequestration in coal fired power plants) then the optimal policy may be to delay full implementation of climate policy until the future.

This chapter also belong to the literature on "green paradox" as proposed by Sinn (2008). The green paradox basically implies that increasing the current price of oil (eg. using carbon taxes) and committing to a high tax in the future can decrease the current international price as oil producers will want to extract as much oil as possible in the present, thus increasing the emissions today. Overall Sinn (2008) argues, the net effect of the carbon tax on GHG concentrations is negligible since we have only traded future emissions for current emissions In that vein the paper closest to our is by Smulders et. al. (2010). Their paper can generate a "green paradox" by pre-annoucing climate policy, instead of relying on exhaustible resources as in the Sinn (2008) paper. However their results rely on a key assumption on the investment versus operating costs of both technologies. In our case we can also generate the "green paradox" however, it is far from being the most likely outcome of the model.

One critique to the approach explored in this chapter is that of time (in)consistency. As Kydland and Prescott (1977) already showed, governments may have incentives to abandon policies that were previously committed to. However in practice once a law has been passed it is not generally easy to reverse. Although policy reversals and changes exist, we do not observe countries reversing policies very often[4]. Moreover since policy uncertainty can be costly in terms of economic growth and political

[4]Fernandez and Rodrik (1991) and Coate and Morris (1999) present two different explanations on policy persistence or status quo bias.

support, governments if possible would rather avoid it. Finally, it is at least plausible that developing countries will tend to adopt climate policy due to international pressure, therefore reducing the possibilities of reversing the policy. In this context, pre-committing to a climate policy for the future may even benefit the country in terms of reduced international pressure, or higher access to markets for their products.

There is also extensive evidence that many policies are carried out in gradual ways, with pre announced commitments. For example there are examples of unilateral policies establishing a timetable for tariff reduction that can last several years and in the case of Free Trade Agreements they often last a decade or longer[5]. In addition, two other examples, the Montreal Protocol on CFC 's and the Kyoto Protocol, both were phased in over a long period of time (of course with substantially different success rate).

3.3 A Simple Model

Assume there is a private investor that chooses to invest in a project with two possible technologies. The first (dirty) technology has a period return of R_D if no climate policy is enacted and zero if climate policy is in place. The second (clean) technology has a period return of $R_C < R_D$ regardless of the existence of climate policy. We further assume that both technologies have the same investment horizon (lifetime) of T, such that it can be used for T periods and then it is abandoned. The period discount factor is δ and assumed constant. It is useful to define the function

[5] A few examples include Chile's unilateral tariff reduction that went from 11% to 6% over a five year period. The FTA's between Chile and USA and Chile and EU had both 10 year reduction schedules, while the agreement with MERCOSUR had a 17 year moratorium for some products.

$V(R, T, \delta)$ as the present discounted value at period 0 of a annuity of R during T periods with a discount factor of δ and the first payment in period 1[6]. That is:

$$V(R, T, \delta) = \sum_{t=1}^{T} R\delta^t = R\frac{\delta - \delta^{T+1}}{1 - \delta}$$

The expected value of the clean technology project is given by:

$$V_C = V(R_C, T, \delta)$$

while the expected value of the dirty technology project will depend on the probability of the country enacting climate policy. To keep the algebra simple assume that in each period the probability status quo is equal to p, so probability of the country deciding to enact climate policy is $1 - p$. Therefore this probability can be combined with the discount factor in a very simple way. Thus the expected value of a dirty investment project will be given by the expression:

$$V_D = V(R_D, T, \delta p)$$

From these two expressions we can find parameter values in order to compare the value of both dirty and clean technology projects, and see how this difference depends on each of the 4 parameters, R_C/R_D, p, δ and T.

[6]The assumption is that the investor chooses which project to undertake in period 0, but only starts receiving payoffs starting in period 1. This is consistent with the fact that normally long lived capital projects are rarely completed in a short period of time.

Remark 1 *(i) Until climate policy is enacted the investor always chooses to invest in the same technology. (ii) Once the climate policy is enacted, the investor chooses always the clean technology.*

The second part is trivial since $V_C > 0 = V_D$ with climate policy. For the first part we observe that given p, δ, T, R_C and R_D constant, then if for a given period $V_C < V_D$, then the investor chooses the dirty technology, and unless the climate policy is enacted, then this will also be true in $t+1$. If the reverse is true, then the investor always chooses the clean technology. Thus, the investor always chooses the same technology until climate policy is in place.

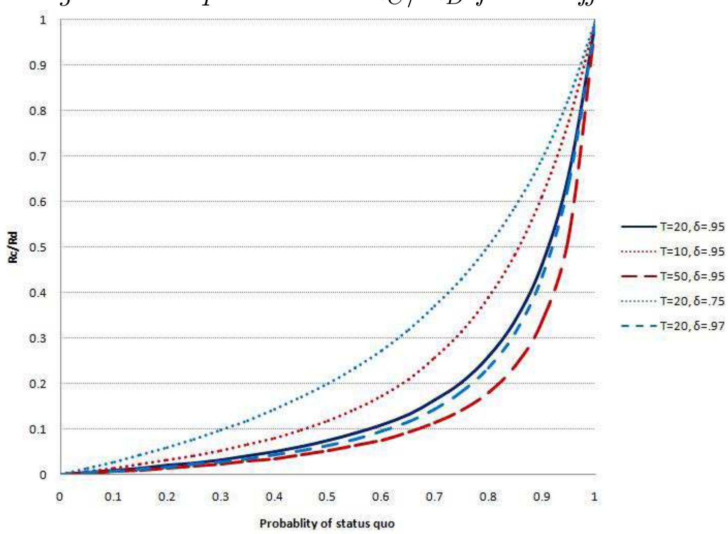
Next we equalize the present discounted values of both technologies and reorganizing terms one can get:

$$\frac{R_C}{R_D} = p\frac{1-\delta}{1-\delta p}\frac{1-(\delta p)^T}{1-\delta^T}$$

We can use this expression to do some comparative statics on each one of the four parameters mentioned before. First, the effect of the flow payoff of both technologies, R_C/R_D, and the probability of keeping the status quo, p, are straightforward in the choice between both technologies. The effect of the probability of the status quo is direct seeing that the right hand side of the previous expression is increasing in p. The effect of the horizon is direct too, a longer horizon will increase the risk of some climate policy change would be implemented, so a higher T is more favorable to a clean technology. This can be seen in the last expression because the right hand side is decreasing in T. Finally, a higher discount factor, that is a more patient investor, increases the relative value of the clean technologies over the dirty ones, because the future risk of a change is more important into the evaluation. Again, in the last expression the right hand side is decreasing in δ, an effect that coincides with the intuition. The Appendix D presents formal proofs of these statements.

Figure 3 presents this comparative static graphically. The figure shows the required ratio R_C/R_D take makes the private investor indifferent between both technologies, for different combinations of the other parameters.

Figure 3: Required ratio R_C/R_D for indifference.



As can be observed, the curvature increases with the timespan of the project and a higher discount factor. Both of these imply that clean technologies will be preferred the longer the project lasts and lower the discounting.

3.4 Eliminating the uncertainty: announcing future policy

Now assume that the government can credibly announce and pre commit to enact climate policy in $T^* < T$ periods, which will be in effect the next period[7]. Of course this does not change the expected value of the clean technology but it does change the expected value of the dirty technology. The present value of the dirty technology with the new announcement is now given by:

$$V_{DA}\left(T^*\right) = V\left(R_D, T^*, \delta\right)$$

That is, the uncertainty is eliminated but the horizon of planning is changed too. Now one can compare this value with the value of the clean technology. One can see that $V_C = V_{DA}\left(T^*\right)$ if and only if:

$$\frac{R_C}{R_D} = \frac{1-\delta^{T^*}}{1-\delta^{T}}$$

Let's define T_C as the announcement term, T^*, that solves the last equation. And solving we find that:

Lemma 3.1 (Existence of a critical time for clean technology) *There exist is a* $0 < T_C < T$ *such that for all* $T^* > T_C$*, then* $V_{DA}\left(T^*\right) > V_C$.

[7] We choose that climate policy is in effect the period after its is enacted, since it simplifies notation. Otherwise, expected values would entail a T-1 term.

Proof. By simple inspection we can see that $V_{DA}(0) < V_C$, while $V_{DA}(T) < V_C$. Thus by continuity there must exist T_C such that $V_{DA}(T_C) = V_C$. Given the monotonicity of the expression with respect to T^* the desired result holds. ∎

What this proposition shows is that there exists a period T_C such that announcing the policy to be effective in T_C leaves the investor indifferent between both technologies. Thus an investor that each period invests in one of these projects will have the following optimal strategy summarized in this corollary.

Corollary *The profile of investment of an investor under a pre announced climate policy will be to choose a dirty technology until the time left to the policy to be in effect is less than T_C. At that moment the investor will begin to invest only in clean technology.*

Proof. It is straightforward from the definition of T_C. ∎

The result above shows that in a pre-announced world the use of a dirty technology will be limited to a certain date. This deadline will be periods before the policy change is implemented, and in this model the number of periods is T_C. So, if the authority wants the technological change to occur at a certain date, the announcement should be made for the policy to be effective in future date. This difference between the date when the policy is implemented and the date when the relevant change is produced is important and depends on the other parameters of the system. For example, as the discount factor increases then the gap between the change in the technology and the implementation of the change increases.

3.5 When is it profitable to eliminate the uncertainty?

Now the question is what changes when an announcement is made. We assume the government is credible, so we do not focus on time inconsistency issues. First, the present discounted value of a dirty technology under announcement can be summarized as the following:

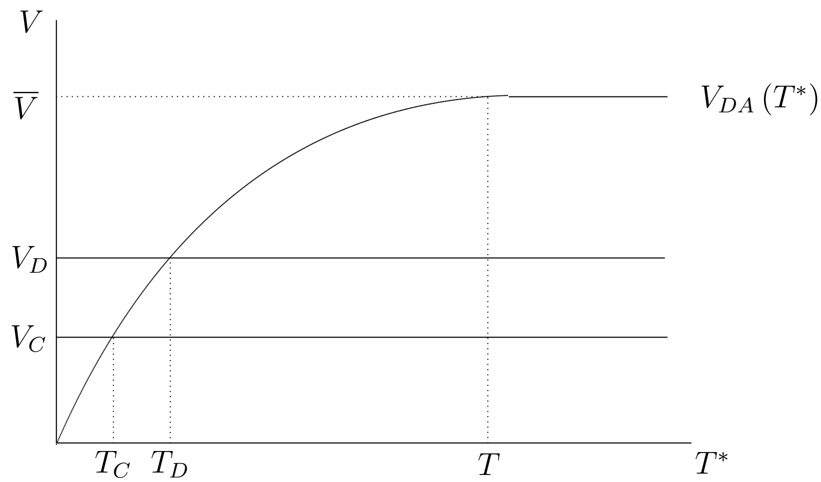
$$V_{DA}\left(T^{*}\right)=\begin{cases} R_D\frac{\delta-\delta^{T^{*}+1}}{1-\delta} & \text{if } T^{*}<T \\ \\ R_D\frac{\delta-\delta^{T+1}}{1-\delta} & \text{if } T^{*}\geq T \end{cases}$$

It is clear that $R_D\frac{\delta-\delta^{T+1}}{1-\delta}$ is greater than V_C and V_D. Let's define T_D as the solution of the following equation:

$$V_{DA}\left(T_D\right)=V_D$$

That is, the term that makes the investor indifferent between the investment in a dirty technology with an announcement and the investment under uncertainty in a dirty technology. Given $V_{DA}\left(T^{*}\right)$ is strictly increasing in the relevant range these equations have a unique solution. The next figure show this relations in the one relevant cases, that is, when $V_D>V_C$.

Figure 4: Values in the case with and without announcement.



One relevant result of the chapter is stated in the following theorem:

Theorem 3.1 (Effect of an announcement) *Suppose R_C/R_D, p, δ and T are such that $V_D > V_C$, then there exists a unique $T_D(R_D, p, \delta, T)$ such that for all $\hat{T} > T_D$ there exists a unique $\tau\left(\hat{T}\right)$ such that:*

i For all periods $t < \tau\left(\hat{T}\right)$ the private investor prefers to invest in a dirty technology

ii For all periods $t > \tau\left(\hat{T}\right)$ the private investor prefers to invest in a clean technology

In particular, at $t = 0$, the private investor in a dirty technology is better off with the announcement at T^* than without it if and only if $\tau(T^*) > T_D - T_C$, where T_C solves $V_{DA}(T_C) = V_C$.

Proof. The existence of a unique $T_D(R_D, p, \delta, T)$ is given by the definition of this variable. Moreover, if $V_D > V_C$ then $T_D > T_C$. Take any $\hat{T} > T_D$, then $\tau\left(\hat{T}\right)$ solves the following equation:

$$\tau\left(\hat{T}\right)+T_C=\hat{T}$$

In this case, for all $t<\tau\left(\hat{T}\right)$ it holds $V_C<V_{DA}\left(\hat{T}-t\right)$, so the investor prefers the dirty technology. At any period where $t\geq\tau\left(\hat{T}\right)$, the last inequality is reversed.

The case $t=0$ is just based on $\tau\left(T^*\right)>T_D-T_C$ is equivalent to $T^*>T_D$. ∎

The previous proposition states that the reduction of uncertainty could make the dirty technology more profitable in the short run, but for sure will make the clean technology more profitable in the future. This result does not include the effect of the reduction of the variance in the expected revenues, since the model assumes risk neutral investors. Including this effect should increase the benefits from the precommittment climate policy, raising the investors' expected return, for both clean and dirty technologies even further.

There are at least three considerations with respect to this last result.

The first one is that one can show that the expected time of the occurrence of the change, $E_p\left[T\right]$, is larger than the minimum time required to make an announcement profitable for the private sector that uses dirty technology today, T_D. That means that if one announces at $t=0$ that the change will be effective at the expected time, the private sector that uses dirty technology today will be better off, and the technological change will start T_C periods sooner. The only sector that is affected is the future investors in dirty technology, controlling for the probability of arrive to certain date without a change.

The second one is related to the restriction of this improvement. A preannouncement required that the commitment is credible. If the country can commit to this change in the future, then there is a space for a gain given the reduction in the uncertainty. By the other hand, if the country cannot commit, for credibility issues, the the only possibility is follow the period by period process. That is, the gains of the preannouncement are based in the credibility of the country regulators.

Finally, it is worth noting that the government has a range of possible timings to choose a policy that both increase the expected return to the private sector and reduces emissions, i.e. is good for the environment. However this relies on the assumption of $V_D > V_C$ initially. If this is not the case we get the following corollary.

Corollary

Green Paradox: Suppose R_C/R_D, p, δ and T are such that $V_D < V_C$, then:

- *if the announcement term $T^* < T_C$ the private investor will be unaffected by the elimination of the uncertainty and will continue preferring the clean technology.*
- *if the announcement term $T^* > T_C$ then the point [i.] of this proposition holds and a green paradox emerges.*

Proof. Straightforward from $T^* < T_C$ implies $V_{DA}(T^*) < V_C$.

Thus if the change in technology had already begun, due to perception of either a high discount factor or low probability of status quo, then we generate the green paradox, which will shift investment back into the dirty sector at least for a while.

Now we can undertake some comparative statics on $\tau\left(\hat{T}\right)$, that is, the period when the clean technologies will be the optimal private choice. Given the definition of this variable one finds the solution solving the following system of equations:

$$\tau\left(\hat{T}\right) = \hat{T} - T_C$$
$$\frac{R_C}{R_D} = \frac{1-\delta^{T_C}}{1-\delta^{T}}$$

Where the second equation comes from the definition of T_C. So, given a fixed $\hat{T}$, $\tau\left(\hat{T}\right)$ is decreasing in R_C/R_D, T, and δ. The implication of this result is that if different investors have different discount factors, even if the announcement is common for both, then one will begin investing in the clean technology before the other. As an extension, this implies that if investors from different countries face different discount factors (for example due to different risk premiums) then countries with lower discount factors (higher risk premium) will delay the deployment of clean technologies compared to countries with higher discount factors (ie. lower risk premiums).

3.6 Discussion and extensions

We can summarize the results obtained here by highlighting the following. As mentioned above the announcement or pre-commitment has a positive environmental outcome (shift from investing in dirty to clean technologies earlier than without the announcement) if the investor was initially facing a high probability of status quo or low discount factors, or the relative return to the clean technology was very low. In addition, we also observe that if the project's lifetime (T) is very short then this plays against clean technologies.

From these elements one can hypothesize about how these different variables might affect the adoption of clean technologies. In principle we should expect poor countries and high risk countries to have fewer incentives to shift to clean technologies. The former due to the fact that the probability of status quo is likely decreasing in income as richer countries will probably face more international pressure to curb emissions. Countries that have low emissions or low emission intensity would also probably face lower international pressure[8]. The latter as was mentioned earlier, since high risk countries will probably face low discount factors and therefore investors that do not care much about the future. We should also expect different adoption of technologies in different sectors. Projects that are short lived have fewer incentives to shift towards cleaner technologies, and of course costly projects will also face smaller incentives. This could for example explain why consumers for example do not care about the electricity consumption of a mobile phone and to some extent why they do not buy hybrid cars, but may be interested in weatherproofing their homes or buying an efficient heat pump, or switching from electric heating towards gas. It could also explain why countries have tended to adopt wind farms rather than solar power plants.

On the other hand one could extend the model to allow for a large range of additions. If we new for certain that the clean technology will become more (relatively) profitable over time, then the space for improvements would be reduced and the timing of the shift in technology would move towards an earlier date. However if the returns to the clean technology are uncertain (in the stochastic sense), then waiting

[8]Thus China would face increasing pressure on the emissions side but somewhat a reduced pressure since the country is still very poor.

might be a better alternative for the firm[9]. In addition to this we could introduce the possibility of a retrofit to the dirty technology at a given cost[10]. This, as it is also shown by Strand et. al. (2011), would "help" the dirty technology in allowing them to operate at a cost and therefore having a positive stream of income even under the climate policy.

An additional point that may be raised as mentioned in the literature review is that of credibility. However, our model to some extent capture both extremes of the credibility issue. If the government is not credible, then the investor would disregard the announcement and behave as if the announcement did not occur. On the other hand if the government is credible then our results follows. Partial credibility should generate some intermediate result given the monotonicity of our results, however it may depend on the structure of beliefs of different agents.

Finally is worth to thinking about in a model where the restriction of the investment per period is relaxed. This model is more general and it is considered in a ongoing research project. The basic structure and preliminary result of this model are the following.

3.6.1 An alternative approach

In this subsection we will relax the restriction of investing in one plant every period. In order to do that we solve the dynamic problem of a monopolist in a market where the demand is growing at a deterministic rate. The monopolist can choose the

[9] Goulder and Mattai (2000) and Strand et. al. (2011) stress this point when they find that higher uncertainty might induce to do less rather than more today.

[10] For example carbon capture and storage technology that could remove (and store) carbon emissions from for example a coal-fired power plant.

amount to invest at each point in time, and the investment has a depreciation rate. We use a discrete time model. The idea is keep the model as simple as possible. The market inverse demand for the final product at time t is given by:

$$P(t) = A(t) - BQ(t)$$

The parameter $A(t)$ reflects the increase in the market size (i.e. growth in the economy). Let's start with the case where there is only one available technology, represented by a constant marginal cost c and a unitary investment cost κ. It is easy to see that if there is no uncertainty, and there is not excess capacity at the first period, then the capacity will never be higher than the quantity that the monopolist sells in each period. Thus one can concentrate in the case where the capacity is equal to the quantity sold in every period. The monopolist problem can be represented by the following Bellman equation:

$$V(K, A) = \max_{Q,I} \{P(Q, A) Q - cQ - \kappa I + \beta V(K + I - \delta K, A + \Delta)\}$$
$$\text{s.t. } Q = K$$

Where β is the discount factor, κ the investment unitary cost, δ the depreciation rate, and Δ the growth of A between one period and the next one[11]. Let denote by $\hat{K}(t, C, \beta)$ the policy function of the previous Bellman equation, that determines the optimal path of investment.

[11] This growth rate could depend on the time or the value of A, but in this simple case this is not relevant and one can omit this dependence.

$$\hat{K}\left(t, C, \beta\right) = \underbrace{\frac{A\left(t\right) - C}{2B}}_{Static\ optimal} - \underbrace{\frac{\kappa}{2B}\left(\frac{1}{\beta} - \left(1 - \delta\right)\right)}_{Dynamic\ effect}$$

That is, the optimal dynamic quantity differs from the static one. Later one needs to add a new technology and evaluate when it is optimal to choose between one or the other. Use $I_D\left(t\right)$ and $I_C\left(t\right)$ to denote the investment in period t on dirty and clean technology respectively. Denote by $K\left(t\right)$ the total installed capacity, and by $K_D\left(t\right)$ and $K_C\left(t\right)$ the installed capacities in dirty and clean technology respectively.

Another way to understand the optimal capacity is the following:

$$K\left(t, C, \beta\right) = \tfrac{A(t) - \tilde{C}}{2B}$$

Where,

$$\tilde{C}\left(\beta\right) = C + \kappa\left(\tfrac{1}{\beta} - \left(1 - \delta\right)\right)$$

That is, is the same solution of the static problem but with an additional cost that summarizes the investment involved. This approach is useful because the last expression can be interpreted in a slightly different way. In the optimal path the private should be indifferent between invest a marginal unit in period t or $t + 1$, because if this is not true then he can adjust the investment path an increase his profits. If one writes this indifference condition, then one obtain that the difference between the marginal revenue and the marginal cost[12] is the same as the additional term in $\tilde{C}$. This way to understand $\tilde{C}$ will be useful in the preannouncement case. Let use $\tilde{C}_D(\beta)$ and $\tilde{C}_C(\beta)$ for the cases where the marginal cost is the dirty or the clean technology respectively.

Let denote by C'_D the marginal cost of the dirty technology after the policy change is enacted. We assume that $C_D < C_C < C'_D$. In the case where there is no announcement the following lemma summarizes the equilibrium path:

Lemma 3.2 (Effects of an unannounced change with flexible investment) *Suppose C_D, C_C, C'_D, p, δ, and β are such that before the announcement is made the private sector invest in dirty technology (the details of this conditions are given in the Appendix D), then if in period t^* the climate policy is changed without a preannouncement the following is the optimal path:*

- *i If $C'_D > \tilde{C}_C$ then for any $t \leq t^*$ the investment level is given by the dirty technology including the risk of a policy change, that is $K(t) = K_D(t) = \hat{K}(t, C_D, \beta p)$.*
- *ii There is no more investment in dirty technology after t^*, that is, for any $t \geq t^*$, $I_D(t) = 0$, and $K_D(t) = K_D(t^*) * (1-\delta)^{t-t^*}$*
- *iii If $\tilde{C}_C(\beta) < C'_D$ then $I_C(t^*) > 0$, that is, the investment in clean technology starts immediately.*
- *iv If $\tilde{C}_C(\beta) > C'_D$ then the solution depends on the installed capacity. If $\hat{K}(t^*+1, C_C, \beta) < K_D(t^*) * (1-\delta)$, then $I_C(t^*) = 0$, that is, there is no*

[12] Given that we analyze the monopolist case the marginal revenue is the relevant function.

investment at t^*. In this case the investment in clean technology starts at the first period t such that $\hat{K}(t, C_C, \beta) > K_D(t^*) * (1-\delta)^{t-t^*}$.

The proof is in the Appendix D. The most important result of the previous proposition is the following, depending on C'_D, that is the marginal cost of the dirty technology after the policy change, there could be or not investment in clean technology immediately after the change is enacted. It is not enough that this cost is higher than the marginal cost of the clean technology. If the new cost of the dirty technology is barely higher that the clean technology, there could be a period where there is no investment in new capacity, so all the production will be based in dirty technology. Here there is a trade off. A low increase in the dirty technology costs could delay the investment in clean technology, but the market prices will increase gradually.

Now, in the case of a preannouncement there are more variables involved in the analysis. We will restrict to the case when after the policy change made is optimal to invest in clean technology immediately. The next proposition summarizes the dynamic for this particular case.

Theorem 3.2 (Effect of an announced change with flexible investment) *Suppose C_D, C_C, C'_D, p, δ, and β are such that $\tilde{C}_C(\beta) < C'_D$, then if in period t^* the climate policy is changed with a preannouncement in the period $\hat{t}$ the following is the optimal path:*

i *For any $\hat{t} < t \leq t^*$ the marginal revenue will be between $\tilde{C}_D(\beta)$ and $\tilde{C}_C(\beta)$.*

ii *If $t^* - \hat{t}$ is high enough, then at the first periods after $\hat{t}$ the marginal revenue will be $\tilde{C}_C(\beta)$ and there will be investment in dirty technology.*

iii *If the marginal revenue is equal to $\tilde{C}_C(\beta)$ there will be investment in clean technology.*

The proof is based on backward induction.

Proof.

The optimal path is constructed in the following way:

i At the period $t = t^*$ there will be investment in clean technology because $\tilde{C}_C(\beta) < C'_D$, that is, is cheaper to invest in clean technology than use the installed dirty one. The investment will be the amount such that the marginal revenue in the next period be equal to $\tilde{C}_C$.

ii One period before, at $t = t^* - 1$, there will be investment in clean technology if the marginal revenue is equal to $\tilde{C}_C$.

iii At $t = \hat{t}$ there will be investment in dirty technology if and only if the present value of the marginal revenue is high enough. It is important to note that the present value is only until t^*, because after that period the dirty technology is not used (we are not including here a limit in the investment per period). The investment will be made until the marginal revenue in the next period be equal to $\tilde{C}_D$.

iv Repeating [ii] and [iii] until the investment is not profitable one gets the optimal investment path.

v Between the periods where there are investment in dirty technology, if any, and the ones with investment in clean one, if any, the marginal revenue will increase up to $\tilde{C}_C(\beta)$.

The idea is the following; the dirty technology could be installed after the announcement if there is enough time to obtain the required profit. If this is something that the authority wants to avoid, then this gives a limit for optimal announcement period.

The model still have preliminary results, but has the advantage to include variables like the investment cost that allow to compare policies like carbon taxes (like the ones used until now) with the ones focused on the investment, like subsidies or restrictions. Another step in the model is estimate the total welfare in the economy in each case. In this step one will need an estimation of the pollution cost.

3.7 Conclusions and further work

We have presented a very simple model in which a private investor in a developing country that will have to face some type of climate policy restriction in the future may prefer to know in advance when this one will be in place. The main idea behind this is that given that investments last for long periods, then if these are going to face a large negative shock it is optimal to know in advance when this would occur, and therefore phase out old technologies in a relatively cost-effective way. Thus the main result of this chapter is that there are private costs avoided when implementing the announced policy.

The model so far has assumed a market in which the return to each investment is exogenous, which may not be completely realistic for some sectors/countries. In particular the power sector has some of the characteristics of long lived capital, but is perhaps organized as oligopolies or regulated monopolies, where their investment decision affect the return on investment. Taking this into account may well change the optimal behavior and thus policy choice. A second extension is to consider that the probability of climate policy is not constant but rather increasing over time since the pressure over developing countries to adopt climate policy is rather unlikely to diminish and very likely to increase.

A few testable prediction that would be interesting to explore is to see if investors in long lived capital projects are more willing to demand some type of climate policy than investors in shorter term projects.

When the investment per period is flexible, one can incorporate the investment decision to this problem. That allows to evaluate alternative measures against the dirty technology, like a a subsidy to the clean technology. Another advantage is one can have an estimation of the total welfare in the economy, so concepts like the optimal time of announcement can be developed.

Appendix A

Equivalence between Markov strategies and belief monotonicity (Theorems 1 and 2)

Theorem 1 states that if the Investor can commit to one period in advance, then one can use Markov strategies to solve the Investor's problem if and only if the sequence $\left\{\tilde{\delta}_t\left(x_t, x_{t+1}\right)\right\}_{t=1}^{\infty}$ is increasing. Call this last property *belief monotonicity.* Just as a reminder:

$$\tilde{\delta}_t\left(x_t, x_{t+1}\right) = \frac{x_t - \pi\left(x_t\right)}{x_{t+1}}$$

As stated in the chapter, the idea behind the use of Markov strategies here is that all the relevant history can be summarized in two state variables. These two state variables should be sufficient in order to update the belief about the type of Associate given that he is still cooperating, and to calculate the next period optimal investment level. In one direction it suffices to show that, under Markov strategies, the equivalent discount factor sequence is increasing.

The belief updating process is just an adjustment of the lowest possible type that is still cooperating. So use this belief as one state variable and denote it by $\hat{\delta}_t$. That is, at the beginning of period t all the types of Associate still cooperating have a discount factor above this level. The other state variable is the investment level at t, because that level was committed at $t-1$. Given that the problem can be solved using only these two variable states then the Associate should not need more information than the next period investment to make his decision to cooperate or quit at t. Having this in mind let use the following Bellman equation to solve the Investor's problem:

$$W\left(\hat{\delta}_t, x_t\right) = \max_{x_{t+1}}\{[1-p_t]\left[\pi\left(x_t\right) + \beta W\left(\hat{\delta}_{t+1}, x_{t+1}\right)\right] - p_t x_t\} \tag{A.1}$$

$$\text{s.t. } p_t \in [0,1]$$

Where p_t is the probability that an Associate quits at t given that he had cooperated at $t-1$. Given that x_{t+1} is sufficient for the Associate for decide to cooperate or to quit at t his best response is:

- The Associate cooperates if $x_t < \pi\left(x_t\right) + \delta x_{t+1}$
- The Associate quits if $x_t > \pi\left(x_t\right) + \delta x_{t+1}$
- The Associate is indifferent between cooperate and quit if $x_t = \pi\left(x_t\right) + \delta x_{t+1}$

So, using a uniform distribution for the Associate's types, the probability of quitting is:

$$p_t = \frac{\frac{x_t - \pi(x_t)}{x_{t+1}} - \hat{\delta}_t}{\delta_h - \hat{\delta}_t}$$

And the updated belief is just:

$$\hat{\delta}_{t+1} = \frac{x_t - \pi\left(x_t\right)}{x_{t+1}}$$

So the belief at time $t+1$ is just the equivalent discount factor at t. Finally, given that the probability should be between 0 and 1 that has two implications for the optimal choice of x_{t+1}

- A probability lower than 1 implies that x_{t+1} should be high enough should that at least the most patient Associate's type cooperate at t
- A probability higher than 0 implies $\hat{\delta}_{t+1} \geq \hat{\delta}_t$

So non negativity of the probability of quitting at t gives the required implication: The sequence $\left\{\tilde{\delta}_t\left(x_t, x_{t+1}\right)\right\}_{t=1}^{\infty}$ is increasing, that is, the belief monotonicity property holds.

In order to prove on other direction it is suffices to show that if the belief monotonicity property holds then the Associate needs to forecast one period in the future for his best response.

Remember that the Associate's best response depends on the relationship between the investment level at t and the highest payoff after t, y_t, where $y_t = \max_{\tau>t}\left\{\sum_{s=t}^{s=\tau-1}\delta^{s-t}\pi\left(x_s\right)+\delta^{\tau-t}x_\tau\right\}$. Focus in particular on when the Associate wants to quit at t, that is if $x_t > y_t$, and show that under belief monotonicity this condition is implied by $x_t > \pi\left(x_t\right)+\delta x_{t+1}$.

i. If $x_t > \pi\left(x_t\right)+\delta x_{t+1}$ then:

$$\delta < \frac{x_t-\pi\left(x_t\right)}{x_{t+1}} = \tilde{\delta}_t$$

ii. Given the belief monotonicity property, $\delta < \tilde{\delta}_\tau$ for each $\tau \geq t$

iii. Now use $\delta < \tilde{\delta}_{t+1}$

$$\delta < \frac{x_{t+1}-\pi\left(x_{t+1}\right)}{x_{t+2}} \Rightarrow x_{t+1} > \pi\left(x_{t+1}\right)+\delta x_{t+2} \Rightarrow \pi\left(x_t\right)+\delta x_{t+1} > \pi\left(x_t\right)+\delta\pi\left(x_{t+1}\right)+\delta^2 x_{t+2}$$

Hence quitting at $t+1$ is better than quitting at $t+2$. Given the condition in [ii], quitting in $t+1$ is better than quitting in any other period in the future. Then $y_t = \pi(x_t) + \delta x_{t+1}$. Finally, given [i] $x_t > y_t$ so the Associate chooses to quit at t.

On the other side, if $x_t < \pi(x_t) + \delta x_{t+1}$, then the Associate's best response at t is to cooperate. That is because it is better to quit at $t+1$ than at t. So in order to solve the problem one can reduce the initial infinite horizon problem to a two period problem. This problem can be solved using the Markov strategies defined at the beginning of the chapter.

Theorem 2 follows directly from the belief monotonicity condition. Given that the next period investment in enough to determine the Associate's decision, then this decision will not be affected by a commitment to a future investment level.

Appendix B

Commitment and non commitment steady states

B.1 Lemma 2

Assume not, that is that there exists t such that $x_{t+1} < x_t$. Then by belief monotonicity:

$$\tilde{\delta}_{t+1} \geq \tilde{\delta}_t \iff \frac{x_{t+1}-\pi(x_{t+1})}{x_{t+2}} \geq \frac{x_t-\pi(x_t)}{x_{t+1}}$$
$$\iff \frac{x_{t+1}-\pi(x_{t+1})}{x_t-\pi(x_t)} \frac{x_{t+1}}{x_{t+2}} \geq 1$$

Given that $x - \pi(x)$ is strictly increasing, if $x_{t+1} < x_t$ then the first term is strictly lower than one. So the only way to have belief monotonicity is $x_{t+2} < x_{t+1}$. That is, a decrease in the investment in one period implies a decrease in the next one. That means that all the investment levels after t should be lower than x_t and that cannot be optimal because is better to choose the initial investment level and keep forever this offer.

B.2 Lemma 5

The idea of the proof is to first show that the proposed equation is the steady state of the problem and after that to show that the conditions are sufficient for the stable steady state.

The commitment problem is:

$$W\left(x_{t-1},x_t\right)=\max_{x_{t+1}}\{\left[1-p_t\right]\left[\pi\left(x_t\right)+\beta W\left(x_t,x_{t+1}\right)\right]-p_t x_t\}$$

$$\text{s.t. } p_t\in[0,1]$$

So, x_{LC} is the value that solves $h_W\left(x_{LC},x_{LC}\right)=x_{LC}$, where $h_W\left(\cdot,\cdot\right)$ is the policy function of the equation $W\left(\cdot,\cdot\right)$.

Denote by $f^{(i)}\left(\cdot\right)$ to the derivative of f with respect to the i-th argument, and denote by p_t the probability that the Associate quits at t, that is, the expression where the first argument is x_{t-1}. Assuming an interior solution, the optimal conditions are:

First Order Condition:

$$-p_t^{(3)}\left[\pi\left(x_t\right)+\beta W\left(x_t,x_{t+1}\right)+x_t\right]+\left[1-p_t\right]\beta W^{(2)}\left(x_t,x_{t+1}\right)=0$$

Envelope Conditions:

$$\{x_{t-1}\}:W^{(1)}\left(x_{t-1},x_t\right)=-p_t^{(1)}\left[\pi\left(x_t\right)+\beta W\left(x_t,x_{t+1}\right)+x_t\right]$$

$$\{x_t\}:W^{(2)}\left(x_{t-1},x_t\right)=-p_t^{(2)}\left[\pi\left(x_t\right)+\beta W\left(x_t,x_{t+1}\right)+x_t\right]+$$

$$\left[1-p_t\right]\left[\pi'\left(x_t\right)+\beta W^{(1)}\left(x_t,x_{t+1}\right)\right]-p_t$$

Combining the two envelope conditions and substituting into the First Order Condition:

$$p_t^{(3)}\left[\pi\left(x_t\right)+\beta W\left(x_t,x_{t+1}\right)+x_t\right]=\left[1-p_t\right]\beta\{-p_{t+1}^{(2)}\left[\pi\left(x_{t+1}\right)+\beta W\left(x_{t+1},x_{t+2}\right)+x_{t+1}\right]+$$
$$\left[1-p_{t+1}\right]\left[\pi'\left(x_{t+1}\right)+\beta\left(-p_{t+2}^{(1)}\left[\pi\left(x_{t+2}\right)+\beta W\left(x_{t+2},x_{t+3}\right)+x_{t+2}\right]\right)\right]-p_{t+1}\}$$

So the "Euler Equation" depends on $x_{t-1},x_t,x_{t+1},x_{t+2},x_{t+3}$ and the function $W(\cdot,\cdot)$. Now in steady state the following condition must hold (abuse of notation by using x as the steady state level):

$$x_t=x \quad p\left(x,x,x\right)=0$$
$$p^{(1)}\left(x,x,x\right)=-\tfrac{1-\pi'(x)}{x[\beta-1]+\pi(x)}$$
$$p^{(2)}\left(x,x,x\right)=\tfrac{1}{x}\tfrac{x[1-\pi'(x)]+[x-\pi(x)]}{x[\beta-1]+\pi(x)}$$
$$p_{(3)}\left(x,x,x\right)=-\tfrac{1}{x}\tfrac{x-\pi(x)}{x[\beta-1]+\pi(x)}$$
$$V\left(x,x\right)=\tfrac{\pi(x)}{1-\beta}$$

Substituting the steady state conditions into the "Euler Equation":

$$-\tfrac{1}{x}\tfrac{x-\pi(x)}{x[\beta-1]+\pi(x)}\left[\tfrac{\pi(x)}{1-\beta}+x\right]=\beta\Big\{-\tfrac{1}{x}\tfrac{x[1-\pi'(x)]+[x-\pi(x)]}{x[\beta-1]+\pi(x)}\left[\tfrac{\pi(x)}{1-\beta}+x\right]+$$
$$\pi'\left(x\right)+\beta\tfrac{1-\pi'(x)}{x[\beta-1]+\pi(x)}\left[\tfrac{\pi(x)}{1-\beta}+x\right]\Big\}$$
$$\Rightarrow\left[\pi\left(x\right)+\left[1-\beta\right]x\right]\left[\pi\left(x\right)+\left[\beta-1\right]x\right]=2\beta x\pi'\left(x\right)\pi\left(x\right)$$

Define:

$$F(x) \equiv 2\beta\pi'(x)\, x\pi(x) - [\pi(x) + x[1-\beta]][\pi(x) - x[1-\beta]]$$

Then $F(x_{LC}) = 0$ is the steady state condition. This equation may have more than one solution.

Now check the condition for the stability of one of the steady states. Consider state variables $(x-\epsilon, x)$ and check the condition for the policy function to be higher than x. Compare the value obtained by increasing the investment in the next period by a positive marginal amount, γ, and remaining at that level forever versus keepping x forever. Denote by $\tilde{W}(y, x, z)$ the value when the state variables are (y, x) and the chosen investment is z from the next period to the future.

Then we need to evaluate:

$$\tilde{W}(x-\epsilon, x, x+\gamma) = [1-q_1]\left[\pi(x) + \beta\left\{[1-q_2]\frac{\pi(x+\gamma)}{1-\beta} - q_2[x+\gamma]\right\}\right] - q_1 x$$
$$\tilde{W}(x-\epsilon, x, x) = [1-q_3]\frac{\pi(x)}{1-\beta} - q_3 x$$

where the probabilities of deviation are:

$$q_1 = \frac{\frac{x-\pi(x)}{x+\gamma} - \frac{x-\epsilon-\pi(x-\epsilon)}{x}}{\beta - \frac{x-\epsilon-\pi(x-\epsilon)}{x}} = \frac{1}{x+\gamma}\frac{x[\pi(x-\epsilon)-\pi(x)]+\gamma[\pi(x-\epsilon)-x]+\epsilon x+\epsilon\gamma}{\beta x-[x-\epsilon-\pi(x-\epsilon)]}$$
$$q_2 = \frac{\frac{x+\gamma-\pi(x+\gamma)}{x+\gamma} - \frac{x-\pi(x)}{x+\gamma}}{\beta - \frac{x-\pi(x)}{x+\gamma}} = \frac{\gamma+\pi(x)-\pi(x+\gamma)}{\beta x+\beta y-[x-\pi(x)]}$$
$$q_3 = \frac{\frac{x-\pi(x)}{x} - \frac{x-\epsilon-\pi(x-\epsilon)}{x}}{\beta - \frac{x-\epsilon-\pi(x-\epsilon)}{x}} = \frac{\epsilon+\pi(x-\epsilon)-\pi(x)}{\beta x-[x-\epsilon-\pi(x-\epsilon)]}$$

So, increasing the investment in the next period is optimal if:

$$\tilde{W}(x-\epsilon, x, x+\gamma) > \tilde{W}(x-\epsilon, x, x)$$
$$\iff \frac{\beta}{1-\beta}\left[\pi(x+\gamma)-\pi(x)\right] - \beta p_2\left[x+\gamma+\frac{\pi(x+\gamma)}{1-\beta}\right] - p_1\left[x+\pi(x)+\beta\frac{\pi(x+\gamma)}{1-\beta}\right] +$$
$$p_1 p_2 \beta\left[x+\gamma+\frac{\pi(x+\gamma)}{1-\beta}\right] > -p_3\left[x+\frac{\pi(x)}{1-\beta}\right]$$

Dividing by γ and taking the limit $\gamma \to 0$, $\epsilon \to 0$ and $\epsilon/\gamma \to \kappa$:

$$\frac{[\pi(x)-[1-\beta]x][\pi(x)+[1-\beta]x]-2\beta x\pi'(x)\pi(x)}{\beta x-[x-\pi(x)]} < 0$$
$$\iff \frac{F(x)}{\beta x-[x-\pi(x)]} > 0$$

Given the definition of $\tilde{x}_\delta$:

$$x < \tilde{x}_\beta \Rightarrow \beta x - [x-\pi(x)] = x\left[\frac{\pi(x)}{x} - [1-\beta]\right] > 0$$

Combining both inequalities and $\gamma > 0$:

$$x < \tilde{x}_\beta \wedge F(x) > 0 \Rightarrow \tilde{W}(x-\epsilon, x, x+\gamma) > \tilde{W}(x-\epsilon, x, x)$$

So, $x = x_{_{LC}}$ and $F'(x) < 0$ is a stable steady state. Because the previous expressions are continuous then the inequality holds for a neighborhood around x.

B.3 Theorem 3:

First obtain the expression for the stable solutions.

In the one shot case the problem to solve is:

$$\max_{x}\{[1-p(x)]\frac{\pi(x)}{1-\beta}-p(x)\,x\}$$

$$s.t.p(x)=\frac{\tilde{\delta}(x,x)-\delta_l}{\delta_h-\delta_l}$$

$$p(x)\in[0,1]$$

Computing the First Order Condition and assuming an interior solution:

$$\pi(x)^2+[1-\beta][1-\delta_l]\,x^2-2\pi'(x)\,x\pi(x)=0$$

So define:

$$K(x,\beta,\delta_l)\equiv 2\pi'(x)\,x\pi(x)-\pi(x)^2-x^2[1-\beta][1-\delta_l]$$

Then $K(x_{_J},\delta_l)=0$

The next step is look for the stable solution, by definition:

$$x_{_{SJ}}=\tilde{x}_{\delta^*}=x_{_J}(\delta^*)$$

Where δ^* should be determined. From the previous definitions:

$$\frac{\pi\left(\tilde{x}_{\delta^*}\right)}{\tilde{x}_{\delta^*}} = 1 - \delta^*$$

$$\pi\left(x_{SJ}\left(\delta^*\right)\right)^2 + x_{SJ}\left(\delta^*\right)^2\left[1-\beta\right]\left[1-\delta^*\right] - 2\pi'\left(x_{SJ}\left(\delta^*\right)\right)x_{SJ}\left(\delta^*\right)\pi\left(x_{SJ}\left(\delta^*\right)\right) = 0$$

Combining both:

$$\pi\left(x_{SJ}\right) + x_{SJ}\left[1-\beta\right] - 2\pi'\left(x_{SJ}\right)x_{SJ} = 0$$

So define:

$$J\left(x\right) \equiv 2\pi'\left(x\right)x - \pi\left(x\right) - x\left[1-\beta\right]$$

Then $J\left(x_{SJ}\right) = 0$

Now is necessary to prove the inequality $x_{SJ} < x_{LC}$. Evaluate $F\left(x_{SJ}\right)$

$$\begin{aligned} F\left(x_{SJ}\right) &= 2\beta\pi'\left(x_{SJ}\right)x_{SJ}\pi\left(x_{SJ}\right) - \left[\pi\left(x_{SJ}\right) + x_{SJ}\left[1-\beta\right]\right]\left[\pi\left(x_{SJ}\right) - x_{SJ}\left[1-\beta\right]\right] \\ &= 2\beta\pi'\left(x_{SJ}\right)x_{SJ}\pi\left(x_{SJ}\right) - 2\pi'\left(x_{SJ}\right)x_{SJ}\left[\pi\left(x_{SJ}\right) - x_{SJ}\left[1-\beta\right]\right] \\ &= 2\pi'\left(x_{SJ}\right)x_{SJ}\left[1-\beta\right]\left[x_{SJ} - \pi\left(x_{SJ}\right)\right] \end{aligned}$$

If $x_{SJ} \in intX$ then all the elements of the last expressions are strictly positive. Given that there exists a unique zero of F, x_{LC}, and $F'\left(x_{SJ}\right) < 0$, then $x_{SJ} < x_{LC}$.

B.4 Lemma 6:

First solve the artificial problem for the non commitment case:

$$V\left(x_{t-1}\right)=\max_{x_t}\{\left[1-\tilde{p}_t\right]\left[\pi\left(x_t\right)+\beta V\left(x_t\right)\right]-\tilde{p}_t x_t\}$$

$$\text{s.t. } \tilde{p}_t=p\left(x_{t-1},x_t,h_V\left(x_t\right)\right)$$

$$p_t\in[0,1]$$

Again, look for x_{LNC} such that $h_V\left(x_{LNC}\right)=x_{LNC}$ if the solution exists. Using the same notation as before and assuming an interior solution, the first order condition and envelope condition are:

First order condition:

$$-\tilde{p}_{t-1}^{(2)}\left[\pi\left(x_t\right)+\beta V\left(x_t\right)+x_t\right]-\tilde{p}_{t-1}+\left[1-p_{t-1}\right]\left[\pi'\left(x_t\right)+\beta V'\left(x_t\right)\right]=0$$

Envelope condition:

$$V'\left(x_{t-1}\right)=-\tilde{p}_{t-1}^{(1)}\left[\pi\left(x_t\right)+\beta V\left(x_t\right)+x_t\right]$$

Where $\tilde{p}_{t-1}^1=p_{t-1}^1$ and $\tilde{p}_{t-1}^2=p_{t-1}^2+p_{t-1}^3h_V'\left(x_{t-1}\right)$ given the definition of $\tilde{p}$. In a steady state, $x_{t-1}=x_t=x$, $p_{t-1}=0$, $V\left(x\right)=\pi\left(x\right)/\left[1-\beta\right]$ and $h_V'\left(x_{t-1}\right)=1$. The partial derivatives of the probabilities are the same as before. Replacing the EC in the FOC and using the steady state conditions:

$$-\frac{1-\pi'(x)}{\pi(x)-[1-\beta]x}\frac{\pi(x)+[1-\beta]x}{1-\beta}+\pi'(x)-\beta\frac{1-\pi'(x)}{\pi(x)-[1-\beta]x}\frac{\pi(x)+[1-\beta]x}{1-\beta}=0$$
$$\iff \pi(x)+[1-\beta]x-2\pi'(x)\pi(x)=0$$

Then, defining $G(x)\equiv 2\pi'(x)\pi(x)-\pi(x)-[1-\beta]x$, one has that $G(x_{LNC})=0$. Again, this equation can have more than one solution.

Now check the stability of one steady state. Again compare the value of one stream when one increases the investment marginally and keeps it at this level, $\tilde{V}(x,x+\epsilon)$, versus keepping the previous level forever, $\tilde{V}(x)$.

$$\tilde{V}(x,x+\epsilon)=\frac{\pi(x+\epsilon)}{1-\beta}-p_1\left[x+\epsilon+\frac{\pi(x+\epsilon)}{1-\beta}\right]$$
$$p_1=\frac{\frac{x+\epsilon-\pi(x+\epsilon)}{x+\epsilon}-\frac{x-\pi(x)}{x+\epsilon}}{\beta-\frac{x-\pi(x)}{x+\epsilon}}=\frac{\epsilon-[\pi(x+\epsilon)-\pi(x)]}{\beta[1-\beta]-[x+\epsilon]}$$
$$\tilde{V}(x)=\frac{\pi(x)}{1-\beta}$$

Then, it is optimal to increase the previous level if:

$$\tilde{V}(x,x+\epsilon)>\tilde{V}(x)$$
$$\iff \pi(x+\epsilon)-\pi(x)>\frac{\epsilon-[\pi(x+\epsilon)-\pi(x)]}{\beta[x+\epsilon]-[x-\pi(x+\epsilon)]}\left[[1-\beta][x+\epsilon]+\pi(x+\epsilon)\right]$$

Again, divide both sides by ϵ and take the limit when $\epsilon\to 0$:

$$\pi'(x) > \frac{1-\pi'(x)}{\beta x - x + \pi(x)}\left[[1-\beta]x + \pi(x)\right]$$
$$\iff \frac{2\pi'(x)\pi(x)}{\beta x - x + \pi(x)} > \frac{[1-\beta]x + \pi(x)}{\beta x - x + \pi(x)}$$

Again, if $x < \tilde{x}_\beta$ then the denominator is positive, so the last inequality is equivalent to $G(x) > 0$.

Finally, if one takes the difference between the polynomial G and the polynomial J:

$$G(x) - J(x) = 2\pi'(x)\pi(x) - \pi(x) - [1-\beta]x - 2\pi'(x)x + \pi(x) + x[1-\beta]$$
$$\Rightarrow G(x) - J(x) = 2\pi'(x)[\pi(x) - x]$$

If $x \in intX$ then the last expression is strictly negative. So, $G(x_{SJ}) < 0$ and $J(x_{LNC}) > 0$. That implies that $x_{SJ} > x_{LNC}$, so x_{SJ} is the steady state.

B.5 Theorem 4:

Given the conditions, by Theorem 3, x_{LC} is the steady state in the commitment case. By Lemma 5, x_{SJ} is the steady state in the non commitment case. Theorem 3 shows that $x_{LC} > x_{SJ}$

B.6 Theorem 5:

Under these conditions the steady state under commitment is given by one zero of the polynomial F, while the steady state without commitment is given by one zero of the polynomial J:

$$F(x) = 2\beta\pi'(x)\,x\pi(x) - [\pi(x) + x[1-\beta]]\,[\pi(x) - x[1-\beta]]$$

$$J(x) = 2\pi'(x)\,x - \pi(x) - x[1-\beta]$$

It is straightforward that when $\beta \to 1$ then $F(x) \to \pi(x)\,J(x)$. So, the zeros of F are the zeros of K and the zeros of π. The zeros of π are two and both are outside the FIS, so the zeros of F and K inside the FIS will converge as β approaches to 1.

Appendix C

Cobb Douglas case

C.1 Lemma 7:

The strictly roots of F can be find by solving the quadratic equation obtained by avoid the solution at $x = 0$. That solutions are:

$$x_{1,2}(\alpha, \beta, \gamma) = \left[\frac{\gamma [2\alpha\beta - 1]}{\beta [\alpha + 1] - 1 \pm \sqrt{[1 - \alpha\beta][1 - \alpha\beta - 2\beta + 2\beta^2]}} \right]^{\frac{1}{1-\alpha}}$$

i. $x_1 < \overline{x}$

First check the case $\tilde{x_\beta} > \hat{x}$

$$F(\tilde{x_\beta}) = 2\beta\pi'(x_\beta) x_\beta \pi(x_\beta) < 0 \qquad \{\pi'(x_\beta) < 0\}$$

$$F(\hat{x}) = -[\pi(\hat{x}) + \hat{x}(1 - \beta)][\pi(\hat{x}) - \hat{x}(1 - \beta)] < 0 \quad \{\pi(\hat{x}) > \hat{x}(1 - \beta)\}$$

Given that $F'(0) > 0$ and $F(0) = 0$, then $\tilde{x_\beta}$ and $\hat{x}$ are between the two roots, so $x_1 < \hat{x} < \tilde{x_\beta} < x_2$.

Now see the case $\tilde{x_\beta} < \hat{x}$. Evaluating $F(x)$ at the critical points:

$$F\left(\tilde{x_\beta}\right) = 2\beta\pi'\left(x_\beta\right)x_\beta\pi\left(x_\beta\right) > 0 \qquad \left\{\pi'\left(x_\beta\right) > 0\right\}$$

$$F\left(\hat{x}\right) = -\left[\pi\left(\hat{x}\right) + \hat{x}\left(1-\beta\right)\right]\left[\pi\left(\hat{x}\right) - \hat{x}\left(1-\beta\right)\right] > 0 \quad \left\{\pi\left(\hat{x}\right) < \hat{x}\left(1-\beta\right)\right\}$$

So both points could be at the left of x_1 or at the right of x_2.

ii. *If* $\tilde{x}_\beta > \hat{x}$ *then* $x_2 > \tilde{x}_\beta$

This is direct from (i) in the case $\tilde{x_\beta} > \hat{x}$.

iii. *If* $\tilde{x}_\beta < \hat{x}$ *then* $x_2 < \tilde{x}_\beta$

This is direct from (i) in the case $\tilde{x_\beta} < \hat{x}$.

iv. *If* $\tilde{x}_\beta = \hat{x}$ *then* $x_2 = \overline{x}$

By definition of $\tilde{x_\beta}$ and $\hat{x}$:

$$\tilde{x_\beta} = \hat{x} \Leftrightarrow \alpha\left(2-\beta\right) = 1 \Leftrightarrow \alpha = 1/\left(2-\beta\right)$$

Replacing in the expression for x_2 one obtains the equality with x_1.

C.2 Lemma 8:

In area I if $\beta > 4\left(1-\alpha\right)$ the steady state investment level is given by:

$$x_{LC} = \left[\frac{\gamma\left[2\alpha\beta - 1\right]}{\beta\left[\alpha+1\right] - 1 + \sqrt{\left[1-\alpha\beta\right]\left[1-\alpha\beta-2\beta+2\beta^2\right]}}\right]^{\frac{1}{1-\alpha}}$$

After a little bit of algebra one can show that x_{LC} decreases with β for lower values of β, that is, when the strategic limit is just above the technological limit. One can show too that when β goes to 1, then x_{LC} is increasing in the Investor's discount factor.

C.3 Lemma 10:

Remember that x_{LC} and x_{SJ} are the zeros of polynomial F and G respectively. That means:

$$2\beta\pi'\left(x_{LC}\right)x_{LC}\pi\left(x_{LC}\right)-\left[\pi\left(x_{LC}\right)+x_{LC}\left[1-\beta\right]\right]\left[\pi\left(x_{LC}\right)-x_{LC}\left[1-\beta\right]\right]=0$$

$$2\beta\pi'\left(x_{SJ}\right)x_{x_{SJ}}-\left[\pi\left(x_{SJ}\right)+x_{SJ}\left[1-\beta\right]\right]=0$$

And $\tilde{x}_0$ holds $\pi\left(\tilde{x}_0\right)=\tilde{x}_0$. One can rewrite $F\left(x\right)$ in the following way:

$$F\left(x\right)=\beta\pi\left(x\right)\left[2\pi'\left(x\right)x-\left[\pi\left(x\right)+x\left[1-\beta\right]\right]\left[\frac{1}{\beta}-\frac{x}{\pi\left(x\right)}\left[\frac{1}{\beta}-1\right]\right]\right]=0$$

So, if $\pi\left(x\right)=x$ the strictly positive zeros of F are the same zeros of G.

Appendix D

Optimal announcement time

D.1 Net present value for dirty and clean technologies:

We start from the following equation.

$$\frac{R_C}{R_D} = p\frac{1-\delta}{1-\delta p}\frac{1-(\delta p)^T}{1-\delta^T}$$

The effect of R_C/R_D is direct. The effect of p is straightforward if one rewrites the right hand side as

$$p\frac{1-\delta}{1-\delta p}\frac{1-(\delta p)^T}{1-\delta^T} = \frac{\delta p-(\delta p)^T}{1-(\delta p)^T}\frac{1-\delta^T}{\delta-\delta^T}$$

The first term is increasing in p and the second term is independent of p. The effect of T can be seen taking the difference of the right hand side evaluated in one period and the period after.

$$p\frac{1-\delta}{1-\delta p}\frac{1-(\delta p)^T}{1-\delta^T}-p\frac{1-\delta}{1-\delta p}\frac{1-(\delta p)^{T+1}}{1-\delta^{T+1}}=$$
$$p\frac{1-\delta}{1-\delta p}\frac{1-(\delta p)}{1-\delta}\left[\frac{\sum_{t=1}^{T-1}(\delta p)^t}{\sum_{t=1}^{T-1}\delta^t}-\frac{\sum_{t=1}^{T}(\delta p)^t}{\sum_{t=1}^{T}\delta^t}\right]=$$
$$p\frac{1-\delta}{1-\delta p}\frac{1-(\delta p)}{1-\delta}\frac{1}{\sum_{t=1}^{T-1}\delta^t\sum_{t=1}^{T}\delta^t}\left[\left(\sum_{t=1}^{T-1}(\delta p)^t\right)\delta^T-\left(\sum_{t=1}^{T-1}\delta^t\right)(\delta p)^T\right]=$$
$$p\frac{1-\delta}{1-\delta p}\frac{1-(\delta p)}{1-\delta}\frac{1}{\sum_{t=1}^{T-1}\delta^t\sum_{t=1}^{T}\delta^t}(\delta p)^T\left[\left(p^{-T}+\ldots+\delta^{T-1}p^{-1}\right)-\left(1+\ldots+\delta^{T-1}\right)\right]$$

Given $p<1$, the last expression is strictly greater than 0. So a longer T is more favorable to the clean technology.

D.2 Proof of Lemma 12

As in chapter 3, let denote by $\hat{K}(t,C,\beta)$ the policy function of the previous Bellman equation, that determines the optimal path of investment.

$$\hat{K}(t,C,\beta)=\underbrace{\frac{A(t)-C}{2B}}_{Static\ optimal}-\underbrace{\frac{\kappa}{2B}\left(\frac{1}{\beta}-(1-\delta)\right)}_{Dynamic\ effect}$$

Let use $I_D(t)$ and $I_C(t)$ for denote the investment in period t on dirty and clean technology respectively. Let denote by $K(t)$ the total installed capacity, and by $K_D(t)$ and $K_C(t)$ the installed capacities in dirty and clean technology respectively. Finally, let denote by $\tilde{C}(\beta)$ the equivalent cost, including the investment, of the technology with marginal cost equal to C.

The problem involves solving the following two value functions:

$$V_{BAU}\left(K_D.K_C,A\right)=Max_{Q_D,Q_C,I_D,I_C}\left\{P\left(Q_D+Q_C,A\right)\left(Q_D+Q_C\right)-C_DQ_D-C_CQ_C\right.$$

$$\left.-\kappa\left(I_D+I_C\right)+\beta\left[pV_{BAU}\left(K'_D,K'_C,A'\right)+\left(1-p\right)V_{tax}\left(K'_D,K'_C,A'\right)\right]\right\}$$

s.t. $Q_D\leq K_D \qquad Q_C\leq K_C$

$$K'_D=K_D\left(1-\delta\right)+I_D \qquad K'_C=K_C\left(1-\delta\right)+I_C$$

$$A'=A+\Delta$$

$$V_{tax}\left(K_D.K_C,A\right)=Max_{Q_D,Q_C,I_D,I_C}\left\{P\left(Q_D+Q_C,A\right)\left(Q_D+Q_C\right)-C'_DQ_D-C_CQ_C\right.$$

$$\left.-\kappa\left(I_D+I_C\right)+\beta V_{tax}\left(K'_D,K'_C,A'\right)\right\}$$

s.t. $Q_D\leq K_D \qquad Q_C\leq K_C$

$$K'_D=K_D\left(1-\delta\right)+I_D \qquad K'_C=K_C\left(1-\delta\right)+I_C$$

$$A'=A+\Delta$$

The proof of part [iii] is based in the following. In the after tax stage, that is $t\geq t^*$, if $\tilde{C_C}<C'_D$ then the optimal solution is to invest only in clean technology. That is, because is better to invest in clean technology than use the dirty one installed. For part [iv], that is if $\tilde{C_C}>C'_D$, is better to use the actual dirty capacity installed, let this capacity depreciates (there is no incentive to invest), and let the marginal revenue raises until $\tilde{C_C}$, where it is optimal to start investing in clean technology.

The prof of part [ii] is based in the assumption $C_C < C'_D$ implies that $\tilde{C_C} < \tilde{C'_D}$. So, is cheaper to invest in clean technology than in dirty one after the change.

The proof of part [i], as well as the assumption that the assumption that before the change is better to invest in dirty technology, is based in the following. If there is no value of the marginal investment in dirty technology after the policy change, then the problem before is only a problem with dirty technology and a discount factor equal to βp.

Formally, if after the change the investor will produce using dirty technology, $\tilde{C_C} > \tilde{C'_D}$, then this investment will have a remanent value after the BAU stage. There is two possible cases:

- The remaining dirty capacity it is not enough to cover the market. This happens if:

$$K_D(t)(1-\delta) > \frac{A(t^*+1) - BC_C}{2} - \frac{\kappa B}{2}\left(\frac{1}{\beta} - (1-\delta)\right)$$

  In this case the marginal revenue in the market will be equal to $\tilde{C_C}$, so the remaining value of the marginal investment in dirty technology is based in this value.

- The remaining dirty capacity it is enough to cover the market. This happens if the previous inequality is reversed. In this case the marginal value will increase until reaches $\tilde{C_C}$, and then will stay at this level.

In both cases $\partial V_{tax}/\partial K_D > 0$, that is, there is value in the marginal investment in dirty technology.

If $\tilde{C_C} > \tilde{C'_D}$ then there is no value in the marginal investment in dirty technology. So, given this condition, the marginal effect of the dirty capacity in the BAU stage becomes simpler, because the marginal effect in the after tax stage is zero. So, under this condition and if the investor chooses the dirty technology before the tax, the optimal capacity and production in the BAU is:

$$K_D\left(t\right) = \tilde{K}\left(t, C_D, \beta p\right)$$

So, if the condition holds, the optimal path for $t \leq t^*$ is given by $\hat{K}\left(t, C_D, \beta p\right)$.

Bibliography

Andreoni, J. and L. Samuelson (2006) , "Building rational cooperation", Journal of Economic Theory 127, pp. 117-154.

Blonski, M. and D. Probst (2004) , "The Emergence of Trust", unpublished manuscript, Goethe University, Frankfurt am Main.

Calvin, K.; Edmonds, J.; Bond-Lamberty, B. ; Clarke, L.; Kim, S. H.; Kyle, P.; Smith, S. J.; Thomson, A. & Wise, M. (2009), "2.6: Limiting climate change to 450 ppm CO2 equivalent in the 21st century", Energy Economics, 31, pp 107-120.

Chisari, O. & Galiani, S. (2010) , "Climate Change: Notes towards the elaboration of a research agenda for the IADB", IDB Technical Note IDB-TN-164.

Coate, S. & Morris, S. (1999) , "Policy Persistence', American Economic Review, 89(5), pp 1327-1336.

Edmonds, J. A.; Clarke, L. E.; Wise, M. A. & Lurz, J. P. (2007) , "International CO2 Stabilization Scenarios with Less-Than-Perfect Where and When Flexibility", Technical report, Pacific Northwest National Laboratory.

Fernandez, R. & Rodrik, D. (1991) , "Resistance to Reform: Status Quo Bias in the Presence of Individual-Specific Uncertainty", American Economic Review, 81(5), pp 1146-1155.

Furusawa, T. and T. Kawakami (2008) , "Gradual cooperation in the existence of outside options", Journal of Economic Behavior & Organization 68, pp. 378-389.

Gerlagh, R.; Kverndokk, S. & Rosendahl, K. (2009) , "Optimal Timing of Climate Change Policy: Interaction Between Carbon Taxes and Innovation Externalities", Environmental & Resource Economics", 43(3), pp 369-390.

Golombek, R.; Greaker, M. & Hoel, M. (2010) , "Climate Policy without Commitment", Technical report, CESifo Group Munich.

Goulder, L. H. & Mathai, K. (2000) , "Optimal CO2 Abatement in the Presence of Induced Technological Change", Journal of Environmental Economics and Management", 39(1), pp 1-38.

Ghosh, P. and D. Ray (1996) , "Cooperation in community interaction without information flows", The Review of Economic Studies, Vol. 63, No. 3, pp. 491-519.

Ghosh, P. and D. Ray (2001) , "Information and enforcement in informal credit markets", working paper.

Halac, M. (2010) , "Relational Contracts and the Value of Relationships", Columbia University, American Economic Review forthcoming.

Hermalin, B. E. (2002) , "Adverse selection, short-term contracting, and the underprovision of on-the-job training", The BE Journal of Economic Analysis & Policy, Volume 1, Issue 1, Article 5.

Jaccard, M. & Rivers, N. (2007) , "Heterogeneous capital stocks and the optimal timing for CO2 abatement", Resource and Energy Economics, 29(1), pp 1-16.

Kydland, F. E. & Prescott, E. C. (1977) , "Rules Rather Than Discretion: The Inconsistency of Optimal Plans", Journal of Political Economy, 85(3), pp 473-91.

Lockwood, B. and J. Thomas (2002) , "Gradualism and irreversibility", The Review of Economic Studies, Vol. 69, No. 2, pp. 339-356.

Nordhaus, W. D. (2009) , "An Analysis of the Dismal Theorem", Technical report, Cowles Foundation for Research in Economics, Yale University.

Rauch, J. E. and J. Watson (2003) , "Starting small in an unfamiliar environment", International Journal of Industrial Organization, 21, pp. 1021-1042.

Requate, T. (2005) , "Timing and Commitment of Environmental Policy, Adoption of New Technology, and Repercussions on R&D", Environmental & Resource Economics, 31(2), pp 175-199.

Sannikov, Y. (2007) , "Agency Problems, Screening and Increasing Credit Lines", working paper.

Shalizi, Z. & Lecocq, F. (2009) , "Climate change and the economics of targeted mitigation in sectors with long-lived capital stock", Technical report, The World Bank.

Sinn, H.-W. (2008) , "Public policies against global warming: a supply side approach", International Tax and Public Finance, 15(4), pp 360-394.

Smulders, S.; Tsur, Y. & Zemel, A. (2010) , "Announcing Climate Policy: Can a Green Paradox Arise without Scarcity?", Technical report, CESifo Group Munich.

Sobel, J. (1985) , "A theory of credibility", The Review of Economic Studies, Vol. 52, No. 4, pp. 557-573.

Strand, J.; Miller, S. & Siddiqui, S. (2011) , "Infrastructure investments under uncertainty with the possibility of retrofit : theory and simulations", Technical report, The World Bank.

Thomas, J. and T. Worrall (1994) , "Foreign Direct Investment and the Risk of Expropriation", The Review of Economic Studies, Vol. 61, No. 1, pp. 81-108.

Watson, J. (1999) , "Starting small and renegotiation", Journal of Economic Theory 85, pp. 52-90

Watson, J. (2002) , "Starting small and commitment", Games and Economic Behavior 38, pp. 176-199

Webster, M. (2002) , "The Curious Role of "Learning" in Climate Policy: Should We Wait for More Data?", The Energy Journal, 23(2), pp 97-120.

Weitzman, M. L. (2009) , "On Modeling and Interpreting the Economics of Catastrophic Climate Change", The Review of Economics and Statistics, 91(1), pp 1-19.

Williams III, R.C. (2010) , "Setting the Initial Time-Profile of Climate Policy: The Economics of Environmental Policy Phase-Ins", NBER Working Paper Series No. 16120, June.