Essays in Business Cycles and International Finance

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Abstract

This dissertation studies financial flows within and across countries. Chapter 1 presents new evidence that gross foreign assets and liabilities in equity investments, measured at market value, are positively correlated over the business cycle in each of the Group of Seven industrialized countries (G7). The close comovement of assets and liabilities, in turn, reflects strong cross-country correlation between equity prices and moderate comovement of gross outflows and inflows. I analyze an international real business cycle (IRBC) model to evaluate possible causes of these correlations. A complete markets model with diminishing returns to capital predicts positive cross-country correlation between equity prices. I show that imperfect substitutability between goods strengthens this correlation, and I show that cross-border financial costs lead to negative correlation between gross capital outflows and inflows.

Chapter 2 seeks to explain why aggregate debt issued and equity payouts are procyclical and positively correlated over the business cycle in the U.S. I develop a real business cycle (RBC) model with an interest tax deduction and costly monitoring of firms, and I use the model to explain the dynamics of debt issued and equity payouts. The key insight is that the marginal benefit of issuing debt is constant, but the marginal cost varies over the business cycle. Economic booms both increase firms' optimal payouts and reduce the cost of borrowing by making firms appear more creditworthy.

Chapter 3 develops a two-country, two-good equilibrium endowment model in which asset trade is limited to two locally denominated real bonds. Unless the elasticity of substitution between goods is exceptionally low, the model predicts that each country will hold a short position in foreign bonds, which appears counterfactual for most advanced countries. I also consider an alternative arrangement in which only equities are traded. Under plausible assumptions, the set of parameters for which equity home bias attains (when only equities are traded) is the same set for which countries are long in foreign bonds (when only bonds are traded).

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Introduction

This dissertation focuses on three observations about aggregate U.S. financial data. First, gross foreign assets and liabilities in cross-border equity investments are strongly positively correlated over the business cycle. Second, aggregate debt issued and net equity payouts are positively correlated with output, positively correlated with investment, and positively correlated with each other. Third, foreign investors hold large amounts of U.S. bonds in their portfolios. Each chapter of my dissertation discusses one of these facts and analyzes possible causes using a dynamic stochastic general equilibrium (DSGE) model.

Chapter 1 explores the relationship between gross foreign assets and liabilities in cross-border equity investments. I present new evidence that gross equity-based foreign assets and liabilities, measured at market value, are positively correlated over the business cycle in the U.S. This finding extends to the other Group of Seven industrialized countries (G7) as well. I show that equity prices, rather than capital flows, are the larger factor behind the positive correlation between assets and liabilities – although equity outflows and inflows also co-move in the U.S. Next, I analyze an international real business cycle (IRBC) model to better understand the positive asset/liability correlation. A complete markets IRBC model performs well in explaining the cross-country correlation between equity prices. Diminishing returns to capital is key to this result: it generates capital spillovers in response to a positive, country-specific productivity shock. If goods are imperfect substitutes, then the terms of trade further strengthen the correlation between equity prices.

I then introduce capital flow dynamics by restricting the menu of financial assets to equity in home and foreign firms, subject to cross-border financial costs. I calibrate the model's steady-state to match the equity home bias of the U.S. This version of the model predicts that gross outflows and inflows should be *negatively* correlated, which is inconsistent with U.S. data. However, the positive correlation between equity prices dominates the negative correlation between capital flows, and gross assets and liabilities are positively correlated, as in the data.

Aggregate cross-border equity investment consists of foreign direct investment (FDI) and portfolio equity (PE). FDI, in turn, can be subdivided into mergers and acquisitions (MA) and greenfield investment – the formation of new fixed capital by foreign investors. I disaggregate gross equity-based capital flows for the G7 and find that PE outflows and inflows are weakly procyclical for most countries, while greenfield FDI outflows and inflows tend to be countercyclical. To explore these differences, I develop an IRBC model of greenfield FDI and PE. I introduce a novel menu of financial assets – claims to long-term and short-term projects in both countries. The model suggests that gross assets and liabilities should be more closely correlated in PE than in FDI.

Chapter 2 documents that when U.S. output is high, U.S. firms as a whole issue more debt, invest more, and pay more to shareholders. To better understand this behavior, I develop a real business cycle model with a novel corporate finance decision. Firms decide how much debt to issue by weighing the costs and benefits of additional debt. Interest payments are assumed to be tax deductible, creating an incentive to borrow. However, I also require bondholders to pay a monitoring cost that is increasing in the firm's debt-to-capital ratio. The tradeoff between the debt tax shield and monitoring costs determines the firm's optimal level of debt.

I calibrate the model to U.S. data for the postwar period 1952–2007. The model correctly predicts that debt issuance and equity payouts are procyclical, positively correlated with investment, and positively correlated with each other. The key insight is that the marginal benefit of issuing debt is constant, but the marginal cost varies over the business cycle. In a boom, the firm's capital stock grows. For any given level of debt, the firm's debt-to-capital ratio is lower, and the cost of borrowing is reduced. Firms therefore borrow more when output and payouts are high. In essence, booms reduce the cost of borrowing for firms by making them appear more creditworthy.

The third and final chapter extends the international diversification literature to bonds. I document that foreign holdings of domestic bonds are large in the U.S. and other advanced countries. I then develop a two-country, two-good equilibrium endowment model in which asset trade is limited to two locally denominated real bonds. I derive an analytical expression relating steady-state equilibrium bond portfolios to structural parameters governing risk aversion, substitutability between goods, and consumption home bias. For plausible parameterizations, the model predicts that countries will hold short positions in foreign bonds, which appears counterfactual.

I also analyze the same model under an alternative assumption: that asset trade is limited to two locally denominated equities. I show that the elasticity of substitution between goods plays a key role in determining whether countries exhibit equity home bias (in the equity regime) and go long in foreign bonds (in the bond regime). In particular, both results obtain if and only if the elasticity of substitution is below a common cutoff value that I compute.

Chapter 1: Cross-Border Equity Investment and the Business Cycle

1 Introduction

This chapter explores the relationship between gross foreign assets and liabilities in cross-border equity investments. A country's gross foreign assets will increase if (i) the market value of those assets rises (valuation effect), or (ii) the country buys more foreign assets (capital outflow). I present new evidence that gross equity-based foreign assets and liabilities, measured at market value, are positively correlated over the business cycle in each of the Group of Seven industrialized countries (G7). The close comovement of assets and liabilities, in turn, reflects strong cross-country correlation between equity prices and moderate comovement of gross outflows and inflows. These positive correlations lack an obvious explanation. In particular, it seems more natural to suppose that gross outflows and inflows are negatively correlated. If investors in different countries continually reallocate capital to countries with the highest expected, risk-adjusted returns, then capital should flow more quickly into – and less quickly out of – countries with the best investment prospects. Furthermore, if wealth effects are important, we would expect a "booming" country to buy up existing equity at home and abroad, leading to larger gross outflows and smaller gross inflows.

I analyze an international real business cycle (IRBC) model under several different assumptions to better understand the close correlation between gross foreign assets and liabilities. A conventional one good complete markets IRBC model performs well in explaining the cross-country correlation between equity prices. Diminishing returns to capital is key to this result: it generates capital spillovers in response to a positive productivity shock in one country. If goods are imperfect substitutes, then the terms of trade further amplify the correlation between equity prices by increasing the value of scarce foreign goods after a positive shock to the home country's productivity.

I then introduce capital flow dynamics by restricting the menu of financial assets to equity in home and foreign firms, subject to cross-border financial costs. I calibrate the model's steady-state to match the equity home bias of the U.S. The incomplete markets model predicts that gross outflows and inflows should be *negatively* correlated. Thus the positive outflow/inflow correlations in the data remain a puzzle. The negative correlation in the model reflects a wealth effect: home households own a disproportionately large share of home firms in the calibrated steady-state, so a positive shock to home firms raises home households' relative wealth. These households invest their extra wealth in both home and foreign equity. Because equity claims are in fixed supply, foreign households must sell both kinds of equity to home households, causing gross inflows to fall while gross outflows rise.

Aggregate cross-border equity investment consists of foreign direct investment (FDI) and portfolio equity (PE). FDI captures cross-border equity investments that result in a 10% or greater stake in the target company; smaller equity transactions are classified as portfolio equity. FDI, in turn, can be subdivided into mergers and acquisitions (MA) and greenfield investment – the formation of new fixed capital by foreign investors. Greenfield projects typically require a long startup phase before production begins. Examples include the development of new oil fields and the construction of new manufacturing plants. Financial investments in such projects are frequently "locked in" for a period of time before they produce any returns. In contrast, acquisitions can typically be unwound (albeit at a cost), and PE can be withdrawn at any point. Greenfield FDI is therefore likely to be a higher-risk asset class. If so, the business cycle properties of greenfield FDI may differ from those of MA and PE.

I disaggregate gross equity-based capital flows for the G7 and find that PE outflows and inflows are weakly procyclical for most countries, while greenfield FDI outflows and inflows tend to be countercyclical. To explore these differences, I develop an IRBC model of greenfield FDI and PE. I introduce a novel menu of financial assets – claims to long-term and short-term projects in both countries. The model suggests that gross assets and liabilities should be more closely correlated in PE than in FDI. However, the qualitative behavior of the two asset classes is similar. The model predicts procyclical gross outflows and countercyclical gross inflows for both types of equity. The subtle differences in cyclicality observed in the data remain a puzzle.

This chapter makes three contributions to the literature. First, on the empirical side, I present new stylized facts about the business cycle behavior of gross foreign assets and liabilities. To my knowledge, all previous business cycle studies of FDI and PE have looked at *flows*; I add an analysis of *positions* at market value and show how they differ from flows.[1] I also examine FDI at a finer granularity by analyzing merger and acquisition flows separately from greenfield FDI flows. Second, I show how different assumptions about preferences and market structure affect the business

[1] Recent business cycle studies of disaggregated capital flows include Contessi et al. (2009) and Smith and Valderrama (2009). See also Albuquerque (2003) and references therein. Lane and Milesi-Ferretti (2007), Gourinchas and Rey (2007), Tille and Van Wincoop (2007) and Devereux and Sutherland (2008) emphasize the importance of valuation effects in understanding foreign assets positions. However, they do not investigate the business cycle properties of these positions.

cycle properties of equity prices, capital flows and foreign asset positions in an IRBC framework. In doing so, I offer new insights into the mechanisms at work in one- and two-good open economy macro models.[2] Third, I develop a model of FDI in new projects and portfolio equity, and I show how the length of the underlying investment project affects the cross-country correlations of equity prices, flows and positions.

The model of FDI and PE contributes to a new but growing literature on the FDI-PE decision. Goldstein and Razin (2005) model a tradeoff between FDI and portfolio equity by introducing an information asymmetry between direct and arms-lengths investors. Albuquerque (2003) develops a small open economy (SOE) model to explain the relative volatilities of FDI and PE inflows to emerging countries. Smith and Valderrama (2009) present an SOE model in which FDI overcomes financial constraints in an emerging country but requires costly search on the part of investors. My main contribution here is to explicitly model *two-way* FDI and portfolio equity investment between similar countries, rather than one-way flows into an emerging country.

The rest of the chapter proceeds as follows. Section 2 presents stylized business cycle facts for cross-border equity investment in the G7. Section 3 presents the benchmark IRBC model and analyzes its predictions for equity prices, capital flows and foreign asset positions under several different assumptions about preferences and market structure. Section 4 develops a model of FDI and PE and analyzes its predictions for equity prices, flows and positions in each asset class. Section 5 offers some concluding remarks. A technical Appendix provides additional details on the

[2] Important examples of such models include (but are not limited to) Backus et al. (1992), Baxter and Crucini (1995), Stockman and Tesar (1995), Baxter and Jermann (1997), Heathcote and Perri (2002), and Heathcote and Perri (2008).

data and models.

2 Cross-border equity investment in the G7: Business cycle facts

This section documents the business cycle properties of equity-based gross foreign assets and liabilities in the G7 countries. I use a data set from Lane and Milesi-Ferretti (2007) that includes gross FDI and portfolio equity (PE) assets and liabilities at annual frequency for a large sample of countries from 1980 – 2004. An advantage of this data is that it measures positions at *market value*. The authors thus account for valuation effects as well as capital flows when constructing positions. Consider gross foreign assets for a given "home" country. Let P_t be a price index for foreign assets at the end of period t, and let A_t be a quantity index of assets held (e.g., number of equity shares) at the end of period t. The change in gross foreign assets can be decomposed as follows:

$$\Delta \text{Gross foreign assets}_t = P_t A_t - P_{t-1} A_{t-1}$$

$$= P_t (A_t - A_{t-1}) + A_{t-1}(P_t - P_{t-1})$$

$$= \text{Capital outflows} + \text{Valuation effect}$$

The first term captures the home country's net new purchases of foreign assets.[3] The second term captures the change in value of the country's prior holdings.

[3]For illustration, I have assumed that all purchases and sales of assets in period t take place at the end-of-period price P_t. A similar decomposition can be made when transactions take place at varying prices over the period; see Lane and Milesi-Ferretti (2007) for details.

Valuation effects in turn arise from two sources: changes in foreign asset prices and changes in exchange rates. Lane and Milesi-Ferretti (2007) show that valuation changes can have large effects on the market value of gross foreign assets and liabilities.

Table 1 (left side) presents business cycle correlations between gross foreign assets and liabilities based on the Lane and Milesi-Ferretti (2007) data set. The first column gives the correlations between total equity-based assets (FDI plus portfolio equity) and liabilities. The correlations are positive and significant for all seven countries.[4] The second column presents correlations between gross FDI assets and liabilities, and the third column presents correlations between gross portfolio equity assets and liabilities. Here FDI captures all cross-border equity investments that result in a 10% or greater stake in the target company. For FDI, gross assets and liabilities are positively correlated for six of the G7 countries, and five of the correlations are significant. (Japan is a notable exception.) For portfolio equity, all seven countries have positive correlations, and six are significant. Overall, gross equity-based assets and liabilities are closely correlated for most of the G7.

Are gross outflows and inflows positively correlated as well? I examine data on capital flows from the IMF's International Financial Statistics (for portfolio equity) and UNCTAD's World Investment Report (for FDI). The fourth column of Table 1 gives the correlation between total equity outflows (FDI plus portfolio equity) and inflows. For three countries (U.K., France and Canada), the outflow/inflow corre-

[4]The original data is in current U.S. dollars. I convert the nominal series to constant 2000 U.S. dollars using a world GDP deflator. I then remove a nonlinear trend using a Hodrick-Prescott filter with a smoothing parameter of 6.25, which Ravn and Uhlig (2002) suggest for annual data. Results using alternative filters (Baxter-King band-pass, Christiano-Fitzgerald random walk, time trend) are similar and available on request. "Significance" refers to a two-sided t-test of the null hypothesis that the correlation coefficient is zero. See the Appendix for details.

	eqa, eql	fda, fdl	pea, pel	eqo, eqi	fdo, fdi	peo, pei
U.S.	0.89*	0.87*	0.91*	0.64*	0.68*	0.27
U.K.	0.84*	0.31	0.84*	0.87*	0.86*	0.09
France	0.82*	0.57*	0.85*	0.86*	0.18	0.68*
Germany	0.86*	0.59*	0.73*	0.39	-0.12	-0.51*
Japan	0.50*	-0.55*	0.86*	-0.22	-0.30	-0.21
Canada	0.82*	0.71*	0.84*	0.82*	0.61*	0.57*
Italy	0.58*	0.69*	0.38	-0.04	0.26	-0.18

Table 1: **Business cycle correlations between gross equity-based foreign assets and liabilities: G7 countries, 1980 – 2004.** *eqa* is the stock of real gross FDI plus portfolio equity (PE) assets at market value, *eql* is the stock of real gross FDI plus PE liabilities at market value, fda is the stock of real gross FDI assets at market value, fdl is the stock of real gross FDI liabilities at market value, *pea* is the stock of real gross PE assets at market value, *pel* is the stock of real gross PE liabilities at market value, *eqo* is real gross equity outflows, *eqi* is real gross equity inflows, fdo is real gross FDI outflows, fdi is real gross FDI inflows, *peo* is real gross PE outflows, and *pei* is real gross PE inflows. See the Appendix for details. Stars denote significance at the 5% level of a two-sided t-test of the null hypothesis that the correlation coefficient is zero.

lation is about the same as the asset/liability correlation. For the other countries, outflows and inflows are less closely correlated. A similar pattern holds for FDI and portfolio equity separately (fifth and sixth columns).

The fact that assets and liabilities are more closely correlated than outflows and inflows suggests that valuation effects are very closely correlated. To confirm this, I construct time series for valuation effects using the data on positions from Lane and Milesi-Ferretti (2007) together with the data on flows from the IMF and UNCTAD.[5] Note that valuation effects simply reflect movements in equity prices, expressed in a common currency – in this case, the U.S. dollar. Table 2 gives the correlations between asset and liability valuation effects for total equity as well as for FDI and portfolio equity separately. The correlations are large and significant for most countries. Valuation effects do indeed co-move strongly over the business cycle. Assets and liabilities therefore co-move more strongly than outflows and inflows.

Do gross foreign assets and liabilities co-move positively or negatively with the home country's real GDP? Table 3 presents business cycle correlations between total equity positions and domestic output (left side) and between total equity flows and domestic output (right side). Gross equity-based assets and liabilities are weakly procyclical for most countries. The positive correlations with GDP are especially strong in the U.S., and they are weakest or absent for Japan and Italy. Equity outflows and inflows are also generally procyclical.

The analysis so far has treated FDI as a single type of investment. However, FDI consists of two conceptually distinct components: mergers and acquisitions

[5]Consider FDI assets as an example. For each year, I compute the change in FDI assets at market value from the end of the prior year to the end of the current year. I then subtract gross FDI outflows that took place during the year. The residual is the valuation effect: the capital gain or loss on the prior year's FDI assets. See the Appendix for details.

	$eqav, eqlv$	$fdav, fdlv$	$peav, pelv$
U.S.	0.94*	0.92*	0.92*
U.K.	0.76*	0.26	0.83*
France	0.79*	0.71*	0.92*
Germany	0.90*	0.46*	0.94*
Japan	0.44*	-0.77*	0.70*
Canada	0.93*	0.81*	0.82*
Italy	0.81*	0.76*	0.73*

Table 2: **Business cycle correlations between equity-based asset and liability valuation effects: G7 countries, 1980 − 2004.** *eqav* is the real capital gain on gross FDI plus portfolio equity (PE) assets, *eqlv* is the real capital gain on gross FDI plus PE liabilities, *fdav* is the real capital gain on gross FDI assets, *fdlv* is the real capital gain on gross FDI liabilities, *peav* is the real capital gain on gross PE assets, and *pelv* is the real capital gain on gross PE liabilities. See the Appendix for details. Stars denote significance at the 5% level of a two-sided t-test of the null hypothesis that the correlation coefficient is zero.

	eqa, y	eql, y	eqo, y	eqi, y
U.S.	0.55*	0.51*	0.45*	0.62*
U.K.	0.25	0.25	0.28	0.19
France	0.20	0.12	0.49*	0.38
Germany	0.31	0.16	0.20	0.32
Japan	0.06	-0.22	0.01	-0.03
Canada	0.34	0.36	0.29	0.35
Italy	0.02	-0.14	0.03	0.19

Table 3: **Business cycle correlations of equity-based positions and flows with domestic output: G7 countries, 1980 − 2004.** *eqa* is the stock of real gross FDI plus portfolio equity (PE) assets at market value, *eql* is the stock of real gross FDI plus PE liabilities at market value, *eqo* is real gross equity outflows, *eqi* is real gross equity inflows, and y is real GDP. See the Appendix for details. Stars denote significance at the 5% level of a two-sided t-test of the null hypothesis that the correlation coefficient is zero.

(MA) and greenfield investment. A merger or acquisition involves an investor in one country acquiring a lasting interest in an existing foreign firm.[6] In contrast, greenfield FDI describes an investor in one country starting a new firm in a foreign country. Do MA and greenfield FDI behave similarly over the business cycle? To address this question, I use detailed FDI data from UNCTAD's World Investment Report from 1987 – 2004. For this time horizon, UNCTAD measures both total FDI outflows and inflows as well as MA outflows and inflows. Following Calderón et al. (2004), I obtain a rough proxy for greenfield flows by subtracting MA from total flows.[7]

Table 4 presents business cycle correlations between MA outflows and inflows (left column) and between greenfield FDI outflows and inflows (right column). (Data on MA and greenfield FDI positions at market value is not available.) Most of the outflow/inflow correlations are positive for both types of FDI. The positive correlations are especially strong in the U.S., U.K. and Canada; they are weaker or absent in Germany and Japan. Overall, outflows and inflows do co-move in both categories for most of the G7.

Table 5 presents business cycle correlations between MA flows and output, between greenfield FDI flows and output, and between PE flows and output. MA outflows and inflows are both procyclical for most of the G7; the only exception is MA inflows for Japan. The pattern is less clear-cut for the other types of equity. Greenfield FDI outflows and inflows are each weakly countercyclical for four out of

[6] A "lasting interest" is typically defined as ownership of 10% or more of a firm's outstanding equity. See the Appendix for details.

[7] The proxy for greenfield flows is imperfect because total FDI flows include a third component: financial transactions between parents and foreign subsidiaries. Some, but not all, of these transactions reflect new capital investment in the foreign country. Despite the limitations, this approach provides a useful first pass at separating greenfield FDI from mergers and acquisitions.

	mao, mai	gro, gri
U.S.	0.86*	0.79*
U.K.	0.92*	0.61*
France	0.60*	0.38
Germany	-0.10	-0.02
Japan	0.47*	0.00
Canada	0.61*	0.49*
Italy	0.22	0.70*

Table 4: **Business cycle correlations between gross outflows and inflows in mergers and acquisitions (MA) and greenfield FDI: G7 countries, 1987 – 2004.** *mao* is real gross MA outflows, *mai* is real gross MA inflows, *gro* is real gross greenfield FDI outflows and *gri* is real gross greenfield FDI inflows. See the Appendix for details. Stars denote significance at the 5% level of a two-sided t-test of the null hypothesis that the correlation coefficient is zero.

	mao, y	mai, y	gro, y	gri, y	peo, y	pei, y
U.S.	0.73*	0.83*	0.08	0.16	0.17	0.40*
U.K.	0.26	0.39	-0.25	-0.32	0.22	0.10
France	0.58*	0.21	-0.16	0.16	0.04	0.21
Germany	0.20	0.37	-0.31	-0.40	0.26	-0.10
Japan	0.41	-0.10	0.24	-0.06	-0.30	-0.02
Canada	0.30	0.53*	-0.22	-0.34	0.23	0.11
Italy	0.28	0.03	0.15	0.21	-0.06	0.05

Table 5: **Business cycle correlations between disaggregated gross capital flows and output: G7 countries, 1987 – 2004.** *mao* is real gross merger and acquisition (MA) outflows, *mai* is real gross MA inflows, *gro* is real gross greenfield FDI outflows, *gri* is real gross greenfield FDI inflows, *peo* is real gross PE outflows, *pei* is real gross PE inflows, and y is real GDP. See the Appendix for details. Stars denote significance at the 5% level of a two-sided t-test of the null hypothesis that the correlation coefficient is zero.

seven countries, while PE flows are weakly procyclical for a majority of countries. Overall, MA is the most procyclical type of equity, and greenfield FDI is the most likely to be countercylical. Studies that focus on aggregate FDI flows will likely miss this subtle distinction.

To summarize the main findings from this section:

1. Gross equity-based assets and liabilities are closely correlated in most of the G7 countries.

2. The correlations between asset and liability valuation effects are generally stronger than the correlations between gross capital outflows and inflows.

3. Total gross positions and flows are procyclical for most of the G7. Merger and acquisition (MA) flows are the most procyclical, while greenfield FDI flows are weakly countercyclical for a majority of countries.

4. Gross MA outflows and inflows are positively correlated in most of the G7, as are gross greenfield FDI outflows and inflows.

3 Benchmark model

What economic channels can account for the close correlation between gross equity-based foreign assets and liabilities in the G7? The preceding analysis reveals two proximate causes. First, valuation effects are closely correlated across countries. Second, gross outflows and inflows are positively correlated in many countries. For most of the G7, the correlation between home and foreign valuation effects is stronger than the correlation between outflows and inflows.

To think about underlying causes, we need a model. An international real business cycle (IRBC) framework is a natural starting point, as it has been shown to be broadly consistent with a large set of international business cycle facts (Backus et al. (1992), Heathcote and Perri (2008)). I analyze an IRBC model under several different assumptions about preferences and market structure. The next section lays out the benchmark model.

3.1 Model

The model features two countries and two traded goods. Perfectly competitive firms in each country produce country-specific output goods using country-specific physical capital. The production function has diminishing returns to capital. Production is also subject to country-specific aggregate productivity shocks each period. These shocks are the only sources of uncertainty in the model. Each period, firms choose how to allocate their output between dividends to shareholders and new investment. Households in each country like to consume bundles of home and foreign output goods, with a possible bias toward domestic goods. Households also trade equity shares in home and foreign firms. An equity share entitles its owner to a fraction of the firm's dividend for as long as the household owns the share.

3.1.1 Households

Each country is populated with a continuum of identical households. Households in the home country have the following preferences:

$$E_t\left[\sum_{j=0}^{\infty}\beta^j\frac{C_{t+j}^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor and $\gamma > 0$ is the constant coefficient of relative risk aversion. C_t denotes home households' consumption of home consumption goods, which are a CES aggregate of home and foreign output goods:

$$C_t = \left[\lambda^{\frac{1}{\phi}}\left(C_t^H\right)^{\frac{\phi-1}{\phi}} + (1-\lambda)^{\frac{1}{\phi}}\left(C_t^F\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{2}$$

Here C_t^H and C_t^F denote the home household's consumption of home and foreign output goods, $\lambda \in (0,1)$ is the weight that home households assign to home output goods, and $\phi > 0$ is the elasticity of substitution between home and foreign output goods. Let P_t^H and P_t^F denote the prices of home and foreign output goods in terms of a numeraire (to be defined shortly). Then the numeraire price indices of home and foreign consumption goods, denoted P_t and $\widehat{P}_t$, are as follows:

$$P_t = \left[\lambda\left(P_t^H\right)^{1-\phi} + (1-\lambda)\left(P_t^F\right)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{3}$$

$$\widehat{P}_t = \left[(1-\lambda)\left(P_t^H\right)^{1-\phi} + \lambda\left(P_t^F\right)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{4}$$

I take the numeraire to be the geometric mean of the home and foreign consumption price indices:[8]

$$P_t^{\frac{1}{2}} \widehat{P}_t^{\frac{1}{2}} = 1 \tag{5}$$

Recall that a household can hold equity shares in home and foreign firms. I assume that it is costly for home (foreign) households to deviate from their long-run holdings of foreign (home) equity. The home household's budget constraint is:

$$\begin{aligned} P_t C_t + P_t^{X,H} A_t^H + P_t^{X,F} A_t^F = \\ (P_t^{X,H} + D_t^H) A_{t-1}^H + (P_t^{X,F} + D_t^F) A_{t-1}^F - \frac{\psi}{2} P_t^F \left(A_t^F - A^F \right)^2 \end{aligned} \tag{6}$$

where A_t^H and A_t^F are the number of shares of home and foreign equity held by the home household at the end of period t; D_t^H and D_t^F are the total dividends paid by home and foreign firms, measured in units of the numeraire; $P_t^{X,H}$ and $P_t^{X,F}$ are the numeraire prices of home and foreign equity; and A^F is the long-run number of foreign equity shares held by the home household (i.e., the steady-state value of A_t^F). The parameter ψ controls the magnitude of the cross-border financial cost, which is paid in units of foreign output goods. The household's problem is to maximize (1) subject to (2) and (6), taking all prices as given.

The foreign household faces a mirror-symmetric problem; the Appendix gives details. The first-order conditions for home and foreign households' equity holdings are:

[8] This choice of numeraire is convenient for exposition, as it imposes symmetry. However, the choice of numeraire does not affect the results.

$$1 = E_t \left[M_{t+1,t} R^{X,H}_{t+1} \right] \tag{7}$$

$$1 = E_t \left[M_{t+1,t} R^{*X,F}_{t+1} \right] \tag{8}$$

$$1 = E_t \left[\widehat{M}_{t+1,t} R^{*X,H}_{t+1} \right] \tag{9}$$

$$1 = E_t \left[\widehat{M}_{t+1,t} R^{X,F}_{t+1} \right] \tag{10}$$

$$\text{where } M_{t+1,t} \equiv \frac{\beta C_{t+1}^{-\gamma} P_{t+1}^{-1}}{C_t^{-\gamma} P_t^{-1}} \tag{11}$$

$$\text{and } \widehat{M}_{t+1,t} \equiv \frac{\beta \widehat{C}_{t+1}^{-\gamma} \widehat{P}_{t+1}^{-1}}{\widehat{C}_t^{-\gamma} \widehat{P}_t^{-1}} \tag{12}$$

$M_{t+1,t}$ is the home household's intertemporal marginal rate of substitution (IMRS) of time-$(t+1)$ numeraire goods for time-t numeraire goods, and $\widehat{M}_{t+1,t}$ is the foreign household's IMRS. The equity returns are given by:

$$R^{X,H}_{t+1} \equiv \frac{P^{X,H}_{t+1} + D^{X,H}_{t+1}}{P^{X,H}_t} \tag{13}$$

$$R^{*X,F}_{t+1} \equiv \frac{P^{X,F}_{t+1} + D^{F}_{t+1}}{P^{X,F}_t + \psi P^F_t \left(A^F_t - A^F \right)} \tag{14}$$

$$R^{*X,H}_{t+1} \equiv \frac{P^{X,H}_{t+1} + D^{H}_{t+1}}{P^{X,H}_t + \psi P^H_t \left(\widehat{A}^H_t - \widehat{A}^H \right)} \tag{15}$$

$$R^{X,F}_{t+1} \equiv \frac{P^{X,F}_{t+1} + D^{F}_{t+1}}{P^{X,F}_t} \tag{16}$$

Here $\widehat{A}^H_t$ is the number of shares of home equity held by the foreign household at the end of period t, and $\widehat{A}^H$ is the long-run (steady-state) value of $\widehat{A}^H_t$. $R^{*X,F}_{t+1}$ and $R^{*X,H}_{t+1}$ are the *effective* returns on foreign and home equity to home and foreign households respectively – that is, the returns adjusted for marginal cross-border financial costs. Note that when the home household holds more than the steady-state level of foreign equity, an additional purchase of foreign equity increases cross-border costs, depressing the effective return $R^{*X,F}_{t+1}$. Conversely, when the home household holds less than the steady-state level of foreign equity, an additional purchase of foreign equity decreases cross-border costs (by bringing the position closer to the steady-state), enhancing the effective return. Analogous reasoning applies to foreign holdings of home equity.

3.1.2 Firms

Each country is populated with a continuum of perfectly competitive firms that produce country-specific output goods using country-specific physical capital. The production function for home firms is:

$$Y^H_t = Z^H_t \left(K^H_{t-1}\right)^{\theta} \ , 0 < \theta < 1 \tag{17}$$

Here K^H_{t-1} is the home firm's capital stock at the end of period $t-1$, available for production in period t; Y^H_t is the output produced by the home firm; and Z^H_t is an aggregate productivity shock affecting all firms operating in the home country. Firms choose capital levels and dividends each period to maximize the expected present discounted value of dividends to shareholders, discounted using the domestic household's IMRS. Formally, home firms solve the following problem:

$$\max_{K_t^H, D_t^H} D_t^H + \sum_{j=1}^{\infty} M_{t+j,t} D_{t+j}^H$$
$$\text{s.t. } D_t^H = P_t^H \left[Y_t^H + (1-\delta)K_{t-1}^H - K_t^H\right] \tag{18}$$

D_t^H denotes total dividends paid by home firms to their shareholders, measured in units of the numeraire; $M_{t+j,t}$ is the home household's intertemporal marginal rate of substitution (IMRS) of time-$(t+j)$ numeraire goods for time-t numeraire goods; and $\delta \in [0,1]$ is the depreciation rate. Note that K_t^H and Y_t^H are measured in units of home output goods. The implicit assumption is that home output goods can be converted one-for-one to home investment goods (and vice versa).[9] The home firm's first-order condition is:

$$1 = E_t \left[M_{t+1,t} \frac{P_{t+1}^H}{P_t^H} \left(\frac{\theta Y_{t+1}^H}{K_t^H} + 1 - \delta \right) \right] \tag{19}$$

3.1.3 Market clearing

Market clearing for output goods requires:

$$K_t^H - (1-\delta)K_{t-1}^H + C_t^H + \widehat{C}_t^H = Y_t^H - \frac{\psi}{2}\left(\widehat{A}_t^H - \widehat{A}^H\right)^2 \tag{20}$$
$$K_t^F - (1-\delta)K_{t-1}^F + C_t^F + \widehat{C}_t^F = Y_t^F - \frac{\psi}{2}\left(A_t^F - A^F\right)^2 \tag{21}$$

[9] It would be straightforward to relax this assumption, either by introducing capital adjustment costs or by requiring that investment goods be composites of home and foreign output goods.

I model the financial costs as "iceberg" costs; that is, no agent derives any benefit from them. I normalize the supply of equity shares in home and foreign firms to one:

$$1 = A_t^H + \widehat{A}_t^H \tag{22}$$

$$1 = A_t^F + \widehat{A}_t^F \tag{23}$$

Note that with this normalization, we can interpret A_t^F as the share of outstanding foreign equity held by home households and $\widehat{A}_t^H$ as the share of outstanding home equity held by foreign households. I will refer to A_t^F and $\widehat{A}_t^H$ as the *cross-border ownership shares*.

3.1.4 Shock processes

The shock vector s_t for this economy is the pair of log productivity shocks $(\ln Z_t^H, \ln Z_t^F)'$. I close the model by specifying a stochastic process for the shock vector:

$$s_t = \rho s_{t-1} + \epsilon_t \ , \ 0 \leq \rho < 1 \tag{24}$$

where $\epsilon_t \equiv (\epsilon_t^H, \epsilon_t^F)'$ is a vector of mean-zero iid innovations with variance-covariance matrix Σ:

$$\Sigma \equiv \begin{bmatrix} \sigma_H^2 & \xi\sigma_H\sigma_F \\ \xi\sigma_H\sigma_F & \sigma_F^2 \end{bmatrix} \tag{25}$$

3.1.5 Equilibrium

A competitive equilibrium is a sequence of prices and allocations such that all markets clear when consumers and firms behave optimally, taking equilibrium prices as given. The Appendix gives more details on the foreign household's problem, first-order conditions, and the solution method.

3.2 A complete markets version with one good

I start by analyzing a frictionless, complete markets version of the benchmark model with one good. I calibrate the model with the U.S. as the home country and the "rest of the world" (ROW) as the foreign country. Assume (for now) that in addition to trading equities, households in both countries can trade a complete set of state-contingent claims. It is then straightforward to show that:

$$M_{t+1,t} = \widehat{M}_{t+1,t} \tag{26}$$

This is the Backus-Smith condition: with complete markets, the IMRS of home and foreign households must equalize in all dates and states. For this exercise, I also set $\psi = 0$, so there are no financial frictions associated with equities. To make the two good model effectively have one good, I set the elasticity of substitution between home and foreign output goods, ϕ, arbitrarily high.[10]

The remaining parameters are calibrated as follows. I set the annual subjective discount factor β to 0.95, corresponding to a steady-state real interest rate of about 5%. The coefficient of relative risk aversion γ is set to 2, a common value in the literature. I set the production function parameter θ to 0.3 and the depreciation rate δ to 0.08.[11] I set ρ to 0.8, and I calibrate σ_H and σ_F to 0.0087 to match the standard deviation of U.S. output. (The standard deviation of global output is similar.) I set the correlation coefficient between home and foreign technology shocks ξ to zero in order to demonstrate the model's ability to generate positive correlation between equity prices without forcing the two countries' output processes to move together. Note that with $\psi = 0$, the parameters A^F and $\widehat{A}^H$ do not enter any of the model's equations and therefore do not need to be calibrated.

Figure 1 (dashed lines) presents impulse responses of capital stocks, dividends and equity prices to a positive, one standard deviation shock to home country technology. In response to the shock, the capital stock of home firms rises on impact, while the capital stock of foreign firms falls. With complete markets, we can interpret the results in terms of a social planner who maximizes the joint welfare of home and foreign households. The social planner invests physical capital so as to equalize the expected marginal products of capital (MPK) across the two countries. With diminishing returns to capital, equalization of expected MPKs requires an increase in the capital stock of home firms following a positive shock to home technology. For this calibration, the planner must also decrease the capital stock of foreign firms

[10] Alternatively, one can rewrite the model explicitly in terms of one good. I have verified that the two approaches give exactly the same results.

[11] There is no labor in the model. However, if I added labor to the production function in a Cobb-Douglas way, θ would be the share of capital in income.

on impact – effectively transferring capital from foreign to home firms. Starting next period, if there are no more shocks, expected technology in home firms decays relative to foreign firms, and the planner gradually transfers capital back from home to foreign firms. Note that the technology shock shifts current and future income up for both countries (thanks to complete risk-sharing). Since households like to smooth consumption, and since home and foreign capital are the only storage technologies, equilibrium requires a higher *global* capital stock than in the steady-state. As the home technology shock decays, some of this "excess" capital accrues to the foreign country, and the foreign capital stock rises above its steady-state level.

On impact, the home firm dividend falls, as home firms conserve on payouts in order to boost their capital stock. Starting next period, however, the home firm dividend rises and remains above steady-state for many periods. This reflects persistently higher output from home firms, which in turn results both from better technology and a larger capital stock. The foreign dividend must rise on impact to allow the foreign capital stock to fall. Starting next period, the foreign dividend drops below its steady-state value and stays low for several periods, reflecting low levels of foreign capital (and unchanged foreign technology). However, as with foreign capital, the foreign dividend eventually recovers and surpasses its steady-state value.

The home and foreign equity prices capture the expected present discounted value of *future* home and foreign dividends, respectively (starting with next period's dividend). The home equity price clearly must rise on impact, because the home firm dividend is always above its steady-state value starting next period. Whether the foreign equity price goes up or down on impact depends on the relative strength of the early decline in dividends (starting next period) versus the eventual rise. It

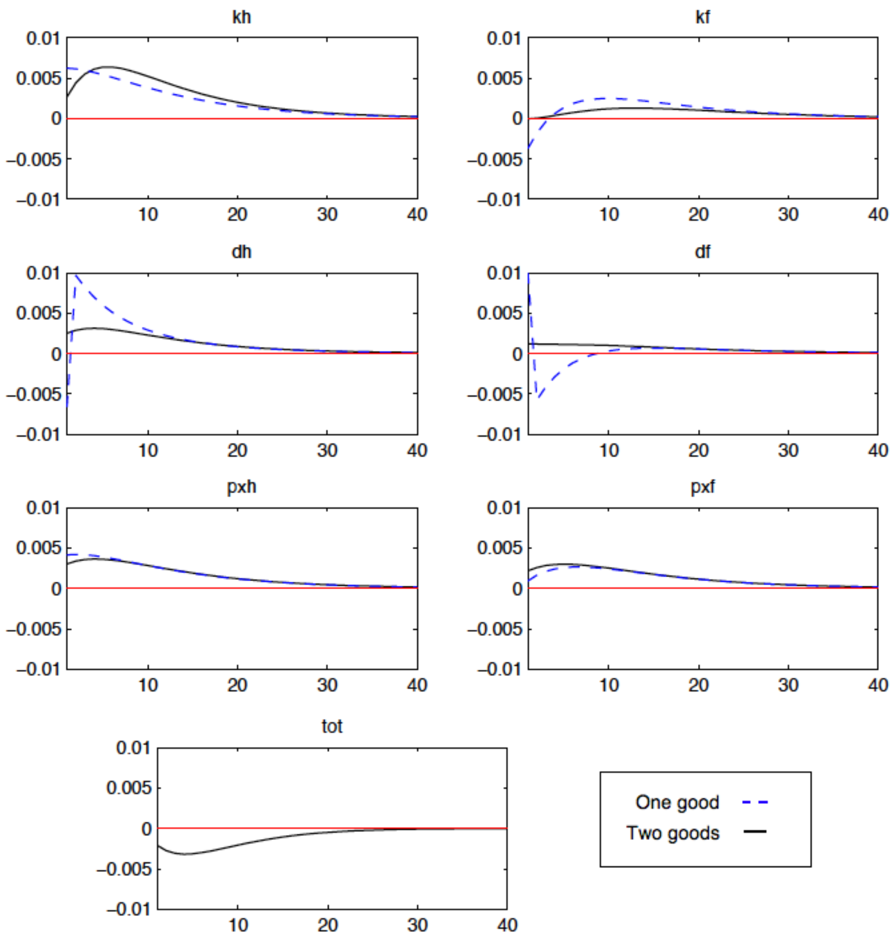

Figure 1: **Impulse responses to a positive, one standard deviation shock to home country technology: complete markets models.** The dashed lines are impulse responses in the one good complete markets version, and the solid lines are responses in the two good complete markets version. kh is the capital stock of home firms, kf is the capital stock of foreign firms, dh is dividends paid by home firms, df is dividends paid by foreign firms, pxh is the price of an equity share in home firms, pxf is the price of an equity share in foreign firms, and tot is the terms of trade (the price of home output goods divided by the price of foreign output goods – two good model only).

also depends on the discount factor. For the calibrated model, the foreign equity price rises on impact, and equity prices are positively correlated across countries. The correlation coefficient (0.89) is quite high.

3.3 A complete markets version with two goods

Next I analyze a complete markets version of the model in which output goods are imperfect substitutes. For this calibration, I set the elasticity of substitution between home and foreign output goods, ϕ, to 2. Estimates of ϕ vary widely. However, Coeurdacier (2009) argues that this elasticity should be greater than unity; otherwise, a model with internationally traded equities would suffer immiserizing growth. I set the weight of home (foreign) output goods in the home (foreign) consumption bundle, λ, to 0.75, for a steady-state import share of 25%. The remaining parameters are calibrated as in the one good complete markets model (Section 3.2).

Figure 1 (solid lines) presents impulse responses of capital stocks, dividends and equity prices to a positive, one standard deviation shock to home country technology in the two good model. In contrast with the one good model (dashed lines), capital in foreign firms does not fall on impact. Recall that home and foreign capital stocks are measured in terms of home and foreign output goods, while dividends and equity prices are measured in terms of the numeraire. The social planner now seeks to equalize expected marginal products of capital (MPK) *in terms of the numeraire.* The planner's optimal capital allocation thus depends not only on expected technology and capital levels but also on relative goods prices. The home technology shock causes a boom in home output goods, which raises the relative price of foreign output goods. The terms of trade – the price of home output goods divided by the price of foreign output goods – fall, as shown in the bottom-most impulse response.[12] Since the shock is persistent, the excess of home goods is expected to persist, and the terms of trade are expected to remain below their steady-state level. This raises the expected numeraire-based MPK of foreign firms relative to the one good case, prompting the planner to allocate more capital to foreign firms on impact.

Dividends paid by foreign firms increase on impact because of the rise in the price of foreign goods. Foreign dividends stay high, both because the foreign goods price stays high and because the planner invests more capital in foreign firms over time. Home capital and dividends rise on impact and stay high as a direct result of the technology shock. Since home and foreign dividends rise on impact and remain high, both equity prices also rise on impact. For this calibration, equity prices are even more closely correlated across countries (0.99) than in the one good complete markets model (0.89).

3.4 Benchmark model with financial costs and two goods

The complete markets model demonstrates that equity prices can be very closely correlated across countries when capital is allocated so as to ensure perfect risk-sharing. However, the complete markets model is less informative about asset ownership. As long as households can trade a complete set of state-contingent claims, the cross-border equity ownership shares are indeterminate. One approach is to prohibit households from trading state-contingent claims, but keep trade in equities frictionless ($\psi = 0$). It is then possible to solve for cross-border ownership shares in the "near-nonstochastic steady-state" using the method of Devereux and Sutherland

[12] In the one good model, the terms of trade are constant, because both countries produce identical goods.

(2008).[13] However, the steady-state ownership shares predicted by the frictionless model imply much more diversification than we see in the data. This is the well known "equity home bias" puzzle. For example, the calibrated two good model predicts that foreigners will own 70% of home equity in the steady-state. This vastly overstates the average foreign ownership share of U.S. equity from 1980 to 2004, which was about 7%.

My goal is not to solve the equity home bias puzzle; rather, I want to analyze how equity holdings respond to shocks when average holdings resemble what we see in the data. To accomplish this, I first drop the set of complete state-contingent claims, retaining equities in home and foreign firms as the only assets traded. I then "turn on" the cross-border financial costs by setting $\psi > 0$. Recall that this imposes a small cost on home (foreign) households to being away from their long-run holdings of foreign (home) equity. The key is that the steady-state cross-border ownership shares are calibrated to reflect the "underdiversification" observed in the real world, rather than the values that would effectively complete markets. This approach generates first-order dynamics in ownership shares around a unique, locally stationary steady-state.

When $\psi > 0$, and the steady-state ownership shares exhibit equity home bias, complete risk-sharing no longer obtains. That is, the home and foreign IMRS's need not be equal. For example, consider a positive productivity shock in the home country at time t. This raises the time-t return on home equity relative to foreign equity. Because home households own a disproportionate share of home

[13] For the symmetric model presented here, the steady-state cross-border ownership shares predicted using the method of Devereux and Sutherland (2008) deliver perfect risk-sharing when $\psi = 0$, to a first-order approximation. As a result, these shares can also be derived by decentralizing the complete markets solution, as in Kollmann (2006) and Heathcote and Perri (2008).

equity in the steady-state, home's income rises relative to foreign income. Home households consume part of this excess income, leading to a low ex-post IMRS ($M_{t-1,t}$). Intuitively, financial costs associated with holding foreign assets render the complete risk-sharing portfolio suboptimal. If ψ were equal to zero, there would be no reason to bias steady-state portfolios towards home equities. In that case, the Devereux and Sutherland (2008) steady-state portfolios would generate complete risk-sharing (to a first-order approximation).

There are three new parameters to be calibrated: the steady-state share of U.S. equity held by foreigners ($\widehat{A}^H$), the steady-state share of foreign equity held by the U.S. (A^F), and the cross-border cost parameter ψ. To get a sense of $\widehat{A}^H$, I use data on U.S. equities outstanding from the Federal Reserve Flow of Funds. For each year (1980 – 2004), I compute the ratio of U.S. portfolio equity liabilities (from Lane and Milesi-Ferretti (2007)) to U.S. equities outstanding. The average value of this ratio is 0.07. To get a sense of A^F, I use data from the World Federation of Exchanges (WFE) to compute annual ratios of U.S. portfolio equity assets to foreign equities outstanding (1990 – 2004).[14] The average value is 0.09. To keep the model symmetric, I set $\widehat{A}^H = A^F = 0.08$. That is, 8% of each country's equity is held by the other country's households in the steady-state.[15]

Recall that the parameter ψ controls the magnitude of the cross-border financial costs. I follow Schmitt-Grohé and Uribe (2003) and calibrate ψ to match the standard deviation of the U.S. current-account-to-GDP ratio (0.011). The calibrated value for ψ is 12. I report results for the two good model only ($\phi = 2, \lambda = 0.75$). The remaining parameters are calibrated as in Section 3.2.

[14] Data from the WFE is only available starting in 1990.

[15] Results are very similar if I set $\widehat{A}^H = 0.07$ and $A^F = 0.09$.

Figure 2 presents impulse responses of capital stocks, dividends and equity prices to a positive, one standard deviation shock to home country technology in the benchmark model with financial costs (solid lines). For comparison, I have also plotted the responses from the two good complete markets model (dashed lines). The dynamics of these variables are very similar across the two models. However, the path of the home equity price is slightly higher – and the path of the foreign equity price slightly lower – in the model with financial costs. This makes home and foreign equity prices somewhat less closely correlated (0.71) than they were under complete markets (0.99). The last cell in Figure 2 shows that complete risk-sharing breaks down in the benchmark model. As mentioned above, after a positive technology shock in the home country, home's ex-post IMRS temporarily drops below the foreign IMRS, reflecting unexpectedly high consumption in the home country.

Figure 3 presents impulse responses of consumption, cross-border ownership shares, and numeraire valued cross-holdings. The top two panels show that the gap between home and foreign consumption is larger in the benchmark model than in the two good complete markets model.[16] This is a result of incomplete risk-sharing. After a positive, persistent technology shock in the home country, the return on home equity exceeds that on foreign equity. This disproportionately benefits home households, since they own the larger stake in home firms. Home households enjoy greater lifetime wealth and a higher consumption path. The middle-left impulse response shows that home households gradually increase their ownership share of

[16]In the two good complete markets model, the impulse responses of home and foreign consumption are not identical because of preferences: home and foreign consumption goods are not the same. In the one good complete markets model, the home and foreign consumption responses are identical.

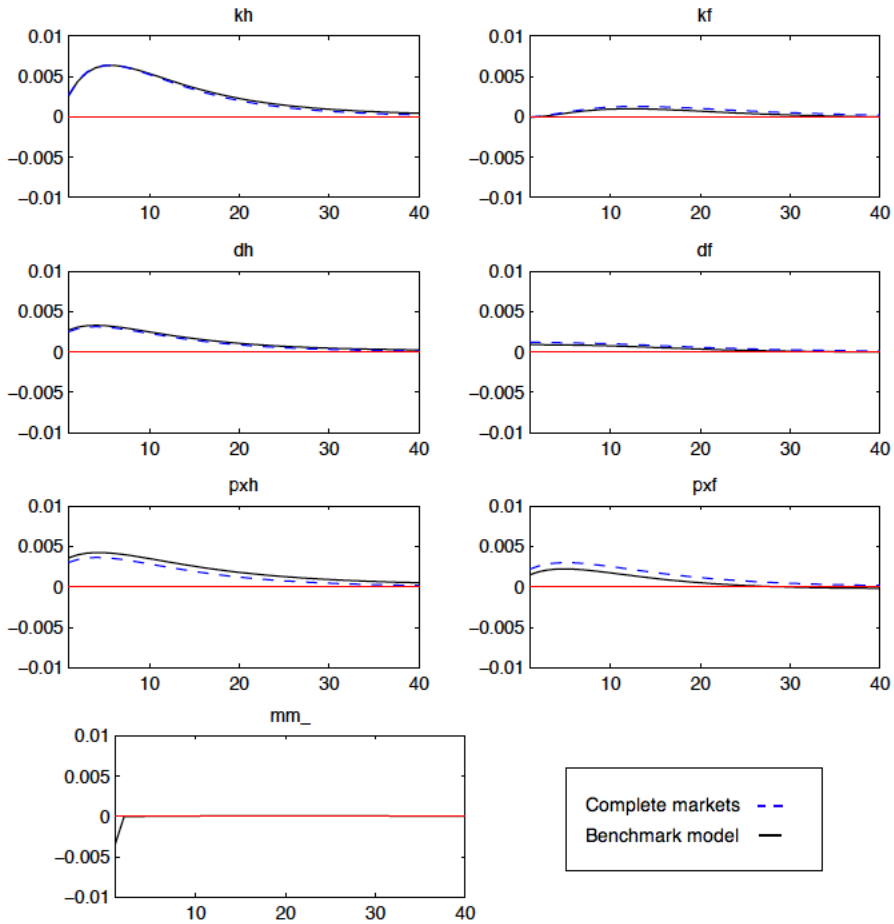

Figure 2: **Impulse responses to a positive, one standard deviation shock to home country technology: two good benchmark model versus two good complete markets model.** The dashed lines are impulse responses in the two good complete markets version, and the solid lines are impulse responses in the benchmark model with financial costs. kh is the capital stock of home firms, kf is the capital stock of foreign firms, dh is dividends paid by home firms, df is dividends paid by foreign firms, pxh is the price of an equity share in home firms, pxf is the price of an equity share in foreign firms, and $mm\_$ is the ratio of the home to foreign IMRS (benchmark model only).

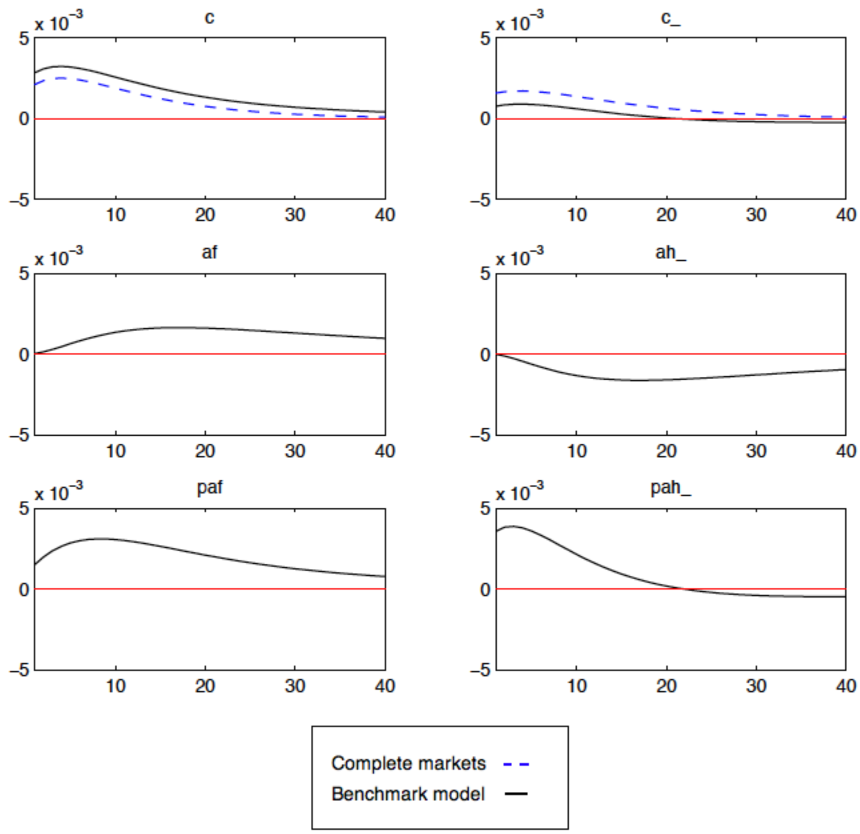

Figure 3: **Impulse responses to a positive, one standard deviation shock to home country technology: two good benchmark model versus two good complete markets model.** The dashed lines are impulse responses in the two good complete markets version, and the solid lines are impulse responses in the benchmark model with financial costs. c is the home household's consumption of home consumption goods, c_{-} is the foreign household's consumption of foreign consumption goods, af is the home household's ownership share of foreign equity (benchmark model only), ah_{-} is the foreign household's ownership share of home equity (benchmark model only), paf is the home household's numeraire-valued holdings of foreign equity ("gross foreign assets" – benchmark model only), and pah_{-} is the foreign household's numeraire-valued holdings of home equity ("gross foreign liabilities" – benchmark model only).

foreign equity.[17] Therefore, there are positive capital outflows from the home country (increased foreign assets) for several periods following the shock. However, home households also increase their ownership share of *home* equity. Since equity is in fixed supply, foreign households must sell home equity to home households. This corresponds to *negative* capital inflows (decreased foreign liabilities) for several periods from the perspective of the home country, as shown in the middle-right panel. Eventually, the financial costs cause cross-border ownership shares to return to their steady-state levels.

It follows from the discussion above that each country's gross capital outflows are negatively correlated with gross capital inflows. How does this impact gross foreign assets and liabilities? The bottom two panels of Figure 3 show the impulse responses of the home country's gross foreign assets (paf) and gross foreign liabilities (pah_{-}). Both assets and liabilities rise on impact and stay high for several periods. The initial jump is mostly due to the jump in equity prices, because the initial capital flows are very small. However, liabilities eventually drop below their steady-state level. This occurs because foreigners continue to hold low levels of home equity even after the home equity price has returned to its steady-state value. The correlation between gross foreign assets and liabilities is still positive (0.62), but it is smaller than the cross-country correlation between equity prices (0.71).

4 FDI and portfolio equity

How do the business cycle properties of equity prices, flows and positions depend on the duration of the underlying investment project? To address this question, I

[17] In the complete markets models, ownership shares are indeterminate.

develop an international real business cycle (IRBC) model of greenfield FDI and portfolio equity based on two simple distinctions. First, greenfield FDI finances investment in new projects, while portfolio equity finances ongoing operations. Second, greenfield FDI is "locked in" for a period of time before it produces any returns, while portfolio equity can be withdrawn at any point. I use the benchmark model from Section 3, modified to allow for investment projects of different durations.

4.1 Model

4.1.1 Firms

Firms in each country have an overlapping generations structure, with new firms born every period. A new firm acquires firm-specific capital for a two-period startup project by issuing new equity in the form of direct ownership claims. When the startup project completes, the firm pays a dividend to its initial owners and reinvests part of its output in an ongoing, single-period project. At this decision point, the firm undergoes an initial public offering: the direct owners sell their stakes, and the firm issues portfolio equity shares. At the end of the ongoing project, the firm pays a dividend to its portfolio owners and exits.

New (N) and ongoing (G) firms in the home country produce home output goods, while new and ongoing firms in the foreign country produce foreign output goods. Goods produced by new and ongoing firms within a country are identical. The only sources of uncertainty are two country-specific aggregate productivity shocks, Z_t^H and Z_t^F. Each country-specific shock impacts all new and ongoing firms in a particular country. Throughout this section, I focus on home firms; foreign firms are symmetric.

Figure 4 illustrates the overlapping-generations structure of firms. Every period, a mass of identical new firms is born and an aggregate productivity shock is realized. A new firm lives for four periods, or "stages". The following describes the actions that a firm born at time t takes at each stage. The firm described here is the bottom-most firm in the figure.

Stage 1: The firm issues equity at price $P_t^{N,H}$. It uses the proceeds to acquire firm-specific capital $K_t^{N,H}$, which it invests in a two-period startup project.

Stage 2: The firm cannot take any action at this stage. Its capital is sunk in the startup project.

Stage 3: The startup project finishes and yields its output. The firm chooses how much of this output to reinvest in a one-period ongoing project, $K_{t+2}^{G,H}$, and how much to pay out in dividends, $D_{t+2}^{N,H}$.

Stage 4: The ongoing project finishes and yields its output. The firm pays all of its output as dividends, $D_{t+3}^{G,H}$, and exits.

A new firm can do only one thing: issue an ownership claim at numeraire price $P_t^{N,H}$ and use the proceeds to acquire capital $K_t^{N,H}$. I assume that the firm can convert home output goods into capital one-for-one. The amount of capital the firm can create through share issuance is:

$$K_t^{N,H} = \frac{P_t^{N,H}}{P_t^H} \tag{27}$$

Now consider an ongoing firm (Stage 3) that has just completed its startup project in period t. This firm is the second from the top in the figure; it was born

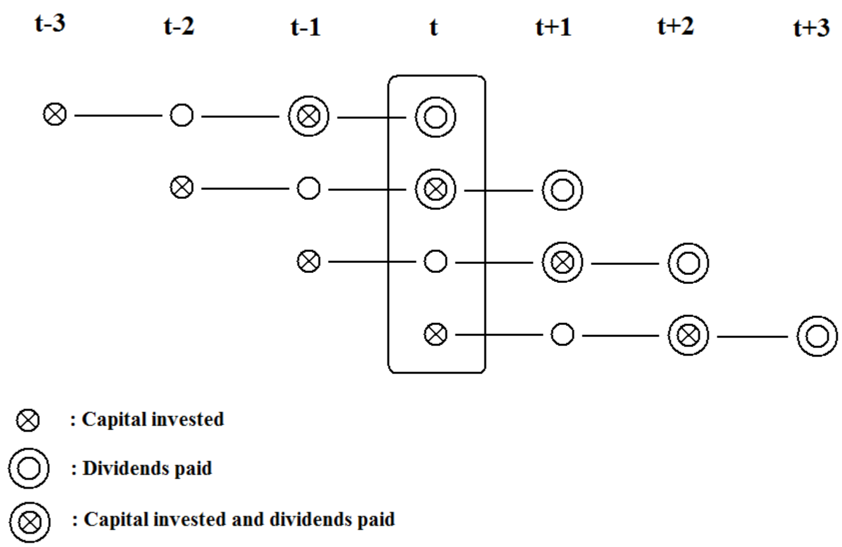

Figure 4: **Illustration of the overlapping generations structure of firms in the model of FDI and portfolio equity.**

in period $t-2$. The output from the startup project is:

$$Y_t^{N,H} = Z_t^H \left(K_{t-2}^{N,H}\right)^\theta \ , \theta \in (0,1) \tag{28}$$

where $K_{t-2}^{N,H}$ is startup capital invested two periods ago and Z_t^H is the aggregate productivity shock affecting all firms in the home country. Given ongoing capital investment $K_t^{G,H}$, output from the ongoing project next period will be:

$$Y_{t+1}^{G,H} = Z_{t+1}^H \left(K_t^{G,H}\right)^\theta \tag{29}$$

Capital in ongoing firms depreciates at the rate $\delta \in [0,1]$ over one period, and capital in new firms depreciates at the rate $\delta_N = 1-(1-\delta)^2$ over two periods. At time t, the firm chooses ongoing capital investment, $K_t^{G,H}$, and today's dividend to the startup owners, $D_t^{N,H}$, to solve the following problem:

$$\max_{K_t^{G,H}, D_t^{N,H}} \left\{ D_t^{N,H} + E_t \left[M_{t+1,t} D_{t+1}^{G,H} \right] \right\}$$

$$\text{s.t. } D_t^{N,H} = P_t^H \left[Y_t^{N,H} + (1-\delta_N) K_{t-2}^{N,H} - K_t^{G,H} \right] \tag{30}$$

$$D_{t+1}^{G,H} = P_{t+1}^H \left[Y_{t+1}^{G,H} + (1-\delta) K_t^{G,H} \right] \tag{31}$$

As before, $M_{t+1,t}$ is the home household's intertemporal marginal rate of substitution (IMRS) of time-$(t+1)$ numeraire goods for time-t numeraire goods. Also recall that output and capital are measured in terms of home output goods, while dividends and prices are measured in terms of the numeraire. The first-order condition for the ongoing firm is:

$$1 = E_t \left[M_{t+1,t} \frac{P_{t+1}^H}{P_t^H} \left(\frac{\theta Y_{t+1}^{G,H}}{K_t^{G,H}} + 1 - \delta \right) \right] \tag{32}$$

4.1.2 Households

Households have the same preferences as in the benchmark model, but the budget constraint is different. Households in both countries can hold claims to four types of firms: new home firms, ongoing home firms, new foreign firms and ongoing foreign firms. The budget constraint for the representative home household is:

$$\begin{aligned} P_t C_t + P_t^{N,H} A_t^{N,H} + P_t^{N,F} A_t^{N,F} + P_t^{G,H} A_t^{G,H} + P_t^{G,F} A_t^{G,F} \\ = \left(P_t^{G,H} + D_t^{N,H} \right) A_{t-2}^{N,H} + \left(P_t^{G,F} + D_t^{N,F} \right) A_{t-2}^{N,F} \\ + D_t^{G,H} A_{t-1}^{G,H} + D_t^{G,F} A_{t-1}^{G,F} \\ - \frac{\psi}{2} P_t^F \left(A_t^{N,F} - A^{N,F} \right)^2 - \frac{\psi}{2} P_t^F \left(A_t^{G,F} - A^{G,F} \right)^2 \end{aligned} \tag{33}$$

where, for example, $A_t^{N,H}$ is the number of new home firms owned by home households at the end of time t. Note that the payoff to a new firm's direct ownership claim includes two components – the new firm's dividend plus the price of the ongoing firm. This reflects the assumption that the new firm's owners can sell their stakes at the end of the startup project. The payoff to a share in an ongoing firm is the final dividend, paid just before the firm exits.

The second-to-last term in (33) represents financial costs that the home household must pay whenever the number of new foreign firms owned differs from its steady-state value. Analogously, the last term represents costs paid whenever the number of shares in ongoing foreign firms differs from its steady-state value. The Appendix states the foreign household's budget constraint; it is the mirror image of the home household's budget constraint. The Appendix also gives the first-order conditions of the model.

4.1.3 Market clearing

Market-clearing for output goods requires:

$$K_t^{N,H} - (1-\delta_N)K_{t-2}^{N,H} + K_t^{G,H} - (1-\delta)K_{t-1}^{G,H} + C_t^H + \widehat{C}_t^H = Y_t^{N,H} + Y_t^{G,H} - \Omega_t^H \tag{34}$$

$$K_t^{N,F} - (1-\delta_N)K_{t-2}^{N,F} + K_t^{G,F} - (1-\delta)K_{t-1}^{G,F} + C_t^F + \widehat{C}_t^F = Y_t^{N,F} + Y_t^{G,F} - \Omega_t^F \tag{35}$$

Here Ω_t^H represents costs paid by foreign households to deviate from their long-run holdings of home equity, and Ω_t^F represents costs paid by home households to deviate from their long-run holdings of foreign equity.[18] In the calibrated model, these terms are very small.

I normalize the supply of each security to one:

[18]These terms are given by:

$$\Omega_t^H \equiv \frac{\psi}{2}\left(\widehat{A}_t^{N,H} - \widehat{A}^{N,H}\right)^2 + \frac{\psi}{2}\left(\widehat{A}_t^{G,H} - \widehat{A}^{G,H}\right)^2$$
$$\Omega_t^F \equiv \frac{\psi}{2}\left(A_t^{N,F} - A^{N,F}\right)^2 + \frac{\psi}{2}\left(A_t^{G,F} - A^{G,F}\right)^2$$

Note that Ω_t^H and Ω_t^F are measured in units of home and foreign output goods, respectively.

$$1 = A_t^{N,H} + \widehat{A}_t^{N,H} \tag{36}$$

$$1 = A_t^{G,H} + \widehat{A}_t^{G,H} \tag{37}$$

$$1 = A_t^{N,F} + \widehat{A}_t^{N,F} \tag{38}$$

$$1 = A_t^{G,F} + \widehat{A}_t^{G,F} \tag{39}$$

I refer to $A_t^{N,F}$ and $\widehat{A}_t^{N,H}$ as the *cross-border ownership shares of new firms*, and I refer to $A_t^{G,F}$ and $\widehat{A}_t^{G,H}$ as the *cross-border ownership shares of ongoing firms.*

The shock processes are the same as in Section 3.

4.1.4 Equilibrium

A competitive equilibrium is a sequence of prices and allocations such that all markets clear when consumers and firms behave optimally, taking equilibrium prices as given.

4.2 Results

I interpret home (foreign) holdings of new foreign (home) firms as the greenfield FDI assets (liabilities) of the home country, and I interpret home (foreign) holdings of ongoing foreign (home) firms as gross portfolio equity assets (liabilities). The calibration is very similar to the benchmark model with financial costs; see Sections 3.2 through 3.4 for a discussion. I set $\widehat{A}^{N,H} = \widehat{A}^{G,H} = A^{N,F} = A^{G,F} = 0.08$. So the cross-border ownership shares are 8%, both for new firms ("greenfield FDI") and for ongoing firms ("portfolio equity"). I report results for the two good version with persistent shocks only ($\phi = 2, \lambda = 0.75, \rho = 0.8$). I recalibrate σ_H and σ_F to 0.010 to

	peo, y	pei, y	gro, y	gri, y	peo, pei	gro, gri
FDI/PE Model	0.22	-0.22	0.38	-0.38	-1.00	-1.00
U.S. Data	0.17	0.40	0.08	0.16	0.27	0.79

Table 6: **Summary of model results for the cyclicality of portfolio equity (PE) and greenfield FDI flows.** *peo* is real gross PE outflows, *pei* is real gross PE inflows, *gro* is real gross greenfield FDI outflows, *gri* is real gross greenfield FDI inflows, and y is real GDP.

match the standard deviation of U.S. GDP, and I recalibrate the cross-border cost parameter ψ to 2.0 to match the standard deviation of the U.S. current-account-to-GDP ratio.

Figure 5 presents impulse responses of physical capital, dividends and equity prices to a positive, one standard deviation shock to home country technology. Responses are shown both for new firms (dashed lines) and for ongoing firms (solid lines). The capital stock of new home firms rises less on impact than the capital stock of ongoing home firms. Because technology shocks are only partially persistent, expected technology for new firms two periods ahead is less than expected technology for ongoing firms tomorrow. In addition, new firms are exposed to greater risk: they face two shocks to technology before production occurs. As a result, new firms invest less on impact. Dividends and equity prices also adjust more gradually for new home firms. Equity prices for new firms are positively correlated across countries (0.61), as are equity prices for ongoing firms (0.49).

Figure 6 presents impulse responses of home and foreign consumption, cross-border ownership shares, and numeraire-valued cross-holdings. *panf* represents the gross greenfield FDI assets of the home country, $panh\_$ represents gross greenfield FDI liabilities, *pagf* represents gross portfolio equity (PE) assets, and $pagh\_$ repre-

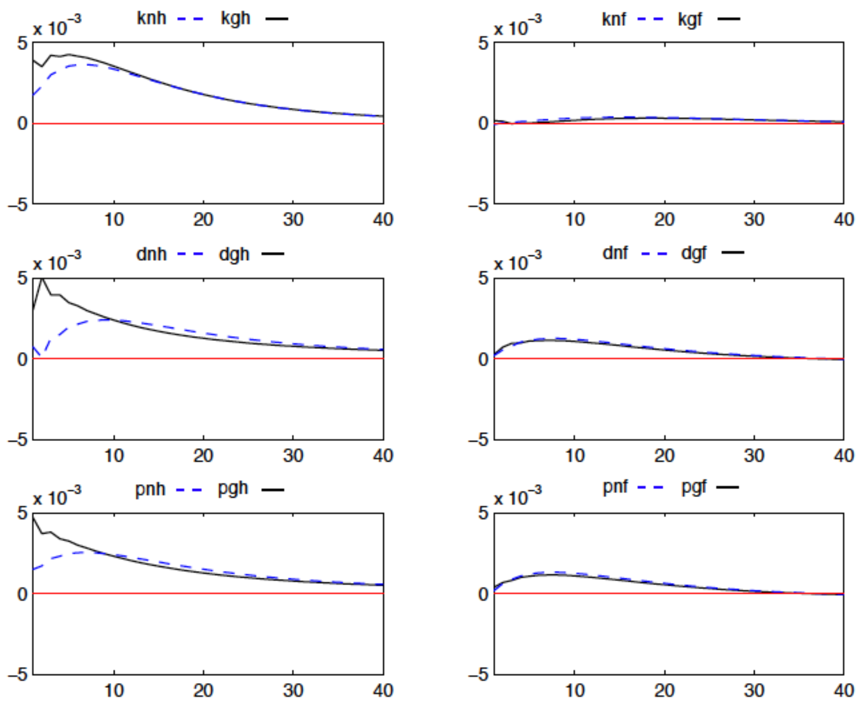

Figure 5: **Impulse responses to a positive, one standard deviation shock to home country technology: FDI and portfolio equity model with two goods.** The dashed lines are impulse responses for new firms, and the solid lines are responses for ongoing firms. knh is the capital stock of new home firms, kgh is the capital stock of ongoing home firms, knf is the capital stock of new foreign firms, kgf is the capital stock of ongoing foreign firms, dnh is dividends paid by new home firms, dgh is dividends paid by ongoing home firms, dnf is dividends paid by new foreign firms, dgf is dividends paid by ongoing foreign firms, pnh is the price of a claim to a new home firm, pgh is the price of an equity share in ongoing home firms, pnf is the price of a claim to a new foreign firm, and pgf is the price of an equity share in ongoing foreign firms.

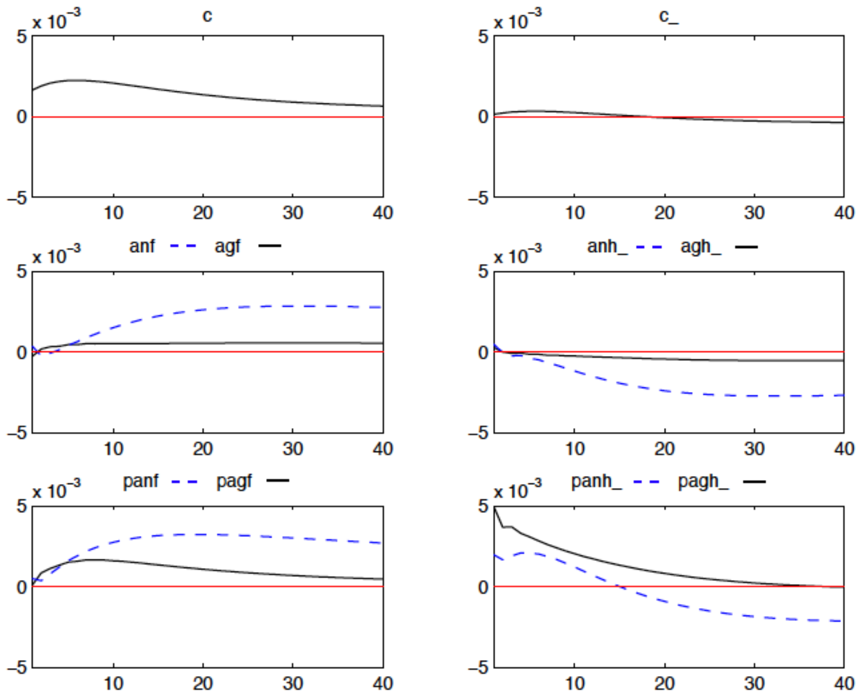

Figure 6: **Impulse responses to a positive, one standard deviation shock to home country technology: FDI and portfolio equity model with two goods.** c is the home household's consumption of home consumption goods, c_{-} is the foreign household's consumption of foreign consumption goods, anf is the home household's ownership share of new foreign firms, agf is the home household's ownership share of foreign ongoing-firm equity, anh_{-} is the foreign household's ownership share of new home firms, agh_{-} is the foreign household's ownership share of home ongoing-firm equity, $panf$ is the home household's numeraire-valued holdings of new foreign firms ("gross greenfield FDI assets"), $pagf$ is the home household's numeraire-valued holdings of foreign ongoing-firm equity ("gross portfolio equity assets"), $panh_{-}$ is the foreign household's numeraire-valued holdings of new home firms ("gross greenfield FDI liabilities"), and $pagh_{-}$ is the foreign household's numeraire-valued holdings of home ongoing-firm equity ("gross portfolio equity liabilities").

sents gross PE liabilities. As in the benchmark model with financial costs, the path of home consumption is significantly higher than the path of foreign consumption. After the shock, home households gradually increase their ownership shares of new and ongoing foreign firms, and foreign households gradually decrease their ownership shares of new and ongoing home firms. These changes reflect wealth effects favoring the home country, as discussed in Section 3.4.[19] However, capital flows are smaller in portfolio equity than in greenfield FDI. Recall that equity prices in the model are positively correlated across countries in both asset classes, while capital flows are negatively correlated. Because capital flows are small in PE, the price effect dominates, and the correlation between gross PE assets and liabilities is positive (0.58). In contrast, for FDI, the capital flow effect dominates, and the correlation between gross FDI assets and liabilities is negative (-0.79).

Although the quantitative effects of capital flows differ for new versus ongoing firms, the results are qualitatively very similar. After a positive productivity shock in the home country, the home household increases its ownership shares of all four types of equity, and the foreign household's ownership shares all fall. Therefore, gross capital outflows and inflows are negatively correlated in the model, both for greenfield FDI and PE. Furthermore, the negative correlation is nearly perfect. Table 6 presents selected business cycle statistics for capital flows in the model. Home's gross capital outflows are positively correlated with domestic output, both for PE (0.22) and for greenfield FDI (0.38). In contrast, home's gross capital inflows are countercyclical, both for PE (-0.22) and for greenfield FDI (-0.38). The procyclical gross outflows predicted by the model are at least qualitatively consistent with U.S.

[19] Although the changes in cross-border ownership shares are very long-lived in this calibration, the shares do eventually return to their steady-state levels.

data; however, the countercyclical gross inflows are not. The model's prediction of a strong negative correlation between outflows and inflows is also counterfactual.

5 Conclusion

This chapter has documented a strong positive correlation between gross equity-based assets and liabilities in the G7. This correlation reflects strong cross-country correlation between equity prices and moderate comovement of gross outflows and inflows. I then analyzed an international real business cycle (IRBC) model under several different assumptions to evaluate possible causes of these correlations: diminishing marginal product of capital, imperfect substitutability of goods, incomplete markets, and investment project duration. I showed that a complete markets model with diminishing returns to capital predicts close cross-country correlation between equity prices. Adding imperfect substitutability between goods strengthened this result, because the terms of trade help to propagate productivity shocks across countries. In contrast, an incomplete markets model with realistically under-diversified ownership shares introduced negative correlation between outflows and inflows, which weakened the correlation between gross foreign assets and liabilities. Finally, I developed a model of greenfield FDI and portfolio equity. The model suggests that assets and liabilities should be more closely correlated in portfolio equity than in greenfield FDI.

Many avenues are open for future research. A natural extension of the FDI/PE model would include debt finance. Bonds are an important component of international asset trade, and they account for a large fraction of U.S. gross liabilities. It would also be useful to extend the framework to emerging countries, where portfolio inflows tend to be very volatile and FDI inflows are often countercyclical. One approach is to introduce financial constraints in one country, as in Albuquerque (2003) or Smith and Valderrama (2009). Such a framework may help explain FDI and portfolio dynamics in emerging countries, like China, that can influence world prices despite having limited domestic financial markets.

Chapter 2: Capital Structure Over The Business Cycle

1 Introduction

When output is high, U.S. firms as a whole issue more debt, invest more, and pay more to shareholders. Debt issued and equity payouts are both positively correlated with investment and positively correlated with each other. These findings suggest that firms use debt financing both to invest in their operations and to increase payments to shareholders. While borrowing to finance investment is well understood, borrowing to pay shareholders is more puzzling. Why would firms systematically transfer resources from bondholders to shareholders over the business cycle?

The workhorse model for analyzing business cycle correlations among macroeconomic variables is the real business cycle (RBC) model. However, traditional RBC models abstract from firm financing decisions. In the standard setup, a representative consumer-firm has access to a neoclassical production function and makes optimal consumption-investment decisions given a known stochastic process for productivity.[20] An implicit assumption is that firms can costlessly reallocate resources in order to maximize the consumer's lifetime expected utility. A consequence of this approach is that conventional RBC models have little to say about financial behavior over the business cycle.

In reality, of course, firms have an array of options for financing their activities. In particular, firms can raise capital by borrowing at a fixed interest rate, by issuing equities, or by using internal funds. The classic paper by Modigliani and Miller (1958) showed that if capital markets are frictionless, firms should be indifferent regarding their capital structure. However, a common view in the corporate finance

[20]For an example of a "standard" RBC model, see Hansen (1997).

literature is that firms pursue a financial mix that balances the costs and benefits of different forms of finance (Leary and Roberts (2005)). According to this "tradeoff theory" of capital structure, financial frictions such as interest tax deductions and bankruptcy costs pin down a firm's optimal debt-equity ratio (see, for example, Scott (1976) and Miller (1977)). In a wide-ranging survey of CFOs conducted by Graham and Harvey (2001), most respondents reported that they have at least a loose target for their firms' debt-equity ratio. Many of the participants stated that interest tax deductions and credit ratings influence decisions about how much debt to issue.

In this chapter I add a corporate finance decision to a standard RBC model. Firms decide how much debt to issue and how much to pay shareholders by weighing the costs and benefits of debt and equity financing. My goal is to explain some stylized facts about aggregate debt and equity flows in U.S. data. First, aggregate corporate borrowing and payouts to shareholders are almost always positive throughout the postwar period (1952 – 2007). Second, debt issuance and net equity payouts are (i) positively correlated with output, (ii) positively correlated with investment, and (iii) positively correlated with each other. Third, real variables have become less volatile in recent years, while financial variables have become more volatile.

The key feature of the model is a set of parameterized financial frictions. On the debt side, interest payments are tax deductible. This makes debt financing preferred to equity financing for low levels of debt. However, I also require bondholders to pay a monitoring cost that is increasing in the firm's debt-to-capital ratio. The tradeoff between the debt tax shield and the monitoring costs determines the firm's optimal level of debt. On the equity side, I assume that it is costly for firms to adjust their payouts to shareholders. This assumption is consistent with evidence that firms tend to smooth dividends over time. It also captures, in a stylized way, the legal and accounting costs associated with issuing and repurchasing equity shares.

I calibrate the model to U.S. data for the postwar period 1952 – 2007. In contrast with a standard RBC model, my model predicts positive borrowing at all points in the business cycle. The model correctly implies that debt issuance and equity payouts are procyclical, positively correlated with investment, and positively correlated with each other. Finally, I calibrate the model to two separate time periods, 1952 – 1983 and 1984 – 2007, in order to explain the stylized facts described by Jermann and Quadrini (2007): namely, that real variables have become less volatile in the second subperiod, while financial variables have become more volatile. By varying both the scale of the technology shocks and the degree of financial frictions, the model can match both facts.

I interpret these results as evidence that the types of frictions modeled – tax incentives, debt monitoring costs, and equity adjustment costs – can help explain the degree to which firms use debt financing to increase payments to shareholders. The results also provide support for a "dynamic tradeoff theory" of capital structure: firms target an optimal debt-equity ratio, but the target evolves over time as economic conditions change.

This chapter contributes to a growing literature on capital structure over the business cycle. The debt friction can be viewed as a reduced form of the debt enforcement problem in Bernanke et al. (1999). Relative to their model, my model allows for equity issuance. Covas and den Haan (2006), Levy and Hennessy (2007) and Jermann and Quadrini (2007) all present models of debt and equity financing over the business cycle. Covas and den Haan have a partial equilibrium model of debt and equity finance where the interest rate process is exogenous. In contrast, the risk-free rate in my model is determined endogenously in general equilibrium. Levy and Hennessy (2007) consider a general equilibrium problem where managers can finance investment with debt or equity but are allowed to divert resources from both bondholders and shareholders. I differ by assuming that the firm's objective is aligned with that of shareholders. In my model, the equity friction reflects adjustment costs rather than an explicit agency problem.

My model is most closely related to Jermann and Quadrini (2007). They present a model of debt and equity finance and use it to explain the reduced volatility of macroeconomic variables and increased volatility of financial variables over the past two decades. My work differs in three main ways. First, the key shock in Jermann and Quadrini's main analysis is an asset price shock, whereas I consider a standard technology shock. Second, Jermann and Quadrini's debt friction takes the form of an endogenous "debt ceiling", above which firms cannot borrow at any price. In practice, however, firms do not face a strict ceiling on debt; rather, the cost of debt may increase if firms borrow excessively. I attempt to capture this phenomenon through monitoring costs. Third, while Jermann and Quadrini also consider a convex cost for equity payouts, their cost function is increasing in the *deviation of today's equity payout from its long-term target.* In contrast, my cost function is an adjustment cost: it is increasing in the *deviation of today's equity payout from yesterday's payout.* I argue that this more naturally captures the legal and financial costs that I am trying to model.

The rest of the chapter is structured as follows. Section 2 documents some stylized facts about debt and equity flows. Section 3 explains the financial frictions that I attempt to model. Section 4 presents the model. Section 5 discusses calibration and presents the results. Section 6 concludes.

2 Debt and Equity Flows in the U.S.

In this section I discuss the empirical features of debt and equity flows that I would like my model to capture. Figure 7 plots aggregate debt issued, equity payouts, and real fixed investment in the U.S. nonfarm sector, measured in billions of 2000 dollars. Data are quarterly flows from the Federal Reserve's Flow of Funds and the BEA's NIPA accounts (for fixed investment) for the period 1952:1 – 2007:4. I convert nominal flows to real flows using the BEA price index for value-added in the nonfarm sector. "Debt Issued" is the net increase in credit market liabilities over the quarter. A negative number reflects a net repayment of debt by businesses. "Equity Payout" equals dividends plus net repurchases of equity shares in the corporate sector (net of new stock issuance), less proprietors' net investment in the noncorporate sector. The idea is to capture *net* flows to shareholders as a group, including small business owners. I view dividends and equity issues as two sides of the same coin: a firm that wishes to "raise capital" through equity may do so by lowering its dividend, by offering new shares, or both. Equivalently, a firm that wishes to "reward shareholders" may do so by increasing its dividend, by repurchasing shares, or both. The analysis reflects this presumed equivalence of dividends and share repurchases.[21]

A number of facts emerge from Figure 7. First, debt issued and net equity payouts are almost always positive over the postwar period. *In aggregate*, the U.S. business sector rarely issues equity or repurchases debt. Positive borrowing, in particular, suggests that some degree of leverage is optimal for firms. A second

[21] Jermann and Quadrini (2007) also consider net equity payouts, measured in a similar way.

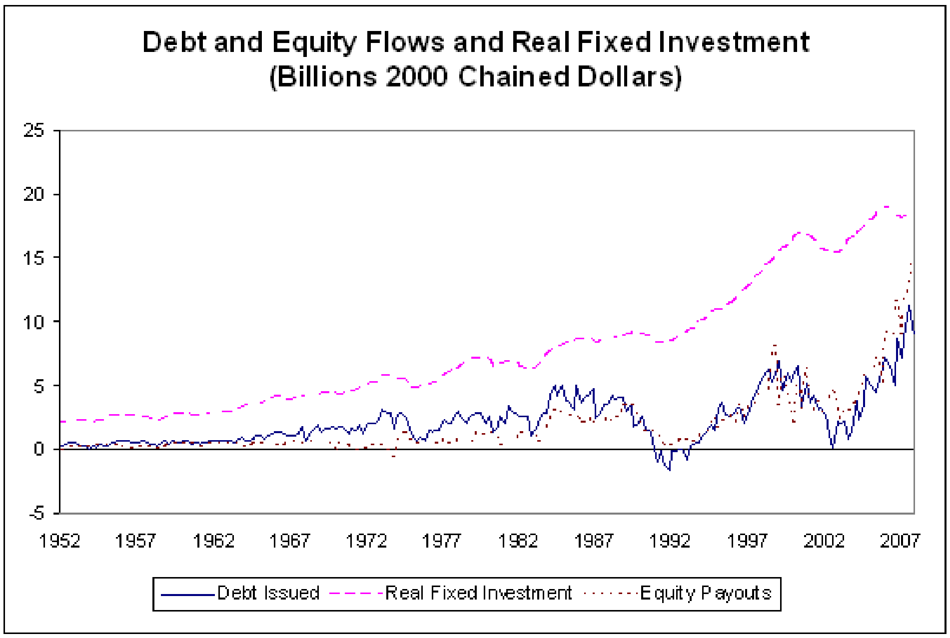

Figure 7: Aggregate debt and equity flows and aggregate real investment in the U.S. nonfarm sector, in billions of chained 2000 dollars. Sources: Flow of Funds, Federal Reserve Board and NIPA Accounts, BEA.

regularity is that debt issued and equity payouts are positively correlated. This close co-movement is especially striking beginning in the late 1980s. Third, both debt issued and equity payouts are positively correlated with investment. Finally, debt issued and equity payouts are considerably more volatile in the second half of the period.

Table 7 computes business cycle correlations for equity payouts, debt issued, real GDP and real fixed investment. I have detrended all variables with a Hodrick-Prescott filter. Debt issued and equity paid are both positively correlated with GDP ("procyclical"), positively correlated with investment, and positively correlated with each other. These findings suggest that firms borrow more heavily during booms. Firms apparently use the proceeds from borrowing both to invest and to finance higher payments to shareholders.

Table 8 presents business cycle volatilities for the two subperiods 1952 – 1983 and 1984 – 2007. Debt issued and equity payouts (as shares of GDP) have become more volatile.[22] In contrast, real GDP and real investment have become less volatile, a well-documented phenomenon known as business cycle moderation.

I summarize the stylized facts from this section as follows. First, aggregate debt issued and equity payments to shareholders are almost always positive over the business cycle. Second, debt issued and equity payouts are (i) positively correlated with output, (ii) positively correlated with investment, and (iii) positively correlated with each other. Third, debt issued and equity payouts have become more volatile starting in 1984, while real variables have become less volatile.

[22] I scale equity payouts and debt issued by nominal GDP (value-added in the nonfarm sector) when computing volatilities. The increase in volatility is even more dramatic when these variables are not scaled by GDP. Measured in billions of chained 2000 dollars, the volatility of equity payouts increased by a factor of 3.78 and the volatility of debt issued increased by a factor of 2.48.

Variables	*Correlation*
(Equity Payout, GDP)	0.16
(Debt Issued, GDP)	0.45
(Real Fixed Investment, GDP)	0.90
(Equity Payout, Real Fixed Investment)	0.19
(Debt Issued, Real Fixed Investment)	0.52
(Equity Payout, Debt Issued)	0.38

Table 7: Business cycle correlations for selected real and financial variables from 1952 – 2007. "Debt Issued" is the net increase in credit market liabilities. A negative number reflects a net repayment of debt by businesses. "Equity Payout" equals dividends plus net repurchases of equity shares in the corporate sector, less proprietors' net investment in the noncorporate sector. *GDP* is real gross value-added in the nonfarm business sector. All variables are detrended using a Hodrick-Prescott filter with a smoothing parameter of 1600. Sources: Flow of Funds, Federal Reserve Board and NIPA Accounts, BEA.

Standard Deviations ($\times$ 100)	*1952–1983*	*1984–2007*	*Late/Early*
Equity Payout / GDP	0.85	1.44	1.69
Debt Issued / GDP	1.32	1.69	1.28
Log Real GDP	2.56	1.18	0.46
Log Real Fixed Investment	5.58	3.63	0.65

Table 8: Changes in selected business cycle statistics for the Nonfarm sector between 1952 – 1983 and 1984 – 2007. All variables are detrended using a Hodrick-Prescott filter with a smoothing parameter of 1600. Sources: Flow of Funds, Federal Reserve Board and NIPA Accounts, BEA.

The next section describes and motivates the financial frictions that I model. Section 4 presents the full model.

3 Financial Frictions

Conceptually, firms can finance their activities with debt, equity, or internal funds. In the model, debt financing takes place through one-period corporate bonds. Corporate bonds are issued by firms and pay a “quoted” interest rate r_t, but they require costly monitoring to ensure repayment. Consumers are the investors in this model, so they pay the monitoring costs. Monitoring costs for a firm’s debt are assumed to be a linear function of the firm’s debt-to-capital ratio L_t, which consumers take as given. The price of a corporate bond to consumers is given by:

$$P_t^{B,C} = \frac{1}{1 + r_t} + \mu L_t \tag{40}$$

μ is an exogenous parameter that controls the scale of the debt friction. Monitoring costs can be motivated as follows. Each period a firm could, in principle, default on its debt and enter a state of bankruptcy. Default is more likely the larger the amount borrowed relative to the firm’s capital stock, which serves as collateral. Assume that bankruptcy is a distress state with unavoidable deadweight losses. In this setting, monitoring costs can be interpreted as a safety buffer against potential bankruptcy costs. Despite this motivation, I do not explicitly model bankruptcy. In particular, there is no default in equilibrium: investors always pay the monitoring costs, and the firm always repays its debt.[23]

[23] A deeper foundation for monitoring costs would require explicitly modeling the

An alternative interpretation of the debt friction is that it reflects a rating effect. There is ample evidence that firms are concerned about credit ratings when deciding how much debt to issue; see Graham and Harvey (2001) and Kisgen (2006). One of the metrics that rating agencies use to evaluate creditworthiness is a firm's debt-to-capital ratio. Figure 8 shows the median debt-to-capital ratio for firms with different credit ratings. Firms with higher debt-to-capital ratios tend to have lower ratings.[24] It is also well known that lower-rated corporate bonds have greater yields than higher-rated bonds. Therefore, it is more costly for firms with low credit ratings to issue corporate debt. The monitoring cost in (40) can thus be interpreted as reflecting the effect of higher debt levels on the cost of debt via a lower credit rating.

Interest paid on corporate debt is tax-deductible to the firm, a concept known as the "debt tax shield". A corporate bond in the model is a promise by the firm to pay one real unit to the bondholder tomorrow. The amount that the firm receives today in exchange for issuing such a bond is given by:

$$P_t^{B,F} = \frac{1}{1 + (1 - \tau) r_t} \tag{41}$$

bankruptcy-inducing event. A standard approach uses an idiosyncratic shock and a debt contract. The debt contract specifies an interest rate and a cutoff value for the shock above which the firm always repays. Examples include Bernanke et al. (1999) and Covas and den Haan (2006). I take a simplified approach to the debt friction in order to avoid the heterogeneity arising from idiosyncratic shocks. Since my focus is on *aggregate* fluctuations over the business cycle, little explanatory power is lost by making this simplification.

[24]Here, "capital" refers to a firm's outstanding debt plus its outstanding equity, which together equal the firm's assets. In the model, a firm's only asset is its capital stock. Therefore, I treat the "debt-to-capital ratio" in the model as the ratio of a firm's outstanding debt to its capital stock.

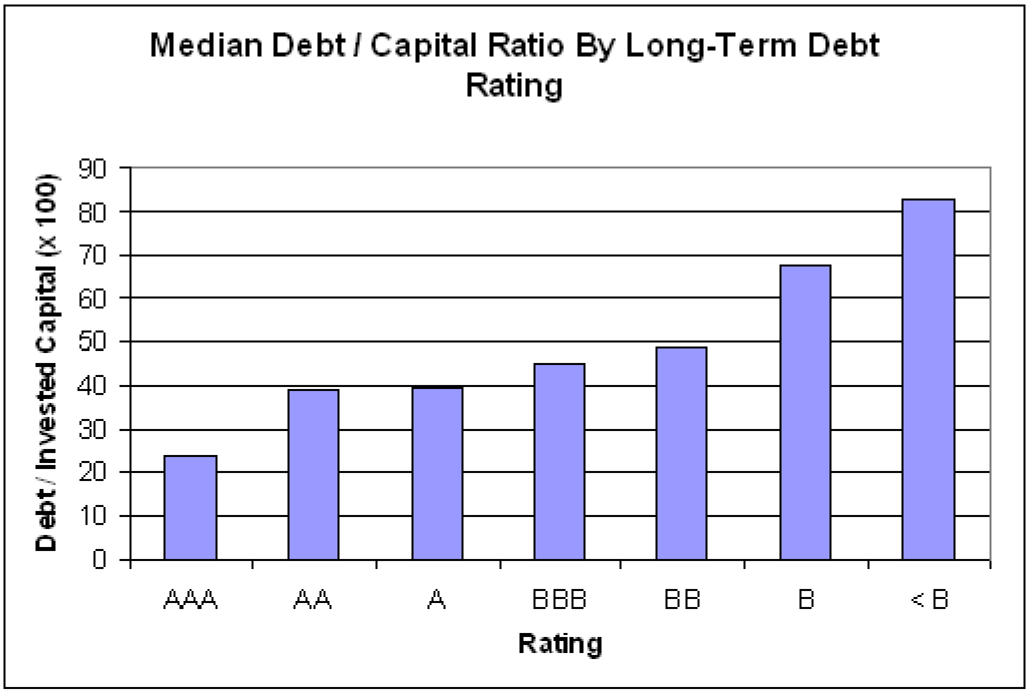

Figure 8: Median debt-to-capital ratios for firms by long-term S&P credit rating. Here "capital" refers to a firm's outstanding debt plus its outstanding equity, which together equal the firm's assets. Each bar in the graph plots the median debt-to-capital ratio among all active Compustat firms with the given credit rating. Source: Compustat, author's calculations.

τ is the marginal corporate tax rate, and the effective cost of debt to the firm is $(1-\tau)r_t$. Note that in general, $P_t^{B,C}$ need not equal $P_t^{B,F}$. For example, in the model, the tax deduction enjoyed by firms is paid for by a lump-sum tax on consumers. This creates a wedge between the proceeds a firm receives from issuing a bond and the price that consumers pay for a bond. Since the firm perceives that it can issue bonds at a favorable price, it will want to issue a positive amount of debt. However, the price of corporate debt to consumers rises with the amount issued because of the monitoring costs described above. As the price of debt increases, consumer demand for corporate bonds declines. The tradeoff between the tax advantage and monitoring costs will pin down the equilibrium level of debt and the "quoted" rate r_t in equilibrium.

On the equity side, I assume that it is costly for firms to adjust their payouts to shareholders. Conceptually, the equity payout in the model reflects *net* flows to shareholders: dividends plus repurchases of shares, net of new issues. I impose the following quadratic cost function for a firm that wishes to pay D_t at time t, given last period's payout, D_{t-1}, and the capital stock carried over from last period, K_{t-1}:

$$c(D_t, D_{t-1}, K_{t-1}) = D_t + \phi K_{t-1}\left(\frac{D_t}{D_{t-1}} - 1\right)^2 \ , \phi > 0 \qquad (42)$$

Note that the adjustment cost is increasing in the percentage change of today's payout from yesterday's payout. The parameter ϕ controls the scale of the adjustment cost. The convex functional form given here is consistent with evidence that firms tend to smooth dividends over time, an observation first made by Lintner (1956). It also consistent with evidence regarding the legal and accounting costs associated with issuing and repurchasing equity shares. For example, Hansen and Torregrosa (1992) and Atlinkilic and Hansen (2000) show that underwriting fees exhibit increasing marginal cost in the size of the equity offering. I note that the adjustment cost here differs from Jermann and Quadrini (2007) in an important way. My cost function is the *deviation of today's equity payout from yesterday's payout*, rather than the *deviation of today's equity payout from its long-run target*. In a model with persistent productivity shocks, the firm's optimal payout in any given period may be very different from its optimal long-run target. For example, in a model that fluctuates around a steady-state, the equilibrium payout is typically above the long-run target whenever productivity is above its steady-state level. Legal and accounting costs associated with share issuance and repurchases suggest that it is costly for firms to *change* their equity payout, but not necessarily costly to be away from the steady-state. I therefore adopt a cost function that depends on the change from yesterday's payout.

Finally, a firm can of course finance its investment through the use of internal funds. In the model, "internal funds" consist of a firm's current output plus its undepreciated capital stock. I do not model *capital* adjustment costs, so it is costless for a firm to use its internal funds to increase or decrease its capital stock.

The financial frictions serve two roles in the model. First, they determine the firm's optimal (positive) level of debt even in the absence of stochastic shocks. Second, they affect the way in which the firm reacts to shocks. In response to a positive, persistent productivity shock, the firm would like to both increase investment (to take advantage of higher expected productivity) and pay more to shareholders (to pass on the unexpected increase in lifetime profitability). If debt is available and the cost of changing the equity payout is low, the firm accomplishes this by borrowing heavily to boost its capital stock and increase payments to shareholders. On the other hand, if the cost of adjusting equity is high, the firm seeks to avoid large swings in its payout. As a result, the firm increases payouts only gradually, and its borrowing is both lower in magnitude and delayed in time.

4 The Model

The model economy consists of a continuum of identical consumers and a continuum of identical, perfectly competitive firms. First I specify the consumer's problem and the firm's problem. I then define an equilibrium and briefly discuss the solution procedure.

4.1 Consumer's problem

A representative consumer can hold one-period risk-free bonds, $\tilde{B}_t$, one-period corporate bonds, B_t, and shares of firms, S_t. Risk-free bonds pay the net interest rate $\tilde{r}_t$ with certainty. Corporate bonds are issued by firms and pay the "quoted" interest rate r_t, but they require costly monitoring in order to ensure repayment. Monitoring costs are assumed to be a linear function of the firm's debt-to-capital ratio L_t, which the consumer takes as given.

Each share pays a dividend D_t and trades in the stock market at the price P_t. Every period, the consumer receives income from her portfolio and decides how much to consume, C_t, and how much to reinvest in the three assets. Denote all variables *chosen* or *realized* in period t with the subscript t. Endogenous variables that are predetermined in period t carry the subscript $t-1$. The consumer is infinitely-lived and maximizes the expected present discounted value of her utility,

$E_t \sum_{j=0}^{\infty} \beta^j u(C_{t+j})$, with $0 < \beta < 1$. The consumer's problem can be written recursively as follows:

$$W(\tilde{B}_{t-1}, B_{t-1}, S_{t-1}) = \max_{\tilde{B}_t, B_t, S_t} \left\{ u(C_t) + E_t \left[\beta W(\tilde{B}_t, B_t, S_t) \right] \right\} \tag{43}$$

$$\text{s.t. } C_t + \left(\frac{1}{1+\tilde{r}_t} \right) \tilde{B}_t + P_t^{B,C} B_t + P_t S_t = \tilde{B}_{t-1} + B_{t-1} + (P_t + D_t) S_{t-1} - T_t \tag{44}$$

T_t is a lump-sum tax; it will be used to finance the tax advantage enjoyed by firms. The consumer's first-order conditions are as follows:

$$(\tilde{B}_t) : \frac{1}{1+\tilde{r}_t} = E_t[M_{t+1}] \tag{45}$$

$$(B_t) : P_t^{B,C} = E_t[M_{t+1}] \tag{46}$$

$$(S_t) : P_t = E_t[M_{t+1}(P_{t+1} + D_{t+1})] \tag{47}$$

$$\text{where } M_{t+1} \equiv \frac{\beta u'(C_{t+1})}{u'(C_t)} \tag{48}$$

Equations (40), (45), (46) relate the corporate interest rate r_t to the firm's debt-to-capital ratio L_t and the risk-free rate $\tilde{r}_t$:

$$\frac{1}{1+r_t} = \frac{1}{1+\tilde{r}_t} - \mu L_t \tag{49}$$

Note that there is no risk of default, so the net return on corporate bonds equals the risk-free return. I assume a standard CRRA utility function:

$$u(C) = \frac{C^{1-\eta}}{1-\eta} \text{ , } \eta > 0 \tag{50}$$

4.2 Firm's problem

A representative firm operates a neoclassical production function:

$$F(K_{t-1}, N_t Z_t) = K_{t-1}^{\alpha}(N_t Z_t)^{1-\alpha} \text{ , } \alpha \in (0,1) \tag{51}$$

K_{t-1} is the firm's capital stock carried over from last period, N_t is labor input, and Z_t is labor-augmenting technological progress. Labor input N_t is normalized to 1 for all t; I abstract from fluctuations in labor supply.[25] Let D_t denote the firm's net equity payout. The firm attains its new capital stock K_t by issuing corporate bonds B_t and by adjusting its net equity payout D_t. Interest paid on corporate bonds is tax-deductible, as given by (41). The firm also faces a quadratic adjustment cost when changing its equity payout, as given by (42). The firm's budget constraint is as follows:

$$K_t = K_{t-1}^{\alpha} Z_t^{1-\alpha} + (1-\delta)K_{t-1} + \frac{B_t}{1+(1-\tau)r_t} - B_{t-1} - D_t - \phi K_{t-1}\left(\frac{D_t}{D_{t-1}} - 1\right)^2 \tag{52}$$

δ is the depreciation rate of capital, and τ is the marginal corporate tax rate.

[25] It would be straightforward to extend the model to incorporate a labor-leisure decision along the lines of Hansen (1997).

Since the firm is owned by consumers, it discounts payments to be made at time $j > t$ using the consumer's stochastic discount factor, $M_{t+j} \equiv \beta u'(C_{t+j})/u'(C_t)$. The firm's problem can be written recursively as follows:

$$V(K_{t-1}, B_{t-1}, D_{t-1}) = \max_{B_t, D_t, K_t} \{D_t + E_t [M_{t+1} V(K_t, B_t, D_t)]\} \tag{53}$$
$$\text{s.t. (52)}$$

Let λ_t be the Lagrange multiplier on the firm's budget constraint (52). In the Appendix I derive three Euler Equations for the firm – one each for bonds, dividends, and capital. The Euler Equations are:

$$(B_t) : \frac{\lambda_t}{1 + (1-\tau) r_t} = E_t [M_{t+1} \lambda_{t+1}] \tag{54}$$

$$(D_t) : 1 + E_t \left[2\phi M_{t+1} \lambda_{t+1} \left(\frac{K_t}{D_t} \right) \left(\frac{D_{t+1}}{D_t} \right) \left(\frac{D_{t+1}}{D_t} - 1 \right) \right] = \lambda_t \left[1 + 2\phi \left(\frac{K_{t-1}}{D_{t-1}} \right) \left(\frac{D_t}{D_{t-1}} - 1 \right) \right] \tag{55}$$

$$(K_t) : E_t \left[M_{t+1} \lambda_{t+1} \left\{ \alpha \left(\frac{Z_{t+1}}{K_t} \right)^{1-\alpha} + (1-\delta) - \phi \left(\frac{D_{t+1}}{D_t} - 1 \right)^2 \right\} \right] = \lambda_t \tag{56}$$

Define the debt-to-capital ratio L_t, which appears in the household's problem, as follows:

$$L(t) \equiv \frac{B_t}{K_{t-1}} \tag{57}$$

The debt-to-capital ratio is defined as the ratio of the firm's outstanding debt to its capital stock, which is the firm's only asset. Note that consumers take the firm's debt-to-capital ratio as given when solving their portfolio allocation problem. This is justified theoretically by the assumption of infinitesimally "small" consumers: each consumer perceives that her decision about how much debt to hold does not affect the *aggregate* debt-to-capital ratio.

4.3 Equilibrium and solution technique

In equilibrium, the tax exemption on corporate bonds is financed by the lump-sum tax on households. Firms receive proceeds $B_t/[1+(1-\tau)r_t]$ from issuing bonds, while households pay only $B_t/(1+r_t)$ (net of monitoring costs) to buy those bonds. Therefore, the lump-sum tax must equal:

$$\begin{aligned} T_t &= \left[\frac{1}{1+(1-\tau)r_t} - \frac{1}{1+r_t}\right] B_t \\ &= \left\{\frac{\tau r_t}{[1+(1-\tau)r_t](1+r_t)}\right\} B_t \end{aligned} \tag{58}$$

The risk-free bond $\tilde{B}_t$ is in zero net supply. On the other hand, corporate borrowing B_t will always be strictly positive because of the tax advantage. I normalize shares S_t to be 1 for all t.[26] In equilibrium, the consumer's budget constraint (44) then reduces to the following:

[26] Recall that conceptually, D_t represents dividends plus net repurchases of shares. However, because I normalize S_t to 1 for all t, D_t equals dividends in the model, and there is technically no issuing or repurchasing activity. If firms were allowed to change both D_t *and* the number of shares S_t, there would be an infinite set of optimal values for (D_t, S_t). To avoid this indeterminacy, I fix the number of shares; but I could just as well have fixed the dividend level and let firms choose the number of shares. In the discussion that follows, I will continue to interpret D_t as dividends plus net repurchases of shares.

$$C_t = D_t + B_{t-1} - \left[\frac{1}{1 + (1-\tau) r_t} + \mu L_t \right] B_t \tag{59}$$

I close the model by specifying a stochastic process for log productivity z_t.

$$z_t = z_{1t} + z_{2t} \tag{60}$$

$$\Delta z_{1t} = (1 - \rho_1) g + \rho_1 \Delta z_{1t-1} + \epsilon_{1t} \ , \ |\rho_1| < 1 \tag{61}$$

$$z_{2t} = \rho_2 z_{2t-1} + \epsilon_{2t} \ , \ |\rho_2| < 1 \tag{62}$$

$$E[\epsilon_{1t}] = E[\epsilon_{2t}] = 0 \ , \ Var[\epsilon_{1t}] = \sigma_1^2 \ , \ Var[\epsilon_{2t}] = \sigma_2^2 \tag{63}$$

$$\epsilon_{1t} \text{ and } \epsilon_{2t} \text{ iid over time and independent of each other} \tag{64}$$

This specification offers the flexibility of using a difference-stationary shock, an AR(1) shock, or a combination of both. An *equilibrium* is a sequence of prices and allocations such that all markets clear when consumers and firms behave optimally, taking equilibrium prices as given.

I characterize the equilibrium in two steps. First, I find the unique nonstochastic balanced growth path (BGP) equilibrium where the variables Z_t, K_t, B_t, D_t, C_t and P_t all grow at a constant rate and the variables L_t, M_t, $\tilde{r}_t$, r_t, and λ_t are constant. Next, I log-linearize the equations of the model around the nonstochastic BGP. This generates a system of linear expectational difference equations in a set of stationary variables. Finally, I solve this system computationally using a technique described by Uhlig (1997). The result is an equilibrium law of motion and an equilibrium policy rule.

5 Results

In this section I present results from simulating the model described in the previous section. First, I explain how the model was calibrated. I then show and discuss impulse response functions for debt and equity flows. Next, I discuss simulation results for the overall period 1952 – 2007. Finally, I calibrate the model to the two subperiods 1952 – 1983 and 1984 – 2007 in order to account for the change in volatilities of real and financial variables over the past two decades.

5.1 Calibration

Table 9 summarizes the calibrated parameter values for the model. I set η, the consumer's coefficient of relative risk aversion, to 2, a commonly used value in empirical macro studies. In the benchmark specification, I use an AR(1) shock only, so there is no trend growth ($g = 0$). I set the persistence of the shock, ρ_2, to 0.95. The share of capital in income, α, is set to 0.40, and the quarterly depreciation rate, δ, is set to 0.02. Following Jermann and Quadrini (2007), I set the marginal corporate tax rate τ to 0.30.

The rest of the parameters are calibrated to match particular moments in quarterly data for the U.S. nonfarm business sector over the period 1952:1 – 2007:4. Given the assumption of no trend growth, the subjective discount factor β pins down the real risk-free rate. I take the real risk-free rate to be the average annualized yield on three-month Treasury bills over the quarter, net of inflation.[27] This rate is 1.30%, which corresponds to a quarterly value of 0.99675 for β. I scale the technology shocks to match the standard deviation of GDP over the time period.

[27] I measure inflation as the annualized percent change in the CPI over the quarter.

Parameter	Value	Calibration Target	Target Value
η	2	Standard in literature	
β	0.99675	Real risk-free rate	1.30%
G	1	Zero growth in steady-state	
α	0.40	Standard in literature	
δ	0.02	Standard in literature	
τ	0.30	Jermann and Quadrini	
ρ_2	0.95	Standard in literature	
σ_2	0.0282	Std dev of GDP (x100)	2.09
μ	0.0384	Mean of Debt / GDP	0.62
ϕ	0.000655	Std dev of Equity Payout / GDP (x100)	1.14

Table 9: Calibration.

Given τ, the debt friction μ determines the steady-state level of corporate borrowing. A higher μ makes the cost of borrowing more sensitive to the amount borrowed, which in turn leads to less borrowing in the steady-state. I calibrate μ to match the average debt-to-GDP ratio in the nonfarm sector. The value of that ratio is 0.62, which corresponds to a value for μ of 0.0384. The equity friction parameter ϕ captures the cost of adjusting the firm's equity payout. A higher ϕ results in "smoother" equity payouts over time. I calibrate ϕ to match the standard deviation of net equity payouts as a share of GDP. The volatility in the data is 1.14, corresponding to a value for ϕ of 0.000655.[28]

[28] I also tried the following variations, none of which significantly altered the results. (i) I tried values for η in the range of 0.5 to 2, including $\eta = 1$ (log utility). (ii) I used a difference stationary shock, first with $g = 0$ and then with $g = \ln(1.5)$. (iii) I set $\alpha = 0.3$. (iv) I set $\tau = 0.2$, recalibrating μ as described above. Results available on request.

5.2 Impulse Responses

To fix intuition, it is helpful to look at impulse response functions for debt and equity flows. First, consider a *frictionless* RBC model with no debt financing and no equity adjustment cost. I represent this in the model by removing B_t as a choice variable and setting $\phi = 0$. Consumption in this model is just equal to the net equity payout.[29] Figure 9 shows the impulse response function for net equity payouts in the frictionless model. All impulse responses are expressed in terms of percentage deviations from steady-state values. In response to a positive, persistent productivity shock, the firm immediately raises its payout. The intuition is that after the shock, the firm's expected lifetime profitability is suddenly higher. Since the firm's objective is to maximize shareholder utility, and since shareholders place positive weight on consumption today, the firm optimally raises its payout immediately. As time goes on, the firm accumulates capital, increases output and raises its equity payout still higher. Eventually, as the shock dies out, the firm's payout peaks and then declines back to its steady-state value. It is important to realize that although the equity payout goes up on impact, the firm is also investing more. Greater investment is optimal because high productivity today forecasts high productivity tomorrow, which increases tomorrow's expected marginal product of capital. Figure 10 shows the impulse response for investment. Since the firm is not raising equity capital (it is instead increasing payments to shareholders), the firm finances its investment through its *stock of internal funds.* The firm prefers to finance using internal funds because issuing equity would detract too much from

[29]See equation (59). Recall that when there are no frictions, debt is indeterminate; this is the classic prediction of Modigliani and Miller (1958). Therefore, in the frictionless model, I do not allow firms to borrow, and I set $B_t = 0$ for all t.

shareholder utility. Note that today's positive productivity shock increases the stock of internal funds available for both investment and equity payouts.

Now consider the full model with both debt and equity financing. Figure 11 shows impulses response for the equity payout. I consider two cases: the dashed line sets $\phi = 0$, and the the solid line sets $\phi = 0.000655$ (its calibrated value). Consider first the case of no equity adjustment cost (labeled "No Friction" in the figures). In response to a positive and persistent productivity shock, the equity payout now increases steeply on impact, then returns quickly to its steady-state value. The intuition is that with access to tax-advantaged debt, the firm can "borrow from bondholders to pay shareholders." Despite the apparent spike in the equity payout, consumers still face a relatively smooth consumption profile, as shown in Figure 13. The reason is that consumers are also the bondholders in general equilibrium, and the net issuance of corporate bonds to consumers largely offsets the increase in equity payouts. Indeed, Figure 12 shows that with no equity adjustment cost, the firm's net issuance of debt on impact nearly equals the change in its equity payout. Unlike equity, however, debt declines gradually back towards its steady-state level. The ongoing issuance of debt finances capital investment; this is illustrated in Figure 14.

The introduction of the equity adjustment cost significantly dampens the firm's equity payout. In particular, Figure 11 shows that the increase of equity payouts on impact is less than one-fifth of the increase in the no-adjustment-cost scenario. Since adjustment costs are a pure deadweight loss, the firm faces a strong incentive to smooth its equity payout. As a result, the firm engages in much less "borrowing from bondholders to pay shareholders." Figure 12 shows that relative to the no-adjustment-cost case, the firm borrows less on impact and reaches peak borrowing

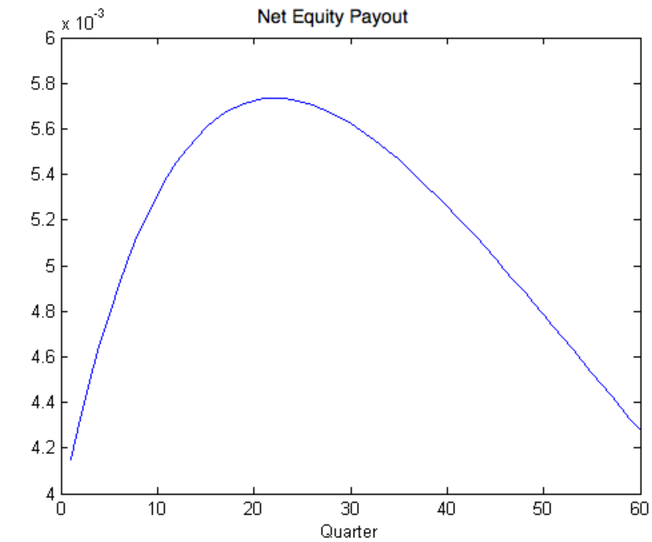

Figure 9: Impulse response for net equity payout in a frictionless model.

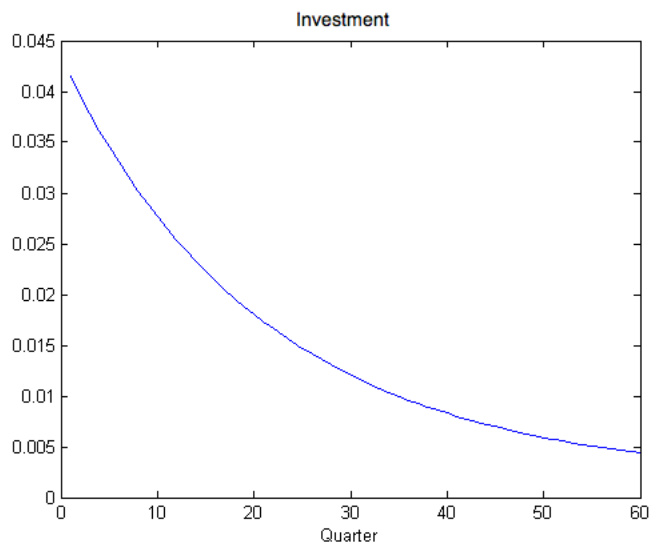

Figure 10: Impulse response for investment in a frictionless model.

only after a delay.

While the impact of the frictions on financial variables is significant, the impact on real allocations is very small. Figures 13 and 14 show that the impulse responses of consumption and investment do not change much when I add the calibrated equity adjustment cost. In addition, these impulse responses are very similar to their counterparts in the frictionless model with no debt. Contrast this result with Jermann and Quadrini (2007), where the severity of financial frictions has a significant effect on the impulse responses of real variables. The Jermann and Quadrini result depends on two key features not present in my model: (i) a debt ceiling above which the firm cannot borrow at any price, and (ii) an asset-price shock that alters the debt ceiling without affecting aggregate productivity. This suggests that the theoretical effect of financial frictions on the real economy depends critically on how both the frictions and the shocks are modeled.

5.3 Volatilities and Cross Correlations

Table 10 presents selected business cycle statistics from simulating the model and compares them with the corresponding moments in the data. I simulate the calibrated model by generating 50 sample paths of 200 quarters each for productivity, discarding the first 100 quarters. I compute business cycle statistics for each sample path and take averages over the 50 samples. I apply an HP filter with a smoothing parameter of 1600 to both actual and simulated data.[30] I consider the model with and without an equity adjustment cost. Looking at the standard deviations, the full model is able to replicate the volatility of the equity-payout-to-GDP ratio observed

[30]Results are similar when the simulated data is left unfiltered. Furthermore, the business cycle statistics in the data don't change much under alternative filters, such as the Baxter-King band-pass filter. Results available on request.

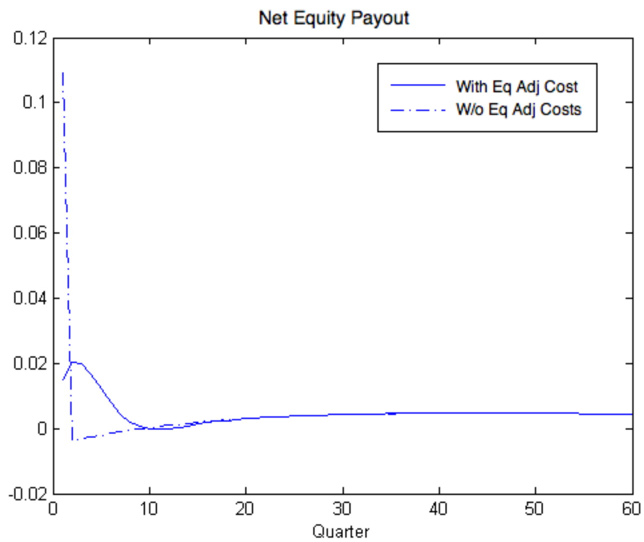

Figure 11: Impulse responses for net equity payout.

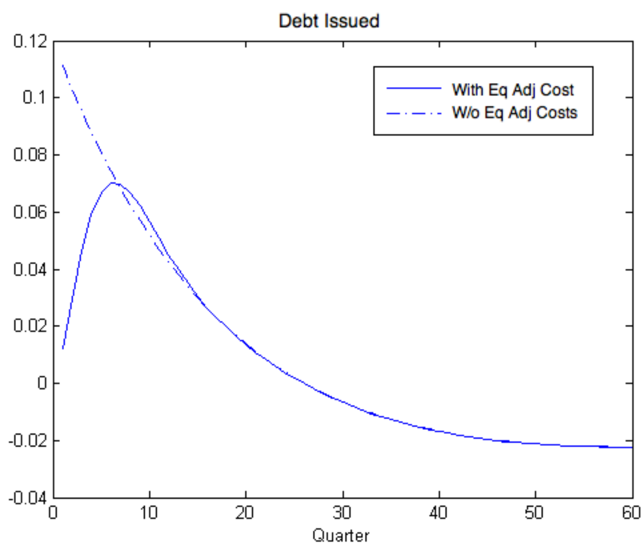

Figure 12: Impulse responses for debt.

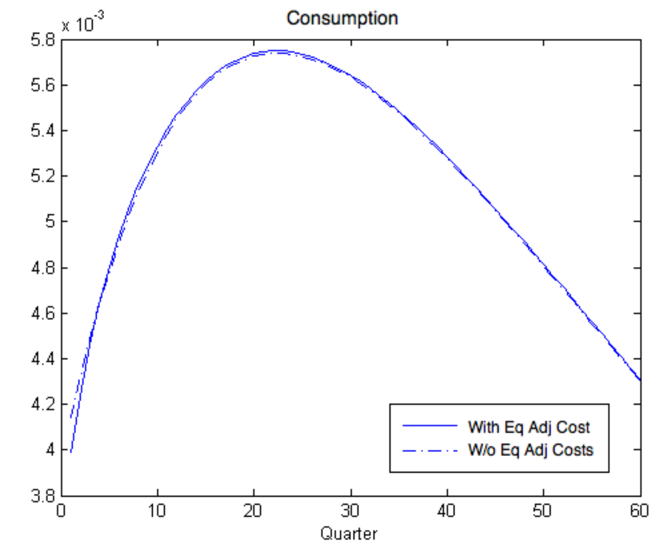

Figure 13: Impulse responses for consumption.

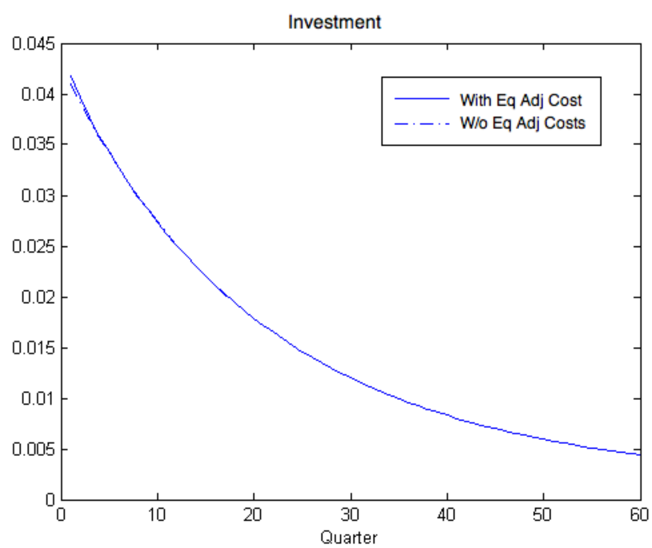

Figure 14: Impulse responses for investment.

in the data. Recall that I calibrated ϕ to 0.000655 in order to match this moment. Although this may appear to be a small friction, it makes a big difference. When I set $\phi = 0$, the implied volatility of equity payouts jumps from 1.14 to 6.51 – almost six times higher than in the data. As suggested by the impulse response functions, the adjustment cost results in "smoother" equity payouts over time.

Looking at correlations, equity payouts and debt issued are (i) positively correlated with output, (ii) positively correlated with investment, and (iii) positively correlated with each other, consistent with the stylized facts described in Section 2. These results hold both with and without equity adjustment costs. The tax advantage on debt, combined with monitoring costs, is sufficient to replicate these correlations. I interpret this as evidence that the interest tax deduction and concerns about excessive leverage influence the aggregate behavior of debt and equity flows over the business cycle.

These results also provide some evidence for a "dynamic tradeoff theory" of capital structure. Given the costs and benefits of debt, firms appear to target an optimal debt-equity ratio. However, the target itself changes over time as shocks impact firms' resources and alter their forecasts for future productivity. Of course, the evidence presented is all at the aggregate level. More convincing evidence of a dynamic tradeoff theory would require an empirical firm-level analysis, which is beyond the scope of this work.

The model is somewhat less successful at matching the absolute values of some key volatilities. Note that the volatility of (log) real GDP was calibrated to match the data. The model's volatility of investment is close to the data, as in standard RBC models. However, the model's volatility of debt is too high; and the model's volatilities for the ratio of outstanding debt to equity ("debt-equity ratio") and the

Standard Deviations (× 100)	*Data*	*Full Model*	ϕ = *0*
Equity Payout / GDP	1.14	1.14	6.49
Debt Issued / GDP	1.49	4.53	7.55
Debt Outstanding / Equity Outstanding	3.17	0.02	0.04
Equity Payout / Equity Outstanding	0.84	0.01	0.03
Log Real GDP	2.09	2.09	2.09
Log Fixed Investment	4.84	5.11	5.05

Correlations	*Data*	*Full Model*	ϕ = *0*
(Equity Payout, GDP)	0.16	0.81	0.39
(Debt Issued, GDP)	0.45	0.60	0.96
(Real Fixed Investment, GDP)	0.90	1.00	1.00
(Equity Payout, Real Fixed Investment)	0.19	0.83	0.40
(Debt Issued, Real Fixed Investment)	0.52	0.57	0.97
(Equity Payout, Debt Issued)	0.38	0.26	0.43

Table 10: Standard deviations and correlations from data and model, HP-filtered.

ratio of equity payments to equity outstanding ("payout-to-market-value") are too low. The latter two ratios depend in large part on the movement of stock prices in the data, which are much more volatile than predicted by most macro models. However, as I show below, the model is more successful at replicating *changes* in these volatilities over time.

5.4 Changes in Volatilities over Time

As documented in Section 2, the business cycle volatilities of output and investment have declined substantially starting in the mid-1980s, a well known phenomenon known as business cycle moderation. Jermann and Quadrini (2007) document that during the same period, the financial structure of firms has become more volatile – a finding that I also replicated in Section 2. In particular, the volatility of equity payouts has increased by over 50% from the period 1952 – 1983 to the period 1984 – 2005. In this section I demonstrate that the model can successfully account for the joint findings of dampened real volatility and increased financial volatility.

Since financial frictions do not have large real effects in my framework, I adopt the position of Arias et al. (2006) and assume that the decline in *real* volatility is a result of less volatile productivity shocks. On the other hand, I assume that the increase in financial volatility was driven by innovations in financial markets that eased the frictions in the model. Such innovations include the wide adoption of securitized assets and SEC rules facilitating greater flexibility in equity offerings and repurchases; see Jermann and Quadrini (2007) for other examples.

I proceed by calibrating the volatility of technology shocks σ_2, the debt friction μ and the equity friction ϕ separately for each subperiod. As before, my calibration targets are the standard deviation of GDP, the average debt-GDP ratio and the standard deviation of the equity-payout-to-GDP ratio. Table 11 presents my results. The model is fairly successful at matching relative volatilities across the two time periods. Given that GDP volatility declined by about 50%, the model predicts a 50% decline in the volatilities of real investment and consumption, consistent with the data. By calibrating μ, I am able to match a 50% *increase* in the average value of the debt-to-GDP ratio. By calibrating ϕ, I also reproduce the roughly 70% increase in the volatility of equity payouts. The model also matches, at least qualitatively, relative volatilities for four variables that were *not* calibration targets: the debt-issued-to-GDP ratio, the debt-equity ratio, the payout-to-market-value ratio, and the real risk-free rate. The results in Table 11 are calculated from HP-filtered data. Results from unfiltered data were similar and are available on request.

The model provides an alternative explanation for the joint findings of dampened

	1952 – 1983		*1984 – 2007*		*Late/Early*	
Mean	*Data*	*Model*	*Data*	*Model*	*Data*	*Model*
Debt Stock / GDP	0.51	0.51	0.78	0.78	1.53	1.53
Standard Deviations (× 100)	*Data*	*Model*	*Data*	*Model*	*Data*	*Model*
Equity Payout / GDP	0.85	0.86	1.44	1.44	1.69	1.67
Debt Issued / GDP	1.32	3.70	1.69	3.76	1.28	1.02
Debt Outst. / Eq. Outst.	2.83	0.02	3.57	0.02	1.26	1.11
Eq. Payout / Eq. Outst.	0.72	0.00	0.98	0.01	1.36	1.76
Log Real GDP	2.56	2.56	1.18	1.18	0.46	0.46
Log Investment	5.58	6.30	3.63	2.86	0.65	0.45
Log Consumption	1.47	0.63	0.73	0.29	0.49	0.46
Real T-Bill Rate	133.04	0.08	124.73	0.03	0.94	0.40

Table 11: Changes in business cycle statistics for the Nonfarm sector between 1952 – 1983 and 1984 – 2007.

real volatility and increased financial volatility over the past two decades. In my framework, the moderation in real business cycles is driven by the "good fortune" of less volatile productivity shocks, while the increase in financial volatility is a result of reduced financial frictions. Note that the reduction in financial frictions is sufficient to increase financial volatility even in the presence of dampened technology shocks, which by themselves would decrease financial volatility. In contrast, Jermann and Quadrini (2007) present a model where financial innovations drive both results. Their model relies on an asset price shock and an endogenous debt ceiling that transmits pure financial shocks to the real sector. In order to discriminate between the two explanations, one would need to pin down the relative importance of asset price shocks and technology shocks in the data. Identifying and quantifying different types of shocks involves many challenges, not the least of which is to arrive at meaningful and agreed-upon definitions. I defer this topic for future research.

6 Conclusion

I have shown that an RBC model with an explicit capital structure decision can explain a number of stylized facts about aggregate debt and equity flows in U.S. data. I developed an augmented RBC model characterized by three financial frictions: a debt tax shield, debt monitoring costs, and (optionally) an equity adjustment cost. The tax shield and costly monitoring pin down an optimal, positive amount of debt issued. The equity adjustment cost allows for more realistic fluctuations of equity payouts in response to technology shocks. In calibrated simulations, the model correctly implies that debt issued and equity payouts are both positively correlated with GDP, positively correlated with investment, and positively correlated with each other. Finally, I use the model to explain the finding of Jermann and Quadrini (2007) that real variables have become less volatile over the last two decades, while financial flows have become more volatile. By varying both the scale of the technology shocks and the degree of financial frictions, I can account for both results.

A number of avenues are available for further research. One straightforward extension would be to estimate the key parameters of the model (μ and ϕ) using a simulated method-of-moments technique. This would potentially generate a better fit with the data. The model has implications for the capital structure decisions of individual firms. For example, firms with high tax exposure and firms that are easily monitored should make greater use of debt financing than other firms. Furthermore, as a firm's tax exposure and other characteristics change over time, its reliance on debt financing should also change. These implications could be tested in firm-level panel data. A more ambitious extension would involve extending the model to an international setting. A two-country version of the model with trade in financial assets and asymmetric financial frictions would have predictions for debt and equity flows across countries. I plan to pursue these ideas in future work.

Chapter 3: International cross-holdings of bonds in a two-good DSGE model

1 Introduction

Evidence suggests that cross-border holdings of bonds are large. For example, at the end of 2007, foreign holdings of U.S. corporate bonds amounted to 28% of the outstanding value of those bonds, and foreign holdings of U.S. Treasuries were 48% of their outstanding value (Federal Reserve Flow of Funds). Large foreign holdings of sovereign domestic debt are also prominent in the U.K. (32%, U.K. Debt Management Office), France (60%, Agence France Trésor) and other OECD countries. This chapter asks a simple question: can a two-country, two-good equilibrium endowment model predict positive cross-holdings of bonds? If the elasticity of substitution between goods is sufficiently low, the answer is yes. However, the cutoff elasticity is at the lower end of estimates reported in the literature. For higher elasticities, the model predicts short foreign bond positions, which appear counterfactual for most advanced economies.

Most theoretical work on international diversification has focused on the "equity home bias puzzle": open economy macro models tend to predict much more cross-country diversification in equities than is observed in the data (see, e.g., Baxter and Jermann (1997)). A number of recent papers introduce bonds and equities together: Engel and Matsumoto (2010), Pavlova and Rigobon (2007), Coeurdacier et al. (2007) and Coeurdacier and Gourinchas (2008). However, all of these studies have more than one kind of shock in order to avoid portfolio indeterminacy. Furthermore, most of these models introduce bonds in order to improve the predictions for *equity* portfolios, rather than to study debt portfolios per se. Instead, I focus explicitly on bond portfolios in the simplest possible two-good model – one where the only shocks are to endowments.

2 Evidence on Foreign Bond Positions

Lane and Shambaugh (2010) offer a framework for evaluating the foreign currency exposure in a country's balance sheet. They compute a country's foreign currency exposure in debt instruments, FXD, as the difference between foreign currency debt assets ($FCAD$) and foreign currency debt liabilities ($FCLD$), divided by the sum of all foreign debt assets (AD) and debt liabilities (LD):

$$FXD = \frac{FCAD - FCLD}{AD + LD} \tag{65}$$

Table 12, column 1, presents this metric for a sample of advanced and emerging economies in 2004. Japan, China, India and Russia have long positions in foreign currency bonds; most other countries have short positions. However, as emphasized by Lane and Shambaugh (2010), FXD is driven primarily by a country's overall indebtedness, regardless of currency. Let $NFAD \equiv (AD - LD)/(AD + LD)$ be a country's net foreign asset position in debt instruments, normalized by the sum of all foreign debt assets and liabilities. Columns 1 and 2 of Table 12 show that with the possible exception of the United Kingdom, all countries with positive (negative) $NFAD$ also have positive (negative) foreign currency exposure.

Since the symmetric model that I analyze features zero net foreign assets in

Table 12: Foreign currency exposure in debt instruments for selected countries, 2004. FXD is the difference between foreign currency debt assets and foreign currency debt liabilities, divided by the sum of all foreign debt assets and liabilities (regardless of currency). $NFAD$ is the net foreign asset position in debt instruments, divided by the sum of all foreign debt assets and liabilities (regardless of currency). $FXD^0 = FXD - NFAD$. Source: Author's calculations based on data from Lane and Shambaugh (2010) and Lane and Milesi-Ferretti (2007).

Country	FXD	$NFAD$	FXD^0
United States	-0.03	-0.31	0.28
United Kingdom	0.00	-0.06	0.06
France	-0.06	-0.04	-0.02
Germany	-0.05	-0.02	-0.04
Japan	0.39	0.38	0.01
Canada	-0.23	-0.34	0.11
Italy	-0.05	-0.20	0.15
China	0.58	0.58	0.00
India	0.10	0.10	0.00
Brazil	-0.41	-0.41	0.00
Russia	0.17	0.17	0.00

the steady-state, it is useful to consider an empirical measure of bond positions that abstracts from overall indebtedness. Again following Lane and Shambaugh (2010), I compute the "centered" foreign currency exposure FXD^0 as the difference between FXD and $NFAD$:

$$FXD^0 = \frac{FCAD - FCLD}{AD + LD} - \frac{AD - LD}{AD + LD} \tag{66}$$

Conceptually, a positive value for FXD^0 indicates that a country would be long in foreign currency debt instruments if it had a zero net foreign asset position, holding the currency composition of assets and liabilities unchanged. Column 3 of Table 12 presents values for FXD^0. Except for France and Germany, the advanced countries in the sample have positive centered positions in foreign currency.[31] I interpret this as evidence that, abstracting from overall indebtedness, advanced countries tend to be long in foreign bonds.

3 The Model

The model economy consists of two countries, home (H) and foreign (F). Each country features a "Lucas tree" that delivers a stochastic endowment of a country-specific good, Y_t^i, with $i \in \{H, F\}$. Country endowments (in logs) are assumed to follow a joint AR(1) process:

[31] The centered positions for China, India, Brazil and Russia are zero because 100% of these countries' foreign (debt-based) assets and liabilities are in foreign currency.

$$\log Y_t^i = \rho \log Y_{t-1}^i + \epsilon_t^i \tag{67}$$

where $0 \leq \rho < 1$ and $\epsilon_t \equiv (\epsilon_t^H, \epsilon_t^F)$ is a vector of zero-mean i.i.d. shocks with variance-covariance matrix Σ. These endowment shocks are the only source of uncertainty in the model.

Each country is populated with a continuum of identical households of mass 1. Households in country i have preferences over a country-specific composite consumption good:

$$E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[\frac{(C_{t+j}^i)^{1-\gamma}}{1-\gamma} \right] \right\} \tag{68}$$

where $0 < \beta < 1$ is the subjective discount factor and $\gamma > 0$ is the (constant) coefficient of relative risk aversion. C_t^i denotes country i's consumption of its composite consumption good, which is a CES aggregate of home and foreign endowment goods:

$$C_t^i = \left[\lambda^{\frac{1}{\phi}} (C_t^{i,i})^{\frac{\phi-1}{\phi}} + (1-\lambda)^{\frac{1}{\phi}} (C_t^{i,j})^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \ , i \neq j \tag{69}$$

Here $C_t^{i,j}$ denotes country i's consumption of endowment good j. $\lambda \in (0,1)$ is the weight that households place on their own country's endowment good, and $\phi > 0$ is the elasticity of substitution between H and F endowment goods.

There is no money in the model; all variables are real. Let P_t^i denote the price of endowment good i in terms of a numeraire (to be specified shortly). The consumer price index in country i, $P^i_{C,t}$, is:

$$P^i_{C,t} = \left[\lambda(P^i_t)^{1-\phi} + (1-\lambda)(P^j_t)^{1-\phi}\right]^{\frac{1}{1-\phi}} \quad , \; i \neq j \tag{70}$$

Let the numeraire be a *world price index*:

$$\left(P^H_{C,t}\right)^{\frac{1}{2}}\left(P^F_{C,t}\right)^{\frac{1}{2}} = 1 \tag{71}$$

Define the terms of trade TOT_t and the real exchange rate RER_t as follows:

$$TOT_t = \frac{P^H_t}{P^F_t} \tag{72}$$

$$RER_t = \frac{P^H_{C,t}}{P^F_{C,t}} \tag{73}$$

The market-clearing conditions for goods are as follows:

$$C^{H,H}_t + C^{F,H}_t = Y^H_t \tag{74}$$

$$C^{F,F}_t + C^{H,F}_t = Y^F_t \tag{75}$$

3.1 Bond Regime

First consider an environment in which the only traded assets are two real, infinitely-lived, locally-denominated bonds ("consols"). The home (foreign) bond offers a stream of constant payoffs of home (foreign) endowment goods. Let $P^i_{B,t}$ denote the price of the bond that delivers good i. Returns are given as follows:

$$R^i_t = \frac{P^i_{B,t} + P^i_t}{P^i_{B,t-1}} \tag{76}$$

Let $A^{i,j}_{t-1}$ denote country i's holdings of the j-good bond at the end of period $t-1$, to be carried into period t. Note that asset holdings, asset prices and asset returns are all expressed in terms of the numeraire. I assume that bonds are in zero net supply:

$$A^{H,H}_t + A^{F,H}_t = 0 \tag{77}$$

$$A^{F,F}_t + A^{H,F}_t = 0 \tag{78}$$

Let $W^i_t \equiv A^{i,H}_t + A^{i,F}_t$ denote country i's financial wealth at the end of period t. Following Devereux and Sutherland (2008), rewrite country i's budget constraint as follows:

$$W^i_t = W^i_{t-1} R^F_t + A^{i,H}_{t-1} \left(R^H_t - R^F_t \right) + P^i_t Y^i_t - P^i_{C,t} C^i_t \tag{79}$$

A representative household in country i maximizes (68) subject to (69) and (79), taking prices as given.

3.2 Equity Regime

I also consider an alternative environment in which the only traded assets are two locally-denominated equities. The home (foreign) equity is a claim to the stochastic stream of payoffs associated with the home (foreign) endowment. Let $P^i_{E,t}$ denote the price of the equity associated with country i's endowment. Returns are given as follows:

$$R^i_t = \frac{P^i_{E,t} + P^i_t Y^i_t}{P^i_{E,t-1}} \tag{80}$$

In this regime, let $A^{i,j}_{t-1}$ denote country i's (value-based) holdings of country-j's equity at the end of period $t-1$. I normalize the (nominal) supply of each equity to 1, so the following resource constraints must hold:

$$A^{H,H}_t + A^{F,H}_t = P^H_{E,t} \tag{81}$$

$$A^{F,F}_t + A^{H,F}_t = P^F_{E,t} \tag{82}$$

Again let $W^i_t \equiv A^{i,H}_t + A^{i,F}_t$ denote country i's financial wealth at the end of period t. Country i's budget constraint in the equity regime can be written as follows:

$$W^i_t = W^i_{t-1} R^F_t + A^{i,H}_{t-1}\left(R^H_t - R^F_t\right) - P^i_{C,t} C^i_t \tag{83}$$

A representative household in country i maximizes (68) subject to (69) and (83), taking prices as given.

3.3 Equilibrium

An *equilibrium* (for either regime) is a sequence of stage-contingent values for consumption, asset holdings, goods prices, and asset prices such that all households behave optimally, taking prices as given, and goods and asset markets clear.

Following Coeurdacier et al. (2007), I further assume that $\gamma > 1$ (households are more risk-averse than log investors) and $1/2 < \lambda < 1$ (countries exhibit consumption home bias). These assumptions are not necessary to solve the model, but they do simplify the interpretation of the equilibrium portfolios.

4 Equilibrium Portfolios

I solve for "near-non-stochastic" steady-state bond portfolios using the methodology of Devereux and Sutherland (2008).[32] The Appendix contains detailed derivations of the following results.

Proposition 1: Let X_B denote the home (foreign) household's holdings of the home (foreign) bond, normalized by the price of the home (foreign) bond. In equilibrium, X_B is given by the following:

[32]The procedure involves taking second-order Taylor approximations of the first-order conditions for portfolios and first-order approximations of the remaining model equations. The "near-non-stochastic" steady-state bond portfolios are defined to be the (unique) constant portfolios that satisfy the approximated model. These locally-accurate portfolios can be interpreted as the true equilibrium portfolios in a world with an arbitrarily small amount of stochastic noise. See Devereux and Sutherland (2008) for details.

$$X_B = \frac{1}{2}\left[\left(1 - \frac{1}{\gamma}\right)(2\lambda - 1) + (\theta - 1)\right] \tag{84}$$

where $\theta \equiv \phi[1 - (2\lambda - 1)^2] + (2\lambda - 1)^2/\gamma$ is the inverse of the elasticity of the terms of trade (in logs) with respect to home's (log) relative endowment (Y_t^H/Y_t^F).[33]
Proof: See Appendix. ■

Recall that bonds are in zero net supply. Therefore, if $X_B > 0$, home households are long the home bond and short the foreign bond. Conversely, if $X_B < 0$, home households are long the foreign bond and short the home bond.

The first term in the equilibrium bond portfolio, $(1/2)(1 - 1/\gamma)(2\lambda - 1)$, is the optimal hedge against real exchange rate fluctuations. For $\gamma > 1$ and $\lambda > 1/2$, this term is positive. The real exchange rate is the relative price of home consumption. When the real exchange rate appreciates (and $\gamma > 1$), home's relative consumption expenditures rise. To finance the additional expenditures required in states of the world in which the real exchange rate is high, home households want to be long in home bonds, because the home bond delivers the higher payoff in these states.

The second term, $(1/2)(\theta - 1)$, is the optimal hedge against fluctuations in (log) relative endowment income $(P_t^H Y_t^H)/(P_t^F Y_t^F)$. After a negative home endowment shock, home's terms of trade rise, since home goods are relatively scarce. Home's relative endowment income may rise or fall depending on the value of θ, which is roughly the value of ϕ. If $\theta > 1$ (the goods are relatively close substitutes), then the terms of trade rise by less than the fall in the endowment, and home's relative

[33] I am grateful to an anonymous referee for suggesting this way of rewriting the bond portfolio and the interpretation that follows.

endowment income falls. To compensate for the income loss in this state of the world, home households want to be long in home bonds, because home bonds have the higher payoff when the terms of trade are high. Conversely, if $\theta < 1$ (the goods are poor substitutes), then the terms of trade rise by *more* than the fall in the endowment, and home's relative endowment income rises. Absent asset trade, the home country would be better off, and the foreign country worse off, after a negative home endowment shock. Home households now want to be long in foreign bonds to optimally share risk.

It is instructive to contrast these results with an environment in which only equity assets are traded.

Proposition 2: Consider an environment in which equities, rather than bonds, are traded. Let Sh_E denote the home (foreign) household's holdings of the home (foreign) equity, normalized by home (foreign) financial wealth. In equilibrium, Sh_E is given by the following:

$$Sh_E = \frac{1}{2}\left[1 - \left(1 - \frac{1}{\gamma}\right)\left(\frac{2\lambda - 1}{\theta - 1}\right)\right] \tag{85}$$

Proof: See Appendix. ■

Households exhibit "equity home bias" when $Sh_E > 0.5$. The first term, 1/2, reflects optimal diversification in a one-good world ($\phi \to \infty$): each country holds half its wealth in home equities and half in foreign equities, as in Lucas (1982). The second term, $-(1/2)(1 - 1/\gamma)(2\lambda - 1)/(\theta - 1)$, is the optimal hedge against real exchange rate fluctuations, as analyzed, for example, in Kollmann (2006) and

Coeurdacier (2009). For $\gamma > 1$ and $\lambda > 1/2$, this term is negative if and only if $\theta > 1$.[34]

From Propositions 1 and 2, it is straightforward to show the following:

Corollary 1: Assume that $\gamma > 1$ and $\lambda \in (1/2, 1)$. Let $\hat{\phi}$ be the unique value of ϕ such that $X_B = 0$. It follows that (i) $X_B < 0$ if and only if $\phi < \hat{\phi}$, and (ii) $Sh_E \in (1/2, 1)$ if and only if $\phi < \hat{\phi}$.

$Sh_E \in (1/2, 1)$ is the case of "realistic" equity home bias: home households hold more than half their wealth in home equities.[35] Corollary 1 shows that the set of elasticities for which realistic equity home bias attains (when only equities are traded) is the same set of elasticities for which countries take a long position in foreign bonds (when only bonds are traded). Both outcomes – equity home bias and long foreign bond positions – require the elasticity of substitution between home and foreign goods to be relatively low.

Table 13 tabulates values for $\hat{\phi}$ for various combinations of $\gamma > 1$ and $\lambda \in (1/2, 1)$. All values of $\hat{\phi}$ in this parameter space are less than 1. For example, if $\gamma = 1.5$ and $\lambda = 0.8$, then for any value of ϕ less than 0.88, each country takes a long position in foreign bonds and a short position in domestic bonds (when only bonds are traded). Estimates for ϕ vary widely in the literature, but most recent studies estimate ϕ to be greater than 1 (Coeurdacier, 2009). If $\phi > 1$, the model predicts that countries will be long in domestic bonds and short in foreign bonds – a pattern that appears

[34] Kollmann (2006) and Coeurdacier (2009) also report (85) as the optimal equity portfolio in two-period versions of the current model. In Coeurdacier (2009), home bias in consumption is replaced with trade costs.

[35] For $\phi > \hat{\phi}$, Sh_E may be greater than 1. In this case, the home portfolio consists of a leveraged bet on home equity and a short position in foreign equity. The short foreign equity position seems empirically implausible.

Table 13: Values for $\hat{\phi}$ such that $X_B(\gamma, \lambda, \hat{\phi}) = 0$. For $\phi < \hat{\phi}$, countries hold a long position in foreign bonds (when only bonds are traded) and exhibit equity home bias (when only equities are traded).

γ/λ	0.6	0.7	0.8	0.9
1.25	0.97	0.94	0.93	0.91
1.5	0.94	0.90	0.88	0.85
2	0.92	0.86	0.81	0.78
5	0.87	0.77	0.70	0.64

counterfactual for most advanced economies.

5 Conclusion

I have shown that a two-country, two-good DSGE endowment model can generate long positions in foreign bonds if the elasticity of substitution between goods is sufficiently low. For commonly estimated elasticities, however, the model predicts that each country will hold a short position in foreign bonds.

Appendix

A Data sources (Chapter 1)

This section describes the data used to compute business cycle statistics for the U.S. and G7 gross equity positions. The source data (time series) for Tables 1 through 3 are as follows:

- fda_n: Gross foreign direct investment (FDI) assets, at market value, in current U.S. dollars. Source: Lane and Milesi-Ferretti (2007) (LM)

- pea_n: Gross portfolio equity (PE) assets, at market value, in current U.S. dollars. Source: LM

- fdl_n: Gross FDI liabilities, at market value, in current U.S. dollars. Source: LM

- pel_n: Gross PE liabilities, at market value, in current U.S. dollars. Source: LM

- fdo_n: Gross FDI outflows in current U.S. dollars. Source: UNCTAD's World Investment Report (WIR)

- peo_n: Gross PE outflows in current U.S. dollars. Source: IMF's International Financial Statistics (IFS)

- fdi_n: Gross FDI inflows in current U.S. dollars. Source: WIR

- pei_n: Gross PE inflows in current U.S. dollars. Source: IFS

- y: Real GDP in constant 2000 U.S. dollars. Source: World Bank's World Development Indicators (WDI)

- y_n^g: World GDP in current U.S. dollars. Source: IMF's World Economic Outlook (WEO)

- y^g: Real world GDP in constant 2000 U.S. dollars. Source: Author's calculations using y_n^g and annual real GDP growth rates from WEO

The n subscripts above indicate that a variable is nominal. Real variables carry no subscript.

All data sets measure FDI in accordance with the IMF's Balance of Payments manual. Accordingly, FDI consists of any cross-border equity transaction that involves an investor in one country acquiring a "lasting interest" in a foreign firm. In practice, an investor is assumed to obtain a lasting interest if he or she acquires 10% or more of a firm's outstanding equity; but other transactions can be counted as FDI if there is evidence that the investor gained "an effective voice in management." Greenfield investments – an investor in one country starting a new firm in a foreign country – are an important form of FDI. Finally, financial transactions between parents and foreign subsidiaries are also counted as FDI. Cross-border equity purchases that do not qualify as FDI (typically because they lead to less than 10% ownership) are classified as portfolio equity.

Data from Lane and Milesi-Ferretti (2007) covers 1970 – 2004 and is only available at an annual frequency. The world GDP series from WEO is available starting in 1980. Most of the data from UNCTAD and IFS covers at least 1980 – 2004. Two exceptions are France, for which portfolio equity flows are only available starting in 1983; and Italy, for which portfolio equity inflows are only available starting in

1989. I restrict my attention to the years 1980 – 2004, using shorter samples when necessary for France and Italy.

I compute total equity-based assets, liabilities, outflows and inflows as follows:

- $eqa_n = fda_n + pea_n$: Gross FDI plus portfolio equity (PE) assets in current U.S. dollars
- $eql_n = fdl_n + pel_n$: Gross FDI plus PE liabilities in current U.S. dollars
- $eqo_n = fdo_n + peo_n$: Gross FDI plus PE outflows in current U.S. dollars
- $eqi_n = fdi_n + pei_n$: Gross FDI plus PE inflows in current U.S. dollars

I compute valuation effects as follows. Consider FDI assets (fda_n) as an example. For each year ($t = 1981 - 2004$), I compute the change in FDI assets from the end of the prior year to the end of the current year: $\Delta fda_{n,t} = fda_{n,t} - fda_{n,t-1}$. I then subtract gross FDI outflows in time t: $fdav_{n,t} \equiv \Delta fda_{n,t} - fdo_{n,t}$. The difference, $fdav_{n,t}$, is the valuation effect: the capital gain or loss on the prior year's FDI assets. I use an analogous technique to compute valuation effects for PE assets ($peav_n$), FDI liabilities ($fdlv_n$) and PE liabilities ($pelv_n$). Finally, I compute total valuation effects for assets and liabilities: $eqav_n = fdav_n + peav_n$ and $eqlv_n = fdlv_n + pelv_n$.

I work with real time series expressed in constant 2000 U.S. dollars. To do so, I first compute a global GDP deflator as follows:

- $p^g = y_n^g / y^g$: Global GDP deflator ($2000 = 1$)

I then divide each nominal variable by the global GDP deflator to obtain a real time series; e.g., $fda = fda_n / p^g$. These series, along with y, are the ones used to compute business cycle statistics. I apply a Hodrick-Prescott filter to each real series using a smoothing parameter of 6.25, which is the value that Ravn and Uhlig (2002) suggest for annual data. Note that I filter actual data only, not simulated model data.

For Tables 4 and 5, from 1987 – 2004, I have two additional source variables:

- mao_n: Gross merger and acquisition (MA) outflows in current U.S. dollars. Source: WIR

- mai_n: Gross MA inflows in current U.S. dollars. Source: WIR

Following Calderón et al. (2004), I obtain a rough proxy for greenfield flows by subtracting MA from total flows:

- $gro_n = fdo_n - mao_n$: Gross greenfield FDI outflows in current U.S. dollars

- $gri_n = fdi_n - mai_n$: Gross greenfield FDI inflows in current U.S. dollars

I then deflate and detrend these variables as described above.

B Additional model details (Chapter 1)

B.1 Benchmark model (Section 3)

B.1.1 Foreign household's problem

The preferences for foreign households are:

$$E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{\widehat{C}_{t+j}^{1-\gamma}}{1-\gamma} \right] \tag{86}$$

where $\widehat{C}_t$ is a CES aggregate of home and foreign output goods:

$$\widehat{C}_t = \left[(1-\lambda)^{\frac{1}{\phi}} \left(\widehat{C}_t^H\right)^{\frac{\phi-1}{\phi}} + \lambda^{\frac{1}{\phi}} \left(\widehat{C}_t^F\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{87}$$

Here $\widehat{C}_t^H$ and $\widehat{C}_t^F$ denote the foreign household's consumption of home and foreign output goods, respectively. The budget constraint for foreign households is:

$$\begin{gathered}\widehat{P}_t\widehat{C}_t + P_t^{X,H}\widehat{A}_t^H + P_t^{X,F}\widehat{A}_t^F = \\ (P_t^{X,H} + D_t^H)\widehat{A}_{t-1}^H + (P_t^{X,F} + D_t^F)\widehat{A}_{t-1}^F - \frac{\psi}{2}P_t^H\left(\widehat{A}_t^H - \widehat{A}^H\right)^2\end{gathered} \tag{88}$$

Here $\widehat{A}_t^H$ and $\widehat{A}_t^F$ denote the number of shares of home and foreign equity held by the foreign household at the end of time t, and $\widehat{A}^H$ is the long-run (steady-state) value for $\widehat{A}_t^H$. Note that the financial costs on foreign short-run holdings of home equity are paid in units of home output goods. The foreign household's problem is to maximize (86) subject to (87) and (88).

B.1.2 Foreign firm's problem

The foreign firm's production function is:

$$Y_t^F = Z_t^F \left(K_{t-1}^F\right)^{\theta} \tag{89}$$

Here K_{t-1}^F is the foreign firm's capital stock at the end of period $t-1$, available for production in period t; Y_t^F is the output produced by the foreign firm; and Z_t^F is an aggregate productivity shock affecting all firms operating in the foreign country. Foreign firms solve the following problem:

$$\max_{K_t^F, D_t^F} D_t^F + \sum_{j=1}^{\infty} \widehat{M}_{t+j,t} D_{t+j}^F$$
$$\text{s.t. } D_t^F = P_t^F \left[Y_t^F + (1-\delta) K_{t-1}^F - K_t^F \right] \tag{90}$$

B.1.3 Additional first-order conditions

The home household's first-order conditions for home and foreign output goods are:

$$\frac{C_t^H}{C_t} = \lambda \left(\frac{P_t^H}{P_t} \right)^{-\phi} \tag{91}$$
$$\frac{C_t^F}{C_t} = (1-\lambda) \left(\frac{P_t^F}{P_t} \right)^{-\phi} \tag{92}$$

The foreign household's first-order conditions for home and foreign output goods are:

$$\frac{\widehat{C}_t^H}{\widehat{C}_t} = (1-\lambda) \left(\frac{P_t^H}{\widehat{P}_t} \right)^{-\phi} \tag{93}$$
$$\frac{\widehat{C}_t^F}{\widehat{C}_t} = \lambda \left(\frac{P_t^F}{\widehat{P}_t} \right)^{-\phi} \tag{94}$$

The foreign firm's first-order condition is:

$$1 = E_t \left[\widehat{M}_{t+1,t} \frac{P_{t+1}^F}{P_t^F} \left(\frac{\theta Y_{t+1}^F}{K_t^F} + 1 - \delta \right) \right] \tag{95}$$

B.2 FDI and portfolio equity (Section 4)

B.2.1 Foreign household's budget constraint

The foreign household's budget constraint is:

$$\begin{aligned} &\widehat{P}_t \widehat{C}_t + P_t^{N,H} \widehat{A}_t^{N,H} + P_t^{N,F} \widehat{A}_t^{N,F} + P_t^{G,H} \widehat{A}_t^{G,H} + P_t^{G,F} \widehat{A}_t^{G,F} \\ &= \left(P_t^{G,H} + D_t^{N,H} \right) \widehat{A}_{t-2}^{N,H} + \left(P_t^{G,F} + D_t^{N,F} \right) \widehat{A}_{t-2}^{N,F} \\ &\quad + D_t^{G,H} \widehat{A}_{t-1}^{G,H} + D_t^{G,F} \widehat{A}_{t-1}^{G,F} \\ &\quad - \frac{\psi}{2} P_t^H \left(\widehat{A}_t^{N,H} - \widehat{A}^{N,H} \right)^2 - \frac{\psi}{2} P_t^H \left(\widehat{A}_t^{G,H} - \widehat{A}^{G,H} \right)^2 \end{aligned} \tag{96}$$

B.2.2 Additional first-order conditions

The first-order condition for the foreign ongoing firm is:

$$1 = E_t \left[\widehat{M}_{t+1,t} \frac{P_{t+1}^F}{P_t^F} \left(\frac{\theta Y_{t+1}^{G,F}}{K_t^{G,F}} + 1 - \delta \right) \right] \tag{97}$$

The home household's first-order conditions for C_t^H and C_t^F – and the foreign household's first-order conditions for $\widehat{C}_t^H$ and $\widehat{C}_t^F$ – are unchanged from the benchmark model. The home household's first-order conditions for equity holdings are:

$$1 = E_t \left[M_{t+2,t} R_{t+2}^{N,H} \right] \tag{98}$$

$$1 = E_t \left[M_{t+2,t} R_{t+2}^{*N,F} \right] \tag{99}$$

$$1 = E_t \left[M_{t+1,t} R_{t+1}^{G,H} \right] \tag{100}$$

$$1 = E_t \left[M_{t+1,t} R_{t+1}^{*G,F} \right] \tag{101}$$

$$\text{where } M_{t+j,t} \equiv \frac{\beta^j C_{t+j}^{-\gamma} P_{t+j}^{-1}}{C_t^{-\gamma} P_t^{-1}} \tag{102}$$

And the foreign household's first-order conditions for equity holdings are:

$$1 = E_t \left[\widehat{M}_{t+2,t} R_{t+2}^{*N,H} \right] \tag{103}$$

$$1 = E_t \left[\widehat{M}_{t+2,t} R_{t+2}^{N,F} \right] \tag{104}$$

$$1 = E_t \left[\widehat{M}_{t+1,t} R_{t+1}^{*G,H} \right] \tag{105}$$

$$1 = E_t \left[\widehat{M}_{t+1,t} R_{t+1}^{G,F} \right] \tag{106}$$

$$\text{where } \widehat{M}_{t+j,t} \equiv \frac{\beta^j \widehat{C}_{t+j}^{-\gamma} \widehat{P}_{t+j}^{-1}}{\widehat{C}_t^{-\gamma} \widehat{P}_t^{-1}} \tag{107}$$

Equity returns are as follows:

$$R_{t+2}^{N,H} \equiv \frac{P_{t+2}^{G,H} + D_{t+2}^{N,H}}{P_t^{N,H}} \tag{108}$$

$$R_{t+2}^{N,F} \equiv \frac{P_{t+2}^{G,F} + D_{t+2}^{N,F}}{P_t^{N,F}} \tag{109}$$

$$R_{t+1}^{G,H} \equiv \frac{D_{t+1}^{G,H}}{P_t^{G,H}} \tag{110}$$

$$R_{t+1}^{G,F} \equiv \frac{D_{t+1}^{G,F}}{P_t^{G,F}} \tag{111}$$

$$R_{t+2}^{*N,H} \equiv \frac{P_{t+2}^{G,H} + D_{t+2}^{N,H}}{P_t^{N,H} + \psi P_t^H \left(\widehat{A}_t^{N,H} - \widehat{A}^{N,H} \right)} \tag{112}$$

$$R_{t+2}^{*N,F} \equiv \frac{P_{t+2}^{G,F} + D_{t+2}^{N,F}}{P_t^{N,F} + \psi P_t^F \left(A_t^{N,F} - A^{N,F} \right)} \tag{113}$$

$$R_{t+1}^{*G,H} \equiv \frac{D_{t+1}^{G,H}}{P_t^{G,H} + \psi P_t^H \left(\widehat{A}_t^{G,H} - \widehat{A}^{G,H} \right)} \tag{114}$$

$$R_{t+1}^{*G,F} \equiv \frac{D_{t+1}^{G,F}}{P_t^{G,F} + \psi P_t^F \left(A_t^{G,F} - A^{G,F} \right)} \tag{115}$$

C Solution method (Chapter 1)

The models are amenable to standard perturbation techniques. I derive the unique non-stochastic steady-state analytically. For the model of FDI and portfolio equity (Section 4), I use a numerical technique to pin down some steady-state values, such as the capital stocks of new firms. I then use DYNARE to take a second-order Taylor approximation and solve for second-order policy functions. I verify that there is a unique stationary transition path. Since the model has no trend, I do not filter simulated model data. The business cycle correlations I report are the theoretical moments reported by DYNARE. Results from simulations using the pruning algorithm of Kim et al. (2008) are broadly similar.

D Derivation of Euler Equations for Firm's Problem (Chapter 2)

The firm's problem can be written as follows:

$$V(K_{t-1}, B_{t-1}, D_{t-1}) = \max_{B_t, D_t, K_t} \{D_t + E_t M_{t+1} V(K_t, B_t, D_t)\}$$
$$+ \lambda_t \left\{ F(K_{t-1}, Z_t) + (1-\delta)K_{t-1} + \frac{B_t}{1+(1-\tau)r_t} - B_{t-1} - c(D_t, D_{t-1}, K_{t-1}) - K_t \right\} \tag{116}$$

Taking first-order conditions:

$$(B_t) : E_t M_{t+1} V_B(K_t, B_t, D_t) + \frac{\lambda_t}{1+(1-\tau)r_t} = 0 \tag{117}$$

$$(D_t) : 1 + E_t M_{t+1} V_D(K_t, B_t, D_t) - \lambda_t c_1(D_t, D_{t-1}, K_{t-1}) = 0 \tag{118}$$

$$(K_t) : E_t M_{t+1} V_K(K_t, B_t, D_t) - \lambda_t = 0 \tag{119}$$

From the envelope conditions, we have:

$$V_K(K_t, B_t, D_t) = \lambda_{t+1} \left[F_K(K_t, Z_{t+1}) + (1 - \delta) - c_3(D_{t+1}, D_t, K_t) \right] \tag{120}$$

$$V_B(K_t, B_t, D_t) = -\lambda_{t+1} \tag{121}$$

$$V_D(K_t, B_t, D_t) = -\lambda_{t+1} c_2(D_{t+1}, D_t, K_t) \tag{122}$$

Substituting the envelope conditions back into the first-order conditions:

$$(B_t) : \frac{\lambda_t}{1 + (1 - \tau) r_t} - E_t M_{t+1} \lambda_{t+1} = 0 \tag{123}$$

$$(D_t) : 1 - E_t M_{t+1} \lambda_{t+1} c_2(D_{t+1}, D_t, K_t) - \lambda_t c_1(D_t, D_{t-1}, K_{t-1}) = 0 \tag{124}$$

$$(K_t) : E_t M_{t+1} \lambda_{t+1} \left[F_K(K_t, Z_{t+1}) + (1 - \delta) - c_3(D_{t+1}, D_t, K_t) \right] - \lambda_t = 0 \tag{125}$$

I impose the following functional forms for the production function and dividend-adjustment cost function:

$$F(K_{t-1}, Z_t) = K_{t-1}^{\alpha} Z_t^{1-\alpha} \ , \ \alpha \in (0, 1) \tag{126}$$

$$c(D_t, D_{t-1}, K_{t-1}) = D_t + \phi K_{t-1} \left(\frac{D_t}{D_{t-1}} - 1 \right)^2 \ , \ \phi > 0 \tag{127}$$

Applying the functional forms above to equations (123), (124) and (125) gives the Euler Equations listed in the main text.

E Equations Characterizing Equilibrium (Chapter 2)

For convenience, all the equations of the model are reproduced here:

$$\frac{1}{1+\tilde{r}_t} = E_t[M_{t+1}] \tag{128}$$

$$P_t = E_t[M_{t+1}(P_{t+1} + D_{t+1})] \tag{129}$$

$$M_t \equiv \frac{\beta u'(C_t)}{u'(C_{t-1})} \tag{130}$$

$$\frac{1}{1+r_t} = \frac{1}{1+\tilde{r}_t} - \mu L_t \tag{131}$$

$$\begin{aligned} K_t = K_{t-1}^{\alpha} Z_t^{1-\alpha} + (1-\delta)K_{t-1} + \frac{B_t}{1+(1-\tau)r_t} - B_{t-1} \\ - D_t - \phi K_{t-1}\left(\frac{D_t}{D_{t-1}} - 1\right)^2 \end{aligned} \tag{132}$$

$$\frac{\lambda_t}{1+(1-\tau)r_t} = E_t M_{t+1}\lambda_{t+1} \tag{133}$$

$$\begin{aligned} 1 + E_t\left\{2\phi M_{t+1}\lambda_{t+1}\left(\frac{K_t}{D_t}\right)\left(\frac{D_{t+1}}{D_t}\right)\left(\frac{D_{t+1}}{D_t} - 1\right)\right\} = \\ \lambda_t\left\{1 + 2\phi\left(\frac{K_{t-1}}{D_{t-1}}\right)\left(\frac{D_t}{D_{t-1}} - 1\right)\right\} \end{aligned} \tag{134}$$

$$E_t M_{t+1}\lambda_{t+1}\left\{\alpha\left(\frac{Z_{t+1}}{K_t}\right)^{1-\alpha} + (1-\delta) - \phi\left(\frac{D_{t+1}}{D_t} - 1\right)^2\right\} = \lambda_t \tag{135}$$

$$L(t) \equiv \frac{B_t}{K_{t-1}} \tag{136}$$

$$C_t = D_t + B_{t-1} - \left[\frac{1}{1+(1-\tau)r_t} + \mu L_t\right] B_t \tag{137}$$

$$z_t = z_{1t} + z_{2t} \tag{138}$$

$$\Delta z_{1t} = (1-\rho_1)g + \rho_1 \Delta z_{1t-1} + \epsilon_{1t} \ , \ |\rho_1| < 1 \tag{139}$$

$$z_{2t} = \rho_2 z_{2t-1} + \epsilon_{2t} \ , \ |\rho_2| < 1 \tag{140}$$

$$E[\epsilon_{1t}] = E[\epsilon_{2t}] = 0 \ , \ Var[\epsilon_{1t}] = \sigma_1^2 \ , \ Var[\epsilon_{2t}] = \sigma_2^2 \tag{141}$$

F Bond and Equity Portfolios (Chapter 3)

My solution approach is based on Devereux and Sutherland (2008). They use second-order approximations of the portfolio equations (i.e., the first-order conditions for portfolio holdings) and first-order approximations of all non-portfolio equations to jointly solve for the first-order behavior of non-portfolio variables and the zero-order values for portfolio holdings. All non-portfolio variables are approximated around the non-stochastic steady-state. The zero-order solutions for portfolio holdings can be interpreted as the true equilibrium portfolios is a “near-non-stochastic” world; i.e., in a world with an arbitrarily small amount of stochastic noise (Devereux and Sutherland, 2008). In this section, I explain in detail how I implemented this approach to derive the solutions for bond and equity holdings (Propositions 1 and 2 in the main text).

F.1 First-Order Conditions

For either asset regime, the first-order conditions for the home household can be written as follows:

$$C_t^{H,H} = \lambda \left(\frac{P_t^H}{P_{C,t}^H} \right)^{-\phi} C_t^H \tag{142}$$

$$C_t^{H,F} = (1-\lambda) \left(\frac{P_t^F}{P_{C,t}^H} \right)^{-\phi} C_t^H \tag{143}$$

$$E_t \left[M_{t+1}^H R_{t+1}^F \right] = 1 \tag{144}$$

$$E_t \left[\left(C_{t+1}^H \right)^{-\gamma} \left(P_{C,t+1}^H \right)^{-1} R_{t+1}^H \right] = E_t \left[\left(C_{t+1}^H \right)^{-\gamma} \left(P_{C,t+1}^H \right)^{-1} R_{t+1}^F \right] \tag{145}$$

$$\text{where } M_{t+1}^H \equiv \beta \left(\frac{C_{t+1}^H}{C_t^H} \right)^{-\gamma} \left(\frac{P_{C,t+1}^H}{P_{C,t}^H} \right)^{-1} \tag{146}$$

The first-order conditions for the foreign household are:

$$C_t^{F,F} = \lambda \left(\frac{P_t^F}{P_{C,t}^F} \right)^{-\phi} C_t^F \tag{147}$$

$$C_t^{F,H} = (1-\lambda) \left(\frac{P_t^H}{P_{C,t}^F} \right)^{-\phi} C_t^F \tag{148}$$

$$E_t \left[M_{t+1}^F R_{t+1}^F \right] = 1 \tag{149}$$

$$E_t \left[\left(C_{t+1}^F \right)^{-\gamma} \left(P_{C,t+1}^F \right)^{-1} R_{t+1}^H \right] = E_t \left[\left(C_{t+1}^F \right)^{-\gamma} \left(P_{C,t+1}^F \right)^{-1} R_{t+1}^F \right] \tag{150}$$

$$\text{where } M_{t+1}^F \equiv \beta \left(\frac{C_{t+1}^F}{C_t^F} \right)^{-\gamma} \left(\frac{P_{C,t+1}^F}{P_{C,t}^F} \right)^{-1} \tag{151}$$

Note that M_{t+1}^i is the one-period-ahead stochastic discount factor for country i

expressed in terms of the numeraire.

F.2 Non-Stochastic Steady-State

In my model I approximate around the *symmetric* non-stochastic steady-state, which can be characterized as follows:

$$M^H = M^F = \beta \tag{152}$$

$$R^H = R^F = \frac{1}{\beta} \tag{153}$$

$$P^H = P^F = P_C^H = P_C^F = TOT = RER = 1 \tag{154}$$

$$C^H = C^F = Y^H = Y^F = 1 \tag{155}$$

$$C^{H,H} = C^{F,F} = \lambda \tag{156}$$

$$C^{H,F} = C^{F,H} = 1 - \lambda \tag{157}$$

Note that steady-state output in each country is normalized to 1. In addition, for the bond regime we have:

$$W^H = W^F = 0 \tag{158}$$

$$P_B^H = P_B^F = \frac{\beta}{1-\beta} \equiv P_B \tag{159}$$

And for the equity regime we have:

$$W^H = W^F = P_E^H = P_E^F = \frac{\beta}{1-\beta} \equiv W \tag{160}$$

F.3 Terms of trade, real exchange rate, and relative consumption expenditures

I begin by deriving first-order (log-linear) approximations relating the terms of trade, the real exchange rate and relative consumption expenditures to relative endowments. The resulting equations will be re-used often to derive both the bond and equity portfolios. This step parallels Coeurdacier (2009) quite closely.

Combine the first-order conditions (142), (143), (147) and (148) with the goods market-clearing conditions (74) and (75) and the definition of the terms of trade (72) to show that:

$$TOT_t^{-\phi} \cdot \Omega \left[\left(\frac{P_{C,t}^F}{P_{C,t}^H} \right)^{\phi} \frac{C_t^F}{C_t^H} \right] = \frac{Y_t^H}{Y_t^F} \tag{161}$$

$$\text{where } \Omega(x) \equiv \frac{\lambda + (1-\lambda)x}{\lambda x + (1-\lambda)}$$

Now define relative (home divided by foreign) consumption expenditures PC_t as follows:

$$PC_t \equiv \frac{P_{C,t}^H C_t^H}{P_{C,t}^F C_t^F}$$

Using this definition and the definition of the real exchange rate (73), rewrite

(161) as follows:

$$TOT_t^{-\phi} \cdot \Omega \left[\frac{RER_t^{1-\phi}}{PC_t} \right] = \frac{Y_t^H}{Y_t^F} \tag{162}$$

Log-linearize (162) around the symmetric steady-state to get:

$$-\phi \widehat{TOT}_t + (2\lambda - 1) \left[(\phi - 1)\widehat{RER}_t + \widehat{PC}_t \right] = \widehat{Y}_t^H - \widehat{Y}_t^F \tag{163}$$

Except where stated otherwise, "hatted" variables denote log-deviations from steady-state: $\widehat{X}_t \equiv \log(X_t/X)$.[36] Next, log-linearize the expressions for the price indices (70):

$$\widehat{P}_{C,t}^H = \lambda \widehat{P}_t^H + (1-\lambda)\widehat{P}_t^F \tag{164}$$

$$\widehat{P}_{C,t}^F = \lambda \widehat{P}_t^F + (1-\lambda)\widehat{P}_t^H \tag{165}$$

Next, express the log-linearized real exchange rate as follows:

$$\widehat{RER}_t = \widehat{P}_{C,t}^H - \widehat{P}_{C,t}^F = \eta \widehat{TOT}_t \tag{166}$$

where $\eta \equiv 2\lambda - 1$

Combining (163) and (166) and solving for $\widehat{TOT}_t$:

[36]The exceptions are $\widehat{R}_t^X$ and $\widehat{W}_t^i$ (for bonds); these will be defined below.

$$\widehat{TOT}_t = \eta\psi\widehat{PC}_t - \psi\widehat{Y}_t^{H,F} \tag{167}$$

$$\text{where } \psi \equiv \left[\phi - \eta^2(\phi - 1)\right]^{-1} \text{ and } \widehat{Y}_t^{H,F} \equiv \widehat{Y}_t^H - \widehat{Y}_t^F$$

Equation (167) provides a link between $\widehat{TOT}_t$ and $\widehat{PC}_t$ (both endogenous) and $\widehat{Y}_t^{H,F}$ (exogenous). We will reuse this expression repeatedly when deriving steady-state bond and equity portfolios.

Next, log-linearize the home and foreign FOCs (144) and (149):

$$E_t\left[-\gamma(\widehat{C}_{t+1}^H - \widehat{C}_t^H) - (\widehat{P}_{C,t+1}^H - \widehat{P}_{C,t}^H) + \widehat{R}_{t+1}^H\right] = 0 \tag{168}$$

$$E_t\left[-\gamma(\widehat{C}_{t+1}^F - \widehat{C}_t^F) - (\widehat{P}_{C,t+1}^F - \widehat{P}_{C,t}^F) + \widehat{R}_{t+1}^H\right] = 0 \tag{169}$$

Now subtract (168) from (169), rearrange, and use the definitions of $\widehat{RER}_t$ and $\widehat{PC}_t$:

$$E_t\left[\gamma\widehat{PC}_{t+1} - (\gamma - 1)\widehat{RER}_{t+1}\right] = \gamma\widehat{PC}_t - (\gamma - 1)\widehat{RER}_t \tag{170}$$

Substitute (166) into (170):

$$E_t\left[\gamma\widehat{PC}_{t+1} - \eta(\gamma - 1)\widehat{TOT}_{t+1}\right] = \gamma\widehat{PC}_t - \eta(\gamma - 1)\widehat{TOT}_t \tag{171}$$

And now substitute (167) into (171) and rearrange to get:

$$E_t[\widehat{PC}_{t+1}] = \widehat{PC}_t + \zeta \widehat{Y}_t^{H,F} \tag{172}$$
$$\text{where } \zeta \equiv \frac{\eta\psi(\gamma-1)(1-\rho)}{\gamma-\eta^2\psi(\gamma-1)}$$

Equation (172) provides an expression for expected one-period-ahead relative consumption expenditures. We will also make repeated use of this equation when deriving steady-state bond and equity portfolios.

F.4 Bond Portfolios

I first consider the bond regime. Recall the assumption that bonds are in zero net supply. As a result, steady-state wealth is zero. As a technicality, it is not possible to characterize wealth in terms of *log* deviations from steady-state. Instead, following Devereux and Sutherland (2008), I describe the dynamics of wealth in terms of level deviations.

Begin by log-linearizing the home and foreign budget constraints for bonds (79):

$$\widehat{W}_{t+1}^H = \frac{1}{\beta}\widehat{W}_t^H + \widehat{P}_{t+1}^H + \widehat{Y}_{t+1}^H - \widehat{P}_{C,t+1}^H - \widehat{C}_{t+1}^H + \tilde{A}^{H,H}\widehat{R}_{t+1}^X \tag{173}$$
$$\widehat{W}_{t+1}^F = \frac{1}{\beta}\widehat{W}_t^F + \widehat{P}_{t+1}^F + \widehat{Y}_{t+1}^F - \widehat{P}_{C,t+1}^F - \widehat{C}_{t+1}^F + \tilde{A}^{F,H}\widehat{R}_{t+1}^X \tag{174}$$
$$\text{where } \tilde{A}^{i,H} \equiv \frac{A^{i,H}}{\beta}\ ,\ \widehat{R}_{t+1}^X \equiv \widehat{R}_{t+1}^H - \widehat{R}_{t+1}^F\ ,$$
$$\widehat{W}_t^i \equiv W_t^i - W = W_t^i$$

$\widehat{R}_{t+1}^X$ is the excess (log) return on home bonds over foreign bonds. Since bonds are in zero net supply, we have:

$$\tilde{A}^{F,H} = -\tilde{A}^{H,H}$$
$$\widehat{W}_t^F = -\widehat{W}_t^H$$

Therefore, write (174) as follows:

$$-\widehat{W}_{t+1}^H = -\frac{1}{\beta}\widehat{W}_t^H + \widehat{P}_{t+1}^F + \widehat{Y}_{t+1}^F - \widehat{P}_{C,t+1}^F - \widehat{C}_{t+1}^F - \tilde{A}^{H,H}\widehat{R}_{t+1}^X \tag{175}$$

Now subtract (175) from (173):

$$2\widehat{W}_{t+1}^H = \frac{2}{\beta}\widehat{W}_t^H + \widehat{TOT}_{t+1} + \widehat{Y}_{t+1}^{H,F} - \widehat{PC}_{t+1} + 2\tilde{A}\widehat{R}_{t+1}^X \tag{176}$$
$$\text{where } \tilde{A} \equiv \tilde{A}^{H,H} \equiv \frac{A^{H,H}}{\beta}$$

Equation (176) is just a linear combination of the home and foreign (log-linearized) budget constraints. It is also a difference equation in $\widehat{W}_t^H$. Following Devereux and Sutherland (2008), note that $\widehat{R}_{t+1}^X$ is a mean-zero iid random variable to a first-order approximation, so it will not affect the eigenvalues of the log-linearized system. I therefore introduce the variable ξ_{t+1}:

$$\xi_{t+1} \equiv \tilde{A}\widehat{R}_{t+1}^X \tag{177}$$

where ξ_{t+1} is also a mean-zero iid random variable. Now solve (176) forward, apply the $E_{t+1}[\cdot]$ operator, invoke the appropriate transversality condition, and solve for $\widehat{PC}_{t+1}$ (using (167) and (172)) to get:

$$\widehat{PC}_{t+1} = \pi_B^{PC,W}\widehat{W}_t^H + \pi_B^{PC,YHF}\widehat{Y}_{t+1}^{H,F} + \pi_B^{PC,\xi}\xi_{t+1} \tag{178}$$

$$\text{where } \pi_B^{PC,W} \equiv \frac{2(1-\beta)}{\beta(1-\eta\psi)}\ ,\ \pi_B^{PC,YHF} \equiv \frac{(1-\psi)(1-\beta)-\beta\zeta(1-\eta\psi)}{(1-\eta\psi)(1-\beta\rho)}\ ,$$

$$\pi_B^{PC,\xi} \equiv \frac{2(1-\beta)}{1-\eta\psi}$$

Equation (178) expresses relative consumption expenditures as a function of the endogenous state variable, $\widehat{W}_t^H$, the exogenous relative endowment, $\widehat{Y}_{t+1}^{H,F}$, and the realized excess return on the home portfolio, ξ_{t+1}. To implement the technique in Devereux and Sutherland (2008), we need a similar expression for the variable $\widehat{CDR}_{t+1}$:

$$\widehat{CDR}_{t+1} \equiv \widehat{C}_{t+1}^H - \widehat{C}_{t+1}^F + \frac{1}{\gamma}\widehat{RER}_{t+1} \tag{179}$$

To do this, note that $\widehat{CDR}_{t+1}$ is related to $\widehat{PC}_{t+1}$ as follows:

$$
\begin{aligned}
\widehat{CDR}_{t+1} &= \widehat{PC}_{t+1} - \left(\frac{\gamma - 1}{\gamma}\right)\widehat{RER}_{t+1} \\
&= \widehat{PC}_{t+1} - \eta\left(\frac{\gamma - 1}{\gamma}\right)\widehat{TOT}_{t+1} \\
&= \kappa\widehat{PC}_{t+1} + \mu\widehat{Y}^{H,F}_{t+1} \qquad (180) \\
\text{where } \kappa &\equiv \frac{\gamma - \eta^2\psi(\gamma - 1)}{\gamma} \text{ , } \mu \equiv \frac{\eta\psi(\gamma - 1)}{\gamma}
\end{aligned}
$$

where the last line above follows from (167). Substituting (178) into (180) gives the desired expression:

$$
\widehat{CDR}_{t+1} = \pi_B^{CDR,W}\widehat{W}_t^H + \pi_B^{CDR,YHF}\widehat{Y}^{H,F}_{t+1} + \pi_B^{CDR,\xi}\xi_{t+1} \qquad (181)
$$

$$
\text{where } \pi_B^{CDR,W} \equiv \kappa\pi_B^{PC,W} \text{ , } \pi_B^{CDR,YHF} \equiv \mu + \kappa\pi_B^{PC,YHF} \text{ ,}
$$

$$
\pi_B^{CDR,\xi} \equiv \kappa\pi_B^{PC,\xi}
$$

The next piece we need is an expression for $\widehat{R}^X_{t+1}$ in terms of state variables. Log-linearizing the returns on bonds (76), we can write:

$$
\widehat{R}^X_{t+1} = \beta(\widehat{P}^H_{B,t+1} - \widehat{P}^F_{B,t+1}) - (\widehat{P}^H_{B,t} - \widehat{P}^F_{B,t}) + (1-\beta)\widehat{TOT}_{t+1} \qquad (182)
$$

Solve (182) forward, apply the $E_t[\cdot]$ operator, invoke the appropriate transversality condition, and use equation (172) to derive:

$$\widehat{P}^H_{B,t} - \widehat{P}^F_{B,t} = \eta\psi\widehat{PC}_t + \left[\frac{\eta\psi\zeta - \rho\psi(1-\beta)}{1-\beta\rho}\right]\widehat{Y}^{H,F}_t \tag{183}$$

Iterate (183) forward one period, substitute it back into (182), and use (167) to get:

$$\widehat{R}^X_{t+1} = -(\widehat{P}^H_{B,t} - \widehat{P}^F_{B,t}) + \eta\psi\widehat{PC}_{t+1} + \left[\frac{\beta\eta\psi\zeta - \psi(1-\beta)}{1-\beta\rho}\right]\widehat{Y}^{H,F}_{t+1} \tag{184}$$

Substituting (178) into (184) gives the solution for $\widehat{R}^X_{t+1}$:

$$\widehat{R}^X_{t+1} = \pi_B^{RX,PHF}\left(\widehat{P}^H_{B,t} - \widehat{P}^F_{B,t}\right) + \pi_B^{RX,W}\widehat{W}^H_t + \pi_B^{RX,YHF}\widehat{Y}^{H,F}_{t+1} + \pi_B^{RX,\xi}\xi_{t+1} \tag{185}$$

$$\text{where } \pi_B^{RX,PHF} \equiv -1 \ , \ \pi_B^{RX,W} \equiv \eta\psi\pi_B^{PC,W} \ ,$$

$$\pi_B^{RX,YHF} \equiv \eta\psi\pi_B^{PC,YHF} + \frac{\beta\eta\psi\zeta - \psi(1-\beta)}{1-\beta\rho} \ , \ \pi_B^{RX,\xi} \equiv \eta\psi\pi_B^{PC,\xi}$$

Equation (185) expresses the excess return $\widehat{R}^X_{t+1}$ as a function of the endogenous state variables, $\widehat{P}^H_{B,t} - \widehat{P}^F_{B,t}$ and $\widehat{W}^H_t$, the relative endowment, $\widehat{Y}^{H,F}_{t+1}$, and the excess return on the home portfolio, ξ_{t+1}.

Devereux and Sutherland (2008) show that a *second-order* approximation of the home and foreign portfolio FOCs (145) and (150) imply:

$$E_t\left[\widehat{CDR}_{t+1}\widehat{R}^X_{t+1}\right] = 0 \tag{186}$$

The steady-state portfolio $\tilde{A}$ is defined to be the one that satisfies (186). To solve for $\tilde{A}$, first set $\xi_{t+1} = \tilde{A}\widehat{R}^X_{t+1}$ in (185) and re-solve for $\widehat{R}^X_{t+1}$:

$$\widehat{R}^X_{t+1} = \frac{\pi_B^{RX,PHF}}{1-\pi_B^{RX,\xi}\tilde{A}}\left(\widehat{P}^H_{B,t} - \widehat{P}^F_{B,t}\right) + \frac{\pi_B^{RX,W}}{1-\pi_B^{RX,\xi}\tilde{A}}\widehat{W}^H_t + \frac{\pi_B^{RX,YHF}}{1-\pi_B^{RX,\xi}\tilde{A}}\widehat{Y}^{H,F}_{t+1} \quad (187)$$

Next set $\xi_{t+1} = \tilde{A}\widehat{R}^X_{t+1}$ in (181):

$$\widehat{CDR}_{t+1} = \pi_B^{CDR,W}\widehat{W}^H_t + \pi_B^{CDR,YHF}\widehat{Y}^{H,F}_{t+1} + \pi_B^{CDR,\xi}\tilde{A}\widehat{R}^X_{t+1} \quad (188)$$

Substituting (187) and (188) into (186), evaluating the time-t expectations, and solving for $\tilde{A}$ gives:

$$\tilde{A} = \frac{\pi_B^{CDR,YHF}}{\pi_B^{CDR,YHF}\pi_B^{RX,\xi} - \pi_B^{CDR,\xi}\pi_B^{RX,YHF}} \quad (189)$$

Actual bond holdings are related to $\tilde{A}$ as follows:

$$A = A^{H,H} = A^{F,F} = \beta\tilde{A} \quad (190)$$

In Proposition 1, I report home (foreign) holdings of the home (foreign) bond, divided by the price of a home (foreign) bond:

$$X_B \equiv \frac{A}{P_B} = \frac{A}{\frac{\beta}{1-\beta}} = (1-\beta)\tilde{A} \tag{191}$$

The final result can be written as follows:

$$X_B = \frac{1}{2}\left[\left(1-\frac{1}{\gamma}\right)(2\lambda-1)+(\theta-1)\right] \tag{192}$$

where $\theta \equiv \phi[1-(2\lambda-1)^2]+(2\lambda-1)^2/\gamma$

F.5 Equity Portfolios

Now consider the equity regime. Recall that equities are assumed to be in positive net supply, so steady-state wealth is positive under this regime. I will therefore characterize wealth dynamics in terms of log deviations from steady-state. The log-linearized budget constraints for equities (83) can be written as follows:

$$\widehat{W}^H_{t+1} = \frac{1}{\beta}\left(\widehat{W}^H_t + \widehat{R}^F_{t+1}\right) - \left(\frac{1-\beta}{\beta}\right)\left(\widehat{P}^H_{C,t+1} + \widehat{C}^H_{t+1}\right) + \tilde{A}^{H,H}\widehat{R}^X_{t+1} \tag{193}$$

$$\widehat{W}^F_{t+1} = \frac{1}{\beta}\left(\widehat{W}^F_t + \widehat{R}^F_{t+1}\right) - \left(\frac{1-\beta}{\beta}\right)\left(\widehat{P}^F_{C,t+1} + \widehat{C}^F_{t+1}\right) + \tilde{A}^{F,H}\widehat{R}^X_{t+1} \tag{194}$$

where $\tilde{A}^{i,H} \equiv \dfrac{A^{i,H}}{\beta W}$ and $\widehat{R}^X_{t+1} \equiv \widehat{R}^H_{t+1} - \widehat{R}^F_{t+1}$

$\widehat{R}^X_{t+1}$ is the excess (log) return on home equity over foreign equity. By combining the equity market-clearing conditions (81) and (82), the definition of wealth ($W^i_t \equiv A^{i,H}_t + A^{i,F}_t$), and the steady-state conditions, we can show the following:

$$\widehat{W}_t^F = \widehat{P}_{E,t}^H + \widehat{P}_{E,t}^F - \widehat{W}_t^H \tag{195}$$

$$\tilde{A}^{F,H} = \frac{1}{\beta} - \tilde{A}^{H,H} \tag{196}$$

By log-linearizing (80), we can also derive an expression for $\widehat{R}_{t+1}^X$:

$$\widehat{R}_{t+1}^X = \beta(\widehat{P}_{E,t+1}^H - \widehat{P}_{E,t+1}^F) - (\widehat{P}_{E,t}^H - \widehat{P}_{E,t}^F) + (1-\beta)\widehat{TOT}_{t+1} + (1-\beta)\widehat{Y}_{t+1}^{H,F} \tag{197}$$

Substitute (195), (196) and (197) into the foreign budget constraint (194) to get:

$$\begin{aligned} 2\widehat{P}_{E,t+1}^F - \widehat{W}_{t+1}^H = \frac{1}{\beta}\left(2\widehat{P}_{E,t}^F - \widehat{W}_t^H\right) + \frac{1}{\beta}\widehat{R}_{t+1}^F - \left(\frac{1-\beta}{\beta}\right)\left(\widehat{P}_{C,t+1}^F + \widehat{C}_{t+1}^F\right) \\ + \left(\frac{1-\beta}{\beta}\right)\widehat{TOT}_{t+1} + \left(\frac{1-\beta}{\beta}\right)\widehat{Y}_{t+1}^{H,F} - \tilde{A}\widehat{R}_{t+1}^X \end{aligned} \tag{198}$$

$$\text{where } \tilde{A} \equiv \tilde{A}^{H,H} \equiv \frac{A^{H,H}}{\beta W}$$

Now subtract (198) from (193):

$$\begin{aligned} 2\widehat{WP}_{t+1}^{H,F} = \frac{2}{\beta}\widehat{WP}_t^{H,F} - \left(\frac{1-\beta}{\beta}\right)\left(\widehat{PC}_{t+1} + \widehat{TOT}_{t+1}\right) \\ - \left(\frac{1-\beta}{\beta}\right)\widehat{Y}_{t+1}^{H,F} + 2\tilde{A}\widehat{R}_{t+1}^X \end{aligned} \tag{199}$$

$$\text{where } \widehat{WP}_t^{H,F} \equiv \widehat{W}_t^H - \widehat{P}_{E,t}^F$$

Equation (199) is a difference equation in $\widehat{WP}_t^{H,F}$. As with the bond regime, let $\xi_{t+1} \equiv \tilde{A}\widehat{R}^X_{t+1}$, and note that ξ_{t+1} is a mean-zero iid random variable to a first-order approximation. Now solve (199) forward, apply the $E_{t+1}[\cdot]$ operator, invoke the appropriate transversality condition, and solve for $\widehat{PC}_{t+1}$ (using (167) and (172)) to get:

$$\widehat{PC}_{t+1} = \pi_E^{PC,WP}\widehat{WP}_t^{H,F} + \pi_E^{PC,YHF}\widehat{Y}_{t+1}^{H,F} + \pi_E^{PC,\xi}\xi_{t+1} \tag{200}$$
$$\text{where } \pi_E^{PC,WP} \equiv \frac{2}{1+\eta\psi}\ ,\ \pi_E^{PC,YHF} \equiv -\left[\frac{\beta\zeta(1+\eta\psi)+(1-\psi)(1-\beta)}{(1+\eta\psi)(1-\beta\rho)}\right]\ ,$$
$$\pi_E^{PC,\xi} \equiv \frac{2\beta}{1+\eta\psi}$$

Note that equation (180), relating the variable $\widehat{CDR}_{t+1}$ to $\widehat{PC}_{t+1}$, remains valid for equities. Therefore the solution for $\widehat{CDR}_{t+1}$ is given by:

$$\widehat{CDR}_{t+1} = \pi_E^{CDR,WP}\widehat{WP}_t^{H,F} + \pi_E^{CDR,YHF}\widehat{Y}_{t+1}^{H,F} + \pi_E^{CDR,\xi}\xi_{t+1} \tag{201}$$
$$\text{where } \pi_E^{CDR,WP} \equiv \kappa\pi_E^{PC,WP}\ ,\ \pi_E^{CDR,YHF} \equiv \mu + \kappa\pi_E^{PC,YHF}\ ,$$
$$\pi_E^{CDR,\xi} \equiv \kappa\pi_E^{PC,\xi}$$

The next piece we need is an expression for $\widehat{R}^X_{t+1}$ in terms of state variables. Start by substituting (167) into (197) to get:

$$\widehat{R}^X_{t+1} = \beta(\widehat{P}^H_{E,t+1} - \widehat{P}^F_{E,t+1}) - (\widehat{P}^H_{E,t} - \widehat{P}^F_{E,t}) + \eta\psi(1-\beta)\widehat{PC}_{t+1} + (1-\psi)(1-\beta)\widehat{Y}^{H,F}_{t+1} \tag{202}$$

Equation (202) is a difference equation in $\widehat{P}^H_{E,t} - \widehat{P}^F_{E,t}$. Solve it forward, apply the $E_t[\cdot]$ operator, and invoke the appropriate transversality condition to get:

$$\widehat{P}^H_{E,t} - \widehat{P}^F_{E,t} = \eta\psi\widehat{PC}_t + \left[\frac{\eta\psi\zeta + \rho(1-\psi)(1-\beta)}{1-\beta\rho}\right]\widehat{Y}^{H,F}_t \tag{203}$$

Iterate (203) forward one period and substitute it back into (202):

$$\widehat{R}^X_{t+1} = -(\widehat{P}^H_{E,t} - \widehat{P}^F_{E,t}) + \eta\psi\widehat{PC}_{t+1} + \left[\frac{\beta\eta\psi\zeta + (1-\psi)(1-\beta)}{1-\beta\rho}\right]\widehat{Y}^{HF}_{t+1} \tag{204}$$

And finally substitute (200) into (204) to get:

$$\widehat{R}^X_{t+1} = \pi^{RX,PHF}_E(\widehat{P}^H_{E,t} - \widehat{P}^F_{E,t}) + \pi^{RX,WP}_E\widehat{WP}^{H,F}_t + \pi^{RX,YHF}_E\widehat{Y}^{H,F}_{t+1} + \pi^{RX,\xi}_E\xi_{t+1} \tag{205}$$

$$\text{where } \pi^{RX,PHF}_E \equiv -1 \ , \ \pi^{RX,WP}_E \equiv \eta\psi\pi^{PC,WP}_E \ ,$$
$$\pi^{RX,YHF}_E \equiv \eta\psi\pi^{PC,YHF}_E + \frac{\beta\eta\psi\zeta + (1-\psi)(1-\beta)}{1-\beta\rho} \ , \ \pi^{RX,\xi}_E \equiv \eta\psi\pi^{PC,\xi}_E \tag{206}$$

The solution for $\tilde{A}$ takes the same form as the solution under the bond regime. In particular, $\tilde{A}$ must solve the equity regime analog of (189):

$$\tilde{A} = \frac{\pi_E^{CDR,YHF}}{\pi_E^{CDR,YHF}\pi_E^{RX,\xi} - \pi_E^{CDR,\xi}\pi_E^{RX,YHF}} \tag{207}$$

In Proposition 2, I report home (foreign) holdings of the home (foreign) equity, divided by home (foreign) financial wealth:

$$Sh_E = \frac{A}{W} = \frac{A^{H,H}}{W^H} = \frac{A^{F,F}}{W^F} = \beta\tilde{A} \tag{208}$$

The final result can be written as follows:

$$Sh_E = \frac{1}{2}\left[1 - \left(1 - \frac{1}{\gamma}\right)\left(\frac{2\lambda - 1}{\theta - 1}\right)\right] \tag{209}$$

where $\theta \equiv \phi[1 - (2\lambda - 1)^2] + (2\lambda - 1)^2/\gamma$

F.6 Additional checks and comparisons

As a double-check, I also obtained equilibrium decision rules for $\widehat{CDR}_{t+1}$ and $\widehat{R}^X_{t+1}$ using a standard first-order software package (for each regime separately, treating ξ_{t+1} as an additional shock). I then recomputed the steady-state equilibrium bond portfolio using (189) and the equity portfolio using (207). The results were identical using the analytical and numerical techniques.

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