

ESSAYS IN MACROECONOMICS

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By

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# ESSAYS IN MACROECONOMICS

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## ABSTRACT

What are the welfare costs of price rigidities when labor mobility is restricted? What are the effects of pecuniary externality in high-leveraged economies? Should Central Banks react to changes in asset prices? How did the credit risk modeling evolve since 1970s? This dissertation uses New Neoclassical Synthesis (NNS) models to answer these questions and presents a literature review of credit risk modeling.

Chapter 1 studies the welfare costs of price rigidities in an economy without labor mobility. Labor immobility plays an allocative role and causes large fluctuations in hours of work. This, in turn, magnifies the welfare costs of nominal rigidities. The welfare costs can be eliminated by strict CPI inflation targeting in a one-sector model. When there are two vertically integrated sectors, strict CPI inflation targeting rule is no longer optimal due to the distinct sectoral inflation rates. In the two-sector model, I show that a modified Taylor rule with two measures of inflation is nearly optimal even when labor mobility is restricted.

Chapter 2 studies capital fire sales of the firms in an economy with financial frictions and price rigidity hit by negative aggregate productivity shocks. Collateral constraints and the fact that asset prices are determined in a competitive market generate pecuniary externality. Inefficiency losses in aggregate quantities are more pronounced in an economy with high leverage because of the higher negative externality stemming from fire sales. As goods price inflation can positively affect borrowers' net worth by reducing the real burden of the entrepreneur's current debt for given interest rate, a Taylor rule with a mild reaction to inflation and a positive reaction to asset prices almost achieves welfare under Ramsey solution.

Chapter 3 presents a literature review of the credit risk models developed since 1970s. These models are divided into two main categories: (a) credit pricing models, and (b) credit value-at-risk (VaR) models. Three main approaches in credit pricing models discussed are (i) first generation structural-form models, (ii) second generation structural-form models, and (iii) reduced form models. Credit VaR models are examined under two main categories: (i) default mode models (DM) and (ii) mark-to-market (MTM) models.

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## INTRODUCTION

This dissertation consists of three essays in macroeconomics. While the first two chapters study the welfare implications of monetary policy rules in models with labor market or financial market frictions, the last chapter reviews the evolution of credit risk modeling since 1970s.

Over the last two decades, there has been general agreement that monetary policy should be designed in order to achieve low and stable inflation. However, the standard DSGE models with sticky prices, New Neoclassical Synthesis (NNS) models, are not satisfactory in supporting that the welfare gains can be substantial from better countercyclical policies. This raises the question how useful it is to study optimal monetary or fiscal policies if gains are actually very small. For this reason, some researchers augmented NNS models with allocative wage rigidities in addition to price rigidities to provide numerical evidence for the efficacy of monetary policy. However, these models are criticized because wages may not play an allocative role. I argue that there are real rigidities one should consider before drawing any clear-cut conclusion about the efficacy of the monetary policy. For this reason, I introduce the real rigidities in the form of labor immobility in Chapter 1. In addition, I solve for the optimal (Ramsey) monetary policy and compare with Taylor-type interest rate rules to see if an implementable rule can bring the welfare level to the one under optimal monetary policy in an economy without labor mobility.

I show that labor immobility is quite costly to the households because it creates

employment dispersion when the economy exhibits price rigidities. When prices are sticky household-firms are constrained to meet the demand at that given price. Those who cannot adjust their prices react by raising the markup and reducing the labor demand. Because labor mobility is restricted, this creates employment dispersion due to the fluctuations in hours of work. Therefore, price rigidities lead to inefficiencies in consumption choices and labor hiring decisions, which are both costly to households.

I examine the welfare costs of sticky prices in two separate cases. I compute the welfare costs as the percentage of consumption that households would be willing to give up to avoid the consequences of price rigidities. I first examine the one-sector model because it provides more intuition about the interactions between labor immobility and price stickiness. In the one-sector model, because price stickiness is the only reason that creates fluctuations in hours of work and consumption, once the monetary authority eliminates the price dispersion, the need for adjusting hours of work by the household-firms disappear along with inefficient consumption decisions. Since price stickiness together with labor immobility plays an allocative role in employment, the elimination of variations in prices helps remove the negative effects of labor immobility. For this reason, an extreme CPI inflation targeting is able to produce the flexible-price solution even labor cannot move freely among household-firms.

In the two-sector model, even though the degree of price rigidities affects the magnitude of welfare costs, what is more effective in magnifying the welfare cost of nominal rigidities is the presence of labor immobility. As labor is firm specific



and cannot move across sectors either, this creates large fluctuations in hours of work, which households do not like. The benevolent central bank faces a trade-off in stabilizing output and relative price gaps, CPI inflation and PPI inflation in the two-sector model. Even though CPI inflation targeting helps reducing the price dispersion in the final goods sector, the dispersion in the intermediate goods sector still exists. Because these two sectors are vertically integrated, the inefficiency in the intermediate goods sector's output leads to inefficient choices and therefore an inefficient level of output in the final goods sector. I show that reacting only to one of CPI or PPI inflation would leave one of these sectors with inefficient choices. By taking price dispersions in both sectors into consideration, I show that the central bank is able to reduce the welfare costs of nominal rigidities noticeably. A modified Taylor rule with two measures of inflation brings the level of welfare much closer to that under optimal monetary policy than the estimated Taylor rule does even when labor mobility is restricted.

Chapter 2 of my dissertation is motivated by the recent credit crisis and the subsequent actions of the Federal Reserve. In particular, is there a need for policy intervention and if yes, how should interest rate policies be designed to stabilize inefficient fluctuations in economic activity and inflation when firms are financially constrained? Chapter 2 addresses this question focusing on a pecuniary externality that arises from the combination of competitive market for real assets of the firms with collateral constraints. I incorporate the systemic risk argument of Lorenzoni (2008)

into an infinite horizon monetary model. In particular, I quantitatively examine the importance of the existence of a pecuniary externality on the dynamic behavior of the aggregate quantities and investigate the need for policy intervention during a credit crisis.

The infinite horizon economy consists of entrepreneurs, households, and retailers. The model assumes that each household owns a firm in the traditional sector as in Lorenzoni (2008). Firms in this sector absorb capital sold in a competitive market by financially troubled entrepreneurs. Both entrepreneurs and traditional sector firms produce the same homogeneous intermediate goods and sell it to the retailers, which are monopolistically competitive. Unlike the households, entrepreneurs have access to a risky but more productive technology in the production process, however; they are also subject to borrowing constraints. As the entrepreneurs have limited external funds and their production is subject to an aggregate productivity shock, they need to sell part of their capital if the economy is hit by a negative aggregate shock to meet debt payments and to finance future production. Because the firms in the traditional sector are less productive and capital is traded in a competitive market, as the supply of capital increases by the financially troubled entrepreneurs, the price of capital will go down. This creates a pecuniary externality: as entrepreneurs want to sell more capital, the price of capital decreases even more, which results in tighter borrowing constraints and inefficiencies in output and the allocation of capital.

Chapter 2 is able to produce capital fire sales of the financially constrained entre-

preneurs in the case of a negative productivity shock. Depending on the loan-to-value ratio, the capital stock of the entrepreneurial sector declines dramatically after the negative shock hits the economy. These results suggest that the fact that asset prices are determined in a competitive market and that entrepreneurs are credit constrained play a major role in the responses of the aggregate quantities to technology shocks. It shows that the strength of borrowing constraints is important in explaining the amount of capital fire sales. The responses of aggregate quantities to a negative shock are larger if borrowing constraints are relatively loose, i.e., if the loan-to-value ratio is higher. Moreover, I show that since asset prices are determined in a competitive market and capital moves from more productive sector to the traditional (less productive) sector following a negative productivity shock, technology shocks generate large fluctuations in asset prices so that their impact on output and capital stock work through credit constraints.

In addition, I investigate whether a central bank with an implementable interest rate rule can reduce the size of the asset sales when the economy is hit by a negative aggregate shock. I follow a Ramsey-type approach to analyze optimal monetary policy in an economy with sticky prices, nominal debt, and borrowing constraints in the presence of a pecuniary externality. Due to sticky prices and the two-way feedback between asset prices and aggregate quantities, one obvious candidate is a modified Taylor rule with goods price inflation and asset prices. Since goods price inflation can positively affect borrowers' net worth by reducing the real burden of the current

debt for a given interest rate, I find that inflation variability is a feature of Ramsey equilibrium. In particular, I find that a flexible inflation targeting rule with a positive reaction to asset prices performs better than a strict inflation targeting policy. The welfare gain from the former policy rule relative to a strict stabilization of inflation is 0.23% of consumption per period.

Chapter 3 of my dissertation is also motivated by the recent credit crisis. We have recently witnessed that financial decisions taken by private firms or institutions can create a big impact in the output and investment in the aggregate economy. Therefore, it is important to understand how credit risk arises and how it can be measured. Because credit valuation process is very important to lending, an accurate credit valuation should result in debt pricing that corresponds to the risks taken. For this reason, I review the credit risk modeling since 1970s in Chapter 3. Credit pricing and the credit value-at-risk models are the two main categories examined. I discuss both theoretical and empirical papers so that one can see how the theoretical models evolved over time to match the empirically observed debt pricing and yield spreads.

# Chapter 1

## How Costly Is CPI Inflation

## Targeting: A Two-Sector Model

## with No Labor Mobility

### 1.1 Introduction

Over the last two decades, there has been general agreement that monetary policy should be designed in order to achieve low and stable inflation. King and Wolman (1999) showed that strict CPI inflation targeting achieves the constrained optimum in a model with price rigidity. However, Erceg, Henderson and Levin (2000) showed that when wage rigidity is added to the model strict inflation targeting is no longer optimal and claimed that the central bank should also respond to movements in the

nominal wage or the output gap. However, Goodfriend and King (2001) argued wages may not play an allocative role as suggested in Erceg, Henderson and Levin (2000). In addition to closed economy models above, Benigno (2004) found that in a two-region currency area with different degree of price rigidities, an inflation targeting monetary policy is proven to be nearly optimal if it assigns higher weight to the inflation rate in the region with higher degree of price rigidity. A more recent paper by Huang and Liu (2005) suggested that in a two-sector model, the monetary authority should conduct a policy in which the interest rate should respond to variations in both CPI and PPI inflation.

However, there also has been a debate about the effectiveness of monetary policies in improving economic efficiency. Lucas (2003) argued that the gains from better countercyclical policies are not larger than one-half of one-tenth of one percent of consumption. This raises the question of how useful it is to study optimal countercyclical policies if gains are actually very small. This paper tries to answer this type of question; however, it is not the first in the literature. For example, Canzoneri, Cumby and Diba (2007) calculated that a single household would be willing to give up one to three percent of its consumption each period to be free of the nominal rigidities in a one sector New Neoclassical Synthesis (NNS) model with allocative wage rigidity, price stickiness and capital formation. The models presented in this paper abstract from allocative wage rigidities and address Lucas' argument by suggesting that there are some real rigidities one should consider before drawing any clear-cut conclusions

about the efficacy of the monetary policy.

The models presented in this paper assume sticky prices but flexible wages and a labor market friction is modeled in the form of labor immobility. I examine the welfare cost of nominal rigidities in two separate cases. In the first case, a one-sector model is calibrated in order to motivate that the welfare cost of nominal rigidities can be significant especially when labor is immobile among household-firms. I calculate the welfare cost of price rigidities as the percentage of consumption households would on average be willing to give up in order to avoid the consequences of those rigidities. The one-sector model with labor immobility shows that the welfare costs of the nominal rigidities are significant when labor is immobile and that the gains from better countercyclical policies are substantial. Labor immobility magnifies the welfare costs because it causes large swings in hours of work. Since prices are rigid and output and employment are demand determined, household-firms' response to a productivity shock is confined to adjusting hours of work inefficiently. But I show that strict CPI inflation targeting is still able to eliminate the welfare cost of price rigidities in the one-sector model even in the presence of labor immobility. Once a benevolent monetary authority eliminates the dispersion in prices the need for adjusting labor demand disappears too, therefore an extreme CPI inflation targeting achieves the flexible-price solution in the one-sector model.

In the second case, I develop a vertically integrated two-sector model with nominal and real rigidities where there is a natural distinction between the rates of inflation

in the final and intermediate goods sectors. In the two-sector model, real rigidities are introduced by restricting labor mobility across sectors and firms. As in the one-sector model, price stickiness is the only nominal rigidity. Each household has two members, each supplying labor to one of the two sectors. Firms in both sectors produce a differentiated good and a composite of the intermediate goods is used to produce the final goods.

I solve the model by taking second-order approximations around the zero-inflation steady state. The welfare costs of nominal rigidities are reported under optimal (Ramsey) monetary policy and optimized Taylor-type interest rate rules. In order to characterize optimal monetary policy, I assume that *ex-ante commitment* is feasible and a Ramsey planner maximizes aggregate welfare subject to the equilibrium constraints in the two-sector economy. Because optimal monetary policy requires the central bank to have the knowledge of past and the current values of the aggregate variables and Lagrange multipliers, I investigate the degree to which it can be achieved through one simple rule for the conduct of monetary policy. Due to price rigidities in both sectors, one obvious candidate would be a modified Taylor rule with two measures of inflation, a lagged interest rate and the output gap.

The vertical structure of the two sectors in this paper is similar to that in Huang and Liu (2005). This paper extends Huang and Liu (2005) by introducing real rigidities into the model. In contrast to one-sector models, I show that a CPI inflation targeting policy is not optimal in a model with two sticky-price sectors and immobile



labor across firms and sectors, even when wages are flexible. The welfare costs of price rigidities are significant under the estimated Taylor rule because reacting only to CPI inflation does not help to reduce the price and employment dispersion in the intermediate goods sector. The intuition for this result is as follows: In the model, labor immobility plays an allocative role and creates employment dispersion due to sticky prices in both the intermediate and final goods sectors. This, in turn, magnifies the welfare costs. The dispersion of prices and employment in the intermediate goods sector leads to inefficiencies in consumption and employment decisions by household-firms. However, since the two sectors are vertically integrated these inefficiencies propagate from the intermediate goods sector to the final goods sector.

As is well-known in the optimal monetary policy literature, the degree of price stickiness in multiple sector models plays an important role in the determination of the optimal weights on the sectoral inflation rates. For this reason, I consider the welfare cost of nominal rigidities for three cases of price stickiness in the two-sector model. First, based on the studies by Blinder et al. (1998) and Bils and Klenow (2004), I assume that prices are more flexible in the final goods sector. Second, following the empirical survey by Taylor (1999), I assume that the intermediate and final goods sector have the same degree of price rigidity. Third, I chose the price rigidities in both sectors to match the responses of log intermediate good prices to a monetary policy shock as in Clark (1999). Clark (1999) reports that in response to a policy tightening input prices fall more rapidly and by a larger amount than output

prices at early stages of production. I find that intermediate goods prices need to be more flexible than final goods prices in order to be consistent with Clark's (1999) finding.

In Section 1.3.2, I show that optimal monetary policy cannot produce the flexible-price solution in the two-sector model with no labor mobility. The benevolent central bank faces a trade-off in stabilizing output and relative price gaps, CPI inflation and PPI inflation. Also, when labor is immobile, welfare costs are substantial and range from 1.62% to 2.33% of consumption per period under an estimated CPI inflation targeting rule for different degree of price rigidities considered above. However, a modified Taylor rule with two measures of inflation brings the level of welfare much closer to that under optimal monetary policy than the estimated Taylor rule does. For all three cases above, we see that inflation in the intermediate goods sector requires some attention by the central bank. This result is similar to the one shown by Huang and Liu (2005). The findings in this paper make this result even stronger by showing how costly CPI inflation targeting can be when labor cannot move freely across firms and sectors and suggest that the welfare gains from better countercyclical policies might actually be very important. Although Huang and Liu (2005) finds that the monetary authority should respond to changes in both CPI and PPI inflation when the two sectors are vertically integrated, it still leaves Lucas' argument partially unanswered. They expressed the welfare losses from a policy as the ratios of actual welfare losses to that under the optimal monetary policy. However, that ratio can

be large even if the welfare cost as a percentage of per period consumption under a CPI inflation targeting monetary policy is small. For this reason, in Section 1.3.2, I also provide the welfare costs of nominal rigidities in the two-sector model with labor mobility. I find that the welfare costs in the two-sector model with labor mobility are about one-fifth of those without labor mobility, which takes us back to Lucas' criticism about better countercyclical policies.

The rest of the paper is organized as follows: Section 1.2 presents the two-sector model in detail; Section 1.3 discusses the calibration of the one and two-sector models with and without labor mobility and compares their implications for the conduct of optimal monetary policy. Section 1.4 concludes.

## **1.2 Model**

### **1.2.1 Household-Firm Maximization Problem**

This section describes an infinite horizon production economy with price rigidity and immobile labor. There are two sectors— intermediate and final goods sectors— and no distinction between households and firms. The economy consists of a continuum of monopolistically competitive household-firm units on the interval  $[0, 1]$ . Each household has two members with one member working and producing for the intermediate goods sector and the other for the final goods sector. Each household-firm of type  $i$  produces a differentiated good for each of the two sectors described above and receives

utility from its consumption of final good and disutility from its work effort in the two sectors. Household  $i$ 's utility function is given by

$$E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \log[C_j(i)] - \frac{\kappa}{1 + \chi_m} N_{m,j}(i)^{1+\chi_m} - \frac{\kappa}{1 + \chi_f} N_{f,j}(i)^{1+\chi_f} \right\}, \quad (1.1)$$

where  $C_t(i)$ ,  $N_{mt}(i)$ , and  $N_{ft}(i)$  denote consumption and labor input in the intermediate and final goods sectors, respectively.  $\beta$  is a subjective discount factor,  $1/\chi_k$  with  $k = \{m, f\}$  is the Frisch elasticity of labor supply in each sector and  $E_t$  denotes the expectation operator conditional on information set at time  $t$ .

The consumption good,  $C_t$ , is a composite of differentiated final goods  $Y_{f,t}(i)$  and is given by

$$C_t = \left[ \int_0^1 Y_{f,t}(i)^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}, \quad (1.2)$$

where  $\sigma > 1$  is the elasticity of substitution between the differentiated final goods.

The production technology for differentiated good  $i$  is summarized by a Cobb-Douglas production function. In particular,

$$Y_{f,t}(i) = Z_{f,t} N_{f,t}(i)^{1-\alpha_f} M_t(i)^{\alpha_f}, \quad (1.3)$$

where  $N_{ft}(i)$  is the labor input of household of type  $i$  and  $M_t(i)$  is the demand by household  $i$  for composite good of the differentiated intermediate goods. Specifically,  $M_t(i)$  is the amount of aggregate intermediate good acquired by household  $i$  for

producing final good  $i$ . The production of the differentiated intermediate good is linear in the labor input of household  $i$  and capital stock is implicitly assumed to be fixed at 1. Therefore, the production technology takes the following form

$$Y_{m,t}(i) = Z_{m,t} N_{m,t}(i), \quad (1.4)$$

where  $Z_{k,t}$ ,  $k = \{m, f\}$  is the sector specific productivity shock with a standard AR(1) process  $\log Z_{k,t} = \rho_k \log Z_{k,t-1} + \varepsilon_t$ . The linkage in equations (1.3) and (1.4) through the composite intermediate good  $M_t$  makes the two sectors vertically integrated.

Following Chari, Kehoe and McGrattan (2000), I assume there exists an artificial bundler in this two-sector model. The bundler's demand for the intermediate good of household  $i$  obtained from his expenditure minimization problem is as follows:

$$Y_{m,t}^d(i) = \left[ \frac{P_{m,t}}{P_{m,t}(i)} \right]^\sigma M_t, \quad (1.5)$$

where the bundler buys each differentiated intermediate good  $Y_{m,t}(i)$  paying the prices  $P_{m,t}(i)$  and uses the Dixit Stiglitz aggregator in order to get the composite intermediate good

$$M_t = \left[ \int_0^1 Y_{m,t}(i)^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}, \quad \sigma > 1. \quad (1.6)$$

The aggregate price level in the intermediate goods sector calculated from the bundler's

expenditure minimization problem is

$$P_{m,t} = \left[ \int_0^1 P_{m,t}(i)^{1-\sigma} di \right]^{1/1-\sigma}. \quad (1.7)$$

In a similar way, the artificial bundler acquires differentiated final good  $i$ ,  $Y_{f,t}(i)$ , from household  $i$  by paying  $P_{f,t}(i)$  to put together the composite final good  $Y_t$  using the following aggregator

$$Y_t = \left[ \int_0^1 Y_{f,t}(i)^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}, \quad (1.8)$$

where the bundler's demand for each differentiated final good  $i$  is given by

$$Y_{f,t}^d(i) = \left[ \frac{P_{f,t}}{P_{f,t}(i)} \right]^{\sigma} Y_t. \quad (1.9)$$

The expenditure minimization implies that the aggregate price level is

$$P_t = \left[ \int_0^1 P_{f,t}(i)^{1-\sigma} di \right]^{1/1-\sigma}. \quad (1.10)$$

In equilibrium,  $C_t = Y_t$  as there is no government and capital formation in the model for simplicity.

Following Calvo (1983), the household-firm unit gets the chance to set a new price with a constant probability  $(1 - \omega_m)$  in the intermediate goods sector and  $(1 - \omega_f)$  in

the final goods sector so that the average duration of a price contract is  $1/(1 - \omega_k)$ ,  $k \in \{m, f\}$ . By the law of large numbers, the fraction of the firms in each sector that gets to announce a new price is  $(1 - \omega_k)$ ,  $k \in \{m, f\}$ .

Household-firm unit  $i$ 's objective is to maximize its utility by choosing its consumption,  $\{C_j\}_{j=t}^{\infty}$ , the prices of its products in the intermediate and final goods sectors,  $\{P_{m,t}(i), P_{f,t}(i)\}$ , its labor input in the two sectors,  $\{N_{m,j}(i), N_{f,j}(i)\}_{j=t}^{\infty}$ , and its demand for the composite intermediate good which will be used in its final good production,  $\{M_j\}_{j=t}^{\infty}$ . Hence, the household-firm unit  $i$ 's utility maximization problem is given by

$$\max E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \log[C_j(i)] - \frac{\kappa}{1 + \chi_m} N_{m,j}(i)^{1+\chi_m} - \frac{\kappa}{1 + \chi_f} N_{f,j}(i)^{1+\chi_f} \right\},$$

subject to

$$E_t [\Delta_{t+1,t} B_{t+1}(i)] + P_t C_t(i) = B_t(i) + P_{f,t}(i) Y_{f,t}(i) - P_{m,t} M_t(i) + P_{m,t}(i) Y_{m,t}(i), \quad (1.11)$$

and equations (1.2)-(1.10).<sup>1</sup> Equation (1.11) represents the household-firm's budget constraint. There are complete financial markets and no obstacles to borrowing against future income. With the complete financial markets assumption, each household-firm unit has access to one period state contingent bond that pays one dollar in a given state. In addition, the household pays  $P_t$  to buy one unit of consumption

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<sup>1</sup> $\Delta_{t+1,t}$  is the stochastic discount factor. For further discussion of complete contingent claims, see Chapter 3 of Cochrane (2001).

of composite final good. There are three sources of income for household-firm unit  $i$  from the sales of its product in the final and intermediate goods sector,  $P_{f,t}(i)Y_{f,t}(i)$  and  $P_{m,t}(i)Y_{m,t}(i)$ , respectively, and from its bond holdings. Each household-firm unit is a price taker in the input market of the composite intermediate good and pays  $P_{m,t}$  to buy one unit of  $M_t$  in order to produce differentiated final good  $Y_{f,t}(i)$ .

The utility maximization problem described above implies the following first order conditions:

$$\frac{1}{C_t} = \lambda_t P_t, \quad (1.12)$$

where  $\lambda_t$  is the household-firm unit  $i$ 's marginal utility of nominal wealth. Although the model is a heterogeneous agent model, the household-firm units are identical in terms of their consumption because of complete contingent claims market assumption. In equilibrium,  $\int_0^1 C_t(i) di = C_t(i) = C_t$  for all  $i \in [0, 1]$ . For equation (1.13) below, consider a one-period bond that costs 1 dollar in period  $t$  and that pays  $(1+i_t)$  dollars in all states of nature in period  $t+1$

$$E_t[\lambda_{t+1}/\lambda_t] = E_t[\Delta_{t+1,t}] = \frac{1}{1+i_t}. \quad (1.13)$$

The equation below determines household-firm unit  $i$ 's demand for the composite intermediate good,  $M_t$ , for its production of the differentiated final good  $i$

$$M_t(i) = \alpha_f \frac{P_{f,t}(i)}{P_{m,t}} Y_{f,t}(i). \quad (1.14)$$



The optimal price for the differentiated final good of household-firm unit  $i$  if it gets to announce a new price is

$$P_{f,t}^*(i)^\phi = \mu \frac{\kappa}{1 - \alpha_f} \frac{E_t \sum_{j=t}^{\infty} (\omega_f \beta)^{j-t} [Z_{f,j}^{-1} P_j^\sigma Y_j M_j(i)^{-\alpha_f}]^{\frac{1+\chi_f}{1-\alpha_f}}}{E_t \sum_{j=t}^{\infty} (\omega_f \beta)^{j-t} \lambda_j Y_j P_j^\sigma}, \quad (1.15)$$

where  $\mu = \frac{\sigma}{\sigma-1}$  and  $\phi = \frac{1+\alpha_f \chi_f + \sigma \chi_f (1-\alpha_f)}{1-\alpha_f}$ .<sup>23</sup>

Similarly, the optimal price of the intermediate good of household-firm  $i$  is given by

$$P_{m,t}^*(i)^{1+\sigma \chi_m} = \mu \frac{\kappa}{1 - \alpha_f} \frac{E_t \sum_{j=t}^{\infty} (\omega \beta)^{j-t} [Z_{m,j}^{-1} P_{m,j}^\sigma M_j]^{1+\chi_m}}{E_t \sum_{j=t}^{\infty} (\omega \beta)^{j-t} \lambda_j M_j P_{m,j}^\sigma}. \quad (1.16)$$

From equations (1.15) and (1.16), it is understood that when a household-firm gets to set a new price for its final or intermediate good products, its choice depends on the current aggregate output and prices, and their expected values in future periods. Moreover, the optimal prices in equations (1.15) and (1.16) can be interpreted as the monopoly markup factor over a weighted average of future expected marginal costs.

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<sup>2</sup> $\mu = \frac{\sigma}{\sigma-1}$  is a markup factor due to the monopolistic competition in the intermediate and final goods sector.

<sup>3</sup>The derivation of the optimal price for the differentiated final good of household-firm unit  $i$  is provided in Appendix in Section 1.5.

## 1.2.2 Aggregate Welfare and Welfare Cost of Nominal Rigidities

### Aggregate Welfare

Recall that the utility for household-firm  $i$  is defined by equation (1.1) in Section 1.2.1. Let  $U_t$  denote the aggregate welfare in the two-sector Yeoman Farmer economy obtained by integrating the above utility function over  $[0, 1]$

$$U_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \log[C_j] - \frac{\kappa}{1 + \chi_m} \int_0^1 N_{m,j}(i)^{1+\chi_m} di - \frac{\kappa}{1 + \chi_f} \int_0^1 N_{f,j}(i)^{1+\chi_f} di \right\}. \quad (1.17)$$

The aggregate welfare in this economy decreases due to the households' work effort in both sectors and the price dispersions. Let  $ALM_t$  define the aggregate disutility of work in the intermediate goods sector. In particular,  $ALM_t = \int_0^1 N_{m,t}(i)^{1+\chi_m} di$ . The aggregate disutility of work is

$$ALM_t = \left( \frac{M_t}{Z_{m,t}} \right)^{1+\chi_m} DGM_t, \quad (1.18)$$

where

$$DGM_t = \int_0^1 \left( \frac{P_{m,t}}{P_{m,t}^*(i)} \right)^{\sigma(1+\chi_m)} di, \quad (1.19)$$

is the extra disutility due to the price dispersion in the intermediate goods sector.

In a flexible price environment, the output in the intermediate goods sector would

be equivalent to  $M_t = Z_{m,t}N_{m,t}$  and therefore equation (1.18) is simply  $ALM_t = \int_0^1 N_{m,t}^{1+\chi_m} di = N_{m,t}^{1+\chi_m}$  as  $DGM_t = 1$  when prices are flexible.

The difference equation governing the dynamics of  $DGM_t$  is as follows:

$$DGM_t = (1 - \omega_m) \left[ \frac{P_{m,t}}{P_{m,t}^*(i)} \right]^{\sigma(1+\chi_m)} + \omega_m \left[ \frac{P_{m,t}}{P_{m,t-1}} \right]^{\sigma(1+\chi_m)} DGM_{t-1}. \quad (1.20)$$

It is straightforward to show that the aggregate disutility of work in the final goods sector is

$$AL_t = \int_0^1 N_{f,t}(i)^{1+\chi_f} di = \left( \frac{P_{m,t}}{P_t} \right)^{\frac{\alpha_f(1+\chi_f)}{1-\alpha_f}} \left( \frac{1}{Z_{f,t}\alpha_f} \right)^{\frac{1+\chi_f}{1-\alpha_f}} Y_t^{1+\chi_f} DG_t, \quad (1.21)$$

where

$$DG_t = \int_0^1 \left[ \frac{P_t}{P_{f,t}^*(i)} \right]^{(1+\chi_f)(\sigma + \frac{\alpha_f}{1-\alpha_f})} di, \quad (1.22)$$

is the extra disutility of work due to the price dispersion in the final goods sector, analogous to  $DGM_t$ . Equation (1.22) can be written as

$$DG_t = (1 - \omega_f) \left( \frac{P_t}{P_{f,t}^*(i)} \right)^{(1+\chi_f)(\sigma + \frac{\alpha_f}{1-\alpha_f})} + \omega_f \left( \frac{P_t}{P_{t-1}} \right)^{(1+\chi_f)(\sigma + \frac{\alpha_f}{1-\alpha_f})} DG_{t-1}. \quad (1.23)$$

Aggregate welfare in this economy is therefore

$$U_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \log[C_j] - \frac{\kappa}{1+\chi_m} ALM_j - \frac{\kappa}{1+\chi_f} AL_j \right\}, \quad (1.24)$$

where  $ALM_j$  and  $AL_j$  are given by equations (1.18) and (1.21).

To summarize, there are two types of rigidities in this model. The first type of rigidity is price rigidity—Calvo style price setting— that creates price dispersion in the intermediate and final goods sector. Price dispersion affects households' choice of consumption basket of goods and their employment decisions. Since output and employment are demand determined, each household's labor input will depend on consumption decision of other households. Therefore, there will be a gap between MRS and MPL. The second type of rigidity is the real rigidities that come from the assumption of labor immobility. Recall that the disutility of work enters in the household-firms' utility separately for the intermediate and final goods sector. The labor immobility together with price stickiness creates employment dispersion even though the model abstracts from wage rigidities. On the one hand, the aggregate output is inefficiently low due to the price dispersion in both sectors. On the other hand, the terms  $DG_t$  and  $DGM_t$  in the aggregate disutilities of work for both sectors summarize the dispersion in the employment due to the price stickiness and labor immobility in the model. Therefore, the model implies that household-firms dislike the volatilities in their consumption and work effort due to the nominal and real rigidities.

## Welfare Cost of Nominal Rigidities

In order to compute the welfare cost of nominal rigidities I start with defining the value functions for aggregate welfare in the flexible and sticky price environment. Let  $V_t$  be the value function for aggregate welfare at time  $t$ , evaluated at the non-stochastic steady state values of the state variables when prices are flexible. Equation (1.24) implies

$$V_t = \log(C_t) - \frac{\kappa}{1 + \chi_m} ALM_t - \frac{\kappa}{1 + \chi_m} AL_t + \beta E_t [V_{t+1}]. \quad (1.25)$$

Similarly, let  $V_t(\omega_m, \omega_f)$  be the value function for aggregate welfare in the presence of price rigidities in both sectors where  $(\omega_m, \omega_f)$  denotes the parameters for the degree of price stickiness in each sector.

Therefore, the welfare cost of the nominal rigidities in this two-sector model can be written as

$$WC_t(\omega_m, \omega_f) = V_t(0, 0) - V_t(\omega_m, \omega_f). \quad (1.26)$$

In particular, let  $\{C_j^*, ALM_j^*, AL_j^*\}$  denote the consumption and average disutility of work in each sector in the flexible price solution, respectively. Similarly, let  $\{C_j, ALM_j, AL_j\}$  denote the consumption and the average disutility of work in each

sector in the sticky price solution. Also, let  $\zeta$  solve

$$\begin{aligned}
V_t(0,0) &= E_t \sum_{j=t}^{\infty} \beta^{j-t} [\log C_j^* - \frac{\kappa}{1+\chi_m} ALM_j^* - \frac{\kappa}{1+\chi_f} AL_j^*] = \\
&= E_t \sum_{j=t}^{\infty} \beta^{j-t} [\log((1+\zeta)C_j) - \frac{\kappa}{1+\chi_m} ALM_j - \frac{\kappa}{1+\chi_f} AL_j] \\
&= \frac{\zeta}{1-\beta} + E_t \sum_{j=t}^{\infty} \beta^{j-t} [\log C_j - \frac{\kappa}{1+\chi_m} ALM_j - \frac{\kappa}{1+\chi_f} AL_j] \\
&= \frac{\zeta}{1-\beta} + V_t(\omega_m, \omega_f)
\end{aligned}$$

$$\frac{\zeta}{1-\beta} = V_t(0,0) - V_t(\omega_m, \omega_f) \quad \Rightarrow \quad \zeta = (1-\beta)WC_t(\omega_m, \omega_f) \quad (1.27)$$

As a result,  $WC_t(\omega_m, \omega_f) = 100 \times \zeta$  expresses the welfare cost of price rigidities in both sectors as a percentage of consumption for  $\beta = 0.99$  (discount factor) and for any Calvo price setting parameters  $(\omega_m, \omega_f)$ .

One should note that the welfare cost computed in this paper is not the same as the one given in Lucas (2003) paper. The disutility of the work effort in the households' utility function and the household heterogeneity due to the firm-specific labor input make hard to compare the welfare cost in this paper with Lucas' calculations. However, one can interpret  $WC_t(\omega_m, \omega_f)$ , the welfare cost of price rigidities in the two sectors, as the percentage of consumption households would be willing to give up in order to avoid the fluctuations in consumption and employment due to nominal price stickiness. In his thought experiment, Lucas calculates that a single consumer would

give up about one-half of one-tenth of a percent if all the variability around a trend in his consumption were to disappear magically. Since the fluctuations in consumption come from different sources such as price rigidity and the frictions in labor and financial markets, Lucas' calculation indicates that the welfare cost of nominal rigidities would be small too in a model, which partly explains the fluctuations in consumption. However, numerical results presented in Section 1.3 will prove otherwise. Section 1.3 presents the welfare cost of nominal rigidities under optimal Taylor-type policy rules for a reasonable parameterization of the one-sector and two-sector models to investigate how costly it can be to society if the monetary authority does not respond properly to the changes in the price levels in the economy. Note that the monetary authority cannot achieve the first-best outcome due to monopolistic competition and the price stickiness in both sectors. However, since the price stickiness and labor immobility create fluctuations in consumption and employment, the welfare cost of nominal rigidities given by  $WC_t(\omega_m, \omega_f)$  will be changing for certain monetary policy rules and therefore might be reduced with an appropriate monetary policy rule.

### **1.3 Calibration and the Model Implications**

I first present the numerical results from the one-sector model in this section. Since the one-sector model is a special case of the two-sector model, I do not specifically discuss its details in the paper. The assumptions about the production technology, nominal and real rigidities are the same as those assumed in the two-sector model. There is

an immense literature about the optimal monetary policy in one-sector models. King and Wolman (1999) reported that strict inflation targeting achieves the constrained optimum in a one-sector model in the presence of price rigidities. However, this result no longer holds when wage rigidities are introduced as in Erceg, Henderson and Levin (2000). Since my one-sector model is a special case of the two-sector model and abstracts from wage rigidities, an inflation targeting monetary policy rule will be appropriate in the calibration. The idea here is simply to see whether an inflation targeting monetary policy is still able to eliminate the welfare cost of the price rigidities in the presence of labor immobility. Then it will be possible to compare the numerical results from the one-sector model with those from the two-sector model in order to make conclusions about the effectiveness and welfare implications of the monetary policy rules studied in this paper.

### **1.3.1 One-Sector Model**

Table 1.1 specifies the parameters used in the one-sector model. The discount rate,  $\beta$ , is 0.99, which implies approximately 4% annual rate of return.  $1 - \alpha$  is the share of labor input in the production technology with capital assumed to be fixed. Another crucial parameter in the calibration of the model is  $\omega$  which measures the degree of price stickiness. In the benchmark calibration of the one-sector model  $\omega = 0.67$  which implies that household-firms adjust their prices, on average, every three quarters. The elasticity of substitution between the differentiated goods,  $\sigma$ , is assumed to be



7.00.<sup>4</sup> Following the standard business cycle literature, the autocorrelation in the productivity shock,  $\rho$  is 0.95, and the variance of innovations to the productivity shock is  $74.10^{-6}$  as in Canzoneri, Cumby, Diba (2007).

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Table 1.1

Parameters for the Benchmark

Calibration of One-Sector Model

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$\beta$	$\alpha$	$\omega$	$\sigma$	$\rho$	$\sigma_\varepsilon^2$
0.99	0.33	0.67	7.00	0.95	$74.10^{-6}$

---

In the calibration of the one-sector model, the monetary policy rule is a simple Taylor rule with which the monetary authority sets the interest rates in response to the changes in price levels and output fluctuations. In particular, the rules can be described in the following forms:

$$\text{Policy Rule (1)} : \quad \log(i_t) = -\log(\beta) + 1.5 \log(\pi_t) + 0.5(y_{act,t} - y_{ss,t}),$$

$$\text{Policy Rule (2)} : \quad \log(i_t) = -\log(\beta) + 100.0 \log(\pi_t),$$

where  $\pi_t = \frac{P_t}{P_{t-1}}$  is gross CPI inflation,  $y_{act,t}$  is the logarithm of aggregate output at time  $t$  with nominal and real rigidities and  $y_{ss,t}$  is the logarithm of steady state level of aggregate output. The welfare costs of price rigidities in the one-sector model

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<sup>4</sup>Price markups estimated in the literature vary across sectors from 11% to 23%. Rotemberg and Woodford (1997) use a markup factor of 15% for manufacturing sector. However, Bayoumi, Laxton, and Pesenti (2003) point out that markups in the other sectors are likely to be higher. Hence, following Canzoneri, Cumby, and Diba (2007), I use  $\sigma = 7$  which gives a markup factor of 17% as a middle value.

under the parameter values specified in Table 1.1 and the monetary policy rules (1) and (2) are given in Table 1.2.

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Table 1.2

Welfare Costs of Price Rigidity in the One-Sector Model

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	With Labor Mobility		Without Labor Mobility	
	$\chi = 1$	$\chi = 3$	$\chi = 1$	$\chi = 3$
Policy 1	0.29	1.16	2.74	5.15
Policy 2	0.00	0.00	0.00	0.00

---

In Table 1.2, the welfare costs of price rigidities in the one-sector model for Frisch elasticities of 1 and 0.33 are presented. The first row of Table 1.2 shows the welfare costs of the nominal rigidities as percentages of consumption under monetary policy rule 1 with or without labor mobility. Let's first consider perfect labor mobility case. An average household-firm is willing to give up 0.29% of its consumption to be free of the nominal rigidities for  $\chi = 1$  whereas this number increases to 1.16% of consumption for  $\chi = 3$  when labor can move freely among household-firm units. Similarly, the second row of Table 1.2 summarizes the welfare costs of nominal rigidities as percentages of consumption under monetary policy rule 2. For both  $\chi = 1$  and  $\chi = 3$ , as can easily be seen from Table 1.2, a policy rule that adjusts the interest rates in response to changes in price levels and output fluctuations are much more costly compared to a strict CPI inflation targeting rule. To be more specific, under perfect mobility assumption, welfare gain from conducting policy rule (2) instead of

policy rule (1) will be 1.16% of consumption each period when  $\chi = 3$ . Note that as the Frisch elasticity of labor supply decreases the welfare costs of nominal rigidities become bigger because the higher  $\chi$  the more household-firms dislike the fluctuations in their labor supply. However, since household-firms are allowed to allocate their labor freely among the firms when labor is perfectly mobile, they can easily move their labor input to another household-firm – in a sense start producing another type of differentiated product depending on the productivity shock that hits the economy and on the requirement for price adjustment. Since prices are not flexible, i.e., nominal rigidities exist; the labor mobility gives some flexibility to the household-firm, which results in less fluctuation in hours of work.

When labor is immobile, as can be seen from the last two columns of Table 1.2, the welfare costs of nominal rigidities are much bigger under policy rule (1). The intuition for this is as follows: Labor immobility causes labor input of the household-firm to fluctuate more. Since output is demand determined and since household-firm cannot adjust its price every time it needs to do so, its response to an increase (decrease) in demand is confined to increase (decrease) the hours of work, which leads to higher welfare cost of nominal rigidities. As in the perfect labor mobility case, higher welfare costs of nominal rigidities are associated with lower Frisch elasticity of labor supply.

In summary, two observations can be made from this one-sector model. First, the welfare costs of price rigidities are significant when labor is immobile across firms. While the welfare cost is 1.16% with perfect labor mobility when  $\chi = 3$ , it becomes

5.15% of consumption if real rigidities are also present in the model. The numerical results from the one-sector model with nominal and real rigidities indicate that the welfare costs of nominal rigidities are very big when the monetary authority adjusts the interest rate in response to changes in price levels and to output fluctuations. When there is no labor mobility, the welfare gains from adopting policy rule (2) are 2.74% and 5.15% of consumption for  $\chi = 1$  and  $\chi = 3$ , respectively. Since the welfare gains are quite significant in either case, how the monetary authority adjusts the interest rate to reduce the fluctuations in consumption and employment will be very important. As a result, one may conclude that welfare gains from better countercyclical policies could be significant with an appropriate monetary policy rule. Second, the monetary authority can eliminate the welfare costs of price rigidity with strict inflation targeting rule (Policy Rule 2) with or without real rigidities in the one-sector model. However, as will be discussed in the next section, this result does not hold in a two-sector model with nominal and real rigidities.

Of course, the question of how big the welfare cost of nominal rigidities is in reality is somewhat subjective. First, since the policy rule (1) includes the output gap and the reaction to the gap is very large, it is not surprising the welfare costs in the one-sector model are very big.<sup>5</sup> Second, even if the economies are subject to some degree of labor immobility in reality, the assumption of no labor mobility might be extreme and might result in relatively higher welfare costs. Although the degree

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<sup>5</sup>Rotemberg and Woodford (1997) showed that smoothing output may reduce household welfare in a model driven by productivity shocks.

of labor immobility changes among countries, it is a well-known fact that labor in different sectors for any given country is subject to some degree of labor immobility. Nevertheless, the welfare cost of price rigidities in a model with some degree of labor immobility would likely be somewhere between 1.16% and 5.15 % of consumption for small values of Frisch elasticity of labor supply. What is more important here is that a strict inflation targeting rule is able to eliminate the welfare cost of nominal rigidities with or without labor immobility. Although we know that the monetary authority cannot achieve the first-best outcome due to monopoly distortions mentioned in the previous section, there are notable welfare gains if it adopts a strict inflation targeting in the one-sector model calibrated above. In sum, the type of the monetary policy conducted in a country might be very significant and might help increase economic efficiency.

### 1.3.2 Two-Sector Model

This section reports the welfare cost of nominal rigidities under optimal (Ramsey) monetary policy and optimal Taylor-type interest rate rules. In order to characterize optimal monetary policy, I assume that *ex-ante commitment* is feasible and a benevolent central bank maximizes aggregate welfare subject to the equilibrium constraints in the two-sector economy. Optimal monetary policy for the two-sector model is computed with the help of a Dynare procedure and associated subroutines, *get\_ramsey*, created for Levin and Lopez-Salido (2004) and Levin, Onatski, Williams,

and Williams (2005). This procedure generates the first-order conditions of the Ramsey policymaker and provides a linear system for obtaining the numerical steady-state of the Lagrange multipliers. When there are no stochastic disturbances, i.e. each of the exogenous variables  $Z_{k,t}$ ,  $k = \{m, f\}$  takes a constant value, the first-order conditions for optimality admit a steady state solution, in which the rates of inflation in both sectors are zero. For small enough exogenous productivity shocks, the optimal policy would require rates of inflation in both sectors to fluctuate around zero.

The implementation of an optimal policy requires the central bank to monitor the past and the current values of two measures of inflation, output gap and of Lagrange multipliers in order to determine how it must act to maximize aggregate welfare subject to the equilibrium constraints in the private economy. A useful question about the optimal policy is the degree to which it can be achieved through one simple rule for the conduct of monetary policy. Due to price rigidities in both sectors, one obvious candidate would be a modified Taylor rule with two measures of inflation, a lagged interest rate and the output gap. The modified Taylor rule includes the lagged interest rate in order to smooth interest rate fluctuations and to avoid hitting the zero-bound for the interest rate. Let  $\pi_{m,t} = \frac{P_{m,t}}{P_{m,t-1}}$  and  $\pi_t = \frac{P_t}{P_{t-1}}$  denote the gross inflation rates in the intermediate and final goods sectors, respectively.

$$\begin{aligned}
\text{Policy Rule (3)} : \log(i_t) = & r_r \log(i_{t-1}) + \\
& (1 - r_r) \left\{ \begin{array}{l} b \left[ (1 - a) \log(\pi_{m,t}) + a \log(\pi_t) \right] \\ + r_y (y - y_{ss}) - \log(\beta) \end{array} \right\},
\end{aligned}$$

While the coefficient  $a \in [0, 1]$  is the weight on CPI inflation, the coefficient  $b \in [1.1, 3]$  measures the responsiveness of the monetary authority to changes in the weighted average of inflation rates in each sector.<sup>6</sup> All reaction coefficients in front of targeting variables are chosen optimally to maximize aggregate welfare. In order to characterize these coefficients in the modified Taylor rule, I used Dynare (see Julliard (2003)) in Matlab to compute the second order approximation of the model and of the value function for aggregate welfare around the zero-inflation steady state.

Table 1.3 summarizes the calibrated values of the parameters in the two-sector model.

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Table 1.3

Parameters for the Benchmark Calibration of Two-Sector Model

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$\beta$	$\alpha_f$	$\sigma$	$\omega'_m$	$\omega'_f$	$\rho_m$	$\rho_f$	$\sigma_{\varepsilon,m}^2$	$\sigma_{\varepsilon,f}^2$
0.99	0.90	7.00	0.75	0.30	0.95	0.95	$74.10^{-6}$	$74.10^{-6}$

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<sup>6</sup>Here, I restrict the coefficient  $b$  to be between 1.1 and 3. For Blanchard-Kahn conditions to be satisfied,  $b$  needs to be greater than 1. Although, in theory,  $b$  could be any number greater than 1, in searching for coefficients in optimal modified Taylor rule, I set the upper bound to be 3. Having a larger coefficient lower the welfare cost of nominal rigidities only marginally but would require large changes in nominal interest rate, which are not supported in the data.

In the benchmark calibration for the two-sector model, there are some additional things to decide. First, how do we break the whole economy into two sectors? In the results presented here, I take the final goods sector to be the retail sector and the intermediate goods sector to be everything else in the economy. According to Bureau of Economic Analysis (BEA) 2007 sectoral data, the size of the retail sector is around 7% percent of the whole economy, therefore it is reasonable to assume the share of the labor in the final goods production to be 0.10. The elasticity of substitution between the differentiated goods in the intermediate and final goods sectors is the same as in the one sector model. Moreover, following Huang and Liu (2005), the autocorrelation coefficients in productivity shocks and the variances of the innovations to the shock are the same in both sectors, i.e.,  $\rho_m = \rho_f = 0.95$  and  $\sigma_{\varepsilon,m}^2 = \sigma_{\varepsilon,f}^2 = 74.10^{-6}$ .

One other important thing that needs to be decided is the price rigidities in the two sectors. I consider three cases to calibrate  $(\omega_m, \omega_f)$ . In the first case, I assume that prices are more flexible in the final goods sector based on the studies by Blinder et al. (1998) and Bils and Klenow (2004) so that  $(\omega'_m, \omega'_f) = (0.75, 0.30)$ . Blinder et al. (1998) survey 2000 firms from manufacturing, services, trade, construction and mining, transportation, communication and utilities industries. They find that a median firm adjusts its price once a year and about 80% of those firms' products are intermediate goods. In the light of this study, I assume the price rigidity parameter is 0.75 in the intermediate goods sector. For the final goods sector, one could expect that prices are more flexible compared to those in the intermediate goods sector. Bils



and Klenow (2004) examine the frequency of price changes for 350 categories of goods and services covering about 70% of consumer spending, based on unpublished data from the BLS over the years 1995 and 1997 and conclude that half of prices last less than 4.3 months. Therefore, it is reasonable to assume the price rigidity parameter to be 0.30 so that prices in the retail sector last around one and a half quarters. In the second case, following the empirical survey by Taylor (1999),  $(\omega''_m, \omega''_f)$  are set to  $(0.75, 0.75)$  so that nominal prices in each sector last on average four quarters. In the third case, I chose the price rigidities in both sectors to match the responses of log intermediate good prices to a monetary policy shock as in Clark (1999). Clark (1999) reports that in response to a policy tightening input prices fall more rapidly and by a larger amount than output prices at early stages of production. In this case,  $(\omega'''_m, \omega'''_f)$  are found to be  $(0.50, 0.75)$  so that intermediate good prices are more flexible to be consistent with Clark's (1999) finding.

Before presenting the welfare cost of nominal rigidities under optimal monetary policy and optimal modified Taylor rule, I consider an empirical specification of monetary policy given in Canzoneri, Cumby, and Diba (2007). They estimate an interest rate rule over the Volcker and Greenspan years (1979.3 - 2003.2) to describe monetary policy:

$$\log(i_t) = 0.222 + 0.824 \log(i_{t-1}) + 0.35552 \log(\pi_t) + 0.032384(y_t - y_{ss}) + \epsilon_{i,t}, \quad (1.28)$$

where  $\pi_t = \frac{P_t}{P_{t-1}}$  and the standard error of the interest rate shock,  $\epsilon_{i,t}$ , is 0.00245.<sup>7</sup>

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Table 1.4

Welfare Cost of Nominal Rigidities under Estimated Interest Rate

Rule when  $\chi = 3$

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$(\omega_m, \omega_f)$	Without Labor Mobility	With Labor Mobility
(0.75, 0.30)	1.62	0.23
(0.75, 0.75)	2.33	0.43
(0.50, 0.75)	2.19	0.31

---

The welfare cost of price rigidities with and without labor mobility under the estimated interest rule are summarized in Table 1.4. The first row reports the welfare costs for  $(\omega_m, \omega_f) = (0.75, 0.30)$  following [Bils and Klenow \(2004\)](#) and [Blinder et al. \(1998\)](#). Because the degree of nominal rigidity in each sector would potentially affect the model dynamics and therefore the welfare cost, the last two rows in Table 1.4 also report the welfare costs for the other two cases of  $(\omega_m, \omega_f)$  following [Taylor \(1999\)](#) and [Clark \(1999\)](#).<sup>8</sup>

One striking fact from Table 1.4 for all three calibrations of price rigidities, the welfare costs are much higher without labor mobility for the estimated rule as in the one-sector model. When labor is immobile, welfare costs are substantial and range

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<sup>7</sup>See Appendix B (available online) in [Canzoneri, Cumby, and Diba \(2007\)](#) for their estimation procedure and data sources.

<sup>8</sup>In the two-sector model, I only present the welfare cost of nominal rigidities for the Frisch elasticity of 1/3. As we already know from the one-sector model, the smaller the Frisch elasticity of labor supply, the larger the welfare cost of nominal rigidities. In this section, I rather investigate the effects of the degree of price rigidity in each sector on the welfare cost and the need for the central bank to react to PPI inflation in setting the short-term interest rate.

from 1.62% to 2.33% of consumption per period. However, they become relatively smaller if labor can freely move across sectors and firms. As in the one-sector model, labor mobility helps to attenuate the fluctuations in the labor input of household-firms, which, in turn, brings about lower welfare cost. Not surprisingly, the higher welfare loss occurs when both sectors are more rigid, i.e.,  $(\omega_m, \omega_f) = (0.75, 0.75)$  regardless of labor immobility. Table 1.4 shows there is clearly room for policy improvement especially when there are real rigidities in the economy. For example, from Tables 1.4 and 1.5, we see that when  $(\omega_m, \omega_f) = (0.75, 0.75)$ , the welfare costs of price rigidities are 2.33% and 0.40% of consumption under the estimated rule and Ramsey policy, respectively. Therefore, the welfare cost of price rigidities relative to Ramsey policy is 1.93% of consumption each period if labor mobility is restricted. This suggests that welfare gains from an appropriate monetary policy rule can be quite significant.

It is well-known that, in the class of one-sector models with price rigidities only, constrained optimum is attainable with a strict inflation targeting. In Section 1.3.1, we have seen that this result can be generalized to one-sector models with real rigidities in the form of labor immobility. The welfare costs of nominal rigidities are completely eliminated with an extreme inflation targeting policy even if labor is immobile in the one-sector model.

In the two-sector model, we know that a first-best solution is not attainable because the monetary authority faces trade-offs in stabilizing price dispersions in both

sectors and output gap. Then one could possibly wonder if there exists a simple implementable policy rule which can mimic the Ramsey solution, which is second-best. If not, it is natural to ask to what degree an implementable rule is able to reduce the welfare cost of nominal rigidities relative to a Ramsey solution without labor mobility?

Table 1.5 summarizes the welfare costs and optimal coefficients in the modified Taylor rule for the two sector model without labor mobility. As expected, optimal monetary policy cannot achieve the flexible price solution for any configuration of price rigidities assumed. However, the welfare costs of nominal rigidities under optimal monetary policy might be used as a benchmark value to evaluate the performance of the simple implementable rules.

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Table 1.5

Welfare Costs ( $WC$ ) under Optimal Monetary Policy (OMP) and Optimal Modified Taylor Rule (OTR) When Labor is *Immobile*

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$(\omega_m, \omega_f)$	$r_r$	$b$	$a$	$WC$ Under OTR	$WC$ under OMP
(0.75, 0.30)	0.79	3	0.1	0.14	0.11
(0.75, 0.75)	0.89	3	0.7	0.47	0.40
(0.50, 0.75)	0.79	3	0.9	0.29	0.24

---

The first row of Table 1.5 presents the welfare costs under the assumption of more flexible prices in the final goods sector. Under the optimal modified Taylor rule, the welfare cost of nominal rigidities are lowered substantially (0.14% of consumption

per period) compared to that is under the estimated rule (1.62% of consumption per period). Since the prices in the final goods sector are quite flexible, the optimal coefficient in front of CPI inflation is quite small and equal to 0.1. On the other hand, PPI inflation requires a bigger reaction and the optimal weight is 0.9 due to more rigid prices in the intermediate goods sector. Note that the welfare cost under optimal monetary policy is 0.11% of consumption and the optimal modified Taylor rule with two measure of inflation is quite successful in bringing the welfare cost of nominal rigidities close to that under the optimal monetary policy.

The second row of Table 1.5 considers the case where the degree of price rigidities are the same in both sectors and prices in each sector last around four quarters. The welfare costs are almost four times larger compared to the previous case for both optimal monetary policy and modified Taylor rule. From Table 1.5 we know that the welfare cost under the estimated rule with more rigid prices and without labor mobility is about 2.33% of consumption per period. Although the optimal modified Taylor rule that reacts to the two measures of inflation cannot achieve the second-best solution, the welfare gain from replacing the estimated rule with the optimal modified Taylor rule is quite substantial (1.86% of consumption).<sup>9</sup> With stickier prices in the final goods sectors, the optimal coefficient in front of CPI inflation rate increases from 0.1 to 0.7 as one would expect. Even though the optimal coefficient in front of PPI inflation goes down to 0.3, it still requires some reaction by the monetary authority.

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<sup>9</sup>The welfare gains are calculated by subtracting the welfare cost under optimal modified Taylor rule from the welfare cost under the estimated rule.

The last row of Table 1.5 reports the welfare costs of nominal rigidities under the assumption that prices in the intermediate goods sectors reacts more rapidly and by a larger amount than those in the final goods sector to a positive monetary policy shock, following Clark (1999). In this case, the results are qualitatively similar to those earlier cases considered above. However, one thing that should be noted is that when prices in the intermediate goods sector are assumed to be more flexible compared to those in final goods sector, the optimal weight in PPI inflation is relatively smaller and equal to 0.1. Although PPI inflation in this case requires relatively little attention by the central bank, it is still not negligible. A policy switch from a standard estimated Taylor rule to a modified rule results in a welfare gain of 1.9% of consumption per period. As before, the modified Taylor rule gets very close to the second-best solution.<sup>10</sup>

For all three cases of price rigidities assumed above, we see that the inflation in the intermediate goods sector requires some attention by the central bank. However, its optimal weight depends on the degree of the price rigidities in both sectors. The optimal weight is the smallest when intermediate goods prices are more flexible compared to the final goods prices. Also, we observe that an estimated standard Taylor rule is very costly to society when labor is immobile. Although an optimal Taylor rule with both CPI and PPI inflation cannot entirely eliminate the welfare cost of nominal

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<sup>10</sup>In the modified Taylor rule, the optimal coefficient in front of the output gap is found to be essentially zero. This is not surprising given that Rotemberg and Woodford (1997) reported that smoothing output may reduce household welfare in a model driven by productivity shocks.

rigidities, it does reasonably well if one takes the welfare cost under optimal monetary policy as a benchmark value. The welfare gains from adopting a modified Taylor rule are quite large under any assumption about the degree of the price rigidity in each sector and range from 1.48% to 1.9% of consumption per period. This suggests that there is potential room for improvement in demand management policies.

This is not the first paper that finds the CPI inflation targeting is not always optimal. A recent paper by Huang and Liu (2005) showed that, in a vertically integrated two-sector model, a benevolent monetary authority should conduct a policy in which the interest rate responds to variations in both CPI and PPI inflation.<sup>11</sup> This paper differs from Huang and Liu (2005) by the introduction of the real rigidities into the model. For comparison purposes, Table 1.6 reports the welfare costs of different degrees of nominal rigidity in a vertically integrated two-sector model with perfect labor mobility. When labor is perfectly mobile, the household-firms are allowed to allocate their labor input freely among the production of the differentiated goods and the labor can also move freely between the final and the intermediate goods sector. In particular, the following numerical results are obtained by replicating the model in Huang and Liu (2005). The only difference is that I calculate the welfare cost of nominal rigidities in consumption equivalence whereas the welfare costs calculated in their paper are expressed as the ratios of the actual welfare losses to that under the optimal monetary policy.<sup>12</sup> However, it is clearly not obvious from these ratios

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<sup>11</sup>In this paper, PPI inflation is equivalent to the inflation in the intermediate goods sector.

<sup>12</sup>See Table 3 for details on Page 1454 in Huang and Liu (2005).

how big the actual welfare losses would be in consumption equivalence.<sup>13</sup> Therefore, the welfare costs based on the model in Huang and Liu (2005) are calculated as the difference between the value function for aggregate welfare at time  $t$ , evaluated at the non-stochastic steady state values of the state variables under flexible prices and that under nominal rigidities with the optimal monetary policy and optimal modified Taylor rule. If one relaxes the assumption of real rigidities in the model, the numerical results from the two-sector model with perfect labor mobility is as follows:<sup>14</sup>

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Table 1.6

Welfare Costs ( $WC$ ) under Optimal Monetary Policy (OMP) and Optimal Modified Taylor Rule (OTR) When Labor is *Mobile*

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$(\omega_m, \omega_f)$	$r_r$	$b$	$a$	$WC$ Under OTR	$WC$ under OMP
(0.75, 0.30)	0.19	3	0.2	0.01	0.01
(0.75, 0.75)	0.59	3	0.6	0.03	0.02
(0.50, 0.75)	0.49	3	0.8	0.02	0.01

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As was the case for the two-sector model without labor mobility, the optimal weight in inflation in the intermediate goods sector is positive and ranges from 0.2 to 0.8 depending on the degree of price rigidity in this sector. This result is consistent with the finding in Huang and Liu (2005). However, note that, with labor mobility,

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<sup>13</sup>There is only one occasion in which Huang and Liu (2005) argues that the welfare loss is 0.3% of consumption under optimal Taylor rule for a reasonable calibration of parameters in the model. However, the standard deviations of the innovations to the productivity shocks are assumed to be 0.02, which are substantially larger than empirical estimates found in the RBC literature. For example, see King and Rebelo (1999) for the estimates of productivity process in RBC models.

<sup>14</sup>The calibrated parameters are the same as given in Table 1.4. The numbers in Table 1.6 differ from those in Table 1.5 due to the assumption of perfect labor mobility.



the welfare costs of price rigidities are fairly low no matter which sector has more flexible prices. When both sectors exhibit quite sticky prices, i.e.,  $(\omega_m, \omega_f) = (0.75, 0.75)$ , the welfare cost of nominal rigidities is 0.03% of consumption.<sup>15</sup> The optimal modified Taylor rule almost achieves the second-best solution. The welfare gain from adopting a modified Taylor rule with two measure of inflation instead of an estimated Taylor rule ranges from 0.22% to 0.40% of consumption per period when labor is mobile. Although these welfare gains are suggestive for researchers to investigate better demand management policies, they are relatively small. However, with the introduction of labor immobility into the model, the welfare gain from adopting a modified Taylor with two measures of inflation is at least 1.48% of consumption per period. The model presented in this paper addresses Lucas's argument by suggesting that there are some real rigidities one should consider before drawing any clear-cut conclusions about the efficacy of the monetary policy. Even without assuming allocative nominal wage rigidities, this paper shows that there are sizeable welfare gains with an appropriate monetary policy which takes into account the changes in price levels in both final and intermediate goods sectors.

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<sup>15</sup>This welfare cost is about one-tenth of the one found in Huang and Liu (2005). The main reason for this is that they assume that the standard deviations of the innovations to productivity shocks to be 0.02, which is way larger than what is reported in RBC literature. In the calibration of the two-sector model, I, on the other hand, set the standard deviation in line with the estimates in King and Rebelo (1999).

### 1.3.3 The Role of Labor (Im)mobility

In this section, I try to characterize the interactions between real and nominal rigidities. In order to investigate the implications of the labor (im)mobility on household welfare and optimal monetary policy it is useful to describe the intuition for the one-sector model at first. In this special case, we observe that labor immobility increases the welfare cost of nominal rigidities. What is more, I show that, in the one-sector model, extreme CPI inflation targeting is able to replicate the flexible-price solution even if labor mobility is restricted.

In standard one-sector models with staggered price setting, the monetary authority does not face a trade-off between stabilizing an output gap and CPI inflation. I show that this result holds in one-sector models with nominal and real rigidities. Labor immobility in this environment is costly to the households because it creates employment dispersion when the one-sector economy exhibits price rigidity. Suppose the one-sector economy is hit by a positive productivity shock. If prices were flexible, since each firm has the same marginal cost and the elasticity of substitution among the differentiated products are constant, each household-firm would choose equal amounts of firms' products and supply the same amount of labor. However, when prices are rigid household-firms are constrained to meet the demand at that given price. Since they are not able to adjust the product's price, they react by raising the markup and reducing the labor demand. Because labor mobility is restricted, this creates employment dispersion due to the fluctuations in hours of work.

In sum, the nominal price rigidity leads to inefficiency in consumption choices and labor hiring decisions, which are both costly to the household-firms. Once the central bank eliminates the variation in prices, the need for adjusting hours of work by the household-firms disappears along with inefficient consumption decisions. Because price stickiness together with labor immobility plays an allocative role in employment, the elimination of variations in prices helps remove the negative effects of labor immobility. For this reason, an extreme CPI inflation targeting is able to produce the flexible-price solution even when labor cannot move freely among household-firms.

Even though the interactions between real and nominal rigidities in the two-sector model are similar to those in the one-sector model, things are a little more complicated due to the allocative role played by the fluctuations in sectoral relative prices. In the two-sector model, the degree of price rigidities affects the magnitude of welfare costs, but it does not change them substantially as can be seen from Tables 1.4, 1.5 and 1.6. What is more effective in magnifying the welfare cost of nominal rigidities is the presence of labor immobility. From Table 1.4, we see that the welfare cost is 2.33% of consumption without labor mobility but it is only 0.43% of consumption with labor mobility under the estimated standard Taylor rule when  $(\omega_m, \omega_f) = (0.75, 0.75)$ . The introduction of labor immobility augments the welfare costs by at least five times under the estimated monetary policy rule.

As stated in Section 1.3.2, optimal monetary policy cannot replicate the flexible-price solution in the two-sector model. The benevolent central bank faces a trade-

off in stabilizing output and relative price gaps, CPI inflation and PPI inflation. Suppose again that there is a positive productivity shock in the economy. Since in this case there is a need for price adjustment, those firms which cannot reset prices due to price rigidities respond to this shock by reducing labor demand and changing markup. Because labor is firm-specific and cannot move across sectors either, this creates large fluctuation in hours of work, which households do not like. Even though CPI inflation targeting helps reducing the price dispersion and therefore employment dispersion in the final goods sector, the dispersion in the intermediate goods sector still exists. This variability in the prices in the intermediate goods sector adversely affects the demand for the composite intermediate good and labor input by the household-firms producing the differentiated intermediate good. Because these two sectors are vertically integrated, the inefficiency in the intermediate goods sector's output would lead to inefficient choices and therefore an inefficient level of output in the final goods sector.

In general, reacting only to one of CPI or PPI inflation would leave one of these sectors with inefficient choices. By taking price dispersions in both sectors into consideration, the central bank is able to reduce the welfare costs of nominal rigidities noticeably as discussed in the previous section. Since the optimal weight on inflation rates depends on the degree of price rigidity in these sectors, the central bank can only partially eliminate the price dispersion. In particular, the two-sector economy without labor mobility would still have some price dispersion along with employment

dispersion in both sectors but at considerably lower levels. A modified Taylor rule with two measures of inflation brings the level of welfare much closer to that under optimal monetary policy than the estimated Taylor rule does. When labor is mobile, however, most of the employment dispersion would disappear because household-firms could easily move their labor input to another household-firm – in a sense start producing another type of differentiated product depending on the productivity shock that hits the economy and on the requirement for price adjustment. Since prices are sticky by assumption, the labor mobility gives some flexibility to household-firms, which results in fewer fluctuations in their labor input. For this reason, we observe that the two-sector model with labor mobility has much lower welfare costs under optimal modified Taylor rule compared to the model without labor mobility.

## 1.4 Conclusion

This paper discusses the welfare cost of nominal rigidities in an economy with nominal and real rigidities. While the sources of nominal rigidities are sticky prices in the final and intermediate goods sectors, the source of real rigidities is labor immobility across firms and sectors. The numerical results from the one-sector model are included to better characterize the linkage between labor immobility and price rigidities and to see what it implies for the monetary authority. Although the model discussed in this paper abstracts from nominal wage rigidities, sizeable fluctuations in hours of work arise because of labor immobility and price stickiness. This, in turn, augments the

welfare losses because household-firms dislike fluctuations in both consumption and hours of work. Hence, the assumption of labor immobility plays a significant role for the quantitative results of this paper. One of the findings is that when there is only one sector, a monetary authority which adopts a strict CPI inflation targeting rule is able to eliminate the welfare cost of nominal rigidities. More specifically, even with the introduction of labor immobility into the one-sector model, strict inflation targeting achieves the flexible-price solution. I address Lucas' criticism about demand management policies by showing the welfare costs of nominal rigidities are significant under a standard Taylor rule and that gains from better countercyclical policies are substantial in the one-sector model when labor is immobile.

As a more realistic case, I then develop a two-sector model with nominal and real rigidities where the two sectors are vertically integrated. I find that the welfare costs of nominal rigidities vary from 1.62% to 2.33% of consumption per period under the estimated rule. However, welfare losses are relatively smaller if the two-sector model abstracts from real rigidities. As in the one-sector model, labor immobility and price stickiness create employment dispersion. Moreover, vertical integration of the two-sectors and the price dispersion in the intermediate goods sector lead to inefficiencies in labor-consumption decisions by the household-firm units in the final goods sector through the intermediate good even if the monetary authority eliminates the dispersion in the final goods sector. By using the welfare level under optimal monetary policy as a benchmark, I show that an optimal modified Taylor rule with

two measures of inflation does substantially better than the estimated monetary policy rule. The optimal weight on PPI inflation depends on the degree of price rigidities and is smallest when prices in the intermediate goods are more flexible than those in the final goods sector.

In this paper, I show that there is substantial room for improvement in countercyclical policies even when wages are flexible. In particular, I find that monetary policy becomes very important when labor cannot freely move across household-firms and sectors. For simplicity, I assumed that capital stock is fixed in the model. One may introduce capital accumulation and firm-specific capital into the model in order to evaluate its effect on the welfare costs of nominal inertia. It would also be interesting to introduce a service sector that sells directly to consumers because, in reality, not every firm sells through the retail sector, as postulated in the two-sector economy in this paper. These are issues for future research.

## 1.5 Appendix

*A1: The optimal price for the differentiated final good of household-firm unit  $i$  (equation (1.15) in the text)–*

The first order condition for the new price of the final good  $i$  is

$$\begin{aligned}
P_{f,t}^*(i) &= \left[ \frac{\sigma}{\sigma-1} \frac{\kappa}{1-\alpha_f} \frac{E_t \sum_{j=t}^{\infty} (\omega\beta)^{j-t} \left[ \frac{P_j^\sigma Y_j}{Z_{f,j} M_j(i)^{\alpha_f}} \right]^{\frac{1+\chi_f}{1-\alpha_f}}}{E_t \sum_{j=t}^{\infty} (\omega\beta)^{j-t} \lambda_j Y_j P_j^\sigma} \right]^{\frac{1-\alpha_f}{\sigma(\alpha_f+\chi_f)+1-\alpha_f}}, \\
&= \left[ \frac{(\sigma\kappa) E_t \sum_{j=t}^{\infty} (\omega\beta)^{j-t} \left[ \frac{P_j^{\sigma(1-\alpha_f)} Y_j^{1-\alpha_f} P_{m,j}^{\alpha_f}}{Z_{f,j} \alpha_f^{\alpha_f}} \right]^{\frac{1+\chi_f}{1-\alpha_f}}}{(\sigma-1)(1-\alpha_f) E_t \sum_{j=t}^{\infty} (\omega\beta)^{j-t} \lambda_j Y_j P_j^\sigma} \frac{P_{f,t}^*(i)^{\frac{(1-\sigma)\alpha_f(1+\chi_f)}{1-\alpha_f}}}{P_{f,t}^*(i)^{\frac{1-\alpha_f}{1-\alpha_f}}} \right]^{\frac{1-\alpha_f}{\sigma(\alpha_f+\chi_f)+1-\alpha_f}}.
\end{aligned}$$

Finally one can get

$$P_{f,t}^*(i)^{\frac{1+\alpha_f\chi_f+\sigma\chi_f(1-\alpha_f)}{1-\alpha_f}} = \frac{\sigma}{\sigma-1} \frac{\kappa}{1-\alpha_f} \frac{E_t \sum_{j=t}^{\infty} (\omega\beta)^{j-t} \left[ \frac{P_j^{\sigma(1-\alpha_f)} Y_j^{1-\alpha_f} P_{m,j}^{\alpha_f}}{Z_{f,j} \alpha_f^{\alpha_f}} \right]^{\frac{1+\chi_f}{1-\alpha_f}}}{E_t \sum_{j=t}^{\infty} (\omega\beta)^{j-t} \lambda_j Y_j P_j^\sigma}. \quad (1.15)$$

The derivation of the optimal price in the final goods sector is similar to that in the intermediate goods sector.

*A2: The aggregate disutility of work in the final goods sector (equation (1.21) in the text)*

Recall that  $AL_t = \int_0^1 N_{f,t}(i)^{1+\chi_f} di$  is the aggregate disutility of work in the final



goods sector and that the final good  $i$  can be written as

$$Y_{f,t}(i) = Z_{f,t}^{1/(1-\alpha_f)} \alpha_f^{\alpha_f/(1-\alpha_f)} N_{f,t}(i) \left( \frac{P_{f,t}(i)}{P_{m,t}} \right)^{\alpha_f/(1-\alpha_f)}.$$

One can rewrite equation above as

$$\begin{aligned} N_{f,t}(i) &= \frac{Y_{f,t}(i)}{Z_{f,t}^{1/(1-\alpha_f)} \alpha_f^{\alpha_f/(1-\alpha_f)} \left( \frac{P_{f,t}(i)}{P_{m,t}} \right)^{\alpha_f/(1-\alpha_f)}}, \\ &= \left( \frac{1}{Z_{f,t} \alpha_f^{\alpha_f}} \right)^{1/(1-\alpha_f)} \left( \frac{P_t}{P_{f,t}^*(i)} \right)^\sigma Y_t \left( \frac{P_{m,t}}{P_{f,t}^*(i)} \right)^{\alpha_f/(1-\alpha_f)}. \end{aligned}$$

I substitute the equation above into  $AL_t = \int_0^1 N_{f,t}(i)^{1+\chi_f} di$  to get the aggregate disutility of work

$$\begin{aligned} AL_t &= \left( \frac{Y_t^{1-\alpha_f}}{Z_{f,t} \alpha_f^{\alpha_f}} \right)^{1+\chi_f/(1-\alpha_f)} \int_0^1 \left( \frac{P_t}{P_{f,t}^*(i)} \right)^{\sigma(1+\chi_f)} \left( \frac{P_{m,t}}{P_{f,t}^*(i)} \right)^{\alpha_f(1+\chi_f)/(1-\alpha_f)} di, \\ &= \left( \left( \frac{P_{m,t}}{P_t} \right)^{\alpha_f} \frac{Y_t^{1-\alpha_f}}{Z_{f,t} \alpha_f^{\alpha_f}} \right)^{1+\chi_f/(1-\alpha_f)} \int_0^1 \left( \frac{P_t}{P_{f,t}^*(i)} \right)^{(\sigma+\alpha_f/(1-\alpha_f))(1+\chi_f)} di, \end{aligned}$$

$$AL_t = \left( \frac{P_{m,t}}{P_t} \right)^{\alpha_f(1+\chi_f)/(1-\alpha_f)} \left( \frac{1}{Z_{f,t} \alpha_f^{\alpha_f}} \right)^{1+\chi_f/(1-\alpha_f)} Y_t^{1+\chi_f} DG_t. \quad (1.21)$$

The derivation for the average disutility of work in the intermediate goods sector is similar to that in the final goods sector.

# Chapter 2

## Fire Sales of the Firms: A

## Monetary Model with Financial

## Frictions

### 2.1 Introduction

In the last two decades, many countries, developed or emerging, have experienced large credit expansions combined with an unprecedented rise in household and firm debt. The rise in asset prices encouraged firms to extract equity from their asset holdings leading to further thereby borrowing against the realized capital gains. In particular, the recent credit crisis and the subsequent actions of the Federal Reserve have generated a renewed interest in the interaction between financial market frictions

and monetary policy. The current financial crisis raises several questions regarding the design of monetary policy in the face of deteriorating economic conditions. In particular, is there a need for policy intervention and if yes, how should interest rate policies be designed to stabilize inefficient fluctuations in economic activity and inflation when firms are financially constrained? This paper addresses this question focusing on a pecuniary externality that arises from the combination of competitive market for real assets of the firms with collateral constraints.

Recently, Lorenzoni (2008) formalized the *systemic risk* argument in a three-period competitive equilibrium model in the presence of state-contingent contracts. He shows that competitive financial contracts can result in excessive borrowing ex ante and excessive volatility ex post. In this paper, I incorporate the *systemic risk* argument of Lorenzoni (2008) into an infinite horizon monetary model. I quantitatively examine the importance of the existence of a *pecuniary externality* on the dynamic behavior of the aggregate quantities and investigate the need for policy intervention during a financial crisis.

The monetary policy literature has developed in the last few years within the framework of the New Neoclassical Synthesis (NNS). This framework builds on micro-founded models with monopolistic competition and nominal rigidities, and has been an important tool for the normative analysis of monetary policy.<sup>1</sup> However, the transmission mechanism of monetary policy is limited to a conventional real interest rate

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<sup>1</sup>See, among many others, Goodfriend and King (1997), Clarida, Gali and Gertler (1999), King and Wolman (1999), Woodford (2003).

channel on aggregate demand in the NNS models. The analysis in this paper follows a different transmission channel and attempts to formalize the *systemic risk* argument, focusing on a *pecuniary externality* working through asset prices in a monetary model with credit market imperfections; collateralized debt linked to the evolution of asset prices.

The paper models a discrete time, infinite horizon economy, populated by entrepreneurs, households, and retailers. Entrepreneurs hire labor from households, purchase/sell capital in a perfectly competitive market, and combine the two to produce a homogenous intermediate good. Unlike the households, they have access to a risky but more productive technology in the production process, however; they are also subject to borrowing constraints. Since the entrepreneurs have limited external funds and their production is subject to an aggregate productivity shock, they need to sell part of their capital if the economy is hit by a negative aggregate shock to meet debt payments and to finance future production. More specifically, because equity financing is costly and there are limits on entrepreneurs' borrowing capacity, the only way left for the entrepreneurs to raise money will be to sell their capital. The model assumes that each household owns a firm in the traditional sector as in Lorenzoni (2008). Firms in this sector absorb capital sold in a competitive market by financially troubled entrepreneurs. Because the firms in the traditional sector are less productive and capital is traded in a competitive market, as the supply of capital increases by the financially troubled entrepreneurs, the price of capital will go down.

This creates a pecuniary externality: as entrepreneurs want to sell more capital, the price of capital decreases even more, which results in tighter borrowing constraints and inefficiencies in output and the allocation of capital.

Households consume, work for entrepreneurs, and produce in their traditional sector. Essentially, both entrepreneurs and traditional sector firms produce the same homogeneous intermediate goods and sell it to the retailers. However, the latter makes a less productive use of capital, which results in an inefficient level of output if the entrepreneurs are forced to sell their capital when borrowing capacity deteriorates. Retailers are monopolistically competitive and the source of nominal rigidities is sticky prices. They buy the intermediate goods produced by households and entrepreneurs and differentiate these products into final goods at no cost.

This paper is able to produce capital *fire sales* of the financially constrained entrepreneurs in the case of a negative productivity shock. Depending on the loan-to-value ratio, the capital stock of the entrepreneurial sector declines by 2.3% to 5.5% after the negative shock hits the economy. The resulting fall in output in the entrepreneurial sector is about 1.7% to 2.7% in response to a 1% negative productivity shock. These results suggest that the fact that asset prices are determined in a competitive market and that entrepreneurs are credit constrained play a major role in the responses of the aggregate quantities to technology shocks. It shows that the strength of borrowing constraints is important in explaining the amount of capital *fire sales*. The responses of aggregate quantities to a negative shock are larger if borrowing constraints are

relatively loose, i.e., if the loan-to-value ratio is higher. The depth of financial crisis and the inefficiency loss in aggregate quantities are more pronounced in an economy with high leverage because of the higher negative externality stemming from the fire sales of capital by highly indebted entrepreneurs.

Shleifer and Vishny (1992) and Kiyotaki and Moore (1997) are two leading examples that focus on the general equilibrium feedback between financial distress and asset prices. In the theoretical framework of Kiyotaki and Moore (1997), the financial multiplier works through a two-way feedback between asset prices and aggregate quantities: a decline in asset prices lowers the collateral value and contracts the entrepreneurs' borrowing capacity which result in lower investment; a fall in investment lowers future output, which depresses the current asset prices and reduces the value of the collateral. Thus, in theory, an economic shock can provide room for generating large fluctuations of aggregate quantities if it can move asset prices. If there is no such economic shock, the financial multiplier will have weak effects. Kocherlakota (2000) and Cordoba and Ripoll (2004) find weak financial multiplier effects of economics shocks in their calibrated models with credit constraints.<sup>2</sup> In this paper, on the other hand, I show that since asset prices are determined in a competitive market and capital moves from more productive sector to the traditional (less productive) sector following a negative productivity shock, technology shocks generate large fluc-

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<sup>2</sup>The magnitude of the amplification of shocks depends on the factor shares in a small open economy with credit constraints in Kocherlakota (2000). In addition, Cordoba and Ripoll (2004) study the effects of monetary shocks on output. They show that output amplification is larger only when debt contracts are not contingent upon the monetary shock.

tuations in asset prices so that their impact on output and capital stock work through credit constraints.

This paper also investigates whether a central bank with an implementable interest rate rule can reduce the size of the asset sales when the economy is hit by a negative aggregate shock. The reduction in the size of the asset sales leads to an increase in asset prices, resulting in reallocation of funds from the traditional sector, who is buying assets, to the entrepreneurial sector, who is selling them. Due to the presence of financial frictions, this reallocation leads to an aggregate welfare gain, which is not internalized by private agents.

Krugman (1998) emphasizes the role of asset *fire sales* during recent episodes of financial crisis. Pulvino (1998) and Aguiar and Gopinath (2005) present systematic evidence on fire sales. Iacoviello (2005) develops and estimates a monetary business cycle model with nominal loans and collateral constraints tied to housing values. He shows that collateral effects dramatically improve the response of aggregate demand to housing price shocks; and nominal debt improves the sluggish response of output to inflation shocks. However, in his paper, the metric adopted for the evaluation of the relative performance of policy rules is an output-inflation volatility frontier. Because, in this case, it is harder to correctly rank alternative specifications for monetary policy, and to safely draw any conclusions about the performance of different monetary policy rules, I follow a Ramsey-type approach to analyze optimal monetary policy in an economy with sticky prices, nominal debt, and borrowing constraints in the

presence of a pecuniary externality. In order to characterize optimal monetary policy, I assume that ex-ante commitment is feasible and a Ramsey planner maximizes aggregate welfare subject to the equilibrium constraints in the two-sector economy. Because optimal monetary policy requires the central bank to have the knowledge of the past and the current values of the aggregate variables and Lagrange multipliers, I investigate the degree to which it can be achieved through one simple rule for the conduct of monetary policy. Due to sticky prices and the two-way feedback between asset prices and aggregate quantities, one obvious candidate is a modified Taylor rule with goods price inflation and asset prices. Since goods price inflation can positively affect borrowers' net worth by reducing the real burden of the current debt for a given interest rate, I find that inflation variability is part of Ramsey equilibrium. In particular, I find that a flexible inflation targeting rule with a positive reaction to asset prices performs better than a strict inflation targeting policy. The welfare gain from the former policy rule relative to a strict stabilization of inflation is 0.23% of consumption per period.

The paper is organized as follows. Section 2.2 describes the model. Section 2.3 presents the numerical results and contains the optimal monetary policy analysis. Section 2.4 concludes.



## 2.2 Model

This section describes a discrete time, infinite horizon economy, populated by entrepreneurs, households, and retailers. Entrepreneurs and households purchase/sell capital in a competitive market, hire labor from households, and combine the two to produce an intermediate homogeneous good. Moreover, entrepreneurs raise external funds by issuing debt and equity. As in Hennessy and Whited (2005) and Jermann and Quadrini (2006), they prefer debt over equity because of its tax advantage. Unlike the households, they have access to a random technology in the production process, however; they are also subject to borrowing constraints as in Kiyotaki and Moore (1997). Households consume, work, and produce using the capital sold in the competitive market. They buy capital from financially troubled firms in a perfectly competitive market in case of a negative productivity shock and produce the same homogeneous good. Retailers are the source of nominal rigidities. They buy the intermediate goods produced by households and entrepreneurs; differentiate it at no cost into final goods. In addition, there is a central bank which adjusts nominal interest rates based on the variations in inflation and the price of capital.

### 2.2.1 Households

The household sector is standard, with the exception that each household owns a firm in the *traditional sector* as in Lorenzoni (2008). Firms in the traditional sector invest capital  $K_t$  in period  $t$  and use  $\bar{L}$  as an input from their leisure time to produce inter-

mediate goods in period  $t + 1$ . The technology of the traditional sector is represented by the production function

$$F(K_{t-1}, \bar{L}) = K_{t-1}^\nu \bar{L}^{1-\nu} \text{ with } 0 < \nu < 1, \quad (2.1)$$

Households maximize a lifetime utility function given by

$$E_t \sum_{j=t}^{\infty} \beta^{j-t} [\ln C_j]$$

where  $\beta \in (0, 1)$  is the discount factor,  $C_t$  is consumption at  $t$ . In addition, households are also entrepreneurial firms' shareholders and own non-contingent bonds along with equity shares. The flow of funds for the households in nominal terms is given by

$$\begin{aligned} P_t C_t + Q_t [K_t - K_{t-1}] + (1 + r_{t-1}) B_{t-1} + \\ s_t Q_t^s + T_t = B_t + W_t L_t + P_t^w F(K_{t-1}) + \Gamma_t + s_{t-1} (P_t^w D_t + Q_t^s), \end{aligned} \quad (2.2)$$

where  $B_t$  is the one-period bond,  $s_t$  the equity share,  $P_t^w D_t$  the equity payment received from their portfolio of shares, and  $Q_t^s$  the market price of the entrepreneurs' share.  $L_t^e$  is the labor supply of households to the firms in the entrepreneurial sector. Since there is no labor leisure choice,  $L_t^e \equiv 1$ ,  $\forall t$  in equilibrium. In addition,  $\Gamma_t$  are lump-sum profits received from the retailers, and  $T_t$  are net transfers from the central bank. Following Bernanke et al. (1999), I assume that output from the traditional and

entrepreneurial sector cannot be transformed immediately into consumption good. I also assume that retailers purchase the intermediate goods from entrepreneurs and traditional sector firms at the wholesale price  $P_t^w$  and transform it into a composite final good, whose price index is  $P_t$ . With this notation,  $X_t \equiv P_t/P_t^w$  denotes the markup of final over intermediate goods. Households and entrepreneurs buy and sell capital good in a perfectly competitive market in which the price of capital,  $Q_t$ , is determined by demand for and supply of capital.

First order conditions for capital, bond holding and equity shares are given by

$$K_t : \frac{q_t}{C_t} = \beta E_t \frac{1}{C_{t+1}} \left[ \frac{F'(K_t)}{X_{t+1}} + q_{t+1} \right], \quad (2.3)$$

$$B_t : \frac{1}{C_t} = \beta E_t \frac{(1+r_t)}{\pi_{t+1} C_{t+1}}, \quad (2.4)$$

$$s_{t+1} : \frac{q_t^s}{C_t} = \beta E_t \frac{1}{C_{t+1}} \left( \frac{D_{t+1}}{X_{t+1}} + q_{t+1}^s \right). \quad (2.5)$$

Equation (2.3) is an intertemporal condition on capital demand by the households. It requires the households to equate the marginal utility of consumption to the marginal gain of incremental investment in capital. Equation (2.4) is the standard Euler equation for households. The last equation determines the market price of equity shares.

## 2.2.2 Entrepreneurs

Entrepreneurs use a Cobb-Douglas constant returns-to-scale (CRS) technology that employs capital and labor as inputs. They produce a homogeneous intermediate good  $Y_t$  according to

$$Y_t = A_t(K_{t-1}^e)^\alpha(L_t^e)^{1-\alpha}, \quad (2.6)$$

where  $0 < \nu < \alpha < 1$  and  $A_t$  is an exogenous technology parameter with a standard AR(1) process  $\log A_t = \rho \log A_{t-1} + \varepsilon_t$ ,  $K_{t-1}^e$  is the amount of aggregate capital carried by the entrepreneur into period  $t$ , and  $L_t^e$  is the labor input hired from the households.

The entrepreneurs raise external funds by issuing debt and equity. As in Hennessy and Whited (2005) and Jermann and Quadrini (2006), entrepreneurs prefer debt over equity financing because of its tax advantage. Given the interest rate,  $r_t$ , and the tax rate,  $\tau$ , the effective cost of debt is  $\frac{r_t(1-\tau_t)}{\pi_t}$ .

Let  $V_t$  denote the nominal value of the firm at the end of period after paying dividends. It is defined as

$$V_t = E_t \sum_{j=t}^{\infty} M_j P_j^w D_j,$$

where  $M_j = \beta^{j-t} \frac{P_t C_t}{P_j C_j}$  is the relevant stochastic discount factor and  $D_{t+j}$  is the net payment to shareholders. The optimization problem is subject to the budget and borrowing constraints. As in Kiyotaki and Moore (1997), since entrepreneurs can default at the end of period and deflect some of their resources I assume there exists a limit on the obligations of the entrepreneurs. Suppose that the lenders can repossess

borrowers' assets by paying a proportional transaction cost  $(1 - m)E_t(Q_{t+1}K_t^e)$  if borrowers disclaim their debt obligations. Therefore, the maximum amount  $B_t$  that an entrepreneur can borrow cannot exceed  $mE_t(Q_{t+1}K_t^e/R_t)$ .

$$B_t^e \leq mE_t(Q_{t+1}K_t^e/R_t), \quad (2.7)$$

where  $R_t = 1 + r_t(1 - \tau)$ . Following Jermann and Quadrini (2006), I assume that the firm's payout is subject to a quadratic adjustment cost in order to capture the frictions associated with issuing and repurchasing shares as well as paying dividends. The total cost of payout,  $D_t$ , in units of intermediate good is:

$$\vartheta(D_t) = D_t + \eta(D_t - \underline{D})^2,$$

where  $\eta \geq 0$ , and  $\underline{D}$  is the long-run payout target level. The parameter  $\eta$  measures the degree of market incompleteness. In particular,  $\eta = 0$  represents a frictionless economy. In this case, a negative productivity shock which requires debt adjustments due to the borrowing limit would costlessly be substituted by issuing equity.

The flow of funds for the entrepreneurs in nominal terms is

$$Q_t [K_t^e - K_{t-1}^e] + (1 + r_{t-1}(1 - \tau_{t-1}))B_{t-1}^e + W_t L_t^e + P_t^w \vartheta(D_t) = P_t^w Y_t + B_t^e. \quad (2.8)$$

Entrepreneurs choose the payout  $D_t$ , the new capital  $K_t^e$ , the new debt  $B_t^e$ , and the

labor input  $L_t^e$  to maximize the nominal value of the firm  $V_t$  subject to equations (2.6), (2.7) and (2.8). Assuming entrepreneurs take all prices as given, including the stochastic discount factor  $M_t$  and the interest rate  $r_t$ , first order conditions for the entrepreneur's maximization problem are

$$L_t^e : \frac{(1 - \alpha)Y_t}{X_t L_t^e} = w_t, \quad (2.9)$$

$$K_t^e : \frac{q_t}{\vartheta_D(D_t)} = E_t \beta \frac{C_t}{C_{t+1}} \frac{1}{\vartheta_D(D_{t+1})} \left\{ \alpha \frac{Y_{t+1}}{X_{t+1} K_t^e} + q_{t+1} \right\} + m \mu_t q_{t+1} \pi_{t+1}, \quad (2.10)$$

$$B_t^e : \frac{1}{\vartheta_D(D_t)} = \mu_t (1 + r_t (1 - \tau)) + E_t M_{t+1} \frac{(1 + r_t (1 - \tau))}{\vartheta_D(D_{t+1})}, \quad (2.11)$$

where  $\mu_t$  is the time  $t$  shadow value of the borrowing constraint and  $\vartheta_D(D_t) = 1 + 2\eta(D_t - \underline{D})$ .

Both the Euler and the capital demand equations differ from the standard formulations due to the presence of  $\mu_t$ . Equation (2.9) is standard and equates the marginal product of labor to the real wage. Note that in equilibrium  $L_t^e$  will be normalized to 1 as there is no labor-leisure choice in the household utility. Equation (2.10) is an intertemporal condition on capital demand by the entrepreneurs. If the borrowing constraint is not binding in the current period, then the shadow value of the borrowing constraint is  $\mu_t = 0$ . If adjustment cost is zero in addition to a non-binding collateral constraint, equation (2.10) states that the marginal productivity of capital plus the realized resale value of the capital purchased in the previous period is equalized to its marginal cost.

The assumption of corporate tax advantage guarantees that entrepreneurs are always constrained in the steady state and the steady-state level of debt is unique and positive. More specifically, in a deterministic steady state, equations (2.4) and (2.11) with zero inflation imply  $\mu = 1/(1+r(1-\tau))-\beta = 1/(1+r(1-\tau))-1/(1+r) = r\tau/[(1+r(1-\tau))(1+r)] > 0$ . With uncertainty, as long as  $\tau$  is sufficiently high, the borrowing constraint holds with equality,

$$b_t^e = mE_t(q_{t+1}K_t^e\pi_{t+1}/R_t). \quad (2.7')$$

Therefore the existence of tax advantage guarantees that entrepreneurs would always borrow up to their limit in and around the steady state.

Following a negative productivity shock, firms will need to sell part of their productive asset (capital) because of the loss in revenues and credit market imperfections. More specifically, because equity financing is costly and there are limits on entrepreneurs' borrowing capacity, the only way left for the entrepreneurs to raise money to make debt payments will be to sell their capital. Because the firms in traditional sector are less productive and the capital is traded in a competitive market, as the supply of capital increases by the financially troubled entrepreneurs, capital price will go down in the current period. Because both entrepreneurs and households are rational, they will also expect lower future capital prices and tighter borrowing constraints. Note that all entrepreneurs are subject to a common technology shock. When an individual entrepreneur has trouble meeting debt payments and sells assets,

the highest valuation buyers of these assets are the other entrepreneurs in the same sector. But these entrepreneurs are themselves likely to have trouble making interest payments due to common negative technology shock, the productive asset (capital) will be sold to the firms in the traditional sector, which are less productive. This will reduce the price of capital below value in best use, leading to more of capital sold by the financially troubled entrepreneurs due to tightening borrowing constraints. This creates a pecuniary externality: as entrepreneurs want to sell more capital, the price of capital decreases even more, which will result in tighter borrowing constraints. In section 2.3, I provide the numerical results to discuss the quantitative effects of these fire-sales on asset prices and the aggregate quantities.

### 2.2.3 Retailers

I assume monopolistic competition at the retail level to motivate sticky prices as in Bernanke et al. (1999). A continuum of retailers of mass 1, indexed by  $i$ , buy intermediate goods  $F(\cdot) = M_t$  from the traditional sector firms and  $Y_t$  from entrepreneurs at price  $P_t^w$  in a competitive market. In order to simplify the notation I define the total intermediate good that a retailer firm buys in period  $t$  as  $Z_t = M_t + Y_t$ . Retailer firm differentiates the intermediate good  $Z_t$  at no cost into  $Z_t(i)$  and sell it at the price  $P_t(i)$ .



Final consumption good is

$$Z_t = \left( \int_0^1 Z_t(i)^{\sigma-1/\sigma} di \right)^{\sigma/\sigma-1},$$

where  $\sigma > 1$  is the elasticity of substitution among the differentiated products. Given this aggregate output index, the consumer price index is

$$P_t = \left( \int_0^1 P_t(i)^{1-\sigma} di \right)^{1/1-\sigma},$$

so that each retailer faces an individual demand curve of

$$Z_t(i) = (P_t(i)/P_t)^{-\sigma} Z_t.$$

Each retailer chooses a sale price  $P_t(i)$  taking  $P_t^w$  and the demand curve as given.

The sale price can be changed in every period only with probability  $1 - \omega$ . Denote with  $P_t^*(i)$  the "reset" price and  $Z_j^*(i) = (P_t^*(i)/P_j)^{-\sigma} Z_j$  the corresponding demand.

The optimal price,  $P_t^*(i)$ , solves:

$$\sum_{j=t}^{\infty} \omega^j E_t \left\{ \Lambda_{t,j} \left( \frac{P_t^*(i)}{P_j} - \frac{X}{X_j} \right) Z_j^*(i) \right\} = 0, \quad (2.12)$$

where  $\Lambda_{tj} = \beta^{j-t}(C_t/C_j)$  is the relevant discount factor and  $X$  is the steady state markup. This condition states that  $P_t^*$  equates expected discounted marginal revenue to expected discounted marginal cost. Profits  $\Gamma_t = (1 - 1/X_t)Z_t$  are finally rebated

to patient households.

As a fraction  $\omega$  of prices stays unchanged, the evolution of aggregate price level can be written as

$$P_t = [\omega (P_{t-1})^{1-\sigma} + (1 - \omega)(P_t^*)^{1-\sigma}]^{1/(1-\sigma)}. \quad (2.13)$$

## 2.2.4 Monetary Authority and Policy Rules

The monetary authority makes lump sum transfers of money to the real sector to implement a Taylor-type interest rate rule. Let  $r_{ss}$  and  $\pi_{ss}$  denote the steady states of the interest rate and inflation. Then the rule takes the form

$$\log [(1 + r_t)/(1 + r_{ss})] = r_\pi \log (\pi_t/\pi_{ss}) + r_q \log (q_t/q_{t-1}) + \varepsilon_t,$$

where  $r_\pi > 1$  and  $-\infty < r_q < \infty$ .  $\varepsilon_t$  is a white noise shock process with zero mean and variance  $\sigma_\varepsilon^2$ . Here, monetary policy responds systematically to current inflation and the change in asset prices following a Taylor-type interest rate rule.

## 2.2.5 Market Clearing and Equilibrium

Equilibrium in the goods market requires that the production of the final goods be allocated to consumption expenditure

$$C_t = Y_t + M_t. \quad (2.14)$$

Equilibrium in the debt, capital and labor markets requires respectively

$$B_t + B_t^e = 0, \tag{2.15}$$

$$K_t + K_t^e = \bar{K}, \tag{2.16}$$

$$s_t = 1, \tag{2.17}$$

$$\bar{L} = L_t = L_t^e = 1. \tag{2.18}$$

Therefore, for any specified policy process  $\{r_t\}$  and exogenous state vector  $\{A_t, \varepsilon_t\}$ , an (imperfectly) *competitive allocation* is a sequence for  $\{L_t, b_t, K_t, K_t^e, C_t, C_t^e, Y_t, M_t, \pi_t, \mu_t, q_t\}$  satisfying equations (2.1),(2.3)-(2.6), (2.7') and (2.8)-(2.18).

## 2.3 Parameterization and Numerical Results

### 2.3.1 Parameterization

This section describes the benchmark parameterization of the model. This will be useful for the quantitative analysis conducted below. The households' discount rate is set to be  $\beta = 0.99$ . This implies an annual real interest rate of 4%.

Throughout I assume that all outstanding debt is collateralized ignoring the role of unsecured debt. The stock of capital is assumed to be fixed throughout the analysis. It does not depreciate and there exists no technology for making it grow. Hence, in

the baseline parameterization, the depreciation rate,  $\delta$ , is zero.

Table 2.1

BENCHMARK PARAMETERS IN THE MODEL

Description	Parameter	Value
Discount rate	$\beta$	0.99
Price elasticity of demand	$\sigma$	7.00
Capital Share (Entrepreneurs)	$\alpha$	0.33
Capital Share (Traditional Sector)	$\nu$	0.10
Loan-to-value ratio	$m$	0.55
Depreciation rate	$\delta$	0.00
Payout Adjustment Costs	$\eta$	0.25
Corporate Tax Rate	$\tau$	0.30
Probability of Fixed Price	$\omega$	0.67

The annual average loan-to-value (LTV) ratio on real assets is roughly  $m = 0.55$ . To see the effects of LTV ratio on fire sales of the entrepreneurs, I also present the results for relatively looser borrowing constraints in response to developments in the financial market after 1980s. In this case,  $m = 0.75$  as the average nonfarm and nonfinancial businesses' loan-asset ratio is 0.75.

The probability of fixed prices  $\omega$  is set to 0.67. The elasticity of substitution between the differentiated goods,  $\sigma$ , is assumed to be 7.00.<sup>3</sup> Following the standard

<sup>3</sup>Price markups estimated in the literature vary across sectors from 11% to 23%. Rotemberg and

business cycle literature, the autocorrelation in the productivity shock,  $\rho$  is 0.95, and the variance of innovations to the productivity shock is  $74.10^{-6}$  as in Canzoneri, Cumby, Diba (2007). In entrepreneurial production,  $\alpha$  is chosen to be 0.33. On the other hand, the share of capital in the traditional sector production,  $\nu$ , is set to be 0.10 following Greenwood and Hercowitz (1991).<sup>4</sup> Table 2.1 summarizes the parameters.

### 2.3.2 Model Dynamics

Whether an economic shock will have significant effects on the dynamics of aggregate quantities and asset prices depends on the model's transmission mechanism. Figure 2-1 presents impulse responses of the entrepreneurial capital, output, debt and asset prices to a 1% negative productivity shock.

For a 1% negative productivity shock, the drop in entrepreneurial capital is immediate and it decreases by 2.26% approximately upon impact. Because there are frictions associated with issuing and repurchasing shares as well as paying dividends, equity financing is limited. In turn, entrepreneurs who have limited access to outside financing are forced to sell part of their capital to the firms in the traditional sector to finance their losses and to repay debt. In order for firms in the traditional sector to absorb the capital sold by the entrepreneurs, the price of the capital goes down and

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Woodford (1997) use a markup factor of 15% for manufacturing sector. However, Bayoumi, Laxton, and Pesenti (2003) point out that markups in the other sectors are likely to be higher. Hence, following Canzoneri, Cumby, and Diba (2007), I use  $\sigma = 7$  which gives a markup factor of 17% as a middle value.

<sup>4</sup>Greenwood and Hercowitz (1991) develop a model that stresses the role of capital in household activities to study the allocation of capital and time across the household and business sector.

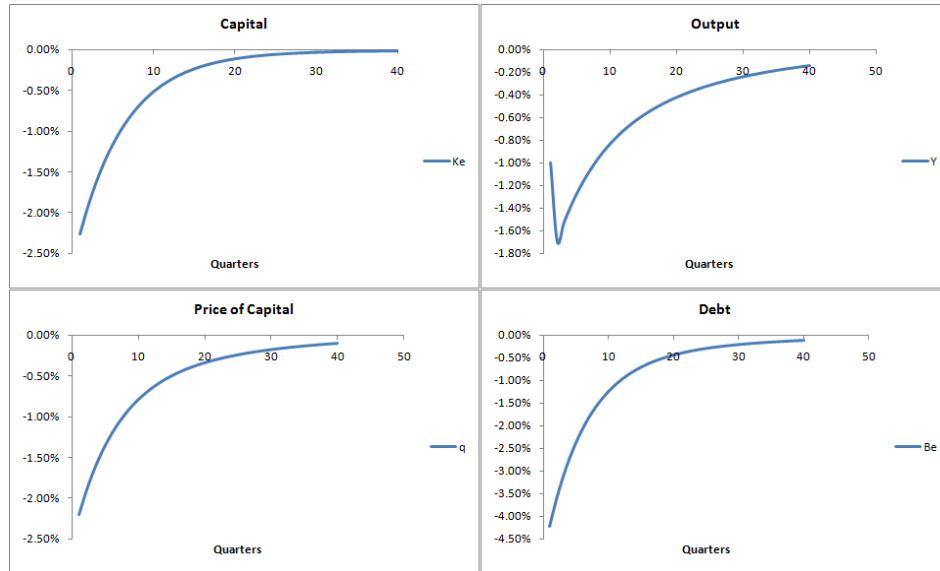


Figure 2-1: Impulse responses to a 1% negative productivity shock. In these simulations monetary policy is described by a strict stabilization of inflation,  $\pi_t = \pi_{ss}, \forall t$ .

starts increasing to its original level as the effects of the negative productivity shock disappear. As the price of capital declines, this contracts the borrowing capacity of the entrepreneurs by lowering the collateral value of productive asset. However, this causes entrepreneurs to sell even more capital. The fall in the productive asset of the entrepreneurs lowers future output and entrepreneurs' net worth today which reduces the current asset prices. Thus, because the productivity shock can move asset prices, it also propagates across the economy and creates large fluctuations in the entrepreneurial output through credit constraints. This finding is mainly driven by the movement of capital from the entrepreneurial sector to traditional sector and the fact that the price of capital is determined in a competitive market. As the marginal product of capital is lower in the traditional sector and as the supply of capital by

the financially troubled entrepreneurs increases, the price of capital needs to go down further to reach the competitive equilibrium in the asset market. Although entrepreneurs are more productive compared to the traditional sector firms, the capital stock is allocated inefficiently between the traditional sector firms and entrepreneurs due to the borrowing constraints on entrepreneurs' side. Therefore, we see a large movement in the price of capital, 2.2% initially, in response to a 1% negative productivity shock. This result is in contrast to Kocherlakota (2000) and Cordoba and Ripoll (2004) that find weak multiplier effects of technology shocks in their calibrated models with credit constraints.<sup>5</sup> In this paper, a technology shock is not only an important source of fluctuations in entrepreneurial output as in a standard real business cycle model, but also generates large movements in assets prices so that it affects entrepreneurial output through credit constraints. As it is clear from Figure 2-1, the initial decline in entrepreneurial debt is about 4.23%, which is driven by the lower collateral value of capital. Because of the decline in borrowing capacity and costly equity financing, entrepreneurs are forced to sell capital and thus the entrepreneurial output drops by 1.7% following the negative shock.<sup>6</sup>

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<sup>5</sup>Liu, Wang, and Zha (2010) emphasize that large financial multiplier effects can exist only in models with credit constraints that can generate large movements in asset prices resulting from financial shocks.

<sup>6</sup>Note that I assumed that the monetary authority strictly stabilizes the inflation in the simulation results of Figure 2-1. As we shall see in later sections, the conduct of monetary policy plays an important role in determining the response of aggregate quantities to the technology shock and therefore affects the households' welfare. Section 3.4 discusses the optimal monetary policy and compares it with implementable interest rate rules.

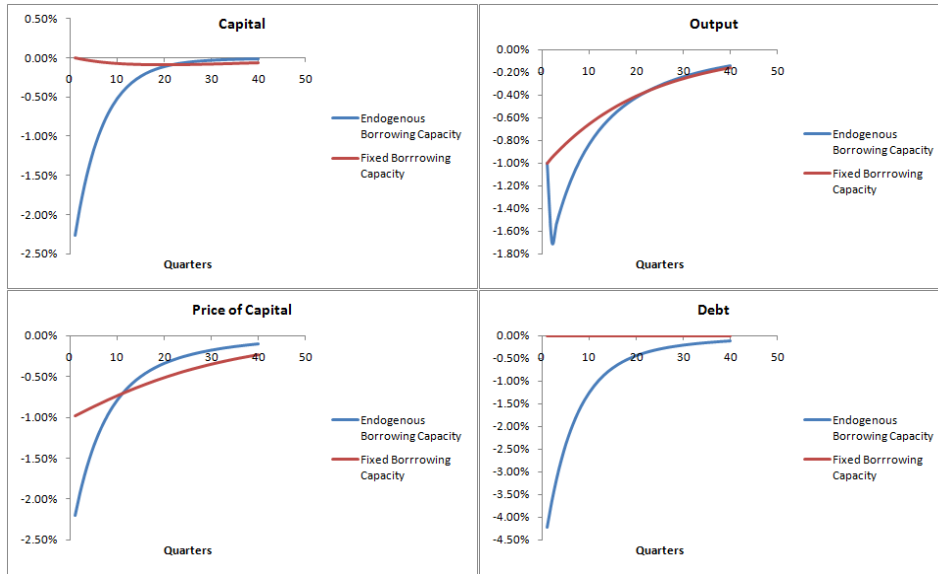


Figure 2-2: Impulse responses to a 1% negative productivity shock. Blue and red lines represent the model with endogenous and exogenous borrowing capacity, respectively. In these simulations monetary policy is described by a strict stabilization of inflation,  $\pi_t = \pi_{ss}, \forall t$ .

### 2.3.3 The Role of Borrowing Constraints

To emphasize the effects of endogenous borrowing limits and the two-way feedback between asset prices and aggregate quantities, Figure 2-2 presents the impulse responses of asset prices and aggregate quantities to a 1% negative technology shock under endogenous and exogenous borrowing limits.

In the case of exogenous borrowing limits, the borrowing limit is exogenously fixed at its steady-state. The drop in asset prices is a little less than 1% when the technology shock hits the economy. It reverts to its steady state over time. Because the entrepreneurs' ability to borrow is not affected by the change in asset prices, declines in output and capital are smaller compared to the model with endogenous



borrowing limits. However, since the borrowing limit is exogenously fixed at its steady-state, the need for selling capital by the entrepreneurs is reduced markedly, therefore the drop in entrepreneurial output becomes smaller as well. The negative productivity shock reduces the entrepreneurial output by 1% initially.

In the case of endogenous borrowing limits, however, the decrease in capital, output and asset prices are more pronounced as the credit limits respond significantly to the technology shock. When the economy is hit by a negative technology shock, entrepreneurs start selling capital in response to lower revenues which depresses the asset prices. Recall that borrowing limits are endogenously determined by asset prices and the capital stock available to entrepreneurs to collateralize. Because of falling asset prices the need for selling capital increases as the drop in asset prices lead to tighter borrowing constraints. In turn, output and entrepreneurial net worth deteriorate and the overall declines in aggregate quantities and asset prices become larger in the case of endogenous credit limits. In the later case, while the entrepreneurial capital and price of capital decreases by more than 2%, the entrepreneurial output declines by 1.7% in response to a 1% negative technology shock. Because the credit limits depend on the amount of the capital available to collateralize and its market price, entrepreneurial debt is reduced by more than 4% following the shock.

The model impulse responses to a negative shock to entrepreneurial productivity are shown in Figure 2-3 when collateral constraints loosen, i.e.,  $m = 0.75$ . The rise in the loan-to-value ratio increases entrepreneurs' ability to borrow for a given amount

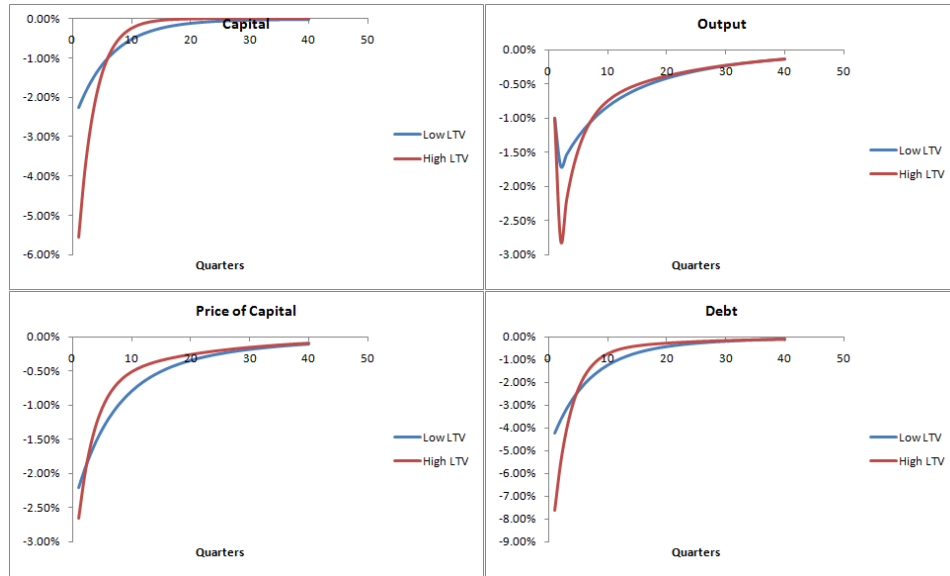


Figure 2-3: Impulse responses to a 1% negative productivity shock. Blue and red lines represent the model with low and high loan-to-value ratio, respectively. In these simulations monetary policy is described by a strict stabilization of inflation,  $\pi_t = \pi_{ss}, \forall t$ .

of capital and capital price. When the economy is hit by a negative technology shock, the entrepreneurs produce less with given amount of capital, which, in turn, makes it harder to meet the interest payments on the nominal debt and to finance losses. This happens because entrepreneurs always borrow up to limit due to the corporate tax advantage. Since entrepreneurs are highly leveraged, a larger amount of capital moves from the entrepreneurial sector to the traditional sector. With pecuniary externality at work, we observe that the decline in asset prices is larger in the case of a high LTV. Under these circumstances, the economy suffers from a severe collapse in asset prices during the crisis due to the large supply of loans to the entrepreneurs as shown in Figure 2-3. Moreover, the output and capital losses in the entrepreneurial sector

are much larger in an economy with loose borrowing constraints. Figure 2-3 points out that the strength of the collateral constraints in an economy affects the depth of a financial crisis.

As can be seen from Figure 2-3, the response of capital owned by the entrepreneurs is larger compared to a low loan-to-value ratio. Because of lower capital in the entrepreneurial sector, the output decreases by 2.8% right after the shock. On the other hand, the decline in output is approximately 1.7% when the loan-to-value ratio is lower.

The scenario described above sheds light on why a credit boom might be inefficient. It explains the inefficiency in output in the entrepreneurial sector focusing on the negative externality by which higher borrowing of some agents may increase *systemic risk*. The depth of financial crisis and the loss in output is more pronounced in an economy with high leverage because of the higher negative externality stemming from the fire sales of capital by highly indebted entrepreneurs if the economy is hit by a negative productivity shock.

### **2.3.4 Optimal Monetary Policy and Policy Implications**

Having laid out the model and its dynamics for a strict stabilization of inflation, this section studies the optimal conduct of monetary policy. In what follows, I describe the optimal monetary policy and compare it with an implementable Taylor-type interest rule.

In order to characterize optimal monetary policy, I assume that ex-ante commitment is feasible and a benevolent central bank maximizes aggregate welfare subject to the equilibrium constraints in the two-sector economy. Assuming ex-ante commitment is feasible, a Ramsey planner maximizes the following utility function:

$$W_t \equiv E_t \sum_{j=t}^{\infty} \beta^{j-t} \ln C_j.$$

The Ramsey problem under commitment is given by the following:

Let  $\{\lambda_{k,t}\}_{t=0}^{\infty}$  ( $k = 1, 2, \dots$ ) represent sequences of Lagrange multipliers on the constraints (1),(3)-(6), (7'), and (8)-(18) respectively. For given stochastic processes  $\{A_t\}_{t=0}^{\infty}$ , plans for the control variables  $\{b_t, b_t^e, M_t, C_t, Y_t, K_t, K_t^e, \pi_t, \mu_t, q_t, r_t\}_{t=0}^{\infty}$ , for the costate variables  $\{\lambda_{k,t}\}_{t=0}^{\infty}$  ( $k = 1, 2, \dots$ ) represent a second-best constrained allocation if they solve the following maximization problem:

$$\max W_t \tag{2.19}$$

subject to equations (2.1),(2.3)-(2.6), (2.7'), (2.8)-(2.18).<sup>7</sup>

The maximization problem given above is not time-invariant. It is non-recursive as a result of some of the constraints in problem (19) containing future expectations of control variables.<sup>8</sup> One may formally rewrite the same problem in a recursive station-

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<sup>7</sup>Note that the monetary authority faces two distortions (i.e., price stickiness and borrowing constraints on the entrepreneurs' side) and one policy instrument, and hence cannot simultaneously stabilize both distortions. For this reason, a first-best solution is not attainable in this economy.

<sup>8</sup>See Kydland and Prescott (1980).

ary form to expand the planner's state space with additional (pseudo) costate variables as in Marcat and Marimon (1999). Such costate variables are used to track the value to the planner of committing to the pre-announced policy plan along the dynamics. This maximization program is recursive saddle-point stationary in the enlarged state space  $\{A_t, \Psi'_t\}$ , where  $\Psi'_t = \{\lambda_{3,t-1}, \lambda_{4,t-1}, \lambda_{5,t-1}, \lambda_{7',t-1}, \lambda_{10,t-1}, \lambda_{11,t-1}, \lambda_{12,t-1}\}$  and with the initial condition  $\Psi'_0 = \bar{\Psi}'$ .

After defining the Ramsey problem in a recursive form, one proceeds in the following way. To determine the deterministic Ramsey steady state, one needs to compute the stationary allocations that characterize the deterministic steady state of the efficiency conditions of problem (19). Then, second-order approximations of the relevant policy functions and the aggregate welfare are computed in the neighborhood of the Ramsey steady state to find the numerical solution to the Ramsey problem.

The numerical solution to Ramsey problem is computed with the help of a Dynare procedure and associated subroutines, `get_ramsey`, created for Levin and Lopez-Salido (2004) and Levin, Onatski, Williams, and Williams (2005). This procedure generates the first-order conditions of the Ramsey policymaker and provides a linear system for obtaining the numerical steady-state of the Lagrange multipliers. When there are no stochastic disturbances, i.e. the exogenous variable  $A_t = 1$ , takes a constant value, the first-order conditions for optimality admit a steady state solution. For small enough exogenous productivity shocks, the optimal policy would require the rate of inflation to fluctuate around its steady state value.

The implementation of an optimal policy requires the central bank to monitor the past and the current value of the inflation rate, the output gap and of Lagrange multipliers in order to determine how it must act to maximize aggregate welfare subject to the equilibrium constraints in the private economy. A useful question about the optimal policy is the degree to which it can be achieved through one simple rule for the conduct of monetary policy. Due to sticky prices and the two-way feedback between aggregate quantities and asset prices through credit constraints, one obvious candidate would be a modified Taylor rule with goods price inflation, the output gap and asset prices.

Inflation is distortionary due to sticky prices in the retail sector. However, because entrepreneurs' obligations are set in nominal terms, it might be optimal for the monetary authority to deviate from price stability. For a given interest rate, an increase in inflation would reduce the real burden of the entrepreneurs' nominal debt. In this case, theoretically, we would expect entrepreneurs to sell less of their existing capital stock in the case of a negative technology shock and therefore, the deterioration in the output, consumption and asset prices would be smaller. To see to what extent the inflation variability is part of Ramsey equilibrium, I present the impulse responses of aggregate quantities to a 1% negative productivity shock under optimal (Ramsey) monetary policy, a Taylor rule with inflation only (TRI) and strict stabilization of inflation (IT).<sup>9</sup>

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<sup>9</sup>Monacelli (2006) finds that inflation variability is optimal in a model with impatient borrowers and patient lenders. He shows that the optimal volatility of inflation is increasing in three key para-

In Figure 2-4, impulse responses of entrepreneurial capital, output, debt, and price of capital are presented. Under optimal (Ramsey) monetary policy, the initial drop in entrepreneurial capital is very small. This is achieved by a sharp increase in inflation following the shock.<sup>10</sup> With the increase in inflation, for a given interest rate, the real burden of the debt is reduced. This leads to a lesser need for selling capital. Because, in this case, we do not observe huge fire-sales of capital, the reductions in the output and the price of capital are smaller. The entrepreneurial output decreases by 1% upon impact, due to 1% negative productivity shock, and then it reverts to its steady state. After the first quarter, the reduction in output is bigger than that of productivity shock because a small amount of entrepreneurial capital is sold to the firms in the traditional sector. The price of capital is reduced by 0.8%. Thus, the borrowing capacity of the entrepreneurs contracts slightly under optimal monetary policy.

In Figure 2-5, impulse responses of inflation and nominal interest rate are presented. Under optimal (Ramsey) monetary policy, the response of the interest rate is smaller than what a Taylor rule with inflation only or strict stabilization of inflation suggests. Lower rates under optimal monetary policy allow inflation to jump more following the shock. Entrepreneurs benefit from this jump due to the reduction in the real burden of the debt. Moreover, relatively small increase in the interest rate under

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eters: (i) the borrower's weight in the planner's objective function; (ii) the borrower's impatience rate; (iii) the degree of price flexibility.

<sup>10</sup>See the left panel of Figure 2-5.

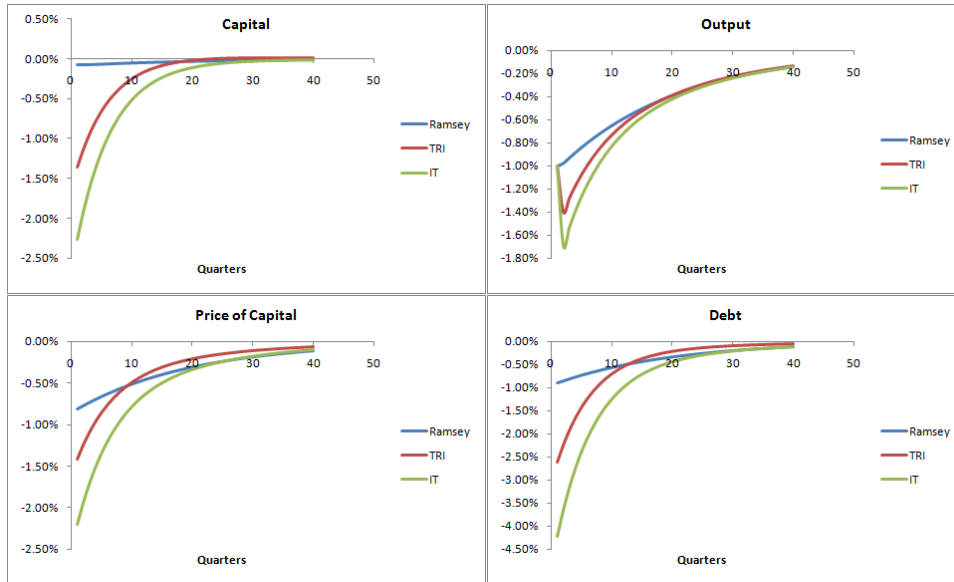


Figure 2-4: Impulse responses to a 1% negative productivity shock. Blue, red and green lines represent the impulse responses under optimal (Ramsey) monetary policy, Taylor rule with inflation only, and strict stabilization of inflation respectively. In these simulations, while a Taylor rule with inflation only (TRI) is given by  $\log((1+r_t)/(1+r_{ss})) = 1.5 \log(\pi_t/\pi_{ss})$ , strict stabilization of inflation (IT) is described by  $\pi_t = \pi_{ss}$ .

optimal monetary policy keep the effective cost of debt small and therefore make borrowing easier for the entrepreneurs compared to the other policy rules. Overall, this improves consumption smoothing, as we shall see in Table 2.2, leading to an increase in households' welfare.

Comparison of the behavior of the aggregate quantities and inflation under optimal monetary policy shows that benefits from inflation variability outweighs its costs. This is mainly because the monetary authority is able to reduce the need for selling capital by allowing inflation to jump markedly and therefore by reducing their real burden of the debt. Although inflation is distortionary due to sticky prices, by affecting



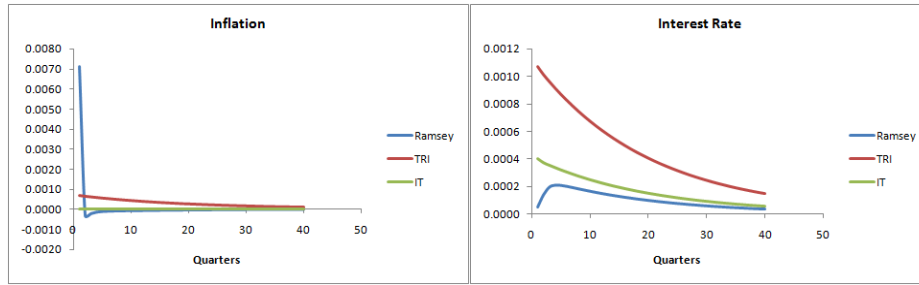


Figure 2-5: Impulse responses to a 1% negative productivity shock. Blue, red and green lines represent the impulse responses under optimal (Ramsey) monetary policy, Taylor rule with inflation only, and strict stabilization of inflation respectively. In these simulations, while a Taylor rule with inflation only (TRI) is given by  $\log((1+r_t)/(1+r_{ss})) = 1.5 \log(\pi_t/\pi_{ss})$ , strict stabilization of inflation (IT) is described by  $\pi_t = \pi_{ss}$ .

the net worth of the entrepreneurs with inflation variability, the optimal monetary policy mitigates the negative effects of the decline in productivity. Because there is almost no need for capital to move from the entrepreneurial sector to the traditional sector, the effect of pecuniary externality is very limited under Ramsey equilibrium. The impulse responses of the aggregate quantities and asset prices under optimal (Ramsey) monetary policy are similar to the model with exogenous borrowing limits in which the price of capital does not affect the borrowing capacity of the entrepreneurs. Because inflation variability is part of Ramsey equilibrium, we see that a Taylor rule (excluding the output gap) with a mild reaction (i.e.,  $r_\pi = 1.5$ ) to inflation brings the impulse responses closer to those under optimal monetary policy.<sup>11</sup>

As noted before, due to the presence of sticky prices, inflation is costly. However,

<sup>11</sup>In Figure 2-7, I compare the performance of various policy rules using the household welfare as a measure.

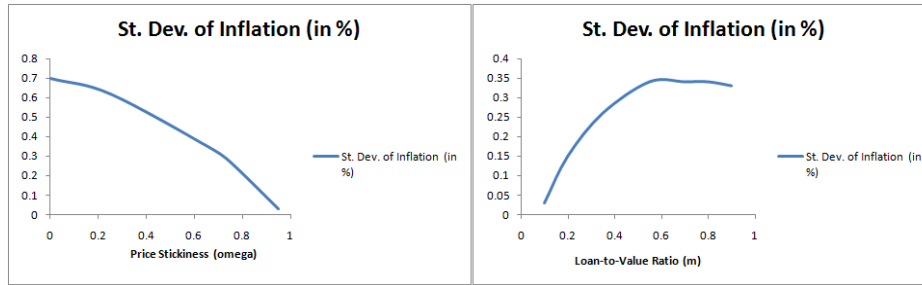


Figure 2-6: Inflation Volatility in the Ramsey equilibrium for various degree of price stickiness,  $\omega$ , and loan-to-value ratios,  $m$ .

we have seen that under the benchmark parameterization, the benefits of variation in inflation become crucial to dampen the declines in aggregate quantities. While the monetary authority has an incentive to offset the price stickiness distortion, it also wishes to relax entrepreneurs' collateral constraint via the redistributive effect of inflation. To explore this trade-off, I plot the inflation volatility in the Ramsey equilibrium for various degrees of price stickiness and various loan-to-value ratios.

In the left panel of Figure 2-6, optimal inflation volatility is shown for various degrees of price stickiness. Note that  $\omega = 0$  corresponds to fully flexible prices. In case of fully flexible prices, the optimal inflation volatility is the highest, around 0.7% quarterly. As  $\omega$  increases, final good prices become more rigid, thereby making inflation more costly to the households. As a result, the Ramsey planner finds optimal to reduce the volatility of inflation and the optimal monetary policy moves towards stable inflation. As final good prices become stickier, the cost of inflation prevails over the benefits from its redistributive effect to ease borrowing constraint of the entrepreneurs.

In the right panel of Figure 2-6, I plot the optimal volatility of inflation for various loan-to-value ratios (LTV). If the loan-to-value ratio,  $m$ , increases the inflation happens to be more volatile in the Ramsey equilibrium for a given degree of price stickiness. As we saw in Figure 2-3, the less binding the collateral effects (the higher  $m$ ), the larger the effects of technology shocks on aggregate quantities. This points that the Ramsey planner will require inflation to be more volatile with a redistributive motive as is evident in Figure 2-6.

### **2.3.5 Should the Central Bank React to the Changes in Asset Prices?**

Because movements in the price of capital constitute an important channel in the propagation of the technology shocks across the economy in this paper, it is natural to ask whether asset prices should be included in the Taylor rule. The issue of whether monetary policy should respond to asset prices has recently been the object of an intense debate in the aftermath of the financial crisis. Bernanke and Gertler (2001a,b) and Iacoviello (2005) argue that stabilization gains from including asset prices as independent arguments in monetary policy rules are negligible. Faia and Monacelli (2007) model monetary policy in terms of simple welfare-maximizing interest rate rules. They find that monetary policy should respond negatively to asset prices. They also show that when monetary policy reacts to inflation strongly, the marginal welfare gain of responding to asset prices vanishes as in Bernanke and

Gertler (2001a,b) and Iacoviello (2005).<sup>12</sup> However, Cecchetti et al. (2002) argue that this gain is likely to depend on the underlying source of shocks.

Before continuing with the welfare analysis for the assessment of alternative interest rate rules, some observations on the computation of welfare should be made clear. One should note that, in an economy like this, distortions due to collateral constraints in the entrepreneurial sector and monopolistic competition in the retail sector have an effect both in the short-run and in the steady-state. Stochastic volatility affects both first and second moments of those variables that are important to welfare. For this reason, to correctly rank the performance of various monetary policy rules, I numerically compute the second-order approximation of the relevant policy functions and of the households' welfare using Dynare.

Figure 2-7 shows the (percent) fraction of consumption required to equate welfare to the one under the optimal monetary policy. In the figure, the monetary policy rules considered are simple Taylor rules (excluding the output gap) with different values of the reaction parameter,  $r_q$ , to asset prices while holding the reaction coefficient to inflation constant at  $r_\pi = 1.5$ .

The main result that emerges in Figure 2-7 is that, when inflation coefficient is low, there exists a positive effect on household welfare of responding positively to asset prices. For  $r_\pi = 1.5$ , the (percent) fraction of consumption required to equate welfare to the one under optimal monetary decreases, with minimum reached for

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<sup>12</sup>In this paper, on the other hand, I show that a strict stabilization of inflation is no longer optimal due to the redistributive effect of the inflation.

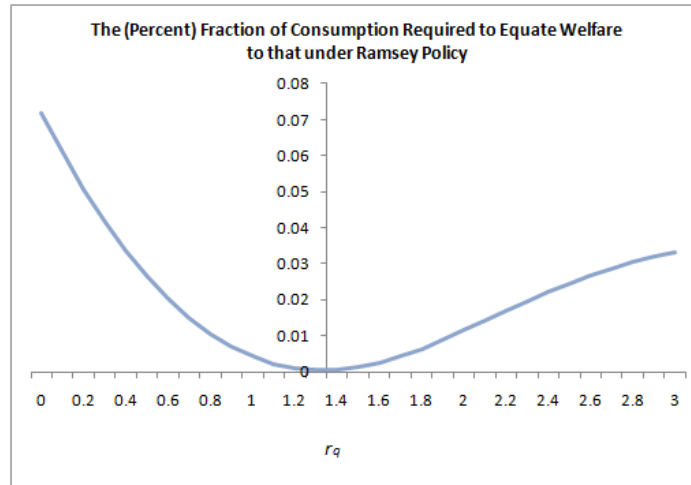


Figure 2-7: Effects of responding to the changes in asset prices in a Taylor rule with fixed inflation coefficient,  $r_\pi = 1.5$  on households' welfare.

a value around 1.3. It starts increasing as  $r_q$  takes values larger than 1.3. The intuition for why monetary policy should respond positively to asset prices is as follows. Recall that, in the model, the fact that the price of capital is determined in a competitive market and that the entrepreneurs are credit constrained induce endogenous movements in the capital price when the economy is hit by the aggregate productivity shock. Endogenous movements (a decline in the case of a negative shock) in the price of capital adversely affect the entrepreneurs' borrowing capacity. When the asset prices decrease, by responding to the change in asset prices the monetary authority keeps the interest rate low, letting inflation to jump right after the shock. This reduces entrepreneurs' debt obligations in real terms and at the same time enables them to borrow easily in the future periods because of a lower effective cost of debt.

I also compare the welfare performance of alternative specifications of the monetary policy rule. The rules are the following:

- (i) Strict inflation stabilization (IT),  $\pi_t = \pi_{ss}, \forall t$ .
- (ii) Taylor rule with inflation only (TRI),  $\log((1+r_t)/(1+r_{ss})) = 1.5 \log(\pi_t/\pi_{ss})$ ;
- (iii) TRI with a positive response to the change in the asset price,  $\log((1+r_t)/(1+r_{ss})) = 1.5 \log(\pi_t/\pi_{ss}) + r_q \log(q_t/q_{t-1})$ , with  $r_q = 1.3$  (optimal coefficient);
- (iv) Standard Taylor rule including the entrepreneurial output gap (STR),  $\log((1+r_t)/(1+r_{ss})) = 1.5 \log(\pi_t/\pi_{ss}) + r_y \log(Y_t/Y_{ss})$ .<sup>13</sup> In addition to the monetary policy rules above, I provide the welfare under optimal (Ramsey) monetary policy rule in the last row of the Table 2.2.

I compare the rules both in terms of conditional welfare  $W_t$  and in terms of a compensating measure given by  $\zeta$  of households' consumption that would be needed to equate conditional welfare  $W_t$  under a generic interest rate policy rule to the level of welfare implied by the optimal (Ramsey) monetary policy (OMP),  $W_t^*$ . Hence one can solve for  $\zeta$  and obtain

$$\begin{aligned} W_t^* &= E_t \sum_{j=t}^{\infty} \beta^{j-t} \ln [(1+\zeta)C_j] \\ &= \frac{\zeta}{1-\beta} + E_t \sum_{j=0}^{\infty} \beta^{j-t} \ln [C_j] \end{aligned}$$

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<sup>13</sup>In the specifications of monetary policy rules, the welfare gain from including interest rate smoothing term was negligible.

$$\zeta = (1 - \beta)(W_t^* - W_t).$$

As a result,  $\gamma = 100 \times \zeta$  expresses the % fraction of consumption required to equate welfare under any given policy rule to the one under the optimal policy. The results are shown in Table 2.2.

Table 2.2 - Welfare Results

Specification of Monetary Policy	Welfare	$\gamma$
IT	173.44	0.23
TRI with inflation only	173.60	0.07
STR	172.63	1.04
TRI with $r_q = 1.3$	173.67	0.001
OMP	173.67	0.00

Notes:  $\gamma$  is the (%) fraction of consumption required to equate welfare under any given policy rule to the one under the optimal (Ramsey) policy. Welfare is calculated as conditional to the initial deterministic steady state.

One striking fact that emerges from Table 2.2, strict stabilization of inflation results in lower welfare compared to a Taylor rule with a mild reaction to inflation. Responding to output gap in a Taylor rule is unfavorable; the percentage of consumption that is needed to equate the welfare under STR to the one under the optimal policy is 1.04. On the other hand, responding to asset prices positively and having a

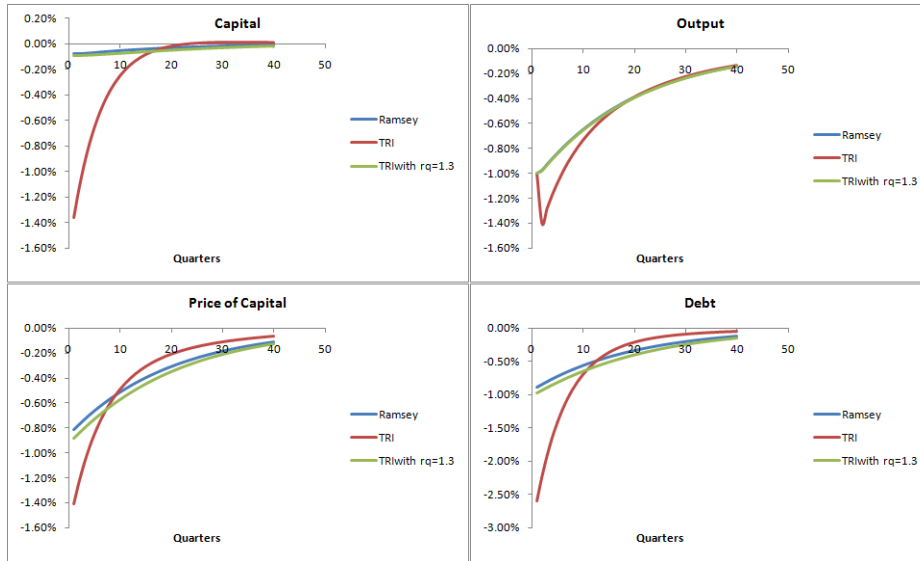


Figure 2-8: Impulse responses to a 1% negative productivity shock. Blue, red and green lines represent the impulse responses under optimal (Ramsey) monetary policy, Taylor rule with inflation only with or without reaction to the change in asset prices. In these simulations, while a Taylor rule with inflation only (TRI) is given by  $\log((1+r_t)/(1+r_{ss})) = 1.5 \log(\pi_t/\pi_{ss})$ , TRI with reaction to asset prices is given by  $\log((1+r_t)/(1+r_{ss})) = 1.5 \log(\pi_t/\pi_{ss}) + r_q \log(q_t/q_{t-1})$ , with  $r_q = 1.3$ .

mild inflation coefficient improve welfare and  $\gamma$  is only 0.001% in this case. It is clear that strict stabilization of inflation in this model is inferior to a Taylor rule with a positive reaction to the changes in asset prices. The percentage of consumption that is needed to equate the welfare under IT to the one under optimal Taylor rule, (i.e., TRI with  $r_q = 1.3$ ) is 0.23% of consumption, which is larger than what has been found in the models that examine the welfare gain from reacting to asset prices.<sup>14</sup>

With a positive reaction to the change in the price of capital, the need for selling capital by the entrepreneurs to the firms in the traditional sector is reduced remark-

<sup>14</sup>See Table 2 on page 3246, for example, in Faia and Monacelli (2007).



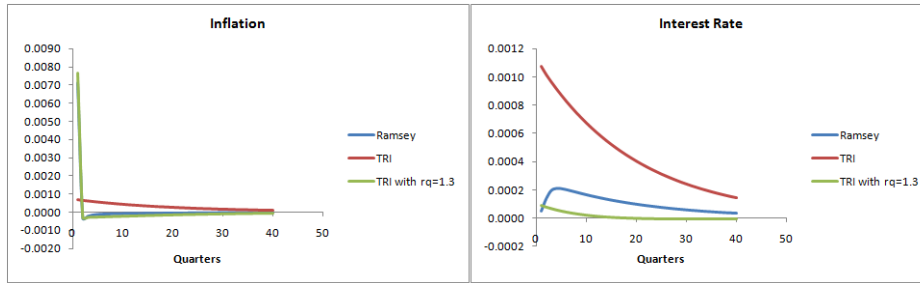


Figure 2-9: Impulse responses to a 1% negative productivity shock. Blue, red and green lines represent the impulse responses under optimal (Ramsey) monetary policy, Taylor rule with inflation only with or without reaction to the change in asset prices. In these simulations, while a Taylor rule with inflation only (TRI) is given by  $\log((1+r_t)/(1+r_{ss})) = 1.5 \log(\pi_t/\pi_{ss})$ , TRI with reaction to asset prices is given by  $\log((1+r_t)/(1+r_{ss})) = 1.5 \log(\pi_t/\pi_{ss}) + r_q \log(q_t/q_{t-1})$ , with  $r_q = 1.3$ .

ably as we see in Figure 2-8. The impulse responses of the aggregate variables mimic those from the optimal (Ramsey) monetary policy. Entrepreneurs' borrowing capacity is reduced, due to the falls in the price of capital and capital owned by the entrepreneurs; however these reductions are smaller compared to a Taylor rule with inflation only (TRI).

We see that the increase in the interest rate is the lowest under the monetary policy rule with a positive reaction to asset prices in Figure 2-9. More specifically, an interest rate rule with a positive reaction to the asset prices increases the interest rate by less than TRI does. With a lower interest rate, inflation jumps more following the negative productivity shock, which is beneficial to the entrepreneurs because of its redistributive role. Intuitively, it is easier to borrow for entrepreneurs when the increase in the interest rate is lower. By looking at equation (2.7'), we observe that

both higher inflation and lower increase in interest rates under TRI with  $r_q = 1.3$  works in favor of entrepreneurs and therefore reduces the fire sales of capital compared to the other interest rate rules considered above. As a result, both higher inflation and lower increase in the interest rate relax the borrowing constraint of entrepreneurs. Because very little capital moves from the entrepreneurial sector to the traditional sector, the distortions in aggregate quantities are small, which brings the welfare level closer to its optimal level.

Note that the monetary authority faces two distortions (i.e., price stickiness and borrowing constraints on the entrepreneurs' side) and one policy instrument, and hence cannot simultaneously stabilize both distortions. However, Table 2.2 shows that the marginal benefit of stabilizing financial market distortion largely outweighs the marginal benefit of stabilizing the price stickiness distortion. As a result, a Taylor rule with a mild reaction to inflation and a positive reaction to the changes in asset prices brings the dynamic responses of the aggregate quantities closer to those under optimal monetary policy and enhances the welfare of the households on a quantitative ground.

## 2.4 Conclusion

This paper develops an infinite horizon monetary model with financial frictions to study capital *fire sales* of the financially constrained entrepreneurs in the case of a negative productivity shock. It shows that the strength of borrowing constraints is

important in explaining the amount of capital *fire sales*. The depth of financial crisis and the inefficiency loss in aggregate quantities are more pronounced in an economy with high leverage because of the higher negative externalities stemming from the *fire sales* of capital by highly indebted entrepreneurs. The inefficiencies occur because of the pecuniary externality that is not internalized by the entrepreneurs and the misallocation of the capital between the households and entrepreneurs.

The paper also investigates whether a central bank with an implementable policy rule can reduce the costs of an economic crisis. For this, I numerically solve a Ramsey planner's problem and compare the dynamic behavior of aggregate variables in the Ramsey equilibrium with those in the private equilibrium with simple implementable Taylor-type interest rate rules. Because inflation is able to redistribute the wealth from households to the entrepreneurs in the case of a negative technology shock, inflation volatility is shown to be a feature of Ramsey equilibrium. As a result, a flexible inflation targeting rule brings the impulse responses of the aggregate variables closer to those in the Ramsey equilibrium compared to strict stabilization of inflation. The welfare gain from the former policy rule relative to a strict stabilization of inflation is 0.23% of consumption per period. However, as price stickiness increases, the cost of inflation outweighs its benefits. In this case, optimal monetary policy moves toward stable inflation. To what extent inflation volatility is optimal also depends on the loan-to-value ratio (LTV). For higher LTV, the collateral constraints bear greater importance in the dynamic behavior of the aggregate variables. As a result, I find

that optimal inflation volatility increases for a given degree of price stickiness.

The paper also examines whether a reaction to asset prices in a Taylor rule brings the welfare closer to the one under the optimal monetary policy. Since it was shown that some inflation variability proves to be useful in the model, I consider a Taylor rule with a mild reaction to inflation and a positive reaction to asset prices. Under this rule, the interest rate increases but less than it does under any other interest rate rule. With lower increase in the interest rate, increase in inflation is more pronounced. Both higher inflation and relatively lower interest rate under a Taylor rule with a positive reaction to asset prices are beneficial to the entrepreneurs. As a result, we observe that a Taylor rule with a mild reaction to the inflation and a positive reaction to the asset prices brings consumer welfare closer to the one under optimal (Ramsey) monetary policy.

The current model lacks a fiscal authority and therefore, does not address fiscal policy. It looks at the Ramsey planner problem to investigate if there is an implementable Taylor-type interest rate rule, which may generate an equilibrium path that is close to the Ramsey solution. However, it would also be interesting to include a fiscal authority to implement a tax scheme on entrepreneurs to avoid over-borrowing, which results in misallocation of capital due to *fire sales* in the case of a bad productivity shock. In the model, the corporate tax advantage was assumed to be constant. With the introduction of the fiscal authority one can therefore investigate the optimal fiscal policy and determine the corresponding optimal tax rate. Increases in the

corporate tax advantage rate in bad times would likely lower the amount of capital fire sales by reducing the interest payments on the entrepreneurial debt. However, in reality, it takes longer time to implement changes in tax rates. For this reason, it would also be interesting to study the optimal fiscal policy through lump-sum transfers to the entrepreneurs and the households in the model. Finally, one may introduce financial intermediaries into the model. In this environment, in bad times, lowering capital requirements on the financial intermediaries which finance the entrepreneurial firms might improve household welfare by reducing the need for selling capital by financially troubled entrepreneurs.

## 2.5 Appendix

### 2.5.1 The Reset Price in terms of the Aggregate Variables in the Model

The optimal  $P_t^*(i)$  solves:

$$\sum_{j=t}^{\infty} \omega^j E_t \left\{ \Lambda_{t,j} \left( \frac{P_t^*(i)}{P_j} - \frac{X}{X_j} \right) Z_j^*(i) \right\} = 0 \quad (2.12)$$

where  $Z_{t+k}^*(i) = (P_t^*(i)/P_{t+k})^{-\sigma} Z_{t+k}$  and  $\Lambda_{t,k} = \beta^k (C_t/C_{t+k})$ .

$$\sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{C_t}{C_j} \left( \frac{P_t^*(i)}{P_j} - \frac{X}{X_j} \right) \frac{(P_t^*)^{-\sigma}}{(P_j)^{-\sigma}} Z_j \right\} = 0$$

$$\sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{C_t}{C_j} \left( \frac{(P_t^*)^{1-\sigma}}{(P_j)^{1-\sigma}} - \frac{X (P_t^*)^{-\sigma}}{X_j (P_j)^{-\sigma}} \right) Z_j \right\} = 0$$

$$\sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{C_t (P_t^*)^{1-\sigma}}{C_j (P_j)^{1-\sigma}} Z_j \right\} = \sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{C_t X (P_t^*)^{-\sigma}}{C_j X_j (P_j)^{-\sigma}} Z_j \right\}$$

$$(P_t^*)^{1-\sigma} C_t \sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_j}{C_j (P_j)^{1-\sigma}} \right\} = (P_t^*)^{-\sigma} X C_t \sum_{j=0}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_j}{C_j X_j (P_j)^{-\sigma}} \right\}$$

$$(P_t^*) \sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_j}{C_j (P_j)^{1-\sigma}} \right\} = X \sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_j}{C_j X_j (P_j)^{-\sigma}} \right\}$$

$$P_t^* = X \frac{\sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_j}{C_j X_j (P_j)^{-\sigma}} \right\}}{\sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_j}{C_j (P_j)^{1-\sigma}} \right\}}$$

$$P_t^* = X \frac{PB_t}{PA_t}$$

where  $P_t^*$  is the reset price and  $PB_t$  and  $PA_t$  are defined below.

$$PB_t = \sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_j}{C_j X_j (P_j)^{-\sigma}} \right\}$$

$$PB_t = \frac{Z_t (P_t)^\sigma}{C_t X_t} + \sum_{j=t+1}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_j (P_j)^\sigma}{C_j X_j} \right\}$$

$$PB_t = \frac{Z_t (P_t)^\sigma}{C_t X_t} + (\theta\beta) \sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_{j+1} (P_{j+1})^\sigma}{C_{j+1} X_{j+1}} \right\}$$

$$PB_t = \frac{Z_t (P_t)^\sigma}{C_t X_t} + (\omega\beta) E_t PB_{t+1}$$

Define

$$pb_t = \frac{PB_t}{P_t^\sigma}$$

$$pb_t = \frac{Z_t}{X_t C_t} + (\omega\beta) E_t pb_{t+1} \frac{P_{t+1}^\sigma}{P_t^\sigma}$$

$$pb_t = \frac{Z_t}{X_t C_t} + (\omega\beta) E_t pb_{t+1} \pi_{t+1}^\sigma$$

$$PA_t = \sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_j}{C_j} (P_j)^{\sigma-1} \right\}$$

$$PA_t = \frac{Z_t}{C_t} (P_t)^{\sigma-1} + \sum_{j=t+1}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_j}{C_j} (P_j)^{\sigma-1} \right\}$$

$$PA_t = \frac{Z_t}{C_t} (P_t)^{\sigma-1} + (\omega\beta) \sum_{j=t}^{\infty} (\omega\beta)^j E_t \left\{ \frac{Z_{j+1}}{C_{j+1}} (P_{j+1})^{\sigma-1} \right\}$$

$$PA_t = \frac{Z_t}{C_t} (P_t)^{\sigma-1} + (\omega\beta) E_t PA_{t+1}$$

Define

$$pa_t = \frac{PA_t}{P_t^{\sigma-1}}$$

$$pa_t = \frac{Z_t}{C_t} + (\omega\beta) E_t pa_{t+1} \frac{P_{t+1}^{\sigma-1}}{P_t^{\sigma-1}}$$

$$pa_t = \frac{Z_t}{C_t} + (\omega\beta) E_t pa_{t+1} \pi_{t+1}^{\sigma-1}$$

$$P_t^* = X \frac{PB_t}{PA_t} = X \frac{pb_t P_t^\sigma}{pa_t P_t^{\sigma-1}}$$

$$\frac{P_t^*}{P_t} = X \frac{pb_t}{pa_t}$$

If prices are fully flexible, i.e.  $\omega = 0$

$$pb_t = \frac{Z_t}{X_t C_t}$$

$$pa_t = \frac{Z_t}{C_t}$$

$$\frac{P_t^*}{P_t} = X \frac{\frac{Z_t}{X_t C_t}}{\frac{Z_t}{C_t}}$$



$$\frac{P_t^*}{P_t} = X \frac{1}{X_t}$$

$$\frac{P_t^*}{P_t} = X \frac{1}{\frac{P_t}{P_t^w}}$$

$$\frac{P_t^*}{P_t} = X \frac{P_t^w}{P_t}$$

## 2.5.2 Baseline Model Steady State Equations

$$F(K) = K^v \tag{2.1'}$$

$$C + (1+r)b/\pi + T = b + w + \frac{F(K)}{X} + \Gamma + \frac{D}{X} \tag{2.2'}$$

$$q = \frac{\beta}{1-\beta} \frac{F'(K)}{X} \tag{2.3'}$$

$$\beta R/\pi = 1 \tag{2.4'}$$

$$q^s = \frac{\beta}{1-\beta} \frac{D}{X} \tag{2.5'}$$

$$Y = (K^e)^\alpha L^{(1-\alpha)} \tag{2.6'}$$

$$b^e(1 + r(1 - \tau)) = \beta m q K^e \pi \quad (2.7'')$$

$$(1 + r(1 - \tau))b^e/\pi + w + \frac{D}{X} = \frac{Y}{X} + b^e \quad (2.8')$$

$$\frac{wL^e}{Y} = \frac{(1 - \alpha)}{X} \quad (2.9')$$

$$q = \beta \left\{ \alpha \frac{Y}{XK^e} + q \right\} + m\mu q \pi \quad (2.10')$$

$$1 = \mu(1 + r(1 - \tau)) + \beta(1 + r(1 - \tau))/\pi \quad (2.11')$$

$$p^* = X \frac{pb}{pa} \quad (2.12')$$

$$C = Y + F(K) \quad (2.14')$$

$$B + B^e = 0 \quad (2.15')$$

$$K + K^e = \bar{K} \quad (2.16')$$

$$s = 1 \quad (2.17')$$

$$\bar{L} = L = L^e = 1 \quad (2.18')$$

# Chapter 3

## A Literature Review of Credit Risk Modeling

### 3.1 Introduction

Credit risk for a lender arises when a borrower does not make payments as promised. The credit risk of a financial asset is affected by the probability of default (PD), the loss given default (LGD), and the exposure at default (EAD). Credit valuation process is very important to lending. A careful credit valuation, therefore, should result in debt pricing that corresponds to the risks taken. For these reasons, it must be a quantitative process, which should not be determined by personal judgement. In fact, credit valuation framework should be based on the borrowers' overall ability to repay. In addition, it is important to have a theory that describes the relationship between a

borrower's assets, capital structure and its potential default. In other words, a credit valuation model should examine the company's operations, its current and future expected cash flows, and assess the future earning power of the firm. If a credit valuation model can explain how the state of the borrower affects its probability of default, it will also be useful to detect a possible deterioration of the creditworthiness.

This paper presents a literature review of how the credit risk models developed since early 1970s.<sup>1</sup> Credit pricing and the credit value-at-risk models are the two main categories examined in the paper. When discussing the credit pricing models three main approaches are considered. The first category of credit pricing models, first generation structural-form models, is that based on Merton (1974) model. In this approach, whether a company defaults or not depends on the value of the company's assets. A firm will default when its market value is lower than the value of its liabilities. The payment to the debt holders at the maturity of debt is therefore the smaller of the face value of the debt or the market value of the firm's assets. Following this basic intuition, Merton derives a formula for risky bonds to estimate the probability of default of a firm and the yield gap between a risky bond and default-free bond. In addition to Merton (1974), Black and Cox's (1976), Geske's (1977), and Vasicek's (1984) models might be classified in the first generation structural-form models. These models try to improve the original Merton framework by relaxing one or more of the unrealistic assumptions.

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<sup>1</sup>Altman, Resti, and Sironi (2003) summarize the credit risk models focusing on the relationship between recovery and default rates.

Even though the research that followed the Merton framework was successful in dealing with the qualitative aspects of pricing credit risks, it has been less successful in practical applications. One of the reasons for this failure is that the firm defaults mostly at the maturity of the debt but not necessarily at the coupon payments along the way in the original Merton framework. However, this assumption is unlikely to hold in reality. Another reason is that the conventional contingent claims models assume a flat risk-free term structure. However, the interest rate risk might have a significant effect on the values of Treasury and corporate debt. In addition, the seniority structures of various debts need to be specified in order to use the model to value risky debt of a firm with a complex capital structures in the first generation structural-form models. While Merton framework assumes that debts are paid-off according to their seniority, empirical evidence shows that absolute-priority rules are very likely to be violated.

To overcome the drawbacks mentioned above, second-generation structural form models assume that a firm may default any time between the issuance and maturity of the debt and specify a stochastic process for the evolution of the short-term rates. In this scenario, the default may occur whenever the market value of the firm goes below a lower limit determined by the lender and borrower in the debt contract. Second generation structural-form models include Kim, Ramaswamy and Sundaresan (1993) and Longstaff and Schwartz (1995).

While the second generation structural-form models provide some improvements

over the first generation models, they still have certain problems which cause them to perform poorly in empirical analysis. The firm's market value still needs to be estimated in these models. This task is difficult as the parameters to determine the value of firm's assets are not necessarily observable. Moreover, these models do not take into account credit-rating changes for risky corporate debt of the firms. Therefore, the second generation structural-form models cannot be used to price various credit derivatives whose payouts depend on the credit rating of the debt issue.

The reduced-form models avoid these two problems. In the reduced-form models, the time of bankruptcy is given exogenously. This approach is more advantageous for two reasons. First, it allows exogenous assumptions to be imposed only on observables. Second, it can easily be modified to include credit rating and therefore can be used to price credit derivatives whose payouts are affected by the credit rating of the debt issue. Credit ratings also allow one to draw conclusions about the financial health of the firm without requiring information about its market value. Jarrow, Lando and Turnbull (1997) study the term structure of credit risk spreads in a model with credit ratings. By incorporating the credit ratings, Duffie and Singleton (1999) present a new approach to modeling the valuation of contingent claims subject to default and focus on the applications to the term structure of interest rates for corporate bonds. Their study differs from other reduced-form models by the way they parameterize the losses in case of default. Last but not least, Duffee (1999) discusses

the empirical performance of the reduced-form models and shows that these models may not be useful in explaining the relatively flat or steeper yields for firms with low credit or higher credit risks, respectively.

During the late 1990s, certain banks developed credit value-at-risk models under two main categories. The first is the default mode models (DM) in which the credit risk is linked to the default risk. While a firm can either default or survive in DM models, the second group of credit value-at-risk models, mark-to-market (MTM) models takes more outcomes into consideration in terms of the creditworthiness of the borrower. Because, in DM models, there are only two possible outcomes; default or survival, credit losses occur only when the firm defaults. However, losses may arise whenever the creditworthiness of the borrower changes.

Credit risk models generate a probability density function (PDF) of the future losses that might occur on a credit portfolio. With the help of this function, it is possible for a financial institution to estimate its expected losses on a given credit portfolio. The credit value-at-risk models include *Credit Suisse Financial Products' Credit Risk (1997)*, McKinsey's *CreditPortfolioView*, J.P. Morgan's *CreditMetrics* and KMV's *CreditPortfolioManager*.

The paper is organized as follows. Section 3.2 and 3.3 review the first generation and second generation structural-form models, respectively. Section 3.4 presents the reduced-form models. Section 3.5 discusses the credit value-at-risk (VaR) models. Section 3.6 concludes.



## 3.2 First Generation Structural-Form Models

As discussed in the introduction, this paper reviews the credit pricing models under three main categories. The first category of credit pricing models, first generation structural-form models, is that based on Merton (1974) model. These models follow the Merton (1974) approach using the principles of option pricing in Black and Scholes (1973). Merton (1974) states that although options are rather less important financial instruments, the option pricing framework in Black and Scholes (1973), later clarified and extended by Merton (1973), can be used to develop a pricing theory for corporate debt of the companies. Merton (1974) develops a basic equation for the pricing of financial instruments based on Black and Scholes framework and later applies this to the discount bonds with no coupon payments.

The basic equation developed in his paper determines the value of any security whose value can be described as a function of the value of the firm and time. The value of the security depends on the risk-free interest rate, the volatility of the firm's value, the payout policy of the firm, and the payout policy to the holders of the security. However, the expected rate of return on the firm, the risk preferences of the investors and the set of other assets available to the investors are not required to determine the value of the security.

In a specific example, Merton (1974) examines the value of a corporate debt where the issuing firm has only a single, homogenous class of debt and equity. Provisions and restrictions on the indenture of the bond issue are as follows: (i) the firm promises

to pay a certain amount of dollars to the bondholders on a specific date; (ii) if this payment is not made, the bondholders take over the firm; (iii) the firm cannot issue new senior debt and cannot make cash dividend payments. In this approach, a firm will default when its market value is lower than the value of its liabilities. On the maturity date, if the firm's value is greater than the face value of the bond then it is optimal for the firm to pay the bondholders because the value of the equity is nonzero in this case. On the other hand, if the firm's value is less than the face value, the firm will default to the bondholders so that the equity holders will not have to make additional payment. The payment to the debt holders at the maturity of debt is therefore the smaller of the face value of the debt or the market value of the firm's assets. Following this basic intuition, Merton derives a formula for risky bonds to estimate the probability of default of the firm and the yield gap between a risky bond and default-free bond.<sup>2</sup> He finds that as the present value of the promised payment approaches the current the value of the firm, the probability of default increases. On the other hand, as the present value of the promised payment becomes very small, the probability of default approaches zero and the value of the risky security approaches to the value of a riskless bond. Therefore, the risk characteristics of the risky and riskless debt become similar. For the yield differential between the risky and risk-free bonds, Merton (1974) shows that yield-to-maturity on risky debt less of the riskless rate for a given maturity is a function of (i) volatility of the firm's operations and (ii)

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<sup>2</sup>In a more complex example, the original formula derived in his paper is then applied to the risky bonds with coupon payments. See Section VI on pages 467-469 in Merton (1974) for details.

the ratio of the present value (at the riskless rate) of the promised payment to the current value of the firm.

In addition to Merton (1974), Black and Cox's (1976), Geske's (1977), and Vasicek's (1984) models might be classified in the first generation structural-form models. These models try to improve the original Merton framework by relaxing one or more of the unrealistic assumptions.

Black and Cox (1976) first describe some solution methods to be applied when the problem of valuation of contingent claims is discrete in time. They then examine the effects of safety covenants, subordination arrangements, and restrictions on the financing of interest and dividend payments on the value of the security. They find that in theory these provisions may have significant effects on the behavior of the firm's securities and may increase the value of the risky bonds.

The original Merton framework assumes that the firm does not go through any sort of reorganization in its financial arrangements even if the value of the firm may reach to an arbitrarily high or low level. Black and Cox (1976), on the other hand, consider a valuation problem in which reorganization occurs in some lower or upper value of the firm. In particular, the firm's securities may take certain values at these lower and upper boundaries. Black and Cox consider the indenture agreements in which these boundaries are determined exogenously as opposed to those determined endogenously in the optimal decision problem. In their approach, each security has four sources of value: (i) its value at the maturity date if the firm is not reorganized

before this date; (ii) its value if the firm is reorganized at the lower boundary; (iii) its value if the firm is reorganized at the upper boundary; and (iv) the value of payouts it will potentially receive.<sup>3</sup> As Black and Cox (1976) consider the valuation problem of a zero-coupon risky bond, the fourth component will essentially be zero before the maturity date. Each of these components contributes to the current value of the claim and is defined by the discounted expected value of that component in a risk neutral world. One should note that the contribution at the reorganization boundaries requires the knowledge about the distribution of the first passage time to the boundary as we know the amount to be received at each boundary but not the time of this transaction. While the expected discounted value at the reorganization boundaries can be calculated by using Cox and Ross (1975, 1976) when the boundaries are specified in the bond indenture agreements, this is no longer the case when they are determined endogenously in the optimal decision problem.

In a more complex example, Black and Cox (1976) propose solving the problems recursively in which the reorganization boundaries are determined endogenously at some discrete points in time. In this recursive solution method, the value of the claim at any time can be found by working backward where the terminal condition at each stage is determined by the solution to the previous stage. However, this method gives an approximate solution when the optimal decision points are continuous in time.

After defining the solution methods above to the valuation problem when there

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<sup>3</sup>Note that the first three of these sources are mutually exclusive.

might be some reorganization in the financial arrangements of the firm before going into default, Black and Cox (1976) next consider the valuation of a risky bond in the presence of safety covenants. A safety covenant is a provision of a bond indenture requiring that if the value of the firm falls to or below a specified level then the bondholders are entitled to some immediate settlement of their claim on the firm. This settlement might give the bondholders the right to force a reorganization of the firm or in the worst case scenario the firm might have to declare bankruptcy. In market practice, we may observe that a firm might be forced to some sort of reorganization or bankruptcy if it misses several interest payments on its debt with coupon payments.<sup>4</sup> On the other hand, for a zero-coupon corporate debt, a safety covenant would be a contractual provision which lets the bondholders take the ownership of the firm's assets if the firm's value falls to a specified level.

Black and Cox (1976) show that the value of risky discount bonds is an increasing function of the firm value and a decreasing function of the business risk of the firm, the risk-free rate, and dividend payments to the stockholders in the presence of safety covenants. These findings are similar to those in Merton (1974) when there are no safety covenants. What is different in the presence of safety covenants is that Black and Cox (1976) provide a floor value for the bond. If the business risks of the firm or the dividend payments to the stockholders increase to unexpected high levels, the value of the bond does not necessarily go to zero. This is because the bondholders

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<sup>4</sup>Note that if the stockholders are allowed to sell the firm's assets to finance interest payments, the safety covenant will not be very effective.

may take over the ownership of the firm as soon as the firm value reaches the specified boundary.

Another form of indenture agreement considered in Black and Cox (1976) is the subordination of junior bonds to the senior bonds. When the debt is subordinated, no payments can be made to the junior debt holders if the full promised payment to the senior holders has not been made at the maturity date of the bonds. If the pre-specified lower boundary for the firm value is low, only the senior bondholders benefit from the presence of a safety covenant. As the lower boundary increases, the junior bondholders begin to receive benefits as well at the expense of stockholders. Subordinated debt has different characteristics from the regular debt. First, while the price of senior debt is always a concave function of the firm value, the price of junior debt is initially a convex function of the value of the firm. The junior debt can be initially convex because even a small increase in the firm value from the pre-specified lower reorganization boundary would reduce the default risk by a large amount for the junior bondholders. For every small increase in the firm value, we may see that the value of the junior bond increases by a larger amount until the firm value reaches a point where the default risk is low enough. After this cut-off point is reached, the junior bondholders would not necessarily benefit from an additional increase in the firm value as much as they did when the firm value was around lower boundary. In fact, the contribution of each marginal increase in the firm value to the junior bond value is reduced. As a result, the value of the junior bond becomes a concave

function for large values of the firm. The value of the junior debt can be an increasing function of the business risk of the firm, which is not the case for the senior debt. Because senior debt holders must keep their right to approve or reject investment policy changes that might increase the business risk of the firm in order to protect the value of their holdings, the value of the senior debt is always a concave function of the firm value. Unlike the senior debt, the value of the junior debt might be an increasing function of time to maturity. This happens because the junior debt can be worthless at the time of the maturity. In this case, it would be optimal for the junior bondholders to extend the maturity date of the whole debt issue. With a longer time to maturity, it could be possible to avoid a potential bankruptcy so that junior bondholders would be entitled to some non-zero payment at the maturity date.

The last modification to the original Merton framework considered in Black and Cox (1976) is the restriction on the financing of interest and dividend payments. They derive the value of interest paying bonds when there is a limit to the sale of the assets of the firm to meet the interest and dividend payments. In an extreme case, they consider the effects of these restrictions on the bond value when selling assets to raise money for these payments is not allowed at all. Black and Cox (1976) describe the implications of the use of junior debt under two different scenarios. First, suppose that because of legal restrictions, the junior bondholders cannot play an active role or cannot change the terms of conditions in their contract. Under these conditions interest and dividend payments must be financed by issuing new equity

or subordinated debt. However, issuing any new junior debt would actually be more beneficial to the senior bondholders than it is to the junior bondholders. Because, in this case, it would be more likely that an interest payment will be missed and the junior bondholders will be paid only if the firm value is higher than the sum of final and interest payments to the senior bondholders. Second, suppose that junior bondholders are allowed to change their status. In particular, consider a junior debt indenture in which the stockholders sign their entire equity over to the junior holders when they cannot make an interest payment. In this case, it will be possible for the junior bondholders to reorganize the firm so that it will have only equity and senior debt. As a result, it will be less likely for the senior bondholders to take the ownership of the firm, which is more beneficial to the junior bondholders.

Geske (1977) modifies the original Merton framework by allowing the risky bond to have discrete interest payments. Although, Black and Cox (1976) looks at a similar problem, in their case, the interest payments are continuous in time and state that in general, there is no closed form solution when the interest payments are discrete in time. However, Geske (1977) derives a general valuation equation for a risky coupon bond with an arbitrary number of discrete coupon payments and a principal payment using the compound option technique developed in Geske (1966).<sup>5</sup> He also discusses the effects of safety covenants, subordinated debt, and payout financing restrictions in the compound option case. In particular, the general valuation equation developed

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<sup>5</sup>Geske (1966) shows that it is possible to find an analytic solution for valuing compound options in either discrete or continuous in time.



using the compound option technique is applied to the subordinated debt.

Geske (1977) follows Rubinstein's (1976) theory to discount uncertain income flows and Geske's (1966) compound option approach in order to value risky coupon bonds in discrete time. In what follows, assuming that the firm has only common stock and coupon bonds outstanding and that the coupon bond has  $n$  identical interest payments of certain dollars each, the common stock can be considered as a compound option. At each coupon date, the stockholders have the option of buying the next option by making the coupon payment or must let the bondholders take over the firm otherwise. Geske (1977) derives the value of the common stock by recursively solving for values at each payment date in terms of the solution to the previous payment date. Then, he calculates the value of the risky coupon bond at each coupon payment date by subtracting the value of the stock from the firm value. The general formula obtained for valuing risky coupon bonds requires the calculation of  $n$ -dimensional integral as there are  $n$  discrete coupon payments in time.<sup>6</sup> In a specific application of this technique, Geske (1977) derives a new formula for which debt is subordinated. This new formula generalizes Black and Cox's (1976) valuation equation so that it can be applied even when the senior and junior debt issues mature at different dates.

In addition to Geske's (1977) paper, Vasicek (1984) discusses the distinction between the long-term and short-term liabilities in valuing credit risk. However, the

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<sup>6</sup>Geske (1977) shows that any  $n$ -dimensional normal integral can be reduced either to  $n/2$ -dimensional normal integral if  $n$  is even or to  $1/2(n-1)$ -dimensional integral if  $n$  is odd with the use of an appropriate correlation matrix. With this convenience, the technique developed by Geske (1977) is easy to apply to the valuation problem of any other risky securities with constant payouts.

valuation of debt becomes more complicated when one considers a debt structure by priority and by term. When all debt matures at the same time, the senior bondholders need not be concerned about any junior debt. Because, in this case, the senior bondholder faces a loss only if the firm's higher priority liabilities are greater than the firm's assets.<sup>7</sup> However, if the maturity dates for the firm's debt differ, the lender should not only be concerned about his claim but also other claims on the firm's asset that mature earlier even if they are junior debt.

The firm might be forced to bankruptcy if the market value of the firm's assets is less than the total short-term maturing debt on the maturity date of the short-term credit. Vasicek (1984) points out that the size of the expected loss will depend on the market value of the firm's assets and that of its total maturing debt and higher priority debt. He considers three cases. First, if the firm's market value is greater than its total maturing debt on the maturity date of the short-term loan, there will be no loss. Second, if the firm value is less than its total maturing debt but larger than its higher priority debt, the value of total maturing debt minus the value of the firm's assets will determine the loss. Note that in this case, if the firm is forced to bankruptcy, the long-term debt would immediately become payable resulting in a lower payment to the short-term lender. As a result, the short-term bondholder would only recover a fraction of the firm's assets after payments are made to the senior bondholders and long-term credit providers. Vasicek (1984) argues that should

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<sup>7</sup>For example, employee wages and benefits, and provisions for taxes can be considered as the most senior claims on the firm.

the short-term bondholder make a partial credit to the firm in this case, the firm would not have to go bankrupt and making payments to the long-term lenders would be avoided. Third, the short-term lender will not be paid at all if the firm value is less than the senior debt. This happens because all the assets will be given to the senior bondholders in this case. Vasicek (1984) finally points out that the long-term debt is as good as the firm's capital.

After describing the effects of debt structure by term on the probability of default and the expected loss, Vasicek (1984) gives a method to find the price of a short-term loan. His method is based on the option pricing theory as in the earlier first generation structural-form models. He states that the price of a short-term loan can be calculated by the difference between the loan face value and the expected loss discounted at the risk-free interest rate.

### **3.3 Second Generation Structural-Form Models**

While the original Merton framework provides insights to the qualitative properties of pricing credit risks, some empirical studies questioned its ability to explain the yield spreads between risky corporate bonds and corresponding risk-free Treasury bonds. One of the reasons for this failure is that the firm defaults mostly at the maturity of the debt but not necessarily at the coupon payments along the way in the original Merton framework. However, this assumption is unlikely to hold in reality. Another reason is that the conventional contingent claims models assume a flat risk-free term

structure. However, the interest rate risk might have a significant effect on the values of Treasury and corporate debt.

To overcome the drawbacks mentioned above, second-generation structural form models assume that a firm may default any time between the issuance and maturity of the debt. In addition, they model the interest rate risk by specifying a stochastic process for the evolution of the short-term rate. Second generation structural-form models include Kim, Ramaswamy and Sundaresan (1993) and Longstaff and Schwartz (1995).

Kim, Ramaswamy and Sundaresan (1993) show that conventional contingent claims models are unsuccessful in generating the credit spreads observed empirically even when excessive debt ratios and high level business risk parameters are used in numerical simulations. Due to this finding, they modify the conventional contingent claims model in two directions. First, they allow the bankruptcy to occur anytime between the issuance and maturity of the bond. In particular, the issuing firm may default on its coupon payment obligations any time. Second, they relax the flat risk-free rate assumption by specifying a stochastic process for the evolution of the short rate. Third, they introduce the call features to examine its effect in the yield spreads between corporate and Treasury bonds.

The net cash flow is the key source to incorporate default risk of coupon and common dividend payments.<sup>8</sup> Assuming that the net cash flow is continuously dis-

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<sup>8</sup>The net cash flow is defined as the cash revenues less expenses and a predetermined investment outlay.

tributed to the shareholders and bondholders and that the firm is not allowed to sell its assets, if the firm does not have enough cash to make the coupon payments, it is forced into bankruptcy.<sup>9</sup> Kim, Ramaswamy and Sundaresan (1993) argue that this assumption is justified in many bond indenture provisions where the lack of cash to make the necessary coupon payments is the key source of bankruptcy. With this modification, in contrast to conventional contingent claims model, a firm might be forced into bankruptcy even if the total value of its assets is higher than its total debt obligations. A lower boundary for the reorganization of the firm is determined by its net cash flow. Therefore, the firm will be bankrupt whenever the firm's net cash flow goes below this threshold. In case of bankruptcy, the bondholders recover either the total value of the firm's assets or a fraction of the value of a comparable default-free Treasury bond.<sup>10</sup>

Because of stochastic interest rates and coupon-bearing corporate debt assumptions, the valuation equation derived in the paper cannot be solved analytically. For this reason, Kim, Ramaswamy and Sundaresan (1993) provide the numerical solutions for valuing corporate debt. They first compare their model without stochastic interest rates with Merton's framework. They find that their model, which assumes a coupon bearing corporate bond, matches observed yield spread between risky corpo-

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<sup>9</sup>Note that this is a strong assumption. However, if the sale of assets is allowed in the model, this would affect the investment policy of the firm and therefore its net cash flow. Since Kim, Ramaswamy and Sundaresan (1993) model the stochastic evolution of the firm value and short-term risk-free interest rate simultaneously, the sale of assets is restricted so that the model would be tractable.

<sup>10</sup>At maturity, this fraction is assumed to be equal to 1 so that the bondholders recover either the promised payment or the total value of the firm's assets, whichever is less.

rate and Treasury bonds better than the Merton's model with no coupon payments and flat term structure. Once the stochastic interest rates are introduced, the capital structure of the firm plays a major role in the shape of the term structure of yield spreads. They find that the yield spread on corporate debt of a firm with low debt ratio is an increasing function of the time to maturity. This happens as more coupons are subject to default risk for a long-term bond, which makes long-term bonds riskier. On the other hand, it is reported that the spread is hump-shaped when the debt ratio is high. In this case, short-term lenders are subject to a higher risk of default on the balloon payment. As a result, this requires short-term corporate bonds to be priced to give a higher yield than long-term bonds.

Kim, Ramaswamy and Sundaresan (1993) argue that although the Treasury and corporate bond yields are significantly affected by the interest rate uncertainty, its effect on the yield spread is quite limited. They find that increasing the variance of the interest rate leads to a very little increase in the spread. However, they note that, the distance between the short-term rate and its long-run mean rate play an important role in the determination of the yield spreads. They argue that if the short-term rate goes to infinity, the value of corporate and Treasury bonds are reduced significantly therefore spreads approaches zero. If it goes to zero, then the yield spread widens because the present value of the default risk is maximized in this case.

Because the majority of corporate debt is callable, Kim, Ramaswamy and Sundaresan (1993) also examine the yield spreads between callable corporate and treasury

bonds. As a first step, they define the total spread as the yield differential between a callable corporate bond and noncallable Treasury bond. The optimal call policy depends on the interest rate and the value of the firm which will need to be determined endogenously. They report that the optimal call policy is less sensitive to the firm value than it is to the interest rate. Because calling the bond requires an instantaneous cash outflow while relieving the firm of its high coupon obligations, it takes longer time for the firm to call when the firm value is low. In the next step, Kim, Ramaswamy and Sundaresan (1993) define two other yield spreads: the yield spread between noncallable corporate bonds and the straight Treasury bond and the yield spread between callable and noncallable Treasury bond. For the latter, they report that the contribution of the call provision to the promised yield to maturity is larger than the one between callable and noncallable corporate bonds. They conclude that the call feature reduces the value of corporate bonds by less than it does that of Treasury bonds. Furthermore, it is reported that the sum of the yield spread between the straight corporate and straight Treasury bonds and that between callable and straight Treasury bond is larger than the yield spread between the callable corporate and straight Treasury bonds. This differential is due to the interaction between the call provision and default risk. The total yield spread of a callable corporate bond is reduced more when interest rates are low and the firm's debt ratio is high. As a result, Kim, Ramaswamy and Sundaresan (1993) conclude that stochastic interest rates are important to the determination of the yield differentials between a callable

corporate and callable Treasury bonds due to the interaction between call provision and default risk.

Another important study among the second generation structural-form models is given by Longstaff and Schwartz (1995). They modify the first generation models in three directions: (i) default can arise anytime between the issuance and the maturity of the bonds; (ii) interest rates are not flat, i.e. there exists interest rate risk; (iii) strict absolute priority is violated.<sup>11</sup>

In contrast to Kim, Ramaswamy and Sundaresan (1993), this paper derives a closed form solution to the valuation equation of risky fixed-rate and floating-rate coupons in a model with complex capital structure. In an application of their model to value risky discount and coupon bonds, they show that credit spreads produced by the model are comparable in magnitude to actual spreads. Furthermore, the model implies that credit spreads may differ among the firms with same default risk. The main reason for this is that the value of these firms' assets may have a different degree of correlation with interest rates. This implication of the model is helpful in explaining the observed differences in credit spreads among the similar rated bonds across various industries.

Longstaff and Schwartz (1995) derive a closed-form solution to the valuation of risky floating-rate debt. They show that the price of a floating-rate coupon payment

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<sup>11</sup>Although Black and Cox (1976) earlier introduced this more general type of default mechanism by defining a threshold for the value of the firms' assets, their paper abstracts from interest rate risk.



might be an increasing function of time-to-maturity. Suppose that the value of one floating-rate coupon payment is determined before the time at which payment is going to be made. The payoff on this claim at the time of this payment equals the short-term rate if the default has not occurred and a given fraction of the short-term rate if it has. When the short-term rate is below its long-run average value; the expected value of the payoff on the claim increases with the maturity due to the mean-reverting property of the short-term rate. However, as time-to-maturity increases, the discount factor applied to the payoff reduces the value of the floating-coupon payment. If the first effect dominates the second, in fact this is the case for small time-to-maturity values; we observe that the value of the floating payment increases with time.

Longstaff and Schwartz (1995) argue that the value of the floating-rate coupon payment may be an increasing function of the interest rate. As discussed above, an increase in the short-term rate results in an increase in the expected payment but a decrease in the discount factor applied to the payoff. For short-term bonds, the first effect might be dominant. Another reason for this result comes from the relationship between the interest rates and the returns of the firm. When they are positively correlated, an increase in the short-term risk-free interest rate implies that firm is less likely to default which leads to an increase in the value of the floating-rate coupon payment. As a result, the value of the risky corporate debt depends on the correlation between interest rates and the returns of the firm.

Longstaff and Schwartz (1995) proposes that one can sum the values of the fixed-

rate coupons and the value of the terminal principal payment in order to value the risky fixed-rate coupon bonds. They also apply this method in the valuation of the risky floating-rate coupon bond by summing the values of the floating-rate coupons and the promised payment at the maturity. With this method, they are able to obtain closed-form solutions to these valuation equations. They discuss that credit spreads for corporate bonds might be results of either an asset value factor or an interest rate factor. Since their model allows interest rate risk, they regress the changes in credit spreads on proxies for these two factors. While they use 30-year Treasury bond yield as a proxy for interest rate factor, the proxy for asset returns are given by the returns computed from S&P industrial, utility and railroad indexes. Using Moody's corporate bond yield averages, they find that there is a negative correlation between the credit spreads and the interest rate levels and the majority of the variation in credit spread come from the interest rate risk. The paper concludes that both default risk and the interest rate risk are important in explaining the observed credit spreads, therefore both must be taken into account in any valuation model for corporate debt.

### **3.4 Reduced-Form Models**

Although the second generation structural-form models provide some improvements over the first generation models, they still face some difficulties if they are to be implemented in practice. The firm's market value still needs to be estimated in these models to determine when a firm is likely to default. This proves to be a difficult task

as the parameters to determine the value of firm's assets are not easily observable. However, a model that incorporates the credit ratings of a debt issue will reveal information about the financial health of the firm without explicitly estimating the firm's assets value. This also makes it possible for the reduced-form models to be used for pricing various credit derivatives whose payouts depend on the credit rating of the debt issue.<sup>12</sup>

Because of the difficulties discussed in the structural form models, the default process of a firm and its timing do not depend explicitly on the market value of the firm in the reduced-form models. Instead, reduced-form models make use of an exogenous Poisson random variable to determine the default probability of a firm.<sup>13</sup> All of the exogenous assumptions in the model are imposed only on observables. The bankruptcy process is specified exogenously and does not depend on the firm's underlying assets, which makes reduced-form models more tractable mathematically than the models that follow the Merton framework. In the reduced-form models the firm goes into default whenever the exogenous random variable shifts and of course this event is unexpected. Hence, at each instance of time, a firm may default on its obligations with some positive probability and this does not require information about the value of the firm's assets.<sup>14</sup>

Jarrow, Lando and Turnbull (1997) study the term structure of credit risk spreads

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<sup>12</sup>For instance, payouts on credit sensitive notes and spread adjusted notes are dependent on the credit rating of the debt issue.

<sup>13</sup>See Litterman and Iben (1991) and Jarrow and Turnbull (1995).

<sup>14</sup>Since there is no direct relationship between the firm's value and these stochastic processes, Duffie and Singleton (1995) call this alternative approach as reduced-form models.

in a model with the bankruptcy process following a discrete state space Markov chain in credit ratings. Their model is the first contingent claims model that explicitly incorporates credit rating information into the valuation methodology.<sup>15</sup> They assume that the interaction between the default-free term structure and the firm's bankruptcy process is statistically independent. Jarrow, Lando and Turnbull (1997) state that this assumption is pretty reasonable for investment grade debt but not necessarily for speculative grade debt.<sup>16</sup> The current model can be used in risk management and to compute two common statistics: the maximum exposure and expected exposure time profiles. In other words, the model is useful to compute the probability of being in a given credit class for a certain time interval starting from a particular credit class. Assuming the recovery rate is determined exogenously, they assume that the bondholders receive a certain amount for sure at the maturity of the contract, if bankruptcy occurs prior to maturity. This is equivalent to saying that the term structure of the risky debt collapses to that of the default-free bonds in case of bankruptcy. Under the assumption that the stochastic process for default-free spot rates and the bankruptcy process are statistically independent, it is sufficient to specify a distribution for the time of bankruptcy to uniquely determine the evolution of the term structure of risky debt with the martingale probabilities. Therefore, the paper contributes to the reduced-forms models literature by explicitly modeling this

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<sup>15</sup>The current model is an extension of the Jarrow and Turnbull (1995).

<sup>16</sup>Jarrow, Lando and Turnbull (1997) impose this assumption as a simplifying assumption although they admit that its accuracy deteriorates for speculative grade debt.

distribution as the first hitting time of a Markov chain where the credit ratings and default are the relevant states. In the examples provided in the paper, it is shown that the probability of default for the second class among three different ratings is higher than that of the first class but smaller than the worst class. This is simply because the probability of default increases as the credit rating decreases. When the recovery rate is set to zero, the hazard rate for a firm with credit class  $i$  at time  $t$  is the rate of default at time  $t$  for a firm that is in class  $i$  at time 0 and has not defaulted up to time  $t$  yet.

Duffie and Singleton (1999) present a new approach to modeling the valuation of contingent claims subject to default and focus on the applications to the term structure of interest rates for corporate bonds. Their approach differs from the other reduced-form models by how they parameterize losses at default in terms of the fractional reduction in the market value when default occurs. As in the other reduced-form models, however, they treat default as an unpredictable event given by a hazard-rate process. They argue that loss-of-market value assumption, compared to a loss-of-face value assumption, generate similar par yield spreads and that the former is analytically more tractable to estimate default hazard rates.

Duffie and Singleton (1999) show that the price of a defaultable claim can be written as the present value of the promised payoff discounted by the default adjusted short rate. The adjusted short rate accounts for both the probability and timing of the

default and for the effect of losses when default occurs.<sup>17</sup> A key feature of the valuation equation presented in the paper is that the mean-loss rate is given exogenously. In other words, neither the default hazard rates nor the fractional recovery depend on the value of the contingent claim. Therefore, by using a default adjusted short rate instead of default-free rate, one can price the securities subject to default risk as in the standard valuation models.

An important application of the Duffie and Singleton (1999) model with exogenous default risk is the valuation of defaultable corporate bonds. They compare recovery of market value (RMV) with the conventional recovery of the face value assumption (RFV) and with the recovery of treasury (RT) formulations to determine the bond pricing errors under RMV.<sup>18</sup>

Under RT, computing the value of a security for a given fractional recovery process will be challenging as one needs to deal with the joint probability distribution of recovery rate, short rate and hazard rate over various horizons. For this reason, Jarrow and Turnbull (1995) assume that the default hazard rate process is independent of the short rate and the fractional loss at default is constant. Duffie and Singleton (1999) argue that the choice between RMV and RFV should be determined by the legal

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<sup>17</sup>  $R_t = r_t + h_t L_t$  is the default adjusted short rate where  $r_t$  is the default-free short rate,  $h_t$  is the hazard rate for default at time  $t$  and  $L_t$  is the expected fractional loss in market value if default occurs at time  $t$ , conditional on the information set at time  $t$ . Moreover, Duffie and Singleton (1999) call  $h_t L_t$  "risk-neutral mean-loss rate."

<sup>18</sup> Duffie and Singleton (1999) note that in reality, bonds prices are computed based on a given fractional recovery of face value. In the paper, RT is given by  $\varphi_t = (1 - L_t)P_t$ , where  $L_t$  is an exogenously specified fractional process and  $P_t$  is the price of a default-free bond whereas RFV is given by  $\varphi_t = (1 - L_t)$ , where  $(1 - L_t)$  represents the fraction of face (\$1) value that the lender recovers in the case of default.

structure of the instrument to be priced. Also, the RMV model is easy to implement in the valuation process of any security as the techniques developed for the standard default-free term-structure modeling will be applicable in this case. However, if the bond indenture agreement allows liquidation at default and absolute priority rules are in effect, bondholders of the same seniority will have equal recovery under RFV which makes it more appropriate. Duffie and Singleton (1999) report that the uses of RMV or RFV assumptions in the calculation of corporate bonds makes little difference especially for a fixed loss rate that is strictly less than one. They also note that the RMV or RFV assumption results in different spread implications for bonds with a significant premium or discount, or with steeply upward or downward sloping term structures of interest rates.

The valuation framework developed in Duffie and Singleton (1999) has been applied by Duffee (1999) in order to price noncallable corporate bonds. Duffee (1999) describe the default-free term structure as a translated two-factor square root diffusion model and extends the Pearson and Sun (1994) model to noncallable corporate bonds. The paper estimates the parameters in the stochastic processes mentioned above using bond issues of investment-grade firms and to see how well the model fits the data given these parameter estimates and what they indicate about the behavior of individual firms' bond yields.

In the model, Duffee (1999) assumes that the instantaneous probability of default for a given firm follows a translated single-factor square-root diffusion process and

that the default process is correlated with the factors driving the default-free term structure.<sup>19</sup> In other words, default is an unpredictable jump in a Poisson process and correlated with the term structure. This setup is important to explain empirical features of corporate bond yield spreads. Duffee notes that the yield spreads fluctuate with the financial health of the firm, are nonzero even for the high-quality firms, and are systematically related to the variations in the risk-free term structure. With the specifications in the model, one can find closed-form solutions to risky zero-coupon bond prices.

The corporate bond data is provided by the Lehman Brothers Fixed Income Database which covers mainly investment-grade firms. Duffee (1999) estimates the model using the extended Kalman filter approach. In the first step, the default-free term structure is estimated using Treasury yields. Then, these estimates are used to separately estimate the parameters of each firm's default process. The paper finds that the error, on average, in fitting corporate bond yields is about 10 basis points. The model is able to produce non-zero yield spreads even for highest quality firms and steeply sloped term structures for lower quality firms.

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<sup>19</sup>Factors that create a spread between Treasury and corporate bonds prices are default risk, liquidity differences, state taxes, and special repo rates. Duffee (1999) consider all these factors in a stochastic default risk process.



### 3.5 Credit Value-at-Risk Models

Value at Risk (VaR) is used to measure the potential loss in the value of a risky portfolio over a defined period for a given confidence interval. With BIS 1998 in place, certain banks developed credit value-at-risk models under two main categories during the late 1990s. The first type of credit VaR models is the default mode models (DM) in which the credit risk is linked to the default risk. While a firm can either default or survive in DM models, the second group of credit value-at-risk models, mark-to-market (MTM) models, takes more outcomes into consideration in terms of the creditworthiness of the borrower. Because, in DM models, there are only two possible outcomes; default or survival, credit losses occur only when the firm defaults. However, losses may arise whenever the creditworthiness of the borrower changes in the MTM models. The credit value-at-risk models include J.P. Morgan's *CreditMetrics*, KMV's *CreditPortfolioManager*, *Credit Suisse Financial Products' Credit Risk + (1997)*, McKinsey's *CreditPortfolioView*. Crouhy, Galai, and Mark (2000) study a comparative analysis of these credit risk models.<sup>20</sup>

Credit risk models generate a probability density function (PDF) of the future losses that might occur on a credit portfolio. With the help of this function, it is possible for a financial institution to estimate its losses on a given credit portfolio. In particular, *CreditMetrics'* analysis takes into account a change in the credit rating might occur in a given time horizon. This model provides the forward distribution

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<sup>20</sup>See Crouhy, Galai and Mark (2000) for a more comprehensive review of these credit VaR models.

of the values of a given portfolio of loans under deterministic interest rates where changes in credit rating may lead to a change in the value of the portfolio. Credit VaR of a portfolio is defined as the percentile of the distribution corresponding to the desired confidence level. KMV's *CreditPortfolioManager* differs from *CreditMetrics* in that it makes use of Expected Default Frequency for every issuer instead of historical frequencies provided by the rating agencies for different credit classes. Both *CreditMetrics* and *CreditPortfolioManager* model the asset value using the Merton (1974) framework but with different simplifying assumptions. In contrast to *CreditMetrics* and *CreditPortfolioManager*, *Credit Suisse Financial Products' Credit Risk +* specializes in default and assumes that default for loans can be described by a Poisson distribution. Because changes in credit ratings are not a feature of this model, it is DM type of model where a firm can either default or survive. Like *Credit Risk +*, McKinsey's *CreditPortfolioView* models default risk by taking macroeconomic variables into account noting that credit cycles coincides with business cycles.

J.P. Morgan developed *CreditMetrics* to (i) create a benchmark for credit risk measurement in a mark-to-market framework; (ii) promote credit risk transparency along with better risk management tools; and (iii) encourage a regulatory capital framework that can closely reflect economic risk. *CreditMetrics* not only provide expected losses but also value-at-risk (VaR). Changes in value due to changes in credit quality including default results in credit VaR. Immediate gains or losses are immediately realized in a mark-to-market framework whenever a loan or bond issuer

faces a credit upgrade or downgrade. This is not the case in book value framework.

Credit risk differs from market risk in that the value of credit portfolios changes little upon up(down)grades but may decrease substantially if default occurs. Therefore, due to the possibility of large losses, the distribution of the value of portfolios is skewed with fatter tails compared to the normally distributed returns observed in market value-at-risk models. Modeling portfolio risk is challenging due to nonnormality of credit portfolio returns and lack of empirical data to compute the correlations across various assets. Note that since VaR is used to measure potential loss before default happens, its calculation requires simulating the full forward distribution, usually one year forward, of the changes in portfolio value. In this case, knowing only mean and standard deviation are not enough to compute the expected losses which require information about the fat and long tail of the distribution. The second challenge is the lack of data for computing the cross asset correlations. One must indirectly derive credit quality correlations from equity prices.

*CreditMetrics* methodology consists of three steps: (i) establishing the exposure profile of each obligor in a portfolio; (ii) computing the volatility in the value of each instrument due to changes in credit rating; and (iii) computing the volatility of aggregate portfolio taking correlations and volatility of the assets in the portfolio into consideration. *CreditMetrics* specify a rating system and the probabilities of migrating from one credit class to another over the risk horizon. The transition matrix for these probabilities is obtained from either Moody's, or Standard & Poor's,

or from J.P. Morgan's the internal sources. *CreditMetrics* assume that all issuers in the same credit class are subject to the same transition and default probabilities.<sup>21</sup> Given that the risk horizon is usually one year, *CreditMetrics* specifies the forward discount curve for the risk horizon for each credit class.<sup>22</sup> If default occurs, the value of the financial instrument is set to certain fraction of the face value of the bond. Then, all this information is used to derive the forward distribution of the changes in portfolio value resulting from a possible credit migration for a given time horizon, usually one year in the future.

In *CreditMetrics* methodology, the two critical assumptions are that firms in the same credit class are identical in terms of their default or migration probabilities and default probability for each class is determined by the historical average default rate as noted. Crouhy, Galai, and Mark (2000) emphasize that this cannot be true as the update of ratings is discrete in time but default risk is continuous in time. KMV shows that historical average default and migration probabilities are substantially different from the actual rates in a Monte Carlo simulation. For this reason, KMV's *CreditPortfolioManager* derives the Expected Default Frequency (EDF) for each firm based on firm's capital structure, the asset return value and its volatility using Merton's (1974) framework. In this method, each value of the EDF can then be used to

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<sup>21</sup>Crouhy, Galai, and Mark (2000) note that KMV's framework, in contrast, assumes that each debt issuer's transition and default probabilities are determined by its own capital structure and asset return distributions.

<sup>22</sup>*CreditMetrics* generalize the Merton's (1974) framework to incorporate changes in credit ratings. To achieve this, one must slice the distribution of asset returns into various bands so that every random draw from this distribution would be compatible with the probability of migrating given in the transition matrix.

specify a credit rating. The default probabilities are derived in three steps. First, the value of the firm's assets is estimated based on a standard geometric Brownian motion as in the Merton's (1974) framework. Second, distance-to-default is computed. The distance-to-default is the number of standard deviations between the mean of the asset value and the default point where the default point is defined as the sum of the short-term debt liabilities and half of the long-term liabilities to be met over the risk horizon. The third and last step is to derive the default probabilities, EDFs, from the distance-to-default index. The probability of default is then the proportion of the firms of a given ranking of distance-to-default which actually defaulted over the risk horizon, usually one year. The EDFs can also be used as an indicator of the creditworthiness of the issuing firms. Based on a sample of 100,000 companies, KMV showed that there would be a sharp increase in the slope of EDF prior to default of those firms that have defaulted or went bankrupt over a 20-year period. With this empirical evidence, each EDF index can be matched one-on-one to one of those conventional credit rating classes. While the lowest EDF corresponds to highest credit rating, it increases as the credit rating goes down implying a negative relationship between the two.

*CreditRisk +* models only default in contrast to the previous two models. This model assumes that a firm either defaults or survives and the probability of default is small and the same in any given time period. It is also assumed that the number of defaults is history independent meaning that large number of defaults in the current

period does not necessarily imply more default occurrences in the future periods. Under these assumptions, the probability distribution of the number of defaults can be estimated by a Poisson distribution. One should be careful about the mean number of defaults when approximating the actual number of defaults. Crouhy, Galai, and Mark (2000) point out that for credit rating of B or lower, the standard deviation of default rate is higher than what is implied by an appropriate Poisson distribution. This, in turn, leads to underestimation of the actual default probability. They also note that as the default rates vary over time, if one assumes stochastic mean default rates, a Poisson distribution can still be used to approximate the number of defaults in a given period. In this case, the introduction of stochastic default rates partially accounts for migration risk.

Crouhy, Galai and Mark (2000) make a comparison of *CreditRisk +* to the other models mentioned above. They state that *CreditRisk +* is easy to apply as it can drive a closed form solution for the loss distribution of a credit portfolio. In addition, since this model abstracts from credit quality and rating, the number of inputs required to implement the model is relatively smaller. As in the earlier models, there is no market risk in *CreditRisk +*. Because both market risk and credit migration are ignored in this model, each borrower's exposure is the same and changes in its credit quality do not affect its exposure. Crouhy, Galai and Mark (2000) also note that all of the models mentioned above including *CreditRisk +* are not designed for nonlinear instruments such as options and foreign currency swaps.

The last model that is worth mentioning among credit VaR models is McKinsey's *CreditPortfolioView*. This model focuses not only on the default probability but also on the credit migration probabilities. It is a multi-factor model in the sense that it provides the joint distribution of default and credit migration conditional on macroeconomic factors.<sup>23</sup> Crouhy, Galai and Mark (2000) suggest that there is a positive relation between the business and credit cycles so it is reasonable to assume that macroeconomic factors play a role in default and migration probabilities.

*CreditPortfolioView* predicts the default probabilities with the help of a logit function. The independent variable is a country specific index whose value determined by a multi-factor model. The multi-factor model consists of current values of the macroeconomic variables for each country or industry. The macroeconomic variables are assumed to be auto-regressive model of order 2 in the calibration of the model to a specific country or industry. As one needs a transition matrix that provides migration and default probabilities in this model, *CreditPortfolioView* makes use of an unconditional Markov transition matrix based on Moody's or Standard & Poor's historical data. Simulation of the transition matrix many times produces the distribution of the cumulative conditional default probability for any rating over a pre-specified time horizon.

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<sup>23</sup>The macroeconomic factors that would affect the default probability or credit quality of an obligor can be unemployment rate, GDP growth rate, long-term interest rates, government expenditures, and foreign exchange rates.

## 3.6 Conclusion

This paper presents a literature review of the credit risk models developed during the last thirty years. In the paper, these models are divided into two main categories: (a) credit pricing models, and (b) credit value-at-risk (VaR) models. Three main approaches in credit pricing models discussed are (i) first generation structural-form models, (ii) second generation structural-form models, and (iii) reduced form models.

The models discussed under first generation structural-form models include Merton (1974), Black and Cox's (1976), Geske's (1977), and Vasicek's (1984) models. These models develop a basic equation for the pricing of financial instruments based on Black and Scholes option pricing framework and later apply this to the discount bonds with or without coupon payments. The second generation structural-form models include Kim, Ramaswamy and Sundaresan (1993) and Longstaff and Schwartz (1995). These models modify the first generation structural-form models to explain the empirically observed yield spreads between risky corporate bonds and corresponding risk-free Treasury bonds. Finally, reduced form models improve the second generation structural-form models by taking changes in credit rating into account in addition to default. This modification is made because we observe that the credit rating of the corporate debt is lowered before they go into default unlike in the structural form models. These models define the default process and its timing with the help of an exogenous Poisson random variable. The bankruptcy process is, therefore, specified exogenously and does not depend on the firm's underlying assets, which is one of the



drawbacks of the first and second generation structural-form models. Reduced form models discussed in this paper include Jarrow, Lando and Turnbull (1997), Duffie and Singleton (1999), and Duffee (1999).

Credit VaR models are examined under two main categories: (i) default mode models (DM) and (ii) mark-to-market (MTM) models. While a firm can either default or survive in DM models, the second group of credit value-at-risk models, mark-to-market (MTM) models, takes more outcomes into consideration in terms of the creditworthiness of the borrower. The credit value-at-risk models include J.P. Morgan's *CreditMetrics*, KMV's *CreditPortfolioManager*, *Credit Suisse Financial Products' Credit Risk + (1997)*, McKinsey's *CreditPortfolioView*. Both *CreditMetrics* and *CreditPortfolioManager* model the asset value using the Merton (1974) framework but with different simplifying assumptions. In contrast to *CreditMetrics* and *CreditPortfolioManager*, *Credit Suisse Financial Products' Credit Risk +* specializes in default, therefore it is a DM model and assumes that default for loans can be described by a Poisson distribution.

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