ESSAYS ON SOVEREIGN DEBT

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ABSTRACT

The first chapter shows how advanced economies often have access to cheap borrowing even when they hold huge levels of debt, whereas emerging economies typically suffer from higher spreads even when they hold relatively low levels of debt. Standard sovereign debt models typically fail to explain both the "debt intolerance" that emerging countries inherit, and the "graduation" to cheaper rates that characterize developed countries. I develop a dynamic small open economy model with reputation acquisition to account for the puzzle. Information revelation is the key mechanism. A competent government wishes to transmit private information about its current income to uninformed lenders who, in turn, update their beliefs about the government's reputation for transparency. When times are bad, governments gain in the short run from misrepresenting the health of their economy, but suffer the long run cost of a lower reputation by doing so. The government cares about its reputation only indirectly because bond markets respond favorably to high reputation countries in equilibrium. The model generates a separating equilibrium in which (i) governments with a lower-than-threshold reputation are trapped with high interest rates even though they hold low levels of debt and (ii) governments with higher reputation are able borrow at lower interest rates even when they hold higher levels of debt.

Chapter two examines the recent proposals of introducing common euro area sovereign securities (Eurobonds). We focus on proposals that include the introduction of guarantees with the objective of reducing the risk of default for Eurobonds, making them virtually default-free. If these proposals were implemented, Eurobonds

would be a new source of financing for European governments in addition to traditional defaultable bonds. We evaluate these proposals using a model of equilibrium sovereign default augmented to allow for both defaultable and non-defaultable debt. Our simulation results indicate that introducing Eurobonds may reduce the spread on defaultable sovereign bonds significantly. However, without restrictions to defaultable debt issuances, this spread reduction is only temporal. Eurobonds do not change significantly the government's willingness to issue defaultable debt and face default risk.

This thesis research is dedicated to my family.

It would not have been possible without them.

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CHAPTER 1

SOVEREIGN DEFAULT AND CHEAP TALK

1.1 INTRODUCTION

Relative to its GDP, Canada's debt is twice as large as that of Mexico. Yet, it receives far more favorable treatment in bond markets than Mexico, despite the fact that both countries had similar growth paths in recent years. The comparison is representative of a broader pattern: emerging economies must pay significantly higher rates than advanced economies on new borrowing, despite carrying substantially lower debt (Table B.1).

A large body of empirical literature has emerged to examine this anomaly. Much of it points toward a process of "graduation" through which some economies make the successful transition from high interest rate spreads to lower ones as markets gain confidence over time in the country's ability to repay. However, the underlying mechanism that drives this "reputation-gaining" process remains a mystery. This paper explores one such mechanism in a dynamic model of sovereign debt with reputation acquisition.

Previous studies have shown that conditioning on macro-indicators is not sufficient enough to explain the graduation. The existing literature on sovereign default and reputation relies heavily upon the government's default-repayment decision. A default decision reveals the type of government once and for all. However, it is a well known fact that among current advanced economies there exist several that were once serial

defaulters. Over time, those countries were able to escape from the "debt trap" and begin to borrow with a lower interest rate (Table B.2). In this paper, I address this problem with an information transmission mechanism. In particular, countries have private information about the current state of the economy. Transparent countries are willing to disclose this information to the public to be perceived accountable in the eyes of the lenders.

This paper develops a dynamic model of sovereign debt with reputation acquisition where the country is subject to aggregate i.i.d. income shocks. I consider an open economy with a benevolent government and competitive lenders that trades one-period zero coupon bonds. In this environment, as in the noble framework of Eaton and Gersovitz [1981], the government is not committed to repay the debt. Reputation acquisition is introduced following Morris [2001]. I assume that the government receives a private signal about the current state of the economy. A government is classified as "competent" if it is better at collecting taxes and receiving an informative signal. A competent government wishes to disclose private information about its current income to uninformed lenders. Current income is fully revealed in the next period and lenders update their belief about the government's transparency. A government cares about its reputation because bond markets respond favorably to high reputation countries. Two strategic decisions are to be considered by a government: (i) whether it decides to repay or not, (ii) if it decides to repay, to consider whether or not to tell the truth about its private information when borrowing on new terms. When times are bad, conveying the true health of the economy may sometimes be costly. For instance, if the government observes a low (L) signal about its current income, announcing L would mean higher spreads and thus costly borrowing. Governments gain from misrepresenting the health of the economy in the short-run, but face the

cost of a lower reputation in the long run. In the event of a default, a government stays in the financial autarky for an exogenous period of time.

A contribution of this paper is to provide an explanation of a country's transition from costly borrowing to cheap borrowing. In particular, this paper shows that as the market's assessment about a government's transparency increases, that government receives favorable interest rates.

It is natural to think that the market usually reacts to information that the governments report. Announcements about current fundamentals of the economy influence agents' expectations and can therefore be a significant source of economic fluctuation. There are many examples of announcements influencing the behavior of the bond prices. For instance, after the announcement that Greece cheated on its national accounts, investors lost their confidence and spreads soared, leading to a deeper crisis (Figure 1.1). It takes time to rebuild the confidence lost by international creditors. In the case of the Argentinean default episode of 2001, it was announced that Argentina's inflation reports were cooked and unreliable. Although it has been over 10 years since the default episode, any announcements from Argentinean government officials regarding macro indicators are not perceived as creditworthy by the international community. The IMF World Economic Outlook (2011) stated that "until the quality of data reporting has improved, IMF staff will also use alternative measures of GDP growth and inflation for macroeconomic surveillance". *The Economist* also stopped publishing deceiving numbers provided by INDEC, the statistical office of Argentina, and harshly criticized the government for cooking the books.(see Economist [2012])

In the theory developed here, lenders form beliefs about a government's type much like private agencies specializing in assessing a government's accountability. In essence, providing truthful data either builds or deteriorates the trust between governments and the lenders. The model predicts that as the government's transparency improves,

the interest rate on the government debt decreases even though a government increases its debt holdings. My model also proposes that in order for a government's report to be anticipated by the lenders, the government's debt holdings have to be lower than a threshold level of debt. Since lenders do not anticipate the government's messages for higher levels of debt, a government cannot improve its reputation and thus cannot graduate.

In line with the results, a majority of the current advanced economies had long periods of low levels of debt during the graduation process as illustrated in Figure B.2, and there is a highly negative correlation between the government's transparency and spreads. For the evaluation of transparency, I chose to use the World Bank's "regulatory quality index". It captures the market's perception of a government's ability to implement sound policies and institutions that boost transparency and development. A higher regulatory quality index translates into lower spreads. Table 1.1 shows a high negative correlation between the market's perception of the government's transparency and spreads. Therefore, one way to influence the creditors' beliefs on a government's transparency is through disclosure of private information.

Table 1.1: Correlation of regulatory quality index and spreads

Emerging Markets	*corr(spread, reg. qual)*
Argentina	-0.56
Mexico	-0.82
Turkey	-0.74
Mexico	-0.82
Mean	-0.68
Advanced Economies	
France	-0.16
Japan	-0.22
Spain	-0.28
Mean	-0.24

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In summary, the current literature fails to explain the fact that some countries receive far more favorable bond prices even though they hold high levels of debt, have a history of default, and have similar GDP growth paths. My paper is the first in the sovereign debt literature to address the graduation puzzle in a dynamic model of sovereign debt with reputation acquisition.

Figure 1.1: 10-year Greek Bond Yields, Source: Bloomberg
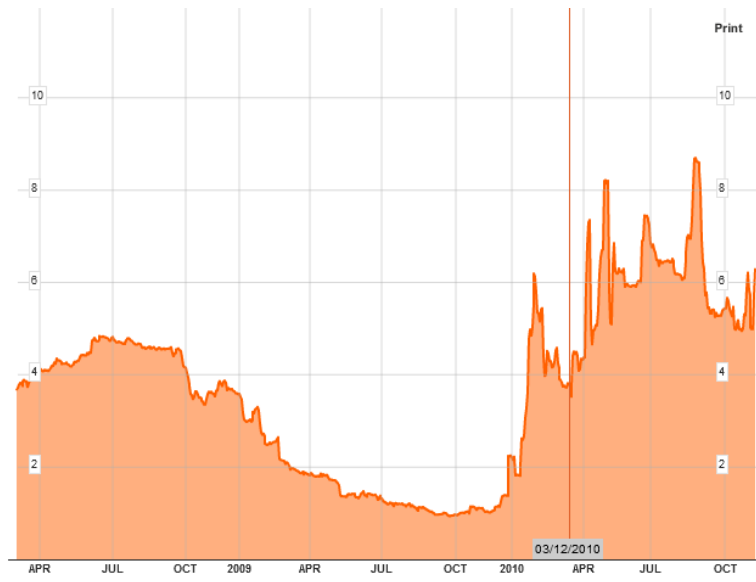
1.2 ENVIRONMENT

This paper studies the sovereign default with a reputation acquisition mechanism in a dynamic model with asymmetric information. There are two agents in the economy; a benevolent government that maximizes the utility of the representative agent, and international creditors who trade one-period zero-coupon non-contingent bonds. The country receives i.i.d. income shock with equal probabilities, a high (H) or a (L) shock. Formally, a country's income is denoted by y, with $y \in \mathcal{Y} = \{y_H, y_L\}$, $y_H > y_L$. Here y_H denotes income under the high shock while y_L denotes income if the low

shock is realized. The country receives an additional transitory negative income shock $z \in \mathcal{Z}$ if it does not repay its debt.

1.2.1 INFORMATION STRUCTURE

Competent government observes an informative signal $s \in \mathcal{S} = \{H, L\}$ about its current period's income with a probability of $\gamma \in (\frac{1}{2}, 1)$, $p(s = y|y, t_c) = \gamma$, whereas a non-competent government does not observe any informative signal, $p(s = y|y, t_{nc}) = 1/2$ where t_c denotes the competent type and t_{nc} denotes the non-competent type. I assume that the government can choose its debt b from a finite set $\mathcal{B} \subset \mathcal{R}$. The set $\mathcal{B}$ contains positive, zero and negative elements. Lenders do not have any information about a government's income but they communicate with a government who may have been partially informed. Lenders do not know what the type of the borrower is but assign a probability λ that it is a competent (informed) type. The government first decides to default or repay its existing debt. If the government repays the debt, it announces a message m from a message space $\mathcal{M} = \{H, L\}$. Given the uncertainty about the type of government, the lenders will interpret the message they receive and take action by specifically setting the bond price q. I restrict q to lie in a compact set $\mathcal{Q}$. After the lenders' action, the state of the world y becomes public and lenders rationally update their beliefs about the government's type, as a function of current debt holdings b', realized state y, last period's reputation λ, and the message sent m. The government's reputation at the beginning of the next period is written as $\lambda' = \mathcal{F}(b', y, \lambda, m)$. [1]

This set up is an example of a cheap talk game in which the government's message does not directly influence its utility; rather it indirectly affects its utility through influencing lenders' beliefs about current period's income. In this sense the government

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[1]For endogenous output please see Mendoza and Yue [2012b]

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has a costless communication with the lenders. (see Crawford and Sobel [1982]). Benabou and Laroque [1992] analyzed a variant of Sobel's game in which the agents were receiving noisy signals. The authors assumed that each type of agent were receiving informative signals. By comparison, in my set up only the competent government is able to observe an informative signal and informativeness of the signal comes from the government's institutions such as tax collection or national statistical agencies.

Every cheap talk game has equilibria where players of the game ignore the messages. If lenders do not infer any meaning in the messages, then there exists no incentive for the competent government to influence the expectations. If sending messages do not affect the lenders' beliefs, then the competent government simply randomizes $50 - 50$ between sending an H and an L message regardless of the signal it has observed. Such equilibria in which no information is conveyed is known as "babbling equilibria." The interesting case, in all cheap talk models, is to focus on equilibria where cheap talk conveys meaning.

1.2.2 TIME LINE

The timing of events can be summarized as follows:

1. Period t begins with a level of debt b. Last period's income y_{t-1}, will be denoted as y_- throughout the paper, is revealed at the beginning of the period, and market's belief of government being a competent type with a probability λ is updated.

2. Government observes a private signal, H or L, about today's income which is going to be public at time $t + 1$.

3. The government chooses whether to default or not:

- If it chooses to default, it will be subject to an additional negative income shock $z \in \mathcal{Z}$ while in default and will come back to the markets with an exogenous probability η next period.

- If the government repays:

  - Government sends a strategic message m about its signal s to the lenders, in particular it decides whether to be truthful or to lie about its signal, and chooses b' at a price $q(b', \lambda, m)$.

4. Period $t+1$ begins with b', realized y and updated reputation λ', which is determined according to Bayes' rule.

1.2.3 GOVERNMENT'S PROBLEM

I will use the noble framework of Eaton and Gersovitz [1981] in modeling the sovereign default. The households are identical and have preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1.1}$$

where E denotes the expectation operator, $0 < \beta < 1$ is the discount factor, c_t denotes consumption at time t, and utility function $u(.) : [0, \infty) \to \mathcal{R}$ is an increasing, strictly concave, continuous, and bounded above by the quantity U given as:

$$u(c) = \frac{c^{1-\rho}}{1 - \rho}$$

where ρ is the constant coefficient of relative risk aversion.

It is a small open economy environment where in each period households receive an i.i.d. income shock, in particular high (H) or low (L) shocks with equal probabilities. The benevolent government's objective is to maximize the expected discounted utility of the representative agent. The government has private information about today's

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income and differs in its ability to receive private signals and trades one-period zero-coupon bonds[2] with international risk-neutral competitive creditors. The government can buy bonds b at price $q(b', \lambda, m)$ which will be determined in equilibrium. The price of the bond does not depend on the transitory income shocks, since shocks are i.i.d, they do not inform the likelihood of future default. Lenders have perfect information on the country's last-period's income and its current asset position. A purchase of a discount bond with a positive face value b' at time t means that the government has entered into a contract to receive $b' \geq 0$ units of goods to be delivered at time $t+1$. A purchase of a discount bond with a negative face value b' at time t means that the government has entered into a contract that the government receives $-q(b', \lambda, m)b'$ units of consumption goods to be delivered $b' < 0$ units of goods at time $t+1$ conditional on not-defaulting. If a government defaults, it does not deliver. Markets are incomplete, government can use one-period zero-coupon bonds to save and borrow. The resource constraint of an economy that chooses to repay its debt would be:

$$c = y + b - q(b', \lambda, m)b' \tag{1.2}$$

If the government opts to default, then it will stay in autarky at least for one period and it is customary in the literature to denote debt as a negative asset.[3] The government's expected income in autarky is strictly less than its expected income when it has access to international credit markets as in Cole and Kehoe [1998]. The resource constraint of an economy that chooses to default is then as follows:

$$c = y + z$$

Here $z \in \mathcal{Z} = [-y_L, 0]$ is a transitory income shock drawn i.i.d each period with continuous cdf $G(z)$. The i.i.d shock z is included for the continuity properties of the

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[2]see Hatchondo and Martinez [2009b] for long-term period bonds

[3]Please see Arellano [2008b] and Aguiar and Gopinath [2006]

utility function when the government defaults. In the numerical exercises to follow, modeling the consumption during a default episode as $c = y^{aut} < y$ yields similar results.

Strategies Given the stationary Markovian structure of the model, I will focus on Markovian strategies. At any point in time t, the signal $s_t \in \mathcal{S}$, reputation $\lambda_t \in \Lambda$, income $y_{t-1} \in \mathcal{Y}$, $z_t \in \mathcal{Z}$ and asset holdings $b_t \in \mathcal{B}$ summarize the relevant history of the game. Strategies map the level of debt b, income y and z, reputation λ and signal s into a choice of actions. A government's strategy is a pair $(\sigma_c, \sigma_{nc}, \theta_c, \theta_{nc})$, each $\sigma_I : \mathcal{B} \times \mathcal{Y} \times \mathcal{Z} \times \Lambda \times \mathcal{S} \to \{0, 1\} \times \mathcal{R}$ and $\theta_I : \mathcal{B} \times \mathcal{Y} \times \mathcal{Z} \times \Lambda \times \mathcal{S} \to [0, 1]$. σ_I takes value 1 if the government repays and borrows b' or takes value 0 if it defaults. θ_I is the probability that government I tells the truth when it observes signal s. Lenders' strategy is function $\chi : \mathcal{B} \times \Lambda \times \mathcal{M} \to \mathcal{R}$ where $\chi(m; b', \lambda)$ is the lenders' action if they receive message m, in particular lenders set the bond prices. At any history, a strategy profile induces an outcome and hence a payoff for each player.

Now we are ready to write down the reputation updating function $\mathcal{F}(b', y, \lambda, m)$ and state inference function $\pi(m)$. According to Bayes' rule, the posterior probability of the government being a competent type if it sends a message m and income y is realized for given level of debt b', will be

$$\mathcal{F}(b', y, \lambda, m) = \lambda' = \frac{\lambda \theta_c(m|y, b')}{\lambda \theta_c(m|y, b') + (1 - \lambda)\theta_{nc}(m|y, b')} \tag{1.3}$$

Let $\pi(m)$ be the lenders' posterior belief that the actual state is H if message H is reported. By Bayes' rule,

$$\pi(m) = \frac{\lambda \theta_c(m|H, b') + (1 - \lambda)\theta_{nc}(m|H, b')}{\lambda \theta_c(m|H, b') + (1 - \lambda)\theta_{nc}(m|H, b') + \lambda \theta_c(m|L, b') + (1 - \lambda)\theta_{nc}(m|L, b')} \tag{1.4}$$

where $\theta_I(m|y,b)$ is the probability that government type I (nc or c) sends a message m given income y, and debt b'. Equation 1.3 is well defined as long as the denominator is nonzero. I adopt the convention that $\mathcal{F}(b',y,\lambda,m) = \lambda$ if

$$\theta_c(m|y=i,b')=\theta_{nc}(m|y=i,b') = 0, \text{ i} \in \{H,L\}.$$

It is assumed that messaging does not exist when a government is excluded from the markets since messages are not informative and lenders are not interested. So $\theta_I(m|y,b)$ is assumed to be 0 while the government is in autarky, thus $\lambda' = \lambda$.

Now consider the maximization problem of a government with $b \in \mathcal{B}$ bonds in arrear, endowment $y \in \mathcal{Y}$, $z \in \mathcal{Z}$, current reputation $\lambda \in \Lambda$ and signal $s \in \mathcal{S}$. Denote the type I (subscript c denotes competent government and subscript nc denotes non-competent government) government's lifetime utility conditional on not defaulting by the function $v_I^{nd}(b,y_-,\lambda,s)$: $\mathcal{B} \times \mathcal{Y} \times \Lambda \times \mathcal{S} \to \mathcal{R}$, its lifetime utility from defaulting by the function $v_I^d(y_-,z,\lambda,s) : \mathcal{Y} \times \mathcal{Z} \times \Lambda \times \mathcal{S} \to \mathcal{R}$ and its unconditional lifetime utility by the function $v_I(b,y_-,z,\lambda,s)$: $\mathcal{B} \times \mathcal{Y} \times \mathcal{Z} \times \Lambda \times \mathcal{S} \to \mathcal{R}$ where v_I^d, v_I^{nd} and v_I are generated by strategy profile (σ_I, θ_I, χ).

If a sovereign country borrows, it receives $q(b',\lambda,m)b'$ units of consumption good today and promises to pay back b' units of good tomorrow. If the government chooses to repay, it decides to tell the truth or to lie about the signal it has observed. So the government's maximization problem can be represented recursively as follows:

$$v_I(b,y_-,z,\lambda,s) = \max_{\{nd,d\}} \{v_I^{nd}(b,y_-,\lambda,s), v_I^d(y_-,z,\lambda,s)\} \tag{1.5}$$

Value of defaulting is given as:

$$v_I^d(y_-,z,\lambda,s) = E(u(y+z)) + \beta E_{(y,s',z'|y_-,s)}\left[\eta v_I(0,y,z',\lambda',s') + (1-\eta)v_I^d(y,z',\lambda,s')\right] \tag{1.6}$$

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Value of not-defaulting can be obtained as follows:

$$v_I^{nd}(b, y_-, \lambda, s) = \max_{m \in \mathcal{M}, b' \in \mathcal{B}} \left\{ E\big(u(y - q(b', \lambda, m)b' + b)\big) + \beta E_{(y, s', z')|y_-, s)} v_I(b', y, z', \lambda', s') \right\}$$

(1.7)

$$\text{s.t. } \lambda' = \mathcal{F}(b', y, \lambda, m)$$

Similar to Morris [2001] and Ottaviani and Sorensen [2006], I focus on the competent government's behavior seeing that a non-competent government cannot observe any informative signal.

In general, there exist equilibria in which the competent government sometimes lies. On observing L signal, the competent government may find it optimal to randomize between telling the truth (to enhance its reputation) and lying (to receive favorable interest rates). Indeed, the states in which the government indeed finds it optimal to tell the truth will be obtained as in equation 1.9.

From the optimal choices above, I can characterize the default set and deviating set as follows, default set $D_I(b, \lambda)$ and deviating set $L_I(b, \lambda)$ are defined as the set of y's and messages m for which default and deviating are optimal respectively, given the reputation of the borrower and indebtedness.

$$D_I(b, \lambda) = \{(y, s) \in \mathcal{Y} \times \mathcal{S} : v_I^{nd}(b, y_-, \lambda, s) < v_I^d(y_-, z, \lambda, s)\}$$

(1.8)

$$L_I(b, \lambda) = \{(y, s) \in \mathcal{Y} \times \mathcal{S} : v_I^{nd}(b, y_-, \lambda, s = m) < v_I^{nd}(b, y_-, \lambda, s \neq m)\}$$

(1.9)

1.2.4 INTERNATIONAL RISK NEUTRAL INVESTORS

Government trades one-period zero coupon bonds with risk-neutral competitive lenders. The opportunity cost of funds is given by the exogenous risk free interest rate r^f.

$$\Omega = -q(b', \lambda, m)b' + \frac{1 - \delta(b', y_-, z, \lambda, m)}{1 + r^f}b'$$

The first term on the right hand side of the equation indicates that when investors lend to the government in the current period, they buy government bonds at price $q(b', \lambda, m)$. The second term illustrates that investors may receive the present value of the face value of a bond with a probability of default. The probability of default correspondence $\delta(b', y_-, z, \lambda, m)$ on a loan b' will be determined endogenously in equation 1.10 using the default sets explained in 1.8 and 1.9. Let $\mathcal{Q}(\mathcal{B} \times \Lambda \times \mathcal{M})$ be the set of all functions on $\mathcal{B} \times \Lambda \times \mathcal{M}$ taking values in $[0, \frac{1}{1+r^f}]$.

What remains now is to describe how to obtain the default probabilities. For most of the states, the government either prefers repaying over default or prefers default over repaying. However, it is possible that the government is indifferent for some states and will randomize between defaulting and repaying for those states. For the sake of proving the existence of price function (please see the appendix for the details), I define an indicator correspondence for default. Let $\psi_I(b, y, z, \lambda, s) \in \Psi_I(b, y, z, \lambda, s)$ be an indicator for default correspondence for debt b in state (y, z, λ, s).

$$\Psi_I(b, y, z, \lambda, s) = \begin{cases} 1 & \text{if } v_I^d > v_I^{nd}, \\ 0 & \text{if } v_I^d < v_I^{nd}, \\ [0,1] & \text{if } v_I^d = v_I^{nd}. \end{cases}$$

Now I am ready to define the probability of default correspondence Δ on a loan b' at state (b, y_-, z, λ) as the set of all $\delta(b', y_-, z, \lambda, m)$ constructed as

$$\delta(b', y_-, z, \lambda, m) = \sum_{D_I(b', \lambda')} \psi_I(b, y, z, \lambda, s)\pi(m) \tag{1.10}$$

for some $\psi_I(b, y, z, \lambda, s) \in \Psi(b, y, z, \lambda, s)$. Since it is a perfectly competitive market for international investors, the expected profit will be zero in equilibrium. For b'

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smaller than 0, the investors lend, and if it is bigger than 0, the lenders borrow. The price of the bonds can be shown as follows:

$$q(b', \lambda, m) = \begin{cases} \frac{1}{1+r^f} & \text{if } b' \geq 0, \\ \frac{1-\delta(b',y_-,z,\lambda,m)}{1+r^f} & \text{if } b' < 0. \end{cases}$$

So $q(b', \lambda, m)$ is the set of prices of a bond for today that pays one unit of good tomorrow which depends on the current state (λ, m) and total borrowing b'.

Since this paper investigates the effect of cheap talk on default decision, I do not explicitly model the renegotiation stage. Benjamin and Wright [2008a], D'Erasmo [2010] and Yue [2010b] endogenizes the renegotiation stage following a default. In my model, countries stay in autarky when they default at least for one period and regain access to international markets next period with an exogenous probability of η and zero-debt b, or keep staying in autarky with an exogenous probability of $1 - \eta$. During an autarky there is no messaging since lenders are not interested in the country's state of the world; therefore, there is no reputation updating process during the exclusion state.

Within the information structure, competent government is assumed to receive a noisy signal γ. A slightly complicated version of the model could reconcile this discussion by allowing governments to receive more precise signals, particularly $\gamma \in \{\underline{\gamma}, \overline{\gamma}\}$. Let's assume that the ability to receive an accurate signal γ is larger in countries where fundamentals are better. This would help explain why it takes so long for some countries to build up their reputation even though they tell the truth every period as for the economies having a larger γ would imply that their reports convey more meaning. Further, I assume that the government's type is chosen once and for all in my model. A slight modification would relax this assumption, in particular a shock to γ with Markov switching probabilities would provide a type change.

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The timing of the messages as to whether to send the message before or after a default decision does not matter. In this paper, the messages are sent after a repayment decision. Alternatively, the messages could have been sent before a default event, however, before making the default-repayment decision, the government already anticipates its messaging strategy and the bond prices. If a government finds it optimal to repay and lie about its signal, it will do so. Furthermore, seeing that the reputation of a government does not change during a default state, its reputation will not be updated even if a government sends its message before default.

1.3 EQUILIBRIUM DEFINITION

Definition 1. *A Markov Perfect Bayesian equilibrium (MPBE) is characterized by a strategy profile $(\sigma_I, \theta_I, \chi)$, collection of value functions (v_I, v_I^d, v_I^{nd}), bond price q and beliefs $(\mathcal{F}, \pi)$ such that:*

1. *Government strategies σ_I and θ_I are optimal and induces v_I, v_I^d, v_I^{nd} given lenders' strategy χ.*

2. *χ maximizes lenders' profits given any q, b', y_-, λ.*

3. *The bond price function q satisfies lenders' zero expected profit condition, that is at $q(b', \lambda, m)$*

$$\Omega(b', y_-, z, \lambda, m) = 0, \forall (b', y_-, z, \lambda, m).$$

4. *Value functions v_I, v_I^d, v_I^{nd} satisfy the equations 1.5, 1.6 and 1.7.*

5. *The functions $\mathcal{F}(b', y, \lambda, m)$, $\pi(m)$ satisfies Bayes' rule and they are defined as in equations 1.3 and 1.4, respectively.*

This equilibrium definition is standard and condition 5 deserves some attention. These functions must be consistent with Bayes' rule wherever possible and off-equilibrium-path beliefs should be well-specified.

Off-Equilibrium-Path Beliefs - Let's rewrite equations 1.4 and 1.3.

$$\mathcal{F}(b', y, \lambda, m) = \frac{\lambda\theta_c(m|y, b')}{\lambda\theta_c(m|y, b') + (1-\lambda)\theta_{nc}(m|y, b')}$$

$$\pi(m) = \frac{\lambda\theta_c(m|H, b') + (1-\lambda)\theta_{nc}(m|H, b')}{\lambda\theta_c(m|H, b') + (1-\lambda)\theta_{nc}(m|H, b') + \lambda\theta_c(m|L, b') + (1-\lambda)\theta_{nc}(m|L, b')}$$

Both equations are well defined as long as the denominators are non-zero. In the computation of the model, I adopt the convention that out-of-equilibrium beliefs are equal to their prior, that is $\mathcal{F}(b', y, \lambda, m) = \lambda$. Allowing for other off-equilibrium-path beliefs does not lead to different equilibrium behavior.

1.4 Characterization and Existence of Equilibrium

The first few results establish some technical but essential properties of the equilibrium and later on I prove the existence of a truthful MPBE and characterize the equilibrium pricing function, prove its existence, and then show some of its properties. (Please refer to the appendix for details). I will now continue with some basic monotonicity and continuity results.

Proposition 1. *Given $q(b', \lambda, m) \in \mathcal{Q}(\mathcal{B} \times \Lambda \times \mathcal{M})$, there exist functions $v_I(b, y_-, z, \lambda, s)$, $v_I^d(y_-, z, \lambda, s)$, $v_I^{nd}(b, y_-, \lambda, s)$ where v_I, v_I^{nd} are continuous in q and v_I^d is continuous in z and collectively solve equations 1.5 - 1.7. Moreover, v_I^{nd} is strictly increasing in b and λ, v_I^d is strictly increasing in z.*

Proof. Intuition - The existence of bounded and continuous value functions v, v^{nd}, and v^d follows from the standard contraction mapping arguments. The strict monotonicty

properties of value functions follow from the strict monotonicity of u. Please see appendix for the details. $\qquad\square$

Proposition 2. *If defaulting is optimal for debt level of b^1 for some values of output y and reputation λ, then it would be optimal to default as well for a debt level of b^2 for the same output y and reputation λ for all $b^2 < b^1$, that is; if $b^2 < b^1$, $D(b^2, \lambda) \geq D(b^1, \lambda)$.*

Proof. It is a standard result that default sets are increasing as debt holdings go up. Proof is similar to Chatterjee et al. [2007], Chatterjee and Eyigungor [2011] and Eaton and Gersovitz [1981]. (Please see appendix) $\qquad\square$

Proposition 3. *Let $\underline{\lambda}$ and $\overline{\lambda} \in [0,1]$ such that $\overline{\lambda} > \underline{\lambda}$. If it is optimal for the higher reputation government $\overline{\lambda}$ to default for given level of debt b, and state y, then it is also optimal for the lower reputation government $\underline{\lambda}$ to default too for the same level of debt holdings b, and state y, that is, $D(b,\overline{\lambda}) \subseteq D(b,\underline{\lambda})$.*

Proof. Please see appendix. $\qquad\square$

The following analysis focuses on truthful Markov Perfect Bayesian Equilibrium. First, it is shown that stationary Markov perfect equilibrium does always exist and then it is shown that for periods in which decision making to tell-the-truth is sufficiently important, truth-telling MPBE will exist.

Proposition 4. *A truthful Markov perfect Bayessian equilibrium exists.*

The intuition for the existence is as follows. Suppose some pair of valuations for the governments and lenders (v_c, v_{nc}, v_l) occur with very low probability ψ. Then presume the government always babbles unless (v_c, v_{nc}, v_l) is not drawn and if (v_c, v_{nc}, v_l) is drawn, then the competent government always tells the truth. The non-competent then mixes and lenders make inferences from the message and set the bond prices.

It can be established that for a small ψ, these proposed strategies will be the best responses to each other.

Proof. Please see appendix. □

Proposition 5. *(Characterization of Equilibrium Prices) In any MPBE: (i) $q^*(b', \lambda, m)$ is increasing in b', and increasing in λ; (ii) $q^*(b', \lambda, m = H) \geq q^*(b', \lambda, m = L)$; (iii) for some $b^1 > b^2$ and for some $\overline{\lambda} > \underline{\lambda}$, $q^*(b^2, \overline{\lambda}, m) \geq q^*(b^1, \underline{\lambda}, m)$.*

Proof. Please see appendix. □

The first property simply suggests that as the government's debt level and reputation increases, the implied interest rate decreases. The second property shows that when a government sends a message H, it receives a lower interest rate. The final property says that there exist levels of reputation such that the government is treated favorably by the market even though the government holds higher debt holdings and has the same income.

1.5 Computational Algorithm

1. Set the grids over assets, endowments and reputation. It is important to set finer grids on assets.

2. Make an initial guess for the bond price schedule $q^0(b', \lambda, m) \in \mathcal{Q}$. I particularly set it for $\frac{1}{r^f}$

3. Given the bond price schedule and reputation, solve the government problem to obtain the value functions and the default interval. This includes the following:

    • Find the set of y's and signal s such that default is optimal;

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- Find the set of y's and message m such that telling the truth is optimal.

4. Using the default sets and truth-telling sets described above, solve for the new schedule of bond prices $q^1(b', \lambda, m)$ until the convergence is satisfied such that $||q^0(b', \lambda, m) - q^1(b', \lambda, m)|| < \epsilon$, otherwise move to 3.

To check for equilibrium, I examined whether the competent and non-competent type will in fact want to use the proposed strategies in each state. It is then straightforward to calculate the current utility and discounted expected utility if any type of the government deviates. The proposed strategies must be best responses to each other for a truthful Markov Perfect Bayesian Equilibrium to exist. Below are the parameters that has been used in the paper.

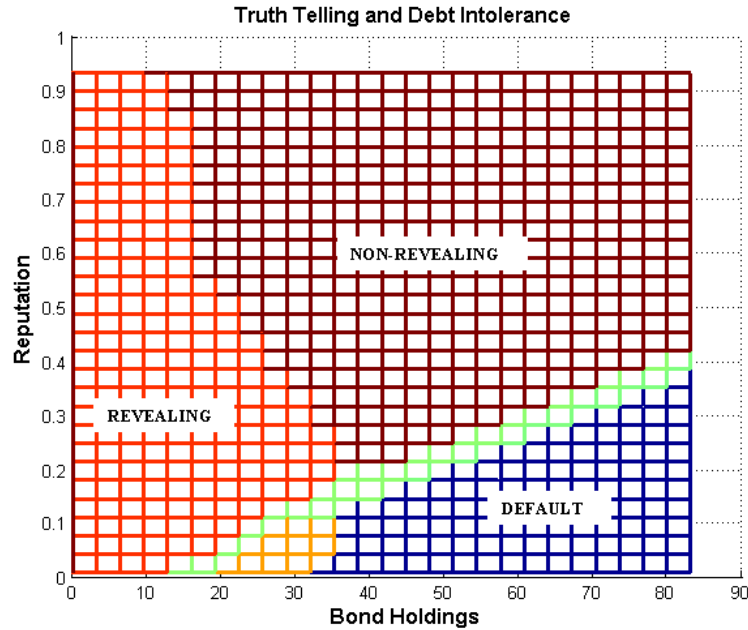| Parameter		Value
Risk aversion rate	ρ	2
Risk free interest rate	r^f	0.015
Discount factor	β	0.97
Output loss	$1 - \epsilon$	0.05
Prob excl ends	η	0.282
Accuracy of the signal	γ	0.75
High accuracy of the signal	$\overline{\gamma}$	0.9
Low accuracy of the signal	$\underline{\gamma}$	0.6
Good shock	y_H	1.05
Bad shock	y_L	0.95

Table 1.2: Model Parameters

Figure 1.2: Blue region represents the states in which it is optimal for the govern-ment to default. Red region identifies the area in which lenders anticipate the gov-ernment's messages and the competent government separates whereas in the brown region lenders do not anticipate the government's messages. The only region where a government can build up its reputation is in the red area. The government holding rel-atively lower levels of debt can therefore increase its reputation by truthfully reporting its signal. Since lenders do not anticipate the government's messages in the brown region, a vertical movement is not possible. The only way for a government in the brown region to graduate is to diminish its debt levels and move to the red region such that its messages can be anticipated. The Green region represents the states in which the government is indifferent between defaulting and not-defaulting.

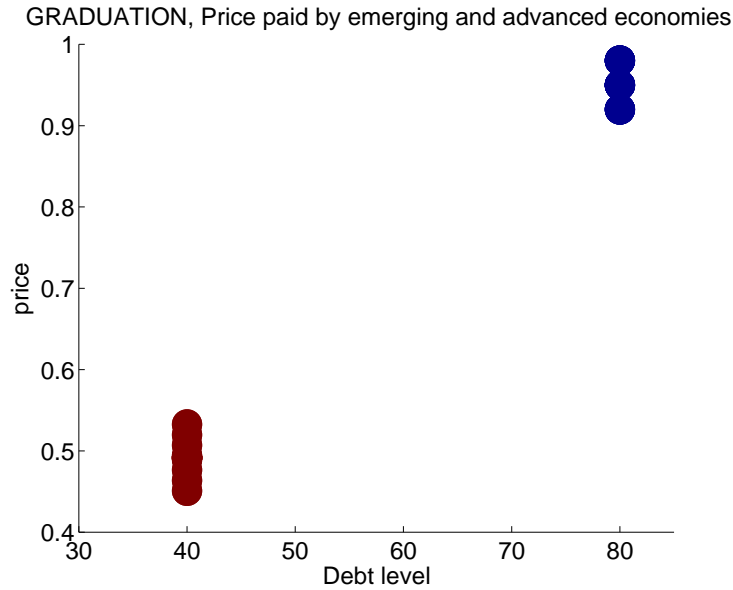GRADUATION, Price paid by emerging and advanced economies

Figure 1.3: Price paid by emerging and advanced economies. The blue bubbles on the upper right corner of the figure represent the advanced economies while the red bubbles on the lower left corner represents the emerging economies. This graph shows that, emerging economies borrow at a lower price (higher interest rate) than the advanced economies even though they hold lower levels of debt and are subject to the same income shock. Emerging economies represent the lower reputation $\underline{\lambda}$ countries, whereas advanced economies represent the higher reputation $\overline{\lambda}$ governments.
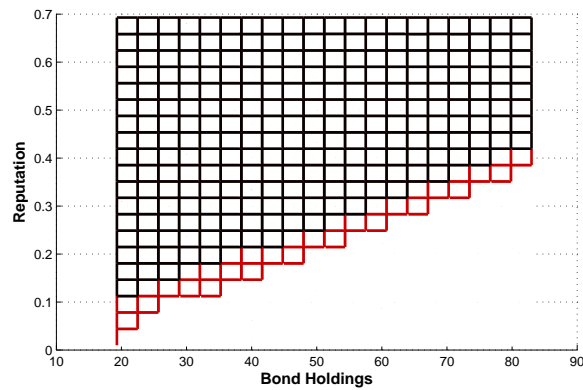

Figure 1.4: This picture shows the combinations of debt holdings and reputation levels for which: (i) the government would default regardless of the signal it has observed (white area), (ii) the government would default only if it receives an L signal (red area), and (iii) the government would not default (black area).
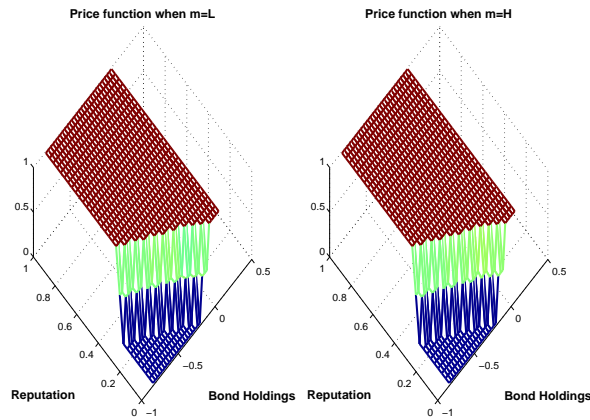
Figure 1.5: This picture shows how the bond-prices behave for each level debt b, and reputation λ for a given message. To obtain a better understanding of how the bond price behaves for a given level of debt and reputation, please check figures (1.8) and (1.9).

Figure 1.6: If the lenders do not place any significance on a government's reputation, and thus the government does not have any reputational concerns similar to Arellano (2008) environment, the model predicts that with i.i.d shocks the prices will be constant up to a certain level of debt.

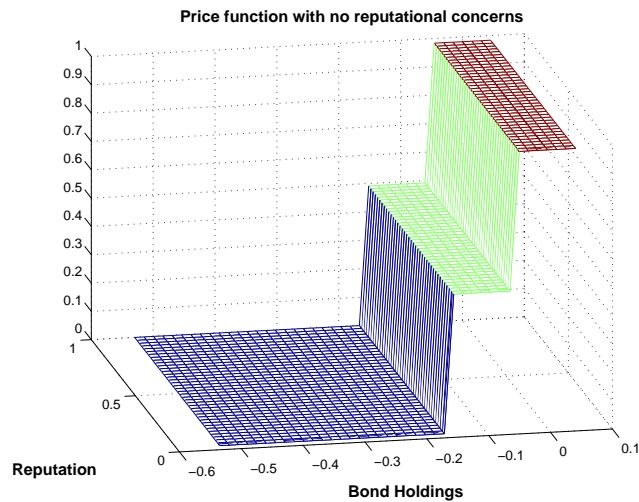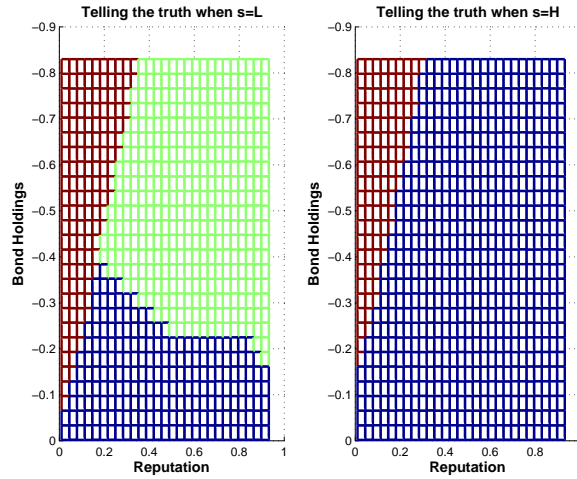Figure 1.7: The left panel presents the government's truth-telling and default decisions with a given level of bond holdings and stock of reputation when it receives a low signal. The blue, green and brown colors represent truth-telling, deviating and default regions respectively. Blue region identifies the separating equilibrium in which a competent government always chooses to tell the truth about its signal and the competent government pools in the green region. The right panel shows the government's decision rules when it receives a high signal and not surprisingly it always reports truthfully when it receives a high signal.
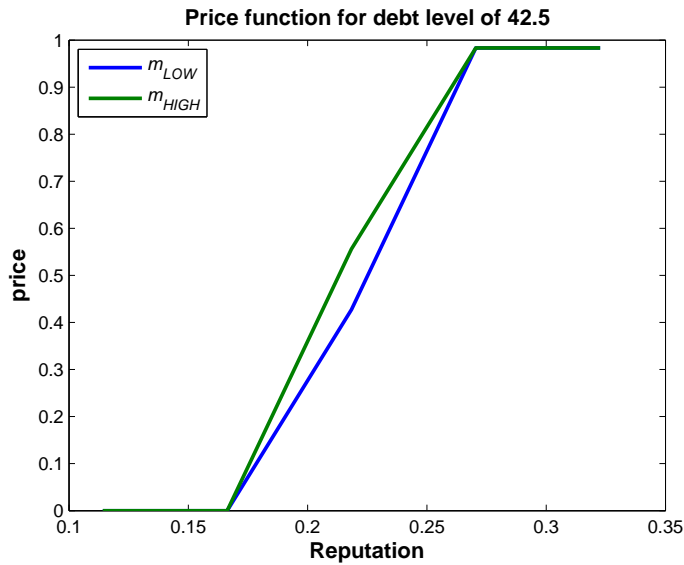
Figure 1.8: Price function for a given level of debt with the government's message. This graph shows that price function is higher with a H message.

Price function for reputation of = 0.22
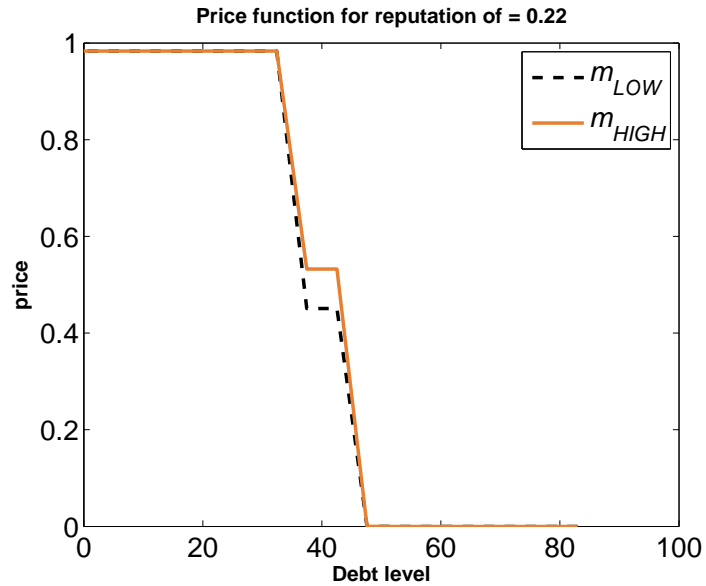
Figure 1.9: This picture shows how a government's price function behaves with a given message. As shown the price is lower (interest rate is higher) when the message is L. Therefore, it is costly for a government to tell the truth when it observes an L signal.
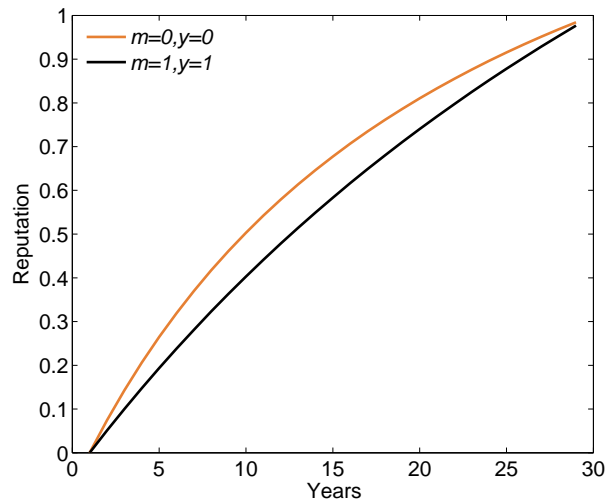
Figure 1.10: This graph shows how a government's reputation is updated if a government's report matches with the actual data. For instance, if a competent government always receives a high shock and it reports this information truthfully, then the market's assessment about a government's reputation is depicted by the black line.
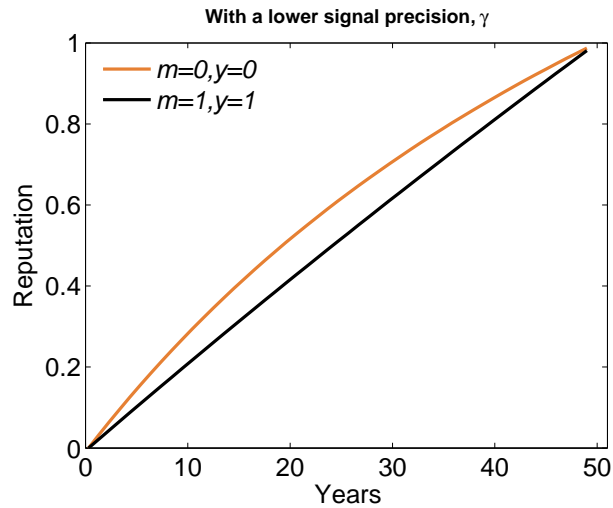
Figure 1.11: This figure shows the time it takes to graduate when the precision of the signal $\gamma = \underline{\gamma}$. Notice that as the precision of the signal goes down, the curvature of the updating function decreases and it takes longer for a government to "graduate".

1.7 Conclusion

Previous sovereign debt models fail to jointly account for graduation and advanced economies' lower borrowing costs given their high debt holdings. In this paper, I provide a model of reputation gaining mechanism to account for these two facts. The model developed in this paper illustrates that lenders form beliefs about the existing government through the government's reports. Current government has private information about today's state of the economy and communicates its information with the creditors. Lenders update their beliefs about the government being a competent type when the state of the economy becomes public next period. The endogenously determined default probabilities and the information structure are essential ingredients to obtain the main results. Figure 1.6 shows how my model would behave with a full information model. In full information models, reputation

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is insignificant as states are observable and the government's repayment decision does not influence the future bond prices. My model with private information sheds light on how some economies are trapped in "debt intolerance" and provides one possible explanation on how a country can graduate. It suggests that in order to graduate, a government has to hold low levels of debt and should be transparent about its economy. Overtime, a government can build trust and thus reputation by truthfully disclosing its private information with the lenders.

Empirically, my results seem to be in line with the transition of countries from costly borrowing to cheap borrowing. As Figure B.2 suggests, the United States and Germany had long periods of low levels of debt, while Australia was suffering from high spreads, it paid the burden of high interest rates, managed to decrease its debt levels over time and built up its reputation by disclosing non-deceiving numbers to the public. Reinhart and Rogoff [2009] illustrate how Chile is in the process of "graduation" and in fact Chile is doing things right; (i) Chile has been keeping its debt levels low as in Figure B.2 and (ii) provides truthful reports which is measured using a transparency index provided by the World Bank.

CHAPTER 2

EUROBONDS

2.1 INTRODUCTION

We evaluate recent proposals of common euro area sovereign securities, henceforth Eurobonds, using a model of equilibrium sovereign default. These proposals are being considered in policy circles as a response to current events in Europe, and also as a long-term key element for fiscal management in currency unions. We intend to inform policy discussions with a formal analysis. We study both the long-run effects of implementing different Eurobond proposals and transitions from states with different levels of default risk.

Most Eurobonds proposals include the introduction of guarantees with the objective of reducing the risk of default for these bonds, making them virtually default-free (see Claessens et al., 2012, and the references therein). Proposals differ on the amount of financing that would be available through Eurobonds and on the circumstances in which these bonds could be issued. For instance, Hellwig and Philippon proposed the mutualization of short-term debt of a member state of up to 10 percent of its GDP. Similarly, the "Blue-Red bondŤ proposal involves the mutualization of the debt of a member state of up to 60 percent of its GDP. In contrast, the German Council of Economic Experts proposed the mutualization of the debt of a member state in excess of 60 percent of its GDP.

We analyze these proposals using a sovereign default framework à la Eaton and Gersovitz (1981). We study a small open economy that receives a stochastic endowment stream of a single tradable good. The governmentŠs objective is to maximize the expected utility of private agents. Each period, the government makes two decisions. First, it decides whether to default on previously issued debt. A defaulting government faces an output cost and is temporarily prevented from issuing defaultable debt. Second, the government decides how much to borrow. The government can borrow by issuing non-contingent long-term defaultable bonds, as in Arellano and Ramanarayanan [2012], Chatterjee and Eyigungor [2012], and Hatchondo and Martinez [2009a]. We study the effects of implementing different Eurobond proposals by incorporating into the model non-defaultable sovereign bonds and making different assumptions about the amount of non-defaultable bonds the government can issue and the circumstances in which these bonds can be issued. In contrast with most previous studies that calibrate models of sovereign defaults using as a reference an emerging economy, we calibrate our model using as a reference European economies currently facing significant sovereign risk. Thus, our benchmark calibration is such that income is less volatile, debt levels are higher, and sovereign risk is lower than in previous work.

Preliminary simulation results indicate that introducing Eurobonds may reduce the spread on defaultable sovereign bonds significantly. However, without restrictions to defaultable debt issuances, this spread reduction is only temporarily. The government first uses the newly available Eurobond financing to reduce the level of its defaultable debt. But after exhausting its new source of financing, the government increases the level of defaultable debt. In the long-run, Eurobonds do not change significantly the government's willingness to issue defaultable debt and face default risk.

We also intend to study the introduction of Eurobonds in conjunction with fiscal rules that limit the government's ability to borrow using defaultable debt. Fiscal rules are likely to play a central role in fiscal management for currency unions, as exemplified by Europe's new Fiscal Compact. We plan to study how Eurobonds could alter the optimal design of fiscal rules (Hatchondo et al., 2011 study fiscal rules in an environment without Eurobonds).

The rest of the article proceeds as follows. Section 2.2 presents the model. Section 2.3 presents a preliminary parameterization. Section 2.4 introduces preliminary results.

2.2   THE MODEL

We assume that the government can issue both defaultable and non-defaultable debt. Issuances of non-defaultable debt, henceforth, Eurobonds, are subject to an exogenous borrowing constraint. The government cannot commit to future (default and borrowing) decisions. Thus, one may interpret this environment as a game in which the government making decisions in period t is a player who takes as given the (default and borrowing) strategies of other players (governments) who will decide after t. We focus on Markov Perfect Equilibrium. That is, we assume that in each period, the government's equilibrium default and borrowing strategies depend only on payoff-relevant state variables.

The government has preferences given by

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right),$$

where $\mathbb{E}$ denotes the expectation operator, β denotes the subjective discount factor, and c_t represents consumption of private agents. The utility function is strictly increasing and concave.

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The timing of events within each period is as follows. First, the government learns its income. After that, the government chooses whether to default on its (defaultable) debt. Before the period ends, the government may change its debt positions, subject to the constraints imposed by its default decision.

The economy's endowment of the single tradable good is denoted by $y \in Y \subset \mathbb{R}_{++}$. This endowment follows a Markov process.

As in Hatchondo and Martinez [2009a] and Arellano and Ramanarayanan [2012], we assume that a defaultable bond issued in period t promises an infinite stream of coupons, which decreases at a constant rate δ. In particular, a defaultable bond issued in period t promises to pay $(1-\delta)^{j-1}$ units of the tradable good in period $t+j$, for all $j \geq 1$. Hence, debt dynamics can be represented as follows:

$$b_{t+1} = (1-\delta)b_t + i_t,$$

where b_t is the number of defaultable coupons due at the beginning of period t, and i_t is the number of defaultable bonds issued in period t.

Each (non-defaultable) Eurobond is a promise to deliver one unit of the good in the next period. There is an limit to the number of Eurobonds the government can issue. This limit may depend on whether the government is in default, the endowment, and the levels of both defaultable and non-defaultable debt.

Bonds are priced in a competitive market inhabited by a large number of foreign investors. Thus, bond prices are pinned down by the foreign investors' zero-expected-profit condition. Foreign investors are risk-neutral and discount future payoffs at the rate r.

When the government defaults, it does so on all current and future defaultable debt obligations. This is consistent with the observed behavior of defaulting governments

and it is a standard assumption in the literature.[1] As in most previous studies, we also assume that the recovery rate for debt in default (i.e., the fraction of the loan lenders recover after a default) is zero.[2]

A default event triggers exclusion from the defaultable-debt market for a stochastic number of periods. Furthermore, income is given by $y - \phi^d(y)$ in every period in which the government is excluded from the defaultable-debt market. Starting the first period after a default, the government regains access to debt markets with constant probability $\psi^d \in [0, 1]$.

2.2.1 RECURSIVE FORMULATION

We now describe the recursive formulation of the government's optimization problem. Let e denote the number of Eurobonds the government must pay, and $\bar{e}^R(b, e, y)$ and $\bar{e}^D(e, y)$ denote the limit to the number of Eurobonds the government can issue when it is not in default and when it is in default, respectively. Let V denote the value function of a government that is not currently in default. For any defaultable bond price function q, the function V satisfies the following functional equation:

$$V(b, e, y) = \max \left\{ V^R(b, e, y), V^D(e, y) \right\}, \tag{2.1}$$

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[1]Sovereign debt contracts often contain an acceleration clause and a cross-default clause. The first clause allows creditors to call the debt they hold in case the government defaults on a payment. The cross-default clause states that a default in any government obligation constitutes a default in the contract containing that clause. These clauses imply that after a default event, future debt obligations become current.

[2]Yue [2010a] and Benjamin and Wright [2008b] present models with endogenous recovery rates.

where the government's value of repaying is given by

$$V^R(b, e, y) = \max_{b' \geq 0, e' \geq 0, c} \left\{ u(c) + \beta \mathbb{E}_{y'|y} V(b', e', y') \right\}, \qquad (2.2)$$

subject to

$$c = y - b - e + q(b', e', y) \left[b' - (1 - \delta)b \right] + \frac{e'}{1 + r},$$

$$e' \leq \bar{e}^R(b, e, y).$$

The value of defaulting is given by:

$$V^D(e, y) = \max_{e' \geq 0, c} u(c) + \beta \mathbb{E}_{y'|y} \left[(1 - \psi^d) V^D(e', y') + \psi^d V(0, e', y') \right], \qquad (2.3)$$

subject to

$$c = y - \phi^d(y) - e + \frac{e'}{1 + r},$$

$$e' \leq \bar{e}^D(e, y).$$

The solution to the government's problem yields decision rules for default $\hat{d}(b, e, y)$, next-period defaultable debt $\hat{b}(b, e, y)$, next-period Eurobonds $\hat{e}^R(b, e, y)$ and $\hat{e}^D(e, y)$, and consumption $\hat{c}^R(b, e, y)$ and $\hat{c}^D(e, y)$. The default rule $\hat{d}(\cdot)$ is equal to 1 if the government defaults, and is equal to 0 otherwise. In a rational expectations equilibrium (defined below), investors use these decision rules to price debt contracts. Because investors are risk neutral, the bond-price function solves the following functional equation:

$$q(b', e', y)(1 + r) = \mathbb{E}_{y'|y}[1 - \hat{d}(b', e', y')][1 + (1 - \delta)q(b'', e'', y')], \qquad (2.4)$$

where

$$b'' = \hat{b}(b', e', y')$$

$$e'' = \hat{e}^R(b', e', y')$$

Equation (2.4) indicates that in equilibrium, for a risk-neutral investor, the value of selling a defaultable bond today and investing in a risk-free asset (left-hand side of

equation (2.4)) has to be equal to the expected value of keeping the bond (right-hand side of equation (2.4)). If the investor keeps the bond and the government does not default next period, he first receives a one unit coupon payment and then sell the bonds at market price, which is equal to $(1 - \delta)$ times the price of a bond issued next period.

2.2.2 RECURSIVE EQUILIBRIUM

A *Markov Perfect Equilibrium* is characterized by

1. a set of value functions V, V^R and V^D,

2. rules for default $\hat{d}$, next-period defaultable debt $\hat{b}$, next-period Eurobonds $\hat{e}^R$ and $\hat{e}^D$, and consumption $\hat{c}^R$ and $\hat{c}^D$,

3. and a bond price function q,

such that:

i. given a bond price function q; $\left\{V, V^R, V^D, \hat{d}, \hat{b}, \hat{e}^R, \hat{e}^D, \hat{c}^R, \hat{c}^D\right\}$ solve the Bellman equations (2.1), (2.2), and (2.3).

ii. given policy rules $\left\{\hat{d}, \hat{b}, \hat{e}^R\right\}$, the bond price function q satisfies condition (2.4).

2.3 PRELIMINARY PARAMETERIZATION

We first solve the model without Eurobonds $(\bar{e}^R(b, e, y) = \bar{e}^D(e, y) = 0)$ and then study the effects of introducing Eurobonds under different assumptions for $\bar{e}^R$ and $\bar{e}^D$. The utility function displays a constant coefficient of relative risk aversion form, i.e.,

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \text{ with} \gamma \neq 1.$$

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The endowment process follows:

$$\log(y_t) = (1 - \rho)\,\mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$, and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$.

Following Arellano [2008a], we assume the cost of defaulting increases more than proportionally with income. In particular, as in Chatterjee and Eyigungor [2012], we assume a quadratic loss function for income during a default episode $\phi^d(y) = d_0 y + d_1 y^2$. This is a property of the endogenous default cost in Mendoza and Yue [2012a] and, as shown by Chatterjee and Eyigungor [2012], allows the equilibrium default model to match the behavior of the sovereign spread (i.e., the difference between the sovereign bond yield and the risk-free interest rate) in the data. Since sovereign defaults are associated with disruptions in the availability of private credit, it is natural to assume that the cost of these events is higher in good times when investment financed by credit is more productive.

Table 2.1 presents the benchmark values given to all parameters in the model. A period in the model refers to a quarter. The coefficient of relative risk aversion is set equal to 2, and the risk-free interest rate is set equal to 1 percent. These are standard values in quantitative business cycle and sovereign default studies. As in Arellano [2008a], we assume that the probability of regaining access to capital markets (ψ) is 0.282.

We choose other parameter values using as a reference European economies that are paying a significant sovereign premium. Parameter values for the endowment process are consistent with the lower GDP volatility of these economies compared with emerging economies.[3] We set $\delta = 3.4\%$. With this value, bonds have an average

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[3]See for instance, Alvarez et al. [2011]. For example, Chatterjee and Eyigungor [2012] calibrate a similar model using as a reference Argentina and assume $\rho = 0.95$ and $\sigma_\epsilon = 2.7\%$.

Table 2.1: Parameter Values.

Risk aversion	γ	2
Risk-free rate	r	1%
Probability of reentry after default	ψ^d	0.282
Income autocorrelation coefficient	ρ	0.85
Standard deviation of innovations	σ_ϵ	1.5%
Mean log income	μ	$(-1/2)\sigma_\epsilon^2$
Debt duration	δ	0.0341
Discount factor	β	0.961
Income cost of defaulting	d_0	-0.69
Income cost of defaulting	d_1	1.017

duration of 5 years in the simulations, which is consisted with the duration of sovereign debt for European countries.[4]

We need to calibrate the value of three other parameters: the discount factor β, and the parameters of the income cost of defaulting d_0 and d_1. As Chatterjee and Eyigungor [2012], we calibrate these parameter values targeting the mean and standard deviation of the sovereign spread, and the mean debt level. Compare with emerging economies, European economies that are paying a significant sovereign premium have higher debt levels, pay lower spreads, and face spreads that are less volatile. We target (for the economy without Eurobonds, i.e., with $\bar{e}^R(b, e, y) = \bar{e}^D(e, y) = 0$) a mean debt-to-income ratio of 46%, a mean spread of 3%, and a standard deviation of the spread of 0.7%.[5]

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[4]We use the Macaulay definition of duration that, with the coupon structure in this paper, is given by $D = \frac{1+r^*}{\delta + r^*}$, where r^* denotes the constant per-period yield delivered by the bond.

[5]For example, Chatterjee and Eyigungor [2012] calibrate a similar model using as a reference Argentina and target a mean debt-to-income ratio of 7%, a mean spread of 8.15%, and a standard deviation of the spread of 4.4%.

In the simulations, in order to compute the sovereign spread implicit in a bond price, we compute the yield i an investor would earn if it holds the bond to maturity and no default is declared. This yield satisfies

$$q_t = \sum_{j=1}^{\infty} \frac{(1-\delta)^{j-1}}{(1+i)^j}.$$

The sovereign spread is the difference between the yield i and the risk-free rate r. We report the annualized spread

$$r_t^s = \left(\frac{1+i}{1+r}\right)^4 - 1.$$

Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate, i.e., $b'(\delta + r)^{-1}$.

2.4   RESULTS

We solve the model using spline interpolation over debt levels and linear interpolation over endowment levels. Hatchondo et al. [2010] discuss the advantage of using interpolation in models of equilibrium default. As discussed by Krusell and Smith [2003], there may be multiple Markov perfect equilibria in infinite-horizon economies. In order to avoid this problem, we solve for the equilibrium of the finite-horizon version of our economy. We then increase the number of periods of the finite-horizon economy until the value and bond-price functions for the first and second periods of this economy are sufficiently close. We use the first-period equilibrium functions as the infinite-horizon-economy equilibrium functions.

2.4.1   IMMEDIATE EFFECT OF THE INTRODUCTION OF EUROBONDS ON DEFAULT RISK

Table 2.2 shows that introducing Eurobonds may reduce the spread on defaultable sovereign bonds significantly, and that introducing Eurobonds reduces the spread

more when default risk is higher. We first solve the model without Eurobonds $(\bar{e}^R(b, e, y) = \bar{e}^D(e, y) = 0)$. Then, we measure the effect on the sovereign spread of an unanticipated announcement explaining that from now on the government will be able to issue Eurobonds. In particular, we assume it is announced that the government can issue Eurobonds for up to 10 percent of trend income if it is not in default. When in default, the government can rollover existing Eurobonds but cannot increase its level of Eurobond debt. That is, it is announced that the constraints on Eurobond levels changes from $\bar{e}^R(b, e, y) = \bar{e}^D(e, y) = 0$ to $\bar{e}^R(b, e, y) = 0.4$ and $\bar{e}^D(e, y) = e$.

Table 2.2 presents results for three states with different levels of the pre-Eurobond spread, which reflect different levels of default risk. For all cases the debt level is equal to 46 percent of trend income. The medium, highest, and lowest risk cases correspond to income levels equal to the mean level, and two standard deviations below and above the mean level, respectively.

Table 2.2: Effects of the introduction of Eurobonds

	Highest risk	Medium risk	Lowest risk
Spread before the introduction of Eurobonds	6.0%	3.7%	2.5%
Spread after the introduction of Eurobonds	3.2%	2.1%	1.9%
Welfare gain from the introduction of Eurobonds	1.2%	1.1%	1.0%

The effects of introducing Eurobonds on the sovereign spread presented in Table 2.2 are computed before the government takes any action. Thus, these effects reflect the lenders' expectations about future declines in defaultable debt. The introduction of Eurobonds facilitates servicing defaultable debt and thus reduces default risk. Figure 2.1 shows how, after the introduction of Eurobonds, the stock of defaultable debt declines while the stock of Eurobonds increases.
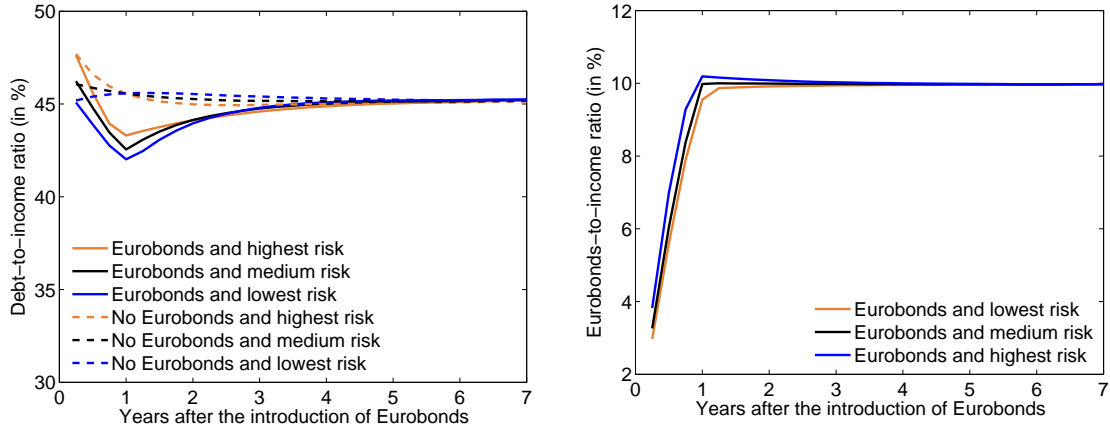
Figure 2.1: Mean defaultable-debt-to-income ratio $(\frac{b'/(\delta+r)}{4y})$ and Eurobond-to-income $(\frac{e'/(1+r)}{4y})$ ratio during transitions that follow the introduction of Eurobonds, for samples without defaults.

Figure 2.2 presents the government's equilibrium default decision in the economy without Eurobonds and in the economy with Eurobonds when the initial level of Eurobonds is equal to zero (i.e., when $e = 0$). The figure shows that the introduction of Eurobonds could help avoid a default: there is a set of combinations of income and defaultable debt levels for which the government would default if Eurobonds are not introduced but would not default after Eurobonds are introduced. Furthermore, there is no combination of income and defaultable debt levels for which the government would default if Eurobonds are introduced but would not default if Eurobonds are not introduced.
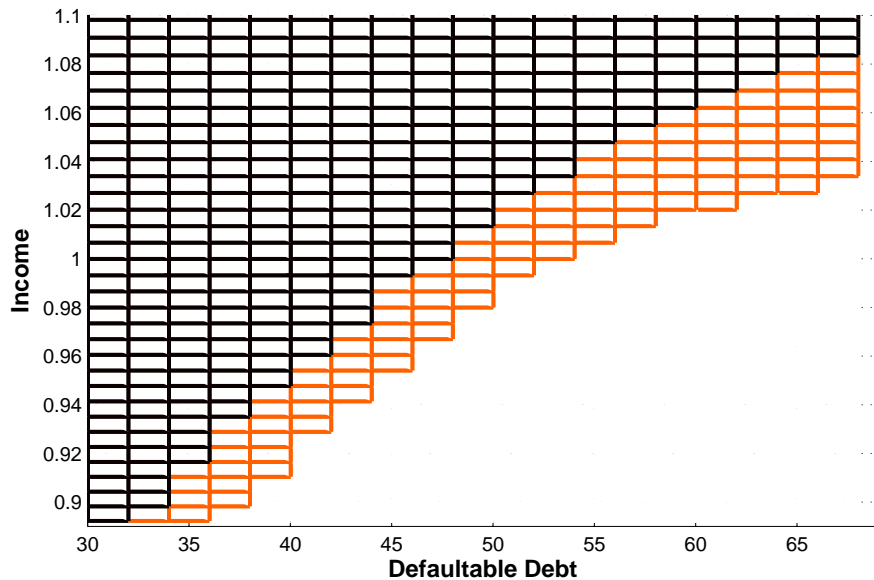
Figure 2.2: Government's equilibrium default decision in the economy without Eurobonds and in the economy with Eurobonds when the initial level of Eurobonds is equal to zero (i.e., when $e = 0$). The figure presents combinations of income and defaultable debt levels for which: (i) the government would not default with or without Eurobonds (black area), (ii) the government would default only if Eurobonds are not introduced (orange area), and (iii) the government would default with or without Eurobonds (white area). (There is no combination of income and defaultable debt levels for which the government would default only if Eurobonds are introduced.)

2.4.2 WELFARE GAINS

Table 2.2 shows that welfare gains from introducing Eurobonds may be significant. We measure welfare gains as the constant proportional change in consumption that would leave a consumer indifferent between living in the economy without Eurobonds and living in the economy with Eurobonds. This consumption change is given by

$$\left(\frac{V^E(b, 0, y)}{V^N(b, y)} \right)^{\frac{1}{1-\gamma}} - 1,$$

where V^E and V^N denote the value functions with Eurobonds ($\bar{e}^R(b, e, y) = 0.4$ and $\bar{e}^D(e, y) = e$) and without Eurobonds ($\bar{e}^R(b, e, y) = \bar{e}^D(e, y) = 0$), respectively—note that in the latter economy we do not need e as a state variable. Thus, a positive welfare gain means that agents prefer the economy with Eurobonds. In this economy, agents benefit from access to cheaper financing through non-defaultable debt. Furthermore, in Table 2.2 welfare gains are larger for initial states with more risk. The higher the initial risk, the more attractive default-free financing is compared with risky financing.

2.4.3 Long-run effect of the introduction of Eurobonds on default risk

Figure 2.3 shows that the effect of the introduction of Eurobonds on default risk (and thus on spreads) declines over time. In particular, the introduction of Eurobonds does not have a significant effect on default risk after four years. As illustrated in Figure 2.1, the first year after the introduction of Eurobonds, the government uses Eurobond financing to reduce the level of its defaultable debt. But after one year, Eurobond financing is exhausted and the government starts increasing the level of defaultable debt. Four years after the introduction of Eurobonds, Eurobonds do not have a significant effect on the level of defaultable debt.

The long-run negligible effect of the availability of Eurobonds on default risk is also illustrated in Table 2.3. The table reports simulation results for the economies with and without Eurobonds (i.e., with $\bar{e}^R(b, e, y) = 0.4$ and $\bar{e}^D(e, y) = e$, and with $\bar{e}^R(b, e, y) = \bar{e}^D(e, y) = 0$). Table 2.3 also shows that the simulations without Eurobonds match the calibration targets reasonably well.

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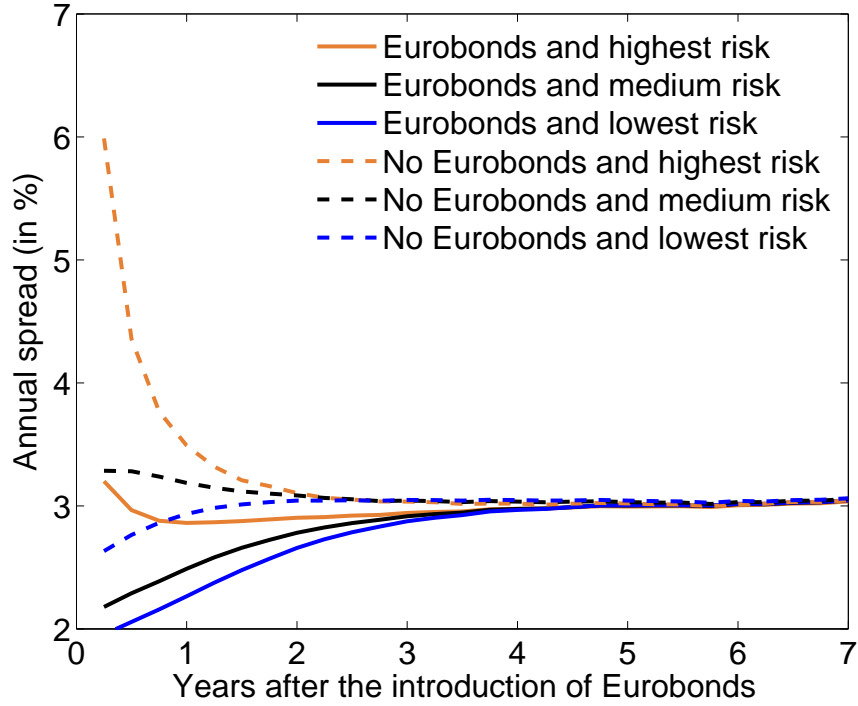
Figure 2.3: Spread during transitions that follow the introduction of Eurobonds, for samples without defaults.

2.4.4 THE LEVEL OF EUROBOND ISSUANCES

In this subsection we document how the gains from introducing Eurobonds change when we change the limit of Eurobonds issuances. In particular, we assume that the government can issue Eurobonds for up to 15 percent of trend income if it is not in default. When in default, the government can rollover existing Eurobonds but cannot increase its level of Eurobond debt. That is, $\bar{e}^R(b, e, y) = 0.6$ and $\bar{e}^D(e, y) = e$. Table 2.4 presents the effects of introducing Eurobonds under these constraints, for the three initial states considered in Table 2.2. The comparison of results presented

Table 2.3: Simulation Results

	With Eurobonds	Without Eurobonds
Mean Defaultable-debt-to-GDP (%)	46	46
Mean r_s	3.03	3.04
$\sigma\left(r_s\right)$	0.69	0.69
Defaults per 100 years	2.30	2.32
Mean Eurobond-debt-to-GDP (%)	10	0

Note: The standard deviation of x is denoted by $\sigma\left(x\right)$. Moments correspond to the mean value of each moment in 250 simulation samples, with each sample including 120 periods (30 years) without a default episode, and starting at least five years after a default.

in Tables 2.2 and 2.4 shows that the spread reduction triggered by the introduction of Eurobonds is larger when the government will be able to issue more Eurobonds. The extra spread reduction gained by increasing the limit to Eurobond issuances is more important when there is more default risk. In all cases, a 50 percent increase in the Eurobond limit produces an increase in the welfare gain from introducing Eurobonds close to 50 percent.

Table 2.4: Effects of the introduction of Eurobonds with a limit of 15 percent

	Highest risk	Medium risk	Lowest risk
Spread before the introduction of Eurobonds	6.0%	3.7%	2.5%
Spread after the introduction of Eurobonds	2.4%	1.8%	1.7%
Welfare gain from the introduction of Eurobonds	1.7%	1.6%	1.5%

2.4.5 EUROBOND ISSUANCES DURING DEFAULTS

In previous subsections, during defaults, we only allow the government to issue enough Eurobonds to rollover previous Eurobond issuances: $\bar{e}^D(e, y) = e$. This

restriction on Eurobond issuances during default is important for the immediate effect of the introduction of Eurobonds on default risk. Without this restriction, introducing Eurobonds could trigger a default. Part of the cost of defaulting is loosing access to credit. This cost would be mitigated if the government can issue Eurobonds after defaulting, making defaults a more attractive option for the government.

Suppose there is no additional restriction for Eurobond issuances during default: $\bar{e}^R(b, e, y) = \bar{e}^D(e, y) = 0.4$. Figure 2.4 shows that the introduction of Eurobonds without this additional restriction could trigger a default: there is a set of combinations of income and defaultable debt levels such that the government would default if Eurobonds are introduced but would not default if Eurobonds are not introduced. Furthermore, there is no combination of income and defaultable debt levels such that the government would default if Eurobonds are not introduced but would not default if Eurobonds are introduced.

This restriction on Eurobond issuances during default is not important for the long-run effect of the introduction of Eurobonds on default risk. This is shown in Table 2.5, which presents simulation results with and without this restriction.

2.4.6 Debt buybacks

In order to reflect the intend of some of the existing Eurobond proposals, we next discuss the effects of introducing Eurobonds under the assumption that the government first issues Eurobonds up to the imposed limit and uses the proceeds of this issuance to finance a buyback of defaultable debt. Suppose before the introduction of Eurobonds, the economy is characterized by (b, y), and that the government does not default in the period in which implements the Eurobond proposal. Let $\bar{e}^R$ denote the initial Eurobond issuance. The proceeds for this
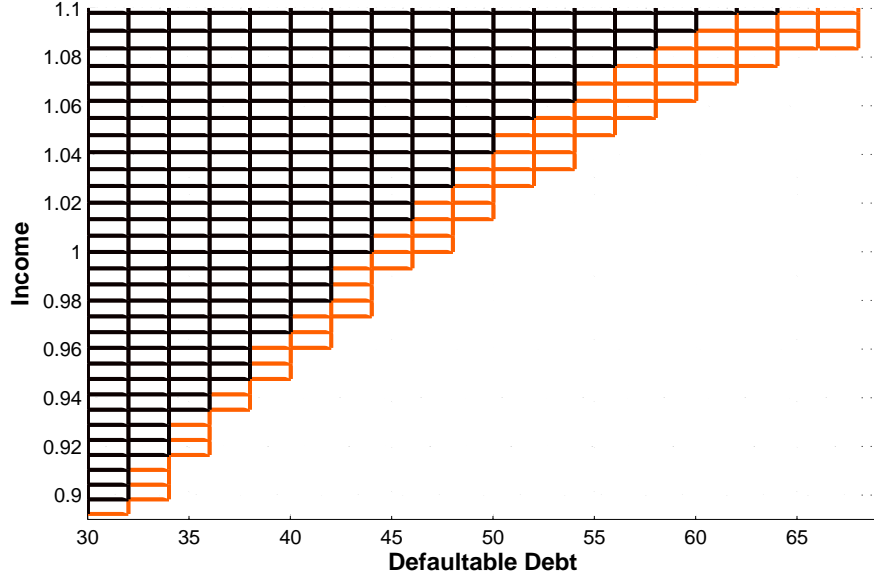
Figure 2.4: Government's equilibrium default decision in the economy without Eurobonds and in the economy with Eurobonds ($\bar{e}^R(b, e, y) = \bar{e}^D(e, y) = 0.4$) when the initial level of Eurobonds is equal to zero (i.e., when $e = 0$). The figure presents combinations of income and defaultable debt levels for which: (i) the government would not default with or without Eurobonds (black area), (ii) the government would default only if Eurobonds are introduced (orange area), and (iii) the government would default with or without Eurobonds (white area). (There is no combination of income and defaultable debt levels for which the government would default only if Eurobonds are not introduced.)

issuance are given by $\bar{e}^R/(1 + r)$. The government uses these proceeds to buy back defaultable bonds. Suppose the government cannot change its Eurobond and defaultable debt positions in the buyback period. The price at which a lender is willing to sell a defaultable bond, $q^B(b, \bar{e}^R, y)$ equals the resources the lender would obtain if it does not sell his bond:

$$q^B(b, \bar{e}^R, y) = 1 + (1 - \delta)q(b^B(b, \bar{e}^R, y), \bar{e}^R, y),$$

Table 2.5: Simulation results with and without restricting Eurobond issuances during defaults

Eurobond limit during defaults	$\bar{e}^D(e,y) = 0.4$	$\bar{e}^D(e,y) = e$
Mean Defaultable-debt-to-GDP (%)	46	46
Mean r_s	3.07	3.03
$\sigma(r_s)$	0.70	0.69
Defaults per 100 years	2.35	2.30
Mean Eurobond-debt-to-GDP (%)	10	10

Note: The standard deviation of x is denoted by $\sigma(x)$. Moments correspond to the mean value of each moment in 250 simulation samples, with each sample including 120 periods (30 years) without a default episode, and starting at least five years after a default.

where

$$b^B(b, \bar{e}^R, y) = b - \frac{\bar{e}^R}{(1+r)q^B(b, \bar{e}^R, y)}$$

denotes the level of defaultable debt after the buyback. We assume $\bar{e}^R(b, e, y) = 0.4$ and $\bar{e}^D(e, y) = e$.

Table 2.6 shows that with the buyback, introducing Eurobonds reduces even more the spread on defaultable sovereign bonds, indicating that the buyback would be the preferred option for creditors. This is not surprising since all Eurobond resources are used immediately for reducing the amount of defaultable debt. Welfare gains are very similar to the ones obtained without the buyback. The table presents results for the three states used for Table 2.2. As with the introduction of Eurobonds without the buyback, the spread reduction is larger when default risk is higher. Figure 2.5 shows how, after the buyback, both the stock of defaultable and the spread come back to the pre-buyback average levels.

Table 2.6: Effects of the introduction of Eurobonds with a defaultable-debt buyback

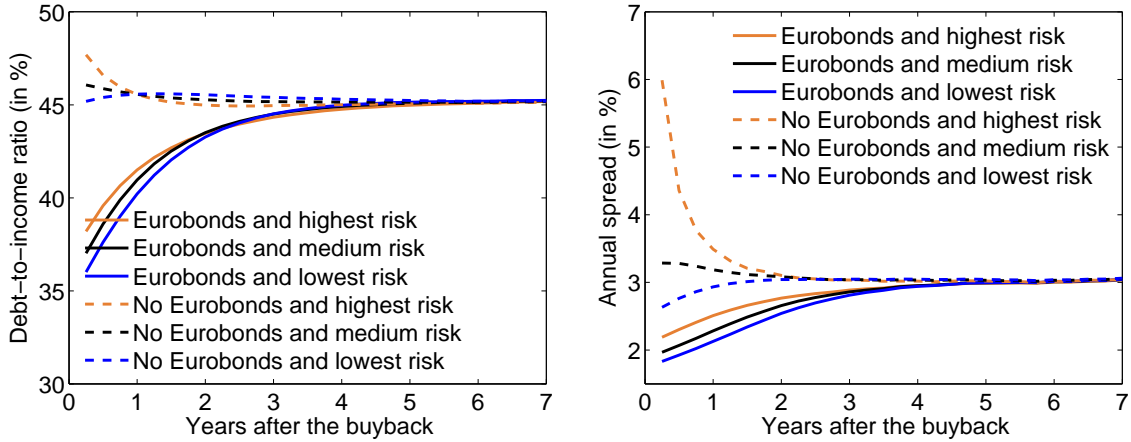|	Highest risk	Medium risk	Lowest risk
Spread before the buyback	6.0%	3.7%	2.5%
Spread after the buyback	2.2%	2.0%	1.8%
Defaultable debt bought back (% of trend income)	10.9%	10.8%	10.7%
Welfare gain from the buyback	1.1%	1.2%	1.2%

Figure 2.5: Mean defaultable-debt-to-income ratio $\left(\frac{b'/(\delta+r)}{4y}\right)$ and spread during transitions that follow the introduction of Eurobonds through a buyback of defaultable debt, for samples without defaults.

2.4.7 DEBT SWAPS WITHOUT BONDHOLDERS' CAPITAL GAINS

Table 2.6 shows that the buyback studied in the previous Subsection produces significant bondholders' capital gains as reflected in the spread decline caused by the buyback. The increase in bond prices caused by the buyback limits the government's ability to reduce its level of indebtedness: with Eurobonds issuances for 10 percent of trend annual income, the government reduces the level of defaultable debt by less than 11 percent of trend annual income.

This limitation of debt buybacks is well understood. Consequently, it is often argued that debt buybacks are only sensible if the government is able to obtain some compensation from bondholders. For instance, Bulow and Rogoff [1988] conclude that "Buybacks can be justified only if the country negotiates substantial concessions or compensation for undertaking the repurchase."

In this Subsection we study the effects of introducing Eurobonds through a debt buyback for which bondholders' compensate the government. In particular, we focus on the extreme case in which the government is able to capture all bondholders' capital gains. Subsection 2.4.6 presents the other extreme case in which bondholders enjoy all capital gains.

Suppose that, at the beginning of the debt swap period, the government extends a take-it-or-leave-it offer to bondholders of swapping each existing bond for $\bar{e}^R/b$ Eurobonds to be paid next period plus b^S/b defaultable bonds that start paying coupons in the current period. If bondholders do not accept this offer, Eurobonds will not be introduced. With these assumptions, the government can choose b^S to make bondholders indifferent between accepting or not the offer and, thus, can capture all gains from introducing Eurobonds. For simplicity, we assume the government cannot borrow in the period of the debt swap. Therefore, the post-swap quantity of defaultable bonds b^S is such that

$$\frac{\bar{e}^R}{b(1+r)} + \frac{b^S[1 + (1-\delta)q(b^S, \bar{e}^R, y)]}{b} = 1 + (1-\delta)q^N(\hat{b}^N(b,y), y), \qquad (2.5)$$

where q^N and $\hat{b}^N$ denote the bond price and government's borrowing functions in the economy without Eurobonds (i.e., where $\bar{e}^R = \bar{e}^D = 0$ and, therefore, we do not need e as a state variable). The left-hand-side of equation (2.5) represents what the holder of one bond would obtain if he accepts the swap. The right-hand-side of equation (2.5) represents what the holder of one bond would obtain if he rejects the swap.

Table 2.7 shows that the reduction in defaultable debt levels is larger with the debt swap described above (i.e., a debt buyback that does not produce bondholders' capital gains) than with the debt buyback presented in Subsection 2.4.6. Gains from doing the swap instead of the buyback are larger when the level of risk is higher, which would imply larger bondholders' capital gains from the introduction of Eurobonds. In particular, for the highest risk case we study, the defaultable debt reduction almost double and the welfare gain more than double when Eurobonds are introduced with a swap instead than with a regular buyback. Figure 2.6 shows that, as in previous cases, both the stock of defaultable and the spread eventually come back to the pre-Eurobond average levels.

Table 2.7: Effects of the introduction of Eurobonds with a debt swap without bond-holders' capital gains

	Highest risk	Medium risk	Lowest risk
Spread before the swap	6.0%	3.7%	2.5%
Spread after the swap	1.8%	1.9%	1.8%
Reduction in defaultable debt (% of trend income)	19.6%	13.5%	12.2%
Welfare gain from the swap	2.3%	1.5%	1.5%

2.4.8 DEBT SWAPS AND DEFAULTABLE DEBT LIMITS

Previous Subsections show that default risk and spreads decline when Eurobonds are introduced but eventually go back to the pre-Eurobond levels. Eurobonds are initially use to reduce the level of defaultable debt but after Eurobond financing is exhausted the government starts issuing defaultable debt again. In this Subsection we study the effects of introducing Eurobonds together with a limit on defaultable debt.[6] We use a defaultable debt limit of 32.5 percent of trend income. With this limit, there is no default risk in the long run. We also assume that Eurobonds are

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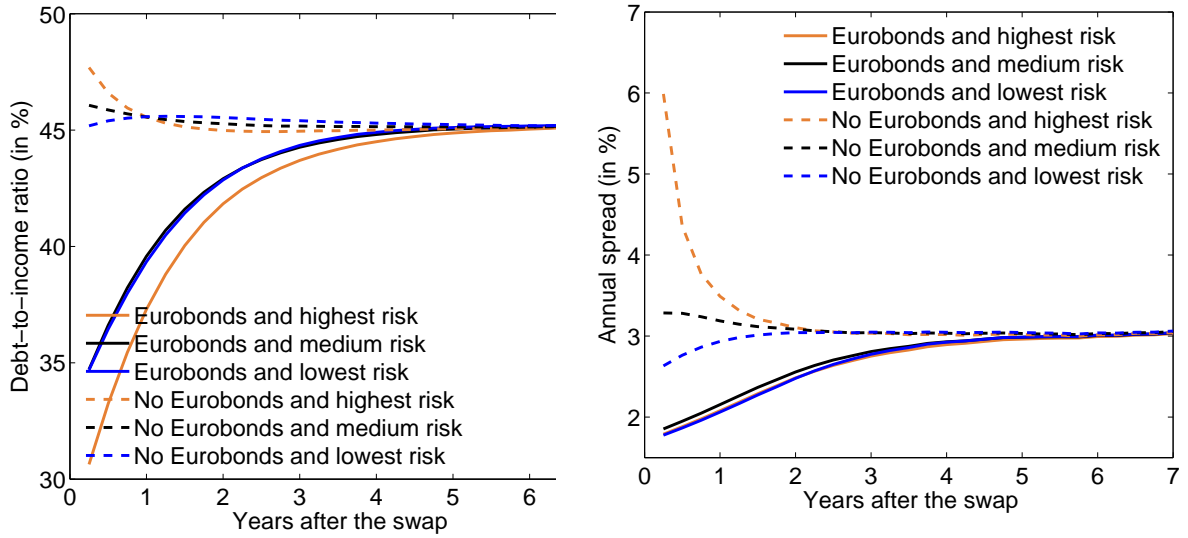[6]Hatchondo et al. [2011] study the gains from committing defaultable-debt limits (in environments without Eurobonds).

Figure 2.6: Mean defaultable-debt-to-income ratio $(\frac{b'/(\delta+r)}{4y})$ and spread during transitions that follow the introduction of Eurobonds through a debt swap without bondholders' capital gains, for samples without defaults.

introduced through a debt swap without bondholders' capital gains, as in Subsection 2.4.7. Table 2.8 shows that the default risk disappears immediately after the introduction of Eurobonds. Furthermore, the debt swap is sufficient to put the government below the defaultable-debt limit.

Table 2.8: Effects of the introduction of Eurobonds and defaultable debt limits

	Highest risk	Medium risk	Lowest risk
Spread before the swap	6.0%	3.7%	2.5%
Spread after the swap	0.0%	0.0%	0.0%
Reduction in defaultable debt (% of trend income)	30.5%	29.6%	24.0%

Cheap talk games through communication and learning are the subject of Lagunoff et al. [2012]. For setting out-of-equilibrium beliefs Athreya et al. [2012] provides a good discussion of why it is essential for computational purposes and Huggett [2003] provides a nice approach to monotone comparative dynamics.

Proof of proposition 1: *Given $q(b', \lambda, m) \in \mathcal{Q}(\mathcal{B} \times \Lambda \times \mathcal{M})$, there exist functions $v_I(b, y_-, z, \lambda, s)$, $v_I^d(y_-, z, \lambda, s)$, $v_I^{nd}(b, y_-, \lambda, s)$ where v_I, v_I^{nd} are continuous in q and v_I^d is continuous in z and collectively solve equations 1.5 - 1.7. Moreover, v_I^{nd} is strictly increasing in b and λ, v_I^d is strictly increasing in z.*

Proof - Let $\mathcal{V}$ be the set of continuous functions on $\mathcal{B} \times \mathcal{Y} \times \mathcal{Z} \times \Lambda \times \mathcal{S}$ whose values are in the interval $[\frac{u(0)}{1-\beta}, \frac{U}{1-\beta}]$. $(\mathcal{V}, \|.\|)$ is a complete metric space when $\mathcal{V}$ is equipped with sup norm $\|.\|_\infty$

Let $v_I^{nd}(b, y_-, \lambda, s; v_I, q)$ be the solution to 1.7 for $v_I \in \mathcal{V}$. The finiteness of $\mathcal{B}$ ensures a solution, $v_I^{nd}(b, y_-, \lambda, s; v_I, q)$ exists. v_I^{nd} is continuous in q for every b' where repayment is feasible since $c = y + b - q(b', \lambda, m)b'$ is continuous in q and u is continuous in c. Therefore, $v_I^{nd}(b, y_-, \lambda, s; v_I, q)$ is continuous in q.

Let $v_I^d(y_-, z, \lambda, s; v_I)$ be the solution to 1.6 for $v_I \in \mathcal{V}$. Equation 1.6 defines a contraction mapping in v_I^d with modulus $\beta\eta$ so the solution exists. It is continuous in z because $c = y + z$ is continuous in z and u is continuous in c. Next, define the operator $T_I(v_I)$ that gives the maximum life-time utility by

$$T_I(v_I)(b, y_-, z, \lambda, s; q) = max \left\{ \max_{\substack{m \in M, b' \in \mathcal{B} \\ s.t.\ c = y - q(b', \lambda, m)b' + b}} \left\{ E\big(u(c)\big) + \beta E_{(y, s', z'|y_-, s)} v_I(b', y, z', \lambda', s'; q) \right\}, \right.$$

$$\left. E(u(y+z)) + \beta E_{(y, s', z'|y_-, s)} \Big[\eta v_I(0, y, z'\lambda', s'; q) + (1-\eta)v_I^d(y, z', \lambda, s'; v_I) \Big] \right\} \quad (A.1)$$

on the space of functions $\mathcal{V}$. $\mathcal{B}$ is a finite set and the Theorem of Maximum implies that the operator T_I is continuous. Furthermore, both $v_I^{nd}(b, y_-, \lambda, s; v_I, q)$ and $v_I^d(y_-, z, \lambda, s; v_I) \in [\frac{u(0)}{1-\beta}, \frac{U}{1-\beta}]$.

First show that the operator is monotone. Let v_I^1, $v_I^2 \in \mathcal{V}$ such that $v_I^1 \geq v_I^2$. Then

$$T(v_I^1)(b, y_-, z, \lambda, s; q) = max \left\{ \max_{m \in M, b' \in \mathcal{B}} \left\{ E\big(u(c)\big) + \beta E_{(y,s',z'|y_-,s)} v_I^1(b', y, z', \lambda', s'; q) \right\}, \right.$$
$$\text{s.t. } c = y - q(b', \lambda, m) b' + b$$

$$\left. E(u(y+z)) + \beta E_{(y,s',z'|y_-,s)} \left[\eta v_I^1(0, y, z', \lambda', s'; q) + (1 - \eta) v_I^d(y, z', \lambda, s'; v_I^1) \right] \right\}$$

$$\geq max \left\{ \max_{m \in M, b' \in \mathcal{B}} \left\{ E\big(u(c)\big) + \beta E_{(y,s',z'|y_-,s)} v_I^2(b', y, z', \lambda', s'; q) \right\}, \right.$$
$$\text{s.t. } c = y - q(b', \lambda, m) b' + b$$

$$\left. E(u(y+z)) + \beta E_{(y,s',z'|y_-,s)} \left[\eta v_I^2(0, y, z', \lambda', s'; q) + (1 - \eta) v_I^d(y, z', \lambda, s'; v_I^2) \right] \right\}$$

$$\geq T(v_I^2)(b, y_-, z, \lambda, s; q)$$

To show that $v_I^d(y_-, z, \lambda, s; v_I^1)$ is greater than $v_I^d(y_-, z, \lambda, s; v_I^2)$,

$$v_I^d(y_-, z, \lambda, s; v_I^1) = E(u(y+z)) + \beta E_{(y,s',z'|y_-,s)} \left[\eta v_I^1(0, y, z', \lambda', s'; q) \right.$$
$$\left. + (1 - \eta) v_I^d(y, z', \lambda, s'; v_I^1) \right] \implies$$
$$v_I^d(y_-, z, \lambda, s; v_I^1) - \beta(1 - \eta) E_{(y,s',z'|y_-,s)} v_I^d(y, z', \lambda, s'; v_I^1) = E(u(y+z)) +$$
$$\beta \eta E_{(y,s',z'|y_-,s)} v_I^1(0, y, z', \lambda', s'; q) \geq$$
$$v_I^d(y_-, z, \lambda, s; v_I^2) - \beta(1 - \eta) E_{(y,s',z'|y_-,s)} v_I^d(y, z', \lambda, s'; v_I^2) = E(u(y+z)) +$$
$$\beta \eta E_{(y,s',z'|y_-,s)} v_I^2(0, y, z', \lambda', s'; q)$$

Now I will show that the operator T satisfies the discounting property. To see this, note that for all $k \geq 0$,

$$v_I^{nd}(b, y_-, \lambda, s; v_I + k, q) = \max_{m \in M, b' \in \mathcal{B}} \left\{ E\big(u(c)\big) + \beta E_{(y,s',z'|y_-,s)} (v_I(b', y, z', \lambda', s'; q) + k) \right\}$$
$$\text{s.t. } c = y - q(b', \lambda, m) b' + b$$

$$= \max_{m \in M, b' \in \mathcal{B}} \left\{ E\big(u(c)\big) + \beta E_{(y,s',z'|y_-,s)} v_I(b', y, z', \lambda', s'; q) \right\} + \beta k$$
$$\text{s.t. } c = y - q(b', \lambda, m) b' + b$$

$$= v_I^{nd}(b', y_-, \lambda, s; q, v_I) + \beta k.$$

$$v_I^d(y_-, z, \lambda, s; v_I + k) = E(u(y+z)) + \beta E_{(y,s',z'|y_-,s)} \left[\eta(v_I(0, y, z', \lambda', s'; q) + k) \right.$$
$$+ (1 - \eta) v_I^d(y, z', \lambda, s'; v_I + k) \right]$$

$$= E(u(y+z)) + \beta E_{(y,s',z'|y_-,s)} \left[\eta v_I(0, y, z', \lambda', s'; q) \right.$$
$$+ (1 - \eta) v_I^d(y, z', \lambda, s'; v_I + k) \right] + \beta \eta k.$$

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and take the term $E_{(y,s',z'|y_-,s)}\left(v_I^d(y,z',\lambda,s';v_I+k)\right)$ to the left hand side,

$$v_I^d(y_-,z,\lambda,s;v_I+k) - \beta(1-\eta)E_{(y,s',z'|y_-,s)}\left(v_I^d(y,z',\lambda,s';v_I+k)\right)$$

$$= E(u(y+z)) + \beta\eta E_{(y,s',z'|y_-,s)}\left(v_I(0,y,z',\lambda',s';q)\right) + \beta\eta k$$

Therefore,

$$T(v_I+k) = max\{v_I^{nd}(b',y_-,\lambda,s;q,v_I) + \beta k, v_I^d(b',y_-,\lambda,s;v_I) + \frac{\beta\eta}{1-\beta(1-\eta)}k\}$$

We have that both β and $\eta \in (0,1)$, so the discounting property is satisfied. Therefore, T is a contraction mapping with modulus τ where $\tau = \max\{\frac{\beta\eta}{1-\beta(1-\eta)},\beta\}$. The existence of a unique solution to equation A.1 in $\mathcal{V}$ follows from the Contraction Mapping Theorem.

For strict monotonicity of $v_I^d(y_-,z,\lambda,s)$ with respect to z follows from z being strictly increasing, $c = y + z$, so c is increasing in z and u is strictly increasing in c and z is independently and identically distributed.

For strict monotonicity of $v_I^{nd}(b,y_-,\lambda,s)$ with regard to b, observe that for $b^1 < b^2$, we have $y + b^1 - q(b',\lambda,m)b^1 < y + b^2 - q(b',\lambda,m)b^2$ for a feasible b'. For strict monotonicity of $v_I^{nd}(b,y_-,\lambda,s)$ with regard to λ, observe that $\lambda' > \lambda$ from the updating rule $\mathcal{F}$ whenever $m = s$ and observe that state inference function $\pi(m)$ is higher as λ goes up which decreases the default probabilities defined in equation 1.10. Hence the price of the bonds would go up. For $\lambda^1 < \lambda^2$, we have $y + b - q(b',\lambda^1,m)b' < y + b - q(b',\lambda^2,m)b'$. $\square$

Proof of proposition 2: *If defaulting is optimal for debt level of b^1 for some values of output y and reputation λ, then it would be optimal to default as well for a debt level of b^2 for the same output y and reputation λ for all $b^2 < b^1$, that is; if $b^2 < b^1$, $D(b^2,\lambda) \geq D(b^1,\lambda)$.*

Proof - To get a contradiction, for some pair y,λ,s,z suppose the following holds: $D(b^2,\lambda) < D(b^1,\lambda)$. Then $D(b^2,\lambda) = 0$ and $D(b^1,\lambda) = 1$. The former implies $v^{nd}(b^2,y_-,\lambda,s) \geq v^d_I(y_-,z,\lambda,s)$ and the latter implies $v^d_I(y_-,z,\lambda,s) > v^{nd}(b^1,y_-,\lambda,s)$. These two inequality imply $v^{nd}(b^2,y_-,\lambda,s) > v^{nd}(b^1,y_-,\lambda,s)$ which is a contradiction since value functions are monotonic in b, showed in the earlier proposition. $\square$

Proof of proposition 3: *Let $\underline{\lambda}$ and $\overline{\lambda} \in [0,1]$ such that $\overline{\lambda} > \underline{\lambda}$. If it is optimal for the higher reputation government $\overline{\lambda}$ to default for given level of debt b, and state y, then it is also optimal for the lower reputation government $\underline{\lambda}$ to default too for the same level of debt holdings b, and state y, that is, $D(b,\overline{\lambda}) \subseteq D(b,\underline{\lambda})$.*

Proof - For all $\{y\} \in D(b,\overline{\lambda})$,

$$u(y+z) + \beta E\left(\eta v(0,y,z',\lambda',s') + (1-\eta)v_I^d(y,z',\lambda',s')\right) >$$

$$\left\{ E\big(u(y - q(b', \overline{\lambda}, m)b' + b)\big) + \beta E v_I(b', y, z', \lambda', s')\right\}.$$

Since

$$y - q(b', \overline{\lambda}, m)b' + b > y - q(b', \underline{\lambda}, m)b' + b,$$
$$u(y - q(b', \overline{\lambda}, m)b' + b) + \beta E v_I(b', y, z', \lambda', s') >$$
$$u(y - q(b', \underline{\lambda}, m)b' + b) + \beta E v_I(b', y, z', \lambda', s')\big\}$$

Hence;

$$u(y + z) + \beta E\Big[\eta v(0, y, z', \lambda', s') + (1 - \eta)v_I^d(y, z', \lambda', s')\Big] >$$
$$\left\{ E\big(u(y - q(b', \underline{\lambda}, m)b' + b)\big) + \beta E v_I(b', y, z', \lambda', s')\right\},$$

that is, $\{y\} \in D(b, \underline{\lambda})$ $\qquad\square$

Next, I will show that the operator T_I defining the value functions is continuous in q.

Lemma 1. $v_I^*(b, y_-, z, \lambda, s; q)$, $v_I^{d*}(y_-, z, \lambda, s; v_I^*)$, $v_I^{nd*}(b, y_-, \lambda, s; v_I^*, q)$ *is continuous in q.*

Proof. The continuity of the operator T_I in q comes from direct adaptation of Lemma A3 from Chatterjee and Eyigungor [2011] which essentially uses Theorem 4.3.6 of Hutson et al. [1980]. To show that $v_I^*(b, y_-, z, \lambda, s; q)$ is continous in q, it is sufficient to show that $T_I(v)(b, y_-, z, \lambda, s; q)$ is continuous in q. (see Hutson et al. [1980] page 117-18) To establish this, it is enough to show that $v_I^{nd}(b, y_-, \lambda, s)$ is continuous in q where it is already showed in Proposition 1. The continuity of the $v_I^{d*}(y_-, z, \lambda, s; v_I^*)$ in q follows from the contraction operator T_I defining $v_I^d(y_-, z, \lambda, s; v_I)$ depends on v_I via the $v_I^{nd}(0, y, \lambda, s)$ where the operator T_I is continuous in $v_I^{nd}(0, y, \lambda, s)$. Using the fact that $v_I^{nd*}(0, y', \lambda', s')$ is continuous in q and the application of Theorem 4.3.6 of Hutson et al. [1980], I can establish that $v_I^{d*}(y_-, z, \lambda, s; v_I^*)$ is continuous in q. $\qquad\square$

Following notation will be handy and save some space. $\hat{u}_I(\pi, s)$ stands for the expected value of type I government if it observed signal s and lenders believe that the true state in fact is 1 with probability π. So $q(1)$ is the value of the bond price when the true state in fact is H with probability π for a given state b' and λ.

$$\hat{u}_c(\pi, 1) = \gamma u_c(q(1), 1) + (1 - \gamma)u_c(q(1), 0),$$

$$\hat{u}_c(\pi, 0) = \gamma u_c(q(0), 0) + (1 - \gamma)u_c(q(0), 1),$$

$$\hat{u}_{nc}(\pi, 1) = \frac{1}{2}u_c(q(1), 1) + \frac{1}{2}u_c(q(1), 0),$$

$$\hat{u}_{nc}(\pi, 0) = \frac{1}{2}u_c(q(0), 0) + \frac{1}{2}u_c(q(0), 1).$$

$\Pi_I^C(s)$ stands for the net current gain to the type I government choosing a message 1 when it observes a signal s and $\Pi_I^R(s)$ stands for the expected reputational gain to the type I government choosing a message 0 when it observes a signal s, given that the lenders follow their optimal strategy, that is,

$$\Pi_c^C(s) = \hat{u}_c(\pi(1), s) - \hat{u}_c(\pi(0), s),$$

$$\Pi_{nc}^C(s) = \hat{u}_{nc}(\pi(1)) - \hat{u}_{nc}(\pi(0)).$$

$$\Pi_c^R(1) = \gamma[w_c(\lambda'(0, 1)) - w_c(\lambda'(1, 1))] + (1 - \gamma)[w_c(\lambda'(0, 0)) - w_c(\lambda'(1, 0))]$$

$$\Pi_c^R(0) = \gamma[w_c(\lambda'(0, 0)) - w_c(\lambda'(1, 0))] + (1 - \gamma)[w_c(\lambda'(0, 1)) - w_c(\lambda'(1, 1))]$$

$$\Pi_{nc}^R(s) = \frac{1}{2}[w_{nc}(\lambda'(0, 1)) - w_{nc}(\lambda'(1, 1)) + w_{nc}(\lambda'(0, 0)) - w_{nc}(\lambda'(1, 0))]$$

Thus a type I government would report 1 if $\Pi_I^C(s)$ exceeds $\Pi_I^R(s)$. Lenders optimal decision depends on how likely a government is willing to report 0 when it observes 0 signal.

In an environment in which competent government always tell the truth, what would be the best response of a non-competent government? Recall that non-competent type cannot draw any informative signal and would like to be perceived as a competent type. If there were no reputational cost of reporting H, the non-competent government would have an incentive to announce H each period.

Thus the non-competent government cannot always send an H message. Let's assume it chooses to report H every period. Then announcing L for any type regardless of the realized state would update the lenders' beliefs of government being a competent type with probability one. More precisely, it can be shown that the non-competent type would like to mimic the competent type and thus mixes. The proposed equilibrium strategy may be summarized as in the Table A.1.

	s = H	s = L
C gov't	H	L
NC gov't	ν	$1 - \nu$

Table A.1: Government's Strategies

Now given the borrower's message, what inferences will the lenders draw about the current state of the economy? If a message, say H, comes from a competent type, lenders will assign probability γ to state H; if it comes from a non-competent type, then lenders will assign probability $\frac{1}{2}$ to state H. Let $\pi(m)$ be the lenders' posterior belief that the actual state is H if message H is reported. By Bayes' rule,

$$\pi(m) = \frac{\lambda\theta_c(m|H,b') + (1-\lambda)\theta_{nc}(m|H,b')}{\lambda\theta_c(m|H,b') + (1-\lambda)\theta_{nc}(m|H,b') + \lambda\theta_c(m|L,b') + (1-\lambda)\theta_{nc}(m|L,b')} \tag{A.2}$$

where $\theta_I(m|y,b')$ is the probability that government type I (nc or c) sends a message m given income y, and debt b'. Equation 1.4 is well defined as long as the denominator is nonzero. I adopt the convention that $\pi(m) = \frac{1}{2}$ if

$$\theta_c(m|y=i,b') = \theta_{nc}(m|y=i,b') = 0, \text{ i} \in \{H, L\}.$$

Since γ is larger than $\frac{1}{2}$, higher the reputation of the government λ, higher the informativeness of the message anticipated by the lender. Informativeness of a

message will play a role on determining the bond prices. Observe that when the signal is uninformative, $\gamma = \frac{1}{2}$, or government's reputation λ is 0, messages do not carry out any information.

After given these strategies, what does the lender infer about the government's type? Let's suppose, for instance, the government announced an H message and an H income is realized. The probability of truth-telling government sends an H message if the true income is in fact H is γ (probability of observing an informative signal). Since a non-competent government cannot observe any informative signal, the probability that a non-competent government reports H when the true state is H is ν. Now by Bayes' rule, the posterior probability of the government being a competent type if it sends a message m and income y is realized for given level of debt b' will be

$$\mathcal{F}(b', y, \lambda, m) = \lambda' = \frac{\lambda \theta_c(m|y, b')}{\lambda \theta_c(m|y, b') + (1 - \lambda) \theta_{nc}(m|y, b')} \tag{A.3}$$

$$\lambda'(\lambda, m = 1, y = 1) = \lambda'(\lambda, 1, 1) = \frac{\lambda \gamma}{\lambda \gamma + (1 - \lambda)\nu}$$

$$\lambda'(\lambda, 0, 0) = \frac{\lambda \gamma}{\lambda \gamma + (1 - \lambda)(1 - \nu)}$$

$$\lambda'(\lambda, 1, 0) = \frac{\lambda(1 - \gamma)}{\lambda(1 - \gamma) + (1 - \lambda)\nu}$$

$$\lambda'(\lambda, 0, 1) = \frac{\lambda(1 - \gamma)}{\lambda(1 - \gamma) + (1 - \lambda)(1 - \nu)}$$

where $\theta_I(m|y, b)$ is the probability that government type I (nc or c) sends a message m given income y, and debt b'. Equation 1.3 is well defined as long as the denominator is nonzero. I adopt the convention that $\mathcal{F}(b', y, \lambda, m) = \lambda$ if

$$\theta_c(m|y = i, b') = \theta_{nc}(m|y = i, b') = 0, \text{ i} \in \{H, L\}.$$

Since $\gamma > \frac{1}{2}$, equation 1.3 implies

$$\lambda'(\lambda, 0, 0) > \lambda'(\lambda, 1, 1); \lambda'(\lambda, 0, 1) > \lambda'(\lambda, 1, 0)$$

Thus a competent government has a strict reputational incentive to tell the truth about its signal. Since a non-competent government is assumed not to receive any informative signals at all, it is useful to focus our discussion on the competent government.

So far, it was assumed that the competent government always told the truth. It is true that government will always announce H whenever it observes an H signal, since this will provide a higher bond prices and enhance its reputation. However, if it observes an L signal and announces an H message, its gain will be cheaper debt for the current period but its reputation will go down in the next period. Thus if its reputational concerns are sufficiently large, truth-telling will be consistent in equilibrium.

Proof of proposition 4: *A truthful Markov perfect Bayessian equilibrium exists.*
proof - The proof will be established similar to Morris [2001] as follows. Consider that some states of the world (b^*, y^*, z^*, s^*) occurs with a very low probability α. Suppose a strategy profile in which both governments always babbles unless (b^*, y^*, z^*, s^*) is drawn. I will establish that when (b^*, y^*, z^*, s^*) is drawn, the competent government will tell the truth and the non-competent government will mix and their strategies will be best responses to each other. Denote $\tilde{q}(\pi(m))$ as being the bond price with a message m.
Consider the following government strategy:

$$\theta_c(s|\lambda, b, y_-, z) = \left\{ \begin{array}{ll} \frac{1}{2} & \text{if } (b, y, z, s) \neq (b^*, y^*, z^*, s^*), \\ s & \text{if } (b, y, z, s) = (b^*, y^*, z^*, s^*). \end{array} \right.$$

and

$$\theta_{nc}(s|\lambda, b, y_-, z) = \left\{ \begin{array}{ll} \frac{1}{2} & \text{if } (b, y, z, s) \neq (b^*, y^*, z^*, s^*), \\ \nu & \text{if } (b, y, z, s) = (b^*, y^*, z^*, s^*). \end{array} \right.$$

where ν is shown in Table A.1. The best response for the lenders is:

$$\chi(m|b', \lambda, y_-) = \left\{ \begin{array}{ll} \tilde{q}(\frac{1}{2}) & \text{if } (b, y, z, s) \neq (b^*, y^*, z^*, s^*), \\ \tilde{q}(\pi(m)) & \text{if } (b, y, z, s) = (b^*, y^*, z^*, s^*). \end{array} \right.$$

The value function for the competent and non-competent government must satisfy $v_c = T[v_c]$ and $v_{nc} = T[v_{nc}]$ where

$$T_c[v_c] = (1-\alpha)[\frac{1}{2}\hat{u}_c(\frac{1}{2}, 1) + \frac{1}{2}\hat{u}_c(\frac{1}{2}, 1) + \beta v_c(b, y, z, \lambda, s)]$$

$$+\alpha\left\{\frac{1}{2}\hat{u}_c(\pi(1), 1) + \frac{1}{2}\hat{u}_c(\pi(0), 0)\right.$$

$$+\beta\left[\frac{1}{2}\gamma v_c(b^*, y^*, z^*, \frac{\lambda\gamma}{\lambda\gamma + (1-\lambda)\nu}, s^*) + \frac{1}{2}(1-\gamma)v_c(b^*, y^*, z^*, \frac{\lambda(1-\gamma)}{\lambda(1-\gamma) + (1-\lambda)(1-\nu)}, s^*)\right.$$

$$+\frac{1}{2}\gamma v_c(b^*, y^*, z^*, \frac{\lambda\gamma}{\lambda\gamma + (1-\lambda)(1-\nu)}, s^*) + \left.\left.\frac{1}{2}(1-\gamma)v_c(b^*, y^*, z^*, \frac{\lambda(1-\gamma)}{\lambda(1-\gamma) + (1-\lambda)\nu}, s^*)\right]\right\}$$

$$T_{nc}[v_{nc}] = (1-\alpha)[\frac{1}{2}\hat{u}_{nc}(\frac{1}{2}, 1) + \frac{1}{2}\hat{u}_{nc}(\frac{1}{2}, 1) + \beta v_{nc}(b, y, z, \lambda, s)]$$

$$+\alpha\left\{\frac{1}{2}\hat{u}_{nc}(\pi(1), 1) + \frac{1}{2}\hat{u}_{nc}(\pi(0), 1)\right.$$

$$+\beta\frac{1}{2}\left[\frac{1}{2}v_{nc}(b^*, y^*, z^*, \frac{\lambda\gamma}{\lambda\gamma + (1-\lambda)\nu}, s^*) + \frac{1}{2}v_c(b^*, y^*, z^*, \frac{\lambda(1-\gamma)}{\lambda(1-\gamma) + (1-\lambda)(1-\nu)}, s^*)\right.$$

$$+\frac{1}{2}v_{nc}(b^*, y^*, z^*, \frac{\lambda\gamma}{\lambda\gamma + (1-\lambda)(1-\nu)}, s^*) + \left.\left.\frac{1}{2}v_{nc}(b^*, y^*, z^*, \frac{\lambda(1-\gamma)}{\lambda(1-\gamma) + (1-\lambda)\nu}, s^*)\right]\right\}$$

$\square$

Both T_c and T_{nc} maps the set of strictly non-decreasing continuous functions on $\mathcal{V}$ continuously onto itself. By proposition 1, observe that monotonicity property and discounting property are satisfied. So T is a contraction mapping with modulus β, and there exists a unique fixed point.

Now we should verify the optimality of the proposed strategies, so we should show that any strategy is always optimal to babbling when $(b, y, s) = (b^*, y^*, s^*)$.

Let $g(\nu)$ be the net utility gain to the non-competent government from announcing 1 (rather than 0).

$$g(\nu) = \frac{1}{2}\left[u(y^* + b^* - q(b', \lambda, m = H)b')\right.$$

$$-u(y^* + b^* - q(b', \lambda, m = L)b')$$

$$+u(y^* + b^* - q(b', \lambda, m = H)b')$$

$$-u(y^* + b^* - q(b', \lambda, m = L)b')$$

$$+v_{nc}\left(b', y, z, \frac{\lambda_2(1-\gamma)}{\lambda_2(1-\gamma) + (1-\lambda_2)\nu}, s'\right)$$

$$-v_{nc}\left(b', y, z, \frac{\lambda_2(1-\gamma)}{\lambda_2(1-\gamma) + (1-\lambda_2)(1-\nu)}, s'\right)$$

$$+v_{nc}\left(b', y, z, \frac{\lambda_2\gamma}{\lambda_2\gamma + (1-\lambda_2)\nu}, s'\right)$$

$$\left.-v_{nc}\left(b', y, z, \frac{\lambda_2\gamma}{\lambda_2\gamma + (1-\lambda_2)(1-\nu)}, s'\right)\right]$$

$g(\nu)$ is strictly decreasing in ν, some terms in the above expression are weakly decreasing and some of them are strictly decreasing. There exists exactly one ν such that $g(\nu) = 0$ and denote $\tilde{\nu}$ for that value of ν.

Now find the competent government's incentive to tell the truth when it observes signal 0 under strategy profile $\sigma_c(0) = 0$, $\sigma_c(1) = 1$, $\sigma_{nc}(0) = \tilde{\nu}$ and $\sigma_{nc}(1) = 1\text{-}\tilde{\nu}$. The competent government will tell the truth if and only if

$$\gamma u(y^* + b^* - q(b', \lambda, m = L)b')$$

$$+(1-\gamma)u(y^* + b^* - q(b', \lambda, m = L)b')$$

$$-\gamma u(y^* + b^* - q(b', \lambda, m = H)b')$$

$$-(1-\gamma)u(y^* + b^* - q(b', \lambda, m = H)b')$$

$$+\gamma v_c\left(b', y, z, \frac{\lambda_2\gamma}{\lambda_2\gamma + (1-\lambda_2)(1-\nu)}, s'\right)$$

$$+(1-\gamma)v_c\left(b', y, z, \frac{\lambda_2(1-\gamma)}{\lambda_2(1-\gamma) + (1-\lambda_2)(1-\nu)}, s'\right)$$

$$-\gamma v_c\left(b', y, z, \frac{\lambda_2(1-\gamma)}{\lambda_2(1-\gamma) + (1-\lambda_2)\nu}, s'\right)$$

$$-(1-\gamma)v_c\left(b', y, z, \frac{\lambda_2\gamma}{\lambda_2\gamma + (1-\lambda_2)\nu}, s'\right) \geq 0$$

Proof of proposition 5: *(Characterization of Equilibrium Prices) In any MPBE: (i)* $q^*(b', \lambda, m)$ *is increasing in* b', *and increasing in* λ; *(ii)* $q^*(b', \lambda, m = H) \geq q^*(b', \lambda, m = L)$; *(iii) for some* $b^1 > b^2$ *and for some* $\overline{\lambda} > \underline{\lambda}$, $q^*(b^2, \overline{\lambda}, m) \geq q^*(b^1, \underline{\lambda}, m)$.

First let's prove the following lemma.

Lemma 2. *For given* $v_I^{nd}(b, y_-, \lambda, s)$ *and* $v_I^d(y_-, z, \lambda, s) \in \mathcal{V}$, *there exists an equilibrium bond price function* $q \in \mathcal{Q}$.

Proof. - For any $q^n \in \mathcal{Q}(\mathcal{B} \times \Lambda \times \mathcal{M})$, the operator T^q is defined as follows. For given q^n, use the operator $T(v)$ until the convergence to $(v^{nd})^n$

$$\psi^n(b, y, z, \lambda, s) = \begin{cases} 1 & \text{if } v^d > (v^{nd})^n, \\ 0 & \text{if } v^d \leq (v^{nd})^n. \end{cases}$$

From that I can now establish the default probability:

$$\delta^n(b', y_-, z, \lambda, m) = \sum_{D(b', \lambda')} \psi^n(b, y, z, \lambda, s)\pi(m) \tag{A.4}$$

and the set of bond prices when the government borrows can be obtained as follows:

$$q^n(b', \lambda, m) = \frac{1 - \delta(b', y_-, z, \lambda, m)}{1 + r^f}$$

Now I can define the sequence $\{q^n\}_{n=0}^{\infty}$ using the operator T^q iteratively beginning with an initial guess of $q^0(b', \lambda, m) = \frac{1}{1+r^f}$. I can now show it is monotone and bounded sequence in $\mathcal{Q}(\mathcal{B} \times \Lambda \times \mathcal{M})$. To see that it is monotone, observe that $q^1 \leq q^0$ when debt increases. As in Benjamin and Wright [2008a], the fixed points of the operator $T_I(v)$ are ordered, and thus we obtain an ordered sequence of $\delta^n(b', y_-, z, \lambda, m)$ which leads a monotonically decreasing sequence of q^n. It is clear that it is bounded below by zero; so q^n converges to a fixed point in $\mathcal{Q}$. □

Next, I can show the properties of the equilibrium prices.
(i) Proof (a) By proposition 2, $D(b', \lambda)$ is increasing in b', thus $q(b', \lambda, m)$ is increasing in b' *(b)* By proposition 3, $D(b', \lambda)$ is increasing in λ, thus $q(b', \lambda, m)$ is increasing in λ.
(ii) Proof - It is sufficient to show that default probabilities are increasing when a message is L for a given level of debt b and reputation λ, that is $\delta(b', y_-, z, \lambda, m = L) \geq \delta(b', y_-, z, \lambda, m = H)$. From the equation 1.10, default probabilities are defined as:

$$\delta(b', y_-, z, \lambda, m) = \sum_{D(b', \lambda')} \psi(b, y, z, \lambda, s)\pi(m)$$

Using the definition of state inference function, π from equation 1.4, I can show that when $m = L$, the probability of the actual state being L increases which in turn increases the likelihood of government falling into default sets, so $D(b, \lambda)$ goes up. *(iii) Proof* - Let $\overline{\lambda} > \underline{\lambda}$ and define b^{sup} as the maximum level of debt a high reputation government $\overline{\lambda}$ can hold where its existence is shown in Eaton and Gersovitz [1981]. It suffices to show that $q(b^2, \overline{\lambda}, m) = q(b^1, \underline{\lambda}, m)$ for some $b^1 > b^2$. It was showed that countries with lower reputation find it optimal to default for some debt b, whereas higher reputation countries do not, that is, there exists λ such that $D(b, \overline{\lambda}) = 0$ and $D(b, \underline{\lambda}) = 1$ for some b. For $\epsilon < 0$ such that $b + \epsilon \leq b^{sup}$, I can have a default correspondence such that $D(b + \epsilon, \overline{\lambda}) = 0$. $\qquad \square$

Let $\mathcal{Q}$ be the set of all nonnegative functions $q(b', \lambda, m)$ defined on $\mathcal{B} \times \Lambda \times \mathcal{M}$ and

let $\mathcal{C} \subset \mathcal{Q}$ be the subset of functions that are increasing in λ, b' and bounded above

by $\frac{1}{1+r^f}$. Now consider the mapping $\mathcal{H}(q)(b', \lambda, m) : \mathcal{C} \to \mathcal{Q}$ as

$$\mathcal{H}(q)(b', \lambda, m) = \frac{1 - \delta(b', y_-, z, \lambda, m)}{1 + r^f}$$

for some $\delta \in \Delta$ where δ is same as in the text and explained again below.

Default Indicator is defined as

$$\Psi_I(b, y, z, \lambda, s) = \begin{cases} 1 & \text{if } v_I^d > v_I^{nd}, \\ 0 & \text{if } v_I^d < v_I^{nd}, \\ [0,1] & \text{if } v_I^d = v_I^{nd}. \end{cases}$$

Using the indicator, define the probability of default $\Delta(b', y_-, z, \lambda, m)$ as the set of

all δ constructed as

$$\delta(b', y_-, z, \lambda, m) = \sum_{D_I(b', \lambda')} \psi_I(b, y, z, \lambda, s)\pi(m), \text{for some } \psi_I \in \Psi_I.$$

This is a closed interval contained in $[0, \frac{1}{1+r^f}]$, so it's a compact valued. A straight

forward adaptation of Lemma A8 from Chatterjee et al. [2007] shows that it is also

upper-hemi continuous.

Tables

Emerging Markets	Public Debt/GDP	Spread
Argentina	48.1	4.05
Brazil	66.1	31.12
Chile	9.19	3
Colombia	36	5.72
Hungary	80.4	2.7
Indonesia	77.4	6.2
Mexico	42.9	4.1
Pakistan	56.2	5.9
Peru	24.5	17.4
Philippines	44.7	4.5
Russia	11.7	4.8
Mean	42.0	7.0
Advanced Economies		
Australia	20.9	2.17
Canada	84	-0.23
Denmark	43.7	-0.38
France	82.4	-0.03
Germany	83.2	-0.42
Greece	143	9.22
Italy	119	1.43
Japan	220	-2.23
Netherlands	344	-0.24
New Zealand	134	2.56
Portugal	217	3.46
Singapore	99	2.5
Spain	154	2.07
US	93.5	0
Mean	81.4	1.1

Table B.1: (Public Debt/GDP, spreads (Dec, 2010)) Source: World Bank, Financial Times, IMF, Bloomberg. The list continues, please ask the author for the complete list. Some countries are cut to fit it in one page.

	# of Defaults (1501 - 1813)	# of Defaults (1814 - 2010)	Avg. Gross Ext. Debt / GDP (1990-2010)
Spain	6	7	84
France	8	0	107
Austria	1	6	132
Germany	4	3	99
Greece	na	5	85
Portugal	na	6	128
Netherlands	1	1	185
Italy	na	1	79
Japan	na	1	34
UK	na	1	252

Table B.2: This table shows that some of the advanced economies once serial defaulters. Source: Reinhart et al. [2010]
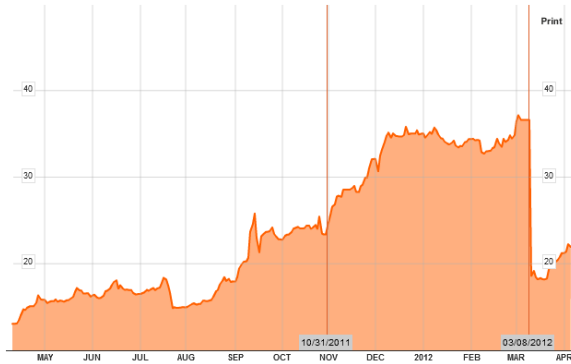
Figures

Figure B.1: 10-year Greek Bond Yields, Bloomberg. The above figure shows how cheap talk may affect the bond yields. European leaders and Greek Prime Minister George Papandreou agreed on a 50 percent debt haircut; however Papandreou put the deal at risk by announcing that he will take the deal to the referendum. Bond yields jumped right after the announcement until the deal had actually passed. This can be translated into that the international lenders lost their confidence on Mr. Papandreou after his announcement.
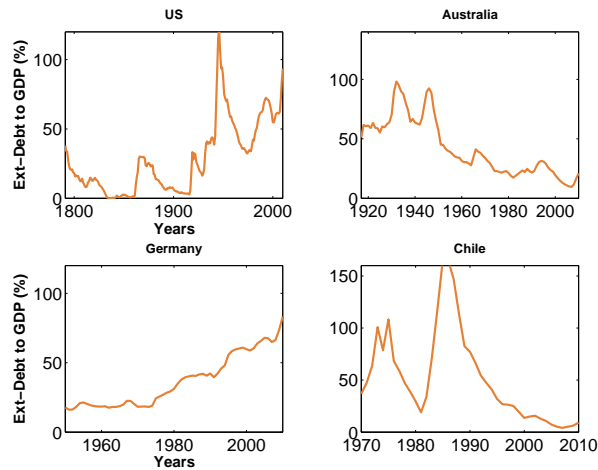

Figure B.2: This graph shows the selected countries' external debt-to-GDP ratios for some period. Advanced economies like the United States and Germany used to have low levels of debt at least for 30 years and after some time they begin increasing their debt holdings. Australia is another advanced economy and had high levels of debt, but Australia managed to take its debt levels down to 20 percent and kept it low for a long -time. I also provided the graph for Chile as well, simply because Reinhart and Rogoff [2009] talk about its graduation process. In fact, it is also in line with my argument. Governments need to hold low-levels of debt in order to convey their information truthfully and thus build up reputation. Remember that lenders would not anticipate the government's report if the government's level of debt is above the threshold level. Source: IMF

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