

ESSAYS ON QUANTITATIVE MACROECONOMICS OF DEFAULT

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Masaki Higurashi, M.A.

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Masaki Higurashi, M.A.

Dissertation Advisor: Jinhui Bai

ABSTRACT

This dissertation consists of two essays, which study agents' default under dynamic stochastic general equilibrium models with heterogeneous agents and financial intermediaries who charge a risk premium corresponding to the agent's default risk. We use computational tools to address the economic issues quantitatively in both chapters.

Chapter 1 studies consumer bankruptcy over the business cycle in the U.S. economy. The number of consumer bankruptcies is countercyclical, which is not obvious in economic models with forward-looking agents. This chapter constructs a heterogeneous-agent general equilibrium model with capital market imperfections and households' default option. Households face aggregate productivity shocks as well as idiosyncratic shocks to labor efficiency, discount factor, and liability loss. The key contribution of this chapter is that our quantitative model successfully replicates the countercyclical property and the high volatility of bankruptcy rate by introducing *surprising* aggregate shocks: households and financial intermediaries cannot observe a forthcoming aggregate shock before making a financial contract.

Chapter 2 studies firm defaults and corporate taxation reforms in the Japanese economy. The motivation of chapter 2 comes from the fact that standard corporate income taxes (SCIT) distort a firm's behavior. To eliminate tax advantages from borrowing over holding equity in SCIT, tax professionals have designed two polar reform proposals, namely the Comprehensive Business Income Tax (CBIT) and the Allowance for Corporate Equity (ACE). Economists traditionally favor an ACE system in the closed economy since it offsets a firm's investment distortion and it

obtains neutrality between debt and equity finance. However, this chapter will cast doubt on the superiority of ACE over CBIT in the closed economy quantitatively by constructing a heterogeneous-firm general equilibrium model with corporate income taxes, financial frictions, firms' default option and idiosyncratic shocks to firm's productivity. Once the firms' heterogeneity and default options are introduced, the neutrality of the ACE system no longer holds. We find that the ACE and CBIT tax reforms significantly decrease the debt ratio and the firm's default rate. We also find that, in our closed economy setup, welfare gains from the reforms are almost the same at 1.08% and 1.10%, respectively, in terms of consumption compensation.

INDEX WORDS: Default, Consumer Bankruptcy, Heterogenous Agents, Business Cycle, Countercyclical Property, Tax Reforms, ACE, CBIT, Welfare Gains

DEDICATION

This dissertation is dedicated to my wife Tomoko and my family. Thank you for your unwavering support and encouragement over the years.

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CHAPTER 1

A QUANTITATIVE STUDY ON CONSUMER BANKRUPTCY OVER THE BUSINESS CYCLE

1.1 INTRODUCTION

1.1.1 MOTIVATION

The past three decades have seen the following observations on the number of bankruptcy filings and indebtedness.

1. The number of consumer bankruptcies and indebtedness has dramatically increased (see figure 1.1)¹.
2. The number of consumer bankruptcies is countercyclical and relatively stable (see figure 1.2, figure 1.3 and table 1.1)^{2 3}.

Many researchers, including Gross and Souleles [18], Athreya [4], Livshits et al. [29] and Athreya et al. [5], recently studied the first observation. For example, Gross

¹The bankruptcy rate is the percentage of households filing for bankruptcy. The data sources are various editions of the Statistical Abstract of the United States. We acknowledge the use of Zagorsky and Lupica [42]'s data set of the bankruptcy rate. We review data before 2005 and note that, from 2004 to 2005, the bankruptcy rate decreased dramatically. Note that the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCA) of 2005 came into effective and does not allow people who have the ability to pay to discharge all of their debts.

²Data on real output was taken from the Bureau of Economic Analysis.

³The data were detrended using an HP filter with a parameter of 100 for annual data and with a parameter of 1600 for quarterly data.

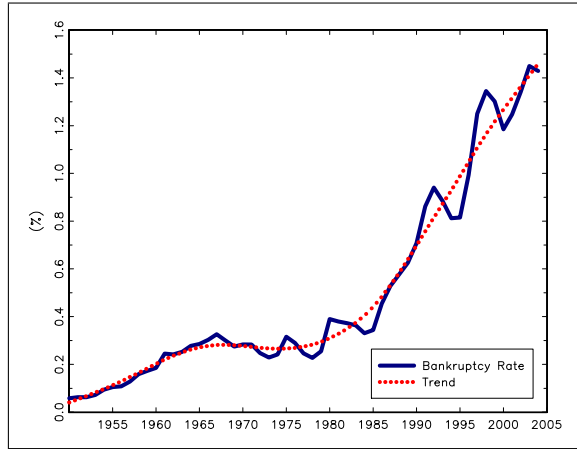


Figure 1.1: Bankruptcy rate (annual) and trend (HP filter with $\lambda = 100$)

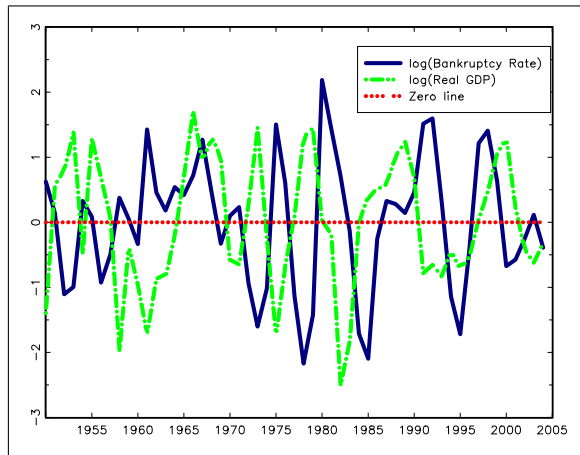


Figure 1.2: Bankruptcy rate and output (annual)—data are detrended with an HP filter ($\lambda = 100$) and standerized using the standard deviation

Table 1.1: Bankruptcy rate and output

	Annual 1950 – 2005	Quarterly 1994 – 2005
Correlation	-0.31	-0.50
Relative S.D.	5.0	9.5

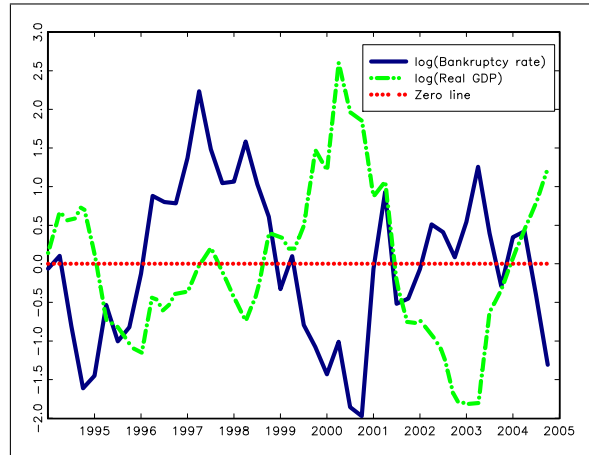


Figure 1.3: Bankruptcy rate and output (quarterly)—data are detrended with an HP filter ($\lambda = 1600$) and standardized using the standard deviation

and Souleles [18] discussed the possibility that, during recent years, people were more willing to default than in the early 1980s. Athreya [4] and Livshits et al. [29] explored the possibility that falling transaction costs lead to an increase in the bankruptcy rate and indebtedness. Athreya et al. [5] explained the possibility that an improvement in the information that lenders have leads to an increase in indebtedness and the bankruptcy rate. However, previous studies on quantitative macroeconomics ignored economic fluctuations and paid little attention to the interaction between consumer bankruptcy and economic fluctuations. This dearth of studies exists not because the issue is not an important, but primarily because constructing the general equilibrium model and solving the model quantitatively is very difficult in general.

To the best of our knowledge, Nakajima and Rios-Rull [33] is the only exception that constructed a heterogeneous-agent general equilibrium model with default options and aggregate shocks, and analyzed the second observation. They intro-

duced aggregate shocks by following Diaz-Gimenez et al. [15]’s approach, assuming the timing of observations of stochastic shocks such that prices are exogenous and do not depend on the entire distribution of agents. In Diaz-Gimenez et al. [15]’s setting, aggregate stochastic shocks are not a *surprising* event in the sense that each agent observes a forthcoming aggregate state before he or she decides how much to save and how much to consume in the current period. Nakajima and Rios-Rull [33] succeeded in replicating the aggregate conditions of the U.S. economy in many dimensions (including wealth distribution and bankruptcy rate) but was not able to replicate the countercyclical property of the bankruptcy rate in their baseline model. Nakajima and Rios-Rull [33] noted, due to the persistency of the aggregate state, in economic expansions the future looks better and the option to borrow is less likely to be needed. This makes incentives to default increase. In addition, a forthcoming expansion induces an increase in loans and debtors where banks charge a risk premium corresponding to the consumer’s default risk, leading to an increase in the number of individuals who default during expansions.

The countercyclical property of default is intuitive but not obvious in economic models with forward-looking agents. Even in literature on sovereign defaults, their countercyclical properties have not been replicated quantitatively until Arellano [3] was written. In the participation constraint models with a complete set of contingent assets (such as Alvarez and Jermann [2], Kehoe and Levine [23] and Kocherlakota [27]), default incentives are typically higher in good economic situations. Arellano [3] was the first paper to replicate the countercyclical property of sovereign default by replacing the contingent debt of the old models with non-contingent debt.

1.1.2 ABOUT THIS CHAPTER

The goal of this chapter is to overcome the computational difficulty and to propose a quantitative model consistent with the second observation, as well as with the first one. We construct a heterogeneous-agent general equilibrium model with capital market imperfections and consumer bankruptcy. In our model, households face aggregate shocks and idiosyncratic shocks to labor efficiency, discount factor and liability loss. Under the condition that banks have *full information* on a household's type, banks offer households individual specific interest rate, which is standard practice in bankruptcy literature after Chatterjee et al. [12]'s pioneering work. The timing of the observation of stochastic shocks in our model is close to that of Krusell and Smith [28]'s approach. Then aggregate shocks are *surprising*: households and financial intermediaries cannot observe a forthcoming aggregate shock before they make financial contracts in each period. Once we incorporate aggregate technology shocks into the economy in this way, prices, including the individual specific interest rate, depend on the distributions over an agent's state and are time variant leading to a large state space and a heavier computational burden.

The key contribution of this chapter is that our quantitative model successfully replicates the countercyclical property and high volatility of bankruptcy rate by introducing the *surprising* aggregate shock. Our quantitative model also successfully replicates the main features of the U.S. statistics, such as statistics on bankruptcy (including average bankruptcy rate), debt, and wealth distribution (high wealth concentration and Gini coefficients).

Given the success of our model in replicating the data, we examine the effects of improvements to banks' information technology on aggregate time series properties and wealth dispersion by comparing two economies. We imagine that the first

economy (FI economy) is the same as previously described, where the lenders have *full information* on household type and offer individual specific interest rates. The FI economy is close to the recent state of financial information technology. In the other economy (NI economy), we assume that banks have *no information* associated with assessing default risk (information asymmetry exists between banks and households) and that banks never make loans to households, who can save their asset but cannot borrow. There are no debtors and no voluntary defaulters in the NI economy. This model's setting is assumed simple but consistent with the early 1980s: default rates and indebtedness are low. Note that, in general, the information asymmetry may endogenously collapse the market. For example, Zhang [43] constructed the standard incomplete market general equilibrium model with agents' default option and single interest rate. He shows that the endogenous borrowing limit becomes tight enough to ensure that agents weakly prefer to repay debts. Athreya et al. [5] also shows that the limitation of the information nearly eliminates the market for unsecured loans under a situation similar to our setting.

In our main quantitative analysis, we find that improvements in financial information technology under aggregate uncertainty (*i*) explain the first observation, (*ii*) increase wealth dispersion, and (*iii*) slightly smooths consumption.

1.1.3 RELATED WORKS

The model in this chapter is constructed along the lines of heterogeneous-agent general equilibrium models originally developed by Bewley [7], Huggett [20], Aiyagari [1] and Krusell and Smith [28]. We incorporate liability shocks, endogenous default decisions, individual specific interest rates and capital market imperfections into the standard heterogeneous-agent models. These features make our model more difficult to solve because the equilibrium individual specific interest rate depends on the individual

state and aggregate capital stock and labor. Therefore, we add additional steps to solve the model quantitatively.

A number of recent papers have studied consumer bankruptcy in quantitative dynamic equilibrium models such as Livshits et al. [29], Chatterjee et al. [12], and Meh and Terajima [30]. These papers focused on stationary equilibrium and ignored business fluctuations. In particular, Chatterjee et al. [12] made theoretical and quantitative contributions in macroeconomic models with a bankruptcy option in which prices of unsecured loans depend on loan size and household characteristics.

This chapter is also related to the literature on wealth distribution. Many researchers, including Huggett [21], Quadrini [36], Cagetti and Nardi [10], and Krusell and Smith [28], studied the source of high wealth concentrations in the U.S. economy and recent increases in wealth dispersion. For example, Quadrini [36] and Cagetti and Nardi [10] incorporated entrepreneur activity into the incomplete market general equilibrium and succeeded in generating high wealth concentration. Also Krusell and Smith [28] replicated the wealth distribution in the U.S. economy by introducing a stochastic discount factor, which is the same setting as ours.

1.1.4 ORGANIZATION OF THE CHAPTER

This chapter is organized as follows. Section 1.2 characterizes the model economy. Section 1.3 defines the recursive competitive equilibrium. Section 1.4 selects the parameter values and discuss the calibration procedures. Section 1.5 describes the main findings and section 1.6 concludes. The appendix to this chapter describes the computation procedures.

1.2 THE MODEL

In this section, we describe the model economy. The model incorporates a recent literature on consumer bankruptcy into the incomplete market model (Bewley [7], Huggett [20], Aiyagari [1] and Krusell and Smith [28]). In the model, households are ex-ante identical but are subject to idiosyncratic shocks as well as aggregate shocks. There is no insurance against these contingencies. In a manner similar to other incomplete market models, precautionary savings works as a type of insurance to smooth consumption over time.

1.2.1 THE ENVIRONMENT AND TIMING OF EVENTS

The model economy is populated by a continuum of infinitely-lived households of measure one. There are four sectors in the economy: households, firms, financial intermediaries, and hospitals. In each period, each household is engaged in production activity, decides whether to declare bankruptcy, and chooses how much to save and how much to consume. The summary of the timing of the events in each period is as follows (see figure 1.4).

(i) **At the beginning of each period t :** Each household with holding asset $a_t \in \mathcal{A}$ observes four types of uninsurable shocks: idiosyncratic labor efficiency shocks ε_t , a value from $\mathcal{E} = \{\varepsilon_1, \dots, \varepsilon_n\}$ where $\varepsilon_1 < \dots < \varepsilon_n$, idiosyncratic liability shocks ζ_t , a value from $\mathcal{Z}_\zeta = \{0, \underline{\zeta}\}$ where $\underline{\zeta} > 0$, idiosyncratic preference shocks β_t , a value from $\mathcal{B} = \{\underline{\beta}, \beta_1, \dots, \beta_n\}$ where $\underline{\beta} \ll \beta_1 < \dots < \beta_n$, and aggregate productivity shocks z_t , a value from $\mathcal{Z} = \{z_1, \dots, z_n\}$ where $z_1 < \dots < z_n$. We assume that labor efficiency shocks ε_t , preference shocks β_t , and aggregate productivity shocks z_t follow the first-order Markov process and liability shocks ζ_t are independent and identically distributed (i.i.d.) over time. As in Chatterjee et al. [12], liability shocks represent hospital bills

from consuming medical services. Additionally, the realization of $\underline{\beta}$ shock captures sudden events that a household have strong incentives for spending during the current period.

(ii) Then, each household supplies its labor to firms engaged in producing composite goods by renting capital from financial intermediaries and hiring workers.

(iii) **At the end of each period t :** Each household decides whether to declare bankruptcy, denoted by $d_t \in \{0, 1\}$ where $d_t = 0$ indicates declaring bankruptcy and $d_t = 1$ indicates not declaring bankruptcy. If the household defaults, all of its liabilities are discharged. If not, the household pays off her debt and interest and pays any existing medical bill.

(iv) Then, each household decides how much to save (or borrow), a_{t+1} , and how much to consume, c_t .

To be efficient with notations, we drop the t subscript, use primes to denote next-period values, and use the subscript -1 to denote previous-period values. We also use $\mathbf{s} = \{\varepsilon, \beta, \zeta, z\}$ to denote a set of stochastic shocks.

1.2.2 LEGAL ENVIRONMENT

We model the default option to reflect Chapter 7 under U.S. bankruptcy law. There are two types of credit records, h from $\mathcal{H} = \{g(\text{good}), b(\text{bad})\}$. Defaulting is an option only for households with *good* credit records.

If a household declares bankruptcy, then all debt is discharged and the household's credit record is *bad* during the next period. The household's current and future earnings are protected. During the periods in which the household's credit record is *bad*, it cannot access a new loan but can save the asset. The credit record changes from *bad* to *good* at an exogenous probability $(1 - \lambda)$. The probabilistic credit record

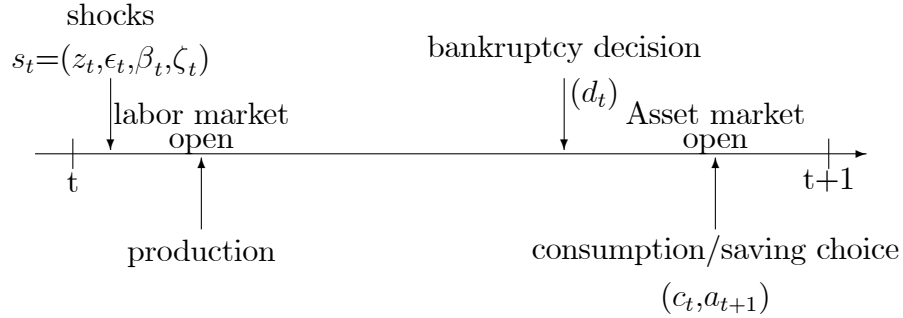


Figure 1.4: Timing of events

recovery captures the effect of the current U.S. bankruptcy law, requiring households to have *bad* credit records in certain years. This simplification significantly saves state space and avoids keeping track of the precise duration of having a *bad* credit record.

Note that the model does not consider the change in the U.S. bankruptcy law during 2002, which limits above-median-income households from filing under Chapter 7. This change will be incorporated into the model in a future study.

1.2.3 THE HOUSEHOLD SECTOR

The preference for households, who value consumption and consider costs of filing bankruptcy, are represented by the expected value of a discounted sum of utility functions:

$$E_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) - \mathcal{I}_h \varrho),$$

where $\beta \in (0, 1)$ is the realization of an idiosyncratic shock on preferences (discount factor), c_t is consumption of composite goods in period t , \mathcal{I}_h is an indicator function where \mathcal{I}_h equals one for a bad credit record and zero otherwise, and ϱ is the nonpecuniary costs of filling bankruptcy in terms of utility—standard assumptions in the literature (see, for example, Livshits et al. [29] and Athreya et al. [5]). We assume that the consumption of medical services is nondiscretionary and nonproductive, and does not positively affect a household’s utility.

Each household is endowed with one unit of time and supplies exogenously \tilde{l} ($0 < \tilde{l} < 1$) units of time to the labor market. Each agent enjoys $(1 - \tilde{l})$ units of time as leisure.

At the beginning of each period, each household observes stochastic shocks, \mathbf{s} , and supplies its labor time, l , to the labor market at the competitive wage rate, w .

At the end of each period, the household’s decision consists of two stages. In the first stage, a household with a good credit record decides whether to declare bankruptcy, and a household with a bad credit record is not eligible to do so. If the household defaults, all of its debt and liability is discharged. If it does not default, it pays off its debt and existing medical bill. In the second stage, the household decides how much to consume and how much to save (or borrow). The budget constraint for a household with a good credit record and a bad credit record at the second stage is written in the following forms.

Case 1: The budget constraint for a household with a good credit record is described by:

$$\begin{aligned}
 c + a' &= ((1 + \mathcal{R}(a, \mathbf{s}_{-1}, z, \Gamma))a - \zeta)d + \tilde{l}\epsilon w, \\
 a' &\geq \begin{cases} 0 & \text{if } d = 0 \text{ (default)} \\ \phi(z_{-1}, \mathbf{s}, \Gamma) & \text{if } d = 1 \text{ (not default)} \end{cases}
 \end{aligned}$$

where ϕ is an individual specific borrowing limit, \mathcal{R} is an individual specific interest rate, and Γ is the distribution of households over $\mathcal{A} \times \mathcal{E} \times \mathcal{B} \times \mathcal{Z}_\zeta \times \mathcal{Z} \times \mathcal{H}$. Both ϕ and \mathcal{R} are endogenously determined as described later. If a household defaults ($d = 0$), net liabilities $(1 + \mathcal{R}(a, \mathbf{s}_{-1}, z, \Gamma))a - \zeta$ are discharged and it cannot borrow in the period. In other words, the borrowing constraint is $a' \geq 0$.

Case 2: The budget constraint for a household with a bad credit record is described by:

$$\begin{aligned} c + a' &= \max \{((1 + r)a - \zeta), 0\} + \tilde{l}\epsilon w, \\ a' &\geq 0 \end{aligned}$$

where $r(z, \Gamma)$ is the interest rate for savers.

We assume that if a household is hit by a liability shock and net liability becomes negative, in other words, $(1 + r)a - \zeta < 0$, then the hospital discharges the liability. We also assume that discharging the bill does not mean "declaring bankruptcy" nor does doing so affect the probability of having good credit in the next period. Even though a household with a bad credit record is not eligible to default, for convenience we allow $d = 0$ if $(1 + r)a - \zeta < 0$ and $d = 1$ otherwise.

We now describe the recursive version of the household problem. We denote Θ_h as a set of state variables $(a, \mathbf{s}, \mathbf{s}_{-1}, \Gamma)$ for a household with a credit record $h = \{g, b\}$. We also denote $v_h(\Theta_h)$ as a value function for a household with state Θ_h and $H(z_{-1}, z, \Gamma)$ as a law of motion for distribution Γ . A household with a good credit record solves the dynamic program:

$$v_g(\Theta_g) = \max_{\{a', c, d\}} u(c) + \beta E_t[(1 - d)v_g(\Theta'_g) + dv_b(\Theta'_b)]$$

subject to:

$$\begin{aligned}
c + a' &= ((1 + \mathcal{R})a - \zeta)d + \tilde{l}\epsilon w \\
h' &= \begin{cases} b(\text{bad}) & \text{if } d = 0 \\ g(\text{good}) & \text{if } d = 1 \end{cases} \\
a' &\geq \begin{cases} 0 & \text{if } d = 0 \\ \phi(z_{-1}, \mathbf{s}, \Gamma) & \text{if } d = 1 \end{cases} \\
c &\geq 0 \\
\Gamma' &= H(z_{-1}, z, \Gamma)
\end{aligned}$$

A household with a bad credit record solves the dynamic program:

$$v_b(\Theta_b) = \max_{\{a', c\}} \left\{ \begin{array}{l} (u(c) - \varrho) + \beta E_t[\lambda v_g(\Theta'_g)] \\ + (1 - \lambda)v_b(\Theta'_b) \end{array} \right\}$$

subject to:

$$\begin{aligned}
c + a' &= \max((1 + r)a - \zeta, 0) + \tilde{l}\epsilon w \\
a' &\geq 0, \quad c \geq 0 \\
\Gamma' &= H(z_{-1}, z, \Gamma)
\end{aligned}$$

1.2.4 THE PRODUCTION SECTOR

There are large firms in the economy that produce composite goods with a production function described by:

$$y = zf(K, L),$$

where K and L are aggregate capital and labor. At the beginning of each period, firms produce goods with capital borrowed from financial intermediaries at rental cost r and hire workers at wage rate w . The firm's profit π_f is described as:

$$\pi_f = \max_{\{k, l\}} zf(K, L) - wL - rK - \delta K,$$

where δ is depreciation rate.

We assume that the market is competitive. In the equilibrium, rental rate r and wage rate w are competitively determined and are given by the marginal products of each factor in production as follows:

$$\begin{aligned} r(K, L, z) &= z f_1(K, L) - \delta \\ w(K, L, z) &= z f_2(K, L). \end{aligned}$$

1.2.5 FINANCIAL INTERMEDIATION

ECONOMY WITH FULL INFORMATION

In the model, the financial intermediation sector collects deposits from households (savers) and promises to pay interest rate r . In the economy with full information (FI economy), financial intermediaries lend their deposits to firms at rental rate r and make loans to households (borrowers) at individual specific interest rate \mathcal{R} under a situation in which financial intermediaries have full information on a household's type and have the ability to access each household's default risks. The financial intermediaries then add the corresponding risk premium to risk-free interest rate.

The individual specific interest rate \mathcal{R} at time t is determined by the financial contract between a financial intermediary and a household (borrower) made at time $t - 1$ through the following two-stage game. At time $t - 1$, we assume that the history of individual states, aggregate states and asset distribution until $t - 1$ and the law of motion for aggregate variables is common information among households and financial intermediaries. In stage one, households announce to financial intermediaries a desired level of asset holding (amount of debt) a at time t . In stage two, financial intermediaries engage in a Bertrand competition and compete in auction in which they simultaneously post the individual specific interest rate \mathcal{R} , which is contingent

on the aggregate state z at time t , for the desired asset level that households want to issue. At time t , the contingent profit of a financial intermediary that entered into a financial contract at time $t - 1$ with a household is described as:

$$\pi_{FI} = a(1 + r) - a(1 + \mathcal{R})d(\Theta_g),$$

where $a(1 + r)$ is the cost at which the financial intermediary collects the asset from households (savers) and $-a(1 + \mathcal{R})d(\Theta_g)$ is the amount of the asset returned by households (borrowers), whose default decision is $d(\Theta_g)$.

Using the law of large numbers, the total profit for entering into a contract with a large number of agents with $(a, \epsilon_{-1}, \beta_{-1})$ under (z_{-1}, z, Γ) is equal to the expectation of π_{FI} conditional on $(a, \mathbf{s}_{-1}, z, \Gamma)$. In equilibrium, competition among financial intermediaries results in zero profits related to any contracts and contingencies. Then, individual specific interest rates $\mathcal{R}(a, \mathbf{s}_{-1}, z, \Gamma)$ are described as:

$$1 + \mathcal{R}(a, \mathbf{s}_{-1}, z, \Gamma) = \frac{(1 + r)}{\Pr(d(\Theta_g)|a, \mathbf{s}_{-1}, z, \Gamma)} \quad \text{for } \forall a, \mathbf{s}_{-1}, z \text{ and } \Gamma. \quad (1.1)$$

From the view of households at time $t - 1$, the individual specific interest rate \mathcal{R} is a function of the status of individuals and aggregate shocks, \mathbf{s}_{-1} , the amount of assets they want to issue, a , and information on distribution, Γ , and is contingent on the aggregate state z .

The borrowing limit is endogenously determined by the financial contract in equilibrium. We assume that financial intermediaries never offer financial contracts to a household that will default with a probability close to one for some contingencies in the next period: $\Pr(d|a, \mathbf{s}_{-1}, z, \Gamma) < \kappa$, where κ is a small positive value. In the computation, we set 10^{-6} for the value of κ . In other words, the borrowing constraint at time t , $\phi(z_{-1}, \mathbf{s}, \Gamma)$, is determined by:

$$\phi(z_{-1}, \mathbf{s}, \Gamma) = \{\phi \in \mathbb{R} \mid \Pr(d'|a', z_{-1}, \mathbf{s}, z', \Gamma) \geq \kappa \text{ for } \forall z' \text{ and } a' \geq \phi\} \quad (1.2)$$

ECONOMY WITH NO INFORMATION

In the economy with no information (NI economy), information asymmetry exists between financial intermediaries and households, and financial intermediaries have *no information* on household type. In the NI economy, we assume that the debt market collapses and financial intermediaries never make loans to households. In this economy, each household faces an interest rate for savers, $r(z, \Gamma)$, and a borrowing limit, ϕ , such as:

$$\phi(z_{-1}, \mathbf{s}, \Gamma) = 0 \quad \text{for any } z_{-1}, \mathbf{s}, \Gamma$$

The assumption that the debt market exogenously collapses is for simplicity. Note that the information asymmetry may endogenously collapse the market in general (see Zhang [43] and Athreya et al. [5]).

We assume that all other conditions in the NI economy are the same as in the FI economy. Once we impose no borrowing conditions in the FI economy, there are no voluntary defaulters in equilibrium. We can assume that the NI economy is a special case of the FI economy.⁴

⁴In the NI economy, a household solves the dynamic program:

$$v^{NI}(\Theta_g) = \max_{\{a', c\}} u(c) + \beta E_t(v^{NI}(\Theta_g))$$

subject to:

$$\begin{aligned} c + a' &= \max((1+r)a - \zeta, 0) + \tilde{\ell}ew \\ a' &\geq 0, \quad c \geq 0 \\ \Gamma' &= H^{NI}(z_{-1}, z, \Gamma), \end{aligned}$$

where $v^{NI}(\Theta_g)$ is a value function for households with state Θ_g and $H^{NI}(z_{-1}, z, \Gamma)$ is the law of motion in this economy.

1.2.6 THE HOSPITAL SECTOR

The hospital sector has the technology to change composite goods into medical goods by one-to-one. Hospitals charge households with medical bills when they are hit by a liability shock. However, the assumption is that hospitals cannot collect the full amount of the bill from a household that defaults. If a household has positive assets (before the liability shock), $(1+r)a > 0$, and then is hit by the liability shock and defaults, then the hospital collects only $(1+r)a$. If a household has negative assets and is hit by liability shocks and defaults, then the hospital does not collect anything. In other words, hospitals receive $\max((1+r)a, 0)$ from a household that defaults. They supply medical service to households in the amount $\frac{\zeta}{m(z_{-1}, z, \Gamma)}$, where $m(z_{-1}, z, \Gamma)$ is the markup. Their profit π_H is described as:

$$\pi_H(z_{-1}, z, \Gamma) = \int \left[d(\Theta_h)\zeta + \max((1+r)a, 0)(1 - d(\Theta_h)) - \frac{\zeta}{m(z_{-1}, z, \Gamma)} \right] d\Gamma$$

We assume that hospitals are not profit maximizers and set $m(z_{-1}, z, \Gamma)$ to make their profits zero in each period.

1.3 EQUILIBRIUM

We now define a competitive equilibrium. A recursive competitive equilibrium is a set of household decision rules $\{a'(\Theta_h), c(\Theta_h), d(\Theta_h)\}$, firm decisions $\{L(z, \Gamma), K(z, \Gamma)\}$, hospital markups $\{m(z_{-1}, z, \Gamma)\}$, prices $\{r(z, \Gamma), w(z, \Gamma), \mathcal{R}(a, \mathbf{s}_{-1}, z, \Gamma)\}$, and the law of motion $\Gamma' = H(z_{-1}, z, \Gamma)$, such that:

1. *Household problem:* Given prices and the law of motion, $\{a'(\Theta_h), c(\Theta_h), d(\Theta_h)\}$ solves the household problem.
2. *Firm problem:* Given prices, $\{L(z, \Gamma), K(z, \Gamma)\}$ solves the firm optimization problem. Then, prices $r(z, \Gamma)$ and $w(z, \Gamma)$ are competitively determined.

3. *Financial intermediary problem*: $\mathcal{R}(a, \mathbf{s}_{-1}, z, \Gamma)$ solves the financial intermediary optimization problem.
4. *Hospital problem*: $m(z_{-1}, z, \Gamma)$ solves the hospital problem.
5. *Market clearing conditions*⁵
 - (a) Labor market clears: $L = \tilde{l} \int \epsilon d\Gamma$.
 - (b) Asset market clears: $K = \int a d\Gamma$.
6. *Consistency*: The law of motion $H(z_{-1}, z, \Gamma)$ is consistent with the decision rules.

1.4 PARAMETERIZATION

In this section, we discuss the parameter values and data targets that we attempt to match in the model.

We assume that the utility function takes the logarithm form:

$$u(c) = \ln(c).$$

The production function is a standard Cobb-Douglas function. We set the capital depreciation rate δ to 10%, the capital income share $(1 - \alpha)$ to 0.36, and the exogenous labor supply \tilde{l} to 0.314 based on standard choices in the literature.

To parameterize the stochastic aggregate productivity, we specify $\ln(z)$ as:

$$\ln(z') = \rho_z \ln(z) + \varepsilon'$$

where ρ_z is the coefficient of persistency and ε' follows a normal distribution with standard deviation σ_ε . Using the results of the empirical work King and Rebelo

⁵Goods market clearing is satisfied by Walras's Law

[25], we set ρ_z to 0.979 and σ_ϵ to 0.0072. We then approximate this process with a Markov chain through the method developed by Tauchen [41]. We calibrate the lower bound and the upper bound to the support of $\ln(z)$ to match the volatility of the model's output to the U.S. data, and take equally spaced points over $\ln(z)$ from the upper bound to the lower bound, which are symmetric at zero. To ensure feasible computational time, we take three points over $\ln(z)$: $\ln(z_L)$, $\ln(z_M)$, and $\ln(z_H)$. We identify the economy as in expansions when productivity is high: $z = z_H$, and as in recessions when productivity is low: $z = z_L$.

Regarding labor efficiency shocks, ϵ , we take two states of ϵ , $(\epsilon_1, \epsilon_2) = (0.09, 1.0)$. A household is unemployed when $\epsilon = \epsilon_1$ and is employed when $\epsilon = \epsilon_2$. For simplicity, we assume that the home-working labor efficiency (unemployment insurance) is 9% of the labor efficiency of the employed. The individual shocks to labor efficiency are correlated with aggregate technology shocks. Following the approach in Krusell and Smith [28], wisely choosing a transition matrix for technology shocks and labor efficiency implies that the unemployment rate as well as aggregate labor always depend on the economic situation. For this to occur, the transition probabilities must satisfy the restrictions such as for any $(z, z', \epsilon, \epsilon')$:

$$\sum_{\epsilon'} \pi_{zz'\epsilon\epsilon'} = \pi_{zz'}$$

and,

$$u_z \frac{\pi_{zz'11}}{\pi_{zz'}} + (1 - u_z) \frac{\pi_{zz'21}}{\pi_{zz'}} = u_{z'}$$

where $\pi_{zz'\epsilon\epsilon'}$ is the probability of transition from state (z, ϵ) today to state (z', ϵ') tomorrow and u_{z_i} is the unemployment rate in z_i . To calibrate the stochastic process, we set 4% as the unemployment rate in economic expansions and 10% in economic recessions, 1.5 quarters as the average duration of an unemployment spell in economic expansions, and 2.5 quarters in economic recessions, and we set the average values

of the unemployment rate and the average duration of an unemployment spell in aggregate state z_M . These settings are similar to those used in Krusell and Smith [28].

We assume that each household's discount factor is ex-ante identical but ex-post heterogeneous and follows Markov process. We set $\beta = (\underline{\beta}, \beta_1 = 0.9858, \beta_2 = 0.9894, \beta_3 = 0.9930)$. In the economy of Krusell and Smith [28], $\underline{\beta}$ shocks never realize and the transition probabilities of preferences is assigned, $\Pr^{KS}(\beta' = \beta_j | \beta = \beta_i)$ for $i, j = 1, 2, 3$, such that the invariant distribution $\Pr^{*KS}(\beta_i)$ has 80% of the population at the middle β_2 and 10% at β_1 and β_3 , immediate transitions between the values of β_1 and β_3 occur with probability zero, and the average duration of the remaining β_1 and β_3 is 50 years. In our model with $\underline{\beta}$ shocks, we define the transition probabilities $\Pr(\beta' | \beta)$ such that (i) each household is hit by a $\underline{\beta}$ shock in the next period with probability $\Pr(\underline{\beta})$, (ii) conditional on avoiding a $\underline{\beta}$ shock in the next period, households with current period preference β_i draws the next period's preference from the transition probabilities, $\Pr^{KS}(\beta' = \beta_j | \beta = \beta_i)$ for $i, j = 1, 2, 3$, and households facing a $\underline{\beta}$ shock in the current period draw one from the invariant distribution $\Pr^{*KS}(\beta_i)$ for $i = 1, 2, 3$.

Regarding the probability of a $\underline{\beta}$ preference shock and a $\underline{\zeta}$ liability shock, no data are associated with these probabilities. We set $\Pr(\underline{\beta})$ and $\Pr(\underline{\zeta})$ to 40 years to match the average length of avoiding these shocks.

According to the Fair Credit Report Act, a bankruptcy filing stays on a household's credit record for 10 years. We calibrate λ to match the average length of having a bad credit record.

We simultaneously calibrate remaining parameters $\{\underline{\beta}, \underline{\zeta}, \varrho\}$ to match three targets: a measure of borrowers equal to 6.7%, defaulters due to liability shocks equal to 0.17%, and defaulters due to preference shocks equal to 0.30%, by minimizing the

sum of the squared distance between the target statistics and the implied results from the model.⁶

Note that the bankruptcy rate, representing the ratio of people who file to the total population, is given as 0.54%.⁷ Chakravarty and Rhee [11] categorizes the reasons for bankruptcy filings into "job loss" (12.2%), "credit misuse" (41.3%), "marital disruption" (14.3%), "health-care bills" (16.4%) and "lawsuit/harassment" (15.9%) from the PSID (1984–1995). We associate "job loss" with the earning shocks; "credit misuse" and "marital disruption" with preference shocks; and "health-care bills" and "lawsuit/harassment" with liability shocks.^{8,9}

The fixed parameters and the endogenously calibrated parameters are listed in Table 1.2.

1.5 RESULTS

This section presents the calibration results and the properties of the model. Table 1.3 reports the target statistics and the implied results from the model. In the FI economy, we successfully replicate the countercyclical property of the bankruptcy rate. The correlation between bankruptcy rate and output is -0.67 . The model also

⁶A value for a measure of borrowers is taken from Chatterjee et al. [12]. This figure excludes households with negative net worth larger than 120% of average income in the 2001 SCF because the debts are likely the result of the entrepreneurial activity from which the model abstracts.

⁷According to the Administrative Office of the U.S. Courts, the total number of filers for personal bankruptcy under Chapter 7 was 1.087 million in 2002. According to the Census Bureau, the total population older than 20 years of age in 2002 was 201 million.

⁸Chatterjee et al. [12] states that marital disruption leads to higher nondiscretionary spending on the part of each partner, which in turn increases the incentive for discretionary spending in the current period.

⁹The targets for each bankruptcy filing reason are as follows. We set 0.066% ($0.54\% \times 12.2\%$) for earning shocks (job loss), 0.30% ($0.54\% \times 55.6\%$) for preference shocks (marital disruption and credit misuse), and 0.17% ($0.54\% \times 32.3\%$) for liability shocks (health-care bills and lawsuits/harassment).

Table 1.2: Calibrated parameters of the model economy

parameters		values
Fixed parameters		
Capital share of income	$1 - \alpha$	0.64
Depreciation rate	δ	0.025
Probability of recovering credit record(%)	$1 - \lambda$	2.5
Probability of ζ liability shock(%)	$\Pr(\underline{\zeta})$	0.625
Probability of $\underline{\beta}$ preference shock(%)	$\Pr(\underline{\beta})$	0.625
Endogenously calibrated parameters		
Upper bound to the support of $\ln(z)$	$\ln(z_H)$	1.0072
Liability shocks	$\underline{\zeta}$	2.53
Preference shocks	$\underline{\beta}$	0.0685
Costs of filling to bankruptcy	ϱ	0.204

successfully replicates the distribution of wealth. The Gini coefficient is close to the target, as is the percentage of wealth of the top 1%. We also successfully replicate the measure of the debtor. Furthermore, the model replicates the bankruptcy rate as well as the relative importance of the reasons cited for default, and we successfully replicate the high relative standard deviation of the bankruptcy rate.

1.5.1 PROPERTIES OF INTEREST RATES AND BORROWING CONSTRAINTS

We now discuss the properties of interest rates and borrowing constraints. For the remainder, interest rates and borrowing limits are determined by probability of default through equations 1.1 and 1.2. Figure 1.5 provides an example of probability of default for the unemployed and the employed with $\beta = \beta_2$, contingent on high and low aggregate shocks in the next period conditional on middle aggregate shocks in the

Table 1.3: Results

statistic	target	model economy	
		FI	NI
Gini coefficient	0.80	0.81	0.71
Wealth of top 1% (%)	34.7	37.7	33.3
Wealth of top 5% (%)	57.8	70.5	61.9
Wealth of top 10% (%)	66.0	78.7	70.3
Total Defaulters(%)	0.54	0.53	—
Liability shocks	0.17	0.17	—
Preference shocks	0.30	0.30	—
Debtor(%)	6.7	6.9	—
Correlation of bankruptcy	−0.50	−0.67	—
Relative S.D. of bankruptcy	9.45	17.9	—

current period, $z = z_M$, and on average capital accumulation, $A = \bar{A}$. The points that we want to make are as follows.

1. **Monotonicity:** The probability of default is weakly increasing as the level of debt increases for each individual because households with higher debt rationally have a weakly stronger incentive to discharge their debt. Additionally, the probability of default is step-formed because households default due to shocks drawing from discrete probability distributions. If household's debt is close to zero, then so is the probability of default. Even if they choose little debt, there is small probability with which they default due to being hit by liability shocks.
2. **The employed versus the unemployed:** Under our parameter values, the probability of default in the next period for the unemployed is higher than that for the employed, conditional on the same individual status and debt level. An

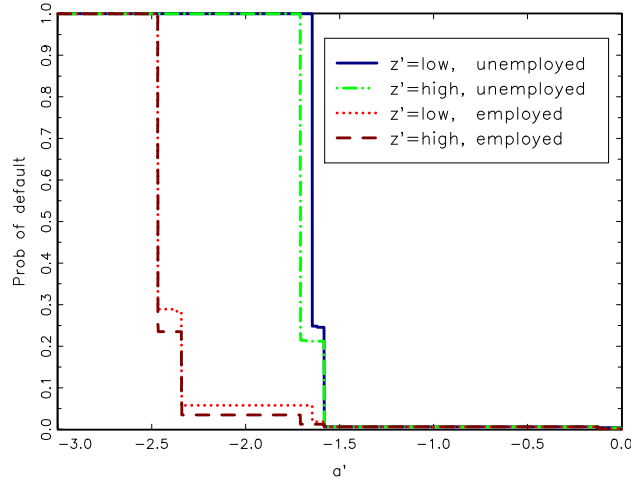


Figure 1.5: Probability of default in the next period ($z = z_M$, $\beta = \beta_2$, $A = \bar{A}$)

individual's employment status in the current period itself does not affect the probability of default in the next period but affects the probability distribution for the individual's status in the next period. This property is related to a higher likelihood for the unemployed to stay unemployed in the next period given persistent earning shocks. Figure 1.5 provides an example of this property, which implies that the unemployed face higher interest rates and tighter borrowing limits than the employed. Table 1.4 shows the endogenously determined borrowing limits for the unemployed and the employed with $\beta = \beta_2$ conditional on high and low aggregate shocks in the current period and average capital accumulation.

3. **Determination of borrowing limits:** Borrowing constraints are determined by the probability of default conditional on *low* aggregate productivity shocks in the next period. For the remainder, financial intermediaries are assumed to

Table 1.4: Endogenous borrowing limits ($\beta = \beta_2$, $A = \bar{A}$)

$\epsilon \setminus z$	<i>High</i>	<i>Low</i>
The unemployed	-1.77	-1.65
The employed	-2.47	-2.53

never make financial contracts to households who have a probability of default of close to one in the next period (see equation 1.2 for the precise definition). Figure 1.5 shows that the probability of default conditional on a *low* productivity shock in the next period is close to one for a lower debt level than the probability conditional on a *high* shock in the next period. We confirm that this phenomenon occurred in other cases. This property relates to the construction of the transition matrix of labor efficiency. A household is more likely to be unemployed conditional on low aggregate productivity shocks in the next period than middle or high productivity shocks in the next period.

1.5.2 COUNTERCYCLICAL PROPERTIES

Figure 1.6 shows the simulation results of output and bankruptcy rate normalized by standard deviation. The results show that the model replicates the countercyclical property of the bankruptcy rate; the correlations between output and the bankruptcy rate in the model's economy and the U.S. economy are -0.67 and -0.50 , respectively.

The key assumption of the model that replicates the countercyclical property of bankruptcy rate is timing of observation of aggregate shocks: an aggregate shock is a *surprising* event for both households and financial intermediaries. Households and financial intermediaries enter into financial contract before they observe the economic

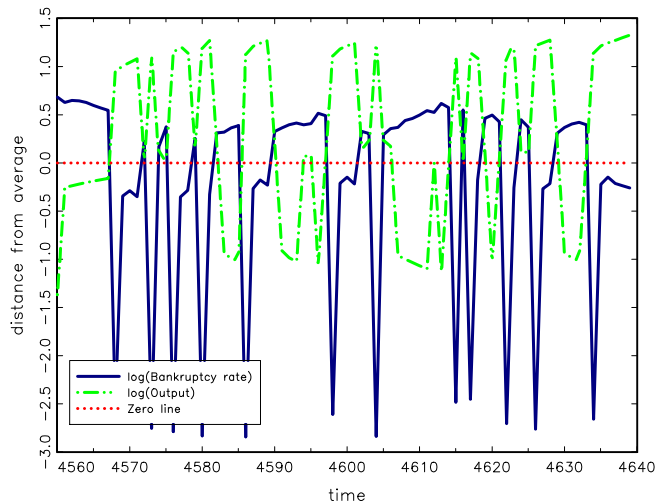


Figure 1.6: Bankruptcy rate and output (model)

situation in the next period. Borrowing limits and the capability of each agent to borrow are not contingent on the aggregate shocks in the next period. When the labor efficiency shock is positively correlated with the aggregate shock, relatively poor agents are likely to borrow more in economic recessions and borrow less in economic expansions to smooth consumption over time. Because the aggregate shock is persistent, a higher number of debtors and a higher level of debt are likely to exist in economic recessions than in economic expansions, which causes higher number of defaulters during recessions. Also, when the economic situation is revealed to good, debtors *surprisingly* face a lower risk premium and a lower cost of repaying debt, leading debtors less likely to default during expansions. This effect is prominently observed when the economic situations are reversed.

The cyclical property is intuitive but not obvious. We note that the difference between our model and the baseline model in Nakajima and Rios-Rull [33] is that, in

their model, bankruptcy rate and the number of debtors are positively correlated to output. In the Nakajima and Rios-Rull [33] model, assuming each agent can observe today's aggregate shocks in the previous period, aggregate shocks are not a *surprising* event. If a recession in the next period is known, then financial intermediaries set higher interest rates and tighter borrowing limits for loans in the current period because they are correctly forecasting more bankruptcies. Financial intermediaries request higher risk premiums from debtors in economic recessions. Given the financial intermediaries' reaction before entering into financial contracts, households cannot borrow as much as in economic expansions; therefore, level of debt and the number of households in debt are lower in recessions, leading to fewer defaulters during recessions in their model.

1.5.3 APPROXIMATE AGGREGATION

For the heterogeneous-agent model with aggregate shocks, we need to keep track of the distribution to compute the general equilibrium. This introduces a technical challenge, given that the distribution Γ is time variant and has infinite dimensions. To solve this class of the model, Krusell and Smith [28] proposed approximating the distribution as a finite number of state variables and the law of motion as a certain functional form, respectively. We choose a log-linear functional form for the law of motion and only the mean of capital holdings (aggregate capital) as a state variable, then define nine equations:

$$\ln K' = \Phi_1(z, z_{-1}) + \Phi_2(z, z_{-1}) \ln K \quad \text{for } z, z_{-1}.$$

where $\Phi_1(z, z_{-1})$ and $\Phi_2(z, z_{-1})$ are coefficients dependent on aggregate shocks in the current period and the previous period. We suppose that the effect of aggregate shocks in the previous period on determining aggregate capital in the next period is

Table 1.5: Results of approximate aggregation

Aggregate shocks $\{z\}$	Coefficients		Measures of fit	
	$\Phi_1(z, \cdot)$	$\Phi_2(z, \cdot)$	R^2	$\hat{\sigma}$
<i>High</i>	0.132	0.946	0.99999	0.00006
<i>Middle</i>	0.134	0.945	0.99999	0.00009
<i>Low</i>	0.133	0.945	0.99996	0.00003

very small because aggregate shocks in the previous period, z_{-1} , affect the individual specific interest rate, which poor people (debtors) should care about. In contrast, wealthy people care about $r(z, \Gamma)$ and $w(z, \Gamma)$ when deciding on how much to save and they pay little attention to z_{-1} . Under the stochastic β model, wealthy people own most of the wealth, and the law of motion for aggregate capital is determined by their savings behavior. Then we impose coefficients $\Phi_2(z, \cdot)$ on the same value for each z_{-1} .

Table 1.5 shows the results of the approximate aggregation, including values for the coefficients and measures of fit: R^2 and the standard deviation of the regression error $\hat{\sigma}$. We confirm that the accuracy of the aggregate law of motion for capital is excellent, because R^2 is sufficiently close to one and $\hat{\sigma}$ is significantly small.

1.5.4 COMPARISON WITH THREE ECONOMIES

Given the success of our model in replicating the data, we examine the effects of improvement to a bank's information technology on aggregate time-series properties and wealth dispersion.

BUSINESS CYCLE

We make comparisons in three economies: *(i)* the FI economy, heterogeneous-agent model, *(ii)* the NI economy, heterogeneous-agent model, and *(iii)* the complete-markets (CM) economy, representative-agent model. In the CM economy, the representative-agent faces only aggregate productivity shocks. Her preference, labor efficiency and liability loss are fixed at the average values of heterogeneous-agents in the FI and NI economies.

Table 1.6 shows the business cycle properties of U.S. data, the FI economy, and the NI economy, respectively. These properties consist of standard deviation of output and bankruptcy rate, relative standard deviation, and correlation of some variables with output. All variables represent quarterly data in logarithms and have been detrended with the HP filter. The asterisk * in table 1.6 indicates data taken from table 1 in King and Rebelo [25], which covers the period 1947 (first quarter) to 1996 (fourth quarter).

The summary of the findings in table 1.6 are as follows. First, the volatility of output in the models matches values in the U.S. economy. Second, the model successfully replicates the volatility of investment, which is about three times more volatile than output. Consumption is substantially smoother than output in the model's economy as well as in the U.S. economy. However, consumption in the model is only half as volatile as output, whereas it is over two-thirds as volatile as output in the U.S. economy. This occurrence is common in most business cycle models (for example, see table 3 in King and Rebelo [25]). Third, the model replicates a strong positively correlation of consumption and investment. Fourth, the model successfully replicates the high volatility of the bankruptcy rate (FI economy), which comes from our setting that aggregate shock is a *surprising* event for each agent. The result

of Nakajima and Rios-Rull [33]’s model, which has no *surprising* aggregate shocks, shows the bankruptcy rate to be 4% as volatile as the output.

Finally, regarding volatility of consumption, consumption in the CM economy is the smoothest, which is obvious because individual idiosyncratic shocks are assumed to be completely insured in the CM economy. Consumption in the FI economy is also smoother than in the NI economy. For the remainder, under the assumption of an incomplete market in the FI and NI economies, people are not able to purchase insurance for stochastic shocks and save in a precautionary manner to smooth consumption. During economic expansions (high productivity shocks), households have an incentive to save more to prepare for economic recessions when they are more likely to lose their jobs. In contrast, during economic recessions, households are more likely to withdraw from their savings. From this point of view, the default option in the FI economy partly works as insurance to ensure smooth consumption over time for each individual (especially for poor households) and leads to less precautionary motives to save.

WEALTH CONCENTRATION

Figure 1.7 shows the wealth distributions (Lorenz Curve) of the U.S. economy, the FI economy, and the NI economy respectively. The FI economy successfully replicates high wealth concentration, in contrast, wealth concentration is lower in the NI economy. The percentage of wealth held by the top 1% is 37.7% in the FI economy and 33.3% in NI economy, and the target is 34.7%. Gini coefficients in the FI economy and the NI economy are 0.81 and 0.71, and the target is 0.80. This result suggests that improvements to financial information technology lead to a significant increase in wealth dispersion.

Table 1.6: Business Cycle Statistics

Statistic	Data	Model economy		
		CM	FI	NI
Standard Deviation(%)				
Output	1.81*	1.84	1.81	1.81
Relative S.D.				
Consumption	0.74*	0.45	0.51	0.52
Investment	2.93*	3.26	2.89	2.92
Bankruptcy rate	9.5	—	17.9	—
Correlation with Output				
Consumption	0.88*	0.64	0.74	0.76
Investment	0.80*	0.95	0.94	0.93
Bankruptcy rate	-0.50	—	-0.67	—

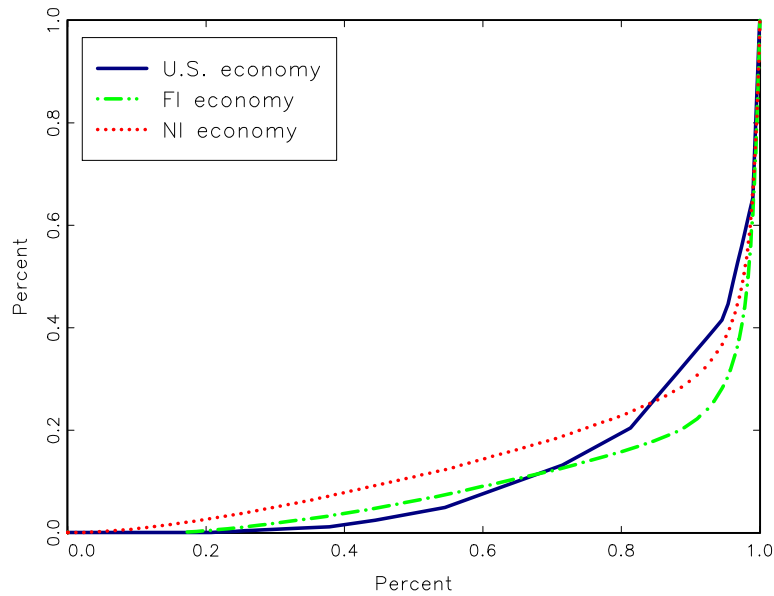


Figure 1.7: Wealth Distribution — Lorenz Curve

1.6 CONCLUSION

This chapter investigated how economic fluctuations interact with consumer bankruptcy under improved financial information technology by constructing a heterogeneous-agent general equilibrium model that incorporates capital market imperfections and consumer bankruptcy. Households face aggregate shocks as well as idiosyncratic shocks on labor efficiency, discount factor and liability loss.

The key contribution of this chapter is that our quantitative model successfully replicates the countercyclical property and high volatility of bankruptcy rate. Our quantitative model also successfully replicates the main features of the U.S. statistics, such as statistics on bankruptcy (including average bankruptcy rate), debt, and wealth distribution (high wealth concentration and Gini coefficients). We also find that improvements to financial information technology under aggregate uncertainty (*i*) explain the first observation, and (*ii*) increase wealth dispersion.

CHAPTER 2

A QUANTITATIVE STUDY ON FIRM DEFAULT AND TAX REFORMS

2.1 INTRODUCTION

2.1.1 MOTIVATION

Almost all developed countries rely on corporate income taxation as an important source of public financing. Auerbach et al. [6] pointed out the potential normative justification for taxing corporations; corporations may offer an easier point of tax collection, the base of taxation may be most easily measured at the corporate level, and taxing corporations may expand the scope of possible tax bases. As a result, many researchers and policymakers have great interest in its design.

The base of corporate income taxation in almost all Organization for Economic Cooperation and Development (OECD) countries corresponds to a measure of company profits and deductions for interest payments (Stukey and Mirrlees [40]). That such a deductibility distorts a firm's behavior in several undesirable ways is well known. For example, deductibility makes borrowing favored over retained profits or new equity as a source of financing for corporate investments. It may also leave firms more exposed to the risk of insolvency and bankruptcy. To equalize the opportunity costs of debt and equity, tax professionals designed two polar reform proposals to eliminate the differential treatment of debt and equity, namely the Comprehensive Business Income Tax (CBIT) and the Allowance for Corporate Equity (ACE). Regarding

tax bases, CBIT removes the deductibility for interest payments. In contrast, ACE proposes treating equity in the same manner as debt.

The IFS Capital Taxes Group [19] elaborated on ACE, which was based on the earlier notion of Boadway and Bruce [8]. Variants of the ACE tax have been implemented in Croatia, Austria, Belgium, Italy, and Brazil (Klemm [26]). Traditionally, researchers noted that, at the company level, ACE would exempt the cost of equity financing from taxation, simply subject profits exceeding a normal rate of return to taxation, and be neutral with respect to marginal investment decisions under certain circumstances.

The CBIT was developed by the U.S. Treasury Department in the beginning of the nineties. No real-world experiments of actual CBIT regimes exist yet. However, countries have imposed reforms that limit the deductibility of interest in some way, usually through thin-capitalization rules that imply that interest is not deductible from profits if the debt-to-equity ratio exceeds a certain threshold. Buttner et al. [9] reported that some form of thin-capitalization rules were introduced by approximately 60% of the European countries in 2005 and were effective in reducing debt-to-equity ratios.

Natural questions to ask include how the ACE and CBIT tax reforms affect welfare, a firm's investment and frequency of default. Many researches analyzed the effect of the introduction of ACE and CBIT (such as Radulescu and Stimmelayr [37], Keuschnigg and Dietz [24] and Mooij and Devereux [32]). Economists traditionally favor an ACE system in the closed economy since it offsets a firm's investment distortion and it obtains neutrality between debt and equity finance. The main critique of introducing ACE in the open economy is that it results in a narrower tax base and hence requires a higher corporate tax rate to balance government revenues, which is not attractive to multinational firms deciding on locations for their investments.

The international distortions implied that CBIT reforms may enhance welfare more than ACE reforms (Devereux and Griffith [14]).

2.1.2 ABOUT THIS CHAPTER

This chapter will cast doubt on the superiority of ACE over CBIT in the closed economy quantitatively. We construct general equilibrium models with heterogeneous firms, endogenous firms' decision between debt financing and equity financing and default options. Firms face idiosyncratic shocks to their productivity.

One difference between previous studies and this chapter is to introduce firm's heterogeneity, which is important in analyzing the effects of tax reform, as emphasized by Gourio and Miao [17] in analyzing dividend taxation. Depending on a firm's productivity and its capital stock, it attempts to make the optimal financing decisions such as issuing new equity, paying dividends, or financing using debt. In other words, heterogeneous firms use different financial instruments. Because of firm heterogeneity, different firms in different financial situations may have different responses to tax reform.

Another difference that this paper adopts is that it incorporates firms' endogenous default option and the corresponding firm-specific interest rate. The existing literature on ACE and CBIT ignores firm defaults although defaults may play an important role in firm's financing decisions and on its welfare.

Once the firm heterogeneity and default options are introduced, ACE system is no longer neutral on firm's investment decision and debt-equity financing decision. We note that the scope of the chapter is not to find an optimal tax system but to design corporate income taxation given that government will rely on a certain amount of financing from this system. We find that the tax reforms eliminate the tax advantages in borrowing over holding equity, which causes dramatically decreases in

debt ratio and default rate. We also find that, in our closed economy setup, welfare gains from the ACE and CBIT tax reforms are almost the same at 1.08% and 1.10%, respectively, in terms of consumption compensation.

2.1.3 RELATED WORKS

This chapter is related to the literature on the ACE and CBIT tax reforms. For example, Radulescu and Stimmelamayr [37] analyzed the switch to an ACE or a CBIT type of tax system from the present German tax system. The model included two countries with a representative firm and an infinitely lived agent. They showed that an ACE type of tax reform financed by an increase in the VAT and not in the profit tax might be preferred to a CBIT. Mooij and Devereux [32] explored the effects of ACE and CBIT using an applied general equilibrium model for the EU and found that if governments adjust statutory corporate tax rates to balance their budgets, profit shifting and a discrete location choice render CBIT more attractive for most individual European countries.

This chapter is also related to the recent literature on firm dynamics (such as Cooley and Quadrini [13], Gourio and Miao [17], Jermann and Quadrini [22] and Gilchrist and Zakrajsek [16]). In particular, Cooley and Quadrini [13] introduced financial market frictions into a heterogeneous firm dynamics model with productivity shocks and default option, and succeeded to account simultaneously for size and age dependence. Gourio and Miao [17] constructed a general equilibrium model with heterogeneous firms subject to idiosyncratic productivity shocks and analyzed the long-run effects of dividend tax reform. They showed that, in an economy with the representative firm, dividend taxation has no effect on long-run capital accumulation, but in an economy with heterogeneous firms, dividend taxation promotes long-run capital accumulation.

2.1.4 ORGANIZATION OF THE CHAPTER

The chapter is organized as follows. Section 2.2 characterizes the model economy. Section 2.3 defines the recursive competitive stationary equilibrium. Section 2.4 selects the parameter values and discusses the calibration procedures. Section 2.5 describes the main findings and section 2.6 concludes. The appendix to this chapter describes the computation procedures and provides some proofs.

2.2 THE MODEL

This section describes the model economy, which is mainly based on the firm dynamics model developed in Cooley and Quadrini [13]. The model's economy has four sectors: a representative household, firms, financial intermediaries, and the government sector. Firms are a continuum of measure one, are owned by the household, and are subject to idiosyncratic productivity shocks. They are ex ante identical but ex post different. Because we assume no aggregate uncertainty, all aggregate quantities and prices are deterministic in stationary equilibrium based on the law of large numbers.

2.2.1 FIRMS

Let $\tilde{\Omega}_t$ be the end-of-period value of a firm before issuing new shares or paying dividends at time t , which is the value of equity described by

$$\tilde{\Omega}_t = E_t \left[\sum_{j=1}^{\infty} m_{t+j} \{d_{t+j}(1 - \lambda(d_{t+j}))\} \right]$$

where m_{t+j} are the firm's discount factors explained later in this section, d_{t+j} are the amount of net payments to the shareholder (household) and $d_{t+j}\lambda(d_{t+j}) \geq 0$ are the adjusting costs associated with issuing new shares or paying dividends.¹ The adjusting

¹If the firm pays dividends, then d_t is positive. Otherwise, if the firm issues new shares, then d_t is negative.

costs $d_{t+j}\lambda(d_{t+j})$ capture the tendency that firms prefer to increasing equity with internally generated funds and issuing new shares or paying dividends occasionally or gradually.² To be efficient with notations, we drop the t subscript, and use primes and double primes to denote the variables for the next-period and two periods from now, respectively.

Firms combine labor and capital to produce goods according to the product function as $y = (z + \epsilon)G(k, l)$, where y is the output, k is capital input, sum of equity e and debt b , and l is labor input. We assume that capital and labor are perfect complements, and are used in fixed-proportion, $\frac{k}{l}$. Then, we express the firm's production function as $y = (z + \epsilon)f(k)$. We assume that z is persistent shocks, which follow the first-order Markov process, and ϵ is individual idiosyncratic distributed (*i.i.d.*). We denote \tilde{r} as the interest rate on debt charged by financial intermediaries described later. The timing of the events is as follows (see also figure 2.1).

1. **At the beginning of each period**, the equity e and debt b levels are already decided, and productivity shock z is already revealed. Then, the firm observes productivity shocks ϵ and is engaged in production. The firm's end-of-period net worth (before the decision to default) $\pi(e, b, z, \epsilon)$ is written as:

$$\pi(e, b, z, \epsilon) = (z + \epsilon)f(k) + (1 - \delta)k - wl - (1 + \tilde{r})b - T_c,$$

where T_c is corporate income tax payment.

²Jermann and Quadrini [22] noted that λ should not be interpreted necessarily as a pecuniary cost. It is a simple way of modeling the speed with which firms can change their source of funds. The possible pecuniary costs associated with share repurchases and equity issuance can also be incorporated as λ .

There are many specifications regarding the cost in previous literature. In this chapter, we model the cost corresponds to how much the firm changes its equity level, similar to Cooley and Quadrini [13] and Gilchrist and Zakrajsek [16].

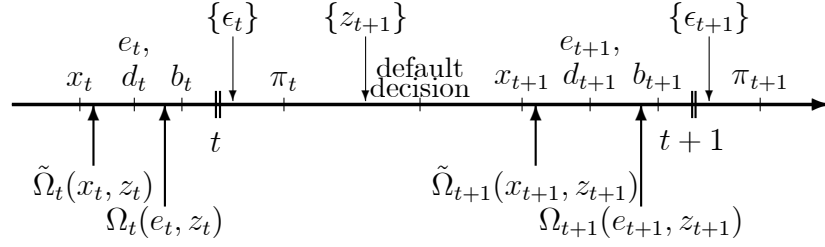


Figure 2.1: Timing of Events

2. **After revenues are realized**, the firm observes productivity shocks in the next period, z' , and decides whether or not to default on its debt. We assume that default does not lead to the firm's exit, but to the renegotiation of debt to the point at which the firm does not default. The firm defaults if its value as continuing entity at that time is less than zero: $\tilde{\Omega}(\pi, z') < 0$. We denote the end-of-period net worth by $\underline{e}(z')$ and the value of productivity shocks by $\underline{e}(e, b, z, z')$, which are thresholds of the firm's default decision. In other words, the firm defaults if $\pi(e, b, z, \epsilon) < \underline{e}(z')$ or $\epsilon < \underline{e}(e, b, z, z')$, where $\underline{e}(z')$ and $\underline{e}(e, b, z, z')$ are determined as

$$\begin{aligned}\tilde{\Omega}(\underline{e}(z'), z') &= 0 \\ \pi(e, b, z, \underline{e}) &= \underline{e}(z').\end{aligned}$$

Note that $\underline{e}(z')$ and $\underline{e}(e, b, z, z')$ are uniquely determined from the monotonicity of $\tilde{\Omega}(\cdot, z)$ and $\pi(e, b, z, \cdot)$ as discussed in Cooley and Quadrini [13]. Then, we denote the firm's equity after default decision and before issuing new shares or

paying dividends by x' , which is expressed by:

$$x'(e, b, z + \epsilon, z') = \max(\pi(e, b, z, \epsilon), \underline{e}(z')). \quad (2.1)$$

Equation 2.1 indicates that a firm's equity before issuing new shares or paying dividends is the same as the firm's end-of-period net worth $\pi(e, b, z, \epsilon)$ when the firm does not default: $\pi(e, b, z, \epsilon) \geq \underline{e}(z')$. The firm's equity becomes $\underline{e}(z')$ when the firm defaults and renegotiates its debt with a financial intermediary, as discussed later.

3. **After observation of productivity shocks z' , the default decision and the realization of x'** , the firm chooses how much to issue new shares or pay dividends, d' . The firm's equity after issuing new shares or paying dividends e' is $(x' - d')$. Then, the firm's problem at this stage can be described as:

$$\tilde{\Omega}(x', z') = \max_{d', e'} d'(1 - \lambda(d')) + \Omega(e', z') \quad (2.2)$$

subject to

$$e' = x' - d'.$$

Now, we denote the value of the firm after issuing new shares or paying dividends by $\Omega(e', z')$.

4. **After issuing new shares or paying dividends**, the firm decides the next period debt b' . The amount of the firm's debt and capital is assumed to have to be positive. The firm's problem at the stage can be written by:

$$\Omega(e', z') = \max_{b'} E_{z'} \left[\int m' \tilde{\Omega}(x''(e', b', z' + \epsilon', z''), z'') f(d\epsilon) \right] \quad (2.3)$$

subject to

$$\begin{aligned}
x''(e', b', z' + \epsilon', z'') &= \max \{ \pi(e', b', z', \epsilon'), \underline{e}(z'') \} \\
e' + b' &\geq 0 \text{ and } b' \geq 0 \\
\tilde{\Omega}(\underline{e}(z''), z'') &= 0,
\end{aligned}$$

where $f(d\epsilon)$ is the probability distribution function of productivity shock ϵ and m' is the firm's discount factor. Under the assumption that all firms are owned by the representative household, a firm's discount factor m' is equivalent to the household's intertemporal marginal rate of substitution. Note that the dividend policy and the choice of the next period's debt are decided simultaneously, but we consider that both of them are decided at different stages for convenience.

2.2.2 FINANCIAL INTERMEDIATION

In the model, the financial intermediation sector collects deposits from households by offering a promising interest rate r . They lend the funds to firms and charge a firm-specific interest rate \tilde{r} . We assume that a firm's type (e, z) is common information among financial intermediaries and firms. We also assume that interest rate \tilde{r} is determined by the financial contract under the two stage game. In stage one, firms name an amount of debt b . In the second stage, financial intermediaries compete in an auction in which they simultaneously post an interest rate \tilde{r} . They are engaged in a Bertrand competition. We denote $\Pi_{FI}(\epsilon | e, b, z)$ as the revenue of the financial intermediary that entered into the financial contract with firms of type (e, z) , offering b and being hit by productivity shock ϵ . If the firm does not default in the next period, $\epsilon \geq \underline{e}(e, b, z, z')$, the financial intermediary will collect its entire claim on the firm. Then, revenue is simply described as:

$$\Pi_{FI}(\epsilon | e, b, z) = (1 + \tilde{r})b \quad \text{for } \epsilon \geq \underline{e}(e, b, z, z').$$

If the firm defaults in the next period, $\epsilon < \underline{\epsilon}(e, b, z, z')$, the financial intermediary will not collect the full amount of its debt. It will discharge its debt and renegotiate for a new loan such that the firm's equity becomes $\underline{\epsilon}(z')$, which makes the continuing value of the firm at this stage zero because the financial intermediary can collect a repayment amount from the firm by renegotiating its debt than by liquidating the firm. If the financial intermediary liquidates a firm that cannot repay its debt, it will lose $-\pi(e, b, z, \epsilon)$. Otherwise, if the financial intermediary renegotiates the debt, it will lose only $\underline{\epsilon}(z') - \pi(e, b, z, \epsilon)$. As a result, the revenue for the financial intermediary in this case is described as

$$\begin{aligned}\Pi_{FI}(\epsilon|e, b, z) &= \pi(e, b, z, \epsilon) + (1 + \tilde{r})b - \underline{\epsilon}(z') \\ &= ((1 + \tilde{r}) - \Theta)b \quad \text{for } \epsilon < \underline{\epsilon}(e, b, z, z')\end{aligned}\quad (2.4)$$

where $\Theta = \frac{\underline{\epsilon}(z') - \pi(e, b, z, \epsilon)}{b}$ is the loss rate for the financial intermediary when the firm defaults. Given the law of large numbers, the total profit for entering into contract with a large number of firms with (e, b, z) is equal to the expectation of revenue Π_{FI} over ϵ minus the cost of lending, which is described as:

$$\begin{aligned}\pi_{FI}(e, b, z) &= E_z(\Pi_{FI}(\epsilon|e, b, z)) - (1 + r + \zeta(b))b \\ &= (1 + \tilde{r})b - E_z \left[\int_{\epsilon < \underline{\epsilon}} \Theta f(d\epsilon) \right] b \\ &\quad - (1 + r + \zeta(b))b,\end{aligned}$$

where cost of lending consists of repayment of the risk-free loan $(1 + r)b$ and the intermediation cost for lending $\zeta(b)b$ with $\zeta'(b) \geq 0$.

In equilibrium, competition among financial intermediaries results in no profits for any of their contracts. Then, the interest rate \tilde{r} is described as

$$\tilde{r}(e, b, z) = r + \zeta(b) + E_z \left[\int_{\epsilon < \underline{\epsilon}} \Theta f(d\epsilon) \right]. \quad (2.5)$$

The third term of the left hand side in equation 2.5 is interpreted as the risk premium. Then, the firm's interest rate is the sum of the risk free rate, intermediation costs and the risk premium.

2.2.3 HOUSEHOLDS

We assume that the representative household derives utility from consumption and leisure according to the standard utility function described as

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \right],$$

where β is the discount factor, C_t is consumption and L_t is labor supply, and $u(.,.)$ satisfies $u_1 > 0$, $u_{11} < 0$, $u_2 < 0$, $u_{12} > 0$, and the Inada conditions. The household owns a firm's share and trades them in the end of each period. The household also holds risk-free asset A as a deposit to financial intermediaries for the promise of receiving interest payment rA .

1. **At the beginning of each period**, the household starts with holding asset A and the firms' share θ , and decides how much to supply its labor L to firms at wage rate w .
2. **At the end of each period**, the household receives interest rA from financial intermediaries, accepts dividends or purchases new issuing shares, and decides how of the firms' share, θ' , and the risk-free asset, A' , to hold in the next period.

The budget constraint is given by:

$$\begin{aligned} C + A' + \int \theta' \Omega' d\mu' \\ = wL + (1+r)A + \int \theta \{d'(1 - \lambda(d')) + \Omega'\} d\mu \end{aligned}$$

where μ is the cross-sectional distribution of firms over the state (x, z) . The household solves the maximization problem by choosing $(A', C, L,)$ each time.

First-order conditions with respect to A' , L and θ' and equilibrium condition $\theta = 1$ imply:

$$\begin{aligned}
A' & : u_1(C, L) = \beta E_{z'} [u_1(C', L')(1 + r')] \\
L & : w = -E_{z'} \left[\frac{u_2(C, L)}{u_1(C, L)} \right] \\
\theta' & : \Omega' = E_{z'} \left[\left(\frac{\beta u_1(C', L')}{u_1(C, L)} \right) \int d''(1 - \lambda(d'')) + \Omega'' d\mu \right]
\end{aligned} \tag{2.6}$$

2.2.4 GOVERNMENT AND TAX SYSTEM

We consider a simple government budget rule in which tax revenues collected by the government are spent in an unproductive manner, G , such as a defence. That the government expenditure to output is a fixed proportion Υ and the budget is balanced in each period is assumed.

For simplicity, that the government levies tax only on corporate income is also assumed. In other words, we ignore government taxation on other objects such as dividends, capital gains, labor income, and interest income. To finance through corporate income taxation, governments needs to determine the tax base and the tax rate. We denote TB_{SCIT} , TB_{ACE} and TB_{CBIT} as the tax bases for SCIT, ACE, and CBIT, respectively. The taxbases for SCIT, ACE, and CBIT are described as:

$$TB_C = (z + \epsilon)f(k) - \delta^*k - wl - \rho_b \tilde{r}b - \rho_e \tilde{r}e$$

$$for C = \{SCIT, ACE, CBIT\}$$

where ρ_b and ρ_e are coefficients that indicate the tax base and δ^* is the tax deduction depreciation rate. As a reminder, SCIT provides a deduction for interest on debt when computing the tax base. In other hands, ACE and CBIT equalize the opportunity cost of debt and equity. The CBIT provides a deduction for interest on debt and the opportunity cost of financing equity, and ACE provides neither. The values of ρ_b and

Table 2.1: Tax Base for SCIT, ACE, and CBIT

	SCIT	ACE	CBIT
ρ_b	1	1	0
ρ_e	0	1	0

ρ_e are described in Table 2.1. We also denote the tax rate for SCIT, ACE and CBIT by τ_{SCIT} , τ_{ACE} and τ_{CBIT} . We assume that the government cannot collect taxes from firms whose tax base is negative, and firms cannot carry back or carry forward its tax loss for keeping the computation manageable. Then, the tax revenue from each firm is simply equal to the tax base multiplied by tax rate, say, $T_c = \tau_C \times \max(TB_C, 0)$ for $C = \{SCIT, ACE, CBIT\}$.

2.3 EQUILIBRIUM

We now define a steady state recursive competitive equilibrium. It is characterized by a set of household's decision rules $\{C, \theta', A', L\}$, a firm's decision $\{e, b, d\}$, prices $\{r, \tilde{r}, w\}$, and a measure μ such that:

1. *Firms' problem:* Given prices, firms solve their problems;
2. *A household's problem:* Given prices, households solve their problem;
3. *A financial intermediary's problem:* \tilde{r} solves financial intermediary's problem;
4. Asset and labor markets clear:

(a) Asset market

$$A = \int b d\mu$$

(b) Labor market

$$L = \int l d\mu$$

5. A balanced government budget:

$$\Upsilon = \frac{\int T_c d\mu}{\int y d\mu}$$

6. Measure is stationary and its transition is consistent with a firm's decision rule.

2.4 PARAMETERIZATION

To study the economy numerically, parameters need to be calibrated.

The utility function takes the following form with a unit Frisch elasticity of labor supply:

$$u(C, L) = \ln(C) - \frac{hL^2}{2},$$

where $h > 0$ is the weight on the disutility of labor supply. The intertemporal discount factor β is set to $\frac{1}{1.03}$ so that the annual interest rate is 3%, which is approximately consistent with the Japanese economy from 1980 and to 2000.

The production function is a Cobb-Douglas form with decreasing returns to scale:

$$(z + \epsilon)A_c k^\nu,$$

where $0 < \nu < 1$. We set ν to 0.95, from Miyagawa et al. [31]. The depreciation rate δ is set to 10%, a standard choice in the literature. Unfortunately, we could not obtain micro evidence for estimating parameters to determine the process of productivity shocks. Following Cooley and Quadrini [13], we specialize in an analysis of the case in which z follows a symmetric two-state Markov process with $\Pr(z_1|z_1) = \Pr(z_2|z_2) = 0.95$. The values of the shock are set as $\{z_1, z_2\} = \{0.9, 1.1\}$. The technology shock ϵ is assumed to be normally distributed with mean zero and standard deviation σ_ϵ .

We simply specify the cost of issuing new equity (paying dividends) $\lambda(d)$ and the intermediation cost $\zeta(b)$: $\lambda(d) = \phi_d d$ and $\zeta(b) = \phi_b b$, respectively.

The corporate tax rate τ_{SCIT} is set at 40%, which represents the sum of the national and local taxes on corporate income in the Japanese economy from 2002 to 2011. We set the ratio of corporate tax collection in output Υ , which is to 5%, consistent with the Japanese economy (Cabinet Office of Japan [35]).

Simultaneously, we calibrate the remaining model parameters $\{A_c, \phi_d, \phi_b, h, \sigma_\epsilon, \delta^*, \frac{k}{l}\}$ to match seven targets. The probability of default is 2.8%, equivalent to the average default ratio from 1997 to 2002, taken from Table 1 of Sakai et al. [39]. The ratio of corporate tax collection to output is 5%. Labor supply is 0.3, standard choice in literature. Aggregate debt-to-capital ratio is 68% and the average debt-to-capital ratio of the top 50% largest firms is 72%, taken from the Survey for Actual Conditions for SMEs 2009. The investment share in output is approximately 22.5% from the JIP(Japan Industry Productivity) Database 2008. The labor's share of income is approximately 52% from 2002 to 2006 (Cabinet Office of Japan [34]).

Table 2.2 shows values of fixed parameters and endogenously calibrated parameters.

2.5 RESULTS

This section presents the calibration results and the properties of the model. Table 2.3 reports the target statistics and the implied results from the model. For the economy with SCIT, we successfully replicate the Japanese economy. The ratio of corporate tax collection to output and labor supply in the model economy completely matches the targets. Investment share, default ratio, debt-to-capital ratio, labor's

Table 2.2: Parameter values

Parameters		Value
Fixed Parameters		
Discount rate	β	$\frac{1}{1.03}$
Return to scale	ν	0.95
Depreciation rate	δ	0.1
Transition matrix	$\Pr(z' z)$	$\begin{pmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{pmatrix}$
Value of z	$\{z_1, z_2\}$	$\{0.9, 1.1\}$
Endogenously Calibrated Parameters		
Firms's productivity	A_c	0.412
Cost of changing level of equity	ϕ_d	0.003
Intermediation cost	ϕ_b	0.00303
Weight on disutility of labor supply	h	9.37
Standard deviation. of productivity shock	σ_ϵ	0.37
Depreciation rate for tax deduction	δ^*	0.155
Fixed proportion of capital and labor	$\frac{k}{l}$	4.15

share, and the average debt ratio in the top 50% largest firm match sufficiently the targets.

2.5.1 PROPERTIES OF RISK PREMIUM AND INTEREST RATE

We now discuss the properties of a firm's interest rates. As a reminder, a firm's interest rate is the sum of the risk free-rate, intermediation costs and the risk premium, as described in equation 2.5. The risk premium corresponds with a firm's default risk, caused by the stochastic shocks ϵ and z' on a firm's productivity. The points that we want to make are as follows.

Table 2.3: Results of the Model Economy

Parameters	Targets	Model (SCIT)
Tax Collection-Output Ratio (%)	5.0	5.0
Labor supply	0.30	0.300
Investment share (I/Y)	0.225	0.230
Default ratio(%)	2.80	2.75
Debt-Capital Ratio (B/K)	0.68	0.74
Labor's share (wL/Y)	0.52	0.59
Average debt ratio in the top 50% firm	0.72	0.67

1. **Monotonicity in debt ratio:** For each capital level, the risk premium increases weakly as the debt ratio increases. Figure 2.2 provides an example of the property, which shows the risk premium for high productivity firms that hold capital 1.5, 2.5, and 3.5, respectively. If a firm's debt ratio is sufficiently low, the risk premium is zero, meaning that firms with low debt ratio will not default even if they are hit by the worst productivity shocks.
2. **High productivity firms vs. low productivity firms:** The risk premium for low productivity firms is equal to or higher than that for high productivity firms when firms choose the same portfolio. For example, figure 2.3 shows that for each debt ratio and capital level (1.5 and 2.5), the risk premium for low productivity firms is equal to or greater than that for high productivity firms. As long as the debt ratio is sufficiently low, the risk premium is zero for both low and high productivity firms.

Appendix B provides the proofs for both properties. Intuitively, from the view of the financial intermediary, firms with *higher* debt ratio or *lower* productivity are more

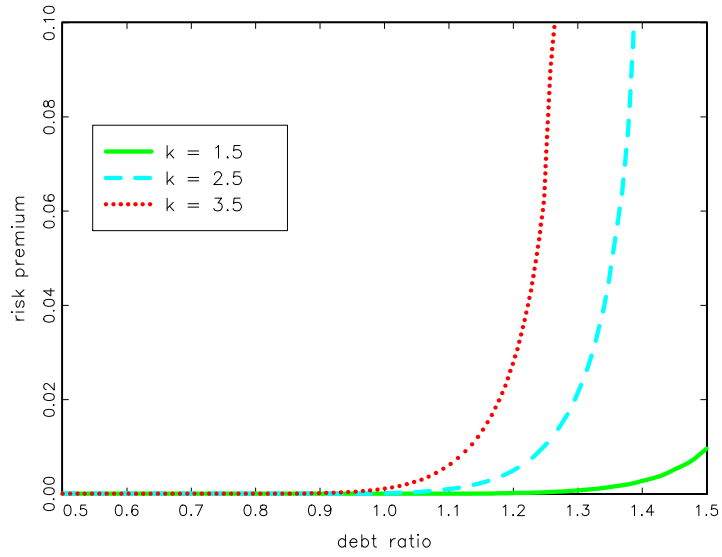


Figure 2.2: Risk Premium and Debt Ratio (b/k) for High Productivity Firms

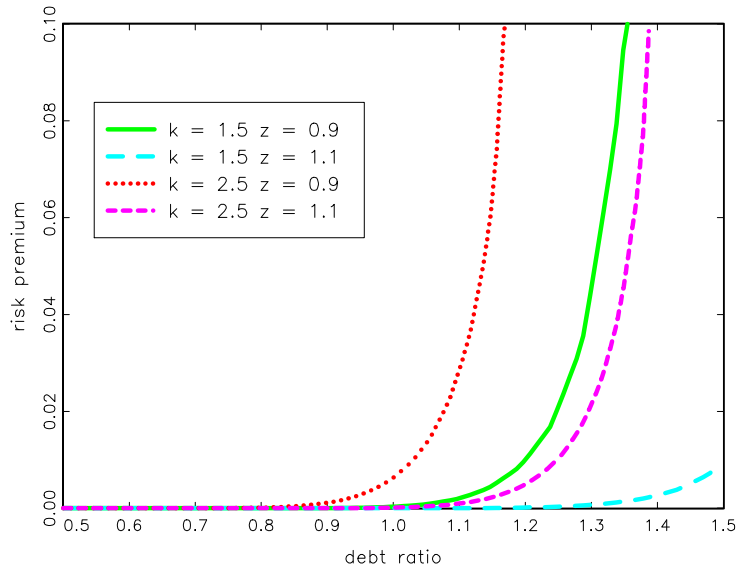


Figure 2.3: Risk Premium and Debt Ratio (b/k) for High and Low Productivity Firms

likely to default and bring *higher* loss rates when they default. Then, the financial intermediary will offer a *higher* risk premium and a *higher* interest rate to compensate for the *higher* loss rate and probability of default.

2.5.2 SCIT vs. ACE vs. CBIT

Given the success of our model with the SCIT system in replicating the data, we examine how the ACE and CBIT tax reforms affect the Japanese economy. In the model economies with ACE and CBIT, the broadness of the tax bases is different from SCIT. To keep the ratio of corporate tax collection to output at 5%, we calibrate the corporate tax rate in the economies with ACE and CBIT, respectively, keeping other parameter values constant.

Table 2.4 shows the results of the model economies with SCIT, ACE, and CBIT. The summary of the findings in table 2.4 is as follows. First, we successfully calibrate the corporate tax rate in the economies with ACE and CBIT. The ratio of corporate tax collection to output matches 5% in both economies. Corporate tax rates increase by 2.9% and decrease by 6.3% after the tax reforms to ACE and CBIT, respectively.

Second, the debt ratio dramatically decreases after the tax reforms. In the economy with SCIT, borrowing is significantly favored over holding equity: the debt ratio is higher than 70%. In contrast, it sharply decreases to 17% and 9% after the ACE and CBIT tax reforms are implemented, implying that these reforms significantly affect a firm's portfolio choices.

Third, the tax reforms result in a zero default rate, which also results from the effect of the tax reforms on a firm's portfolio choices. The tax reforms eliminate the tax advantages in borrowing over holding equity. Then, firms will not risk default or will not borrow as more as to pay the risk premium.

Table 2.4: Model Economy Results

parameters	SCIT	ACE	CBIT
Corporate Tax			
Tax Collection-Output Ratio(%)	5.0	5.0	5.0
Corporate tax rate(%)	40.0	42.9	33.7
Other Statics			
Output	0.539	0.537	0.535
Consumption	0.383	0.385	0.385
Capital	1.24	1.24	1.23
Labor Supply	0.300	0.299	0.298
Debt Ratio	0.73	0.17	0.09
Wage rate	1.076	1.097	1.092
Default rate(%)	2.8	0	0
Welfare improvement(%)	–	1.08	1.10

Finally, the ACE and CBIT tax reforms improve social welfare by 1.08% and 1.10% respectively. The welfare improvement from the tax reforms is calculated in terms of consumption compensation over time to make households in the economy with SCIT have the same welfare as in economies with ACE and CBIT. Under our parameter values, the economy with CBIT attains almost the same welfare improvement as the economy with ACE. This result is slightly surprising because previous studies showed that ACE will not distort a firm's investment decision and is better than CBIT in a closed economy with the representative firm. Note that once we introduce idiosyncratic productivity shocks into the model, ACE, as well as SCIT and CBIT, will create distortions in a firm's investment behavior.

2.6 CONCLUSION

This chapter investigated how the ACE and CBIT tax reforms from SCIT affect a firm's decision and social welfare. We constructed a heterogeneous-firm general equilibrium model with endogenous firms' decisions between debt financing, equity financing, and the default option. Firms face idiosyncratic productivity shocks and are subject to corporate taxation.

The model can replicate the main features of the Japanese economy with an SCIT system. We find that the ACE and CBIT tax reforms significantly affect a firm's behavior. The tax reforms eliminate the tax advantages in borrowing over holding equity, which dramatically decreases in the debt capital ratio and the default rate. We also find that, in our closed economy setup, welfare gains from the ACE and CBIT tax reforms are almost the same at 1.08% and 1.10%, respectively, in terms of consumption compensation.

We believe that this quantitative study will be helpful for policymakers in designing effective corporate taxation. Because this chapter is one of the first attempts to analyze the effect of the ACE and CBIT tax reforms under a firm's heterogeneity, we believe significant scope exists for enhancements. First, we need to analyze transitional problems related to moving to ACE and CBIT. For example, CBIT brings a significant disadvantage for firms relying on debt. Analyzing the dynamic effect of tax reforms made gradually over time might be interesting. Second, determining the optimal deductibility of debt and equity and the corresponding tax rate under a firm's heterogeneity may also be of interest. Third, introducing secured loans into our model may be interesting and may significantly affect a firm's behavior. We plan to address these potential analyses in our future work.

APPENDIX A

APPENDIX TO CHAPTER 1

A.1 COMPUTATION OF THE MODEL

This appendix describes the algorithm used to solve the model. Most of the difficulty comes from getting convergence many times for the law of motion and individual specific interest rates, which have large state spaces. Additionally, the computational task is burdensome because we need to solve thousands of equilibriums to find parameter values that satisfactorily match the target statistics.

As is well known, the state variable Γ is a high dimensional object. Following Krusell and Smith [28], we approximate the distribution, Γ , only using the mean of capital holdings (aggregate capital). In other words, each agent perceives that prices depend only on the subset of moments. Additionally, in order to solve the model numerically, the law of motion must be specified in certain forms. We start with the log-linear family:

$$\ln K' = \Phi_1(z, z_{-1}) + \Phi_2(z, z_{-1}) \ln K \quad \text{for } \forall z, z_{-1},$$

where $\Phi_1(z, z_{-1})$ and $\Phi_2(z, z_{-1})$ are coefficients that depend on aggregate shocks in the current and previous periods.

As previously described, we choose a transition matrix for aggregate shocks and labor efficiency shocks to ensure that aggregate labor supply (or unemployment rate)

only depends on aggregate productivity shocks (economic situations). Then, the risk-free interest rate and wage rate are determined by aggregate state z and aggregate capital K .

We now provide more details on each step.

Step 1. Make a guess of law of motion by setting the initial values of coefficients, $\Phi_1(z, z_{-1})$ and $\Phi_2(z, z_{-1})$.

Step 2. Make a guess of the probability of default in the next period and compute corresponding individual specific interest rates, $\mathcal{R}(a, \mathbf{s}_{-1}, z, K)$, and endogenously determined borrowing limits, $\phi(z_{-1}, \mathbf{s}, \Gamma)$.

Step 3. Given prices $r(z, K)$, $w(z, K)$ and $\mathcal{R}(a, \mathbf{s}_{-1}, z, K)$, and the law of motion, a household's decision rule for the state $\Theta_h = \{a, \mathbf{s}_{-1}, \mathbf{s}, K\}$ solves the dynamic problem by value function iterations. No theory guarantees the concavity of value functions. We need to use a non-derivative dependent maximization solver. A household's decision rules consist of the saving rule $a'(\Theta_h)$ and the bankruptcy rule $d(\Theta_h)$.

Step 4. Given decision rules, $a'(\Theta_h)$ and $d(\Theta_h)$ and the law of motion, compute a new probability of default and corresponding new individual specific interest rates $\mathcal{R}^{new}(a, \mathbf{s}_{-1}, z, K)$.

Step 5. Set an evolution for the path of aggregate shocks z from time -1 to time 5000 and take an arbitrary distribution $\Gamma_0(a, s, s_{-1}, h)$ and aggregate capital $K_0 = \int a d\Gamma_0$ at time 0. We simulate the economy using decision rules $a'(\Theta_h)$ and $d(\Theta_h)$. Because decision rule functions are interpolated, they may not fall in the thin grid. Following the method suggested by Rios-Rull [38], for any Θ_h, h such that $a'(\Theta_h)$ with $a_i \leq a'(\Theta_h) \leq a_{i+1}$, where $\{a_i\}$ is a thin grid, we assume $a'(\Theta_h) = a_{i+1}$ with probability $\frac{a'(\Theta_h) - a_i}{a_{i+1} - a_i}$ and $a'(\Theta_h) = a_i$ with probability $\frac{a_{i+1} - a'(\Theta_h)}{a_{i+1} - a_i}$. In other words, we keep track of the probability distribution function $\Gamma_t(a, s, s_{-1}, h)$ instead of simulating a large number of households to determine the cross-sectional distribution. Regress

K' on K to compute the new law of motion, represented by coefficients $\Phi_1^{new}(z, z_{-1})$ and $\Phi_2^{new}(z, z_{-1})$.

Step 6. If the new individual specific interest rate and new law of motion are satisfactorily close to the old ones, respectively, then go to **Step 7**. Otherwise, update the probability of default and the law of motion, $\Phi_1(z, z_{-1})$ and $\Phi_2(z, z_{-1})$, and go back to **Step 1**. Once these converge, evaluate the fit of the laws of motions. Following Krusell and Smith [28], we use the R^2 linear fit measure and the standard errors of the regression. Check whether R^2 is close to one and the standard error is close to zero. If not, we need to add moments to the specification of the law of motion.

Step 7. If the model's statistics corresponding to the targets is satisfactorily close to one, then **Stop**. Otherwise, update the values of each calibrating parameter and go back to **Step 1**.

APPENDIX B

APPENDIX TO CHAPTER 2

B.1 COMPUTATION OF THE MODEL

This appendix describes the algorithm used to solve the model. The major difficulty in solving the model is to obtain convergence many times in a firm's specific interest rate $\tilde{r}(e, b, z)$, which has a large state space. We take 100 gridpoints in the e and b directions. We also take 101 grid points in the ϵ direction to sufficiently smooth firm's specific interest rate. Our computational task is very burdensome because we need to solve thousands of equilibriums to calibrate seven parameter values that satisfactorily match the target statistics. The model in chapter 2 only focuses on the stationary equilibrium in which all aggregate variables and prices are deterministic. The algorithms proceed in five steps.

Step 1. First, we set calibrating parameter values. We set $\frac{1}{\beta} - 1$ for the risk-free interest rate r , and make a guess on wage rate w . Then, we set initial value functions $\tilde{\Omega}(x, z)$ and solve $\underline{e}(z')$ such that $\tilde{\Omega}(\underline{e}(z'), z') = 0$ for each z' . Given $\underline{e}(z')$, we compute the corresponding firm's specific interest rate $\tilde{r}(e, b, z)$. Because there are no analytical solutions for $\tilde{r}(e, b, z)$, we compute $\tilde{r}(e, b, z)$ through an iterative method using equation 2.5.

Step 2. Given prices r , \tilde{r} and w , we solve the dynamic programming problem using recursive equations 2.2 and 2.3 through value function iterations. No theory guarantees the concavity of the value functions. We need to use a nonderivative dependent

maximization solver. In each iteration, we update the value function $\tilde{\Omega}(x, z)$, solve $\underline{e}(z')$, and compute the corresponding firm's specific interest rate $\tilde{r}(e, b, z)$. After the convergence in $\tilde{\Omega}(x, z)$ and $\underline{e}(z')$, we stop the iterations. Then, we obtain a firm's decision rules, $b(e, z)$ and $e(x, z)$.

Step 3. We set the initial $\mu(x, z)$ and simulate the economy using a firm's decision rules $b(e, z)$ and $e(x, z)$. We compute a stationary distribution by iterating forward until convergence, and the corresponding total labor demand $\int ld\mu$ and total debt $\int bd\mu$.

Step 4. We plug the equilibrium conditions, $A = \int bd\mu$, $L = \int ld\mu$, and $\theta = \theta' = 1$, into the household's budget constraint. Then, we compute consumption C and the new wage rate from the first-order condition in equation 2.6. If the new wage rate is satisfactorily close to old one, then go to **Step 5**. Otherwise, update the wage rate and go back to **Step 1**.

Step 5. If the model's statistics are satisfactorily close to corresponding targets, then **stop**. Otherwise, update the values of each calibrating parameter and go back to **Step1**.

B.2 PROOF: MONOTONICITY OF THE RISK PREMIUM IN THE DEBT RATIO

From equation 2.4, the ratio of the revenue for the financial intermediary when the firm defaults to the amount of debt is described as:

$$\frac{\Pi_{FI}(\epsilon|e, b, z)}{b} = (1 + \tilde{r}) - \Theta \quad \text{for } \epsilon < \underline{\epsilon}. \quad (\text{B.1})$$

Taking the partial derivative of both sides of equation B.1 with respect to b (keeping k constant) results in:

$$\frac{\frac{\partial \Pi_{FI}}{\partial b}|_k b - \Pi_{FI}(\epsilon|e, b, z)}{b^2} = \frac{\partial \tilde{r}}{\partial b}|_k - \frac{\partial \Theta}{\partial b}|_k. \quad (\text{B.2})$$

Because $\Pi_{FI}(\epsilon|e, b, z)$ is positive and $\frac{\partial \Pi_{FI}}{\partial b}|_k$ is zero for any $\epsilon < \underline{\epsilon}$ as long as k is constant, then the left-hand side of equation B.2 is negative. Then, $\frac{\partial \Theta}{\partial b}|_k > \frac{\partial \tilde{r}}{\partial b}|_k$ holds for any $\epsilon < \underline{\epsilon}$. Taking the partial derivative of equation 2.5 with respect to b results in:

$$\frac{\partial \tilde{r}}{\partial b}|_k = \zeta'(b) + E_z \left[\int_{\epsilon < \underline{\epsilon}} \frac{\partial \Theta}{\partial b}|_k f(d\epsilon) \right]. \quad (\text{B.3})$$

Now, plug the conditions $\frac{\partial \Theta}{\partial b}|_k > \frac{\partial \tilde{r}}{\partial b}|_k$ and $\zeta'(b) \geq 0$ into equation B.3 to obtain:

$$\frac{\partial \tilde{r}}{\partial b}|_k > \frac{\zeta'(b)}{\left(1 - E_z \left[\int_{\epsilon < \underline{\epsilon}} f(d\epsilon) \right]\right)} \geq 0.$$

Since $\frac{\partial \Theta}{\partial b}|_k > \frac{\partial \tilde{r}}{\partial b}|_k$, $\frac{\partial \Theta}{\partial b}|_k > 0$ also holds. Therefore, $E_z \left[\int_{\epsilon < \underline{\epsilon}} \frac{\partial \Theta}{\partial b}|_k f(d\epsilon) \right]$ must be positive, which means that the risk premium increases as the debt ratio increases.

B.3 PROOF: MONOTONICITY OF THE RISK PREMIUM IN PRODUCTIVITY

Suppose $\tilde{r}(e, b, z_1) < \tilde{r}(e, b, z_2)$ (or $E_{z_1} \left[\int_{\epsilon < \underline{\epsilon}} \Theta f(d\epsilon) \right] < E_{z_2} \left[\int_{\epsilon < \underline{\epsilon}} \Theta f(d\epsilon) \right]$) for some (e, b) and $z_1 < z_2$. The revenue for the financial intermediary when the firm does not default in the next period is simply $(1 + \tilde{r})b$, which implies $\Pi_{FI}(\epsilon|e, b, z_2) > \Pi_{FI}(\epsilon|e, b, z_1)$ for $\epsilon > \underline{\epsilon}$. Because the revenue for the financial intermediary when the firm does not default in the next period is independent from interest rate \tilde{r} :

$$\Pi_{FI}(\epsilon|e, b, z_2) - \Pi_{FI}(\epsilon|e, b, z_1) = (z_2 - z_1)f(k) > 0$$

is satisfied for any $\epsilon \leq \underline{\epsilon}$. In other words, $\Pi_{FI}(\epsilon|e, b, z_2) > \Pi_{FI}(\epsilon|e, b, z_1)$ for any ϵ , which implies $E_z(\Pi_{FI}(\epsilon|e, b, z_2)) > E_z(\Pi_{FI}(\epsilon|e, b, z_1))$. This contradicts the zero profit condition for both z_1 and z_2 . Therefore, the risk premium for low productivity firms must be greater than for high productivity firms: $\tilde{r}(e, b, z_1) \geq \tilde{r}(e, b, z_2)$. Inequality holds if there exist some probabilities with which firms with low productivity default in the next period.

BIBLIOGRAPHY

- [1] S. Rao Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684, 1994.
- [2] Fernando Alvarez and Urban J. Jermann. Efficiency, equilibrium, and asset pricing with risk of default. *Econometrica*, 68(4):775–798, July 2000.
- [3] Cristina Arellano. Default risk and income fluctuations in emerging economies. *American Economic Review*, 98(3):690–712, 2008.
- [4] Kartik Athreya. Shame as it ever was : stigma and personal bankruptcy. *Economic Quarterly, Federal Reserve Bank of Richmond*, issue Spr:1–19, 2004.
- [5] Kartik Athreya, Xuan S. Tam, and Eric R. Young. A quantitative theory of information and unsecured credit. *American Economic Journal: Macroeconomics*, 4(3):153–83, July 2012.
- [6] Alan Auerbach, Michael Devereux, and Helen Simpson. Dimensions of tax design: The mirrlees review. In Institute for Fiscal Studies, (IFS), editor, *Dimensions of Tax Design*, chapter 9, pages 837–904. Oxford University Press, 2010.
- [7] Truman Bewley. A difficulty with the optimum quantity of money. *Econometrica*, 51(5):1485–1504, 1983.
- [8] Robin Boadway and Neil Bruce. A general proposition on the design of a neutral business tax. *Journal of Public Economics*, 24(2):231–239, July 1984.

- [9] Thiess Buttner, Michael Overesch, Ulrich Schreiber, and Georg Wamser. The impact of thin-capitalization rules on multinationals? financing and investment decisions. Discussion Paper Series 1: Economic Studies 2008,03, Deutsche Bundesbank, Research Centre, 2008.
- [10] Marco Cagetti and Mariacristina De Nardi. Wealth inequality: Data and models. *Macroeconomic Dynamics*, 12(Supplement S2):285–313, September 2008.
- [11] Sugato Chakravarty and Eun-Young Rhee. Factors affecting an individual’s bankruptcy filing decision. URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=164351. May 1999.
- [12] Satyajit Chatterjee, Dean Corbae, Makoto Nakajima, and Jose-Victor Rios-Rull. A quantitative theory of unsecured consumer credit with risk of default. *Econometrica*, 75(6):1525–1589, November 2007.
- [13] Thomas F. Cooley and Vincenzo Quadrini. Financial markets and firm dynamics. *American Economic Review*, 91(5):1286–1310, 2001.
- [14] Michael P. Devereux and Rachel Griffith. Taxes and the location of production: evidence from a panel of us multinationals. *Journal of Public Economics*, 68(3):335–367, June 1998.
- [15] Javier Diaz-Gimenez, Edward C. Prescott, Terry Fitzgerald, and Fernando Alvarez. Banking in computable general equilibrium economies. *Journal of Economic Dynamics and Control*, 16(3-4):533–559, 1992.
- [16] Simon Gilchrist and Egon Zakrajsek. Credit spreads and business cycle fluctuations. *American Economic Review*, 102(4):1692–1720, June 2012.

- [17] Francois Gourio and Jianjun Miao. Firm heterogeneity and the long-run effects of dividend tax reform. *American Economic Journal: Macroeconomics*, 1:131–168, 2010.
- [18] David B. Gross and Nicholas S. Souleles. An empirical analysis of personal bankruptcy and delinquency. *Review of Financial Studies*, 15(1):319–347, March 2002.
- [19] IFS Capital Taxes Group. *Equity for Companies: A Corporation Tax for the 1990s*. IFS commentary. Institute for Fiscal Studies, 1991.
- [20] Mark Huggett. The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5-6):953–969, 1993.
- [21] Mark Huggett. Wealth distribution in life-cycle economies. *Journal of Monetary Economics*, 38(3):469 – 494, 1996.
- [22] Urban Jermann and Vincenzo Quadrini. Macroeconomic effects of financial shocks. NBER Working Papers 15338, National Bureau of Economic Research, Inc, September 2009.
- [23] Timothy J Kehoe and David K Levine. Debt-constrained asset markets. *Review of Economic Studies*, 60(4):865–88, October 1993.
- [24] Christian Keuschnigg and Martin Dietz. A growth oriented dual income tax. *International Tax and Public Finance*, 14(2):191–221, April 2007.
- [25] Robert G. King and Sergio T Rebelo. Resuscitating real business cycles. In J. B. Taylor and M. Woodford, editors, *Handbook of Macroeconomics*, volume 1, Part B, chapter 14, pages 927–1007. Elsevier, 1 edition, 1999.

- [26] Alexander Klemm. Allowances for corporate equity in practice. IMF Working Papers 06/259, International Monetary Fund, November 2006.
- [27] Narayana R Kocherlakota. Implications of efficient risk sharing without commitment. *Review of Economic Studies*, 63(4):595–609, October 1996.
- [28] Per Krusell and Jr. Smith, Anthony A. Income and wealth heterogeneity in the macroeconomy. *The Journal of Political Economy*, 106(5):867–896, 1998.
- [29] Igor Livshits, James MacGee, and Michele Tertilt. Consumer bankruptcy: A fresh start. *American Economic Review*, 97(1):402–418, 2007.
- [30] Césaire A. Meh and Yaz Terajima. Unsecured debt, consumer bankruptcy, and small business. Working Papers 08-5, Bank of Canada, 2008.
- [31] T. Miyagawa, Y. Sakuragawa, and M. Takizawa. Productivity and business cycles in Japan: Evidence from Japanese industry data. *The Japanese Economic Review*, 57(2):161–186, 2006.
- [32] Ruud Mooij and Michael Devereux. An applied analysis of ACE and CBIT reforms in the EU. *International Tax and Public Finance*, 18(1):93–120, February 2011.
- [33] Makoto Nakajima and Jose-Victor Rios-Rull. default and aggregate fluctuations in storage economies. In Timothy J. Kehoe, T. N. Srinivasan, and John Whalley, editors, *Frontiers in Applied General Equilibrium Modeling*, chapter 7, pages 127–150. Cambridge University Press, 2005.
- [34] Cabinet Office of Japan. *Annual Report on the Japanese Economy and Public Finance (in Japanese)*. Printing Bureau, Ministry of Finance, 2008.

- [35] Cabinet Office of Japan. *Annual Report on the Japanese Economy and Public Finance (in Japanese)*. Printing Bureau, Ministry of Finance, 2010.
- [36] Vincenzo Quadrini. Entrepreneurship, saving, and social mobility. *Review of Economic Dynamics*, 3(1):1–211, 2000.
- [37] Doina Maria Radulescu and Michael Stimmelayr. ACE versus CBIT: Which is better for investment and welfare? *CESifo Economic Studies*, 53:294–328, 2007.
- [38] Jose-Victor Rios-Rull. Computation of equilibria in heterogeneous agent models. Staff Report 231, Federal Reserve Bank of Minneapolis, 1997.
- [39] Koji Sakai, Ichiro Uesugi, and Tsutomu Watanabe. Firm age and the evolution of borrowing costs: Evidence from japanese small firms. *Journal of Banking & Finance*, 34(8):1970–1981, August 2010.
- [40] I.F. Studies and J. Mirrlees. *Tax By Design: The Mirrlees Review*. Oxford University Press, USA, 2009.
- [41] George Tauchen. Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20(2):177–181, 1986.
- [42] Jay L Zagorsky and Lois R Lupica. Study of consumers’ post-discharge finances: Struggle, stasis, or fresh-start? *American Bankruptcy Institute Law Review*, 16(1):283–319, 2008.
- [43] Harold H. Zhang. Endogenous borrowing constraints with incomplete markets. *The Journal of Finance*, 52:2187–2209, 1997.