THE EFFECTS OF ANALYZING STUDENT WORK AND THINKING ON PRESERVICE TEACHERS' KNOWLEDGE AND BELIEFS: A MIXED METHODS STUDY

By

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2014

For my wife, Sarah, my parents, Pete and Karen, and my brother, Tim,
who supported me throughout the whole process

ACKNOWLEDGMENTS

There are many people I am tremendously thankful for regarding the success of this dissertation. From family and spousal support, to committee and advisor support, to fellow graduate student support, I owe each and every one of you a sincere thank you that I will try to put into words here. However, know that this brief thank you only begins to capture my boundless gratitude to you all.

First, I would like to thank my family for everything they have done. I owe a huge thank you to my wife, Sarah. I will always be grateful for the countless number of times you talked me off the ledge as I was conducting and writing this dissertation. I thank you and love you from the bottom of my heart. Your unwavering support made this process both possible and worth it. I also owe a huge thank you to my mother, Karen and my father, Pete. Without their support (both financial and moral) and guidance, I can honestly say I would never have been in the position to write this dissertation in the first place. Thank you both for everything you have done for me. You are wonderful people, I love you both, and I am proud to say I am your son. I would also like to thank my brother, Tim. You always talked as though you believed I would complete this journey even when my own belief wavered. The distractions you provided (including football games, nights out, and low key holiday adventures) were integral to my sanity. I love you and appreciate everything greatly.

Next, I would like to extend a sincere thank you to my dissertation committee for all of the insight and help you provided me with. Your suggestions and feedback helped shape this document and my thinking about mathematics education research. Thank you for your time and genuine willingness to help me succeed. Furthermore, I would like to thank Dr. Tim Jacobbe. Your role as chairperson for my study was no easy task, but

your guidance through my graduate program was paramount for my success. Thank you for everything you did for me.

I am also extremely appreciative to my fellow graduate students (both at and outside of the University of Florida) who allowed me to bounce ideas off them and who provided feedback and support throughout. In particular, I would like to thank Kristen Apraiz, Julie Brown, and Mark Hogue for everything that you contributed – formally and informally – to this dissertation. I cannot thank you enough.

There remain countless other individuals who, in one way or another, helped shape this dissertation. Please know that I am forever grateful to everyone not listed here for your contributions. No matter how small, every piece of the puzzle is necessary for the whole picture to be seen. To everyone, I say thank you.

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May 2014

Chair: Tim Jacobbe
Major: Curriculum and Instruction

Using a situated theoretical perspective, this dissertation, presented in manuscript format, examined the effects of analyzing student work in the university classroom on the content knowledge and mathematical beliefs of preservice teachers (PSTs). Forty-two PSTs participated in the study. Quantitative data were collected from all 42 participants (randomly assigned to the treatment group [n=21] and control group [n=21]) using existing instruments designed to measure common content knowledge (CCK) and mathematical beliefs for effective teaching. These data were collected in a pretest/posttest format over the course of the eight-week study. Qualitative data were collected from four treatment group participants (selected using intensity sampling) through retrospective interviews.

By utilizing a mixed methods approach, this study was able to measure the effects on content knowledge and beliefs while also investigating the role that pedagogical content knowledge (PCK) played in PSTs' ability to analyze student work. The findings of this study highlight the importance of using context in the preparation of PSTs to teach mathematics effectively. Significant impacts were discovered on PSTs'

beliefs about the effective teaching and learning of mathematics. Furthermore, evidence of the elicitation of PCK was discovered in those participants who were exposed to the student work analysis.

The treatment and findings of this study were then used to inform two scholarly articles regarding the practical use of student work analysis. One article was directed towards teacher educators and shares how to select student work and use it to structure lessons for PSTs about the teaching and learning of mathematics. The second article was directed towards practicing teachers and shares how the use of student work can be leveraged for teacher development of knowledge and beliefs. Overall, the lessons learned from the effects of student work analysis on PSTs' knowledge and beliefs provided insight into how PST education programs can work to best prepare prospective teachers to teach mathematics in effective ways.

CHAPTER 1
INTRODUCTION

The Program for International Student Assessment (PISA) of the Organization for Economic Cooperation and Development (OECD) described mathematical literacy as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (Schleicher, 1999, p. 41). The need for increasing such mathematical competencies in U.S. citizens has been a point of focus in the literature over the past few decades (e.g., California Space Education and Workforce Institute, 2008; Gardner, 1983; NCATE, 2010). An identified lack of mathematical literacy in the United States has been a major factor driving this focus.

For example, Phillips (2007) reported that high numbers of adults struggled with daily tasks involving mathematics, including such tasks as computing interest paid on a loan (78% of those involved), calculating miles per gallon when traveling (71%), and determining a 10% gratuity for a lunch bill (58%). These deficiencies are likely due, at least in part, to the mathematics education they received during their days as primary and secondary students. Despite these alarming percentages, students can and should learn mathematics in deep, conceptual ways that lead to mathematical literacy (NCTM, 2000), which has been called the new literacy necessary for success in the world (Schoenfeld, 1995). So then, where do we look to improve the mathematical literacy of our citizenry as we work towards utilizing individuals' potential?

Teacher education has been researched as a way to increase K-12 students' understanding of mathematics (e.g., Hill, Rowan, & Ball, 2005). Research has revealed that teachers are among the most influential elements on students' success and mathematical literacy attainment (NCATE, 2010). Debates exist, however, over the specific types of knowledge and beliefs teachers must possess in order to teach mathematics effectively (National Mathematics Advisory Panel, 2008). As a result, how teacher education programs develop the appropriate knowledge and beliefs for effective mathematics teaching is also an element of debate. This dissertation will explore the potential for analyzing students' work and thinking in the development of mathematical knowledge for teaching (MKT) among preservice teachers (PSTs) (Figure 1-1) and their beliefs about what constitutes effective mathematics teaching.

Existing studies on the use of student work (e.g., Crespo, 2000; Kazemi & Franke, 2004) as well as the sociocultural and situative learning perspectives provide both the conceptual and theoretical rationale for the potential success of this study. Student work and thinking analysis have been shown to have the potential to provide vicarious opportunities for PSTs to learn about effective mathematics teaching while remaining situated in an authentic teaching context (e.g., Crespo & Nicol, 2006; Philipp, Armstrong, & Bezuk, 1993).

More specifically, this dissertation collected and analyzed quantitative data in an attempt to measure changes in common content knowledge (CCK) (one element of MKT) and beliefs about effective mathematics teaching that PSTs experience as a result of the study's treatment. Furthermore, it collected and analyzed qualitative data in an attempt to explore the elements of pedagogical content knowledge (PCK) being accessed by PSTs as they complete the treatment tasks. An undergraduate content course for prospective elementary school teachers served as the setting for the analysis of student work and thinking treatment. Specifically, this treatment asked PSTs to diagnose students' understanding and plan next steps for instruction based on the observations of the students' work and thinking. Through these activities, it was hypothesized that PSTs would develop CCK and the beliefs necessary to effectively teach mathematics and develop students' mathematical literacy.

Working Definitions

In the mathematics education literature, many terms are not universally defined. Therefore, several terms are defined here for the purpose of clarification for the reader. Unless otherwise indicated in the writing, the working definitions presented here are the intended meanings throughout this dissertation:

1. Conceptual understanding – knowledge that is rich in relationships. This can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network (Hiebert & Lefevre, 1986, pp. 3-4).

2. Procedural understanding – a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols. This type of understanding consists of rules or procedures for solving mathematical problems, many of which are chains of prescriptions for manipulating symbols (Hiebert & Lefevre, 1986, pp. 7-8).

3. Effective teaching – teaching that yields students with high levels of conceptual understanding and mathematical literacy.

4. Effective instruction – used interchangeably with effective teaching.

5. Student work – the responses students provide to a posed mathematical problem, which may include calculations, drawings, verbalizations, use of manipulatives, videos, etc.

6. Student thinking – the oral or written accompaniment that students use to justify or explain their work.

7. Justification – the demonstration or proof that something is just, right, or valid.

8. Explanation – giving a clear and detailed account of one's actions.

9. Reform-based mathematics – mathematics taught by reform-minded teachers.

10. Reform-minded teachers – those who pose problems and encourage students to think deeply about possible solutions, promote making connections to other ideas within mathematics and other disciplines, ask students to furnish proof or explanations for their work, use different representations of mathematical ideas to foster students' greater understanding, and ask students to explain the mathematics (Stiff, 2001).

Theoretical Perspective

Shulman (1986a, 1986b, 1987) conceptualized a specialized knowledge base required for the teaching profession as he built on earlier works in the field (e.g., Dewey, 1902; Schwab, 1978). Since then, continued work has been done within the field of mathematics education to more fully unpack the complex elements of this unique knowledge base. Ball, Thames, and Phelps (2008), Hill, Ball, and Schilling (2004), Hill, Ball, and Schilling (2008), and Hill, Rowan, and Ball (2005) have provided theoretical insight and empirical evidence to suggest that effective mathematics teachers draw upon several specific and definable components of knowledge when teaching. These elements align strongly with and build upon Shulman's original conceptualizations. The resulting theory of Mathematical Knowledge for Teaching (MKT) is often referred to as the "knowledge egg" and contains both subject matter knowledge (SMK) and pedagogical content knowledge (PCK) – six individual components in all (Hill, Ball, & Schilling, 2008, p. 377; Figure 1-1 below). This dissertation utilized this theoretical framework for the knowledge necessary for effective mathematics teaching and explored an intervention for developing CCK while also investigating whether the participants drew upon any of the elements of PCK during the intervention.

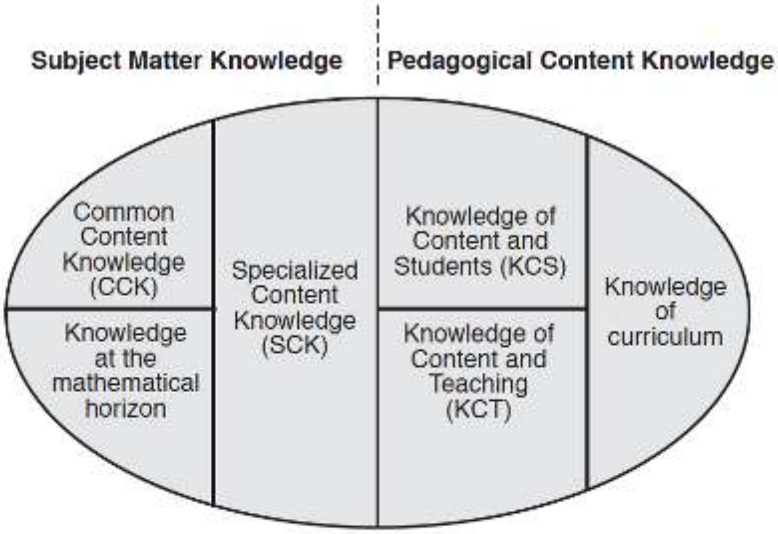


Figure 1-1. Mathematical knowledge for teaching.

This dissertation also drew upon existing teacher belief theory. Philipp (2008) utilized a theory for belief changes in prospective mathematics teachers. The theory known as "circles of caring" was derived from Noddings (1984). According to Philipp (2008), changing PSTs' beliefs about the importance of understanding mathematical concepts is paramount as a prerequisite to knowledge growth and development. Philipp (2008) explained the following:

> When my colleagues and I approached the issue of teaching mathematics to PSTs, we asked ourselves what it is they care about in relation to mathematics teaching and learning. We decided that fundamentally, PSTs entered teaching because they cared deeply about children, and rather than try to get PSTs to care about mathematics for mathematics sake, we took the approach that we wanted PSTs to care about mathematics for the sake of the children they would one day teach. Our Circles of Caring model highlights how their thinking about children may lead to PSTs' learning mathematics. (p. 8)

The model for this theory (Philipp, 2008, p. 9; Figure 1-2 below) shows the focus of children at the core. This theory maintains that PSTs' initial concern is to protect children and keep them comfortable, safe, and happy. Research studies (Philipp et al., 2007; Philipp, 2008) have shown that although PSTs initially associate their caring for children with the belief that they should avoid challenging them, with instructor support and opportunities for visual evidence (through viewing videos) PSTs can expand their circles of caring to include the mathematical thinking of children. Moreover, when PSTs learned about children's mathematical thinking, they began to redefine their circles of caring to include mathematics as they realized their own need to grapple with mathematics to prepare for supporting their students' learning. Philipp et al. (2007) also found that developing these beliefs does not require that PSTs be in direct contact with students in order for such changes to take place. Analyzing videos and vignettes of students' work and thinking was sufficient, and, in some cases, was actually more productive than field placements. Both this theory of PST beliefs about effective mathematics teaching and the previous theory of MKT were viewed through a sociocultural and situative lens.

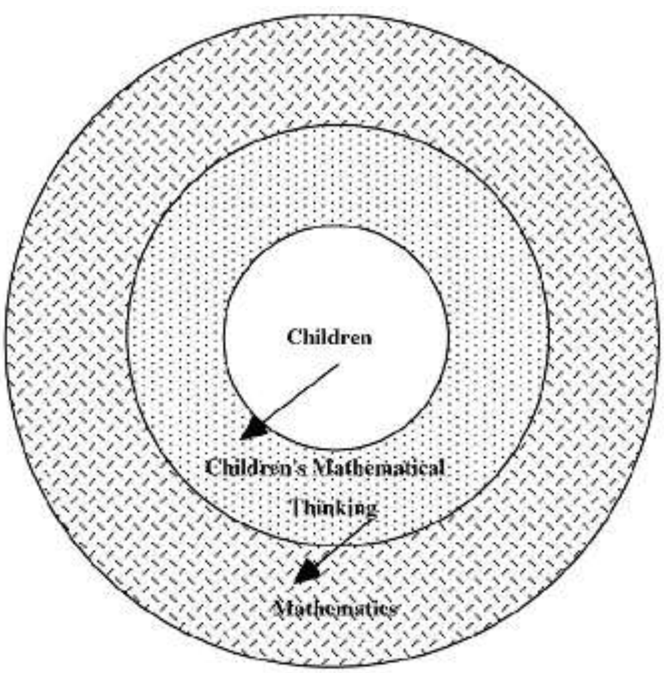


Figure 1-2. Circles of caring.

Sociocultural learning theories claim that “learning, thinking, and knowing are relations among people in activity in, with, and arising from the socially and culturally structured world” (Lave, 1991). Vygotsky (1978) stated that learning is embedded within social events, and social interaction plays a fundamental role in the improvement of learning. The notion of zones of proximal development (ZPD) helped to explain how this social interaction leads to learning. Nuthall (1997) further realized sociocultural theory by opining that “we understand a word by knowing how it is used, who uses it, and in what physical, social, and historical context it gets used. These are what we know when we ‘understand’ a word rather than some mental entity called ‘meaning’ (p. 731).” He added that “the words used to talk about mental processes refer to nothing more than the things we do in interaction with others when we are engaged in ‘thought-related’ activities (p. 732).”

Furthermore, Rogoff (1994) described a sociocultural framework for learning that has had considerable impact on the conceptualization of this dissertation. This theory, transformation through participation, says learning takes place when people participate in shared endeavors. It represents neither a sole focus on the learner nor the teacher, but rather a joint and collective effort. Involvement in social activities produces true learning. However, this theory fails to recognize what is happening within the individual during the participation.

Borko (2004) outlined an additional, necessary layer beyond Rogoff’s theory. Scholars have argued that learning has individual and sociocultural features (e.g., Borko, 2004; Cobb, 1994; Driver et al, 1994). They claim that the learning process is one of enculturation *and* construction (i.e., situative theory). This theory views learning both as “changes in participation in socially organized activities and individuals’ use of knowledge as an aspect of their participation in social practices” (Borko, 2004, p. 4). Both the individual and the group can be held as the unit of analysis. Although learning occurs through participation and social activities, individual knowledge is constructed and can thus be measured on an individual basis. However, all learning within the situative perspective is heavily tied to context and situation.

Situative theory holds that learning is grounded in everyday situations. Knowledge, therefore, is acquired through experiencing social situations, and is transferred only to similar situations. Instructors need to immerse learners in authentic activities that are relevant and applicable to the world outside of the classroom. Without authentic context, social learning that takes place will not be long lasting or internalized. Educators also need to make their classrooms places where inquiry is valued and reinforced (Wilson & Myers, 1999).

Vicarious or observational learning (Bandura, 1986) explains why learning can take place even when PSTs are not directly involved with students in K-12 classroom settings. According to Bandura, learning also happens when individuals are not involved in a learning situation themselves. Watching another individual solve a problem, interact with his or her peers, or interact with his or her teacher can vicariously produce learning experiences for those observing. Asking PSTs to analyze and discuss student work and thinking as they diagnose understanding and plan next steps utilizes, in part, a vicarious element of learning. It also allows PSTs to gain understanding and knowledge in context that otherwise could not happen within the confines of a university classroom. Together these theories of learning (i.e., situative, transformation through participation, vicarious learning, teacher knowledge, and teacher beliefs) provided the grounding for using student work and thinking to impact PSTs knowledge and beliefs for effective mathematics teaching.

According to the theoretical frameworks underpinning this study, PSTs must participate as well as socially negotiate, discuss, and reflect during their preparation programs in order to meaningfully learn. Analyzing student work and thinking has been shown to create such vicarious, social opportunities while remaining situated in an authentic teaching context (e.g., Crespo, 2000; Crespo & Nicol, 2006; Kazemi & Franke, 2004; Philipp, Armstrong, & Bezuk, 1993; Philipp et al., 2007; Son & Crespo, 2009; Stacey et al., 2001; Vacc & Bright, 1999). This dissertation focused on using this sociocultural theoretical framework to guide the development of student work and thinking analysis activities for PSTs designed to affect their CCK and beliefs about effective mathematics teaching. Furthermore, the research examined the elements of PCK that PSTs may use to diagnose student work and thinking and plan next steps for instruction through qualitative data analysis.

Teacher Links to Student Success

Teachers' Knowledge

The Conference Board of the Mathematical Sciences (CBMS) (2012) asserted that there are two critical pillars necessary for a strong K-12 student education. The first of these pillars was a well-qualified, knowledgeable teacher in every classroom. Several research studies support this claim by documenting the strong relationship between teachers' mathematical content knowledge and student achievement (e.g., Hill, Rowan, & Ball, 2005). Hill et al. (2005) studied the effects of various types of teachers' knowledge on student achievement in elementary grades. They reported that teachers' content knowledge was in fact a significant predictor of student achievement in mathematics.

Rowan, Correnti, and Miller (2002) also found significant correlations between teachers' knowledge and the achievement of students. Their study was in response to the methodological and conceptual issues that they saw in the research literature using large-scale survey data. It was found that specific teacher characteristics accounted for the discovered effects. Among these were content knowledge, teaching strategies, and patterns of content coverage – all are elements of MKT.

More results supporting the linkage between teacher knowledge and student achievement were found by Monk (1994). This large-scale study was based on data from the Longitudinal Study of American Youth that began in 1987. The results of the study showed a significant correlation between teachers' coursework and student achievement. The findings of this study support the claim of linkages between the content knowledge of teachers and student achievement.

These research studies have found that, in order to best promote student learning, teachers must be able to integrate many different areas of knowledge that align tightly to those described for MKT. In light of this, these authors have strongly suggested that any and all student achievement gains be approached through teachers' content knowledge development.

Teachers' Beliefs

Several studies also relay the importance of teacher beliefs about what constitutes effective mathematics teaching on student achievement. During the Cognitively Guided Instruction (CGI) project, results showed that when teachers' beliefs were more consistent with the principles of CGI (which included a focus on building instruction from observing student thinking), students' performed better on tasks related to various mathematical topics including whole numbers, fractions, and counting strategies (Carpenter et al., 1988; Carpenter et al, 1996; Carpenter et al., 1989). This research found that beliefs are not an entirely separate element from teacher knowledge, but that teacher belief changes translated into a greater focus on student thinking, problem solving, promoting conceptual understanding, and multiple solution strategies. This suggests that teachers' beliefs could affect knowledge of mathematics, students, and pedagogy. The specific belief changes necessary to promote increased student learning will be discussed in a later section. The CGI studies are not alone in these findings regarding the importance of teacher beliefs.

Several publications have documented the ability of beliefs to affect the acquisition of content and pedagogical knowledge by PSTs (e.g., Ambrose, 2004; CBMS, 2012). For example, Ambrose (2004) suggested a strong impact of beliefs on knowledge acquisition among PSTs. Furthermore, these publications suggested that PSTs' beliefs affect the way they teach, what subject matter they feel comfortable teaching, and they predict an effect on future student achievement.

The studies cited here highlight the empirical links between teacher knowledge, teacher beliefs and student achievement generally found in the mathematics education literature. They support the notion that increasing student achievement is a direct result of knowledgeable, well-prepared teachers. We must look to preservice teacher (PST) education programs to serve the role of identifying and developing the types of knowledge and beliefs necessary to best promote student achievement if we are to realize a more mathematically literate citizenry. However, an immediate roadblock arises because the knowledge and beliefs necessary for teachers to best promote student achievement is not well defined. Also contributing to this roadblock is the fact that best practices of PST education programs for developing knowledge and beliefs are equally ambiguous. There are, however, recent documents and literature that may help us hurdle the roadblock.

Teacher Knowledge and PST Education: Bypassing the Roadblocks

Whether one agrees or disagrees with the current curricula and standards reform in the U.S. (i.e., the Common Core State Standards for Mathematics [CCSSM]), these reforms help to define the skills and competencies students will need to survive in today's society and in the increasingly global economy (Horizon Research, 2011). Students will be required to perform on high stakes testing that will accompany this reform. With the introduction of the CCSSM, teacher education programs have a new, clearer goal. The new standards claim the number of topics teachers will be asked to teach is narrowing, while the focus on depth and conceptual understanding is increasing (CCSSI, 2010). National organizations continue to provide updated recommendations for teacher knowledge and beliefs (e.g., NCTM, 2000; NCTM, 2006). Teachers' knowledge and beliefs must now embrace conceptual understanding and multiple solution strategies over the traditional computation fluency towards a common procedure and answer (Ambrose, 2004). In light of all this, PST education programs must adapt and be able to define and develop in their prospective teachers the types of knowledge and beliefs necessary for promoting student success in the era of CCSSM and the current mathematics education reform. Literature exists that specifies some recommendations for alterations in current PST education programs; their focus is on changing the current organization and goals. Those pertinent to this argument are examined next.

What do PST education programs need to accomplish today? The National Council for Accreditation of Teacher Education (NCATE) Blue Ribbon Panel report (2010), the National Mathematics Advisory Panel (NMAP) final report (2008), and Darling-Hammond and Baratz-Snowden (2007) have made several recommendations for current programs in an effort to define what PST education should involve. Beginning teachers need opportunities to develop mathematical proficiency in the context of K-12 educational situations. Moreover, teachers need to learn to be "adaptive experts" since every classroom and student situation will be different (Darling-Hammond and Baratz-Snowden, 2007, p. 115). Student thinking must also be a focus in order to prepare teachers to grapple with and make use of the conceptions, misconceptions, prior knowledge, and unique solution strategies that students possess (NCTM, 2000). A deeper focus on the goals of using student work and thinking will appear later in this dissertation as one of the major elements informing the intervention.

PSTs also need a plethora of mathematical and pedagogical knowledge as well as knowledge of the intersection between the two. NCATE (2010) calls for teachers to have a robust knowledge base capable of informing them about content, how to teach it, and how to be innovative when working with students. They also call for a shift in emphasis from simply acquiring knowledge to using it to develop practice that effectively addresses the needs of students and promotes student achievement. According to NMAP (2008), all efforts to increase student achievement are in vain without "an adequate supply of mathematically knowledgeable and properly trained teachers" (p. 4).

It has been reported that PST education programs have not always historically provided the types of experience, knowledge and belief development necessary to prepare teachers for effective mathematics teaching (NCATE, 2010). As early as 1991, the Mathematics Association of America (MAA) published *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics* (Leitzel, 1991), and the National Council of Teachers of Mathematics made similar recommendations when writing their standards for teaching (NCTM, 1991). These documents called for changes in how PST education programs approached teacher preparation in light of the shift from procedural to conceptual understandings in mathematics. These recommendations are ideologically consistent with that of the CCSSM, which is bringing with it new financial backing linked to testing aimed at helping to realize these long-standing goals.

The ties between student achievement and teacher knowledge and beliefs remain prevalent. As a result, PST education programs must continue to seek best practices for developing knowledge and beliefs (Darling-Hammond et al., 1999). The National Governors Association (2011) reported that U.S. students – both high achieving and low achieving – are falling behind their counterparts in other developed nations. Students' mathematical performance in the U.S. must be faced head on. The goal in conducting this dissertation study is to explore the use of student work and thinking in the development of PSTs' MKT (CCK specifically) and beliefs about effective mathematics teaching.

Statement of the Problem

There are multiple current conceptualizations of the types of knowledge teachers need to teach mathematics effectively. At the forefront of the mathematical education literature is the knowledge “egg,” or MKT, conceptualization (Hill, Ball, & Schilling, 2008), which divides necessary knowledge into either mathematical content knowledge or pedagogical content knowledge. In many ways, this egg is theoretically supported and empirically grounded. However, the empirical evidence and ability to quantitatively measure gains in teachers and PSTs have been limited to the elements of pure mathematical knowledge. When PCK is the focus, the instruments created have failed to be reliable or valid (Hill, 2010).

Perhaps this lack of validation points to a need for more sophisticated or refined instruments, or perhaps the constructs of the PCK base are themselves the problem. But, it is more likely that the elements of PCK do not lend themselves to the quantitative measures that have been used. Thus, one purpose of this study is to examine how PSTs go about diagnosing understanding and planning the next steps in instruction when analyzing student work and thinking through qualitative data analyses. Thematic analysis (Aronson, 1994) will be used to help uncover what elements of PCK are used and to what extent PSTs are drawing upon them to complete treatment activities. These treatment tasks are fully explained in Chapter 3.

Although reliable content knowledge instruments have been created with sound psychometric values (Schilling, Blunk, & Hill, 2007), the research base remains thin in regards to interventions for increasing this knowledge and the related beliefs in PSTs. Studies exist that utilize qualitative methodologies to investigate teachers’ and PSTs’ experiences with analyzing student work and thinking (e.g., Crespo, 2000; Kazemi & Franke, 2004). Many studies have also included proxies for mathematical knowledge, including student achievement outcomes (e.g., Carpenter et al., 1989; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996). This dissertation sought to add a study to the literature that examines how analyzing students' work and thinking impacts PSTs' MKT and beliefs about effective mathematics teaching.

The literature base also remains thin in empirical quantitative studies that have investigated best practices for using student work and thinking to impact PSTs' CCK. Therefore, the second purpose of this study was to create and empirically test student work and thinking analysis activities as they apply to affecting PSTs' CCK. This intervention asked PSTs to both diagnose the levels of understanding students have as well as plan next steps in instruction. These activities will be fully explained in Chapter 3.

Finally, research has shown (e.g., Ambrose, 2004; Philipp et al., 2007; Sowder, 2007) that a focus on developing PSTs knowledge without a focus on their beliefs is a very counter-productive endeavor. The research base remains thin in regards to studies that simultaneously monitor knowledge and beliefs in PSTs who analyze student work and thinking. This dissertation therefore also sought to fill this gap by examining the direct effect of analyzing student work and thinking on PSTs' mathematical beliefs about what constitutes effective mathematics teaching.

Research Questions

The following research questions will guide the dissertation study:

- What is the influence of analyzing student work and thinking (by way of diagnosing understanding and planning next steps for instruction) on PSTs' CCK?
- What is the influence of analyzing student work and thinking (by way of diagnosing understanding and planning next steps for instruction) on PSTs' beliefs about effective mathematics teaching?

- What types of PCK do PSTs draw upon when diagnosing student understanding and planning next steps for instruction while analyzing student work and thinking?

Structure of the Dissertation

The written presentation of this dissertation will include five chapters. This chapter (Chapter 1) serves as an introduction and includes an overview of the literature and a theoretical framework, which together provide context for the study. Chapter 2 provides an in-depth review of the germane literature necessary for grounding and justifying this study. Chapter 3 is a formal methods section that will fully articulate the instruments, methodologies, interventions, data collection methods, and data analyses methods used. The results of the study are presented in Chapter 4 through a research article written for publication independent of the dissertation document. Chapter 5 and Chapter 6 are practitioner articles written to disseminate the treatment activities of this study through publication. A final chapter (Chapter 7) consists of conclusions, implications, limitations, and possible future research directions. Appendices include informed consent documents, treatment group course schedule, student work and thinking analysis activities, student work examples, an interview protocol, quantitative results tables, and quantitative instrument samples.

CHAPTER 2
REVIEW OF THE LITERATURE

In response to the issues and discussions about the knowledge and beliefs necessary for effective mathematics teaching and the best practices for the development of PSTs, many research studies have been conducted to help shine light on this area of mathematics education (e.g., Hill, 2010; Hill, Rowan, & Ball, 2005; Shulman, 1986a; Shulman, 1986b; Shulman, 1987). A portion of this research base involves the use of student work and thinking in aiding teacher knowledge development (e.g., Ambrose, 2004; Crespo, 2000; Fennema et al., 1996; Kazemi & Franke, 2004; Philipp et al., 2007). The following chapter of this dissertation will investigate the literature around teacher knowledge, teacher beliefs, and the use of student work and thinking in the development of mathematics teachers.

Mathematical Knowledge for Teaching (MKT): Knowledge for Success

Introduction to Necessary Knowledge

What types of knowledge do teachers need in order to produce mathematically literate students? According to the National Council of Teachers of Mathematics, effective teachers must possess several kinds of knowledge for teaching (NCTM, 1991). Among these are knowledge of the challenges students are likely to encounter in learning, knowledge about how ideas can be represented to teach effectively, and knowledge about how students' understanding can be assessed (NCTM, 2000). This requires teachers to, among other things**,** understand mathematics content, pedagogical strategies, and their students as learners.

Aligned well with NCTM, the Conference Board of Mathematical Sciences (CBMS) (2012) provided two recommendations for the knowledge preparation of PSTs:

(1) prospective teachers need mathematics courses that develop a good understanding of the mathematics they will teach (i.e., the development of content knowledge), and (2) coursework that allows time to engage in reasoning, explaining, and making sense of the mathematics they will teach (i.e., the development of teaching skills and pedagogical content knowledge (PCK)). Teachers need both types of courses in order to avoid relying on their past experiences as learners of mathematics during teaching (CBMS, 2012). Moreover, several studies have provided the grounding for the existence, conceptualization, and assessment of a robust knowledge base for effective teaching (Ball, Thames, & Phelps, 2008; Carpenter et al., 1989; Cobb et al., 1991; Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005; Saxe et al., 2001; Shulman, 1986a; Shulman, 1986b; Shulman, 1987). Next, I examine the history of how we have arrived at conceptualizing the necessary teacher effective knowledge in this manner as well as introduce a framework for unpacking teacher knowledge related to content, pedagogy, and students as learners. This history will ultimately lead us to the MKT framework.

The Early Years

Defining what teachers must know to be effective began as early as the 1900s. John Dewey, in his essay *The Child and the Curriculum*, wrote extensively about the difference between logical understanding (the knowledge of the "scientist") and psychological understanding (the knowledge necessary for “teachers”) (Shulman, 2008). Dewey constructed a notion of a specialized body of knowledge necessary for the teaching profession. He defined a category of professional knowledge that separated teachers from other professionals who might know a subject well, but who lacked opportunities to develop the knowledge needed for teaching that subject. Although this marks a milestone in the conceptualization of teacher knowledge, Dewey's ideas laid relatively dormant for many years. It was not until Lee Shulman's work in the 1980s that the idea of a specialized knowledge base for teaching once again received highlighted attention.

Shulman (1986a), Shulman (1986b), and Shulman (1987) were influential in determining the knowledge necessary to develop mathematical literacy in students. To be clear, these articles couch teacher knowledge in a more general sense but form the basis for many disciplines' views of teacher knowledge, including that of mathematics education. In these seminal pieces, Shulman discussed the then current conceptions of the teaching professional and the reform that would be necessary to fix them. Among those conceptions was the idea of what knowledge (or lack thereof) was necessary for teachers to perform and students to achieve. The specific content of these writings are explored next.

In what would later become the basis for his 1986 article, *Those Who Understand: Knowledge Growth in Teaching* (Shulman, 1986a), Shulman's 1985 presidential address to the American Educational Research Association (AERA) unveiled his conceptions about the knowledge base necessary for effective teaching. Also during this talk, he advanced the idea of PCK and called for the field to work on better understanding the categories that comprised the content and pedagogical knowing required for teaching (Fenstermacher, 1994). He set research in motion by commenting on two components of effective teachers' knowing: (1) the pedagogical structures of students' conceptions and misconceptions, and (2) the necessity for understanding the features that make particular topics easy or difficult to learn. After feedback and continued discussion about this address, Shulman's (1986a) article was published.

Shulman (1986a, 1986b) drew out and attacked the dreadful societal philosophy summed up by the quote of George Bernard Shaw, "He who can does. He who cannot teaches." According to Shulman, the teaching profession required very detailed and sophisticated ways of knowing. Teachers carry carefully constructed knowledge that involved both content and pedagogy. He proposed that the two were intertwined and thus formed an extremely complex base. Shulman stated, "As we have begun to probe the complexities of teacher understanding and transmission of content knowledge, the need for a more coherent theoretical framework has become rapidly apparent" (Shulman, 1986a, p. 9). Here, Shulman identified a teacher's complex way of knowing which involved many components related both to content and pedagogy. This call for a framework to help better conceptualize and unpack teachers' knowledge was the catalyst for a plethora of future work including that which led to the development of MKT and the knowledge "egg" (Hill, Ball, & Schilling, 2008) – the explanation of this "egg" will culminate in this section.

Shulman further refuted George Bernard Shaw's statement by rewording Aristotle's idea of the philosophy of teaching: "what distinguishes the man who knows from the ignorant man is an ability to teach, and this is why we hold that art and not experience has the character of genuine knowledge" (Shulman, 1986a, p. 7); this is not the only historical instance that portrayed teaching as a highly coveted profession. Medieval universities named their highest-ranking degree of "master" or "doctor" (they were often used interchangeably) because they meant "teacher." To be considered an expert historically was to be considered worthy of teaching the material to others. Even the term “bachelor” was used to indicate an apprentice teacher who was in the process of the long reign of practice teaching and knowledge development necessary to become truly knowledgeable. Maybe most importantly to Shulman, the medieval universities made no distinction between content and pedagogy – an important distinction compared to the way this has been viewed more recently.

Shulman (1986a, 1986b) accomplished a second task. He examined the history of the cleavage between content and pedagogy and called to action teacher education that addressed the two equally. This was in response to the pendulum swing he noticed in the history of teacher examinations. For example, in the 1870s, teacher preparation programs focused almost exclusively on content knowledge (during this era only 50 out of the total 1000 possible points were pedagogically related on a typical teacher certification examination). This reflected the philosophy of the time that understanding content was both necessary and sufficient for effective teaching. Shulman (1986a, 1986b) reported a significant shift in teacher certification examinations during the 1980s.

During this time teachers were tested much more heavily on their abilities to teach, and content knowledge fell to mere basic questions to determine basic competencies. Some examples of teacher certification categories in California in the 1980s were organization in preparing and presenting instructional plans, evaluation, recognition of individual differences, cultural awareness, understanding youth, management, and educational policies and procedures (Shulman, 1986a, p. 5). This left Shulman and many others wondering where the content had gone. However, asking for the content back and requiring a better balance of teacher preparation entailed the articulation of knowledge needed for effective teaching.

As a result, Shulman (1986a, 1986b) suggested that teacher knowledge had three parts – subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. These three elements serve as the basis for most current day conceptions of teacher knowledge (including in mathematics education), making them some of the most prolific developments in teacher education. Shulman (1986a) also conceptualized what these three types of knowledge entailed. The second article (Shulman, 1986b) also contributed to the conceptions of teacher knowledge, but in slightly less detail.

Subject matter content knowledge was defined as the amount and organization of knowledge in a teacher's mind. A teacher must not only know that something is so, but also must further understand why it so, as well as on what grounds its warrant can be asserted and under what circumstances it could fail to be justified (Shulman, 1986a, p. 9). PCK was described as a form of content knowledge that goes beyond general subject matter to subject matter specifically in the interest of teaching. This knowledge allowed teachers to employ the most useful forms or representations of ideas, the most powerful analogies during explanations, and well-conceptualized illustrations, examples, and demonstrations, "in a word, the ways of representing and formulating the subject to make it comprehensible to others" (Shulman, 1986a, p. 9). PCK represents a deep understanding of what makes a subject easy or difficult to learn. Curricular knowledge, on the other hand, deals with understanding materials used in teaching (texts, software, manipulatives, etc.) and understanding the vertical and lateral curriculum. The lateral curriculum entailed understanding how the subject being taught fits with other subjects students are learning simultaneously, while the vertical curriculum referred to understanding how the current material fits with what students have already learned as well as what they will learn in years to come. Having laid the groundwork, Shulman continued to fine-tune his ideas of teacher knowledge during the following year.

Shulman (1987) revisited most of the information presented in 1986. He also added two major components to this publication regarding the knowledge necessary for effective teaching. First, he elaborated considerably on his idea of PCK, calling it "the category most likely to distinguish the understanding of the content specialist from that of the pedagogue" (Shulman, 1987, p. 8). Teachers hold a very specific type of knowledge that allows them to blend their understanding of content and pedagogy. By doing so, teachers possess a unique understanding of how to adapt to the interests and abilities of their learners and of how topics should be presented for the learning to occur. Shulman (1987) recognized four sources for this specialized body of knowledge within the teaching profession. These sources were identified as:

- scholarship in content (knowing, understanding, skills, and dispositions to be learned by school children);
- materials and settings of the institutionalized educational process (from curricula, textbooks, school organization, and the structure of the teaching profession);
- research (involving schooling, human development, teacher development, and social and cultural phenomena);
- and the wisdom of practice itself (deemed the least codified, it was the maxim that guided or provided reflective rationalizations for teaching).

Preservice teacher education should and can provide the opportunities for the exposure of PSTs to all these sources. By 1987, the teacher education community had a much more sound and developed idea of the types of knowledge that lead to effective teaching. The activities and opportunities needed to develop this knowledge were (and to some extent still are) up for debate.

Shulman (1986a, 1986b, 1987) left his readers with a firm foundation of the types of knowledge he believed was necessary for teachers to effectively produce mathematically literate students – content knowledge, PCK, and curricular knowledge. As I continue to progress through the history of teacher knowledge, this foundation remains both constant and prevalent.

The Middle Years

Between 1987 and 2008 many educational scholars (both inside and outside of mathematics education) worked to more fully unpack and conceptualize the knowledge that Lee Shulman spoke and wrote about. For example, Berliner and Rosenshine (1987) opined that teacher education was on the verge of pinning down the specialized knowledge needed for effective teaching. They also encouraged rigorous research to further support this notion of a specialized knowledge base and move it beyond a presumption, which Shulman (1987) had begun to do with case studies of experienced teachers.

Researchers such as Elbaz (1991) and Schön (1991) qualitatively examined teachers' practices to determine the types of working knowledge present. Their conclusions suggested that effective teachers in fact had a specialized knowledge base that encompassed students' attributes, strengths, and weaknesses along with a deep repertoire of instructional techniques and management skills. Moreover, effective teachers seemed to possess knowledge of themselves, milieu, subject matter, curriculum development, and instruction. It is important to note that these knowledge observations are very well aligned with those of Shulman (1986a, 1986b), namely curricular, subject matter, and pedagogical content. Schön (1991) deemed this qualitative approach necessary for determining effective teacher knowledge because teachers work in messy social situations that do not necessarily lend themselves to hard, empirical science.

Furthermore, the National Research Council (NRC) (2001) examined both theoretical and empirical evidence of effective teacher knowledge and concluded that effective teachers needed to have a deep understanding of the subject, of the many approaches students might take on a problem, and of ways to guide students at different levels of understanding. Again, these recommendations closely resemble the explanations given by Shulman (1986a) when he described curricular, subject matter, and pedagogical content knowledge. One study examined by NCR (2001) was created by Saxe, Gearhart, and Nasir (2001). This study found a strong positive correlation between teachers' knowledge and students' achievement in mathematics. Teachers' levels of knowledge were determined by their participation in development activities that focused on student thinking, student motivation, and teachers' understandings of fractions. This empirical evidence suggested that teachers must have knowledge of both content and pedagogical content in order to be effective.

Carpenter et al. (1988) provided further empirical support for effective teacher knowledge. During their Cognitively Guided Instruction (CGI) project, they determined that all teachers had some level of knowledge about the subject, about their students (primarily of students' thinking), and about ways to teach. They further determined that this could and should be built upon. When teachers worked to develop these types of knowledge, Carpenter et al. (1988) reported increased student achievement in mathematics. This revealed again that knowledge of how students are learning (a form of Shulman's PCK) and knowledge about mathematics (subject matter knowledge (SMK)) are necessary for effective teaching.

Ball (undated) also demonstrated the need for knowledge beyond common mathematical content. During her days as a third grade teacher, she compiled the now famous "Shea Numbers Case." A lesson about odd and even numbers sparked a conversation among students about how to apply their newly acquired definitions. One student, Shea, claimed that six was an odd number because it was made up of three groups of two. His rationale seemed to be that a number made up of odd numbers must itself be odd. Also, Shea seemed to believe (and even convinced some classmates) that three groups of two would produce a number that would have a remainder when split into two groups (two to one person, two to a second person, and two left over). This way of thinking caused him to violate the class's working definition of "even" and thus classify the number six as odd. Based on the class discussions and Shea's persistence, Ball (undated) realized that this problem was not a small matter that a simple mathematical definition or explanation could solve.

The situation caused her to draw on several other types of knowledge for teaching. Schoenfeld (2002) opined that Ball was forced to adapt and co-construct knowledge with her students (as Darling-Hammond and Baratz-Snowden (2007) had suggested was necessary), involve large numbers of students, interpret students' language and mathematical ideas, encourage students' reflection, and also engage in reflection herself. Knowing how to approach these tasks required pedagogical knowledge, PCK, and knowledge of students. It was also clear that these types of knowledge were being used in addition to and in conjunction with mathematical content knowledge. Using this plethora of knowledge allowed Ball to effectively manage the lesson and produce elevated levels of understanding in her students (Schoenfeld, 2002, p. 151).

At this point in the history of classifying effective teacher knowledge, the conceptual foundation had been laid (Shulman 1986a, 1986b), and a supportive research base had begun to emerge (e.g., Shulman, 1987; Elbaz, 1991; Schon, 1991; Saxe, Gearhart, & Nasir, 2001). Next, I will examine how mathematics education in particular continued this progression into a well-articulated and researched framework regarding the teacher knowledge necessary to teach mathematics effectively and yield mathematically literate students.

Recent Years: The Framework

There has been a recent movement in mathematics education aimed at unpacking and defining the specific components of the knowledge necessary for effective mathematics teaching (e.g., Ball, 1999; Ball & Bass, 2000; Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Brown, McGatha, & Karp, 2006; Hill, 2010; Hill, Ball, & Schilling, 2004; Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005). This movement originated at the University of Michigan and went beyond simply categorizing teacher knowledge as either content, PCK, or curricular as Shulman had done. The details of several of these studies will now be addressed to demonstrate how they ultimately build to the knowledge “egg” conceptualized by Hill, Ball, and Schilling (2008).

Hill, Rowan, and Ball (2005) and Ball, Hill, and Bass (2005) studied the effects of teachers’ MKT on students’ achievement in elementary grades. Both studies were extensions of the Study of Instructional Improvement (SII) project. This project sought to determine the effects of widely disseminated reform programs on teachers, students, and schools in the elementary setting.

Drawing from the work of Dewey, Shulman, and others, Hill, Rowan, and Ball (2005) and Ball, Hill, and Bass (2005) embarked on a mission to find the exact components that made up an effective mathematics teachers' knowledge base, with a specific focus on content knowledge. They began by defining MKT as "the mathematical knowledge used to carry out the work of teaching mathematics. Examples of this "work of teaching" included explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs" (Hill, Rowan, & Ball, 2005, p. 373); and "a kind of professional knowledge of mathematics different from that demanded by other mathematically intensive occupations, such as engineering, physics, accounting, or carpentry" (Ball, Hill, & Bass, 2005, p. 35). MKT built on previous studies (Shulman, 1986a; Shulman, 1986b) but researchers began to dig deeper and tease out individual components in a much more practical and assessable way.

A meta-analysis of previous research studies (e.g., Begle, 1972, 1979; Greenwald et al., 1996; Hanushek, 1981, 1996) has shown that job experience, certification status, and postsecondary coursework are poor proxies of this teacher knowledge base (Hill, Rowan, & Ball, 2005). The authors therefore worked to develop and validate instruments to measure MKT in more reliable ways. When measured with their questionnaire and survey instruments (79% teacher-observer agreement reliability), the MKT of 699 teachers was a significant predictor of the 1,190 grade 1 and the 1,773 grade 3 students' success on the McGraw-Hill's Terra Nova Battery exams (Ball, Hill, & Bass, 2005; Hill, Rowan, & Ball, 2005).

What exactly did these instruments measure? To answer this question we must turn to their conceptualization of teachers' content knowledge. They created assessments that more intricately analyzed teachers' mathematical content knowledge. Content knowledge was seen as being comprised of common knowledge of mathematics as well as "specialized content knowledge" (SCK) (Ball, Hill, & Bass, 2005, p. 22; Hill, Rowan, & Ball, 2005, p. 377). SCK was described as the ability to represent mathematical material using diagrams, knowing how to provide careful explanations of mathematical rules, and understanding how to appraise the validity of alternative solutions strategies to a given problem. In contrast, common content knowledge (CCK) was described as the knowledge of mathematics that any well-educated adult would possess (Ball, Hill, & Bass, 2005, p. 22). The dichotomy of CCK and SCK marked the first extension from Shulman's ideas of teacher knowledge towards the knowledge "egg."

Ball, Hill, and Bass (2005) provided a well-conceived example of SCK that helps us to understand what these instruments look like. Given a problem like 25x35, the instrument may ask teachers to analyze students answers in the following ways: determine the correctness of a given answer, determine the validity of a given answer, determine what a student's strategy appears to be, determine if a strategy is generalizable, or determine if the given numbers (25 and 35) are good choices to unveil conceptual underpinnings of an algorithm (Appendix A). By creating and validating questions that address both CCK and SCK (like that of the above example), these instruments are capable of assessing components of MKT needed for effective teaching. This is evidenced by the instrument's ability to demonstrate the correlation between teachers' MKT and student achievement (Hill, Ball, & Bass, 2005). These conceptualizations of MKT served as the bases for the theoretical and empirical articles (e.g., Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008) that produced the current framework for MKT (i.e., the knowledge "egg").

In their fully theoretical piece, Ball, Thames, and Phelps (2008, p. 2) hypothesized an even greater refinement of Shulman's original classifications of teacher knowledge (Figure 2-1). This list represents a slightly more in depth configuration of teacher knowledge than did his 1986 works discussed earlier. Ball et al. began to

- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter
- Knowledge of learners and their characteristics
- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds
- Content knowledge
- Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers
- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

Figure 2-1. Shulman's major categories of teacher knowledge.

work on the ideas in Figure 2-1 together with their emerging conceptions of MKT. Figure 2-2 (Ball, Thames, & Phelps, 2008, p. 5; see below) maps Shulman's theory of PCK and SMK onto their own, creating the first version of what has been called the knowledge "egg." Both SMK and PCK are viewed as subsets of the umbrella term MKT.

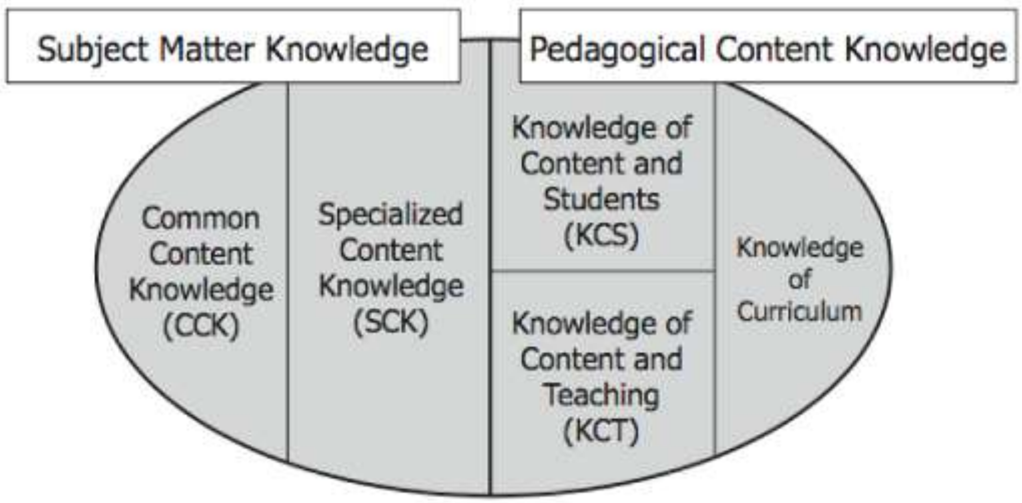


Figure 2-2. Shulman's original category scheme from 1985.

Based on their analysis of the mathematical demands of teaching, Ball, Thames, and Phelps (2008) hypothesized that Shulman's categories of content knowledge and PCK should be subdivided. The divisions came in the form of SMK into CCK and SCK, on the one hand, and PCK into knowledge of content and students (KCS) and knowledge of content and teaching (KCT), on the other. Their biggest interest was in the addition of SCK to Shulman's original theory. Like PCK, it is closely related to practice but does not require additional knowledge of students or teaching. SCK is distinctly mathematical knowledge that is not necessarily mathematical knowledge familiar to well-educated adults. The justification for SCK existing separately from PCK is provided in Figure 2-3 below. Each of these is something effective teachers routinely do and, taken collectively, they create unique mathematical knowledge. Such tasks imply that teachers need to know a body of mathematics not typically taught to students.

> Teachers need to understand different interpretations of the operations in ways that students do not. They need to know the difference between "take away" and "comparison" models of subtraction, and between "measurement" and "partitive" models of division. They also need to know features of mathematics that they may never teach to students, such as a range of non-standard methods or the mathematical structure of student errors. These knowledge demands are distinct from those described by Shulman under the label of pedagogical content knowledge (Ball, Thames, & Phelps, 2008, p. 6).

Presenting mathematical ideas
Responding to students' "why" questions
Finding an example to make a specific mathematical point
Recognizing what is involved in using a particular representation
Linking representations to underlying ideas and to other representations
Connecting a topic being taught to topics from prior or future years
Explaining mathematical goals and purposes to parents
Appraising and adapting the mathematical content of textbooks
Modifying tasks to be either easier or harder
Evaluating the plausibility of students' claims (often quickly)
Giving or evaluating mathematical explanations
Choosing and developing useable definitions
Using mathematical notation and language and critiquing its use
Asking productive mathematical questions
Selecting representations for particular purposes
Inspecting equivalencies

Figure 2-3. Mathematical tasks of teaching.

Ball, Thames, and Phelps (2008) supplied definitions for each of these subdivisions of MKT. CCK was defined here in the same manner as Ball, Hill, & Bass (2005, p. 22), that is the mathematical knowledge that one would expect any well-educated adult to have. CCK also includes "knowing when students have answers wrong, recognizing when the textbook gives an inaccurate definition, and being able to use terms and notation correctly when speaking and writing at the board. In short, it is the knowledge teachers need in order to be able to do the work that they are assigning their students" (Ball, Thames, Phelps, 2008, p. 6). The remaining mathematics content knowledge was classified as SCK.

SCK was again defined in the same manner as was done by Ball, Hill, and Bass (2005), that is, knowledge about mathematics which goes beyond that of a well-educated adult but does not yet require knowing about students or teaching. Teaching mathematics requires many tasks that fit this category (Figure 2-3). Overall, SCK refers to teachers' abilities to unpack elements of mathematics to make their features apparent and to teachers' abilities to coherently explain or justify mathematical ideas.

The need to draw on knowing about teaching, pedagogy, curricula, or students separates PCK from content knowledge alone. The PCK side of the "egg" is comprised of KCS, KCT, and knowledge of curriculum. KCS is defined as a type of PCK that intricately combines knowing about students and knowing about mathematics. There are several examples of this type of knowledge in action.

Teachers need to predict what students will find interesting and motivating about given problems or topics. When assigning a task, teachers need to anticipate what students are likely to do with it, as well as if they will find it easy or difficult (which was part of the definition of PCK given by Shulman, 1987) (Ball, Thames, & Phelps, 2008). Teachers must also be able to interpret students' emergent thinking, even if it is incomplete. Each of these tasks "requires an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking" (Ball, Thames, & Phelps, 2008, p. 9).

KCT is a closely related subcategory to KCS. This form of knowledge strategically combines knowing about teaching with knowing about mathematics. Teachers of mathematics are often faced with tasks that require their mathematical knowledge to interact with the design of instruction. Often, they must decide upon the ordering of content as they plan instruction and select which examples to start with and which are capable of pushing students' understanding to deeper levels. Teachers also need to evaluate the instructional advantages and disadvantages of potential representations of mathematical material and decide when to ask for more clarification, when to use students' remarks as teaching moments, and when to interject new questions or tasks to push students' learning forward (Ball, Thames, & Phelps, 2008). "Each of these requires an interaction between specific mathematical understanding and an understanding of pedagogical issues that affect student learning" (p. 9).

Finally, Ball, Thames, and Phelps, (2008) offer no novel description of curriculum knowledge. They instead rely on the definition put forward by Shulman (1987): knowledge that portrays a particular grasp of the materials and programs involved in teaching that serve as the "tools of the trade" for teachers. Having covered definitions and examples of the domains of MKT, let us now examine empirical support for this framework of effective teacher knowledge.

Hill (2010) assessed mathematics teacher knowledge based on the domains of MKT defined in the "egg" (Ball, Thames, & Phelps, 2008). A geographically stratified sample of 1200 U.S. elementary schools were chosen by a research database. From this sample, 1090 schools agreed to participate in the study. One mathematics teacher was randomly selected from each elementary school and offered $50 to complete a survey. The study ultimately surveyed the MKT of 625 teachers – a 59% response rate. Weights calculated to take both sampling and nonresponse into account were applied to the analysis (Hill, 2010).

The survey used in this study was designed to test the MKT that each teacher possessed. It was reported that the MKT framework (Ball, Thames, & Phelps, 2008) was used over other potential views of teacher knowledge because:

> …it was based in the real work teachers do in classrooms, with children. In fact, it was developed from a grounded study of mathematics teaching that involved observing teachers, students, and their interactions with mathematical content in real classrooms. The MKT framework also specifies a way of thinking about the various mathematics-related tasks teachers are asked to complete in classrooms, as opposed to a list of topics that teachers should master and upon which they should be assessed. Finally, MKT incorporates multiple forms of teacher knowledge that may affect instruction, in line with Shulman and colleagues' observations about the nature of teacher knowledge (Hill, 2010, p. 521).

The survey assessment of MKT contained questions that involved numbers and operations (arithmetic), specifically with rational numbers, whole numbers, and integers. This content was chosen because it constitutes 50% of the instruction delivered by elementary mathematics in the U.S. (Hill, 2010). In terms of the domains of MKT, the survey assessment contained 6 CCK items, 23 SCK items, 1 KCS item, and 7 KCT items. The item numbers stemmed from two issues: a heightened interest in SCK and a lack of success in writing KCS and KCT items. A variety of validation and pilot work was completed on the survey items before the study was conducted.

A set of cognitively tracing interviews were conducted that verified that the answers teachers gave on these survey items did in general reflect their underlying thinking. Furthermore, a study of 10 elementary teachers revealed that the quality of their classroom mathematics instruction (as evidenced by the analysis of 9 videotaped lessons per teacher) correlated highly with their MKT score on the survey assessment ($r = 0.74$). A similar study with similar results was conducted on 26 middle school mathematics teachers.

After achieving this validation, Hill (2010) analyzed the survey assessment data using factor analysis and item response theory. The results showed a variety of relationships. First, teacher experience was moderately related to MKT scores. Also, teacher self-perceptions and background coursework were very weakly correlated to MKT, and higher elementary grade level teachers tended to have higher MKT scores than lower grade level teachers. Finally, teachers had great difficulty with many of the SCK, KCT, and KCS questions, which suggested that development is needed by both PST and inservice teachers regarding types of knowledge outside CCK.

Both empirical and theoretical literature supports the usefulness of the MKT "egg" framework (Figure 2-2). We now turn to the final seminal piece of literature that completes the MKT framework outlining the knowledge necessary to increase student achievement and understanding to produce a more mathematically literate society.
Hill, Ball, and Schilling (2008) reported that the field was beginning to see widespread agreement that effective teachers have many kinds of knowledge, including knowledge in mathematics, students' thinking, and pedagogy. This study further developed the subcategories of MKT that were started by Ball, Thames, and Phelps (2008) (Hill, Ball, & Schilling, 2008, p. 377; Figure 2-4 below). Knowledge at the mathematical horizon is defined as "a recent development of an aspect of our theory that centers on a kind of

mathematical ‘peripheral vision,’ a view of the larger mathematical landscape, that teaching requires. We call this kind of vision horizon knowledge of mathematics and we consider it a part of mathematical knowledge for teaching” (Ball & Bass, 2000, p. 1). With the exception of this additional domain, the remaining elements of the knowledge “egg” are defined and arranged in an identical manner similar to that outlined by Ball, Thames, and Phelps (2008). However, special attention is paid to KCS and KCT in this study.

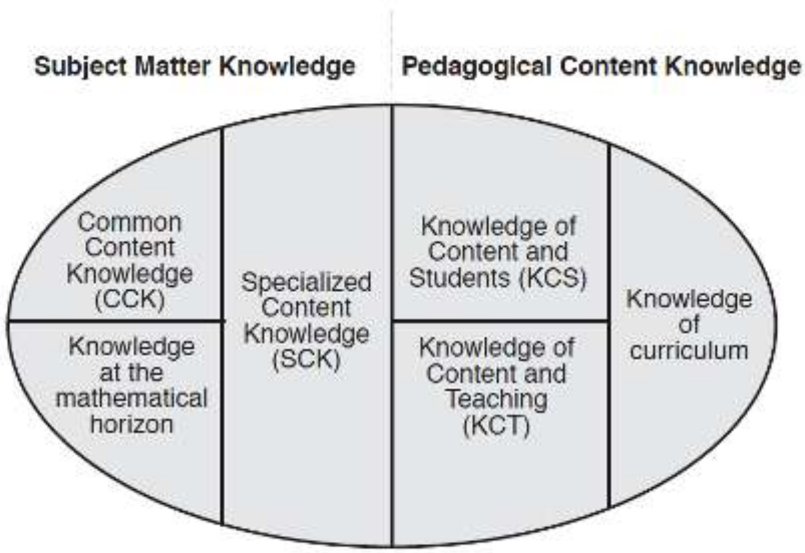


Figure 2-4. Mathematical knowledge for teaching.

The types of knowledge that comprise the PCK domains of KCS and KCT (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008; Shulman, 1986a; Shulman, 1986b) are salient and empirically supported features of effective teachers (Carpenter et al., 1989; Cobb et al., 1991; Hill, Ball, & Schilling, 2008; Saxe et al., 2001). However, few have successfully assessed teachers regarding them (including Hill, 2010). Simply knowing they exist and qualitatively observing them was not enough. This article was a self-proclaimed first attempt at measuring teachers KCS and KCT through assessments that can be practically used and administered.

Many previous studies examined how students solve problems and develop mathematically (e.g., Behr et al., 1992; Carpenter et al., 1989; Carpenter, Franke, & Levi, 2003; Fuson, 1992; Kamii, 1986; Lamon, 1999). They have informed this study's approach and instrument development. Hill, Ball, and Schilling, (2008) reaffirmed that KCS was viewed as requiring respondents to use knowledge of students' thinking about a given topic as well as their own mathematical knowledge. Therefore, KCS assessment items were designed to in part require respondents to draw on their knowledge of mathematics but not solely on this knowledge, as CCK or SCK items might. These assessment items also exhibited a focus on measuring teachers' abilities to reason about student work rather than simply "knowing that" students may develop in certain ways or make certain mistakes.

To help illustrate the type of knowledge KCS and KCT entail, the authors "found it helpful to think about what mathematically able individuals who do *not* teach children would *not* know" (Hill, Ball, & Schilling, 2008, p. 379). The authors noticed, a priori, that their assessment items fell into four categories:

- Common student errors: identifying and providing explanations for errors, having a sense of what errors arise with what content, etc.
- Students understanding of content: interpreting student productions as sufficient to show understanding, deciding which student productions indicated better understanding, etc.
- Student developmental sequences: identifying the problem types, topics, or mathematical activities that are easier/more difficult at particular ages, knowing what students typically learn first, having a sense of what third graders might be able to do, etc.

- Common student computational strategies: being familiar with landmark numbers, fact families, etc.

Both pilot testing at the California Mathematics Professional Development Institutes (CMPDIs) and data collected for the study showed two results. First, KCS is a distinguishable and important form of knowledge for effective teaching. The research also showed that it is possible to write items capable of identifying, at least in part, the levels of KCS that teachers possess. However, there was a fair amount of overlap between CCK, SCK, KCS, and KCT. Respondents also reported utilizing strategic test-taking skills to bolster their scores. The final conclusion was that more work must be done to better conceptualize the domain of PCK and better create items that can assess it. That work is continuing today, although much of it has yet to be published.

Although valid and reliable assessments are still a work in progress, the knowledge "egg" framework put forth by Hill, Ball, and Schilling (2008) is well documented and supported for describing the types of knowledge necessary for effective mathematics teaching. However, a MKT focus alone is insufficient in developing PST to teach mathematically effectively. As is typical in educational practice, no single element occurs in isolation. The beliefs that coincide with knowledge development must also be addressed in order to achieve mathematical literacy in students.

The Necessary Beliefs for Teachers

Definitions and Measurement

Beliefs are psychologically held understanding, premises, or propositions about the world that are thought to be true – they are lenses through which we see the world, dispositions towards our actions, and are held to varying degrees of conviction (Philipp, 2007). Developing PST knowledge without addressing the beliefs they hold is a very counterproductive endeavor (Ambrose, 2004; Philipp et al. 2007). Fundamental PST beliefs about teaching must typically be challenged in order for them to strengthen their knowledge for teaching mathematics (Sowder, 2007). Many PSTs are products of an education system that required only procedural mathematics knowledge that focused solely on the correctness of final answers. In addition, many PSTs see only procedural requirements during field placements and student teaching. This may create a belief structure that does not hold conceptual understanding as important (Eisenhart et al., 1993). When this occurs, PST education efforts to develop MKT may be in vain. "Teaching itself is seen by beginning teachers as the simple and rather mechanical transfer of information" (Wideen et al., 1998, p. 143). Weinstein (1989) determined that this homogeneous and simplistic view of teaching causes PSTs to undervalue the importance of their subject-matter preparation.

It is extremely important to identify and affect the beliefs that PSTs hold. MKT can only be developed if PSTs believe that effective teaching and learning of mathematics necessitates insights about students' thinking (KCS), about alternative methods and sound explanations (SCK), about basic mathematics (CCK), about the ordering and progression of materials and problems (KCT), about the progression of mathematics and how it fits with other subjects (knowledge at the mathematical horizon, curricular knowledge), etc. However, the belief held by many PSTs is that mathematics is a fixed set of rules and procedures that is best learned by being shown in a prescribed, step-by-step way (Philipp et al., 2007). This stands in sharp contrast to many of the reform efforts of PST education programs and the philosophy of learning that underlies MKT. Thankfully, beliefs have been measured and subsequently affected on many occasions in mathematics education research.

A typical belief instrument has traditionally been Likert-scale surveys (Philipp, 2007). Although many researchers have concerns about the validity of such instruments (due in large to the self-reporting nature and honesty requirement), many studies have used Likert scales to identify and track changes in teacher beliefs. A beliefs instrument was created and used for the CGI study (Fennema, Carpenter, & Loef, 1990; Carpenter et al., 1989). This instrument was a 48 question Likert-scale assessment designed to address beliefs related to the role of the learner, the relationship between skills and understanding, the sequencing of topics, and the role of the teacher. Many other studies have also used this instrument (e.g., Vacc & Bright, 1999; Fennema et al., 1996).

Zollman and Mason (1992) and Enochs, Smith, and Huinker (2000) have also developed widely-used belief instruments. Zollman and Mason (1992) based their instrument on the Curriculum and Evaluations Standards for School Mathematics (NCTM, 1989). Their hope was to determine and track changes in teachers' beliefs about the NCTM standards as part of teacher development. Enochs, Smith, and Huinker (2000), on the other hand, created beliefs assessments involving teaching efficacy and expectancy outcomes. One question used on this survey was, "Even if I try very hard, I will not teach mathematics as well as I will most subjects." This type of question has raised concerns about whether or not teachers' reports are accurate. There remains considerable debate about whether Likert scales are valid measures from which to draw inferences about teachers' beliefs (Philipp, 2007). There is, however, another school of thought regarding the assessment of beliefs.

Ambrose, Clement, Philipp, and Chauvot (2004) developed a web-based, open-ended survey for assessing teachers' beliefs as part of the Integrating Mathematics and Pedagogy (IMAP) project. This effort was in response to three major issues the research team saw in Likert-scale instruments: problems inferring how respondents interpret the wording of items, lack of information for determining the importance of the issue to respondents, and the provision of little to no context. Take, for example, a Likert-scale item that states "It is important for a child to be a good listener in order to learn how to do mathematics." How would the researcher determine the way the respondent is interpreting a "good listener" if they only report where they fall on a Likert scale (Philipp, 2007)? Ambrose et al. (2004) worked to create an instrument capable of overcoming this issue. The IMAP beliefs survey utilized four critical elements of beliefs identified in the literature. They are:

1. Beliefs influence perceptions – they shape individuals' interpretations of events (Pajares, 1992).
2. Beliefs draw one towards a particular position or direction regarding a given issue (McGuire, 1969; Rokeach, 1968).
3. Beliefs are not all-or-nothing entities – they are held with differing intensities (Pajares, 1992).
4. Beliefs tend to be context specific, arising in situations with specific features (Cooney, Shealy, & Arvold, 1998).

The researchers viewed these elements as accounting for the vital role that beliefs play in the teaching and learning of mathematics. As a result, they strongly considered these elements as they developed their instrument.

The IMAP team addressed these elements in many ways (Ambrose et al., 2004). Their open-ended questions required respondents to interpret complex situations that would necessitate drawing on one's beliefs (in response to critical element 1). Moreover, they provided respondents with opportunities to make decisions about teaching. These opportunities were general enough to be asked of both practicing and prospective teachers (in response to critical element 2). The answers to these questions provided the researchers with enough information to infer the underlying belief (Philipp, 2007). To address critical element 3, the IMAP team provided tasks with multiple interpretation points. They also carefully ordered the questions to avoid giving away the preferred answers. Finally, their beliefs survey situated segments in contexts and inferred a respondent's belief based on his or her interpretation of that context (in response to critical element 4). The resulting survey provided the researchers with the opportunity to capture PSTs' beliefs through the authentic short answers they provided.

This instrument did not go without validation. Because of the nature of beliefs, the validation procedures relied on the individual testaments of experts. Six mathematics education researchers with expertise in teachers' belief and six mathematics education graduate students completed and examined the survey. They attested to the validity of the items as measures of the specific beliefs, as well as the validity of the rubric used to score the data (Philipp et al., 2007, p. 451). As a result of the validation process, the researchers were confident that their instrument provided insights into the beliefs and interpretations of its respondents. The instrument was also said to yield numerical scores capable of discerning differences among groups in different treatments during statistical analyses. PSTs' mathematical belief changes were therefore traceable for research studies that intended to measure change. This is immensely important for studies that seek to influence beliefs to align with knowledge development. Only by creating this alignment can PST education programs hope to develop MKT and improve the mathematical literacy of students. But, how would a program approach such belief changes?

Belief Changes

Philipp (2007) recognized two schools of thought in changing an individual's beliefs. If we are to accept that beliefs act as filters affecting what we are capable of seeing (Pajares, 1992), then belief changes must occur before changes in teaching can be realized. There exists a research base that supports this notion (e.g., Glaeser, Leuer, & Grant, in press; Ertmer, 2005; Stipek et al., 2001). Gutskey (1986) investigated how experiences with students affect beliefs. He suggested that teaching beliefs change only after witnessing strong evidence that alternative views are producing better student learning outcomes. So, a teacher (or PST) is unlikely to change their beliefs about teaching and learning mathematics without opportunities to implement, view, discuss, and reflect on instructional practices and student outcomes. Student outcomes can be analyzed through the examination of student work and thinking. Next, I will outline studies that have attempted to change beliefs about teaching and learning mathematics.

Grant, Hiebert, and Wearne (1998) measured and tracked teachers' beliefs while showing them models of effective teaching. The authors' speculation was that, instead of telling teachers how to be effective (which they accused many studies of doing), showing real teaching and real student outcomes may be more effective in changing teachers' beliefs. The twelve participants' initial beliefs were placed on a continuum ranging from a focus on skills/teacher-responsibility (traditional, procedural instruction) to a focus on process/student-responsibility (reform, conceptual instruction). The participants were then observed, and strong relationships were discovered between beliefs and classroom practices. Belief (and subsequently teaching) changes did occur, leaving Grant, Hiebert, and Wearne (1998) to conclude that observing effective teaching was not enough. Beliefs acted as filters that prevented teachers from “seeing” certain aspects of the lesson that were incompatible with their current beliefs. Teachers needed the opportunities to interact with and reflect on the teaching they observed, as well as participate in peer discussions in order for their beliefs to change. Consequently, analyzing student work and thinking provides an interactive context from which reflective and conversational activities can arise.

Benbow (1995) also spoke of the challenges of changing teachers’ beliefs. In his study, PSTs’ belief changes were tracked through early field experiences that included teaching lessons. He found that PSTs’ current beliefs did affect how they interpreted their school experiences as evidenced by his Likert scale and open-ended surveys. However, PST instructional practices and interpretations of student outcomes did affect their beliefs – albeit mostly about pedagogy and self-efficacy. This study further confirms that changing teacher beliefs is possible through interactive experiences, and that belief changes occur slowly and require deliberate exposure and reflection to happen.

Philipp et al. (2007) and Ambrose (2004) supported Benbow’s (1995) findings, but also revealed another layer to the complexity of changing teacher beliefs. Most studies attempt to change beliefs about the teaching and learning of mathematics during methods courses, which typically happen after all subject matter courses have been completed (Ambrose, 2004, p. 92). Their view was that belief changes and methods courses must be taught in conjunction with mathematical subject matter. Only through this combination can real changes in beliefs happen and provide the footing for effective pedagogical and content knowledge to be learned.

Ambrose (2004) used a variety of data sources including surveys, interviews, prospective teachers' written work, and field notes. PSTs were given the opportunity to work with elementary students on learning fractions and whole number concepts. Coursework provided PSTs with suggestions and discussions about children's informal knowledge (i.e., KCS), children's tendencies in solving problems (i.e., KCS), how the thought processes of children often differ from adult's thinking (i.e., KCT), the importance of multiple representations and answers (i.e., SCK), and the importance of a student-centered approach (i.e., KCS/KCT). Their methodology stemmed from their recognition that belief changes in PSTs happen through one of four mechanisms: (a) participation in emotion packed, vivid experiences that leave an impression; (b) immersion in a community such that they become enculturated into new beliefs through cultural transmission; (c) reflection on their beliefs so that hidden beliefs become overt; and (d) participation in experiences or reflections that help them to connect beliefs to one another and, thus, to develop more elaborated attitudes (Ambrose, 2004, p. 95).

The survey data collected for this study came in the form of six open-ended questions that the PSTs were asked to complete at the beginning and end of the study. This instrument was piloted and used during both the Children's Mathematical Thinking Experience (CMTE) and IMAP projects. The inferences drawn from the survey answers were further validated through a member-checking process. Data were also collected from interviews at the commencement and culmination of the study. These interviews included questions about the prospective teachers' attitudes toward mathematics, their thoughts about teaching and learning mathematics, and follow-up questions about the belief survey.

Other data were collected through observing and discussing participants' field notes. The tasks participants used while working with students were designed to build conceptual understanding and to reduce the emphasis on prescribed procedural work that can lead children to misconceptions (Mack, 1995, as cited in Ambrose, 2004). The prospective teachers were encouraged to make instructional decisions and log their experiences. The analyses of this data suggested that the participants' beliefs about what effective teaching entailed changed during the study. Participants began to believe that teaching involved much more than simply presenting information and that providing children time to think and struggle with concepts was very important. This represented a shift from the predominately teacher-centered views the PSTs possessed at the beginning of the study. This and the other belief change studies mentioned here provide support that PSTs beliefs can be changed to align with the principles of MKT, making effective mathematics teaching knowledge acquirable.

I have, at this point, established a framework for conceptualizing MKT, reviewed research that suggested teacher knowledge acquisition is affected by the beliefs they hold about the teaching and learning of mathematics, and reviewed studies that suggested how PST education programs might affect teacher beliefs in ways that align with the MKT framework. It remains, however, necessary for me to suggest how PST education programs can attempt to develop MKT in prospective teachers. Much work has been done involving the use of student work and student thinking in the development of teachers. Given the explicit references of student work and student thinking in the MKT framework (specifically within the domains of KCT and KCS), their appearance in the recommendations of NCTM, CMBS, and NMAP, and current literature that explores their use regarding teacher knowledge and beliefs, student work and student thinking appear to be logical tools for developing MKT and beliefs in PST.

Student Work and Thinking: Developing Knowledge and Beliefs

Knowledge

The ability to understand and use students' thinking has been extensively endorsed in the education community, including its appearance as one of the central tasks of mathematics teaching (NCTM, 1991). Stacey et al. (2001) studied the content knowledge and PCK that PSTs possessed. They found that PSTs struggled with mathematical content knowledge that integrates different aspects of number knowledge (a major component of SCK) and PCK that included a thorough understanding of common student difficulties (a component of KCS). Several studies have examined the use of student work and student thinking specifically in the development of these types of teacher knowledge (e.g., Crespo, 2000; Crespo & Nicol, 2006; Kazemi & Franke, 2004; Son & Crespo, 2009; Stacey et al., 2001). It has been reported that recognizing the meaning of students' work and making sense of students' mathematical thinking are challenging tasks for prospective teachers (Son & Crespo, 2009). This becomes even more challenging when PSTs are asked to examine work that involves nontraditional strategies. Crespo (2000), and Crespo and Nicol (2006) conducted similar studies that qualitatively observed the types of knowledge development PSTs experienced from analyzing student work and thinking, while Son and Crespo (2009) provided evidence for the need.

Son and Crespo (2009) presented PSTs with the task of analyzing students' nontraditional responses. This task was couched within a larger teaching task, to simulate how nontraditional answers and approaches may (and should) surface during instruction. The process involved PSTs responding to prompts about how they had already addressed and dealt with a nontraditional division by fractions response. These included: How would you respond to this student? Do you think this strategy would work in all cases? Explain in as much detail as you can. The goal of these prompts was to force the PSTs to consider how they had approached the nontraditional strategy and what may have influenced this.

Thirty-four PSTs participated in this study, 17 of whom were seeking middle or secondary mathematics certification and 17 of whom were seeking elementary certification. The data were collected from a survey administered towards the end of a methods course; this survey included the two prompts listed above.

The PSTs responses were divided into teacher-centered (e.g., the teachers told, explained, or showed how the students' nontraditional response would or would not work) and student-centered (e.g., teachers provided their hypothetical student with opportunities to explain and justify thoughts, and provided a guide for students to figure out if the response was correct). It was found that the majority of elementary PSTs produced teacher-centered responses to the nontraditional student work. These responses also revealed that the most of the elementary PSTs (75%) analyzed the nontraditional student response at a surface level only (failed to establish the generalizability of the response). In addition, almost twenty-five percent of the elementary PSTs failed to categorize the student response, which was completely mathematically sound, as correct.

The authors concluded that PSTs' knowledge about mathematics and pedagogy was directly tied to their responses and interpretations of students' work and thinking. Moreover, the authors argued that traditional PST education programs often fail to prepare PSTs for analyzing students' work, especially that which involves nontraditional elements. The following studies demonstrate the types of benefits that have been documented when teachers are prepared to do so.

Being prepared to use students' mathematical thinking is the hallmark of reform-minded visions of mathematics teaching (Crespo, 2000). However, Deborah Ball (as cited in Crespo (2000)) reminded us that effectively attending to students' thinking and work is no easy task. The thirteen PSTs that participated in this study were therefore given opportunities to examine 4^{th} grade students' work and thinking through a letter writing activity. This activity was grounded in an understanding of situated cognition, which views knowledge as being inseparable from the context or activity through which it is constructed (Crespo, 2000, p. 157).

Letter writing was chosen because it provided a context that encapsulated the interactive nature of teaching while easing the cognitive demands that are found in a live classroom. The letter writing spanned eleven weeks as part of a mathematics methods course of an undergraduate elementary education degree program. Each PST corresponded with one 4^{th} grade student (with a few exceptions where PSTs corresponded with two students). The student letters were read and responded to during class time. This was done in groups of four PSTs to encourage discussion and collaboration as they constructed their responses.

The data for this study were collected from a variety of sources: (1) all written work associated with the course, (2) videotaped class sessions (used for descriptive purposes), (3) six letters exchanged between preservice teachers and elementary students regarding three common mathematics problems, (4) one teacher-directed letter from the students about the role of calculators and computers in their classroom, (5) journals about deliberations and reflections on the interactions with students, and (6) a case report written at the end of the course about the learning experiences related to the work with the students (Crespo, 2000, p. 159). The journal entries and case reports were the main sources of data used to capture what was gleaned from the experience. The analysis of the journal entries and case reports focused on inferring the changes in how PSTs interpreted the letters and class discussions regarding student work and student thinking. Occasionally, the PSTs' letters and videos of the methods course were referenced as secondary data sources.

The results of this study are salient to the notion that engaging PSTs in the examination of student work and thinking may help to develop elements of MKT. Initially, the PSTs seemed to be focused on the correctness of the students' answers in the letters that they received. PSTs also made remarkably quick judgments about the level of understanding their student possessed. This judgment was based almost entirely on the correctness of the final answer. Furthermore, short or incomplete answers were quickly interpreted as a lack of motivation and ultimately a lack of mathematical ability. The patterns of noticing began to change around the fifth week of the semester.

After five weeks of writing letters (one per week) and discussing students' thinking and work during class, the PSTs began to show developmental changes in their understanding of the mathematical work (SMK), as well as changes in their understanding of students as learners (KCS). PSTs reported taking it upon themselves to develop a deeper understanding of the mathematical topics to be prepared to evaluate the wide range of solution strategies they were seeing in the letters. They also began to create more open-ended questions to elicit more in depth answers from their students. The PSTs were now interested in providing return letters contingent upon the students' understanding and thinking. To do this, they needed to carefully elicit detailed, information-rich responses from their 4th grader.

Overall, their journal entries and case report assignments became much more analytical of the mathematics that appeared in the student's answer; the PSTs were also much more in tune with what the students may potentially have difficulty with and began to predict mistakes the students might make. The latter two developments are aligned closely with the definition of KCS as presented by Shulman (1986) and Hill, Ball, and Schilling (2008). Also, the PSTs changed how they viewed the mathematical abilities and motivation of the students. The incompleteness of answers were now viewed as necessitating a change in pedagogical strategies, thoughts about how old topics might need to be retaught, and thoughts about how new topics might be introduced (i.e., KCT).

Crespo and Nicol (2006) also documented PSTs' knowledge development as a result of analyzing student work and thinking during a methods course. This study designed and implemented two tasks aimed to extend and challenge PSTs understanding of division by zero. One of the major goals of the study was to promote curiosity and a disposition to explore taken-for-granted mathematical knowledge. The topic, "division by zero" was chosen because the authors saw it as a rich context for mathematical and pedagogical inquiry.

Crespo and Nicol (2006) explicitly stated the two research questions that drove the study: (1) How do prospective elementary teachers respond to the question of division by 0 before they have opportunities to discuss their ideas or investigate the topic? (What kinds of explanations do they use to justify their answers? What kinds of explanations do they use to explain to young students?), and (2) How do prospective teachers participating in two different instructional interventions respond to the question of "division by 0" after their explorations? (What can we learn from their responses that might help prospective teachers of mathematics develop the "mathematical attitudes" and "sensitivity to students' thinking" needed in and for teaching?) (Crespo & Nicol, 2006, p. 86). The participants were PSTs enrolled in one of two mathematics methods courses taught by the authors. One course was part of a five-year undergraduate teacher preparation program, and one course was part of a one-year post-baccalaureate program for prospective teachers. There were 32 participants in all, with 18 of 28 post-baccalaureate PSTs and 14 of 20 undergraduate PSTs participating.

The mathematical tasks presented to the PSTs were couched in students' erroneous answers to division by zero in lieu of a straight mathematical question. This presentation was designed to allow PSTs to develop and practice the problem solving and decision-making skills needed for teaching. One version of the task was used in the post-baccalaureate course. This form consisted of a short video that captured students answering an interviewer's question about division by zero. The children regularly answered that five divided by zero is zero, and they justified the response with the rule that "anything divided by 0 is 0" or offered, "My teacher taught me that," as an explanation (Crespo & Nicol, 2006, p. 87). The preservice teachers were instructed to individually construct a set of focus questions, which would later serve as the basis of a discussion within their group and with the whole class.

Another version was used with the undergraduate PST course. In this version, the task was comprised of a written prompt based on the video used in the post-baccalaureate course. The classwork portion was identical to that in the first version; PSTs were asked to construct focus questions individually and later discuss them within their groups and as a whole class. The undergraduate version, however, did contain an extra component. The PSTs in this course were asked to address an out-of-course task and write up their reflections and findings in a journal. The additional task read as follows:

> A student in your Grade 5 class thinks that 5 / 0 = 0. When asked to justify the answer, the student replies: "My teacher taught me that." What do you think of this student's response? When did you learn about division by 0 and what do you remember about it? Is the answer really 0? Can you prove it right (or wrong)? Would the answer still be the same when you divide 0/0? What happens when you divide 0 by 5? Use words or pictures to prove your answers. How would you teach students about division by 0 so that they can make sense of it rather than memorize the answer? (Crespo & Nicol, 2006, p. 88).

It is important to note that both versions of the task indirectly engaged PSTs with mathematical content through the challenge of being able to create answers and explanations that would help and make sense to students who were struggling.

The data for this study involved the PSTs' individual focus questions as well as the instructors' observation notes during group and class discussions. These data sets were used to determine the PSTs' initial ideas about division by zero and what explanations they would provide to students who struggled. Class discussions were also video- and audio-taped (undergraduate course) and journal entries were collected (post-baccalaureate course) for extra data to investigate changes in the quality of PSTs' explanations as the course progressed.

Before the PSTs engaged in the study's tasks, only 15% (5 of 32) were able to provide a reasoned explanation for dividing by zero. Only 47% answered that division by zero is undefined (15 of 32). Other responses included five and zero. Four participants who failed to answer correctly did show promise in their attempt to reason mathematically about how to think of division by zero. PSTs also struggled with what they would do to help a student learn about this topic. Two of the participants provided a verbatim textbook-type rule as a teaching tool, and twenty-one of the participants provided no explanation at all. Instead, they provided responses such as "not sure what I would say to teach this to my students" (Crespo & Nicol, 2006, p. 89). Furthermore, only two of the PSTs attempted to relate division by zero to the way that division works and is defined with other numbers.

These initial data provided two major insights. First, the PSTs showed a disconnected understanding of mathematics. They seemed to view division by zero as a rule that explained an isolated case in mathematics that was not related to other division concepts. Also, they showed issues from a PCK standpoint. The PSTs decided on their answers to the tasks without considering the sources of the students' struggles. They did not consider offering alternative explanations and did not seem to consider how students might misunderstand or misinterpret the explanations they were providing. These deficiencies indicated insensitivity towards students' thinking (Crespo & Nicol, 2006, p. 90).

After the PSTs were introduced to the division by zero tasks, it became evident (in the transcripts of class recordings and entries in journals) that they were beginning to grapple with their own misunderstandings of the subject matter as well as their understanding of students' thinking and misconceptions. The PSTs' later discussions and responses showed their negotiations of division by zero, and showed drastic shifts in how they went about forming explanations and questions for students. However, it was observed that PSTs often moved onto a student focus before solidifying their own understanding of the material. Overall, the study found that the PSTs' understanding of division by zero (a component of SMK), understanding of students' misconceptions (a component of KCS), and understanding of how to identify difficulties and better explain this topic to students (components of KCS and KCT) developed extensively (Crespo & Nicol, 2006, p. 94).

Kazemi and Franke (2004) did not focus on PSTs but did provide interesting results related to the use of student work and thinking on teacher knowledge development. In this study, participants collectively examined student work as part of a teacher development project. This setup was based on the situated view of learning, more specifically the transformation through participation view presented by Rogoff (1997). This view maintains that learning is evidenced by the changes in participation of individuals within a larger social context. Kazemi and Franke (2004) stated, "The transformation of participation view takes neither the environment nor the individual as the unit of analysis. Instead, it holds activity as the primary unit of analysis and accounts for individual development by examining how individuals engage in interpersonal and cultural historical activities" (p. 205).

Ten teachers participated in this yearlong study. The methodology was loosely based on the premise of the cognitively guided instruction (CGI) professional development project. The study facilitated workgroup meetings centered on students' written or oral mathematical work. The researchers also observed and conducted informal interactions with teachers in their classrooms. Participants were encouraged to bring their own students' work to the workgroup sessions for discussions and analyses. The researchers introduced the ideas of CGI (which included the importance of students' thinking in the teaching of mathematical topics), and asked the participants to consider them in the discussions and analyses of the work they brought.

The data collected came from seven workgroup meeting transcripts (audio recorded), written teacher reflections, copies of student work shared by the teachers, and end-of-the-year teacher interviews. The teachers were asked to reflect on the work they brought at the beginning of each workgroup meeting. After reflecting, the teachers were encouraged to discuss their reflections with the group while keeping the CGI principles in mind. These data were analyzed through case studies and grounded theory. The data were categorized and interpreted in terms of what they told about the participation in workgroups and the use of CGI principles.

Two major shifts in participation were discovered (Kazemi & Franke, 2004). The first shift was in the attempt to elicit student thinking. Teachers came to the first meetings rather uncertain of the different ways students could solve problems and how rich their explanations could be. They began to discuss ways in which they might be able to better pose questions or select problems to elicit the understanding of their students (a component of KCS). They also began to report on their pleasure of seeing advanced thinking in their students when they were given the chance to explain their work. The second shift in the teachers' participation was in their development of possible instructional trajectories in response to the student thinking they were analyzing (a component of KCT and knowledge at the horizon). Teachers also subsequently began to participate differently in their approach to discussing the mathematical knowledge needed to compose and decompose numbers efficiently (a component of SCK).

Examining and discussing student work and thinking enabled teachers to develop their knowledge and understanding of mathematics and students as learners (Kazemi & Franke, 2004, p. 216). Focusing on the details of students' thinking enabled the teachers to transform how they viewed and participated in conversations about student learning and thinking. The authors reported this significant knowledge development as evidenced by the transformation in participation view of learning (p. 213). They found student work and a focus on student thinking to be very useful tools in the development of teacher knowledge that tacitly aligns with several elements of MKT.

The studies reviewed here provide extensive support for the ability of a focus on student work and thinking to affect the knowledge development of PSTs (and in one case practicing teachers). The methodologies consisted of qualitative designs that documented teacher knowledge development through the interpretations of a variety of data including interviews, surveys, field notes, journal entries, and classroom observations. However, as was discussed earlier, an intervention must address teacher beliefs in order to truly effect teacher knowledge. Examples of support for student work and thinking to address teachers' beliefs will now be examined.

Beliefs

Several studies have shown that a focus on student work and student thinking can also be powerful in changing teachers' beliefs (Philipp, Armstrong, & Bezuk, 1993; Philipp et al., 2007; Vacc & Bright, 1999). This focus represents a necessary element to changing teachers' practice and developing teachers' knowledge (Ambrose, 2004; Benbow, 1995; Grant, Hiebert, & Wearne, 1998; Philipp, 2007; Philipp et al., 2007).

Philipp, Armstrong, and Bezuk (1993) conducted a case study of one PST who was exposed to CGI principles (in class and by observation of a master teacher) that encouraged her to link student thinking and student work to instruction throughout her final year of teacher preparation courses and experiences. The majority of the data collected came from the first semester of her student teaching placement. Data included scores on beliefs scales, interviews, observations of her student teaching, and transcripts of discussions about her observations of a master teacher. Her belief development was documented as well as the effects on her subsequent instructional practices.

The interviews and belief instrument scores indicated that the participant possessed beliefs that individuals construct knowledge, that mathematics should be taught through understanding and problem solving, and that a student discussing her or his thinking translates to meaningful understanding of mathematics. Despite these promising belief characteristics, the participant oftentimes did not connect student thinking to instructional practices in the beginning of the study. She did begin to make more connections and view herself more as a CGI teacher as the year went on. This suggests that focusing on student work and thinking has the ability to influence PSTs beliefs about themselves as teachers. The study concluded with one culminating finding, “This study, which provides an existence proof that preservice teachers *can* utilize pedagogical content knowledge about how children think in such a way that it influences her practice, carries important implications for teacher preparation” (Philipp, Armstrong, & Bezuk, 1993, p. 494). This study is one example of how student work and thinking has been shown to influence PST beliefs in practical ways.

Vacc and Bright (1999) and Ambrose (2004) suggested that changing beliefs in PSTs was difficult but attainable. Both suggested the importance of student work and thinking in bringing about this change. Vacc and Bright (1999) tracked PSTs’ belief changes during a two-year undergraduate teacher preparation program. They reported that little change was found during the majority of their coursework and internship experiences as determined by the CGI belief scale discussed earlier in this paper. Nevertheless, a methods course focused on students’ mathematical work and thinking provided a catalyst for significant changes in their beliefs about mathematics teaching and learning.

Ambrose (2004) found that motivating students to recognize the importance of mathematics through working with children and their mathematical thinking was a promising avenue towards belief change. She reported that providing experiences that intimately involved PSTs with children's thinking could positively affect their beliefs (p. 117). Ambrose (2004) also discovered that belief changes often do not indicate abandonment of old beliefs. Instead, PSTs tend to hold and build onto their original beliefs. PST education courses should therefore avoid trying to influence belief reversals. It is far more productive to approach PSTs' beliefs with the plan to build them towards those necessary for alignment with MKT and effective teaching. D'Ambrosio and Campos (1992) suggested that creating disequilibrium could help PSTs to build on their current beliefs. Changes occur when PSTs reflect on what they learned from analyzing students' work and thinking as it is often very contradictory to the beliefs held about the learning and understanding of mathematics.

Philipp et al. (2007) extensively documented belief changes (related to the use of student work and thinking) through experiences that created a disequilibrium. In Dewey's influential essay "The Child and the Curriculum" as cited in Philipp et al. (2007), Dewey felt that students should be viewed as the curriculum. Schools could then be focused on children's interests and capacities. Based on this view, the authors were interested in determining how to prepare PSTs in a way that encouraged the importance of students' interests and capacities. Their theory for this approach was based on Figure 2-5 below (Philipp, 2008, p. 9). The authors believed that children themselves are the focus of most PSTs' caring who gravitate toward a teaching career.

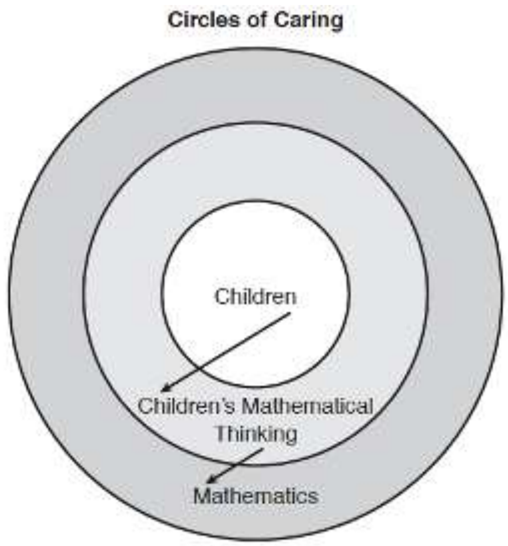


Figure 2-5. Circles of caring.

If deep conceptual knowledge for effective teaching and a mathematically literate society are to be realized, PSTs must care about learning and teaching mathematics. To do this, PST education programs must draw upon prospective teachers' natural feelings of caring towards children. PSTs are more likely to care about children's mathematical thinking if they see students struggle with mathematics. It is when they believe they need to understand mathematics in new ways to help their students that they begin to care about mathematics. This theory is referred to as "Circles of Caring" (Philipp et al., 2007, p. 441).

The authors argued that, by and large, PSTs do not know the mathematics they need to teach effectively and many are not open to learning the material in a deeper and more conceptual way. This stems from their elementary experiences. PSTs often view their traditional, fragmented learning of mathematics as sufficient, claiming, "If I, a college student, do not know something, then children would not be expected to know it" (Philipp et al., 2007, p. 439). This view is exemplary of a belief that mathematics is something one either understands or does not understand and that knowing a procedure without knowing how or why it applies represents “knowing” mathematics. This study set out to address and change PSTs’ beliefs about mathematics.

The participants for this study were 159 PSTs enrolled in the first of four mathematics content courses of their undergraduate teaching program. When placing participants in groups, the authors utilized modified random assignment. The PSTs’ class schedules, work schedules, and the times of possible school visits confined the group assignment process. However, most participants were available for at least two potential groups.

The course’s purpose was to develop PSTs’ mathematical knowledge (it appears CCK, KCS, and SCK were addressed) through conceptual approaches that included a focus on students’ work and thinking. Participants were divided into one of five groups (4 treatment and one control). The treatment groups were labeled CMTE-L (children’s mathematical teaching experience, live), CMTE-V (children’s mathematical teaching experience, video), MORE-S (mathematical observation and reflection experience, select), and MORE-C (mathematical observation and reflection experience, convenient). All groups (control included) covered the same core mathematical content; they only differed in their exposure (types and quantity) to children’s mathematical thinking.

The CMTE treatments were categorized as laboratory models because the exposure to students’ thinking came primarily during class time through the analysis and discussion of videos. Both of these treatments differed from typical content and methods courses (Philipp et al., 2007). The mathematics was introduced through the interactions with students (either live or via video) instead of being given directly (typical content course). No attempts were made at developing the PSTs skills at instructing groups of students as in a typical methods course.

The CMTE-L treatment provided the participants (n=50) with opportunities to watch and analyze videos of children solving problems. These videos were created for this study and highlighted students' mathematical strengths (inventing strategies, flexibility with numbers and operations, alternative reasoning strategies) and weaknesses (mistakes and misconceptions).

This group also conducted six problem-solving experiences with local elementary students. PSTs visited local elementary schools where they worked in pairs to tutor and interview individual students. The interviews consisted of carefully chosen prompts for the PSTs to follow. The main goal of these interviews was to make apparent children's views of mathematics and build on teacher beliefs as outlined in the circles of caring (Figure 2-5). In particular, the authors wanted PSTs to see the strategies and conceptions students have, how students often make sense of mathematical problems using ways not formally taught in school, and how students often do not understand the mathematics they have been formally taught in school. PSTs participated in class discussions about their interviews and tutoring experiences.

The CMTE-V group (n=27) met on a college campus. PSTs analyzed mathematical problems and student work and discussed students' mathematical thinking and possible solution strategies. They were given unedited videotapes of the CMTE-L interviews on six occasions throughout the semester. The PSTs were charged with viewing and answering questions about what they saw on the videos. The questions involved analyzing students' thinking, skills, misconceptions, mistakes, and strategies. This group spent considerably more time discussing students' thinking and work in class due to the absence of planning for and discussing interviews as the CMTE-L group did.

The MORE groups were designed to be analogous to Dewey's apprenticeship model of teacher education (Philipp et al., 2007, p. 448). Participants in these two treatments made 14 weekly visits to elementary schools. After their first seven visits, PSTs were reassigned to a second classroom for the remainder of the semester. The weekly visits lasted 90 minutes. PSTs observed mathematics lessons for the majority of the time, but observed other subjects in the event that the mathematics lesson did not last the full 90 minutes. No specific arrangements were made for the PSTs to meet or debrief with the classroom teachers, although this did happen unofficially in some cases. A one- to two-page reflection paper was required by each participant for each weekly observation. Furthermore, each participant was required to submit a midterm and end-of-semester reflection about their experiences in the classroom. The MORE groups were created to determine if selecting classroom teachers would significantly affect PSTs experiences and subsequent learning and beliefs.

The MORE-S participants were assigned to classrooms of teachers who were identified as being enthusiastic towards reform-based professional development opportunities. The authors relied on recommendations by colleagues when making this classification. The MORE-C participants, on the other hand, were assigned to classrooms of teachers that were conveniently located close to the university where this study took place. This group was deemed to be representative of the "typical early field experiences" of PSTs where university faculty have little to no control over the quality of the type of teaching that PSTs observe (Philipp et al., 2007, p. 448). It was expected that PSTs in the MORE-S group would experience greater positive changes in belief scores due to the reform-minded nature and focus on conceptual understandings likely exhibited in their placements.

The control group was not well defined in the study. It was reported that control PSTs were enrolled in a mathematics content course designed for prospective teachers and that similar mathematical topics as were presented in the CMTE groups were covered. No field experiences or videos of children were used with the control PSTs. However, details about how the course was taught were not shared.

Philipp et al. (2007) report that measuring PSTs beliefs is a difficult task. They also report that their instrument makes measuring beliefs manageable due to the context provided for each response and the presence of rubrics that make scoring consistent. This instrument is a web-based beliefs survey that was created with the idea that beliefs must be inferred in mind (Pajares, 1992). The survey addressed PSTs beliefs about mathematics and what it means to understand and learn mathematics; it was intended to be used to, “(a) derive a common metric for measuring change in individuals and for comparing individuals to one another, and (b) obtain qualitative data that could be used for more holistic analysis” (Philipp et al., 2007, p. 451). Open-ended question types were chosen to allow for respondents to construct more authentic answers that could be used to better infer beliefs. This study reported a plethora of results, but only those involving PSTs’ belief changes are discussed here.

The results indicated that PSTs in the CMTE groups developed much more sophisticated beliefs about mathematics understanding and learning than did the MORE or control groups. Also, no significant differences or data patterns were found between the CMTE-L and CMTE-V groups on any of the beliefs represented on the survey. This suggested that live student interactions do not guarantee better belief influences than that of video interactions. Differences did appear between the two MORE groups. The MORE-C group did not experience any significant change in beliefs. Moreover, their overall belief changes were less than that of even the control group. The MORE-S PSTs did experience significant positive belief changes for some of the beliefs represented in the survey. However, they did not experience the levels of change exhibited by the CMTE groups.

These results suggest a number of implications for PST education. Observations of practicing teachers, even of those who are reform-minded, are not as powerful as university coursework that provides opportunities to analyze students' mathematical thinking or attend guided field experiences. Time in the field can be very valuable, but it must be guided and experiences debriefed. Maybe most importantly, field experiences with random convenient classroom teachers have the ability to stifle belief developments aligned with effective teaching and MKT. According to this study, no field experience is more productive than haphazardly chosen ones. Finally, integrating a focus on students' mathematical work and thinking into PST education courses can be powerful in positively affecting PSTs beliefs about understanding and learning mathematics – even if that focus is based solely on secondhand interactions with the work and thinking.

The literature outlined above has shown ways in which student work and student thinking have promoted the development of PSTs' knowledge (several domains of MKT) and beliefs when used as a tool by teacher education programs. However there are still research niches to be filled regarding ways to integrate student work and thinking into PST education courses. For example, few research studies have examined the effects of asking PSTs to both diagnose and plan the next steps in teaching while analyzing student work and thinking. This dissertation study is aimed, in part, at helping to fill this gap and move the field forward in terms of best practices for developing PSTs' knowledge and beliefs for effective mathematics teaching.

However, there remains some disagreement about the elements of MKT as put forward by Hill, Ball, and Schilling (2008). One major concern is that the three elements of PCK have yet to be measured with much success. This dissertation study therefore also seeks to explore what types of knowledge PSTs use to diagnose students' understanding and plan the next steps for instruction when analyzing student work and thinking.

CHAPTER 3
METHODS

Overview

The purpose of this study was to understand how a particular series of activities for analyzing student work and thinking affected PSTs' CCK and beliefs about teaching and learning mathematics. In addition, the qualitative portion of this study sought to understand what elements of PSTs' PCK were drawn upon when completing these analyses activities. The following questions guided this investigation:

- What is the influence of analyzing student work and thinking (by way of diagnosing understanding and planning next steps for instruction) on PSTs' CCK?
- What is the influence of analyzing student work and thinking (by way of diagnosing understanding and planning next steps for instruction) on PSTs' beliefs about effective mathematics teaching?
- What types of PCK do PSTs draw upon when diagnosing student understanding and when planning next steps for instruction, while analyzing student work and thinking?

This study adopted a mixed methodology approach and utilized existing quantitative beliefs and knowledge measurement instruments (Hill, Rowan, & Ball, 2005; Philipp et al., 2007) and a qualitative interview protocol (Sudman, Bradburn, & Schwartz, 1996). The study was conducted through a sociocultural learning theory lens (Lave, 1991; Vygotsky, 1978). Participants were randomly assigned from an incoming cohort of PSTs to participate in either the control or treatment group. Data were collected through the aforementioned instruments from all 42 participants as well as through interviews with four individuals from the treatment group and field notes taken by the researcher. The quantitative data were analyzed using an ANOVA (CCK) model and a Chi-Square (beliefs) test, while the qualitative data were analyzed using the thematic analysis method described by Aronson (1994).

Methodology

This dissertation was a mixed method study of the use of student work and thinking analyses on PSTs' CCK and beliefs about effective mathematics teaching that was undertaken through the lens of a sociocultural paradigm (see Vygotsky, 1978). Sociocultural learning theory assumes that learning and knowing are relations among people and therefore require activity in, with, and arising from a socially and culturally structured world (Lave, 1991). Learning is embedded within social events and social interaction plays a fundamental role in learning (Vygotsky, 1978).

According to the theoretical frameworks underpinning this study, PSTs must participate as well as socially negotiate, discuss, and reflect during their preparation programs in order to learn in a meaningful manner. Analyzing student work and thinking has been shown to create such vicarious social opportunities while remaining situated in an authentic teaching context (e.g., Crespo, 2000; Crespo & Nicol, 2006; Kazemi & Franke, 2004; Philipp, Armstrong, & Bezuk, 1993; Philipp et al., 2007; Son & Crespo, 2009; Stacey et al., 2001; Vacc & Bright, 1999). This dissertation focused on using this sociocultural theoretical framework to guide the development of student work and thinking analyses activities.

A mixed methodology was chosen because it allowed for a qualitative exploration of PSTs' PCK accessed for teaching tasks (i.e., the analyses of student work and thinking) as well as a quantitative measurement of the effects of a treatment created for this study. An ANOVA was used in light of the goal to compare the mean CCK posttest scores of the two groups while controlling for pretest scores. A Chi-Square analysis was used to compare the pre- to posttest change scores of each of the seven beliefs for the two groups.

A thematic analysis methodology was chosen because it allowed the data to be collected in a retrospective manner (Aronson, 1994). When individuals answer interview questions based on recall of past experiences, these individuals will be more accurate and reliable than when they are forced to infer and construct answers to general hypothetical questions (Ericsson & Simon, 1993). In addition, answers to specific questions about past events and experiences can be evaluated for accuracy by the interviewee during the interview process, whereas the validity of hypothetical answers and evaluations is difficult to define.

Procedures

Participants

Identification of participants

The participants for this study were PSTs enrolled in an undergraduate mathematics content course for prospective teachers at a large research university during the Spring 2013 semester. The quantitative knowledge and beliefs data were collected from PSTs in the control ($n = 21$) and treatment ($n = 21$) groups. The participants were randomly assigned to one of the two groups to help control for confounding differences. One section was randomly selected as the control group and the other as the treatment group. The researcher taught the treatment group while another mathematics faculty member taught the control group.

The retrospective interview data were collected from four treatment group participants – two purposefully selected by the researcher because, relative to other group members, they had high changes in their pretest to posttest scores on the quantitative instruments, while two were selected for having low changes. Patton (1990) provides rationale for several purposeful sampling procedures within qualitative research. One of these purposeful sampling methods, “intensity,” refers to information-rich cases that provide evidence of powerful experiences, such as high achieving or struggling students. The sampling procedure for the qualitative data participants in this study took the form of purposeful intensity sampling.

All students enrolled in the content course were invited to participate in the study. Those willing to participate were provided with an informed consent form (Appendix B) that provided them with a description of the research and required their signatures. Forty-two PSTs participated in the pretest, posttest, and gave informed consent.

Description of participants

All participants (2 male, 40 female) were undergraduate students who were enrolled in their first semester of a preparation program for prospective elementary school teachers. This program leads to the degrees of Bachelor of Arts in Education and Master of Education as well as recommendation for state teaching certification for grades K-6. It requires 124 (B.A.E.) credit hours and 36 (M.E.) credit hours. Prior to entering the study, each participant has completed a mathematics course at or above the college algebra level. In addition, all participants are required to have four mathematics education courses – the course used for this study is the first in this sequence.

Setting

The student work and thinking analysis treatment took place during class meetings throughout the first eight weeks of the spring 2013 semester. This timeline was based on timelines of similar studies involving student work and thinking (e.g., Crespo, 2000 [11 weeks]; Crespo & Nicol, 2003 [12 weeks]; Kazemi & Franke, 2004 [9 meetings]). The classes met once per week for a three-hour session. Meetings for both the treatment and control groups were held in a university classroom conducive to small group arrangements of PSTs. The classroom also had access to manipulatives for teaching elementary mathematics. Both groups met at the same day and time in separate locations.

The quantitative knowledge and beliefs data were collected during the first class meeting (pretest) and on the eighth class meeting (posttest). The collection of all quantitative data took place through a paper and pencil format in the same classroom that the treatment activities were held. Qualitative interview data were collected during the tenth week of the semester to allow for the quantitative data to be scored and participants to be selected. The interviews were conducted in an open, common area during a convenient day/time for the researcher and participant.

Treatment

Treatment group

The treatment group participants were exposed to a series of student work and thinking analysis activities – some designed for classwork and others for homework. All of the activities were modified versions of the National School Reform Faculty's (NSRF) "ATLAS – Learning From Student Work" protocol (http://www.nsrfharmony.org/protocol/doc/atlas_lfsw.pdf). This protocol calls for selecting and discussing student work samples as a way to help teachers discover what students understand and how they are thinking, which fits soundly with the sociocultural framework guiding this study. The student work samples for the treatment activities were chosen as suggested by the ATLAS protocol to capture a variety of solution strategies and a combination of successful and unsuccessful solutions. Furthermore, class discussions in the form of structured dialogue in both small and large groups were used to help guide participants in identifying what students understood, what they did not understand, and finally to identify next steps for instruction based on what was gleaned from the work. The dialogue took place verbally for in-class activities and in the form of online forums for take-home assignments.

The common theme of these activities was to introduce and explore mathematical topics through the analysis of student work and student thinking. In the university classroom, eight lesson plans (one for each week of treatment) were used to guide the study. Participants were introduced to each week's mathematical topic through viewing student work and thinking. This included watching videos, reading vignettes, viewing written student work, etc. Once the topic was introduced within the context of elementary students' performance, participants were asked to analyze the levels of understanding as well as plan next steps for instruction based on the analysis and resulting conclusions. Participants were also asked to write informal objectives describing what they believed would demonstrate understanding of each mathematical topic. Lenges (2010) asserted that such written objectives were a successful method for eliciting and building mathematical content knowledge in elementary school teachers. An example lesson plan for integrating the student work treatment is attached in Appendix C.

The take-home portion of the treatment was comprised of two assignments – one due during week four and the second due in week eight. These assignments covered the same mathematical topics as in class but participants were asked to analyze new work for understanding, as well as to plan next steps for instruction based on the analysis and resulting conclusions. An example take-home assignment for integrating the student work treatment is attached in Appendix D.

Control group

The control group for the study followed the typical format for a content and methods course at the university. This format included direct instruction, group problem solving, and an absence of student work analysis. Participants were taught the same core topics in both the control and treatment groups. However, the control group did not use the take-home student work projects or introduce mathematical topics through student work examples. The control participants were not asked to diagnose student work or plan next steps for instruction, although an occasional student work example was covered in class and in the homework problem sets. A detailed schedule of weekly topics and assignments for the treatment group can be found in Appendix E. The control group followed a very similar schedule and covered the same mathematical topics listed there.

Data Sources

The data sources for this study were fourfold – quantitative data coming from an established mathematical beliefs about effective mathematics teaching survey (Philipp et al., 2007), quantitative data coming from an established mathematical CCK exam (Hill, Rowan, & Ball, 2005), qualitative data coming from retrospective interviews investigating the types of PCK that PSTs draw upon to complete the treatment tasks, and finally field notes from treatment group class meetings. The quantitative data were pencil and paper responses while the qualitative data were audio recordings and written field notes.

Data Collection

Knowledge instrument

A pretest and posttest were used to capture any effects the student work and thinking analyses treatment had on the PSTs' CCK over the course of the study. Researchers at the University of Michigan and Harvard University created the knowledge instrument using funding from the National Science Foundation. Variations of the instrument exist that cover different grade bands, different mathematical topics, and different types of mathematical knowledge. The study utilized the elementary level CCK for number concepts and operations. This version provides two equivalent forms, which addressed a possible confounding issue with repeated testing.

The elementary number concepts and operations version (both form A and form B) contained fifteen questions designed to determine the level of mathematical content knowledge for elementary school teachers. It was also designed to take approximately thirty minutes to complete, although its authors suggested that a one-hour time block be given. The participants in this study were given one full hour. The questions addressed concepts such as base ten manipulations, addition, subtraction, multiplication, division, and word problems. An example question from this instrument is provided in Appendix F.

The instrument has been piloted on a large-scale through the California's Mathematical Professional Development Institute and is available for use by researchers. Form A was piloted with n = 629 and produced a reliability coefficient of 0.84. Form B was piloted with n = 620 and produced a reliability coefficient of 0.85. Hill, Rowan, and Ball (2005) reported that the items used in both forms were subjected to a content validity check and contained adequate coverage across the number concepts, operations, and patterns suggested as necessary by the NCTM standards – which have directly informed the Common Core State Standards for Mathematics (NCTM, 2010). Furthermore, Hill, Ball, and Schilling (2004) reported that the items represented teaching-specific mathematical skills and could reliably discriminate among teachers, and could meet basic validity requirements for measuring teachers' mathematical content knowledge including CCK.

The instrument was administered to participants during the last hour of class time during the first (pretest) and eighth (posttest) week of the Spring 2013 semester. The researcher attended a training course for administering the instrument and followed that protocol with the utmost fidelity. Each participant was provided a paper version of the instrument. Answers to each item were recorded directly on the instrument by circling the selected multiple-choice response. Participants were only identified by study IDs, which they recorded directly on the instrument as well as on a confidential reference sheet with their names (this sheet was never seen by the researcher scoring the data).

Beliefs instrument

A pretest and posttest were used to capture any effects the student work and thinking analyses treatment had on the PSTs' mathematical beliefs for effective teaching over the course of the study. Researchers at San Diego State University created the beliefs instrument using funding from the National Science Foundation. There is only one form of this instrument that was created as a web-based design only. The researcher modified this format to allow participants to respond to the items in a paper and pencil format. No alterations to the items, instrument layout, or directions were made. Each question originally appeared online as a single screen and appeared on the paper and pencil format as a single page. All spacing and answer formats/spaces were preserved.

The beliefs instrument contained sixteen questions designed to determine the mathematical beliefs participants possess in relation to seven beliefs categories deemed important in the research literature (e.g., Ambrose, 2004) for elementary mathematics teaching. The seven beliefs that the instrument was built upon are:

Beliefs About Mathematics

1. Mathematics is a web of interrelated concepts and procedures (and school mathematics should be too).

Beliefs About Learning or Knowing Mathematics, or Both

2. One’s knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts.

3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.

4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

Beliefs About Children's (Students') Learning and Doing Mathematics

5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect.

6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children’s initial thinking whereas symbols do not.

7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

There is no time restraint suggested by the authors. However the participants were given one hour to complete the assessment. A selection from this instrument is provided in Appendix G.

Six mathematics education researchers with expertise in teachers' beliefs, and six mathematics education graduate students, completed and examined the belief survey. They attested to the validity of the items as accurate measures of the listed beliefs as well as the validity of the rubric used to score the data (Philipp et al., 2007, p. 451). As a result of the validation process, the researchers were confident that their instrument provided insights into the beliefs and interpretations of the respondents. However, no reliability reports about this instrument were found.

The authors have determined that the scoring rubrics that accompany the instrument are trustworthy and easily and reliably useable by other researchers. The bulk of the instrument development work was focused on fine-tuning the rubrics, specifically assigning scores to categories, and clearly defining categories so that most responses would fit one and only one category. One of the most important aspects of their work was testing for reliability. After a team refined a rubric and developed a scoring system, several rounds of issues led to further clarification and to the final rubrics.

The instrument was administered to participants during a one-hour block of class time on the first and eighth week of the Spring 2013 semester. The researcher administered the instrument as instructed in the manual (with the exception of the web-based to paper and pencil modification). Each participant was provided a paper version of the instrument. Answers to each item were recorded directly on the instrument by circling the selected multiple-choice responses or filling in the short answer responses. Video viewing was also part of this instrument. As a result, the researcher provided the videos to the groups of participants as a modification of the original self-initiated online viewing. Again, participants were only identified by study IDs.

Interviews

Retrospective interviews were used to collect the qualitative data during the tenth week of the study (Ericsson & Simon, 1993). The interviews with participating PSTs focused on exploring their experiences with the student work and thinking analysis treatment. A semi-structured interview protocol (Knox & Burkard, 2009) was used to elicit what types of PCK the participants drew upon when completing the treatment analyses (Appendix H). Of particular interest was an effort to determine if the participants were using any types of knowledge described in the knowledge egg (Hill, Ball, & Schilling, 2008). These formal interviews (one per group – high and low change) lasted approximately one hour each. They were scheduled at a time convenient for the participant and will be audio recorded and transcribed verbatim by the researcher. DiCicco-Bloom and Crabtree (2006) contend that a single interview is both appropriate and common when the researcher has previously established rapport with the participants, as was the case in this study.

Moreover, informal interviews took place during the course of the study. The researcher asked participants on several occasions to explain their thinking or repeat parts of in-class conversations that arose from the treatment activities. These informal interactions revealed information about the types of knowledge being used to complete the treatment tasks, and the researcher recorded these data through field notes (Wolfinger, 2002). These notes served as an outline and were written up formally immediately after the class concluded. This secondary source of data served in the triangulation of the findings from the formal interviews.

Observations

Each class session for the control and treatment group was video-recorded to help ensure the fidelity of both the treatment and control implementation. After each session, the researcher watched the videos and took notes about similarities and differences in course content and student work use.

Data Analysis

The data collected from the beliefs and knowledge instruments were analyzed with an ANOVA using the Statistics Package for the Social Sciences (SPSS) 18.0 to determine if there are significant differences between the means of the treatment and control groups while controlling for the pretest scores. The three assumptions of this test (Shavelson, 1996) were taken into consideration: those are independence (crucial), homogeneity of variance (robust to violations if group sizes are similar), and normality (robust to violations). The data sets for this study presented no violations to these assumptions.

The transcribed interview data were analyzed using the thematic analysis approach described by Aronson (1994). There are four steps in this analysis method:

1. The first step is to transcribe conversations and begin to look for patterns of experiences that exist in the data. The researcher should read through the transcriptions several times to familiarize him or herself with the data before the initial patterns are identified. These patterns can come from direct quotes or paraphrasing common ideas that are seen.

2. The second step to a thematic analysis is to identify all data that relate to the initial classified patterns from step 1. The identified patterns are expounded on by coding all data that further explains it. In this step it is important that all of the talk that fits into a specific pattern is identified and placed with the corresponding pattern.

3. The third step to a thematic analysis is to combine and catalogue related patterns into themes. Themes are defined as units derived from patterns and are identified by bringing together components of initial patterns, which are often

meaningless when viewed alone. Themes that emerge from the participant's words are pieced together to form a comprehensive picture of the collective experience. It is up to the researcher to be familiar with the data and to understand it as a whole in order to provide coherence and accuracy in the themes.

4. Once the themes are established, thematic analysis suggests a fourth step of member checking to ensure that the themes are accurate depictions of the information given by the participants. This can be done by asking the participants to provide feedback on the established themes and then incorporating this feedback to adjust the themes as necessary. Any changes must be reflective of a misinterpretation by the researcher.

These four steps were used to analyze the audio-recorded interviews with the participants. As themes fit with an element of the knowledge egg (Hill, Ball, & Schilling, 2008), it was named accordingly. If a theme did not fit with an existing element of knowledge, the researcher named it. The ultimate goal is to determine what types of knowing make it possible to attend to the treatment activities in effective ways.

Limitations of the Study

In education research, it is important to ensure that a study is both rigorous and trustworthy. Although many precautions have been taken and many previous research findings considered, there are limitations to this study. To begin, the sample for this study was a convenience sample of PSTs based on those enrolled in the mathematics content course of interest. This may have limited the generalizability of the study to PSTs in other settings. The participants of this study were also members of two cohorts, one comprising the treatment group and the other the control. These individuals were in close proximity to one another on a daily basis. Although they did not have inter-cohort requirements or classwork, these individuals could have interacted on their own accord, causing a diffusion threat to the internal validity.

The instruments themselves also caused some limitations. The beliefs survey instrument has only one form, so a testing threat to the internal validity was also a possibility. This survey was also validated as a web-based survey. Because it is no longer available in a web-format, the researcher produced an identical instrument in paper and pencil form. The instructions, item order, item content, and scoring rubrics have been reformatted with the utmost fidelity. However, it is still possible that the validity of this study was compromised due to the lack of validation of the new instrument format.

The knowledge instrument is still being developed and tweaked by researchers to increase the reliability and validity. The current attributes are acceptable for educational research, but they do allow for the possibility of low statistical power (a threat to the statistical conclusion validity) and a construct confounding issue in terms of defining the knowledge being measured (a threat to the construct validity). This instrument, however, is one of the most developed and piloted within the field of mathematics education.

Finally, there exists the possibility of researcher bias in this study. The researcher was the instructor of the treatment group. There were extensive attempts made to stay true to the implementation of the treatment as described in the methods section. However this design structure produced the possibility that some researcher bias could have been introduced. This may have resulted in an additional threat to internal validity.

Conclusion

Knowing how activities used in PST education programs influence teachers' mathematical beliefs and mathematical knowledge for teaching is crucial in preparing teachers to teach effectively. The literature suggests that context-based experiences within the university classroom, such as analyzing student work and thinking, could help advance PSTs' knowledge and beliefs (e.g., Ambrose, 2004; Crespo, 2000; Kazemi & Franke, 2004; Philipp et al., 2007). However, continued research can produce clearer definitions of the types of activities that can be used to increase PSTs' mathematical knowledge and beliefs. This study was designed with the goal of determining how the use of student work and thinking analyses and activities (as described in this chapter) influence PSTs' knowledge and beliefs for teaching elementary mathematics. Moreover, this study sought to determine the types of knowledge (particularly as they relate to the "knowledge egg") that the PSTs drew upon to complete the treatment tasks. Determining the effects of these activities serves to inform the field of the usefulness of this form of student work and thinking analysis in preservice mathematics teacher education.

CHAPTER 4
ARTICLE 1 – THE IMPACT OF USING STUDENT WORK AND THINKING ON PRESERVICE TEACHERS' KNOWLEDGE AND BELIEFS FOR EFFECTIVE MATHEMATICS TEACHING

The need for increasing mathematical competencies among our citizens has been a point of focus in the literature over the past few decades (e.g., California Space Education and Workforce Institute, 2008; Gardner, 1983; NCATE, 2010). An identified lack of mathematical literacy in the United States has been a major factor driving this focus. For example, Phillips (2007) reported that high numbers of adults struggled with daily tasks involving mathematics, including computing interest paid on a loan (78% of those involved), calculating miles per gallon when traveling (71%), and determining a 10% gratuity for a lunch bill (58%). These deficiencies are likely due, at least in part, to the mathematics education they received during their days as primary and secondary students. Despite these alarming percentages, students can and should learn mathematics in deep, conceptual ways that lead to mathematical literacy (NCTM, 2000), which has been called the new literacy necessary for success in the world (Friedman, 2005; Schoenfeld, 1995).

Aligned well with NCTM, the Conference Board of Mathematical Sciences (CBMS) (2012) provided two recommendations for the knowledge preparation of preservice teachers (PSTs): (1) PSTs need mathematics courses that develop a good understanding of the mathematics they will teach (i.e., the development of content knowledge), and (2) coursework that allows time to engage in reasoning, explaining, and making sense of the mathematics they will teach (i.e., the development of teaching skills and pedagogical content knowledge (PCK)). PSTs need courses that develop both kinds of knowledge in order to avoid relying on their past experiences as learners of mathematics during teaching (CBMS, 2012). Moreover, several studies have provided grounding for the existence, conceptualization, and assessment of this robust knowledge base for effective teaching (e.g., Ball, Thames, & Phelps, 2008; Carpenter et al., 1989; Cobb et al., 1991; Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005; Saxe et al., 2001; Shulman, 1986a; Shulman, 1986b; Shulman, 1987).

Research also supports the strong relationship between teachers' complex knowledge base and student achievement (e.g., Hill, Rowan, & Ball, 2005; Rowan, Correnti, & Miller, 2002). There are multiple conceptualizations of the types of knowledge teachers need in order to teach mathematics effectively and to promote student achievement. At the forefront of mathematics education literature is the mathematical knowledge for teaching (MKT) conceptualization (Hill, Ball, & Schilling, 2008, p. 377), which divides necessary knowledge into either subject matter knowledge or pedagogical content knowledge (Figure 4-1).

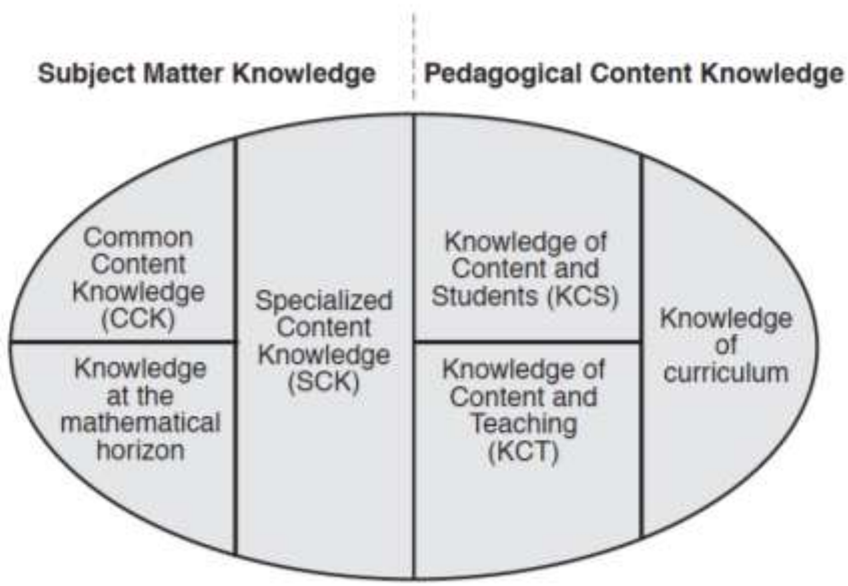


Figure 4-1. Mathematical knowledge for teaching (MKT).

However, an MKT focus alone is insufficient in developing PSTs to teach mathematics effectively. As is typical in educational practice, no single element occurs in isolation. The beliefs that PSTs hold about mathematics must also be addressed. Research has shown that a focus on developing knowledge for PSTs without focusing on their beliefs as well is counter-productive (e.g., Ambrose, 2004; Philipp et al., 2007; Sowder, 2007).

Beliefs are psychologically held understandings, premises, or propositions about the world that are thought to be true – they are lenses through which we see the world, dispositions towards our actions, and are held to varying degrees of conviction (Philipp, 2007). In many instances, PSTs see only procedural requirements during their preparation (Eisenhart et al., 1993). When this occurs, PST education efforts to develop MKT may be in vain. “Teaching itself is seen by beginning teachers as the simple and rather mechanical transfer of information” (Wideen et al., 1998, p. 143). Ambrose (2004) suggested a strong impact of beliefs on content knowledge and PCK acquisitions in PSTs. The results suggested that PSTs’ beliefs affected the way they taught as well as what subject matter they felt comfortable teaching. Since beliefs have such an impact, it is important to identify and develop those necessary for effective mathematics teaching.

There are four critical elements of beliefs (Ambrose, 2004) that must be acknowledged in order to define the necessary beliefs for effective mathematics teaching. They are:

1. Beliefs influence perceptions – they shape individuals’ interpretations of events (Pajares, 1992).
2. Beliefs draw one towards a particular position or direction regarding a given issue (McGuire, 1969; Rokeach, 1968).

3. Beliefs are not all-or-nothing entities – they are held with differing intensities (Pajares, 1992).

4. Beliefs tend to be context specific, arising in situations with specific features (Cooney, Shealy, & Arvold, 1998).

With these in mind, Philipp et al. (2007) defined the following as the beliefs necessary for effective mathematics teaching:

Beliefs About Mathematics

1. Mathematics is a web of interrelated concepts and procedures (and school mathematics should be too).

Beliefs About Learning or Knowing Mathematics, or Both

2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding the underlying concepts.

3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.

4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

Beliefs About Children's (Students') Learning and Doing Mathematics

5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect.

6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not.

7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

It is important that beginning and prospective teachers are afforded opportunities to develop mathematical proficiency in the context of K-12 educational situations (Darling-Hammond and Baratz-Snowden, 2007), and the development of content knowledge and mathematical beliefs are no exception. Many factors influence PSTs' beliefs and knowledge for teaching mathematics; however, using student work and thinking as a catalyst for their development has shown promise with both PSTs and inservice teachers (Crespo, 2000; Kazemi & Franke, 2004; Son & Crespo, 2009). Philipp et al. (2007) found that this K-12 educational context can be meaningfully recreated in the university classroom setting by viewing students' work and thinking through videos and vignettes.

The ability to understand and use students' thinking has been extensively endorsed in the education community, including its appearance as one of the central tasks of mathematics teaching (NCTM, 1991). Several studies have examined the use of student work and thinking specifically in the development of content knowledge and PCK (e.g., Crespo, 2000; Crespo & Nicol, 2006; Kazemi & Franke, 2004; Son & Crespo, 2009; Stacey et al., 2001). It has been reported that recognizing the meaning of students' work and making sense of students' mathematical thinking are challenging tasks for prospective teachers (Son & Crespo, 2009). Therefore, it is important for PST preparation programs to include such opportunities.

Other studies have shown that a focus on student work and thinking can also be powerful in changing teachers' beliefs (Philipp, Armstrong, & Bezuk, 1993; Philipp et al., 2007; Vacc & Bright, 1999). Philipp et al. (2007) found that PSTs developed much more sophisticated beliefs about mathematical understanding and learning when exposed to student work and thinking. In both knowledge and belief instances, student work and thinking has shown promise as a powerful intervention for helping PSTs prepare to teach effectively; yet the research base remains thin with regard to studies that simultaneously monitor knowledge and beliefs in PSTs who analyze student work and thinking. Therefore, one purpose of this study was to create and empirically test student work and thinking analyses activities as they apply to affecting PSTs' MKT and related mathematical beliefs.

The MKT framework encompasses a multitude of knowledge types. However, this study will focus on CCK and PCK. The CCK focus was chosen as the sole focus from the subject matter knowledge (SMK) half of the MKT framework because a current instrument exists for measuring the levels of knowledge teachers have for this element of MKT. Currently, no instruments have been developed to measure specialized content knowledge (SCK) or mathematics on the horizon (the remaining two elements of SMK) in a reliable or valid manner. Furthermore, no quantitative measures have been developed for use with the elements of PCK. Another purpose of this study was thus to collect and examine qualitative data to help determine the role PCK plays in PSTs' abilities to diagnose understanding and plan the next steps in instruction when analyzing student work and thinking.

The third purpose of this study was to determine the impacts of analyzing student work and thinking on the development of PSTs' beliefs about effective mathematics teaching. Because a valid, reliable instrument exists for collecting quantitative belief data, this study examined the impacts on the seven beliefs described above through the analysis of quantitative data used to determine belief levels held by PSTs. Specifically, three research questions guided the investigation of CCK, beliefs, and PCK:

1. What is the influence of analyzing student work and thinking (by way of diagnosing understanding and planning next steps for instruction) on PSTs' CCK?
2. What is the influence of analyzing student work and thinking (by way of diagnosing understanding and planning next steps for instruction) on PSTs' beliefs about effective mathematics teaching?

3. What types of PCK do PSTs draw upon when diagnosing student understanding and planning next steps for instruction while analyzing student work and thinking?

These research questions were developed through inquiries from the researcher's own teaching experiences and work with MKT. It was hypothesized that several elements of MKT, including PCK and CCK might be influenced through the use of student work and thinking analyses. Furthermore, the nature of these questions influenced the methodology of the study. Because the first two questions were best answered through quantitative data and the last question through qualitative data, a mixed methods approach was used.

Theoretical Perspective

Sociocultural learning theory claims "learning, thinking, and knowing are relations among people in activity in, with, and arising from the socially and culturally structured world" (Lave, 1991). Vygotsky (1978) stated that learning is embedded within social events and social interaction plays a fundamental role in the improvement of learning. Furthermore, Rogoff (1994) described a sociocultural framework for learning that has had considerable impact on the conceptualization of this study. This theory, "transformation through participation," says learning takes place when people participate in shared endeavors. There is neither a sole focus on the learner nor the teacher, but rather a joint and collective effort. Involvement in social activities produces true learning. However, this theory fails to recognize what is happening within the individual during their participation.

Borko (2004) outlined an additional, necessary layer beyond Rogoff's theory. Scholars have argued that learning has individual and sociocultural features (e.g., Borko, 2004; Cobb, 1994; Driver et al, 1994). They claim the learning process is one of enculturation *and* construction (i.e., situative theory). This theory views learning both as "changes in participation in socially organized activities and individuals' use of knowledge as an aspect of their participation in social practices" (Borko, 2004, p. 4). Both the individual and the group can be held as the unit of analysis. Although learning occurs through participation and social activities, individual knowledge is constructed and can thus be measured on an individual basis. However, all learning within the situative perspective is heavily tied to context and situation.

Based on sociocultural theory, PSTs must participate as well as socially negotiate, discuss, and reflect during their preparation programs in order to learn in a meaningful fashion. Analyzing student work and thinking has been shown to create such vicarious, social opportunities while remaining situated in an authentic teaching context (e.g., Crespo, 2000; Crespo & Nicol, 2006; Kazemi & Franke, 2004; Philipp, Armstrong, & Bezuk, 1993; Philipp et al., 2007; Son & Crespo, 2009; Stacey et al., 2001; Vacc & Bright, 1999). This study focused on using the sociocultural framework described above to guide the development of student work and thinking analyses activities for PSTs.

Research Methods

This study utilized a mixed methods approach where PSTs' growth in CCK and beliefs about effective teaching were examined through quantitative data analyses while PCK was examined through a qualitative data analysis. A content and methods course at a major university served as the setting for both the control and treatment groups. The 42 participants in the study were randomly assigned to either the treatment (n=21) or the control group (n=21). All participants (2 male, 40 female) were undergraduate students enrolled in their first semester of a preparation program for prospective elementary school teachers. This program leads to the degrees of Bachelor of Arts in Education and Master of Education as well as recommendation for state teaching certification for grades K-6. Prior to entering the study, each participant has completed a mathematics course at or above the college algebra level.

The treatment for this study was the participants' involvement in a student work and thinking analysis protocol (Appendix I) developed by modifying an existing professional development protocol from the National School Reform Faculty (NSRF). Outside of this protocol and the use of student work analyses, the treatment and control groups were very similar. However, the original protocol was designed for professional development purposes and has not been validated or subjected to reliability checks for research purposes.

Field notes were written to document the treatment and control group class meetings. The treatment and control class meetings were video recorded and later annotated by the researcher to document the mathematical topics covered by each group as well as the modes of instruction used by each instructor. These field notes served as a way to monitor the similarities and differences between the treatment and control groups. The treatment group focused on student work as the catalyst for all mathematical content discussions, while the control group focused on direct instruction and group discussion to introduce mathematical topics. However, both groups covered the same content (i.e., number and operation topics that included the base ten system, numbers in other bases, and addition, subtraction, multiplication, and division in base ten). Because both groups (created by random assignment) focused on the same material and were held at the same time of day in similar settings, the treatment activity of analyzing student work through the modified protocol was the main difference. However, there exists the possibility that the treatment instructor had additional influences on the treatment participants through teaching philosophy that went beyond that of the treatment protocol activities.

This treatment protocol guided the discussion around each new mathematical topic that was introduced. Student work was carefully selected to represent various levels of students' mathematical understanding as well as address beliefs as outlined by Philipps et al., 2007. For example, a video of a student struggling to use an algorithm but succeeding with a drawing was chosen to help address belief 6 (the ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking whereas symbols do not).

To learn a topic, the treatment group first analyzed several pieces of student work in small groups by answering questions about the students' perceived level of understanding and planning the next steps for instruction based on that understanding. For example, participants might choose to identify and suggest the re-teaching of an underlying concept that a student misunderstands, create a more challenging problem for a student who shows good understanding, or create a new problem to expose a potential misconception. The student work presented to them represented many different levels of understanding and many different solution strategies. These included examples of correct standard algorithm use, incorrect standard algorithm use, common misconceptions, invented strategies, and unusual strategies – both correct and incorrect. After the small group analysis, a whole group discussion was used to debrief the treatment group on everything that was discussed about a given piece of student work. Moreover, the student work came in the form of vignettes, videos, and written work selected from PST education textbooks and local schools.

Several data sources were utilized for this study. First, quantitative data came in part from an existing survey instrument aimed at collecting data for the seven beliefs previously discussed (Philipp et al., 2007) (Appendix G) that was administered to all 42 participants. The researcher was trained to score this survey using the practice modules provided by the survey developers. These modules contained a rubric, examples of rubric use, and multiple survey responses to be recorded for scorer calibration purposes. The researcher read through all rubrics and examples of rubric use, and then participated in the practice-scoring portion of the module. This process produced a reliability coefficient of 0.93 between the researcher and standards set forth and validated by the survey developers.

The beliefs instrument contained sixteen questions designed to determine the mathematical beliefs participants possessed in relation to the seven beliefs categories discussed earlier. Six mathematics education researchers with expertise in teachers' belief and six mathematics education graduate students examined the belief survey. They attested to the validity of the items as accurate measures of the listed beliefs as well as the validity of the rubric used to score the data (Philipp et al., 2007). As a result of the validation process, the researchers were confident that their instrument provided insights into the beliefs and interpretations of its respondents. However, no reliability reports about this instrument were found.

The ordinal data from the beliefs survey were analyzed with a Chi-Squared test using the Statistics Package for the Social Sciences (SPSS) 18.0. Each participant received a pretest (week 1) and posttest (week 8) score for each of the seven beliefs discussed earlier. Since this instrument lacked parallel forms, the beliefs data were collected using the same form. These initial pre- and posttest integer scores ranged from 0 – 4 and were determined from the rubrics validated for the instrument. After all scores were given, change scores were calculated for each participant for each of the seven beliefs. The final possible change scores were zero (no change from pretest to posttest or a decrease in score), one (an increase of one point from pretest to posttest), and two (an increase of more than one point from pretest to posttest).

Quantitative data also came from an established mathematical CCK exam (Hill, Rowan, & Ball, 2005) (for a sample, Appendix F) that was administered to all 42 participants. The instrument has been piloted on a large scale through California's Mathematical Professional Development Institute (CMPDI). Form A was piloted with 629 participants and produced a reliability coefficient of 0.84. Parallel form B was piloted with 620 participants and produced a reliability coefficient of 0.85. Form B contained no questions identical to form A. Hill, Rowan, and Ball (2005) reported that the items used in both forms were subjected to a content validity check and contained adequate coverage across the number concepts, operations, and patterns suggested as necessary by the NCTM standards – which have directly informed the Common Core State Standards for Mathematics (NCTM, 2010). Furthermore, Hill, Ball, and Schilling (2004) reported that the items represented teaching-specific mathematical skills and could reliably discriminate among teachers and meet basic validity requirements for measuring teachers' CCK.

The CCK data were collected from form A (pretest, week 1) of the instrument and form B (posttest, week 8) of the instrument. The two forms of this instrument contained 22-24 questions that were scored as either correct or incorrect using a multiple-choice format. Some questions required participants to select all correct answers from the list of possible choices. In those cases, the question was only scored as correct if the participant selected all correct choices and no incorrect choices. Each participant was given a raw score and then a scaled score out of 100. The scaled score was used for analysis purposes. Unlike the beliefs data, no change scores were calculated with the CCK data, so the pretest CCK knowledge data were only used to control for group differences in the ANOVA model. The data collected from the CCK instrument were also analyzed with an ANOVA using SPSS 18.0 to determine if there were significant differences between the means of the treatment and control groups while controlling for the pretest scores.

Qualitative data were collected primarily from interviews held with four participants from the treatment group. Retrospective interviews were used to collect the data after the completion of the study, which allowed participants to be more accurate and reliable than if they were forced to infer and construct answers to general hypothetical questions (Ericsson & Simon, 1993). The interviews with participating PSTs focused on exploring their experiences with the student work and thinking analysis treatment. They did so by asking participants to analyze student work as they had done during the study and to share their thinking aloud in a conversation with the researcher (Appendix H). A semi-structured interview protocol (Knox & Burkard, 2009) was used to attempt to elicit what types of PCK the participants drew upon when completing the treatment analyses. DiCicco-Bloom and Crabtree (2006) contend that a single interview is both appropriate and common when the researcher has previously established rapport with the participants, as was the case in this study.

The four participants were broken into two groups. The first interview was held with the two participants who showed the lowest collective changes in their CCK and beliefs over the course of the study, as determined by the quantitative instruments described above. The second interview was held with the two participants who showed the highest collective changes in their CCK and beliefs over the course of the study, again as determined by the quantitative instruments described above. Patton (1990) provides rationale for several purposeful sampling procedures within qualitative research. One of these purposeful sampling methods, "intensity," refers to information-rich cases that provide powerful evidence of the experience. The sampling procedure for the qualitative data participants in this study took on the form of purposeful intensity sampling by choosing participants with the highest and lowest belief change scores.

The transcribed interview data were analyzed using the thematic analysis approach described by Aronson (1994). There are four steps in this analysis method:

1. Transcribe conversations and begin to look for patterns of experiences that exist in the data. The researcher reads through the transcriptions several times to familiarize him or herself with the data before the initial patterns are identified. These patterns can come from direct quotes or paraphrasing common ideas that are seen.

2. Identify all data that relate to the initial classified patterns from Step 1. The identified patterns are expounded on by coding all data that further explains it. In this step it is important that all the talk that fits under a specific pattern is identified and placed with the corresponding pattern.

3. Combine and catalogue related patterns into themes. Themes are defined as units derived from patterns and are identified by bringing together components of initial patterns, which often are meaningless when viewed alone. Themes that emerge from the participant's words are pieced together to form a comprehensive picture of the collective experience.
4. Use member checking to ensure that the themes are accurate depictions of the information given by the participants. This is done by asking the participants to provide feedback on the established themes and then incorporating this feedback to adjust the themes as necessary. Any changes must be reflective of a misinterpretation by the researcher.

These four steps were used to analyze the audio-recorded interviews with the participants. First, the researcher read through the transcribed interview data three times to become familiar with it. Then, initial patterns were identified both by direct quotes and common ideas found throughout the transcripts. All data were then coded as support for one of the initial patterns or put into a miscellaneous category. Themes were then identified by bringing together components of the initial patterns to form a comprehensive picture of the participants' experiences with the student work analyses activities. Finally, conversations took place with all four participants to complete the member checking component. The participants were asked to verify whether or not they felt their experiences with the treatment activities were accurately portrayed. The ultimate goal of the thematic analysis was to determine what types of knowing made it possible to attend to the treatment activities in effective ways.

Findings

To answer the first research question, an ANOVA was run on the CCK data. The results revealed that no significant difference was present ($p = 0.599 > 0.05$) between the groups when controlling for the pretest scores (Table 4-1). As the mean scores show, the growth was nearly the same in both groups. In fact, a dependent samples t-test revealed that both the control group ($p=0.002$) and the treatment group ($p<0.001$)

made significant gains from pre- to posttest, respectively. While the student work analysis treatment did not produce significant gains beyond that of the control group, it is important to note that a wider focus on PCK and beliefs did not hinder the development of CCK in the treatment participants compared to the control group.

Table 4-1. CCK mean scores

Group	Pretest Score	Posttest Score
Treatment	42.2	54.6
Control	37.8	50.1

In order to answer the second research question, a Chi-Square analysis was used to analyze the data from the beliefs survey. This analysis revealed a significant difference between the treatment and control groups for six of the seven beliefs (Table 4-2). Only belief 3 (understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures), $p=0.465$, saw no significant difference between the treatment and control groups.

The cross-tabulation in Table 4-3 below allows for further interpretation beyond the p-value and significance. The actual counts reveal that the treatment group experienced larger positive changes in their beliefs about effective mathematics teaching. While only four treatment participants experienced no belief changes (compared to 12 for the control group), seven treatment participants experienced an increase of two or more belief levels (compared to no one in the control group). This finding suggests a significant impact from the treatment activities. This lopsided pattern of larger numbers of participants with high change in the treatment group and larger numbers of no change in the control group was true for all seven of the beliefs measured except belief 3. The remaining six change score cross-tabulation tables are provided in Appendix J.

Table 4-2. Belief change score significance values.

Belief	Pearson Chi-Square Value	Degrees of Freedom	p-value
1	11.053	2	0.004
2	12.185	2	0.002
3	1.533	2	0.465
4	7.795	2	0.020
5	10.462	2	0.005
6	27.300	2	< 0.000
7	9.333	2	0.009

Table 4-3. Belief 1 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	8	4	9.5	10	3.5	7
Control	8	12	9.5	9	3.5	0

To ensure that the control group did not have a ceiling effect on their change scores, a Chi-Square analysis was also run on the pretest belief scores for all participants (Table 4-4). These results show that the pretest scores were not significantly different between the control and treatment groups for beliefs 1, 3, 4, 5, and 7. Furthermore, the treatment group had significantly higher pretest scores for belief 2 (Table 4-5). Only with belief 6 did the control group have a higher pretest score and thus a higher potential for a ceiling effect on their change scores (Table 4-6). Overall, the belief data show that participants in the treatment group changed significantly more towards having beliefs consistent with effective mathematics teaching than did the control participants.

Table 4-4. Beliefs pretest significance values.

Belief	Pearson Chi-Square Value	Degrees of Freedom	p-value
1	6.795	2	0.079
2	6.985	2	0.030
3	3.887	2	0.274
4	6.467	2	0.091
5	0.220	2	0.896
6	9.674	2	0.022
7	1.024	2	0.500

Table 4-5. Belief 2 pretest crosstabulation.

Group	Score 0, expected count	Score 0, actual count	Score 1, expected count	Score 1, actual count	Score 2+, expected count	Score 2+, actual count
Treatment	16.5	13	4	7	0.5	1
Control	16.5	20	4	1	0.5	0

Table 4-6. Belief 6 pretest crosstabulation.

Group	Score 0, expected count	Score 0, actual count	Score 1, expected count	Score 1, actual count	Score 2+, expected count	Score 2+, actual count
Treatment	8.5	13	8.5	5	4	3
Control	8.5	4	8.5	12	4	5

In order to answer the third and final research question, the qualitative data were analyzed using a thematic analysis approach (Aronson, 1994). To begin, the initial patterns identified were:

1. PCK playing a role in PSTs' understanding of student work;
2. Parsing out conceptual vs. procedural understanding in student work;
3. Belief dissimilarities between the high change group and low change group;
4. Knowledge dissimilarities between the high change group and low change group.

The identification of these four initial patterns happened after the researcher became familiar with the data and found common ideas emerging.

Pattern 1 occurred on 54 occasions in the data where participants talked about their student work analysis approach in ways consistent with the definitions of the three elements of PCK. For example, one participant said she found herself wanting to give students more examples to help with concepts before she asked them to solve the problem they were working on (consistent with knowledge of content and teaching within PCK).

Pattern 2 occurred on 47 occasions in the data where participants talked about the underlying understanding of the students. In all cases, participants tried to determine the levels of conceptual and procedural understanding that was being demonstrated in the work. They found this to be an important part of analyzing students' work and imperative in determining the next steps for instruction. A representative example came when one participant shared that she wanted to ask a student to draw out the solution to a multiplication problem so she could determine if the student understood what multiplication really was as opposed to just being proficient with the steps of the traditional algorithm.

Patterns 3 and 4 arose from the realization that the high and low change interview groups were approaching the first two patterns in different ways. The beliefs and types of knowledge that emerged during the conceptual/procedural discussions and PCK involvement (i.e., patterns 1 and 2) gave way to a stark distinction between the two interview groups. The high change participants drew heavily on both CCK and PCK as they analyzed the work and answered interview questions. They revealed beliefs that were consistent with the seven outlined by Philipp et al. (2007). The low change group, on the other hand, shared beliefs that mathematics is more about final answers and drew on much more limited knowledge that appeared to be procedural in and of itself. Although some PCK use came from the low change group, it was much more limited than that of the high change group.

These four initial patterns were then further reduced to give way to themes that cut across the original four pattern groupings of the data. Pattern 1 and pattern 2 had several crosscutting ideas that allowed their data to be merged into the first of the final two themes. Also, patterns 3 and 4 shared several crosscutting ideas that allowed their data to be merged into the second of the final two themes. These themes were identified as:

1. KCS and KCT present in PSTs' thinking
2. Belief and knowledge links

The first major theme, "Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) present in PSTs' thinking," was a result of the numerous appearances of these two domains in the interview responses. After reducing the data into the major themes, it was discovered that the PCK drawn upon was limited to the domains of KCS and KCT. All four participants spoke about why they thought a student might struggle as well as why they thought a particular topic was difficult to learn, albeit in very different ways from group to group. Moreover, all four participants commented on other examples or questions they might use to help students further their understanding, with the high change group containing much more justification and detail. One PST commented on her student work analysis thinking by saying, "I kept thinking about why the student would have struggled with that question. I mean if they are struggling with it maybe it's because it's hard for a lot of people and I could use that to help my whole class." A second student shared, "So, with the division problem we looked at it seemed that the student was forgetting the place holder. I don't think it's the idea of division as much as it is the setup. I guess I was thinking about how to avoid that problem and draw some pictures like you said to help the student see what was going on and not let them mess it up doing it the regular way." These quotes highlight such reactions which continually appeared in the data representing the KCS domain of PCK.

The KCT domain was also well represented in the data. One PST remarked, "I really wanted to ask this student about their first multiplication step. I think their order [of multiplication] was wrong, and if that was the case I would have had them do a one digit problem to see if they could find their own mistake." Another participant commented, "I don't think this problem set is a good one to check for understanding. I look here and I really don't know if the student understands or not. I would want them to show me on a number line what the subtraction means in a real situation. If they could do that I would be way more likely to say they understand. Here it just seems like they know the formula well." These quotes help display the consistent level of KCT that was present in the PSTs' responses.

The second major theme, "belief and knowledge links," revealed two key findings. First, all four participants talked about mathematical knowledge necessary to complete and grade mathematical work. However, the high change and low change groups presented their responses in very different ways. The low change group talked about KCS and KCT, but they spent a majority of their time focusing on whether the answers were right or wrong, as one participant in this group explained, "I don't know if this student could do, say, a word problem involving division like this. But, they do understand how to do subtraction because they have solved all the problems correctly. It might be better if I asked them to do something else, but I wouldn't be too concerned because again they are getting the answers right." Here, the PST was flirting with the idea of using other examples to push a student's thinking but seemed satisfied with the correctness of the answer and may be unlikely to push the student to a more conceptual understanding. Perhaps this was due, at least in part, to the misalignment between the PST's beliefs and that of effective mathematics teaching.

The high change group, on the other hand, seemed to use PCK but also wanted to push students well beyond right or wrong answers. A participant in this group stated, "The student got these ones right so it could be like an 80 or 90% [on an assessment]. But that answer there outweighs the right ones. It looks to me like they are following the rules when there are not two digits in the bottom number. I would like to see them draw out a multiplication problem and explain to me what multiplication means. If they could do that then it is probably just a simple error in their setup." Here, it becomes clear that the high change PSTs felt more comfortable predicting and talking about why answers were wrong. They also were quick to search for other examples or questions that might help the students improve their understanding.

Overall, the results of the qualitative data show that PCK (in particular KCS and KCT) was being elicited in the participants as they analyzed student work for understanding and planned next steps for instruction. However, the beliefs that the PSTs hold greatly affect how they approach knowledge development in students. Their beliefs seemed to create a comfort level to determine how far the students should be pushed. Those PSTs with high change (more alignment with beliefs about effective mathematics teaching) seemed more comfortable in pushing students towards a conceptual understanding. They suggested the use of questioning strategies and deliberately chosen examples, where the low change group seemed much more comfortable with a focus on correct answers. The low change group exhibited a more procedural understanding of the mathematics content themselves, and this bled through to their expectations for students' understanding.

To help ensure the accuracy of these results, member checking was used as a final step in the analysis. An email conversation took place between all four participants and the researcher in a one-on-one fashion. First, each participant was told briefly about the elements of KCS and KCT (definitions and one example) and asked if they agreed that their responses were drawing upon these types of knowledge. Next, the participants were told about how their beliefs were being interpreted in relation to the seven beliefs about effective mathematics teaching. Each participant then agreed that the data were accurate in describing their beliefs and knowledge.

For example, the low change group agreed that right/wrong responses were more indicative of understanding and that it was too difficult (for them and the students) to undertake alternative approaches to the problems. The high change group also agreed that the right/wrong answers were important, but they expressed the opinion that pushing students to explain their thinking and expose misconceptions was important and well worth the time spent to do so. The member checking experience further supported the data interpretation and helps to validate the qualitative findings.

Discussion

One purpose of this study was to determine whether asking PSTs to analyze student work and thinking by way of the modified protocol explained earlier could have an impact on the CCK and beliefs about effective mathematics teaching that the participants possessed. The study also sought to determine if any elements of PCK were being elicited from the participants during the treatment activities. More specifically the research questions that drove the study were:

1. What is the influence of analyzing student work and thinking (by way of diagnosing understanding and planning next steps for instruction) on PSTs' CCK?

2. What is the influence of analyzing student work and thinking (by way of diagnosing understanding and planning next steps for instruction) on PSTs' beliefs about effective mathematics teaching?

3. What types of PCK do PSTs draw upon when diagnosing student understanding and planning next steps for instruction while analyzing student work and thinking?

The results of the mixed methods analysis showed that, first, the student work and thinking analysis protocol treatment had the same effect on the CCK of the treatment group as the control activities had on the group. It is important to recognize that the treatment group experienced gains in CCK equal to that experienced by the control group (both significant from pre- to posttest). This may have been in part because of the initial belief filters (i.e., an effect of beliefs that allows PSTs to interpret experiences or information in their courses in ways different from those their instructors intended) prevented growth beyond that of the control group as has been suggested by Ambrose (2004).

The PST's beliefs about effective mathematics teaching, however, saw a significant increase in the treatment group as compared to the control group. Of the seven beliefs measured, six were found to be significantly impacted by the treatment activities. Only belief three (understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures) did not turn out to be significant. This suggests that the treatment activities were able to create a context for developing mathematical beliefs in the university classroom as opposed to being in the field. This supports the work of Philipp and others (2007) who indicated that field placements that are not supportive of the change being taught in classrooms might actually be counterproductive in developing beliefs for effective mathematics teaching.

Finally, the qualitative data revealed that the PSTs in the treatment group drew upon elements of PCK (as defined by Hill, Ball, and Schilling, 2008) when they were asked to complete the treatment activities. Not only did they grapple with KCS and KCT topics in their responses, but they also revealed that their beliefs acted as a filter towards their chosen focus when analyzing student work and planning next steps for instruction, which supports Ambrose (2004). Member checking helped to validate these two major themes which emerged from the data.

Despite these positive findings, future iterations of research are still needed to continue to refine the role of student work and thinking in the preparation of PSTs. Although the content knowledge was not significantly impacted in this study, it is entirely possible that the initial belief filters prevented the knowledge growth. Future research can help to discover if knowledge gains may be latent until after belief changes have taken root, or if it may be necessary to refine the student work examples chosen for use. Additionally, some miscellaneous qualitative data pointed to a preference in the type of student work example. The low change group seemed more drawn to the video examples over the written student work. Perhaps context-rich student work coupled with verbalized student thinking may have provided more scaffolding for PSTs who were still grappling with their own belief changes. Although the high change group appeared to be indifferent about the type of student work example, it may be necessary to conduct future research to determine if different types of student work examples have different impacts on knowledge and belief growth.

Another reason to conduct future research is the limiting nature of certain elements of this study. To begin, the small group sizes (n=21) may have affected the outcomes of the quantitative data analyses. The Chi Square analysis, for instance, had very low expected values in some cells of the cross-tabulations. Furthermore, the length of the study could have caused two issues. Exposing the treatment group to the modified protocol for student work analyses for only eight weeks may have limited the knowledge growth if the initial belief filters prevented the accumulation of new CCK. Also, the brevity of the study could potentially lead to non-lasting effects on the PSTs' beliefs. A longer study with follow-up components would be needed to determine if the treatment activities are capable of creating lasting effects on PSTs' beliefs about the effective teaching and learning of mathematics.

There was also a possibility for a threat to the design validity of the study that could have affected the results. Specifically, there was a diffusion threat to the study's construct validity due to possible interactions between the control and treatment groups. Although these groups were not in any other courses together (they were members of separate cohorts of students at the university), they took classes in the same buildings and classrooms.

Finally, the qualitative analysis may have been influenced by the researchers' own bias and background. Although steps were taken to follow the thematic analysis process (Aronson, 1994) and member check with interview participants, it is difficult to completely separate the researcher from the findings. It is possible that because the researcher was involved in the study, the qualitative findings were bolstered to match the hypotheses that existed going into the study.

Even with the limitations to the study, findings suggest that using student work and thinking in the preparation of PSTs can have a positive impact on the development of the knowledge and beliefs necessary for effective mathematics teaching. The Using Student Work (Appendix I) protocol can be adapted to fit the needs of individual university classrooms, and the results suggest that it is a worthwhile undertaking for PST instructors to provide student work analysis opportunities to prospective teachers. Although future iterations of research are certainly necessary to refine its role, this study shows that student work and thinking analyses should be an integral part of the PST classroom aimed at helping prospective teachers learn and teach mathematics in effective ways.

CHAPTER 5
ARTICLE 2 – USING STUDENT WORK IN THE PRESERVICE TEACHER CLASSROOM TO DEVELOP KNOWLEDGE AND CHANGE BELIEFS

Over the past several decades, one of the most prevalent educational topics in the United States has been preparing teachers to teach mathematics effectively. A common thread through the years has been that the single greatest intervention a school can provide for increasing student achievement is an effective teacher (NCATE, 2010). For example, the National Research Council (NRC) (2001) identified teacher knowledge as one of the most influential factors on student achievement. However, research has also shown (e.g., Ambrose, 2004; Philipp et al., 2007) that belief change and alignment are necessary co-requisites to teachers' knowledge development. Alignment between beliefs and knowledge for teaching mathematics is pertinent for preservice teacher (PST) growth. To this end, it becomes critical for PST education programs to help PSTs see the interconnected, conceptual nature of mathematics and to develop the knowledge and beliefs necessary to teach in that manner. This article focuses on explaining and developing the beliefs necessary for PSTs to teach mathematics in effective ways. A mechanism for developing these beliefs is allowing PSTs to analyze student work in ways that have been shown to develop these beliefs (Chapter 4).

There are four critical elements of beliefs (Ambrose, 2004) that must be acknowledged in order to define the beliefs necessary for effective mathematics teaching. They are:

1. Beliefs influence perceptions – they shape individuals' interpretations of events (Pajares, 1992).
2. Beliefs draw one towards a particular position or direction regarding a given issue (McGuire, 1969; Rokeach, 1968).

3. Beliefs are not all-or-nothing entities – they are held with differing intensities (Pajares, 1992).

4. Beliefs tend to be context specific, arising in situations with specific features (Cooney, Shealy, & Arvold, 1998).

With these in mind, Philipp et al. (2007) defined the following as the beliefs necessary for effective mathematics teaching:

Belief About Mathematics

1. Mathematics is a web of interrelated concepts and procedures (and school mathematics should be too).

Beliefs About Learning or Knowing Mathematics, or Both

2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts.

3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.

4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.

Beliefs About Children's (Students') Learning and Doing Mathematics

5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect.

6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking, whereas symbols do not.

7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

Many factors influence PSTs' beliefs and knowledge for teaching mathematics. However, using student work and thinking as a catalyst for their development has shown promise (Crespo, 2000; Kazemi & Franke, 2004; Son & Crespo, 2009).

More specifically, analyzing student work in structured ways has been shown to help PSTs build beliefs about the effective teaching and learning of mathematics while maintaining steady growth in CCK (Chapter 4). In that article, a treatment group was exposed to a student work analysis protocol (Appendix I) that provided a structured question and discussion format around students' mathematical work and thinking. This treatment was found to have a significant impact on PSTs' beliefs about effective mathematics (six of the seven beliefs listed above). These findings further suggested that student work and thinking analyses could be a catalyst for PSTs' belief development. This article shares details about the protocol, about collecting student work, and about using the protocol to teach PSTs. This protocol supports teaching mathematical content in ways that help develop beliefs about effective mathematics teaching.

The Student Work Protocol

The protocol developed for analyzing student work in the PST classroom is a modification from the National School Reform Faculty's (NSRF) ATLAS protocol for discussing student work with practicing teachers. The core components of the original protocol are structured dialogue, guiding questions, and reflection. Although the protocol was adapted for the PST classroom, these core components were kept intact and served as the core components of the new protocol as well.

The main modifications came in the wording of questions (to make them more appropriate for individuals who have not yet had their own classroom) and changing how student work is collected. For example, the original protocol asked practicing teachers to analyze student work and predict effective teaching strategies, think about their own practice in the classroom, and alter existing assignments they have given.

These questions were removed and a greater focus was placed on diagnosing the understanding students displayed and planning the next steps for instruction when the PST worked one-on-one with the student. This focus allows for a meaningful analysis of the mathematical content without asking purely pedagogical questions that would require some experience in a K-12 classroom setting.

This protocol provides structure and support when writing lessons that present mathematics content to PSTs in the context of student thinking and learning. The development of a lesson plan is shared to show how this protocol can be a guide to using student work as the basis for teaching mathematical content. Next, information on how to collect and use student work in conjunction with this protocol is shared for a unit on numbers and operations.

Collecting Student Work

Using student work analyses in the PST classroom presents the need for the PST educator to provide the work to be analyzed. These work examples can, and should, come from a variety of places. Again, the work collected for each mathematical topic should present a variety of levels of understanding as well as a variety of solution strategies.

Student work can be collected from several places including local schools, preservice teacher textbooks, online resources, and other research projects. For the number and operations unit, student work was collected from a local third grade classroom, two preservice teacher textbooks, and a video series from an existing research project with preservice teachers. The example unit discussed here deals specifically with number and operations, although collecting student work for other mathematical topics can happen in the same manner.

The researchers contacted a local building principal and gained permission to meet with teachers. From there, meetings were set up with teachers in their classrooms to discuss students' work. The teachers provided recommendations about which students they believed would demonstrate misconceptions, show advanced strategies, provide invented strategies, and convey fluency in using standard algorithms. These recommendations were used to select student workbooks, which were borrowed for the day. Each workbook was examined for solution strategies (advanced strategies, invented strategies, and algorithm fluency) and misconceptions. As examples were found, researchers covered up all student identifiers to ensure anonymity and then made copies of the material. These work copies were then taken back to the university classroom to be used with PSTs and the workbooks were returned to the students. An example is provided in Figure 5-1. This example was chosen because of the misconception shown for the lattice multiplication algorithm. This student multiplied correctly but placed the results in the incorrect boxes, which created incorrect answers. This piece of student work represents a misconception that can spark discussion when analyzed. Moreover, it helped to address Belief 3 (understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures) and Belief 4 (if students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them; if they learn the procedures first, they are less likely ever to learn the concepts).

Other work examples were collected from PST education textbooks. The two books used for this sample unit were written by Sowder, Sowder, and Nickerson (2009) and Ashlock (2010). The work examples in these textbooks were already aligned with

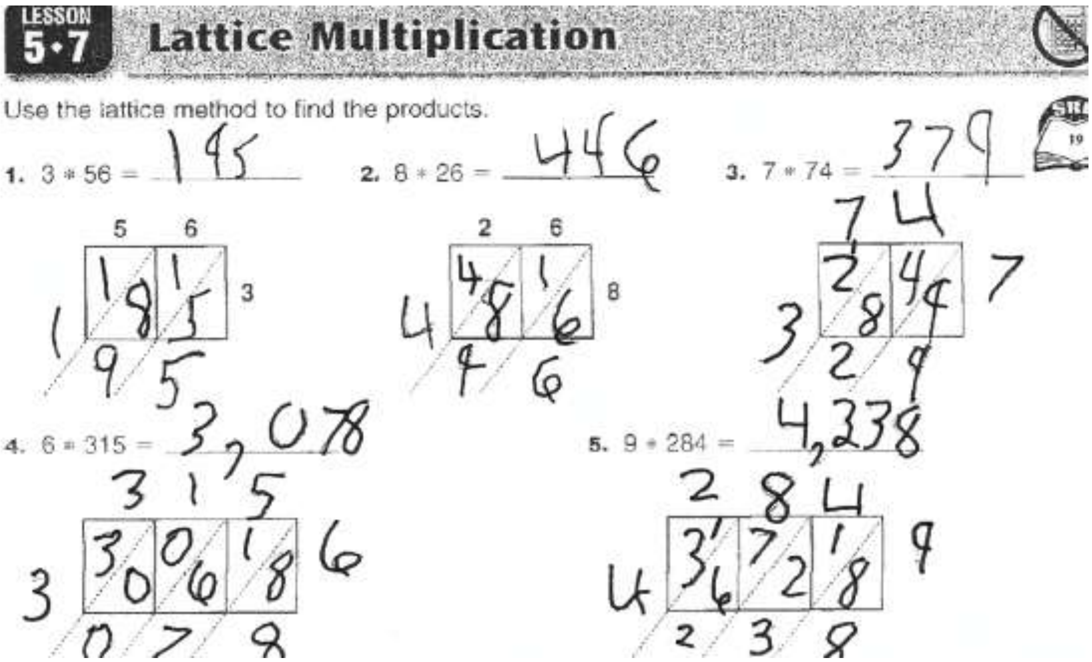


Figure 5-1. Student work example from classroom collection.

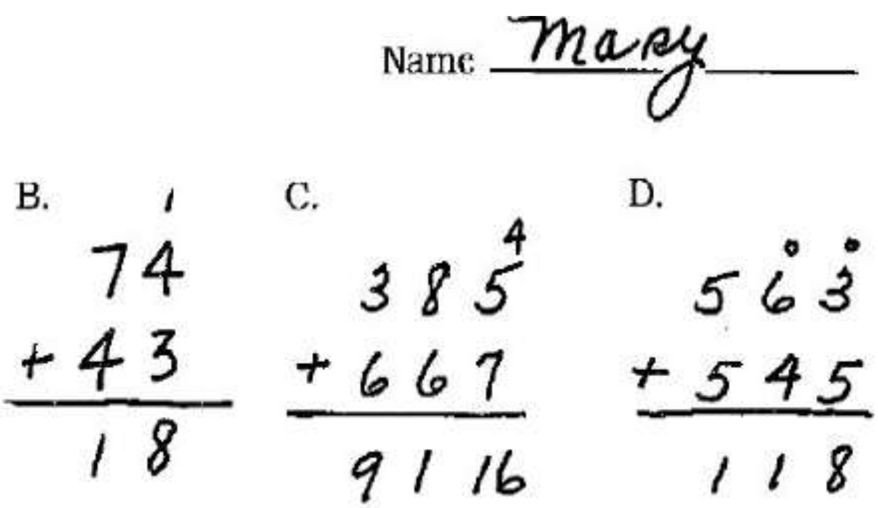


Figure 5-2. Traditional algorithm misconception.

specific mathematical topics; however, the numbers of examples found were limited. Although many student work examples were present in these texts, only four examples were chosen for this unit on number and operation. Figure 5-2 provides an example from Ashlock (2010, p. 20). This example was chosen to follow the student work sample above on lattice multiplication. The student in Figure 5-2 is making a similar mistake, only with the traditional algorithm. By pairing these examples, PSTs were prompted to compare the two solution methods as well as to discuss the similarities and differences of the misconceptions being presented. This further bolstered the importance of beliefs 3 and 4.

Still, other work examples were collected from two teacher education resources, namely the Integrating Mathematics and Pedagogy (IMAP) project and the Annenberg Learner website. The IMAP project produced a series of videos that captured elementary-aged students solving an assortment of problems with a variety of results. These videos range from thirty seconds to over six minutes in length. They also cover a variety of mathematical topics, student understanding, and grade levels. For this numbers and operations unit, three videos were selected from Annenberg and six videos from the IMAP project. The selection criteria included variety of student understanding and coverage of number and operations concepts.

For example, one video was selected from the IMAP project that presented a student with a very flexible, conceptual understanding of mathematics. He was able to do most problems in his head and explain multiplication through partial products and the relationship to anchor points (i.e., 5x6 = 5x5+5 since he knew 5x5 but not 5x6). This video helped develop Beliefs 1 (mathematics is a web of interrelated concepts and procedures and school mathematics should be too) and 5 (children can solve problems in novel ways before being taught how to solve such problems, and children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect), because it portrays flexible thinking that was not likely taught directly, as well as highlighting the connectedness of numbers (through anchor points) and operations (connects addition to multiplication).

As another example, a video was selected from the Annenberg Learner website that captured a second grade lesson. Students were asked to calculate a missing addend problem (how many more objects did one person have than another person). As students solved the problem, many chose to subtract while others chose to count up or draw pictures to represent the situation. Discussions with the students revealed that several thought about the problem in very different ways than the teacher had expected. Further discussion showed that the real world context of the problem allowed students to explore solutions that deviated from the algorithms they were taught. The context provided a platform for their thinking about novel situations. This video helped develop Beliefs 6 (the ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics; for example, real-world contexts support children’s initial thinking whereas symbols do not) and 7 (during interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible) by depicting the power of context and the varied thinking strategies students used.

Finally, a video was selected from the IMAP project to show that procedural fluency does not necessary imply conceptual understanding (i.e., Belief 2 - one’s knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts). In this video, a student is using the subtraction algorithm to solve a two-digit subtraction problem. The student misinterprets subtraction from zero but otherwise has a very good handle on the algorithm. What appears to be a solid understanding of subtraction is brought into question when the student is asked to draw the example and use manipulatives to check the original answer. It becomes clear that this student has a procedural understanding of subtraction with an almost complete lack of conceptual understanding. This video has proved to be very powerful for PSTs because many admit to originally thinking the student "understood" and should not have been probed further. However, they also admit to rethinking the use of procedures as the baseline of mathematical understanding.

These examples were then used to build lesson plans that covered number and operation topics while also exposing PSTs to students' thinking in order to help develop mathematical beliefs for effective teaching. Next, a specific plan for one lesson within the number and operations unit is shared to demonstrate how the protocol can be used to guide the development of lessons for PSTs. The chosen topic was subtraction of whole numbers.

Using the Protocol to Develop Lessons

Instead of preparing a lesson plan that investigates the conceptual meaning of subtraction with PSTs as "students" of mathematics, the goal of this lesson plan was to immerse PSTs in the work and thinking of elementary students to help unpack the types of knowing one must have in order to conceptually understand subtraction. This goal requires that PSTs be exposed to student work in meaningful ways that require them to make sense of solutions, both correct and incorrect. Through this process, PSTs must draw from and build on their own understanding of subtraction in order to diagnose students' understandings and plan next steps for instruction.

The modified protocol becomes the vehicle that moves the student work examples (including those shared earlier) into position to develop and strengthen PSTs' understanding of subtraction as well as their beliefs about how to effectively teach it. The structured dialogue, guiding questions, and reflection components bring the student work to life by requiring the PSTs to interact with it and with each other. It is through these interactions that PSTs test and bolster their understanding of subtraction (Chapter 4). Appendix K contains this lesson plan in its entirety, but the use and development will be discussed in detail next.

Student work examples were chosen for this lesson plan to represent student misconceptions of the subtraction algorithm and invented strategies that are non-traditional in nature. In all, nine pieces of student work were selected for this lesson. Six multi-digit subtraction problems (Figure 5-3) were selected from Sowder, Sowder, and Nickerson (2009, p. 55).

These problems were chosen because they represent a wide range of solution strategies as well as both correct and incorrect solutions. More specifically, the strategies shown here address Beliefs 5 (children can solve problems in novel ways before being taught how to solve such problems, and children in primary grades generally understand more mathematics and have more flexible solution strategies than adults expect), 6 (the ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics; for example, real-world contexts support children’s initial thinking whereas symbols do not), and 7 (during interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible). These six problems collectively demonstrate students’ thinking about problems in novel ways, many of which may not have been taught or may not have been the PSTs’ first choice of solution strategy.

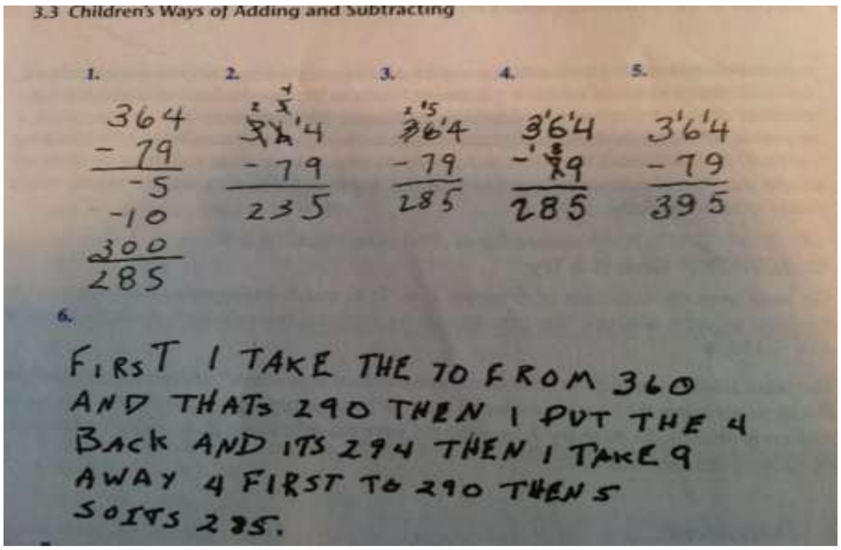


Figure 5-3. Student work examples for subtraction to address Beliefs 5, 6, and 7.

Furthermore, the next student work sample (Figure 5-4) addresses Beliefs 3 (understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures) and 4 (if students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them; if they learn the procedures first, they are less likely ever to learn the concepts). This piece, created by the researcher, shows a misconception with place value in a situation that is not truly a base ten example (i.e., hours and minutes). The misuse of the traditional algorithm for subtraction suggests that learning the procedure of subtracting may not transfer to novel situations that require an adjustment in the regrouping phase. Learning procedures without learning the underlying concepts is not a generative approach.

"A person leaves their house at 7:26am bound for their workplace. They arrive there at 8:17am. How long did it take them to travel to work?"

A student solves this problem using the traditional algorithm for subtraction:

$$\begin{array}{r} 8{:}17 \\ -\ 7{:}26 \\ \hline 91 \text{ minutes} \end{array}$$

They explain that they borrowed from the 8 to make the 1 an "eleven"

Figure 5-4. Student work example for subtraction to address beliefs 3 and 4.

The final two student work examples were videos selected from the IMAP project. They were selected specifically to address beliefs 1 (mathematics is a web of interrelated concepts and procedures (and school mathematics should be too)) and 2 (one's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts) while also covering the topic of subtraction. The first video was of three students (each working separately) on a missing addendum problem. They were given the scenario that a person had six fish but wanted a total of 13, and the question asked how many more they would need to acquire. One solution, in particular, created fertile ground for discussing belief 1. A student drew out the problem but misheard the goal of 13. After calculating an incorrect response, he mentally adjusted his answer by adding to it as opposed to starting over with the new information.

The second video showed a student who correctly uses the subtraction algorithm several times before coming across a situation that involved a zero. When an incorrect answer was reported, the teacher asked the student to solve without the algorithm. It quickly becomes clear that the apparent understanding of subtraction was purely procedural and that the student lacked an understanding of subtraction. She viewed subtraction as a take-away modeled by the algorithm, but had not previously viewed subtraction as distance between numbers. This was evident as she checked over her incorrect response.

After these examples were selected, the lesson was planned around the examples using the modified protocol to plan instructional time. With each student work example, PSTs were first asked to individually identify how well they believed the student understood the concept of subtraction. They were encouraged to write down talking points as they thought about the examples. After a few minutes, the question of understanding was posed again for small groups to discuss. The groups were given 3-5 minutes to discuss their thoughts. This individual/group alternating process was repeated for each of the nine student work samples chosen for this lesson.

When posed to the small groups, the question of understanding sparked much debate over the students' understanding as well as the PSTs' understanding. For example, several groups initially struggled with identifying the error in the problem involving time. The discussions around this problem quickly shifted to why the algorithm had failed and why a conceptual understanding of subtraction was necessary for student success.

During the small group time, PSTs were also asked to decide how they would move forward with the student. Their efforts often included attempting to alleviate misconceptions and reworking algorithm procedures for incorrect work, while creating more challenging problems to push students who did seem to understand. For example all small groups identified the misconception with the time subtraction (Figure 5-4) and reported that the student did not understand the concept of regrouping. They suggested an array of techniques, from drawing pictures to using blocks, for the regrouping process. Moreover, one group suggested the student in Problem 4 (Figure 5-3 above) would need to solve and explain a few more problems using that unusual method of regrouping before they would be comfortable commenting about his understanding. That group also wanted this student to model his or her method using the base ten blocks. They felt it would help them and the student better understand how much of this strategy was procedural and how much showed a deeper understanding.

Diagnosing understanding and planning next steps for instruction during the small group sessions produced several mathematical knowledge and belief discussions. PSTs grappled with conceptual versus procedural understanding, models for subtraction, and beliefs about what was necessary to know about subtraction. These discussions showed PSTs specifically grappling with each of the seven beliefs for effective mathematics teaching.

A large group reflection period followed the small group setting for each student work example used. The purpose of the large group discussion was primarily to allow for groups to share their ideas with the class and receive feedback. In many cases, the groups approached the student work differently. The large group discussion was rich in dialogue and many ideas were exchanged. This time also allowed the instructor to interject any additional thoughts about the mathematical topic or the student understanding that had not yet been discussed. For this lesson on subtraction, the goals were to cover why the traditional algorithm works, model subtraction on a number line, model subtraction using base ten blocks, discuss the relationship between addition and subtraction, and discuss the relationship between subtraction and division. Some of these topics came out of the conversations that PSTs had in both the small and large group settings. However, the relationship to division and modeling subtraction on the number line did not. The instructor used the large group setting to pose these ideas and allow for PSTs to practice modeling on a number line and to discuss how and why division is based on subtraction (repeated subtraction). This supplemental discussion finished the subtraction lesson and set the stage for the next topic on division.

The use of this structured-dialogue protocol based on student work creates lessons that differ from a traditional content course for prospective teachers. However, it allows for the same mathematical topics to be covered while also providing an additional layer of context. In addition, using this protocol has led to significant mathematical belief changes in PSTs while allowing them to maintain steady growth in content knowledge and PCK (Chapter 4).

Discussion

Research has shown that using student work to contextualize mathematical learning for PST is a worthwhile endeavor (e.g., Crespo, 2000; Son & Cresop, 2009). Furthermore, the specific use of the modified ATLAS protocol has produced significant results in terms of mathematical belief change while supporting continued positive growth in CCK, along with strong evidence of eliciting PCK. In the continued attempt to refine and bolster the way PSTs are prepared to teach mathematics, these results are promising.

Using lesson plans like the one shared in this article provide a structure for teaching mathematical content within the greater context of teaching. PSTs involved in this learning showed deep reflection regarding their understanding of the mathematics as well as an ability and a willingness to talk about the others' understanding. Learning within the greater teaching context helps PSTs enrich their knowledge and beliefs about what is necessary to teach mathematics effectively. By purposefully selecting the student work examples to expose the seven beliefs, PSTs grappled with and changed their own beliefs about mathematics. The seven beliefs about effective mathematics teaching (Philipp et al., 2007) help us understand how teachers need to view the teaching and learning of mathematics in order to teach effectively. Drawing from this, promising interventions such as the modified student work analysis protocol discussed here help PST education programs prepare PSTs to teach mathematics in effective ways.

CHAPTER 6
ARTICLE 3 – USING A STUDENT WORK PROTOCOL TO DEVELOP TEACHERS' CONTENT KNOWLEDGE AND BELIEFS ABOUT EFFECTIVE TEACHING

Teachers' preparation for teaching mathematics takes on many forms and covers a variety of topics at the elementary school level. Because teachers' mathematical understanding is a critical component of K-12 students' acquisition of mathematical knowledge (Darling-Hammond & Baratz-Snowden, 2007), it is important to provide teachers with opportunities to develop mathematical knowledge and beliefs that are consistent with effective teaching (Philipp et al., 2007), regardless of the topic and setting.

The analysis of student work and thinking has been shown to provide such experiences within the context of K-12 mathematical learning (Ambrose, 2004; Crespo, 2000, Philipp et al., 2007). This article shares a student work and thinking analysis protocol developed to advance preservice teachers' abilities to effectively teach mathematics. The protocol will be of interest to professional development (PD) providers who are trying to change the way teachers view the discipline of mathematics. Although inservice teachers are capable of selecting their own work for professional development activities, the protocol presented in this article will allow teachers to be introduced to the importance of analyzing students' work. Initially, presenting student work will be especially useful at the elementary school level because elementary teachers teach more than one subject and generally do not consider themselves to be experts in mathematics.

The main focus of this protocol is to elicit responses that force teachers to grapple with their own mathematical understanding and beliefs through the evaluation of students' understanding. The protocol scaffolds from two guiding questions:

1. What does this student's work show about his or her understanding and/or misunderstanding of the topic?
2. Based on your response to Question 1, what are the next steps for instruction that should be used with this student?

This protocol and its guiding questions have been shown to elicit and develop elements of teachers' mathematical knowledge that align with Hill, Ball, and Schilling's (2008) knowledge framework. Specifically, common content knowledge (CCK) and pedagogical content knowledge (PCK) have been positively affected (Busi, 2014). Hill, Ball, and Schilling (2008) defined CCK as knowledge that teachers use in ways similar to how it is used in many other professions or occupations. PCK, on the other hand is knowledge that is a teacher's transformation of content knowledge in the context of facilitating student learning (Hill, Ball, & Schilling, 2004). These elements (among others) are an important component of preparing PSTs to teach mathematics in effective ways.

According to Hiebert & Lefevre (1986), effective mathematics teaching is an approach that prepares students to build high levels of conceptual understanding, which refers to an:

> understanding that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network" (pp. 3-4).

This is in contrast to a procedural understanding that is characterized by:

> a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols. It consists of rules or procedures for solving mathematical problems, many of which are chains of prescriptions for manipulating symbols (Hiebert & Lefevre, 1986, pp. 7-8).

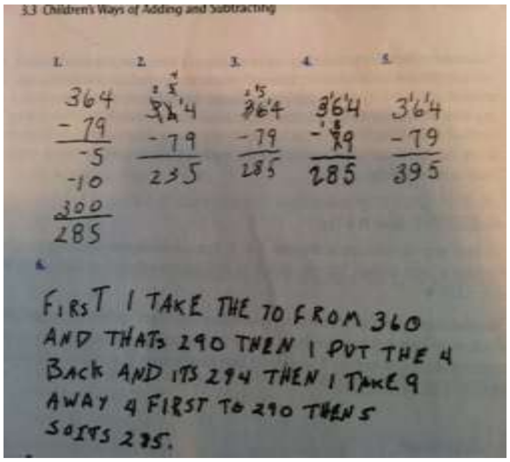
Organizations such as the National School Reform Faculty (NSRF) continue to make great efforts to provide teachers and teacher educators with the tools necessary to promote this type of learning in students.

The NSRF has provided a variety of protocols to the education community for a number of years covering an assortment of topics. The ATLAS protocol, in particular, was created to guide a professional development session aimed at using student work analyses to help teachers better understand what their students know and how they are thinking. ATLAS utilizes a structured dialogue format that prompts teachers to examine their own students' work in groups, as well as to answer questions about the students' understanding. Rounds of prompts and questions exist for describing what the work is about, and interpreting what the students know in order to inform changes for the classroom. As with most protocols, ATLAS finishes with a reflecting and debriefing process. This protocol fits experienced practicing teachers very well and provides a useful structure for bringing students' work from the participants' classrooms. However, inservice teachers may not be prepared to initially identify a variety of misconceptions given the significant demands on their time as well as their own preparation in the discipline of mathematics. Therefore, teachers would benefit from participating in professional development where the provider collects the student work and aligns it with the curriculum. This initial experience can serve as the basis for teachers' to select their own students' work for ongoing professional development.

The protocol created and shared here is based on the ATLAS protocol but contains the major modification of the PD provider selecting the work and integrating into the PD. Places to get this work include local elementary schools, videos from research projects such as the Integrating Mathematics and Pedagogy (IMAP) project, classroom videos from education sites such as Annenberg Learner (http://www.learner.org/), etc. As suggested in the ATLAS protocol, this work should be selected to represent a variety of ways to solve problems. Examples of traditional algorithms, student invented strategies, and common misconceptions should be included. The modified protocol is provided in its entirety in Appendix I.

To demonstrate the practical use of this protocol, an example of how it might be used follows. This example is from an undergraduate content/methods course for prospective elementary teachers who did not have access to students. A session focused on subtracting integers would start with a collection of student work covering the traditional subtraction algorithm, drawings of subtraction, subtraction with a number line, student invented strategies for subtraction, etc. Again, this work can be written or can be accessed from video vignettes that capture students working on subtraction problems. A PD provider is limited to the student work he or she can find on the topic, but as much variety as possible should be included. For this example, student work was collected from a local elementary school as well as compiled from videos (IMAP project) and a preservice teacher education textbook (Sowder, Sowder, & Nickerson, 2009, p. 55). Figure 6-1 shows the specific examples that were chosen.

Once the student work is selected, the PD provider should select the piece of work (Figure 6-2) that will be used to introduce the topic. This can be an exemplar or one that has a misconception. Regardless, the example should be shown and the teachers should familiarize themselves with what is happening in the written work or on the video. With the teachers in small groups, they can address the questions for each students' work shown in Figure 6-2 (Sowder, Sowder, & Nickerson, 2009, p. 55).



A student is given a word problem as follows:

"A person leaves their house at 7:26am bound for their workplace. They arrive there at 8:17am. How long did it take them to travel to work?"

A student solves this problem using the traditional algorithm for subtraction:

  8:17
- 7:26
  91 minutes

They explain that they borrowed from the 8 to make the 1 an "eleven" and then subtracted each column.

IMAP video #3 covers an error using the traditional subtraction algorithm with a zero (70 – 23). Also shows manipulatives and hundreds charts.

IMAP video #1, covers a missing addendum problem that is solved using counting up as well as manipulatives to find a difference.

Figure 6-1. Student work examples chosen for the onset of the example lesson.

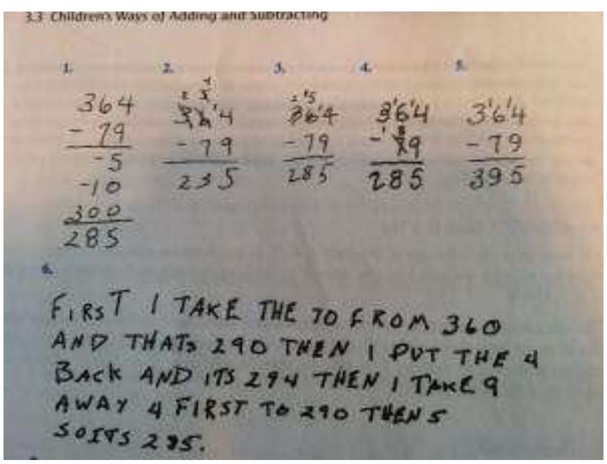


Figure 6-2. Student work example.

1. Is the solution correct? If not, what mistake is the student making? Explain your thinking.

2. Analyze the level of understanding the student has. What has the student done well? What concepts or understanding is the student lacking? Explain your thinking.

3. What should the next steps in instruction be for teaching this student? How would you expand their understanding of the concepts mastered and/or help them improve the missing concepts or understanding? Explain your thinking.

The variety of students' responses was chosen so that teachers can identify students who understand subtraction at different levels. Many examples provide rich discussion points for these three questions. Teachers may need about one minute to think about the first student response and how they would answer the three questions. Then, each teacher in the group can take one minute to share his or her responses. After this analysis and discussion takes place, the group can move into the reflection process.

Reflections are designed to take place in a large group setting and are intended to share the analysis activities that took place in small groups. The PD provider can pose the following two questions to the group and ask for volunteers to share what their group discussed:

1. What is one thing that you learned while talking over the student work at your table? Why is this significant to you?
2. What new perspectives about the student, mathematical understanding, and/or mathematical content did your colleagues provide you?

At this point, the PD provider can use the responses from the reflection questions to springboard into a discussion about the content of subtraction.

During the remainder of the session, additional student work examples can be used to demonstrate other important points about the topic, as well as provide more practice for analyzing student work. Common questions might include, "What has this student done wrong?" for misconception examples, and "Will this student's method always work?" or "What has this student done?" for unique or self-invented solution strategy examples. From Figure 6-1, the time problem is an example of a misconception that sets the stage for a place value and borrowing discussion while the

IMAP videos provide examples of unique solutions outside of tradition algorithms that teachers can analyze.

This example shows how student work can be used to get teachers in the habit of analyzing students' work. It also presents an opportunity for PD providers to introduce content, in this case subtraction, in a non-threatening manner. Regardless of the topic, using student work as a platform for building mathematical and pedagogical knowledge in teachers is an effective approach. Gradually, teachers will be prepared to select their own examples to highlight various levels of understanding during follow-up sessions.

CHAPTER 7
CONCLUSION AND DISCUSSION

This study examined how the use of student work and thinking through a specific analysis protocol impacted PSTs CCK (Hill, Ball, & Schilling, 2008), PCK (Hill, Ball, & Schilling, 2008), and beliefs (Philipp et al., 2007) about effective mathematics teaching. It used a mixed methodology approach to study CCK and beliefs through quantitative data analyses while investigating PCK through qualitative data analysis. A randomized control design was also employed. The study utilized a situated theoretical framework of learning along with the MKT theoretical framework of necessary mathematical knowledge for effective teaching. Furthermore, the study operated under the theory that belief changes are co-requisite to knowledge gains (Ambrose, 2004, Philipp et al., 2007).

Forty-two PSTs enrolled in an undergraduate mathematics content course for prospective teachers participated in this study; twenty-one represented the control group, and twenty-one represented the treatment group (assigned randomly). Quantitative data were collected from all forty-two participants in a pretest-posttest format that occurred during the first and eighth week of the study. Qualitative data were collected from four participants who were chosen based on their quantitative belief change scores. The two participants with the lowest changes in beliefs were selected for the low change qualitative interview group, while the two participants with the greatest changes in beliefs were selected for the high change qualitative interview group.

The treatment and control groups were exposed to the same mathematical topics (number and operation concepts) for the same length of time. However, the treatment group was taught using the modified student work analyses protocol, which introduced and explored each mathematical topic through the examination of student work and thinking. The control group learned through lecture, discussions, and group work that were absent of student work analyses. Again, both qualitative and quantitative data were collected during the study. Quantitative data were collected with the goal of determining the effects of the treatment activities on participants' common content knowledge (CCK) and beliefs about effective mathematics teaching. Qualitative data were collected to help determine the role that pedagogical content knowledge (PCK) played in the ability of PSTs to analyze student work and thinking.

The quantitative pretest data were collected using established instruments. The CCK instrument (Form A) contained 15 questions designed to measure the participants' mathematical content knowledge. It was validated during a large-scale study (n=629) with the California Mathematical Professional Development Institute (CMPDI) and produced a reliability coefficient of 0.84 (Hill, Rowan, & Ball, 2005). The beliefs instrument contained 16 questions designed to measure the participants' beliefs about effective mathematics teaching and learning. Leading beliefs experts in the field provided validation of the instrument (Philipp et al., 2007).

The quantitative posttest data were collected using the same instruments. The CCK instrument had a parallel form (Form B) that was used for the posttest data collection, which also contained 15 questions. This form was also validated with the CMPDI (n=620) and produced a reliability coefficient of 0.85. The beliefs data were collected using the same form as the pretest given that no parallel forms were developed for this instrument.

The qualitative data were collected only after the study was completed using retrospective interviews (Ericsson & Simon, 1993). This allowed the participants to recall their experiences over the course of the study. Interviews were held with four treatment group participants. These participants were split into two groups based on changes in their belief score from pre- to posttest. The two participants with the largest belief changes were placed in one group (the high change group), while the two participants with the smallest belief changes were placed in the other group (the low change group). The two groups were interviewed separately, and the interviews lasted approximately one hour each. During the interviews, participants were asked to talk about their experiences with the student work analysis activities as well as re-analyze pieces of student work while sharing their thinking in detail.

This study produced a myriad of findings from both the qualitative and quantitative data analyses. These findings and their interpretations are discussed next. Moreover, contributions to the field of mathematics education as well as the limitations of the study's design are shared. Finally, future research directions are discussed.

Interpretations and Contributions

To begin, the CCK data were analyzed using an ANOVA with SPSS 18.0. This analysis revealed that the treatment group made equal CCK gains to that of the control group. Although the mean of the treatment group (mean score of 54.6) was higher than the mean of the control group (mean score of 50.1), the difference was not significant once the pretest scores for each group were included in the ANOVA model. This suggests that the treatment activities, namely analyzing student work guided by the modified protocol, did not have significant impacts on the growth and development of CCK beyond that of the control activities.

Still, there are several contributions to the field of mathematics education in terms of CCK development. It is important to note that although the results were not significant, they do show that a wider focus on beliefs about effective mathematics teaching and PCK did not detract from the growth of CCK. The student work analyses protocol and student work selection process (which intentionally selected student work to represent various levels of mathematical understanding and address beliefs as outlined by Philipp et al., 2007) did not prevent the treatment group from making the same significant CCK gains from pre- to posttest as did the control group. Overall, the results suggest that CCK can be affected simultaneously with other elements of MKT and beliefs about effective mathematics teaching and learning.

The beliefs data were analyzed using a Chi-Square analysis also with SPSS 18.0. This analysis was run on both change score (posttest minus pretest scores for each participant on a belief by belief basis resulting in a score of 0 [no change], 1 [increase of one belief level], or 2 [increase of more than one belief level]) and pretest scores. The change score analysis revealed that the treatment and control groups differed significantly on six of the seven beliefs measured (only Belief 3 was not significant). Follow up interpretations of the Chi-Square cross-tabulation cells showed that the treatment group had much greater numbers of high change participants (change score of 2) while the control group had much higher numbers of no change participants (change score of 0). In other words, it was much more likely that a treatment group member made positive changes in their beliefs about effective mathematics teaching and learning. This indicates a strong relationship between the treatment activities and PSTs' beliefs about mathematics.

Several contributions to the field can be drawn from the findings regarding beliefs. First, the treatment activities and student work selection were able to encompass most of the beliefs deemed necessary for effective mathematics teaching. This suggests that selecting student work to address each of the necessary beliefs is a possible endeavor. However, the treatment activities did not have a significant effect on Belief 3 (understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures) for two reasons. The cross-tabulation values from the Chi-Square analysis revealed that the non-significant finding for this belief was due in part to several more 0 change score participants from the treatment group than for the other beliefs, as well as in part to several more 2 change score participants from the control group than for the other beliefs. This combination of factors contributed to the lack of significant difference between the two groups, and thus of the treatment activity. In general, it appears that beliefs about effective mathematics teaching and learning can indeed be changed through the use of student work analyses activities.

Finally, the qualitative interview data were analyzed using a thematic analysis approach (Aronson, 1994). This analysis produced two major themes present in the data. First, participants drew on elements of PCK to successfully analyze the student work presented to them. More specifically, they drew on their knowledge of content and students (KCS) and knowledge of content and teaching (KCT). This shows that the treatment activities for the study are capable of eliciting PSTs' KCS and KCT. This is the first step in fully understanding the types of experiences that can help build and develop these types of knowledge. Hill, Ball, and Schilling (2008) have documented the presence of this type of knowledge in the teaching profession. However, the field is still working on fully conceptualizing and measuring it. Studies such as this help to move the field towards these goals.

The second major theme from the thematic analysis was the presence of links between PSTs' knowledge and beliefs about mathematics. The high change interview group possessed beliefs that were more closely aligned to the seven necessary beliefs for effective mathematics teaching and learning. This group showed a much more developed sense of KCT and KCS and drew heavily on them to perform the treatment activities. The low change group, on the other hand, showed almost no evidence of drawing on KCS or KCT in their responses. The direction of the correlation was not clear. Perhaps the presence of KCT and KCS leads to more developed beliefs or, perhaps, more developed beliefs lead to higher levels of KCT and KCS. In either case, a clear link was present between knowledge and beliefs.

The second theme further supports the idea that the treatment activities are capable of developing elements of MKT in PSTs, in this case elements of PCK specifically. However, this theme also suggests that lower levels of PCK often accompany ineffective beliefs about mathematics. Furthermore, the types of student work used for the study may not have impacted all participants equally. The low change group seemed to gravitate towards the video examples and away from the written work. Perhaps all types of student work are not equal in terms of the ability to affect PSTs' beliefs and knowledge of mathematics.

Limitations of the Study

In education research, it is important to ensure that a study is both rigorous and trustworthy. Although the study was designed with great care, there are many limiting factors on the findings presented in this dissertation. To begin, the sample for this study used a convenience sample of PSTs based on enrollment in a mathematics content course. This may have limited the generalizability of the study to PSTs in other settings. The participants of this study were also members of two cohorts (one comprising the treatment group and the other the control cohort). These individuals were in close proximity to one another on a daily basis. Although they did not have inter-cohort requirements or classwork, these individuals could have interacted on their own accord causing a diffusion threat to the internal validity.

The instruments themselves also caused some limitations. The beliefs survey instrument was validated as a web-based survey. Because it is no longer available in a web-format, the researcher produced an identical instrument in paper and pencil form. The instructions, item order, item content, and scoring rubrics were reformatted with the utmost fidelity. However, it is still possible that the validity of this study was compromised due to the lack of validation of the new instrument format.

The knowledge instrument is still being developed and tweaked by researchers to increase the reliability and validity. The current attributes are acceptable for educational research, but they do allow for the possibility of low statistical power (a threat to the statistical conclusion validity) and a construct confounding issue in terms of defining the knowledge being measured (a threat to the construct validity). This instrument, however, is one of the most developed and piloted within the field of mathematics education.

Finally, there exists the possibility of researcher bias in this study. The researcher was the instructor of the treatment group. There were extensive attempts made to stay true to the implementation of the treatment as described in the methods section. However, this design structure produced the possibility that some researcher bias could have been introduced. The video recordings of both groups helped to monitor the study, but this element of the research design may have resulted in an additional threat to internal validity.

Future Research

In closing this dissertation, I would like to share the future research trajectories that became evident to me as a result of the study. First, the data analysis revealed that the treatment activities had no significant impact on the CCK of the treatment group as compared to the control group. A natural question to this is whether or not the initial beliefs filters may have prevented the treatment activities from developing CCK to the fullest of its ability. Future research is needed to determine if analyzing student work and thinking can have a bigger impact on the SMK portion of MKT after belief changes are initiated. To this end, future research is also needed to determine if the analysis of student work as outlined in this study can impact PSTs' SCK. SCK is another important element to effective mathematics teaching, and future research can help determine if the current treatment can have an effect.

The impacts on the PSTs' beliefs were much more significant. Of the seven beliefs necessary for effective mathematics teaching, the treatment activities were able to significantly affect six of them. Although this is a promising finding, it raises the question of why the one belief (Belief 3 – understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures) was not significantly influenced. Perhaps different types of student work or different protocol questions could better address the development of this belief. In particular, maybe collecting work from the same student over time to demonstrate the struggles that accompany a lack of conceptual understanding would be a good addition to the treatment activities. Future research is needed to refine the use of student work and thinking for developing Belief 3 as well as determining the transferability and sustainability of the results. Future iterations can help determine if these results would be typical with new groups of PSTs as well as if the belief changes remain after the participants stop using the treatment activities.

Furthermore, this study only examined the impact on beliefs and CCK within the topic of number and operation. Future research is needed to determine if and how the elements of MKT are impacted by the use of student work and thinking for other mathematical topics and concepts. It is hypothesized that the positive findings presented in this dissertation are not unique to number and operation, but more iterations of research are necessary to make that claim.

Qualitative data analysis also produced several questions that lead to future research. During the thematic analysis, it was discovered that the PSTs with low belief changes struggled with interpreting the students' written work. They seemed to gravitate to the video examples. They shared that the video examples provided more information and made them feel more comfortable when performing the treatment activities. Future research can help determine if certain formats of student work are better able to help PSTs build beliefs for effective mathematics teaching and learning. Perhaps more informative examples such as videos are better representations of students' work for initial PST belief changes should be used before more implicit, complex written examples are used.

Overall, this dissertation helped to answer the three research questions that guided the study. However, many questions were also raised that necessitate future iterations of research to help the field fully understand the potential of using student work to develop PSTs' MKT and beliefs within a situated context. Even with the limitations and additional questions raised, this dissertation suggests that using student work to contextualize the learning of PSTs' MKT and beliefs for effective mathematics teaching is a promising endeavor worthy of future investigation.

APPENDIX A
SPECIALIZED CONTENT KNOWLEDGE ITEM

Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35	35	35
× 25	× 25	× 25
125	175	25
+ 75	+ 700	150
875	875	100
		+ 600
		875

Which of these students is using a method that could be used to multiply any two whole numbers?

APPENDIX B
INFORMED CONSENT

UF | UNIVERSITY of FLORIDA

College of Education
School of Teaching and Learning

2403 Norman Hall
PO Box 117048
Gainesville, FL 32611-7048
352-373-4215
http://education.ufl.edu/school

Dear Potential Participant:

The Unified Elementary Proteach program is very interested in your experience and learning in the program. The faculty want to know how the various experiences you have in the program affect your knowledge and beliefs about teaching as well as your teaching practices and effectiveness as a teacher. The purpose of this letter is to secure your consent for participation in a study of your development as a teacher from the point of entry into the program through your first year of teaching. The following types of information may be collected:

- Survey: A variety of instruments may be used to document your perspectives and practices.
- Artifacts: A variety of artifacts may be collected to document your perspectives and practices. These may include but are not limited to lesson plans, reflective journal, papers, tests, and projects completed to meet course requirements, and samples of children's work with names removed.
- Online archives: Some UEP classes and field experiences use technology to facilitate student learning. Posting may be archived.
- Observations: Observation data in the form of field notes or as recorded with an observation instrument may be collected in college classrooms and in elementary classrooms to document your teaching practice. Observations may include video recordings.
- Interviews: Interviews lasting approximately 30-60 minutes may be conducted to gain insight into your perspectives and practices. Interviews will be audio or video recorded and transcribed. Interviews will focus on the following kinds of questions
  - Describe what you are expected to do in this course or field experience.
  - What has been challenging for you? Please explain.
  - What have you learned as a result of your participation?
  - How has your participation made a difference in the way you think about teaching or in your development as a teacher?
  - How has your participation made a difference in the way you teach or intend to teach?
  - How has your participation affected your students' learning (in elementary classrooms).

The Foundation for The Gator Nation
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Approved by
University of Florida
Institutional Review Board 02
Protocol # 2010-U-0973
For Use Through 09-22-2013

- Quantitative academic data: Data such as GPA, grades, and standardized test data such as SAT, GRE, or Florida Teacher Certification Examination performance data.

Your participation is strictly voluntary. There is no compensation to you for participating in the study. Non-participation or denied consent to collect some or all of the data listed above will not affect your grades or your status as a UEP student. In addition, you may request at any time that your data not be included. The data that are collected will be analyzed by a team of researchers consisting of university faculty and/or doctoral students who work with UEP students. Your identity will be protected through use of pseudonyms, and your confidentiality will be protected to the full extent provided by law. I do not perceive that there are any risks for your participation in the study. In fact, UEP students generally enjoy the opportunity to reflect on their own learning and have a voice in shaping the UEP program.

Please sign and return to me this copy of the letter. A second copy is for your records. If you have any questions about this study of the procedures for data collection, please contact me (colvin@coe.ufl.edu, 273-4218). If you have questions about the rights of research participants, you can contact the University of Florida Institutional Review Board Office, P.O. Box 112250, University of Florida Gainesville, FL 32611.

Sincerely,

Suzanne M. Colvin

Suzanne M. Colvin, Principal Investigator
Director, Unified Elementary Proteach Program

Approved by
University of Florida
Institutional Review Board 02
Protocol # 2010-U-0973
For Use Through 09-22-2013

I have read the procedure described above for the study of student learning in the Unified Elementary Proteach Program. I agree to participate, and I have received a copy of this description.

Signature of Participant Date

APPENDIX C
WEEKLY LESSON PLAN TREATMENT EXAMPLE

Topic: Understanding Whole Number Operations (Addition/Subtraction)

From last week (30 minutes):
Discuss reading assignment and online posts that are due today – reading was from Van de Walle and Lovin (2006) pages 74-86. This covered an introduction to basic facts and number relations as they apply to addition and subtraction.

Online post prompt: "Do you think teaching and using subtraction is necessary? Why or why not? Why is understanding how numbers are related to one another helpful for elementary students? How do you see this skill translating to middle and high school mathematics?"

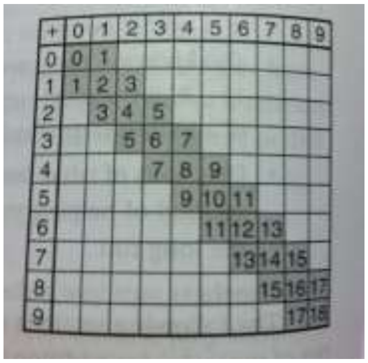
PSTs will discuss their reading/post thoughts/questions at their table (small group, 4-6) for 15 minutes. One member of each group will take notes to summarize the conversation. For the next 15 minutes, each group will then share their summary and the floor will be open to any points of discussion for the whole group by any PST. Instructor may ask follow up questions to group summaries.

Basic Facts Activity for Adding and Subtracting (45 minutes):
Present students with printouts of ten frames and foam circles for marking. Begin by watching students use ten frames on video (http://www.youtube.com/watch?v=aQ0cpiD_dO4&feature=relmfu). Ask PSTs to discuss what they saw in the video and what they belief the purpose of ten frames to be. Prompt them with the following questions:

> What are ten frames and how do you envision using them? How did the students use them? What did it show about their understanding of basic facts?

Ask them to be a student and use the ten frames to solve a few problems. Discuss as a whole group how this activity pertains to the 5 & 10 anchors (for conceptual understanding) from their previous reading assignment.

Share the building of basic facts (doubles, plus one, plus zeo, etc.) and discuss how/why they should appear in teaching elementary mathematics:



+	0	1	2	3	4	5	6	7	8	9
0	0	1								
1	1	2	3							
2		3	4	5						
3			5	6	7					
4				7	8	9				
5					9	10	11			
6						11	12	13		
7							13	14	15	
8								15	16	17
9									17	18

Van de Walle and Lovin (2006), p. 80

How are building and teaching facts this way different than simply memorizing them? How do these relate to the ten frames and ten frames activities we did?

Break (15 minutes)

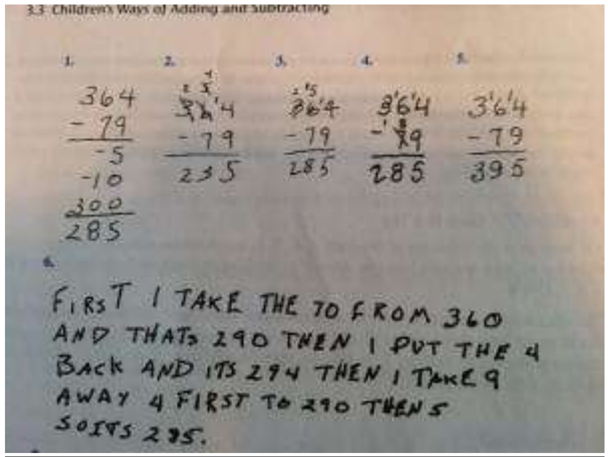
Understanding Addition and Subtraction (90 minutes):
Show IMAP videos (Video #2 and #3). Video #2 covers addition and subtraction strategies used by a young elementary student when responding to word problems. Video #3 covers a student thinking dissonance example caused by using both a traditional subtraction method (incorrectly) and an alternative subtraction method (correctly).

Allow PSTs time to discuss what they noticed about the students' work and thinking in the videos. Prompt them with questions for discussion including: (a) discuss and justify how well you think the students understand addition and/or subtraction (b) what would you do if you were tutoring this student, that is how would you extend their correct thinking or help to improve their incorrect thinking (c) discuss what knowledge a student must have in order to be successful with addition and subtraction, be specific. Instructor will listen and add to discussions. Each group will record and share a summary of their group's responses with the whole class. Floor will be opened for any follow up questions or comments about the videos.

Again in small groups, PSTs will work to define what addition and subtraction are and to solve additional addition and subtraction problems (some word problems). During this time, the focus will be on producing multiple representations of the solution. PSTs will be encouraged to produce both traditional algorithm solutions as well as multiple alternative methods. PSTs will share and discuss their strategies. Instructor will build from what PSTs produce to ensure inclusion of methods such as manipulatives (base ten blocks), pictures, counting up, distance between, adjusting numbers, breaking apart (to relate to algebra/distributing) etc. – scaffolding and guiding only as much as necessary. Example:

> $29 + 13 \rightarrow (20 + 9) + (10 + 3) \rightarrow (20 + 10) + (9 + 3) \rightarrow 30 + 12 = 42$
>
> Do you like this method? How does it work? What is the goal? How does it relate to future mathematics students that will be exposed to? What properties justify this moving around and regrouping of numbers?

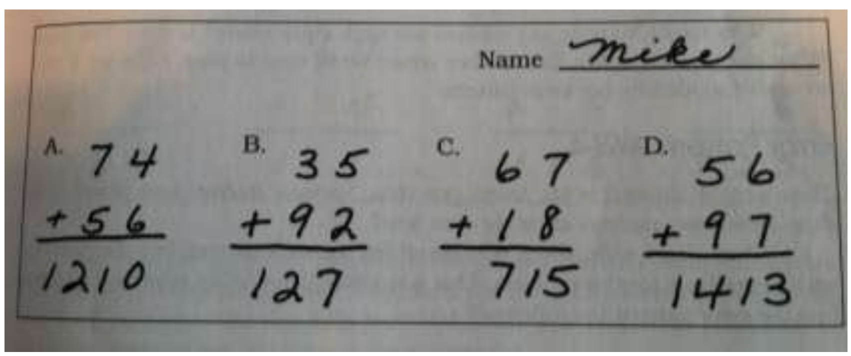
PSTs will then be asked to compare and contrast traditional and nontraditional methods. What does each strategy reveal about the student's understanding? Which strategies are more advanced? Less advanced? They will begin with discussing and examining the strategies they produced, and finish by examining Sowder, Sowder, and Nickerson (2009) page 55 and 57, a portion is included below.

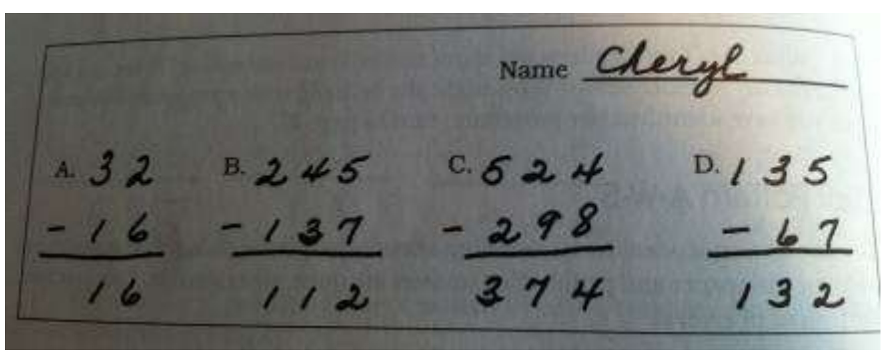


PSTs will analyze each student's response (#1-6) as well as similar problem types on pages 56 and 57.

Assignment: Read pages 87-97 in Van de Wall and Lovin (2006), which covers basic facts and number sense for multiplication and division.

PSTs complete Ashlock pg. 19 & 22:





PSTs complete Sowder, Sowder, and Nickerson (2009), page 51, activity 4, and page 58 #1-3:

ACTIVITY 4 Writing Story Problems

1. Pair up with someone in your class. Write problem situations (not necessarily in the order given) that illustrate these different views of addition and subtraction:
   a. addition that involves putting together two quantities
   b. addition that involves thinking about two quantities as one quantity
   c. take-away subtraction
   d. comparison subtraction
   e. missing-addend subtraction
   f. addition arising from a comparison situation
2. Share your problems with another group. Each group should identify the types of problems illustrated by the other group, and discuss whether each group can correctly identify the problem situations with the description. In each problem, identify whether the quantities are discrete or continuous.

Supplementary Learning Exercises for Section 3.3

1. A child is learning addition of one-digit numbers. But he writes results like 1 + 6 = 16 and 1 + 2 = 12. What might the child be thinking?
2. Use "empty" number lines (Learning Exercise 5 in Section 3.3) to calculate these sums and differences.
   a. 189 + 43 b. 936 + 89 c. 1000 − 732 d. 381 − 62
3. Write down the calculations this child is working.
   a. "400, then 140 is 540, then ten more is 550. 150."
   b. "25 more to get to 100 and 300 more to get to 400, so 325."

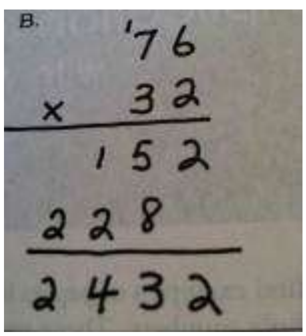
APPENDIX D
TAKE HOME ASSIGNMENT TREATMENT EXAMPLE

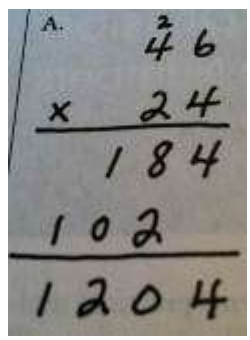
Please do all your work in a neat and organized manner; make sure to staple. You may handwrite, type, or utilize a combination for your responses. Be thoughtful and thorough in your answers. You may work in small groups – please ensure all group members contribute equally.

1) Use the traditional algorithm for multiplication to compute 44 x 29. Describe and justify each step in your process. Why do we use placeholder zeroes in answers when we use this method? Explain fully.

2) Compute 44 x 29 using another method that you think is useful (do not use the traditional algorithm…or a calculator). Describe and justify each step in your process. Then, compare this method with the method from #1. Describe some advantages and disadvantages of each method. Which do you think is a more effective way to multiply? A more conceptual way? Explain.

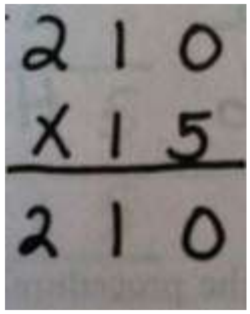
3) A student completes the multiplication problems as follows:

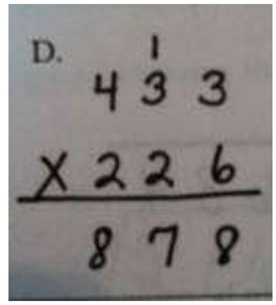




a) What error is this student making? Explain.
b) How would this student likely compute 25 x 12?
c) What mathematical ideas does this student need to understand in order to be successful with multi-digit multiplication problems? If you were tutoring this student, what are the next steps you would take to teach this student those mathematical ideas? Describe in detail.

4) A student completes the multiplication problems as follows:

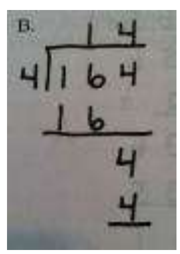




a) What error is this student making? Explain.
b) Use a drawing to illustrate what 210 x 15 looks like (think 210 children each get 15 M&Ms).
c) What mathematical ideas does this student need to understand in order to be successful with multiplication? If you were tutoring this student, what are the next steps you would take to teach this student about those mathematical ideas? Describe in detail.

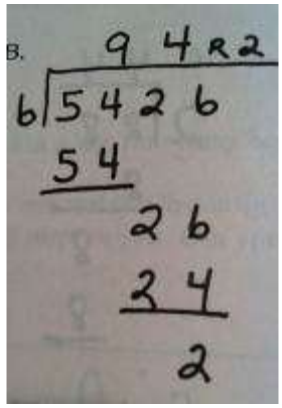
5) A second grader asks you "What is division?" How would you respond? How would you respond if your university instructor asked you? Be thorough and detailed in your answers.

6) A student completes the division problem as follows:



a) What error is the student making? Explain.
b) How do you think this student might compute 1614 ÷ 12?
c) What mathematical ideas are missing from the child's thinking that may have led to this way of doing division? If you were tutoring this student, what are the next steps you would take to teach this student those mathematical ideas? Describe in detail.

7) A student completes the division problems as follows:



a) What error is the student making? Explain.
b) How do you think this student might compute 171 ÷ 12?
c) What mathematical ideas are missing from the child's thinking that may have led to this way of doing division? If you were tutoring this student, what are the next steps you would take to teach this student those mathematical ideas? Describe in detail.

8) A student completes a division problem 520 ÷ 15 as follows:

"I can take away ten 15's, which is 150, and can do this three times to get 450, which is thirty 15's total. I have 70 left and that is four 15's with 10 left over. The answer is 34, remainder 10."

a) Is the answer correct? Is the solution strategy? Will this way of doing division work for all division problems?
b) Assess how well this student understands division. Design a division problem that could help you further test how well this student understands division. Explain why and how you developed the problem.
c) Do you think this student's approach represents a more or less conceptual understanding of mathematics than using the traditional algorithm for division? Explain your thoughts in detail.

APPENDIX E
COURSE SCHEDULE – FOR 8 WEEKS OF THE STUDY

Week	Date	Topic	Assignment Given (Due the following week unless noted)
1	1/11	Syllabus, Learning Mathematics, Conceptual Learning, CCSS, and Study Pretest	Read Van de Walle, ch. 1. Read Sowder, sect. 1.4. Complete online post. Solve Sowder, p.19, #1-9.
2	1/18	Number Sense, Basic Facts, Problem Solving	Read Van de Walle, ch. 3. Complete online post. Do Sowder p.7 act. 3, p.9 act. 4. Student Work #1 (due 2/1).
3	1/25	Number Systems, Intro to Other Bases	Read articles on Moodle (Ten Frames and Chinese Numbers). Complete online post. Do Sowder, p.31-32, #4,5,6,8,9, 12,13,15,17.
4	2/1	Continue to Work Other Bases, Extensions to Base 10, Place Value (base 10 vs. other bases)	Read Sowder p.34-36 & p.39-40. Complete online post. Do Sowder p.34 (‘One’ Activity). Do Sowder p.38, #1,2,4,5,6,9.
5	2/8	Review For Exam, Continue Place Value Understanding (base 10)	Read Sowder section 2.2. Complete online post. Do Sowder p.25, #1,3,6,8,9. Student Work #2 (due 3/1).
6	2/15	Content Exam 1, Adding and Subtracting with Whole Numbers	Read Van de Walle p.100-113. Complete online post. Do Sowder p. 46, #1,3,5,8 and p.51, #2,3,7.
7	2/22	Finish Adding/Subtracting Whole Numbers, Multiplying and Dividing Whole Numbers	Read Van de Walle p.113-128. Complete online post. Do Sowder p. 64, #5,7,9,15 and p.69, #1,3,4,6,8.
8	3/1	Finish Multiplying/Dividing Whole Numbers, Using Numbers in Sensible Ways	Read Sowder ch. 5. Complete online post. Do Sowder p.89 #1-5, p. 93 #1-8, p. 96 # 1,2,4,5, p. 98 #1,2, 3a,4a,8.

All data collected will be regarding the participants’ knowledge and beliefs about number and operation. This material will be covered during the first eight weeks of the semester. Week nine will begin the topic of algebra and is not included in the data collection.

APPENDIX F
KNOWLEDGE INSTRUMENT SAMPLE

INSTRUCTIONS

- **Answer the questions by circling your choice, e.g.**

  1. During a unit on functions, Ms. Lopez asks her students to write journal entries on exponential growth. Which of the following journal entries illustrate exponential growth? (For each item below, circle EXPONENTIAL, NOT EXPONENTIAL, or I'M NOT SURE.)

	Exponential	Not Exponential	I'm not sure
a) An example of exponential growth would be if you got 1% raise each year.	(1)	2	3
b) An example of exponential growth would be if a car increases in speed by 10 miles per hour ever second.	1	(2)	3
c) Exponential growth is when the y-axis increases faster than the x-axis. For example, if each time the x-coordinate goes up by 2, the y-coordinate goes up by 3.	(1)	2	3

APPENDIX G
BELIEFS INSTRUMENT SAMPLE

4. Here are two approaches that children used to solve the problem 635 – 482.

Lexi	Ariana
$\begin{array}{r} {}^{5}6\,{}^{1}3\;5 \\ -\;4\;8\;2 \\ \hline 1\;5\;3 \end{array}$ Lexi says, "First I subtracted 2 from 5 and got 3. Then I couldn't subtract 8 from 3, so I borrowed. I crossed out the 6, wrote a 5, then put a 1 next to the 3. Now it's 13 minus 8 is 5. And then 5 minus 4 is 1, so my answer is 153."	$\begin{array}{rcl} 635 - 400 & = & 235 \\ 235 - 30 & = & 205 \\ 205 - 50 & = & 155 \\ 155 - \underline{2} & = & 153 \\ 482 & & \end{array}$ Ariana says, "First I subtracted 400 and got 235. Then I subtracted 30 and got 205, and I subtracted 50 more and got 155. I needed to subtract 2 more and ended up with 153."
4.1 Does Lexi's reasoning make sense to you? ○ Yes ○ No	**4.2 Does Ariana's reasoning make sense to you?** ○ Yes ○ No

a. Which child (Lexi or Ariana) shows the greater mathematical understanding? Why?

APPENDIX H
FORMAL INTERVIEW PROTOCOL

1. During class, you were asked to analyze the student work/thinking example in IMAP video #2 and #3. Could you talk generally about how you analyzed each of these? [show each video separately, allowing time for response after each]

2. Can you describe what you were thinking as you tried to determine the student's level of understanding [show each video again, pausing it as the participant explains]?

3. Can you describe what you were thinking as you tried to determine what the next steps in instruction should be [show each video again, pausing it as the participant explains]?

4. In several instances this semester, you have been asked to describe what skills/ways of knowing are necessary for students to be successful with given topics [show participant's response to student work project 2, #6]. Can you describe what you were thinking as you constructed this response?

5. I am going to show you a new piece of student work [show participant Ashlock (2010) example not used in class]. Can you tell me if the work is correct or what mistake the student is making? Verbalize and justify all of your thinking.

6. Analyze the level of understanding the student has. What concepts or understanding is the student lacking? Verbalize and justify all of your thinking.

7. What should the next steps in instruction be for teaching this student the missing concepts you said they were lacking above? Verbalize and justify all of your thinking.

The researcher will probe for additional comments from the participants if they do not fully verbalize, justify, or explain their thinking as it relates to the knowledge they are drawing upon to complete the analyses activities.

APPENDIX I
USING STUDENT WORK PROTOCOL (ADAPTED FROM THE NSRF)

Selecting Student Work to Share
The selected student work will be used as the focal point of course lessons and in class discussions. The work itself will provide the mathematical topics as well as the teaching context for each lesson.

Choose student work that covers a variety of mathematical topics with a variety of solution types (i.e., traditional solution strategies, student invented algorithms, common errors, unique correct responses, etc.).

The key is to have enough artifacts and enough variety to drive the discussions and create situations that make PSTs examine their own understanding of the topic. Remember, student work comes in a variety of forms including videos (e.g., the Integrating Mathematics and Pedagogy [IMAP] project, Annenberg Learner website), written work collected from local schools, written work from PST education textbooks, etc.

Sharing and Discussing Student Work
Discussing student work requires a guide to help PSTs feel comfortable in sharing their thoughts about students' understanding as well as their own. Since learning is best accomplished through hands-on interactions, a structured dialogue format works well to promote thinking and learning about students' understanding and mathematical topics.

Ask the PSTs to assume that the students who completed the work or answered the questions in the videos were putting forth their best effort. Any mistakes or misconceptions are most likely honest.

Using the Protocol

Getting Started
The instructor should provide the student work example to the class and briefly introduce the mathematical topic of focus. If the example is written, the PSTs should have the opportunity to familiarize themselves with the work. If the example is a video, the instructor should play through the video two times to allow the PSTs to familiarize themselves with the work and the situation.

Small Group Session
Next, the following questions should be posed to the PSTs to discuss in small groups:

1. Is the solution correct? If not, what mistake is the student making? Explain your thinking.

2. Analyze the level of understanding the student has. What has the student done well? What concepts or understanding is the student lacking? Explain your thinking.

3. What should the next steps in instruction be for teaching this student? How would you expand their understanding of the concepts mastered and/or help them improve the missing concepts or understanding? Explain your thinking.

PSTs should be given 5 minutes at their small group (groups of three or four) setting to discuss their answers to the three questions. The instructor should ask the PSTs to read the questions and think about their responses for one minute. After that, each group member should take one minute to describe his or her thoughts to the group.

Reflecting on the Responses

After the small group discussions are complete, the instructor should bring the group back together as a whole. Debriefing should take place by posing the following questions to the whole group:

1. What is one thing that you learned while talking over the student work at your table? Why is this significant to you?
2. What new perspectives about the student, mathematical understanding, and/or mathematical content did your classmates provide you?

This discussion should be opened up to the entire group for volunteers to speak. If any major insights about the student work have been missed, the instructor should pose those questions and ask for ideas from the whole group.

The instructor should finish the protocol by providing a brief summary of the mathematical topic shown in the work example, the possible misunderstanding or exemplary understandings, and possible next steps for instruction.

APPENDIX J
CHANGE SCORE CROSSTABULATIONS

Table J-1. Belief 1 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	8	4	9.5	10	3.5	7
Control	8	12	9.5	9	3.5	0

Table J-2. Belief 2 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	10.5	5	6.5	9	4	7
Control	10.5	16	6.5	4	4	1

Table J-3. Belief 3 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	10	8	6	7	10	6
Control	10	12	6	5	10	4

Table J-4. Belief 4 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	13	9	6	8	2	4
Control	13	17	6	4	2	0

Table J-5. Belief 5 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	13	9	4	4	4	8
Control	13	17	4	4	4	0

Table J-6. Belief 6 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	10	2	4	5	7	14
Control	10	18	4	3	7	0

Table J-7. Belief 7 crosstabulation values.

Group	Change score 0, expected count	Change score 0, actual count	Change score 1, expected count	Change score 1, actual count	Change score 2, expected count	Change score 2, actual count
Treatment	15	11	2.5	3	3.5	7
Control	15	19	2.5	2	3.5	0

APPENDIX K
SUBTRACTION LESSON PLAN USING STUDENT WORK

Introduction to Subtraction: PSTs will be in groups of 3-5. In their group, they should answer the three questions from the protocol (correctness?, level of understanding? what now?). These questions should be answered about each of the 9 questions listed below. However, for each individual question, a small group discussion (5 minutes) should be followed by a large group reflection period (5 minutes). During the large group, encourage PSTs to share one thing that they learned about subtraction, one thing about student thinking, and one new perspective they gained about subtraction.

For these problems, a particular focus should be on what the student is trying to do and whether they understand the concepts allowing their method to be justified. For incorrect answers, push groups to clearly define what is going on and think about the generalizability of the strategy. For the videos, play them twice to allow full immersion in the situation (each is less than two minutes). During the large group reflection, probe for understanding about the place value issues that are arising in video 3 and the time problem. Introduce ten frames and base ten blocks and allow PSTs to investigate how these manipulatives can help with place value understanding (15 minutes).

With remaining time, provide PSTs with subtraction related problems listed below to work on in their small groups. They should strive to solve the problems and explain how/why they are thinking.

1. Subtract 351 – 298 in at least two ways that are not the traditional algorithm. Why did you choose these strategies? How do they compare to the traditional algorithm?
2. Create and solve (using the traditional algorithm) a subtraction problem that involves “borrowing”. Very specifically explain how and why the traditional algorithm works. Why are the steps happening?
3. Create a subtraction word problem that you think might be challenging to elementary-aged students. Explain why you think it would be difficult and how you would help fix the errors you anticipate.

Closure: Ask for any questions about subtraction or ways to think about subtraction (both traditional and non-traditional). What does a conceptual understanding of subtraction look like?

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BIOGRAPHICAL SKETCH

Richard P. Busi was born in 1982 in western Pennsylvania. He and his brother Tim are the two children of Pete and Karen Busi. Rich graduated from Slippery Rock Area High School in 2001, from Slippery Rock University in 2006 with a B.S. in mathematics, from Slippery Rock University in 2007 with a M.Ed. in mathematics Education, and finally from the University of Florida in May 2014 with a Ph.D. in curriculum and instruction. His major area of concentration for his Ph.D. was mathematics education.

After completing his M.Ed. in 2007, Richard took a teaching job with the Clark County School District in Las Vegas, Nevada. There he taught secondary mathematics for one year before returning to Pennsylvania to take a job with the Cranberry Area School District. During his two years at Cranberry, Rich again taught secondary mathematics as well as coached multiple sports and advised student organizations. In fall 2010, he began study at the University of Florida after receiving an alumni fellowship to support his pursuit of a terminal degree.

Rich is a peer-reviewed author in the field of mathematics education. He has multiple articles approved for publication and has served as both a co- and lead author. He has also presented his work at a variety of local, regional, and national conferences including the Florida Council of Teachers of Mathematics, the National Council of Teachers of Mathematics, and the Association of Mathematics Teacher Educators.

Rich married Sarah Swiatek Busi on September 15, 2012. They have two dogs (Rita and Dexter) and two cats (Nubbie and JD). In April 2014, they moved to Harrisonburg, Virginia for Rich to accept a position at James Madison University.