ALTERNATIVELY CERTIFIED TEACHERS' USE OF MATHEMATICAL DISCOURSE

By

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To my wonderful husband

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The importance of education will continue to impact our world, and I hope that I am afforded the opportunity to light the pathway of others as they seek their own

educational journeys. The possibility to encourage and support others to achieve their educational and professional goals is an honor I look forward to for many years to come. Nothing is gained without sacrifice and support of others.

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KEY TERMINOLOGY

ALTERNATELY CERTIFIED TEACHER	in this study is an individual who attained teacher certification through an alternate pathway instead of the traditional university teacher program route.
COMMUNICATION	in this study is a process by which information e disc exchanged between individuals through a common system of system of symbols, signs or behavior (Merriam-Webster.com, 2012).
DISCOURSE	in this study is a classroom discussions or conversations that may be general in nature or related to mathematics (Sfard & Kiernan, 2001).
DISCOURSE COMMUNITY	in this study is a group of individuals who are part of a specific group based on the sharing common language, symbols and practices. The common language includes visual mediators and routines that support the discourse community (Sfard, 2007).
MATHEMATICAL DISCOURSE	in this study is teacher-to-student and student-to-student discussions that revolve solely around mathematics. The discussions may include the use of symbols (Sfard, 2000).
SCAFFOLDING	in this study is the use of focused questions between an adult (teacher) and student to assist with understanding a concept (Wood, Bruner, & Ross, 1976).
ZONE OF PROXIMAL DEVELOPMENT (ZPD)	in this study is the zone in which an individual is able to achieve more with assistance than he or she can do alone (Vygotsky, 1978; Goos, 2004).

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Educational reform efforts such as the National Council of Teachers of Mathematics' (NCTM) Principles and Standards for School Mathematics and the implementation of the Common Core State Standards for mathematics have placed a new emphasis on student success. One of NCTM's ten standards is communication. It should come as no surprise that the term discourse is featured prominently in educational research and is the focus of recent teacher education and professional development efforts.

I designed this research to examine how three alternatively certified teachers in two rural school districts conducted mathematical discourse using the Advancing Children's Thinking (ACT) framework. The ACT framework categorizes teacher-student mathematical discourse into three groups: eliciting student engagement, supporting student thinking, and extending mathematical discourse. All three participants engaged in mathematical discourse in each of the three categories of the ACT framework, which is important given the dependence of alternatively certified teachers by school districts nationwide. These participants regularly elicited and supported student thinking; however, participants extended mathematical discourse infrequently.

None of the participants had more than three years of teaching experience. No specific curriculum or professional development was a part of the study. Participants taught regularly-scheduled mathematics lessons. I recorded, transcribed and coded observations; I conducted post-observation interviews; and I completed one stimulated recall interview as a final component of the study.

The use of scenarios as a method of professional development for currently employed teachers, particularly those alternatively certified, to support implementation of extended mathematical discourse is one of several recommendations. I advise mentoring in the establishment of classroom routines and a strong intellectual environment as a part of the new teacher implementation process. Finally, I recommend professional development for alternatively certified teachers in the area of effective questioning techniques that facilitate discussion in all three areas of the ACT framework. Implementing strong mathematical discourse in the classroom is no easy task, and teachers must be prepared to meet the challenge of engaging students to learn mathematics.

CHAPTER 1
DESCRIPTION OF THE STUDY

Introduction

Current approaches to reform in mathematics education emphasize the need for classroom communication and application of discourse (National Council of Teachers of Mathematics [NCTM], 2000; No Child Left Behind [NCLB], 2001; American Restoration and Recovery Act [ARRA], 2009). The NCTM signaled the importance of mathematical discourse by designating Communication as one of the ten standards in Principles and Standards for School Mathematics (PSSM) (NCTM, 2000). The standard indicates the need for teachers to implement whole class communication, select meaningful tasks that promote and allow discourse (teacher-student and student-student) to occur, and monitor students' learning to guide discussions (NCTM, 2000). Clearly, great importance has been placed on classroom discourse regarding mathematics.

Discourse became a part of educational research decades ago (Mehan, 1979) and continues to be an integral part of educational reform. The need for communication to be a natural part of mathematical learning in the classroom became central when it appeared in the PSSN (NCTM, 2000). Discourse is also noted to have influences at the individual and community levels (Gee, 2012) playing a role in the development of the knowledge. Over the past thirty years, discourse has made its way into the various disciplines. In fact, due to the very specific language, text, and symbols used in mathematics, researchers have begun to view mathematics as a discourse (Sfard, 2008; Ball, 1991).

Mathematical discourse is multifaceted and covers a broad range of interactions that include classroom, teacher, and student discourse (Walshaw & Anthony, 2008).

The teacher's role in each of these situations is important to the learning experience provided to students. Because discourses are intimately related to the distribution of social power and hierarchical structure in society (Gee, 1991), it is important for teachers to be skilled in conducting mathematical discourse. The classroom is a place with specialized structure that requires the teacher and students to have a clear understanding of how they will work together for learning to take place.

In society, those who are knowledgeable of skills and accepted norms are more likely to successfully navigate the world. On a classroom level, students equipped to articulate mathematical ideas and analyze conjecture will be in a position to influence the mathematical community. Therefore, successful establishment of mathematical discourse that fosters an environment of discussion, debate and reflection where students learn to think and act mathematically will promote deeper understanding of mathematics (Ball, 1991).

Implementing strong mathematical discourse in the classroom is no easy task because of the diverse experiences each student brings to the classroom community. To establish mathematical discourse, teachers must engage in an array of activities. First, teachers need to develop a classroom environment that will encourage active participation in mathematical discourse (Manoucheheri & Enderson, 1999; Yackel & Cobb; 1996; Williams & Baxter, 1996). Established and consistent norms with acceptance of all perspectives lay the foundation for an environment where learning is safe, open, and valued by everyone. Teachers also need to assess each student's current mathematical understanding and prepare tasks that will challenge thinking and enhance learning (Goos, 2004). Developing engaging tasks can be the basis for establishing mathematical discourse that promotes student learning if properly planned. Tasks too easy or too difficult can hinder mathematical discourse and limit, or even stifle learning by causing students boredom or frustration. Finally, teachers need to assume the role of facilitator in order to guide mathematical discourse and subsequent learning for all students (Walshaw & Anthony, 2008; Goos, 2004). After decades of the teacher as the focal point of learning, acting as a facilitator is not instinctive for most teachers. However, mathematical discourse as an integral part of whole group instruction it is believed expands student understanding of the curriculum. The role of the teacher needs to be understood as highly creative, very flexible, and much more challenging that has been traditionally espoused (Zack & Graves, 2001).

All teachers must be able to integrate mathematical discourse into the classroom (NCTM, 2000) irrespective of the pathway to teaching. Embedded in education reform discussions is the ongoing use of alternatively certified teachers, particularly in the domain of mathematics education. With 500,000 alternatively certified teachers serving the educational needs of students (National Center for Alternative Certification [NCAC], 2010), it is important to have an understanding of the teacher's role in mathematical discourse for both traditionally trained teachers and those who have earned a teaching certificate through an alternate pathway. Today's classroom required teachers and students to take on new roles; teachers must teach in ways they have never taught before (Darling-Hammond & McLaughlin, 1995). Accountability has forced teaching to become more complex, therefore all major subgroups of teachers are important to research as we strive to improve student learning in mathematics and indeed in all disciplines. Having defined the key terminology, the following two sections give an overview of mathematical discourse, scaffolding, and alternative teacher certification as intended for this study.

Discourse and the Role of the Teacher

Discourse is central to how we learn about our world. When students interact with one another and with the teacher, knowledge is shared for everyone to evaluate and analyze. The interactivity of discourse builds knowledge among students by encouraging the sharing of private thoughts with others in the learning community. The teacher's role in the establishment of discourse comes in the form of instituting classroom routines, cultivating an intellectual environment, and providing a responsive approach to mathematical discourse as it takes place in the classroom. Discourse guided by teachers provides an opportunity for students to present thinking and evaluate one another's thinking and provides a rich environment for learning mathematical reasoning (NCTM, 2000).

Teacher understanding and use of mathematical discourse does not happen automatically (Stigler, 1988). Careful orchestration of classroom routines must occur if quality communication with and about mathematics is to take place in the classroom. "Daily rituals" (Yackel & Cobb, 1996) are needed to establish mathematical discourse. Students should have a clear understanding of rules regarding social norms as they relate to the structure of the learning environment. Understanding basics such as speaker order, yielding points, topic change, and how to conduct oneself in whole and small group settings is foundational to establishing mathematical discourse that leads to advancing student learning (Leinhardt & Steele, 2005; Manoucheheri & Enderson, 1999).

Once classroom routines are established, groundwork can be laid to begin discourse specifically related to the mathematics experience. One strategy used by teachers to engage students in mathematical discourse is to present tasks that challenge students, but with critical thinking and analysis, are attainable. Interesting problems that "go somewhere" mathematically can often be catalysts for rich mathematical discourse (NCTM, 2000). In order to develop these tasks, teachers must know each student's Zone of Proximal Development (ZPD) and plan extensively prior to each lesson so activities benefit the educational growth of all students in the classroom. Woodward and Irwin (2005) and Ding, Li, Piccolo, & Kulm (2007) noted in separate studies that when students are engaged in problem solving tasks and whole group discussions overall learning is increased.

With the development of appropriate tasks for learning, teachers then can begin to guide mathematical discourse centered on these problems. This is a challenging skill for even experienced teachers, and requires "thinking on your feet." Planning appropriate lessons that include questions to guide discourse is the first step. Then, teachers must be able to identify key instructional moments to expand the mathematical discourse. Without teacher knowledge and skills to promote and sustain dialogue, the importance of these tasks is under-recognized. Better mathematical understanding is dependent upon developing a culture of discourse that elicits clarification and produces consensus in the classroom community (Walshaw & Anthony, 2008). Understanding how to expand a conversation in those "teachable moments" is an important pedagogical skill to be acquired and perfected through application and practice.

While teachers need to plan detailed lessons and identify important points in the discourse, teachers are also charged with assisting students in developing the skills needed to evaluate other students' statements for accuracy. To accomplish this, teachers must do more listening and guiding, and students more reasoning for quality mathematical discourse to enhance student learning (NCTM, 2000). This reiterates the need for teachers to become facilitators of mathematical discourse. An instructional approach utilizing carefully planned lessons and tasks that engage students allows teachers to become more of a facilitator in the learning process (Manoucheheri & Enderson, 1999) which is contrary to the role of the traditional teacher as experienced by most in their own formal education. As teachers develop the ability to help students scaffold new knowledge with prior knowledge, mathematical skills will increase (Goos, 2004; Morrone, Harkness, D'Ambrosio, & Caulfield, 2004).

Possessing the ability to guide mathematical discourse is an important pedagogical skill for educators. Teachers who earned their credentials through a traditional certification route have the potential opportunity to practice these skills in the classroom environment during required internships. Conversely, teachers credentialed through an alternative certification route rarely have an internship requirement prior to employment (Wilson, Floden, & Ferrini-Mundy, 2001; Zeichner & Schulte, 2001). With this practice being widely the case, some students across the United States are being taught by teachers who have little or no practice in using mathematical discourse, may not have the background to apply scaffolding practices as a part of the mathematical discourse, and may not be establishing an environment conducive to promoting mathematical discussions to increase student learning.

As teachers identify gaps in student knowledge during mathematical discourse, the use of scaffolding can assist students with connecting prior knowledge with new skills being introduced (Goos, 2004; Cazden, 2001). While this is often part of traditional teacher preparation programs and modeled during teaching internships, the alternatively certified teacher may not have the same exposure to this pedagogical skill. Scaffolding uses questioning techniques to guide student thinking to connect new knowledge with existing skills to understand a new concept. The questions often start out simple and are frequently offered to guide student learning. As the student gains a greater understanding of the concept, the expert (teacher) decreases the amount and type of questions until they are no longer needed. Teachers with established intellectual environments that embrace the application of scaffolding and mathematical discourse have higher student engagement (Nathan & Knuth, 2003) and increased learning (Hatano & Inagaki, 1991).

Examples of Classroom Discourse

Research and instructional literature have provided examples of mathematical discourse as it should be conducted in the classroom. The PSSM (NCTM, 2000) provides a vision for mathematics education where all students have access to high quality, engaging mathematics instruction that prepares students for all facets of life, including everyday life and the workplace. Teachers who provide lessons that involve challenging tasks paired with mathematical discourse on an on-going basis become increasingly comfortable with the process. A second distinctive feature is the establishment of a learning environment that embraces a sense of community for the individual and the group as a whole. The following excerpt adapted from the PSSM

(NCTM, 2000; p.268-269), describes what mathematical discourse should look like for middles grades (6-8):

> **Problem:** A certain rectangle has length and width that are whole numbers of inches, and the ratio of its length to its width is 4 to 3. Its area is 300 square inches. What are its length and width?
>
> Students begin working on the problem with the teacher circulating the room responding to questions and noting different approaches taken to solve the problem. After most had completed the problem, the teacher asked two students to present their method to the class. The students briefly restated the problem; the students indicated they needed "a number that both 3 and 4 would go into." The teacher asked why they multiplied 3 by 4. Student 1 replied that the ratio of the length to the width was given as "4 to 3" in the problem. Student 2 went on to say that they had determined that "3 goes into 15 five times and 4 goes into 20 five times." Since 15 times 20 equals 300, the area of the given rectangle, they concluded that 15 inches times 20 inches were the width and length of the rectangle. Can you explain to us how you selected the numbers 15 and 20? The pair could not provide any insights.
>
> The teacher asked if there were questions for the two students. A third student seemed to question their solution as well. He wanted to know where the 12 came from and how that would help solve the problem. Neither of the two students could explain how this connected to their solution to the problem. A fourth student also quizzed them on how they got the number 5. Finally, a fifth student jumped in and said, "Did you guys just guess and check?" to which they responded, "Yeah!"
>
> To address the confusion, the teacher solicited another response to the problem. She called another pair of students she knew had used another method to solve the problem. These two students were able to explain the problem and justify their answer to the class.

As noted in the description of the vignette (NCTM, 2000), the teacher's role in creating an opportunity for mathematical discourse and facilitating the discussion through the entire process is apparent. The teacher had all students engaged in the learning process and was able to identify who had solved the problem correctly. Further, the teacher was aware of who had used different methods to solve the task under discussion. Beyond this, it was evident that the teacher had selected a meaningful task for the students to challenge their thinking, but a task that was also solvable with their knowledge and skills with some critical thinking. Finally, the teacher monitored the discourse at the student and whole group levels. She scaffolded the first two students through the task, and when it became apparent that they did not understand the problem or answer it correctly, she enlisted the assistance of peers. She selected students who had solved the problem correctly and could articulate this to the group to dispel the confusion that was created by the first pair of students (NCTM, 2000). This is an exemplary model of implementing and managing mathematical discourse to promote student learning.

Educational research on mathematical discourse also supports the importance of knowing where students are mathematically as a whole and with a specific task. Teachers should have the ability to select appropriate and engaging tasks that challenge students, while creating an environment that embraces all thoughts and perspectives in the solution process to encourage student participation. The following is a scenario adapted from Meyer and Turner (2002; p.23) on mathematical discourse that involves scaffolding to assist student learning:

> **Problem:** Student had factored 180 and was asked by the teacher to explain the tree and evaluate her solution.
>
> Teacher: Right, OK, explain it to us.
> Student: I factored 30 and 6 and I factored 30 into 5 times 6, and I factored 6 into 2 times 3, and I took the 5 down, and I factored the other 6 into 3 times 2.
>
> Teacher: And how do you know you are all done?
>
> Student: ‘Cause they are all prime.
>
> The teacher then went to another student to model another factor tree for 180 but found she could not yet do this on her own. The teacher then

> scaffolded by suggesting a strategy for choosing two initial factors, which allowed the student to be successful in solving the problem.
>
> Teacher: What are you going to factor into? (Writes 180 on the overhead.)
>
> Student: Ummm. I don't know.
>
> Teacher: Want somebody to help you out? How would you do it if you were at home?
>
> Student: Ummm (thinking)
>
> Teacher: Get a calculator or piece of paper?
>
> Student: Piece of paper.
>
> Teacher: OK. Let's go. Starting writing factors of 180. Start with 1 over here like I showed you, and the other over here on the far end, what are you going to put?
>
> The teacher continued to navigate the student through the task steps, allowing the student to do what she could and helping her to understand steps she did not.
>
> Teacher: So, what are you going to break 180 into?
>
> Student: 60 and 3?
>
> Teacher: OK. Go ahead.
>
> The student slowly proceeds to complete the problem, factoring until all prime numbers are across the bottom of the problem.

Mathematical discourse is varied but has specific characteristics if it is significant and contributes to overall learning. First, an environment must be created that acknowledges and respects all students and their perspectives. A safe intellectual environment is the foundation for developing discourse, particularly with middle school students (Bennett, 2010). The teacher should have an understanding of individual students' mathematical ability and zone of proximal development. Using this knowledge, the teacher must then develop tasks that challenge student thinking and create situations to move learning forward using discourse to drive the learning process.

Finally, the teacher should act as a facilitator of mathematical discourse, having an understanding of how to help students think through the problem and formulate critical responses.

Influence of Alternative Certification

The focus of reform efforts on teachers is obvious with the direct impact teachers have on student learning. In fact, teacher quality has been one of the most hotly contested topics in modern education (Darling-Hammond, 2001; Walsh, 2001). A facet of this debate has been the broad use of alternatively certified teachers by school districts nationwide (Darling-Hammond, 2010; NCES, 2010). Teacher attrition, an aging teaching workforce, and fewer individuals selecting the teaching profession as a career fuel this need (Ingersoll, 2003).

A University of Washington research report (2002) by the Center for the Study of Teaching and Policy presented findings highlighting the challenges school districts faced in finding highly qualified teachers, saying “contemporary educational theory holds that one of the pivotal causes of inadequate student achievement, especially in disadvantaged schools, is the inability of schools to adequately staff classrooms with qualified teachers” (Ingersoll, 2003). As employment rates of alternatively certified teachers have soared in the last 25 years, teacher accountability has risen to new heights under No Child Left Behind (United States Congress, 2001) and the American Recovery and Rehabilitation Act (United States Congress, 2009), hereafter referred to as Race to the Top. Educational reform has placed the need for highly qualified teachers under the microscope. Thus, the debate of alternative certification has been renewed.

Over the past two-and-a-half decades, there has been an increased need and use of alternatively certified teachers in all levels and subject areas. In 1986, there were 18 states with alternative credentialing in place that more than doubled by 1992, to 40 states (Stoddart & Floden, 1995). More recent statistics indicate approximately 62,000 alternatively certified teachers were employed in our nation's schools in 2007-2008 alone, and more than 500,000 have been employed in the last 25 years (NCES, 2010). Of particular concern is the increase in alternatively certified teachers employed in critical shortage areas such as mathematics and science and often in low socioeconomic communities (Ingersoll, 1999).

Alternative certification programs vary significantly around the nation, though generally most provide access to full-time teaching positions without the need for certification prior to employment. Individuals with little or no experience are hired by school districts under this pathway and provided temporary certification while completing the requirements for full certification. For example, the Georgia Professional Standards Commission (GPSC) provides a 2-year alternate route for individuals desiring to teach. Individuals who possess a four-year degree or higher from an accredited institution can be accepted into the Teacher Alternative Preparation Program (TAPP). Once hired by a Georgia school through TAPP, the individual must complete specific requirements over the next two years to receive a renewable teaching certificate from the state. The state of South Carolina uses the American Board for Certification of Teacher Excellence (ABCTE) as one of the official alternative certification routes to teach in this state. The ABCTE is a self-paced online program, which takes an average of 8 to 10 months to complete (ABCTE, 2012). South Carolina also provides a college coursework route, as do most states, to achieve certification. The State of Florida's Department of Education provides multiple pathways to teacher certification through alternate routes. Individuals with 4-year degrees may seek certification through a school district's individualized training program, by completing specified coursework outlined in a Statement of Eligibility letter, or through an Educator Preparation Institute (EPI) at approved colleges and universities. Potential teachers have 2 years to complete these requirements and other related state tests in order to become certified to teach. Alternative certification programs provide abbreviated classroom preparation and quick access to employment, but with the growing concern about teacher quality in educational reform, it is important to understand how this subgroup of teachers impacts student learning, specifically regarding mathematical discourse.

Statement of the Problem

While there is extensive research regarding teacher and student mathematical discourse (Mehan, 1979; Shulman, 1986; Ball, 1990, 1993; Yackel & Cobb, 1996; Lampert, 1990, 1992; Kazemi, 1998; Nathan & Knuth, 2003; Kazemi & Stipek, 2001; Whitenack & Yackel, 2002; Sfard, 2006), no studies have been conducted to determine how the subgroup of alternatively certified teachers conduct mathematical discourse. The purpose of this study was to examine what specific actions alternatively certified teachers take to develop a culture of mathematical discourse and guide students' thinking in middle school mathematics. Research questions for this study were as follows:

1. How do alternatively certified teachers elicit student engagement in and contributions to mathematical discourse?

2. What tools (i.e. routines, establishment of classroom climate/community, tasks) do alternatively certified teachers use to establish a foundation for mathematical discourse?

3. How do alternatively certified teachers use mathematical discourse to facilitate scaffolding of students' mathematics learning and evaluate ZPD?

Justification of the Study

This study was significant and contributed to the body of literature surrounding the teacher's role in discourse, particularly in mathematics education. First, educational researchers continue to seek more information regarding the relationship between mathematical discourse and student learning (Yackel & Cobb, 1996; Ball, 1990; Goos, 2004; Woodward & Irwin, 2005; Gillies & Boyle, 2006). This is an area of research in need of more comprehensive studies in order to understand the full impact of how mathematical discourse enhances student learning. Educational research in general is difficult due to the complex interactions and factors that influence human interactions. Studying teacher knowledge about mathematics and how they infuse this knowledge into their teaching and student interactions is important to understanding the social and pedagogical skills needed in teaching (Lampert, 2009). If we can understand these practices and how they support mathematical discourse we can better prepare alternatively certified teachers and pre-service teachers before they enter the classroom and create professional development opportunities for those already in the field.

Secondly, continuing low graduation numbers from teacher preparation programs (NCES, 2010) indicates that alternative certification will continue to have a strong influence on the education of our nation's students. Established statistics (NCES, 2010; NCAC, 2010) document the broad use of alternatively certified teachers in the United States. Because improving teaching of mathematics is a principal aim of the reform movement (Sfard, 2000; Nathan & Knuth, 2003), how this large subgroup of educators enacts discourse is important to educational efforts as researchers strive to improve student learning through research and reform-based efforts.

Finally and most importantly, many teachers find it challenging to integrate discourse as a primary teaching strategy on a daily basis. While there is some understanding of how teachers conduct discourse, studies on mathematical discourse examine teachers as a group, irrespective of their preparation pathway (Manoucheheri & Enderson, 1999; Williams & Baxter, 1996; Turner, et al., 2003; Ball, 1993; Whitenack & Yackel, 2002; Lampert, 1992). Yet a multitude of subgroups comprise this teaching population. Examining the implementation and use of mathematical discourse specifically in the subgroup of alternatively certified teachers will contribute to an important gap in mathematics education literature.

Theoretical Framework

Social constructivism, developed by Lev Vygotsky (1978), focuses on the individual learner's role in the learning process and the strong influences of social interaction with an expert. Social constructivist learning theory is grounded three core notions. First, there is the role of social interaction. Vygotsky believed that as individuals we learn about the world around us through interactions with others. Through these interactions, or in the case of the classroom-collaboration, individuals change themselves and those around them. The social constructivist learning theory also concentrates on growth as an ongoing process. As humans we are constantly coming into contact with new knowledge and experiences to assimilate into existing knowledge. Social constructivism uses tasks to support this process. Finally, while there is a focus on individual growth, the social aspects of this learning theory also indicate the important of others in the community as an Important to the development of the individual. As individuals we are part of a larger society with whom we must come into contact with on daily basis. There are three ways these three parts of social constructivism support the work in this study.

One major premise of social constructivism is the social aspect that impacts individual learning. Social interactions play a vital role in cognitive development. For education, and mathematics in particular, interactions within the classroom allow students to be exposed to the thoughts and experiences of others. Discourse provides the opportunity for teachers to challenge students' use of problem solving approaches and guide student learning. Through the use of worthwhile tasks and the sharing of ideas teachers can monitor student thinking as the mathematical discourse evolves.

As individuals, we all participate in a variety of discourse communities that impact our knowledge and perspectives of existing and new information. For example, a middle school student participates in discourse communities related to each subject area, possibly a chorus or physical education class, and peers. In each of these settings, the discourse community shapes information that is accepted or denied by the individual. Specifically addressing mathematics, students are a collections of their past mathematics experiences. These differences affect the way they process and assimilate new information shared by others in the group. Because every student belongs to numerous discourse communities, this raises the argument that as students participate as individuals in whole group mathematical discourse they are contributing the experiences of others who are participating in the learning experience (Gee, 1991).

There is value to what other students have learned during their personal and educational development. Teachers should have the skills to help individuals make connections with the curriculum and resulting mathematical discourse. Discourse is the medium in which an exchange of knowledge can take place within the classroom setting. In order for promote an environment that supports mathematical discourse the teacher must establish a supportive structure. This structure includes a feeling of safety, guidelines for who speaks and when, and expectations of participation by all in the group. If this is foundation is provided, the opportunity to express mathematical thinking to a different audience for evaluation and feedback enhances the learning of all. Using discourse in the educational development of the individual learner through an established intellectual environment connects this study to the social constructivism.

A second important premise of social constructivism is the role of the teacher and students in the learning process. Vygotsky (1978) clearly placed importance on a community of experts and novices, which could be interpreted as teacher-to-student or student-to-student. A more knowledgeable individual (usually the teacher) helps to guide the learning of individuals and the group as a whole simultaneously. In examining the teacher's role in this study, it is important to emphasize facilitation of knowledge and information. Teachers must adapt to the role of facilitators (Bauersfeld, 1995) in classroom discussions and activities, differing from the traditional view of a teacher whose sole responsibility is to share knowledge. In classrooms where mathematical discourse is fully integrated, observers should witness students engaged in critically examining the mathematical thoughts and ideas of others contributing to the discourse. Further, the teacher is acting in the role of facilitator during the process, keeping the mathematical discourse moving in a direction that promotes learning and further student engagement.

As teachers promote mathematical discourse, they must be aware of individual student ability in order to assist students challenged by the task. When a student is unable to complete a problem, the teacher should step in a facilitate learning through scaffolding. Scaffolding, introduced by Wood, Bruner, and Ross (1976) and closely related to Vygotsky's ZPD, is the use of focused questions to help a student understand a concept or skill. Initially, there may be a need for the teacher to pose questions at each step in the process in order for the student to acquire the new skill or concept. Over time, the teacher is able to reduce the questions needed to support the student until there is full understanding. Teachers who have an understanding of where their students are developmentally can ask challenging questions to promote individual and group thinking, awakening mental functions that have not yet matured (Goos, 2004). Assuming the role of facilitator to guide students' learning through mathematical discourse also links this theoretical framework with this study.

The final component of the social constructivist theory related to this study is the use of interactive tasks and problems. In order for people to learn, they much have real experiences that are meaningful and can be processed in a way that they understand (Piaget, 1968; Vygotsky, 1978). An important support for mathematical discourse is the use of tasks requiring interaction and collaboration among students and with the teacher. In order for teachers to create tasks that are challenging to all students in the group, they must know where the students are in their understanding of the concept being taught and mathematical ability in general. This knowledge was identified by

Vygotsky (1978) as the Zone of Proximal Development (ZPD). Teachers with this understanding can provide tasks that challenge all learners, implementing scaffolding to assist those as needed. Use of teacher guided mathematical discourse generated around tasks geared to raise students' ZPD thus provides a third link to Vygotsky's social constructivist learning theory. The use of language to promote individual and whole group learning is the focus of this research.

Organization of the Study

A review of relevant literature regarding mathematics as a discourse, establishing mathematical discourse, and alternatively certified teachers' use of mathematical discourse will be presented in Chapter 2. The methodology used for the study is reported in Chapter 3, followed by an analysis of the data and limitations of the study in Chapter 4. Finally, Chapter 5 provides a summary of the results, implications, and recommendations for future research.

CHAPTER 2
REVIEW OF THE LITERATURE

Overview

I conducted a review of the relevant literature with a focus on three areas supporting this study. First, I present a review of the literature as it relates to establishing a foundation for mathematical discourse. Understanding the development of discourse and the establishment of classroom norms by the teacher are important to the implementation of mathematical discourse. What the research says regarding classroom routines, student engagement, and the overall intellectual environment is shared. A second important aspect I covered in this review of the literature is the teacher's role in mathematical discourse. Establishing expectations of justifying responses, challenging others' thinking, and guiding student learning as facilitators are important facets of this study. Finally, I reviewed literature regarding the use of mathematical discourse by alternatively certified teachers. The broad use of alternatively certified teachers in mathematics classrooms coupled with the educational reform focus on mathematical discourse solidifies this subgroup's importance in educational research.

Establishing a Foundation for Mathematical Discourse

The teaching and learning of mathematics involves the activities of reading and writing, listening and discussing (Pimm & Keynes, 1994). In order for students to master each of these aspects of communication, the teacher must provide an environment conducive to learning, be prepared to provide well-thought out lessons and activities, and have the skills necessary to move learning forward while considering the varying skill levels of students as individuals. Conversely, teachers must also consider the contributions of the larger group and how tasks and whole group mathematical discourse can propel student learning to new levels of understanding (Leinhardt & Steele, 2005; Bennett, 2010).

Over the past two decades, discourse research has expanded from linguists to other areas and disciplines of research. Discourse has matured from the basics of analyzing language and understanding of language to more complex and specialized subject matter, including mathematical discourse (Gee, 2012; Sfard, 2006). As the understanding of discourse has evolved a variety of frameworks have been developed to examine characteristics aligned with mathematical discourse (Knuth & Peressini, 2001; Fraivillig, et al., 1999). The establishment of frameworks through educational research has provided insights into common threads in how teachers make mathematical discourse a core part of each lesson. Researchers have identified classroom management skills such as establishing a safe intellectual environment for students, creating social norms that outline expectations for student participation in the learning process, and the implementation of classroom routines as key to the foundation of mathematical discourse in the classroom setting. (Yackel & Cobb, 1996; Manoucheheri & Enderson, 1999; Leinhardt & Steele, 2005).

Gee (1991, 2012) establishes the individuality and community aspects of discourse and the contributions each play in the development of the individual. This development begins with discourse at home and continues to advance the literacy and understanding of the world at the individual level through influences of the community and culture. Because of these connections, some scholars relate discourse to literacy and literacy to discourse with an emphasis on the social interactions of the family and community as part and parcel of individual development in similar ways. Research indicates mathematical discourse Language and literacy are greatly intertwined with all types of learning, including mathematics.

Researchers in the field of mathematics education now acknowledge the importance of classroom discourse and how collaboratively constructing knowledge for the analysis of the whole group contributes to learning at several levels (Sfard, 2000; Cazden, 2001; Knuth & Peressini, 2001; Fraivillig, et al., 1999; Ball, 1991; Manoucheheri & Enderson, 1999). The following two sections are a review of the perspectives of linguists whose work provides a foundation on which mathematics discourse can be constructed and is relevant to today's classroom through establishing discourse as shaped by the individual and community.

The Individuality of Discourse

As individuals, we are exposed to numerous discourses as we develop and learn. Discourse has seemingly simple beginnings: the first discourse we experience comes directly from the family and the home environment. From infancy to preschool our main method of learning is through spoken language and referred to as the primary discourse (Gee, 2007). The primary discourse is your "identity kit," according to Gee (1991), where you are taught how to act and speak in a particular role so that others will recognize you. This "identity kit" expands as speaking, writing, and behavior mature and as we develop knowledge, values, beliefs, history, and citizenship.

The primary discourse serves as the "framework" or "base" for all other acquisition and learning of other discourses throughout life (Gee, 2007). The development of an individual's primary discourse is influenced by the way family members' discourses have also developed and been influenced (Gee, 2007). Parents can unintentionally pass on to children their own primary discourse, affecting the newly developing primary discourse at very early stages (Gee, 2007) in both positive and negative ways. Development of the primary discourse lays the foundation for learning, as individuals expand their discourse to include others in the communities in which they live. As we come into contact with the discourses of others, we expand our own knowledge and experiences. Thus, Gee (1991) argues discourse to be directly tied to literacy based on the use of language as a means for acquiring knowledge. It is important for teachers to understand that every individual in the class brings entirely different experiences, knowledge and mathematical skills to the discourse.

Lotman (1990) provides a different perspective on how people communicate with one another to achieve meaning and understanding. Lotman's view of discourse, more technical in nature, focused less on the development of the individual and more on the meaning of the text being shared. The terms univocal and dialogic were part of Lotman's (1990) "functional dualism." He believed that univocal voice was how we as individuals conveyed meaning adequately, ideally in a perfect or exact message from one individual to another or a group. For example, a teacher giving directions to the class to prepare for a lesson would be an opportunity for univocal voice. The second voice, dialogic, Lotman (1990) viewed as discourse that would generate meaning. This "give-and-take" discourse (Nathan & Knuth, 2001) relates ideally to what types of communications should take place in the classroom regarding mathematical discourse. Discourse should develop new or enhance current understanding of mathematics as the dialogic discourse takes place.

Nathan and Knuth (2001) examined discourse patterns in a longitudinal study using Lotman's framework. While the generation of new understanding through mathematical discourse functions mainly through the dialogic voice, Nathan and Knuth found there is a role for the univocal voice as well. When an exact message, or univocal voice, is misunderstood, it is at this juncture that an opportunity for mathematical discourse begins. Therefore, the teacher (or another student) may have conveyed what they believed to be a message that was understood, only to find this was not the case. Both of these voices have a place teaching mathematics, depending on the instructional goals of the day (Nathan & Knuth, 2001). Teachers need to understand how these two voices function together as they plan lessons and identify speakers to contribute to the overall mathematical discourse.

Other researchers have differing perspectives on the links between literacy, learning, and ultimately mathematics. The views have common threads with slightly varying views of the same terminology. Wertsch (1991) uses the terms monologic and dialogic to describe individual development. Wertsch (1991), building on those proposed by Lotman (1990), believes monologic refers to how individuals interpret the meaning of the text; however, Wertsch (1991) views the information as available for future use or reflection, as in a cultural or community sense. The passing on of cultural values and beliefs through the monologic voice, one to another. A belief of the individual development a result of the cultural influences on literacy and also giving back to the same community. Related to the educational experience, Wells and Arauz (2006) tied the monologic voice to instruction in the classroom. These researchers see monologic instruction as direct or traditional instruction, where the teacher maintains control of all learning. It is important to note that researchers (Gee, 1991; Lotman, 1990; Wertsch, 1991; Nathan & Knuth, 2001) believe there is a role for both of these voices or discourses in the classroom. At times the teacher may need to take on the monologic role to guide discussions to advance learning in a more traditional fashion. There is still a role for traditional instruction in the classroom setting.

The individuality of discourse must be acknowledged to understand how students interact with and learn from the larger classroom community. Developing a strong intellectual environment that promotes student learning becomes an increasingly complex task when considering the various cultural experiences and levels of mathematical knowledge of students and their experiences with parents, teachers, and peers. The individuality of discourse must be acknowledged to understand how students interact with and learn from the larger classroom community. In the next section, I discuss literature regarding how the community influences individual learning.

A Community of Discourse

Some scholars also believe there is a broader sense of discourse that promotes literacy and learning of the individual (Gee, 1991, 2012; Wells & Arauz, 2006). This expanded view of discourse encompasses the community at large in a local and even global sense, as well as the technology used to conduct discourse. Gee (2007) identifies a second level of discourse, secondary discourse*,* to denote other social groups involved in the individual development process. There are many places considered to be secondary discourse groups, including where we work, shop and go to school. At the secondary discourse level, the individuals we come into contact with we know little about and may have little in common (Gee, 2007). Through interactions with these groups in the community we gain some understanding of how to successfully interact with them and gain some shared knowledge. Gee placed emphasis on the influence of these groups on the development of the individual. Secondary discourse groups expand who we are as individuals and influence our interactions and methods of communication, and therefore impact our own literacy (Gee, 2007).

Functioning beside individuals in the secondary discourse group forces us to examine their thoughts, actions, beliefs, and understandings. We must learn to “get along with” these individuals, particularly in places where we come into close contact such as the classroom. Sometimes this inexperience may not be an issue. Other times there can be considerable challenges when mediating a new secondary discourse group. Interaction within these groups forces us to build upon the primary discourse of the family, in which it may or may not be compatible (Gee, 2007). In other words, what you have been taught at home about acceptable ways to communicate with others may come into conflict with what society deems correct and appropriate responses and behaviors. Because of differences in norms among groups, secondary discourse is important to individual growth and maturation. Interactions with secondary discourse groups provide opportunities for analysis and critique of new information, ideas, behaviors, and social expectations. This is important for teachers to remember as they develop lessons and guide learning.

Gee (2012) says that mastery of the secondary discourse is the true definition of literacy and the ability to navigate society successfully. In the book Social Linguistics and Literacies: Ideology in Discourses (2012; p.174), Gee discusses two different types of knowledge principles that occur both inside and outside the classroom related to discourse and literacy. The two principles define the difference between acquisition and learning of which discourse is the central component. The principles also indicate how knowledge is shared and applied in all settings.

> The Acquisition Principle:
> Any discourse (primary or secondary) is for most people most of the time only mastered through acquisition, not learning. Thus, literacy is a product of acquisition, not learning, that is, it requires being exposed to models in natural, meaningful, and functional settings.

Having shared this principle, it is important to note that in some circumstances and settings Gee (1991) believes knowledge through acquisition meets many individual and community needs in society. With regard to mathematics skills, many students can acquire the “act” of mathematics, or the steps necessary to determine the answer to a problem based on a process or procedure. There are times when acquiring knowledge is appropriate for the learner and will serve them well. One does not need to know all the workings of a car in order to drive it. However, there is also a need for deep learning to take place in order to meet the needs of the individual, which leads to Gee’s (2012; p.174) learning principal, where deeper knowledge or “meta-knowledge” is developed.

> The Learning Principle:
> One cannot critique one discourse with another one unless one has a meta-level knowledge about both discourses. This meta-level knowledge is best developed through learning, though often learning applied to a discourse one has to a certain extent already acquired.

Returning to the example of driving the car, if you were asked to discuss how the engine ignites when the car starts you may be at a loss. Understanding how or why is not needed in order for you to obtain a driver’s license and operate a car. The knowledge acquired in learning to drive gained access to the group.

This is not the case with mathematical discourse. In asking students why they performed certain steps or how they arrived at the answer, teachers are able to determine a presence or absence of “meta-knowledge” (Gee, 1991) or learning based on the response. Further, students lacking a deep understanding of the mathematical concept being discussed cannot analyze conjectures presented by peers, nor can they contribute to the whole group discussion, thus minimizing their own gains in learning. According to Gee (1991), in order for individuals in society to have the ability to critically analyze and evaluate information one is exposed to on a constant basis, one must have "meta-knowledge" about the discourse. Thus, students must learn mathematics on a conceptual level, and not just a procedural level, in order to participate in a mathematical discourse community and move learning forward at the individual level as well as contributing to the learning of the larger classroom mathematics community.

In the previous section I discussed the monologic voice of Lotman (1990) and Wertsch (1991) and their views on literacy and learning as individuals. These researchers also had a second voice, dialogic voice, they believed played a major role in learning. The dialogic voice, part of both Lotman (1990) and Wertsch (1991) views of conveying meaning through text, is the community view of learning. These researchers consider the dialogic voice as the cultural influences and broader exposure to others as the community voice influencing individual development. Wells and Arauz (2006) liken the dialogic voice of Lotman and Wertch at the classroom level as playing a primary role in learning, indicating an importance of dialogue in the classroom. Dialogic interactions are the thoughts and beliefs of others in the classroom that contribute to the learning of the entire group. Wells and Arauz (2006) note knowledge is shared and misunderstandings discussed and corrected by anyone in the group. Dialogic voice is the opportunity for individuals to share knowledge and experiences for discussion, analysis and reflection. With regard to mathematics and discourse, students are able to share their thoughts, while critiquing the views of others to gain new understandings of the concepts and skills being shared. The power of group knowledge propels learning forward for all through the contributions of individuals. Teachers need to be well trained in asking questions to which theore are multiple answers to build upon the whole group's knowledge (Wells & Arauz, 2006) to expand the use of the dialogic voice. It is important to note that researchers (Gee, 1991; Lotman, 1990; Wertsch, 1991; Nathan & Knuth, 2001) believe there is a role for both of these voices or discourses in the classroom. At times the teacher may need to take on the monologic role to guide discussions to advance learning in a more traditional fashion, while simultaneously incorporating the dialogic voice for the contributions of the larger group. There is a place for both monologic and dialogic voices or primary and secondary discourses in mathematical discourse and student learning.

Regardless of the connections between discourse and literacy, one belief of these scholars (Gee, 1991; Wells & Arauz, 2006) is that cultural and individual experiences are a major factor in shaping the learning of individuals and their ability to engage in learning experiences. Experiences from their family and the community in which they live influence their knowledge and beliefs. These, in turn, shape discussions within the classroom - as mathematics concepts are - debated. Consequently, our own individual development is influenced by the knowledge we develop with and through others (Vygotsky, 1981), therefore mathematical knowledge is constantly constructed through classroom interactions. Navigating this large and varied area is a challenge for teachers as they strive to shape students' mathematical knowledge toward a conceptual level of understanding shared by the community, while increasing individual meta-knowledge.

Regarding mathematical discourse specifically, these principles indicate the importance of teaching that leads initially to acquisition of knowledge at the foundational level, which is then followed by mastery and the development of meta-knowledge when learning has taken place. The teacher, in full awareness of a student's ZPD (Vygotsky, 1978), scaffolds the students' abilities (acquisition), withdrawing as understanding develops (Cazden, 2001). Once mastery (learning) is achieved, students should be well equipped to articulate explanations and provide analyses of tasks when asked higher order questions, provided related tasks, or on hearing incorrect comments or responses from the teacher or peers. At this point, the teacher is developing student learning by increasing meta-knowledge through mathematical discourse. Teaching for acquisition and teaching for learning are two different things (Gee, 2012), and good teachers do both (Cazden, 2001). The challenge for mathematics teachers is to move beyond procedural knowledge and processes to real comprehension demonstrated through the ability to participate in mathematical discourse at analytical and evaluative levels.

In the next section I review the literature on mathematical discourse through the lens of scholars and researchers in the field of mathematics education. Sfard (2000) and Fraivillig, Murphy and Fuson (1999) have developed their own theories on how students learn mathematics and the relationship that this learning has to discourse. Their research connects discourse and mathematical discourse with current educational reform efforts.

Mathematical Discourse

As previously explained in this chapter, we are exposed and must learn to navigate the discourses of others in relation to our own knowledge and experiences. Classroom discourse is considered one of many discourses that comprise who we are both socially and intellectually (Gee, 1991; Cazden, 2001). This new communication system is highly social in nature, and school is typically the first place we are expected to participate individually and publicly in a large-scale group (Cazden, 2001) where our own discourses may be called into question.

One valuable component, or sub discourse (Gee, 1991), of classroom discourse is mathematical discourse. Using mathematical discourse provides all students the chance to participate in the discussion and critically evaluate what others may contribute to maximize learning for the whole group. In this sense, according to Lave & Wenger (1991), language is the "tool" for development of the individual (to participate in the mathematical discourse community effectively and for the development of others in their community through participation (Wells & Arauz, 2006). Teachers need to embrace a reform-oriented classroom, which is collaborative in nature, to maximize mathematical learning for all students. Using the mathematical knowledge and skills of individuals to contribute to and expand the learning of the entire group increases opportunities for positive learning experiences and educational growth of all students in mathematics.

Mathematics is a Discourse

Sfard (2000) shifts the focus from Gee's (1991) acquisition of learning to one of encouraging students to participate in a certain discourse. Within the discourse framework of research it is understood that mathematical discourse is one of the discourse communities that is characterized by distinctive features (Sfard, 2007). These features are: mathematical use of words, the use of symbols or visual mediators specifically for communicating about mathematics; special discursive routines with well-defined tasks; and hypothesis and conjecture produced through throughout the mathematical discourse regarding the task. Depending on the lesson, the teacher may use one or more to achieve instructional goals.

Along with the features Sfard (2007) uses to define mathematical discourse is the term norm. Sfard (2007) identifies classroom norms as important to the success of students as they participate in mathematical discourse. Gee (2007) suggests community norms as a part of the secondary discourse directly influence our growth and development. In comparing the two perspectives, participation in the mathematical discourse is the responsibility of the teacher by establishing classroom norms that embrace all learners (secondary discourses). The more individuals will contribute to the secondary discourse within the classroom, the greater they are influenced by others.

Gee (2007) and Sfard (2007) appear to agree that the influence of norms, or secondary discourses (Gee, 2007), can be a predictor of success. Gee (2007) says that as one expands primary discourse through input from the secondary discourse, successful individuals are able to acquire more knowledge and ultimately power through the acquisition and increase of financial and materialistic assets. For Sfard (2007), norms are more related to meta-discursive practices that are important for all students to learn. Sfard (2007) notes the importance of the teacher being the “carrier of tradition” and obliged to ensure students learning mathematics in the classroom are able to also participate in the broader community. Thus, students who have gained the skills can influence their position in the classroom and beyond. Those who do not attain the skills will find it difficult to participate in mathematical discourse as the curriculum level becomes more challenging. As students who fall into this category fall further behind mathematically each year, they are less likely to become part of the broader community (Sfard, 2007), much as Gee (1991) sees failure to negotiate secondary discourses successfully limits access to financial freedom and social power of the individual.

Access for all to the mathematical discourse group is an important role of the teacher. Students must be taught to develop a relationship with mathematics. Classrooms where mathematical discourse is a part of the “daily rituals” (Yackel & Cobb, 1996) conducted in a safe learning environment can provide the opportunity to embrace mathematics. How to establish routines that support mathematical discourse need to be a part of teacher preparation for alternatively certified teachers.

Evaluating Mathematical Discourse

One framework used to examine mathematical discourse focuses on the instructional patterns of the teacher. Fraivillig, Murphy, and Fuson (1999) developed a pedagogical framework, Advancing Children’s Thinking (ACT), to examine the teaching practices of eliciting, supporting, and extending musing mathematical discourse of students during whole group discussions. The framework allows researchers to document instructional actions by teachers using mathematical discourse. The ACT framework is comprised of three components, eliciting, supporting, and extending. (Table 2.1). Below is a list of ACT framework components identified in the study.

Table 2-1. Advancing Children's Thinking Framework

Instructional Components		
Eliciting	Supporting	Extending
Facilitates Students' Responding • Waits for and listens to solution or solution method • Conveys accepting attitude towards students' errors or problem-solving efforts • Encourages elaboration of responses Orchestrates Classroom Discussions • Decides who will speak and what methods are shared • Monitors student engagement	Supports Describer's Thinking • Assist individuals with clarifying answers • Reminds students of a similar problem already solved • Provides background knowledge Supports Listeners' Thinking • Demonstrates teacher-selected solution method Supports Describer's and Listeners' Thinking • Records solutions on the board Supports Individuals in Private Help Sessions • Encourages individuals to seek private assistance	Maintains High Standards and Expectations for all Students • Asks all students to attempt all problems Encourages Mathematical Reflection • Encourages students to draw generalizations • Encourages students to justify responses

(Adapted from Fraivillig, et al., 1999)

In the study, Fraivillig, et al., (1999), 18 teachers are observed twice weekly. From this group, six teachers were identified as being skilled in the integration of mathematical discourse in daily mathematics lessons. Drilling down further, one teacher was identified as an embedded case study and reported on in the article. The teacher was observed additional times and interviewed to identify patterns of practice. The act framework was used as a part of the analysis. The broader findings indicated that there was little evidence of teachers eliciting or extending mathematical discourse. Most teacher efforts were identified consistently in the supporting category of the ACT framework. The embedded case study had participation in all three areas of the framework, including extending student thinking. Though this was also observed the least, it was concluded by the researchers that her contributions to student learning were significant in comparison to the larger group.

The role of discourse has been central to education and learning (Knuth & Peressini, 2001), and is inherently social in nature (Vygotsky, 1981; Gee, 1991). Understanding how teachers create an environment for meaningful discussions and implement effective mathematical discourse is important to professional development and reform initiatives, including alternative teacher preparation. In the next section, I examine what the literature says about establishing mathematical discourse. Having a clearly defined classroom environment with rules and expectations lays the groundwork for mathematical discourse.

Establishing Mathematical Discourse in the Classroom

Research on mathematical discourse has focused on both teacher and student interactions (Mehan, 1979; Shulman, 1986; Ball, 1990, 1993; Yackel & Cobb, 1996; Lampert, 1990, 1992; Kazemi, 1998; Kazemi & Stipek, 2001; Whitenack & Yackel, 2002; Sfard, 2006) in an effort to gain a deeper understanding of how students learn mathematics from teachers and peers. This body of research indicates using mathematical discourse provides a deeper understanding of concepts and allows individuals to contribute to the learning of the entire group. As teachers, we bear the responsibility of supporting mathematical discourse to enhance student learning.

In the following sections, I review studies supporting the importance of establishing classroom routines, student engagement, and a positive intellectual environment. As indicated by the research, the teacher plays a central role in establishing the mathematical quality in the classroom environment and in establishing mathematical norms (Yackel & Cobb, 1996).

Classroom Routines

Mathematical discourse and the tasks that support has been a central theme to engage students in the (Spillane & Zeuli, 1999; Leinhardt & Steele, 2005). A classroom promoting mathematical discourse has established routines and an intellectual climate that encourages student engagement and supports instructional dialogue (Leinhardt & Steele, 2005; Ball, 1993). Regarding classroom routines, teachers must establish a structured environment that addresses speaker order, the opening and closing topics, and students' voice control as the very foundation for establishing mathematical discourse. By doing this, students have a clear understanding of the boundaries in which the discussion must take place. I was able to identify several studies highlighting the importance of classroom routines as part of the groundwork for establishing mathematical discourse. The studies indicate that implementation of a positive learning environment in the classroom is key to active participation of students in mathematical discourse.

Manoucheheri and Enderson (1999) reported on a case study of 25 middle school mathematics students in an inclusion classroom. The study focused on the establishment of social and mathematical norms and the positive impact on mathematical discourse. The researchers indicated students clearly understood how and when to engage in mathematical discussions and were clear on teacher expectations. The teacher was attempting to integrate an inquiry-based learning model for mathematics. Lessons were videotaped at the beginning of the year and again six months into the school year. After analyzing the lessons, researchers concluded the teacher had successfully implemented an inquiry-based learning environment in mathematics. The teacher had protocols established for whole and small group interactions. The study report the teacher reminded students to raise their hands and called on specific individuals to share group findings. The teacher also asked consensus type questions to gauge where students were mathematically as the lesson progressed. A classroom environment of active engagement as a participant and listener was clearly established as a requirement of the mathematics lesson on a daily basis as evidenced by student behaviors.

In a 2005 study, Seeing the Complexity of Standing to the Side: Instructional Dialogues¸ Leinhardt and Steele identify classroom routines as one of three important characteristics in supporting mathematical discourse. The study observed mathematical scholar and researcher Magdalene Lampert teaching a 10 lesson unit to fifth graders. Dr. Lampert was interviewed before and after each lesson, kept a daily journal of teachings, and each lesson was audiotaped and analyzed. The study found that Lampert’s use of instructional dialogue was a unique pattern of actions and responses that served to set overarching valued goals by the students (Leinhardt & Steele, 2005). The teacher worked with the students to develop their own methods for handling various parts of a lesson. Having this natural understanding or set of processes allowed for the teacher and class to focus on the lesson. These “daily rituals” (Yackel & Cobb, 1996) help to provide organization to the classroom and flow to the lessons.

Classroom routines are of foundational importance to mathematical discourse. Yackel and Cobb (1996) noted that daily rituals of the classroom with regard to the rights and obligations of participation play an important role in how students perceive and learn mathematics. In their studies of second and third grade teachers and the inclusion of mathematical discourse as a regular part of whole group instruction showed that effective teaching demands that these routines be established not only for classroom discipline, but also for a quality learning experience. Yackel and Cobb (1996) noted that the students clearly understood when it was appropriate to contribute to the mathematical discourse of the group, actively listening to what the teacher and their peers where saying and analyzing these thoughts against their own perspectives. Important to the success of the mathematical discourse was an understanding of established social cues on how and when it was appropriate to join in the conversation. Teachers need to remember that intellectual climate and classroom culture have a large influence on how students engage in the learning of mathematics and participate in mathematical discourse in small and whole group settings (Schoenfeld, 1989; 1992).

Intellectual Climate

Paired with a structured environment is an expectation of more than providing correct answers; one of justifying conjecture, questioning from both teacher and peers, and promoting a safe learning environment in which ideas can be shared for consideration and evaluation by peers (NCTM, 2000; NRC, 2001). Historically, mathematics instruction has focused on procedural knowledge (Nystrand & Gamoran, 1991) or calculation discourse (Cobb in Sfard et al., 1998), which is commonly found in traditional teaching. This involves the teacher as the focal point of the learning process teaching students the “steps” to solving problems. Educational reform efforts no call for a deeper understanding of mathematics, conceptual knowledge (Kazemi & Stipek, 2001), or conceptual discourse (Cobb in Sfard, et al., 1998). This modern form of collaborative learning engages students in discourse to discuss the reasoning behind the “steps” or procedural knowledge, in an effort to build understanding of mathematical concepts and how to articulate that understanding to others. Establishing an intellectual climate that embraces conceptual knowledge and conceptual discourse helps students apply mathematical discourse to a variety of tasks and lays the foundation to learn more challenging mathematical concepts.

One important reason for establishing a positive intellectual environment is for students to feel comfortable sharing personal thoughts in the large group setting. In order to do this, a teacher must guarantee safety for all students to share thoughts openly (Leinhardt & Steele, 2005; Bennett, 2010; Fraivillig, et al., 1999). By providing an environment that embraces the thoughts of the entire community, students can reveal private thoughts without fear of ridicule or embarrassment, which is often a challenge at the secondary level (Bennett, 2010). Research indicates that teachers who require students to support their responses during mathematical discourse increase student participation and contribution to the community of learners (Woodward & Irwin, 2005; Nathan & Knuth, 2003; Leinhardt & Steele, 2005).

Leinhardt and Steele (2005) reported on a study of Dr. Magdalene Lampert, a well-known mathematics researcher. One of the findings was her establishment of a safe intellectual environment. The mathematical discourse data indicated students were comfortable being wrong, challenging the thinking of their peers and even correcting themselves publically during lessons. The study inferred that first and second year teachers are so busy focusing on understanding the curriculum and meeting employment requirements that establishing an intellectual environment centered on discourse is not a focus and considered too challenging to consider as a new teacher .

Making the students' ideas and thoughts the focus of the discussion while keeping the lesson moving forward intellectually, continues to be challenging for all teachers, particularly teachers new to the classroom.

In a similar study of mathematical discourse, Zack and Graves (2001) note an important role for the teacher is to provide a safe classroom environment where ideas can be explored together as teacher and class. The case study presented in the article focuses on the teacher's role as an inquirer in the learning process with her students, Zack and Graves (2001) note that the fifth grade teacher is not only an inquirer with the students but also demonstrated her expertise in the role of listener. The teacher left the position of provider of knowledge to that of knowledgeable orchestrator (Zack & Graves, 2001) to assist the students in the formation of knowledge and advancing their level of understanding of mathematics. The teacher uses reform-oriented instruction that is collaborative in nature on a daily bas is. Of particular note in the study was the teacher's use of acknowledgement of students and giving credit to students for answers throughout the lesson. This set the tone for community and a safe learning environment embracing mathematical discourse and the contributions of the group. The trust established by the teacher developed a sense of community for all students. Mathematical discourse connected students and the teacher at the individual and whole group level (Zack & Graves, 2001).

Establishing a positive intellectual climate within the classroom also provides a framework for enhanced mathematical discourse with critical and evaluative thinking. Some research studies found teachers become much more adept at negotiating whole group mathematical discourse when they integrated an expectation for justifying responses, required supporting information, and expected students to use correct terminology during discussion (Morrone, Harkness, D'Ambrosio, & Caulfield, 2004; Woodward & Irwin, 2005; Nathan & Knuth, 2003). How teachers promote student learning by extending mathematical discourse continues to be of importance to educational reform efforts.

Woodward and Irwin (2005) shared a multi-year study using the Numeracy Development Project (NDP) to train teachers to facilitate discourse during mathematics lessons. One teacher was highlighted in the article because of her integration of key instructional moves to promote mathematical discourse. This teacher asked open-ended questions, allowed time for student responses, and acted more as a facilitator of the learning process than others in the study (Woodward & Irwin, 2005). The study used the ACT model (Fraivillig, et al., 1999) to evaluate teacher-led mathematical discourse decisions and instructional. Sessions were videotaped and coded using the ACT framework with no specific mathematics curricula used. Both whole group and small group instruction were recorded. One small group was simultaneously recorded and analyzed in each of the five while group recorded sessions. Findings of their study indicated that there were a higher portion of students that engaged at the upper levels of the ACT framework through the NDP, indicating higher order thinking and reasoning skills and the ability to analyze discussions and contribute to the overall discussions. An embedded case study shared in the report indicated that one teacher utilized the ACT framework components. The study noted many teachers incorporated actions like allowing more time for students to respond, probing for better explanations and using challenging questions (Woodward & Irwin, 2005).

Similarly, a study conducted by Kazemi and Stipek (2001) examined four teachers in fourth and fifth grades, all teaching a lesson on fractions. The study focused on a shift to conceptual learning instead of computational learning of mathematics as has been past practice by many educators. Teachers were trained on reform-oriented curriculum (not specifically named) before the study began. Four teachers took part in the study, with only one having minimal experience. The study recorded the levels at which teachers "pressed" the students' understanding of fractions. The study revealed a significant positive correlation between episodes of high press and the degree of student understanding and the enactment of sociomathematical norms. Kazemi and Stipek (2001) argued how teachers establish classroom culture and their own teaching practices impact how students learn mathematics. Classroom practices that are characterized by a high press for conceptual thinking allow for the mathematics to drive the students' engagement of activities (Kazemi & Stipek, 2001). Interestingly, the least experienced teacher used the most procedural questions in the study, indicating a need for professional development of teachers in integrating mathematical discourse to advance student's mathematical thinking.

It is important to note that more talk in the classroom does not necessarily equate to more learning (Walshaw & Anthony, 2008). In general, we know as educators that quality is often more important than quantity. This is certainly true with regard to mathematical discourse. When teachers provide an intellectual climate promoting the expectation of student engagement and justification of responses students have opportunities to begin to understand how to think and speak mathematically as independent learners that can also contribute to the whole group (Leinhardt & Steele, 2005; Sfard, 2006). Students need to understand that it is not only appropriate to challenge another student's thinking, but how to go about it constructively. Developing discourse requires a safe environment where it is acceptable to be wrong, to challenge or correct another, and more importantly, to correct oneself (Leinhardt & Steele, 2005). Crafting an intellectual environment in which students present their thinking, oppose other views, and evaluate their own learning while simultaneously maintaining a focus on instructional goals is a challenge for many teachers.

Because it is the responsibility of teachers to set the tone for classroom discourse and extended student engagement, how they choose facilitate discussions directly influences the student learning that takes place. Establishing an environment that is conducive for mathematical discourse is important to increase student learning (Schoenfeld, 1989; 1992; Yackel & Cobb, 1996. Gee (1991; p.9) connects the importance of acquisition of knowledge and true learning intimately with discourses:

> Learning should lead to the ability for all children-mainstream and non-mainstream-to critique their primary and secondary discourses. This requires exposing children to a variety of alternative primary and secondary ones not so they acquire them, but so they can learn about them.

In the context of mathematics, mathematical discourse is one of many subdiscourses that contribute to the ability to develop through critique and learning from the knowledge of others in the mathematics community. As students share mathematical knowledge with others, they are exposing and being exposed to the mathematical discourses and participation in the discourse community.

After establishing classroom routines and a structured, intellectual learning environment, teachers have the foundation in place to begin building mathematical discourse. In the next section, I review literature focusing on the teacher's role in mathematical discourse.

The Teacher's Role in Mathematical Discourse

Teachers must become skilled in leading mathematical discourse to provide learning opportunities for students and an environment that promotes various viewpoints (Thames & Ball, 2004; Zack & Graves, 2001). As students gain deeper understandings of mathematics concepts, the teacher is able to assume the role of facilitator, assisting only when there is an obvious misunderstanding or to refocus the conversation if the discussion becomes unrelated. Achieving the role of facilitator can be challenging for educators of all levels of experience (Bennett, 2010; Leinhardt & Steele, 2005).

Research indicates when teachers extend mathematical discourse through pressing for additional responses and apply scaffolding practices students benefit from the increased interactions (Goos, 2004; Morrone, Harkness, D'Ambrosio, & Caulfield, 2004). In studies examining the teacher's role in mathematical discourse there is evidence that when teachers extend the discussions, student participation (Nathan & Knuth, 2003) and learning (Hatano & Inagaki, 1991) increases. Teachers can accomplish this by pressing students to expand on initial response, not accepting simply right answers or correcting wrong answers. Further, through encouraging and eventually requiring students to engage in conversations on mathematical concepts by justifying responses and challenging the thinking of others, teachers can extend the mathematical discourse and provide additional opportunity for student learning. In order for this to take place, the teacher must assume the role of facilitator (Goos, 2004), using scaffolding and higher order thinking questions to lead students to understanding mathematical concepts.

In 2004, Goos described the study of a high school mathematics teacher in Australia who guided mathematical discourse and used scaffolding to promote student thinking and engagement. Weekly videotaped observations over two years, along with interviews of students and teachers framed the extensive data collection for this study. Goos (2004) developed nine categories that expressed actions the teacher took to promote mathematical discourse:

1. The teacher models mathematical thinking.
2. The teacher asks students to clarify and justify their responses.
3. The teacher emphasizes sense-making.
4. The teacher explicitly references mathematical conventions and symbolism.
5. The teacher encourages reflection and self-monitoring.
6. The teachers uses students' ideas as starting points for discussions.
7. The teacher structures students' thinking.
8. The teacher encourages exploratory discussion.
9. The teacher structures students' social interactions.

From this list you can begin to see the development of a teacher's role in facilitating mathematical discourse and advancing student learning. Teacher decisions and instructional actions the basis for what types of mathematical discussions will take place and the direction of the overall conversation. The teacher in this case study had a well-developed idea of how to facilitate mathematical discourse in his high school classroom and develop the mathematical proficiency of the students by creating a sense of community of inquiry (Goos, 2004).

In A Study of Whole Classroom Mathematical Discourse and Teacher Change, Nathan and Knuth (2003) reported on the discourse of the first two years of new middle school mathematics teacher. Professional development was provided to teachers in the summer. These trainings were recorded to determine the teacher's reflection of her own teaching. Weekly videotaped lessons during the year were coded and analyzed for teacher-to-student and student-to-student interactions. In year one there was less student-to-student interactions than year two. Students in year two also used and mathematical terminology expressed themselves more coherently. This indicates the teacher assumed more of a role of facilitator in the learning process (Nathan & Knuth, 2003). The teacher assuming the role of facilitator allowed for increased student engagement and expression of mathematical thoughts. -

Morrone, Harkness, D'Ambrosio and Caulfield (2004) conducted a study which focused on pre-service teachers through an experimental mathematics course. The course was taught using a social constructivist approach designed to expand students' understanding of problem-solving and mathematics in general. Classes were videotaped and provided examples of teachers pressing students for more detailed responses to demonstrate higher-order thinking and deeper understanding. In the study scaffolding indicated low understanding on the part of the student, while the ability to express conceptual ideas and conjecture as a high level of understanding. As the class progressed pre-service teachers became more adept at participating in mathematical discourse though in the evaluations some expressed frustration with the process. The instructor was committed to pressing the students for more information in an effort to extend the discourse and advance mathematical thinking for the individual and the group (Morrone, Harkness, D'Ambrosio, & Caulfield, 2004).

In an important mathematical discourse study conducted by Williams and Baxter (1996), focused on discourse-oriented teaching through the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project. The study examined one middle school teacher's lessons over a period of three years as an embedded case study of a larger multi-school and multi-teacher study. The QUASAR project promoted the philosophy of student talk to develop meaningful mathematical discourse in the whole group setting. Students were encouraged to work together to explain and justify mathematical concepts extensively. The identified teacher reported on in the article embraced mathematical discourse as a part of her daily lessons. She defined roles for the students as listeners and problem solvers for clarity on participation. The teacher relied on discourse to help students build social norms. Though this study was conducted in earlier years of mathematics reform efforts (after the PSSM) regarding mathematical discourse, the teacher seemed to have understood and integrated mathematical discourse at a deeper level than prior to the QUASAR project. The study indicates the importance of the teacher's role in the production of knowledge (Williams & Baxter, 1996).

Delving into student thinking through discourse allows students to process not only their thinking, but also the thinking of their peers. Teachers who expect students to justify thinking, explain solutions, and use precise vocabulary will gain a better mathematical understanding of the concept being discussed (Hatano & Inagaki, 1991) and become better equipped to evaluate mathematical premises presented by others. Having the expectation for students to expand on their response and share how they arrived at a particular answer, whether correct or incorrect, provides peers with additional opportunities to understand the concepts being discussed. Expanding on basic answers also provided the teacher with insights into student understanding and current ZPD (Vygotsky, 1978), thus allowing them to guide mathematical discourse for further scaffolding as appropriate.

Mathematical discourse has become essential to the classroom at all levels of education. The teacher as facilitator has the responsibility of guiding conversations and providing a classroom environment that provides a quality mathematical experience (Walshaw & Anthony, 2008; Goos, 2004; Williams & Baxter, 1996). Understanding what questions to ask and when, knowing when to redirect a conversation that may have gone off topic, or preparing lessons and tasks that will generate mathematical discourse are all areas of development for many in-service and pre-service teachers.

Approaches used by alternatively certified teachers to enact mathematical discourse are important and worthy of examination. A pedagogical approach that advances students' thinking forward begins with an environment of respect and trust, allowing for discussion and problem solving. This type of intellectual environment enhances the inner capabilities of the individual and emphasizes the dispositions of self-discipline, both of which allow for the development of the individual (Popkewitz, 1988). In the next section, I present a review of the literature regarding the use of alternatively certified teachers and how they conduct mathematical discourse is shared as well as a review of the literature regarding the use of alternatively certified teachers.

Alternatively Certified Teachers

For more than two decades many states have had in place some form of emergency credentialing to address the issue of teacher shortage. Although not a new phenomenon, alternative certification has grown in use by states across the nation in response to increasing student enrollments, legislation requiring class size reductions, teacher attrition of various types, and negative perceptions of the profession among the general public that have impacted high school graduates from pursuing this profession as a career (Claycomb, 2000; Ingersoll, 1996; 2002). Federal policy initiatives set forth by the No Child Left Behind Act (2001) have also encouraged alternative certification programs to allow individuals to enter the classroom before they have completed formal training (Darling-Hammond, 2010).

The use of alternatively certified teachers and the training of teachers in general is important to educational reform. School districts will be more likely to retain all teachers, traditionally trained and alternatively certified if they are provided quality training before and support upon entering the workforce. In doing this, teachers have time to mature in their knowledge and skills and hopefully remain in the classroom to become effective classroom facilitators (Darling-Hammond, 2010; Scribner & Akiba, 2009) The more stable teachers are in the classroom, the more effective they become as teachers.

In the following section, I deliver a review of the literature that examines the impact of teacher certification on success in mathematics. Studies indicate a direct correlation between teacher certification and student success in mathematics (Darling-Hammond & Youngs, 2002; Wayne & Youngs, 2003). When considering that fact that at least one-third of all secondary mathematics teachers do not hold a major or minor in mathematics (Ingersoll, 1999), this is of concern to education of students and the nation's overall reform agenda. Education researchers need a deeper understanding of this cross-section of teachers to support their preparation.

Teacher Certification and Mathematics Success

Literature on alternative certification helps to breakdown the myriad of pieces to this complex route to teacher certification. It is clear that there is no one explanation as to what influences student success with regard to teacher preparation. Some researchers believe career experience, types of degrees, and pass rates of licensure exams are good indicators of success with students (Ballou & Podgursky, 1997; Walsh, 2001).Others believe equity and access to highly qualified teachers explains disparities in teacher attrition and lack of qualified teachers in certain regions and school districts in the nation (Darling-Hammond, 2000; Cochran-Smith, 2005; Ingersoll, 1999).

Despite the varying interpretations of why alternative certification is used by states cross the nation, the fact remains that individuals do enter the classroom under varying types of emergency credentials and alternate pathways. The arguments supporting the use of alternative certification are compelling. Some factors include increasing the diversity of the workforce, recruitment of individuals with professional experiences that support student learning, and apply real world knowledge in meaningful ways for students (Adams & Dial, 1993). So what does the research say about alternatively certified teachers and student success in mathematics?

Two studies, both by Goldhaber and Brewer (1997, 2000), examined teacher certification and the relationship to student success in the four major subject areas- history, science, mathematics, and English. Findings in the 1997 study indicated that students taught by teachers holding the subject related certification in mathematics or science has a positive relationship to student gains within the subject area. Certification was not specifically asked as a part of this study, though results are important to note since the study was replicated. In the second study (2000), certification type was more closely examined with regard to mathematics. Teachers with “temporary, provisional, or emergency certification” were noted as a subgroup. The findings were inconclusive but suggested that alternatively certified and traditionally trained teachers were equally effective.

A related study conducted by Raymond, Fletcher, and Lugue (2001) compared a specific route of alternative certification, Teach For America (TFA) teachers to all beginning teachers, using data from a Texas school district. TFA teachers have degrees in science, technology, engineering, and mathematics (STEM) and have agreed to teach in low-socioeconomic areas of the country. The study indicated students who were taught by alternatively teachers with a STEM background had slightly higher gains of about three months in mathematics on the state exam over teacher with traditional certification.

Researchers have begun to examine more than degrees, career experience, and pass rates on state licensure exams. The focus has now shifted to mathematical content knowledge with recent studies conducted by Deborah Ball, Heather Hill, and colleagues (2004; 2005). Ball and others have begun to examine the relationship between teacher preparation, mathematical content knowledge and mathematical knowledge needed for teaching. These studies do not cite alternatively certified teachers as an issue, but rather suggest all mathematics teachers need to have a deep understanding of mathematics as well has specific knowledge and skills on how to teach the subject.

A recent research study by Boyd, et al. (2012) focused on the alternative certification program established in New York City (NYC) and several other urban cities to meet the need for highly qualified mathematics teachers. Like many school districts in America, New York City struggles to identify and employ qualified mathematics candidates and must train them using their Math Immersion component (Boyd, et al., 2012). School districts in the study acknowledge alternatively certified teachers in particular need specialized training in order to by effective mathematics teachers. The study examined student achievement gains of teachers who entered the profession through the traditional route to those who earned licensure through this alternative certification program. The study found that on average students in grades 6-8 of alternatively certified teachers with Math Immersion training had smaller gains than students of teachers prepared through a traditional route or who had a strong background in mathematics.

Questions of equity and teacher quality highlight the importance of understanding more concerning alternatively certified teachers and how they teach. The success of students in mathematics in relation to teacher experience, training, and other potential influencing factors continues to be debated by researchers (Ingersoll, 1999; Goldhaber & Brewer, 2001; Walsh, 2001). As the nation continues to focus on educational reform at all levels, the need to understand the impact of alternative teacher certification on student success becomes increasingly important as more are employed to fill employment gaps.

Use of Mathematical Discourse

With regard specifically to alternatively certificated teachers and how they enact mathematical discourse, specific studies on this topic are absent from the literature. The subgroup of alternative certification within teachers and how they implement mathematical discourse does not appear to be addressed by the literature.

I located one article that examined two teachers with two years or less experience but was not specifically related to alternative certification. It's Hard Getting Kids to Talk About Math: Helping New Teacher Improve Mathematical Discourse examined two first year teachers mentored by the researcher on their implementation and use of mathematical discourse and questioning techniques (Bennett, 2010). Three lessons from the first part of the year where videotaped and analyzed. Both teachers used basically the same teaching approach when initially observed-lecture followed by class time to complete problems and see teacher assistance as needed, what would be considered the traditional teaching model. Neither teacher had a high use of questioning during their lessons. If a question was asked, there were no follow up or probing questions to the responding student or class as a whole.

After feedback from the mentor/researcher on the initial findings at the mid-term part of the year on the lack of mathematical discourse and student engagement, three more sessions were videotaped and analyzed during the second part of the year. Both teachers had dramatic differences in terms of instructional time, questioning and student engagement in the second set of observations. Notable differences included a 60% reduction in the time one teacher spent "instructing" the class on the lesson and shifting this time to applying skills and engaging students in whole and small group conversations about the task. The increased teacher and student interactions allowed for two follow up questions for every initial question posed by the teacher. The second teacher had an increase in the number of questions asked of students as well, though teaching time was reduced by only five minutes. The second participant increased overall in whole group participation, which was significant when the first three recorded lessons indicated students were expected to work individually and silently once instruction was complete. An interesting point, this teacher had a mathematics background and was well versed in the understanding of mathematics, whereas teacher one had a science background and was teaching mathematics.

During the last quarter century there has been an ever-increasing use of alternatively certified teachers in public schools. Further, it has become increasingly difficult for school districts to identify and employ traditionally trained mathematics teachers. While the debate once considered licensure exam pass rates, career experience and degrees to define highly qualified, new research indicates there is much more to consider before we claim a teacher to be skilled and competent to teach mathematics. Mathematical content knowledge and how mathematics is taught is something worthy of additional research no matter the teacher certification route; however, one cannot ignore a potentially significant gap in knowledge and ability of those who are alternatively certified to teach mathematics.

As noted by the lack of studies for review, the use of mathematical discourse by alternatively certified teachers is needed in the literature. The literature review of alternative certification with regard to mathematical discourse confirms the milieu facing public education regarding the need for high quality mathematics instruction and the increasing demand for qualified teachers. Clearly alternative certification will continue to play a role in 21^{st}-century education, indicating a need for researchers, educators, and policymakers to be informed on how this subgroup contributes to the mathematics classroom and overall educational reform initiatives. This study will help to fill this identified gap and contribute to the research on teacher education.

CHAPTER 3
RESEARCH DESIGN AND METHODOLOGY

Participants and Setting

Research Objective

The purpose of this study was to investigate the use of mathematical discourse by alternatively certified middle school teachers using three key questions. First, how do alternatively certified teachers elicit student engagement in and contributions to mathematical discourse? Secondly, what tools were used to establish a foundation for mathematical discourse? Tools examined as a part of this study refer to classroom routines, type of learning environment established, and tasks teachers required of the students. The final question in the study was how do alternatively certified teachers use mathematical discourse to facilitate scaffolding of students' mathematical learning and evaluate ZPD?

Participants

Three middle school teachers participated in the study (Table 3-1). I selected participants from a pool of teachers identified as currently teaching mathematics in the middle school setting with a minimum of one year and no more than three years of teaching experience and who had achieved alternative certification through any state-approved pathway. To control for factors that might influence study findings, such as outside professional development opportunities and natural maturation of teachers as they develop within the profession, I considered the participants' range of teaching experience (Nathan & Knuth, 2003; Clotfelter, Ladd, & Vigdor, 2007). I confirmed each participant's credentials and experience with each school district's Human Resources (HR) department and by using a series of questions at a pre-study meeting (Appendix

A). Each HR department identified eligible mathematics teachers employed in the district after being provided the criteria for participation in the study. From this list, the HR director or school principal contacted the teachers for possible interest in the study. One teacher accepted from one county, and two teachers accepted from another nearby county to comprise the three required for the study. Upon confirmation of interest in the study, I contacted each participant to schedule a time for a pre-study meeting where the official study letter was signed and conducted the Pre-Study Interview.

After conducting the pre-study meeting, the following is a broader description of each participant. I used pseudonyms for each participant, as were any references to students by name to protect identities.

Mary (pseudonym) is a second year teacher with a Bachelor’s degree in Public Relations. She was alternatively certified through a state approved alternative route and felt the program did little to prepare her for teaching mathematics, but was helpful for teaching in general. Mary had no prior experience specifically in mathematics and took only the minimally required mathematics (one course) in college for her major. Prior to the study she had not participated in any professional development related to scaffolding, questioning or related topics. She expressed a love of mathematics all through school and has always embraced the subject.

Patrick (pseudonym) is a second year teacher with a Bachelor’s degree in Health Sciences. Patrick was an Emergency Medical Technician before seeking his degree and stated he enjoyed mathematics as a student and great mathematics teachers in his view. Patrick was also alternatively certified through a state approved teacher education program, which he felt did not help prepare him for the classroom or to teach mathematics specifically. Prior to the study Patrick had not received any professional development in scaffolding or questioning.

Timothy (pseudonym) is completing is third year as a mathematics teacher and has a degree in International Studies. Timothy is a retired Air Force service man and has traveled extensively around the world. He was alternatively certified through a state approved program, which he believed did not help to prepare him to teach mathematics, though portions of the program helped him with teaching in general. Timothy shared a love of mathematics and a frustration for engaging students. Timothy had no prior professional development that would influence the outcomes of the study.

Setting

The study took place at two middle schools serving grades 6-8 in two rural counties in the southeast region of the United States. For study purposes the schools I refer to School A in the Liberty County School District (pseudonym) and School B in the Rockwood County School District (pseudonym), with statistics summarized in Table 3-2. School A was the only middle school in the Liberty County School District, with 47.44% of students qualified for free or reduced lunch, indicating a high number of low socioeconomic students enrolled in the school. Three of the 11 or 27% of teachers have earned certification through an alternate route and have taught less than five years.

School B was one of three public middle schools in the Rockwood County School District. School B employs five regular mathematics teachers across the three grades represented at the school, with three of the five currently employed teachers, or about 60%, earning a teaching certificate in middle school mathematics through an alternative pathway. One of those three (33%) has less than three years teaching experience.

Table 3-1. Overview of Teachers in the Study

	Grade(s) Taught	Number of Years Teaching	Degree Discipline	Alternatively Certified
Patrick	6th	2	Health Science/EMT	Yes
Mary	7th	2	Public Relations	Yes
Timothy	7th and 8th	3	International Studies	Yes

Table 3-2. Overview of School Statistics

		School A	School B
Title I School		No	Yes
Population		1,099 students	506 students
Gender	Male	50.86%	47.23%
	Female	49.14%	52.77%
Ethnicity	Caucasian	86.26%	84.39%
	African American	11.19%	8.50%
	Hispanic	2.00%	6.32%
	Asian	.55%	.80%
Mathematics Teachers	Total	11	5
	Alternatively Certified	5	3
	Alternatively Certified with Three or Less Years of Experience	3	1

Curriculum

I asked each participant to continue teaching their normally scheduled lessons during the study; therefore, I observed a variety of mathematics topics. Since the focus of the study was participants' facilitation of mathematical discourse and guidance of student learning, I deemed it unnecessary to focus on a specific mathematics topic or series of mathematics topics during the planning stage. Topics ranged from adding and subtracting polynomials to solving multi-step equations, all of which are required by the state's mandated curriculum standards.

Data Collection

The study consisted of three case studies yielding qualitative data using the conceptual ACT framework Fraivillig, et al., (1999). The three pedagogical components of this framework, eliciting, supporting, and extending, supported the goals of this study. Eliciting relates to the participant's efforts to have the student explain their solution method. Supporting indicates participant assistance with students' thinking through the learning process. Extending depicts the participant's efforts to continue mathematical discourse to promote whole group student learning and move individual student learning forward. More detail is provided on the categories and indicators within each category in Table 2-1, introduced in Chapter 2.

I designed the study to evaluate how alternatively certified teachers conduct mathematical discourse, the use of scaffolding to assist students with the learning process, and the type of learning environment established in the classroom to support mathematical discourse. I collected data in the form of classroom observations, through three different types of participant interviews, and both teacher and student artifacts. I used methodological triangulation through the convergence of multiple data sources (observations, interviews, artifacts) to control researcher bias, ensure the research accurately reflected the data collected during the study, and increase credibility of the findings (Mathison,1988).

Pre-Study Interviews

I collected the initial data for this study through the pre-study interview (Appendix A). The goal of this interview was to confirm that each teacher met the study's participant criteria, determine if the teacher had attended or participated in professional development that may influence findings of the study, and to gain insights into the teacher's perspective on how alternative certification had prepared them to teach mathematics. This interview was brief and direct, capturing the information easily with seven open-ended questions. I conducted this interview in person with Patrick and Mary and via phone with Timothy prior to the first observation.

Classroom Observations

I conducted classroom observations during the spring term, with dates and times agreed upon between the researcher and each of the participating teachers. I used the passive participation method to videotape lessons and take field notes. I observed and videotaped each of the three participants five to seven times for one hour each time teaching lessons of their choice (Table 3-3). Scheduling conflicts with two of the participants reduced their observations by two over the third participant. The camera remained on the participant at all times. Occasionally, I recorded students if interaction occurred at the board as a part of instruction or the participant used an instructional technique that required the camera to focus on or include students. The intent of focusing on the participant was to capture mathematical discourse used to initiate, support and extend discussions with students as a part of the instruction. Additionally, emphasis on the participants' mathematical discourse helped to identify the use of scaffolding as a part of teacher-to-student interactions in support of student mathematical learning. I took notes during the observations to help identify videotape segments for the intensive interviews and key teacher and student interactions for post observation interview discussions.

Post-Observation Interviews

I conducted and recorded post-observation interviews (Appendix B) after each observation whenever possible with three goals in mind. First, the interviews provided insights into the planning process each teacher used prior to the lesson. I asked participants about the planning of tasks and problems for the lesson, anticipated challenges and possible student misconceptions that may arise during instruction, and how they planned to address these challenges as a part of the lesson. The post-observation interviews also delved into the impact the lesson had on

Table 3-3. Post Observation and Intensive Interviews

Participant	Topics Taught During Observations	Number of Observations	Number of Post-Observation Interviews	Intensive Interview with Video-Stimulated Recall
Timothy	Adding/Subtracting Polynomials, Adding/Subtracting Polynomials with Fractional Coefficients, Graphing Quadratic Equations	7	4	Yes
Mary	Multi-Step Equations, Equations with Variables on Both Sides	5	4	Yes
Patrick	Integers/Absolute Value, Comparing and Ordering Integers, Adding Integers	5	4	Yes

the participant's thought processes for planning and instructional actions for subsequent lesson plans based on student responses and understanding of the current lesson. The interviews helped me to understand the participants' perspective on students and their ZPD, how the lesson proceeded in relation to the planning process, and what next steps would be based on that day's interactions. Lastly, the post observation interviews provided a time for the participants discuss any significant interactions or learning that occurred during the observation. Participants shared their perspective on the students' understanding of the lesson in general and any particular significant individual interactions. While conducting these interviews, I took notes on important points about the class interaction as a whole and any individual or small group interactions mentioned by the participant. At times, participants would share some of these thoughts informally while students worked on tasks when post-observation interviews were not possible due to scheduling conflict. When this occurred, I took detailed notes to record as much information as possible.

Intensive Interviews with Video-Stimulated Recall

I asked each participant to set aside one block of 30 minutes for an intensive interview. This type of post-observation interview used "stimulated recall" supported by the use of a videotaped teaching segment of the participant as a frame of reference to initiate a deeper discussion surrounding a teaching episode (Lyle, 2003; Wilcox & Trudel, 1998). I selected the video segment at the end of the study in order to have the greatest opportunity to select a highly interactive segment of mathematical discourse. The segments I selected were two to three minutes in length and allowed the participant the opportunity to share thought processes and internal decision making used to extend the mathematical discourse. I played the segment and asked questions based on the interaction (Appendix C). I used the video segment to stimulate the participant's recall of the teacher-student interaction to probe for decisions and thoughts influencing the direction of the interaction, reasons for pursuing the interaction, and how responses impacted instructional actions. I designed questions to prompt the participant to recall their own internal thought processes during the specific videotaped instructional interaction to gain insights into the mathematical discourse and teaching strategies at that moment in the lesson. By incorporating video segments of instruction paired with specific questions in the intensive interview, the participant could address why certain mathematical discourse was beneficial (or not) during the interaction and how their view of the student's knowledge and ZPD changed (or not) as a result of the lesson interaction.

Intellectual Environment

One question in the study analyzed the type of tools (i.e. routines, establishment of classroom climate/community, tasks) each participant used to establish an intellectual environment that supported mathematical discourse. I analyzed classroom observations for smooth transitions within the lesson, speaker order, whole and large group interaction rules, and positive responses from the teacher and/or peers. I used lesson plans and classroom observations to examine the types of tasks each participant used in the lesson to cultivate a safe learning environment that valued all perspectives for consideration by the group.

Artifacts

As a part of the study, I collected two types of artifacts whenever possible. First, some participants supplied copies of their lesson plans. Lesson plans provided anecdotal information and insights into the depth of teacher planning prior to the lesson in written form and thought given to mathematical misconceptions that may arise during whole group instruction. Student work was shared by the participants and served as a discussion point during post observation and intensive interviews. Student work provided me the opportunity to evaluate application of mathematical concepts taught during observed lessons and initial student understanding of the mathematical concepts covered in class compared to participants' perceptions of student understanding. This was of particular value when discussing student ZPD and scaffolding that occurred during the lesson.

Data Analysis Procedure

Using qualitative analysis methods, I reviewed the videotaped observations, two types of interviews (post and intensive), and various artifacts obtained during the data collection phase to address each research question in the study. The first question, How do alternatively certified teachers elicit student engagement in and contributions to mathematical discourse? was answered using the videotaped observations and an analysis of the instructional tasks incorporated into each lesson. I used classroom observations and field notes to answer the second question, What tools (i.e. routines, establishment of classroom climate/community, tasks) do alternatively certified teachers use to establish a foundation for mathematical discourse? The final research question in this study, How do alternatively certified teachers use mathematical discourse to monitor students' zone of proximal development? was answered through videotape analysis of classroom observations, post-observation and intensive interviews using video-stimulated recall, and various artifacts (student work, lesson plans, field notes). Data analysis was an ongoing process that occurred during and after the collection phase. I used the ACT framework (Fraivillig, et al., 1999) to analyze videotaped observations. I transcribed recorded interviews and I prepared interview notes with increased detail when interviews could not be recorded in an effort to capture and retain as much information as possible for analysis. I completed all of these analyses in multiple steps and describe in more detail in the following sections.

Pre-Study Interview Analysis

Of all data collected, this was most concise. The pre-study interview served to confirm each participant met the minimum criteria for the study. The interview also provided the specific degree of each participant and any related professional development that may influence the study findings. I took the opportunity to delve into getting to know each participant's perspective on the alternative certification pathway they took and how it prepared them (or not) for the classroom in general and mathematics specifically. This interview lasted approximately 10-15 minutes, depending on the responses of the participant. The information gathered was straightforward and not recorded or transcribed in any way. I took detailed notes with each participant in this data collection phase of the study.

Post Observation Interview Analysis

The post observation interview took place after the observation when possible. The scheduled observation time for one participant was the last class period of the school day, thus making post observation interviews easier to conduct than with the other two participants in the study. When possible, I recorded the interviews and later transcribed them as the data collection process took place. I incorporated specific questions into the post observation protocol (Appendix B) to elicit from each participant the planning and preparation in advance of the lesson, what mathematical misconceptions students may have coming into the lesson (if considered), how the participant planned in advance to address these misconceptions, and what tasks were planned to accomplish the learning outcomes of the lesson.

Post observation interview analysis was a multi-phase process. For those interviews that I did not record, a detailed account of my notes was prepared on the day an interview took place to capture as much information from the interaction as possible. I transcribed recorded post observation interviews with participant responses marked during transcription if comments made were valuable to the research study. Once I transcribed all interviews or prepared detail notes, I reviewed for common statements indicating participant consideration of the following elements as they planned for the lesson: student performance the previous day related to the lesson or skills taught, types of tasks used during and assigned after the instructional phase of the lesson, student misconceptions anticipated by the participant, how misconceptions were addressed in the planning phase of the lesson, and if participants used daily mathematical discourse to gauge student progress within their zone of proximal development.

Intensive Interview with Video-Stimulated Recall Analysis

Each participant engaged in one intensive interview with video-stimulated recall during the study. I reviewed videotaped lessons in advance of this interview for a key interaction the participant shared with a student or students during a particular lesson. I viewed the videotaped episode with the participant and asked specific protocol questions (Appendix C) that related to the identified episode. During an observation I can see and hear the interactions between the participant and the student, but I can only guess or infer participant thought processes to lead mathematical discourse in a particular direction with a student. The intensive interview with video-stimulated recall provided the opportunity to discuss what decisions the participant made inherently to propel the student's learning forward or to address a misconception held by the class regarding the concept being taught.

I recorded, transcribed and analyzed each of the three interviews based on the protocol questions as well as questions related to the specific instructional episode discussed using descriptive coding methods. The intensive interview allowed me to have a rich conversation with each participant regarding the thought processes of the moment, why certain questions were asked, and whether or not student ZPD had changed as a result of the mathematical discourse interaction.

Classroom Observation Analysis

During the data collection phase, 17 videotaped classroom observations occurred. I analyzed the observations multiple times on two separate levels, each examining for specific data and confirmed with two reviews I conducted in all areas of data analysis process for intracoder reliability. I conducted the first analysis to identify and code mathematical discourse into two broad categories of scaffolded or non-scaffolded (Meyer & Turner, 2002). When engaging a student, the teacher has the option of scaffolded or non-scaffolded interactions, according to a qualitative study conducted by Meyer and Turner (2002). Scaffolded interactions are identified as instructional scenarios where mathematical discourse was promoted by the teacher to engage a student or class in understanding a specific mathematical concept, problem, or step in a procedure using questions or hints. For example, if the participant asked, “Why did you place your point in quadrant three? followed by “Why don’t you go back and look at your x and y in the question?” This would be coded as a scaffolded question series because the participant is attempting to guide the student through a problem to obtain a correct response or encourage the student to re-evaluate their answer by examining the problem. I identified non-scaffolded mathematical discourse as discourse lacking support or relation to the topic or a question related to moving through the task. For example, if a participant asked, “What is the answer to number 12?” this would be coded as non-scaffolded. Another example of an instance that would be coded as non-scaffolded would be if the participant did not pry for an answer from a student and moved onto another for a response, such as: “Why did you place your point in quadrant three, Sue?” (No response) “John, can you tell us why you placed your point in quadrant three for number 15?”

After I designated interactions as scaffolded or non-scaffolded, I conducted a second descriptive analysis using a priori coding and a second coder for intercoder reliability (Saldana, 2009) to provide a deeper understanding of the types of mathematical discourse that took place in each lesson Using the ACT framework’s three instructional components, *eliciting*, *supporting*, and *extending,* from the Fraivillig, et al., (1999) study to establish the codes used to analyze scaffolded interactions, as detailed in Table 2-1. Each of these components has designated criteria related to the mathematical discourse established in the ACT framework, which established the coding scheme for the analysis process.

Eliciting is defined as the opportunity and encouragement a teacher provides a student to express his or her ideas about mathematics (Fraivillig, et al., 1999). This is the first step to get students talking about their mathematical thinking in order to develop a whole group or individual learning episode. For example, if the participant pauses and allows additional time for the student to respond to the question, this is considered a form of eliciting in the ACT framework. Another example is the question, “Can you explain to me the steps you took to arrive at your answer?” Eliciting initiates engagement of a student with the teacher to begin mathematical discourse.

The ACT framework considers the assistance of students in their ZPD for the subgroup called supporting. Once a teacher has elicited a response, the teacher makes pedagogical decisions on how to assist the student in answering the question. These instructional techniques often include the use of scaffolding as a means to support the student in solving the problem. Also seen as a part of the supporting subcategory of the ACT framework are instructional actions such as restating the steps correctly achieved thus far in the problem, referencing a similar problem answered earlier in the lesson, and asking another student to explain the solution. Examples of questions being coded as supporting are: “I understand how you got y + 4= 2. You have solved the problem correctly to this point. Can you tell me what would be the next step to solve this equation?” or “Do you remember how Ethan did his problem on the board?” Use the same step here to complete this equation.”

The third and final category in the ACT framework is extending. Participant interactions viewed as extending the mathematical discourse are visible and invisible; visible in that the participant follows up with more in-depth questions and invisible with regard to the intrinsic decisions made on what should be asked, when, and to whom. The invisible aspect of extending mathematical thinking is specific in nature because the participant must be aware of each student’s ZPD in order to ask questions that will challenge thinking based on the application of familiar mathematical methods. The visible aspect is the instructional technique the participant chooses to make based on their understanding of the student’s ZPD. One example of an extended response may look like this if the participant questions the student’s understanding of isolating and solving for one variable:

> **Teacher:** Riley, thank you for raising your hand. Can you explain to the class how you arrived at -2 for your answer? (y + 4= 2)
>
> **Riley:** I subtracted 4 from both sides of the equation and that left negative 2.
>
> **Teacher:** Why did you subtract 4 from both sides of the equation?

Riley: I know that what you do to one side of the equation you have to do to the other, so I knew it was minus 4.

Teacher: Why did you choose 4? Why not subtract 2?

Riley: I had to get the y by itself.

Teacher: And why is that?

Riley: Because that is the letter in the problem we are finding.

Teacher: Excellent, Riley! You are correct in how you have explained the problem. Because we are solving for y in this equation, we have to subtract 4 from both sides and leave the y by itself.

A second example of an extended response may look like this:

Teacher: Ethan, share with the class how you solved the problem on the board. (x + 6 + 2x = 15)

Ethan: First, I added the x's together. Then, I minused the 6 on both sides. Then, I divided by 3 and got 5.

Teacher: I like how you solved that problem, Ethan! You did each step exactly right. (Repeats his steps.) Did anyone else do it differently? Yes, Jan?

Jan: I minused 6 on both sides first and then added my x's. I still got 5 when I divided.

Teacher: And that is perfectly fine. You did the same steps as Ethan, but in a different order for steps one and two. Excellent work!

Analyzing classroom observations using the ACT framework will provide insights into how often alternatively certified mathematics teachers employ instructional strategies to promote mathematical discourse in their classrooms in these three varying levels of interaction.

For interactions falling into the non-scaffolded category, I used the subgroups teacher controlled responses and non-supportive motivational responses. Teacher controlled responses are considered to be primarily negative in nature. Examples include asking questions to which the answer is known, emphasizing participant knowledge over the collective group, and offering confirmation of right and wrong answers with no focus on the conceptual knowledge behind the response (Meyer & Turner, 2002). An example of this is, “No, Rob, the answer is not -12xy. Who can tell me the correct answer?”

I identified non-supportive motivational responses as discourse negative in nature and provided no support of students’ mathematical learning. I coded all sarcasm regarding student abilities, highlighting mathematical tasks as difficult, confusing, or boring, and positive superficial statements with no link to the context of the lesson in the non-supportive motivational subgroup. An example of a non-supportive response would be: “Nice try, Liz, but why don’t you think a little longer before responding to the question?” These two subgroups helped to identify mathematical discourse used to establish a safe intellectual environment by the participant.

Intellectual Environment Analysis

I also used classroom observations and recorded interviews to identify established classroom routines and intellectual environment. I categorized these interactions into supportive and non-supportive subgroups using the coding of field notes. Supportive learning environments consisted of smooth transitions in the classroom between activities and within the lesson, established classroom routines with student understanding of whole and small group interactions, and a safe learning environment where thoughts could be shared openly by all. I considered a learning environment as non-supportive if the teacher made disconnected transitions, openly criticized students’ thoughts, or no rules for discourse engagement were in place. I used anecdotal information in field notes and post lesson interviews to elaborate on findings.

Artifact Analysis

I collected and examined participant lesson plans and student work when possible. I reviewed lesson plans for evidence of planning tasks and questions to address misconceptions students may have with the lesson, scaffolding students' knowledge through the lesson and practice activities, and overall preparation of well-developed lessons that incorporate mathematical discourse. I examined student work for student understanding of the mathematical concept taught and level of detail in the work provided. I used anecdotal information from post observation interviews to support findings in lesson plans and student work.

CHAPTER 4
RESULTS

Extending Mathematical Discourse

I designed this study to investigate alternatively certified teachers' use of mathematical discourse by examining three areas: how teachers elicit student engagement in and contributions to mathematical discourse, what routines and intellectual climate are established to support mathematical discourse, and the use of scaffolding to facilitate mathematical learning. The study results presented in this chapter discuss the types of questions participants used to elicit, support, and extend mathematical discourse during instruction and the classroom routines and intellectual climate supporting these interactions. The chapter also provides insights into each participant's thought processes in preparation for a lesson, during the instruction, and following the lesson.

Engaging Students in Mathematical Discourse

In the analysis of the observational and interview data from all three classrooms concerning each participant's instruction, I identified mathematical discourse episodes aligned with all three categories of the ACT framework. As identified in the ACT framework, I established episodes of eliciting student responses, supporting student thinking, and extending mathematical thinking using discourse. I also identified observational data concerning the development of a supportive learning environment. Participants used a variety of instructional strategies to establish an environment that promoted mathematical discourse and an acceptance of others. Mathematical discourse episodes paired with established classroom routines and intellectual climate describe successful mathematics instruction observed in the study.

Level of Scaffolded Interactions

For this study, the first analysis of videotaped observations identified scaffolded and non-scaffolded episodes (Meyer & Turner, 2000). Scaffolded instructional episodes were mathematical discourse interactions promoted by the participant to engage a student or the class in understanding a specific mathematical concept or a problem or step in a procedure related to the concept. Non-scaffolded episodes were interactions that lacked student support by the teacher or relation to the topic or question currently under discussion. I analyzed and coded each participant's lessons twice (intracoder reliability) to determine the frequency in which scaffolded and non-scaffolded interactions occurred (Table 4-1).

Table 4-1. Calculations of Scaffolded and Non-Scaffolded Mathematical Discourse

Participant	Scaffolded Interactions (All Observations)	%	Non-Scaffolded Interactions (All Observations)	%	Total Questions
Mary	78	85.71	13	14.29	91
Patrick	68	91.89	6	8.10	74
Timothy	239	93.73	16	6.27	255

Identifying the frequency and level of scaffolded and non-scaffolded interactions observed during the study provided the foundation needed to establish the amount and type of mathematical discourse taking place in the classroom. Participants' mathematical discourse was almost exclusively in the scaffolding category, indicating a high volume of mathematical questions and teacher-student interaction, as well as a positive learning environment. I further analyzed the coded interactions of mathematical discourse using the ACT framework and discussed in detail by each of the three framework components, including the use of scaffolding within the lesson. The ACT framework also provided data regarding the establishment of a safe intellectual environment. This is supported with anecdotal information I noted and by post-observation and intensive interviews. Results are included as appropriate.

Types of Mathematical Discourse Interactions

This study used the three components of the ACT framework (eliciting, supporting, and extending) to provide a structure in which to organize the use of mathematical discourse by participants. From the identified episodes considered to be scaffolded interactions, I again reviewed and analyzed teacher-student mathematical discourse to determine the related ACT framework component. I reviewed observations with 100% agreement between the two codings. A second individual analyzed and coded the observation transcripts. We discussed identified differences in coding and reached consensus. The analysis used strengthens the study through intracoder and intercoder reliability. The following describes each component of the ACT framework and examples of how participants established a supportive classroom environment and engaged students in mathematical discourse.

Table 4-2. Frequency of Engagement According to ACT Framework

Participant	Eliciting	Supporting	Extending
Mary	44 (45.36)	45 (46.39)	8 (8.25)
Patrick	39 (59.09)	14 (21.21)	13 (19.70)
Timothy	76 (35.35)	129 (60.00)	10 (4.65)

*Percentage of category as compared to overall questions indicated in parentheses.

Overview of Participants

Mary. In the study Mary had a balanced approach in the eliciting and supporting portions of the ACT framework. Extending has observed the least in the five of her classroom observations. Of the three participants, Mary played more of a facilitator role and would refrain from sharing her approach to solving a problem. Mary waited until students were at an impasse in the discussion or possible solutions had been shared and her method would contribute to the group's discussion.

Patrick. The observations of Patrick revealed a very traditional teaching style was used to teach mathematics. Students took notes and copied sample problems, which were explained by him as the lesson was taught. This is reflected in the overall numbers reflected in the framework. Patrick had the least overall interactions of the three participants. He also had very few interactions in the supporting and extending categories of the ACT framework. Though he had 13 noted in the extending category, the highest of all three participants, this fell almost exclusively in one category-encouraging students to complete all problems.

Timothy. Timothy had the highest percentage of mathematical discourse of the three participants. He was high energy and very engaging with his students using humor and stories to keep students attentive and participatory. Timothy used both facilitator and instructor approaches in his lessons. Timothy had the highest portion of interactions in the supporting category and the least in the extending area of the framework. He had no trouble getting his middle school students to share their thoughts on mathematics.

Eliciting Student Solutions

The first component, eliciting solution methods (eliciting), focused on the engagement of students by providing an intellectual environment encouraging the expression of mathematical ideas and the opportunity to share those thoughts with peers. Understanding where students are mathematically as individuals and collectively as a group is important to teachers when they develop lessons and attempt to create mathematical discourse opportunities. (Vygotsky, 1978; Fraivillig, et al., 1999; Ball, 1991; Woodward & Irwin, 2005). Comprising this category is mathematical discourse that facilitates students responding to questions and sharing solution methods as well as the participants' efforts to orchestrate mathematical discourse opportunities. When teachers engage students in mathematical discourse they create opportunities for individual learning and expanding the knowledge of the entire class. (Yackel & Cobb, 1996).

In the analysis of the three participants in the study I observed many examples of eliciting student responses in the subcategories of Facilitates Students' Responding and Orchestrates Classroom Discussions. I describe each of these contexts and provide an example for clarity of this component of the ACT framework following the Fraivillig, et al., (1999) study model to help the reader understand the various categories and subcategories examined in this study.

Facilitates Students' Responding

Waits for and listens to students' solution methods. All three participants provided varying amounts of wait time for students to respond to a posed question or task. Two participants elicited excitement in students to respond to questions by talking fast and sometimes following one question quickly with another if an immediate response was not provided. For example, in the first observation of Mary it was noted Mary would ask a student a question, and if the student did not begin a response within a second or two Mary would answer the question for the student. The wait time improved in subsequent observations, but still was limited in length for this participant. The following exchange took place during an observation of Mary as a student, where Mary demonstrated more wait time for a response than previously observed. The student teaching this part of the lesson is at the board in the role of teacher:

T: OK This is??? (Pauses and students begin to respond with answers.)

S7: It's 2d + 9.

S8: It's a negative 2d.

S4: No it's not. It's 2d + 9.

S3: It's 2d + 7.

S8: It's a negative.

T: Is that right? (Refers to answer he was given, which was incorrect).

S8: I think it's a negative 2d. (sticking to original answer).

S4: What are you doing? (Said to teacher when he changes it to another incorrect response that was given.)

T: I don't know. What's the right answer?

S5: 2d + 9

T: (Stepping in now). Well, wait a second.

Encourages elaboration. Two participants encouraged students to describe their solution method in more detail, often prompting students to use correct mathematical terminology in the process. At times participants asked questions that provided insight into the student's level of understanding regarding the problem highlighting their advanced understanding of the lesson. In the following mathematical discourse interaction the student recalled information from the previous lesson and comprehended where the participant was going with the lesson before instruction began.

S1: It's a binomial.

T: Bring it! (Excited)

S1: r^3

T: Hmmmm. Wait a minute. What does r to the 3rd power mean?

S1: r x r x r

T: Everyone understands what she just said? She's ahead of me.

Conveys accepting attitude of wrong answers and problem solving. Teacher attitudes greatly impact mathematical discourse (Fraivillig, et al., 1999). Participants in the study provided a supportive and safe learning environment where wrong answers were viewed as an opportunity for the entire class to learn and not an individual's lack of knowledge about the concept. Participants called on students to share responses who they knew had an incorrect solution. The students were sometimes aware and other times not aware as to where the problem had gone wrong. Often the participant would scaffold a student through this situation, asking questions to guide thinking to support identification of where the error occurred so the student could make the correction to the problem. The following episode of mathematical discourse depicts the acceptance of wrong answers and supporting the intellectual environment to advance student learning.

T: Alright #12 (Student). Point L. Walk them through it.

S6: Start at the origin. Go to -4. And then down 5.

T: Does everyone agree with that? Raise your hand if you agree with that. Raise your hand if you disagree with that. (Teacher chooses a student who raised their hand to respond why he disagrees.) You disagree with that? Why?

S7: Because it should be, it should be uhhhhh. It should be….(Goes quiet)

S8: Ohhh I know.

T: Pay attention to the coordinates (pointing to the problem on the board.)

Yes, sir. (Calling on another student.)

S9: It should be in the fourth thing (meaning quadrant).

> **S7/8:** It should be in the second (quadrant). Student walks away from the board without plotting final correct coordinate. Student appeared fine that she had not gotten the answer correct in its entirety.
>
> **T:** You were completely right about going back -4 horizontally. (Teacher reinforced the correct portion of the student's answer and went onto explain the incorrect part.)

Orchestrates Mathematical Discourse

Decides which students need to speak publicly or which methods should be discussed. All three participants in the study revealed in their post observation and intensive observation interviews they selected key times for specific students to respond to questions or share solution methods to maximize mathematical discourse. Participants, through circulating the classroom, knew where each student was mathematically (ZPD) with the concept being taught and strategically called on certain students to respond in order to create opportunities for mathematical discourse during whole group instruction. Participants also revealed that they asked questions at strategic points in the lesson to check student understanding. Identified students in these orchestrated mathematical discourse opportunities often did not know the correct answer or demonstrated limited understanding to the participant, thus allowing the participant during small group work time to check individual progress during whole group instruction. I noted that each participant would often ignore students' raised hands and call on a student, whose hand was not raised, using the partially correct or incorrect response to create a mathematical discourse interaction that would benefit the individual student and entire class.

Monitors student engagement. Participants monitored student engagement during the lesson using multiple strategies. Students understood whole group participation was an expectation when the teacher or a peer was speaking, and not only when the student was speaking or engaged in a problem. Participants called on students identified as possibly drifting from the lesson to redirect their attention to the mathematical discourse taking place. Timothy appeared to excel in this area with his middle school students. Timothy seemed to know exactly how engaged every student was in the lesson and who needed to have their engagement level increased or attention redirected to the lesson, calling upon them to contribute to the discussion by calling a student by name and asking, “Did you agree with that answer?” or stating “Eyes up here” to reengage wandering attention.

Results of Eliciting Mathematical Discourse

Eliciting student engagement is the first step in establishing an opportunity for mathematical discourse (Fraivillig, et al., 1999). When teachers take the time to engage students in discourse and examine their mathematical thinking the opportunities for learning are expanded for the individual and the whole group. Teachers must be prepared to ask questions, understanding where their students are mathematically as individuals (ZPD) and collectively as a class. Eliciting mathematical discourse allows the teacher to engage students in classroom interactions and orchestrate opportunities to advance mathematical thinking.

Participants in this study had the second greatest portion of questions in the eliciting student engagement component of the ACT framework (Table 4-2). There was continuity between two of the participants in the number of eliciting questions posed during the observed lessons. Mary and Patrick had similar numbers overall in terms of the total overall interactions, with 44 and 39 respectively. The third participant, Timothy, had a higher frequency of eliciting interactions with 76 coded. Timothy, however, was observed two more times than the other two participants, which likely accounts for this difference. I found eliciting questions to be used by participants the most through conveying an accepting attitude of wrong answers and solution methods. Encouraging elaboration of student responses was the area of eliciting that was practiced the least by all three participants. A complete overview of the results follows.

Table 4-3. Frequency of Eliciting Mathematical Discourse

	Frequency of Eliciting Mathematical Discourse		
	Facilitates Students' Responding *M/P/T	Orchestrates Classroom Discussions *M/P/T	Total *M/P/T
Waits for and listens to students' solution methods. (WS)	9/6/16		31
Encourages elaboration. (EE)	13/3/9		25
Conveys accepting attitude of wrong answers and problem solving. (AA)	8/12/17		37
Decides which students speak publicly or which methods are discussed. (DW)		8/11/17	36
Monitors student engagement. (ME)		6/7/17	30
TOTAL RESPONSES			159

*M/P/T = Mary/Patrick/Timothy

Comparison of Use of Eliciting Mathematical Discourse between Participants

Teachers play a pivotal role in the acquisition of knowledge and actual learning that takes place. As discussed in Chapter 2, Zack and Graves (2001) indicated teachers must become skilled in leading conversations regarding mathematics, and when teachers extend mathematical discourse student learning is increased (Nathan & Knuth, 2003; Hatano & Inagaki, 1991). During post-observation interviews and identified in the coded questions eliciting mathematical discourse, each participant clearly articulated their awareness of students who had correct or incorrect answers. Further, participants indicated that they planned who would speak and when based on this knowledge, gained when circulating the classroom during small group work time. Strategizing who spoke and when demonstrated each participant's value of knowledge and experience through individual discourse to influence the mathematical discourse of the entire learning community, thus advancing the mathematical thinking of the community discourse as indicated by Gee, Sfard and Cazden in Chapter 2. I found an example in the analysis of Patrick's intensive interview with simulated recall. Patrick noted mentally that because several students had come to the board and correctly plotted ordered pairs on the coordinate plane, he specifically decided to call on student he knew had the incorrect answer on a specific problem to plot her point. The following is a portion of Patrick's discussion regarding this episode of eliciting student engagement between me (the researcher) and the teacher:

> **Researcher:** And so what were you thinking as she was working that problem on the board?
>
> **Patrick:** I actually knew. I had looked at her paper, and I knew that she had the wrong answer. I called her up there specifically because I knew she had the wrong answer and this is a hard concept. (Plotting ordered pairs with four quadrants and negative numbers.) There is a few types of problems that are exceptions, they are just weird and you need to see them so you can say, "Oh, that is how you do it." This one involved zero actually being on the axis. That is why it was a weird one and people need to see it. Everyone else who was coming up to the board was getting their problems right so I was like, OK, it's time to get one wrong so I can prove a point, especially on these weird ones. So I actually called on her because I knew she had the wrong answer. I knew I would be <u>safe</u> doing that with (Student) because I knew she would think it was funny and that I am not picking on her.
>
> **Researcher:** She could handle it?
>
> **Patrick:** Exactly. So I choose (Student) specifically because I knew she had the wrong answer so that gave me the ability to say, "Hey, who agrees with this and who disagrees with this?" You know what I am saying? That way they (students) saw it happen wrong and then they (students) got to decide if it was wrong or not. Then we got to discuss the right way to do it.

Interwoven in deciding who will speak and when is the importance of conveying an accepting attitude of wrong answers and solutions methods provided by students. Participants were strongest in this component of the eliciting category. I observed repeatedly various indications of conveying an accepting attitude and ultimately a safe intellectual environment. Students asked to come to the board to work problems, requested to attempt a solution to a problem prior to instruction, and a willingness to go back to the board to work another problem if a student got theirs incorrect in the first attempt. The participants conveyed their accepting attitude in a variety of ways as well, including the use of humor, allowing students to converse among themselves and bring forward new solution ideas, and scaffolding students when challenged to complete a problem. Below is an example of the use of humor with a student who used incorrect terminology during a mathematical discourse episode.

T: So now we already see what we pick in the middle, right?

S5: Yes.

T: Which two are they?

S6: -2 and -14

T: There you go! Now. What do I do next? (Class quiet) (Teacher waits)

S5: You rewrite the thingy.

T: Embracing response with humor-I do! I rewrite the "thingy." It is exactly the way as it was before. Any suggestions?

S5: Yeah. But you have to figure out which one goes on which side, don't you?

T: You do. So what am I looking for when I am trying to figure out which number goes on which side? (Quiet class) (Teacher waits)

The areas of the eliciting component observed the least were monitoring student engagement and encouraging elaboration. All three participants had established safe intellectual environments (Bennett, 2010; Leinhardt & Steele, 2005; Fraivillig, et al., 1999) that were supported by their acceptance of wrong answers, which was the most frequently observed interaction in the eliciting component. The positive interactions I witnessed along with the attitudes and statements made by each participant conveyed this message clearly to the students. Further, participants knew when students had incorrect answers and strategically called on these individuals at key times to create learning opportunities for the entire class. The expectation of elaboration was the subcategory I identified the least in the eliciting component. There were many opportunities I noted where students could have been asked to elaborate on a solution method or answer that were not seized by the participants. This is likely influenced by multiple factors, including years of experience, alternative certification, and overall planning of the lesson. While as teachers we can prepare for many scenarios, one can never anticipate all directions a lesson will go when instruction occurs.

The final area of eliciting students' thinking within the ACT framework identified in the study is monitoring student engagement. Participants in the study did well with monitoring the engagement of the class as a whole, redirecting students who had drifted from the lesson. Each participant dealt with a student who was not engaged in the lesson as soon as it was noted. Timothy did this by asking the student a question regarding the lesson, but was careful to ask something the student could answer to avoid detracting from the discussion that was taking place. Patrick and Mary asked students to move to another sear or refocused attending on the board and continued on with the discussion. Two participants had one student who had to have follow-up more than one time. Most students acknowledged they were off task and rejoined the whole group discussion as listening participants.

In the eliciting subcategory, participants in general did not elicit multiple solution methods from students before advancing mathematical discourse. When a student solution and/or method was shared, the participant tended to use this as the basis for supporting and extending the discourse if any additional dialogue took place. Additional solutions were sought if a student gave a wrong answer, but students were not asked how they solved the problem if another method was used. Adding this type of questioning to the mathematical discourse would have provided greater opportunities to share mathematical knowledge and to extend mathematical discourse and student thinking.

Eliciting student engagement creates opportunities for extending mathematical discourse. Through eliciting students’ thoughts, participants were able to determine individual and whole group understanding of mathematical concepts. During this process teachers made deliberate instructional decisions to support the extension of the discourse. The next section will examine how participants used these opportunities to support and extend students’ understanding of mathematics.

Supporting Engaged Students

The second component of the ACT framework, supporting conceptual understanding (supporting), examined mathematical discourse when a student responded to a question or provided a solution method. Research shows teacher decisions on how to extended mathematical discourse is critical to maintaining engagement and advancing learning (Woodward & Irwin, 2005; Fraivillig, et al.,1999). If a teacher chooses to support student thinking about a particular problem, the opportunity to extend discourse is increased. There are two groups identified by the ACT framework who are a part of the discourse. One is the individual doing the actual speaking, who is called the “describer.” This is the student engaged with the teacher in mathematical discourse at the moment. This can transition at any time to another student who enters the conversation. The second is the remainder of the class or “listeners.” The remainder of the class should be “actively engaged” in the mathematical discourse even though not current contributing to the discussion of the concept or task.

The category of supporting conceptual understanding examined the types of mathematical discourse participants used once a student was engaged with a question or problem. I observed evidence of supporting students’ conceptual understanding in the lessons of all three participants. Examples in the context of supporting that I identified in the data are provided for clarity of the second component of the ACT framework following the Fraivillig, et al., (1999) study model to help the reader understand the various categories and subcategories examined in this study.

Supports Describer’s Thinking

Assists individual students in clarifying solution methods. I observed two of the three participants in the study assisting individual students in clarifying their own proposed solution methods. By clarifying solution methods, participants modeled appropriate responses for students to use when responding to questions and contributing to the discourse. This type of interaction helps to increase the vocabulary of the students and the degree of sophistication in responses to the teacher and peers. Participants accomplished this by restating the student’s description with correct sentence structure and vocabulary or by asking another student to explain how the problem was solved. Key to implementing this second mathematical discourse strategy was identifying which students understood the concept and had the ability to articulate this to the class. The following is a mathematical discourse episode demonstrating this component of supporting.

T: What is the equation of the graphs below?

S1: (Raised his hand and teacher pointed to him to respond). I got 5, 8. I got 5 as the x and 8 as the y.

T: Now how would I say that? You're right, but how do I say that?

S1: I don't remember how to say it.

T: Watch. Let's go to this line (A graph on the board). When y is 0, what is X?

Sg: 5

T: When y is 5, what is X?

S1: Confusion!

T: OK When y is -5, what is X?

Sg: 5

T: When y is 1, what is X?

S1: 5

T: So what is X?

S2: Wait! It is on the 5 line so X is 5! Oh my gosh! That was the easiest thing ever!

T: (Excited face and body language.) Yes! Do you remember when I said what is this? (Holding arms out horizontally).

S2: But it confused me when you said what is X.

T: Ok What is X?

S1: X=5

T: X=5.

(The teacher practices more with the class.)

T: I'm being picky but I want you to get used to saying that.

Reminds students of similar problems already successfully solved. Two participants reminded students of similar problems already solved to support the continuation of mathematical discourse. Reminding students that one of their peers solved a similar problem or a reference to the previous day's lesson provided an alternate frame of reference for mathematical reflection to potentially obtain a correct response without having to provide more specific background information. Through this strategy, participants assisted a student or the class when struggling with a problem in an attempt to advance mathematical thinking towards a successful response. The following is an example of Patrick referencing a previous lesson where the same skill was applied.

> **T:** Multiply. Remember what we talked about the other day? We could do addition-275 + 275+275+275+275. But what is a quick way of doing addition?

Provides background knowledge. Reviewing vocabulary, mathematical rules, or other conceptual information helped continue mathematical discourse for students struggling during a response. All three participants in the study at some point in classroom observations attempted to extend mathematical discourse by sharing background information important to the problem, such as a mathematical rule, in an effort to support mathematical discourse. Mary had the highest frequency of use in this particular component of supporting. The following is an example from one of the observations in her classroom.

> **T:** I have a question, then. The directions said to put the like terms on one side of the equation and simplify. It didn't say solve. So, should we stop at the -28 = 10x or should we solve for x? (Silence for several seconds)

> **S1:** I would say solve.

T: Simplify means to bring things together. Solve means get x alone.
(Waits a couple of seconds) So we should?

S1: Simplify.

Supports Listeners' Thinking

Demonstrates teacher-selected solution methods. One participant provided an example of how they would solve the problem *after* instruction of the mathematical concept. Mary modeled her solution method, but did not work the problem until after the class had an opportunity to devise their own solutions and whole group discussions had taken place. Facilitating the solutions of the students as a "guide on the side" (Nathan & Knuth, 2003) indicates a willingness by the participant to allow students to lead the mathematical discourse and guide mathematical thinking and the overall learning.

Supports Describer's and Listeners' Thinking

Records student responses of solution method on board. All three participants used visual references to support mathematical discourse so students could follow discussions more closely as they process responses and begin to assimilate the mathematical concept. Recording student responses reaffirms all students' thoughts are of value to the group and should be considered, thus demonstrating a safe intellectual environment when thoughts are shared openly without hesitation during a lesson.

Supports Individuals in Private Help Sessions

Encourages students to request assistance. Participants in the study used varying methods to answer students' questions following a lesson. Students understood assistance was available and used it openly. I observed this interaction more than recorded in transcription. The study results only indicated what was presented in the transcribed portion of the observations, which was not a focus of the transcription and more a behavior I observed. All three participants encouraged students to seek private assistance when needed if formal whole group instruction was not taking place.

Results of Supporting Mathematical Discourse

Once a teacher has engaged a student in mathematical discourse, the possibility the dialogue will continue increases Teachers must understand where each student is mathematically to guide discourse (Ball, 1993; Ball & Bass, 2000). How teachers treat students' elicited responses and support students within their ZPD is complex and multifaceted because of the need to advance both individual learning and the group as a whole. In order to navigate mathematical learning, teachers must be ready to make quick decisions regarding student responses to orchestrated mathematical discourse.

Table 4-4. Frequency of Supporting Mathematical Discourse

	Frequency of Supporting Mathematical Discourse				
	Supports Describer's Thinking *M/P/T	Supports Listener's Thinking *M/P/T	Supports Describers' and Listeners' Thinking *M/P/T	Supports Individuals in Private Help Sessions *M/P/T	Total *M/P/T
Assists individual students in clarifying solution methods or answers. (AS)	14/0/3				17
Reminds students of similar problems already successfully solved. (RP)	2/4/14				20
Provides background knowledge. (BK)	10/4/2				16
Demonstrates teacher-selected solution methods. (TM)		6/0/0			6
Records student responses of solution methods on board. (RR)			13/5/110		128
Encourages students to request private assistance. (PA)				0/1/0	1
TOTAL RESPONSES					188

*M/P/T = Mary/Patrick/Timothy

Participants in the study had the largest number (188) interactions identified in the second component of the ACT framework (Table 2-1), supporting students' conceptual understanding (supporting) of mathematics. One of the participants, Timothy, had a significantly higher percentage (60.00) of mathematical discourse in this category. Mary and Patrick had 46.39% and 21.21% of their mathematical discourse identified as supportive in nature. The significantly higher frequency found in Timothy's observations may be influenced by the two additional observations I conducted with this participant. A second factor may be tied to the type of instruction that took place during the observations. For example, Mary had her students teaching during two of the five observations I conducted, so students were at the board recording responses and not the participant. With regard to the instruction observed in Patrick's class, there was a very traditional type of note taking and direct instruction that limited student responses, thus reducing this component within the supporting category for this participant. I observed demonstrating teacher-selected solutions the least (6 times) in the supporting category, while I observed recording student responses the most (129 times). A complete overview of the results follows by subcategory.

Comparison of Use of Supporting Mathematical Discourse between Participants

While I observed all three participants providing supporting instruction, the study found the three participants varied in the amount of support provided during student engagement in mathematical discourse. As noted in the Fraivillig, et al., (1999) study, teachers focused the greatest amount of mathematical discourse interaction around the supporting category. For two of the three participants in this study the same was true. Mary and Timothy had the most supporting interactions, with 46.39 and 60.00

respectively. The third participant, Patrick, had the least amount of interaction in the supporting category of the three participants. This may suggest alternatively certified teachers may find the same types of questioning and instructional decision making easier to focus on as with traditionally certified teachers. Further, the level of traditional instructional approach used by the participants seemed to parallel the amount of supporting mathematical discourse observed in the classroom. One participant, Patrick, had significantly less supportive mathematical discourse (21.21%) than the other two participants. The same participant also used a traditional instructional style in his lessons, including note taking and copying examples from the board as they were modeled. Mary, on the other hand, used a more collaborative instructional style and tangentially had a higher number of supportive questioning used to extend mathematical discourse during whole group instruction. Because Timothy had such a high volume in recording student's responses, his ranking was first in this category, though it may be considered somewhat of an outlier. If this were closer to the 13 or 5 recorded from the other two participants, Timothy would be more in line with the other two participants. This, too, aligns with his moderately traditional teaching approach that modeled procedural learning with some imbedded conceptual learning. This subcategory of supporting students' thinking may need to be more clearly defined in the framework with regard to coding transcriptions. This would provide for clarity on its intended role in mathematical discourse and ensure appropriate inclusion in data and data analysis.

I observed supporting describers' and listeners' thinking frequently during the study. The three participants in the study supported the continuation of mathematical discourse by writing student responses regarding answers or steps to a problem on the board. Participants would sometimes continue with a problem even though they knew it was incorrect, to allow for the student to begin to question their response. Student acknowledgement of error did not always occur, and the participant would take the student back through the problem, scaffolding the student through the steps not yet mastered.

Recording students' responses on the board was used to support mathematical discourse in several ways. First, recorded answers provided the entire class the opportunity to reflect on the mathematical discussion and solution method being presented. Students were able to compare and contrast responses to formulate their own opinion and evaluate that of their peers. Timothy took recording student responses to a different level in his class to support and extend mathematical discourse, with 110 identified responses. During one observation Timothy recorded student responses to support the development of two formulas in a lesson on adding polynomials. Unique to Timothy was his ability to use written responses to engage and excite students about the mathematics by incorporating stories to establish a frame of reference for students as they learned the new concept. Students were well-versed in Timothy's application of real-world analogies to establish an understanding of a new mathematical concept. Immediately upon opening the lesson by telling the class he was coming home from work and needed to make a stop to buy some things the students were engaged. Timothy prompted students with questions regarding what he bought, recording their responses on the board and drawing pictures for increased visual impact and engagement. Students viewed this as fun and knew the participant was going to connect the food purchases with the mathematics lesson. Below is an example of one such episode. The interaction begins after the grocery bags have been drawn and the number of items has been determined.

T: How many apples go on the shelf?

Sg: (Shout number)

T: How many Hershey bars go on the shelf?

Sg: (Shout number)

T: How many Kit Kats go on the shelf?

Sg: (Shout number)

T: How many chickens go on the shelf?

Sg: (Shout number)

T: Let's write an equation for this. (Items in bags). Can I just say 5, 7, 8, 9 (Adding everything up together)?

Cl: No!

T: Why?

S1: No, they aren't the same.

S2: Can't create a chicken.

T: Right, These are not the same things. Can't create a chicken, can I? Maybe in the future a chocolate chicken apple.

Cl: Chuckles

T: Can't put them together because they are not the same.

S3: No

T: NO (Emphasis) Ok. We've done the boring stuff. (Laying groundwork for problem) Now let's do the fun stuff. I go to the store and bring home a bag of stuff. That is what math is-a bunch of "stuff" and we have to keep track of our "stuff."

$(5x^3 + 3x + 7) + (6x^2 + 5x + 5)$ evolved from the bags into a problem.

Finally, as noted in Chapter 2, maintaining a written record of student responses reminded students of answers that have already been provided, thus allowing the discussion to move forward or reference back to earlier thoughts as the group came to a consensus on the answer. Timothy recorded student answers and left previously worked problems on the board to provide a reference for students when he asked supporting questions or scaffolded students through a response to a problem. A possible added benefit of extending the mathematical discourse by maintaining a written record of responses may be increased participation throughout the discussion. Students can reference earlier responses to form new thoughts and possible solutions to the problem or discredit previous responses of their peers during the mathematical discourse episode.

The third most frequently observed use of supporting mathematical discourse was participants assisting students with clarifying their mathematical responses. Participants assisted students in articulating responses at different phases in the lesson and for different reasons. Mary would often support through clarification when the student appeared to be struggling. Mary had students review the solution procedure aloud articulate and determine if the problem was correct to that point, before asking key questions to support the mathematical discourse. Patrick, however, would assist the student after the problem was complete, whether correct or not, supporting students in the formation of a more concise response using correct mathematical terminology. Timothy helped students clarify responses by using questions to prompt students as the solution was being determined. In the following example students were working on tasks created by Timothy, who asked students to write equations for each of the graphs provided. Students as a whole were struggling with the concept, so the participant called on a student to share his solution.

> **T:** What is the equation of the graphs below? (Points to student to respond)
>
> **S1:** Alright. I got 5 and 8. I got 5 as the x and 8 is the y.
>
> **T:** Now how would I say that? You are right.
>
> **S1:** I don't remember how to say it.
>
> **T:** Watch. Let's go to this line. Right.
>
> **T:** Teacher turns to the class and begins asking students questions by pointing to different values of Y on the graph. When y is 0, then X is what?
>
> **S1:** 5
>
> **T:** When y is 5, then X is what?
>
> **Sg:** 5
>
> **S2:** Confusion!
>
> **T:** When y is -5, then X is what?
>
> **Sg:** 5
>
> **T:** When y is 1, then X is what?
>
> **Sg:** 5
>
> **T:** So what is X?
>
> **S3:** Wait! It is on the 5 line so it is 5. X=5 (The correct response).
>
> **T:** Gives an excited expression and says, Yes! Addresses original student and says, X = 5.

Timothy did not merely assist the one student on how to articulate the correct response. He posed supporting questions to extend the mathematical discourse so all students would benefit from how to arrive at and articulate the correct response because he had assessed the lack of understanding in the entire group.

Reminding students of conceptually similar problems that have been solved successfully and providing background knowledge to students who are engaged in a response are two other areas participants helped to support the students' mathematical thinking. Reminding students of similar problems that have already been solved helps to generate reflection by the speaker as well as the whole group (Fraivillig, et al., 1999). Participants extended mathematical discourse by reminding students of similar problems frequently in the study, with this component having the second highest frequency in the supporting category. There were occasions that a participant referenced by student name a problem solved earlier in the lesson or the previous day. This type of support was often observed during the review of the previous day's homework or the warm-up exercises that started class almost daily in each of the participant's classrooms. Participants used statements like, "Remember how we did this a couple of months ago?" or "This is similar to this problem already on the board." On occasion, one participant used a game reference that was implemented in class while teaching a specific skill. When students were reminded of the game they immediately connected with the problem being discussed.

All three participants provided background knowledge to students responding to a question or by sharing their solution method. At the middle school level the common mathematical discourse I noted the citation of a mathematical rule to support the student in solving the problem. For example, in separate observations both Timothy and Mary referenced the distributive property in helping students understand why signs became negative when solving an equation involving subtraction. Both of these participants were at similar points in their curriculum even though they served different schools and school districts, so it was interesting to observe them both support the extension of mathematical discourse using a rule to encourage reflective thinking by the student.

One of the two areas of supporting mathematical discourse used the least by participants was the encouragement of students to seek private assistance. Participants had cultivated an environment where all students appeared to be comfortable seeking additional assistance from their teacher. Each participant used a different approach, depending on how they structured their classroom (Timothy and Patrick) or what the goals were for the day (Mary). Timothy sought out each student individually, wheeling down the aisle in his desk chair. This was followed by standing at the board and waiting for specific assistance requests. Depending on the student’s preference, Timothy assisted at the board or went to their seat. Patrick used the same method for all observations, which consisted of circulating around the room and assisting students as requested. Mary, like Timothy, differed in her approach. Her changes were driven by the goals she had established for the lesson. On days students worked in groups to prepare to teach their lesson, Mary circulated the room and then met with each group individually at a table before class dismissed. Other days, she circulated the room while students worked on their own or in pairs to assist with questions.

Demonstrating the teacher-selected solution method was used by one participant in the study. Mary shared her solution method to model a concise procedure after all students had finished teaching their lesson. Using her solution method, Mary would assist students with generating a procedure that would help them to solve conceptually similar problems in a succinct manner. Students were not required to use the method, but I noted students often preferred the method Mary presented involving the instruction of new mathematical concepts. The acceptance of her response seemed to be generated by the concise nature of the steps and thorough explanation presented to the class. Students at times seemed unsure of their responses and even frustrated on occasion. Once Mary felt students could not proceed any further, she would close the discussion and step in to share her solution method. During the intensive interview with stimulated recall it was apparent that Mary held her students to a high standard and believed they could and should do any mathematics. The following is an excerpt from the interview:

> When I get to somewhere I think they (students) may have a question (not understand) I will ask them (group), “Ok, well, what do we do next?” or “Why would you do this?” or “Explain this to me.” They (students) should be doing the work. They (Students) think that I am the math teacher, and I should just do all the work. But I let them (students) know they (students) should be able to figure it out.

While this comment may sound harsh standing alone, it was evident that Mary cared deeply for her students. She was an active teacher who was willing to be a facilitator of the learning process, as discussed in the literature review as important to increasing mathematical discourse (Nathan & Knuth, 2003; Goos, 2004; Walsh & Anthony, 2008). Mary was comfortable allowing her students to take the lead in a lesson, but did use teacher led instruction when appropriate.

Timothy and Patrick used a traditional instruction style that provided students with a structured and procedural way to complete problems, thus students were taught the teacher’s solution method during the lesson. When new material was presented, the students were expected to follow the procedure shared by the participants during the lesson. During the observations where participants were reviewing for the state standardized tests, it should be noted these two participants reminded students there was more than one way to complete the assigned problems. When the class reviewed the problems, however, the teacher-selected solution was the only one discussed as a whole group. Patrick's teaching was engrained in traditional approaches with note taking and copying example problems. Timothy also used traditional approaches, but did include application of the procedural knowledge to concepts by providing opportunities for students to see where the mathematical skills would be used. For example, students learned how to find the area and volume of geometric shapes by factoring binomials.

In the subcategory of supporting, participants had opportunities to continue mathematical discourse by supporting students' thinking. Had participants solicited multiple solution methods from students in the eliciting phase, those discussions could have climaxed with the teacher solution method being shared and why the particular method is best (short cuts, important steps included, etc.) and affirmation provided to other solution methods and their value to the mathematical discussions taking place. A second area in supporting participants could expand on is assisting students with clarifying their responses. Use of appropriate terminology and speaking in complete sentences and thoughts was not practiced regularly or expected of students. This seemed to be more of a focus in a lesson here and there. Finally, engaging peers in the discourse through asking them to explain another student's explanation could have been incorporated into lessons. I did not observe this in any lessons, and could be used by a teacher to confirm if a student understands the concept being taught. This could also be a strategy used to ensure a student is engaged in the lesson.

Supporting students' conceptual thinking through mathematical discourse provides teachers with the opportunity to help students understand new mathematical concepts and skills. Students often do not make connections between existing knowledge and new ideas shared by teachers or peers in the traditional classroom setting. Teachers who use the deliberate practice of supporting students' mathematical discourse assist students in focusing their mathematical thinking and reflection. In doing so, teachers create opportunities for extending mathematical discourse and expanding student understanding of the discipline.

Extending Mathematical Discourse

The third and final component of the ACT framework builds upon the initial elicitation of the student and supporting the individual learning process by extending mathematical thinking through the use of discourse. This component captures the methods and questions teachers employ to challenge or extend students' mathematical thinking (Fraivillig, et al., 1999; Nathan & Knuth, 2003; Woodward & Irwin, 2005; Bennett, 2010). This is the most complex level of mathematical discourse that takes place during a lesson and involves processes that are both visible and imperceptible to an observer. Teachers who conduct mathematical discourse at this level have multiple thought processes occurring simultaneously. Imperceptible processes include the on-going evaluation of the individual student ZPD during discourse as well as the understanding of the entire class, based on responses and interactions. Another important instructional method not easily observed is the decision of how and when to follow up with a student's response or lack of one. Knowing when to wait longer for a response, simplify a question to guide a student to the correct response, or call on a peer for assistance is challenging for seasoned teachers as discussed in Chapter 2.

There are also observable, visible processes such as asking challenging follow-up questions or justification for a response that was provided.

The category of extending mathematical thinking examined mathematical discourse used by the participants to extend the discourse to higher levels of mathematical thinking (Table 4-5). I observed evidence of maintaining high standards and expectations for all students and encouraging mathematical reflection, subcategories of the extending component. Explanations and examples are provided for this third and final component of the ACT framework following the Fraivillig, et al., (1999) study model to help the reader understand the various categories and subcategories examined in this study.

Maintains High Standards and Expectations for All Students

Asks all students to attempt to solve difficult problems. Study participants expected students to not only attempt to solve difficult problems, but all problems. Despite the expectation by participants for students to attempt all problems, the classroom environment did not seem visibly strained or stressed. Students were willing to attempt the problems and shared openly in discussions.

Encourages Mathematical Reflection

Encourages students to draw generalizations. All participants urged students to generalize mathematical knowledge after a few examples had been discussed in class. Textbook company generated assignments often provide more challenging problems in the assignment than is presented in the explanation of the concept. With this in mind, participants assigned a variety of problems using the textbook or a worksheet, but did not specifically instruct on how the concept generalized to increasingly difficult questions. The following is an interaction observed in Timothy’s class. The formal instruction of the lesson had ended and the participant was attempting to have students use factoring of polynomials to find an appropriate fence perimeter for cattle.

> **T:** Yes, what I said was these pictures are the real life application of (factoring and grouping) those problems. What if I need to put a fence around my cows? How do I need to figure to do that?
>
> **S1:** Why would I need to know how much fence is in between each post?
>
> **T:** Let's say I have a certain amount of fence. 500 yards of fence. I need to put it up against a river. I need to know the best way to use that length of fence to get the largest amount of area. I could go 1 yard here and here (ends) and 498 yards here. My cows aren't going to be very happy. The way I use my fence is very important.
>
> **S1:** So we basically do the same thing here (pointing to the last problem where solving the equation for the volume of the cube) as we did over there (pointing to the first problem solved that was a basic factoring problem).
>
> **T:** Exactly. (Students attempt to solve the problem.)

Encourages students to justify responses to solution method. Participants asked students to justify responses regarding their solution methods to extend the mathematical discourse into higher levels of critical thinking. Questions such as "Why do you combine like terms?" and "How do you know when a negative means a subtraction sign?" were identified in the study.

Table 4-5. Frequency of Extending Mathematical Discourse

	Frequency of Extending Mathematical Discourse		
	Maintains High Standards and Expectations for All Students	Encourages Mathematical Reflection	Total
	*M/P/T	*M/P/T	*M/P/T
Asks all students to attempt to solve difficult problems. (DP)	1/10/3		14
Encourages students to draw generalizations. (DG)		4/2/5	11
Encourages students to justify responses to solution method. (JR)		3/1/2	6
TOTAL RESPONSES			31

*M/P/T = Mary/Patrick/Timothy

Comparison of Use of Extending Mathematical Discourse between Participants

After analyzing video-taped lessons, observable mathematical discourse found the three participants asked the least amount of questions at the extending level of the ACT framework. Significant, though, regarding this finding is the Fraivillig, et al. study (1999), found that teachers were observed infrequently extending mathematical discourse into this category, suggesting alternatively certified teachers may perform at similar levels of traditionally trained teachers with regard to extending mathematical discourse in the classroom. The results further indicate that alternatively certified teachers may find it challenging to “guide discussion and activities and the seeding of ideas” (Nathan & Knuth, 2003) as do many traditionally trained teachers.

Two of the participants in the study rarely moved beyond the supporting component of the ACT framework and into extending mathematical thinking, which is consistent with the Fraivillig, et al. (1999) study findings. Patrick had the highest number of identified episodes of extending mathematical discourse, which is interesting with the strong traditional teaching approach he uses. In reviewing the data I noted that Patrick had a high number of instances encouraging students to attempt all problems. This appears to have inflated his coding in the extending category in a single component. While this is subcategory of extending is correctly situated according to the ACT framework, this particular subcategory may be better suited in the eliciting or supporting categories as it more encouraging in nature and not truly an opportunity to extend mathematical discourse and student thinking. In reviewing the coding in the other two categories I only identified a total of three interactions, which is more indicative of the traditional teaching style Patrick used. I observed the subcategory encourages students to justify responses infrequently (6) in extending mathematical thinking, while I observed drawing generalizations slightly more often (11). All three participants expected students to attempt all assigned tasks.

Participants were consistent with the expectation of attempting to solve all problems, encouraging students to give it a try and asking them to think about the problem. Mary verbalized her faith in their ability to do the math and would continue to encourage individuals and small groups as appropriate. During an observation in Mary's class on solving equations with variables on both sides, some students found it challenging to decide where to begin, while others seemed to have disputes within their small group on which method was correct when they started differently but resulted in the same solution. Students indicating the first step was difficult were given specific directions on what to examine and where to begin. At their current ZPD, providing a more procedural approach helped them overcome this initial hurdle. Students in other groups had different methods with the same answer. For these, Mary posed different questions to help them understand you could start on either side of the equation. These students were more advanced in their thinking of this mathematical concept. During whole group instruction Mary strategically brought these differences out to the class to help students challenged by a starting point to understand either side of the equation will result in the same solution.

Timothy waited the least amount of time among participants to begin assisting students by beginning to provide “hints” to the students to stimulate mathematical thinking. Students appeared to know this and began to ask questions almost as soon as the lesson ended. I also observed prodding for hints during the warm-up exercises in Timothy and Mary’s classes. Again, Timothy “caved” before Mary did in providing responses to student questions. Eventually, due to time constraints established for this in the lesson plans, participants provided assistance to students and used scaffolding at varying levels, depending on individual ZPD.

While observing in Patrick’s class during a standardized test review session, I noted the participant reminded students there were multiple ways to solve a lengthy, multi-step problem. However, instead of calling on students to share their responses, Patrick proceeded to work the problem on the board using the most efficient method asking only eliciting level questions throughout the completion of the problem. After Patrick completed the problem students raised their hand and give a brief explanation of how they solved the problem if it deviated from the given solution method, which was affirmed by the participant as acceptable though not explained in detail or demonstrated for the class. Mary, Timothy, and Patrick expected all students to attempt to work assigned problems in their classroom.

I observed encouraging students to generalize their mathematical thinking five times in Timothy's class, the most of the three participants. Timothy provided a variety of challenging problems in his lesson, but refrained from explaining and demonstrating the problems. While Timothy's instructional methods were traditional per se, he integrated scaffolding throughout the problem-solving process, challenging mathematical thinking and waiting for student responses to questions to extend mathematical reflection through discourse. Timothy excelled at developing a non-mathematical context for students to generalize to in order for them to understand the mathematical concept. The following is an example of this type of discourse interaction using groceries to set the foundation of the lesson. The story line below was two stops on the way home where the participant purchased the same as well as different items at each stop.

> **T:** I go to the store and bring home a bag of stuff from each stop. (Refers to board drawings of the items fictitiously purchased where he lead a discussion on transitioning the food to a formula and now is guiding the addition of the two formulas.) That's what math is, a bunch of stuff, and we have to keep track of our stuff. $(5x^3 + 3x + 7) + (6x^2 + 5x + 5)$ will eventually be the formula.
>
> **T:** So if it's 5 + 7, what is that?
>
> **S1:** 12
>
> **T:** 12 stuff, but this is all the same stuff or we wouldn't add them together, would we?
>
> **S2:** You can add apples and bananas together, or apples and oranges.
>
> **T:** (Encouragingly) So what do I have?
>
> **S3:** You have 8 somethings cubed, $8x^3$. (Incorrectly adding $5x^3 + 3x$ together)
>
> **T:** Where do I have 8 somethings cubed?
>
> **S4:** (Shouts) 3x + 3

T: Wait! (directed at shout) Where do I put $8x^{3?}$

S3: On the left side.

T: Where? Show me.

S3: $5x + 3x$

T: (Corrects student) That's not $5x + 3x$. (Pointing to the board.)

S3/4: $5x^3 + 3x = 8x^3$

T: That's what I'm getting at. They are not the same thing. Teacher erases $8x^3$ and the class gets quiet.

S4: $5x + 5x^2$ wait you add $5x^3 + 6x$…..slowly becomes quiet again.

T: (Now continues the mathematical discourse by posing another question.) What is that? (Points to $5x^3$ in the formula.) Is it an apple, a banana, what?

S5: It's a banana.

T: What's this? (Points to 3x in the formula.)

Sg: It's an apple.

T: Wait! You just said they are different things. Can I add them?

Sg: No

T: No, I can't add them. So since I can't add them, what do I do with them?

S4: $5x^3$ + student continues to develop a formula and respond to the question.

T: That's it. (Excitedly.) (Slowing the response down.)

S4: $5x^3 + 8x + 12$.

T: But there's something that has a higher degree. (Bring in another student.) Kacey, what's a higher degree? What degree is this one? (Points to $5x^3$.)

S5: Uh, the 3.

T: Yes, it's cubed, cubic or third degree.

S4: $6x^2$ (Ignored by teacher because other student had the floor.)

T: Kacey, what is it?

S5: Uh, 3.

T: But what’s the next highest degree in the answer?

S5: $6x^2$

T: $6x^2$ Now we are going to get to what you guys wanted to do. The fun part.

The class continued combining the two formulas without additional confusion of the process. Timothy then presented tasks with other degrees for students to generalize what was appropriate to combine and add in order to solve a problem correctly.

Mary provided the least traditional instructional methods of the three participants in the study. She allowed students to solve problems on their own or in small groups before she would bring the class together for a whole group discussion. Mary presented her solution method after students shared their thoughts. Like Timothy and Patrick, Mary assigned textbook work, but expected students to generalize the whole group discussion to complete similar but increasingly difficult problems intentionally included in the assignments.

I observed that the final subcategory of extending mathematical discourse encouraged students to justify responses more frequently than generalizing mathematical thinking. All three participants asked students to justify their mathematical thinking to extend mathematical discourse and present ideas for evaluation to the larger group. Justification of responses was accomplished by encouraging students to provide reasoning for their response or by evaluating a claim.

During the observations of students teaching lessons in Mary's class, I noted that the participant in the role of student was able to examine individual student knowledge about the concept they were teaching. The following is an excerpt from a mathematical discourse episode involving Mary as a student, and other students in the class where students provided reasoning behind their response:

S3: Gets up and puts a problem on the board.

S3: Is there anything we can distribute?

T: What does distribute mean?

S3: Distribute, you know the properties, commutative, associative and distributive. Commutative is the order. Associative is the groups, and distribute is divide and share.

T: So you can give it to something?

S3: Yes.

T: So how do you know you are going to distribute? Is there a sign or something?

S3: Yes, parentheses.

T: What's beside the parentheses? Is there a number right beside it or does it say multiply?

S3: It's inside the parentheses.

T: OK

S3: First, you need to combine like terms.

S4: Why do you combine like terms?

S3: I'm about to show you. So you combine like terms, the 2n and 7n. This is negative so you subtract.

S4: This is serious. Where do you see negative at?

S3: Well, it's the minus right here. I didn't mean to. See this right here. It is at the bottom.

S4: So that is the negative?

S3: Yes.

S4: So it's negative. OK.

T: Miss Mason, isn't it when you combine like terms whatever you circle you do what's in the problem so it would be 7 – 2?

S3: HmmmHmmmm.

S5: How do you know when it is negative for subtraction sign?

S3: For this you just basically it has the subtraction sign and you would use that (unclear explanation by the student)

T: Aren't a subtraction sign and a negative sign the same thing?

S3: Yes, so on this we are going to subtract 2 and cancel this out. Subtract 2 from this. What does 7-2 equal?

S4: 5

S3: Right. You bring down what's left. On this step we would keep the variable alone, so on this one we won't mess with the variable. Robin, what would we do with the one?

S4: You would add 1 to both sides.

S3: 14 plus 1 gets us to…

S4: 15 Wait. How did you get the one there?

T: Why did you add 1 to both sides?

S3: Because that is how you get rid of it. You have to do the inverse operation.

S5: But if you got rid of it wouldn't you put the 6 plus 14.

S3: Remember, like I just said we have to leave the variable alone. We can't mess with it. Alright. So, Hope, what would we do right here?

S6: You would divide by 5 the 5n and 15.

S3: Alright. (works step) And what would that give us, Mark?

S7: 2. Can't see it well.

S3: 15/5?

S7: oh, 3.

S3: And that is how you do that one.

Mathematical discourse of this type allowed Mary to check for student understanding and support the instruction the students were presenting to the class by creating mathematical discourse for the benefit of the entire class. The participant allowed students to take the lead on the discussion as both the teacher and students, while the participant interjected where she felt appropriate to extend the mathematical discourse and to redirect the interaction if she determined it may be going in the wrong direction.

I observed the category of extending students' thinking the least, and yet there were ample opportunities for all three participants to inquire of students to do just that-defend how and why they solved a particular problem using a specific method. I rarely observed this despite the strong interactions that took place in two of the three classrooms. Asking students how and why certain solution methods were used provides the greatest opportunity for challenging students' thinking and understanding where they are mathematically (NCTM, 2000). Extended sessions such as this can be difficult to create and extend if not properly trained; however, fruitful for the individuals involved in the discussion as well as the teacher if they take place. Individuals can assess ideas and thoughts shared based on questions and responses from their peers. The teacher is able to assess many individual's ZPD and understanding of the mathematical concept being presented.

Routines Supporting Mathematical Discourse

As discussed in the research in Chapter 2 (Leinhardt & Steele, 2005; Ball, 1993; Manoucheheri & Enderson, 1999; Yackel & Cobb, 1996) in order for teachers to promote mathematical discourse a classroom must have established routines and procedures. Having a structured environment allows teachers to create mathematical discourse opportunities for individuals and the whole class. Participants in the study had established routines that contributed to the mathematical discourse of whole group.

All three participants began each class period with a warm-up exercise while required bookkeeping and attendance were handled. The warm-up exercises usually consisted of problems from the previous day's lesson and were reviewed by the participant with the class before moving into the formal lesson. Warm-up exercises were also used by one participant to review mathematical concepts that were covered done throughout the year. This kept all mathematics fresh in the students' minds as they moved closer towards formal state standardized testing dates. Mary, in a post-observation interview noted that she liked to collect this work to review and see how much the students understood when they went home and came back to see it again the next day. She viewed this as another opportunity for students to practice and master a mathematical concept and for her to monitor their progress.

Upon completing the warm-up exercises, participants moved into the lesson. Lessons varied by participant in approach, but there was clear structure in two of the three participants' management of the learning process. Mary and Patrick expected students to raise their hand and wait to be called upon. Both would ignore students who attempted to call out answers and would make general statements reminding the class to raise their hand if the mathematical discourse became excited many students suddenly had a response they wanted to share. Timothy, on the other hand, would often "accept" answers that were shouted out by students, particularly in moments where the discourse may stall, such as the one student having a long pause, providing multiple incorrect answers, or unable to respond to the participant's questioning and scaffolding. While an observer may criticize Timothy for this type of classroom management, he seemed to enjoy the challenge of answering many students at one time and maintaining a high level of energy with the group to inspire engagement and evaluative thinking. The classroom appeared to be one great conversation and all were willing to participate.

Finally, participants had clearly established rules for whole and small group interactions as well as acceptable behavior during class work time, including voice control and movement around the classroom. Students could elect to work alone or with one or two others on the tasks assigned by the teacher. The participants had guidelines for asking for assistance and closing out the class period. All of these aspects of classroom routines appeared to support mathematical learning by providing boundaries for the students in various situations and providing a foundation for the participants to build a positive intellectual environment.

Intellectual Environment Supporting Mathematical Discourse

Important to the creation of mathematical discourse opportunities is the establishment of a positive intellectual environment. (Manoucheheri & Enderson, 1999; Yackel & Cobb, 1996; William & Baxter, 1996). How a student perceives his or her answer will be received is paramount to establishing extended mathematical discourse with individual students and across the classroom mathematical discourse community. Facilitating individual knowledge, or one's primary discourse (Gee, 2007), for participation in the whole group, or secondary discourse (Gee, 2007), is pivotal to advancing mathematical learning for the entire group. As discussed in Chapter 2, when students have the opportunity to hear many primary discourses they develop a meta-knowledge about the discourse, in this case mathematics, and the ability to critique those discourses. Providing an environment that embraces the knowledge and experiences of the individual as well as the group as a whole leadership to guide conversations in a manner that evaluates constructively the ideas and mathematical experiences brought forward for the whole group to consider for assimilation or rejection into their knowledge. Because teachers establish and maintain classroom discourse, they play a pivotal role in the acquisition of knowledge and actual learning that takes place. In order to increase student learning, a classroom environment that supports engagement and discussion must be established (Schoenfeld, 1989; 1992; Yackel & Cobb, 1996).

Participants in the study had established acceptance of all thoughts and ideas regarding mathematics as a part of the learning process. The established social norm provided an environment where mathematical discourse was fluent and unfettered. Students of all ability levels sought participation in the discourse of both small and whole group discussions. Further, there was obvious acceptance of primary and secondary discourses as individual students were willing to share their thoughts without reservation in large part. One humorous example of a participant reassuring a student in a “weak moment” before he would share his response occurred in Timothy’s class. A student who had volunteered to share his answer was called upon, but before responding he said laughing as he spoke to the participant, “You are going to make me look stupid with the answer, aren’t you?” Timothy replied laughingly, “I don’t think so. Tell you what, put your hands on your face like this.” The student did so. Timothy said, “Now I have made you look silly, but let’s hear your math answer.” The student laughed and gave his response and the class moved forward with discussions.

Students should know incorrect answers will not be opportunities for peers to make fun of the response, which is a sensitive issue particularly at the middle school level. Social acceptance with adolescents is paramount to generating whole group mathematical discourse (Bennett, 2010). As discussed in Chapter 2, creating an intellectual environment that embraces the thoughts of all learners is important to establishing mathematical discourse. Students must believe it is alright to be wrong and that the teacher guarantees their “safety” from ridicule in order for thoughts to be shared openly (Bennett, 2010; Leinhardt & Steele, 2005; Fraivillig, et al., 1999). Study participants indicated a strong establishment of a safe intellectual environment.

There was an obvious respect between each participant and their students as well as among students in each class. I observed many instances of wrong answers, incorrect or vague use of mathematical vocabulary, and correct answers. In each instance there was mutual respect and a willingness to share mathematical thinking when asked. During all 17 observations there was only one instance where a student made an inappropriate remark to another student answering a question at the board. The participant did not ignore the comment even though it was intended as a joke between friends. The remark was handled immediately and gracefully to save face for the student making the remark as well as the one to whom the remark was made, thus reminding the class this was not an acceptable behavior and would not be tolerated.

Participants used a variety of strategies to establish a positive intellectual environment conducive to learning for all students. Patrick and Timothy used humor widely to connect with their students. Their humor relaxed students and reduced mental barriers some may have had regarding their ability to learn mathematics. Timothy also used humor to excite students about mathematics. This not only engaged more students, but extended the duration of their engagement during the lesson. Another indication from recorded observations and anecdotal records that students felt safe was their desire to answer questions and come to the board to share their solution methods. While generally teachers prefer to have students raise their hand and wait to be called upon, Timothy appeared to use the desire expressed by the students to participate in the mathematical discourse to carry the excitement for the lesson. He also allowed students to talk during the whole group discussion if they were deciding on possible new answers to contribute to the larger discussion. Timothy provided constant feedback to students individually and as a whole group.

Mary and Patrick used a more structured approach during whole group mathematical discourse. This did not stall discussions and provided these two participants with the opportunity to have more individualized mathematical discourse and more time with each student who responded. From the intensive interview with stimulated recall, Mary and Patrick appeared to be more methodical in their instructional decision-making regarding who would speak and when. As with Timothy, Mary and Patrick provided constant feedback to their students.

Finally, all three participants encouraged students to seek assistance from their peers or the teacher, which contributed to the positive learning environment. Providing an “all-access” pass to the participant or their peers encouraged students to share openly and discuss differing views on solving a problem. Students in Timothy’s class were willing to come to the board and discuss their problem, writing their response in full view of the class. This was something that I had not seen, but was obviously common practice, as the other students paid no attention to the interaction at the board. During one observation, there was one student who was still unsure on how to combine like terms when adding polynomials. The participant made up his own problem on the board for the student to solve while he assisted other students at their desk. When he returned to the board the student had solved the problem correctly, but transposed the letters “e” and “z” in the answer. The answer she had was “2ze” but in reality the answer was “2ez” which was Timothy’s use of humor to relax the student and open their mind to the mathematics. The student laughed and went to her seat to continue working on problems. A safe intellectual environment would have to be established for this type of interaction to take place between teacher and student.

CHAPTER 5
CONCLUSION

Discussion and Implications

When teachers ask authentic questions (eliciting), they open the door to what students have to say; when they engage in the uptake (supporting), they build upon what students have said; and when their evaluation of student responses is high (extending), they certify new turns in the discussion occasioned by student answers (Nystrand & Gamoran, 1991). With the extensive use of alternatively certified teachers the need to focus on how this subgroup, used nationwide, implements mathematical discourse in the classroom setting is important to educational research. There is extensive research on how teachers establish an intellectual environment and incorporate mathematical discourse in the classroom setting (Mehan, 1979; Shulman, 1986; Ball, 1990, 1993; Yackel & Cobb, 1996; Fraivillig, et al., 1999; Lampert, 1990, 1992; Kazemi, 1998; Kazemi & Stipek, 2001; Whitenack & Yackel, 2002; Manoucheheri & Enderson, 1999; Spillane & Zeuli, 1999; Leinhardt & Steele, 2005; Woodward & Irwin, 2005; Sfard, 2006), but there is no research specifically on alternatively certified teachers' use of mathematical discourse. The ACT framework (Fraivillig, et al., 1999) was used to examine this subgroup of teachers to identify how they create an intellectual environment and enact mathematical discourse, specifically with middle school students.

The desire to establish research examining alternatively certified teachers' use of mathematical discourse was the focus this study. In the final chapter, the findings from the study will be summarized and implications for future research studies will be shared in hopes of contributing to the gap in the literature regarding alternatively certified teachers' use of mathematical discourse.

Discussion

This study used the Advancing Children's Thinking framework (Fraivillig, et al., 1999) to assess the types of mathematical discourse used by alternatively certified teachers in three middle school classrooms. The ACT framework consists of three categories in which teachers have the opportunity to extend mathematical discussions. The categories were eliciting students' solutions, supporting conceptual understanding, and extending children's thinking. Within each of these three categories I identified subcategories considered as mathematical discourse that were analyzed and coded.

Noteworthy in this study is the identification of mathematical discourse in all three core categories established within the framework. This indicates the ACT framework is broad and comprehensive in nature. Further, the framework appears to document many types of mathematical discourse occurring in the classroom, judging by the data representation in many subcategories and categories. Within each of the three major categories are subcategories the authors designated as evidence of each action taken by the teacher to promote learning and extend the mathematical discourse during whole group instruction. In the study, I did not identify data in each of the subcategories of the ACT framework. Factors influencing the lack of representation in all subcategories are varied. First, the mathematical topics observed may not have promoted mathematical discourse between teacher and student. The curricula observed was selected by each teacher and was not a focus of the study. Varying topics within the mathematics discipline will generate varying levels of discussion. Some of this discussion may be seen more in a small group setting, or peer-to-peer, which was not a focus of this study. Secondly, the alternatively certified teachers may have planned poorly for the lesson, not establishing effective questions to engage students. This is a good possibility since all three participants are alternatively certified teachers and did not have the benefits of traditional teacher preparation. The length of the study and amount of data collected was limited to a short timeframe. This may have influenced the number of opportunities to observe whole group mathematical discourse. The types of mathematical discourse episodes witnessed in the study were seen at the middle school level, and, most importantly, in the classrooms of traditionally trained teachers in current research. Conversely, the alternatively certified teachers in this study had instructional skills and strategies supporting mathematical discourse well planned for specific areas of the ACT framework (unplanned as a part of the study) as did traditionally trained teachers, indicating possible specific strengths of these teachers with regard to mathematical discourse despite their certification route to licensure.

A second area in the study deserving focus is the indication of the data that the three participants in the study had a larger number of mathematical discourse interactions with students in the second area of the ACT framework, supporting children’s thinking, which is the same as the original study from which the ACT framework emerged (Fraivillig, et al., 1999). This may suggest alternatively certified teachers find the supporting of students’ thinking through mathematical discourse an area that comes more naturally as did traditional teachers in the Fraivillig, et al. (1999) study and others noted in the literature review. While this study is not generalizable due to the small number of participants, this is of interest and worthy of further research and discussions within this population of teachers as well as in comparison to those trained through the traditional route

The lack of data identified in the third category of the ACT framework, extending, is an area of concern for all teachers. Other studies using the ACT framework identified this category of extending as rarely observed in the classroom setting no matter the teacher training pathway. Ball (1993) indicated extending mathematical discourse as important to advancing mathematical thinking and understanding. Data in this study indicates alternatively certified teachers may not have the skills needed to plan and carryout lessons that incorporate mathematical discourse at levels appropriate with higher order thinking, which is needed to participate in mathematical discourse. The study seems to support the ongoing call for professional development of all teachers in this area to increase episodes in the extending category of the ACT framework.

The three participants in this study had high frequencies of interactions in the eliciting category of the ACT framework. This is a deviation from the original study, which had low frequency from its participants. This study indicates the three alternatively certified teachers planned and implemented eliciting questions and planned who would speak and when to guide student learning through orchestrating the mathematical discourse that would take place as indicated in the interviews. This is encouraging data given the alternative certification background of all three. Additionally, through the post observation and intensive interviews with stimulated recall it became apparent the participants planned and adjusted instruction as the lesson evolved. This is a finding of interest and worthy of further discussions for this population of teachers on a larger scale and in comparison to traditionally trained teachers.

This study supports past research (Bennett, 2010; Leinhardt & Steele, 2005; Zack & Graves, 2001; Fraivillig, et al., 1999) citing the need for a safe intellectual environment as a foundation to mathematical discourse. Participants' data in all three categories suggests mathematical discourse materialized due to their ability to establish a learning environment that valued all perspectives. This indicates a need for continued support of new teachers, both traditional and alternatively certified, in the first years of teaching. Ensuring teachers have the skills to establish an intellectual environment that encourages discourse is foundational to increasing mathematical understanding. Further, the episodes of student-to-student mathematical discourse in two of the classrooms indicate a sense of primary and secondary discourse (Gee, 2007) at a high level. The sense of community was clearly established by participants in the study. Participants had an obvious rapport with their students. This sense of community was articulated by the humor and openness in which all instruction and discourse took place. Students were at ease with asking questions or admitting a lack of understanding with a statement as simple as "I don't get what you just did." This type of intellectual climate is seen in traditionally trained teachers, and I was pleased that it was present in the three participant's classrooms.

Finally, through this study I provided initial research data on the use mathematical discourse to advance children's thinking by alternatively certified teachers. The study revealed that these three participants, who have not been formally trained as teachers or mathematicians, did utilize various levels and forms of mathematical discourse to engage students and help students to organize their mathematical thinking. The study found strong use of the supporting category, indicating efforts by the participants to scaffold students' thinking and encourage the communication of solutions and ideas. Conversely, coded data in the analysis indicated that participants in the study did not have a large number of interactions in the extending category of the framework. The Advancing Children's Thinking framework provided a comprehensive outline to examine mathematical discourse and can continue to serve as a guide for further research studies. While this data provides a starting point for research and data regarding alternatively certified teachers, additional research studies larger in scale should be conducted to truly capture a clear picture on how this subgroup of teachers conducts varying levels of mathematical discourse.

Implications

Establishing Qualified Alternatively Certified Teachers

The study findings have several implications for developing qualified alternatively certified teachers prior to employment in the classroom setting. First, individuals seeking teacher certification should be provided professional development and mentorship of experienced teachers to support their own development of mathematical discourse. As noted in the study, the foundation for mathematical discourse is the establishment of a safe intellectual climate for all students in order to foster engagement, participation, and ultimately extended mathematical discourse (Bennett, 2010; Leinhardt & Steele, 2005; Fraivillig, et al., 1999). Establishing this type of environment can be challenging even for traditional teachers; therefore, it is paramount alternatively certified teachers are taught, observed and supported to implement an environment that embraces discourse in the classroom and integrates it fully in lessons. Having administrative and mentorship support during the critical first year, but also beyond into year's two and three will help ensure this subgroup has the knowledge and skill to manage their classroom and establish a strong intellectual environment for mathematical discourse and overall student learning. Observations of highly skilled master teachers utilizing these strategies would be ideal for alternatively certified teachers to observe regularly on a variety of topics, as well as be observed and given constructive feedback on progress by administrators and mentors alike. Finally, providing a community of learners among colleagues would further support the development of alternatively certified teachers. The opportunity to share in discussions regarding lesson plans, teaching strategies, classroom management, and content knowledge would enhance confidence and expand teaching abilities. While these suggestions are not new, the evidence provided in this study supports the importance of continued professional develop and localized school support to develop skilled alternatively certified teachers for mathematics.

Second, study findings suggest the importance of educating alternatively certified teachers on how to move beyond eliciting and supporting student thinking to the most challenging component of extending student thinking. As evidenced in the data and previous studies (Fraivillig, et al., 1999; Kazemi & Stipek, 2001; Woodward & Irwin, 2005), teachers infrequently extend mathematical discourse during lessons. Alternatively certified teachers need education and practice on research-based techniques that will empower their students to conceptualize mathematics in meaningful ways. Shifting to the role of facilitator will require thorough professional development. One possible new teaching strategy is the use of scenarios that allow for role play and detailed discussions and analysis to assist with the development of skills and confidence to fully integrate mathematical discourse at all levels of the ACT framework. Scenario education may include but not be limited to:

1. Developing an understanding of individual student abilities and anticipating possible responses and misconceptions students may have and how to address them. Teachers need to understand the importance of planning a lesson thoroughly, including specific questions be asked and when.

2. Activities to develop the confidence, patience, and teaching strategies of teachers to engage and extend all students in mathematical discourse. Timothy used stories to do this with his students in the study. There are other ways this skill can be developed in teachers.

3. Role play for teachers to practice engaging students in explaining, justifying, and defending one's ideas to stimulate a deeper understanding of mathematics. Having groups of teachers present lessons and engage in attempts of authentic mathematical discourse may increase comfort level and therefore usage in the classroom.

4. Practicing effective questioning techniques in all three subcategories of the ACT framework to engage students at all levels in the whole group discussion. Alternative certification programs rarely include questioning techniques as a part of teacher preparation, so the inclusion of this in

programs or at least at the professional development level may increase opportunities teachers would provide students to extend mathematical discourse.

5. Help teachers to establishment of routines, rules and language that support the belief all ideas are valued and respected. While teachers are becoming increasingly culturally sensitive, there continues to be room for development in this area. Certainly how to establish a safe learning environment conducive to mathematical discourse should be a focus of teacher preparation programs and professional development.

Professional Development

With the nation's schools employing alternatively certified teachers since the 1980's, professional development should continue to be used to educate those already in the workforce. Partnering with universities to educate alternatively certified teachers on how to increase the use of mathematical discourse in the classroom setting would help address the gap in teacher knowledge and skills related to mathematical discourse. A focus should be placed on all important phases of the lesson planning process, including the planning of appropriate tasks, anticipating questions, knowing individual student ZPD, and using reflection to evaluate and improve lessons and overall instruction. The development of tasks appropriate for whole and small group mathematical discourse should also be emphasized in professional development. Alternatively certified teachers would benefit from having a deeper understanding of how to extend discourse by incorporating tasks that stimulate mathematical thinking and provide a starting point for deeper discussions. Additionally, teachers need professional development that increases content knowledge of the curriculum (Hill, Schilling, & Ball, 2004; 2005) in order to be more effective facilitators of mathematical discourse. This is particularly critical for alternatively certified teachers with no mathematical background.

Participants in this study all professed in at least one post observation interview to have a “love of mathematics.” This is rare in teachers in general and even more so in alternatively certified educators. School districts and states should consider identifying individuals with STEM backgrounds to recruit for alternative certification to increase the pool of individuals who have more content knowledge and have a “love for mathematics.” Certainly having those excited about mathematics in the classroom is an important milestone in identifying highly qualified persons for teaching.

Recommendations for Future Research

The results from the study indicate several possibilities for future research. While the study helped to provide some information for the literature on how alternatively certified teachers conduct mathematical discourse, there are several related areas that would enhance research in the literature. Studies examining how the curriculum is used would be beneficial to the literature. Research is needed to understand what the current curriculum contains, specifically tasks and support provided to teachers to extend mathematical discourse. Research on how questions and tasks need to be included and modified to elicit participation, support mathematical thinking and extended mathematical discourse be conducted

Studies should be conducted on instructional actions by additional alternatively certified teachers of mathematics to include teachers and alternatively certified teachers of all experience levels. This would provide insights on how alternatively certified teachers progress through their skill attainment in relation to their traditionally trained peers and provide a clearer picture on internalized models of teaching that could guide professional development and alternative teacher certification education. The ACT framework may need to be expanded or categories adjusted to reflect findings in further studies.

Finally, research that examines content knowledge of teachers, including the specific subgroup of alternatively certified, in conjunction with the levels of mathematical discourse that take place in the classroom between the teacher and students. Having a highly skilled, highly trained teacher workforce is critical to -closing achievement gaps in American education. Mathematics, as an identified STEM area is central to these initiatives for America to remain globally competitive. Changing teachers' views on how the embrace and implement classroom discourse is significant to success of educational initiatives. Dialogue composed of diverse viewpoints and perspectives is essential to learning (Gee, 2007). Teachers who can create an intellectual environment incorporating tasks to create mathematical discourse provide new and expanded learning opportunities for all their students. This should main a major focus of teacher training and professional development in the coming decade. If learning teaching is about preparing novices to teach and preparing veterans to teach better, teacher preparation needs to be about thoughts and actions and how these simultaneously impact student learning (Lampert, 2009).

APPENDIX A
CERTIFICATION PREPARATION

Pre-Study Meeting

1. How many years have you been teaching?

2. What alternative certification route did you use to acquire teacher status?

3. In what ways did the alternative certification pathway prepare you for teaching in general?

4. In what ways did the alternative certification pathway prepare you for teaching mathematics?

5. What prior experience or training do you have specifically in mathematics, if any?

6. Have you participated in any professional development (PD) related to mathematical discourse? If so, share the PD experience.

7. Have you participated in professional development related to scaffolding? If so, share the PD experience.

APPENDIX B
POST LESSON INTERVIEW PROTOCOL

Class Characteristics (first interview only)
1. What grade level are the students in the observed class?
2. How many students are in the class? (Note number present for observations)
3. Which of these best describes the overall ability of the students in the class?
    - The students in this class have a lower range of ability levels.
    - The students in this class have a middle range of ability levels.
    - The students in this class have higher range of ability levels.
    - The students in this class have a wide range of ability levels.
4. *How do you develop an understanding of each student's zone of proximal development?
5. *How do you track their changes as the year progresses?

Lesson Observation (every interview)
1. What was the topic of the lesson?
2. What was the purpose of the lesson, and how did it relate to state requirements in mathematics for this grade level?
3. What instruction had you provided in this topic prior to today?
4. What questions did you plan to ask as a part of your lesson today?
5. What tasks did you plan as a part of your lesson today?
6. *What misconceptions or issues did you anticipate may arise from today's lesson?
7. *How did you arrive at these misconceptions to plan for them?
8. *How to you address them in your planning of the lesson?
9. *How did you assess the understanding of students overall at the end of the lesson?
    Probe for specific examples of students who understood and did not understand and why. Also, probe specific instances of extended discourse sessions and how the teacher use discourse to scaffold student learning.
10. *How did your overall lesson go with regarding to your preplanning?
11. *How will you use today's information to prepare for the next lesson?

APPENDIX C
INTENSIVE REFLECTION INTERVIEW PROTOCOL

One interview with video clip

1. Explain why you think the interaction you had with this student/these students led to the extended mathematical discourse response.

2. Why do you think you were able to engage this student/these students in an extended mathematical discourse response?

3. Why did you ask the student ___________? (Pull from the video) Ask additional questions (record) based on the Teacher’s response.

4. What were you thinking when you asked the student _________? (Pull from the video) Ask additional questions (record) based on the Teacher’s response.

5. Based on your interaction in the lesson with this student/these students, do you believe he/she/they increased their mathematical understanding of the skill being taught? Why or why not?

6. How does this interaction alter your belief of the student’s/students’ ZPD?

7. Will this interaction help you plan future related lessons? If yes, how? If no, why not?

APPENDIX D
OBSERVATION DATA TOTALS

All observations

	CODE	Mary	Patrick	Timothy	Totals
Waits for and listens to student solutions	WS	9	6	16	31
Encourages elaboration	EE	13	3	9	25
Conveys accepting attitude of wrong answers and solution methods	AA	8	12	17	37
Decides who will speak and what methods are shared	DW	8	11	17	36
Monitors student engagement	ME	6	7	17	30
Assists individuals with clarifying answers	AS	14	0	3	17
Reminds students of similar problem already solved successfully	RP	2	4	14	20
Provides background knowledge	BK	10	4	2	16
Demonstrates teacher-selected solution method	TM	6	0	0	6
Records student solutions on board (Observed)	RR	13	5	110	128
Encourages students to seek private assistance (Observed)	PA	0	1	0	1
Asks all students to attempt to solve all problems	DP	1	10	3	14
Encourages students to draw generalizations	DG	4	2	5	11
Encourages students to justify responses	JR	3	1	2	6

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