Three Essays on Investment-Specific Technical Change

Max Elger

STOCKHOLM SCHOOL OF ECONOMICS

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AKADEMISK AVHANDLING

Som för avläggande av ekonomie doktorsexamen
vid Handelshögskolan i Stockholm
framläggs för offentlig granskning
14 September, 2007, klockan 15:15
i sal 750 Handelshögskolan, Sveavägen 65

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Max Elger

Dissertation for the Degree of Doctor of Philosophy, Ph.D, in Economics
Stockholm School of Economics, 2007

KEYWORDS: Economic Growth, Endogenous Growth, Productivity Growth, Research and Development, Investment-Specific Technical Change, Search, Matching, Wage Inequality

ISBN NR 978-91-7258-728-1

PRINTED BY:
Elanders Gotab, Stockholm, 2007

DISTRIBUTED BY:
EFI, The Economic Research Institute
Stockholm School of Economics
P O Box 6501, SE-113 83 Stockholm
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Acknowledgements

A thesis is a joint effort. I am grateful for the support from many. Without whom this thesis would not have been. I want to thank you all for helping me through this. But as importantly: I have had the time of my life. I want to thank you all for letting me.

I am especially indebted to *Lars Ljungqvist*, my advisor. His effort has been instrumental for the completion of this thesis. His support and encouragement invaluable. And patience unfathomable. Lars. Thank you!

David Domeij and *Martin Flodén* have taken much time to discuss research ideas. And many other things. For this I am very grateful. I have truly appreciated our talks. They also put up with me as their teaching assistant for all these years, which I really enjoyed. Thank you.

Paul Segerstrom inspired me to think about endogenous growth theory, and took much time to discuss this side of my research. Outside the School, *Per Krusell*, has given me important advice and a lot of support and encouragement. Thank you.

Research can be lonely. Being a PhD-student is not. Because there are others. And they are great. You are great. Despite the crashing computers, absent ideas, missing results and general stress, I have always cherished being at work. At the Cuckoo's Nest on the ninth floor. At Saltis. And the Holländargatan-dungeon.

Peter Gustafsson, my room-mate for four years had to endure more Max than most. Always ready to talk. Always ready not to talk. The perfect room-mate.

The first year. Courses. It certainly wouldn't have been as fun without the company of class-mates and room-mates *Anna Breman* and *Fredrik Wilander*. And the problem sets would have been disastrous. *Elena Paltseva* and *Witness Simbanegavi*, in the room next door, also lent a helping hand.

The PhD-parties and Friday-beers of the early years would not have been as memorable without *Rudolfs Bems*, *Harald Edqvist*, *Kristian Hartelius*, *Nina* and *Daniel Waldenström*, *Niclas Damsgaard* and *Therese Lindahl*.

Holländargatan became the PhD-student stronghold. To start with, *Andreas Madestam* was next door. Without him, the dungeon-summer '05 would have been very lonely. Then came *Henrik Lundvall* and *Kristin Magnusson* on the other side. Music and Macro. A great combination. At the far end of the corridor, *Per Sonnerby* and *Björn Tyrefors* had an open door for chats on politics and poker.

Erik Höglin and *Ola Granström* dropped in. Always a great distraction. Chats about micro theory and labour policy with *Robert Östling* and *Erik Mohlin* was always fun. Econometrics with *Erik Lindqvist* enjoyable and educating. As was South American grapes with *Damian Migueles*, and South American life with *Ola Melander.*

Eva Ranehill, Emma von Essen, Johan Almenberg, Anna Dreber, Sara Formai, Karen Khachatryan, Caterina Mendicino, Marcus Salomonsson, Ignat Stepanok and *Robertas Zubrickas* all contributed to the great atmosphere in the corridor and the coffee-room. *Erika Färnstrand, Andreas Müller* and *Gökhan Buturak* were fellow TA-s in the undergraduate macro course, and made that duty a pleasure. From the University, *Daria Finocchiaro, Anders Fredriksson, Virginia Queijo, Anna Larsson* and *José Mauricio Prado Jr* helped making the course-work fun during the first years. I probably would not have gone sanely through the Job Market without the support from fellow candidates: *Cedric Argenton, Martin Bøg* and especially *Milo Bianchi* helped a lot.

Boys and girls. Thank you!

The administrative staff at the Department is fantastic. A warm thank you goes to *Carin Blanksvärd, Anneli Sandladh* and *Lilian Öberg* for their friendliness and help. I would like to say a special thank you to *Ritva Kiviharju* who has given me much assistance over these years. Also, financial support from the Wallander foundation is gratefully acknowledged.

Two children and a thesis would have been an impossible challenge, without the help from my family. I am deeply grateful to my parents, *Hans* and *Birgitta*, for all help over the past years. And for their support and encouragement over all my years. I also want to express my deep gratitude to my parents in law, *Sören* and *Brenda.*

Michelle. You have seen the ups and downs of this. You were always there with love. Thank you.

Stockholm, August 2007

Max Elger

Summary

The relative price of durable goods fell at an accelerating rate during the last half of the twentieth century. A computer became cheaper in terms of apples or milk, at an increasing rate. From this we can learn that the rate of productivity growth in the sector that produces durables grew, relative to that of the non-durable sector. But why did it happen? And what consequences did it have? On the wages of college-graduates relative to those that did not go to college? On the dispersion of wages among identical workers?

These are the topics of this thesis. In chapter one we address the first question. In chapters two and three the two last.

Chapter 1 - Endogenous Growth and Investment-specific Innovations

In chapter 1, we ask if more research and development (RnD) yields faster productivity growth. The defining studies[1] in RnD-based endogenous growth theory all share one central prediction: Economies that exert more effort in RnD experience faster growth. Yet, when this hypothesis was first put to a test, by Jones (1995), it was strongly rejected. From the sixties through the eighties employment in RnD increased substantially. Productivity growth, however, did not accelerate. If anything, it declined.

In contrast, we find that productivity growth did increase during the post-war era in the US. Jones studied growth in total factor productivity (TFP). We instead focus on growth in investment-specific productivity. We find no reason to reject the scale-effect in investment-specific productivity growth. This is the empirical contribution of chapter 1.

The quantitative analysis by Greenwood, Hercowitz and Krusell (GHK,1997) "suggests that investment-specific technological change accounts for the major part of growth". Thus, even though investment-specific productivity does not account for all productivity change over the period, the understanding of the sources of investment-specific technical change is clearly paramount to the understanding of economic growth.

We, therefore, create a two-sector endogenous growth model, where RnD augments the productivity of Equipment-production. We use our empirical results to calibrate the correspondence between RnD and productivity; the production function for ideas. In a quantitative experiment we let the model predict RnD and investment-specific technical change during the post-war period. The predicted RnD-employment from

[1]see Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992)

this experiment is on average 3 per cent higher than in the data. We conclude that RnD-based endogenous growth theory does offer an explanation to investment-specific productivity growth.

Chapter 2 - Assortative matching and investment-specific technical change

What is the skill-bias[2] of technical change? In chapter 2, we focus on the contribution of accelerating investment-specific technical change to an increase in the skill-premium. In the equilibrium of our model[3], workers are positively assortatively matched to a vintage-capital structure. Skilled workers operate younger - and hence more productive - machines than unskilled workers. Faster investment-specific technical change implies that productivity differences between machines of different ages increase. Thus, an increase in the skill-premium results.

We study this mechanism in two settings: with and without frictions in the labor market.

In the economy without frictions in the labor-market, workers are perfectly and positively assortatively matched to machines. Skilled workers operate the younger machines, and unskilled the older. In the economy with frictions, parties are not perfectly assortatively matched. For intermediate ages, machines are operated by both skilled and unskilled workers. Furthermore, with labor-market frictions, there is wage-inequality within groups of workers. Identical workers that operate machines of different ages may earn different wages.

We quantitatively evaluate the contribution of assortative matching and faster investment-specific technical change to the increase in the skill-premium. We calibrate the models - with and without labor-market frictions - to match some moments of the US economy prior to 1975. Then, in static experiments, we increase the rate of investment-specific technical change and the productivity and supply of skilled labor. While accounting for the increase in the supply of skilled labor, we find that vintage capital and faster investment-specific technical change can account for forty per cent of a ten per cent increase in the skill-premium.

Interestingly - and contrary to previous claims - the model with frictions predicts a simultaneous increase in both within- and between-group inequality. Moreover, and consistent with recent observations, it predicts larger within-group inequality among the skilled than among the unskilled.

Chapter 3 - Inequality in Economies with Vintage Capital and Stochastic Skill Accumulation

In chapter 2, we discussed accelerating investment-specific technical change as a cause of increasing wage differentials between college-graduates and workers without a college-degree. But inequality within groups of observationally identical individuals also increased during the same period[4]. Previous studies report that the majority of the increase in wage inequality is residual - meaning that it is explained by unobserved characteristics among workers within observationally identical groups.

[2] Returns to education increased from the mid-seventies, as documented in the work by Katz and Murphy (1992). They conclude that there was a skill-bias in technical change.

[3] The models in Chapter 2 and 3 are extensions of the framework studied in Hornstein, Krusell and Violante (2007)

[4] This was documented by Juhn, Murphy and Pierce (1993). Moreover, Gottschalk and Moffitt (1994) find that a sizable fraction of this increase reflects increased wage instability for single individuals.

Therefore, in chapter 3, we focus on the contribution of accelerating investment-specific technical change to an increase residiual inequality. Workers are positively assortatively matched to a vintage-capital structure. More productive workers operate younger - and hence more productive - machines than less productive workers. Faster investment-specific technical change implies that the productivity-differential between machines of different ages grows. Larger wage-dispersion results.

In chapter 3, we assume that labor-productivity evolves stochastically. While working, lucky workers become more productive. Unlucky workers less productive.

We study economies with and without frictions in the labor market. In the economy without frictions in the labor-market, workers are perfectly positively assortatively matched to machines. The most productive worker operates the new machine. The least productive the oldest. In the economy with frictions, parties are not perfectly assortatively matched. For intermediate ages, machines are operated by workers of both types.

The models with and without frictions are calibrated to match some moments of the US economy around 1975. Interestingly, and consistent with recent observations, our models predict a larger increase in wage-dispersion in the upper tail of the wage-distribution than in the lower tail. Intuitively: Less productive workers operate older machines. As the rate of investment-specific technical change increases, the productivity of all machines fall, relative to that of a new machine. The older the machine, the greater the decline. It follows that the relative wage of less productive workers declines more than that of more productive workers.

Moreover, the introduction of stochastic skill increases inequality - as compared to the economy studied in Chapter 2.

However, the observed acceleration in the rate of investment-specific technical change does not suffice to explain the increase in wage-inequality. The predicted increase in the log-variance of residual wages is approximately one fifth of the corresponding increase in the data.

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Chapter 1

Endogenous Growth and Investment-Specific Innovations

Abstract

In contrast to the previous literature, we document that more employment in research and development (RnD) yields faster productivity growth. Therefore, we do not reject the 'scale effect' in growth. Previous studies have investigated the impact of RnD-input on total factor productivity growth. We instead focus on growth in investment-specific productivity. Besides criticizing the prevailing view of no 'scale effects' in growth, we conduct the first quantitative study of endogenous growth with a structural model and time series data. Using quality-adjusted equipment price indexes to identify investment-specific technological progress, our calibrated two-sector endogenous growth model can explain RnD-input in the US over the post-war period. It follows that the model explains the major part of US growth.

1.1 Introduction

Does more research and development (RnD) yield faster growth? The defining studies in RnD-based endogenous growth theory, notably Romer (1990), Aghion and Howitt (1992) and Grossmann and Helpman (1991), all share one central prediction: Economies that exert more effort in RnD experience faster growth.

Yet, when this hypothesis was first put to a test, by Jones (1995), it was strongly rejected. From the sixties through the eighties employment in RnD increased substantially. Productivity growth, however, did not accelerate. If anything, it declined.

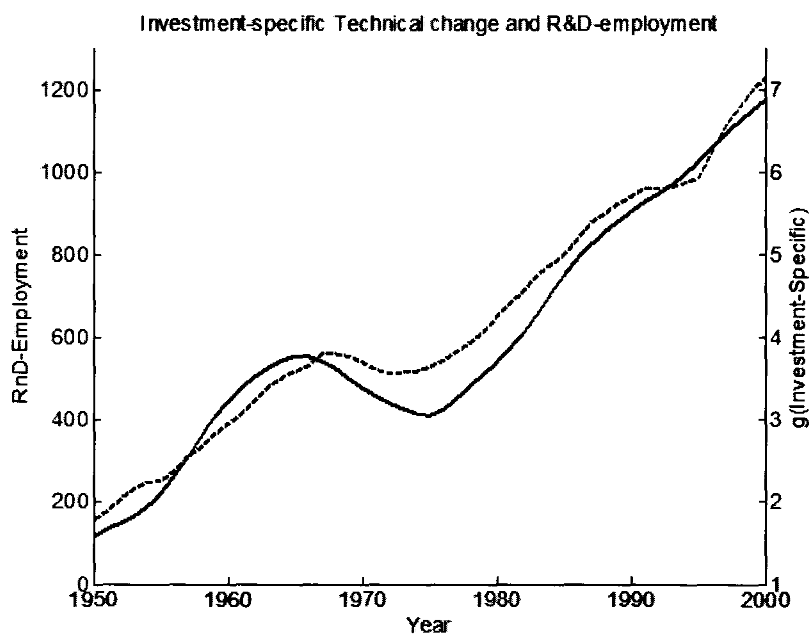
In contrast, we find that productivity growth did increase during the post-war era in the US. Furthermore, we document that changes in productivity growth coincide with changes in RnD-input in the same way as predicted by early endogenous growth theory. We find no reason to reject the scale-effect in productivity growth. This is our empirical contribution.

Jones studied growth in total factor productivity (TFP). We instead focus on growth in investment-specific productivity.

The quantitative analysis by Greenwood, Hercowitz and Krusell (GHK,1997) "suggests that investment-specific technological change accounts for the major part of growth". Thus, even though investment-specific productivity does not account for all productivity change over the period, the understanding of the sources of investment-specific technical change is clearly paramount to the understanding of economic growth.

RnD-employment and the growth rate of investment-specific technical change do not only share a positive time-trend. In fact, employment in research-activities declined between the mid-sixties and mid-seventies. The growth rate of investment-specific technical change also decreased during the same period. This is depicted in Figure 1. Evidently, there exists a correlation between RnD-employment and investment-specific productivity in excess of a common trend.

Figure 1



R&D Employment (000s) in the USA on the left-hand axis (grey, dashed). The growth-rate of investment-specific technical change on the right-hand axis. RnD-employment from Jones (2002),updated as described in Appendix A. Productvity growth is from Cummins and Violante (2003). Producitivity growth is the trend from a decomposition, using a Hodrick-Prescott filter with loss-parameter ($\lambda = 100$)

Why do economies do so much RnD? In endogenous growth models, RnD-input and ensuing productivity are equilibrium objects. Thus, a correctly specified model predicts RnD-effort and productivity outcomes with accuracy. Yet, endogenous growth theory has not previously been applied quantitatively for this purpose.

Previously, endogenous growth theory has been employed to predict productivity outcomes – given observed RnD-input. These reduced-form attempts have been moderately successful. Howitt and Ha (2006) investigate several specifications of the process by which RnD is translated into productivity. They note that the proposed formulations of this relationship do not perform "much better than a linear time trend in explaining the time series of productivity".

It is, thus, not surprising that attempts to predict both RnD-input and productivity growth by the use of a structural model are absent from the literature. Our empirical findings allow us to remedy this deficiency. This is our second contribution.

We create a two-sector endogenous growth model, where RnD augments the productivity of Equipment-production. The model is designed to be simple and congruent with our empirical findings. We follow GHK in the disaggregation of the economy into two sectors. One sector produces industrial equipment and the other produces consumption goods. We assume that technical progress in the Equipment-producing sector is endogenous. Like Romer (1990) we assume that RnD increases the variety of intermediate goods, used in the production of Equipment.

We use our empirical results to calibrate the correspondence between RnD and productivity; the production function for ideas. We pick remaining parameters from Greenwood et al. In the quantitative experiment we let the model predict RnD and investment-specific technical change, as the economy transits from one balanced growth path to another. We assume that the economy grew along a balanced growth path before 1950 and that it will again arrive on another balanced growth path some time after 2050. Shocks to aggregate employment during the transition are assumed to be exogenous. We let our model solve for equilibrium allocations and prices during the transition. The predicted RnD-employment from this experiment is on average 3 per cent higher than in the data.

The remainder of this paper is organized as follows: In Section 2, Background, we review the preceding literature with regard to empirical studies on RnD and productivity growth, research-based endogenous growth models and two-sector macroeconomic models. In section 3 we test the hypothesis that there exists a positive relationship between RnD-effort and productivity growth. We focus our investigation on investment-specific productivity growth. We use price-based measures of this productivity, as suggested by GHK. We find that there exists a positive correlation between RnD-employment and the growth-rate of productivity. In section 4 we create a two-sector endogenous growth model that is congruent with our empirical findings. We extend the framework suggested by GHK with endogenous RnD-based investment-specific productivity. Like Romer (1990) we let RnD increase the range of available intermediate goods used in production. In section 5 we calibrate our model and undertake a quantitative experiment in which we let the model predict RnD-employment and productivity and all other equilibrium varialbles. We assess the sensitivity of the results from our experiment to our model-specification and parameterisation. Suggestions for other specifications are discussed. Section 6 concludes.

1.2 Background

The first wave of endogenous growth models shared the prediction that more RnD yields faster productivity growth. This is commonly referred to as the 'scale-effect' in growth (see Jones (1999) for a thorough review of the debate on scale-effects in growth, and a full discussion of the distinguishing properties of the competing theories). In these models RnD produces ideas[1] that their owners can supply monopolistically to goods-producers. The process by which RnD-input is converted to ideas is called 'the production-function for ideas'. The cost for RnD is recouped from the profits stemming from these monopolies. In short, larger economies offer larger profits to intermediate goods producers. Therefore, larger economies devote more resources to RnD, and thus experience faster growth.

When tested, by Jones (1995), this hypothesis was rejected. Growth in TFP did not accelerate from the sixties to the eighties, even though RnD-employment three-folded.

Another attempt to assess the validity of the early endogenous growth models was undertaken by Kortum (1997). Instead of measuring RnD-output by TFP he studies output of patents per researcher. Kortum reaches the same conclusion as Jones. The number of granted patents did not accelerate in response to increased RnD-employment.

In a response to these findings, several studies – notably Jones (1995b), Kortum (1997) and Segerstrom (1998) – present models in which larger economies do not experience faster growth. Common to these studies is that RnD is assumed to become progressively more difficult, as productivity increases. Then, a constant growth rate necessitates constantly growing RnD-input. Consequently, the equilibrium productivity growth rate is pinned down by the growth rate of the population and other exogenous parameters. Therefore, these models are often referred to as 'semi-endogenous' growth models.

Another strand of endogenous growth models (for example Young (1998), Aghion and Howitt (1998) and Peretto (1998)) assumes that the variety of intermediate goods increases with the size of the economy, and that the quality of these goods is augmented by RnD. In these models, productivity growth is invariant to scale. Larger economies face a more difficult task, since more products have to be updated in order to uphold constant growth. In these models, growth is not determined by nature only. They are therefore referred to as the second generation endogenous growth models.

Howitt and Ha (2006) find that the semi-endogenous formulation consistently under-predicts productivity outcomes. Also, performing various cointegration-tests they find that the second generation endogenous growth formulation of the production function for ideas is more consistent with the data.

[1] In the literature, the terms ideas, patents and blue-prints are used inter-changeably. All these terms refer to an object that is cost-less to use, costly to produce and augments the productivity of some other factor of production.

We find that RnD-employment and investment-specific productivity are positively correlated. One important implication of this finding is that we do not find reason to assume that RnD becomes more difficult, regardless of the source of increasing difficulty. We, thus, do not reject the scale-effect in growth.

Consequently, our findings imply that long-term productivity growth is possible even if the population is non-increasing. In fact, constant productivity growth necessitates a constant population. Naturally, our view of the long-term prospects for the US Economy is thoroughly different from the view stemming from the models referred to above. We predict accelerating productivity growth as long as the US population is increasing, whereas Jones (2002) predicts a significant slow-down in productivity growth in the long run.

We focus on the relationship between investment-specific technical change and RnD-input. Greenwood et al document that investment-specific technological change accounts for the major part of productivity growth. Furthermore, during the post-war era in the U.S RnD-employment has been concentrated to manufacturing industries - and in particular to Equipment producing industries. For most of our period the share of RnD-employment in Equipment producing industries was above 70 per cent[2].

The idea that RnD affects the production of capital-goods is not new to the literature. The suggestion by Romer (1990) that RnD increases the available variety of intermediate goods indeed implies that technical change is embodied. Our addition is that RnD only augments the variety of intermediate goods that are used in the production of durable goods.

Furthermore, our study is not the first to document a positive relationship between RnD-input and productivity. In a cross-sectional analysis, Wilson (2002) shows that durable goods industries that undertake more RnD experience larger price declines in their output on average. We add to this finding the relationship between aggregate RnD-employment and the aggregate price decline of durable goods.

Other studies have stressed that productivity change need not only stem from RnD-effort. Comin (2004) examplifies that changes in managerial and organisational practices and also learning by doing could increase TFP without any corresponding investment in RnD. If, as Wilson documents, RnD affects the productivity of the industry at which it is directed, sector-specific productivity is a better measure of RnD-output than TFP. Furthermore, if changing managerial practices and learning by doing occur in all industries, the relative productivity of one sector does not change in response to these events.

Krusell (1998) proposes a model where RnD augments the productivity of investment. Krusell assumes that there is a fixed amount of intermediate-goods varieties. Intermediate goods producers supply one variety monopolistically. They improve their own products by conducting RnD. In our model, the quality of intermediate goods is constant – but the variety expands. Moreover, there is free entry into research and development in our model. Both models share the feature that RnD reduces the relative price of capital goods.

Gertler and Comin (2006) incorporate endogenous investment-specific technical change in their study of medium-term cycles. Our work complements this study by providing an explanation of the long-term trend

[2]See Appendix A Data

in investment-specific productivity.

GHK derive their measure of investment-specific technical change from the quality-adjusted price of durable goods relative to that of non-durables. They create a simple two-sector model of the economy. One sector produces consumption goods and structures. The other durable goods. If factor markets clear, increased productivity in one sector yields a corresponding decline in the relative price of goods from this sector.

The quality-adjusted relative price of durables, used in GHK, is the ratio between quality-adjusted deflator for durables reported by Gordon (1990) and the standard NIPA-deflator for non-durable consumption and services. Gordon uses hedonic techniques to create the deflator for durable goods[3]. In our empirical work, we use the same method to infer investment-specific productivity as GHK do. We, however, also infer investment-specific technical change from NIPA-deflators only. This change in methodology does not alter our conclusion: There exists a significant positive correlation between RnD and investment-specific technical progress. We do not reject the scale-effect in RnD.

1.3 Time-series tests of two-sector endogenous growth models

Previous empirical work in endogenous growth theory has focused on documenting a relationship between TFP-growth and RnD-input. Howitt and Ha (2006) note that "the central component of any R&D-based growth model is a knowledge-creation function, according to which the flow of new knowledge ΔQ depends on RnD input and other variables. In these models Q measures productivity as well as knowledge, so that specifying a knowledge-creation function is equivalent to specifying a productivity-growth function, according to which the growth rate $g_Q \equiv \frac{\Delta Q}{Q}$ of TFP is a function of RnD input and other variables"[4].

In contrast to Howitt and Ha (and other previous studies) we do not limit our study to the relationship between aggregate TFP-growth and RnD-input. On the contrary, we study the relationship between investment-specific productivity growth and RnD-input. We will later formulate a framework where investment-specific innovations are endogenous and the result of RnD.

Using a different productivity-measure, we investigate several specifications of the relationship between RnD and productivity growth - the knowledge-greation function or the production function for ideas - that have been suggested previously. Thus, while we agree with previous studies on the importance of documenting a relationship between RnD-input and productivity growth, we differ in our measure of productivity growth.

[3]This measure of investment-specific technical change has been used in several studies. See, for example, Krusell, Ohanian, Rios-Rull & Violante (2002), Hornstein, Krusell &Violante (2006) and Cummins & Violante (2003) for studies that explore implications of investment-specific technical change as measured from declining relative price of capital-goods.

[4]In the quote we have changed the notation. We denote productivity by Q instead of A. Moreover, in our study time is discrete.

1.3.1 The production function for ideas

An essential part of any research-based endogenous growth model is a function that maps RnD-effort (and possibly other variables) into productivity outcomes. This is commonly referred to as 'The Production Function for Ideas'. Since the study by Jones(1995) much theoretical work has focused on the parameterization of this function. In particular, several models (as referred above) have been developed to reconcile Jones' empirical findings with endogenous growth theory[5].

A generalized form of the production function for ideas, that nests all differing views of the functional form of the production function for ideas is:

$$\Delta_{t+1}(\cdot) = \varphi \left(\frac{L_{Qt}}{L_t^{\eta}} \right)^{\gamma} Q_t^{\phi} \tag{1.1}$$

Δ_{t+1} is the incremental change in productivity Q from time t. L_Q is employment in research and development. All endogenous growth models have $0 < \gamma \leqq 1$ so that productivity is increasing in research employment. If $\gamma < 1$ there are decreasing returns to scale in RnD. Howitt and Ha (2006) classify endogenous growth theory into First-generation fully endogenous, Semi-endogenous and Second generation fully endogenous growth models. The parameter-values assumed in these theories are summarized in table 1 below (from Howitt and Ha).

Table 1. Classifying various growth models

	γ	η	ϕ
Neo-classical	$=0$	$-$	$=1$
Fully endogenous I	>0	$=0$	$=1$
Semi-endogenous	>0	$=0$	<1
Fully endogenous II	>0	$=1$	$=1$

Before we discuss these different specifications, it will be useful to re-arrange the production function for ideas. We define $g_Q \equiv \frac{\Delta_{t+1}(\cdot)}{Q_t}$ which, along a balanced growth path, is constant. We have;

$$g_Q = \varphi \left(\frac{L_{Qt}}{L_t} \right)^{\gamma} \frac{Q_t^{\phi-1}}{L_t^{\gamma(\eta-1)}}$$

The first generation fully endogenous growth models assumed $\phi = 1$ (and implicitly $\eta = 0$). This implies that the growth rate of productivity is independent of the current state of technology. Moreover, it

[5]For a thorough overview of this debate, see Jones (1999) and Howitt and Ha (2006)

also implies that more R&D-employment (L_Q) yields faster productivity growth, also in the long run. We immediately see that balanced growth is incompatible with population growth. A growing work-force implies more research-employment, since the share of RnD-workers has to be constant.

Jones (1995) found that over the period 1960-1987 the growth rate of TFP did not increase, despite a massive contemporaneous increase in RnD-employment. Jones (1995) suggests that the producitivty of researchers is decreasing in the current state of technology. Correspondingly, Jones suggested $\phi < 1$ (and implicitly $\eta = 0$). We see that the growth rate along a balanced growth path will be pinned down by the growth rate of the labor force[6].

If we, however, have $\phi = 1$ and $\eta = 1$ we see that the growth rate of productivity is independent of the scale of the economy, yet fully endogenous. These are the assumptions of the second generation fully endogenous growth models. Productivity growth depends on the share of the economy that is devoted to RnD-activities.

Howitt and Ha undertake several empirical tests to discriminate between the competing views of the production function for ideas. They find that the second generation fully endogenous growth formulation is more consistent with the data than alternative formulations. Yet, they note that the predictive success of this class of models is limited.

Like previous studies, our empirical work aims at discriminating between these competing views of the correspondence between RnD-input and productivity growth. In contrast to previous studies, our measure of productivity growth is investment-specific technical progress.

Greenwood, Hercowitz and Krusell (1997) decompose the economy into two sectors. One sector produces durable goods, and the other produces non-durables, services and structures. They demonstrate how technical change that only affects the productivity of the production of durables yields a corresonding change in the relative price of theses goods. Investment-specific productivity can, thus, be inferred from changes in the relative price between durables and non-durables. We use the same measure of investment-specific technical change as Greenwood et al do.

In their study, GHK account for quality-changes in durables. They infer the quality-adjusted relative price of durables from the hedonic deflator for durable goods constructed by. Gordon (1990) and deflators from the National Income and Product Accounts (NIPA) for non-durables, services and structures. Changes in the ratio between these deflators reflect changes in sector-specific productivities. During the period studied in GHK, the quality-adjusted price of durables declines. This implies that investment-specific technical change was substantial.

Gordon constructs the quality-adjusted deflator for durables for the years 1947-1983. GHK use econometric techniques to extend the deflator reported by Gordon until 1990. More recently, Cummins & Violante (2003) use similar techniques to extend Gordon's series from 1983 to 2000. Cummins and Violante also report

[6] Along a balanced growth path $\frac{Q_t^{\phi-1}}{L_t^{\gamma(\eta-1)}}$ is constant. Then, for $\phi < 1$, the growth rate of Q is a function of the growth rate of L. This, obviously, holds for all $0 \leqq \eta < 1$.

growth-rates in relative productivity (investment-specific technical change) for these years. We use Cummins and Violante's series as our data for relative prices, and thus for investment-specific technical change.

Jones (1995) finds no correlation between employment in RnD-activities and growth-rates in TFP. Jones uses data from the National Science Foundation on the number of scientists and engineers engaged in RnD-activities during the period 1950 to 1993. We use Jones' data as our measure of RnD-employment. We extend Jones' series to 2000 using data from the OECD. As a point of reference, we also include TFP-data, as reported by Jones (1995)[7].

Before we undertake empirical tests, we describe the data.

Table 2. Description of the data

	g_q	L_Q	g_{TFP}
t^a	0.0918**	18.9077**	−0.0008
	(4.6299)	(33.1957)	(−2.0139)
α_{ADF} ^{b}	0.1258	5.5608	0.1685
ϕ_{ADF}	−7.5264**	2.0513	−4.9546**

a. The reported coefficient in the first row is β from a regression:
$x_t = \alpha + \beta t + \epsilon_t$
where x is the variable in the column. t−statistic in parenthesis below.
b. The reported coefficient α_{ADF} is the estimated autoregressive coefficient from an augmented Dickey-Fuller test with one lagged difference. ϕ_{ADF} is the ADF test-statistic. The critical value at the one-per cent level is −3.56.

Research-employment shows a positive and significant trend. Our measure of investment-specific technical change also shows a positive and significant trend. In other words, investment-specific productivity grows faster over time. This contrasts to the TFP-growth rate that shows no significant time-trend. We also see that there is considerable short-term variation in the two productivity growth-series.

The positive time-trend in the rate of investment-specific productivity growth is inconsistent with the balanced growth path versions of the endogenous growth models without scale-effects discussed above.

Greenwood et al decompose productivity growth into investment-specific and neutral productivity growth. They find that the two measures of productivity evolve very differently over time. Whereas investment-specific productivity is increasing over their studied period, neutral productivity is falling from the seventies on. This slowdown explains how growth in TFP from a one-sector aggregation can lack a positive time-trend, while investment-specific productivity growth from a two-sector aggregation can possess such a trend. Investment-specific productivity does not explain all of the change in US productivity growth over the past-war period, but Greenwood et al. note that it explains the major part.

Despite the strong positive time-trend, we reject non-stationarity for our measure of investment-specific technical change. We conclude that there is considerable short-term variation in our growth series. This

[7] See Appendix A (Data) for details.

variation is of sufficient magnitude to mute the observed positive trend, when testing for stationarity. The endogenous growth models that we have referred to above do not explain such short-term variation. They do not include frictions and do not aim at explaining any cycles. Therefore, in our tests below, we filter our data using the Hodrick-Prescott filter with a loss-parameter of 100. We argue that these filtered data better correspond to the phenomenon that the endogenous growth models under study are designed to explain; economic growth over the longer term.

Taking logs of the generalized production-function for ideas (1.1), we have;

$$\log g_Q = \log \varphi + \gamma \log L_{Qt} - \gamma\eta \log L_t + (\phi - 1) \log Q_t \tag{1.2}$$

We test different versions of this equation. Results are reported in Table 3.

Table 3. Tests of production function for ideas

	$\log g_Q^{HP}$	$\log g_Q^{HP}$	$\log g_Q^{HP}$	$\log g_Q^{HP}$	$\log g_Q^{HP}$	$g_Q^{HP,NIPA}$	$g_Q^{HP,NIPA}$
$\log L_{Qt}$	0.90**		0.71**	0.73**	0.84**	3.33**	2.46**
$\log L_t$	−1.28**						
$\log Q_t$	0.57**	1.99**	0.02				
$\log\left(\frac{L_{Qt}}{L_t}\right)$		1.10**					
detrended	*no*	*no*	*no*	*no*	*yes*	*no*	*yes*

Tests of different versions of the regression (1.2). The columns refer to the dependent variable. Rows report coefficients for the regressors. *) and **) refer to significance on the 5 and 1 per cent level, respectively. A constant term is included in all regressions that use data that is not detrended. Superscript HP indicates that the data has been filtered, using the HP-filter with a loss-parameter of 100. Superscript $NIPA$ indicates that growth has been inferred from NIPA-deflators directly - instead of using the quality-adjusted series reported by Cummins and Violante.

When all variables are included, and we impose no restrictions on the parameters, only the coefficient on research labor is within the range suggested by theory. The coefficient on technology is positive. This is inconsistent with any endogenous growth model that possesses a balanced growth path. We also note that our implied estimate of η is greater than 1, which is at odds with the formulation of the production function for ideas suggested by the second generation endogenous growth models.

If we - as suggested by the second generation endogenous growth models - restrict η to be 1, our coeffictient on current productivity is well above 0, implying absence of a balanced growth path. Semi-endogenous growth theory proposes that the coefficient on the current state of technology is negative. In our regressions our estimate of the coefficient is never negative and significantly different from zero.

The proposal of the first generation fully endogenous growth models - that the coefficient on RnD-employment is positive holds true for all our regressions. In fact, the simplest specification, that only includes the R&D-labor force and a constant term, explains the variation in productivity growth very well, which can be seen in Figure 1 above. The R^2 from this regression is 0.96.

Interestingly, the coefficient for the state of productivity (Q) is insignificant when the semi-endogenous formulation of the production function for ideas is tested (table 3, column 3).

Since our measure of RnD-labor and our measure of productivity growth are trending positively, correlation could be spurious. Yet, when we remove a time trend from both series, the explanatory power of RnD-input is large ($R^2 = 0.87$) and the coefficient on RnD-labor is positive and significant. We conclude that there exists a covariation between these series in excess of a common time-trend. Visually, this can be seen from Figure 1.

We infer changes in investment-specific productivity from changes in relative prices. We measure these changes using the investment-specific productivity growth series reported by Cummins and Violante (2003). They update Gordon's hedonic deflator for durables. There are two main objections to this procedure:

First, durables and non-durables are deflated with different methodologies. Since the deflator for non-durables does not account for quality-changes in the same way as the deflator for durables, the decline in the relative price of non-durables could be over-stated. Furthermore, this bias need not have a specific time-trend. Thus, any movements in the relative price of durables could be remnants of an erroneous statistical procedure.

Second, Gordon only reports the deflator for durables until 1983. After this year, some estimate of an equivalent of Gordon's deflator has to be used. We use the deflator reported by Cummins and Violante. Thus, even if we assume that the ratio between Gordon's deflator and the NIPA-deflator for non-durables does reflect the true quality-adjusted relative price, we may measure the deflator for durables erroneously for the period after 1983 .

We address this criticism by including an alternative relative price of durables. We derive the relative price of durables from NIPA-deflators only. This price also declines over time, albeit at a slower pace than when Gordon's deflator is used. Nevertheless, the rate of this decline is also increasing, implying that investment-specific technical change is accelerating. Moreover, as is the case with the series reported by Cummins and Violante, the relative price decline deduced only from NIPA-deflators slows down between the mid-sixties to the mid-seventies. Hence, we document that the correlation between RnD and investment-specific productivity is not unique to the use of data stemming from Gordon's deflator with extensions.

In column 8 and 9 we report results from regressions where we infer investment-specific technical change from the NIPA-data directly. We infer relative prices from the deflators for Equipment and Software and a Törnqvist-aggregate for non-durable consumption goods and non- housing-services. We find a positive and significant co-variation between research-input and productivity growth. Moreover, also when we remove a time-trend, the coefficient remains positive and significant.

Evidently, the choice of measure of productivity growth - $TFP-$growth or investment-specific technical progress - does influence our conclusions with regard to the relationship between research and development and productivity growth. The significant positive correlation between productivity growth and research cannot be detected if TFP is the chosen measure of productivity.

We have found that there exists a positive correlation between the level of RnD employment and the rate of investment-specific technological progress. The relationship between the level of RnD-employment and productivity growth is well described by the production function for ideas suggested in the first generation endogenous growth models. We do not reject the scale-effect in productivity growth.

1.4 A model with endogenous investment-specific productivity growth

We have found that there exists a positive correlation between RnD-employment and investment-specific technological progress. In this section we create a model that is consistent with this finding. There are two goods-producing sectors in the economy - the consumption goods sector and the investment goods sector. Output in each sector is a function of the input of labor, capital and the current state of technology in each of the sectors. Productivity specific to the investment-goods sector is the outcome of historical R&D-effort.

Our model is an extension of the basic framework suggested by Greenwood, Hercowitz and Krusell. We endogenize investment-specific productivity. We assume that RnD increases the variety of available intermediate goods that are used for production of investment goods. This is the same mechanism as Romer (1990) suggests.

For expositional ease, R&D is assumed to be undertaken by firms specialized in this activity. Also, we assume that there are financial intermediaries that supply households with financial assets.

1.4.1 The Environment

We assume that there are two different kinds of goods; consumption-goods (C) and investment-goods (I). Capital (K) is accumulated through the production of investment goods. Capital can - costlessly - be converted into intermediate goods (x).

Consumption goods are produced using capital (K) and labor (L) only. Investment goods are produced with intermediate goods (x) and labor.There is a continuum (Q) varieties of intermediate goods. Every variety corresponds to a unique idea that facilitates the production of this variety. A greater variety (Q) in intermediate goods, hence, corresponds to more ideas being available in the economy. Neutral productivity (Z) affects the produduction in both goods-producing sectors equally. Neutral productivity is exogenous to our model.

Ideas are produced using labor only.

For simplicity, we assume that goods-production in both sectors is characterized by Cobb-Douglas functions. In the calibration section we will return to the specific form of the production function for ideas.

Production in our economy is , thus given by:

$$C_t = Z_t K_{Ct}^{\alpha} L_{Ct}^{(1-\alpha)} \tag{1.3}$$

$$I_t = Z_t L_{It}^{(1-\alpha)} \int_0^{Q_t} x_t(\omega)^{\alpha} d\omega \tag{1.4}$$

$$\Delta Q_{t+1} = \Delta(L_{Qt}, Q_t) \tag{1.5}$$

We denote by ΔQ_{t+1} the change in the range of available ideas between time t and the following period. Labor can be allocated to either of the three sectors; production of investment goods or consumption goods or to research and development, which is production of ideas. Capital is used in the two goods-producing sectors. Thus, the resource constraints of the economy are:

$$L_{Ct} + L_{It} + L_{Qt} \leq L_t \tag{1.6}$$

$$K_{Ct} + K_{It} \leq K_t \tag{1.7}$$

$$\text{where} \quad K_{It} = \int_0^{Q_t} x_t(\omega) d\omega \tag{1.8}$$

Raw capital is used in consumption goods production. Capital that has been converted into intermediate goods (x) is used in the production of investment goods. There are (Q) varieties of intermediate goods, and thus all capital used in production of (I) is the sum of all intermediate goods input over all varieties. The aggregate capital stock obeys the law of motion;

$$K_{t+1} = (1-\delta) K_t + I_t \tag{1.9}$$

Note, also, that the production function for ideas specifies how the stock of ideas evolves over time.

1.4.2 Households

Households maximize the present-value of utility. Households provide labor inelastically, and only derive utility from consumption c. Households have access to a financial asset A. They, maximize discounted utility given their budget constraint.

$$U = \max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1.10}$$

$$s.t. \quad A_{t+1} = ((R_t - 1)(1-\tau) + 1) A_t + w_t - c_t + T_t \tag{1.11}$$

R_t is the gross financial interest rate and β is the discount rate. τ is the tax-rate on capital income. All tax-revenue is distributed back to households in lump-sum transfers T_t. The first order conditions from the households optimization problem yield the familiar Euler-equation;

$$\frac{u_{ct}}{u_{ct+1}} = \beta((R_{t+1} - 1)(1-\tau) + 1) \tag{1.12}$$

1.4.3 Financial intermediaries

Financial intermediaries can either invest in capital or patents[8]. We assume free entry into financial intermediation. This implies that financial intermediaries make no profits. The free entry assumption yields two no-arbitrage conditions; one with regard to investment in capital, and one with regard to investment in ideas.

Financial assets yield interest R_{t+1}. We normalize the price of consumption goods to 1 at all times. We denote by $\frac{1}{q_t}$ the relative price of capital in period t . Since there are no profits in financial intermediation, we - in equilibrium - have the following no-arbitrage condition:

$$R_{t+1} = \left(\frac{(1-\delta)}{q_{t+1}} + r_{t+1}\right) q_t \tag{1.13}$$

One unit of consumption goods can be exchanged for q_t units of capital at time t. Next period, the remaining (after depreciation) $q_t(1-\delta)$ units of capital can be exchanged for $\frac{q_t(1-\delta)}{q_{t+1}}$ consumption goods. Furthermore, q_t units of capital are compensated by $q_t r_{t+1}$ when used in production.

We assume that there is no infringement on patents. We also assume that patent-protection is eternal. An owner of a patent, then, is the sole receiver of the profit stemming from the productive use of the protected

[8] Throughout this paper we will refer to intermediate-goods varieties as patents or ideas. In the literature, blue-prints, patents and ideas refer to very similar concepts. In our paper, holding an idea/patent/blue-print yields the owner the right to control the use of it in goods-production, and to claim the profit stemming from this use.

idea. We denote by Π_{xt} the monopoly-profit - an expression we will return to shortly. We denote by V_t the value of a patent at time t. Then, in equilibrium, we have the following no-arbitrage condition:

$$R_{t+1} = \frac{1}{V_t}\left(\Pi_{xt+1} + V_{t+1}\right) \tag{1.14}$$

One consumption-good can be exchanged for $\frac{1}{V_t}$ patents at time t. Next period, these yield $\Pi_{xt+1}\frac{1}{V_t}$ profit. Also, the acquired patents can be exchanged for $V_{t+1}\frac{1}{V_t}$ consumption goods at this time.

1.4.4 Goods producing firms

We assume that there is perfect competition in the production of consumption-goods. Firms maximize profits Π_{Ct} taking prices for labor and capital-services - w_t and r_t - given. Firms, maximize

$$\Pi_{Ct} = \max_{K_{Ct}, L_{Ct}} \left\{ Z_t K_{Ct}^{\alpha} L_{Ct}^{(1-\alpha)} - w_t L_{Ct} - r_t K_{Ct} \right\} \tag{1.15}$$

The first order conditions from the optimization problem are:

$$w_t = (1-\alpha)\frac{C_t}{L_{Ct}} \quad ; \quad r_t = \alpha \frac{C_t}{K_{Ct}} \tag{1.16}$$

We assume that there is perfect competition in the production of investment-goods. There is a continuum of Q_t varieties of intermediate goods available to the economy at time t. Firms maximize profits Π_{It} taking prices of investment goods, labor and intermediate goods - q_t, w_t and $p(\omega)_t$ - as given. We have;

$$\Pi_{It} = \max_{L_{It}, x_t(\omega); \omega \in [0, Q_t]} \left\{ \frac{1}{q_t} Z_t L_{It}^{(1-\alpha)} \int_0^{Q_t} x_t(\omega)^{\alpha} d\omega - w_t L_{Ct} - \int_0^{Q_t} p(\omega)_t x_t(\omega) d\omega \right\} \tag{1.17}$$

Optimization yields;

$$w_t = (1-\alpha)\frac{I_t}{q_t L_{It}} \quad ; \quad \alpha \frac{1}{q_t} Z_t L_{It}^{(1-\alpha)} x_t(\omega)^{\alpha-1} = p(\omega)_t \qquad \nabla\omega \in [0, Q_t] \tag{1.18}$$

Intermediate-goods producers rent capital at r_t. They cost-lessly amend capital by the use of an idea. They let these amended - and thus differentiated - intermediate goods to investment-goods producers. Differentiated intermediate goods are payed $p(\omega)_t$ for their services. Since the technology used for the production of intermediate goods is proprietary, producers operate as monopolists.

The demand for intermediate goods is given by the first order condition with regard to intermediate goods from optimizing investment-goods producers.

Hence, producers of intermediate goods maximize profits, given the demand function from the investment-goods sector:

$$\begin{aligned} \Pi_{xt} &= \max_{x_t(\omega)} \{p_t(\omega)\, x_t(\omega) - r_t x_t(\omega)\} && (1.19) \\ s.t.\ p_t(\omega) &= \alpha \frac{1}{q_t} Z_t L_{It}^{(1-\alpha)} x_t(\omega)^{\alpha-1} \end{aligned}$$

Profit maximization implies;

$$p_{xt}(\omega)\, x_t(\omega) + p_t(\omega) = r_t \qquad \forall \omega \in [0, Q_t] \tag{1.20}$$

It follows that all producers of intermediate goods set the same mark-up price over the rental rate (r):

$$p_t(\omega) = \frac{1}{\alpha} r_t \qquad \forall \omega \in [0, Q_t] \tag{1.21}$$

1.4.5 Research and Development

Firms undertake R&D activities in order to acquire ideas that can be sold to intermediate-goods producers. The value of an idea is the discounted present value of all future monopoly profits stemming from the use of the patent. We assume that there is free entry into R&D-activities. Thus, R&D-firms make no profits. Aggregate profits of R&D-firms in the economy are;

$$\Pi_{Qt} = V_t \Delta(L_{Qt}, Q_t) - w_t L_{Qt} \tag{1.22}$$

Each new patent has value V_t at time t. An increment of ΔQ_{t+1} patented ideas are produced at this time. The total cost of this operation is $w_t L_{Qt}$. Free entry implies that we in equilibrium have;

$$V_t \frac{\Delta (L_{Qt}, Q_t)}{L_{Qt}} = w_t \tag{1.23}$$

1.4.6 Equilibrium

An equilibrium in our economy consists of quantities $\{c_t, A_t, K_{Ct}, K_{It}, L_{Ct}, L_{It}, L_{Qt}\}_{t=0}^{\infty}$ and prices $\{w_t, p(\omega)_t, q_t, r_t, R_t, V_t\}_{t=0}^{\infty}$ such that;

I. Consumers maximize intertemporal utility, and the Euler-equation (1.12) holds.

II. Firms maximize profits and conditions (1.16) , (1.18) and (1.20) hold.

III. The free entry condition in RnD (1.23) holds.

IV. There are no arbitrage-opportunities in financial intermediation, so that (1.13) and (1.14) hold.

V. Factor markets clear, implying that (1.6) and (1.7) hold with equality.

VI. The laws of motion for capital (1.9), assets (1.11) and ideas (1.5) hold.

VII. Goods production is given by functions (1.3) and (1.4).

This completes our description of the equilibrium. Below we use the model to derive the price-based measure of investment-specific technical change, that we used in our empirical work above. We also study some of the qualitative properties of the equilibrium, that aid us in understanding the outcomes of our quantitative exercises in section 5.

1.4.7 A price-based measure of investment-specific technical change

In a competitive equilibrium, where factors move freely between sectros, marginal products will be equal across sectors. This implies that, in equilibrium, relative prices will change in response to changes in sector-specific productivity. The relatively more productive sector will experience a fall in the relative price of its product. We see this by combining the first order conditions from profit maximization in the goods producing sectors. It is straight-forward to derive[9]:

$$\frac{q_{t+1}}{q_t} = \left(\frac{Q_{t+1}}{Q_t}\right)^{(1-\alpha)} \tag{1.24}$$

[9]Note that (1.21) implies that all intermediate-goods producing firms charge the same price. Thus, in all industries (ω) they supply the same quantity, x_t. Then $K_{It} = Q_t x_t$. From the profit maximization conditions in goods production it follows that the ratio between the capital-labor ratios in our two sectors is $\frac{1}{\alpha}$. If we, then, divide the first order conditions w.r.t labor in our in the consumption-producing sector with the equivalent condition from the investment-producing sector, we have q_t as a function of Q_t and parameters. Dividing two consecutive periods, the expression for our measure of investment-specific productivity results.

We see that growth in the productivity of the investment-producting sector results in a decline in the relative price $\left(\frac{1}{q}\right)$ of investment goods. This is the expression we have used to infer investment-specific technical change in our empirical work above.

1.4.8 Qualitative remarks

In Appendix C below, we study the properties of a balanced growth path equilibrium. In our quantitative experiment - section 5 below - we study the transition from one balanced growth path (assumed to characterize the US economy in 1950) to another (which is assumed to characterize the US economy some time after year 2050). During this transition the economy undergoes an exogenous shock to the working population (L) and neutral productivity (Z). Therefore we here focus on the consequences on the labor-market allocation of shocks to neutral productivity Z_t and the working population, L_t.

Researchers are remunerated for their production of new patents. Total wages in RnD are equal to the sum of the value of all new patents, as seen in the free-entry condition (1.23):

$$w_t L_{Qt} = V_t \Delta\left(L_{Qt}, Q_t\right)$$

The value of a patent is given by the no-arbitrage condition governing patents (1.14). Profits from holding a single patent follow from the first-order condition of investment-goods producers (1.18), the pricing rule (1.21) and the definition $K_{It} = Q_t x_t$:

$$\Pi_{xt} = (1-\alpha)\,\alpha \frac{\frac{1}{q_t} I_t}{Q_t} = \alpha w_t \frac{L_{It}}{Q_t}$$

The second equality follows from the first order condition with regard to labor in investment production. Here, we see that shocks to Z affect profits and wages proportionally. We substitute fro R_{t+1} from the arbitrage-condition from the capital-market (1.13) into the no-arbitrage-condition for RnD (1.14), re-arrange and have:

$$V_t = \Pi_{xt} \frac{\frac{\Pi_{xt+1}}{\Pi_{xt}}}{\left(\frac{(1-\delta)}{q_{t+1}} + r_{t+1}\right) q_t - \frac{V_{t+1}}{V_t}} \tag{1.25}$$

Combining this expression, the free-entry condition in RnD (1.23) and dividing by the first order condition with regard to labor in investment production we get:

$$\frac{L_{Qt}}{L_{It}} = \alpha \left(\frac{\frac{\Pi_{xt+1}}{\Pi_{xt}}}{\left(\frac{(1-\delta)}{q_{t+1}} + r_{t+1}\right) q_t - \frac{V_{t+1}}{V_t}} \right) \frac{\Delta\left(L_{Qt}, Q_t\right)}{Q_t} \tag{1.26}$$

The denominator only contains forward-looking variables, the state variable Q_t and the relative price q_t which is a function of Q_t as seen in (1.24). Hence, at unchanged allocations, a temporary positive shock in Z_t decreases the numerator by increasing current profit. A shock to Z only affects future variables through effects on the capital-stock. Consequently, temporary positive shocks in Z_t will re-allocate labor from RnD-activities to investment-production.

Along a balanced growth path, the level of Z will not affect the allocation between investment-production and RnD-activities. Along a balanced growth path, the financial interest rate $R_{t+1}$is a function of the constant growth rate in consumption, as seen from the Euler-equation. Thus, changes in Z only have temporary effects on allocations.

If employment shares do not change, an increase in L does not change the ratio L_{Qt}/L_{It}. As L_{Qt} increases, however, $\frac{\Delta(L_{Qt},Q_t)}{Q_t}$ increases. Then, along a balanced growth-path, L_Q/L_I increases. This holds if the long-run growth rate $\frac{\Delta(L_{Qt},Q_t)}{Q_t}$ is not exogenously determined[10].

We conclude that permanent changes to neutral productivity only have temporary effects on the labor-market allocation, whereas shocks to the working population (L) have lasting effects.

1.5 Experiment

Above, we have found that the first generation endogenous growth models provide us with a good description of the correspondence between R&D-input and investment-specific productivity growth. We now turn to assessing the relevance of our model by evaluating it's predictive power. We will conduct an experiment where neutral productivity (Z) and the working population (L) are exogenous variables. We will let the model predict both investment-specific productivity (Q) and research-employment (L_Q), besides all other endogenous variables.

In order to undertake our experiment, we first need data on neutral productivity (Z). We will decompose the US growth experience into changes in investment-specific productivity (Q) and neutral productivity (Z) using the same methodolgy as Greenwood, Hercovitz and Krusell. After calibrating the model, we proceed to quantitative experiments.

The original model by GHK also accounts for capital in the form of structures. We have, in the above, abstracted from this. In Appendix B we extend the model to account for structures capital. Below, in the sensitivity analysis, we report some findings from this extended model.

1.5.1 Decomposition

The rate at which investment-specific technical change is growing increases over the studied period. Yet, the traditional measure of productivity-growth - growth in TFP - is non-increasing during the same period. The cause of this dissimilarity is that the rate of neutral technical change, in our two-sector model, is

[10]The growth rate is exogenously determined in semi-endogenous growth models.

non-increasing over time. This has previously been documented by Greenwood, Hercovitz and Krusell who decompose productivity into investment-specific and neutral productivity, using a two-sector model that is very similar to the one proposed in this paper. Their decomposition shows that neutral productivity increased until the mid-seventies, and decreased thereafter.

In the following, we use our model to decompose productivity change into changes in investment-specific productivity and neutral productivity. We will use the series for neutral productivity (Z in the model above) as an exogenous input in our quantitative experiment. But by decomposing the growth experience we also illustrate quantitatively how the growth rate of investment-specific productivity can accelerate, without there being an equivalent acceleration in the growth rate of TFP from a one-sector aggregation.

Data

We use data from the National Income and Product Accounts (NIPA) of the BEA. Our model has two sectors; one that produces investment goods - that are not consumed. And one that produces consumption goods - that are not used for investment purposes. Our measure of consumption-production is the NIPA data of non-durable consumption and non-housing services. We use the corresponding BEA deflators aggregated by a Törnqvist-procedure to find the deflator for our consumption-data.

Our measure of investment-production is investment in Equipment and Software. We use the implied deflator from Cummins and Violante (2003) as our deflator for the investment-data.

We take data on employment from BLS; 'Current Employment Statistics survey'.

Procedure

In our decomposition we abstract from the allocation of labor to RnD activities. We assume that our data on employment reflect employment in the goods-producing sectors. We use the series for the relative price of capital q as reported by Cummins and Violante (2003). We calculate Q as described above;

$$\frac{1}{q_t} = \left(\frac{1}{\alpha}\right)^{\alpha} \frac{1}{Q_t^{(1-\alpha)}} \tag{1.27}$$

For our decomposition we need an estimate of α. In their two-sector model of the US economy, GHK report that the share of Equipment and software in total production is 0.17. They also report the share of structures to be 0.13. In our base-line decomposition, we abstract from the production and use of structures. We, accordingly, set $\alpha = 0.17/(1 - 0.13)$. In Appendix B, we describe an equilibrum which also takes structures into account. This extended model accounts for all sectors of the economy that GHK account for.

Dividing our first order conditions in goods-production w.r.t. labor we have;

$$\frac{\frac{1}{q_t}I_t}{C_t} = \frac{L_{It}}{L_{Ct}} \tag{1.28}$$

The ratio of labor-input in our respective goods-producing sectors is the same as the ratio between nominal production in the same sectors. Nominal production is readily observable. We assume that labor is distributed across sectors according to this expression.

Re-arranging the first order conditions from the goods-producing sectors we get:

$$\frac{\frac{K_{Ct}}{L_{ct}}}{\frac{K_{It}}{L_{It}}} = \frac{1}{\alpha} \tag{1.29}$$

We distribute capital across sectors accordingly.

The capital stock evolves according to the law of motion for capital. We follow GHK in setting the depreciation rate for Equipment $\delta = 0.124$. We assume that the economy initially is on a balanced growth path where all technical change is neutral, and neutral technical change is 2 per cent per year. We solve for the initial capital stocks using the balanced growth path conditions from the laws of motion for captial of this economy[11].

We estimate $\{Z\}_{1950}^{2000}$ by iterating over the following procedure;

i) We observe $\frac{\frac{1}{q_t}I_t}{C_t}$ and L_t. We calculate L_{It} and L_{Ct} according to (1.28) assuming that the labor market clears.

ii) We calculate K_{Ct} and K_{It} according to (1.29) given K_t

iii) We use our production functions to calculate production.

iv) We calculate Z_t by dividing the observed real production of consumption goods, C_t, (or investment goods I_t) with our predicted values

v) We calculate K_{t+1} from the law of motion for capital.

Having, thus, obtained our series for neutral productivity, Z, we can assess the relative contributions of neutral and investment-specific productivity to growth in real GDP. First, we note that in a two-sector model real GDP is an aggregate of output from the two sectors. In the NIPA, chain-weighted indexes are used to aggregate output from the different sectors of the US economy into the measure of real GDP. These chain-weighted indexes can be approximated by a Törnqvist-index. We choose this approximation.

In the Törnqvist-index, the growth rate of GDP is defined as the weighted average of the growth rates in the two sectors respectively. The weights are the average nominal shares in GDP. For our model this implies;

[11] We lack data on investment-specific technical change before 1947. Our assumption is that technical change evolved at the same pace in consumption- and investment-production. Note that this assumption only affects our estimate of the initial capital stocks. For Equipment, this is of limited consequence - since the depreciation rate is high.

$$\frac{Y_{t+1}}{Y_t} = \overline{s}_t \frac{C_{t+1}}{C_t} + (1-\overline{s}_t)\frac{I_{t+1}}{I_t} \tag{1.30}$$

$$\overline{s}_t = \frac{1}{2}\left(\frac{C_t}{C_t+\frac{1}{q_t}I_t} + \frac{C_{t+1}}{C_{t+1}+\frac{1}{q_{t+1}}I_{t+1}}\right) \tag{1.31}$$

Inserting our production-functions into (1.30) and re-arranging we get;

$$\frac{Y_{t+1}}{Y_t} = \frac{Z_{t+1}}{Z_t} * \frac{\left(\overline{s}_t \frac{K_{Ct+1}^{\alpha} L_{Ct+1}^{(1-\alpha)}}{K_{Ct}^{\alpha} L_{Ct}^{(1-\alpha)}} + (1-\overline{s}_t)\frac{K_{It+1}^{\alpha}(Q_{t+1}L_{It+1})^{(1-\alpha)}}{K_{It}^{\alpha}(Q_t L_{It})^{(1-\alpha)}}\right)}{\left(\overline{s}_t \frac{K_{Ct+1}^{\alpha} L_{Ct+1}^{(1-\alpha)}}{K_{Ct}^{\alpha} L_{Ct}^{(1-\alpha)}} + (1-\overline{s}_t)\frac{K_{It+1}^{\alpha} L_{It+1}^{(1-\alpha)}}{K_{It}^{\alpha} L_{It}^{(1-\alpha)}}\right)} * \left(\overline{s}_t \frac{K_{Ct+1}^{\alpha} L_{Ct+1}^{(1-\alpha)}}{K_{Ct}^{\alpha} L_{Ct}^{(1-\alpha)}} + (1-\overline{s}_t)\frac{K_{It+1}^{\alpha} L_{It+1}^{(1-\alpha)}}{K_{It}^{\alpha} L_{It}^{(1-\alpha)}}\right) \tag{1.32}$$

The first factor in this expression is the contribution to real GDP-growth from neutral technical change. The second is the contribution of investment-specific change. We define;

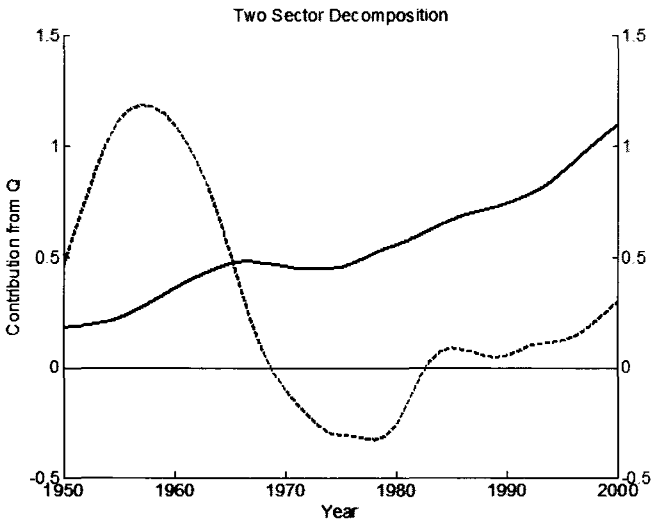
$$\widetilde{G}_Q \equiv \frac{\left(\overline{s}_t \frac{K_{Ct+1}^{\alpha} L_{Ct+1}^{(1-\alpha)}}{K_{Ct}^{\alpha} L_{Ct}^{(1-\alpha)}} + (1-\overline{s}_t)\frac{K_{It+1}^{\alpha}(Q_{t+1}L_{It+1})^{(1-\alpha)}}{K_{It}^{\alpha}(Q_t L_{It})^{(1-\alpha)}}\right)}{\left(\overline{s}_t \frac{K_{Ct+1}^{\alpha} L_{Ct+1}^{(1-\alpha)}}{K_{Ct}^{\alpha} L_{Ct}^{(1-\alpha)}} + (1-\overline{s}_t)\frac{K_{It+1}^{\alpha} L_{It+1}^{(1-\alpha)}}{K_{It}^{\alpha} L_{It}^{(1-\alpha)}}\right)}$$

The third factor is the contribution from changes in factor inputs.

Results from the decomposition

Greenwood, Hercowitz and Krusell decompose productivity growth into neutral and investment-specific technical change. They find that neutral productivity contributes negatively to growth during the seventies and the eighties. In our decomposition neutral technological change slows down considerably after the mid-sixties. This can be seen in Figure 2. During the seventies the contribution from neutral technological change is negative. From the eighties and on, the contribution from neutral technological change is modest but positive. GHK account for capital in the form of structures. This explains the different results for the eighties.

Figure 2



Contributions (in percentage points) to growth from neutral technical change (Z,grey/dashed) and investment-specific technical change (Q). The series depict the percentage changes corresponding to the first and second factors of equation (1.32). The contribution from Z is $(\frac{Z_{t+1}}{Z_t} - 1)*100$. The contribution from Q is $\left(\tilde{G}_Q - 1\right)*100$. The series are the trends from the data, using a Hodrick-Prescott filter with loss-parameter ($\lambda^{HP} = 100$).

The contribution of investment-specific technical change to productivity growth is increasing over the post-war period. In absolute terms, the contribution is around a quarter of a percentage point during the fifties, and more than three quarters during the nineties. In relative terms, the contribution of investment-specific technical change is increasing. During the fifties, the contribution from investment-specific technical change to total productivity growth was approximately 20 per cent (0.25 percentage points out of a total of 1.25). During the nineties, the contribution is around 80 per cent. And during the seventies, only investment-specific technical change contributes positively to aggregate productivity growth.

We see how the increasing rate of investment-specific productivity growth can be concealed by large variations in neutral technical change if a one-sector model is used to aggregate the economy. This is likely to explain why the positive correlation between R&D-input and productivity growth that we document has remained undetected previously.

In our decomposition we have studied a subset of the American economy. We have abstractred from the consumption of durable goods, and the investment in structures. Below, in Appendix B, we extend our model to include investment in structures. We thus create an endogenous growth verision of GHK. In the extended model investment-specific technical change is the sole contributor to positive productivity growth from the late sixties through the eighties, as has previously been documented by GHK. Neutral productivity growth picks up and is positive during the nineties. When investment in structures is included, neutral technical change is growing somewhat slower during the fifties and the early sixties than in the reduced model.

1.5.2 Calibration

Above, we have not specified the consumption-utility function. As Greenwood et al. we assume that utility is logarithmic.

$$u(c_t) = \log c_t \tag{1.33}$$

Our base-line model, then, has six parameters to calibrate. We choose all parameters that govern preferences and goods-production from GHK. As in our decomposition above, we set $\alpha = 0.195$ and $\delta = 0.124$. We apply the discount rate suggested by the same authors, $\beta = 0.95$. We also set the capital income tax rate to 42 per cent, as GHK do.

Table 3. Calibration - Baseline model

Parameter	Value	Comment
α	0.195	Adjusted Equipment share from GHK
δ	0.124	GHK
β	0.95	GHK
γ	0.73	OLS-estimate
φ	$4.31 * 10^{-4}$	OLS-estimate
τ	0.42	GHK

In our empirical work, we found that the production function for ideas is well described by the first generation endogenous growth models. Productivity growth is a function of research-labor and parameters only. There is no increasing difficulty in RnD.

The production-function for ideas is:

$$\Delta(L_{Qt}, Q_t) = \varphi L_Q^{\gamma} Q_t$$

We calibrate the production-function for ideas using our ordinary least squares estimates of γ and φ from the corresponding logarithmic regression. As above, we find the data on $\frac{\Delta(L_{Qt}, Q_t)}{Q_t}$ by converting the relative-price-series reported by Cummins&Violante. Before running the regression, we filter the series using the HP-filter ($\lambda = 100$) . We use the same data as Jones (2002) for RnD-employment, but we extend the series to year 2000 as described above.

1.5.3 Quantitative experiment

We predict the growth experience of the US Economy during the post-war period. Our model predicts the RnD-labor force as a function of the size of the economy - in terms of the employment level (L), current neutral productivity (Z), current investment-specific productivity (Q) and the capital-stock at hand (K). The RnD-effort, in turn, determines investment-specific productivity next period. Agents choose consumption optimally to maximize discounted utility. Wages are set so that the labor market clears, and labor is allocated across goods-production and research activities according to optimizing behaviour of firms.

Our experiment is the following:

The economy is assumed to grow along a balanced growth path in 1949. This assumption allows us to solve for the capital stock at hand in 1950. Along this balanced growth path, TFP-growth is assumed to be 2 per cent per annum.

In 1950 the economy is hit by a composite shock. Agents become perfectly informed that the working population (L_t) will evolve exactly as measured in our data during the period 1950 – 2000. Furthermore, neutral productivity during this period will assume the values that were reported from our decomposition above.

After year 2000, and until 2050, employment is assumed to grow at an annual rate of 2.1 per cent. This is the average employment growth rate during 1950 – 2000. During the same period, neutral productivity is assumed to grow at 0.6 per cent per annum. After 2050,employment is assumed to be constant. Neutral productivity is also assumed to be constant.

Our experiment is to solve for the transition between one balanced growth path, that characterized the economy in 1950 to another balanced growth path - to which the economy will have converged some time after 2050.

Table 4. Composite shock

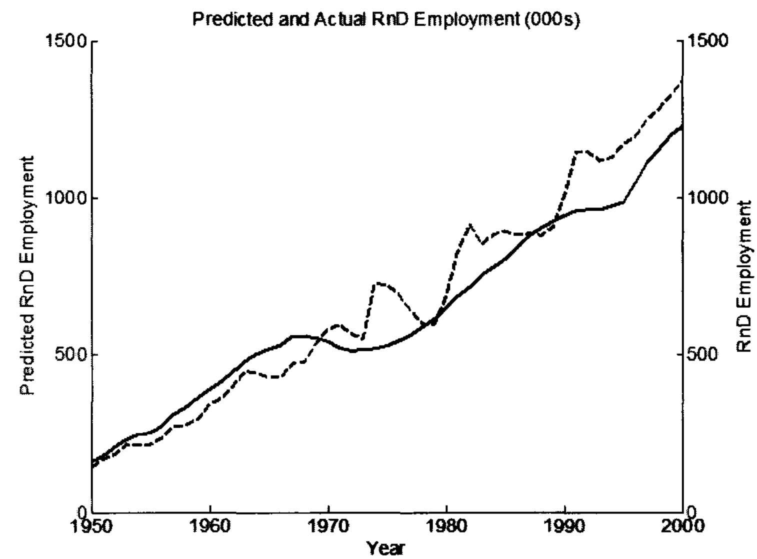
Exogenous variation		Source
$\{L_t\}_{t=1950}^{2000}$		Current Employment Statistics survey
$n_{2000-2050}$	2.1%	Average employment growth 1950-2000
$\{Z_t\}_{t=1950}^{2000}$		Two-sector decomposition above
$g_{Z;2000-2050}$	0.6%	Average neutral productivity growth 1950-2000

When solving the described problem, we add 50 periods after 2050, during which employment and netural productivity are constant, in order to allow the economy to converge to the new balanced growth path. This takes approximately ten years.

1.5.4 Predictions from the experiment

As we have seen in our qualitative remarks above, the model predicts that a larger working population (L) implies more employment in research and development. The model has been calibrated using parameter estimates from the study by Greenwood et al. Additionally, the production-function for ideas was calibrated using data on RnD-employment and investment-specific productivity outcomes only. Note that data on the working population or neutral productivity have not been used in our calibration of the model. Employment in research-activities is an equilibrium outcome of our model, as described above. One way of assessing the relevance of our model is to compare our predictions with the data. If the predictions are in the vicinity of the data, we conclude that our model does provide a relevant description of the US economy.

Figure 3



Predicted (grey,dashed) and Actual Employment in Research and Development, in thousands. See Appendix A (Data) for a description of the data-source.

The model does predict a level of RnD-employment that is close to the observed level (see fig. 3). On average and for the whole period, the model overstates RnD-employment by 3 per cent. The model predicts a faster increase in RnD-employment than we observe in the data, as can be seen from figure 3. Our model under-estimates RnD-employment until the early seventies, and overestimates it thereafter.

Research and Development only constitutes a small fraction of total employment. In the data RnD-employment as a share of the working population grows from one third of a percentage point to slightly less than one percentage point. In terms of the working population, our model predicts that RnD-employment is on average and for the whole period 0.03 percentage points higher than in the data. We consider this deviation small.

Given our documented relationship between RnD-employment and the decline in the relative price of durables, it is not surprising that our model explains the decline in the relative price of durables well. Cummins and Violante report that, on average, the relative price declines at a rate of 3.91 per cent per annum. Our model predicts an average decline of 4.08 per cent.

In the data RnD-employment and the decline in the relative price of durables share a common cycle around the long-term trend. The model does not predict this cyclicality. There are no frictions in this model, and hence absence of cyclicality in our predictions is not surprising. In the previous literature, Gertler and Comin (2006) have developed a model that accounts for medium-term cycles and entails endogenous productivity in the production of capital-goods. We complement this work with an explanation of the long-term trend, that is exogenous to their model.

1.5.5 Sensitivity analysis

We regress the actual and predicted levels of RnD-employment on a constant and a time-trend. We report the coefficients from these regressions in the first two rows of Table 5. We report the same measures from all our sensitivity tests.

Mendoza, Razin and Tesar (MRT) report an average effective tax rate on capital income in the US of 42.9 per cent over the period 1965 to 1988. Carey and Tchilingurian, using the MRT methodology, report somewhat lower estimates for the period 1980 to 1997. Their average estimate for this period is 39.9 per cent. Using other revised methodologies, they report estimates between 24.9 per cent ant 51.0 per cent depending on the time period under study. Domeij and Heathcote, using the MRT method, report an estimate of 39.7 per cent. In our study, we do not aim at discriminating between these estimates. We note that the reported range of estimates is wide.

As a sensitivity check to our results, we solve for the tax rate at which our model predicts average RnD-employment exactly. At a tax-rate of 43.9 per cent, the model predicts average employment in RnD correctly. This tax-rate falls in the range of reported estimates. As above, the model understates RnD-employment at the beginning of our period, and overstates it towards the end. We see that the predictions of RnD-employment are sensitive to our estimate of the tax-rate.

Table 5. Sensitivity Analysis

	γ_1	γ_2
data	635.9	18.9
base-line calibration	678.7	23.4
adjusting τ		
$\tau = 0.439$	635.9	22.3
No employment growth from year:		
2030	678.7	23.4
2010	678.7	23.4
No neutral technical change ever:	693.1	24.0
Extending the model, to allow for structures (GHK with endogenous growth)		
base-line (See Appendix B)	553.1	19.9
adjusting τ		
$\tau = 0.379$	635.9	22.1

The γs are the estimated coefficients from the regression $L_{Qt} = \gamma_1 + \gamma_2 t$, where $t = 0$ is set at year 1975. Thus, $\gamma_1$is average RnD-employment over the period. In these regressions we insert the actual or predicted values of $\{L_{Qt}\}_{t=1950}^{2000}$

The model was parameterized using parameter-estimates from GHK and our empirical work above.

In our experiment we assume that employment grows at 2.1 per cent per annum until year 2050. We investigate the consequences of instead assuming that employment ceases to grow year 2030 and year 2010. These changes are of very limited quantitative importance for our predictions.

In our experiment, we let neutral productivity follow from the two-sector decomposition undertaken above. Naturally, productivity could be measured with error. In our qualitative remarks above, we noted that shocks to Z would have only temporary effects. We therefore, counterfactually, let our model predict RnD-employment, assuming that neutral productivity does not change at all. As expected, we see that this assumption has limited consequences for our predictions of the average level and time-trend of RnD-employment.

In the above, we have aimed at presenting a simple model, that provides transparency to the analysis. The original model by Greenwood et al. accounts for capital in the form of structures. Structures capital is not subject to swift embodied technical change. The productivity of construction production has increased at a rate very similar to the production of non-durables and services. We therefore, in Appendix B, extend our model, to include capital in the form of structures. This extended model is the original model by GHK augmented with endogenous productivity growth in the production of Equipment. We calibrate the model using parameters from GHK. We assume that the production function for ideas is as above. In Appendix B we present our calibration.

Extending the model in this manner alters our view on the historical productivity shocks. As in GHK, neutral technical change is no longer positive during the eighties. It, however, picks up during the nineties.

Adding structures capital decreases the share of the economy that uses durables. Therefore, there is less incentive to undertake RnD. Our extended model predicts less Research and Development than does the simple model. In the base-line calibration of the extended model, the average level of RnD-employment is under-estimated by approximately 13 per cent.

As above, we adjust the tax-rate to match average RnD employment exactly. This implies a tax-rate of 37.9 per cent, which is well in the range of estimates in the literature.

1.5.6 Additional extensions

Our base-line model predicts the US growth experience well over the past fifty years. It is, however, a simple model that does not incorporate many features of the real world economy that could be demanded from a richer model. We have, above and in Appendix B, described one extension of the model - where investment in structures is included. Here, we discuss other extensions that could be of qualitative and quantitative consequence.

Whelan (2003) suggests a two-sector model where households also consume durable goods. Durable consumption is non-negligable. Extending the model according to the suggestions from Whelan constitutes a reasonable and relevant extension of the model. We leave this extension for future research.

We have, above, assumed that research and development only alters the productivity of investment production. This is clearly a simplification. In the US economy, RnD is undertaken also in the production of non-durables. RnD-employment is concentrated in Equipment-producing sectors. For most of our period it is above 70 per cent of total RnD-employment (see Appendix A Data). Yet, substantial RnD-effort goes into the production of non-durable goods, notably in the chemical industry. A richer model would separately account for RnD-effort both in the durable and in the non-durable sector. We find the investigation of multi-sector models with endogenous sector-specific productivity a promising avenue for future research.

We have assumed, above, that new ideas yield perpetual profits. This is clearly not the case in the real world, where patents expire and close (or better) substitutes to patented products appear from competing firms. Since we abstract from copying and improvements, our model is likely to overstate RnD-employment. The addition of copying or creative destruction of ideas could increase the predictive performance of the model.

In our model we describe the US economy as a closed economy. We assume that productivity is an outcome of research effort in the US only. This is clearly not the case, since intermediate products created with ideas from the whole world are being used for productive purposes in the USA. We limit our study to the US, since we do not have data on quality-adjusted prices of capital goods from other parts of the world. Yet, a richer model should incorporate world RnD in the specification of the production function for ideas. This would imply a re-calibration of the production function for ideas.

Neutral productivity is assumed to be exogenous. It shows no time-trend. But neutral productivity in our model calls for an explanation. This could be an interesting research-project.

We leave these extensions for future research.

1.6 Conclusion

Previous studies, pioneered by Greenwood, Hercowitz & Krusell (1997), have found that investment-specific technical change accounts for the major part of productivity growth in the US during the post-war period. Therefore, the study of the sources of investment-specific technical change is important to the understanding of the sources of productivity growth.

We document that the rate of investment-specific technical progress varies positively with RnD-input. This finding is the empirical contribution of our paper. Previous studies, notably Jones (1995), have instead focused on growth in total factor productivity. These studies have not detected any relationship between RnD and productivity growth. The number of researchers and engineers active in RnD has grown substantially since the 1950s, whereas TFP-growth has not. We, thus, contribute new empirical support for endogenous growth theory.

The absence of empirical support for early endogenous growth models has spurred much interesting research. The assumption that research and development becomes progressively more difficult does allow for increasing RnD-effort without accelerating productivity growth. Several endogenous growth models without 'scale-effects' have been developed. Since, however, investment-specific technical change accelerates over the post-war period, we find the assumption of increased difficulty superfluous. We do not reject the hypothesis of a 'scale-effect' in growth that was suggested by the first generation of endogenous growth models.

Endogenous growth theory has not before been successfully employed to predict RnD-input and productivity growth simultaneously. In endogenous growth models, both are equilibrium objects. Previous empirical studies have, however, had limited success in establishing a relationship between RnD and productivity growth. Our empirical findings allow us to undertake the first quantitative study of endogenous growth with a structural model and time series data. We conjecture that the absence of such efforts in the preceding literature stems from the absence of empirical findings with regard to the relationship between RnD-input and productivity growth.

Our base-line model, calibrated using parameters from Greenwood et al and our empirical work, predicts both RnD-input and productivity growth with accuracy. This strenghens us in the view that our specification of the nexus between RnD and productivity is relevant.

The model that we present is simple. Several extensions are reasonable and interesting.

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1.7 Appendices

1.7.1 Appendix A; Data

We use the following data-sources for our quantitative work:

g_q	Rate of decline in the relative price of capital. We use the rate of decline depicted in. Cummins & Violante (2002), figure 1
L_Q	RnD-employment. For the period 1950 to 1993, we use the employment series underlying Jones (2002), available on(http://www.econ.berkeley.edu/~chad/Sources50.asc.) From 1994 to 2000 we use data on Total Researchers (FTE), as reported by; SourceOECD; Science and Technology Statistics- Main Science and Technology Indicators Vol 2006 release 01. (http://www.sourceoecd.org .) The OECD data states that RnD-employment in 1993 was 5.31 per cent larger than the data from Jones states. We therefore multiply all entries in the OECD data with 0.946.
L	Total Employment. We use the data from BLS, Current Employment Statistics survey, Not Seasonally Adjusted,Total nonfarm Employment, All Employees, Annual Data, as available from: (http://data.bls.gov/PDQ/servlet/SurveyOutputServlet)
g_{TFP}	As a reference, we include the TFP-data used by Jones (1995). It is available on (http://www.econ.berkeley.edu/~chad/TimeEGM.asc)
RnD – Employment in Equipment industries	We use the historical statistics on FTE-employment in RnD provided by the NationalScience Foundation (NSF) available on: (http://www.nsf.gov/statistics/iris/excel-files/historical%20tables/h-19.xls) For Equipment employment we sum employment in Fabricated Metal Products, Electrical Equipment, Machinery, Transportation equipment and professional and scientific instruments.We impute missing numbers. In 1992 the NSF apply a new sample design, at which time the employment share in Equipment is lower and RnD-employment in services, notably computer and data processing services, increases. Before 1992 the NSF did not conduct large sample surveys annually, but used sub-sample surveys and a wedging methodology to update data from the previous large sample survey. The last large sample survey before 1992 was in 1987. Before 1987 the share of RnD-employment in Equipment-industries is stable around the average of 72 per cent. This share drops from 70 per cent in 1987 to 55 per cent 1992, at which level it stays reasonably stable.

1.7.2 Appendix B; The Extended model

In the above discussion we presented a simple model, that provided transparency to the analysis. One obvious deviation from the original framework by Greenwood et al is that they account for investment in structures, whereas we do not. We therefore extend the model, allowing for capital in the form of structures.

The households problem is unchanged. The production-functions (1.3) and (1.4) are augmented with structures-capital, in exactly the same way as Greenwood, Hercowitz & Krusell propose. We let sub-script S denote Structures and subscript E denote Equipment. Then our production functions are

$$C_t = Z_t K_{SCt}^{\alpha_S} K_{ECt}^{\alpha_E} L_{Ct}^{(1-\alpha_S-\alpha_E)} \tag{1.34}$$

$$I_t = Z_t K_{SIt}^{\alpha_S} L_{It}^{(1-\alpha_S-\alpha_E)} \int_0^{Q_t} x_t(\omega)^{\alpha_E} d\omega \tag{1.35}$$

We add a law of motion for Structures-capital:

$$K_{St+1} = (1-\delta_S) K_{St} + C_t - c_t^N \tag{1.36}$$

There are now two types of capital, and we have two markets for capital;

$$\begin{aligned} K_{ECt} + K_{EIt} &\leq K_{Et} && (1.37) \\ where \quad K_{EIt} &= \int_0^{Q_t} x_t(\omega)\, d\omega \\ K_{SCt} + K_{SIt} &\leq K_{St} && (1.38) \end{aligned}$$

We have an additional no-arbitrage-condition in financial intermediation;

$$R_{t+1} = (1-\delta_S) + r_{t+1}^S \tag{1.39}$$

Goods-producing firms use structures optimally;

$$r_t^S = \alpha_S \frac{C_t}{K_{SCt}} \quad ; \quad r_t^S = \alpha_S \frac{1}{q_t} \frac{I_t}{K_{SIt}} \tag{1.40}$$

An equilibrium in the modified economy consists of quantities $\{c_t^N, A_t, K_{SCt}, K_{ECt}, K_{SIt}, K_{EIt}, L_{Ct}, L_{It}, L_{Qt}\}_{t=0}^{\infty}$ and prices $\{w_t, p(\omega)_t, q_t, r_t, r_t^S, R_t, V_t\}_{t=0}^{\infty}$ such that;

I. Consumers maximize intertemporal utility. The Euler-equation (1.12) holds for non-durable consumption goods.

II. Firms maximize profits and conditions (1.16) , (1.18) and (1.20) hold. Additionally, (1.40) holds.

III. The free entry condition in RnD (1.23) holds.

IV. There are no arbitrage-opportunities in financial intermediation, so that (1.13) and (1.14) hold. Furthermore (1.39) holds.

V. Factor markets clear, implying that (1.6), (1.37) and (1.38)hold with equality.

VI. The modified laws of motion for capital (1.9) and (1.36), assets (1.11) and ideas (1.5) hold.

VII. Goods production is given by the modified functions (1.34) and (1.35).

This is the model that we will be using in the quantitative exercises below.

Decomposition using the extended model

For our decomposition we use data from the National Income and Product Accounts (NIPA) of the BEA. We, here, decompose the US growth experience using the extended model presented above. Our measure of consumption-production is non-durable consumption, services (less housing services) and investment in non-residential structures. We deflate non-durables with the corresponding BEA deflators aggregated by a Törnqvist-procedure. Our measure of investment-production is investment in Equipment and Software. We use the implied deflator from Cummins and Violanteas our deflator for production of durables.

As above, we take data on employment from BLS; 'Current Employment Statistics survey'.

Procedure

In our decomposition we abstract from RnD-employment. We assume that our data on employment reflect employment in the goods-producing sectors. We use the series for the relative price of capital q as derived from Cummins & Violante. We calculate Q;

$$\frac{1}{q_t} = \left(\frac{1}{\alpha_E}\right)^{\alpha_E} \frac{1}{Q_t^{(1-\alpha_E)}} \tag{1.41}$$

For our decomposition we need estimates of α_E and α_S. We follow GHK and set: $\alpha_E = 0.17$ and $\alpha_S = 0.13$. As above, we distribute labor according to (1.28). Re-arranging the first order conditions from the goods-producing sectors we get:

$$\frac{\frac{K_{ECt}}{L_{ct}}}{\frac{K_{EIt}}{L_{It}}} = \frac{1}{\alpha_E} \quad (1.42)$$

$$\frac{\frac{K_{SCt}}{L_{ct}}}{\frac{K_{SIt}}{L_{It}}} = 1 \quad (1.43)$$

We distribute capital across sectors accordingly.

Capital stocks evolve according to the laws of motion for capital. We follow GHK and set the depreciation rate for Equipment , $\delta_E = 0.124$ and $\delta_S = 0.056$. We assume that the economy initially is on a balanced growth path where all technical change is neutral, and neutral technical change is 2 per cent per year. We solve for the initial capital stocks using the balanced growth path conditions from the laws of motion for captial of this economy[12].

We find our series for Z iterating over the following procedure;

i) We observe $\frac{\frac{1}{q_t} I_t}{C_t}$ and L_t. We calculate L_{It} and L_{Ct} according to (1.28) assuming that the labor market clears.

ii) We calculate K_{SCt} and K_{SIt} according to (1.43) given K_{St}, and we calculate K_{ECt} and K_{EIt} according to (1.42) given K_{Et}

iii) We use our production functions to predict production.

iv) We calculate Z_t by dividing the observed real production of consumption goods, C_t, (or investment goods I_t) with our predicted values

v) We calculate K_{Et+1} and K_{St+1} from their respective laws of motion.

We proceed as above when we calculate the contributions from neutral and investment-specific technical change. Inserting our production-functions into (1.30) and re-arranging we get;

....

$$\begin{aligned}\frac{Y_{t+1}}{Y_t} &= \frac{Z_{t+1}}{Z_t} * \frac{\left(\bar{s}_t \frac{K_{ECt+1}^{\alpha_E} K_{SCt+1}^{\alpha_S} L_{Ct+1}^{(1-\alpha_E-\alpha_S)}}{K_{ECt}^{\alpha_E} K_{SCt}^{\alpha_S} L_{Ct}^{(1-\alpha_E-\alpha_S)}} + (1-\bar{s}_t) \frac{K_{EIt+1}^{\alpha_E} K_{SIt+1}^{\alpha_S} L_{It+1}^{(1-\alpha_E-\alpha_S)} Q_{t+1}^{(1-\alpha_E)}}{K_{EIt}^{\alpha_E} K_{SIt}^{\alpha_S} L_{It}^{(1-\alpha_E-\alpha_S)} Q_t^{(1-\alpha_E)}}\right)}{\left(\bar{s}_t \frac{K_{ECt+1}^{\alpha_E} K_{SCt+1}^{\alpha_S} L_{Ct+1}^{(1-\alpha_E-\alpha_S)}}{K_{ECt}^{\alpha_E} K_{SCt}^{\alpha_S} L_{Ct}^{(1-\alpha_E-\alpha_S)}} + (1-\bar{s}_t) \frac{K_{EIt+1}^{\alpha_E} K_{SIt+1}^{\alpha_S} L_{It+1}^{(1-\alpha_E-\alpha_S)}}{K_{EIt}^{\alpha_E} K_{SIt}^{\alpha_S} L_{It}^{(1-\alpha_E-\alpha_S)}}\right)} \\ &* \left(\bar{s}_t \frac{K_{ECt+1}^{\alpha_E} K_{SCt+1}^{\alpha_S} L_{Ct+1}^{(1-\alpha_E-\alpha_S)}}{K_{ECt}^{\alpha_E} K_{SCt}^{\alpha_S} L_{Ct}^{(1-\alpha_E-\alpha_S)}} + (1-\bar{s}_t) \frac{K_{EIt+1}^{\alpha_E} K_{SIt+1}^{\alpha_S} L_{It+1}^{(1-\alpha_E-\alpha_S)}}{K_{EIt}^{\alpha_E} K_{SIt}^{\alpha_S} L_{It}^{(1-\alpha_E-\alpha_S)}}\right)\end{aligned} \quad (1.44)$$

The first factor in this expression is the contribution to real GDP-growth from neutral technical change. The second is the contribution of investment-specific change. The third factor is the contribution from

[12]We lack data on investment-specific technical change before 1947. Our assumption is that technical change evolved at the same pace in consumption- and investment-production. Note that this assumption only affects our estimate of the initial capital stocks. For Equipment, this is of limited consequence - since the depreciation rate is substantial.

changes in factor inputs. Like GHK, we find that the contribution of neutral productivity to growth is negative on average during the seventies and eighties. It, however, picks up during the nineties.

Calibration

We calibrate the model using parameters from GHK and our estimates of the production function for ideas. Consumption-utility is logarithmic as in GHK.

Table 4. Calibration - Extended model

Parameter	Value	Comment
α_E	0.17	GHK
α_S	0.13	GHK
δ_E	0.124	GHK
δ_S	0.056	GHK
β	0.946	GHK
γ	0.73	OLS-estimate
φ	$4.31 * 10^{-4}$	OLS-estimate

1.7.3 Appendix C; A balanced growth path equilibrium

We let G_j denotes the gross growth rate of variable j along a blanced growth path. We consider an economy without taxes, population-growth and growth in neutral productivity. From the production function for investment-goods it is evident that $G_I = G_K = G_Q$. From the production function for consumption goods, we have $G_C = G_Q^{\alpha}$. Since nominal shares are constant, we have that $G_q = G_Q^{(1-\alpha)}$. It also follows that $G_q = 1/G_r$. Consequently, we have $G_{\Pi_x} = G_Q^{-(1-\alpha)}$. From (1.14) it is evident that $G_V = G_{\Pi_x}$.

The balanced growth version of the no-arbitrage condition in RnD is

$$V_t = \Pi_{xt}\frac{G_{\Pi_x}}{R - G_V} = \Pi_{xt}\frac{1}{R/G_V - 1}$$

We divide the free-entry condition (1.23) with the first order condition w.r.t labor in investment goods production, and get the balanced growth version of (1.25):

$$\frac{L_Q}{L_I} = \alpha\frac{1}{R/G_V - 1}\frac{\Delta\left(L_{Qt}, Q_t\right)}{Q_t} = \alpha\frac{G_Q - 1}{R/G_V - 1} \tag{1.45}$$

Without taxes; $G_C = \beta R$ from the Euler-equation, and hence; $R/G_V = \frac{1}{\beta}G_Q$. Evidently economies on growth-paths with higher growth devote more labor to RnD relative to investment-goods production than economies with lower growth.

Combining the first order condition with regard to intermediate goods (1.18) and the pricing-rule (1.21) in intermediate goods production, we have:

$$rq = \alpha^2\frac{I}{K_I}$$

Along a balanced growth path, the employment shares in different sectores are constant. Re-arranging the first order conditions from the goods-producing sectors we get:

$$\frac{K}{K_I} = \frac{1}{\alpha}\frac{L_C}{L_I} + 1$$

Then, the balanced growth version of the law of motion for capital can be re-written:

$$G_Q = (1-\delta) + \frac{I}{K}$$

We insert these expressions in the balanced-growth version of the no-arbitrage condition on the capital-market (1.13) and re-arrange:

$$\frac{RG_q - (1-\delta)}{G_Q - (1-\delta)} = \alpha^2 \left(\frac{1}{\alpha}\frac{L_C}{L_I} + 1\right) \tag{1.46}$$

Along a balanced growth path without taxes $RG_q = R/G_V = \frac{1}{\beta}G_Q$. Evidently economies on growth-paths with higher growth devote more labor to investment-production relative to consumption-goods production than economies with lower growth. We substitute for labor-market clearing, insert (1.45) and re-arrange

$$\frac{\frac{1}{\beta}G_Q - (1-\delta)}{G_Q - (1-\delta)} + \alpha(1-\alpha) = \alpha^2 \frac{(L-L_Q)}{L_Q}\frac{G_Q - 1}{\frac{1}{\beta}G_Q - 1} \tag{1.47}$$

This is the condition that yields RnD-employment along a balanced growth path. The left-hand side is decreasing in G_Q and positive. The right hand side is decreasing in L_Q for any specification of the production function for ideas that has decreasing marginal return to RnD-employment (i.e $\gamma < 1$) Thus, economies who experience higher growth devote more labor to RnD-activities.

Note that if G_Q is a function of parameters only, as is the case in semi-endogenous growth models, this expression pins down the fraction of workers active in RnD.

Existence and uniqueness of equilibrium

The RHS of (1.47) is decreasing in L_Q. Furthermore it spans $(\infty : 0)$ as L_Q goes from 0 to L. The LHS of (1.47) is always positive and less than ∞. Thus, at least one equilibrium exists.

The LHS is decreasing;

$$\frac{\partial}{dL_Q}(LHS) = (1-\delta)\left(1 - \frac{1}{\beta}\right)\frac{1}{(G_Q - (1-\delta))^2}\frac{dG_Q}{\partial L_Q} < 0$$

Note that

$$\begin{aligned}
\frac{\partial^2}{\partial L_Q^2}(LHS) &= (1-\delta)\left(1-\frac{1}{\beta}\right) * \\
&\quad \frac{1}{(G_Q-(1-\delta))^4}\left[\left(\frac{\partial^2 G_Q}{\partial L_Q^2}\right)(G_Q-(1-\delta))^2 - \left(\frac{\partial G_Q}{\partial L_Q}\right)^2 2(G_Q-(1-\delta))\right] \\
&> 0
\end{aligned}$$

All terms in the square brackets are negative, which implies that $\frac{d}{dL_Q}(LHS)$ is increasing and converging to zero, as L_Q approaches infinity. Evidently, LHS converges to $\frac{1}{\beta} + \alpha(1-\alpha)$ as L_Q approaches infinity. Then, a sufficient condition for a single crossing and thus uniqueness is that the second derivative of the RHS is positive. We define:

$$\begin{aligned}
A &\equiv (L - L_Q) > 0 \\
B &\equiv \tfrac{1}{G_Q-\beta} > 0 \\
D &\equiv \tfrac{G_Q-1}{L_Q} > 0
\end{aligned}$$

Then,

$$RHS = \alpha^2\beta ABC$$

Note that, with the general production function for ideas described above, we have:

$$\frac{dG_Q}{dL_Q} = \frac{\gamma}{L_Q}(G_Q-1)$$

And hence;

$$\begin{aligned}
\tfrac{dA}{dLQ} &= -1 < 0 \\
\tfrac{dB}{dL_Q} &= -\tfrac{1}{(G_Q-\beta)^2}\tfrac{dG_Q}{dL_Q} < 0 \\
\tfrac{dD}{dL_Q} &= \tfrac{1}{L_Q^2}(\gamma-1)(G_Q-1) < 0
\end{aligned}$$

Evidently, RHS is decreasing in L_Q, for any specification of the production function for ideas that has non-increasing returns to RnD-employment. Furthermore, .

$$\frac{1}{\alpha^2\beta}\frac{d^2}{dL_Q^2}(RHS) = \begin{matrix}
\frac{d^2A}{dL_Q^2}BD & +\frac{dA}{dL_Q}\frac{dB}{dL_Q}D & +\frac{dA}{dL_Q}B\frac{dD}{dL_Q} \\
+\frac{dA}{dL_Q}\frac{dB}{dL_Q}D & +A\frac{d^2B}{dL_Q^2}D & +A\frac{dB}{dL_Q}\frac{dD}{dL_Q} \\
+\frac{dA}{dL_Q}B\frac{dD}{dL_Q} & +A\frac{dB}{dL_Q}\frac{dD}{dL_Q} & +AB\frac{d^2D}{dL_Q^2}
\end{matrix}$$

Naturally, all off-diagonal elements are positive. Trivially, $\frac{d^2A}{dL_Q^2} = 0$. We also have

$$\frac{d^2B}{dL_Q^2} = -\frac{1}{\left(G_Q - \beta\right)^4}\left[\frac{d^2G_Q}{dL_Q^2}\left(G_Q - \beta\right)^2 - \left(\frac{dG_Q}{dL_Q}\right)^2 2\left(G_Q - \beta\right)\right] > 0$$

This is easily seen, since $\frac{d^2G_Q}{dL_Q^2} < 0$ and $\left(\frac{dG_Q}{dL_Q}\right)^2 2\left(G_Q - \beta\right) > 0$. Finally;

$$\frac{d^2D}{dL_Q^2} = (\gamma - 1)(\gamma - 2)\frac{(G_Q - 1)}{L_Q^3} > 0$$

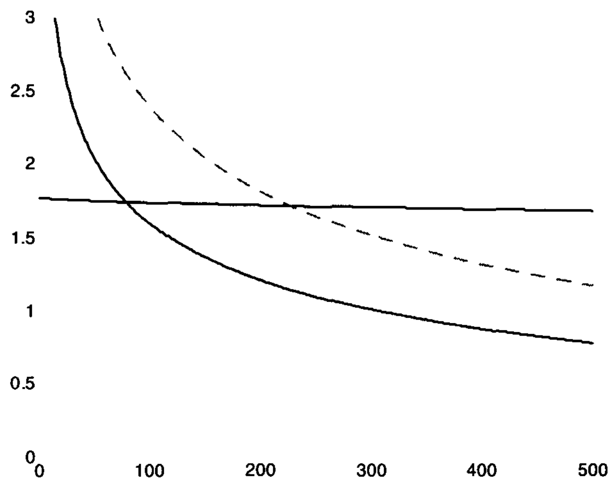
Thus, all diagonal elements are positive or zero for any production function of ideas that has non-increasing marginal return to RnD-employment. The second derivative of the RHS is positive. Hence, we have established that the equilibrium is unique.

Comparative statics

In our quantitative experiment we study the effects of shocks to (Z) and (L). In the sensitivity analyses we study changes to (τ).Therefore, we here compare balanced growth path equilibriua where we change these exogenous variables.

The RHS is increasing in L for all values of L_Q.The LHS is constant w.r.t L. This implies that an increase in L will increase equilibrium L_Q. Visually, the RHS pivots around the point where $L_Q = L$ and $RHS = 0$ Hence, a larger labor-force will imply a larger RnD-labor-force and thus faster investment-specific technical change. This effect is transitionary if growth is semi-endogenous. If growth is purely endogenous ($\phi = 1$), this is the 'scale-effect' in growth.

Appendix Figure 2



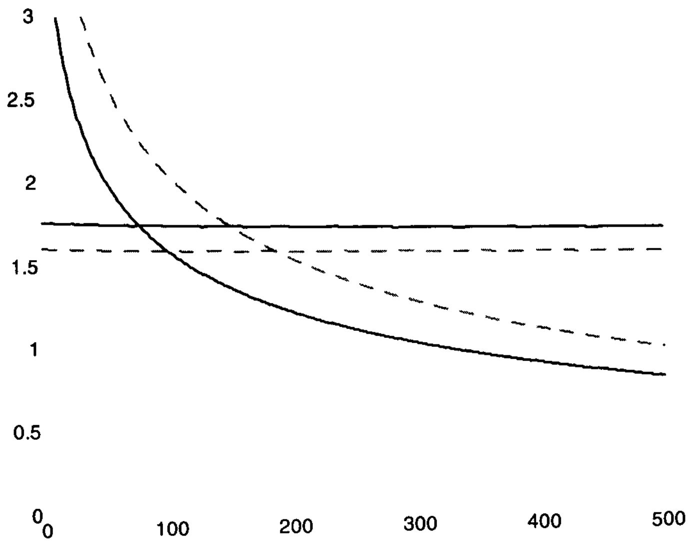
The solid lines depict the RHS and LHS of (1.47)above.
L is aggregate employment in 1950. The calibration is base-line.
The dashed line depicts the RHS after a 50 per cent increase
in aggregate employment L.

Z does not enter into (1.47) and our balanced growth path allocation is, thus, unaffected by the level of neutral productivity.

It follows from the Euler equation that the tax-rate increases the financial interest rate R for any given research-intensity. Then, with higher τ the RHS is lower for any given level of RnD. This reflects that the value of a patent is decreasing in the financial interest rate. Visually, the RHS shifts down.

As R is higher for any given research-intensity, the LHS is higher for all L_Q. This reflects that the value of investment goods is decreasing in the financial interest rate. Visually the LHS pivots closer to the origin. Thus, Equilibrium RnD-intensity is decreasing in τ.

Appendix Figure 3



The solid lines depict the RHS and LHS of (1.47)above.
L is aggregate employment in 1950. The calibration is base-line.
The dashed lines depicts the RHS and LHS after a reduction of τ from 0.40 to 0.25.

Chapter 2

Assortative matching and investment-specific technical change

Abstract

We study a Diamond-Mortensen-Pissarides model with vintage capital and heterogenous workers. In equilibrium there is positive assortative matching. The skill-premium not only reflects differences in productivity between workers, but also their accepted set of machines. More skilled workers operate younger - and therefore more productive - machines.

The rate of investment-specific technical change has a skill-bias. Faster investment-specific technical change implies a larger skill-premium. This results since workers and machines are positively assortatively matched.

In our calibrated model, while accounting for an increase in the supply of skilled workers, a ten per cent increase in the skill premium results from a six per cent increase in the productivity of the skilled. The interaction between faster investment-specific technical change and assortative matching yields the remainder.

Contrary to previous studies we find that both within- and between-group inequality increase in response to skill-biased technical change.

2.1 Introduction

What is the skill-bias of technical change? We focus on the contribution of accelerating investment-specific technical change to an increase in the skill-premium. Workers are positively assortatively matched to a vintage-capital structure. Skilled workers operate younger - and hence more productive - machines than unskilled workers. Faster investment-specific technical change implies that productivity differences between machines of different ages increase. Thus, an increase in the skill-premium results.

The mechanism under study has three principal components; vintage capital, decentralized production into worker-machine pairs and complementarity in output between capital and labor. Capital is costly to install and there is investment-specific technical change, such that newer investment-goods - or machines - are more productive than older. Capital, in other words, has a vintage-structure. One machine can be operated by one worker only. Within a pair capital and labor are complements in production, so that there is positive assortative matching in equilibrium[1].

In equilibrium, the life-span of a machine is determined so that the discounted profit from a new machine equals its price. Skilled workers operate younger machines than unskilled workers. The skill-premium thus depends on the raw productivity differential between skilled and unskilled workers and on the age-differential between machines operated by the skilled and the unskilled respectively.

Faster investment-specific technical change implies that the productivity-differential between machines of different ages grows. An increase in the skill-premium results.

We study the above mechanism in two settings: with and without frictions in the labor market.

In the economy without frictions in the labor-market, workers are perfectly and positively assortatively matched to machines. Skilled workers operate the younger machines, and unskilled the older. At a critical age, when the proceeds to the firm is equal from employing a skilled or an unskilled worker, the firm switches from employing skilled to unskilled workers. There is no inequality within groups of workers. All workers within a group earn the wage of the worker within the group that operates the least productive machine. The unskilled workers earn the production of the oldest machine in the economy.

We introduce labor-market frictions in this economy, in the Diamond-Mortensen-Pissarides fashion. Unemployed workers and vacant machines randomly search for a match. The flow of matches is a function of aggregate unemployment and the stock of vacancies. In this economy, parties are not perfectly assortatively matched. For intermediate ages, machines are operated by both skilled and unskilled workers. Furthermore, with labor-market frictions, there is wage-inequality within groups of workers. Identical workers that operate machines of different ages may earn different wages.

We quantitatively evaluate the contribution of assortative matching and faster investment-specific technical change to the increase in the skill-premium. We calibrate the models - with and without labor-market frictions - to match some moments of the US economy prior to 1975. Then, in a quantitative experiment, we increase the rate of investment-specific technical change and the productivity and supply of skilled labor and compare steady state outcomes. While accounting for the increase in the supply of skilled labor, we find that vintage capital and faster investment-specific technical change can account for forty per cent of a ten per cent increase in the skill-premium.

Interestingly, in our experiments with the model with frictions in the labor-market, we find that both within- and between-group inequality simultaneously grow in response to an increase in the rate of investment-specific technical change. This contradicts a previous claim in the literature that Mortensen-Pissarides-style models are unable to generate a simultaneous increase in both these measures of inequality[2]. Moreover - as has been observed - the model with frictions predicts larger within-group-inequality among the skilled, than among the unskilled.

[1] In Becker's (1973) neoclassical marriage market model, matching is positively assortative if types are complements: i.e. match output is supermodular in inputs.

[2] Wong (2003) argues that "the (Mortensen-Pissarides) model fails to account for the wage inequality pattern over time found in the data." In that study, an increase in the skill-premium coincides with less variation in wages within groups.

This article is organized as follows: Section II discusses previous studies on investment-specific technical change, assortative matching and skill-biased technical change. Section III describes the economy without frictions, and section IV the economy with frictions. In section V we calibrate the models to match some features of the US economy prior to 1975. We then undertake quantitative experiments. Section VI concludes.

2.2 Overview of the literature

Returns to education increased from the mid-seventies, as documented in the work by Katz and Murphy (1992). They conclude that there was a skill-bias in technical-change during this period. The supply of workers with a college-degree increased over the same period. Therefore, supply-factors can be excluded as an explanation to the increase in the relative wage of the educated. In our study, we focus on one explanation to the skill-bias of technical change.

Our focus is the interaction between assortative matching, vintage capital and an acceleration in the rate of investment-specific technical change. In this section, we discuss our model and how it relates to previous studies. We first discuss our assumptions on the structure of capital in the economy and then our assumption on worker-heterogeneity. We discuss how workers and capital are matched in the economy, and relate this to the literature on assortative matching. Finally, we relate our work to other frameworks that have connected investment-specific technical change and the increase in wage-inequality.

Our first assumptions concern the structure of capital in the economy. We assume that there is investment-specific technical change. This implies that efficient capital-goods become progressively cheaper, relative to consumption goods. Gordon (1990) documents that the relative price of durable goods was falling from 1947 until 1983. In an important study, Greenwood, Hercovitz and Krusell (1997) infer investment-specific technical change from Gordon's series of quality-adjusted prices. They document that the major part of US productivity growth in the post-war period stems from investment-specific technical change. More recently, Cummins and Violante (2002), using econometric techniques, update the series in Gordon (1990) to year 2000.

Central to our study is the finding in Cummins and Violante that the rate of investment-specific technical change is increasing over time. We investigate the consequences on the skill-premium of this acceleration in our economy.

Capital is costly to install in our economy, so that - like in Solow (1960) - the capital stock in the economy has a vintage-structure. Last, we assume that production is decentralized into worker-machine pairs. This assumption is common in models of economies with labor-market-frictions. This implies that, for equally productive workers, a worker-machine pair with a younger machine produces more than a pair with an older machine.

Workers in our economy are either skilled or unskilled. Skilled workers are more productive than unskilled workers.

Since production is decentralized into pairs, a production-unit consists of a machine of some age and a worker that is either skilled or unskilled. We find that, in equilibrium, there is positive assortative matching. Skilled workers operate younger machines than unskilled workers.

Jovanovic (1998) studies an economy without frictions, where capital has a vintage-structure. In his framework capital ages and workers are heterogeneous. On the worker side, productivity is distributed according to a continuous function. In equilibrium there is sorting. The most productive (youngest) machine is operated by the most productive worker. Every instant the machine switches workers to a less productive worker. Our friction-less economy is a simplified version this framework.

We proceed to introducing frictions in the labor-market in the setting that we have described above. We assume search frictions that give rise to quasi-rents within matched pairs and induce unemployment. The flow of new matches is determined by a matching-function, which takes aggregate unemployment and the stock of vacancies as inputs. Wages are the outcome of Nash-bargaining between the worker and the owner of the machine. There is free entry of firms. All these assumptions are standard in the study of frictional labor-markets and define a stylized Diamond-Mortensen-Pissarides (DMP) matching-model[3].

Hornstein, Krusell and Violante (2007, henceforth HKV) study the same economy, except that workers in their setting are identical. Our model is an extension of their frame-work. This extension allows us to study wage-differentials between groups of workers.

HKV document how the introduction of vintage capital into the DMP-framework introduces variation in wages across workers. The mechanism is the following: Capital is costly to install, and there is free entry to investment. In equilibrium, therefore, the value of a new vacant machine is equal to the cost of investment. With age, the relative productivity of the machine and its remaining life-span falls. Lower production and shorter life-span implies that the surplus of a match with an older machine is lower than a match with a newer machine. Since wages are the outcome of Nash-bargaining over match-surplus, wages fall with the age of the machine.

Since workers are identical, assortative matching is no issue in the HKV-framework. However, the mechanism that generates wage-inequality in the HKV-framework, also generates wage-inequality within groups of workers in our model. However, the mapping between age and wage is obscured for the unskilled workers - since it may be more lucrative for young machines to search for skilled workers than to match with unskilled workers.

Violante (2002) analyses the response of residual inequality to an acceleration in investment-specific technical change. Our models are similar, and the variation that we are interested in is the same, but the object of study is different. In the Violante-framework there is no ex ante-heterogeneity among workers. Therefore, the increase in the skill-premium is not addressed.

[3]In our study, we use the term 'Mortensen-Pissarides' framework to describe a class of models created for the study of frictional labor markets. This class of models include a search friction that produces quasi-rents and induces unemployment, a matching function that determines the flow of new matches, a wage that is determined by bargaining between the matched parties and free entry of firms. Seminal contributions to this literature are Diamond (1982), Mortensen (1982) and Pissarides (1990).

Without frictions, and as in Jovanovic (1998), positive assortative matching is perfect. Skilled workers operate machines up to a critical age, after which unskilled workers are employed. At this age, the proceeds to the firm is equal from employing any worker. The introduction of frictions alters this pattern. With frictions, for a range of intermediate ages, workers of both skills accept machines. Positive assortative matching, thus, is not perfect. This finding is similar to results reported in Shimer and Smith (2000).

Shimer and Smith (2000) study assortative matching in a frame-work with frictions, with two-sided heterogeneity. They introduce search-frictions into Becker's (1973) neoclassical marriage market model. Parties are heterogenous by some continuously distributed variable. They find that in equilibrium agents accept a range of matches within this distribution. Positive assortative matching holds on average under some conditions. More productive parties accept a range of matches that is more productive, than the range accepted by less productive parties. In our model, firms are heterogenous by age - which is a continuous variable. As in Shimer and Smith, workers accept a range of ages. Positive assortative matching holds on average. The skilled, on average, accept younger machines than the unskilled.

Ours is the first frame-work to study assortative matching with labor-market frictions and vintage capital. Previously, however, assortative matching has been studied as a source of wage-differentials. We now relate our model to the work of Acemoglu (1999), Wong (2003) and Sattinger (1995).

Acemoglu (1999) links assortative matching and growing between-group wage differentials. In his frame-work, there are frictions in the labor market. Entrepreneurs choose the level of capital they invest in a vacancy. When skilled labor is scarce - or when the productivity of the skilled is not much larger than that of the unskilled - entrepreneurs only create vacancies that are acceptable to all workers. This is a 'middling' equilibrium, where skilled and unskilled workers operate the same technology. A larger skilled labor force induces entrepreneurs to create vacancies that are only acceptable to skilled or unskilled workers. In this sense, skilled labor creates its own demand.

In the setting of Acemoglu (1999), and increase in the supply of skilled labor may shift the economy from the 'middling' equilibrium to a sorted equilibrium. Then, skilled workers operate more capital than unskilled workers do. The skill-premium increases. The increase in the skill-premium stems from the shift from a 'middling' to a sorted equilibrium.

A fundamental difference between our frame-work and Acemoglu (1999) is that firms are heterogeneous by age. Firms choose the level of investment in new machines - which in turn determines the supply of older machines in the economy. Firms cannot create special capital-goods for skilled or unskilled workers. But they can condition their decision to accept matches on the age of the vacancy at their disposal. In equilibrium, these acceptable ages are different for the skilled and unskilled. This compositional effect affects the skill-premium.

In our framework, an increase in the supply of skilled labor does increase the level of investment - but the effect on the skill-premium is negative. In our frame-work there is, thus, no 'creation of demand' for skilled labor. Moreover, in our model the economy does not shift from one equilibrium regime to another.

Smooth changes in parameters change equilibrium decisions smoothly.

In the economy studied by Acemoglu (1999) there is no within-group inequality, since workers of one type all operate the same technology. In a similar framework, Wong (2003) studies a frictional labor-market where firms choose to invest in either 'high-tech' or 'low-tech' capital goods. Workers are skilled or unskilled. For some parameter settings - when productivity differentials are moderate - workers of both types accept matches to both high- and low-tech firms. There is between-group and within-group inequality. Interestingly, Wong documents that as the skill-premium increases in her framework, within-group inequality does not increase. As the productivity of the skilled grows, so does their value of unemployment. Hence, in all types of matches, their wage increases. Therefore, within-group inequality does not increase.

Wong concludes that "the (Mortensen-Pissarides) model fails to account for the wage inequality pattern over time found in the data." In our calibrated model - a version of the Mortensen-Pissarides framework - between- and within-group inequality do increase simultaneously, demonstrating a limitation to the generality of this claim.

In a similar setting - where heterogeneity in productiviy is distributed according to a discrete distribution - Sattinger (1995) shows that multiple equilibria can arise. For the same parameter-values an economy could maintain a mixed or a sorted equilibrium. In our model, changes in equilibrium outcomes do not stem from such shifts between equilibrium regimes.

Lemieux (2006) decomposes the change in residual inequality into within group effects and composition effects. He finds that residual inequality increases the most among college-graduates, and more among the experienced than workers with less experience. Within-group wage-dispersion actually decreases for groups with little experience and education. Interestingly, in our quantitative experiments, we find a similar response in the wage-distribution to the acceleration in investment-specific technical change.

Ours is not the first study to link investment-specific technical change and an increase in the skill-premium. Krusell et al. (2000, KORV) account for investment-specific technical change in their investigation of the skill-premium. In their frame-work the relative price of efficient capital falls. This implies that the capital-stock grows faster than if investment-specific technical change is not accounted for. In the KORV-economy skilled workers and capital are complements, and substitute for unskilled workers. A larger capital stock, thus, implies a larger skill-premium.

2.3 The friction-less economy

In this section, we discuss the economy without frictions. The results will help us understand the more elaborate model with frictions, and will serve as a benchmark in our quantitative experiments. This is our environment:

Time is continuous. We assume a stationary measure 1 of workers in the economy. Workers are heterogenous by type, denoted h. A type h worker supplies χ_h units labor inelastically. There are L_h workers of type h.

Capital is installed at cost $\iota(t)$ and can be scrapped at no cost. Machines age. They depreciate at rate δ. Furthermore, new machines become more productive at rate γ - the rate of embodied technical change. Production is decentralized into worker-machine pairs. Moreover, the productivity of a worker-machine pair grows at rate ψ of neutral technical change.

2.3.1 Production

The efficency units of a machine age a at time t is given by:

$$k(a,t) = k(0,0)e^{\gamma(t-a)-\delta a} \tag{2.1}$$

A machine of age a that is observed at time t was installed at time $t-a$. At installation the state of embodied productivity was $e^{\gamma(t-a)}$. After installation, the machine is not improved upon. With age it depreciates.

A production unit consists of a worker with skill h that is paired with a machine aged a. Neutral productivity grows at rate ψ. The output of one production unit at time t is, thus:

$$y_h(a,t) = e^{\psi t}\chi_h k(a,t)^{\alpha} = e^{(\psi+\alpha\gamma)t}\chi_h e^{-(\gamma+\delta)\alpha a} \tag{2.2}$$

where, without loss of generality, we have normalized $k(0,0) = 1$.

Installation of capital is costly. The installation of one machine at time t costs $\iota(t)$. There is free entry into investment - and at time t we denote the volume of investment $\nu(t)$.

2.3.2 Agents

Workers supply labor inelastically, and there are no frictions in the labor market. Firms maximize profits, by choosing if and what type of worker to hire. The flow profit, or rent, at time t of a machine aged a is:

$$\pi_h(a,t) = \max_h \{y_h(a,t) - w_h(t)\} \tag{2.3}$$

The present value of a new machine is, thus:

$$\Pi(t) = \max_{\overline{a}_{ht}} \left\{ \sum_h \int_{\underline{a}_{ht}}^{\overline{a}_{ht}} e^{-\rho a}\{y_h(a,t+a) - w_h(t+a)\}\, da \right\} \tag{2.4}$$

where $\underline{a}_{ht}$ denotes the age at which firms choose to start hiring workers of type h, and $\overline{a}_{ht}$ is the age at which firms choose to hire some other type of workers. Naturally, if at time $\overline{a}_{ht}$ firms start hiring workers of type j we have the condition: $\overline{a}_{ht} = \underline{a}_{jt}$. For the workers that operate the youngest machine workers $\underline{a}_h = 0$. The first-order condition from the firms decision-problem is:

$$y\left(\overline{a}_{ht}, h, t\right) - w\left(h, t\right) = y\left(\underline{a}_{jt}, j, t\right) - w\left(j, t\right) \tag{2.5}$$

At the critical ages the firm is indifferent between hiring workers of different two types. We assume that the machine can be scrapped costlessly. Then, for the type of workers operating the oldest machine, we have:

$$y\left(\overline{a}_{ht}, h, t\right) = w\left(h, t\right) \tag{2.6}$$

The firm operates a machine, until production of the pair is equal the lowest wage in the economy. Thereafter, the machine is scrapped. Some type of workers operates the new machine. For this type we have $\underline{a}_{ht} = 0$. Free entry implies that the present value of investment equals the cost of investment. We, thus, have the free entry condition:

$$\iota\left(t\right) = \Pi\left(t\right) = \max_{\overline{a}_{ht}} \left\{ \sum_h \int_{\underline{a}_{ht}}^{\overline{a}_{ht}} e^{-\rho a} \left\{y_h\left(a, t+a\right) - w_h\left(t+a\right)\right\} da \right\} \tag{2.7}$$

The cost of investment is equal to the present value of a new machine.

2.3.3 Equilibrium

An equilibrium is a set of wages $w\left(h, t\right)$ and allocations $\overline{a}_{ht}, \nu\left(t\right)$ for all times, such that the free entry condition (3.7) holds and the labor market clears at all times. Labor market clearing is given by:

$$\int_{\underline{a}_{ht}}^{\overline{a}_{ht}} \nu\left(a\right) da = L_h \qquad \forall h$$

where a denotes the age of machines operated at time t.

2.3.4 Analysis

As in Jovanovic (1998), all equilibria exhibit positive assortative matching. The profit maximization condition (3.5) characterizes the ages at which firms choose to change the type of worker it hires. Firms that have machines just younger than $\overline{a}_{ht}$ choose to hire workers of type h. This implies that $y_h(a,t) - y_j(a,t) > w_h(t) - w_j(t)$ for every $a < \overline{a}_{ht}$. Production in a pair is given by (3.2). Thus, $y_h(a,t) - y_j(a,t) = (\chi_h - \chi_j) e^{(\psi+\alpha\gamma)t} e^{-(\gamma+\delta)\alpha a}$. This decreases in a if and only if $(\chi_h > \chi_j)$. Thus, we have positive assortative matching. Workers and machines in a pair are complements. Therefore, this result is a corollary of the finding in Becker (1973).

The most productive worker operates the newest machine, and that the oldest machine is operated by the least productive worker. Moreover, we find that the least productive worker is payed his or her marginal productivity. Better workers are payed the marginal productivity of the oldest machine their type operates, less the flow-profit firms would have received from hiring a less productive worker with the same machine.

2.3.5 A balanced growth path equilibrium with two types of workers

The focus of our study is the wage-differential between skilled and unskilled workers. Therefore, in this section, we study a balanced growth path equilibrium with two types of workers - skilled (S) and unskilled (U). Along a balanced growth path the volume of investment $\nu(t)$ is constant ν. Furthermore, the age at

which firms to hire another type of worker is independent of time $\overline{a}_{ht} = \overline{a}_h$. Labor market clearing, thus, implies that

$$\int_0^{\overline{a}_S} \nu d\tilde{a} = L_S \quad ; \quad \int_{\overline{a}_S}^{\overline{a}_U} \nu d\tilde{a} = L_U$$

The firms decisions yield:

$$\begin{aligned} w_U(t) &= e^{(\psi+\alpha\gamma)t} \chi_U e^{-(\gamma+\delta)\alpha\overline{a}_U} \\ w_S(t) &= e^{(\psi+\alpha\gamma)t} \left[(\chi_S - \chi_U) e^{-(\gamma+\delta)\alpha\overline{a}_S} + \chi_U e^{-(\gamma+\delta)\alpha\overline{a}_U} \right] \end{aligned}$$

The free-entry condition is:

$$\begin{aligned} \iota(t) &= \int_0^{\overline{a}_S} e^{-\rho\tilde{a}} \{ y_S(\tilde{a}, t+\tilde{a}) - w_S(t+\tilde{a}) \} d\tilde{a} + \int_{\overline{a}_S}^{\overline{a}_U} e^{-\rho\tilde{a}} \{ y_U(\tilde{a}, t+\tilde{a}) - w_U(t+\tilde{a}) \} d\tilde{a} = \\ & e^{(\psi+\alpha\gamma)t} \int_0^{\overline{a}_S} e^{-\rho\tilde{a}} e^{(\psi+\alpha\gamma)\tilde{a}} \left\{ \chi_S e^{-(\gamma+\delta)\alpha\tilde{a}} - \left[(\chi_S - \chi_U) e^{-(\gamma+\delta)\alpha\overline{a}_S} + \chi_U e^{-(\gamma+\delta)\alpha\overline{a}_U} \right] \right\} d\tilde{a} + \\ & e^{(\psi+\alpha\gamma)t} \int_{\overline{a}_S}^{\overline{a}_U} e^{-\rho\tilde{a}} e^{(\psi+\alpha\gamma)\tilde{a}} \chi_U \left\{ e^{-(\gamma+\delta)\alpha\tilde{a}} - e^{-(\gamma+\delta)\alpha\overline{a}_U} \right\} d\tilde{a} \end{aligned}$$

Evidently, along a balanced growth path, the cost of investment grows at rate $(\psi + \alpha\gamma)$. We define the growth rate of the economy $g \equiv (\psi + \alpha\gamma)$ and render the model stationary by dividing wages, production and the investment cost by e^{gt}. Note that this implies that we have normalized the productivity of a new machine (of age 0) to 1 at all times. We - for notational convenience - define $\partial \equiv (\gamma + \delta)\,\alpha$. The stationary version of the balanced growth path equilibrium is, thus, characterized by:

$$
\begin{aligned}
w_U &= \chi_U e^{-\partial \bar{a}_U} \\
w_S &= (\chi_S - \chi_U)\, e^{-\partial \bar{a}_S} + \chi_U e^{-\partial \bar{a}_U} \\
\iota &= \int_0^{\bar{a}_S} e^{-(\rho-g)\tilde{a}} \left\{ \chi_S e^{-\partial \tilde{a}} - \left[(\chi_S - \chi_U)\, e^{-\partial \bar{a}_S} + \chi_U e^{-\partial \bar{a}_U} \right] \right\} d\tilde{a} + \\
& \quad + \int_{\bar{a}_S}^{\bar{a}_U} e^{-(\rho-g)\tilde{a}} \left\{ \chi_U e^{-\partial \tilde{a}} - e^{-\partial \bar{a}_U} \right\} d\tilde{a} \\
\nu \bar{a}_S &= L_S \\
\nu (\bar{a}_U - \bar{a}_S) &= L_U
\end{aligned}
$$

2.3.6 Comparative statics in the economy without frictions

In our study of the economy with frictions we focus on the response of the skill-premium to faster embodied technical change (an increase in γ) and to an increase in the supply of skilled labor, L_S. In the friction-less economy, the equilibrium skill-premium is:

$$
\frac{w_S}{w_U} = \left(\frac{\chi_S - \chi_U}{\chi_U} \right) e^{\partial(\bar{a}_U - \bar{a}_S)} + 1 \tag{2.8}
$$

The skill-premium is larger than the raw productivity differential $\left(\frac{\chi_S}{\chi_U}\right)$ since workers are positively sorted.

Proposition 1. The skill-premium is increasing in the rate of embodied technical change, γ.

Proof: The free entry-condition states that the present value of all future profits equals the cost of investment. An increase in γ does not affect the future production of a machine that is installed - since it is not subject to investment-specific technical change. Wages, however, grow faster in response to an increase in the rate of investment-specific technical change. This implies that wages - in terms of the production of a new machine - have to decrease in order for the free-entry condition to hold. Thus, in equilibrium, $\partial \bar{a}_U$ is increasing in γ. Since $\frac{w_S}{w_U} = \left(\frac{\chi_S}{\chi_U} - 1\right) e^{\partial(\bar{a}_U - \bar{a}_S)} + 1 = \left(\frac{\chi_S}{\chi_U} - 1\right) e^{\partial L_u \bar{a}_U} + 1$ it follows that the skill-premium is increasing in γ.

Proposition 2. The skill-premium is increasing in the share of the labor-force that is unskilled

Proof: Profit-maximization implies that the profit-flow is higher for production with skilled than with unskilled workers. Therefore, an increase in the share of unskilled workers reduces the present value of profits from a new machine - at unchanged decisions. It follows that wages have to be lower in order for the free-entry condition to hold. Wages decrease in the rejection-age $\bar{a}_U$. It follows that, in equilibrium, $\bar{a}_U$ is increasing in L_u. Since $\frac{w_S}{w_U} = \left(\frac{\chi_S}{\chi_U} - 1\right) e^{\partial L_u \bar{a}_U} + 1$, it follows that the skill-premium, in equilibrium, is increasing in L_u.

We also note that as the investment-cost ι converges to 0, the skill-premium converges to $\left(\frac{\chi_S}{\chi_U}\right)$ since the age-differential between the marginal machine operated by workers of different categories decreases as both ages converge to 0.

2.4 The economy with frictions

We now assume that the labor market works imperfectly. A firm owns a machine. This is either operated by a worker - or the firm posts a vacancy. Unemployed workers search for employment. Firms and workers are matched according to a matching function. All parties conduct random search. If a firm and a worker encounter each other on the labor market, they bargain over the ensuing surplus, should it be positive. We assume Nash-bargaining. There is free entry into investment.

We assume that there are only two types of workers, skilled and unskilled. Furthermore, we let workers of all types enjoy utility b when not working.

2.4.1 Matching

As in the economy without frictions, a production-unit is a pair of a machine and a worker and output follows the production function (3.2) above. In the economy with frictions, workers and machines are matched according to the function;

$$M(u, v) = Au^{\omega}v^{1-\omega} \tag{2.9}$$

The flow of matches at every instant is increasing in aggregate unemployment, u, and the stock of vacancies; v.

Vacancies are vacant machines, and are thus heterogenous with regard to their age a. As in the economy without frictions, machines over some age $\overline{a} = \max\{\overline{a}_S, \overline{a}_U\}$ will not be operated. Thus:

$$v = \int_0^{\overline{a}} v(a)da$$

where $v(a)$ is the density of vacancies age a. Unemployment $u = u_S + u_U$ is simply the sum of unemployed skilled and unemployed unskilled workers. The probability of a vacancy being matched with a worker is $\frac{M(u,v)}{v} = M(\frac{u}{v},1) \equiv M(\theta,1) \equiv q(\theta)$ and the probability of a worker being matched with a machine is

$\frac{M(u,v)}{v}\frac{v}{u} \equiv \theta q(\theta)$. Moreover, the probability to the vacant machine, of being mathed with a worker of a specific type h is

$$p_h(\theta) = q(\theta)\frac{u_h}{\sum_h u_h} = q(\theta)\frac{u_h}{u} \tag{2.10}$$

The probability to the worker of being matched with a machine age a is

$$p_a(\theta) = \theta q(\theta)\frac{v(a)}{\int_0^{\overline{a}} v(a)da} = \theta q(\theta)\frac{v(a)}{v} \tag{2.11}$$

2.4.2 Values

Vacancies are machines, currently not operated by a worker. As in the economy without frictions, above some age, $\overline{a}$, no workers accept a match to the machine. At this age the machine is cost-lessly scrapped. The value of a vacancy depends on the age of the machine a and the probability of finding a worker, which in turn is a function of labor-market tightness, θ. Moreover, the value depends on the availability of skilled and unskilled workers among the unemployed, u_U and u_S. Furthermore, the value of a vacancy is a function of the decisions to reject matches, the critical ages $\overline{a}_S$ and $\overline{a}_U$. We denote the value of a vacancy $V \equiv V(a,\theta,u_U,u_S,\overline{a}_S,\overline{a}_U)$.

We will see below that, in equilibrium, u_U, u_S and θ are functions of the decisions $\overline{a}_S$ and $\overline{a}_U$. Now, for ease of notation, we suppress all economy-wide arguments, except a, so that $V(a,\theta,u_U,u_S,\overline{a}_S,\overline{a}_U) \equiv V(a)$. Along a balanced growth path, the stationarized value $V(a)$ of a vacancy age a is given by:

$$(\rho - g)V(a) = \max\left\{q(\theta)\left[\left(\frac{u_S}{u}J_S(a) + \frac{u_U}{u}J_U(a)\right) - V(a)\right] + V_a(a), 0\right\} \tag{2.12}$$

$J_h(a)$ is the value to the firm of being matched with a worker type h. $\frac{u_h}{u}$ is the probability that the unemployed worker that the firm encounters is type h.Then, $q(\theta)\left[\left(\frac{u_S}{u}J_S(a)+\frac{u_U}{u}J_U(a)\right)-V(a)\right]$ is the expected gain of being matched to a worker. The value of a vacancy is non-negative, since the firm can always choose to scrap the machine at no cost. $V_a(a)$ is the change in the value of having an open vacancy due to aging. This derivative is negative, since an older machine has a shorter remaining productive life-span and is less productive than a younger machine. The firm, thus, scraps the machine when $V(a)=0$.

The value to an age a firm of being matched to a type h worker; $J_h(a)$; is given by

$$(\rho-g)\,J_h(a)=\max\left\{y_h(a)-w_h(a)+\sigma\left(V(a)-J_h(a)\right)+J_{ha}(a),(\rho-g)\,V(a)\right\} \tag{2.13}$$

The firm receives a profit-flow $y_h(a)-w_h(a)$, where $w_h(a)$ is the wage to a type h worker at age a. With exogenous probability σ the match is dissolved. The firm, then, gains the value of a vacancy and loses the value of being matched. The firm choses whether or not to remain in the match. Therefore, the value of employing a worker with a machine age a is never smaller than the value of a vacancy age a . At age $\overline{a}_h$ the firm chooses to dissolve the match. At this age we have $V(\overline{a}_h)=J_h(\overline{a}_h)$. For the type of workers that operate the oldest machine, we have $\overline{a}_h=\overline{a}$.

The value of unemployment - U_h - is given by

$$(\rho-g)\,U_h=b+\frac{\theta q(\theta)}{v}\int_0^\infty v(a)\left[W_h(a)-U_h\right]da \tag{2.14}$$

The worker receives instant utility b from being unemployed. With probability $\theta q(\theta)$ the firm is matched with a machine, which yields the expected gain $\frac{1}{v}\int_0^\infty v(a)\left[W_h(a)-U_h\right]da$, where $W_h(a)$ is the value to the type h worker of being employed with a machine age a. Note that, for ages above $\overline{a}$, we have $v(a)=0$, since firms choose not to post vacancies for machines that will never be accepted by workers.

The worker who is employed at a firm age a holds value $W_h(a)$, given by:

$$(\rho-g)\,W_h(a)=\max\left\{w_h(a)-\sigma\left(W_h(a)-U_h\right)+W_{ha}(a),(\rho-g)\,U_h\right\} \tag{2.15}$$

The worker only accepts matches of age $a\leq\overline{a}_h$. At $\overline{a}_h$ we have $W_h(\overline{a}_h)=(\rho-g)\,U(h)$.

2.4.3 Bargaining

We assume, as is standard in the literature, that the joint surplus from a match is allocated to the parties in Nash-bargaining. The surplus of a match is:

$$S_h(a) = (J_h(a) - V(a)) + (W_h(a) - U(h)) \tag{2.16}$$

The bargaining weights are ϕ and $(1-\phi)$ of the worker and firm, respectively. We, hence, have that:

$$W_h(a) = U(h) + \phi S_h(a)$$

$$J_h(a) = V(a) + (1-\phi) S_h(a)$$

Parties maximize the joint surplus. In equilibrium, they choose to dissolve matches at the age when $S_h(a) = 0$.

2.4.4 Surplus functions

The ages at which agents choose to dissolve matches - $\overline{a}_h$ - are equilibrium objects and functions of joint surplus-maximization. We now proceed to finding decision-rules that determine these ages, the Job-destruction conditions. First, it will be helpful to state the surplus-functions explicitly. We substitute for our expressions for the value of a vacancy (3.20), a producing machine (3.21), an unemployed worker (3.22) and an employed worker (3.24) into our definition of the surplus (3.25). We, then, for both types of workers have the surplus-function;

$$(\rho - g) S_h(a) = \max\left\{y_h(a) - \sigma S_h(a) - (\rho - g) U^h - q(\theta)(1-\phi)\left[\frac{u_S}{u} S_S(a) + \frac{u_U}{u} S_U(a)\right] + S_{h_a}(a), 0\right\} \tag{2.17}$$

Hence, for ages when both types of workers accept matches, we have the following system of differential-equations:

$$\begin{aligned}(\rho - g)\begin{bmatrix} S_S(a) \\ S_U(a)\end{bmatrix} &= \max\{e^{-\partial a}\begin{bmatrix} \chi_S \\ \chi_U\end{bmatrix} - (\rho - g)\begin{bmatrix} U_S \\ U_U\end{bmatrix} - \sigma\begin{bmatrix} S_S(a) \\ S_U(a)\end{bmatrix} \\ &\quad -q(\theta)(1-\phi)\begin{bmatrix} \frac{u^S}{u} & \frac{u^U}{u} \\ \frac{u^S}{u} & \frac{u^U}{u}\end{bmatrix}\begin{bmatrix} S_S(a) \\ S_U(a)\end{bmatrix} + \begin{bmatrix} S_{S_a}(a) \\ S_{U_a}(a)\end{bmatrix}, 0\}\end{aligned} \tag{2.18}$$

Some type of workers, which we denote e for early, may choose to reject matches at a lower age $\overline{a}_e$ than the other type of workers, which we denote type l for late. For notational ease, we define $\varrho \equiv (\rho - g + \sigma)$

and $\varphi_j \equiv q(\theta)(1-\phi)\frac{u^j}{u}$. The surplus for type e workers above $\overline{a}_e$ is zero. It follows from (3.31) that the surplus-function for older matches - above $\overline{a}_e$- is given by:

$$\varrho S_l(a) = \chi_l e^{-\partial a} - (\rho - g)\,U_l - \varphi_l S_l(a) + S_{la}(a) \quad ;S_l(a) > 0 \tag{2.19}$$

Production is decreasing in a. Therefore $S_l(a)$ is decreasing in a.

2.4.5 Joint maximizaion

The parties choose the separation ages in order to maximize match-surplus. Since $S_l(a)$ is decreasing in a we have $S_l(\overline{a}) = 0$. A match is operated only as long as it yields a positive surplus. We use this end-value-condition when solving the differential equation (2.19). The solution is

$$S_l(a) = e^{(\varrho+\varphi_l)a}\int_a^{\overline{a}} e^{-(\varrho+\varphi_l)\tilde{a}}\left[\chi^l e^{-\partial\tilde{a}} - (\rho - g)\,U^l\right]d\tilde{a}; \quad a > \overline{a}_e \tag{2.20}$$

Furthermore, joint maximization implies that the parties choose the separation-age - $\overline{a}$ - to maximize the value $S_l(a)$. The first order condition from this maximization is:

$$\chi^l e^{-\partial\overline{a}} = (\rho - g)\,U^l \tag{2.21}$$

Thus, at age $\overline{a}$ the worker is exactly compensated for not being unemployed. The firm is indifferent between producing and scrapping the machine and receives no compensation. All production is paid to the worker. As we will see below, there may or may not exist an age $\underline{a}_U$ before which firms reject matches with unskilled workers. In this case, the loss of the opportunity to search for a skilled worker with a sufficiently young machine - younger than $\underline{a}_U$ – is too large. Below, we define this age. For ages between $\underline{a}_U$ and $\overline{a}_e$ the surplus functions are given by (3.31). The solution, which we derive in Appendix 1, to this system of differential equations is:

$$\mathbf{S} = \begin{bmatrix} 1 & \frac{u_U}{u} \\ 1 & -\frac{u_S}{u} \end{bmatrix} * \left(\begin{bmatrix} e^{-(\varrho+\varphi_S+\varphi_U)(\overline{a}_e-a)}S_A(\overline{a}_e) \\ e^{-\varrho(\overline{a}_e-a)}S_D(\overline{a}_e) \end{bmatrix} + \int_a^{\overline{a}_e}\begin{bmatrix} e^{-(\varrho+\varphi_S+\varphi_U)(\tilde{a}-a)}\left(\chi_A e^{-\partial\tilde{a}} - (\rho-g)\,U_A\right) \\ e^{-\varrho(\tilde{a}-a)}\left(\chi_D e^{-\partial\tilde{a}} - (\rho-g)\,U_D\right) \end{bmatrix} d\tilde{a}\right) \tag{2.22}$$

Where $S_A = \frac{u^S}{u}S_S + \frac{u^U}{u}S_U$ is the expected - taken over shares in unemployment - surplus of a match and S_D is the difference in surplus between the skilled and the unskilled. S_A and S_D follow first order linerar differential equations - as is immediatedly seen from (3.31) above.

The first term within brackets corresponds to the end-value condition; that matches of the type of workers that do operate older machines is positive. The oldest machine could be operated by a skilled or an unskilled worker. If the unskilled operate the oldest machine - the end-value conditions are $S_A(\overline{a}_S) = \frac{u_U}{u} S_U(\overline{a}_S)$ and $S_D(\overline{a}_S) = -S_U(\overline{a}_S)$.If, on the other hand, the skilled operate the oldest machine, we have the end-value conditions: $S_A(\overline{a}_S) = \frac{u_S}{u} S_S(\overline{a}_U)$ and $S_D(\overline{a}_U) = S_S(\overline{a}_U)$. For these end-values, $S_j(\overline{a}_e)$ is given by (2.20), evaluated at $\overline{a}_e$.

Joint optimization - setting $\frac{d}{d\overline{a}_e}\mathbf{S} = \mathbf{0}$ - implies that, upon separation, the production of matches that separate at age $\overline{a}_e$ is allocated to the worker and the firm according to:

$$\chi_e e^{-\partial \overline{a}_e} = q(\theta)(1-\phi)\frac{u_l}{u} S_l(\overline{a}_e) + (\rho - g) U^e \tag{2.23}$$

The worker is exactly compensated for not choosing unemployment. The firm is exactly compensated for not vacating the machine and search for another worker.

We now return to the critical age $\underline{a}_U$ before which only skilled workers are accepted for employment. We see that $\underline{a}_U$ - if positive - solves the problem:

$$\begin{aligned} \begin{matrix} S_S(\underline{a}_U) \\ 0 \end{matrix} &= \begin{bmatrix} 1 & \frac{u_U}{u} \\ 1 & -\frac{u_S}{u} \end{bmatrix} * \\ &* \left(\begin{bmatrix} e^{-(\varrho+\varphi_S+\varphi_U)(\overline{a}_h - \underline{a}_U)} S_A(\overline{a}_h) \\ e^{-\varrho(\overline{a}_h - \underline{a}_U)} S_D(\overline{a}_h) \end{bmatrix} + \int_{\underline{a}_U}^{\overline{a}_h} \begin{bmatrix} e^{-(\varrho+\varphi_S+\varphi_U)(\tilde{a}-\underline{a}_U)} \left(\chi_A e^{-\partial\tilde{a}} - (\rho-g) U_A\right) \\ e^{-\varrho(\tilde{a}-\underline{a}_U)} \left(\chi_D e^{-\partial\tilde{a}} - (\rho-g) U_D\right) \end{bmatrix} d\tilde{a} \right) \\ \textit{and} \quad \underline{a}_U &> 0 \\ \textit{or} \quad \underline{a}_U &= 0 \end{aligned} \tag{2.24}$$

Maximization implies that, $S_U(a) = 0$ if $a < \underline{a}_U$. For ages lower than $\underline{a}_U$ we have that $S_S(a)$ is given by:

$$S_S(a) = e^{(\varrho+\varphi_S)(a-\underline{a}_U)} S_S(\underline{a}_U) + e^{(\varrho+\varphi_S)a} \int_a^{\underline{a}_U} e^{-(\varrho+\varphi_S)\tilde{a}} \left[\chi^S e^{-\partial\tilde{a}} - (\rho-g) U^S\right] d\tilde{a} \quad ; a < \underline{a}_U \tag{2.25}$$

We have described the surplus-functions that characterize our economy for all ages during which production is undertaken, and the decision-rules that follow from joint maximization. We now turn to discussing the job-creation and job-destruction conditions.

2.4.6 Job Creation and Job-destruction

Firms create jobs by investing in new machines. There is free-entry to investment. Thus, the value of a vacancy age 0 in equilibrium equals the cost of investment ι. Hence, the Job-Creation condition is:

$$\iota = V(0) = q(\theta)(1-\phi)\int_0^{\overline{a}} e^{-(\rho-g)\widetilde{a}} S_A(\widetilde{a})\, d\widetilde{a} \tag{2.26}$$

where $S_A(\widetilde{a})$, as above, is the expected surplus of a match age $\widetilde{a}$. The value of a new vacancy is the discounted expected gains from search (See Appendix 2 for details).

Matches are dissolved at two ages - one at the oldest age at which machines are operated, and one at the intermediate age after which only one type of workers accept matches. There are, thus, two Job-destruction conditions, which we get by combining the rules for separation (2.21) and (3.35) with the corresponding values of unemployment (3.22):

$$\chi_l e^{-\partial\overline{a}} = b + \frac{\theta q(\theta)}{v}\phi\int_0^{\overline{a}} v(a)S_l(a)da \tag{2.27}$$

$$\chi_e e^{-\partial\overline{a}_e} - q(\theta)(1-\phi)\frac{u_l}{u}S_l(\overline{a}_e) = b + \frac{\theta q(\theta)}{v}\phi\int_0^{\overline{a}_e} v(a)S_e(a)da \tag{2.28}$$

The marginal productivity of the worker that operates the oldest machine equals his or her value of unemployment. For the other type of workers the flow value of unemployment equals production upon separation less the compensation the employer receives for not vacating the machine.

2.4.7 The stationary distributions

In equilibrium, the distribution of vacancies (and matched machines) over ages is stationary. We here, in order to characterize the equilibrium, solve for these stationary distributions in terms of the endogenous variables. As in the economy without frictions, we denote by η the constant measure of machines that is installed at every instant. We denote by $m(a)$ the measure of machines age a that are matched to a worker. $v(a)$ is is the measure of vacant machines age a. We have $\eta = m(a) + v(a)$. The measure of matches evolves according to the law of motion:

$$\begin{aligned} m_a(a) &= -\sigma m(a) + q(\theta)\frac{u_S}{u}v(a) \quad if \quad 0 \le a < \underline{a}_U \\ m_a(a) &= -\sigma m(a) + q(\theta) v(a) \quad if \quad \underline{a}_U \le 0 < \overline{a}_e \\ m_a(a) &= -\sigma m(a) + q(\theta)\frac{u_l}{u}v(a) \quad if \quad \overline{a}_e \le a < \overline{a} \end{aligned}$$

The solution to this problem is:

$$\begin{aligned} m(a) &= e^{-(\sigma+\widetilde{q})(a-a_o)}m_o + e^{-(\sigma+\widetilde{q})a}\eta\widetilde{q}\int_{a_o}^{a} e^{(\sigma+\widetilde{q})\widetilde{a}}d\widetilde{a} \quad ; \quad where \\ \widetilde{q} &= q(\theta)\frac{u_S}{u} \quad ; \quad m_o = 0 \quad ; \quad a_o = 0 \quad ; \quad if \quad 0 \le a < \underline{a}_U \\ \widetilde{q} &= q(\theta) \quad ; \quad m_o = m(\underline{a}_U) \quad ; \quad a_o = \underline{a}_U \quad ; \quad if \quad \underline{a}_U \le a < \overline{a}^e \\ \widetilde{q} &= q(\theta)\frac{u_l}{u} \quad ; \quad m_o = m_l(\overline{a}_e) \quad ; \quad a_o = \overline{a}_e \quad ; \quad if \quad \overline{a}_e \le a \end{aligned}$$

where m_l denotes the matches involving workers type l. If $\underline{a}_U = 0$, which implies that all workers accept a match to a new machine, $m_l(\overline{a}^h) = \frac{u_l}{u}m(\overline{a}^h)$. Otherwise, matches - for ages that are accepted by both types of workers - are given by:

$$\begin{aligned} \begin{bmatrix} m_S(a) \\ m_U(a) \end{bmatrix} &= m_S(\underline{a}_U)\begin{bmatrix} pe^{-(q+\sigma)(a-\underline{a}_U)} + e^{-\sigma(a-\underline{a}_U)}(1-p) \\ (1-p)e^{-(q+\sigma)(a-\underline{a}_U)} - e^{-\sigma(a-\underline{a}_U)}(1-p) \end{bmatrix} \\ &\quad + q\eta\int_{\underline{a}_U}^{a}\begin{bmatrix} pe^{(\widetilde{a}-a)(q+\sigma)} \\ (1-p)e^{(\widetilde{a}-a)(q+\sigma)} \end{bmatrix}d\widetilde{a} \end{aligned}$$

The vacancy distribution is merely $v(a) = \eta - m(a)$. It follows that factor markets clear when

$$L_S - u_S = \int_0^{\overline{a}_S} m(a)da \quad ; \quad L_U - u_U = \int_{\max\{\underline{a}_U 0\}}^{\overline{a}_U} m(a)da \quad ; \quad \eta\overline{a} = v + \int_0^{\overline{a}} m(a)da$$

The measure of matches at any age is increasing in the probability that a vacancy encounters a worker. Thus, the Right-hand sides of the expressions from the labor market are increasing in their respective measures of unemployment. The Left-hand sides are obviously decreasing in their respective measures of unemployment. It follows that the quatruplet $\{\overline{a}, \overline{a}_h, \underline{a}_U, \theta\}$ determines unique levels of unemployment. From the definition of θ it follows that the supply of vacancies is also uniquely determined by the same quatruplet. The capital-market condition, thus, determines η, the investment level. Hence, our equilibrium is characterized by the quatruplet $\{\overline{a}, \overline{a}_h, \underline{a}_U, \theta\}$.

2.4.8 Equilibrium

We can characterize the equilibrium with production in our economy as a measure of tightness $\{\theta\}$ and decisions $\{\overline{a}, \overline{a}_h, \underline{a}_U\}$ such that;

i) The Job-Destruction conditions (2.28) and (3.40) hold.

ii) The Job-Creation condition (3.39) holds.

iii) The condition (3.37) holds.

2.4.9 Analysis of equilibria

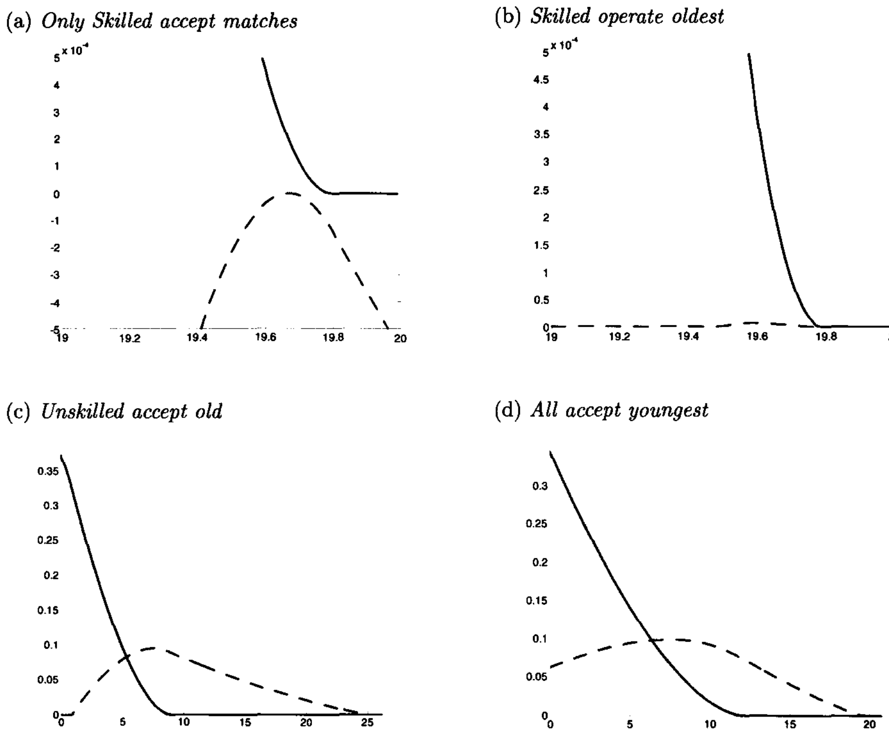
In our economy, there are three classes of equilibria with production. First, we may have an equilibrium in which only the skilled work. This will be the case if the value of leisure is sufficiently high, so that unskilled workers accept no matches. Second, we may have an equilibrium in which the skilled workers operate the oldest machine, and the unskilled operate old machines. Third, we have an equilibrium where the oldest machine is operated by an unskilled worker - and skilled workers operate younger machines. In this section we describe the intuition behind the conditions that determine the equilibrium setting.

As a thought-experiment, in order to illustrate the different classes of equilibria, we vary χ_U while holding all other parameters constant. As we let χ_U increase, we move from the equilibrium where the unskilled do not work, via the equilibrium where the unskilled operate a range of old machines, but where the oldest machine is operated by a skilled worker, to the equilibrium where the unskilled operate a range of older machines than the skilled.

Only the skilled accept matches

First, we note that there will be no production in the economy if $\chi_S < b$. In this case, not even the youngest machine produces more than the unemployed skilled worker. Thus, even if investment were costless - there would be no investment.

Figure 1 - Classes of equilibria



Balanced growth equilibrium Surplus-functions of the skilled (black) and unskilled (grey/dashed).
In panel (a) $\widetilde{S}_U$ from equation (2.29) is depicted for $\underline{\chi}$ as defined in (2.30).

By the same argument, for small enough χ_U, only skilled workers will accept matches. (For example, if $\chi_U < b$ any machine would produce less with an unskilled worker than the utility the unemployed worker derives from not accepting any matches). If the productivity of the unskilled is large enough, however, both types of workers will accept some matches.

There, thus, exists some critical level of $\chi_U \equiv \underline{\chi}_U$ at which unskilled workers begin accepting matches, taking all other parameters given. Here, we describe this critical level of productivity.

We start by analysing the economy in which only skilled workers accept matches. We easily see - from (2.28) - that the value of unemployment to the unskilled is given by $(\rho - g)\, U_U = b$. The unemployed receives the discounted value of being unemployed forever.

For the economy in which only skilled workers accept matches, we can define the function:

$$\widetilde{S}_U(a) = \chi_U e^{-\partial a} - q(\theta)(1-\phi)\frac{u_S}{u}S_S(a) - b \tag{2.29}$$

We have merely re-written the Job-Destruction-condition (2.28). By definition, this function is negative for all a (otherwise, the unskilled would accept matches of some age). We note that this function attains a maximum when

$$\partial \chi_U e^{-\partial a_{\max}} = -q(\theta)(1-\phi)\frac{u_S}{u}S_{aS}(a_{\max})$$

Maximization implies that the Right Hand Side of this expression converges to 0 as $a_{\max}$ approaches $\overline{a}_S$. The Left Hand Side is positive for any $a_{\max}$. Thus, $a_{\max} < \overline{a}_S$. Evidently, $\widetilde{S}_U$ is increasing in χ_U. Thus, for larger χ_U , the maximum attainable value of $\widetilde{S}_U$ is larger. For some $\chi_U \equiv \underline{\chi}_U$ this value is exactly 0 so that:

$$\begin{aligned} b + q(\theta)(1-\phi)\frac{u_S}{u}S_S(a_{\max}) &= \underline{\chi}_U e^{-\partial a_{\max}} \\ where \quad a_{\max} \quad satisfies \quad \partial \underline{\chi}_U e^{-\partial a_{\max}} &= q(\theta)(1-\phi)\frac{u_S}{u}S_{aS}(a_{\max}) \end{aligned} \tag{2.30}$$

S_S is given by equilibrium conditions for the economy where only skilled workers operate machines[4]. In figure 1, panel (a), we depict the surplus function of the skilled, and the function $\widetilde{S}_U$ for $\underline{\chi}_U$ in the equilibrium in which only the skilled work[5]

Interestingly, we note that - in contrast to the economy with homogenous workers - even if the unskilled workers could produce more in the market than as unemployed - i.e $\chi_U > b$ - they may choose not to accept any matches. Firms that hold young vacancies may choose to reject matches with unskilled workers, since this means refraining from searching for a skilled worker.

Firms only accept matches to unskilled workers if the production is larger than the sum of the flow value of unemployment and the expected surplus from meeting a skilled worker; $q(\theta)(1-\phi)\frac{u_S}{u}S_S(a)$. This follows immediately from (2.29). Evidently, the choice to participate in the market or not could be analyzed in a calibrated version of our model. We here, however, leave this for future research.

All workers accept matches

If $\chi_U > \underline{\chi}_U$, both types of workers accept some matches. For χ_U marginally larger than $\underline{\chi}_U$ we have that the oldest machine is still operated by a skilled worker, whereas the unskilled workers operate machines of

[4]Hornstein, Krusell and Violante analyse the equilibrium with homogenous agents thoroughly, and we refer to their study for analysis of this setting.

[5]We use all parameters that are assumed to describe the US economy after 1975, investment-specific technical change as well as the supply of skilled labor is higher than in the base-line calibration. We change the productivity of the unskilled workers. In this setting $\underline{\chi}_U$ is 0.1201 which is larger than the unemployment benefit $b = 0.05$.

lower ages. This equilibrium is illustrated in Figure 1, panel b) above. The oldest - and least productive - machine in the economy cannot compensate the unskilled for not being unemployed. Younger, and more productive, machines can. For a range of ages it is more lucrative to the firm to employ unskilled workers, than to search for skilled workers. Yet, the machines operated by the unskilled are, in the calibrations we have investigated, on average older than the machines operated by the skilled, since $\underline{a}_U$ is large.

As we let χ_U increase, we reach the equilibrium where the unskilled operate the oldest machine. The oldest machine is operated by an unskilled worker. The youngest machine may not be operated by the unskilled - as depicted in panel (c). For small differences in χ the youngest machine is operated by both types of workers - as depicted in panel (d) above.

In (2.22), we see that if the skilled and unskilled are equally productive, they operate the same range of machines. End-values are 0, S_D is 0, the surplus of both types of workers is S_A and they enjoy the same value of unemployment.

2.4.10 Wages

We have seen that we solve for the equilibrium without explicitly solving for wages. Nevertheless, since wage-heterogeneity is the focus of our investigation, we here derive the expression for wages. The value to the employed worker is given by (3.24). This expression, combined with the Nash-bargaining assumption, yields:

$$
\begin{aligned}
w_h(a) &= (\rho - g)\,U(h) + \phi\left((\rho - g + \sigma)\,S_h(a) - S_a(a,h)\right) = \\
&= (\rho - g)\,U(h) + \phi\left(y^h(a) - (\rho - g)\,U^h - q\,(\theta)\,(1-\phi)\,[E_h S_h(a)]\right)
\end{aligned}
$$

The term $\left(y^h(a) - (\rho - g)\,U^h - q\,(\theta)\,(1-\phi)\,[E_h S_h(a)]\right)$ is the production that exceeds the flow value of unemployment and the gain from search for a vacant machine - the quasi-rent from the match. The wage, thus, is the sum of a compensation for not being unemployed and the bargaining weight of the quasi-rent in the match.

2.5 Quantitative experiments

We study the response of the skill-premium to changes in the rate of investment-specific technical change (γ) and changes in the stock of skilled workers (L_S). Cummins and Violante (2003) document that the rate of investment specific technical increases over the post-war period. We assume - as do Vilante (2002) and HKV - that this rate shifts from a lower level to a higher level in 1975. We, therefore, calibrate the model to match some moments from the US economy prior to 1975. Given this parameterisation we quantitatively investigate the consequences of shifts in γ and L_S. We find that these shifts alone do not allow our model to replicate the increase of the skill-premium. In addition we, therefore, investigate the consequences of skill-biased technical change (an increase in χ_S).

First, we calibrate the model and make observations in the calibrated model. Second, we conduct static experiments, where we change the rate of investment-specific technical change and the productrivity of the skilled, respectively. We analyse our findings, and relate them to previous studies.

2.5.1 Calibration

We calibrate our model to match some features of the US economy prior to 1975. There are 14 parameters for this economy: $\{\alpha, \delta, \phi, \rho, \sigma, \psi, \omega, A, b, \chi^U, \chi_S^{t<1975}, \gamma^{t<1975}, L_u^{t<1975}, \iota\}$ .Our calibration-strategy is straightforward. We use those parameter values from Hornstein-Krusell and Violante that they choose from existing estimates. This allows us to calibrate $\{\psi, \alpha, \gamma^{t<1975}, \omega, \rho, b\}$.

Eight parameters remain. We let our model match some moments of the US economy prior to 1975 to calibrate $\{\delta, \phi, \sigma, A, \iota\}$. We match the same moments that HKV match for the calibration of remaining parameters that are common to the models. Some parameters relate to the studied heterogeneity between skilled and unskilled workers. We choose these parameters; $\{\chi_S^{t<1975}, \chi^U, L_u^{t<1975}\}$; to match stylized facts from the US labor market prior to 1975.

Table 1. Technical Change

ψ	0.008	*disembodied technical change*
$\gamma^{t<1975}$	0.04	*decline of relative price of investment-goods*
α	0.3	*growth rate of output per capita*

We follow Hornstein, Krusell and Violante (2004) in calibrating the parameters that govern technical change. In their, as well as in our, model there are two sources of productivity growth - disembodied (or neutral) technical change - which we denote ψ - and capital-embodied (or investment-specific) technical change, which we denote γ. HKV - with reference to Hornstein and Krusell (1996) set the rate of disembodied technical change to $\psi = 0.008$.

Along a balanced growth path, the fixed cost of investment ι grows at rate $g = \psi + \alpha\gamma$. The productivity of raw capital goods grows at γ. This implies that the relative price of capital goods - the amount of efficiency units of capital that one final good can be transformed into - changes at rate $g - \gamma$. Cummins and Violante document that the quality adjusted relative price of capital $(g - \gamma)$ declined at an average rate of 2 per cent prior to 1975. The average growth rate of per capita production (g) prior to 1975 was 2 per cent. We use these observations to calibrate $\gamma^{t<1975} = 2 - (-2) = 4$ per cent per annum.

The definition of $g = \psi + \alpha\gamma = 0.02$ and our values of $\psi = 0.008$ and $\gamma^{t<1975} = 0.04$ imply that $\alpha = 0.3$. This concludes the calibration of the parameters that govern technical change. Below, we will study the consequences of changes in γ as documented by Cummins and Violante.

Table 2. Labor market

ω	0.5	*micro-estimates*
L_u	0.87	*Katz & Murphy (1992)*

We set the elasticity of the matching-function with regard to vacancies $(1-\omega)$ to 0.5. This is the average of the estimates reported in the overview by Petrongolo and Pissardes (2001) and is also the value that HKV use. We solve our model for a balanced-growth path. The supply of skilled labor, however, is not stable and increases from the sixties on. The relative supply of workers without a college-degree in 1963, as measured by hours in the CPS, was 87 per cent as reported by Katz & Murphy (1992), from which level it declines. To break ties[6], we set $L_u = 0.87$.

We, like HKV, choose to calibrate the bargaining-weight by matching the equilibrium outcome of the model to the observed labor-share in the US economy.

Table 3. Other external parameters

ρ	0.04	*interest rate*
b	0.05	*HKV*
χ^U	1	*Normalization*

We set the subjective discount-rate to 0.04 to match an equilibrium real interest rate of 4 per cent. We, as HKV, set unemployment benefits to 0.05. We normalize the human capital of the unskilled to 1. Below, in Appendix 4 we, as a sensitivity-analysis, investigate consequences of our choice of b by assigning a wide range of values for this parameter. We find that our assumption on the level of the unemployment benefit does not drive our results with regard to the wage-structure in any significant manner. The remaining parameters we calibrate by matching the equilibrium outcome of the model to moments of the US economy prior to 1975.

Table 4. Internally calibrated parameters

δ	0.072
σ	0.20
A	12.4
$\chi_S^{t<1975}$	1.34
I	4.35
ϕ	0.83

We, like HKV, simultaneously choose $\{\delta, \sigma, A, \chi_S, I, \phi\}$ to match some characteristics of the US economy prior to 1975. The depreciation rate δ is chosen to match an average age of capital goods of 11.5 years. Abrahams and Shimer (2002) report that the average duration of unemployment is 2 months. We choose the scale-parameter of the matching-function - A - to have the model satisfy this observation. We choose the separation rate, σ, to match an unemployment rate of 4 per cent. This implies a separation rate of 25

[6] By choosing the lowest relative supply of skilled workers we maximize the size of the supply-shock in skilled labor. With regard to our predictions of the increase in the skill-premium, we - thus - minimize the impact of changes in investment-specific technical change. However, below, in Appendix 4, as a sensitivity-analysis we try out how a range of alternative values for L_u and its changes alter results with regard to the skill-premium in our quantitative experiments.

per cent. Note that σ is lower, since some separations occur endogenously - when matches reach the critical age $\overline{a}$ or $\overline{a}_e$. The fixed cost of investment, I, affects vacancy-creation. As HKV, we choose this parameter to match the observation of Hall (2005) that vacancies on average have a duration of 1 month. We choose the Nash-weight of the employees, ϕ, to match a labor share of 70 per cent.

Furthermore, our model has skill-heterogeneity. We choose the productivity of the skilled, χ_S, to match a college-premium of 57 per cent, the average for the period 1963 to 1975 as reported by Katz and Murphy (1992). Note that χ_S is lower than 1.57 - since workers are sorted according to ability.

2.5.2 Observations in the calibrated model

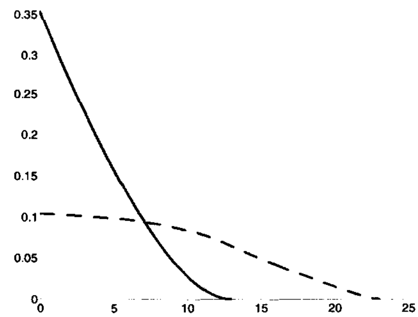
Table 5. Observations in the calibrated model with frictions

$\overline{a}_U$	23
$\overline{a}_S$	13.0
$average\ a_U$	12.3
$average\ a_S$	6.5
$\overline{w}_S/\overline{w}_U$	1.57
$var(\log(w_S/\overline{w}_S))$	0.0015
$var(\log(w_U/\overline{w}_U))$	0.00012

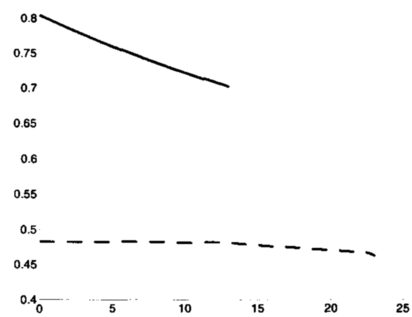
Before we undertake experiments in our model economy, we observe some descriptive statistics of the economy. The equilibrium in our preferred parameterisation is positively sorted - the unskilled operate the oldest machine. Both types of workers are accepted by a new machine. The skilled reject machines that are older than 13 years - whereas the unskilled accept machines until the age of 23 years[7]. The skill-premium is 1.57 by construction.

[7] The rejection age of the unskilled is 23 years by construction. Since - along a balanced growth path - machines are uniformly distributed over ages, an average age of 11.5 years (one of our calibration targets) implies a highest rejection age of 23 years.

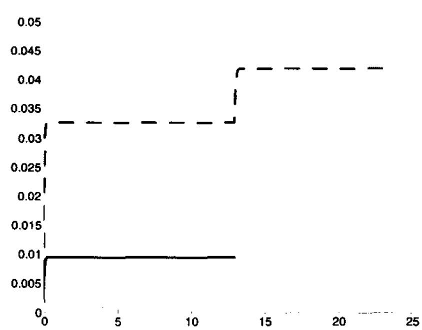
Figure 2 - Baseline Calibration
(a) Surplus



(b) Wages



(c) Matches



Surplus, wages and match-densities of the skilled (black) and the unskilled (dashed/grey) in the base-line equilibrium

The average age of machines operated by skilled workers is slightly more than 6.5 years. The average age of machines operated by unskilled workers is 12.3 years. The probability of finding a job is high (since the average duration of unemployment is 2 months). Thus, the skilled workers that are employed are almost uniformly distributed over the age-span of machines that they operate. The distribution of matches for the unskilled is skewed toward older machines, as can be seen in Figure 2, panel (c). For younger ages, vacancies also are accepted by the skilled. Therefore, a larger portion of the unskilled are matched with older machines in equilibrium.

The variance of wages of the skilled is significantly greater than the variance of wages of the unskilled. Wage heterogeneity stems from heterogeneity in age across matches. The level of within-group variance is much lower than their empirical counterparts[8]. Nevertheless, it is a virtue of the model that it predicts larger within-group variance among the skilled than among the unskilled. The surplus of the skilled is large for low ages. This yields compression in wages among the unskilled for these ages. Firms are compensated for not vacating their machines in search of a skilled worker.

2.5.3 Quantitative comparative statics

We have calibrated the model to match the US economy prior to 1975. In this section, we investigate the consequences of an increase in the rate of investment-specific technical change (γ) and an increase in the supply of skilled labor (L_S). Furthermore we study the effects of an increase in the productivity of the skilled workers (χ_S). Finally, we study combinations of these changes. Our aim is to quantify - in our frame-work - the relative importance of these modifications in explaining changes in the skill-premium. We also study consequences for within-group inequality of the same operations.

As a point of reference, we include corresponding changes in the friction-less economy[9].

Table 6. Comparative Statics *relative changes*

	base-line	γ	L_S	χ_S	γ ; L_S	γ; L_S ; χ_S
$\overline{a}_U$	23	0.86 (0.86)	0.99 (0.99)	0.998 (0.997)	0.86 (0.85)	0.85 (0.85)
$\overline{a}_S$	13.0 (3.91)	0.84 (0.86)	1.11 (1.53)	0.95 (0.997)	0.94 (1.32)	0.90 (1.31)
average a_U	12.3	0.86 (0.86)	1.05 (1.07)	1.004 (0.997)	0.91 (0.92)	0.91 (0.92)
average a_S	6.5	0.84 (0.86)	1.11 (1.53)	0.95 (0.997)	0.96 (1.32)	0.90 (1.31)
$\overline{w}_S/\overline{w}_U$	1.57	1.03 (1.034)	0.99 (0.972)	1.10 (1.11)	1.01 (1.002)	1.10 (1.09)
$var(\log(w_S/\overline{w}_S))$	0.0015	1.29	0.88	1.09	1.14	1.21
$var(\log(w_U/\overline{w}_U))$	0.00012	1.04	0.88	0.89	1.34	2.46

(Results from the model without frictions in parenthesis)

[8] Lemieux (2006) finds that, as an average over the period 1973 − 75, the variance of log wages ranges from 0.1 to 0.16, depending on experience, for groups without a college degree. For college-graduates the same range was 0.13 to 0.28.

[9] We calibrate the friction-less economy to match the same average age of capital, skill-premium and have the same relative supply of skilled workers. The resulting parameters for the friction-less economy are $\chi_S = 1.30; I = 4.78$.

An increase in the rate of investment-specific technical change

Cummins and Violante observe that the relative price of capital goods has been declining faster during the period following 1975 than during the preceding period. In particular, they note that the relative price decline of capital goods after 1975 has, on average, been -4.5%. Since the relative price-decline is given by $\left(g - \gamma^{t>1975}\right) = \psi - (1-\alpha)\gamma^{t>1975}$ we find $\gamma^{t>1975} = 0.076$. The rate of investment-specific technical change has, hence, increased significantly. We solve for the steady-state equilibrium in our calibrated model under the new growth-regime. We report our results in Table 6. An increase in the rate of investment-specific technical change implies that the relative productivity of machines falls faster. This, in turn, reduces the horizon over which investors discount their investments. In equilibrium, wages as a share of production have to be lower for firms to re-coup their fixed cost of investment. Also, the equilibrium probability for vacancies to find workers (q) is higher. Thus, unemployment is higher for both types of workers.

Our focus is effects on the wage-structure. An increase in γ reduces the ages at which workers reject matches. The separation-age falls more for the skilled than for the unskilled. This contributes to an increase in the skill-premium with 3 per cent. Inequality among the skilled increases. Faster investment-specific technical change implies that relative production declines faster with age. Consequently, the surplus of the skilled declines faster with age.

Since wages are functions of the surplus, the wage profile of the skilled becomes steeper - implying increased within-group inequality. Within-group inequality also increases among the unskilled - but to a lesser degree. As we have seen above, within-group inequality is small compared to the empirical counterparts. Nevertheless, our model predicts larger increases in within-group inequality among the skilled than among the unskilled.

An increase in the supply of skilled labor

We increase the supply of skilled labor to 26.3 per cent - the value reported for 1987 by Katz and Murphy (1992). In the friction-less economy, the rejection age of the unskilled is reduced. Therefore, the wage of the unskilled increases in equilibrium. Free entry implies that more firms enter as the supply of skilled workers increases, since more skilled workers increase the present value of a new machine at unchanged decision-rules. Increased investment reduces the maximum age of a machine that is used in production. This in turn determines the wage of the unskilled.

The same mechanism is operating in the economy with frictions. The oldest machine, which is operated by the unskilled, is younger in the new equilibrium. The age of the oldest machine determines the value of unemployment to the unskilled. Thus, unskilled wages increase.

Skilled workers on average work with older machines, and skilled wages are lower on average. The skill-premium, thus, contracts. This effect is stronger in the friction-less economy than in the economy with frictions. With frictions the increase in the average and maximum ages operated by the skilled increases less than in the friction-less case. This implies that their wages decline by less than in the friction-less case.

The within-group variance of both the skilled and the unskilled decreases. More skilled labor increases the likelihood for the firm to find a skilled worker. This affects the outside-option for firms that are not older than the rejection-age of the skilled. In a match it allows them to extract a larger fraction of production. This yields a flatter wage-schedule for the skilled. Equivalently, unskilled wages for ages that are accepted by the skilled become more compressed. Also, both types of workers on average operate older machines for which production-outcomes (in absolute numbers) are less disperse.

An increase in the productivity of the skilled

We have targeted our model to the average male skill-premium for the period 1963 – 1975. Following a fall during the seventies, the skill-premium has increased over the eighties and the nineties. In the late eighties the skill-premium had increased with approximately 10 per cent from this level. We, therefore, choose to study an increase in the productivity of the skilled that corresponds to an increase in the skill-premium of this magnitude[10]. Note that this implies an increase in the productivity of the skilled of 6.8 per cent in the economy with frictions. The remaining 3.2 per cent result since workers are sorted.

The skilled reject machines at a younger age. The rejection-age of the unskilled declines very little. This implies that more unskilled operate machines of older ages - causing the average age of machines operated by the unskilled to increase somewhat. Wages of the unskilled are virtually unchanged - and increase only by 0.08 per cent. In the friction-less economy wages of the unskilled increase more. As skilled workers become more productive, more firms enter. Investment is larger and consequently the rejection-age is lower. This implies higher wages to the unskilled.

Within-group inequality among the skilled increases. We can understand the mechanism by studying the friction-less economy. Increasing productivity of the skilled implies that a larger fraction of the discounted profits that firms invest to reap are produced by the skilled. Similarly, a larger fraction of the discounted surplus in the economy with frictions will stem from expected matches with skilled workers. This implies that the surplus-function of the skilled is skewed to the left and starts at a higher level. Since wages are a function of the surplus of the match, the wage distribution of the skilled, also becomes steeper. Unskilled workers, on the other hand, on average come to operate older machines - for which production-levels are less disperse.

A combined effect of supply and investment-specific technical change

The two observed changes to the US economy that we have studied above - the increase in investment-specific technical change and the increase in the supply of skilled labor - have occured at the same time. As we have

[10] In our exercise, we compare balanced growth paths. As is familiar, the skill-premium does not increase monotonically during the post-war period. We have calibrated the model to match the average skill-premium from 1963 to 1975. This is not very different from the average skill-premium for the period 1975 to 1987. Yet, the skill premium increases by more than 20 per cent during the eighties.

In our exercises below, we choose to target an increase of 10 per cent - which is the approximate increase from our targeted average level to the skill-premium in the last three years of the eighties.

seen, the technological change increases the skill-premium and the within-group variance of log wages. An increase in the supply of skilled labor works in the opposite direction for both between-group and within group measures of inequality. We now study the equilibrium outcome when we let both these changes occur simultaneously.

The skill-premium increases by 1 per cent in the economy with frictions, and by less in the friction-less economy. In the economy with frictions, the average and maximum age acceptable to the unskilled decrease. The same holds true for the skilled, but to a lesser degree. This is not true for the friction-less economy, in which the rejection-age of the skilled increases due to the increasing supply of skilled workers. Thus, frictions reduce the negative effect of increased supply on the wage of the skilled.

Within-group inequality increases more among the unskilled than among the skilled. The increased supply of skilled workers reduce the surplus of the unskilled at young ages. Furthermore, increased investment-specific technical change reduces the life-time of a machine, which also works in the direction of reducing the surplus of young unskilled matches. The combined effect surpresses unskilled wages in the younger end of the distribution. The wage-distribution for the unskilled is hump-shaped. This increases the variance of wages among the unskilled.

A combined effect

We now study changes in labor-supply, investment-specific technical change and an increase in the productivity of the skilled. As above, we study a change in the skill-premium of 10 per cent. In this experiment it implies an increase in χ_S with 5.9 per cent. In the economy with frictions, all critical ages decrease - as in the previous exercise. In the friction-less economy the rejection-age of the skilled increases due to the increase in supply of skilled labor. Then, vintage capital, assortative matching and the acceleration in investment-specific technical change explains 40 per cent of a ten per cent increase in the skill-premium, while accounting for the increase in the supply of skilled labor.

Within-group inequality increases, and the relative incree is larger among the unskilled. As in the previous exercise, wage outcomes in the younger tail decrease to the degree that the dispersion of wages increases. The level of within-group inequality is still, however, considerably larger among the skilled.

Discussion

We have undertaken several experiments with our model. We find that an increase in investment-specific technical change increases the skill-premium in both the friction-less economy and the economy with frictions. We also find that an increase in the supply of skilled labor compresses the skill-premium. Both these effect are stronger in the friction-less economy. The net effect of a simultaneous shift in both these parameters is positive with regard to the skill-premium, in both the friction-less economy and the model with frictions. It is more positive in the model with frictions, where the supply effect is of less consequence.

An increase in the productivity of the skilled, naturally, increases the skill-premium. In our frameworks, an increase in the productivity of the skilled has a disproportionate impact on the skill-premium. This follows since workers are sorted. When investment-specific technical change and labor supply shift simultaneously, a 5.9 per cent increase in the residual productivity of skilled workers implies a 10 per cent increase in the skill-premium.

Lemieux (2006) documents that within-group inequality is larger among college-graduates than among those with less education. This is consistent with the predictions of our model. The predicted levels of within-group inequality are, however, considerably smaller than the empirical counterparts. Shifts in the rate of investment-specific technical change or in the productivity of the skilled increase inequality among the skilled more than among the unskilled. This is what Lemieux documents in the data. Our simultaneous change of these parameters and the supply of skilled workers, however, predicts a larger increase in the inequality among the unskilled, than among the skilled.

Within-group heterogeneity in our model stems from *ex-ante* heterogeneity among firms. Their productivity is given by their age. Previous studies of matching models with *ex-ante* heterogeneous agents include Sattinger (1995), Acemoglu (2000), Wong (2003) and Shimer and Smith (2000). Sattinger and Wong assume that the matching parties are heterogeneous by an exogenous discrete productivity-variable. In Acemoglu, firms are free to choose the productivity (or size) of the capital-stock in a match, but in equilibrium at the most choose one productivity per type of worker. In Shimer and Smith the productivities of the matching parties are continuously distributed.

In the models with discrete equilibrium distributions of productivities - Sattinger, Wong and Acemoglu - within-group inequality arises in 'mixed' equilibria, that is when workers of one type accepts matches with different types of firms. Mixed equilibria occur when productivity differences between workers are sufficiently small - so that the gain to skilled workers of searching for a high-quality match is small. Therefore they also settle with low quality matches. Similarily, high-quality firms may accept a match with a low-skilled worker if the production is not very different from that in a high-skill match.

Wong (2003) studies an economy with a (2×2) matrix of productivities. Firms are either high-tech or low-tech. Workers are either skilled or unskilled. Within-group inequality among the skilled exists if skilled workers accept both high-tech and low-tech jobs. In the calibrated model, an increase in the productivity of the skilled workers does not increase within-group inequality among workers, while the skill-premium increases. Wong finds that "the (Mortensen-Pissarides) model fails to account for the wage inequality pattern over time found in the data." We have above illustrated that the generality of this claim is limited. In our version of the Mortensen-Pissarides framework, between- and within-group inequality do increase simultaneously.

There are differences between the setting in Wong and ours. The fundamental difference is that firms in our model are not high-tech or low-tech by nature. Today's high-tech firms are tomorrow's low-tech firms. A more direct comparison between our work and Wong would be to assume time (and therefore productivity) discrete. We leave such efforts for future research.

As documented in Sattinger, there may exist multiple equilibria in settings where productivities are distributed discretely. For the same parameters an economy could sustain both a mixed equilibrium and a sorted equilibrium. It is important to note that our quantitative results do not stem from discrete shifts between different equilibria. Smooth changes in parameter-values generate smooth changes in the decision-variables; the rejection ages. For all exercises above, we remain in the equilibrium where the youngest firm accepts workers of both types, and the oldest machine is operated by an unskilled worker. Furthermore, transitions between the equilibria described in section IV above do not imply discrete jumps in the decision-variables.

Shimer and Smith (2000) study a model with search-frictions, where agents on both sides are heterogenous. Agents are heterogeneous according to some continuous function - as are the firms in our model. They describe an equilibrium where agents accept a range of match-qualities that includes the quality that would have been accepted under no frictions. In a labor-market setting, this is equivalent to observing wage-heterogeneity among observationally identical agents. In our calibrated model, workers accept a range of firms. In this respect, our equilibrium is similar to that described by Shimer and Smith. As opposed to Shimer and Smith, the productivity distribution of firms is endogenous in our model - and results from the vintage-structure of capital. Shimer and Smith do not discuss multiplicity of equilibria in their setting.

Krusell, Ohanian, Rios-Rull and Violante (2000) explain the increase in the skill-premium over the seventies and eighties in a framework where skilled labor and capital are complements, and substitute for unskilled labor. Investment-specific technical change, in their setting decreases the relative price of productive capital - and thus increases the rate at which the capital-stock grows.

Our mechanism is different. Skilled workers are not more or less complementary with capital than unskilled workers. Both complement capital perfectly in a production-unit. An increase in the skill-premium is, thus, not driven by the increase in the stock of efficient capital. An increase in the rate of investment-specific technical change increases the difference between the new machine and the least productive machine in equilibrium. Since unskilled workers operate older machines than skilled workers - this implies that the skill-premium increases in equilibrium.

2.6 Conclusion

We have studied a model with frictional labor markets, where workers have different levels of skill and capital is subject to ageing. We extend the framework presented by Hornstein, Krusell and Violante (2007) to include differences in productivity between groups of workers.

In the calibrated version of our model, skilled and unskilled workers both accept a wide range of firms. Inequality among the skilled is larger than that among the unskilled. This is consistent with empirical findings comparing residual inequality across different educational groups. Unskilled workers operate older machines, for whom production - at a given time-span - is less disperse. For young machines firms have the opportunity to match with skilled workers. This puts a negative pressure on the wage of an unskilled worker operating a young machine.

Contrary to other versions of the Mortensen-Pissarides framework, our model predicts both increased within-group inequality and an increased skill-premium as the result of skill-biased technical change. In our framework firms are heterogeneous by age and not by choice. This is a general difference between our framework and other.

Faster investment-specific technical change implies a larger skill-premium in our framework. Previous studies have studied complementarity between college-education and the capital-stock. The mechanism in our study is different from these frame-works. Skilled and unskilled workers are equally complementary to capital - a production unit is merely a machine and a worker. The skill-premium arises from raw differences in productivity of skilled and unskilled workers - and from assortative matching. Skilled workers on average operate younger machines than unskilled workers do.

In our quantitative exercises, we account for an increase in the supply of skilled labor and an increase in the rate of investment-specific technical change, while studying a ten per cent increase in the skill-premium. A six per cent increase in the productivity of the skilled is sufficient to explain the widening of the wage-gap between skilled and unskilled workers.

The levels of within-group inequality that our model predicts are much lower than their empirical counterparts. Evidently, other sources of heterogeneity are important for a full understanding of the wage-structure. The addition of match-specific heterogeneity to our framework could further increase the level of inequality among observationally identical workers. Another possible source of within-group heterogeneity is stochastic accumulation of experience. We leave these extensions for future research.

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2.7 Appendices

2.7.1 Appendix 1 - Solution to the system of differential equations

We rewrite the system of differential-equations (3.31)

$$\mathbf{S}_a = \mathbf{AS} - \left(\chi e^{-\partial a} - \mathbf{U}\right) \tag{2.31}$$

where

$$\mathbf{S}=\left[\begin{array}{c} S_S(a) \\ S_U(a) \end{array}\right];\mathbf{A}=\left[\begin{array}{cc} \varrho+\varphi_S & \varphi_U \\ \varphi_S & \varrho+\varphi_U \end{array}\right];\boldsymbol{\chi}=\left[\begin{array}{c} \chi_S \\ \chi_U \end{array}\right];\mathbf{U}=\left[\begin{array}{c} (\rho-g)\,U_S \\ (\rho-g)\,U_U \end{array}\right]$$

It follows that the solution to the homogenous system $\mathbf{S}_a = \mathbf{AS}$ is $\mathbf{S} = \mathbf{\Phi}(a)\,\mathbf{C}$ where

$$\mathbf{\Phi}(a)=\left[\begin{array}{cc} 1 & -\varphi_U \\ 1 & \varphi_S \end{array}\right]\left[\begin{array}{cc} e^{(\varrho+\varphi_S+\varphi_U)a} & 0 \\ 0 & e^{\varrho a} \end{array}\right]$$

and $\mathbf{C}$ is a column of arbitrary constants. The particular solution to our system is, thus:

$$\mathbf{S}_p=-\mathbf{\Phi}(a)\int \mathbf{\Phi}^{-1}(\widetilde{a})\left(\boldsymbol{\chi}e^{-\delta\widetilde{a}}-\mathbf{U}\right)d\widetilde{a}$$

where;

$$\mathbf{\Phi}^{-1}(a)=\frac{1}{\varphi_S+\varphi_U}\left[\begin{array}{cc} e^{-(\varrho+\varphi_S+\varphi_U)a} & 0 \\ 0 & e^{-\varrho a} \end{array}\right]\left[\begin{array}{cc} \varphi_S & \varphi_U \\ -1 & 1 \end{array}\right]$$

At age $\overline{a}_h$ the surplus for workers type h is 0, since they reject older matches. The value for workers not type h is $S_j(\overline{a}_h)$, given by (2.20). The final-value condition is, thus $\mathbf{F}=\left\{\begin{array}{c} S_S(\overline{a}_U) \\ 0 \end{array} \text{ or } \begin{array}{c} 0 \\ S_U(\overline{a}_S) \end{array}\right\}$ depending on what type of workers operate the oldest machine in the economy. The solution to our problem is, thus:

$$\mathbf{S}=\mathbf{\Phi}(a)\,\mathbf{\Phi}^{-1}(\overline{a}_h)\,\mathbf{F}+\mathbf{\Phi}(a)\int_a^{\overline{a}_h}\mathbf{\Phi}^{-1}(\widetilde{a})\left(\boldsymbol{\chi}e^{-\delta\widetilde{a}}-\mathbf{U}\right)d\widetilde{a} \tag{2.32}$$

It will be convenient to re-write this expression. We define χ_A as the average skill among the unemployed workers and χ_D as the difference in productivity between the skilled and the unskilled workers. Equivalently, we define $(\rho-g)\,U_A$ and $(\rho-g)\,U_D$ as the average of and difference in flow values of unemployment. Then,

$$\mathbf{S}=\left[\begin{array}{cc} 1 & \frac{u_U}{u} \\ 1 & -\frac{u_S}{u} \end{array}\right]*\left(\left[\begin{array}{c} e^{-(\varrho+\varphi_S+\varphi_U)(\overline{a}_h-a)}S_A(\overline{a}_h) \\ e^{-\varrho(\overline{a}_h-a)}S_D(\overline{a}_h) \end{array}\right]+\int_a^{\overline{a}_h}\left[\begin{array}{c} e^{-(\varrho+\varphi_S+\varphi_U)(\widetilde{a}-a)}\left(\chi_A e^{-\delta\widetilde{a}}-(\rho-g)\,U_A\right) \\ e^{-\varrho(\widetilde{a}-a)}\left(\chi_D e^{-\delta\widetilde{a}}-(\rho-g)\,U_D\right) \end{array}\right]d\widetilde{a}\right)$$

where, if the oldest machine is operated by a skilled worker $S_A(\overline{a}_h) = (1-p)\,S_S(\overline{a}_U)$ and $S_D(\overline{a}_h) = S_S(\overline{a}_U)$, and $S_A(\overline{a}_h) = pS_U(\overline{a}_S)$ and $S_D(\overline{a}_h) = -S_U(\overline{a}_S)$ if the oldest machine is operated by an unskilled worker.

As in the case when only one type of workers are employed, the parties choose $\overline{a}_h$ optimally, so that $\frac{d}{d\overline{a}_h}\mathbf{S}(\overline{a}_h) = \mathbf{0}$. We, thus, have the condition:

$$\begin{bmatrix} -(\varphi_S+\varrho) & -\varphi_U \\ -\varphi_S & -(\varphi_U+\varrho) \end{bmatrix}\mathbf{F} + \frac{d}{d\overline{a}}\mathbf{F} + \chi e^{-\partial\overline{a}} - \mathbf{U} = \mathbf{0}$$

or

$$\begin{bmatrix} \chi_S e^{-\partial\overline{a}_S} - (\rho-g)\,U_S - \varphi_U S_U(\overline{a}_S) \\ S_{Ua}(\overline{a}_S) + \left(\chi_U e^{-\partial\overline{a}_S} - (\rho-g)\,U_U\right) - (\varrho+\varphi_U)\,S_U(\overline{a}_S) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ or}$$
$$\begin{bmatrix} S_{Sa}(\overline{a}_U) + \left(\chi_S e^{-\partial\overline{a}_U} - (\rho-g)\,U_S\right) - (\varrho+\varphi_S)\,S_S(\overline{a}_U) \\ \chi_U e^{-\partial\overline{a}_U} - (\rho-g)\,U_U - \varphi_S S_S(\overline{a}_U) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

We see that at $\overline{a}_h$ the production of matches that are dissolved are shared between workers and firms so that workers are compensated exactly for not being unemployed. Firms are indifferent between producing and holding a vacancy. The term $\varphi_j S_j(\overline{a}_h) = q(\theta)(1-\phi)\frac{u_j}{u}S_j(\overline{a}_h)$ is equal to the expected gain from search to the firm at $\overline{a}_h$.As in the economy without frictions, the workers that do not operate the oldest machine are payed less than their productivity on the margin. Firms are compensated for not searching for workers of a different type.

2.7.2 Appendix 2 - The Job-Creation condition

The value of a vacancy for machines that could produce is given by:

$$(\rho-g)\,V(a) = q(\theta)\left[E_h J^h(a) - V(a)\right] + V_a(a) = q(\theta)(1-\phi)\,S_A(a) + V_a(a)$$

where $S_A(a)$ is the unemployment-weighted average surplus. The value of a vacancy age a is thus:

$$V(a) = e^{(\rho-g)a}\int_a^{\overline{a}} e^{-(\rho-g)\tilde{a}}q(\theta)(1-\phi)\,S_A(\tilde{a})\,d\tilde{a} \qquad (2.33)$$

Evaluating the above expression at age 0 and setting it equal to the investment cost yields the Job-Creation condition.

2.7.3 Appendix 3 - The stationary distributions if $\underline{a}_U > 0$

We begin by defining by $p \equiv \frac{u_S}{u}$, the share of the unemployed that are skilled. Our system is, then:

$$\mathbf{m}_a = \mathbf{Am}+q\eta\mathbf{p} \tag{2.34}$$

where

$$\mathbf{m} = \begin{bmatrix} m_S(a) \\ m_U(a) \end{bmatrix}; \mathbf{A} = \begin{bmatrix} -(qp+\sigma) & -qp \\ -q(1-p) & -(q(1-p)+\sigma) \end{bmatrix}; \mathbf{p} = \begin{bmatrix} p \\ (1-p) \end{bmatrix}$$

It follows that the solution to the homogenous system $\mathbf{m}_a = \mathbf{Am}$ is $\mathbf{m} = \mathbf{\Phi}(a)\,\mathbf{C}$ where

$$\mathbf{\Phi}(a) = \begin{bmatrix} p & -1 \\ (1-p) & 1 \end{bmatrix} \begin{bmatrix} e^{-(\sigma+q)a} & 0 \\ 0 & e^{-\sigma a} \end{bmatrix}$$

and $\mathbf{C}$ is a column of arbitrary constants. The particular solution to our system is, thus:

$$\mathbf{m}_p = \mathbf{\Phi}(a)\, q\eta \int \mathbf{\Phi}^{-1}(\widetilde{a})\, \mathbf{p} d\widetilde{a}$$

where;

$$\mathbf{\Phi}^{-1}(a) = \begin{bmatrix} e^{(\sigma+q)a} & 0 \\ 0 & e^{\sigma a} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -(1-p) & p \end{bmatrix}$$

At age $\underline{a}_U$ the matches involving skilled workers is $m_S(\underline{a}_U)$. The initial-value condition is, thus $\mathbf{F} = \begin{bmatrix} m_S(\underline{a}_U) \\ 0 \end{bmatrix}$. The solution to our problem is, thus:

$$\mathbf{m} = \mathbf{\Phi}(a)\, \mathbf{\Phi}^{-1}(\underline{a}_U)\, \mathbf{F} + q\eta \mathbf{\Phi}(a) \int_{\underline{a}_U}^{a} \mathbf{\Phi}^{-1}(\widetilde{a})\, \mathbf{p} d\widetilde{a}$$

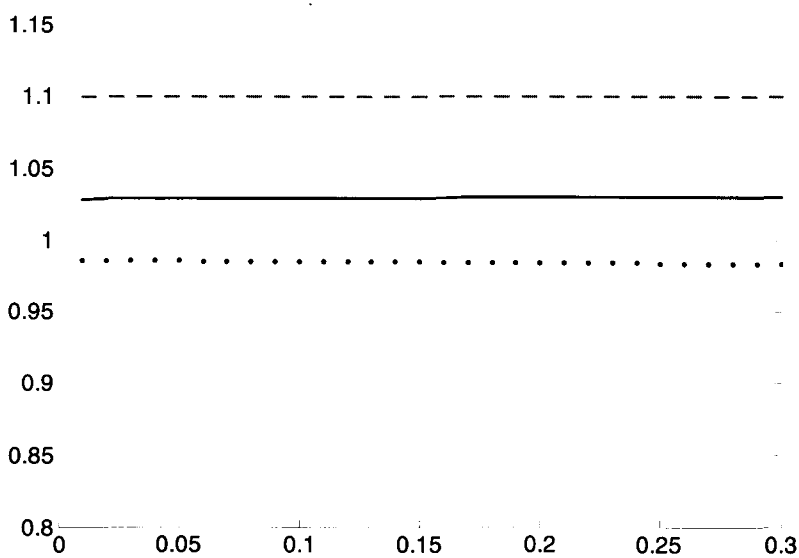
Which simplifies to;

$$\begin{aligned} \mathbf{m} &= m_S(\underline{a}_U) \begin{bmatrix} pe^{-(q+\sigma)(a-\underline{a}_U)} + e^{-\sigma(a-\underline{a}_U)}(1-p) \\ (1-p)e^{-(q+\sigma)(a-\underline{a}_U)} - e^{-\sigma(a-\underline{a}_U)}(1-p) \end{bmatrix} + \\ & q\eta \int_{\underline{a}_U}^{a} \begin{bmatrix} pe^{(\widetilde{a}-a)(q+\sigma)} \\ (1-p)e^{(\widetilde{a}-a)(q+\sigma)} \end{bmatrix} d\widetilde{a} \end{aligned}$$

2.7.4 Appendix 4 - Sensitivity Analysis

In the base-line model, we assume that the unemployment benefit b is 0.05 - a value taken from Hornstein-Krusell and Violante. Here, we investigate whether or not the results from our comparative statics are sensitive to the value of b.

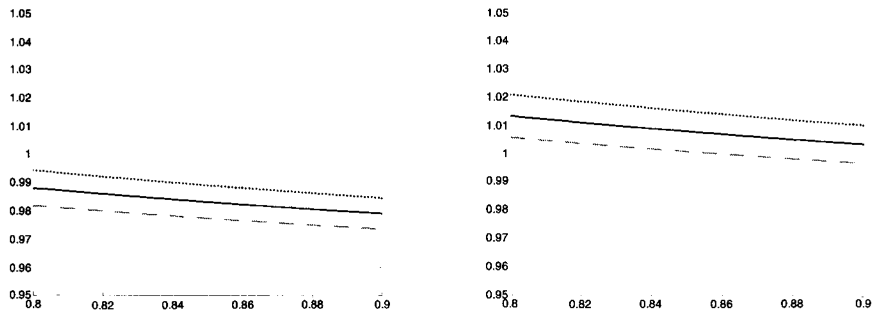
Appendix figure 1. Alternative values of b



Response of skill-premium for changes in χ_S (grey, top) γ (black, middle) and L_S (dotted, bottom) at levels of b (x-axis)

In our quantitative exercises above, we study responses for the wage-structure of changes in the productivity of the skilled (χ_S), the rate of investment-specific technical change (γ) and the supply of skilled workers (L_S). We now do the same exercises for values of b between 0.01 and 0.3. In Appendix figure 1 we see that the responses of the skill-premium to changes in our studied variables are very similar for the range of b under study. Note that we recalibrate the internal parameters $\{\delta, \sigma, A, \chi_S^{t<1975}, I, \phi\}$for every assumption on b. Thus, the level of b is of little importance for our predictions with regard to changes in the skill-premium when we hold the calibration-targets constant.

Appendix figure 2. Alternative values of L_U



Response of skill-premium for changes in L_U to $L_U^{t>1975} = 0.65$ (grey, bottom); $L_U^{t>1975} = 0.7$ (black, middle) and $L_U^{t>1975} = 0.75$ (dotted, top) at different levels of $L_U^{t<1975}$ (x-axis). In the left panel, all other parameters are unchanged. In the right panel the rate of investment-specific technical change is increased from 0.04 to 0.076 as in the experiments above. In the base-line parameterisation .$L_U^{t<1975}$=0.87 and $L_U^{t>1975}$=0.737.
In the base-line experiment changes to the skill-premia are 0.99 (for a supply shock only)and 1.01 for the combined shock

In our base-line calibration we choose to match our steady states to the relative supply of skilled labor in the years 1963 and 1987 respectively. Since the relative supply of skilled labor was increasing steadily during the post-war period - the steady-state approximation is crude. Here we investigate the quantitative responses to the skill-premium of a range of alternative assumptions on the supply of skilled labor. As expected, assuming a smaller initial skilled labor-force implies that the skill-premium decreases less in response to an increase in the stock of skilled labor. Over all - responses in the skill-premium to changes in the skilled labor-supply and the rate of investment-specific technical change are small.

Chapter 3

Inequality in Economies with Vintage Capital and Stochastic Skill Accumulation

Abstract

We study an economy with vintage capital and stochastic accumulation of skill - with and without frictions in the labor market. Faster embodied technical change increases wage-dispersion. Furthermore, an acceleration in the rate of embodied technical change increases the relative wage more for more productive workers. Thus, upper-tail wage-inequality grows more than wage-dispersion in the lower-tail. This is consistent with recent empirical findings.

In the economy with frictions, inequality is larger with stochastic skill accumulation, than in the same economy with ex ante worker heterogeneity.

In our calibrated models, we find that an acceleration in the rate of embodied technical change does not suffice to replicate the observed increase in inequality. The log-variance of wages increases by about one fifth of the observed increase.

3.1 Introduction

Wage dispersion in the United States increased during the last decades of the last century. Wage differentials between college-graduates and workers without a college-degree expanded. But inequality within groups of observationally identical individuals also increased. Juhn, Murphy and Pierce (1993) report that the majority of the increase in wage inequality is residual - meaning that it is explained by unobserved characteristics among workers within observationally identical groups. Furthermore, Gottschalk and Moffitt (1994) document that a sizable fraction of this increase in residual inequality is transitory - it reflects increased instability in the wage of single individuals. At the same time, the rate of capital-embodied technical change increased.

The first purpose of this study is to investigate the contribution of accelerating investment-specific technical change to an increase the transitory component of residual inequality. We assume that labor-productivity evolves stochastically. While working, lucky workers become more productive. Unlucky workers less productive. Workers are positively assortatively matched to a vintage-capital structure. More productive workers operate younger - and hence more productive - machines than less productive workers. Faster investment-specific technical change implies that the productivity-differential between machines of different ages grows. Larger wage-dispersion results.

We study the above mechanism in two settings: with and without frictions in the labor market.

In the economy without frictions in the labor-market, workers are perfectly positively assortatively matched to machines. The most productive worker operates the new machine. The least productive the oldest. When the proceeds to the firm is equal from employing a more or less productive worker, the firm switches workers to the less productive. The least productive worker earns the production of the oldest machine.

Labor-market frictions are introduced into this economy, in the Diamond-Mortensen-Pissarides fashion. Unemployed workers and vacant machines randomly search for a match. The flow of matches is a function of aggregate unemployment and the stock of vacancies. Matched parties Nash-bargain over match-surplus. We study an economy with two different levels of labor-productivity. As in the economy without frictions, workers may become more productive while at work. Workers, however, stand the risk of losing productivity if they are exogenously separated from a job.

Recently, Autor, Katz and Kearney (2007), have documented that residual inequality grew more in the upper tail of the wage-distribution, than in the lower tail, during the last decades of the twentieth century. The second purpose of this study is to investigate the contribution of an acceleration in the rate of investment-specific technical change to this observation. Intuitively: Under assortative matching, less productive workers operate older machines. As the rate of investment-specific technical change increases, the productivity of all machines fall, relative to that of a new machine. The older the machine, the greater the decline. It follows that the production of less productive workers declines more than that of more productive workers. Then, less productive workers experience larger relative wage losses than more productive workers.

Third, we compare the model with frictions to the model studied in Chapter 2. Stochastic skill accumulation alters the decisions of agents. Less productive workers accept older machines, since they by working gain the option of becoming more productive. More productive workers reject younger machines, since they face the risk of losing productivity if they are fired. Moreover, workers are compensated in accordance with these risks. Thus, wage-inequality is larger in the economy with stochastic skill than in the same economy with static labor productivity.

We calibrate the models with and without frictions to match some moments of the US economy around 1975. We target a measure of residual inequality. Our models predict a larger increase in wage-dispersion in the upper tail of the wage-distribution than in the lower tail. We, however, find that the observed acceleration in the rate of investment-specific technical change does not suffice to explain the increase in wage-inequality.

The predicted increase in the log-variance of residual wages is approximately one fifth of the corresponding increase in the data. In our calibrated model with frictions, the introduction of stochastic skill accumulation increases inequality moderately, as compared to the model with static labor productivity.

This article is organized as follows: Section II relates our study to previous studies on investment-specific technical change, assortative matching and residual inequality. Section III describes the economy without frictions, and section IV the economy with frictions. In section V we calibrate the models to match some features of the US economy prior to 1975. We then undertake quantitative experiments. Section VI concludes.

3.2 Overview of the literature

Residual inequality increased during the last decades of the last century, as documented in the work by Juhn, Murphy and Pierce (1993). Furthermore, Gottschalk and Moffitt (1994) document that a sizable fraction of this increase is transitory. More recently, Autor, Katz and Kearney (2007, AKK) have documented that this increase was larger in the upper tail of the residual wage distribution than in the lower tail. We focus on one explanation to these changes in the wage-structure.

Our focus is the interaction between assortative matching between labor and capital, vintage capital and an acceleration in the rate of investment-specific technical change. In this section, we discuss our model and how it relates to previous studies.

We first discuss our assumptions on worker-heterogeneity, then on the structure of capital in the economy. We discuss how workers and capital are matched in the economy, and relate this to the literature on assortative matching. Finally, we relate our work to other frameworks that have connected investment-specific technical change and the increase in wage-inequality.

The productivity of workers evolves stochastically. While at work, lucky workers become more productive. Unlucky workers become less productive. In the friction-less economy, there is no unemployment. Workers always face the hazard of becoming more ore less productive. In the economy with frictions in the labor market, we adopt the same mechanism as in Ljunqvist and Sargent (1998). They present a framework where individuals stochastically accumulate skills on the job. When becoming unemployed or while unemployed, their skills may depreciate. Thus, the productivity of individuals varies over time, due to their luck in accumulating skill on the job - or their misfortune of losing it while unemployed or when being fired.

We assume that there is investment-specific technical change. This implies that efficient capital-goods become progressively cheaper, relative to consumption goods. Gordon (1990) documents that the relative price of durable goods was falling from 1947 until 1983. Greenwood, Hercovitz and Krusell (1997) infer investment-specific technical change from Gordon's series of quality-adjusted prices. They document that the major part of US productivity growth in the post-war period stems from investment-specific technical change. More recently, Cummins and Violante (2002), using econometric techniques, update the series in Gordon (1990) to year 2000.

Central to our study is the finding in Cummins and Violante that the rate of investment-specific technical change is increasing over time. We investigate the consequences on residual inequality of this acceleration in our economy.

Capital is costly to install in our economy, so that - like in Solow (1960) - the capital stock in the economy has a vintage-structure. Last, we assume that production is decentralized into worker-machine pairs. This assumption is common in models of economies with labor-market-frictions. This implies that, for equally productive workers, a worker-machine pair with a younger machine produces more than a pair with an older machine.

Since production is decentralized into pairs, a production-unit consists of a machine of some age and a worker that has some productivity. We find that, in equilibrium, there is positive assortative matching. More productive workers operate younger machines than less productive workers.

Jovanovic (1998) studies an economy without frictions, where capital has a vintage-structure. In his framework capital ages and workers are heterogeneous. On the worker side, productivity is distributed according to a continuous function. In equilibrium there is sorting. The most productive (youngest) machine is operated by the most productive worker. Our friction-less economy resembles this framework. However, while worker heterogeneity in Jovanovic (1998) stems from variation in purposeful human capital investment, in our framework it is driven by luck only. Moreover, in Jovanovic (1998), the relative productivity of a worker does not change. There is, thus, no transitory inequality.

In our model with frictions in the labor market, we assume search frictions that give rise to quasi-rents within matched pairs and induce unemployment. The flow of new matches is determined by a matching-function, which takes aggregate unemployment and the stock of vacancies as inputs. Wages are the outcome of Nash-bargaining between the worker and the owner of the machine. There is free entry of firms. All these assumptions are standard in the study of frictional labor-markets and define a stylized Diamond-Mortensen-Pissarides (DMP) matching-model[1].

Hornstein, Krusell and Violante (2007, henceforth HKV) study the same economy, except that workers in their setting are ex post identical. HKV document how the introduction of vintage capital into the DMP-framework introduces variation in wages across workers. The mechanism is the following: Capital is costly to install, and there is free entry to investment. In equilibrium, therefore, the value of a new vacant machine is equal to the cost of investment. With age, the relative productivity of the machine and its remaining life-span falls. Lower production and shorter life-span implies that the surplus of a match with an older machine is lower than a match with a newer machine. Since wages are the outcome of Nash-bargaining over match-surplus, wages fall with the age of the machine.

Our model is an extension of the HKV frame-work. HKV find that inequality, which is residual and

[1] In our study, we use the term 'Mortensen-Pissarides' framework to describe a class of models created for the study of frictional labor markets. This class of models include a search friction that produces quasi-rents and induces unemployment, a matching function that determines the flow of new matches, a wage that is determined by bargaining between the matched parties and free entry of firms. Seminal contributions to this literature are Diamond (1982), Mortensen (1982) and Pissarides (1990).

transitory, in their model is very small. By introducing an additional source of heterogeneity - stochastic accumulation of skill - our model allows for a more reasonable level of inequality. The mechanism that generates wage-inequality in the HKV-framework also generates wage-inequality in our model with frictions. However, the bulk of the variation in wages stems from the difference in productivity between workers.

Without frictions positive assortative matching is perfect. More productive workers operate younger machines than less productive workers. At a critical age, when the proceeds to the firm is equal from workers of two different productivity levels, the firm switches workers to the less productive. The introduction of frictions alters this pattern. With frictions, for a range of intermediate ages, workers of both levels of productivity accept machines. Positive assortative matching, thus, is not perfect. This finding is similar to results reported in Shimer and Smith (2000). For a more thourough discussion on the literature on assortative matching, and how it relates to our general framework, we refer to Chapter 2 above.

Our study is not the first to investigate the nexus between the acceleration in investment-specific technical change and the increase in residual inequality. Violante (2002) investigates the same issue. In Violante (2002) workers accumulate skill stochastically, as in our framework. Furthermore, skills are vintage-specific - which introduces additional richness in the evolution of the productivity of an individual worker. Additionally, Violante (2002) allows for more than two levels of skill. However, for computational reasons, the standard assumption of Nash-bargaining between matched parties is dropped.

We choose a different route. Our framework allows for less heterogeneity across individuals, and the law of motion for individual productivity is simpler. But we retain the standard assumption of Nash-bargaining.

Violante finds that the bulk of the increase in inequality in response to an acceleration of investment-specific technical change, stems from resulting changes in the equilibrium distribution of labor productivity. In our framework this effect is not present. The increase in inequality that derives from larger productivity-dispersion across firms is, in Violante's framework, moderate. This source of variation is present in our framework. Like Violante, we do not find that the increased productivity-dispersion of firms yields a large increase in inequality. Therefore, with regard to the quantitative predictions of our model, we do not find that our choice of modelling is of any greater consequence, vis-a-vis the Violante framework.

AKK find that residual inequality increased more in the upper tail than in the lower tail of the wage-distribution. This is the prediction of our model without frictions and the calibrated model with frictions. Intuitively; assortative matching implies that less productive workers operate older machines. As the rate of investment-specific technical change increases, the productivity of all machines fall, relative to that of a new machine. The older the machine, the greater the decline in relative productivity. It follows that the relative wage of less productive workers declines more than that of more productive workers. The increase in upper-tail inequality relative to wage-dispersion in the lower tail results.

In Chapter 2 we studied an economy where agents are ex ante heterogeneous by productivity. Here, we introduce stochastic skill accumulation, in order to study transitory inequality. We find that the decisions of agents are altered, if skill is accumulated stochastically. Less productive workers accept older machines, since they by working gain the option of becoming more productive. More productive workers reject younger machines, since they face the risk of losing productivity if they are fired. Wages compensate workers for these risks. Therefore, wage inequality is larger in the economy with stochastic skill than in the same economy with static labor productivity.

3.3 The friction-less economy

Time, t, is continuous. We assume that there is a measure 1 workers in the economy, that supply labor inelastically. There is an infinite set of types of workers, denoted $h \in \{0, 1, ..., \infty\}$. While working, workers type h face hazards x and $\sigma\tau$ of becoming type $(h+1)$ and $(h-1)$ respectively. A worker type h has productivity χ_h, where $\chi_{h+1} > \chi_h$ for all h.

Machines age. They depreciate at rate δ. Furthermore, new machines become more productive at rate γ - the rate of embodied technical change. Capital costs $\iota(t)$ to install at time t. There is free entry to investment, and capital can be scrapped cost-lessly. Production is decentralized into worker-machine pairs. In addition to investment-specific technical change, the productivity of a worker-machine-pair grows at neutral rate ψ.

It follows that:

$$k(a,t) = k(0,0)e^{\gamma(t-a)-\delta a} \tag{3.1}$$

At time t an age a machine was installed at time $t-a$. At installation the state of embodied productivity was $e^{\gamma(t-a)}$. The machine was, by assumption, not improved upon, and with age it has depreciated.

A production pair consists of a worker type h and an age a machine. The output of one pair at time t is, thus:

$$y_h(a,t) = e^{\psi t}\chi_h k(a,t)^\alpha = e^{(\psi+\alpha\gamma)t}\chi_h e^{-(\gamma+\delta)\alpha a} \tag{3.2}$$

where we, without loss of generality, have normalized $k(0,0) = 1$.

3.3.1 Agents

Workers supply labor inelastically, and there are no frictions in the labor market. Firms maximize profits, by choosing if and what type of worker to hire. The flow profit, or rent, at time t of a machine aged a is:

$$\pi(a,t) = \arg\max_h \{y_h(a,t) - w_h(t)\} \tag{3.3}$$

The present value of a new machine is:

$$\Pi(t) = \sum_h \int_{\underline{a}_{ht}}^{\overline{a}_{ht}} e^{-\rho a} \arg\max_h \{y_h(a, t+a) - w_h(t+a)\} da \tag{3.4}$$

where $\underline{a}_{ht}$ denotes the age at which firms choose to start hiring workers type h, and $\overline{a}_{ht}$ is the age at which firms choose to hire some other type of workers. Naturally, if firms at $\overline{a}_{ht}$ start hiring workers of type j we have the condition: $\overline{a}_{ht} = \underline{a}_{jt}$. For the workers that operate the youngest machine workers $\underline{a}_{ht} = 0$. Profit maximization implies:

$$y_h(\overline{a}_{ht}, t) - w_h(t) = y_j(\underline{a}_{jt}, t) - w_j(t) \tag{3.5}$$

At the critical ages the firm is indifferent between hiring workers of different two types. Since capital can be scrapped at no cost, for the type of workers operating the oldest machine, we have:

$$y_h(\overline{a}_{ht}, t) = w_h(t) \tag{3.6}$$

Some type of workers operates the new machine. For this type we have $\underline{a}_{ht} = 0$. Free entry implies that the present value of a new machine equals the cost of investment. We, thus, have the free entry condition:

$$\iota(t) = \Pi(t) = \sum_h \int_{\underline{a}_{ht}}^{\overline{a}_{ht}} e^{-\rho a} \arg\max_h \{y_h(a, t+a) - w_h(t+a)\} da \tag{3.7}$$

The cost of investment is equal to the discounted profits over the time of operation of the machine. At time t, $\eta(t)$ machines are installed.

We denote by $L_h(t)$ the measure of type h workers at t. $L_h(t)$ evolves according to

$$L_{ht}(t) = (\sigma\tau)(L_{h+1}(t) - L_h(t)) - x(L_h(t) - L_{h-1}(t)) \tag{3.8}$$

3.3.2 A balanced growth path equilibrium

We study a balanced growth path equilibrium. Trivially, along a balanced growth path, $\eta(t)$ is constant η, and decisions $\overline{a}_{ht}$ are independent of t. The labor market clears if $\eta \int_{\underline{a}_{ht}}^{\overline{a}_{ht}} da = L_h$ where L_h is given by $(\sigma\tau) L_{h+1} = xL_h$ for all h, and the sum L_h over h is 1. A balanced growth path equilibrium is a set of wages $w_h(t)$, allocations $\overline{a}_h$ and η, such that, at all t, the free entry condition (3.7) holds, and the labor market clears.

Discussion

The balanced growth path equilibrium exhibits positive assortative matching. The profit maximization condition (3.5) characterizes the ages at which firms choose to change the type of worker it hires. Firms that have machines just younger than $\overline{a}_{ht}$ choose to hire workers of type h. This implies that $y(a,h,t)-y(a,j,t) > w(h,t)-w(j,t)$ for every $a < \overline{a}_{ht}$. We have $y(a,h,t)-y(a,j,t) = (\chi(h)-\chi(j))\, e^{(\psi+\alpha\gamma)t}e^{-(\gamma+\delta)\alpha a}$, which is decreasing in a if and only if $(\chi(h) > \chi(j))$. Thus, we have positive sorting.

The most productive worker operates the newest machine, and that the oldest machine is operated by the least productive worker. Moreover, we find that the least productive worker is payed his or her marginal productivity. Better workers are payed the marginal productivity of the oldest machine their type operates, less the flow-profit firms would have received from hiring a less productive worker with that machine.

In a stationary equilibrium, the ratio between workers of two consecutive types is $L_{h-1} = \frac{(\sigma\tau)}{x}L_h \equiv \frac{1}{\varsigma}L_h$. We denote the type of workers that have the lowest productivity L_0. Since there are measure 1 workers, it follows that $L_0 = (1-\varsigma), L_1 = \varsigma(1-\varsigma), L_2 = \varsigma^2(1-\varsigma)$ etc.

Evidently, a stationary equilibrium only exists if $1 > \varsigma$, or $\sigma\tau > x$. Then, workers are geometrically distributed over types h. Otherwise, the productivity of all workers converges to infinity - and there is not stationary distribution of workers over productivities.

Assuming that workers with the lowest skill reject machines aged $\overline{a}_0$, then rejection-ages are also geometrically distributed, so that

$$\overline{a}_h = \overline{a}_0\varsigma^h \quad ; h \geqq 0 \tag{3.9}$$

In a stationary equilibrium, investment is constant. The labor market clears if;

$$1 = \eta\overline{a}_0 \tag{3.10}$$

From the profit maximization condition (3.5) we have that $w(h,t) = y(\overline{a}_{ht},h,t) - y(\overline{a}_{ht},h-1,t) + w(h-1,t) = e^{(\psi+\alpha\gamma)t}\left(\chi_h - \chi_j\right)e^{-(\gamma+\delta)\alpha\overline{a}_h} + w(j,t)$. Starting with the profit maximizing-condition $w(0,t) = e^{(\psi+\alpha\gamma)t}\chi_0 e^{-(\gamma+\delta)\alpha\overline{a}_0}$, and substituting from (3.9) it follows that

$$w_h(t) = e^{(\psi+\alpha\gamma)t}\sum_{\widetilde{h}=0}^{h}\left(\chi_{\widetilde{h}} - \chi_{\widetilde{h}-1}\right)e^{-(\gamma+\delta)\alpha\varsigma^{\widetilde{h}}\overline{a}_0} \quad where \;\; \chi_{-1} = 0 \tag{3.11}$$

The free entry condition is, thus:

$$\iota(t) = \Pi(t) = e^{(\psi+\alpha\gamma)t}\sum_{h=0}^{\infty}\int_{\varsigma^{h+1}\overline{a}_0}^{\varsigma^h\overline{a}_0} e^{-(\rho-(\psi+\alpha\gamma))a}\left\{\chi_h e^{-(\gamma+\delta)\alpha a} - \sum_{\tilde{h}=0}^{h}\left(\chi_{\tilde{h}} - \chi_{\tilde{h}-1}\right)e^{-(\gamma+\delta)\alpha\varsigma^{\tilde{h}}\overline{a}_0}\right\}da \quad (3.12)$$

The term in large brackets is always positive. The present value of a new machine is finite[2] for a large class of functions χ_h of h.

Evidently, along a balanced growth path, the cost of investment grows at rate $(\psi + \alpha\gamma)$ as do wages. We define the growth rate of the economy $g \equiv (\psi + \alpha\gamma)$ and render the model stationary by dividing wages and the investment cost by e^{gt}. Note that this implies that we have normalized the productivity of a new machine (of age 0) to 1. We - for notational convenience - define $\partial \equiv (\gamma + \delta)\alpha$. The stationary version of the balanced growth path equilibrium is, thus, characterized by:

$$\begin{aligned} \iota &= \sum_{h=0}^{\infty}\int_{\varsigma^{h+1}\overline{a}_0}^{\varsigma^h\overline{a}_0} e^{-(\rho-g)a}\left\{\chi_h e^{-\partial a} - \sum_{\tilde{h}=0}^{h}\left(\chi_{\tilde{h}} - \chi_{\tilde{h}-1}\right)e^{-\partial\varsigma^{\tilde{h}}\overline{a}_0}\right\}da \\ 1 &= \eta\overline{a}_0 \end{aligned} \quad (3.13)$$

We see that a balanced growth path equilibrium can be reduced to a decision $(\overline{a}_0)$ and an investment level (v), such that the the free-entry condition (3.13) holds and factor markets (3.10) clear.

3.3.3 Comparative statics in the economy without frictions

In our study of the economy with frictions we focus on the response of inequality to faster embodied technical change (an increase in γ). In the friction-less economy, the ratio of the wage of an agent type h to an agent of type $h+1$ is:

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[2]The value of the integral is positive for any h. The first (and positive) term is $\chi_h \int_{\varsigma^{h+1}\overline{a}_0}^{\varsigma^h\overline{a}_0} e^{-(\rho-(\psi+\alpha\gamma)+(\gamma+\delta)\alpha)a}da \equiv$ $\chi_h \int_{\varsigma^{h+1}\overline{a}_0}^{\varsigma^h\overline{a}_0} e^{-\xi a}da = \chi_h\frac{1}{\xi}e^{-\xi\varsigma^h\overline{a}_0}\left(e^{\xi\varsigma^h\overline{a}_0(1-\varsigma)} - 1\right)$
The ratio between two consecutive h of this term is:
$\frac{\chi_{h+1}}{\chi_h}e^{\xi\varsigma^h\overline{a}_0(1-\varsigma)}\frac{\left(e^{\xi\varsigma^{h+1}\overline{a}_0(1-\varsigma)}-1\right)}{\left(e^{\xi\varsigma^h\overline{a}_0(1-\varsigma)}-1\right)}$
$e^{\xi\varsigma^h\overline{a}_0(1-\varsigma)}$converges to 1 as h goes to infinity. By l'Hôpital's rule we see that $\frac{\left(e^{\xi\varsigma^{h+1}\overline{a}_0(1-\varsigma)}-1\right)}{\left(e^{\xi\varsigma^h\overline{a}_0(1-\varsigma)}-1\right)}$ converges to ς as h approaches infinity. Thus, the ratio between two consecutive terms approaches a constant iff $\frac{\chi_{h+1}}{\chi_h}$ converges to a constant that is less than $\frac{1}{\varsigma}$. This is true for any polynomial in h and for any function $\chi_h = B^h$ where $B < \frac{1}{\varsigma}$.

$$\frac{w_{h+1}}{w_h} = \frac{\sum_{\tilde{h}=0}^{h+1}\left(\chi_{\tilde{h}} - \chi_{\tilde{h}-1}\right) e^{-\partial\varsigma^{\tilde{h}}\bar{a}_0}}{\sum_{\tilde{h}=0}^{h}\left(\chi_{\tilde{h}} - \chi_{\tilde{h}-1}\right) e^{-\partial\varsigma^{\tilde{h}}\bar{a}_0}} = \frac{\left(\chi_{h+1} - \chi_h\right) e^{-\partial\varsigma^{h+1}\bar{a}_0}}{\sum_{\tilde{h}=0}^{h}\left(\chi_{\tilde{h}} - \chi_{\tilde{h}-1}\right) e^{-\partial\varsigma^{\tilde{h}}\bar{a}_0}} + 1 \tag{3.14}$$

We note that the wage-premium is larger than the raw productivity differential $\left(\frac{\chi_{h+1}}{\chi_h}\right)$ since workers are positively assortatively matched to capital goods[3]. We also note see the wage-premium converges to the raw productivity differential, as h approaches infinity. Workers with high productivity operate young machines. As the productivity of workers becomes higher, the age of the machine converges to zero. This implies that the difference in production between workers that operate different machines converges to the raw productivity difference of workers.

For every h the probability of observing a worker of any higher type is constant. Therefore, in the upper tail where age-differences between machines are negligable, the probability of observing a wage that is $\left(\frac{\chi_{h+1}}{\chi_h}\right), \left(\frac{\chi_{h+2}}{\chi_h}\right), \ldots$ times larger than w_h is constant for any h. The wage-distribution in the upper tail, thus, is the discrete equivalent of the Pareto distribution if $\chi_h = \chi^h$. Therefore, we make this assumption[4]. Under this assumption, a stationary equilibrium only exists if $\varsigma\chi < 1$. Otherwise, the present value of production does not go to 0 as h approaches infinity. We assume that this holds.

Proposition 1. The wage-premium is increasing in the rate of embodied technical change (γ)

Proof: The free entry-condition states that the present value of all future profits equals the cost of investment. An increase in γ does not affect the future production of a machine that is installed - since it is not subject to investment-specific technical change. Wages, however, grow faster than previously. This implies that wages in terms of the production of a new machine - have to decrease in order for the free-entry condition to hold. Thus, in equilibrium, $\partial\bar{a}_0$ is increasing in γ.

The wage premium of two types h and $h+1$ can be re-written $\frac{w_{h+1}}{w_h} = \frac{(\chi_{h+1}-\chi_h)e^{\partial\bar{a}_0\left(1-\varsigma^{h+1}\right)}}{\sum_{\tilde{h}=0}^{h}(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})e^{\partial\bar{a}_0\left(1-\varsigma^{\tilde{h}}\right)}} + 1$, and since $\left(1-\varsigma^{h+1}\right) > \left(1-\varsigma^{\tilde{h}}\right)$ for any $\tilde{h} \leq h$ we see that the numerator increases faster than the denominator in response to increases in $\partial\bar{a}_0$. Thus, the wage-premium between two types h and $h+1$ is increasing in $\partial\bar{a}_0$. It follows that the wage-premium for any pair of types is increasing in γ.

[3] We see this by noting that $\frac{(\chi_{h+1}-\chi_h)e^{-\partial\varsigma^{h+1}\bar{a}_0}}{\sum_{\tilde{h}=0}^{h}(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})e^{-\partial\varsigma^{\tilde{h}}\bar{a}_0}} > \frac{(\chi_{h+1}-\chi_h)e^{-\partial\varsigma^{h+1}\bar{a}_0}}{e^{-\partial\varsigma^{h}\bar{a}_0}\sum_{\tilde{h}=0}^{h}(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})} = \frac{(\chi_{h+1}-\chi_h)}{\chi_h}e^{\partial\bar{a}_0\varsigma^h(1-\varsigma)} > \frac{(\chi_{h+1}-\chi_h)}{\chi_h}$.

[4] That the upper-tail of wage-distributions is well approximated by a Pareto-distribution is a well-established observation. For example, Lydall (1959), notes that "the Pareto hypothesis has been strikingly successful in fitting the 'high tail' of the distribution of incomes".

Proposition 2. For any pair of workers, h and $h+1$, *the wage-premium is increasing more in the rate of embodied technical change* (γ) *for larger* h.

Proof: In Appendix 4, below.

In proposition 1 and 2 we have proven that wage-inequality is increasing in the rate of embodied technical change, and that wage-inequality in the upper tail grows more than in the lower tail of the wage-distribution. We see that the stationary wages of all workers decrease in response to an increase in the rate of embodied technical change. The wage of a worker is the sum of the wage-rates of all less productive workers and a fraction of the production of a newer machine than any of the machines that less productive workers operate. Since the productivity of this machine decreases less than older machines, in response to faster embodied technical change, the wage decreases less than wages of less productive workers. This helps us to understand why wage inequality is increasing in the rate of embodied tecnical change.

In our quantitative experiments we investigate the response of wage-inequality to changes in the transition probabilities x and $\sigma\tau$. We, therefore, stud study the response of relative wages in the to such variation. First, we normalize the productivity of the least productive workers (type 0) to 1. The relative wage $\widetilde{w}_h$ of a worker, type h, to the least productive workers is:

$$\widetilde{w}_h = \frac{w_h}{w_0} = \sum_{\widetilde{h}=0}^{h} \left(\chi_{\widetilde{h}} - \chi_{\widetilde{h}-1}\right) e^{\partial \overline{a}_0 \left(1-\varsigma^{\widetilde{h}}\right)}$$

Proposition 3: The wage of a type $h > 0$ *worker relative to the least productive worker is decreasing in* ς.

Proof: An increase in ς (an increase in x or a decrease in $\sigma\tau$) implies that the workforce becomes more productive. For the job-creation-condition to hold, we must have that wages are higher. This implies that $\overline{a}_0$ is lower. Moreover, $\left(1-\varsigma^{\widetilde{h}}\right)$ is decreasing in ς for any $\widetilde{h}$. Thus, from our expression for $\widetilde{w}_h$ we see that a higher ς implies lower wages for all workers relative to the least productive workers.

In Proposition 3, we document that any parametric changes that re-allocate workers to more productive states compress wages. We refer to this as the 'price-effect' stemming from the re-allocation in the labor market. A more productive workforce implies that wages are compressed. Wage differentials come closer to the raw productivity differentials - and differences in age of the machines operated become smaller.

However, it does not follow from Proposition 3 that residual inequality decreases in ς. A larger ς implies that there are more workers with higher productivity in the economy. The distribution of workers over types becomes flatter, which - at constant wages - would imply larger wage-dispersion. We refer to this as a 'composition-effect'. In the economy with infinitely many levels of productivity on the worker side, this composition effect on wage-dispersion is always positive.

3.3.4 A balanced growth path equilibrium with two types of workers

When we study the economy with frictions in the labor-market, we will focus on the case when there are two types of workers - that either have high (H) or low (L) productivity. It is, therefore, instructive to study the friction-less economy with only two types of workers.

In a stationary equilibrium, with two levels of skill, the labor-market clears if $L_L = \frac{(\sigma\tau)}{x} L_H$. It follows that, in our economy with two types of skill, we have the following wages and allocations:

$$\begin{array}{ll} L_L = \frac{\sigma\tau}{x+\sigma\tau} & L_H = \frac{x}{x+\sigma\tau} \\ w_L = \chi_L e^{-\partial \bar{a}_L} & w_H = (\chi_H - \chi_L)\, e^{-\partial L_H \bar{a}_L} + \chi_L e^{-\partial \bar{a}_L} \end{array}$$

In our study of the economy with frictions, we focus on measures of wage-dispersion. The wage-premium in the economy with two skills is:

$$\frac{w_H}{w_L} = (\chi_H - \chi_L)\, e^{\partial L_L \bar{a}_L} + 1 \tag{3.15}$$

As in the economy with infinitely many productivity levels, an increase in $\sigma\tau$ or decrease in x increases the dispersion of wage-rates. Thus, as in the economy with infinitely many skills, the price-effect is always negative. More highly productive labor implies a more compressed distribution of wage-rates.

We now study the variance of relative wages. We normalize the wage of the less productive workers to 1. The variance of relative wages ($\widetilde{w}$) is:

$$V(\widetilde{w}) = L_L\,(1 - L_L)\left(e^{\partial L_L \bar{a}_L}\,(\chi_H - \chi_L)\right)^2 \tag{3.16}$$

An increase in the supply of L_L (a decrease in x or an increase in $\sigma\tau$) has two effects on the variance of wages. First, it alters the relative quantities of workers ($L_L\,(1 - L_L)$). In the economy with two levels of skill, this composition effect can be either positive or negative. If L_L is larger than $\left(\frac{1}{2}\right)$ this effect is negative, and positive otherwise.

Second, the relative wages of workers are altered. This price-effect is always positive. The variance of relative wages goes to 0 as L_L approaches 1 or 0. An increase in the supply of less productive workers, therefore, could have a positive or a negative effect on the variance of relative wages.

In our study of the economy with frictions, we will analyse the variance of log-wages. This is another measure of variaton in relative wages. However, the same effects - the composition-effect and the price-effect - operate in the same directions as above.

3.4 The Economy with frictions

We introduce search frictions in the labor market into the economy with two levels of skill, discussed above. Firms and workers conduct random search. The flow of matches is governed by an aggregate matching function in unemployment and the aggregate measure of vacancies. Search-frictions give rise to quasi-rents, over which the matched parties Nash-bargain. Search is cost-less, but capital is costly to install. Risk-neutral workers supply labor inelastically and enjoy utility b when unemployed.

We assume that low-skilled workers stochastically accumulate skill, while working. Matches are exogenously separated at rate σ. On separation from a job, high-skilled workers lose productivity with probability τ. This is the turbulence in the economy.

3.4.1 The Environment

As in the economy without frictions, a production-unit is a pair of a machine and a worker. Production is given by (3.2). In the economy with frictions, workers and machines are matched according to the function;

$$M(u,v) = Au^{\omega}v^{1-\omega} \tag{3.17}$$

The measure of matches at every instant is increasing in the stock of unemployed workers u and the total amount of vacancies in the economy; v. Search is random and not directed toward any specific type of workers - or any specific age of machines.

Vacancies are vacant machines. They are heterogenous with regard to their age a. High-skilled workers operate machines until they reach the age $\overline{a}_H$, and low-skilled until the machine reaches the age $\overline{a}_L$. Then, machines older than $\overline{a} = \max\{\overline{a}_H, \overline{a}_L\}$ will not be operated. They are scrapped, costlessly, at age $\overline{a}$. The total amount of vacancies is thus:

$$v = \int_0^{\overline{a}} v(a)da$$

Unemployment $u = u_H + u_L$ is simply the sum of high-skilled and low-unskilled workers. The probability of a vacancy being matched with a worker is $\frac{M(u,v)}{v} = M(\frac{u}{v},1) \equiv M(\theta,1) \equiv q(\theta)$. The probability of a worker being matched with a machine is $\frac{M(u,v)}{v}\frac{v}{u} \equiv \theta q(\theta)$. Moreover, for the vacant firm the probability of being mathed with a worker of a specific type h is

$$p(h,\theta) = q(\theta)\frac{u_h}{\sum_h u_h} = q(\theta)\frac{u_h}{u} \tag{3.18}$$

Similarly, for the unemployed worker the probability of being matched with a machine that is of a specific age a is

$$p(a,\theta) = \theta q(\theta) \frac{v(a)}{\int_0^{\overline{a}} v(a)da} = \theta q(\theta) \frac{v(a)}{v} \tag{3.19}$$

where $v(a)$ denotes the measure of vacancies aged a.

The productivity of workers evolves over time. On the job, workers accumulate skill. With probability x workers with low productivity become workers with high productivity. Matches are dissolved exogenously with probability σ. Upon an exogenous separation, a fraction τ of the high-skilled workers loose productivity and become low-skilled. We refer to this destruction of skills as turbulence.

3.4.2 Agents

Firms cost-lessly post a vacancy if their machine is not matched to a worker. As in the economy without frictions, above some age, $\overline{a}$, no workers accept a match to the machine. At this age the machine is cost-lessly scrapped. The value of a vacancy depends on the age of the machine a and the probability of finding a worker, which in turn is a function of labor-market tightness, θ. Moreover, the value depends on the availability of high-productive and low-productive labor among the unemployed, u_H and u_L, respectively. Furthermore, the value of a vacancy is a function of the decisions to reject matches, the critical ages $\overline{a}_H$ and $\overline{a}_L$. We denote the value of a vacancy $V \equiv V(a, \theta, u_H, u_L, \overline{a}_H, \overline{a}_L)$. For ease of notation, we suppress all economy-wide arguments, so that $V(a, \theta, u_H, u_L, \overline{a}_H, \overline{a}_L) \equiv V(a)$. The same applies to the other value-functions, discussed below. Along a balanced growth path, the stationarized value $V(a)$ of a vacancy age a is given by:

$$(\rho - g)\, V(a) = \max \left\{ q(\theta) \left[\left(\frac{u_H}{u} J_H(a) + \frac{u_L}{u} J_L(a) \right) - V(a) \right] + V_a(a), 0 \right\} \tag{3.20}$$

The firm - that owns a machine - posts a vacancy at no cost. $J_h(a)$ is the value to the firm of being matched with a worker type h. The value of a vacancy is non-negative, since the firm can always choose to scrap the machine at no cost. Above $\overline{a}$, the machine is not productive enough to compensate any workers for not being unemployed. We assume that firms scrap machines at age $\overline{a}$. $V_a(a)$ is the change in the value of holding a vacancy due to aging. This derivative is negative, since an older machine has a shorter remaining productive life-span and is less productive than a younger machine.

The value to the firm of employing a worker type h, $(J_h(a))$ is given by

$$\begin{aligned} (\rho - g)\, J_H(a) &= \max \{ y_H(a) - w_H(a) - \sigma\,(J_H(a) - V(a)) + J_H(a), (\rho - g)\, V(a) \} \qquad (3.21) \\ (\rho - g)\, J_L(a) &= \max \{ y^L(a) - w_L(a) - \sigma\,(J_L(a) - V(a)) + x\,(J_H(a) - J_L(a)) + J_{La}(a), (\rho - g)\, V(a) \} \end{aligned}$$

where sub-scripts H and L denote high and low productivity, respectively. The firm receives the rent $y_h(a) - w_h(a)$, where $w_h(a)$ is the wage of a type h worker operating an age a machine. With exogenous probability σ the match is dissolved. The firm, then, gains the value of a vacancy and loses the value of being matched. With probability x the low-skilled worker becomes more productive. The firm then loses the current value, and gains the value of being matched with a more productive worker.

The firm choses whether or not to remain in the match. Therefore, the value of employing a worker with a machine age a is never smaller than the value of a vacancy age a . At age $\overline{a}_h$ the firm chooses to dissolve the match. At this age $J_h(a) = V(a)$.

The value of unemployment - U_h - is given by:

$$(\rho - g) U_H = b + \frac{\theta q(\theta)}{v} \int_0^\infty v(a) [W_H(a) - U_H] da \tag{3.22}$$

$$(\rho - g) U_L = b + \frac{\theta q(\theta)}{v} \int_0^\infty v(a) [W_L(a) - U_L] da \tag{3.23}$$

The worker receives instant utility b from being unemployed. With probability $\theta q(\theta)$ the firm meets a machine of some age. This yields the expected gain $\frac{1}{v} \int_0^\infty v(a) [W_h(a) - U_h] da$, where $v(a)$ is the measure of vacant machines age a. Note that, for ages above $\overline{a}$ we have $v(a) = 0$.Firms do not post vacancies for machines that will never be accepted by any workers.

The worker, type h, who is employed at a firm age a holds value $W_h(a)$, given by:

$$(\rho - g) W_H(a) = \max\{w_H(a) - \sigma (W_H(a) - U_H) - \sigma\tau (U_H - U_L) + W_{Ha}(a), (\rho - g) U_H\} \tag{3.24}$$

$$(\rho - g) W_L(a) = \max\{w_L(a) - \sigma (W_L(a) - U_L) + x (W_H(a) - W_L(a)) + W_{La}(a), (\rho - g) U_L\}$$

The worker receives wage $w_h(a)$. With probability σ the match is exogenously dissolved. A fraction τ of the skilled worker become less productive. They lose the value of being unemployed as high-productivity workers, and instead are unemployed with low productivity. With probability x the low-productivity workers become more productive. They gain the difference in value between being employed with high productivity and being employed with low productivity.

The worker only accepts matches of age $a \leq \overline{a}_h$. At $\overline{a}_h$ we have $W_h(\overline{a}_h) = U_h$. Workers are indifferent between being employed and unemployed.

3.4.3 Bargaining

We assume that the joint surplus from a match is allocated to the parties in Nash-bargaining. The surplus of a match is:

$$S_h(a) = (J_h(a) - V(a)) + (W_h(a) - U_h) \tag{3.25}$$

The bargaining weights are ϕ and $(1-\phi)$ of the worker and firm, respectively and we thus have that:

$$W_h(a) = U_h + \phi S_h(a) \tag{3.26}$$

$$J_h(a) = V(a) + (1-\phi)\, S_h(a)$$

We see that, since both parties maximize the joint surplus, they optimally choose to dissolve matches at the same age.

3.4.4 Surplus functions

The ages at which agents choose to dissolve matches - $\overline{a}_h$ - are equilibrium objects, and outcomes of joint surplus-maximization. In the next section, we proceed to finding decision-rules that determine these ages; the Job-destruction conditions. In this section, we derive explicit solutions to the surplus-functions described above.

First, it will be helpful to state the surplus-functions explicitly. We substitute for our expressions for the value of a vacancy (3.20), a producing machine (3.21), an unemployed worker (3.22) and an employed worker (3.24) into our definition of the surplus (3.25). This yields the following system of differential equations:

$$(\rho - g)\, S_H(a) = \max\left\{ \begin{pmatrix} y_H(a) - (\rho - g)\, U_H - \sigma\tau\,(U_H - U_L) \\ -\sigma S_H(a) - q(\theta)\,[E_h\{J_h(a)\} - V(a)] + S_{H_a}(a) \end{pmatrix}, 0 \right\} \tag{3.27}$$

$$(\rho - g)\, S^L(a) = \max\left\{ \begin{pmatrix} y_L(a) - (\rho - g)\, U_L + x\,(U_H - U_L) - \sigma S_L(a) + \\ x\,(S_H(a) - S_L(a)) + S_{L_a}(a) - q(\theta)\,[E_h\{J({}_h a)\} - V(a)] \end{pmatrix}, 0 \right\} \tag{3.28}$$

At ages $a < \overline{a}$ the machine may be accepted by both types of workers, or only by one type. We begin by solving the surplus functions for those ages when only one type of workers accept employment, and the other type of workers only accepts younger machines.

The oldest machine may be operated by a high productivity or a low productivity worker. We first discuss the case when the high productivity workers operate the oldest machine. Then, we proceed to the case when the oldes machine is operated by a worker with low productivity.

First, the oldest machine in the economy may be operated by a worker with high productivity. In this case, we have $\bar{a} = \bar{a}_H > \bar{a}_L$. Matches to low-productivity workers are rejected, and firms instead choose to hold their machines vacant, in search of a high-productivity worker. We, hence, have $J_L(a) = V(a)$ for ages $\bar{a}_L < a \leqq \bar{a}_H$. It follows that, for these ages, the surplus-function for type H workers is given by:

$$\left(\rho - g + \sigma + q(\theta)(1-\phi)\frac{u_H}{u}\right) S^H(a) = y_H(a) - (\rho - g) U^H - \sigma\tau\left(U^H - U^L\right) + S_a^H(a)$$

For notational ease, we define $\varrho \equiv (\rho - g + \sigma)$ and $\kappa \equiv q(\theta)(1-\phi)$. Also, we let $p \equiv \frac{u^H}{u}$. At $\bar{a}_H$ the surplus is 0. The solution to this first-order differential equation is, thus:

$$S^H(a) = e^{(\varrho+\kappa p)a} \int_a^{\bar{a}} e^{-(\varrho+\kappa p)\tilde{a}} \left(\chi^H e^{-\partial\tilde{a}} - (\rho - g) U^H - \sigma\tau\left(U^H - U^L\right)\right) d\tilde{a} \tag{3.29}$$

Otherwise, the oldest machine in the economy is operated by a worker with low productivity. We, then, have $\bar{a} = \bar{a}_L > \bar{a}_H$. Matches to high-productivity workers are rejected, since firms cannot compensate them for remaining out of unemployment. Firms instead choose to hold their machines vacant, in search of a low-productivity worker. We, hence, have $S^H(a) = 0$ and $J^H(a) = V(a)$ for ages $\bar{a}_H < a \leqq \bar{a}_L$. In this case, it follows that the surplus-function, for these ages, for type L workers is given by:

$$(\varrho + \kappa(1-p) + x) S^L(a) = y^L(a) - (\rho - g) U^L + x\left(U^H - U^L\right) + S_a^L(a)$$

Since the surplus is 0 for ages $\bar{a}_L$ and above, we have that $S^L(a)$ is given by:

$$S^L(a) = e^{(\varrho+\kappa(1-p)+x)a} \int_a^{\bar{a}} e^{-(\varrho+\kappa(1-p)+x)\tilde{a}} \left(\chi^L e^{-\partial\tilde{a}} - (\rho - g) U^L + x\left(U^H - U^L\right)\right) d\tilde{a} \tag{3.30}$$

Below, we will discuss the possible existence of an age $\underline{a}_L > 0$, before which no machines accept matches to the low productivity workers. Now, we discuss those ages a at which both types of workers accept matches, i.e. $a \in [\max\{\underline{a}_L, 0\}, \min\{\bar{a}_L, \bar{a}_H\}]$. For these ages , the surplus-functions (3.27) and (3.28) describe a system of differential equations. We re-write this system in matrix form:

$$\varrho \begin{bmatrix} S^H(a) \\ S^L(a) \end{bmatrix} = e^{-\partial a} \begin{bmatrix} \chi^H \\ \chi^L \end{bmatrix} - \begin{bmatrix} (\rho - g) + \sigma\tau & -\sigma\tau \\ -x & (\rho - g) + x \end{bmatrix} \begin{bmatrix} U^H \\ U^L \end{bmatrix} - \begin{bmatrix} \kappa p & \kappa(1-p) \\ \kappa p - x & \kappa(1-p) + x \end{bmatrix} \begin{bmatrix} S^H(a) \\ S^L(a) \end{bmatrix} + \begin{bmatrix} S^H_a(a) \\ S^L_a(a) \end{bmatrix} \tag{3.31}$$

We denote the earliest rejection age $\overline{a}_e = \min\{\overline{a}_L, \overline{a}_H\}$. For ages between $\underline{a}_L$ and $\overline{a}_e$ the surplus functions are given by (3.31). The solution, which we derive in Appendix 2, to this system of differential equations is:

$$\mathbf{S} = \frac{\kappa}{\kappa - x} \begin{bmatrix} 1 & (1-p) \\ 1 & -\left(p - \frac{x}{\kappa}\right) \end{bmatrix} * \\ * \left(\begin{bmatrix} e^{-(\kappa+\varrho)(\overline{a}_e - a)} S_1(\overline{a}_e) \\ e^{-(x+\varrho)(\overline{a}_e - a)} S_2(\overline{a}_e) \end{bmatrix} + \int_a^{\overline{a}_e} \begin{bmatrix} e^{-(\tilde{a}-a)(\kappa+\varrho)} \left(\chi_1 e^{-\partial\tilde{a}} - U_1\right) \\ e^{-(\tilde{a}-a)(x+\varrho)} \left(\chi_2 e^{-\partial\tilde{a}} - U_2\right) \end{bmatrix} d\tilde{a} \right) \tag{3.32}$$

where

$\chi_1 \equiv \left(\chi_H \left(p - \frac{x}{\kappa}\right) + \chi_L (1-p)\right)$ $\quad \chi_2 \equiv (\chi_H - \chi_L)$

$S_1(a) \equiv \left(\left(p - \frac{x}{\kappa}\right) S_H(a) + (1-p) S_L(a)\right)$ $\quad S_2(a) \equiv (S_H(a) - S_L(a))$

$U_1 \equiv (\rho - g)\left[(1-p) U_L + \left(p - \frac{x}{\kappa}\right) U_H\right]$ $\quad U_2 \equiv (\rho - g + x + \sigma\tau)(U_H - U_L)$

S_1 and S_2 follow first order linerar differential equations. The first term within brackets corresponds to the end-value condition. The surplus of matches with the type of workers that does operate older machines is positive. The oldest machine could, as discussed above, be operated by a type H or a type L worker.

If a low productive worker operates the oldest machine - the end-value conditions are $S_1(\overline{a}_H) = \frac{u_L}{u} S_L(\overline{a}_H)$ and $S_2(\overline{a}_L) = -S_L(\overline{a}_H)$. Since a low productive worker operates the oldest machine, $S_L(\overline{a}_H)$ is given by (3.30).If, on the other hand, a high productivity worker operates the oldest machine, the end-value conditions are: $S_1(\overline{a}_H) = \left(\frac{u_S}{u} - \frac{x}{\kappa}\right) S_H(\overline{a}_H)$ and $S_2(\overline{a}_H) = S_H(\overline{a}_H)$. In this case, $S_H(\overline{a}_H)$ is given by (3.29).

3.4.5 Joint optimization

We have assumed Nash-bargaining, and thus workers and firms jointly maximize the surplus in a match. This implies that they choose the separation age $\overline{a}^h$ optimally. We begin by studying the period when only one type of workers accept matches, and the surplus is given by (3.29) and (3.30), respectively.

If a worker with high productivity operates the oldest machine in the economy, the first order condition of the surplus-function (3.29) w.r.t $\overline{a}_H$ is:

$$(\rho - g) U^H = \chi^H e^{-\delta \overline{a}_H} - \sigma\tau \left(U^H - U^L\right) \quad or \tag{3.33}$$

The value of unemployment to the skilled worker is given by the production of the oldest machine, less the expected loss in value due to turbulence.

Alternatively, the oldest machine in the economy may be operated by un unskilled worker. Then, the first order condition of (3.30) w.r.t $\overline{a}_L$ is:

$$(\rho - g) U^L = \chi^L e^{-\delta \overline{a}_L} + x \left(U^H - U^L\right) \tag{3.34}$$

The value of unemployment to the unskilled worker is the sum of the production of the oldest machine and the expected gain in value from accumulating experience on the job.

In both cases, the firm is indifferent between producing and not producing, and receives no compensation from the match. All production is payed to the worker.

Note that the employed worker with low productivity (L) accepts matches that produce less than the flow-value of unemployment. They accept a wage lower than the flow-value of unemployment, since employment offers the option to become skilled. Similarly, in our setting, the worker with high productivity (H) rejects matches that produce more than the flow-value of unemployment. The employed worker stands the risk of being fired, and may experience a loss in productivity. As unemployed, the high-skilled worker - in our setting - does not risk losing productivity.

For ages when both types of agents accept employment, surplus-functions are given by the system (3.32). Joint optimization - $\frac{d}{d\overline{a}_e}\mathbf{S} = \mathbf{0}$ - implies that the optimal separation ages are given by:

$$\chi^H e^{-\delta \overline{a}_H} = \kappa (1-p) S^L (\overline{a}_H) + (\rho - g) U^H + \sigma\tau \left(U^H - U^L\right) \qquad if\ \overline{a}_L > \overline{a}_H \ \ or \tag{3.35}$$

$$\chi^L e^{-\delta \overline{a}_L} = p\kappa S^H (\overline{a}_L) + (\rho - g) U^L - x \left(U^H - U^L\right) - x S^H (\overline{a}_L) \qquad otherwise \tag{3.36}$$

Matches are being dissolved when production just suffices to compensate both parties for not leaving the match. This implies that the parties are compensated according to their options outside the match.

When type H workers are the first to reject machines, $\overline{a}_H$ is given by (3.35). Production compensates the firm for not vacating the machine in search of a less skilled worker - which corresponds to the term $(\kappa (1-p) S^L (\overline{a}_H))$. The worker is compensated for not being unemployed $((\rho - g) U^H)$, and for incurring the risk of losing productivity $(\sigma\tau \left(U^H - U^L\right))$

If the low-skilled workers choose to reject machines earlier than the high-skilled workers, $\overline{a}_e = \overline{a}_L$ is given by (3.36). Production is sufficient to compensate the firm for not vacating the machine in search of a high-skilled worker $(p\kappa S^H(\overline{a}_L))$, less the expected gain from accumulation of productivity $((1-\phi)\, x S^H(\overline{a}_L))$. The worker is compensated for not being unemployed $((\rho - g)\, U^L)$, less the expected gain from accumulation of productivity within the match $(\phi x S^H(\overline{a}_L))$ and the expected gain that only accrues to the worker from being unemployed as a more productive morker $x\left(U^H - U^L\right)$.

As mentioned above, there may exist an age $\underline{a}_L > 0$ before which firms reject matches with workers with low productivity. In this case, the loss to the firm with a sufficiently young machine - younger than $\underline{a}_L$ – of the opportunity to search for a highly productive worker is too large. We now define this age. After $\underline{a}_L$ both types of workers accept matches. The surplus-functions are given by the system (3.32). Then we have that $\underline{a}_L$ - if positive - solves the problem:

$$\begin{aligned}
\begin{matrix} S_H(\underline{a}_L) \\ 0 \end{matrix} &= \frac{\kappa}{\kappa - x}\begin{bmatrix} 1 & (1-p) \\ 1 & -\left(p - \frac{x}{\kappa}\right) \end{bmatrix} * \\
&\quad * \left(\begin{bmatrix} e^{-(\kappa+\varrho)(\overline{a}_h - a)} S_1(\overline{a}_h) \\ e^{-(x+\varrho)(\overline{a}_h - a)} S_2(\overline{a}_h) \end{bmatrix} + \int_{\underline{a}_L}^{\overline{a}_h} \begin{bmatrix} e^{-(\tilde{a}-a)(\kappa+\varrho)}\left(\chi_1 e^{-\partial\tilde{a}} - U_1\right) \\ e^{-(\tilde{a}-a)(x+\varrho)}\left(\chi_2 e^{-\partial\tilde{a}} - U_2\right) \end{bmatrix} d\tilde{a} \right) \\
\textit{where} \quad \underline{a}_L &> 0 \\
\textit{or} \quad \underline{a}_L &= 0
\end{aligned} \tag{3.37}$$

Maximization implies that, $S_L(a) = 0$ for any $a < \underline{a}_L$. Thus, for ages lower than $\underline{a}_L$ we have that $S_S(a)$ is given by:

$$S^H(a) = e^{(\varrho+\kappa p)(a-\underline{a}_U)} S_H(\underline{a}_L) + e^{(\varrho+\kappa p)a} \int_a^{\underline{a}_L} e^{-(\varrho+\kappa p)\tilde{a}} \left(\chi^H e^{-\partial\tilde{a}} - (\rho - g)\, U^H - \sigma\tau\left(U^H - U^L\right)\right) d\tilde{a} \tag{3.38}$$

We have described the surplus-functions that characterize our economy for all ages during which production is undertaken. We now turn to the decisions to create and destruct jobs.

3.4.6 Job Creation and Job-destruction

Firms create jobs by investing in new machines. There is free-entry to investment. Thus, the value of a vacancy age 0 in equilibrium equals the cost of investment ι. The Job-Creation condition is:

$$\iota = V(0) = q(\theta)(1-\phi)\int_0^{\overline{a}} e^{-(\rho-g)\tilde{a}}\left(pS_H(\tilde{a}) + (1-p)S_L(\tilde{a})\right)d\tilde{a} \tag{3.39}$$

The value of a new vacancy is simply the expected value of all discounted gains from search for workers, during the life-span of the machine.

Matches are dissolved at two ages - one at the oldest age at which machines are operated, and one at the intermediate age after which only one type of workers accept matches. In equilibrium, there are, thus, two Job-destruction conditions.

First, We may have an equilibrium in which the high-skilled workers operate the oldest machine. For this equilibrium, we combine the expressions (3.33) and (3.36) that determine the separation-ages with the expressions for the value of unemployment (3.22). This yields the following job-destruction conditions:

$$\chi_H e^{-\partial\overline{a}} - \sigma\tau(U_H - U_L) = b + \frac{\theta q(\theta)}{v}\phi\int_0^{\overline{a}} v(a)S_H(a)da \tag{3.40}$$

$$\chi_L e^{-\partial\overline{a}_L} - (p\kappa - x)S_H(\overline{a}_L) + x(U_H - U_L) = b + \frac{\theta q(\theta)}{v}\phi\int_0^{\overline{a}_L} v(a)S_L(a)da \tag{3.41}$$

Otherwise, the low-skilled operate the oldest machine. Then, we combine the separation-decisions (3.34) and (3.35) with the expressions for the value of unemployment (3.22). Our job-destruction conditions are:

$$\chi_H e^{-\partial\overline{a}_H} - \kappa(1-p)S_L(\overline{a}_H) - \sigma\tau(U_H - U_L) = b + \frac{\theta q(\theta)}{v}\phi\int_0^{\overline{a}_H} v(a)S_H(a)da \tag{3.42}$$

$$\chi_L e^{-\partial\overline{a}_L} + x(U_H - U_L) = b + \frac{\theta q(\theta)}{v}\phi\int_0^{\overline{a}} v(a)S_L(a)da \tag{3.43}$$

Jobs are being destructed when production is exactly sufficient to compensate the parties for their out-side options. The outside option of the skilled unemployed worker is affected by the hazard of becoming unskilled. The unskilled worker, on the other hand, loses the opportunity of becoming skilled if unemployed. Note that, from the definition of U^h (3.22) we can solve for both values of unemployment in terms of the surplus function and the rejection ages. Our job-destruction conditions, thus, are functions of market-tightness θ and the decision rules - $\overline{a}, \overline{a}_e$ and $\underline{a}_L$ if greater than 0.

3.4.7 The stationary distributions

In equilibrium, the distribution of vacancies (and matched machines) over ages is stationary. We, here, solve for these stationary distributions in terms of the endogenous variables. Thereafter, we will characterize the equilibrium. As in the economy without frictions, we denote by η the constant measure of machines that is installed at every instant in a stationary equilibrium. We denote by $m(a)$ the measure of machines age a that are matched to a worker. $v(a)$ is is the measure of vacant machines age a. We have $\eta = m(a) + v(a)$.

If a match between a new machine and a less skilled worker is rejected - that is if $(0 < \underline{a}_L)$ - the youngest matches are only skilled. The measure of matches evolves according to:

$$m_{Ha}(a) = -\sigma m_H(a) + qpv(a) = qp\eta - (\sigma + qp)\, m_H(a)$$

For notational ease, we suppress $q \equiv q(\theta)$. Workers only flow into employment from unemployment. No unskilled workers gain experience and become skilled in this age-segment - since there are no employed unskilled workers operating machines of these ages. The density of matches for any age $0 < a < \underline{a}_L$ is given by:

$$m_H(a) = qp\eta e^{-(\sigma+qp)a} \int_0^a e^{(\sigma+qp)\widetilde{a}} d\widetilde{a} \tag{3.44}$$

.

For ages $\underline{a}_L < a < \bar{a}_e$ both types of workers accept machines. Matches evolve according to the system of differential equations:

$$\begin{aligned} m_{Ha}(a) &= qp\eta - (\sigma + qp)\, m_H(a) - qpm_U(a) + xm_L(a) \\ m_{La}(a) &= q(1-p)\,\eta - q(1-p)\, m_H(a) - (\sigma + q(1-p))\, m_L(a) - xm_L(a) \end{aligned}$$

The initial condition is that $m_H(\underline{a}_L)$ is given by (3.44) and that there are no low-skilled matches at this age. The solution to our system is:

$$\begin{bmatrix} m_H(a) \\ m_L(a) \end{bmatrix} = m_H\left(\underline{a}_L\right) \begin{bmatrix} e^{-\left(a-\underline{a}_L\right)(x+\sigma)} \frac{q-qp}{q-x} + e^{-\left(a-\underline{a}_L\right)(q+\sigma)} \frac{(qp-x)}{q-x} \\ \frac{q-qp}{q-x}\left(e^{-\left(a-\underline{a}_L\right)(q+\sigma)} - e^{-\left(a-\underline{a}_L\right)(x+\sigma)}\right) \end{bmatrix}$$
$$+q\eta\frac{q}{q-x}\int_{\underline{a}_L}^{a} \begin{bmatrix} \left(p-\frac{1}{q}x\right)e^{-(a-\widetilde{a})(q+\sigma)} + (1-p)\frac{1}{q}xe^{-(a-\widetilde{a})(x+\sigma)} \\ (1-p)e^{-(a-\widetilde{a})(q+\sigma)} - (1-p)\frac{1}{q}xe^{-(a-\widetilde{a})(x+\sigma)} \end{bmatrix} d\widetilde{a}$$

After $\overline{a}_e$ either the skilled or the unskilled operate machines. The density of matches evolves according to

$$\begin{aligned} m_{Ha}(a) &= qp\eta - (\sigma+qp)\, m_H(a) & \quad if\ \overline{a}_H > \overline{a}_L\ \ or \\ m_{La}(a) &= q(1-p)\,\eta - (\sigma+x+q(1-p))\, m_L(a) & \quad otherwise \end{aligned} \tag{3.45}$$

with solutions

$$\begin{aligned} m_H(a) &= e^{-(\sigma+qp)(a-\overline{a}_L)} m_H(\overline{a}_L) + qp\eta e^{-(\sigma+qp)a}\int_{\overline{a}_L}^{a} e^{(\sigma+qp)\widetilde{a}} d\widetilde{a} \\ if\ \overline{a}_H &> \overline{a}_L\ \ or \\ m_L(a) &= e^{-(\sigma+x+q(1-p))(a-\overline{a}_H)} m_L(\overline{a}_H) + q(1-p)\,\eta e^{-(\sigma+x+q(1-p))a}\int_{\overline{a}_L}^{a} e^{(\sigma+x+q(1-p))\widetilde{a}} d\widetilde{a} \\ & \quad otherwise \end{aligned} \tag{3.46}$$

The vacancy distribution is merely $v(a) = \eta - m(a)$, where $m(a) = m_H(a) + m_L(a)$. Factor markets clear when

$$1-u = \int_0^{\overline{a}} m(a)da \quad (3.47)$$

$$\frac{\int_0^{\overline{a}_H} m_H(a)da}{\int_0^{\overline{a}_L} m_L(a)da} = \frac{x}{\sigma\tau} \quad (3.48)$$

$$\eta\overline{a} = v + \int_0^{\overline{a}} m(a)da \quad (3.49)$$

First, in (3.47) we merely have that the sum of all unemployed and employed workers equal the workforce 1. Second, workers flow between states of productivity. Workers only transit to other states if they are employed. Every instant $\sigma\tau\int_0^{\overline{a}_H} m_H(a)da$ workers with high productivity lose skill. This flow, in equilibrium, equals the flow of low skilled workers that become more productive, i.e. $x\int_0^{\overline{a}_L} m_L(a)da$. This is the condition (3.48). Then, in a stationary equilibrium where investment in capital goods is constant, the total measure of machines in the economy is $\eta\overline{a}$ which has to equal the total measure of vacancies v and all matches $\int_0^{\overline{a}} m(a)da$. Thus, the capital market clears when (3.49) holds.

The measure of matches at any age is increasing in the probability that a vacancy encounters a worker. Thus, the Right-hand side of (3.47) is increasing in aggregate unemployment. The left-hand side is, evidently, decreasing. Hence, the $\{\overline{a}, \overline{a}_e, \underline{a}_U, \theta\}$ uniquely determines aggregate unemployment (u).

From the definition of θ it follows that aggregate vacancies (v) are pinned down by the same quatruplet. Then, the investment-level η follows immediately from (3.49).

In a stationary equilibrium, the ratio between high-productivity matches and low-productivity matches is constant ($\frac{x}{\sigma\tau}$). Given $\{\overline{a}, \overline{a}_e, \underline{a}_U, \theta\}$, this implies that the unemployment-levels u_H and u_L are uniquely pinned down from the flow-conditions within each productivity state.

Hence, our equilibrium is characterized by the quatruplet $\{\overline{a}, \overline{a}_e, \underline{a}_U, \theta\}$.

3.4.8 Equilibrium

We can characterize the equilibrium with production in our economy as a measure of tightness $\{\theta\}$ and decisions $\{\overline{a}, \overline{a}_e, \underline{a}_U\}$ such that;

i) The Job-Destruction conditions, either (3.41) & (3.40) or (3.42) & (3.43), hold.

ii) The Job-Creation condition (3.39) holds.

iii) The condition (3.37) holds.

3.4.9 Wages

We have seen that explicit expressions for wages are not necessary to characterize an equilibrium in our model. Nevertheless, wage-distributions are the core of our interest in this investigation. Therefore, we here derive expressions for wages in our economy. We deduct the value of unemployment from the value function for the employed worker (3.24), and substitute for the Nash-bargaining-assumption, as expressed in (3.26) to get:

$$\begin{aligned} w_H(a) &= (\rho - g)\, U_H + \sigma\tau\,(U_H - U_L) + \phi\,(y_H(a) - ((\rho - g)\, U_H + \sigma\tau\,(U_H - U_L)) - q\,(\theta)\,[E_h\,\{J_h(a)\} - V(a)]) \\ w_L(a) &= (\rho - g)\, U_L - x\,(U_H - U_L) + \phi\,(y_L(a) - ((\rho - g)\, U_L - x\,(U_H - U_L)) - q\,(\theta)\,[E_h\,\{J_h(a)\} - V(a)]) \end{aligned} \tag{3.50}$$

Workers are compensated for not being unemployed. In addition, high productivity workers are compensated for incurring the risk of becoming less skilled. Low productivity workers, on the other hand, accept a lower wage than their flow-value of unemployment, since they - by remaining employed - could accumulate skill. This is the compensation workers receive to cover their options outside the match. Additionally, the quasi-rents from the match are split according to the bargaining weights of the parties.

3.4.10 A comparison to the model with ex ante heterogeneity

In Chapter 2, above, we investigate a Mortensen-Pissarides model with vintage capital and workers that are ex ante heterogeneous by productivity. The introduction of stochastic skill-accumulation affects the decisions to accept matches. With ex ante heterogeneity, unskilled workers cannot accumulate skill by working, and therefore reject younger matches. Skilled workers do not face the risk of losing productivity while working, and therefore accept older matches. Thus, the introduction of stochastic skill-accumulation increases the age-differential (and therefore productivity-differential) between machines operated by skilled and unskilled workers.

Moreover, as seen in (3.50) workers are compensated for the risks they incur while employed. More skilled workers are compensated for assuming the risk of productivity-loss. Less skilled workers accept lower wages if they have the option of becoming more productive. Hence, the introduction of stochastic skill-accumulation implies larger wage-inequality. Below, we investigate this quantitatively.

3.5 Quantitative Experiments

In this section, we undertake quantitative experiments with our models. First, we calibrate them to match some moments of the US economy prior to 1975. Then, we vary the rate of embodied technical change, γ, and the productivity of the more productive workers, χ_H, and evaluate the response of the wage-distribution.

3.5.1 Calibration of the model with frictions

There are 15 parameters in our economy with frictions: $\{\alpha, \delta, \phi, \rho, \sigma, \psi, \omega, A, b, \chi_L, \chi_H, \gamma^{t<1975}, \iota, x, \tau\}$ .This is our calibration-strategy: We use those parameter values from Hornstein, Krusell and Violante that they choose from existing estimates. Thus, we use estimates from the literature, and simple calculations to calibrate $\{\psi, \alpha, \gamma^{t<1975}, \omega, \rho, b\}$.

We let our model match some moments of the US economy prior to 1975 to calibrate the remaining parameters. First, we match the same moments that HKV match. This allows us to calibrate $\{\delta, \phi, \sigma, A, \iota\}$. Remaining parameters; $\{\chi_L, \chi_H, x, \tau\}$; relate to wage-heterogeneity. We choose these parameters to match stylized facts from the US labor market around 1975.

Table 1. Technical Change

ψ	0.008	*disembodied technical change*
$\gamma^{t<1975}$	0.04	*decline of relative price of investment-goods*
α	0.3	*growth rate of output per capita*

We follow HKV in calibrating the parameters that govern technical change. In their, as well as in our, model there are two sources of productivity growth - disembodied (or neutral) technical change - which we denote ψ - and capital-embodied (or investment-specific) technical change, which we denote γ. HKV - with reference to Hornstein and Krusell (1996) - set the rate of disembodied technical change to $\psi = 0.008$.

Along a balanced growth path, the fixed cost of investment ι grows at rate $g = \psi + \alpha\gamma$. The productivity of raw capital goods grows at γ. This implies that the relative price of capital goods - the amount of efficiency units of capital that one final good can be transformed into - changes at rate $g - \gamma$. Cummins and Violante (2002) document that the quality adjusted relative price of capital $(g - \gamma)$ declined at an average rate of 2 per cent prior to 1975. The average growth rate of per capita production (g) prior to 1975 was 2 per cent. We use these observations to calibrate $\gamma^{t<1975} = 2 - (-2) = 4$ per cent per annum.

The definition of $g = \psi + \alpha\gamma = 0.02$ and our values of $\psi = 0.008$ and $\gamma^{t<1975} = 0.04$ imply that $\alpha = 0.3$. This concludes the calibration of the parameters that govern technical change. Below, we will study the consequences of changes in γ as documented by Cummins and Violante.

Table 2. Labor market

ω	0.5	*micro-estimates*

We set the elasticity of the matching-function with regard to vacancies $(1 - \omega)$ to 0.5. This is the average of the estimates reported in the overview by Petrongolo and Pissardes (2001) and is also the value that HKV use.

Table 3. Other external parameters

ρ	0.04	*interest rate*
b	0.05	*HKV*
χ^U	1	*Normalization*

We set the subjective discount-rate to 0.04 to match an equilibrium real interest rate of 4 per cent. We, as HKV, set unemployment benefits to 0.05. We normalize the human capital of the unskilled to 1. The remaining parameters we calibrate by matching the equilibrium outcome of the model to moments of the US economy prior to 1975.

Table 4. Internally calibrated parameters (base-line)

δ	0.093	*average age of* 11.5 *years*
σ	0.18	*separation rate of* 25 *per cent*
A	11.9	*average unemployment spell,* 2 *months*
ι	4.8	*average vacancy spell,* 1 *month*
ϕ	0.79	*labor share,* 70 *per cent*
χ^H	1.34	*log-variance of* 0.053
x	0.065	*Average wage-growth of* 3 *per cent*
τ	0.71	*Average wage-loss of* 23 *per cent, upon separation*

We simultaneously choose the parameters $\{\delta, \sigma, A, I, \phi\}$ to match some characteristics of the US economy prior to 1975. The depreciation rate δ is chosen to match an average age of capital goods of 11.5 years. Abrahams and Shimer (2002) report that the average duration of unemployment is 2 months. We choose the scale-parameter of the matching-function - A - to have the model satisfy this observation. We follow HKV, in choosing the separation rate, σ, to match an unemployment rate of 4 per cent. This implies an average separation rate of 25 per cent. Note that σ is lower than the target, since some separations occur endogenously - when matches reach the critical age $\bar{a}$ or $\bar{a}_e$. The fixed cost of investment, I, affects vacancy-creation. We choose this parameter to match the observation of Hall (2005) that vacancies on average have a duration of 1 month. We choose the Nash-weight of the employees, ϕ, to match a labor share of 70 per cent.

Furthermore, in our model workers are heterogeneous with regard do their productivity. Agents accumulate productivity on the job, and lose it on separations. Violante (2001) reports that wages within jobs on average grew with 3 per cent on an annual basis. We set x so that wages of low-productivity workers, that are not exogenously separated from their jobs, grow at this rate on average over a year. Violante (2001) uses a targeted wage-loss on separation of 23 per cent in the calibration of his model with vintage-specific human capital. We use the same target in calibrating the turbulence-parameter τ to an average wage loss of 23 per cent of the high-productivity workers that are exogenously separated from their jobs. Finally, we set the productivity of type H workers (χ^H) to 1.34 to match a log-variance in wages of 0.053, the transitional component of residual inequality, as used by Violante (2001), based on the study by Gottschalk and Moffitt (1994)[5].

[5]We target the stationary equilibrium to match the transtional component of residual inequality. In empirical studies, the

3.5.2 Observations in the base-line model with frictions

Table 5. Observations in the base-line equilibrium

$\bar{a}_H$	13.1
$\bar{a}_L$	23
$\frac{w_H}{w_L}$	1.62
(90/50)	0.48
(50/10)	0.04

Before we undertake experiments in our model economy, we observe some descriptive statistics of the economy. The equilibrium in our preferred parameterisation shows positive assortative matching. The unskilled operate the oldest machine. Both types of workers are accepted by a new machine. The skilled reject machines that are older than 13.1 years - whereas the unskilled accept machines until the age of 23 years. The wage-premium of type H workers is larger than the raw productivity-differential. This is due to positive assortative matching.

The median worker has low productivity. Therefore, wages are more disperse in the upper tail of the wage-distribution, than in the lower tail. The wage-gap between the wage of the ninetieth percentile and the median wage is 48 log-points - which corresponds almost exactly to the wage-premium of type H-workers. The wage-gap between the median worker and the wage of the tenth percentile is 0.04 log-points. There is, thus, little variation among lower wage-outcomes. Autor, Katz and Kearney report a 90/50 gap in residual wages of 0.47 log-points on average over the period 1973 – 1977, and a 50/10 gap of 0.53 for the same period. Clearly, our model does not generate variation in wages in the lower tail, that are on par with these observations.

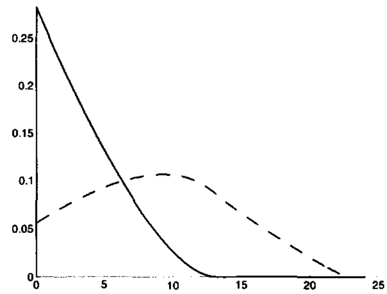
transitional component is measured as the residual variance while controlling for individual fixed effects. As the sample-period becomes longer, this transitional measure will converge to the measure of the stationary distribution. Another procedure would be to simulate data in our stationary equilibrium, and match the transitional component of that data to our target.

We, instead, in Appendix 5, assign a range of values to (χ^H) and consequently a range of stationary distributions with larger variance than our target. Even though the resulting variance of wages does increase with (χ^H), the relative change of this variance in response to an acceleration in the rate of embodied technical change is almost constant - around 13 per cent.

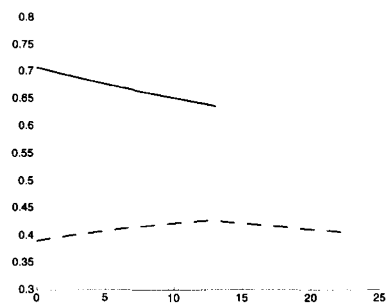
In Appendix 5, we also try out a range of values for x and τ. The results, with regard to the increase in residual inequality resulting from an increase in the rate of embodied technical change (γ) are very similar.

Figure 1 - Baseline Calibration

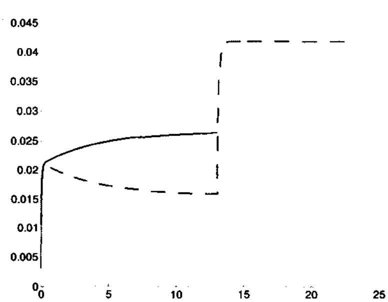
(a) Surplus



(b) Wages



(c) Matches



Surplus, wages and match-densities of type H (black) and type L (dashed/grey) in the base-line equilibrium. Ages on the x-axis.

The average age of machines operated by workers with high productivity is 6.8 years, whereas the less productive workers operate machines that are on average 13.9 years. The variation of wages among the highly productive workers is significantly larger than the variation among the less productive workers. Wage heterogeneity within groups of workers with the same productivity stems from variation in age across matches.

3.5.3 Calibration of the models without frictions

We have discussed two models without frictions. One with two levels of labor productivity, and one with infinitely many. We calibrate both. We pick the values for $\{\alpha, \delta, \rho, \psi, \chi_L, \gamma^{t<1975}\}$ from our calibration of the economy with frictions above.

In the economy with two levels of skill, we pick $\{x, \tau\}$ from the model with frictions. Then, we choose $\{\iota, \chi_H\}$ to match an average age of a machine of 11.5 years and a log-variance of wages of 0.053. These are the same targets that were used in the calibration of the model with frictions. We get $\chi_H = 1.34$ and $\iota = 5.4$.

In the economy with infinitely many levels of productivity, we choose $\{\iota, \chi\}$ to match the same targets. Recall that $\chi_h = \chi^h$ so that upper-tail wages are distributed according to the discrete equivalent of a Pareto-distribution. We get $\chi_H = 1.09$ and $\iota = 5.4$.

3.5.4 Quantitative Experiments

We have calibrated our model to match some features of the US economy in the mid-seventies, including the transitional component of residual inequality, as reported by Violante (2002). In this section, we study how our different measures of residual inequality respond to an acceleration in the rate of embodied technical change (γ). Then, we investigate the response of our inequality-measures to variation in the productivity of type H workers. We solve for the stationary equilibrium of the model at different parameter constellations. Our experiments, thus, compare stationary equilibria. We do not study transitional dynamics.

Table 6. Experiments

	With frictions			
		relative changes		
	base-line	γ	χ_H	γ & χ_H
$var(\log(w/\overline{w}))$	0.053	0.130	0.680	0.680
(90/50)	0.48	0.027	0.142	0.140
(50/10)	0.04	0.008	0.013	0.019

	Without frictions, $H = 2$			
		relative changes		
	base-line	γ	χ_H	γ & χ_H
$var(\log(w/\overline{w}))$	0.053	0.12	0.680	0.680
(90/50)	0.48	0.059	0.29	0.29
(50/10)	0	–	–	–

	Without frictions, $H = \infty$			
		relative changes		
	base-line	γ	χ_H	γ & χ_H
$var(\log(w/\overline{w}))$	0.053	0.134	0.680	0.680
(90/50)	0.34	0.072	0.292	0.301
(50/10)	0.14	0.052	0.330	0.309

An increase in the rate of embodied technical change

Cummins & Violante (2002) report that the rate of embodied technical change, as inferred from a decline in the relative price of capital, (γ) increased from an average of 4 per cent per annum during the seventies, to 7.6 per cent per annum thereafter. In our investigation of the model without frictions, above, we found that faster embodied technical change yields a more disperse wage-distribution. We alter the rate of embodied technical change, and solve for the stationary equilibrium.

We find that an increase in γ increases all our measures of inequality - as seen in Table 6. The changes are, however, smaller than observed changes in the US economy. The variance of log wages increases by 13 per cent, less than the 60 per cent increase reported by Violante (2001). The (90/50) gap increases by 2.7 per cent. In the data - for the period 1973 to 2005 - AKK report that the (90/50) gap grew by approximately 6.2 per cent[6]. The corresponding increase in the (50/10) gap was 2.3 per cent in the data - as compared to 0.8 per cent, the prediction from the model with frictions.

The models without frictions perform similarily to the model with frictions. A change in the rate of investment-specific technical change increases the variance of log-wages, but less than the observed variation. Upper-tail inequality increases more than in the lower tail. In the economy with only two levels of skill, the median worker earns the same wage as all workers with low productivity. Hence, there is no (50/10)-gap.

[6]AKK control for the composition of the labor force. At constant composition, the (90/50) gap grows between 5.7 and 6.8 per cent, during the period 1973 – 2005, depending on the base-year from which the composition is chosen. 6.2 is the increase when the composition in the center year - 1989 - is chosen.

In response to an increase in the rate of embodied technical change, our model with frictions predicts changes in inequality that go in the same direction as the data, but of a smaller magnitude. Therefore, we investigate the consequences of changes to other parameters in the model.

An increase in the productivity of more productive workers

The acceleration in the rate of investment-specific technical change does not suffice to explain the increase in residual inequality. We, therefore, proceed to analysing a more disperse distribution of productivities across workers. We increase the productivity of the more productive workers (χ_H). Evidently, observation of an increase in the productivity of some workers that are observationally identical is not possible. The increase in inequality, however, is. Therefore, we target an increase in the variance of log wages of 68 per cent, which is what Violante (2001) observes.

We find that a 10 per cent increase in (χ_H) yields the targeted increase in residual inequality in the economy with frictions. In response to this change, the (90/50) gap increases by 14 per cent, which is above target. The (50/10) gap increases 1.3 per cent, which is below target.

The expected wage-loss of high-productivity workers increases from 23 per cent to 28 per cent - an increase with 22 per cent. This is less than the 50 per cent increase, on average, in wage-losses upon separations that is reported in Violante (2001). Wages of low-productivity workers, that are not exogenously separated from their jobs, grow at 4 per cent on average over a year, as compared to 3 per cent in the base-line calibration. However, within-job wage growth during the eighties, as reported by Violante (2001), was 5 per cent.

In the models without frictions, the (90/50)$-$ gap increases considerably more than the observed increase. In the friction-less economy with many types of workers this is also true for the (50/10)-gap.

A change in (χ_H), that targets the change in the variance of log-wages, has consequences for other measures of inequality that go in the same direction as the data.

A combined effect

In the previous section, we targeted the change in the log-variance of wages while increasing the productivity of type H workers. In this section, we study a combined effect of an increase in (χ_H) and an acceleration in the rate of embodied technical change (γ). We still target the log-variance of wages. This implies that (χ_H) has to grow by 7 per cent in the economy with frictions in the labor market.

The results are similar to those obtained from the experiment with an isolated change in (χ_H). The (90/50) gap is still predicted to grow more than in the data. The (50/10) gap comes closer to the data equivalent. Measures of wage-growth and wage-losses are of the same magnitude as in the experiment with an isolated change in (χ_H).

Excluded variation

In their study of European unemployment, Ljungqvist and Sargent (1998) focus on the impact of an increase in turbulence. In their calibrated model, the wage-distribution becomes more disperse, as a consequence of increased turbulence. In our model, however, an increase in turbulence reduces variation in wages.

As we have seen in the economy without frictions, an increase in turbulence has two effects on the variance of log wages. First, an increase in turbulence skews the distribution of workers toward the less productive workers - the composition effect. This only increases the variance of log-wages if the skilled workers are more than half of the labor-force. Otherwise, the compisition-effect is negative. In our calibrated model with frictions, less productive workers are more common than more productive workers. This implies that the composition-effect is negative in our model with frictions. Second, an increase in turbulence always reduces the supply of high-productivity workers, and increases the wage-premium of high-productivity workers. This price-effect implies a more disperse wage-distribution. In our calibrated model with frictions, the composition-effect dominates the price-effect. This implies that an increase in (τ) reduces the variance of log wages[7]. This is clearly not consistent wity the experience in the US economy.

Note that turbulence can not be measured directly, but is inferred from wage-losses upon separation. Thus, any changes in parameters that increase the wage-losses on separations - including faster embodied technical change - could cause the phenomenon from which turbulence is inferred. However, as we have seen above, we do not get an increase in the wage-loss that is on par with the data.

The flip-side of our argument with regard to the composition-effect is an increase in x. Given the allocation of workers over types, an increase in x would skew the distribution of workers toward the more skilled. In isolation, this would increase the variance of log-wages. On the other hand, the price-effect - stemming from an increase in the supply of high productive workers - would reduce the difference between average wages of more and less productive workers. This implies lower wage-losses on separation of the more productive workers - which goes contrary to observations. We, therefore, do not investigate increases in x.

A comparison to the model with exogenous labor productivity

In Chapter 2, above, we investigate a model where workers are ex ante heterogeneous by productivity, and in which this productivity is constant. Here, we have introduced stchastic accumulation of skill. This affects the separation decisions. More productive workers choose to separate from matches earlier, since they risk losing productivity if their match is exogenously separated. On the other hand, less productive workers accept older matches. By working, they hold the option of becoming more productive. Thus, the introduction of stochastic skill-accumulation increases the age-differential - and hence productivity-differential - of machines operated by more and less productive workers.

Moreover, as seen in (3.50) above, workers in a match are compensated for the risks the incur by working.

[7] In the sensitivity analysis, in Appendix 5 below, we report the resulting variance for wages when x and τ are assigned a range of values.

More productive workers are compensated for the risk of losing productivity. Less productive workers accept wages lower than their flow-value of Unemployment, since working gives them the chance of accumulating skill.

We now, quantitatively, assess the effects of letting the productivity of workers evolves stochastically over time. We construct an economy with ex ante heterogenous workers, that has no stochastic skill-dynamics. In this economy, we assume that the ratio between the supply of high-productive and low-productive labor is $\frac{x}{\sigma\tau}$, so that the allocation of labor across types is the same as in the economy with stochastic labor productivity. We hold all parameters constant, but let τ and x both be zero.

In this calibrated economy, with ex ante heterogeneous workers, the less productive (or unskilled) workers choose to reject matches that are 22.7 years, less than the 23 years in our base-line economy. The more productive workers choose to reject matches that are 14.2 years, as compared to 13.1 in the base-line economy. The new machine accepts both types of workers. The introduction of stochastic skill-accumulation, thus, implies a somewhat wider spread in the productivity of machines that workers choose to operate.

The log-variance of wages in the economy with exogenous skill is 94 per cent of the log-variance in the economy with stochastic skill. An acceleration in the rate of investment-specific technical change yields an increase in the log-variance of wages of 12 per cent - a little less than the 13 per cent from the economy with stochastic skill accumulation.

3.6 Conclusion

Residual inequality, and its transitory component, has increased over the past decades. Upper-tail inequality has increased more than wage-dispersion in the lower tail. At the same time, the rate of embodied technical change has increased. We have studied the extent to which this acceleration in the growth-rate can explain the increased dispersion of wages.

In all our models, workers experience shocks to their own productivity. They may become more productive while working. Or they may become less productive. Machines, due to aging, become less productive. In equilibrium there is positive assortative matching. Younger machines are paired with more productive workers. In the models without frictions this is true for all worker-machine pairs. At critical ages, firms change workers to less productive workers. In the economy with frictions, more productive workers on average operate younger machines than less productive workers.

Wage heterogeneity stems from differences in productivity across workers, and is amplified by assortative matching. Wage-differentials are, hence, larger than the raw differentials in labor-productivity.

In the economy with frictions, there is an additional source of wage-heterogeneity. Frictions in the labor-market give rise to rents to the pairs of machines and workers that do work together. These rents are split through bargaining between the firm and the worker. Therefore, identical workers may receive different wages, if they operate machines of different ages.

An acceleration in the rate of embodied technical change increases the productivity-differential between machines of different ages. Since workers are assortatively matched, this, in turn, yields more wage dispersion between workers of different productivity. In the economy with frictions, faster embodied technical change also alters the wage-distribution among equally productive workers. Faster embodied technical change implies that the relative productivity of a machine falls faster. This affects the wage-distribution within groups of workers with equal productivity.

In our calibrated models, the acceleration in the rate of embodied technical change is not a sufficient source of variation to explain the observed increase in residual inequality. In all our models, it raises the variance of log-wages with approximately one fifth of the observed increase.

Violante (2002) reports a similar finding. In his model, the bulk of the increase in residual inequality stems from changes in the productivity-distribution of workers. This in turn is an outcome of assumed vintage-specificity of skill. We do not have any such mechanism. Instead, in our model, residual inequality increases as the productivity distribution of capital becomes more disperse. The same effect is present in Violante's setting. He finds that it only contributes "mildly to the increase in residual inequality." One difference between our framework and Violante's is that we assume Nash-bargaining over the surplus within a match. On a quantitative level, this alternative assumption does not alter the above quantitative conclusion from Violante (2002).

In our models, an increase in the rate of embodied technical change increases inequality in the upper tail more than in the lower tail. This is consistent with recent observations. The calibrated model with frictions predicts increases in the (90/50)-gap and the (50/10)-gap that are approximately one third of the observed changes.

In Chapter 2, above, we investigated a Mortensen-Pissarides model with vintage capital and workers that are ex ante heterogeneous by productivity. The introduction of stochastic skill-accumulation increases the productivity-differential between machines operated by skilled and unskilled workers. Less skilled workers accept older machines, since they may become more productive. More productive workers reject younger machines, if they risk productivity-loss at work. Moreover, workers in a match are compensated for these risks. Hence, the introduction of stochastic skill-accumulation implies larger wage-inequality.

In our settings, we have found that faster embodied technical change does not suffice to generate the observed increase in inequality. Mechanisms that allow for larger changes to the equilibrium distribution of workers over labor productivity - such as vintage-specificity of human capital, as suggested by Violante (2002) - would improve the performance of the general framework. We leave such efforts for future research.

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3.7 Appendices

3.7.1 Appendix 1 - Solution of the single differential equation

We have two types of equilibria - one where the skilled operate the oldest match and one where the unskilled do. We begin by solving the differential equation when the skilled operate the oldest match. We let $(\rho - g + \sigma) \equiv \varrho$; $q(\theta)(1-\phi) \equiv \kappa$; $\frac{u^H}{u} \equiv p$. The solution to the first order linear differential equation

$(\varrho + \kappa p) S^H(a) = y_H(a) - (\rho - g) U^H - \sigma\tau \left(U^H - U^L\right) + S^H_a$; is:

$$S^H(a) = e^{(\varrho+\kappa p)a} \int_a^{\overline{a}_H} e^{-(\varrho+\kappa p)\widetilde{a}} \left(\chi^H e^{-\partial\widetilde{a}} - (\rho - g) U^H - \sigma\tau \left(U^H - U^L\right)\right) d\widetilde{a} \tag{3.51}$$

While the unskilled work, they gain experience, and may see their productivity increase. After the age $(\overline{a}_H)$ at which skilled matches are being rejected, the surplus of a match with an unskilled worker is given by: $(\varrho + \kappa(1-p) + x) S^L(a) = y^L(a) - (\rho - g) U^L + x \left(U^H - U^L\right) + S^L_a(a)$ and the solution is

$$S^L(a) = e^{(\varrho+\kappa(1-p)+x)a} \int_a^{\overline{a}_L} e^{-(\varrho+\kappa(1-p)+x)\widetilde{a}} \left(\chi^L e^{-\partial\widetilde{a}} - (\rho - g) U^L + x \left(U^H - U^L\right)\right) d\widetilde{a} \tag{3.52}$$

The matched parties choose the separation-age optimally, implying that

$$\begin{array}{ll} (\rho - g) U^H = \chi^H e^{-\partial \overline{a}_H} & if\ \overline{a}_H > \overline{a}_L \\ (\rho - g) U^L = \chi^L e^{-\partial \overline{a}_L} + x \left(U^H - U^L\right) & otherwise \end{array}$$

The skilled earn their marginal productivity in the oldest match that they operate.

3.7.2 Appendix 2 - solution of the system of differential equations

We have the system of differential equations:

$$\begin{aligned} (\rho - g + \sigma) \begin{bmatrix} S^H(a) \\ S^L(a) \end{bmatrix} &= e^{-\partial a} \begin{bmatrix} \chi^H \\ \chi^L \end{bmatrix} - \begin{bmatrix} (\rho - g) + \sigma\tau & -\sigma\tau \\ -x & (\rho - g) + x \end{bmatrix} \begin{bmatrix} U^H \\ U^L \end{bmatrix} \\ &- \begin{bmatrix} q(\theta)(1-\phi)\frac{u^H}{u} & q(\theta)(1-\phi)\frac{u^L}{u} \\ q(\theta)(1-\phi)\frac{u^H}{u} - x & q(\theta)(1-\phi)\frac{u^L}{u} + x \end{bmatrix} \begin{bmatrix} S^S(a) \\ S^U(a) \end{bmatrix} + \begin{bmatrix} S^S_a(a) \\ S^U_a(a) \end{bmatrix} \end{aligned} \tag{3.53}$$

Which we re-wrtie

$$\mathbf{S}_a = \mathbf{AS} - \left(\boldsymbol{\chi} e^{-\partial a} - \mathbf{U}\right) \tag{3.54}$$

or

$$\mathbf{S} = \begin{bmatrix} S^H(a) \\ S^L(a) \end{bmatrix}; \mathbf{A} = \begin{bmatrix} \varrho + \kappa p & \kappa(1-p) \\ \kappa p - x & \varrho + \kappa(1-p) + x \end{bmatrix}; \boldsymbol{\chi} = \begin{bmatrix} \chi^H \\ \chi^L \end{bmatrix}; \mathbf{U} = \begin{bmatrix} (\rho - g) U^H + \sigma\tau\left(U^H - U^L\right) \\ (\rho - g) U^L - x\left(U^H - U^L\right) \end{bmatrix}$$

It follows that the solution to the homogenous system $\mathbf{S}_a = \mathbf{AS}$ is $\mathbf{S} = \mathbf{\Phi}(a)\,\mathbf{C}$ where

$$\mathbf{\Phi}(a) = \begin{bmatrix} 1 & (1-p)\kappa \\ 1 & -\kappa\left(p - \frac{x}{\kappa}\right) \end{bmatrix} \begin{bmatrix} e^{(\varrho+\kappa)a} & 0 \\ 0 & e^{(\varrho+x)a} \end{bmatrix}$$

and $\mathbf{C}$ is a column of arbitrary constants. The particular solution to our system is, thus:

$$\mathbf{S}_p = -\mathbf{\Phi}(a) \int \mathbf{\Phi}^{-1}(\widetilde{a}) \left(\boldsymbol{\chi} e^{-\partial \widetilde{a}} - \mathbf{U}\right) d\widetilde{a}$$

where;

$$\mathbf{\Phi}^{-1}(a) = \frac{1}{\kappa - x}\begin{bmatrix} e^{-(\varrho+\kappa)a} & 0 \\ 0 & e^{-(\varrho+x)a} \end{bmatrix}\begin{bmatrix} \kappa\left(p - \frac{x}{\kappa}\right) & \kappa(1-p) \\ 1 & -1 \end{bmatrix}$$

$$\begin{matrix} \kappa\left(p - \frac{x}{\kappa}\right) & \kappa(1-p) \\ 1 & -1 \end{matrix} \begin{bmatrix} U^H(\rho - g + \sigma\tau) - U^L\sigma\tau \\ U^L(x - g + \rho) - U^H x \end{bmatrix} = \begin{bmatrix} \left(p - \frac{x}{\kappa}\right)\left(U^H(\rho - g) + \sigma\tau\left(U^H - U^L\right)\right) + \left((\rho - g)U^L - x\left(U^H - \right.\right. \\ U^H(\rho - g + \sigma\tau + x) - U^L(x - g + \rho + \sigma\tau) \end{bmatrix}$$

At age $\overline{a}_h$ the surplus for workers type h is 0, since they reject older matches. The the other type of workers it $S_j(\overline{a}_h)$, as given by (3.51) or (3.52) above. The end-value condition is, thus $\mathbf{F} = \left\{\begin{bmatrix} S^H(\overline{a}_L) \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ S^L(\overline{a}_H) \end{bmatrix}\right\}$ depending on what type of workers operate the oldest machine in the economy. It follows that the solution to our problem is:

$$\mathbf{S} = \mathbf{\Phi}(a)\,\mathbf{\Phi}^{-1}(\overline{a}_h)\,\mathbf{F} + \mathbf{\Phi}(a)\int_a^{\overline{a}_h} \mathbf{\Phi}^{-1}(\widetilde{a})\left(\chi e^{-\partial\widetilde{a}} - \mathbf{U}\right) d\widetilde{a} \tag{3.55}$$

which we re-write:

$$\mathbf{S} = \frac{\kappa}{\kappa - x}\begin{bmatrix} 1 & (1-p) \\ 1 & -\left(p - \frac{x}{\kappa}\right) \end{bmatrix}\left(\begin{bmatrix} e^{-(\kappa+\varrho)(\overline{a}_h - a)}S_1(\overline{a}_h) \\ e^{-(x+\varrho)(\overline{a}_h - a)}S_2(\overline{a}_h) \end{bmatrix} + \int_a^{\overline{a}_h}\begin{bmatrix} e^{-(\widetilde{a}-a)(\kappa+\varrho)}\left(\chi_1 e^{-\partial\widetilde{a}} - U_1\right) \\ e^{-(\widetilde{a}-a)(x+\varrho)}\left(\chi_2 e^{-\partial\widetilde{a}} - U_2\right) \end{bmatrix} d\widetilde{a}\right) \tag{3.56}$$

where $\chi_1 \equiv \left(\chi^H\left(p - \frac{x}{\kappa}\right) + \chi^L(1-p)\right)$;$\chi_2 \equiv \left(\chi^H - \chi^L\right)$; $S_1 \equiv \left(S^H\left(p - \frac{x}{\kappa}\right) + S^L(1-p)\right)$;$S_2 \equiv \left(S^H - S^L\right)$; $U_1 \equiv \left((1-p)(\rho - g)U^L + \left(p - \frac{x}{\kappa}\right)(\rho - g)U^H\right) - (1-p)x\left(U^H - U^L\right)$ and$U_2 \equiv (\rho - g + x)\left(U^H - U^L\right)$ respectively.

As in the case when only one type of workers are employed, the parties choose $\overline{a}_h$ optimally, so that $\frac{d}{d\overline{a}_h}\mathbf{S}(\overline{a}_h) = \mathbf{0}$. We, thus, have the condition:

$$\begin{bmatrix} -(\varrho + p\kappa) & -\kappa(1-p) \\ -p\kappa + x & -(\kappa(1-p) + \varrho) - x \end{bmatrix}\mathbf{F} + \frac{d}{d\widetilde{a}}\mathbf{F} + \chi e^{-\partial\widetilde{a}} - \mathbf{U} = \mathbf{0}$$

which we re-write:

$$\begin{bmatrix} S_a^H(\overline{a}_L) + \left(\chi^H e^{-\partial\overline{a}_L} - (\rho - g)U^H\right) - (\varrho + p\kappa)S^H(\overline{a}_L) \\ \left(\chi^L e^{-\partial\overline{a}_L} - (\rho - g)U^L\right) - p\kappa S^H(\overline{a}_L) + xS^H(\overline{a}_L) + x\left(U^H - U^L\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \textit{or}$$

$$\begin{bmatrix} \left(\chi^H e^{-\partial\overline{a}_H} - U^H(\rho - g)\right) - \kappa(1-p)S^L(\overline{a}_H) \\ S_a^L(\overline{a}_H) + \left(\chi^L e^{-\partial\overline{a}_H} - (\rho - g)U^L\right) - (\varrho + \kappa(1-p))S^L(\overline{a}_H) - xS^L(\overline{a}_H) + x\left(U^H - U^L\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

3.7.3 Appendix 3 - the stationary distributions

Matches are formed in the labor market, when firms and workers that accept a match meet. Young firms ($0 \leq a \leq \underline{a}_L$) only accept high-skilled workers. For intermediate ages, both types of workers accept matches, and only one type of workers accept old machines. Thus, there are different sets of differential equations that govern the evolution of the stocks of matches. We describe and solve these below.

For ages when only skilled workers accept matches the stock of skilled matches $m_H(a)$ evolves according to:

$$m_{Ha}(a) = -\sigma m_H(a) + qpv(a) = qp\eta - (\sigma + qp)\, m_H(a)$$

The solution of which is:

$$m_H(a) = qp\eta e^{-(\sigma+qp)a} \int_0^a e^{(\sigma+qp)\widetilde{a}} d\widetilde{a}$$

When both types of workers accept matches, these evolve according to:

$$\begin{aligned} m_{Ha}(a) &= qp\eta - (\sigma + qp)\, m_H(a) - qpm_U(a) + xm_L(a) \\ m_{La}(a) &= q(1-p)\eta - q(1-p)\, m_H(a) - (\sigma + q(1-p))\, m_L(a) - xm_L(a) \end{aligned}$$

which we re-write

$$\mathbf{m}_a = \mathbf{A}\mathbf{m} + q\eta\mathbf{p} \tag{3.57}$$

where

$$\mathbf{m} = \begin{bmatrix} m_H(a) \\ m_L(a) \end{bmatrix}; \mathbf{A} = \begin{bmatrix} -(\sigma + qp) & x - qp \\ -q(1-p) & -(\sigma + q(1-p)) - x \end{bmatrix}; \mathbf{p} = \begin{bmatrix} p \\ (1-p) \end{bmatrix}$$

It follows that the solution to the homogenous system $\mathbf{m}_a = \mathbf{A}\mathbf{m}$ is $\mathbf{m} = \mathbf{\Phi}(a)\,\mathbf{C}$ where

$$\mathbf{\Phi}(a) = \begin{bmatrix} p - \frac{x}{q} & -1 \\ 1-p & 1 \end{bmatrix} \begin{bmatrix} e^{-(\sigma+q)a} & 0 \\ 0 & e^{-(\sigma+x)a} \end{bmatrix}$$

and $\mathbf{C}$ is a column of arbitrary constants. The particular solution to our problem is given by:

$$\mathbf{m}_p = q\eta\mathbf{\Phi}(a)\int\mathbf{\Phi}^{-1}(a)\,\mathbf{p}$$

where $\frac{d}{da}$

$$\mathbf{\Phi}^{-1}(a) = \frac{q}{q-x}\begin{bmatrix} e^{(\sigma+q)a} & 0 \\ 0 & e^{(\sigma+x)a}\end{bmatrix}\begin{bmatrix} 1 & 1 \\ -(1-p) & -\left(\frac{x}{q}-p\right)\end{bmatrix}$$

Finally, we have the initial condition that at time $\underline{a}_L$ the high-quality matches are $m_H(\underline{a}_L) = qp\eta e^{-(\sigma+qp)\underline{a}_L}\int_0^{\underline{a}_L} e^{(\sigma+qp)\tilde{a}}d\tilde{a}$ and thus the solution to our problem is:

$$\mathbf{m} = \mathbf{\Phi}(a)\mathbf{\Phi}^{-1}(\underline{a}_L)\mathbf{F} + q\eta\mathbf{\Phi}(a)\int_{\underline{a}_L}^{a}\mathbf{\Phi}^{-1}(\tilde{a})\,\mathbf{p}d\tilde{a}$$

where $\mathbf{F} = \begin{bmatrix} m_H(\underline{a}_L) \\ 0\end{bmatrix}$. We re-write this:

$$\begin{bmatrix} m_H(a) \\ m_L(a)\end{bmatrix} = m_H(\underline{a}_L)\begin{bmatrix} e^{-(a-\underline{a}_L)(x+\sigma)}\frac{q-qp}{q-x} + e^{-(a-\underline{a}_L)(q+\sigma)}\frac{(qp-x)}{q-x} \\ \frac{q-qp}{q-x}\left(e^{-(a-\underline{a}_L)(q+\sigma)} - e^{-(a-\underline{a}_L)(x+\sigma)}\right)\end{bmatrix}$$
$$+q\eta\frac{q}{q-x}\int_{\underline{a}_L}^{a}\begin{bmatrix}\left(p-\frac{1}{q}x\right)e^{-(a-\tilde{a})(q+\sigma)} + (1-p)\frac{1}{q}xe^{-(a-\tilde{a})(x+\sigma)} \\ (1-p)e^{-(a-\tilde{a})(q+\sigma)} - (1-p)\frac{1}{q}xe^{-(a-\tilde{a})(x+\sigma)}\end{bmatrix}d\tilde{a}$$

3.7.4 Appendix 4, Proof of Proposition 2

First, note that the wage of any worker h relative to the worker with the lowest wage is: $\widetilde{w}_h = \frac{w_h}{w_0} = \sum_{\tilde{h}=0}^{h}\left(\chi_{\tilde{h}} - \chi_{\tilde{h}-1}\right)e^{\partial\overline{a}_0\left(1-\varsigma^{\tilde{h}}\right)}$. Thus,

the elasticity of the relative wage with regard to $\partial\overline{a}_0$ is $\frac{\frac{d}{d(\partial\overline{a}_0)}(\widetilde{w}_h)}{\widetilde{w}_h} = \frac{\sum_{\tilde{h}=0}^{h}\left(1-\varsigma^{\tilde{h}}\right)\left(\chi_{\tilde{h}}-\chi_{\tilde{h}-1}\right)e^{\partial\overline{a}_0\left(1-\varsigma^{\tilde{h}}\right)}}{\sum_{\tilde{h}=0}^{h}\left(\chi_{\tilde{h}}-\chi_{\tilde{h}-1}\right)e^{\partial\overline{a}_0\left(1-\varsigma^{\tilde{h}}\right)}}$. The ratio of elasticities for a pair of workers h and $h+1$ is thus:

$$\frac{\frac{\frac{d}{d(\partial\overline{a}_0)}(\widetilde{w}_{h+1})}{\widetilde{w}_{h+1}}}{\frac{\frac{d}{d(\partial\overline{a}_0)}(\widetilde{w}_h)}{\widetilde{w}_h}} = \frac{\sum_{\tilde{h}=0}^{h}\left(\chi_{\tilde{h}}-\chi_{\tilde{h}-1}\right)e^{\partial\overline{a}_0\left(1-\varsigma^{\tilde{h}}\right)}\sum_{\tilde{h}=0}^{h+1}\left(1-\varsigma^{\tilde{h}}\right)\left(\chi_{\tilde{h}}-\chi_{\tilde{h}-1}\right)e^{\partial\overline{a}_0\left(1-\varsigma^{\tilde{h}}\right)}}{\sum_{\tilde{h}=0}^{h+1}\left(\chi_{\tilde{h}}-\chi_{\tilde{h}-1}\right)e^{\partial\overline{a}_0\left(1-\varsigma^{\tilde{h}}\right)}\sum_{\tilde{h}=0}^{h}\left(1-\varsigma^{\tilde{h}}\right)\left(\chi_{\tilde{h}}-\chi_{\tilde{h}-1}\right)e^{\partial\overline{a}_0\left(1-\varsigma^{\tilde{h}}\right)}} =$$

$$\frac{1}{1+\frac{(\chi_{h+1}-\chi_h)e^{\theta\bar{a}_0\left(1-\varsigma^{h+1}\right)}}{\sum_{\tilde{h}=0}^{h}(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})e^{\theta\bar{a}_0\left(1-\varsigma^{\tilde{h}}\right)}}}\ \frac{1+\frac{\left(1-\varsigma^{h+1}\right)(\chi_{h+1}-\chi_h)e^{\theta\bar{a}_0\left(1-\varsigma^{h+1}\right)}}{\sum_{\tilde{h}=0}^{h}\left(1-\varsigma^{\tilde{h}}\right)(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})e^{\theta\bar{a}_0\left(1-\varsigma^{\tilde{h}}\right)}}}{1}$$

The elasticity is larger for larger h iff this ratio is greater than 1. Then it is true that:

$$\frac{\left(1-\varsigma^{h+1}\right)(\chi_{h+1}-\chi_h)e^{\theta\bar{a}_0\left(1-\varsigma^{h+1}\right)}}{\sum_{\tilde{h}=0}^{h}\left(1-\varsigma^{\tilde{h}}\right)(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})e^{\theta\bar{a}_0\left(1-\varsigma^{\tilde{h}}\right)}} > \frac{(\chi_{h+1}-\chi_h)e^{\theta\bar{a}_0\left(1-\varsigma^{h+1}\right)}}{\sum_{\tilde{h}=0}^{h}(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})e^{\theta\bar{a}_0\left(1-\varsigma^{\tilde{h}}\right)}}$$

We re-write this expression:

$$\left(1-\varsigma^{h+1}\right) > 1 - \frac{\sum_{\tilde{h}=0}^{h}\varsigma^{\tilde{h}}(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})e^{\theta\bar{a}_0\left(1-\varsigma^{\tilde{h}}\right)}}{\sum_{\tilde{h}=0}^{h}(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})e^{\theta\bar{a}_0\left(1-\varsigma^{\tilde{h}}\right)}}\text{ ; This implies:}$$

$$1 < \frac{\sum_{\tilde{h}=0}^{h}\varsigma^{\tilde{h}}(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})e^{\theta\bar{a}_0\left(1-\varsigma^{\tilde{h}}\right)}}{\varsigma^{h+1}\sum_{\tilde{h}=0}^{h}(\chi_{\tilde{h}}-\chi_{\tilde{h}-1})e^{\theta\bar{a}_0\left(1-\varsigma^{\tilde{h}}\right)}}$$

This is true, since $\varsigma^{h+1} < \varsigma^{\tilde{h}}$ for any $\tilde{h} < h+1$.

3.7.5 Appendix 5, Sensitivity analysis

In our preferred calibration, we target a log-variance of wages of 0.053. This target corresponds to a measure of the transitional component of residual inequality. Our model depicts a steady state. Emprically, the transitional component of residual inequality is the variation in wages, when controlling for individual fixed effects. For long enough sample periods, the transitional component will converge to the variance of the stationary distribution.

An alternative approach would have been to simulate wages of individuals in our stationary equilibrium, to generate a measure of transitory residual inequality. We do not do this. Instead, we here document that our results are insensitive to the choice of target-variance. We have targeted the log-variance of wages by varying the productivity of the more productive workers. Therefore, we lift the target for the variance and vary this (χ_H) parameter.

The parameters that govern the distribution of workers across states of productivity are x - the probability of becoming more productive - and τ the probability of losing productivity on separation. We vary these parameters, while lifting their respective targets and holding χ_H constant at 1.34. We report values for these parameters, the resulting variance when investment-specific productivity growth is low and the resulting change when we increase the rate of investment specific technical change.

		$var(\log(w/\overline{w}))$	relative change in $var(\log(w/\overline{w}))$ in response to increase in γ
	1.44	0.090	0.14
χ_H	1.34	0.053	0.13
	1.24	0.029	0.13
	0.91	0.047	0.13
τ	0.71	0.053	0.13
	0.51	0.056	0.13
	0.085	0.056	0.13
x	0.065	0.053	0.13
	0.045	0.046	0.13

We see that the response in the wage-dispersion to a change in the rate of investment-specific technical change is insensitive to our parameterisation, and to the target variance.

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ISBN 978-91-7258-728-1