

**Four Essays on Building
Conditional Correlation GARCH Models**



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Four Essays on Building Conditional Correlation GARCH Models

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*To my family
for their patience and encouragement*

Contents

Acknowledgements	xi
1 Introduction	1
2 Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model	9
2.1 Introduction	12
2.2 The extended constant conditional correlation GARCH model	13
2.2.1 Definition	13
2.2.2 Stationarity of the ECCC-GARCH process	14
2.3 Likelihood function and asymptotic properties	15
2.3.1 The log-likelihood function	15
2.3.2 The score and the Hessian of the log-likelihood function	15
2.3.3 Asymptotic behaviour of the quasi maximum likelihood estimator	17
2.3.4 The fourth-order moment condition	17
2.4 Test for volatility interactions	18
2.5 Bivariate illustration of the test statistic	19
2.5.1 The bivariate ECCC-GARCH(1, 1) model and the partial derivatives of the conditional variance equations	19
2.5.2 Components of the test statistic	21
2.6 Simulation experiments of the LM_{ECCC} test statistic	22
2.6.1 Size simulations	22
2.6.2 Power simulations	23
2.6.3 The test under changing conditional correlations	23
2.7 Applications to daily return series	24
2.7.1 Data	24
2.7.2 Results of CCC/ECCC-GARCH estimation	25
2.8 Concluding remarks	26
Appendix 2.A Mathematical derivations and proofs	32
2.A.1 Proof of Lemma 2.5	32
2.A.2 Proof of Lemma 2.6	33
2.A.3 Proof of Lemma 2.7	35

3	An Alternative Test for Causality in Variance in the Conditional Correlation GARCH Models	47
3.1	Introduction	50
3.2	The conditional correlation GARCH model	51
	3.2.1 The model specification	51
	3.2.2 Estimation strategies	53
3.3	Tests for causality in conditional variance	54
	3.3.1 Alternative test for causality in conditional variance	54
	3.3.2 Relations to similar tests	57
3.4	Simulation experiments	58
	3.4.1 Design of the experiments	58
	3.4.2 Size properties	58
	3.4.3 Power properties	59
3.5	Applications	60
3.6	Concluding remarks	61
4	Positivity Constraints on the Conditional Variances in the Family of Conditional Correlation GARCH Models	77
4.1	Introduction	80
4.2	The model and the positive definiteness of the covariance matrix . . .	81
4.3	Main results	82
	4.3.1 CC-GARCH(p, q) model	82
	4.3.2 CC-GARCH(1, 1) model	83
4.4	Empirical example	84
4.5	Concluding remarks	84
	Appendix 4.A Reparameterisation of the model and proof of Corollary 4.4	89
5	ccgarch: An R Package for Building Multivariate GARCH Models with Conditional Correlations	93
5.1	Introduction	96
5.2	The model	96
	5.2.1 The volatility component of the model	97
	5.2.2 The correlation component of the model	98
5.3	The package: ccgarch	100
	5.3.1 Overview	100
	5.3.2 Simulation with dcc.sim	101
	5.3.3 Estimation with dcc.estimation	102
	5.3.4 Utility functions	105
5.4	Illustrative example	107
	5.4.1 The data	107
	5.4.2 Fitting a DCC-GARCH(1,1) model	107
	5.4.3 Cautionary remarks	109
5.5	Concluding remarks	109
	Appendix 5.A Mathematical derivations	114
	5.A.1 Partial derivatives of the volatility component	114

5.A.2 Partial derivatives of the correlation component 114

5.A.3 Asymptotic covariance matrix of the QML estimator 115

Appendix 5.B A User's Manual of **ccgarch** 116

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Aarhus, March 2010

Tomoaki Nakatani

Chapter 1

Introduction

This thesis consists of four research papers. The main focus is on building multivariate Generalised Autoregressive Conditional Heteroskedastic (GARCH) models with explicit modelling of conditional correlations. The general formulation of the model is as follows. Assume an N -dimensional vector of stochastic process such that

$$\boldsymbol{\varepsilon}_t = \mathbf{D}_t \mathbf{z}_t \quad (1.1)$$

where \mathbf{D}_t is a diagonal matrix with elements $h_{i,t}^{1/2}$, $i = 1, \dots, N$, the terms in the sequence of the stochastic vectors $\{\mathbf{z}_t\} = [z_{1,t}, \dots, z_{N,t}]'$ are independently distributed with $E[\mathbf{z}_t | \mathcal{F}_{t-1}] = \mathbf{0}$ and $E[\mathbf{z}_t' \mathbf{z}_t | \mathcal{F}_{t-1}] = \mathbf{P}_t$ where \mathcal{F}_t is the information set up to and including time t , and $\mathbf{P}_t \in \mathcal{F}_{t-1}$. Then we have $E[\boldsymbol{\varepsilon}_t | \mathcal{F}_{t-1}] = \mathbf{0}$ and $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' | \mathcal{F}_{t-1}] = \mathbf{D}_t \mathbf{P}_t \mathbf{D}_t = \mathbf{H}_t$. The elements of matrix \mathbf{H}_t are defined as follows

$$[\mathbf{H}_t]_{ij} = \begin{cases} h_{i,t} & i = j, \\ h_{i,t}^{1/2} h_{j,t}^{1/2} \rho_{ij,t} & i \neq j. \end{cases} \quad (1.2)$$

The equations (1.1) and (1.2) constitute a multivariate GARCH model with explicit modelling of conditional correlations. We shall call this class of multivariate GARCH models the family of the Conditional Correlation (CC-) GARCH models. In these models, \mathbf{H}_t and \mathbf{P}_t are the time-varying conditional covariance matrix and the time-varying positive definite conditional correlation matrix of the process $\boldsymbol{\varepsilon}_t$, respectively.

The most attractive feature of the family of the CC-GARCH models is that the definitions of \mathbf{H}_t and \mathbf{P}_t make it possible to define separate parametric structures for the conditional variances and the conditional correlations. The basic framework of the CC-GARCH model was introduced by Bollerslev (1990) with the assumptions that $\mathbf{P}_t \equiv \mathbf{P}$, that is, the conditional correlations are constant over time and that each diagonal element of \mathbf{H}_t follows the standard univariate GARCH(p, q) model of Bollerslev (1986). Following this Constant CC- (CCC-) GARCH representation, much attention has been paid on building more flexible models for describing the dynamics of the conditional correlations. Examples include the Dynamic Conditional Correlation (DCC-) GARCH of Engle (2002), the Varying Correlation GARCH of Tse and Tsui (2002), the Smooth Transition Conditional Correlation (STCC-) and Double STCC (DSTCC-) GARCH of Silvennoinen and Teräsvirta (2005, 2009a) and the generalised DCC-GARCH of Hafner and Franses (2009), to name a few.

Modelling the conditional variances and the conditional correlations separately leads to gains in flexibility. But then, one has to deal with model building issues on both the conditional variance and the conditional correlation components. As Silvennoinen and Teräsvirta (2009b) pointed out, however, compared to the efforts made on modelling the correlations less work has been done on modelling of the conditional variance component in the CC-GARCH models. The few exceptions include Jeantheau (1998), Ling and McAleer (2003) and He and Teräsvirta (2004)¹. They all

¹Another extension was recently proposed by McAleer, Hoti, and Chan (2009). Their model, a Vector ARMA-Asymmetric GARCH model, specifies the conditional variance component as a multivariate extension of the univariate GJR-GARCH model of Glosten, Jagannathan, and Runkle (1993).

consider the following vector formulation of the conditional variance component:

$$\mathbf{h}_t = \mathbf{a}_0 + \sum_{i=1}^q \mathbf{A}_i \boldsymbol{\varepsilon}_{t-i}^{(2)} + \sum_{j=1}^p \mathbf{B}_j \mathbf{h}_{t-j} \quad (1.3)$$

where $\mathbf{h}_t = [h_{1,t}, \dots, h_{N,t}]'$, $\boldsymbol{\varepsilon}_t^{(2)} = [\varepsilon_{1,t}^2, \dots, \varepsilon_{N,t}^2]'$, \mathbf{a}_0 is an $(N \times 1)$ vector of constants, and \mathbf{A}_i and \mathbf{B}_j are $(N \times N)$ matrices with elements such that $h_{k,t}$, $k = 1, \dots, N$, in \mathbf{h}_t are positive for all t .

The equation (1.3) is a multivariate counterpart of the univariate GARCH(p, q) representation. This equation differs from the univariate GARCH(p, q) model in that it allows for interaction effects or causality in conditional variance through off-diagonal elements of \mathbf{A}_i and \mathbf{B}_j . Therefore, a conditional variance $h_{k,t}$ of the k th variable in the system is not only a function of $(\varepsilon_{k,t-i}, h_{k,t-j})$ but also of $(\varepsilon_{l,t-i}, h_{l,t-j})$, $l \neq k$. He and Teräsvirta (2004) showed that the CCC-GARCH model with (1.3) can describe a richer autocorrelation structure of the squared observation than does the standard CCC-GARCH model. They called this specification an Extended CCC (ECCC-) GARCH model.

Nevertheless, many of papers in the existing literature merely use a variant of univariate GARCH models in characterising the conditional variances. Despite the fact that a number of different specifications are possible for the univariate models, incorporating interaction effects or causality in conditional variances into the model is not possible. Consequently, this thesis mainly focuses on issues of building the CC-GARCH models with the conditional variance of the form (1.3). In the following three chapters, misspecification testing and parameter restrictions in these models are discussed. In the final chapter, a computer package for building major variants of the CC-GARCH models is presented. A brief summary of each of these chapters follows next.

Chapter 2, entitled “*Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*,”² proposes a misspecification test for modelling of the conditional variance part of the ECCC-GARCH model. The test proposed in this chapter is to test of the nullity of the off-diagonal elements of the matrices \mathbf{A}_i and \mathbf{B}_j in (1.3). If the null hypothesis that the off-diagonal elements are all zero is true, then each conditional variance in (1.3) may be described by the standard GARCH(p, q) model. Since the standard CCC-GARCH(p, q) model is numerically much easier to estimate than the alternative, the test is constructed on the Lagrange Multiplier (LM) or the score principle that only requires the estimation of the null model.

Finite sample properties of the test are investigated in the bivariate case through Monte Carlo experiments. The results show that the test is well behaved (the empirical size of the test is reasonably close to nominal one) when the sample size reaches 2000. Although the test is derived under the assumption of the constant conditional

²This chapter is a joint work with Timo Teräsvirta. An abridged version was published in *Econometrics Journal* as Nakatani and Teräsvirta (2009): Copyright (2009) The Authors, Royal Economic Society and Wiley. Materials are used in this thesis with permission from John Wiley and Sons.

correlation, the simulation experiments suggest that the test is also applicable to building CC-GARCH models with changing conditional correlations.

Chapter 3, entitled “*An Alternative Test for Causality in Variance in the Conditional Correlation GARCH Models,*” provides yet another misspecification test for modelling of the conditional variance component of the CCC-GARCH model. One of the properties of the CC-GARCH model is its method of estimation. A CC-GARCH model is often estimated as follows. First decompose the log-likelihood function into two parts, of which the first part is a function of the parameters in the conditional variance component, whereas the second one includes, given the estimates of the parameters in the first part, the parameters associated with the conditional correlation component. Then carry out the optimisation in two steps: first maximise the GARCH component, and then, conditionally on the GARCH parameters, the conditional correlations.

The estimator obtained in these two steps is a two-stage quasi-maximum likelihood estimator (2SQMLE). In the case of the ECCC-GARCH model, conditions for the asymptotic normality of the entire estimator vector were established by Ling and McAleer (2003). Their results, combined with the existing formulae with respect to 2SQMLE, lead to the asymptotic normality of the first stage estimators. Taking advantage of this asymptotic result, the test considered in Chapter 3 is formulated using the LM principle, which, as already pointed out, requires only the estimation of univariate GARCH models. It is also shown that the test statistic may be computed by using an auxiliary regression. A robust version of the new test is available through another auxiliary regression. All of this amounts to a substantial simplification in computations compared with the test proposed in Chapter 2.

Finite sample properties of the test are examined in up to five-dimensional models by simulation, and its performance is compared with the test in Chapter 2 and a similar test proposed by Hafner and Herwartz (2006). The test of Hafner and Herwartz (2006) was derived in the bivariate framework and is generalised to higher dimensions accordingly. It is shown that, under both normality and under leptokurtotic innovations, as well as under changing conditional correlations, the new test has reasonable size and power properties.

When modelling the conditional variance, it is necessary to keep a sequence of conditional covariances positive almost surely for any time horizon. For the standard univariate GARCH(p, q) model of Bollerslev (1986), Nelson and Cao (1992) and Tsai and Chan (2008) worked out necessary and sufficient conditions for the conditional variances to be positive almost surely. In multivariate models, the positivity of the conditional variance is replaced by the positive definiteness of the conditional covariance matrix. Yet, as pointed out by He and Teräsvirta (2002), it is not simple to establish such conditions for the parameters in (1.3). As a result, the past literature assumed a trivial sufficient condition that all the parameters in (1.3) were non-negative. In Chapter 4, “*Positivity Constraints on the Conditional Variances in*

the *Family of Conditional Correlation GARCH Models*,³ it is demonstrated that under certain conditions some of the parameters of the model can take negative values while the conditional covariance matrix remains positive definite almost surely. It is also shown that even in the simplest vector GARCH(1, 1) representation, that is $p = q = 1$ in (1.3), the relevant parameter space can contain negative values for some parameters, which was not possible in the univariate model. This finding makes it possible to incorporate negative volatility spillovers into the CC-GARCH framework⁴. The usefulness of this extension is illustrated by estimating a bivariate extended CCC-GARCH model with negative volatility spillovers with a pair of daily Japanese stock returns.

As is clear from above, many new GARCH models and misspecification testing procedures have been recently proposed in the literature. When it comes to applying these models or tests, however, there do not seem to exist many options for the users to choose from other than creating their own computer programmes. This is especially the case when one wants to apply a multivariate GARCH model.

The last chapter, “**ccgarch**: An R Package for Building Multivariate GARCH Models with Conditional Correlations,” provides a workable environment for building CC-GARCH models. The package is open source, freely available on the Internet, and designed for use in the open source statistical environment R (R Development Core Team, 2009). It can estimate (E)CCC- and (E)DCC-GARCH models as well as simulate data from (E)CCC-, (E)DCC- and (E)STCC-GARCH data generating processes with multivariate normal or Student’s t innovations. In addition, the package is equipped with the necessary functions for conducting diagnostic tests such as those to be discussed in Chapter 3 of this thesis.

A detailed derivation of the ECCC-GARCH model and the relevant statistical tests will follow next.

³This chapter is a joint work with Timo Teräsvirta. It was published in *Finance Research Letters* as Nakatani and Teräsvirta (2008), and is contained in this thesis under the Authors’ rights granted by Elsevier.

⁴Conrad and Karanasos (2009) recently considered similar but more general conditions for the positive definiteness of the conditional covariances of the CC-GARCH model.

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Chapter 2

Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model

Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model^{1†}

Abstract

In this paper we propose a Lagrange multiplier (LM) test for volatility interactions among markets or assets. The null hypothesis is the Constant Conditional Correlation (CCC) GARCH model of Bollerslev (1990) in which volatility of an asset is described only through lagged squared innovations and volatility of its own. The alternative hypothesis is an extension of that model in which volatility is modelled as a linear combination not only of its own lagged squared innovations and volatility but also of those in the other equations while keeping the conditional correlation structure constant. This configuration enables us to test for volatility transmissions among variables in the model. We derive an LM test of the null hypothesis. Monte Carlo experiments show that the proposed test has satisfactory finite sample properties. The size distortions become negligible when the sample size reaches 2500. The test is applied to pairs of foreign exchange returns and individual stock returns. Results indicate that there seem to be volatility interactions in the pairs considered, and that significant interaction effects typically result from the lagged squared innovations of the other variables.

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2.1 Introduction

During the last few decades, considerable attention has been paid to the conditional second moments of financial time series. Models of generalised autoregressive conditional heteroskedasticity (GARCH), either univariate or multivariate, have become standard tools for studying such series. In financial econometrics, the analysis of interdependence in volatility is important for portfolio risk managements on the one hand, and is necessary for research on the degree of market integration on the other. A large number of researchers have found ample evidence that the conditional variances of financial time series are interacting. Such empirical studies include Hamao, Masulis, and Ng (1990), Baillie and Bollerslev (1990), Cheung and Ng (1996), Hong (2001) and Cifarelli and Paladino (2005) among others. For surveys of multivariate GARCH models, see Bauwens, Laurent, and Rombouts (2006) and Silvennoinen and Teräsvirta (2009b).

The Extended Constant Conditional Correlation (ECCC-) GARCH model that Jeantheau (1998) introduced, offers a platform for modelling volatility interactions between markets or assets. The model nests the Constant Conditional Correlation (CCC-) GARCH model by Bollerslev (1990). This extension of the CCC-GARCH model allows the interaction in the form of both lagged squared observations and lagged conditional variances from the other equations of the system. The CCC-GARCH model only allows contemporaneous dependence through conditional correlations, which is not sufficient for volatility interaction.

Since the ECCC-GARCH model nests the CCC-GARCH one, a natural idea would be to test for volatility interactions in the ECCC-GARCH framework. In fact, Wong and Li (1997) and Wong, Li, and Ling (2000) employed the ECCC-GARCH model for describing volatility interactions between the daily Standard and Poor 500 index and the Sydney All Ordinaries index returns, and among three major foreign exchange rates, albeit without first testing the hypothesis of no interactions. In order to save computational efforts, a test of this hypothesis should merely involve estimating the CCC-GARCH model. Estimating the ECCC-GARCH model would become an issue only when the null hypothesis were rejected. Consequently, the aim of this paper is to construct such a test using the score or Lagrange multiplier (LM) principle and to investigate its finite-sample properties.

The existing misspecification tests of the CCC-GARCH model are designed for testing the constancy of correlations. Tse (2000) and Bera and Kim (2002) formed tests of the assumption of constant conditional correlations against an unspecified alternative. Berben and Jansen (2005) and Silvennoinen and Teräsvirta (2005, 2009a) provided LM tests of the CCC-GARCH model against parametric alternatives with time-varying correlations that are variants of the Smooth Transition Conditional Correlation (STCC-) GARCH model. Misspecification of the GARCH structure of this model has received less attention in the literature, and the contribution of this paper will lie in that direction.

The paper is organised as follows. In Section 2.2 the ECCC-GARCH model is defined and its stationarity conditions are mentioned. Section 2.3 contains the first and second partial derivatives of the log-likelihood function of the ECCC-GARCH

model as well as the asymptotic properties of the quasi maximum likelihood estimator of its parameter vector. The LM test is derived in Section 2.4, and its structure is illustrated by a bivariate example in Section 2.5. Finite sample properties of the test are studied by Monte Carlo simulations in Section 2.6. In Section 2.7 the proposed test is applied to pairs of daily foreign exchange returns and daily stock returns. Section 2.8 concludes.

2.2 The extended constant conditional correlation GARCH model

2.2.1 Definition

Following Jeantheau (1998) and He and Teräsvirta (2004), consider the following vector stochastic process:

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t \quad (2.1)$$

$$\boldsymbol{\varepsilon}_t = \mathbf{D}_t \mathbf{z}_t \quad (2.2)$$

where \mathbf{y}_t is a stochastic $(N \times 1)$ vector, $\boldsymbol{\mu}$ is an $(N \times 1)$ intercept vector and $\mathbf{D}_t = \text{diag}(h_{1,t}^{1/2}, \dots, h_{N,t}^{1/2})$ is a diagonal matrix of conditional standard deviations of $\boldsymbol{\varepsilon}_t$. The above formulation is a special case of the vector ARMA-GARCH model in Ling and McAleer (2003). The sequence $\{\mathbf{z}_t\}$ with the stochastic vector $\mathbf{z}_t = [z_{1,t}, \dots, z_{N,t}]'$, is a sequence of independent and identically distributed variables with mean $\mathbf{0}$ and time-invariant positive definite covariance matrix $\mathbf{P} = [\rho_{ij}]$ with ones on the main diagonal. With these assumptions,

$$\mathbb{E}[\boldsymbol{\varepsilon}_t | \mathcal{F}_{t-1}] = \mathbf{0} \quad \text{and} \quad \mathbb{E}[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' | \mathcal{F}_{t-1}] = \mathbf{H}_t = \begin{cases} h_{i,t} & i = j \\ h_{i,t}^{1/2} h_{j,t}^{1/2} \rho_{ij} & i \neq j \end{cases}$$

where \mathcal{F}_t is the information set up to and including time t , and $\mathbf{H}_t = \mathbf{D}_t \mathbf{P} \mathbf{D}_t$. Matrices \mathbf{H}_t and \mathbf{P} are the conditional covariance matrix and the constant conditional correlation matrix of the process $\{\boldsymbol{\varepsilon}_t\}$, respectively.

The vector GARCH(p, q) process of $\boldsymbol{\varepsilon}_t$ is defined as follows:

$$\mathbf{h}_t = [h_{1,t}, \dots, h_{N,t}]' = \mathbf{a}_0 + \sum_{i=1}^q \mathbf{A}_i \boldsymbol{\varepsilon}_{t-i}^{(2)} + \sum_{j=1}^p \mathbf{B}_j \mathbf{h}_{t-j} \quad (2.3)$$

where $\boldsymbol{\varepsilon}_t^{(2)} = (\varepsilon_{1,t}^2, \dots, \varepsilon_{N,t}^2)'$, \mathbf{a}_0 is an $(N \times 1)$ vector, and \mathbf{A}_i and \mathbf{B}_j are $(N \times N)$ matrices with elements such that $h_{i,t}$ in \mathbf{h}_t are positive for all t . The superscript within the parentheses in a vector or a matrix denotes an elementwise exponent. A sufficient condition for \mathbf{h}_t to have positive elements for all t is that all elements in \mathbf{a}_0 are positive and all elements in \mathbf{A}_i and \mathbf{B}_j for each i and j are nonnegative³.

³Nakatani and Teräsvirta (2008) show that off-diagonal elements in \mathbf{B}_j can assume negative values while positive definiteness of \mathbf{H}_t is retained. This extension does not affect the asymptotic distribution of the test statistic in Section 2.4 because off-diagonal elements are all zero under the null hypothesis. Therefore, we maintain the simple sufficient condition throughout this chapter.

This guarantees that, together with the positive definiteness of \mathbf{P} , the conditional variance matrix \mathbf{H}_t is positive definite almost surely for all t . Necessary and sufficient conditions for the positive definiteness of \mathbf{H}_t in the general (p, q) case were recently worked out in Nakatani and Teräsvirta (2008) and Conrad and Karanasos (2009).

Equations (2.1), (2.2) and (2.3) jointly define the N -dimensional Extended CCC (ECCC)-GARCH(p, q) model. Note that if both \mathbf{A}_i and \mathbf{B}_j are diagonal for all i and j , the ECCC-GARCH(p, q) model collapses into the CCC-GARCH(p, q) model of Bollerslev (1990). If, furthermore, $\mathbf{B}_j = \mathbf{0}$, $j = 1, \dots, p$, the model is the CCC-ARCH(q) model of Cecchetti, Cumby, and Figlewski (1988). Wong and Li (1997) applied the ECCC-ARCH(1) model to the S&P 500 and Sydney's All Ordinaries index returns. Wong, Li, and Ling (2000) considered the first-order ECCC-GARCH model with the restriction that \mathbf{B}_1 be diagonal, and applied it to the Hang Seng index and the S&P 500 index.

For simplicity and given the fact that the first-order models describe many heteroskedastic time series well in a vast majority of empirical applications, we restrict our discussion to the case of $p = q = 1$ unless otherwise stated. Excluding $\boldsymbol{\mu}$, there are, in the ECCC-GARCH(1, 1) model, $N(5N + 1)/2$ parameters to be estimated, of which $N(2N + 1)$ parameters appear in \mathbf{h}_t and the remaining $N(N - 1)/2$ in \mathbf{P} .

2.2.2 Stationarity of the ECCC-GARCH process

In the context of vector GARCH processes with constant conditional correlations, a sufficient condition for weak and strict stationarity of an ECCC-GARCH(p, q) process with a constant conditional mean was established in Jeantheau (1998). Ling and McAleer (2003) found a more general condition that allows the process to have an ARMA structure in a conditional mean. He and Teräsvirta (2004) derived a sufficient condition for the ECCC-GARCH(2, 2) process defined by (2.2) and (2.3) to be weakly and strictly stationary. This was done in a fashion different from Jeantheau (1998) and Ling and McAleer (2003). The technique of He and Teräsvirta (2004) can be used for deriving conditions for the existence of the fourth-order moments of this process and its special cases as well as the autocorrelation function of the squared observations.

A sufficient condition for weak and strict stationarity of an ECCC-GARCH(1, 1) model with normal errors is immediate from Theorem 1 of He and Teräsvirta (2004). Define a sequence of i.i.d. stochastic matrices $\{\mathbf{C}_t\}$ such that

$$\mathbf{C}_t = \mathbf{A}_1 \mathbf{Z}_t^2 + \mathbf{B}_1 \quad (2.4)$$

where $\mathbf{Z}_t = \text{diag}(z_{1,t}, \dots, z_{N,t})$. The ECCC-GARCH(1,1) process is weakly and strictly stationary if

$$\lambda(\boldsymbol{\Gamma}_C) < 1 \quad (2.5)$$

where $\boldsymbol{\Gamma}_C = E[\mathbf{C}_t]$ and $\lambda(\boldsymbol{\Gamma})$ is the spectral radius, or the modulus of the largest eigenvalue, of $\boldsymbol{\Gamma}$. If $N = 1$, the inequality (2.5) collapses into the condition for the univariate GARCH(1, 1) process with unit variance to be weakly stationary.

2.3 Likelihood function and asymptotic properties

2.3.1 The log-likelihood function

The aim of this paper is to construct a test for testing the hypothesis that \mathbf{A}_1 and \mathbf{B}_1 are diagonal matrices in the model (2.1), (2.2) and (2.3) with $p = q = 1$. For this we need the (quasi) log-likelihood function of the model and its first two partial derivatives. Without loss of generality, we can assume $\boldsymbol{\mu} = \mathbf{0}$. Let then $\boldsymbol{\theta}' = [\boldsymbol{\omega}', \boldsymbol{\rho}']$ where $\boldsymbol{\omega}$ contains the parameters in \mathbf{h}_t and $\boldsymbol{\rho} = \text{vecl}(\mathbf{P})$. Operator vecl stacks the lower off-diagonal elements of a symmetric $(N \times N)$ matrix into an $N(N-1)/2$ vector. The quasi log-likelihood function for observation t is given by

$$\begin{aligned} \ell_t(\boldsymbol{\theta}) &= -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{D}_t \mathbf{P} \mathbf{D}_t| - \frac{1}{2} \boldsymbol{\varepsilon}_t' \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \\ &= -\frac{N}{2} \ln(2\pi) - \ln |\mathbf{D}_t| - \frac{1}{2} \ln |\mathbf{P}| - \frac{1}{2} \boldsymbol{\varepsilon}_t' \mathbf{H}_t^{-1} \boldsymbol{\varepsilon}_t. \end{aligned} \quad (2.6)$$

The quasi maximum likelihood estimator (QMLE) $\hat{\boldsymbol{\theta}}$ equals

$$\hat{\boldsymbol{\theta}} = \arg \max_{\boldsymbol{\theta}} \sum_{t=1}^T \ell_t(\boldsymbol{\theta}). \quad (2.7)$$

The following assumptions are made to ensure the asymptotic normality of the QMLE $\hat{\boldsymbol{\theta}}$ in (2.7), see Ling and McAleer (2003):

Assumption 2.1 *The spectral radius $\lambda(\mathbf{P})$ has a positive lower bound over the parameter space Θ that is a compact subspace of the Euclidean space such that all true parameters lie in interior of Θ . In addition, each element of \mathbf{a}_0 has positive lower and upper bounds over Θ .*

Assumption 2.2 *All the roots of $\det(\mathbf{I}_N - \sum_{i=1}^q \mathbf{A}_i x^i - \sum_{j=1}^p \mathbf{B}_j x^j)$ lie outside the unit circle where \mathbf{I}_N denotes the identity matrix of order N . Furthermore, $\mathbf{I}_N - \sum_{j=1}^p \mathbf{B}_j L^j$ and $\sum_{i=1}^q \mathbf{A}_i L^i$ are left coprime where L is the lag operator.*

Assumption 2.3 *The identifiability conditions discussed in Jeantheau (1998) are satisfied.*

Assumption 2.4 $E|\varepsilon_{i,t}^6| < \infty$, $i = 1, \dots, N$.

2.3.2 The score and the Hessian of the log-likelihood function

We next define the first and second partial derivatives of (2.6). Let $\mathbf{S}_t(\boldsymbol{\theta}) = \partial \ell_t(\boldsymbol{\theta}) / \partial \boldsymbol{\theta}$ be the score vector for observation t , and let $\bar{\mathbf{S}}(\boldsymbol{\theta}) = (1/T) \sum_{t=1}^T \mathbf{S}_t(\boldsymbol{\theta}) = (1/T) \mathbf{S}(\boldsymbol{\theta})$ be the average score. We use the notation $\mathbf{S}_t(\hat{\boldsymbol{\theta}})$ for the score evaluated at $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$.

The population information matrix is given by the expectation of the outer product of the score evaluated at the true parameter $\boldsymbol{\theta}_0$, that is,

$$\mathcal{I}(\boldsymbol{\theta}_0) = \frac{1}{T} E[\mathbf{S}(\boldsymbol{\theta}_0) \mathbf{S}(\boldsymbol{\theta}_0)'] = E[\mathbf{S}_t(\boldsymbol{\theta}_0) \mathbf{S}_t(\boldsymbol{\theta}_0)']. \quad (2.8)$$

Using the similar notation, the Hessian of the log-likelihood function evaluated at $\boldsymbol{\theta}_0$ equals

$$\mathcal{H}(\boldsymbol{\theta}_0) = \sum_{t=1}^T \mathcal{H}_t(\boldsymbol{\theta}_0) = \sum_{t=1}^T \left. \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'} \right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}. \quad (2.9)$$

We further define the negative of the expected Hessian evaluated at $\boldsymbol{\theta}_0$ by

$$\mathcal{J}(\boldsymbol{\theta}_0) = -\frac{1}{T} \mathbb{E}[\mathcal{H}(\boldsymbol{\theta}_0)] = -\mathbb{E}[\mathcal{H}_t(\boldsymbol{\theta}_0)]. \quad (2.10)$$

The next two lemmas give expressions for $\mathbf{S}_t(\boldsymbol{\theta})$ and $\mathcal{J}(\boldsymbol{\theta})$, namely, the first and second order partial derivatives of (2.6) with respect to the parameters of interest.

Lemma 2.5 (the score vector) *The score vector for observation t of (2.6) has the following form*

$$\mathbf{S}_t(\boldsymbol{\theta}) = \begin{bmatrix} \frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega}} \\ \frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho}} \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} \nabla \mathbf{D}_t \text{vec} (2\mathbf{D}_t^{-1} - \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \mathbf{D}_t^{-1} - \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t') \\ \nabla \mathbf{P} \text{vec} (\mathbf{P}^{-1} - \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1}) \end{bmatrix} \quad (2.11)$$

where

$$\nabla \mathbf{D}_t = \frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \quad \text{and} \quad \nabla \mathbf{P} = \frac{\partial \text{vec}(\mathbf{P})'}{\partial \boldsymbol{\rho}}.$$

Proof. See Appendix 2.A.1. ■

Lemma 2.6 (the negative of the expected Hessian) *The negative of the expected Hessian for observation t has the form*

$$\begin{aligned} \mathcal{J}(\boldsymbol{\theta}) &= -\mathbb{E}[\mathcal{H}_t(\boldsymbol{\theta})] = -\mathbb{E} \begin{bmatrix} \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega} \partial \boldsymbol{\omega}'} & \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho}' \partial \boldsymbol{\omega}} \\ \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega}' \partial \boldsymbol{\rho}} & \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho} \partial \boldsymbol{\rho}'} \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} \nabla \mathbf{D}_t \left\{ 2(\mathbf{D}_t^{-1} \otimes \mathbf{D}_t^{-1}) \right. & \nabla \mathbf{D}_t (\mathbf{D}_t^{-1} \mathbf{P}^{-1} \otimes \mathbf{I}_N \\ \quad \left. + \mathbf{H}_t^{-1} \otimes \mathbf{P} + \mathbf{P} \otimes \mathbf{H}_t^{-1} \right\} \nabla \mathbf{D}_t' & \quad \left. + \mathbf{I}_N \otimes \mathbf{D}_t^{-1} \mathbf{P}^{-1} \right) \nabla \mathbf{P}' \\ \nabla \mathbf{P} (\mathbf{P}^{-1} \mathbf{D}_t^{-1} \otimes \mathbf{I}_N & \nabla \mathbf{P} (\mathbf{P}^{-1} \otimes \mathbf{P}^{-1}) \nabla \mathbf{P}' \\ \quad + \mathbf{I}_N \otimes \mathbf{P}^{-1} \mathbf{D}_t^{-1}) \nabla \mathbf{D}_t' & \end{bmatrix}. \quad (2.12) \end{aligned}$$

where \otimes denotes the Kronecker product of two matrices.

Proof. See Appendix 2.A.2. ■

Expressions (2.11) and (2.12) are rather general in that the conditional variances in \mathbf{D}_t are not specified in detail. For this reason a number of different specifications are possible for \mathbf{h}_t . Eklund and Teräsvirta (2007) derived similar formulas for partial derivatives of the log-likelihood function of a vector autoregressive model with time-varying covariances. Their aim was to test constancy of the error covariance matrix of a vector autoregressive model.

2.3.3 Asymptotic behaviour of the quasi maximum likelihood estimator

The consistency and the asymptotic normality of the QMLE $\hat{\boldsymbol{\theta}}$ were established by Ling and McAleer (2003) for a class of vector ARMA-GARCH models with constant conditional correlations. Since our ECCC-GARCH model defined by (1) through (3) falls into this class, we can make use of their results.

To begin with, it follows from their Lemma 5.2 that, under Assumptions 1 through 3 and the existence of the fourth-order moment of $\{\boldsymbol{\varepsilon}_t\}$,

$$\sqrt{T}\bar{\mathbf{S}}(\boldsymbol{\theta}_0) = \frac{1}{\sqrt{T}}\mathbf{S}(\boldsymbol{\theta}_0) \xrightarrow{D} \mathbf{N}(\mathbf{0}, \mathcal{I}(\boldsymbol{\theta}_0)). \quad (2.13)$$

The asymptotic normality of the score (2.13) serves as the basis for constructing the LM test which will be developed later.

The consistency and the asymptotic normality of the QMLE are proved in Theorems 4.1 and 5.1 of Ling and McAleer (2003). The consistency requires Assumptions 1 through 3 whereas the asymptotic normality additionally involves Assumption 4. The latter results in

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \xrightarrow{D} \mathbf{N}(\mathbf{0}, \mathcal{J}^{-1}(\boldsymbol{\theta}_0)\mathcal{I}(\boldsymbol{\theta}_0)\mathcal{J}^{-1}(\boldsymbol{\theta}_0)). \quad (2.14)$$

If we further assume that $\mathbf{z}_t \sim \mathbf{N}(\mathbf{0}, \mathbf{P})$, (2.6) is an exact log-likelihood function, so that $\mathcal{I}(\boldsymbol{\theta}_0) = -\mathcal{J}(\boldsymbol{\theta}_0)$ holds. It then follows that

$$\sqrt{T}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \xrightarrow{D} \mathbf{N}(\mathbf{0}, \mathcal{I}^{-1}(\boldsymbol{\theta}_0)). \quad (2.15)$$

In either case, Lemmas 5.5 and 5.6 of Ling and McAleer (2003) show that $\mathcal{I}(\boldsymbol{\theta}_0)$ and $\mathcal{J}(\boldsymbol{\theta}_0)$ can be consistently estimated by

$$\mathcal{I}(\hat{\boldsymbol{\theta}}) = \frac{1}{T} \sum_{t=1}^T \mathbf{s}_t(\hat{\boldsymbol{\theta}})\mathbf{s}_t(\hat{\boldsymbol{\theta}})' \quad (2.16)$$

and by

$$\mathcal{J}(\hat{\boldsymbol{\theta}}) = -\frac{1}{T} \sum_{t=1}^T \mathcal{H}_t(\hat{\boldsymbol{\theta}}), \quad (2.17)$$

respectively.

2.3.4 The fourth-order moment condition

As we saw in the previous subsection, the existence of the fourth- and the sixth-order moments of $\{\boldsymbol{\varepsilon}_t\}$ is necessary in developing the asymptotic theory for the quasi maximum likelihood estimator of the ECCC-GARCH model. However, finding these conditions, in particular those for the sixth-order moment, seems an involved task.

It seems that the only available results are in Ling and McAleer (2003) and He and Teräsvirta (2004). Their results are general in the sense that the distribution of \mathbf{z}_t need not be normal.

To introduce notation, let $\mathbf{a} = (a_1, \dots, a_N)'$ be an $(N \times 1)$ vector and define the $(N \times N)$ diagonal matrix $\text{diag}(\mathbf{a}) = \text{diag}(a_1, \dots, a_N)$. Operator diag creates the $(N \times N)$ diagonal matrix from an $(N \times 1)$ vector. Then under the assumption of normality we have from Corollary 2 in He and Teräsvirta (2004) the following sufficient condition for the existence of the unconditional fourth-moment matrix $\mathbf{E}[\boldsymbol{\varepsilon}_t^{(2)} \boldsymbol{\varepsilon}_t^{(2)'}]$ of $\{\boldsymbol{\varepsilon}_t\}$ generated by an ECCC-GARCH(p, q) process:

Lemma 2.7 *Assume that (2.5) holds, and that $\mathbf{z}_t \sim \mathbf{N}(\mathbf{0}, \mathbf{P})$. Then $\mathbf{E}[\boldsymbol{\varepsilon}_t^{(2)} \boldsymbol{\varepsilon}_t^{(2)'}]$ exists if*

$$\lambda(\boldsymbol{\Gamma}_{C \otimes C}) < 1 \quad (2.18)$$

where

$$\begin{aligned} \boldsymbol{\Gamma}_{C \otimes C} &= \mathbf{E}[\mathbf{C}_t \otimes \mathbf{C}_t] \\ &= (\mathbf{A}_1 + \mathbf{B}_1) \otimes (\mathbf{A}_1 + \mathbf{B}_1) + 2(\mathbf{A}_1 \otimes \mathbf{A}_1) \text{diag}(\text{vec}(\mathbf{P}) \odot \text{vec}(\mathbf{P})) \end{aligned} \quad (2.19)$$

where \odot denotes the Hadamard (elementwise) product of two matrices or vectors.

Proof. See Appendix 2.A.3. ■

2.4 Test for volatility interactions

Assuming $p = q = 1$, we now construct an LM test for volatility interactions with the hypothesis

$$H_0 : \mathbf{A}_1 \text{ and } \mathbf{B}_1 \text{ in (2.3) are diagonal matrices}$$

against the alternative

$$H_1 : \text{either } \mathbf{A}_1 \text{ or } \mathbf{B}_1 \text{ or both are non-diagonal matrices.}$$

We denote the test statistic LM_{ECCC} . The null hypothesis defines a CCC-GARCH(1, 1) model, and the alternative is an ECCC-GARCH(1, 1) model. The test may best be viewed as a test of no volatility interaction among the variables in the model, while conditional correlations between them are allowed.

Let $\tilde{\boldsymbol{\theta}} = [\tilde{\boldsymbol{\omega}}', \tilde{\boldsymbol{\rho}}']'$ be the ML estimator of $\boldsymbol{\theta}$ under the null. Since $\boldsymbol{\rho}$ is a vector of nuisance parameters, the average score evaluated at $\tilde{\boldsymbol{\theta}}$ equals

$$\bar{\mathbf{S}}(\tilde{\boldsymbol{\theta}}) = \frac{1}{T} \sum_{t=1}^T \left[\begin{array}{c} \frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega}} \\ \frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho}} \end{array} \bigg|_{\boldsymbol{\theta}=\tilde{\boldsymbol{\theta}}} \right] = \left[\begin{array}{c} \bar{\mathbf{S}}_{\boldsymbol{\omega}}(\tilde{\boldsymbol{\theta}}) \\ \mathbf{0}_M \end{array} \right] \quad (2.20)$$

where $\mathbf{0}_M$ denotes an $(M \times 1)$ null vector with $M = N(N-1)/2$. $\bar{\mathbf{S}}_{\boldsymbol{\omega}}(\tilde{\boldsymbol{\omega}})$ has $(2N^2 + N)$ elements, of which the ones corresponding to the other nuisance parameters \mathbf{a}_0 and the diagonal elements in \mathbf{A}_1 and \mathbf{B}_1 , are equal to zero. To keep the notation tractable, we leave these $3N$ zero elements in $\bar{\mathbf{S}}_{\boldsymbol{\omega}}(\tilde{\boldsymbol{\theta}})$ and do not define a separate block for them.

As already mentioned, the information matrix can be consistently estimated either by (2.16) or (2.17). Due to the fact that the score under the null (2.20) has zero elements, we only require the relevant part of the inverse of the information matrix to derive the LM_{ECCC} statistic. Applying the formula for the inverse of a partitioned matrix to (2.17), the inverse of the relevant block equals $\mathcal{J}_{11}^{-1}(\tilde{\theta}) = (\tilde{\mathbf{J}}_{11} - \tilde{\mathbf{J}}_{12}\tilde{\mathbf{J}}_{22}^{-1}\tilde{\mathbf{J}}'_{12})^{-1}$ where

$$\mathcal{J}(\tilde{\theta}) = \begin{bmatrix} \tilde{\mathbf{J}}_{11} & \tilde{\mathbf{J}}_{12} \\ \tilde{\mathbf{J}}'_{12} & \tilde{\mathbf{J}}_{22} \end{bmatrix}. \quad (2.21)$$

In (2.21), $\tilde{\mathbf{J}}_{11}$ and $\tilde{\mathbf{J}}_{22}$ correspond to the second partial derivatives with respect only to ω and to ρ , respectively, and $\tilde{\mathbf{J}}_{12}$ contains the cross derivatives, all evaluated at $\theta = \tilde{\theta}$. The partitioning in (2.21) corresponds to the one in (2.12).

We are now able to state the main result:

Theorem 2.8 (the LM test statistic) *Let Assumptions 1–3 hold and assume that the fourth-order moment matrix of $\{\varepsilon_t\}$ exists. Then, the LM test statistic of testing H_0 , given by the quadratic form*

$$\begin{aligned} \text{LM}_{ECCC} &= T\bar{\mathbf{S}}'(\tilde{\theta})\mathcal{J}^{-1}(\tilde{\theta})\bar{\mathbf{S}}(\tilde{\theta}) \\ &= T\bar{\mathbf{S}}'_{\omega}(\tilde{\theta})\mathcal{J}_{11}^{-1}(\tilde{\theta})\bar{\mathbf{S}}_{\omega}(\tilde{\theta}) \end{aligned} \quad (2.22)$$

has an asymptotic χ^2 distribution with $2N(N-1)$ degrees of freedom when the null hypothesis is valid.

2.5 Bivariate illustration of the test statistic

Definition (2.22) of the LM_{ECCC} statistic is quite general and therefore not very illuminating. Because of this, we shall illustrate the structure of the statistic by a bivariate example. First we set up the model, then apply Theorem 2.8 to obtain the LM_{ECCC} statistic for the bivariate model.

2.5.1 The bivariate ECCG-GARCH(1, 1) model and the partial derivatives of the conditional variance equations

The bivariate ECCG-GARCH(1, 1) model has its conditional variance equation (2.3) of the form

$$\begin{aligned} \mathbf{h}_t &= \begin{bmatrix} h_{1,t} \\ h_{2,t} \end{bmatrix} = \mathbf{a}_0 + \mathbf{A}_1\varepsilon_{t-1}^{(2)} + \mathbf{B}_1\mathbf{h}_{t-1} \\ &= \begin{bmatrix} a_{10} \\ a_{20} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} h_{1,t-1} \\ h_{2,t-1} \end{bmatrix} \\ &= \begin{bmatrix} a_{10} + a_{11}\varepsilon_{1,t-1}^2 + a_{12}\varepsilon_{2,t-1}^2 + b_{11}h_{1,t-1} + b_{12}h_{2,t-1} \\ a_{20} + a_{21}\varepsilon_{1,t-1}^2 + a_{22}\varepsilon_{2,t-1}^2 + b_{21}h_{1,t-1} + b_{22}h_{2,t-1} \end{bmatrix}. \end{aligned} \quad (2.23)$$

To compute the LM_{ECCC} statistic, we require the partial derivatives of (2.23) with respect to parameters therein. Let $\boldsymbol{\omega} = [\boldsymbol{\omega}'_1, \boldsymbol{\omega}'_2]'$ where $\boldsymbol{\omega}_1 = [a_{10} \ a_{11} \ a_{12} \ b_{11} \ b_{12}]'$ and $\boldsymbol{\omega}_2 = [a_{20} \ a_{21} \ a_{22} \ b_{21} \ b_{22}]'$, and let $\mathbf{v}_t = [1 \ \varepsilon_{1,t}^2 \ \varepsilon_{2,t}^2 \ h_{1,t} \ h_{2,t}]'$. The partial derivatives of $h_{1,t}$ with respect to $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ are

$$\frac{\partial h_{1,t}}{\partial \boldsymbol{\omega}_1} = \mathbf{v}_{t-1} + b_{11} \frac{\partial h_{1,t-1}}{\partial \boldsymbol{\omega}_1} + b_{12} \frac{\partial h_{2,t-1}}{\partial \boldsymbol{\omega}_1} \quad (2.24)$$

and

$$\frac{\partial h_{1,t}}{\partial \boldsymbol{\omega}_2} = b_{11} \frac{\partial h_{1,t-1}}{\partial \boldsymbol{\omega}_2} + b_{12} \frac{\partial h_{2,t-1}}{\partial \boldsymbol{\omega}_2}. \quad (2.25)$$

The partial derivatives of $h_{2,t}$ with respect to $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ have a similar representation. Under the null hypothesis, \mathbf{A}_1 and \mathbf{B}_1 are jointly diagonal, *i.e.*,

$$H_0 : \boldsymbol{\omega}_1 = [a_{10} \ a_{11} \ 0 \ b_{11} \ 0] \quad \text{and} \quad \boldsymbol{\omega}_2 = [a_{20} \ 0 \ a_{22} \ 0 \ b_{22}]'. \quad (2.26)$$

Evaluating both (2.24) and (2.25) at $\boldsymbol{\omega}_1 = \tilde{\boldsymbol{\omega}}_1$ and $\boldsymbol{\omega}_2 = \tilde{\boldsymbol{\omega}}_2$, the ML estimator under H_0 , yields

$$\frac{\partial \tilde{h}_{1,t}}{\partial \boldsymbol{\omega}_1} = \tilde{\mathbf{v}}_{t-1} + \tilde{b}_{11} \frac{\partial \tilde{h}_{1,t-1}}{\partial \boldsymbol{\omega}_1} \quad \text{and} \quad \frac{\partial \tilde{h}_{1,t}}{\partial \boldsymbol{\omega}_2} = \mathbf{0}. \quad (2.27)$$

Similarly,

$$\frac{\partial \tilde{h}_{2,t}}{\partial \boldsymbol{\omega}_1} = \mathbf{0} \quad \text{and} \quad \frac{\partial \tilde{h}_{2,t}}{\partial \boldsymbol{\omega}_2} = \tilde{\mathbf{v}}_{t-1} + \tilde{b}_{22} \frac{\partial \tilde{h}_{2,t-1}}{\partial \boldsymbol{\omega}_2}. \quad (2.28)$$

Non-zero elements in (2.27) and (2.28) are

$$\frac{\partial \tilde{h}_{i,t}}{\partial a_{ij}} = \tilde{\varepsilon}_{j,t-1}^2 + \tilde{b}_{ii} \frac{\partial \tilde{h}_{i,t-1}}{\partial a_{ij}} \quad \text{and} \quad \frac{\partial \tilde{h}_{i,t}}{\partial b_{ij}} = \tilde{h}_{j,t-1} + \tilde{b}_{ii} \frac{\partial \tilde{h}_{i,t-1}}{\partial b_{ij}}.$$

For these recursions to be tractable, initial values are needed. Therefore, we set $\tilde{\boldsymbol{\varepsilon}}_0^{(2)} = \tilde{\mathbf{h}}_0 = (1/T) \sum_{t=1}^T \tilde{\boldsymbol{\varepsilon}}_t^{(2)}$, and $\partial \tilde{h}_{i,0} / \partial \boldsymbol{\omega}_j = \mathbf{0}$, $i, j = 1, 2$, following the suggestion by Fiorentini, Calzolari, and Panattoni (1996). The recursions of the non-zero elements in (2.27) and (2.28) then proceed as

$$\frac{\partial \tilde{h}_{i,1}}{\partial a_{ij}} = \tilde{\varepsilon}_{j,0}^2, \quad \frac{\partial \tilde{h}_{i,1}}{\partial b_{ij}} = \tilde{h}_{j,0} \quad \text{for } t = 1, \quad (2.29)$$

and

$$\frac{\partial \tilde{h}_{i,t}}{\partial a_{ij}} = \tilde{\varepsilon}_{j,t-1}^2 + \tilde{b}_{ii} \tilde{\varepsilon}_{j,t-2}^2, \quad \frac{\partial \tilde{h}_{i,t}}{\partial b_{ij}} = \tilde{h}_{j,t-1} + \tilde{b}_{ii} \tilde{h}_{j,t-2} \quad \text{for } t > 1 \quad (2.30)$$

where \tilde{b}_{ii} , $\tilde{\varepsilon}_{j,t-1}^2$ and $\tilde{h}_{j,t-1}$ in (2.30) are estimated from the null model.

2.5.2 Components of the test statistic

We shall now provide analytical expressions for the components of LM_{ECCC} in a bivariate framework and begin by for $\bar{\mathbf{S}}(\boldsymbol{\theta})$ and $\mathcal{J}(\boldsymbol{\theta})$. To simplify notation, set $\mathbf{k}_{ij,t} = h_{i,t}^{-1} \partial h_{i,t} / \partial \boldsymbol{\omega}_j$ and $\tilde{\mathbf{k}}_{ij,t} = \tilde{h}_{i,t}^{-1} \partial \tilde{h}_{i,t} / \partial \boldsymbol{\omega}_j$, $i, j = 1, 2$. Then we have the following corollaries regarding the score and the relevant part of the Hessian.

Corollary 2.9 *In the bivariate case, the average score has the form*

$$\begin{aligned} \bar{\mathbf{S}}(\boldsymbol{\theta}) &= \begin{bmatrix} \bar{\mathbf{S}}_{\omega_1}(\boldsymbol{\theta}) \\ \bar{\mathbf{S}}_{\omega_2}(\boldsymbol{\theta}) \\ \bar{\mathbf{S}}_{\rho}(\rho) \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{2T} \sum \left[\mathbf{k}_{11,t} \left\{ 1 - \frac{1}{(1-\rho^2)} z_{1,t} (z_{1,t} - \rho z_{2,t}) \right\} + \mathbf{k}_{21,t} \left\{ 1 - \frac{1}{(1-\rho^2)} z_{2,t} (z_{2,t} - \rho z_{1,t}) \right\} \right] \\ -\frac{1}{2T} \sum \left[\mathbf{k}_{12,t} \left\{ 1 - \frac{1}{(1-\rho^2)} z_{1,t} (z_{1,t} - \rho z_{2,t}) \right\} + \mathbf{k}_{22,t} \left\{ 1 - \frac{1}{(1-\rho^2)} z_{2,t} (z_{2,t} - \rho z_{1,t}) \right\} \right] \\ \frac{\rho}{(1-\rho^2)} - \frac{\rho}{T(1-\rho^2)^2} \sum (z_{1,t}^2 + z_{2,t}^2) + \frac{(1+\rho^2)}{T(1-\rho^2)^2} \sum z_{1,t} z_{2,t} \end{bmatrix} \end{aligned} \quad (2.31)$$

where \sum denotes the summation from $t = 1$ to T . Under H_0 , $\bar{\mathbf{S}}_{\rho}(\tilde{\rho}) = 0$ so that

$$\bar{\mathbf{S}}(\tilde{\boldsymbol{\theta}}) = \begin{bmatrix} \bar{\mathbf{S}}_{\omega_1}(\tilde{\boldsymbol{\theta}}) \\ \bar{\mathbf{S}}_{\omega_2}(\tilde{\boldsymbol{\theta}}) \\ 0 \end{bmatrix} = -\frac{1}{2T} \begin{bmatrix} \sum \tilde{\mathbf{k}}_{11,t} \left\{ 1 - \frac{1}{(1-\tilde{\rho}^2)} \tilde{z}_{1,t} (\tilde{z}_{1,t} - \tilde{\rho} \tilde{z}_{2,t}) \right\} \\ \sum \tilde{\mathbf{k}}_{22,t} \left\{ 1 - \frac{1}{(1-\tilde{\rho}^2)} \tilde{z}_{1,t} (\tilde{z}_{2,t} - \tilde{\rho} \tilde{z}_{1,t}) \right\} \\ 0 \end{bmatrix}. \quad (2.32)$$

Remark 2.10 *In (2.32), only its third, fifth, seventh and ninth elements do not equal zero. These non-zero elements correspond to the zero restriction in (2.26). Accounting explicitly for this would complicate the notation and is therefore not done here.*

Corollary 2.11 *The relevant upper left block of $\mathcal{J}^{-1}(\boldsymbol{\theta})$ equals $\mathcal{J}_{11}^{-1}(\boldsymbol{\theta}) = (\mathbf{J}_{11} - \mathbf{J}_{12} \mathbf{J}_{22}^{-1} \mathbf{J}'_{12})^{-1}$ where*

$$\mathbf{J}_{11} = \begin{bmatrix} \frac{2-\rho^2}{4T(1-\rho^2)} \sum \mathbf{k}_{11,t} \mathbf{k}'_{11,t} & -\frac{\rho^2}{4T(1-\rho^2)} \sum \mathbf{k}_{11,t} \mathbf{k}'_{22,t} \\ -\frac{\rho^2}{4T(1-\rho^2)} \sum \mathbf{k}_{22,t} \mathbf{k}'_{11,t} & \frac{2-\rho^2}{4T(1-\rho^2)} \sum \mathbf{k}_{22,t} \mathbf{k}'_{22,t} \end{bmatrix} \quad (2.33)$$

$$\mathbf{J}_{12} = \begin{bmatrix} -\frac{\rho}{2T(1-\rho^2)} \sum \mathbf{k}_{11,t} \\ -\frac{\rho}{2T(1-\rho^2)} \sum \mathbf{k}_{22,t} \end{bmatrix} \quad (2.34)$$

and

$$\mathbf{J}_{22} = \frac{1 + \rho^2}{(1 - \rho^2)^2}. \quad (2.35)$$

Consequently,

$$\mathcal{J}_{11}(\boldsymbol{\theta}) = \frac{1}{4T} \begin{bmatrix} \frac{2-\rho^2}{(1-\rho^2)} (\sum \mathbf{k}_{11,t} \mathbf{k}'_{11,t}) & -\frac{\rho^2}{(1-\rho^2)} (\sum \mathbf{k}_{11,t} \mathbf{k}'_{22,t}) \\ -\frac{\rho}{T(1+\rho^2)} (\sum \mathbf{k}_{11,t}) (\sum \mathbf{k}'_{11,t}) & -\frac{\rho}{T(1+\rho^2)} (\sum \mathbf{k}_{11,t}) (\sum \mathbf{k}'_{11,t}) \\ -\frac{\rho^2}{(1-\rho^2)} (\sum \mathbf{k}_{22,t} \mathbf{k}'_{11,t}) & \frac{2-\rho^2}{(1-\rho^2)} (\sum \mathbf{k}_{22,t} \mathbf{k}'_{22,t}) \\ -\frac{\rho}{T(1+\rho^2)} (\sum \mathbf{k}_{22,t}) (\sum \mathbf{k}'_{11,t}) & -\frac{\rho}{T(1+\rho^2)} (\sum \mathbf{k}_{22,t}) (\sum \mathbf{k}'_{22,t}) \end{bmatrix}. \quad (2.36)$$

Finally, replacing the true parameters and relevant terms with the ML estimators under the null, namely $\boldsymbol{\theta}$ with $\tilde{\boldsymbol{\theta}}$, \mathbf{z}_t with $\tilde{\mathbf{z}}_t$, and $\mathbf{k}_{ij,t}$ with $\tilde{\mathbf{k}}_{ij,t}$, $i, j = 1, 2$, gives the LM_{ECCC} statistic (2.22). The statistic in the bivariate case has an asymptotic χ^2 distribution with four degrees of freedom when the null hypothesis is valid. In practice, we are able to compute the test statistic numerically through the relevant expressions.

2.6 Simulation experiments of the LM_{ECCC} test statistic

In this section we conduct simulation experiments in a bivariate case to see how the proposed test behaves in finite samples. In both size and power simulations, we use the sample sizes $T = 1000, 2500, 5000$ and 10000 for each data generating process (DGP). Additional experiments are carried out to see how the test performs when we remove the assumption of constant conditional correlation. To minimise initial effects, the first 500 observations generated are discarded. The number of replications equals 5000. The empirical rejection frequencies are compared with the nominal significance levels 5% and 10%. All numerical calculations are carried out in the free statistical environment R 2.3.1 (R Development Core Team, 2005) with the author's own source codes⁴.

2.6.1 Size simulations

The size simulations are carried out for five different CCC-GARCH data generating processes (DGPs) whose parameter values can be found in Table 2.1. We set the intercept term in the conditional variance equation equal to $\mathbf{a} = [0.1 \ 0.2]'$. The DGPs have the following dynamic properties. DGP 1 has moderate persistence in volatility ($a_{ii} + b_{ii} = 0.9$) with low correlation ($\rho = 0.3$) in \mathbf{z}_t . The parameter values for \mathbf{A}_1 and \mathbf{B}_1 are common to both DGPs 2 and 3 with very high persistence ($a_{ii} + b_{ii} = 0.99, 0.95$ for $i = 1, 2$). DGP 2 has high correlation ($\rho = 0.9$) whereas DGP 3 has low correlation ($\rho = 0.3$). DGPs 4 and 5 have low persistence in volatility with high and low correlations, respectively. The parameter values satisfy the stationarity condition (2.5) and the fourth-order moment condition (2.18).

⁴R is freely available at <http://www.r-project.org> for various operating systems. In producing source codes, we used C routines from *GNU Scientific Library* (GSL) ver. 1.8.

The actual rejection frequencies are reported in Panel (a) of Table 2.2 for the nominal 5% level and for the nominal 10% level in Panel (b), respectively. As may be expected, the rejection frequencies are approaching the nominal significance levels as the number of observations increases. There are size distortions, however, in DGPs 1 through 3 when $T \leq 5000$. In DGPs 4 and 5, the actual sizes are close to the nominal size already when $T = 1000$, which is a modest number in many GARCH applications.

The size properties of the test suggest that at least a couple of thousands observations are required for empirical analyses. This requirement does not appear an obstacle for implementing the test since these amounts of observations are readily available in financial time series. However, our results are only valid for bivariate models, and longer series may be needed for higher-dimensional processes.

2.6.2 Power simulations

The DGPs for the power simulations are listed in Table 2.3. The weak stationarity and the fourth-order moment conditions are satisfied for all DGPs. DGP 6 has high persistence in volatility ($a_{ii} + b_{ii} = 0.99, 0.93$ for $i = 1, 2$) and a moderate correlation coefficient ($\rho = 0.7$). In DGP 7, one of the off-diagonal coefficients has a large value ($b_{21} = 0.02$) with high persistence as in DGP6. DGP 8 has a design similar to DGP 7 but the off-diagonal elements have a small value ($a_{ij} = b_{ij} = 0.001$). DGPs 9 and 10 have a rather unusual structure in the sense that both DGPs have moderate persistence and large values for all off-diagonal elements.

The results are summarised in Table 2.4. It can be seen that the power of the test is low for DGPs 6 and 8 for all T . This is expected, however, because the true parameters under test are close to zero. Despite that, small changes in the value of a_{21} and b_{21} may already bring an increase in the power of the test. In all other cases, the power reaches a reasonable level as T increases. In DGPs 7 the power is already high for $T = 1000$, and the same is true for DGP 9 when $T = 2500$. DGP 10 with less variable conditional variances constitutes an exception: the power is not yet high for $T = 5000$. Obtaining higher power for ECCC-GARCH processes such as DGP 10 requires a sample of 10000 observations, which corresponds to a daily time series of about 40 years of data. It should be noted, however, that observed time series are typically quite different from realisations generated by such DGPs.

2.6.3 The test under changing conditional correlations

We conduct size and power simulations under changing conditional correlations using the DCC- and the STCC-GARCH models and their extended versions (abbreviated by the EDCC- and ESTCC-GARCH, respectively). The definitions of the DGPs can be found in Table 2.5. For the EDCC- and the ESTCC-GARCH processes, the simulated design is such that both processes have the same parameter matrices in the conditional variance equation as the DGP7. In the STCC and the ESTCC processes, the exogenous transition variable s_t is driven by a GARCH(1, 1) process following Silvennoinen and Teräsvirta (2005).

The actual rejection frequencies based on the 5% and 10% levels of significance can be found in Table 2.6. The size of the test is distorted when the data are generated by the DCC-GARCH process, but the degree of distortion is not a monotonically increasing function of T . It is also worth emphasising that when the DGP is an STCC-GARCH model, the size of the test converges towards the nominal level when T increases. Overall, the results suggest that the power of the test does lie in the non-zero off-diagonal elements of the parameter matrices, and that non-constancy of the conditional correlation matrix only has a minor role to play in rejecting the null hypothesis. Therefore, the test seems reasonably applicable to the models with changing conditional correlations.

2.7 Applications to daily return series

2.7.1 Data

In this section, the LM_{ECCC} statistic is applied to pairs of foreign exchange rates as well as stock prices. Before fitting GARCH models, the level series are first transformed to the continuously compounded rate of returns by $100 \ln(p_t/p_{t-1})$, whereafter the sample means of returns are subtracted from the series.

The considerations include daily foreign exchange rates and stock price series. The exchange rates are daily noon buying rates in New York of the Japanese yen (JPY) and the Swiss franc (CHF) against the U.S. dollars certified by the Federal Reserve Bank of New York⁵. The foreign exchange rate series extend from 2 January 1975 to 2 December 2005, with the total of 7766 observations in each series. Fig. 2.1 depicts the level and the mean-subtracted continuously compounded return series for the two exchange rates. The stock prices are the daily closing prices of General Motors (GM) and IBM traded at the New York Stock Exchange, and of two Japanese leading electronic firms, NEC and Hitachi, traded at the Tokyo Stock Exchange. The sample stretches from 2 January 1962 to 28 February 2006 with 11116 observations for the U.S. stock data, and from 4 January 1983 to 1 March 2006 with 5914 observations for the Japanese data⁶. The levels and the demeaned continuously compounded returns are plotted in Figs. 2.2 and 2.3.

Descriptive statistics for all return series can be found in Table 2.7. As is typical for many financial time series, one can see strong excess kurtosis (KR) and non-zero skewness (SK). The Japanese stock returns are positively skewed, whereas the U.S. stocks and the foreign exchange returns contain negative skewness. The Lomnicki-Jarque-Bera (LJB) test of normality suggests non-normality of the return series. The McLeod-Li (1983) test indicates that higher-order dependence is present in the return series. As mentioned in Section 2.3.3, one can still obtain the quasi-maximum likelihood estimates of the parameters even when the data do not follow a multivariate normal distribution. An alternative method is to use a leptokurtic distribution such

⁵The data are downloadable from *Economic Research and Data* section at the internet page of the Board of Governors of the Federal Reserve System (<http://www.federalreserve.gov/>).

⁶The data are downloaded from *Yahoo! Finance* and *Yahoo! Japan Finance* for the U.S. and Japanese data, respectively.

as a multivariate Student's t density to take into account the non-normality of the data, see for example Kawakatsu (2006). However, the outlier-robust versions of the skewness (Rob.SK) and the excess kurtosis (Rob.KR) measures described in Kim and White (2004) yield different results. In particular, the robust excess kurtosis values are much smaller than the non-robust ones. These outcomes strongly suggest that large values of the standard measures of skewness and kurtosis and, consequently, those of the LJB normality statistic, are caused by a small number of outliers. Apart from them, the marginal distributions of the returns do not appreciably deviate from the normal distribution.

2.7.2 Results of CCC/ECCC-GARCH estimation

Given the fact that the robust skewness and kurtosis measures do not contain much evidence against normality of the return distributions, we assume that $\mathbf{z}_t \sim \mathbf{N}(\mathbf{0}, \mathbf{P})$ in the applications. This assumption is supported by the descriptive statistics of the standardised residuals reported in Table 2.8. According to the conventional measures of skewness and kurtosis, the standardised residuals are skewed and have fat tails. In contrast, the robustified statistics, Rob.SK and Rob.KR, do not suggest non-normality.

Estimation of parameters is carried out with the nonlinear optimisation function “optim” in R 2.3.1. We use the BFGS algorithm for the optimisation. Convergence is typically achieved after a few hundred iterations. During iterations, only the positivity constraints on \mathbf{a}_0 , \mathbf{A}_1 and \mathbf{B}_1 are imposed to alleviate numerical difficulties in parameter restrictions. The weak stationarity and the fourth-order moment conditions are checked *ex post*. All the estimation results are summarised in Panels A, B and C of Table 2.9. Within each panel, the upper half contains the results of the CCC-GARCH model, whereas the lower half has to do with the ECCC-GARCH model

Foreign exchange returns

The estimation results of the CCC-GARCH(1, 1) and the ECCC-GARCH(1, 1) model fitted to the foreign exchange series appear in Panel A of Table 2.9. Values of the LM_{ECCC} statistic are reported in the third column from the right. In Panel A (the pair of JPY and CHF returns) the test statistic has the p-value 3×10^{-6} , so the null hypothesis of no interaction is rejected at any conventional significance level. The estimates of the ECCC-GARCH(1, 1) model conform to this outcome. It is seen from Panel A that both $\hat{a}_{12} = 0.0038$ and $\hat{b}_{21} = 0.0080$ deviate significantly from zero at conventional levels of significance. This means that the lagged volatility in JPY has a positive effect on the current day's volatility in CHF, and that the squared innovation of CHF of day $t - 1$ has a positive influence on the volatility of JPY at day t .

Although in both cases a statistically significant spillover exists, the magnitude of the contributions of one currency to the other is small. Accordingly, the estimates of the conditional variance from the two models are similar to each other (see Fig. 2.4), and the same is true for the estimates of the conditional correlation coefficients.

Furthermore, both models satisfy the conditions for the stationarity and the existence of the unconditional fourth moments.

Stock returns

The estimation results of the models applied to the pair of the U.S. and another pair of Japanese stock returns can be found in Panels B and C of Table 2.9. The values of the LM_{ECCC} statistics are large enough to reject the null hypothesis of no volatility interaction for both pairs of returns. None of the estimated off-diagonal elements in $\hat{\mathbf{B}}_1$ is statistically significant, whereas both off-diagonal estimates in $\hat{\mathbf{A}}_1$ are significant in the equation for the two U.S. stocks, and $\hat{a}_{12} = 0.0132$ is significant in the one for the Japanese pair of returns. Interestingly, the magnitude of the significant off-diagonal elements in $\hat{\mathbf{A}}_1$ is not much less than that of the diagonal elements. By comparison, it is much larger than in the exchange rate examples. The dynamic behaviour of volatility in stock returns is affected by the lagged squared innovation but not by the lagged conditional variance of the other asset. The estimate of the conditional correlation coefficient is positive and significant for both pairs of returns.

The diagonal elements of $\hat{\mathbf{B}}_1$ are smaller in magnitude in the ECCC-GARCH model than in the CCC-GARCH one. This may be a consequence of the nonzero off-diagonal elements of $\hat{\mathbf{A}}_1$ in the ECCC-GARCH model. However, even here the changes in parameter estimates have left the estimated conditional variances and correlations unaffected. The graphs of the estimated conditional variances in Figs. 2.5 and 2.6 are similar for the extended and the standard CCC-GARCH model. Finally, the estimated stock return models satisfy the conditions for the stationarity and the existence of the fourth moments.

2.8 Concluding remarks

In this paper we propose an LM test for detecting the presence of volatility interactions or transmission in the context of the CCC-GARCH model. Simulation experiments indicate that the test statistic has favourable finite sample properties. Its empirical size is typically close to the nominal one for time series with over a couple of thousands observations. The test also has reasonable power, although, as usual, counterexamples can be found through some extreme parameterisations. According to the power simulations under changing conditional correlations, the null of no interaction in volatility is not often rejected when DGPs have no volatility interaction. This indicates that the test is reasonably robust against time-varying conditional correlations.

All three pairs of daily return series analysed in the paper seem to have volatility interactions. In the exchange rate example, the interactions appear through the lagged conditional variance, whereas in the two ECCC-GARCH models for pairs of stock returns the lagged squared innovations form the channel for interactions. Although the interaction effects found are small, they are detected by the tests. It may thus not be a good idea to exclude such interactions a priori, which makes our test useful. Since it also works reasonably well in the case of time-varying conditional correlations, the test is a practical tool in modelling volatility in multivariate financial time series.

The test is derived assuming constant conditional correlations, which is not always a realistic restriction. Although the test seems applicable to the models with changing conditional correlations, it is of interest to extend the current test to cover the CC-GARCH models with time-varying conditional correlations. Such extensions, however, are left for future research.

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2.A Mathematical derivations and proofs

Throughout this appendix, vector and matrix derivatives, and some properties of special matrices are intensively used. These results can be found in Lütkepohl (1996) and Magnus and Neudecker (1998). See also Eklund and Teräsvirta (2007).

2.A.1 Proof of Lemma 2.5

Since $\boldsymbol{\theta}' = [\boldsymbol{\omega}', \boldsymbol{\rho}']$, the partial derivative of the log-likelihood function for observation t with respect to $\boldsymbol{\theta}$ is partitioned into

$$\frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \begin{bmatrix} \frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega}} \\ \frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho}} \end{bmatrix}. \quad (2.37)$$

The upper block in (2.37) has $N(2N+1)$ entries while the lower one has $N(N-1)/2$ elements.

For the upper block in (2.37), we can use the chain rules of vector derivatives to have

$$\begin{aligned} \frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega}} &= -\frac{\partial \ln |\mathbf{D}_t|}{\partial \boldsymbol{\omega}} - \frac{1}{2} \frac{\partial (\boldsymbol{\varepsilon}_t' \mathbf{H}_t^{-1} \boldsymbol{\varepsilon}_t)}{\partial \boldsymbol{\omega}} \\ &= -\frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \frac{\partial \ln |\mathbf{D}_t|}{\partial \text{vec}(\mathbf{D}_t)} - \frac{1}{2} \frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \frac{\partial \text{vec}(\mathbf{D}_t^{-1})'}{\partial \text{vec}(\mathbf{D}_t)} \frac{\partial \text{vec}(\mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1})'}{\partial \text{vec}(\mathbf{D}_t^{-1})} \\ &\quad \times \frac{\partial (\boldsymbol{\varepsilon}_t' \mathbf{H}_t^{-1} \boldsymbol{\varepsilon}_t)}{\partial \text{vec}(\mathbf{H}_t^{-1})} \\ &= -\frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \text{vec}(\mathbf{D}_t^{-1}) + \frac{1}{2} \frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} (\mathbf{D}_t^{-1} \otimes \mathbf{D}_t^{-1}) (\mathbf{P}^{-1} \mathbf{D}_t^{-1} \otimes \mathbf{I}_N \\ &\quad + \mathbf{I}_N \otimes \mathbf{P}^{-1} \mathbf{D}_t^{-1}) \text{vec}(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') \\ &= -\frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \text{vec} \left(\mathbf{D}_t^{-1} - \frac{1}{2} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \mathbf{D}_t^{-1} - \frac{1}{2} \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \right). \end{aligned} \quad (2.38)$$

By similar manipulations for the lower block in (2.37), we obtain

$$\begin{aligned} \frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho}} &= -\frac{1}{2} \frac{\partial \ln |\mathbf{P}|}{\partial \boldsymbol{\rho}} - \frac{1}{2} \frac{\partial (\boldsymbol{\varepsilon}_t' \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t)}{\partial \boldsymbol{\rho}} \\ &= -\frac{1}{2} \frac{\partial \text{vec}(\mathbf{P})'}{\partial \boldsymbol{\rho}} \text{vec}(\mathbf{P}^{-1} - \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1}). \end{aligned} \quad (2.39)$$

The equations (2.38) and (2.39) constitute (2.11). ■

2.A.2 Proof of Lemma 2.6

The second partial derivative of the log-likelihood function for observation t with respect to $\boldsymbol{\theta}$, or the Hessian, can be partitioned in the following way:

$$\frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}' \partial \boldsymbol{\theta}} = \begin{bmatrix} \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega}' \partial \boldsymbol{\omega}} & \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho}' \partial \boldsymbol{\omega}} \\ \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega}' \partial \boldsymbol{\rho}} & \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho}' \partial \boldsymbol{\rho}} \end{bmatrix}. \quad (2.40)$$

In the subsequent sections, we supply blockwise derivations of the Hessian (2.40). The final results (2.12) is attained by taking the conditional expectations of derived blocks with the relations $\mathbf{E}[\mathbf{z}_t \mathbf{z}_t'] = \mathbf{P}$ and $\mathbf{H}_t = \mathbf{D}_t \mathbf{P} \mathbf{D}_t'$.

The upper left block of the Hessian

The upper left block of (2.40) is given by

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{\omega}'} \left(\frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega}} \right) &= - \frac{\partial}{\partial \boldsymbol{\omega}'} \left(\frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \text{vec}(\mathbf{D}_t^{-1}) \right) \\ &\quad + \frac{1}{2} \frac{\partial}{\partial \boldsymbol{\omega}'} \left(\frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \text{vec}(\mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1}) \right) \\ &\quad + \frac{1}{2} \frac{\partial}{\partial \boldsymbol{\omega}'} \left(\frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \text{vec}(\mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \mathbf{D}_t^{-1}) \right). \end{aligned} \quad (2.41)$$

The first term in (2.41) is decomposed to

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{\omega}'} \left(\frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \text{vec}(\mathbf{D}_t^{-1}) \right) &= - \frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} (\mathbf{D}_t^{-1} \otimes \mathbf{D}_t^{-1}) \frac{\partial \text{vec}(\mathbf{D}_t)}{\partial \boldsymbol{\omega}'} + \left[\text{vec}(\mathbf{D}_t^{-1})' \otimes \mathbf{I}_k \right] \frac{\partial^2 \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}' \partial \boldsymbol{\omega}}. \end{aligned} \quad (2.42)$$

The second term in (2.41) is developed to

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{\omega}'} \left(\frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \text{vec}(\mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1}) \right) &= \left[\text{vec}(\mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1})' \otimes \mathbf{I}_k \right] \frac{\partial^2 \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}' \partial \boldsymbol{\omega}} \\ &\quad + \frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \frac{\partial \text{vec}(\mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1})}{\partial \text{vec}(\mathbf{D}_t^{-1})'} \frac{\partial \text{vec}(\mathbf{D}_t^{-1})}{\partial \text{vec}(\mathbf{D}_t)'} \frac{\partial \text{vec}(\mathbf{D}_t)}{\partial \boldsymbol{\omega}'} \\ &= \left[\text{vec}(\mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \mathbf{D}_t^{-1})' \otimes \mathbf{I}_k \right] \frac{\partial^2 \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}' \partial \boldsymbol{\omega}} - \frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} (\mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \otimes \mathbf{D}_t^{-1} \\ &\quad + \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1} \otimes \mathbf{z}_t \mathbf{z}_t' + \mathbf{D}_t^{-1} \otimes \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \mathbf{D}_t^{-1}) \frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}'} \end{aligned} \quad (2.43)$$

By similar operations, the third term in (2.41) is rewritten as

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{\omega}'} \left(\frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \text{vec}(\mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \mathbf{D}_t^{-1}) \right) \\ = \left[\text{vec}(\mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t')' \otimes \mathbf{I}_k \right] \frac{\partial^2 \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}' \partial \boldsymbol{\omega}} - \frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} (\mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \mathbf{D}_t^{-1} \otimes \mathbf{D}_t^{-1} \\ + \mathbf{z}_t \mathbf{z}_t' \otimes \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1} + \mathbf{D}_t^{-1} \otimes \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t') \frac{\partial \text{vec}(\mathbf{D}_t)}{\partial \boldsymbol{\omega}'}. \end{aligned} \quad (2.44)$$

Substituting back into (2.41) and rearranging yields

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{\omega}'} \left(\frac{\partial \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega}} \right) = - \left\{ \left[\text{vec}(\mathbf{D}_t^{-1})' \otimes \mathbf{I}_k \right] - \frac{1}{2} \left[\text{vec}(\mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \mathbf{D}_t^{-1})' \otimes \mathbf{I}_k \right] \right. \\ \left. - \frac{1}{2} \left[\text{vec}(\mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t')' \otimes \mathbf{I}_k \right] \right\} \frac{\partial^2 \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}' \partial \boldsymbol{\omega}} \\ + \frac{1}{2} \frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} \left\{ 2(\mathbf{D}_t^{-1} \otimes \mathbf{D}_t^{-1}) - (\mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \otimes \mathbf{D}_t^{-1} \right. \\ \left. + \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1} \otimes \mathbf{z}_t \mathbf{z}_t' + \mathbf{D}_t^{-1} \otimes \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \mathbf{D}_t^{-1}) - (\mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \mathbf{D}_t^{-1} \otimes \mathbf{D}_t^{-1} \right. \\ \left. + \mathbf{z}_t \mathbf{z}_t' \otimes \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{D}_t^{-1} + \mathbf{D}_t^{-1} \otimes \mathbf{D}_t^{-1} \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t') \right\} \frac{\partial \text{vec}(\mathbf{D}_t)}{\partial \boldsymbol{\omega}'}. \end{aligned} \quad (2.45)$$

The lower right block of the Hessian

The lower right block of (2.40) can be written as

$$\frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho}' \partial \boldsymbol{\rho}} = -\frac{1}{2} \frac{\partial}{\partial \boldsymbol{\rho}'} \left(\frac{\partial \text{vec}(\mathbf{P})'}{\partial \boldsymbol{\rho}} \text{vec}(\mathbf{P}^{-1}) \right) + \frac{1}{2} \frac{\partial}{\partial \boldsymbol{\rho}'} \left(\frac{\partial \text{vec}(\mathbf{P})'}{\partial \boldsymbol{\rho}} \text{vec}(\mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1}) \right). \quad (2.46)$$

Noticing that $\partial^2 \text{vec}(\mathbf{P})' / \partial \boldsymbol{\rho}' \partial \boldsymbol{\rho} = \mathbf{0}$, the first term in (2.46) is reduced to

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{\rho}'} \left(\frac{\partial \text{vec}(\mathbf{P})'}{\partial \boldsymbol{\rho}} \text{vec}(\mathbf{P}^{-1}) \right) &= \frac{\partial \text{vec}(\mathbf{P})'}{\partial \boldsymbol{\rho}} \frac{\partial \text{vec}(\mathbf{P}^{-1})}{\partial \text{vec}(\mathbf{P})'} \frac{\partial \text{vec}(\mathbf{P})}{\partial \boldsymbol{\rho}'} \\ &= -\frac{\partial \text{vec}(\mathbf{P})'}{\partial \boldsymbol{\rho}} (\mathbf{P}^{-1} \otimes \mathbf{P}^{-1}) \frac{\partial \text{vec}(\mathbf{P})}{\partial \boldsymbol{\rho}'}, \end{aligned} \quad (2.47)$$

and the second term is equal to

$$\begin{aligned} \frac{\partial}{\partial \boldsymbol{\rho}'} \left(\frac{\partial \text{vec}(\mathbf{P})'}{\partial \boldsymbol{\rho}} \text{vec}(\mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1}) \right) \\ = \frac{\partial \text{vec}(\mathbf{P})'}{\partial \boldsymbol{\rho}} \frac{\partial \text{vec}(\mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1})}{\partial \text{vec}(\mathbf{P}^{-1})'} \frac{\partial \text{vec}(\mathbf{P}^{-1})}{\partial \text{vec}(\mathbf{P})'} \frac{\partial \text{vec}(\mathbf{P})}{\partial \boldsymbol{\rho}'} \\ = -\frac{\partial \text{vec}(\mathbf{P})'}{\partial \boldsymbol{\rho}} (\mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \otimes \mathbf{P}^{-1} + \mathbf{P}^{-1} \otimes \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1}) \frac{\partial \text{vec}(\mathbf{P})}{\partial \boldsymbol{\rho}'}. \end{aligned} \quad (2.48)$$

Combining all together produces

$$\begin{aligned} \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho}' \partial \boldsymbol{\rho}} &= \frac{1}{2} \frac{\partial \text{vec}(\mathbf{P}')'}{\partial \boldsymbol{\rho}} \left\{ (\mathbf{P}^{-1} \otimes \mathbf{P}^{-1}) - (\mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \otimes \mathbf{P}^{-1}) \right. \\ &\quad \left. - (\mathbf{P}^{-1} \otimes \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1}) \right\} \frac{\partial \text{vec}(\mathbf{P})}{\partial \boldsymbol{\rho}'}. \end{aligned} \quad (2.49)$$

The lower left and upper right blocks of the Hessian

Next consider the lower left block that is the cross derivatives. Using the facts that

$$\frac{\partial}{\partial \boldsymbol{\omega}'} \left(\frac{\partial \text{vec}(\mathbf{P}')'}{\partial \boldsymbol{\rho}} \text{vec}(\mathbf{P}^{-1}) \right) = \mathbf{0} \quad \text{and} \quad \frac{\partial}{\partial \boldsymbol{\omega}'} \text{vec} \left(\frac{\partial \text{vec}(\mathbf{P}')'}{\partial \boldsymbol{\rho}} \right) = \mathbf{0}, \quad (2.50)$$

we have

$$\begin{aligned} \frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\omega}' \partial \boldsymbol{\rho}} &= \frac{1}{2} \frac{\partial \text{vec}(\mathbf{P}')'}{\partial \boldsymbol{\rho}} \frac{\partial (\mathbf{P}^{-1} \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \mathbf{D}_t^{-1} \mathbf{P}^{-1})}{\partial \text{vec}(\mathbf{D}_t^{-1})'} \frac{\partial \text{vec}(\mathbf{D}_t^{-1})}{\partial \text{vec}(\mathbf{D}_t)'} \frac{\partial \text{vec}(\mathbf{D}_t)}{\partial \boldsymbol{\omega}'} \\ &= -\frac{1}{2} \frac{\partial \text{vec}(\mathbf{P}')'}{\partial \boldsymbol{\rho}} (\mathbf{P}^{-1} \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \otimes \mathbf{P}^{-1} + \mathbf{P}^{-1} \otimes \mathbf{P}^{-1} \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t') (\mathbf{D}_t^{-1} \otimes \mathbf{D}_t^{-1}) \\ &\quad \times \frac{\partial \text{vec}(\mathbf{D}_t)}{\partial \boldsymbol{\omega}'} \\ &= -\frac{1}{2} \frac{\partial \text{vec}(\mathbf{P}')'}{\partial \boldsymbol{\rho}} (\mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t' \otimes \mathbf{P}^{-1} \mathbf{D}_t^{-1} + \mathbf{P}^{-1} \mathbf{D}_t^{-1} \otimes \mathbf{P}^{-1} \mathbf{z}_t \mathbf{z}_t') \frac{\partial \text{vec}(\mathbf{D}_t)}{\partial \boldsymbol{\omega}'}. \end{aligned} \quad (2.51)$$

The upper right block of (2.40) is obtained by transposing (2.51), so that

$$\frac{\partial^2 \ell_t(\boldsymbol{\theta})}{\partial \boldsymbol{\rho}' \partial \boldsymbol{\omega}} = -\frac{1}{2} \frac{\partial \text{vec}(\mathbf{D}_t)'}{\partial \boldsymbol{\omega}} (\mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1} \otimes \mathbf{D}_t^{-1} \mathbf{P}^{-1} + \mathbf{D}_t^{-1} \mathbf{P}^{-1} \otimes \mathbf{z}_t \mathbf{z}_t' \mathbf{P}^{-1}) \frac{\partial \text{vec}(\mathbf{P})}{\partial \boldsymbol{\rho}'}. \quad (2.52)$$

Finally, taking the conditional expectations of (2.41), (2.49), (2.51) and (2.52) with the relations $E[\mathbf{z}_t \mathbf{z}_t'] = \mathbf{P}$ and $\mathbf{H}_t = \mathbf{D}_t \mathbf{P} \mathbf{D}_t$, and transposing them yields (2.12). \blacksquare

2.A.3 Proof of Lemma 2.7

Since (2.18) is immediate from Corollary 2 in He and Teräsvirta (2004), we verify that (2.19) holds under the assumption of normality. First note that

$$E[\mathbf{C}_t \otimes \mathbf{C}_t] = (\mathbf{A}_1 \otimes \mathbf{A}_1) E[\mathbf{Z}_t^2 \otimes \mathbf{Z}_t^2] + (\mathbf{A}_1 + \mathbf{B}_1) \otimes (\mathbf{A}_1 + \mathbf{B}_1) - \mathbf{A}_1 \otimes \mathbf{A}_1. \quad (2.53)$$

Assume $\mathbf{z}_t \sim N(\mathbf{0}, \mathbf{P})$ with $\mathbf{P} = [\rho_1 \dots \rho_N]$ and let $\mathbf{M} = [\mathbf{M}_{ij}] = \mathbf{Z}_t^2 \otimes \mathbf{Z}_t^2$ where the blocks \mathbf{M}_{ij} are of size $N \times N$. Then,

$$\begin{aligned} \mathbf{M}_{ii} &= z_{i,t}^2 \text{diag}(\mathbf{z}_t \odot \mathbf{z}_t) \\ &= z_{i,t}^2 \text{diag}(z_{1,t}^2, \dots, z_{N,t}^2), \quad i = 1, \dots, N \end{aligned} \quad (2.54)$$

and $\mathbf{M}_{ij} = \mathbf{0}$, $i \neq j$. Then (see Nabeya, 1951),

$$\mathbb{E}z_{i,t}^2 z_{j,t}^2 = \begin{cases} 3 & i = j \\ 1 + 2\rho_{ij}^2 & \text{otherwise.} \end{cases} \quad (2.55)$$

It thus follows that

$$\mathbb{E}\mathbf{M}_{ii} = \mathbf{I}_N + 2 \operatorname{diagv}(\boldsymbol{\rho}_i \odot \boldsymbol{\rho}_i), \quad i = 1, \dots, N \quad (2.56)$$

and

$$\mathbb{E}[\mathbf{Z}_t^2 \otimes \mathbf{Z}_t^2] = \mathbf{I}_N \otimes \mathbf{I}_N + 2 \operatorname{diagv}(\operatorname{vec}(\mathbf{P}) \odot \operatorname{vec}(\mathbf{P})). \quad (2.57)$$

Inserting (2.57) into (2.53) yields (2.19). ■

Table 2.1 Data generating processes for size simulations for test of CCC-GARCH(1,1)

	DGP 1	DGP 2	DGP 3	DGP 4	DGP 5
ARCH parameters	$\begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0.04 & 0 \\ 0 & 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.04 & 0 \\ 0 & 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$	$\begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$
GARCH parameters	$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.7 \end{bmatrix}$	$\begin{bmatrix} 0.95 & 0 \\ 0 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.95 & 0 \\ 0 & 0.9 \end{bmatrix}$	$\begin{bmatrix} 0.45 & 0 \\ 0 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.45 & 0 \\ 0 & 0.6 \end{bmatrix}$
Correlation	0.30	0.90	0.30	0.90	0.30
$\lambda(\mathbf{\Gamma}_C)$	0.90	0.99	0.99	0.80	0.80
$\lambda(\mathbf{\Gamma}_{C \otimes C})$	0.89	0.98	0.98	0.72	0.72

Notes: The constant term in the conditional variance equation is set to $\mathbf{a} = [0.10.2]'$ for all DGPs. $\lambda(\mathbf{\Gamma}_C)$ and $\lambda(\mathbf{\Gamma}_{C \otimes C})$ denote the stationarity and the fourth-order moment conditions, respectively. See Section 2.2.2 for the definitions.

Table 2.2 Size simulations for test of CCC-GARCH(1,1) model against ECCC-GARCH(1,1) model

T	DGP 1	DGP 2	DGP 3	DGP 4	DGP 5
(a): nominal 5%					
1000	0.060	0.107	0.075	0.052	0.057
2500	0.062	0.064	0.073	0.062	0.053
5000	0.055	0.061	0.063	0.056	0.051
10000	0.055	0.063	0.058	0.053	0.057
(b): nominal 10%					
1000	0.112	0.170	0.135	0.106	0.100
2500	0.117	0.117	0.133	0.111	0.103
5000	0.102	0.114	0.116	0.104	0.099
10000	0.106	0.124	0.112	0.099	0.105

Note: The number of replications for each simulation is 5000.

Table 2.3 Data generating processes for power simulations for test of CCC-GARCH(1,1)

	DGP 6	DGP 7	DGP 8	DGP 9	DGP 10
ARCH parameters	$\begin{bmatrix} 0.05 & 0.001 \\ 0.004 & 0.04 \end{bmatrix}$	$\begin{bmatrix} 0.09 & 0.001 \\ 0.004 & 0.04 \end{bmatrix}$	$\begin{bmatrix} 0.05 & 0.001 \\ 0.001 & 0.04 \end{bmatrix}$	$\begin{bmatrix} 0.07 & 0.01 \\ 0.03 & 0.04 \end{bmatrix}$	$\begin{bmatrix} 0.25 & 0.01 \\ 0.02 & 0.1 \end{bmatrix}$
GARCH parameters	$\begin{bmatrix} 0.9 & 0.004 \\ 0.002 & 0.85 \end{bmatrix}$	$\begin{bmatrix} 0.9 & 0.004 \\ 0.02 & 0.89 \end{bmatrix}$	$\begin{bmatrix} 0.9 & 0.001 \\ 0.001 & 0.85 \end{bmatrix}$	$\begin{bmatrix} 0.8 & 0.04 \\ 0.03 & 0.75 \end{bmatrix}$	$\begin{bmatrix} 0.35 & 0.01 \\ 0.04 & 0.2 \end{bmatrix}$
Correlation	0.7	0.95	0.7	0.8	0.3
$\lambda(\mathbf{\Gamma}_C)$	0.95	0.992	0.95	0.814	0.604
$\lambda(\mathbf{\Gamma}_{C \otimes C})$	0.9084	0.9996	0.9076	0.8978	0.4881

Notes: The constant term in the conditional variance equation is set to $\mathbf{a} = [0.1 \ 0.2]'$ for all DGPs. $\lambda(\mathbf{\Gamma}_C)$ and $\lambda(\mathbf{\Gamma}_{C \otimes C})$ denote the stationarity and the fourth-order moment conditions, respectively. See Section 2.2.2 for the definitions.

Table 2.4 Power simulations for test of CCC-GARCH(1,1) model against ECCC-GARCH(1,1) model

T	DGP 6	DGP 7	DGP 8	DGP 9	DGP 10
(a): nominal 5%					
1000	0.113	0.820	0.088	0.427	0.182
2500	0.116	0.996	0.083	0.813	0.293
5000	0.143	1.000	0.103	0.982	0.497
10000	0.221	1.000	0.107	1.000	0.783
(b): nominal 10%					
1000	0.184	0.883	0.151	0.544	0.266
2500	0.184	0.998	0.134	0.883	0.399
5000	0.219	1.000	0.163	0.992	0.603
10000	0.326	1.000	0.165	1.000	0.850

Note: The number of replications for each simulation is 5000.

Table 2.5 Data generating processes for simulations under changing conditional correlations

Constants				
DCC and EDCC		STCC and ESTCC		
$[0.02 \ 0.02]'$		$[0.01 \ 0.01]'$		
ARCH parameters				
DCC		EDCC		STCC
$\begin{bmatrix} 0.04 & 0 \\ 0 & 0.06 \end{bmatrix}$		$\begin{bmatrix} 0.09 & 0.001 \\ 0.004 & 0.04 \end{bmatrix}$		$\begin{bmatrix} 0.04 & 0 \\ 0 & 0.05 \end{bmatrix}$
				ESTCC
				$\begin{bmatrix} 0.09 & 0.001 \\ 0.004 & 0.04 \end{bmatrix}$
GARCH parameters				
DCC		EDCC		STCC
$\begin{bmatrix} 0.95 & 0 \\ 0 & 0.93 \end{bmatrix}$		$\begin{bmatrix} 0.9 & 0.004 \\ 0.02 & 0.89 \end{bmatrix}$		$\begin{bmatrix} 0.94 & 0 \\ 0 & 0.92 \end{bmatrix}$
				ESTCC
				$\begin{bmatrix} 0.9 & 0.004 \\ 0.02 & 0.89 \end{bmatrix}$
Other parameters and assumptions				
DCC and EDCC		STCC and ESTCC		
$\bar{Q} = 0.6$		$\rho_{(1)} = 0, \rho_{(2)} = 0.5$		
		$s_t = h_{s,t}^{1/2} u_t, u_t \sim N(0, 1)$		
$[\alpha_1 \ \beta_1] = [0.05 \ 0.8]$		$h_{s,t} = 0.02 + 0.04s_{t-1}^2 + 0.95h_{s,t-1}$		
		$\gamma = 5, c = 0$		

Note: For definitions of these models, see Engle and Sheppard (2001), Silvennoinen and Teräsvirta (2005, 2009b).

Table 2.6 Size and power simulations under changing conditional correlations

T	Size		Power	
	DCC	STCC	EDCC	ESTCC
(a): nominal 5%				
1000	0.127	0.083	0.979	0.807
2500	0.110	0.063	1.000	0.994
5000	0.118	0.060	1.000	1.000
10000	0.159	0.057	1.000	1.000
(b): nominal 10%				
1000	0.212	0.146	0.990	0.865
2500	0.191	0.122	1.000	0.997
5000	0.205	0.110	1.000	1.000
10000	0.259	0.101	1.000	1.000

Note: The numbers represent actual rejection frequencies in 5000 replications based on the nominal 5% and 10% levels of significance.

Table 2.7 Descriptive statistics for the mean subtracted return series

	JPY	CHF	GM	IBM	NEC	Hitachi
Minimum	-5.619	-4.400	-23.653	-26.857	-19.012	-15.466
Maximum	3.569	5.836	16.644	12.327	13.446	11.122
Std.dev	0.656	0.735	1.691	1.636	2.229	2.041
SK	-0.488	-0.016	-0.093	-0.320	0.282	0.266
KR	4.349	2.843	8.026	13.054	3.364	2.854
Rob.SK	0.010	-0.011	0.009	0.017	-0.078	-0.081
Rob.KR	0.318	0.224	0.147	0.150	0.080	0.161
LJB	6427.97 [0.000]	2614.75 [0.000]	29846.14 [0.000]	79106.37 [0.000]	2865.89 [0.000]	2075.84 [0.000]
$Q^2(25)$	1338.69 [0.000]	952.21 [0.000]	1499.38 [0.000]	902.19 [0.000]	1174.04 [0.000]	1798.48 [0.000]
T	7765	7765	11115	11115	5913	5913

Notes: SK and KR denote the skewness and the excess kurtosis, respectively. Rob.SK and Rob.KR are outlier-robust versions of SK and KR described as SK_2 and KR_2 in Kim and White (2004). LJB is the test of normality by Lomnicki (1961) and Jarque and Bera (1980). The numbers in square brackets are p-values. $Q^2(25)$ is the McLeod-Li (1983) portmanteau test statistic for serial correlation up to lag 25 in the squared return series.

Table 2.8 Skewness and kurtosis for the standardised residuals

	JPY	CHF	GM	IBM	NEC	Hitachi
SK	-0.472	-0.142	-0.065	-0.103	0.287	0.268
KR	3.049	1.487	4.933	4.719	1.749	1.269
Rob.SK	-0.017	-0.015	0.009	0.013	-0.055	-0.043
Rob.KR	0.222	0.134	0.086	0.071	0.060	0.069

Notes: SK and KR denote the skewness and the excess kurtosis, respectively. Rob.SK and Rob.KR are outlier-robust versions of SK and KR described as SK_2 and KR_2 in Kim and White (2004). LJB is the test of normality by Lomnicki (1961) and Jarque and Bera (1980). The numbers in square brackets are p-values.

Table 2.9 Estimation results of bivariate CCC/ECCC-GARCH(1,1) models

Model	Asset	a_0	A_1	B_1	ρ	-LogLik	LM_{ECCC}	$\lambda(\Gamma_C)$	$\lambda(\Gamma_{C \otimes C})$	
Panel A	CCC	JPY	0.0022 (0.0002)	0.0522 (0.0018)	0.9450 (0.0032)	0.5413 (0.0064)	13923.07	40.56 [3×10^{-8}]	0.9972	0.9998
		CHF	0.0079 (0.0018)	0.0580 (0.0008)	0.9278 (0.0042)					
ECCC	JPY	0.0012 (0.0002)	0.0459 (0.0012)	0.0038 (0.0017)	0.9482 (0.0026)	3×10^{-9} (0.0036)	13904.03		0.9962	0.9961
	CHF	0.0068 (0.0018)	6×10^{-8} (0.0019)	0.0592 (0.0008)	0.0080 (0.0034)	0.9223 (0.0050)				
Panel B	CCC	GM	0.0266 (0.0019)	0.0553 (0.0024)	0.9369 (0.0020)	0.3809 (0.0066)	39720.62	147.35 [8×10^{-31}]	0.9933	0.9960
		IBM	0.0263 (0.0022)	0.0684 (0.0018)	0.9249 (0.0022)					
ECCC	GM	0.0259 (0.0022)	0.0567 (0.0011)	0.0131 (0.0026)	0.9240 (0.0015)	8×10^{-10} (0.0026)	39652.12		0.9946	0.9947
	IBM	0.0203 (0.0025)	0.0108 (0.0036)	0.0698 (0.0025)	1×10^{-6} (0.0028)	0.9147 (0.0039)				
Panel C	CCC	NEC	0.0992 (0.0111)	0.0658 (0.0055)	0.9139 (0.0032)	0.6316 (0.0066)	23238.84	83.87 [3×10^{-17}]	0.9894	0.9841
		Hitachi	0.0419 (0.0042)	0.0512 (0.0046)	0.9382 (0.0034)					
ECCC	NEC	0.1011 (0.0119)	0.0672 (0.0041)	0.0132 (0.0058)	0.9006 (0.0019)	0.6309 (0.0068)	23218.93		0.9866	0.9782
	Hitachi	0.0386 (0.0051)	0.0087 (0.0081)	0.0519 (0.0066)	2×10^{-9} (0.0038)	0.9281 (0.0054)				

Notes: -LogLik is the negative value of the log-likelihood function at the maximum. The numbers in parenthesis are ML standard errors while those in square brackets are p-values for LM_{ECCC} statistics. The stationarity and the fourth-order moment conditions are satisfied if $\lambda(\Gamma_C) < 1$ and $\lambda(\Gamma_{C \otimes C}) < 1$, respectively.

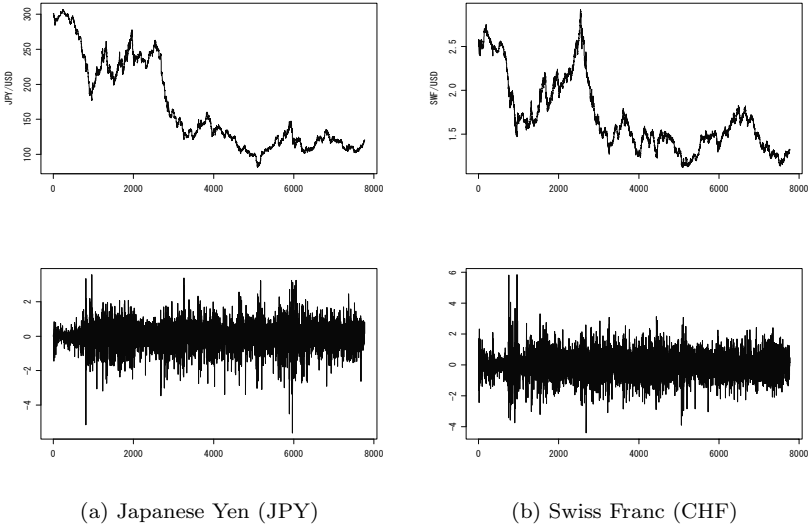


Figure 2.1 The two foreign exchange rates against U.S. dollars between 2 Jan, 1975 and 2 Dec, 2005: the upper panels depict the level series, and the lower ones are the mean subtracted return series.

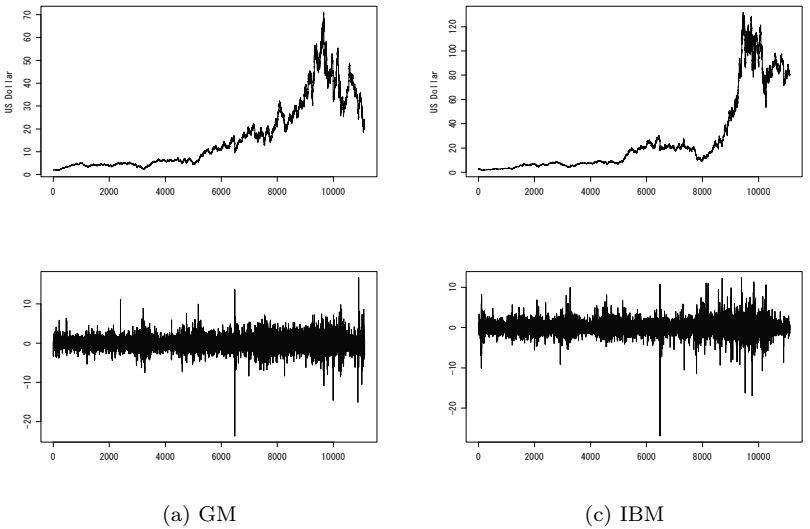


Figure 2.2 The three U.S. stocks between 2 Jan, 1962 and 28 Feb, 2006: the upper two panels depict the level series, and the lower ones are the mean subtracted return series.

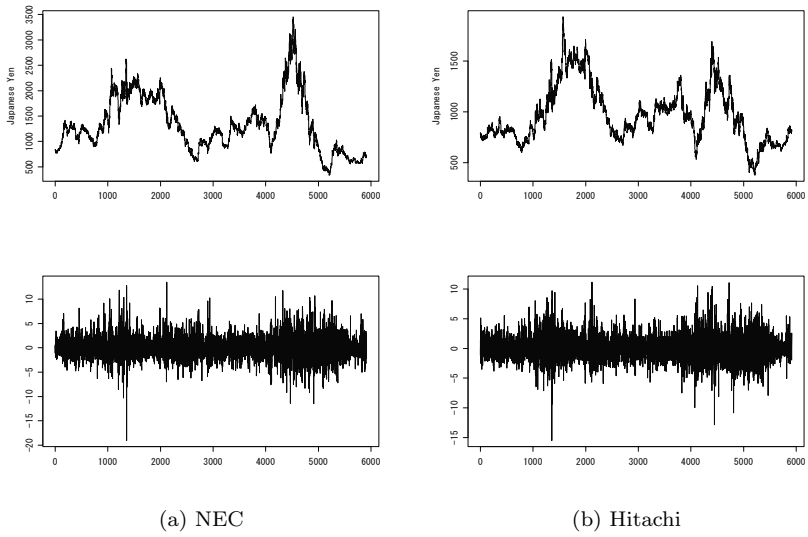


Figure 2.3 The two Japanese stocks between 4 Jan, 1983 and 1 Mar, 2006: the upper panels depict the level series, and the lower ones are the mean subtracted return series.

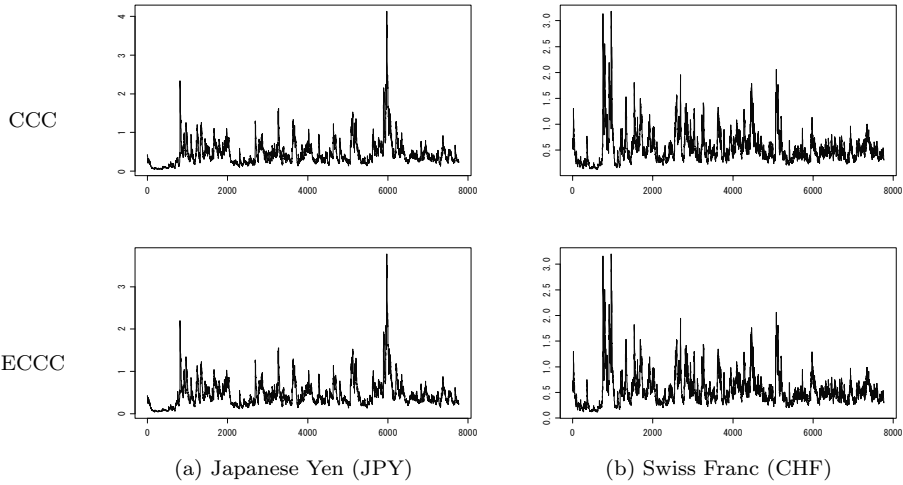


Figure 2.4 Estimated conditional variances in the CCC-/ECCC-GARCH(1, 1) models.

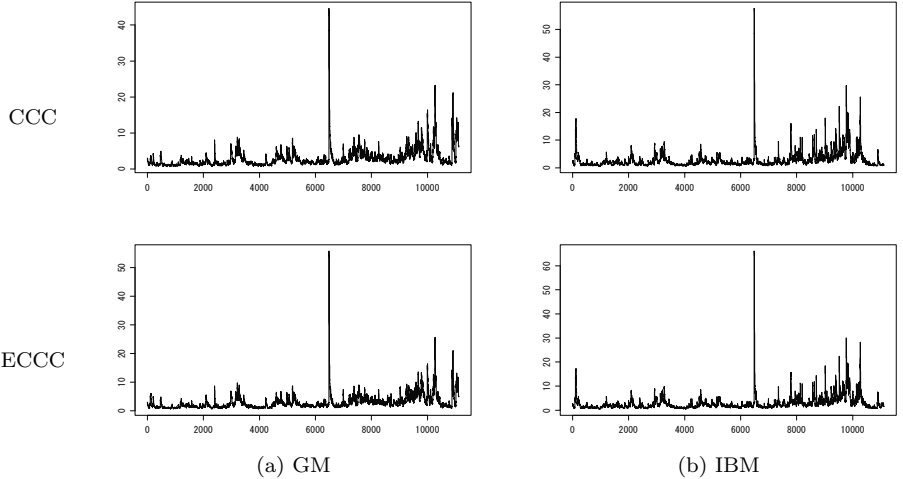


Figure 2.5 Estimated conditional variances in the CCC-/ECCC-GARCH(1, 1) models.

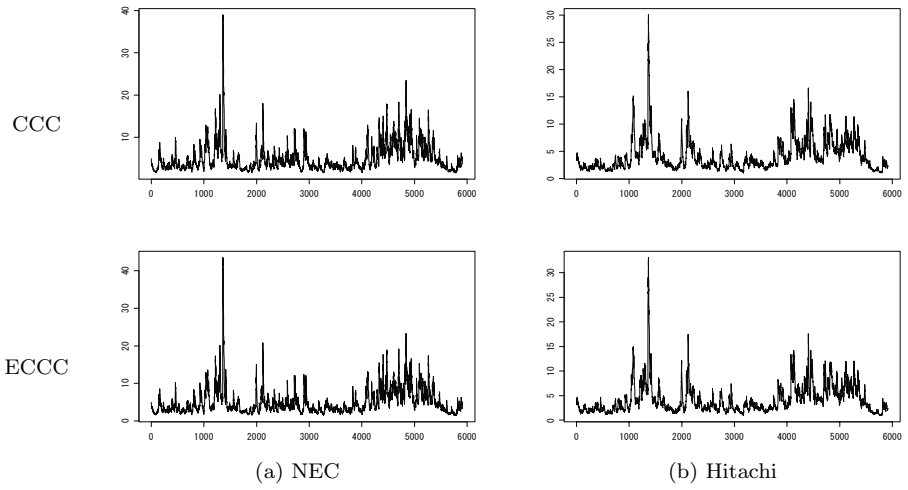


Figure 2.6 Estimated conditional variances in the CCC-/ECCC-GARCH(1, 1) models.

Chapter 3

An Alternative Test for Causality in Variance in the Conditional Correlation GARCH Models

An Alternative Test for Causality in Variance in the Conditional Correlation GARCH Models*

Abstract

In this paper, we propose an alternative Lagrange multiplier test for volatility interactions or causality in conditional variance in the multivariate GARCH models with constant conditional correlations. Although a similar test has recently been suggested by the authors, the test necessitates estimation of the constant conditional correlation GARCH model. Our new test, on the other hand, can be computed only through univariate GARCH estimations. In addition, a robust version of the new test is provided. Finite sample properties of the new test are investigated through Monte Carlo simulations. The results show that the new test has reasonable size and power properties under the normally and leptokurtotically distributed innovations as well as under changing conditional correlations. Usefulness of the new test is illustrated by empirical examples.

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3.1 Introduction

Since the introduction of a multivariate generalised autoregressive conditional heteroskedastic (MGARCH) model by Bollerslev, Engle, and Wooldridge (1988), there have been efforts to find parsimonious parametric models for the conditional second moments or conditional covariance matrix of returns of multivariate financial time series. See survey articles such as Palm (1996), Bauwens, Laurent, and Rombouts (2006) and Silvennoinen and Teräsvirta (2009b) for details. In particular, attention has been paid on modelling the conditional covariance and conditional correlation matrices separately since the introduction of the constant conditional correlation (CCC-) GARCH model of Bollerslev (1990). Nakatani and Teräsvirta (2008b) called the ensuing family of MGARCH models the conditional correlation (CC-) GARCH models. Due to the so-called “dimensionality” in the other types of MGARCH models and the need for assuring positive definiteness of the conditional covariance matrix, the CC-GARCH models have attracted much interest and generated plenty of research. In recent years, a number of extensions to the CCC-GARCH model have been proposed. Examples include the dynamic conditional correlation (DCC-) GARCH of Engle (2002), the varying correlation GARCH of Tse and Tsui (2002), the smooth transition conditional correlation (STCC-) and double STCC (DSTCC-) GARCH of Silvennoinen and Teräsvirta (2005, 2009a) and the generalised DCC-GARCH of Hafner and Franses (2009), to name a few. All of these models try to capture time-varying nature of the conditional correlations.

As Silvennoinen and Teräsvirta (2009b) pointed out, however, less attention has been put on modelling of the GARCH equations in the CC-GARCH models. Indeed, most of the above-mentioned CC-GARCH models merely use a variant of univariate GARCH models in modelling of the conditional variances. Although a number of different univariate GARCH specifications exist, incorporating interaction effects or causality in conditional variances into the univariate model has not been considered.

The extended CCC- (ECCC-) GARCH model introduced by Jeantheau (1998) provides a framework for modelling causality in conditional variances. The model is applied, for instance, by Wong and Li (1997), Wong, Li, and Ling (2000), Nakatani and Teräsvirta (2008b, 2009) for modelling volatility spillovers in the returns of stock prices and foreign exchange rates. Theoretical properties of the model have also been investigated. For instance, He and Teräsvirta (2004) considered fourth-moment structure of the model while Ling and McAleer (2003) established consistency and asymptotic normality of the quasi maximum estimators of the parameters. Recently, Nakatani and Teräsvirta (2008b) and Conrad and Karanasos (2009) worked out conditions for negative volatility spillovers effects while keeping the conditional covariance matrix positive definite almost surely.

As to modelling with the CC-GARCH models, misspecification tests have been proposed for testing the constancy of conditional correlations. Tse (2000), Bera and Kim (2002) and Silvennoinen and Teräsvirta (2005) are such examples. Regarding the volatility component of the CC-GARCH models, a need for developing diagnostic tests was also addressed by Silvennoinen and Teräsvirta (2009b). To the best of our knowledge, Nakatani and Teräsvirta (2009) is the only effort to consider a mis-

specification test for the volatility component of the CC-GARCH framework. These authors derived a Lagrange multiplier (LM) test of the CCC-GARCH model against the ECCC-GARCH model. Besides the CC-GARCH models, Comte and Lieberman (2000) dealt with second-order causality or causality in conditional variance by testing the appropriate parameter restrictions in the BEKK-GARCH model. Hafner and Herwartz (2006) employed the idea of the ‘ARCH in GARCH’ test of Lundbergh and Teräsvirta (2002) to obtain an LM test of noncausality in conditional variance. These tests test of particular parameter restrictions to detect causality in conditional variances. Cheung and Ng (1996), on the other hand, applied the cross-correlation function and proposed a Portmanteau test for causality in variance of the univariate GARCH residuals.

The aim of this paper is to propose an alternative test similar to, but simpler than, the one suggested by Nakatani and Teräsvirta (2009). The similarity is due to the fact that the new test is constructed in the same spirit as that of Nakatani and Teräsvirta (2009), whereas the simplicity comes from the property that carrying out the test only requires estimation of univariate GARCH models.

The rest of the paper is organised as follows. In Section 3.2, the ECCC-GARCH model is defined and some estimation strategies as well as asymptotic properties of the maximum likelihood estimator are described. The new test is derived in Section 3.3 and compared with existing ones. Section 3.4 contains simulation experiments with the purpose of investigating finite sample properties of the test statistic. Usefulness of the test is illustrated in Section 3.5 through an application. Finally, Section 3.6 concludes.

3.2 The conditional correlation GARCH model

3.2.1 The model specification

Following Nakatani and Teräsvirta (2009), we begin by defining the constant conditional correlation (CCC-) GARCH model such that

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t \quad (3.1)$$

$$\boldsymbol{\varepsilon}_t = \mathbf{D}_t \mathbf{z}_t \quad (3.2)$$

where \mathbf{y}_t is a stochastic $(N \times 1)$ vector, $\boldsymbol{\mu}$ is an $(N \times 1)$ intercept vector, $\mathbf{D}_t = \text{diag}(h_{1,t}^{1/2}, \dots, h_{N,t}^{1/2})$ is a diagonal matrix whose diagonal elements consist of conditional standard deviations of $\boldsymbol{\varepsilon}_t$. The sequence of the stochastic vector \mathbf{z}_t is independently distributed with $E[\mathbf{z}_t | \mathcal{F}_{t-1}] = \mathbf{0}$ and $E[\mathbf{z}'_t \mathbf{z}_t | \mathcal{F}_{t-1}] = \mathbf{P} = [\rho_{ij}]$ where \mathcal{F}_{t-1} is the information set up to and including time $t - 1$. $\rho_{ij} = 1$ for $i = j$ so that each $z_{i,t}$ in \mathbf{z}_t has unit variance. With this formulation, we have

$$E[\boldsymbol{\varepsilon}_t | \mathcal{F}_{t-1}] = \mathbf{0} \quad \text{and} \quad E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_t | \mathcal{F}_{t-1}] = \mathbf{D}_t \mathbf{P} \mathbf{D}_t = \mathbf{H}_t = \begin{cases} h_{i,t} & i = j \\ h_{i,t}^{1/2} h_{j,t}^{1/2} \rho_{ij} & i \neq j \end{cases}, \quad (3.3)$$

and therefore the matrices \mathbf{H}_t and \mathbf{P} are the time-varying conditional covariance matrix and the constant conditional correlation matrix of the process $\boldsymbol{\varepsilon}_t$, respectively.

In the original CCC-GARCH model of Bollerslev (1990), each $h_{i,t}$ is described by a univariate GARCH(p, q) model. Later on, Jeantheau (1998) proposed the extended CCC- (ECCC-) GARCH specification allowing for causality in conditional variances. He considered a model of the conditional variances as follows:

$$\mathbf{h}_t = \mathbf{a}_0 + \sum_{i=1}^q \mathbf{A}_i \boldsymbol{\varepsilon}_{t-i}^{(2)} + \sum_{j=1}^p \mathbf{B}_j \mathbf{h}_{t-j} \quad (3.4)$$

where $\boldsymbol{\varepsilon}_t^{(2)} = (\varepsilon_{1,t}^2, \dots, \varepsilon_{N,t}^2)'$, \mathbf{a}_0 is an $(N \times 1)$ vector, and \mathbf{A}_i and \mathbf{B}_j are $(N \times N)$ matrices with elements such that $h_{i,t}$ in \mathbf{h}_t are positive for all t . The equation (3.4) can be seen as a direct augmentation of the univariate GARCH (p, q) model to an N -dimensional system.

Theoretical characteristics of the ECCC-GARCH model have been studied by several authors. He and Teräsvirta (2004) investigated moment conditions of the ECCC-GARCH model and found that the model is capable of describing richer auto-correlation structure for the squared observations than the CCC-GARCH one. Ling and McAleer (2003) considered asymptotic properties of the estimators of the ECCC-GARCH model. Recently, Nakatani and Teräsvirta (2008b) and Conrad and Karanasos (2009) worked out that the ECCC-GARCH model can allow for negative spillover effects in conditional variance while keeping the positive definiteness of the conditional covariance matrix \mathbf{H}_t almost surely for all t .

Compared to the univariate GARCH model, there is the difference that the equation (3.4) allows for interactions or causality in conditional variance either through the past squared residuals of the other variables or through the past conditional variances or both. This interactions or causality in conditional variance is parameterised by the off-diagonal elements of the parameter matrices \mathbf{A}_i and \mathbf{B}_j in (3.4).

Assuming \mathbf{A}_i and \mathbf{B}_j diagonal simply reduces (3.4) to a vector of univariate GARCH (p, q) equations. Nakatani and Teräsvirta (2009) proposed a Lagrange Multiplier (LM) test, which they called the LM_{ECCC} test, for volatility interactions or nullity of the off-diagonal elements of \mathbf{A}_i and \mathbf{B}_j in (3.4) with $p = q = 1$. Using the test they found that there are statistically significant spillover effects in the conditional variance in stock and foreign exchange returns.

In this paper, we propose an alternative test similar to, but simpler than, the LM_{ECCC} test of Nakatani and Teräsvirta (2009). The similarity refers to the fact that the new test has the same null hypothesis as the LM_{ECCC} test, whereas the simplicity comes from the property that the new test only requires estimation of univariate GARCH models while the the LM_{ECCC} test necessitates estimation of a CCC-GARCH model.

For simplicity and given the fact that the first-order models describe many heteroskedastic time series well in a vast majority of empirical applications, we assume $\boldsymbol{\mu} = \mathbf{0}$ in (3.1) and restrict our discussion to the case of $p = q = 1$ in (3.4). In addition, we restrict the following theoretical derivations on the case of the constant conditional correlation. The validity of the proposed test under both constant and dynamic conditional correlations is examined through Monte Carlo simulations.

3.2.2 Estimation strategies

Let $\theta' = [\omega', \rho']$ where ω contains the parameters in \mathbf{h}_t and ρ constant conditional correlations in \mathbf{P} . The quasi log-likelihood function for observation t of the ECCC-GARCH model is given by

$$\ell_t(\theta) = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{D}_t^2| - \frac{1}{2} \ln |\mathbf{P}| - \frac{1}{2} \boldsymbol{\varepsilon}_t' \mathbf{H}_t^{-1} \boldsymbol{\varepsilon}_t. \quad (3.5)$$

The quasi maximum likelihood estimator (QMLE) $\hat{\theta}$ of θ is in principle given by

$$\hat{\theta} = \arg \max \sum_{t=1}^T \ell_t(\theta) \quad (3.6)$$

where T denotes the number of observations. The consistency and asymptotic normality of $\hat{\theta}$ were worked out by Ling and McAleer (2003) under mild regularity conditions.

There has been a couple of methods proposed for finding $\hat{\theta}$ in the (extended) CCC-GARCH model. In order to reduce the computational burden, Bollerslev (1990) recommended the use of a concentrated likelihood function. In this approach, the conditional correlation matrix \mathbf{P} is concentrated out and some other simplifying assumptions are imposed.

For the sake of deriving our Lagrange multiplier test, we consider yet another strategy, namely the two-stage quasi maximum likelihood (2SQML) estimation of $\theta' = [\omega', \rho']$. For brevity, let $\mathbf{V}_t = \mathbf{D}_t^2$. To apply the 2SQML technique, the log-likelihood function (3.5) has to be decomposed into two components such that

$$\ell_t(\omega, \rho) = \ell_{v,t}(\omega) + \ell_{c,t}(\omega, \rho) \quad (3.7)$$

where

$$\ell_{v,t}(\omega) = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{V}_t| - \frac{1}{2} \boldsymbol{\varepsilon}_t' \mathbf{V}_t^{-1} \boldsymbol{\varepsilon}_t \quad (3.8)$$

$$\ell_{c,t}(\omega, \rho) = -\frac{1}{2} \ln |\mathbf{P}| - \frac{1}{2} \mathbf{z}_t' \mathbf{P}^{-1} \mathbf{z}_t + \frac{1}{2} \mathbf{z}_t' \mathbf{z}_t. \quad (3.9)$$

By this decomposition, estimation of the parameter θ is carried out in two steps. The first stage consists of finding

$$\hat{\omega} = \arg \max \bar{\ell}_v(\omega) \quad (3.10)$$

whereas the second one of obtaining, given the first stage estimator $\hat{\omega}$,

$$\hat{\rho} = \arg \max \ell_c(\hat{\omega}, \rho). \quad (3.11)$$

The estimator $\hat{\theta}' = [\hat{\omega}', \hat{\rho}']$ is regarded as the two-stage quasi maximum likelihood estimator (2SQMLE). This decomposition and the estimation strategy are analogous to the ones suggested for the DCC-GARCH model by Engle (2002).

Since the consistency and asymptotic normality of the QMLE were established by Ling and McAleer (2003), the resulting 2SQML estimator is also consistent and asymptotically normal, albeit not efficient. Therefore, by Theorem 6.11 of White (1994), we have

$$\sqrt{T}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \xrightarrow{d} N(\mathbf{0}, \mathbf{G}^{-1} \boldsymbol{\Omega} \mathbf{G}'^{-1}) \quad (3.12)$$

where $\boldsymbol{\theta}_0$ is the true parameter,

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix} = E \begin{bmatrix} \nabla_{\boldsymbol{\omega}}^2 \ell_v & \mathbf{0} \\ \nabla_{\boldsymbol{\omega} \boldsymbol{\rho}} \ell_c & \nabla_{\boldsymbol{\rho}}^2 \ell_c \end{bmatrix} \quad (3.13)$$

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} \\ \boldsymbol{\Omega}_{21} & \boldsymbol{\Omega}_{22} \end{bmatrix} = E \begin{bmatrix} \nabla_{\boldsymbol{\omega}} \ell_v \nabla_{\boldsymbol{\omega}} \ell'_v & \nabla_{\boldsymbol{\omega}} \ell_v \nabla_{\boldsymbol{\rho}} \ell'_c \\ \nabla_{\boldsymbol{\rho}} \ell_c \nabla_{\boldsymbol{\omega}} \ell'_v & \nabla_{\boldsymbol{\rho}} \ell_c \nabla_{\boldsymbol{\rho}} \ell'_c \end{bmatrix} \quad (3.14)$$

with $\nabla_{\boldsymbol{\omega}}^2 \ell_v = \nabla_{\boldsymbol{\omega}}^2 \ell_v(\boldsymbol{\omega}_0)$ and so on. ∇ and ∇^2 represent the first and second order partial derivatives with respect to the elements of variables in the subscript. It follows from White (1999, Corollary 4.24, p. 71) that the asymptotic normality of the GARCH parameter estimators $\widehat{\boldsymbol{\omega}}$ also holds, that is,

$$\sqrt{T}(\widehat{\boldsymbol{\omega}} - \boldsymbol{\omega}_0) \xrightarrow{d} N(\mathbf{0}, \mathbf{G}_{11}^{-1} \boldsymbol{\Omega}_{11} \mathbf{G}_{11}'^{-1}). \quad (3.15)$$

We use this property to derive an LM test for causality in conditional variance in the next section.

Putting this differently, the first stage objective function (3.8) is the log-likelihood function with the assumption that $\mathbf{z}_t \sim N(\mathbf{0}, \mathbf{I}_N)$ where \mathbf{I}_N denotes the N dimensional identity matrix or, equivalently, $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{V}_t)$. It is then quite natural that the asymptotic normality (3.15) holds.

Given the fact that \mathbf{V}_t is diagonal, (3.8) is further simplified to

$$\ell_t(\boldsymbol{\omega}) = -\frac{1}{2} \sum_{i=1}^N \left(\ln h_{i,t} + \frac{\varepsilon_{i,t}^2}{h_{i,t}} \right). \quad (3.16)$$

The equation (3.16) is equal to the sum of the N individual log-likelihood functions of $\varepsilon_{i,t}$. When no causality in conditional variance is allowed in the model and $h_{i,t}$ is specified as a member of univariate GARCH models, typically as a univariate GARCH (1, 1), maximisation of (3.8) or (3.16) is the same as maximising a univariate GARCH log-likelihood function of each of N time series and summing them up. Therefore, computing the test statistic for causality in variance only requires univariate GARCH estimation. This greatly alleviates the computational burden in applications.

3.3 Tests for causality in conditional variance

3.3.1 Alternative test for causality in conditional variance

The vector GARCH equation (3.4) shows that the off-diagonal elements of the parameter matrices \mathbf{A}_1 or \mathbf{B}_1 characterise causality in conditional variance. Thus, a

test for causality in conditional variance is a test of parametric restrictions on the GARCH parameters ω . We state the null and alternative hypothesis as follows:

$$H_0 : \mathbf{A}_1 \text{ and } \mathbf{B}_1 \text{ in (3.4) are jointly diagonal}$$

against

$$H_1 : \text{at least one of off-diagonal elements of } \mathbf{A}_1 \text{ and } \mathbf{B}_1 \text{ is non-zero.}$$

Under the null hypothesis H_0 , the model is the standard CCC-GARCH(1, 1) model, whereas the extended CCC-GARCH(1, 1) model results from the alternative hypothesis H_1 .

As explained in the previous section, the GARCH parameters ω are asymptotically normal and can be consistently estimated by maximising the first-stage objective function (3.8) or (3.16), regarded as a quasi-log-likelihood function of ε_t based on the assumption that $\varepsilon_t \sim N(\mathbf{0}, \mathbf{V}_t)$. It is then straightforward to construct an LM test for causality in conditional variance. The LM statistic for testing H_0 is given by

$$LM = T\bar{s}'(\tilde{\omega})\bar{\mathcal{I}}(\tilde{\omega})^{-1}\bar{s}(\tilde{\omega}) \tag{3.17}$$

where $\tilde{\omega}$ denotes the first-stage estimator under the restricted (null) model, $\bar{s}(\tilde{\omega}) = T^{-1} \sum_{t=1}^T \nabla_{\omega} \ell_{v,t}$ and $\bar{\mathcal{I}}(\tilde{\omega}) = T^{-1} \sum_{t=1}^T E [\nabla_{\omega_i} \ell_{v,t} \nabla'_{\omega_i} \ell_{v,t}]$ are the average score and information matrix evaluated at $\tilde{\omega}$, respectively. It should be mentioned that $\bar{\mathcal{I}}(\tilde{\omega})$ is equal to the sample average of Ω_{11} evaluated at $\tilde{\omega}$. Under H_0 , the test statistic (3.17) is asymptotically distributed as a χ^2 distribution with $2N(N - 1)$ degrees of freedom.

Under the assumption that $\varepsilon_t \sim N(\mathbf{0}, \mathbf{V}_t)$, the score of (3.8) or (3.16) is given by

$$\begin{aligned} \nabla_{\omega} \ell_{v,t} &= -\frac{1}{2} \nabla_{\omega} \mathbf{V}_t \text{vec} (\mathbf{V}_t^{-1} - \mathbf{V}_t^{-1} \varepsilon_t \varepsilon_t' \mathbf{V}_t^{-1}) \\ &= \frac{1}{2} \sum_{i=1}^N (z_{i,t}^2 - 1) h_{i,t}^{-1} \nabla_{\omega} h_{i,t} \end{aligned} \tag{3.18}$$

and the information matrix becomes block diagonal with respect to the parameters in each univariate GARCH equation. The analytic expression of each block is written as

$$E [\nabla_{\omega_i} \ell_{v,t} \nabla'_{\omega_i} \ell_{v,t}] = \frac{1}{2} h_{i,t}^{-2} \nabla_{\omega_i} h_{i,t} \nabla'_{\omega_i} h_{i,t} \tag{3.19}$$

where ω_i , $i = 1, \dots, N$, denote the parameters in the univariate GARCH equation for the i th variable, that is, ω_i consist of elements in the i th row of \mathbf{A}_1 and \mathbf{B}_1 .

To work out the analytic expression of (3.17), we rearrange the order of the elements of ω in such a way that, while suppressing the notation for transpose, $\omega = [\omega_1^d, \dots, \omega_N^d, \omega_1^o, \dots, \omega_N^o]$ where $\omega_i = [\omega_i^d, \omega_i^o]$ and ω_i^d is a 3×1 vector containing the univariate GARCH(1, 1) parameters for $\varepsilon_{i,t}$ and ω_i^o is a $2(N - 1) \times 1$

vector whose entries correspond to the off-diagonal elements in the i th row of \mathbf{A}_1 and \mathbf{B}_1 .

Under H_0 we have $\omega_i^o = \mathbf{0}$, so that $\tilde{\omega} = [\tilde{\omega}_1^d, \dots, \tilde{\omega}_N^d, \mathbf{0}]$. With this partitioning and rearrangement, the LM statistic (3.17) is computed by the relevant parts of $\bar{\mathbf{s}}(\tilde{\omega})$ and $\bar{\mathcal{I}}(\tilde{\omega})$. Due to block-diagonality of the information matrix, the LM statistic (3.17) can be obtained by

$$LM = \sum_{i=1}^N LM_i \quad (3.20)$$

and each LM_i is given by

$$LM_i = \frac{1}{2} \left(\sum_{t=1}^T \tilde{u}_{i,t} \tilde{\mathbf{k}}_{i,t}^{o'} \right) \bar{\mathcal{I}}_{\omega_i^o}(\tilde{\omega})^{-1} \left(\sum_{t=1}^T \tilde{u}_{i,t} \tilde{\mathbf{k}}_{i,t}^o \right) \quad (3.21)$$

where $\tilde{u}_{i,t} = \tilde{z}_{i,t}^2 - 1$, $\tilde{\mathbf{k}}_{i,t}^o = \tilde{h}_{i,t}^{-1} \tilde{\nabla}_{\omega_i^o} h_{i,t}$, $\tilde{\mathbf{k}}_{i,t}^d = \tilde{h}_{i,t}^{-1} \tilde{\nabla}_{\omega_i^d} h_{i,t}$ and

$$\bar{\mathcal{I}}_{\omega_i^o}(\tilde{\omega}) = \sum_{t=1}^T \tilde{\mathbf{k}}_{i,t}^o \tilde{\mathbf{k}}_{i,t}^{o'} - \left(\sum_{t=1}^T \tilde{\mathbf{k}}_{i,t}^o \tilde{\mathbf{k}}_{i,t}^{d'} \right) \left(\sum_{t=1}^T \tilde{\mathbf{k}}_{i,t}^d \tilde{\mathbf{k}}_{i,t}^{d'} \right)^{-1} \left(\sum_{t=1}^T \tilde{\mathbf{k}}_{i,t}^d \tilde{\mathbf{k}}_{i,t}^{o'} \right).$$

Alternatively, the test statistic is conveniently computed by the so-called “ TR^2 ” form through auxiliary regressions. The procedure takes the following steps:

1. Estimate an univariate GARCH(1, 1) model for each $\varepsilon_{i,t}$, and compute the conditional variance $\tilde{h}_{i,t}$, $\tilde{u}_{i,t}$, $\tilde{\mathbf{k}}_{i,t}^d$ and $\tilde{\mathbf{k}}_{i,t}^o$ for $i = 1, \dots, N$.
2. Regress $\tilde{u}_{i,t}$ on $\tilde{\mathbf{k}}_{i,t}^d$ and $\tilde{\mathbf{k}}_{i,t}^o$ to obtain $LM_i = TR_i^2$ for each i where T is the number of observations and R_i^2 is the coefficient of determination in this regression.
3. Under H_0 , $LM = \sum_{i=1}^N LM_i$ is asymptotically equal to the test statistic in (3.17).

The partial derivatives $\nabla_{\omega_i^d} h_{i,t}$ and $\nabla_{\omega_i^o} h_{i,t}$ are computed recursively. The details can be found in Nakatani and Teräsvirta (2008a). We shall call this LM test the NT test.

The NT test relies on the assumption that $\varepsilon_t \sim \mathbf{N}(\mathbf{0}, \mathbf{V}_t)$. Since the covariance matrix \mathbf{V}_t is diagonal, the possibility of non-zero conditional correlations is ruled out from the first-stage objective function (3.8). Assuming zero conditional correlations is not a realistic assumption, and the failure of this assumption may lead to a misspecification of the model. Therefore, it is of importance to have a test that is robust to such a misspecification. A robust version of the NT test is readily available through the method suggested by Wooldridge (1990, 1991). The robust version (the robust NT test) can be computed as follows:

1. Estimate an univariate GARCH(1, 1) model for each $\varepsilon_{i,t}$, and compute the necessary components $\tilde{h}_{i,t}$, $\tilde{u}_{i,t}$, $\tilde{\mathbf{k}}_{i,t}^d$ and $\tilde{\mathbf{k}}_{i,t}^o$ for $i = 1, \dots, N$.
2. Regress $\tilde{\mathbf{k}}_{i,t}^o$ on $\tilde{\mathbf{k}}_{i,t}^d$ to have the $2(N-1)$ vector of residuals $\tilde{\mathbf{s}}_{i,t}$ in this multivariate regression.
3. Regress 1 on $\tilde{u}_{i,t}\tilde{\mathbf{s}}_{i,t}$ and compute the residual sum of squares $SSR_{i,0}$.
4. Repeat the above steps and compute $LM_{r,i} = T - SSR_{i,0}$ for each $i = 1, \dots, N$.
5. $LM_r = \sum_{i=1}^N LM_{r,i}$ is a robust version of the test (3.17) and is asymptotically distributed under the null as a χ^2 distribution with $2N(N-1)$ degrees of freedom.

3.3.2 Relations to similar tests

In the original GARCH article, Bollerslev (1986) proposed, based on the LM principle, a misspecification test for the presence of the higher order ARCH or GARCH terms in the univariate GARCH (p, q) model. In one way or the other, the NT test can be seen as a generalisation of the Bollerslev's LM test. Instead of considering higher order ARCH or GARCH effects, the NT tests examines the influence of cross-terms from other GARCH equations. In fact, when $N = 1$, a univariate case, the test statistic (3.17) is reduced to the Bollerslev's ξ_{LM} test for testing the existence of higher-order ARCH and/or GARCH components in the univariate GARCH (p, q) model.

A similar test for causality in conditional variance was proposed by Hafner and Herwartz (2006). They used a framework of the 'ARCH in GARCH' test introduced by Lundbergh and Teräsvirta (2002). Their test (the HH test) is also derived on the basis of the LM principle, and may also be computed through auxiliary regressions. They consider the following model:

$$\varepsilon_{i,t} = z_{i,t}\sqrt{h_{i,t}g_{i,t}}, \quad g_{i,t} = 1 + \boldsymbol{\pi}'_i\boldsymbol{\eta}_{j,t}, \quad \boldsymbol{\eta}_{j,t} = (\varepsilon_{j,t-1}, h_{j,t-1})' \quad (3.22)$$

for $i, j = 1, \dots, N$, $i \neq j$ where $\boldsymbol{\pi}_i$ is a (2×1) vector of parameters. By construction, causality in conditional variance in the HH test is incorporated multiplicatively through the term $\sqrt{h_{i,t}g_{i,t}}$. The null hypothesis of no causality in conditional variance is $\boldsymbol{\pi}_i = \mathbf{0}$, so that the HH test is asymptotically χ^2 distributed with 2 degrees of freedom under the null.

According to their setup, the HH test seems a pairwise test for the variables in the N dimensional system. It is not immediately obvious how one can join the HH tests to form a more general one. To make performance comparisons of the HH test with ours, we generalise their formulation by using $\boldsymbol{\eta}_{-i,t} = (\boldsymbol{\varepsilon}'_{-i,t-1}, \mathbf{h}'_{-i,t-1})'$ in stead of $\boldsymbol{\eta}_{j,t}$ in (3.22), where the subscript $-i$ means that the vector contains all elements but the i th one. The null hypothesis remains $\boldsymbol{\pi}_i = \mathbf{0}$, but now it is a $2(N-1) \times 1$ vector. The generalised HH test is asymptotically distributed as $\chi^2\{2(N-1)\}$ under the null. It may still be computed through auxiliary regressions.

3.4 Simulation experiments

3.4.1 Design of the experiments

In this section we conduct simulation experiments to investigate finite sample properties of the NT and the robust NT tests in comparison with the HH test and the LM_{ECCC} test. In what follows, the HH test serves as a benchmark. We use the DCC- and CCC-GARCH models for size simulations and their extended versions for power simulations. The extended versions have non-diagonal parameter matrices in the vector GARCH equation (3.4) as data generating processes (DGPs). Note that asymptotic normality of the maximum likelihood estimators of the parameters of the (E)DCC- GARCH model has not been proven. Here, as has been explained, we ignore the correlations, however. In both size and power simulations, we use the sample sizes $T = 1000, 2500, 5000$ and 10000 for each DGP. The number of dimensions varies from $N = 2$ to 5 .

The model parameters \mathbf{a} , \mathbf{A} , \mathbf{B} , \mathbf{P} and $[\alpha, \beta]$ are summarised in Table 3.1. We use principal submatrices (the $N \times N$ upper-left block) of them for different N . For the size study, the diagonal entries of \mathbf{A} and \mathbf{B} are used as model parameters. We assume that the innovation process \mathbf{z}_t follows a multivariate t -distribution with four different degrees of freedom, namely $df = 8, 10, 15$ and ∞ . The last case is equivalent to assuming that \mathbf{z}_t follows a multivariate normal distribution. The matrix \mathbf{P} has two roles. First, it serves as a covariance matrix of \mathbf{z}_t in the (E)CCC-GARCH DGPs. Second, in the (E)DCC-GARCH DGPs it is a sample correlation matrix of \mathbf{z}_t which is denoted as \mathbf{Q} in Engle (2002). In this case the dynamic conditional correlation matrix \mathbf{P}_t is constructed through the method suggested by Engle (2002).

A total of 5000 replications are conducted for each DGP. To minimise initial effects, the first 1000 generated observations are discarded. All numerical calculations are implemented in the free statistical environment R 2.10.0 (R Development Core Team, 2009) or later with the `ccgarch` package (Nakatani, 2010) for modelling multivariate GARCH models with conditional correlations¹.

3.4.2 Size properties

The results of the size simulations are summarised graphically in Figures 3.1 and 3.2 for the CCC-GARCH DGPs and in Figures 3.3 and 3.4 for the DCC-GARCH DGPs, respectively. In all of these figures, the size discrepancies (the actual size minus the nominal one) are plotted against the nominal sizes from 0.0001% to 10.0% by 0.001 up to the 0.01% nominal size and by 0.005 thereafter. The format of the graphs follows a suggestion by Davidson and MacKinnon (1998).

Figures 3.1 and 3.3 contain the results when \mathbf{z}_t follows a multivariate t -distribution with 8 degrees of freedom and a multivariate normal distribution while Figures 3.2 and 3.4 do so for \mathbf{z}_t following a multivariate t -distribution with 10 and 15 degrees of

¹The latest version of the `ccgarch` package includes commands for computing the NT, robust NT and HH tests.

freedom. To gain visibility of the figures, only the results of the robust NT, HH and LM_{ECCC} tests with $T = 2500$ and 10000 are plotted².

In most of panels in Figures 3.1 and 3.2, one can observe that the LM_{ECCC} test is severely size-distorted. Except for the bivariate CCC-GARCH case when \mathbf{z}_t follows a normal distribution (the right uppermost panel in Figure 3.1), the value of size discrepancy often exceeds 10% at the 0.001% nominal size and grows thereafter when $N \geq 3$. Due to this large distortion, many of panels in Figures 3.1 and 3.2 do not contain markers corresponding to the LM_{ECCC} test. This consequence implies that the LM_{ECCC} test is effective only when $N = 2$ and is not suitable for testing of non-causality in conditional variance in higher dimensional models. In other cases, the empirical distribution of the test statistic has to be constructed by simulation. The same interpretation applies to the outcomes of the DCC-GARCH DGPs (Figures 3.3 and 3.4).

Looking at the results of the robust NT and HH tests, the size of both tests is distorted to some extent, but the actual size approaches to the nominal one as T becomes larger. Although the benchmark HH test shows in most of the cases better size properties than the robust NT test, the performance of the latter test seems acceptable because the level of the discrepancy in the robust NT test is largely less than 5%. However, when a DGP is t -distributed with $N \geq 4$ and $T \leq 2500$, size discrepancy of the robust NT test goes slightly beyond 5% around the 5% nominal size. The results of the simulations for the DCC-GARCH DGPs do not differ much from those for the CCC-GARCH case.

Overall, the asymptotic null distribution of the robust NT test statistics is a good approximation to the unknown finite-sample null distribution for $T > 2500$ and $N \geq 2$, and for normally- and t -distributed innovations, when the results are compared to those of the HH test. The constancy of the conditional correlations plays almost no role in determining the finite-sample performance of the robust NT test: The LM_{ECCC} test has limited effectiveness for testing of non-causality in variance.

3.4.3 Power properties

The actual rejection frequencies (power curves) are plotted against the nominal significance levels from 0.001% to 10.0% in Figures 3.5 and 3.6 for the extended CCC-(ECCC-) GARCH DGPs, while Figures 3.7 and 3.8 contain outcomes for the extended DCC (EDCC-) GARCH DGPs. As before, only the results of the robust NT, HH and LM_{ECCC} tests with $T = 2500$ and 10000 are plotted. The assumption for the innovation \mathbf{z}_t is the same as in the size study. We did not make size-adjustment for the power curves since the tests of interest, namely, the robust NT and HH tests, have acceptable size properties. In what follows, the power properties of the LM_{ECCC} test are not considered, because the test is severely size-distorted.

One can see it on each graph of Figures 3.5 through 3.8 that, for fixed N , the power of both tests is an increasing function of T . For fixed T , the power of both tests is increasing as N becomes larger. When $N \geq 3$, the two tests have good power in $T > 2500$.

²Remaining results are available upon request to the author.

It is interesting to note that, for fixed N and T , the power of both tests increases when the number of the degrees of freedom of t -distribution decreases. This is most apparent in Figures 3.5 and 3.7 where comparisons are made between a graph in the left column and a corresponding one on the right. In other words, the fatter the tails of an innovation distribution are, the more powerful are both tests.

If we compare panels in Figure 3.5 with those in Figure 3.7, or panels in Figure 3.6 with those in Figure 3.8, it can be seen that assuming dynamic conditional correlations slightly lowers the power of both tests. However, this effect is marginal and is vanishing as N or T or both become larger. Therefore, assuming dynamic conditional correlations has little effect in the power of the tests.

3.5 Applications

In this section, we apply the proposed NT test (standard and robust ones) as well as the benchmark HH test to a tri-variate system of stock returns. The data are downloaded from Yahoo!Finance Japan (<http://finance.yahoo.co.jp/>) for the three leading milk companies in Japan, namely, Snow Brand (Yukijirushi), Meiji and Morinaga. The sample period spans from April 1, 1993 to March 25, 2009, giving 3930 observations for each series. The descriptive statistics of the demeaned return series are summarised in Table 3.2.

We proceed to applying our proposed test to the return series. If the null hypothesis of no causality is rejected, the ECCC- and EDCC-GARCH models are estimated. We computed three test statistics, namely the NT, robust NT and HH tests, and the resulting test statistics and relevant p -values are reported in Table 3.3. Since the number of dimensions of the system at our hands is three ($N = 3$), the statistics are distributed, under the null of no causality in conditional variance, as a χ^2 distribution with $2N(N - 1) = 12$ degrees of freedom. The results show that the null of no causality in variance is rejected by all of the tests with very small p -values.

Given the fact that the null of no causality in conditional variance is rejected, the system is thought to be better explained by the extended version of the CCC- or DCC-GARCH model. We fit extended ECCC- and EDCC-GARCH models to the demeaned returns, and summarise the estimation results in Table 3.4. In Table 3.4, Panel A contains the estimates, their robust standard errors and the value of the log-likelihood function evaluated at the estimates for the ECCC-GARCH model, whereas Panel B does so for the EDCC-GARCH model. In both Panels A and B, $\lambda(\Gamma_C) < 1$, so that the stationarity condition of the GARCH equation (3.4) is satisfied. As for the ECCC-GARCH model, one can conclude that the estimate $\hat{a}_{23,ECCC} = 0.0360$ (0.0015) is statistically significant at the conventional levels of significance (robust standard errors in parenthesis). If we assume asymptotic normality of the estimators in the EDCC-GARCH model, the estimate $\hat{a}_{23,EDCC} = 0.0365$ (0.0016) is also significant. As such, the above-mentioned examples show the empirical usefulness of the proposed test.

3.6 Concluding remarks

In this paper, we propose an alternative test, the NT test, for causality in conditional variance. It is an extension and simplification of the LM_{ECCC} test by Nakatani and Teräsvirta (2009). We consider asymptotic properties of the 2SQML estimator of the parameters in the ECCC-GARCH model, through which the NT test has an asymptotic χ^2 distribution. Its finite sample properties are investigated through Monte Carlo experiments and compared with those of the LM_{ECCC} test as well as the HH test by Hafner and Herwartz (2006).

It turns out that the LM_{ECCC} test is size-distorted, meaning that it has limited effectiveness for testing of non-causality in variance. On the other hand, the simulation results revealed that the NT test has good finite sample properties both in size and power in larger dimensional systems. In addition, size and power of the NT test was little affected by distributional assumptions. Therefore, we can conclude that the NT test is robust against fatter tailed distributions and changing conditional correlations. Finally, empirical usefulness of the NT test was illustrated by applying it to a trivariate stock return series.

As mentioned earlier, procedures for computing test statistics of the NT test as well as the HH test are included in the package **ccgarch** (Nakatani, 2010) for the free statistical environment R. Therefore, the test should be readily available to everyone and it is hoped that the NT test would be a necessary diagnostic tool as a misspecification test of the GARCH equation in the CC-GARCH models.

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Table 3.1 Parameter Matrices for Simulation

$\mathbf{a} =$	$[0.02 \ 0.01 \ 0.03 \ 0.05 \ 0.09]'$
$\mathbf{A} =$	$\begin{bmatrix} 0.0400 & 0.0020 & 0.0030 & 0.0002 & 0.00005 \\ 0.0020 & 0.0200 & 0.0040 & 0.0040 & 0.00100 \\ 0.0010 & 0.0020 & 0.0600 & 0.0030 & 0.00400 \\ 0.0070 & 0.0024 & 0.0040 & 0.0100 & 0.00300 \\ 0.0010 & 0.0019 & 0.0040 & 0.0080 & 0.07000 \end{bmatrix}$
$\mathbf{B} =$	$\begin{bmatrix} 0.9100 & 0.0008 & 0.0008 & 0.0010 & 0.00050 \\ 0.0002 & 0.9300 & 0.0005 & 0.0020 & 0.00050 \\ 0.0009 & 0.0070 & 0.8900 & 0.0010 & 0.00040 \\ 0.0006 & 0.0001 & 0.0006 & 0.9400 & 0.00070 \\ 0.0006 & 0.0003 & 0.0005 & 0.0001 & 0.88000 \end{bmatrix}$
$\mathbf{P} =$	$\begin{bmatrix} 1.000 & & & & \\ 0.489 & 1.000 & & & \\ 0.853 & 0.289 & 1.000 & & \\ 0.534 & 0.323 & 0.399 & 1.000 & \\ 0.105 & 0.658 & 0.221 & 0.368 & 1.000 \end{bmatrix}$
$[\alpha \ \beta] =$	$[0.05 \ 0.93]$

Note: For the size studies, the diagonal entries of \mathbf{A} and \mathbf{B} are used.

Table 3.2 Descriptive statistics of the return series

	Snow Brand	Meiji	Morinaga
T	3930	3930	3930
Mean	0.000	0.000	0.000
Max	27.535	20.140	18.571
Min	-24.645	-14.607	-13.307
S.E.	2.407	2.407	1.951
Std. Sk	0.080	0.683	0.424
Rob. Sk	-0.072	-0.043	-0.068
Std. Kr	17.130	8.075	7.982
Rob. Kr	0.221	0.217	0.250

Note: S.E. stands for standard errors. Std. Sk and Std. Kr denote the standard Skewness and the standard excess Kurtosis, whereas Rob. Sk and Rob. Kr represent robust Skewness and robust excess Kurtosis based on quantiles; See Kim and White (2004) for details.

Table 3.3 Test statistics and associated p -values of the NT and HH tests

	NT	Robust NT	HH
Statistics	33.78	34.01	23.43
p -values	7×10^{-4}	7×10^{-4}	0.02

Note: Under the null of no causality in variance, all the test statistics are distributed as a χ^2 distribution with 12 degrees of freedom. The 5% critical value is 21.03.

Table 3.4 Estimation results for the trivariate CC-GARCH(1,1) models for the Japanese stock returns

Panel A: The ECCG-GARCH(1,1) Model											
$\begin{bmatrix} h_{1,t} \\ h_{2,t} \\ h_{3,t} \end{bmatrix}$	$=$	$\begin{bmatrix} 0.0118 \\ 0.0388 \\ 0.3154 \end{bmatrix}$	$+$	$\begin{bmatrix} 0.0483 \\ 4 \times 10^{-10} \\ 0.0138 \end{bmatrix}$	$\begin{bmatrix} 0.0052 \\ 0.0724 \\ 0.0360 \end{bmatrix}$	$\begin{bmatrix} 3 \times 10^{-16} \\ (1 \times 10^{-2}) \\ 0.0232 \\ (0.0144) \\ 0.1213 \\ (0.0024) \end{bmatrix}$	$+$	$\begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{2,t-1}^2 \\ \varepsilon_{3,t-1}^2 \end{bmatrix}$	$\begin{bmatrix} 0.9480 \\ 0.0011 \\ 3 \times 10^{-8} \end{bmatrix}$	$\begin{bmatrix} 4 \times 10^{-17} \\ (6 \times 10^{-3}) \\ 0.9055 \\ (0.0180) \\ 0.0399 \\ (0.0263) \end{bmatrix}$	$\begin{bmatrix} 8 \times 10^{-8} \\ (5 \times 10^{-3}) \\ 5 \times 10^{-7} \\ (2 \times 10^{-2}) \\ 0.6716 \\ (0.0733) \end{bmatrix}$
$\hat{\rho}_{12} = 0.3192,$	$\hat{\rho}_{13} = 0.2934,$	$\hat{\rho}_{23} = 0.4250,$	$\text{LogLik} = -24574.61,$	$\lambda(\Gamma_C) = 0.9975$							
(0.0190)	(0.0197)	(0.0170)									
Panel B: The EDCC-GARCH(1,1) Model											
$\begin{bmatrix} h_{1,t} \\ h_{2,t} \\ h_{3,t} \end{bmatrix}$	$=$	$\begin{bmatrix} 0.0198 \\ 0.0373 \\ 0.2303 \end{bmatrix}$	$+$	$\begin{bmatrix} 0.0550 \\ 7 \times 10^{-6} \\ 0.0091 \end{bmatrix}$	$\begin{bmatrix} 0.0086 \\ 0.0865 \\ 0.0365 \end{bmatrix}$	$\begin{bmatrix} 3 \times 10^{-5} \\ (1 \times 10^{-2}) \\ 0.0250 \\ (0.0176) \\ 0.1174 \\ (0.0036) \end{bmatrix}$	$+$	$\begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{2,t-1}^2 \\ \varepsilon_{3,t-1}^2 \end{bmatrix}$	$\begin{bmatrix} 0.9359 \\ 0.0011 \\ 3 \times 10^5 \end{bmatrix}$	$\begin{bmatrix} 5 \times 10^7 \\ (4 \times 10^3) \\ 0.8777 \\ (0.0174) \\ 0.0059 \\ (0.0224) \end{bmatrix}$	$\begin{bmatrix} 0.0011 \\ (0.0049) \\ 0.0183 \\ (0.0201) \\ 0.7548 \\ (0.0559) \end{bmatrix}$
$\hat{\alpha} = 0.0154,$	$\hat{\beta} = 0.9601,$	$\text{LogLik} = -31921.33,$	$\lambda(\Gamma_C) = 0.9937$								
(0.0033)	(0.0111)										

Note: The numbers in parenthesis are the robust standard errors. LogLik denotes the value of the log-likelihood function evaluated at the estimates. The stationarity condition for the GARCH equation is satisfied if $\lambda(\Gamma_C) < 1$; See Nakatani and Teräsvirta (2009) for details.

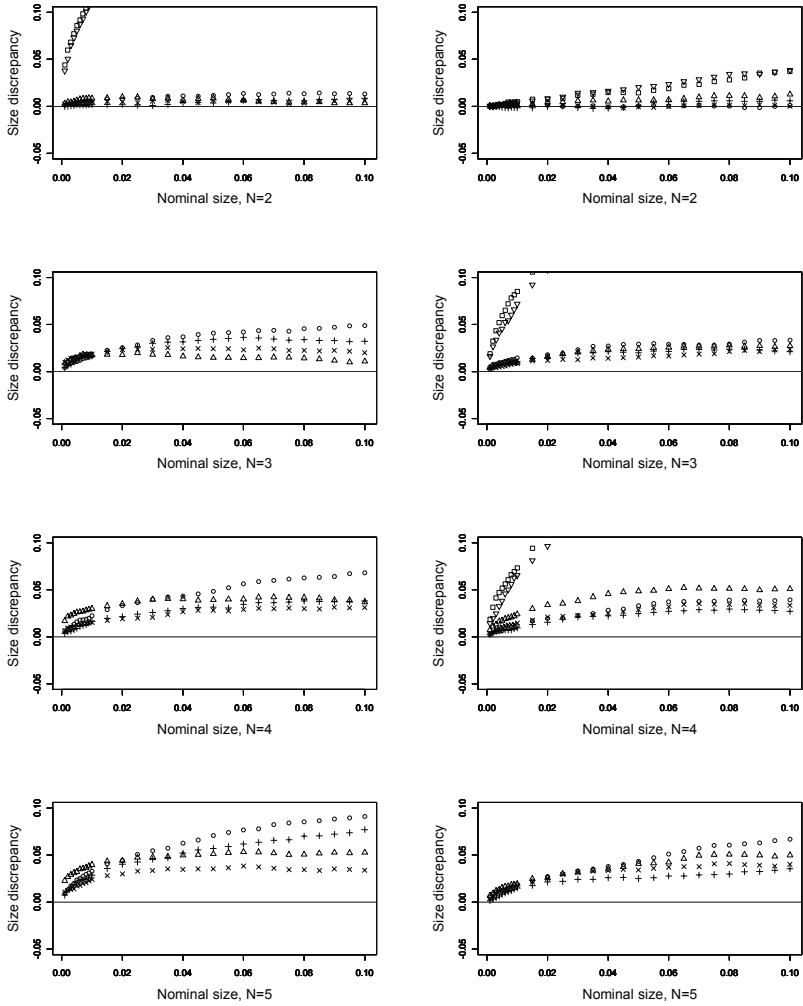


Figure 3.1 Results of size simulations of the tests using the CCC-GARCH(1,1) DGPs with $\mathbf{z}_t \sim t_8(\mathbf{0}, \mathbf{P})$ (left panels) and $\mathbf{z}_t \sim N(\mathbf{0}, \mathbf{P})$ (right panels) for four different numbers of dimensions (N). The size discrepancy is plotted against the nominal size. Markers correspond to Robust NT (\circ : $T = 2500$, $+$: $T = 10000$), HH (\triangle : $T = 2500$, \times : $T = 10000$) and LM_{ECCC} (\square : $T = 2500$, ∇ : $T = 10000$).

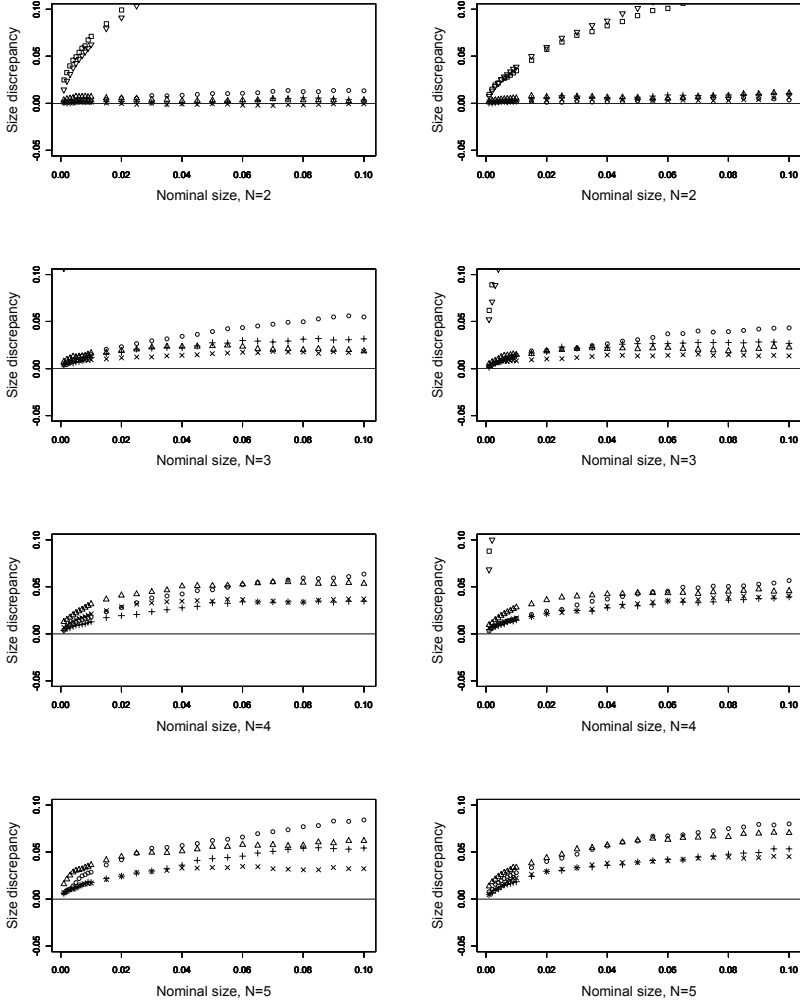


Figure 3.2 Results of size simulations of the tests using the CCC-GARCH(1,1) DGPs with $\mathbf{z}_t \sim t_{10}(\mathbf{0}, \mathbf{P})$ (left panels) and $\mathbf{z}_t \sim t_{15}(\mathbf{0}, \mathbf{P})$ (right panels) for four different numbers of dimensions (N). The size discrepancy is plotted against the nominal size. Markers correspond to Robust NT (\circ : $T = 2500$, $+$: $T = 10000$), HH (\triangle : $T = 2500$, \times : $T = 10000$) and LM_{ECCC} (\square : $T = 2500$, ∇ : $T = 10000$).

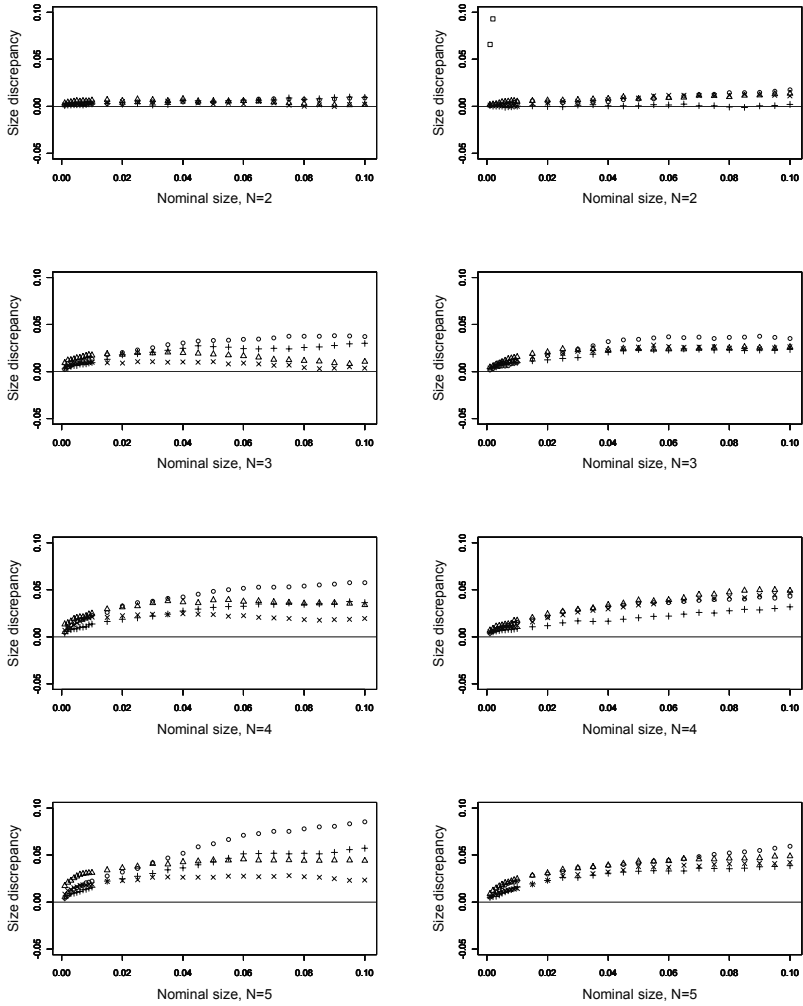


Figure 3.3 Results of size simulations of the tests using the DCC-GARCH(1,1) DGPs with $\mathbf{z}_t \sim t_8(\mathbf{0}, \mathbf{P}_t)$ (left panels) and $\mathbf{z}_t \sim N(\mathbf{0}, \mathbf{P}_t)$ (right panels) for four different numbers of dimensions (N). The size discrepancy is plotted against the nominal size. Markers correspond to Robust NT (\circ : $T = 2500$, $+$: $T = 10000$), HH (Δ : $T = 2500$, \times : $T = 10000$) and LM_{ECCC} (\square : $T = 2500$, ∇ : $T = 10000$).

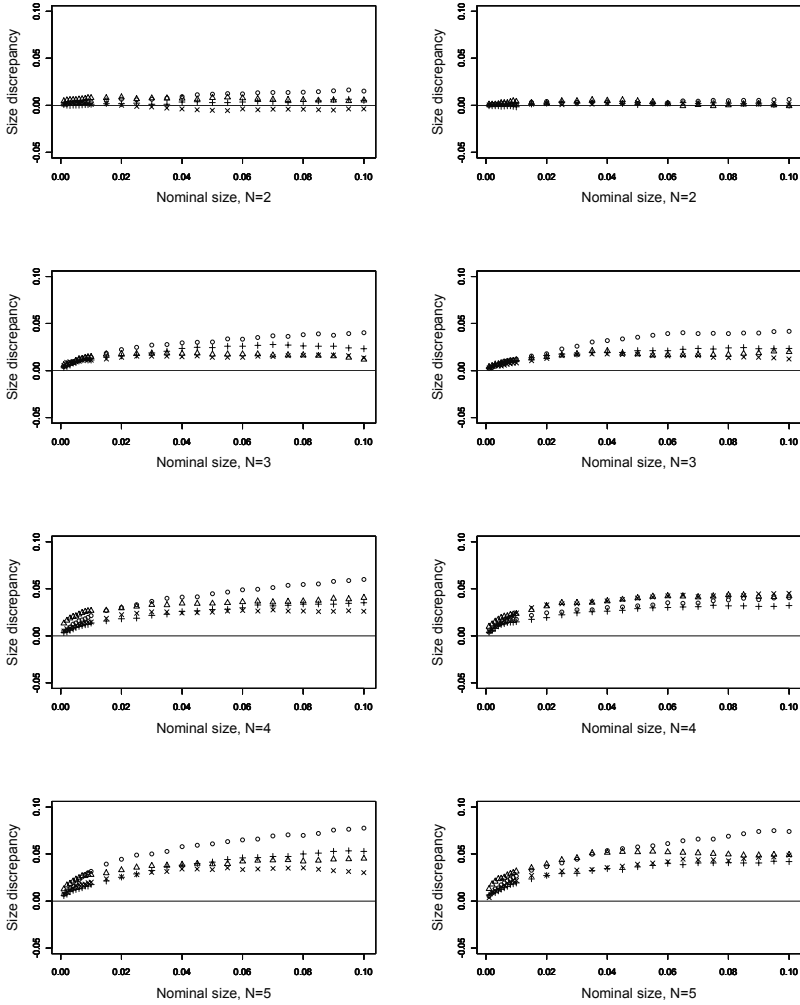


Figure 3.4 Results of size simulations of the tests using the DCC-GARCH(1,1) DGPs with $\mathbf{z}_t \sim t_{10}(\mathbf{0}, \mathbf{P}_t)$ (left panels) and $\mathbf{z}_t \sim t_{15}(\mathbf{0}, \mathbf{P}_t)$ (right panels) for four different numbers of dimensions (N). The size discrepancy is plotted against the nominal size. Markers correspond to Robust NT (\circ : $T = 2500$, $+$: $T = 10000$), HH (\triangle : $T = 2500$, \times : $T = 10000$) and LM_{ECC} (\square : $T = 2500$, ∇ : $T = 10000$).

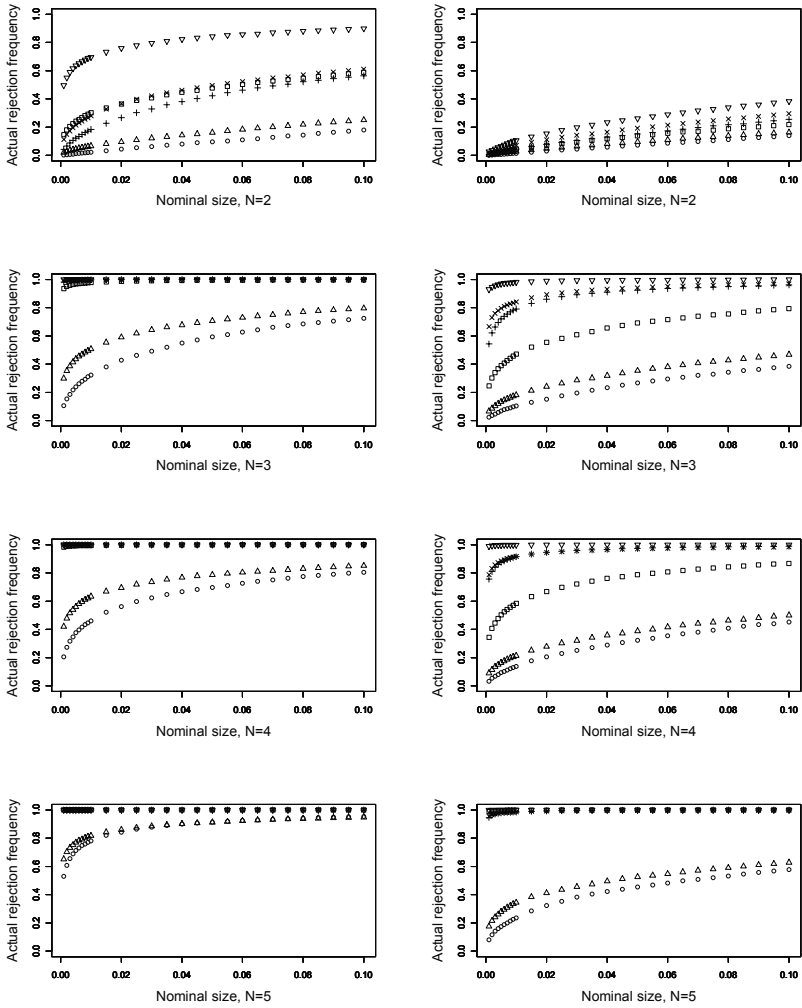


Figure 3.5 Results of power simulations of the test using the ECCC-GARCH(1,1) DGPs with $\mathbf{z}_t \sim t_8(\mathbf{0}, \mathbf{P})$ (left panels) and $\mathbf{z}_t \sim N(\mathbf{0}, \mathbf{P})$ (right panels) for four different numbers of dimensions (N). The actual rejection frequency is plotted against the nominal size. Markers correspond to Robust NT (\circ : $T = 2500$, $+$: $T = 10000$), HH (\triangle : $T = 2500$, \times : $T = 10000$) and LM_{ECCC} (\square : $T = 2500$, ∇ : $T = 10000$).

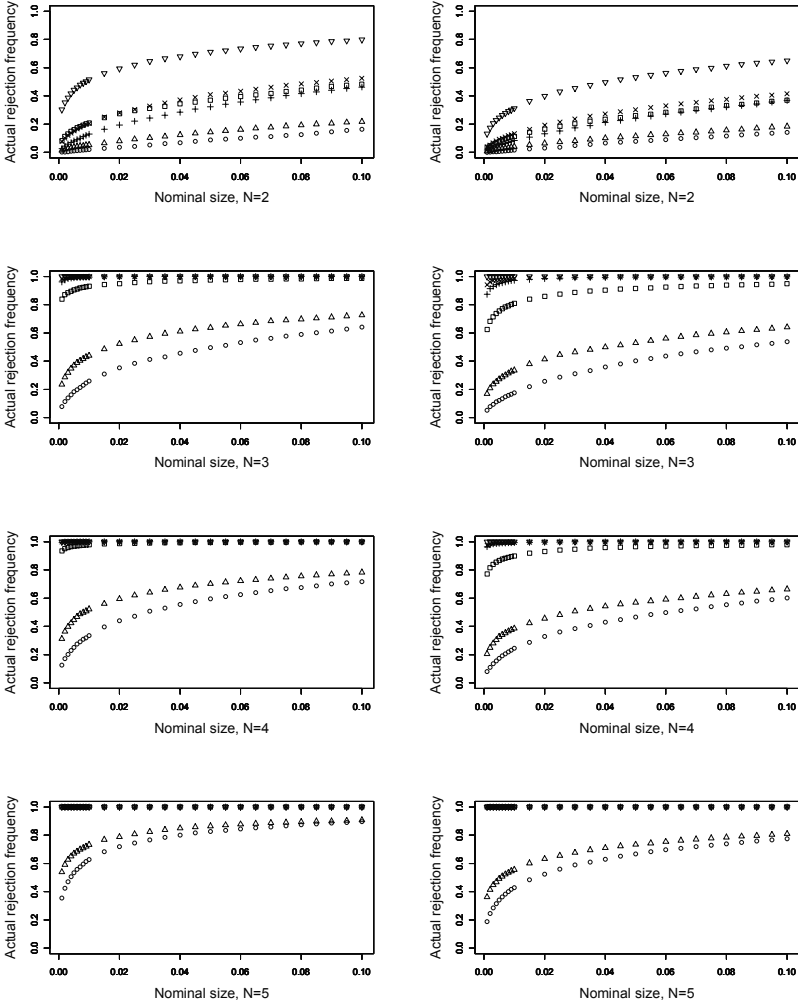


Figure 3.6 Results of power simulations of the tests using the ECCC-GARCH(1,1) DGPs with $\mathbf{z}_t \sim t_{10}(\mathbf{0}, \mathbf{P})$ (left panels) and $\mathbf{z}_t \sim t_{15}(\mathbf{0}, \mathbf{P})$ (right panels) for four different numbers of dimensions (N). The size discrepancy is plotted against the nominal size. Markers correspond to Robust NT (\circ : $T = 2500$, $+$: $T = 10000$), HH (\triangle : $T = 2500$, \times : $T = 10000$) and LM_{ECCC} (\square : $T = 2500$, ∇ : $T = 10000$).

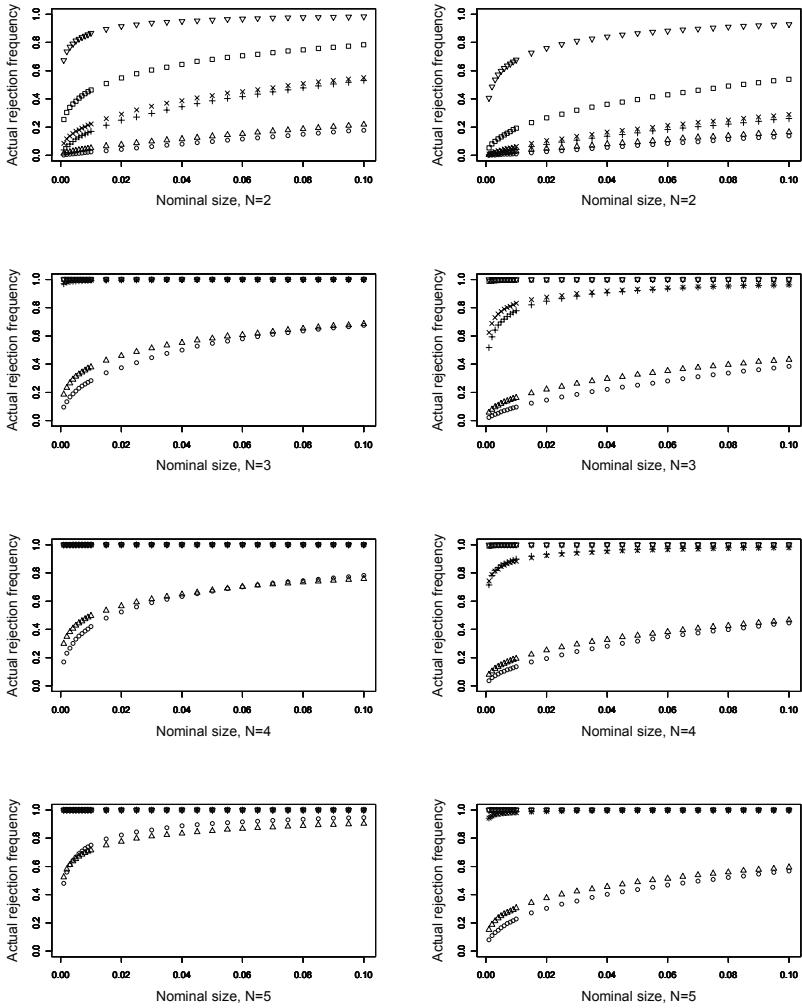


Figure 3.7 Results of power simulations of the test using the EDCC-GARCH(1,1) DGPs with $\mathbf{z}_t \sim t_8(\mathbf{0}, \mathbf{P}_t)$ (left panels) and $\mathbf{z}_t \sim N(\mathbf{0}, \mathbf{P}_t)$ (right panels) for four different numbers of dimensions (N). The actual rejection frequency is plotted against the nominal size. Markers correspond to Robust NT (\circ : $T = 2500$, $+$: $T = 10000$), HH (\triangle : $T = 2500$, \times : $T = 10000$) and LM_{ECC} (\square : $T = 2500$, ∇ : $T = 10000$).

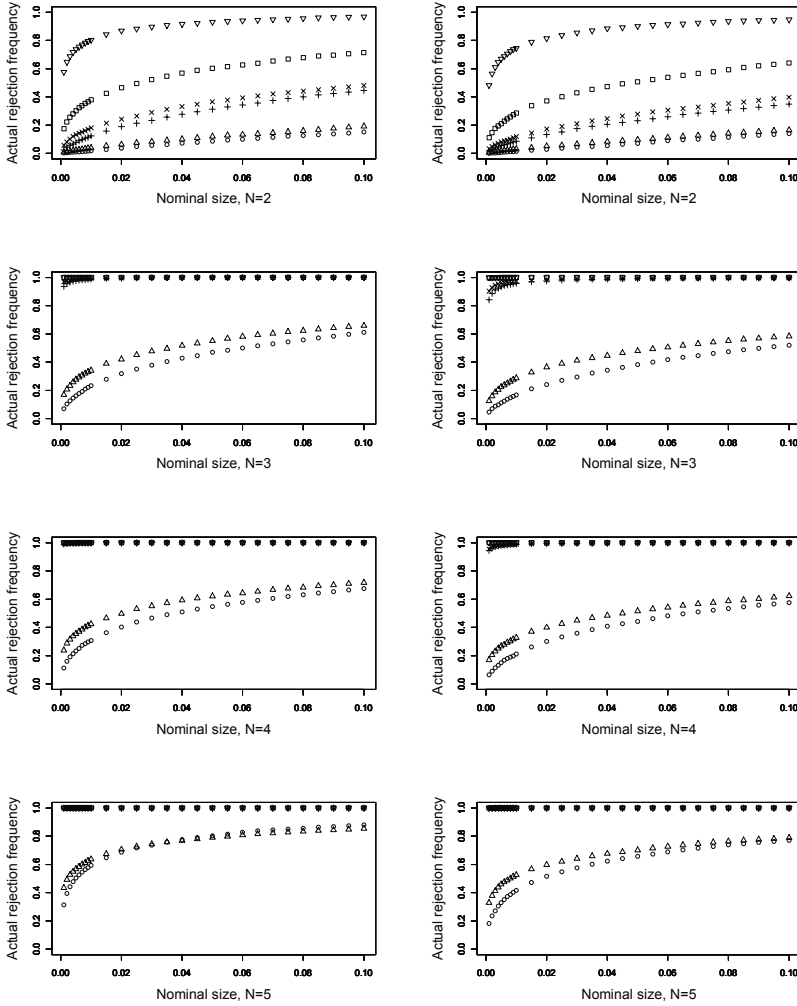


Figure 3.8 Results of power simulations of the tests using the EDCC-GARCH(1,1) DGPs with $\mathbf{z}_t \sim t_{10}(\mathbf{0}, \mathbf{P}_t)$ (left panels) and $\mathbf{z}_t \sim t_{15}(\mathbf{0}, \mathbf{P}_t)$ (right panels) for four different numbers of dimensions (N). The size discrepancy is plotted against the nominal size. Markers correspond to Robust NT (\circ : $T = 2500$, $+$: $T = 10000$), HH (\triangle : $T = 2500$, \times : $T = 10000$) and LM_{ECCC} (\square : $T = 2500$, ∇ : $T = 10000$).

Chapter 4

Positivity Constraints on the Conditional Variances in the Family of Conditional Correlation GARCH Models

Positivity Constraints on the Conditional Variances in the Family of Conditional Correlation GARCH Models^{1†}

Abstract

In this article, we derive a set of necessary and sufficient conditions for positivity of the vector conditional variance equation in multivariate GARCH models with explicit modelling of conditional correlation. These models include the constant conditional correlation GARCH model of Bollerslev (1990, *Review of Economics and Statistics*, 72, 498) and its extensions. Under the new conditions, it is possible to introduce negative volatility spillovers in the model. An empirical example illustrates usefulness of having such conditions in practice.

¹This is a joint work with Timo Teräsvirta. The chapter was published as Nakatani and Teräsvirta (2008), and is contained in this thesis under the Authors' rights granted by Elsevier.

[†]**Acknowledgements:** We wish to thank Christian Hafner for helpful comments on an earlier version of the paper. Responsibility for any errors and shortcomings remains ours. This research has been supported by the Jan Wallander and Tom Hedelius Foundation, Project No. P2005-0033:1, and the Danish National Research Foundation. Part of the research was conducted while I was visiting CREATES, Aarhus University, whose kind hospitality is gratefully acknowledged.

4.1 Introduction

Restrictions on parameters in univariate GARCH models are necessary in order to ensure the positivity of the conditional variance. Nelson and Cao (1992) and Tsai and Chan (2008) obtained necessary and sufficient conditions for such a positivity restriction in the standard GARCH(p, q) model of Bollerslev (1986). The positivity of the conditional variance is replaced with the positive definiteness of the conditional covariance matrix in multivariate GARCH models. However, except for particular models such as the BEKK-GARCH of Engle and Kroner (1995) and the matrix exponential GARCH of Kawakatsu (2006) that by construction provide the positive definiteness, establishing analytic necessary and sufficient conditions in general multivariate GARCH models seems complicated (He and Teräsvirta, 2002).

The constant conditional correlation GARCH of Bollerslev (1990) is yet another model with built-in positive definiteness of the conditional covariance matrix. The volatility part of this model is a vector analogue of Bollerslev's original GARCH(p, q) representation with the restriction that a conditional variance is modelled as a function of its own lagged squared innovations and conditional variances. Therefore, each conditional variance is usually described by a standard univariate GARCH(p, q) model, and because of this, the necessary and sufficient positivity conditions derived for the univariate model are directly applicable. This property is carried over to other multivariate GARCH representations with explicit modelling of conditional correlations, such as the varying correlation GARCH of Tse and Tsui (2002), the dynamic conditional correlation GARCH of Engle (2002), and the smooth transition conditional correlation GARCH model of Silvennoinen and Teräsvirta (2005).

A natural extension of this type of multivariate GARCH models is to allow for volatility interactions or spillovers; see, for instance, Jeantheau (1998) and Nakatani and Teräsvirta (2009). However, the necessary and sufficient positivity conditions for the standard univariate GARCH model are no longer valid because each of the extended conditional variance equations now contains terms for spillover effects as well that are described by lagged squared innovations and conditional variances of other variables in the system.

The aim of this paper is to provide necessary and sufficient positivity conditions of the extended conditional variance equation. Together with a positive definite conditional correlation matrix that can be time-varying, the derived conditions guarantee the positive definiteness of the conditional covariance matrix in multivariate conditional correlation (CC-) GARCH models.

The new “positivity” conditions relax a set of trivial sufficient conditions that all parameters in the extended conditional variance equations in a CC-GARCH model are non-negative. It will be shown that certain parameters associated with volatility interactions can take negative values while the conditional correlation matrix remains positive definite. The new conditions thus allow the existence of negative volatility spillover effects in the CC-GARCH framework. In this respect, the paper is best viewed as a multivariate extension of the work of Nelson and Cao (1992).

The organisation of the paper is as follows. Section 4.2 defines the general expression for the CC-GARCH model. The main results are derived in Section 4.3. As an

empirical illustration, we fit a bivariate CC-GARCH model with constant conditional correlation to stock returns in Section 4.4. Finally, Section 4.5 concludes.

4.2 The model and the positive definiteness of the covariance matrix

Consider the following vector stochastic process:

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t \quad (4.1)$$

$$\boldsymbol{\varepsilon}_t = \mathbf{D}_t \mathbf{z}_t \quad (4.2)$$

where $\mathbf{D}_t = \text{diag}(h_{1,t}^{1/2}, \dots, h_{N,t}^{1/2})$, the sequence of the stochastic vector $\mathbf{z}_t = [z_{1,t}, \dots, z_{N,t}]'$ is independently distributed with $E[\mathbf{z}_t | \mathcal{F}_{t-1}] = \mathbf{0}$ and $E[\mathbf{z}_t' \mathbf{z}_t | \mathcal{F}_{t-1}] = \mathbf{P}_t$ where \mathcal{F}_t is the information set up to and including time t , and $\mathbf{P}_t \in \mathcal{F}_{t-1}$.

With these assumptions, $E[\boldsymbol{\varepsilon}_t | \mathcal{F}_{t-1}] = \mathbf{0}$ and $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' | \mathcal{F}_{t-1}] = \mathbf{D}_t \mathbf{P}_t \mathbf{D}_t = \mathbf{H}_t$, so that \mathbf{H}_t and \mathbf{P}_t are the time-varying conditional covariance matrix and the time-varying positive definite conditional correlation matrix of the process $\boldsymbol{\varepsilon}_t$, respectively. Matrix \mathbf{H}_t has its elements given by

$$\mathbf{H}_t = \begin{cases} h_{i,t} & i = j, \\ h_{i,t}^{1/2} h_{j,t}^{1/2} \rho_{ij,t} & i \neq j. \end{cases} \quad (4.3)$$

Let $\mathbf{h}_t = [h_{1,t}, \dots, h_{N,t}]'$. Define the vector GARCH(p, q) process such that

$$\mathbf{h}_t = \mathbf{a}_0 + \sum_{i=1}^q \mathbf{A}_i \boldsymbol{\varepsilon}_{t-i}^{(2)} + \sum_{j=1}^p \mathbf{B}_j \mathbf{h}_{t-j} \quad (4.4)$$

where $\boldsymbol{\varepsilon}_t^{(2)} = [\varepsilon_{1,t}^2, \dots, \varepsilon_{N,t}^2]'$, \mathbf{a}_0 is an $(N \times 1)$ vector of constants, and \mathbf{A}_i and \mathbf{B}_j are $(N \times N)$ matrices with elements such that $h_{i,t}$ in \mathbf{h}_t are positive for all t . When \mathbf{A}_i and \mathbf{B}_j are diagonal, the positivity conditions for \mathbf{h}_t collapse into conditions for $h_{i,t} > 0$, given in Nelson and Cao (1992). Furthermore,

$$\mathbf{P}_t = \mathbf{P}(\mathbf{x}_t, \boldsymbol{\phi}) \quad (4.5)$$

where $\mathbf{P}(\cdot)$ is a function such that \mathbf{P}_t remains a positive definite correlation matrix, $\mathbf{x}_t \in \mathcal{F}_{t-1}$ is a vector of explanatory variables, and $\boldsymbol{\phi}$ is a vector of parameters. Equations (4.1) through (4.5) define a class of N -dimensional CC-GARCH(p, q) models.

A number of different specifications are possible for both \mathbf{h}_t and \mathbf{P}_t such that \mathbf{H}_t and \mathbf{P}_t are positive definite for all t . Modelling the dynamic behaviour of the conditional correlation matrix (4.5) has recently attracted much attention. However, we leave the form of \mathbf{P}_t unspecified and simply assume that \mathbf{P}_t is positive definite for all t .

He and Teräsvirta (2002) considered conditions for \mathbf{H}_t in a general vector ARCH model to be positive definite. Recently, Gouriéroux (2007) derived necessary and

sufficient conditions for the positive definiteness of the conditional covariance matrix in a bivariate autoregressive specification. Although in principle their results can be extended to a general vector GARCH model, the authors point out that the extension seems infeasible because of the tedious algebra involved. But then, in the CC-GARCH model, the situation is less complicated as the following proposition (Bollerslev, 1990) demonstrates:

Proposition 4.1 *Suppose that the conditional correlation matrix \mathbf{P}_t is positive definite for all t . Matrix \mathbf{H}_t is positive definite for all t if and only if \mathbf{D}_t is well defined, that is, $\mathbf{h}_t > \mathbf{0}$ elementwise³ for all t .*

By Proposition 4.1, the positive definiteness of \mathbf{H}_t in the CC-GARCH model solely rests on the positivity of \mathbf{h}_t , provided that \mathbf{P}_t is positive definite. A set of trivial sufficient conditions for $\mathbf{h}_t > \mathbf{0}$ is that $\mathbf{a}_0 > \mathbf{0}$, $\mathbf{A}_i \geq \mathbf{0}$ and $\mathbf{B}_j \geq \mathbf{0}$ for each i and j . As we shall show, considerations analogous to the ones in Nelson and Cao (1992) enable us to find necessary and sufficient conditions for $\mathbf{h}_t > \mathbf{0}$ in the class of CC-GARCH models.

4.3 Main results

4.3.1 CC-GARCH(p, q) model

We begin by rewriting the CC-GARCH(p, q) equation (4.4) as

$$\mathbf{B}(L)\mathbf{h}_t = \mathbf{a}_0 + \mathbf{A}(L)\varepsilon_t^{(2)} \quad (4.6)$$

where $\mathbf{B}(L) = \mathbf{I}_N - \sum_{j=1}^p \mathbf{B}_j L^j$, $\mathbf{A}(L) = \sum_{i=1}^q \mathbf{A}_i L^i$, L is the lag operator and \mathbf{I}_N denotes the N -dimensional identity matrix. For (4.6) to have the vector ARCH(∞) form, we make the following assumptions:

A1: $|\mathbf{B}(z)| \neq 0$ for $|z| \leq 1$.

A2: The CC-GARCH(p, q) model of (4.6) is minimal⁴ in the sense of Jeantheau (1998, Definition 3.3).

A3: $|\mathbf{I}_N - \sum_{j=1}^p \mathbf{B}_j z^j - \sum_{i=1}^q \mathbf{A}_i z^i| \neq 0$ for $|z| \leq 1$.

Under these assumptions, the ARCH(∞) representation of (4.6) is available and is given by $\mathbf{h}_t = \mathbf{a}_0^* + \Psi(L)\varepsilon_t^{(2)}$ where $\mathbf{a}_0^* = \mathbf{B}(1)^{-1}\mathbf{a}_0$ and $\Psi(L) = \sum_{k=0}^{\infty} \Psi_k L^k = \mathbf{B}(L)^{-1}\mathbf{A}(L)$. With this representation, necessary and sufficient conditions for $\mathbf{h}_t > \mathbf{0}$ are obvious and summarised in Theorem 4.2.

³Inequalities associated with matrices or vectors denote elementwise relation throughout.

⁴Assumption A2 is the identifiability condition of the model (4.6). It rules out, among others, a common left divisor of $\mathbf{A}(L)$ and $\mathbf{B}(L)$.

Theorem 4.2 *Let $\mathbf{A}_k = \mathbf{0}$ for $k > q$, $\mathbf{B}_k = \mathbf{0}$ for $k > p$ and $\Psi_k = \mathbf{0}$ for $k \leq 0$ and assume that A1, A2 and A3 hold. Then $\mathbf{h}_t > \mathbf{0}$ if and only if for all $k \in \mathbb{N}$*

$$\mathbf{a}_0^* = \mathbf{B}(1)^{-1} \mathbf{a}_0 > \mathbf{0} \quad (4.7)$$

$$\Psi_k = \mathbf{B}_1 \Psi_{k-1} + \mathbf{B}_2 \Psi_{k-2} + \cdots + \mathbf{B}_{k-1} \Psi_1 + \mathbf{A}_k \geq \mathbf{0}. \quad (4.8)$$

It follows from (4.8) with $k = 1$ that the elements of \mathbf{A}_1 have to remain positive regardless of the orders, p and q . Since $k \in \mathbb{N}$, Theorem 4.2 involves infinitely many difference equations. Expressing these conditions by a finite number of parameter constraints is possible in the CC-GARCH(1,1) model. However, tedious algebra makes it hard to generalise them for higher-order models. We shall consider the simplest case which is the bivariate CC-GARCH(1,1) model. For this model, analytic results are readily available.

4.3.2 CC-GARCH(1,1) model

The conditional variance equation of the CC-GARCH(1,1) model has the following form:

$$\mathbf{h}_t = \mathbf{a}_0 + \mathbf{A}_1 \varepsilon_{t-1}^{(2)} + \mathbf{B}_1 \mathbf{h}_{t-1}. \quad (4.9)$$

The next corollary is an immediate consequence of Theorem 4.2.

Corollary 4.3 *Let A1, A2 and A3 hold for $p = q = 1$. In the CC-GARCH(1,1) model, $\mathbf{h}_t > \mathbf{0}$ for all t if and only if*

$$(\mathbf{I} - \mathbf{B}_1)^{-1} \mathbf{a}_0 > \mathbf{0} \quad (4.10)$$

$$\mathbf{B}_1^{k-1} \mathbf{A}_1 \geq \mathbf{0}, \forall k \in \mathbb{N}. \quad (4.11)$$

If \mathbf{B}_1 and \mathbf{A}_1 are assumed to be diagonal, the trivial sufficient conditions are also necessary. In contrast, Corollary 4.3 allows some entries in \mathbf{B}_1 to take negative values once the diagonality assumption is removed.

Further simplification is available in the bivariate case. With the reparameterisation shown in the Appendix, it is summarised as follows:

Corollary 4.4 *In addition to the assumptions in Corollary 4.3, let the following assumptions hold:*

A4: The diagonal elements b_{11} and b_{22} of \mathbf{B}_1 are positive, and $|\mathbf{B}_1| > 0$,

A5: \mathbf{B}_1 has two distinct real eigenvalues.

Then (4.11) is satisfied if and only if

$$\begin{cases} c_{ij,1} > 0 \\ c_{ij,1} + c_{ij,2} > 0 \end{cases} \quad i, j = 1, 2 \quad (4.12)$$

where $c_{ij,1}$ and $c_{ij,2}$, $i, j = 1, 2$, are defined in the Appendix. The proof can be found in the Appendix. For higher-dimensional models, an analogous simple structure does

not seem to be available. Instead, our recommendation is to check the validity of both (4.10) and (4.11) numerically during optimisation.

It is not unrealistic to assume that b_{11} and b_{22} be positive and that $|\mathbf{B}_1| = b_{11}b_{22} - b_{12}b_{21} > 0$, because in applications off-diagonal elements typically take values one magnitude smaller in absolute value than the positive diagonal ones. In addition to the conditions required for satisfying (4.10), there are four restrictions in the first-order bivariate model. This is useful in applications because the restrictions can then easily be incorporated in an appropriate algorithm for maximum likelihood estimation.

4.4 Empirical example

As an empirical illustration, we fit a bivariate CC-GARCH(1, 1) model with the constant conditional correlation to a pair of daily Japanese stock returns under the two different sets of conditions. The series consist of daily closing prices of Toshiba and Hitachi, two leading companies in the Japanese heavy industry, traded at the Tokyo Stock Exchange. A return is computed by the log-price difference for two consecutive trading days multiplied by 100. The time series extend from January 4, 1983 to March 1, 2006, which amounts to 5913 observations for each return series.

The estimation results⁵ can be found in Table 4.1. All the estimates satisfy both (4.10) and (4.11). The maximum of the log-likelihood increases when the restrictions requiring all coefficients to be non-negative are no longer binding. The most conspicuous change occurs in the estimate for b_{21} . Under the trivial restrictions, it is a small positive number (3×10^{-8}) and statistically insignificant but turns negative (-0.047) and significant under our new conditions. At the same time, the estimates of the other off-diagonal elements both in \mathbf{A}_1 and \mathbf{B}_1 become larger, whereas the estimate of the constant conditional correlation remains unchanged (0.611 for both sets of conditions).

4.5 Concluding remarks

In this article, we consider necessary and sufficient conditions on parameter matrices to ensure the positive definiteness of the conditional covariance matrix in the class of CC-GARCH models. These models can be extended to have parameters that describe interactions between volatilities. With any such extension, assuming all parameters positive is a trivial sufficient condition for the positive definiteness of the covariance matrix. We derive a set of necessary and sufficient conditions for the positive definiteness by considering the positivity of the individual conditional variances. We show that these conditions are expressed in a relatively simple analytic form when the model is a bivariate CC-GARCH(1,1) model. The conditions become more complicated with the increasing dimension of the model. Validity of these conditions is checked in estimation, either analytically in the bivariate model or numerically in higher-order models.

⁵Numerical optimisations are carried out with the free statistical environment R ver.2.5.1 (R Development Core Team, 2007).

As an empirical example, we fit a bivariate CC-GARCH(1,1) model with the constant conditional correlation to a pair of daily Japanese stock return series. The results show that the model estimated with our necessary and sufficient conditions fits the data better than the model estimated under the trivial conditions. It also captures both negative and positive volatility spillover effects. Consequently, we conclude that in modelling and forecasting volatility it is important to take into account the interaction effects among volatilities, and, in particular, to allow for negative as well as positive effects.

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4.A Reparameterisation of the model and proof of Corollary 4.4

We begin by presenting the reparameterisation. Applying the Jordan decomposition to the 2×2 matrix \mathbf{B}_1 yields

$$\begin{aligned} \mathbf{B}_1^{k-1} \mathbf{A}_1 &= \mathbf{\Gamma} \mathbf{\Lambda}^{k-1} \mathbf{\Gamma}^{-1} \mathbf{A}_1 \\ &= \begin{bmatrix} 1 & \gamma_2 \\ \gamma_1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1^{k-1} & 0 \\ 0 & \lambda_2^{k-1} \end{bmatrix} \begin{bmatrix} 1 & \gamma_2 \\ \gamma_1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \\ &= \left[\sum_{n=1}^2 c_{ij,n} \lambda_n^{k-1} \right] \end{aligned} \quad (4.13)$$

where $\mathbf{\Gamma}$ is a matrix of eigenvectors of \mathbf{B}_1 with ones on diagonal entries and $\mathbf{\Lambda}$ is a diagonal matrix of eigenvalues ($\lambda_1 > \lambda_2$) of \mathbf{B}_1 . Furthermore, denote

$$c_{11,1} = (a_{11} - a_{21}\gamma_2)/(1 - \gamma_1\gamma_2) \quad (4.14)$$

$$c_{11,2} = \gamma_2 c_{21,2} \quad (4.15)$$

$$c_{21,1} = \gamma_1 c_{11,1} \quad (4.16)$$

$$c_{21,2} = (a_{21} - a_{11}\gamma_1)/(1 - \gamma_1\gamma_2) \quad (4.17)$$

$$c_{12,1} = (a_{12} - a_{22}\gamma_2)/(1 - \gamma_1\gamma_2) \quad (4.18)$$

$$c_{12,2} = \gamma_2 c_{22,2} \quad (4.19)$$

$$c_{22,1} = \gamma_1 c_{12,1} \quad (4.20)$$

$$c_{22,2} = (a_{22} - a_{12}\gamma_1)/(1 - \gamma_1\gamma_2). \quad (4.21)$$

The proof proceeds as follows. By A1, A4 and A5, we have $0 < \lambda_2 < \lambda_1 < 1$. It follows that the terms in the sequence $(\lambda_2/\lambda_1)^{k-1}$, $k = 1, 2, \dots$, are positive and converge monotonically to zero as $k \rightarrow \infty$. Therefore,

$$c_{ij,1}\lambda_1^{k-1} + c_{ij,2}\lambda_2^{k-1} > 0 \iff c_{ij,1} + c_{ij,2}(\lambda_2/\lambda_1)^{k-1} > 0.$$

When $c_{ij,2} > 0$,

$$c_{ij,2} > c_{ij,2}(\lambda_2/\lambda_1) > \dots > c_{ij,2}(\lambda_2/\lambda_1)^{k-1} > 0$$

so that by adding $c_{ij,1} > 0$

$$c_{ij,1} + c_{ij,2}(\lambda_2/\lambda_1)^{k-1} > c_{ij,1} > 0.$$

But then, for $c_{ij,2} < 0$ one obtains

$$c_{ij,2} < \dots < c_{ij,2}(\lambda_2/\lambda_1)^{k-1},$$

and adding $c_{ij,1} > 0$ gives

$$0 < c_{ij,1} + c_{ij,2} < \dots < c_{ij,1} + c_{ij,2}(\lambda_2/\lambda_1)^{k-1}.$$

This proves the sufficiency of (4.12).

To show that these conditions are necessary, let $c_{ij,2} > 0$. Then

$$c_{ij,1} + c_{ij,2} > \cdots > c_{ij,1} + c_{ij,2}(\lambda_2/\lambda_1)^{k-1} > 0$$

and

$$\liminf_{k \rightarrow \infty} \{c_{ij,1} + c_{ij,2}(\lambda_2/\lambda_1)^{k-1}\} = c_{ij,1} > 0.$$

When $c_{ij,2} < 0$

$$c_{ij,1} > c_{ij,1} + c_{ij,2}(\lambda_2/\lambda_1)^{k-1} > \cdots > c_{ij,1} + c_{ij,2} > 0.$$

This concludes the proof. ■

Table 4.1 Estimation results of the bivariate CC-GARCH(1,1) model for the Japanese stock returns

Panel A: the trivial sufficient conditions

$$\begin{bmatrix} h_{1,t} \\ h_{2,t} \end{bmatrix} = \begin{bmatrix} 0.0997 \\ (0.0205) \\ 0.0416 \\ (0.0128) \end{bmatrix} + \begin{bmatrix} 0.0845 & 0.0295 \\ (0.0097) & (0.0282) \\ 0.0201 & 0.0528 \\ (0.0325) & (0.0126) \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} 0.8509 & 0.0197 \\ (0.0097) & (0.0226) \\ 3 \times 10^{-8} & 0.9154 \\ (0.0079) & (0.0198) \end{bmatrix} \begin{bmatrix} h_{1,t-1} \\ h_{2,t-1} \end{bmatrix}$$

$$\hat{\rho} = 0.6107, \quad \lambda(\mathbf{\Gamma}_C) = 0.9873, \quad \text{LogLik} = -23018.11$$

(0.0091)

Panel B: the necessary and sufficient conditions

$$\begin{bmatrix} h_{1,t} \\ h_{2,t} \end{bmatrix} = \begin{bmatrix} 0.1288 \\ (0.0252) \\ 0.0541 \\ (0.0135) \end{bmatrix} + \begin{bmatrix} 0.1018 & 0.0350 \\ (0.0100) & (0.0257) \\ 0.0341 & 0.0394 \\ (0.0315) & (0.0118) \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} 0.8093 & 0.0353 \\ (0.0087) & (0.0197) \\ -\mathbf{0.0467} & 0.9627 \\ (0.0056) & (0.0142) \end{bmatrix} \begin{bmatrix} h_{1,t-1} \\ h_{2,t-1} \end{bmatrix}$$

$$\hat{\rho} = 0.6109, \quad \lambda(\mathbf{\Gamma}_C) = 0.9909, \quad \text{LogLik} = -23007.27$$

(0.0089)

Note: Numbers in parenthesis are the robust standard errors. $\lambda(\mathbf{\Gamma}_C) < 1$ is the stationarity condition, see Chapter 2 for details. LogLik denotes the value of the log-likelihood function evaluated at the estimates.

Chapter 5

ccgarch: An R Package for Building Multivariate GARCH Models with Conditional Correlations

ccgarch: An R Package for Building Multivariate GARCH Models with Conditional Correlations*

Abstract

This paper contains an introduction to the package **ccgarch** that is developed for use in the open source statistical environment R (R Development Core Team, 2009). **ccgarch** can estimate certain types of multivariate GARCH models with explicit modelling of conditional correlations (the CC-GARCH models). The package is also capable of simulating data from major types of the CC-GARCH models with multivariate normal or Student's t innovations. Small Monte Carlo simulations are conducted to see how the choice of initial values affects the parameter estimates in maximum likelihood estimation. The usefulness of the package is illustrated by fitting a trivariate Dynamic CC-GARCH model to daily stock returns series.

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5.1 Introduction

Multivariate GARCH models with explicit modelling of conditional correlations (hereafter called CC-GARCH models) have been widely used in modelling high-frequency financial time series. Examples include the Constant Conditional Correlation (CCC-) GARCH of Bollerslev (1990), the Dynamic Conditional Correlation (DCC-) GARCH of Engle (2002), the Varying Correlation (VC-) GARCH of Tse and Tsui (2002), the Smooth Transition Conditional Correlation (STCC-) and the Double STCC (DSTCC-) GARCH of Silvennoinen and Teräsvirta (2005, 2009a) and their extensions to allowing for volatility spillovers (see Nakatani and Teräsvirta, 2009). For recent reviews of multivariate GARCH literature, see, for example, Bauwens, Laurent, and Rombouts (2006) and Silvennoinen and Teräsvirta (2009b).

This paper contains an introduction to the package **ccgarch** (Nakatani, 2010) that is developed for use in the open source statistical environment R (R Development Core Team, 2009). In applying the CC-GARCH models, one generally has to resort to existing commercial packages or has to spend a substantial amount of time to create his or her own codes. The purpose of the package **ccgarch** is to reduce such an effort and give the users free access to software with which to handle a number of CC-GARCH models.

Currently, there are some contributed R packages that can handle the standard univariate GARCH model and its variants¹. As for multivariate GARCH models, on the other hand, **ccgarch** and **gogarch** (Pfaff, 2009) are the only registered packages at CRAN.

The paper is organised as follows. The next section reviews the CC-GARCH models considered in **ccgarch**. The contents of **ccgarch** are described in Section 5.3. How to use the package is illustrated in Section 5.4 by fitting a trivariate DCC-GARCH model to daily Japanese stock market data. Section 5.5 concludes.

5.2 The model

Nakatani and Teräsvirta (2008b) define the family of CC-GARCH models as follows. Consider the following vector stochastic process:

$$\mathbf{y}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t \quad (5.1)$$

$$\boldsymbol{\varepsilon}_t = \mathbf{D}_t \mathbf{z}_t \quad (5.2)$$

where $\mathbf{D}_t = \text{diag}(h_{1,t}^{1/2}, \dots, h_{N,t}^{1/2})$, the sequence of the stochastic vector $\mathbf{z}_t = [z_{1,t}, \dots, z_{N,t}]'$ is independently distributed with $E[\mathbf{z}_t | \mathcal{F}_{t-1}] = \mathbf{0}$ and $E[\mathbf{z}_t' \mathbf{z}_t | \mathcal{F}_{t-1}] = \mathbf{P}_t = [\rho_{ij,t}]$ where \mathcal{F}_t is the information set up to and including time t , and $\mathbf{P}_t \in \mathcal{F}_{t-1}$. With these assumptions, $E[\boldsymbol{\varepsilon}_t | \mathcal{F}_{t-1}] = \mathbf{0}$ and $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' | \mathcal{F}_{t-1}] = \mathbf{D}_t \mathbf{P}_t \mathbf{D}_t = \mathbf{H}_t$, so that \mathbf{H}_t and \mathbf{P}_t are the time-varying conditional covariance matrix and the time-varying positive definite conditional correlation matrix of the process $\boldsymbol{\varepsilon}_t$, respectively. Matrix \mathbf{H}_t has its (i, j)

¹See Comprehensive R Archive Network (CRAN) Task View: Empirical Finance at <http://cran.r-project.org/web/views/Finance.html> for details

element defined as $h_{i,t}$ if $i = j$ and $h_{i,t}^{1/2} h_{j,t}^{1/2} \rho_{ij,t}$ otherwise. A time-varying conditional correlation matrix at time t is of a general form given by

$$\mathbf{P}_t = \mathbf{P}(\mathbf{x}_t, \phi) \quad (5.3)$$

where $\mathbf{P}(\cdot)$ is a function such that \mathbf{P}_t remains a positive definite correlation matrix, $\mathbf{x}_t \in \mathcal{F}_{t-1}$ is a vector of observable explanatory variables², and ϕ is a vector of parameters. Equations (5.1) through (5.3) define a class of N -dimensional CC-GARCH(p, q) models. For simplicity, we assume $p = q = 1$ in the rest of the paper.

The log-likelihood of the CC-GARCH model at time t is in general given by

$$\ell_t(\theta) = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{D}_t \mathbf{P}_t \mathbf{D}_t| - \frac{1}{2} \varepsilon_t' \mathbf{D}_t^{-1} \mathbf{P}_t^{-1} \mathbf{D}_t^{-1} \varepsilon_t. \quad (5.4)$$

A number of different specifications are possible for both $\mathbf{h}_t = [h_{1,t}, \dots, h_{N,t}]'$ and \mathbf{P}_t such that \mathbf{H}_t and \mathbf{P}_t are positive definite for all t .

In the subsequent sections, we first look at the volatility component of the model, and then consider three major representations of the conditional correlation matrix (5.3).

5.2.1 The volatility component of the model

Here we focus on its simplest specification, namely, a multivariate counterpart of the univariate GARCH(1, 1) model. Define the vector GARCH(1, 1) process such that

$$\mathbf{h}_t = \mathbf{a}_0 + \mathbf{A} \varepsilon_{t-1}^{(2)} + \mathbf{B} \mathbf{h}_{t-1} \quad (5.5)$$

where $\varepsilon_t^{(2)} = [\varepsilon_{1,t}^2, \dots, \varepsilon_{N,t}^2]'$, \mathbf{a}_0 is an $(N \times 1)$ vector of positive constants, and \mathbf{A} and \mathbf{B} are $(N \times N)$ matrices with elements such that the elements of $\mathbf{h}_t = (h_{1,t}, \dots, h_{N,t})'$ are positive for all t . Nakatani and Teräsvirta (2008b) give conditions that allow negative off-diagonal elements in \mathbf{B} . If \mathbf{A} and \mathbf{B} are diagonal and if the model is of order (1, 1) as in (5.5), $h_{i,t}$ are then GARCH (1, 1) processes, and the necessary and sufficient positivity conditions already appeared in Bollerslev (1986) ($a_{0i} > 0$, $a_{1i} > 0$ and $b_{1i} > 0$, $i = 1, \dots, N$). For simplicity, however, we assume that all the elements in \mathbf{A} and \mathbf{B} are positive.

Assuming \mathbf{A} and \mathbf{B} diagonal in (5.5) rules out the possibility of volatility spillovers. In the bivariate case, the model can then be written as

$$\mathbf{h}_t = \begin{bmatrix} a_{01} \\ a_{02} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{1,t-1} \\ h_{2,t-1} \end{bmatrix}. \quad (5.6)$$

Most of the previous literature adopts this diagonal specification in the context of CC-GARCH models. On the other hand, a natural extension of (5.6), proposed by Jeantheau (1998), does allow volatility spillovers:

$$\mathbf{h}_t = \begin{bmatrix} a_{01} \\ a_{02} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} h_{1,t-1} \\ h_{2,t-1} \end{bmatrix}. \quad (5.7)$$

²This formulation therefore excludes the Regime Switching Dynamic Correlation GARCH model of Pelletier (2006).

Spillover effects are described through the off-diagonal elements of \mathbf{A} and \mathbf{B} . Using Jeantheau’s CCC-GARCH model, Nakatani and Teräsvirta (2009) find statistically significant volatility spillovers in both stock and foreign exchange markets. Their results justify such an extension of the standard CCC-GARCH model. The package `ccgarch` covers extended CC-GARCH models as well.

5.2.2 The correlation component of the model

The CCC-GARCH model

The simplest way of modelling the correlation structure is to assume a constant relationship among variables in the model. The CCC-GARCH model is proposed by Bollerslev (1990) and is later extended by Jeantheau (1998) as described in the previous section. In the (Extended) CCC-GARCH model the conditional correlation matrix is constant over time, that is,

$$\mathbf{P}_t \equiv \mathbf{P}. \tag{5.8}$$

Bollerslev (1990) suggests a two-step procedure for estimating a CCC-GARCH model. First, estimate the parameters of the GARCH equation. Then, conditionally on the GARCH parameter estimates, estimate the correlations. Asymptotic normality of the estimators can be achieved through the two step estimation method, so that it has become a standard procedure for estimating CC-GARCH models. One can also estimate all parameters of an (E)CCC-GARCH model simultaneously. The latter method gains efficiency. The log-likelihood function of the CCC-GARCH model at time t is obtained by replacing \mathbf{P}_t in (5.4) with \mathbf{P} . This facilitates numerical optimisation of the log-likelihood function because inversion of the correlation matrix, hence the covariance matrix, is required only once in each iteration.

The package `ccgarch` carries out estimation of the (E)CCC-GARCH(1,1) model. In addition, it includes a function for simulating data from an (E)CCC-GARCH(1,1) process.

The DCC-GARCH model

It turns out that the assumption of constant conditional correlations over time is often too restrictive in practice, and for this reason many authors have proposed models of time-varying conditional correlations. The DCC-GARCH model of Engle (2002) is one of the most popular CC-GARCH models with time-varying conditional correlations. Engle (2002) applies GARCH-type dynamics in modelling the conditional correlations. Its correlation structure is defined as

$$\mathbf{P}_t = (\mathbf{Q}_t \odot \mathbf{I}_N)^{-1/2} \mathbf{Q}_t (\mathbf{Q}_t \odot \mathbf{I}_N)^{-1/2} \tag{5.9}$$

$$\mathbf{Q}_t = (1 - \alpha - \beta) \mathbf{Q} + \alpha \mathbf{z}_{t-1} \mathbf{z}'_{t-1} + \beta \mathbf{Q}_{t-1} \tag{5.10}$$

$$\alpha + \beta < 1 \quad \text{and} \quad \alpha, \beta > 0 \tag{5.11}$$

where \odot denotes the Hadamard or elementwise product of two conformable matrices and \mathbf{Q} is a sample covariance matrix of \mathbf{z}_t . In this formulation, the correlation process

is driven by the two parameters, α and β . This is one of the advantages of the DCC-GARCH model in the sense that the number of parameters to be estimated for conditional correlations does not depend on the number of variables in the model. With this property, one can alleviate the computational burden and yet obtain large-dimensional conditional correlations. But then, the simple structure of the DCC parameterisation may be seen as a weakness because all correlation processes are assumed to have the same dynamic behaviour.

The log-likelihood function of the DCC-GARCH model is in principle obtained by defining \mathbf{P}_t in (5.4) as in (5.9). It can be decomposed into two parts, namely the volatility component and the correlation component. The volatility component at time t is given by

$$\ell_{v,t}(\boldsymbol{\omega}) = -\frac{N}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{V}_t| - \frac{1}{2} \boldsymbol{\varepsilon}'_t \mathbf{V}_t^{-1} \boldsymbol{\varepsilon}_t \quad (5.12)$$

where $\mathbf{V}_t = \mathbf{D}_t^2$, and the correlation component at time t is

$$\ell_{c,t}(\boldsymbol{\omega}, \boldsymbol{\phi}) = -\frac{1}{2} \ln |\mathbf{P}_t| - \frac{1}{2} \mathbf{z}'_t \mathbf{P}_t^{-1} \mathbf{z}_t + \frac{1}{2} \mathbf{z}'_t \mathbf{z}_t. \quad (5.13)$$

By applying this decomposition, the estimation of a DCC-GARCH model can be carried out in two steps. First, maximise (5.12) with respect to $\boldsymbol{\omega}$, the parameters in the volatility component. Then maximise (5.13) with respect to $\boldsymbol{\phi}$, the parameters in the correlation component, given the estimates from the preceding stage. It is worth mentioning that the constraints on α and β must be satisfied throughout the iterations because otherwise $\{\mathbf{Q}_t\}$ may become an explosive sequence.

The package **ccgarch** provides functions for simulating data from and estimating the parameters of the (E)DCC-GARCH(1,1) model of an arbitrary dimension. Estimation is carried out using the aforementioned two-step procedure.

The STCC-GARCH model

Another variant of the CC-GARCH models is proposed by Silvennoinen and Teräsvirta (2005). They assume time-invariant two extreme states in the correlation structure, $\mathbf{P}_{(1)}$ and $\mathbf{P}_{(2)}$, and a conditional correlation matrix at time t is described by a convex combination of the two regimes:

$$\mathbf{P}_t = (1 - G_t) \mathbf{P}_{(1)} + G_t \mathbf{P}_{(2)}, \quad \mathbf{P}_{(1)} \neq \mathbf{P}_{(2)} \quad (5.14)$$

$$G_t(s_t) = [1 + \exp\{-\gamma(s_t - c)\}]^{-1}, \quad \gamma > 0. \quad (5.15)$$

The correlation structure is smoothly changing over time according to the transition function $G_t(\cdot)$ that is bounded between 0 and 1. Silvennoinen and Teräsvirta (2005) use the logistic function (5.15), though one can define other functions for $G_t(\cdot)$ as well. The parameters γ and c are the slope and the location parameters, of which γ determines the speed of transition and c the location. The conditional correlations become constant either if $s_t \equiv c$, $\mathbf{P}_{(1)} = \mathbf{P}_{(2)}$ or $\gamma \rightarrow 0$, whereas the transition between the two states becomes abrupt when $\gamma \rightarrow \infty$.

The transition function (5.15) is driven by the transition variable s_t . Silvennoinen and Teräsvirta (2005) list typical candidates for s_t such as time, functions of past values of the return series and a stochastic exogenous variable. The STCC-GARCH model has an advantage in the sense that the model builder can choose the transition variable conforming to his or her research problem. In their application, for example, Silvennoinen and Teräsvirta (2005) investigate effects of market turbulence on conditional correlations with lagged seven-day averages of absolute Standard and Poors 500 returns as transition variable.

The log-likelihood function of the STCC-GARCH model has the same form as in (5.4) with \mathbf{P}_t now given by (5.14). Estimation of an STCC-GARCH model is implemented iteratively by concentrating the likelihood. The optimisation is performed by sequentially iterating over the three sets of parameters, namely, those in the volatility component, the ones in the correlation component and the two parameters in the transition function. Their experience is that by splitting every iteration into these three stages, numerical problems are avoided. For more details on an estimation procedure, see Silvennoinen and Teräsvirta (2005).

Silvennoinen and Teräsvirta (2009a) further extend the STCC-GARCH to the Double STCC (DSTCC-) GARCH in which the time-varying conditional correlation matrix \mathbf{P}_t is described as a convex combination of four constant correlation matrix of extreme states. The current version of **ccgarch** is able to simulate data from the STCC-GARCH with the transition variable following an univariate GARCH(1,1) process. A future version of **ccgarch** will include functions for dealing with estimation and simulation of the STCC- and DSTCC-GARCH models with other types of transition variables.

5.3 The package: **ccgarch**

5.3.1 Overview

The first public release of the package **ccgarch** was dated on 29 August 2008 with initial version 0.1.0. The current version (as of March 20, 2009) is 0.2.0 and is continuously updated. **ccgarch** has been registered as a contributed extension package for R. Since its first release, it has been distributed under the terms of the GNU General Public License (GPL) Version 2 or higher. The source codes (for Unix or Linux) and the pre-compiled binaries (for Windows and Mac OS) are available from CRAN (<http://cran.r-project.org/package=ccgarch>) or its mirror sites. Running the package requires R Version 2.6.1 or later. The installation can proceed in the usual manner.

Due to the autoregressive nature of GARCH models, it is not easy to avoid using loops in the codes. One can write all the necessary programs for the CC-GARCH models in the R language, but this would potentially slow down the execution. Therefore, to gain faster implementation, main engines for estimation and simulation heavily rely on the external C language through the interface function `.Call`. Most of the C codes in the package then use the Basic Linear Algebra Subprograms (BLAS) libraries that come along with R. The header file `R.home/include/R_ext/BLAS.h` includes declarations of the BLAS functions.

In what follows, we keep in mind the users of the Windows version of R. Note that `>` denotes the prompt on the R console window, followed by associated commands.

Once the package is installed, the users can invoke it by a command such as

```
> library(ccgarch)
```

The HTML help files are available from the R console window.

Currently, **ccgarch** can carry out the following tasks:

- Simulation of the following first-order models:
 - CCC-GARCH, ECCC-GARCH
 - DCC-GARCH, EDCC-GARCH
 - STCC-GARCH, ESTCC-GARCH
- Estimation of the following first-order models:
 - CCC-GARCH, ECCC-GARCH
 - DCC-GARCH, EDCC-GARCH
- Checking for the stationarity conditions of the GARCH component.
- Misspecification tests for causality in conditional variances
 - a test proposed by Hafner and Herwartz (2006)
 - a test proposed in Chapter 3 and its robust version
- Computing basic statistics:
 - the Ljung-Box test statistic
 - the Jarque-Bera test statistic
 - skewness and excess kurtosis measures and their robust versions

The subsequent sections describe briefly about usage of commands for the tasks above.

5.3.2 Simulation with `dcc.sim`

Inputs

The current version of **ccgarch** is capable of simulating data from a (E)CCC-GARCH(1, 1) by the function `eccc.sim`, a (E)DCC-GARCH(1, 1) by `dcc.sim`, a (E)STCC-GARCH(1, 1) by `stcc.sim`. In the default set-up, innovations are assumed to follow a multivariate normal distribution. One can alternatively use a multivariate Student's t -distribution by setting the argument `d.f=nu` $\in (4, \infty)$. The dimension N and the length of the time series T are restricted by the available memory space of the computer.

Table 5.1 contains the required arguments for simulating data from a DCC-GARCH(1,1) process by the function `dcc.sim`. The subsequent R commands illustrate how data are simulated from a DCC-GARCH(1,1) process:

```

# Defining the parameters
> nobs <- 5000; nu <- 8
> a <- c(0.003, 0.005, 0.001)
> A <- diag(c(0.2,0.3,0.15))
> B <- diag(c(0.75, 0.6, 0.8))
> uncr <- matrix(c(1.0, 0.4, 0.3, 0.4, 1.0, 0.12, 0.3, 0.12, 1.0),
                 3, 3)
> dcc.para <- c(0.01,0.98)
# for normally distributed innovations
> dcc.data <- dcc.sim(nobs, a, A, B, uncr, dcc.para,
                    model="diagonal")
# for t distributed innovations
> dcc.data.t <- dcc.sim(nobs, a, A, B, uncr, dcc.para, d.f=nu,
                      model="diagonal")

```

The first line after the comment (a comment line begins with the symbol #) defines the number of observations to be simulated and the degrees of freedom parameter for the t -distribution. The values of the parameters of the model parameters are specified in the next five lines. The number of dimensions in the data is automatically determined by the number of rows in `uncr`. When the parameter matrices `A` and `B` are full matrices and `model="extended"`, `dcc.sim` generates data from an EDCC-GARCH(1,1) process. One can also simulate data with t -distributed innovations by setting `d.f=nu`.

Outputs

The output from `dcc.sim` is summarised in a list with variables in Table 5.2. In the sample codes above, `dcc.data$eps` is a $(T \times N)$ matrix containing realisations from the DCC-GARCH(1,1) process. The simulated dynamic conditional correlations and conditional variances are stored in `dcc.data$dcc` and `dcc.data$h`, respectively. Each row of `dcc.data$dcc` contains $\text{vec}(\mathbf{P}_t)'$ where $\text{vec}()$ denotes an operator that transforms an $(N \times N)$ matrix into an $(N^2 \times 1)$ vector by stacking its columns. The list `dcc.data` includes the standardised residuals (`dcc.data$std.z`) and the random draws (`dcc.data$z`) as well.

`*.sim` functions have an analogous structure, and the help pages of `ccgarch` can be consulted for detailed implementation.

5.3.3 Estimation with dcc.estimation

Inputs

The current version of `ccgarch` can estimate the first-order (E)DCC-GARCH model by `dcc.estimation`, and the (E)CCC-GARCH model by `eccc.estimation`. We shall now use `dcc.estimation` as an example to explain how users can estimate a DCC-GARCH model.

`dcc.estimation` requires nine arguments that are listed in Table 5.3. Neither one of them, except for `method`, `gradient` and `message`, has default values, so one has to explicitly provide the values. The first four arguments, namely `a` (length N), `A` ($N \times N$), `B` ($N \times N$) and `dcc.para` (length 2), contain initial values for iterations that have to be fed in an appropriate way. Section 5.3.3 discusses further remarks regarding a choice of initial values.

The next argument, `dvar`, contains the data to be analysed. Since the conditional mean is assumed to equal zero, the data must be pre-whitened by fitting, for instance, a vector ARMA model in advance. The argument `model` is a character string that indicates which model is to be estimated, *i.e.*, either “diagonal” or “extended”. The arguments `method` and `gradient` specify the optimisation algorithm used at the first and second stage in `optim` and `constrOptim`, respectively. The default is “BFGS” for both stages.

In the default setup, `dcc.estimation` returns a message to tell the user that the estimation has been completed. When the function is used in simulation studies, it is annoying to see such a message every time the optimisation is finished. The last argument `message` is designed for controlling the display of the message. The user can turn it off by setting `message=0`.

Remarks on the choice of initial values

The user must feed these initial values in an appropriate way. Otherwise the program does not run correctly. For example, `A` and `B` must be supplied as an $N \times N$ matrix for both diagonal and extended specifications. In choosing initial values, the following conditions must be met to avoid an unexpected termination of the estimation:

positivity: $\mathbf{a}_0 > 0$, $\mathbf{A} > 0$, and $\mathbf{B} > 0$ elementwise

stationarity 1: the stationarity of the GARCH process (5.5)

stationarity 2: the stationarity of the DCC process (5.10).

It is easy to make sure that **positivity** is satisfied. **stationarity 1** can be verified by the utility function `stationarity`. A brief explanation about **stationarity 1** is given in Section 5.3.4. When an extended model is estimated, it is always recommended to check whether or not **stationarity 1** is fulfilled. To satisfy **stationarity 2**, set the elements of the DCC parameters (`dcc.para`) according to (5.11), that is, `sum(dcc.para)<1` and `min(dcc.para)>0`. It should be noted that the above conditions also apply to simulating data from a DCC-GARCH process.

It is worth mentioning that a choice of initial values does not seem to affect resulting parameter estimates in `dcc.estimation`. Small Monte Carlo experiments are carried out in such a way that a DCC-GARCH(1,1) model is fitted to the data simulated from a DCC-GARCH(1,1) process with two sets of initial values, one with the true parameter values and the other with ones that deviate from the true values. The DGP is defined in Section 5.3.2 with $T = 1000$. Descriptive statistics of the parameter estimates in 1000 replications are summarised in Table 5.4. The numbers under the heading “True initial values” contain the mean, standard deviations (SD)

and mean squared errors (MSE) of estimates with the true parameter values as initial values in the optimisation, while those under “Untrue initial values” include outcomes with untrue values as starting values. The means of estimates in both cases are very close to each other as well as to the true values although those in the untrue initial values have larger SD and MSEs. Regarding the average number of iterations for the first stage optimisation to converge, it is 162.8 with the untrue initial values, and 112.5 with the true initial values. For the second stage estimation, the corresponding figures do not differ much from each other: 18.6 iterations with the true values and 18.7 with the untrue values.

Outputs

Once the estimation process is completed, the results are stored in a list with components summarised in Table 5.5. The first argument `out` contains the quasi-maximum likelihood (QML) estimates of the parameters and their robust standard errors. The value of the log-likelihood function evaluated at the estimates are saved in `loglik`. The estimated conditional variances and the dynamic conditional correlations are stored in `h` and `DCC`, respectively. The outcomes of the first and second stage optimisation can be retrieved through the list variables `first` and `second`. The procedure is further illustrated in Section 5.4.

There are two major possibilities of erroneous termination of the estimation process. One is that the initial parameter values do not satisfy conditions listed above. For example, if **stationarity 1** is not met, the estimation process may be terminated immediately with the following error message:

```
Error in optim(par = init, fn = loglik.dcc1, method = method,
control = list(maxit = 10^5, :
  initial value in 'vmin' is not finite
```

This message shows that the objective function in the first stage estimation (`loglik.dcc1`) does not attain a finite value with the initial parameter values supplied.

The other possibility is that the numerical optimisation does not end successfully. When convergence is not achieved, the following error messages appear in the console window. For the first stage, the message is

```
*****
* The first stage optimization has failed. *
* See the list variable "first" for details. *
*****
```

whereas for the second stage, it has the form

```
*****
* The second stage optimization has failed. *
* See the list variable "second" for details. *
*****
```

By looking at the value of `first$convergence` or `second$convergence`, one may pinpoint a cause of an error. It is often the case in GARCH estimation that the number of iterations performed exceeds a pre-determined limit. In `ccgarch`, however, this convergence error is unlikely to occur because the limit of iterations is set to 10^5 . Error messages due to other types of unfavourable behaviour such as a singular Hessian matrix depend on internal messages in R. Therefore, the user is requested to read associated help files of the internal functions when other types of error message appear.

Some technical issues

There are technical issues that deserve an explanation. As was mentioned in Section 5.2.2, estimation of a DCC-GARCH model is performed in two steps. The function `dcc.estimate` internally calls two other functions, `dcc.estimate1` and `dcc.estimate2`, that carry out the first and second stage optimisation. `optim` is called in `dcc.estimate1`, whereas `constrOptim` is invoked in `dcc.estimate2`. Past experiences show that the optimisation at the first stage converges without numerical problems in almost all cases. On the other hand, second stage optimisation often leads to an overflow due to the violation of the restriction $\alpha + \beta < 1$. Therefore, the constraint is explicitly imposed during iterations, and this is done through `constrOptim`. As was mentioned earlier, the default optimisation algorithm is set to BFGS at both stages. For the first stage, the user can change it either to the Nelder and Mead (1965) method (“Nelder-Mead”) or to a conjugate gradients method (“CG”) by setting the argument `method` in `dcc.estimate`. If the user wants to change the algorithm at the second stage optimisation, setting `gradient=0` implies using the Nelder-Mead algorithm. One can further change the optimisation algorithm, convergence criteria and other parameters both in `optim` and `constrOptim`. To do this, the user must modify the relevant source codes. This is not recommended, however, unless the user is familiar with optimisation techniques and R programming convention. For discussion of various optimisation algorithms see, for example, Fletcher (1987), Hendry (1995, Appendix A5) or Teräsvirta, Tjøstheim, and Granger (2010, Chapter 12).

5.3.4 Utility functions

`ccgarch` includes some utility functions for carrying out misspecification tests, reporting basic statistics and checking restrictions on parameters.

Misspecification tests

As was described in Chapter 3, there are a couple of misspecification tests proposed for testing causality in conditional variance. From version 0.2.0, `ccgarch` includes two functions for carrying out such misspecification tests. One is `hh.test`, and the other is `nt.test`. The former computes a test statistic proposed by Hafner and Herwartz (2006) and the latter is for the test discussed in Chapter 3. `nt.test` returns the

standard (non-robust) and heteroskedasticity robust test statistics and associated p -values. Both functions are designed for computing test statistics for N -dimensional time series data. Their usage is illustrated in Section 5.4.

Skewness and excess kurtosis

In financial econometrics, it is well-known that stock returns exhibit negative skewness and large excess kurtosis. This is often regarded as evidence for non-normality of stock return distribution. Kim and White (2004), however, find in their Monte Carlo simulations that the conventional measures of skewness and kurtosis are extremely sensitive to a small number of outliers. Therefore, they propose alternative measures based on quantiles that are robust against the existence of outliers. Further discussions about stylised facts and robust measures can be found, for instance, in Teräsvirta and Zhao (2007). The functions `rob.sk` and `rob.kr` return both conventional and robustified measures of skewness and excess kurtosis, respectively. The examples of their use are shown in Section 5.4.

Stationarity condition

When formulating the CC-GARCH model, the GARCH equation (5.5) is assumed to be stationary. Parameter matrices \mathbf{A} and \mathbf{B} are therefore constrained in such a way that the stationarity of (5.5) holds. Such a condition has been worked out by He and Teräsvirta (2004) for a general error distribution and by Nakatani and Teräsvirta (2009) under the assumption of normality. The function `stationarity` checks the stationarity condition of Nakatani and Teräsvirta (2009) for the GARCH equation (5.5). This is done by computing the modulus of the largest eigenvalue of $\mathbf{A} + \mathbf{B}$. The stationarity condition is satisfied if the modulus of the largest eigenvalue of this matrix is strictly less than one. This function is useful in the extended CC-GARCH models. In the diagonal models one can directly examine the condition by looking at the largest element in $\mathbf{A} + \mathbf{B}$, as $a_{ii} + b_{ii} < 1$ is the stationarity condition of the i th univariate GARCH (1, 1) model.

Tests for autocorrelations and non-normality of residuals

The Ljung-Box (LB) test statistic for serial correlations (Ljung and Box, 1978) can be calculated by `ljung.box.test`. `ljung.box.test` can only handle a univariate time series. The LB test is often applied to squared residuals to detect evidence for ARCH effects in the time series. When this is the case, the LB test is equivalent to the McLeod and Li (1983) test. However, Li and Mak (1994) found that the asymptotic null distribution of the McLeod and Li (1983) test statistic is not a χ^2 distribution when the test is applied to the residuals of an estimated GARCH equation. Therefore, the McLeod and Li (1983) test is not suitable for this purpose. For further discussions, see, Li and Mak (1994) and Silvennoinen and Teräsvirta (2009b, Section 4).

Another function in the package carries out the Lomnicki-Jarque-Bera (LJB) test for non-normality (Lomnicki, 1961; Jarque and Bera, 1987). The LJB test is imple-

mented by `jb.test`. `jb.test` simultaneously returns test statistics and associated p-values for as many time series as desired.

5.4 Illustrative example

5.4.1 The data

In this section, we demonstrate how the user can fit a DCC-GARCH (1,1) model to financial data. The data set consists of the daily closing stock prices of three Japanese leading electronic firms, NEC, Hitachi and Toshiba, traded at the Tokyo Stock Exchange. The sample period spans from 4 January 1983 to 1 March 2006, giving us 5913 observations of log-returns for each series.³ These return data are stored in the variable called `eps`. The graphs of the three Japanese stock returns are presented in the top row of the panels in Figure 5.1.

Basic descriptive statistics such as mean, minimum, maximum and standard deviations can be calculated by standard functions in the base package of R. The conventional (standard) and the robustified skewness and excess kurtosis are computed by functions in the `ccgarch` as follows:

```
> rob.sk(eps)
      series 1      series 2      series 3
standard 0.28244996 0.30720786 0.26612296
robust   -0.07751916 -0.03743396 -0.08137347
> rob.kr(eps)
      series 1      series 2      series 3
standard 3.36349611 2.5790472 2.8534653
robust   0.07961675 0.1982779 0.1612939
```

One can recognise from the output that the difference between the standard statistics and the robust ones are large, meaning that the conventional measures are affected by a small number of outliers.

5.4.2 Fitting a DCC-GARCH(1,1) model

Before fitting a DCC-GARCH model, misspecification tests for causality in conditional variance were performed by the two functions, `nt.test` and `hh.test`. Under the null of no causality in conditional variance, all the test statistics are distributed as χ^2 with $2N(N-1)$.

The outcomes of `hh.test` and `nt.test` are as follows:

```
> hh.test(eps)
      Test Stat      p-Value
5.121297e+01 8.542556e-07
> nt.test(eps)
```

³Part of the data was investigated in Nakatani and Teräsvirta (2009).

	Test stat	p-value
NT	106.70618	2.676155e-17
robNT	66.60557	1.374528e-09

According to the reported p -values, the null of no causality in conditional variance is rejected with very very small significance levels in any of three tests. Therefore, it is recommended to use the extended version of the conditional variance.

For the estimation of the extended DCC (EDCC-) GARCH model, the initial parameter matrices are set by

```
a <- c(0.003, 0.005, 0.001)
A <- matrix(c(0.2, 0.002, 0.001,
              0.001, 0.3, 0.0005,
              0.003, 0.005, 0.15), 3, 3, byrow=T)
B <- matrix(c(0.75, 0.0001, 0.0015,
              0.002, 0.6, 0.0003,
              0.001, 0.008, 0.8), 3, 3, byrow=T)
dcc.para <- c(0.1, 0.8)
```

and the estimation is carried out by the following command:

```
> dcc.results <- dcc.estimation(inia=a, iniA=A, iniB=B,
                              ini.dcc=dcc.para, dvar=eps,
                              model="extended")
```

To estimate the EDCC-GARCH model, the variable “model” is set to “extended”. In the above code the method of optimisation is not specified, so that the optimisation is performed through the default algorithm “BFGS”. As already mentioned, the user can specify another algorithm, namely, either “Nelder-Mead” or “CG”.

When the estimation is successfully completed, the following messages appear in the console window:

```
*****
* Estimation has been completed. *
* The outputs are saved in a list with components: *
* out : the estimates and their standard errors *
* loglik : the value of the log-likelihood at the estimates *
* h : a matrix of estimated conditional variances *
* DCC : a matrix of DCC estimates *
* std.resid : a matrix of the standardised residuals *
* first : the results of the first stage estimation *
* second : the results of the second stage estimation *
*****
```

The parameter estimates and their robust standard errors are retrieved by:

```
> round(dcc.results$out,3)
      a1      a2      a3      A11      A21      A31      A12      A22      A32
```



```

estimates 0.095 0.087 0.033 0.061 0.013 0.008 0.018 0.077 0.020
std.err   0.024 0.011 0.012 0.012 0.026 0.040 0.038 0.026 0.011
          A13  A23  A33  B11  B21  B31  B12  B22  B32
estimates 0.010 0.031 0.050 0.889 0.003 0.001 0.00 0.847 0.005
std.err   0.014 0.011 0.023 0.041 0.038 0.017 0.01 0.011 0.010
          B13  B23  B33 dcc alpha dcc beta
estimates 0.006 0.012 0.906      0.025      0.957
std.err   0.022 0.030 0.028      0.002      0.005

```

The robust standard errors are computed by the analytic first and second partial derivatives of the log-likelihood function of the DCC-GARCH model. The analytic expressions for these derivatives are summarised in Appendix 5.A. The estimated conditional variances and the dynamic conditional correlations are saved in `dcc.results$h` and `dcc.results$DCC`, respectively. They are depicted on the panels in the second and the third rows of Figure 5.1.

5.4.3 Cautionary remarks

If either the first or the second stage optimisation algorithm does not converge, the estimation process is terminated with an error message explained in Section 5.3.3. There are two ways of solving the non-convergence problem. The first option is to change the initial parameter values and try again. The second one consists of fine tuning of the optional variables in `optim` for the first stage or `constrOptim` for the second stage. Then each optimisation is implemented separately by `dcc.estimation1` or by `dcc.estimation2`. In the latter option, one can manually follow the steps that are executed in `dcc.estimation`. Therefore, the user can use the functions such as `dcc.results` to obtain standard errors of the estimates, `vector.garch` to have the conditional variances, and `dcc.est` to compute the dynamic conditional correlations. When this happens, the user is asked to redefine the relevant parts of the associated functions in `ccgarch`. The contents of the functions can be seen by typing their names without argument in the R console window.

5.5 Concluding remarks

This paper deals with usage of the package `ccgarch` that is developed for use in the open source statistical environment R. `ccgarch` is capable of estimating certain types of the CC-GARCH models and generating data from them by simulation. Before documenting the contents of the package, three representative members, namely, the CCC-, the DCC- and the STCC-GARCH, of the CC-GARCH models and their extended forms are briefly reviewed. In applying the CC-GARCH models, one has hitherto been forced to resort to commercial packages or has been compelled to spend a substantial amount of time to create his or her own codes. The purpose of the package `ccgarch` is to narrow the gap between the demand for estimation and evaluation of the CC-GARCH models and the accessibility of software available for performing these tasks.

The package **ccgarch** is still in its infancy, which means that there is room for improvements and extensions in many directions. One of the most urgent tasks is to incorporate procedures for various diagnostic or misspecification tests into the package. Including a function for estimating an STCC-GARCH model in **ccgarch** is also an imminent task. In fact, the work is already underway to include those and other useful functions in future versions of the package.

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5.A Mathematical derivations

In this appendix, derivatives of the likelihood function and the robust covariance matrix of the QML estimators of the (E)DCC-GARCH model are summarised since the package relies on those derivatives in estimating the robust covariance matrix. Further details of the covariance matrix estimation can be found in Engle (2002) and references therein.

It is straightforward to derive all the results below through vector and matrix derivatives, so that proofs are omitted. Similar results can be found, for example, in Hafner and Herwartz (2008) and Nakatani and Teräsvirta (2009). To save space, $\nabla_{\boldsymbol{\omega}}$ is used for denoting partial derivatives with respect to $\boldsymbol{\omega}$, and so on.

5.A.1 Partial derivatives of the volatility component

The first partial derivative of (5.12) with respect to its parameter vector $\boldsymbol{\omega}$ is given by

$$\nabla_{\boldsymbol{\omega}} \ell_{v,t} = -\frac{1}{2} \nabla_{\boldsymbol{\omega}} \mathbf{V}_t \text{vec} (\mathbf{V}_t^{-1} - \mathbf{V}_t^{-1} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' \mathbf{V}_t^{-1}). \quad (5.16)$$

The negative of the conditional expectation of the second partial derivatives of (5.12) is equal to

$$-E [\nabla_{\boldsymbol{\omega}}^2 \ell_{v,t}] = \frac{1}{2} \nabla_{\boldsymbol{\omega}} \mathbf{V}_t (\mathbf{V}_t^{-1} \otimes \mathbf{V}_t^{-1}) \nabla_{\boldsymbol{\omega}} \mathbf{V}_t' = \sum_{i=1}^N h_{i,t}^{-2} \nabla_{\boldsymbol{\omega}} h_{i,t} \nabla_{\boldsymbol{\omega}} h_{i,t}'. \quad (5.17)$$

The expression for $\nabla_{\boldsymbol{\omega}} h_{i,t}$ can be found in Nakatani and Teräsvirta (2008a).

5.A.2 Partial derivatives of the correlation component

Partial derivatives of the correlation component are available in a similar way. The first partial derivative of (5.13) is given by

$$\nabla_{\boldsymbol{\phi}} \ell_{c,t} = -\frac{1}{2} \nabla_{\boldsymbol{\phi}} \mathbf{P}_t \text{vec} (\mathbf{P}_t^{-1} - \mathbf{P}_t^{-1} \mathbf{z}_t \mathbf{z}_t' \mathbf{P}_t^{-1}) \quad (5.18)$$

where

$$\nabla_{\boldsymbol{\phi}} \mathbf{P}_t = \nabla_{\boldsymbol{\phi}} \mathbf{Q}_t \left[\tilde{\mathbf{Q}}_t^{-1} \otimes \tilde{\mathbf{Q}}_t^{-1} - \frac{1}{2} \text{dg} \left(\text{vec} \left(\tilde{\mathbf{Q}}_t^{-1} \right) \right) \left(\mathbf{P}_t \otimes \tilde{\mathbf{Q}}_t^{-1} + \tilde{\mathbf{Q}}_t^{-1} \otimes \mathbf{P}_t \right) \right] \quad (5.19)$$

and

$$\nabla_{\boldsymbol{\phi}} \mathbf{Q}_t = - \begin{bmatrix} \text{vec}(\mathbf{Q})' \\ \text{vec}(\mathbf{Q})' \end{bmatrix} + \begin{bmatrix} \text{vec}(\mathbf{z}_{t-1} \mathbf{z}_{t-1}')' \\ \text{vec}(\mathbf{Q}_{t-1})' \end{bmatrix} + \beta \nabla_{\boldsymbol{\phi}} \mathbf{Q}_{t-1}. \quad (5.20)$$

The operator dg creates an $(N \times N)$ diagonal matrix from its argument vector of length N .

The negative of the conditional expectation of the second and cross partial derivatives are

$$-E[\nabla_{\phi}^2 \ell_{c,t}] = \frac{1}{2} \nabla_{\phi} \mathbf{P}_t (\mathbf{P}_t^{-1} \otimes \mathbf{P}_t^{-1}) \nabla_{\phi} \mathbf{P}'_t \quad (5.21)$$

$$\begin{aligned} -E[\nabla_{\phi\omega} \ell_{c,t}] &= \frac{1}{2} \nabla_{\phi} \mathbf{P}_t (\mathbf{P}_t^{-1} \otimes \mathbf{P}_t^{-1}) \nabla_{\omega} \mathbf{P}'_t \\ &\quad + \frac{1}{2} \nabla_{\phi} \mathbf{P}_t (\mathbf{P}_t^{-1} \mathbf{D}_t^{-1} \otimes \mathbf{I}_N + \mathbf{I}_N \otimes \mathbf{P}_t^{-1} \mathbf{D}_t^{-1}) \nabla_{\omega} \mathbf{D}'_t \end{aligned} \quad (5.22)$$

where

$$\nabla_{\omega} \mathbf{P}_t = \nabla_{\omega} \mathbf{Q}_t \left[\tilde{\mathbf{Q}}_t^{-1} \otimes \tilde{\mathbf{Q}}_t^{-1} - \frac{1}{2} \text{dg} \left(\text{vec} \left(\tilde{\mathbf{Q}}_t^{-1} \right) \right) \left(\mathbf{P}_t \otimes \tilde{\mathbf{Q}}_t^{-1} + \tilde{\mathbf{Q}}_t^{-1} \otimes \mathbf{P}_t \right) \right], \quad (5.23)$$

$$\nabla_{\omega} \mathbf{Q}_t = -\alpha \nabla_{\omega} \mathbf{D}_{t-1} (\mathbf{P}_{t-1} \otimes \mathbf{D}_{t-1}^{-1} + \mathbf{D}_{t-1}^{-1} \otimes \mathbf{P}_{t-1}) + \beta \nabla_{\omega} \mathbf{Q}_{t-1} \quad (5.24)$$

and using the fact that $E[\mathbf{z}_{t-1} \mathbf{z}'_{t-1}] = \mathbf{P}_{t-1}$.

$$(5.25)$$

5.A.3 Asymptotic covariance matrix of the QML estimator

The asymptotic covariance matrix of the QMLE $\hat{\theta}$ of the (E)DCC-GARCH model is given by

$$\Sigma = \mathbf{G}^{-1} \Omega \mathbf{G}^{-1} \quad (5.26)$$

where

$$\mathbf{G} = E \begin{bmatrix} \nabla_{\omega}^2 \ell_v & \mathbf{0} \\ \nabla_{\phi\omega} \ell_c & \nabla_{\phi}^2 \ell_c \end{bmatrix} \quad (5.27)$$

$$\Omega = E \begin{bmatrix} \nabla_{\omega} \ell_v \nabla_{\omega} \ell'_v & \nabla_{\omega} \ell_v \nabla_{\phi} \ell'_c \\ \nabla_{\phi} \ell_c \nabla_{\omega} \ell'_v & \nabla_{\phi} \ell_c \nabla_{\phi} \ell'_c \end{bmatrix} \quad (5.28)$$

with $\nabla_{\omega} \ell_v = \sum_{t=1}^T \nabla_{\omega} \ell_{v,t}$ evaluated at the true parameters, and so on. \mathbf{G} and Ω are consistently estimated by their sample counter parts.

5.B A User's Manual of **ccgarch**

The following pages contain a user's manual of **ccgarch** that is accompanied with the package bundle. When the user installed the package into the R system on her/his PC, the same contents are available either in the HTML format or in the plain texts that can be accessible from the R console window. Alternatively, the PDF version of the manual is downloadable from <http://cran.r-project.org/package=ccgarch>.

Package ‘ccgarch’

March 22, 2010

Version 0.2.0

Date 2010-03-20

Title Conditional Correlation GARCH models

Author Tomoaki Nakatani (naktom2@gmail.com)

Maintainer Tomoaki Nakatani (naktom2@gmail.com)

Depends R (>= 2.6.1)

Description Functions for estimating and simulating the family of the CC-GARCH models.

License GPL (>=2)

R topics documented:

analytical.grad	2
analytical.Hessian	3
d2lv	4
dcc.est	4
dcc.estimation	5
dcc.estimation1	7
dcc.estimation2	8
dcc.results	9
dcc.sim	10
dlc	11
dlv	12
dlv.est	13
eccc.estimation	14
eccc.sim	15
fourth	17
grad.dcc.full	18
grad.dcc2	18
hh.test	19
jb.test	20
ljung.box.test	21
loglik.dcc	22
loglik.dcc1	23
loglik.dcc2	24
loglik.eccc	24

nt.test 25
 p.mat 26
 rob.kr 26
 rob.sk 27
 stationarity 28
 stcc.sim 28
 tr.func 30
 uni.vola 31
 uni.vola.sim 31
 vdR 32
 vec.garch.derivative 33
 vector.garch 34

Index **35**

<code>analytical.grad</code>	<i>Analytical gradient of the log-likelihood function of the (E)CCC-GARCH(1,1) model</i>
------------------------------	--

Description

This function returns the analytical gradient of the log-likelihood function of the (E)CCC-GARCH(1,1) model.

Usage

```
analytical.grad(a, A, B, R, u, model)
```

Arguments

- `a` a vector of constants in the vector GARCH equation ($N \times 1$)
- `A` an ARCH parameter matrix in the vector GARCH equation ($N \times N$)
- `B` a GARCH parameter matrix in the vector GARCH equation ($N \times N$)
- `R` a constant conditional correlation matrix ($N \times N$)
- `u` a matrix of the data used for estimating the (E)CCC-GARCH(1,1) model ($T \times N$)
- `model` a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

a $npar \times T$ matrix of gradients

Note

In the output, each column (not row) corresponds to the gradient at observation t .

analytical.Hessian

3

References

Nakatani, T. and T. Teräsvirta (2009), “Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model”, *Econometrics Journal*, **12**, 147–163.

Nakatani, T. and T. Teräsvirta (2008), “Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*” Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

analytical.Hessian Analytical Hessian of the (E)CCC-GARCH

Description

This function computes the analytical Hessian of the log-likelihood function of the (E)CCC-GARCH model.

Usage

```
analytical.Hessian(a, A, B, R, u, model)
```

Arguments

- a a vector of constants in the vector GARCH equation ($N \times 1$)
- A an ARCH parameter matrix in the vector GARCH equation ($N \times N$)
- B a GARCH parameter matrix in the vector GARCH equation ($N \times N$)
- R a constant conditional correlation matrix ($N \times N$)
- u a matrix of the data data used for estimating the (E)CCC-GARCH(1,1) model ($T \times N$)
- model a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

a $npar \times npar$ Hessian matrix of the log-likelihood function of the (E)CCC-GARCH model

References

Nakatani, T. and T. Teräsvirta (2009), “Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model”, *Econometrics Journal*, **12**, 147–163.

Nakatani, T. and T. Teräsvirta (2008), “Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*” Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

d2lv *Hessian of the DCC log-likelihood function*

Description

This function returns the analytical Hessian of the volatility part of the DCC log-likelihood function.

Usage

```
d2lv(u, B, h, model)
```

Arguments

u	a matrix of the data data used for estimating the (E)DCC-GARCH(1,1) model ($T \times N$)
B	a GARCH parameter matrix ($N \times N$)
h	a matrix of the conditional variances ($T \times N$)
model	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

the Hessian of the volatility part of the DCC log-likelihood function ($T \times N^2$)

References

Engle, R.F. and K. Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

Hafner, C.M. and H. Herwartz (2008), "Analytical Quasi Maximum Likelihood Inference in Multivariate Volatility Models." *Metrika* **67**, 219–239.

dcc.est *Dynamic conditional correlations*

Description

This function returns dynamic conditional correlations based on the parameters specified.

dcc.estimation

5

Usage

```
dcc.est(dvar, param)
```

Arguments

- dvar a matrix of the standardised residuals ($T \times N$)
- param a vector of the DCC parameters (2×1)

Value

a list with components:

- DCC a matrix of the dynamic conditional correlations ($T \times N^2$)
- Q a matrix of the Q_t ($T \times N^2$)

Note

a constant matrix Q in the DCC equation is computed by $Q = cov(dvar)$.

References

Engle, R.F. and K. Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

<code>dcc.estimation</code>	<i>Estimating an (E)DCC-GARCH model</i>
-----------------------------	---

Description

This function carries out the two step estimation of the (E)DCC-GARCH model and returns estimates, standardised residuals, the estimated conditional variances, and the dynamic conditional correlations.

Usage

```
dcc.estimation(inia, iniA, iniB, ini.dcc, dvar, model,
               method="BFGS", gradient=1, message=1)
```

Arguments

<code>inia</code>	a vector of initial values for the constants in the GARCH equation <code>length(inia)=N</code>
<code>iniA</code>	a matrix of initial values for the ARCH parameter matrix ($N \times N$)
<code>iniB</code>	a matrix of initial values for the GARCH parameter matrix ($N \times N$)
<code>ini.dcc</code>	a vector of initial values for the DCC parameters (2×1)
<code>dvar</code>	a matrix of the data ($T \times N$)
<code>model</code>	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model
<code>method</code>	a character string specifying the optimisation method in <code>optim</code> . There are three choices, namely, Nelder-Mead, BFGS (default) and CG.
<code>gradient</code>	a switch variable that determines the optimisation algorithm in the second stage optimisation. If <code>gradient=0</code> Nelder-Mead is invoked. Otherwise BFGS is used (default).
<code>message</code>	a switch variable to turn off the display of the message when the estimation is completed. If <code>message=0</code> , the message is suppressed. Otherwise, the message is displayed (default)

Value

a list with components:

<code>out</code>	the parameter estimates and their standard errors
<code>loglik</code>	the value of the log-likelihood at the estimates
<code>h</code>	a matrix of the estimated conditional variances ($T \times N$)
<code>DCC</code>	a matrix of the estimated dynamic conditional correlations ($T \times N^2$)
<code>std.resid</code>	a matrix of the standardised residuals ($T \times N$). See <i>Note</i> .
<code>first</code>	the results of the first stage estimation
<code>second</code>	the results of the second stage estimation

Note

The standardised residuals are calculated by dividing the original series `dvar` by the estimated conditional standard deviations `sqrt(h)`. See Engle (2002), in particular the equations (2) and (14), for details.

`dcc. estimation` calls `dcc. estimation1` and `dcc. estimation2` for the first and second stage estimation, respectively.

The details of the first and second stage estimation are also saved.

The switch variable `simulation` is useful when one uses `dcc. estimation` for simulation. It suppresses the display of the completion message.

References

Engle, R.F. and K. Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

`dcc.estimation1`

7

See Also

`dcc.estimation1`, `dcc.estimation2`, `loglik.dcc1`, `loglik.dcc2`, `vector.garch`, `dcc.est`

Examples

```
# Simulating data from the original DCC-GARCH(1,1) process
nobs <- 1000; cut <- 1000
a <- c(0.003, 0.005, 0.001)
A <- diag(c(0.2, 0.3, 0.15))
B <- diag(c(0.75, 0.6, 0.8))
uncR <- matrix(c(1.0, 0.4, 0.3, 0.4, 1.0, 0.12, 0.3, 0.12, 1.0), 3, 3)
dcc.para <- c(0.01, 0.98)
dcc.data <- dcc.sim(nobs, a, A, B, uncR, dcc.para, model="diagonal")

# Estimating a DCC-GARCH(1,1) model
dcc.results <- dcc.estimation(inia=a, iniA=A, iniB=B, ini.dcc=dcc.para,
                             dvar=dcc.data$eps, model="diagonal")

# Parameter estimates and their robust standard errors
dcc.results$out
```

`dcc.estimation1` *Maximising the first stage log-likelihood function of the (E)DCC-GARCH model*

Description

This function carries out the first stage (volatility part) estimation of the (E)DCC-GARCH model.

Usage

```
dcc.estimation1(dvar, a, A, B, model, method="BFGS")
```

Arguments

<code>dvar</code>	a matrix of the data used for estimating the (E)DCC-GARCH(1,1) model ($T \times N$)
<code>a</code>	a vector of constants in the vector GARCH equation ($N \times 1$)
<code>A</code>	an ARCH parameter matrix in the vector GARCH equation ($N \times N$)
<code>B</code>	a GARCH parameter matrix in the vector GARCH equation ($N \times N$)
<code>model</code>	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model
<code>method</code>	a character string specifying the optimisation method in <code>optim</code> . There are three choices, namely, "Nelder-Mead", "BFGS" (default) and "CG".

Value

a list of the estimation results. See the explanations in `optim`.

8

dcc.estimation2

References

Engle, R.F. and K. Sheppard (2001), “Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH.” *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), “Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models.” *Journal of Business and Economic Statistics* **20**, 339–350.

See Also

optim, dcc.estimation2, dcc.estimation

dcc.estimation2 *Maximising the second stage log-likelihood function of the (E)DCC-GARCH model*

Description

This function carries out the second stage (DCC part) estimation of the (E)DCC-GARCH model.

Usage

```
dcc.estimation2(dvar, para, gradient=0)
```

Arguments

dvar	a matrix of the standardised residuals ($T \times N$)
para	a vector of the DCC parameters (2×1)
gradient	a switch variable whether to use the gradient in the constraint optimisation. passed to constrOptim

Value

a list of the estimation results. See the explanations for constrOptim.

Note

dcc.estimation2 is a wrapper to constrOptim. The restrictions are $\alpha + \beta \leq 1$ and $\alpha, \beta \geq 0$ in the DCC equation.

References

Engle, R.F. and K. Sheppard (2001), “Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH.” *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), “Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models.” *Journal of Business and Economic Statistics* **20**, 339–350.

See Also

constrOptim, dcc.estimation1, dcc.estimation

`dcc.results`

9

<code>dcc.results</code>	<i>Computing robust standard errors of the estimates in the (E)DCC-GARCH model</i>
--------------------------	--

Description

This function computes the robust standard errors of the estimates of a DCC-GARCH model.

Usage

```
dcc.results(u, garch.para, dcc.para, h, model)
```

Arguments

- `u` a matrix of the data used for estimating the (E)DCC-GARCH model ($T \times N$)
- `garch.para` a vector of the estimates of the volatility parameters
- `dcc.para` a vector of the estimates of the DCC parameters (2×1)
- `h` a matrix of the estimated conditional variances ($T \times N$)
- `model` a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

A matrix with the estimates in the first row, and the standard errors in the second row.

Note

`dcc.results` is called from `dcc.estimation`. When `model="diagonal"`, only the diagonal entries in A and B are used.

References

Engle, R.F. and K. Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

See Also

`dcc.estimation`

10

dcc.sim

dcc.sim

Simulating an (E)DCC-GARCH(1,1) process

Description

This function simulates data either from the original DCC-GARCH by Engle (2002) or from the Extended DCC-GARCH that has non-zero off-diagonal entries in the parameter matrices in the GARCH equation, with multivariate normal or student's t distributions.

The dimension (N) is determined by the number of elements in the a vector.

Usage

```
dcc.sim(nobs, a, A, B, R, dcc.para, d.f=Inf, cut=1000, model)
```

Arguments

nobs	a number of observations to be simulated (T)
a	a vector of constants in the vector GARCH equation ($N \times 1$)
A	an ARCH parameter matrix in the vector GARCH equation ($N \times N$)
B	a GARCH parameter matrix in the vector GARCH equation ($N \times N$)
R	an unconditional correlation matrix ($N \times N$)
dcc.para	a vector of the DCC parameters (2×1)
d.f	the degrees of freedom parameter for the t -distribution
cut	the number of observations to be thrown away for removing initial effects of simulation
model	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

A list with components:

z	a matrix of random draws from $N(\mathbf{0}, \mathbf{I})$. ($T \times N$)
std.z	a matrix of the standardised residuals. $std.z_t \sim N(0, \mathbf{R}_t)$ where \mathbf{R}_t is the DCC matrix at t . If d.f is set to a finite positive real number, $\mathbf{z}_t \sim t_{d.f}(0, \mathbf{R}_t)$ ($T \times N$)
dcc	a matrix of the simulated dynamic conditional correlations ($T \times N^2$)
h	a matrix of the simulated conditional variances ($T \times N$)
eps	a matrix of the simulated time series with DCC-GARCH process ($T \times N$)

Note

When d.f=Inf, the innovations (the standardised residuals) follow the standard normal distribution. Otherwise, they follow a student's t -distribution with d.f degrees of freedom.

When model="diagonal", only the diagonal entries in A and B are used. If the ARCH and GARCH matrices do not satisfy the stationarity condition, the simulation is terminated.

dlc

11

References

Engle, R.F. and K. Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

See Also

stcc.sim, eccc.sim

Examples

```
# Simulating data from the original DCC-GARCH(1,1) process
nobs <- 1000; cut <- 1000; nu <- 8
a <- c(0.003, 0.005, 0.001)
A <- diag(c(0.2,0.3,0.15))
B <- diag(c(0.75, 0.6, 0.8))
uncR <- matrix(c(1.0, 0.4, 0.3, 0.4, 1.0, 0.12, 0.3, 0.12, 1.0),3,3)
dcc.para <- c(0.01,0.98)

# for normally distributed innovations
dcc.data <- dcc.sim(nobs, a, A, B, uncR, dcc.para, model="diagonal")

# for t distributed innovations
dcc.data.t <- dcc.sim(nobs, a, A, B, uncR, dcc.para, d.f=nu,
model="diagonal")
```

dlc	<i>Various partial derivatives of the DCC part of the log-likelihood function</i>
-----	---

Description

This function computes various analytical derivatives of the second stage log-likelihood function (the DCC part) of the (E)DCC-GARCH model.

Usage

```
dlc(dcc.para, B, u, h, model)
```

Arguments

- dcc.para the estimates of the (E)DCC parameters (2×1)
- B the estimated GARCH parameter matrix ($N \times N$)
- u a matrix of the used for estimating the (E)DCC-GARCH model ($T \times N$)

12

dlv

h a matrix of the estimated conditional variances ($T \times N$)

model a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

a list with components:

d1c the gradient of the DCC log-likelihood function w.r.t. the DCC parameters ($T \times 2$)

dvecP the partial derivatives of the DCC matrix, P_t w.r.t. the DCC parameters ($T \times N^2$)

dvecQ the partial derivatives of the Q_t matrices w.r.t. the DCC parameters ($T \times N^2$)

d21c the Hessian of the DCC log-likelihood function w.r.t. the DCC parameters ($T \times 4$)

dfdwd21c the cross derivatives of the DCC log-likelihood function ($T \times npar.h + 2$) $npar.h$ stand for the number of parameters in the GARCH part, $npar.h = 3N$ for "diagonal" and $npar.h = 2N^2 + N$ for "extended".

References

Engle, R.F. and K. Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

Hafner, C.M. and H. Herwartz (2008), "Analytical Quasi Maximum Likelihood Inference in Multivariate Volatility Models." *Metrika* **67**, 219–239.

dlv *Gradient of the GARCH part of the log-likelihood function of an (E)DCC-GARCH model*

Description

This function returns the analytical partial derivatives of the volatility part of the log-likelihood function of the DCC-GARCH model. The function is called from `dcc.results`.

Usage

```
dlv(u, a, A, B, model)
```

`dlv.est`

13

Arguments

- `u` a matrix of the data used for estimating an (E)DCC-GARCH model ($T \times N$)
- `a` a vector of the constants in the volatility part ($N \times 1$)
- `A` an ARCH parameter matrix ($N \times N$)
- `B` a GARCH parameter matrix ($N \times N$)
- `model` a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

A matrix of partial derivatives. ($T \times npar.h$) where *npar.h* stand for the number of parameters in the GARCH part, *npar.h* = $3N$ for "diagonal" and *npar.h* = $2N^2 + N$ for "extended".

References

Engle, R.F. and K. Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

Hafner, C.M. and H. Herwartz (2008), "Analytical Quasi Maximum Likelihood Inference in Multivariate Volatility Models." *Metrika* **67**, 219–239.

See Also

`dcc.estimation`

<code>dlv.est</code>	<i>Gradient of the GARCH part of the log-likelihood function of an (E)DCC GARCH model</i>
----------------------	---

Description

This function returns the gradient of the volatility part of the log-likelihood function of the DCC.

Usage

```
dlv.est(par, dvar, model)
```

Arguments

- `par` a vector of the parameters in the vector GARCH equation
- `dvar` a matrix of the data used for estimating an (E)DCC-GARCH model ($T \times N$)
- `model` a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

14

eccc. estimation

Value

A vector of the gradient. $(3N \times 1)$ for "diagonal" and $(2N^2 + N \times 1)$ for "extended".

Note

The function can be called from `optim` in `dcc. estimation1`. For obtaining the gradient for all t , use `dlv` instead.

References

Engle, R.F. and K. Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

Hafner, C.M. and H. Herwartz (2008), "Analytical Quasi Maximum Likelihood Inference in Multivariate Volatility Models." *Metrika* **67**, 219–239.

See Also

`dcc. estimation1`, `dlv`

`eccc. estimation` *Estimating an (E)CCC-GARCH model*

Description

This function estimates an (E)CCC-GARCH(1,1) model and returns estimates, estimated volatility and various diagnostic statistics.

Usage

```
eccc. estimation(a, A, B, R, dvar, model, method="BFGS")
```

Arguments

<code>a</code>	initial values for constants ($N \times 1$)
<code>A</code>	initial values for an ARCH parameter matrix ($N \times N$)
<code>B</code>	initial values for a GARCH parameter matrix ($N \times N$)
<code>R</code>	initial values a constant conditional correlation matrix ($N \times N$)
<code>dvar</code>	a matrix of data used for (E)CCC-GARCH estimation ($T \times N$)
<code>model</code>	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model
<code>method</code>	a character string specifying the optimisation method in <code>optim</code> . There are three choices, namely, Nelder–Mead, BFGS (default) and CG.

`eccc.sim`

15

Value

A list with components:

<code>out</code>	a $(4 \times npar)$ matrix. The estimates are contained in the first row. The remaining rows report standard errors based on three different methods of estimating the asymptotic covariance matrix
<code>h</code>	the estimated conditional variances $(T \times N)$
<code>std.resid</code>	a matrix of the standardised residuals $(T \times N)$. See <i>Note</i> .
<code>opt</code>	the detailed results of the optimisation
<code>para.mat</code>	vectorised parameter estimates

Note

The standardised residuals are calculated through dividing the original series by the estimated conditional standard deviations. See, for instance, p.303 of Bollerslev (1990) for details.

References

Bollerslev, T. (1990), “Modelling the Coherence in Short-run Nominal Exchange Rates: A Multivariate Generalized ARCH Model”, *Review of Economics and Statistics*, **20**, 498–505.

Nakatani, T. and T. Teräsvirta (2009), “Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model”, *Econometrics Journal*, **12**, 147–163.

Nakatani, T. and T. Teräsvirta (2008), “Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*” Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

`eccc.sim`*Simulating an (E)CCC-GARCH(1,1) process*

Description

This function simulates data either from the original CCC-GARCH by Bollerslev (1990) or from the Extended CCC-GARCH that has non-zero off-diagonal entries in the parameter matrices in the GARCH equation. The innovations (the standardised residuals) can be either a normal or student's t distribution.

The dimension (N) is determined by the number of elements in the `a` vector.

Usage

```
eccc.sim(nobs, a, A, B, R, d.f=Inf, cut=1000, model)
```


16

eccc.sim

Arguments

nobs	a number of observations to be simulated (T)
a	a vector of constants in the GARCH equation ($N \times 1$)
A	an ARCH parameter matrix in the GARCH equation. A can be a diagonal matrix for the original CCC-GARCH model or a full matrix for the extended model ($N \times N$)
B	a GARCH parameter matrix in the GARCH equation. B can be a diagonal matrix for the original CCC-GARCH model or a full matrix for the extended model ($N \times N$)
R	a constant conditional correlation matrix ($N \times N$)
d.f	the degrees of freedom parameter for the t -distribution
cut	the number of observations to be thrown away for removing initial effects of simulation
model	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

A list with components:

h	a matrix of the simulated conditional variances ($T \times N$)
eps	a matrix of the simulated time series with (E)CCC-GARCH process ($T \times N$)

Note

When `d.f=Inf`, the innovations (the standardised residuals) follow the standard normal distribution. Otherwise, they follow a student's t -distribution with `d.f` degrees of freedom equal.

When `model="diagonal"`, only the diagonal entries in **A** and **B** are used. If the ARCH and GARCH matrices do not satisfy the stationarity condition, the simulation is terminated.

References

Bollerslev, T. (1990), "Modeling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Approach", *Review of Economics and Statistics*, **72**, 498–505.

Nakatani, T. and T. Teräsvirta (2009), "Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model", *Econometrics Journal*, **12**, 147–163.

Nakatani, T. and T. Teräsvirta (2008), "Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*" Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

See Also

dcc.sim, stcc.sim

fourth

17

Examples

```
# Simulating data from the original CCC-GARCH(1,1) process
nobs <- 1000; cut <- 1000; nu <- 10
a <- c(0.003, 0.005, 0.001)
A <- diag(c(0.2,0.3,0.15))
B <- diag(c(0.79, 0.6, 0.8))
R <- matrix(c(1.0, 0.4, 0.3, 0.4, 1.0, 0.12, 0.3, 0.12, 1.0),3,3)
ccc.data <- eccc.sim(nobs,a, A, B, R, model="diagonal")
ccc.data.t <- eccc.sim(nobs,a, A, B, R, d.f=nu, model="diagonal")
```

fourth

*Fourth-order moment condition for the vector GARCH equation***Description**

This function computes the fourth-order moment condition for the vector GARCH equation in the (E)CCC-GARCH models.

Usage

```
fourth(A, B, R)
```

Arguments

A	an ARCH parameter matrix ($N \times N$)
B	a GARCH parameter matrix ($N \times N$)
R	a constant conditional correlation matrix ($N \times N$)

Value

a scalar. If strictly less than unity, the condition is satisfied.

References

He, C. and T. Teräsvirta (2004): “An Extended Constant Conditional Correlation GARCH model and its Fourth-moment Structure”, *Econometric Theory*, **20**, 904–926.

Nakatani, T. and T. Teräsvirta (2009), “Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model”, *Econometrics Journal*, **12**, 147–163.

Nakatani, T. and T. Teräsvirta (2008), “Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*” Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

See Also

stationarity

18

grad.dcc2

grad.dcc.full	<i>Numerical gradient of the full log-likelihood function of the (E)DCC-GARCH model</i>
---------------	---

Description

This function computes numerical gradient of the full log-likelihood function of the (E)DCC-GARCH(1,1) model with respect to its parameters.

Usage

```
grad.dcc.full(a, A, B, dcc.param, dvar, d=1e-5, model)
```

Arguments

a	a constant vector in the vector GARCH equation ($N \times 1$)
A	an ARCH parameter matrix in the vector GARCH equation ($N \times N$)
B	a GARCH parameter matrix in the vector GARCH equation ($N \times N$)
dcc.param	a vector of the DCC parameters (2×1)
dvar	a matrix of the data used for estimating the (E)DCC-GARCH model ($T \times N$)
d	a step size for computing numerical gradient
model	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

a matrix of partial derivatives ($T \times npar$)

Note

this function is currently not in use.

grad.dcc2	<i>Numerical gradient of the DCC part of the log-likelihood function</i>
-----------	--

Description

This function computes numerical gradient of the second stage log-likelihood function of the DCC-GARCH model w.r.t. its parameters.

Usage

```
grad.dcc2(param, dvar, d=1e-5)
```

`hh.test`

19

Arguments

`param` a vector of DCC parameters (2×1)
`dvar` a matrix of the standardised residuals ($T \times N$)
`d` a step size for computing numerical gradient

Value

a vector of partial derivatives (2×1)

Note

The function is used only from `dcc.estimation2` when the switch variable `gradient!=0`. `dlc$dlc` in `dlc` contains the analytical gradient of the second stage log-likelihood function.

See Also

`dlc`, `dcc.estimation`

`hh.test`*Carrying out the test of Hafner and Herwartz*

Description

This function computes the test statistic and the associated p-value of the test for causality in conditional variance in the CC-GARCH models.

Usage

```
hh.test(dvar)
```

Arguments

`dvar` ($T \times N$)

Value

A vector containing the test statistic and the associated p-value

References

Hafner, C.M. and H. Herwartz (2006), "A Lagrange Multiplier Test for Causality in Variance." *Economics Letters* **93**, 137–141.

See Also

`nt.test`

20

jb.test

`jb.test`*The Lomnicki-Jarque-Bera Test of normality (JB test)*

Description

This function performs the Lomnicki-Jarque-Bera Test of normality and returns test statistics and associated p-values.

Usage

```
jb.test(x)
```

Arguments

`x` a vector or matrix of variables to be tested

Value

Vector of test statistics and p-value

References

Jarque, C.M. and A.K. Bera (1987), "A Test for Normality of Observations and Regression Residuals", *International Statistical Review*, **55**, 163–172.

Lomnicki, Z.A. (1961), "Tests for Departure from Normality in the Case of Linear Stochastic Processes", *Metrika*, **4**, 37–62.

See Also

```
rob.sk, rob.kr, ljung.box.test
```

Examples

```
# for a vector
x <- rnorm(1000)
jb.test(x)

# for a matrix
X <- matrix(rnorm(10000), 5000, 2)
jb.test(X)
```

ljung.box.test

21

<code>ljung.box.test</code>	<i>The Ljung-Box Test statistic</i>
-----------------------------	-------------------------------------

Description

This function performs the Ljung-Box Test for a univariate time series.

Usage

```
ljung.box.test(x)
```

Arguments

`x` a vector of variables to be tested

Value

LB test statistics and associated p-values for lags 5, 10, ..., 50.

Note

Argument `x` must be a vector. When `x` is squared residuals, the test is equivalent to the McLeod and Li (1983) test.

References

Ljung, G.M. and G.E.P. Box (1978): "On a Measure of Lack of Fit in Time-Series Models", *Biometrika*, **65**, 297–303.

McLeod, A.I., and W.K. Li (1983): "Diagnostic checking ARMA time series models using squared-residual autocorrelations", *Journal of Time Series Analysis*, **4**, 269–273.

See Also

`rob.sk`, `rob.kr`, `jb.test`

Examples

```
x <- rnorm(1000)
ljung.box.test(x)           # returns the LB Test statistic
ljung.box.test(x^2)        # returns the McLeod-Li Test for no-ARCH effect
```

loglik.dcc

*The log-likelihood function for the (E)DCC GARCH model***Description**

This function returns a log-likelihood of the (E)DCC-GARCH model.

Usage

```
loglik.dcc(param, dvar, model)
```

Arguments

param	a vector of all the parameters in the (E)DCC-GARCH model
dvar	a matrix of the data used for estimating the (E)DCC-GARCH model ($T \times N$)
model	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

the negative of the full log-likelihood of the (E)DCC-GARCH model

Note

param must be made by stacking all the parameter matrices.

References

Robert F. Engle and Kevin Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Robert F. Engle (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalised Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

Examples

```
# Simulating data from the original DCC-GARCH(1,1) process
nobs <- 1000; cut <- 1000
a <- c(0.003, 0.005, 0.001)
A <- diag(c(0.2, 0.3, 0.15))
B <- diag(c(0.75, 0.6, 0.8))
uncR <- matrix(c(1.0, 0.4, 0.3, 0.4, 1.0, 0.12, 0.3, 0.12, 1.0), 3, 3)
dcc.para <- c(0.01, 0.98)
dcc.data <- dcc.sim(nobs, a, A, B, uncR, dcc.para, model="diagonal")

# Estimating a DCC-GARCH(1,1) model
dcc.results <- dcc.estimation(inia=a, iniA=A, iniB=B, ini.dcc=dcc.para,
  dvar=dcc.data$eps, model="diagonal")

# Parameter estimates and their robust standard errors
```

`loglik.dcc1`

23

```

    dcc.results$out

# Computing the value of the log-likelihood at the estimates
loglik.dcc(dcc.results$out[1,], dcc.data$eps, model="diagonal")

```

`loglik.dcc1`*The 1st stage log-likelihood function for the (E)DCC GARCH***Description**

This function returns a log-likelihood of the (E)DCC-GARCH model in the first stage estimation.

Usage

```
loglik.dcc1(param, dvar, model)
```

Arguments

<code>param</code>	initial values for a vector of the parameters ($npar \times 1$)
<code>dvar</code>	a matrix of the data ($T \times N$)
<code>model</code>	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

the negative of the first stage log-likelihood

Note

The function is used in `optim` in `dcc.estimation1`.

References

Engle, R.F. and K. Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

See Also

`dcc.estimation`, `dcc.estimation1`

24

loglik.eccc

`loglik.dcc2`*The 2nd stage log-likelihood function for the (E)DCC GARCH*

Description

This function returns a log-likelihood of the (E)DCC-GARCH model in the 2nd step estimation.

Usage

```
loglik.dcc2(param, dvar)
```

Arguments

<code>param</code>	initial values for the DCC parameters (2×1)
<code>dvar</code>	a matrix of the standardised residuals ($T \times N$)

Value

the negative of the second stage log-likelihood

Note

The function is used in `constrOptim` in `dcc.estimation2`.

References

Engle, R.F. and K. Sheppard (2001), "Theoretical and Empirical Properties of Dynamic Conditional Correlation Multivariate GARCH." *Stern Finance Working Paper Series* FIN-01-027 (Revised in Dec. 2001), New York University Stern School of Business.

Engle, R.F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models." *Journal of Business and Economic Statistics* **20**, 339–350.

See Also

```
dcc.estimation, dcc.estimation2
```

`loglik.eccc`*The log-likelihood function of the (E)CCC-GARCH model*

Description

This function computes a log-likelihood of the (E)CCC-GARCH(1,1) model.

Usage

```
loglik.eccc(param, dvar, model)
```

`nt.test`

25

Arguments

`param` a vector of all the parameters in the (E)CCC-GARCH model
`dvar` a matrix of the data used for estimating the (E)DCC-GARCH model ($T \times N$)
`model` a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

the negative of the (E)CCC-GARCH log-likelihood

References

Nakatani, T. and T. Teräsvirta (2009), "Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model", *Econometrics Journal*, **12**, 147–163.

Nakatani, T. and T. Teräsvirta (2008), "Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*" Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

`nt.test`*Carrying out the test of Nakatani and Teräsvirta*

Description

This function computes the test statistic and the associated p-value of the test for causality in conditional variance in the CC-GARCH models.

Usage

```
nt.test(dvar)
```

Arguments

`dvar` ($T \times N$)

Value

A matrix containing the test statistics of the standard (non-robust) test and the robust version, and the associated p-values

References

Nakatani, T and T. Teräsvirta (2010), "An Alternative Test for Causality in Variance in the Conditional Correlation GARCH models." *mimeo*, Stockholm School of Economics.

See Also

`hh.test`

26

rob.kr

p.mat *Re-arranging a vector into parameter matrices*

Description

A utility function that re-arranges a vector of parameters into parameter matrices in the CC-GARCH(1,1) model.

Usage

```
p.mat(para, model, ndim)
```

Arguments

para	a vector of parameters to be re-arranged into parameter matrices
model	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model
ndim	the number of dimension of the model

Value

A list with components:

a	a vector of constants in the vector GARCH equation
A	an ARCH parameter matrix
B	a GARCH parameter matrix
R	a constant conditional correlation matrix

References

Nakatani, T. and T. Teräsvirta (2009), "Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model", *Econometrics Journal*, **12**, 147–163.

Nakatani, T. and T. Teräsvirta (2008), "Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*" Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

rob.kr *Computing standard and robustified excess kurtosis*

Description

This function computes standard and robustified excess kurtosis of a vector or matrix of variables.

Usage

```
rob.kr(x)
```

`rob.sk`

27

Arguments`x` vector or matrix of variables**Value**

Vector of excess kurtosis and robustified excess kurtosis

ReferencesKim, T-H. and H. White (2004), “On More Robust Estimation of Skewness and Kurtosis”, *Finance Research Letters*, **1**, 56–73.**See Also**`rob.sk`, `ljung.box.test`, `jb.test`**Examples**

```
x <- matrix(rnorm(1000), 100, 10)
rob.kr(x)
```

`rob.sk`*Computing standard and robustified skewness*

Description

This function computes standard and robustified skewness measures of a vector or matrix of variables.

Usage`rob.sk(x)`**Arguments**`x` a vector or matrix of variables**Value**

Vector of skewness and robustified skewness

ReferencesKim, T-H. and H. White (2004), “On More Robust Estimation of Skewness and Kurtosis”, *Finance Research Letters*, **1**, 56–73.**See Also**`rob.kr`, `ljung.box.test`, `jb.test`**Examples**

```
x <- matrix(rnorm(1000), 100, 10)
rob.sk(x)
```

28

stcc.sim

stationarity

*The stationarity condition in Extended CC-GARCH models***Description**

A utility function that checks if the two parameter matrices in a vector GARCH model satisfy the stationarity condition.

Usage

```
stationarity(A,B)
```

Arguments

A an ARCH parameter matrix in the vector GARCH equation ($N \times N$)
 B a GARCH parameter matrix in the vector GARCH equation ($N \times N$)

Value

a scalar. If strictly less than unity, the condition is satisfied.

References

He, C. and T. Teräsvirta (2004): “An Extended Constant Conditional Correlation GARCH model and its Fourth-moment Structure”, *Econometric Theory*, **20**, 904–926.
 Nakatani, T. and T. Teräsvirta (2009), “Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model”, *Econometrics Journal*, **12**, 147–163.
 Nakatani, T. and T. Teräsvirta (2008), “Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*” Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

See Also

fourth

stcc.sim

*Simulating Data from an STCC-GARCH\$(1,1)\$ process***Description**

This function simulates data either from the original STCC-GARCH by Silvennoinen and Teräsvirta (2005) or from the Extended STCC-GARCH that has non-zero off-diagonal entries in the parameter matrices in the GARCH equation, with multivariate normal or student's t distribution.

The dimension (N) is determined by the number of elements in the a vector.

`stcc.sim`

29

Usage

```
stcc.sim(nobs, a, A, B, R1, R2, tr.par, st.par, d.f=Inf,
         cut=1000, model)
```

Arguments

<code>nobs</code>	a number of observations to be simulated (T)
<code>a</code>	a vector of constants in the vector GARCH equation ($N \times 1$)
<code>A</code>	an ARCH parameter matrix in the vector GARCH equation. ($N \times N$)
<code>B</code>	a GARCH parameter matrix in the vector GARCH equation. ($N \times N$)
<code>R1</code>	a conditional correlation matrix in regime 1 ($N \times N$)
<code>R2</code>	a conditional correlation matrix in regime 2 ($N \times N$)
<code>tr.par</code>	a vector of scale and location parameters in the transition function (2×1)
<code>st.par</code>	a vector of parameters for the GARCH(1,1) transition variable (3×1)
<code>d.f</code>	the degrees of freedom parameter for the t -distribution
<code>cut</code>	the number of observations to be thrown away for removing initial effects of simulation
<code>model</code>	a character string describing the model. "diagonal" for the diagonal model and "extended" for the extended (full ARCH and GARCH parameter matrices) model

Value

A list with components:

<code>h</code>	a matrix of conditional variances ($T \times N$)
<code>eps</code>	a matrix of time series with DCC-GARCH process ($T \times N$)
<code>tr.var</code>	a vector of the transition variable
<code>st</code>	a vector of time series of the transition function
<code>vecR</code>	a ($T \times N^2$) matrix of Smooth Transition Conditional Correlations

Note

When `d.f=Inf`, the innovations (the standardised residuals) follow the standard normal distribution. Otherwise, they follow a student's t -distribution with `d.f` degrees of freedom equal.

When `model="diagonal"`, only the diagonal entries in **A** and **B** are used. If the ARCH and GARCH matrices do not satisfy the stationarity condition, the simulation is terminated.

References

Silvennoinen, A. and T. Teräsvirta (2005), "Multivariate Autoregressive Conditional Heteroskedasticity with Smooth Transitions in Conditional Correlations." *SSE/EFI Working Paper Series in Economics and Finance* No. 577, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0577.htm>.

See Also

`dcc.sim`, `eccc.sim`

30

*tr.func***Examples**

```

# Simulating data from the original STCC-GARCH(1,1) process
nobs <- 1000; cut <- 1000
a <- c(0.003, 0.005, 0.001)
A <- diag(c(0.2,0.3,0.15))
B <- diag(c(0.79, 0.6, 0.8))
# Conditional Correlation Matrix for regime 1
R1 <- matrix(c(1.0, 0.4, 0.3, 0.4, 1.0, 0.12, 0.3, 0.12, 1.0),3,3)
# Conditional Correlation Matrix for regime 2
R2 <- matrix(c(1.0, 0.01, -0.3, 0.01, 1.0, 0.8, -0.3, 0.8, 1.0),3,3)
# a parameter vector for the scale and location parameters
# in the logistic function
tr.param <- c(5,0)
# a parameter vector for a GARCH(1,1) transition variable
st.param <- c(0.02,0.04, 0.95)
nu <- 15
stcc.data <- stcc.sim(nobs, a, A, B, R1, R2,
                     tr.param=tr.param, st.param=st.param, model="diagonal")
stcc.data.t <- stcc.sim(nobs, a, A, B, R1, R2,
                       tr.param=tr.param, st.param=st.param, d.f=nu,
                       model="diagonal")

```

*tr.func**Logistic transition function*

Description

This function computes values from a Logistic transition function.

Usage

```
tr.func(tr.par, tr.var)
```

Arguments

```
tr.par      a vector of parameters ( $2 \times 1$ )
tr.var      a vector of transition variable ( $T \times 1$ )
```

Value

a vector of transition function ($T \times 1$)

Note

this function is used in `stcc.sim`

References

Teräsvirta, T. (1994): "Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models", *Journal of the American Statistical Association*, **89**, 208–218.

See Also

`stcc.sim`

`uni.vola`

31

`uni.vola`*Computing univariate GARCH(1,1) conditional variances*

Description

This function returns an univariate GARCH(1,1) conditional variances.

Usage

```
uni.vola(a, u)
```

Arguments

`a` a vector of parameters in the GARCH(1,1) equation (3×1)
`u` a vector of the data ($T \times 1$)

Value

a vector of GARCH(1,1) conditional variances ($T \times 1$)

References

Bollerslev, T. (1986): "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, **31**, 307–327.

See Also

```
uni.vola.sim
```

Examples

```
a <- c(0.01, 0.04, 0.95)
u <- rnorm(1000)
h <- uni.vola(a, u)
```

`uni.vola.sim`*Simulating a series with univariate GARCH(1,1) conditional variances*

Description

This function simulates an univariate time series with GARCH(1,1) conditional variances.

Usage

```
uni.vola.sim(a, nobs, d.f=Inf, cut=1000)
```


32

vdR

Arguments

a	a vector of parameters (3×1)
nobs	a number of observations simulated (T)
d.f	degrees of freedom parameter for t -distribution
cut	a number of observations to be removed to minimise the initial effects

Value

A list with components:

h	GARCH(1,1) conditional variances ($T \times 1$)
eps	a series of error term with the conditional variances "h" ($T \times 1$)

Note

When `d.f=Inf`, the innovations (the standardised residuals) follow the standard normal distribution. Otherwise, they follow a student's t -distribution with `d.f` degrees of freedom.

References

- Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, **31**, 307–327.
- Florentini, G., G. Calzolari and L. Panattoni (1996), "Analytic Derivatives and the Computation of GARCH Estimates", *Journal of Applied Econometrics*, **11**, 399–417.

See Also

`uni.vola`

Examples

```
nobs <- 1000
nu <- 8
a <- c(0.1, 0.2, 0.7)
# with normal innovations
eps <- uni.vola.sim(a, nobs)
# with t innovations
eps.t <- uni.vola.sim(a, nobs, d.f = df)
```

vdR

Computing partial derivatives of the CCC matrix

Description

This function computes partial derivatives of the CCC matrix with respect to its correlation coefficients.

Usage

```
vdR(n)
```

`vec.garch.derivative`

33

Arguments

`n` the number of dimension of the model

Value

a matrix of zeros and ones $((N(N - 1))/2 \times N^2)$

References

Nakatani, T. and T. Teräsvirta (2009), “Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model”, *Econometrics Journal*, **12**, 147-163.

Nakatani, T. and T. Teräsvirta (2008), “Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*” Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

`vec.garch.derivative`

Computing partial derivatives of a vector GARCH(1, 1) equation

Description

This function computes partial derivatives of a vector GARCH(1, 1) equation with respect to its parameters.

Usage

`vec.garch.derivative(dvar, B, h)`

Arguments

`dvar` a matrix of the data used for estimating an ECCC or DCC GARCH model ($T \times N$)

`B` a GARCH parameter matrix in the vector GARCH equation ($N \times N$)

`h` a matrix of conditional variances ($T \times N$)

Value

a vector of partial derivatives ($T \times N * npar.h$)

References

Nakatani, T. and T. Teräsvirta (2009), “Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model”, *Econometrics Journal*, **12**, 147-163.

Nakatani, T. and T. Teräsvirta (2008), “Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*” Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

vector.garch	A vector GARCH(1,1) conditional variances
--------------	---

Description

This function computes a vector GARCH(1,1) conditional variances.

Usage

```
vector.garch(dvar, a, A, B)
```

Arguments

dvar	a matrix of the data, used as epsilon ($T \times N$)
a	initial values for constants in the vector GARCH equation ($N \times 1$)
A	initial values for an ARCH parameter matrix in the vector GARCH equation ($N \times N$)
B	initial values for a GARCH parameter matrix in the vector GARCH equation ($N \times N$)

Value

a matrix of conditional variances ($T \times N$)

References

Nakatani, T. and T. Teräsvirta (2009), “Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model”, *Econometrics Journal*, **12**, 147-163.

Nakatani, T. and T. Teräsvirta (2008), “Appendix to *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model*” Department of Economic Statistics, Stockholm School of Economics, available at <http://swopec.hhs.se/hastef/abs/hastef0649.htm>.

Index

- *Topic **algebra**
 - fourth, 17
- *Topic **array**
 - p.mat, 26
- *Topic **datagen**
 - dcc.sim, 10
 - eccc.sim, 16
 - stcc.sim, 29
 - uni.vola.sim, 32
- *Topic **htest**
 - jb.test, 20
 - ljung.box.test, 21
 - rob.kr, 27
 - rob.sk, 27
- *Topic **models**
 - analytical.grad, 2
 - analytical.Hessian, 3
 - d2lv, 4
 - dcc.est, 5
 - dcc.results, 9
 - dcc.sim, 10
 - dlc, 12
 - dlv, 13
 - dlv.est, 14
 - eccc.estimation, 15
 - eccc.sim, 16
 - grad.dcc.full, 18
 - grad.dcc2, 19
 - hh.test, 19
 - loglik.dcc, 22
 - loglik.dcc1, 23
 - loglik.dcc2, 24
 - loglik.eccc, 24
 - nt.test, 25
 - stcc.sim, 29
 - uni.vola.sim, 32
 - vdR, 33
 - vec.garch.derivative, 33
 - vector.garch, 34
- *Topic **multivariate**
 - analytical.Hessian, 3
 - d2lv, 4
 - dcc.est, 5
 - dcc.estimation, 6
 - dcc.estimation1, 7
 - dcc.estimation2, 8
 - dcc.results, 9
 - dcc.sim, 10
 - dlc, 12
 - dlv, 13
 - dlv.est, 14
 - eccc.estimation, 15
 - eccc.sim, 16
 - grad.dcc.full, 18
- dcc.estimation, 6
- dcc.estimation1, 7
- dcc.estimation2, 8
- dcc.results, 9
- dcc.sim, 10
- dlc, 12
- dlv, 13
- dlv.est, 14
- eccc.estimation, 15
- eccc.sim, 16
- grad.dcc.full, 18
- grad.dcc2, 19
- hh.test, 19
- loglik.dcc, 22
- loglik.dcc1, 23
- loglik.dcc2, 24
- loglik.eccc, 24
- nt.test, 25
- stationarity, 28
- stcc.sim, 29
- vdR, 33
- vec.garch.derivative, 33

- grad.dcc2, 19
- hh.test, 19
- jb.test, 20
- ljung.box.test, 21
- loglik.dcc, 22
- loglik.dcc1, 23
- loglik.dcc2, 24
- loglik.eccc, 24
- nt.test, 25
- rob.kr, 27
- rob.sk, 27
- stationarity, 28
- stcc.sim, 29
- tr.func, 30
- uni.vola, 31
- uni.vola.sim, 32
- vdR, 33
- vec.garch.derivative, 33
- vector.garch, 34
- *Topic univar**
 - uni.vola, 31
- *Topic utilities**
 - p.mat, 26
- analytical.grad, 2
- analytical.Hessian, 3
- d2lv, 4
- dcc.est, 5, 7
- dcc.estimation, 6, 8–10, 13, 19, 23, 24
- dcc.estimation1, 7, 7, 9, 14, 23
- dcc.estimation2, 7, 8, 8, 19, 24
- dcc.results, 9
- dcc.sim, 10, 17, 30
- dlc, 12, 19
- dlv, 13, 14
- dlv.est, 14
- eccc.estimation, 15
- eccc.sim, 11, 16, 30
- fourth, 17, 28
- grad.dcc.full, 18
- grad.dcc2, 19
- hh.test, 19, 25
- jb.test, 20, 21, 27, 28
- ljung.box.test, 20, 21, 27, 28
- loglik.dcc, 22
- loglik.dcc1, 7, 23
- loglik.dcc2, 7, 24
- loglik.eccc, 24
- nt.test, 20, 25
- p.mat, 26
- rob.kr, 20, 21, 27, 28
- rob.sk, 20, 21, 27, 27
- stationarity, 18, 28
- stcc.sim, 11, 17, 29, 31
- tr.func, 30
- uni.vola, 31, 32
- uni.vola.sim, 31, 32
- vdR, 33
- vec.garch.derivative, 33
- vector.garch, 7, 34

Table 5.1 Arguments for `dcc.sim`

Name	Description
<code>nobs</code>	number of observations to be simulated.
<code>a</code>	vector of constants in the GARCH equation (length N).
<code>A</code>	ARCH parameter in the GARCH equation ($N \times N$).
<code>B</code>	GARCH parameter in the GARCH equation ($N \times N$).
<code>R</code>	unconditional correlation matrix ($N \times N$).
<code>dcc.para</code>	vector of the DCC parameters (length 2).
<code>d.f</code>	degrees of freedom parameter for the t -distribution (optional).
<code>model</code>	character string, “diagonal” or “extended”.

Table 5.2 Output from `dcc.sim`

Name	Description
<code>z</code>	random draws from $\mathbf{N}(\mathbf{0}, \mathbf{I})$ ($T \times N$).
<code>std.z</code>	standardised residuals. <code>std.z[t,]</code> $\sim \text{ID}(0, \mathbf{P}_t)$ ($T \times N$).
<code>dcc</code>	dynamic conditional correlations ($T \times N^2$).
<code>h</code>	simulated conditional variances ($T \times N$).
<code>eps</code>	time series with DCC-GARCH process ($T \times N$).

NOTE: ID denotes an independent distribution, either a multivariate normal or Student’s t distribution.

Table 5.3 Arguments for `dcc.estimation`

Name	Description
<code>a</code>	initial values for the constants (length N).
<code>A</code>	initial values for the ARCH parameter matrix ($N \times N$).
<code>B</code>	initial values for the GARCH parameter matrix ($N \times N$).
<code>dcc.para</code>	initial values for the DCC parameters (length 2).
<code>dvar</code>	a matrix of the data ($T \times N$).
<code>model</code>	a character string, “diagonal” or “extended”.
<code>method</code>	a character string specifying the optimisation method in <code>optim</code> . There are three choices, namely, “Nelder-Mead”, “BFGS” (default) and “CG”.
<code>gradient</code>	a switch variable that determines the optimisation algorithm in the second stage optimisation. If <code>gradient=0</code> Nelder-Mead is applied. Otherwise BFGS is used (default).
<code>message</code>	a switch variable that controls if the message is displayed when the estimation is completed. If <code>message=0</code> , the message is suppressed. Otherwise, the message is displayed (default)

Table 5.4 Effects of initial values for parameter estimates in the DCC-GARCH(1, 1) model

Coefficients	True initial values				Untrue initial values		
	True	Mean	SD	MSE	Mean	SD	MSE
a_1	0.003	0.0033	0.0010	0.0344	0.0033	0.0011	0.0345
a_2	0.005	0.0052	0.0013	0.0406	0.0053	0.0014	0.0446
a_3	0.001	0.0011	0.0004	0.0126	0.0012	0.0007	0.0234
a_{11}	0.200	0.1997	0.0340	1.0749	0.1991	0.0346	1.0937
a_{22}	0.300	0.2996	0.0453	1.4321	0.3008	0.0465	1.4709
a_{33}	0.150	0.1501	0.0283	0.8941	0.1520	0.0293	0.9269
b_{11}	0.750	0.7427	0.0409	1.3137	0.7435	0.0416	1.3302
b_{22}	0.600	0.5940	0.0530	1.6845	0.5914	0.0566	1.8095
b_{33}	0.800	0.7927	0.0384	1.2344	0.7877	0.0552	1.7877
α	0.010	0.0109	0.0070	0.2225	0.0114	0.0078	0.2512
β	0.980	0.9507	0.0854	2.8546	0.9404	0.0998	3.3928

NOTE: Based on 1000 replications. SD and MSE stand for standard deviations and mean squared errors of parameter estimates, respectively.

Table 5.5 Outputs from `dcc.estimation`

Name	Description
<code>out</code>	the estimates and their robust standard errors
<code>loglik</code>	the value of the log-likelihood function at the estimates
<code>h</code>	a matrix of the estimated conditional variances ($T \times N$)
<code>DCC</code>	a matrix of dynamic conditional correlation estimates ($T \times N^2$)
<code>std.resid</code>	a matrix of the standardised residuals ($T \times N$)
<code>first</code>	the results of the first stage estimation
<code>second</code>	the results of the second stage estimation

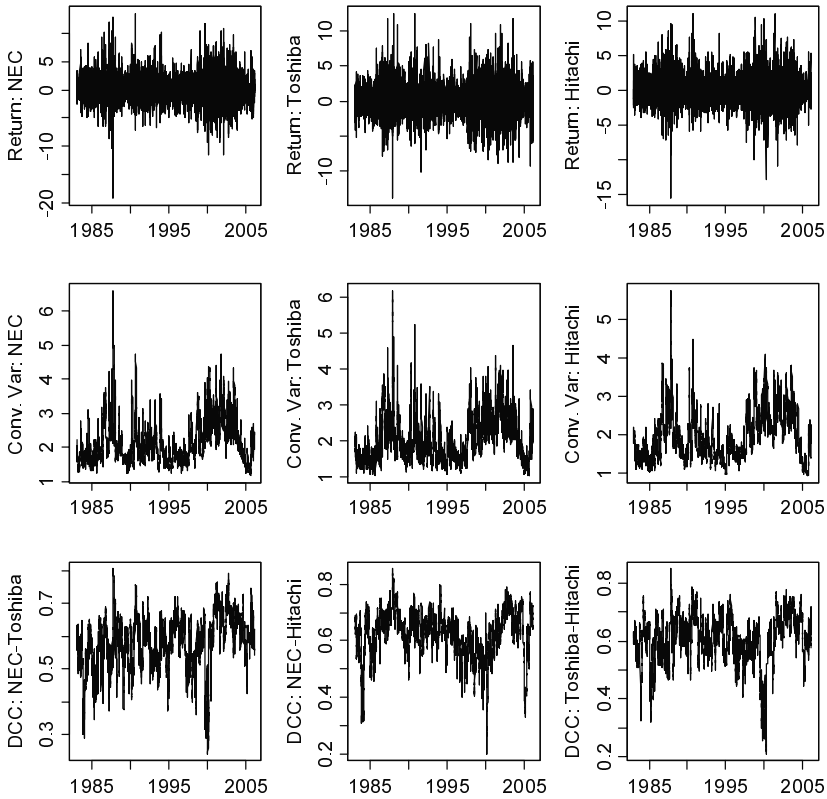


Figure 5.1 Returns, estimated conditional variances and dynamic conditional correlations of the three Japanese stock return series.

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