ESSAYS IN APPLIED GAME THEORY

Marcus Salomonsson

STOCKHOLM SCHOOL OF ECONOMICS

In this thesis, game theoretical methods are applied in two distinct areas. First, the strategic interaction between players in financial markets is studied. In particular, the strategic interaction between informed traders, noise traders, and market makers is studied. A bid-ask spread is introduced into the Kyle model, and it is also shown that noise traders can be endogenized within the Glosten-Milgrom framework.

Second, evolutionary game theory is used to study how preferences, especially social preferences, have been formed. A group selection model based on reproductive externalities is developed, and it is shown that it can account not only for altruism, but also for other social preferences, such as spite and willingness to undertake costly punishment. The literature on group selection is also surveyed. The early contributions and the group selection controversy are described. The main approaches to group selection in the recent literature - fixation, assortative group formation, and reproductive externalities - are also described and discussed.

Marcus Salomonsson finished his Ph.D. thesis at the SSE Department of Economics in 2009.

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Dissertation for the Degree of Doctor of Philosophy
Stockholm School of Economics

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KEYWORDS: Market microstructure, spread; market maker; no-trade theorems, adverse selection; group selection; social preferences; altruism; fairness; altruism; spite; externalities; conformity; fixation; signalling

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To Ylva

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Stockholm in April 2009
Marcus Salomonsson

Chapters

CHAPTER 1

Introduction

In game theory, everybody is a player. That is, everybody engages in strategic interactions. In many classical analyses of economic problems this is not the case. Although it is typically assumed that people maximize their utility, it is often assumed that an individual is an atom in an infinitely large population. This means that one can disregard the effect of that individual's behavior on other individuals' behavior. This is often a good first approach to many problems, but in some cases it is too simplistic.

In this thesis I have applied game theoretical methods in two distinct areas. First, I have studied the strategic interaction between players in financial markets. Second, I have used evolutionary game theory to study how preferences, especially social preferences, have been formed.

1. Information on financial markets

In traditional models of financial markets, it is often assumed that information is symmetric and markets are efficient. This is clearly not the case: people do have different information. Nevertheless, it may be that this fact is not important for understanding how financial markets function. However, the assumption that markets are efficient can easily be shown to result in a paradox. If market were efficient, in the sense that every piece of information is incorporated into prices immediately, then nobody would gain from doing costly information gathering. The reason is that it would be impossible to cover costs, since markets had already incorporated the information into prices. But if there exists no incentive for people to get information, then prices should at most only incorporate completely costless information, which means that markets can not be efficient in the sense just described. This is known as the Grossman-Stiglitz paradox. The paradox suggests that markets can not be efficient, at least not to such a degree that every piece of information is immediately incorporated in prices. Instead it should be possible for people with information to gain by trading on financial markets.

But this raises another question. If two people trade, then one player's expected profit will be the other player's expected loss. This seems odd, since it would suggest that some people trade although they on expectation will make a loss. The explanation may be that one of the parties is ready to accept to trade at an expected loss because he or she has some other reason to trade, for example a need for liquidity. This type of traders, since they are typically just represented by a statistical noise term, are usually called noise traders, and sometimes liquidity traders.

There are two basic models to understand the interaction between informed traders and noise traders. Both have three types of agents; informed traders, noise traders, and market makers. In the first model, the Kyle model, the traders submit orders to the market makers, who adjust prices depending on the orders they get. That, is, if the net order is a buy order, then he increases the price; if the net order is instead a sell order, then he lowers the price. In the other model, the Glosten-Milgrom model, market makers announce a bid price and an ask price. They buy at the bid price and sell at the ask price. The difference, called the spread, results in a gain when trading with noise traders. This gain can compensate the losses the market makers incur when they trade with informed traders.

In chapter two I have merged these two models. This means that market makers both adjust prices depending on the net order and make a profit on the spread. The advantage of this merger is threefold. First, it is more realistic. Second, it makes it possible to examine the market makers' trade off between adjustment of prices and spreads. Third, and finally, it results in a more robust model in the sense that it has an equilibrium under more circumstances than previous formulations.

However, the noise traders are still exogenous. That is, their reasons for trading are not explained within the model. In chapter three I consider an approach towards explaining noise traders within the Glosten-Milgrom model. As described earlier, noise traders lose on expectation. The standard interpretation is that, if they own an asset, they must sometimes sell it for some exogenous reason. The reason could, for example, be that the money is needed elsewhere. However, if they were rational when buying the asset, then they would only buy it if they were given a discount to compensate for this expected loss. I introduce this feature into the model, and show that it means that prices must be increasing on expectation for this type of endogenous noise traders to participate in the market.

I also extend the model to compare a monopolistic informed trader with a competitive informed trader. It turns out that the noise traders' losses are larger if the informed trader is competitive. This is in stark contrast to comparable studies within the Kyle model. In the Kyle model increased competition between informed traders results in prices revealing more information. As a consequence, the market makers adjust prices more accurately and can afford to allow the noise traders to trade at better prices. Within the Glosten-Milgrom model, however, market makers announce prices before the traders trade. In addition, competition between informed traders implies that they will acquire better information. As a result, market makers will make larger losses if informed traders are competitive, which means that they must give the noise traders worse prices.

2. Social preferences

A common misconception is that economists believe that individuals maximize their profits. However, already in 1738 the swiss mathematician Daniel Bernoulli showed that profit maximization would lead to unreasonable results. The argument, originally formulated by his cousin Nicolas Bernoulli in 1713, stemmed from something called the St. Petersburg paradox. The argument in this paradox is that one can construct a lottery that would have an infinitely high expected value, but also a sufficiently high risk, so that real people would only bid a finite amount for it. Since the St. Petersburg paradox showed that real people not only cared about expected profit, Daniel Bernoulli introduced the concept of utility and argued that individuals strived to maximize expected utility.

As the St. Petersburg paradox illustrated, risk should play a role in utility. However, one can also argue that other measures may play a role. For example, prestige or status may be more important than monetary remunerations. A related issue is whether people should only care about themselves. In economic models, it is typically assumed that individuals are selfish. While this approach has met with tremendous success, there are also well known examples when people do not seem to be selfish. One quite obvious example is their behavior in relation to their offspring. Even the most casual observer would note that most normal human parents care not only for themselves, but also for their children. This is an empirical fact, but has also been convincingly argued to be selfish in theoretical models. The argument, popularized by the evolutionary biologist Richard Dawkins, but originally formulated by the evolutionary biologists Bill Hamilton, is that selection is for genes, and that organisms are the vehicles for genes to spread. This means that if a gene exists in two individuals, then, from the gene's point of view, it does not matter which organism survives and which dies, as long as the surviving organism carries the gene. The argument can be pushed even further: If the gene is present in one old individual, who will not get any offspring; and in another individual, who may get offspring, then the gene that pushes the old organism to help the young would spread faster than other genes.

This example seems quite obvious. Even if they would not believe in evolution, few would think that not helping their offspring would be normal human behavior. There are other instances, though, when people are not relatives, and still seems to have social preferences. One example is repeated interactions. The reason is that then they invest in their own reputation. This means that even if an individual would not expect to ever meet the other individual again, he could still gain from helping him if news of his behavior would spread to others. This mechanism for creating social preferences is called reciprocity. It can be both direct, when an individual interacts with the same individual in repeated interactions; and indirect, when he interacts with different individuals.

In chapter four and five, I study another mechanism that could foster altruism: group selection. The basic argument was formulated already by Darwin. He argued that groups with members that are ready to sacrifice themselves for each other should be more successful than other groups, and thus spread. Group selection has been widely discussed, although no clear consensus regarding the exact mechanism and its importance have yet been established. In chapter four, written with Jörgen Weibull, I study a model of group selection based on reproductive externalities.

An externality is essentially a spillover effect. If the spillover effect is positive, then whoever is producing it may produce too little if he is not rewarded for it. It also opens up for free-riders. A well known example is defence. While defending a group or a territory, a spillover effect is that also others are defended. To illustrate how individual selection and group selection may move in different directions, consider preparations to defend against an attacker. Being prepared if attacked is naturally a good thing. At the same time, it costs time and resources. This means that if the group is never attacked, then the individuals that did not prepare have not wasted time and resources, and are therefore better off. Thus, on an individual level it is better to not prepare. However, whenever the group is attacked it will pay off to be prepared. As a result, the attack has the effect of rewarding groups where many are prepared, and in that sense internalizes the externality.

We generalize this argument and show that it can be used to explain both altruism and spite, and that it makes it possible to explain a number of empirical findings regarding social preferences.

In chapter five, I survey the literature on group selection. Despite the fact that already Darwin mentioned it, group selection has caused quite a lot of controversy. In particular, the controversy has been about an extreme interpretation of group selection. This interpretation has not been formalized in any stringent mathematical model, but proponents seemed to argue that when two levels of selection, for example individual selection and group selection, moved in different directions, then the higher level would trump the lower level. This was forcefully rejected in the 1960's by the evolutionary biologists John Maynard Smith and George C. Williams

Other formalizations have been more stringent. John Maynard Smith formulated a group selection model that became known as the Haystack model. In this model, he showed that group selection could indeed foster altruism. However, he also argued that the driving mechanisms were unlikely to play an important role empirically, and that this type of group selection thus would only have a minor influence on preferences. The evolutionist David Sloan Wilson formulated another theory based on a correlation between being an altruist and being matched with other altruists. This theory begs the question of how this matching occurs. I discuss two possibilities. First the possibility that an innate urge to conform could lead to assortative matching. Second, that signalling could achieve it. Finally, various authors have formulated models of group selection that in effect are equivalent to the reproductive externality approach described above.

CHAPTER 2

Introducing a Spread into the Kyle Model

Marcus Salomonsson[1]

Abstract. The Kyle (1985) model is extended to take into account market maker competition and the spread. It is shown that with a spread the Kyle model has a Nash equilibrium also with two market makers, not only with three or more, as shown in earlier research. The spread is endogenized, and two testable predictions of the model are generated. The first is that the spread is increasing in the standard deviation of the fundamentals. The second is that it is independent of the standard deviation of noise trades.

1. Introduction

A well known robustness problem with the Kyle model, due to Kyle (1985), is that it has no equilibrium when there are fewer than three market makers. This was shown first by Dennert (1993) in the one period setting, and then by Bondarenko (2001) in the multiperiod setting. In this chapter it is shown that the Kyle model has an equilibrium also with two market makers if the model is extended to incorporate the spread.

The rationale for introducing the spread into the Kyle model is threefold. Firstly, spreads are indeed a reality on most markets.[2]

Secondly, although Bernhardt and Hughson (1997) have shown that an equilibrium exists with both one and two market makers if noise trader demand falls as prices increase - provided it does not fall too much, in practice this may often not be the case. For example, in the literature on predatory trading, e.g. Brunnermeier and Pedersen (2005), it has been noted that traders may be pushed to exit their positions if prices move too far in the harmful direction. An example would be that somebody who is short an asset may have to buy it back if the price is pushed sufficiently high, which of course would mean that noise trader demand is increasing in the price. Thus,

[1] I am grateful to the Wallenberg Foundation for financial support. I am also grateful for comments from Stefano Rossi, Francesco Sangiorgi, Andrei Simonov and Jörgen Weibull. Any errors are my own.

[2] Recently, Bollen et al (2004) showed empirically that the spread during three recent periods on the Nasdaq dependeded on the minimum tick size, the order processing cost, the level of competition, inventory holding costs, and adverse selection costs.

especially in times of crisis, Bernhardt and Hughson's (1997) approach may suggest that markets are much more vulnerable than they really are.

Thirdly, introducing spreads into the Kyle model improves our understanding of how market makers cover costs. In the original Kyle model they do it by using pricing rules that results in overshooting prices, whereas they do it by using the spread in Glosten and Milgrom (1985). In this merger of the two models, we see that they use a combination of both approaches.[3]

The perfect competition between market makers assumed in Kyle (1985) has been interpreted in at least two ways. First, as argued in Kyle (1984), it can be interpreted as an ideal case resulting from competition between infinitely many market makers. Second, as argued in Bernhardt and Hughson (1997), it can be interpreted as the result from a winner-takes-all contest between two, or more, market makers.

Dennert (1993) looks at both possibilities. First he looks at market maker competition in the setting of Glosten and Milgrom (1985). In that setting the market makers announce bid ask prices and the quantities they offer at those prices. The noise traders then take the best offer available, given their exogenous need to trade. It is assumed that the market maker with the best price can always satisfy the noise traders' entire liquidity need. As a result, the other market makers will not trade at all with the noise traders. This implies that there exists no pure strategy equilibrium in this setting. However, Dennert shows that a symmetric mixed strategy equilibrium always exists. In this mixed equilibrium all market makers make zero profits. In addition, the best prices are actually offered to noise traders when there are only two market makers. The reason is that as the number of market makers increase, the risk of only trading with the informed trader - who is unconstrained when it comes to the size of the order he can trade - increases. To compensate, the market makers must use a wider spread.

Second, Dennert looks at market maker competition within the Kyle (1985) model. Since market makers announce linear pricing rules, orders will always be split among market makers - as long as they use the same intercept. As a result, the winner-takes-all structure, that existed in the Glosten and Milgrom setting, is no longer relevant. Instead there exists a symmetric pure equilibrium when there are more than two market makers - and no equilibrium otherwise. In addition, the noise traders get better and better prices as the number of market makers increase. In addition, as the number of market makers approach infinity, their own profits approaches zero.

Bernhardt and Hughson (1997) expands on the winner-takes-all case discussed by Dennert and considers the effect if applied on the Kyle (1985) model. They show that

[3] In a different approach Back and Baruch (2004) show that the Glosten-Milgrom model converges to the Kyle model when the informed trader is allowed to optimize his times of trading.

if the noise trader is not allowed to split trades between market makers, then the model has a mixed equilibrium - just as in the Glosten and Milgrom (1985) setting. However, they argue that this result is not robust in the sense that if it is allowed for the noise traders to split their orders, then the mixed equilibrium breaks down. Instead, when we have two market makers, we get the no equilibrium result that Dennert established.

The results in this chapter are consistent with the notion that perfect competition is interpreted as an ideal case resulting from competition between infinitely many market makers. Thus, noise traders are allowed to split their trades and the equilibrium is a pure strategy equilibrium.

The chapter is organized as follows. The baseline model is defined in Section 2. There is an arbitrary number of noise traders, one informed trader, and two market makers. The market makers use a pricing rule that is linear in the net order flow and where an exogenous bid-ask spread has been added. To make the model more tractable the intercept in the pricing rule is set equal to the expected fundamental value of the asset, which is an equilibrium result in both Kyle (1985) and Dennert (1993). Since the spread is exogenously given the market makers compete only by choosing the slope of the pricing rule.

In Section 3 the model is solved and it is shown that a Nash equilibrium exists even with only two market makers. The reason is that the introduction of a spread increases the market makers' potential profit per trade, and thus increases the competition between market makers. The perpetual overbidding found in Dennert (1993) will thus eventually stop. Some comparative statics are then performed and it is demonstrated that if the spread is sufficiently high, then the noise traders would gain from pooling their trades and, if possible, clearing them with each before approaching the market makers.

In Section 4 the spread is endogenized and it is again shown that a Nash equilibrium exists. Furthermore, it is shown that the spread is increasing in the volatility of the fundamentals. This is a prediction of the model that, to my knowledge, has neither been shown formally in previous theoretical models, nor been tested empirically. Another prediction, possibly less robust, of the model is that the optimal spread is independent of the volatility of noise trades.

In Section 5 the baseline model is extended to allow for any arbitrary number of market makers. As soon as we have more than two market makers there is sufficient competition for an equilibrium to exist even without a spread. However, when the spread is also taken into account, the competition increases even further, and the prices are pushed even lower. Some comparative statics when going from two to three market makers are considered. The market makers always lose as a group, while the noise traders and the informed traders always gain. Finally, in Section 6 the related literature is discussed and in Section 7 the chapter is concluded.

2. The Model

The model is the Kyle model extended to take into account competition between market makers, and to incorporate a spread. The timing is as follows.

(1) Two market makers simultaneously announce their pricing functions. The market maker $m \in \{1, 2\}$ announces a pair of pricing functions

$$p_m^+ = \alpha + \beta_m \tilde{y}_m + \Delta \tag{2.1}$$

$$p_m^- = \alpha + \beta_m \tilde{y}_m - \Delta, \tag{2.2}$$

where he sells at the higher price and buys at the lower one. The net order flow, $\tilde{y}_m$, is given by $\tilde{y}_m = \tilde{x}_m + \sum_n \tilde{z}_{nm}$, where $\tilde{x}_m$ is the order from the informed trader and $\tilde{z}_{nm}$ is the order from noise trader n. Both $\alpha \in \mathbb{R}_+$ and $\Delta \in \mathbb{R}_{++}$ are exogenous. Let market maker m's strategy be $\beta_m \in \mathbb{R}_+$. The market makers' strategy profile is written $\boldsymbol{\beta} = (\beta_1, \beta_2)$.

(2) Nature draws $\tilde{v} \sim N(\alpha, \sigma_v^2)$ and $\tilde{u}_n \sim N(0, \sigma_u^2/N)$ independently.

(3) The informed trader and the noise traders trade simultaneously.
   - The informed trader, i, gets information on the realization of the fundamental value $\tilde{v}$, and submits an order $\tilde{x}_m \in \mathbb{R}$ to each market maker m. His strategy $\xi : \mathbb{R}_{++}^2 \times \mathbb{R} \to \mathbb{R}^2$ results in the orders

$$\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2) = \xi(\boldsymbol{\beta}, \tilde{v}). \tag{2.3}$$

   - Each noise trader $n \in \{1, ..., N\}$ observes $\tilde{u}_n$ and submits an order $\tilde{z}_{nm}$ to each market maker where $\tilde{z}_{n1}$ and $\tilde{z}_{n2}$ both add up to, and have the same sign as, $\tilde{u}_n$. His strategy $\phi_n : \mathbb{R}_{++}^2 \times \mathbb{R} \to \mathbb{R}^2$ results in the orders

$$\tilde{\mathbf{z}}_n = (\tilde{z}_{n1}, \tilde{z}_{n2}) = \{\phi_n(\boldsymbol{\beta}, \tilde{u}_n) : \tilde{z}_{n1} + \tilde{z}_{n2} = \tilde{u}_n \ \cap \ |\tilde{z}_{n1}|, |\tilde{z}_{n2}| \le |\tilde{u}_n|\}. \tag{2.4}$$

   The noise traders' strategy profile is written $\boldsymbol{\phi} = \{\phi_1, \phi_2, ..., \phi_N\}$.

(4) The market makers observe their respective net order flows and set the prices according to the prespecified rule.

(5) The payoffs are realized. Let us use the notation

$$\tilde{x}_m^+ = \max\{0, \tilde{x}_m\} \tag{2.5}$$

$$\tilde{x}_m^- = \min\{0, \tilde{x}_m\} \tag{2.6}$$

$$\tilde{z}_{nm}^+ = \max\{0, \tilde{z}_{nm}\} \tag{2.7}$$

$$\tilde{z}_{nm}^- = \min\{0, \tilde{z}_{nm}\}. \tag{2.8}$$

- Market maker m's payoff $\tilde{\pi}_m : \mathbb{R}_{++} \to \mathbb{R}$ is defined by

$$\begin{aligned} \tilde{\pi}_m(\beta_m) &= \left(p_m^+ - \tilde{v}\right)\left(\tilde{x}_m^+ + \sum_n \tilde{z}_{nm}^+\right) + \left(p_m^- - \tilde{v}\right)\left(\tilde{x}_m^- + \sum_n \tilde{z}_{nm}^-\right) && (2.9) \\ &= \left(p_m - \tilde{v}\right)\left(\tilde{x}_m + \sum_n \tilde{z}_{nm}\right) + \Delta\left(\left|\tilde{x}_m\right| + \sum_n \left|\tilde{z}_{nm}\right|\right), && (2.10) \end{aligned}$$

where

$$p_m = \alpha + \beta_m \tilde{y}_m. \tag{2.11}$$

- The informed trader's payoff $\tilde{\pi}_i : \mathbb{R}^2 \to \mathbb{R}$ is defined by

$$\begin{aligned} \tilde{\pi}_i(\tilde{x}_1, \tilde{x}_2) &= \sum_{m=1}^{2} \left[\left(\tilde{v} - p_m^+\right)\tilde{x}_m^+ + \left(\tilde{v} - p_m^-\right)\tilde{x}_m^-\right] && (2.12) \\ &= \sum_{m=1}^{2} \left[\left(\tilde{v} - p_m\right)\tilde{x}_m - \Delta\left|\tilde{x}_m\right|\right]. && (2.13) \end{aligned}$$

- The noise trader's payoff $\tilde{\pi}_n : \mathbb{R}^2 \to \mathbb{R}$ is defined by

$$\begin{aligned} \tilde{\pi}_n(\tilde{z}_{n1}, \tilde{z}_{n2}) &= \sum_{m=1}^{2} \left[\left(\tilde{v} - p_m^+\right)\tilde{z}_{nm}^+ + \left(\tilde{v} - p_m^-\right)\tilde{z}_{nm}^-\right] && (2.14) \\ &= \sum_{m=1}^{2} \left[\left(\tilde{v} - p_m\right)\tilde{z}_{nm} - \Delta\left|\tilde{z}_{nm}\right|\right]. && (2.15) \end{aligned}$$

Thus, a strategy profile is $s = (\boldsymbol{\beta}, \xi, \boldsymbol{\phi})$. The approach will be to propose that a certain strategy profile is a Nash equilibrium, and then prove that nobody can unilaterally gain by deviating if everybody else is playing the proposed strategies. If the proposed Nash equilibrium is $s^* = (\boldsymbol{\beta}^*, \xi^*, \boldsymbol{\phi}^*)$, then we will use the notational convention that $s = \left(\boldsymbol{\beta}^*_{-m}, \xi^*, \boldsymbol{\phi}^*\right)$ means that everybody except market maker m is playing the proposed strategy. Similarly, $\boldsymbol{\phi}^*_{-n\boldsymbol{.}}$ will denote the situation when every noise trader except noise trader n plays the proposed strategy.

3. Analysis

Let us consider the strategy profile $s^* = (\boldsymbol{\beta}^*, \xi^*, \boldsymbol{\phi}^*)$, where

$$\beta_m^* = 4\frac{A(\Delta)}{B(\Delta, N)}, \tag{3.1}$$

$$\xi_m^*(\boldsymbol{\beta}, \tilde{v}) = \begin{cases} \frac{\tilde{v}-(\alpha-\Delta)}{2\beta_m} & \text{if } \tilde{v} < \alpha - \Delta \\ 0 & \text{if } \alpha - \Delta < \tilde{v} < \alpha + \Delta \\ \frac{\tilde{v}-(\alpha+\Delta)}{2\beta_m} & \text{if } \alpha + \Delta < \tilde{v} \end{cases}, \tag{3.2}$$

$$\phi_{nm}^*(\boldsymbol{\beta}, \tilde{u}_n) = \frac{\beta_{-m}}{\beta_m + \beta_{-m}}\tilde{u}_n, \tag{3.3}$$

for every market maker $m = \{1, 2\}$. The functions $A : \mathbb{R}_{++} \to \mathbb{R}_{++}$ and $B : \mathbb{R}_{++} \times \mathbb{N} \to \mathbb{R}_{++}$ are given by

$$A(\Delta) = \frac{(\sigma_v^2 + \Delta^2)}{2}\left(1 - F\left(\frac{\Delta}{\sigma_v}\right)\right) - \frac{\Delta\sigma_v}{2}F'\left(\frac{\Delta}{\sigma_v}\right), \tag{3.4}$$

$$B(\Delta, N) = \Delta\sqrt{\frac{2N\sigma_u^2}{\pi}}, \tag{3.5}$$

where $F(x)$ is the standard normal cumulative distribution function, and $F'(x)$ is the standard normal density function given by

$$F'(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right). \tag{3.6}$$

The main result in this section is that this strategy profile is a Nash equilibrium Formally, we have the following proposition:

PROPOSITION 1. *The strategy profile $s^* = (\boldsymbol{\beta}^*, \xi^*, \boldsymbol{\phi}^*)$ is a Nash equilibrium.*

The proof can be found in the Appendix. In the remainder of this section, we outline the intuition behind the results through some comparative statics.

3.1. Comparative statics.

3.1.1. *The market makers' best replies.* Driving the results is that the market maker's best reply will be given by

$$\frac{A(\Delta)}{\beta_m^2} + \frac{\beta_{-m}^{*2}\left(\beta_{-m}^* - \beta_m\right)}{\left(\beta_m + \beta_{-m}^*\right)^3}\sigma_u^2 - \frac{\beta_{-m}^*}{\left(\beta_m + \beta_{-m}^*\right)^2}B(\Delta, N) = 0 \tag{3.7}$$

instead of as in Dennert (1993) where it was given by

$$\frac{\sigma_v^2}{4\beta_m^2} + \frac{\beta_{-m}^{*2}\left(\beta_{-m}^* - \beta_m\right)}{\left(\beta_m + \beta_{-m}^*\right)^3}\sigma_u^2 = 0. \tag{3.8}$$

Thus, in Dennert (1993) it is always best for the market maker to announce a higher slope than the competitor, whereas this is not the case with a spread. This is reflected in the figure below, where we plot the best replies both with no spread and with a strictly positive spread. It can be seen that without a spread the best replies never intersect, whereas they do with a spread.

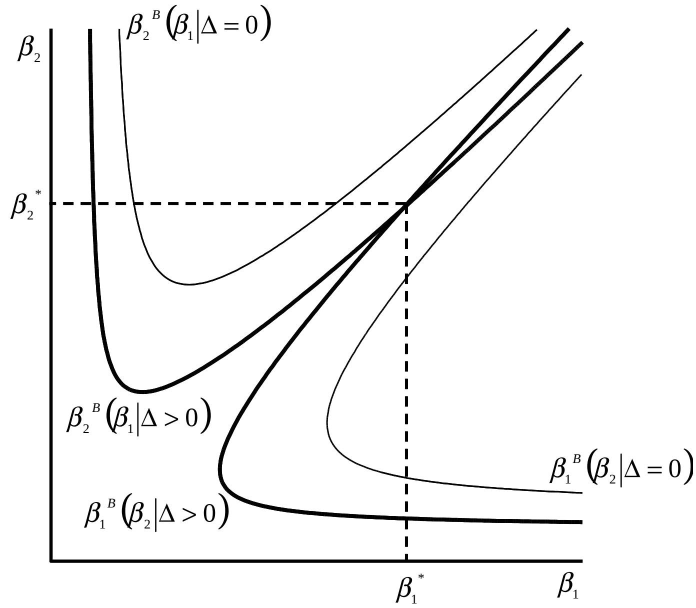

Figure 1: The best replies with and without a spread.

Thus, even an infinitesimal spread will increase the competition between the two market makers so that an equilibrium exists.

3.1.2. *The equilibrium slope.* The equilibrium slope varies depending on the spread. As the spread increases, the market makers' potential profit increases, which lead to increased competition, and thus a lower slope. However, the relationship is not linear. Instead, the spread's effect on the equilibrium slope is diminishing, as the figure below suggests.

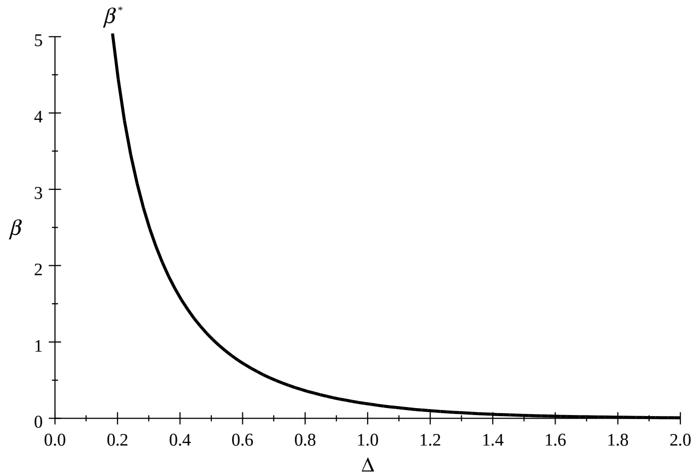

Figure 2: The equilibrium slope as the spread changes, for $\sigma_v^2 = \sigma_u^2 = N = 1$.

3.1.3. *Unconditional expected profits when* $N = 1$. Let us now consider the unconditional profits of the three types of players as the spread changes. Note that we have

$$N \cdot \mathbb{E}\left[\pi_n \mid S = (\boldsymbol{\beta}^*, \xi^*, \boldsymbol{\phi}^*)\right] = -2\frac{A(\Delta)}{B(\Delta, N)}\sigma_u^2 - B(\Delta, N) \tag{3.9}$$

$$\mathbb{E}\left[\tilde{\pi}_i \mid S = (\boldsymbol{\beta}^*, \xi^*, \boldsymbol{\phi}^*)\right] = \frac{B(\Delta, N)}{2} \tag{3.10}$$

$$2\mathbb{E}\left[\tilde{\pi}_m \mid S = (\boldsymbol{\beta}^*, \xi^*, \boldsymbol{\phi}^*)\right] = 2\frac{A(\Delta)}{B(\Delta, N)}\sigma_u^2 + \frac{B(\Delta, N)}{2}. \tag{3.11}$$

The noise trader's loss can thus be decomposed into two component. The first is the loss due to the slope. This part of the loss goes to the market makers. The second part of the loss is due to the spread. This part of the loss is evenly split between the informed trader and the market makers. As we have seen, the optimal slope is inversely proportional to the spread. This results in the informed trader's expected profit actually being positively related to the spread. The reason is that as the spread increases, the optimal slope will decrease. The total effect is a gain for the informed trader. Below we plot the unconditional expected profits in equilibrium, aggregated for each of the three types, when $N = 1$.

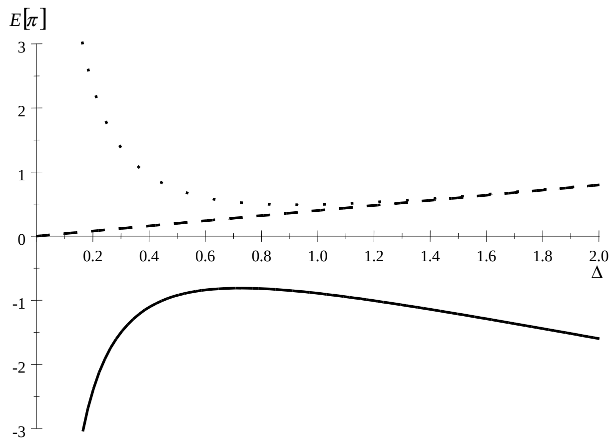

Figure 3: The unconditional expected profits, aggregated over type, when $\sigma_v^2 = \sigma_u^2 = N = 1$. The dotted line is the aggregated profits of market makers. The dashed line is the informed trader's profit. The solid line is the noise trader's profit.

Note that when the spread is very low, the noise trader's loss is very high. The reason is that the market makers' prices will be very sensitive to order flow. Since the noise trader has to trade at all prices, his loss will be very high. The informed trader's profit, on the other hand, will be very low. He can chose when to trade, but will trade in small quantities since the prices are so sensitive to order flow. As the spread increases, competition between the market makers increase and the prices become less sensitive to order flow. As a consequence, the noise trader's loss decreases. However, at a certain level the costs the spread entails for the noise traders outweigh the less sensitive prices. As a result the noise traders' loss increases again.

3.1.4. *Unconditional expected profits when N changes from 1 to 2.* Let us look at how a change in the number of noise traders affect the profits of the three types of agents. In the figure below we plot the aggregated unconditional expected profits when we go from $N = 1$ to $N = 2$.

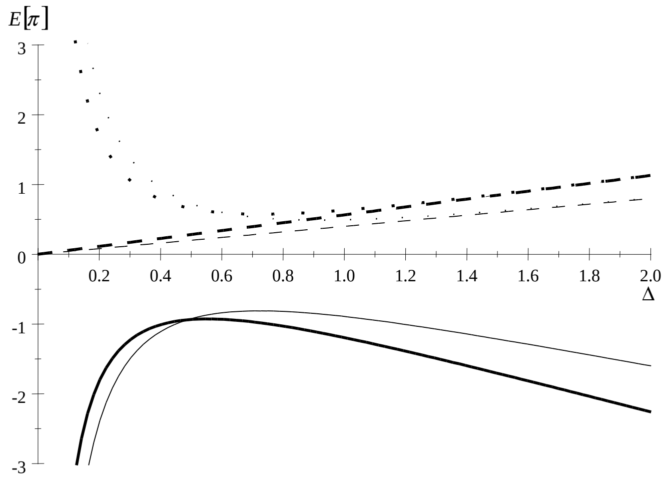

Figure 4: Aggregated unconditional expected profits. The thin curves correspond to $N = 1$. The thick curves correspond to $N = 2$.

Increasing the number of noise traders, we see that the noise traders benefit when the spread is low. Their total cost will then be lower compared to when there was only one noise trader. The reason is that with two noise traders there exists a possibility for the market makers to offset two opposing trades with each other and thus make a net gain. This results in a higher collective loss for the noise traders. However, when the spread is low, the net effect is actually positive. The reason is that the possibility of making a larger gain increases the competition between the market makers, and the prices become less sensitive to order flow. However, as the spread increases, the effect on competition, and thus on the price sensitivity, diminishes, while the noise traders still make their collective loss. Thus, the overall effect is that when the spread is high, then the noise traders as a group are actually worse off the more numerous they are.

Individually, however, a noise trader gains if more noise traders join the market. The reason is that the cost the spread entails remains constant for the individual noise trader, whereas the effect on the price sensitivity results in a gain. Below we plot the case with one and two noise traders. The solid curve is the noise trader's unconditional expected profit when there is only one noise trader. The dashed curve is the same noise trader's cost when there are two noise traders. The dotted curve is the aggregated cost for noise traders when there are two noise traders. Thus, we clearly see that an individual noise trader gains from having another noise trader joining the market, at a constant total noise trading variance. However, as a group, the noise traders are worse off if the spread is sufficiently high.

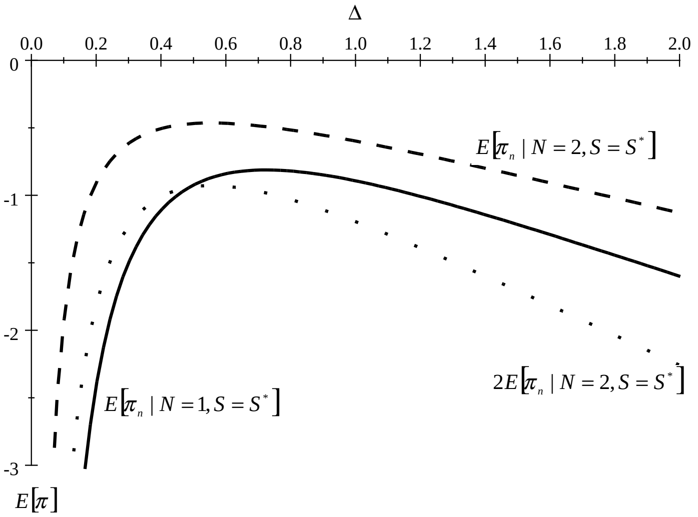

Figure 5: Comparing the cost of being a noise trader when we go from one to two noise traders, and $\sigma_u = \sigma_v = 1$.

In the next section we will endogenize the spread. It then turns out that the optimal spread in this case would be 0.8, which is large enough to imply that the noise traders as a group is worse off if another noise trader joins the market. This then suggests that the noise traders would gain from pooling their trades and first try to offset them with each other before they approach the market makers.

4. Endogenous spread

We will now endogenize the spread. Thus each market maker m can choose a spread $\Delta_m \in \mathbb{R}_+$, which implies that market maker m's strategy is now $\gamma_m = (\beta_m, \Delta_m) \in \mathbb{R}^2_+$. The informed trader's strategy $\xi : \mathbb{R}^4_{++} \times \mathbb{R} \to \mathbb{R}^2$ now results in the orders

$$\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2) = \xi(\boldsymbol{\gamma}, \tilde{v}), \tag{4.1}$$

while noise trader n's strategy $\phi_n : \mathbb{R}^4_{++} \times \mathbb{R} \to \mathbb{R}^2$ results in the orders

$$\tilde{\mathbf{z}}_n = (\tilde{z}_{n1}, \tilde{z}_{n2}) = \{\phi_n(\boldsymbol{\gamma}, \tilde{u}_n) : \tilde{z}_{n1} + \tilde{z}_{n2} = \tilde{u}_n \ \cap \ |\tilde{z}_{n1}|, |\tilde{z}_{n2}| \leq |\tilde{u}_n|\}. \tag{4.2}$$

Again we will propose a strategy profile to be a Nash equilibrium, and then show that it is indeed the case.

The proposed strategy profile is $s^{**} = (\boldsymbol{\gamma}^{**}, \xi^{**}, \boldsymbol{\phi}^{**})$, where

$$\beta^{**}_m = 4\frac{A(\Delta_m)}{B(\Delta_m, N)}; \tag{4.3}$$

Δ_m^{**} is the unique solution to the fixed point equation $\Delta_m = G(\Delta_m)$, where[4]

$$G(\Delta_m) = \frac{4}{N}\left(\frac{A(\Delta_m)}{\Delta_m} - A'(\Delta_m)\right); \tag{4.4}$$

the informed trader's strategy is

$$\xi_m^{**}(\boldsymbol{\gamma}, \tilde{v}) = \begin{cases} \frac{\tilde{v}-\alpha+\Delta_m}{2\beta_m} & \text{if} \quad \tilde{v} < \alpha - \Delta_m \\ 0 & \text{if} \quad \alpha - \Delta_m < \tilde{v} < \alpha + \Delta_m \\ \frac{\tilde{v}-\alpha-\Delta_m}{2\beta_m} & \text{if} \quad \alpha + \Delta_m > \tilde{v} \end{cases}; \tag{4.5}$$

and the noise trader's strategy is

$$\phi_{nm}^{**}(\boldsymbol{\gamma}, \tilde{u}_n) = \frac{\beta_{-m}}{\beta_m + \beta_{-m}}\tilde{u}_n - \frac{\Delta_m - \Delta_{-m}}{2\left(\beta_m + \beta_{-m}\right)}\frac{\tilde{u}_n}{|\tilde{u}_n|}, \tag{4.6}$$

which also satisfies

$$\begin{array}{ll} z_{nm}^{**} \in [0, u_n] & \text{if} \quad u_n > 0 \\ z_{nm}^{**} \in [u_n, 0] & \text{if} \quad u_n < 0. \end{array} \tag{4.7}$$

The main result in this section is the following proposition

PROPOSITION 2. *The strategy profile* $s^{**} = (\boldsymbol{\gamma}^{**}, \xi^{**}, \boldsymbol{\phi}^{**})$ *is a Nash equilibrium.*

The proof can be found in the Appendix.

4.1. Comparative statics.

4.1.1. *The optimal slope and spread as functions of the standard deviations.* In the figure below we plot the optimal slope and spread as a function of the standard deviation of fundamentals. As we can see, they are both increasing in σ_v.

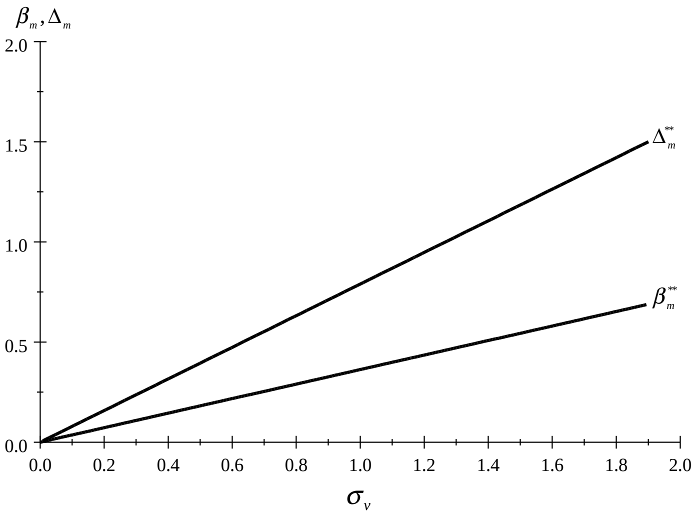

Figure 6: The endogenous spread and slope as functions of the standard deviation of fundamentals when $\sigma_u = N = 1$.

[4] It is straight forwards to show that $\lim_{\Delta \to 0} G(\Delta) = \infty$ and $G'(\Delta) < 0$, $\forall \Delta \in \mathbb{R}_+$. This implies both existence and uniqueness of Δ^{**}.

To my knowledge, it has not been shown in earlier theoretical models that the spread is increasing in the standard variation of fundamentals, nor has it been tested empirically.[5]

In the figure below we plot the optimal slope and spread as a function of the standard deviation of noise trades. The optimal slope is downward sloping. However, the optimal spread is independent of the standard variation of noise trades. The reason for this result has its roots in the noise trader's strategy (4.6). In a symmetric equilibrium the spreads will not influence the noise traders' choice of market maker. As a result, the market maker will not care about noise trader demand when setting the spread, which makes the optimal spread independent of the standard deviation of noise trades.

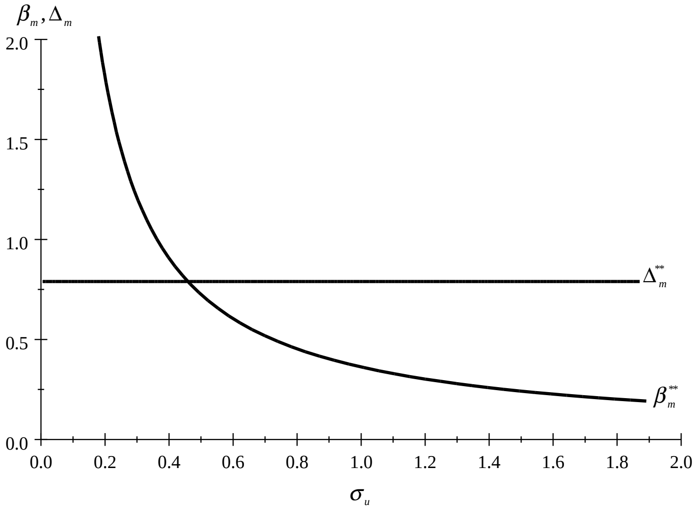

Figure 7: The endogenous spread and slope as functions of the standard deviation of noise trades when $\sigma_v = 1$. Note that the spread does not depend on the standard deviation of noise trades, whereas the slope is inversely proportional to the standard deviation of noise trades.

It should be noted that this result may not hold if noise trader demand is price elastic. Then the absolute level of the spread, instead of only the relative level, may influence noise trader demand. However, this issue is outside the scope of this model.

4.1.2. *The effects of changes in the number of noise traders.* In the figure below we have plotted β_m^{**} and Δ_m^* as N changes. Note that the spread, when there are few noise traders, is higher than the slope. As the number of noise traders increase, the spread falls and the slope rises.

[5] A somewhat related empirical paper is Jayaraman (2008). He tests the relationship between the difference between the volatility of earnings and the volatility of cash flows, and the spread, and finds a U-shaped relationship. However, he does not test directly whether the spread is positively correlated with either the volatility of earnings of cash flows.

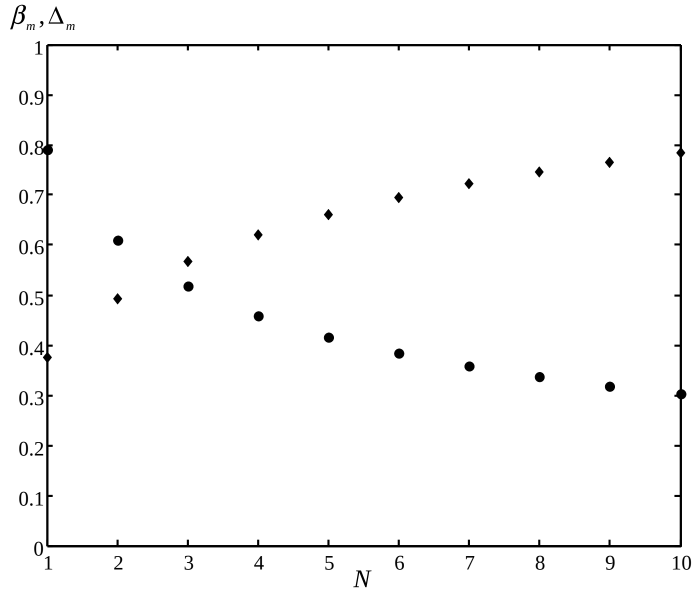

Figure 8: β_m^{**} (circles) and Δ_m^{**} (diamonds) as the number of noise traders changes, for $\sigma_v^2 = \sigma_v^2 = 1$.

5. Extension and robustness

In this section we will first extend the baseline model to take into account an arbitrary number of market makers. The assumption that the intercept is equal to the expected value of the fundamental value is then briefly discussed.

5.1. M **market makers.** Let us now extend the baseline model, i.e. with an exogenous spread, to allow for M market makers. The market makers are indexed by $m \in \{1, ..., M\}$. We will again propose a strategy profile that will be played in a Nash equilibrium. The proposed strategy profile is $s^{***} = (\boldsymbol{\beta}^{***}, \xi^{***}, \boldsymbol{\phi}^{***})$, where

$$\beta_m^{***} = \begin{cases} 4\frac{A(\Delta)}{B(\Delta,N)} & \text{if} \quad M = 2 \\ \left(\begin{array}{c} -\frac{M(M-1)}{2(M-2)\sigma_u^2} B(\Delta, N) \\ +\sqrt{\frac{M^3}{(M-2)\sigma_u^2} A(\Delta) + \frac{M^2(M-1)^2}{4(M-2)^2\sigma_u^4} B(\Delta, N)^2} \end{array} \right) & \text{if} \quad M > 2 \end{cases}, \tag{5.1}$$

$$\xi_m^{***}(\boldsymbol{\beta}, \tilde{v}) = \begin{cases} \frac{\tilde{v}-\alpha+\Delta}{2\beta_m} & \text{if} \quad \tilde{v} < \alpha - \Delta \\ 0 & \text{if} \quad \alpha - \Delta < \tilde{v} < \alpha + \Delta \\ \frac{\tilde{v}-\alpha-\Delta}{2\beta_m} & \text{if} \quad \alpha + \Delta > \tilde{v} \end{cases}, \tag{5.2}$$

$$\phi_{nm}^{***}(\boldsymbol{\beta}, \tilde{u}_n) = \frac{\prod_{j\neq m}^{M} \beta_j}{\sum_{k}^{M} \prod_{j\neq k}^{M} \beta_j} \tilde{u}_n, \tag{5.3}$$

for every $m \in \{1, ..., M\}$ and $n \in \{1, ..., N\}$ .The main result in this section is the following proposition.

PROPOSITION 3. *The strategy profile* $s^{***} = (\boldsymbol{\beta}^{***}, \xi^{***}, \boldsymbol{\phi}^{***})$ *is a Nash equilibrium.*

5.1.1. *Comparative statics.* In the figure below we plot the equilibrium slope depending on the number of market makers. As we can see, prices are very sensitive to order flows when there are only two market makers, whereas the sensitivity decrease substantially as soon as there are at least three market makers. However, as the number of market makers increase the sensitivity again increases. The reason is that a larger number of market makers have to divide a given number of noise trading among themselves.

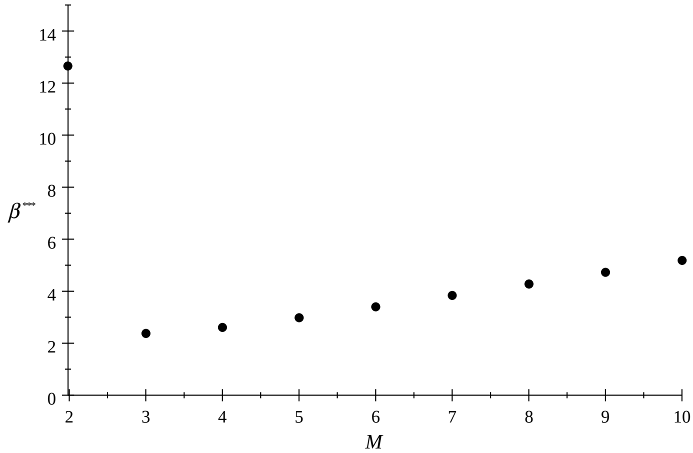

Figure 9: The equilibrium slope β^{***} as M changes, for $\sigma_u^2 = \sigma_v^2 = N = 1$, $\Delta = 0.1$.

Let us now look at the total profits for each type as we go from two to three market makers, and as the spread changes. The aggregated expected profits of each of the three types are

$$N \cdot \mathbb{E}\left[\pi_n \mid s^{***} = (\boldsymbol{\beta}^{***}, \xi^{***}, \boldsymbol{\phi}^{***})\right] = -\frac{1}{M}\beta^{***}\sigma_u^2 - B(\Delta, N) \tag{5.4}$$

$$\mathbb{E}\left[\tilde{\pi}_i \mid s^{***} = (\boldsymbol{\beta}^{***}, \xi^{***}, \boldsymbol{\phi}^{***})\right] = M\frac{A(\Delta)}{\beta^{***}} \tag{5.5}$$

$$M \cdot \mathbb{E}\left[\tilde{\pi}_m \mid s^{***} = (\boldsymbol{\beta}^{***}, \xi^{***}, \boldsymbol{\phi}^{***})\right] = \frac{1}{M}\beta^{***}\sigma_u^2 + B(\Delta, N) - M\frac{A(\Delta)}{\beta^{***}}. \tag{5.6}$$

Note again the noise traders' loss can be decomposed into two components, one depending on the slope, the other depending on the spread. Both components corresponds to gains for the market makers. However, the market makers also make a loss to the informed trader. This loss is increasing in the number of market makers.

Below we have plotted the aggregated unconditional expected profits when $M = 2$ and $M = 3$. With three market makers there is an equilibrium also without a spread.

As a result the market makers' payoff is always lower when there are three market makers rather than two. However, when the spread is low, the competition is fairly weak, which makes it possible for the market makers as a group to get a higher profit than the informed trader. Nevertheless, as the spread increases, the market makers' profit initially decreases, while the informed trader's profit increases linearly. As a result the informed trader eventually receives a higher profit than the market makers.

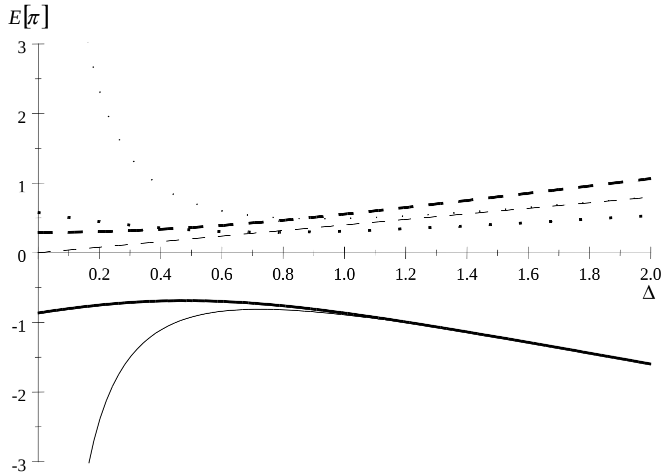

Figure 10: Comparing the aggregated unconditional expected profits when we go from $M = 2$ (thin lines) to $M = 3$ (thick lines), keeping the number of noise traders constant at 1.

5.2. The intercept in the pricing function. In the baseline model we made the simplifying assumption that the intercept in the pricing function is equal to the expected fundamental value. As already noted, this an equilibrium result in both Kyle (1985) and Dennert (1993). In the present model the noise traders are only allowed to perform limited arbitrage. For example, a noise trader is not allowed to buy from one market maker and sell to the other. However, even when the noise traders' best replies are independent of the intercept, as they are in this model, the market makers will not have an incentive to change the intercept. The reason is that the informed trader performs some arbitrage. From (3.2) we can see that if a market maker increases his intercept, then the informed trader will sell more often to that market maker and buy less often from him. As a result, the profit expression of the market maker is independent of the intercept. It is thus not possible for a market maker to improve his payoff by unilaterally changing the intercept, which means that the intercept being equal to the expected value of the fundamentals is indeed a Nash equilibrium.[6]

[6] This is not a full analysis of the problem though. Ideally, we would let the noise traders buy from one market maker and sell to another, with best replies depending also on the intercept. However, this would reduce the tractability of the model, while it seems unlikely that it would change the result.

6. Discussion

The two workhorse models in market microstructure, Kyle (1985) and Glosten and Milgrom (1985), both assumed perfect competition between market makers. As discussed in Section 1, this assumption was relaxed in Dennert (1993), Bondarenko (2001), and Bernhardt and Hughson (1997).[7] Dennert showed that if noise traders did not split trades, then a perfectly competitive mixed equilibrium existed with only two market makers. However, in the Kyle model, where the noise traders may split trades, no equilibrium exists with only two market makers. An equilibrium exists though with three or more market makers, and the perfect equilibrium corresponds to the ideal case when the number of market makers approach infinity. Bernhardt and Hughson (1997) showed that if the noise trader demand was decreasing in price then an equilibrium exists in the Kyle model both with one and two market makers. However, noise trader demand can not be too price elastic, then trade breaks down. While it is in most cases realistic to assume that noise trader demand is decreasing in price, it is unrealistic to assume that it is always is the case - for example under predatory trading as described by Brunnermeier and Pedersen (2005).

A quite substantial literature has been devoted to studying the spread and its components. Typically, three components are stressed. The first is an order processing cost. The second is an inventory holding cost, as described by Demsetz (1968), Stoll (1978), Ho and Stoll (1981), and Ho and Stoll (1983). The third component is adverse selection as demonstrated by Bagehot (1971), Copeland and Galai (1983), and Glosten and Milgrom (1985).

Empirical studies of the composition of the spread has been made by Glosten and Harris (1988), Stoll (1989), George et al (1991), Lin et al (1995), Huang and Stoll (1997), Ahn et al (2002), and Bollen et al (2004). Bollen et al (2004) show that the bid-ask spread is a function of the minimum tick size, the inverse of the trading volume, competition between market makers, and the expected inventory holding premium.

The effect of decimalization has been investigated by Bacidore et al (2001), Bessembinder (2003), Chung et al (2004), Gibson et al (2003), and Serednyakov (2005), who all found that the spread decreased substantially when decimalization was introduced on the NYSE. According to Serednyakov (2005) it appears to be primarily due to order processing and inventory holding costs going down, while Gibson et al (2003) find that the reduction in spreads is due to lower order processing costs. Giouvris and Philippatos (2008) studied the components of the bid-ask spread when the London Stock

[7] Glosten (1989) compared perfect competition with a supervised monopoly. He noted that a supervised monopolist may sometimes trade at a loss due to the fact that he can average gains and losses over time. This may help restore trading when it has broken down.

Exchange changed from a quote driven to an order driven market and found that the adverse selection component was reduced.

Another issue is that collusion between market makers may be a factor, as demonstrated by Christie and Schultz (1994) and Christie, Harris and Schultz (1994). Godek (1996) argues that preference trading may result in collusion, whereas Kandel and Marx (1997) show that market makers can use odd-tick avoidance as a coordination device to increase spreads. Their argument is especially valid if the tick size is large relative to the spreads being charged. Dutta and Madhavan (1997), on the other hand, show that market makers may engage in implicit collusion if they are sufficiently patient and if there are barriers to entry. Price discreteness is thus not necessary in their model.

7. Conclusion

In this chapter it was shown that a Nash equilibrium exists in the Kyle model with two market makers if it is extended to take into account the spread. A side effect is that the Kyle model has also been extended to take into account gross order flow instead of only considering the net order flow. Thus, whereas market makers in the original Kyle model only cares about adverse selection, they here also care about whether they can offset opposing trades with each other. Conceptually this means that the price sensitivity can no longer be interpreted as a measure of the spread, which it often is in the original Kyle model. This also opens up for further developments of the Kyle model. We briefly looked at how the number of noise traders will influence how sensitive prices will be to order flow, but other extensions may also be possible. For example, although we did extend the baseline model, i.e. the one with an exogenous spread, to take into account an arbitrary number of market makers, one could also envision such an extension with an endogenous spread. In addition, a dynamic extension of this static model is also called for.

Appendix A

A.1. Proof of Proposition 1. We now proceed to show that the proposed Nash equilibrium is indeed a Nash equilibrium. This is achieved by showing that no player can gain by unilaterally deviating, given that all other players are playing the proposed strategy.

A.1.1. *The noise traders.* If everybody else is playing the proposed Nash equilibrium, then noise trader n's expected profit is

$$\mathbb{E}\left[\pi_n \mid s = \left(\boldsymbol{\beta}^*, \xi^*, \boldsymbol{\phi}^*_{-n\cdot}\right), \tilde{u}_n = u_n\right] = \\ = -\beta_1^* z_{n1}^2 - \Delta\left|z_{n1}\right| - \beta_2^*\left(u_n - z_{n1}\right)^2 - \Delta\left|u_n - z_{n1}\right|. \tag{A.1}$$

The first order condition is

$$\frac{\partial \mathbb{E}\left[\pi_n \mid s=\left(\boldsymbol{\beta}^*, \xi^*, \boldsymbol{\phi}^*_{-n\bullet}\right), \tilde{u}_n=u_n\right]}{\partial z_{n1}} = -2\beta_1^* z_{n1}^B + 2\beta_2^*\left(u_n - z_{n1}^B\right) = 0. \tag{A.2}$$

The second order condition is

$$\frac{\partial^2 \mathbb{E}\left[\pi_n \mid s=\left(\boldsymbol{\beta}^*, \xi^*, \boldsymbol{\phi}^*_{-n\bullet}\right), \tilde{u}_n=u_n\right]}{\partial z_{n1}^2} = -2\left(\beta_1^*+\beta_2^*\right) \leq 0, \tag{A.3}$$

which is always satisfied. Thus, the best replies are

$$\left(z_{n1}^B, z_{n2}^B\right) = \left(\frac{\beta_2^*}{\beta_1^*+\beta_2^*}u_n, \frac{\beta_1^*}{\beta_1^*+\beta_2^*}u_n\right), \tag{A.4}$$

which corresponds to playing the strategy in the proposed Nash equilibrium.

A.1.2. *The informed trader.* If everybody else is playing the proposed Nash equilibrium, then the informed trader's expected profit is

$$\mathbb{E}\left[\tilde{\pi}_i \mid s=\left(\boldsymbol{\beta}^*, \xi, \boldsymbol{\phi}^*\right), \tilde{v}=v\right] = \sum_{m=1}^{2}\left[\left(v-\alpha\right)x_m - \beta_m^* x_m^2 - \Delta\left|x_m\right|\right]. \tag{A.5}$$

The first order condition in

$$\frac{\partial \mathbb{E}\left[\tilde{\pi}_i \mid s=\left(\boldsymbol{\beta}^*, \xi, \boldsymbol{\phi}^*\right), \tilde{v}=v\right]}{\partial x_m} = v-\alpha-2\beta_m^* x_m^B - \Delta\frac{x_m^B}{\left|x_m^B\right|} = 0. \tag{A.6}$$

The second order condition is

$$\frac{\partial^2 \mathbb{E}\left[\tilde{\pi}_i \mid s=\left(\boldsymbol{\beta}^*, \xi, \boldsymbol{\phi}^*\right), \tilde{v}=v\right]}{\partial x_m^2} = -2\beta_m^* \leq 0, \tag{A.7}$$

which is always satisfied. Thus, the best reply is

$$x_m^B = \begin{cases} \frac{v-\alpha+\Delta}{2\beta_m^*} & \text{if} \quad v<\alpha-\Delta \\ 0 & \text{if} \quad \alpha-\Delta<v<\alpha+\Delta \\ \frac{v-\alpha-\Delta}{2\beta_m^*} & \text{if} \quad \alpha+\Delta<v \end{cases}, \tag{A.8}$$

which again corresponds to playing the strategy in the proposed Nash equilibrium.

A.1.3. *The market makers.* Let us derive market maker m's expected profit. First it is straight-forward to show that

$$\mathbb{E}\left[\tilde{\pi}_m | s=\left(\boldsymbol{\beta}^*_{-m}, \xi^*, \boldsymbol{\phi}^*\right)\right] =$$

$$= \mathbb{E}\left[\left(\alpha+\beta_m\tilde{x}_m^*-\tilde{v}\right)\tilde{x}_m^*\right] + \Delta\mathbb{E}\left[\left|\tilde{x}_m^*\right|\right] + \beta_m\sum_n \mathbb{E}\left[\tilde{z}_{nm}^{*2}\right] + \Delta\sum_n \mathbb{E}\left[\left|\tilde{z}_{nm}^*\right|\right]. \tag{A.9}$$

Let us consider the terms in this expression. Using the proposed strategy for noise trader n, (3.3), we get

$$\sum_n \mathbb{E}\left[\tilde{z}_{nm}^{*2}\right] = \frac{\beta_{-m}^2}{\left(\beta_m + \beta_{-m}\right)^2}\sigma_u^2, \tag{A.10}$$

and

$$\sum_n \mathbb{E}\left[\left|\tilde{z}_{nm}^*\right|\right] = \frac{\beta_{-m}}{\beta_m + \beta_{-m}}\sqrt{\frac{2N\sigma_u^2}{\pi}}. \tag{A.11}$$

Using the proposed strategy for the informed trader, (3.2), we get

$$\mathbb{E}\left[\left|\tilde{x}_m^*\right|\right] = \frac{1}{2\beta_m^*\sqrt{2\pi\sigma_v^2}}\left(\begin{array}{c} \int\limits_{-\infty}^{\alpha-\Delta} -\left(\tilde{v} - (\alpha - \Delta)\right)\exp\left(-\frac{(\tilde{v}-\alpha)^2}{2\sigma_v^2}\right)d\tilde{v} \\ +\int\limits_{\alpha+\Delta}^{\infty}\left(\tilde{v} - (\alpha + \Delta)\right)\exp\left(-\frac{(\tilde{v}-\alpha)^2}{2\sigma_v^2}\right)d\tilde{v} \end{array}\right) \tag{A.12}$$

and

$$\mathbb{E}[\left(\alpha + \beta_m^*\tilde{x}_m^* - \tilde{v}\right)\tilde{x}_m^*] =$$

$$= \frac{1}{2\beta_m^*\sqrt{2\pi\sigma_v^2}}\left(\begin{array}{c} \int\limits_{-\infty}^{\alpha-\Delta}\left(\alpha + \frac{\tilde{v}-\alpha+\Delta}{2} - \tilde{v}\right)\left(\tilde{v} - \alpha + \Delta\right)\exp\left(-\frac{(\tilde{v}-\alpha)^2}{2\sigma_v^2}\right)d\tilde{v} \\ +\int\limits_{\alpha+\Delta}^{\infty}\left(\alpha + \frac{\tilde{v}-\alpha-\Delta}{2} - \tilde{v}\right)\left(\tilde{v} - \alpha - \Delta\right)\exp\left(-\frac{(\tilde{v}-\alpha)^2}{2\sigma_v^2}\right)d\tilde{v} \end{array}\right) \tag{A.13}$$

Using the expressions $(A.12)$ and $(A.13)$ we get

$$\mathbb{E}\left[\left(\alpha + \beta_m^*\tilde{x}_m^* - \tilde{v}\right)\tilde{x}_m^*\right] + \Delta\mathbb{E}\left[\left|\tilde{x}_m^*\right|\right] = -\frac{A\left(\Delta\right)}{\beta_m^*}. \tag{A.14}$$

Inserting the terms $(A.10)$, $(A.11)$, and $(A.14)$ into $(A.9)$ we get

$$\mathbb{E}\left[\tilde{\pi}_m | S = \left(\boldsymbol{\beta}_{-m}^*, \xi^*, \boldsymbol{\phi}^*\right)\right] =$$

$$= -\frac{A\left(\Delta\right)}{\beta_m} + \beta_m\frac{\beta_{-m}^{*2}}{\left(\beta_m + \beta_{-m}^*\right)^2}\sigma_u^2 + \frac{\beta_{-m}^*}{\beta_m + \beta_{-m}^*}B\left(\Delta, N\right). \tag{A.15}$$

The first order condition is

$$\frac{\partial\mathbb{E}\left[\tilde{\pi}_m | S = \left(\boldsymbol{\beta}_{-m}^*, \xi^*, \boldsymbol{\phi}^*\right)\right]}{\partial\beta_m} =$$

$$= \frac{A\left(\Delta\right)}{\beta_m^2} + \frac{\beta_{-m}^{*2}\left(\beta_{-m}^* - \beta_m\right)}{\left(\beta_m + \beta_{-m}^*\right)^3}\sigma_u^2 - \frac{\beta_{-m}^*}{\left(\beta_m + \beta_{-m}^*\right)^2}B\left(\Delta, N\right) = 0. \tag{A.16}$$

In the proposed Nash equilibrium, the second order condition is satisfied, i.e.

$$\frac{\partial^2\mathbb{E}\left[\tilde{\pi}_m | S = \left(\boldsymbol{\beta}^*, \xi^*, \boldsymbol{\phi}^*\right)\right]}{\partial\beta_m^2} = -\frac{1}{64}\frac{2A\left(\Delta\right)\sigma_u^2 + B\left(\Delta, N\right)^2}{A\left(\Delta\right)^2}B\left(\Delta, N\right) < 0 \tag{A.17}$$

Inserting the proposed strategy for market maker $-m$ and rewriting the first order condition as a cubic equation, we get

$$(4A(\Delta) - B(\Delta, N)\beta_m)\Psi(\beta_m) = 0, \tag{A.18}$$

where $\Psi(\beta_m)$ is given by

$$\Psi(\beta_m) = a_2\beta_m^2 + a_1\beta_m + a_0, \tag{A.19}$$

and

$$a_2 = 16A(\Delta)\sigma_u^2 + 3B(\Delta,1)^2 \tag{A.20}$$

$$a_1 = 16A(\Delta)B(\Delta,1) \tag{A.21}$$

$$a_0 = 16A(\Delta)^2. \tag{A.22}$$

Note that $\Psi(0) = a_0$ and

$$\frac{\partial\Psi(0)}{\partial\beta_m} = a_1, \tag{A.23}$$

and that all three coefficients are positive. This implies that the two roots must be negative and thus not admissible.

Thus, the only admissible solution to the first order condition is

$$\beta_m = 4\frac{A(\Delta)}{B(\Delta,N)}, \tag{A.24}$$

i.e. the proposed solution.

Finally, note that

$$\lim_{\beta_m\to 0}\mathbb{E}\left[\tilde{\pi}_m | S = \left(\boldsymbol{\beta}^*_{-m}, \xi^*, \boldsymbol{\phi}^*\right)\right] = -\infty. \tag{A.25}$$

$$\lim_{\beta_m\to \infty}\mathbb{E}\left[\tilde{\pi}_m | S = \left(\boldsymbol{\beta}^*_{-m}, \xi^*, \boldsymbol{\phi}^*\right)\right] = 0. \tag{A.26}$$

A.2. Proof of Proposition 2.

A.2.1. *The noise trader.* If everybody else is playing the proposed Nash equilibrium, then noise trader n's expected profit is

$$\mathbb{E}\Big[\pi_n \mid s = \left(\boldsymbol{\gamma}^{**}, \xi^{**}, \boldsymbol{\phi}^{**}_{-n\cdot}\right), \tilde{u}_n = u_n\Big] = \\ = -\beta_1^{**} z_{n1}^2 - \Delta_1^{**}|z_{n1}| - \beta_2^{**}(u_n - z_{n1})^2 - \Delta_2^{**}|u_n - z_{n1}|. \tag{A.27}$$

The first order condition w.r.t. z_{n1} is

$$-2\beta_1^{**} z_{n1}^B + 2\beta_2^{**}\left(u_n - z_{n1}^B\right) - \Delta_1^{**}\frac{z_{n1}^B}{|z_{n1}^B|} + \Delta_2^{**}\frac{u_n - z_{n1}^B}{|u_n - z_{n1}^B|} = 0. \tag{A.28}$$

Using the assumption that

$$\frac{z_1}{|z_1|} = \frac{u_n - z_1}{|u_n - z_1|} = \frac{u_n}{|u_n|}, \tag{A.29}$$

the best reply is

$$z_{n1}^B = \frac{\beta_2^{**}}{\beta_1^{**}+\beta_2^{**}}u_n - \frac{1}{2}\frac{\Delta_1^{**}-\Delta_2^{**}}{\beta_1^{**}+\beta_2^{**}}\frac{u_n}{|u_n|}, \tag{A.30}$$

while also satisfying

$$\begin{array}{ll} z_{n1}^B \in [0,u_n] & \text{if} \quad u_n > 0 \\ z_{n1}^B \in [u_n,0] & \text{if} \quad u_n < 0 \end{array}, \tag{A.31}$$

which corresponds to playing the proposed strategy.

The second order condition is

$$\frac{\partial^2 \mathbb{E}\left[\pi_n | s=\left(\boldsymbol{\gamma}^{**},\boldsymbol{\xi}^{**},\boldsymbol{\phi}_{-n.}^{**}\right),\tilde{u}_n=u_n\right]}{\partial z_{n1}^2} =$$

$$= -2\left(\beta_1^{**}+\beta_2^{**}\right) - \frac{\Delta_1^{**}-\Delta_2^{**}}{\beta_1^{**}+\beta_2^{**}}\delta\left(u_n\right) \leq 0, \tag{A.32}$$

where $\delta(u)$ is the Dirac delta function. Note that the second order condition must be satisfied when everybody else is playing the proposed Nash equilibrium. The proposed strategy is thus a best reply.

A.2.2. *The informed trader.* If everybody else is playing the proposed Nash equilibrium, then the informed trader's expected profit is

$$\mathbb{E}\left[\tilde{\pi}_i \mid s=\left(\boldsymbol{\gamma}^{**},\boldsymbol{\xi},\boldsymbol{\phi}^{**}\right),\tilde{v}=v\right] = \sum_{m=1}^{2}\left[v-\alpha-\beta_m^{**}x_m^2-\Delta_m^{**}\left|x_m\right|\right]. \tag{A.33}$$

The first order condition is

$$\frac{\partial \mathbb{E}\left[\tilde{\pi}_i \mid s=\left(\boldsymbol{\gamma}^{**},\boldsymbol{\xi},\boldsymbol{\phi}^{**}\right),\tilde{v}=v\right]}{\partial x_m} = v-\alpha-2\beta_m^{**}x_m^B-\Delta_m^{**}\frac{x_m^B}{\left|x_m^B\right|}=0. \tag{A.34}$$

The second order condition is

$$\frac{\partial^2 \mathbb{E}\left[\tilde{\pi}_i \mid \left(\boldsymbol{\gamma}^{**},\boldsymbol{\xi},\boldsymbol{\phi}^{**}\right),\tilde{v}=v\right]}{\partial x_m^2} = -2\beta_m^{**} \leq 0, \tag{A.35}$$

which is always satisfied. Thus, the best replies are

$$x_m^B = \begin{cases} \frac{v-\alpha+\Delta_m}{2\beta_m^{**}} & \text{if} \quad v < \alpha-\Delta_m \\ 0 & \text{if} \quad \alpha-\Delta_m < v < \alpha+\Delta_m \\ \frac{v-\alpha-\Delta_m}{2\beta_m^{**}} & \text{if} \quad \alpha+\Delta_m > v \end{cases}. \tag{A.36}$$

A.2.3. *The market maker.* If everybody else is playing the proposed Nash equilibrium, then market maker m's expected profit is

$$\mathbb{E}\left[\tilde{\pi}_m | s=\left(\boldsymbol{\gamma}_{-m}^{**},\xi^{**},\boldsymbol{\phi}^{**}\right)\right] =$$

$$= -\frac{A\left(\Delta_m\right)}{\beta_m} + \beta_m C\left(\Delta_m,\beta_m\right) + \Delta_m\frac{D\left(\Delta_m,\beta_m\right)}{\left(\beta_m+\beta_{-m}^{**}\right)}, \tag{A.37}$$

where

$$C\left(\Delta_m,\beta_m\right)=\left(\beta_m+\beta_{-m}^{**}\right)^2\sum_n \mathbb{E}\left[\tilde{z}_{nm}^{**2}\right], \tag{A.38}$$

and

$$D\left(\Delta_m,\beta_m\right)=\left(\beta_m+\beta_{-m}^{**}\right)\sum_n \mathbb{E}\left[\left|\tilde{z}_{nm}^{**}\right|\right]. \tag{A.39}$$

These expressions can easily be calculated, but are fairly cumbersome. For our purposes we note that in the proposed equilibrium we have

$$C\left(\gamma_m \mid s=\left(\boldsymbol{\gamma}^{**},\xi^{**},\boldsymbol{\phi}^{**}\right)\right) = \beta_{-m}^{**2}\sigma_u^2 \tag{A.40}$$

$$D\left(\gamma_m \mid s=\left(\boldsymbol{\gamma}^{**},\xi^{**},\boldsymbol{\phi}^{**}\right)\right) = \beta_{-m}^{**}\sqrt{\frac{2N\sigma_u^2}{\pi}}. \tag{A.41}$$

Market maker m's first order condition w.r.t. β_m is

$$\frac{\partial\mathbb{E}\left[\tilde{\pi}_m|s=\left(\boldsymbol{\gamma}_{-m}^{**},\xi^{**},\boldsymbol{\phi}^{**}\right)\right]}{\partial\beta_m}=$$

$$=\left(\begin{array}{c}\frac{A(\Delta_m)}{\beta_m^2}+\frac{\left(\beta_{-m}^{**}-\beta_m\right)C(\gamma_m)}{\left(\beta_m+\beta_{-m}^{**}\right)^3}+\beta_m\frac{\partial C(\gamma_m)}{\partial\beta_m}\\ +\frac{\Delta_m}{\left(\beta_m+\beta_{-m}^{**}\right)}\frac{\partial D(\gamma_m)}{\partial\beta_m}-\frac{\Delta_m D(\gamma_m)}{\left(\beta_m+\beta_{-m}^{**}\right)^2}\end{array}\right)=0, \tag{A.42}$$

In the proposed equilibrium, we have

$$\frac{\partial C\left(\gamma_m^{**} \mid s=\left(\boldsymbol{\gamma}^{**},\xi^{**},\boldsymbol{\phi}^{**}\right)\right)}{\partial\beta_m} = 0 \tag{A.43}$$

$$\frac{\partial D\left(\gamma_m^{**} \mid s=\left(\boldsymbol{\gamma}^{**},\xi^{**},\boldsymbol{\phi}^{**}\right)\right)}{\partial\beta_m} = 0. \tag{A.44}$$

It is then straight forward to see that the first order condition is satisfied in the proposed equilibrium.

The first order condition w.r.t. Δ_m is

$$\frac{\partial\mathbb{E}\left[\tilde{\pi}_m|s=\left(\boldsymbol{\gamma}_{-m}^{**},\xi^{**},\boldsymbol{\phi}^{**}\right)\right]}{\partial\Delta_m}$$

$$=\left(\begin{array}{c}-\frac{A'(\Delta_m)}{\beta_m}+\frac{\beta_m}{\left(\beta_m+\beta_{-m}^{**}\right)^2}\frac{\partial C(\gamma_m)}{\partial\Delta_m}\\ +\frac{D(\gamma_m)}{\beta_m+\beta_{-m}^{**}}+\frac{\Delta_m}{\beta_m+\beta_{-m}^{**}}\frac{\partial D(\gamma_m)}{\partial\Delta_m}\end{array}\right)=0. \tag{A.45}$$

In the proposed equilibrium, we have

$$\frac{\partial C\left(\gamma_m^{**} \mid s=\left(\boldsymbol{\gamma}^{**},\xi^{**},\boldsymbol{\phi}^{**}\right)\right)}{\partial\Delta_m} = -\beta_{-m}^{**}\sqrt{\frac{2N\sigma_u^2}{\pi}} \tag{A.46}$$

$$\frac{\partial D\left(\gamma_m^{**} \mid s=\left(\boldsymbol{\gamma}^{**},\xi^{**},\boldsymbol{\phi}^{**}\right)\right)}{\partial\Delta_m} = -\frac{1}{2}N. \tag{A.47}$$

Then it is straight forward to see that the proposed equilibrium does indeed satisfy the first order condition.

An analytical proof that the second order conditions are satisfied has not yet been established. However, a numerical proof is presented in the figure below. Let us consider the case $\sigma_v^2 = \sigma_u^2 = N = 1$. Then we have $\left(\beta_{-m}^{**}, \Delta_{-m}^{**}\right) \approx (0.38, 0.79)$. the figure shows market maker m's expected profit depending on β_m and Δ_m. As expected, the function only has one maximum, at the proposed strategy.

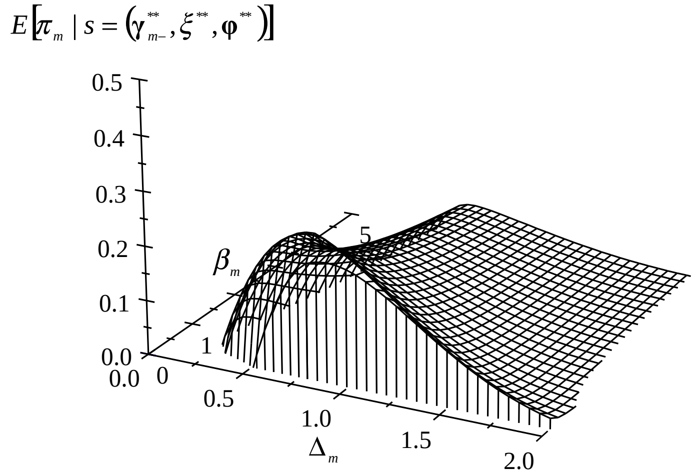

Figure 11: Market maker m's expected profit, depending γ_m, when everybody else is playing the proposed strategy.

In addition, this strategy results in the expected profit

$$\mathbb{E}\left[\tilde{\pi}_m | s = (\boldsymbol{\gamma}^{**}, \xi^{**}, \boldsymbol{\phi}^{**})\right] = 0.25475. \tag{A.48}$$

A.3. Proof of Proposition 3.

A.3.1. *The noise traders.* Noise trader n's first order condition is

$$\frac{\partial \mathbb{E}\left[\pi_n | s = \left(\boldsymbol{\beta}^{***}, \boldsymbol{\xi}^{***}, \boldsymbol{\phi}_{-n,.}^{***}\right), \tilde{u} = u\right]}{\partial z_{nm}} =$$

$$-2\beta_m^{***} z_{nm}^B + 2\beta_M^{***}\left(u_n - z_{nm}^B - \sum_{k \neq m}^{M-1} z_{nk}\right) = 0, \tag{A.49}$$

for $\forall m \in \{1, ..., M-1\}$. The second order condition is always satisfied since

$$\frac{\partial^2 \mathbb{E}\left[\pi_n \mid s = \left(\boldsymbol{\beta}^{***}, \boldsymbol{\xi}^{***}, \boldsymbol{\phi}_{-n,.}^{***}\right), \tilde{u} = u\right]}{\partial z_{nm}^2} = -2\left(\beta_m^{***} + \beta_M^{***}\right) < 0, \tag{A.50}$$

$\forall m \in \{1, ..., M-1\}$.

Reshuffling the first order condition, we get

$$z_{nm}^B = \begin{cases} \frac{\beta_M^{***}\left(u_n - \sum_{k \neq m}^{M-1} z_{nk}\right)}{\beta_m^{**} + \beta_M^{**}} & \text{for} \quad m = \{1, ..., M-1\} \\ u_n - \sum_{m}^{M-1} z_{nm} & \text{for} \quad m = M \end{cases} \tag{A.51}$$

Solving this system of equations, we get $z_{nm}^B = z_{nm}^{***}$ for $\forall m \in \{1, ..., M\}$, i.e. the best reply is indeed the proposed strategy.

A.3.2. *The informed trader.* The informed trader's expected profit is

$$\mathbb{E}\left[\tilde{\pi}_i \mid s = (\boldsymbol{\beta}^{***}, \xi, \boldsymbol{\phi}^{***})\right] = \sum_{m=1}^{M} \left[v - \alpha - \beta_m^{***} x_m^2 - \Delta \left|x_m\right|\right]. \tag{A.52}$$

which only differs from $(A.5)$ in the number of market makers. Thus, the best replies are $x_m^B = x_m^{***}, \forall m \in \{1, ..., M\}$, i.e. the best reply is indeed the proposed strategy.

A.3.3. *The market makers.* Market maker m's expected profit is

$$\mathbb{E}\left[\tilde{\pi}_m \mid s = \left(\boldsymbol{\beta}_{-m}^{***}, \boldsymbol{\xi}, \boldsymbol{\phi}^{***}\right)\right] = \begin{pmatrix} -\frac{A(\Delta)}{\beta_m} + \beta_m \left(\frac{\beta^{***}}{\beta^{***}+\beta_m(M-1)}\right)^2 \sigma_u^2 \\ +\frac{\beta^{***}}{\beta^{***}+\beta_m(M-1)} B\left(\Delta, N\right) \end{pmatrix}. \tag{A.53}$$

Taking, the first order condition, we get

$$\frac{\partial \mathbb{E}\left[\tilde{\pi}_m | s = \left(\boldsymbol{\beta}_{-m}^{***}, \boldsymbol{\xi}, \boldsymbol{\phi}^{***}\right)\right]}{\partial \beta_m} =$$

$$= \begin{pmatrix} \frac{A(\Delta)}{\beta_m^2} + \frac{\beta^{***3}\sigma_u^2 - \beta_m \beta^{***2}(M-1)\sigma_u^2}{(\beta^{***}+\beta_m(M-1))^3} \\ -\frac{\beta^{***}}{(\beta^{***}+\beta_m(M-1))^2} \left(M-1\right) B\left(\Delta, N\right) \end{pmatrix} = 0. \tag{A.54}$$

Let us first show that the proposed strategy is a local maximum. Setting $\beta_m = \beta^{***}$, we get

$$\frac{M^3 A\left(\Delta\right) - \left(M-2\right) \beta^{***2} \sigma_u^2 - M\left(M-1\right) \beta^{***} B\left(\Delta, N\right)}{M^3 \beta^{***2}} = 0. \tag{A.55}$$

Simplifying and multiplying by M, the first order condition can be expressed as

$$M^4 A\left(\Delta\right) = M\left(M-2\right) \beta^{***2} \sigma_u^2 + M^2 \left(M-1\right) \beta^{***} B\left(\Delta, N\right). \tag{A.56}$$

Taking the second order condition and setting $\beta_m = \beta^{***}$, we get

$$M^4 A\left(\Delta\right) > \beta^{***2} \left(M-1\right)\left(M-3\right) \sigma_u^2 + \beta^{***} M \left(M-1\right)^2 B\left(\Delta, N\right). \tag{A.57}$$

Thus, if the first order condition is satisfied, then also the second order condition must be satisfied. The proposed strategy is thus clearly a local maximum.

Rewriting the first order condition, we get

$$\Phi\left(\beta_m\right) = c_3 \beta_m^3 + c_2 \beta_m^2 + c_1 \beta_m + c_0 = 0, \tag{A.58}$$

where

$$c_0 = -A\beta^3 \tag{A.59}$$

$$c_1 = -3A\beta^2(M-1) \tag{A.60}$$

$$c_2 = -\beta\left((3A(M-1)-\beta)(M-1)+\beta^2\sigma_u^2\right) \tag{A.61}$$

$$c_3 = -(M-1)\left(A-\beta(M-1)+AM(M-2)-\beta^2\sigma_u^2\right). \tag{A.62}$$

Note that $\Phi(0) = c_0$ and

$$\frac{\partial\Phi(0)}{\partial\beta_m} = c_1, \tag{A.63}$$

and that $c_0, c_1 \in \mathbb{R}_{--}$ whereas $b_2, b_3 \in \mathbb{R}$. As a result at maximum two roots can be positive. By continuity only one of those roots can correspond to a maximum. Thus, there can only be one solution that satisfies both the first and second order conditions. The proposed strategy is thus a best reply.

CHAPTER 3

Endogenous Noise Traders

Marcus Salomonsson[1]

Abstract. The Copeland-Galai framework is extended to construct the simplest model possible to endogenize noise traders. An endogenous noise trader anticipates that future shocks may force him to exit his position and will thus only enter a position at a discount - or a required return. We show that the original seller of the asset pays the required return. This can only be optimal if the seller has access to an investment opportunity that gives a sufficiently high return, compared to the noise trader's investment opportunities. We also show that, if the informed trader's cost function is convex in the precision, then the required return is higher when the informed trader is competitive, compared to when he is a monopolist.

1. Introduction

Bagehot (1971) was one of the first to note that market makers face a problem when trading with informed traders. Since informed traders can choose not to trade if the prices do not suit them, market makers will never gain from trading with them - and might sometimes lose. This adverse selection problem may lead to market making not being viable, and markets may break down. However, Bagehot[2] also suggested that exogenously motivated traders, or so called noise traders, could provide the market maker with enough gains to compensate for the losses on informed traders.

Noise traders have ever since played an important role in the market microstructure literature. Indeed Black (1986) concluded that "[n]oise makes trading in financial markets possible". In the model of Copeland and Galai (1983), and in the dynamic extension of Glosten and Milgrom (1985), they are needed for the market maker to finance losses to informed traders. In Kyle (1985) noise traders provide camouflage for a monopolistic informed trader. Noise traders, under the guise of an exogenously

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[2] The article was written pseudonymously by Jack Treynor.

changing supply, ensure that prices are somewhat inefficient in Grossman and Stiglitz (1980). This allows for informed traders to recover information costs.[3]

The need for noise traders in models of financial markets can also be understood as a way of side-stepping the various no-trade theorems, among which Milgrom and Stokey (1982) and Tirole (1982) are the most well-known. The principle behind these theorems follows from Aumann (1976): "If two people have the same priors, and their posteriors for an event A are common knowledge, then these posteriors are equal." Translated into the world of financial markets, this means that, given the above assumptions, these two people must agree on the price of an asset. As a result, they may trade - even if they start off from a Pareto optimal allocation - but they will be indifferent between trading and not trading. Thus, as soon as any costs of trading are introduced, there will be no trade. At least one participant will lose from trading, and will then prefer not to trade. The costs may be transaction costs, information costs, or, as in Milgrom and Stokey (1982), remuneration for risk.

A problem with the noise trader approach is that noise traders systematically lose money. As discussed in Dow and Gorton (2006), this has led to a literature trying to endogenize noise traders as rational agents. Various approaches have been considered. Diamond and Verrecchia (1981) suggest that noise traders may trade for insurance reasons. De Long, Shleifer, Summers, and Waldman (1990) consider the possibility that arbitrageurs have a limited time horizon. This results in a limited arbitrage that makes it possible for noise traders to survive. Shleifer and Vishny (1997) elaborate on this idea and consider agency problems. They consider a situation where an arbitrageur borrows money from an uninformed investor. As the investor is uninformed, he tries to conjecture whether the arbitrageur's positions are sound or not by observing returns. A temporary shock can then lead to the investor recalling the money, although the arbitrageur's position is fundamentally sound. Another approach, also based on the agency problem, has been developed by Dow and Gorton (1997). They note that a fund manager might trade excessively to appear informed to his investors. This excessive trade results in a systematic loss that would correspond to the loss of noise traders.

In this chapter, yet another approach is considered, based on the notion of *required return*. According to this approach, the noise trader anticipates his future trading losses when selling the asset, and thus demands a discount when buying the asset. The argument can be traced back to Amihud and Mendelson (1986) who argued that, since the bid-ask spread is a trading cost, it should be positively correlated with the expected return. Gârleanu and Pedersen (2004) further developed the argument by arguing that

[3] Shleifer and Summers (1990) give an outline of the noise trader approach to finance. Recent surveys of the market microstructure literature are O'Hara (1995), Madhavan (2000), Brunnermeier (2001), Stoll (2003), and Biais, Glosten, and Spatt (2005).

if the bid-ask spread was caused by adverse selection, as claimed by Copeland and Galai (1983), then traders would demand a higher return if they expected to exit their position due to a liquidity shock, but a lower return if they expected to exit due to private information. As a result, Gârleanu and Pedersen argued, the two effects can cancel each other, and only when the liquidity signal and the informational signal are in conflict will an actual cost be incurred.

The model of Copeland and Galai (1983) is extended to construct the simplest model possible to endogenize noise traders. In section 2, the adverse selection problem is illustrated in a simple model with risk neutral players, but where informative signals are costly. It is shown that without noise traders, no costly information will be acquired. However, if exogenous noise traders are introduced, then information costs can be covered. In section 3, the model is extended to endogenize the noise traders. Anticipating that they may have to exit their position at a loss, they will only enter it at a discount, or at a required return. Furthermore, it is possible to analyze the effect of the competitive structure among informed traders. It turns out that if the informed trader is a monopolist, then the required return has to finance his profit. However, a monopolistic informed trader will choose a lower precision than if he were competitive. Thus, the unambiguous result, if information costs are convex in the precision, is that the required return increases if the informed trader is competitive compared to if he is a monopolist. The reason is not the competition per se, but that competition between informed traders increases the precision of their signal, thus worsening the adverse selection problem. This is in contrast with other result on the effect of competition between informed traders, who have noted that competition between informed traders will lead to prices containing more information, and thus resulting in a lower adverse selection cost for noise traders. The relationship with this literature is discussed in Section 4, and the chapter is concluded in Section 5.

2. Adverse selection

2.1. The model. The adverse selection problem on financial markets can be illustrated as follows. Let us assume that we have an informed trader (IT) and a competitive market maker (M),[4] and that the timing of the game is as follows:

(1) Nature draws a fundamental value for the asset $v \in \{0, 1\}$ with equal probability. The realization is not observed by anybody.

[4] We do not model why the market maker is competitive. However, as discussed in Glosten and Milgrom (1985) it may be a result of competition with other limit orders or with another market maker at the same, or another exchange.

(2) The informed trader chooses a precision $\lambda \in [1/2, 1]$ for a signal to be received in 4. The signal costs $c(\lambda)$, where $c\left(\frac{1}{2}\right) = c'\left(\frac{1}{2}\right) = 0$, and $c'(\lambda), c''(\lambda) > 0$.

(3) The competitive market maker observes λ and announces ask and bid prices $p_A, p_B \in [0, 1]$.

(4) The informed trader gets the signal $s \in \{0, 1\}$, where

$$\begin{aligned}\Pr(v = 1 \mid s = 1) &= \Pr(v = 0 \mid s = 0) = \lambda \\ \Pr(v = 0 \mid s = 1) &= \Pr(v = 1 \mid s = 0) = 1 - \lambda.\end{aligned}$$

He observes the bid-ask prices and chooses whether to buy, sell, or do nothing. His strategy set is thus $S_{IT} = \{B, S, N\}$.

(5) The asset expires and all trades clear at the fundamental value v. More precisely, the market maker and the informed trader reverse their trade, if there was a trade, at the price v.

The players are rational and risk neutral. There is no time discounting and the reservation value is 0. We allow for both long and short positions, and for futures contracts, if a prospective short seller can not borrow an asset.

2.2. Analysis. The only subgame perfect Nash equilibrium in this game is for the informed trader to not pay an information cost and thus receive an uninformative signal. To see this, let us solve the game by backward induction.

In 4, IT's conditional payoffs are

$$\begin{aligned}\mathbb{E}[\pi_{IT} \mid s = 1, B] &= \lambda(1 - p_A) + (1 - \lambda)(0 - p_A) - c \\ \mathbb{E}[\pi_{IT} \mid s = 1, S] &= \lambda(p_B - 1) + (1 - \lambda)(p_B - 0) - c \\ \mathbb{E}[\pi_{IT} \mid s = 1, N] &= -c \\ \mathbb{E}[\pi_{IT} \mid s = 0, B] &= \lambda(0 - p_A) + (1 - \lambda)(1 - p_A) - c \\ \mathbb{E}[\pi_{IT} \mid s = 0, S] &= \lambda(p_B - 0) + (1 - \lambda)(p_B - 1) - c \\ \mathbb{E}[\pi_{IT} \mid s = 0, N] &= -c.\end{aligned}$$

Thus, if IT gets the signal $s = 1$, then with probability λ it is correct, and by buying he would get the payoff $(1 - p_A)$. With probability $1 - \lambda$ it is incorrect and then he would get the payoff $(0 - p_A)$. The information costs are sunk, so if IT does not trade, then his payoff is $-c$.

Thus, IT weakly gains from buying if

$$p_A \leq \lambda \quad \text{and} \quad s = 1, \tag{2.1}$$

and from selling if

$$p_B \geq 1 - \lambda \quad \text{and} \quad s = 0. \tag{2.2}$$

In 3, the competitive market maker will set bid-ask prices so that they by themselves result in an expected profit of zero. He takes the reversed position of the informed trader - half of the time on each side. Thus, the bid-ask prices must satisfy

$$-\frac{1}{2}(\lambda - p_A) = 0 \tag{2.3}$$

$$-\frac{1}{2}(p_B - (1-\lambda)) = 0. \tag{2.4}$$

Reshuffling, we get the equilibrium bid-ask prices

$$p_A^* = \lambda \tag{2.5}$$

$$p_B^* = 1-\lambda. \tag{2.6}$$

It follows immediately that these bid-ask prices satisfy conditions $(2.1)-(2.2)$. In other words, if the market maker sets prices so as to achieve zero profits when trading, then the informed trader will trade according to his signal.[5]

In 2, IT's expected profit is

$$\mathbb{E}[\pi_{IT}] = \lambda - \frac{1}{2} - \frac{1}{2}(p_A - p_B) - c(\lambda). \tag{2.7}$$

Inserting the bid-ask prices, we get

$$\mathbb{E}[\pi_{IT}] = -c(\lambda). \tag{2.8}$$

Thus, the only information cost that IT can cover is $c = 0$. The reason is that M will never announce prices so that he makes a loss on expectation, but then IT can never cover strictly positive information costs.

2.2.1. *Exogenous noise traders.* The standard approach to solving this dilemma is to assume that some traders trade for exogenous reasons. This type of traders are usually called liquidity traders or noise traders. In this chapter we have chosen the term noise traders.

Let us assume that noise traders trade with the probability $\alpha > 0$ on the ask side, and with probability $\beta > 0$ on the bid side. Then the competitive market maker faces a different problem in 3. The bid-ask prices must now satisfy

$$\alpha\left(p_A - \frac{1}{2}\right) - \frac{1}{2}(\lambda - p_A) = 0 \tag{2.9}$$

$$\beta\left(\frac{1}{2} - p_B\right) - \frac{1}{2}(p_B - (1-\lambda)) = 0. \tag{2.10}$$

[5] Anticipating $IT's$ trading decision in 4, the market maker can also achieve zero profits by setting $p_A \geq \lambda$ and $p_B \leq 1-\lambda$, where at least one inequality is strict. Then profits will be zero for the simple reason that IT will not trade on at least one side. Since we interpret the market maker's zero profit condition as resulting from competition, we disregard this possibility.

The equilibrium bid-ask prices are

$$p_A = \frac{\alpha+\lambda}{2\alpha+1} \tag{2.11}$$

$$p_B = \frac{\beta+1-\lambda}{2\beta+1}. \tag{2.12}$$

Again it is easy to see that these bid-ask prices satisfy conditions $(2.1)-(2.2)$. Furthermore, in 2, IT's expected profit is

$$\mathbb{E}\left[\pi_{IT}\right] = \lambda - \frac{1}{2} - \frac{1}{2}\left(\frac{\alpha+\lambda}{2\alpha+1} - \frac{\beta+1-\lambda}{2\beta+1}\right) - c\left(\lambda\right).$$

It is weakly positive if

$$\frac{(2\lambda-1)}{2}\frac{(\alpha+\beta+4\alpha\beta)}{(2\beta+1)(2\alpha+1)} \geq c\left(\lambda\right). \tag{2.13}$$

Thus, if only $\lambda > 1/2$, i.e. if the signal is informative, then a positive information cost can be covered - provided it is sufficiently low.

Thus the introduction of exogenous noise traders makes it possible to finance costly information acquisition. On the other hand, since these noise traders trade for exogenous reasons, we have pushed the actual source of financing outside the model. The objective with the concept of required return is to bring it inside the model. In the next section we will show how that may be done.

3. Required return

The main idea with the concept of required return is that if the noise trader anticipates that he might have to exit the asset at a cost, then he will only enter at a discount. Note, however, that this implies that only the noise trader on the bid side in the previous example can be endogenized. A noise trader on the ask side, if he is rational, would still sell for some exogenous reason.

3.1. The model. Let us imagine a game with four representative types; a seller (S), a noise trader (NT), an informed trader (IT), and a competitive market maker (M). The timing of the model is as follows.

(1) The seller gives the noise trader a take-it-or-leave-it offer to buy one unit of an asset for $p_0 \in [0,1]$. The seller will use the funds to finance a project with an expected return $\mathbb{E}[r_S] > 0$.
(2) The noise trader decides whether to accept the seller's offer or not.
(3) Nature draws a fundamental value for the asset $v \in \{0,1\}$ with equal probability. The realization is not observed by anybody.

(4) The informed trader chooses a precision $\lambda \in [1/2, 1]$ for the signal that he will receive in 6. The signal costs $c(\lambda)$, where $c\left(\frac{1}{2}\right) = c'\left(\frac{1}{2}\right) = 0$, and $c'(\lambda), c''(\lambda) > 0$.
(5) The competitive market maker observes λ and announces ask and bid prices $p_A, p_B \in [0, 1]$.
(6) The informed trader gets the signal $s \in \{0, 1\}$, where

$$\begin{aligned} \Pr(v = 1 \mid s = 1) &= \Pr(v = 0 \mid s = 0) = \lambda \\ \Pr(v = 0 \mid s = 1) &= \Pr(v = 1 \mid s = 0) = 1 - \lambda. \end{aligned}$$

He observes the bid-ask prices and chooses whether to buy, sell, or do nothing. His strategy set is thus $S_{IT} = \{B, S, N\}$. With probability β the noise trader gets a shock and must sell for p_B. With probability $1 - \beta$ the noise trader gets no shock and keeps the asset.
(7) The asset expires and all trades clear at the fundamental value v. More precisely, the market maker and the informed trader reverse their trade, if there was a trade, at the price v. The asset is bought by the seller for the price v.

Thus, compared to the previous model we have added two players, who each make decisions before trading occurred in the previous model.

We have an endogenous noise trader. He might receive a shock in 6, which hinders him to keep the asset to maturity. However, he anticipates this shock when deciding at which price he is ready to buy the asset in 2.

We also have a seller of the asset, who might sell the asset to the noise trader in 2. The seller can be interpreted either as the issuer of the asset, for example at an IPO, or as any owner of the asset who considers selling the asset to use the funds for some other investment opportunity.[6]

As before, all players are rational and risk neutral. There is no time discounting and the reservation value is 0 - except for S. We allow for both long and short positions, and for futures contracts, if a prospective short seller can not borrow an asset.

3.2. Analysis. Solving the model by backward induction, we now note that the market maker will announce the bid-ask prices

$$p_A^* = \lambda \tag{3.1}$$

$$p_B^* = \frac{\beta + 1 - \lambda}{2\beta + 1}. \tag{3.2}$$

It is easy to verify that these bid-ask prices satisfy conditions $(2.1) - (2.2)$.

[6] The fact that the seller possibly buys back the asset in 7 is simply a technical device to calculate the expected profit from his initial sale.

If IT is a monopolist, then he maximizes profits with respect to λ. Inserting equations (3.1) and (3.2) into (2.7) and taking the first order condition, we can conclude that the equilibrium precision is given by

$$\lambda^* = \lambda_{MON} = c'^{-1}\left(\frac{\beta}{2\beta+1}\right).$$

On the other hand. if the profits are pushed to zero, for example due to competition, then the equilibrium precision is given by $\lambda^* = \lambda_{COMP}$, where λ_{COMP} satisfies

$$\frac{2\lambda_{COMP}-1}{2}\frac{\beta}{2\beta+1} = c\left(\lambda_{COMP}\right). \tag{3.3}$$

Since $c(\lambda)$ is convex, we know that $\lambda_{COMP} \geq \lambda_{MON}$.

In 2, the expected profit of NT is

$$\mathbb{E}\left[\pi_{NT}\right] = \beta p_B^* + (1-\beta)\,\mathbb{E}\left[v\right] - p_0. \tag{3.4}$$

He will thus accept any offer that satisfies

$$p_0 \leq \beta p_B^* + (1-\beta)\,\mathbb{E}\left[v\right], \tag{3.5}$$

and reject other offers.

Moving back to 1, the seller's expected profit is

$$\mathbb{E}\left[\pi_S\right] = \left(1+\mathbb{E}\left[r_S\right]\right)p_0 - \mathbb{E}\left[v\right].$$

Thus, he will only give offers that satisfy

$$p_0 \geq \frac{\mathbb{E}\left[v\right]}{1+\mathbb{E}\left[r_S\right]}.$$

Thus for

$$\frac{\mathbb{E}\left[v\right]}{1+\mathbb{E}\left[r_S\right]} \leq \beta p_B^* + (1-\beta)\,\mathbb{E}\left[v\right],$$

the seller will give the noise trader an offer that he will accept. Reshuffling, and inserting (3.2), and $\mathbb{E}[v] = 1/2$, the condition becomes

$$\frac{(2\lambda^*-1)\,\beta}{1+2\beta-\beta\,(2\lambda^*-1)} \leq \mathbb{E}\left[r_S\right]. \tag{3.6}$$

If condition (3.6) is satisfied, then the seller uses his first mover advantage and gives the offer p_0^*, where

$$p_0^* = \beta p_B^* + (1-\beta)\,\mathbb{E}\left[v\right]. \tag{3.7}$$

The equilibrium required return, or equivalently, the seller's cost of capital, is thus

$$\begin{aligned} r^* &= \frac{\mathbb{E}\left[v\right]-p_0^*}{p_0^*} \\ &= \frac{(2\lambda^*-1)\,\beta}{3\beta-2\beta\lambda^*+1}. \end{aligned}$$

If condition (3.6) is not satisfied, then the seller can not give the noise trader an offer he will accept. In this case, the asset will thus not be sold. Since the left hand side is strictly positive, it is not enough if $\mathbb{E}[r_S]$ simply is strictly positive - it must be sufficiently high. Thus, the seller and the noise trader must have sufficiently asymmetric investment opportunities.

Calculating the sensitivity of the required return to the precision of the signal, we get

$$\frac{\delta r^*}{\delta \lambda^*} = \frac{2(2\beta+1)\beta}{(2\beta\lambda^* - 3\beta - 1)^2} > 0. \tag{3.8}$$

In other words, a more competitive informed trader implies that the required return increases. Thus, although the seller may have to pay the monopolist informed trader's positive profits, he still prefers that to having to pay for the increased adverse selection cost when the informed traders is competitive.

Also note that the bid-ask spread depends on the precision λ, but only indirectly on the information cost. This complements the result of Gârleanu and Pedersen (2004) that transaction costs increase the bid-ask spread.

3.3. An example of $c(\lambda)$. Let us explore an example where the cost function is

$$c(\lambda) = \ln\left(\frac{1}{2(1-\lambda)}\right) + 1 - 2\lambda. \tag{3.9}$$

The cost function is plotted in Figure 1 below. We have also plotted IT's revenues when $\beta = 1$.

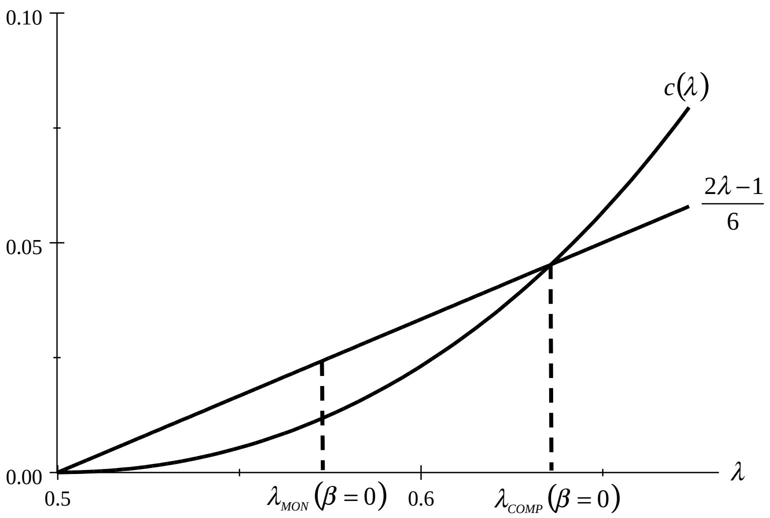

Figure 1: The cost function and IT's revenue when $\beta = 1$.

Using (3.9), λ_{MON} is given by

$$\lambda_{MON} = c'^{-1}\left(\frac{\beta}{2\beta+1}\right) = \frac{3\beta+1}{5\beta+2}. \tag{3.10}$$

Note that since

$$\frac{\delta\lambda_{MON}}{\delta\beta} = \frac{1}{(5\beta+2)^2} > 0 \qquad (3.11)$$

$$\frac{\delta\lambda_{MON}}{\delta\beta} = -\frac{10}{(5\beta+2)^3} < 0, \qquad (3.12)$$

λ_{MON} is increasing and concave in β. Figure 2 below shows how the optimal precision changes as a function of β.

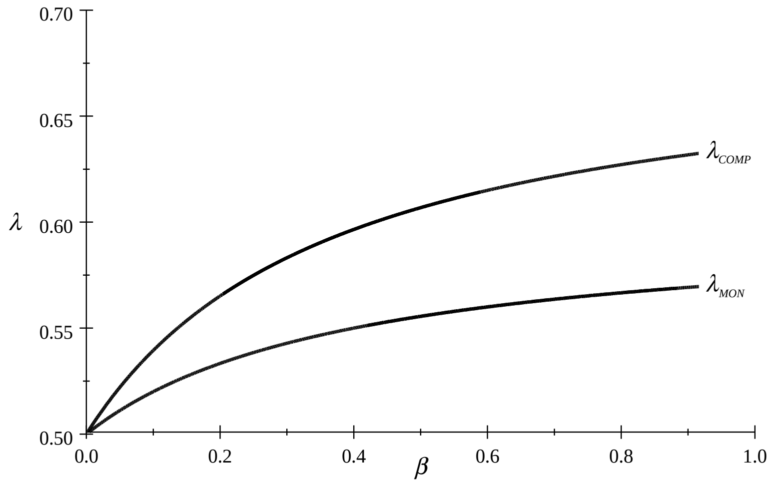

Figure 2: The precisions as a function of β.

Using Equations (3.1) – (3.2) we thus get

$$p_B^* = \frac{4\beta + 5\beta^2 + 1}{(5\beta+2)(2\beta+1)} \qquad (3.13)$$

$$p_A^* = \frac{3\beta+1}{5\beta+2}. \qquad (3.14)$$

It follows that

$$p_0^* = \frac{1}{2}\frac{(3\beta+1)(3\beta+2)}{(5\beta+2)(2\beta+1)}. \qquad (3.15)$$

Figure 3 shows equilibrium bid-ask prices as β increases, as well as p_0^* (the thin downward sloping curve).

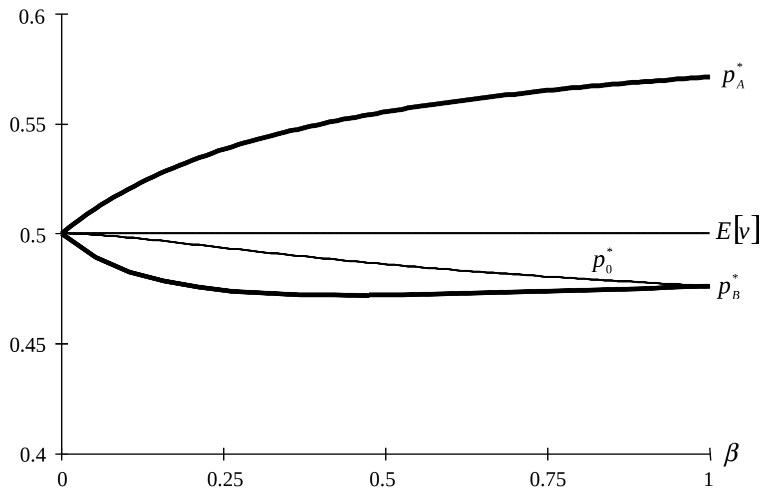

Figure 3: The bid ask spread, p_0^*, and $E[v]$ as a function of β.

The horizontal line $\mathbb{E}[v]$ shows the expected value of the asset. Note that the spread is not symmetric around this value. This is because noise traders exit their positions on the bid side. Since this reduces the adverse selection problem, the bid price increases.

As β increases, M gets higher trading gains vis-à-vis NT. As a result, the possibility to make trading losses vis-à-vis IT increases. IT can thus finance more costly information. As the precision of signals rise, the bid-ask spread also rises. The bid-ask spread as a function of β is concave for two reasons. First, since the cost function is convex, an increase in β allows a less than proportional increase in λ from a cost perspective. Second, as the bid-ask spread increases in λ, the gain from a higher precision is also less than proportional.

Finally, the required return is given by

$$r^* = \frac{\beta^2}{(3\beta+1)(3\beta+2)}.$$

4. Discussion

In this extension of the Copeland and Galai (1983) and Glosten and Milgrom (1985) framework, we have noted that in an intertemporal world, also noise traders can be endogenized. They do not have to have exogenous reasons for trading if they are able to enter their positions at a discount. This gives rise to a required return that must be a feature of all traded assets - at least if noise traders are to be endogenous.

In addition, it was shown that in this framework, if there is perfect competition between informed traders, then this increases the adverse selection problem for the noise traders - and thus the required return. The reason is that in the Copeland-Galai framework, the market maker can not adjust prices after having received his orders.

Instead he must increase the spread, and thus the cost for the noise trader, if he believes that the informed trader will have better information than usual.

In the Kyle framework, the mechanism is completely different. There the market maker can adjust prices after he has received his orders. As a consequence, if he believes that the informed trader has submitted a particularly large order, then he would also adjust his price accordingly, resulting in a more informative price, and thus a lower adverse selection cost for the noise trader.

Kyle (1984), was the first to consider strategic interactions among several informed traders in this framework. He considers the situation when the noise increases. At a constant number of informed traders this leaves the informational content of prices unchanged - the informed traders only increase their volume and as a result their profits. If entry is allowed then the increase in profits will result in more informed traders entering the market. This will result in increased competition, and the total effect is that prices are more informative than before the noise increased.[7]

Admati and Pfleiderer (1988) noted that if noise traders could trade somewhat at their discretion, then they would choose to trade at the same time as other noise traders. Noise trading thus attracts both other noise traders and informed traders. In addition, when informed traders are attracted to the market, then prices become more informative, reducing the cost for noise traders, which implies that even more noise traders will be attracted to the market. Thus Admati and Pfleiderer argue that this may explain the observed tendency for trades to cluster, and thus may also explain intraday patterns.

However, Subrahmanyam (1991) shows that risk aversion may have important effects on market liquidity. In particular, he shows that increasing the number of informed traders may decrease liquidity if the informed traders are risk averse. The reason is that if noise trading increases, then the informed traders will not fully compensate for this effect by trading more. As a result, with no entry allowed, the prices will actually be less informative compared to before. On the other hand, an influx of new informed traders now not only increases competition, it also reduces the aggregate risk for informed traders. The informed traders will thus trade more, and the prices will be more informative. This effect is especially large when there are few informed traders to start with. Thus, there are two conflicting effects in play. When there are few informed traders, the first effect will dominate, whereas the second effect will dominate when there are many traders. As a result, when there are few informed traders an increase in noise will lead to prices becoming more informative, whereas it will lead to prices becoming less informative when there are many informed traders.

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[7] Also see Kyle (1989).

Both Holden and Subrahmanyam (1992) and Foster and Viswanathan (1993) extend the Kyle (1985) model to consider price adjustment with several informed traders, with identical information. In particular, Holden and Subrahmanyam (1992) show that in a dynamic environment, competition will be very fierce between informed traders, even when they are very few. Thus, information will be incorporated into prices very quickly. In the limit, as the number of informed traders goes to infinity, the information will be revealed in the first period.

However, Back et al (2000) extend the analysis to let informed traders have uncorrelated signals. They then show that information will be incorporated in prices much slower. In fact, had a monopolist had access to both traders information, there would be some date after which prices would reveal more information under a monopolist than under two competing informed traders, provided that their information is not perfectly correlated.

5. Conclusion

We have aimed to create a simple way to extend Copeland and Galai (1983) to incorporate endogenous noise traders. We have shown that the concept of required return implies that the adverse selection cost ultimately is paid by the seller of the asset. It is thus part of his cost of capital. We have framed the model so that the seller is any owner of the asset. However, he could also be interpreted as the original issuer of the asset. We showed that for the required return argument to be consistent with rational profit maximizing agents, the seller must have a reservation value that is sufficiently above that of the buying noise trader's.

As we have seen, in the Copeland-Galai framework the noise trader's adverse selection cost increases if competition between informed traders increase, whereas the opposite has be found to be the case in the Kyle framework. This difference is due to the fact that in the Copeland-Galai framework the market maker has to announce bid-ask prices before the traders trade, whereas he can adjust his price depending on the order flow in the Kyle framework.

A straight-forward extension would be to let noise traders be distributed over β. The required return would then depend on that distribution, and on the seller's need for funds. One could also envision a model where traders are two-dimensional in the sense that they have both an β and a precision λ. This would allow for the possibility noted by Gârleanu and Pedersen (2004), i.e. that a trader may trade for both informational or liquidity reasons, and that this affects the required return.

This would thus open up for situations where informed traders may have to exit their positions prematurely. The next step would then be to endogenize also when this exit will take place. Such an endogenization would in turn open up for predatory traders. Such traders, discussed in Brunnermeier and Pedersen (2005), actively try to push other traders across their liquidity thresholds with the objective to enter new positions at fire sale prices.

CHAPTER 4

Natural Selection and Social Preferences

Jörgen Weibull and Marcus Salomonsson[1,2]

Abstract. A large number of individuals are randomly matched into groups, where each group plays a finite symmetric game. Individuals breed true. The expected number of surviving offspring depends on own material payoff, but may also, due to cooperative breeding and/or reproductive competition, depend on the material payoffs to other group members. The induced population dynamic is equivalent with the replicator dynamic for a game with payoffs derived from those in the original game. We apply this selection dynamic to a number of examples, including prisoners' dilemma games with and without a punishment option, coordination games, and hawk-dove games. For each of these, we compare the outcomes with those obtained under the standard replicator dynamic. By way of a revealed-preference argument, our selection dynamic can explain certain "altruistic" and "spiteful" behaviors that are consistent with individuals having social preferences.

1. Introduction

One of the longest standing controversies in evolutionary game theory is the group selection controversy. The group selection idea, which traces its origins all the way back to Darwin, essentially says that groups with internal cooperation will be more successful than other groups, and that this may cause altruistic behaviors — individual sacrifices for the common good of the group — to survive and in some circumstances thrive:

> "There can be no doubt that a tribe including many members who, from possessing in a high degree the spirit of patriotism, fidelity, obedience, courage, and sympathy, were always ready to give aid to each other and to sacrifice themselves for the common good, would be victorious over most other tribes, and this would be natural selection." (Darwin, 1871, page 166.)

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2 We are grateful for comments from two anonymous referees and an editor of Journal of Theoretical Biology, and from Milo Bianchi, Ernst Fehr, Jens Josephson, Olof Leimar, Bill Sandholm, Yannick Viossat, Franjo Weissing and participants in presentations at the PED Institute (Harvard), the Henri Poincaré Institute and the London School of Economics. Marcus Salomonsson thanks the Wallenberg Foundation for financial support of his research.

The controversy was long believed to have been finally settled after an exchange between Wynne-Edwards (1962) and Maynard Smith (1964). The exchange was ignited by Wynne-Edwards, who argued in favor of group selection. His argumentation was informal and based on examples. In response, Maynard Smith argued that Wynne-Edwards' examples were explicable without reference to group selection, and went on to formulate what a more precise model of group selection might look like. Based on this model sketch, called the *haystack model*, Maynard Smith dismissed group selection.

In the haystack model, groups are randomly reshuffled at given time intervals. Between each such reshuffle, a one-shot prisoners' dilemma game is played recurrently in every group. A crucial assumption is that the population state in every group converges to a limit state before groups are reshuffled. The process is thus *adiabatic*: individual selection within groups is an order of magnitude faster than group selection. This feature of the model implies that all cooperators in mixed groups become extinct before it is time to reshuffle the groups. Only cooperators in groups that consist exclusively of cooperators survive. The fact that such groups must be pure led Maynard Smith to conclude that circumstances for group selection to be effective were so special that group selection was unlikely to play an important role.

This model was viewed as a sounding rejection of the concept. Consequently, after Maynard-Smith's and Wynne-Edwards' exchange, and after a passionate criticism of the concept by Williams (1966), the group selection idea all but disappeared from the evolutionary literature.[3] When it was mentioned, it was rather as a cautionary tale of how natural selection does *not* work. In later years, however, group selection has had a vivid revival. The literature has in fact become much too large to be fairly treated here. Surveys of the group selection literature are given in Bergstrom (2002) and Wilson and Sober (1994), and recent contributions, with extensive discussions, are given in Kerr and Godfrey-Smith (2002) and Henrich (2004).

The aim of this study is not to provide arguments for or against group selection, but instead to suggest a parsimonious, operational and simple population selection model that allows for group selection forces without the adiabatic assumption of the haystack model. In a nutshell, our model is as follows. A large population of individuals are randomly matched into groups of fixed size. The interaction in each group takes the form of a finite symmetric game. The game can be simple or complex, and may consist of one or many stages — as, for example, in finitely repeated games. All individuals use pure strategies. The play of the game in a group results in material payoffs to all group members. Each individual breeds true, and the expected number of surviving offspring depends on the individual's own material payoff, but may also

[3] Wilson (1983) gives a more detailed description of this period, from a proponent's point of view.

depend on other group members' material payoffs. This dependence can be positive, as under cooperative breeding of offspring—where group members to some extent help each other in rearing and guarding the group's offspring—or negative—as under group members' competition for mates, food or territory for own offspring (for a recent analysis of cooperative breeding, see Pen and Weissing (2000)). It seems to us that such "reproductive externalities" are the rule rather than the exception for many species, including our own.

As a canonical numerical example, we will consider the case when the expected number of surviving offspring is proportional to own material payoff multiplied by the group's material payoff sum. This particular form of reproduction externality arises if the number of offspring is proportional to own material payoff, as in the standard replicator dynamic, and, in addition, the survival of the group's offspring depends on group members taking turns guarding them against some hazard, such as a predator, an infection, or adverse weather condition. We also consider a few other functional forms and derive these as special cases of a reproduction formula that includes both a cooperative and a competitive component. The key assumption in our population dynamic is weaker, however. It rests on the assertion that the expected number of surviving offspring may depend, in part, on other group members' material well-being. We here take this dependence as a primitive, although it, in turn, may have arisen from the long-run interplay between biological, physical and social forces. For instance, as humans turned from hunting and gathering to agriculture, the dependence most likely changed.

An important strand of the group selection literature has used generalizations of the Price equation in order to analyze multi-level selection. An example is Henrich (2004), mentioned above. A nice treatment of the Price equation is also given in Frank (1998).[4] The Price equation is a convenient tool to show that there must be differences between groups for group selection to have an effect. In our model, this is by construction always the case; since groups are formed by random matching, differences between groups arise stochastically. We take the population to be very large (technically a continuum), while groups are of a fixed finite size. The group size is treated as an exogenous parameter, while the population shares of individuals using the different pure strategies are endogenous state variables. Like in the haystack model, there is no direct competition between groups in our model. Instead, groups are compared indirectly, by way of their members' reproductive success, in comparison with the population aggregate. At an abstract level, this is similar to how perfect competition

[4] Other examples are Okasha (2004), who discusses three different formulations of the Price equation; and Roze and Michod (2001), who stress the importance of differences between groups. Like we, Roze and Michod consider random group formation.

is modelled in economics: firms' do not compete directly with each other, but their profits depend on aggregate supply and demand, aggregates that depend on all firms' behaviors but that no individual firm can influence.

We show that the deterministic flow approximation of our population process—its mean-field equation—is identical with the Taylor and Jonker (1978) replicator dynamic for a derived game, the payoffs of which are functions of the material payoffs in the underlying game. Relying on established results for the standard replicator dynamic, predictions for long-run population states in our selection dynamic can thus readily be made. In particular, if the dependence of reproductive success on other group members' material payoffs is sufficiently strong, "altruistic" and/or "spiteful" behaviors among group members may emerge. The standard replicator dynamic is the special case when the reproductive externalities are absent.

We illustrate the implications of our approach by way of a number of examples, including prisoners' dilemma games, coordination games, hawk-dove games, and a prisoners' dilemma with the possibility for a cooperator to afterwards punish a defector. For each of these games we provide conditions under which the long-run outcomes in our selection dynamic are distinct from, or identical with, those in the standard replicator dynamic. As expected, the effect of positive reproduction externalities, such as cooperative breeding, is promotion of "altruistic" behaviors that benefit the common good. However, in some games and for weak forms of cooperative breeding, the long-run effect is nil—the outcome is identical with that under the standard replicator dynamic. Conversely, if the competitive element in reproduction is strong enough, then selection will favor certain "spiteful" behaviors. In particular, so-called altruistic punishment (Fehr and Gächter, 2002) is promoted both by reproductive cooperation and competition, but in qualitatively distinct ways.

Utility theory in economics is based upon a revealed-preference principle; human behavior is interpreted as the result of rational choice according to some underlying binary preference relation over outcomes, or, more generally, lotteries over outcomes. If choice behaviors meet certain regularity conditions with respect to variations of the set of alternatives, there exists a *utility function* for the decision-maker such that his or her behavior is consistent with the maximization of the expected value of that function. Such a mathematical representation allows for powerful analysis and prediction of how behaviors adapt to new environments—a pillar upon which most economic analyses rest.

We argue that the revealed-preference argument can be brought one step further, allowing for the interpretation of stable long-run behaviors in the present selection dynamic as choices made by rational individuals with utility functions given by the payoffs of the derived game. More precisely, irrespective of whether or not the population state converges over time, strategies that are strictly dominated in the derived game will be wiped out in the long run.[5] It is then as if individuals in the (fluctuating) long-run population were rational and this rationality were common knowledge among them.[6] Moreover, if the population state converges, then the limit population state constitutes a symmetric Nash equilibrium of the derived game.[7] Hence, individuals then behave as if they were rational and had consistent expectations about each others' behaviors. Since the payoffs in the derived game in general depend on all players' material payoffs, the so revealed preferences are "other-regarding" or "social" — combining a concern for one's own material payoff with some concern for the material payoffs to others. In this sense, the present model provides an evolutionary explanation of the combined hypothesis of game theoretic rationality and social preferences — a common hypothesis in much "behavioral" and experimental game theory.

The rest of the chapter is organized as follows. The model is formalized in section 2 and applied to examples in section 3. Section 4 discusses briefly the evolutionary support for rewards and punishments. Implications for "as if" rationality and social preferences are discussed in section 5. Related literature is discussed in section 6 and section 7 concludes.

2. Model

Consider a finite and symmetric two-player game G with pure strategy set $S = \{1, 2, ..., m\}$ and payoff matrix $\Pi = (\pi_{hk})$, where π_{hk} is the material payoff to pure strategy $h \in S$ when played against pure strategy $k \in S$. Let $u(x, y)$ denote the expected material payoff to mixed strategy $x \in \Delta(S)$ when played against mixed strategy $y \in \Delta(S)$:

$$u(x, y) = \sum_{h \in S} \sum_{k \in S} x_h \pi_{hk} y_k \tag{2.1}$$

Suppose this game is played recurrently at times $t = 0, \Delta, 2\Delta, ...$ in randomly matched groups of size $n = 2$, drawn from a finite population in which every individual is "programmed" to play a certain pure strategy. Let $N(t)$ be the population size at

[5] A pure strategy is *strictly dominated* for a player if there exists another (pure or mixed) strategy that always gives a higher payoff to the player. A pure strategy is *iteratively strictly dominated* if it can be eliminated by a finite number of rounds of elimination of strictly dominated strategies from the players' pure strategy sets.

[6] An event is *common knowledge* (Lewis (1969), Aumann (1976)) in a group of individuals if it is known by all group members, if this knowledge is known by all group members, etc. *ad infinitum.*

[7] A *Nash equilibrium* is a strategy combination such that each player's strategy is optimal against the others.

time t, and for each pure strategy $h \in S$, let $N_h(t)$ be the number of "h-strategists" in the current population. For each individual, all matches with others are equally likely.

Every group plays the game G once, and each individual breeds true; all offspring inherit their single parent's pure strategy. The expected number of surviving offspring to a h-strategist in a group where the other member plays $k \in S$ is $\Delta \cdot \phi(\pi_{hk}, \pi_{kh})$, where $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}_+$. Hence, $\phi(\pi_{hk}, \pi_{kh})$ is the *fitness* obtained by using pure strategy h against pure strategy k. At the end of each period, a fixed fraction $\Delta \cdot \theta \geq 0$ of all old individuals die, where $\theta \geq 0$ is the death rate (this rate turns out to play no role), and all remaining individuals, from all groups, are brought together.

2.1. The induced selection dynamic. The mean-field equation for the induced stochastic population process can be derived as follows. Assume that the population is non-extinct at some time $t = 0, \Delta, 2\Delta, ...$ For every pure strategy $h \in S$ in the game, let $x_h(t)$ denote the population share of h-strategists: $x_h(t) = N_h(t)/N(t)$. This is the fraction of the population who use pure strategy h when randomly paired to play the game in question. Hence, the probability that a h-strategist will be matched with a k-strategist is $N_k(t)/[N(t)-1]$ if k is another strategy than h, while it is $[N_k(t)-1]/[N(t)-1]$ if $k = h$. Hence, for each pure strategy h, and for any even number $N(t)$ of individuals in the population, the expected number of h-strategists in the next period is

$$\mathbf{E}[N_h(t+\Delta) \mid N_1(t), ..., N_m(t)] =$$
$$= \left(1 + \Delta \sum_{k \in S} \left[\frac{N_k(t) - \delta_{hk}}{N(t)-1} \phi(\pi_{hk}, \pi_{kh})\right] - \Delta\theta\right) N_h(t), \tag{2.2}$$

where δ_{hk} is Kronecker's delta.[8] In other words, the expected number of new h-strategists is the expected fitness of pure strategy h in the current population state multiplied by $\Delta N_h(t)$. The former is obtained by adding up matching probabilities, multiplied with the corresponding fitness, for all pure strategies. The fraction of surviving old individuals is $1 - \Delta\theta$.

For $N(t)$ large, the probability of being matched with a k-strategist is approximately $x_k(t)$. By moving one term from the right-hand side to the left-hand side of equation (2.2), and dividing both sides by $\Delta > 0$, we obtain

$$\frac{\mathbf{E}[N_h(t+\Delta) \mid N_1(t), ..., N_m(t)] - N_h(t)}{\Delta} \approx \left[\sum_{k \in S} x_k(t) \phi(\pi_{hk}, \pi_{kh}) - \theta\right] N_h(t). \tag{2.3}$$

[8] That is, $\delta_{hk} = 1$ if $h = k$, otherwise $\delta_{hk} = 0$. For $N(t)$ odd, the denominator is $N(t) - 2$ instead of $N(t) - 1$.

In the limit as the time period shrinks to zero, $\Delta \to 0$, the left-hand side becomes the time derivative, $\dot{N}_h(t)$, of the number of h-strategists in the population.[9] Using the identity $N(t)\,x_h(t) \equiv N_h(t)$, and dropping the time argument (for the sake of brevity), we obtain the following deterministic approximation of the evolution of population shares:[10]

$$\dot{x}_h = \left[\hat{u}\left(e^h, x\right) - \hat{u}(x, x)\right] x_h, \tag{2.4}$$

where e^h is the unit vector in direction h ($e^h_k = 0$ for all coordinates $k \neq h$ and $e^h_h = 1$) and

$$\hat{u}(x, y) = \sum_{h \in S} \sum_{k \in S} x_h \phi(\pi_{hk}, \pi_{kh})\, y_k. \tag{2.5}$$

The equations (2.4), one for each pure strategy $h \in S$, are the so-called *mean-field equations* for the stochastic population dynamic, and the function $\hat{u}$, defined in equation (2.5), is the *derived payoff function,* generated from the pure-strategy payoff matrix $\hat{\Pi} = (\hat{\pi}_{hk})$ defined by

$$\hat{\pi}_{hk} = \phi(\pi_{hk}, \pi_{kh}). \tag{2.6}$$

The selection dynamic (2.4) is thus nothing but the Taylor and Jonker (1978) replicator dynamic for the derived game. In the special case when $\phi(\pi_{hk}, \pi_{kh}) \equiv \pi_{hk}$, the derived game is identical with the original game and (2.4) is the standard replicator dynamic. This model easily generalizes to a population playing any finite and symmetric n-player game, for any $n \geq 2$; a group is simply defined as a random match between n individuals who play the game in question.

2.2. A class of fitness functions. The present model's predictions clearly depend on the fitness function ϕ. To identify this function is ultimately an empirical question, but we here derive such a function for a stylized reproduction environment that include the following three elements: competition, cooperation and fertility.

Suppose that, keeping other group members' material resources fixed, the expected number of offspring to each group member i is proportional to i's material resources. However, due to competition for food, territory and perhaps also for mates, the proportionality factor may be decreasing in other group members' material resources. More specifically, in the case $n = 2$, if the material resources available to the two group

[9] This limit is taken just for the sake of analytical convenience, in order to obtain a system of ordinary differential equations rather than a system of difference equations. For any fixed period length $\Delta > 0$, equation (2.3) defines a discrete-time version of our selection dynamic, with similar qualitative properties.

[10] To see this, note that the chain-rule applied to the mentioned identity gives $\dot{N}x_h + N\dot{x}_h = \dot{N}_h$, where $\dot{N}_h = \left[\sum_{k \in S} x_k \phi(\pi_{hk}, \pi_{kh}) - \theta\right] x_h N$ by (2.3), and where $\sum_{h \in S} x_h = 1$ gives $\dot{N} = \sum_h \dot{N}_h = \left[\sum_{h,k \in S} x_k \phi(\pi_{hk}, \pi_{kh})\, x_h - \theta\right] N$.

members are $v_1 > 0$ and $v_2 > 0$, let the expected number of offspring to individual i be given by

$$\left[\alpha + \frac{\beta f(v_i)}{f(v_1) + f(v_2)}\right] v_i,$$

for some $\alpha, \beta \geq 0$ and some non-decreasing function f.[11] As a consequence, the expected number of offspring decreases when the other group member's material resources increase. The parameters α and β reflects the intensity of this reproductive competition, with $\alpha > 0$ and $\beta = 0$ representing the special case of no competition; then the expected number of offspring is proportional to own material resources.

Suppose, moreover, that the group's offspring is exposed to some collective hazard, such as a predator or bad weather. The survival of the offspring may require one or both group members' defence. If only one group member is required, and that group member has material resources v, then the probability that his or her own offspring will survive is $p(v)$ and the survival probability for the other group member's offspring is $\gamma p(v)$, where p is a non-decreasing function and, plausibly, $\gamma \in [0, 1]$. Hence, if an offspring's parent happens to be the guardian, then its survival probability is not smaller than if the the other group member were the guardian. The parameter γ reflects the degree to which breeding is cooperative, with $\gamma = 0$ representing the case of purely individualistic breeding and $\gamma = 1$ the polar case of fully cooperative breeding. If both group members are required for the defence of the group's offspring, and they have material resources v_i and v_j, then the probability of successful protection is taken to be $p(v_i)\, p(v_j)$. In other words, both group members need to be successful and their successes are statistically independent events.

Under statistical independence between fertility and breeding, the expected number of surviving offspring to each group member $i = 1, 2$ is (for $j \neq i$):

$$\phi(v_i, v_j) = [\sigma p(v_i) + \sigma\gamma p(v_j) + (1 - 2\sigma)\, p(v_i)\, p(v_j)] \cdot \left[\alpha + \frac{\beta f(v_i)}{f(v_i) + f(v_j)}\right] v_i, \quad (2.7)$$

where $\sigma \in [0, 1/2]$ is the probability that the hazard requires a single guardian (with equal probability for both).

In sum: reproductive competition and cooperation within groups gives rise to (positive or negative) reproductive externalities, in the sense that the material resources of other group members may influence (increase or decrease) each individual's expected

[11] This formula is inspired by the following observation. Suppose that n competitors compete for a prize, where the competitor who makes the best performance wins, and where the perfomance of each competitor i is $x_i = \ln f(v_i) + \varepsilon_i$, where ε_i are i.i.d. Gumbel distributed random variables. Then the probability that i wins is $f(v_i) / \left(\sum_j f(v_j)\right)$.

number of surviving offspring. Equation (2.7) defines ϕ as a generalized fitness function that allows for such externalities. It seems plausible that a variety of reproductive externalities exist for many species, including *homo sapiens*.

2.3. 2×2 **games.** Applied to a symmetric 2×2 game, our approach gives

$$\hat{\Pi} = \begin{pmatrix} \phi(\pi_{11}, \pi_{11}) & \phi(\pi_{12}, \pi_{21}) \\ \phi(\pi_{21}, \pi_{12}) & \phi(\pi_{22}, \pi_{22}) \end{pmatrix}. \tag{2.8}$$

The best-reply correspondence, weak and strict dominance, risk dominance, and the replicator dynamic are all unaffected by the addition or subtraction of a constant to a column of the payoff matrix (see e.g. Weibull (1995)).[12] Hence, the derived game is equivalent in these respects with the *normalized* derived game

$$\tilde{\Pi} = \begin{pmatrix} \phi(\pi_{11}, \pi_{11}) - \phi(\pi_{21}, \pi_{12}) & 0 \\ 0 & \phi(\pi_{22}, \pi_{22}) - \phi(\pi_{12}, \pi_{21}) \end{pmatrix}. \tag{2.9}$$

This game, and hence also the derived game, is a (strict) *coordination (CO) game* if both diagonal entries are positive, a (strict) *hawk-dove (HD) game* if both diagonal entries are negative, and a (strictly) *dominance-solvable (DS) game* if the diagonal entries have opposite signs. In the case of a CO-game, the pure-strategy pair $(1,1)$ strictly *risk dominates* the pure-strategy pair $(2,2)$ if and only if the first diagonal entry, $\tilde{\pi}_{11}$, exceeds the second, $\tilde{\pi}_{22}$. In case of a DS-game, the derived game $\hat{\Pi}$, but not necessarily the normalized derived game $\tilde{\Pi}$, is a *prisoners' dilemma (PD) game* if and only if the dominant pure strategy earns less against itself than the other pure strategy earns against itself, in terms of derived payoffs. In the opposite case, we call the derived game $\hat{\Pi}$ an *efficient dominance-solvable (ED) game*.

The replicator dynamic in a generic symmetric 2×2 game converges from all initial states. Moreover, the limit point is a best reply to itself (the strategy of a symmetric Nash equilibrium) if the initial state is interior. Strict CO-games have two attractors — the whole population playing one of the two pure strategies — and their basins of attraction are separated by the unique (but unstable) mixed Nash equilibrium strategy. Strict HD-games have one attractor, the population mix defined by the unique mixed Nash equilibrium strategy. Strict DS-games, finally, also have a unique attractor — the whole population playing the dominant pure strategy — irrespective of whether this is socially efficient or not.

[12] The *best-reply correspondence* maps strategy profiles to optimal strategies. A strategy *strictly dominates* another if it always gives a higher payoff, and it *weakly dominates* another strategy if it never gives a lower payoff and sometimes gives a higher payoff. The notion of *risk dominance*, due to Harsanyi and Selten (1988), captures the strategic risk associated with strict equilibria in coordination games.

3. Examples

3.1. Prisoners' dilemmas, coordination and hawk-dove games. Consider symmetric 2×2-games with material payoffs

$$\Pi = \begin{pmatrix} 2 & a \\ b & 1 \end{pmatrix}, \tag{3.1}$$

for arbitrary constants a and b. Such a game is a HD-game when $a > 1$ and $b > 2$, a ED-game when $a > 1$ and $b < 2$, a CO-game when $a < 1$ and $b < 2$, and a PD-game when $a < 1$ and $b > 2$, as discussed in section 2.2 above. These conditions cut the (a, b)-plane into four regions, oriented clockwise around the point $(1, 2)$, see the orthogonal straight cross in Figure 1 below.

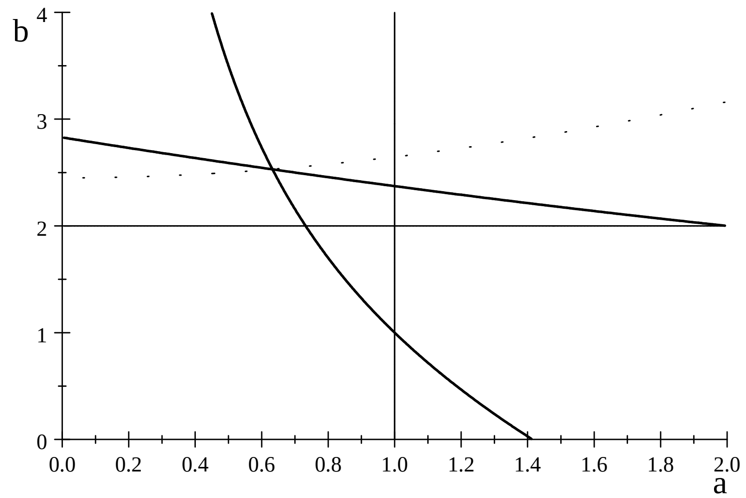

Figure 1: Parameter combinations (a, b) and the nature of the two games Π (straight lines) and $\hat{\Pi}$ (curves), for the fitness function in (3.2).

Suppose, first, that there is no reproductive competition but cooperative breeding, such that the probability of successful guarding of offspring is proportional to the guarding individual's material payoff, and such that fertility is proportional to own material payoff. Assume further that only one group member is required for the defence of offspring and that the material resources available to each group member consists entirely of his or her material payoff from playing the game. Neglecting constant factors, this gives

$$\phi(v_i, v_j) = (v_i + v_j)\, v_i. \tag{3.2}$$

The resulting derived game is

$$\hat{\Pi} = \begin{pmatrix} 8 & a(a+b) \\ b(a+b) & 2 \end{pmatrix}. \tag{3.3}$$

The two downward sloping curves in Figure 1 divide the (a, b)-plane into the four regions that determine the nature of the derived game. The regions are oriented in the same way as for the original game: HD-games being located north-east, CO-games south-west, and dominance solvable games south-east (ED-games) and north-west (PD-games) of the two curves' intersection. We see, in particular, that if the game Π, defined in terms of material payoffs, is a PD game, then the derived game $\hat{\Pi}$ can be any one of the four generic game types. Suppose, for example, that $a = 0.8$ and $b = 3$. In this case, the long-run population state is a certain interior state — the mixed strategy in the derived HD-game — for all interior initial states. As another example, suppose $a = 0.6$ and $b = 2.4$. Then the derived game is a CO-game. Hence, the long-run population state depends on the initial state. In particular, if there are sufficiently many cooperators in the initial population state, then defectors will be asymptotically wiped out from the population — although the game is a PD-game in terms of material payoffs.

In the presence of perpetual random mutations, as modelled in Kandori, Mailath, and Rob (1993), the population process becomes ergodic, and, for generic values of a and b, its invariant distribution places virtually all probability mass on one of the symmetric Nash equilibria of the derived game. In particular, $a = 0.6$ and $b = 2.4$, it places virtually unit probability on the population state in which all individuals play pure strategy 1 ("C" in the original PD game). This follows from the observation that the (1,1) equilibrium risk dominates (2,2) in the derived game for these parameter values.[13] The strict equilibrium (C,C) is risk dominant for all parameter pairs below the upward-sloping curve in Figure 1.

Secondly, suppose instead that the cooperative breeding situation is such that the hazard requires both group members' simultaneous defence, that is, that $\sigma = 0$, but otherwise things are as in the preceding example. We then obtain

$$\phi(v_i, v_j) = v_j v_i^2. \tag{3.4}$$

The derived game becomes

$$\hat{\Pi} = \begin{pmatrix} 8 & a^2 b \\ ab^2 & 1 \end{pmatrix}, \tag{3.5}$$

resulting in a different, but qualitatively similar, division of the game parameter space, see Figure 2 below.

[13] In the derived game $\hat{\Pi}$ in (3.3), the strategy profile (1,1) strictly risk dominates (2,2) if and only if $b^2 < a^2 + 6$, an inequality that holds for $a = 0.6$ and $b = 2.4$.

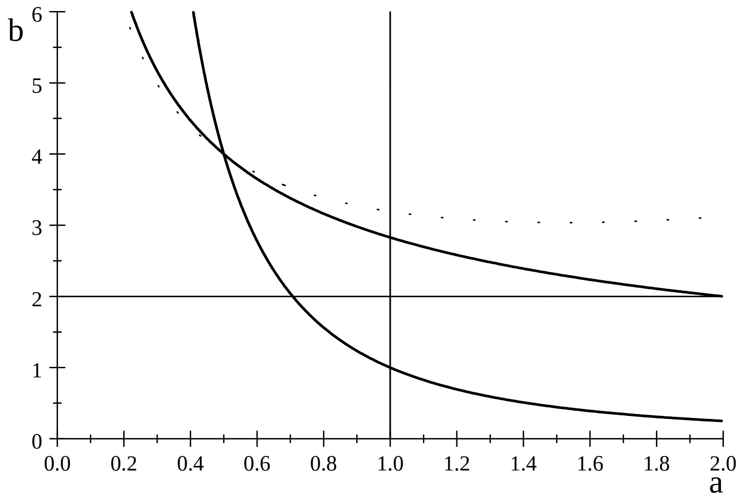

Figure 2: Parameter combinations (a, b) and the nature of the two games Π (straight lines) and $\hat{\Pi}$ (curves), for the fitness function in (3.4).

Thirdly, suppose that we are in the polar case of reproductive competition and individualistic breeding, such that reproductive success is proportional to one's share of the group's total material payoff ($\alpha > 0$, $\beta = 0$ and f linear). Again neglecting constant factors, we obtain

$$\phi(v_i, v_j) = \frac{v_i}{v_i + v_j} v_i, \tag{3.6}$$

a fitness function that is increasing in own material payoff but decreasing in the other group member's material payoff. The derived game is then

$$\hat{\Pi} = \begin{pmatrix} 1 & a^2/(a+b) \\ b^2/(a+b) & 1/2 \end{pmatrix}, \tag{3.7}$$

inducing yet another, but still qualitatively similar, division of the game parameter space, see Figure 3 below. We note that all PD-games in material payoffs remain PD games also in the derived game. Hence, this selection dynamic leads the population, from any interior population state, toward the state where all individuals play D, in all such games. Moreover, this is true for all nearby games in parameter space.[14]

[14] More exactly, for all games with parameters $a < 1 + \varepsilon$ and $b > 2 - \varepsilon$, for all $\varepsilon > 0$ sufficiently small.

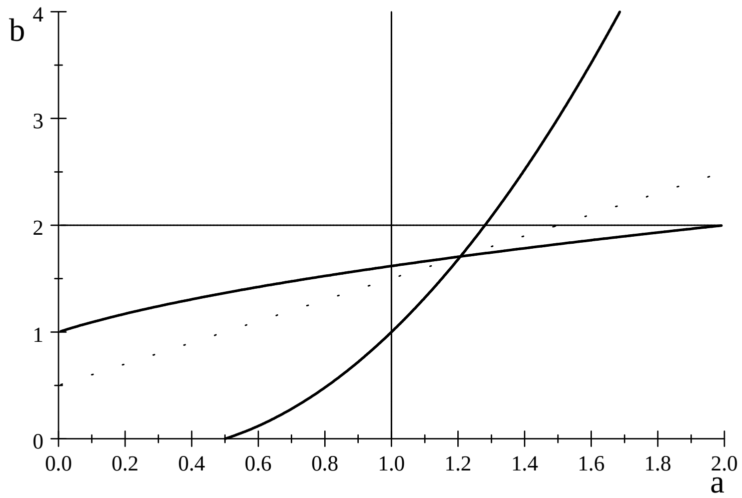

Figure 3: Parameter combinations (a, b) and the nature of the two games Π (straight lines) and $\hat{\Pi}$ (curves), for the fitness function in (3.6).

3.2. Punishing defectors and rewarding cooperators. There is experimental evidence, see Fehr and Gächter (2002), that human subjects punish defectors in public-goods provision interactions, even when such punishment is costly to the punisher. The threat of such punishment enhances cooperation and hence welfare in interacting groups of human subjects. However, to implement such punishment violates rationality, as defined in terms of material payoffs. Can "altruistic" punishing behaviors nevertheless be explained by the present model?

Figure 4 below shows the extensive-form representation of a symmetric two-player public-goods provision game that allows cooperators to punish defectors. The first stage of this game is a simultaneous-move prisoners' dilemma, where each player chooses C or D, with material payoffs according to (3.1), for $a < 1$ and $b > 2$. In the second stage, a player who cooperated in stage one has the option to punish defection by the other player. The cost of punishing is $c > 0$ and the effect of punishment is a reduction of the other player's payoff by $d > 0$. The unique subgame perfect equilibrium of this extensive-form game is not to punish— since punishment reduces the punisher's material payoff—and hence for both players to defect in the first stage.[15]

[15] A Nash equilibrium is *subgame perfect* (Selten (1965)) in an extensive-form game if it prescribes a Nash equilibrium in all subgames.

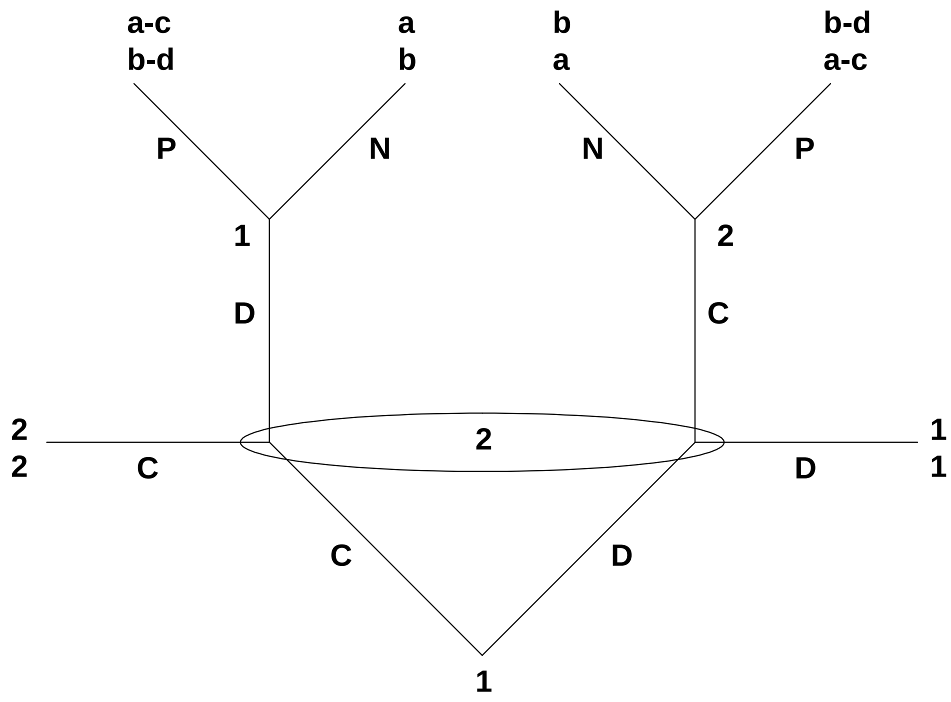

Figure 4: A two-stage prisoner's dilemma game with punishment option.

In the normal form of the game, each player has four pure strategies 1=CN, 2=CP, 3=DN and 4=DP at his or her disposal, and the material payoff matrix of this symmetric game is

$$\Pi = \begin{pmatrix} 2 & 2 & a & a \\ 2 & 2 & a-c & a-c \\ b & b-d & 1 & 1 \\ b & b-d & 1 & 1 \end{pmatrix}. \tag{3.8}$$

Not surprisingly, the pure strategy CN weakly dominates strategy CP; CN gives the same material payoff as CP if the other plays C and a lower payoff if the other plays D). However, if the punishment is not too harsh, $b-d>2$, then each of the two (behaviorally equivalent) strategies DN and DP strictly dominates CN; they always result in a higher material payoff. In such cases, the game has a unique Nash equilibrium component, where both players play arbitrary mixes between DN and DP, and this component attracts all interior solution orbits of the replicator dynamic.[16] However, if punishment is harsh, $b-d<2$, then there exists a "cooperative" component of symmetric Nash equilibria, namely all mixed strategies $pCN+(1-p)\,CP$ with

$$p \le 1-(b-2)/d. \tag{3.9}$$

A fairly large set of initial population states lead asymptotically to this cooperative Nash equilibrium component, see Figure 5 below, computed for $a=0.5$, $b=3$, $c=0.3$

[16] A Nash equilibrium *component* is a closed and connected set of Nash equilibria.

and $d = 2.7$. The diagram shows replicator solution trajectories for the game with payoff matrix Π, in the unit simplex with vertices CP (cooperate and punish defectors), CP (cooperate and do not punish defectors) and D (defect, and punish or do not punish defectors).[17] The "cooperative" Nash equilibrium component is marked with a thicker line; this line segment consists of those boundary points, between the vertices CN and CP, where $pCN + (1-p)\,CP$ for $p \leq \bar{p} \approx 0.63$. Each of these points, except the end-point, $\bar{p}CN + (1-\bar{p})\,CP$, are Lyapunov stable in the selection dynamic; that is, small pushes do not lead the population far away. The reason for the stability of these population states is that near the cooperative component D is rarely played and hence punishment is not very costly; CP gives almost the same expected payoff as CN. Hence, defectors "learn" that defections are likely to be followed by punishments (approximately with probability $1-p$), and punishers "learn" that CP is somewhat more costly than CN. These two adaptations occur at comparable time rates in the replicator dynamic, and hence the population state moves back toward the cooperative equilibrium component, except when the population state is close to its end-point. Similar dynamic phenomena have been observed in Binmore and Samuelson (1999), in the context of ultimatum bargaining; and in Sethi and Somanathan (1996) for the tragedy of the commons.

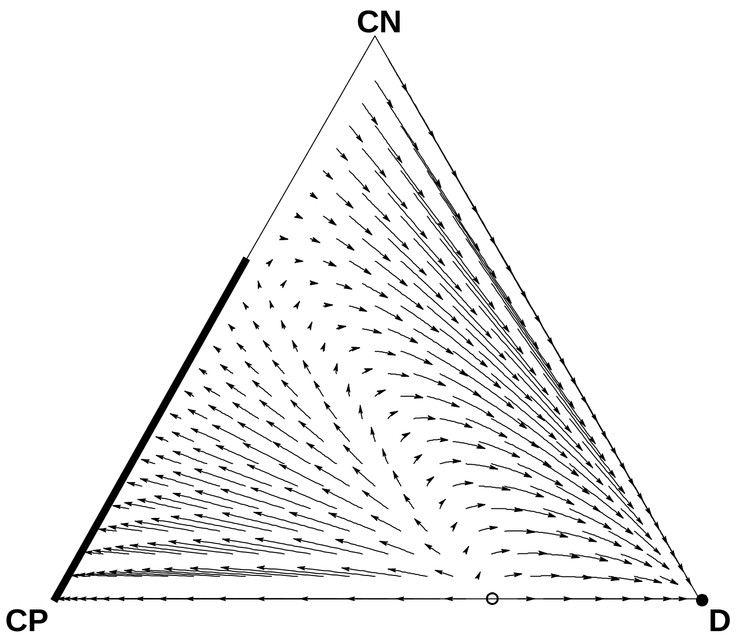

Figure 5: Solutions to the replicator dynamic in the original game Π.

[17] We thank Bill Sandholm for providing the software for this and the next two diagrams. The software is available at http://www.ssc.wisc.edu/~whs/.

We now turn to the present selection dynamic. The derived payoff matrix is

$$\hat{\Pi} = \begin{pmatrix} \phi(2,2) & \phi(2,2) & \phi(a,b) & \phi(a,b) \\ \phi(2,2) & \phi(2,2) & \phi(a-c,b-d) & \phi(a-c,b-d) \\ \phi(b,a) & \phi(b-d,a-c) & \phi(1,1) & \phi(1,1) \\ \phi(b,a) & \phi(b-d,a-c) & \phi(1,1) & \phi(1,1) \end{pmatrix}. \quad (3.10)$$

If the fitness function ϕ is strictly increasing in both arguments, that is, a higher material payoff to any one of the group members increases the expected number of surviving offspring to both members, then strategy CP is weakly dominated by strategy CN also in the derived game. This is, for example, the case under fitness specifications (3.2) and (3.4). For an arbitrary fitness function ϕ, all mixed strategies $pCN + (1-p)\,CP$ with

$$p \leq \frac{\phi(2,2) - \phi(b-d,a-c)}{\phi(b,a) - \phi(b-d,a-c)} \quad (3.11)$$

are best replies to themselves in the derived game. Hence, if

$$\phi(b-d,a-c) \leq \phi(2,2),$$

then there is a "cooperative" symmetric Nash equilibrium component also in the derived game. Indeed, under cooperative breeding and not too strong reproductive competition, we would expect this component to be larger, and attract a larger set of initial population states, than in the game defined in terms of material payoffs. It is not difficult to confirm this conjecture under bilinear fitness (3.2).

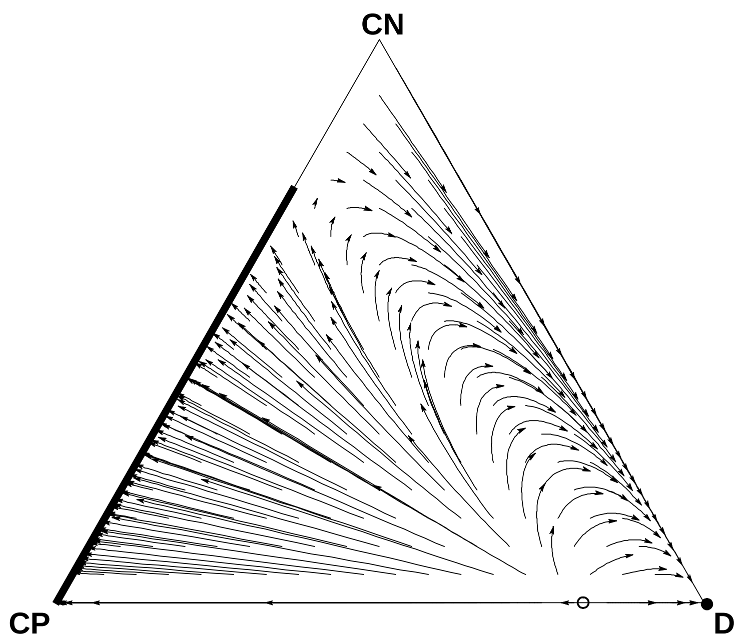

Figure 6: Solutions to the selection dynamic (2.4), for the fitness function in (3.2).

Figure 6 shows solution trajectories for the derived game $\hat{\Pi}$ based on this fitness specification, for the same material payoffs as used in Figure 5. As expected, the end-point of the cooperative equilibrium component has moved up, from about 0.63 to about 0.75, and the set of initial states leading towards the cooperative component has increased. However, in the presence of perpetual random mutations, the unique outcome in the "ultra-long run" is still that everyone defects.

In order for the cooperative component to become an attractor, and hence an outcome in the ultra-long run (see Kandori, Mailath, and Rob (1993) and Benaïm and Weibull (2003)), there needs to be a sufficiently strong competitive component in the reproduction process. Figure 7 below shows solution trajectories for the derived game when the fitness specification (3.2) is replaced by that in (3.6). The cooperative equilibrium component has become an attractor, absorbing all population states in a large basin of attraction. In other words: it takes reproductive competition in order for "altruistic punishment" (Fehr and Gächter, 2002) to become asymptotically stable.

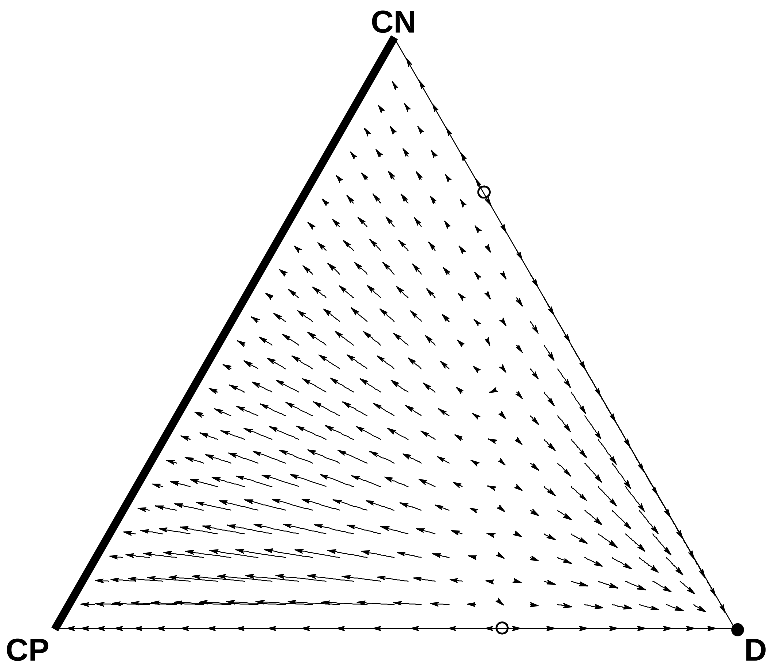

Figure 7: Solutions to the selection dynamic (2.4), for the fitness function in (3.6).

4. The evolutionary logic of rewards and punishments

We here make some general remarks about the rationality of, and evolutionary support for, punishments and rewards. Consider, thus, a behavior strategy profile in a finite extensive-form game and an information set on the path of this profile, that is, an information set that is reached with positive probability when the profile is played. A behavior strategy is *sequentially rational* (Kreps and Wilson, 1982) at such an information set if its conditionally expected payoff, conditioned on the induced probabilities at its nodes, cannot be exceeded by any other behavior strategy when used from this information set on. If the payoffs in the game tree are the derived payoffs as defined here, then Lyapunov stability in the present selection dynamic implies sequential rationality at all information sets that are reached with positive probability by play.[18]

Consider now a decision node in a finite extensive-form game where the player has perfect information about what others have done before his or her move (thus, a singleton information set) and where each move at the node is immediately followed by a terminal node in the game tree. Suppose, that the player i in question has the option to reward another player j. Let v_i and v_j be the two player's material payoffs if i chooses not to reward j, and let the payoffs be $v_i - c$ and $v_j + r$ if i chooses to reward, where $c > 0$ is the cost of rewarding and r is the reward. As an alternative scenario, suppose that i instead has the option to punish j. We also cover this case, by setting $r < 0$.

In terms of material payoffs, it is never sequentially rational to reward/punish, since this is costly. However, in terms of derived payoffs, it is sequentially rational to do so if and only if

$$\phi(v_i - c, v_j + r) \geq \phi(v_i, v_j). \tag{4.1}$$

In particular, if reproductive cooperation "dominates" reproductive competition, so that ϕ is increasing in its second argument, then it is *never* sequentially rational to punish but *may* be sequentially rational to reward, while the converse holds if reproductive competition dominates reproductive cooperation.

This asymmetry can be illustrated by means of the parametric specification in section 2.2. Figure 8 below shows inequality (4.1) for the three fitness functions in (3.2), (3.4) and (3.6), applied to a situation where both individuals initially have the same material payoff: $v_1 = v_2 = 1$. The highest curve represents the case (3.2) and the second highest curve the case (3.4). To reward/punish is sequentially rational for all parameter pairs (c, r) in the region above (or on) the curve in question, and only there. Hence, punishments are never sequentially rational, and rewards are sequentially rational only if the reward is sufficiently large in comparison with the cost. The lowest curve represents the case (3.6) of reproductive competition and individualistic breeding. To reward/punish is sequentially rational in the region below that curve, and only there. Hence, rewards are never sequentially rational under reproductive competition,

[18] This follows from the two facts that (a) Lyapunov stability in the replicator dynamic implies Nash equilibrium play, and (b) a behavior strategy profile in a finite extensive form game is a Nash equilibrium if and only if it prescribes sequentially rational play at all information sets on its path, see Damme (1987).

but punishments are sequentially rational if the punishment is sufficiently harsh in comparison with the cost.

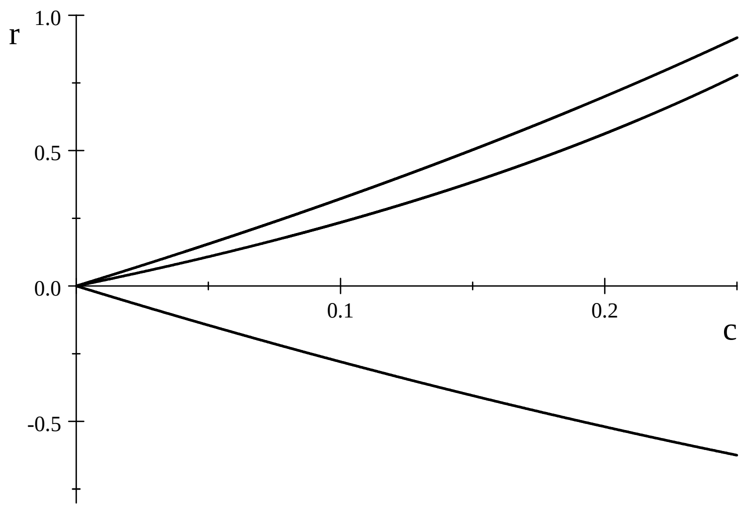

Figure 8: Regions in parameter space where rewarding and punishing behaviors are sequentially rational.

5. Social preferences

The analysis so far has assumed that individuals do not choose a strategy; they are "programmed" to a pure strategy from birth. However, the long run of the present model can be interpreted in terms of rational individual choice and expectations formation as follows. Let the underlying material payoffs be given by the matrix Π and consider the derived payoff matrix $\hat{\Pi}$ as a representation of rational individuals' von Neumann Morgenstern utilities. In other words, we now view the derived payoffs to each player as defining a goal function the expected value of which that player strives to maximize.[19]

It is well-known that a pure strategy in a finite two-player game is strictly dominated if and only if it is not optimal under any probabilistic belief of the other player's strategy choice (Pearce, 1984). It is also known that the replicator dynamic asymptotically wipes out all iteratively strictly dominated pure strategies, from any interior initial population state and in all finite games.[20] Hence, observed play in the present model, after the population has evolved over a long time span from an arbitrary interior initial

[19] More precisely, von Neumann-Morgenstern utilities are real numbers $u(\omega)$ attached to outcomes ω in such a way that the decision maker's choices among lotteries over outcomes are consistent with the maximization of the expected value of the function u.

[20] Akin (1980) showed that all strictly dominated strategies are wiped out in the standard replicator dynamic in all finite and symmetric two-player games. This result was generalized to iteratively strictly dominated strategies in arbitrary finite two-player games, for both the Taylor (1979) and Maynard Smith (1982) versions of the multi-population replicator dynamic by Samuelson and Zhang

state, is consistent with the hypothesis that all individuals are rational and have von Neumann-Morgenstern utilities (2.6), and, moreover, that this is common knowledge in the population.[21]

Let us consider the examples in section 3.1 in this light. Assume $a = 0.8$ and $b = 2.2$. From Figure 1 we deduce that in terms of material payoffs, the game is a prisoners' dilemma with strategy 1 being "cooperate", C. However, in terms of the derived payoffs, strategy C strictly dominates strategy D. The present selection dynamic thus takes the population state from any interior state to the state in which everybody plays C. Hence, to an outside observer who sees this long-run outcome, and who knows the material payoffs Π, this observation is consistent with the hypothesis that all individuals are rational and have von Neumann Morgenstern utilities (2.6) as detailed in (3.2). Such preferences are "social" or "other-regarding:" they depend both on own material payoff and on the other player's material payoff.

It is known that if the replicator dynamic converges to some population state from an interior initial state, then the limit state is a Nash equilibrium.[22] Hence, observation of play after the population has evolved for a long time along some interior convergent solution trajectory is consistent with the hypothesis that all individuals have preferences according to (2.6) and play (approximately) a Nash equilibrium. Consider the example in section 3.2 in this light, for the parameter combination used in Figures 5 and 6. If the population behavior converges to some point in the cooperative equilibrium component, and an observer sees this long-run behavior, then this observation is consistent with the hypothesis that all individuals have von-Neumann-Morgenstern utilities (3.10), and play (close to) one of the Nash equilibria in that component—where few individuals defect and a significant population fraction punishes defectors. Again, it is as if individuals were rational, had social preferences, and consistent expectations about each others' behaviors.

The above observations substantiate the first part of the above claim, namely that the present model can be viewed as a model of rational choice and, sometimes, Nash equilibrium play. In order to substantiate the claim that individuals behave as if they had *social* preferences, it remains to examine more closely the nature of the fitness function ϕ, which in turn depends on the "reproduction technology" of the population

(1992), and to arbitrary finite games and a wider class of single- and multi-population selection dynamics by Hofbauer and Weibull (1996).

[21] If the game payoffs to all players, and their rationality, is common knowledge among the players, then these will not use iteratively strictly dominated strategies.

[22] This was first proved by Nachbar (1990) and later generalized to a wide class of selection dynamics in all finite games by Weibull (1995).

under study. In particular, as this technology changes, for example when humans turn from hunter-gatherer societies to agriculture, so may the induced social preferences.

Assume, first, that the fitness function ϕ is as specified in equation (3.2). The figure below shows a contour map of this function. Hence, these are the *indifference curves* of an individual with such preferences, with own material payoff on the horizontal axis and the other player's material payoff on the vertical. The thin straight lines have slopes plus and minus 45 degrees, respectively.

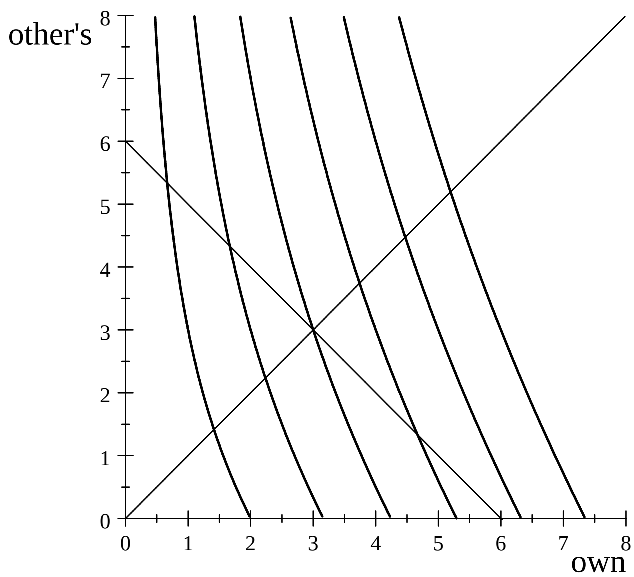

Figure 9: Indifference curves induced by the fitness function in (3.2).

We see that the indifference curves are not the vertical ones of *homo oeconomicus*—the selfish species studied in most of economics. Indeed, an individual with ϕ as his or her utility function prefers the "fair" payoff allocation $(3, 3)$ to one where he/she gets 4 material payoff units and the other individual zero. In this sense, individuals have a certain "preference for fairness." However, as the -45^o line shows: if there is a given material payoff sum to be divided (here 6 units), then each individual prefers to get the whole "pie" for him- or herself. It is as if others' material well-being is of some concern, but less so than one's own, in particular when others are better off (there the indifference curves are steeper).

What about more general fitness functions ϕ? Let us briefly reconsider the class defined in equation (2.7), in the special case of $\gamma = 1$ (fully cooperative breeding), and f and p power functions:

$$\phi(v_i, v_j) = \left[\sigma\left(v_i^\kappa + v_j^\kappa\right) + (1 - 2\sigma) v_i^\kappa v_j^\kappa\right] \cdot \left[\alpha + \frac{\beta v_i^\lambda}{v_i^\lambda + v_j^\lambda}\right] \cdot v_i, \tag{5.1}$$

for $\kappa, \lambda \geq 0$. The first, "cooperative," factor is a *social welfare function*: a symmetric and increasing function of individual payoffs. For $\sigma = 1/2$, this factor is a utilitarian welfare function—the sum of the group members' material payoffs raised to some power. For $\sigma = 0$, it is a version of Nash's bargaining-based welfare function—the product of the group members' material payoffs. The cooperative factor is a constant when $\kappa = 0$; the case of purely individualistic breeding. The second, "competitive," factor is a logistic function of both individuals' material resources, increasing from α to $\alpha + \beta$ as v_i increases from zero towards plus infinity. The larger λ is, the closer the graph of this logistic function is to the graph of the step function that "jumps" from β to $\alpha + \beta$ as v_i moves from below to above v_j. The limit case $\lambda \rightarrow +\infty$ thus represents rank-dependent reproductive success (when $\beta > 0$). This factor is a constant when $\beta = 0$ and/or $\lambda = 0$; the case of no reproductive competition. We obtain the special case studied in equation (3.2) by setting $\beta = 0$, $\kappa = 1$ and $\sigma = 1/2$. Likewise, equation (3.4) obtains when $\beta = 0$, $\kappa = 1$ and $\sigma = 0$, and equation (3.6) when $\alpha = \kappa = 0$ and $\lambda = 1$. In sum: the class of fitness functions ϕ in equation (5.1) generalize the earlier examples and induces social preferences that are non-decreasing in own absolute and relative material payoffs, and in social welfare in the group.

A contour map of such a fitness function is shown in Figure 10 below (for $\kappa = 1$, $\sigma = 0$, $\alpha = 0.2$, $\beta = 0.8$ and $\lambda = 10$). An individual with this function as his or her utility function still prefers the "fair" payoff allocation $(3, 3)$ to one where he/she gets 4 material payoff units and the other zero. Indeed, she prefers $(3, 3)$ over all allocations where the other individual gets nothing (since her offspring would then be defenceless against the hazard). If a material payoff sum were to be divided between the two individuals (here 6 units), then each individual prefers to get approximately two thirds of the "pie" for herself. Moreover, now there is an element of "spite:" at some allocations, in particular when the other individual is better off, individuals prefer that material resources be taken away from the other (because of the reproductive competition). This phenomenon is particularly clear when material resources are not close to zero, as in situations where the group members' material payoffs from playing the game are added to positive material resources that they already posses (a form of background fitness). In such cases, an individual may well prefer an allocation where both individuals get zero material payoffs over an allocation where the other gets a much larger material payoff. Such preferences qualitatively agree with those in Fehr and Schmidt (1999) and Charness and Rabin (2002), and are consistent with the rejections of "unfair" offers that have been observed in ultimatum bargaining experiments (see e.g. Zamir (2001)).

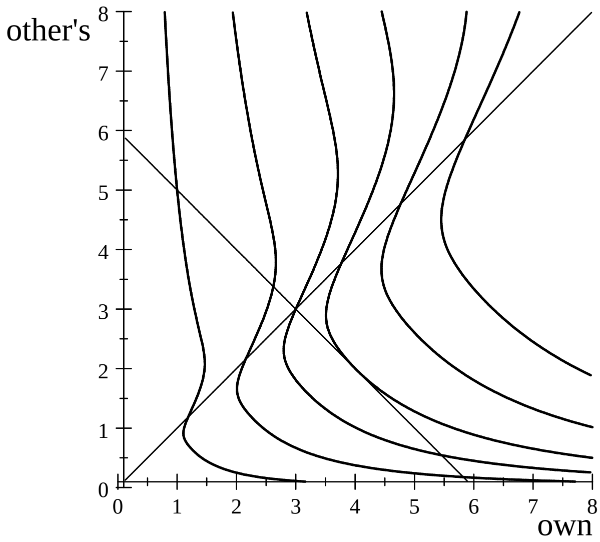

Figure 10: Indifference curves induced by a fitness function of form (5.1).

6. Related literature

As mentioned above, the model in Maynard Smith (1964) is adiabatic, with individual selection working on a faster time scale than group selection. Behaviors within each group thus first converges to a steady state determined by individual selection, before selection among groups take place.[23] By contrast, Kerr and Godfrey-Smith (2002) allow individual and group selection to operate on the same time scale, just as we do here, and discuss individualist and multi-level perspectives on natural selection. In their vocabulary, the present model is an example of the *contextual* approach, where individuals are the bearers of fitness and fitness is sensitive to the context of the individual. They contrast this approach with what they call the *collective* approach, where instead collectives (groups) are fitness-bearing entities in their own right. Their model is not game-theoretic and their analysis is only remotely related to ours.

Another strand of the literature is the so-called *indirect evolutionary* approach, pioneered by Güth and Yaari (1992).[24] In that approach, individuals are randomly drawn from large populations to play a game defined in terms of material payoffs, just as here. However, individuals have different preferences, and, by assumption, play some Nash equilibrium (either of the game defined in terms of the drawn individuals' preferences or in the game defined by the population distribution of preferences). Preferences

[23] A similar effect is obtained in the present model by setting $\phi(\pi_{hk}, \pi_{kh}) = \pi_{hk}$ if $\pi_{hk} > \pi_{kh}$, $= \pi_{hk}$ if $\pi_{hk} = \pi_{kh}$ and $= 0$ otherwise. Such a fitness function is obtained from (5.1) for $\alpha = \kappa = 0$ and $\beta > 0$, in the limit as $\lambda \to +\infty$.

[24] Se also Bester and Güth (1998), Huck and Oechssler (1999), Sethi and Somanathan (2001), Güth and Peleg (2001) and Herold (2004).

that result in higher material payoffs in the corresponding equilibrium are selected for. The indirect evolutionary approach is thus quite distinct from the present—we make no assumption that a Nash equilibrium be played and we do not consider dynamics in preference space. Closest to our approach among those models is that in Herold (2004), who studies preferences for rewarding altruistic behaviors and punishing spiteful and selfish behaviors (c.f. Section 3 above). He shows that such preferences can survive in the indirect evolutionary approach with randomly formed groups, when group members condition their strategy choice on the preference distribution in their own group, an assumption not made in the present approach.

Kuzmics (2003) develops a model of simultaneous individual and group selection for symmetric two-player games. Like here, he considers the mean-field equation in a large population. However, unlike here, the number of groups is finite, so each group becomes large in the mean-field approximation. Groups may thus be rather be thought of as *subpopulations*. Individual selection operates within each subpopulation and group selection is driven by migration. More exactly, the model operates in continuous time by way of a Poisson arrival process of migration-*cum*-imitation opportunities for individuals. When an individual gets such an opportunity, she migrates to another group with a probability that is increasing in that group's current average material payoff. Whether or not she migrated, she thereafter imitates a randomly drawn individual in the chosen subpopulation, with a higher probability to imitate an individual with a higher current material payoff. When applied to CO-games, the long-run prediction is that all individuals play the pure strategy that yields the highest material payoff. This contrasts with the predictions of the current model, where other outcomes are possible. The reason for the tendency towards joint payoff maximization in Kuzmics' model is migration, which, by assumption, is directed towards subpopulations with high average payoffs. When applied to PD-games, the long-run prediction diverges. Individual selection in favor of strategy D is counter-acted by migration to groups where many play C, resulting in perpetual fluctuations in the population state. Again, the prediction differs from that of our model.

In Vega-Redondo (1996), a finite population of individuals are recurrently and randomly matched in pairs to play a prisoners' dilemma. Time is divided into an infinite sequence of discrete periods, and with each such period, every individual is matched with another individual, just as in the present model. Each matched pair forms a group, and there is both individual and group selection at the end of each time period. First, individual selection takes place: each individual switches to the strategy that yielded the highest payoff in the individual's group. Thereafter, a mutation may take place: with a small probability the selected strategy is replaced by the other pure strategy. Third, group selection takes place: each group is disbanded with a positive probability, and the members of disbanded groups switch to the strategies used in those groups that earned the highest payoff sum in that period. This defines an ergodic population process, and Vega-Redondo (1996) shows that if the mutation rate is very small and the population large, its invariant distribution places virtually all probability mass on the population state in which all individuals cooperate. This result contrasts with ours, where this is the long-run outcome only for certain parameter values (see section 3.1). The reason for Vega-Redondo's drastic result seems to be that if the population initially is in the pure-C state, and, say, a pair of individuals mutates to (D,D), then group selection will bring the pair back to (C,C) as soon as it disbands, and this happens with a probability that is "infinitely" higher than a mutation. Hence, while Maynard Smith's haystack model can be said to be tilted in favor of individual selection by letting this operate at a faster time scale than group selection, Vega-Redondo's model can be said to be tilted in favor of group selection by giving group selection full swing as soon as a single group has mutated to a "better" strategy profile.

The model in Sjöström and Weitzman (1996), finally, is not strictly game-theoretic but deals with the same issues as here, in the context of the internal efficiency of competing firms. There are infinitely many firms and each firm has the same finite number of workers. Within a firm, each worker is "programmed" to some effort level. All workers in each firm are paid a wage that depend on their average effort. The (owner of the) firm sets the wage so as to keep the workers indifferent between staying and leaving. Individual selection drives efforts down towards zero within each firm, a force counteracted by a form of "firm selection." Sjöström and Weitzman (1996) show that the ratio between the individual and firm selection rates is crucial for the long-run outcome. The firms in Sjöström and Weitzman (1996), when viewed as groups, are asymmetric: one group member absorbs all excess payoff from the others.

7. Concluding remarks

The aim of this study was to construct a parsimonious population selection model that allows for both individual and group selection forces, without the adiabatic feature of the haystack model. Group selection operates on the same time scale as individual selection.

Some earlier models have modelled group selection as a pairwise contest between groups; out of two randomly selected groups, the one with highest group fitness "wins," in terms of future population shares. Instead, we have a very large number of groups who all "compete" indirectly with each other: groups with higher fitness "win," in terms of future population shares, over those with lower group fitness. In this respect, our approach is similar to how perfect competition is modelled in economics; where no individual firm can affect the market conditions of any other firm, while aggregate firm (and consumer) behavior determines the market conditions of all firms. We hope to have shown the power of this approach for analyses of relevance to evolutionary, experimental and behavioral game theory.

The present analysis is but a first step in a new direction. Extensions and elaborations are called for. In particular, it would be valuable to develop exact stochastic models of selection and mutation in large but finite populations, thereby accounting more accurately for population states where some pure strategies are near extinction. Likewise, empirical and theoretical analyses of reproductive cooperation and competition, and of the influence of the natural environment of these, would be valuable (see e.g. Pen and Weissing (2000) for an analysis of cooperative breeding). One could even conceive of evolutionary selection of forms of reproduction and breeding in different natural environments, thus rendering the fitness function ϕ itself endogenous in the very long run. We leave these extensions for future research.

CHAPTER 5

Group Selection: The Quest for Social Preferences

Marcus Salomonsson[1]

ABSTRACT. This chapter surveys the literature on group selection. I describe the early contributions and the *group selection controversy.* I also describe the main approaches to group selection in the recent literature; fixation, assortative group formation, and reproductive externalities.

1. Introduction

Individual selection has been extremely powerful in explaining both human and animal behavior. However, both empirical and experimental studies have found evidence that humans and animals to some degree have social preferences.[2] Several mechanisms to foster social preferences have been advanced in the literature. Nowak (2006) discusses five of them.[3,4] They are kin selection, direct and indirect reciprocity, networks,[5] and group selection. All five mechanisms, at least in their modern forms, are within the selfish paradigm. They all amount to saying that an organism may appear to have social preferences, but that these preferences can be explained by appealing to selfishness.

[1] I am grateful to the Wallenberg Foundation for financial support. I am also grateful for comments from Cédric Argenton, Christoph Kuzmics, Olof Leimar, Erik Mohlin, Arne Traulsen, and Jörgen Weibull. Any errors are my own.

[2] The evidence on social preferences is surveyed in Dawes and Thaler (1988), Ledyard (1995), and Gintis, Bowles, Boyd, and Fehr (2003).

[3] Another recent overview of the evolution of cooperation and altruism is provided by Leimar and Hammerstein (2006).

[4] Regardless of how social preferences have been selected for, it should be noted that they are probably not calibrated for all conceivable situations. Instead, they should rather be suited for situations that are, or have been, common to the organism. In fact, to preserve resources, preferences may not even be perfectly tuned for such situations. It may be more economical to have *rule of thumb preferences.* For example, a predisposition towards cooperation is not always beneficial. However, in most cases it may be. If the predisposition is not sufficiently costly, compared to the resources saved, it may be preserved. Thus, in a broader sense, when resources are taken into account, the preferences may still be optimal.

[5] Nowak uses the term network reciprocity. I will instead only call these models network models, to emphasize that repeated interactions are not needed. These models are also called *spatial* models. Nowak argues that the more general term *network* is better suited.

Kin selection was formally analyzed by Hamilton (1964). The idea, popularized by Dawkins (1976), is that organisms are vehicles for genes to spread. Genes thus have selfish reasons to influence their organism into helping other organisms with the same gene. This implies that it is rational for parents to help their offspring, and for siblings to help each other etc. The principle is illustrated by Haldane's famous quote that he would sacrifice his life for "two brothers or eight cousins".

Direct and indirect reciprocity concern repeated interactions. Direct reciprocity, advanced by Trivers (1971), refers to a situation where these interactions take place between the same individuals. Indirect reciprocity instead refers to situations where the interactions take place between new individuals but where each individual has a reputation acquired through earlier interactions.[6]

Network models build on the idea that if individuals interact within smaller clusters, then this may foster cooperation. Typically, as in Eshel et al (1998), it both reduces the gain from deviating and the cost of cooperating by reducing the number of interactions.[7]

Group selection also involves interactions in smaller clusters, however, there is more to it than just a local interaction. There is also some kind of competition between groups. In this survey I will consider three basic approaches to group selection. The first approach is the fixation approach, where the crucial element is the existence of pure groups. The second approach is the assortative group formation approach, where there is a correlation between being a certain type and being matched with others of that type. The third approach is the reproductive externalities approach, where group competition makes it possible to internalize externalities.[8]

There is some disagreement in the literature as to what the exact definition of kin selection should be. Lehmann et al (2007) write that "kin selection operates whenever interactions occur among genetic relatives, that is, among individuals who tend to share a more recent common ancestor than individuals sampled randomly from the whole population." This is a very broad definition which implies that all of Nowak's five mechanisms are kin selection as soon as the interaction is "among individuals who tend to share a more recent common ancestor than individuals sampled randomly from the whole population". For example, two siblings engaged in repeated interactions would

[6] In game theory direct and indirect reciprocity are often treated as variants of the *Folk theorems,* formally analyzed by Aumann (1959), Friedman (1971), Rubinstein (1979), and Fudenberg and Maskin (1986). The Folk theorems imply that if a Prisoner's Dilemma is repeated indefinitely, and with a sufficiently high discount factor, then there exists several subgame perfect Nash equilibria, some of which consist of cooperation in every period.

[7] Other examples are Ellison (1993) and Ohtsuki and Nowak (2006).

[8] The term externality is widely used in economics. In biology the term by-product is typically used. An externality is a side effect of an action. The side effect is also inconsequential to the initiator of the action. If an externality is internalized, then the initiator of the action is made to take into account the side effect.

always fall under kin selection and never under direct reciprocity.[9] This definition thus makes it impossible to disentangle different effects from each other whenever relatives are engaged in an interaction. In the papers treated in this survey this is not an issue. In fact, although the word "offspring" is often used, the possibility that individuals may have the same gene, and may help each other for that reason, is simply neglected. This implies that for all practical purposes the individuals are not related and kin selection is not an issue. This interpretation of kin selection is in line with the definition implied by Nowak (2006): "When evaluating the fitness of the behavior induced by a certain gene, it is important to include the behavior's effect on kin who might carry the same gene."

The term inclusive fitness has come to be more general than kin selection. For example, Ricklefs and Miller (2001) define inclusive fitness as "the fitness of an individual plus the fitness of its relatives, weighted according to the coefficient of relatedness". In addition, Grafen (2007) argues that relatedness should be understood as "genetic similarity, however caused, whether by common ancestry, assortation of genotypes or kin recognition". Thus, according to this definition, two individuals having a larger genetic similarity than the rest of the population, but not sharing the same ancestry in any higher degree than the rest of the population, should fall under inclusive fitness, but not under kin selection. Note that this definition excludes two individuals having a larger phenotypic similarity than the rest of the population, but different relevant genotypes.[10] As with kin selection, also the broader concept of inclusive fitness is irrelevant in the papers treated in this survey. In fact, even in the papers treating assortative group formation inclusive fitness is disregarded, and is thus irrelevant.

The chapter is organized as follows. First I discuss two symmetric $2 \times 2-$games - Coordination Games and Prisoner's Dilemma games - where group selection is especially relevant. Group selection may of course also be relevant for more complex games, but these games illustrate neatly the two issues that group selection can address. The first issue is the problem of equilibrium selection: If there are several equilibria, can group selection tell us anything about which equilibrium will be played? The second issue is the problem of social preferences: Can group selection lead to locally altruistic behavior being viable? After going through the games and briefly discussing the main arguments of the various approaches, I go on and describe the early literature on group selection, up to the *group selection controversy.* Then I consider the more recent

[9] In particular, Lehmann et al argue that kin selection and group selection is the same process when groups are rarely reshuffled.

[10] The genotype is the genetic constitution of an organism, whereas the phenotype is the observable characteristic of the organism. Thus, different genotypes may result in the same phenotype.

literature, and consider the three main approaches to group selection that have been formally explored.[11] Finally, I conclude.

2. Preliminaries

To fix ideas, let us consider two symmetric 2×2 games where group selection is of special interest.[12] Originally, the payoffs in these games were considered to be von Neumann-Morgenstern utilities, and still generally are. However, here we interpret the payoffs as fitness, typically the number of offspring.

2.1. Coordination Games. In this class of games there are two Nash equilibria in pure strategies. The game (2.1) is an example of a Coordination Game.

	A	B
A	1,1	0,0
B	0,0	2,2

(2.1)

One Nash equilibrium is for both players to play A. Then they both get a payoff of 1 each. The other pure equilibrium is for both players to play B. Then they both get a payoff of 2 each.

In evolutionary settings, we often think of a large population. We randomly draw two individuals from the population and let them play the game against each other. The payoffs are then the offspring that the individuals get. Thus, if sufficiently many in the population play A, then individuals playing A will get a higher payoff than individuals playing B. Thus, everybody playing A can be an equilibrium outcome despite the fact that the equilibrium where everybody plays B yields more.

Potentially, a group selection model could eliminate the low-yield equilibrium. A very basic model would be to randomly divide the population into groups, and then randomly draw two individuals from each group and let them play a Coordination Game. The individuals and their offspring are then returned to the group they came from. If the population is large and groups are formed randomly, then some groups will converge to the high-yield equilibrium while others will converge to the low-yield equilibrium. Since individuals in the groups that converge to the high-yield equilibrium earn more, they will outgrow the individuals in the low-yield groups.

[11] I can unfortunately not claim to have covered all papers treating group selection. For other surveys of the vast group selection literature, see Sober and Wilson (1998), Bergstrom (2002), and chapter 6 in Okasha (2006). Dugatkin (2002) and Henrich (2004) also have extensive discussions of the concept.

[12] For a complete categorization of symmetric $2 \times 2-$games, see Weibull (1995).

This sketchy model is in fact only a network model. There is a group structure, but no group selection. However, if we would dissolve groups at some probability - and let them be replaced by new groups formed by individuals from high yielding groups - then we effectively have a group selection model. Canals and Vega-Redondo (1998) have constructed such a model. They consider a subset of Coordination Games, called Stag Hunt games. The game (2.2) is an example of a Stag Hunt game.

	A	B
A	4,4	0,3
B	3,0	3,3

(2.2)

Again, one of the equilibria, namely when everybody plays A, has a higher payoff. This equilibrium is the Pareto dominant equilibrium. However, the low-yield equilibrium is now a bit different from before. Imagine that player 1 has no idea what player 2 will play, and thus assumes that he will play A or B with equal probability. Then player 1 would achieve a higher expected payoff by playing B. This equilibrium is called the risk dominant equilibrium. Another way of understanding the concept is to say that the risk dominant equilibrium has the largest basin of attraction.

In two very influential papers, Kandori, Mailath, and Rob (1993) and Young (1993) showed that, in a standard one-population setting with mutations, the population would spend most of its time close to the risk dominant equilibrium. The reason is that fewer mutations are needed for a population to shift from the Pareto optimal basin of attraction to the risk dominant basin of attraction than vice versa. As a result, a population process with mutations would spend most of its time close to the risk dominant equilibrium. Since there is no group selection in a network model, this will also be the case in these models, as demonstrated by Ellison (1993). Indeed Ellison showed that convergence to the risk dominant equilibrium will be faster in a network model.

However, in a group selection model, the result may be completely different. Groups in the Pareto optimal equilibrium would spread faster than groups in the risk dominant equilibrium. This may compensate for the fact that mutations are more likely to take the group from the Pareto optimal equilibrium to the risk dominant equilibrium, rather than the other way around. As we will see, Canals and Vega-Redondo (1998) show that this is indeed the case when the mutation rate approaches zero.[13]

[13] One could also assume that individuals would migrate from low yielding to high yielding groups, which would favor the Pareto optimal equilibrium rather than the risk dominant equilibrium. Oechssler (1997), Ely (2002), and Kuzmics (2003) have shown that this is the case.

2.2. Prisoner's Dilemma games. In this class of games, group selection has been more controversial. The game (2.3) is a Prisoner's Dilemma game. Here both players will play D in equilibrium, despite the fact that if they could commit to playing C, they would both earn more.

	A	B
A	4,4	2,5
B	5,2	3,3

(2.3)

The strategy C is often referred to as an altruistic strategy. The reason is that by playing C a player confers a benefit on the other player while inflicting harm on himself.

A network model in this context would again be to divide the population into groups. Again high-yield groups, i.e. those with many altruists, should grow faster than low-yield groups. However, now the situation is different compared to the Coordination Games. Within each group the best response is to defect. This means that although groups with many altruists may earn more than other groups, each altruist will earn less than a defector in his group. As a result, in the long run the population should converge to everybody defecting.

A theoretical possibility to avoid such an outcome is if neither migration nor mutations are allowed and all players in at last one group play the pure strategy C. Then that group will grow much faster than the other groups and eventually overtake them. Although such models are really network models, they have historically been called group selection models or Haystack models. To somewhat adhere to this tradition I will call them group selection models based on fixation.

Yet another example of a network model often presented as a group selection model is what I will call group selection models based on assortative group formation. In these models there is again no actual competition between groups, but rather a correlation between playing C and ending up in groups where others also play C.

An easy way to extend a network model into a group selection model is to assume that groups are wiped out - with everybody in the group receiving the payoff 0 - with a probability negatively dependent on the sum of payoffs in the group. For example, consider groups of two players. Suppose that groups with only cooperators never are wiped out, whereas mixed groups are wiped out with the probability 0.5, and defecting groups always are wiped out. Then the expected profit of a cooperator in a pure group is 4, whereas it is 1 in mixed groups. The expected profit of a defector is 2.5 in mixed groups and 0 in pure groups. If we start off with an equal proportion of cooperators and defectors in the population, and group formation is completely random, then a player has an equal probability of ending up in a pure group or a mixed group. As a result, a cooperator's expected payoff is 2.5, while a defector's expected payoff is 1.25. Thus, a cooperator has a higher expected payoff than a defector. Models of this type will be called group selection models based on reproductive externalities.

Before we turn to these three approaches to group selection I will briefly discuss the early group selection literature and the group selection controversy.

3. Early contributions

Fittingly, it was Darwin who made the first allusion to group selection in The Descent of Man and Selection in Relation to Sex.[14] The following sentence is often quoted:

> "There can be no doubt that a tribe including many members who, from possessing in a high degree the spirit of patriotism, fidelity, obedience, courage, and sympathy, were always ready to give aid to each other and to sacrifice themselves for the common good, would be victorious over most other tribes, and this would be natural selection." (Darwin (1871), page 166.)

Thus, Darwin saw that social preferences could be developed through natural selection. In addition he seems to have held the view that it was some form of competition between groups that would foster these social preferences.

Carr-Saunders (1922) thought that group selection fostered social evolution only among humans. He argued that it came about in primitive societies. These societies were nominally thought to be nomad societies, but Carr-Saunders argued that they were in fact restricted to a territory, within which they evolved social conventions to optimize the potential for long-term survival. Through abstention, abortion, and infanticide, they restricted their numbers so that an optimum number was reached. At this number of individuals, the income per head was maximized. These social conventions served to avoid more Malthusian methods of population control, the objective being to avoid social instability.[15]

Among the early contributions, Wright (1945), was the only one to construct a mathematical model. The aim was to show that a "character of value to the population, but disadvantageous at any given moment to the individual" could survive. He noted that if such a *character* - or *strategy* in game theoretical terminology - completely

[14] Allee (1943) traces the idea "of natural cooperation" all the way back to the Greek philosopher Empedocles (ca. 490–430 BC).

[15] Williams (1966) discusses the advantage of a low variability in numbers. A high variability implies a higher risk of going extinct.

dominated a group, then that group would grow much faster than other groups. However, if a selfish strategy appeared in the group - either by mutation or migration - it would take over the group and the altruistic strategy would disappear.

Wright thus noted that isolation and fixation of the group was necessary for an altruistic trait to be preserved. His idea seems to be based on several groups that are isolated for long times. He argued that drift - i.e. random occurrences in reproduction - could lead to complete domination of altruists in a group. Once fixation had occurred, there would be no way for the selfish trait to reappear in the group, barring mutations and migration. The group would then reproduce much faster than other groups. Eventually, after a long period of isolation, a migration phase would ensue. The group would then spread over the world and a new phase of isolation would follow. A group would eventually drift into fixation, and the cycle would restart.

By stressing isolation and fixation, and the trade-off between migrating altruists and resident egoists, Wright anticipated the haystack model of Maynard Smith (1964). The main conceptual difference is that, as we will see, Maynard Smith considered group formation to be the phase where the altruists had the largest chance of completely dominating a group.

Wynne-Edwards (1962), somewhat like Carr-Saunders (1922), was concerned with the problem of resource management. In particular he wanted to construct a theory to explain how a species, or a group from a species, managed resources to avoid extinction. He drew the parallel to the North Pacific Sealing Convention of 1911. This treaty was an attempt to curb the overfishing of seal in the North Pacific. It came into place after several years of conflict between fishing vessels from various countries, spurred by the scarcity of seals.

If humans could come up with such a treaty - Wynne-Edwards thought - would not animals also be able to construct similar conventions to avoid excessive consumption of resources?[16] He argued that such conventions could also explain why animals were often not living on subsistence levels. He noted that on the fringes of a species' territory they often did. However, in the centers they were in good health - "and sometimes actually fat".[17] Thus, he wanted to find a theory that did not depend on famine, predators, or diseases, but instead depended on social conventions within a territory, group, or species, that optimized the potential for long-term survival within the territory, or of the group or species.

[16] Although he saw humans as able to form social conventions to manage resources, Wynne-Edwards (1963) argued that modern societies had ceased to manage their own numbers, and in that sense provided "a spectacular exception to the general rule."

[17] This would not surprise present day economists, who would argue that individuals on the fringe are the *marginal individuals*.

Interestingly, it seems that he did not think of these conventions as being necessarily upheld by strictly altruistic behavior. Instead, he hypothesized that enforcement of population control would not depend on the parents in each case, but instead on other individuals in the group. By interpreting his theory in this way, group selection would not be needed. Instead it would suffice to say that when resources become scarce, then parents have an interest in killing other parent's offspring, which squares well with individual selection. On the other hand, Wynne-Edwards explicitly states that social conventions are based on group selection. He noted that group selection and individual selection might be in conflict, but then argued that group selection would always prevail. The reason, as he saw it, was that if it did not, then the species would go extinct.

4. The controversy

The *group selection controversy* broke out as a result of Wynne-Edwards (1962). Maynard Smith (1964) commented on a companion paper, Wynne-Edwards (1963), and dismissed Wynne-Edwards' theory. Maynard Smith argued that most of the observations that Wynne-Edwards claimed corroborated his theory in fact could just as easily, if not more easily, be explained by individual selection. However, he also went ahead and constructed a mathematical model as to how group selection could work. This model became known as the *Haystack model*.

The model consists of a field with haystacks. A pregnant mouse is placed under each haystack as it is created. The mice then procreate in the haystacks without any migration between them. In the end of each period the haystacks are collected, and the mice are put into one common population. New haystacks are then constructed and a new pregnant mouse, drawn from the new population, is placed under each. The mice can be of two types; aggressive mice, A, who breeds at the same rate irrespectively of the group size; and timid mice, a, who stop breeding when the group reaches a certain size.

Groups starting off with only aggressive mice will eventually run out of resources and will have a period of starving. However, Maynard Smith excludes the possibility that the mice actually go extinct. In groups with both aggressive and timid mice, both breed at the same rate until the population reaches the timid mice's limit. At this limit, the timid mice stop breeding and the aggressive mice take over the group completely. Since their resource management has been slightly better than the group with only aggressive mice, starving starts later in these groups. Finally, in groups that start off with only timid mice, they breed optimally and avoid starving towards the end of the period.

Thus, the model has some features that stand out. First, as already noted, aggressive mice breed aggressively enough to starve, but not aggressively enough to go extinct, which already Wynne-Edwards' noted in his comment to the paper. Second, in mixed groups the timid mice go completely extinct. This feature may be a shortcut to illustrate that the period under the haystack is very long, or at least sufficiently long for suboptimal strategies to be crowded out.

Thus, since groups of aggressive mice do not go extinct, and since timid mice in mixed groups go extinct, the model is biased against timid mice. Nevertheless, the result is that if only isolation is long enough, then the ratio of timid mice in the population will increase. The reason being that groups with only timid mice will grow very large. However, the general feeling among readers was that these long periods of absolute isolation were unrealistic, at least if this type of group selection should be a feature with wide applicability.

A further blow to the group selection argument was Williams (1966). Williams elaborated on Maynard Smith's argument that most of the phenomena ascribed to group selection could be explained by individual selection. Just like Maynard Smith, Williams did not reject group selection flat out, but insisted that the limited possibility of finding phenomena that group selection, but not individual selection could explain, suggested that it was a very weak force.

Williams argued that natural selection required a certain stability among the entities being selected for. Genes, for example, are fairly stable. There might be mutations, but overall they are fairly rare. Groups, on the other hand, are in constant flux, primarily due to migration. Furthermore, an individuals is fairly short-lived, while groups have longer time-spans. This also limits the possibility of more fit groups to replace less fit ones.

In sum, the contributions of Maynard Smith and Williams were very influential. They effectively dismissed group selection in the form advanced by Carr-Saunders and Wynne-Edwards. Maynard Smith's Haystack model also for a long time became the benchmark group selection model.

5. Fixation

The contributions of Wright (1945), and Maynard Smith (1964) stressed the importance of fixation, i.e. pure groups, for group selection to have any effect. The underlying idea is that mixed groups will degenerate over time. Pure groups are thus needed for cooperators to survive and eventually thrive. Wright thought of these pure groups coming into existence through drift, while Maynard Smith thought of them coming into existence when groups were formed.

Eshel (1972) extended the fixation approach to focus on migration. He confirmed Wright's conjecture that altruistic traits will dominate the population if migration is sufficiently low. Eshel also argued that an innovation that increases mobility would lead to less altruism.

Traulsen and Nowak (2006) provide a link between fixation models and the reproductive externality models treated later.[18] They divide a population into groups and let the individuals in each group play an n-person prisoner's dilemma game. A single individual is chosen for reproduction with a probability proportional to its fitness. The offspring is put into the same group. When a group reaches the size n two things can happen: With probability q the group is split into two groups and another group is eliminated. With probability $1 - q$ the group is not split up. Instead a randomly selected individual in the group is eliminated.

The fact that the group can split in two only after it has reached the size n implies that the model is a reproductive externality model. However, to make it mathematically tractable, Traulsen and Nowak focus on the special case when q is very small, instead stressing the fixation effect. The authors analytically show that the smaller the group size and the more numerous the groups, the higher will the probability be that altruists will dominate the entire population.

Traulsen and Nowak then substantially extend Wright's model. They show that the probability that altruists will dominate the population increases with q. The reason is that as q increases it pays off relatively more to be in a group that has reached the size n, which groups with many altruists reach faster than other groups. In that sense the fixation result is an extreme result, giving a lower bound to the probability of altruists thriving in the population. Traulsen and Nowak also extend the model to include migration and multilevel selection. As in Wright (1945) and Maynard Smith (1964), increased migration makes it more difficult to sustain cooperation.

It is interesting to note that the fixation approach to group selection implies that somewhere in the human species' distant past there was a golden era for the evolution of social preferences. However, as mobility increased the circumstance under which altruism was created ceased to exist. In a somewhat related empirical study, Henrich et al (2001) let subjects from 15 small scale societies play the ultimatum game.[19] They found that the modal offers were between 15 and 50 percent, while it is typically 50 percent in industrialized societies. Mean offers varied between 26 and 58 percent, while

[18] That link is not intrinsic. Both Wright (1945) and Maynard Smith (1964) disregard reproductive externalities.

[19] This is a two player game where one player is given a sum of money and the possibility to divide it with the other player. If the other player rejects the division, then neither player gets anything. If he accepts, then that division is executed.

it is typically 44 percent in industrialized societies. Thus, Henrich et al's findings do not indicate that industrialized societies have less altruists than other societies. If anything, the results rather suggest the contrary.

6. Assortative group formation

Wilson (1975) was the first to analyze how assortative group formation could lead to altruism being sustainable. While earlier models argued that isolation was a precondition for altruism to thrive, Wilson's model can instead be interpreted as a model were groups are reshuffled in every period. However, for altruists to survive in such a model, being an altruist and being matched with an altruist should be correlated.

Wilson notes that this can be achieved through various venues. Group formation based on kin is one of them, but this is only a special case. In subsequent work, e.g. in Sober and Wilson (1998) and Wilson and Dugatkin (1997), the author has discussed various structures that can result in assortative group formation.

Here we will discuss two mechanisms through which assortative groups may arise. First we will discuss the possibility of there being a conformist bias in imitation. Second we will discuss signalling.

6.1. Conformist bias. Experiments in social psychology has established that there seems to be a tendency for individuals to conform to other individuals' opinions.[20] Such conformity can be of two types. It can be an informative influence in the sense that other individuals' opinions may reveal useful information. It can also be a normative influence in the sense that individuals may attach an intrinsic value to conforming.[21]

If the conformist bias is important then it would lead to a fairly strong positive correlation between cooperating and being in groups that cooperate. This would thus lead to these groups performing better than other groups. However, to the extent that it is still better to deviate, or not be a conformist, this behavior would not be evolutionary viable.

Boyd and Richerson (1985) argued that individuals using an imitation rule with a conformist bias will be selected for. The reason, as they see it, is that once a majority in the population has adopted a new and better behavior, then it is better to imitate that majority rather than to use an unbiased imitation rule.

[20] See Aronson, Wilson, and Akert (2007) and DeLamater and Myers (2007) for textbook treatments of conformity.

[21] Henrich and Boyd (2001) argues that conformist transmission amounts to "using the popularity of a choice as an indirect measure of its worth". This would thus amount to informative conformism discounted by the risk of the measure being incorrect.

Henrich and Boyd (1998) considered both individual learning and social learning. They compared an unbiased random rule with a conformist rule in a computer simulation, and found that the conformist rule would survive if the environment did not change too often. Individual learning is in general better than social learning if a new and better technology is only used by a small fraction of the population. However, this is also when a conformist bias would perform badly compared to an unbiased imitation rule. Thus a conformist bias is not really better than no bias per se. It is simply that in situations when no bias are better than conformity, then social learning itself will not be successful.[22]

In essence, a conformist rule is better than an unbiased rule if a majority uses the best strategy. Then individuals using the conformist rule will shift over to the best strategy faster than individuals using an unbiased rule. However, the converse is also true. If the majority uses a bad strategy, then individuals using the conformist rule will shift to the best strategy slower than individuals using an unbiased rule. Eriksson et al (2007) modified Henrich and Boyd's model to prolong the period when a new and better technology is used by a minority. As a result, a conformist bias seems never to be an evolutionary stable strategy - although it seemed to be under some parameter values in Henrich and Boyd's setting.

The idea of a conformist bias being the crucial mechanism to uphold altruism is an interesting one. In particular its proponents have noted that it may serve to favor cooperation also in very large groups. However, it should be noted that although conformism may lead to cooperation, it is also costly for the conformist. Thus, if we invoke exogenous conformity then we somewhat push the problem with defectors outside the model. Henrich and Boyd (2001) have attempted to somewhat address this issue. They consider a Prisoner's dilemma augmented by i stages of punishments. I.e. in the first stage those that did not cooperate in the Prisoner's dilemma are punished. In all stages after that people who did not punish in the previous stage are punished. The payoff difference between a payoff biased rule and a conformity biased rule decreases for every stage. Eventually, in stage i, if everybody has at least some conformity in them, this will outweigh the urge to consider the payoff, and everybody will conform - and punish. Since everybody punishes in stage i, everybody will also punish in stage $i-1$. The backward induction is not perfect though. If there are sufficiently large benefits from defection, then everybody will still defect in the Prisoner's dilemma. Nevertheless, if these gains are not large enough, then cooperation will be sustained. The result is that the game has two Nash equilibria, one with defection and no punishment, and the

[22] Also see Wakano and Aoki (2007) and Nakahashi (2007) for a fuller treatment of Henrich and Boyd (1998).

other with cooperation and punishment to the i^{th} level. Adding group selection to the model, the authors argue that the last equilibrium would be chosen.

Guzmán et al (2007) perform simulations to compare an imitation rule with a payoff bias with an imitation rule with a conformist bias. Individuals are only allowed to imitate other individuals within their own group. This means that the group selection effect disappears completely for payoff imitating individuals, who will never cooperate. Conformity imitating individuals, on the other hand, will sometimes end up in groups with many cooperators, and will then reap the group selection benefits through their imitating rule. The crucial mechanism in Guzmán et al is thus that individuals with a payoff bias will only imitate other group members. Thus, the players in this model seem to have fairly limited cognitive abilities. It would be interesting to let the players mental abilities grow, so that they also can imitate other players outside their own group. It seems likely that then a payoff bias would be favored instead of a conformist bias, suggesting that this type of conformity may have been an advantage early in human development, but since then has become a liability.

Richerson and Boyd (2005) argue that culture would tend to favor a conformist rule. Nevertheless, it seems that a conformist biased rule does not seem to be particularly conducive to technological innovations compared to a payoff biased rule. The incentives to innovate will be lower, as will the possibilities to spread new innovations. It is only after an innovation has been made, and after a majority has accepted it, that a conformist bias will perform as well as a payoff biased imitation rule. Thus, it could indeed be the case that a conformist bias is particularly ill suited for a species where innovations play such an important role as for humans. In addition, to the extent that new and better technologies make groups more successful, it seems that group selection itself would favor a payoff biased rule rather than a conformist biased rule.

6.2. Signalling. Another approach towards assortative group formation is to consider signalling before groups are formed. To my knowledge a formal analysis of such a game has not been made.[23] However, Grégoire and Robson (2003) analyze a somewhat similar game. They add the possibility of sending a costly signal before playing a Prisoner's Dilemma, and then add a group structure to the game.

The problem with signalling before playing a prisoner's dilemma, is of course that individuals will be prone to lie. A signal is not a commitment. Even if it is a costly signal, it will be subgame perfect to deviate from it. To make it possible to avoid being taken advantage off by the defectors, Grégoire and Robson allows for a strategy called the *secret handshake*. This is a promise to cooperate that is only understood by other

[23] Gintis, Smith, and Bowles (2001) consider a model where providing a public good is a costly signal.

secret handshakers. In addition, somebody who plays the secret handshake will indeed cooperate when the game is played. The authors also allow for a strategy that takes advantage of secret handshakers. This strategy is called the *sucker punch*. Somebody who plays the sucker punch sends the secret handshake signal, but then deviates when the game is played.

The authors start off with a one-population game. Each player plays a round robin and gets the average payoff. All players then imitate the strategy that earns the highest payoff. Thus, if the entire population is playing defect without a signal, then the population can be invaded by two secret handshake players. The reason is that the secret handshakers will cooperate when playing with each other, but defect when playing with everybody else. The defectors, on the other hand, will always defect. The secret handshakers will thus earn a higher payoff than the defectors, and take over the population. However, this makes it possible for one sucker punch player to invade the population. Finally, when the population is dominated by sucker punch players, then one non-signaling defector may invade the population. As these defectors take over the population, it again becomes possible for two secret handshake players to invade it. Given that the secret handshake needs two mutations to invade a population, while both the sucker punch and the non-signalling defectors only need one mutation, the population will most of the time consist of defectors.

The authors then add a group structure to the model. Players are first playing within the group. They play against all other players in the group and receive the average payoff. All players in each group then imitate the highest yielding strategy in that group. Players are then playing a second round, again within the group, but now they instead imitate the highest yielding strategy in the entire population. Thus, if at least one group happens to be in a cooperative state, then the entire population will switch to that state. As a result, Grégoire and Robson (2003) show that if there are at least three groups, then all stochastically stable states involve cooperation.

To see this, suppose that the entire population is defecting, without sending any signals. A mutation to two secret handshakers within a group would then transform that group into secret handshakers and then transform the entire population to secret handshakers. To leave this state it is required that each group has a simultaneously mutation to at least one sucker puncher. Naturally, the more groups there are, the less likely is this to happen.

Although the signal is made after groups are formed, it seems that the logic of Grégoire and Robson (2003) would translate also into a setting where individuals first signal, then are put into groups, and finally play a Prisoner's Dilemma. In fact, then it seems it would suffice with two mutations to secret handshakers in the entire population

- as opposed to within the same group - to transform first one group and then the entire population into secret handshakers. Such a population would be difficult to invade for sucker punchers though. A sucker puncher would transform his group into sucker punchers, but that group would then be transformed back to secret handshakers in the group stage. Thus, for sucker punchers to take over the population there must first be sufficiently many mutations to sucker punchers, and then at least one would have to end up in each group. To my knowledge such a variant of Grégoire and Robson's model has not yet been explored formally.

An example of signalling that seems be at work both at the group formation stage and at the strategy selection stage is quorum sensing. This phenomenon has been observed especially in bacteria, but also in some social insects. Bacteria that use quorum sensing secrete a signalling substance as a function of how much of the substance it can sense in its vicinity. This can create a feed-back loop attracting and inducing ever more bacteria to secrete the substance. At a certain threshold, the bacteria can sense that they are in sufficient numbers to achieve a certain effect, e.g. bioluminescence, and can then trigger the necessary behavior to achieve that effect.[24] The phenomenon has been quite thoroughly empirically studied, but it at this point it is not certain whether it is altruistic or if group selection is involved.[25]

7. Reproductive externalities

Many activities create externalities for others. That is, an organism's behavior will affect also other organisms' fitness, without this effect on others affecting the first organism's fitness. In economics this is called an externality. Externalities can be *internalized* through different avenues. In some cases, such as air pollution, extended property rights - i.e. rights to pollute - makes this possible. In other cases, such as defense or public roads, a central authority can at least partially ensure that such public goods are provided. In other cases, although much less explored, competition between companies may lead to higher efficiency within each company.[26] In biology, a natural way to internalize externalities is through group selection. Groups that provide the public good will be more successful than others, which will lead to provision of that public good to spread. Weibull and Salomonsson (2006) call externalities relevant for organisms' fitness *reproductive externalities*.[27]

[24] I am gratful to Olof Leimar for the pointer.

[25] See Joint, Downie, and Williams (2007) for a full issue of Philosphical Transactions B dedicated to quorum sensing.

[26] Examples are Boyer and Orléan (1992), Vega-Redondo (1993), Sjöström and Weitzman (1996) and Weibull (2000).

[27] In biology the term by-product has sometimes been used to describe externalities, see e.g. Connor (1995) and Leimar and Connor (2003).

Several group selection papers have implicitly used reproductive externalities. Typically, as in Canals and Vega-Redondo (1998), Vega-Redondo (1996), and Traulsen and Nowak (2006), a group is disbanded at some exogenous probability, and high-yielding groups then have a possibility to colonize the disbanded group's niche or territory. Being a high-yield group is thus a public good. To the extent that they affect the provision of this public good, individual behavior consequently have reproductive externalities - and they can be at least partly internalized through group selection. Other papers, such as Weibull and Salomonsson (2006), and Killingback et al (2006) are more explicitly concerned with externalities and public goods.

Canals and Vega-Redondo (1998) generalize the stochastic evolutionary model of Kandori, Mailath, and Rob (1993) into a group selection model. Individuals are subject both to individual selection and a small mutation rate. The individuals are divided into groups, and these groups are disbanded at some rate and replaced by new groups. The members in the new groups can either imitate strategies that earn the highest average payoff in the general population, or imitate the strategies in the group with the highest payoff. Any weight on the latter imitation rule is thus a weight on group selection. Canals and Vega-Redondo then look at Coordination Games, in particular Stag Hunt games exemplified in game (2.2) earlier. As Kandori, Mailath and Rob showed, there will be a mutation force within each group leading to the evolutionary process spending most of its time close to the risk dominant equilibrium. The strength of that mutation force will be very weak if the mutation rate is very small. In Kandori, Mailath and Rob's setting this does not matter. However, with Canals and Vega-Redondo's group structure there is a counter balancing force. Each time a group is disbanded a proportion of the new members will imitate the strategies played in the groups with the highest payoff. Letting the mutation rate go to zero, the authors show that group selection will be a much stronger force, and the system will spend most of its time at the Pareto dominant equilibrium, rather than at the risk dominant equilibrium.

The setup in Vega-Redondo (1996) is similar to Canals and Vega-Redondo (1998),[28] but he only lets new group members imitate strategies from groups with the highest payoff. Furthermore, instead of considering a Coordination Game, a Prisoner's Dilemma is considered. As long as the mutation rate is sufficiently small, Vega-Redondo's model result in a stochastic process spending most of its time with all groups composed of cooperating individuals.

In similar models, Vega-Redondo (1993) and Sjöström and Weitzman (1996) discuss the effect of competition on firms' efficiency. Within companies, employees have an

[28] The first version of Canals and Vega-Redondo (1998) was published in 1994 as a Universidad de Alicante working paper.

incentive to shirk. However, if they do so, then their companies run a larger risk of going bankrupt, implying unemployment for the staff. Vega-Redondo (1993) considers a stag hunt game, whereas Sjöström and Weitzman (1996) consider a prisoner's dilemma. In both cases the total outcome is that competition between firms hinders shirking within firms. In the setting of Sjöström and Weitzman (1996) a technical issue arises with a finite number of firms. Then it is possible that simultaneous degeneration across all firms lead to a long run degeneration of the entire population. However, they show that with an infinitesimal probability of exogenous mutations in favor of cooperation, this degenerative tendency will lead to the same qualitative results as with an infinite number of firms. Weibull (2000) extends the basic textbook Cournot model to take into account managerial owners' trade-off between profit and effort. The result is that stiffer competition leads to a higher effort from managerial owners, i.e. a higher internal efficiency.

Weibull and Salomonsson (2006) consider both positive and negative reproductive externalities. When externalities are positive, for example when a parent protects the group's young, they foster altruism. When they are negative, for example in competition for mates or food, they foster spite.

The model consists of a finite, but large population. Groups of two players are randomly, and non-assortatively, formed in every period. The group members play the game once and receive the material payoffs in terms of offspring. Survival of the offspring, or effective payoff, depends on a function ϕ that depends on both group member's material payoffs. This is thus the group selection element in the model. The group members and their surviving offspring is then returned to the population and a new round starts. The function ϕ can be specified to include both within group competition, fostering spite; and within group cooperation, fostering altruism.

Weibull and Salomonsson first consider symmetric $2\times 2-$games, and then show that when breeding is cooperative, then some games that are Prisoner's Dilemma games in material payoffs are transformed to either Coordination Games, Hawk-Dove games or Efficient Dominance Solvable games in effective payoffs. As a result, an observer only looking at material payoffs would conclude that the players were altruistic, whereas he would conclude that they were selfish if he looked at effective payoffs. In contrast, when breeding is competitive, then some Coordination Games, some Hawk-Dove games, and some Efficient Dominance Solvable games are instead transformed to Prisoner's Dilemma games.

In a more general formulation, and taking into account both cooperative and competitive breeding, Weibull and Salomonsson derive social preferences reflecting both altruism and spite. The derived preferences qualitatively agree with those discussed in Fehr and Schmidt (1999) and Charness and Rabin (2002). They are, for example, consistent with costly punishment.[29,30] Furthermore, they are also consistent with rejections of "unfair" offers that have been observed in ultimatum game experiments.

Since groups are reshuffled in every period, these social preferences will be robust to migration. In contrast to the fixation approach, the reproductive externalities approach does not hinge on groups being isolated for long periods of time. Nevertheless, Williams (1966) also argued that migration would result in all groups being so similar that selection between them would at best be a very weak force. This argument has been empirically refuted by Bowles (2006) who, based on genetic data from recent hunter-gatherer populations, argues that "genetic differences between early human groups are likely to have been great enough so that lethal intergroup competition could account for the evolution of altruism".

Instead social preferences based on reproductive externalities hinge on the existence of public goods. The existence of such goods have presumably varied extensively during history, but can also be assumed to vary across current societies. Thus, a cultural interpretation of the theory could be used to explain the variability of social preferences between societies, as studied by Henrich et al (2001). Note that, in contrast to the fixation approach, there is nothing in this approach that indicates that social preferences should deteriorate as migration increases. In fact, to the extent that an increase in migration coincides with more public goods, social preferences could in fact become even stronger. Interestingly, in the study by Henrich et al, it was found that in societies where payoffs to cooperation was high, offers in the ultimatum game were also higher. This result is in line with social preferences being fostered in a cultural model of reproductive externalities.

Killingback et al (2006) allow for different group sizes. They consider a public goods game. Individuals are asked to contribute to a common pool. The contributions are then multiplied by a sufficiently large factor and distributed equally among the individuals. The factor is sufficiently large in the sense that in small populations it makes it a unique evolutionary stable strategy to contribute, while it does not make it an evolutionary stable strategy to contribute in large populations. Instead, in large populations, the evolutionary stable strategy is to not contribute. Killingback et al

[29] Also see Herold (2004) for an account of how costly punishment and rewarding may be fostered in a group selection model based on reproductive externalities. Technically, Herold's model is based on reproductive externalities since the players know the composition of their groups. Boyd et al (2003) have looked at a similar model using simulation methods.

[30] There is a fairly large literature on costly, or altruistic, punishment. E.g. Boyd et al (2003) has argued that costly punishment is altruistic since it may serve to uphold cooperation. However, Dreber et al (2008) present experimental evidence suggesting that punishment rather triggers retaliations and that punishers earn less than others.

(2006) simulate the results in a model where group sizes are variable. The fact that the number of small groups is non-zero leads to contributions in the overall population being non-zero.[31]

As Killingback et al's (2006) results illustrate, the ability to internalize reproductive externalities will be smaller in larger groups. With that in mind, it may seem puzzling that we can observe cooperation within very large human groups. However,despite the fact that humans can be organized in very large groups, sociological studies typically report that humans have remarkably few close friends. McPherson et al (2006) reported that the average American in 2004 only had 2.08 close friends with whom to discuss important matters. Although the number was down from 2.94 in 1985, it is striking that this number is much smaller than the number of acquaintances that our brains are thought to be able to handle. Dunbar (1998) argues that our neocortical processing capacity limits the number of acquaintances we are able to keep track of to 150 individuals. It may be that by allowing for networks and hierarchies it would be possible to reconcile these apparent contradictions.

8. Conclusion

Group selection has typically been used to address two issues: equilibrium selection and altruism. The equilibrium selection issue - illustrated by the Coordination Games - has never really been controversial. The recent literature has instead focused on more complex issues, like whether group selection can lead to the Pareto dominant equilibrium, instead of the risk dominant equilibrium, being stochastically stable. As Canals and Vega-Redondo (1998) have shown, that is indeed the case.

Group selection as an explanation of altruism - illustrated by the Prisoner's Dilemma games - has been more controversial. The first formal models by Wright (1945) and Maynard Smith (1964) argued that group selection could only promote altruism if pure groups of altruists were allowed to be isolated for long periods. However, since then group selection has also been found to work through assortative group formation and reproductive externalities.

Group selection based on assortative group formation begs the question of how assortion comes about. A strand of the literature has focused on a conformity bias in imitation which could create assortion, whereas signalling as a means of obtaining assorted groups seems to have been somewhat neglected. An interesting aspect with

[31] Grafen (2007) interpreted Killingback et al (2006) to mean that Simpson's paradox is driving the results. Instead he argued that kin selection was the crucial factor. However, Killingback et al (2006) do not allow for kin selection. The mechanism at work is reproductive externalities. They are internalized in small groups, but not in large groups.

signalling is that the interpretation of signals will involve culture. E.g. different cultures may develop different signals for the same behavior.

Group selection based on reproductive externalities has been more successful. Vega-Redondo (1996) has shown that group selection with reproductive externalities will make altruism viable, whereas Weibull and Salomonsson (2006) have shown that reproductive externalities can result in social preferences discussed by Fehr and Schmidt (1999) and Charness and Rabin (2002). In addition Bowles (2006) have shown empirically that genetic differences between early human groups seems to have been sufficient to foster altruism.

Much empirical work remains to be done. For example, it would be interesting to compare the predictions of the fixation approach to the approach of reproductive externalities. The fixation approach predicts that altruism will increase if migration decreases, while the reproductive externalities approach suggests that altruism will increase if the positive externalities increases - as long as they can be internalized through group selection. It would be interesting to see whether these predictions are borne out, and, if so, which approach has more explanatory value. This may also calm some fears that our societies' increased mobility will lead to less altruism, and thus eventually the demise of civilized society. If the increased mobility also creates more positive externalities that we can try to internalize, then it may in fact lead to more altruism rather than less.

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