Essays on the Term Structure of Interest Rates and Long-Run Risks

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To my family and the
loving memory of my mother

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Henrik Hasseltoft
Stockholm, July 2009

Contents

Chapter 1

Introduction

All three papers in this thesis are at the intersection of macroeconomics and finance. Emphasis is put on understanding how assets are priced and which forces determine movements in asset prices over time. Understanding the link between the macroeconomy and financial markets is not only of interest for academics but is also important for investors and all of us who have some capital invested in asset markets. The first two papers are theoretical and take a consumption-based approach to asset pricing in which fundamental macro variables such as consumption and inflation determine movements in asset prices. The third paper is more empirical in nature and focuses on the predictability of international bond returns.

In the first paper, *Stocks, Bonds, and Long-Run Consumption Risks*, I extend the framework of Bansal and Yaron (2004) to the term structure of interest rates. Bansal and Yaron (2004) show that a representative agent asset pricing model featuring Epstein-Zin and Weil recursive preferences and persistent shocks to the first and second moments of consumption growth can account for key moments of equity markets using a plausible level of risk aversion. I show that a calibrated version of the model can account for key features of bond markets such as deviations from the expectations hypothesis, the upward sloping nominal yield curve, and the predictive power of the nominal yield curve. Positive shocks to the volatility of consumption growth lead to a steepening of the yield curve. Since the volatility follows a mean-reverting process, volatility is expected to decline which increases the expected excess return on long bonds. As a result, the slope of the yield curve and bond excess returns become positively correlated which allows the model to match the so-called expectation hypothesis puzzle. A negative correlation between inflation and consumption growth turns nominal bonds into risky assets as their payoffs are procyclical. The nominal yield curve therefore slopes up. However, an estimated version of the model is shown to have difficulties matching key moments of asset prices without resorting to a high level of risk aversion since the estimation yields a too low persistence of consumption shocks. The sensitivity of the model to small changes in parameter values is emphasized.

In the second paper, *The "Fed-model" and the changing correlation of stock and bond returns: An equilibrium approach*, I analyze how well an equilibrium model featuring Epstein-Zin and Weil recursive preferences and time-varying first and second moments of consumption growth, dividend growth, and inflation can explain two features of data that have been considered puzzling. First, the model suggests that dividend yields on equity and nominal interest rates are positively correlated, a relation often called the Fed-model, when high inflation signals low future consumption growth. Investors' dislike positive inflation shocks as they lead to lower economic growth and accordingly demand a positive risk premium for holding assets that are poor inflation hedges, such as equity and nominal bonds. Shocks to inflation therefore induce common movements in risk premiums on both equity and nominal bonds, making dividend yields and nominal yields positively correlated through a risk premium channel. Second, the model predicts that the correlation between stock and bond returns moves together with macroeconomic volatility, as high macro volatility lead to low stock and bond returns. The highly positive correlations observed in the late 1970s and early 1980s are therefore attributed to high levels of economic risk. The drop in correlations observed since the 1980s are attributed to lower macro volatility, a period often denoted The Great Moderation. Including the covariances between the three macro variables as state variables allows the model to produce negative correlations. In particular, the negative correlations observed in the late 1990s are partly attributed to low volatility in conjunction with a positive shock to the covariance of dividend growth and inflation, resulting in high stock returns and low bond returns.

The third paper, *International Bond Risk Premia*, is more empirical in nature and builds on the analysis of Cochrane and Piazzesi (2005, CP) who construct a return forecasting factor for US bond returns that consists of five contemporaneous forward rates. The factor is shown to predict bond returns with significantly higher explanatory power compared to classical regressions that uses the slope of the yield curve. We extend their analysis to international bond markets by constructing return forecasting factors for bond excess returns across different countries. While the international evidence for predictability is weak using the slope of the yield curve as predictor, we document that local CP factors have significant predictive power. We also contribute to the literature by constructing a global CP factor which is shown to predict bond returns with even higher explanatory power than the local factors. Including local and global factors jointly increases the predictive power further, indicating that bond excess returns are driven by both country-specific and global factors. Our results suggest that shocks to US bond risk premia are particularly important determinants for international bond premia. Motivated by these findings, we estimate a no-arbitrage affine term structure model in which risk premia are assumed to only be driven by one local and one global CP factor. We find that local CP factors are similar to local slope factors while the global factor is more similar to a world interest rate level factor. The literature is still silent on what type of information the CP factor captures. The counter-cyclical nature of the factor suggests that it is related to business cycles in the economy. Understanding the link between the CP factors and the underlying economy opens up many research questions which are sure to keep me busy for many years to come.

Chapter 2

Stocks, Bonds, and Long-Run Consumption Risks

Abstract

Bansal and Yaron (2004) show that long-run consumption risks and time-varying economic uncertainty in conjunction with recursive preferences can account for important features of equity markets. I bring the model to the term structure of interest rates and show that a calibrated version of the model can simultaneously explain properties of bonds and equities. Specifically, the model accounts for deviations from the expectations hypothesis, the upward sloping nominal yield curve, and the predictive power of the nominal yield spread. However, an estimation of the model using Simulated Method of Moments yields less convincing results and illustrates the difficulty of precisely estimating parameters of the model. Real (nominal) interest rates in the model are positively (negatively) correlated with consumption growth and real stock returns move inversely with inflation. The cyclicality of nominal interest rates and yield spreads is shown to depend on the relative values of the elasticity of intertemporal substitution and the correlation between real consumption growth and inflation.

2.1 Introduction

The literature has established several intriguing facts including the well-known equity premium puzzle (Mehra and Prescott, 1985), the expectations hypothesis puzzle indicating time-varying bond risk premiums (e.g., Fama and Bliss, 1987, and Campbell and Shiller, 1991), the cyclicality of risk premiums in equity and bond markets (Fama and French, 1989), and the ability of interest rates to predict real economic activity (e.g., Estrella and Hardouvelis, 1991). The quest for justifying these findings has produced a large number of theoretical models. Bansal and Yaron (2004) demonstrate that long-run consumption risks and time-varying economic uncertainty in conjunction with recursive preferences go a long way in explaining important aspects of equity markets. I investigate whether the same model can explain and match properties of bonds and equity simultaneously.

This paper shows that properties of interest rates and equity can jointly be explained within the framework of Bansal and Yaron (2004). A calibrated version of the model can account for several empirical observations such as deviations from the expectations hypothesis of interest rates, the upward sloping nominal yield curve, the downward sloping term structure of volatility and the predictive power of the yield spread. However, an estimation of the model using Simulated Method of Moments yields less convincing results and highlights the difficulty of precisely estimating parameters of the model. A positive nominal yield spread predicts future real consumption growth and excess stock returns with a positive sign and inflation with a negative sign. This is in line with empirical evidence provided in the paper. Deviations from the Fisher hypothesis produce countercyclical nominal yields and real stock returns that move inversely with realized inflation. The cyclicality of nominal interest rates and yield spreads is shown to depend on the relative values of the elasticity of intertemporal substitution (EIS) and the correlation between expected inflation and expected real consumption growth.

The long-run risk model contains three main features. First, the representative agent has Epstein and Zin (1989) and Weil (1989) recursive preferences which allows the risk aversion coefficient to be separated from the EIS.[1] Second, expected consumption is subject to highly persistent shocks. These shocks represent long-run risks of consumption as they affect the distribution of risk over time. Third, the variance of consumption growth is time varying as it is also affected by persistent shocks. This is referred to as volatility risk and produces a time-varying risk premium on assets. Consumption growth being non-i.i.d. is an important feature of the model.

Parameters governing the exogenously specified law-of-motion for consumption growth, dividend growth and inflation are estimated using Simulated Method of Moments. Only moments of macro data are used in the estimation. The preference parameters are later calibrated to examine whether features of asset price data can be explained for reasonable values of risk aversion, the EIS, and the discount factor. Using the point estimates, the model has difficulties in explaining observed risk premiums without resorting to a high level of risk aversion and fails to produce enough time variation in bond risk premiums. However, the parameters of the model are estimated imprecisely which highlights the problem of detecting the long-run risk component (e.g. Bansal and Yaron, 2004, Hansen, 2007, and Hansen et al. 2008). Due to the imprecise estimation and to provide a sensitivity analysis, the model is calibrated using parameter values that all lie within two standard errors from the estimated values. The model is now capable of explaining important features of asset price data while matching macro moments reasonably well.

Positive shocks to expected consumption are associated with higher real rates. Real bonds therefore act as a hedge, generating positive returns in periods of negative shocks to expected consumption growth. Real bonds also serve as a hedge against periods of increased economic uncertainty as an increase in the variance of future consumption growth lead to lower real yields. As a result, real bonds carry negative risk premiums since the agent is provided with insurance against periods of high marginal utility. The negative slope of the real yield curve is supported by empirical evidence from UK index-linked bonds

[1] Other papers that make use of recursive preferences in asset-pricing include Campbell (1993, 1996, 1999), Duffie et al. (1997), and Restoy and Weil (1998).

(e.g., Evans, 1998, and Piazzesi and Schneider, 2006).[2]

I introduce an exogenously specified inflation process in order to model nominal yields. The specification allows for a correlation between the real and nominal sides of the economy. An estimation of the model yields a negative relationship between shocks to expected real consumption and the level of expected inflation, which produces a positive inflation risk premium. It also implies a deviation from the Fisher neutrality assumption which has been documented by several studies.[3] The non-neutrality of inflation has two effects. First, higher expected consumption raises nominal yields through the same channel as for real rates but also lowers them as high expected consumption growth is associated with low expected inflation. Nominal yields become countercyclical when the latter effect dominates, which means that nominal bonds no longer serve as a hedge against bad times. Second, long nominal yields depend positively on volatility shocks, while short-term bonds remain a safe haven in times of economic turbulence. The nominal yield curve therefore steepens as the level of uncertainty in the economy increases. The agent accordingly demands a positive risk premium for holding nominal bonds, which increases with the maturity of the bond. This allows the model to match the positive unconditional slope of the nominal yield curve.

In order to replicate deviations from the expectations hypothesis, bond-risk premiums should co-vary positively with the slope of the yield curve. Variations in the uncertainty about future consumption accomplish this. Positive shocks to economic uncertainty raise the slope of the nominal yield curve while also raising the expected excess return on long bonds. The latter effect arises as the conditional variance of consumption growth is expected to revert back to

[2]Data for US index-linked bonds only date back to 1997 and indicates a positively sloped real yield curve on average. This evidence should be interpreted with caution as the time series is rather short and the market was illiquid at the inception of trading.

[3]For example, Fama (1981) finds that real stock returns correlate negatively with inflation; Fama and Gibbons (1982) provide evidence that expected real returns on nominal bonds vary inversely with expected inflation; and Boudoukh (1993) and Evans (1998) document a negative relation between real interest rates and inflation rates.

its mean, lowering long yields. Bond excess returns therefore becomes predictable, matching the findings of Fama and Bliss (1987) and Campbell and Shiller (1991).

The procyclicality of real yields in the model is in line with the empirical findings of Chapman (1997) and relates to Harvey (1988), who finds that the term structure of real interest rates contains information about future consumption growth. The countercyclical feature of nominal yields is consistent with findings in Rendu de Lint and Stolin (2003) and Ang et al. (2007). The ability of the nominal yield spread to predict future real activity is well established (e.g., Stock and Watson, 1989, Estrella and Hardouvelis, 1991, Estrella 2005, and Ang et al. , 2006). The long-run risk model matches the observed positive correlation between nominal yield spreads and subsequent real consumption growth. Furthermore, nominal term spreads inside the model predict future excess stock returns with a positive sign and future realized inflation with a negative sign. This is consistent with empirical evidence provided in the paper. The negative correlation between inflation and consumption growth also produces real stock returns that move inversely with inflation. This has been documented by several empirical and theoretical studies (e.g. Fama, 1981, Stulz, 1986, and Lee, 1992). The relationship between nominal interest rates and economic activity is shown to depend on the relative values of the EIS parameter and the correlation between real consumption and inflation.

This paper relates to the vast literature on the term structure of interest rates. A number of studies have used general equilibrium models to explain and enhance our understanding of interest rate dynamics; early contributions include Cox et al. (1985), Dunn and Singleton (1986) and Campbell (1986).[4] Backus et al. (1989) demonstrate that a standard power-utility model with heteroscedastic consumption growth cannot generate enough time variation in risk premiums to match observed deviations from the expectations hypothesis. Donaldson et al. (1990) demonstrate that the neoclassical

[4]Other papers addressing features of asset markets include Abel (1990, 1999), Campbell and Cochrane (1999), Constantinides (1990), Constantinides and Duffie (1996), Heaton (1995), and Sundaresan (1989).

stochastic growth model generates countercyclical real interest rates and procyclical real term spreads. Brandt and Wang (2003), Wachter (2006) and Buraschi and Jiltsov (2007) provide evidence that variants of consumption-based habit models are able to match observed interest rates while replicating deviations from the expectations hypothesis. Piazzesi and Schneider (2006) explore the role of surprise inflation as a message of lower future real consumption growth. They highlight that a drop in the real payoff of nominal bonds in bad times leads investors to demand a positive risk premium for holding long-term nominal bonds. Their model is able to generate realistic moments for interest rates but the expectation hypothesis holds and they do not consider equity. Gallmeyer et al. (2007) include a Taylor rule in a setup related to the long-run risk model and demonstrate that it can produce realistic moments for interest rates. Eraker (2007) demonstrates that a continuous-time version of Bansal and Yaron (2004) can match observed yield curve moments. However, he does not consider the expectations hypothesis puzzle and the cyclical properties of the model. In a contemporaneous paper, Bansal and Shaliastovich (2007) provide evidence that the long-run risk model is able to simultaneously generate rejections of the expectations hypothesis and match the forward-premium puzzle. This paper differs from theirs in several aspects. First, I estimate the model formally and highlight the uncertainty surrounding several of the key parameters and its implication for matching features of both equity and bonds. Second, I study the cyclical properties of the model and emphasize the interplay between the value of the EIS and the correlation between consumption and inflation for matching established empirical facts.

2.2 The Model

This section provides dynamics of the model's economic variables, the preferences of the representative agent and the solutions for bond prices. For simplicity, I use the notation of Bansal and Yaron (2000, 2004) throughout the section.

2.2.1 Dynamics

The real economy is subject to three main processes:

$$\begin{aligned} g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1}, && (2.1)\\ x_{t+1} &= \rho x_t + \varphi_e \sigma_t \varepsilon_{t+1}, && (2.2)\\ \sigma_{t+1}^2 &= \sigma^2 + v_1 \left(\sigma_t^2 - \sigma^2\right) + \sigma_w w_{t+1}, && (2.3)\\ \eta_{t+1}, \varepsilon_{t+1}, w_{t+1} &\sim N.i.i.d. \left(0, 1\right). && (2.4) \end{aligned}$$

The log growth rate of consumption is denoted g_{t+1} and is determined by the unconditional mean μ, a persistent component x_t, and a shock η_{t+1}, which represents short-run risks to consumption. The persistent part serves as a state variable and causes the one-step-ahead expected growth rate to deviate from its unconditional mean. It is affected by shocks whose persistence is governed by ρ, producing uncertainty about the conditional mean of growth rates. Persistent shocks affect the conditional mean of consumption growth far into the future and therefore represent long-run risks of consumption. Consider the revision of the conditional mean of consumption growth for a horizon of n periods, $E_t\left(g_{t+n}\right) - E_{t-1}\left(E_t\left(g_{t+n}\right)\right) = \rho^{n-1}\varphi_e\sigma_{t-1}\varepsilon_t$. This revision is zero when φ_e equals zero. The second state variable is the conditional variance of consumption growth, σ_{t+1}^2. It is also subject to shocks, which produce time-varying economic uncertainty. This is referred to as volatility risk. Consumption growth being non-i.i.d. is a crucial feature of the model.

The nominal side of the economy is governed by the following processes:

$$\begin{aligned} \pi_{t+1} &= \mu_\pi + x_t^\pi + \delta_1 \sigma_t \eta_{t+1}^\pi, && (2.5)\\ x_{t+1}^\pi &= \rho_\pi x_t^\pi + \delta_2 \sigma_t \varepsilon_{t+1} + \delta_3 \sigma_t \varepsilon_{t+1}^\pi, && (2.6)\\ \eta_{t+1}^\pi, \varepsilon_{t+1}^\pi &\sim N.i.i.d. \left(0, 1\right). && (2.7) \end{aligned}$$

The log inflation rate is denoted π_{t+1} and is governed by its unconditional mean μ_π, expected inflation x_t^π, and a shock term $\delta_1\sigma_t\eta_{t+1}^\pi$. Expected inflation is modeled as an autoregressive process that is affected by shocks to expected consumption growth through δ_2. Shocks to both realized and expected inflation are heteroscedastic. All shocks in the economy, real and nominal, are uncorrelated. For parsimonious reasons, the volatility of inflation and consumption are governed by the same process.[5] The notion of heteroscedasticity in inflation is a well established empirical fact; early contributions include Engle (1982) and Bollerslev (1986). The specification of the nominal side allows for a deviation from the Fisher hypothesis and is similar to the dynamics used in for example Campbell and Viceira (2001) and Piazzesi and Schneider (2006). However, in contrast to them, I allow for heteroscedasticity. The Fisher hypothesis holds when δ_2 equals zero.

2.2.2 Investor Preferences

The representative agent in the economy has Epstein and Zin (1989) and Weil (1989) recursive preferences:

$$U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \tag{2.8}$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, $\gamma \geq 0$ denotes the risk aversion coefficient and $\psi \geq 0$ the elasticity of intertemporal substitution (EIS). The discount factor is represented by δ. This preference specification allows time preferences to be separated from risk preferences. This stands in contrast to time-separable expected utility in which the desire to smooth consumption over states and over time are interlinked. The agent prefers early (late) resolution of risk when the risk aversion is larger (smaller) than the reciprocal of the EIS. A preference for early resolution and an EIS above one imply that $\theta < 1$. This specification nests the time-separable power utility model for $\gamma = \frac{1}{\psi}$ (i.e., $\theta = 1$).

The agent is subject to the following budget constraint:

$$W_{t+1} = R_{a,t+1}\left(W_t - C_t\right), \tag{2.9}$$

where the agent's total wealth is denoted W_t, $W_t - C_t$ is the amount of wealth invested in asset markets and $R_{a,t+1}$ denotes the unobservable

[5] Introducing a separate volatility process for inflation would add one more state variable but is straightforward. Derivations are available upon request.

gross return on the total wealth portfolio. This asset delivers aggregate consumption as its dividends each period. Epstein and Zin (1989) show that this economy implies an Euler equation for asset return $R_{i,t+1}$ in the form of:

$$E_t[\underbrace{\delta^{\theta} G_{t+1}^{\frac{-\theta}{\psi}} R_{a,t+1}^{-(1-\theta)}}_{M_{t+1}} R_{i,t+1}] = 1, \tag{2.10}$$

where G_{t+1} denotes the aggregate gross growth rate of consumption and M_{t+1} denotes the intertemporal marginal rate of substitution (IMRS). The logarithm of the IMRS can be written as:

$$m_{t+1} = \theta \ln(\delta) - \frac{\theta}{\psi} g_{t+1} - (1-\theta)\, r_{a,t+1}, \tag{2.11}$$

where $\ln R_{a,t+1} = r_{a,t+1}$ and $\ln G_{t+1} = g_{t+1}$. Note that the IMRS depends on both consumption growth and on the return from the total wealth portfolio. Recall that $\theta = 1$ under power utility, which brings us back to the standard time-separable IMRS.

2.2.3 Solving the model

The return on the aggregate wealth portfolio is approximated using the analytical solutions found in Campbell and Shiller (1988):

$$r_{a,t+1} = k_0 + k_1 z_{t+1} - z_t + g_{t+1}, \tag{2.12}$$

where z_t denotes the log price-consumption ratio and constants k_0 and k_1 are functions of the average level of z_t, denoted $\bar{z}$.[6] Specifically, the constants are:

$$k_1 = \frac{\exp(\bar{z})}{1 + \exp(\bar{z})} \tag{2.13}$$

$$k_0 = \ln(1 + \exp(\bar{z})) - k_1 \bar{z}. \tag{2.14}$$

[6]Bansal et al. (2007) show that the approximate analytical solution for the wealth return is close to the numerical solution and delivers similar model implications.

Bansal and Yaron (2004) conjecture that the log price-consumption ratio z_t is a linear function of the two state variables x_t and σ_t^2:

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2. \tag{2.15}$$

Using the standard Euler equation together with the dynamics of consumption and uncertainty, Bansal and Yaron (2004) demonstrate that the solution is given by:

$$\begin{aligned} A_0 &= \frac{1}{1-k_1}\Bigg[\ln(\delta) + (1-\frac{1}{\psi})\mu + k_0 + \qquad (2.16)\\ &\quad k_1 A_2 \sigma^2 (1-v_1) + \frac{\theta}{2}(k_1 A_2 \sigma_w)^2\Bigg], \\ A_1 &= \frac{1-\frac{1}{\psi}}{1-k_1\rho}, \qquad (2.17)\\ A_2 &= \frac{1}{2}\frac{\left[(\theta-\frac{\theta}{\psi})^2 + (\theta A_1 k_1 \varphi_e)^2\right]}{\theta(1-k_1 v_1)}. \qquad (2.18) \end{aligned}$$

Ignoring the term A_0, the first coefficient, A_1, measures the sensitivity of the price-consumption ratio to changes in expected consumption growth. The coefficient is positive when the EIS exceeds one, which implies an increase in the ratio in response to higher expected consumption growth. The higher the persistence, captured by ρ, the greater the effect as shocks to expected consumption growth last longer. The second coefficient, A_2, governs the response of the price-consumption ratio to changes in economic uncertainty. The coefficient is negative when θ is negative, for example, when the risk aversion coefficient and the EIS exceed one. An increase in the variance of growth rates then pushes down the price of the consumption claim. Again, a high persistence amplifies the effect of volatility shocks.

Consider the following expression for the innovation to the real pricing kernel, where the vector λ represents market prices of risk:

$$m_{t+1} - E_t(m_{t+1}) = -[\lambda_\eta \ \lambda_\varepsilon \ \lambda_w][\sigma_t \eta_{t+1} \ \sigma_t \varepsilon_{t+1} \ \sigma_w w_{t+1}]', \tag{2.19}$$

$$\begin{aligned}\lambda_\eta &= \gamma, &(2.20)\\ \lambda_\varepsilon &= (1-\theta)k_1 A_1 \varphi_e, &(2.21)\\ \lambda_w &= (1-\theta)k_1 A_2. &(2.22)\end{aligned}$$

The crucial feature of this model is that long-run risk ε, and volatility risk w, are priced in addition to short-run risk η. The price of long-run risk λ_ε, is positive when the agent prefers early resolution of uncertainty and $\psi > 1$. Volatility risk on the other hand have a negative price if the agent prefers early resolution of uncertainty and ψ and γ exceed one. Recall that $\theta = 1$ under power utility, which means that only short-run risk is priced.

The logarithm of the nominal pricing kernel is determined by the difference between the real pricing kernel and the inflation rate:

$$m^{\$}_{t+1} = m_{t+1} - \pi_{t+1}. \tag{2.23}$$

2.2.4 Model Implications for Bond Prices

In this subsection, I derive analytical expressions and analyze model implications for real and nominal bonds. Later, the model is estimated and calibrated using Simulated Method of Moments. The model solution for equity is reported in Appendix A.1.

Real Bonds

Log prices of real bonds with a maturity of n periods are linear functions of the state variables:

$$q_{t,n} = D_{0,n} + D_{1,n}x_t + D_{2,n}\sigma_t^2. \tag{2.24}$$

Let $y_{t,n} = -\frac{1}{n}q_{t,n}$ denote the n-period continuously compounded yield. Then:

$$y_{t,n} = -\frac{1}{n}\left(D_{0,n} + D_{1,n}x_t + D_{2,n}\sigma_t^2\right). \tag{2.25}$$

Using the Euler equation of the agent, the log price of a bond can be written as:

$$q_{t,n} = E_t\left[m_{t+1} + q_{t+1,n-1}\right] + \frac{1}{2}Var_t\left[m_{t+1} + q_{t+1,n-1}\right]. \tag{2.26}$$

Using this recursive structure, Bansal and Yaron (2000) show that:

$$\begin{aligned} D_{0,n} &= \theta \ln(\delta) - \frac{\theta}{\psi}\mu + (\theta-1)(k_0 + k_1 A_0 + \\ &\quad k_1 A_2 \sigma^2 (1-v_1) - A_0 + \mu) + D_{0,n-1} + \\ &\quad D_{2,n-1}\sigma^2(1-v_1) + \frac{1}{2}\sigma_w^2((\theta-1)k_1 A_2 + D_{2,n-1})^2, \end{aligned} \tag{2.27}$$

$$D_{1,n} = \rho D_{1,n-1} - \frac{1}{\psi}, \tag{2.28}$$

$$\begin{aligned} D_{2,n} &= v_1 D_{2,n-1} + (\theta-1)A_2(k_1 v_1 - 1) + \frac{1}{2}\left(\lambda_\eta^2 + \right. \\ &\quad \left.(-\lambda_\varepsilon + \varphi_e D_{1,n-1})^2\right), \end{aligned} \tag{2.29}$$

where $D_{0,0} = D_{1,0} = D_{2,0} = 0$. These loadings determine the response of real bonds to movements in the expected mean and variance of real consumption growth. $D_{1,n}$ is negative and increasingly so with maturity which means that the price of real bonds decreases in response to higher expected consumption growth. Lowering the EIS amplifies the effect and increasing the persistence ρ, makes long bonds react more strongly than short bonds. The sign of $D_{2,n}$ depends on the preference parameters in a less straightforward way. However, the term is positive for reasonable values of the risk aversion and the EIS which implies that bond prices increase as economic uncertainty increases. The magnitude of the coefficient is increasing in the level of risk aversion and in the maturity n, of the bond which means that the prices of long bonds react more to changes in economic uncertainty than short bonds.

Nominal Bonds

Nominal bonds are a function of expected inflation, in addition to the conditional mean and variance of consumption. Let nominal bond prices and yields be denoted by superscript \$. The log price of a nominal bond then takes the form:

$$q_{t,n}^{\$} = D_{0,n}^{\$} + D_{1,n}^{\$} x_t + D_{2,n}^{\$} \sigma_t^2 + D_{3,n}^{\$} x_t^{\pi}. \tag{2.30}$$

The nominal yield can be written as:

$$y^{\$}_{t,n} = -\frac{1}{n}\left(D^{\$}_{0,n} + D^{\$}_{1,n}x_t + D^{\$}_{2,n}\sigma^2_t + D^{\$}_{3,n}x^{\pi}_t\right). \tag{2.31}$$

Using the nominal pricing kernel in (2.23) together with the Euler equation, we have that:

$$q^{\$}_{t,n} = E_t[m_{t+1} - \pi_{t+1} + q^{\$}_{t+1,n-1}] + \frac{1}{2}Var_t[m_{t+1} - \pi_{t+1} + q^{\$}_{t+1,n-1}], \tag{2.32}$$

I show in Appendix A.2 that the loadings are defined as follows:

$$\begin{aligned}
D^{\$}_{0,n} &= \theta\ln(\delta) - \frac{\theta}{\psi}\mu + (\theta-1)(k_0 + k_1A_0 + \qquad (2.33)\\
&\quad k_1A_2\sigma^2(1-v_1) - A_0 + \mu) - \mu_\pi + D^{\$}_{0,n-1} + \\
&\quad D^{\$}_{2,n-1}\sigma^2(1-v_1) + \frac{1}{2}\sigma^2_w((\theta-1)k_1A_2 + D^{\$}_{2,n-1})^2, \\
D^{\$}_{1,n} &= \rho D^{\$}_{1,n-1} - \frac{1}{\psi}, \qquad (2.34)\\
D^{\$}_{2,n} &= v_1D^{\$}_{2,n-1} + (\theta-1)A_2(k_1v_1 - 1) + \qquad (2.35)\\
&\quad \frac{1}{2}\left(\lambda^2_\eta + (-\lambda_\varepsilon + \varphi_e D^{\$}_{1,n-1} + \delta_2 D^{\$}_{3,n-1})^2 + \right.\\
&\quad \left.(D^{\$}_{3,n-1}\delta_3)^2 + \delta^2_1\right), \\
D^{\$}_{3,n} &= D^{\$}_{3,n-1}\rho_\pi - 1. \qquad (2.36)
\end{aligned}$$

where $D^{\$}_{0,0} = D^{\$}_{1,0} = D^{\$}_{2,0} = D^{\$}_{3,0} = 0$. The new term, $D^{\$}_{3,n}$, governs the response of nominal bonds to inflation. The term is negative and increasingly so for longer maturities. The response of nominal yields to changes in real consumption growth is the same as for real yields, i.e. $D_{1,n} = D^{\$}_{1,n}$. Furthermore, the introduction of inflation affects the loading on volatility as the last term in (2.35) is different from the case of real bonds. The term $(-\lambda_\varepsilon + \varphi_e D^{\$}_{1,n-1} + \delta_2 D^{\$}_{3,n-1})^2$ is important for the asset pricing implications discussed later. A negative dependence between inflation and real consumption growth, $\delta_2 < 0$, decreases the value of the squared expression which may lead to a decline in the price of nominal bonds in response to higher uncertainty in consumption growth. Hence, that term determines whether nominal bonds are a hedge or not against increases in economic uncertainty, which has implications for the slope of the nominal yield curve.

2.3 Data, Estimation, and Calibration of Model

2.3.1 Data

It is common in the term-structure literature to focus on the period after 1952 as yields were not market determined prior to the Fed-Treasury accord. A sample of real consumption and inflation data for the period 1953-2005 is therefore used. To mitigate the effect of seasonality and measurement errors in consumption data (Wilcox, 1992), annual aggregate real consumption data of nondurables and services from Bureau of Economic Analysis are used. Value-weighted market returns (NYSE/AMEX/NASDAQ) are retrieved from CRSP. Nominal interest rates are collected from the Fama-Bliss file in CRSP. The CPI is collected from Bureau of Labor Statistics. Monthly dividends are constructed as in Campbell and Shiller (1988) and Bansal et al. (2005), using monthly CRSP market returns including and excluding dividends to compute dividend yields. A series of real annual dividend growth is then formed by summing monthly dividends for the last 12 months, adjusting for inflation. Table 1 compares macro moments for the period 1953-2005, to the period commonly used in the long-run risk literature, 1930-2005. Moments for the shorter sample period are harder to match when simultaneously trying to match the level of risk premiums in the economy as both the volatility and persistence of consumption growth are lower. The unconditional contemporaneous correlation between annual consumption and dividend growth is also lower for the shorter sample, 0.33 vs. 0.57. Consumption growth and dividend growth exhibit a statistically significant amount of persistence for up to one year while inflation show evidence of persistence for up to three years. The correlation between inflation and real consumption growth for the shorter time period is negative, -0.19, but statistically insignificant. Inflation data are only used for the shorter time period to match the sample length for nominal bonds.

2.3.2 Estimation and Calibration

Economic models can be identified through either formal statistical estimation techniques (e.g. method of moments) or calibration. I choose to both estimate the model using Simulated Method of Moments and calibrate it in order to identify potential differences in parameter values and model implications. See Hansen and Heckman (1996) for a discussion of estimation vs. calibration and sensitivity analyses.

The fact that reported aggregated consumption measures consumption expenditures over a period rather than at a fixed point in time gives rise to a temporal-aggregation effect. Working (1960) shows that the time averaging of an i.i.d. process automatically induces positive autocorrelation and produces a less volatile series compared to the original one. To account for temporal-aggregation, the decision interval of the optimizing agent in the model is assumed to be monthly while targeted data consist of annual moments of observed data.[7] Annual moments implied by the model are computed by aggregating monthly observations.

The coefficients, k_1 and k_0, stemming from the log-linear approximation in Section 2.2.3 are endogenous as the average price-consumption ratio in the economy changes when the parameter configuration of the model changes. This is accounted for in the simulations and calibrations by first writing $\bar{z} = \ln(\frac{k_1}{1-k_1})$ and $k_0 = -\ln((1-k_1)^{1-k_1} k_1^{k_1})$. Solving the equation $A_0 + A_2\sigma^2 - \bar{z} = 0$ yields k_1 which then is used to compute k_0.

Simulated Method of Moments

Simulated methods of moments (SMM) is an estimation procedure that makes it possible to account for effects stemming from time-aggregation and allows for simulation of long samples. The proce-

[7] Heaton (1995) and Campbell and Cochrane (1999) are examples in which the agent's decision interval is of a higher frequency than the targeted data.

dure, which is similar to General Methods of Moments, is described in Lee and Ingram (1991) and Duffie and Singleton (1993) and aims at minimizing the distance between actual sample moments and simulated model moments. Appendix A.3 describes the SMM procedure in detail.

Parameters governing the law-of-motion for consumption, inflation, and dividends are estimated using moments of macro data exclusively. Preference parameters are left out in the estimation as they have no impact on the exogenously specified macro dynamics. The three preference parameters are later calibrated with the purpose of matching moments for stock returns and interest rates. The estimation uses moments of real consumption, dividend growth and inflation for the period 1953-2005. Parameters to be estimated are $[\mu, \mu_d, \rho, \varphi_e, v_1, \sigma_w, \sigma, \phi, \varphi_d, \mu_\pi, \delta_1, \rho_\pi, \delta_2, \delta_3]$. The moments to match are: $[E(\mu_g), E(\mu_{g_d}), E(\mu_\pi), E(\sigma_g^2), E(\sigma_{g_d}^2), E(\sigma_\pi^2), E(g_t - \mu_{g_t})(g_{t-i} - \mu_{g_{t-i}}), E(g_{d,t} - \mu_{g_{d,t}})(g_{d,t-i} - \mu_{g_{d,t-i}}), E(\pi_t - \mu_{\pi_t})(\pi_{t-i} - \mu_{\pi_{t-i}}), E(g_t - \mu_g)(g_{d,t} - \mu_{g_{d,t}}), E(g_t - \mu_g)(\pi_t - \mu_{\pi_t})]$ for $i = 1, 2, 3$ years. There are in total 14 parameters to estimate and 17 moments to match, which means there are three overidentifying restrictions. The real dynamics for the longer sample period, 1930-2005, are also estimated using moments of consumption growth and dividend growth.

Table 2 presents the estimation results. Focussing first on the shorter time period, the point estimate of ρ is close to, but less than, one (0.955). It is lower than the calibrated values commonly used in the long-run risk literature. This is partly a result of the lower persistence and volatility of consumption growth for the chosen time period compared to the longer sample period. The associated standard error of 0.053, highlights the difficulty of estimating the parameter precisely.[8] The long-run risk component, φ_e, is estimated to 0.041 with an associated standard error of 0.036. This illustrate the problem of separating the non-i.i.d. specification of consumption growth from the i.i.d. case. This is discussed in Bansal and Yaron (2004), Hansen (2007), and Hansen et al. (2008). The volatility parameters

[8] The long estimation sample of 210,000 months should greatly mitigate the well-known downward bias of the persistence parameter in an autoregressive process (Kendall, 1954).

of the model are also imprecisely measured, where the persistence of the volatility is estimated to 0.981 for the time period 1953-2005. Expected inflation is estimated to be highly persistent, where ρ_π equals 0.970. Parameter δ_2 governs mainly the covariance between consumption and inflation and is estimated to be negative, -0.069. The estimation results for the longer sample period also report a lower estimated persistence of long-run risks (0.961) than what is commonly used in the long-run risk literature. Later on, it is shown that the model carries significantly different asset pricing implications when using the estimated parameter values compared to using values that all lie within two standard errors from the estimated values.

The estimated parameter values are used to simulate the model 2,000 times, each using 636 months for the period 1953-2005 and 912 months for the period 1930-2005. The columns labeled I 1930-2005 and II 1953-2005 in Table 3 report the distribution of simulated moments. The volatility and persistence of consumption growth are particularly close to their sample moments, which is important for evaluating the model's ability to match risk premiums in the economy. The correlation between consumption and inflation for the shorter time period is also close to the sample value, -0.18 vs. -0.19. Generally, simulated moments for inflation and consumption growth are closer to their sample counterparts than for the case of dividend growth. This stems from the use of an optimal weighting matrix which downweights sample moments that are measured imprecisely, which in turn has an effect on the estimated parameter values.[9] Judging from the J-test, with a p-value of 0.003, the model specification can be rejected at conventional significance levels. Having matched the macro dynamics reasonably well, the preference parameters $[\gamma, \psi, \delta]$ are calibrated to match moments of stock returns and interest rates. The risk aversion, γ, is set to 50, the EIS, ψ, to 1.5 and the discount factor, δ, to 0.9993. Section 2.4 below motivates the chosen values and discusses the model implications.

[9]Table 1 reports that the mean and volatility of dividend growth are measured more imprecisely compared to the other macro variables.

Calibration

In response to the imprecise estimation results, the model is calibrated using parameter values that all lie within two standard errors from the estimated values. The calibration exercise also provides a sensitivity analysis of the model. Section 2.4 below shows that the model implications are significantly different compared to the estimated model. The rightmost column in Table 2 reports the calibrated values. The mean parameters are set to match the unconditional means of the sample series. The persistence parameters are increased for both long-run shocks, volatility shocks and shocks to expected inflation. The persistence of long-run risk shocks is increased from 0.955 to 0.983, the calibrated persistence of volatility shocks is set to 0.994 compared to the estimated value 0.981, and the persistence of shocks to expected inflation is calibrated to 0.996 vs. the estimated value 0.970. Parameter δ_2 is lowered to -0.090. The risk aversion γ, is set to 14, the EIS ψ, to 1.5 and the discount factor δ, to 0.9994. There is some controversy surrounding plausible values for the EIS. For example, Bansal et al. (2007) and Attanasio and Vissing-Jorgensen (2003) estimate it to be in excess of one while Hall (1988) and Campbell (1999) document that it is close to zero. Section 2.4.4 demonstrates that the value of the EIS together with the correlation between real consumption growth and inflation are the most important determinants of the model's cyclical properties.

The column labeled III 1953-2005 in Table 3 reports the distribution of simulated macro moments using the calibrated parameters values. The unconditional means are almost exactly matched. The average volatility and persistence of consumption growth and inflation exceed the ones observed in data. The average correlation between consumption growth and dividend growth is 0.16, which is lower than the observed value of 0.33. The model implied correlation between consumption growth and inflation is more negative than what is observed in the data, -0.46 vs. -0.19. Observed moments all lie within the 5th-95th percentiles of simulated data.

The preference parameters used throughout the paper imply that the agent prefers early resolution of uncertainty. Furthermore, the use of Epstein-Zin utility implies that long-run risk and volatility risk are priced. The former source of risk has a positive price, $\lambda_\varepsilon > 0$, which means that positive shocks to expected consumption lower the IMRS. The latter risk has a negative price, $\lambda_w < 0$, which means that positive shocks to the uncertainty of consumption increases the IMRS.

2.4 Implications for Asset Prices

This section describes the dynamics of asset prices, generated from the estimation and calibration exercises described above. Columns labeled I 1930-2005 and II 1953-2005 in the tables referred to below use estimated parameter values when simulating the model while the columns labeled III 1953-2005 use calibrated parameter values.

2.4.1 Real Term Structure

Real yields are generated from the real dynamics in Section 2.2.1. Consider the innovation to yields:

$$y_{t+1,n} - E_t(y_{t+1,n}) = -\frac{1}{n}(D_{1,n}\varphi_e\sigma_t\varepsilon_{t+1} + D_{2,n}\sigma_w w_{t+1}). \tag{2.37}$$

Real bond prices are negatively related to long-run risk, i.e. $D_{1,n}$ is less than zero, which leads to higher yields in response to positive shocks to expected consumption growth. Real yields are therefore procyclical. This means that real bonds act as a hedge against periods of low consumption growth. The loadings on volatility risk, $D_{2,n}$, are positive which indicates that real bonds also act as a hedge against positive shocks to economic uncertainty with long bonds being more sensitive than short bonds. Accordingly, the agent demands a negative risk premium for holding real bonds as they provide insurance against periods of high marginal utility. Let $h_{t+1,n} = q_{t+1,n-1} - q_{t,n}$ denote the one period log holding period return for a bond with a maturity of n periods. The risk premium can be written as:

$$\begin{aligned} &E_t(h_{t+1,n} - r_{f,t}) + \frac{1}{2}Var_t(h_{t+1,n}) = -Cov_t(m_{t+1}, h_{t+1,n}), \\ &= \lambda_\varepsilon\varphi_e D_{1,n-1}\sigma_t^2 + \lambda_w D_{2,n-1}\sigma_w^2, \end{aligned} \tag{2.38}$$

where the variance term on the left-hand side is a Jensen's inequality term. The risk premium depends on the market prices of risk and the loadings on long-run and volatility risks. Note that the risk premium is independent of short-run risks. A positive price of long-run risks and a negative value of $D_{1,n-1}$ imply a negative risk premium. Similarly, a negative price of volatility risk and a positive value of $D_{2,n-1}$ also imply a negative expected excess return.[10] The stochastic volatility of consumption growth σ_t^2, gives rise to a risk premium that varies over time. Recall that both λ_ε and λ_w equal zero under power utility, implying that expected excess returns are constant.

Next, consider the unconditional slope of the real yield curve measured as the long rate (60 months) minus the short rate (1 month):

$$E(y_{t,60} - y_{t,1}) = \left(D_{0,1} - \frac{D_{0,60}}{60} \right) + \left(D_{2,1} - \frac{D_{2,60}}{60} \right) \sigma^2, \tag{2.39}$$

which is mainly determined by the average level of uncertainty in the economy and the difference in loadings across maturities on volatility shocks. A higher sensitivity of long yields to volatility shocks contributes to a negative slope. Table 4 reports implications for the term structure of real interest rates. It shows that both the estimated and calibrated model produce a downward sloping real yield curve which is supported by Evans (1998) and Piazzesi and Schneider (2006) who document a downward sloping yield curve for UK index-linked bonds. Data for US index-linked bonds indicate a positive unconditional slope but the time series only dates back to 1997 and the market was illiquid at the beginning of the sample. Ang et al. (2007) estimate a regime-switching model on US data for the period 1952-2004 and find the real yield curve to be fairly flat with some regimes in which the curve is downward sloping. The long-run risk model also produces a downward sloping term structure of volatility and highly persistent real yields. This is consistent with data from both the US and the UK.

[10] Bansal and Yaron (2000) briefly mention that their model generates negative risk premiums for real bonds but they do not elaborate further on the issue.

2.4.2 Nominal Term Structure

Nominal yields are generated using the dynamics in Section 2.2.1. Consider the innovation to nominal yields:

$$\begin{aligned} y^{\$}_{t+1,n} - E_t\left(y^{\$}_{t+1,n}\right) &= -\frac{1}{n}\left(\left(D^{\$}_{1,n}\varphi_e + D^{\$}_{3,n}\delta_2\right)\sigma_t\varepsilon_{t+1} + \right. \\ &\quad \left. D^{\$}_{2,n}\sigma_w w_{t+1} + D^{\$}_{3,n}\delta_3\sigma_t\varepsilon^{\pi}_{t+1}\right). \end{aligned} \tag{2.40}$$

The response of nominal rates to long-run consumption risks ε_{t+1}, depends on both the original loading $D^{\$}_{1,n}$, and on the term $D^{\$}_{3,n}\delta_2$. Recall that a negative value of δ_2 implies that periods of high expected consumption are associated with low expected inflation. This causes nominal yields to decrease or increase less than real yields as expected consumption is revised upwards. The unconditional correlation between nominal yields and real consumption growth becomes negative when the effects of deviations from the Fisher hypothesis dominate the real effect. Recall from (2.35) that the loading on volatility risk for nominal bonds is different than for real bonds. Long yields now rise or fall less than the short rate as future consumption growth becomes more uncertain. As a result, long nominal bonds do not provide insurance against bad times which warrants a positive risk premium. Furthermore, positive inflation shocks ε^{π}_{t+1}, increase nominal yields since $D^{\$}_{3,n}\delta_3$ is negative. The average nominal slope can be written as for real bonds, but with nominal coefficients:

$$E(y^{\$}_{t,60} - y^{\$}_{t,1}) = \left(D^{\$}_{0,1} - \frac{D^{\$}_{0,60}}{60}\right) + \left(D^{\$}_{2,1} - \frac{D^{\$}_{2,60}}{60}\right)\sigma^2. \tag{2.41}$$

As short maturity bonds provide a better hedge against volatility shocks than long bonds, the last term is positive. Implications for the nominal yield curve are reported in Table 5. The estimated model produces a yield curve with a positive slope of 19 basis points, measured as the difference in yields between the five-year and the one-month bond. This is less than the observed slope of 132 basis points. The calibrated model performs better with a slope of 119 basis points. The main difference lies in the parameter δ_2 which governs the correlation between inflation and real consumption growth. This parameter is more negative in the calibrated case and therefore produces a higher inflation risk premium. This comes at the expense of producing an average correlation between consumption and inflation that exceeds the observed value. However, the simulated 5th-95th percentiles include the observed value. The slope is also a function of the risk aversion, as with any risk premium in the economy. For example, setting the risk aversion to five in the estimated model produces a flat curve. Both versions of the model produce a term structure that is more linear than what is observed in data. The relatively large observed difference between the one-year and the one-month rate is particularly hard to match. The model also generates a downward sloping term structure of volatility and highly persistent yields as they inherit the persistence from the state variables. The higher persistence of shocks in the calibrated model results in a smaller difference in volatility between long and short bonds. Finally, nominal yields are more volatile than real yields reflecting the volatility of inflation.

The Inflation Risk Premium

The yield on a nominal bond with a maturity of n periods can be expressed as the sum of the corresponding real yield, the expected inflation over the bond's maturity, the inflation risk premium, and a Jensen's inequality term:

$$\begin{aligned} y^{\$}_{t,n} \quad = \quad & y_{t,n} + \frac{1}{n}E_t\left(\sum_{i=1}^{n}\pi_{t+i}\right) + \frac{1}{n}Cov_t\left(\sum_{i=1}^{n}m_{t+i}, \sum_{i=1}^{n}\pi_{t+i}\right) - \\ & \frac{1}{2n}Var_t\left(\sum_{i=1}^{n}\pi_{t+i}\right). \end{aligned} \tag{2.42}$$

The inflation risk premium is positive if inflation is high during periods of high marginal utility, leading to higher yields. This occurs in the model as negative shocks to real consumption growth are estimated to have a positive effect on expected inflation. The covariance is zero in the model when n equals one month implying a zero inflation risk premium for the short rate. However, the covariance becomes positive and increasing as n increases and δ_2 becomes more negative, resulting in an upward sloping term structure of the inflation risk premium.

2.4.3 Cyclical Properties of Interest Rates

Table 6 documents lead, contemporaneous, and lag correlations between three-month, one-year, and five-year interest rates and the one-year real consumption growth rate. The population values for real rates are all positive which matches the empirical findings of Chapman (1997). It also relates to Harvey (1988) who finds that the term structure of real interest rates contains information about future consumption growth. Correlations are higher for the calibrated model as consumption shocks are more persistent. Nominal rates in both versions of the model are unconditionally countercyclical, correlating negatively with consumption growth.[11] The correlation coefficients found in data are also negative but not significantly different from zero, except for the correlation between the three-month rate and the contemporaneous consumption growth rate. Countercyclical nominal yields are also found in Rendu de Lint and Stolin (2003) and Ang et al. (2007).[12] The calibrated model produces correlations that are more negative than what is observed in data as a result of the lower value of δ_2. See Section 2.4.4 below for a fuller discussion.

Table 7 reports results of predicting annual real consumption growth rates, excess stock returns, and inflation using the nominal yield spread. The model produces positive population coefficients when predicting future annual consumption growth. This matches the well established empirical fact, reported in the same table, that a positively sloped nominal yield curve tends to predict an increase in future real economic activity (e.g., Stock and Watson, 1989, Estrella and Hardouvelis, 1991, Estrella 2005, and Ang et al. , 2006). Furthermore, the model-implied nominal yield spread predicts future excess stock returns with a positive sign and future inflation with a negative sign.

[11] The Monte-Carlo distributions for the estimated model suggest that the correlations are not statistically different from zero.

[12] Correlations seem to vary over time. For example, Rendu de Lint and Stolin (2003) report more negative correlations for the period 1960-1998.

This matches the empirical evidence, also provided in the same table. Both the calibrated and estimated model produce population regression coefficients with the same sign as in data. The real yield spread in the model also contains predictive power. The numerical results are not reported but a positive real yield spread predicts negative subsequent consumption growth.

2.4.4 The Role of the EIS and Deviations from the Fisher Hypothesis

This section illustrates how the cyclicality of nominal interest rates and yield spreads depends on the EIS and on deviations from the Fisher hypothesis. For brevity, numerical results will only be reported for the calibrated model but same principles hold for the estimated model.

Table 8 demonstrates that lowering the EIS from 1.5 to 0.2 leads to nominal rates being procyclical. Increasing δ_2 to -0.01, i.e. reducing the negative correlation between consumption and inflation, has a similar effect. The same table demonstrates that the sign of the slope coefficients when predicting consumption growth also depends on the value of the EIS. As the EIS is lowered from 1.5, the coefficient approaches zero and eventually switches sign. An EIS equal to 0.2 generates slope coefficients with the opposite sign of what is observed in data. Again, increasing δ_2 has a similar effect. Hence, reducing the negative dependence between inflation and consumption and/or reducing the EIS makes nominal bonds (yield spreads) behave more like real bonds (yield spreads).

The underlying mechanism can be illustrated by considering the real and nominal short rate. Consider first the real short rate:

$$y_{t,1} = constant + \frac{1}{\psi}x_t - \sigma_t^2\left[(\theta-1)\,A_2(k_1 v_1 - 1) + \frac{1}{2}\left(\lambda_\eta^2 + \lambda_\varepsilon^2\right)\right], \tag{2.43}$$

which is derived from the results in Section 2.2.4. In consumption-based models of this sort, it is well known that higher expected real consumption growth leads to an increase in real yields for which the magnitude is determined by the reciprocal of the EIS. The unconditional covariance between the one period real yield at time t and the consumption growth at time $t+1$ is always positive since:

$$Cov\left(y_{t,1}, g_{t+1}\right) = Cov\left(-D_{1,1}x_t, x_t\right) = \frac{1}{\psi}Var\left(x_t\right) > 0. \tag{2.44}$$

The same conclusion holds for a n-period bond and consumption growth over the period t to $t+n$. Changing the EIS therefore only affects the magnitude to which real yields are procyclical, not the sign of the correlation.

This is not true for nominal bonds due to deviations from the Fisher hypothesis. Consider the nominal short rate:

$$\begin{aligned} y_{t,1}^{\$} &= constant + \frac{1}{\psi}x_t - \sigma_t^2\left[\left(\theta-1\right)A_2(k_1 v_1 - 1) + \right. \\ &\quad \left. \frac{1}{2}\left(\lambda_\eta^2 + \lambda_\varepsilon^2 + \delta_1^2\right)\right] + x_t^\pi. \end{aligned} \tag{2.45}$$

An increase in expected consumption x_t affects the nominal short rate in two ways. First, it affects the nominal short rate positively through the same channel as in the real case. Second, it affects the short rate negatively as high realizations of x_t are associated with periods of low x_t^π. The first effect dominates for low values of the EIS or high values of δ_2 while the second effect dominates for high values of the EIS or low values of δ_2. This determines whether nominal yields are pro- or countercyclical, which can be seen from the covariance between the one period nominal yield and the consumption growth at time $t+1$:

$$\begin{aligned} Cov\left(y_{t,1}^{\$}, g_{t+1}\right) &= Cov\left(-D_{1,1}^{\$}x_t - D_{3,1}^{\$}x_t^\pi, x_t\right) = \\ & \quad Cov\left(\frac{1}{\psi}x_t + x_t^\pi, x_t\right) \gtrless 0. \end{aligned} \tag{2.46}$$

The same conclusion holds for a n-period bond and consumption growth over the period t to $t+n$. This suggests that instability of the relationship between inflation and consumption over time leads to changes in the cyclicality of nominal interest rates over time. A similar logic holds for the covariance between the nominal yield spread at time t and consumption growth at $t+1$:

$$Cov\left[\left(\frac{D^{\$}_{1,12}}{12}-\frac{D^{\$}_{1,60}}{60}\right)x_t+\left(\frac{D^{\$}_{3,12}}{12}-\frac{D^{\$}_{3,60}}{60}\right)x^{\pi}_t, x_t\right], \tag{2.47}$$

where the loadings on expected consumption and inflation are both negative. The same two counteracting effects are at work. A high EIS and/or a low δ_2 is needed to produce regression coefficients with the same sign as observed in data. This suggests that the documented differences in the predictive power of the yield spread across countries and time periods (e.g., Campbell, 2003) is partly due to differences in the EIS and/or in the relationship between consumption and inflation. A recent paper by Campbell et al. (2007) explores the latter.

2.4.5 Expectations hypothesis

The expectations hypothesis can be expressed in different forms (e.g. Cox et al. ,1981, and Campbell et al. ,1997). One version states that log excess holding period returns for bonds differ across maturities but are constant through time. Evidence documented in Fama and Bliss (1987) and Campbell and Shiller (1991) indicate that risk premiums for US nominal bonds in fact seem to vary over time. While most of the empirical literature has been focusing on nominal yields, Evans (1998, 2003) document time-varying risk premiums also for real bonds using data from the UK.

Campbell and Shiller (1991) run the following regression:

$$y^{\$}_{t+m,n-m}-y^{\$}_{t,n}=\alpha_{n,m}+\beta_{n,m}\left(\frac{m}{n-m}\right)\left(y^{\$}_{t,n}-y^{\$}_{t,m}\right)+\epsilon_{t+m,n},$$

where m denotes the time step forward. The expectations hypothesis says that the current yield spread is a perfect predictor of the change in long-term yields. This assumes that excess returns on bonds are constant and the regression should therefore yield a $\beta_{n,m}$ of one. To see this, write the excess period log return for $m=1$ as:

$$\begin{aligned} hx_{t+1,n} &= q_{t+1,n-1}-q_{t,n}-y_{t,1}=(y_{t,n}-y_{t,1})- \\ &\quad (n-1)(y_{t+1,n-1}-y_{t,n}). \end{aligned}$$

Taking expectations of both sides gives that the current yield spread depends on the expected change in yields and on expected excess returns. The expectations hypothesis holds when the latter is constant. However, we know that risk premiums in the model vary over time due to the stochastic volatility of consumption growth.

Table 10 reports regression results for nominal interest rates, both from inside the model and from the data. The well-known negative slope coefficients obtained in data imply that a steepening of the yield curve tends to predict lower long yields in the future. Alternatively, an increase in the slope predicts higher expected excess returns on long bonds. This can be seen by rewriting the Campbell and Shiller regression, regressing excess returns for a n-period bond onto the yield spread. Such a regression would have a slope coefficient of $1 - \beta_{n,m}$. The same table shows that the calibrated model is able to produce similar coefficients as in data. We know from (2.41) that the nominal term spread steepens as volatility of consumption growth goes up. After a positive volatility shock, the uncertainty in the economy is expected to revert back to its mean. This implies an increase in the expected excess returns of long bonds over short bonds. Hence, movements in the slope of the yield curve and expected excess returns are positively correlated. A high persistence of shocks is needed for the model to match the observed coefficients. The estimated model exhibits too low persistence and fails to match violations of the expectations hypothesis. Table 9 reports that the calibrated model also generates rejections of the expectations hypothesis for real interest rates, but to a smaller extent than for nominal rates.

2.4.6 Equity

Moments for equity are reported in Table 11. Both the estimated and calibrated model produce a similar equity premium as observed in data. However, the estimated model needs a risk aversion of 50 in order to compensate for the lower persistence of shocks. The estimated model produces a less volatile excess return and therefore a higher Sharpe ratio due to the lower volatility of dividend growth and the lower degree of persistence.

Several empirical and theoretical studies document and attempt to explain the negative relationship between real stock returns and inflation (e.g. Fama, 1981, Stulz, 1986, and Lee, 1992). As shown in Table 12, the model reproduces this finding. This can be seen from the innovation to real market returns:

$$r_{m,t+1} - E_t(r_{m,t+1}) = k_{1,m}A_{1,m}\varphi_e\sigma_t\varepsilon_{t+1} + k_{1,m}A_{2,m}\sigma_w w_{t+1} + \varphi_d\sigma_t u_{t+1}.$$

See Appendix A.1 for the model solution for equity. Since the innovation to real stock returns depends positively on long-run consumption shocks ε_{t+1}, the estimated negative relationship between ε_{t+1} and the level of expected inflation x^{π}_{t+1} causes real stock returns to move inversely with inflation.

2.5 Conclusion

I show that a calibrated model featuring recursive preferences, long-run consumption risks, time-varying uncertainty, and deviations from the Fisher hypothesis simultaneously can explain the dynamics and cyclical properties of interest rates and the level and volatility of equity returns. While matching moments of consumption growth, inflation and real stock market returns reasonably well, the model can account for deviations from the expectations hypothesis, the upward sloping nominal yield curve, the downward sloping term structure of volatility and the predictive power of the nominal yield curve. Nominal bonds in the model are subject to positive risk premiums as deviations from the Fisher hypothesis turn them into risky assets. Furthermore, an increase in economic uncertainty steepens the nominal yield curve while also raising expected excess returns on long bonds. As a result, the model matches violations of the expectations hypothesis. Consistent with empirical findings, nominal term spreads have predictive power of future consumption growth, excess stock returns and inflation. The cyclical properties of nominal interest rates and yield spreads are shown to depend on the relative values of the elasticity of intertemporal substitution and the correlation between real consumption growth and inflation.

An estimation of the model using Simulated Method of Moments illustrates the difficulty of accurately estimating parameters of the long-run risk model using only macro data. Using the point estimates, the model fails to match violations of the expectations hypothesis and needs a high level of risk aversion to match risk premiums in the economy. The sensitivity of the model for given parameter values is discussed.

Appendix

A.1 Equity

The dividend growth process follows the following process:

$$g_{d,t+1} = \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1}, \tag{2.48}$$

where μ_d denotes the average dividend growth rate, ϕ the leverage of the dividend paying claim onto expected consumption and $u_{t+1} \sim N.i.i.d.(0,1)$. The log return on the market portfolio $r_{m,t}$, is approximated using the analytical solutions found in Campbell and Shiller (1988):

$$r_{m,t+1} = k_{0,m} + k_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1}, \tag{2.49}$$

where $z_{m,t}$ denotes the log price-dividend ratio and constants $k_{0,m}$ and $k_{1,m}$ are functions of the average level of z_m.

Bansal and Yaron (2004) conjecture that z_m is a linear function of the two state variables:

$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2. \tag{2.50}$$

Using the standard Euler equation together with the dynamics of consumption and uncertainty, Bansal and Yaron (2004) demonstrate that the solution is given by:

$$\begin{aligned} A_{0,m} &= \frac{1}{1-k_{1,m}}[\theta \ln(\delta) - \frac{\theta}{\phi}\mu + (\theta - 1)(k_0 + k_1 A_0 + \\ &\quad k_1 A_2 \sigma^2 (1 - v_1) - A_0 + \mu) + k_{0,m} + \\ &\quad k_{1,m} A_{2,m} \sigma^2 (1 - v_1) + \mu_d + \frac{1}{2}\sigma_w^2((\theta - 1) k_1 A_2 + \\ &\quad k_{1,m} A_{2,m})^2], \end{aligned} \tag{2.51}$$

$$A_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - k_{1,m}\rho}, \tag{2.52}$$

$$A_{2,m} = \frac{(1-\theta)A_2(1 - k_1 v_1) + \frac{1}{2}H_m}{1 - k_{1,m} v_1}, \tag{2.53}$$

where $H_m = \lambda_\eta^2 + (-\lambda_\varepsilon + k_{1,m} A_{1,m} \varphi_e)^2 + \varphi_d^2$. The risk premium on the market portfolio can be written as:

$$\begin{aligned} E_t(r_{m,t+1} - r_{f,t}) + \frac{1}{2} Var_t(r_{m,t+1}) &= -Cov_t(m_{t+1}, r_{m,t+1}) = \\ k_{1,m} A_{1,m} \varphi_e \lambda_\varepsilon \sigma_t^2 + k_{1,m} A_{2,m} \lambda_w \sigma_w^2, \end{aligned} \tag{2.54}$$

which varies over time as σ_t^2 fluctuates.

A.2 Pricing nominal bonds

The conditional moments in the Euler equation (2.32) can be found using the real and nominal dynamics:

$$\begin{aligned} & E_t(m_{t+1} - \pi_{t+1} + q^\$_{t+1,n-1}) = \\ & \quad \theta \ln(\delta) - \frac{\theta}{\psi}(\mu + x_t) + (\theta - 1)(k_0 + k_1(A_0 + A_1 \rho x_t + \\ & \quad A_2(\sigma^2 + v_1(\sigma_t^2 - \sigma^2))) - A_0 - A_1 x_t - A_2 \sigma_t^2 + \mu + x_t) - \\ & \quad \mu_\pi - x_t^\pi + D^\$_{0,n-1} + D^\$_{1,n-1} \rho x_t + \\ & \quad D^\$_{2,n-1}(\sigma^2 + v_1(\sigma_t^2 - \sigma^2)) + D^\$_{3,n-1} \rho_\pi x_t^\pi, \end{aligned} \tag{2.55}$$

$$\begin{aligned} & Var_t(m_{t+1} - \pi_{t+1} + q^\$_{t+1,n-1}) = \\ & \quad Var_t[\sigma_t \eta_{t+1}(-\frac{\theta}{\psi} + \theta - 1) + \sigma_t \varepsilon_{t+1}((\theta - 1) k_1 A_1 \varphi_e + \\ & \quad D^\$_{1,n-1} \varphi_e + D^\$_{3,n-1} \delta_2) + \sigma_w w_{t+1}((\theta - 1) k_1 A_2 + \\ & \quad D^\$_{2,n-1}) + \sigma_t \varepsilon_t^\pi (D^\$_{3,n-1} \delta_3) - \sigma_t \eta^\pi_{t+1} \delta_1] \\ & = \sigma_t^2 (-\frac{\theta}{\psi} + \theta - 1)^2 + \sigma_t^2((\theta - 1) k_1 A_1 \varphi_e + \\ & \quad D^\$_{1,n-1} \varphi_e + D^\$_{3,n-1} \delta_2)^2 + \sigma_t^2 (D^\$_{3,n-1} \delta_3)^2 + \\ & \quad \sigma_t^2 \delta_1^2 + \sigma_w^2((\theta - 1) k_1 A_2 + D^\$_{2,n-1})^2. \end{aligned} \tag{2.56}$$

Collecting terms for the state variables, using Equation (2.30), yields the loadings in (2.33)-(2.36).

A.3 Simulated Method of Moments

This appendix describes how Simulated Method of Moments (SMM) is implemented. SMM is based on generalized method of moments and aims at minimizing the distance between observed and simulated moments. Following the notation in Lee and Ingram (1991), let $\{x_t, t = 1, ..., T\}$ denote the vector of annually observed data where $T = 53$ for the time period $1953 - 2005$ and $T = 76$ for the time period $1930 - 2005$. Let $\{y_j(\beta), j = 1, ..., N\}$ denote the simulated time series obtained from the model where $N = nT$. Annual moments implied by the model are computed by aggregating simulated monthly observations. A simulated sample of 210,000 months is used for each estimation, which are aggregated to $N = 17,500$ annual observations. The vector β contains the free parameters to be estimated. Define:

$$\begin{aligned} H_T(x) &= \frac{1}{T}\sum_{t=1}^{T} h(x_t), && (2.57) \\ H_N(y(\beta)) &= \frac{1}{N}\sum_{j=1}^{N} h(y_j(\beta)), && (2.58) \end{aligned}$$

where $H_T(x)$ is a s x 1 vector of statistics based on the observed data and $H_N(y(\beta))$ is a corresponding vector based on the simulated series. The moment conditions used are described in the text. The simulated estimator $\hat{\beta}_{TN}$ is the solution to:

$$\min_{\beta}[H_T(x) - H_N(y(\beta))]' W_T [H_T(x) - H_N(y(\beta))], \tag{2.59}$$

where W_T is a random s x s weighting matrix having a rank of at least $dim(\beta)$. See Lee and Ingram (1991) for conditions that establish consistency and asymptotic normality of the estimator. It is required that W_T converges in probability to matrix W. Based on Hansen (1982), the optimal choice for W is:

$$
\begin{aligned}
W &= [(1+\frac{1}{n})\Omega]^{-1}, \\
\Omega &= \sum_{i=-\infty}^{\infty} R_x(i), \\
R_x(i) &= E\left\{[h(x_t)-E(h(x_t))][h(x_{t-i})-E(h(x_{t-i}))]'\right\}.
\end{aligned}
$$

I use the optimal weighting matrix throughout the paper. In that case, the following holds:

$$
\sqrt{T}(\hat{\beta}_{TN}-\beta_0) \xrightarrow{d} N(0,(B'(1+\frac{1}{n})^{-1}\Omega^{-1}B)^{-1}),
$$

where $B = E[\frac{\partial h(y_j(\beta))}{\partial \beta}]$, which must be of full rank. I estimate the covariance matrix Ω using the Newey and West (1987) estimator with 4 lags. The minimization of (2.59) is done numerically. Importantly, the random errors for the simulated series should be held fixed for every iteration in order to not violate the continuity assumption concerning the objective function.

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Table 1: Sample Moments

	1930–2005		1953–2005	
	Moment	SE	Moment	SE
Consumption growth, g				
Mean	1.98	(0.32)	2.19	(0.18)
Std.Dev.	2.17	(0.53)	1.09	(0.08)
AC1	0.45	(0.11)	0.33	(0.11)
AC2	0.15	(0.14)	–0.03	(0.17)
AC3	–0.11	(0.14)	–0.09	(0.15)
Dividend growth, g_d				
Mean	0.93	(1.18)	1.52	(0.89)
Std.Dev.	11.25	(2.37)	5.80	(0.92)
AC1	0.19	(0.15)	0.28	(0.10)
AC2	–0.22	(0.16)	0.04	(0.07)
AC3	–0.16	(0.10)	–0.17	(0.11)
Inflation rate, π				
Mean	3.21	(0.74)	3.77	(0.70)
Std.Dev.	4.16	(0.72)	2.86	(0.53)
AC1	0.62	(0.11)	0.77	(0.08)
AC2	0.27	(0.14)	0.48	(0.12)
AC3	0.09	(0.11)	0.36	(0.09)
Correlations				
g and g_d	0.57	(0.18)	0.33	(0.32)
g and π	0.46	(0.18)	–0.19	(0.27)

The table presents sample moments for annual data on consumption growth, dividend growth, and inflation for two different sub-samples, 1930–2005 and 1953–2005. AC1, AC2, and AC3 refer to the first, second, and third autocorrelation. Standard errors, denoted SE, are computed as in Newey and West (1987), using four lags, and given in parentheses.

Table 2: Estimated and Calibrated Parameters

	SMM 1930–2005		SMM 1953–2005		Calibration 1953–2005
	Value	SE	Value	SE	Value
Real Parameters					
μ	0.00143	(0.00071)	0.00192	(0.00016)	0.00182
μ_d	0.00036	(0.00242)	0.00024	(0.00105)	0.00126
ρ	0.961	(0.089)	0.955	(0.053)	0.983
φ_e	0.068	(0.202)	0.041	(0.036)	0.043
v_1	0.982	(0.944)	0.981	(0.886)	0.994
$\sigma_w * 10^{-5}$	0.384	(2.075)	0.211	(2.079)	0.200
σ	0.0047	(0.0621)	0.0026	(0.0483)	0.0015
ϕ	3.673	(1.524)	6.274	(2.679)	3.100
φ_d	3.240	(3.041)	2.020	(3.118)	7.600
Inflation Parameters					
μ_π			0.00291	(0.00055)	0.00314
δ_1			0.116	(1.579)	0.010
ρ_π			0.970	(0.025)	0.996
δ_2			–0.069	(0.096)	–0.090
δ_3			0.147	(0.049)	0.050
Preference Parameters					
δ			0.9993		0.9994
γ			50		14
ψ			1.5		1.5

The table presents estimated parameters for two different sub-samples, 1930–2005 and 1953–2005, and calibrated parameters for the latter sub-period. Simulated Method of Moments (SMM) is used for the estimation. Preference parameters are calibrated. Annual data is used throughout the estimation. The sample covariance matrix in the SMM procedure is computed as in Newey and West (1987), using four lags. Standard errors are given in parentheses. A sample length of 210,000 months is used in the estimation. The moments used for the longer period estimation are the mean, variance, and 1-3 year autocovariance of real consumption and dividend growth and the covariance between the two macro variables. Estimation results for the shorter time period make use of the same moments plus the mean, variance, and 1-3 year autocovariance of inflation and the covariance between inflation and real consumption growth.

Table 3: Simulated Macro Moments

	I 1930–2005			II 1953–2005			III 1953–2005		
	Mean	5%	95%	Mean	5%	95%	Mean	5%	95%
Consumption growth, g									
Mean	1.71	1.09	2.25	2.30	1.94	2.64	2.19	1.43	2.92
Std.Dev.	1.99	1.54	2.45	1.09	0.75	1.33	1.37	0.90	1.92
AC1	0.48	0.34	0.61	0.31	0.07	0.52	0.45	0.18	0.68
AC2	0.23	0.02	0.42	0.04	–0.22	0.32	0.21	–0.11	0.52
AC3	0.06	–0.13	0.26	0.01	–0.24	0.27	0.15	–0.16	0.43
Dividend growth, g_d									
Mean	0.41	–1.98	2.88	0.29	–1.28	1.82	1.52	–1.84	4.83
Std.Dev.	6.82	5.19	8.55	3.54	2.40	4.59	8.55	5.58	11.88
AC1	0.49	0.29	0.64	0.56	0.36	0.72	0.25	0.02	0.48
AC2	0.20	–0.04	0.43	0.24	–0.04	0.49	0.01	–0.24	0.28
AC3	0.11	–0.12	0.34	0.09	–0.19	0.37	0.00	–0.27	0.25
Inflation rate, π									
Mean				3.55	2.10	4.82	3.76	–1.19	8.95
Std.Dev.				2.38	1.54	3.27	3.40	1.62	5.45
AC1				0.72	0.55	0.84	0.87	0.74	0.95
AC2				0.43	0.13	0.65	0.71	0.44	0.89
AC3				0.24	–0.09	0.51	0.58	0.22	0.82
Correlations									
g and g_d	0.47	0.25	0.66	0.38	0.08	0.61	0.16	–0.13	0.45
g and π				–0.18	–0.49	0.13	–0.46	–0.72	–0.13
SMM Statistics									
J-stat	8.79			13.78					
p-val	0.012			0.003					

The table presents simulated macro moments. Columns I 1930-2005 and II 1953-2005 use estimated parameter values obtained from Simulated Methods of Moments and column III 1953-2005 uses calibrated parameter values. AC1, AC2, and AC3 refer to the first, second, and third autocorrelation. The reported statistics refer to 2,000 Monte-Carlo simulations, each using 912 months for the period 1930–2005 and 636 months for the period 1953–2005. Monthly observations are time-aggregated to 76 and 53 annual observations, respectively.

Table 4: Term structure of real interest rates

	II 1953–2005			III 1953–2005		
	Mean	Std	AC1	Mean	Std	AC1
Maturity						
1m	1.48	0.56	0.95	1.09	0.67	0.97
1y	1.26	0.52	0.95	0.93	0.65	0.97
2y	1.04	0.49	0.96	0.76	0.63	0.97
3y	0.85	0.46	0.96	0.58	0.62	0.97
4y	0.68	0.42	0.96	0.41	0.61	0.97
5y	0.53	0.39	0.96	0.23	0.61	0.97

The table presents the model-implied term structure of real interest rates. Column II 1953-2005 uses estimated parameter values obtained from Simulated Methods of Moments and column III 1953-2005 uses calibrated parameter values. All yields are in annualized percentages. Autocorrelation coefficients (AC1) are computed for a lag of one month. Reported statistics refer to 2,000 Monte-Carlo simulations, each using 636 months.

Table 5: Term structure of nominal interest rates

	II 1953–2005			III 1953–2005			Data 1953-2005		
	Mean	Std	AC1	Mean	Std	AC1	Mean	Std	AC1
Maturity									
1m	5.00	2.42	0.95	4.84	2.82	0.98	4.88	2.78	0.97
1y	5.01	2.06	0.95	5.00	2.79	0.98	5.60	2.90	0.98
2y	5.03	1.75	0.95	5.20	2.73	0.98	5.81	2.84	0.99
3y	5.08	1.50	0.94	5.44	2.70	0.98	5.98	2.77	0.99
4y	5.13	1.31	0.93	5.72	2.66	0.98	6.11	2.73	0.99
5y	5.19	1.15	0.93	6.03	2.64	0.97	6.20	2.70	0.99
5y–1m	0.19	0.45	0.96	1.19	0.43	0.97	1.32	1.08	0.87

The table reports the model-implied and observed nominal term structure of interest rates. Column II 1953-2005 uses estimated parameter values obtained from Simulated Methods of Moments and column III 1953-2005 uses calibrated parameter values. All yields are in annualized percentages. Autocorrelation coefficients (AC1) are computed for a lag of one month. Reported statistics refer to 2,000 Monte-Carlo simulations, each using 636 months.

Table 6: Correlations between consumption growth and interest rates

	II 1953–2005				III 1953–2005				Data 1953–2005
	Mean	5%	95%	Pop	Mean	5%	95%	Pop	Value
Correlations									
$g_{t:t+12}$ and $y_{t,3}$	0.26	–0.01	0.50	0.28	0.53	0.26	0.75	0.59	
$g_{t:t+12}$ and $y_{t,12}$	0.22	–0.05	0.47	0.23	0.49	0.21	0.72	0.56	
$g_{t:t+12}$ and $y_{t,60}$	0.14	–0.14	0.41	0.14	0.35	–0.01	0.64	0.40	
$g_{t:t+12}$ and $y^{\$}_{t,3}$	–0.11	–0.37	0.19	–0.12	–0.39	–0.63	–0.10	–0.41	–0.25*
$g_{t:t+12}$ and $y^{\$}_{t,12}$	–0.12	–0.38	0.19	–0.13	–0.39	–0.63	–0.11	–0.41	–0.19
$g_{t:t+12}$ and $y^{\$}_{t,60}$	–0.13	–0.39	0.17	–0.14	–0.40	–0.64	–0.12	–0.42	–0.12
$g_{t:t+12}$ and $y_{t-12,3}$	0.16	–0.09	0.40	0.20	0.43	0.13	0.68	0.52	
$g_{t:t+12}$ and $y_{t-12,12}$	0.13	–0.11	0.38	0.17	0.40	0.09	0.67	0.48	
$g_{t:t+12}$ and $y_{t-12,60}$	0.08	–0.18	0.34	0.10	0.28	–0.09	0.59	0.35	
$g_{t:t+12}$ and $y^{\$}_{t-12,3}$	–0.02	–0.27	0.24	–0.08	–0.27	–0.54	0.02	–0.35	–0.04
$g_{t:t+12}$ and $y^{\$}_{t-12,12}$	–0.01	–0.27	0.24	–0.09	–0.27	–0.54	0.01	–0.35	–0.01
$g_{t:t+12}$ and $y^{\$}_{t-12,60}$	0.02	–0.22	0.25	–0.10	–0.28	–0.54	0.01	–0.36	–0.06
$g_{t:t+12}$ and $y_{t+12,3}$	0.18	–0.09	0.45	0.21	0.45	0.15	0.69	0.52	
$g_{t:t+12}$ and $y_{t+12,12}$	0.16	–0.12	0.42	0.18	0.42	0.11	0.67	0.49	
$g_{t:t+12}$ and $y_{t+12,60}$	0.10	–0.17	0.37	0.11	0.29	–0.06	0.59	0.36	
$g_{t:t+12}$ and $y^{\$}_{t+12,3}$	–0.10	–0.39	0.20	–0.13	–0.39	–0.65	–0.09	–0.42	–0.21
$g_{t:t+12}$ and $y^{\$}_{t+12,12}$	–0.10	–0.39	0.19	–0.13	–0.39	–0.65	–0.10	–0.43	–0.17
$g_{t:t+12}$ and $y^{\$}_{t+12,60}$	–0.11	–0.40	0.18	–0.14	–0.40	–0.65	–0.10	–0.43	–0.10

The table presents unconditional correlations between consumption growth, $g_{t:t+12}$, and n-months real yields $y_{t,n}$, and nominal yields $y^{\$}_{t,n}$. Column II 1953-2005 uses estimated parameter values obtained from Simulated Methods of Moments and column III 1953-2005 uses calibrated parameter values. The maturities of yields used are 3, 12, and 60 months. Consumption data is observed annually and correlated with end-of-year interest rates. Lead/lag and contemporaneous correlations are considered. Reported model-statistics refer to 2,000 Monte-Carlo simulations, each using 636 months. Pop refers to the population correlation coefficient obtained from simulating one sample of 210,000 months. *, **, and *** refer to 10%, 5%, and 1% significance levels.

Table 7: Predicting economic variables with the nominal yield spread

	II 1953–2005					III 1953–2005					Data 1953–2005		
	Mean β	5%	95%	R^2	Pop β	Mean β	5%	95%	R^2	Pop β	β	t-stat	R^2
Economic variables													
Real consumption growth	0.09	–0.18	0.37	0.03	0.09	0.09	–0.77	0.94	0.04	0.13	0.31	2.63	0.11
Excess stock returns	0.56	–1.27	2.27	0.02	0.66	4.42	–2.13	11.76	0.04	2.77	3.44	2.00	0.05
Inflation	–2.07	–2.46	–1.66	0.69	–2.15	–1.62	–5.00	1.72	0.17	–2.90	–1.00	–2.90	0.14

The table reports results from predicting economic variables with the nominal yield spread. Column II 1953-2005 uses estimated parameter values obtained from Simulated Methods of Moments and column III 1953-2005 uses calibrated parameter values. The yield spread equals the 5-year yield minus the 3-month yield. The following regression is run: $E_{i,t:t+1} = \alpha + \beta(y^{\$}_{t,5y} - y^{\$}_{t,3m}) + \epsilon_t$, where E_i for $i = 1, 2, 3$ refers to the three economic variables being forecasted. All forecast horizons are one year. Annual data is used for forecasting consumption growth and monthly data for forecasting excess stock returns and inflation. In the case of monthly data, standard errors are computed as in Newey and West (1987), using 12 lags. Reported model-statistics refer to 2,000 Monte-Carlo simulations, each using 636 months. Pop refers to the population slope coefficient obtained from simulating one sample of 210,000 months.

Table 8: Cyclicality, the EIS, and deviations from the Fisher hypothesis

	III 1953–2005			
	$y^{\$}_{t,3}$	$y^{\$}_{t,12}$	$y^{\$}_{t,60}$	Mean β
Correlation $(g_{t:t+12}, y^{\$}_{t,n})$				
$\psi = 1.5,\ \delta_2 = -0.09$	–0.39	–0.39	–0.40	
$\psi = 0.2,\ \delta_2 = -0.09$	0.26	0.21	0.02	
$\psi = 1.5,\ \delta_2 = -0.01$	0.07	0.06	0.02	
Predicting real consumption growth				
$\psi = 1.5,\ \delta_2 = -0.09$				0.13
$\psi = 0.2,\ \delta_2 = -0.09$				–0.47
$\psi = 1.5,\ \delta_2 = -0.01$				–1.58

The table reports implications for the cyclicality of nominal interest rates and yield spreads when changing the EIS and the correlation between real consumption growth and inflation. Column III 1953-2005 refers to the use of calibrated parameter values. The upper part reports correlations between the one year consumption growth rate and nominal yields with maturities 3, 12, and 60 months. Consumption data is observed annually and correlated with end-of-year interest rates. The lower part runs the regression: $g_{t:t+12} = \alpha + \beta(y^{\$}_{t,5y} - y^{\$}_{t,3m}) + \epsilon_t$. Forecast horizon is one year. Reported results refer to population coefficients obtained from simulating one sample of 210,000 months.

Table 9: Testing the expectations hypothesis for real interest rates

	II 1953–2005				III 1953–2005			
	Mean $\beta_{n,m}$	5%	95%	Pop $\beta_{n,m}$	Mean $\beta_{n,m}$	5%	95%	Pop $\beta_{n,m}$
Maturity								
2y	1.58	0.18	3.15	1.26	–0.19	–1.62	1.62	–0.30
3y	1.84	0.37	3.45	1.49	–0.27	–1.80	1.59	–0.31
4y	2.07	0.53	3.76	1.68	–0.34	–2.00	1.60	–0.32
5y	2.24	0.68	4.04	1.83	–0.40	–2.21	1.63	–0.33

The table presents results from testing the expectations hypothesis for model-implied real interest rates. Column II 1953-2005 uses estimated parameter values obtained from Simulated Methods of Moments and column III 1953-2005 uses calibrated parameter values. The following regression is run: $y_{t+m,n-m} - y_{t,n} = \alpha_{n,m} + \beta_{n,m}(\frac{m}{n-m})(y_{t,n} - y_{t,m}) + \epsilon_{t+m,n}$. The time step $m = 12$ months and $n = 2 - 5$ years. Reported statistics refer to 2,000 Monte-Carlo simulations, each using 636 months. Pop $\beta_{n,m}$ refers to the population beta obtained from simulating one sample of 210,000 months.

Table 10: Testing the expectations hypothesis for nominal interest rates

	II 1953–2005				III 1953–2005				Data 1953–2005	
	Mean $\beta_{n,m}$	5%	95%	Pop $\beta_{n,m}$	Mean $\beta_{n,m}$	5%	95%	Pop $\beta_{n,m}$	$\beta_{n,m}$	t-stat
Maturity										
2y	1.31	0.31	2.63	0.89	–0.25	–2.24	1.89	–0.39	–0.68	–3.79
3y	1.30	0.35	2.55	0.89	–0.43	–2.49	1.74	–0.46	–1.01	–3.77
4y	1.28	0.38	2.46	0.90	–0.61	–2.74	1.57	–0.52	–1.39	–4.01
5y	1.26	0.41	2.38	0.90	–0.79	–3.04	1.44	–0.58	–1.38	–3.60

The table presents results from testing the expectations hypothesis for model-implied and observed nominal interest rates. Column II 1953-2005 uses estimated parameter values obtained from Simulated Methods of Moments and column III 1953-2005 uses calibrated parameter values. The following regression is run: $y^{\$}_{t+m,n-m} - y^{\$}_{t,n} = \alpha_{n,m} + \beta_{n,m}(\frac{m}{n-m})(y^{\$}_{t,n} - y^{\$}_{t,m}) + \epsilon_{t+m,n}$. The time step $m = 12$ months and $n = 2 - 5$ years. T-stat values correspond to $H_0 : \beta_{n,m} = 1$. Standard errors are computed as in Newey and West (1987), using 12 lags. Reported statistics refer to 2,000 Monte-Carlo simulations, each using 636 months. Pop $\beta_{n,m}$ refers to the population beta obtained from simulating one sample of 210,000 months.

Table 11: Equity Moments

	II 1953–2005	III 1953–2005	Data 1953–2005
	Mean	Mean	Mean
Moments			
$E(r_m - r_f)$	5.63	5.41	5.72
$\sigma(r_m - r_f)$	7.15	13.38	14.88
Sharpe ratio	0.79	0.40	0.38

The table presents model-implied and observed moments for equity. Column II 1953-2005 uses estimated parameter values obtained from Simulated Methods of Moments and column III 1953-2005 uses calibrated parameter values. All moments are annualized. The short rate, r_f, refers to the 1-month rate. Reported statistics for columns SMM 1953–2005 and Calibration 1953–2005 refer to 2,000 Monte-Carlo simulations, each using 636 months and aggregated to 53 annual observations.

Table 12: Correlations between real stock returns and realized inflation.

	II 1953–2005				III 1953–2005			
	Mean	5%	95%	Pop	Mean	5%	95%	Pop
Correlations								
$r_{m,t}$ and π_t	–0.21	–0.44	0.02	–0.21	–0.17	–0.39	0.06	–0.15
$r_{m,t+1}$ and π_t	–0.13	–0.37	0.12	–0.14	–0.07	–0.29	0.17	–0.09
$r_{m,t}$ and π_{t+1}	–0.14	–0.37	0.10	–0.15	–0.15	–0.38	0.08	–0.14

The table presents model-implied correlations between annual real stock returns and realized inflation. Column II 1953-2005 uses estimated parameter values obtained from Simulated Methods of Moments and column III 1953-2005 uses calibrated parameter values. Reported statistics refer to 2,000 Monte-Carlo simulations, each using 636 months. Pop refers to population coefficients obtained from simulating one sample of 210,000 months.

Chapter 3

The "Fed-Model"and the Changing Correlation of Stock and Bond Returns: An Equilibrium Approach

Abstract

This paper presents an equilibrium model that provides a rational explanation for two features of data that have been considered puzzling: The positive relation between US dividend yields and nominal interest rates, often called the Fed-model, and the time-varying correlation of US stock and bond returns. Key ingredients are time-varying first and second moments of consumption growth, inflation, and dividend growth in conjunction with Epstein-Zin and Weil recursive preferences. Historically in the US, inflation has signaled low future consumption growth. The representative agent therefore dislikes positive inflation shocks and demands a positive risk premium for holding assets that are poor inflation hedges, such as equity and nominal bonds. As a result, risk premiums on equity and nominal bonds comove positively through their exposure to macroeconomic volatility. This generates a positive correlation between dividend yields and nominal yields and between stock and bond returns. High levels of macro volatility in the late 1970s and early 1980s caused stock and bond returns

to comove strongly. The subsequent moderation in aggregate economic risk has brought correlations lower. The model is able to produce correlations that can switch sign by including the covariances between consumption growth, inflation, and dividend growth as state variables.

3.1 Introduction

The correlation between US stock and bond returns has varied substantially over time, reaching highly positive levels in the late 1970s and early 1980s while turning negative in the late 1990s. Several statistical models have been put forward to model the time variation but little work has been done on explaining the phenomenon within an equilibrium model. A second feature of data that has been considered puzzling is the highly positive correlation between US dividend yields and nominal interest rates, a relation often referred to as the Fed-model.[1] From the Gordon growth formula, dividend yields are given by the real discount rate on equity minus the real dividend growth rate. Since changes in expected inflation and bond risk premiums have been the main source of variation in nominal yields (e.g. Campbell and Ammer, 1993, and Best et al., 1998), the positive correlation observed in data implies that one or both of these components must either be positively associated with real discount rates or negatively associated with real dividend growth rates.[2] However in the literature, it has been considered implausible for inflation to have rational effects on any of these two real components of dividend yields. Instead, a behavioral explanation in the form of inflation illusion (e.g., Modigliani and Cohn, 1979, and Campbell and Vuolteenaho, 2004) has been put forward.

I propose a representative agent asset pricing model that provides a rational explanation for these two features of data. Key ingredients of the model are exogenous consumption growth, inflation, and dividend growth in conjunction with Epstein and Zin (1989) and Weil (1989) recursive preferences. In post 1952 US data, inflation has signaled low future consumption growth. The representative agent therefore

[1]The Fed-model is not endorsed by the Federal Reserve. The name comes from Ed Yardeni at Prudential Securities in the mid 1990s following research reports at the Federal Reserve describing the relation.

[2]While the Gordon growth formula also can be written in nominal terms, changes in expected inflation is expected to have offsetting effects on the nominal parts of the discount rate and the dividend growth rate. This leaves the real components to explain the correlation.

dislikes positive inflation shocks. As a result, the agent demands a positive risk premium for holding assets that are poor hedges against inflation, for example equity and nominal bonds. This mechanism causes risk premiums on equity and nominal bonds to comove positively through their common exposure to macroeconomic volatility. Dividend yields and nominal yields therefore become positively associated, allowing the model to match correlations observed in data. The comovement of risk premiums is consistent with evidence in for example Fama and French (1989) and Campbell and Ammer (1993). In particular, the model suggests that inflation volatility plays a key role for determining risk premiums on both stocks and bonds. Similarly, stock and bond returns move together through common changes in risk premiums. Changes in macroeconomic risk, measured as time-varying second moments of consumption growth, inflation, and dividend growth, are shown to account for a large part of changes in realized correlations between stock and bond returns. The high correlations seen in the late 1970s and early 1980s are attributed to high volatility of inflation and consumption growth. The subsequent drop in correlations is explained by lower macroeconomic volatility. During the late 1990s, correlations turned sharply negative. The model is able to produce correlations that can switch sign by including the covariances between consumption growth, inflation, and dividend growth as state variables. The negative correlations are partly captured by the model as a result of low economic volatility together with a positive covariance between dividend growth and inflation. A series of small positive shocks to inflation generated negative bond returns while positive cash flow shocks raised stock prices through both higher dividend growth and a lower equity risk premium. The discount rate effect on equity arises since positive cash flow shocks that occur during times of positive inflation shocks (bad times) imply that equity is a hedge against periods of high marginal utility. While the model performs well in capturing low-frequency movements in realized correlations, it is more challenging for the model to predict quarter-to-quarter changes in correlations. Model-implied conditional correlations predict realized quarterly correlations significantly and with an R^2 of 13%. Including the lagged realized correlation as explanatory variable does not drive out the significance of the model forecast.

The dynamics of the model are presented in two specifications. In the first specification, I build on Piazzesi and Schneider (2006) and model consumption growth and inflation as a VARMA(1,1) process written in state-space form with all shocks in the economy being homoscedastic. I extend their setup by also modeling dividend growth and introducing equity in to the model. I also allow for the elasticity of intertemporal substitution (EIS) to be different from one and I provide approximate analytical solutions to the model.[3] The codependence of consumption growth and inflation is important for the results of the paper as it makes real asset prices and the price-dividend ratio functions of expected inflation. This generates a direct channel through which the Fed-model is explained. Both expected and unexpected inflation are estimated to have a negative effect on future consumption growth which therefore leads to positive risk premiums on risky assets such as equity and nominal bonds due to inflation shocks. When the EIS is above one, model-implied price-dividend ratios are negatively related to expected inflation. This is supported by empirical evidence provided in the paper.[4] The specification also draws on the so called long-run risk model of Bansal and Yaron (2004) in which expected real consumption growth contains a small persistent component. However, their model does not consider inflation and its effect on real consumption growth, the real pricing kernel, and therefore on risk premiums in the economy. I show that the real effects of inflation are important to consider and therefore draws an important distinction between the two models. Dividend growth is also modeled differently than in Bansal and Yaron (2004). Rather than assuming expected dividend growth

[3]Bansal et al. (2007a) and Attanasio and Vissing-Jorgensen (2003) estimate the EIS to be in excess of one while Hall (1988) and Campbell (1999) document that it is close to zero. Lustig et al. (2008) provide evidence that the wealth-consumption ratio varies over time, indicating that the EIS is not equal to one.

[4]It is well known in the literature that the so called long-run risk model of Bansal and Yaron (2004) needs an EIS in excess of one to capture the negative relation between consumption volatility and price-dividend ratios, and the positive relation between expected consumption growth and price-dividend ratios. The model in this paper needs an EIS above one for a different reason, namely to capture the negative relation between price-dividend ratios and expected inflation.

to be a function of expected consumption growth, I allow expected dividend growth to be a separate state variable for price-dividend ratios. This is shown to be important for explaining the two features in data as it drives a wedge between stock and bond prices. Together with expected dividend growth, the conditional means of consumption growth and inflation serve as state variables in the economy. In the second specification, I extend the first by introducing heteroscedasticity. The volatility of shocks to the three macro variables and their covariances are allowed to change over time, which enables me to compute a time-varying conditional correlation between stock and bond returns. In addition to the conditional first moments, the conditional variance of consumption growth, inflation, and dividend growth, and their covariances serve as state variables in the second specification.

The main contributions of the paper are twofold. First, I show that the so called Fed-model can be explained within a rational consumption-based equilibrium model. This stands in contrast to the hitherto dominant explanation in the form of inflation illusion. Second, I show that the changing correlation between stock and bond returns to a large extent can be explained using the same simple equilibrium model. To the best of my knowledge, this is the first equilibrium model to provide a rational explanation of the Fed-model and the first consumption-based equilibrium model that is able to account for a large part of changes in realized correlations between stock and bond returns.

This paper builds on the literature of pricing stocks and bonds in equilibrium. Early contributions include Cox et al. (1985), Mehra and Prescott (1985), Campbell (1986), and Dunn and Singleton (1986). The recursive preferences of Epstein and Zin (1989) and Weil (1989) have been used extensively in the asset-pricing literature (e.g., Campbell, 1993, 1996, 1999, Duffie et al., 1997, and Restoy and Weil, 1998). Bansal and Yaron (2004) show that recursive preferences in conjunction with a time-varying first and second moment of consumption growth can explain the level of the equity risk premium and its variation over time. Hasseltoft (2008) extends the long-run risk model to the term structure of interest rates and shows that a calibrated version of the model is capable of explaining deviations from the expectations hypothesis, the upward sloping nominal yield curve, and the predictive power of the yield curve. It is also shown that the cyclicality of nominal interest rates and yield spreads depend on the relative values of the EIS and the correlation between real consumption growth and inflation. Bansal and Shaliastovich (2008) explain violations of both the expectation hypothesis in bond markets and the uncovered interest rate parity in currency markets using the long-run risk model. Piazzesi and Schneider (2006) make use of recursive preferences and show that the nominal yield curve slopes up if inflation is bad news for future consumption growth. They also explore the role of learning about macroeconomic fundamentals.

The positive correlation of US dividend yields and nominal interest rates, 0.30 for the period 1952-2007 and 0.74 for the period 1965-2007, is commonly known as the Fed-model. It is widely used among practitioners as a tool for comparing the relative valuations of the stock and bond markets and has received academic interest as rational explanations have been considered implausible. From the Gordon growth model, the dividend yield is given by the real discount rate on equity minus the real dividend growth rate. Similarly, variations in nominal yields can be decomposed into changes in real interest rates, inflation, and future excess returns. Evidence suggest that the latter two components are the dominant factors (e.g. Campbell and Ammer, 1993, and Best et al., 1998). For the positive correlation in data to arise, changes in inflation and bond risk premiums must be negatively associated with real dividend growth rates and/or positively associated with real discount rates. If the Gordon growth formula is written in nominal terms, changes in expected inflation is expected to have offsetting effects on the nominal parts of dividend growth rates and discount rates which leaves the real components to explain the high correlation. Surprisingly, none of these explanations have found empirical support in the literature until recently. Instead an explanation in the form of inflation illusion, originally put forward by Modigliani and Cohn (1979), has found empirical support (e.g., Ritter and Warr, 2002, Asness, 2003, Campbell and Vuolteenaho, 2004, and Cohen et al., 2005). This explanation suggests that investors are irrational and fail to properly adjust the expected nominal dividend growth rate with changes in expected inflation but fully adjust the nominal discount rate. Alternatively, one can view it as investors are discounting real cash flows with nominal interest rates. This implies that stocks are undervalued in periods of high inflation from the viewpoint of a rational investor. In a recent empirical paper, Bekaert and Engstrom (2008) argue that rational mechanisms are at work and ascribe the high correlation to the large incidence of stagflation in US data. They show that the correlation between equity yields and bond yields is mainly driven by a correlation between expected inflation and the equity risk premium as periods of high expected inflation are associated with periods of high risk aversion and high economic uncertainty. Hence, it seems that any rational equilibrium model that would like to explain the Fed-model must contain a link between inflation and the equity risk premium. The model in this paper contains exactly that.

The relation between stock and bond returns has received great academic interest and is of central importance for asset allocation decisions. Early contributions focus on the unconditional correlation. Shiller and Beltratti (1992) fail to match the observed comovement using a present-value model. Campbell and Ammer (1993) decompose the variance of stock and bond returns and find offsetting effects from changes in real interest rates, excess returns, and expected inflation. Barsky (1989) explore the role of changes in risk and productivity growth for the behavior of stock and bond returns within a general equilibrium framework. Recently, the focus has shifted towards understanding the conditional correlation of stock and bond returns which displays large time variation. Several statistical models have been put forward to shed light on the comovement. Scruggs and Glabadanidis (2003) find that models that impose a constant correlation restriction on the covariance matrix between stock and bond returns are strongly rejected. Connolly et al. (2005) document a negative relation between stock market uncertainty and the future correlation of stock and bond returns. Baele et al. (2007) attempt to explain the time-varying correlation using macro factors but conclude that their factors fit the reality poorly and argue that liquidity factors help explain the time variation. Campbell et al. (2008) specify an exogenous process for the real stochastic discount factor and estimate a term structure model using inflation and asset price data. One of their state variables is the covariance between inflation and the real economy which they link to the changing covariance of stock and bond returns and changes in bond risk premia. Nominal bonds are expected to be a poor hedge against economic fluctuations when inflation is countercyclical since it implies pro-cyclical bond returns. Investors therefore demand a positive risk premium to hold them. David and Veronesi (2008) explore the role of learning about inflation and real earnings for the second moments of stock and bond returns. They estimate their model using asset price data as well and forecast the covariance of stock and bond returns quite accurately. In contrast to these papers, I build on the traditional literature on consumption-based asset pricing and address the issue of changing correlations using a consumption-based equilibrium model. This implies a real stochastic discount factor that is based on consumption growth. Furthermore, only macro data is used to estimate the model which provides a test of how much fundamental macro variables can account for features of asset prices.

This paper is also related to the literature on whether stocks provide a hedge or not against inflation. Several articles have established a negative relation between inflation and stock returns. Fama and Schwert (1977) document that common stock returns are inversely related to expected inflation. Fama (1981) argues that inflation proxies for real activity. He argues that higher expected output increases both stock prices and the demand for real money. If the latter is not accommodated with a similar change in money supply, the quantity theory of money suggests a drop in the price level. Hence, a negative relation between stock returns and inflation. Kaul (1987) stresses the combination of money demand and counter-cyclical money supply effects for the stock return-inflation relation. Boudoukh et al. (1994) explore the cross-sectional relation between stock returns and inflation and find that non-cyclical industries tend to covary positively with inflation while the reverse holds for cyclical industries. Pilotte (2003) documents that dividend yields and capital gains are differently related to expected inflation.

3.2 The Model

This section describes the dynamics of consumption growth, inflation, and dividend growth, which are presented in two specifications. Specification I models the first moments, assuming all macro shocks have constant second moments. Specification II allows for heteroscedasticity and models the second moments. Section 3.2.2 describes the preferences of the representative agent. Section 3.2.3 solves the model and provides solutions for equity and bond prices.

3.2.1 Dynamics

Specification I: Homoscedasticity

Consumption growth, inflation, and dividend growth are modeled in state-space form. Let $\mathbf{z}_{t+1} = [\Delta c_{t+1}, \pi_{t+1}, \Delta d_{t+1}]'$ denote the logarithmic consumption growth, inflation, and dividend growth and let $\mathbf{x}_t = [x_c, x_\pi, x_d]'$ represent the time-varying part of the conditional means. The components of $\mathbf{x}_t$ serve as state variables in the economy. The following dynamics are assumed:

$$\begin{aligned}
\mathbf{z}_{t+1} &= \boldsymbol{\mu} + \mathbf{x}_t + \boldsymbol{\eta}_{t+1}, && (3.1)\\
\mathbf{x}_{t+1} &= \boldsymbol{\beta}\mathbf{x}_t + \boldsymbol{\delta}\boldsymbol{\eta}_{t+1}, && (3.2)\\
\boldsymbol{\eta}_{t+1} &\sim N.i.i.d.\,(0, \boldsymbol{\Omega}),\\
\boldsymbol{\mu} &= [\mu_c, \mu_\pi, \mu_d]',\\
\boldsymbol{\eta} &= [\eta_c, \eta_\pi, \eta_d]',\\
\boldsymbol{\Omega} &= \begin{pmatrix} \sigma_c^2 & \sigma_{c\pi} & \sigma_{cd} \\ \sigma_{c\pi} & \sigma_\pi^2 & \sigma_{\pi d} \\ \sigma_{cd} & \sigma_{\pi d} & \sigma_d^2 \end{pmatrix}.
\end{aligned}$$

All shocks in the economy are assumed to be homoscedastic, subject to the variance-covariance matrix $\boldsymbol{\Omega}$. Shocks to inflation and consumption growth are correlated through $\sigma_{c\pi}$, shocks to consumption and dividend growth through σ_{cd}, and shocks to dividend growth and inflation through $\sigma_{\pi d}$. The persistence of shocks and their effect on the conditional means are governed by $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$:

$$\begin{aligned}\boldsymbol{\beta} &= \begin{pmatrix} \beta_1 & \beta_2 & 0 \\ \beta_3 & \beta_4 & 0 \\ \beta_5 & 0 & \beta_6 \end{pmatrix}, \\ \boldsymbol{\delta} &= \begin{pmatrix} \delta_1 & \delta_2 & 0 \\ \delta_3 & \delta_4 & 0 \\ \delta_5 & 0 & \delta_6 \end{pmatrix}.\end{aligned}$$

The state-space system can be written as a VARMA(1,1):

$$\mathbf{z}_{t+1} - \boldsymbol{\mu} = \boldsymbol{\beta}(\mathbf{z}_t - \boldsymbol{\mu}) + \boldsymbol{\eta}_{t+1} + \boldsymbol{\alpha}\boldsymbol{\eta}_t, \tag{3.3}$$

where $\boldsymbol{\alpha}$ is a 3-by-3 matrix. Taking the conditional mean of (3.3) and letting $\mathbf{x}_t = \boldsymbol{\beta}(\mathbf{z}_t - \boldsymbol{\mu}) + \boldsymbol{\alpha}\boldsymbol{\eta}_t$ denote the conditional mean yields the state-space system above where $\boldsymbol{\delta} = \boldsymbol{\beta} + \boldsymbol{\alpha}$.

Consider first the relation between expected consumption growth and inflation, which follows Piazzesi and Schneider (2006). The two conditional means are interdependent through β_2 and β_3. This allows real asset prices and valuation ratios, for example the price-dividend ratio, to be a function of expected inflation since the conditional mean of $x_{c,t+1}$ depends on $x_{\pi,t}$. So if one conjectures that asset prices are a function of expected consumption growth, it implies they also are a function of expected inflation. Hence, $x_{c,t}$ and $x_{\pi,t}$ both serve as state variables. This implies a direct link between expected inflation and the real pricing kernel which therefore have implications for risk premiums in the economy. This way of capturing the real effects of inflation is important for explaining the Fed-model and the changing correlation of stock and bond returns. Specification I nests dynamics used in other consumption-based equilibrium models. For example, Wachter (2006) models consumption growth and inflation as two separate ARMA(1,1) processes with correlated shocks. This translates into setting $\beta_2 = \beta_3 = \delta_2 = \delta_3 = 0$. Bansal and Yaron (2004) assume that shocks to realized and expected consumption growth are different and uncorrelated. However, the specification above nests their dynamics if one set the two shocks equal, leave out realized inflation completely, and then set $\beta_2 = \beta_3 = \beta_4 = \delta_2 = \delta_3 = \delta_4 = 0$. Hasseltoft (2008) models expected inflation as an AR(1) process with lagged consumption shocks but keeping expected consumption growth only a function of its own lag and shocks. This implies setting $\beta_2 = \beta_3 = \delta_2 = 0$.

The realized dividend growth rate is modeled analogously to consumption growth and inflation, with an unconditional mean, μ_d, and a time-varying part, x_d.[5] This specification is different from the common approach in consumption-based models of modeling dividend growth rates as a function of expected consumption growth times a leverage parameter (e.g., Abel, 1999, Bansal and Yaron, 2004). Modeling dividend growth in such a way has the less desirable property of producing a correlation equal to one between expected consumption growth and expected dividend growth. The specification used in this paper breaks that link which makes the expected dividend growth a state variable for price-dividend ratios. This drives a wedge between price-dividend ratios and nominal interest rates in the model as it captures cash flow shocks affecting equity. This allows the model to produce a realistic correlation between price-dividend ratios and nominal interest rates. It also improves the ability of the model to match the observed correlation between stock and bond returns as it contributes to a low or even negative correlation between asset returns as discussed in Section 3.6.

Expected dividend growth rates are allowed to depend on lagged expected consumption growth through β_5. The effect of expected dividend growth on future consumption growth and inflation, entries (1,3) and (2,3) of $\boldsymbol{\beta}$, are both set to zero. Relaxing the restriction would make expected dividend growth rates a state variable for bond prices. The economic rationale for why changes in cash flow growth rates should affect prices of Treasury bonds is not clear, and I therefore choose to restrict the dynamics. The effect of expected inflation on future expected dividend growth rates, entry (3,2) of $\boldsymbol{\beta}$, is set to zero. This is done for two reasons. First, it emphasizes the effect inflation has on risk premiums through its relation with future consumption growth. That is, the emphasis is on one of the two rational channels

[5]Consumption and dividends are not cointegrated. Considering the long-run relation between consumption and dividends is potentially important, as for example argued in Bansal et al. (2007b).

that can generate a positive correlation between dividend yields and nominal interest rates. Second, the unconditional correlation between dividend growth and inflation is close to zero in data, indicating that the second possible rational channel of explaining the Fed-model is less important. Empirical evidence provided in Bekaert and Engstrom (2008) indicate that the relation between expected inflation and the cash flow component of dividend yields plays a minor role.[6] Similarly, the matrix $\boldsymbol{\delta}$ is subject to the same restrictions.

Specification II: Heteroscedasticity

Empirical evidence suggests that inflation (e.g., Engle, 1982, and Bollerslev, 1986), consumption growth (e.g., Kandel and Stambaugh 1990, and Bansal et al., 2005), and dividend growth (e.g., Bansal and Yaron, 2000) are all subject to heteroscedasticity. The volatilities of shocks to the economy are therefore modeled as time varying, incorporating a notion of changing economic uncertainty. The conditional second moments of the shocks are later used to compute the conditional correlation between stock and bond returns.

The dynamics for the first moments are kept the same as above, but the conditional variance-covariance matrix of the shocks are allowed to change over time. The variances and the covariances are modeled as separate autoregressive processes subject to random shocks that are assumed to be normally distributed and uncorrelated. Assume the macro shocks $\boldsymbol{\eta}_{t+1}$ to be normally distributed with mean zero and subject to the conditional variance-covariance matrix $\boldsymbol{\Omega}_t$, which takes the form:

$$\boldsymbol{\Omega}_t = \begin{pmatrix} \sigma^2_{c,t} & \sigma_{c\pi,t} & \sigma_{cd,t} \\ \sigma_{c\pi,t} & \sigma^2_{\pi,t} & \sigma_{\pi d,t} \\ \sigma_{cd,t} & \sigma_{\pi d,t} & \sigma^2_{d,t} \end{pmatrix},$$

and let the variances and covariances follow:

[6] I have estimated the system allowing for an interaction between expected inflation and future expected dividend growth rates. The coefficient turns out to be negative but is not statistically different from zero at usual significance levels.

$$\begin{aligned}
\sigma^2_{c,t+1} &= \alpha_c + \phi_c(\sigma^2_{c,t} - \alpha_c) + \tau_c \epsilon_{c,t+1}, && (3.4)\\
\sigma^2_{\pi,t+1} &= \alpha_\pi + \phi_\pi(\sigma^2_{\pi,t} - \alpha_\pi) + \tau_\pi \epsilon_{\pi,t+1}, && (3.5)\\
\sigma^2_{d,t+1} &= \alpha_d + \phi_d(\sigma^2_{d,t} - \alpha_d) + \tau_d \epsilon_{d,t+1}, && (3.6)\\
\sigma_{c\pi,t+1} &= \alpha_{c\pi} + \phi_{c\pi}(\sigma_{c\pi,t} - \alpha_{c\pi}) + \tau_{c\pi} \epsilon_{c\pi,t+1}, && (3.7)\\
\sigma_{cd,t+1} &= \alpha_{cd} + \phi_{cd}(\sigma_{cd,t} - \alpha_{cd}) + \tau_{cd} \epsilon_{cd,t+1}, && (3.8)\\
\sigma_{\pi d,t+1} &= \alpha_{\pi d} + \phi_{\pi d}(\sigma_{\pi d,t} - \alpha_{\pi d}) + \tau_{\pi d} \epsilon_{\pi d,t+1}, && (3.9)
\end{aligned}$$

where α denotes the unconditional mean of each process, the ϕ parameters govern the persistence of shocks to the second moments, and the τ parameters determine the volatility of the second moments. These dynamics are extensions of Bansal and Yaron (2004) who model consumption volatility as in (3.4). The assumption of conditionally normal second moments is made out of convenience since it allows for closed-form affine solutions of asset prices. These dynamics are therefore assumed for modeling purposes. When estimating the second moments in Section 3.3.2, I use the diagonal VEC-model of Bollerslev et al. (1988). The estimated parameters are then mapped into the dynamics above.

3.2.2 Investor Preferences

The representative agent in the economy has Epstein and Zin (1989) and Weil (1989) recursive preferences:

$$U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}, \tag{3.10}$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, $\gamma \geq 0$ denotes the risk aversion coefficient and $\psi \geq 0$ the elasticity of intertemporal substitution (EIS). The discount factor is represented by δ. This preference specification allows time preferences to be separated from risk preferences. This stands in contrast to time-separable expected utility in which the desire to smooth consumption over states and over time are interlinked. The agent prefers early (late) resolution of risk when the risk aversion is larger (smaller) than the reciprocal of the EIS. A preference for early resolution and an EIS above one imply that $\theta < 1$. This specification nests the time-separable power utility model for $\gamma = \frac{1}{\psi}$ (i.e., $\theta = 1$).

The agent is subject to the following budget constraint:

$$W_{t+1} = R_{c,t+1} \left(W_t - C_t \right), \tag{3.11}$$

where the agent's total wealth is denoted W_t, $W_t - C_t$ is the amount of wealth invested in asset markets and $R_{c,t+1}$ denotes the gross return on the agents total wealth portfolio. This asset delivers aggregate consumption as its dividends each period. Epstein and Zin (1989) show that this economy implies an Euler equation for asset return $R_{i,t+1}$ in the form of:

$$E_t[\underbrace{\delta^{\theta} G_{t+1}^{\frac{-\theta}{\psi}} R_{c,t+1}^{-(1-\theta)}}_{M_{t+1}} R_{i,t+1}] = 1, \tag{3.12}$$

where G_{t+1} denotes the aggregate gross growth rate of consumption and M_{t+1} denotes the intertemporal marginal rate of substitution (IMRS). The logarithm of the IMRS can be written as:

$$m_{t+1} = \theta \ln \left(\delta \right) - \frac{\theta}{\psi} \Delta c_{t+1} - \left(1 - \theta \right) r_{c,t+1}, \tag{3.13}$$

where $\ln R_{c,t+1} = r_{c,t+1}$ and $\ln G_{t+1} = \Delta c_{t+1}$. Note that the IMRS depends on both consumption growth and on the return from the total wealth portfolio. Recall that $\theta = 1$ under power utility, which brings us back to the standard time-separable IMRS.

3.2.3 Solving the model

This section solves the model for the case of constant and time-varying second moments of the macro shocks. Common to the two cases are the returns on the aggregate wealth portfolio and the market portfolio, which are approximated using the analytical solutions found in Campbell and Shiller (1988):

$$r_{c,t+1} = k_{c,0} + k_{c,1} pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{3.14}$$

$$r_{m,t+1} = k_{d,0} + k_{d,1} pd_{t+1} - pd_t + \Delta d_{t+1}, \tag{3.15}$$

where pc_t and pd_t denote the log price-consumption ratio and the log price-dividend ratio, and the constants k_c and k_d are functions of the average level of pc_t and pd_t, denoted $\bar{pc}$ and $\bar{pd}$.[7] Specifically, the constants are:

$$k_{c,1} = \frac{\exp(\bar{pc})}{1+\exp(\bar{pc})}, \tag{3.16}$$

$$k_{c,0} = \ln(1+\exp(\bar{pc})) - k_{c,1}\bar{pc}, \tag{3.17}$$

and similarly for the k_d coefficients.

Specification I: Homoscedasticity

All asset prices and valuation ratios are conjectured to be functions of the time-varying conditional means of the three macro variables. Starting with the log price-consumption ratio, it is conjectured to be a linear function of expected consumption growth, x_c, expected inflation, x_π, and expected dividend growth, x_d:

$$pc_t = A_{c,0} + A_{c,1}x_{c,t} + A_{c,2}x_{\pi,t} + A_{c,3}x_{d,t}. \tag{3.18}$$

Using the standard Euler equation together with the dynamics of consumption growth and inflation one can solve for the coefficients. Appendix A.1 explains how to solve for the A-coefficients and reports the expression for $A_{c,0}$. The remaining coefficients are given by:

$$A_{c,1} = \frac{1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - \frac{1}{\psi}}{1 - k_{c,1}\beta_1},$$

$$A_{c,2} = \frac{k_{c,1}\beta_2(1-\frac{1}{\psi})}{(1-k_{c,1}\beta_4)(1-k_{c,1}\beta_1) - k_{c,1}^2\beta_2\beta_3},$$

$$A_{c,3} = 0.$$

First note that $A_{c,3}$ equals zero as a result of the restrictions imposed on the dynamics, which were discussed in Section 3.2.1. Coefficient,

[7] Bansal et al. (2007a) show that the approximate analytical solutions for the returns are close to the numerical solutions and deliver similar model implications.

$A_{c,1}$, measures the response of the price-consumption ratio to changes in expected consumption growth. The denominator is positive given that $\beta_1 < 1$ so its sign depends on whether expected consumption growth implies good or bad news for future inflation, β_3, on whether the price-consumption ratio responds positively or negatively to inflation expectations, $A_{c,2}$, and on whether the EIS, ψ, is above or below one. Setting β_3 and/or $A_{c,2}$ equal to zero gives the expression for $A_{c,1}$ found in Bansal and Yaron (2004). However, the influence of inflation on real variables is key to the paper and represents an important distinction from the long-run risk model. The denominator of $A_{c,2}$ is positive for plausible parameter values so its sign depends on β_2 and the EIS. For example, high inflation expectations will depress the price-consumption ratio ($A_{c,2} < 0$) when high inflation signals low future consumption growth, $\beta_2 < 0$, and the EIS is above one. An EIS in excess of one implies that the intertemporal substitution effect dominates the wealth effect. As high inflation signals low future returns, agents sell risky assets which leads to lower valuation ratios. In the case of expected utility ($\frac{1}{\psi} = \gamma$), a risk aversion coefficient above one instead implies that the wealth effect dominates which results in a positive value of $A_{c,2}$ given that $\beta_2 < 0$. Section 3.4 provides empirical evidence that expected inflation is negatively related to the price-consumption ratio and accounts for a large part of its variation.[8]

The following expression represents innovations to the real pricing kernel, where vector λ represents the market prices of risk:

$$
\begin{aligned}
m_{t+1} - E_t(m_{t+1}) &= \lambda_{\eta_c}\eta_{c,t+1} + \lambda_{\eta_\pi}\eta_{\pi,t+1}, && (3.19)\\
\lambda_{\eta_c} &= -\frac{\theta}{\psi} + (\theta - 1)(k_{c,1}A_{c,1}\delta_1 + && (3.20)\\
&\quad k_{c,1}A_{c,2}\delta_3 + k_{c,1}A_{c,3}\delta_5 + 1), \\
\lambda_{\eta_\pi} &= (\theta - 1)(k_{c,1}A_{c,1}\delta_2 + && (3.21)\\
&\quad k_{c,1}A_{c,2}\delta_4).
\end{aligned}
$$

The model allows both shocks to consumption growth and inflation to be priced. For example, $\lambda_{\eta_\pi} > 0$ implies that the representative

[8]The unobservable price-consumption ratio is proxied by the wealth-consumption ratio measured in Lustig et al. (2008).

agent dislikes positive inflation shocks and therefore requires a higher risk premium for assets that perform badly in periods of high inflation. Hence, this represents an additional part of risk premiums in the economy compared to models in which only consumption shocks are priced, e.g., Bansal and Yaron (2004). Recall that $\theta = 1$ under power utility, which means that inflation risk is not priced and the price of consumption risk collapses to $-\frac{1}{\psi} = -\gamma$.

The log price-dividend ratio is conjectured to be a linear function of the same three state variables as above:

$$pd_t = A_{d,0} + A_{d,1}x_{c,t} + A_{d,2}x_{\pi,t} + A_{d,3}x_{d,t}. \tag{3.22}$$

Again, the coefficients are solved for using the standard Euler equation. Appendix A.2. describes the derivations and reports the expression for $A_{d,0}$. The remaining coefficients are given by:

$$\begin{aligned}
A_{d,1} &= \frac{-\frac{\theta}{\psi} + (\theta-1)(k_{c,1}(A_{c,1}\beta_1 + A_{c,2}\beta_3 + A_{c,3}\beta_5) - A_{c,1} + 1)}{1 - k_{d,1}\beta_1} + \\
&\quad \frac{k_{d,1}(A_{d,2}\beta_3 + A_{d,3}\beta_5)}{1 - k_{d,1}\beta_1}, \\
A_{d,2} &= \frac{(1 - k_{d,1}\beta_1)X + Y}{(1 - k_{d,1}\beta_4)(1 - k_{d,1}\beta_1) - k_{d,1}^2\beta_2\beta_3}, \\
A_{d,3} &= \frac{(\theta-1)A_{c,3}(k_{c,1}\beta_6 - 1) + 1}{1 - k_{d,1}\beta_6}, \\
X &= (\theta-1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}), \\
Y &= k_{d,1}\beta_2\left[-\frac{\theta}{\psi} + (\theta-1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 - A_{c,1} + 1) + \right. \\
&\quad \left. k_{d,1}A_{d,3}\beta_5\right],
\end{aligned}$$

Since $A_{c,3}$ equals zero, $A_{d,3}$ equals $\frac{1}{1-k_{d,1}\beta_6}$ which is positive given that $\beta_6 < 1$. This implies that higher expected dividend growth naturally raises price-dividend ratios. Coefficient $A_{d,2}$ is in general negative when the EIS is above one and when high expected inflation is a signal of low future consumption growth, $\beta_2 < 0$. Expected consumption growth and price-dividend ratios are in general positively associated ($A_{d,1} > 0$) for high values of the EIS and negative values of β_3, provided that $A_{d,2}$ is negative. As for the price-consumption ratio, Section 3.4 provides empirical evidence that expected inflation is negatively related to price-dividend ratios.

Real and nominal log bond prices are conjectured to be functions of the same state variables:

$$
\begin{aligned}
q_{t,n} &= D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}x_{d,t}, && (3.23) \\
q^{\$}_{t,n} &= D^{\$}_{0,n} + D^{\$}_{1,n}x_{c,t} + D^{\$}_{2,n}x_{\pi,t} + D^{\$}_{3,n}x_{d,t}. && (3.24)
\end{aligned}
$$

Let $y_{t,n} = \frac{1}{n}q_{t,n}$ and $y^{\$}_{t,n} = \frac{1}{n}q^{\$}_{t,n}$ denote the n-period continuously compounded real and nominal yield. Then:

$$
\begin{aligned}
y_{t,n} &= -\frac{1}{n}(D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}x_{d,t}), && (3.25) \\
y^{\$}_{t,n} &= -\frac{1}{n}(D^{\$}_{0,n} + D^{\$}_{1,n}x_{c,t} + D^{\$}_{2,n}x_{\pi,t} + D^{\$}_{3,n}x_{d,t}), && (3.26)
\end{aligned}
$$

where the D-coefficients determine how yields respond to changes in expected consumption growth, inflation, and dividend growth. Solving for nominal log bond prices requires the use of the nominal log pricing kernel which is determined by the difference between the real log pricing kernel and the inflation rate:

$$
m^{\$}_{t+1} = m_{t+1} - \pi_{t+1}. \qquad (3.27)
$$

Appendix A.3 and A.4 show how to solve for the coefficients and report the expressions for $D_{0,n}$ and $D^{\$}_{0,n}$. The remaining coefficients are given by:

$$
\begin{aligned}
D_{1,n} &= -\frac{\theta}{\psi} + (\theta-1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 - A_{c,1} + 1) + \\
&\quad D_{1,n-1}\beta_1 + D_{2,n-1}\beta_3, \\
D_{2,n} &= (\theta-1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) + D_{1,n-1}\beta_2 + \\
&\quad D_{2,n-1}\beta_4, \\
D_{3,n} &= 0, \\
D^{\$}_{1,n} &= -\frac{\theta}{\psi} + (\theta-1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 - A_{c,1} + 1) + \\
&\quad D^{\$}_{1,n-1}\beta_1 + D^{\$}_{2,n-1}\beta_3, \\
D^{\$}_{2,n} &= (\theta-1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) + D^{\$}_{1,n-1}\beta_2 + \\
&\quad D^{\$}_{2,n-1}\beta_4 - 1, \\
D^{\$}_{3,n} &= 0,
\end{aligned}
$$

where $D_{i,0}$ and $D^{\$}_{i,0}$ equal zero for $i = 1, 2, 3$. For plausible parameter values, real yields in the model increase in response to positive shocks to consumption growth. This suggests that $D_{1,n}$ is negative and means that real bonds act as a hedge against bad times. They are therefore subject to negative risk premiums in the economy which makes the real yield curve slope downwards. This is supported by empirical evidence from UK-index linked bonds which have been trading since the mid 1980s (e.g., Evans, 1998, and Piazzesi and Schneider, 2006) Unfortunately, data for US index-linked bonds only date back to 1997 but indicate a positively sloped yield curve. However, the rather short sample period and the fact that the market was illiquid at the inception of trading warrants some caution in interpreting the data. The pro-cyclical nature of real yields is also consistent with the empirical findings of Chapman (1997) and Ang et al. (2008). Real yields decrease in response to higher expected inflation if high inflation is bad news for future consumption growth, i.e., $D_{2,n}$ is then positive. This is consistent with earlier studies such as Fama and Gibbons (1982), Pennacchi (1991), and Boudoukh (1993). Ang et al. (2008) also document a negative relation between real rates and expected inflation but find the correlation to be positive for longer horizons.

As expected, high expected inflation depresses nominal bond prices leading $D^{\$}_{2,n}$ to be negative. The reaction of nominal bonds to changes in expected consumption growth depends on the relation between expected consumption growth and future inflation captured by β_3. A positive β_3 implies a negative $D^{\$}_{1,n}$ since high consumption growth then signals high future inflation which is bad news for nominal bond returns. The economic intuition for why changes in dividend growth rates should affect bond prices is not clear, so both real and nominal bonds have a zero loading on x_d. The zero loadings arise due to the restrictions imposed on the dynamics in Section 3.2.1..

Specification II: Heteroscedasticity

Having solved for asset prices using the conditional means as state variables, this section conjectures that asset prices also are functions of the conditional variances and covariances of consumption growth, inflation, and dividend growth. The coefficients in front of the conditional means remain the same as above, wherefore only the solutions for the second moments are reported.

The log price-consumption ratio is conjectured to be a linear function of the following state variables:

$$\begin{aligned} pc_t \;=\; & A_{c,0} + A_{c,1} x_{c,t} + A_{c,2} x_{\pi,t} + A_{c,3} x_{d,t} + A_{c,4} \sigma^2_{c,t} + \\ & A_{c,5} \sigma^2_{\pi,t} + A_{c,6} \sigma^2_{d,t} + A_{c,7} \sigma_{c\pi,t} + A_{c,8} \sigma_{cd,t} + A_{c,9} \sigma_{\pi d,t}. \end{aligned} \tag{3.28}$$

Appendix A.1 shows that the solutions are given by:

$$
\begin{aligned}
A_{c,4} &= \frac{-0.5X^2}{\theta(\phi_c k_{c,1}-1)}, \\
A_{c,5} &= \frac{-0.5Y^2}{\theta(\phi_\pi k_{c,1}-1)}, \\
A_{c,6} &= 0, \\
A_{c,7} &= \frac{-XY}{\theta(\phi_{c\pi} k_{c,1}-1)}, \\
A_{c,8} &= 0, \\
A_{c,9} &= 0, \\
X &= \left[\lambda_{\eta_c} + k_{c,1}(A_{c,1}\delta_1 + A_{c,2}\delta_3 + A_{c,3}\delta_5 + 1)\right], \\
Y &= \left[\lambda_{\eta_\pi} + k_{c,1}(A_{c,1}\delta_2 + A_{c,2}\delta_4)\right],
\end{aligned}
$$

where λ_{η_c} and λ_{η_π} are the market prices of risk, found in (3.20)-(3.21). For negative values of θ, that is when the risk aversion and the IES both are above one, $A_{c,4}$ and $A_{c,5}$ are both negative since ϕ_c, ϕ_π, and $k_{c,1}$ are all positive and less than one. Increased volatility of consumption growth and inflation therefore depresses the log price-consumption ratio. The need for an IES above one to capture the negative relations between the log price-consumption ratio and macro volatility is the same as in the long-run risk model. The response of the price-consumption ratio to the covariance of consumption growth and inflation, $\sigma_{c\pi,t}$, is determined by X and Y which are closely related to the market prices of risk of consumption and inflation shocks. For example, $A_{c,7}$ tends to be positive when positive consumption shocks lower the marginal utility of the agent while positive inflation shocks increase the marginal utility. A pro-cyclical inflation process is then associated with a higher price-consumption ratio. The remaining coefficients are zero as a result of the restrictions imposed on the dynamics.

The log price-dividend ratio is conjectured to be a function of same state variables:

$$
\begin{aligned}
pd_t &= A_{d,0} + A_{d,1}x_{c,t} + A_{d,2}x_{\pi,t} + A_{d,3}x_{d,t} + A_{d,4}\sigma^2_{c,t} + \\
&\quad A_{d,5}\sigma^2_{\pi,t} + A_{d,6}\sigma^2_{d,t} + A_{d,7}\sigma_{c\pi,t} + A_{d,8}\sigma_{cd,t} + A_{d,9}\sigma_{\pi d,t}.
\end{aligned} \tag{3.29}
$$

Appendix A.2 shows that the solutions are given by:

$$
\begin{aligned}
A_{d,4} &= \frac{0.5X^2 + (\theta-1)A_{c,4}(k_{c,1}\phi_c - 1)}{(1-k_{d,1}\phi_c)}, \\
A_{d,5} &= \frac{0.5Y^2 + (\theta-1)A_{c,5}(k_{c,1}\phi_\pi - 1)}{(1-k_{d,1}\phi_\pi)}, \\
A_{d,6} &= \frac{0.5Z^2 + (\theta-1)A_{c,6}(k_{c,1}\phi_d - 1)}{(1-k_{d,1}\phi_d)}, \\
A_{d,7} &= \frac{XY + (\theta-1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1)}{(1-k_{d,1}\phi_{c\pi})}, \\
A_{d,8} &= \frac{XZ + (\theta-1)A_{c,8}(k_{c,1}\phi_{cd} - 1)}{(1-k_{d,1}\phi_{cd})}, \\
A_{d,9} &= \frac{YZ + (\theta-1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1)}{(1-k_{d,1}\phi_{\pi d})}, \\
X &= \left[\lambda_{\eta_c} + k_{d,1}(A_{d,1}\delta_1 + A_{d,2}\delta_3 + A_{d,3}\delta_5)\right], \\
Y &= \left[\lambda_{\eta_\pi} + k_{d,1}(A_{d,1}\delta_2 + A_{d,2}\delta_4)\right], \\
Z &= \left[k_{d,1}A_{d,3}\delta_6 + 1\right].
\end{aligned}
$$

As for the price-consumption ratio above, a high value of the EIS is needed to produce a negative relation between price-dividend ratios and macroeconomic volatility. The last three coefficients determine how the price-dividend ratio responds to changes in the covariances between the three macro variables.

Similarly, the real and nominal log bond prices are conjectured to be functions of the same state variables:

$$
\begin{aligned}
q_{t,n} &= D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}x_{d,t} + D_{4,n}\sigma^2_{c,t} + \qquad (3.30)\\
&\quad D_{5,n}\sigma^2_{\pi,t} + D_{6,n}\sigma^2_{d,t} + D_{7,n}\sigma_{c\pi,t} + D_{8,n}\sigma_{cd,t} + D_{9,n}\sigma_{\pi d,t}, \\
q^\$_{t,n} &= D^\$_{0,n} + D^\$_{1,n}x_{c,t} + D^\$_{2,n}x_{\pi,t} + D^\$_{3,n}x_{d,t} + D^\$_{4,n}\sigma^2_{c,t} + \qquad (3.31)\\
&\quad D^\$_{5,n}\sigma^2_{\pi,t} + D^\$_{6,n}\sigma^2_{d,t} + D^\$_{7,n}\sigma_{c\pi,t} + D^\$_{8,n}\sigma_{cd,t} + D^\$_{9,n}\sigma_{\pi d,t}.
\end{aligned}
$$

Appendix A.3 and A.4 show how to solve for the coefficients. The coefficients for real bonds are for brevity reported in Appendix A.3

while the coefficients for nominal bonds are given by:

$$
\begin{aligned}
D_{4,n}^{\$} &= (\theta-1)A_{c,4}(k_{c,1}\phi_c - 1) + D_{4,n-1}^{\$}\phi_c + 0.5X^2, \\
D_{5,n}^{\$} &= (\theta-1)A_{c,5}(k_{c,1}\phi_\pi - 1) + D_{5,n-1}^{\$}\phi_\pi + 0.5Y^2, \\
D_{6,n}^{\$} &= 0, \\
D_{7,n}^{\$} &= (\theta-1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + D_{7,n-1}^{\$}\phi_{c\pi} + XY, \\
D_{8,n}^{\$} &= 0, \\
D_{9,n}^{\$} &= 0, \\
X &= \left[\lambda_{\eta_c} + D_{1,n-1}^{\$}\delta_1 + D_{2,n-1}^{\$}\delta_3 + D_{3,n-1}^{\$}\delta_5\right], \\
Y &= \left[\lambda_{\eta_\pi} - 1 + D_{1,n-1}^{\$}\delta_2 + D_{2,n-1}^{\$}\delta_4\right],
\end{aligned}
$$

where $D_{i,0}$ and $D_{i,0}^{\$}$ equal zero for $i = 4, 5, ..., 9$. An increase in the volatility of consumption growth and inflation has a negative effect on bond prices, provided a high EIS and high values of the persistence parameters. In that case, nominal bonds do not provide a hedge against periods of high economic turbulence and higher macroeconomic volatility therefore raises nominal yields. Coefficient $D_{7,n}^{\$}$ determines how nominal yields response to changes in the covariance of consumption growth and inflation and depends on the market prices of risk of consumption and inflation shocks, the first terms in X and Y, and on whether bond prices increase or decrease in response to the shocks, the remaining terms in X and Y. A positive $D_{7,n}^{\$}$ implies higher bond prices and lower yields when inflation is procyclical. The volatility of dividend growth and the covariance terms involving dividend growth have no effect on real and nominal bond prices. This is due to the restrictions imposed on the δ matrix in Section 3.2.1..

3.3 Data and Estimation

This section explains the data used in the paper and the estimation of the homoscedastic and heteroscedastic dynamics specified in Section 3.2.1. Preference parameters are calibrated to match unconditional moments of asset prices and are therefore discussed in Section 3.4.

3.3.1 Data

Quarterly aggregate US consumption data on nondurables and services is collected from Bureau of Economic Analysis for the period 1952-2007. Inflation is computed as in Piazzesi and Schneider (2006) using the price index corresponding to the consumption data. Value-weighted market returns (NYSE/AMEX/NASDAQ) are retrieved from CRSP. Nominal interest rates are collected from the Fama-Bliss file in CRSP and from the website of J. Huston McCulloch. The former set of yields are used to match unconditional moments and the latter are used for computing quarterly bond returns since they make it possible to compute quarterly returns on a five year bond. Daily stock returns and daily 5-year nominal interest rates for the period January 1962 - December 2007 are collected from CRSP and the Federal Reserve Bank in St. Louis, respectively. The daily data is used for computing the correlation between stock and bond returns within each quarter. Dividend growth is computed using monthly CRSP returns including and excluding dividends. The procedure follows Bansal et al. (2005) among others. Quarterly dividends are formed by summing dividends for each quarter. To mitigate seasonalities, a moving four-quarter average is used. Real dividend growth rates are found by taking the log first difference and deflating using the constructed inflation series.

Table 1 provides summary statistics of the data. Dividend growth displays the highest volatility of the three series, followed by inflation and consumption growth. Inflation displays positive autocorrelations over both one and two years while the autocorrelations for consumption growth and inflation are not statistically different from zero. The unconditional correlation between inflation and consumption growth is negative, -0.35, and statistically significant at the 10% level. The correlation between consumption growth and dividend growth and between dividend growth and inflation have a positive and negative sign, respectively.

3.3.2 Estimation

Specification I

The state space system for consumption growth, inflation, and dividend growth is estimated using maximum likelihood (e.g., Hamilton, 1994) and quarterly US data for the period 1952:2 - 2007:4. The three series are demeaned. The assumption of Gaussian error terms, $\boldsymbol{\eta}$, gives a log-likelihood function that is the sum of Gaussian densities. The following objective function is maximized:

$$\sum_{t=1}^{T}\left[-\frac{1}{2}n\ln(2\pi)-\frac{1}{2}\ln\mid\boldsymbol{\Omega}\mid-\frac{1}{2}\boldsymbol{\eta}_t'\boldsymbol{\Omega}^{-1}\boldsymbol{\eta}_t\right], \tag{3.32}$$

where $\boldsymbol{\eta}_t = \mathbf{z}_t - \boldsymbol{\mu} - \mathbf{x}_{t-1}$, T equals the number of observations and n equals the dimension of vector $\mathbf{z}_t$, 223 and 3 respectively. The likelihood function is evaluated by finding the state vector recursively from $\mathbf{x}_{t+1} = \boldsymbol{\beta}\mathbf{x}_t + \boldsymbol{\delta}\boldsymbol{\eta}_{t+1}$, where $\mathbf{x}_0$ is set equal to its unconditional mean of zero.

Table 2 presents the estimated parameter values. Negative values of β_2 and δ_2 imply that both expected and unexpected inflation lead to lower future consumption growth. As a result, the agent demands a positive risk premium on assets that are poor hedges against inflation, e.g. equity and nominal bonds. It also makes price-dividend ratios negatively related to expected inflation. The covariance of shocks to consumption growth and inflation is negative and statistically significant. Using the estimated parameter values, the model is simulated 2,000 times in which each simulation contains 223 quarters. The model-implied macro moments are displayed in Table 1 together with the sample moments. The model provides an overall good fit to data.

Figure 1 depicts time series of realized and expected consumption growth, inflation, and dividend growth using the fitted values from the estimation. The time-varying part of the conditional means are extracted from data. The solid lines represent the sum of μ and x for each macro variable and the dashed lines are the realized values. The stagflation period in the 1970s is evident from the spike in inflation and the sharp drop in consumption growth. Noteworthy is also the gradual decline in inflation from 1980 up the beginning of the 2000s. The spike in dividend growth at the end of the sample period is the result of a large difference between cum and ex-dividend market returns obtained from CRSP for November 2004.[9]

Using the estimated parameters, it is straightforward to back out the implied macro shocks. Figure 2 displays the squared shocks to the three macro variables, which provides an understanding of how the macroeconomic volatility has changed over time. Consumption growth was subject to several large shocks during the early part of the sample period while the magnitude of the shocks have decreased over time, with the exception of spikes in the early 1980s and early 1990s. Inflation experienced a period of large shocks during the 1970s and 1980s after which the size of the shocks has decreased.

Figure 3 depicts the cross products of shocks to consumption growth and inflation, shocks to consumption growth and dividend growth, and shocks to inflation and dividend growth. This provides an understanding of how the covariances have changed over time. The period of stagflation in the 1970s is evident from the first graph in which shocks to consumption growth and inflation had opposite signs. After that period, the cross products have stayed close to zero. One can also note that the product of shocks to consumption growth and inflation has stayed negative throughout most of the sample period. The relation between shocks to consumption growth and dividend growth has on the other hand stayed mostly positive, with the exception of the late 1980s. Shocks to inflation and dividend growth were negatively correlated in the mid 1970s and positively correlated in the late 1990s. As will be discussed later, the positive shocks to both dividend growth and inflation in the late 1990s lead to negative correlations in the model between stock and bond returns since higher dividend

[9] The large shock to dividend growth stems from a very large special payment by Microsoft.

growth increases stock returns while bond returns suffer from positive inflation shocks.

Specification II

The objective of this section is to estimate processes for the time-varying conditional second moments. I choose to model the conditional variances and covariances as in the diagonal VEC-model of Bollerslev et al. (1988). Let the macro shocks, $\boldsymbol{\eta}_{t+1}$, be normally distributed with mean zero and subject to the conditional variance-covariance matrix $\mathbf{H}_t$. Let vech($\mathbf{H}_t$) be an operator that stacks the columns of the lower triangular of the variance-covariance matrix. Then, every element of vech($\mathbf{H}_t$) follows a univariate process:

$$h_{c,t} = c_c + a_c \eta_{c,t}^2 + b_c h_{c,t-1}, \quad (3.33)$$

$$h_{c\pi,t} = c_{c\pi} + a_{c\pi} \eta_{c\pi,t}^2 + b_{c\pi} h_{c\pi,t-1}, \quad (3.34)$$

$$h_{cd,t} = c_{cd} + a_{cd} \eta_{cd,t}^2 + b_{cd} h_{cd,t-1}, \quad (3.35)$$

$$h_{\pi,t} = c_{\pi} + a_{\pi} \eta_{\pi,t}^2 + b_{\pi} h_{\pi,t-1}, \quad (3.36)$$

$$h_{\pi d,t} = c_{\pi d} + a_{\pi d} \eta_{\pi d,t}^2 + b_{\pi d} h_{\pi d,t-1}, \quad (3.37)$$

$$h_{d,t} = c_d + a_d \eta_{d,t}^2 + b_d h_{d,t-1}. \quad (3.38)$$

The system is estimated using maximum likelihood and the shocks extracted from data in Specification I above are used as inputs. Table 3 presents the estimation results. The b parameters are all estimated to be in excess of 0.80, indicating that the second moments are persistent processes. Using the estimated parameters and the shocks, I compute time series of the implied conditional second moments. Figure 4 plots the conditional volatilities and Figure 5 plots the conditional covariances. The conditional volatility of consumption growth has been lower in the second half of the sample with notable spikes in the early 1980s and 1990s. Inflation displayed high volatility in the late 1970s and early 1980s after which the volatility declined. The current decade has seen an increase in the volatility of inflation, up to levels last seen in the 1980s. Dividend growth experienced a period of increased volatility from the mid 1980s to the mid 1990s and in the early 2000s. The conditional covariance between inflation and dividend growth has been mostly negative with the exception of the late 1950s and the late 1990s. The covariance between inflation and consumption growth is estimated to have been negative throughout the entire sample period with a sharp drop in the mid 1970s. Finally, the covariance between consumption growth and dividend growth has been positive throughout the sample period.

The estimated parameter values are used when simulating the model. The mapping between the VEC-model and the model's dynamics for the second moments is done in the following way. A GARCH model of the form $h_t = c + bh_{t-1} + a\eta_t^2$ can be rewritten as $h_t = c+(a+b)h_{t-1}+a(\eta_t^2-h_{t-1})$ where the last term can be viewed as a shock to volatility with mean zero, conditional on information at time $t-1$. Assuming for simplicity that the shock is normally distributed, the expression maps into the model's volatility dynamics specified in Section 3.2.1.. I therefore use the sum of the estimated a and b for each second moment as persistence parameters in the model. The unconditional mean of the second moments in the model, that is the α parameters in Section 3.2.1., are set as $\alpha = \frac{c}{1-a-b}$ for each process. The remaining parameter which governs the volatility of volatility, τ, is set as to match the unconditional variance of the model's second moments with the ones estimated from data. The unconditional variance of the second moments within the model are given by $\frac{\tau^2}{1-\phi^2}$. The τ parameters are therefore set as $\tau = \sqrt{Var(h)(1-\phi^2)}$ for each of the second moments. Table 4 presents the parameter values used for the model's second moments.

3.4 Implications for Asset prices

Having estimated the dynamics, the three preference parameters remain to be determined. It is well known from Bansal and Yaron (2004) that models using Epstein-Zin recursive preferences in conjunction with persistent shocks to expected consumption growth can match the observed equity premium for plausible values of risk aversion. However, as is discussed in for example Bansal and Yaron (2000) and Hasseltoft (2008) these models are sensitive to the persistence of macro shocks. A high persistence allows the model to lower the required risk aversion in order to match risk premiums in the economy. The well-known downward bias of estimated persistence parameters in finite samples (Kendall, 1954) also leads to a higher risk aversion needed for matching the level of risk premiums (Bansal et al., 2007a).

I choose to use a risk aversion coefficient of 10 which seems to be a plausible level of risk aversion in the literature. The discount factor, δ, is set to 0.997 and the EIS, ψ, is set to 1.5. The magnitude of the EIS is subject to controversy as mentioned earlier in the paper. It is know in the literature that the so called long-run risk model needs an EIS above one to be able to explain features of asset price data. The model presented in this paper needs an EIS in excess of one for a different reason. Given that high inflation expectations are estimated to signal low future consumption growth ($\beta_2 < 0$), an EIS above one is needed to produce a negative relation between expected inflation and the price-consumption ratio and the price-dividend ratio. This implies that the intertemporal substitution effect dominates the wealth effect and agents therefore sell risky assets in anticipation of lower future returns, yielding lower valuation ratios. The negative signs are supported by empirical evidence by running a contemporaneous regression of a proxy for the unobservable price-consumption ratio, stemming from Lustig et al. (2008), and the observed price-dividend ratio onto the extracted state variables. Table 5 reports the results. Starting with the price-consumption ratio, the results report a positive and statistically significant coefficient when only x_c is used as explanatory variable. This is also the case in the model if one set $A_{c,2}$ equal to zero and keeping the EIS above one. However, adding x_π as explanatory variable drives out the significance of x_c and changes the sign of the coefficient to negative while the coefficient for x_π is negative and strongly significant. The R^2 also increases from 13% to 32%. Results for the price-dividend ratio are similar. Using only x_c as explanatory variable yields a positive and statistically significant slope coefficient. However, adding expected inflation drives out the significance of expected consumption growth and changes it sign to negative. Expected inflation enters negative and highly significant while the R^2

increases from 3% to 13%. Expected dividend growth enters positive, as it is in the model, but is not statistically significant.

Table 6 reports the implied unconditional moments of asset prices. The average equity excess return implied by the homoscedastic model when using the estimated parameter values to simulate the model, is 0.24% which is much lower than the 5.52% observed in data. The average yield spread between a 5-year bond and a 3-month bond implied by the model is 0.15% compared to 0.96% in data. However, Table 6 shows that by increasing the persistence parameter of inflation, β_4, to 1.055 which is less than one standard error away from the point estimate, the model generates an average equity excess return of 4.16% and a slope of the yield curve of 1.50%.[10] The risk aversion is kept at 10 for the calibrated model. This highlights the sensitivity of the model to the persistence in the macro variables and also shows that the model is able to give a reasonable match to data using parameter values that are close to the point estimates. Model-implied dividend yields have a correlation of 0.34 with observed dividend yields and the correlation of model yields with actual yields are 0.72, 0.65, and 0.30 for the short rate, the 5-year rate, and the yield spread respectively.

Turning to the heteroscedastic model (Specification II), I set the preference parameters as above. Again, I evaluate the model implications using both the estimated parameter values and the calibrated value of β_4. The last two columns of Table 6 report the asset pricing implications. Using the point estimates, the model generates an equity premium of 0.14% and a yield curve slope of 0.23%. Both values are smaller than those observed in data. However, the sensitivity of the model to a small increase in inflation persistence is highlighted in the last column. Setting β_4 equal to 1.055 results in an average equity premium of 5.03% and a yield curve slope of 1.66%. The unconditional moments of the model are sensitive to small alterations in other parameters as well, suggesting that a more extensive calibration exercise is likely to yield an even better match to data. For example, Hasseltoft (2008) shows that these type of models also are sensitive to

[10]The model-implied autocorrelations of inflation when setting β_4 equal to 1.055 are 0.77 and 0.66 for a one-year and two-year horizon, respectively.

changes in the persistence of volatility shocks.

3.5 Explaining the Fed-model

The so called Fed-model refers to the positive correlation between US dividend yields and US nominal interest rates. Figure 6 plots the US dividend yield and the 5-year nominal Treasury yield for the period 1952-2007. The two series display an unconditional correlation of 0.30 for the entire sample period, but 0.74 for the period 1965-2007. This phenomenon has been considered puzzling since it implies that changes in expected inflation and bond risk premiums, which have been the main drivers of nominal interest rates (e.g. Campbell and Ammer, 1993, and Best et al., 1998), should be associated with movements in dividend yields. The puzzle can be illustrated by considering the Gordon growth model that expresses the dividend-price ratio in steady state as:

$$\frac{D}{P} = R - G, \tag{3.39}$$

where R is the real discount rate on equities and G is the real dividend growth rate. The real discount, R, can be decomposed into the real risk free rate and the equity risk premium. For the positive correlations in data to arise, expected inflation and bond risk premiums must be either positively associated with real interest rates and equity risk premiums or negatively correlated with dividend growth rates. Surprisingly, the literature has considered any of these explanations to be unlikely. Instead, an explanation in the form of inflation illusion has found wide support (e.g., Ritter and Warr, 2002, Asness, 2003, Campbell and Vuolteenaho, 2004, and Cohen et al., 2005). This entails irrational investors who fail to adjust the expected nominal dividend growth for changes in inflation but they adjust the nominal discount rate. In effect, investors discount real cash flows using nominal interest rates, leading equities to be undervalued from the viewpoint of a rational investor in times of high inflation. However in a recent empirical paper, Bekaert and Engstrom (2008) argue that rational mechanisms are at work. Using a vector autoregressive framework, they ascribe the high correlation of dividend yields and bond yields to mainly a positive relation between expected inflation and the equity risk premium. They proxy the equity risk premium with a measure of economic uncertainty and a consumption-based measure of risk aversion and find that they are both positively correlated with expected inflation. Using cross-country data, they further argue that the correlation between dividend yields and bond yields are higher in countries with a higher average incidence of stagflation. Hence, any equilibrium model that tries to explain the Fed-model seems to need a link between expected inflation and the equity risk premium. The model presented in this paper contains such a link.

As the objective of this section is explain the unconditional correlation between dividend yields and nominal yields, it suffices to analyze the homoscedastic case of the model. Consider the unconditional correlation between the log dividend yield, dp_t and the nominal yield on a bond with a maturity of n periods, $y^{\$}_{n,t}$:

$$\begin{aligned} Cov(dp_t, y^{\$}_{t,n}) &= Cov(-pd_t, y^{\$}_{t,n}), \\ &= Cov(-A_{d,1}x_{c,t} - A_{d,2}x_{\pi,t} - A_{d,3}x_{d,t}, \\ &\quad -\frac{1}{n}(D^{\$}_{1,n}x_{c,t} + D^{\$}_{2,n}x_{\pi,t} + D^{\$}_{3,n}x_{d,t})), \\ &= A\sigma^2_{x_c} + B\sigma^2_{x_\pi} + C\sigma^2_{x_d} + D\sigma_{x_c x_\pi} + \\ &\quad E\sigma_{x_c x_d} + F\sigma_{x_\pi x_d}, \end{aligned} \tag{3.40}$$

where:

$$
\begin{aligned}
A &= (-A_{d,1})(-\frac{D^{\$}_{1,n}}{n}), \\
B &= (-A_{d,2})(-\frac{D^{\$}_{2,n}}{n}), \\
C &= (-A_{d,3})(-\frac{D^{\$}_{3,n}}{n}), \\
D &= \left((-A_{d,1})(-\frac{D^{\$}_{2,n}}{n}) + (-A_{d,2})(-\frac{D^{\$}_{1,n}}{n})\right), \\
E &= \left((-A_{d,1})(-\frac{D^{\$}_{3,n}}{n}) + (-A_{d,3})(-\frac{D^{\$}_{1,n}}{n})\right), \\
F &= \left((-A_{d,2})(-\frac{D^{\$}_{3,n}}{n}) + (-A_{d,3})(-\frac{D^{\$}_{2,n}}{n})\right).
\end{aligned}
$$

Given the estimated and calibrated parameters, the second term is dominating which means that shocks to inflation serve as the main determinant for the covariance between dividend yields and nominal yields. Specifically, B is positive indicating that the covariance is increasing in the volatility of inflation. Coefficient A is also positive meaning that macroeconomic volatility in general is suggested to play a key role for explaining the Fed-model. The loading on dividend volatility is zero since expected dividend growth only affects dividend yields and not bond prices, i.e. C equals zero. Coefficient D is positive while both E and F are negative. For example, periods in which both dividend growth and inflation increase imply lower dividend yields through higher cash flows and higher nominal yields through higher inflation. This generates a negative covariance between the two variables. As mentioned above, variations in dividend yields can be decomposed into changes in real interest rates, risk premiums, and dividend growth while changes in nominal yields can be decomposed into changes in real interest rates, inflation, and risk premiums. While variations in real interest rates is likely to induce a positive correlation between the two, it has been shown that changes in real rates contributes little to changes in asset prices (e.g. Campbell and Ammer, 1993). Instead a likely channel for the positive correlation to arise is through common changes in risk premiums.

Consider the equity risk premium for the homoscedastic case:

$$
\begin{aligned}
E_t[r_{m,t+1} - r_{f,t}] + \frac{1}{2}Var_t[r_{m,t+1}] &= -Cov_t[m_{t+1}, r_{m,t+1}] \qquad (3.41)\\
&= -Cov_t[A\eta_{c,t+1} + B\eta_{\pi,t+1}, C\eta_{c,t+1} + D\eta_{\pi,t+1} + E\eta_{d,t+1}],\\
&= -[AC\sigma_c^2 + BD\sigma_\pi^2 + (AD + BC)\sigma_{c\pi} + AE\sigma_{cd} + BE\sigma_{\pi d}],\\
A &= \lambda_{\eta_c},\\
B &= \lambda_{\eta_\pi},\\
C &= k_{d,1}A_{d,1}\delta_1 + k_{d,1}A_{d,2}\delta_3 + k_{d,1}A_{d,3}\delta_5,\\
D &= k_{d,1}A_{d,1}\delta_2 + k_{d,1}A_{d,2}\delta_4,\\
E &= k_{d,1}A_{d,3}\delta_6 + 1,
\end{aligned}
$$

which is determined by the conditional covariance between the real pricing kernel and the real market return. As positive inflation shocks signal worse economic conditions, the marginal utility is increasing in shocks to inflation. The representative agent therefore dislikes higher inflation and the market price of inflation risk, λ_{η_π}, has therefore a positive sign. The market return on the other hand decreases in response to inflation shocks, $D < 0$, implying that stocks are a poor hedge against inflation. Inflation shocks therefore cause a negative conditional covariance between m_{t+1} and $r_{m,t+1}$ and contributes to a positive equity risk premium. This part of the equity risk premium is absent in models that do not consider the real effects of inflation, for example the long-run risk model.

The yield on a nominal bond can be written as the sum of the corresponding real yield, the expected inflation over the bond's maturity, the inflation risk premium, and a Jensen's inequality term. The following holds for the nominal short rate:

$$
y_{t,3m}^{\$} = y_{t,3m} + E_t(\pi_{t+1}) + Cov_t(m_{t+1}, \pi_{t+1}) - \frac{1}{2}Var_t(\pi_{t+1}),
$$

where the covariance term represents the inflation risk premium. Positive inflation shocks that occur during periods of high marginal utility imply a positive inflation risk premium as nominal bonds then perform badly in bad times. Nominal yields therefore increase as a result. Solving for the inflation risk premium in the model yields:

$$\begin{aligned} Cov_t(m_{t+1}, \pi_{t+1}) &= Cov_t(A\eta_{c,t+1} + B\eta_{\pi,t+1}, \eta_{\pi,t+1}), \\ &= B\sigma_\pi^2 + A\sigma_{c\pi}, \\ A &= \lambda_{\eta^c}, \\ B &= \lambda_{\eta^\pi}. \end{aligned} \tag{3.42}$$

Recall that the price of inflation risk, λ_{η^π}, is positive, so higher inflation volatility raises risk premiums on nominal bonds. The price of consumption risk, λ_{η^c}, is negative so a counter-cyclical inflation process contributes to a higher inflation risk premium. This arises since bonds then perform badly in periods of low consumption growth. That is, bond returns are procyclical. Analyzing risk premiums on equity and bonds jointly suggests that risk premiums on both assets load positively on the unconditional volatility of inflation. In fact, dividend yields and nominal yields become positively correlated in the model mainly through the positive correlation of risk premiums on equity and bonds.

Table 7 reports observed and model-implied correlations for both the homoscedastic and heteroscedastic case. The model produces a high comovement between dividend yields and 5-year nominal interest rates. The correlation coefficients for the homoscedastic case are 0.82 for the whole sample period and 0.81 for the period 1965-2007 compared to 0.30 and 0.74 in data. The heteroscedastic model generates similar coefficients, 0.73 and 0.72. The model reproduces the high correlations observed in data solely through rational channels, mainly the common effect of inflation on equity and bond risk premiums. Turning off the effect of inflation being bad news for consumption growth, i.e., setting $\beta_2 = 0$, reduces the model-implied correlation drastically to 0.17. Recall the expressions for the log price-dividend ratio and for the 5-year log nominal bond price:

$$\begin{aligned} pd_t &= A_{d,0} + A_{d,1}x_{c,t} + A_{d,2}x_{\pi,t} + A_{d,3}x_{d,t}, \\ q^\$_{5y,t} &= D^\$_{0,5y} + D^\$_{1,5y}x_{c,t} + D^\$_{2,5y}x_{\pi,t} + D^\$_{3,5y}x_{d,t}. \end{aligned}$$

The fact that an increase in both expected and unexpected inflation imply bad news for future consumption growth leads to positive risk premiums on equity and a negative relation between inflation and price-dividend ratios. That is, coefficient $A_{d,2}$ is negative. As expected, nominal bond prices decline in response to higher inflation which implies that $D^{\$}_{2,5y}$ is negative. Hence, dividend yields and nominal yields become positively associated.

3.6 Explaining the correlation of stock and bond returns

The unconditional correlation of US stock and bond returns for the period 1952:2-2007:4 is slightly positive, 0.10. However, it has varied substantially through time. Figure 7 displays a 20-quarter rolling correlation between nominal US stock returns and nominal returns on the 5-year US Treasury bond. The 1950s and early 1960s experienced negative correlations which turned positive during the 1970s and early 1980s. The comovement of stock and bond returns declined sharply in 1987 at the time of the stock market crash. The correlation subsequently turned positive in the early 1990s before the stock market boom in the late 1990s evolved with associated negative correlations. The correlations remained negative in the early 2000, at the time stock prices were falling and the Federal Reserve were lowering short rates in response to lower economic activity. This section makes use of the heteroscedastic case of the model since the objective is to explain the changing conditional correlations. Risk premiums on equity and bonds are therefore time varying in this section. Before exploring the implications of the model for the correlation of stock and bond returns, consider the equity risk premium for the heteroscedastic case:

$$
\begin{aligned}
E_t[r_{m,t+1} - r_{f,t}] + \frac{1}{2}Var_t[r_{m,t+1}] &= -Cov_t[m_{t+1}, r_{m,t+1}] \qquad (3.43)\\
&= -[AC\sigma^2_{c,t} + BD\sigma^2_{\pi,t} + (AD+BC)\sigma_{c\pi,t} + AE\sigma_{cd,t} + \\
&\quad BE\sigma_{\pi d,t} + F], \\
F &= (\theta - 1)k_{c,1}k_{d,1}(A_{c,4}A_{d,4}\tau_c^2 + A_{c,5}A_{d,5}\tau_\pi^2 + A_{c,6}A_{d,6}\tau_d^2 + \\
&\quad A_{c,7}A_{d,7}\tau_{c\pi}^2 + A_{c,8}A_{d,8}\tau_{cd}^2 + A_{c,9}A_{d,9}\tau_{\pi d}^2),
\end{aligned}
$$

where coefficients A-E are the same as in (3.41). Risk premiums on equity vary over time in response to changes in the second moments of consumption growth, inflation, and dividend growth. The dominant factors for determining risk premiums on equity is inflation volatility, $\sigma^2_{\pi,t}$, followed by the covariance between dividend growth and inflation, $\sigma_{\pi d,t}$. Expected excess returns increase as the volatility of inflation increases. That is, stocks are risky assets as they perform badly in periods of high macroeconomic volatility. This relates to the so called long-run risk model in which consumption volatility plays an important role. However in contrast to that model, inflation volatility turns out to be the major driver of risk premiums when the real effects of inflation are taken into account. A positive conditional covariance between dividend growth and inflation contributes to a lower equity risk premium since high dividend growth in bad inflationary times implies that stocks are a good hedge. As will be discussed below, a positive $\sigma_{\pi d,t}$ is suggested to have played an important role for making stock and bond returns negatively correlated in the late 1990s.

Solving for the inflation risk premium in the heteroscedastic case yields:

$$
\begin{aligned}
Cov_t(m_{t+1}, \pi_{t+1}) &= Cov_t(A\eta_{c,t+1} + B\eta_{\pi,t+1}, \eta_{\pi,t+1}), \\
&= B\sigma^2_{\pi,t} + A\sigma_{c\pi,t}, \qquad (3.44)
\end{aligned}
$$

where A and B are the same as in (3.42). Again, inflation volatility is the dominant factor. As discussed in Section 3.5, inflation volatility moves risk premiums on equity and bonds in the same direction suggesting that their returns should be positively correlated through their common exposure to macroeconomic risk. A highly volatile inflation rate implies that macro risk becomes the main determinant for changes in risk premiums. When inflation is less volatile, its effect on risk premiums is less dominant which leaves room for other factors to affect the comovement between asset returns. One such factor is changes in dividend growth that only affects equity in the model. I choose to analyze the correlation between stock and bond returns in two different ways. First, I analyze the model-implied conditional correlations. Second, I compute rolling correlations of model-implied realized returns and compare to rolling correlations of actual returns.

The heteroscedastic dynamics in Specification II allow me to compute time-varying conditional correlations implied by the model. First consider the model-implied quarterly conditional covariance between nominal stock returns and returns on a 5-year nominal bond:

$$\begin{aligned} Cov_t(r_{m,t+1} + \pi_{t+1}, h^{\$}_{t+1,60m}) &= A + B\sigma^2_{c,t} + C\sigma^2_{\pi,t} + \\ & \quad D\sigma_{c\pi,t} + E\sigma_{cd,t} + F\sigma_{\pi d,t}, \end{aligned} \tag{3.45}$$

where $h^{\$}_{t+1,60m} = q^{\$}_{t+1,57m} - q^{\$}_{t,60m}$ and where the expressions for the coefficients A to F are different from the ones used above. The two most important factors to consider are again the volatility of inflation and the covariance between inflation and dividend growth. Coefficient C is by far the largest indicating that changes in inflation volatility has the strongest effect on the covariance between asset returns. This stems from its common effect on equity and bond risk premiums. The covariance of returns react negatively to periods in which dividend growth and inflation are positively associated since higher dividend growth raises stock returns while bond returns suffer from an increase in inflation, i.e., coefficient F is negative. A period of rising dividend growth together with an increase in inflation does not only lead to an outperformance of stocks versus bonds due to higher cash flows. A positive covariance between the two also lowers the equity risk premium since high dividend growth in bad inflationary times imply that stocks are a good hedge against less favorable times. Given that inflation volatility is low, a positive covariance between dividend growth and inflation can therefore generate a negative correlation between stock and bond returns. This is what the model suggests happened during the late 1990s. Note that the variance of the macroeconomic variables only contributes to a positive covariance in the model. The covariance terms are therefore important as they allow the model-implied correlations to switch sign.

Dividing the conditional covariance by the product of the conditional volatility of stock and bond returns allows me to compute conditional correlations. I use the extracted second moments from Section 3.3.2. to compute a time series of model-implied correlations. Figure 8 plots the model-implied quarterly conditional correlations. The model generates low conditional correlations in the beginning of the sample, highly positive correlations during the late 1970s and early 1980s and subsequently lower correlations. The model predicts correlations close to zero in the late 1990s, falling short of the sharply negative realized correlations observed for the same period. Similarly, the model did not foresee the negative correlations that occurred during the late 1950s. To evaluate the predictive ability of the model, I regress observed correlations within each quarter, that is between time t to $t+1$, computed using daily data onto the model-implied conditional correlation at time t. Table 8 reports the results. The regression yields a statistically significant coefficient and an R^2_{adj} of 13%. For comparison, regressing the observed quarterly correlation onto its own lag yields a statistically significant slope coefficient and an R^2_{adj} of 38%. The last regression in the table includes the model forecast as an independent variable together with the lagged observed correlation. Including the model's prediction increases the R^2 from 38% to 41% while the model coefficient remains statistically significant. The model therefore seems to contain information beyond what is included in lagged correlations. Restricting the time period to 1970-2007 improves the model's ability to explain variations in future correlations, raising the R^2 from 13% to 20%.[11]

Next I consider the implications of the model for realized stock and bond returns. Nominal quarterly stock returns are formed as:

[11] Regression results for the subperiod are not reported in order to conserve space, but are available upon request.

$r^{\$}_{m,t+1} = k_{0,m} + k_{1,m}pd_{t+1} - pd_t + \Delta d_{t+1} + \pi_{t+1}$ and nominal quarterly returns for the 5-year bond are computed as the difference between the 57-month log bond price and the 60-month log bond price, $h^{\$}_{t+1,60m} = q^{\$}_{t+1,57m} - q^{\$}_{t,60m}$. I then form 20-quarter rolling correlations of the returns. Figure 9 displays model correlations vs. correlations of actual returns. The model correlations fit data quite well with a correlation between the two series of 0.71. The model therefore seems to capture low-frequency movements in realized correlations quite accurately. The late 1970s and early 1980s experienced high levels of volatility in both consumption growth and inflation which made both stocks and bonds risky, generating a high positive correlation of returns. Volatility levels gradually decreased during the 1980s with the result of a gradual decline in the correlation. The downturn in model-implied correlations in the late 1980s are mainly due to an increase in the volatility of stock returns as the volatility of dividend growth increased sharply during that period. The early 1990s saw a spike in the correlations as consumption growth volatility increased sharply together with lower stock return volatility. Subsequently, up to year 2000, macroeconomic volatility decreased to historically low levels with the effect of lower correlations. The negative model correlations in the late 1990s arise as a result of low economic volatility together with a positive covariance between dividend growth and inflation. This positive covariance has two effects on equity returns in the model. For example, consider a period in which both dividend growth and inflation increase. First, stock returns respond positively through higher cash flows. Second, it lowers the equity risk premium since positive cash flow shocks that occur in bad times (rising inflation) imply that stocks are a hedge against bad inflationary times. At the same time, a slowly increasing inflation rate in the late 1990s made bonds perform badly, yielding negative bond returns. The model therefore attributes the negative correlations to an outperformance of stocks vs. bonds through higher cash flows and a lower equity risk premium together with a series of small positive inflation shocks. The fact that the model does not predict negative correlations in the late 1990s but capture some of the negative realized correlations is due to a higher covariance of dividend growth and inflation than expected.

The model-implied correlations increased sharply in the early 2000s as the volatility of inflation started to pick up. However, in data the correlations remained negative as stock prices fell and the Federal Reserve started cutting interest rates to stimulate the economy, yielding positive bond returns. This is not the first paper to have difficulties explaining and matching the extent of the negative correlations observed throughout the current decade. The unconditional correlation of stock and bond returns is 0.25 in the model, which is somewhat higher than the 0.10 observed in data.

3.7 Conclusion

This paper proposes a consumption-based equilibrium model that to a large extent can explain two features of data that have been considered puzzling. First, the paper shows that the strikingly high correlation between US dividend yields and nominal interest can be explained within a rational model through a risk-premium channel. This stands in contrast to the hitherto dominant explanation in the form of inflation illusion. Second, the model attributes a large part of changes in realized correlations between stock and bond returns to changes in macroeconomic risk. High volatility of consumption growth and inflation caused stock and bond returns to comove strongly in the late 1970s and early 1980s. Risk premiums on both equity and bonds in the model react similarly to changes in macroeconomic volatility, making their returns positively correlated. The subsequent decline in aggregate economic risk from the early 1980s until 2000 is suggested to have brought correlations lower. The negative correlations observed in the late 1990s are partly attributed to low levels of macroeconomic volatility in conjunction with a positive covariance between dividend growth and inflation.

However, there are still some unresolved issues from the perspective of the model. First, the estimated negative relation between consumption growth and inflation is a statistical relation. The model is silent on what the actual underlying mechanisms are. A possible explanation would be monetary policy through which a central bank, keen on bringing down inflation expectations, raise short rates such that consumption growth contracts in the following periods. An interesting area for future research would be to examine the role of monetary policy for explaining the so called Fed-model and correlations between asset returns, but also its role for determining risk premiums in general.

Second, starting in year 2000, inflation volatility started to increase and has today reached levels last seen in the early 1980s. This implies a positive correlation between stock and bond returns in the model. However, correlations in data have remained negative throughout the current decade which suggests that other forces are at work. The extent of the negative correlations have puzzled many others in the literature as well and warrants a further investigation. A particularly interesting period to analyze is the recent financial crisis in which stock and bond returns have tended to be negatively correlated. Expecting consumption-based models to fully explain asset correlations during such extreme periods is perhaps too much to hope for. Instead, modeling so called liquidity factors jointly with macro factors is likely to yield more insights into so called “flight-to-safety”periods.

Appendix

Sections A.1 - A.4 solve the model using approximate analytical solutions for the case of heteroscedasticity (Specification II). Coefficients for the conditional first moments, x_c, x_π, x_d, are the same for the homoscedastic Specification I and for the heteroscedastic Specification II.

A.1 The price-consumption ratio

The coefficients governing the price-consumption ratio are derived using the logarithm of the intertemporal marginal rate of substitution, $m_{t+1} = \theta \ln(\delta) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}$, together with the dynamics of consumption growth, inflation, and volatility in Section 3.2.1, and the approximation of the return on the consumption paying asset, $r_{c,t+1} = k_{c,0} + k_{c,1} pc_{t+1} - pc_t + \Delta c_{t+1}$, where $pc_t = A_{c,0} + A_{c,1} x_{c,t} + A_{c,2} x_{\pi,t} + A_{c,3} x_{d,t} + A_{c,4} \sigma^2_{c,t} + A_{c,5} \sigma^2_{\pi,t} + A_{c,6} \sigma^2_{d,t} + A_{c,7} \sigma_{c\pi,t} + A_{c,8} \sigma_{cd,t} + A_{c,9} \sigma_{\pi d,t}$. Consider the Euler equation for the consumption claim:

$$E_t \left[\exp(\theta \ln(\delta) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1} + r_{c,t+1}) \right] = 1.$$

Due to the conditional normality of Δc and the state variables, and therefore also r_c, the log Euler condition can be written as:

$$E_t \left[m_{t+1} + r_{c,t+1} \right] + \frac{1}{2} Var_t \left[m_{t+1} + r_{c,t+1} \right] = 0.$$

Appendix

The conditional mean is given by:

$$
\begin{aligned}
&E_t\left[m_{t+1}+r_{c,t+1}\right]=\theta\ln(\delta)-\frac{\theta}{\psi}\mu_c+\theta(k_{c,0}+k_{c,1}(A_{c,0}+\\
&A_{c,4}\alpha_c(1-\phi_c)+A_{c,5}\alpha_\pi(1-\phi_\pi)+\\
&A_{c,6}\alpha_d(1-\phi_d)+A_{c,7}\alpha_{c\pi}(1-\phi_{c\pi})+A_{c,8}\alpha_{cd}(1-\phi_{cd})+\\
&A_{c,9}\alpha_{\pi d}(1-\phi_{\pi d}))-A_{c,0}+\mu_c)+\\
&x_{c,t}\left[-\frac{\theta}{\psi}+\theta(k_{c,1}A_{c,1}\beta_1+k_{c,1}A_{c,2}\beta_3+k_{c,1}A_{c,3}\beta_5-A_{c,1}+1)\right]+\\
&x_{\pi,t}\left[\theta(k_{c,1}A_{c,1}\beta_2+k_{c,1}A_{c,2}\beta_4-A_{c,2})\right]+x_{d,t}\left[\theta A_{c,3}(k_{c,1}\beta_6-1)\right]+\\
&\sigma_{c,t}^2\left[\theta A_{c,4}(k_{c,1}\phi_c-1)\right]+\sigma_{\pi,t}^2\left[\theta A_{c,5}(k_{c,1}\phi_\pi-1)\right]+\\
&\sigma_{d,t}^2\left[\theta A_{c,6}(k_{c,1}\phi_d-1)\right]+\sigma_{c\pi,t}\left[\theta A_{c,7}(k_{c,1}\phi_{c\pi}-1)\right]+\\
&\sigma_{cd,t}\left[\theta A_{c,8}(k_{c,1}\phi_{cd}-1)\right]+\sigma_{\pi d,t}\left[\theta A_{c,9}(k_{c,1}\phi_{\pi d}-1)\right].
\end{aligned}
$$

and the conditional variance is given by:

$$
\begin{aligned}
&Var_t\left[m_{t+1}+r_{c,t+1}\right]=\sigma_{c,t}^2X^2+\sigma_{\pi,t}^2Y^2+\sigma_{d,t}^2Z^2+\\
&2\sigma_{c\pi,t}XY+2\sigma_{cd,t}XZ+2\sigma_{\pi d,t}YZ+\\
&(\theta k_{c,1}A_{c,4}\tau_c)^2+(\theta k_{c,1}A_{c,5}\tau_\pi)^2+(\theta k_{c,1}A_{c,6}\tau_d)^2+(\theta k_{c,1}A_{c,7}\tau_{c\pi})^2+\\
&(\theta k_{c,1}A_{c,8}\tau_{cd})^2+(\theta k_{c,1}A_{c,9}\tau_{\pi d})^2,
\end{aligned}
$$

$$
\begin{aligned}
X &= \left[-\frac{\theta}{\psi}+\theta(k_{c,1}A_{c,1}\delta_1+k_{c,1}A_{c,2}\delta_3+k_{c,1}A_{c,3}\delta_5+1)\right],\\
Y &= \left[\theta(k_{c,1}A_{c,1}\delta_2+k_{c,1}A_{c,2}\delta_4)\right],\\
Z &= \left[\theta(k_{c,1}A_{c,3}\delta_6)\right].
\end{aligned}
$$

Setting the conditional moments equal to zero and solving for the A_c-coefficients yield the following expressions:

$$
\begin{aligned}
A_{c,0} &= (\theta(1-k_{c,1}))^{-1}\Big[\theta\ln(\delta) - \frac{\theta}{\psi}\mu_c + \theta(k_{c,0} + \\
&\quad k_{c,1}(A_{c,4}\alpha_c(1-\phi_c) + A_{c,5}\alpha_\pi(1-\phi_\pi) + \\
&\quad A_{c,6}\alpha_d(1-\phi_d) + A_{c,7}\alpha_{c\pi}(1-\phi_{c\pi}) + \\
&\quad A_{c,8}\alpha_{cd}(1-\phi_{cd}) + A_{c,9}\alpha_{\pi d}(1-\phi_{\pi d})) + \mu_c) + \\
&\quad 0.5((\theta k_{c,1}A_{c,4}\tau_c)^2 + (\theta k_{c,1}A_{c,5}\tau_\pi)^2 + (\theta k_{c,1}A_{c,6}\tau_d)^2 + (\theta k_{c,1}A_{c,7}\tau_{c\pi})^2 + \\
&\quad (\theta k_{c,1}A_{c,8}\tau_{cd})^2 + (\theta k_{c,1}A_{c,9}\tau_{\pi d})^2)\Big], \\
A_{c,1} &= \frac{1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - \frac{1}{\psi}}{1-k_{c,1}\beta_1}, \\
A_{c,2} &= \frac{k_{c,1}A_{c,1}\beta_2}{1-k_{c,1}\beta_4}, \\
A_{c,3} &= 0, \\
A_{c,4} &= \frac{-0.5X^2}{\theta(k_{c,1}\phi_c - 1)}, \\
A_{c,5} &= \frac{-0.5Y^2}{\theta(k_{c,1}\phi_\pi - 1)}, \\
A_{c,6} &= \frac{-0.5Z^2}{\theta(k_{c,1}\phi_d - 1)}, \\
A_{c,7} &= \frac{-XY}{\theta(k_{c,1}\phi_{c\pi} - 1)}, \\
A_{c,8} &= \frac{-XZ}{\theta(k_{c,1}\phi_{cd} - 1)}, \\
A_{c,9} &= \frac{-YZ}{\theta(k_{c,1}\phi_{\pi d} - 1)},
\end{aligned}
$$

where X,Y, and Z are determined as above. Coefficients $A_{c,6}, A_{c,8}$, and $A_{c,9}$ are zero since $A_{c,3}$ equals zero. Note that $A_{c,1}$ and $A_{c,2}$ are determined jointly. Solving the simultaneous equation system by substituting $A_{c,1}$ into $A_{c,2}$ returns:

$$A_{c,2} = \frac{k_{c,1}\beta_2(1-\frac{1}{\psi})}{(1-k_{c,1}\beta_4)(1-k_{c,1}\beta_1)-k_{c,1}^2\beta_2\beta_3}.$$

The homoscedastic dynamics (Specification I) has a different $A_{c,0}$ term, namely:

$$A_{c,0} = \frac{\Big[\theta\ln(\delta)-\frac{\theta}{\psi}\mu_c+\theta(k_{c,0}+\mu_c)}{\theta(1-k_{c,1})} + \frac{0.5(\sigma_c^2X^2+\sigma_\pi^2Y^2+\sigma_d^2Z^2+2\sigma_{c\pi}XY+2\sigma_{cd}XZ+2\sigma_{\pi d}YZ)\Big]}{\theta(1-k_{c,1})},$$

where X,Y, and Z are determined as above.

A.2 The price-dividend ratio

The coefficients governing the price-dividend ratio are found in an analogous manner. The Euler condition for the market return, $r_{m,t+1}$, is written as:

$$E_t\left[m_{t+1}+r_{m,t+1}\right]+\frac{1}{2}Var_t\left[m_{t+1}+r_{m,t+1}\right],$$

where $r_{m,t+1}=k_{d,0}+k_{d,1}pd_{t+1}-pd_t+\Delta d_{t+1}$ and $pd_t=A_{d,0}+A_{d,1}x_{c,t}+A_{d,2}x_{\pi,t}+A_{d,3}x_{d,t}+A_{d,4}\sigma_{c,t}^2+A_{d,5}\sigma_{\pi,t}^2+A_{d,6}\sigma_{d,t}^2+A_{d,7}\sigma_{c\pi,t}+A_{d,8}\sigma_{cd,t}+A_{d,9}\sigma_{\pi d,t}$. Coefficients $k_{d,0}$ and $k_{d,1}$ are defined as:

$$\begin{aligned} k_{d,0} &= \ln(1+\exp(\bar{pd}))-k_{d,1}\bar{pd}, \\ k_{d,1} &= \frac{\exp(\bar{pd})}{1+\exp(\bar{pd})}. \end{aligned}$$

Using the dynamics of consumption growth, inflation, dividend growth, and the second moments, the conditional mean is given by:

$$
\begin{aligned}
&E_t\left[m_{t+1} + r_{m,t+1}\right] = \\
&\quad \theta\ln(\delta) - \frac{\theta}{\psi}\mu_c + (\theta-1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,4}\alpha_c(1-\phi_c) + \\
&\quad A_{c,5}\alpha_\pi(1-\phi_\pi) + A_{c,6}\alpha_d(1-\phi_d) + A_{c,7}\alpha_{c\pi}(1-\phi_{c\pi}) + \\
&\quad A_{c,8}\alpha_{cd}(1-\phi_{cd}) + A_{c,9}\alpha_{\pi d}(1-\phi_{\pi d})) - A_{c,0} + \mu_c) + \\
&\quad + k_{d,0} + k_{d,1}(A_{d,0} + A_{d,4}\alpha_c(1-\phi_c) + A_{d,5}\alpha_\pi(1-\phi_\pi) + \\
&\quad A_{d,6}\alpha_d(1-\phi_d) + A_{d,7}\alpha_{c\pi}(1-\phi_{c\pi}) + A_{d,8}\alpha_{cd}(1-\phi_{cd}) + \\
&\quad A_{d,9}\alpha_{\pi d}(1-\phi_{\pi d})) - A_{d,0} + \mu_d + \\
&\quad x_{c,t}\Bigg[-\frac{\theta}{\psi} + (\theta-1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - \\
&\quad A_{c,1} + 1) + k_{d,1}A_{d,1}\beta_1 + k_{d,1}A_{d,2}\beta_3 + k_{d,1}A_{d,3}\beta_5 - A_{d,1}\Bigg] + \\
&\quad x_{\pi,t}\left[(\theta-1)(k_{d,1}A_{d,1}\beta_2 + k_{d,1}A_{d,2}\beta_4 - A_{d,2}) + k_{d,1}A_{d,1}\beta_2 + \right. \\
&\quad \left. k_{d,1}A_{d,2}\beta_4 - A_{d,2}\right] + \\
&\quad x_{d,t}\left[(\theta-1)A_{c,3}(k_{c,1}\beta_6 - 1) + k_{d,1}A_{d,3}\beta_6 - A_{d,3} + 1\right], \\
&\quad \sigma^2_{c,t}\left[(\theta-1)A_{c,4}(k_{c,1}\phi_c - 1) + A_{d,4}(k_{d,1}\phi_c - 1)\right], \\
&\quad \sigma^2_{\pi,t}\left[(\theta-1)A_{c,5}(k_{c,1}\phi_\pi - 1) + A_{d,5}(k_{d,1}\phi_\pi - 1)\right], \\
&\quad \sigma^2_{d,t}\left[(\theta-1)A_{c,6}(k_{c,1}\phi_d - 1) + A_{d,6}(k_{d,1}\phi_d - 1)\right], \\
&\quad \sigma^2_{c\pi,t}\left[(\theta-1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + A_{d,7}(k_{d,1}\phi_{c\pi} - 1)\right], \\
&\quad \sigma^2_{cd,t}\left[(\theta-1)A_{c,8}(k_{c,1}\phi_{cd} - 1) + A_{d,8}(k_{d,1}\phi_{cd} - 1)\right], \\
&\quad \sigma^2_{\pi d,t}\left[(\theta-1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1) + A_{d,9}(k_{d,1}\phi_{\pi d} - 1)\right].
\end{aligned}
$$

The conditional variance is given by:

$$
\begin{aligned}
Var_t\left[m_{t+1}+r_{m,t+1}\right] = {} & \\
& \sigma_{c,t}^2 X^2+\sigma_{\pi,t}^2 Y^2+\sigma_{d,t}^2 Z^2+2\sigma_{c\pi,t}XY+ \\
& 2\sigma_{cd,t}XZ+2\sigma_{\pi d,t}YZ+ \\
& ((\theta-1)k_{c,1}A_{c,4}\tau_c+k_{d,1}A_{d,4}\tau_c)^2+ \\
& ((\theta-1)k_{c,1}A_{c,5}\tau_\pi+k_{d,1}A_{d,5}\tau_\pi)^2+ \\
& ((\theta-1)k_{c,1}A_{c,6}\tau_d+k_{d,1}A_{d,6}\tau_d)^2+ \\
& ((\theta-1)k_{c,1}A_{c,7}\tau_{c\pi}+k_{d,1}A_{d,7}\tau_{c\pi})^2+ \\
& ((\theta-1)k_{c,1}A_{c,8}\tau_{cd}+k_{d,1}A_{d,8}\tau_{cd})^2+ \\
& ((\theta-1)k_{c,1}A_{c,9}\tau_{\pi d}+k_{d,1}A_{d,9}\tau_{\pi d})^2, \\
X = {} & \left[-\frac{\theta}{\psi}+(\theta-1)(k_{c,1}A_{c,1}\delta_1+k_{c,1}A_{c,2}\delta_3+k_{c,1}A_{c,3}\delta_5+1)+\right. \\
& \left. k_{d,1}A_{d,1}\delta_1+k_{d,1}A_{d,2}\delta_3+k_{d,1}A_{d,3}\delta_5\right], \\
Y = {} & \left[(\theta-1)(k_{c,1}A_{c,1}\delta_2+k_{c,1}A_{c,2}\delta_4)+k_{d,1}A_{d,1}\delta_2+k_{d,1}A_{d,2}\delta_4\right], \\
Z = {} & \left[(\theta-1)(k_{c,1}A_{c,3}\delta_6)+k_{d,1}A_{d,3}\delta_6+1\right].
\end{aligned}
$$

Setting the conditional moments equal to zero and solving for the A_d-coefficients yield the following expressions:

$$
\begin{aligned}
A_{d,0} &= (1-k_{d,1})^{-1}\Bigg[\theta\ln(\delta)-\frac{\theta}{\psi}\mu_c+(\theta-1)(k_{c,0}+k_{c,1}(A_{c,0}+\\
&\quad A_{c,4}\alpha_c(1-\phi_c)+A_{c,5}\alpha_\pi(1-\phi_\pi)+A_{c,6}\alpha_d(1-\phi_d)+\\
&\quad A_{c,7}\alpha_{c\pi}(1-\phi_{c\pi})+A_{c,8}\alpha_{cd}(1-\phi_{cd})+\\
&\quad A_{c,9}\alpha_{\pi d}(1-\phi_{\pi d}))-A_{c,0}+\mu_c)+k_{d,0}+\\
&\quad k_{d,1}(A_{d,4}\alpha_c(1-\phi_c)+A_{d,5}\alpha_\pi(1-\phi_\pi)+A_{d,6}\alpha_d(1-\phi_d)+\\
&\quad A_{d,7}\alpha_{c\pi}(1-\phi_{c\pi})+A_{d,8}\alpha_{cd}(1-\phi_{cd})+A_{d,9}\alpha_{\pi d}(1-\phi_{\pi d}))+\mu_d+\\
&\quad 0.5(((\theta-1)k_{c,1}A_{c,4}\tau_c+k_{d,1}A_{d,4}\tau_c)^2+((\theta-1)k_{c,1}A_{c,5}\tau_\pi+\\
&\quad k_{d,1}A_{d,5}\tau_\pi)^2+((\theta-1)k_{c,1}A_{c,6}\tau_d+k_{d,1}A_{d,6}\tau_d)^2+\\
&\quad ((\theta-1)k_{c,1}A_{c,7}\tau_{c\pi}+k_{d,1}A_{d,7}\tau_{c\pi})^2+\\
&\quad ((\theta-1)k_{c,1}A_{c,8}\tau_{cd}+k_{d,1}A_{d,8}\tau_{cd})^2+((\theta-1)k_{c,1}A_{c,9}\tau_{\pi d}+\\
&\quad k_{d,1}A_{d,9}\tau_{\pi d})^2)\Bigg],\\
A_{d,1} &= \frac{-\frac{\theta}{\psi}+(\theta-1)(k_{c,1}A_{c,1}\beta_1+k_{c,1}A_{c,2}\beta_3+k_{c,1}A_{c,3}\beta_5-A_{c,1}+1)}{+}\\
&\quad \frac{k_{d,1}A_{d,2}\beta_3+k_{d,1}A_{d,3}\beta_5}{1-k_{d,1}\beta_1},\\
A_{d,2} &= \frac{(\theta-1)(k_{c,1}A_{c,1}\beta_2+k_{c,1}A_{c,2}\beta_4-A_{c,2})+k_{d,1}A_{d,1}\beta_2}{1-k_{d,1}\beta_4},\\
A_{d,3} &= \frac{(\theta-1)A_{c,3}(k_{c,1}\beta_6-1)+1}{1-k_{d,1}\beta_6},\\
A_{d,4} &= \frac{(\theta-1)A_{c,4}(k_{c,1}\phi_c-1)+0.5X^2}{1-k_{d,1}\phi_c},\\
A_{d,5} &= \frac{(\theta-1)A_{c,5}(k_{c,1}\phi_\pi-1)+0.5Y^2}{1-k_{d,1}\phi_\pi},\\
A_{d,6} &= \frac{(\theta-1)A_{c,6}(k_{c,1}\phi_d-1)+0.5Z^2}{1-k_{d,1}\phi_d},\\
A_{d,7} &= \frac{(\theta-1)A_{c,7}(k_{c,1}\phi_{c\pi}-1)+XY}{1-k_{d,1}\phi_{c\pi}},\\
A_{d,8} &= \frac{(\theta-1)A_{c,8}(k_{c,1}\phi_{cd}-1)+XZ}{1-k_{d,1}\phi_{cd}},\\
A_{d,9} &= \frac{(\theta-1)A_{c,9}(k_{c,1}\phi_{\pi d}-1)+ZY}{1-k_{d,1}\phi_{\pi d}},
\end{aligned}
$$

where X,Y, and Z are determined as above. Similar to the consumption paying asset, $A_{d,1}$ and $A_{d,2}$ are determined jointly. Substituting $A_{d,1}$ into $A_{d,2}$ yields:

$$\begin{aligned} A_{d,2} &= \frac{(1-k_{d,1}\beta_1)X + Y}{(1-k_{d,1}\beta_4)(1-k_{d,1}\beta_1) - k_{d,1}^2\beta_2\beta_3}, \\ X &= (\theta-1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}), \\ Y &= k_{d,1}\beta_2\left[-\frac{\theta}{\psi} + (\theta-1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - \right. \\ & \left. A_{c,1} + 1) + k_{d,1}A_{d,3}\beta_5\right]. \end{aligned}$$

The homoscedastic dynamics (Specification I) has a different $A_{d,0}$ term, namely:

$$\begin{aligned} & A_{d,0} = ((1-k_{d,1}))^{-1}[\theta\ln(\delta) - \frac{\theta}{\psi}\mu_c + (\theta-1)(k_{c,0} + k_{c,1}A_{c,0} - \\ & A_{c,0} + \mu_c) + k_{d,0} + \mu_d + 0.5(\sigma_c^2X^2 + \sigma_\pi^2Y^2 + \sigma_d^2Z^2 + 2\sigma_{c\pi}XY + \\ & 2\sigma_{cd}XZ + 2\sigma_{\pi d}YZ)] \end{aligned}$$

where X,Y, and Z are determined as for conditional variance above.

A.3 Real bonds

The Euler condition for a real bond takes the form:

$$Q_{t,n} = E_t\left[M_{t+1}Q_{t+1,n-1}\right],$$

where $Q_{t,n} = \exp(D_{0,n} + D_{1,n}x_{c,t} + D_{2,n}x_{\pi,t} + D_{3,n}x_{d,t} + D_{4,n}\sigma_{c,t}^{\$} + D_{5,n}\sigma_{\pi,t}^{\$} + D_{6,n}\sigma_{d,t}^{\$} + D_{7,n}\sigma_{c\pi,t} + D_{8,n}\sigma_{cd,t} + D_{9,n}\sigma_{\pi d,t})$. Again, using the conditional lognormality of the state variables:

$$q_{t,n} = E_t\left[m_{t+1} + q_{t+1,n-1}\right] + \frac{1}{2}Var_t\left[m_{t+1} + q_{t+1,n-1}\right].$$

The conditional mean is given by:

$$
\begin{aligned}
&E_t\left[m_{t+1}+q_{t+1,n-1}\right]= \\
&\quad \theta\ln(\delta)-\frac{\theta}{\psi}\mu_c+(\theta-1)(k_{c,0}+k_{c,1}(A_{c,0}+ \\
&\quad A_{c,4}\alpha_c(1-\phi_c)+A_{c,5}\alpha_\pi(1-\phi_\pi)+A_{c,6}\alpha_d(1-\phi_d)+ \\
&\quad A_{c,7}\alpha_{c\pi}(1-\phi_{c\pi})+A_{c,8}\alpha_{cd}(1-\phi_{cd})+ \\
&\quad A_{c,9}\alpha_{\pi d}(1-\phi_{\pi d}))-A_{c,0}+\mu_c)+D_{0,n-1}+ \\
&\quad D_{4,n-1}\alpha_c(1-\phi_c)+D_{5,n-1}\alpha_\pi(1-\phi_\pi)+D_{6,n-1}\alpha_d(1-\phi_d)+ \\
&\quad D_{7,n-1}\alpha_{c\pi}(1-\phi_{c\pi})+D_{8,n-1}\alpha_{cd}(1-\phi_{cd})+D_{9,n-1}\alpha_{\pi d}(1-\phi_{\pi d})+ \\
&\quad x_{c,t}\Bigg[-\frac{\theta}{\psi}+(\theta-1)(k_{c,1}A_{c,1}\beta_1+k_{c,1}A_{c,2}\beta_3+k_{c,1}A_{c,3}\beta_5-A_{c,1}+1)+ \\
&\quad D_{1,n-1}\beta_1+D_{2,n-1}\beta_3+D_{3,n-1}\beta_5\Bigg]+ \\
&\quad x_{\pi,t}\left[(\theta-1)(k_{c,1}A_{c,1}\beta_2+k_{c,1}A_{c,2}\beta_4-A_{c,2})+D_{1,n-1}\beta_2+D_{2,n-1}\beta_4\right]+ \\
&\quad x_{d,t}\left[(\theta-1)A_{c,3}(k_{c,1}\beta_6-1)+D_{3,n-1}\beta_6\right]+ \\
&\quad \sigma^2_{c,t}\left[(\theta-1)A_{c,4}(k_{c,1}\phi_c-1)+D_{4,n-1}\phi_c\right]+ \\
&\quad \sigma^2_{\pi,t}\left[(\theta-1)A_{c,5}(k_{c,1}\phi_\pi-1)+D_{5,n-1}\phi_\pi\right]+ \\
&\quad \sigma^2_{d,t}\left[(\theta-1)A_{c,6}(k_{c,1}\phi_d-1)+D_{6,n-1}\phi_d\right]+ \\
&\quad \sigma_{c\pi,t}\left[(\theta-1)A_{c,7}(k_{c,1}\phi_{c\pi}-1)+D_{7,n-1}\phi_{c\pi}\right]+ \\
&\quad \sigma_{cd,t}\left[(\theta-1)A_{c,8}(k_{c,1}\phi_{cd}-1)+D_{8,n-1}\phi_{cd}\right]+ \\
&\quad \sigma_{\pi d,t}\left[(\theta-1)A_{c,9}(k_{c,1}\phi_{\pi c}-1)+D_{9,n-1}\phi_{\pi c}\right],
\end{aligned}
$$

and the conditional variance by:

$$
\begin{aligned}
&Var_t\left[m_{t+1} + q_{t+1,n-1}\right] = \\
&\quad \sigma^2_{c,t}X^2 + \sigma^2_{\pi,t}Y^2 + \sigma^2_{d,t}Z^2 + 2\sigma_{c\pi,t}XY + 2\sigma_{cd,t}XZ + \\
&\quad 2\sigma_{\pi d,t}YZ + ((\theta-1)k_{c,1}A_{c,4}\tau_c + D_{4,n-1}\tau_c)^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,5}\tau_\pi + D_{5,n-1}\tau_\pi)^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,6}\tau_d + D_{6,n-1}\tau_d)^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,7}\tau_{c\pi} + D_{7,n-1}\tau_{c\pi})^2 + ((\theta-1)k_{c,1}A_{c,8}\tau_{cd} + D_{8,n-1}\tau_{cd})^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,9}\tau_{\pi d} + D_{9,n-1}\tau_{\pi d})^2, \\
&\quad X = \left[-\frac{\theta}{\psi} + (\theta-1)(k_{c,1}A_{c,1}\delta_1 + k_{c,1}A_{c,2}\delta_3 + k_{c,1}A_{c,3}\delta_5 + 1) + \right. \\
&\quad \left. D_{1,n-1}\delta_1 + D_{2,n-1}\delta_3 + D_{3,n-1}\delta_5\right], \\
&\quad Y = \left[(\theta-1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4) + D_{1,n-1}\delta_2 + D_{2,n-1}\delta_4\right], \\
&\quad Z = \left[(\theta-1)(k_{c,1}A_{c,3}\delta_6) + D_{3,n-1}\delta_6\right].
\end{aligned}
$$

Matching the coefficients gives:

$$
\begin{aligned}
&D_{0,n} = \\
&\quad \theta \ln(\delta) - \frac{\theta}{\psi}\mu_c + (\theta-1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,4}\alpha_c(1-\phi_c) + A_{c,5}\alpha_\pi(1-\phi_\pi) + \\
&\quad A_{c,6}\alpha_d(1-\phi_d) + A_{c,7}\alpha_{c\pi}(1-\phi_{c\pi}) + A_{c,8}\alpha_{cd}(1-\phi_{cd}) + A_{c,9}\alpha_{\pi d}(1-\phi_{\pi d})) - \\
&\quad A_{c,0} + \mu_c) + D_{0,n-1} + D_{4,n-1}\alpha_c(1-\phi_c) + D_{5,n-1}\alpha_\pi(1-\phi_\pi) + \\
&\quad D_{6,n-1}\alpha_d(1-\phi_d) + D_{7,n-1}\alpha_{c\pi}(1-\phi_{c\pi}) + \\
&\quad D_{8,n-1}\alpha_{cd}(1-\phi_{cd}) + D_{9,n-1}\alpha_{\pi d}(1-\phi_{\pi d}) + \\
&\quad 0.5(((\theta-1)k_{c,1}A_{c,4}\tau_c + D_{4,n-1}\tau_c)^2 + ((\theta-1)k_{c,1}A_{c,5}\tau_\pi + D_{5,n-1}\tau_\pi)^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,6}\tau_d + D_{6,n-1}\tau_d)^2 + ((\theta-1)k_{c,1}A_{c,7}\tau_{c\pi} + D_{7,n-1}\tau_{c\pi})^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,8}\tau_{cd} + D_{8,n-1}\tau_{cd})^2 + ((\theta-1)k_{c,1}A_{c,9}\tau_{\pi d} + D_{9,n-1}\tau_{\pi d})^2), \\
&\quad D_{1,n} = -\frac{\theta}{\psi} + (\theta-1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - A_{c,1} + 1) + \\
&\quad D_{1,n-1}\beta_1 + D_{2,n-1}\beta_3, \\
&\quad D_{2,n} = (\theta-1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) + D_{1,n-1}\beta_2 + D_{2,n-1}\beta_4, \\
&\quad D_{3,n} = (\theta-1)A_{c,3}(k_{c,1}\beta_6 - 1) + D_{3,n-1}\beta_6, \\
&\quad D_{4,n} = (\theta-1)A_{c,4}(k_{c,1}\phi_c - 1) + D_{4,n-1}\phi_c + 0.5X^2, \\
&\quad D_{5,n} = (\theta-1)A_{c,5}(k_{c,1}\phi_\pi - 1) + D_{5,n-1}\phi_\pi + 0.5Y^2, \\
&\quad D_{6,n} = (\theta-1)A_{c,6}(k_{c,1}\phi_d - 1) + D_{6,n-1}\phi_d + 0.5Z^2, \\
&\quad D_{7,n} = (\theta-1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + D_{7,n-1}\phi_{c\pi} + XY, \\
&\quad D_{8,n} = (\theta-1)A_{c,8}(k_{c,1}\phi_{cd} - 1) + D_{8,n-1}\phi_{cd} + XZ, \\
&\quad D_{9,n} = (\theta-1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1) + D_{9,n-1}\phi_{\pi d} + YZ.
\end{aligned}
$$

The coefficients are computed recursively using the fact that $D_{i,0} = 0$ for $i = 0, 1, 2$. Note that coefficients $D_{3,n}, D_{6,n}, D_{8,n}$, and $D_{9,n}$ are zero since $A_{c,3}, A_{c,6}, A_{c,8}$, and $A_{c,9}$ equal zero. The homoscedastic dynamics (Specification I) has a different $D_{0,n}$ term, namely:

$$
\begin{aligned}
&D_{0,n} = \theta\ln(\delta) - \frac{\theta}{\psi}\mu_c + (\theta-1)(k_{c,0} + k_{c,1}A_{c,0} - A_{c,0} + \mu_c) + D_{0,n-1} + \\
&0.5(\sigma_c^2X^2 + \sigma_\pi^2Y^2 + \sigma_d^2Z^2 + 2\sigma_{c\pi}XY + 2\sigma_{cd}XZ + 2\sigma_{\pi d}YZ)
\end{aligned}
$$

where X,Y, and Z are determined as for conditional variance above.

A.4 Nominal bonds

The Euler condition for the real price of a nominal bond is:

$$\begin{aligned} \frac{Q^{\$}_{t,n}}{\Pi_t} &= E_t\left[M_{t+1}\frac{Q^{\$}_{t+1,n-1}}{\Pi_{t+1}}\right], \\ Q^{\$}_{t,n} &= E_t\left[M_{t+1}\frac{Q^{\$}_{t+1,n-1}\Pi_t}{\Pi_{t+1}}\right], \end{aligned}$$

where the following is conjectured: $Q^{\$}_{t,n} = \exp(D^{\$}_{0,n} + D^{\$}_{1,n}x_{c,t} + D^{\$}_{2,n}x_{\pi,t} + D^{\$}_{3,n}x_{d,t} + D^{\$}_{4,n}\sigma^2_{c,t} + D^{\$}_{5,n}\sigma^2_{\pi,t} + D^{\$}_{6,n}\sigma^2_{d,t} + D^{\$}_{7,n}\sigma_{c\pi,t} + D^{\$}_{8,n}\sigma_{cd,t} + D^{\$}_{9,n}\sigma_{\pi d,t})$. Taking logs and again using the conditional lognormality yields:

$$q^{\$}_{t,n} = E_t\left[m_{t+1} - \pi_{t+1} + q^{\$}_{t+1,n-1}\right] + \frac{1}{2}Var_t\left[m_{t+1} - \pi_{t+1} + q^{\$}_{t+1,n-1}\right].$$

The conditional mean is given by:

$$
\begin{aligned}
&E_t\left[m_{t+1}-\pi_{t+1}+q^{\$}_{t+1,n-1}\right]= \\
&\quad \theta\ln(\delta)-\frac{\theta}{\psi}\mu_c+(\theta-1)(k_{c,0}+k_{c,1}(A_{c,0}+A_{c,4}\alpha_c(1-\phi_c)+ \\
&\quad A_{c,5}\alpha_\pi(1-\phi_\pi)+A_{c,6}\alpha_d(1-\phi_d)+A_{c,7}\alpha_{c\pi}(1-\phi_{c\pi})+A_{c,8}\alpha_{cd}(1-\phi_{cd})+ \\
&\quad A_{c,9}\alpha_{\pi d}(1-\phi_{\pi d}))-A_{c,0}+\mu_c)-\mu_\pi+D^{\$}_{0,n-1}+ \\
&\quad D^{\$}_{4,n-1}\alpha_c(1-\phi_c)+D^{\$}_{5,n-1}\alpha_\pi(1-\phi_\pi)+ \\
&\quad D^{\$}_{6,n-1}\alpha_d(1-\phi_d)+D^{\$}_{7,n-1}\alpha_{c\pi}(1-\phi_{c\pi})+ \\
&\quad D^{\$}_{8,n-1}\alpha_{cd}(1-\phi_{cd})+D^{\$}_{9,n-1}\alpha_{\pi d}(1-\phi_{\pi d})+ \\
&\quad x_{c,t}\left[-\frac{\theta}{\psi}+(\theta-1)(k_{c,1}A_{c,1}\beta_1+k_{c,1}A_{c,2}\beta_3+k_{c,1}A_{c,3}\beta_5-A_{c,1}+1)+\right. \\
&\quad \left. D^{\$}_{1,n-1}\beta_1+D^{\$}_{2,n-1}\beta_3\right]+ \\
&\quad x_{\pi,t}\left[(\theta-1)(k_{c,1}A_{c,1}\beta_2+k_{c,1}A_{c,2}\beta_4-A_{c,2})-1+D^{\$}_{1,n-1}\beta_2+D^{\$}_{2,n-1}\beta_4\right], \\
&\quad x_{d,t}\left[(\theta-1)A_{c,3}(k_{c,1}\beta_6-1)+D^{\$}_{3,n-1}\beta_6\right]+ \\
&\quad \sigma^2_{c,t}\left[(\theta-1)A_{c,4}(k_{c,1}\phi_c-1)+D^{\$}_{4,n-1}\phi_c\right]+ \\
&\quad \sigma^2_{\pi,t}\left[(\theta-1)A_{c,5}(k_{c,1}\phi_\pi-1)+D^{\$}_{5,n-1}\phi_\pi\right]+ \\
&\quad \sigma^2_{d,t}\left[(\theta-1)A_{c,6}(k_{c,1}\phi_d-1)+D^{\$}_{6,n-1}\phi_d\right]+ \\
&\quad \sigma_{c\pi,t}\left[(\theta-1)A_{c,7}(k_{c,1}\phi_{c\pi}-1)+D^{\$}_{7,n-1}\phi_{c\pi}\right]+ \\
&\quad \sigma_{cd,t}\left[(\theta-1)A_{c,8}(k_{c,1}\phi_{cd}-1)+D^{\$}_{8,n-1}\phi_{cd}\right]+ \\
&\quad \sigma_{\pi d,t}\left[(\theta-1)A_{c,9}(k_{c,1}\phi_{\pi c}-1)+D^{\$}_{9,n-1}\phi_{\pi c}\right],
\end{aligned}
$$

and the conditional variance by:

$$
\begin{aligned}
&Var_t\left[m_{t+1} - \pi_{t+1} + q^{\$}_{t+1,n-1}\right] = \\
&\quad \sigma^2_{c,t}X^2 + \sigma^2_{\pi,t}Y^2 + \sigma^2_{d,t}Z^2 + 2\sigma_{c\pi,t}XY + 2\sigma_{cd,t}XZ + \\
&\quad 2\sigma_{\pi d,t}YZ + ((\theta-1)k_{c,1}A_{c,4}\tau_c + D^{\$}_{4,n-1}\tau_c)^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,5}\tau_\pi + D^{\$}_{5,n-1}\tau_\pi)^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,6}\tau_d + D^{\$}_{6,n-1}\tau_d)^2 + ((\theta-1)k_{c,1}A_{c,7}\tau_{c\pi} + D^{\$}_{7,n-1}\tau_{c\pi})^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,8}\tau_{cd} + D^{\$}_{8,n-1}\tau_{cd})^2 + ((\theta-1)k_{c,1}A_{c,9}\tau_{\pi d} + D^{\$}_{9,n-1}\tau_{\pi d})^2, \\
&\quad X = \left[-\frac{\theta}{\psi} + (\theta-1)(k_{c,1}A_{c,1}\delta_1 + k_{c,1}A_{c,2}\delta_3 + k_{c,1}A_{c,3}\delta_5 + 1) + \right. \\
&\quad \left. D^{\$}_{1,n-1}\delta_1 + D^{\$}_{2,n-1}\delta_3 + D^{\$}_{3,n-1}\delta_5\right], \\
&\quad Y = \left[(\theta-1)(k_{c,1}A_{c,1}\delta_2 + k_{c,1}A_{c,2}\delta_4) - 1 + D^{\$}_{1,n-1}\delta_2 + D^{\$}_{2,n-1}\delta_4\right], \\
&\quad Z = \left[(\theta-1)(k_{c,1}A_{c,3}\delta_6) + D^{\$}_{3,n-1}\delta_6\right].
\end{aligned}
$$

Matching the coefficients gives:

$$
\begin{aligned}
&D^{\$}_{0,n} = \\
&\quad \theta\ln(\delta) - \frac{\theta}{\psi}\mu_c + (\theta-1)(k_{c,0} + k_{c,1}(A_{c,0} + A_{c,4}\alpha_c(1-\phi_c) + \\
&\quad A_{c,5}\alpha_\pi(1-\phi_\pi) + A_{c,6}\alpha_d(1-\phi_d) + A_{c,7}\alpha_{c\pi}(1-\phi_{c\pi}) + A_{c,8}\alpha_{cd}(1-\phi_{cd}) + \\
&\quad A_{c,9}\alpha_{\pi d}(1-\phi_{\pi d})) - A_{c,0} + \mu_c) - \mu_\pi + D^{\$}_{0,n-1} + \\
&\quad D^{\$}_{4,n-1}\alpha_c(1-\phi_c) + D^{\$}_{5,n-1}\alpha_\pi(1-\phi_\pi) + \\
&\quad D^{\$}_{6,n-1}\alpha_d(1-\phi_d) + D^{\$}_{7,n-1}\alpha_{c\pi}(1-\phi_{c\pi}) + \\
&\quad D^{\$}_{8,n-1}\alpha_{cd}(1-\phi_{cd}) + D^{\$}_{9,n-1}\alpha_{\pi d}(1-\phi_{\pi d}) + \\
&\quad 0.5((((\theta-1)k_{c,1}A_{c,4}\tau_c + D^{\$}_{4,n-1}\tau_c)^2 + ((\theta-1)k_{c,1}A_{c,5}\tau_\pi + D^{\$}_{5,n-1}\tau_\pi)^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,6}\tau_d + D^{\$}_{6,n-1}\tau_d)^2 + ((\theta-1)k_{c,1}A_{c,7}\tau_{c\pi} + D^{\$}_{7,n-1}\tau_{c\pi})^2 + \\
&\quad ((\theta-1)k_{c,1}A_{c,8}\tau_{cd} + D^{\$}_{8,n-1}\tau_{cd})^2 + ((\theta-1)k_{c,1}A_{c,9}\tau_{\pi d} + D^{\$}_{9,n-1}\tau_{\pi d})^2), \\
&\quad D^{\$}_{1,n} = -\frac{\theta}{\psi} + (\theta-1)(k_{c,1}A_{c,1}\beta_1 + k_{c,1}A_{c,2}\beta_3 + k_{c,1}A_{c,3}\beta_5 - A_{c,1} + 1) + \\
&\quad D^{\$}_{1,n-1}\beta_1 + D^{\$}_{2,n-1}\beta_3, \\
&\quad D^{\$}_{2,n} = (\theta-1)(k_{c,1}A_{c,1}\beta_2 + k_{c,1}A_{c,2}\beta_4 - A_{c,2}) - 1 + D^{\$}_{1,n-1}\beta_2 + D^{\$}_{2,n-1}\beta_4, \\
&\quad D^{\$}_{3,n} = (\theta-1)A_{c,3}(k_{c,1}\beta_6 - 1) + D^{\$}_{3,n-1}\beta_6, \\
&\quad D^{\$}_{4,n} = (\theta-1)A_{c,4}(k_{c,1}\phi_c - 1) + D^{\$}_{4,n-1}\phi_c + 0.5X^2, \\
&\quad D^{\$}_{5,n} = (\theta-1)A_{c,5}(k_{c,1}\phi_\pi - 1) + D^{\$}_{5,n-1}\phi_\pi + 0.5Y^2, \\
&\quad D^{\$}_{6,n} = (\theta-1)A_{c,6}(k_{c,1}\phi_d - 1) + D^{\$}_{6,n-1}\phi_d + 0.5Z^2, \\
&\quad D^{\$}_{7,n} = (\theta-1)A_{c,7}(k_{c,1}\phi_{c\pi} - 1) + D^{\$}_{7,n-1}\phi_{c\pi} + XY, \\
&\quad D^{\$}_{8,n} = (\theta-1)A_{c,8}(k_{c,1}\phi_{cd} - 1) + D^{\$}_{8,n-1}\phi_{cd} + XZ, \\
&\quad D^{\$}_{9,n} = (\theta-1)A_{c,9}(k_{c,1}\phi_{\pi d} - 1) + D^{\$}_{9,n-1}\phi_{\pi d} + YZ.
\end{aligned}
$$

Appendix

The coefficients are computed recursively using the fact that $D^{\$}_{i,0} = 0$ for $i = 0, 1, 2$. Note that coefficients $D^{\$}_{3,n}, D^{\$}_{6,n}, D^{\$}_{8,n}$, and $D^{\$}_{9,n}$ are zero since $A_{c,3}, A_{c,6}, A_{c,8}$, and $A_{c,9}$ equal zero. The homoscedastic dynamics (Specification I) has a different $D^{\$}_{0,n}$ term, namely:

$$D^{\$}_{0,n} = \theta \ln(\delta) - \frac{\theta}{\psi}\mu_c + (\theta - 1)(k_{c,0} + k_{c,1}A_{c,0} - A_{c,0} + \mu_c) + D^{\$}_{0,n-1} +$$
$$0.5(\sigma_c^2 X^2 + \sigma_\pi^2 Y^2 + \sigma_d^2 Z^2 + 2\sigma_{c\pi} XY + 2\sigma_{cd} XZ + 2\sigma_{\pi d} YZ)$$

where X,Y, and Z are determined as for conditional variance above.

A.5 Conditional covariance of stock and bond returns

The conditional covariance between nominal stock and bond returns can be written as:

$$
\begin{aligned}
&Cov_t[r_{m,t+1} + \pi_{t+1}, h^{\$}_{t+1,n-1}] = Cov_t[k_{d,1}pd_{t+1} + \Delta d_{t+1} + \pi_{t+1}, q^{\$}_{t+1,n-1}] = \\
&Cov_t[k_{d,1}(A_{d,1}x_{c,t+1} + A_{d,2}x_{\pi,t+1} + A_{d,3}x_{d,t+1} + A_{d,4}\sigma^2_{c,t} + \\
&A_{d,5}\sigma^2_{\pi,t} + A_{d,6}\sigma^2_{d,t} + A_{d,7}\sigma_{c\pi,t} + A_{d,8}\sigma_{cd,t} + A_{d,9}\sigma_{\pi d,t}) + \\
&\eta_{d,t+1} + \eta_{\pi,t+1}, D^{\$}_{1,n-1}x_{c,t} + D^{\$}_{2,n-1}x_{\pi,t} + D^{\$}_{3,n-1}x_{d,t} + D^{\$}_{4,n-1}\sigma^2_{c,t} + \\
&D^{\$}_{5,n-1}\sigma^2_{\pi,t} + D^{\$}_{6,n-1}\sigma^2_{d,t} + D^{\$}_{7,n-1}\sigma_{c\pi,t} + D^{\$}_{8,n-1}\sigma_{cd,t} + D^{\$}_{9,n-1}\sigma_{\pi d,t}] = \\
&Cov_t[\eta_{c,t+1}(k_{d,1}\delta_1 A_{d,1} + k_{d,1}\delta_3 A_{d,2} + k_{d,1}A_{d,3}\delta_5) + \\
&\eta_{\pi,t+1}(k_{d,1}\delta_2 A_{d,1} + k_{d,1}\delta_4 A_{d,2} + 1) + \\
&\eta_{d,t+1}(k_{d,1}\delta_6 A_{d,3} + 1) + k_{d,1}(A_{d,4}\tau_c\epsilon_{c,t+1} + A_{d,5}\tau_\pi\epsilon_{\pi,t+1} + A_{d,6}\tau_d\epsilon_{d,t+1} + \\
&A_{d,7}\tau_{c\pi}\epsilon_{c\pi,t+1} + A_{d,8}\tau_{cd}\epsilon_{cd,t+1} + A_{d,9}\tau_{\pi d}\epsilon_{\pi d,t+1}), \\
&\eta_{c,t+1}(D^{\$}_{1,n-1}\delta_1 + D^{\$}_{2,n-1}\delta_3 + D^{\$}_{3,n-1}\delta_5) + \eta_{\pi,t+1}(D^{\$}_{1,n-1}\delta_2 + D^{\$}_{2,n-1}\delta_4) + \\
&\eta_{d,t+1}(D^{\$}_{3,n-1}\delta_6) + D^{\$}_{4,n-1}\tau_c\epsilon_{c,t+1} + D^{\$}_{5,n-1}\tau_\pi\epsilon_{\pi,t+1} + D^{\$}_{6,n-1}\tau_d\epsilon_{d,t+1} + \\
&D^{\$}_{7,n-1}\tau_{c\pi}\epsilon_{c\pi,t+1} + D^{\$}_{8,n-1}\tau_{cd}\epsilon_{cd,t+1} + D^{\$}_{9,n-1}\tau_{\pi d}\epsilon_{\pi d,t+1}] = \\
&A + BVar_t[\eta_{c,t+1}] + CVar_t[\eta_{\pi,t+1}] + DCov_t[\eta_{c,t+1}, \eta_{\pi,t+1}] + \\
&ECov_t[\eta_{c,t+1}, \eta_{d,t+1}] + FCov_t[\eta_{\pi,t+1}, \eta_{d,t+1}],
\end{aligned}
$$

where

$$
\begin{aligned}
A &= k_{d,1}(A_{d,4}D^{\$}_{4,n-1}\tau_c^2 + A_{d,5}D^{\$}_{5,n-1}\tau_\pi^2 + A_{d,6}D^{\$}_{6,n-1}\tau_d^2 + \\
&\quad A_{d,7}D^{\$}_{7,n-1}\tau_{c\pi}^2 + A_{d,8}D^{\$}_{8,n-1}\tau_{cd}^2 + A_{d,9}D^{\$}_{9,n-1}\tau_{\pi d}^2), \\
B &= (k_{d,1}\delta_1 A_{d,1} + k_{d,1}\delta_3 A_{d,2} + k_{d,1}A_{d,3}\delta_5)(D^{\$}_{1,n-1}\delta_1 + \\
&\quad D^{\$}_{2,n-1}\delta_3 + D^{\$}_{3,n-1}\delta_5), \\
C &= (k_{d,1}\delta_2 A_{d,1} + k_{d,1}\delta_4 A_{d,2} + 1)(D^{\$}_{1,n-1}\delta_2 + D^{\$}_{2,n-1}\delta_4), \\
D &= (k_{d,1}\delta_1 A_{d,1} + k_{d,1}\delta_3 A_{d,2} + k_{d,1}A_{d,3}\delta_5)(D^{\$}_{1,n-1}\delta_2 + D^{\$}_{2,n-1}\delta_4) + \\
&\quad (k_{d,1}\delta_2 A_{d,1} + k_{d,1}\delta_4 A_{d,2} + 1)(D^{\$}_{1,n-1}\delta_1 + D^{\$}_{2,n-1}\delta_3 + D^{\$}_{3,n-1}\delta_5), \\
E &= (k_{d,1}\delta_1 A_{d,1} + k_{d,1}\delta_3 A_{d,2} + k_{d,1}A_{d,3}\delta_5)(D^{\$}_{3,n-1}\delta_6) + \\
&\quad (k_{d,1}\delta_6 A_{d,3} + 1)(D^{\$}_{1,n-1}\delta_1 + D^{\$}_{2,n-1}\delta_3 + D^{\$}_{3,n-1}\delta_5), \\
F &= (k_{d,1}\delta_2 A_{d,1} + k_{d,1}\delta_4 A_{d,2} + 1)(D^{\$}_{3,n-1}\delta_6) + \\
&\quad (k_{d,1}\delta_6 A_{d,3} + 1)(D^{\$}_{1,n-1}\delta_2 + D^{\$}_{2,n-1}\delta_4).
\end{aligned}
$$

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Table 1: Macro Moments

	Sample		Model		
	Moment	SE	Mean	5%	95%
Consumption growth, Δc					
Mean	0.81	(0.05)	0.81	0.72	0.90
Std.Dev.	0.47	(0.04)	0.46	0.42	0.50
AC(4)	0.04	(0.07)	0.07	–0.06	0.20
AC(8)	–0.13	(0.07)	0.00	–0.12	0.13
Dividend growth, Δd					
Mean	0.45	(0.19)	0.45	–0.01	0.91
Std.Dev.	1.91	(0.28)	1.92	1.72	2.13
AC(4)	0.02	(0.17)	0.17	0.04	0.32
AC(8)	0.08	(0.10)	0.07	–0.07	0.23
Inflation, π					
Mean	0.92	(0.09)	0.92	0.54	1.30
Std.Dev.	0.62	(0.08)	0.56	0.43	0.73
AC(4)	0.70	(0.10)	0.60	0.40	0.77
AC(8)	0.50	(0.11)	0.43	0.17	0.67
Correlations					
Δc and Δd	0.15	(0.14)	0.08	–0.05	0.21
Δc and π	–0.35	(0.19)	–0.36	–0.48	–0.23
Δd and π	–0.16	(0.14)	–0.01	–0.21	0.19

This table presents unconditional moments of observed and model-implied data. Means and percentiles for the model are computed over 2,000 simulations each containing 223 quarters. AC(k) denotes the autocorrelation for k lags. Standard errors, denoted SE, are computed as in Newey West (1987), using four lags. The sample period is 1952:2 to 2007:4.

Table 2: Estimated Parameters: Specification I

	Estimate	SE
β_1	0.533	(0.157)
β_2	–0.104	(0.052)
β_3	0.281	(0.122)
β_4	1.019	(0.038)
β_5	0.564	(0.413)
β_6	0.799	(0.071)
δ_1	0.245	(0.068)
δ_2	–0.107	(0.092)
δ_3	0.076	(0.050)
δ_4	0.495	(0.064)
δ_5	–0.203	(0.235)
δ_6	0.295	(0.055)
σ_c^2	0.183	(0.017)
σ_π^2	0.100	(0.009)
σ_d^2	2.873	(0.271)
$\sigma_{c,\pi}$	–0.039	(0.009)
$\sigma_{c,d}$	0.089	(0.048)
$\sigma_{\pi,d}$	–0.062	(0.036)

This table presents results from estimating the parameters in Section 3.2.1 using maximum likelihood. The sample period is 1952:2 to 2007:4.

Table 3: Estimated Parameters: Specification II

	Estimate	SE
c_c	0.003	(0.004)
$c_{c\pi}$	–0.003	(0.003)
c_{cd}	0.013	(0.162)
c_π	0.003	(0.002)
$c_{\pi d}$	–0.004	(0.009)
c_d	0.086	(0.052)
b_c	0.916	(0.052)
$b_{c\pi}$	0.874	(0.115)
b_{cd}	0.816	(2.286)
b_π	0.881	(0.039)
$b_{\pi d}$	0.841	(0.297)
b_d	0.810	(0.052)
a_c	0.072	(0.041)
$a_{c\pi}$	0.051	(0.042)
a_{cd}	0.012	(0.113)
a_π	0.090	(0.034)
$a_{\pi d}$	0.054	(0.078)
a_d	0.170	(0.042)

This table presents results from estimating the parameters of the diagonal VEC-model in Section 3.3.2. using maximum likelihood. The sample period is 1952:2 to 2007:4.

Table 4: Parameters governing the model's second moments

	Value
α_c	0.0000260
$\alpha_{c\pi}$	–0.0000034
α_{cd}	0.0000073
α_π	0.0000114
$\alpha_{\pi d}$	–0.0000037
α_d	0.0004284
ϕ_c	0.988
$\phi_{c\pi}$	0.924
ϕ_{cd}	0.828
ϕ_π	0.970
$\phi_{\pi d}$	0.896
ϕ_d	0.980
τ_c	$1.16*10^{-6}$
$\tau_{c\pi}$	$8.35*10^{-7}$
τ_{cd}	$4.79*10^{-7}$
τ_π	$1.08*10^{-6}$
$\tau_{\pi d}$	$1.59*10^{-6}$
τ_d	$9.17*10^{-5}$

This table presents the parameter values used for the second moments within the model. They are set as to match the first two moments of the model's second moments with the ones estimated from data. Section 3.3.2. describes in detail how the parameters are set.

Table 5: Regressing observed valuation ratios on estimated state variables

	pc_t		pc_t		pd_t		pd_t	
	β	t-stat	β	t-stat	β	t-stat	β	t-stat
Explanatory variables								
$x_{c,t}$	0.35	3.73	–0.00	–0.03	0.41	2.17	–0.20	–0.71
$x_{\pi,t}$			–0.18	–5.76			–0.31	–3.26
$x_{d,t}$							0.01	0.15
R^2_{adj}	0.13		0.32		0.03		0.13	

This table presents regression results from regressing the observed log price-consumption ratio (pc) and the log price-dividend ratio (pd) onto the extracted state variables, x_c, x_π, and x_d. The regressions are run contemporaneously. As a proxy for the price-consumption ratio, the wealth-consumption ratio in Lustig et al. (2008) is used. All variables are measured on a quarterly basis. The sample period is 1952:2 to 2006:4. T-stats are based on heteroscedasticity robust standard errors.

Table 6: Unconditional Asset Price Moments

	Sample	Estimated Model Specification I	Calibrated Model Specification I	Estimated Model Specification II	Calibrated Model Specification II
Stock market excess returns					
$E(r_m - r_f)$	5.52	0.24	4.16	0.14	5.03
$\sigma(r_m - r_f)$	16.30	8.96	10.89	11.54	12.91
Price-dividend ratios					
$E(\exp(pd))$	36.38	63.34	20.45	68.16	17.94
$\sigma(pd)$	0.39	0.08	0.14	0.11	0.18
Real and nominal interest rates					
$\mathrm{E}(\mathrm{y}_{3m})$		3.28	2.74	3.28	2.62
$\sigma(y_{3m})$		0.49	0.49	0.50	0.65
$\mathrm{E}(\mathrm{y}_{60m} - y_{3m})$		–0.07	–0.24	–0.05	–0.23
$\mathrm{E}(\mathrm{y}^{\$}_{3m})$	5.13	7.07	6.79	7.07	6.66
$\sigma(y^{\$}_{3m})$	2.87	1.87	2.02	1.88	1.96
$\mathrm{E}(\mathrm{y}^{\$}_{60m} - y^{\$}_{3m})$	0.96	0.15	1.50	0.23	1.66

This table presents unconditional moments of observed and model-implied asset prices. Specification I and Specification II refer to the homoscedastic and heteroscedastic model, respectively. Moments for the estimated model stem from using the parameter values estimated from data. The calibrated model increases the persistence parameter of inflation, β_4, with less than one standard error from the point estimate. The log price-dividend ratio is denoted pd. A risk aversion of 10, an elasticity of intertemporal substitution of 1.5, and a discount factor of 0.997 are used. All moments are annualized. The sample period is 1952:2 to 2007:4.

Table 7: Explaining the Fed Model

	Sample	Specification I	Specification II
$Corr(DP, y_{60m})$ 1952-2007	0.30	0.82	0.73
$Corr(DP, y_{60m})$ 1965-2007	0.74	0.81	0.72

This table presents correlation coefficients between dividend-price ratios and 60-month nominal yields in data and from the model. Specification I and Specification II refer to the estimated homoscedastic and heteroscedastic model, respectively. The sample periods are 1952:2 to 2007:4 and 1965:1 to 2007:4.

Table 8: Predicting quarterly correlations

	$\text{Corr}_{Data,t:t+1}$		$\text{Corr}_{Data,t:t+1}$		$\text{Corr}_{Data,t:t+1}$	
	β	t-stat	β	t-stat	β	t-stat
Explanatory variables						
$\text{Corr}_{Model,t}$	0.78	4.84			0.40	3.93
$\text{Corr}_{Data,t-1:t}$			0.63	9.00	0.57	7.21
R^2_{adj}	0.13		0.38		0.41	

This table presents regression results from regressing observed quarterly correlations between US stock and bond returns for time t to $t+1$ ($CORR_{Data,t:t+1}$) onto its own lagged value ($CORR_{Data,t-1:t}$) and the model-implied conditional correlation at time t ($CORR_{Model,t}$). Observed quarterly correlations are computed using daily stock and bond returns within that particular quarter. The sample period is 1962:1 to 2007:4. T-stats are based on heteroscedasticity robust standard errors.

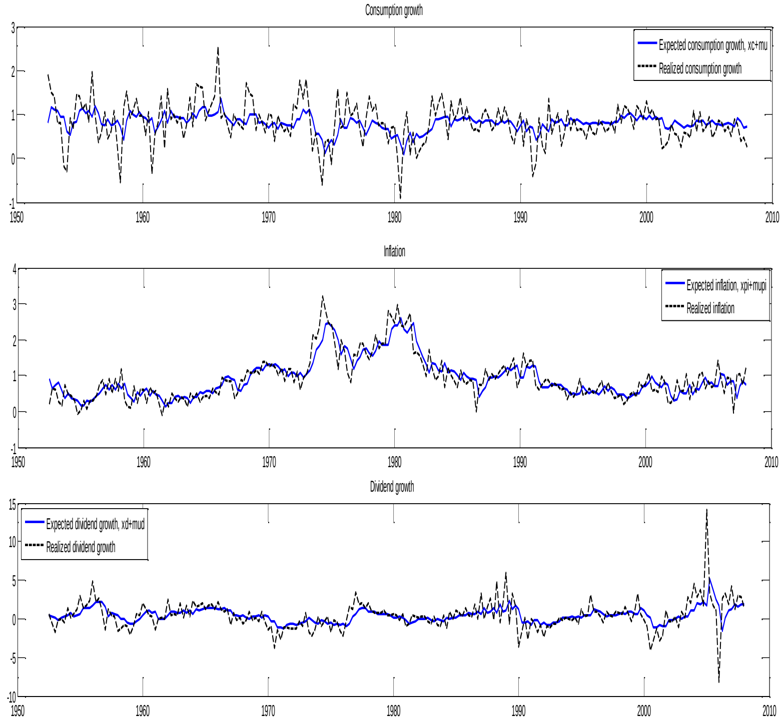

Figure 1: Consumption growth, inflation, and dividend growth. The figure displays realized quarterly consumption growth, inflation, and dividend growth and the extracted conditional means using the estimated parameters from the maximum likelihood estimation. Growth rates are expressed in quarterly units.

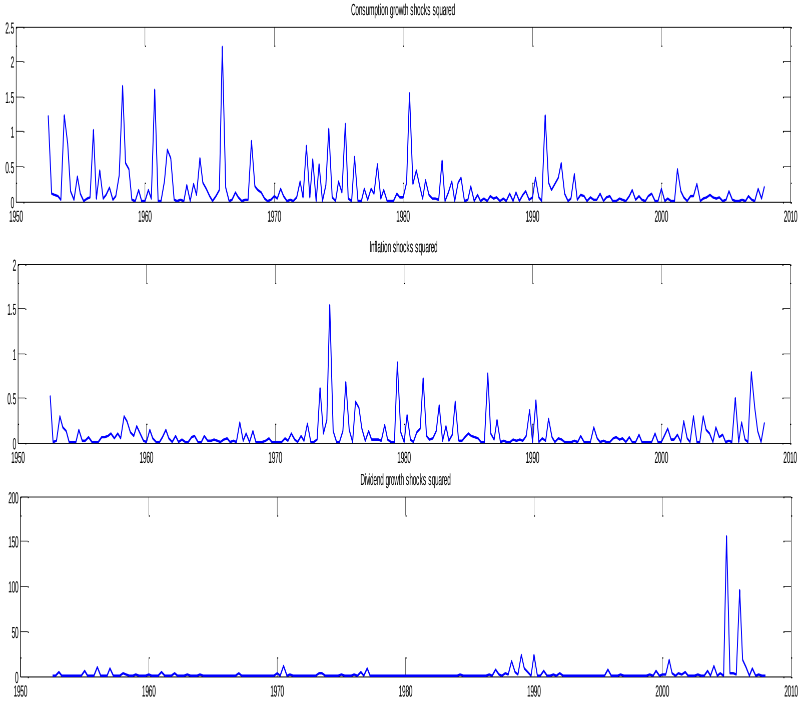

Figure 2: Squared shocks to consumption growth, inflation, and dividend growth. The figure displays squared shocks to quarterly consumption growth, inflation, and dividend growth. All shocks are extracted from the estimated dynamics in Section 3.3.2.

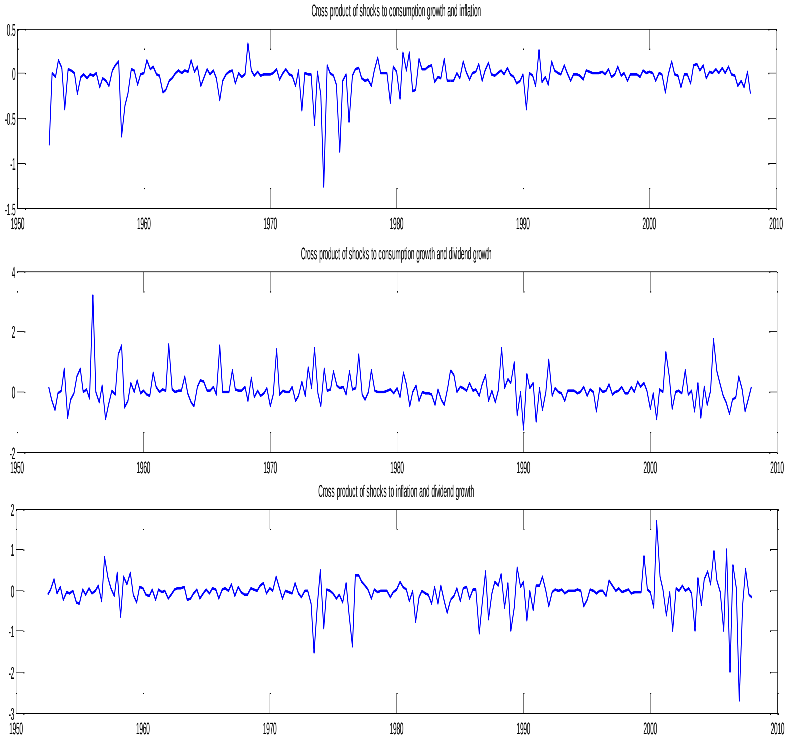

Figure 3: Cross product of shocks to consumption growth, inflation, and dividend growth. The figure displays cross products of shocks to consumption growth and inflation, to consumption growth and dividend growth, and to inflation and dividend growth. All shocks are extracted from the estimated dynamics in Section 3.3.2.

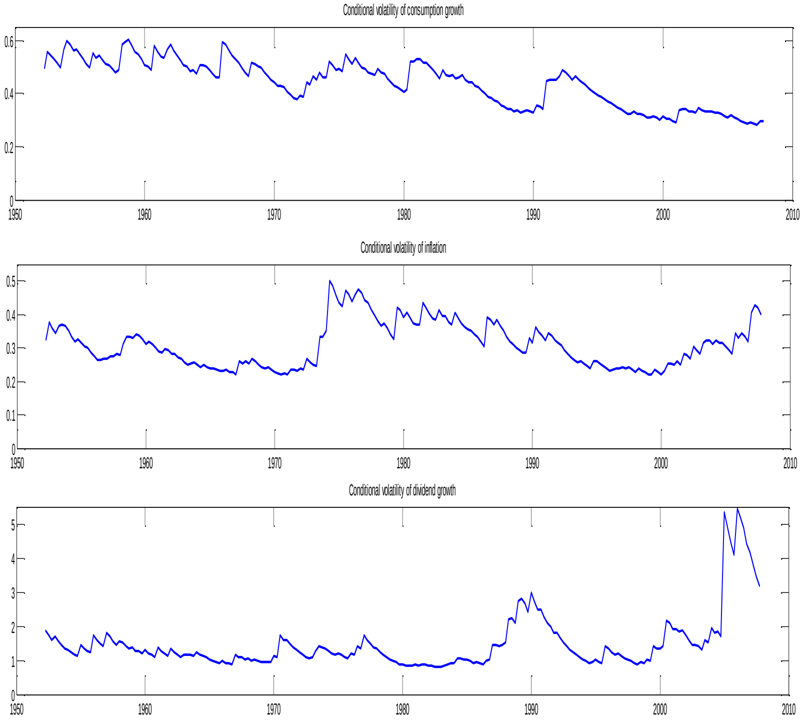

Figure 4: Conditional volatilities of macro variables. The figure displays the conditional standard deviation of consumption growth, inflation, and dividend growth. The volatilities are expressed in quarterly units and stem from the estimated diagonal VEC-model in Section 3.3.2.

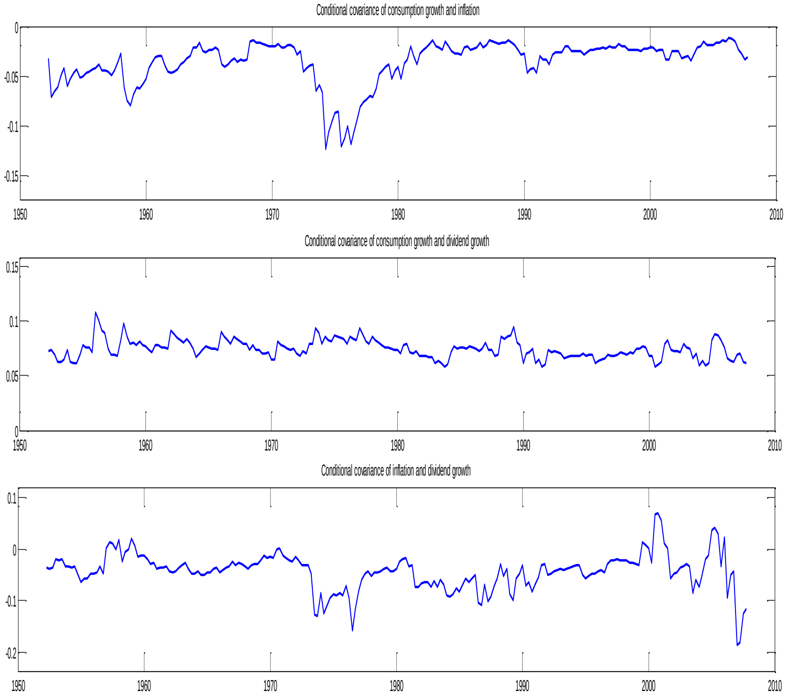

Figure 5: Conditional covariances of macro variables. The figure displays the conditional covariance of consumption growth and inflation, of consumption growth and dividend growth, and of inflation and dividend growth. The covariances are expressed in quarterly units and stem from the estimated diagonal VEC-model in Section 3.3.2.

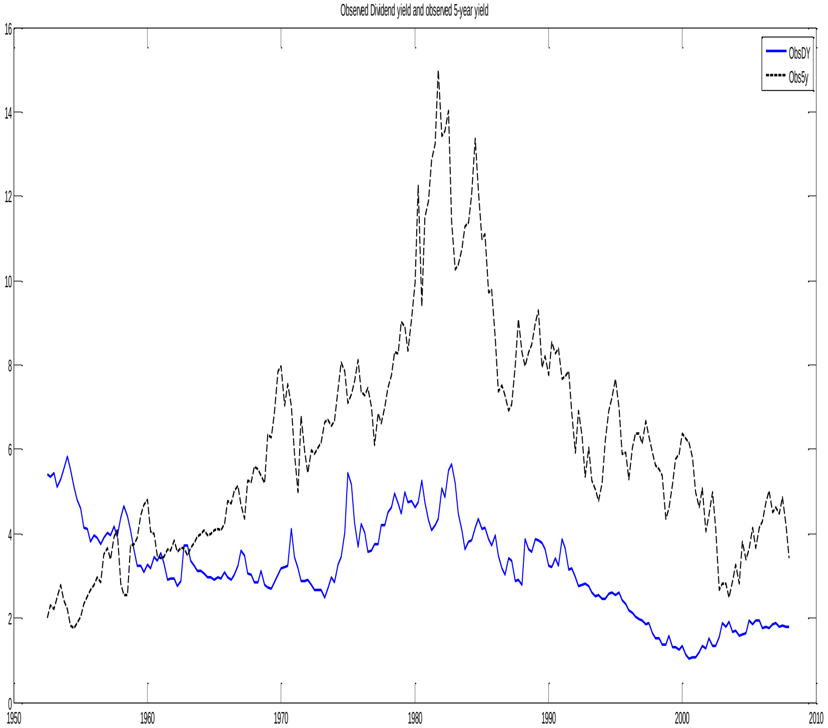

Figure 6: Dividend yields and the 5-year Treasury rate. The figure displays the nominal US 5-year Treasury rate and the US aggregate dividend yield.

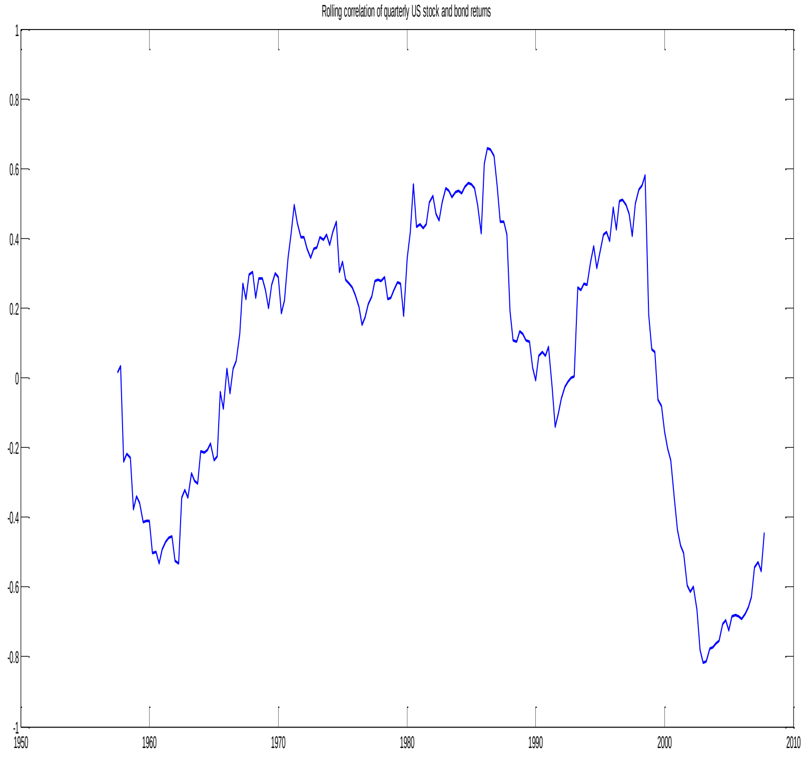

Figure 7: Observed correlation of stock and bond returns. The figure displays a 20-quarter rolling correlation of returns on US stocks and 5-year US Treasury bonds.

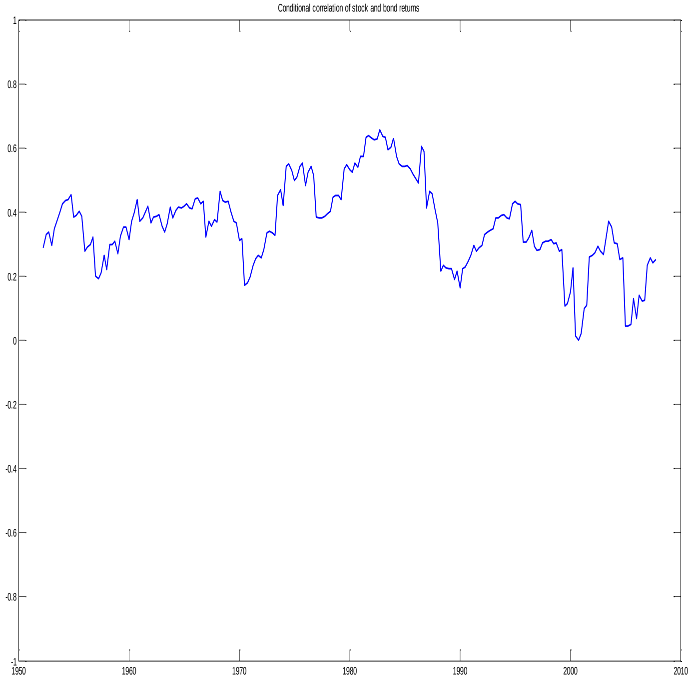

Figure 8: Model-implied conditional correlation of stock and bond returns. The figure displays quarterly model-implied conditional correlations of stock and bond returns.

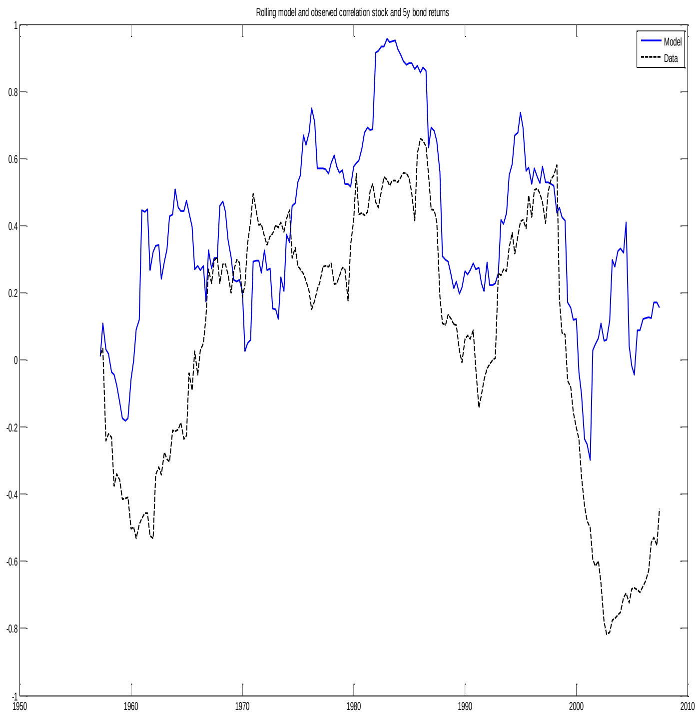

Figure 9: Observed and model-implied correlations of stock and bond returns. The figure displays 20-quarter rolling correlations of stock and bond returns implied from the model and observed in data. The solid line represents model correlations and the dashed line sample correlations.

Chapter 4

International Bond Risk Premia

Joint work with Magnus Dahlquist

Abstract

We extend Cochrane and Piazzesi (2005, CP) to international bond markets by constructing forecasting factors for bond excess returns across different countries. While the international evidence for predictability is weak using Fama and Bliss (1987) regressions, we document that local CP factors have significant predictive power. We also construct a global CP factor and provide evidence that it predicts bond returns with high R^2s across countries. The local and global factors are jointly significant when included as regressors, which suggests that variation in bond excess returns are driven by country-specific factors and a common global factor. Shocks to US bond risk premia seem to be particularly important determinants for international bond premia. Motivated by these results, we estimate a parsimonious no-arbitrage affine term structure model in which risk premia are driven by one local and one global CP factor. We find that international bond risk premia are driven by a local slope factor and a world interest rate level factor.

4.1 Introduction

The expectation hypothesis of interest rates states that bond risk premia are constant over time. However, ample evidence suggests that risk premia in bond markets do vary over time. For example, Fama and Bliss (1987, FB) and Campbell and Shiller (1991, CS) show that US bond excess returns are predictable using the forward-spot rate differential and the slope of the yield curve. A steep yield curve has historically predicted lower future long yields and positive excess returns on long bonds over short bonds. Cochrane and Piazzesi (2005, CP) establish even stronger evidence for predictability when more information from the yield curve is incorporated. Using five contemporaneous forward rates as predictors, they document significantly higher R^2 compared to the commonly used FB or CS regressions.

We extend the setup of CP to an international setting and construct local CP factors for Germany, Switzerland, the UK, and the US for the period January 1976 to December 2007. The local factors are shown to have significant forecasting power for bond excess returns while FB regressions show weak or no evidence of predictability for countries outside the US. Next, we construct a global CP factor and show that it predicts bond returns with similar or higher explanatory power compared to local CP factors. The local and global CP factors are jointly significant when included as regressors and increase the explanatory power even further. Our results suggest that there exists a common global return-forecasting factor that predicts bond returns across countries and that bond risk premia are driven by both a country-specific factor and a common global factor. Motivated by this finding, we propose and estimate a parsimonious no-arbitrage affine term structure model in which risk premia for each country vary with the local and global CP factor. Shocks to the CP factors and to the level of interest rates are found to be significantly priced across all four countries. Our estimation results suggest that international bond risk premia are driven by a local slope factor and a global interest rate level factor.

The local CP factors are positively associated with the slope of local yield curves and with the fourth principal component of local yields and are shown to be positively correlated over the sample period. Correlations are higher during the second half of the sample period compared to the first half which indicates an increasing comovement of international bond risk premia over time. The global CP factor is computed as a GDP-weighted average of the local CP factors and is positively associated with the level and slope of local term structures. The global factor is close to perfectly correlated with the US CP factor. The fact that the global factor predicts bond returns with high R^2 for countries outside the US indicates that shocks to US bond risk premia are important determinants for international bond premia.

Our evidence of predictable bond returns across countries stands in contrast to the existing literature which finds weak or no evidence of predictability internationally. For example, Hardouvelis (1994) and Bekaert and Hodrick (2001) find it hard to reject the expectation hypothesis for countries outside the US. In contrast, we show that both a local and global CP factor predict returns significantly in countries for which FB regressions finds no or weak evidence of predictability. Flamini and Veronesi (2008) document similar results using a common return forecasting factor. In a recent paper, Kessler and Scherer (2009) also construct CP factors across countries and find significant forecasting power. However, the focus of their paper is different from ours as they are mainly interested in evaluating different trading strategies. Our finding that bond returns are governed by a country-specific and a global factor is related to Dahlquist (1995), who find that variations in forward term premia are to a great extent captured by the shape of domestic and world term structures, and Driessen et al. (2003), who find that a world interest rate level factor accounts for nearly half of the variation in bond returns. Furthermore, Perignon et al. (2007) find that US bond returns share only one common factor with German and Japanese bond returns which they link to changes in the level of interest rates. Ilmanen (1995) also examines the predictability of international bond returns and find that global factors predict bond returns across countries. Our work is also related to Cochrane and Piazzesi (2008) who estimate an affine model on US data using the local CP factor plus three latent variables. Only level shocks are assumed to be priced in their model where risk premia vary with the

CP factor. Our estimations show that not only level shocks are priced but also shocks to the CP factor itself. Koijen et al. (2009) find that the CP factor is able to price the cross-section of US stock returns.

Several equilibrium models have been put forward to explain the mechanics of time-varying bond risk premia, linking macroeconomic variables to changing expected excess returns. For example, Brandt and Wang (2003), Wachter (2006), and Buraschi and Jiltsov (2007) all build on the habit-formation model of Campbell and Cochrane (1999) and show that it can generate rejections of the expectation hypothesis. Bansal and Shaliastovich (2008) and Hasseltoft (2008) build on the long-run risk model of Bansal and Yaron (2004) and argue that changing bond risk premia are driven by time-varying volatility of consumption growth. Ludvigson and Ng (2008) provide empirical evidence that macro factors do predict bond returns. By using common factors from a large set of macro variables, they document R^2s up to 26% when predicting US bond excess returns. They find that including the CP factor increases the R^2s up to over 40% with all coefficients being statistically significant. Our paper is also related to work on term structure models such as Dai and Singleton (2000), Duffee (2002), and Dai and Singleton (2002). Diebold et al. (2008) builds on Nelson and Siegel (1987) and document the existence of global yield curve factors which appear to be linked to global macroeconomic factors such as inflation and real activity. Related is also the literature on global factors in other asset markets. For example, Harvey (1991), Campbell and Hamao (1992), and Ferson and Harvey (1993) use global risk factors to predict international stock returns while Backus et al. (2001) and Lustig et al. (2009) address the forward premium puzzle using affine models including country-specific and common factors.

Our paper proceeds as follows. In Section 4.2 we describe the data, present summary statistics, and provide the key results related to predictability regressions of bond returns. In Section 4.3 we propose an affine term structure model with local and global factors. We present the results of estimating these models in Section 4.4, and discuss implications for yields in terms of yield loadings, impulse response functions, and variance decompositions. In Section 4.5 we discuss how the affine model can be linked to structural models and outline future research in light of our results. We conclude in Section 4.6.

4.2 Predictability of bond returns

4.2.1 Data

Our data set covers monthly zero-coupon interest rates for Germany, Switzerland, United Kingdom, and United States and spans the time period January 1976 to December 2007. One-to-five year zero-coupon yields for Germany are collected from Bundesbank, Swiss yields are derived from forward rates up to December 2003 after which yields from the Swiss National Bank are used, yields for the UK are retrieved from Bank of England, while yields for the US are collected from the Fama-Bliss discount bond file in CRSP. The one-month interbank rate, collected from Datastream, is used as short rate for Germany and Switzerland. For UK, the one-month interbank rate is used until February 1997 and then one-month yields from Bank of England. The Fama one-month yield from CRSP is used for the US. Quarterly data on GDP, computed using purchasing power parity, is retrieved for each country from Datastream. As the GDP data are quarterly, the weights applied to the monthly CP factors are constant within each quarter. Table 1 presents summary statistics for yields across countries. Yield curves tend to be upward sloping on average while yields on short-maturity bonds tend to be more volatile than yields on long-maturity bonds. Yield levels are positively correlated across countries with correlations being higher among yields on longer-term bonds. Annual bond excess returns on 2-5 year bonds are also positively correlated across countries as indicated in Table 2.

4.2.2 Constructing local and global Cochrane-Piazzesi factors

We construct local CP factors as in Cochrane and Piazzesi (2005) for each country c in our sample. Define the annual return on a n-period bond in excess of the one-year yield as $rx^n_{c,t+1} = p^{n-1}_{c,t+1} - p^n_{c,t} - y^1_{c,t}$, where

p denotes the log bond price and y denotes the log yield, computed as $y^n_{c,t} = -p^n_{c,t}/n$. Define the one-year forward rate between periods $n-1$ and n as the differential in log bond prices, $f^n_{c,t} = p^{n-1}_{c,t} - p^n_{c,t}$. A CP factor is constructed by regressing average excess returns across maturity at each time t on the one-year yield and four forward rates:

$$\overline{rx}_{c,t+1} = \gamma_{c,0}+\gamma_{c,1}y^1_{c,t}+\gamma_{c,2}f^2_{c,t}+\gamma_{c,3}f^3_{c,t}+\gamma_{c,4}f^4_{c,t}+\gamma_{c,5}f^5_{c,t}+\bar{\epsilon}_{c,t+1}, \quad (4.1)$$

where $\overline{rx}_{c,t+1} = \sum_{n=2}^{5} rx^n_{c,t+1}/4$. Let the right hand side variables, including the constant term, for each country be collected in the vector $f_{c,t}$ and let the corresponding estimated coefficients be collected in the vector $\hat{\gamma}_c$. A local CP factor $CP_{c,t}$ is then given by $\hat{\gamma}'_c f_{c,t}$.[1]

We construct a global CP factor defined as the GDP-weighted average of each local CP factor at time t. That is:

$$GCP_t = \sum_{c=1}^{C} w_{c,t} CP_{c,t}, \quad (4.2)$$

where $w_{c,t} = GDP_{c,t}/\sum_{c=1}^{C} GDP_{c,t}$, and where $C = 4$. The average weights over the sample period is 0.70 for US, 0.12 for UK, 0.16 for Germany, and 0.02 for Switzerland. Our size-weighted global risk factor is hence dominated by the US.[2]

[1]Cochrane and Piazzesi (2005) find the γs to form a tent-shaped pattern. We find a similar shape for the US, using the same data source as CP but for a different sample period. The shapes are different for the remaining countries. Dai et al. (2004) emphasize that different ways of smoothing yield curves give rise to different patterns. Yields that are choppy and less smoothed produce patterns that are more similar to tents. While the US yields that we use are unsmoothed Fama-Bliss yields, yields for the remaining countries are smoothed by each country's central bank. Hence, the patterns are different. However, including only the one-year yield, the three-year forward rate, and the five-year forward rate on the right hand side produces tent shapes also for smoothed yields without changing the dynamics of the CP factor to any great extent.

[2]We have considered alternative ways of constructing a global CP factor; for example, we have elaborated with an equal-weighted factor and a factor given by the first principal component of the covariance matrix of local CP factors. Our main result that bond risk premia are determined by both a local and a global factor remains.

Table 3 presents correlations of the local CP factors as well as the global CP factor. While the US factor is only weakly positively correlated with the others, the European factors display higher correlations among each other. Correlations are higher for the second half of the sample period with correlations in excess of 0.5. This suggests that international bond risk premia have become more correlated over time. This can also be seen in Figure 1, which plots the four local CP factors. The table also shows that the US factor and the global factor are almost perfectly correlated, while correlations are lower than 0.5 for the other countries. Figure 2 plots the global factor together with NBER contractions. The global factor tends to increase during US recessions, indicating that it is countercyclical and closely related to US economic conditions.

4.2.3 Predictability regressions

We start by running Fama and Bliss (1987) regressions for each country. We regress excess returns on a n-period bond onto a constant and the forward rate-spot rate differential:

$$rx^n_{c,t+1} = a^n_c + b^n_c(f^n_{c,t} - y^1_{c,t}) + \epsilon^n_{c,t+1}, \tag{4.3}$$

where a^n_c and b^n_c are parameters and $\epsilon^n_{c,t+1}$ is an error term. Table 4 displays the results. Consistent with earlier evidence in the literature, we find that a positive forward-spot rate spread predicts US returns positively with R^2s ranging between 5% and 13%. Slope coefficients for maturities of two to four years are statistically significant at the 1% level, while the coefficient for the five-year bond is statistically significant at the 10% level. However, none of the predictability coefficients for UK and Germany are statistically different from zero while for Switzerland only slope coefficients for the two- and three-year bonds are significant at conventional levels. The explanatory power of the regressions are lower than for the US. This finding goes in line with existing evidence that the expectation hypothesis is more difficult to reject for countries outside the US.

Next, we predict bond returns using our constructed local CP factors and run the following regression for each country:

$$rx^n_{c,t+1} = b^n_{c,CP} CP_{c,t} + \epsilon^n_{c,t+1}. \tag{4.4}$$

Table 4 presents also these results. Predictability coefficients are all highly significant across the four countries. The explanatory power is higher for the US compared to the other countries. However, the R^2 is substantially higher for the CP regressions than for the earlier FB regressions. For countries in which the FB regressions pointed to no or weak evidence of predictability, the CP regressions suggest that international bond risk premia are indeed predictable. This is likely due to the fact that CP regressions make use of more information from the yield curve, compared to the FB regressions.

To put the explanatory power of the local CP factors further in context, we contrast the results with the ones using the first three principal components of yield levels to predict returns. It is common in the term-structure literature to summarize the information in yields using these components as they explain virtually all of the variation in yields. See for example Litterman and Scheinkman (1991). The first three components are often labeled level, slope, and curvature. We do a principal component analysis of yield levels for each country.[3] We then run the following regression for each country:

$$\begin{aligned} rx^n_{c,t+1} \;&=\; a^n_c + b^n_{c,Level} Level_{c,t} + b^n_{c,Slope} Slope_{c,t} + \\ &\quad\; b^n_{c,Curvature} Curvature_{c,t} + \epsilon^n_{c,t+1}. \end{aligned} \tag{4.5}$$

The results from these regressions are presented in Table 5. Judging from the statistical significance of the coefficients, the slope and curvature factors seem important for predicting returns. Furthermore, the explanatory power is higher than for the FB regressions for all countries. However, the R^2 are all lower compared to using the local CP factors with the exception of Switzerland, where the explanatory power of the two regressions are similar.

[3]The principal component (PC) analysis is done through an eigenvalue decomposition of the variance-covariance matrix of demeaned yield levels. The first PC accounts for 97.9–98.9% of the yield variance across countries, the second accounts for 1.0–1.9% of the variance, while the third only accounts for 0.02–0.12% of the variance.

To sum up the results so far, the local CP factors all predict bond returns with significantly higher R^2 than the commonly used FB regressions and they seem to contain more information than the first three principal components, with the possible exception of Switzerland.

Based on our earlier discussion of international bond risk premia being positively correlated, we investigate whether there exists a common global factor that predicts returns for each country. Using our constructed global CP factor, GCP, we predict excess returns by running the following regression:

$$rx^n_{c,t+1} = b^n_{c,GCP} GCP_t + \epsilon^n_{c,t+1}. \tag{4.6}$$

Table 6 presents the results. Interestingly, the R^2 is higher for the European countries compared to using the local CP factors. The explanatory power is, however, less for the US. Since the global factor is highly correlated with the US factor, our results suggests that shocks to US bond risk premia have great predictive power for bond returns outside the US. The lower R^2 for the US signifies that incorporating information from other countries is less important for predicting US bond returns.[4]

Having established that both a local and global CP factor predict returns significantly with high R^2 we include the local and global factors jointly and run the following regression:

$$rx^n_{c,t+1} = b^n_{c,CP} CP_{c,t} + b^n_{c,GCP} GCP_t + \epsilon^n_{c,t+1}. \tag{4.7}$$

These results are also presented in Table 6. The results for US suffer from multicollinearity which makes the individual regression coefficients insignificant. However, p-values from Wald tests suggest that the coefficients are jointly significant. Both coefficients are individually and jointly significant for the other three countries. The R^2 are also higher compared to the individual regressions. The joint significance of the coefficients suggests that bond risk premia are driven by both global and local factors.

[4]Running the predictability regression using the US factor confirms the importance of US risk premia for predicting international bond risk premia.

4.3 An affine model with local and global factors

Motivated by our finding that international bond risk premia seem to be driven by a common global factor as well as a country-specific factor, we explore in this section how CP factors drive expected excess returns. We are interested in finding out how shocks affect yields and whether there are differences across countries. We do so by estimating a parsimonious no-arbitrage term structure model for each country. The model consists of three factors for countries outside the US: The local CP factor, the global CP factor, plus the first principal component of yields which is related to the level of yields. The fact that the US factor and the global factor are close to perfectly correlated renders inference problems, so we instead choose to estimate a two factor model for the US consisting of the global CP factor and the first principal component of yields. The level component is orthogonalized with respect to the CP factors by regressing yields of maturities one-five year on a constant and the CP factors. The level factor is then the first principal component of the residuals.[5] Following the results from our predictive regressions, we assume that risk premia are only driven by the local and global CP factors. The level factors lower pricing errors and serve as country-specific interest rate factors that are not priced. Including more factors such as slope and curvature factors naturally leads to lower pricing errors, as discussed in Section 4.4.2 on robustness. However, including incremental factors do not change our main results so we choose instead parsimony.

4.3.1 Setup of the model

The model is described for one country with K state variables. For simplicity, we suppress the country subscript c. Assume that the vec-

[5]That is, the principal components are computed using yields of maturities one-to-five years as to match the maturities used in the predictability regressions. Yields on one-month bonds are merely used in the affine model to pin down the short end of the term structure.

tor of state variables follows:

$$X_t = \mu + \rho X_{t-1} + \eta_t, \tag{4.8}$$

where $\eta_t \sim N(0, \Sigma)$, and X, μ, and η are $K \times 1$ vectors, and ρ and Σ are $K \times K$ matrices. The state vector contains $CP_{c,t}, GCP_t$, and $Level_{c,t}$ for countries outside the US, and GCP_t and $Level_{c,t}$ for the US. Assume that the one-month yield follows:

$$r_t = \delta_0 + \delta_1' X_t, \tag{4.9}$$

where δ_0 is a scalar and δ_1 is a $K \times 1$ vector. The discount factor is specified as an exponential affine function of the three factors:

$$M_{t+1} = \exp\left(-\delta_0 - \delta_1' X_t - \lambda_t' \eta_{t+1} - \frac{1}{2}\lambda_t' \Sigma \lambda_t\right), \tag{4.10}$$

where λ_t are the time-varying market prices of risk. The process for λ_t is assumed to be affine: $\lambda_t = \lambda_0 + \lambda_1 X_t$, where λ_0 is a $K \times 1$ vector and λ_1 is a $K \times K$ matrix. The price of an asset satisfies standard no-arbitrage conditions, such that bond prices can be computed as:$P_t^{n+1} = E_t(M_{t+1} P_{t+1}^n)$. Bond prices become exponential affine functions of the state variables: $P_t^n = \exp(A_n + B_n' X_t)$, where A_n is a scalar and B_n is a $K \times 1$ vector. A and B satisfy:

$$\begin{aligned} A_{n+1} &= A_n + B_n' \mu^* + \frac{1}{2} B_n' \Sigma B_n - \delta_0, && (4.11) \\ B_{n+1}' &= B_n' \rho^* - \delta_1', && (4.12) \end{aligned}$$

where $A_0 = B_0 = 0$ and $\mu^* = \mu - \Sigma\lambda_0$ and $\rho^* = \rho - \Sigma\lambda_1$ are the mean vector and transition matrix under the risk neutral measure. The continuously compounded yield y_t^n is given by: $y_t^n = -\ln(P_t^n)/n = -A_n/n - B_n' X_t/n$. Model yields are subject to constant second moments since the state vector is assumed to be homoscedastic. This is counterfactual to data but simplifies our analysis. Expected log excess return on a n-period bond over the short rate is given by:

$$E_t(rx_{t+1}^n) = -Cov_t(m_{t+1}, rx_{t+1}^n) - \frac{1}{2} Var_t(rx_{t+1}^n), \tag{4.13}$$

where $rx_{t+1}^n = p_{t+1}^{n-1} - p_t^n - y_t^1$ denotes the log excess return, p denotes the log bond price, m denotes the log discount factor, and where the variance term is a Jensen's inequality term. Recognizing that the covariance term can be written as:

$$\begin{aligned} -Cov_t(m_{t+1}, rx_{t+1}^n) &= Cov_t(\eta_{t+1}, p_{t+1}^{n-1})\lambda_t \\ &= B_{n-1}^{'}\Sigma\lambda_t, \end{aligned} \tag{4.14}$$

and that the variance term can be written as:

$$\frac{1}{2}Var_t(rx_{t+1}^n) = \frac{1}{2}B_{n-1}^{'}\Sigma B_{n-1}, \tag{4.15}$$

the log excess return can be written as:

$$\begin{aligned} E_t(rx_{t+1}^n) = {} & B_{n-1}^{'}\Sigma\lambda_0 + B_{n-1}^{'}\Sigma\lambda_1 X_t - \\ & \frac{1}{2}B_{n-1}^{'}\Sigma B_{n-1}. \end{aligned} \tag{4.16}$$

Risk premia vary over time due to the time-varying market price of risk, λ_t, rather than through time-varying volatility of the state vector and are equal to zero when $\lambda_0 = 0$ and $\lambda_1 = 0$, ignoring the Jensen's inequality term. Equation (4.16) shows that λ_1 governs the price of time-varying risk. The sign of the time-varying part of the risk premium depends on the sign of the market price of risk and on the product of yield loadings and the variance-covariance matrix $B'_{n-1}\Sigma$. The usual intuition holds: the risk premium is positive if a positive shock to the state variables raises the pricing kernel while lowering bond prices as it implies low excess returns in bad times. As a result, the bond is considered risky by the investor who accordingly demands a positive risk premium for holding the asset.

4.3.2 Impulse responses and variance decompositions

Impulse response functions and variance decompositions are useful for analyzing the impact of economic shocks on yields and to gauge the relative importance of shocks for the variance of yields. See for example Hamilton (1994) for details. Starting with impulse response functions, write the state dynamics in vector MA(∞) form:

$$X_t = \sum_{i=0}^{\infty} \Psi_i \eta_{t-i}. \tag{4.17}$$

As the state dynamics are given by a VAR(1) process, $\Psi_i = \rho^i$. Shocks are orthogonalized using a Cholesky decomposition of the variance-covariance matrix Σ, which returns the lower triangular matrix P where $PP' = \Sigma$. Define a new shock vector v_t as $P^{-1}\eta_t$, so that $\eta_t = v_t P$. Then $E(v_t) = 0$ and $E(v_t v_t') = I_K$. Then redefine (4.17) as :

$$X_t = \sum_{i=0}^{\infty} \Psi_i P v_{t-i}. \tag{4.18}$$

Impulse responses can now be interpreted as the response of the system to a one standard deviation shock. Considering that yields are linear functions of the state variables, $y_t^n = -A_n/n - B_n' X_t/n$, they can be written as:

$$y_t^n = -\frac{A_n}{n} - \sum_{i=0}^{\infty} \frac{B_n'}{n} \Psi_i P v_{t-i}. \tag{4.19}$$

Hence, $-\frac{B_n'}{n}\Psi_i P_j$ is the impulse response for a n-period yield at a horizon of i months given a one standard deviation shock to state variable j at time zero, were P_j is the jth column of P.

The variance of yields is decomposed as follows. Using the vector MA(∞) form of the state dynamics, the error in forecasting the state VAR s periods ahead can be written as:

$$X_{t+s} - \hat{X}_{t+s|t} = \sum_{i=0}^{s-1} \Psi_i \eta_{t+s-i}. \tag{4.20}$$

Using (4.19), the s-period forecast error of the yield on an n-maturity bond can be written as:

$$y_{t+s}^n - \hat{y}_{t+s|t}^n = -\sum_{i=0}^{s-1} \frac{B_n'}{n} \Psi_i \eta_{t+s-i} = -\sum_{i=0}^{s-1} \frac{B_n'}{n} \Psi_i P v_{t+s-i}. \tag{4.21}$$

Then the mean squared error, MSE, of the forecast is:

$$\begin{aligned} MSE &= E[(y^n_{t+s} - \hat{y}^n_{t+s|t})(y^n_{t+s} - \hat{y}^n_{t+s|t})'] = \qquad (4.22) \\ &= \frac{B_n'}{n}\Sigma\frac{B_n}{n} + \frac{B_n'}{n}\Psi_1\Sigma\Psi_1'\frac{B_n}{n} + ... + \frac{B_n'}{n}\Psi_{s-1}\Sigma\Psi_{s-1}'\frac{B_n}{n}, \end{aligned}$$

since $Var(v_t) = I$. As we are interested in the contribution of shocks to each one of the K state variables, (4.22) can be rewritten as:

$$\begin{aligned} MSE &= \sum_{j=1}^{K}\left[\frac{B_n'}{n}P_jP_j'\frac{B_n}{n} + \frac{B_n'}{n}\Psi_1P_jP_j'\Psi_1'\frac{B_n}{n} + ... \right. \qquad (4.23) \\ &\left. + \frac{B_n'}{n}\Psi_{s-1}P_jP_j'\Psi_{s-1}'\frac{B_n}{n}\right], \end{aligned}$$

using the fact that $Var(v_{j,t}) = 1$ and where $v_{j,t}$ denotes the jth element in the v vector and where p_j denotes the jth column in matrix P. The relative contribution of a shock to the jth state variable for the variance of an n-period yield and for a horizon of s months is therefore:

$$\frac{\frac{B_n'}{n}P_jP_j'\frac{B_n}{n} + \frac{B_n'}{n}\Psi_1P_jP_j'\Psi_1'\frac{B_n}{n} + ... + \frac{B_n'}{n}\Psi_{s-1}P_jP_j'\Psi_{s-1}'\frac{B_n}{n}}{MSE}. \qquad (4.24)$$

4.4 Estimation

We estimate in a first step the risk-neutral dynamics of the state variables directly from observed yields. We then estimate the market prices of risk in λ_1 in a second step such that the model matches the slope coefficients of the in-sample predictability regressions that includes the local and global CP factors jointly.

The risk-neutral dynamics of the state variables is estimated by matching model-implied yields with observed yields. All state variables are demeaned prior to estimation, that is $\mu = 0$. The condition $\mu^* = -\Sigma\lambda_0$ is imposed in the estimation to make sure that the model reproduces state variables with a sample mean of zero. We use an estimate of Σ from an OLS estimation of the state dynamics in Equation (4.8). We estimate $\lambda_0, \rho^*, \delta_0$, and δ_1 with the generalized method of moments (GMM) framework of Hansen (1982), using the identity matrix as weighting matrix. The sample counterpart of the following moment condition is used:

$$E \quad (\nu_t \otimes z_t) = 0, \tag{4.25}$$

where ν_t is a vector of yield errors with a typical element given by $y_t^{n,data} - y_t^{n,model}$, where we consider bonds with maturities one month, and one to five years. Vector z_t contains a constant and the state variables. For countries outside the US, $z_t = [1, CP_{c,t}, GCP_t, Level_{c,t}]$, while $z_t = [1, GCP_t, Level_{c,t}]$ for the US. In total there are 16 parameters to estimate for countries outside the US, consisting of δ_0, the three elements in δ_1, the three elements in λ_0, and the nine elements in ρ^*. The number of moment conditions are 24 since ν_t has dimension 6×1. For the US, there are nine parameters to estimate and 18 moment conditions. The risk-neutral dynamics of the state variables are restricted to be stationary throughout the estimations by requiring the eigenvalues of ρ^* to lie inside the unit circle.

Parameters in λ_1 are estimated in a second step which provides an understanding of how shocks to each factor are priced. Based on results from our predictive regressions, we restrict λ_1 so that risk premia in the model only are driven by the local and global CP factors. We therefore set the column in λ_1 that refers to level shocks equal to zero. We also impose restrictions such that each CP factor only price shocks to itself, in addition to level shocks. This is done for simplicity and relaxing the restrictions does not change our results. This means that:

$$\lambda_1 \quad = \quad \begin{pmatrix} \lambda_{11} & 0 & 0 \\ 0 & \lambda_{22} & 0 \\ \lambda_{31} & \lambda_{32} & 0 \end{pmatrix}, \tag{4.26}$$

for countries outside the US while the corresponding matrix for the US is:

$$\lambda_1 \quad = \quad \begin{pmatrix} \lambda_{11} & 0 \\ \lambda_{21} & 0 \end{pmatrix}, \tag{4.27}$$

since only the global CP factor is assumed to drive risk premia for the US market. Based on our regressions, expected excess returns can be written as $E_t(rx^n_{c,t+1}) = b^n_{c,CP} CP_{c,t} + b^n_{c,GCP} GCP_t$ for $n = 2, 3, 4, 5$. The estimated slope coefficients are therefore 4×1 vectors. The corresponding expression for model-implied log excess returns are as in Equation (4.16).[6] We have estimated loadings B, the variance-covariance matrix Σ, and λ_0 from the first-step so the only unknown parameters are the λ_1 parameters. We estimate these by matching estimated expected returns in data with model-implied expected returns. For the US, we would like the model to match the global CP regression in Table 6. Let the 4×1 vector ϵ_t denote the difference $E_t(rx_{t+1})^{data} - E_t(rx_{t+1})^{model}$. We form 16 moments conditions and estimate four parameters in λ_1 for countries outside the US and form 12 moment conditions and estimate two parameters in λ_1 for the US. We estimate the system with GMM using the identity matrix as weighting matrix. The moment conditions are:

$$E \quad (\epsilon_t \otimes z_t) = 0, \tag{4.28}$$

where $z_t = [1, CP_{c,t}, GCP_t, Level_{c,t}]$ for Germany, Switzerland, and the UK while $z_t = [1, GCP_t, Level_{c,t}]$ for the US. Given the earlier estimated ρ^*, we impose stationarity on the implied physical dynamics of the state variables by requiring the eigenvalues of $\rho = \rho^* + \Sigma\lambda_1$ to lie inside the unit circle.

4.4.1 Estimation results

The estimation results for each country are reported in Table 7 which consists of parameter estimates and standard errors. All but one element of the λ_1 matrices across countries are statistically significant which indicates that shocks to all three state variables are priced and that the local and global CP factor are significant drivers of risk premia. All significant estimates of λ_1 are negative which means that

[6]Since we have demeaned the CP factors for estimation purposes, we are matching the in-sample slope coefficients obtained using demeaned CP factors. They are very similar to the ones reported in Table 6.

positive shocks to the state variables raise the pricing kernel. Whether this give rise to positive or negative risk premia depends on the sign of the yield loadings and on the variance-covariance matrix of the shocks. Figure 3 shows estimated yield loadings across countries. First, yields load positively on the level factor with loadings on the short end being somewhat higher. Second, the local CP factor takes the form of a slope factor in all countries which is consistent with local CP factors being highly positively correlated with the second principal component in each country.[7] In the UK, however, the slope is less pronounced and the CP factor is more similar to a level factor. This is in line with the UK factor being highly positively correlated with the first principal component of yields. The yield loadings of the global factor have similar shapes as the level factor in each country, which is consistent with the global factor being positively correlated with the first principal component. The global factor acts as a combination of a slope and curvature factor in the US which of course is a result of the global factor being dominated by the US factor.

Pricing errors of the model are reported in Table 8 and are lowest for Switzerland with a root mean squared error (RMSE) of 0.22% and highest for the UK with a RMSE of 0.50%. The pricing error for US of 0.36% seems reasonable considering we estimate a two-factor model. The variation of pricing errors is highest for the one-month yield which is known to be difficult to model. In the next sub-section, we discuss the effect of including the second and third principal component as additional factors.

To sum up our estimation results, we find that shocks to all state variables are priced and that local and global CP factors are significant drivers of risk premia. While the local CP factors have yield loadings that are similar to slope factors, the global factor is more similar to a level factor. The predictive power of the local CP factor suggests

[7] The correlations between local CP factors and local slope factors are 0.83, 0.88, and 0.40 for Germany, Switzerland, and the UK respectively. The corresponding correlations between local CP factors and local level factors are 0.35, 0.32, and 0.57. Correlations in Germany, Switzerland, the UK, and the US between the global factor and local level and slope factors are 0.36, 0.44, 0.33, and 0.32 for the level factor and 0.30, 0.29, 0.26, and 0.73 for the slope factor.

that a steeper and more curved term structure imply higher expected excess return while the predictive ability of the global factor implies that there exists a global level factor that drives international bond risk premia.

4.4.2 Robustness

We have chosen to use only the first principal component of yields in our affine model. However, it is common in the literature to also use a slope and curvature factor in addition to a level factor. Here we discuss how the inclusion of two additional factors affects our results.[8]

Including the first three principal components in addition to the global and local CP factors produces a RMSE for Germany, Switzerland, and the UK of 0.17%, 0.22%, and 0.21% compared to 0.29%, 0.22%, and 0.50% in the original specification. Hence, the pricing error for both the UK and Germany are reduced while the pricing error for Switzerland is unchanged. Even though more variables are added and shocks to the slope and curvature factor also are priced, the GCP and local CP factors still retain their status as level and slope factors respectively. Adding two more factors to the US specification of GCP plus a level factor lowers the RMSE to 0.30% which is somewhat lower than the original 0.35%. The reduction in pricing error is not dramatic since the original specification is close to a level factor plus a slope factor due to the highly positive correlation of 0.73 between GCP and the US slope factor. Including a second and third principal component does not change the slope-like shape of GCP yield loadings for US yields.

Hence, including a second and third principal component lowers pricing errors but it does not change the main message of the paper: The local CP and global CP factors act as slope and level factors and are important for pricing shocks, and determining risk premia in the economy. Motivated by this conclusion, we choose parsimony and focus on affine models with only two and three factors.

[8] The numerical robustness results are not reported in tables and figures for brevity but are available in full form upon request.

4.4.3 Impulse responses and variance decompositions

Figure 4 depicts impulse response functions for yields on one-month and five-year bonds, given a one standard deviation shock to the state variables. In Germany, positive shocks to the level factor and the local CP factor raise short-maturity yields both in the short and long run while long-maturity yields also increase except for very long horizons where the effect of the shocks turns negative. Shocks to the German factor immediately increase the slope of the yield curve by 44 basis points after which the slope decreases, reaching zero two years after the initial shock, and then becomes negative. It is evident that the global factor only has a small impact on long yields while the effect on short yields is larger and negative, leading to a steepening of the yield curve. The figure shows that the impulse responses do not settle down after ten years. This is since the ρ matrix for Germany contains an eigenvalue very close to one, resulting in shocks that lasts for a very long time. For Switzerland, it is evident that the global factor again has little effect on yields as the impulse responses are close to zero throughout the horizons. In contrast, positive shocks to the local CP factor lower short yields while raising long yields initially, indicating that the CP factor acts as a slope factor. The yield curve steepens initially by 50 basis points after which the slope decreases and reaches zero less than two years after the initial shock. In the UK, positive shocks to local and global CP factors have an initial effect on short-maturity yields that is negative but small while the long-run response of the short yield is virtually the same for the two shocks. However, shocks to the local CP factor have a stronger effect on long-maturity yields, raising the five-year yield by 28 basis points initially. As a result, positive innovations to the local CP factor lead to an initial steepening of the yield curve of 30 basis points after which the curve gradually flattens. The slope effect due to the local CP factor is only five basis points two years after the shock and reaches zero three years after the initial shock. In the US, the global factor acts as a slope factor as it lowers short yields and raises long yields, producing an initial slope of 56 basis points. The yield curve flattens subsequently with a slope of only five basis points after one and a half years. An eigenvalue very close to one for the US ρ matrix results in impulse response functions that decay very slowly towards zero.

The reason why a shock to the local or global CP factors can have an initial negative effect on yields even though their yield loadings may be strictly positive is the negative correlation between shocks to CP factors and the level factor. For Germany, the correlation of shocks between the level factor and the local and global CP factors are -0.33 and -0.74 respectively. For Switzerland, the corresponding correlations are -0.14 and -0.83. The correlations for the UK are -0.67 and -0.43. In the US, the correlation between shocks to GCP and the level factor is -0.79. The negative correlations also imply that a shock to the GCP factor has less of an impact on yields than the GCP yield loadings suggest since positive GCP shocks are accompanied by offsetting negative level shocks.

To sum up, an increase in local CP factors leads to an initial steepening of yield curves which lasts between one and two years while shocks to the global factor has a muted impact on yields except for the German one-month yield over very long horizons. The former effect is consistent with the positive correlation between local CP factors and the corresponding slope factor for each country. The results are robust to the ordering of the state variables, which otherwise is known to impact the results (see, for example, Bikbov and Chernov, 2008, for a discussion).

Table 9 shows results from the variance decomposition, illustrating the contribution of each shock to the variance of yield forecast errors. In Germany, the local CP factor contributes with 39% and 37% of the short and long-yield variance respectively, for a one-month horizon. Its impact on long-run variance is similar for long yields but drops down to 12% for the short yield. The global factor is not important for determining variance in yields as its largest share of variance is only 6%. In Switzerland, the impact of the local CP factor increases further as it accounts for over half of the variance of five-year yields and between 30% and 49% of the variance in one-month yields. The GCP factor has virtually zero impact on the variance of yields, underlining its little importance for determining the dynamics of yield levels. In the UK, the local CP factor is more important for the variance of long yields than short yields, accounting for 31% of the variance of long yields over a horizon of one month. Shocks to the GCP factor have a rather limited impact on the variance. For example, it accounts for 15% of the long-run variance in one-month yields. In the US, the global factor accounts for half of the short-term variance in short yields while its impact on five-year yields is tiny. As is commonly found in the literature, the bulk of the variance across countries is accounted for by the level factor. Our results are again robust to the ordering of state variables.

The results suggest that the global factor is not important for the dynamics of yield levels as it contributes very little to the variance of yields, except for the US where it is important for the variance of short-maturity yields. In contrast, the local CP factors account for a sizeable part of the forecast error variance and most notably so for Germany and Switzerland.

4.5 Where does the CP factor come from?

The ability of the CP factor to predict returns is intriguing and naturally raises the question of where it is coming from. The literature is still silent on what the CP factor actually represents. Cochrane and Piazzesi (2005) show that the US factor is correlated with business cycles, high in troughs and low in peaks. However, we still do not know exactly what type of information the CP factor captures. The natural starting point would be to consider the link between macroeconomic conditions and the CP factor. We know from asset pricing theory that risk premia should be positive on average for assets whose return covary positively with investors' well being. Furthermore, risk premia have been found to vary over time in a counter-cyclical fashion (e.g., Fama and French, 1989). Using the intuition from consumption-based models, bond risk premia are positive on average if inflation is counter-cyclical since nominal bonds then have low payoffs in bad times. To get time-variation in risk premia, one option is to consider time-varying macroeconomic volatility. For example, Bansal and Shaliastovich (2008) and Hasseltoft (2008) build on the long-run risk model of Bansal and Yaron (2004) and show that time-varying volatility of consumption growth induces time variation in bond risk premia. Using a similar model, Hasseltoft (2009) shows that also inflation volatility is an important determinant for changes in bond risk premia. Using the habit-formation model of Campbell and Cochrane (1999), Brandt and Wang (2003) and Wachter (2006) show that variation in the consumption surplus ratio induces time variation in bond risk premia. These theoretical models suggest a tight link between macroeconomic variables and risk premia. This would imply a link between the CP factor and the macroeconomy.

The reason the predictive power of the CP factor had gone unnoticed until Cochrane and Piazzesi (2005) is that it is common to focus on the first three principal component of yields which account for virtually all of the variation in yields. Even though the CP factors are highly positively correlated with the second principal component, it is also positively associated with the fourth principal component which explains a negligible part of yield variations but which has considerable forecasting power. Duffee (2008) discusses how a factor can have zero effect on current yields but be important for bond risk premia. Since yields of any maturity can be written as the sum of expected future short yields and a risk premium, such a factor must have offsetting effects on these two components. Duffee (2008) estimates a five-factor term structure model and uncovers a factor that has a negligible effect on current yields but contains substantial information about expected future short yields and expected excess bond returns. He finds the factor to be negatively associated with survey-based expected future short yields and positively associated with bond risk premia. The factor is also found to be negatively associated with industrial production, consistent with counter-cyclical risk premia. Ludvigson and Ng (2008) document using US data that the CP factor contains incremental information beyond macroeconomic variables such as inflation and real output.

The mystery of the CP factor remains. We intend to explore, in a global context, where it is coming from. Results from our affine model suggest that macro variables which affect the level and slope of the yield curve also drive risk premia. Natural candidates are inflation, real output, macroeconomic volatility/uncertainty, and monetary policy. We are also interested in understanding the nature of the global factor. Our results seem to suggest that global macro variables have predictive power across countries. A model including both local and global macro variables is likely to match the evidence of predictability. The close relation between the US factor and the global factor suggest that US macro variables or US monetary policy has implications for global bond risk premia. We would like to explore also this aspect in the future.

4.6 Conclusion

We find that bond excess returns outside the US are predictable using locally constructed forecasting factors as in Cochrane and Piazzesi (2005). The explanatory power is significantly higher than when using Fama and Bliss (1987) regressions. We also provide evidence that a global CP factor, closely related to US bond risk premia, has considerable forecasting power for international bond returns. Furthermore, the local and global CP factors are jointly significant, indicating that bond risk premia are driven by both country-specific and global factors.

Having established the predictive power of international CP factors, we propose and estimate a parsimonious no-arbitrage term structure model in which risk premia are assumed to be driven by one local and one global CP factor. The estimation reveals that the local CP factors act as a slope factor while the global factor is similar to a level factor. Hence, risk premia across countries seem to be driven by a local slope factor and a world interest rate level factor.

It is still considered a mystery where the CP factors are coming from. We hope to shed further light on the link between the macroeconomy and the CP factors in the future. Specifically, we think it is worthwhile exploring how macro variables such as inflation, real output, macroeconomic uncertainty, and monetary policy are related to the CP factors across countries, while also exploring the link between the global factor and the world economy. Our results suggest that US macro variables have considerable forecasting power for international bond risk premia.

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Table 1: Summary Statistics

	Maturity	Mean	Std.Dev.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Germany	1 month	5.15	2.50	1.00	0.97	0.94	0.91	0.88	0.86	1.00	0.88	0.69	0.61
	1 year	5.33	2.35	0.97	1.00	0.99	0.97	0.95	0.93	1.00	0.90	0.76	0.70
	2 years	5.57	2.20	0.94	0.99	1.00	0.99	0.98	0.97	1.00	0.89	0.82	0.75
	3 years	5.80	2.10	0.91	0.97	0.99	1.00	1.00	0.99	1.00	0.90	0.85	0.79
	4 years	5.98	2.01	0.88	0.95	0.98	1.00	1.00	1.00	1.00	0.91	0.87	0.81
	5 years	6.12	1.94	0.86	0.93	0.97	0.99	1.00	1.00	1.00	0.91	0.88	0.83
Switzerland	1 month	3.44	2.57	1.00	0.98	0.96	0.93	0.90	0.88	0.88	1.00	0.65	0.47
	1 year	3.75	2.30	0.98	1.00	0.99	0.97	0.95	0.93	0.90	1.00	0.69	0.53
	2 years	3.93	2.05	0.96	0.99	1.00	0.99	0.98	0.97	0.89	1.00	0.71	0.59
	3 years	4.12	1.89	0.93	0.97	0.99	1.00	1.00	0.99	0.90	1.00	0.75	0.64
	4 years	4.29	1.78	0.90	0.95	0.98	1.00	1.00	1.00	0.91	1.00	0.77	0.67
	5 years	4.41	1.69	0.88	0.93	0.97	0.99	1.00	1.00	0.91	1.00	0.78	0.69
UK	1 month	8.39	3.52	1.00	0.98	0.95	0.93	0.91	0.89	0.69	0.65	1.00	0.75
	1 year	8.20	3.11	0.98	1.00	0.99	0.97	0.96	0.95	0.76	0.69	1.00	0.82
	2 years	8.32	3.02	0.95	0.99	1.00	1.00	0.99	0.98	0.82	0.71	1.00	0.85
	3 years	8.41	2.99	0.93	0.97	1.00	1.00	1.00	0.99	0.85	0.75	1.00	0.86
	4 years	8.48	2.99	0.91	0.96	0.99	1.00	1.00	1.00	0.87	0.77	1.00	0.87
	5 years	8.54	3.01	0.89	0.95	0.98	0.99	1.00	1.00	0.88	0.78	1.00	0.87
US	1 month	5.70	2.94	1.00	0.98	0.95	0.93	0.92	0.90	0.61	0.47	0.75	1.00
	1 year	6.45	3.08	0.98	1.00	0.99	0.98	0.97	0.96	0.70	0.53	0.82	1.00
	2 years	6.73	2.98	0.95	0.99	1.00	1.00	0.99	0.98	0.75	0.59	0.85	1.00
	3 years	6.92	2.88	0.93	0.98	1.00	1.00	1.00	0.99	0.79	0.64	0.86	1.00
	4 years	7.09	2.80	0.92	0.97	0.99	1.00	1.00	1.00	0.81	0.67	0.87	1.00
	5 years	7.19	2.74	0.90	0.96	0.98	0.99	1.00	1.00	0.83	0.69	0.87	1.00

The table presents means and standard deviations of yields on zero-coupon bonds with maturities between one month and five years. The sample period is January 1976 to December 2007. Columns 1-10 present correlations: Columns 1-6 between yields of bonds with different maturities; columns 7-10 between yields of bonds with same maturity across countries.

Table 2: Correlations between Excess Returns

	n		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.
Germany	2	1.	1.00	0.98	0.96	0.93	0.81	0.81	0.81	0.81	0.68	0.67	0.67	0.67	0.62	0.61	0.61	0.61
	3	2.	0.98	1.00	0.99	0.98	0.79	0.80	0.81	0.82	0.70	0.70	0.70	0.71	0.63	0.63	0.63	0.63
	4	3.	0.96	0.99	1.00	1.00	0.76	0.79	0.81	0.82	0.70	0.71	0.72	0.72	0.63	0.63	0.64	0.64
	5	4.	0.93	0.98	1.00	1.00	0.75	0.78	0.80	0.81	0.69	0.71	0.72	0.73	0.63	0.63	0.64	0.65
Switzerland	2	5.	0.81	0.79	0.76	0.75	1.00	0.97	0.95	0.94	0.64	0.63	0.62	0.61	0.43	0.45	0.46	0.46
	3	6.	0.81	0.80	0.79	0.78	0.97	1.00	0.99	0.98	0.66	0.67	0.67	0.67	0.46	0.49	0.50	0.51
	4	7.	0.81	0.81	0.81	0.80	0.95	0.99	1.00	0.99	0.67	0.68	0.68	0.68	0.46	0.48	0.50	0.51
	5	8.	0.81	0.82	0.82	0.81	0.94	0.98	0.99	1.00	0.67	0.70	0.70	0.70	0.48	0.50	0.52	0.53
UK	2	9.	0.68	0.70	0.70	0.69	0.64	0.66	0.67	0.67	1.00	0.98	0.95	0.92	0.43	0.43	0.42	0.42
	3	10.	0.67	0.70	0.71	0.71	0.63	0.67	0.68	0.70	0.98	1.00	0.99	0.98	0.50	0.51	0.51	0.51
	4	11.	0.67	0.70	0.72	0.72	0.62	0.67	0.68	0.70	0.95	0.99	1.00	1.00	0.54	0.55	0.55	0.55
	5	12.	0.67	0.71	0.72	0.73	0.61	0.67	0.68	0.70	0.92	0.98	1.00	1.00	0.55	0.57	0.57	0.57
US	2	13.	0.62	0.63	0.63	0.63	0.43	0.46	0.46	0.48	0.43	0.50	0.54	0.55	1.00	0.99	0.97	0.95
	3	14.	0.61	0.63	0.63	0.63	0.45	0.49	0.48	0.50	0.43	0.51	0.55	0.57	0.99	1.00	0.99	0.98
	4	15.	0.61	0.63	0.64	0.64	0.46	0.50	0.50	0.52	0.42	0.51	0.55	0.57	0.97	0.99	1.00	0.99
	5	16.	0.61	0.63	0.64	0.65	0.46	0.51	0.51	0.53	0.42	0.51	0.55	0.57	0.95	0.98	0.99	1.00

The table presents correlations between one-year returns on bonds with maturities of two, three, four, and five years in excess of the return on a one-year bond for Germany, Switzerland, the UK, and the US. The sample period is January 1976 to December 2007.

Table 3: Correlations between Local and Global CP Factors

	Germany	Switzerland	UK	US	Global
1976:01–2007:12					
Germany	1.00	0.63	0.25	0.32	0.46
Switzerland	0.63	1.00	0.55	0.27	0.41
UK	0.25	0.55	1.00	0.12	0.27
US	0.32	0.27	0.12	1.00	0.98
Global	0.46	0.41	0.27	0.98	1.00
1976:01–1991:06					
Germany	1.00	0.35	-0.15	0.23	0.31
Switzerland	0.35	1.00	0.39	0.12	0.21
UK	-0.15	0.39	1.00	-0.10	0.01
US	0.23	0.12	-0.10	1.00	0.99
Global	0.31	0.21	0.01	0.99	1.00
1991:07–2007:12					
Germany	1.00	0.82	0.63	0.38	0.53
Switzerland	0.82	1.00	0.75	0.53	0.65
UK	0.63	0.75	1.00	0.64	0.73
US	0.38	0.53	0.64	1.00	0.98
Global	0.53	0.65	0.73	0.98	1.00

The table presents correlations between local CP factors for Germany, Switzerland, the UK, and the US, and the global CP factor based on data for the full sample period (1976:01–2007:12) and two sub-sample periods (1976:01–1991:06 and 1991:07–2007:12).

Table 4: Fama-Bliss and Cochrane-Piazzesi Regressions

	n	a_c^n	b_c^n	R^2	$b_{c,CP}^n$	R^2
Germany	2	0.35	0.37	0.03	0.42	0.12
		(0.33)	(0.42)		(0.09)	
	3	0.50	0.61	0.05	0.85	0.15
		(0.70)	(0.50)		(0.17)	
	4	0.59	0.74	0.05	1.21	0.17
		(1.03)	(0.57)		(0.24)	
	5	0.69	0.81	0.05	1.52	0.17
		(1.32)	(0.63)		(0.30)	
Switzerland	2	0.16	0.61	0.09	0.44	0.16
		(0.28)	(0.24)		(0.12)	
	3	0.43	0.57	0.04	0.86	0.18
		(0.58)	(0.34)		(0.21)	
	4	0.66	0.58	0.03	1.23	0.19
		(0.88)	(0.46)		(0.28)	
	5	0.93	0.54	0.02	1.47	0.18
		(1.21)	(0.61)		(0.34)	
UK	2	0.33	0.38	0.03	0.42	0.14
		(0.27)	(0.27)		(0.14)	
	3	0.61	0.49	0.03	0.85	0.18
		(0.46)	(0.36)		(0.25)	
	4	0.90	0.47	0.02	1.22	0.19
		(0.64)	(0.45)		(0.37)	
	5	1.24	0.41	0.01	1.51	0.18
		(0.82)	(0.50)		(0.47)	
US	2	0.12	0.87	0.10	0.46	0.29
		(0.34)	(0.30)		(0.06)	
	3	-0.03	1.18	0.13	0.87	0.32
		(0.61)	(0.35)		(0.11)	
	4	-0.15	1.33	0.13	1.24	0.34
		(0.90)	(0.45)		(0.16)	
	5	0.37	0.97	0.05	1.42	0.31
		(1.21)	(0.57)		(0.21)	

The table presents results from Fama-Bliss (1987) and Cochrane-Piazzesi (2005) regressions, corresponding to regression equations (4.3) and (4.4). The sample period is January 1976 to December 2007. Point estimates with Newey and West (1987) standard errors, accounting for conditional heteroscedasticity and serial correlation up to twelve lags, in parentheses are reported together with adjusted R-squares.

Table 5: Level, Slope, and Curvature Regressions

	n	a_c^n	$b_{c,Level}^n$	$b_{c,Slope}^n$	$b_{c,Curvature}^n$	R^2
Germany	2	0.51	5.70	70.51	36.22	0.10
		(0.23)	(4.91)	(34.06)	(187.21)	
	3	1.02	10.54	148.99	119.16	0.12
		(0.43)	(8.51)	(64.86)	(343.41)	
	4	1.41	13.05	226.42	242.61	0.13
		(0.59)	(11.54)	(91.19)	(466.14)	
	5	1.73	14.30	299.69	386.75	0.14
		(0.74)	(14.32)	(113.78)	(569.16)	
Switzerland	2	0.36	4.85	104.12	-278.45	0.18
		(0.25)	(5.38)	(27.25)	(93.77)	
	3	0.80	11.00	199.53	-334.49	0.17
		(0.44)	(10.50)	(45.91)	(179.17)	
	4	1.17	15.11	300.69	-295.05	0.18
		(0.59)	(14.33)	(61.66)	(254.32)	
	5	1.44	20.17	350.13	-266.93	0.17
		(0.73)	(17.62)	(75.21)	(321.04)	
UK	2	0.40	5.89	47.24	249.49	0.10
		(0.25)	(3.61)	(34.78)	(135.69)	
	3	0.76	12.03	76.52	510.69	0.11
		(0.44)	(6.71)	(63.27)	(239.08)	
	4	1.08	17.89	104.79	656.49	0.11
		(0.61)	(9.85)	(89.48)	(338.73)	
	5	1.40	23.08	135.13	679.22	0.10
		(0.77)	(12.70)	(112.79)	(429.02)	
US	2	0.58	6.72	115.34	342.28	0.22
		(0.26)	(4.00)	(32.64)	(110.37)	
	3	0.94	9.13	223.22	685.48	0.21
		(0.48)	(7.62)	(63.93)	(202.86)	
	4	1.28	10.64	337.00	950.36	0.23
		(0.64)	(10.43)	(87.76)	(277.73)	
	5	1.34	11.61	420.57	1099.99	0.23
		(0.77)	(12.69)	(106.45)	(334.09)	

The table presents results from principal-component regressions, corresponding to regression equation (4.5). The sample period is January 1976 to December 2007. The first three principal components of the yield covariance matrix are referred to as level, slope, and curvature. Point estimates with Newey and West (1987) standard errors, accounting for conditional heteroscedasticity and serial correlation up to twelve lags, in parentheses are reported together with adjusted R-squares.

Table 6: Local and Global Cochrane-Piazzesi Regressions

	n	$b^n_{c,CP}$	R^2	$b^n_{c,GCP}$	R^2	$b^n_{c,CP}$	$b^n_{c,GCP}$	R^2	Wald
Germany	2	0.42	0.12	0.46	0.28	0.13	0.39	0.29	[0.00]
		(0.09)		(0.05)		(0.10)	(0.06)		
	3	0.85	0.15	0.86	0.27	0.35	0.67	0.29	[0.00]
		(0.17)		(0.10)		(0.20)	(0.11)		
	4	1.21	0.17	1.19	0.25	0.56	0.88	0.29	[0.00]
		(0.24)		(0.15)		(0.29)	(0.17)		
	5	1.52	0.17	1.46	0.24	0.73	1.06	0.28	[0.00]
		(0.30)		(0.19)		(0.37)	(0.21)		
Switzerland	2	0.44	0.16	0.41	0.20	0.25	0.28	0.24	[0.00]
		(0.12)		(0.09)		(0.13)	(0.10)		
	3	0.86	0.18	0.81	0.22	0.48	0.56	0.27	[0.00]
		(0.21)		(0.17)		(0.23)	(0.19)		
	4	1.23	0.19	1.14	0.23	0.71	0.78	0.28	[0.00]
		(0.28)		(0.24)		(0.32)	(0.27)		
	5	1.47	0.18	1.40	0.23	0.80	0.99	0.28	[0.00]
		(0.34)		(0.29)		(0.38)	(0.34)		
UK	2	0.42	0.14	0.40	0.16	0.28	0.28	0.23	[0.00]
		(0.14)		(0.11)		(0.12)	(0.10)		
	3	0.85	0.18	0.78	0.19	0.58	0.53	0.27	[0.00]
		(0.25)		(0.19)		(0.22)	(0.16)		
	4	1.22	0.19	1.14	0.21	0.81	0.80	0.29	[0.00]
		(0.37)		(0.27)		(0.32)	(0.22)		
	5	1.51	0.18	1.49	0.22	0.95	1.09	0.29	[0.00]
		(0.47)		(0.32)		(0.40)	(0.27)		
US	2	0.46	0.29	0.57	0.29	0.26	0.26	0.29	[0.00]
		(0.06)		(0.08)		(0.35)	(0.47)		
	3	0.87	0.32	1.06	0.30	0.85	0.03	0.32	[0.00]
		(0.11)		(0.15)		(0.61)	(0.81)		
	4	1.24	0.34	1.50	0.32	1.30	-0.08	0.34	[0.00]
		(0.16)		(0.22)		(0.79)	(1.04)		
	5	1.42	0.31	1.72	0.29	1.55	-0.15	0.31	[0.00]
		(0.21)		(0.27)		(1.01)	(1.32)		

The table presents results from local and global Cochrane-Piazzesi (2005) regressions, corresponding to regression equations (4.4), (4.6), and (4.7). The sample period is January 1976 to December 2007. Point estimates with Newey and West (1987) standard errors, accounting for conditional heteroscedasticity and serial correlation up to twelve lags, in parentheses are reported together with adjusted R-squares. P-values from Wald tests of joint significance are given in square brackets.

Table 7: Estimations of Affine Models with Local and Global Factors

	δ_0	δ_1		ρ^*		λ_0		λ_1	
Germany	0.435	-0.007	0.994	-0.019	-0.018	-1.296	-0.270	0.000	0.000
	(0.007)	(0.005)	(0.052)	(0.045)	(0.063)	(3.651)	(0.058)		
		0.032	0.255	0.650	-0.342	-3.158	0.000	-0.699	0.000
		(0.003)	(0.927)	(0.798)	(1.057)	(8.879)		(0.217)	
		0.046	-0.110	0.236	1.229	-0.695	0.043	-0.343	0.000
		(0.001)	(0.657)	(0.559)	(0.750)	(1.421)	(0.042)	(0.051)	
Switzerland	0.290	-0.015	0.996	0.011	-0.006	-0.037	-0.059	0.000	0.000
	(0.004)	(0.004)	(0.034)	(0.007)	(0.009)	(0.016)	(0.006)		
		0.048	-0.968	0.992	0.319	-0.547	0.000	-0.717	0.000
		(0.002)	(0.883)	(0.123)	(0.210)	(0.341)		(0.196)	
		0.051	0.909	-0.020	0.699	-0.230	-0.075	-0.215	0.000
		(0.001)	(0.769)	(0.104)	(0.184)	(0.051)	(0.019)	(0.036)	
UK	0.707	0.067	0.434	-0.101	0.044	0.802	-0.099	0.000	0.000
	(0.013)	(0.008)	(0.591)	(0.204)	(0.047)	(1.074)	(0.008)		
		0.018	1.558	1.245	-0.066	-2.613	0.000	-0.341	0.000
		(0.006)	(1.382)	(0.469)	(0.111)	(2.669)		(0.024)	
		0.047	0.113	0.050	0.942	-0.536	-0.055	-0.118	0.000
		(0.002)	(0.666)	(0.152)	(0.039)	(0.704)	(0.001)	(0.004)	(0.000)
US	0.480	0.010		0.857	-0.019	0.040		-0.137	0.000
	(0.007)	(0.004)		(0.031)	(0.007)	(0.033)		(0.011)	
		0.039		0.205	1.017	-0.146		-0.081	0.000
		(0.001)		(0.035)	(0.009)	(0.010)		(0.013)	

The table presents estimation results of affine models with local and global factors (see Section 4.4.1) for yields on bonds with maturities of one month, and one to five years. Germany, Switzerland, and the UK have three state variables ($CP_{c,t}$, GCP_t, and $Level_{c,t}$), whereas the US has two state variables (GCP_t and $Level_{c,t}$). The parameters in δ_0, δ_1, ρ^*, and λ_0 are estimated in a first step by GMM, using the sample counterparts of the moment conditions in (4.25). The parameters in λ_1 are estimated in a second step by GMM, using the sample counterparts of the moment conditions in (4.28). The λ_1 matrix for Germany, Switzerland, and the UK is restricted as in equation (4.26), and for the US it is restricted as in equation (4.27). See Section 4.4 for details. Point estimates and standard errors, accounting for conditional heteroscedasticity and serial correlation up to twelve lags as in Newey and West (1987), are reported. Estimates of parameters in δ_0 and δ_1 are multiplied with 100. The sample period is January 1976 to December 2007.

Table 8: Yield Diagnostics of the Estimated Affine Models

	1 month	1 year	2 years	3 years	4 years	5 years	RMSE	MAD
Germany	-0.028	0.007	0.007	-0.003	-0.007	0.006	0.295	0.155
	{0.654}	{0.224}	{0.064}	{0.080}	{0.100}	{0.153}		
Switzerland	-0.011	-0.008	-0.020	0.001	0.014	0.006	0.222	0.120
	{0.481}	{0.080}	{0.128}	{0.033}	{0.112}	{0.167}		
UK	0.002	-0.005	0.006	0.000	-0.003	0.002	0.504	0.310
	{1.050}	{0.468}	{0.135}	{0.095}	{0.230}	{0.361}		
US	-0.017	0.055	-0.026	-0.034	0.005	0.017	0.362	0.227
	{0.732}	{0.323}	{0.155}	{0.080}	{0.167}	{0.293}		

The table presents diagnostics of the estimated affine models with local and global factors (see Section 4.4.1) for yields on bonds with maturities of one month, and one to five years. Averages and standard deviations (in curly brackets) of yield errors, are reported in the first six columns. The last two columns report a root mean squared error (RMSE) and a mean absolute deviation (MAD) of yield errors. All statistics are expressed in % per year.

Table 9: Variance Decompositions

	Variable	Horizon	1 month	5 year
Germany	Local CP	1	0.39	0.37
		120	0.12	0.38
	Global CP	1	0.00	0.02
		120	0.06	0.03
	Level	1	0.61	0.62
		120	0.82	0.59
Switzerland	Local CP	1	0.30	0.49
		120	0.49	0.68
	Global CP	1	0.00	0.00
		120	0.02	0.00
	Level	1	0.70	0.51
		120	0.49	0.32
UK	Local CP	1	0.00	0.31
		120	0.13	0.13
	Global CP	1	0.09	0.02
		120	0.15	0.09
	Level	1	0.91	0.67
		120	0.72	0.78
US	Global CP	1	0.50	0.02
		120	0.12	0.09
	Level	1	0.50	0.98
		120	0.88	0.91

The table presents variance decompositions of yield forecast errors, attributed to each state variable at horizons of one month and 120 months for yields on a one-month bond and a five-year bond.

Figure 1: Local CP Factors

The figure shows the local CP factors (in % per year) for Germany, Switzerland, the UK, and the US.

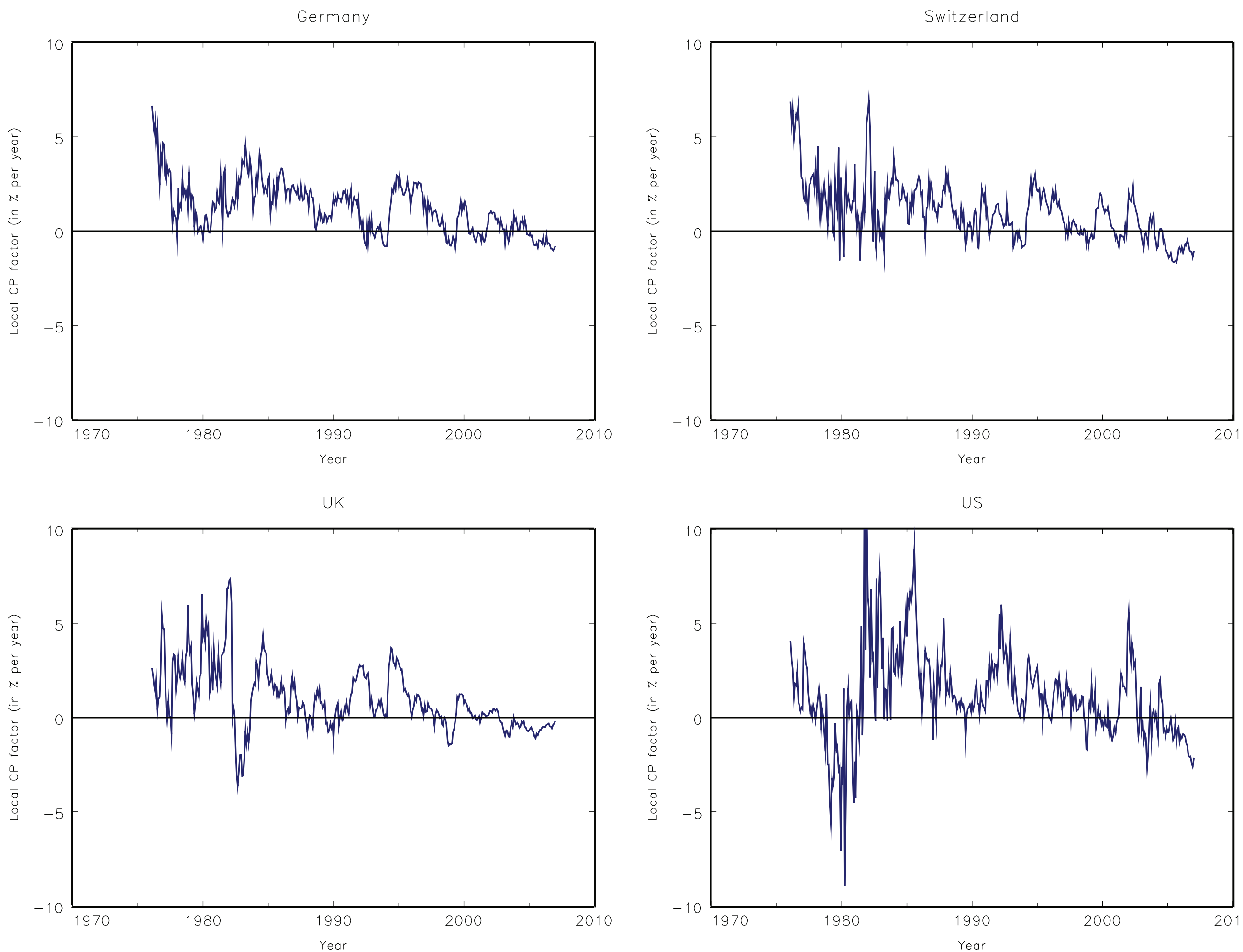


Figure 2: Global CP Factor

The figure shows the global CP factor (in % per year), which is a GDP-weighted average of the local CP factors (for Germany, Switzerland, the UK, and the US). The shaded areas mark US contractions (peaks to troughs) as dated by the NBER.

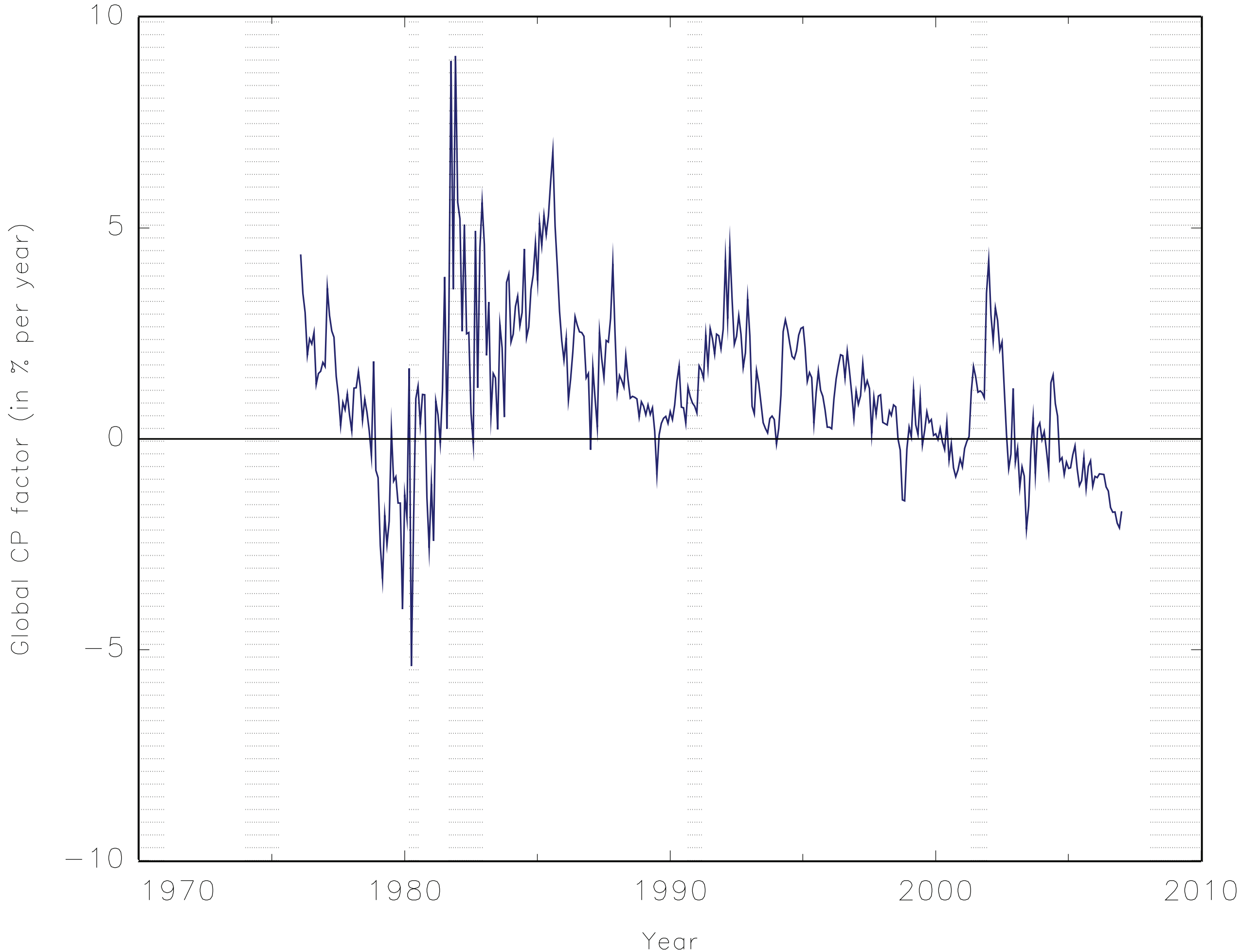


Figure 3: Yield Loadings

The figure shows the yield loadings (in % per year) for Germany, Switzerland, the UK, and the US. The loadings are $-\frac{B_n'}{n}$, presented in Equation(4.12). The solid line refers to the local CP factor, the dashed line to the global CP factor, and the short-dashed line to the local level factor.

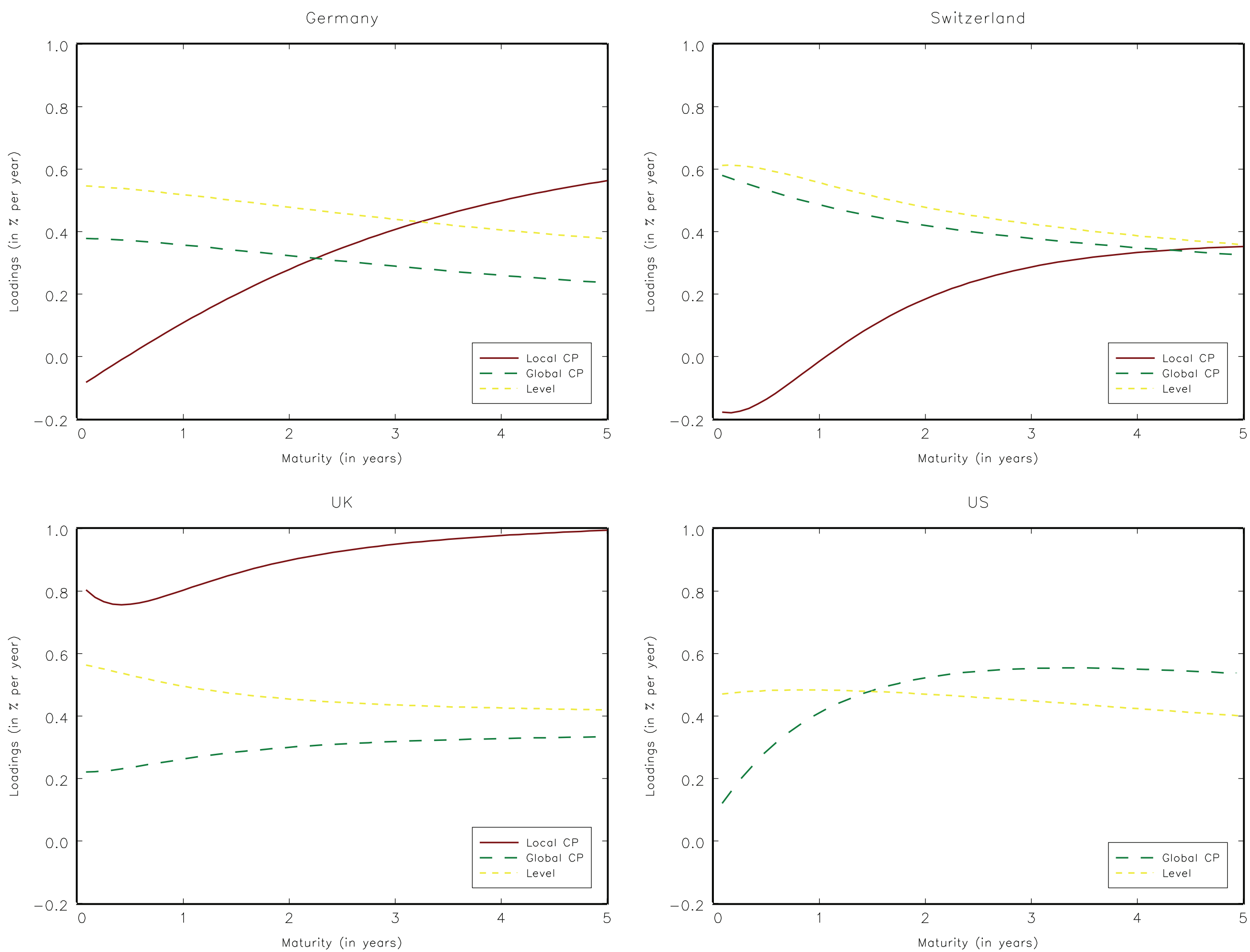


Figure 4: Impulse Response Functions

The figure shows the impulse response functions (in % per year) for the yield on a one-month bond and the yield on a five-year bond given a one standard deviation shock to each state variable. The solid line refers to impulses from the local CP factor, the dashed line to the global CP factor, and the short-dashed line to the local level factor.

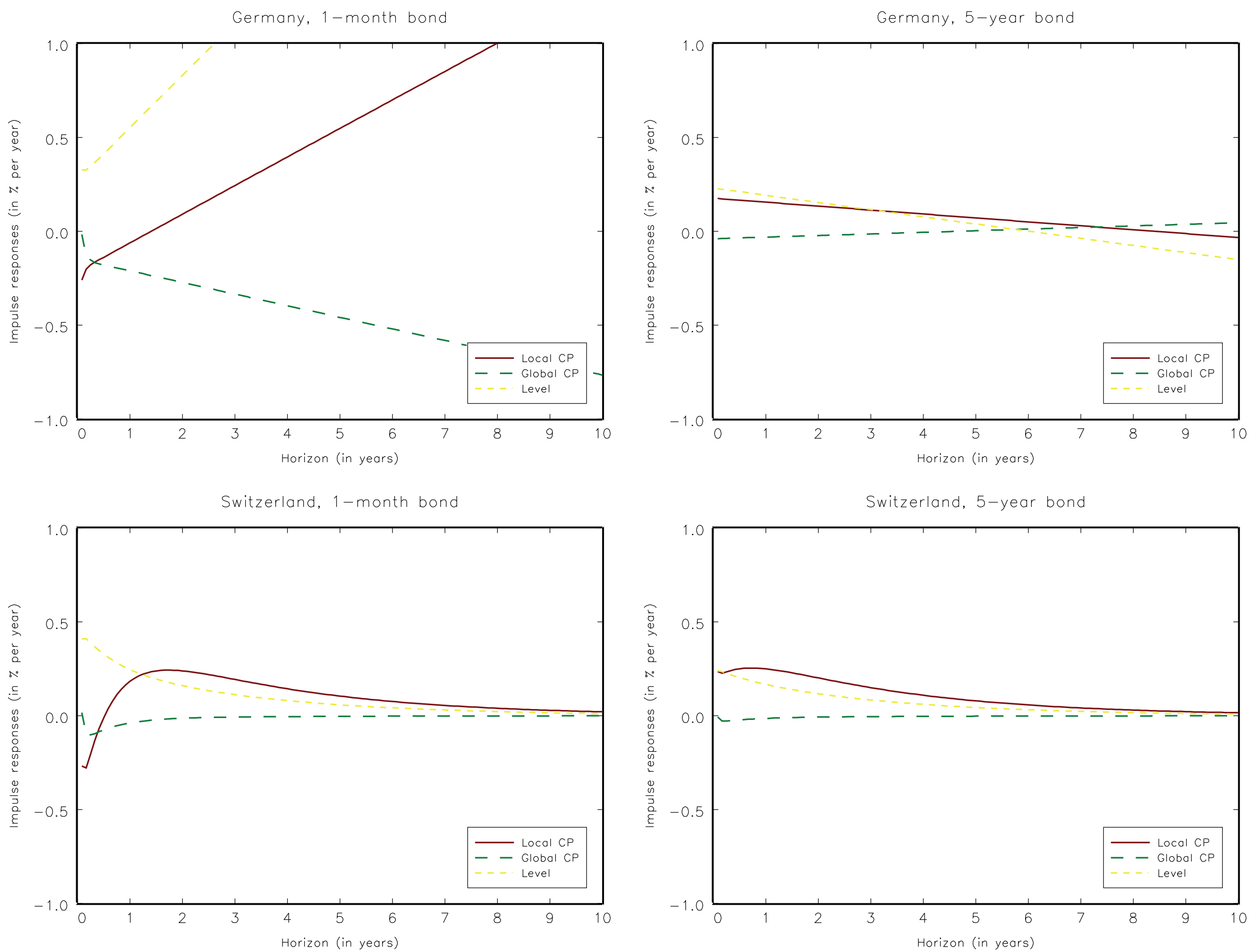
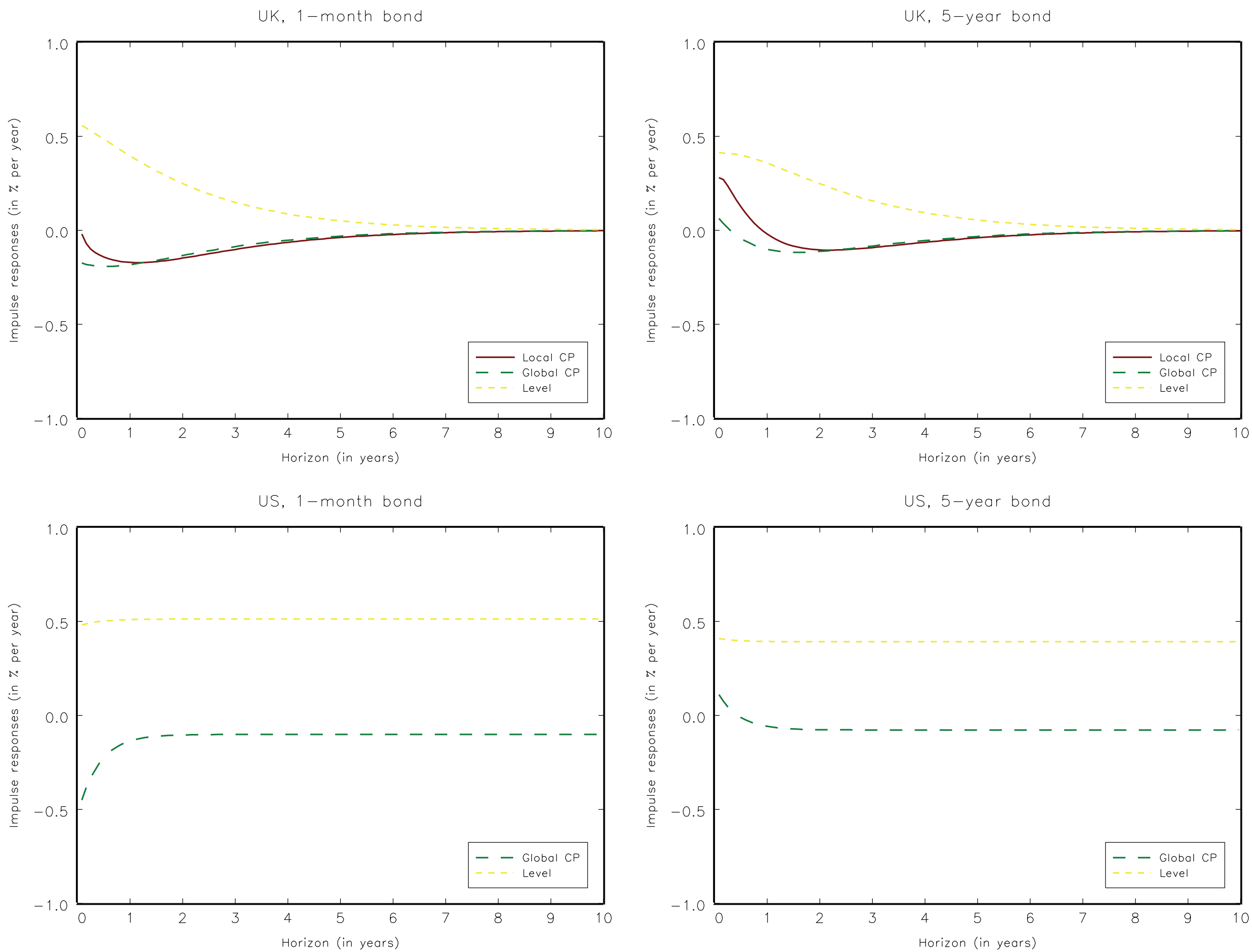


Figure 4: Impulse Response Functions (Cont'd)



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