POVERTY AND THE DYNAMICS OF EQUILIRBIUM UNEMPLOYMENT

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Poverty and the Dynamics of Equilibrium Unemployment

to my parents

Contents

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CHAPTER 1

Introduction

Imagine that you have been unemployed for some time, and that one morning you find out that you have run out of savings. Would your financial difficulties affect your efforts to find a new job? The theory of job search has a relatively straightforward answer to this question. It predicts that you would become less picky about which job offers you accept, and that you would increase your efforts to search for job openings. In more technical language, search models with risk averse agents and incomplete markets predict that the reservation wage of an unemployed worker increases in the level of her wealth, and that her search intensity should fall in wealth (Danforth 1979 and Lentz and Tranas 2005). By implication, these models also predict that if you control for other relevant worker characteristics, such as educational attainment, work experience and other, you would find a positive relationship between an unemployed worker's wealth and the duration of her unemployment spell. A number of empirical studies have found evidence for such a positive effect of workers' wealth on unemployment duration.[1] All three studies in this volume deals with the effect of wealth on workers' reservation wages. Chapter 2 and 3 are concerned with the effects of wealth on the aggregate level and duration of unemployment. Are these effects of quantitative importance; do we need to include them when accounting for aggregate outcomes in the labor market? Even if these two chapters do not explicitly deal with normative issues, it is clear that the answer to these questions may have important implications for the design of economic policy. Traditionally, the welfare analysis of publicly provided unemployment insurance (UI) have contrasted the positive insurance effects with the negative substitution effects: unemployment benefits afford insurance to workers, but they also affect economic efficiency negatively because they tend to reduce the supply

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[1] See Bloemen and Stancanelli (2001), Algan et al.(2003), Alexopoulos and Gladden (2004), Chetty (2008) and references therein.

of labor.[2] Chetty (2008) shows that a third and potentially important effect need to be included in this analysis: when benefits are increased, borrowing constrained workers gain some liquidity, and they may therefore want to decrease their search intensity. Importantly, the decrease in search intensity that comes through this mechanism is beneficial to welfare. Chetty (2008) points out that the extent to which this 'liquidity effect' on unemployment durations is quantitatively important depends on the elasticity of the search intensity with respect to savings. It thus seems that there is a strong motivation, from the perspective of economic policy, for models that can jointly and realistically account for workers' savings and their job search behavior.

The last chapter makes an effort to explain a model prediction that I encountered by chance, when working on the studies presented in chapter 2 and 3. When skill dynamics are introduced into the model of Danforth (1979), workers may, under some specific circumstances, choose to decrease their reservation wage the more savings they have. When this happens, the model predicts that workers find jobs quicker the more savings they have. The study does not assess the likelihood of this kind of behavior. It is tempting to speculate, however, that the job-market decisions of young people could relatively often depend on the wealth of their parents. Some low-paid jobs may be important stepping stones towards a better career. If there is enough money around to compensate for that low wage, then such a career investment may look more appealing.

1.1. Aggregate Wealth and Unemployment

Of the three studies presented in this volume, that of chapter 2 is in some sense the most ambitious. In *Workers' Wealth and Unemployment: Do Poor Workers Affect Aggregate Outcomes* I try to account for both the aggregate wealth distribution and the aggregate labor market outcomes of the U.S. economy. In general equilibrium, the model of chapter 2 uses heterogeneity in workers' skills and in their educational attainment in order to explain, to some degree, the very high level of observed U.S. wealth inequality. Like the model in chapter 3, this model also includes a simple life cycle dynamics, a public unemployment insurance program and a rudimentary public

[2] See, for example, Krueger and Meyer (2002).

pension system. In order to evaluate the aggregate effects of workers' wealth on the unemployment rate, I use an out-of-equilibrium experiment where workers are forced to set their reservation wage as if they had the median level of wealth of their respective educational group. The results from this exercise indicate that the effect of wealth inequality on unemployment is negative but very small. Because poor workers chose low reservation wages, the aggregate unemployment rate is lower when workers are free to set what reservation wage they want, compared to the simulation with forced decisions. However, an inspection of the equilibrium wealth distribution reveals that workers in the model economy have unrealistically high levels of savings, compared to the median income. Though the model makes several reasonable predictions, it cannot account for the low levels of savings adequacy observed among American households.

1.2. Blue Collar Workers, Unemployment, and Savings Adequacy

In several ways, the lessons drawn from the analysis in chapter 2 have guided the work presented in chapter 3. *Poverty and the Dynamics of Equilibrium Unemployment* is concerned with the same basic question as chapter 2: do poor workers affect the average level and duration of unemployment? The focus, however, is different. Instead of trying to account for the U.S. distribution of wealth, this study looks only at the behavior of American blue collar workers. American blue collar households hold considerably less wealth in relation to their income than do white collar workers, and there is recent evidence that blue collar workers are less able than white collar workers to insure themselves against negative income shocks.[3] Like the study in chapter 2, this model relies partly on skill heterogeneity to explain differences in workers' savings rate. Unlike the previous chapter, however, chapter 3 accounts only for workers holdings of liquid wealth. The model makes realistic predictions concerning the levels of savings adequacy of unemployed workers, and in contrast to the findings of chapter 2, risk aversion and incomplete insurance play a quantitatively important role for the average level and duration of unemployment.

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[3] See Budria Rodríguez et al.(2002) and Blundell et al.(2008).

Since chapter 3 has the same title as the entire volume, this may be a good place to make some comments about that title. 'Poverty' is a word charged with ideological and other references, and it's use may be controversial. In this work, 'poverty' refers to poverty in wealth, and it primarily relates to inequality. Ultimately, chapter 2 and 3 focus on the behavior of workers at the very bottom of the wealth distribution, trying to understand how their wealth poverty affects aggregate outcomes in the labor market. To some people, the term 'equilibrium unemployment' may also be controversial, perhaps because it seems to suggest that there is also such a thing as 'out-of-equilibrium unemployment'. If some readers take offence at this use of the words, I apologize. All that is intended here with 'equilibrium unemployment' is the average long-run level of unemployment.

1.3. Reservation Wages that Decrease in Wealth

The last chapter is concerned only with the behavior of individual workers, not with aggregate outcomes or equilibrium distributions. When a McCall search model is augmented with skill dynamics, unemployed workers may sometimes choose to lower their reservation wages the higher is their wealth. It turns out that this prediction can be understood much in the same way as can the prediction of the simpler model: workers that search for jobs make risky investment decisions, and wealthy workers may find such investments more appealing than do less wealthy workers. In the McCall search model, this investment concerns the expected wage of a future job. In the skill-augmented model, workers must decide how much to invest in higher future wages (per efficiency unit of labor), and how much to invest in quicker, expected skill appreciation.

References

Alexopoulos, M and T Gladden (2004), *The Effects of Wealth, and Unemployment Benefits on Search Behavior and Labor Market Transitions* (Mimeo, University of Toronto)

Algan, Y, A Cheron, J O Hairault and F Langot (2003), Wealth Effects on Labor Market Transitions, *Review of Economic Dynamics* **6**, 156-178.

Bloemen, H G and E Stancanelli (2001), Individual Wealth, Reservation Wages, and Transitions into Employment, *Journal of Labor Economics* **19**, 400-439.

Blundell, R, L Pistaferri and I Preston (2008), Consumption Inequality and Partial Insurance, *American Economic Review* **98**, 1887-1921.

Budría Rodríguez, S, J Díaz-Giménez, V Quadrini and J-V Ríos-Rull (2002), Updated Facts on the U.S. Distributions of earnings, Income, and wealth, *Federal Reserve Bank of Minneapolis Quarterly Review* **26**, 2-35.

Chetty, R (2008), Moral Hazard vs. Liquidity and Optimal Unemployment Insurance, *Journal of Political Economy* **116**, 173-234.

Danforth, J P (1979), On the role of consumption and decreasing absolute risk aversion in the theory of job search, in S A Lippman and J J McCall (eds.), *Studies in the Economics of Search*, North Holland.

Krueger, A, and B D Meyer (2002), Labor Supply Effects of Social Insurance, in A Auerbach and M Feldstein (eds.), *Handbook of Public Economics* **4**.

Lentz, R and T Tranas (2005), Job Search and Savings: Wealth Effects and Duration Dependence, *Journal of Labor Economics* **3**, 467-490.

CHAPTER 2

Workers' Wealth and Unemployment:
Do Poor Workers Affect Aggregate Outcomes?

Henrik Lundvall

ABSTRACT. It is well known that risk averse workers who are imperfectly insured against shocks to their labor income set reservation wages that depend on their asset holdings. Much less is known about the quantitative importance of this effect on aggregate outcomes of the labor market. Does the distribution of wealth matter for a country's unemployment rate? I address this question using a general equilibrium model of frictional unemployment, where all transitions back and forth between employment and unemployment are endogenous. The results show that the wealth distribution does affect the equilibrium unemployment rate, although the effect is modest in size. Poor workers set reservation wages that are drastically lower than those of more wealthy workers, thereby contributing to a lower unemployment rate.

2.1. Introduction

Risk aversion and binding borrowing constraints affect the optimal search policy of unemployed workers, as well as the wage demands of employed agents. Unemployed workers who are financially poor and borrowing constrained are forced to lower their consumption, thereby making the utility cost of search high. For this reason, search models with risk averse agents generally predict that workers' reservation wages increase in wealth holdings (Danforth 1979).[1] The risk of facing an extended spell of unemployment, without the savings needed to sustain a decent level of consumption, will

[1] In models where the variable of choice is the search intensity instead lof the reservation wage, a similar result hold: wealthy workers choose to exert less effort in search than do poor workers (Lentz and Tranas 2005).

affect also employed workers who are financially poor, inducing them to lower their demands for compensation. These interactions between workers' wealth and search policies have recently motivated new studies of optimal unemployment insurance (Shimer and Werning 2005) and of the welfare consequence of various labor market policies (Alvarez and Veracierto 2001; Gomes, Greenwood and Rebelo 2001; and others). A number of studies have found empirical evidence supporting the prediction that higher levels of savings are associated with longer unemployment spells.[2]

One question that has received relatively little attention, however, is whether or not these wealth effects on search policies are important for our understanding of the aggregate labor market outcomes. Does a country's distribution of wealth affect its equilibrium unemployment rate?[3] The empirical literature does not give any clear guidance on this issue. On the one hand, Algan et al.(2003) and Chetty (2008) argue that the effect of wealth on unemployment durations are large in size. For example, Chetty (2008) uses data on unemployment durations and on severance payments in the United States to decompose the effect UI benefits on durations. He concludes that 60% of the increase in unemployment durations caused by UI benefits come from the liquidity effect that allow borrowing constrained households to extend the duration of their search. On the other hand, Bloemen and Stancanelli (2001) and Alexopoulos and Gladden (2004) find that the effect of wealth on transition rates is small in size.

This paper uses a general equilibrium model of frictional unemployment to investigate whether or not wealth matters to the aggregate unemployment rate. More precisely, the aim is to study how the distribution of wealth affect the aggregate unemployment rate through the decisions of individual workers. The focus is on workers' choice of reservation wages, and how it affects the long run, average unemployment rate. The main contribution of this paper is to investigate how heterogeneity in workers' skills and educational attainment can help explain the dispersion of wealth and the behavior of unemployed workers. Because workers have different levels of skills

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[2] See Alexopoloues and Gladden (2002), Algan et al. (2003), Bloemen and Stancanelli (2001) and Chetty (2008).

[3] Krusell et al.(2010) and Shao and Silos (2007) do investigate the impact of risk aversion and missing insurance markets on aggregate labor market outcomes. See the following section.

and education, they behave differently when they are unemployed, even if their asset holdings are the same. Most of the previous general equilibrium search models with endogenous savings feature agents that, once they are unemployed, differ only with respect to their wealth.[4]

2.2. Related Literature

A number of papers have used a setup with frictional labor markets, missing insurance markets and *ad hoc* borrowing constraints to investigate, in general equilibrium, the consequences of various labor market policies. Alvarez and Veracierto (2001) look at the effects of severance payments on aggregate unemployment and welfare; Gomes, Greenwwod and Rebelo (2001) study the consequences of changes in the UI replacement rate and the cost of business cycles; and Alonso-Borrego et al.(2005) use the same basic setup to investigate the effects of temporary labor contracts in Spain. The authors of these studies argue that agents' ability to self-insure through savings need to be modeled explicitly because it is such an important determinant of workers' welfare.

In addition to the assumption of incomplete insurance markets and frictional labor markets, the three models mentioned above all share two important features: (1) workers are *ex ante* identical; and (2), conditional on being unemployed, workers differ only with respect to their wealth holdings. Ljungqvist and Sargent (2007) add to the richness of this structure by introducing human capital dynamics into the model of Alvarez and Veracierto (2001), with the consequence that unemployed agents differ also with respect to their skill level. The main point of Ljungqvist and Sargent (2007) is to show that the interactions of human capital dynamics and public unemployment insurance can create substantial long-term unemployment in the presence of turbulence.

Recently, Krusell et al.(2010) and Shao and Silos (2007) have contributed by systematically comparing the outcomes of a real business cycle model, extended with Diamond-Mortensen-Pissarides (DMP) search and matching frictions, to the outcomes of the same models when they are augmented by risk aversion and missing insurance

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[4] Ljungqvist and Sargent (2007) do analyse an island search model with endogenous savings and human capital dynamics, where unemployed workers have different sills. Their search model is an extension of Alvarez and Veracierto (2001).

markets.[5] Shao and Silos (2007) focus on the business cycle properties of their model, and they conclude that under plausible calibrations of the model, risk aversion and missing insurance markets do not substantially change the dynamics of the labor market over the cycle. Krusell et al.(2010) reach similar conclusions using a more elaborate model. Krusell et al.(2010) also investigate the welfare consequences of UI benefits in their model, and they show that there are important, negative welfare consequences of unemployment benefits that work through the demand for work: higher benefits strengthen workers' bargaining position and thus make it less profitable for firms to create vacancies.

Like this study, Krusell et al.(2010) and Shao and Silos (2007) are interested in the effects of risk aversion and missing insurance markets on aggregate labor market outcomes. Using the DMP setup, these two studies incorporate the effects of workers' wealth on firms' decisions to post vacancies and on wage bargaining. Such effects of wealth on the demand for labor and on wage bargaining are not present in my model, which assumes, in essence, that wages are given by an exogenous distribution. Instead, I focus on the supply of labor through the behavior of individual workers, and their choice of reservation wages. In Krusell et al.(2010) and in Shao and Silos (2007), unemployed workers always accept the first job offer they receive. Their models thus abstract from the job-search behavior of individual workers, and how this behavior is affected by workers' asset positions.

A different strand of the literature tries to jointly account for the distributions of income and wealth. Krusell and Smith (1998) find that income inequality and wealth inequality are hard to reconcile, but argue that the fit of the wealth distribution can be substantially improved upon by allowing for heterogeneity in agents' time preferences. The model of Castañeda et al.(2003) produces realistic levels of both wealth and income inequality. It is uncertain, however, if the income dynamics of their model economy can be reconciled with data.

Lise (2010) makes an interesting contribution to this literature, using a model of on-the-job search and precautionary savings. In a partial equilibrium setting, his model

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[5] See Diamond (1981), Pissarides (1985) and Mortensen and Pissarides (1994).

jointly generates income and wealth distributions that closely resembles those of the 1979 youth cohort of the National Longitudinal Survey of Labor Market Experience (NLSY79). In his model, workers typically start out at jobs paying low wages, and then gradually move up the wage ladder. In the beginning of their careers, agents anticipate higher future wages, and therefore save little. Later on, when they have moved to better paid jobs, they become more and more concerned with the risk of losing their jobs - thereby losing their position high up on the wage ladder - and start saving at considerably higher rates. By this mechanism, the model of Lise (2010) generates substantial wealth inequality from a distribution of income that has realistic features.

Reichling (2007) extends Moretensen and Pissarides (1994) to allow for risk aversion and endogenous savings. Workers and firms bargain over the wage rate and over the number of hours supplied by the worker. In partial equilibrium, Reichling (2010) shows that financially poor workers work unusually long hours to be able to quickly build a buffer stock of wealth. Like most previous models, unemployed workers differ only with respect to their level of assets.

2.3. The Economy

My model builds on the work of Gomes et al.(2001), with two major extensions. First, workers are different with respect to education and skills. Differences in educational attainment are modeled exogenously, with two types of workers: 'educated workers' have high permanent productivity, while 'uneducated workers' have low permanent productivity. In addition to this *ex ante* difference between worker types, I introduce human capital dynamics à la Ljungqvist and Sargent (1998): agents are *ex ante* identical in their skill level, but skills will differ between workers *ex post*, depending on the labor market history of individual workers. The second major extension is the inclusion, in a stylized manner, of life cycle dynamics.

2.3.1. Basic Assumptions. The economy is populated by a continuum of agents, all of which are either workers or retired citizens. The total mass of these agents is normalized to unity. With per-period probability λ an active agent will start out the following period as retired. When this happens, an agent can no longer earn any

labor income, and must therefore live off past savings and government transfers. The
only choice of a retired agent is how much of disposable income she will consume
and how much she will save to the next period. Death occurs to retired agents with
probability κ. Deceased agents are immediately replaced with an offspring who inherits
any assets that are left behind and who enters active life as unemployed. All agents
enjoy consumption and dislike work effort. They maximize the expected, discounted
sum of future utility, with per period utility defined as follows:

$$u(c_t, 1 - l_t) = \frac{\left[\left(c_t - \frac{l_t^{1+\theta}}{1+\theta}\right)\right]^{1-\sigma} - 1}{1 - \sigma}, \qquad \theta > 0, \quad \sigma > 0. \tag{2.1}$$

Here, $(1 - l_t)$ represents the amount of leisure the agent enjoys in period t, σ is the
coefficient of relative risk aversion, and $1/\theta$ designates the elasticity of labor supply.
All agents discount future streams of utility at the constant factor $\beta \in (0, 1)$.

The productivity of working agents depend on the level of their education, the
amount of skills they have accumulated while working and the attractiveness of the job
at which they are currently employed. A worker's educational attainment is determined
before she enters the labour market and does not change during the course of her
working life. There is thus an *ex ante* difference between worker's productivity that is
permanent in nature. On the other hand, workers are *ex ante* identical with respect
to their skill level: all agents enter active life at the lowest possible level of skills. The
skill level of a worker then evolves stochastically as she moves through active life. The
probability of losing and gaining skills varies depending on her employment status.
Finally, workers differ with respect to the attractiveness of the jobs at which they are
employed.

Denote by $(\phi + \gamma_t)$ the level of productivity specific to an individual agent at time
t. Here, ϕ takes on the value ϕ_{nc} if the agent does not have a college education,
and ϕ_c if she does. At any point in time, her skill level γ_t can take on one out of
G different levels, depending on her work history. Following Ljungqvist and Sargent
(1998), I assume that skills evolve differently for employed and for unemployed agents,

so that $H_e(\gamma, \gamma') = prob\left\{\gamma_{t+1} \leq \gamma' \,|\gamma_t = \gamma\right\}$ represents the distribution function of γ conditional on employment in period t. If an agent is unemployed in period t, her skill level in period $(t+1)$ will be distributed according to $H_u(\gamma, \gamma')$. At the beginning of each period, active agents have at hand a job opportunity that is characterized by an idiosyncratic level of productivity, ε_t. ε_t is the realization of a continuous random variable whose support is the set of real numbers. The job opportunity allows the agent to work in the present period with production technology O:

$$O(k_t, l_t; \varepsilon_t + \phi + \gamma_t) = \exp(\varepsilon_t + \phi + \gamma_t)k_t^{\alpha}l_t^{1-\alpha}. \tag{2.2}$$

where $\alpha \epsilon (0, 1)$ and where l_t and k_t are, respectively, the inputs of labor and capital. A worker supplies her own labor to the job and rent capital from a competitive spot market. There is one homogenous type of good, which can be used either for consumption or as capital in production. When the good is used as capital, the rate of depreciation is δ.

When a new period begins, active agents observe their own productivity $(\phi + \gamma_t)$ and that of the job currently at hand, ε_t. Given their level of savings, they then decide whether to work at the available job or to become unemployed and search for a new job. If an agent chooses to work, she rents capital in the competitive capital market at rental rate $(r + \delta)$. Agents who decide to work choose optimal levels of k_t and l_t, and they pay for the capital used in production and are subject to a per period flat tax rate on labor income, τ. Output net of rental and tax payments is divided between consumption and savings. In the following period, a new realization of the job-specific productivity, ε_{t+1}, is drawn from the distribution $I(\varepsilon, \varepsilon')$. Together with a new realization of the individual productivity, $(\phi + \gamma_{t+1})$, ε_{t+1} will determine the attractiveness to the agent of keeping the job. If the agent decides not to retain the job, she becomes unemployed in period $t+1$ and can then start to search for new jobs.

Agents who decide not to work in period t are considered to be unemployed in that period. This means they have no income besides the interest on their savings

and the unemployment benefits. Unemployed agents pay no taxes. Like other agents, they decide how much to consume and how much to save. After each period of search, unemployed agents receive a new job opportunity with the level of productivity, ε_{t+1}, drawn from $J(\varepsilon)$. Just like employed agents, the unemployed then decide either to retain the new job opportunity and to work or to remain unemployed for one more period, waiting to draw a new productivity ε_{t+2} in the period after that. The search technology specified here follows Gomes et al.(2001). Unemployed agents cannot decide how much search effort to exert; their only choice in terms of a search strategy is an optimal reservation productivity.

Strictly speaking, agents in this economy are self-employed; they make decisions concerning a reservation productivity, not a reservation wage. I regard this mostly as a convenient abstraction, however, I and will make no distinction between a worker's 'reservation productivity' and the associated 'reservation wage', which is the labor earnings associated with that level of productivity.

The government pays unemployment benefits as a lump sum transfer b to all unemployed agents in every period. The government also runs a pay-as-you-go pension scheme, with a transfer s paid to all retired agents in all periods. These undertakings are financed by a proportional income tax τ levied on all labor earnings. In every period, the government balances its budget.

2.3.2. Equilibrium. Define $Y_j(\varepsilon, \gamma)$ as the income net of rental and tax payments and of the disutility of work of an employed agent whose permanent productivity is ϕ_j:

$$Y_j(\varepsilon, \gamma) = \max_{k,l} \left\{ (1 - \tau) \left[O(k, l; \varepsilon + \phi_j + \gamma) - (r + \delta)k \right] - D(l) \right\}. \qquad (2.3)$$

where $D(l) = \frac{l^{1+\theta}}{1+\theta}$. Further, let $\widetilde{c}$ refer to an agents' consumption net of the disutility of work. This notation is convenient since the agent's optimization over k and l is independent of her intertemporal optimization over c and a'. The functional form of the per-period utility implies that agents make no distinction between units of consumption and utils derived from leisure. Workers are thus indifferent between two consumption baskets with the same level of $\widetilde{c}$ but with different compositions of consumption, c, and leisure. For future reference, let $K_j(\varepsilon, \gamma)$ and $L_j(\varepsilon, \gamma)$ be the policy functions for capital and labor inputs that solve 2.3, given that the agent's permanent productivity is ϕ_j. In order to keep track of tax payments to the government, I also define $\bar{Y}_j(\varepsilon, \gamma)$ to be the income, net of rental payments, of a working agent with permanent productivity ϕ_j:

$$\bar{Y}_j(\varepsilon, \gamma) = O \left\{ K_j(\varepsilon, \gamma), L_j(\varepsilon, \gamma); \varepsilon + \phi_j + \gamma \right\} - (r + \delta) K_j(\varepsilon, \gamma). \qquad (2.4)$$

Now, let $W^j(a, \varepsilon, \gamma)$ be the value function of a working agent whose permanent, individual productivity is $(\phi_j + \gamma)$, who has at hand a job opportunity with productivity level ε and whose level of savings is a. Further, let $S^j(a, \gamma)$ be the value function of an unemployed worker and denote by $R(a)$ the value function of a retired agent.

The worker's problem is then:

$$W^j(a, \varepsilon, \gamma) = \max_{\widetilde{c}, a'} \Big\{ U(\widetilde{c}) + \lambda \beta R(a') + \tag{2.5}$$

$$(1 - \lambda)\beta \int \sum_{\gamma'} \max \left[W^j(a', \varepsilon', \gamma'), S^j(a', \gamma') \right] h_e(\gamma' \,|\gamma) dI(\varepsilon, \varepsilon') \Big\}$$

s.t.

$$\widetilde{c} + a' = Y_j(\varepsilon, \gamma) + (1 + r)a,$$

$$a' \geq \bar{a},$$

where $h_e(\gamma' \,|\gamma)$ is the conditional probability of γ', given γ, and where $\bar{a}$ is a borrowing limit. The searcher's problem is:

$$S^j(a, \gamma) = \max_{\widetilde{c}, a'} \Big\{ U(\widetilde{c}) + \lambda \beta R(a') + \tag{2.6}$$

$$(1 - \lambda)\beta \int \sum_{\gamma'} \max \left[W^j(a', \varepsilon', \gamma'), S^j(a', \gamma') \right] h_u(\gamma' \,|\gamma) dJ(\varepsilon') \Big\}$$

s.t.

$$\widetilde{c} + a' = (1 + r)a + b,$$

$$a' \geq \bar{a}.$$

Finally, the recursive problem of a retired agent reads:

$$R(a) = \max_{c,a'} \left\{ U(c) + (1 - \kappa)\beta R(a') \right\} \tag{2.7}$$

s.t.

$$c + a' = (1 + r)a + s,$$

$$a' \geq \bar{a}.$$

Part of the solution to 2.5 is an optimal policy function for savings, $a' = A_j^W(a, \varepsilon, \gamma)$. Similarly, the policy functions that solve the searcher's and the retiree's programs are $A_j^S(a, \gamma)$ and $A^R(a)$, respectively. In the beginning of a period, an agent with permanent productivity ϕ_j and state (a, ε, γ) will choose to work if $W^j(a, \varepsilon, \gamma) \geq S^j(a, \gamma)$ and will search otherwise. The optimal policy of an active agent with respect to the employment decision will be represented by the function $\Omega_j(a, \varepsilon, \gamma)$, where:

$$\Omega_j(a, \varepsilon, \gamma) = \begin{cases} 1 \text{ if } W^j(a, \varepsilon, \gamma) \geq S^j(a, \gamma) \\ 0 \text{ otherwise.} \end{cases} \tag{2.8}$$

Given the employment policy Ω of active agents, an optimal savings policy for active agents, $A_j(a, \varepsilon, \gamma)$, reads:

$$A_j(a, \varepsilon, \gamma) = \Omega_j(a, \varepsilon, \gamma) A_j^W(a, \varepsilon, \gamma) + \left[1 - \Omega_j(a, \varepsilon, \gamma) \right] A_j^S(a, \gamma).$$

In a steady state equilibrium, the distribution of agents across different states of productivity and wealth is time invariant. Define $z_j^A(a, \varepsilon, \gamma)$ and $z^R(a)$ to be measures of active and retired agents over the state space. $z_j^A(a, \varepsilon, \gamma)$ is then the measure of active

agents with permanent productivity ϕ_j who enjoy the transitory productivity $(\varepsilon + \gamma)$ and who holds a units of savings. With these definitions, the market clearing condition of the capital market reads:

$$\int \left\{ \sum_j \sum_\phi z_j^A(a, \varepsilon, \gamma) A_j(a, \varepsilon, \gamma) + z^R(a) A^R(a) - \right. \tag{2.9}$$

$$\left. \sum_j \sum_\phi \Omega_j(a, \varepsilon, \gamma) z_j^A(a, \varepsilon, \gamma) K_j(\varepsilon, \gamma) \right\} da d\varepsilon = 0.$$

The government runs a balanced budget, with tax receipts exactly offsetting the payments to the unemployed and the retired. A balanced policy $\{b, s, \tau\}$ satisfies:

$$\tau \int \sum_j \sum_\phi \Omega_j(a, \varepsilon, \gamma) z_j^A(a, \varepsilon, \gamma) \bar{Y}_j(\varepsilon, \gamma) da d\varepsilon = \tag{2.10}$$

$$\int \left\{ \sum_j \sum_\phi [1 - \Omega_j(a, \varepsilon, \gamma)] \, b z_j^A(a, \varepsilon, \gamma) + s z^R(a) \right\} da d\varepsilon.$$

A *recursive equilibrium* consists of:

1) A collection of value functions $\{W^j(a, \varepsilon, \gamma), S^j(a, \gamma)\}_{j \in \{nc,c\}}$ and $R(a)$, and associated policy functions such that:

$W^j(a, \varepsilon, \gamma)$ and $A_j^W(a, \varepsilon, \gamma)$ solves 2.5 for $j = nc, c$.

$S^j(a, \gamma)$ and $A_j^S(a, \gamma)$ solves 2.6 for $j = nc, c$.

$R(a)$ and $A^R(a)$ solves 2.7.

2) A policy function $\Omega_j(a, \varepsilon, \gamma)$ as defined in (2.8).

3) An interest rate r such that (2.9) holds.

4) A government policy $\{b, s, \tau\}$ such that (2.10) is satisfied.

The model is calibrated to American data and solved numerically. The calibration is outlined and motivated in the following section, and a brief explanation of the solution method can be found in the appendix.

2.4. Calibration

The functional form of agents' utility and several parameters of that function are the same as in Gomes et al.(2001). Specifically, the coefficient of relative risk aversion, σ, is set to 2 and the elasticity of labour supply, $\frac{1}{\theta}$, to 0.1. Further, a time period of the model is half a quarter. The parameters governing the life cycle dynamics (λ and κ) are fixed so as to make the average agent be active for 50 years and retired for 20 years. The addition to the model of life cycle dynamics and a relatively generous public pension scheme has the effect, in equilibrium, to increase the interest rate. In order to avoid an unrealistically high equilibrium interest rate, I therefore set the subjective discount rate, β, to 0.9950 instead of 0.9927, which is the value chosen by Gomes et al.(2001).

2.4.1. Human Capital and Technology. According to OECD (2004), 38% of American workers held some form of college education in 2002. In the calibrated model, then, 38% of all active agents are assigned the higher of the two values of permanent productivity, ϕ_c. In what follows, these agents will be referred to as 'workers with college' or 'educated workers'. Furthermore, in 2003, workers with tertiary education earned an hourly wage that on average was 70% higher than workers without such education (OECD 2005). Assuming that the two different values of ϕ are symmetric around zero, this statistic identifies the two values of ϕ.

Turning to the transitory component of worker-specific productivity, γ, this variable evolves over time as the individual worker gains experience and suffers spells of unemployment. Here, the calibration of γ roughly follows that of the corresponding skill variable in Ljungqvist and Sargent (1998). In particular, it is assumed a) that an agent with the highest (transitory) skill level earns exactly twice as much as an

agent with the lowest skill level, and b) that working agents face a certain probability that their skills will appreciate, while unemployed agents are subject to stochastic skill losses.[6]

The last part of a worker's productivity state, ε, is specific to the job opportunity that the agent currently has at hand. For employed agents, ε, is assumed to be determined as the outcome of an AR(1) process:

$$\varepsilon' = \rho\varepsilon + \xi, \qquad \xi \sim N(0, \sigma_\varepsilon^2) \quad and \quad \rho \in (0, 1). \tag{2.11}$$

For searching agents, on the other hand, ε is determined as a random draw from a normal distribution:

$$\varepsilon = \upsilon, \qquad \upsilon \sim N(\mu_\nu, \sigma_\upsilon^2). \tag{2.12}$$

For comparability, the benchmark calibration has the same value of ρ, 0.9, as Gomes et al.(2001). In that study, σ_ε^2 and σ_υ^2 are chosen so as to make the model match the average rate of unemployment in the U.S. (5,9%) and the average duration of unemployment spells (13 weeks). Gomes et al. (2001) report that with $\sigma_\varepsilon = 0.052$ and $\sigma_\upsilon = 0.085$, the corresponding unemployment rate and duration of the model are 6,1% and 11 weeks, respectively. Again striving for comparability, I calibrate σ_ε^2 and σ_υ^2 so as to match an unemployment rate of 5,9% and a duration of 11 weeks. In all calibrations reported in this paper, changes in σ_ε^2 and σ_υ^2 have the same qualitative effect on the

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[6] Ljungqvist and Sargent (1998) focus their analysis on the interaction of human capital accumulation and unemployemnt insurance. They therefore allow for a fine grid of 21 different skill levels. In the present model, where much analytical effort is spent on precautionary savings, the grid for transitory skill levels must necessarily be much coarser. Thus, γ can take one out of three different values. The chosen transition probabilities imply that an employed agent, who suffers no unemployment, is expected to advance from the lowest to the highest skill level in ten years. When unemployed, the rate of skill depreciation is twice as fast. (In Ljungqvist and Sargent (1998), it takes on average 7 years and 8 months for a worker to double her earnings ability. Skill depreciation is twice as fast.)

model's unemployment rate and duration. An increase in the variance of the shocks to a worker's productivity, σ_ε^2, produces an increase in the unemployment rate and a decrease in the average duration of spells. Intuitively speaking, an increase in σ_ε^2 makes it more likely that a worker's job-specific productivity will suffer an unfavorable shock, thereby making it more likely that agents will quit their jobs and start searching. The upshot is an increased flow of workers into unemployment. The increased volatility of wages also affect the search policy of unemployed agents, since it decreases the incentive to wait for a good wage offer. An increase in σ_ε^2 therefore induce agents to lower their reservation productivity, thereby lowering the expected duration of each spell.

An increase in the variance of the job-offers received by searchers, σ_υ^2, also induces an increase in the unemployment rate, but such a change also causes an *increase* in the average duration. This result is a well known feature of search models: increasing the dispersion of the wage offers received by unemployed agents increases the benefit of continued search, inducing agents to become more picky about which jobs to accept.

While rather small changes in σ_ε^2 and σ_υ^2 bring about large effects on the unemployment rate, it takes considerably larger changes in these two parameters to achieve a similar percentage change in the duration of unemployment. This is the reason I target 11 weeks of unemployment duration instead of 13 weeks. If the target of the calibration exercise would be to achieve 13 weeks, the parametrization of shocks would be very different from a calibration with 11 weeks of duration.

2.4.2. Government Policy. A government policy consists of: 1) the transfer to the unemployed, b; 2) the transfer to the retired agents, s; and 3) a tax rate τ on labor income. Given the focus on steady states with a balanced budget, it is only the level of the two transfers that will have to be calibrated; it will then be the duty of the solution algorithm to find a corresponding tax rate.

In a study of the wealth holdings of unemployed American workers, Gruber (2001) uses a sample consisting of all spells of unemployment between 1984 and 1992 that are recorded in the Survey of Income and Program Participation. Using a simulation program to proxy for UI eligibility among sample households, the author finds that the

Variable		Value
β	Subjective Discount Rate	0,995
σ	Relative Risk Aversion	2
$1/\theta$	Elasticity of Labor Supply	0,10
λ	Probability of Retirement	0,0025
κ	Probability of Death	0,00625
Φ	Support of Permanent Productivity	[-0,1544 0,1544]
Γ	Support of Transitory Skills	[-0,2016 0,0343 0,2016]
ρ	Persistence in Shocks to Worker Prod.	0,9
σ_ε	Std of Shocks to Worker Prod.	0,0235
σ_υ	Std of Searcher's Prod. Distr.	0,2180
b/w^{NC}	Replacement Rate UI	30%
s/w	Replacement Rate Social Security	45%

w refers to the average wage rate, net of taxes, of all working agents, while w^{NC} refers to the average wage rate of working agents who do not have a college education.

Table 1: Calibration Summary

average worker receives benefits amounting to roughly 45% of the previously held wage rate. However, because workers with low educational attainment face higher unemployment rates than do more educated workers, there are reasons to believe that this number overstates the replacement rate faced by the average American worker (OECD 2005). The sample used in Gruber (2001) is representative of actually unemployed workers, while here we are looking for the average replacement rate of agents in the workforce. Furthermore, most American states limit the number of weeks that benefits can be collected to 26. Because agents in the model can collect benefits for an unlimited number of periods, the model UI system is considerably more generous than it's real world counterpart, for any given replacement rate. Based on these considerations, I set the replacement rate of the benchmark calibration to 30% of the average wage rate of uneducated workers. Transfers to the retired citizens, on the other hand, are defined as a fraction of the average wage of all workers. Following Eisensee (2006), this fraction is set to 45%.[7] The benchmark calibration is summarized in table 1.

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[7] Concerning both the unemployment insurance and the pension system, the calibrated replacement rates refer to the average wage rate net of taxes and rental payments to capital owners.

Variable	Model	Data
Replacement Rate UI	30%	45%[1]
Social Security	45%	45%[2]
Tax Rate	13%	23%[3]
Interest Rate (yearly)	4.9%	
Capital-to-Output Ratio (yearly)	3.7	3,3[4]
Mean-to-Median Ratio of Wealth	1.6	4,03[5]
Gini Coefficient of Wealth	0.47	0,80[5]
Unemployment Rate	6.2%	5,9%[6]
Unemployment Rate No College	7.1%	6,9%[7]
Unemployment Rate College	4.8%	3,4%[7]
Average Duration (weeks)	11	13[6]

[1]Gruber (2000).
[2]Eisensee (2006).
[3]Carey and Tchilinguirian (2000).
[4]Cooley and Prescott (1995).
[5]Rodríguez et al. (2002).
[6]Gomes et al. (2001).
[7]OECD (2005).

Table 2: Benchmark (American) Equilibrium

2.5. Results

The main aggregate variables of the benchmark economy are presented in Table 2. The model make predictions concerning the distribution of wealth and the unemployment rates specific to uneducated and educated workers, predictions that are well suited to evaluate the models ability to match statistics in the data. In order to investigate these predictions and to explore the economic forces at work in the model, I will take some time out to discuss, in turn, the distribution of wealth and the incentives of workers. Following that, I present an out-of-equilibrium exercise that quantify the effect of the wealth distribution on the aggregate unemployment rate.

2.5.1. The Distribution of Wealth. Not surprisingly, increased heterogeneity in workers' productivity produces increased dispersion in wealth. While the Gini coefficient of wealth in Gomes et al.(2001) is 0.38, the corresponding statistic in the benchmark economy is 0.47. Even so, the dispersion in the model economy is still far

	1st	2nd	3d	4th	5th
U.S.	-0,30	1,3	5,0	12	82
Model	0,35	4,6	13	25	57

The first column shows the fraction of total wealth
held by the poorest 20% of households, etc.
The numbers in the first row are taken from table 7
in Burrida-Rodríguez et al. (2002).

Table 3: Wealth Distribution by Quintiles

from the Gini coefficient of 0.80 that characterizes the U.S. distribution of wealth. In
Table 3, the American and model wealth distributions are decomposed by quintiles.

Behind the statistics of the aggregate wealth distributions, there exist important differ-
ences across the different educational and skill groups. The average white collar worker
in the model is twice as wealthy as the average blue collar worker, and the difference in
median wealth is even larger: the median wealth of white collar workers is larger than
that of blue collar workers by a factor of 2.2. Differences of similar magnitudes appear
when the wealth distribution is decomposed by skills. Table 4 presents the distribution
of active agents across the three different skill levels, as well as a decomposition of
wealth by skills.

The upper part of Table 4 reveals small but non-negligible differences in skills between
the two groups, which might lead one to expect that the difference in average earnings
between the two groups would be larger than the 70% targeted in the calibration. It
turns out, however, that the average labor income of white collar workers is 66% higher
than that of blue collar workers, indicating that the average job-specific productivity
of blue collar workers is actually higher than that of educated workers. As will become
obvious in the next subsection, blue collar workers have better average job-specific
productivity because they are more picky about which jobs they accept. Why, then, do
white collar workers build twice as large assets as blue collar workers, when the average
earnings difference is only about 70%? The reason lies in the redistributive character of
the government policies, which fixes the transfers to all unemployed and retired agents
at the same absolute levels, b and s. Because the effective replacement rates of the two

Distribution of Skills

(Percentage of Total by Educational Attainment.)

Skill Level:	1	2	3	
no college	15,1	19,9	65,0	100
college	14,7	16,8	68,8	100

Median Wealth Holdings by Skill Group

(Percentage of the Median Wealth of All Active Agents.)

Skill Level:	1	2	3
no college	15,0	41,6	91,6
college	19.1	67,3	218
all	16,7	48,6	127

The upper pane presents the share of workers with different skill levels,
by educational attainment. The lower pane presents the median wealth
of workers in different skill groups, as a percentage of the median
wealth of all working agents.

Table 4: Skills and Wealth.

Length of spell (in model periods)	1	2	3	4	5	6	7	≥ 8
Median wealth	25.7	25.4	25.0	23.8	23.6	23.3	23.0	22.8

Table 5: Median Wealth of Unemployed Workers by Spell Duration. Wealth holdings were measured in the period preceding the start of the spell, and were normalized by the avregae, monthly wage, net of taxes.

government programs are lower for educated than for non-educated workers, educated workers have a stronger incentive to build savings in order to smooth consumption between states of work, unemployment and retirement. For the same reasons, there are large differences in wealth holdings across workers with different skills. All workers start their working life with low skills, and then gradually accumulate skills during their employment spells. When agents find themselves in a state of high skills, they increase their saving rates in order to build a buffer stock of savings that can be used in case of unemployment, loss of skills or retirement.

Another interesting aspect of the wealth distribution is the unconditional correlation between the wealth holdings of unemployed workers, and the duration of their spell.

Gruber (2001) reports that in the cross section of American unemployment spells, median wealth falls monotonically with the length of the unemployment spell, from 1320$ for those with a spell equal to or shorter than 1 month, to 676$ for workers with a spell greater than 12 months.[8] Table 5 shows the median wealth holdings of unemployed agents in the period prior to the start of the spell, by duration of the spell. In the table, median wealth was normalized by the average, monthly wage, net of taxes. There are two things to note about this result. First, the model is able to rationalize the negative, unconditional correlation of wealth and spell duration observed by Gruber (2001). This happens even though individual workers in the model set reservation wages that increase in the level of their wealth. The second thing to note about Table 5 is the high ratio of median wealth to the average wage. It is obvious that workers in the model economy hold much more liquid assets, in relation to their income, than do American workers.

2.5.2. Incentives to Work and to Search. As reported in table 2, the model does a reasonable job at predicting the unemployment rates specific to workers with and without education. For blue collar workers, the benchmark calibration produces an unemployment rate of 7,1%, while that of white collar workers is 4,8%. In 2003, the unemployment rates of American workers without and with tertiary education stood at 6.9% and 3.4%, respectively.[9] Note that while the model was calibrated to yield an aggregate unemployment rate of 6%, the jobless rates specific to the two educational groups where not targeted in the calibration. Why does the model produce different unemployment rates for these two groups of workers? The answer, again, lies in the design of government policy. All unemployed workers receive a per-period transfer of

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[8]   Gruber (2001) reports the median real wealth holdings, in 1994 dollars, of the household of the unemployed, by duration of the spell. Wealth is defined as 'groos financial wealth' and information on the houshold's asset positions are collected at the interwiev that preceded the beginning of the unemployment spell.

[9]   OECD (2005) reports American unemployment rates, for the year 2003, by three groups of educational attainment: lower secondary education or less, upper secondary education and tertiary education. The first two of these groups are identified as uneducated workers in the model, and the group with tertiary education is identified as educated workers. In order to obtain an unemployment rate for the whole group of uneducated workers, the unemployment rates of the first two groups of workers in OECD (2005) are weighted by the shares of the population with the corresponding educational attainments, reported for the year 2002 in OECD (2004).

b, irrespective of their previous earnings history. As a consequence, educated workers face a lower effective replacement rate than do blue-collar workers, implying that the opportunity cost of search is greater for educated workers. In agents' policy functions, this difference in opportunity costs translates into differences in reservation productivity, leading educated workers to accept jobs with lower job-specific productivity, ε. Because white collar workers have higher permanent productivity, they nevertheless receive higher wages than do blue collar workers.

Obviously, the design of government policy in the model is a simplification of the policies put in place in the United States, where the level of benefits paid to an individual worker depends on her previous income. Even so, the policy of the model economy does retain one important feature of it's real world counterpart: all public unemployment schemes in the U.S. contain caps on their benefit levels, implying that benefits raise with previous income only to the level of the cap. Therefore, American workers with moderately high and high wages do face lower effective replacement rates than do low-income workers.

Summarizing the differences in behavior that pertain to educated and non-educated workers, we have seen that white collar workers have higher saving rates and that they set lower reservation productivity than do blue-collar workers. The differences in incentives that arise because workers have different skill levels are somewhat more involved, owing to the dynamic character of the skill variable.

Figure 1 displays the hazard rates out of unemployment as a function of agents' wealth.

The horizontal axis represent workers' savings, with units normalized to the average, monthly earnings net of taxes.[10] The vertical axis represent the per period probability of accepting a job. These hazard rates are determined jointly by agents' reservation productivity policies, and the distribution of jobs to the unemployed (as defined by 2.12). The same reservation productivity policies, combined with the distribution of shocks to

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[10]   This normalization was done separately for each of the two educational groups. Thus, one unit on the horizontal axis of the upper pane in Figure 1 corresponds to 3.1 units of the good, which is the average, monthly earnings, net of taxes, for blue collar workers. One unit on the horizontal axis of the lower pane represents 5.2 units of the good, which is equal to the average, monthly earnings, net of taxes, of white collar workers.
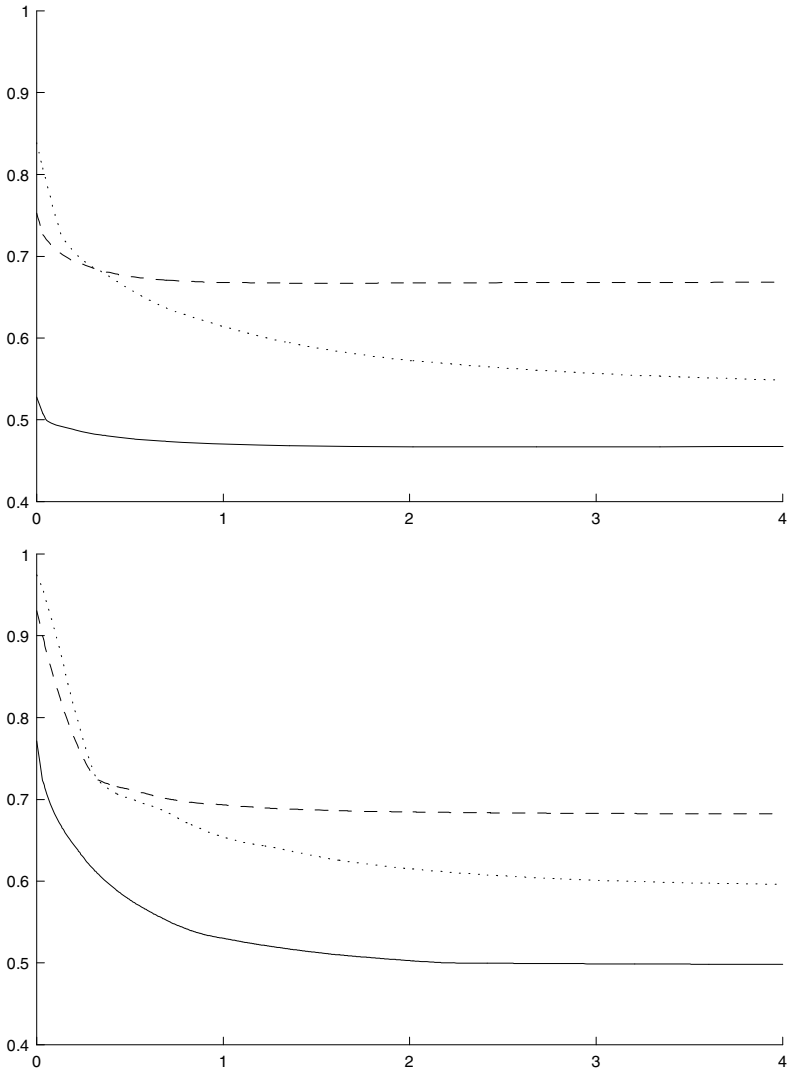
Figure 2.1: Hazard rates as functions of wealth. The upper pane shows the hazard rates of blue collar workers, while the lower pane refers to white collar workers. Solid lines refer to workers with the lowest skill level, and dashed lines represent workers at the intermediate skill level. Highly skilled workers are represented by dotted lines. The scale on the horizontal axes was normalized by the average, monthly wage, net of taxes, of the respective educational group.

employed agents (as defined by 2.11) determine the flow of workers into unemployment. First, let us focus on the hazard rate of a worker with a given level of assets, and see how her incentives change depending on her skill level. Disregarding the very bottom of the wealth distribution, Figure 1 reveals an inverted, v-shaped relationship between the hazard rate of an unemployed worker and her skill level. Low-skilled workers set the highest reservation productivity, translating into low hazard rates. Workers with intermediate skills set the lowest reservation productivity, while highly skilled workers fix their reservation productivity at an intermediate level. This non-monotonic relationship between skills and reservation wage policies was first discussed by Ljungqvist and Sargent (1998). As in their model, there are two opposing economic forces at work in this model: the incentive of workers to accumulate skills, and the incentive to find a job with a high wage rate. At the lowest skill level, workers need not fear that they loose skills, and therefore the incentive to find a job with a high productivity dominates. At the intermediate level, agents care more about preserving the skills they have and about gaining new ones, and they accordingly set relatively low reservation productivities. At the highest level of skills, the incentive to find a job which pays again dominates.

When agents choose how to vary their reservation productivity with their level of assets, a third economic force comes into play, namely the desire to use savings as a buffer stock against income fluctuations. In a McCall search model with declining absolute risk aversion and incomplete markets, an unemployed agent's reservation wage always increase in the level of her wealth (Danforth [1979]). The intuition for this result is straightforward. A higher reservation wage is an investment in a higher expected, future wage. The cost of this investment is a longer expected duration of the unemployment spell, something which increases the expected income loss associated with that spell. More wealthy individuals chose to invest more in job search than do less wealthy agents. The same mechanism is active in this model, although here the worker chooses a reservation productivity rather than a reservation wage: Figure 1 reveals that hazard rates decrease in savings, as workers become more picky about wage offers

the more wealthy they are.[11] The figure also show, however, that the strength of this effect varies considerably with the level of assets at hand. Already at relatively low levels of savings, equivalent to two monthly wages, the elasticity of the hazard with respect to wealth is visibly quite low, for most groups of workers. But close to the borrowing constraint, hazard rates decline sharply in wealth, indicating that workers wage demands increase steeply in the level of their savings. At very low wealth levels, the liquidity effect is strong enough to alter the ordering of hazard rates among workers with different skill levels. The v-shaped relationship between reservation wages and skills disappears and hazard rates instead increase monotonically in agents' skill level. In this state, the effective replacement rate of the UI system dominates other forces and determines the ordering of hazard rates: workers with the highest skill level face the lowest effective replacement rate, while workers with the lowest skill level receive a relatively generous unemployment benefit.

2.5.3. The Quantitative Importance of Wealth to Aggregate Unemployment.

What is the effect of agents' wealth holdings on the aggregate unemployment rate? What we would like to know, more precisely, is how the equilibrium unemployment rate would change if agents' positions in the wealth distribution did not affect their decision to work or to search. To answer this question, I perform an out-of-equilibrium simulation of the economy, where all agents are forced to make this decision as if they held the median level of wealth. One interpretation of this exercise is that it collapses the benchmark equilibrium to a model with a representative level of wealth. However, since the model contains two types of agents, workers with and without education, each agent will be forced to set her reservation productivity as if she held the median level of wealth of her type.[12]

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[11] Lundvall (2010) shows that when the McCall search model is augmented with skill dynamics, reservation wages may both increase and decrease in wealth, depending on the state of the unemployed agent. Also in this model, reservation productivities may decrease in agents' wealth. However, because of the relatively low UI replacement rate adopted in the calibration, reservation productivities that decrease in wealth do not play an important role in this equilibrium.

[12] The statistics reported in this section are based on a synthetic sample of 1600 agents simulated during 12000 model periods.

Unemployment Rates (percentage)		
All Active Agents	No College	College
Benchmark Equilibrium 6.24	7.11	4.79
Forced Decisions 6.28	7.11	4.88

Table 6: Out-of-Equilibrium Simulation with Forced Decisions, Compared to Becnhmark Equilibrium.

As can be seen in Table 5, the net effect of the distribution of wealth on the aggregate unemployment rate is negative and very small. In the out-of-equilibrium simulation, the aggregate unemployment rate is 6.28%, relative to 6.24% in the benchmark. A decomposition by educational groups reveals that for blue collar workers, there is no measurable effect of wealth dispersion on the unemployment rate. Instead, the difference in aggregate unemployment rates between the two simulations comes solely from the group of educated workers, whose unemployment rate increases from 4.79% to 4.88%. How come that wealth dispersion does not play a bigger role in determining workers' decisions? Figure 1 did show that agents that are close to the borrowing constraint make drastic changes to their reservation wages. The answer to this question lies in the joint distribution of savings and skills. Inspecting the distribution of savings of the two educational groups, it turns out that only 2.7% of all blue collar workers have savings that are smaller than the average monthly earning of all blue collar workers. For white collar workers, the corresponding share is 1.3%. Looking back at Figure 1, it is evident that few agents find themselves in a state (of low savings) where their level of savings is an important determinant in their choice of reservation productivity. Furthermore, of these few agents that have savings smaller than one month of average earnings (of their respective group), 88% are at the lowest skill level. As an implication, most of the effects of wealth on the aggregate unemployment rate can be understood by looking at the two hazard rates in Figure 1 that pertain to workers at the lowest skill level. For low-skilled, white collar workers who are close to the borrowing constraint, a relatively modest change in wealth can have a considerable impact on the per period probability of accepting a job. For example, if such a worker has savings equal

to one month of the average wage (of white collar workers), the per period probability of accepting a job is 0.53. If the same worker runs her savings down to zero, the per period probability of accepting a job will jump to 0.77. For a low-skilled blue collar worker with savings equal to one average monthly wage, the per period hazard is 0.47. If all savings are run down, the hazard rate increases to 0.53. To understand these differences in behavior, it is instructive to look at the average, effective replacement rate of the UI system for these two categories of workers. Among low-skilled blue collar workers, the average monthly wage, net of taxes, is 2.91 units of the good. With the UI benefit, b, fixed at 1.41, the average effective replacement rate for this group of agents is approximately 48%. For low-skilled white collar workers, the average wage is 4.80, which makes for an average, approximate effective replacement rate of 29%. For blue collar workers without skills, the UI system is relatively generous, and it therefore attenuates the role of 'self-insurance'. For unskilled white collar workers, the public UI system affords considerably less insurance relative to the expected loss of income, and the importance of own savings for shaping the decisions of these workers is therefore greater.

2.6. Concluding Remarks

This study has introduced heterogeneity in workers' education and skills in order to study, in general equilibrium, how the distribution of wealth among workers affect their choice of reservation wages, and, by implication, the aggregate unemployment rate. The model incorporates two government programs: a public UI benefit system and a pay-as-you-go pension system. The interaction of these programs with worker heterogeneity creates several realistic predictions. Thus, white collar workers are less likely to be unemployed than are blue collar workers, and they accumulate more assets in comparison to their labor income. Although individual workers set higher reservation wages the higher are their savings, the aggregate, unconditional correlation between wealth and unemployment duration is negative. Furthermore, heterogeneity in workers skills and educational attainment may help to explain dispersion in wealth, although

the wealth dispersion of the model economy still falls short of the extreme levels of wealth inequality observed in the U.S.

An out-of-equilibrium exercise shows that the effect of risk aversion and imperfect insurance on the aggregate unemployment rate is negative but small. However, inspection of the equilibrium wealth distribution also shows that the level of wealth held by unemployed agents is unrealistically high, at least when compared to observed measures of liquid wealth holdings. This result suggests a tension between the aim of the study, which is to understand the impact of workers' wealth on the aggregate unemployment rate, and the basic model setup. In the model economy, all wealth is perfectly liquid and can be used to smooth consumption between states of employment and unemployment. In real life, much of private sector wealth is held in highly illiquid forms, such as in private houses and commercial constructions, and in business equipment and machines. It thus seems that the level of abstraction of the model's capital market makes it hard to adequately measure the effects of wealth on labor market outcomes. Indeed, there is an important lesson to be drawn from an inspection of the policy functions of unemployed agents in the model: when unemployed workers decide on a job-search strategy, wealth matters quantitatively only for workers with very little savings.

References

Alexopoulos, M and T Gladden (2004), *The Effects of Wealth, and Unemployment Benefits on Search Behavior and Labor Market Transitions* (Mimeo, University of Toronto).

Algan, Y, A Cheron, J O Hairault and F Langot (2003), Wealth Effects on Labor Market Transitions, *Review of Economic Dynamics* **6**, 156-178.

Alonso-Borrego, C, J Fernández-Villaverde and J E Galdón-Sánchez (2005), *Evaluating Labor Market Reforms: A General Equilibrium Approach*, NBER Working Paper 11519.

Alvarez, F and M Veracierto (2001), Severance payments in an economy with frictions, *Journal of Monetary Economics* **47**, 477-498.

Bloemen, H G and E Stancanelli (2001), Individual Wealth, Reservation Wages, and Transitions into Employment, *Journal of Labor Economics* **19**, 400-439.

Carey, D and H Tchilinguirian (2000), *Average effective tax rates on capital, labour and consumption*, OECD Working Paper 258.

Castanẽda, A, J Díaz-Giménez and J-V Ríos-Rull (2003), Accounting for the U.S. Earnings and Wealth Inequality, *Journal of Political Economy* **111**, 818-857.

Chetty, R (2008), Moral Hazard vs. Liquidity and Optimal Unemployment Insurance, *Journal of Political Economy* **116**, 173-234.

Cooley, T F and E C Prescott (1995), Economic Growth and Business Cycles, in T F Cooley (ed.), *Frontiers of Business Cycle Research*, Princeton University Press.

Danforth, J P (1979), On the role of consumption and decreasing absolute risk aversion in the theory of job search, in S A Lippman and J J McCall (eds.), *Studies in the Economics of Search*, North Holland.

Diamond, P A (1981), Mobility Costs, Frictional Unemployment, and Efficiency, *Journal of Political Economy* **89**, 798-813.

Eisensee, T (2006), *Essays on Public Finance: Retirement Behavior and Disaster Relief*, Stockholm University.

Gomes, J, J Greenwood and S Rebelo (2001), Equilibrium Unemployment, *Journal of Monetary Economics* **48**, 109-142.

Gruber, J (2001), The Wealth of the Unemployed, *Industrial and Labor Relations Review* **55**, 79-94.

Krusell, P and A Smith, Jr (1998), Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy* **106**, 867-896.

Krusell, P, T Mukoyama and A Sahin (2010), Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations, *Review of Economic Studies* **77**, 1477-1507.

Lentz, R and T Tranas (2005), Job Search and Savings: Wealth Effects and Duration Dependence, *Journal of Labor Economics* **3**, 467-490.

Lise, J (2010), On-the-Job Search and Precautionary Savings: Theory and Empirics of Earnings and Wealth Inequality (Mimeo, University College London).

Ljungqvist, L and T Sargent (1998), The European Unemployment Dilemma, *Journal of Political Economy* **106**, 514-550.

Ljungqvist, L and T Sargent (2007), Understanding European unemployment with matching and search-island models, *Journal of Monetary Economics* **54**, 2139-2179.

Lundvall, H (2010), Why Reservation Wages May Fall in Wealth, chapter 3 of this volume.

Mortensen, D and C Pissarides (1994), Job Creation and Job Destruction in the Theory of Unemployment, *Review of Economic Studies* **61**, 397-415.

OECD (2004), *Education at a Glance 2004: OECD Indicators*, OECD Publishing.

OECD (2005), *OECD Employment Outlook 2005*, OECD Publishing.

Pissarides, C (1985), Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages, *American Economic Review* **75**, 676-690.

Reichling, F (2007), Optimal Unemployment Insurance in Labor Market Equilibrium when Workers can Self-Insure (Mimeo, Stanford University).

Shao, E and P Silos (2007), Uninsurable Individual Risk and the Cyclical Behavior of Unemployment and Vacancies (Mimeo, Federal Reserve Bank of Atlanta).

Shimer, R and I Werning (2008), Liquidity and Insurance for the Unemployed, *American Economic Review* 98, 1922-1942.

Appendix - Solution Algorithm

The solution algorithm computes the model's equilibria in two stages. For a given guess on the policy variables $\{b, s, \tau\}$, an inner loop of the algorithm finds the corresponding equilibrium of the capital market as a fixed point in r, the interest rate. Once approximations to the value functions and policy functions are found, the economy is simulated for a large number of periods and the excess supply of capital is computed. A new guess on the interest rate is picked, after which the process is repeated. When a capital market equilibrium is found, the program finds out whether or not the government's budget constraint is satisfied. If this is not the case, a new guess on $\{b, s, \tau\}$ is initiated and the process of finding an equilibrium interest rate starts over. A policy $\{b, s, \tau\}$ is accepted as an equilibrium policy when the difference between the government's receipts and expenses is smaller than some predetermined level of tolerance.[1]

The functions $S^j(a, \gamma)$ and $R(a)$ are approximated by a set of one-dimensional, piecewise cubic splines, the argument of which is a, the level of savings. The chosen interpolation method is shape-preserving in the sense that it preserves monotonicity.[2] Given G different levels of skills ($\gamma \in \{\gamma_1, \gamma_2, ..., \gamma_G\}$) and P different levels of educational attainment ($\phi \in \{\phi_1, \phi_2, ..., \phi_P\}$), $(G * P)$ different splines are needed to approximate $S^j(a, \gamma)$.

In a similar fashion, $(G * P)$ different functions are used to approximate the value function of a working agent, $W^j(a, \varepsilon, \gamma)$. In this case, however, the original function to be approximated has two arguments, a and ε. The approximand chosen for this interpolation problem is a two-dimensional spline that is piecewise cubic in a-space and linear in ε-space.

The process of finding good approximands for $W^j(a, \varepsilon, \gamma)$, $S^j(a, \gamma)$ and $R(a)$ is initiated with a concave guess on each of these functions. These guesses are then used to evaluate the continuation values of the corresponding Bellman equations. For

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[1] Note that since both b and s are fractions of the average wage, net of taxes, a specific government policy is completely defined by two variables, either τ and b or τ and s.

[2] For an exhaustive explanation of this interpolation method, the interested reader is refered to Judd (1998).

each grid point in a-space, a maximization algorithm finds the optimal level of savings to carry to the next period. The right-hand side of the Bellman equations are then evaluated at the grid points in a-space using the optimal policy, producing a new and up-dated guess on the functions $W^j(a, \varepsilon, \gamma)$, $S^j(a, \gamma)$ and $R(a)$. The process is repeated until convergence is achieved.

CHAPTER 3

Poverty and the Dynamics of Equilibrium Unemployment

Henrik Lundvall

ABSTRACT. This paper uses a calibrated search model to quantitatively evaluate the effects of risk aversion and missing insurance markets on the equilibrium level and duration of unemployment. In order to generate realistically low levels of savings adequacy among unemployed workers, the model incorporates skill dynamics and state-dependent unemployment benefits. Already when risk aversion is relatively low (interpreted as a coefficient of relative risk aversion of 2), workers in some states make drastic changes to their reservation wages in response to relatively small changes in their liquid wealth. However, at that level of risk aversion, an endogenous, negative correlation between asset levels and effective replacement rates assures that relatively few workers find themselves in those states. When workers are more risk averse (c.r.r.a. at 3), missing insurance markets have relatively important effects on the equilibrium level and average duration of unemployment, suggesting that workers' wealth be an important variable to consider in the design of public policy.

3.1. Introduction

The ability of U.S. workers to buffer income losses from unemployment with own savings varies greatly across households. Although the average household wealth of blue collar workers is more than five times as large as their average yearly wage, one quarter of American workers that become unemployed suffer associated income losses that are greater than the total net worth of his or her household. Almost every second household that experience unemployment suffer income losses that exceed the household's total liquid wealth.[1] Economic theory predicts, and empirical studies confirm, that the size of

[1] See Gruber (2001), table 3. What Gruber (2001) defines as 'Gross Financial Wealth' is what I label 'liquid wealth'.

a worker's savings affects her behavior in the labor market: wealthy workers set higher reservation wages and search less intensively for new jobs than do workers with small savings.[2] What is not clear, however, is whether or not these effects are quantitatively important: Is household wealth an important determinant of the aggregate outcomes of the labor market, such as the incidence and duration of unemployment?

A considerable amount of research has recently been devoted to assessing the impact of risk aversion and missing insurance markets on labor market outcomes. A number of empirical studies have found evidence for the prediction that search intensities decrease, and reservation wages increase, in workers' wealth. However, the estimated size of this effect varies widely across studies. Bloemen and Stancanelli (2001) and Alexopoulos and Gladden (2004) find only negligible effects, while Algan et al. (2002) and Chetty (2008) estimate large effects of wealth on workers' transition rates.

In the theoretical and quantitative literature, efforts have been made to integrate different kinds of search and marching models into the framework of Bewley (undated), Huggett (1993) and Aiyagari (1994), in which agents are risk averse and insurance markets are missing. Thus, a number of studies have reexamined well-known policy questions, related to the labor market, using models that share the basic Bewley-Huggett-Aiyagari (BHA) assumptions. Alvarez and Veracierto (2001), Gomes et al. (2001) and Alosno-Borrego et al.(2005) are early examples of this modelling approach. More recently, Krusell et al.(2010) and Shao and Silos (2007) have contributed by systematically comparing the outcomes of a standard business cycle model, extended with matching frictions à la Diamond, Mortensen and Pissarides (DMP), to the outcomes of similar models that are augmented with the BHA framework.[3] Shao and Silos (2007) focus on the business cycle properties of their model, and they conclude that under plausible calibrations of the model, risk aversion and missing insurance markets do not substantially change the dynamics of the labor market over the cycle. Krusell et al.(2010) reach similar conclusions using a more elaborate model. Krusell et al. (2010) also investigate the welfare consequences of UI benefits in their model, and they show

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[2] Danforth (1979) and Lentz and Tranas (2005).
[3] See Diamond (1981), Pissarides (1985) and Mortensen and Pissarides (1994).

that there are important, negative welfare consequences of unemployment benefits that work thorough the demand for work: higher benefits strengthen workers' bargaining position and thus make it less profitable for firms to create vacancies.

The aim of this paper is to gauge the quantitative importance of risk aversion and missing insurance markets to the level and duration of unemployment. The focus is on the long-run determinants of labor supply and the chief contribution made here, relative to other studies, is the focus on the realism of the left-hand tail of the wealth distribution. More specifically, I concentrate on the adequacy of workers' saving with respect to the risk associated with their labor income, and primarily the risk of unemployment. Although the risk of unemployment is known to vary considerably over the business cycle, the complexity of adding aggregate uncertainty would make it hard to model with detail and realism the decisions of borrowing-constrained households, and this study therefore deals only with steady-state, long-run equilibria. Like the studies mentioned in the previous paragraph, the model used here features search frictions in the labor market, and it adopts the basic BHA framework. However, the focus on the behavior of workers and on the realism of workers' wealth holdings motivate a number of modelling choices that are different from the ones made, for example, by Krusell et al.(2010) and by Shao and Silos (2007). This study deals with the supply of labor and does not include the vacancy-posting, matching and bargaining mechanisms that are central to the DMP framework. Furthermore, empirical evidence suggest that savings inadequacy and the associated inability to buffer large income shocks is a greater concern to blue collar workers than it is to workers with a college education, and I therefore concentrate on the former group. Because the savings of blue collar workers make up such a small fraction of total U.S. wealth, changes in the savings behavior of this group will arguably have a limited impact on the equilibrium interest rate, and I therefore treat the interest rate as exogenously given.

These and other modelling choices are more thoroughly motivated in the following section. Section three and four present the theoretical model, while section five explains the calibration procedure. Section six contains the main results, and the last section offers some concluding remarks. The combined effects of a relatively large state space

and a non-convex constraint set complicate the numerical solution of the model. For this reason, the computational algorithm makes use of an auxiliary model that includes wealth lotteries. The details of this lottery model and the algorithm are discussed and outlined in the first appendix.

3.2. Savings Adequacy and Worker Heterogeneity

Given the objective to achieve an equilibrium with realistic levels of savings adequacy, some basic facts about the U.S. wealth distribution need to be considered. First of these is the strikingly strong correlation between wealth and age. A household whose head is between 61 and 65 years old is, on average, almost 35 times as wealthy as a household with a head who is 25 years or younger.[4] More generally, average household wealth increases monotonically with the age of the head up until retirement age, after which it starts to decline. To allow for a correlation between age and wealth in the model economy, I introduce a simple life cycle structure, where working agents face a constant per period probability of becoming retired citizens, and where retired agents face a constant, per period probability of death. Deceased agents will be immediately replaced with newborn agents that enter the labor force as unemployed. Assets left behind by the deceased will be distributed lump-sum to all surviving agents. With this simple life cycle structure, the model is able to replicate one of the fundamental regularities of the U.S. economy: young workers are poor, middle-aged and old workers are relatively rich.

A second well known and basic fact concerning the U.S. wealth distribution is its strong skewness. More than 80% of total wealth is concentrated in the hands of the richest 20% of the population. Related to this is the fact that self-employed persons are on average 5.6 times as wealthy as salaried workers. These circumstances alone suggest a relatively weak link between, on the hand, the savings behavior of ordinary workers and, on the other hand, the determination of the aggregate level of wealth in the economy. As shown by Quadrini (1999) and Quadrini (2000), entrepreneurship is a key determinant of the level of aggregate wealth. Given the aim of this study, which is not

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[4] All statistics on the U.S. wealth distribution refered to in this section are taken from tables 7 and 8 in Budría Rodríguez et al.(2002).

to explain the distribution of wealth *per se*, I choose to study only partial equilibria of the capital market, and to assume an exogenously given interest rate. I further constrain the analysis by modeling only blue collar workers. Blue collar households hold considerably less wealth than do white collar households, a difference that remains also when wealth is normalized by average incomes. This is a reason to suspect that blue collar workers have a harder time to buffer income shocks than do white collar workers. A recent study by Blundell et al. (2008) confirms this hypothesis: the authors use longitudinal data in order to explicitly estimate the ability of U.S. households to smooth consumption, and they find that blue collar households are less able than white collar households to shield their consumption from adverse income shocks.

In order to focus on the behavior of workers, I use a McCall-type search model, where agents receive wage offers from an exogenous wage offer distribution. The subjective discount rate of the model will be calibrated so as to achieve a realistic median ratio of wealth to income of employed agents. Although uninsurable idiosyncratic risk appears to be present in most labor markets, there is not a total lack of risk-sharing. Indeed, most industrialized countries have set up more or less generous public unemployment insurance systems, and the U.S. is no exception. Especially for blue collar workers, the U.S. public unemployment benefits provide a non-trivial amount of insurance, and an important part of the analysis must therefore be to provide a reasonably realistic modelling of those benefits. I therefore include a history-dependent unemployment benefit system in the model economy, where workers' previous earnings determine the amount of benefits paid and where unemployed workers face the risk of exhausting their benefits. Another key feature of the model will be it's ability to produce realistic levels of savings (in)adequacy among unemployed workers. In order to improve the fit of a generic McCall search model in this dimension, I introduce skill heterogeneity in the fashion of Ljungqvist and Sargent (1998). Heterogeneity in workers' productivity is one plausible reason for why real-world wealth distributions exhibit more dispersion

than that of standard search and search-and-matching models.[5] However, when preferences are characterized by constant relative risk aversion, heterogeneity in workers' productivity does not automatically result in increased dispersion in the *adequacy* of workers' savings. Skill dynamics à la Ljungqvist and Sargent (1998) is one way to achieve increased dispersion in savings adequacy.

3.3. The Economy

A continuum of agents are indexed on the unit interval. Their identical preferences over bundles of consumption, c_t, and work effort, e_t, can be summarized by the return function $E_t \sum_{i=t}^{\infty} \beta^{i-t} u(c_i, e_i)$, where $u(\cdot)$ is assumed to be increasing and concave in its first argument, and decreasing in the second argument. $\beta \in (0,1)$ represents agents' subjective discount rate. All agents are either workers, i.e. belonging to the labor force, or they are retired citizens. Retired agents receive a transfer s from the government in each period; any consumption over and above s must be financed through private savings. With per period probability κ, retired agents die, and are then immediately replaced by an offspring who enters the labor force as unemployed. Any assets left behind by the deceased are divided between all remaining agents and distributed as a lump-sum payment. With per period probability λ, an agent who belongs to the labor force in period t will be retired in period $(t+1)$.

3.3.1. Wage Offers and Skill Dynamics. In the beginning of every period, each agent who remains with the labor force has at hand a job offer that allows her to work in that period and earn wage income W_t. The size of the wage income depends on three variables: the wage rate w_t that is specific to the worker's current job; her current skill level, denoted γ_t; and, finally, the amount of work effort, e_t, that she decides to exert in that period:

$$W_t = \gamma_t w_t e_t.$$

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[5] For a discussion of the characteristics and realism of the wealth distribution produced by such models, see Krusell et al.(2010).

In other words, w_t can be interpreted as a wage rate per efficiency unit of labor. If the worker accepts the job offer, she collects her income and pays any income taxes imposed by the government. She also decides how much of the resources at hand to consume, and how much to save for coming periods. Unless the worker is laid off at the beginning of period $(t+1)$, a new wage offer w_{t+1} will be determined by the realization of a random variable, with conditional distribution function $J(w'|w) = prob\{w_{t+1} \leq w'|w_t = w\}$. If instead the worker decides to be unemployed in period t, she forgoes the wage income W_t. In the subsequent period, she receives a wage offer from a stationary distribution $O(w')$. Search is assumed to be effortless, so that $e = 0$ for all the unemployed.

Since the focus of this study is on the behavior of workers, layoffs will be treated as exogenous events. With constant probability δ, a worker who was employed in period t will be laid off at the beginning of period $(t + 1)$. When a worker willingly leaves employment, by declining a wage offer from the distribution $J(w'|w)$, I will say that she quit.

Agents' skills evolve stochastically over time, with different transition probabilities for employed workers and for workers that decide to separate from their jobs. Suppose that in period t a worker with skill level γ_t was employed, and suppose she keeps her job in period $(t+1)$. At the beginning of period $(t+1)$, her new skill level, γ_{t+1}, will then be determined as a random draw from the distribution $T_E(\gamma'|\gamma) = prob\{\gamma_{t+1} \leq \gamma'|\gamma_t = \gamma\}$. However, if the worker is unlucky enough to be laid off at the beginning of period $(t + 1)$, she risks an instantaneous loss of skills. The probability distribution of skills for such an agent, conditional on the level γ_t of skills that she had before the lay-off, is denoted $T_S(\gamma'|\gamma)$. Workers that quit do not suffer instantaneous skill losses. However, all unemployed workers face a risk of some gradual skill losses during their spell. A third distribution function, $T_U(\gamma'|\gamma)$, governs the risk of such gradual skill losses. The reason to distinguish between the two distributions T_S and T_U is to allow for possibly drastic skill losses at the time of separation. One way to interpret this setup is to think of T_S as governing the loss of firm specific human capital at the time of separation, while T_U governs the gradual loss of general human capital during the spell.

3.3.2. Asset Markets and Government Policy. All agents, workers and re-
tired citizens, can save in a risk free asset that pays a net return of r units of consump-
tion per period; they have no access to any private markets for contingent claims. The
rate of return on the safe asset will be specified exogenously, implying that the analysis
will abstract from any general equilibrium effects that might be channelled through
changes in the interest rate.

The government of the model economy runs two public insurance programs, an un-
employment insurance system and a pay-as-you-go pension program. The government
also needs to finance some public consumption each period, the value of which is de-
noted G. Agents do not derive any utility from public consumption. From a modelling
perspective, the purpose of G is simply to allow for a realistic calibration of both the
replacement rates of the two government programs, and of the income tax rate. In the
benchmark calibration, G will be determined residually, so as to close the government
budget. All government expenses are financed through a tax schedule $\tau(W_t)$ on labor
income.

The two U.S. government programs considered here, the public pension and unem-
ployment insurance programs, are both complex systems. Given the relatively large
state space of the model economy, some simplification is necessary. I have chosen to
model the UI program with some care to details, while representing the pension system
with a simple transfer of s units of the good per period to each retired agent. Two
considerations motivated this choice. First, the model's skill dynamics will generate
non-trivial dispersion in the endogenous wage distribution. With a very simplistic
rendering of the UI benefit system, such as a simple transfer of b units of the good
to all unemployed agents, it would be hard or impossible to avoid unrealistically high
effective replacement rates at the very bottom of the wage distribution. With unreal-
istically high replacement rates for workers with low skills, it is hard to imagine that
the model would yield a realistic savings behavior for those same workers. Second, the
U.S. public pension system has a strong redistributive character, with relatively high
replacement rates for households with low incomes, and lower replacement rates for

workers with high wages.[6] This redistributional component will be captured by the model pension system, where all workers receive the same level of benefits.

There are three characteristics of the U.S. unemployment insurance program that I believe are important to include in the model UI system. First, the level of benefits an individual worker can claim depends on her previous earnings history. In almost all U.S. states, monetary entitlement is a function of a worker's wage earnings in four of the five quarters that precede an unemployment spell. Second, all states specify caps, or maximum benefit amounts, implying that effective replacement rates for workers with relatively high earnings are lower than the systems' nominal replacement rates would imply. Finally, all states restrict the duration of benefit payments, the most common benefit period being 26 weeks.[7] To capture these features in the model economy, let $b_t \geq 0$ denote a worker's entitlement level in period t: if the worker is laid off in that period, the government makes her a transfer of value b_t. In the following period, the worker will keep her entitlement with probability χ, implying $b_{t+1} = b_t$; with probability $(1-\chi)$, she looses her entitlement, in which case $b_{t+1} = b_w$, where b_w represent a welfare payment. If an agent looses her entitlement between periods t and $(t+1)$, I will say the worker exhausted her benefits in period t. Workers who quit will immediately loose any entitlement they may have had. To allow workers' entitlement levels to depend on their previous earnings history, let $\widehat{w}_{it}$ denote worker i:s highest period wage earnings, net of taxes, since she last exhausted her benefits. For a newborn worker who never experienced any unemployment, $\widehat{w}_{it}$ will simply be the highest period labor income

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[6] A number of studies have argued that the amount of redistribution built into Social Security is in effect quite small. See Gustman and Steimeier (2001); Liebman (2002); and Coronado et al. (2000). These studies point to the fact that while the formulas of the system seem to imply substantive redistribution, a number of other factors work in the opposite direction. These factors include the longer life expectancy of high income households, and the fact that members of such households generally start to work later in life than do people with lower incomes. Furthermore, high income households are more likely than others to receive spouse benefits. However, the first two of these factors are not relevant to the modelling choices that are here under consideration. It may be that long life expectancy and late entry in to the labor force of high income households strongly dampen the net distributional effect of Social Security. But because the model abstracts from such features of the real economy, what is interesting to this study are the actual benefit formulas.

[7] For an overview of the provisions of the U.I. systems of different American states, see U.S. Department of Labor (2007). Follwoing the recent great recession, Congress has enacted legislation in order to extend the duration of benefit payments. These changes to the U.S. public UI system are not considered here.

she earned so far in her working life. The government chooses a nominal replacement rate, η, a 'target entitlement', b_t^T, and a maximum benefit level, $b_{\max}$, which together determine a worker's actual entitlement b_t in the following way. The target entitlement is set according to: $b_t^T = \min\{\eta \widehat{w}_{it}, b_{\max}\}$. If the difference between b_t^T and the worker's entitlement in the previous period, b_{t-1}, is not too great, the workers is assigned the target entitlement, b_t^T. However, if $b_t^T - b_{t-1}$ is greater than a certain number, the actual entitlement is a number between b_{t-1} and b_t^T. The reason to distinguish between the target entitlement level, b_t^T, and the actual entitlement, b_t, is to prevent workers from climbing from the lowest to the highest entitlement level in just one or two periods.

3.3.3. Equilibrium. Since the aim of this study is to understand the determinants of equilibrium, or long-run, unemployment, it is natural to focus on steady state equilibria. By implication, the interest rate, r, and all variables that define government policy, $\{\tau(W_t), s, \chi, \eta, b_{\max}\}$, will be constants. The relevant state of a retired agent is $\{a\}$, the level of savings with which she enters a new period. A worker's state will be comprised of four variables, $\{a, \gamma, w, b\}$: the level of savings, the skill level, the wage offer at hand and the current entitlement level. Denoting by x' next periods value of variable x, the value function of a retired agent is:

$$R(a) = \max_{c, a'} \left\{ u(c, 0) + (1 - \kappa)\beta R(a') \right\}, \tag{3.1}$$

$$\text{s.t.}$$

$$c + a' \leq (1 + r)a + q + s,$$

$$a' \geq \overline{a},$$

where $\overline{a}$ is a borrowing limit and q is the *per capita* accidental bequest made by deceased agents.

The value function of a worker who decides to work in the current period is:

$$W(a, \gamma, w, b) = \max_{c,e,a'} \{u(c, e) + \lambda\beta R(a') + \delta(1 - \lambda)\beta E_d[S(a', \gamma', b')|\gamma]+ \qquad (3.2)$$

$$(1 - \delta)(1 - \lambda)\beta E_w\left(\max[W(a', \gamma', w', b'), S(a', \gamma', b_w)]|\gamma, w, b)\right\},$$

s.t.

$$c + a' \leq \gamma w e - \tau(\gamma w e) + (1 + r)a + q,$$

$$a' \geq \overline{a},$$

where the subscript d on the first expectations operator is meant to signify that workers who are laid off face the risk of instantaneous and drastic skill losses. The subscript w on the second expectations operator indicates that the wage offer and skill level of employed workers evolve according to the distributions $J(w'|w)$ and $T_E(\gamma'|\gamma)$, respectively. $S(a, \gamma, b)$ denotes the value of search. This value function, in turn, is defined in the following way:

$$S(a, \gamma, b) = \max_{c,a'} \{u(c, 0) + \lambda\beta R(a')+ \qquad (3.3)$$

$$+(1 - \lambda)\beta E_s\left(\max[W(a', \gamma', w', b'), S(a', \gamma', b')]|\gamma, w, b)\right\},$$

s.t.

$$c + a' \leq (1 + r)a + q + b,$$

$$a' \geq \overline{a}.$$

Unemployed workers form expectations regarding future wage offers and skills according to the distributions $O(w')$ and T_U. Unemployed workers also risk loosing their UI. entitlement. As a reminder of this, the expectations operators of unemployed workers carry the subscript s[8]. Associated with the solution of 3.1 is an optimal savings policy, $A^R(a)$, and a consumption policy, $C^R(a) \equiv (1+r)a + s + q - A^R(a)$. Analogously, the policy functions that are part of the solution to 3.2 and 3.3, and which pertain to the choice of saving and consumption, will be denoted $A^W(a, \gamma, w, b)$, $C^W(a, \gamma, w, b)$, $A^S(a, \gamma, b)$ and $C^R(a, \gamma, b)$, respectively. Active agents also decide whether to work or to search, with an optimal policy $\Omega(a, \gamma, w, b)$ defined in the following way:

$$\Omega(a, \gamma, w, b) = \begin{cases} 1 \text{ if } W(a, \gamma, w, b) > S(a, \gamma, 0), \\ 0 \text{ otherwise.} \end{cases} \tag{3.4}$$

The optimal savings policy of an active agent, $A(a, \gamma, w, b)$, can thus be written:

$$A(a, \gamma, w, b) = \Omega(a, \gamma, w, b)A^W(a, \gamma, w, b) + [1 - \Omega(a, \gamma, w, b)]A^S(a, \gamma, b).$$

Agents that decide to work also decide how much effort to exert. The optimal policy for this choice will be denoted $L^e(a, \gamma, w, b)$. For any aggregate state of the economy, let $Z(a, w \,|\, \gamma, b)$ denote the fraction of agents with skills γ and entitlement level b that has savings smaller than or equal to a, and who, in the beginning of the period, had at hand a wage offer smaller than or equal to w. Analogously, let $Z^R(a)$ denote a distribution function for retired agents, and define $R \equiv \int_{\underline{a}}^{\infty} dZ^R(a)$, the total mass of retired agents. In a steady state, the per period flow of agents from active life to retirement must equal the flow of retired agents to death:

$$(1 - R)\lambda = \lambda \sum_{\gamma,b} \int_{\overline{a}}^{\infty} dZ(a, w \,|\, \gamma, b) = \kappa R. \qquad (3.5)$$

The budget of the public sector will be in balance if:

$$\sum_{\gamma,b} \int_{-\infty}^{+\infty} \int_{\overline{a}}^{+\infty} \{ \tau \,[w\gamma L(a, \gamma, w, b)] - b \} \, dZ(a, w \,|\, \gamma, b) - s \int_{\overline{a}}^{+\infty} dZ^R(a) - G = 0. \qquad (3.6)$$

Finally, *per capita* bequest transfers, q, must balance the actual bequests left behind by deceased agents:

$$\kappa \int_{\overline{a}}^{\infty} A^R(a) dZ^R(a) = q. \qquad (3.7)$$

A recursive equilibrium consists of:

Value functions and associated policy functions $R(a)$, $A^R(a)$, $W(a, \gamma, w, b)$, $A^W(a, \gamma, w, b)$, $C^W(a, \gamma, w, b)$, $L^e(a, \gamma, w, b)$, $S(a, \gamma, b)$, $A^S(a, \gamma, b)$ and $C^S(a, \gamma, b)$;

A policy function $\Omega(a, \gamma, w, b)$ as defined in 3.4;

Distribution functions $Z(a, w \,|\, \gamma, b)$ and $Z^R(a)$;

An interest r, and;

A government policy $\{\tau(W_t), s, \chi, \eta, b_{\max}\}$ such that:

1) Given r and $\{\tau(W_t), s, \chi, \eta, b_{\max}\}$,

$R(a)$ and $A^R(a)$ solve 3.1;

$W(a, \gamma, w, b)$, $A^W(a, \gamma, w, b)$, $C^W(a, \gamma, w, b)$ and $L^e(a, \gamma, w, b)$ solve 3.2;

$S(a, \gamma, b)$, $A^S(a, \gamma, b)$ and $C^R(a, \gamma, b)$ solve 3.3.

2) The government budget restriction, 3.6, holds.

3) Aggregate bequests equal the *per capita* bequest transfer (eq. 3.7 holds).

4) $Z(a, w \,|\, \gamma, b)$ and $Z^R(a)$ are time invariant and together satisfy 3.5.

3.4. Calibration

I assume per period utility $u(\cdot)$ is of the CRRA type, with the following functional specification:

$$u(c_t, e_t) = \frac{\left[c_t - \frac{e_t^{1+\theta}}{1+\theta} \right]^{1-\sigma} - 1}{1 - \sigma}, \qquad \theta > 0, \qquad \sigma > 0.$$

θ can be interpreted as the inverse of the Frisch elasticity of labor supply and is set to 10, which is within the standard ranges obtained by the empirical literature. σ corresponds to the coefficient of relative risk aversion. The degree to which workers dislike risk will obviously have a first order impact on the results. In the benchmark calibration, σ is set to 2, a value that can arguably be interpreted as a moderately low level of risk aversion. In the following section, however, I will explore the impact of setting σ to 3, which I interpret as a moderately high level of risk aversion.

The specific functional form of $u(\cdot)$, used by Gomes et al. (2001), substantially simplifies the solution of the model, since the work effort exerted by employed agents, e_t, will not depend on their level of assets, a_t; given a worker's choice to accept the wage offer at hand, the choice of e_t will not be subject to any intertemporal trade-off.[9] The assumption that work effort is independent of agents' asset holdings is probably not a realistic one. However, this choice of functional form allows me to include in the

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[9] To see this point, substitute the budget constraint into the utllity function and differentiate the right-hand side of the Bellman equation w.r.t. e. The first order condition is:

$$\widetilde{c}^{-\sigma} \left[\gamma w \left(\frac{\partial \tau}{\partial W} - 1 \right) + e^{\theta} \right] = 0,$$

where $\widetilde{c} = c - \frac{e^{1+\theta}}{1+\theta}$ and $W = \gamma w e$. With $\sigma > 0$, an interior solution must satisfy:

$$e^{\theta} = \gamma w \left[1 - \frac{\partial \tau}{\partial W} \right].$$

analysis the possibly important adverse effects of income taxation on work effort, while at the same time greatly simplifying the solution of the model.[10]

One model period corresponds to half a quarter. The model's stylized life cycle dynamics are governed by the two parameters λ and κ, denoting, respectively, the per period probability of retirement and death. Following Castañeda et al. (2002), I set these values so as to achieve an expected duration of a working life of 45 years, and an expected duration of retirement of 18 years.

The risk free interest rate r is set to 0.49 % per period, corresponding to a yearly net return of 4%, a standard value in the macroeconomic literature. To pin down the subjective discount rate, β, I target the median ratio of wealth to weekly income among employed, tenured workers. In the sample of Gruber (2001), this ratio is 10.2.

3.4.1. Wage Offers. The labor market opportunities of workers, and their associated risk, will be determined jointly by 1) the distribution of wage offers to employed workers, $J(w'|w)$; 2) the distribution of wage offers to unemployed workers, $O(w')$; and, 3), the stochastic evolution of the skill variable, γ. Concerning $J(w'|w)$, I assume that the logarithm of wages of employed workers follow an AR(1) process with Gaussian innovations:

$$\ln(w') = \rho \ln(w) + \varepsilon, \qquad \rho \in (0,1), \quad \varepsilon \sim N(0, \sigma_e^2).$$

An important feature of this process is persistence, which will create incentives for workers to search for good wage offers, and to separate from jobs offering low wages. Using American data, Storesletten et al.(2002) find evidence of a highly persistent, possibly permanent, component in idiosyncratic shocks to earnings. Due to details of the solution techniques that I use, I restrict ρ to the open interval $(0,1)$, and I therefore set ρ to 0,999 in all calibrations. Also the distribution of wage offers received by unemployed workers is assumed to be log-normal:

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[10] For an interesting quantitative analysis of the relationship between workers' savings and hours worked, see Reichling (2007).

$$\ln(w') \sim N(0, \sigma_u^2).$$

σ_e^2 and σ_u^2 will be set in order to achieve an unemployment rate of 7%, and an average duration of unemployment spells of 13 weeks.

3.4.2. Skill Dynamics. Turning to the models' skill dynamics, these are governed by three distributions: 1) T_E determines the probability that an employed agent will see an appreciation of skills from one period to the next; 2) T_S allows for the possibility that laid off agents will face an instantaneous loss of skills; 3) and T_U, finally, determines the gradual skill losses suffered by unemployed agents as their spells evolve. To calibrate these distributions, I draw on the work of Ljungqvist and Sargent (1998) in two ways. First, the general structure of these distributions will be the same as that of Ljungqvist and Sargent (1998). Second, I follow them in assuming that the data set of Jacobson et al. (1993), on earnings losses faced by displaced workers in Pennsylvania in the early and mid-1980's, is representative of the conditions faced by all blue collar workers in the U.S. Krebs (2007) provides an overview of the empirical literature on the long-term earnings losses of displaced workers and notes that, regarding such earnings losses in the U.S., Jacobson et al. (1993) remains one of the most thorough studies.

Employed agents face a constant per period probability ω_e of a one level skill upgrade. Similarly, unemployed agents face a constant per period probability ω_u of a one level skill loss. Assume γ takes on H different values that evenly partitions the interval $[1, 2]$: $\gamma \epsilon [1, \gamma_2, ..., \gamma_{H-1}, 2]$. This implies that for a given wage level, and for a given level of work effort, a highly skilled worker can at most earn double the wage rate of a worker with the lowest skill level. A worker who was employed in period t with skill level $h < H$, will then with probability ω_e have skill level $(h + 1)$ in period $(t + 1)$. If the agent was employed in period t with skill level $h = H$, she will retain that skill level with certainty in period $(t + 1)$. In a similar fashion, an agent who was unemployed in period t with skill level $h > 1$ will, with probability ω_u, see her skills depreciate to level $(h - 1)$ between periods t and $(t + 1)$. The immediate skill losses of separating workers, in turn, will be governed by the truncated left half of a Normal

distribution, the standard deviation of which is denoted ω_d. The exact definition of the transition probabilities in T_S are deferred to Appendix B. Here, I just note that, like in Ljungqvist and Sargent (1998), the skill loss distribution can be summarized by a single parameter, ω_d: the higher is ω_d, the higher the probability that a laid-off worker suffer immediate skill losses; and the higher is ω_d, the more skills is a laid-off worker expected to loose, conditional on such skill losses occurring.

Given this structure, and conditional on the employment status of a particular worker, there are in all three parameters that determine the distributions of skills: $\{\omega_e, \omega_u, \omega_d\}$. Three statistics are used as corresponding calibration targets: 1) the average real wage growth of U.S. workers, which Topel (1991) estimates to be 3% yearly; 2) the average earnings losses of displaced workers one year after displacement, compared to their earnings three years prior to displacement, which Jacobson et al. (1993) report to be 33% in their Pennsylvania data set; 3) the average earnings losses of displaced workers 6 years after displacement, estimated by Jacobson et al. (1993) to be 25%. However, this exercise is complicated by the fact that transition rates back and forth between the two states of unemployment and employment are endogenous. As a result, the exact mapping between the calibrated skill distributions, summarized by $\{\omega_e, \omega_u, \omega_d\}$, and the resulting wage distribution, is in itself an equilibrium object. When deciding whether to work or to search, agents take into account all the information contained in their state (a, γ, w, b). The wage distribution of the model, therefore, is the combined result of the skill distribution, the wage offers available to employed and unemployed workers, and of workers' reservation wage policies. To overcome these problems, I adopt the following calibration strategy. Starting with an initial guess on the parameters governing the wage offers, σ_e^2 and σ_u^2, and on the subjective discount rate, β, I specify a coarse grid in the space of the skill parameters $\{\omega_e, \omega_u, \omega_d\}$. A simple loss function finds the parameter combination $\{\omega_e, \omega_u, \omega_d\}$ that minimizes the distance between the three statistics on the U.S. wage distribution that were mentioned above, and the corresponding statistics computed from model simulations. Once the parameters of the skill distributions are fixed, I fine tune the values of σ_e^2, σ_u^2 and β, so as to achieve the targets on the unemployment rate (7%), the average spell duration

(13 weeks) and the median ratio of wealth to weekly earnings (10.2). It turns out that small changes in the parameters σ_e^2, σ_u^2 and β have only a negligible impact on the targeted wage dynamics, a result that lends some support to this calibration approach.

3.4.3. Government Policy. The aim, when calibrating the policy variables, particularly the UI system and the income tax function, will be to achieve realistic renderings of the corresponding U.S. variables. To accommodate any discrepancies between what the model government collects in taxes and what it spends, I introduce the residual variable G, 'public consumption'; in the benchmark calibration, government budget balance will be guaranteed by the definition of G.

The model's public pension system consists of a transfer of s units of the good to each retiree in each period. Denoting by $\overline{y}_{NT}$ the average labor income, net of taxes, the value of s is set so as to make the ratio $\frac{s}{\overline{y}_{NT}}$ equal to 0.45.[11]

To solve the model, I discretize the interval $[b_w, b_{\max}]$, assuming that entitled workers receive one out of $(B+1)$ different benefit levels: $b \in [b_w, ..., b_{\max}]$. Let I be an indicator that takes on values in the set $\{0, 1, ..., B\}$, where $I = 0$ denotes 'no entitlement' or 'welfare payment', $I = 1$ indicates entitlement to a transfer b_1 per period of unemployment, et cetera. The government determines a worker's entitlement based on her previous earnings through a replacement rate policy. Let η denote the government's 'target replacement rate', and let $\widehat{w}$ be an agent's highest period labor income since she last exhausted her benefits. If $0 < \eta\widehat{w} \le b_B$, the government assigns to that agent a target entitlement level I_T such that $b_{I_T-1} > \eta\widehat{w} \ge b_{I_T}$. If the worker did not earn any income, $I_T = 0$; and if $\eta\widehat{w} > b_B$, the worker gets target entitlement $I_T = B$. The actual entitlement level, I, is the maximum of, on the one hand, I_T and, on the other, the agent's entitlement level in the previous period, plus one entitlement level. The reason to distinguish between a target entitlement level and the actual entitlement level is to slow down the process whereby agents gain entitlement: in most U.S. states, entitlement is calculated from the income earned during the first four of the five

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[11] Eisensee (2006) finds that the average replacement rate, net of taxes, of the U.S. Socail Security system is approximately 45%. He also argues that the system has a strong redistributional component. For a brief discussion of the redistributional character of Social Security, see note 4.

quarters that preceded the unemployment spell. Finally, assume that the government randomly withdraws entitlement from unemployed workers, so that agents with $I > 0$, who are unemployed in period t, face a certain probability χ that they will enter period $(t + 1)$ with $I = 0$. Unemployed agents without entitlement receive a transfer $b_w \geq 0$ which could be thought of as representing the combined effects of welfare payments and food stamps that are available to citizens with no other income.

The majority of U.S. states set their nominal replacement rates at 50%, with a maximum benefit period of around half a year.[12] In keeping with these numbers, η is set at 0.50, and χ is fixed so that the expected time of benefit payments is two quarters, or four model periods. Looking at all spells of unemployment recorded in the Survey of Income and Program Participation (SIPP) between 1984 and 1992, Gruber (2001) simulates the actual (or effective) replacement rates received by insured workers in the sample. He finds that insured workers receive benefits equal to roughly 45% of the previously held wage rate, which will be the calibration target adopted here. The target for the transfer b_w to unemployed agents with no entitlement is set equal to 25% of the average wage, net of taxes.

Finally, I use the same functional form for the U.S. income tax schedule as do Gouveia and Strauss (1994), and I target an average income tax of 22%.[13]

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[12] See footnote 7.

[13] See Castan͂eda et al.(2003) for an explanation of this approach.

Table 1: Main Calibration Targets and Outcomes

	Target	Model
Unemployment rate	7.0%	7.0%
Average spell duration	13 weeks	13 weeks
Ratio of wealth to		
average weekly wage (median)	10.2	10.2
Average yearly wage growth	3.0%	2.9%
Skill losses of displ. workers:		
1 year after displ.	33%	34%
6 years after displ.	25%	24%
Government policy:		
average UI. repl. rate	45%	45%
ratio of welfare payment to average wage	25%	24%
average repl. rate public pension	45%	45%
average tax rate	22%	22%

Note: For a motivation and description of the calibration targets, see section 5.

3.5. Results

Table 1 summarizes the calibration targets and the corresponding outcomes in the benchmark equilibrium.[14] Of course, the close match between targets and model outcomes is an artifact of the calibration process; Table 1 serves mainly as a reminder of how the model was calibrated. Table 2 offers some summary statistics on the equilibrium distributions of earnings and wealth in the benchmark equilibrium and in the U.S. The model distribution of earnings referred to here is the distribution of W_t, which is the total income from labor earned by employed workers.[15] What strikes the eye in Table 2 is the difference in skewness (as measured by the ratio of the average-to-median) and in dispersion (as measured by the Gini coefficient) between the model and U.S. distributions; by both measures, the U.S. economy is more unequal than the model economy. Given the modeling choices, that were discussed in section 2, this should

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[14] For a discussion and motivation of the targets, please refer back to the previous section.

[15] The statistics on the U.S. distributions of earnings and wealth were taken from Budría Rodríguez et al. (2002). Note that the statistics on wealth in Budría Rodríguez et al.(2002) refer to households' net worth, while the model is calibrated to match the level of liquid savings among workers.

Table 2: Benchmark and U.S. Distributions of Earnings and Wealth

	Earnings Distribution		Wealth Distribution	
	Model	U.S.	Model	U.S.
Average	1.69	-	2.52	-
Median	1.58	-	1.34	-
Ratio of average to median	1.07	1.61	1.89	4.03
Gini coefficient:	-	-	0.65	0.80
before taxes	0.21	0.61	-	-
after taxes	0.20	-	-	-

Notes: The statistics on the U.S. distributions of earnings and wealth reproduced
here are taken from Burdía Rodríguez et al. (2002), tables 1 and 2. The wealth
statistics in Budría Rodríguez et al. (2002) refer to households' net worth.

come as no surprise, however. The U.S. economy has several dimensions of worker het-
erogeneity that were deliberately left out of the model. Two examples are innate ability
and educational attainment. The fact that the model economy does not include any
entrepreneurial agents, with particular incentives to accumulate savings, also makes
for a less skewed wealth distribution.

One important determinant of labor earnings in the model economy is the endoge-
nous distribution of skills. The upper part of Table 3 shows the distribution of workers
over the five different skill levels in the benchmark distribution. This distribution is
determined jointly by the parameters of the skill distribution, $\{\omega_e, \omega_u, \omega_d\}$, by agents'
reservation wages and by the probability δ of exogenous layoffs. The parameters of
the skill distribution were chosen by an iterative calibration procedure, and their val-
ues imply that, conditional on no job loss, it takes a worker on average 6.25 years to
accumulate one level of skills. In the unlikely event that an individual worker never
experiences any unemployment, she would be expected to advance from the lowest to
the highest skill level in a time-span of 25 years. The calibration procedure assigns zero
probability to skill losses that occur gradually during the course of an unemployment

Table 3: Equilibrium Distribution of Workers over Skills and U.I. Entitlement

	1	2	3	4	5	$\sum$
Skills (share of agents, percent)						
all workers	24.1	23.8	19.5	14.2	18.4	100
the unemployed	34.0	26.1	18.8	12.0	9.2	100
UI. entitlement (share of agents, percent)						
all workers	14.0	12.7	12.7	11.5	49.2	100
the unemployed	57.6	5.8	6.1	5.8	24.8	100

spell ($\omega_u = 0$), with the implication that, in the benchmark calibration, all skill losses occur instantaneously, at the time of exogenous layoffs (what Ljungqvist and Sargent [1998] refer to as 'turbulence'). In return, the value of ω_d implies that such instantaneous and drastic skill losses are relatively frequent events. A worker with the highest skill level, who is laid off, faces a 68.3% probability of instantaneous skill losses. This probability decreases with the level of initial skills, and is 28.6% for a worker at the second skill level. Together, these transition probabilities, and the reservation wage policy of workers, result in an equilibrium distribution of skills were the lowest skill level is the most common, and were only a little more than 18% of all workers are at the highest skill level. Table 3 also shows the skill levels of unemployed workers, which are lower, on average, than those of employed workers.

The lower part of Table 3 shows the fraction of workers with different UI entitlement levels. Here, '1' refers to the case of no entitlement, in which case the worker, should she become unemployed, receives only a welfare payment from the government. The numbers ranging from 2 to 5 refer to UI. entitlements for which workers much qualify through their previous earnings, and which may be exhausted in case of unemployment. The highest entitlement level is the most common among employed workers, while the 'no entitlement' state is the most common among the unemployed. Obviously, this outcome is a consequence of the relatively short duration of benefit payments.

3.5.1. Savings and the Expected Duration of Unemployment. Together with the exogenous probability δ of layoffs, agents' reservation wage policy determines

the incidence and duration of equilibrium unemployment. In the benchmark equilibrium, the reservation wage is everywhere decreasing in workers' skill level, and everywhere increasing in the level of UI. entitlement. The negative relationship between skill level and reservation wage is a result of the higher opportunity cost of search that comes with higher skill levels. For a given wage per efficiency unit of labor, w_t, a skilled worker will have higher total earnings per hour worked, and will therefore find it less worthwhile to spend time in unemployment in order to find a job paying a high w_t.[16] Of course, the positive relationship between UI. entitlement and reservation wages is to be expected.

Turning now to the relationship between savings and reservation wages, Figure 3.1 depicts the reservation wage as a function of wealth and UI. entitlement for workers with two different skill levels, the lowest and the highest. The units on the axis representing the savings dimension were normalized by the average, monthly wage, net of taxes, and ranges from 0 to 6. The upper pane of Figure 3.1 shows the policy of a worker at the lowest skill level, while the lower pane looks at the optimal policy of a worker at the highest skill level. To facilitate comparisons, the scale on the vertical axes were kept the same in the two panes. The reservation wage of the low-skilled worker increases in UI. entitlement; as was just noted, this is true for workers in all states. It also increases in wealth, although for most levels of entitlement and for most of the wealth levels shown in the figure, this effect is small. Only for workers with no entitlement, and with savings equal to half the average, monthly wage, or less, is the wealth effect clearly visible. The reason behind this muted wealth effect lies in the relatively high effective replacement rate faced by low-skilled workers. For low-skilled workers, the degree of insurance afforded by the public UI. program is relatively high, and the need to use own savings to buffer income shocks is relatively small. For high-skilled workers, the situation is quite different. Highly skilled workers face low effective replacement rates, and if they have little or no wealth, they will be forced to lower their level of

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[16] This result differs from the U-shaped relationship between skill level and reservation wages that Ljungqvist and Sargent (1998) obtain, and which depend on the incentives to accumulate skills and the risk of loosing skills. Those forces are muted here because the parameters ω_e and ω_u take on smaller values. If the model is solved with values that resemble the Ljungqvist and Sargent (1998) calibration, the U-shaped relationship reappears.

consumption considerably in case they become unemployed. Since workers are risk averse, they prefer to lower their reservation wage quite substantially. To take an example, consider the situation of a worker with the highest skill level, who has no entitlement (perhaps because benefits were exhausted) and with savings equal to two average monthly wages. Such a worker chooses a (log-)reservation wage of -0.011, which corresponds to a per period probability of accepting a job of 52%. If instead the same worker had no savings at all, she would set her reservation wage to -0.453, with a per period probability of job-acceptance of 96%. That higher replacement rates diminish the effect of wealth on reservation wages is clearly visible in the lower pane of Figure 3.1, where the elasticity of the reservation wage is seen to decrease with higher levels of UI. entitlement.

The same effect is also strikingly visible when comparing the two panes in Figure 3.2. The upper pane of Figure 3.2 shows reservation wages as a function of wealth and skills for workers with no UI. entitlement, while the lower pane shows reservation wages for workers with the highest entitlement level. The upper pane again shows how workers with high skills, and therefore low effective replacement rates, are more sensitive to their financial situation than are workers with little skills. For workers with the highest entitlement level, however, the effect of wealth on reservation wages is barely visible, except for workers with the highest skill level.

3.5.2. Savings Adequacy. Gruber (2001) analyzes the savings adequacy with respect to the income risk of unemployment among U.S. households. Using a large and representative sample of unemployment spells from the S.I.P.P. , the author calculates, for each spell, the ratio of household wealth to realized income loss. Household wealth is defined as 'gross financial assets' and Gruber uses the last observation that was recorded prior to the beginning of the unemployment spell[17]. The income loss from unemployment is defined as the difference between what the household would have earned during the spell, had the unemployed worker not lost her job and kept her previous wage, and the income she actually earned during the spell. The second and

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[17] Gruber (2001) calculates the corresponding statistics also for two other definitions of wealth: 'net financial wealth' and 'total net worth'.

Figure 3.1: Reservation wage policies as functions of wealth and U.I. entitlement. The scale in the wealth dimension was normalized by the average, monthly wage, net of taxes. The upper pane shows the policy of workers at the lowest skill level; the lower pane represents workers at the highest skill level.
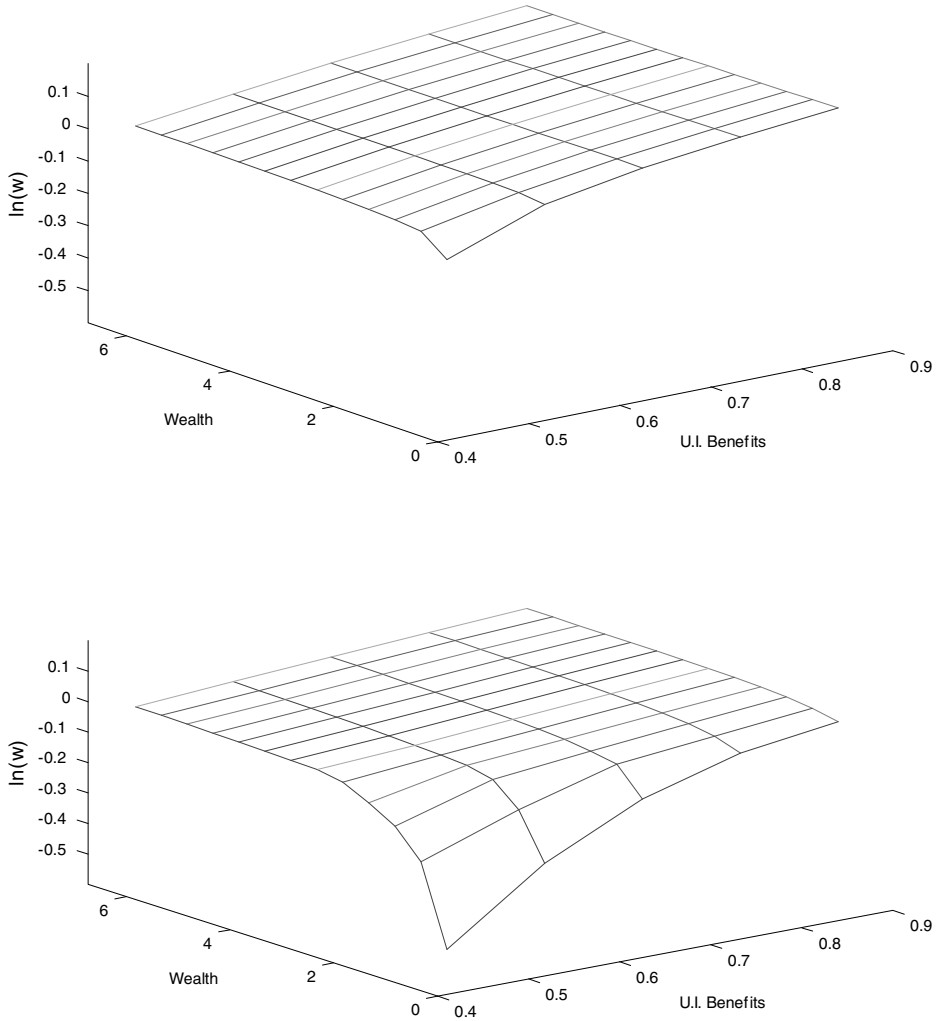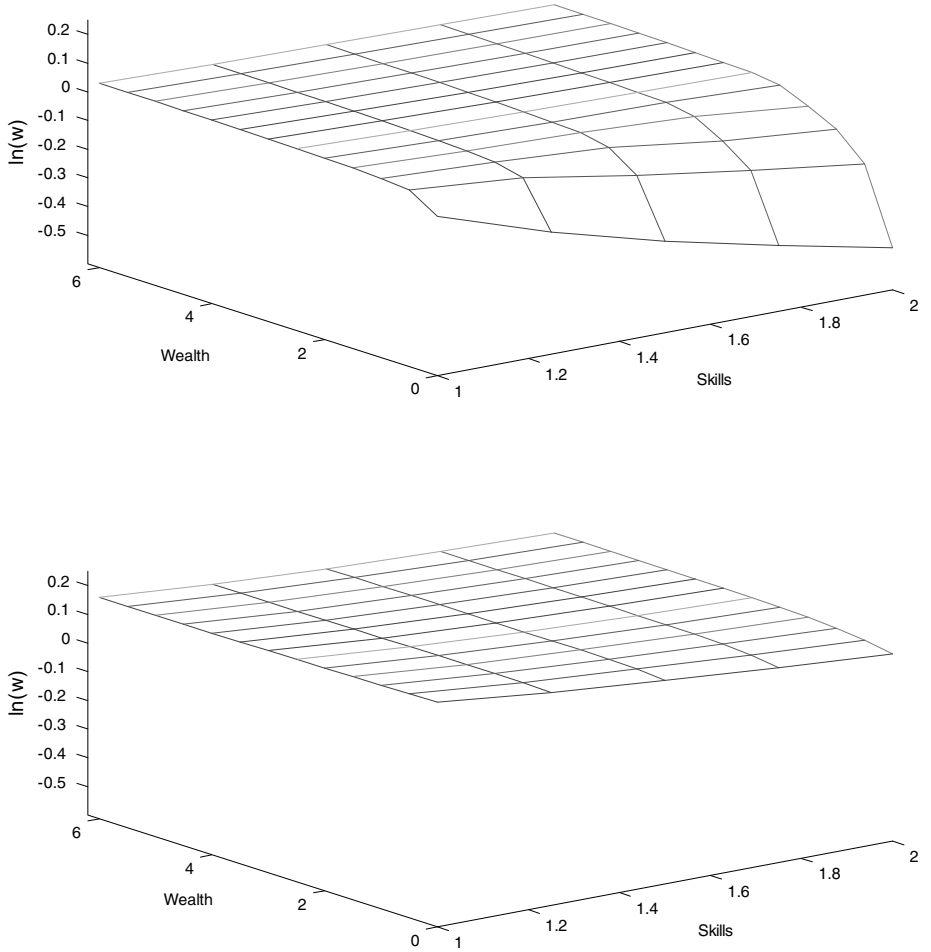
Figure 3.2: Reservation wage policies as functions of wealth and skill level. The scale in the wealth dimension was normalized by the average, monthly wage, net of taxes. The upper pane shows the policy of workers with no U.I. entitlement; the lower pane represents workers with the highest entitlement level.

Table 4: Wealth Holdings of the Unemployed Relative to Income Loss

Percentage loss	UI.				
	Earnings	Eligibility	Benchmark	Basic	Basic Skills
median	0.56	1.19	0.55	0.58	0.43
< 25% of loss	0.33	0.28	0.25	0.20	0.35
< 50% of loss	0.49	0.41	0.47	0.43	0.55
< 100% of loss	0.59	0.48	0.72	0.71	0.77

Notes: Columns 2 and 3 reproduce results from Gruber (2001) on the ratio of wealth to income loss using two different definitions of income loss. Wealth is measured as 'gross financial wealth'. The first row shows the median, and the following three rows show the fraction of agents with wealth less than a certain percentage of the incurred loss. The three last columns shows corresponding statistics for the benchmark equilibrium and for two other model versions. For full details, see text.

third columns of Table 4 reproduces the relevant results in Gruber's study.[18] In order to evaluate the realism of workers' savings adequacy in the benchmark equilibrium, a large sample of unemployment spells were drawn from model simulations. Data from this synthetic sample was used to compute statistics on the ratio of wealth to income loss that correspond as closely as possible to the statistics calculated by Gruber (2001). The results from this exercise are displayed in column 4 of Table 4.

When comparing the ratio of wealth to income loss in the sample in Gruber (2001) and in the synthetic sample, it is important to keep in mind that not all unemployed

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[18] See Table 3 in Gruber (2001). The numbers in Table 4 above are taken from columns 2 and 3 in Gruber's study and uses 'gross financial assets' as a measure of workers' wealth.

Americans that are entitled to payments from the UI. program actually claim their benefits. Estimates of the take-up rate ranges from 0.53 to 0.71.[19] To tackle this problem, Gruber (2001) calculates the adequacy statistic using several different definitions, the results from two of which are reproduced in Table 4. The numbers in column 2 ignore unemployment benefits altogether, with the implication that these numbers overestimate the size of the income loss incurred by unemployed households. The numbers in column 3 were calculated under the assumption that all eligible workers actually claim their benefits, a strategy that will underestimate the size of the income loss. These two sets of numbers can thus be viewed as a lower and upper bound, respectively, on the true ratio of wealth to income loss of unemployed U.S. workers. The first row of table 4 shows the median ratio of wealth to income loss in the S.I.P.P. sample and in the benchmark equilibrium, with the median of the synthetic distribution at 0.55, close to the lower bound of 0.56 calculated by Gruber (2001). The following three rows show the fraction of all spells where household wealth, prior to the beginning of the spell, corresponded to less than 25% of the incurred income loss, less than 50% of the income loss etc. As can be seen in the third column, the model economy has somewhat too many agents whose wealth prior to the spell was smaller than 100% of the lost income, and somewhat too few agents whose wealth was smaller than 25% of the lost income. Even so, the numbers in the last column line up surprisingly well with the statistics computed from Gruber's (2001) sample; although the fit is not perfect, the model clearly produces levels of savings adequacy that are at a realistic level. It should be noted that none of these statistics were targeted in the calibration process. The only statistic of the wealth distribution that was pinned down in the calibration was the median level of savings-to-income of employed, tenured workers.

3.5.3. The Role of Heterogeneity. Compared to standard models of frictional unemployment, the benchmark economy has additional layers of worker heterogeneity. It is reasonable to ask whether this added complexity is needed to achieve realistic predictions, notably in the dimension of savings adequacy. To answer that question, this subsection takes a look at two alternative and simpler versions of the benchmark

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[19] See Anderson and Meyer (1997), note 2.

model. The simplest of these alternative models, labeled 'the basic model', has no skill dynamics and no state dependence of UI. entitlements. As in the benchmark model, agents can be either workers or retired citizens. Workers in the basic model differ with respect to their level of savings, and workers with a job also differ with respect to the wage rate (per efficiency unit of labor) they currently receive.[20] In many respects, the problem solved by workers in this simple model resembles that of the steady state of models such as Gomes et al. (2001), Krusell et al. (2010) and Shao and Silos (2007), although those models also include equilibrium effects not accounted for here. For comparability, the parameters of the basic model were recalibrated in order for the equilibrium outcomes to match, when applicable, the same target statistics as the benchmark equilibrium does. In particular, the equilibrium of the basic model has an average unemployment rate of 7%, an average spell duration of 13 weeks and a median ratio of savings to weekly wages of tenured workers of 10.2[21]. The 5th column in Table 4 shows results on savings adequacy computed from a synthetic sample generated by the basic model. Judged by these statistics, the basic model appears to do a reasonable job in accounting for the savings adequacy of unemployed workers. Even so, the fraction of unemployed workers who have very low levels of savings adequacy, with savings smaller than 25% of the incurred income loss, is considerably lower than in the benchmark model.

The left-hand tails of the distributions of workers' wealth in the two models are displayed in Figure 3.3, where the distribution of the benchmark equilibrium is represented by the solid line, and where the dashed line represents the basic model. For comparability in terms of savings adequacy, the units on the horizontal axis were normalized by the average, monthly wage rate, net of taxes, in the respective models. The multimodal shape of the wealth distribution of the benchmark model is a result of the small number of skill levels in the calibrated model. The first peak of the distribution, at around 0.8 units of the average wage, correspond, loosely, to agents with the lowest skill level. At the second peak, agents' average skill level is close to 2, etc. The figure

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[20] Refering back to the model definition in section 3, the basic model is obtained by fixing the state variables (γ, b) at constant values.

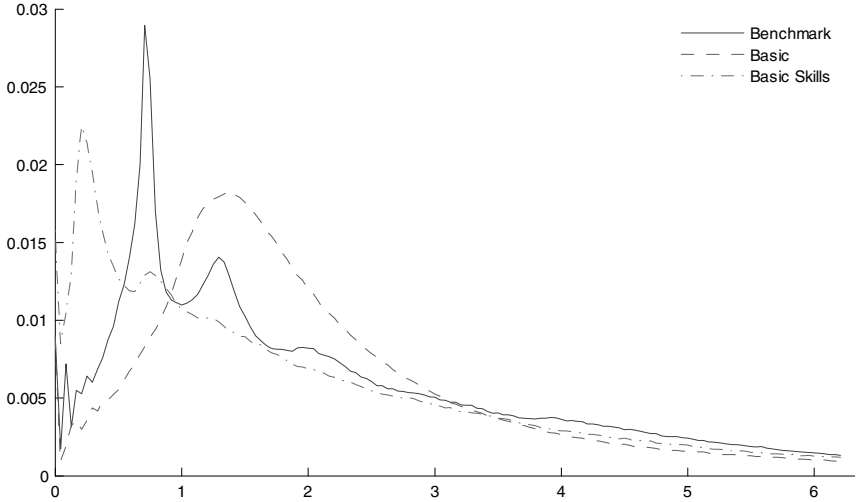[21] The parameters of the basic model are listed in the last Appendix.

Figure 3.3: Equilibrium Wealth Distributions. Note: One unit on the horizontal axis corresponds to one average, monthly wage in each respective model.

reveals that, compared to the basic model, the benchmark model has a larger fraction of workers that have savings the equivalent of one monthly wage, or less. In return, the basic model has a larger mass of agents with savings in a range that goes from one to approximately three average, monthly wages.

The increased dispersion in savings adequacy in the benchmark model, relative to the basic model, is due to the change in the incentives to save caused by the introduction of skill dynamics.[22] Workers with relatively little skills expect higher wages in future periods, due to the expected accumulation of skills, and they therefore see little reason to save. However, as these workers accumulate more skills, the potential for further wage increases becomes smaller, at the same time as the risk of loosing skills in case

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[22] The intuition provided here is due to Lise (2010), who studies the joint determination of the wage and wealth distribution in a wage-ladder model. Though the mechanism that causes wages to increase over the life cycle in his model is different from the skill dynamics used here, the intuition for why savings dispersion increases is valid in both models.

of a lay-off becomes larger. When workers gain skills, their incentives to save therefore become stronger, and savings rates increase.

Figure 3.3 also displays the wealth distribution of a third model, which I will refer to as 'the basic skill model', the second of the two simplified models referred to above. The basic skill model includes the skill dynamics of Ljungqvist and Sargent (1998), specified and calibrated exactly as in the benchmark model, but it has the same rudimentary UI. program as the basic model: in every period, all unemployed workers receive the same level of unemployment benefits from the government, irrespective of their previous earnings history. The wealth distribution of the basic skill model is represented by the dash–dotted line Figure 3.3, and this distribution exhibits even more dispersion than that of the benchmark model. Relative to the benchmark equilibrium, low-skilled workers who inhabit the basic skill model face higher replacement rates, and highly skilled workers face lower replacement rates. In this way, the rudimentary UI. schedule in the basic skill model reinforces the mechanism described in the previous paragraph: with high effective replacement rates, low-skilled workers have even smaller incentives to save, while high-skilled workers save at even higher rates.

3.5.4. The Quantitative Importance of Missing Insurance Markets. By assumption, workers in the model economy are unable to adequately insure against the risk of income losses. This is the reason why agents let their level of wealth influence their reservation wages, and thereby the incidence and duration of unemployment. But how important are the missing insurance markets to the aggregate outcomes of the labor market? To answer this question, this section will contrast the outcomes of the benchmark equilibrium to outcomes from a model with risk neutral agents. This 'linear model' has the same wage offer distributions, the same skill dynamics and, save for the risk aversion, the same parameters as the benchmark model. Because the model abstracts from the demand side of capital markets, there is no reason to include asset markets into the linear model. Consequently, there are no bequests either. The formal specification of the linear model is spelled out in Appendix C.

The benchmark model was calibrated, in section 5, so as to match 10 different statistics that characterize the U.S. labor market. The second column of Table 5 reproduces the corresponding 10 statistics of the benchmark calibration, while column 3 shows the same statistics, computed from the equilibrium distribution of the linear model. The risk neutral workers who inhabit the linear model maximize the present discounted sum of expected future income. Workers in the benchmark model care not only about their expected income, they also worry about variation in their consumption path. As a result, they set lower reservation wages, particularly if their wealth position is small in relation to their expected income. This difference in behavior shows up in the average duration of unemployment, which is roughly 3 weeks higher in the linear model, compared to the benchmark. Because the risk neutral workers invest more time in job search activity, the average level of unemployment is 7.6% in the linear model, 0.6 percentage points higher than in the benchmark model. In the equilibrium of the benchmark model, 27% of all workers have savings that are smaller than one monthly wage, net of taxes. As was shown in Figures 1 and 2, this is a level of savings at which you would expect many agents to make quite drastic changes to their reservation wages in order to avoid large drops in consumption. However, Figures 1 and 2 also revealed that at these low levels of wealth, the elasticity of the reservation wage with respect to wealth depends critically on the agents entitlement and skill-level. Highly skilled agents with low UI entitlement make dramatic changes to their search policies when their savings are run down; agents with low skills and/or high UI entitlement do not. It is thus interesting to know the composition of UI entitlements and skills that are most common among agents at the bottom of the wealth distribution. Table 3 shows that in the cross section of all workers, about 27% have entitlements at levels 1 and 2 (that is, no entitlement at all or low UI entitlement). Of these workers, 78% have assets that are smaller than the average monthly wage. In other words, poor workers are highly overrepresented among workers with low UI entitlements. However, it also turns out that there are unproportionally few workers that have both small savings and high skills. Of all workers in the model economy, 33% have skills at levels 4 or

5. However, of these highly skilled workers, little more than 3% have assets that are smaller than the average monthly wage.

Table 5: Equilibria With and Without Risk Aversion

	Model	Linear	Model	Linear
Risk aversion (σ)	2	–	3	–
Unemployment rate	7.0%	7.6%	7.0%	8.1%
Average spell duration	13.0 weeks	15.6 weeks	13.0 weeks	16.2 weeks
Ratio of wealth to				
average weekly wage (median)	10.2	–	10.2	–
Average yearly wage growth	2.9%	2.8%	3.1%	2.8%
Skill losses of displ. workers:				
1 year after displ.	34%	35%	35%	37%
6 years after displ.	24%	24%	25%	25%
Government policy:				
average UI. repl. rate	45%	48%	45%	49%
ratio of welfare payment to average wage	24%	24%	24%	23%
average repl. rate public pension	45%	–	45%	–
average tax rate	22%	22%	22%	22%

In the benchmark calibration, the coefficient of relative risk aversion, σ, was set to 2. As a robustness exercise, the model was recalibrated under the assumption that $\sigma = 3$. Thus, the subjective discount factor, β, the variance of the wage-offer distributions (σ_e and σ_u) and the parameters defining government policy were changed so that the model with higher risk aversion would still match the main target statistics. However,

to avoid an unreasonable computational burden, the parameters governing the distribution of skills, $\{\omega_e, \omega_u, \omega_d\}$, were kept the same as in the benchmark calibration. The parameters of this second calibration can be found in Appendix D.

The outcome of this robustness exercise are shown in columns 4 and 5 in Table 5. With a higher degree of risk aversion, the difference between the models with and without risk aversion increases, as could be expected. Thus, the unemployment rate increases from 7% to 8.1%, and the average duration of unemployment increases from 13 weeks to 16.2 weeks.

3.6. Concluding Remarks

This study uses a model of job search in order to evaluate the quantitative importance of risk aversion and missing insurance markets to the level and duration of unemployment among blue collar workers. Key features of the model are skill dynamics and an unemployment insurance system with state-dependent benefit payments. When calibrated to replicate the median ratio of liquid wealth to earnings among tenured American workers, the model makes realistic predictions concerning the levels of savings adequacy among unemployed workers. Skill dynamics help to explain why a considerable fraction of unemployed workers have low levels of wealth adequacy.

In the benchmark calibration, workers have a moderately low level of risk aversion, with a coefficient of relative risk aversion at 2. Compared to a model with risk neutral agents, the model with risk aversion has an average unemployment rate that is 0.6 percentage points lower, and an average duration of unemployment that is the 2.6 weeks shorter. In the benchmark calibration, there are relatively many workers with little savings, and some of these workers make dramatic changes to their reservation wages in response to relatively small changes in savings. Based on these results, it would perhaps be reasonable to expect larger differences between the aggregate outcomes of the models with and without risk aversion. The explanation for the relatively small effects on the average level and duration of unemployment lies in the policy functions of poor workers, and in the composition of workers who are poor. Unemployed workers

with little skills face relatively high effective replacement rates from the public UI system. Therefore, these workers do not have great difficulties to smooth consumption between states of employment and unemployment. As a result, the elasticity of the reservation wage with respect to savings is relatively low for such workers. In contrast, highly skilled workers face relatively low effective replacement rates, and their need to use own savings for 'self-insurance' is therefore greater. Accordingly, the reservation wages of highly skilled but poor workers are very responsive to relatively small changes in their assets. However, an inspection of the joint distribution of wealth and skills reveals that the proportion of workers with both little wealth and high skills is very low. Because highly skilled workers face relatively low effective replacement rates, they have a strong incentive to build a buffer stock of savings in order to avoid large drops in their consumption. The simulations also show, not surprisingly, that when workers risk aversion is higher, there are greater effects of risk aversion and incomplete markets on the average level and duration of unemployment. When the coefficient of relative risk aversion is 3, the model with risk aversion has an unemployment rate that is more than one percentage point lower than in the model with risk neutral agents.

There are several reasons why the quantitative effects of risk aversion and incomplete markets on the aggregate level of unemployment are interesting to policy makers. Chetty (2008) focuses on the increased duration of unemployment caused by UI benefits. In general, more generous benefits may cause longer unemployment spells either because the benefits affect workers' substitution between work and leisure, or because higher benefits increase the liquidity of borrowing constrained, unemployed workers. The welfare consequences of these two effects are very different, and in order to take these differences into account when designing optimal policy, it is essential to have models that produce realistic predictions concerning the savings adequacy of unemployed workers. A similar argument can be made concerning the cost of business cycles. In economic downturns, when jobs are hard to come by, workers' welfare is likely to be more severely affected if many workers are liquidity constrained and therefore unable to use own savings to buffer the income loss associated with unemployment. In order

to design adequate economic policy in order to dampen such downturns, and in order to attenuate their negative effects on workers' welfare, policy makers would need to understand and quantify the effects of risk aversion and incomplete markets on labor market outcomes.

References

Aiyagari, S R (1994), Uninsured Idiosyncratic Risk and Aggregate Saving, *Quareterly Journal of Economics* **109**, 659-684.

Alexopoulos, M and T Gladden (2004), *The Effects of Wealth, and Unemployment Benefits on Search Behavior and Labor Market Transitions* (Mimeo, University of Toronto).

Algan, Y, A Cheron, J O Hairault and F Langot (2003), Wealth Effects on Labor Market Transitions, *Review of Economic Dynamics* **6**, 156-178.

Alonso-Borrego, C, J Fernández-Villaverde and J E Galdón-Sánchez (2005), *Evaluating Labor Market reforms: A General Equilibrium Approach*, NBER Working Paper 11519.

Alvarez, F and M Veracierto (2001), Severance payments in an economy with frictions, *Journal of Monetary Economics* **47**, 477-498.

Anderson, M and B D Meyer (1997), Unemployment Insurance Takeup Rates and the After-Tax Value of Benefits, *Quarterly Journal of Economics* **112**, 913-938.

Barillas, F and J Fernández-Villaverde (2007), A generalization of the endogenous grid method, *Journal of Economic Dynamics and Control* **31**, 2698-2712.

Bewley, T (undated), *Interest Bearing Money and the Equilibirum Stock of Capital*, (Mimeo).

Bloemen, H G and E Stancanelli (2001), Individual Wealth, Reservation Wages, and Transitions into Employment, *Journal of Labor Economics* **19**, 400-439.

Blundell, R, L Pistaferri and I Preston (2008), Consumption Inequality and Partial Insurance, *American Economic Review* **98**, 1887-1921.

Budría Rodríguez, S, J Díaz-Giménez, V Quadrini and J-V Ríos-Rull (2002), Updated Facts on the U.S. Distributions of earnings, Income, and wealth, *Federal Reserve Bank of Minneapolis Quarterly Review* **26**, 2-35.

Castanẽda, A, J Díaz-Giménez and J-V Ríos-Rull (2003), Accounting for the U.S. Earnings and Wealth Inequality, *Journal of Political Economy* **111**, 818-857.

Chetty, R (2008), Moral Hazard vs. Liquidity and Optimal Unemployment Insurance, *Journal of Political Economy* **116**, 173-234.

Coronado, J, B Fullerton and T Glass (2000), *The Progressivity of Social Security*, NBER Working Paper 7520.

Danforth, J P (1979), On the role of consumption and decreasing absolute risk aversion in the theory of job search, in S A Lippman and J J McCall (eds.), *Studies in the Economics of Search*, North Holland.

Diamond, P A (1981), Mobility Costs, Frictional Unemployment, and Efficiency, *Journal of Political Economy* **89**, 798-813.

Eisensee, T (2006), *Essays on Public Finance: Retirement Behavior and Disaster Relief*, Stockholm University.

Gomes, J, J Greenwood and S Rebelo (2001), Equilibrium Unemployment, *Journal of Monetary Economics* **48**, 109-142.

Gouveia, M and R Strauss (1994), Effective Federal Individual Income Tax Functions: An Exploratory Empirical Analysis, *National Tax Journal* 47, 317-339.

Gruber, J (2001), The Wealth of the Unemployed, *Industrial and Labor Relations Review* **55**, 79-94.

Gustman, A and T Steinmeier (2001), How Effective is Redistribution under the social Security Benefit Formula?, *Journal of Public Economics* **82**, 1-28.

Jacobson, L, R LaLonde and D Sullivan (1993), Earnings Losses of Displaced Workers, *American Economic Review* **83**, 685-709.

Krebs, T (2007), Job Displacement Risk and the Cost of Business Cycles, *American Economic Review* **97**, 664-686.

Krusell, P, T Mukoyama and A Sahin (2010), Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations, *Review of Economic Studies* **77**, 1477-1507.

Lentz, R and T Tranas (2005), Job Search and Savings: Wealth Effects and Duration Dependence, *Journal of Labor Economics* **3**, 467-490.

Liebman, J (2002), Redistribution in the Current U.S. Social Security System, in M Feldstein and J Liebman (eds.), *Distributional Aspects of Social Security and Social Security Reform*, University of Chicago Press.

Lise, J (2010), On-the-Job Search and Precautionary Savings: Theory and Empirics of Earnings and Wealth Inequality (Mimeo, University College London).

Ljungqvist, L and T Sargent (1998), The European Unemployment Dilemma, *Journal of Political Economy* **106**, 514-550.

Mortensen, D and C Pissarides (1994), Job Creation and Job Destruction in the Theory of Unemployment, *Review of Economic Studies* **61**, 397-415.

Pissarides, C (1985), Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages, *American Economic Review* **75**, 676-690.

Quadrini, V (1999), The Importance of Entrepreneurship for Wealth Concentration and Mobility, *Review of Income and Wealth* **45**, 1-19.

Quadrini, V (2000), Entrepreneurship, Saving, and Social Mobility, *Review of Economic Dynamics* **3**, 1-40.

Reichling, F (2007), Optimal Unemployment Insurance in Labor Market Equilibrium when Workers can Self-Insure (Mimeo, Stanford University).

Shao, E and P Silos (2007), Uninsurable Individual Risk and the Cyclical Behavior of Unemployment and Vacancies (Mimeo, Federal Reserve Bank of Atlanta).

Storesletten, K, C Telmer and A Yaron (2004), Consumption and Risk Sharing over the Life Cycle, *Journal of Monetary Economics* **51**, 609-633.

Topel, R (1991), Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority, *Journal of Political Economy* **99**, 145-176.

U.S. Department of Labor (2007), *The Comparison of State Unemployment Insurance Laws*, available at www.workforcesecurity.doleta.gov.

Appendix A - Solution Algorithm and Lottery Model

Due to the discrete choice between employment and search, the choice set of active agents in the model is non-convex. As a consequence, the first order conditions associated with the two corresponding Bellman equations are not sufficient conditions for an optimal solution. From a numerical point of view, this is a serious drawback because it precludes the use of so called endogenous grid methods, which could otherwise substantially reduce the time needed to find good approximations to the value functions.[23] The state space of the model is relatively large and computational speed is therefore a serious constraint when carrying out the calibration. In order to address this problem, the solution algorithm makes use of an alternative model specification, one that includes wealth lotteries. In what follows, I will refer to the model presented in the main body of the paper as the 'original model', while referring to the alternative specification as the 'lottery model'. The use of lotteries to convexify otherwise non-convex constraint sets is well known to the literature. Gomes et al.(2001) discuss the problem of non-convex constraint sets in search models with endogenous savings and reservation wage policies. Lentz and Tranas (2005) use wealth lotteries in a model with endogenous savings and variable search intensity in order to derive certain properties of the optimal search policy. In this paper, I use wealth lotteries in the alternative lottery model in order to gain computational speed.

The remaining part of this Appendix is organized in the following way. The next section defines the alternative model specification with lotteries, and section 3.6.2. describes the solution algorithm. The last section discusses the method used to find agents' preferred lotteries.

3.6.1. Economy with Lotteries.

In the original model, active agents make at most three choices per period: 1) they decide whether to work or to search; 2) if they chose to work, active agents also choose how much work effort to exert; and 3) all agents decide how much to save and how much to consume. In the lottery model, active agents make five choices per period.

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[23] Barillas and Férnandez-Villaverde (2007) develop an endogenous grid method that is suitable to solve Neoclassical growth models.

In addition to the first two choices, the one regarding their employment status, and the decision about work effort, agents choose three variables that together define a wealth lottery. These three variables will be denoted a_1, a_2 and μ_a, and they represent, respectively, the two possible outcomes of the lottery, and the expected value of the outcome. The lottery works in the following way. With some probability, call it π, an agent who buys a lottery $\{a_1, a_2, \mu_a\}$ in period t starts out in period $(t+1)$ with wealth $(1+r)a_1$. With probability $(1-\pi)$ her wealth will instead be $(1+r)a_2$. π is defined by the condition: $\mu_a = \pi a_1 + (1-\pi)a_2$. Three constraints replace the borrowing constraint of the original model, namely $a_1 \geq \bar{a}$, $\mu_a \geq a_1$ and $\mu_a \leq a_2$. The last two of these constraints imply $\pi \epsilon [0,1]$. Note that an agent can always choose $\mu_a = a_1$ or $\mu_a = a_2$, in which case the lottery is trivial, and her level of savings in the beginning of next period will be $(1+r)\mu_a$ with certainty. The cost to a bank of providing any given lottery is μ_a. Because there is assumed to be perfect competition in the banking industry, this will also be the equilibrium price of the lottery.

Let $\overline{W}(a, \gamma, w, b)$ denote the value function of an agent who inhabits the lottery model, and who is currently employed at the wage rate w, has savings a, skill level γ and who, in case of unemployment, is entitled to UI benefits at the level of b. $\overline{S}(a, \gamma, b)$ will denote the value to an agent, in the lottery model, of being unemployed in state (a, γ, b). For ease of exposition, I define the two functions $\Phi_W(a', \gamma, w, b)$ and $\Phi_S(a', \gamma, b)$:

$$\Phi_W(a', \gamma, w, b) \equiv \lambda \beta R(a') + \delta(1-\lambda)\beta E_d[S(a', \gamma', b')|\gamma] \qquad (3.\text{A}.1)$$

$$+(1-\delta)(1-\lambda)\beta E_w\left(\max[\overline{W}(a', \gamma', w', b'), \overline{S}(a', \gamma', 0)]\big|\gamma, w, b\right),$$

$$\Phi_S(a', \gamma, b) \equiv \lambda \beta R(a') + (1-\lambda)\beta E_s\left(\max[\overline{W}(a', \gamma', w', b'), \overline{S}(a', \gamma', b')]\big|\gamma, w, b\right). \quad (3.\text{A}.2)$$

Under the lottery specification, the worker's program can be written:

$$\overline{W}(a, \gamma, w, b) = \max_{c, e, a_1, a_2, \mu_a} \left\{ u(c, e) + \pi \Phi_W(a_1, \gamma, w, b) + (1 - \pi) \Phi_W(a_2, \gamma, w, b) \right\},$$
(3.A.3)

s.t.

$$c + \mu_a \leq \gamma w e - \tau(\gamma w e) + (1 + r)a + q,$$

$$a_1 \geq \overline{a}, \quad \mu_a \geq a_1, \quad a_2 \geq \mu_a.$$

The searcher's program reads:

$$\overline{S}(a, \gamma, b) = \max_{c, a_1, a_2, \mu_a} \left\{ u(c, 0) + \pi \Phi_S(a_1, \gamma, b) + (1 - \pi) \Phi_S(a_2, \gamma, b) \right\},$$

s.t.

$$c + \mu_a \leq (1 + r)a + q + b,$$

$$a_1 \geq \overline{a}, \quad \mu_a \geq a_1, \quad a_2 \geq \mu_a.$$

For any state where the optimal choice $\{a_1, a_2, \mu_a\}$ is a non-trivial lottery, i.e. when $a_1 < \mu_a < a_2$, the necessary first order conditions associated with 3.A.3 imply:

$$\frac{\Phi_W(a_2, \gamma, w, b) - \Phi_W(a_1, \gamma, w, b)}{a_2 - a_1} = \frac{\partial \Phi_W(a_2, \gamma, w, b)}{\partial a_2} = \frac{\partial \Phi_W(a_1, \gamma, w, b)}{\partial a_1} + \frac{\phi}{\pi}, \quad (3.A.4)$$

where ϕ is the Lagrange multiplier associated with the constraint $a_1 \geq \overline{a}$. An analogous condition characterizing the lotteries chosen by searchers is associated with the

searcher's program. A retired citizen of the lottery model will always choose a trivial lottery; there is no point, therefore, to restate the problem of retired agents.

3.6.2. Solution Algorithm. The decision problems of both the original model and the lottery model are solved for by value function iteration. The calibration of either model includes the specification of a set Γ of possible skill levels γ, and a set B of possible UI transfer levels b. Let $I \equiv \Gamma \bullet B$ denote the Cartesian product of these two sets. For each point $i\epsilon I$, a collection of functions, $\{f_i^W(a,w), f_i^S(a)\}$, are used as approximands of the value functions. $R(a)$ is approximated by one function, $f^R(a)$. In the savings dimension, these value functions exhibit high curvature close to the borrowing constraint, while in the wage dimension, the interpolation problem is less demanding. For these reasons, $f_i^S(a)$ and $f^R(a)$ are chosen to be piecewise cubic splines, and $f_i^W(a,w)$ are piecewise cubic in a-space and piecewise linear in w-space. The particular piecewise cubic spline algorithm that is used preserves monotonicity, and is due to Judd (1998). Once good approximations of the value functions are obtained, the model is simulated for a large number of periods, until aggregate variables are constants. All model statistics presented in the main text were derived from large samples of observations drawn from such stationary distributions.

The endogenous grid method is explained in detail by Barillas and Fernández-Villaverde (2007). The idea is to fix a grid, call it A_X, over agents' choices of asset holdings for period $(t+1)$, and to use the first order conditions of the Bellman equation to back out the corresponding asset holdings in period t. In each iteration, this procedure results in an 'endogenous' grid A_N over asset holdings in period t. It is A_N that is used to interpolate the value functions. The important advantage of the endogenous grid method is that it requires only one evaluation of agents' continuation values in each function iteration. The algorithm that was used to solve for the value functions of the lottery model differs in two ways from the one outlined in Barillas and Fernandéz-Villaverde (2007). One is the inclusion of a subroutine that finds the lotteries preferred by active agents. This routine is outlined in some detail in the following section. The second and computationally much less cumbersome subroutine deals with

the proper location and spacing of the endogenous grid in a-space, A_N. Because the model has a binding borrowing constraint, A_N must have some desirable properties in order for the interpolation to work smoothly. First, A_X should be specified in such a way that the resulting grid points in A_N are all greater than or equal to $\bar{a}$. (Note that since agents are born into the model with non-negative wealth, and since in all periods $a_2 \geq \mu_a \geq a_1 \geq \bar{a}$ must be satisfied, the constraint $a \geq \bar{a}$ must also hold.) Second, the value functions tend to exhibit strong curvature close to $\bar{a}$, while being almost linear at higher wealth levels. For this reason, it is desirable that A_N is dense close to $\bar{a}$ and more sparse at higher asset levels. In order to achieve these two objectives, the main function iteration algorithm includes a subroutine that effectively carries out a series of iterations on a small number of grid-points. The first order conditions are solved at these points, which together constitute a subset of A_X, after which the resulting endogenous grid points, a subset of A_N, are checked against some predetermined criteria. When these criteria are met, the algorithm modifies A_X in a suitable way, and then performs one function iteration on the full set A_X.

3.6.3. Finding the Preferred Lotteries. Consider first the problem of an employed agent, and let $\Phi_W(a', \gamma, w, b)$ be as defined in equation 3.A.1. For any state (γ, w, b) and price $\mu_a \geq \bar{a}$ of a lottery, define the function $\Lambda^W(\mu_a \,|\, \gamma, w, b)$:

$$\Lambda^W(\mu_a \,|\, \gamma, w, b) \equiv \max_{a_1, a_2}\{\pi\Phi_W(a_1, \gamma, w, b) + (1-\pi)\Phi_W(a_2, \gamma, w, b)\} \qquad (3.A.5)$$

s.t.

$$a_1 \geq \bar{a}, \quad \mu_a \geq a_1, \quad a_2 \geq \mu_a.$$

As before, $\pi = \frac{a_2 - \mu_a}{a_2 - a_1}$. With $\bar{a} = 0$, $a_1, a_2 \,\epsilon\, \mathbb{R}_+$.

An analogous function $\Lambda^S(\mu_a \,|\gamma, b)$ constitutes the continuation value of an unemployed agent:

$$\Lambda^S(\mu_a \,|\gamma, b) \equiv \max_{a_1, a_2}\{\pi\Phi_S(a_1, \gamma, b) + (1 - \pi)\Phi_S(a_2, \gamma, b)\} \qquad (3.A.6)$$

s.t.

$$a_1 \geq \overline{a}, \quad \mu_a \geq a_1, \quad a_2 \geq \mu_a.$$

What follows is a description of the method used to find $\Lambda^W(\mu_a \,|\gamma, w, b)$ and its associated policy functions $a_1(\mu_a \,|\gamma, w, b)$ and $a_2(\mu_a \,|\gamma, w, b)$. The method that finds $\Lambda^S(\mu_a \,|\gamma, b)$ and its policy functions is analogous, with $\Phi_S(\cdot, \gamma, b)$ substituted for $\Phi_W(\cdot, \gamma, w, b)$.

The lottery algorithm uses the fact that $\Lambda^W(\mu_a \,|\gamma, w, b)$ is concave in μ_a; it also relies on the following assumption:

Assumption: At any $x\epsilon\mathbb{R}_+$ and state (γ, w, b) where $\Lambda^W(x \,|\gamma, w, b) = \Phi_W(x, \gamma, w, b)$, $\Phi_W(x, \gamma, w, b)$ is once continuously differentiable.

For any state (γ, w, b) and for any $x\epsilon\mathbb{R}_+$, define the two linear functions f^x and g^x that are tangent, respectively, to $\Lambda^W(x \,|\gamma, w, b)$ and $\Phi_W(x, \gamma, w, b)$:

$$f^x : \mathbb{R}_+ \to \mathbb{R}, \text{ with } f^x(z) = \alpha_0^x + \alpha_1^x z, \quad \alpha_1^x = \frac{\partial\Lambda^W(\mu_a \,|\gamma, w, b)}{\partial\mu_a}\Big|_{\mu_a = x} \text{ and } \alpha_0^x = \Lambda^W(x \,|\gamma, w, b) - \alpha_1^x x,$$

$$g^x : \mathbb{R}_+ \to \mathbb{R}, \text{ with } g^x(z) = \beta_0^x + \beta_1^x z, \quad \beta_1^x = \frac{\partial\Phi_W(\mu_a, \gamma, w, b)}{\partial\mu_a}\Big|_{\mu_a = x} \text{ and } \beta_0^x = \Phi_W(x, \gamma, w, b) - \alpha_1^x x.$$

For the first of these two functions, the following condition must hold:

$$\text{for any } z\epsilon\mathbb{R}_+, \quad f^x(z) \geq \Lambda^W(z \,|\gamma, w, b) \geq \Phi_W(z, \gamma, w, b). \qquad (3.A.7)$$

The first inequality in 3.A.7 follows from the concavity of $\Lambda^W(\cdot\,|\gamma,w,b)$, and the second follows from the definition of $\Lambda^W(\cdot\,|\gamma,w,b)$. Now, at any state (γ,w,b) and for any choice $x\epsilon\mathbb{R}_+$ of savings, an agent will either prefer to hold a trivial lottery, with $a_1 = \mu_a$ or $a_2 = \mu_a$, or she will chose a non-trivial lottery with $a_1 < x < a_2$. In the case of a trivial lottery, $\alpha_1^x = \frac{\partial\Lambda^W(\mu_a|\gamma,w,b)}{\partial\mu_a}\big|_{\mu_a=x} = \frac{\partial\Phi_W(\mu_a,\gamma,w,b)}{\partial\mu_a}\big|_{\mu_a=x}$ and $\alpha_0^x = \Lambda^W(x\,|\gamma,w,b) - \alpha_1^x x = \Phi_W(x,\gamma,w,b) - \alpha_1^x x$, i.e. $f^x(z) = g^x(z)$, all $z\epsilon\mathbb{R}_+$. Conversely, suppose that for a state (γ,w,b) and a choice $x\epsilon\mathbb{R}_+$, there is at least one $z\epsilon\mathbb{R}_+$ such that $g^x(z) < \Phi_W(z,\gamma,w,b)$, implying that $g^x(z) < f^x(z)$; then in that state (γ,w,b) and for that choice of savings x, it must be that agents prefer a non-trivial lottery. The algorithm uses this result to find intervals in $\mathbb{R}_+$ where agents prefer non-trivial lotteries. For each state (γ,w,b), a dense grid $A_L^0 \subset \mathbb{R}_+$ is specified, and for each grid-point $x\epsilon A_L$, the algorithm finds the coefficients β_0^x and β_1^x that define g^x. Each function g^x is then evaluated at all grid-points $z\epsilon A_L^0$, and checked against the following condition, which is an operational analog of 3.A.7:

$$g^x(z) - \Phi_W(z,\gamma,w,b) \geq -\delta_L, \qquad (3.A.8)$$

where δ_L is a small, positive real number that serves as a convergence criteria. Whenever a function $g^x(z)$ violates 3.A.8 for one or more points $z\epsilon A_L^0$, it is concluded that at the corresponding state (γ,w,b) and savings level x, agents want to hold a non-trivial lottery. For each such grid-point, or interval of grid-points, the algorithm also needs to find the numbers a_1^x and a_2^x that constitute the possible outcomes of the lottery. This is done by an iterative process, where each iteration i involves the specification of a new grid A_L^i, with grid-points located at intervals where agents switch between trivial and non-trivial lotteries.

In order to achieve a good approximation to $\Lambda^W(\mu_a\,|\gamma,w,b)$, the grid A_L^0 needs to be dense. In fact, if A_L^0 where to cover more than a small subspace of A_X, the computational cost of the lottery algorithm would be prohibitively high. In practice, however, agents hold non-trivial lotteries only at low levels of savings. This is because

at moderately high and at high level of savings, agents' reservation wage function is relatively inelastic with respect to savings.

Before initiating the algorithm, specify a grid A_L^0, with grid-points located close to the borrowing constraint, and chose a number $\delta_L \epsilon \mathbb{R}_+$ to serve as convergence criteria.

1) For each $x \epsilon A_L^i$, evaluate $\Phi_W(x, \gamma, w, b)$ and $\frac{\partial \Phi_W(\mu_a, \gamma, w, b)}{\partial \mu_a} \big|_{\mu_a = x}$, and compute the coefficients β_0^x and β_1^x.

2) Evaluate all functions g^x at all grid-point $z \epsilon A_L^i$ (and possibly, if $i > 1$, at all $z \epsilon A_L^i \cup A_L^{i-1} \cup ... \cup A_L^{i-N}$ for some choice of N). Check condition 3.A.8; for each g^x that violates 3.A.8, conclude that the corresponding state (γ, w, b) and savings level x is one where agents prefer non-trivial lotteries. This procedure yields a separation of A_L^i into two mutually exclusive sets NT^i and T^i that contain, respectively, all points $x \epsilon A_L^i$ where agents prefer non-trivial lotteries, and all points that are candidates for points where agents prefer trivial lotteries.

3) For each $x \epsilon NT^i$, find two elements a_1^x and a_2^x in T^i such that:

$$|x - a_1^x| \leq |z - a_1^x| \qquad \text{all } z \epsilon T^i \text{ such that } z < x, \text{ and}$$

$$|x - a_2^x| \leq |z - a_2^x| \qquad \text{all } z \epsilon T^i \text{ such that } z > x.$$

a_1^x and a_2^x are candidates for the optimal policy associated with $\Lambda^W(\mu_a | \gamma, w, b)$ at $\mu_a = x$.

4) For each $x \epsilon NT^i$, specify a function $h^x(z) = \gamma_0^x + \gamma_1^x z$, with $\gamma_1^x = \frac{\Phi_W(a_2^x, \gamma, w, b) - \Phi_W(a_1^x, \gamma, w, b)}{a_2^x - a_1^x}$ and $\gamma_0^x = \Phi_W(a_1^x, \gamma, w, b) - \gamma_1^x x$. Evaluate each function $h^x(z)$ at all gridpoints $z \epsilon A_L^i$ (and possibly, if $i > 1$, at all $z \epsilon A_L^i \cup A_L^{i-1} \cup ... \cup A_L^{i-N}$ for some choice of N) and check if $h^x(z) - \Phi_W(z, \gamma, w, b) \geq -\delta_L$. If a_1^x and a_2^x are good approximations to

$a_1(\mu_a \,|\gamma, w, b)$ and $a_2(\mu_a \,|\gamma, w, b)$ at $\mu_a = x$, then $h^x(z)$ is a first-order Taylor expansion of $\Lambda^W(\mu_a \,|\gamma, w, b)$ around x, and should satisfy 3.A.7.

5) If, for all $x\epsilon NT^i$, the corresponding function $h^x(z)$ satisfies $h^x(z) - \Phi_W(z, \gamma, w, b) \geq -\delta_L$ at all grid-points $z\epsilon A_L^i$, stop iteration. If $h^x(z) - \Phi_W(z, \gamma, w, b) \geq -\delta_L$ is violated for at least one function $h^x(z)$ at one $z\epsilon A_L^i$, specify a new grid A_L^{i+1}. A_L^{i+1} should be dense close to all points a_1^x and a_2^x that were not accepted as good approximations to $a_1(\mu_a \,|\gamma, w, b)$ and $a_2(\mu_a \,|\gamma, w, b)$. Restart the procedure from 1).

Appendix B - Calibration of the Skill Loss Distribution

Section 5.2 of the main text deals with the calibration of the skill dynamics, which is governed by the following three distributions: T_E, T_S and T_U. T_S relates to the drastic skill losses of laid-off workers, and is the subject of this appendix. Assume first that for any original skill level h, all laid-off workers will face the same probability of loosing one skill level, two skill levels etc. Let d_h denote the distance between two adjacent skill levels; since these levels partitions the interval $[1, 2]$ evenly, any two levels are separated by the same distance d_h. Find the probability that a random variable with distribution $N(2, \omega_d^2)$ realizes within each of the H intervals $[\gamma_1 - \frac{d_h}{2}, \gamma_1 + \frac{d_h}{2}], ..., [\gamma_H - \frac{d_h}{2}, \gamma_H + \frac{d_h}{2}]$. Normalize these probabilities so that they sum to one, call them $[\pi_d^1, ..., \pi_d^H]$, and let them represent the probability skill distribution, in period $(t+1)$, of a worker with the highest skill level γ_H, who was laid off between periods t and $(t+1)$. As an example, π_d^{H-1} will represent the probability that such a high-skilled worker looses one skill level between the two periods. However, in keeping with the above assumption of equal loss probabilities, this number will also represent the probability that a separating worker with skill level $(H-1)$ will see her skills immediately depreciate to level $(H-2)$, etc. For each level of original skills, the probability that a worker keeps that original skill level will be adjusted so that the resulting distribution sums to one. To summarize, let TT represent a transition matrix of skills for newly laid-off workers, with entry TT_{ij} denoting the probability that a laid-off worker with skill level i starts out in the next period with skill level j. For any standard deviation ω_d of a Normal distribution, there will be a corresponding transition matrix TT_{ω_d}, defined in the following way:

$$TT_{\omega_d} = \begin{bmatrix} 1 & 0 & . & . & 0 \\ \pi_d^{H-1} & 1 - \pi_d^{H-1} & 0 & . & 0 \\ . & . & . & . & . \\ \pi_d^1 & . & \pi_d^{H-1} & 1 - \sum_{k=1}^{H-1} \pi_d^k & 0 \\ \pi_d^1 & . & . & \pi_d^{H-1} & \pi_d^H \end{bmatrix}.$$

Appendix C - Linear Model and Calibrated Parameters

The first part of this Appendix outlines the linear model, and the second part document the parameters of all calibrations referred to in the main text.

3.6.4. Linear Model. The linear model has the following utility function:

$$u_L(c, e) = c - \frac{e^{1+\theta}}{1+\theta}.$$

There are no asset markets in the linear model, and consequently no bequests either. The program of an employed agents is:

$$W_L(\gamma, w, b) = \max_{c,e,} \{u_L(c, e) + \delta(1-\lambda)\beta E_d[S_L(\gamma', b')|\gamma]$$

$$+(1-\delta)(1-\lambda)\beta E_w \left(\max[W_L(\gamma', w', b'), S_L(\gamma', 0)]|\gamma, w, b\right)\},$$

s.t.

$$c \leq \gamma we - \tau(\gamma we).$$

Here, $S_L(\gamma', b')$ is the value function of an unemployed worker. This function is defined as follows:

$$S_L(\gamma, b) = \max_c \{u_L(c, 0) + (1-\lambda)\beta E_s \left(\max[W_L(\gamma', w', b'), S_L(\gamma', 0)]|\gamma, w, b\right)\},$$

s.t.

$$c \leq b.$$

Table 1: Calibrated Parameters

		Benchmark	High Risk Av.	Basic	Basic Skills
β	subjective discount rate	0.9705	0.938	0.9630	0.9615
σ	coefficient of rel. risk aversion	2	3	2	2
θ	inverse of Frisch el. of labor supply	10	10	10	10
δ	per period prob. of lay-off	0.0174	0.0174	0.0174	0.0174
ρ	persistence of wage of empl. workers	0.9990	0.9990	0.9990	0.9990
σ_e	std. of shock to wage of empl. workers	0.0245	0.0350	0.0455	0.0430
σ_u	std. of wage distr. unemployed workers	0.2600	0.3640	0.2800	0.2800
ω_e	prob. of skill accumulation (empl. workers)	0.0200	0.0200	—	0.0200
ω_u	prob. of skill loss (unemployed workers)	0	0	—	0
ω_d	std. of shock governing skill losses	0.3000	0.3000	—	0.3000
b_w	welfare payment	0.4100	0.4800	0.44	0.59
b_4	highest UI benefit	0.8400	1.050	—	—
s	transfer to retired workers	0.7600	0.9000	0.5600	0.8300
a_0	parameter of tax function	0.2580	0.2580	0.2580	0.2580
a_1	”	0.7680	0.7680	0.7680	0.7680
a_2	”	2.2000	1.9000	2.2000	2.2000

CHAPTER 4

Why Reservation Wages May Fall in Wealth

Henrik Lundvall

ABSTRACT. In a McCall search model with declining absolute risk aversion and incomplete markets, unemployed workers' reservation wages increase in the level of their savings (Danforth [1979]). The intuition for this result is straightforward: search is an investment in higher expected future wages, and more wealthy individuals chose to invest more than do poor individuals. However, this paper shows that when the model is augmented with stochastic, on-the-job skill accumulation, so that more work experience is associated with higher wages, the reservation wage of an unemployed worker may be either increasing or decreasing in her wealth. In the model with skill accumulation, the unemployed worker weigh two different investment decisions against each other: she may invest in more search in order to find a job that pays a higher wage per efficiency unit of labor, or she may lower her reservation wage in order to invest in quicker (expected) skill accumulation, thereby increasing her total wage income. Reservation wages are more likely to decrease in wealth when the potential gain associated with skill accumulation is high and when the replacement rate is high.

4.1. Introduction

This paper makes a simple point concerning the reservation wage policy of unemployed workers. The context is a simple search model with risk averse workers and incomplete insurance markets. Danforth (1979) showed that in such a model, the reservation wage of the unemployed worker must increase in the level of her savings. This paper shows that when the model is augmented with on-the-job skill accumulation, so that wages are expected to increase in work experience, the reservation wages set by unemployed workers may either increase or decrease in wealth. Contrary to the results obtained by

Danforth (1979), the sign of the slope of the reservation wage with respect to savings thus cannot be determined *a priori.*

The intuition for this result is analogous to the intuition for why reservation wages always increase in wealth in the model with no skills. The problem facing the agent in the model with no skills can be thought of as an investment decision: a higher reservation wage increases the expected future wage at the cost of a longer expected duration of unemployment. Search is an investment in higher future wages, and because the worker is unable (by assumption) to smooth consumption perfectly between the states of unemployment and employment, wealthier agents choose to invest more time in search than do less wealthy agents. When the model is augmented with skill dynamics, the unemployed worker faces not one but two investment opportunities. Like in the model without skills, a higher reservation wage represents more investment in search and in a higher expected wage. However, a higher reservation wage now also imply, in expectations, that the worker will delay the time at which she starts working, thereby delaying the time at which she will start accumulating skills. In this sense, a higher reservation wage represents less investment in quicker skill accumulation. In some situations, the second investment decision will dominate the first, so that agents choose lower reservation wages the larger is their buffer stock of savings. These situations coincide with those where the potential for future skill accumulation induce unemployed workers to set reservation wages that are lower than the level of unemployment benefits, net of the cost associated with search.

The results obtained by Danforth (1979) imply that the more savings an unemployed worker has, the longer she is expected to be unemployed. An analogous result is obtained by Lentz and Tranas (2005) in a model where the control variable of unemployed workers is a search intensity instead of a reservation wage: wealthier agents choose to exert less search effort, so that higher levels of savings are associated with longer expected durations of unemployment. These two theoretical findings have motivated a

number of empirical studies that test the relationship between wealth and unemployment durations. (See Algan et al. [2002], Alexopoulos and Gladden [2004], Bloemen and Stancanelli [2001] and Chetty [2008].) While all these studies find that more wealth (on average) leads to longer expected durations, the estimated size of this effect varies considerably. The results presented in this paper suggest one reason why existing estimates of the effect of wealth on expected durations may be imprecise and/or biased. In the model presented here, the slope of the reservation wage with respect to wealth depends crucially on the potential for wage increases associated with on-the-job skill accumulation. This characteristic of the model suggests that the effects of wealth on the probability of gaining employment depend on the type of job an unemployed worker is looking for, as it is reasonable to assume that the potential for skill accumulation differs between jobs. As pointed out by Chetty (2008), the curvature of the policy function in the savings dimension has important policy implications: the total effect on unemployment durations from changes in the UI replacement rate can be decomposed into a 'substitution effect' (consumption vs. leisure) and a 'liquidity effect', where the latter effect stems from the inability of workers to perfectly smooth consumption. An increase in unemployment durations that is due to the substitution effect has negative welfare implications, while effects coming through the liquidity channel do not. The findings in Chetty (2008) therefore stress the importance of correctly measuring the strength of these liquidity effects, effects that depend directly on the curvature of the policy function. The model presented in the following section shows that the curvature of the reservation wage function looks quite different for different workers, depending on whether or not they face potential wage increases associated with on-the-job skill accumulation.

The following section describes the search model. Skill accumulation is introduced in a very simple way: by varying one key parameter in the model, the potential for skill accumulation can be varied from zero to infinity. This modelling approach has two benefits. First, the stylized nature of the model makes it easier to focus on the basic economic forces that are in play. Second, by keeping the skill dynamics of the model

as simple as possible, the model can be thought of as a straightforward generalization of Danforth's (1979) model. When the key parameter is set to one, so that there is no potential for skill accumulation, the model collapses to that of Danforth (1979).[1] The main section, number 3, uses numerical methods to approximate the solution of the model under different parametrization. Section 4 offers some concluding remarks, while the appendix discusses the methods used for the numerical approximation.

4.2. Model

As in Danforth (1979), I assume that a an infinitely lived worker maximizes the expected discounted sum of present and future utility. The per period utility function $u(c)$ exhibits decreasing absolute risk aversion. Workers discount future utility at the per period factor $\beta \in (0, 1)$. In any given period, the worker is either unemployed or she has a job. Once a worker accepts a job offer, she remains employed forever. While unemployed, the worker receives a wage offer in the beginning of each period. The wage offer is a random draw from the stationary distribution $F(w) = \Pr\{W \leq w\}$, where $w \in [0, B]$. If the worker declines the offer at hand, she remains unemployed for one more period and then receives a new wage offer w' in the following period. An offer w that was declined in a previous period cannot be recalled. In every period of unemployment, the worker must pay a search cost c, but she also receives a transfer $\widehat{b}$ from a public unemployment program. I will denote by $b \equiv \widehat{b} - c$ the unemployment transfer net of search costs.[2]

Financial markets are incomplete in the sense that there is only one financial asset available to workers, a risk-free savings account that pays a net return r per period. Throughout this paper, it is assumed for simplicity that $r = 1/\beta - 1$. Because of some

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[1] There is one difference between the model used by Danfoth (1979) and the model presented here, a difference pertaining to the formulation of the borrowing constraint. This is explained in footnote 3 below.

[2] Danforth (1979) presents two versions of a search model, one with a finite time horizon and one with an infinite horizon. In this paper I consider only agents that are infinitely lived. In Danforth's (1979) model, the resources available to an unemployed agent consists of the assets she carried from last period, minus a search cost, denoted s. Of course, there is no intrinsic difference between that interpretation and the one adopted here, where the resources available to an unemployed agent equal her savings plus a transfer, b, interpreted as an unemployment benefit net of search costs. Danforth (1979) imposes no sign restrictions on the value of s.

kind of market friction, the savings account is constrained by an exogenous borrowing limit $\bar{a}$, so that in each period, the savings a of a worker must satisfy $a \geq \bar{a}$. An unemployed worker who saved an amount a in the previous period, and who chooses to decline the wage offer at hand faces the following budget constraint, where a' denotes the savings that will be carried to the next period:

$$c + a' \leq (1 + r)a + b. \tag{4.1}$$

An unemployed agent makes two decisions in each period: she decides whether to accept or reject the wage offer at hand, and she decides how much to save. Given the budget constraint, the latter decision determines her current level of consumption.[3]

When a worker accepts a wage offer w, she immediately starts working at the per period wage w. Just as b is thought of as the UI benefit net of search costs, I will think of w as the wage rate net of work effort.[4] Conditional on a specific wage offer w, the wage rate in the first period of work is thus deterministic: the worker knows that she will receive the wage w in that period. The worker also knows that she will never again be unemployed. However, in each subsequent period, there is a per period probability π that the wage rate will increase from w to γw, where $\gamma \geq 1$. I will refer to a worker with wage w as an 'unskilled worker' and to a worker with wage γw as a 'skilled worker'. Once skill accumulation has occurred, there is no more uncertainty in the life of the worker: she will continue to earn the wage γw forever more.

4.2.1. Timing and Solution. High-skilled workers solve an easy problem: they face no uncertainty, and the gross return they receive on savings exactly offsets the rate at which they discount future utility. As they prefer to smooth consumption over

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[3] While I assume that the same, exogenous borrowing constraint applies to both unemployed workers and workers that have a job, Danforth (1979) rationalizes the borrowing constraint of his model in terms of the maximum amount of money that an agent could possibly pay back with certainty, if she was to refrain from consumption in all future periods. Because this amount depends on the income of the agent, all unemployed agents face the same borrowing constraint, while workers with a job face a constraint that depends on the level of their wage.

[4] Let $\widehat{w}$ denote the actual wage rate paid by the employer, and let e be the dollar value of the effort a worker needs to exert on-the-job: $w = \widehat{w} - e$.

time, high-skilled workers set consumption equal in all periods. Denote by $W_H(a, w)$ the value function of a high-skilled worker who earns the wage rate w per efficiency unit of labor. It follows that:

$$W_H(a, w) = \frac{u(ra + \gamma w)}{1 - \beta}.$$

Low-skilled workers face one source of uncertainty, the timing of their transition from a low-skilled to a high-skilled worker. In the beginning of each period, a worker that was low-skilled in the previous period learns if skill accumulation has occurred or not. If skill accumulation did occur, her value function in the present period is that of a high-skilled worker. If skill accumulation did not occur, her problem can be described by the following Bellman equation:

$$W_L(a, w) = \max_{a'} \left\{ u[(1 + r)a + w - a'] + (1 - \pi)\beta W_L(a', w) + \pi\beta W_H(a, w) \right\}, \quad (4.2)$$

$$\text{s.t.}$$

$$a' \geq \bar{a}.$$

Unemployed workers face two sources of uncertainty. First, they do not know what their next job offer will be like (i.e. they do not know what wage rate w' they will be offered in the following period). Second, unemployed workers do not know how much time they will have to spend as low-skilled workers once they accept a job-offer. Both these sources of uncertainty affect the choices unemployed workers make concerning their saving and their reservation wage. In the beginning of each period, an unemployed worker learns the wage rate w of the job offer currently at hand. If she accepts that offer, her problem is that of a low-skilled worker described above. If she rejects the offer, she must wait one period until she can again sample the wage-offer distribution. Her value function, $S(a)$, is:

$$S(a) = \max_{a'} \left\{ u[(1+r)a + b - a'] + \beta \int\limits_{0}^{B} \max[S(a'), W_L(a', w')]dF(w') \right\}, \quad (4.3)$$

$$\text{s.t.}$$

$$a' \geq \overline{a}.$$

The problem defined in 4.3 has the reservation wage property, meaning that an unemployed worker (with a certain level of savings, a) will accept all wage offers above a certain amount, and will reject all other offers. I will refer to this cut-off point in the wage-offer distribution as the worker's reservation wage, $\overline{w}$. The model is solved using numerical techniques that involve iterations on the first order conditions associated with 4.2 and 4.3. These first order conditions and a brief description of the numerical methods are deferred to the appendix. Because unemployed agents are confronted with a discrete choice - to accept or reject a wage offer - the constraint set associated with their program is not convex. As a result, the first order conditions associated with the Bellman equation in 4.3 are not sufficient conditions for a solution. It is well known that search problems exhibit this difficulty when they are set in a context of incomplete markets. However, Gomes et al. (2001), Lentz and Tranas (2005) and Chetty (2008) all report that in numerical simulations using plausible parameter values, these non-convexities never arise. Here, I follow Chetty (2008) and simply assume that the first order conditions are in fact sufficient conditions for the optimal solution.

4.2.2. Calibration. A model period will be thought of as half a quarter, or 6 weeks. To make the yearly net return on the savings account 4%, the subjective discount rate, β, is set to 0.9951. I assume that the per period utility function, $u(c)$, is characterized by constant relative risk aversion, with a coefficient of relative risk aversion of 4. The borrowing constraint, $\overline{a}$, is set to 0. To keep the model as simple as possible, the wage offer distribution, $F(w)$, is assumed to be uniform on $[0, 1]$.

The analysis in the following section focuses on the reservation wage policy of unemployed workers. In particular, the idea is to start from a benchmark calibration,

and to study how the shape of the reservation wage policy changes when key para-
meter values are varied around their benchmark values. One parameter will be front
and center: γ, the gain in wages associated with skill accumulation. Besides γ, the
parameters that remains to be pinned down are b, the UI benefit (net of search costs),
and π, the per period probability that skill accumulation occurs. Because I want to
focus the analysis on the incentives that may induce workers to lower reservation wages
when their wealth increase, the benchmark values of these three parameters (γ, b, π)
will be set to values that make such behavior likely. Thus, γ is set to 2, implying that
skill accumulation doubles the wage rate, net of the cost of work effort. π is set to 0.4,
so that low-skilled workers are expected to become skilled after 2.5 model periods, or
15 weeks. Finally, the value of b is set to 0.98. The calibration of π and b may strike
the reader as unrealistic and some comments are therefore warranted.

Concerning the calibration of the UI benefit, remember, first, that w and b represent,
respectively, the wage rate *net of work effort,* and the UI benefit *net of search costs.*
The model replacement rate that correspond to observable measures of UI generosity
is $\frac{\widehat{b}}{\widehat{w}} = \frac{b+c}{w+e}$, where c represent the dollar value of search costs while e is the dollar
value of work effort. Assuming, which is reasonable, that e is considerably greater
than c, it is also reasonable to assume that $\frac{\widehat{b}}{\widehat{w}} = \frac{b+c}{w+e} < \frac{b}{w} = 0.98$, i.e. that the
actual, calibrated replacement rate is well below 0.98.[5] Another important distinction
is that between gross replacement rates and replacement rates net of taxes and benefits.
Martin (1996) computes net UI replacement rates for 18 OECD countries in 1994/95,
taking into account tax rules, housing benefits and child-related benefits. He finds that
net replacement rates were higher than the corresponding gross replacement rates in all
countries except Italy. According to Martin (1996), in the mid 90:s several countries
in Western Europe offered net replacement rates around 80%. The model economy
abstracts altogether from taxes and from child care and housing benefits. This is a

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[5] In a recent study of the time use of unemployed American workers, Krueger and Mueller (2010)
find that the average unemployed worker devotes little more than 40 minutes per weekday to search
for a new job.

second reason why the value of $\frac{b}{w}$ in the model economy should be set higher than the gross replacement rates observed in actual economies.

Contrary to the case of the UI benefit, the benchmark calibration of the stochastic skill process is obviously unrealistic. In the model economy, a low-skilled worker can expect her wage income to double after 15 weeks of work experience. As mentioned in the introduction, the search model with skill dynamics ($\gamma > 1$) offers two different investment opportunities to the unemployed worker. The worker can increase her reservation wage in order to invest more time in searching for a good wage offer. Or she can decrease her reservation wage, thereby investing in quicker skill accumulation. By shortening the expected duration of unemployment, a lower replacement rate shortens the expected time it will take before the worker becomes high-skilled. The cost of this investment is that the worker must be willing to accept lower wage offers. However, the stylized structure of the model will inevitably give an unproportionately high weight to the first of these two investment opportunities, thus obscuring the trade-off that real world workers must make between search and skill accumulation. In the labor market of the model economy, each worker is unemployed only once in her lifetime, and she holds only one single job in her entire career. As agents are infinitely lived, these simplifying assumptions will strongly emphasize workers' incentives to look for a good wage offer. A more realistic model would have the average worker transit several times between employment and unemployment, so that most workers would hold more than one single job in her lifetime. Such a model, therefore, would also make it relatively less important for agents to look for a single, good wage offer. At the same time, the incentives to accumulate (and keep) skills would be given a relatively stronger weight. However, as I argued in the introduction, the primary purpose of this study is not to gauge the quantitative importance of negatively sloped reservation wages. Instead, the focus is on the basic economic forces at work, a motivation that strongly speaks in favor of the stylized model. As was also pointed out in the introduction, this modelling approach has the additional benefit of keeping a close resemblance between the model used here and the model investigated by Danforth (1979).

4.3. Results

The top pane in Figure 1 shows the reservation wages chosen by unemployed workers at different values of γ. The vertical axis shows the value of wage offers, w, while the horizontal axis represents the level of wealth held by the worker. Both axes show values in units of the consumption good. At the top of the figure is the reservation wage corresponding to $\gamma = 1$, the case of no skill dynamics. Although not visible because of the scale on the vertical axis, this reservation wage increases in wealth at all levels of wealth. This monotonically increasing reservation wage illustrates the result obtained by Danforth (1979): when there is no potential for skill accumulation, more wealthy agents always set higher reservation wages than do less wealthy agents. Further down in Figure 1 are reservation wages that correspond to higher values of γ, with values ranging from $\gamma = 1.2$ (directly below the top reservation wage just discussed) to $\gamma = 3$, at the bottom of the figure. As can be seen clearly in the figure, the first order effect of a higher gain from skill accumulation is for the worker to lower her reservation wage at all levels of wealth. Already when $\gamma = 1.2$, an unemployed worker with relatively little wealth chooses a reservation wage that is lower than the benefit she can collect as unemployed. The second effect of a higher γ is a change in the shape of the reservation wage: the curvature of the reservation wage function increases, and close to the borrowing constraint, the function starts to exhibit a section where the slope is negative. To illustrate more clearly these changes in the shape of the policy function, the bottom pane of Figure 1 shows the same reservation wages as the ones shown in the top pane, but shifted vertically so that they all share the same intercept. The higher is γ, the larger is the range close to the borrowing constraint where reservation wages decrease in wealth, and the larger is the positive slope further to the right in the picture.

To understand these changes in the shape of the reservation wage function, it helps to consider how the expected path of future incomes change when γ increases. When $\gamma = 1$, there is only two employment states in the model: the agent can be either unemployed or she can have a job. At the high level of UI benefits considered here
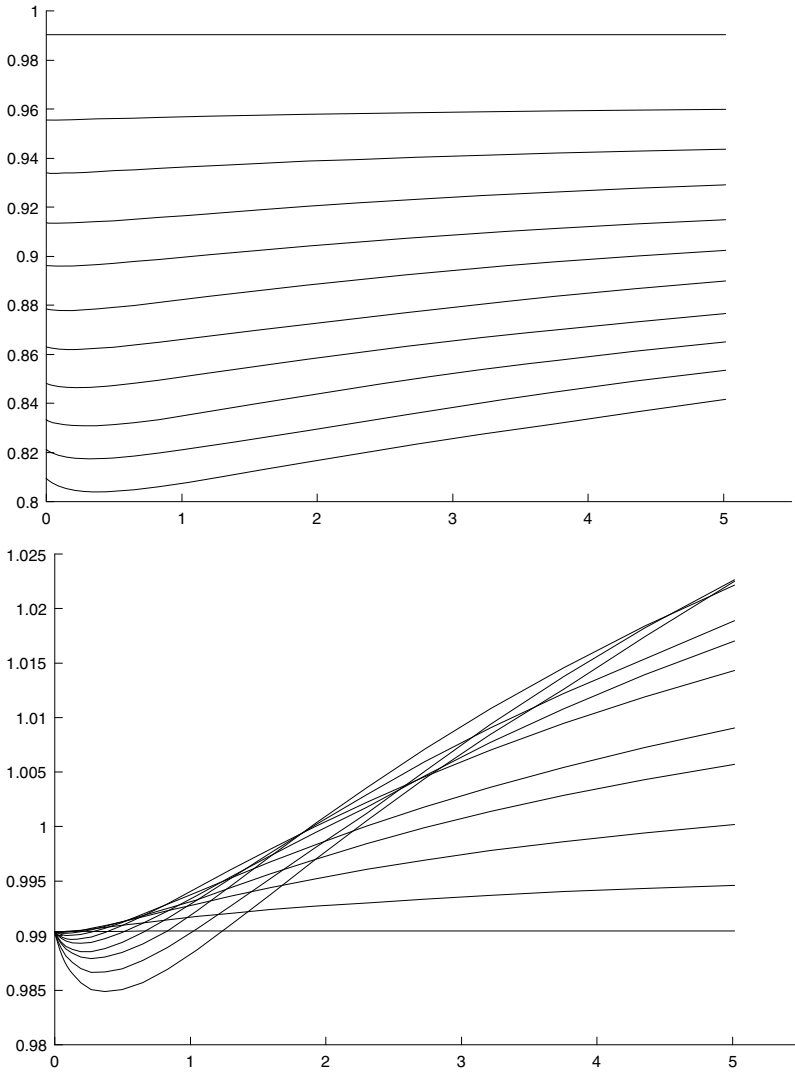
Figure 4.1: Reservation wages at different values of γ, as functions of wealth. The top pane shows reservation wages corresponding to 11 different values of γ, with $\gamma = 1$ at the top of the picture and higher values of γ further down. The bottom pane shows the same 11 reservation wages, but with the intercepts normalized to that of the reservation wage corresponding to $\gamma = 1$. Other parameters are set to their benchmark values. For further details, see text.

($b = 0.98$), an unemployed worker with $\gamma = 1$ can expect to receive a relatively even stream of income across the two states. As long as she stays unemployed, she receives b every period. As there is no reason to accept any wage offers below b, and since b is close to the upper bound of the wage offer distribution ($B = 1$), the worker knows that whenever she receives an acceptable wage offer, that wage will be at a level close to b. It is not surprising then that with this calibration, the reservation wage increases only modestly with higher levels of wealth. An unemployed worker who finds herself in this situation does no find it very difficult to smooth consumption between different states.

4.3.1. Reservation Wages Below b. When γ increases above 1, the unemployed agent faces an expected path of future incomes that is both higher and more uneven. The model now has three distinct employment states: the worker can be unemployed, she can have a job and be low-skilled, or she can have a job and be high-skilled. Because the third state involves an income that is potentially well above that of the first two states, a worker with $\gamma > 1$ finds it more difficult to smooth consumption. As an example, consider the case when $\gamma = 3$ and the worker has no initial wealth. With these characteristics, an unemployed worker sets her reservation wage to 0.809; her expected wage, once she accepts a wage offer, is 0.904[6]. When she accepts a job and starts working, she can expect her skills to appreciate after 2.5 periods, at which time her wage income will triple. The problem of 'self-insurance' facing this worker is considerably more difficult than that of a worker with $\gamma = 1$, and it therefore is no wonder that her level of savings will be more important to the choices she makes. As shown in the bottom pane in Figure 1, the reservation wage function increases more steeply in wealth, for most levels of wealth, when γ is close to 3 compared to the case when $\gamma = 1$. Consider now the section close to the borrowing constraint where reservation wages decrease in wealth. Also this effect has to do with the worker's desire to smooth consumption across states. As mentioned above, workers with $\gamma > 1$ may find it desirable to set their reservation wage below b. Contrary to the case when $\gamma = 1$, a worker with $\gamma > 1$ can benefit from setting $\overline{w} < b$: with a low $\overline{w}$, the worker

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[6] The expected wage rate was calculated as the average accepted wage in a simulation comprising 160 000 agents.

brings closer the point in time when she is expected to enter the third employment state, that of being a high-skilled worker, and the gain associated with such a strategy may outweigh the expected loss from accepting a lower wage rate. For a risk-neutral worker, the decision of a reservation wage boils down to finding a $\overline{w}$ that maximizes the discounted sum of expected future wages. When the worker is risk averse, however, she also cares about the profile of the income stream, shunning states with very low incomes. The strategy of setting $\overline{w} < b$ thus carries an extra cost to the risk averse worker: even if such a strategy may entail a higher discounted sum of expected future wages than if $\overline{w} \geq b$, it also decreases the lowest possible income she may encounter, making her income more uneven across different states. For this reason, the fact of having a little more savings may induce a worker to lower her reservation wage: if the worker accepts a wage $w < b$, she can draw on her savings to prop up consumption in that state. Loosely speaking, the worker's concern to smooth consumption across the two states of unemployment and low-skilled employment may sometimes induce her to lower her reservation wage at higher levels of wealth. However, at higher levels of wealth (above 0.4 units of the good for this calibration), the agent has enough resources to tackle income differences between states 1 and 2, and her concerns concentrate on the income differences between, on the hand, states 1 and 2 and, on the other hand, state 3. At this level of savings, the trade-off between a higher expected future wage (a higher $\overline{w}$) and a longer expected time spent in states 1 and 2 dominates the worker's choice of a reservation wage, and the reservation wage function starts increasing in wealth.

4.3.2. Unemployment Benefits and Impatience. The preceding paragraph argued that reservations wages will only decrease in wealth when agents set reservation wages such that $\overline{w} < b$. To explore this a bit further, the top pane of Figure 4.2 shows how reservation wages vary with different values of b. At the top of the picture is the reservation wage function that agents choose when $b = 0.99$. Reservation wage functions further down in the picture correspond to lower values of b, the lowest line corresponding to $b = 0.5$. As the unemployment benefit becomes less generous, it

becomes less and less likely that the worker chooses a reservation wage below b. The fourth line from the top shows the reservation wage function corresponding to $b = 0.9$. At this level of the benefit, the worker still sets $\overline{w}$ below b (for the ranges of wealth shown here), and there is a small section where the reservation wage function has a negative slope (although this is hard to distinguish in the picture) . However, as b decreases to 0.8 (the fifth line from the top), the worker no longer finds it optimal to set $\overline{w} < b$, and the reservation wage function is now monotonically increasing in wealth. At lower levels of b, the positive slope of the function increases: the lower is b, the greater is the need to 'self-insure', and the more important is the size of the worker's buffer stock of savings.

The subjective discount factor, β, is another parameter that can shed light on the incentives that govern workers' actions. It was argued in section 2.2 that the stylized nature of the model's skill dynamics will tend to overemphasize a worker's incentive to search for a good wage offer, at the expense of her incentive to quickly accumulate skills. Since each worker only hold one single job in her career, the wage rate at that job plays a tremendous role in determining her expected income. At the same time, skill accumulation occurs to all agents sooner or later; the only thing the worker can do is to slightly affect the timing of that event. However, by making the worker less patient (through manipulation of β) it is possible to strengthen her incentives for quicker skill accumulation: with a lower β, the agent attaches relatively less weight to consumption levels later in her life, which are entirely determined by the wage rate w, and instead puts relatively more weight to consumption levels early in her life, which are affected by the exact timing of her skill accumulation. The bottom pane of Figure 4.2 shows workers' reservation wages, as a function of wealth, for 5 different values of β. At the top of the picture is the reservation wage corresponding to $\beta = 0.995$. As β is successively lowered, the policy function shifts down, illustrating how the agent becomes successively less concerned about finding a high w, and instead cares more about achieving quick skill accumulation. As the worker lowers her reservation wage,
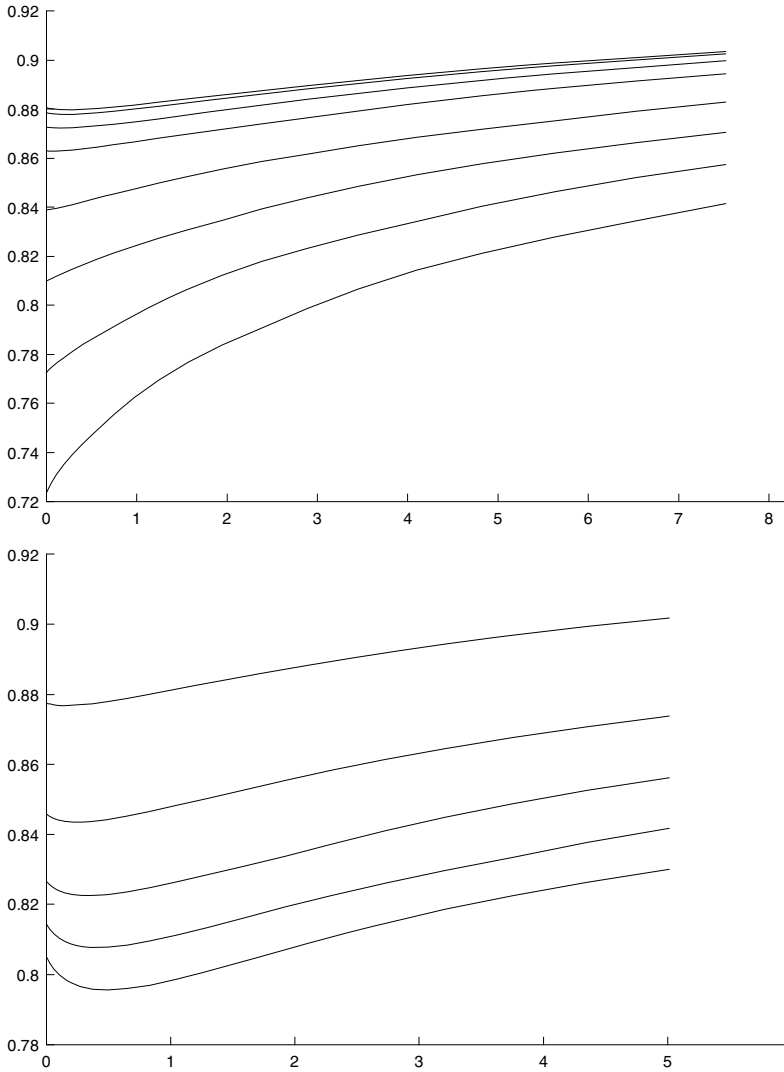
Figure 4.2: Reservation wages at different values of b and β, as functions of wealth. The top pane shows reservation wages corresponding to 9 different values of b, with $b = 0.99$ at the top of the pane and lower values of b further down. The bottom pane shows reservation wage functions for 5 different values of β. At the top of the pane is the reservation wage function corresponding to $\beta = 0.995$, and further down are functions corresponding to lower values of β, with $\beta = 0.975$ at the very bottom. Other parameters are set to their benchmark values. For further details, see text.

the low expected income in state 2 (having a job as low-skilled) becomes more of a concern from the point of view of consumption smoothing, and the reservation wage function starts exhibiting a larger section with a negative slope. The lowest line in the picture corresponds to a β of 0.975.

4.4. Concluding Remarks

This paper has shown that when a simple search model is augmented with stochastic skill dynamics, the reservation wage function of an unemployed worker may either increase or decrease in the level of her wealth. It is assumed that workers are risk averse and that insurance markets are incomplete. This finding stands in contrast to the results obtained in models with no skill dynamics, where the reservation wage function is known *a priori* to increase in wealth (Danforth [1979]). On-the-job skill accumulation and incomplete insurance markets are realistic features of actual labor markets. This paper highlights, in a general way, the interaction of these two elements in determining workers' incentives. Larger returns from investments in skills make it harder for workers to smooth consumption over the life cycle, thereby increasing the shadow-value of wealth. Large potential gains from skill accumulation may also make it worthwhile for workers to search for and accept jobs that are relatively low-paid, as such jobs may be regarded as a stepping-stone to more qualified and skill-intense employment. To put up with such an investment may be more bearable if the savings account is not empty. These findings suggest that workers' behavioral response to various labor market policies may be highly heterogenous, depending on the individual persons work history and her willingness and ability to learn new skills. To take but one example, young workers are often poor, borrowing constrained and unskilled, and under some circumstances they may therefore be expected to respond more strongly to cash grants and unemployment benefits than do older workers.

What this paper does not do is to answer the following question: how likely are workers to find themselves in a situation where more savings would induce them to lower their wage demands? One way to answer that question would be to build a more elaborate

and quantitatively realistic model, one which allowed for repeated spells of unemployment and employment, and where skills could be accumulated gradually over time. Another, more empirical approach is also possible. Most empirical studies of the effect of wealth on transition probabilities out of unemployment rely on estimations where the transition probability of the individual worker is a function of a number of controls, including wealth. The model analyzed in this paper suggest that such empirical specifications take into account interactions between a worker's level of savings and her skill level.

References

Alexopoulos, M and T Gladden (2004), *The Effects of Wealth, and Unemployment Benefits on Search Behavior and Labor Market Transitions* (Mimeo, University of Toronto).

Algan, Y, A Cheron, J O Hairault and F Langot (2003), Wealth Effects on Labor Market Transitions, *Review of Economic Dynamics* **6**, 156-178.

Bloemen, H G and E Stancanelli (2001), Individual Wealth, Reservation Wages, and Transitions into Employment, *Journal of Labor Economics* **19**, 400-439.

Chetty, R (2008), Moral Hazard vs. Liquidity and Optimal Unemployment Insurance, *Journal of Political Economy* **116**, 173-234.

Danforth, J P (1979), On the role of consumption and decreasing absolute risk aversion in the theory of job search, in S A Lippman and J J McCall (eds.), *Studies in the Economics of Search*, North Holland.

Krueger, A B and A Mueller (2010), Job Search and Unemployment Insurance: New Evidence from Time Use Data, *Journal of Public Economics* **94**, 298-307.

Lentz, R and T Tranas (2005), Job Search and Savings: Wealth Effects and Duration Dependence, *Journal of Labor Economics* **3**, 467-490.

Appendix - Solution Algorithm

This appendix describes the solution, first, to the problem of a low-skilled and employed agent and, second, to that of an unemployed agent. Since employed agents face no risk of becoming unemployed, the solution to the first of these two programs is independent of that of the second. The solution to the employed agents' problem consists of a value function, $W_L(a, w)$, and two policy functions. Denote by $A_L(a, w)$ and $C_L(a, w)$, respectively, the savings policy and the consumption policy of a low-skilled employed agent with savings a and wage w. Also, let μ_L denote the Lagrange multiplier associated with the borrowing constraint. The conditions that characterize the solution to the first problem are:

$$-\frac{\partial u(c_L)}{\partial a'} + (1 - \pi)\beta\frac{\partial W_L(a', w)}{\partial a'} + \pi\beta\frac{\partial u(c'_H)}{\partial a'} + \mu_L = 0, \qquad (4.A.1)$$

$$\mu_L(a' - \bar{a}) = 0,$$

$$\mu_L \geq 0,$$

$$a' \geq 0.$$

Here, c_L and c'_H is shorthand for $C_L(a, w)$ and for $C_H(a', w)$, respectively, and $C_H(a, w)$ denotes the savings policy of a high-skilled, employed agent: $C_H(a', w) = ra + \gamma w$. Using the Envelope theorem and the assumption that $r = \frac{1}{\beta} - 1$, 4.A.1 can be written:

$$\frac{\partial u(c_L)}{\partial a'} = (1 - \pi)\frac{\partial u(c'_L)}{\partial a'} + \pi\frac{\partial u(c'_H)}{\partial a'} + \mu_L. \qquad (4.A.2)$$

The approximate, numerical solution is found by iteration on the policy functions, $A_L(a, w)$ and $C_L(a, w)$, using the above first order condition. The policy functions are approximated by splines that are piecewise cubic in the savings dimension and piecewise

linear in the wage dimension. For a motivation of this choice, see the appendix to Lundvall (2010).

Once a solution to the above problem is found, the policy functions and value function of an unemployed agent can be found in a similar way. Denote by $A_S(a)$ and $C_S(a)$ the two functions that describe an unemployed agent's decision to save and to consume, respectively, and denote by $\overline{w}(a)$ the reservation wage policy. Let μ_S represent the multiplier associated with the unemployed agent's borrowing constraint. The conditions that characterize the solution to this problem are:

$$-\frac{\partial u(c_S)}{\partial a'} + \beta \int_0^{\overline{w}'} \frac{\partial S(a')}{\partial a'} dF(w') + \beta \int_{\overline{w}'}^{B} \frac{\partial W_L(a', w')}{\partial a'} dF(w') + \tag{4.A.3}$$

$$\beta f(\overline{w}) \left[S(a') - W_L(a', \overline{w}') \right] \frac{d\overline{w}'}{da'} + \mu_S = 0,$$

$$\beta f(\overline{w}) \left[S(a') - W_L(a', \overline{w}') \right] = 0, \tag{4.A.4}$$

$$\mu_S(a' - \overline{a}) = 0,$$

$$\mu_S \geq 0,$$

$$a' \geq 0.$$

4.A.4 simply states that when the reservation wage of the next period, $\overline{w}'$, is chosen optimally, the value of accepting to work at that wage must equal the value of continued search. Using this condition together with the Envelope theorem and the assumption on r, 4.A.3 can be simplified:

$$\frac{\partial u(c_S)}{\partial a'} = \frac{\overline{w}'}{B} \frac{\partial u(c_S')}{\partial a'} + \int_{\overline{w}'}^{B} \frac{\partial u(c_L')}{\partial a'} dF(w') + \mu_S. \tag{4.A.5}$$

Again, the numerical solution is found by iteration on the policy functions $(A_S(a)$, $C_S(a)$ and $\overline{w}(a))$ using the above, modified first order condition.[1]

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[1] Both problems are solved using the Matlab optimization routine *csolve*. This routine passes guesses on the policy $(a', \overline{w}')$ to a function that evaluates the two first order conditions. This function penalices values on a' that violate the borrowing constraint.