Biased Beliefs and Heterogeneous Preferences

Essays in Behavioral Economics

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Karen Khachatryan

Dissertation for the Degree of Doctor of Philosophy, Ph.D.
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Preface

This report is the result of a research project carried out at the Department of Economics at the Stockholm School of Economics (SSE).

This volume is submitted as a doctor's thesis at SSE. The author has been entirely free to conduct and present his research in his own ways as an expression of his own ideas.

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Filip Wijkström
Associate Professor
SSE Director of Research

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Last but not least, I would like to extend my gratitude to my friends and family. My mother, father, and sister, although at times from far away, have always been there for me. They have provided me with much needed love, support and encouragement. I dedicate this thesis to them.

Stockholm, November 2011
Karen Khachatryan

Contents

Summary of Dissertation

This dissertation is a collection of essays (chapters) on behavioral economics. Behavioral economics—arguably one of the most influential innovations in economics over the last 20 years—is a research paradigm introducing psychologically more realistic assumptions into economics. A common theme throughout the dissertation is the focus on either biased beliefs, or heterogeneous preferences, or both. The first chapter serves as an introduction to some themes in behavioral economics and its implications for market outcomes in industrial organization settings. The next two chapters are theoretical papers on entrepreneurial and managerial overconfidence that can also be thought of as contributions to this newly emerging field of behavioral industrial organization. The last chapter is an empirical contribution on gender differences in preferences and economic behavior at a young age.

Behavioral Industrial Organization

Recent advances in behavioral economics include developments of a number of models of richer choice behavior that can be readily applied (in place of *simple* utility maximization) in various environments, including those that belong to the traditional domain of industrial organization (Ellison, 2006; Rabin, 2002).

Chapter 1, "Behavioral Industrial Organization: A Survey", selectively reviews this newly emerging field of behavioral industrial organization, i.e., applications of behavioral economics in industrial organization settings. The deviations from perfectly rational decision making that result in this research being classified as "behavioral" can come in principle from any of the economic agents involved in the market. Recent research, however, usually attributes the "irrational" behavior to consumers and not to firms. Viewed from this narrower point of view behavioral industrial organization investigates the impact of psychological (ir)regularities in consumer behavior on the strategic choices of firms and the functioning of markets. The main goal of this selected review is to demonstrate the potential of behavioral approaches in industrial organization.

Entrepreneurial and Managerial Overconfidence

Much of economic life is conducted in an uncertain environment. The rational expectations hypothesis postulates that economic agents make decisions as if they held correct probabilistic beliefs, given their information. It is argued (Alchian, 1950; Friedman, 1953) that the rational-expectations hypothesis is justified, because agents holding rational expectations are more likely to survive the market test than those with nonrational expectations. Yet, psychological and empirical research on judgment under uncertainty provides extensive evidence that people tend to be *optimistic* (Peterson, 2006; Sharot, 2011; Taylor and Brown, 1988), exhibit *overconfidence* in judgement (DeBondt and Thaler, 1995; Weinstein, 1980, 1982, 1984, and the reviews therein), and are often *overconfident* about their own relative *ability* (Svenson, 1981). Psychologist Shelley E. Taylor sums up much of the evidence on optimistic biases in her comprehensive book on the subject, Taylor (1989), and even argues that unrealistic optimism is an indispensable trait of the healthy mind.

Chapter 2, "Entrepreneurial Overconfidence and Market Selection", which is a joint work with my advisor, Jörgen Weibull, examines the possibility that somewhat optimistic expectations may survive the market test, indeed, that agents holding such expectations may do better than those with rational expectations. More specifically, we explore whether competition between firms owned and run by entrepreneurs favors overconfident entrepreneurs. We study this question in a variety of settings, all based on Cournot duopoly in the product market. In the basic model, entrepreneurs choose their own firm's output and may have more or less optimistic beliefs about their own firm's (random) production costs. We study both the case of complete and incomplete information about the competitor's type. We also analyze a model with endogenous costs in the complete-information setting in which entrepreneurs make efforts to reduce their firm's production costs. For each of the model versions, we show that, if market selection is driven by firms' absolute and/or relative profit performance, somewhat overconfident entrepreneurs will be selected for, and that this tendency is stronger the more emphasis is placed on relative performance.

Chapter 3, "Overconfidence, Imperfect Competition, and Evolution", takes the theme from the previous chapter to a differentiated goods duopoly setting. In this chapter each of the two firms is run by a manager-*cum*-owner who may have incorrect beliefs about their firm's demand. We interpret a tendency to overestimate one's own firm's product demand—or the extent of product differentiation—as an expression for overconfidence. We show that when there is complete information about the competitor's type, overconfident managers enjoy a competitive edge when (1) either the firms compete by setting quantities and the products are imperfect substitutes; or (2) the firms compete by setting prices and the products are imperfect complements. However, further analysis reveals that evolutionary market selection forces will always favor a positive degree of managerial overconfidence. We also study the case of incomplete information about the competitor's type under quantity competition only and show that evolutionary forces may still favor overconfident managers if market selection is driven by relative rather than absolute profit performance.

Gender Differences in Preferences

Although women have made important strides in catching up with men in the workplace, a gender gap persists both in wages and in prospects for advancement. A number of reasons for this have been proposed in the economics literature including taste based or statistical discrimination, as well as gender differences in preferences, with the focus on competitiveness, risk preferences and social preferences. Women are in general found to be less risk taking than men, and sometimes also less competitive and more altruistic (see, for instance, Bertrand, 2010; Croson and Gneezy, 2009; Eckel and Grossman, 2008a,b; Engel, 2011).

Meanwhile, relatively little is known about the development of the gender gaps in economic preferences, to what extent children exhibit the same type of gender differences in preferences as adults do, and to what extent culture and context might matter. **Chapter 4,** "Gender Differences in Preferences at a Young Age? Experimental Evidence from Armenia", aims to contribute to further this understanding.

In this chapter we explore the gender gap in preferences among a large sample of children and adolescents in Armenia. We focus on competitiveness, risk preferences and altruism since these are the three areas in which gender differences are often found. Competitiveness is typically measured by either the performance change in response to a competitive setting compared to a noncompetitive setting, or the choice of whether to compete or not when given the choice between a competitive setting and a noncompetitive setting. We study competitiveness using both measures in running, skipping rope, math and word search, as well as competitiveness at the choice whether to compete or not in math and word search. We study risk preferences by having the children choose between different certain amount of points and a gamble. Finally, we study altruism through a dictator game where the recipient is a charity.

We find that boys and girls are equally competitive when it comes to performance change in skipping rope, math and word search, whereas girls are more competitive than boys in running. The latter result is different from what has previously been found among children, however this difference is only present among older children in our sample. There is no gender gap in the choice to compete or not in math or word search. We find that boys are more risk taking than girls and that this gap appears around the age of 12. We also find that girls are significantly more altruistic than boys.

Our results add to the literature on the importance of studying different samples, cultures and age groups when exploring the foundations of (gender differences in) economic preferences. Understanding the gender gap in competitiveness, risk preferences and altruism will hopefully further our understanding of the gender gaps observed in important outcomes such as those related to the labor market (see, e.g., Bertrand, 2010, for further discussion). Armenia is different than most previous countries previously studied in that it was part of the Soviet Union and that perhaps the communist policies aimed at influencing the position of women in Armenia during the Soviet era had a long lasting and deep impact on behavior.

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Chapters

Chapter 1

Behavioral Industrial Organization: A Survey*

Karen Khachatryan†

Abstract

This paper reviews the newly emerging field of Behavioral Industrial Organization, i.e., applications of Behavioral Economics to Industrial Organization settings. We present a few select recent models developed in the Behavioral Economics paradigm and then explore the implications of these models for a monopoly pricing problem. These models generalize consumer preferences and judgments to allow for: (1) preference for instant gratification (quasi-hyperbolic discounting); (2) overconfidence and biased beliefs; and (3) sensitivity to reference points and loss aversion. The main goal of this selected review is to demonstrate the potential of behavioral approaches in Industrial Organization.

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1. Introduction

Modern behavioral economics, also known as economics and psychology, enriches the standard economic models to account for psychological properties of preference and judgment, which create limits on rational calculation, willpower and greed, (see Camerer and Loewenstein, 2004; Fudenberg, 2006; Pesendorfer, 2006; Rabin, 1998, 2002, for recent reviews). The modified economic theory aims at providing parsimonious and psychologically sound explanations for empirical findings that the standard model has a tough time explaining. From a methodological perspective, behavioral economics is simply a humble approach to economics, which respects the comparative empirical advantages of neighboring social sciences and sees neighboring sciences as trading partners.

Recent advances in behavioral economics include the development of a number of models of richer choice behavior that can be readily applied (in place of *simple* utility maximization) in various environments, including those that belong to the domain of industrial organization. The deviations from perfectly rational decision making that result in this research being classified as "behavioral" can come in principle from any of the economic agents involved in the market. Recent research, however, usually attributes the "irrational" behavior to consumers and not to firms. The reason is that a firm is more likely to be careful in decision making, several people are often involved in making important decisions, and a firm can hire consultants to help it overcome its irrationality if such irrationality exists. Moreover, irrationality on the side of the firm should *presumably* reduce its profits, hurt the firm's position both in its product markets and in the capital markets in which it obtains financing, and eventually is likely to lead to the firm's bankruptcy due to more rational firms driving it out of business. Therefore it is *assumed* to be hard to believe that firms can behave significantly in an irrational fashion and still survive the competition for a long time.

It is much more plausible that consumers behave irrationally, since the above considerations do not apply to them. Consumers do not disappear if they make biased decisions, they just do not obtain a utility level as high as they could get with optimal decision making. A significant focus in this field, however, is to understand how firms can take advantage of the biases that consumers exhibit. For example, what would be the optimal strategy of firms in terms of pricing or product choice, given that consumers procrastinate and have present-biased preferences or other deviations from expected utility maximization?

Viewed from this narrower point of view behavioral industrial organization investigates the impact of psychological (ir)regularities in *consumer behavior* on the strategic choices of firms and the functioning of markets. Such consumers are clearly boundedly rational. Ellison (2006) distinguishes three kinds of bounded rationality in industrial organization models: (1) agents who use simple rules to make choices; (2) agents who find it costly to make decisions; and (3) agents who exhibit behavioral biases. The first two are closely related since agents with high decision or information costs typically find it optimal to adopt simple rules-of-thumb. Both of these approaches have a long tradition in industrial organization. The last approach is new and is driven by the recent advances in behavioral economics as we know it today. A typical paper in this new behavioral industrial organization literature retains the maximizing calculus of the rational economic models but replaces the consumer's utility functions with objective functions that incorporate a behavioral trait or bias.

In the rest of this paper, we selectively review some of the seminal research in this area.[1] By concentrating only on a few topics we are able to discuss them in some detail and show the merits of this approach. We begin each section by presenting a recent model developed in the behavioral economics paradigm, discussing a detailed application of that model in an industrial organization setting, and conclude with a brief of related research. First, in Section 2, we look at consumers who are time-inconsistent and have self-control problems, present a formal model that captures such behavior and then embed it in a contracting model by DellaVigna and Malmendier (2004), whose work nicely exhibits the potential of behavioral approaches in industrial organization. Then in Section 3 we look at consumers who are overconfident or myopic and present a model of contracting with overconfident consumers due to Grubb (2009) and a model of pricing in add-on markets due to Gabaix and Laibson (2006). In Section 4 we discuss loss aversion, present a model of reference dependent preferences due to Koszegi and Rabin (2006, 2007) that captures loss aversion and then present a model by Heidhues and Koszegi (2005, 2008), exploring the implications of consumer loss aversion for market outcomes.

2. Time Inconsistency and Self-Control Problems

People are impatient—they like to experience rewards soon and to delay costs until later. Economists almost always capture impatience by assuming that people discount streams of utility over time exponentially. An important qualitative feature of exponential discounting is that it implies that a person's intertemporal

[1] In this survey we focus on how poor or non-standard decision making by consumers might affect market outcomes. For a recent survey, that instead focuses on non-standard approaches to firm behavior, see Armstrong and Huck (2010). For a textbook treatment of the literature surveyed in this paper, and other models of interactions of "boundedly rational" consumers and rational firms, the interested reader is recommended to consult Spiegler (2011), who tries to synthesize the literature in this area.

preferences are time-consistent. A person feels the same about a given intertemporal trade-off no matter when she is asked.

Casual observation, introspection, and psychological research all suggest that the assumption of time consistency is importantly wrong (Frederick et al., 2002; Loewenstein and Prelec, 1992). The evidence suggests that discounting is steeper in the immediate future than in the further future. Intertemporal preferences with these features capture *self-control problems.* When evaluating outcomes in the distant future, individuals are patient and make plans to go to gym and exercise, stop smoking, and look for a better job. As the future gets near, the discounting gets steep, and the individuals engage in binge eating, light another (last) cigarette, and stay put on their job. Preferences with these features therefore induce time inconsistency.[2]

There is a growing literature in economics that assumes people have self-control problems, conceived as time-inconsistent taste for immediate gratification. We now present an economic model that formalizes self-control.

2.1. Modeling Self-Control Problems

To examine dynamic choice given time-inconsistent preferences, researchers have converged on a simple modeling strategy[3]: a single individual is modeled as many separate "selves," one for each period. Each period's self chooses her current behavior to maximize her current preferences, where the person's future selves will control her future behavior.[4]

Building on Strotz (1956), Phelps and Pollak (1968) and Akerlof (1991), Laibson (1997) and O'Donoghue and Rabin (1999a,b) formalize self-control problems conceived as time-inconsistent taste for immediate gratification using the (β, δ) framework. Labeling as u_t the instantaneous utility a person gets in period t, an individual's intertemporal preferences at time t, U_t, can be represented by the following utility function:

$$U_t = u_t + \beta\delta u_{t+1} + \beta\delta^2 u_{t+2} + \beta\delta^3 u_{t+3} + \dots$$

[2]While the rubric of "hyperbolic discounting" is often used to describe such preferences, the qualitative feature of the time inconsistency is more general, and more generally supported by empirical evidence, than the specific hyperbolic functional form.

[3]For economics papers on time-inconsistent discounting, see, for example, Goldman (1979, 1980); Schelling (1978); Strotz (1956); Thaler and Shefrin (1981), and Laibson (1997).

[4]Other models different from the one presented here have been proposed to capture self-control problems, including axiomatic models that emphasize preferences over choice sets (Gul and Pesendorfer, 2001) and models of the conflict between two systems, a planner and a doer (e.g. Fudenberg and Levine, 2006; Thaler and Shefrin, 1981, among others).

The only difference from the standard model (with δ as the discount factor) is the parameter $\beta \leq 1$, capturing the self-control problems. For $\beta < 1$, the discounting between the present and the future is higher than between any future time periods. For $\beta = 1$, this reduces to the standard model.[5]

A second key ingredient in this model is the modeling of expectations about future time preferences and awareness of the self-control issues. A way to capture a person's self-awareness about her self-control is to introduce beliefs about her own future behavior. O'Donoghue and Rabin (2001, 2003) allow a person to be partially naive (that is, *overconfident*) about future self-control problems. A partially naive (β, δ) person expects in the future period $t + s$ to have the utility function

$$\hat{U}_{t+s} = u_{t+s} + \hat{\beta}\delta u_{t+s+1} + \hat{\beta}\delta^2 u_{t+s+2} + \hat{\beta}\delta^3 u_{t+s+3} + \dots$$

with $\hat{\beta} \geq \beta$, where $\hat{\beta}$ denotes a person's belief about β. The person may be fully sophisticated about the self-control problem and fully aware of her time inconsistency $\left(\beta = \hat{\beta} < 1\right)$, and make decisions that rationally anticipate this problem. The person may also be fully naive and totally unaware of her self-control problems $\left(\beta < \hat{\beta} = 1\right)$, or somewhere in between $\left(\beta < \hat{\beta} < 1\right)$. This model, therefore, combines self-control problems with a form of overconfidence—naïveté about future self-control.

Whether people are naive or sophisticated is an empirical question. There seem to be elements of both sophistication and naïveté in people. Some degree of sophistication is implied by the fact that people often pay to commit themselves to smaller choice sets (e.g., joining fat farms or Christmas clubs, or buying small rather than large packages of enticing goods). A naive person would never worry that her tomorrow's self might choose an option that she doesn't like today, and she therefore would find committing herself unattractive. On the other hand, people do seem to overestimate the degree to which they will abide by their plans for the future. For example, people who repeatedly do not have the "willpower" to forgo tempting foods or quit smoking predict that tomorrow they will.

How do consumers with $\left(\beta, \hat{\beta}, \delta\right)$ preferences make choices? To illustrate and facilitate exposition for what follows, consider a good with an immediate payoff (relative to comparison activity) b_1 at $t = 1$ and a delayed payoff b_2 at $t = 2$. An investment good, like exercising or mastering a new software, has the features $b_1 < 0$ and $b_2 > 0$: it requires effort at present and delivers rewards tomorrow. A leisure good, on the other hand, like consumption of unhealthy food or using a credit card, has the features $b_1 > 0$ and $b_2 < 0$: it provides an immediate reward at a future cost.

[5]Although, see Koszegi and Szeidl (2011), who provide a focus-based foundation for present bias that complements the hyperbolic-discounting-based mechanism presented here.

Now, from an ex ante perspective, how often or how much does a person want to consume? If the person could set her consumption one period in advance, at $t = 0$, she would consume if her discounted stream of payoffs is nonnegative, i.e., $\beta\delta b_1 + \beta\delta^2 b_2 \geq 0$, or

$$b_1 + \delta b_2 \geq 0. \tag{1}$$

How much does the person *actually* consume when next period arrives, i.e., at $t = 1$? The person consumes if

$$b_1 + \beta\delta b_2 \geq 0. \tag{2}$$

Comparing to the desired, optimal consumption in (1), we see that a (β, δ) individual consumes too little of the investment good ($b_2 > 0$) and too much of the leisure good ($b_2 < 0$). This is the self-control problem in action. In response, a sophisticated person—well aware of her self-control problem—will look for commitment devices to increase the consumption of investment goods and to reduce the consumption of leisure goods.

Finally, how much does a person *expect* to consume? The person expects to consume in the future if

$$b_1 + \hat{\beta}\delta b_2 \geq 0,$$

with $\hat{\beta} \geq \beta$. Compared to the actual consumption in (2), the person overestimates the consumption of the investment good ($b_2 > 0$) and underestimates the consumption of the leisure good ($b_2 < 0$) . Naïveté therefore leads to mispredictions of future usage.

If real life consumers behave like this what does this imply about observed market prices? How should a monopolist price procrastinators? This is the question we take up next.

2.2. Selling to Consumers with Self-Control Problems

Hyperbolic discounting is most likely to be found for products that involve either immediate costs with delayed benefits (visits to gym, medical checkups, saving for retirement) or immediate benefits with delayed costs (smoking, credit card usage, binge eating, and temptation). DellaVigna and Malmendier (2004) (henceforth DM) examine a monopolist firm's optimal pricing strategy in the presence of *overconfident* and *time-inconsistent* consumers. We will now outline their model for investment goods and illustrate the main result.[6]

[6]For a more recent model see Heidhues and Koszegi (2010), that analyzes contract choices, loan-repayment behavior, and welfare in a model of a competitive credit market in which borrowers have a taste for immediate gratification.

Consider a monopolistic firm that interacts with a *homogeneous* pool of consumers with $\left(\beta, \hat{\beta}, \delta\right)$ preferences.[7] The monopolist sells a good which, in the language of previous section, has a payoff structure $b_1 = -c < 0$ and $b_2 = b > 0$. At the contracting stage the consumer knows the benefit b and does not know the cost c but knows that it is distributed according to some distribution F. At $t = 0$ the firm offers a two-part tariff $T(q)$ with fixed fee A and a per unit price p. If the consumer rejects, the firm makes zero profits, and the consumer attains reservation utility $\bar{u}$ in $t = 1$. If the consumer accepts, she pays the lump-sum fee A at $t = 1$ and then has to decide. Upon accepting, the consumer learns her cost type c and then chooses wether to actually consume the good (C) or not (NC) at $t = 1$. If she chooses C, she pays p to the firm and incurs cost c in $t = 1$ and receives V in $t = 2$. If she chooses NC, she attains payoff 0 in both $t = 1$ and $t = 2$. The firm incurs a setup cost $K \geq 0$ at $t = 1$ whenever a consumer accepts the contract, and a per-unit cost $a \geq 0$ incurred at $t = 1$ whenever the consumer chooses C. Finally, suppose that the firm has complete knowledge of the individual preferences and that, as of $t = 0$, it knows the cost distribution F. So, essentially there is no screening problem for the firm.

At $t = 0$, the consumer evaluates consumption C as follows. She discounts by $\beta\delta$ the cost of consumption c and the per-usage fee p due at $t = 1$, and by $\beta\delta^2$ the benefits V accruing at $t = 2$. She therefore assigns discounted utility $\beta\delta\left(\delta b - p - c\right)$ to C and utility 0 to NC. Thus, she would like her future self to choose C, upon learning her type c, whenever $c \leq \delta b - p$.

A *naive* time-inconsistent consumer, though, will choose C less often than her previous self wishes. At the moment of deciding between C and NC, the net payoff from C equals $\beta\delta b - p - c$. Therefore, at $t = 1$ she chooses C if $c \leq \beta\delta b - p$, i.e., with probability $F\left(\beta\delta b - p\right)$. The parameter of short-run impatience, β, determines the difference between desired and actual consumption probability $F\left(\delta b - p\right) - F\left(\beta\delta b - p\right)$. This difference is zero for individuals with time-consistent preferences ($\beta = 1$). The smaller is β, the larger this difference, and the more serious the self-control problems.

A *partially naive* time-inconsistent individual is not fully aware of her time inconsistency. Therefore, as of $t = 0$, she overestimates the probability that her future self will consume C at $t = 1$. She expects that she will consume if $c \leq \hat{\beta}\delta b - p$, i.e., with probability $F(\hat{\beta}\delta b - p)$. The difference between forecast and actual consumption probability, $F(\hat{\beta}\delta b - p) - F\left(\beta\delta b - p\right) \geq 0$, is a measure of the consumer's *overconfidence*. Time-consistent and sophisticated agents have rational expectations about their future time preferences and display no overconfidence.

[7]For simplicity, it is assumed that the firm is time consistent and has the same long run discount factor δ.

Thus, a consumer who accepts the contract (A, p) expects at time 0 to attain the net benefit $\beta\delta\left[-A + \int_{-\infty}^{\hat{\beta}\delta b-p} (\delta b - p - c)\, dF(c)\right]$[8].

The firm is rational, completely and correctly anticipates consumer behavior and maximizes its discounted profits. Therefore, its maximization problem is

$$\max_{A,p} \delta\left[A - K + F(\beta b - p)(p - a)\right]$$

subject to consumer's participation constraint

$$\beta\delta\left[-A + \int_{-\infty}^{\hat{\beta}\delta b-p} (\delta b - p - c)\, dF(c)\right] \geq \beta\delta\bar{u}.$$

By the familiar argument at the optimum this constraint is binding and we can substitute A in the objective function and obtain the unconstrained maximization problem for the firm:

$$\max_{p} \left\{\int_{-\infty}^{\beta\delta b-p} (\delta b - c - a)\, dF(c) + \int_{\beta\delta b-p}^{\hat{\beta}\delta b-p} (\delta b - p - c)\, dF(c)\right\} \tag{3}$$

The first term of (3) is the *actual social surplus* generated by the market interaction of the two parties (from the perspective of $t = 0$). The integrand consists of the net benefit to the consumer, $\delta b - c$, minus the cost for the firm, a. The per-usage price p does not appear in the integrand since it is a mere transfer between the two parties. The second term in (3) is the *fictitious consumer surplus* that reflects the overconfidence of the consumer. At $t = 0$ the consumer erroneously expects to attain this additional surplus. This term is null for time-consistent $\left(\hat{\beta} = \beta = 1\right)$ and sophisticated users ($\hat{\beta} = \beta < 1$) and increases with naïveté as measured by $\hat{\beta} - \beta$.

DellaVigna and Malmendier (2004) show that the first order condition of (3) satisfies[9]

$$p^* - a = -\left(1 - \hat{\beta}\right)\delta b \frac{f\left(\hat{\beta}\delta b - p^*\right)}{f(\beta\delta b - p^*)} - \frac{F\left(\hat{\beta}\delta b - p^*\right) - F(\beta\delta b - p^*)}{f\left(\hat{\beta}\delta b - p^*\right)}. \tag{4}$$

What are the pricing implications of (4) for different consumer types?

First, for time-consistent users ($\hat{\beta} = \beta = 1$), the first order condition (4) implies *marginal cost pricing*. Clearly for $\hat{\beta} = \beta = 1$, the right hand side of (4) is zero so that $p^* = a$.

[8] Note that if $\hat{\beta} \neq \beta$, DM assume that the consumer does not learn from current willpower nor from the firm's offer.

[9] So that a profit-maximizing contract (A^*, p^*) exists but in general is not unique.

Second, for time-inconsistent users ($\beta < 1$) it implies *below marginal cost pricing*, $p^* < a$. Furthermore, for a sophisticated consumer who is aware of her self control problems ($\beta = \hat{\beta}$), $p^* = a - (1 - \beta)\,\delta b$. Therefore, for a sophisticated consumer the outcome is efficient. Pricing corrects perfectly for lack of willpower (which would not be so under uncertainty about b). Below-marginal cost pricing occurs for two distinct reasons.

The first is a commitment rationale. An individual who is at least partially aware of her time inconsistency looks for ways to increase the probability of future investment. Choosing a contract with low p is one such way. The first term in (4) expresses this rationale. The firm lowers p^* below a to the extent that the user is conscious about her future time inconsistency, as measured by $1 - \hat{\beta}$. The term $(1 - \hat{\beta})\delta b$ is the additional utility that the time 0 self places on consumption C, relative to the (anticipated) utility of period 1 self. It represents the value of commitment for an additional unit of consumption.

The second reason for pricing below marginal cost is consumer overconfidence. The firm knows that a (partially) naive consumer overestimates future consumption. Therefore, it offers a contract with a discount on p and an increase in A relative to the contract for a time consistent-agent. While the user would be indifferent between the two contracts if both were offered, the actual welfare is lower than she anticipates, and the firm will make higher profits. The firm sets p so as to increase this *fictitious surplus*, the second term in (3). To a first approximation, this fictitious surplus depends on $F(\hat{\beta}\delta b - p) - F(\beta\delta b - p)$, the overconfidence of future consumption. The higher is the density of F in that area, the higher are the profits from the overestimation of consumption. By choosing p the firm maximizes fictitious surplus.

These results generalize to the case of perfect competition, since competition only alters the equilibrium fee A^*. DM's model thus rationalizes the presence of contracts with no payment per visit in health clubs ($b_2 > 0$), the presence of high interest rates but no annual fees for credit cards ($b_2 < 0$), and cheap room rates and buffets for gamblers in Las Vegas ($b_2 < 0$).

In sum: in the equilibrium both sophisticated and naive hyperbolic agents pay a lower per-usage price than time-consistent consumers. To the extent that the individual is aware of the time inconsistency, she prefers a low per-usage price as a commitment device that increases the usage in the future. To the extent that the consumer is overconfident about the strength of her future willpower, the firm exploits the misperception of the consumption probability by tilting the pricing toward the lump-sum fee A.

As for consumer welfare and firm profits**,** DM show that (1) for sophisticated individuals, consumer welfare and firm profits and the joint consumer-firm surplus are unaffected by the degree of time inconsistency $1-\beta$; (2) for partially naive individuals, firm profits are strictly increasing in the degree of naiveté $\hat{\beta}-\beta$; (3) partially naive individuals have lower surplus and consumer welfare than sophisticated individuals with the same β and δ; and (4) the loss in consumer welfare due to monopoly power becomes larger in absolute value as naïveté increases.

2.2.1. Discussion and Some Empirical Evidence

The above results suggest a clear distinction between non-standard preferences (time inconsistency) and non-rational expectations (naïveté). The first type of deviation does not necessarily affect surplus, profits, and welfare. Firms offer the contract that maximizes the joint surplus, and this contract may neutralize the behavioral effect of the non-standard feature. This result generalizes beyond the specific application of time inconsistency. For example, if consumers have limited computational abilities and are aware of it, firms may offer simple contracts or devices that help consumers perform the computations. In equilibrium the limited computational power is likely to be non-binding.

This conclusion changes if consumers have non-rational expectations. If consumers *misperceive* their objective function or their constraints, the firms do not offer surplus maximizing contract, but rather a contract that accentuates the effects of the non-rational expectations so as to increase profits. Under monopoly the firm keeps the additional profits from contract distortions, while under perfect competition consumers themselves receive the "returns" to naïveté. Competition therefore tempers the adverse effects of naïveté on consumer welfare.

One major weakness of the DM model is consumer homogeneity. Eliaz and Spiegler (2006) generalize this analysis to allow for heterogeneity in naïveté and a more general form of time-inconsistency in preferences. They show that firms offer two types of contracts: perfect commitment devices that cater to time-inconsistent agents that are sufficiently sophisticated, and contracts that take advantage of the consumers that are sufficiently naive. Interestingly, the fully sophisticated agents do not exert any informational externality on the naive types. Thus, the provision of the perfect commitment device does not reduce the gains that the monopolist can extract from naive types.

DellaVigna and Malmendier (2004) also offer extensive empirical evidence on two key predictions of their model. For investment activities, it predicts pricing below marginal cost and a lump-sum fee larger than setup cost. For leisure goods it predicts pricing above marginal cost and (under perfect competition) a lump-sum fee smaller than the set-up cost.

Investment Goods The most convincing evidence on investment goods comes from *health clubs* which are discussed at length in DellaVigna and Malmendier (2006). Contracts with no fee per visit are prevalent. For a large fraction of gym clubs in their sample, a monthly contract is the most frequent contract offered to consumers (40 out of 64 companies offer a monthly contract), followed by an annual contract for less that half of them (27 out of 64), and the pay per visit contract for only 2 companies out of 67. DellaVigna and Malmendier (2006) estimate the marginal cost of a visit at $5, which includes the cost of providing towels, personnel, and replacing broken machines, but excludes congestion effects which are sizable at peak hours. They estimate that on average health clubs price attendance at least $3 to $6 below marginal cost. Additional support for the hypothesis of time inconsistency comes from behavior of consumers in health clubs. DellaVigna and Malmendier (2006) show that consumers who pick monthly or annual contracts would on average have saved paying per-visit, a behavior consistent with demand for commitment or overestimation of attendance.

DM also cite (less compelling) evidence from *vacation time-sharing* plans as supporting their model for investment goods. The typical contract in the time-sharing industry involves a large initial fee (on average, $11,000 in the United States) to become a member and only a small fee ($140) for each week of holiday used.

Leisure Goods *Credit cards* have the intertemporal features of a leisure good: they allow credit-constrained individuals to increase current consumption at the expense of future consumption. Naive individuals underestimate the usage of the credit line and thus DM's model predicts that credit card companies should respond by charging an interest rate above marginal cost together with a low initial fee or even offer a bonus. Indeed, at least in the USA, most banks charge an interest rate on outstanding balances that exceeds the prime rate by as much as 10 percentage points. This high interest rate reflects above-marginal cost pricing and DM argue that it cannot be explained by high default costs as credit card debt resells on the private market at a 20 percent premium. Despite sizable costs, net of the interchange fee, of keeping a credit card account, the issuers typically require no annual fee, and offer extra benefits such as car rental insurance and cash back. As a result, credit card companies experience losses on users with no outstanding balances, who can use the card for transactions, enjoy the extras and borrow for up to 30 days at no cost. For an alternative, more relevant model using both sophisticated and myopic consumers see Gabaix and Laibson (2006), reviewed in Section 3.2 of this paper, and especially Heidhues and Koszegi (2010), who develop a model especially for credit markets with time-inconsistent borrowers..

DM also argue that *mobile phone* companies tempt users with limited self-control to spend time on the telephone rather that other more productive activities. naive users underestimate the number of future calls when they choose the monthly airtime. Cellular phone companies can extract profits from naïveté by setting high marginal prices for minutes beyond the monthly allowance. In the typical contract, the consumers choose a monthly airtime allowance within a menu. The marginal price for minutes beyond the allowance is typically two to four times higher that the average price of a call within the limit. The model with naive consumers does not explain the zero marginal price for minutes within the airtime allowance. Again, for a more applicable model using consumer overconfidence, see Grubb (2009), reviewed in Section 3.1 of this paper, and especially Grubb (2011), who develops a model of inattentive consumption based on biased beliefs.

Additional empirical evidence that rational firms are aware of consumer's present biased preferences and incorporate it in their pricing structure comes from Wertenbroch (1998) and especially Oster and Scott Morton (2005). Wertenbroch (1998) examines the response to price cuts on different sizes of investment and leisure—or, in his terminology, virtue and vice—supermarket products. Vice goods are those such as cigarettes or potato chips where the pleasure is now but the cost is in the future, while a virtue good has less current payoff but future benefits, e.g. laundry detergent. He finds that consumers respond much less to a cut in the price of a large size of a vice good than a large size of a virtue good. This makes sense if a forward-looking consumer with self-control problems rationally does not want to have a large bag of potato chips in her house. She will require a larger price cut to purchase such a product than she would to purchase the large size of laundry detergent. The expensive small package serves as a commitment device.

Using data from American magazines, Oster and Scott Morton (2005) explore the relationship between newsstand and subscription prices and magazine characteristics. In particular, they distinguish between magazines that provide benefits in the future ("investment magazines", such as New York Review of Books or Foreign Policy) versus those that are simply fun to read now (leisure magazines, perhaps The National Enquirer or Star). A time-inconsistent consumer with a present bias at the newsstand discounts the future payoff of the investment good but fully values the leisure good. This difference does not exist for subscriptions. Thus, the ratio of the subscription to newsstand willingness to pay for a magazine should differ between investment and leisure goods. Indeed, they find that for magazines whose payoff is in the future, subscriptions are relatively more costly, ceteris paribus. They find that moving from a non-investment genre to one that has investment characteristics increases the ratio of subscription to newsstand prices by about 12%. This finding suggests that publishers reflect the present bias preferences of consumers in their price setting behavior.[10]

Meier and Sprenger (2010) provide a recent field evidence on whether present-biased time preferences correlate with credit card borrowing. In a field study, they elicit individual time preferences with incentivized choice experiments, and match resulting time preference measures to individual credit reports and annual tax returns. Their results indicate that present-biased individuals are more likely to have credit card debt, and to have significantly higher amounts of credit card debt, controlling for disposable income, other socio-demographics, and credit constraints.

2.2.2. Related Theoretical Research

Sarafidis (2005) studies the interaction of naive time-inconsistent consumers with a slightly modified version of the (β, δ) preferences with a rational durable good monopolist who is aware of the consumers' true preferences and their naïveté. His objective is to explore how consumer naïveté affects the monopolist's ability to screen consumers through intertemporal price discrimination and what, in turn, this implies for monopoly profits, the path of prices and consumer welfare. Nocke and Peitz (2003) have also looked at the durable good monopoly problem in the presence of time-inconsistent hyperbolic discounters with the emphasis on the secondary markets.

Esteban and Miyagawa (2006); Esteban et al. (2007) also analyze non-linear pricing in the face of consumers with self-control problems as formalized in Gul and Pesendorfer (2001).

3. Overconfidence, Projection Bias and Myopia

Many psychological studies show that people are often overconfident (Peterson, 2006; Sharot, 2011; Taylor, 1989; Taylor and Brown, 1994). One can define overconfidence as the tendency (1) to *overestimate* one's own performance or abilities, (2) to *overplace* oneself relative to others, and (3) to have excessive confidence in the accuracy of own beliefs, or *overprecision*.[11]

People also exaggerate the degree to which their future tastes will resemble their current tastes. Loewenstein et al. (2003) present a variety of laboratory and

[10] Esteban and Miyagawa (2006); Esteban et al. (2007) also analyze non-linear pricing in the face of consumers with self-control problems as formalized in Gul and Pesendorfer (2001).

[11] While the overplacement or better than average effect is well established, Benoit and Dubra (2011) have recently questioned its significance.

survey evidence demonstrating the prevalence of such *projection bias*, develop a formal model of it, and use the model to demonstrate its importance in economic environments. Conlin et al. (2007) document projection bias in the field using catalog sales and returns data. Individuals who exhibit this bias overestimate the degree to which their future tastes will resemble their current tastes, and therefore tend to underestimate the variance of their future demand. Moreover, a significant body of experimental evidence shows that individuals are overconfident about the precision of their own predictions when making difficult forecasts. In other words, individuals tend to set overly narrow confidence intervals relative to their own confidence levels.

A typical psychology study might pose the following question to a group of subjects: "What is the shortest distance between England and Australia?" Subjects would then be asked to give a set of confidence intervals centered on the median. A typical finding is that the true answer lies outside a subject's 98% confidence interval about 30% to 40% of the time. Ex post tariff-choice "mistakes" made by cellular phone customers are consistent with such overconfidence.

We now broadly outline the model of Grubb (2009) and discuss the main result. We will concentrate mainly on the case of monopoly since the situation with perfect competition is broadly similar.

3.1. Selling to Overconfident Consumers

Firms commonly offer three-part tariffs, or menus of three-part tariffs, in a variety of contexts. A three-part tariff consists of a fixed fee, an included allowance of units for which marginal price is zero, and a positive marginal price for additional usage beyond the allowance. A prime example is the US cellular phone services market in which firms typically offer consumers plans consisting of a fixed monthly fee, an allowance of minutes, and an overage rate for minutes beyond the allowance. Other examples of three-part tariffs include wireless services, car leasing contracts, which often include a mileage allowance and charge per mile thereafter. In a variety of rental markets, contracts charge a flat rate for a specified period followed by steep late fees.

Traditional Industrial Organization literature on nonlinear pricing does not provide a compelling explanation for such pricing patterns.[12] If consumers have biased beliefs, are *overconfident* and systematically underestimate their demand at the time of contracting, however, then three-part tariffs will emerge. This is the essence of Grubb (2009).[13]

[12]For perfect competition one expects prices to be driven down to cost, while standard nonlinear pricing models suggest the highest demand consumer will pay the lowest marginal price.

[13]Three recent alternative explanations have been put forward to explain three-part tariff pricing, two

Grubb (2009) develops a model of firm pricing when consumers are *overconfident* in a sense of having a tendency to underestimate the variance of their future demand when choosing a tariff. Two important biases lead to this tendency: *forecasting overconfidence*, which has been well documented in the psychology literature, and *projection bias*, mentioned above. Intuitively, underestimating variance of future demand may lead to tariffs of the form observed because consumers do not take into account the risk inherent in the convexity of the tariffs on the menu. This is because although the tariffs have a high average cost per unit for consumers who consume far above or far below their allowance, consumers are overly certain that they will choose a tariff with an allowance that closely matches their consumption. Thus consumers expect to pay a low average price per unit, but sellers profit ex post when consumers make large revisions in either direction.

3.1.1. A Model of Pricing with Overconfident Consumers

Consider a monopolist who sells to a continuum of ex-ante homogeneous consumers. Timing of events differs from a standard screening model. In the first period (at the contracting stage) firms and consumers are homogenous and do not know their future demand type θ. The firm offers tariff $\{q(\theta), P(\theta)\}$—purchase quantity and payment pair intended for each type θ. Consumers accept or reject based on their prior belief over θ at the ex-ante contracting stage. In the second period, consumers privately learn θ and given prior acceptance, purchase quantity $q(\theta')$ and pay bill $P(\theta')$.

The base assumptions about production and preferences match those of a standard screening model. Therefore, a firm's profits are given by revenues P less production costs $C(q)$, which are increasing and convex in quantity delivered q. Consumer's utility U is equal to the value of consuming q^c units, $V(q^c, \theta)$, less the payment to the firm, P. Consumers' marginal value of consumption V_q is strictly decreasing in consumption q^c, and strictly increasing in θ. The outside option of all consumers is the same and is normalized to zero: $V(0, \theta) = 0$.

behavioral and one rational: (1) the "flat-rate bias", which encompasses demand overestimation, risk aversion, and the taxi-meter effect (Lambrecht and Skiera, 2006); (2) demand underestimation, which is related to quasi-hyperbolic discounting; and (3) a monopoly price discrimination explanation (Courty and Li, 2000). Although the first two potential alternatives have important effects on pricing, neither can explain three-part tariff offerings. However, the monopoly price discrimination model does predict three-part tariff pricing given the right type distribution. As the overconfidence and price discrimination models cannot be distinguished based on observed (monopoly) prices, one also needs to observe consumer's choices and this is largely and empirical issue. Grubb (2009) compares the two explanations using both observed prices and tariff and quantity choices in a particular setting: cellular phone services. Although a monopoly price discrimination explanation predicts three-part tariff pricing given the right type distribution, it appears to be inconsistent with consumer usage patterns.

With overconfident consumers one has to make an additional assumption concerning consumer preferences, which would not be relevant in the standard model. Consumers are assumed to have a finite satiation point, $q^S(\theta) \equiv \arg\max_{q^c \geq 0} V(q^c, \theta)$, beyond which they may freely dispose of unwanted units. Hence, given free disposal, consumer type θ who purchases $q(\theta')$ units will only consume the minimum of $q(\theta')$ and $q^S(\theta)$, and will receive consumption value $V\left(\min\{q^S(\theta), q(\theta')\}, \theta\right)$ and utility $U(\theta, \theta') \equiv V\left(\min\left\{q^S(\theta), q(\theta')\right\}, \theta\right) - P(\theta')$.[14]

The key assumption of Grubb's model, which deviates sharply from a standard model, is that consumers *underestimate the variance of their future demand* θ. This is either because they are overconfident about the accuracy of their forecasts of θ, or because they are subject to projection bias. Thus while the firm knows that consumer demand θ follows cumulative distribution $F(\theta)$, consumers have the prior belief that θ follows $F^*(\theta)$. Moreover, the firm knows that consumers are overconfident, so will take this into account when designing its tariff offering. The following overconfidence assumption formalizes this: $F^*(\theta)$ crosses $F(\theta)$ once from below at θ^*.[15]

What is the monopolist's profit maximizing contract?

Invoking the standard revelation principle, the monopolist's profit maximization problem is

$$\max_{P(\theta), q(\theta) \geq 0} \mathbb{E}\left[P(\theta) - C(q(\theta))\right]$$

subject to consumers' incentive compatibility and participation constraints:

$$U(\theta, \theta) \geq U(\theta, \theta') \quad \forall \theta, \theta' \in [\underline{\theta}, \overline{\theta}]$$
$$\mathbb{E}^*[U(\theta)] \geq 0$$

Note, that there are two important deviations from a standard screening model. First, free disposal is explicitly incorporated through consumer preferences, which depend on the consumed quantity $\min\{q^S(\theta), q(\theta')\}$ rather than the purchased quantity $q(\theta')$. Second, consumers' first-period beliefs differ from those of the firm. Thus the ex ante participation constraint requires that consumers' perceived expected utility $\mathbb{E}^*[U(\theta)]$ must be positive, but puts no constraint on their true expected utility $\mathbb{E}[U(\theta)]$. The difference in priors between consumers and the firm creates the wedge separating the expected utility consumers believe they are receiving from the expected utility the firm believes it is actually providing.

[14]The full characterization of the profit maximizing contract requires several more technical assumptions on costs, valuation of the good, and the distribution of types. These can be found in the original paper.

[15]An interesting special case of this overconfidence assumption is where consumers and the firm agree on the mean of θ, in which case $F(\theta)$ is a mean preserving spread of $F^*(\theta)$ and consumers underestimate the variance of their future demand.

Solving this constrained optimization problem seems like a daunting task. But once one recognizes that there will never be any reason for the firm to induce a consumer to purchase beyond her satiation point the task is simplified and can be solved following the standard approach.[16] The firm doesn't need to induce the consumer to buy more units than her satiation point because she would simply dispose of unwanted additional units. By initially selling the consumer her satiation quantity at the same price, the consumer would have been equally well off, the incentive compatibility constraint of other consumers would have been weakly relaxed and the firm could have reduced production costs.

Grubb shows that the equilibrium allocation $\hat{q}(\theta)$ maximizes an expected virtual surplus $\mathbb{E}[\Psi(q(\theta), \theta)]$, consisting of expected true surplus $S(q(\theta), \theta) \equiv V(q(\theta), \theta) - C(q(\theta))$ and a "fictitious surplus", which is the difference between expected utility $\mathbb{E}^*[U(\theta)]$ consumers believe they are receiving and the expected utility $\mathbb{E}[U(\theta)]$ the firm believes it is delivering. Moreover, virtual surplus is give by

$$\mathbb{E}[\Psi(q(\theta), \theta)] = \mathbb{E}[S(q(\theta), \theta)] + \mathbb{E}^*[U(\theta)] - \mathbb{E}[U(\theta)]$$

$$\mathbb{E}^*[U(\theta)] - \mathbb{E}[U(\theta)] = \mathbb{E}\left[V_\theta(q(\theta), \theta)\frac{F(\theta) - F^*(\theta)}{f(\theta)}\right]$$

When consumers and the firm share the same prior ($F^*(\theta) = F(\theta)$) fictional surplus is zero, so the equilibrium tariff maximizes true expected surplus $\mathbb{E}[S(q(\theta), \theta)]$. This implies first best allocation $q^{FB}(\theta) \equiv \underset{q}{\operatorname{argmax}}[V(q, \theta) - C(q)]$ and marginal payment equal to marginal cost. When consumers are overconfident, however, fictional surplus meed not be zero, and may distort the equilibrium allocation away from first best, and marginal pricing away from marginal cost.[17]

What are the pricing implications of the profit maximizing contract? Grubb shows that the equilibrium payment $\hat{P}(q)$ paid by each type θ is continuous and piece-wise smooth function of quantity. There may be kinks in the payment function where marginal price increases discontinuously. These kinks occur where the monotonicity constraint binds and an interval of types "pool" at the same quantity. For quantities at which there is no pooling, marginal price is given by

$$\frac{d\hat{P}(q)}{q} = V_q(q, \hat{\theta}(q)) = \max\left\{0, C_q(q) + V_{q\theta}(q, \hat{\theta}(q))\frac{F^*(\hat{\theta}(q)) - F(\hat{\theta}(q))}{f(\hat{\theta}(q))}\right\}$$

As it is assumed that $V_{q\theta}$ is strictly positive and $f(\theta)$ is finite, this allows marginal price to be compared to marginal cost based on the sign of $[F^*(\theta) - F(\theta)]$. In

[16]For an excellent textbook exposition see Bolton and Dewatripont (2005).

[17]These distortions, and thus the equilibrium allocation will be identical under monopoly and perfect competition.

particular, the sign of $\left[P_q^*(q) - C_q(q)\right]$ is equal to the sign of $[F^*(\theta) - F(\theta)]$ except when $F^*(\theta) < F(\theta)$ and marginal cost is zero, as then marginal price is also zero. This is informative about equilibrium pricing, as the overconfidence assumption dictates the sign of $[F^*(\theta) - F(\theta)]$ above and below θ^*.

When marginal costs are zero, marginal price will be zero below some included allowance Q, and positive thereafter. When marginal costs are strictly positive, marginal price will initially be positive, but will fall below marginal cost and may be zero for some early range of consumption. It is reasonable to assume that the marginal cost of providing an extra minute of call time to a cell phone customer is small. Therefore, given overconfident consumers, the equilibrium tariff bears a striking qualitative resemblance to those offered by cell-phone service providers. Both predicted equilibrium tariffs and observed tariffs involve zero marginal price up to some included limit Q and become positive thereafter.[18]

The intuition for the result is as follows. If consumers are overly confident that their future consumption will be near Q minutes, they will underestimate both the probability of extremely low and extremely high consumption. Thus a firm cannot charge these consumers for extremely high consumption through a fixed fee ex ante. Instead, the firm must wait until consumers learn their true values and charge a marginal fee for high consumption above Q. A firm can, however, charge consumers for low levels of consumption through a fixed fee ex ante. By setting a zero marginal price, the firm avoids paying a refund to those consumers who are later surprised by a low level of demand below Q.[19]

Are these results robust to competition? The equilibrium allocation is the same under both monopoly and perfect competition. However, as expected, the payment is lower than under monopoly and differs only by a fixed fee.

What about welfare? To evaluate welfare Grubb assumes, as is more plausible, due to experience with consumers and other factors, that the firm's prior $F(\theta)$ is correct. Under perfect competition consumers receive all the surplus generated. However, while consumers with correct priors receive the efficient allocation, overconfident consumers receive an allocation that is distorted away

[18]In contrast to observed tariffs, the model predicts that marginal price is always equal to marginal cost for the last unit sold. However this result is not robust to ex-ante heterogeneity. The primary difference is that beyond the included limit Q, marginal price is constant for observed tariffs. Grubb conjectures that this simpler pricing structure approximates optimal pricing and is more practical to implement. The fact that marginal price does not fall to marginal cost at the top may also be due to binding period-one incentive compatibility constraints relevant to the unmodeled self-selection among tariffs at time one.

[19]Whether or not a three part tariff is an optimal approximation depends on complexity costs, consumer preferences and beliefs. However, Grubb shows that a three part tariff is an excellent approximation whenever overconfident consumers are primarily uncertain about the quantity of desirable units and tend to have a consistent an certain value for units that are desirable.

from the first best. As a result, overconfident consumers must be worse off. This suggest that educating consumers or regulating constant marginal prices could potentially improve consumer welfare, and therefore total welfare, as firm profits are always zero. Under monopoly, total welfare is also lower when consumers are overconfident, but in general it is ambiguous as to whether consumers or the firm are better or worse off.

Further analysis in Grubb (2009) extends the model to allow for ex ante heterogeneity and screening at the contracting stage via a menu of multiple tariffs. He characterizes a monopolist's optimal two-tariff menu given two ex ante types. The qualitative pricing results of the single tariff model discussed above are robust as long as overconfidence is sufficiently high relative to ex ante heterogeneity in average demand.

In sum: Grubb has shown that given overconfident consumers, low marginal costs, and free disposal, optimal pricing involves included units at zero marginal price followed by high marginal charges. This provides a promising explanation for the three-part tariff menus observed in the cellular phone services market. His theory ignores tariff complexity costs, and hence does not necessarily predict overage rates to be constant as observed. When consumers are primarily uncertain about the volume of desirable units, relative to a fairly consistent value for units that are desirable, three-part tariffs will be a good approximation for optimal pricing in the sense that they provide a large improvement over two-part tariffs relative to the remaining approximation losses. An example with linear demand and uniform beliefs suggests that this is true for other reasonable cases as well. For another behavioral model applicable to the mobile phone billing industry, see Grubb (2011).

Grubb (2009) incorporates overconfidence by relaxing the common prior assumption. There are a few related papers which also investigate firm pricing when consumers have biased priors about their future demand. Using a somewhat different framework Eliaz and Spiegler (2008) are concerned with screening of consumer's optimistic beliefs. They consider a model where consumers and the seller do not share the same prior and argue that menus of non-linear pricing schemes in monopolistic environments can be usefully interpreted as a consequence of the monopolists's attempt to screen the consumer's type regarding his future willingness to pay.[20] Elias and Spiegler's consumers have only two types of ex post demand: high or low, in contrast to the continuous demand case studied by Grubb (2009). The difference is important because although Eliaz and Spiegler (2008) can capture consumers who over- or underestimate average demand, they cannot capture overconfident consumers who underestimate the likelihood of both upper

[20] However, note that Eliaz and Spiegler (2008)'s interpretation of biased priors is somewhat different.

and lower tails of demand by overweighting the center of the distribution. Allowing for more than two ex post types is also crucial (along with free disposal and consumer satiation) to generate three-part tariff pricing.

Uthemann (2005) considers a model that like Grubb (2009) has continuous ex post demand, but unlike Grubb (2009) (and similar to Eliaz and Spiegler, 2008), does not allow for free disposal or satiation. As a result, the optimal tariffs under both monopoly and competition are qualitatively different than those in Grubb (2009), and in particular have strictly positive marginal prices everywhere.

3.2. Selling to Myopic Consumers

Another source of irrationality and its exploitation by firms is related to the pricing of add-ons (goods or services that are purchased as part of the consumption of other goods and services, such as ink for a printer). In the market for complementary add-on goods firms often choose to hide information from consumers. For example, banks prominently advertise the virtues of their accounts, but the marketing materials do not highlight the costs of such an account which include ATM usage fees, bounced check fees, minimum balance fees, etc. Banks could compete on these costs, but they instead choose to shroud them. Indeed, many bank customers do not learn the details of the fee structure until long after they have opened their accounts (e.g. Cruickshank, 2000).

At first glance, shrouding add-on prices seems like a natural marketing strategy. However, if consumers are all rational (or aware), shrouding should actually hurt the bank because rational consumers will expect the worse from a firm that shrouds (see, for instance, Jovanovic, 1982). Any information that is hidden in the fine print—or excluded from marketing materials altogether—is not likely to be favorable to consumers. Rational consumers will infer that hidden prices are likely to be high prices. Such reasoning creates an incentive for information revelation and unraveling of shrouding. Indeed, all firms choose to unshroud their prices in equilibria with rational consumers.

Gabaix and Laibson (2006), on the other hand rely on the existence of some irrational consumers —naive consumers, who are not aware of marketing schemes—and explain why information shrouding and high add-on markups will persist in markets even with numerous competitors and free advertising.

We now present the model of Gabaix and Laibson (2006) and illustrate their main result.[21]

[21] For a related model, see Miao (2010).

3.2.1. A Model of Shrouded Attributes and Consumer Myopia

Gabaix and Laibson (2006) analyze a market in which firms can shroud attributes of their products. These shrouded attributes are not taken into consideration by some potential consumers.[22] For the purposes of the formal model, a shrouded attribute is a product attribute that is hidden by a firm, even though the attribute could be nearly costlessly revealed. Shrouded attributes may include surcharges, fees, penalties, accessories, options, or any other hidden feature of the ongoing relationship between a consumer and a firm, and can be divided into two mutually exclusive categories: (avoidable) add-ons and (unavoidable) surcharges. Gabaix and Laibson discuss add-ons, the first type of shrouded attribute. Add-ons are complementary goods that consumers have the option to avoid.[23]

Gabaix and Laibson (2006) motivate the model via an example of a bank. Consider a bank that sells two kinds of services. For price p a consumer can open a bank account ("base good"). If the consumer violates a minimum balance requirement (an "add-on") she has to pay a fee $\hat{p}$ (price of the add-on). Without loss of generality they assume that the true cost to the bank of providing all of its services is zero and that $\hat{p}$ is effectively bounded above by $V > e$. The bank sets and potentially shrouds prices. If the bank chooses to shroud $\hat{p}$, the minimum balance fee will not be seen by potential consumers. The bank can alternatively unshroud its add-on and costlessly advertise $\hat{p}$.

There are two types of consumers. Sophisticated consumers—fraction $1 - \alpha$ of the population—are aware of the add-on fee and always take the add-on and its price into consideration, forming Bayesian posteriors about the add-on when its price is shrouded. A consumer who anticipates or observes high add-on prices can exert costly effort $e > 0$ and thereby substitute away from the add-on, by for example, being more vigilant not to violate the minimum balance requirement. Therefore a sophisticated consumer will exert substitution effort e if $e < \mathbb{E}(\hat{p})$, where $\mathbb{E}(\hat{p})$ is the rational expectations about the add-on price of the firm.

Myopic consumers—fraction α of the population—on the other hand, come in two varieties. If a firm does not advertise its add-on price $\hat{p}$, they are all *uninformed* and not aware of it. *Uninformed* myopes do not fully anticipate the fee $\hat{p}$, they may completely overlook the aftermarket or they may mistakenly believe that $\hat{p} < e$. Uninformed myopes neglect the add-on when deciding where to open their

[22]For example, a bank might suppress information about minimum balance fees in the bank's marketing materials. Some consumers will neglect to consider such fees when picking a bank.

[23]For example, hotel guests can avoid paying telephone charges if they instead use cell phones. Likewise, hotel guests can avoid paying for room service meals by finding local restaurants. Both hotel phone use and hotel room service complement a hotel stay. Such complementary (voluntary) goods are referred to as add-ons.

bank account and do not consider exerting substitution effort. If a firm chooses to advertise $\hat{p}$ it is still observed and noted by only a λ fraction of myopic consumers, with $0 < \lambda \leq 1$, making them *informed* myopes. Informed myopes behave exactly like sophisticates. Hence, when information is unshrouded by one or more firms, a fraction λ of myopes is informed—and therefore becomes sophisticated—and a fraction $1-\lambda$ of myopes is uninformed. Hence unshrouding by any firm increases the sophistication of the pool of potential customers shared by all firms.

What causes consumers to be myopic? One interpretation is overconfidence, people believe they will not fail prey to small print penalties or underestimate the use of the add-on. When add-on prices are unshrouded, a fraction λ of myopes becomes informed. These informed myopes behave just like sophisticates with $\mathbb{E}(\hat{p}) = \hat{p}$. A myope who was educated by firm i becomes sophisticated in his behavior *vis-a-vis* all firms. However, even when add-on information is unshrouded, a fraction $1 - \lambda$ of myopes are uninformed.

Gabaix and Laibson characterize symmetric sequential equilibrium of the game played by firms and consumers. Here we will only present and discuss their main finding. Let x_i represent the anticipated net surplus from opening an account at bank i less the anticipated net surplus from opening an account at the best alternative bank.

For sophisticated consumers,

$$x_i = [-p_i - \min\{\mathbb{E}(\hat{p}), e\}] - [-p^* - \min\{\mathbb{E}(\hat{p}^*), e\}],$$

where $\mathbb{E}(\hat{p})$ and $\mathbb{E}(\hat{p}^*)$ are the rational expectations about the add-on price of the firm and its competitors, respectively.

For uninformed consumers,

$$x_i = -p_i + p^*.$$

Let $D(x_i)$ represent the probability that a consumer opens an account at bank i. This demand function D is strictly increasing, bounded below by zero, and bounded above by one.[24] This demand function allows one to flexibly parametrize the degree of competition in the industry, via the quantity $\mu = D(0)/D'(0)$, which turns out to be equal to average profit per consumer. In particular, perfect competition corresponds to $\mu = 0$.

There are two sequential equilibria, depending on whether or not the share of myopic consumers in the marketplace is large enough or not. Gabaix and Laibson

[24] A microfoundation for such a demand function can be found in the random utility theory. See, for example, Anderson et al. (1992).

show that:[25]

(a) If the fraction of myopic consumers is large enough ($\alpha > \alpha^* \equiv e/V$), there exists a symmetric equilibrium in which firms shroud the add-on price. The prices of the base good and the add-on in this Shrouded Price Equilibrium are, respectively,

$$p = -\alpha V + \mu \quad \text{and} \quad \hat{p} = V$$

and only naive myopes consume the add-on. The allocation is inefficient since sophisticates substitute away from the add-on (pay effort cost e to substitute away from consumption).

(b) However, if the fraction of myopic consumers is small ($\alpha \leq \alpha^*$), there exists a symmetric equilibrium in which firms do not shroud the add-on price. The prices in this Unshrouded Price Equilibrium are

$$p = -e + \mu \quad \text{and} \quad \hat{p} = e$$

and all agents consume the add-on which is efficient.

In each of these equilibria the beliefs of sophisticated consumers and educated myopes are $\hat{p} = V$ for the add-on price at a firm that shrouds, independently of its base good price. Also, the total profits of the industry are μ (do not depend on the add-on structure (α, V, e)).

Gabaix and Laibson's model reproduces the well-known result that high markups for the add-on are offset by low or negative markups on the base good (see, e.g., Ellison, 2005).[26] This is easiest to see when the market is approximately competitive (i.e., the demand curve is highly elastic and hence μ is close to zero). In a relatively competitive market with small μ, the base good is always a loss leader with a negative markup: $p^* \approx -\alpha V < 0$ or $p^* \approx -e < 0$. The model implies that the add-on will be the "profit-center" and the base good will be the "loss leader."

[25]Since the add-on can be produced at zero social cost, it is socially efficient for the add-on to be consumed. If a consumer substitutes away from thee add-on (at effort cost e), then an equilibrium is socially inefficient. In the case with inefficiency ($\alpha > \alpha^*$), the welfare losses increase as the fraction of sophisticates increases; in this case sophisticates do not consume the (high-priced) add-on.

[26]Gabaix and Laibson's explanation for add-on shrouding and high markups depends crucially on the presence of myopic consumers. Existing rational actor models that were developed to explain why add-ons have high markups do not explain, however, why firms gratuitously shroud add-on prices. Two types of explanations for high markups figure prominently in the literature: high exogenous search costs and an inability to commit. Ellison (2005) proposes a rational price-discrimination model in which add-on pricing enables firms to charge high demand consumers relatively more than low demand consumers. In Ellison's model, exogenous search costs make it costly for consumers to observe add-on prices. High add-on markups raise profits by facilitating price discrimination.

Will advertising solve the problem? Previous literature in IO has conjectured that the availability of inexpensive advertising would drive down aftermarket prices and eliminate shrouding. For example, Shapiro (1995) describes the inefficiency caused by high mark-ups in the aftermarket and then observes that competition and advertising should drive them away.

> Furthermore, manufacturers in a competitive equipment market have incentives to avoid even this inefficiency by providing information to consumers. A manufacturer could capture profits by raising its [base good] prices above market levels (i.e., closer to cost), lowering its aftermarket prices below market levels (i.e., close to cost), and informing buyers that its overall systems price is at or below market. In this fashion, the manufacturer could eliminate some or all of the deadweight loss, attract consumers by offering a lower total cost of ownership, and still capture as profits some of the eliminated deadweight loss. In other words, and unlike traditional monopoly power, the manufacturers have a direct incentive to eliminate even the small inefficiency caused by poor consumer information (Shapiro, 1995, p. 495).

In the Shrouded Price Equilibrium, Shapiro's intuition about inefficiency applies, but his anticipated unshrouding effects are overturned by other forces. In general, high markups in the aftermarket do not go away as a result of competition or advertising.

However, the findings above show that this competitive effect is overturned by a "curse of debiasing." Specifically, educated consumers would prefer to frequent firms with high add-on prices that they can avoid rather than defecting to firms with marginal cost pricing of the base good and the add-on.

Again, consider the illustrative case of perfect competition, $\mu = 0$. If a sophisticated consumer gives her business to a firm with shrouded market prices, the sophisticated consumer's surplus will be

$$\textit{sophisticated surplus} = -p - e = \alpha V - e > \alpha V - \alpha V = 0.$$

By contrast, if the sophisticated consumer gives her business to a firm with zero markups on both the base-good and the add-on, the sophisticate's surplus will be exactly 0. So, the sophisticated/educated consumer is strictly better off at the firm with shrouded prices (and high add-on markups), that at the firm with marginal cost pricing.

Sophisticated consumers would rather pool with the naive consumers at firms with high add-on prices than defect to firms with marginal cost pricing of both the base good and the add-on. By contrast, if a sophisticate gives her business to a firm with zero mark-ups on both the base good and the add-on, the sophisticate surplus will be 0. This preference for pooling reflects the sophisticate cross-subsidization by naive consumers. Sophisticates benefit from "free gifts" (such as $25 start-up deposit, DVD player, etc) and avoid high fees.

Educating uninformed myopes enables them to get more value out of their relationships with high markup firms. After education, myopes anticipate the high add-on prices, and hence substitute away from add-ons while still enjoying loss-leader prices on the base good. The newly educated consumers benefit from the "free gifts" and avoid high fees.

This generates the curse of debiasing. A firm does not benefit by debiasing uninformed myopic consumers. Newly educated consumers (i.e., sophisticates) are not profitable to any firm. Specifically, sophisticates prefer to patronize—and, in particular, exploit—firms that offer loss-leader prices on base goods.

Gabaix and Laibson's no-advertising result does not change if $\alpha > \alpha^*$. In the paper, they generalize this to the case of continuous choices and imperfect add-on substitutes. Are there any forces that favor unshrouding? The answer is somewhat mixed: learning will accelerate unshrouding, however, innovation will create new add-ons and new opportunities for shrouding. Also, consumer heterogeneity caused by informative advertising, and third party consumer education, i.e. consumer reports subject to some caveats, will favor unshrouding.

In essence, Gabaix and Laibson (2006) show that there are two kinds of exploitation. Firms exploit myopic consumers. In turn, when consumers become sophisticated, they take advantage of these exploitative firms. Finally, when sophisticated consumers exist, firms cannot rid themselves of them. In equilibrium, nobody has an incentive to deviate except the myopic consumers. But the myopes do not know any better, and often nobody has an incentive to show them the error of their ways. Educating a myopic consumer turns him into a (less profitable) sophisticated consumer who prefers to go to firms with loss-leader base-good pricing and high-priced (but avoidable) add-ons. Hence, education does not help the educating firm.[27]

In summary, Gabaix and Laibson (2006) show that the presence of uninformed

[27] Gabaix and Laibson (2006) argue that this mechanism applies to a wide range of markets. An educated banking customer gets the benefit of a $50 gift for opening an account and avoids paying some of the fees that snare myopic consumers. An educated credit card holder gets convenience, float, and miles and avoids paying interest charges and late payment fees. An educated home printer buyer gets a loss-leader price and avoids paying for frequent cartridge replacements (by printing in black and white instead of color, printing in draft mode, or printing fewer large jobs at home). In such markets educated consumers prefer to stick with the firms that feature high add-on prices, since these firms have loss-leader base-good prices, and the educated consumer can partially substitute away from the overpriced add-ons.

myopic consumers incentivizes firms to adopt pricing schedules that have the unintended consequences of subsidizing sophisticates. Making more myopes sophisticated will not help any firm. Because of this curse of debiasing, no firm has an incentive to educate myopes, even when education is costless.

Is there any empirical evidence that shrouding increases firm revenues? Extant empirical literature on price shrouding mostly suggests that shrouding raises profitability. Ellison and Ellison (2009) examine the competition between a group of Internet retailers who operate in an environment where a price search engine plays a dominant role (online firms selling computer memory chips).They find that for some products in this environment, the easy price search makes demand tremendously price sensitive and that shrouding add-ons is a profitable strategy. Brown et al. (2010) conducted field experiments using leading online auction platforms to compare revenues for identical items while varying both the amount and the disclosure level of the shipping charge. They find that shrouding affects revenues: for low shipping charges, a seller is better off disclosing; and increasing shipping charges boosts revenues when shipping charges are shrouded.

4. Reference Dependence and Loss Aversion

In standard utility theory, the attractiveness of a choice alternative depends only on the final outcome that results from that choice. Experimental investigations of small scale trading decisions, as well as monetary risk taking, however, indicate that preferences are reference-dependent: people compare economic outcomes to relevant "reference points" and not only evaluate them according to absolute measures. One of the most robust and quantitatively significant regularities related to reference-dependent preferences is loss aversion—people are more sensitive to losses relative to their reference point than to gains relative to it (Kahneman et al., 1990, 1991; Kahneman and Tversky, 1979; Rabin, 2000).

The concept of reference-dependent preferences has also been extended to the analysis of choice without risk (Tversky and Kahneman, 1991). In a classic experiment that has been replicated many times, one group of objects is endowed with a simple consumer good, such as a coffee mug or an expensive pen. The subjects who are endowed with the good are asked the least amount of money they would accept to sell the good. Subjects who are not endowed with the good are asked how much they would pay to buy one. Most studies find a striking "instant endowment effect." Subjects who are endowed with the good name selling prices which are about twice as large as the buying prices. This endowment effect (Thaler, 1980) is thought to be due to a disproportionate aversion to giving up or losing from one's endowment, compared to the value of gaining, an asymmetry called “loss aversion”. Endowing an individual with an object shifts one’s reference point to a state of ownership and the difference in valuations demonstrates that the disutility of losing a mug is greater than the utility of gaining it.

Like other concepts in economic theory, loss-aversion appears to be general in that it spans domains of data (field and experimental) and many types of choices (Camerer, 2001, 2005).

The domain of most interest to industrial organization economists is the asymmetry of price elasticities (sensitivity of purchases to price changes) for price increases and decreases. Elasticities are larger for price increases than for decreases, which means that demand falls more when prices go up than it increases when prices go down. Loss-averse consumers dislike the pain associated with paying increased prices more that they like the pleasure associated with paying decreased prices. Hence, these consumers should often be more more sensitive to price increases than to price decreases. Putler (1992) was the first to look for such asymmetry in consumer purchases in eggs and did in fact find an asymmetry. Hardie et al. (1993) did a similar (though somewhat more sophisticated) analysis for orange juice, and also found an asymmetry. In addition, the authors estimated the ratio of price elasticities to price increases versus decreases, and found it to be 2.4.

All this evidence relates to consumer behavior in response to price changes. But the fact that consumers behave this way should have implications for firm behavior as well. Suppose that when deciding whether to buy a product, consumers compare the market price to relevant “reference prices” given by past prices and the prices of competing products, with losses counting much more than same-sized gains. For example, when considering whether to buy a pair of jeans for $60, a consumer may be influenced both by how much she is used to paying for jeans and how much the competitors are charging. Paying more than last time, and paying more than other market prices, can both be painful.

What implications would this consumer behavior have on firms’ pricing strategies? It predicts that firms will *reduce price variation both over time* and *between products*. Since consumers are really sensitive to increases in the price above past prices or those of competitors, such increases really hurt demand, so firms do not do it. But lowering one’s price attract fewer new consumers, so lowering one’s price below past prices and those of others is not worth it, either. This means that firms will try to keep stable prices.

This prediction is supported by a lot of evidence. Regular prices in a supermarket or a movie theater, for example, often stay the same for months, even though there is very little cost to changing them. In addition, firms subject to all kinds of shocks often charge an identical, “focal,” price for non-identical products. For example, Coke and Pepsi often sells for the same regular price. And a retailer selling a number of differentiated products often sets the same, "uniform," price for them. Even though demand for different movies is very different, theaters rarely charge different prices for them. While clothes of different sizes are consumed by different people, typically all sizes sell at the same price. We pay the same to see for a Warrior game independently of the opponent, even though popular teams often sell out the arena and many teams do not. Online music vendors price all songs uniformly. And so on.

In what follows we present a very successful and portable recent model capturing loss aversion and reference dependent preferences with endogenous reference points due to Koszegi and Rabin (2006, 2007) and then present a model that explores the implications of consumer loss aversion *a la* Heidhues-Koszegi-Rabin for market outcomes.

4.1. Modeling Reference Dependent Preferences

Empirical evidence suggests that a realistic model of preferences should capture the following three empirical regularities: (1) outcomes are evaluated as changes with respect to a reference point with positive changes framed as gains and negative changes as losses; (2) decision makers are risk-averse in gain domains and risk-seeking in loss domains (the "reflection effect"); and (3) decision makers are loss-averse, i.e, losses generate proportionally more disutility than equal-sized gains.

Prospect theory (Kahneman and Tversky, 1979) is the first formal model of choice that captures these three empirical regularities. Extending their insight, Koszegi and Rabin (2006, 2007) (henceforth KR) develop a model of reference-dependent preferences where "gain-loss" utility is derived from standard "consumption utility" and the reference point is determined endogenously by the economic environment.

In KR model individual utility for a riskless outcome is specified as $u(c|r)$, where $c = (c_1, c_2, \ldots, c_K) \in \mathbb{R}^K$ is consumption and $r = (r_1, r_2, \ldots, r_K) \in \mathbb{R}^K$ is a "reference level" of consumption. The model also allows both consumption and the reference point to be stochastic. In that case if c is drawn according to the probability measure F and the person's reference point is the probability measure G over $\mathbb{R}^K$, then utility is

$$U(F|G) = \iint u(c|r)\, dG(r)\, dF(c).$$

This formulation captures the notion that the sense of gain or loss from a given consumption outcome derives from comparing it with all outcomes possible under the reference lottery. For example, if the reference lottery is a gamble between 0 and 1000, an outcome of 500 feels like a gain relative to 0, and like a loss relative to 1000, and the overall sensation is a mixture of these two feelings.

While preferences are reference-dependent, gains and losses are clearly not all that people care about. The sensation of gain or avoided loss from having more money does significantly affect our utility—but so does the absolute pleasure of consumption we purchase with the money. A usable economic theory must have both. Therefore, the overall utility is assumed to have two components:[28]

$$u(c|r) = m(c) + n(c|r) = \sum_{k=1}^{K} m_k(c_k) + \sum_{k=1}^{K} n_k(c_k|r_k),$$

where $m(c)$ is the classical "consumption utility" and $n(c|r)$ is "gain-loss utility." Beyond saying that a person cares about both consumption and gain-loss utility, KR propose a strong relationship between the two:

$$n_k(c_k|r_k) \equiv \mu\left(m_k\left(c_k\right) - m_k\left(r_k\right)\right),$$

where $\mu(\cdot)$ is a "universal gain-loss function." KR assume that $\mu(\cdot)$ satisfies several simple properties corresponding to Kahneman and Tversky (1979)'s explicit or implicit assumptions about their "value function" defined on $c - r$. The crucial property of $\mu(\cdot)$ is that $\mu'_-(0)/\mu'_+(0) \equiv \lambda > 1$, where $\mu'_+(0) = \lim_{x\to 0}\mu'(|x|)$ and $\mu'_-(0) = \lim_{x\to 0}\mu'(-|x|)$. The parameter λ is the coefficient of loss-aversion: it measures the marginal utility of going from a small loss to zero, relative to the marginal utility of going from zero to a small gain. In a conventional (differentiable) utility function $\lambda = 1$. If $\lambda > 1$ then there is a "kink" at the reference point.

But what is the reference point? The literature has historically fluctuated between imprecision and status quo. A typical approach to this issue is in Tversky and Kahneman (1991):

> "A treatment of reference-dependent choice raises two questions: what is the reference state, and how does it affect preferences? The present analysis focuses on the second question ... the determinants of the reference state lies beyond the scope of the present article. Although the reference state usually corresponds to the decision maker's position, it can also be influenced by aspirations, expectations, norms, and other social comparisons."

[28]In combination with loss aversion, this separability is at the crux of many implications of reference-dependent utility, including the endowment effect.

Other candidates that were proposed in the literature include lagged consumption or endowment, aspirations and goals, social comparisons and expectations. KR assume that the reference-point is fully determined by the *expectations* a person held in the *recent past*. Specifically, a person's reference point is her probabilistic beliefs about the relevant consumption outcome held between the time she first focused on the decision determining the outcome and shortly before consumption occurs. While KR's model of how utility depends on expectations could be combined with any theory of how these expectations are formed, they assume that expectations are *fully rational*. More specifically, they assume that a person's reference point is her rational expectations held in the recent past about outcomes, which are determined in a *personal equilibrium* by the requirement that they must be consistent with optimal behavior given expectations.[29] For recent evidence in support of expectations based reference dependent preferences, see Abeler et al. (2011); Crawford and Meng (2011); Pope and Schweitzer (2011); Sprenger (2011) and references therein.[30]

A simple and striking feature of the KR general model is that when a loss-averse decision maker's choice set is deterministic and all choices in it are deterministic, the predictions of personal equilibrium are identical to those of a model based solely on consumption utility. In a more recent work, Koszegi and Rabin (2009), KR have extended their work to allow for a general dynamic model of reference-dependent utility.

It is easy to demonstrate how KR theory explains the endowment effect. Let the two dimensions of consumption c be mugs and money, respectively, and let

$$m(c) = m_1(c_1)+m_2(c_2) = 6c_1+c_2, \;\; \mu(x) = x \text{ for } x \geq 0, \text{ and } \mu(x) = -2x \text{ for } x < 0,$$

that is, the the intrinsic consumption value of the mag is 6 and $\mu(\cdot)$ is a simplified version of Kahneman and Tversky's value function with a loss aversion coefficient

[29]The formal definitions the reader is refered to the original paper(s). There is a big conceptual issue here related to the notion of consistent plans. The punchline of KR approach is that: (1) departures from classical predictions if and only if there is surprise or uncertainty; (2) because economic settings influence expectations, preferences *seemingly* depend on context; and (3) some desirable plans may not be consistent.

[30]In a real-effort experiment, Abeler et al. (2011) manipulate the rational expectations of subjects and check whether this manipulation influences their effort provision. Abeler et al. (2011) find that effort provision is significantly different between treatments in the way predicted by models of expectation-based, reference-dependent preferences: if expectations are high, subjects work longer and earn more money than if expectations are low. Crawford and Meng (2011) develop a model of labor supply with reference dependence *a la* KR and test it using data from New York City cab drivers. Furthermore, Pope and Schweitzer (2011) document expectations based loss aversion among professional golf players in tournaments. Finally, Karle et al. (2011) find some evidence for expectations-based reference dependence in a controlled consumer choice setting with real consumption and Sprenger (2011) presents evidence from a laboratory experiment on risk aversion.

2[31]. If there was only consumption utility, 6 is the most the decision maker would be willing to pay. With endowment being status quo, the selling price p^S is the lowest price at which the person is just indifferent between keeping the good and selling it when her reference point is to have it: $r = (r_1, r_2) = (1, 0)$. Thus p^S solves

$$m_1(1) + m_2(0) = m_1(0) + m_2\left(p^S\right) + \mu\left(m_1(0) - m_1(1)\right) + \mu\left(m_2(p^S) - m_2(0)\right),$$

so

$$6 + 0 = 0 + p^S + \mu(-6) + \mu\left(p^S\right) = 2p^S - 12,$$

implying that $p^S = 9$.

Similarly, buying price p^B solves

$$m_1(0) + m_2(0) = m_1(1) + m_2\left(-p^B\right) + \mu\left(m_1(1) - m_1(0)\right) + \mu\left(m_2(p^B) - m_2(0)\right),$$

or

$$0 + 0 = 6 - p^B + \mu(6) + \mu\left(-p^B\right) = 12 - 3p^B,$$

yielding $p^B = 4$.

For further illustrations of how the model works with expectations as a reference point, the interested reader is referred to the shoe-shopping application in the paper.

4.2. Selling to Loss-Averse Consumers

We now present a monopoly pricing model that incorporates consumer loss aversion *a la* KR.

Heidhues and Koszegi (2005)[32] analyze the strategic pricing behavior of a profit maximizing monopolist with an uncertain costs of production facing loss averse consumers by applying the theory of reference-dependent preferences in Koszegi and Rabin (2006, 2007).

The timing of the baseline model with commitment is as follows. First, the firm publicly commits to price distribution $F(p)$, then the (representative) consumer forms expectations (beliefs) about behavior. Next, the cost c and price p are realized and the consumer observes a shock $\tilde{w}$. Finally, consumer makes a purchase decision and production takes place.

The shock essentially makes demand continuous and renders uniqueness. The consumer, whose initial wealth is normalized to zero, makes the simple decision

[31] I.e., the person dislikes losses twice as much as she likes same sized gains.

[32] Note that Heidhues and Koszegi (2011) has replaced this version of the paper.

of whether to purchase a single item of a single good. Let $k_1, r_1 \in \{0, 1\}$ be her consumption and reference point in the good, and $k_2, r_2 \in \mathbb{R}$ her consumption and reference point in money, v her intrinsic value for the consumption good and let $k = (k_1, k_2)$, $r = (r_1, r_2)$. Then, following Koszegi and Rabin (2006) the consumer's utility is

$$u = m(k) + n(k|r) = vk_1 + k_2 + \mu(vk_1 - vr_1) + \mu(k_2 - r_2),$$

where $m(k)$ is intrinsic consumption utility and $n(k|r)$ is gain-loss utility. They formalize loss aversion in the simplest possible way: μ is two-piece linear, and has a slope of 1 for gains, and a slope of $\lambda > 1$ for losses. Hence the consumer is more sensitive to losses relative to her reference point than she is to equal sized gains over it. For analytical tractability it is assumed that a consumer doesn't experience gain loss utility in the shock $\tilde{w}$. This implies, that

$$u(k|r) \equiv m(k) + n(k|r) + k_1 w.$$

Consumer's reference point is her lagged probabilistic beliefs about what she is going to get. The consumer faces a probability distribution F of non-negative prices, and can decide whether to buy at each price. Let $\sigma : \mathbb{R}_+ \times (a, b) \to [0, 1]$ be her buying strategy, which assigns a probability of buying at each price-shock pair. Consumer's reference point is the distribution $\Gamma_{\sigma,F}$ induced by σ and F over consumption good-money pairs. To deal with the resulting interdependence between behavior (σ) and expectations ($\Gamma_{\sigma,F}$), Heidhues and Koszegi (2005) require the strategy that generates expectations to be optimal conditional on these expectations.

More formally: $\sigma : \mathbb{R}_+ \times (a, b) \to [0, 1]$ is a *personal equilibrium* for the price distribution F if for all $p \in \mathbb{R}, \tilde{w} \in (a, b)$,

$$\sigma(p, \tilde{w}) \in \arg\max_{s \in [0,1]} s \cdot U(1, -p, \tilde{w}|\Gamma_{\sigma,F}) + (1 - s) \cdot U(0, 0, \tilde{w}|\Gamma_{\sigma,F}).$$

This captures the notion that expectations must be consistent with future behavior.

Firm's pricing strategy is a function from marginal cost to price. Any pricing strategy $P : [\underline{c}, \bar{c}] \to \mathbb{R}_+$ chosen by the firm induces a distribution of prices F_P faced by the consumer. For a strategy σ, let $\bar{\sigma}(p) \equiv \int_a^b \sigma(p, \tilde{w}) dH(\tilde{w})$; $\bar{\sigma}(p)$ is the consumer's probability of buying the good at price p.

The pricing strategy $P(\cdot)$ and the strategies $\sigma_F : \mathbb{R}_+ \times (a, b) \to [0, 1]$ for all price distributions F constitute *pricing equilibrium* if

(a) For any price distribution F, σ_F is a personal equilibrium for F.

(b) $P(\cdot)$ maximizes the firm's expected profits:

$$P \in \arg\max_{\hat{P}(\cdot)} \int_0^\infty (\hat{P}(c) - c) \cdot \bar{\sigma}_{F_{\hat{P}}} \hat{P}(c) d\Theta(c)$$

A pricing equilibrium is a situation in which (1) for any pricing strategy chosen by the firm, the consumer plays a personal equilibrium in the "continuation game," and (2) the firm chooses an optimal pricing distribution correctly anticipating the continuation play of the consumer. The sole source of uncertainty that drives price changes in this model is uncertainty about marginal cost.

Heidhues and Koszegi (2005) characterize this equilibrium. Some interesting effects arise because this commitment allows the firm to manage consumer expectations. One observation is that prices will be sticky and firms may set a constant price (or choose prices from a discrete set) even when cost shocks are continuously distributed. So, they get a menu cost effect without assuming menu costs. The key force behind price stickiness is the "comparison effect" rising from the consumer's loss aversion in money. The disutility that loss-averse consumers feel when they pay more than they were expecting is greater than the utility they derive from paying symmetrically less than they were expecting. This provides an incentive to keep prices constant. By not exposing the consumer to small price movements the firm increases her overall demand and hence its sales revenue.

A related observation is that markups are countercyclical. An economist who studies the firm's behavior and the realized market demand curve, but ignores that demand depends on the consumer's expectations, would conclude that the firm systematically deviates from profit maximization in a number of ways. Most importantly, she would find that the firm's Lerner Index varies less than that of a standard monopolist. When the firm decides how to price in low cost states, it needs to take into consideration that the low price it would like to charge would form a basis for an (unfavorable) comparison should prices be higher, decreasing the consumer's demand in these latter states of the world. Therefore, the monopolist is reluctant to aggressively cut its price in response to low cost realizations.

Another fundamental conclusion is that there is substantial scope for indeterminacy: as in Koszegi and Rabin (2006) the purchase decision contingent on the price is not necessarily unique because the consumer has a higher valuation when she expects to buy, and even apart from this there can be multiple pricing equilibria when the firm lacks commitment power. This implies that even a firm with deterministic cost may offer sales to get consumers used to the idea of buying.

One of the main contributions of the Heidhues and Koszegi (2005), besides shedding new light on three stylized facts about the distribution and time pattern of prices for consumer goods (price stickiness, countercyclical markups, commonness of temporary sales and promotions), is that they develop general techniques for embedding consumer loss aversion in firm's decision problem.

Spiegler (forthcoming) has recently reformulated and simplified the model of Heidhues and Koszegi (2005). Spiegler (forthcoming) analyzes optimal pricing when consumers are loss averse in the sense that an unexpected price hike lowers their willingness to pay. The main message of the Heidhues-Koszegi model, namely that this form of consumer loss aversion leads to rigid price responses to cost fluctuations, carries over in his model. He is further able to show that loss aversion lowers expected prices and the firm's incentive to adopt a rigid pricing strategy is stronger when fluctuations are in demand rather than in costs.

Heidhues and Koszegi (2008) extend their monopoly pricing paper to an oligopolistic setting. They introduce consumer loss aversion into the Salop (1979) model of price competition with differentiated products. In the formal model, firms face uncertain costs of production and after observing their own cost functions simultaneously set prices. One of the main findings is that consumers' sensitivity to losses in money increases her price sensitivity of demand—and hence the intensity of competition—at higher relative to lower market prices, reducing or eliminating price variation in a number of senses consistent with observed pricing regularities. Also they show that under some conditions there exists an equilibrium in which all firms always charge the same "focal" price. Another finding is that when firms face common stochastic costs, in any symmetric equilibrium the markup is strictly decreasing in cost, and the price may be constant over parts or over all of the range of possible costs. Because a change in the price responsiveness of demand affects competition more when margins are high, the above tendencies are stronger in less competitive industries. Finally, because the loss in product satisfaction she would suffer makes a consumer difficult to attract form a competitor, loss aversion decreases competition and increases prices.

Koszegi and Rabin (2006, 2007) and Heidhues and Koszegi (2005, 2008) have kick started a growing literature in Industrial Organization with elements of consumer loss aversion and reference dependence. Two such interesting studies are those of Herweg and Mierendorff (2011) and Karle and Peitz (forthcoming).[33]

Herweg and Mierendorff (2011) consider a model of firm pricing and consumer choice, where consumers are loss averse and uncertain about their future demand. Consumers in their model possibly prefer a flat rate to a measured tariff, even though this choice does not minimize their expected billing amount—a behavior in line with ample empirical evidence. They show that the profit-maximizing two-part tariff is a flat rate if (1) marginal costs are not too high, (2)

[33] See also Zhou (2011) for an extension of standard duopoly price competition model with horizontal product differentiation and loss averse consumers.

loss aversion is intense, and (3) there are strong variations in demand. Moreover, they analyze the optimal nonlinear tariff and show that is has a large flat part when a flat rate is optimal among the class of two-part tariffs.

Karle and Peitz (forthcoming) explore the effect of contextual consumer loss aversion on firm strategy in imperfectly competitive markets. In their model all consumers are fully informed about match value and price at the time they make their purchasing decision. However, a share of consumers are initially uninformed about their tastes and form a reference point consisting of an expected match-value and price distribution, while other consumers are perfectly informed all the time. Karle and Peitz (forthcoming) provide a detailed account of price effects of consumer loss aversion in three settings: a general symmetric duopoly, an asymmetric duopoly and an oligopoly. Their main finding is that a larger share of informed consumers leads to a less competitive outcome of the asymmetry between duopolists is sufficiently large. They also show that narrowing the consideration set of consumers leads to a more competitive outcome.

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Chapter 2

Entrepreneurial Overconfidence and Market Selection*

Karen Khachatryan and Jörgen W. Weibull

Abstract

We explore whether competition between firms owned and run by entrepreneurs favors overconfident entrepreneurs. We study this question in a variety of settings, all based on Cournot duopoly in the product market. In the basic model, entrepreneurs choose their own firm's output and may have more or less optimistic beliefs about their own firm's (random) production costs. We study both the case of complete and incomplete information about the competitor's type. We also analyze a model with endogenous costs in the complete-information setting in which entrepreneurs make efforts to reduce their firm's production costs. For each of the model versions, we show that, if market selection is driven by firms' absolute and/or relative profit performance, somewhat overconfident entrepreneurs will be selected for, and that this tendency is stronger the more emphasis is placed on relative performance.

Keywords: Entrepreneurs, overconfidence, market selection.

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1. Introduction

Much of economic life is conducted in an uncertain environment. The rational expectations hypothesis postulates that economic agents make decisions as if they held correct probabilistic beliefs, given their information. It is argued (Alchian, 1950; Friedman, 1953) that the rational-expectations hypothesis is justified, because agents holding rational expectations are more likely to survive the market test than those with non-rational expectations. Yet, psychological and empirical research on judgment under uncertainty provides extensive evidence that people tend to be *optimistic* (Taylor and Brown, 1988), exhibit *overconfidence* in judgement (DeBondt and Thaler, 1995; Weinstein, 1980, 1982, 1984, and the reviews therein), and are often *overconfident* about their own relative *ability* (Svenson, 1981). Psychologist Shelley E. Taylor sums up much of the evidence on optimistic biases in her comprehensive book on the subject, Taylor (1989), and even argues that unrealistic optimism is an indispensable trait of the healthy mind. In the present paper, we examine the possibility that somewhat optimistic expectations may survive the market test, indeed, that agents holding such expectations may do better than those with rational expectations.

More precisely, we study whether product-market competition between firms owned and run by entrepreneurs may favor overconfident entrepreneurs, that is, entrepreneurs who hold optimistic beliefs about how good they and/or their firms are. The hypothesis that entrepreneurs may tend to be overconfident in this sense has some empirical support, see Busenitz and Barney (1997); Cooper et al. (1988) and Camerer and Lovallo (1999). In Cooper et al.'s sample of about three thousand entrepreneurs, 81% believe that their chances of success are at least 70% and 33% believe their chances are as high as 100%. In reality, only about 25% of new businesses still exist after 5 years. Camerer and Lovallo (1999) experimentally studied whether optimistic biases could plausibly and predictably influence entry into competitive interactions. They find that when subjects' earnings depend on skill, individuals tend to overestimate their chances of success and enter more frequently than when earnings do not depend on skill. This overconfidence is even stronger when subjects could self-select into the experimental sessions, well aware that their success would depend partly on their skill and that their competitors had self-selected too.[1]

[1]The empirically established bias toward overoptimism and overconfidence is most evident in connection with areas of self-declared or self-selected expertise (DeBondt and Thaler, 1995). Thus, the decisions of *entrepreneurs* are more likely to reflect even more *overconfidence* than the population at large. Pessimists, who might tend to be excessively conservative as entrepreneurs, would be likely to select other occupations whose outcomes are more predictable and thus less subject to their own pessimistic expectations; that is, they might prefer to be employees rather than entrepreneurs (see de Mezza

Our analysis is carried out for a variety of simple competitive settings, all based on Cournot duopoly in a homogenous product market. Each firm has a constant but random unit cost, unknown at the time of production decisions. The demand for the product is linearly decreasing in its price. In the basic model, each entrepreneur chooses his or her firm's output volume, on the basis of her beliefs about her own firm's random unit production cost. We study both the case of complete and incomplete information about one's competitor's beliefs. In the complete-information case, both entrepreneurs know each other's cost expectations. In small economies, with a relatively few entrepreneurs, this is arguably a fairly realistic assumption — competitors then often know or recognize each other. In the incomplete-information case, the two competitors in a duopoly do not know or recognize each other, but they know the probability distribution of "entrepreneurial types" in the pool of potential entrepreneurs in the economy.

In the extended model, the probability distribution of a firm's unit production cost is not exogenous and fixed but depends on the entrepreneur's effort. Hence, in this model version, entrepreneurs not only choose output levels but also an effort that probabilistically reduces their own firm's unit production cost. We analyze this extended model version only in the complete-information setting. In essence, each entrepreneur's overconfidence then amounts to a disagreement between the entrepreneurs about their respective skills (they so to speak agree to disagree as in Morris, 1996).

In each of the model versions, we analyze the market outcome in terms of output volumes, market shares and profits, and analyze the potential effect of market selection on the types of entrepreneurs who survive, both when the market test is based on absolute performance — profits — and when it is based on relative performance — profit compared with competitors' profits. We find that somewhat overconfident entrepreneurs will be selected for in such settings, and that this tendency is stronger the more emphasis is placed on relative, as opposed to absolute, performance. In particular, entrepreneurs of the *homo oeconomicus* variety, holding correct expectations about their own firm's cost or their own skill, as a rule earn lower profits.

The rest of the paper, which is a work report from an ongoing research project, is organized as follows. Section 2 introduces the basic setting. The basic model version, with exogenous entrepreneurial effort, is analyzed under complete information in Section 3 and under incomplete information in Section 4. Section 5 develops an explicit dynamic for market selection in these two cases. Section 6 extends the analysis to the case of endogenous entrepreneurial effort. Section 7 discusses some related literature and Section 8 offers concluding remarks and

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and Southey, 1996, for a discussion).

suggestions for future research.

2. The Model

2.1. The Market Interaction

Consider a Cournot duopoly for a homogeneous good, with linear demand,

$$P(Q) = 1 - Q,$$

where $p = P(Q)$, for $0 \leq Q \leq 1$, is the market price when total output is $Q = q_1 + q_2$. Each firm i has a constant unit cost $\tilde{c}_i$, a random variable that takes values in the interval $C = \left(c^L, c^H\right)$, where $c^L = 0$ and $c^H = 1/2$. Each firm i is owned and run by an entrepreneur. The two entrepreneurs choose their respective firms' outputs simultaneously and do not know their true costs at that point in time. We intentionally keep the market interaction this simple, in order to focus better on the subtle issue of entrepreneurs' potential over- or underconfidence.

2.2. The Entrepreneurs

Each entrepreneur i holds a probabilistic belief, ν_i, about his or her own firm's unit cost, and strives to maximize his or her firm's accordingly expected profit,

$$\pi_i(q_1, q_2) = \mathbb{E}_{\nu_i}\left[(P(q_1 + q_2) - \tilde{c}_i) \cdot q_i\right] = (1 - Q - c_i)\, q_i \tag{1}$$

for $i = 1, 2$ and $j \neq i$, where $c_i = \mathbb{E}_{\nu_i}[\tilde{c}_i]$ is the mathematical expectation under entrepreneur i's probabilistic belief about her own firm's unit cost.

When two entrepreneurs meet in our simple market interaction, their types, ν_1 and ν_2, are first independently drawn by "nature" according to some probability measure μ. A type is thus a probabilistic belief about one's own cost. After their types have been drawn, each entrepreneur chooses an output quantity for his firm, according to his or her probabilistic beliefs. Finally, the product market clears, and true costs and profits are realized.[2] We consider two informational settings.

Remark 1 *The present approach to entrepreneurial beliefs can alternatively, and more generally (but at the cost of heavier notation), be cast in terms of current Choquet expected utility (CEU) theory. In particular,* Chateauneuf et al.

[2] To be more precise, let Ω be a sample space, $\mathcal{A}$ a sigma-algebra in Ω and $\tilde{c}_i : \Omega \to C$ an $\mathcal{A}$-measurable random variable. Let V be the space of probability measures on $\mathcal{A}$, with $\rho, \nu_1, \nu_2 \in V$, where ρ is the probability by which true costs are (independently) drawn, with $\mathbb{E}_\rho[\tilde{c}_i] = c$. Likewise, $\mathbb{E}_{\nu_i}[\tilde{c}_i] = c_i$ for $i = 1, 2$, are the subjectively expected own costs. Finally, let μ be a probability measure on some sigma-algebra on V.

(2007) analyze a certain class of such representations of preferences over uncertain prospects, a class that is additive on non-extreme outcomes (or neo-additive). Unlike standard expected-utility maximizers, such decision-makers may pay special attention to extreme outcomes, here corresponding to extreme overconfidence — that one's unit cost is c^L — and extreme underconfidence — that one's unit cost is c^H.

3. Complete Information

Suppose that the entrepreneurs know each others' types, and hence cost expectations, c_1 and c_2. This is arguably realistic in smaller economies and in economies with few entrepreneurial types. The two profit functions in (1) together define a simultaneous-move game, $G(c_1, c_2)$, in quantities $q_1, q_2 \in [0, 1]$. Its unique Nash equilibrium is

$$q_i^* = \frac{1 - 2c_i + c_j}{3}$$

for $i = 1, 2$ and $j \neq i$. For all (subjective) own-cost expectations in C, this results in a market price, $p^* = 1 - Q^*$, that exceeds each participant's (subjectively) expected own unit cost.[3]

3.1. One Overconfident Entrepreneur

Suppose that the true unit costs are i.i.d. with mean value $c \in C$, to be called the average cost. Let ρ be the probability distribution for the true unit costs: $\mathbb{E}_\rho[\tilde{c}_1] = \mathbb{E}_\rho[\tilde{c}_2] = c$. Consider the case when $c_1 = c$ and $c_2 < c$. In other words, entrepreneur 1 holds a correct expectation about her firm's unit cost, while entrepreneur 2 is overconfident about his firm's unit cost. We will refer to entrepreneur 1 as *homo oeconomicus* — since we, as analysts, know that her cost expectation is correct — and we will call entrepreneur 2 *overconfident* — since we know that he underestimates his own cost.

Writing τ_i for c_i/c, the average profits, in duopolies where firm 1 is run by *homo oeconomicus* and firm 2 by an overconfident manager, are

$$\pi_1^* = \mathbb{E}_\rho\left[(P(q_1^* + q_2^*) - \tilde{c}_1)q_1^*\right] = \left(\frac{1 - (2 - \tau_2)c}{3}\right)^2,$$

and

$$\pi_2^* = \mathbb{E}_\rho\left[(P(q_1^* + q_2^*) - \tilde{c}_2)q_2^*\right] = \left(\frac{1 - (2 - \tau_2)c}{3}\right)^2 + \frac{1 - (2 - \tau_2)c}{3}(1 - \tau_2)c.$$

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[3] To see this, note that $p^* = (1 + c_1 + c_2)/3 > c_i$ iff $c_i < \left(1 + c_j\right)/2$ for $i = 1, 2$ and $j \neq i$.

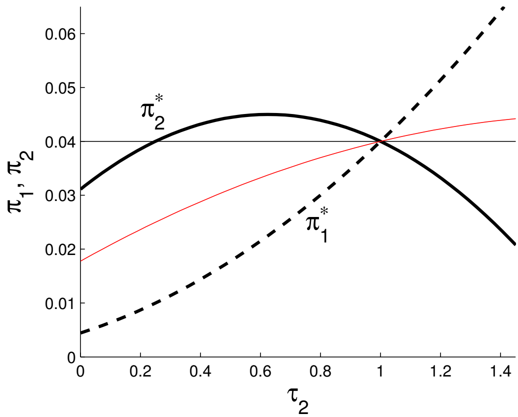

FIGURE 1. PROFITS

Hence,

$$\pi_2^* - \pi_1^* = \frac{1 - (2 - \tau_2)c}{3}(1 - \tau_2)c. \tag{2}$$

It follows that $\pi_2^* > \pi_1^*$ for all $\tau_2 < 1$.[4] The diagram in Figure 1 shows how π_2^* (solid curve) and π_1^* (dashed curve) depend on τ_2 (the horizontal axis), for $c = 0.4$. The thin curve is the average of the two curves — half the industry profit.

We see that:

(a) For all $\tau_2 < 1$ the overconfident entrepreneur 2 produces more and makes a higher profit than entrepreneur 1, *homo oeconomicus*,

(b) There is an optimal degree of overconfidence for entrepreneur 2, $\tau_2 \approx 0.6$,

(c) The industry profit is increasing in τ_2 — the more correct 2's cost expectations are, the higher is the industry profit,

(d) For moderate degrees of overconfidence ($0.25 \lesssim \tau_2 < 1$), entrepreneur 2 earns a higher profit than had he also been *homo oeconomicus* (had $\tau_2 = \tau_1 = 1$).

[4]Note that $(2 - \tau_2)\,c < 1$ since $c < 1/2$ by assumption.

3.2. Two Arbitrarily Confident Entrepreneurs

Suppose, again, that the true unit costs are i.i.d. with mean value $c \in C = \left(c^L, c^H\right)$, and let ρ be the probability distribution for the true unit costs; $\mathbb{E}_\rho\left[\tilde{c}_i\right] = c$ for $i = 1, 2$. By a slight generalization of the above calculations, one easily finds that the true average equilibrium profit to each firm i then is

$$\pi_i^* = \mathbb{E}_\rho\left[(P\left(q_1^* + q_2^*\right) - \tilde{c}_i)q_i^*\right] = \frac{1 - 3c + (\tau_1 + \tau_2)\,c}{3} \cdot \frac{1 - \left(2\tau_i - \tau_j\right)c}{3} \tag{3}$$

for $i = 1, 2$ and $j \neq i$. We make three general observations.

First, as a function of own confidence, τ_i, the average profit π_i^* is a concave function with maximum at

$$\hat{\tau}_i = \frac{1}{4}\left(6 - \frac{1}{c} - \tau_j\right). \tag{4}$$

In particular, if $\tau_1 = 1$ and $c = 0.4$, as in the preceding subsection, then $\hat{\tau}_2 = 0.625$, in broad agreement with the above approximate observation. More generally, for

$$\frac{1}{6} < c < \frac{1}{2} \tag{5}$$

this defines $\hat{\tau}_i$ as a number in the open unit interval. We henceforth assume this inequality to hold throughout Sections 3 to 5.

Second, suppose that the prevalent degree of confidence among entrepreneurs in the industry in question were $\tau = 1$ — that of *homo oeconomicus* — and suppose that entrepreneurs survive in the market according to how well their firms do in terms of average profit. Then *homo oeconomicus* would be selected against. If a small fraction of start-up firms were run by entrepreneurs with higher self-confidence, then these would be favored by the market, and this favoring would be strongest for entrants with τ close to 0.625. An economy with firms run by *homo oeconomicus* would not be sustainable in the long run.

Third, a degree of entrepreneurial confidence would be robust against such market selection if it were the optimal degree of confidence when pitted against itself. In other words, a robust degree of confidence τ would need to satisfy equation (4) for $\hat{\tau}_i = \tau_j = \tau$. The unique degree of confidence with this robustness property is evidently

$$\tau^R = \frac{6}{5} - \frac{1}{5c}. \tag{6}$$

This degree of confidence is increasing in the unit cost c, from $\tau^R = 0$ at $c = 1/6$ to $\tau^R = 4/5$ at $c = 1/2$. In other words, an economy with firms run by entrepreneurs of type τ^R would be sustainable in the long run. In particular, it would be robust to small fractions of start-ups run by *any* other type.

When both entrepreneurs in the duopoly in question have this robust degree of confidence, each firm's expected profit is

$$\pi^R = \frac{2}{25}(1-c)^2,$$

which is less than had both entrepreneurs instead been *homo oeconomicus*; then their profits would have been $(1-c)^2/9$. Hence, market selection tends to select overconfident entrepreneurs and thereby reduce industry profit. However, overconfidence among entrepreneurs is good for consumers — firms run by such entrepreneurs supply more output than firms run by *homo oeconomicus*, and thus press down the market price. For all $c \in (1/6, 1/2)$:

$$p^R = \frac{1}{15} + \frac{4c}{5} < p^* = \frac{1}{3} + \frac{2c}{3},$$

where p^R is the market price when both entrepreneurs have the robust degree of confidence, τ^R, and p^* is the market price when both entrepreneurs are *homo oeconomicus* ($\tau = 1$).

In sum:

Proposition 2 *Assume (5). If market selection of entrepreneurs is driven by their firms' average profits, then the market will select somewhat overconfident entrepreneurs. The unique robust degree of confidence would be* $\tau^R \in (0, 4/5)$*, given in (6). The associated equilibrium market price, and firms' profits, will be lower than had all entrepreneurs been homo oeconomicus.*

We conclude by a comment on market selection based on *relative* performance. Suppose, thus, that entrepreneurs are selected on the basis of the difference between their firm's (average) profit and the average industry profit. Such "yardstick" selection of entrepreneurs, as applied to our simple duopoly market, tends to favor overconfident entrepreneurs even more than market selection based on absolute (average) profits.

To see this, first suppose that the prevalent degree of confidence among entrepreneurs in the industry in question is that of *homo oeconomicus* and suppose that entrepreneurs are selected for according to how well their firms do in terms of the difference between their and their competitor's average profits. Using the formulae from Section 3.1, we obtain that the market would again favor overconfident entrepreneurs over *homo oeconomicus*, see equation (2). More generally, the expected profit difference between two duopoly firms run by entrepreneurs with arbitrary degrees of confidence is

$$\pi_i^* - \pi_j^* = \frac{1 - 3c + (\tau_1 + \tau_2)\, c}{3} \left(\tau_j - \tau_i\right) c.$$

Hence, the entrepreneur with the higher degree of confidence (lower τ) will always make a higher average profit than his or her competitor. Thus, market selection based on relative performance tends to drive up the degree of confidence of entrepreneurs to such a degree that profits eventually turn negative, see equation (3).

4. Incomplete Information

In the preceding section, we assumed that the two entrepreneurs in the duopoly market in question knew each other's types. What can be said if they don't, if each entrepreneur i only knows his or her own cost expectation, but not that of the competitor? In line with standard game theory, we assume, however, that the entrepreneurs are sophisticated in the sense that they both know the type distribution μ from which (they and) their opponent is drawn.

To be more precise: first, "nature" makes statistically independent draws of the two entrepreneur's types, ν_1 and ν_2. Then each entrepreneur $i = 1, 2$ forms his or her expectation, $c_i \in C$, about his or her own unit cost:

$$c_i = \mathbb{E}_{\nu_i}[\tilde{c}_i] = \tau_i c,$$

where $0 < c < 1/2$, and c is the true average cost for each firm, $c = \mathbb{E}_\rho[\tilde{c}_1] = \mathbb{E}_\rho[\tilde{c}_2]$. As before, both entrepreneurs simultaneously choose their output quantities, under the (correct) belief that the other entrepreneur's type has been drawn according to the distribution μ. Thereafter, the market clears and profits are realized.

A pure strategy for an entrepreneur i is a function $s_i : C \to [0, 1]$ that maps his or her cost expectation c_i to an output quantity, $q_i = s_i(c_i)$. The subjectively expected profit to entrepreneur i, when his or her type is ν_i is

$$\Pi_i(s_1, s_2) = \mathbb{E}_{\nu_i}\left[(1 - s_i(c_i) - \mathbb{E}_\mu\left[s_j\left(c_j\right)\right] - \tilde{c}_i) \cdot s_i(c_i)\right] \tag{7}$$

for $i = 1, 2$ and $j \neq i$. Here $s_j\left(c_j\right)$ is the competitor's output, a random variable determined by the competitor's strategy s_j and type ν_j (a draw from the type distribution μ). We solve for (Bayesian) Nash equilibrium and focus on *symmetric pure-strategy equilibria*. Such an equilibrium is defined by a strategy $s \colon C \to$

$[0,1]$ that is a best reply to itself, that is, that for all probabilistic beliefs $\nu_i \in V$ in the support of the type distribution μ satisfies

$$s(c_i) \in \arg\max_{q_i} \left(1 - q_i - \mathbb{E}_\mu \left[s\left(c_j\right)\right] - c_i\right) \cdot q_i, \tag{8}$$

where $c_i = E_{\nu_i}[\tilde{c}_i]$. Let $\bar{c} = E_\mu[c_1] = E_\mu[c_2]$, the mean-value of all entrepreneurial types' own-cost expectations.

Proposition 3 *There exits a unique symmetric pure-strategy Bayesian Nash equilibrium. In this equilibrium, all entrepreneurs expect positive profits, and they all use the strategy*

$$s^*(c_i) = \frac{1}{3} + \frac{\bar{c}}{6} - \frac{c_i}{2} > 0 \qquad \forall c_i \in C. \tag{9}$$

Proof. From (8) we obtain that a necessary first-order condition for a strategy s to maximize i's expected profit against $s_j = s$, when i's cost expectation is c_i, is

$$1 - 2s(c_i) - \mathbb{E}_\mu \left[s\left(c_j\right)\right] - c_i = 0. \tag{10}$$

Since this has to hold for all c_i, we can take the expectation of both sides of the equality, with respect to the type distribution μ, which gives

$$1 - 2 \cdot \mathbb{E}_\mu [s(c_i)] - \mathbb{E}_\mu \left[s\left(c_j\right)\right] - \mathbb{E}_\mu [c_i] = 0,$$

and thus, since $\mathbb{E}_\mu \left[s\left(c_j\right)\right] = \mathbb{E}_\mu [s(c_i)]$,

$$\mathbb{E}_\mu [s(c_i)] = \frac{1 - \bar{c}}{3},$$

where $\bar{c} = \mathbb{E}_\mu [c_i]$. Inserting this in (10) results in (9). Moreover, since the maximand in (8) is concave in q_i, the necessary first-order condition is also sufficient. Finally, an entrepreneur i with own-cost expectation c_i, expects the market price to be

$$1 - s^*(c_i) - \mathbb{E}_\mu [s^*(c_i)] = \frac{1}{3} + \frac{\bar{c}}{6} + \frac{c_i}{2}.$$

This expected price exceeds i's expected own unit cost, c_i, if and only if $c_i < 2/3 + \bar{c}/3$, an inequality that holds for all $c_i \in C$. ■

We note that the more self-confident an entrepreneur is, the more he or she produces in equilibrium; $s^*(c_i)$ is decreasing in c_i. Since the pair of entrepreneurs in a duopoly face the same market price, this implies that the more confident entrepreneur of the two earns a higher profit than the other. Assume that the true unit costs are i.i.d., with support in C. Then the more confident entrepreneur of the two in the duopoly will earn a higher average profit if the type distribution μ is not too dispersed:

Proposition 4 *Suppose that the true unit-cost distribution has support in* $C = (0, 1/2)$*. If, moreover,* $1/3 < c_i < 1/2$ *for all entrepreneurial types* $v_i \in V$*, then the more confident entrepreneur, in any pair of duopolists, always earns a higher average profit than the less confident one.*

Proof. Consider the unique symmetric pure-strategy equilibrium under incomplete information, given in Proposition 3, and suppose that entrepreneur i holds cost expectation $c_i \in C$, then (true) average profit to firm i, in equilibrium, is

$$[1 - s^*(c_1) - s^*(c_2) - c] \cdot s^*(c_i) = \left(\frac{1}{3} - \frac{\bar{c}}{3} + \frac{c_1 + c_2}{2} - c\right)\left(\frac{1}{3} + \frac{\bar{c}}{6} - \frac{c_i}{2}\right).$$

The second factor is positive for all $\bar{c}, c_i \in C$, and the first factor is positive if $1/3 < c_1, c_2, \bar{c} < 1/2$. To see this, note that then $1/3 - \bar{c}/3 - c > -1/3$ and $c_1 + c_2 > 2/3$. Thus, both factors are positive under the hypothesis of the proposition.[5] Likewise, the (true) average profit to the other firm j in the duopoly is

$$\left(\frac{1}{3} - \frac{\bar{c}}{3} + \frac{c_1 + c_2}{2} - c\right)\left(\frac{1}{3} + \frac{\bar{c}}{6} - \frac{c_j}{2}\right),$$

again two positive factors. It follows that the firm run by the entrepreneur with the lowest own-cost expectation will earn the highest average profit. ■

In sum: for modest degrees of overconfidence and not too dispersed type distributions, the more confident entrepreneur in a duopoly earns a higher expected profit than the competitor. Note, however, that this does *not* imply that more confident entrepreneurs *on average,* over all potential duopoly matchings in the entrepreneurial population, earn a higher expected profit than less confident ones. Indeed, the unique entrepreneurial type that earns the highest average profit is *homo oeconomicus*:

Proposition 5 *The expected average equilibrium profit to an entrepreneur i with own-cost expectation* $c_i \in C$*, in a random match with another entrepreneur from the type distribution* μ*, is*

$$\mathbb{E}_\mu [\pi^*(c_i)] = \left(\frac{1}{3} - \frac{\bar{c}}{3} + \frac{c_i + \bar{c}}{2} - c\right)\left(\frac{1}{3} + \frac{\bar{c}}{6} - \frac{c_i}{2}\right),$$

and this profit is maximized at $c_i = c$.

[5] This algebra shows that the given sufficient condition can in fact be somewhat weakened (at the cost of becoming less transparent).

Proof. From (9) and (7) we obtain

$$\begin{aligned}\mathbb{E}_{\mu}[\Pi_1(s^*(c_1), s^*(c_2))|c_1 = c_i] &= \left(1 - s^*(c_i) - \mathbb{E}_{\mu}[s^*(c_2)] - c\right) \cdot s^*(c_i) \\ &= \left(\frac{1}{3} - \frac{\bar{c}}{3} + \frac{c_i + \bar{c}}{2} - c\right)\left(\frac{1}{3} + \frac{\bar{c}}{6} - \frac{c_i}{2}\right).\end{aligned}$$

The right-hand side is quadratic in c_i, with derivative

$$\frac{1}{2}\left(\frac{1}{3} + \frac{\bar{c}}{6} - \frac{c_i}{2}\right) - \frac{1}{2}\left(\frac{1}{3} - \frac{\bar{c}}{3} + \frac{c_i + \bar{c}}{2} - c\right) = \frac{1}{2}(c - c_i).$$

Thus $c_i = c$ is the unique maximand. ■

Like in the case of complete information, we briefly discuss market selection. Suppose, first, that entrepreneurs are selected for according to how well their firms do in terms of their expected profit. A degree of entrepreneurial confidence τ would be robust against such market selection if it were the optimal degree of confidence, in terms of average profit earned, in a random match against a type drawn from the type distribution μ. All other types would be selected against and eventually disappear from the population of active entrepreneurs. Only the optimal degree of confidence would prevail.

According to Proposition 5, the unique robust degree of confidence, that the market will select for, would be $\tau = 1$, granted this belongs to the support of the type distribution (which we presume). In the notation of the preceding section: $\tau^R = 1$ under incomplete information. *Homo oeconomicus* would thus prevail. This contrasts sharply with the outcome of market selection according to absolute performance under complete information, where we found that a certain degree of overconfidence, $\tau^R < 1$, was selected for, see Proposition 2.

Secondly, suppose that entrepreneurs are selected for according to how well their firm fares in comparison with their duopoly competitor. From Proposition 4 it follows, roughly, that only the most overconfident of entrepreneurs would survive. More precisely, suppose that the true unit-cost distribution ρ has support in $(1/3, 1/2)$ and that all the subjective own-cost beliefs $\nu \in V$ in the support of the type distribution μ have the same finite support $C^* \subset (1/3, 1/2)$. In other words: all entrepreneurial types hold probabilistic beliefs over the same finite set of potential unit costs, C^*. Propositions 3 and 4 then both apply. Let $c^* = \min C^*$, the lowest subjective own-cost expectation in the type pool. Then this would be the unique robust degree of confidence, since, by Proposition 4, such entrepreneurs would always earn more than any duopoly competitor they meet. This conclusion is qualitatively the same as that for the case of complete information.

In sum: if market selection sometimes is based on absolute performance, sometimes on relative performance, and if the duopolists sometimes know each others' types, sometimes do not, then our analysis of the four pure cases suggests that market selection will favor some degree of overconfidence, and in three cases out of four select against *homo oeconomicus*. The next section provides a precise underpinning for this broad and general claim.

5. Explicit Market-Selection Processes

In the spirit of the last section, let the number of potential degrees of confidence, τ, be finite: $\tau \in T = \{\tau_1, ..., \tau_n\}$ for some positive integer n and "types" $1 = \tau_1 > \tau_2 > \ldots > \tau_n > 0$. Imagine a heterogeneous population of potential entrepreneurs. For each type $\tau_i \in T$, let $x_i \in [0, 1]$ be the population share of entrepreneurs of type τ_i, and let $x = (x_1, ..., x_n)$ be called the *population state*. We then have $x \in \Delta(T) = \left\{x \in \mathbb{R}^n_+ : \sum_{i=1}^n x_i = 1\right\}$ and

$$\bar{\tau} = \sum_{i=1}^{n} x_i \tau_i.$$

Suppose that now and then firms are screened by creditors, who either look at a sampled firm's absolute profit or at its relative profit in comparison with the competitor in its duopoly market. Write π_{ij} for the expected profit to a firm of type τ_i when matched, in a Cournot duopoly as modelled above, against a firm of type τ_j, and write

$$\pi(i, x) = \sum_{j=1}^{n} \pi_{ij} x_j, \qquad \pi(x, j) = \sum_{i=1}^{n} x_i \pi_{ij}, \qquad \pi(x, x) = \sum_{i=1, j=1}^{n} x_i \pi_{ij} x_j.$$

Suppose that a firm of type i, after a duopoly match against a firm of type j survives with probability

$$\eta \pi_{ij} + (1 - \eta)\left[\pi_{ij} - \pi_{ji}\right],$$

where $\eta \in [0, 1]$ is the probability that the evaluation will be based on absolute performance and $1-\eta$ the probability that it will be based on relative performance. Hence, in any population state x, the probability of survival after a random match is

$$h_i(x) = \eta \pi(i, x) + (1 - \eta)[\pi(i, x) - \pi(x, i)] = \pi(i, x) - (1 - \eta)\pi(x, i),$$

If the credit screenings is a stationary Poisson process with intensity (time rate) 1, then the *death rate* of firms of type i is $\delta_i(x) = 1 - h_i(x)$. With a birth-rate of $\beta_i(x)$ for firms of type i, we obtain the following mean-field approximation of the stochastic population process:

$$\dot{x}_i = (\beta_i(x) - \delta_i(x) - [\beta(x) - \delta(x)])\, x_i,$$

where the dot signifies time derivative, $\beta(x) = \sum_i x_i \beta_i(x)$ and $\delta(x) = \sum_i x_i \delta_i(x)$. In this deterministic approximation (which holds better the larger the population is, see Benaim and Weibull (2003)), probabilities have been replaced by flow shares.

If all entrepreneurial types would have the same exogenous birth rate $b > 0$, then the population dynamic becomes

$$\begin{aligned}\dot{x}_i &= (\delta(x) - \delta_i(x))\, x_i = (h_i(x) - h(x))\, x_i \\ &= (\pi(i,x) - (1-\eta)\pi(x,i) - \pi(x,x) + (1-\eta)\pi(x,x))\, x_i \\ &= (\pi(i,x) - \pi(x,i) - \eta[\pi(x,x) - \pi(x,i)])\, x_i.\end{aligned}$$

For $\eta = 1$, this dynamic boils down to

$$\dot{x}_i = [\pi(i,x) - \pi(x,x)]\, x_i,$$

while for $\eta = 0$ it boils down to

$$\dot{x}_i = [\pi(i,x) - \pi(x,i)]\, x_i.$$

In the special case when $n = 2$ and $\tau_1 > \tau_2$, we have a one-dimensional dynamic:[6]

$$\begin{aligned}\dot{x}_1 &= [\pi(1,x) - \pi(x,1) - \eta[\pi(x,x) - \pi(x,1)]]\, x_1 \qquad (11) \\ &= (1-x_1)(\pi_{12} - \pi_{21} - \eta[(\pi_{12} - \pi_{11})x_1 + (\pi_{22} - \pi_{21})(1-x_1)])\, x_1.\end{aligned}$$

Thus, the population state $x_1 = 1$ is then asymptotically stable if and only if

$$\pi_{21} - \pi_{12} < \eta(\pi_{11} - \pi_{12}). \qquad (12)$$

(We know from the analysis above that the quantity on the left-hand side is positive.) Likewise, the population state $x_1 = 0$ is asymptotically stable[7] if and only if

$$\pi_{21} - \pi_{12} > \eta(\pi_{21} - \pi_{22}). \qquad (13)$$

[6] Recall that $x_2 \equiv 1 - x_1$.

[7] A population state is *Lyapunov stable* if no small perturbation can carry it away and it is *asymptotically stable* if, moreover, it attracts all initial population states in a neighborhood.

Generically, there can exist at most one interior stationary state, and this has to be the unique solution $x_1^* \in (0,1)$ to the equation $\dot{x}_1 = 0$, or, equivalently:

$$x_1^* = \frac{\pi_{12} - \pi_{21} - \eta\,(\pi_{22} - \pi_{21})}{\eta\,(\pi_{12} + \pi_{21} - \pi_{11} - \pi_{22})}. \tag{14}$$

We are now in a position to apply this selection dynamic to each of the two cases analyzed above, complete and incomplete information, respectively.

5.1. Complete Information

In light of Proposition 2 it is of particular interest to see how a firm run by an entrepreneur with the robust degree of confidence $\tau^R < 1$ defined in (6) fares against a firm run by an entrepreneur with a different degree of confidence. Assume that $\tau^R \in T$ and consider bimorphic populations, $T = \{\tau_1, \tau_2\}$. We first consider selection against more overconfident entrepreneurs.

Proposition 6 *Suppose* $\tau_1 = \tau^R > \tau_2$. *Then*

(a) $x_1^* = 1$ *is the only asymptotically stable state if* $\eta = 1$.

(b) $x_1^* = 0$ *is the only asymptotically stable state if* $\eta = 0$.

Proof. If $\eta = 1$, condition (12) reduces to $\pi_{11} - \pi_{21} > 0$. After some algebraic manipulations it is easily verified that

$$\begin{aligned}\pi_{11} - \pi_{21} &= -\frac{2}{9}c^2\left(\tau^R - \tau_2\right)\left(\frac{1}{5c} - \frac{6}{5} + \tau_2\right)\\ &= \frac{2}{9}c^2\left(\tau^R - \tau_2\right)\left(\frac{6}{5} - \frac{1}{5c} - \tau_2\right) = \frac{2}{9}c^2\left(\tau^R - \tau_2\right)^2 > 0.\end{aligned}$$

If $\eta = 0$, condition (13) reduces to $\pi_{21} - \pi_{12} > 0$. But we already know from previous analysis that π_{21} is always greater than π_{12}, so the only asymptotically stable state is $x_1 = 0$ $(x_2 = 1)$ in this case. ■

In other words, Proposition 6 says that if market selection is driven solely by absolute performance ($\eta = 1$), then entrepreneurs who are more confident than those with the robust degree, τ^R, will be driven out of the market, and only those with the robust degree will prevail. By contrast, if selection is based only on relative performance ($\eta = 0$), then entrepreneurs with the robust degree of confidence, τ^R, will be driven out of the market and only the more confident type of an entrepreneur will survive in the long run.

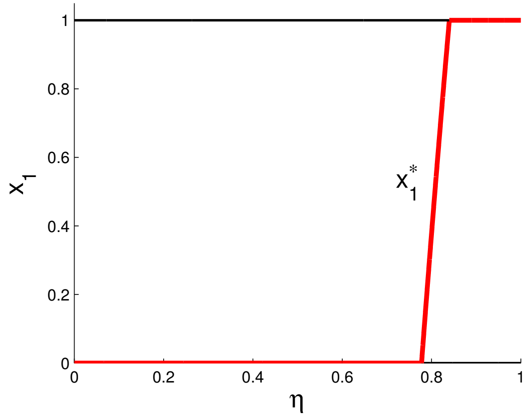

FIGURE 2. STATIONARY AND ASYMPTOTICALLY STABLE POPULATION STATES, COMPLETE INFORMATION

What happens if $\eta \in (0,1)$, i.e., if the evaluation of market performance is based on a mixture of absolute and relative performance? We illustrate this case by way of a numerical example. Suppose thus that $c = 0.4$ and $\tau_1 = 0.7$, the robust degree of confidence, and $\tau_2 = 0.5$. The diagram in Figure 2 gives the graph of the solution correspondence from η to the corresponding set of stationary states. This graph consists of the horizontal lines $x_1 = 0$ and $x_1 = 1$ and the positively sloped curve that connects these. The asymptotically stable states are marked in boldface, these are the state $x_1^*(\eta) = 0$ for $\eta \leq 7/9 \approx 0.78$, $x_1^*(\eta) = 1$ for $\eta \geq 0.84$ and some mixed state $x_1^*(\eta) \in (0,1)$ for intermediate η-values represented by the positively sloped curve.

In other words, if market selection is based mostly on absolute performance ($\eta \geq 0.84$), then the entrepreneurs with the robust degree of confidence will survive, while if the weight placed on absolute performance is less than $7/9$, then only more confident entrepreneurs will survive. In the intermediate range, when the weight on absolute performance is between $7/9$ and 0.84, both entrepreneurial types will prevail.

Next, we turn to selection against less confident entrepreneurs.

Proposition 7 *Suppose $\tau_1 > \tau_2 = \tau^R$. Then $x_1^* = 0$ is the only asymptotically stable state for any $\eta \in [0,1]$.*

Proof. We have to show that condition (13) is satisfied for all η, or that

$$\pi_{21} - \pi_{12} - \eta(\pi_{21} - \pi_{22}) > 0.$$

First, we can easily verify that $\pi_{21} > \pi_{22}$. This implies that the left hand side of the inequality is decreasing in η. Therefore, if it is satisfied for $\eta = 1$, it is satisfied for all $\eta \in [0, 1]$. For $\eta = 1$, the condition (13) boils down to $\pi_{22} - \pi_{12} > 0$, and we have

$$\begin{aligned}
\pi_{22} - \pi_{12} &= \frac{1}{9} c\left(\tau_1 - \tau_2\right)\left(2c\tau_1 - 6c + 3c\tau_2 + 1\right) \\
&= \frac{2}{9} c^2 \left(\tau_1 - \tau^R\right)\left(\tau_1 - 3 + \frac{3}{2}\tau_2 + \frac{1}{2c}\right) \\
&= \frac{2}{9} c^2 \left(\tau_1 - \tau^R\right)\left(\tau_1 - \frac{6}{5} + \frac{1}{5c}\right) = \frac{2}{9} c^2 \left(\tau_1 - \tau^R\right)^2 > 0.
\end{aligned}$$

Hence $x_1^* = 0$ is the only asymptotically stable state in this case. ■

In other words, entrepreneurs who are less confident than those with the robust degree of confidence, τ^R, will be wiped out of the market, and only those with the robust degree of confidence will prevail. In particular, entrepreneurs of the *homo oeconomicus* variety, will vanish. Moreover, this is true irrespective of whether market selection is driven by absolute performance, relative performance, or any mix of the two.

5.2. Incomplete Information

From Section 4 we know that, under certain regularity conditions, *homo oeconomicus* earns the highest expected profit of all types, while overconfident entrepreneurs earn a higher expected profit than their duopoly opponent. Hence, we expect *homo oeconomicus* to have the highest growth rate if $\eta = 1$ (i.e., if selection is based on absolute performance), while entrepreneurs of the most overconfident type, τ_n, may well have the highest growth rate if $\eta = 0$ (i.e., if selection is based on relative performance). In order to study this, we analyze a numerical example for all values of η. Suppose, thus, that $c = 0.3$, $\tau_1 = 1$ and $\tau_2 = 8/15$, the robust degree of confidence under complete information. Figure 3 shows the graph of the correspondence that maps the market-selection parameter η to the corresponding set of stationary states under the market selection dynamics. This graph consists of the horizontal lines $x_1 = 0$ and $x_1 = 1$ and a negatively sloped curve that connects these. The asymptotically stable states are marked in boldface: the stationary state $x_1 = 0$ is asymptotically stable for all $\eta \leq 7/13$ and the stationary state $x_1 = 1$ is asymptotically stable for all $\eta \geq 5/11$. For $\eta \leq 5/11$, the state $x_1 = 0$ attracts the whole interior of the state space, while for $\eta \geq 7/13$, the state $x_1 = 1$ attracts the whole interior of the state space. For intermediate η-values, both pure states are asymptotically stable and each pure state attracts all mixed

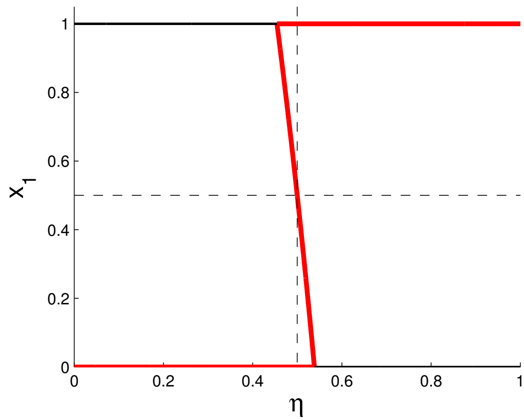

FIGURE 3. STATIONARY AND ASYMPTOTICALLY STABLE POPULATION STATES, INCOMPLETE INFORMATION

initial states with population share greater than $1/2$ of their own type. Hence, in this intermediate range, "history matters".

In sum: when applied to a two-type distribution in which one type is homo oeconomicus and the other type is overconfident of the degree selected for under complete information, the present selection dynamic lends support to our broad theoretical conclusion based on robustness: when selection is mostly based on absolute performance, ($\eta \leq 5/11 \approx 0.45$) homo oeconomicus will (uniquely) prevail while when it is mostly based on relative performance ($\eta \geq 7/13 \approx 0.54$) the overconfident type will (uniquely) prevail. In the intermediate range of selection dynamics ($5/11 < \eta < 7/3$), the outcome is history dependent; if the initial type distribution contains enough of homo oeconomicus, this entrepreneurial type will prevail, while if the initial distribution contains enough of the overconfident type, then that will prevail.

6. A Model with Endogenous Production Costs

In our base-line model outlined above, the unit cost of production is exogenous to each firm. However, in practice, part of an entrepreneur's task is to find efficient ways to produce the desired output. Weibull (2000) develops a model in which entrepreneurs decide how much effort to spend in order to increase their own firm's profit. We here develop a model of entrepreneurial overconfidence based on that approach.

Suppose that each entrepreneur i can influence the probability distribution of his or her own firm's unit cost by way of making some effort. More specifically, let $0 \leq y_i < 1$ be the cost-reducing effort made by entrepreneur i and assume that entrepreneurs have preferences over profit π_i and leisure, $z_i = 1 - y_i$, given by

$$u_i = \pi_i \cdot (1 - y_i)^{\lambda_i} ,$$

where $\lambda_i > 0$ is the intensity in the entrepreneur's preference for leisure. While effort, and hence leisure, is decided single-handedly by the entrepreneur, the resulting profit is a random variable that depends on own output, the competitor's output, and on the realization of one's random unit cost. We here focus on the case when the (true) expected value of the unit cost is linearly decreasing in own effort,

$$c_i = \mathbb{E}[\tilde{c}_i] = 1 - \sigma_i y_i$$

for some $\sigma_i > 0$, the entrepreneur's *skill* — the rate at which his or her effort reduces his or her firm's expected unit cost. Allowing for the possibility that entrepreneurs may have incorrect (typically optimistic) expectations about their own skill, entrepreneur i is assumed to strive to maximize

$$\mathbb{E}_i\left[(1 - q_1 - q_2 - \tilde{c}_i)\, q_i\, (1 - y_i)^{\lambda_i}\right],$$

where his or her subjective expectation is

$$\mathbb{E}_i[\tilde{c}_i] = 1 - \kappa_i \sigma_i y_i$$

for some $\kappa_i > 0$, the entrepreneur's degree of self-confidence. An overconfident entrepreneur i has $\kappa_i > 1$ and one with correct self-image has $\kappa_i = 1$. The parameter κ_i thus plays a similar role as $1/\tau_i$ did in the case of exogenous unit costs.

In sum, an entrepreneur i is characterized by three positive parameters: his or her *taste for leisure* λ_i, *skill* σ_i, and *degree of self-confidence* κ_i. We will refer to the triplet $\theta_i = (\lambda_i, \sigma_i, \kappa_i)$ as entrepreneur i's *type*.

6.1. Entrepreneurs with Correct Self-Image

Consider two entrepreneurs, $i = 1, 2$, who compete in a duopoly product market, as analyzed above. However, now their unit costs are not fixed and given, but determined by their efforts. They choose their efforts and output levels simultaneously, before unit costs are realized. They have correct beliefs about their own skill: $\kappa_1 = \kappa_2 = 1$. Each entrepreneur i then faces a decision problem that can be written as

$$\max_{q_i, y_i > 0} \left[\ln(\sigma_i y_i - q_1 - q_2) + \ln q_i + \lambda_i \ln(1 - y_i)\right].$$

The two first-order conditions for each entrepreneur i give

$$q_i = \frac{\sigma_i y_i - q_j}{2} \quad \text{and} \quad y_i = 1 - \frac{\lambda_i q_i}{\sigma_i}.$$

Combining these, we obtain

$$q_i = \frac{\sigma_i - q_j}{2 + \lambda_i}$$

for $i = 1, 2$ and $j \neq i$. Hence, each entrepreneur responds to the opponent's expected output level with a higher own output, the more skillful the entrepreneur is and the less taste he or she has for leisure.

In Nash equilibrium each firm i is producing

$$q_i^* = \frac{\sigma_i \left(2 + \lambda_j\right) - \sigma_j}{(2 + \lambda_1)(2 + \lambda_2) - 1}.$$

We note that equilibrium output is increasing in own skill and decreasing in own taste for leisure. The equilibrium efforts are, by contrast, decreasing in own skill:

$$y_i^* = 1 - \frac{\left(2 + \lambda_j\right) - \sigma_j/\sigma_i}{(2 + \lambda_1)(2 + \lambda_2) - 1} \lambda_i.$$

The expected unit cost becomes

$$c_i^* = 1 - \sigma_i + \lambda_i q_i^*$$

and equilibrium profits are thus given by

$$\pi_i^* = \left(\frac{\sigma_i (2 + \lambda_j) - \sigma_j}{2(\lambda_1 + \lambda_2) + \lambda_1 \lambda_2 + 3} \right)^2.$$

In the special case of two identical entrepreneurs, we obtain

$$\pi_1^* = \pi_2^* = \left(\frac{\sigma}{\lambda + 3} \right)^2.$$

As one would expect, this is an increasing function of their skill, σ, and decreasing function of their taste for leisure, λ.

6.2. Entrepreneurs with Incorrect Self-Image

Suppose that the entrepreneurs may have incorrect self-images in the sense of believing that their own skill is different from what it actually is. In order to focus on this aspect, suppose that the two entrepreneurs are otherwise identical: $\lambda_1 = \lambda_2 = \lambda$, $\sigma_1 = \sigma_2 = 1$. Along the same lines as in the above analysis one readily obtains that in equilibrium

$$q_i^* = \frac{(2+\lambda)\,\kappa_i - \kappa_j}{\lambda^2 + 4\lambda + 3},$$

$$c_i^* = \frac{\lambda}{\kappa_i} q_i^*,$$

and

$$\pi_i^* = \frac{(2+\lambda)\,\kappa_i - \kappa_j}{\left(\lambda^2 + 4\lambda + 3\right)^2} \cdot \left(3 + 2\lambda - (1+\lambda)\left(\kappa_i + \kappa_j\right) + \lambda \frac{\kappa_j}{\kappa_i}\right). \tag{15}$$

In the special case when entrepreneur 1 is *homo oeconomicus*, $\kappa_1 = 1$, we obtain

$$\pi_2^* = \frac{(2+\lambda)\,\kappa_2 - 1}{\left(\lambda^2 + 4\lambda + 3\right)^2} \cdot (2 + \lambda - (1+\lambda)\,\kappa_2 + \lambda/\kappa_2)$$

and

$$\pi_1^* = \left(\frac{2 + \lambda - \kappa_2}{\lambda^2 + 4\lambda + 3}\right)^2.$$

We note that $\pi_2^* > \pi_1^*$ if and only if

$$(2\kappa_2 + \lambda\kappa_2 - 1) \cdot (2 + \lambda - (1+\lambda)\,\kappa_2 + \lambda/\kappa_2) > (2 + \lambda - \kappa_2)^2. \tag{16}$$

For such parameter combinations, entrepreneur 2, who has self-confidence of degree κ_2, earns a higher profit than entrepreneur 1, *homo oeconomicus*. The diagram in Figure 4 shows the set of parameter pairs (κ_2, λ) that meet this inequality; this is the quasi-triangular area. We see that firm 2 (run by an entrepreneur with self-confidence κ_2) makes a higher expected profit than firm 1 (run by *homo oeconomicus*) if and only if (a) $1 < \kappa_2 < 2$ and (b) their common taste for leisure, λ, is not too strong.

The diagram in Figure 5 plots the equilibrium profits, π_1^* and π_2^*, as functions of the degree of confidence of entrepreneur 2, κ_2, when the common taste for leisure is $\lambda = 0.5$. The solid curve is the profit to firm 2, the dashed to firm 1, and the thin horizontal line the common profit when also entrepreneur 2 is *homo oeconomicus*, $\kappa_2 = 1$. The thin (red) curve is the average industry profit. We see that the profit to the firm run by *homo oeconomicus*, firm 1, is decreasing in the degree of confidence of the competitor, entrepreneur 2. We also note that firm 2 earns a higher profit than firm 1 when entrepreneur 2 has a degree of confidence between 1 and approximately 1.75, and that his or her firm's profit increases as

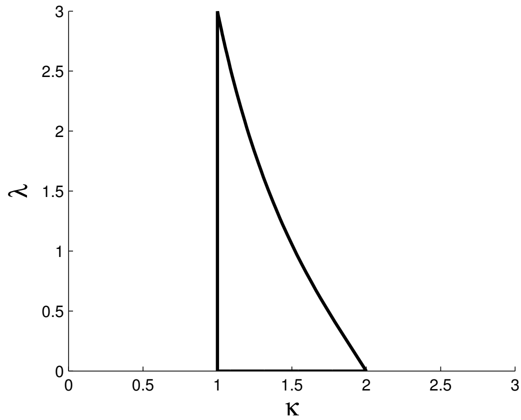

FIGURE 4. THE PARAMETER REGION WHERE CONDITION (16) IS MET

κ_2 increases slightly from $\kappa_2 = 1$ and achieves its maximum roughly at $\kappa_2 = 1.1$ — it is better, in terms of expected profit, to be somewhat overconfident when competing in duopoly against an entrepreneur with a correct self-image.

6.3. Market Selection of Degrees of Confidence

Consider two entrepreneurs with identical skills and taste for leisure, but with potentially differing degrees of self-confidence.

First, suppose that the prevalent degree of confidence among entrepreneurs in the industry in question is $\kappa = 1$, that of *homo oeconomicus*, and suppose that entrepreneurs are selected for according to how well their firms do in terms of their expected profit. Then *homo oeconomicus* would be selected against; the market would instead favor slightly overconfident entrepreneurs, as we saw in Figure 5 above. In this sense, entrepreneurs with correct self-images are selected against. From the same diagram we see that this conclusion holds with a vengeance if selection instead is based on relative performance: the profit difference goes strongly in the favor of the overconfident entrepreneur.

Second, just as in the base-line model, a degree of entrepreneurial confidence would be robust against such market selection if it were the optimal degree of confidence when pitted against itself. What degree of confidence κ, if any, has this property? In order to find this out, consider an entrepreneur with confidence κ' who is matched against an entrepreneur with confidence κ. The expected profit

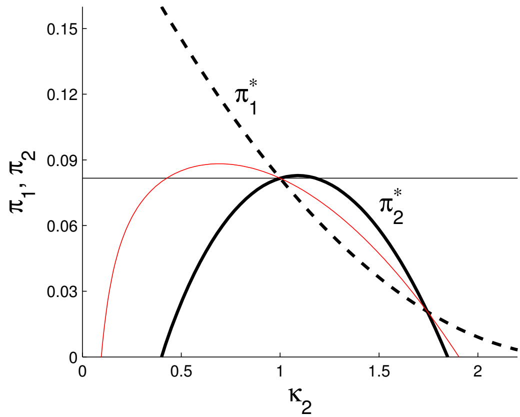

FIGURE 5. PROFITS

to the first entrepreneur is

$$\pi^* = v(\kappa', \kappa) = \left(1 - \frac{2+\lambda-\kappa/\kappa'}{(2+\lambda)^2-1}\lambda - \frac{\kappa'+\kappa}{3+\lambda}\right)\left(\frac{\kappa'(2+\lambda)-\kappa}{(2+\lambda)^2-1}\right)\sigma^2$$
$$= \left(\frac{\sigma}{\lambda^2+4\lambda+3}\right)^2 \cdot ((2+\lambda)\kappa'-\kappa)\cdot\left(3+2\lambda-(1+\lambda)(\kappa'+\kappa)+\lambda\frac{\kappa}{\kappa'}\right).$$

A degree of self-confidence, κ, is robust under market selection based on expected profits if

$$v(\kappa', \kappa) \le v(\kappa, \kappa) \text{ for all } \kappa' > 0 \tag{17}$$

with strict inequality for all $\kappa' \neq \kappa$.[8]

We illustrate this in Figure 6, showing the contour map of the function v, with κ' on the horizontal axis and κ on the vertical, when $\lambda = 0.5$ and $\sigma = 1$. A degree of confidence κ^R is robust against market selection if and only if the tangent of the isoquant through the point (κ, κ) (the thick curve) is horizontal at that point, and no other point on the horizontal line through that point has a higher v-value. Inspecting the graph we conclude that this degree is approximately $\kappa = 1.07$ (indicated by the thin vertical and horizontal lines).

More generally, we have:

Proposition 8 *The unique degree of confidence that is robust to market selection driven by expected profit is*

$$\kappa^R = \frac{2\lambda+6}{3\lambda+5}. \tag{18}$$

[8]Note that condition (17) is independent of the entrepreneurs' common skill, σ.

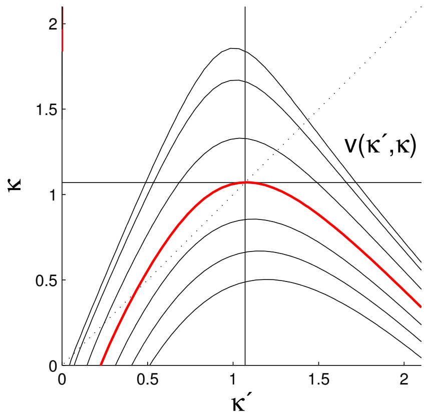

FIGURE 6. PROFIT ISOQUANTS

Proof. A necessary first-order condition for κ' to be optimal against κ is

$$\frac{\partial v(\kappa', \kappa)}{\partial \kappa'} = 0,$$

or, equivalently,

$$(2+\lambda)\left(3+2\lambda-(1+\lambda)(\kappa'+\kappa)+\frac{\lambda\kappa}{\kappa'}\right) = ((2+\lambda)\kappa'-\kappa)\cdot\left(1+\lambda+\frac{\lambda\kappa}{(\kappa')^2}\right).$$

The unique κ that solves this equation for $\kappa' = \kappa$ is κ^R, given in (18). Moreover,

$$\frac{\partial^2 v(\kappa', \kappa)}{\partial (\kappa')^2} = -2(2+\lambda)\cdot\left(1+\lambda+\frac{\lambda\kappa}{(\kappa')^2}\right)+\frac{2\lambda\kappa}{(\kappa')^3}\cdot((2+\lambda)\kappa'-\kappa)$$

and

$$\left.\frac{\partial^2 v(\kappa', \kappa)}{\partial (\kappa')^2}\right|_{\kappa'=\kappa^R} = -2(2+\lambda)(1+\lambda)-\frac{2\lambda}{\kappa^R} < 0,$$

which implies that $\kappa' = \kappa^R$ is the unique optimal degree of confidence against κ^R. ■

We note that $\kappa^R > 1$ if and only if $\lambda < 1$. For $\lambda > 1$, overconfidence is thus a "negative asset" and hence not selected for. Such entrepreneurs have such a strong taste for leisure that their overconfidence induces them to take out more leisure instead of making more effort.

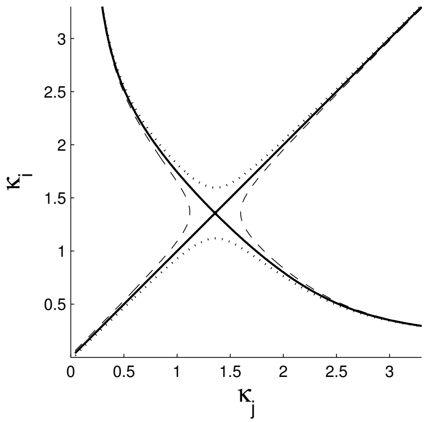

FIGURE 7. THE SADDLE-POINT OF ROBUST SELF-CONFIDENCE

Let us finally briefly consider market selection based on relative performance, that is, selection driven by the expected profit difference with respect to one's duopoly competitor.

For $\sigma = 1$ and arbitrary degrees of confidence, $\kappa_1, \kappa_2 > 0$, and recalling (15), we obtain

$$\pi_i^* - \pi_j^* = \frac{(2+\lambda)\,\kappa_i - \kappa_j}{(\lambda^2 + 4\lambda + 3)^2} \cdot \left(3 + 2\lambda - (1+\lambda)\left(\kappa_i + \kappa_j\right) + \lambda \frac{\kappa_j}{\kappa_i}\right) - \frac{(2+\lambda)\,\kappa_j - \kappa_i}{(\lambda^2 + 4\lambda + 3)^2} \cdot \left(3 + 2\lambda - (1+\lambda)\left(\kappa_i + \kappa_j\right) + \lambda \frac{\kappa_i}{\kappa_j}\right). \quad (19)$$

Figure 7 below shows a contour map for the profit difference $\pi_i^* - \pi_j^*$, for $\lambda = 0.5$. This difference is zero on the full curves, positive on the dashed curves and negative on the dotted curves. It thus appears that there is a unique robust degree of confidence when market selection is based on relative performance, and that this is the saddle-point solution to the equation $\pi_i^* = \pi_j^*$, approximately at $\kappa = 1.4$ in this numerical example. In fact, the saddle-point is not difficult to find for arbitrary λ.

Proposition 9 *The saddle-point solution to the equation $\pi_i^* = \pi_j^*$ is (κ^*, κ^*), where*

$$\kappa^* = \frac{\lambda + 9}{2\lambda + 6}.$$

Proof. Using (19), the equality $\pi_i^* = \pi_j^*$ can be re-written as

$$\begin{aligned} 0 &= (2+\lambda)(3+2\lambda)\left(\kappa_i-\kappa_j\right) - (2+\lambda)(1+\lambda)\left(\kappa_i+\kappa_j\right)\left(\kappa_i-\kappa_j\right) \\ &\quad - (2+\lambda)\lambda\left(\kappa_i-\kappa_j\right) + (3+2\lambda)\left(\kappa_i-\kappa_j\right) - (1+\lambda)\left(\kappa_i+\kappa_j\right)\left(\kappa_i-\kappa_j\right) \\ &\quad + \frac{\lambda}{\kappa_i\kappa_j}\left(\kappa_i-\kappa_j\right)\left(\kappa_i^2+\kappa_i\kappa_j+\kappa_j^2\right). \end{aligned}$$

Factoring out the solution $\kappa_i = \kappa_j$, we obtain

$$\begin{aligned} 0 &= (2+\lambda)(3+2\lambda) - (2+\lambda)(1+\lambda)\left(\kappa_i+\kappa_j\right) \\ &\quad - (2+\lambda)\lambda + 3 + 2\lambda - (1+\lambda)\left(\kappa_i+\kappa_j\right) \\ &\quad + \frac{\lambda}{\kappa_i\kappa_j}\left(\kappa_i^2+\kappa_i\kappa_j+\kappa_j^2\right). \end{aligned}$$

The saddle point (κ^*, κ^*) sits on the diagonal, so $\kappa_1 = \kappa_2 = \kappa^*$ has to solve this equation, or, equivalently,

$$0 = (2+\lambda)(3+2\lambda) - (6+2\lambda)(1+\lambda)\kappa^* - \lambda^2 + 3 + 3\lambda,$$

which gives

$$\begin{aligned} \kappa^* &= \frac{(2+\lambda)(3+2\lambda) - \lambda^2 + 3 + 3\lambda}{(6+2\lambda)(1+\lambda)} \\ &= \frac{(9+\lambda)(1+\lambda)}{(6+2\lambda)(1+\lambda)} \\ &= \frac{\lambda+9}{2\lambda+6}. \end{aligned}$$

■

Hence, there seems to exist a unique degree of entrepreneurial confidence that is robust against market selection based on relative performance.

Conjecture 10 *The unique degree of confidence that is robust to market selection driven by the expected profit difference is*

$$\kappa^* = \frac{\lambda+9}{2\lambda+6}.$$

Figure 8 below shows that this conjectured robust degree κ^* (thick red curve) is higher than when market selection is based upon absolute performance, κ^R (thin curve). In particular, overconfidence becomes a “negative asset” only for quite strong tastes for leisure: $\kappa^* < 1$ if and only if $\lambda > 3$.

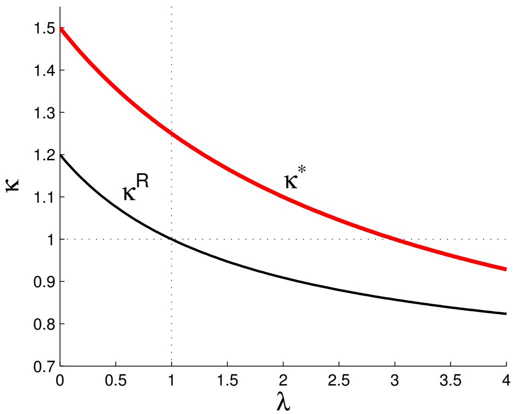

FIGURE 8. THE ROBUST DEGREE OF CONFIDENCE UNDER MARKET SELECTION BASED ON ABSOLUTE AND RELATIVE PERFORMANCE, RESPECTIVELY

7. Related Literature

Our research is related to a few branches of the recent literature in economics and finance.[9] A branch of behavioral economics and finance studies the effects of behavioral biases, including overconfidence, on market outcomes. How overconfidence affects a financial market depends on who in the market is overconfident and on how information is distributed. Odean (1998) examines markets in which price-taking traders, a strategic-trading insider, and risk-averse market-makers are overconfident and finds that overconfidence increases expected trading volume and market depth while lowering the expected utility of those who are overconfident. However, its effect on volatility depends on who is overconfident. Overconfident traders can cause markets to under-react to the information of rational traders. Market actors also under-react to abstract, statistical, or highly relevant information, while they overreact to salient, anecdotal, or less relevant information.

[9] The idea that it may be strategically advantageous to have a known objective function that differs from one's actual payoff function is not new. In their model of strategic delegation, Fershtman and Judd (1987) show that owners of firms engaged in Cournot competition may gain from distorting the objective functions of their hired managers away from pure profit maximization towards a mix of profit and volume maximization. The strategic advantage of such a contract is that, if known by the competitors, it will make them reduce their outputs. This approach has been criticised for not being robust to the introduction of secret side-contracts. Our approach is not vulnerable to this critique since our firms are run by entrepreneur-owners with given personality traits.

Kyle and Wang (1997) study a duopoly model of informed speculation, and show that overconfidence may strictly dominate rationality since an overconfident trader may not only generate higher expected profit and utility than his rational opponent, but also higher than if he were also rational, much like our findings.[10] Manove and Padilla (1999) and Landier and Thesmar (2009) study how entrepreneurs' overconfidence affects financial contracting. Van den Steen (2004) develops a choice-driven mechanism for overconfidence based on the non-common prior assumption and shows how it can generate several other well-known judgment biases. Compte and Postlewaite (2004) study optimal beliefs when confidence enhances performance in a decision theoretic model. They show that in a world where performance depends on emotions, biases in information processing enhance welfare. Sandroni and Squintani (2007) embed overconfidence in a model of insurance markets and show that compulsory insurance makes low-risk agents worse off.

Our paper contributes to the literature on market selection. As mentioned in the opening paragraph of Section 1, the traditional economic view is that profit-driven market dynamics will select for firms that, for whatever reason, maximize profits. According to this argument, those who do not act as profit maximizers will be driven out of the market. However, the more recent literature shows that these statements need to be elaborated and qualified. For example, Dutta and Radner (1999) directly take up the question of whether markets select for firms that maximize expected profits. The answer they find is no: the decision rules within firms that maximize the long-run probability of survival are not those that maximize (the present value of) expected profits.

A growing body of research studies the evolution of preferences and emergence of perception biases. Viewed in that perspective, our paper is related to Heifetz et al. (2007). They develop a general methodology for characterizing the dynamic evolution of preferences in a wide class of strategic interactions and apply their results to study the evolutionary emergence of overconfidence and interdependent preferences. By contrast, we focus on a narrow class of strategic interactions and study in detail how market selection may favor or work against overconfidence. Using a different framework — a herding model — Bernardo and Welch (2001) explain why seemingly irrationally overconfident behavior can persist.

[10] For field evidence on managerial overconfidence see Malmendier and Tate (2005, 2008) and Barber and Odean (2001) for interaction of gender, overconfidence and common stock investment.

8. Conclusion

As mentioned in the introduction, this is work in progress, a report from an ongoing research project. Of particular interest for the continuation of this project would seem to be to consider market selection of multi-dimensional entrepreneurial types, along the lines of the model version in Section 6. While so far we have focused exclusively on the parameter for self-confidence, κ, each entrepreneur in that model version is characterized by a triplet $\theta = (\lambda, \sigma, \kappa)$, where λ is the entrepreneur's taste for leisure, σ his or her skill. We have shown tendency for selection of overconfident types, when entrepreneurs are otherwise identical. Likewise, one would expect selection of more skilled entrepreneurs, and entrepreneurs with less taste for leisure, when they have the same degree of confidence. One might also conjecture a trade-off between these three traits; more skill and less taste for leisure might compensate for lack of confidence etc. We hope to be able so say something about this potential trade-off later on in this research project.

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Chapter 3

Overconfidence, Imperfect Competition, and Evolution*

Karen Khachatryan†

Abstract

This study explores whether market competition between firms owned and run by managers favors overconfident managers. We study this question in a simple duopoly setting with differentiated products and show that when there is complete information about the competitor's type, overconfident managers enjoy a competitive edge when (1) either the firms compete by setting quantities and the products are imperfect substitutes; or (2) the firms compete by setting prices and the products are imperfect complements. However, further analysis reveals that evolutionary market selection forces will always favor a positive degree of managerial overconfidence. We also study the case of incomplete information about the competitor's type under quantitiy competition and show that evolutionary forces may still favor overconfident managers if market selection is driven by relative rather than absolute profit performance.

Keywords: Overconfidence, product differentiation, imperfect competition, evolution, market selection.

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1. Introduction

Humans show many psychological biases, of which overconfidence is one of the most documented. In this paper we examine the possibility that biased beliefs regarding self-confidence, and in particular overconfidence, may be an asset in competition. More specifically, that firms run by overconfident managers may earn higher profits than competing firms run by managers with correct beliefs about their own firm's demand, as in text-book models. Will market competition select for a certain degree of overconfidence that will prevail in the long run?

We study this possibility in a simple oligopoly setting with differentiated products. In our model each of the two firms is run by a manager-*cum*-owner who may have incorrect beliefs about the relative merits of their own firm's products. We interpret a tendency to overestimate one's own firm's product demand—or the extent of product differentiation—as an expression for overconfidence.

Our main contribution is to formalize these phenomena exactly in a few simple market settings, and to show under which conditions market competition selects overconfident managers. Indeed, we shown that a firm run by a moderately overconfident manager will earn higher profits than its competitor if the latter is run by a manager with correct beliefs about own firm's demand if either the firms compete by setting prices and the products are complementary or the firms compete by setting quantities and the products are imperfect substitutes.

The intuition is simple. Under quantity competition the best response against an overconfident competitor is to cut down one's own production when the goods are substitutes and increase production when the goods are complementary. Furthermore, overconfidence has an asymmetric effect on prices. When the goods are imperfect substitutes this will favor the overconfident manager relatively more. However, when the goods are imperfect complements this will favor the rational manager. Similarly, under price competition the best response against an overconfident competitor is to lower your own price when the products are imperfect substitutes and raise your own price when the products are complementary. However, the effect on demand is asymmetric because your own price has a bigger impact on your own profits than your competitor's price. The second effect will dominate when the goods are imperfect substitutes, and the first effect will dominate when the goods are imperfect complements.

Moreover, under quantity competition two competitors run by equally overconfident managers will earn higher profits than two competitors run by managers with correct beliefs when the goods are imperfect complements. Similarly, under price competition two competitors run by equally overconfident managers will earn higher profits than two competitors run by managers with correct beliefs when the goods are substitutable. Given this tendency towards overconfidence one may ask what degree of confidence, if any, will prevail in the long run in a given market context, if there is free entry and exit of managers with different degrees of self-confidence. We identify and characterize this degree of self-confidence, which we call the evolutionarily robust degree of self-confidence.

Overconfidence has been attracting much attention recently from psychologists and economists. The literature on judgment under uncertainty has found that people tend to be overconfident about the information they have, in that their subjective probability distributions on relevant events are too tight (Cesarini et al. (2006); Kahneman et al. (1982)). Overconfidence has also been found in various professions such as entrepreneurs (Busenitz and Barney (1997); Cooper et al. (1988)), policy experts (Tetlock (1999)), and security analysts (Chen and Jiang (2006)). The implications of overconfidence for economic choices and especially for financial markets have recently been studied by numerous researchers (Gervais and Odean (2001); Kyle and Wang (1997); Scheinkman and Xiong (2003)). Hvide (2002) offers a theoretical argument for the endogenous emergence of overconfidence. Recently, Johnson and Fowler (2011) have developed an evolutionary model demonstrating that overconfident populations are evolutionarily stable in a wide range of environments.

In order to model the level of confidence, we allow for asymmetric beliefs about the extent of product differentiation. Recent papers that involve asymmetric beliefs (differing priors) include Admati and Pfleiderer (2004); de la Rosa (2011); Fang and Moscarini (2005); Van den Steen (2004, 2007), and Grubb (2009), among others. Admati and Pfleiderer (2004) study an information transmission game where the sender can be overconfident in his ability to observe the true state. Fang and Moscarini (2005) consider the workers' overconfidence in their skills on wage policies, and de la Rosa (2011) studies the effects of overconfidence on incentive contracts in a moral hazard framework. Van den Steen (2004) models how overconfidence can arise from heterogeneous priors when individuals have a choice over projects, while Van den Steen (2007) examines worker incentives when a worker may disagree with the manager regarding the best course of action. Grubb (2009) develops a model of screening with consumers who overestimate the precision of their demand forecasts. Heller (2011) presents an evolutionary foundation for overconfidence based on diversification of risk.

The paper is organized as follows. Section 2 introduces the model setup. Section 3 analyzes the model in which firms compete by setting quantities and there is complete information about managerial types. Section 4 extends the analysis to an incomplete information scenario. Section 5 instead focuses on price competition with complete information about managerial types. Section 6 concludes with a summary and a discussion of some directions for further research. Some graphical illustrations and the proofs of Propositions 6 and 14, are given in appendixes at the end of the paper.

2. The Model

Consider two firms that produce differentiated goods. The representative consumer's utility, following Dixit (1979) and Singh and Vives (1984), is a function of the consumption of the two differentiated goods and the composite good I and is given by

$$U(q_1, q_2, I) = \alpha(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + I.$$

Thus utility is quadratic in the consumption of q-goods and linear in the consumption of other goods, I. The parameter $\gamma \in [-1, 1]$ measures the substitutability between the two products. When goods are substitutes, the degree of substitutability could be interpreted in terms of horizontal product differentiation. If $\gamma = 0$, the products are unrelated and each firm has a monopolistic market power, while if $\gamma = 1$, the products are perfect substitutes. A negative γ implies that the goods are complementary. Finally, $\alpha > 0$ measures quality in a vertical sense.[1]

Consumers maximize utility subject to the budget constraint $\sum_i p_i q_i + I \leq m$, where m denotes income and the price of the composite good is normalized to one.

This utility function gives rise to a linear demand structure. The first-order condition determining the optimal consumption of good i is

$$\frac{\partial U}{\partial q_i} = \alpha - q_i - \gamma q_j = p_i \tag{1}$$

for $i = 1, 2,\ j \neq i$. Therefore, the inverse demand is given by

$$p_1 = \alpha - q_1 - \gamma q_2$$

for firm 1 and

$$p_2 = \alpha - \gamma q_1 - q_2$$

for firm 2 in the region of quantity space where prices are positive.

[1]Note that an increase in the degree of product differentiation (a decline in γ) shifts the demand curves for both firms outwards.

Inverting, we get the following direct demand system

$$\begin{cases} q_1 = \dfrac{1}{1-\gamma^2}\left(\alpha\left(1-\gamma\right)-p_1+\gamma p_2\right) \\ q_2 = \dfrac{1}{1-\gamma^2}\left(\alpha\left(1-\gamma\right)+\gamma p_1-p_2\right) \end{cases} \tag{2}$$

in the region of price space where quantities are positive. Demand for good i is always downward sloping in its own price and increases (decreases) with increase in the price of the competitor if the goods are imperfect substitutes (complements).

Both firms have the same constant marginal cost, u and there are no fixed costs of production. We consider from now on prices net of marginal cost. This is without loss of generality since if marginal costs are positive, we may replace α by $\alpha - u$.

Each firm i is owned and run by a manager. Each manager i can be overconfident or underconfident, which is modeled as follows. Each manager i is allowed to have different beliefs (perceptions) about the parameter γ. Manager i believes γ to be $\gamma_i = k_i\gamma$, where $k_i \in K = (0, 1/|\gamma|)$[2]. We will refer to k_i as manager i's degree of self-confidence or simply the type of the manager. We assume that the managers correctly perceive their opponents' beliefs about the market, but do not adjust theirs accordingly. In other words, we assume that the type of the manager is observable and the managers simply "agree to disagree" about the representative consumers' preferences (parameter γ).[3] We will also consider the case where a managers' type is not observable.

First, suppose the goods are imperfect substitutes ($0 < \gamma < 1$). Then a manager with self-confidence $k_i < 1$ is *overconfident*, while a manager with $k_i > 1$ is *underconfident*.[4] An overconfident manager perceives the products to be less substitutable (more horizontally differentiated) than they actually are, while an underconfident manager perceives the products to be more substitutable (less horizontally differentiated) than they actually are.

Second, suppose the goods are complementary ($-1 < \gamma < 0$). Then a manager with self-confidence $k_i > 1$ is *overconfident,* while a manager with $k_i < 1$ is

[2]In other words we need to have $\gamma_i^2 < 1$ to ensure concavity of the consumer's utility function.

[3]Strictly speaking we do not have to endow our managers with point beliefs. We could alternatively assume, that there is uncertainty about demand (consumers' preferences). One may think of a lottery over different levels of γ in the representative consumer's utility function, keeping α fixed. An overconfident manager underestimates the risk of a high realization of γ and therefore expects a higher demand (price). Owning to risk neutrality, all the results would go through if γ represents the true expectation. Then this could be a model of non-common priors, for example.

[4]To see this note that a higher perceived $|\gamma_i|$ would imply a higher (perceived) marginal utility of q_i for the representative consumer.

underconfident. An overconfident manager believes that the products are more complementary than they actually are, while an underconfident manager believes the products to be less complementary than they actually are.

Now, when the two managers disagree about γ and manager 1 has self-confidence k_1 and manager 2 has self-confidence k_2 then manager 1 thinks that the indirect demand system is given by

$$\begin{cases} p_1 = \alpha - q_1 - k_1 \gamma q_2 \\ p_2 = \alpha - k_1 \gamma q_1 - q_2 \end{cases} \tag{3}$$

in the quantity space where prices are positive, while the direct demand is given by

$$\begin{cases} q_1 = \dfrac{1}{1 - k_1^2 \gamma^2} \left(\alpha \left(1 - k_1 \gamma \right) - p_1 + k_1 \gamma p_2 \right) \\ q_2 = \dfrac{1}{1 - k_1^2 \gamma^2} \left(\alpha \left(1 - k_1 \gamma \right) - p_2 + k_1 \gamma p_1 \right) \end{cases} \tag{4}$$

in the price space where quantities are positive.

Similarly, from the perspective of manager 2, the inverse demand is given by

$$\begin{cases} p_1 = \alpha - q_1 - k_2 \gamma q_2 \\ p_2 = \alpha - k_2 \gamma q_1 - q_2 \end{cases}, \tag{5}$$

in the quantity space where prices are positive, and the direct demand is given by

$$\begin{cases} q_1 = \dfrac{1}{1 - k_2^2 \gamma^2} \left(\alpha \left(1 - k_2 \gamma \right) - p_1 + k_2 \gamma p_2 \right) \\ q_2 = \dfrac{1}{1 - k_2^2 \gamma^2} \left(\alpha \left(1 - k_2 \gamma \right) - p_2 + k_2 \gamma p_1 \right) \end{cases} \tag{6}$$

in the price space where quantities are positive.

3. Quantity Competition

In this section, we analyze the game in which firms compete by setting quantities under complete information, i.e., when both managerial types are mutually known. For expository purposes, we start with the case of one over- or underconfident manager and then move on to the more general case of two arbitrarily confident managers.

3.1. One Overconfident Manager

Consider the case when manager 1 holds correct beliefs about γ, $k_1 = 1$, while manager 2 is biased and has self-confidence $k_2 \neq 1$. Manager 2 is overconfident if either $\gamma > 0$ and $k_2 < 1$ or $\gamma < 0$ and $k_2 > 1$. These beliefs are common knowledge. Each manager strives to maximize his or her perceived profits.

The manager of firm 1 solves

$$\max_{q_1 \geq 0} \{(\alpha - q_1 - \gamma q_2)\, q_1\}$$

taking q_2 as given. The first-order condition for profit maximization is[5]

$$\alpha - 2q_1 - \gamma q_2 = 0,$$

from which we get the following best-response function

$$q_1\,(q_2) = \frac{1}{2}\,[\alpha - \gamma q_2]\,. \tag{7}$$

The manager of firm 2 solves

$$\max_{q_2 \geq 0} \{(\alpha - k_2 \gamma q_1 - q_2)\, q_2\}$$

taking q_1 as given. The first-order condition for profit maximization is

$$\alpha - 2q_2 - k_2 \gamma q_1 = 0,$$

from which we get the following best-response function

$$q_2\,(q_1) = \frac{1}{2}\,[\alpha - k_2 \gamma q_1]\,. \tag{8}$$

Solving the best response functions (7) and (8) simultaneously, we get that in the unique interior Nash equilibrium of the quantity competition game[6]

$$q_1^* = \frac{\alpha\,(2 - \gamma)}{4 - k_2 \gamma^2}$$

---

[5]Throughout the paper, in the first-order conditions for profit maximization and the associated best response functions, we suppress the nonnegativity constraint(s) but always make sure that none of them is violated.

[6]Note that the associated second-order conditions for a maximum are trivially satisfied and the resulting equilibrium quantities and prices are indeed positive given the parameter ranges for γ and k.

and

$$q_2^* = \frac{\alpha\,(2 - k_2\gamma)}{4 - k_2\gamma^2}.$$

These quantity choices imply the following (true) equilibrium prices

$$p_1^*\,(q_1^*, q_2^*) = \frac{\alpha\,(2 - \gamma)}{4 - k_2\gamma^2},$$

and

$$p_2^*\,(q_1^*, q_2^*) = \frac{\alpha\left(2 - (2 - k_2)\,\gamma + (1 - k_2)\,\gamma^2\right)}{4 - k_2\gamma^2}.$$

The (true) equilibrium profits are, respectively,

$$\pi_1^* = \frac{\alpha^2\,(2 - \gamma)^2}{(4 - k_2\gamma^2)^2},$$

and

$$\pi_2^* = \frac{\alpha^2\,(2 - k_2\gamma)\left(2 - (2 - k_2)\,\gamma + (1 - k_2)\,\gamma^2\right)}{(4 - k_2\gamma^2)^2}.$$

Hence, we have the following differences:

$$\begin{aligned} q_1^* - q_2^* &= \frac{\alpha\,(k_2 - 1)\,\gamma}{4 - k_2\gamma^2} \\ p_1^* - p_2^* &= \frac{\alpha\,(1 - k_2)\,\gamma\,(1 - \gamma)}{4 - k_2\gamma^2} \\ \pi_1^* - \pi_2^* &= \frac{\alpha^2\gamma^2\,(k_2 - 1)\,(1 + k_2\,(1 - \gamma))}{(4 - \gamma^2 k_2)^2} \end{aligned}$$

First, note that when both managers are rational and so have neutral self-confidence, these differences vanish. Second, note that a manager's self-confidence affects quantities and prices in an opposite manner. Which effect dominates for profits? To answer this question, we now look at the case of imperfect substitutes and imperfect complements separately.

3.1.1. Substitutes

Proposition 1 *Suppose the goods are imperfect substitutes* ($\gamma > 0$) *and manager 2 is overconfident* ($k_2 < 1$). *In the unique interior Nash equilibrium of the quantity competition game we have the following:*

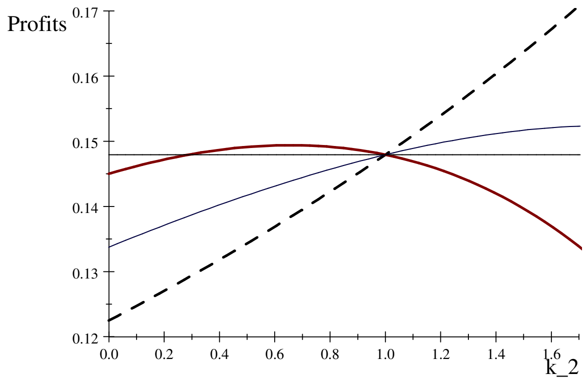

FIGURE 1. PROFITS (SUBSTITUTES)

(a) $q_2^* > q_1^*$

(b) $p_2^* < p_1^*$

(c) $\pi_2^* > \pi_1^*$

In other words, firm 2—run by an overconfident manager—produces more and earns a higher profit than its unbiased competitor, firm 1.

The diagram in Figure 1 shows how π_2^* (solid curve) and π_1^* (dashed curve) depend on k_2 (the horizontal axis), for $\gamma = 0.6$ and $\alpha = 1$. The thin curve is the average of the two curves—half the industry profit.

We see that:

(a) For all $k_2 < 1$ the overconfident manager 2 produces more and makes a higher profit than unbiased manager 1,

(b) There is an optimal degree of overconfidence for manager 2, $k_2 \approx 0.67$,

(c) The industry profit is increasing in k_2,

(d) For moderate degrees of overconfidence $(0.3 < k_2 < 1)$[7], manager 2 earns a higher profit than had she also had correct perceptions (had $k_2 = k_1 = 1$).

[7]For other γ, this interval would be $\left(\min\left\{0, \frac{4\gamma+2\gamma^2-4}{2\gamma^2+\gamma^3-4}\right\}, 1\right)$.

3.1.2. Complements

Proposition 2 *Suppose the goods are imperfect complements* ($\gamma < 0$) *and manager 2 is overconfident* ($k_2 > 1$). *In the unique interior Nash equilibrium of the quantity competition game, we have the following:*

(a) $q_2^* > q_1^*$

(b) $p_2^* < p_1^*$

(c) $\pi_2^* < \pi_1^*$

In other words, the firm run by an overconfident manager produces more, but earns *lower* profit, than its unbiased competitor. In this case, the value of being overconfident is not positive. Similarly,

Proposition 3 *Suppose the goods are imperfect complements* ($\gamma < 0$) *and manager 2 is underconfident* ($k_2 < 1$). *In the unique interior Nash equilibrium of the quantity competition game, we have the following:*

(a) $q_2^* < q_1^*$

(b) $p_2^* > p_1^*$

(c) $\pi_2^* > \pi_1^*$

In other words, the firm run by an *underconfident* manager produces less and earns a *higher profit*, than its unbiased competitor. When the goods are imperfect complements it pays off to be *underconfident*.

The diagram in Figure 2 shows how π_2^* (solid curve) and π_1^* (dashed curve) depend on k_2 (the horizontal axis), for $\gamma = -0.6$ and $\alpha = 1$. The thin curve is the average of the two curves—half the industry profit.

We see that:

(a) For all $k_2 > 1$ the overconfident manager 2 produces more and makes a lower profit than unbiased manager 1,

(b) However, for all $k_2 < 1$ the underconfident manager 2 makes a higher profit,

(c) The industry profit is increasing in k_2,

(d) For moderate degrees of overconfidence ($1 < k_2 < 1.6$)[8], manager 2 earns a higher profit than had she also had correct perceptions (had $k_2 = k_1 = 1$),

[8] For other γ, the exact interval is $\left(1, \frac{4\gamma+2\gamma^2-4}{2\gamma^2+\gamma^3-4}\right)$.

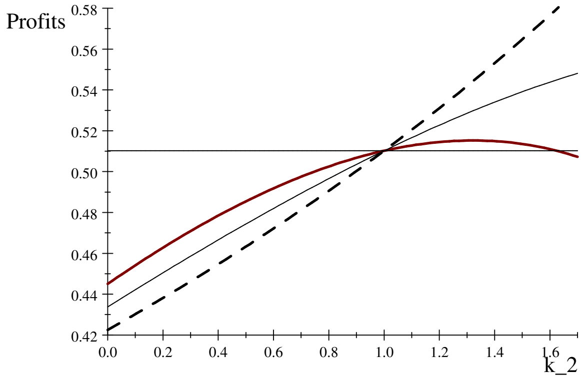

FIGURE 2. PROFITS (COMPLEMENTS)

(e) For all degrees of underconfidence manager 2 earns a lower profit than had she also had correct perceptions (had $k_2 = k_1 = 1$).

3.1.3. Discussion and Intuition

First, note that manager 2, who has biased self-confidence $k_2 \neq 1$, is maximizing a payoff, which can be written in the following form:

$$\begin{aligned} \tilde{\pi}_2(q_1, q_2) &= (\alpha - k_2 \gamma q_1 - q_2) q_2 \\ &= (\alpha - \gamma q_1 - q_2) q_2 + (1 - k_2) \gamma q_1 q_2, \end{aligned} \tag{9}$$

a sum of two terms. The first term is the (true) payoff, $\pi_2(q_1, q_2) = (\alpha - \gamma q_1 - q_2) q_1$ and the second term is due to biased self-confidence and is positive if the manager is overconfident and negative if underconfident.

Second note that

$$\frac{\partial \pi_2(q_1, q_2)}{\partial q_1} = -\gamma q_2 \text{ and } \frac{\partial \pi_2(q_1, q_2)}{\partial q_2 \partial q_1} = -\gamma.$$

Thus when $\gamma > 0$ the managers impose negative externalities on one another (i.e., the larger is the action of manager 2, q_2, the lower is manager 1's payoff, and visa versa. Moreover, the actions, i.e., quantity choices in this case, are strategic substitutes in the sense of Bulow et al. (1985) (the best response functions are decreasing in the (q_1, q_2) space). In contrast, when $\gamma < 0$, the managers impose positive externalities on one another, and their actions are strategic complements.

Now, consider the full perceived payoff and note that

$$\frac{\partial \tilde{\pi}_2(q_1, q_2)}{\partial q_1} = -k_2 \gamma q_2 \text{ and } \frac{\partial \tilde{\pi}_2(q_1, q_2)}{\partial q_2 \partial q_1} = -k_2 \gamma.$$

Suppose the goods are imperfect substitutes, $\gamma > 0$. Thus, an overconfident manager 2 not only overestimates the returns from her own action for any action taken by the other manager ($(1 - k_2)\gamma q_1 > 0$), but also *underestimates* the returns from the opponent's action. Thus biased-beliefs operate through two different channels that reinforce each other. An overconfident manager acts more aggressively than a rational manager as he not only exaggerates the impact of his actions on his payoffs but also because he underestimates the impact of other's actions on his payoffs. When goods are imperfect substitutes, the players' actions are strategic substitutes, so the aggressive behavior of an overconfident manager induces the rival to play soft. In other words, a managers' actions impose negative externalities on their opponent, so the soft behavior of his opponents benefits the overconfident player.

Next, suppose the goods are complementary, $\gamma < 0$. An overconfident manager 2 still overestimates the returns from her own action for any action taken by the other manager ($(1 - k_2)\gamma q_1 > 0$), but she also overestimates the returns from the opponent's action. Thus biased-beliefs operate through two different channels that work against each other. When $\gamma < 0$, actions are strategic complements, so aggressive behavior of an overconfident manager induces rivals to play aggressively as well. Since now the actions of managers impose positive externalities on each other, the aggressive behavior of an overconfident manager benefits both managers in this case.

Of course, being overconfident is also costly, since a manager is not making an optimal decision. When $\gamma > 0$, the benefit of overconfidence gained through strategic advantage outweighs the cost, so in relative terms the overconfident manager is still better off. However when $\gamma < 0$, the benefit of overconfidence gained through strategic advantage is higher for the opponent, so in relative terms the overconfident manager is worse off.

3.2. Two Arbitrarily Confident Managers

We now turn to the analysis of the case of two arbitrarily confident managers. Throughout the analysis remember that $k_1, k_2 \in K = (0, |1/\gamma|)$.

The manager of firm 1 solves

$$\max_{q_1 \geq 0} \{(\alpha - q_1 - k_1 \gamma q_2) q_1\}$$

taking q_2 as given. The first-order condition for profit maximization is

$$\alpha - 2q_1 - k_1\gamma q_2 = 0,$$

from which we get the following best-response function

$$q_1(q_2) = \frac{1}{2}\left[\alpha - k_1\gamma q_2\right]. \tag{10}$$

Similarly, the manager of firm 2 solves

$$\max_{q_2 \geq 0} \{(\alpha - k_2\gamma q_1 - q_2) q_2\}$$

taking q_1 as given. The first-order condition for profit maximization is

$$\alpha - 2q_2 - k_2\gamma q_1 = 0,$$

from which we get the following best-response function

$$q_2(q_1) = \frac{1}{2}\left[\alpha - k_2\gamma q_1\right]. \tag{11}$$

Note that when the goods are imperfect substitutes ($\gamma > 0$) we have downward sloping best response functions and quantities are strategic substitutes. However, when the goods are complementary ($\gamma < 0$), the best response functions are upward sloping and the quantities are strategic complements.

Solving the best response functions (10) and (11) simultaneously, we get that in the unique interior Nash equilibrium of the quantity competition game[9]

$$q_1^* = \frac{\alpha(2 - k_1\gamma)}{4 - k_1k_2\gamma^2}$$

and

$$q_2^* = \frac{\alpha(2 - k_2\gamma)}{4 - k_1k_2\gamma^2}.$$

These equilibrium quantities are indeed positive for all $k_1, k_2 \in K$.

These quantity choices by the managers' imply the following (true) equilibrium prices

$$p_1^*(q_1^*, q_2^*) = \frac{\alpha\left(2 - (2 - k_1)\gamma + (1 - k_1)k_2\gamma^2\right)}{4 - k_1k_2\gamma^2},$$

[9] Note that the first-order conditions for profit maximization are also sufficient.

and

$$p_2^*(q_1^*, q_2^*) = \frac{\alpha\left(2 - (2 - k_2)\gamma + (1 - k_2)k_1\gamma^2\right)}{4 - k_1 k_2 \gamma^2}.$$

The (true) equilibrium profits are, respectively,

$$\pi_1^*(k_1, k_2) = \frac{\alpha^2 (2 - k_1\gamma)\left(2 - (2 - k_1)\gamma + (1 - k_1)k_2\gamma^2\right)}{(4 - k_1 k_2 \gamma^2)^2},$$

and

$$\pi_2^*(k_1, k_2) = \frac{\alpha^2 (2 - k_2\gamma)\left(2 - (2 - k_2)\gamma + (1 - k_2)k_1\gamma^2\right)}{(4 - k_1 k_2 \gamma^2)^2}.$$

Note that these prices, and hence profits, are always positive given parameter restrictions on γ and k_i.

Comparing equilibrium quantities we see that

$$q_1^* - q_2^* = \frac{\alpha\gamma(k_2 - k_1)}{4 - k_1 k_2 \gamma^2}.$$

The denominator is always positive, therefore

$$\text{sign}(q_1^* - q_2^*) = \text{sign}(\gamma(k_2 - k_1)).$$

This implies that $q_1^* > q_2^*$ if and only if:

(a) $\gamma > 0$ and $k_1 < k_2$, i.e. the more self-confident manager will produce more when the goods are imperfect substitutes.

(b) $\gamma < 0$ and $k_1 > k_2$, i.e. the more self-confident manager will produce more when the goods are imperfect complements.

In other words, irrespective of whether the goods are imperfect substitutes or imperfect complements, the more self-confident managers will produce more. In particular, we already saw in the previous section that when an overconfident and a rational manager interact, the overconfident manager will produce more.

Comparing equilibrium prices we see that

$$p_1^*(q_1^*, q_2^*) - p_2^*(q_1^*, q_2^*) = \frac{\alpha\gamma(1 - \gamma)(k_1 - k_2)}{4 - k_1 k_2 \gamma^2},$$

implying that

$$\text{sign}(p_1^* - p_2^*) = \text{sign}(\gamma(k_1 - k_2)).$$

We note that self-confidence moves prices and quantities into opposite directions. We have that $p_1^* > p_2^*$ if and only if:

(a) $\gamma > 0$ and $k_1 > k_2$, i.e. the less self-confident managers will face a higher market price when the goods are imperfect substitutes.

(b) $\gamma < 0$ and $k_1 < k_2$, i.e. the less self-confident managers will face a higher market price when the goods are imperfect complements.

In other words, irrespective of whether the goods are imperfect substitutes or imperfect complements, more self-confident managers will face a lower market price than they have perceived. In particular, when an overconfident and a rational manager interact, the overconfident manager will face a lower price.

We have seen that the manager's level of self-confidence affects prices and quantities in an opposite manner. Which effect will dominate? What is the overall effect on equilibrium profits?

The (true) equilibrium profit difference is

$$\pi_1^* - \pi_2^* = \frac{\alpha^2\gamma^2 (k_2 - k_1)(k_1 - k_1 k_2 \gamma + k_2)}{(4 - k_1 k_2 \gamma^2)^2}.$$

It is easy to verify that the last factor is always positive for all $k_1, k_2 \in K$. Therefore, $\pi_1^* > \pi_2^*$ if and only if $k_1 < k_2$.

We summarize the findings above in the following two propositions:

Proposition 4 *Suppose the goods are imperfect substitutes* ($\gamma > 0$) *and manager i's degree of self-confidence is $k_i \in K$, $i = 1, 2$. Then in the unique interior Nash equilibrium of the quantity competition game, the following are equivalent:*

(a) $k_i < k_{-i}$

(b) $q_i^* > q_{-i}^*$

(c) $p_i^* < p_{-i}^*$

(d) $\pi_i^* > \pi_{-i}^*$

Proposition 5 *Suppose the goods are imperfect complements* ($\gamma < 0$) *and manager i's degree of self-confidence is $k_i \in K$, $i = 1, 2$. Then in the unique interior Nash equilibrium of the quantity competition game, the following are equivalent:*

(a) $k_i < k_{-i}$

(b) $q_i^* < q_{-i}^*$

(c) $p_i^* > p_{-i}^*$

(d) $\pi_i^* > \pi_{-i}^*$

In other words, the manager with a higher degree of confidence will produce more and make higher profits when the goods are imperfect substitutes but will make lower profits when the goods are imperfect complements. In particular, when two underconfident managers interact, the one who is less underconfident will make a higher profit when the goods are imperfect substitutes but will make a lower profit if the goods are imperfect complements. When two overconfident managers interact, the more overconfident manager will earn higher profits when the goods are imperfect substitutes but will make lower profits when the goods are imperfect complements.

Next, is there an optimal degree of self-confidence for a manager to have? Before we answer that question, we make the following observation.

Proposition 6 *Suppose* $\gamma \in \left(-\frac{6}{7}, 1\right)$ *and* $k_1, k_2 \in K$*. As a function of own confidence,* k_i*, the equilibrium profit* π_i^* *to manager i is a strictly concave function with a unique maximum at*

$$\hat{k}_i\left(k_j\right) = \frac{4\left(1 - k_j\gamma\right)}{4 - 2k_j\gamma - k_j\gamma^2}. \tag{12}$$

Proof. See appendix. ■

In particular if $k_1 = 1$ and $\gamma = 0.6$ (-0.6) as in the preceding subsection, then $\hat{k}_2 = 0.656$ (1.322), in broad agreement with the above approximate observation. More generally, (1) for $\gamma > 0$, equation (12) defines $\hat{k}_i$ as a number in $(0, 1)$; and (2) for $\gamma < 0$, equation (12) defines $\hat{k}_i$ as a number larger than 1. Furthermore, if $\gamma \in \left(-\frac{2}{3}, 1\right)$, then $\hat{k}_i < \left|\frac{1}{\gamma}\right|$. We henceforth assume that $\gamma \in \left(-\frac{2}{3}, 1\right)$.

Next, we turn to market selection based on absolute profits and robustness analysis.

3.3. Market Selection: Evolutionarily Robust Self-Confidence

Consider a manager with confidence k' who is matched against a manager with confidence k. The profit to the first manager is

$$\pi^* = v\left(k', k\right) = \frac{\alpha^2\left(2 - k'\gamma\right)\left(\left(1 - k'\right)k\gamma^2 + \left(k' - 2\right)\gamma + 2\right)}{\left(4 - k'k\gamma^2\right)^2}$$

We will call a degree of self-confidence k *evolutionarily robust* if there for every $k' \neq k$ exists an $\bar{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \bar{\varepsilon})$:

$$(1 - \varepsilon)\, v\left(k, k\right) + \varepsilon v\left(k, k'\right) > (1 - \varepsilon)\, v\left(k', k\right) + \varepsilon v\left(k', k'\right) \tag{13}$$

In words: consider a pool of managers who all have the same degree of self-confidence, k. Suppose that a small population fraction of these would "mutate" to some other degree of self-confidence, k'. Let $0 < \varepsilon < 1$ be the population share of such "mutants". The expected profit that a manager of the "incumbent" type k would make when matched against another manager of the same incumbent type would be $v(k,k)$, while the expected profit to the same manager if matched against a manager of the "mutant" type k' would be $v(k,k')$. Hence, the left-hand side in (13) is the expected profit to a manager of the incumbent type when randomly matched against another manager in the pool of managers. Likewise, the right-hand side of (13) is the expected profit to a manager of the mutant type when randomly matched against another manager in the pool.[10] The incumbent degree of self-confidence is thus defined as evolutionarily robust if there exists no other degree of self-confidence that, if appearing in a sufficiently small share of the manager pool, would earn at least the same expected profit as the managers of the incumbent type do, in the mixed population.

For a degree of self-confidence k to be evolutionarily robust in this sense, it is clearly necessary that

$$v(k',k) \leq v(k,k) \text{ for all } k' > 0.$$

In other words, k should be optimal against itself.[11] Conversely, suppose that k is optimal against itself. If, moreover, no other k' is also optimal against k, then it follows that k is evolutionarily robust. Hence, it is of interest to find those k that are optimal against themselves.

A robust degree of confidence k would need to satisfy equation (12) for $\hat{k}_i = k_j = k$. The unique degree of confidence with this robustness property is evidently[12],

$$k^r = \frac{2}{2+\gamma}.$$

For the case of substitutes, we illustrate this in Figure 3, showing the contour map of the function v, with k' on the horizontal axis and k on the vertical, when $\alpha = 1$ and $\gamma = 0.6$. Figure 4 the same illustration for the case of complements, when $\alpha = 1$ and $\gamma = -0.4$. A degree of confidence k^r is robust against market selection if and only if the tangent of the isoquant through the point (k,k)—the thick curve—is horizontal at that point, and no other point on the horizontal line

[10] We assume that the pool is so large that we do not have to adjust the matching probabilities by first subtracting the manager himself from the pool, before randomizing.

[11] For if there would exist a manager type k' that would earn more against k than k does, that is, $v(k',k) > v(k,k)$, then, for $\varepsilon > 0$ sufficiently small, (13) would be violated.

[12] Note that (12) has two fixed points: $-\frac{2}{\gamma}$ and $\frac{2}{2+\gamma}$. The first one falls out of the interval K.

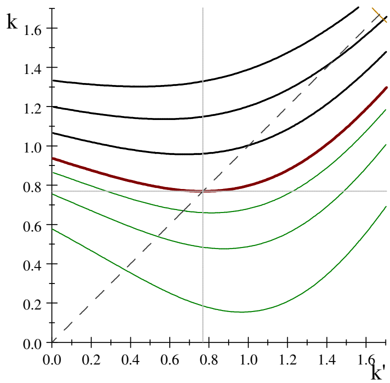

FIGURE 3. EVOLUTIONARILY ROBUST DEGREE OF SELF-CONFIDENCE WHEN THE GOODS ARE IMPERFECT SUBSTITUTES.

through that point has a higher v-value. Inspecting the graph we conclude that this degree is approximately $k = 0.77$ when the goods are imperfect substitutes and $k = 1.25$ when the goods are imperfect complements (indicated by the vertical and horizontal lines).

More formally, we have[13]:

Proposition 7 *Suppose* $\gamma \in \left(-\frac{2}{3}, 1\right)$. *Under quantity competition the unique degree of confidence that is robust to market selection driven by absolute profits is*

$$k^r = \frac{2}{\gamma + 2}, \tag{14}$$

Proof. Recalling Proposition 6, the proof is evident. ■

We illustrate the evolutionarily robust degree of self-confidence under quantity competition in Figure 5. We note that this evolutionarily robust degree of confidence is of the *overconfident* type and is independent of whether the goods are imperfect substitutes or imperfect complements, and hence whether the choice variables are strategic stubstitutes or strategic complements.

Note that, when the goods are imperfect substitutes, the degree of overconfidence is higher the more substitutable the goods are. When the goods are independent, and each firm has a monopoly power, there is no strategic or selection

[13]Remember that we need to restrict γ to $\left(-\frac{2}{3}, 1\right)$ to ensure that the elovutionarily robust self-confidence is less than $\left|\frac{1}{\gamma}\right|$.

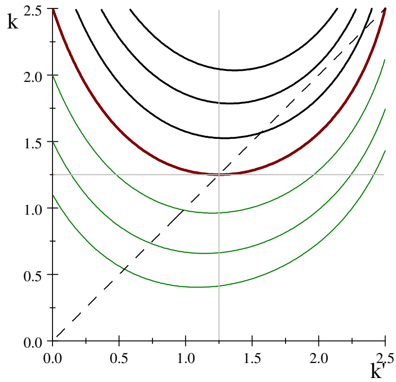

FIGURE 4. EVOLUTIONARILY ROBUST DEGREE OF SELF-CONFIDENCE WHEN THE GOODS ARE IMPERFECT COMPLEMENTS.

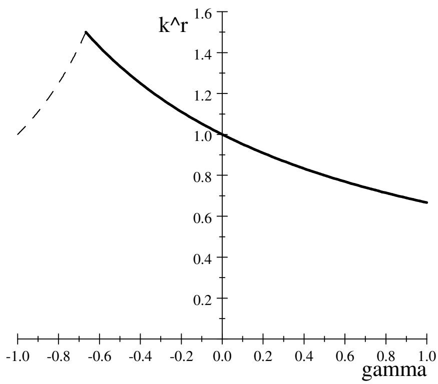

FIGURE 5. EVOLUTIONARILY ROBUST DEGREE OF CONFIDENCE UNDER QUANITITY COMPETITION

value to being overconfident. This implies that overconfident managers should be more prevalent in industries where product differentiation is lower and there is more room for strategic interaction.

When the goods are imperfect complements, the degree of overconfidence is higher the more complementary the goods are. This implies that overconfident managers should be more prevalent in industries where product complementaries are higher.

3.3.1. Profit Comparison

Although in the evolutionarily robust equilibrium all the firms would be run by overconfident managers, it is instructive to compare profits with the case where all the managers were rational. The common profits when both managers have the evolutionarily robust degree of confidence is

$$\begin{aligned}\pi^r &= \frac{\alpha^2 (2 - k'\gamma)}{(4 - k'k\gamma^2)^2} \left((1 - k') k\gamma^2 + (k' - 2)\gamma + 2\right)\Big|_{k=\frac{2}{2+\gamma}, k'=\frac{2}{2+\gamma}} \\ &= \frac{\alpha^2 \left(4 - \gamma^2\right)}{16 (1 + \gamma)},\end{aligned}$$

whereas the common profits when both managers are rational is

$$\pi^* = \left(\frac{\alpha}{2 + \gamma}\right)^2$$

Comparing these profits, we see that the difference is

$$\begin{aligned}\pi^r - \pi^* &= \frac{\alpha^2 \left(4 - \gamma^2\right)}{16 (1 + \gamma)} - \left(\frac{\alpha}{2 + \gamma}\right)^2 \\ &= -\frac{1}{16} \frac{\alpha^2 \gamma^3 (4 + \gamma)}{(1 + \gamma)(2 + \gamma)^2},\end{aligned}$$

which implies that

$$\operatorname{sign}(\pi^r - \pi^*) = -\operatorname{sign}(\gamma).$$

Thus the profit difference is negative if the products are imperfect substitutes ($\gamma > 0$) and positive if the products imperfect complements ($\gamma < 0$).

4. Quantity Competition under Incomplete Information

In the previous section we analyzed the quantity competition game under the assumption that the managers in a duopoly could observe each other's types or levels of self-confidence. What can be said if they do know or cannot observe each other's types? To analyze this scenario, we proceed in two steps. In the first step, for expository purposes, we will look at the case when there is a large population of potential managers who are randomly matched and there are only two types of managers: rational, with self-confidence $k = 1$, and biased with self-confidence $k \neq 1$. Then we will allow for arbitrarily levels of self-confidence for all managers.

4.1. One Overconfident Manager

Suppose there is a mutually incomplete information about the manager's type. Let $1 - \varepsilon$ be the population share of rational managers with self-confidence $k = 1$ and ε the population share of managers with self-confidence $k \neq 1$ (or alternatively, each manager will hold the belief that his opponent has self-confidence $k = 1$ with probability $1 - \varepsilon$ and self-confidence $k \neq 1$ with probability ε). Suppose there are only these types and the type of a manager is a private knowledge. We note that the average degree of confidence in the population, $\bar{k}$, is

$$\begin{aligned}
\bar{k} &= (1 - \varepsilon) \cdot 1 + \varepsilon \cdot k \\
&= 1 + \varepsilon (k - 1) \\
&= 1 - \varepsilon + \varepsilon k < 1 / |\gamma| ,
\end{aligned}$$

Also note that $\bar{k} > k$ if $k < 1$ and $\bar{k} < k$ if $k > 1$.

We are looking for a pure strategy Bayesian Nash equilibrium. At a Bayesian Nash equilibrium of the quantity competition game we must have that

$$\begin{aligned}
q^k &\in \arg\max_{q^i \geq 0} \mathbb{E}[\pi|k], \\
q^1 &\in \arg\max_{q^i \geq 0} \mathbb{E}[\pi|1],
\end{aligned}$$

where q^k is the quantity choice if a manager is of type $k \neq 1$ and q^1 is the quantity choice of a rational manager.

More precisely, we must have that

$$\begin{aligned}
q^k &\in \arg\max_{q^i \geq 0} \left\{ \varepsilon(\alpha - q^i - k\gamma q^k)q^i + (1 - \varepsilon)(\alpha - q^i - k\gamma q^1)q^i \right\} \\
q^1 &\in \arg\max_{q^i \geq 0} \left\{ \varepsilon(\alpha - q^i - \gamma q^k)q^i + (1 - \varepsilon)(\alpha - q^i - \gamma q^1)q^i \right\}
\end{aligned}$$

or more simply

$$q^k \in \arg\max_{q^i \geq 0} \left\{ (\alpha - q^i - \varepsilon k\gamma q^k - (1-\varepsilon)k\gamma q^1)q^i \right\}$$
$$q^1 \in \arg\max_{q^i \geq 0} \left\{ (\alpha - q^i - \varepsilon \gamma q^k - (1-\varepsilon)\gamma q^1)q^i \right\}$$

The first-order conditions for maximization (which are also sufficient) yield:

$$\begin{cases} \alpha - 2q^i - \varepsilon k\gamma q^k - (1-\varepsilon)k\gamma q^1 = 0 \\ \alpha - 2q^i - \varepsilon \gamma q^k - (1-\varepsilon)\gamma q^1 = 0 \end{cases}$$

These first-order conditions have to be satisfied on the equilibrium path, so we must have that

$$\begin{cases} \alpha - 2q^k - \varepsilon k\gamma q^k - (1-\varepsilon)k\gamma q^1 = 0 \\ \alpha - 2q^1 - \varepsilon \gamma q^k - (1-\varepsilon)\gamma q^1 = 0 \end{cases},$$

or

$$\begin{cases} (2+\varepsilon k\gamma)\, q^k + (1-\varepsilon)k\gamma q^1 = \alpha \\ \varepsilon\gamma q^k + (2+(1-\varepsilon)\gamma)\, q^1 = \alpha \end{cases}.$$

The solution to this system of equations is given by

$$q^k = \frac{\alpha\,(2+(1-\varepsilon)(1-k)\,\gamma)}{2\,(2+(1-\varepsilon)\,\gamma+\varepsilon k\gamma)} = \frac{\alpha\left(2+\left(\bar{k}-k\right)\gamma\right)}{2\left(2+\bar{k}\gamma\right)}$$

and

$$q^1 = \frac{\alpha\,(2-\varepsilon\,(1-k)\,\gamma)}{2\,(2+(1-\varepsilon)\,\gamma+\varepsilon k\gamma)} = \frac{\alpha\left(2+\left(\bar{k}-1\right)\gamma\right)}{2\left(2+\bar{k}\gamma\right)}$$

We note that both of these quantities are positive and that the difference is

$$q^k - q^1 = \frac{\alpha\,(1-k)\,\gamma}{2\left(2+\bar{k}\gamma\right)},$$

which is positive if either $\gamma > 0$ and $k < 1$ or $\gamma < 0$ and $k > 1$. In other words, like under complete information, an overconfident manager will act more aggressively (produce more) and this is irrespective of whether the products are imperfect complements or imperfect substitutes.

True equilibrium prices, in a match between a rational manager and a manager with self-confidence k are, respectively

$$p(k,1) = \alpha - q^k - \gamma q^1 = \frac{\alpha(2+\gamma(k-1)(\varepsilon-\gamma\varepsilon+1))}{2(2+\gamma(k\varepsilon+1-\varepsilon))}$$
$$= \frac{\alpha\left(2+\gamma\left[\bar{k}+k-2+\left(1-\bar{k}\right)\right]\gamma\right)}{2\left(2+\bar{k}\gamma\right)} > 0,$$

and

$$p(1,k) = \alpha - q^1 - \gamma q^k = \frac{\alpha(2+\gamma(k-1)(\gamma+\varepsilon-\gamma\varepsilon))}{2(2+(1-\varepsilon)\gamma+\varepsilon k\gamma)}$$
$$= \frac{\alpha\left(2+\gamma\left[\bar{k}-1+\left(k-\bar{k}\right)\gamma\right]\right)}{2\left(2+\bar{k}\gamma\right)} > 0$$

We note that

$$p(k,1) - p(1,k) = \frac{\alpha\gamma(1-\gamma)(k-1)}{2\left(2+\bar{k}\gamma\right)},$$

which is negative if either $\gamma > 0$ and $k < 1$ or $\gamma < 0$ and $k > 1$. In other words, like under complete information, an overconfident manager will face a lower market price than its unbiased competition irrespective of whether the products are imperfect substitutes or imperfect complements.

For completeness, we also note that prices in other matches are given by[14]

$$p(k,k) = \alpha - q^k - \gamma q^k = \frac{\alpha(2+\gamma(k-1)(\gamma+\varepsilon-\gamma\varepsilon+1))}{2(2+(1-\varepsilon)\gamma+\varepsilon k\gamma)}$$
$$= \frac{\alpha\left(2+\gamma\left[\bar{k}+k-2+\left(k-\bar{k}\right)\gamma\right]\right)}{2\left(2+\bar{k}\gamma\right)} > 0,$$

and

$$p(1,1) = \alpha - q^1 - \gamma q^1 = \frac{\alpha(2+\gamma(k-1)\varepsilon(1-\gamma))}{2(2+(1-\varepsilon)\gamma+\varepsilon k\gamma)}$$
$$= \frac{\alpha\left(2+\left(\bar{k}-1\right)(1-\gamma)\gamma\right)}{2\left(2+\bar{k}\gamma\right)} > 0.$$

[14] We also note that both prices at the equilibrium quantities are positive from the (ex ante) perspective of any type of a manager.

We see again, that self-confidence affects equilibrium quantities and prices in an opposite manner. Which effect dominates for (true) equilibrium profits?

Let $\pi^*(1,k)$ and $\pi^*(k,1)$ denote the (true) equilibrium profits to a rational and an overconfident manager, respectively, in a match between them. Then we have that

$$\begin{aligned}\pi^*(1,k) &= \frac{\alpha^2(2+\gamma(k-1)(\gamma+\varepsilon-\gamma\varepsilon))(2-\varepsilon(1-k)\gamma)}{4(2+(1-\varepsilon)\gamma+\varepsilon k\gamma)^2}\\ &= \frac{\alpha^2\left(2+\gamma\left[\bar{k}-1+\left(k-\bar{k}\right)\gamma\right]\right)\left(2+\left(\bar{k}-1\right)\gamma\right)}{4\left(2+\bar{k}\gamma\right)^2}\end{aligned}$$

and

$$\begin{aligned}\pi^*(k,1) &= \frac{\alpha^2(2+\gamma(k-1)(\varepsilon-\gamma\varepsilon+1))(2+(1-\varepsilon)(1-k)\gamma)}{4(2+(1-\varepsilon)\gamma+\varepsilon k\gamma)^2}\\ &= \frac{\alpha^2\left(2+\gamma\left[\bar{k}+k-2+\left(1-\bar{k}\right)\gamma\right]\right)\left(2+\left(\bar{k}-k\right)\gamma\right)}{4\left(2+\bar{k}\gamma\right)^2}.\end{aligned}$$

The profit difference is

$$\pi^*(k,1)-\pi^*(1,k) = \frac{\alpha^2\gamma^2\left(1-k^2\right)}{4\left(2+\bar{k}\gamma\right)^2},$$

which is positive if $k<1$.

In other words, when the goods are imperfect substitutes, in a match between an overconfident and a rational manager, the overconfident manager earns a higher profit. When the goods are complementary, then an overconfident manager earns less. This effect again, is qualitatively the same as under complete information. However, this does not imply that the type that earns the highest profits on average is the overconfident type when the goods are imperfect substitutes and the underconfident type when the goods are imperfect complements.

The profits when two rational managers are matched and the profits when two managers with self-confidence k are matched, are, respectively

$$\begin{aligned}\pi^*(1,1) &= \frac{\alpha^2(2+\gamma(k-1)\varepsilon(1-\gamma))(2-\varepsilon(1-k)\gamma)}{4(2+(1-\varepsilon)\gamma+\varepsilon k\gamma)^2}\\ &= \frac{\alpha^2\left(2+\gamma(1-\gamma)\left(\bar{k}-1\right)\right)\left(2+\left(\bar{k}-1\right)\gamma\right)}{4\left(2+\bar{k}\gamma\right)^2},\end{aligned}$$

and

$$\begin{aligned}\pi^*(k,k) &= \frac{\alpha^2(2+\gamma(k-1)(\gamma+\varepsilon-\gamma\varepsilon+1))(2+(1-\varepsilon)(1-k)\gamma)}{4(2+(1-\varepsilon)\gamma+\varepsilon k\gamma)^2} \\ &= \frac{\alpha^2\left(2+\gamma\left(\bar{k}+k-2+\gamma\left(k-\bar{k}\right)\right)\right)\left(2+\left(\bar{k}-k\right)\gamma\right)}{4\left(2+\bar{k}\gamma\right)^2}.\end{aligned}$$

Therefore, the average expected profit to a rational manager, over all possible matchings is

$$\begin{aligned}\mathbb{E}(\pi|1) &= \varepsilon\pi(1,k)+(1-\varepsilon)\pi(1,1) \\ &= \frac{\alpha^2(k\gamma\varepsilon-\gamma\varepsilon+2)^2}{4(\gamma-\gamma\varepsilon+k\gamma\varepsilon+2)^2} \\ &= \frac{\alpha^2\left(2+\left(\bar{k}-1\right)\gamma\right)^2}{4\left(2+\bar{k}\gamma\right)^2},\end{aligned}$$

while the average expected profit to a manager with self-confidence k, over all possible matchings is

$$\begin{aligned}\mathbb{E}(\pi|k) &= \varepsilon\pi(k,k)+(1-\varepsilon)\pi(k,1) \\ &= \frac{\alpha^2(2+\gamma-k\gamma-\gamma\varepsilon+k\gamma\varepsilon)(2-\gamma+k\gamma-\gamma\varepsilon+k\gamma\varepsilon)}{4(\gamma-\gamma\varepsilon+k\gamma\varepsilon+2)^2} \\ &= \frac{\alpha^2\left(2+\left(\bar{k}-k\right)\gamma\right)\left(2+\left(\bar{k}+k-2\right)\gamma\right)}{4\left(2+\bar{k}\gamma\right)^2}\end{aligned}$$

This expected profit difference is

$$\mathbb{E}(\pi|1)-\mathbb{E}(\pi|k) = \frac{\alpha^2\gamma^2(k-1)^2}{4\left(2+\bar{k}\gamma\right)^2} > 0. \tag{15}$$

We see from (15), that $\mathbb{E}(\pi|1) > \mathbb{E}(\pi|k)$ for all k. In other words, the type that earns highest average expected profits is the *homo oeconomicus* irrespective of whether the goods are imperfect substitutes or imperfect complements.

We gather the findings above in the following proposition.

Proposition 8 *Suppose the managers could have only two possible degrees of self-confidence, one of which is the neutral one, the type of the manager is a private knowledge and the firms compete by setting prices. There exists a unique* *pure-strategy Bayesian Nash equilibrium. If the goods are imperfect substitutes, in equilibrium, in a pair of duopolists, the overconfident type produces more and earns higher profits. If the goods are imperfect complements, then in this equilibrium, in a pair of duopolists, the underconfident type produces more and earns higher profits. However, the type that earns highest average expected profits, over all possible matchings, is the homo oeconomicus.*

We next move on to the case of arbitrarily self-confident managers.

4.2. Arbitrarily Self-Confident Managers

Consider again a large population of potential managers and let μ be a probability measure representing the distribution of their confidence degrees k with support on $K = (0, |1/\gamma|)$. Denote $\mathbb{E}_{\mu}[k] = \bar{k}$ and note that $\bar{k} \in K$. Suppose that the managers are pair-wise randomly matched and that the type of one's competitor is not observable. Assume that the population is so large that we can approximate the conditional distribution of one's opponent's type, given one's own type, by μ.

We are looking for a pure strategy symmetric Bayesian Nash equilibria of the quantity competition game. At such an equilibrium we must have that[15]

$$q(k_i) \in \arg\max_{q_i \geq 0} \{(\alpha - q_i - k_i\gamma\bar{q}) \cdot q\} \text{ for } \forall k_i \in K,$$

where $\bar{q} = \mathbb{E}[q(k_i)]$ is the average quantity choice in the population.

The first-order conditions for profit maximization yield (for each k):

$$\alpha - 2q(k_i) - k_i\gamma\bar{q} = 0,$$

or

$$q(k_i) = \frac{\alpha - k_i\gamma\bar{q}}{2} \text{ for } \forall k_i \in K.$$

Since this holds for all k, we can take the expectation of both sides

$$\mathbb{E}[q(k_i)] = \bar{q} = \frac{\alpha - \bar{k}\gamma\bar{q}}{2},$$

which gives

$$\bar{q} = \frac{\alpha}{2 + \bar{k}\gamma}. \tag{16}$$

[15]Note that all the equilibrium quanitites and both resulting equilibrium prices in any match are positive.

Inserting (16) back into the original first-order condition, gives

$$q^*(k_i) = \frac{\alpha\left(2 + \left(\bar{k} - k_i\right)\gamma\right)}{2\left(2 + \bar{k}\gamma\right)} \text{ for } \forall k_i \in K.$$

We note that, as expected, $q^*(k_i)$ is decreasing in k when the goods are imperfect substitutes (having a low k is like being more self-confident) and increasing in k if the goods are imperfect complements (having a high k is like being more self-confident):

$$q^*(k) - q^*(k') = \frac{\alpha(k' - k)\gamma}{2\left(2 + \bar{k}\gamma\right)}.$$

Hence, when a k and a k' type managers are randomly matched, the more self-confident type will produce more than the less confident type and this is irrespective of whether the goods are substitutes or complements.

The resulting equilibrium prices, in a match between a k and a k' type manager are, respectively,

$$p^*(k, k') = \frac{\alpha\left(2 + \gamma\left(\bar{k} + k - 2 + \left(k' - \bar{k}\right)\gamma\right)\right)}{2\left(2 + \bar{k}\gamma\right)}$$

and

$$p^*(k', k) = \frac{\alpha\left(2 + \gamma\left(\bar{k} + k' - 2 + \left(k - \bar{k}\right)\gamma\right)\right)}{2\left(2 + \bar{k}\gamma\right)}.$$

We see again, that the difference is

$$p^*(k, k') - p^*(k', k) = \frac{\alpha\gamma(1 - \gamma)(k - k')}{2\left(2 + \bar{k}\gamma\right)},$$

implying that the more confident manager of the pair, irrespective of the nature of the goods, will face a lower market price. Like under complete information, the degree of self-confidence affects equilibrium quantities and prices in an opposite manner. Which effect will dominate for profits?

For completeness, the prices in other matches are, respectively,

$$p(k, k) = \frac{\alpha\left(2 + \left(\bar{k} + k - 2 + \gamma\left(k - \bar{k}\right)\right)\gamma\right)}{2\left(2 + \bar{k}\gamma\right)}$$

and

$$p(k',k') = \frac{\alpha\left(2+\left(\bar{k}+k'-2+\gamma\left(k'-\bar{k}\right)\right)\gamma\right)}{2\left(2+\bar{k}\gamma\right)}.$$

In a match between a k and k' type manager, the equilibrium profits are

$$\pi^*(k,k') = \frac{\alpha^2\left(2+\gamma\left(\bar{k}+k-2+\left(k'-\bar{k}\right)\gamma\right)\right)\left(2+\left(\bar{k}-k\right)\gamma\right)}{4\left(2+\bar{k}\gamma\right)^2}$$

to manager k, and

$$\pi^*(k',k) = \frac{\alpha^2\left(2+\gamma\left(\bar{k}+k'-2+\left(k-\bar{k}\right)\gamma\right)\right)\left(2+\left(\bar{k}-k'\right)\gamma\right)}{4\left(2+\bar{k}\gamma\right)}$$

to manager k', respectively.

The difference is

$$\begin{aligned}\pi^*(k,k') - \pi^*(k',k) &= \frac{\alpha^2\gamma^2\left((k')^2-k^2\right)}{4\left(2+\bar{k}\gamma\right)^2}\\ &= \frac{\alpha^2\gamma^2(k'-k)(k'+k)}{4\left(2+\bar{k}\gamma\right)^2},\end{aligned}$$

which implies that

$$\operatorname{sign}(\pi^*(k,k') - \pi^*(k',k)) = \operatorname{sign}(k'-k).$$

This condition says that the manager who earns a higher profit is the more confident one when the goods are imperfect substitutes and the less confident one when the goods are imperfect complements.

However, this does not imply that the type that earns the highest profits on average is the overconfident type when the goods are imperfect substitutes and the underconfident type when the goods are incomplete complements.

The expected profit to a manager of type k_i, over all possible matchings is

$$\begin{aligned}\mathbb{E}_\mu[\pi^*(k_i)] &= (\alpha - q(k_i) - \gamma\bar{q})\cdot q(k_i)\\ &= \left(\alpha - \frac{\alpha\left(2+\left(\bar{k}-k_i\right)\gamma\right)}{2\left(2+\bar{k}\gamma\right)} - \gamma\frac{\alpha}{2+\bar{k}\gamma}\right)\frac{\alpha\left(2+\left(\bar{k}-k_i\right)\gamma\right)}{2\left(2+\bar{k}\gamma\right)}\\ &= \frac{\alpha^2\left(2+\left(\bar{k}-k_i\right)\gamma\right)\left(2+\left(\bar{k}+k_i-2\right)\gamma\right)}{4\left(2+\bar{k}\gamma\right)^2},\end{aligned}$$

a quantity that is maximized at $k_i = 1$. Hence, the type k that earns the highest expected profit is $k = 1$, or, in other words, *homo oeconomicus*.

To see this, let's take the derivative of $\mathbb{E}_\mu[\pi^*(k_i)]$ with respect to k_i.

$$\frac{\partial \mathbb{E}_\mu[\pi^*(k_i)]}{\partial k_i} = \frac{\alpha^2\gamma^2(1-k_i)}{2\left(2+\bar{k}\gamma\right)^2} = 0$$

implies that $k_i = 1$. Moreover,

$$\frac{\partial^2 \mathbb{E}_\mu[\pi^*(k_i)]}{(\partial k_i)^2} = -\frac{\alpha^2\gamma^2}{2\left(2+\bar{k}\gamma\right)^2} < 0$$

so, this is indeed a maximum. In sum:

Proposition 9 *There exists a unique symmetric pure-strategy Bayesian Nash equilibrium. In this equilibrium:*

(a) All the managers expect positive profits and they all use the strategy

$$q^*(k_i) = \frac{\alpha\left(2+\left(\bar{k}-k_i\right)\gamma\right)}{2\left(2+\bar{k}\gamma\right)} \text{ for } \forall k_i \in K$$

(b) In a pair of duopolists, the more confident manager earns a higher profit if the goods are substitutes, and the less confident manager earns a higher profit if the goods are complementary.

(c) The expected average equilibrium profit to a manager i with confidence k_i, in a random match with another manager from the type distribution μ, is

$$\mathbb{E}_\mu[\pi^*(k_i)] = \frac{\alpha^2\left(2+\left(\bar{k}-k_i\right)\gamma\right)\left(2+\left(\bar{k}+k_i-2\right)\gamma\right)}{4\left(2+\bar{k}\gamma\right)^2}$$

and this profit is maximized at $k_i = 1$.

We conclude with a brief discussion of market selection. Suppose, first that managers are selected for according to how well their firms do in terms of their expected profit. A degree of managerial confidence k would be robust against such market selection if it were the optimal degree of confidence, in terms of average earned profit, in a random match against a type drawn from the type distribution μ. All other types would be selected against and eventually disappear from the population of active managers. Only the optimal degree of confidence would prevail.

According to Proposition 9, the unique robust degree of confidence that the market will select for, would be $k = 1$, granted this belongs to the type distribution. In the terminology of the previous section, $k^r = 1$, no matter the nature of the goods. *Homo oeconomicus* would thus prevail. This contrasts sharply with the outcome of market selection according to absolute performance under complete information, where we found that a certain degree of overconfidence was selected for, see Proposition 7.

Secondly, if selection is based on relative profits, instead, then the rational manager will be selected against. In this case it seems that the most overconfident type in the population will be selected for when the goods are imperfect substitutes and the least underconfident type when the goods are imperfect complements.

5. Price Competition

In this section instead of a quantity competition game, we imagine that firms compete by setting prices. The steps in the analysis in this section follow closely to that of the quantity competition case under complete information (mutually observable types).

5.1. One Overconfident Manager

Consider the case when manager 1 holds correct beliefs about γ, $k_1 = 1$, while manager 2 is biased and has self-confidence $k_2 \neq 1$. Manager 2 is overconfident if either $\gamma > 0$ and $k_2 < 1$ or $\gamma < 0$ and $k_2 > 1$. These beliefs are common knowledge. We also assume that $k_2 \in (0, |1/\gamma|)$ when the goods are imperfect complements ($-1 < \gamma < 0$) and that

$$\max\left\{0, \frac{\gamma^2 + 2\gamma - 2}{\gamma}\right\} < k_2 < \frac{\sqrt{3+\gamma} - 1}{\gamma} \tag{17}$$

when the goods are imperfect substitutes ($0 < \gamma < 1$). The first constraint in (17) ensures that equilibrium quantities at the interior equilibrium will indeed be positive, even if $\gamma > \gamma^* = \sqrt{3} - 1$. The second constraint guarantees that even the *perceived* demands are both positive at the equilibrium prices from either of the managers' perspective.

The manager of firm 1 solves

$$\max_{p_1 \geq 0} \left\{ p_1 \cdot \frac{1}{1-\gamma^2} \left(\alpha\left(1-\gamma\right) - p_1 + \gamma p_2\right) \right\}$$

taking p_2 as given. The first-order condition for profit maximization, which is also sufficient, reads

$$\alpha\left(1-\gamma\right) - 2p_1 + \gamma p_2 = 0,$$

from which we get the following best-response function

$$p_1\left(p_2\right) = \frac{1}{2}\left[\alpha\left(1-\gamma\right) + \gamma p_2\right]. \tag{18}$$

The manager of firm 2 solves

$$\max_{p_2 \geq 0} \left\{ p_2 \cdot \frac{1}{1-k_2^2\gamma^2} \left(\alpha\left(1-k_2\gamma\right) + k_2\gamma p_1 - p_2\right) \right\}$$

taking p_1 as given. The first order condition for profit maximization is

$$\alpha\left(1-k_2\gamma\right) + k_2\gamma p_1 - 2p_2 = 0,$$

from which we get the following best-response function

$$p_2\left(p_1\right) = \frac{1}{2}\left[\alpha\left(1-k_2\gamma\right) + k_2\gamma p_1\right]. \tag{19}$$

Note that when the goods are imperfect substitutes ($\gamma > 0$) we have upward sloping best response functions and prices are strategic complements. However, when the goods are imperfect complements ($\gamma < 0$), the best response functions are downward sloping and the prices are strategic substitutes. This problem is the dual of the quantity competition case, with prices playing the same role as quantities in the quantity competition case and visa versa.

Solving the best response functions (18) and (19) simultaneously, we get that in the unique interior Nash equilibrium of the price competition game

$$p_1^* = \frac{\alpha\left(2-\gamma-k_2\gamma^2\right)}{4-k_2\gamma^2}$$

and

$$p_2^* = \frac{\alpha\left(2-k_2\gamma-k_2\gamma^2\right)}{4-k_2\gamma^2},$$

which are indeed positive.

These price choices imply that (true) equilibrium demands are

$$q_1^*(p_1^*, p_2^*) = \frac{\alpha\left(2-\gamma-k_2\gamma^2\right)}{(1-\gamma^2)(4-k_2\gamma^2)},$$

and

$$q_2^*(p_1^*, p_2^*) = \frac{\alpha\left(2-(2-k_2)\gamma-\gamma^2\right)}{(1-\gamma^2)(4-k_2\gamma^2)}.$$

The (true) equilibrium profits, are

$$\pi_1^* = \frac{\alpha^2\left(2-\gamma-k_2\gamma^2\right)^2}{(1-\gamma^2)(4-k_2\gamma^2)^2},$$

and

$$\pi_2^* = \frac{\alpha^2\left(2-k_2\gamma-k_2\gamma^2\right)\left(2-(2-k_2)\gamma-\gamma^2\right)}{(1-\gamma^2)(4-k_2\gamma^2)^2}.$$

Hence, we have the following differences:

$$\begin{aligned} p_1^* - p_2^* &= \frac{\alpha\gamma(k_2-1)}{4-k_2\gamma^2} \\ q_1^* - q_2^* &= \frac{\alpha\gamma(1-k_2)}{(1-\gamma)(4-k_2\gamma^2)} \\ \pi_1^* - \pi_2^* &= \frac{\alpha^2\gamma^2(1-k_2)\left(3-k_2-k_2\gamma-k_2\gamma^2\right)}{(1-\gamma^2)(4-k_2\gamma^2)^2} \end{aligned} \tag{20}$$

We note that $3-k_2-k_2\gamma-k_2\gamma^2 > 0$ for all $k_2 < \frac{3}{1+\gamma+\gamma^2}$, $\gamma \in (-1,1)$.

First, note that if both managers were rational, then these differences would vanish. Second note that, again, the level of a manager's self-confidence affects prices and quantities in an opposite manner. Which effect dominates for profits? To answer this question, we now look at the case of imperfect substitutes and imperfect complements separately.

5.1.1. Substitutes

Proposition 10 *Suppose the goods are imperfect substitutes* ($\gamma > 0$). *In the unique interior Nash equilibrium of the price competition game we have:*

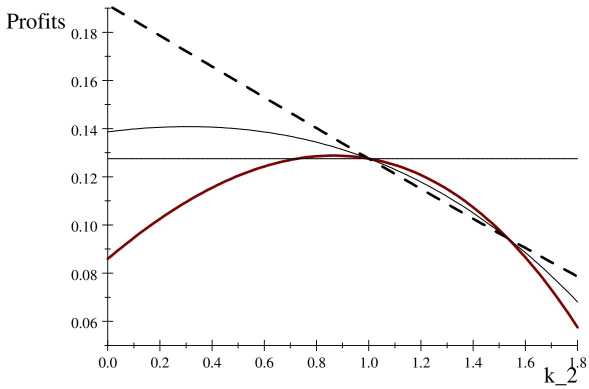

FIGURE 6. PROFITS (SUBSTITUTES)

(a) *If* $\max\left\{0, \frac{\gamma^2+2\gamma-2}{\gamma}\right\} < k_2 < 1$, *then*

(a) $p_2^* > p_1^*$
(b) $q_2^* < q_1^*$
(c) $\pi_2^* < \pi_1^*$

(b) *If* $1 < k_2 < \min\left\{\frac{3}{1+\gamma+\gamma^2}, \frac{\sqrt{3+\gamma}-1}{\gamma}\right\}$, *then*

(a) $p_2^* < p_1^*$
(b) $q_2^* > q_1^*$
(c) $\pi_2^* > \pi_1^*$

In other words, firm 2—if run by an overconfident manager—would set a higher price, but nevertheless earn lower profits than its unbiased competitor, firm 1. Under price competition it does not pay off to be overconfident when the goods are substitutable. If firm 2 is run by an underconfident manager, it will set a lower price and earn a higher profit. The diagram in Figure 6 shows how π_2^* (solid curve) and π_1^* (dashed curve) depend on k_2 (the horizontal axis), for $\gamma = 0.6$. The thin curve is the average of the two curves—half the industry profit.

We see that:

(a) For all $k_2 < 1$ the overconfident manager 2 charges a higher price but makes less profit than unbiased manager 1,

(b) For moderate degrees of overconfidence, $0,73 < k_2 < 1$, manager 2 earns a higher profit that had she also held correct perceptions (had $k_2 = k_1 = 1$). In that sense, there is an optimal degree of overconfidence for manager 2, $k_2 \approx 0.86$,

(c) The industry profit first increases and then decreases in k_2,

(d) For all degrees of *underconfidence* $(k_2 > 1)$, manager 2 ears less profit than had she also had correct perceptions (had $k_2 = k_1 = 1$). However, for all admissible levels of *underconfidence* $(1 < k_2 < 1.5)$ manager 2 earns higher profits than manager 1. The relative profit difference is maximized for $k_2 \approx 1.27$.[16]

5.1.2. Complements

Proposition 11 *Suppose the goods are imperfect complements* ($\gamma < 0$) *and* $k_2 \in K_2 = \left(1, \min\left\{\frac{3}{1+\gamma+\gamma^2}, -\frac{1}{\gamma}\right\}\right)$. *Then, in the unique interior Nash equilibrium of the price competition game, we have the following:*

(a) $p_2^* > p_1^*$

(b) $q_2^* < q_1^*$

(c) $\pi_2^* > \pi_1^*$

In other words, firm 2—run by an overconfident manager—sets a higher prices and nevertheless earns higher profits, than his or her rational competitor, firm 1. Under price competition with complementary goods an underconfident manager will always earn less profits than his or her rational competitor. The diagram in Figure 7 shows how π_2^* (solid curve) and π_1^* (dashed curve) depend on k_2 (the horizontal axis), for $\gamma = -0.6$ and $\alpha = 1$. The thin curve is the average of the two curves—half the industry profit.

We see that:

(a) For all $k_2 > 1$ the overconfident manager 2 sets a higher price and makes a higher profit than unbiased manager 2,

(b) There is an optimal degree of overconfidence for manager 2, $k_2 \approx 1.5$,

(c) The industry profit is decreasing in k_2,

[16] The more exact value would be $k_2 = \frac{8-2\gamma}{2\gamma+\gamma^2-\gamma^3+4}$.

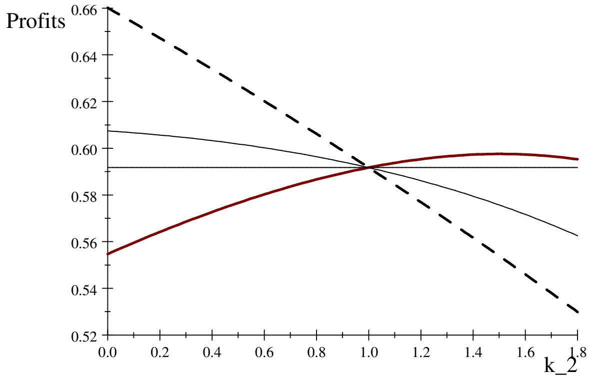

FIGURE 7. PROFITS (COMPLEMENTS)

(d) For all degrees of overconfidence, manager 2 earns a higher profit than had she also had correct perceptions (had $k_2 = k_1 = 1$).

In general, this last point is true for other values of negative γ if $1 < k_2 < \frac{4\gamma-2\gamma^2-4}{2\gamma^2+\gamma^3-\gamma^4-4}$. For example, if $\gamma = -0.4$, then this upper bound for overconfidence would be about 1.4.

5.2. Two Arbitrarily Confident Managers

We now turn to the analysis of the more general case of an interaction between two arbitrarily confident managers.

The manager of firm 1 solves

$$\max_{p_1 \geq 0} \left\{ p_1 \cdot \frac{1}{1-k_1^2\gamma^2} \left(\alpha\left(1-k_1\gamma\right) - p_1 + k_1\gamma p_2\right) \right\}$$

taking p_2 as given. The first-order condition for profit maximization is

$$\alpha\left(1-k_1\gamma\right) - 2p_1 + k_1\gamma p_2 = 0,$$

from which we get the following best-response function

$$p_1\left(p_2\right) = \frac{1}{2}\left[\alpha\left(1-k_1\gamma\right) + k_1\gamma p_2\right]. \tag{21}$$

The manager of firm 2 solves

$$\max_{p_2 \geq 0} \left\{ p_2 \cdot \frac{1}{1-k_2^2\gamma^2} \left(\alpha\left(1-k_2\gamma\right) + k_2\gamma p_1 - p_2\right) \right\}$$

taking p_1 as given. The first-order condition for profit maximization is

$$\alpha (1 - k_2\gamma) + k_2\gamma p_1 - 2p_2 = 0,$$

from which we get the following best response function

$$p_2 (p_1) = \frac{1}{2} [\alpha (1 - k_2\gamma) + k_2\gamma p_1] . \tag{22}$$

Note that when the goods are imperfect substitutes ($\gamma > 0$) we have upward sloping best response functions and the prices are strategic complements. However, when the goods are imperfect complements ($\gamma < 0$), the best response functions are downward sloping and the prices are strategic substitutes.

Solving the best response functions (21) and (22) simultaneously, we get that in the unique interior Nash equilibrium

$$p_1^* = \frac{\alpha \left(2 - k_1\gamma - k_1k_2\gamma^2\right)}{4 - k_1k_2\gamma^2}$$

and

$$p_2^* = \frac{\alpha \left(2 - k_2\gamma - k_1k_2\gamma^2\right)}{4 - k_1k_2\gamma^2}.$$

Note that these prices are indeed positive given that $k_i \in K = (0, |1/\gamma|)$ for $i = 1, 2$.

These prices choices imply that true (rather than *perceived*) equilibrium demands are

$$q_1^* (p_1^*, p_2^*) = \frac{\alpha \left(2 - (2 - k_1) \gamma - k_2\gamma^2\right)}{(1 - \gamma^2) (4 - k_1k_2\gamma^2)},$$

and

$$q_2^* (p_1^*, p_2^*) = \frac{\alpha \left(2 - (2 - k_2) \gamma - k_1\gamma^2\right)}{(1 - \gamma^2) (4 - k_1k_2\gamma^2)}.$$

The (true) equilibrium profits are, respectively,

$$\pi_1^* = \frac{\alpha^2 \left(2 - k_1\gamma - k_1k_2\gamma^2\right)\left(2 - (2 - k_1) \gamma - k_2\gamma^2\right)}{(1 - \gamma^2) (4 - k_1k_2\gamma^2)^2},$$

and

$$\pi_2^* = \frac{\alpha^2 \left(2 - k_2\gamma - k_1k_2\gamma^2\right)\left(2 - (2 - k_2) \gamma - k_1\gamma^2\right)}{(1 - \gamma^2) (4 - k_1k_2\gamma^2)^2}.$$

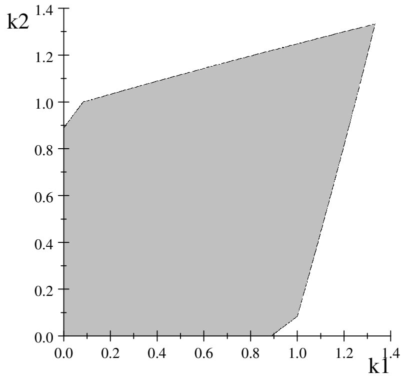

FIGURE 8. THE SET, WHERE CONDITION (23) IS SATISFIED, $\gamma = 0.75$

Note that for negative γ, these quantities indeed define an interior Nash equilibrium for any degree of permissible self-confidence $k_1, k_2 \in K$. However, when the goods are imperfect substitutes, we need additional restrictions on the admissible levels of self-confidence, namely we need to impose the following technical condition:

$$\begin{cases} 2-(2-k_1)\gamma - k_2\gamma^2 > 0 \\ 2-(2-k_2)\gamma - k_1\gamma^2 > 0 \\ 2-(2k_1-k_2)\gamma - k_1^2\gamma^2 > 0 \\ 2-(2k_2-k_1)\gamma - k_2^2\gamma^2 > 0 \end{cases} \tag{23}$$

The first two inequalities ensure that the true equilibrium demand is positive for both firms. The first two inequalities are not binding unless $\frac{2}{3} < \gamma < 1$.

The last two inequalities require that even perceived demands at equilibrium prices be positive for either of the firm. We illustrate condition (23) in Figure 8 for $\gamma = 0.75$. For further illustrations, see Appendix A. In essence condition (23) requires the beliefs not to be too far apart, when both managers are underconfident, and also in case $\gamma > 2/3$, the beliefs cannot diverge too much even in case of overconfidence.

Comparing equilibrium prices we see that

$$p_1^* - p_2^* = \frac{\alpha\gamma(k_2 - k_1)}{4 - k_1k_2\gamma^2}.$$

The denominator is always positive, therefore

$$\operatorname{sign}(p_1^* - p_2^*) = \operatorname{sign}(\gamma(k_2 - k_1)).$$

This implies that $p_1^* > p_2^*$ if and only if

(a) $\gamma > 0$ and $k_1 < k_2$, i.e. the more confident manager will set a higher price when the goods are imperfect substitutes.

(b) $\gamma < 0$ and $k_1 > k_2$, i.e. the more confident manager will set a higher price when the goods are imperfect complements.

In other words, irrespective of whether the goods are imperfect substitutes or complements, more confident managers will set higher prices. We see that prices play the same role in the price competition scenario, as quantities did under quantity competition.

Comparing equilibrium quantities we see that

$$q_1^*(p_1^*, p_2^*) - q_2^*(p_1^*, p_2^*) = \frac{\alpha\gamma(k_1 - k_2)}{(1-\gamma)(4 - k_1 k_2 \gamma^2)},$$

implying that

$$\operatorname{sign}(q_1^* - q_2^*) = \operatorname{sign}(\gamma(k_1 - k_2)) = -\operatorname{sign}(p_1^* - p_2^*).$$

We not that the degree of self confidence affects equilibrium prices and demands in an opposite manner.

We have that $q_1^* > q_2^*$ if and only if

(a) $\gamma > 0$ and $k_1 > k_2$, i.e. the less self-confident manager of the pair will face a higher demand when the goods are imperfect substitutes,

(b) $\gamma < 0$ and $k_1 < k_2$, i.e. the less self-confident manager of the pair will face a higher demand when the goods are imperfect complements.

In other words, irrespective of whether the goods are imperfect substitutes or complements, more confident managers will face a lower demand than anticipated. Again, the duality with the quantity competition case is self-evident.

We have seen that, again, the managers' level of self-confidence moves the prices and quantities in opposite directions. Which effect will dominate? What is the overall effect on equilibrium profits?

The (true) equilibrium profit difference is

$$\pi_1^* - \pi_2^* = \frac{\alpha^2\gamma^2 (k_1 - k_2)\left(4 - k_1 - k_2 - k_1k_2\gamma - k_1k_2\gamma^2\right)}{(1-\gamma^2)(4 - k_1k_2\gamma^2)^2} \tag{24}$$

While in general terms it seems difficult to determine the sign of

$$(k_1 - k_2)\left(4 - k_1 - k_2 - k_1k_2\gamma - k_1k_2\gamma^2\right),$$

we can establish the following:

Proposition 12 *Suppose the goods are imperfect substitutes* ($\gamma > 0$) *and manager i's has self-confidence* $k_i \leq 1$, $i = 1, 2$, *and condition (23) is satisfied. Then in the unique interior Nash equilibrium of the price competition game, the following are equivalent:*

(a) $k_i < k_{-i}$

(b) $p_i^* > p_{-i}^*$

(c) $q_i^* < q_{-i}^*$

(d) $\pi_i^* < \pi_{-i}^*$

Proof. The proof follows easily from noting that in the relevant range, $4 - k_i - k_{3-i} - k_ik_{3-i}\gamma - k_ik_{3-i}\gamma^2 < 0$. ■

In other words, when the goods are imperfect substitutes, both managers are overconfident, then under price competition the less overconfident manager of the pair will set lower prices and earn higher profits. Similarly,

Proposition 13 *Suppose the goods are imperfect complements* ($-1 < \gamma < 0$) *and manager i's degree of self-confidence is* $k_i \geq 1$, $i = 1, 2$. *Then in the unique interior Nash equilibrium of the price competition game, the following are equivalent:*

(a) $k_i < k_{-i}$

(b) $p_i^* < p_{-i}^*$

(c) $q_i^* > q_{-i}^*$

(d) $\pi_i^* < \pi_{-i}^*$

In other words, when the goods are imperfect complements, both managers are overconfident, then the more overconfident manager will set a higher price and earn more profits.

Is there an optimal level of self-confidence for a manager in this set-up, in the sense of being robust to market selection forces? Before we can answer that question, we make the following observation.

Proposition 14 *Assume $\gamma \in \left(-\frac{2}{3}, 1\right)$. As a function of own confidence, k_i, the equilibrium profit π_i^* to manager i under price competition is a strictly concave function with a unique maximum at*

$$\hat{k}_i = \frac{4}{4 + 2k_j\gamma - k_j\gamma^2 - k_j^2\gamma^3} \tag{25}$$

Proof. See appendix. ■

In particular if $k_1 = 1$ and $\gamma = 0.6$ (-0.6) as in the preceding subsection, then $\hat{k}_2 = 0.866$ (1.506), in broad agreement with the above approximate observation.

More generally, (1) for $\gamma > 0$ (25) defines $\hat{k}_i$ as a number in $(0, 1)$; and (2) for $-\frac{1}{2} < \gamma < 0$ equation (25) defines $\hat{k}_i$ as a number in $(1, 1/|\gamma|)$. We henceforth assume that $\gamma \in \left(-\frac{1}{2}, 1\right)$ and turn to market selection and evolutionarily robustness analysis under price competition.

5.3. Market Selection: Evolutionarily Robust Self-Confidence

Consider again the setup from Section 3.3 but instead of quantity competition, assume the firms compete by setting prices. Also, assume that $-\frac{1}{2} < \gamma < 1$ and that $\gamma \neq 0$, $k_i \in K$, $i = 1, 2$, and that condition (23) is satisfied.

Consider a manager with self-confidence k' who is matched against a manager with self-confidence k. The profit to the first manager is

$$\pi^* = v(k', k) = \frac{\alpha^2 \left(2 - k'\gamma - k'k\gamma^2\right)\left(2 - (2 - k')\gamma - k\gamma^2\right)}{(1 - \gamma^2)(4 - k'k\gamma^2)^2}$$

Again, a degree of self-confidence k is evolutionarily robust if condition (13) holds. A robust degree of self-confidence k would need to satisfy equation (25) for $\hat{k}_i = k_j = k$. The unique degree of confidence with this robustness property,

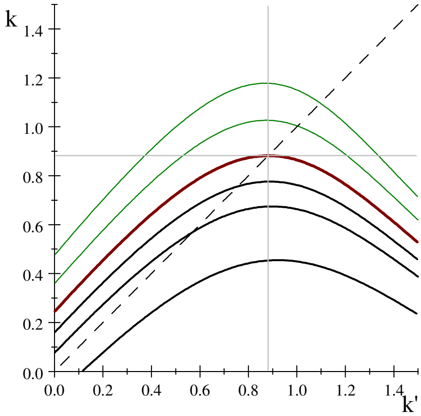

FIGURE 9. EVOLUTIONARILY ROBUST DEGREE OF SELF-CONFIDENCE WHEN THE GOODS ARE IMPERFECT SUBSTITUTES.

under price competition, is[17]

$$k^R = \frac{1}{2\gamma^2}\left(\gamma - \sqrt{4\gamma - 7\gamma^2 + 4} + 2\right).$$

For the case of substitutes, we illustrate this in Figure 9, showing the contour map of the function v, with k' on the horizontal axis and k on the vertical, when $\alpha = 1$ and $\gamma = 0.7$. Figure 10 illustrates the same for the case of complements, $\alpha = 1$ and $\gamma = -0.3$. A degree of confidence k^R is robust against market selection if and only if the tangent of the isoquant through the point (k, k)—the thick curve—is horizontal at that point, and no other point on the horizontal line through that point has a higher v-value. Inspecting the graph we conclude that this degree is approximately $k^R = 0.88$ when the goods are imperfect substitutes and $k^R = 1.26$ when the goods are imperfect complements (indicated by the vertical and horizontal lines).

Formally, we have:

Proposition 15 *Suppose* $\gamma \in \left(-\frac{1}{2}, 1\right)$. *The unique degree of confidence that is*

[17]Note that (25) has three fixed points: $-\frac{2}{\gamma}$, $\frac{\gamma+\sqrt{4\gamma-7\gamma^2+4}+2}{2\gamma^2}$ and $\frac{\gamma-\sqrt{4\gamma-7\gamma^2+4}+2}{2\gamma^2}$. The first two are outside of the interval $\left(0, \left|\frac{1}{\gamma}\right|\right)$.

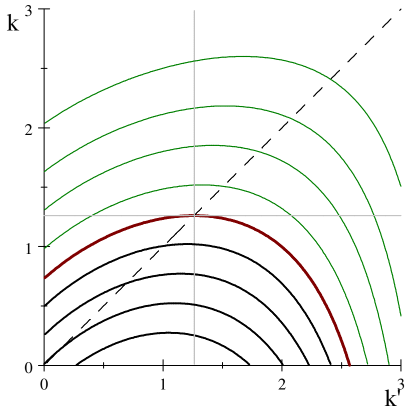

FIGURE 10. EVOLUTIONARILY ROBUST SELF-CONFIDENCE UNDER PRICE COMPETITION WITH IMPERFECT COMPLEMENTS.

robust to market selection driven by absolute profits under price competition is

$$k^R = \frac{1}{2\gamma^2}\left(\gamma - \sqrt{4\gamma - 7\gamma^2 + 4} + 2\right).$$

Proof. The proof follows from Proposition (14). ■

We illustrate this in the figure 11.

We note, that the evolutionarily robust self-confidence is again of the *overconfident* type, and this does not depend on whether the goods are imperfect substitutes or imperfect complements. Note, that when the goods are complementary (but not too complementary), then as in the case of quantity competition the degree of overconfidence is higher the more complementary the goods are.

When the goods are imperfect substitutes, the relationship between product differentiation and robust level of self-confidence is a U-shaped one.

5.4. Profit Comparison

When both managers in a duopoly have this evolutionarily robust degree of confidence, each firm's profit is

$$\pi^R = \frac{2\alpha^2\gamma\left(\gamma + \sqrt{4\gamma - 7\gamma^2 + 4} - 2\right)}{(1+\gamma)\left(3\gamma + \sqrt{4\gamma - 7\gamma^2 + 4} - 2\right)^2}.$$

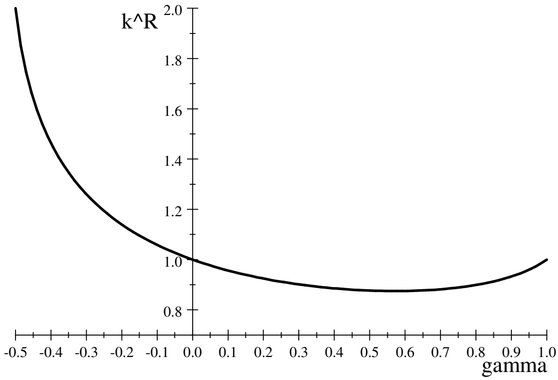

FIGURE 11. EVOLUTIONARILY ROBUST LEVEL SELF-CONFIDENCE UNDER PRICE COMPETITION.

When both managers in a duopoly instead are *homo oeconomicus*, then their profits are

$$\pi^* = \frac{\alpha^2 (1-\gamma)}{(1+\gamma)(2-\gamma)^2}.$$

The profit difference is

$$\pi^R - \pi^* = \frac{\alpha^2}{(1+\gamma)} \left(\frac{2\gamma\left(\gamma + \sqrt{4\gamma - 7\gamma^2 + 4} - 2\right)}{\left(3\gamma + \sqrt{4\gamma - 7\gamma^2 + 4} - 2\right)^2} - \frac{1-\gamma}{(2-\gamma)^2} \right),$$

which is negative if $\gamma < 0$ and positive if $\gamma > 0$. Hence, market selection tends to select overconfident managers and thereby reduce industry profit if the goods are complementary and increase industry profits if the goods are substitutable.

6. Conclusion

In this paper we examined the role of managerial self-confidence in product market competition. We have studied a market interaction between two firms in a differentiated goods duopoly run by owner-managers who have asymmetric beliefs ("confidence") about the degree of product differentiation. We have shown that under quantity competition, more confidence entails more aggressive play irrespective of whether the products are imperfect substitutes or imperfect complements. Subsequently, firms run by more confident managers gain a competitive edge when the goods are imperfect substitutes and the less confident managers gain a competitive edge when the goods are complementary. On the other hand, under price competition more confident managers gain a competitive edge when the goods are complementary and the less confident managers gain a competitive edge when the goods are substitutable. So, in essence overconfident managers get a competitive edge if the choice variables are strategic substitutes. However, the analysis also suggests that market selection forces will always favor some degree of managerial overconfidence, irrespective of whether the choice variables are strategic substitutes or strategic complements.

In this paper we modeled overconfidence as asymmetric beliefs about a particular parameter, the extent of horizontal product differentiation with symmetric firms. It would be interesting so see whether the conclusions reached in this paper would be robust to alternative models of self-confidence in an oligopoly setting, say about market size. It is also possible to envision a model with endogenous product differentiation, in which managers invest in horizontal product differentiation by exerting some effort, and that this could depend on both skill and luck. Then there are more dimensions to consider, and it would be interesting to look at the interaction of skill and various forms of overconfidence.

Appendix A

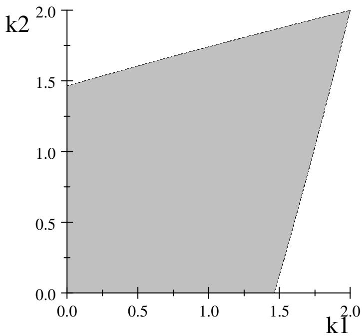

FIGURE 12. THE SET, WHERE CONDITION (23) IS SATISFIED, $\gamma = 0.5$

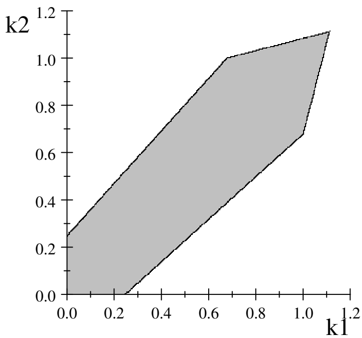

FIGURE 13. THE SET, WHERE CONDITION (23) IS SATISFIED, $\gamma = 9/10$.

Appendix B

Following are the proofs of Propositions 6 and 14.

Proof of Proposition 6. Recall that

$$\pi_i^*\left(k_i, k_j\right) = \frac{\alpha^2\left(2 - k_i\gamma\right)}{\left(4 - k_ik_j\gamma^2\right)^2}\left(\left(1 - k_i\right)k_j\gamma^2 + \left(k_i - 2\right)\gamma + 2\right) \text{ for } \forall k_i, k_j \in K,$$

where $K = (0, |1/\gamma|)$, $i = 1, 2$ and $j \neq i$. Differentiating $\pi_i^*\left(k_i, k_j\right)$ with respect to k_i

$$\frac{\partial\pi_i^*\left(k_i, k_j\right)}{\partial k_i} = \frac{\alpha^2\gamma^2\left(2 - k_j\gamma\right)}{\left(4 - k_ik_j\gamma^2\right)^3}\left(k_ik_j\gamma^2 + 2k_ik_j\gamma - 4k_j\gamma - 4k_i + 4\right)$$

and equating the resulting expression to zero, implies that

$$\hat{k}_i\left(k_j\right) = \frac{4\left(1 - k_j\gamma\right)}{4 - 2k_j\gamma - k_j\gamma^2}.$$

Furthermore,

$$\frac{\partial^2\pi_i^*\left(k_i, k_j\right)}{\partial k_i^2} = \frac{2\alpha^2\gamma^2\left(2 - k_j\gamma\right)}{\left(4 - k_ik_j\gamma^2\right)^4} \cdot \left(\begin{array}{c} 4\gamma k_j - 6\gamma^3k_j^2 + 8\gamma^2k_j - 4\gamma^2k_ik_j \\ +2\gamma^3k_ik_j^2 + \gamma^4k_ik_j^2 - 8 \end{array}\right).$$

The first factor is positive. Hence, if we can show that the last factor is negative everywhere on the relevant range, then we woud have that

$$\frac{\partial^2\pi_i^*\left(k_i, k_j\right)}{\partial k_i^2} < 0$$

and the proof would be complete. The derivative of the last factor with respect to k_i is

$$\gamma^2k_j\left(\gamma k_j\left(\gamma + 2\right) - 4\right) < 0 \text{ for } \forall k_j \in K,$$

so that the whole expression is decreasing in k_i. Therefore, it remains to show that

$$\begin{aligned} &4\gamma k_j - 6\gamma^3k_j^2 + 8\gamma^2k_j - 4\gamma^2k_ik_j + 2\gamma^3k_ik_j^2 + \gamma^4k_ik_j^2 - 8\Big|_{k_i=0} \\ &= -6\gamma^3k_j^2 + 8\gamma^2k_j + 4\gamma k_j - 8 \\ &= -2\left(3\gamma^3k_j^2 - 2\gamma k_j - 4\gamma^2k_j + 4\right) \\ &< 0, \end{aligned}$$

on the relevant range. First, consider the case when $0 < \gamma < 1$. Then we have that

$$\begin{aligned} 3\gamma^3 k_j^2 - 2\gamma k_j - 4\gamma^2 k_j + 4 &= \gamma^3 k_j^2 + 2\gamma^3 k_j^2 - 2\gamma k_j - 4\gamma^2 k_j + 4 \\ &= \gamma^3 k_j^2 - 2\gamma k_j \left(1 - \gamma^2 k_j\right) + 4\left(1 - \gamma^2 k_j\right) \\ &= \gamma^3 k_j^2 + 2\left(1 - \gamma^2 k_j\right)\left(2 - \gamma k_j\right) \\ &> 0. \end{aligned}$$

Second, consider the case when $-\frac{6}{7} < \gamma < 0$. Then $3\gamma^3 k_j^2 - 2\gamma k_j - 4\gamma^2 k_j + 4$ is a downward "facing" parabola with

$$\begin{aligned} 3\gamma^3 k_j^2 - 2\gamma k_j - 4\gamma^2 k_j + 4\Big|_{k_j=0} &= 4 > 0, \\ 3\gamma^3 k_j^2 - 2\gamma k_j - 4\gamma^2 k_j + 4\Big|_{k_j=-\frac{1}{\gamma}} &= 7\gamma + 6 > 0. \end{aligned}$$

Therefore $3\gamma^3 k_j^2 - 2\gamma k_j - 4\gamma^2 k_j + 4 > 0$ for $\forall\, k_j \in K$. This concludes the proof. ■

Proof of Proposition 14. Recall that,

$$\pi_i^*\left(k_i, k_j\right) = \frac{\alpha^2\left(2 - k_i\gamma - k_i k_j \gamma^2\right)\left(2 - (2 - k_i)\,\gamma - k_j\gamma^2\right)}{(1-\gamma^2)\left(4 - k_i k_j \gamma^2\right)^2} \text{ for } \forall k_i, k_j \in K,$$

where $K = (0, |1/\gamma|)$, $i = 1, 2$ and $j \neq i$. Differentiating $\pi_i^*\left(k_i, k_j\right)$ with respect to k_i

$$\frac{\partial \pi_i^*\left(k_i, k_j\right)}{\partial k_i} = \frac{\alpha^2\gamma^2\left(2 + k_j\gamma\right)}{(1-\gamma^2)\left(4 - k_i k_j\gamma^2\right)^3}\left(k_i k_j^2 \gamma^3 + k_i k_j \gamma^2 - 2k_i k_j \gamma - 4k_i + 4\right)$$

and equating the resulting expression to zero, implies that

$$\hat{k}_i = \frac{4}{4 + 2k_j\gamma - k_j\gamma^2 - k_j^2\gamma^3}.$$

Furthermore,

$$\frac{\partial^2 \pi_i^*\left(k_i, k_j\right)}{\partial k_i^2} = \frac{2\alpha^2\gamma^2\left(2 + k_j\gamma\right)}{(1-\gamma^2)\left(4 - k_1 k_2 \gamma^2\right)^4} \cdot \left(\begin{array}{c} 2\gamma^3 k_j^2 - 4\gamma k_j + 8\gamma^2 k_j - 4\gamma^2 k_i k_j \\ -2\gamma^3 k_i k_j^2 + \gamma^4 k_i k_j^2 + \gamma^5 k_i k_j^3 - 8 \end{array}\right).$$

The first factor is positive. Hence, if we can show that the last factor is negative everywhere on the relevant range, then we would have that

$$\frac{\partial^2 \pi_i^*\left(k_i, k_j\right)}{\partial k_i^2} < 0$$

and the proof would be complete. The derivative of the last factor with respect to k_i is (factoring out a positive multiplier)

$$\gamma^3 k_j^2 + \gamma^2 k_j - 2\gamma k_j - 4 = -\left(1 + k_j\gamma\right)\left(2 - k_j\gamma^2\right) - 2$$
$$< 0 \text{ for } \forall k_j \in K,$$

so that the whole expression is decreasing in k_i. Therefore, it remains to show that

$$\left(2\gamma^3 k_j^2 - 4\gamma k_j + 8\gamma^2 k_j - 4\gamma^2 k_i k_j - 2\gamma^3 k_i k_j^2 + \gamma^4 k_i k_j^2 + \gamma^5 k_i k_j^3 - 8\right)\Big|_{k_i=0}$$
$$= 2\left(\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4\right) < 0$$

on the relevant range. First, consider the case when $0 < \gamma < 1$. Then $\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4$ is an "upward" facing parabola in k_j with

$$\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4\Big|_{k_j=0} = -4 < 0,$$
$$\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4\Big|_{k_j=\frac{1}{\gamma}} = 5\gamma - 6 < 0.$$

Next consider the case when $-\frac{2}{3} < \gamma < 0$. Then $\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4$ is a "downward" facing parabola with roots

$$k_{j1} = \frac{1}{\gamma^2}\left(1 - 2\gamma - \sqrt{4\gamma^2 + 1}\right) \text{ and } k_{j2} = \frac{1}{\gamma^2}\left(1 - 2\gamma + \sqrt{4\gamma^2 + 1}\right).$$

Note that $k_{j1} = -\frac{1}{\gamma}$ when $\gamma = -\frac{2}{3}$ and we subsequently have that $0 < k_j < -\frac{1}{\gamma} < k_{j1} < k_{j2}$ for all $\gamma \in \left(-\frac{2}{3}, 0\right)$, which implies that $\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4 < 0$ for all $k_j \in K$. This completes the proof. ■

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Chapter 4

Gender Differences in Preferences at a Young Age? Experimental Evidence from Armenia*

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Abstract

We look at gender differences in competitiveness, risk preferences and altruism among a sample of 824 children and adolescents aged 8 to 16 in Armenia. Exploring four different competition tasks, girls are significantly more competitive in one task when it comes to performance change, and there are no gender differences in the other tasks or in the propensity to choose to compete. We find that girls are more altruistic and less risk taking than boys, and that the latter gap appears around the age of puberty. These results suggest that gender gaps in competitiveness are not always present.

Keywords: Gender differences, development of preferences, experiments with children, competitiveness, risk preferences, dictator game, altruism.

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1. Introduction

Although women in many countries are as likely as men to pursue higher education and participate in the labor market, men still dominate top positions in most sectors in most societies. A number of reasons for this have been proposed in the economics literature including taste based or statistical discrimination, as well as gender differences in preferences, with the focus on competitiveness, risk preferences and social preferences. Women are in general found to be less risk taking than men, and sometimes also less competitive and more altruistic (see, e.g., Bertrand, 2010; Croson and Gneezy, 2009; Eckel and Grossman, 2008a,b; Engel, 2011).

Meanwhile, relatively little is known about the development of the gender gaps in economic preferences, to what extent children and adolescents exhibit the same type of gender differences in preferences as adults do, and to what extent culture and context might matter. This paper aims to contribute to further this understanding.

In this paper we explore the gender gap in preferences among children and adolescents in Armenia. We focus on competitiveness, risk preferences and altruism since these are the three areas in which gender differences are often found. Competitiveness is typically measured by either the performance change in response to a competitive setting compared to a noncompetitive setting, or the choice of whether to compete or not when given the choice between a competitive setting and a noncompetitive setting. We study competitiveness using both measures in running, skipping rope, math and word search, as well as competitiveness at the choice whether to compete or not in math and word search. We study risk preferences by having the subjects choose between different certain amounts and a gamble. Finally, we study altruism through a dictator game where the recipient is a charity.

We find that boys and girls are equally competitive when it comes to performance change in skipping rope, math and word search, whereas girls are more competitive than boys in running. The latter result is different from what has previously been found among children. However, this difference is only present among older children in our sample. There is no gender gap in the choice to compete or not in math or word search. We find that boys are more risk taking than girls and that this gap appears around the age of 12. We also find that girls are significantly more altruistic than boys.

When it comes to competitiveness, most previous related studies find that if there is a gender gap in any of the measures, men and boys tend to be more competitive than women and girls (e.g. Gneezy et al., 2003; Gneezy and Rustichini, 2004b; Gupta et al., forthcoming; Niederle and Vesterlund, 2007; Sutter and Rutzler, 2010). However, the gap can be influenced by the sample in which competitiveness is studied, concerning both country and age group. Among adults, the impact of culture on the gender gap in competitiveness has been shown by Gneezy et al. (2009), who find that men are more competitive than women in a patriarchal society in Tanzania whereas this gender gap is reversed in a matrilineal society in India. The task performed also seems to matter. In some cases (Gneezy and Rustichini, 2004a; Gunther et al., 2009; Kamas and Preston, 2010; Niels and Reiner, 2010; Shurchkov, forthcoming) but not others (Wozniak et al., 2011) the gender gap in competitiveness among adults diminishes when the task performed is word related compared to, for example, solving mazes or simple math.

Among children, tasks and culture have also been shown to influence the size and existence of a gender gap in competitiveness. However, the results are somewhat mixed (Andersen et al., 2011; Booth and Nolen, 2011a; Sutter and Rutzler, 2010; Zhang, 2011). For example, Dreber et al. (2011b) find no difference in the gender gap between running, skipping rope and dancing, whereas Cardenas et al. (forthcoming) find some influence of the task but only in Sweden. When it comes to cultures, the results are also mixed. For example, Gneezy and Rustichini (2004b) find that boys but not girls are competitive when it comes to performance change in running in Israel, whereas Dreber et al. (2011b) find no gender gap in Sweden and Cardenas et al. (forthcoming) find no gender gap in Colombia or Sweden with this measure in the same task.

Studies on risk taking among adults typically find that women are less risk taking than men (Croson and Gneezy, 2009). When it comes to children, Harbaugh et al. (2002) find no gender gap, whereas Borghans et al. (2009); Dreber et al. (2011b); Sutter et al. (2010) and Cardenas et al. (forthcoming) find that boys are more risk taking than girls. As for competitiveness, there is some evidence that the gender gap in risk taking seems also to be influenced by the context or sample studied. Booth and Nolen (2011b) compare children around the age of 15 in single sex and mixed schools and find that boys are more risk taking than girls in mixed schools but that there is no gender gap when comparing boys to girls from single sex schools. Girls are also more risk taking when assigned to all-girl groups than when assigned to mixed groups.

When it comes to altruism, Engel (2011) performs a meta-analysis of the experiments on the dictator game among adults and finds that women are more altruistic than men. The results from studies on children are thus far mixed, where some find that girls are more altruistic (Dreber et al., 2011a; Gummerum et al., 2010; Harbaugh et al., 2003), some that there is no gender gap (Benenson et al., 2007; Blake and Rand, 2010), and one study finds that girls are less altruistic

(Fehr et al., 2011).[1]

Our results add to the literature on the importance of studying different samples, cultures and age groups when exploring the foundations of (gender differences in) economic preferences. Understanding the (development of) gender gaps in competitiveness, risk preferences and altruism will hopefully further our understanding of the gender gaps observed in important economic outcomes such as those related to the labor market (see, e.g., Bertrand, 2010, for further discussion). Armenia is different from most countries previously studied in that it was part of the Soviet Union and that perhaps the communist policies aimed at influencing the position of women in Armenia during the Soviet era had a long lasting and deep impact.

The rest of the paper, which reports the results of two large scale economic experiments, is organized as follows. We present the experimental setup and procedures in Section 2. Results for the competitiveness part of the study are presented in Section 3, for risk preferences in Section 4 and altruism in Section 5. We finish by a discussion in Section 6.

2. Experimental Design and Procedures

Our experiment was divided into two major parts. The experimental design and procedures closely follow to that of Cardenas et al. (forthcoming).[2] The experiments took place in two nearby secondary schools in the capital of Armenia, Yerevan, during a four week period in April–May 2010.[3] Overall 824 students aged 8 to 16 (in school years 2 to 10, 428 boys and 396 girls) participated in the study.

The first study took place during regular physical education classes with students in years 2 to 9 and used two experimental tasks: running and skipping/jumping rope.[4] Before the study began the subjects were told that they would participate in a series of tasks. In this part of the experiment, no extrinsic compensation was

[1]Even though there is no gender gap in the amount given in the dictator game in Blake and Rand (2010), girls are more likely than boys to give something compared to nothing.

[2]In particular, for younger children the experimental design and procedures are identical. This was chosen in order to facilitate potential future comparison with the data gathered in Colombian and Sweden.

[3]To help run the experiments, we recruited and trained four young female experimenters. For the first study, they all worked together as a team overseeing different tasks, each specializing in one or two tasks, i.e., running times were always taken by the same experimenter. For the second study (classroom part), the experimenters worked in teams of two.

[4]The students also participated in a cooperation task, but this is outside the scope of the current paper.

awarded and we relied on the intrinsic motivation that comes from winning a race or a competition, as in Gneezy and Rustichini (2004b) and Dreber et al. (2011b).

The subjects performed the tasks in a random order and in the presence of their classmates. Both the running and skipping rope tasks consisted of two rounds. In the first round, the subjects performed the task alone. In the second round the subjects performed the task in pairs. While performing the tasks in the first round the subjects were unaware of the existence of the second round. In the second round, after all the students had completed all first-round tasks, the students were matched with someone who had a similar performance to themselves in the first round. If more than two students obtained the same result in the first round, the matching was random. The students were made aware of the exact matching procedure. We also let a separate group of students perform the task alone in the second round to serve as control for unobservable characteristics that might differ between genders.

Performance in running was based on how fast the children ran 4 times 13 meters (for a total of 52 meters) and was the same for all the subjects. When competing in running in the second round, the students started at the same time and ran parallel to each other. The skipping rope task was different for younger and older children. In the skipping rope task, students in years 2 to 6 jumped with a long rope that one teacher or experimenter and one child turned and performance was measured by the number of jumps until the child first missed. Students in years 7 to 9 skipped a regular jumping rope for two minutes and performance was measured by the number of jumps in these two minutes. When competing in skipping rope in the second round, two ropes were put next to each other and the students were instructed to start jumping at the same time. Our main measure of competitiveness in running and skipping rope is the absolute change in performance between the first and second rounds, the most common measure of the reaction to competition.

The second study took place during regular class hours, in their own classrooms, with students in years 2 to 10. At the start of the experiment the students were told that they are taking part in an economic experiment and can earn prizes or money by earning points through various tasks and that more points would correspond to more prizes or more money. As an extrinsic motivation the younger children — students in years 2 to 6 — were rewarded with pens, while older subjects — students in years 7 to 10 — were rewarded with money. Information was always revealed sequentially as the experiment progressed and at no point in time did the subjects get any feedback about their performance at any stage.

The students started with either a math or a word search task. Examples of these tasks can be found in Appendix A. The order of these tasks was randomized for each class and year. Each of these tasks consisted of three main stages. Performance in both tasks was measured as the number of correct answers: the number of correctly solved problems in the math task and the number of correct words found in the word search task. The students were sequentially informed of the incentive structure of each stage, which was as follows.

Stage 1: Piece Rate. Students were asked to solve as many problems as possible in 2 minutes. They received 3 points for each correctly solved problem.

Stage 2: Tournament. Students were again asked to solve as many problems as possible in 2 minutes. They received 6 points for each correctly solved problem if they solved at least as many problems as a randomly selected student from their own class with whom they would be paired, otherwise they received 0 points.

Stage 3: Choice of Tournament or Piece Rate. Students were asked to choose either Option One or Option Two and then solve as many problems as possible in 2 minutes. If a student chose Option One, she would get 3 points per solved problem. If she chose Option Two, she would be randomly paired with another student in her own class and she would get 6 points per correctly solved problem if she solved at least as many problems as the other student, otherwise she would get 0 points as in Stage 2.

Comparing performance in the second stage with performance in the first stage gives us a measure of competitiveness as absolute performance change or reaction to competition, whereas the choice in the third stage gives as a measure of competitiveness as a preference for competition. In the last stage of each task, Stage 4, we asked the children to guess how many children they believed had performed better than they did for both the first stage (piece-rate scheme) and the second stage (forced tournament).[5]

After the math and word search tasks the students participated in a risk task. The risk task consisted of six choices where the students could choose between a 50/50 gamble, a lottery in the form of a coin flip that gave 10 or 0 points and a safe option, where the certain amount increased successively in points, from 2 to 7.5. Our first measure of risk preferences relies on the unique switching point where the subject switches from preferring the gamble to preferring the safe option. This measure excludes inconsistent subjects, i.e. those with multiple switching points. Since some of our subjects are inconsistent in their choices, as in most other studies with similar measures, we also employ an alternative measure of

[5] Since we do not observe differential gender selection into competitive environments, performance beliefs, or under and overconfidence, are not explored in this version of the paper.

risk preferences in order not to exclude the inconsistent subjects. This alternative measure is defined by the number of times an individual chooses a risky option, the gamble, over a safe option.

Next, the students took part in a dictator game, where they were asked to allocate 100 experimental points between themselves and a well known charity organization, Zeitoun Orphanage in Yerevan. The children were informed that the points they allocated to the charity would be converted to gifts and money and sent by the experimenters to the orphanage at the end of the study. The amount that a student donates to the charity is our measure of altruistic behavior.

Finally, at the end of the study, the students were administered an "exit" survey in order to measure different beliefs concerning the different tasks, motivation, cooperation, competition as well as collect some demographics.[6] After the experiment was over the experimenters collected and corrected all the papers and the students received their rewards.

Summarizing, the experimental design allows as to analyze (1a) competitiveness as performance change in running, skipping rope, math and word search, (1b) competitiveness as preference for competition in math and word search, (2) risk preferences through incentivized choices over lotteries and safe options, and (3) altruistic behavior via a dictator game.

3. Results: Competition

In this section we test whether there is a gender gap in competitiveness among children and adolescents in Armenia and whether the nature of the experimental task affects the size and direction of the gender gap. We start by looking at gender differences in competitive behavior in the running and skipping rope tasks. Then we address the effect of the gender composition in the competitive setting in these same tasks and present a robustness test. Thereafter we turn to looking at gender differences in competitive behavior in math and word search tasks.

Throughout the paper all tests of the differences in means are analyzed using the nonparametric Wilcoxon-Mann-Whitney U test and a two sided t test. Only the $p-$values for the Mann-Whitney U tests are reported unless otherwise noted.[7] All the error bars in the graphs are 95% confidence intervals based on a normal

[6] In this classroom part of the study the students also participated in a public goods game and a time preference task, but analysis of these data are outside the scope of the current paper and will appear in a future paper.

[7] We have also compared whether the distributions for each reported variable differ between boys and girls using a Kolmogorov-Smirnov test. The results are similar to those reported for mean values, unless otherwise noted.

distribution.[8]

3.1. Competitiveness in Running and Skipping Rope

For the analysis in this subsection and for descriptive purposes, we find it useful to divide the whole sample into three age groups, based on the year in school: years 2 to 4; years 5 to 6; and years 7 to 9.[9] We will refer to these samples as the younger, middle, and older samples, respectively. As noted before the skipping rope task was different for the older sample. This division allows us to compare performance distributions between genders and also across age more conveniently. Throughout the analysis, we also report results based on the whole sample.

3.1.1. Competitiveness in the Running Task

In the running and skipping rope tasks we had two groups of school children performing the task (competitive and control treatments). Since all conditions were identical in the first round, we can pool the outcomes to test for a gender difference in speed. Figure 1 presents a summary plot of the distribution of running times by gender and by age group is presented. We find that in the individual setting, in all the samples, boys ran on average faster than girls. The $p-$values for a significant gender difference are less than 0.001, see Table 1.[10] We note that this result is different from Gneezy and Rustichini (2004b) and similar to Dreber et al. (2011b).

When it comes to competitiveness, Table 2 shows that average performance in the competitive Round 2 differs significantly from performance in the noncompetitive Round 1 (Wilcoxon matched pairs signed-rank test, henceforth SR test in the tables). In all the samples both genders improve their performance significantly in running.

Figure 2 shows the distribution of the performance change in running for all the samples under study. In the whole sample, there are significant gender differences ($p < 0.001$): girls compete more. This result is mainly driven by older girls. In the younger sample of school children this gender difference is not significant ($p = 0.257$). The $p-$value for a significant gender difference in the middle and older samples of school children are 0.012 and 0.002, respectively.

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[8] We should note that none of the relevant variables are normally distributed when employing standard tests of normality, e.g., Shapiro-Wilk and Skewness/Kurtosis tests.

[9] Note that students in year 10 did not participate in this part of the study.

[10] Additional regression analysis of time in the first round on age and a female dummy shows that girls ran on average 1.62 seconds slower compared to boys, and an additional year in age resulted in 0.49 seconds of improvement in speed.

TABLE 1 – SPEED IN THE FIRST ROUND

Age Group	Boys	Girls	All	M–W test (p−value)
Years 2 to 4	17.76 (1.68) 136	19.14 (1.60) 113	18.39 (1.78) 249	< 0.001
Years 5 to 6	16.80 (1.16) 63	18.47 (1.42) 69	17.68 (1.54) 132	< 0.001
Years 7 to 9	15.03 (1.50) 124	16.74 (2.08) 122	15.88 (2.01) 246	< 0.001
All	16.53 (1.96) 323	18.03 (2.08) 304	17.25 (2.15) 627	< 0.001

Notes: The table shows the means as the main number and the standard deviations in parentheses. Number of observations are below the means.

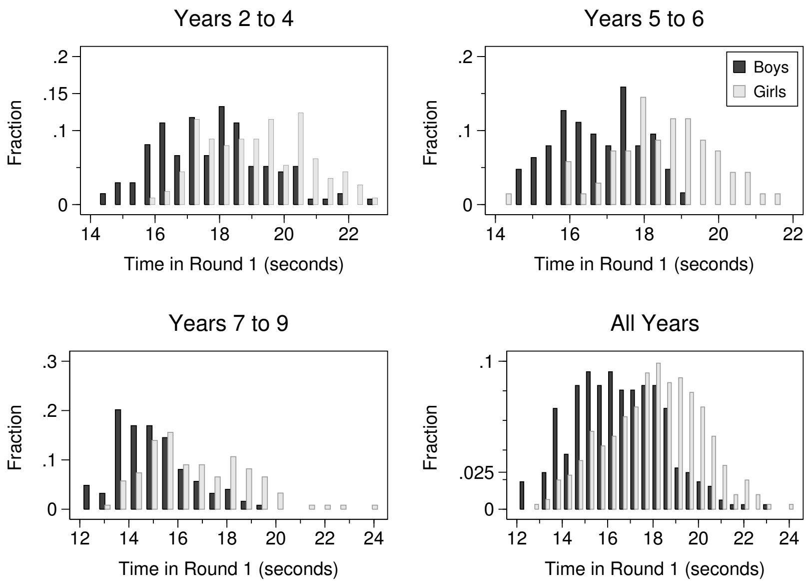

FIGURE 1. DISTRIBUTION OF RUNNING TIMES IN THE FIRST ROUND, BY GENDER AND AGE GROUP

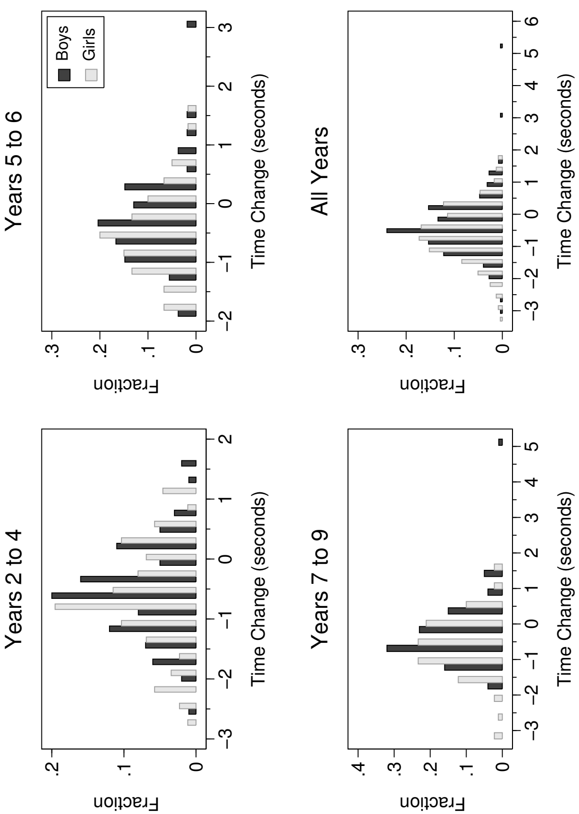

FIGURE 2. DISTRIBUTION OF RUNNING TIME CHANGES, BY GENDER AND AGE GROUP

TABLE 2 – PERFORMANCE IN THE RUNNING TASK (SECONDS)

Age Group	Sex	Round 1	Round 2	SR test (p−value)	Number of observations
Years 2 to 4	Boys	17.67	17.14	< 0.001	100
	Girls	19.27	18.60	< 0.001	87
Years 5 to 6	Boys	16.89	16.67	0.005	54
	Girls	18.50	17.93	< 0.001	60
Years 7 to 9	Boys	15.13	14.92	0.001	100
	Girls	16.94	16.27	< 0.001	91
All	Boys	16.50	16.17	< 0.001	254
	Girls	18.19	17.54	< 0.001	238
	Total	17.32	16.83	< 0.001	492

Even though we find no significant gender difference in mean change in performance in the sample of younger children, there may be difference in the variances of the performance distributions. We test this and find no significant difference in the variance of change in performance between boys and girls in this sample. This is also true for all the other samples.[11]

The pattern for gender similarities and differences for different age groups are displayed in an aggregated manner in Figure 3. The plot shows the average change in performance by each gender within each sample. Girls on average improved 0.68 seconds in the younger sample, 0.57 seconds in the middle sample, 0.66 seconds in the older sample, and 0.64 seconds overall, or about 3.33%, 3%, 3.82%, and 3.43%, respectively. This can be compared to the average improvement for boys. Boys on average improved 0.52 seconds in the younger sample, 0.21 seconds in the middle and older samples, and 0.33 seconds overall, or about 2.86%, 1.26%, 1.3%, and 1.91%, respectively. As stated, the difference in average change in performance between boys and girls is not statistically significant only in the younger sample. The results are qualitatively the same for average relative change in performance.[12]

[11] Since the most common test for comparison of standard deviations, the F−test for homogeneity of variances, relies on the assumption that the data are drawn from an underlying normal distribution and since none of our relevant variables are normally distributed, we also performed a robust test (Levene's test with mean, median and 10% trimmed mean). None of these tests indicated significant difference in the variances of the distributions.

[12] Average relative change in performance is measured as ((performance in round 2 – performance in

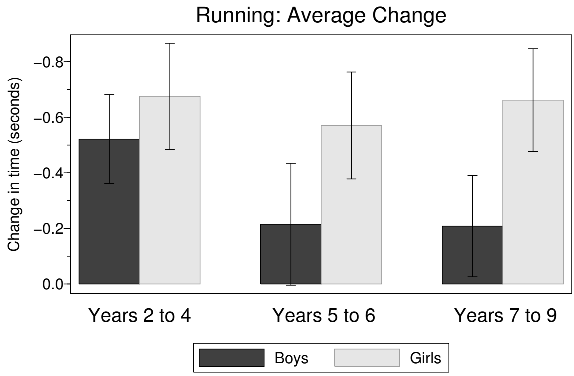

FIGURE 3. AVERAGE CHANGE IN TIME, BY GENDER AND AGE GROUP

3.1.2. Competitiveness in the Skipping Rope Task

Since all the conditions are again identical in the first round we can pool the data to test for a gender difference in skipping rope. A summary plot of the distribution of number of jumps by gender and for each sample is presented in Figure 4. We find that in the individual setting, girls skip rope better than boys do, like in Dreber et al. (2011b). The $p-$value for a significant gender difference is 0.007, see Table 3. However, this gender difference is not significant in our sample of younger school children. It turned out that the younger children were not familiar with the task and had difficulty to perform it. Skipping rope used to be a common activity for young children in Armenia, but this is no longer true. Also, remember that the skipping rope task was not exactly the same for older and younger children, hence the much higher performance of the older children.

When it comes to competitiveness, Table 4 shows that average performance in the competitive setting (Round 2) differs significantly from average performance in the noncompetitive setting (Round 1) only in the older sample. Again, this could be due to the fact that the younger children had difficulty to perform an unfamiliar task.

Figure 5 shows the distribution of performance change in skipping rope for all the three age groups and overall. There are no significant gender differences in the average change in performance in any of the samples under consideration (p = 0.941, p = 0.724, and p = 0.527, respectively). However, the variance of change in performance is significantly higher for girls in the younger sample (Levene's

round 1)/performance in round 1).

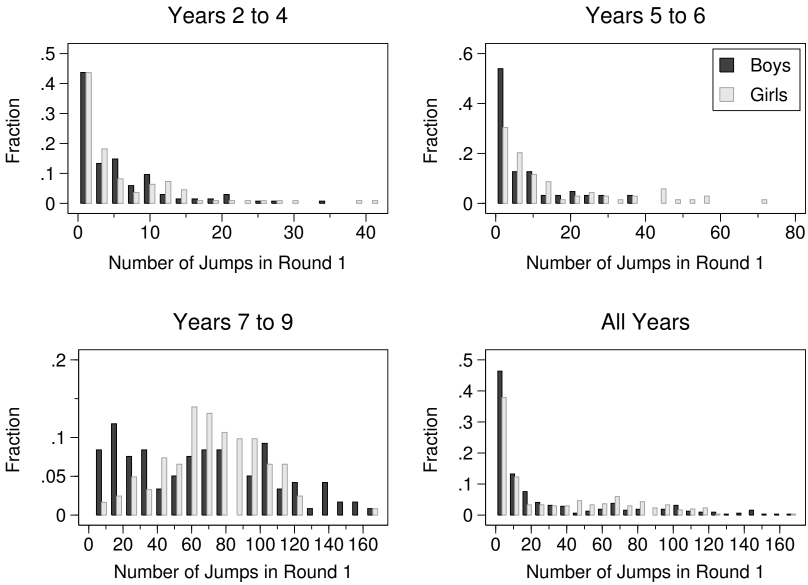

FIGURE 4. DISTRIBUTION OF JUMPS IN THE FIRST ROUND

TABLE 3 – NUMBER OF JUMPS IN THE FIRST ROUND

Age Group	Boys	Girls	All	M–W test (p−value)
Years 2 to 4	5.31 (6.23) 135	6.17 (7.87) 110	5.70 (7.01) 245	0.565
Years 5 to 6	7.24 (9.39) 63	14.64 (17.30) 69	11.11 (14.52) 132	0.013
Years 7 to 9	64.96 (42.93) 119	71.17(28.69) 122	68.10 (36.48) 241	0.087
All	28.09 (39.27) 317	34.46 (36.80) 301	31.19 (38.19) 618	0.007

Notes: The table shows the means as the main number and the standard deviations in parentheses. Number of observations are below the means.

TABLE 4 – PERFORMANCE IN THE SKIPPING ROPE TASK

Age Group	Sex	Round 1	Round 2	SR test (p−value)	Number of observations
Years 2 to 4	Boys	5.14	6.52	0.084	99
	Girls	4.68	6.16	0.126	85
Years 5 to 6	Boys	7.35	6.17	0.539	54
	Girls	13.62	8.47	0.255	60
Years 7 to 9	Boys	67.10	72.27	< 0.001	96
	Girls	70.92	79.45	< 0.001	88
All	Boys	29.66	31.79	< 0.001	249
	Girls	32.50	34.43	< 0.001	233
	Total	31.04	33.07	< 0.001	482

test, $p = 0.033$ at the median) and older sample (Levene's test, $p = 0.086$ at the median) but this difference also vanishes when we look at the variance of relative change in performance.

The pattern for gender similarities for different year groups are displayed in an aggregated manner in Figure 6. The plot shows the average change in performance by each gender within each sample. Girls on average improved by 1.48 jumps in the younger sample, deteriorated by 5.15 jumps in the middle sample, and improved by 8.93 jumps in the older sample, or about 111%, 49%, and 22.6% respectively.[13] This can be compared to the average improvement for boys. Boys on average improved by 1.37 jumps in the younger sample, deteriorated by 5.15 jumps in the middle sample, and improved by 5.51 jumps in the older sample, or about 107%, 113%, and 18.5%, respectively. As stated, the difference in average and relative changes in performance between boys and girls is not statistically significant in any of the three samples.

3.1.3. Impact of Opponent Gender on Competitive Behavior

Since the children could observe each other while performing in the second round, the gender of the opponent is known in both running and skipping rope. We find that both boys and girls compete significantly more against girls with the excep-

[13]Note, that this relative improvement measure ignores those subjects who made 0 jumps in the first round. 31 boys out of 198 and 23 girls out of 176 in years 2 to 6 performed 0 jumps.

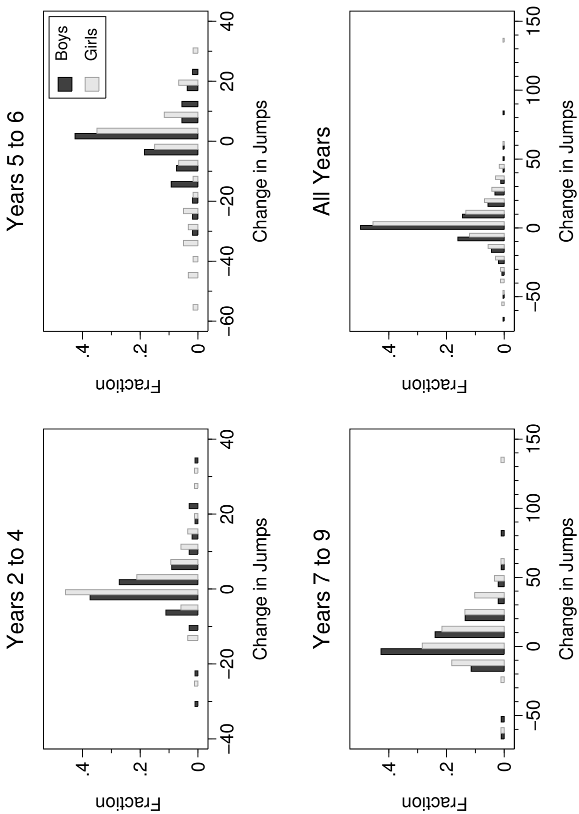

FIGURE 5. DISTRIBUTION OF CHANGES IN JUMPS, BY GENDER AND AGE GROUP

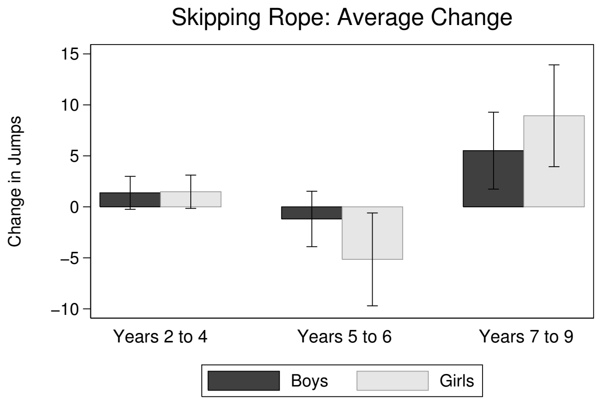

FIGURE 6. AVERAGE CHANGE IN JUMPS, BY GENDER AND AGE GROUP

tion of the younger sample, in which we found no gender difference in reaction to competition. Table 5 gives an overall summary for the whole sample for the different pair compositions in our study for the running task and Table 6 presents the same information for the skipping rope task.

In running, both boys and girls improve the most when competing against a girl. This is also true for any of the samples under investigation. This difference in competitive behavior when facing the same vs. the opposite gender is statistically significant for both girls ($p = 0.006$) and boys ($p = 0.008$). However, as before, these significant differences are driven mainly by the older school children. In the younger and middle samples, the difference in competitive behavior when facing the same vs the opposite gender is statistically insignificant for both boys and girls ($p = 0.389$ and $p = 0.073$ for boys and girls, respectively in the combined sample of children in years 2 to 6).

In skipping rope, again both boys and girls improve the most when competing against a girl, but none of these differences is statistically significant in any of the samples for either gender.

Note that the extant research on the opponent gender effect is mixed. For example, among adults Gupta et al. (forthcoming) and Gneezy et al. (2003) find that women compete more against women and men more against men. Among children, on the contrary, Gneezy and Rustichini (2004b) find that boys are not affected by the gender composition of the competing pairs but girls compete more against boys, while Dreber et al. (2011b) find that neither boys nor girls are influenced by the gender of their opponent.

Table 5 – Running: Performance Change Based on the Gender Composition of the Competing Pairs

Sample	Number of Obs.	Change in Time	Standard Error
Total	490	-0.484	0.039
Total Boys	253	-0.335	0.054
Total Girls	237	-0.644	0.055
Boys with boys	162	-0.248	0.066
Girls with girls	143	-0.764	0.073
Boys in mixed pairs	91	-0.489	0.092
Girls in mixed pairs	94	-0.461	0.083

Table 6 – Skipping Rope: Performance Change Based on the Gender Composition of the Competing Pairs

Sample	Number of Obs.	Change in Jumps	Standard Error
Total	479	2.499	0.739
Total Boys	246	2.415	1.340
Total Girls	233	2.588	1.206
Boys with boys	129	2.116	1.090
Girls with girls	116	2.673	1.371
Boys in mixed pairs	117	2.744	1.340
Girls in mixed pairs	117	2.603	1.996

3.1.4. Robustness Check for the Running and Skipping Rope

As stated in the beginning of this section, we also let a separate group of children perform the task alone in the second stage, serving as a control group. We thereby control for unobservable factors that could cause differences in the outcome, such as one gender exerting more effort initially and getting tired faster than the other. The control group includes 135 subjects in the running task (69 boys and 66 girls) and 132 children in the skipping rope task (67 boys and 65 girls).

Children performing the tasks alone in the first and second rounds showed, on average, either no improvement or a slight improvement in the second round, depending on the task and the sample under investigation. Most importantly though, the difference in change of performance between genders in not significant in any of the tasks or samples.

We end this subsection by summarizing the results on competitiveness in running and skipping rope. When measuring competitiveness as a performance reaction to a competitive setting we found that girls compete more in running, but there is no gender gap in skipping rope. We also found significant opponent effects in running: both girls and boys improve their performance the most when competing against a girl. However, we should also note that the gender gap in running for the younger children was not significant.

3.2. Competitiveness in Math and Word Search

For the analysis in this subsection, we find it useful to divide the whole sample into three different age groups, based on the year in school: years 2 to 5; years 6 to 7; and years 8 to 10. We will refer to these samples as the younger, middle, and older samples, respectively. Note that this division differs slightly from the division in the previous subsection. This division is based on the premise that the level of the difficulty of the math task, and hence performance, was the same within each of the three samples. Therefore comparing and interpreting differences in performance between genders is more straightforward in this manner. Throughout the analysis, we also report results based on the whole sample.

3.2.1. Competitiveness in Math and Word Search: Performance Change

Table 7 shows the average performance of boys and girls in Stage 1 (piece rate) and Stage 2 (compulsory tournament) of the math task. Generally, we see that boys are slightly better in the math task than girls. However, this gender difference in performance is not statistically significant in both stages ($p-$values for the first stage are 0.311, 0.298, 0.151, and 0.102 for the younger, middle, older, and the

TABLE 7 – PERFORMANCE IN THE MATH TASK

Age Group	Sex	Stage 1 (piece rate)	Stage 2 (tournament)	SR test ($p-$value)	Number of observations
Years 2 to 5	Boys	19.64	22.06	< 0.001	177
	Girls	18.50	21.75	< 0.001	156
Years 6 to 7	Boys	11.19	15.88	< 0.001	95
	Girls	12.35	17.24	< 0.001	100
Years 8 to 10	Boys	6.03	7.46	< 0.001	116
	Girls	4.91	6.39	< 0.001	121
All	Boys	13.50	16.18	0.018	388
	Girls	12.50	15.62	0.001	377

whole sample, respectively).[14]

In all the samples, both boys and girls are competitive in math in terms of reacting to competition (Wilcoxon matched-pairs signed rank tests, see Table 7). The increase from Stage 1 to Stage 2 is perhaps not only due to the higher incentives under the tournament scheme, but also to some learning going on. Figure 7 shows the distribution of performance change in math for boys and girls in all the samples. The figure shows that there are no gender differences in any of the distributions ($p-$values are 0.158, 0.703, 0.725, and 0.195 for the younger, middle, older samples, and the whole sample, respectively).[15] The pattern for gender similarities for different age groups are displayed in an aggregated manner in Figure 8. The plot shows the average change in performance by each gender within each age group.

Table 8 shows the average performance of boys and girls in Stage 1 (piece rate) and Stage 2 (compulsory tournament) of the word search task. Generally, we see that girls are better in word search than boys. This gender difference in performance is statistically significant in both stages for the whole sample (p = 0.001 and p = 0.003 in the first and second stages, respectively).[16]

[14]When we look at gender differences in Stage 1 performance within each year, we find some evidence that boys in year 10 are significantly better than girls in math. However, this sample size is quite small (41 subjects: 24 girls and 17 boys).

[15]In the sample of older students boys have a significantly higher variance in performance change using the robust test, but the difference vanishes when we look at the distribution of relative performance change. Also boys in year 10 appear to be significantly more competitive than girls.

[16]When we look at gender differences in Stage 1 performance within each year, we find that there is no gender gap in ability in word search only in year 10. However, we should keep in mind that this

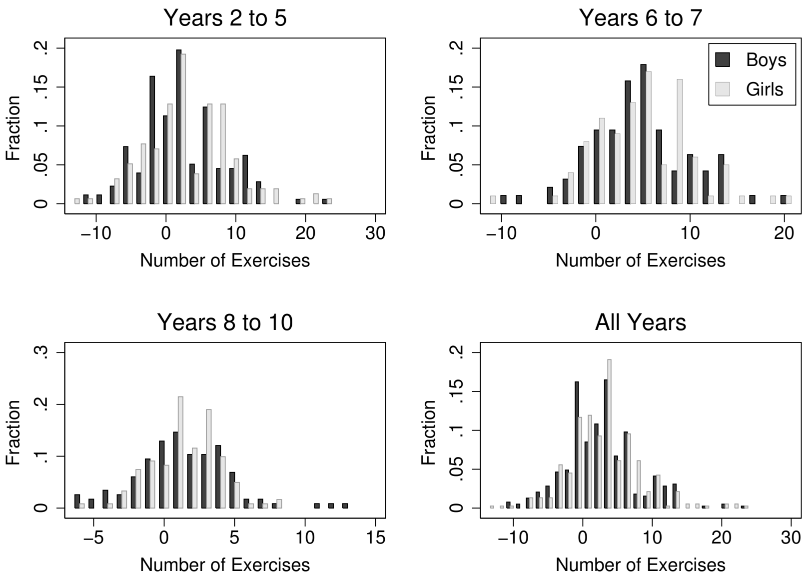

FIGURE 7. DISTRIBUTION OF PERFORMANCE CHANGE IN THE MATH TASK

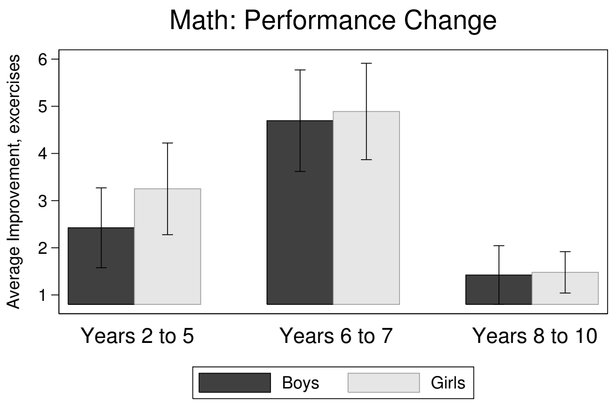

FIGURE 8. AVERAGE CHANGE IN MATH PERFORMANCE, BY GENDER AND AGE GROUP

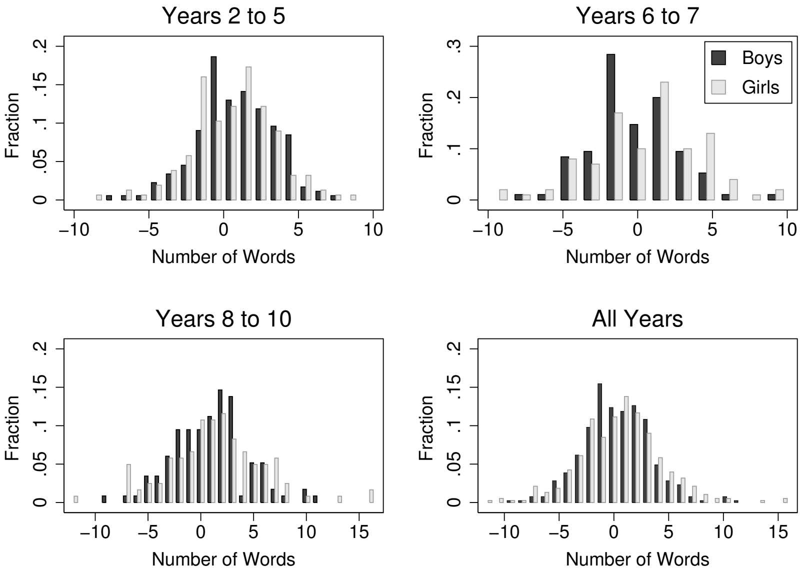

FIGURE 9. DISTRIBUTION OF PERFORMANCE CHANGE IN THE WORD SEARCH TASK

When it comes to reacting to competition both boys and girls are competitive in word search (Wilcoxon matched-pairs signed rank tests, see Table 8). Figure 9 shows the distribution of performance change in word search for boys and girls in all the samples. The histograms show that there are no gender differences in any of the distributions ($p-$ values are $0.418, 0.716, 0.415$, and 0.324 for the younger, middle, older samples and the whole sample, respectively). This result is also robust to using a relative performance change as the measure of competitiveness. The pattern for gender similarities for different age groups are displayed in an aggregated manner in Figure 10. The plot shows the average change in performance by each gender within each age group in the word search task.

3.2.2. Competitiveness in Math and Word Search: Choice

In Stage 3 of math and word search tasks we let the subjects choose their compensation scheme. We find that boys and girls are equally likely to choose to compete in math and word search (math: $p = 0.497$, word: $p = 0.548$; one sided

sample is quite small (41 subjects: 24 girls and 17 boys).

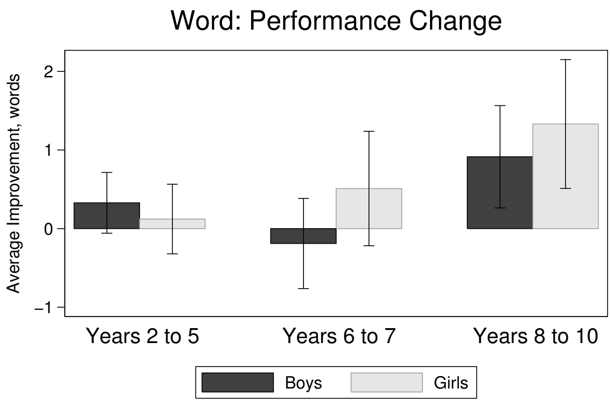

FIGURE 10. AVERAGE CHANGE IN WORD PERFORMANCE, BY GENDER AND AGE GROUP

TABLE 8 – PERFORMANCE IN THE WORD SEARCH TASK

Age Group	Sex	Stage 1 (piece rate)	Stage 2 (tournament)	SR test (p−value)	Number of observations
Years 2 to 5	Boys	4.91	5.24	0.068	177
	Girls	5.51	5.63	0.542	156
Years 6 to 7	Boys	7.31	7.12	0.385	95
	Girls	8.65	9.16	0.109	100
Years 8 to 10	Boys	9.78	10.69	0.008	116
	Girls	10.70	12.03	0.001	121
All	Boys	6.95	7.33	0.018	388
	Girls	8.01	8.62	0.001	377

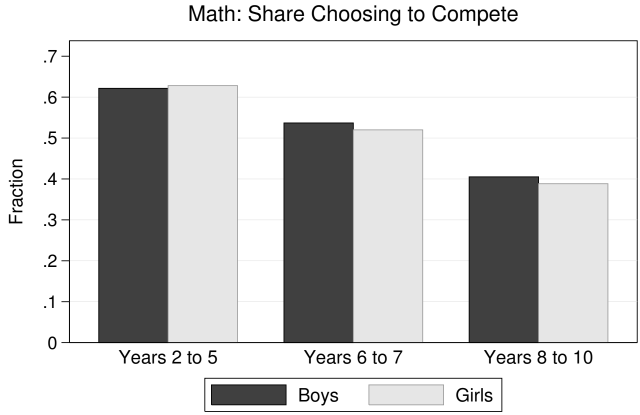

FIGURE 11. MATH: SHARE CHOOSING TO COMPETE

Fisher's exact test for the whole sample). 53.61% of boys and 52.25% of girls chose to compete in math, while 57.36% of boys and 55.97% of girls chose to compete in word search. Figures 11 and 12 shows the entry rates of girls and boys into the tournament payment scheme in Stage 3 of the experiment for each of the age groups. Within any of the samples there are no gender differences in competitiveness in terms of choice.

The only pattern that emerges is that in the math task younger children chose to compete significantly more than older ones. 62.46% of children in the younger sample chose to compete, compared with 52.82% in the middle, and 39.66% in the older samples respectively. These differences between each age group, and for each gender between each age group are very significant using a χ^2 test. These differences could be explained by the difficulty of the task. The level of the math task was significantly more difficult for the older sample than for the younger sample (see Appendix A).[17] To summarize this subsection on competitiveness in math and word search, we found no gender gaps in competitiveness measured as either performance change in reaction to competition or by preference to enter into a competitive environment in either task.

[17] Additional regression analysis for choice in math and word search tasks reveals no significant gender differences. The only significant variables affecting probability of choosing to compete are ability and risk aversion, and also age in the math task only.

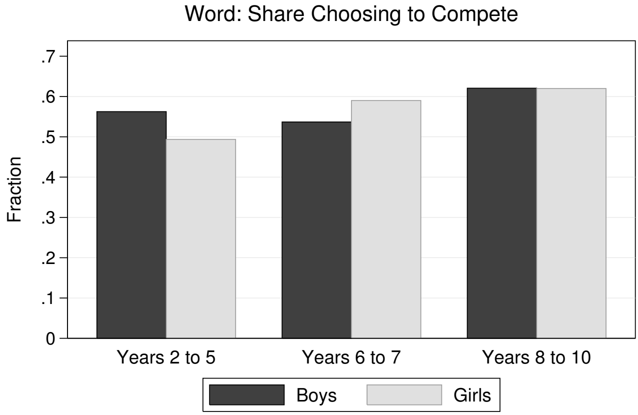

FIGURE 12. WORD: SHARE CHOOSING TO COMPETE

3.3. Additional Analysis

At the end of the experiment we administered an "exit" survey, in which, among other things, we elicited perceptions of how boyish/girlish the subjects considered running, skipping rope, math and word search to be. We further asked how boyish/girlish they considered competing in these tasks to be. We used an eleven point scale from 0 to 10 where a lower number indicates rating the task as more girlish and a higher number as more boyish (0 corresponded to being very girlish, 5 corresponded to being gender neutral, 10 corresponded to being very boyish). We used a similar scale to elicit how important the subjects considered competing against a boy and against a girl to be (with 0 being not at all important and 10 being very important).

3.3.1. Do Subjects Perceive Competing to be Important?

On average, boys rate competing as more important than girls. However, this gender difference is not statistically significant ($p = 0.231$).[18] Boys believe it is more important to compete against a boy than against a girl ($p < 0.001$ in all age groups and overall). Girls believe it is more important to compete against a boy than against a girl, but the difference in ratings is not significant ($p = 0.079$). Interestingly enough, this does not correspond to what we observe in

[18]However, in the sample of students in years 8 to 10 there is some evidence of significant gender difference: $p = 0.026$ for a t-test, $p = 0.059$ for Mann-Whitney U test, and $p = 0.850$ for Kolmogorov-Smirnov test.

actual performance change in running and skipping rope. In both tasks, both boys and girls change their performance more when competing against a girl, see Tables 5 and 6.

3.3.2. Do Subjects Perceive the Tasks to be Gendered?

Boys perceive running to be significantly more boyish than girls do ($p < 0.001$), but the gap in ratings narrows with age. Boys and girls perceive skipping rope differently. Boys rate skipping rope as more gender neutral, while girls consider it to be more girlish and the difference in ratings is statistically significant ($p < 0.001$). When it comes to math boys consider it to be more boyish, while girls consider it to be more gender neutral ($p < 0.001$), and the gap in rating somewhat narrows with age. When it comes to word search, both boys and girls perceive it to be closer to gender neutral but still differ in their opinions ($p < 0.001$).

4. Results: Risk Preferences

In this section we test whether there are gender differences in risk preferences among children and adolescents in Armenia. We look at the gender gap in risk preferences measured from incentivized lotteries in the classroom.

For the analysis in this section (and the next), we again find it useful to divide the subjects into roughly three equally sized groups based on age and year in school: years 2 to 4 (average age just under 9 years old); years 5 to 7 (average age of around 12 years old); and years 8 to 10 (average age just under 15 years old). We will refer to these groups as the younger, middle, and older samples, respectively.

Our main measure of risk taking relies on the unique switching point when subjects switch from preferring the safe option to a lottery. We measure risk taking as the certainty equivalent at the switching point, and take it to be the midpoint of the switching interval. In our sample of 762 schoolchildren, 18.63% of the subjects are inconsistent in their choices of the safe option versus the lottery (coin flip). However, most importantly, there is no gender difference in being inconsistent in the whole sample ($p = 0.274$, equality of proportions test) or any of the subsamples. There is some evidence though that the share of inconsistent subjects is disproportionately higher among younger children. Therefore, we also measure risk preferences in terms of the number of risky options chosen, in order not to exclude those subjects with inconsistent choices. Using this outcome measure, the results are qualitatively similar to those presented here and are relegated to an appendix, see Appendix B.

TABLE 9 – SUMMARY OF RISK MEASURES

Variable	Sample	Mean	St. Dev.	Median	Min	Max	*N*
Risk	Boys	4.75	2.62	4.5	1	8.75	309
	Girls	3.97	2.36	3.5	1	8.75	311
	Total	4.36	2.52	4.5	1	8.75	620
Inconsistent	Boys	0.20	0.40	0	0	1	387
	Girls	0.17	0.38	0	0	1	375
	Total	0.19	0.39	0	0	1	762
Risky choices	Boys	3.02	1.94	3	0	6	387
	Girls	2.50	1.81	2	0	6	375
	Total	2.76	1.89	3	0	6	762

Table 9 presents summary statistics for the risk measures. In the aggregate, we find significant risk aversion in our sample, with a mean (median) measure of risk of 4.36 (4.5) ($p < 0.001$, Wilcoxon signed-rank test, testing whether it is different from 5). We find that boys take 16 percent more risk than girls do ($p < 0.001$). Figure 13 shows the raw distribution of sure amounts at which the subjects started to prefer the safe option by each gender for each of the samples.[19] The distribution of certainty equivalents will look visually the same. The histograms reveal that the distributions for risk preferences look markedly different for different age groups. The distribution for the older sample is remarkably different from the distribution of the younger sample.[20]

We find a gender gap in risk taking in all the age groups, with boys being more risk taking. However, the information presented in Table 10 and the histograms in Figure 13 clearly demonstrate that the gender gap in risk taking is not statistically significant in the younger sample, and that we cannot reject the hypothesis that the distributions of risk preferences for boys and girls are the same in this age group (mean age 8.83). On the other hand, while there is mixed evidence that the gender gap in the middle sample is statistically significant, in the sample of older

[19]The sure amount is coded as 10 for the subjects who always chose the risky option, and as 2 for the subjects who always chose the safe option.

[20]Perhaps younger children have difficulty in processing information about chance, or simply have different risk preferences.

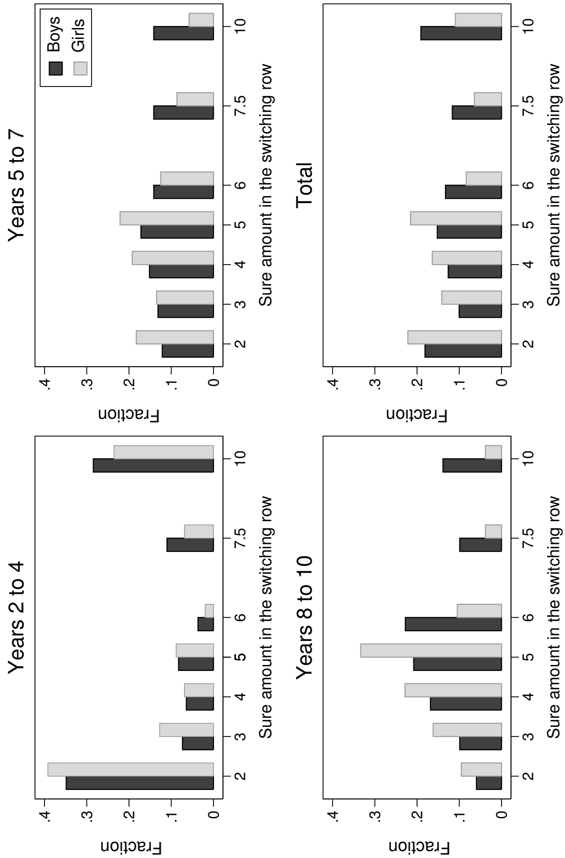

FIGURE 13. DISTRIBUTION OF RISK PREFERENCES

TABLE 10 – SUMMARY OF RISK TAKING

Age Group	Boys	Girls	All	M–W test (p–value)	K–S test (p–value)	t test (p–value)
Years 2 to 4	4.56 (0.31) 109	3.98 (0.31) 102	4.27 (0.22) 211	0.230	0.512	0.184
Years 5 to 7	4.72 (0.24) 99	3.96 (0.20) 104	4.33 (0.16) 203	0.026	0.137	0.016
Years 8 to 10	4.97 (0.21) 101	3.97 (0.16) 105	4.46 (0.14) 206	< 0.001	< 0.001	< 0.001
All	4.75 (0.15) 309	3.97 (0.13) 311	4.36 (0.10) 620	< 0.001	< 0.001	< .0001

Notes: The table shows the mean as the main number and the standard deviation in parentheses. Number of observations are below the means.

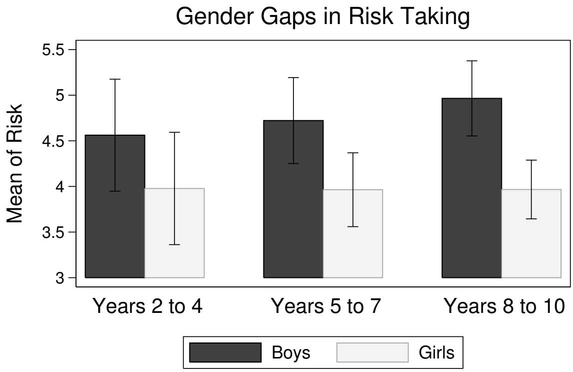

FIGURE 14. AVERAGE RISK TAKING, BY GENDER AND AGE GROUP

children (mean age 14.53) this gender gap is already highly significant. Older boys are much more risk taking than older girls. Figure 14 presents the gender gaps in an aggregated manner for the three samples.[21] We can see from the figure, that while on average risk preferences of girls does not change, there is almost a linear increase in average risk taking for boys with age. Comparing older children with younger ones we find that younger children take less risk than older children. This result is driven by the difference between older and younger boys, since girls are equally risk taking in the two groups.

In sum, we find that boys are more risk taking than girls and that this gender gap gets bigger during adolescence.

5. Results: Altruism

In this section we look at gender differences in altruism as measured via donations in a dictator game. We find that girls are significantly more altruistic than boys in our sample of students ($p < 0.001$). Girls on average donate 60 experimental points and boys 51 points out of 100 to the charity organization (an orphanage) that is the recipient in our dictator game (see Table 11 for a summary). The distribution of donations by gender is presented in Figure 15. The modal allocation is the 50–50 split.[22]

Upon a closer look we find that the gender gap is mainly driven by the behavior of older and younger boys. In the younger and older samples, the gender gap

[21] Doing the analysis for each age separately, shows that a significant gender gap appears at the age of 12 in our sample. Additional regression analysis of risk taking on being female and age shows a significant gender effect. When comparing gender gaps in risk taking between older and younger

TABLE 11 – DONATIONS IN THE DICTATOR GAME

Sex	Number of observations	Average donation	Standard deviation
Boys	387	51.48	33.98
Girls	376	60.16	32.21
Total	763	55.75	33.38

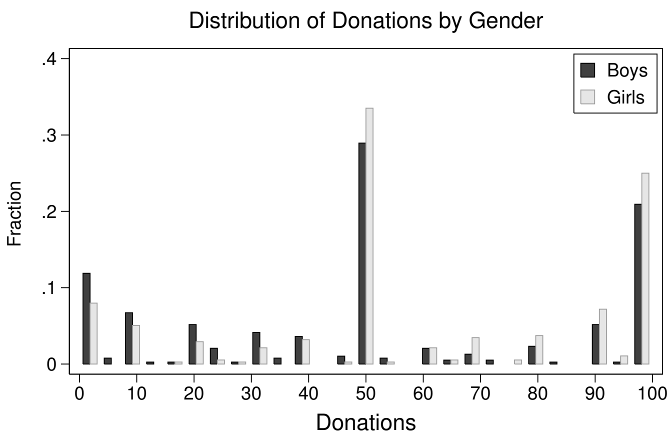

FIGURE 15. DISTRIBUTION OF DONATIONS IN THE DICTATOR GAME, BY GENDER

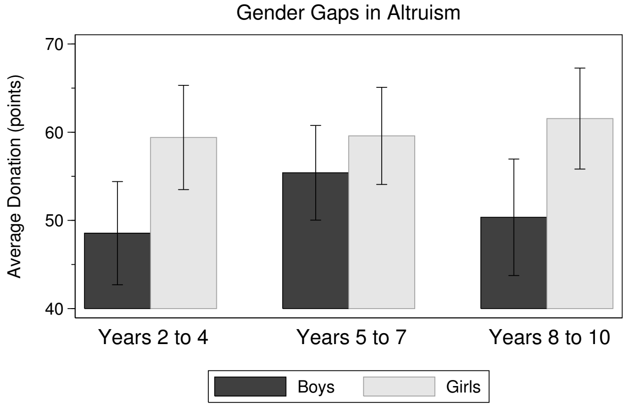

FIGURE 16. AVERAGE DONATIONS, BY GENDER AND AGE GROUP

is significant (p = 0.002 and p = 0.013, respectively), while in the middle sample the gender gap is not significant (p = 0.216), see Figure 16.

Figure 17 shows the average donations for boys and girls in different years in school. The data show that there are clear gender differences in altruism for older and younger students, and this is mainly due to the behavior of the boys.

6. Discussion and Conclusion

Recent papers have explored to what extent gender differences vary across cultures, contexts and age groups. There is some evidence that culture matters, but not always in predictable ways. For example, Gneezy and Rustichini (2004b) find that among children in Israel, only boys run faster when competing against another child compared to running alone, whereas Dreber et al. (2011b) find no gender gap in competitiveness with this measure in Sweden. Israel typically scores lower on gender equality indices than Sweden, which typically scores in the global top five. Another example that suggests that culture and norms, for example, about gender equality might matter in order to explain gender differences in preferences is Andersen et al. (2011). Comparing competitiveness in terms of the choice to compete or not among children aged 7 to 15 in a matrilineal society and patriarchal society in India, they find no gender gap in the matrilineal society,

children in a regression framework, we find that the gap is significant only at the 10% level.

[22] Regression analysis controlling for age confirms a strong gender effect, although the coefficient of age is insignificant.

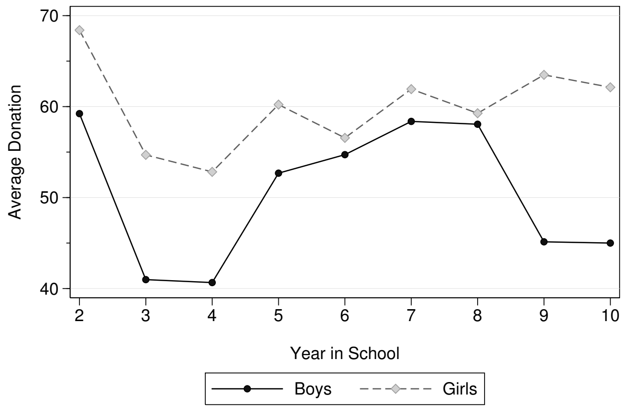

FIGURE 17. AVERAGE DONATION, BY GENDER AND YEAR IN SCHOOL

but that boys become more competitive than girls in the age group 13 to 15. However, Cardenas et al. (forthcoming) find gender differences in competitiveness in Sweden but not in Colombia, a country that scores substantially lower on gender equality, and Zhang (2011) finds no gender gap in the choice to compete in Chinese Han children but that girls are less competitive in both a matrilineal and a patrilineal group in China.

Contributing to these somewhat puzzling results, we find that in Armenia, a country that scores lower on macro gender equality indices than any of the above mentioned countries (see, for example, Hausmann et al., 2010)[23], girls are in one task more competitive than boys when it comes to performance change, and in the other three tasks there are no gender differences. There is no gender difference in preferences for competition in any age group. As most previous studies, we find that girls are less risk taking than boys. Moreover, we find that girls are more altruistic than boys, something that is not always found among children. The risk taking results replicate what most other studies find on children and adolescents (and adults). Boys appear to become more risk taking than girls around the age of puberty in our sample, perhaps suggesting that the gender gap in risk taking is to an extent related to the hormones (Apicella et al., 2008; Sapienza et al., 2009), though see Zethraeus et al. (2009). Previous results on gender differences in altruism among children are mixed, thus more studies are needed on this topic.

Why girls are if anything more competitive and also less risk taking and more

[23]Global Gender Gap Report 2010 puts Armenia in rank 84, lower than China (61), Colombia (55), Israel (52), Austria (37) and Sweden (4).

altruistic than boys in Armenia is for us somewhat of a puzzle. A natural extension of our study would be to perform a similar study in other countries of the former Soviet Union, in order to explore whether similar results would be observed. Another possible extension is to also study adults in a country such as Armenia, and also to study subjects living in the countryside rather in the capital, since gender related norms might differ between these areas. Moreover, it has previously been shown that the gender gap in competitiveness and risk taking partly depends on the institutional framework of the experiment (see Balafoutas and Sutter, 2010; Booth and Nolen, 2011a,b; Cason et al., 2010; Cotton et al., 2009; Ertac and Szentes, 2011; Niederle et al., 2010; Niederle and Yestrumskas, 2008; Wozniak et al., 2011). Testing these different setups in a large number of countries would provide an interesting venue for future research.

In sum, our results provide further evidence that it is important to perform studies in different samples, cultures and contexts in order to increase our understanding of (the development of) the gender gaps in economic preferences.

TABLE 12 – EXAMPLES OF MATH TASKS FOR VARIOUS YEARS

Years 2 to 3	Years 4 to 5	Years 6 to 7	Years 8 to 10
1 + 12 = . . .	82 + 18 = . . .	93 + 67 = . . .	96 + 93 + 3 = . . .
3 + 5 = . . .	48 + 10 = . . .	63 – 38 = . . .	33 – 9 – 85 = . . .
11 + 4 = . . .	47 + 14 = . . .	2 – 38 = . . .	83 + 97 + 14 = . . .
17 + 18 = . . .	39 + 6 = . . .	71 + 52 = . . .	31 – 39 + 28 = . . .
13 + 8 = . . .	65 + 7 = . . .	58 – 72 = . . .	47 – 11 + 5 = . . .
9 + 14 = . . .	99 + 1 = . . .	51 + 27 = . . .	63 + 17 – 72 = . . .
10 + 23 = . . .	68 + 16 = . . .	89 – 46 = . . .	9 – 41 – 75 = . . .

Appendix A.

In this appendix we present examples of the experimental tasks used in the competitiveness part of the second study (classroom part).

Table 12 shows a few examples of the math exercises that the children solved for various years. Students in years 2 to 5 had only to sum a random sequence of two two-digit numbers, while students in years 6 to 7 had to both add and subtract a random sequence of two two-digit numbers. Students in years 8 to 10 had to both add and subtract a random sequence of three two-digit numbers. All the numbers and mathematical operations were randomly generated to insure that the level of difficulty of the math task was the same throughout all the stages of the experiment for each of the year categories.

Figure 18 shows an example of the word search task that was used in the experiment. The students had to find and circle words in any direction on a straight line. Since these word search puzzles were not randomly generated by a computer there might have been slight differences in the difficulty of the word search task in different stages of the experiment. We used the same puzzle within each stage of the experiment for all the subjects.

Թ	Վ	Ի	Ց	Ե	Ղ	Ե	Կ	Ե	Ջ	Ճ	Ի	Յ
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Ն	Շ	ՈՒ	Դ	Գ	Օ	Թ	Զ	Ք	Բ	ՈՒ	Չ	Յ
Գ	Ո	Ր	Գ	Ե	Ր	Լ	Մ	Փ	Ֆ	Ս	Ե	Օ
Ա	Կ	Ա	Ն	Ա	Ղ	Ե	Կ	Ի	Ր	Ե	Ս	Խ
Ր	Ե	Բ	Ա	Ր	Դ	Ի	Հ	Ծ	Զ	Ր	Ո	Ն
Ա	Դ	Ա	Ս	Ա	Ր	Ա	Ն	Ի	Խ	Ե	Փ	Ա
Ն	Վ	Ֆ	Ղ	Ա	Յ	Ո	Ե	Շ	ՈՒ	Ն	Ի	Հ
Ա	Ն	Թ	Բ	Հ	Փ	Օ	Դ	Վ	Ք	ՈՒ	Մ	ՈՒ
Հ	Գ	Ի	Ն	Ե	Ր	Ո	Ջ	Ն	Խ	Ռ	Ե	Տ
Ա	Չ	ՈՒ	Հ	Բ	Թ	Ա	ՈՒ	Վ	Ա	Ա	Ճ	Ա
Ք	Ա	Ր	Զ	Շ	Յ	Զ	Կ	Ը	Ժ	Ղ	Ի	Պ
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FIGURE 18. EXAMPLE OF A WORD SEARCH TASK

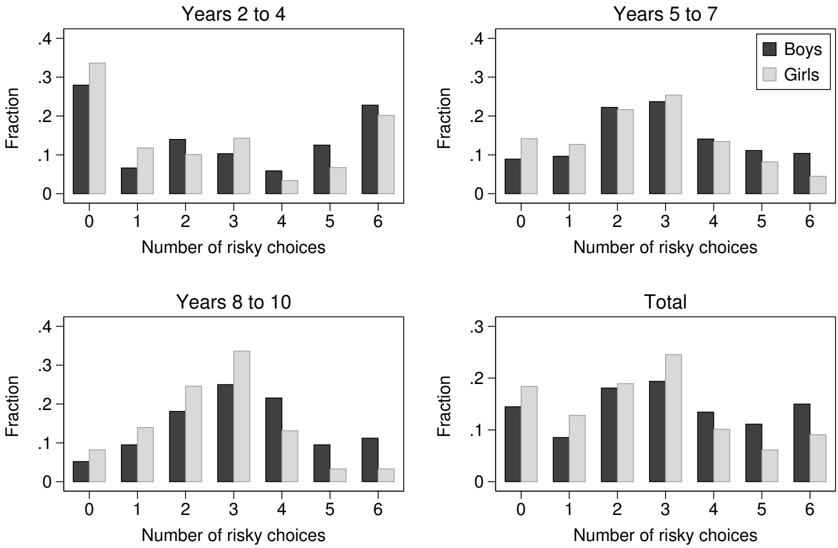

FIGURE 19. DISTRIBUTION OF RISK AS MEASURED BY THE NUMBER OF RISKY CHOICES

Appendix B.

In this section we redo the analysis for the risk preferences employing the number of risky choices as a measure of risk taking. We replicate all the findings on risk taking in the main body of the paper. In the aggregate we find a significant gender gap in risk taking: boys take 20.89 percent more risk than girls do ($p < 0.001$). This gender gap, again, gets bigger with age. In the younger sample of school children (average age 8.83) the gap is 15.74 percent while in the older sample of school children (average age 14.72) it is 27.37 percent. The histograms in Figure 19 and information presented in Table 13 reveal that while in our sample of younger children the gender gap in mean risk taking is not significant, it is quite significant in the sample of older school children.

Figure 20 presents the mean gender gaps in risk taking in an aggregated manner for all the three samples under study. Comparing older children (in years 8 to 10) with younger children (in years 2 to 4) we find that older children are more risk taking (Mann-Whitney U test: $p = 0.088$; Kolmogorov-Smirnov test: $p = 0.002$). While comparing older girls with younger girls and older boys with

Table 13 – Summary of Risk as Measured by the Number of Risky Choices

Age Group	Boys	Girls	All	M–W test (p–value)	K–S test (p–value)	t test (p–value)
Years 2 to 4	2.88 (0.20) 136	2.43 (0.21) 119	2.67 (0.15) 255	0.155	0.402	0.124
Years 5 to 7	2.99 (0.15) 135	2.54 (0.14) 134	2.77 (0.10) 269	0.040	0.535	0.027
Years 8 to 10	3.22 (0.15) 116	2.52 (0.12) 122	2.86 (0.10) 238	< 0.001	0.004	< 0.001
All	3.02 (0.10) 387	2.50 (0.09) 375	4.76 (0.07) 762	< 0.001	0.001	$0 < .001$

Notes: The table shows the mean as the main number and the standard deviation in parenthesis. Number of observations are below the means.

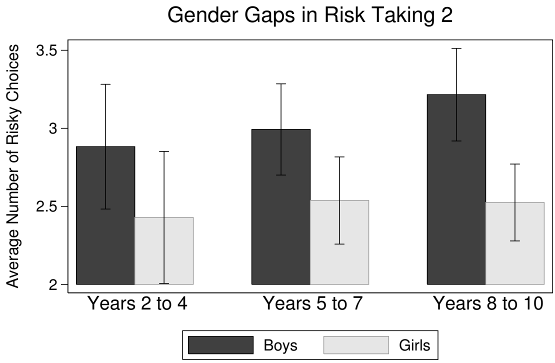

FIGURE 20. AVERAGE NUMBER OF RISKY CHOICES, BY GENDER AND AGE GROUP

younger boys only the Kolmogorov-Smirnov test returns a significant test statistic ($p = 0.002$ and $p = 0.001$, respectively; Mann-Whitney $p-$values are 0.161 and 0.203, respectively).

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