THE EFFECTS OF TEACHING MATHEMATICS THROUGH PROBLEM-SOLVING CONTEXTS ON SIXTH-GRADE STUDENTS' PROBLEM-SOLVING PERFORMANCE AND REPRESENTATION USE

By

JONATHAN D. BOSTIC

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To my grandfather Fred Holt, EdD

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LIST OF ABBREVIATIONS

ANOVA	Analysis of Variance
CCSSO	Council of Chief State School Officers
CFA	Confirmatory Factor Analysis
CFI	Confirmatory Factor Index
DBR	Design-Based Research
DIF	Differential Item Functioning
DTA	Direct Translation Approach
ELL	English Language Learner
FCAT	Florida Comprehensive Assessment Test
FLDOE	Florida Department of Education
GPD	Gradual Program Design
IRT	Item Response Theory
NCTM	National Council of Teachers of Mathematics
NGSSS	Next Generation Sunshine State Standards
PCM	Partial Credit Model
RMSEA	Root Mean Square Error of Approximation
RPD	Realistic Program Design
SPSS	Statistics Package for the Social Sciences
TIMSS	Trends in Mathematics and Science Study
TLI	Tucker-Lewis Index
VIF	Variance Inflation Factor

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By

Jonathan D. Bostic

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Chair: Stephen J. Pape
Co-chair: Tim Jacobbe
Major: Curriculum and Instruction

This study examined sixth-grade students' problem-solving performance and representation use as a result of an instructional intervention. It responds to recently adopted mathematics standards (i.e., Next Generation Sunshine State Standards (Florida Department of Education, 2007); Standards for Mathematical Content and Standards for Mathematical Practice (Chief of Council State School Officers, 2010)) that indicate problem solving needs more prominence within mathematics instruction. The instructional intervention aims to supplement current efforts to enhance students' problem-solving performance and number of representations used to solve word problems.

Multiple scholars designed instructional interventions intending to improve K-8 students problem-solving performance (e.g., Charles & Lester, 1984; Sigurdson, Olson, & Mason, 1994; Verschaffel et al., 1999) or their facility with representations (Verschaffel & De Corte, 1997; Klein, Beishuizen, & Treffers, 1998). A literature review indicated that investigators have not concurrently examined students' problem-solving performance and representation use.

Three sections of sixth-grade mathematics were sampled from a school in Florida; one section was randomly assigned to experience the instructional intervention. The author developed mathematics lessons intended to support students' mathematics learning of rates, ratios, and data analysis by working on word problems and engaging in content-focused and problem-solving discourse. These lessons were enacted for one month while two comparison classrooms received their typical instruction. Participants completed a pretest, posttest, and a unit test.

Data analyses within groups indicate that the intervention had a positive effect on students' problem-solving performance (d = .48) and number of representations used on the posttest (d = .42) whereas the comparison group experienced no changes. Results from multiple regression analyses indicate that intervention students solved more word problems (d = .26) and used more representations on the posttest (d = .18) than their peers. The comparison group had a higher average unit test score than the intervention group (d = .34). Implications for these results as well as limitations of this study and future research are discussed.

CHAPTER 1
INTRODUCTION

There have been recent changes in state (e.g., Next Generation Sunshine State Standards [NGSSS] [Florida Department of Education [FLDOE], 2007]) and national mathematics standards (i.e., Standards for Mathematical Practice and Standards for Mathematical Content [Council of Chief State School Officers [CCSSO], 2010]) that respond to the National Council of Teachers of Mathematics (NCTM) advocacy for making problem solving a priority in everyday instruction (NCTM, 1980, 1989, 2000, 2006, 2009). Proponents maintain that problem solving must remain part of day-to-day instruction because solving problems is central to doing and learning mathematics (Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Schaar, 2005; Kilpatrick, Swafford, & Findell, 2001; Lester, 1994).

> Problem solving is central to mathematics. Problem solving should be the site in which all of the strands of mathematics proficiency converge. It should provide opportunities for student to weave together the strands of proficiency and for teachers to assess students' performance on all of the strands (Kilpatrick et al., 2001, p. 421).

This study aimed to investigate the effects of an instructional intervention (i.e., teaching mathematics through problem-solving contexts) on adolescents' problem-solving performance and representation use when solving word problems. Sixth-grade students' performance on a test of word problems as well as their use of different representations was examined and compared to peers experiencing their everyday instruction from their classroom teacher. Instruction in the intervention classroom emphasized student-to-student discourse and participants examined problems on a daily basis. The intervention's intent was to enhance mathematics learning by examining, solving, and reflecting on word problems.

Students need frequent opportunities to engage in problem solving so that they can become mathematically proficient. Mathematical proficiency characterizes learning mathematics successfully in such a way that one develops (a) conceptual understanding, (b) procedural fluency, (c) strategic competence, (d) adaptive reasoning, (e) and a productive disposition toward mathematics (Kilpatrick et al., 2001). Creating the next generation of educators, scientists, and researchers depends on whether K-12 schooling promotes mathematical proficiency (NCTM, 2009; Kilpatrick et al., 2001; Stein, Remillard, & Smith, 2007). Mathematically proficient students exhibit problem-solving behaviors such as reading problems carefully and understanding them, creating models, and making conjectures about strategies and solutions (Kilpatrick et al., 2001). On the other hand, children lacking mathematical proficiency demonstrate ineffective mathematical behaviors such as attempting to solve problems without making sense of the problem's context. Moreover, they are less likely to use their knowledge of mathematics content while problem solving (CCSSO, 2010). The Standards for Mathematical Practice (CCSSO, 2010) discuss problem solving at length. This study heeds their call for supporting mathematics learning and problem-solving performance by engaging students in daily mathematics instruction that integrates problem-solving features.

Problem solving goes beyond the typical thinking and reasoning students employ while solving exercises (Polya 1945/2004; Verschaffel, Greer, & De Corte, 2000). It means thinking deeply about concepts, their associated representations, viable solution procedures, related context or cultural knowledge, and creating problem models (English & Halford, 1995; Mayer, 1992; Mayer & Wittrock, 2006; Verschaffel et al., 2000). The problem-solving model constructed by Verschaffel et al. (2000) guides this study. Verschaffel et al. suggest that effective problem solvers go through six stages of problem solving. A brief outline of these stages follows, but the model will be fully described in further detail in Chapter 2. First, individuals read the problem and work to understand the text. Understanding leads to a situation model, which adequately characterizes the mathematical and nonmathematical elements of the problem. It supports individuals to generate an appropriate representation of the problem, which facilitates mathematical analysis. This analysis is the same notion as implementing a set of procedures (Verschaffel et al., 2000). After employing procedures, problem solvers arrive at a result, termed the derivation. Individuals interpret this derivation in light of their situation model to generate an interpreted result that might become part of the final answer and is later reported as the final answer, which completes the process. An incorrect situation model that does not reflect the problem may lead problem solvers to believe that the result is correct. Hence, creating an accurate situation model is critically important. If the interpreted result does not match the expectation of the situation model, an effective problem solver revisits the situation model and begins the process again. Many problem solvers tend to skip steps throughout this process, which often leads to incorrect results (Verschaffel et al., 2000). Superficial problem solving is characterized by four steps: (1) reading the problem's text, (2) creating a mathematical model, (3) implementing a representation and set of procedures, and (4) reporting results (Verschaffel et al., 2000) and many students solve problems in this fashion (Anthony, 1996; Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Pape, 2004).

According to Lesh and Zawojewski (2007),

> Problem solving as viewed from a mathematics education perspective is the process of interpreting a situation mathematically, which usually involves several iterative cycles of expressing, testing and revising mathematical interpretations – and of sorting out, integrating, modifying, revising, or refining clusters of mathematical concepts from various topics within and beyond mathematics. (p. 782)

It is a complex activity that requires individuals to maintain focus and both rationally and effectively proceed through the problem. Students' ineffective problem-solving behaviors and disengagement in the process is exacerbated by teacher-directed instruction that frequently uses too many exercises and not enough problems (Lesh & Zawojewski, 2007; Pittman, 2006). One way to foster students' success in the problem-solving process is to provide them with frequent opportunities to engage in problem solving in a student-centered environment that scaffolds students to successfully complete each stage of the process (Verschaffel & De Corte, 1997; Verschaffel et al., 1999). Evidence from prior research indicating the benefits of problem-solving instruction on student-related outcomes will be discussed in detail in Chapter 2.

Multiple studies have demonstrated that when daily mathematics instruction is integrated or supplemented with problem-solving activity, it enhances students' problem-solving capabilities (e.g., Charles & Lester, 1984; Sigurdson, Olson, & Mason, 1994; Verschaffel et al., 1999). Moreover, there is some evidence that students' learning in classroom environments where problem solving is a regular part of mathematics instruction outperform their peers in traditional learning environments on mathematics achievement tests (Sigurdson et al., 1994; Verschaffel et al., 1999). Success on problem-solving and achievement measures is also influenced by the degree to which students are supported to gain facility with representations and procedures.

Strategy (i.e., representation and procedure) use is a critical component of problem solving (Verschaffel et al., 2000). Effective problem solvers actively monitor their actions while implementing a strategy (Mayer & Wittrock, 2006). They consider a variety of procedures and representations that are suitable for completing a task and monitor their progress while completing the procedures (Lesh & Doerr, 2003; Lesh & Zawojewski, 2007). Instruction that allows students to consider a variety of representations and procedures to complete a task and share them has been shown to have positive effects on students' achievement (Klein, Beishuizen, & Treffers, 1998). Creating an instructional context that stimulates mathematical discussions among problem solvers enhances their ability to solve problems and use a variety of representations and procedures. The teacher is the critical factor in making such a learning environment (Lampert, 1990).

The teacher is an important person in the classroom because this individual makes instructional decisions such as their choice of materials and instruction that influence students' mathematics learning and problem-solving performance (Good & Grouws, 1977/2003). Teachers decide whether to enact teacher-directed instruction or foster student-centered instruction. Teacher-directed instruction is characterized by lecture stemming from the teacher's knowledge and a lack of discussion about mathematics (Franke, Kazemi, & Battey, 2007; Good & Grouws, 1977/2003). Children are expected to watch passively, listen to their teacher, and later practice what the teacher showed them (Boaler, 1998; Boaler & Staples, 2008; Good & Grouws, 1977/2003). Discourse in teacher-directed classrooms tends to follow a three-turn interaction termed Initiate-Respond-Evaluate (IRE) (Franke et al., 2007), which includes a teacher's inquiry, student's response, and teacher's evaluative statement (Doyle, 1985). Students in these teacher-centered classrooms tend to think that

> doing mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question, and mathematical truth is determined when the answer is ratified by the teacher. (Lampert, 1990, p. 32)

On the other hand, student-centered instruction involves attending to students' knowledge and building on these ideas in a meaningful way that promotes learning about concepts and procedures (Cobb, 1994; Donovan & Bransford, 2005).

> The teacher works to orchestrate the content, representations of the content, and the people in the classroom in relation to one another. Students' ways of being, their forms of participation, and their learning emerge out of these mutually constitutive relationships. (Franke et al., 2007, p. 227).

The teacher in the student-centered classroom plays the role of learning facilitator and guide during mathematics instruction (Franke et al., 2007). Students discuss mathematics, make conjectures, and construct mathematical arguments and proofs in student-centered classrooms (Lampert, 1990). "Generally it [student-centered instruction] implies an approach in which learners are given opportunities to offer their own ideas and to become actively involved in their learning" (Boaler, 2008). During an observation of a student-centered classroom, one might notice students in small groups trading ideas and making sense of a problem, a teacher and students collaborating to solve a problem, and there are likely established social and sociomathematical norms for doing mathematics in the problem-solving oriented, student-centered mathematics classroom (Bostic & Jacobbe, 2010; Cooke & Buchholz, 2005).

Studies published in the previous two decades provide a good foundation for implementing effective mathematics instruction. Research on mathematics instruction has shown that (a) practicing procedures over and over does not develop students' mathematical understanding (Carpenter et al., 1993; Hiebert & Wearne, 1992); (b) less time spent practicing procedures does not hinder elementary students' ability to solve routine problems (Hiebert & Wearne, 1992); and (c) spending more time on one problem in conjunction with meaningful mathematical discourse creates an opportunity for reflection and analytical thinking that facilitates students' mathematical proficiency (Cobb et al., 1991; Hiebert & Wearne, 1992). With this in mind, mathematics education researchers seek to support students to become effective problem solvers by implementing instructional interventions.

A variety of research designs have been implemented to explore how students solve problems and to enhance their problem-solving performance and representation use. Mathematics educators such as Kantowski (1977) suggest that conducting problem-solving research in the classroom, with students from a variety of grade-levels, and focusing on instruction is needed to enhance researchers and practitioners' understanding of problem solving. Early large-scale problem-solving studies supplemented mathematics teaching by adding a problem-solving component to existing fifth- and seventh-grade instruction (Charles & Lester, 1984). Similarly, Sigurdson et al. (1994) infused problem solving into eighth-grade students' daily instruction. They compared the effects of ten minutes of problem-solving work with instruction that promoted understanding the underlying meaning of concepts (i.e., problem-process teaching) and the same instruction without a problem-solving component (i.e., meaning teaching). Verschaffel et al. (1999) created 20 problem-solving oriented lessons, which were implemented over four months in multiple fifth-grade classrooms. These researchers aimed to determine whether students receiving the intervention had better problem-solving performance and achievement, and whether they developed more effective problem-solving behaviors compared to their peers experiencing their typical instruction. These studies characterize supplementing mathematics instruction with problem-solving components, but their conclusions do not shed light on effects of teaching mathematics through problem-solving contexts on a daily basis. Moreover, these studies do not characterize daily mathematics instruction that teaches state- or country-wide mathematics standards through problem-solving contexts. Prior investigations provide a foundation for examining teaching mathematics associated with the Standards (CCSSO, 2010; FLDOE, 2007) through problem-solving contexts on students' problem-solving performance and representation use. Research is necessary to determine whether students' outcomes from this type of instruction differ from prior problem-solving interventions or everyday instruction. One way to begin such investigations is for a researcher to become an instructor in the classroom.

Lampert's (1990) results from her teaching experiment provide insight into mathematics teaching and learning in the elementary classroom as a result of infusing problem-solving features in daily mathematics instruction. To meet this goal, she became the classroom teacher in one fifth-grade mathematics classroom. Years later, Verschaffel and De Corte (1997) worked with fifth-grade students over two-and-a-half weeks to improve their ability to create mathematical models for solving complex word problems. Verschaffel became the teacher during his study and implemented five lessons that used nonroutine word problems as a means for teaching mathematics through problem solving. These investigators set out with a goal of better understanding the teaching and learning process and they also expected to improve students' problem-solving performance and problem-solving behaviors. Neither Lampert's study, Verschaffel and De Corte's, or others' research examined how problem-solving instruction learning influenced students' use of multiple representations or the type of representations they used. To begin to fill this gap, two curricula were created by Klein and his colleagues (1998) and implemented over one academic year in a second-grade classroom. One group of students learned one representation and procedure at a time and completed exercises to improve their efficiency. A similar group of students experienced instruction that encouraged them to generate multiple representations and procedures to solve problems, and the teacher presented the class with more than one representation to solve these problems. Together, these studies provide a foundation for examining the effects of teaching mathematics through problem-based contexts in a student-centered, discourse-rich classroom on sixth-grade students' problem-solving success and students' use of representations. The present study aims to bridge the areas of problem solving and representation use.

Statement of the Problem

The mathematics education research and teaching community is making positive steps in helping students become better problem solvers. The 2007 Trends in Mathematics and Science Study [TIMSS] examined fourth-grade students' content knowledge as well as knowing, reasoning, and applying capabilities across industrialized nations (Gonzales et al., 2008). U.S. fourth grade students' performed above the median but they still lag behind peers in Hong Kong, Chinese Taipei, Japan, Singapore, England, and Latvia (Gonzales et al., 2008). The upward trend during the last decade is promising, but there is more that can be done to support young problem solvers' mathematical development.

Although U.S. students are improving, they tend to have difficulty completing problems (Greer, 1993), and little is known about problem solver's success on problems drawing on students' knowledge of out-of-classroom (i.e., realistic) contexts (English & Sriraman, 2010). A problem is a task such that a path to the solution is not readily apparent to a problem solver (Kantowski, 1977; Mayer & Wittrock, 2006). Most word problems in textbooks are verbal translations of symbolic exercises that are transparent and easily solved without much struggle (Grischenko, 2009), which make them routine translation problems (Mayer & Wittrock, 2006). Textbooks are a significant factor influencing how teachers conduct their instruction (Tarr, Chavez, Reys, & Reys, 2006), yet teachers may not have the resources to support students' work on complex problems if there are few problems within the textbook.

The Standards for Mathematical Practice, Standards for Mathematical Content (CCSSO, 2010) and NGSSS (FLDOE, 2007) include learning outcomes related to problem solving. Students are expected to "make sense of problems" (CCSSO, 2010, para. 2) and solve real-world problems (FLDOE, 2007), which are more complex than exercises. Every Big Idea in the Next Generation sixth-grade Sunshine State Standards has at least one benchmark that includes students being able to solve problems in real-world contexts. These state Standards include fewer objectives overall and place more attention on problem solving than the prior standards (FLDOE, 1996). Recently adopted standards place a larger emphasis on problem solving in mathematics Standards (CCSSO, 2010; FLDOE, 2007) and instructors are not provided with resources indicating how to blend problem solving and mathematics into daily instruction so that students are prepared to solve realistic and complex problems drawing on current situations. Ideas for instruction stemming from research conducted in classrooms may support mathematics teachers to assist students to become effective problem solvers who solve complex problems as part of their day-to-day instruction.

Students are expected to “understand the approaches of others to solving complex problems” (CCSSO, 2010, para. 2). Thus, learning about alternate representations and procedures is critical to their success. Students learn several ways to solve problems over an academic year but it is not always clear whether a previously learned approach could be applied in a new situation or when one is more efficient than another. Developing productive problem-solving behaviors during classroom mathematics instruction includes promoting the idea that problems can be solved in multiple ways (Hiebert, 2003), often times using previously learned methods.

Representation use is a critically important element of solving problems. Algorithms have been and continue to be a focus in many mathematics classrooms (Boaler, 2008). They are important tools for solving mathematics problems and should be part of mathematics instruction (National Mathematics Advisory Panel, 2008), but focusing mathematics instruction on learning algorithms does not orient students to determine the essential parts of a problem’s situation or enhance their problem-solving performance (Thompson, 2008). Instruction that allows students to manipulate tasks into more manageable or useful representations and employ a variety of representations and procedures facilitates children’s development of mathematical proficiency (Hiebert, 2003; Packer, 2003; Van de Walle, 2003).

There is typically more than one way to solve a word problem (Harel, 1998; Schoenfeld, 1992). Implementing pictorial, tabular, or verbal (i.e., nonsymbolic) representations to solve problems can often be more efficient (Preston & Garner, 2003) and just as effective as symbolic approaches (Bostic & Pape, 2010; Herman, 2007). Much of the literature on students' representation use focuses on secondary students. Young students are capable of learning various ways to solve problems (Cooke & Buchholz, 2005; Klein et al., 1998). They are able to recognize the limitations and benefits of mathematical representations (Perry & Atkins, 2002) and can develop strategic competence that makes them more efficient problem solvers (Klein et al., 1998). Unfortunately many findings from prior research that show improvements in students' representation use as a result of an intervention are not linked to learning grade-level objectives from state or national standards. Hence, there is a need for research that characterizes mathematics instruction that supports students' use of a variety of representations within the context of these state-mandated standards.

Finally, prior investigations of problem-solving interventions have shown inconsistent effects on students' achievement. Verschaffel et al. (1999) found that fifth-grade students who experienced one lesson each week that supported mathematics learning through problem-solving contexts had slightly better scores on an achievement than their peers experiencing traditional instruction. Conversely, incorporating problem solving into daily instruction resulted in negative or nonsignificant effects on some students' achievement (Sigurdson et al., 1994). That is, the instructional intervention may influence the depth of students' understanding related to specific content areas (e.g., rates, ratios, and data analysis). This study provides insight on students' knowledge of mathematics procedures and concepts related to rates, ratios, and data analysis by examining how an intervention group performs on a unit test and compares their outcomes to peers in a comparison group. In conclusion, research must continue to explore students' outcomes from teaching mathematics through problem-solving contexts. Results from studies examining problem solving, representation use, and achievement may provide insight into ways to support students' to become mathematically proficient problem solvers.

Purpose of the Study

The purpose of this study was to investigate the effects of student-centered, discourse-rich mathematics instruction (i.e., instructional intervention) on students' problem-solving performance as well as their representation use when solving word problems. Creating a supportive instructional context that used word problems as the focal activity was intended to support students' opportunities for learning mathematics content and procedures. A fundamental desired outcome of the instructional intervention was to assist sixth-grade students in becoming more effective and efficient problem solvers within the context of mastering mathematics content and demonstrating effective mathematical practices found in the Standards (CCSSO, 2010; FLDOE, 2007). A secondary desired outcome was to demonstrate that teaching mathematics from the Standards (FLDOE, 2007) through problem-solving contexts was possible in the midst of this critical time with new standards for mathematics content and practice.

A sixth-grade mathematics teacher and her students from three sections of sixth-grade mathematics volunteered to participate in this study. Two sixth-grade classrooms continued to experience their instruction with the classroom teacher (i.e., comparison condition) while the researcher became an instructor to one section (i.e., intervention condition). Instruction in the intervention classroom encouraged individual work, small-group collaboration, and whole-class discussions of mathematics content and procedures within the context of problem solving. Participants in both groups completed a word problem pretest and posttest as well as a unit test. The pre- and posttest were scored for accuracy and students' solution methods were coded based on the representation employed. The unit test measured students' understanding of content taught during the month (i.e., rates, ratios, and data analysis) and was also scored for accuracy. Within-group and between-group differences were examined as well as whether there was an association between nonsymbolic representation use on the posttest and membership in the intervention condition.

CHAPTER 2
LITERATURE REVIEW

Problem-Solving Process

The pathways individuals use to solve problems have been investigated extensively and various models of the problem-solving process have been proposed (Mayer, 1992; Polya, 1945/2004; Verschaffel et al., 2000). Verschaffel and his colleagues (2000) created a model of the problem-solving process that builds upon prior frameworks and identifies both appropriate and superficial pathways for solving problems (Figure 2-1). A brief description of the appropriate problem-solving process is provided below. This is followed by a description of the superficial pathway students often employ. Finally, a detailed description of the problem-solving stages is provided.

The problem-solving process begins with an individual reading and understanding a problem's text (i.e., first stage). The text indicates the task and provides the reader with information about the problem. At times, the task is unclear from an initial reading of the problem so an individual reread the problem. This requires being metacognitively active about his/her understanding so he/she can maintain engagement in the task. Understanding includes decoding the text into more manageable chunks in order to create a situation model (Verschaffel et al., 2000).

This situation model is the second stage in the problem-solving process. It is typically an internal representation encompassing mathematical, contextual, and other non-essential aspects of the problem, but some problem solvers create external representations of their models (Ainsworth, 1999). This model is an intermediate model that links the problem's text and mathematical analysis phase of problem solving. As students discuss the problem, they form relational representations connecting internal and external representations. An example might clarify how situation models may be internal, external, or relational. Imagine a circumstance where an individual is given a text with an embedded problem. This person understands the text and creates a representation in his/her mind. In order for this individual to share his/her situational model with another, the internal representation must be transformed into an external representation (e.g., words), which requires a relational representation (Ainsworth, 1999; Goldin, 2002). Regardless of whether an individual uses an internal, relational, and/or external representation to model the situation, effective problem solvers subsequently develop a more mathematically-focused model called a mathematical model and move to the third problem-solving stage.

The mathematical model contains only mathematical aspects that can be acted on using mathematical analysis techniques (Verschaffel et al., 2000). Some examples of mathematical models include graphs and pictures, symbolic expressions, tables, and verbal statements (Lesh & Doerr, 2003). Representation use during problem solving is crucially important if a student expects to find the correct solution (Greeno & Hall, 1997). Effective problem solvers recognize that some representations are more appropriate or lead to the solution quicker than others, depending on the task (Greeno & Hall, 1997; Preston & Garner, 2003; Verschaffel et al., 1999). Furthermore, factors that might impact the mathematical model are more obvious to problem solvers who fully engage in the problem-solving process. Those who take the necessary time and energy to understand the text and develop a situation model are likely to solve the problem (Verschaffel et al., 2000), but that does not guarantee success. The present study examines students' representation use (i.e., mathematical modeling) within the context of the problem-solving process. Mathematical modeling is a critical step in the process because it leads to the mathematical analysis technique (i.e., procedures) used to answer the problem.

The analysis procedures are dependent upon the mathematical model's representation. After implementing an analysis technique, the individual arrives at the fourth stage, derivations from analysis. This is not the final answer, but rather the outcome from carrying out a set of procedures on a mathematical representation. The derivation is just a number or another representation that has not had meaning ascribed to it by the problem solver. The result is important yet it needs to be interpreted within the problem's context. For example, word problems and real-life problems require units in order to make sense of the result. Effective problem solvers evaluate their result with the situation model, judge their alignment, and the outcome is the interpreted result. This evaluation requires a learner to self-monitor his or her mathematical thinking, being careful to consider whether the result aligns with the situation model and if not, to return to the appropriate problem-solving stage and reevaluate his or her work (De Corte, Verschaffel, & Op't Eynde, 2000). Interpreting a problem's result is the fifth problem-solving stage (Verschaffel et al., 2000). Problem solvers who externalize their situation models have something visible to verify their interpreted result whereas others have to revisit their working memory for the situation model. If the result aligns with the situation model then the problem solver communicates the answer. Reporting a solution is the sixth and final problem-solving stage. It occurs when a student effectively answers the questions such as by writing a summary statement or sharing the final solution with a peer. There are many steps to solving a mathematics problem and each stage is critically important to the learner's success.

Successful problem solvers typically go through all six stages of the problem-solving process whereas unsuccessful problem solvers typically take at least one shortcut. Shortcuts are more likely to lead to inappropriate mathematical models, incorrect use of procedures, and reporting the wrong answer to the problem (Verschaffel et al., 2000). Some of the common missteps are discussed here. At the first stage of the superficial problem-solving process, students read the text and create a mathematical model. This leap in the problem-solving process does not facilitate adequately understanding the text or determining the key aspects of the problem. At the third problem-solving stage, some learners employ mathematical representations that are inappropriate for a problem's context. For example, Santos-Trigo (1996) noticed that high school students often tried using symbolic representations and algorithms to solve complex word problems. They were frequently unsuccessful and Santos-Trigo argued that if they had better facility with multiple representations then they might have shown better problem-solving performance. The role of mathematical representations is critically important for problem solver's success and it is a focus of this study.

During mathematical analysis, learners often combine numbers inappropriately because they do not consider alternate representations or their situation model is inaccurate (Verschaffel et al., 2000). Another common mistake is that problem solvers employ a representation, conduct procedures, and report the result as the problem's solution without interpreting it. For example, an individual might indicate 16 as a word problem's solution; however, the correct response requires meaningful units such as dollars, blocks, or people. This expedited problem-solving process takes less time but it also leads to far more incorrect answers (Verschaffel et al., 2000). A common error that can be made at any stage of the problem-solving process is not devoting the necessary cognitive energy to each stage of the process. One error made by many students is not taking time and cognitive energy to sufficiently understand a problem's text (Pape, 2004).

Understanding Text

Actively reading a problem supports individuals to make sense of it; however, the depth and quality of students' decoding and subsequent understanding of the text impacts their success (Pape, 2004). To solve a word problem, individuals must manage both the text and the mathematics encoded within the text (Vilenius-Tuohimaa, Aunola, & Nurmi, 2008). One's reading ability influences how likely an individual will solve a word problem (Vilenius-Tuohimaa et al., 2008) and similarly, one's knowledge of mathematics influences how well an individual deciphers mathematics texts (Pape, 1998). Consequentially a subset of one's mathematical knowledge is one's ability to make sense of mathematics text. The depth and quality of understanding the text are two factors influencing how problem solvers approach a word problem (Okamoto & Case, 1996). It is essential to sufficiently decode a problem's text into meaningful representations so that the task's elements are clear (Staulters, 2006; Verschaffel et al., 2000). Text difficulty can also impact how efficiently an individual solves a word problem. More difficult texts require more cognitive energy to decode, which may influence students' problem-solving behaviors (Staulters, 2006).

Prior studies have examined the influence of a problem's language on students' problem-solving behaviors and performance and provide evidence of students' troubles with word problems (e.g., Lewis & Mayer, 1987; Pape, 1998, 2004). Pape (2004) examined 98 sixth- and seventh-grade students' problem-solving behaviors and drew on a reading comprehension perspective for his analyses. Eight word problems were presented to participants during a think-aloud interview. The problems varied in type and used different types of language (i.e., consistent and inconsistent). Participants were reminded to think and read aloud and occasionally asked questions during the interview such as "What are you doing right now?" (Pape, 2004, p. 195). The interviewer also asked participants to share their struggles when they could not solve a problem. Data were analyzed using a constant-comparative method to investigate students' problem-solving behaviors as well as the types of errors committed.

Two-thirds of the students used a direct translation approach (DTA), which included (1) reading the problem, (2) executing a strategy, and (3) reporting the result. This approach did not foster success among participants with multistep and inconsistent language problems, but some were able to solve straightforward consistent language problems in this fashion. DTA may suffice for simple word problems or translation tasks (e.g., symbolic expressions written as verbal statements) but it is insufficient for solving nonroutine or multi-step word problems (Pape, 2004). When tasks contain unfamiliar terminology or more words than typically seen in translation problems, students using DTA may not adequately read and make sense of the problem.

Reading and understanding a text influences which schemata are activated to solve the problem; hence this initial step in the problem-solving process is important

(Pape, 2004; Verschaffel et al., 2000). Pape's (2004) results describe students' struggle with solving word problems, regardless of the problem's language and the representation and procedure employed. They often used ineffective approaches, which includes insufficiently reading and understanding a problem's text. Mathematics instruction should teach students to completely understand the problem before moving forward in the problem-solving process. Pape's study provides an essential piece of the foundation for investigating students' problem-solving behaviors.

There are other factors that contribute to problem-solving ability such as familiarity with mathematical terminology (Cummins, Kintsch, Reusser, & Weimer, 1988) and contexts (Verschaffel & De Corte, 1997; Verschaffel et al., 1999; Verschaffel et al., 2000). Students with a sufficient understanding of conventional mathematics terminology are apt to solve problems because they understand the meaning of the words they read (Ball & Bass, 2003). Mathematics is a language that relies on symbols but it also includes graphs, charts, and texts to decode. "Reading completely depends on being able to understand the structures of texts and nuances of language; to interpret authors' ideas; and to visualize, evaluate, and infer meanings" (Ball & Bass, 2003; p. 29).

Only recently has evidence begun to quantify the relationship between students' reading comprehension and problem-solving performance. One study investigated the relationship between achievement on a test of word problems and reading comprehension (Vilenius-Tuohimaa et al., 2008). Two hundred and five 9-10 year old students from heterogeneous classes completed reading comprehension tests that assessed their understanding of an expository and a narrative passage. The simplified word problems found on the mathematics test were similar to translation problems found in most textbooks. Data related to students' gender were gathered as well to facilitate analyses that examined the effects of reading ability (i.e., good readers and poor readers) on problem-solving performance.

The fairly strong positive correlations suggested that students' success on each type of word problem (i.e., compare, change, combine, and focus) were associated with most aspects of reading comprehension as well as students' technical reading ability. Individuals with good reading comprehension skills were more likely to solve word problems than poor readers and there were no gender-related differences in problem solving (Vilenius-Tuohimaa et al., 2008). Clearly, reading comprehension and mathematics knowledge are woven together, but further explorations with word problems that are not translation tasks are necessary.

Such investigations might verify whether students' reading comprehension impacts their ability to read and interpret word problems that do not follow "word problemese" (Lave, 1993, p. 77). Word problemese is a problem structure that uses cuing language indicating how to work toward the solution thus making the problem more like an exercise. Many word problems in textbooks use this type of structure (Grischenko, 2009). If reading comprehension influences students' ability to solve simplified problems then it likely might impact students' ability to solve word problems. Investigations and analyses are necessary to confirm this hypothesis. Successfully solving a word problem depends on a problem solver's ability to initially read a problem's text, decode it, and understand the task. If problem solvers effectively read and understand the text, which is the first problem-solving stage, then they are more likely to move to the second problem-solving stage, which involves creating an effective situation model.

Situation Modeling

The situation model is a representation of the text that characterizes the problem in a way that makes sense to the problem solver (English & Halford, 1995; Verschaffel et al., 2000). It contains the mathematical and nonmathematical elements in a manageable representation that facilitate creating the mathematical model (Verschaffel et al., 2000) and clarifies the task (Case, 1996; English & Halford, 1995). Adequate models support efficient and appropriate mathematical thinking leading towards a solution. These models are often internal representations, but sometimes learners formulate external representations such as drawing a picture of the situation or reconstructing the text using simpler words (Goldin, 2002; Lesh & Doerr, 2003). Problem solvers need these representations to solve word problems because they clarify the task and necessary elements of the problem.

Effective problem solvers often reread texts and make and revise their situation models before settling on one (Verschaffel et al., 2000). Critical thinking skills and active online cognitive monitoring support problem solvers to decipher text and make effective situation models (Verschaffel et al., 2000). Critical thinking helps individuals make decisions about the text and decode it into useful representations (English & Halford, 1995). More specifically, it is "reasonable and reflective thinking that is focused on deciding what to believe or do, and is an important part of problem solving" (Ennis & Norris, 1990, p. 1). Critical thinking requires students to reflect on how well they understand information and act on the information to create a model of the situation (De Corte et al., 2000). Active online cognitive monitoring of task completion helps students make strategic adjustments toward reaching a desirable goal (De Corte et al., 2000). Self-monitoring is also important while students create and refine their situation models (Verschaffel et al., 2000).

Students are apt to skip creating a situation model when they perceive it as unnecessary, especially for routine problems (Verschaffel et al., 2000). Nonroutine word problems typically do not follow the typical language and structure associated with textbook word problems (Grischenko, 2009). They require students to read the text carefully and decode it into an adequate situation model. Instruction should encourage model creation regardless of the question's simplicity. Students may need assistance deciphering the text and determining what parts of the problem are important as well as what representations might be appropriate. The instruction in this dissertation study was intended to benefit problem solvers by encouraging them to create adequate representations of the situation so that they could generate appropriate mathematical models.

Representations are absolutely necessary for any mathematical activity to occur because mathematics typically uses sequences of symbolic characters that convey shared meanings among individuals (Kaput & Educational Technology Center, 1989). They provide a means to link two or more configurations of an idea or concept (Goldin, 2002). In the context of word problems, students create representations that "(a) reproduce the action of a story problem; (b) strip away the context, attending only to numerical aspects of the problem; or (c) combine some of both approaches" (Smith, 2003, p. 263). Furthermore, "as individuals or groups work on problems, they may make drawings, write notes, or construct tables or equations. These representations help them keep track of ideas and inferences they have made and also serve to organize their continuing work" (Greeno & Hall, 1997, p. 361). They are one of the initial steps taken by problem solvers to proceed toward a solution. After carrying out the second problem-solving stage, effective problem solvers arrive at the third stage (i.e., mathematical modeling) and the subsequent fourth stage (i.e., derivations from analysis).

Mathematical Modeling, Analysis, and Derivations

This next section combines two stages and an important process because the representation of the mathematical model influences what procedure is employed, which in turn impacts the derivation from analysis. A strategy includes the mathematical model (i.e., representation) and computational steps (i.e., procedures) hence it is necessary to characterize these stages and process together. The present study explicitly focused on students' use of representations to solve word problems.

Students' mathematical modeling has been studied extensively for decades (Charles & Lester, 1984; English, 2009; Lesh & Harel, 2003; Verschaffel et al., 1999; Webb, 1979). Solving word problems requires thinking about possible mathematical models, selecting an appropriate representation for the situation, and determining the mathematical elements (English & Halford, 1995). Individuals with well-developed mathematical proficiency often consider multiple mathematical models before proceeding with one (English & Halford, 1995). Adequately describing problems in precise terms using mathematical models may take several iterations and practice but the payoff is worth the effort (Lesh & Doerr, 2003). Careful reexamination of previous mathematical models leads to more efficient problem solving on future tasks (Chamberlin & Moon, 2008; Lesh & Zawojewski, 2007; Polya 1945/2004; Schoenfeld, 1985). Students' beliefs about doing mathematical activity influence their representation use (Herman, 2007; Lampert, 1990; Schoenfeld, 1992). At times these beliefs and students' dispositions hinder their success or efficiency (Herman, 2007; Perry & Atkins, 2002).

Generating a mathematical model relies on several factors including an individual's comfort with different ways to represent the mathematical elements of a problem. Many young children use picture-oriented mathematical models (Case, 1996) but at some point during their elementary school career they begin to employ symbolic-oriented representations more frequently (Perry & Atkins, 2002). This change may develop into a preference for symbolic representations that continues into middle school (Preston & Garner, 2003), then into high school (Bostic & Pape, 2010), and later into students' college-age years (Herman, 2007).

Representations

Representations include (a) experience-based scripts, (b) manipulative models, (c) pictures and diagrams, (d) graphs (e) verbal or spoken language, (f) written symbols, and (g) tables (Lesh & Doerr, 2003; Lesh, Post, & Behr, 1987). Learners should be able to transform each type of representation into another one that is similar in representation but unique in other ways. Problem solvers can also translate a representation into another one (e.g., symbolic expression into verbal statement) (Lesh et al., 1987). Teachers can support problem solvers working on word problems by scaffolding students during the translation process. While many teachers use representations to help children learn mathematics, it is important that these instructor-generated representations are developmentally appropriate (Murphy, 2004). Overly complex representations beyond a learner's developmental grasp are likely to be confusing or inhibit future growth hence a teacher's scaffolding is a critical to facilitating an individual's cognitive growth (Dufour-Janvier, Bednarz, & Belanger, 1987; Murphy, 2004).

Students who simply produce mathematical models and use representations without reflecting often misinterpret a task's goal or create insufficient mathematical models that fail to account for the key components of the problem (Lesh & Doerr, 2003; Verschaffel et al., 1999; Verschaffel et al., 2000). From a young age, students can develop the misconception that solving mathematics problems should occur quickly and without having to reexamine the task or their models (Lampert, 1990; Mayer, 1992; Schoenfeld, 1985, 1992). Effective problem solvers select a representation and implement procedures based on their appropriateness for the context and efficiency (Lesh & Zawojewski, 2007; Mayer & Wittrock, 2006; Schoenfeld, 1985; Verschaffel et al., 2000).

In Preston and Garner's (2003) study, a mathematics educator and middle school teacher partnered to examine students' representation use to solve an open and complex word problem. Garner, a classroom teacher, asked her seventh-grade students to solve a word problem that drew on students' real-life experiences. Groups of three to five students worked collaboratively and then reported their result to the class. Garner indicated that the goal of the task was to give her students an opportunity to try different representations to mathematically model the problem and consider the benefits and limitations of each representation. Students used equations, graphs, charts, and wrote verbal statements that characterized the mathematical elements of the problem. The first three representations were more likely to help students solve it.

During a whole-class discussion, students shared their mathematical models and offered their rationales for selecting their model. Preston and Garner's (2003) analysis of students' work and engagement in the discussion indicated that students understood the text, crafted situation models, and created mathematical models. As a result of the task and instruction, students "began to explicitly express connections and offer their opinions as to the best representations for this activity" (p. 42). Participants recognized that choosing to employ one mathematical model over another gave them slight advantages in solving the problem. Allowing students to choose their own representation to solve a complex word problem provided a context for the entire class to examine ways of solving problems by using different mathematical models. This brief investigation into one teaching episode provides evidence that students are creative problem solvers, and when given rich tasks they are able to generate approaches that answer the question. Instruction should support students to learn nonsymbolic representations and require them to give a mathematically appropriate rationale for using a specific representation to mathematically model a situation (Preston & Garner).

Many students think that representations are useful for a specific problem-type and rarely consider employing one representation to another problem-type (Murphy, 2004; Pittman, 2006). Furthermore, many children learn representation but do not necessarily know which one to implement unless cues or clues make it obvious (Kilpatrick et al., 2001; Schoenfeld, 1992). Students need to learn when to employ a representation, set of procedures, and gain strategic competence, to become effective and efficient problem solvers.

Strategic competence is critical to a problem solver's success. Students need to know representations, suitable procedures for each representation, as well as when to use the representation (Kilpatrick et al., 2001). Specifically, it is "the ability to formulate mathematical problems, represent them, and solve them" (Kilpatrick et al., 2001, p. 124). Problem solving helps to develop an individual's strategic competence because nonroutine or authentic tasks require a learner to consider multiple perspectives of the problem, select a viable representation, and perform the necessary steps to sufficiently carry out the procedures (Lambdin, 2003; Van de Walle, 2003). Knowing multiple viable ways of solving a class of problems leads to individuals becoming more efficient and effective problem solvers (Kilpatrick et al., 2001; Polya 1945/2004). An individual who has developed strategic competence typically has a flexible approach for solving word problems (Kilpatrick et al., 2001). Students with well-developed knowledge of representations, procedures, and a robust conceptual understanding are likely to know how to solve problems using more than one representation, which includes employing the same mathematical model but using a different and perhaps more efficient set of procedures.

Multiple representations

Students who know more than one way to solve a word problem are more likely to give the correct answer to a word problem (Bostic & Pape, 2010). Schoenfeld (1985) cautions that this knowledge of multiple approaches is useful only if individuals recognize the limitations of each method and have an understanding of when a representation and set of procedures are appropriate. To that end, one must develop strategic competence in order to efficiently solve problems (Kilpatrick et al., 2001).

Many students are unsure how to proceed when presented with word problems (Pittman, 2006; Santos-Trigo, 1996). Santos-Trigo (1996) investigated tenth-grade students' facility with multiple representations to solve word problems using think-aloud interviews. Thirty-five 10th-grade students from two schools volunteered to participate in semi-structured interviews. Students were asked to think aloud while solving five word problems. A content validation team consisting of one mathematics education professor (i.e., the author) and two graduate students determined that the problems could be solved using multiple representations. Interviews were audiotaped and the interviewer occasionally asked students clarifying questions similar to those used by Pape (2004).

Santos-Trigo (1996) noticed that participants did not spend much time trying to understand the problem's text and typically used a symbolic representation. After quickly scanning the text, they immediately created a symbolic-oriented mathematical model without considering alternative representations. Similar to Pape's (2004) participants, they seemed to employ DTA fairly often. They lacked adequate facility with multiple representations and struggled to produce more than one symbolic approach. Several participants alluded to an inability to solve a problem because they could not remember the necessary algorithm or formula. Participants indicated nonsymbolic as well as other symbolic approaches might exist but were reticent to explore this possibility.

Implementing a strategy requires knowledge about procedures and representations. Many students need opportunities to learn about alternatives to algorithms and formulas (Santos-Trigo, 1996; Schoenfeld, 1985), the rationale for using various representations, and their associated limitations (Pittman, 2006; Preston &

Garner, 2003). An investigation such as the present study could help teachers foster effective problem-solving behaviors among students, including developing knowledge related to mathematical models, and improving students' problem-solving performance.

Interpreting and Reporting the Results

Interpreting results implies examining the outcome from an implemented strategy, thinking about what it means, and reflecting on the result's reasonableness given the problem's context (Verschaffel et al., 2000), yet many ineffective problem solvers offer the outcome from a completed strategy as the final solution without interpreting it or reflecting on its appropriateness (Cummins et al., 1988; Greer, 1993). This critical fifth stage can be accomplished by considering the accuracy of the result and asking questions about it (e.g., Does this make sense? Is this possible?). Effective problem solvers are metacognitively active throughout the problem-solving process and typically decide whether the situation model and interpreted result align before reporting it (De Corte et al., 2000; Verschaffel et al., 2000). Active cognitive self-monitoring helps learners make these judgments and successfully answer problems (De Corte et al., 2000; Verschaffel et al., 2000). Common errors such as reporting an incorrect response that is impossible given the context of the problem could be remedied by attention to this near final stage (Greer, 1993).

Greer (1993) explored students' responses to word problems that focused on the topic of proportionality and division. He asked 100 thirteen and fourteen year-old students to solve eight word problem pairs. Each pair had one problem with a feasible solution without adjusting for realistic constraints whereas another had a solution that needed adjustment because of an unrealistic answer (e.g., "If there are 14 pizzas for 4 children at a party, how should they be shared out? If there are 14 balloons for 4 children at a party, how should they be shared out?") (Greer, 1993, p. 243). Students' performance was examined and nearly every child solved the straightforward problem but the majority did not adjust for realistic conditions with the second one thus demonstrating a lack of attention to interpreting the results. Greer explained that some students answered both of the example items because they drew on their contextual knowledge about an inability to give each child three and a half balloons thus highlighting the importance of contextual knowledge. A goal of the present study was to support adolescents to interpret their results with the problem's context and accurately communicate their solution to others. Reporting the solution links both nonmathematical and typical mathematical language and facilitates the development of mathematically proficient students. It is the sixth and final problem-solving stage (Verschaffel et al., 2000).

Summary of the Problem-solving Process

The present study was guided by a cognitive problem-solving framework. It characterizes the effective pathway for solving a problem as well as the superficial approach typically used by students. During the problem-solving process, individuals must also continue to be metacognitively active and pay attention to each stage of the process (De Corte et al., 2000; Verschaffel et al., 2000).

The first phase of work toward solving a nonroutine word problem is reading and understanding the text (Verschaffel et al., 2000). This stage means more than merely reading the text and moving to the next phase (Pape, 2004). It includes making sense of the problem's text and understanding the task (Verschaffel et al., 2000). This leads to developing a situation model, which contains the mathematical and nonmathematical elements of the problem in more manageable problem representations such as pictures, diagrams, or verbal texts. After reflecting on the situation model and task, effective problem solvers construct and subsequently refine their mathematical models. This model captures the essential mathematical elements needed to solve the problem and facilitates working towards a solution (Preston & Garner, 2003; Verschaffel et al., 2000). These models may appear in a variety of representations including symbolic, verbal, pictorial, graphical, and tabular (Lesh & Doerr, 2003) and are the initial step of implementing a strategy. Effective problem solvers construct and reflect on their mathematical models, which facilitate efficiently working toward the solution, and eventually a result (Preston & Garner, 2003). Moreover, they choose efficient representations that facilitate the development of situation and mathematical models during problem solving. Others rely on algorithms, heavy-handed approaches that may require significant cognitive effort, or know only one way to solve a problem (Santos-Trigo, 1996; Schoenfeld, 1985). Interpreting the result requires reexamining the situation model and determining the solution's reasonableness given the problem's context (Greer, 1993). If the solution seems appropriate, then the result should be reported in a clear statement that responds to the question (Verschaffel et al., 2000). Research has shown that students have difficulty at many of these stages and often perform poorly on word problems (Lesh & Zawojewski, 2007; Mayer & Wittrock, 2006; Pape, 2004).

There is evidence that students can learn to develop productive problem-solving behaviors through instructional interventions, thus improving their problem-solving performance. Problem-solving instruction provides support to children so that they develop appropriate problem-solving behaviors (Charles & Lester, 1984; Verschaffel &

De Corte, 1997; Verschaffel et al., 1999), solve more exercises and word problems correctly (Charles & Lester, 1984; Klein et al., 1998; Sigurdson et al., 1994; Verschaffel & De Corte, 1997; Verschaffel et al., 1999), and consider a wide range of possible representations to use during the mathematical analysis phase (Klein et al., 1998; Lampert, 1990). Tasks are a critical element of effective problem-solving instruction. Model Eliciting Activities (MEAs) have the potential to help students develop appropriate mathematical models and collaborate while solving open, complex, and realistic problems (Chamberlin & Moon, 2008; English, 2009; Lesh & Harel, 2003). The next section describes evidence from studies that implemented instruction aimed at improving students' problem-solving performance and representation use, which was the aim of the present study.

Problem-solving Instruction

Effectively engaging in the problem-solving process requires individuals to maintain their focus on a number of factors and work through each stage (Pape, 2004; Verschaffel et al., 2000). Mathematics instruction that contains problem-solving elements can support students to engage in each stage of the process (Lambdin, 2003; Verschaffel et al., 1999). Teachers often model the process or pathways to solve problems during instruction (Boaler & Staples, 2008; Lampert, 1990; Smith, 2003). Students generate valuable intuitive or idiosyncratic processes for solving problems (Gravemeijer & van Galen, 2003). Effective teachers push their students to try alternative mathematical models while problem solving (Bostic & Jacobbe, 2010; Perry & Atkins, 2003; Preston & Garner, 2003), and discuss successful and unsuccessful representations and procedures (Klein et al., 1998; Lampert, 1990; Preston & Garner, 2003).

Instruction should encourage students to understand the problem, create models, and consider multiple ways to solve problems (Cooke & Buchholz, 2005; Charles & Lester, 1984; English, 2009; Klein et al., 1998; Lampert, 1990; Verschaffel & De Corte, 1997). It should build upon students' prior knowledge and experiences and facilitate creating a network of mathematical topics, skills, and strategies (Kilpatrick et al., 2001; Schoenfeld, 1985). Problem-solving practice during mathematics instruction enhances students' mathematical understanding and in turn, well-developed mathematical understanding supports individuals to become more efficient and effective problem solvers (Lambdin, 2003). The next section characterizes how instruction can enhance students' problem-solving performance as well as their representation use during problem solving. Discussion of problem-solving instruction begins with investigations that supplemented everyday instruction with problem solving and transition to studies that blended problem solving and mathematics instruction to positively impact students' representation-use and problem-solving performance and behaviors. One instructional intervention that enhanced students' use of multiple representations is discussed. Finally, one recent investigation that used MEAs is examined in order to provide a context for the type of problems used in the present study.

Supplementing Daily Mathematics Instruction with Problem Solving

A number of instructional interventions supplemented mathematics instruction with problem-solving elements in order to give students an opportunity to practice applying their mathematics knowledge to problems; hence word problems have been called application problems at times (e.g., Charles & Lester, 1984). Charles and Lester designed an instructional program for fifth- and seventh-grade mathematics students intending to improve students' problem-solving performance and behaviors. Twelve fifth-grade and ten seventh-grade mathematics teachers implemented the Mathematical Problem Solving program (MPS) over 23 weeks while a similar number of teachers conducted their usual instruction in comparison classrooms. The MPS program focused on each stage of Polya's (1945/2004) four-stage problem-solving process (i.e., understand the problem, devise a plan, implement the plan, and look back), emphasized extensive experiences with solving word problems, provided experiences related to different ways to solve problems, and implemented an instructional format for teaching problem solving.

Teachers gave students problem-solving activities for approximately 10-25 minutes each day. Some examples were designing a word problem given a context, solving a problem, and examining a complex translation problem. These translation problems required a learner to effectively move from one representation to another during the problem solving process. Teachers posted a problem-solving guide on the wall that mirrored Polya's (1945/2004) four steps, were told to follow a before-during-after teaching format, and to sufficiently discuss students' work during whole-class discussions. A typical lesson began with the teacher giving a problem and orchestrating a discussion focused on understanding the problem followed by students sharing possible ways to solve the problem. Next, students worked independently or in small groups and finally, they shared their representations, procedures, and solutions with the class. Participants gave each other feedback, reflected on the problem-solving process, and shared these reflections during the whole-class discussion. Skill activities and simple translation problems were completed individually whereas problem-solving activities were typically done in small-group interactions.

Students completed a pretest and posttest 23 weeks later. Intermediate tests were given to the intervention classrooms after eight and sixteen weeks of problem-solving instruction. The tests consisted of two problems and two complex translation tasks. Charles and Lester (1984) scored students' work on three dimensions using a three-point rubric: understanding of the problem, planning, and performance. Pretest scores were used as a covariate in the analyses and each class was the unit of analysis. During the study, researchers conducted classroom observations and teacher interviews.

The intervention groups in both fifth- and seventh grade had better outcomes than their respective comparison groups. In both grade levels, students performed better on word problems, $p < .05$, and demonstrated improved understanding and planning related to problem solving. Teachers said they liked having the problem-solving process posted in the classroom and daily problem-solving activities. Some instructors commented that students were more frequently drawing a picture while problem solving, working backwards, creating a list of known information, and discussing problems with a peer during problem solving. This study provides evidence that problem-solving instruction enhances students' problem-solving behaviors and performance. It is the foundation for feasibility and other efficacy studies that examined students' outcomes from supplementing typical mathematics instruction with problem-solving components.

Ten eighth-grade mathematics classrooms were part of a yearlong study investigating the effects of three instructional methods (Sigurdson et al., 1994). In earlier research (Sigurdson & Olson, 1992), students' outcomes from two instructional methods called algorithmic teaching and meaning teaching were examined. Algorithmic teaching was "an attempt to articulate traditional, textbook teaching prevalent in schools" (Sigurdson & Olson, 1992, p. 38). It deemphasized the rationale behind procedures and representations unlike the other two instructional methods: meaning and problem-process. Meaning teaching emphasized that students needed to fully understand concepts and procedures and instruction attempted to facilitate students' cognitive connections between these two areas. Participants learned representations and procedures, but there were many more mathematically relevant student-to-teacher and student-to-student discussions in the meaning teaching classrooms than the algorithmic group. In the earlier study, students' performance in the meaning teaching group was vastly higher than their peers experiencing algorithmic teaching. The third approach used by was termed the problem-process approach (Sigurdson et al., 1994). It incorporated ten minutes of daily problem-solving work into meaning teaching. The purpose of the later study was to determine whether there were any improvements in students' outcomes after supplementing meaning teaching. The three features of the problem-process instructional approach were "(1) simple, content-related problems, (2) interactive discussions of solutions, and (3) a focus on the processes used to solve the problem" (p. 368). Students were frequently encouraged to create models, craft explanations of their results, and provide multiple solution ways of problem solving.

Ten teachers from each group adequately implemented the assigned instructional program. A mathematics achievement test was administered as a pretest and again five months later near the end of the intervention. Students in both meaning-teaching and problem-process groups outperformed peers in the algorithmic-process group on the achievement test. Average-achieving students in the problem-process and meaning-teaching group performed similarly. Low-achieving students in the problem-process group had a ten percent increase in their achievement over the algorithmic-practice teaching group whereas similar students in the meaning-teaching group made only a two percent increase. These results informed the present study as well as others with ways to implement problem-solving instructional interventions as well as areas for improvement.

These explorations provide a substantial foundation for problem-solving research intending to use an instructional intervention, but several unanswered questions and gaps remain. One aspect that supports the present study was the lesson plan format implemented with participants. It included use of small group discussions to foster mathematical thinking, scoring scheme, and analysis techniques for the dissertation study. One of the unanswered questions; however, is whether students' background characteristics or prior knowledge impacted their problem-solving performance. A second question stems from the idea of maintaining only ten minutes of problem-solving instruction (Sigurdson et al., 1994). This surely limited the adequacy of influencing students' problem-solving behaviors and performance. Finally, this study uses work by researchers (Charles & Lester, 1984; Sigurdson et al., 1994) to examine problem-solving performance and students' representation use during problem solving as measured by a test of word problems. The previous investigations offer evidence of participant's performance as measured by achievement tests and simple word problems, yet students' performance on open, complex, and realistic word problems is uncertain. Again, the research provides information about critical aspects for designing instructional interventions that focus on improving problem-solving performance and more insight about such interventions is provided in the discussion of the next study.

Verschaffel and his colleagues (1999) hoped to improve Dutch fifth-grade students' use of modeling and performance on word problems. They employed a similar research design as others (Charles & Lester, 1984; Sigurdson et al., 1994). The Dutch investigators developed, piloted, and implemented an instructional program that was conducted by fifth-grade classroom teachers. The word problems used in the present study were adapted from this investigation. The intervention group was composed of four classes while seven control classes followed their typical mathematics instruction. The researchers believed they might influence students' development of effective problem-solving behaviors by supplementing current instruction with problem-solving lessons.

The purpose of the intervention was to make students (a) more aware of the different phases of the problem-solving process, (b) develop an ability to monitor and evaluate oneself during problem solving, and (c) master eight ways to solve problems (Verschaffel et al., 1999). These problem-solving approaches were (1) draw a picture, (2) make a list or table, (3) distinguish necessary information from irrelevant material, (4) use real-word knowledge, (5) make a flowchart, (6) guess and check, (7) look for a pattern, and (8) simplify the numbers. Researchers designed 20 lessons that were implemented over a four-month period. Lessons were guided by three "pillars" to construct a successful mathematics learning environment (Verschaffel et al., 1999, p. 202).

The first pillar was that problem-solving instruction should use realistic, complex, and open problems. Problems related to students' real-life experiences so that solving these problems might feel meaningful. Complex problems facilitated engagement in the problem-solving process. Open problems permitted a variety of problem-solving approaches including multiple representations. For example, a simple translation task does not satisfy the first pillar. The second pillar was the use of a variety of instructional techniques including (a) short, whole-class introductions to the problem, (b) small-group collaborative problem solving, (c) individually completed independent work, and (d) concluding whole-class discussions to wrap up instruction and reflect on the concepts and skills learned that day. The final pillar was establishing a classroom culture with social and sociomathematical norms for teaching and learning mathematics and problem solving. The authors describe the classroom culture as one that used (a) stimulating activities, (b) holding discussions with students about what counts as a good mathematics problem and response, as well as (c) appropriate mathematical procedures. Another feature of this culture was repositioning the role of teacher and students so that the teacher was not perceived as the holder of knowledge but rather a guide and mentor (Verschaffel et al., 1999). The teacher was the driving force in developing these classroom norms for mathematics teaching and learning but the students were involved in making decisions as well.

A before-during-after format similar to Charles and Lester's study (1984) was employed. Lessons activated students' prior knowledge, engaged them in a problem-solving activity, and ended with an opportunity for reflection and synthesis. Researchers administered reliable versions of a pretest, posttest, and retention test composed of ten word problems that had similar problems across all three measures. The word problems were designed to be open, complex, and realistic. Two raters scored each test; responses were denoted as correct, incorrect, technical error, or no answer. A technical error meant that a student conducted the appropriate steps but made a mistake in procedures during problem solving. Because the aim of the study was students' engagement in the problem-solving process, technical error responses were considered to be correct. Students also completed an achievement test. Finally, a sample of four lessons from intervention classrooms was selected to determine treatment fidelity.

Students in both groups improved their problem-solving performance but the intervention group made greater gains. The intervention group also outperformed the control group on the achievement test. The addition of twenty problem-solving lessons enhanced students' mathematics learning and problem-solving behaviors.

Problem-solving investigators offer advice for researchers who plan to conduct problem-solving research in classrooms. "Presumably, the results would have been better . . . if we could have integrated the learning environment better within the regular mathematics lessons" (Verschaffel et al., 1999, p. 226). That is, more consistent exposure to problem solving might have improved students' outcomes on the problem-solving measures. Students frequently encounter complex real-world problems, and solving exercises as well as problems will prepare them for these challenges. A concern with separating problem-solving instruction from everyday instruction as it had been done in the three previous studies is the connotation that it presents to students: problem solving is distinct from mathematics. This issue informs the decision to thoroughly integrate problem-solving instruction into typical daily mathematics instruction.

Integrating Problem Solving into Daily Mathematics Instruction

The previous studies (i.e., Charles & Lester, 1984; Sigurdson et al., 1994; Verschaffel et al., 1999) supplemented typical mathematics instruction with components of problem-solving instruction. Results were fairly positive except for one finding (Sigurdson et al., 1994) that the additional problem-solving instructional component did not necessarily support average-achieving and above average-achieving students' achievement. Evidence did not indicate students' outcomes as a result of continuous mathematics instruction delivered through problem-solving contexts. To more fully integrate problem-solving instruction within typical mathematics teaching, Verschaffel and De Corte (1997) aimed to determine the value of teaching mathematics in problem-solving contexts during a two-and-a-half week investigation. In an earlier study, Lampert (1990) immersed herself in the classroom to better understand the teaching and learning process. She integrated components of problem solving into her daily mathematics instruction. Verschaffel and De Corte as well as Lampert's study provide a foundation for becoming the instructor during an investigation as well as ideas for implementing effective mathematics and problem-solving instruction on a daily basis.

Verschaffel and De Corte (1997) examined whether 10-12 year old children might develop positive dispositions towards mathematical modeling as a result of experiencing mathematics instruction through problem-solving contexts. They encouraged students to think about problem solving as a multistage process (Verschaffel et al., 2000) and used word problems during instruction. The instructional intervention lasted approximately two-and-a-half hours each day over two-and-a-half continuous weeks at an elementary school in the Netherlands. One fifth-grade classroom was the intervention group while two similarly sized sixth-grade classrooms were the comparison group.

Verschaffel became a fifth-grade teacher during the study and taught five lessons that drew on students' knowledge of real-world situations such as purchasing comic books and going to a swimming pool. The student-centered instruction typically incorporated discussion during its multiple phases. There was a "central role of interactive and cooperative learning through small-group work and whole-class discussions" (Verschaffel & De Corte, 1997, p. 582). In phase one, students worked on a problem in mixed-ability groups of three to four students and then responded to a reflection question (e.g., "What did you learn from solving this problem?" [Verschaffel & De Corte, 1997, p. 582]). In phase two, Verschaffel and the students discussed the problem-solving process and the result from working on the problem. Students' misconceptions and ideas were explored so that the students could learn from each other. Students returned to their original groups and worked on approximately four similar problems during phase three and then engaged in another whole-class discussion during phase four. During phase five, students solved one nonroutine word problem for homework that encouraged them to engage in the problem-solving process. A final whole-class discussion allowed students to share their reactions to the assignment and reflections completed the instructional process. With so many discussions, Verschaffel had to establish new social and sociomathematical norms in the classroom if they were going to be effective.

Norms are critically important for mathematical discourse to be productive (Lampert & Cobb, 2003; Yackel & Cobb, 1996). Relevant mathematical discourse among participants makes group activities during instruction worthwhile (Franke et al., 2007; Stigler & Hiebert, 1999/2009). It allows individuals to convey mathematical ideas and helps them to make sense of mathematical notions (Cobb, Yackel, & Wood, 1992; Yackel & Cobb, 1996). Mathematically relevant communication "helps build meaning and permanence for ideas and makes them public" (NCTM, 2000, p. 60). Norms for discourse that encourage and perpetuate meaningful mathematics discussions must be established before it can have a lasting and meaningful impact on students' learning (Franke et al., 2007; Yackel & Cobb, 1996). Teachers influence students' use and type of communication based on the classroom norms and their behaviors (Williams & Baxter, 1996; Yackel & Cobb, 1996). Those who behave as the holder of knowledge or fail to allow students to justify solutions contribute to a learning environment where students are not responsible for their own learning (Boaler & Staples, 2008; Huang, Normandia, & Greer, 2005). In Verschaffel and De Corte's study (1997), Verschaffel explained to students that his role and actions in the classroom might appear different than they had previously experienced, such as becoming a co-problem solver during lessons. He enacted norms that he believed might facilitate productive mathematical behaviors and dispositions. These sociomathematical norms included determining what counts as (a) a good mathematical word problem, (b) a reasonable solution way to solve a problem, (c) an appropriate response, and (d) a satisfactory explanation (Verschaffel & De Corte). It was expected that students would show dramatic improvement in their problem-solving performance after learning in this environment.

A pretest with ten word problems and five simpler translation tasks was administered, and participants completed a similarly constructed posttest. Participants in the intervention and comparison groups also completed a retention test one month following the study. Students' responses were classified as realistic, nonrealistic, technical error, no answer, or other answer. A realistic response indicated a correct answer whereas no answer was an incorrect response, likely attributable to using the problem's information in an inappropriate manner. A technical error indicated that students' answered the problem correctly except for a slight error such as forgetting a decimal point or accidentally adding an extra zero when previous work indicates the correct answer (e.g., 100 instead of 10). The other answer category was applied when a student's response could not be classified. To complement the data from the measures, one videorecording of each classroom was made yet the tapes were not analyzed.

Participants from the intervention group responded with realistic responses the fewest number of times on the pretest but the most often on the posttest (i.e., 7% and 51% respectively). Students in the intervention group improved but the higher performing students experienced the greatest benefits. Those who experienced these novel lessons continued to outperform their peers on the retention test one month following the intervention. Students in the intervention group also developed positive dispositions towards nonroutine word problems whereas the control groups did not. This study provides evidence that students can learn to solve word problems, effectively engage in the problem-solving process, and develop positive dispositions towards word problems after a brief intervention. The short time period informs the dissertation research because it indicates that students quickly learn to adapt to a potentially novel learning environment and experience positive benefits from engaging in daily problem-solving instruction. This investigation also demonstrates that students can behave like mathematicians who regularly engage in problem solving. Feasibility studies like Verschaffel and De Corte (1997) and Lampert (1990) provide a context for mathematics education researchers to better understand the teaching-learning process.

Lampert (1990) immersed herself in one fifth-grade mathematics classroom for an academic year, hoping to make students' understanding of mathematics more aligned with how mathematicians perceive understanding mathematics. She established appropriate norms for classroom discourse that facilitated mathematics learning. For example, encouraging students to use concept-oriented language meant shifting students' comments from an individual (e.g., I don't agree with you) to the individual's idea (e.g., I don't agree with your idea). Similarly, "I think" (p. 54) statements were encouraged because they are powerful indicators of student's demonstrating ownership of learning and indicate whether students have made sense of mathematical ideas rationally. Students in her classroom routinely discussed their thinking in small-group and whole-class discussions. Lampert asked students to continually revise their thinking and share these revisions during whole-class discussions. She frequently questioned students and used their justifications of their solution as an opportunity to assess their thinking. Through these discussions, students were expected to become more comfortable using proof and conjecture-oriented language. To ensure that students had opportunities to engage in mathematics like mathematicians, she planned and enacted lessons around tasks that required concerted effort, thinking about a concept and its essential skills, and building upon prior knowledge.

She chose problems that encouraged students to think carefully and investigate multiple representations of a mathematical model and frequently reflect on the accuracy and appropriateness of the result from mathematical analysis. Students were encouraged to write their solutions on the board and the class was asked whether anyone wanted to “question so-and-so’s hypothesis” (Lampert, 1990, p. 40). Solutions were hypotheses until they had been mathematically justified using mathematical language. Lampert’s interactions with students were intended to promote that the teacher was a model of what it means to think like a mathematician. She co-explored problems with students, engaged in mathematical discourse, and encouraged them to challenge her ideas and ask for justification.

She used several instructional methods and often implemented a think-pair-share type of instruction. She posed a problem and took time to make certain each student understood the task and its text. Next, she observed students working independently and then listened to their peer-to-peer discourse about the problem. Finally, she initiated a whole-class discussion that clarified terms, symbols, and definitions for students, followed by collaborative problem exploration. Lampert (1990) examined teaching episodes individually and then looked for patterns of change in students’ outcomes. As a result of her instruction, students developed effective problem-solving behaviors, indicated more positive feelings about doing mathematics, and learned to work collaboratively to solve challenging problems.

Lampert's (1990) study provides clarity on a number of issues relating to the teaching and learning of mathematics. First, students are able to think and behave like mathematicians when given the opportunity. To attain this goal, the teacher must choose rich problems that require concentrated thinking and revising of ideas, and encourage collaboration and discourse that focuses on mathematics topics. Second, focusing on a few concepts provides a foundation for examining multiple ways to solve problems. When students master a few concepts during an academic year, they have opportunities to learn a wide variety of representations and procedures that likely will benefit their achievement and problem-solving performance in the long term. Lampert described students' thinking about the viability of multiple ways to solve problems as a result of her instruction. Her study delineates a rationale for making rich problems a focus of instruction as well as utilizing discourse to promote mathematics learning and effective problem-solving behaviors.

Model-eliciting Activities

Model-eliciting activities (MEAs) are related to the types of activities used in this dissertation study. These tasks are ill-structured, open-ended, complex, realistic tasks (Chamberlin & Moon, 2008; English & Sriraman, 2010). They are typically small-group activities meant to support students' mathematical modeling for the present and future problems (Lesh & Doerr, 2003; Lesh & Harel, 2003). An MEA differs from a traditional word problem in three ways: (1) the process and product are both important elements instead of the product only, (2) problem solvers can judge the adequacy of their mathematical model and solution by its relevancy and appropriateness for a problem's context whereas it may not be feasible with a word problem, and (3) the mathematical model is usually employed for other similarly structured problems whereas problem solvers might use the mathematical model for a word problems once (Lesh & Harel, 2003). MEAs also precede formal mathematics instruction (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). Investigations have drawn on MEAs as a means to explore their effect on students' higher order thinking skills (Lesh et al., 2003) and metacognition (Lesh, Lester, & Hjalmarson, 2003). Furthermore, MEAs often connect students' knowledge of mathematics, other disciplines, and outside of school (English, 2009; English & Sriraman, 2010).

English (2009) investigated students' outcomes as a result of implementing an MEA with one class of seventh-grade students. The activity's purpose was to support students to extend, explore, and refine ideas gained while solving previous modeling problems. Students were presented with a situation about a summer reading program for secondary students as well as a data set, asked to determine an appropriate solution, and finally explained and justified their response in a verbal statement. Problem solvers worked in groups of three to four students over three consecutive 50-minute class periods. Participants created pictorial models, which helped them develop symbolically-oriented mathematical models. They also showed an ability to quantify elements of a context in order to solve the problem. The problem's aspects included the length of a text and the text's readability level. Students solved the open, complex, and realistic problem and the lesson provided evidence that it is possible to weave problem solving, mathematics, and other subject areas. English (2009) advocates that MEAs and other similar problem-solving activities "should not be viewed as additional activities that add further load to an already crowded curriculum and overburdened teacher… they should be used to introduce, develop, consolidate, and enrich core concepts and processes" (p. 172-173). The present study heeded this message and participants experienced MEA-type activities as part of their daily mathematics instruction rather than as a supplement.

Summary of Problem-solving Instruction

Several studies have shown that supplementing mathematics instruction with problem-solving aspects enhanced students' problem-solving performance and assisted with developing productive problem-solving behaviors (Charles & Lester, 1984; Sigurdson et al., 1994; Verschaffel et al., 1999). Supplementing mathematics instruction looked slightly different across studies. In Charles and Lester's (1984) investigation, intervention group teachers devoted five to twenty-five minutes of instruction to problem solving. A subsequent study indicated that dedicating 10 instructional minutes each day was also sufficient for improving eighth-grade students' problem-solving performance (Sigurdson et al., 1994). Results from this study also indicated that outcomes are not necessarily positive for all students. More specifically, average-achieving and above average-achieving participants from the problem-process group had lower achievement scores than their peers in the other groups. Finally, Dutch fifth-grade students provided more correct responses to word problems after experiencing 20 problem-solving lessons over the course of an academic year than their peers in comparison classrooms (Verschaffel et al., 1999). Furthermore, the intervention participants had better achievement scores than their peers, contrary to earlier findings (Sigurdson et al., 1994). These studies provide evidence that problem-solving interventions generally produce positive problem-solving outcomes but achievement-related effects are more uncertain.

There is also some evidence that integrating problem solving into daily mathematics instruction positively influences adolescents' problem-solving performance. Lampert (1990) encouraged her students to discuss and critique each other's ideas about solving complex problems. Moreover, she incorporated problems into daily instruction so that students had rich problems that could be solved in multiple ways. Similarly, Verschaffel and De Corte (1997) employed complex, open, and realistic problems as part of their mathematics instruction. Both investigations provide evidence that teaching mathematics through problem-solving contexts on a regular basis supports students' problem-solving performance.

Several researchers have explored students' experiences with MEAs (Chamberlin & Moon, 2008; English, 2009; English & Sriraman, 2010; Lesh & Harel, 2003). MEAs provide experience working with open, complex, and realistic problems and support students' problem-solving development (English & Sriraman, 2010; Lesh & Harel, 2003). English (2009) indicates that MEAs can be used as the central component of mathematics instruction, precede formal explicit instruction, and are not intended to supplement typical mathematics instruction. These rich activities when implemented in the context of discourse-rich, student-centered mathematics instruction lead to improving students' problem solving, engagement in mathematical modeling, and a number of other positive outcomes (Chamberlin & Moon, 2008; English, 2009; Lesh & Zawojewski, 2007).

As indicated in the introduction, changes in students' representation use were examined in the present study. Detailed analyses are necessary to convey to the research and teaching communities what changes occurred in students, possible explanations for the changes, and any differential effects of these changes. Students' representation use is an indicator of whether students are engaging in effective problem-solving behaviors and thinking (Verschaffel et al., 2000). None of the studies described thus far aimed to improve the number and types of representations used by elementary or middle-grades students. The next section discusses the limited research conducted in elementary and middle-grades classrooms within the area of instruction promoting multiple ways to solve problems.

Instruction Focusing on Multiple Representations

Students who are able to implement more than one representation to solve a word problem are more likely to solve it than peers who know only one way (Bostic & Pape, 2010; Herman, 2007). Knowing multiple ways to solve a class of problems also indicates that a learner has developed strategic competence in an area (Kilpatrick et al., 2001). Two instructional programs were created around the empty number line representation, implemented them in matched second-grade classrooms, and examined student-related outcomes (Klein et al., 1998). Empty number lines are an excellent representation for teaching students about addition and subtraction with whole numbers up to 100 (Klein et al., 1998). One instructional program called the Realistic Program Design (RPD) was created so that students were encouraged to share their ideas and investigate multiple representations and procedures for adding and subtracting numbers. Participants learned about the strengths and weaknesses of these representations in a discourse-rich, student-centered instructional environment. Word problems used during instruction drew on contexts relevant to students. The RPD program featured whole-class instruction and discussion about representations and procedures to solve word problems during approximately one third of the instructional period. In the other instructional program, Gradual Program Design (GPD), the teacher taught students how to use one representation and procedure at a time. There was less discussion in the GPD classroom and students spent more time completing exercises than problems. Word problems were a part of the GPD program but were treated as opportunities to practice applying a known representation and set of procedures. It was hypothesized that the GPD program might benefit the entire class' achievement more whereas the RPD program might improve low-ability students' achievement.

The programs were implemented in 10 second-grade classrooms at nine comparable primary schools. Classes were matched based on their prior achievement and randomly assigned to GPD and RPD conditions. An arithmetic test composed of 21 total tasks was administered on two occasions approximately two months apart. Students were asked to solve each problem and show their work. Paired t-tests were used to determine whether there were any differences between students' outcomes on the tests.

The interventions did not necessarily result in differential performance but there were differences in students' problem-solving behaviors. Students in the RPD program "changed their use of procedures according to the characteristics of the problem" (Klein et al., 1998, p. 457) whereas participants in the GPD group tended to use the taught representation and procedure, indicating that RPD participants developed greater strategic competence as a result of the instruction. "There were almost no differences in procedural competence between the two groups of pupils. When significant differences were found, they were mostly in favor of RPD pupils, especially in the case of subtraction problems" (Klein et al., 1998, p. 460). At the end of the academic year, children in the GPD program “still lagged far behind the RPD pupils in [strategic] flexibility” (p.460) and “the RPD pupils attained and sustained a higher level of flexible problem solving than did the GPD pupils” (p. 461). This investigation provides evidence that instruction encouraging multiple ways to solve tasks leads to young students development of effective problem-solving behaviors.

Summary of the Problem-solving Literature

Problem-solving instruction can enhance students’ problem-solving performance and encourage developing effective problem-solving behaviors (Klein et al., 1998, Lampert, 1990; Verschaffel et al., 1999). This instruction is best characterized by (a) problems that encourage critical thinking and reflection (Chamberlin & Moon, 2008; Lampert, 1990; Verschaffel & De Corte, 1997; Verschaffel et al., 1999), (b) small-group and class-wide mathematical discussions (Charles & Lester, 1984; Lampert, 1990), (c) opportunities for students to try different representations and talk about possible methods of analysis (English, 2009; Klein et al., 1998; Lampert, 1990; Lesh & Harel, 2003), (d) a teacher who behaves more like a facilitator and co-problem solver than someone who disseminates mathematical ideas (Lampert, 1990; Preston & Garner, 2003; Verschaffel & De Corte, 1997), and (e) students frequently justifying their ideas to one another rather than waiting for the teacher’s confirmation (English, 2009; Lampert, 1990). Many of these studies described establishing sociomathematical norms in the classroom, which are crucially important to educators who expect their students to discuss mathematics and make sense of new ideas from this peer-to-peer discourse.

Discussions about the tasks and possible representations involved in working towards a solution supported students’ use of multiple representations (Klein et al., 1998), improved participants’ mathematics achievement (Sigurdson et al., 1994;

Verschaffel et al., 1999), and enhanced how students solve problems (Charles & Lester, 1984; Lampert, 1990; Verschaffel & De Corte, 1997; Verschaffel et al., 1999). These studies provide evidence that it is feasible to implement instruction that teaches mathematics content and problem solving while also supporting learners to develop strategic competence and become comfortable with using multiple representations to solve word problems. A necessary step forward for this type of research is conducting this type of instruction and drawing on the Standards (CCSSO, 2010; FLDOE, 2007). Students in classrooms where instruction encouraged multiple representation use were more efficient problem solvers, and at times more effective than their peers experiencing traditional instruction (Klein et al., 1998). The review of relevant literature informs the present study that aims to improve sixth-grade students' problem-solving performance and representation use.

Connections

This section will address the connections between prior literature and the present study, gaps and limitations of problem-solving research, and ways the present research managed these issues. Investigations into problem solving often draw on several aspects of the problem-solving process. For instance, Pape's (2004) examination includes several steps of the problem-solving process, albeit his research greatly informs the research and teaching community about how students read and understand a problem's text. Reading and understanding text is important; however, most studies did not collect students' reading comprehension scores, including Pape (2004) and Greer (1993). A strong correlation between reading comprehension and problem-solving performance (Vilenius-Tuohimaa et al., 2008) justifies including reading comprehension as a covariate in future problem-solving performance analyses. A similar argument can be constructed for including prior mathematics achievement and free-and-reduced lunch status based on the literature (Pape; Vilenius-Tuohimaa et al., 2008). As a result of their work, several covariates were included in the regression analyses.

The focus of the present investigation is to implement an instructional intervention to enhance sixth-grade students' problem-solving performance and representation use. The literature informs ways to maintain high quality research as well as areas for improvement. First, prior research suggests that approximately 20 lessons are sufficient to improve students' problem-solving performance and behaviors (Verschaffel & De Corte, 1997; Verschaffel et al., 1999). Verschaffel & De Corte (1997) were successful after two-and-a-half weeks of consecutive problem-solving instruction that lasted two-and-a-half hours, yet their study did not include teaching mathematics content required by their national standards and their instructional approach is not feasible in most classrooms in the United States because of the lessons' duration. The present study demonstrates ways to align mathematics standards and mathematics instruction with a problem-solving focus. There were positive outcomes after twenty lessons that were periodically delivered to students over four months (Verschaffel et al., 1999). Fifth- and seventh-grade students improved their problem-solving behaviors fairly quickly as well (Charles & Lester, 1984), even when problem-solving instruction was limited to ten minutes each day and separated from everyday instruction. As a result, there is some evidence that brief and/or limited interventions support students' problem-solving performance.

Second, rich tasks that support problem solving and mathematical discourse provide a context for learning mathematics (English, 2009; English & Sriraman, 2010; Lesh & Harel, 2003). MEAs have been used in middle-grade classrooms to enhance students' conceptual and procedural understanding (English, 2009). The present study builds upon prior research by developing and implementing MEA-type activities linked to the Standards (CCSSO, 2010; FLDOE, 2007) as the focus of classroom instruction.

Instruction in several studies employed a before-during-after-type lesson plan format (Charles & Lester, 1984; Lampert, 1990; Verschaffel et al., 1999). Lampert's (1990) think-pair-share format aligns with the before-during-after format and provides structure for delivering problem-solving instruction. Lessons for the present study followed this format and included opportunities for students to think independently, engage peers in small groups or pairs, and discuss ideas as a class. It has been suggested that reflection should be added to any lesson because it enhanced students' synthesis from the tasks and was integral to fostering effective problem-solving behaviors (Verschaffel et al., 1999). The choice to use one representation over another while problem solving should be guided by the effectiveness of the representation for solving the problem as well as its efficiency and this was a part of discussions with participants in this study.

Children are capable of learning multiple ways to solve problems, discussing mathematics and problem solving, and developing effective problem-solving behaviors (Klein et al., 1998; Lampert, 1990). Some have employed word problems as well as exercises in their research (Klein et al., 1998). Tasks were not necessarily problematic enough for second-grade students and more like simple translation problems that use “word problemese” (Lave, 1993, p. 77). A translation problem typically requires a student to translate from one representation into a symbolic expression and then execute algebraic or arithmetic computations. For example, a symbolic expression written as a verbal statement without the complexity usually associated with the characterization of a problem is a translation problem. Lampert’s (1990) study did not make word problems a central component of instruction, albeit her results suggest that rich tasks, including word problems, combined with discourse-rich student-centered instruction are likely to facilitate students’ mathematics learning and problem-solving behaviors, which includes learning multiple ways to solve problems. Two studies indicate that students benefit when instruction uses word problems (Klein et al., 1998; Lampert, 1990).

Instruction that facilitates learning mathematics and problem solving aligns with recent expectations for learning mathematics, specifically the Standards of Mathematical Practice (CCSSO, 2010) and recently adopted Next Generation Sunshine State Standards (FLDOE, 2007). Prior investigations where a researcher immersed him/herself in the study’s setting as an instructor have been successful at improving instructional outcomes for students and provide a context for this investigation’s intervention. One drawback to work such as Lampert (1990) and Verschaffel and De Corte’s (1997) is that it can be more difficult to generate conclusions that impact classrooms with grade-level or content teachers who manage a multitude of factors and make several instructional decisions during one academic period. Although this seems like a limitation, it is an essential step because feasibility research like this dissertation study may eventually lead to scaling up to a larger group of students and include providing professional development to classroom teachers so that they might enact an intervention similar to this one.

The problem-solving process as written is too complex for students to manage (Verschaffel et al., 1999) hence students need versions with simplified language that makes sense. Polya's (1945/2004) seminal text *How to Solve it* discusses appropriate ways to scaffold students engaged in problem solving at great length. "Text → Situation Model" does not give students enough information to know what to do. Students need a better context in order to engage in these stages of problem solving. Characterizing the process in more descriptive terms along with questions such as "What is this problem asking you to find? What do you know?" provide students scaffolding necessary to successfully complete the problem-solving process and solve nonroutine word problems (Polya, 1945/2004). Students need assistance at times so that the process and finding a problem's solution are within their developmental grasp (Charles & Lester, 1984; Lampert, 1990). Charles and Lester (1984) recommend that students have visible reminders such as posters and desk-sized models of the process while problem solving. In line with these recommendations, students in the intervention group were provided with a poster-sized and desk-sized model of the problem-solving process (Appendix A and Appendix B, respectively) and guiding questions were a part of everyday instruction. A discussion of how the questions and model were developed and implemented in the intervention classroom is provided in Chapter 3.

Adequately capturing the instruction during an instructional intervention is critical for framing what influenced participants' outcomes. One study videotaped a few lessons (Verschaffel et al., 1999) whereas in another one an observer took fieldnotes about instruction in the control classrooms (Sigurdson et al., 1994). Many authors share that a limitation of their research was that not enough observations of the instruction were made in intervention and control classrooms (Verschaffel et al., 1999; Charles & Lester, 1984). In the present study, evidence from frequent recordings of instruction as well as a trained observer facilitated accurately characterizing instructional differences between the intervention and comparison classrooms and ascertained treatment fidelity in the intervention classroom. Furthermore, multiple cameras were used to capture classroom activities. Observations and recordings of the present study indicated how much instructional time was typically dedicated to individual, small-group, and whole-class discussions, which was an improvement over the very rough estimates provided by previous researchers (e.g., Charles & Lester, 1984; Sigurdson et al., 1994; Verschaffel et al., 1999). Furthermore, observations and videorecordings assisted in making comparisons between instruction in intervention and control classrooms. An observer enhanced this study because her fieldnotes captured the instruction through the eyes of a trained mathematics educator.

Research related to elementary and middle school students' use of representations in conjunction with problem-solving performance still needs to be conducted. Studies that examined problem solvers' representation use provide insight into students' problem solving, yet there are still gaps in this area specifically related to whether young students could learn to successfully implement alternate representations to solve word problems. This study aims to examine whether students from the intervention group use more representations and if there is a relationship between using nonsymbolic representations (i.e., pictorial, tabular, graphical, or verbal) and membership in the intervention group.

Some studies used valid sampling techniques (i.e., purposeful or representative) when studying students' behaviors so that results would be generalizable. Only one study (i.e., Santos-Trigo, 1996) examined students' multiple representation use, yet convenience sampling from two schools was used, which limited the generalizability of the results. The current investigation improved upon Santos-Trigo's work by convenience sampling students from one school rather than convenience sampling from two schools. Furthermore, classrooms were selected so that differences (e.g., reading achievement, mathematics achievement, and free-and-reduced lunch status) between the intervention and comparison group were minimized. Nearly every student from the three selected classrooms volunteered, which provided a better representative sample than self-selected volunteers from two schools as in Santos-Trigo's research (1996).

Prior analyses considered strategies that were representationally unique to be different (i.e., Bostic & Pape, 2010; Santos-Trigo, 1996). This dissertation study followed this approach by coding students' strategy use based on its representation. One category was added, which had not been examined by the previous investigators: mixed representations. Employing two distinct representations that drew on the same type of representation were counted as two unique representations (i.e., mixed). Thus, in the present study, the focus was on the number and types of representations used.

Every investigation discussed here focused on problem solving, but the way students' problem-solving performance was measured is somewhat concerning. Mayer and Wittrock (2006) suggest that a learner cannot readily solve a problem, but instead the individual needs to think deeper than usual, hence routine word problems (i.e., word problems with cuing and scaffolding-type language) as well as verbal translations of exercises, do not satisfy Mayer and Wittrock's characterizations of a problem. Exercises have their place in instruction: to increase students' efficiency while problem solving and to improve procedural competence (Kilpatrick et al., 2001; Mayer & Wittrock). Routine problems and translation tasks were prevalent on instruments measuring students' problem-solving performance (e.g., Charles & Lester, 1984; Klein et al., 1998; Sigurdson et al., 1994). Complex and nonroutine word problems characterize the same types of tasks. Items on the measures in the current study are open, realistic, and complex word problems and meant to be similar to prior measures (Verschaffel et al., 1999). Thus, the question of whether a problem-solving oriented instructional intervention improves students' performance on a test of word problems was addressed.

Another issue with word problem tests comes up in Greer's (1993) study. The problem pairs on his instrument frequently used different language, and the second item in the pair typically had more complex sentence structures, which would be more problematic for students. In one item pair, the first problem had 20 words whereas the second one had 40 words. Problems were also not examined for their readability. In the present study, readability of the pre- and posttest problems was examined and results indicated that measures were appropriate for middle school students. Finally, students' contextual knowledge and reading comprehension were not measured, which may contribute to the variance in results. Improvements were made upon these studies by using word problems, making certain that problem pairs are similar in readability and content, and an expert panel examined the items prior to their administration.

A comprehensive expert panel is able to judge whether the tasks on the instruments are grade-level appropriate, problematic, realistic, allow for multiple representations, and that item pairs on the pretest and posttest are similar. The expert panel consisted of two mathematics educators and one experienced classroom teacher.

Finally, studies (i.e., Klein et al., 1998; Sigurdson et al., 1994; Verschaffel et al., 1999) examined the effect of the intervention on students' achievement. Klein and his colleagues (1998) used an arithmetic test given annually to Dutch elementary students whereas others designed reliable achievement tests (Sigurdson et al., 1994; Verschaffel et al., 1999). In these studies, the problem-solving interventions enhanced some or all students' achievement but no study used prior mathematics achievement scores as a covariate when analyzing problem-solving performance. Further studies are necessary to confirm whether the effects of interventions like those discussed here impact students' content-specific achievement as measured by a unit test. Considering that there is a positive correlation between mathematics achievement and problem-solving performance (Verschaffel et al., 1999), including prior mathematics achievement as a factor might improve the explanatory power of this study's results.

The literature provided a foundation for exploring ways to enhance students' problem-solving performance and representation use. A number of investigators constructed and implemented instructional interventions and some conducted the instruction themselves. Prior investigations supported further exploring approaches to support students' outcomes after experiencing problem-solving oriented mathematics instruction. Research also provided a foundation for conducting a study with such aims as theirs.

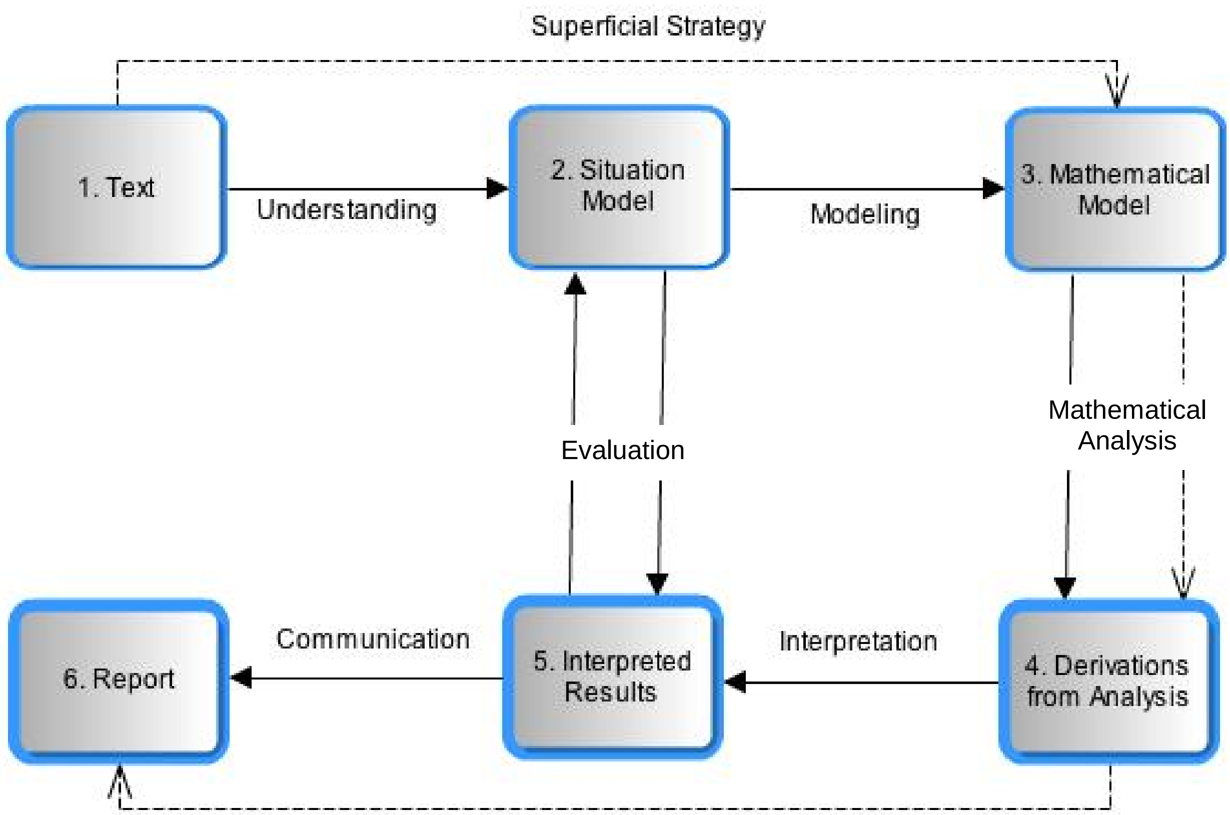

Figure 2-1. A model of the problem-solving process.

Each textbox represents a stage of the problem-solving process. Black lines indicate the pathways followed by effective problem-solvers whereas the dashed lines indicate a superficial pathway. Adapted from *Making Sense of Word Problems* (p. xi), by L. Verschaffel, B. Greer, and E. De Corte, 2000, Lisse: Swetz & Zeitlinger.

CHAPTER 3
METHOD

Overview

The goal of the present study is to examine the effects of an instructional intervention on sixth-grade students' problem-solving performance and representation use. The intervention involves teaching mathematics through problem-solving contexts. There are six research questions for this study.

Questions and Hypotheses

(1) Does the intervention influence students' performance on a test of word problems?

(1H) Students in the intervention group will show improved performance on a test of word problems after one month of the intervention.

(2) Does the intervention influence the total number of representations students use on a word problem test?

(2H) Students in the intervention group will use more total representations on the posttest than the pretest.

(3) Does performance on a test of word problems differ between students from the intervention and comparison groups?

(3H) Students in the intervention group will perform better on a word problem test than their peers in the comparison group.

(4) Does the total number of representations used on a test of word problems differ between students from the intervention and comparison groups?

(4H) Students from the intervention group will use more representations on a word problem test than their peers in the comparison group.

(5) Is there a relationship between intervention status and students' use of nonsymbolic representations on the problem-solving posttest?

(5H) There is a relationship between intervention status and use of nonsymbolic representations (i.e., pictorial, tabular, and mixed) on the problem-solving posttest.

(6) Does performance on a unit test differ between students from the intervention and comparison group?

(6H) There will be no differences between groups related to students' performance on the unit test.

Pilot Study

In order to gather data to answer these questions, several steps were taken to create problem-solving tasks for two measures (i.e., pretest and posttest). Measure construction followed the steps for test development outlined by Gall, Gall, and Borg (2007), which adhere to the *Standards for Educational and Psychological Testing* (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education, 1999). Those steps are (1) define the construct of interest, (2) define the target population, (3) review related instruments, (4) develop a prototype, (5) pilot the prototype and review results of analysis, (6) revise the instrument, and (7) collect data related to reliability and validity (Gall et al., 2007). A thorough review of the problem-solving literature was conducted after deciding to investigate sixth-grade students' (i.e., target population) problem solving (i.e., construct of interest).

A pilot study was conducted in November 2010 to determine how prototype problem-solving measures might function with sixth-grade students in the United States, to confirm each measure's dimensionality, explore item parameters, and investigate measure reliability (i.e., internal consistency and alternate-forms reliability). Analyses of students' responses led to revising the measures for the dissertation study.

Context

The pre- and posttest were administered in a school district in Florida different from the school selected for the dissertation study. This school was purposefully selected because its students had similar achievement compared to the school selected for the dissertation study. The pilot and dissertation school 2009-2010 demographic information are provided in Table 3-1. There were ten sections of sixth-grade mathematics. Tests were administered during students' mathematics period.

Participants. All sixth-grade mathematics students except for English Language Learners (ELLs) and children with a learning disability were asked for their assent as well as their parent's consent to participate. A total of 169 sixth-grade students completed both measures, which was 77% of the sixth-grade population at the school.

Measures

Problem-solving measures administered to Dutch fifth-grade students (Verschaffel et al., 1999) best fit the parameters of this investigation. An individual who is fluent in both Dutch and English translated the measures. Each measure had eight items. For example, one of the problems that was translated from the Dutch instruments was

> A group of 150 tourists wants to take a cable car to the top of a mountain. Each time a maximum of 9 tourists and a driver can go up. How many times will the cable car need to go up to the top of the mountain to take everyone there? (translated task from Verschaffel et al., 1999)

The translated items were then modified for readability, cultural relevance, and realistic contexts. The problems were intended to be complex in nature and it was expected that a problem solver could solve each task using a variety of representations. The sample item shown above was revised to create the following item:

> A group of 150 tourists were waiting for a shuttle to take them from a parking lot to a theme park's entrance. The only way they could reach the park's entrance was by taking this shuttle. The shuttle can carry 18 tourists at a time. After one hour, everyone in the group of 150 tourists reached the theme park's entrance. What is the fewest number of times that the shuttle picked tourists up from the parking lot?

This resulted in eight open-ended, items on the pre- and posttests that covered a variety of topics such as combinations and ratios (Appendices C and D). A Flesh-Kincaid grade-level readability analysis (Kincaid, Fishburne, Rogers, & Chissom, 1975) was conducted on both measures. Values to the left of the decimal point indicate grade-level appropriateness. The average readability score for the pre- and posttest was 5.81 and 7.01, which provides evidence that the items are fairly appropriate for sixth-grade students. A mathematics teacher from a school not participating in the pilot or dissertation verified that items were sufficiently complex for sixth-grade students, could be solved in multiple ways, and drew on realistic contexts.

Procedure

All participants completed the posttest one week after they completed the pretest. Participants were directed to solve each problem using multiple approaches, if known. The directions were read aloud to students. The teacher or researcher did not answer any questions related to the tasks. Students completed each measure during their mathematics period. A small group (i.e., less than 10% of the sample) was provided additional time during their lunch period to complete each measure.

Data analysis

Scoring protocol. Students' performance was scored as correct or incorrect using a scoring protocol (Appendix E). Since the focus of the analysis was on accurate representation rather than accurate computation, responses that contained an arithmetic error or other slight error were scored as correct. This is similar to prior analyses of students' problem-solving performance (Verschaffel & De Corte, 1997; Verschaffel et al., 1999).

Representation coding protocol. The representation coding protocol was created after reviewing prior research with students in this age range (Preston & Garner, 2003). A representation was counted when the student made a distinct and successful attempt at solving the problem. Lesh and Doerr's (2003) strategy categorization scheme influenced the decision to code the ways students solved a task based on the representations used. Students were expected to employ one of the following types of representations: (a) symbolic, (b) pictorial, (c) tabular, and (d) verbal. Employing a unique representation but committing an arithmetic error was counted as successful use of a representation. The number of representations within a category a student employed and the total number of representations was calculated.

Scoring and coding procedures. Students' responses were analyzed similarly for accuracy and representation use. A second coder assisted the researcher in scoring and coding students' responses to the tasks. Initially, one coder and the researcher practiced analyzing tests together in order to ensure calibration. After scoring ten tests, the pair agreed that they were familiar with the protocol and the process. Next, the researcher randomly selected 20% of the tests and each person scored or coded the tests individually. Interrater agreement was calculated using the r_{wg} function (James, Demaree, & Wolf, 1984). Interrater agreement is more appropriate for this study than interrater reliability because it is beneficial to show that two raters' decisions are similar and that ratings would be identical for the rest of the tests (LeBreton & Senter, 2008). The minimum value suggested for interrater agreement is $r_{wg} = .90$ (James et al., 1984).

Analyses. To examine the test and item properties, three analyses were conducted: Confirmatory Factor Analysis (CFA), Item Response Theory (IRT) analysis, and internal consistency and alternate-forms reliability analyses. CFA was conducted using Mplus version 6.0 (Muthén & Muthén, 2006) to determine whether one latent factor (i.e., problem-solving ability) accounted for the collinearity among multiple variables (Bryant & Yarnold, 1995/2005). Several criteria must be met for a CFA to converge: (a) the number of observations must be greater than or equal to the number of free parameters, (b) the factor must have a scale, and (c) there must be at least three indicators (Kline, 1998). Bryant and Yarnold (1995/2005) suggest a minimum of ten participants per item for conducting a factor analysis. The number of participants (i.e., observations) was larger than the total number of variances, covariances, and number of factor loadings because 169 students completed both pilot measures. Weighted least squares with adjusted means and variances was selected for the CFA. The factor loadings and variances were set to one for simplicity. Finally, there were eight items on the pre- and posttest thus the criteria for convergence were met in the present study. CFA was conducted to determine whether a one-factor model was appropriate, and IRT analyses were employed to calculate and examine item parameters.

Model fit was measured using the chi-square statistic, Root Mean Square Error of Approximation (RMSEA), Comparative Fit Index (CFI) and Tucker-Lewis Index (TLI). Nonsignificant chi-square values (Ullman & Bentler, 2009), RMSEA less than or equal to .06 (Bryant & Yarnold, 1995/2005), and CFI and TLI greater than .90 provide evidence of good fit (Ullman & Bentler, 2009). Good fit indicates that the covariance matrix sufficiently aligns with the structure of the model under examination (Ullman & Bentler, 2009).

Next, IRT analysis was conducted using R. Two R programs, eRm (Mair & Hatzinger, 2007) and ltm (Rizopoulos, 2006), were employed. IRT methods provide more useful approximations of item difficulty and discrimination than Classical Testing Theory approaches (de Ayala, 2009; Embretson & Reise, 2000). Item difficulties and discrimination were examined using the 1-PL model with unconstrained discrimination (i.e., discrimination varies freely). Item difficulty characterizes the necessary ability level needed to have a 50% likelihood of correctly answering an item (de Ayala, 2009). Item difficulties typically fall within a six-unit interval (i.e., [-3.0,3.0]), but items located below -2.0 or above 2.0 are characterized as easy and hard items, respectively (de Ayala, 2009). Item discrimination was not fixed initially. The fit of 1-PL logistic and 2-PL logistic model was compared using a likelihood ratio test and results indicated that the 1-PL logistic model sufficiently fit the data as well as a 2-PL model. Furthermore, a 1-PL model was appropriate because of sample size limitations. Other models (i.e., 2-PL or 3-PL) are not necessarily suitable for samples with fewer than 500 participants (de Ayala, 2009). As item discrimination increases, the problem provides more precise information over narrower range of abilities (Embretson & Reise, 2000). Finally, item information was found by calculating the product of the probability of a correct and incorrect response (Embretson & Reise, 2000). The sum of item information on each test determined the test information (Embretson & Reise, 2000).

Finally, internal consistency and alternate-forms reliability were calculated using a covariance structure model (Raykov, 2001) and the Pearson correlation, respectively. The covariance structure approach is a better internal consistency measure than Cronbach's alpha, which typically underestimates reliability (Raykov, 2001). To calculate internal consistency reliability, ρ, factor loadings, factor variance, and error variances are needed. Factor loadings of the items are summed and then this sum is squared. The result is multiplied by the variance of the scale's factor. This product is divided by the quantity of the same value plus the sum of the error variances of the scale's items. The alternate-forms reliability indicates the degree to which two forms of a test are equivalent (Ary, Cheser-Jacobs, Sorenson, & Razavieh, 2009). The minimum criterion for linking scores across tests measuring the same latent trait is .60 (Ary et al., 2009). Modest reliability (i.e., $\rho = .60$; $r = .60$) is sufficient for exploratory studies using newly developed tests and measures with few consequences for participants (Ary et al., 2009). Researchers should strive, however, for reliability near $\rho = .80$ and $r = .80$ or higher (Gall et al., 2007).

Results

Interrater agreement. After scoring the pretest and posttest, the pair had $r_{wg} = .99$ and $r_{wg} = 1$, respectively. Interrater agreement related to coding students' representations was $r_{wg} = .96$ for both measures. The interrater agreement exceeded the minimum criterion.

Descriptive information. Mean performance on the pre- and posttest are located in Table 3-2. Scores were slightly lower on the pretest than the posttest, but they were not significantly different. Students were most successful at solving question one on the pretest (i.e., 104 students) and posttest (i.e., 111 students). On average, students used symbolic representations on most pre- and posttest tasks with one exception. There were roughly an equal number of symbolic and pictorial representations for responses to question one on both measures. In total, there were 29 and 39 tabular representations on the pre- and posttest, respectively. Participants tended not to use verbal representations. There were a total of 11 and 14 verbal representations on the pre- and posttest, respectively. No participant employed a graphical representation.

Model fit. Results from the CFA for the pretest revealed a nonsignificant chi-square value $\chi^2(20) = 21.07$, $p = .39$ and the RMSEA indicated close fit, *RMSEA* = .03. CFI and TLI were both .99. Similarly, the factor analysis for the posttest resulted in a nonsignificant chi-square value, $\chi^2(20) = 21.42$, $p = .37$, *RMSEA* = .02, and CFI and TLI were both .99. There was sufficient evidence of good fit for one latent variable on both measures. CFA parameter estimates and thresholds are provided in Tables 3-3 and 3-4. After gathering evidence of good fit, item parameters for the pre- and posttest were examined.

Item parameters. Pre- and posttest item difficulties and discriminations were estimated using the 1-PL model with unconstrained discrimination. These values as well as item information are presented in Tables 3-3 and 3-4. The pretest items' difficulty ranged from -0.49 to 2.82. Standard error related to item difficulty was greater than 0.17 and less than 0.39. The first item had a negative item difficulty, which suggests an easier item. Items two and three had larger item difficulties, indicating fairly difficult tasks. Items four, five, six, seven, and eight had fairly similar item difficulties and might be characterized as somewhat difficult items. The discrimination for each item on the pretest was constrained by the program to 1.24, indicating that the items provide adequate information over a fairly wide range of abilities. Pretest item information ranged from 0.03 to 0.23 and test information was 0.93.

The results from analyses of the posttest were similar to those from the pretest for some items: item difficulties ranged from -0.66 to 3.47 and standard errors were as small as .17 and as large as .51. Item one on the posttest was the only task that had a negative item difficulty. Items two and three had fairly large item difficulties, indicating that most students were unable to solve them. Items five, six, seven, and eight had fairly similar item difficulties. Item discrimination for the posttest was 1.28, which was similar to the pretest. Posttest item information ranged from 0.01 to 0.21 and test information was 1.16. These results from the pretest and posttest informed instrument construction for the dissertation study.

Reliability. Internal consistency for the pre- and posttest was .79 and .88, respectively. Alternate-forms reliability was calculated using participants' overall performance on each test, which was $r = .60$.

Revisions to Measures and Protocols

Decisions related to final problem-solving measures were guided by four factors: improving (1) internal consistency and (2) alternate-forms reliability; (3) creating shorter instruments; and (4) maintaining items with item difficulties between negative one and two. The tests were shorter than the originals and provided sufficient information resulted from a pre- and posttest with five problems each. Items one, five, six, seven, and eight were retained for the dissertation measures. The internal consistency for the five-item pre- and posttest was estimated to be .79 and .72, respectively. The justification for the five-item measures used in the dissertation study is provided below.

Tasks with item difficulty greater than two were perceived to be too difficult for an average-achieving sixth-grade student. Items two and three on both measures were above 2.0 and were not retained for the dissertation measures. Item difficulty for the fourth task on the pretest indicated that only exceedingly above-average students solved it whereas slightly above-average students were able to solve the fourth item on the posttest. The fourth item was deleted from both measures since they did not have similar item difficulties. After deleting items two, three, and four from both the pre- and posttest, each measure had five items.

Results from this pilot study also supported refining how students' representations were coded. There were occasions where students employed multiple representations (e.g., a picture and a symbolic expression). As a result of the analyses, the representation coding protocol was modified so that it contained a category for a mixture of representations. This new category (i.e., mixed) contained several subcategories indicating which representations were used during problem solving. For example, the symbolic-pictorial code might be applied when a student used an approach that included elements of a symbolic- and pictorial-orientation. There was no pictorial-symbolic code because it was indeterminable whether the student used the pictorial or symbolic representation first. The Representation Coding Protocol (Appendix F) used in the dissertation study was identical to the one used in the pilot study except for the addition of the mixed category.

Research Design for the Dissertation

This study employed a nonequivalent control-group quasi-experimental research design (Gall et al., 2007). Quasi-experimental designs are necessary when participants cannot be randomly assigned to an intervention thus there are multiple potential threats to internal validity (Gall et al., 2007). The researcher who designed the instructional intervention was also the instructor in that classroom therefore "the researcher" or "the instructor" describe the same person.

Context of the Study

Sixth-grade students were chosen as the population of interest because of developmental and academic-related reasons as well as prior investigations conducted with students of approximately this age (e.g., Charles & Lester, 1984; Verschaffel & De Corte, 1997; Verschaffel et al., 1999). Early adolescence has been theorized as an important developmental time for students because many children develop academic-related beliefs that are maintained throughout their K-12 academic career (Zusho & Pintrich, 2002). Students at this age-level have developed sufficient content knowledge that allows them to solve a variety of problems using multiple representations (Preston & Garner, 2003). The sixth-grade mathematics standards provide ample opportunities to solve word problems and explore representation use (FLDOE, 2007). Four sixth-grade benchmarks from the NGSSS (FLDOE, 2007) were the focus of instruction during the study:

- MA.6.A.2.1 Use reasoning about multiplication and division to solve ratio and rate problems
- MA.6.A.2.2 Interpret and compare ratios and rates
- MA.6.S.6.1 Determine the measures of central tendency (mean, median, and mode) and variability (range) for a given set of data
- MA.6.S.6.2 Select and analyze the measures of central tendency or variability to represent, describe, analyze and/or summarize a data set for the purposes of answering questions appropriately.

A K-12 school located in Florida was selected for the dissertation study using convenience sampling. The participating school has roughly 1,100 students and its student body represents the diversity in the state of Florida (Public School Review, 2010). Its students tend to perform well on high-stakes tests but the school did not make Adequate Yearly Progress in the year prior to this study (FLDOE, 2010b). The student population at the school level is approximately 51% Caucasian, 25% African-American, 16% Hispanic, 5% Multiracial, 3% Asian/Pacific Islander, and less than 1% are American Indian (FLDOE, 2010a). Roughly 17% of the students are eligible for the free-or-reduced lunch program and there are no ELLs at this school. About 10% of the student body are identified as having a disability and receive accommodations.

Layout in the Classroom. All sixth-grade students received mathematics instruction in the same classroom resulting in identical classroom layout across sections. There were two whiteboards approximately ten feet in length located along the front and side of the classroom. A projector and document camera were situated near the front of the classroom. There were six clusters of three or four desks spread around the room. Each desk had an empty name placard that contained multiplication tables for the numbers zero through twelve as well as a ruler with inch and centimeter markings. Homework assignments were located on the front whiteboard. Copies of mathematics textbooks and workbooks were located in the back of the classroom. Each child had his/her own workbook and could use a textbook from the class set during class. Mathematical manipulative materials (e.g., bi-color counters) and four-function calculators were located in the back of the classroom.

Participants

The sixth-grade mathematics teacher teaches five sections of sixth-grade mathematics with approximately 22 students in each class. Students from three sixth-grade mathematics classrooms and their mathematics teacher were invited to participate in the study. Three sections that met on the same day were selected. One section was randomly assigned to receive the intervention.

Twenty students from each comparison classroom and eighteen students from the intervention classroom volunteered for the study. None of the participants received services for a disability. Tables 3-5 and 3-6 provide participants' demographic and prior achievement data. More than half of students from each group were white and most students were not receiving free-or-reduced lunch. There were more female than male participants and this was most participants' first year at the school. On average, students from the comparison group scored ten points higher than their peers in the intervention group on the reading FCAT and four points higher on the mathematics FCAT, but there were no significant differences in prior achievement between the comparison and intervention group. Chi-square analyses were also conducted to determine whether there were differences between groups' gender, ethnicity, and free-and-reduced lunch status prior to the intervention. Results indicated that the groups had similar demographics.

Instrumentation

Three instruments facilitated data collection during this study including a Problem-Solving Pretest (Appendix G), Problem-Solving Posttest (Appendix H), and a unit test (Appendix I). The two problem-solving measures were designed to capture students' problem-solving performance and their representation use during problem solving. The unit test was used to collect data about students' learning of rates, ratios, and data analysis.

Problem-Solving Measures

Development of the problem-solving pre- and posttest for this study resulted from the process described earlier. The pre- and posttest had five items each. Items were presented individually on separate sheets of paper. Participants were asked to solve each problem and provide additional solution methods, if known.

Unit Test

The unit test was adapted from the assessment materials that accompanied the class textbook: *Big Ideas: Math 6* (Larson & Boswell, 2010). The test was two pages long and focused on (1) ratios, (2) rate, and (3) mean, median, mode, and range. The test had sixteen items total, five of those items required two or more correct responses in order to receive full credit. For example, one item asked students to find the mean, median, mode, and range of a nine-item data set as well as explain which measures best represent the data set. Three items required students to explain or justify their response. There was no available reliability information for the unit test. Therefore, internal consistency was calculated using data collected during the dissertation study.

Reliability. The internal consistency of the problem-solving measures was calculated using the data from the dissertation study. It was $\rho = .79$ for the pretest and $\rho = .72$ for the posttest. The alternate forms reliability was $r = .77$. This value satisfies the requirement to link participants' problem-solving performance scores across tests (Ary et al., 2009).

All participants' overall scores on the unit test were collected; however, responses to individual items from students in the comparison group were not gathered during data collection. Responses from students in the intervention group were used to calculate the unit test's internal consistency, which was $\rho = .82$. The first item was excluded from analysis because it had zero variance since every student gave the correct response.

Procedures

A timeline for the dissertation study is presented in Table 3-7. The researcher approached faculty and administrators at the school about the study in October 2010. Classroom instruction in three sections of sixth-grade mathematics was regularly observed November 29 – December 14 to give students an opportunity to become more familiar with the researcher's presence in their classroom. On January 3, 2011, the classroom teacher and the researcher briefly discussed the study with students in three sixth-grade classrooms. Consent paperwork for students' parents or guardians was sent home following this visit. Students completed the pretest January 5 and the instructor took over instruction in the intervention classroom following test administration. The researcher became the instructor in one classroom while the classroom teacher continued her instruction in two comparison classrooms. This instruction is characterized as the teaching, learning environment, and instructional actions that occurred in the absence of the intervention. The classroom teacher was not present in the intervention classroom during the study.

Data Collection

Student demographic information

Students' gender, ethnicity, fifth-grade mathematics and reading FCAT scores, free-and-reduced lunch status, and years attending this K-12 school were collected from students' records by school faculty.

Pretest and posttest

The pretest and posttest were administered during students' regular mathematics class on January 5 and 26, respectively. The teacher did not participate in test administration. The researcher read the directions aloud to students and then students began working on the measure. When students asked for assistance the researcher said, “I’m sorry, I cannot answer any questions about the problems. I encourage you to reread the problem and answer the question.” Participants across all sections were given one hour to complete each measure. Students in every section completed the pretest in approximately 20 minutes. Students in the comparison classrooms took approximately 30 minutes to complete the posttest whereas participants from the intervention group took approximately 45 minutes. Those absent on test administration days completed the measures during their lunch period when they returned to school.

Unit test

Participants in every section were administered a unit test by the classroom teacher on January 28. Students received 90 minutes to complete the test. Most were done within 60 minutes. The classroom teacher gave students a copy of the test and asked them to show all of their work. During test administration, she answered students’ questions about items but did not give them the answer. The classroom teacher indicated that her responses were meant to provide scaffolding. Students were instructed to quietly work on an unrelated mathematics assignment when they finished the test.

Classroom Instruction Observations

Instruction was videotaped in the intervention and one comparison classroom everyday. Multiple cameras were placed around the classroom in order to capture small-group and whole-class discourse. A third-year graduate student observed instruction and took fieldnotes in these classrooms one day during the second, third, and fourth week for a total of three observations in each classroom. She had completed one course in qualitative research methods and had a background in mathematics education.

A review of the literature was conducted to locate an instrument to focus her observations on specific facets of instruction. The Reformed Teaching Observation Protocol (Sawada et al., 2002) was selected because it provided the observer an opportunity to rate aspects of the instruction (e.g., teaching for conceptual and procedural understanding as well as classroom culture) as well as make comments. It was slightly modified for this study, for example one item was modified from “The lesson involved fundamental concepts of the subject” (Sawada et al., 2002) to “Lesson involved fundamental concepts of mathematics” so that the protocol focused exclusively on areas pertaining to mathematics classrooms and instruction. The protocol’s purpose was to (a) focus the graduate student’s observations on key facets of the classroom and instruction as well as (b) provide another perspective that would inform characterizations of mathematics instruction in each classroom. A copy of the Observation Protocol used in this study is located in Appendix J. Besides having measures and protocols for the study, other preparations were made to conduct this study.

Preparations for Implementing the Intervention

The teacher and instructor discussed the standards that needed to be taught during the month-long intervention. The sixth-grade mathematics classes that participated in the study met for 45 minutes on Mondays and 90 minutes on Wednesdays and Fridays. The content in the intervention and comparison classrooms focused on the same content from four sixth-grade mathematics standards:

- MA.6.A.2.1 Use reasoning about multiplication and division to solve ratio and rate problems
- MA.6.A.2.2 Interpret and compare ratios and rates
- MA.6.S.6.1 Determine the measures of central tendency (mean, median, and mode) and variability (range) for a given set of data
- MA.6.S.6.2 Select and analyze the measures of central tendency or variability to represent, describe, analyze and/or summarize a data set for the purposes of answering questions appropriately.

Students in both groups had equal access to mathematical manipulatives as well as other materials during the intervention. This school uses the *Everyday Mathematics* (Bell et al., 2004) series in elementary classrooms, and *Big Ideas: Math 6* (Larson & Boswell, 2010) in sixth grade. Students in the comparison group continued to experience their instruction from their classroom teacher. Instructional materials in the intervention classroom typically included one or two word problems that built upon students' prior knowledge and supported learning new ideas as well as discussions about ways to solve problems. Students were expected to spend most of the instructional time exploring and discussing problems.

Classroom instruction with the intervention group drew on the cognitive model of problem solving (Verschaffel et al., 2000) and encouraged students to (1) read and understand the problem, (2) make sense of the task's situation with a model, (3) craft a mathematical model, (4) use a strategy, (5) check to see whether their result matches their model of the situation presented in the task, and (6) finally report their solution. A poster-sized version of the "Guide to the six stages of problem solving" (Appendix A) was displayed on the intervention classroom's walls. Index-card sized versions of the "Six stages of problem solving" (Appendix B) were placed on students' desks and served as reminders to students receiving the intervention. The six problem-solving steps found in each guide were adapted from the problem-solving model (Verschaffel et al., 2000). The poster-sized guide included questions intended to stimulate students' ideas while problem solving and focus their attention on critical aspects at each problem-solving stage. Questions were adapted from ideas discussed by Verschaffel et al. (2000) and Polya (1945/2004). Two mathematics educators and one classroom teacher examined the guides for their appropriateness with sixth-grade students. The poster and index-card sized descriptions were removed after each lesson so that comparison group students and their teacher did not have access to them during instruction.

Students in the intervention group had homework regularly that was assigned by the instructor. The purpose of the homework was for students to master content through solving problems and to gain efficiency by completing exercises. Students typically had assignments based on content seen during class that involved three to six exercises as well as one word problem. Homework assignments were designed with the intention that students might spend more time working on the problem rather than exercises. Homework tasks were drawn from the classroom textbook, other instructional resources, and instructor-created materials. It was expected that students might complete homework assignments in 20-30 minutes.

The instructor developed and administered two quizzes to students in the intervention group using the quiz given to students in the comparison group as a template. These quizzes evaluated students' learning related to the mathematics standards and informed instruction. Students did not receive grades from the instructor, yet assignment completion was recorded for each child and submitted to the classroom teacher.

One feature of the instructional intervention was that mathematical discourse was intended to support mathematics learning through problem-solving contexts. During the study, the instructor attempted to establish social and sociomathematical norms in the intervention classroom with the aim of creating a student-centered, discourse-rich learning environment.

Instructional norms

On the second instructional day, students and the instructor discussed expectations for mathematics learning, including appropriate social and sociomathematical norms. Some examples of social norms are to (a) actively listen to each person's contribution, (b) speak after an individual has finished talking, (c) give each person your attention, and (d) not participate in side conversations while someone is speaking (Lampert, 1990; Lo & Wheatley, 1994). Some sample sociomathematical norms include (a) giving explanations followed by a mathematical justification, (b) using "I think" statements, and (c) commenting on individual's ideas rather than the individual (Cobb, Boufi, McClain, & Whitenack, 1997; Yackel & Cobb, 1996). These social and sociomathematical norms were established by the instructor and transcribed onto posters, which were placed in a prominent location in the classroom and removed at the end of the intervention group's mathematics period. Copies of these posters are found in Appendices K and L. When these norms were first brought to students' attention, they practiced following them through role play and students were asked to practice adhering to them while working on their first problem that day. Students who were doing an exceptional job of following the norms were pointed out during class as being good models for others to follow. Students were reminded to adhere to these expectations while working on classwork and at times the class convened to review the expectations in class.

During the second week of the study, students asked to revise the norms. A student suggested that no electronics except calculators should be permitted in class. Another student added that students should not sit on desks during class. Students and the instructor agreed that the revisions were fair and appropriate. There were no visible reminders of class expectations in the comparison group.

Instructional tasks and materials

Teaching mathematics through problem-solving contexts does not necessarily require tasks that are very different from the textbook (Russell, Esten, Rook, Scott, & Sweeney, 2003). Kilpatrick (1987) suggests that selecting “structured problems requiring productive thinking” facilitates procedural and conceptual understanding of mathematics (p. 134). The process related to creating problems for each lesson began by examining the benchmarks, searching for a model task in one of the resource materials, and reflecting on ways to turn the task into an open and complex problem that drew on realistic contexts.

Modifications were made to textbook problems and supplemental materials so that problem solving became a more prominent focus during instruction. A sample lesson that includes problems about movies, pizza, local weather, and keyrings is provided in Appendix M. For instance, the idea of solving a problem that involved examining the pizza prices of local restaurants came from the class’ textbook. The problem in the textbook was originally structured so that students were expected to translate a verbal statement (i.e., “You pay $27 for 3 pizzas” (Larson & Boswell, 2010,

p. 198) into a symbolic expression describing this ratio and unit rate. A picture of three pizzas accompanied the verbal statement. Informal discussions with students at the school prior to the study suggested that many students were familiar with purchasing pizza from many local establishments. Next, tasks such as this one were modified so that they became problems. A description of this process is provided. First, the data from the old task was augmented with additional information such as the prices of various types of pizza (e.g., cheese only, pepperoni, and any five toppings) as well as each restaurant's dimensions of a large pizza. The original textbook task required students to interpret the verbal statement and respond to one question with two parts. Next, additional questions were added to the task that required students to think about multiple issues related to data analysis as well as rates and ratios. This revised problem asked students to find the smallest ratio describing the cost of one slice of cheese pizza and to think about appropriate ways to describe the data set (i.e., what is the typical value for a large pepperoni pizza?). Third, names of restaurants in the original problem were updated to simulate a realistic context. With these changes to the original task, students might draw on their feelings about pizza from each restaurant as a determining factor for ordering pizza. Problems were meant to be complex, open, and relevant to students' experiences. Additionally, critical components of the instruction included time for students to analyze the problems on the their own, in small groups, and as a class.

Instructional format

The instructional goals in the intervention classroom were to (a) support students to become more effective problem solvers; (b) enhance their facility with multiple representations; and (c) teach mathematics required by the NGSSS. Lessons in the intervention classroom conducted during block scheduled periods tended to follow this order: (1) check homework, (2) complete warm-up task, (3) discuss issues related to homework, (4a) individual work on one problem, (4b) examine the problem with a partner or in a small group, (4c) discuss the problem with the entire class, and (5) and complete a concluding activity meant to stimulate reflection. Roughly 35-50 minutes in total were spent discussing a problem in small groups and as class.

Discourse

Conversations about mathematics were a noticeable feature in the intervention classroom. Questions promoted and initiated mathematical discussions. A list of general questions is provided in the problem-solving guide (Appendix A), but it was augmented during instruction. General questions such as those provided by Schoenfeld (1987) assisted students to reflect on their thinking, “What (exactly) are you doing? (Can you describe it precisely?), (b) Why are you doing it? (How does it fit into the solution?), (3) How does it help you? (What will you do with the outcome when you obtain it?)” (p. 206). Some questions posed to students by the instructor during the study that are related to problem solving are “What is this problem telling you?”, “What are you picturing in your mind after reading this problem?”, “Why do you think that way helps you to solve the problem?”, “How do you know that’s the answer?”, “Is there another way to solve this problem?”, and “What new ideas did you learn that connect to what you learned earlier?”. These questions and others were projected onto the classroom’s whiteboard during problem-solving activity as a way to stimulate dialogue and students were encouraged to ask each other questions while collaborating. Discourse was intended to support students’ learning mathematics content through problem solving and foster greater facility with representations.

Data Analysis

Description of Classroom Instruction

Two sources of data were used to describe the classroom instruction: videotapes of the instruction and the observer's fieldnotes from the protocol. The following procedures facilitated describing classroom instruction in the intervention classroom and one section from the comparison group.

First, the researcher felt that lessons in the intervention classroom occurring during the initial week might differ from others because students and the instructor needed to become familiar with new norms, tasks, and instruction. Thus, descriptions of instruction in the intervention classroom during the first and subsequent weeks were constructed separately. Since instruction in the comparison classroom was a continuation from prior class meetings, there was no expectation that instruction during the first week might differ from the others. Next, data collected using the videorecorder were reviewed and the researcher made notes about aspects of instruction in both classrooms. These notes were intended to be descriptive rather than analytic. Following this, the researcher examined the completed protocols and integrated ratings and the observer's descriptions into the instructor's notes.

Then, the researcher developed an outline that broadly characterized instructional format and activities (e.g., station work) in each classroom. It also facilitated keeping track of the amount of time that was spent on each activity. After creating the outline, three observations from the 90-minute periods were randomly selected and examined, again. The decision to view 90-minute classes excluded Mondays, which were only 45 minutes.

While viewing the videorecorded data, the researcher counted the amount of time spent on an activity and described it within the outline. Discourse was an important element in the classroom. Hence, the researcher also attended to the ways that discourse occurred in both groups. Broad descriptions of common discourse patterns were added to the outline as well. An instructional summary for each classroom was created after examining three observations. Regardless of the attempts made to be objective and fair during this process, the instructional summaries were colored by the researcher's perspective. In order to maintain a fair and appropriate description of each classroom, the summaries of instruction in both classrooms were sent to the observer, and the classroom teacher read and provided feedback on the description of her instruction. Revisions were discussed with the observer and classroom teacher, and modified summaries were created using their feedback and descriptions of the instruction are provided in the results section. These summaries provide a context for understanding the effects of the intervention on students' problem-solving performance and representation use.

Coding and Scoring Measures

Pretest and posttest

Students' performance was scored as correct or incorrect and procedures followed the same format as the pilot study. Appendix E contains the Scoring Protocol. The sum of students' scores indicated their overall performance on the test. The maximum score on either test was five points. The researcher and a second coder scored 20% of the tests and interrater agreement was calculated using the results. The interrater agreement for scoring students' performance on each measure was $r_{wg} = 1$.

A second coder assisted the researcher in coding students' representation use. The number of representations each participant employed on the pre- and posttest was coded using identical procedures to the pilot study. The number of representations within a category and total number of representations employed on each test were calculated. The Representation Coding Protocol (Appendix F) facilitated coding students' approaches for solving problems. Students' responses were coded using the following five representation categories: symbolic, pictorial, tabular, verbal, and mixed. The mixed category had six subcategories to characterize the two representations used to solve a problem. Again, 20% of the tests were randomly selected and interrater agreement was calculated using the results. The researcher and coder had high interrater agreement coding students' representation use on the pretest and posttest ($r_{wg} = 1$). These statistics provide evidence that the pairs agreed on every score and code applied during the independently coded sample of the pre- and posttests.

Unit test

The classroom teacher scored the unit tests for the three participating sections of sixth-grade mathematics. Each response was scored as correct or incorrect and was equally weighted. The maximum possible score was 25 points.

Analyses

Problem-solving measures

Data analyses were conducted using Statistics Package for the Social Sciences (SPSS) 18.0. Descriptive statistics of students' demographic information as well as performance and representation use on the problem-solving measures and scores on the unit test are presented in Table 3-8. Data were examined for their distribution and homogeneity of variance (Shavelson, 1996). The degree to which assumptions of the tests were met is discussed in the results section.

Initially, a repeated measures t-test was employed to determine whether the intervention was effective at improving problem-solving performance and total number of representations used within each group. Next, students' performance and representation use on the posttest were investigated using Poisson regression. Poisson regression is appropriate when the dependent variable is a count rather than a continuous variable (Agresti & Finlay, 2009). The pretest score, number of representations employed on the pretest, students' free-and-reduced lunch status, gender, ethnicity, years attending the school, and students' scale scores from the fifth-grade mathematics and reading FCAT served as covariates for multiple regression analyses. Verschaffel et al. (1999) found significant interaction effects between students' prior problem-solving performance and intervention status thus an interaction effect containing pretest performance and intervention status was added to the regression model. Categorical variables were given values or dummy coded to facilitate regression analyses (Table 3-9). Prior evidence showed that mathematics and reading achievement were correlated (Vilenius-Tuohimaa et al., 2008) thus issues with multicollinearity were explored using the Variance Inflation Factor (VIF) statistic. VIF is a measure that represents the increase in variance due to an independent variable being correlated with other predictors thus causing multicollinearity (Agresti & Finlay, 2009). Multicollinearity exists if VIF > 10 (Agresti & Finlay).

Model building examining the influence of the predictor variables on the outcomes was conducted by using backwards elimination, which sequentially deletes variables one at a time from the model that contains all of the variables (Agresti & Finlay, 2009). This procedure is best for creating a good set of predictors that explain a significant amount of variance in students' outcome (Agresti & Finlay, 2009). Furthermore, backwards elimination procedures support investigations for a parsimonious model and also attend to statistical and theoretical implications (Hamilton, 2009). The criterion to drop a variable was set at .05. A focus of this study was to explore whether the instructional intervention and other covariates impacted students' outcomes and not necessarily make strong causal claims. Therefore, automated procedures such as those found in SPSS are permissible (Agresti & Finlay, 2009). The initial model containing intervention status, all covariates, and the interaction effect is shown below.

Posttest performance = a + β_1(Pretest Performance) + β_2(Intervention Status x Pretest Performance) + β_3(Total Representation Use_Pretest) + β_4(Intervention Status) + β_5(Reading Scale Score) + β_6(Mathematics Scale Score) + β_7(Free-and-Reduced Lunch Status) + β_8(Gender) + β_9(Years of Attendance) + β_{10}(Hispanic) + β_{11}(African American) + β_{12}(Multiracial) + β_{13}(Asian American)

Analyses were examined to make certain that main effects were not removed if the interaction effect was significant. Covariates that were not significant at *p* = .05 were deleted and the model was re-estimated. This process continued until only significant covariates remained in the model. Finally, a chi-square test was performed to determine whether the final model was significant compared to an intercept-only model.

A similar set of analyses was conducted using total number of representations employed on the posttest as the outcome variable. The initial model shown below characterizes the number of representations students used on the posttest as a function of their pretest outcomes, intervention status, and background. Covariates were eliminated and the model was subsequently reanalyzed until only significant regression coefficients remained.

Total Representation Use_Posttest = a + β_1(Total Representation Use_Pretest) + β_2(Pretest Performance) + β_3(Intervention Status x Pretest Performance + β_4(Intervention Status) + β_5(Reading Scale Score) + β_6(Mathematics Scale Score) + β_7(Free-and-Reduced Lunch Status) + β_8(Gender) + β_9(Years of Attendance) + β_{10}(Hispanic) + β_{11}(African American) + β_{12}(Multiracial) + β_{13}(Asian American)

When there was a significant difference between groups, Cohen's *d* was used to explain the size of the difference in terms of standard deviations. Each group's mean was calculated from the multiple regression equation (i.e., regression coefficients). Measures of effect size were compared to the average annual gains in effect size associated with sixth-grade students' performance on nationally normed tests (Bloom, Hill, Black, & Lipsey, 2008) and mean effect size on achievement measures for students in fourth- through sixth-grade (Hill, Bloom, Black, & Lipsey, 2008). These types of comparisons provide a more appropriate context for interpreting effect size rather than Cohen (1988) or Lipsey's (1990) context-free benchmarks.

A chi-square analysis was conducted using two categorical variables: intervention status and students' use of nonsymbolic representation (i.e., pictorial, tabular, and mixed) on the posttest. Table 3-10 shows the means and standard deviations for each group's posttest representation use. Verbal representations were omitted from the chi-square analyses because participants did not use them on the posttest.

Unit test

The mean and standard deviation for each group's performance on the unit test were calculated and are located in Table 3-8. Next, the residual plots and spread of the data were examined to provide evidence that the normality assumption had been met.

A Levene's test was conducted to determine whether the variance of scores in each group were equal. Scores are nested within a section, yet students' performance was being treated as independent of others within the section. After exploring whether the assumptions were met, differences between the intervention and comparison students' scores were analyzed using ordinary least squares multiple regression. Prior mathematics achievement was included as the only covariate.

Summary

The design of this study was based on prior problem-solving research conducted in classrooms with adolescents (i.e., Verschaffel et al., 1999). A pilot study was conducted to construct the pre- and posttest measures for the dissertation study. The responses to the tasks were scored and coded with respect to the type of representation used during problem solving. After scoring and coding, the data were analyzed to examine whether there were within-group and/or between-group differences on the measures following a four-week intervention. Secondary analyses were conducted to explore the relationships between intervention status and type of representation used on the posttest.

Table 3-1. Demographic information for schools

School	Overall Grade	% Meeting High Standards in Reading	% Meeting High Standards in Math	% Making Learning Gains in Reading	% Making Learning Gains in Math	% Minority	% Receiving Free-and-Reduced Lunch
Pilot Study	A[b]	87	82	69	63	39	50
Dissertation	A[b]	78	85	66	79	50	15

[a] (FLDOE, 2010b) [b] Florida schools are graded each year using a number of factors. The majority of students at "A" schools earned a passing grade on the FCAT and the bottom quartile of students made adequate scholarly progress.

Table 3-2. Mean and standard deviations of pilot study participants' performance

	Mean	Std. Dev.
Pretest[a]	1.68	1.52
Posttest[a]	2.11	1.68

[a] N = 169

Table 3-3. Pilot study pretest item information

Test Parameters	Item							
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
CFA								
Estimate	0.22	0.72	0.59	0.65	0.59	0.83	0.70	1.12
Std. Error	0.22	0.10	0.13	0.09	0.12	0.06	0.09	0.05
Threshold	1.80	1.38	1.02	0.67	0.97	0.97	0.88	0.64
Std. Error	0.26	0.20	0.17	0.15	0.16	0.16	0.16	0.15
IRT								
Difficulty	-0.49	2.82	2.47	1.74	0.99	1.65	1.69	1.53
Std. Error	0.17	0.39	0.34	0.25	0.19	0.24	0.24	0.23
Discrimination	1.24	1.24	1.24	1.24	1.24	1.24	1.24	1.24
Std. Error	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
Item Information	0.23	0.03	0.05	0.10	0.18	0.11	0.11	0.12

Table 3-4. Pilot study posttest item information

Test Parameters	Item							
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
CFA								
Factor Loading	0.38	0.37	0.38	0.78	0.65	0.56	0.52	0.83
Std. Error	0.13	0.23	0.16	0.08	0.09	0.10	0.11	0.07
Threshold	-0.40	1.98	1.39	0.80	0.49	0.55	0.70	0.57
Std. Error	0.10	0.21	0.14	0.11	0.10	0.10	0.11	0.10
IRT								
Difficulty	-0.66	3.47	2.34	1.32	0.80	0.91	1.15	0.94
Std. Error	0.17	0.51	0.31	0.21	0.17	0.18	0.20	0.18
Discrimination	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28
Std. Error	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
Item Information	0.21	0.01	0.05	0.14	0.20	0.19	0.16	0.18

Table 3-5. Demographic information for participants

	Intervention Group N = 18 Number (Percent)	Comparison Group N = 40 Number (Percent)
Ethnicity		
White	11 (.61)	20 (.50)
Hispanic	3 (.17)	11 (.28)
African-American	3 (.17)	6 (.15)
Multiracial	1 (.03)	2 (.05)
Asian-American	0 (0)	1 (.02)
Gender		
Male	7 (.39)	18 (.45)
Female	11 (.61)	22 (.55)
Free-or-Reduced Lunch		
Yes	5 (.28)	7 (.18)
No	13 (.72)	33 (.82)
Years Attending School		
Zero	8 (.44)	19 (.48)
One	1 (.06)	0 (0)
Two	2 (.11)	5 (.12)
Three	0 (0)	0 (0)
Four	2 (.11)	0 (0)
Five	0 (0)	0 (0)
Six	5 (.28)	16 (.40)

Table 3-6. Group means and standard deviations related to fifth-grade FCAT scores

	Intervention Group N = 18 Mean (SD)	Comparison Group N = 40 Mean (SD)
Reading Scale Score[a]	330 (36)	340 (45)
Mathematics Scale Score[a]	350 (32)	354 (33)

[a] N = 53

Table 3-7. Dissertation timeline

Date	Procedure	Instrumentation
Week of November 1	Administer pretest to 169 participants during pilot study	Pretest with eight word problems
Week of November 8	Administer posttest to 169 participants during pilot study	Posttest with eight word problem
Week of November 15	Score pretest and posttest and conduct factor analysis	Pretest and Posttest
Week of December 1	Submit instruments to IRB for approval	Pretest and posttest with five items each
January 5	Administer pretest to three sections of sixth-grade mathematics	Pretest with five items
January 5 – 26	Conduct instruction in one sixth-grade mathematics classroom	Intervention
January 26	Administer posttest to three sections of sixth-grade mathematics	Posttest with five items

Table 3-8. Group means and standard deviations related to problem-solving performance, representation use, and unit test performance

	Intervention Group[a]		Comparison Group[b]	
Factor	Mean	Std. Dev.	Mean	Std. Dev.
Pretest				
Performance	2.22	1.17	1.66	1.51
Total Representation Use	2.78	1.73	1.66	1.92
Posttest				
Performance	2.83	1.34	1.73	1.28
Total Representation Use	3.50	1.69	2.20	1.64
Unit Test	17.11	3.69	19.88	3.07

[a] N = 18; [b] N = 40

Table 3-9. Values for categorical predictor variables

Covariate	Code
Gender	
Girls	0
Boys	1
Free-or-Reduced	
No	0
Yes	1
Years Attending School	
Zero	0
One	1
Two	2
Three	3
Four	4
Five	5
Six	6

Table 3-10. Group means and standard deviations of type of representation used on the posttest

Representation	Intervention[a] Mean (SD)	Comparison[b] Mean (SD)
Symbolic	2.78 (1.35)	1.45 (1.15)
Pictorial	0.44 (0.62)	0.48 (0.64)
Tabular	0.17 (0.38)	0.23 (0.42)
Verbal	0 (0)	0 (0)
Mixed	0.11 (0.32)	0.08 (0.27)

[a] N = 18, [b] N = 40

CHAPTER 4
RESULTS

A description of instruction in the intervention and comparison classroom is provided before quantitative analyses are discussed. Results from repeated measures t-tests examined differences in the outcome variables within each group. Assumptions related to normality, homoscedasticity, linearity, and multicollinearity were investigated. Results from examining the residual plots and statistical analyses are provided to justify use of multiple regression. This type of analysis supports investigations that explore pre-intervention differences between groups and explore possible interaction effects. The results from chi-square tests indicated whether there was a relationship between intervention status and nonsymbolic representation use on the posttest. Finally, outcomes from regression analysis with one covariate are provided to characterize the intervention's impact on students' achievement.

Description of Instruction

The descriptions of instruction that are provided were created after examining multiple instructional days. Most students were present for every lesson during the study. The characterizations are not intended to describe one single lesson; instead they depict instruction that typically occurred in each classroom. Examples within each description may come from different days and are provided to support the reader's understanding of the instruction in each classroom.

Intervention Classroom

Initial lessons in the intervention classroom differed somewhat from the others because the participants needed time to adjust to the new instructor and the expectations for problem solving therefore descriptions of instruction during the first week and the rest of the intervention are provided separately.

Week one

During the first class meeting, participants completed the pretest and the remaining time was allocated for examining expectations for class and co-exploring a word problem. During the second meeting, the instructor modeled the social and sociomathematical norms by offering examples and demonstrations of appropriate problem-solving behaviors and discourse. Furthermore, he pointed out students during class who were doing an excellent job of following them so that others might be more likely to adhere to the expectations. The instructor and students also co-solved word problems and became familiar with the problem-solving model (Appendix A) during the first week.

The instructor initiated a discussion about problem solving and encouraged students to think about how the problem-solving guide with six steps might support them when they worked on word problems during the second class. Next, students were provided with a word problem and individually used the model as a guide. The co-solving process was initiated when the instructor handed out a problem, asked students to read the text on their own, and to describe their understanding, which provided a context in which they externalized their situation models to a peer (i.e., “Describe what’s going on in this problem to a classmate.”). Occasionally students were asked to pause their work and reconvene as a class to discuss their progress and clarify each stage of the process. Students who seemed to grasp the intent of the problem-solving model were asked to share their thinking periodically as a model for others to follow. After listening to students discussing their situation models, students were informed that descriptions of the mathematical task and a problem's context were "situation models", which facilitated effective problem solving. Unsuccessful students had another opportunity to develop a situation model. During a whole-class discussion, selected students were asked to share their situation models, comment on the model's appropriateness for the task, and describe how situation models differ from a mathematical model. Some students described perceiving the situation model as a diagram, which included various features of the problem whereas others preferred to create a simplified version of the problem, which contained only the relevant information for solving the problem. After students engaged in small-group discussions, the class reconvened and the instructor led a discussion focusing on the mathematics topics, problem-solving process, and lessons learned from the task. The mathematics was decontextualized, but during the first week the instructor explicitly examined each step of the problem-solving process with students.

Mathematics content was the foundation for these discussions and provided a context to discuss problem solving. The idea of teaching mathematics content fueled lesson implementation, yet during the first week, the discussion tended to focus on the process that students employed while working on word problems (e.g., understanding the text, examining the problem's context, and interpreting the result). This was different from the other weeks when each stage of the problem-solving process was not an explicit topic of conversation. Problem solving was the focus of instruction and the instructor did not explicitly teach mathematics concepts and procedures on a regular basis during the intervention.

During mathematical discussion in the first week, students collaboratively scanned the text for difficult or unfamiliar mathematics terminology and explored alternative representations that might lead to a solution. Incorrect representations of the problem were taken up for discussion as well so that students were clear why these methods were not feasible or appropriate. As a result of establishing norms in the classroom, instruction in the intervention classroom during the rest of the study tended to look fairly similar.

Characterizing a lesson

The expectations for class were hung on the sidewall (Appendices K and L) and the problem-solving poster (Appendix A) was placed near the front whiteboard on top of the classroom teacher's word wall. Desk-sized descriptions of the six stages of problem solving were attached to each desk using Velcro. Bells signaled the start and end of class at 10:55 am and 12:25 pm, respectively. A copy of one lesson is located in Appendix M.

An agenda was projected on the front whiteboard, which indicated tasks to accomplish during the first five minutes of class as well as the benchmarks that were the focus for that day. Tasks were highlighted on screen in yellow and the homework solutions were displayed just below them. Students usually checked homework first and then began a warm-up task. Students were typically asked to examine their homework on their own and consider tasks, procedures, or concepts for discussion. After five minutes the instructor asked students whether they wanted to review any homework tasks. Multiple students usually raised their hands. The teacher gathered students' questions before any were explored. Students often had similar queries. Following the collection, the instructor invited students to resolve each other's inquiries, “Does someone have ideas about this problem?” Several students usually raised their hands and one was selected who proceeded to explain his or her thoughts and/or approaches for solving the problem. Students frequently described noticing aspects of the problem as critical features necessary for solving the problem. After the explanation, the instructor asked the student who originally posed the question whether the uncertainty was resolved, “Does that make sense?” or ”Would you like him/her to describe it in another way?”. It was typical for students to indicate their peer’s response was sufficient. Students’ acceptance of a response was influenced by their pre-intervention mathematical discussions in the classroom as well as the novel social and sociomathematical norms. The instructor followed up by probing students for other ways to solve that same problem - usually one student shared an alternate representation. This process continued until students’ questions about the homework were resolved, which took ten to fifteen minutes.

Following the homework discussion the instructor reminded students to complete the warm-up task, which was projected on the front whiteboard. Students usually completed the warm-up task in ten minutes or less. On one occasion, students examined a list of statistical terms and were directed to think about whether they could define each word and provide an accurate example. Students had a few minutes to work independently and then the instructor led a discussion aimed at exploring terminology that was relevant to the unit. In the sample lesson, the instructor asked students to share terms that were unfamiliar and whether others might offer their (mis)conceptions. The instructor validated correct ideas and indicated that the class would re-examine the list at the end of class.

Following the warm up, the instructor posed a question related to the context of a problem students would examine during class, such as “What was the name of the last restaurant where you purchased a pizza?”. Multiple students mentioned restaurants from the problem and some shared their pizza preferences. The instructor elaborated that the problem of the day involved investigating pizza prices from various local establishments and he distributed individual copies of the typed problem to students.

Initially, the instructor encouraged students to work independently for a few minutes. Students were reminded that they could work together on the problem after they worked independently. Students usually spent five to ten minutes on their own before forming small groups.

When the instructor announced that independent work time was over, students formed pairs or triads on their own. Some students were reticent to form groups so the instructor facilitated collaboration by encouraging these participants to form their own group. During the second week, students worked with peers at their group of desks. They formed new groups during the third and fourth weeks. Some sat in chairs while others worked on the floor. Small-group work among students typically began with peer-to-peer questions, such as “What do we need to do?” and “What do you think about this [problem]?”. After chatting about the problem’s context and goal of the task, students discussed how to solve the problem and at times, challenged each other to justify their ideas, “Why are you doing that?”. After agreeing on a representation, they carried out a set of procedures and talked about interpreting the result. For instance, two students discussing the pizza problem mentioned earlier reexamined the problem’s context and later included appropriate units with the result. Students continued to share ideas in small groups for 15-25 minutes, depending on the problem's complexity. The mathematics content was still the foundation of mathematics learning, yet the instructor did not make the problem-solving process as explicit as he did during the first week. The instructor used questions to encourage students to think about various steps of the problem-solving process when students had trouble. During small-group work, the instructor walked around the classroom, observing students' work and responding to requests for assistance with questions such as "What do you think you're supposed to do?" and "What do you think is important in the problem?". The class reconvened to discuss the problem when most students were finished

The instructor began the whole-class instruction by posing an open-ended inquiry such as "What is going on in this problem?" or "What do we need to find?". The discussion involved multiple children, albeit one of four students usually was the first to speak. Of these four students, two were boys and two were girls. The instructor encouraged other students to share their answers and representations during the whole-class discussion by asking one of the usual initial speakers to hold his/her comments until others had a chance to think and share their ideas. Some transcribed their work onto the whiteboard whereas others used the document camera to project their work. Presenters typically discussed their representation, procedures necessary to solve the problem, thoughts during problem solving, and the result. The instructor asked whether students had questions about the student's presentation, which usually resulted in a couple of student-initiated inquiries. Some asked for assistance such as "Can you explain it again?" whereas others posed more probing questions, "Why did you do it that way?". Students were encouraged to share their representations and there was at least one alternate representation presented for each problem. Discussion ended when students' questions were answered and the problem was solved. The whole-class discussion usually took 20-25 minutes.

The instructor rarely made mathematics procedures an explicit topic of conversation. When it did occur, there was usually disagreement in the room among several students and they were unable to reach a consensus. Typically, the instructor confirmed one or more students' suggestions for carrying out a procedure and moved on with instruction. Explicit mathematics instruction, on the rare occasions when it did occur, was focused on mathematics procedures and lasted about one minute.

Finally the instructor briefly synthesized students' work and offered a summary of concepts and procedures that came up while solving the word problem. Mathematics topics were often the focus of the synthesis. For example, the instructor discussed how range provides different information about a data set than measures of central tendency. Students raised their hands on occasion to add to the instructor's synthesis and share what they learned from solving the problem. The individual, small-group work time, and whole-class discussion usually lasted 40-65 minutes.

Students received an instructor-created worksheet for homework near the end of class. Closure activities such as exit slips, reexamining warm-up tasks, and writing summaries of the lesson were completed during the last five minutes of class. Summaries stayed in students' notebooks whereas exit slips were handed to the instructor as students left the room. Materials attached to desks and walls were taken down at the end of class and the whiteboard was erased.

Comparison Classroom

As a result of reviewing the observer's notes and examining videotapes of classroom instruction, the following description of instruction in the comparison classroom was created to provide a context for students' outcomes. Unfortunately, lesson plans for this unit were not available.

The teacher took roll and made class-related announcements during the first five minutes of class. Announcements usually indicated the necessary materials for class and instructions to check homework from the previous class and to copy the homework assignment from the board. Students gathered the necessary materials for the start of class from the back of the room (e.g., workbook) and began comparing answers to homework with the answers posted on the whiteboard. After students settled down, the teacher read most of the answers aloud from the board and briefly explained how to solve each task. She typically spent about ten seconds describing one solution strategy for each homework task. There were usually a couple of student-initiated questions about the homework, which were answered by the teacher. Preparing for class and checking homework usually took about fifteen minutes. Next, the teacher posed a warm-up task for students.

Warm-up activities were usually exercises taken from the textbook or resource materials associated with the textbook *Big Ideas: Math 6* (Larson & Boswell, 2010). For example, during one class meeting students examined a data set with seven values and found the mean. After one student shared his idea, the teacher confirmed the solution and asked whether this value was appropriate for the data set. Another student responded that it was, the teacher verified her response, and moved on with the lesson.

This was typical when students provided the correct response, which occurred most of the time. The warm-up task generally lasted approximately seven minutes.

Vocabulary instruction was part of one class meeting each week. There was a word wall near the front of the room, which contained terms from previous units. The teacher started vocabulary instruction by asking students to open their textbooks to a page in the textbook that defined terms pertaining to the lessons for the week. She chose a student to read the definition from the textbook, and then she provided an explanation of the term. The teacher usually asked comprehension questions after each term (e.g., “How do you find the median?”). If a student answered the question, she evaluated their response (e.g., “that’s correct”) and continued with the lesson. When a student offered an incorrect result, she evaluated the student’s statement (e.g., “not quite”) and selected another volunteer. This process continued until the solution was stated. She frequently drew on realistic scenarios to facilitate students’ understanding of the term. For example, while discussing the term “range” she mentioned that the range of students’ ages in the classroom would be small whereas the range of ages across the school would be larger. Vocabulary instruction typically lasted five to ten minutes.

Next, the teacher conducted a lesson focusing on one benchmark from the NGSSS. Instruction typically involved (1) a teacher-led demonstration, (2) students co-solving one task with the teacher, and (3) students working independently on a few exercises as practice. During the second stage of the lesson, the teacher assigned a task and asked students to work on it briefly. She walked around the room examining students’ work and eventually returned to solving the problem. She answered students’ questions directly when they asked for assistance. After three to five minutes, one student was selected to give his/her answer to the task. The teacher affirmed the response and proceeded to offer insights into solving similar tasks for five to ten minutes. When a student offered an incorrect result to the task, she indicated that his/her response was incorrect and asked for another volunteer to provide the answer. The third stage was characterized by the teacher assigning a series of exercises for students to complete on their own. After five minutes, the teacher selected students to report their answers. Students' responses to the tasks were correct most of the time. There were typically one or two student-initiated questions during this part of the instruction and the teacher answered students' inquiries. This portion of the lesson typically lasted 15 – 30 minutes. The next phase of the lesson provided students with opportunities to complete activities at stations in the room.

The teacher dedicated 40-50 minutes to small-group work, which was completed at stations. She divided the class into groups of four to five students and indicated the order for rotating through stations. Stations typically involved (1) laptop computers, (2) working on homework, (3) completing pages from the mathematics workbook, and (4) solving exercises with the teacher. Students had 10-15 minutes at each station. Music played through her computer marked the end of activity at a station and students had one to two minutes to move to the next station. The laptop station involved web-based software (i.e., FCAT Explorer) that provided opportunities for students to practice solving a variety of tasks that were similar to ones students might see on the sixth-grade mathematics FCAT. Students occasionally asked each other mathematics-related questions while working at the homework and workbook stations. The workbook had word problems and exercises. A preliminary review of it suggests there were more exercises than word problems. There was no penalty for not finishing the classwork that day but students had to complete all of it (i.e., workbook pages) by the date of the test. The teacher-directed station usually involved personal-sized whiteboards (i.e., slates) and exercises selected or adapted from the textbook. The teacher posed a task and asked students to solve it; they showed her their slates when they arrived at the solution and she verified their solutions. If a student did not have the correct answer, he/she was told it was incorrect and encouraged to reexamine the problem. If the correct answer was shown then the student's response was confirmed and the teacher asked the individual to erase the whiteboard and wait for others to finish. The tasks posed by the teacher at this station, exercises from the workbook, as well as the examples used during direct instruction tended to feature a similar problem structure. Students could usually solve these tasks using the same procedures.

During the last ten minutes of class, students returned materials used during class to their original locations and the teacher made announcements about future assignments or assessments. The announcements also included a brief summary of the topics covered that day. She encouraged students to do their homework and to see her outside of class if students needed extra help. If there was any remaining time after the announcements, students were allowed to socialize until the end of the period.

Conclusion

The goal of providing the descriptions of instruction in the intervention and comparison classrooms was to briefly characterize the format and activities of a lesson, how time was used during the lesson, and the student-to-teacher and student-to-student interactions. The descriptions provide a background for exploring students' outcomes as a function of their classroom instruction and background.

Assumptions for Multiple Regression

A brief discussion related to assumptions necessary for conducting multiple regression analyses is shared followed by the relationship between students' outcomes and the independent variables that were further explored using multiple regression. There are four assumptions for multiple regression: (a) a participant's outcome is randomly sampled and independent of all other participants' outcomes, (b) the residuals are normally distributed for each potential combination of independent variables, (c) the residuals have equal variances for every possible combination of predictors (i.e., homoscedasticity), and (d) there is a linear relationship between the dependent and independent variable when all other independent variables are held constant (i.e., linearity) (Shavelson, 1996). Students' outcomes are nested within each classroom (Raudenbush & Bryk, 2002) thus without conducting multilevel analyses, the degree to which the independence assumption was met is uncertain. Evidence indicating that the remaining assumptions were met comes from examining residual plots. Residual plots of the outcome variable were examined to determine whether normality, homoscedasticity (i.e., variances of the dependent variable for each possible combination of the independent variables are equal), and if a polynomial term might be necessary to accurately model the data. Violations of normality look like unequal spread of data around the mean whereas violations of homoscedasticity might appear as a fan- or trumpet-like appearance within the scatterplot (Shavelson, 1996). If a polynomial term is necessary then there is usually curvature in the scatterplot.

The independence assumption has been met in good faith and multilevel analysis was not appropriate because there were only two teachers. Levene's test was not significant for problem-solving performance, p = .75 and number of representations used on the posttest, p = .61. These nonsignificant findings suggest that the variances in students' outcomes were equal. Furthermore, there was no fan-like shape in the scatterplot. The frequency distribution had a normal curve appearance, thus the data were roughly normal. Finally, the scatterplot did not have any curvature thus polynomial terms were not added to the model. In conclusion, the assumptions for multiple regression analyses using problem-solving performance and representation use had been met.

The independence assumption for multiple regression analyses using participants' unit test scores assumed that scores were independent from others and multilevel analyses were not performed for the same reasons described earlier. Visual inspections of residual plots indicated that (a) most data were within two standard deviations of the mean and there was roughly the same number of data above and below the mean, (b) there was no fanning, and (c) no curvature. The frequency distribution showed that 72% and 85% of the unit test scores from the intervention and comparison group, respectively, were within one standard deviation of their respective group's mean. Levene's test was not significant, p = .50, which provided evidence that there was homoscedasticity. There was also no curvature in the scatterplot. There was sufficient evidence for conducting a multiple regression analysis with the unit test data. Results from these analyses suggest that there was that assumptions necessary to conduct multiple regression had been met.

Group Characteristics

Descriptive information related to students' demographic information was provided in the Chapter 3, in Tables 3-5 and 3-6. The minimum and maximum possible scale scores on the fifth-grade FCAT were 100 and 500, respectively. There was no difference between the intervention and comparison groups' mean FCAT reading, $F(1, 51) = .62$, $p = .44$, or mathematics scores, $F(1, 51) = .17$, $p = .68$. Six academic years was the maximum number of years that participants attended this school. The maximum score on the pretest was five points. There was no difference between groups' pretest performance, $F(1, 56) = 2.01$, $p = .16$ or representation use, $F(1, 56) = 1.42$, $p = .24$.

Within-Group Comparisons

To answer the first research question, results from a paired samples t-test indicated that the students in the intervention group had better performance on the posttest than the pretest, $t(17) = 2.65$, $p = .02$, $d = .48$ whereas their peers did not improve, $t(39) = 0.52$, $p = .61$. The intervention group's problem-solving growth is approximately 17% greater than the finding related to annual gains in mathematics achievement (Bloom et al., 2008). Analyses related to the second research question revealed that intervention participants employed more representations on the posttest compared to the pretest, $t(17) = 2.60$, $p = .02$, $d = .42$. This is a slight improvement over the average annual gain in mathematics achievement, approximately 2%. There were no significant changes in their peers' total representation use, $t(39) = 0.22$, $p = .83$. Thus, students in the intervention group improved their performance and used more representations on the posttest than the pretest, but their peers in the comparison group did not demonstrate changes related to their problem solving.

Between-Group Comparisons

Problem-solving Performance

Backwards selection procedures were conducted with the regression equations to explore factors that best predicted students' outcomes. The following results answered research questions three and four. Figures 4-1 and 4-2 characterize each group's growth over time.

Backwards selection in SPSS was employed for creating a model that explained the intervention's impact on the outcomes, problem-solving performance and total number of representations used on the posttest. First, a model examining the relationship between problem-solving performance, group status, all of the covariates, and the interaction terms was entered into SPSS, which is shown here.

Posttest performance = a + β_1(Pretest Performance) + β_2(Intervention Status x Pretest Performance) + β_3(Total Representation Use_Pretest) + β_4(Intervention Status) + β_5(Reading Scale Score) + β_6(Mathematics Scale Score) + β_7(Free-and-Reduced Lunch Status) + β_8(Gender) + β_9(Years of Attendance) + β_{10}(Hispanic) + β_{11}(African American) + β_{12}(Multiracial) + β_{13}(Asian American)

Nonsignificant coefficients were deleted until only significant coefficients remained, which were pretest performance and intervention status. After iterative analyses, it was determined that pretest performance and intervention status alone best predicted students' posttest performance, $\chi^2(2) = 33.06$, $p < .001$. Pretest performance and intervention status explained 64% of variance in students' performance on the posttest. Moreover, 5% of the total variance in posttest performance was uniquely associated with intervention status. The squared semi-partial correlation associated with pretest performance was approximately 0.50. VIF for both pretest performance and intervention status were approximately one, well below the threshold suggested by Agresti and Finlay (2009). Table 4-1 provides detailed information about the relationship between the variables. The difference in expected posttest performance was 0.37 log units higher for intervention participants than their peers. Another finding was that two participants in the same group with a one-unit difference in problem-solving pretest performance had a 0.32 log unit difference in their problem-solving posttest score. The intervention's effect size was *d* = .26. This is not quite as large as the average annual sixth-grade student gains in effect size for nationally normed mathematics tests, *d* = .41, yet it is 18% larger than the mean effect size on achievement measures for students in fourth- through sixth-grade (*d* = .22) (Hill et al., 2008).

Representation Use

A similar procedure was conducted to determine the influence of intervention status, covariates, and an interaction effect on the total number of representations employed on the posttest. Again, all covariates were entered into SPSS to create a complex model and variables that were not significant were deleted one-by-one until only significant ones remained. The model with all predictor variables is shown below.

Total Representation Use_Posttest = a + β_1(Total Representation Use_Pretest) + β_2(Pretest Performance) + β_3(Intervention Status x Pretest Performance) + β_4(Intervention Status) + β_5(Reading Scale Score) + β_6(Mathematics Scale Score) + β_7(Free-and-Reduced Lunch Status) + β_8(Gender) + β_9(Years of Attendance) + β_{10}(Hispanic) + β_{11}(African American) + β_{12}(Multiracial) + β_{13}(Asian American)

Analyses revealed that pretest performance and intervention status explained the most variance in students' posttest representation use, $\chi^2(2) = 44.00$, $p < .001$. The independent variable and two covariates explained 66% of the variance in the total number of representations employed on the posttest. Furthermore, 4% and 54% of the variance in students' posttest representation use is uniquely explained by intervention status and pretest performance, respectively. VIF for both pretest performance and total number of representations employed on the pretest were near two and intervention status was approximately one, which suggest there was no multicollinearity issue. Detailed information related to the independent variable and covariates is located in Table 4-2. Results from this analysis indicate that controlling for pretest performance, a participant from the intervention group used more representations (i.e., 0.31 log units) compared to his/her matched peer. Furthermore, one-unit differences in pretest performance were associated with 0.25 log unit differences in participants' posttest representation use. Intervention status was associated with an effect size of d = .18. This is lower than the average annual sixth-grade student gain in effect size for nationally normed mathematics tests and also smaller than the mean effect size associated with achievement gains in grades four through six.

Finally, a chi-square test was used to answer the fifth research question: whether there was a relationship between intervention status and students' use of nonsymbolic representations on the posttest (i.e., pictorial, tabular, and mixed). Table 3-10 provides the means and standard deviations of each group's posttest representation use. There was no relationship between intervention status and students' nonsymbolic representation use, $\chi^2(1) = 0.62$, $p = .43$. These results suggest that membership in the intervention group was not associated with use of nonsymbolic representations on the posttest.

Unit Test

Analyses were performed using data from the unit test to examine the intervention's effect on students' unit test performance. Results from multiple regression with the unit test data and students' prior mathematics achievement provided insight into the sixth research question (Table 4-3). There was a significant difference in the groups' mean scores on the unit test, $F(2, 50) = 17.45$, $p < .001$. The average of the intervention group on the unit test was lower than the comparison group (Table 3-8). Intervention status and prior mathematics achievement explained 41% of the variance in students' unit test performance. Approximately 12% of the variance in students' unit test performance was uniquely associated with intervention status. The effect size (d = .34) is smaller than the annual gain in mathematics achievement but 55% larger than the mean effect size related to fourth- through sixth-grade students' achievement gains. A comparison participant experienced a 0.34 standard deviation unit advantage over his/her equally matched peer in the intervention group. A one-unit difference in prior mathematics achievement between participants from the same group resulted in a 0.53 standard deviation unit advantage. In short, students in the comparison group had higher unit test scores than their peers experiencing the intervention and prior mathematics achievement influenced this relationship.

Summary

Four of the six hypotheses presented in Chapter 3 were confirmed. Students in the intervention group had higher scores on the posttest than the pretest. They also used more representations on the posttest than they did on the pretest. There was a statistically significant difference in students' performance and the total number of representations used on the posttest, favoring the intervention group. There was no relationship between intervention status and use of nonsymbolic representations on the posttest. Finally, there was a difference in groups' scores on the unit test favoring the comparison group. The results are discussed further in relation to the conclusions of the study, including implications, limitations, and directions for further research in Chapter 5.

Table 4-1. Problem-solving performance predictors

Variable	Coefficient				
	B	Std. Error	Wald Chi-Square	Sig.	Wald CI
Constant	0.26	0.22	1.40	0.237	[-0.17, 0.68]
Intervention Status	0.37	0.19	3.90	0.048	[0.01, 0.73]
Pretest Performance	0.32	0.06	27.14	0.001	[0.20, 0.44]

Table 4-2. Predictors related to representation use on the posttest

Variable	Coefficient				
	B	Std. Error	Wald Chi-Square	Sig.	Wald CI
Constant	0.44	0.19	2.82	0.093	[0.06, 0.82]
Intervention Status	0.33	0.17	4.02	0.045	[0.01, 0.66]
Pretest Performance	0.33	0.05	36.44	0.001	[0.22, 0.44]

Table 4-3. Unit test performance predictors

Variable	Coefficient				
	B	Std. Error	β	Sig.	CI
Constant	0.45	4.05	--	0.913	[-7.69, 8.58]
Intervention Status	-2.55	0.81	-0.34	0.003	[-4.18, -0.91]
Prior Math Achievement	0.55	0.01	0.53	0.001	[0.03, 0.08]

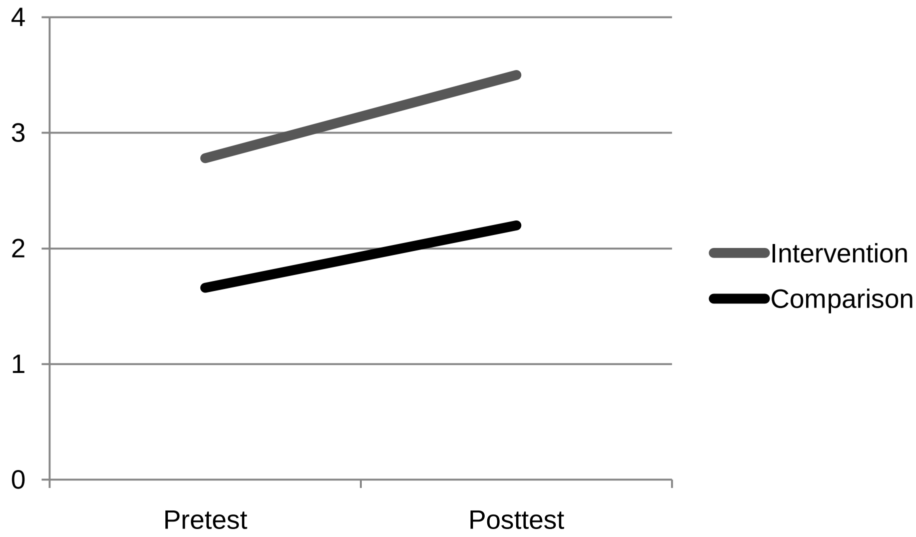

Figure 4-1. Mean problem-solving performance.

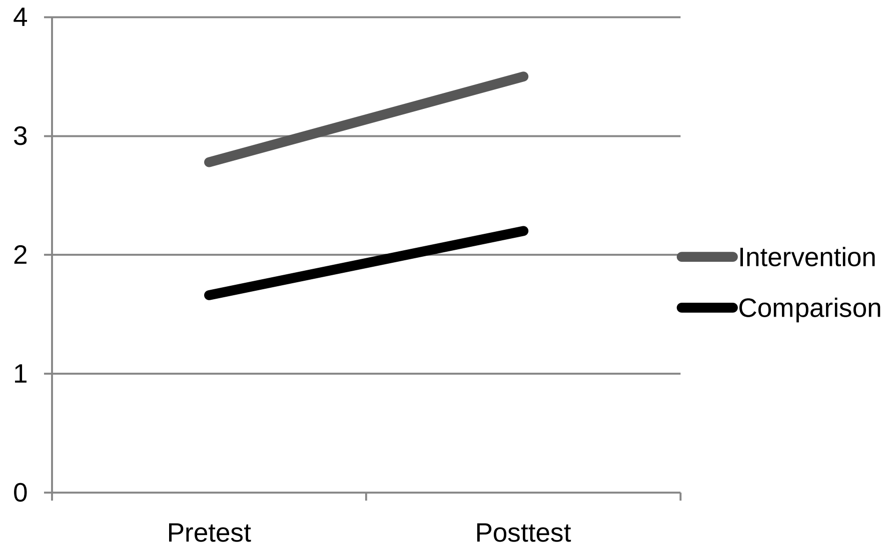

Figure 4-2. Mean number of representations used.

CHAPTER 5
DISCUSSION

Summary of the Findings

The primary purpose of this study was to examine students' problem-solving performance and representation use as a result of an instructional intervention. This investigation was guided by prior research indicating that students learning mathematics through problem-solving contexts might demonstrate enhanced problem-solving performance (e.g., Charles & Lester, 1984; Sigurdson et al., 1994; Verschaffel et al., 1999) as well as improved representation use while problem solving (Klein et al., 1998) compared to their peers experiencing their typical instruction. The open, complex, and realistic word problems were critical to the instructional intervention. The instructor in the intervention classroom aimed to maintain a student-centered, discourse-rich classroom with the goal of supporting students' problem solving as well as their mastery of concepts and procedures associated with the sixth-grade Standards (FLDOE, 2007). Data related to several predictor variables were collected and later investigated in relation to students' outcomes.

Instructional Comparison

There are some critical differences between the instruction in the intervention and comparison group that likely influenced students' outcomes. Researchers have used rich problems that could be solved in multiple ways and offered multiple entry points for learners and found beneficial results (Lampert, 1990; Verschaffel & De Corte, 1997; Verschaffel et al., 1999).

First, the types of materials used during instruction were different (Appendix N). Open, complex, realistic problems were the main focus of instructional materials for intervention participants. These students completed a few exercises for homework, but the assignments were structured so that they were expected to spend more time solving one or two problems than the other tasks. The materials in the comparison classroom tended to focus on correctly completing procedures and solving verbal translations of exercises. For example, the workbook associated with the textbook was always a part of students' station work. The directions preceding workbook tasks usually encouraged students to solve multiple similar exercises. The classroom teacher also gave students exercises to solve using whiteboards and markers. The comparison classroom tasks were usually exercises and problems were the focus in the intervention classroom.

A second difference between the intervention and comparison classroom was the social and sociomathematical norms. Creating a learning environment that promotes mathematical discourse and encourages students to exchange ideas does not happen overnight (Yackel & Cobb, 1996). Prior research has shown that establishing social and sociomathematical norms that support effective mathematics learning are critical components (Verschaffel et al., 1999). Much like the Dutch classrooms in the study conducted by Verschaffel and his team (1999), the instructor in the intervention classroom established and reminded students about social and sociomathematical norms. These adolescents learned quickly to adhere to these norms and were able to engage in sustained, focused mathematical discussions during instruction. For example, one student posed a question during a whole-classroom discussion, which led to a sustained student-to-student discussion about a way (i.e., representation and procedure) to solve a problem. The instructor did not intervene in the discussion except to remind interrupting students to hold their comments until the speaker was done.

Students seemed to take these norms seriously, as evidenced by the fact that they asked to add more expectations to the list after a few days of instruction. There were no visible indications of social or sociomathematical norms in the comparison classroom. The observer noted that discourse in the comparison classroom tended to involve responding to the teacher's questions and occasionally asking or responding to a peer's question during station activities. It appeared that this discourse pattern was expected during instruction. As a result, it seemed that the expectation for doing mathematics was to complete work assigned by the teacher and answer comprehension and recall questions. Similarly, the social norms in the comparison classroom were not obvious during the month-long observation. As a result of establishing norms in the intervention classroom, students were more engaged in mathematical discourse compared to their peers in the comparison classroom.

A third difference between classrooms was the discussions during instruction. Cameras were placed throughout the room during instruction to capture some of the small-group discussions. In the intervention group, these discussions tended to focus on mathematics, appropriate mathematical representations to solve problems, and content and procedures while problem solving. They frequently chatted about their interpretations of the problem, realistic constraints within the problem's context, and useful representations to support problem solving. During the small-group discussions, some participants worked independently on a problem and then talked about their solution(s) and representations. Other small groups co-solved problems and students worked together to complete a problem. This type of instruction is similar to that described by others (Lampert, 1990; Verschaffel & De Corte, 1997; Verschaffel et al., 1999). These authors stress the critical importance of discourse for promoting mathematics understanding and results from the present study provide further evidence related to the positive effects of discourse-rich learning environments. A discourse-rich learning environment is characterized by frequent student-to-student mathematical discussions about procedures and content. An example from the intervention classroom is provided followed by a nonexample from the comparison classroom.

The average (i.e., mean) number of letters in students' and instructor's first names was examined during the second week. Students thought about the task independently for a couple minutes and then discussed it for seven minutes in small groups. Next, participants presented their representations, procedures, and answers. During the whole-class discussion, one student explained from the front of the classroom how she arrived at the result (e.g., six remainder three). Another participant asked her to explain her answer further because he felt her arithmetic was incorrect. The student thought about her answer and stated that he was right and her answer was incorrect. She added that her problem-solving approach was appropriate for this task even though a slight mistake was made. Students continued to offer their representations, procedures, and solutions, and their peers proceeded to critique each other's ideas. At times, the instructor asked students to clarify (e.g., "What do you mean by that?" and "Can you explain that?") and justify their response (e.g., "How can the average number of letters be 22 when there is no one in the room with that many letters in their first name?"). The whole-class discussion lasted for thirteen minutes and was largely led by students. This specific student-led discussion lasted for approximately four minutes and was not interrupted by the instructor except to ask students to speak louder and clarify their responses. This is one instance of the type of rich discourse that was part of the intervention classroom.

On the same day, the classroom teacher conducted instruction focused on rates and unit rates. Students worked at six stations for approximately ten minutes each. Students completed exercises found in their workbooks or on worksheets at five of the six stations. At the teacher-led station, students completed FCAT practice problems. The teacher showed a task using the LCD projector and asked students to respond with the unit rate by showing their white boards with the solution. She provided students with a couple minutes to respond with the answer and then proceeded to complete the task. Occasionally, she asked students for their input (e.g., "What is five times thirty?"). Multiple students responded with the correct answer. She acknowledged that they were correct and continued to solve the problem. Later, she worked another task and posed a similar question to one individual (i.e., "What does four feet nine seconds represent? Distance, speed, or time?"). The student hesitated and the teacher immediately called on another student, who gave the correct answer (i.e., "Speed."). The teacher evaluated the student's response (i.e., "Yes, speed.") and continued solving the problem. These types of conversations were frequent during the classroom teacher's instruction and were nonexamples of rich discourse.

The small-group discussions during station activities in the comparison classroom tended to lack focus on mathematics or problem solving. Students generally asked each other procedural-oriented questions about exercises and spent much of the small-group time talking about material that was unrelated to mathematics. The tasks at each station provided a context for students' discourse, but the discourse did not develop in such a way that might lead to in-depth discussions. The teacher was also a factor in the degree to which mathematical dialogue supported students' mathematics learning. The intervention instructor was a discussion facilitator whereas the classroom teacher gave the expectation that the purpose of students' mathematical discourse was to provide a problem's solution. In other words, discourse patterns and the focus of the conversations was different between groups.

The intervention instructor also asked students to explain and justify their responses after offering a mathematical model, solution, set of procedures, or idea. A cursory review of the classroom videotapes indicates that this type of discourse was not common in the comparison classroom. The teacher in the comparison group conducted direct instruction about mathematics topics or procedures during each class and also led one of the stations. A preliminary review of the three randomly sampled videos from each group suggests mathematical discussions were typically shorter than those in the intervention classroom and teacher initiated. Students did not work collaboratively or discuss the exercises following the station activities. The teacher usually asked a recall or comprehension question, called on a student to respond, and evaluated the students' comment. If the student gave an incorrect response, then the teacher indicated it was wrong and the student returned to working the task. If the student gave a correct response, then the student erased his/her work and solution, and sat quietly waiting for the others to finish. Initiate-Respond-Evaluate (Franke et al., 2007) discourse patterns have been found frequently in US classrooms (Hiebert et al., 2005; Hiebert et al., 2003). This type of discourse conveys the perception that the teacher is responsible for students' learning (Franke et al., 2007). The comparison group teacher verified students' responses thus the teacher was central in determining correct solutions and representations for problem solving. On the other hand, the intervention instructor asked students to make these types of decisions. The present study provides some evidence that a teacher's role as discussion leader can support positive student outcomes, and more detailed analyses of classroom interactions will support this conclusion.

A fourth difference was the instructor and classroom teacher's prior knowledge of students, unit test, and Standards (FLDOE, 2007). The instructor had been a middle grades mathematics teacher in another state previously and became familiar with participants prior to the study by spending time in the classroom with them and asking questions before, during, and after class. Also, the instructor closely examined the NGSSS and Common Core State Standards and discussed them with the classroom teacher prior to the study. These experiences helped to frame the instructor's lessons and instructional behaviors. The classroom teacher had more knowledge from prior interactions with students, the unit test, and content that may have impacted students' outcomes. She spent the present and prior academic year with this group of students. Thus, she had a more robust understanding of her students than the instructor who spent 15 instructional days prior to the study with students. The teacher was also more familiar with the unit test, which was not examined by the instructor until after the third week of the intervention. This familiarity likely guided her instruction, and she was able to frame her instruction to support students' unit test performance. Finally, the instructor was not as familiar with the NGSSS and students' prior knowledge as the classroom teacher. The classroom teacher was more aware of students' understanding of mathematics content and procedures from summative and formative assessments during the prior year as well as the first four-and-a-half months of sixth-grade mathematics instruction. This knowledge may have guided the classroom teacher's implementation of instruction aimed to develop students' mathematics knowledge as measured by the unit test. Evidence from this study indicates that a teacher's prior knowledge of Standards, students, and the unit test may have impacted students' performance on unit test.

A fifth critical difference was the role of the instructor in each learning environment. This contributes to characterizing the comparison instruction as teacher-directed (i.e., weakly student-centered) and intervention classroom's learning environment as decidedly student-centered. The intervention instructor took on the role of discussion facilitator and directly answered students' questions only when participants did not agree on how to correctly carry out a mathematical procedure. For example, the instructor called on students during a whole-class discussion to answer peers' questions rather than explicitly answering a student's question. Lampert (1990) describes her role similarly, "I did not explain how to get the answers…I also expected them to answer questions about mathematical assumptions and the legitimacy of their strategies" (p. 38). The intervention instructor in the present study did not explicitly give answers or tell students how to solve a problem and instead was a problem solver while students worked in small groups. There was usually a brief summary of students' comments following each problem.

The classroom teacher's role was to disseminate content knowledge by providing students' opportunity to practice solving exercises. Her instruction tended to focus on correctly applying procedures and finding the problem’s solution. For example, she confirmed students’ responses at the whiteboard station. Students did not validate each other’s work and did not determine for themselves whether they were correct. Furthermore, she made the mathematics content and procedures much more explicit than the instructor in the intervention classroom. Evidently the instructor and classroom teacher played different roles during instruction thus contributing to the instructional environment and students’ outcomes.

Four critical differences between the intervention and comparison learning environments were (1) materials, (2) norms for the mathematics classroom, (3) the way(s) that discourse was a part of everyday mathematics instruction, and (4) role of the teacher and the learning environment. The intervention materials were mostly problems that were modified tasks from the curricular materials whereas students in the other classroom tended to spend more time solving exercises. The learning environment in the intervention classroom fostered student-to-student discourse because there were visible reminders as well as discussions of social and sociomathematical norms. Students in the intervention group were expected to talk to each other, ask questions, consider alternative representations and procedures and justify their response. There were no reminders or indications regarding the social or sociomathematical norms in the comparison classroom. Finally, the classroom teacher attempted to disseminate knowledge frequently, but the instructor sought ways for students to construct their own ideas. These differences contributed to the disparities between students’ outcomes.

Within-Group Comparisons

Problem-solving performance

The intervention group improved their problem-solving performance after the intervention whereas the comparison group did not. These findings are consistent with Sigurdson and Olson (1992) and provide information about the impact of problem-solving interventions. The effect size (d = .48) associated with the pre- and posttest differences gives an indication of the intervention's positive effect. As seen in Figure 4-1, the comparison group participants' problem-solving performance did not increase significantly between test administrations. The results indicate that the intervention supported students' problem-solving performance.

Representation use

Similarly, the intervention supported participants from the intervention group to use more representations on the posttest than the pretest. More specifically, they provided approximately one additional representation on the posttest. The comparison group participants did not use more representations after four weeks of their usual instruction. The effect size (d = .42) suggests that the intervention somewhat impacted students' representation use. The statistic provides an initial result, which could be confirmed by future research. Prior research has shown that teaching through problem-solving contexts supports students' ability to create appropriate mathematical models to solve word problems (Verschaffel & De Corte, 1997). The evidence from the present study and Verschaffel and De Corte tell a similar story: Using open, complex, and realistic problems with adolescents in student-centered discourse-rich instructional contexts enhances the number of representation used to solve problems whereas traditional instruction does not.

This finding related to adolescents' representation use from the present study is also consistent with explorations with younger children in the Netherlands (Klein et al., 1998). Second-grade students who experienced instruction that encouraged them to utilize a variety of representations were more likely to successfully employ more representations to solve exercises and problems than their peers in comparison classrooms. Drawing on evidence from these three studies, there is a growing body of evidence indicating that student-centered, discourse-rich instruction positively impacts students' use of representations on problem-solving tasks.

Between-Group Comparisons

Problem-solving performance

Results supported the hypothesis that the intervention group would show better problem-solving performance than the comparison group after one month of the instructional intervention. More specifically, controlling for pretest performance students experiencing the intervention solved more word problems on the posttest than their peers. The effect size (d = .26) was not greater than the average annual gain in mathematics achievement but it was greater than typical effect sizes associated with achievement tests administered to students in grades four through six. The difference in the two groups' problem-solving performance confirmed previous findings that students who experienced instruction that includes problem-solving features have better problem-solving performance than their peers in classrooms with traditional instruction (Charles & Lester, 1984, Sigurdson et al., 1994; Verschaffel et al., 1999).

The results from the present study are an extension and confirmation of prior problem-solving research. Charles and Lester (1984) noted that adolescents' problem-solving related outcomes were enhanced when problem-solving instruction was a piece of overall daily mathematics instruction (i.e., 10 - 25 minutes). Similarly, eighth-grade students who experienced 10 minutes of problem-solving activity had better performance on a test that included problem-solving oriented questions than their peers receiving instruction that typically involved algorithmic practice and exercises (Sigurdson et al., 1994). The present study along with others provides evidence that students experiencing mathematics instruction that draws on problem-solving components are likely to solve more problems than their peers experiencing their usual instruction.

The specific mechanism (i.e., materials, instructor, instructional actions, and mathematical discourse) that produced the desirable effect of improving students' problem-solving performance unfortunately cannot be determined, but each component likely impacted students' outcomes. It is hypothesized that several components may have impacted students' outcomes. These components (e.g., tasks, norms, discourse) have been discussed as critical instructional elements prior literature as factors impacting students' outcomes. Thus, manipulating multiple components within classroom contexts such as was done in this study, likely affected students' problem-solving performance and representation use. For example, the materials in the intervention classroom provided a context for students to discuss mathematics content and procedures whereas the workbook exercises were fairly straightforward and did not offer complexities that students might need to co-examine with a peer. The norms in the intervention classroom created a learning environment for sustainable mathematical discourse. A number of investigators (e.g., Cobb et al., 1992; Lo & Wheatley, 1994; Yackel & Cobb, 1996) have argued that social and sociomathematical norms are important factors that influence students' outcomes. Lampert (1990) noted that fifth-grade students engaged in student-centered, discourse-rich mathematics instruction had opportunities to learn mathematics with understanding. Intervention participants from the present study were able to solve more word problems compared to their peers following the instructional intervention. The benefits of having positive instructional components are demonstrated in the present study.

Fifth-grade students who experienced lessons similar to those from the dissertation study approximately once each week for four months had significantly better performance on a test of word problems than their peers who were provided with more traditional instruction (Verschaffel et al., 1999). The intervention in the present study generated a positive effect on students' problem-solving performance (i.e., d = .26), which verified previous findings (d = .31) (Verschaffel et al., 1999). It is possible that daily mathematics instruction using problem-solving contexts might lead to stronger effects if the intervention took place over a longer period of time, but this hypothesis must be further examined in efficacy studies. Following the intervention, students showed better problem-solving performance than their comparison group peers.

Representation use

Prior to the intervention, there was no difference between groups' pretest representation use. After one month of the instructional intervention, participants used approximately one more representation on the posttest than their peers in the comparison group (d = .18). Similar to problem-solving performance differences, it was a mixture of several instructional aspects that likely led to this result.

The first aspect was the use of open, complex, and realistic word problems. Drawing on realistic problems may foster links between students' prior knowledge including their mathematical knowledge and knowledge gained from experiences in the community (Boaler, 2002). For example, problems about pizza prices from local restaurants, movie watching habits of local households, and sports-related statistics from a famous athlete provide a context for students to use their knowledge of real life in conjunction with their mathematics knowledge. Boaler (2002) has argued that realistic problems encourage children to draw on their knowledge from nonacademic situations, which may help them solve problems using novel approaches. Tasks such as those from the present study align with prior research and led to improving students' representation use on problem-solving tasks.

The second factor was a student-centered, discourse-rich learning environment with supportive social and sociomathematical norms. Students in student-centered, discourse-rich learning environments tended to be more effective with their representation use as well as use more representations during problem solving than their peers experiencing more teacher-directed instruction that focused on appropriately employing specific approaches to solve tasks in both the present study and one conducted by Klein and his et al. (1998). Teaching students to choose the "most appropriate and efficient strategy or procedure given the (number) characteristics of the problem at hand" (RPD) (Klein et al., 1998, p. 449) resulted in better representation and procedure use than their peers in the GPD condition. Boaler (2002) suggested that instruction similar to that employed in this dissertation study, (i.e., use of student-centered, discourse-rich mathematics instruction that employs reform-oriented materials) creates a context for learning that supports all learners. Lampert (1990) noted a similar finding: Students who engaged in discourse while problem solving recognized the strengths and limitations of representations. Participants in the present study employed viable representations to solve problems and tended not to use ineffective representations. Encouraging students to share their representations while problem solving resulted in their ability to implement more representations on a problem-solving test than their peers who experienced fairly teacher-directed instruction.

There was no significant relationship between nonsymbolic representation use on the posttest and intervention status. Instruction in the intervention classroom typically encouraged students to think about whether there were alternate representations and procedures to solve a problem. This nonsignificant finding may be influenced by several factors including students' ability to use nonsymbolic representations (Preston & Garner, 2003) as well as their perceptions related to employing nonsymbolic representations to solve word problems (Bostic & Pape, 2010; Herman, 2007). Herman noticed that Algebra students perceived pictorial and tabular representations as backup methods to verify a symbolic-oriented representation. Further research is necessary to explain why no significant relationship was produced by the intervention.

Unit test

In the present study, a unit test was selected from the assessment materials accompanying the textbook and administered to all participants. All items were constructed response and some asked students to provide an explanation of their response. The average unit test score from the comparison group was approximately two points higher than the intervention group (i.e., $d = .34$). This study builds upon others by providing evidence related to students' achievement for specific mathematics topics after experiencing mathematics instruction in problem-solving contexts. Most investigations (Sigurdson et al., 1994; Verschaffel et al., 1999) used summative mathematics achievement tests to measure students' overall mathematics understanding and results have been inconsistent.

There were differential effects of incorporating problem-solving contexts into daily mathematics instruction on eighth-grade students' achievement (Sigurdson et al., 1994). That is, "except for low-achievement students, the 10 minutes devoted to problems each day appears, simply, to detract from students' achievement" (p. 380). It is unclear why all students from their study did not benefit equally; however, it is possible that explicit mathematics teaching that focuses on procedures and content may be a critical element linked to students' unit test performance. For example, comparison participants in the dissertation study routinely practiced exercises similar to those on the unit test during their time at each station. Their homework was also intended to provide opportunities to practice solving these types of problems. In a previous study, participants who experienced both explicit mathematics instruction as well as lessons focused on problem solving had better achievement scores than those in comparison classrooms (Verschaffel et al., 1999). "Greater attention on mathematical problem solving in the experimental classes (at the expense of the other subject-matter topics in mathematics) had no negative side effect and even a small positive (transfer) effect on pupil's mathematical knowledge and skills" (Verschaffel et al., 1999, p. 218). The present investigation offers evidence that teacher-led instruction may be a necessary component of teaching mathematics through problem-solving contexts. Thus, further research is needed to understand the impact of incorporating explicit teacher-led mathematics instruction into problem-solving interventions on students' achievement.

The current study extends problem-solving research by examining sixth-grade students' outcomes following a one-month instructional intervention that supplanted the typical classroom instruction. Students' problem-solving performance and number of representation used on a word problem test was enhanced by the intervention. Participants in the intervention group had lower performance on the unit test than their peers. The results of this study have both theoretical and practical implications, which are discussed further.

Theoretical Implications

The goal of this study was to examine sixth-grade students' outcomes as a result of teaching mathematics through problem-solving contexts. Findings have more implications for practice but there are a few theoretical implications. First, teaching mathematics through problem-solving contexts was linked to both negative and positive outcomes for sixth-grade students. This study confirms previous findings that interventions such as this one support students' problem-solving performance whereas the comparison classroom instruction was not linked with such gains in problem solving. Moreover, tasks on the problem-solving measures were not explicitly constructed to focus on the instructional topics (i.e., rates, ratios, and data analysis). This finding provides evidence that this type of instruction supports students' problem-solving outcomes, regardless of the grade-level content embedded within problem-solving tasks. There may be limits to the positive effects of mathematics instruction that primarily draws on open, complex, and realistic word problems.

Teaching through problem-solving contexts may not necessarily lead to greater achievement outcomes. The intervention appeared not to support students' achievement on the unit test. It is possible that this type of instructional intervention that places little emphasis on solving exercises might not support students' mathematics achievement as measured by a unit test. Another potential explanation of this finding is that mathematics procedures were not synthesized for students or made explicit during instruction in the intervention classroom. The classroom teacher provided explicit instruction focusing on applying mathematical procedures, which might explain comparison students' performance. Below-average performing students benefitted from discussions, syntheses, and explicit examination of mathematics content and procedures that were part of the problem-process instructional program (Sigurdson et al., 1994). Abstracting mathematics from word problems may be a crucial aspect for supporting students' academic growth in classrooms employing instruction similar to this intervention. The findings provide some confirming evidence about teaching through problem-solving contexts and a need for explicit mathematics instruction.

Second, the findings confirm prior research (i.e., Verschaffel et al., 1999) that students' background characteristics were not a significant factor in predicting students' problem-solving outcomes. Analyses revealed that students' demographics (e.g., ethnicity, gender, and free-and-reduced lunch status) were not significantly related to problem-solving performance or representation use. Moreover, students' prior FCAT reading and mathematics achievement were also not significant predictors of problem-solving outcomes. Another set of multiple regression models were examined to provide more evidence that these covariates were not significant predictors when added to the final models examining problem-solving performance and posttest representation use. Specifically, a predictor (e.g., gender) was placed in the final model and its significance was examined. This procedure was conducted for both models of the dependent variable and each covariate. None of the covariates were significant when entered into the model in this fashion. In an era when investigators (e.g., Gonzales et al., 2008) frequently examine the relationship between students' performance and covariates such as these, there is a growing body of evidence suggesting that students' ability to solve open, complex, and realistic problems is dependent on their prior problem-solving related outcomes (i.e., performance and representation use), but not necessarily other factors.

Third, this feasibility study characterizes one way that teaching mathematics through problem-solving contexts might occur. Prior investigations similar to this one (e.g., Verschaffel & De Corte, 1997; Verschaffel et al., 1999) provided guidance for teaching mathematics through problem-solving contexts but instruction was not conducted on a regular basis or drawing on state or national standards. This exploratory study demonstrates that teaching mathematics through problem-solving contexts is feasible during this critical era with new Standards (CCSSO, 2010; FLDOE, 2007). This feasibility study provides one possible way to create and implement such an intervention that teaches state-mandated mathematics standards, which supports future investigators seeking to conduct similar types of research.

Fourth, Cohen (1988) and Lipsey's (1990) guidelines for interpreting *d* provide are unit-free and "there is no inherent practical or substantive meaning to standard deviation units" (Bloom et al., 2008, p. 2). Thus, it is more helpful to interpret effect sizes within a meaningful context, such as usual growth over time (Bloom et al., 2008) or typical effect size estimates of an educational intervention on students' achievement (Hill et al., 2008). Data from 13 standardized assessments and estimated the annual growth in effect size for fifth- and sixth-grade students to be approximately 0.41 (Bloom et al., 2008). Seventy-six research reports and publications contributed to calculating the mean effect size on achievement measures for students in grades four through six (Hill et al., 2008). The intervention's impact on students' problem-solving performance (d = .26) was somewhat lower than the average annual gain in effect size for nationally normed mathematics tests (d = .41), but it was slightly larger the mean effect size related to achievement differences as a result of educational interventions in grades 4-6 (d = .22) (Hill et al., 2008). The effect sizes from the present study may suggest "what might be attainable" (Hill et al., 2008, p. 176) is different for an intervention's impact on problem-performance than achievement. It is possible that the mean effect size of an intervention's impact on problem-solving performance is slightly larger than those associated with achievement.

In conclusion, the findings have some implications for theory. There is a growing body of evidence suggesting the strength and limitations of teaching through problem-solving contexts. This study confirms prior results suggesting that students' characteristics and prior achievement as measured by a standardized test have no significant influence on students' problem-solving outcomes. The description of teaching mathematics through problem-solving contexts offers the research community a classroom-based instructional intervention that might be improved. Finally, the effect of the intervention may encourage others to further examine mean effect sizes related to problem-solving performance and compare them to achievement-related outcomes. A useful metanalysis might indicate the average gains in problem-solving performance over several grade levels or age-ranges. As mentioned at the beginning of this section, there are few theoretical implications but far more consequences for teacher's and teacher educator's practices.

Practical Implications

This study supports classroom teachers as well as researchers. First, the instructional intervention may be a model for conducting mathematics instruction that teaches the middle-grades mathematics Standards (CCSSO, 2010; FLDOE, 2007) in problem-solving contexts. The intervention was not as successful as the comparison teaching with respect to unit-test performance, yet students demonstrated understanding content and procedures fairly well. Teachers might consider allowing some instructional time for explicit mathematics instruction as part of teaching mathematics through problem-solving contexts. Findings provide evidence that this type of instruction (i.e., intervention instruction conducted in the dissertation study) benefits students' problem-solving ability and the number of representations used to solve word problems. Furthermore, instruction from the intervention classroom led to greater problem-solving outcomes compared to typical instruction. If a goal of mathematics instruction is to develop competent problem solvers who are able to solve realistic problems (CCSSO, 2010), then teachers might consider teaching mathematics through problem-solving contexts.

The findings also indicate a problematic issue related to teaching mathematics through problem-solving contexts. One study indicated that participants experiencing mathematics instruction through problem-solving contexts had better achievement, as measured by a comprehensive test, than their peers experiencing business-as-usual instruction (Verschaffel et al., 1999). Participants in the present study who experienced the intervention did not perform as well on the unit test as their peers who received their usual instruction. Considering the present study's findings and those from prior research (i.e., Sigurdson et al., 1994), it appears that teaching mathematics through problem-solving contexts as it was done in this study has limited effects on students' achievement. As mentioned earlier, classroom teachers might consider allowing for some instructional time, albeit limited, to conduct explicit instruction focusing on mathematics skills, procedures, and concepts. One way to provide explicit instruction might be to make skills, procedures, or concepts an explicit topic of discussion as part of a synthesis after students finish working on an open, complex, realistic word problem. Teachers should be cautious about implementing this intervention as constructed as an aim to enhance students' content learning until further research is conducted.

Students who experienced the intervention were able to implement more than one representation to solve at least one of the word problems on the posttest. This somewhat meager increase provides evidence that students were gaining mathematical proficiency because they were able to represent and solve problems in multiple ways, which is an indicator of strategic competence (Kilpatrick, 2001). This result has two practical implications. First, few traditional commercially published textbooks use problems, as they were defined in this study (Grischenko, 2009). Grischenko's (2009) analyses indicated that many textbooks' word problems are exercises translated from symbolic to verbal representations. Tasks meant to encourage comparison participants to problem solve were selected from textbook materials and not adapted or made complex, open, or realistic. Hence, the comparison participants might have had limited exposure to problems. Teachers and mathematics coaches might notice that participants from this study were able to solve challenging problems and consider adding problems such as those used with the intervention group to mathematics instruction. Instructional materials should also provide tasks that can often be solved in multiple ways, that is, representationally- and procedurally different strategies. Pittman (2006) argued that many curricular materials provide insufficient support for students to develop an ability to solve problems in multiple ways and she advocated for changes in the types of tasks that are presented to students. To that end, teachers and mathematics coaches might incorporate open problems like those from this study as a context for mathematics-related dialogue about solving problems using different representations.

The findings may not necessarily influence mathematics instruction district- or state-wide but after presenting preliminary findings to the school faculty, a discussion was sparked among middle-grades teachers about ways to incorporate problem solving into their mathematics instruction. The process of developing a student-centered, discourse-rich classroom environment as well as constructing lessons suited for problem-solving contexts must be shared with the practitioner community. For instance, evidence from videotapes could provide a context to initiate discussions about classroom practices. The classroom teacher from this study indicated that she appreciated the opportunity to learn how others perceived her instruction and how her instruction influenced students' problem solving. The videotapes need to be formatted before showing them to other teachers but after such editing, they may become materials to use during professional development with mathematics teachers in order to support reflection that leads to changes in their classroom practices aimed at improving students' problem-solving performance and representation use.

This research offers examples of teaching mathematics through problem-solving contexts, which might stimulate others to create and enact lessons for other grade levels and/or with different content. The lessons may provide a context for pre- and inservice teachers to learn about teaching through problem-solving contexts and how it differs from more traditional mathematics instruction. This may support their thinking about ways to (a) foster mathematics learning and effective problem-solving behaviors in the classroom, (b) create a student-centered learning environment, and (c) initiate and sustain problem-solving discourse among students. Tapes of the intervention classroom (1) show classroom instruction conducted in a nontraditional way and (2) students engaging in collaborative problem solving. These videos need refining before showing them to pre- and inservice teachers, but there is potential that such video evidence, within the context of professional development or undergraduate or graduate coursework, might support mathematics teacher education development. They could reflect on the extent to which instructional decisions impact student outcomes as part of their undergraduate coursework. There is an abundance of research indicating that these facets of reform-oriented instruction positively impact students' outcomes yet still, mathematics teaching is typically not characterized this way (Hiebert et al., 2005). The tapes, used as part of a comprehensive teacher education program, might support preservice teachers' growth as practitioners.

This study offers suggestions for K-12 instructional practices and has the potential to positively affect inservice teacher's professional development as well as preservice mathematics teacher education. Limitations of the present study impact the validity and generalizability of the results. More problem-solving investigations are needed, which effectively manage issues that arose in this study.

Limitations of the Study and Suggestions for Future Research

Research Design and Instructional Intervention

This study has some limitations that affect the generalizability of the results. First, a limitation of any quasi-experimental study is the inability to randomly assign individual participants to each group. Furthermore, participants were not randomly selected from the greater population of sixth-grade students. A group of participants (i.e., one section of sixth-grade mathematics) was randomly assigned to the intervention, and the comparison group consisted of the other two sections that met on the same day. The intervention and comparison groups had similar problem-solving performance and representation use prior to the study thus differences between groups prior to the intervention were controlled. However, there is the possibility that the groups were significantly different in other ways.

This sample represented a wide variety of students with different backgrounds, yet a larger and more diverse sample might provide more evidence about students' outcomes. The sample size may have limited the statistical power to produce statistically significant coefficients for any variable related to students' background (e.g., ethnicity, free-or-reduced lunch status, and gender). Therefore, large-scale explanatory (i.e., efficacy) research is necessary to confirm findings from the present study indicating that students' background characteristics (e.g., ethnicity and gender) and prior reading and mathematics achievement do not account for a significant amount of variance in their problem-solving outcomes. With two comparison classrooms and one intervention classroom, it was not possible to separate the effects of the instructor, intervention, and classroom. Future researchers might consider two instructors conducting instruction in two classrooms each in order to better separate the classroom and intervention effects. The instructor is a critical aspect of the intervention and it is not feasible to distinguish between the instructor and intervention effects without a different research design. Finally, results cannot be generalized to ELLs and exceptional learners since they were not part of the participant sample that completed the measures. Future research should heed the suggestions provided later in this section and consider using a larger and more diverse sample.

Another limitation of the study was that the intervention was not tested and refined and content experts (i.e., mathematics education researchers and classroom teachers) did not examine the lessons prior to their implementation in the dissertation study. Pilot testing the intervention and refining it may have enhanced the effectiveness of the intervention. It is possible that the intervention's capacity to improve students' achievement as well as problem-solving performance and representation use has not reached its full potential. Statistical power of the intervention may be lower than its potential because it was not implemented and improved based on a pilot study. Examining the strengths and weaknesses of the lessons using a lesson study approach improves a lesson's effectiveness (Fernandez & Yoshida, 2004) and might benefit future explorations similar to this one. With support and guidance from classroom teachers, a future study might have better lessons that lead to supporting students' problem-solving outcomes and unit test performance.

As mentioned in the methods section, the study lacks some ecological validity because it does not describe how a classroom teacher might enact the instructional intervention. The researcher was the instructor in the intervention classroom. The differences in the teacher and instructor's mathematical and pedagogical content knowledge (Hill, Ball, & Schilling, 2008; Shulman, 1986) likely influenced the format and content of the instruction. This weakness could be resolved through an efficacy trial with similar teachers.

This study is a feasibility study that might lead to conducting a subsequent efficacy trial with classroom teachers delivering a revised instructional intervention. "Efficacy trials assess the value or worth of a treatment or instructional program" (Sloane, 2008, p. 625) whereas "effectiveness trials provide tests of whether the formally tested and efficacious treatment does more harm than good when it is delivered under real-world conditions" (Sloane, 2008, p. 625). The intent of the intervention was to create an instructional environment that facilitated student collaboration while problem solving, share their representations, and to engage them in mathematics through open and complex word problems, which draw on realistic contexts. In a previous study, researchers (Klein et al., 1998) perfected their instructional intervention over the course of a year-long pilot study before it was implemented as an efficacy trial. A team of mathematics education researchers and classroom teachers collaborated to develop, implement, and refine lessons based on students' feedback. This likely increased the overall effectiveness of their intervention.

Investigators might consider refining the intervention and scaling it up for the next round of implementation as an efficacy trial. Results from an efficacy trial using an improved instructional intervention might reveal why students in the intervention group did not have similar performance on the unit test to their peers in the comparison group. Furthermore, investigators ought to consider having a similar number of comparison and intervention classrooms and again randomly assign groups to each condition.

Future researchers might also consider drawing on design-based research (DBR) methods (Design-Based Research Collective, 2003) to refine and implement the intervention. A DBR methodology requires iterative cycles of planning, implementation, and data collection and analysis (Barab & Squire, 2004). "The goal of DBR is to use the close study of a single learning environment…and as it occurs in naturalistic contexts, to develop new theories, artifacts, and practices that can be generalized to other schools and classrooms" (Barab, 2006, p. 153). Interventions meant to enhance instructional practices and support students' outcomes best fit this methodology (Barab, 2006; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Design-Based Research Collective, 2003). This is a methodology for revising and implementing the current instructional intervention as part of a future study. Teaching experiments that are enacted, revised, and re-enacted are an example of design-based research (Cobb et al., 2003). A research team composed of educational researchers, classroom teachers, and curriculum specialists might facilitate developing and conducting a stronger study. One logical next step for this study is to refine the measures and intervention and invite one classroom teacher to implement the intervention. If the results confirm prior findings then more teachers should be invited to participate and a district-wide efficacy study might be implemented. By appropriately scaling up classroom-based research and going through the iterative process of DBR (Barab & Squire, 2004), it is possible to reach the goal of developing “usable knowledge” (Lagemann, 2002, p. 8) for researchers, practitioners, policymakers, and other educational stakeholders.

For too long, problem solving has been treated “as an isolated topic akin to algebra or geometry. We need better integration of problem solving within all topic areas across the mathematics curriculum, and ... across disciplines” (English & Sriraman, 2010, pp. 267-268). An aim of this study was to heed this message and provide an example of one way to teach mathematics through problem-solving contexts. The mathematics concepts were de-emphasized during instruction and learning to solve problems and apply mathematics concepts and procedures to complete problems were an important aspect of each lesson. As a result, students learned mathematics, albeit not as well as their peers, and developed effective problem-solving behaviors. This is a foray into teaching mathematics content and procedures required by the Standards (FLDOE, 2007) through problem-solving contexts, drawing on open, complex, and realistic word problems. It provides a foundation for conducting similar instructional intervention using the Standards for Mathematical Content (CCSSO, 2010). Furthermore, this study demonstrates ways to develop the varieties of expertise that mathematics educators should develop in their students (i.e., Standards for Mathematical Practice) (CCSSO, 2010). It is possible that interventions similar to this the one may implicate other issues besides achievement. For example, Boaler (2000) and Lubienski (1998) suggest that some reform-oriented interventions might promote equity.

Equity is a critically important principle of mathematics education (Boaler, 2002; NCTM, 2000). A future direction for research similar to this one is to co-examine students' problem-solving outcomes in conjunction with equity issues. Lubienski examined students' outcomes as a function of teaching with Connected Mathematics Project materials in a student-centered learning environment and found that reform curriculum alone does not support equity in the classroom. She noted that educators must consider "how to adapt instruction to meet lower SES students' needs" (Lubienski, 2000, p. 480). In another equity-focused study, Boaler (2002) examined the positive effects of reform-oriented materials used in conjunction with student-centered and content-focused instructional practices. Instructional format, classroom setting, as well as task's contexts have the propensity to (a) support students of color as well as (b) children living in poverty and (c) foster greater mathematical proficiency for all learners (Boaler, 2002). Boaler and Lubienski have differing perceptions of reform curricula's impact on equity in the classroom. More specifically, their findings show a difference in marginalized students' outcomes as a result of instruction using non-traditional instruction. Both agree though, that realistic contexts are critically important for supporting students' learning and positively impact students' instructional outcomes. More specifically, Lubienski (2000) advocated that students' outcomes as a result of using "open, contextualized problems" (p. 479) need more examination through classroom-based research, especially in classrooms with a high number of students of color and/or students living in poverty. There is tentative evidence suggesting that students of color living in poverty benefit from teaching mathematics through problem-solving contexts (Bostic & Jacobbe, 2010), but explanatory research is needed.

Investigators might revise and implement the intervention as part of an efficacy trial in a school with a large number of students of color and/or who are living in poverty and compare outcomes to peers receiving their usual instruction. Data from the present study could also be further investigated to determine the extent of equity promotion within the classroom. In conclusion, problem-solving researchers who are keenly aware of equity issues should consider the possibly positive effects of interventions like the one from this study.

Measures

The measures also limit generalizations about students' problem-solving performance and representation use. No demographic information was collected during the pilot study. Therefore, Differential Item Functioning (DIF) analyses could not be conducted. The differences between the tests' parameters are attributed to random variance (Embretson & Reise, 2000), but it is also possible that there was uniform or nonuniform DIF. DIF analyses with the dissertation data are inappropriate considering the sample size and number of participants representing each category (e.g., African-American, Hispanic, male, female, etc.) (de Ayala, 2009; Embretson & Reise, 2000). DIF should be investigated with a larger sample in future studies. It is uncertain whether DIF was an issue with the pilot measures therefore no definitive statement can be made about DIF on the dissertation measures.

There is also the potential for the tasks on the problem-solving measures to have uneven step difficulties. As an example, it is possible that arriving at the mathematical modeling stage of problem solving might be more difficult than carrying out procedures and interpreting the result. Students earned full credit when they carried out procedures appropriately but made a minor arithmetic error while working with their representation.

More than one minor error committed during problem solving was coded as an incorrect solution and the representation was not counted. A dichotomous scoring procedure was employed for two reasons: (1) there was evidence from the pilot study that few students' responses fit a partially-correct category and (2) Partial Credit Models (PCMs) from IRT are complex and require that every task have identical item parameters (Embretson & Reise, 2000). For these reasons it was not possible to use PCMs with the data. Future investigators ought to revise the measures and consider PCMs.

As mentioned previously, investigators might consider conducting a longer investigation (e.g., one semester or one academic year) and administering multiple unit tests as well as an overall achievement test. The unit tests might come from a textbook's assessment materials and the achievement test might be composed of items from sources such as the National Assessment for Educational Progress (National Center for Education Statistics, 2009). Results might explain the degree to which instructional interventions such as this one impacts students' specific content knowledge and overall mathematics achievement. Researchers might also investigate the strength of correlations between students' scores on unit tests and an overall achievement test. This type of investigation would provide confirming or contradictory evidence about the effects of teaching mathematics through problem-solving contexts on students' achievement.

Representation Coding Protocol and Strategic Behaviors

Coding students' strategies by their representations was another limitation. Mixing representations during problem solving was coded generally (e.g., symbolic-pictorial), which did not characterize whether a participant used the symbolic or pictorial representation first. Retrospective interviews with participants may be necessary to determine which representation was used initially. Furthermore, only representations were coded, but procedures were not examined. A different coding scheme that combines the representations as well as the procedures might shed light on students' strategy use rather than their representation use. An emergent protocol that stems from students' solution procedures might lead to a more robust and comprehensive coding scheme. The representation coding protocol may be an area to further explore with these data and in the future.

Think-aloud data were collected as part of this study but were not analyzed for this dissertation. These data will provide evidence related to students' strategic behaviors while problem solving. Moreover, these data will supplement the present study because participants were selected from both comparison and intervention groups based on their prior fifth-grade mathematics FCAT score. Researchers noticed that incorporating problem solving into mathematics instruction appeared to benefit below-average performing students more than others, but more evidence is needed to confirm their findings (Sigurdson et al., 1994). Future investigators might compare students' outcomes and also conduct in-depth investigations into the effects of the intervention on students' strategic behaviors using the think-aloud data. Think-aloud data may offer insights into the ways students solve problems and the degree to which the intervention changed how students solved problems. Participants' think-aloud data were not analyzed in time for this dissertation, but these data will be examined and explored in relation to students' outcomes on the problem-solving measures and unit test. Students' explanations and externalized thoughts during the think aloud also provide substantial data for future analyses.

Discourse, Explanation, and Justification

Participants' outcomes were likely influenced by the mathematical discourse during instruction. The descriptions and comparisons were based on analyses of videorecordings and the observer's ratings and fieldnotes, but a more systematic discourse-coding procedure is necessary. Discourse analysis methods such as Gee's (2011) provide one possible means to analyze classroom discourse. Another approach requires coding teacher/instructor's and students' discourse and examining frequency of types of discourse during an instructional episode (Pape et al., 2010). Both approaches would add rigor to the way that discourse was examined.

Students in the intervention classroom frequently discussed problem solving, ways to solve problems, potential issues related to solving problems, and mathematics content. Video evidence was collected from several perspectives in the classroom, which might facilitate examining problem-solving discourse between two students as well as whole-classroom dialogue. Researchers might carefully examine the tapes and conduct discourse analyses as well as investigate students' cooperative learning behaviors. Further research is needed to examine the ways in which mathematical discourse supported and/or limited mathematics learning in the two classrooms.

Intervention students frequently provided explanations and justifications during instruction, as suggested by the graduate student observer's notes. Students' explanations are descriptions of their problem solving whereas justifications constituted comments related to a rationale for engaging in specific problem-solving actions and behaviors (NCTM, 2000). Justification and reasoning are critical elements of problem solving (NCTM, 2000). Researchers should consider examining whether the instructional intervention influenced students' explanations and justifications of their problem-solving approaches and solutions. The way that discourse was used to facilitate students' learning within problem-solving contexts should be examined further.

Summary

In summary, future researchers might explore four potential areas that stem from this investigation and its limitations. First, an improved research design and refined instructional intervention as well as one delivered over a longer time period might provide further understanding of the effects of such instruction. Specifically, findings might elucidate the positive and negative outcomes associated with teaching mathematics through problem-solving contexts. Second, employing a variety of achievement measures in a longer study might clarify the link between an instructional intervention and achievement. Next, explorations into the ways that students solve problems ought to consider coding schemes that consider students' representation and procedure use. Think-aloud data and classroom data as well as a new protocol will support explorations into students' strategic behaviors and whether the intervention affected students' problem solving. Finally, data from this study also support investigations into discourse as well as the ways that students engaged in explanations and justified their mathematical thinking. There are assuredly other research trajectories but those discussed here are four possible areas for future researchers to explore.

Final Thoughts

The central aim of this study was to explore whether the intervention enhanced students' performance and representation use on word problem tests. In summary, intervention participants had better problem-solving performance and representation use between test administrations. Similarly, they also performed better on the measures than their peers in the comparison group. There was no relationship between nonsymbolic representation use and intervention status. Finally, comparison participants had better achievement on the unit test than their intervention peers. This investigation demonstrates that it is possible to teach mathematics from the Standards (FLDOE, 2007) through problem-solving contexts and in ways that develop effective mathematical practices (CCSSO, 2010).

This study is an extension of multiple studies, most specifically one conducted by Verschaffel and his research team (1999). It improves upon Verschaffel and De Corte's (1997) feasibility study and provides suggestions for a future investigation. The study offers implications for theory and practice. Multiple studies including this one show that including problem solving in everyday instruction benefits students' problem-solving performance. There are greater benefits to problem-solving outcomes when instruction such as the type implemented in this study is delivered on a regular basis. Similarly, results from the present study confirm findings from prior research (Klein et al., 1998) that rich discussion about strategy (i.e., representations and procedures) use during instruction benefits students' outcomes. Future researchers ought to employ several videorecorders during instructional interventions so that various instructional aspects can be closely examined. The videotapes are a rich data source that will be further explored for intervention students' discourse, engagement in problem solving, strategic behaviors, and compared to their peers in the comparison classroom. This investigation also offers multiple aspects that might inform teacher's instructional practices. For example, a description of teaching mathematics content from the Standards (FLDOE, 2007) through problem-solving contexts is provided, which may be a model for future implementers working with different mathematics standards. This description might inform the ways that teacher's deliver mathematics instruction focusing on other Standards (e.g., CCSSO, 2010). Furthermore, the process of creating materials intended to support students' learning that are similar to the ones used from this study will be shared so that teachers have more ideas for teaching mathematics content. Data from this study will continue to be examined to enhance both theory and practice.

APPENDIX A
A GUIDE TO THE SIX STAGES OF PROBLEM SOLVING

1. Reading the problem.
   a. Did you read the entire problem?
   b. Were there any words that you need help understanding?
   c. Do you understand what you are supposed to find?

2. Describing the situation
   a. What is happening in this problem?
   b. Can you represent the situation presented in the problem?

3. Creating a mathematical model
   a. What information is necessary to solve the problem?
   b. What information is unnecessary to solve the problem?
   c. Think about whether this problem is similar to others you have seen before.
   d. Is there more than one way to begin solving this problem?

4. Using a strategy and finding the result.
   a. Think about some possible strategies and choose one that will work with what you created in the previous stage.
   b. Look at your work thus far. Did you make any mistakes with your arithmetic or carrying out the strategy?
   c. Does your result make sense when you look at your mathematical model?

5. Interpreting your result
   a. What are the units for your result?
   b. Does your result answer the original question?
   c. Does your result fit with your situation? Is it a realistic answer?

6. Reporting your answer
   a. Did you write a sentence that clearly answers the question with the final solution?

** Is there another strategy that might answer the problem? Does your strategy use different steps to calculate the result? **

APPENDIX B
DESK-SIZED MODEL OF THE SIX STAGES OF PROBLEM SOLVING

The six stages of problem solving

1. Reading the problem
2. Describing the situation
3. Creating a mathematical model
4. Using a strategy and finding the result
5. Interpreting your result
6. Reporting your answer

APPENDIX C
PILOT STUDY PROBLEM-SOLVING PRETEST

Directions: Please answer each question and show all of your work. If you are able to show other ways to solve the problem, there is space below each one to show your work. If you need more space, you are welcome to use the back of each paper or additional sheets of paper.

1) Ruth is planning to serve ice cream sundaes to guests at her birthday party. She purchased 3 flavors of ice cream: vanilla, chocolate, and strawberry, 2 different sauces: chocolate and caramel, and 4 different toppings: bananas, nuts, sprinkles, and whipped cream. How many different types of sundaes can be made if every guest selects only one ice cream flavor, one type of sauce, and one topping?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

2) Jerome needs 1 gallon of paint in order to paint a bedroom ceiling that is shaped like a square and measures 12 feet on each side. How many gallons of paint would he need to paint a living room ceiling that is shaped like a square and each side measures 24 feet?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

3) Bill is making a gate for a wooden fence to keep his dogs in his yard. He bought four boards of wood from the home improvement store. Each board measures 10 feet in length. He needs 3 foot 6 inch pieces of wood to build the gate. How many pieces can Bill make from his four boards?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

4) A youth group and their adult chaperones want to visit a water park. The admission fees for this water park are listed below:

Children: $6.00 Adults: $10.50

The total cost for all 17 people in the group to enter the park is $129.00. How many children were in this group?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

5) A group of 150 tourists were waiting for a shuttle to take them from a parking lot to a theme park's entrance. The only way they could reach the park's entrance was by taking this shuttle. The shuttle can carry 18 tourists at a time. After one hour, everyone in the group of 150 tourists reached the theme park's entrance. What is the fewest number of times that the shuttle picked tourists up from the parking lot?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

6) Aunt Marie purchased 80 Silly Bandz for her two nephews Elijah and Jordan. She gave Elijah 10 more Silly Bandz than Jordan. How many Silly Bandz did Elijah and Jordan each receive?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

7) A family is planning a camping trip to a national park and receives the following information about the costs per day:

Camping Fee	
Children 12 years and younger	$3.00 per day
All others	$7.00 per day
Parking for trailer	$9.00 per day
Use of common areas	$1.50 per person per day

The family will camp for 10 days and need to park their trailer each day. The family consists of 4 people including a father, mother, 8 year-old child, and a 14 year-old child. Each person will need to use the common areas on a daily basis. How much will they pay for their camping trip?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

8) Maria wanted a bicycle so she started saving all of her money. For every $6.00 that Maria saved, her mother gave her $2.00. Maria had $56.00 after three months. How much money did Maria's mother give her?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

***** Please review your test and make certain you respond to every problem. If you can show another way to solve any problem, please show it. Please raise your hand after you have completed the test and reviewed your responses. *****

APPENDIX D
PILOT STUDY PROBLEM-SOLVING POSTTEST

Directions: Please answer each question and show all of your work. If you are able to show other ways to solve the problem, there is space below each one to show your work. If you need more space, you are welcome to use the back of each paper or additional sheets of paper.

1) Students at Sandhill Elementary School purchase their lunches from the cafeteria. There are 3 choices for a main dish: hamburger, slice of pizza, or a turkey sandwich, 4 different fruit options: apple, banana, orange, or peach, and 2 drink options: milk or juice. How many different types of lunches can be made if every student selects only one main dish, one piece of fruit, and one drink?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

2) It takes Jeff 1 hour to mow a lawn that is shaped like a square and is 200 feet on each side. How many hours would it take him to mow a lawn that is shaped like a square if each side measures 400 feet?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

3) Mr. Lee wants to make jump ropes for his students to use on the playground. He purchases four packages of rope from the home improvement store. Each package contains one piece of rope that measures 25 feet. Each jump rope needs to measure 8 feet 6 inches. He can cut the rope but cannot join pieces together. How many jump ropes can Mr. Lee make?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

4) A youth group and their chaperones want to visit the Butterfly Rainforest exhibit at the museum. The admission fees for the Butterfly Rainforest exhibit are listed below:

Children: $6.00

Adults: $10.50

The total cost for all 17 people in the group to enter the museum is $129.00. How many adults were in this group?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

5) A group of 150 people were waiting for a glass bottom boat to take them on a trip through a nature preserve. The boat can carry 18 people on each trip. After several hours, everyone in the group of 150 people had gone through the nature preserve. What is the fewest number of trips made by the boat?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

6) Natasha and Marianne went to a theme park. Together, they spent $80.00. Natasha spent $10.00 more than Marianne. How much money did Natasha and Marianne each spend?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

7) A family is planning to leave their pets at Animal Day Care while they are on vacation. They receive the following information about costs per day:

Kennel costs	
Dog	$11.00 per day
Cat	$9.00 per day
Food	$1.50 per animal per day
Walk	$3.00 per dog per day

The family will need to leave their pets at Animal Day Care for 10 days. They have 1 dog and 3 cats. All of the animals need to receive food on a daily basis and the dog must be walked each day. How much will the family pay for their pets' stay at Animal Day Care?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

8) Janice went shopping at the grocery store and saw the following special offer. “If you buy 6 oranges, you get 2 free.” She purchased her groceries and left the grocery store with 56 oranges. How many oranges did she get for free?

Can you show another way to solve the same problem? If so, please show it in the space below.

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APPENDIX E
SCORING PROTOCOL

Score	Score description	Description
1	Correct	Solution answers the problem. Representation(s) support the solution. Solutions with slight rounding or arithmetic errors are scored as correct.
0	Incorrect/No Response	Solution is not correct or provided, representation(s) led to an unrealistic solution, or an incorrect representation or set of procedures produced the solution by chance.

APPENDIX F
REPRESENTATION CODING PROTOCOL

Category	Representation	Description
A	Symbolic	Expressions that utilize numeric, symbolic or a combination of numeric and symbolic characters
B	Pictorial	Drawings that represent values, symbols, or real-life objects
C	Tabular	Stem-and-leaf plots, frequency tables, or charts that categorize and organize data
D	Verbal	Written statements that use words to represent numbers and mathematical operations
E	Mixed	A combination of representations from two or more categories was employed. One representation might lead to utilizing a second representation. Further code cases using subcases shown below

Note: Mixed representations should be coded using one of the subcases.

F1	Symbolic-Pictorial
F2	Symbolic-Tabular
F3	Symbolic-Verbal
F4	Pictorial-Tabular
F5	Pictorial-Verbal
F6	Verbal-Tabular

APPENDIX G
PROBLEM-SOLVING PRETEST

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Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

2) A group of 150 tourists were waiting for a shuttle to take them from a parking lot to a theme park's entrance. The only way they could reach the park's entrance was by taking this shuttle. The shuttle can carry 18 tourists at a time. After one hour, everyone in the group of 150 tourists reached the theme park's entrance. What is the fewest number of times that the shuttle picked tourists up from the parking lot?

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Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

3) Aunt Marie purchased 80 Silly Bandz for her two nephews Elijah and Jordan. She gave Elijah 10 more Silly Bandz than Jordan. How many Silly Bandz did Elijah and Jordan each receive?

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APPENDIX H
PROBLEM-SOLVING POSTTEST

Directions: Please answer each question and show all of your work. If you are able to show other ways to solve the problem, there is space below each one to show your work. If you need more space, you are welcome to use the back of each paper or additional sheets of paper.

1) Students at Sandhill Elementary School purchase their lunches from the cafeteria. There are 3 choices for a main dish: hamburger, slice of pizza, or a turkey sandwich, 4 different fruit options: apple, banana, orange, or peach, and 2 drink options: milk or juice. How many different types of lunches can be made if every student selects only one main dish, one piece of fruit, and one drink?

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Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

2) A group of 150 people were waiting for a glass bottom boat to take them on a trip through a nature preserve. The boat can carry 18 people on each trip. After several hours, everyone in the group of 150 people had gone through the nature preserve. What is the fewest number of trips made by the boat?

Can you show another way to solve the same problem? If so, please show it in the space below.

Can you show another way to solve the same problem? If so, please show it in the space below or on the back of this paper.

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The family will need to leave their pets at Animal Day Care for 10 days. They have 1 dog and 3 cats. All of the animals need to receive food on a daily basis and the dog must be walked each day. How much will the family pay for their pets' stay at Animal Day Care?

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APPENDIX I
UNIT TEST

1. For a class trip, the teachers would like to have one adult for every 10 students. There are 190 students on the trip. How any adults should go on the trip?

2. The ratio of cement to water when mixing concrete is 4 : 1.
   a. Write the ratio in two other ways.

   b. How much water should you add to 12 cubic feet of cement?

Write the ratio in the simplest form.

3. $\frac{18}{12}$

4. $\frac{2}{6}$

5. You buy a sandwich and a fountain drink for lunch. The ratio of the cost of the drink to the cost of the sandwich is 1 : 3. The total cost of the sandwich and drink is $8. What is the cost of the sandwich? Explain how you found your answer

Write a unit rate for the situation.

6. 12 goals in 3 hours

7. 1200 calories in 3 liters

8. A pint of strawberries costs $2.25. For a sale, the store offers a third pint free if you buy two pints at the regular price. Find the unit price of a pint of strawberries when you buy three on sale.

9. An alligator can run at 13 feet per second on land. At this rate, how far can it run in 3 seconds?

10. A bat flies 180 feet in 20 seconds. At this rate, how far can it fly in 30 seconds?

11. You run at a pace of 1 mile every 9 minutes. At this rate, how far can you run in 45 minutes?

12. You read 16 pages of an assignment in 20 minutes. At this rate, how many pages can you read in 45 minutes?

13. The table gives the number of students in a homeroom of 32 who brought a lunch from home each day.

Day	Mon	Tues	Wed	Thurs
Brought Lunch	18	14	20	16

a. Find the mean of the data.

b. Friday is a class field trip and all 32 students bring their lunch. Will the mean for the whole week be greater than or less than the mean for Monday through Thursday? Explain. Then find the new mean.

Find the median and mode(s) of the data.

14. 4, 6, 5, 4, 4, 5, 4, 8

15. 95, 90, 80, 90, 85, 95, 75

16. Find the mode(s) of the data.

Favorite Elective		
Music	Computers	Art
Music	Art	Computers
Computers	Music	Art
Art	Music	Music
Art	Music	Art

17. The data are the number of minutes students spend studying for a test.
35, 32, 38, 34, 36, 69, 32, 25, 41

a. Find the mean, median, mode and range.

b. Does the *mean*, the *median*, or the *mode* represent the data best? Explain your reasoning.

APPENDIX J
OBSERVATION PROTOCOL

Name of teacher: ______________________ **Topic**: ________________

Date of observation: ___________________ **Start time**: ______ **End time**: _____

Lesson Design and Implementation

1. The instructional strategies and activities respected students' prior knowledge and their preconceptions.

Comments

Not at All		**Somewhat**		**Very Descriptive**
0	**1**	**2**	**3**	**4**

2. The lesson was designed to engage students as members of a learning community.

Comments

Not at All		**Somewhat**		**Very Descriptive**
0	**1**	**2**	**3**	**4**

3. Students' exploration preceded formal presentation.

Comments

Not at All		**Somewhat**		**Very Descriptive**
0	**1**	**2**	**3**	**4**

4. The lesson encouraged students to seek and value alternative modes of problem solving.

Comments

Not at All		**Somewhat**		**Very Descriptive**
0	**1**	**2**	**3**	**4**

5. The focus and direction of the lesson was often determined by ideas originating with students.

Comments

Not at All		**Somewhat**		**Very Descriptive**
0	**1**	**2**	**3**	**4**

Content - Conceptual Knowledge

6. Lesson involved fundamental concepts of mathematics.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

7. Lesson promoted strongly coherent conceptual understanding.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

8. Teacher had a solid grasp of mathematics content inherent in the lesson.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

9. Elements of abstraction (i.e., symbolic representations and theory building) were explored and valued as part of the lesson.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

10. Connections with other disciplines and/or real world phenomena were explored and valued.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

Content – Procedural Knowledge

11. Students used a variety of means (i.e., models, drawings, graphs, concrete materials, and manipulatives) to represent phenomena.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

12. Students made predictions, estimations, and/or hypotheses and devised means for testing them.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

13. Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

14. Students were reflective about their learning

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

15. Intellectual rigor, constructive criticism, and the challenging of ideas were valued.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

Classroom Culture – Communicative Interactions

16. Students were involved in the communication of their ideas to others using a variety of means and/or media.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

17. The teacher's questions triggered divergent modes of thinking.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

18. There was a high proportion of mathematically-oriented student talk and a significant amount of it occurred between and among students.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

19. Student questions and comments often determined the focus and direction of classroom discourse.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

20. There was a climate of respect for what others had to say.

Comments

Not at All		Somewhat		Very Descriptive
0	1	2	3	4

Classroom Culture – Student/Teacher Relationships

21. Active participation of students was encouraged and valued.

Comments

Not at All		**Somewhat**		**Very Descriptive**
0	**1**	**2**	**3**	**4**

22. Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.

Comments

Not at All		**Somewhat**		**Very Descriptive**
0	**1**	**2**	**3**	**4**

23. In general, the teacher was patient with students.

Comments

Not at All		**Somewhat**		**Very Descriptive**
0	**1**	**2**	**3**	**4**

24. The teacher acted as a resource person, working to support and enhance student investigations/thinking.

Comments

Not at All		**Somewhat**		**Very Descriptive**
0	**1**	**2**	**3**	**4**

25. The metaphor “teacher as listener” was characteristic of this classroom.

Comments

Not at All		**Somewhat**		**Very Descriptive**
0	**1**	**2**	**3**	**4**

Additional comments about this lesson

APPENDIX K
SOCIAL NORMS

Expectations for behavior in class
We will be respectful of each other’s time and space and work efficiently. We will actively listen to each other by giving others our attention and not speaking when someone else is talking. We will ask questions when we don’t understand. We will work at our desk and may earn the privilege to work where we feel comfortable but will be respectful if asked to move. We will not sit on desks. We will not use electronics except for calculators while in the classroom.

APPENDIX L
SOCIOMATHEMATICAL NORMS

Expectations for doing mathematics
We may use calculators when we check our work. We will make an educated guess about the result after reading and understanding the problem. We will use pictures, graphs, tables, symbols, numbers, manipulatives, and words to assist us while doing mathematics. If we disagree with someone, we will ask a question about his or her idea and describe why we disagree. We will look for more than one way to answer a problem.

APPENDIX M
SAMPLE LESSON PLAN

Lesson Plan Day 7, January 19 (100 minutes)

- Complete the warmup.
- Check your solution to the BAND problem.

AFTER CHECKING YOUR WORK, PLACE IT IN THE FOLDER NEAR THE PROJECTOR.

FCAT Standard

MA.6.S.6.1 Determine the measures of central tendency (mean, median, and mode) and variability (range) for a given set of data

MA.6.S.6.2 Select and analyze the measures of central tendency or variability to represent, describe, analyze and/or summarize a data set for the purposes of answering questions appropriately.

Agenda

Warm-up

PROBLEM: Viewing habits of Gainesville residents!

PROBLEM: Pizza in Gainesville (if time)

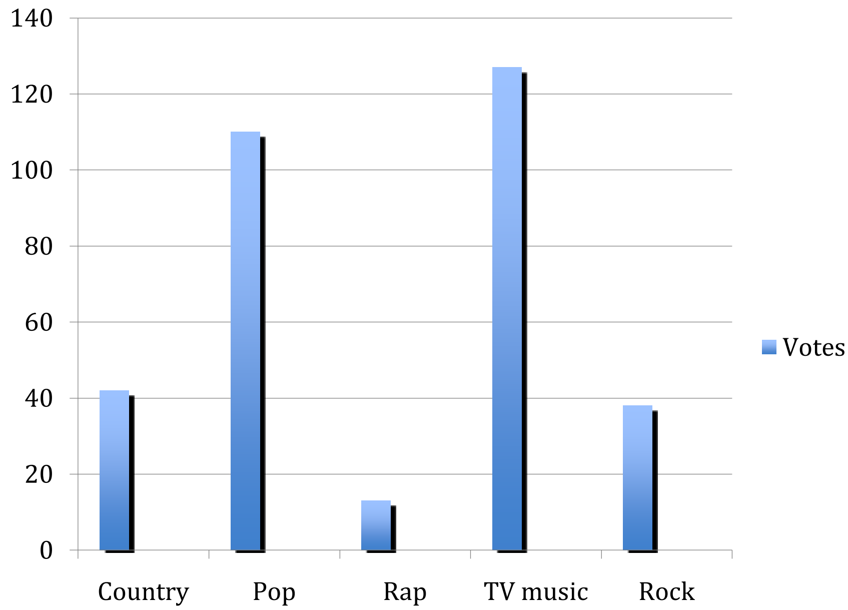
Reflection

Homework

- Page. 228 #1, 4, 7, 9, 13
- Weather and Kwikset Problems
- Quiz on Monday covering data analysis

Complete the warmup.

- Check your solution to the BAND problem with five people. AFTER CHECKING YOUR WORK, PLACE IT IN THE FOLDER NEAR THE PROJECTOR.



Warmup

Each of these words has a meaning related to statistics and data representations. Write down the words that are unfamiliar to you and ask someone for an example or description. Write down their description.

i. Clusters/Groups and gaps (in data)
ii. Distribution of data
iii. Outlier (within a data set)
iv. Range (of a data set)
v. Symmetry (of a data set)
vi. Line Plot
vii. Bar Graph
viii. Mean, Median, Mode (of a data set)
ix. Stem-and-leaf plot

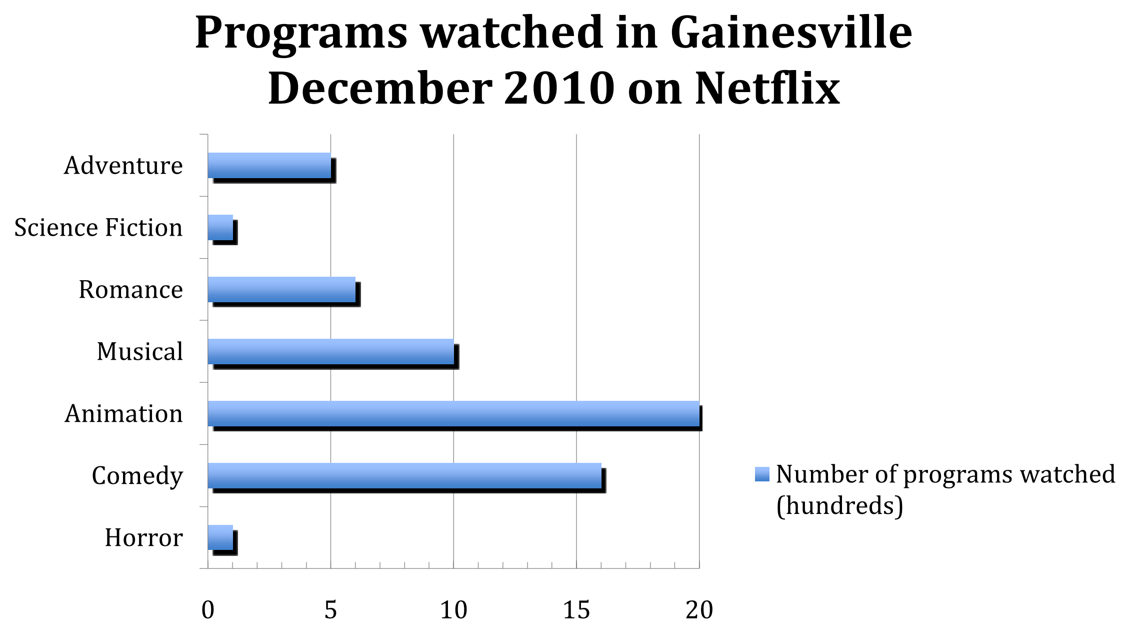
Reflection

What terms are still unfamiliar to you? Write down the terms you want to discuss on Monday and give it to Mr. Bostic as you leave.

Problem **NAME:** ___________________

Directions: Complete each question and then share your ideas with someone else. Write down their ideas below yours if they are different.

Netflix is interested in knowing the viewing habits of residents in Gainesville, Florida. The company randomly collects data during one week in December 2010. The data are shown below.



1. **What can you say about Gainesville residents' viewing habits during this week?** Write down as many things as you can.

____

____

____

Someone else said____

____

2. **What are some questions that cannot be answered using these data?** Write down as many things as you can.

____

____

____

Someone else said____

____

3A. **Which type(s) of average (i.e., mean, median, and mode) is/are appropriate? Justify your response with 1-2 sentences.**

____

____

____

____

Someone else said____

____

3B. **Which type(s) of average (i.e., mean, median, and mode) is/are NOT appropriate? Justify your response with 1-2 sentences.**

____

____

____

Someone else said____

____

4. **What does the spread of the data tell us?**

____

____

____

Someone else said____

____

5A. **What other data displays are appropriate?**

____

____

Someone else said____

____

5B**. What data displays are NOT appropriate?**

____

____

Someone else said____

HWK: Page. 228 #1, 4, 7, 9, 13 **NAME**: ____________________

Weather and Kwikset Problems **DATE**: January ____, 2011

1) The average temperatures in for Ocala, Florida are listed below.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Max	70.4	72.8	78.3	82.9	88.2	92.2	91.6	89.6	83.9	77.4	71.7	82.5
Min	45.7	47.0	52.3	56.0	63.1	69.2	71.1	70.8	68.7	61.0	53.4	47.3
Average												

Create a (1) bar graph or (2) line plot using these data on the grid below.

B) What version of average is MOST appropriate? __________

Calculate the average that is most appropriate and fill in the boxes.

C) How spread out are the max and min temperatures?

____

____

D) What month has the largest spread in its temperature?

____

____

2) Kwikset manufactures keyrings and wants to know whether they should change their current key ring design. The current design holds 20 keys comfortably. Survey 10 adults about the number of keys on their key ring.

A) Create a tally chart (also called a frequency chart) or display for your survey data such as bar graph or pictograph in the box below.

B) Next, think about these terms as you analyze your data:

- Are there any outliers?
- What are the extreme values?
- What is the mean? ____________ Is it useful? (Yes/No)_____________
- What is the median? ___________ Is it useful? (Yes/No)_____________
- What is the mode?_____________ Is it useful? (Yes/No)______________
- What is the range? ____________ Is it useful? (Yes/No)_____________
- Are the data clustered or grouped? (Yes/No)_____________

C) What measure (or measures) of average is/are appropriate for your data set?

____

Write a paragraph (minimum of 3 sentences) (1) describing your analysis and (2) tell Kwikset whether they should change their keyring design.

____

____

____

____

____

____

____

____

____

Problem **NAME:** ___________________
DATE: ___________________

Directions: Complete the problem below using the problem-solving template. Every step must be carried out. Use your knowledge of ratios, rates, unit rates, data representations, and data analysis to answer the questions below.

The city of Gainesville has many places to purchase a pizza. Jeremy decides to create a website to provide residents with information that may help them decide where to purchase their pizza. The following data provide the cost of a cheese pizza, a pepperoni pizza, a large pizza with five toppings, the diameter of a large pizza, and the number of slices on a large pizza:

Pizza Restaurant	# of Slices on Large pizza	Diameter of Large Pizza (in.)	Cost of Large Cheese Pizza (dollars)	Cost of Large Pepperoni Pizza (dollars)	Cost of Large Pizza with 5 Toppings (dollars)
Pizza Hut	8	14	10.00	10.00	10.00
Papa Johns	8	14	8.99	9.99	12.99
Domino's	8	14	9.99	7.99	15.06
Five Star	8	14	8.99	10.49	12.99
Leonardo's	8	14	8.75	10.95	16.50
Hungry Howie's	8	14	10.55	12.95	16.05
Pizza Vito	8	14	10.95	12.70	19.95

(The questions are also listed later on the worksheet.)

What is the best value for one slice of cheese pizza?
What is the average value for a large pepperoni pizza?
Are there any outliers that influence your results?
Is there an average price for any of the pizza types?
What data are most important for consumers?
Create a data representation that Jeremy might display on his website.

1. READ AND UNDERSTAND THE PROBLEM

a) What information is needed to complete each question? Write it here.

I need to

____

____

____

2. DESCRIBE THE SITUATION

a) Highlight or underline the important parts of the problem.

- **CREATE A MATH MODEL AND USE A STRATEGY TO FIND THE RESULT FOR EACH QUESTION.**
  a. Carry out your work here and use the back of the paper.

Q1: What is the best value for one slice of cheese pizza?

Q2: What is the average value for a large pepperoni pizza?

Q3: Are there any outliers that influence your results?

Q4: Is there an average price for any of the pizza types?

Q5: What data are most important for consumers? Why?

Q6: Create a data representation that Jeremy might display on his website.
(The grid may help you if you want to create a bar graph.)

- **INTERPRET THE RESULT**
  b. Do your results match what you might expect to find? If so, write YES. If not, go back and review your work. _______

- **REPORT YOUR ANSWER**
  c. Write 2 or more complete sentences describing the best value for a pizza that your family might be interested in purchasing. Write in a way that a 6th grade student might understand.

*****Check your work with one other person. If they have something different, write it in pen near your answer because we will discuss them later.*****

APPENDIX N
SAMPLE PROBLEMS FROM INTERVENTION CLASSROOM

1. Mr. Lee wants to make jump ropes for his students to use on the playground. He purchases four packages of rope from the home improvement store. Each package contains one piece of rope that measures 25 feet. Each jump rope needs to measure 8 feet 6 inches. He can cut the rope but cannot join pieces together. How many jump ropes can Mr. Lee make?

2. The Domino's Pizza on 13th Street that delivers to P.K. Yonge has three employees who make pizzas during the lunch hours. Jane works Mondays and Fridays, Thomas works Tuesdays and Thursdays, and Sandra works on Wednesdays. On Monday, Mrs. Flavin orders 3 pizzas and it takes Jane 19 minutes 30 seconds to make them. It takes the same amount of time to make a pizza of any size and any number of toppings. On Wednesday, the ninth grade class orders 14 pizzas, which takes Sandra 1 hour and 24 minutes to make them. The middle school faculty orders 5 pizzas, which takes Thomas 35 minutes to make them. The store manager wants to know who is the fastest at making one pizza. What do you tell the manager?

3. The quarterback for the University of Florida in 2008 was Tim Tebow. The table below shows the total number of yards he earned for running and passing the ball during each game.

A. On average, how many running yards did Tim Tebow gain per game?

B. On average, how many passing yards did Tim Tebow gain per game?

Opponent	Running Yards	Passing Yards
Hawaii	37	137
Miami	55	256
Tennessee	26	96
Ole Miss	7	319
Arkansas	32	217
Louisiana State University	22	210
University of Kentucky	48	180
University of Georgia	39	154
Vanderbilt	88	171
University of South Carolina	39	173
The Citadel	34	201
Florida State	80	185
University of Alabama	57	216
University of Oklahoma	109	231

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BIOGRAPHICAL SKETCH

Jonathan Bostic graduated from the College of William and Mary in August 2004 with a Bachelors of Science in mathematics. After graduation, he taught several mathematics classes at Phillip Michael Pennington School in Manassas, Virginia, including algebra, pre-algebra, and mathematics for seventh-grade students. Jonathan also served as the mathematics department chair and mathematics remediation chair for the school, providing support for the other faculty as well as conducting mathematics remediation courses for seventh- and eighth-grade students. While teaching mathematics at Pennington School, he also earned a Master of Education in curriculum and instruction from George Mason University in May 2007. Shortly after graduation, he enrolled at the University of Florida to begin working on a Doctor of Philosophy in curriculum and instruction with an emphasis in mathematics education. Jonathan also completed minors in educational psychology and research and evaluation methodologies, focusing in quantitative methods. In April 2010 he passed his qualifying exams and began working on his dissertation study. His dissertation proposal was reviewed by a committee and approved November 2010. Jonathan graduated in August 2011 and joined the School of Teaching and Learning in the College of Education at Bowling Green State University as an assistant professor of mathematics education. He continues to research ways to promote effective teaching and learning through problem-solving contexts, individuals' uses of representations and procedures to solve problems, and teach mathematics education to pre- and in-service mathematics teachers.