Essays in Financial Mathematics

# Essays in Financial Mathematics

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To you

# Foreword

This volume is the result of a research project carried out at the Department of Finance at the Stockholm School of Economics (SSE). This volume is submitted as a doctoral thesis at SSE. In keeping with the policies of SSE, the author has been entirely free to conduct and present his research in the manner of his choosing as an expression of his own ideas. SSE is grateful for the financial support provided by the Jan Wallander and Tom Hedelius Foundation and the fund Carl Silfvén which has made it possible to fulfill the project.

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Copenhagen, August 2013
Kristoffer Lindensjö

# Contents

# Introduction

This doctoral thesis consists of three independent papers in financial mathematics.

### Paper 1: Optimal investment and consumption under partial information

We present a unified approach for partial information optimal investment and consumption problems in a general non-Markovian Itô process market. The main assumption is that volatility is a nonanticipative functional of the asset price trajectory. The local mean rate of return process and the Wiener process cannot be observed by the agent, whereas the asset price volatility, the stochastic interest rate and the asset prices can be observed. The market is shown to be complete in the sense that any contingent claim adapted to the observable filtration generated by the asset prices can be replicated. Utility functions are general. We tackle this stochastic optimal control problem under partial information in two steps. First, we solve the corresponding full information problem. The market is shown to be complete using a nonstandard martingale representation result, for which we need the volatility to be a nonanticipative functional. We then use the martingale approach to study the optimal investment and consumption problem. We characterize the agent's optimal consumption process and optimal portfolio weights process, as well as the resulting value function and wealth process. We also study some common utility function specifications and relate the solutions to the price processes of certain financial derivatives. Second, we translate the original partial information problem into a corresponding full information problem using filtering theory. Using the solutions of the full information problem, we then derive solutions to the original partial information problem. The main contribution of the paper is the unified approach for partial information optimal investment and consumption problems in complete non-Markovian Itô process markets with stochastic interest rate and stochastic volatility.

## Paper 2: Option pricing under jump-diffusion dynamics for asset prices and interest rates

In this paper we study option pricing when stock prices and an instantaneous forward rate curve, with a corresponding continuum of bonds, follow certain jump-diffusions with random jump sizes. Given specifications of these processes, we derive reasonably explicit formulas for the price and the Greeks of a specific derivative payoff. The payoff of the derivative may depend on two assets which can be either bonds or stocks. We show that many of the option payoffs studied in the literature can be represented as linear combinations of our derivative payoff and hence our pricing formula can be used to price these options. Examples of such options are the European call, put, min, max, exchange and digital options on either stocks or bonds. The main contribution of the paper is to provide a reasonably explicit pricing formula for the above mentioned options in a setting where both the price of the underlying asset and the instantaneous forward rate curve follow jump-diffusions with random jump sizes.

## Paper 3: The end of the month option and other embedded options in futures contracts

The end of the month option allows the holder of the short end of a futures contract to deliver the underlying at any time during the last week of the contract period at a fixed price determined at the start of the last week. We derive a formula for this price within a general and incomplete market framework. An approximation method to calculate this price and some special cases in which explicit solutions for the price exist are also presented. We also study the futures price process of a futures contract with a quality option that first has no option active, then it has an active timing option and lastly it has an active end of the month option. This is the natural setup for embedded options in futures contracts. We show that the futures price process in our setting is dominated by a standard futures price process with maturity at the time of the activation of the end of the month option. Moreover, we show that the futures price process during the time when the timing option is active coincides with the futures price process of a certain other futures contract that only has a timing option. The main contribution of the paper is to properly define the end of the month option and to derive the futures price process of a futures contract with the combination of options described above.