Heterogeneous Firms, International Trade and Institutions

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Preface

This report is a result of a research project carried out at the Department of Economics at the Stockholm School of Economics (SSE).
This volume is submitted as a doctors thesis at SSE. The author has been entirely free to conduct and present her research in her own ways as an expression of her own ideas.
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Stockholm

Filip Wijkström
Associate Professor
SSE Director of Research

To Emilio

Contents

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Introduction

This thesis consists of three independent papers, ordered chronologically with respect to when they were initiated. Empirical research has established that there are large and persistent productivity differences among firms in narrowly defined industries (Bartelsman and Doms, 2000). Other studies, in particular Bernard and Jensen (1999), have shown the existence of a causal link running from ex-ante firm productivity to export decisions. Furthermore, exposure to trade has been found to enhance growth opportunities only for some firms, reallocating market shares and resources toward the more productive ones and contributing thus to aggregate productivity growth (Clerides, Lach and Tybout, 1998; Bernard and Jensen, 2004). These findings have led to the development of new theoretical models emphasizing the interaction between firm heterogeneity and fixed market entry costs in generating international trade and inducing aggregate productivity growth. The first and third chapters of this thesis extend the framework developed by Melitz (2003) to analyze the implications of firm heterogeneity for old and new issues in international trade. The first paper studies the effect of trade liberalization between countries that differ in their relative endowment of skilled workers when growth-promoting R&D activities are skill intensive with respect to goods production. In particular, the analysis focuses on the changes that falling trade costs induce on consumer welfare and on the number of firms active in the different markets. The third paper uses the heterogeneous firm framework to study the interaction between financial constraints and the market entry behavior of firms. It also analyzes whether the impact of trade liberalization on average firm productivity and on individual welfare is affected by the presence of credit frictions. The second chapter presents an empirical work that contributes to the recent but fast growing literature that studies how different institutions and their level of development affect countries comparative advantage. The analysis presented in this paper focuses on the role of legal and financial institution in driving the specialization in contract-intensive goods and on how the degree of institutional development interacts with the propensity of firms to vertical integrate with their suppliers.

1. Firm Heterogeneity in a North-South Trade Model

This paper studies the effects of trade liberalization between countries that differ in their relative endowment of skilled workers, emphasizing the changes that falling trade costs induce on consumer welfare and on the number of firms active in the market. To tackle this issue, I develop an improved version of Melitz (2003) with R&D driven productivity growth and where R&D activities are skill intensive compared to manufacturing activities. A heterogeneous firm model with country asymmetry has been developed by Bernard, Redding and Schott (2007). The authors embed heterogeneous firms and scale economies in a model with endowment-based comparative advantage. Their model assumes different factor intensities only across production sectors while, within the same industry, production and R&D share the same factor intensity. The analysis is thus concerned with how the effects of trade liberalization differ across sectors and it is almost silent on how the effects differ across countries. In this paper I present a model of North-South trade where firm self-selection into exporting and where resources reallocation following trade liberalization varies with the characteristics of the trading partners. Moreover, differently from Bernard, Redding and Schott (2007), the model exhibits positive productivity growth and this has important implication on the welfare effects of trade liberalization.

The main insight of the analysis is that the opportunity cost of innovation with respect to production depends on both relative market sizes and relative factor costs. The relative market size, via its effect on firm selection, magnifies the Norths factor-based cost advantage in variety innovation with respect to good production. Trade liberalization causes reallocation of resources from the least to the most productive firms but the long run welfare effects can be ambiguous and depend on the intertemporal spillovers that characterize the knowledge production function. The Southern region is more likely to gain from a reduction in trade costs and the model thus predicts welfare convergence following trade liberalization. Nevertheless, another interesting finding is that, for plausible parameter values, the model predicts surprisingly small effects of trade liberalization on aggregate welfare.

2. Institution-Driven Comparative Advantage, Complex Goods and Organizational Choice

A substantial body of empirical work in the fields of economic growth and international trade has established that the quality of a country's institutions has a profound effect on its economic performance and, in particular, on its trade patterns. On the other hand, the theory of the firm suggests that firms can respond to a poor institutional environment by vertically integrating their production process, shifting thus transactions from outside to inside the organizations borders. The purpose the second chapter is to examine whether firms' integration opportunities affect the way institutions determine international trade patterns, focusing on legal and financial institutions.

We test two ways that vertical organization choice affects institution-driven comparative advantage in producing complex goods. First, we test if the beneficial effect of a country's legal institutional quality on its comparative advantage in complex goods industries is diminished for industries that also have a high propensity to vertically integrate. This should hold if firms are vertically integrating around the problem of contract incompleteness resulting from poor legal institutions. Second, we test whether or not financial development within a country enhances the comparative advantage of complex goods industries that are more inclined to vertically integrate. This should depend on whether good financial institutions enable firms to finance vertical integration and alleviate thus the hold up problem, more severe in complex goods industries.

The main methodological contribution of this paper is that we use a new measure of industry-level "vertical integration propensity" based on the observed vertical integration outcomes from U.S. firm-level data. This measure, developed in Acemoglu, Johnson and Mitton (2009), has the advantage that it is a direct measure of vertical integration based solely on sector characteristics. We first test our hypotheses with a cross-section, which exploits cross-country variation in institutional quality and cross-industry variation in complexity and vertical integration propensity. We then test our hypotheses with panel and event study analyses, exploiting the available time variation in financial development provided by capital account liberalizations that occurred in several countries during the years 1984-2000.

We find that vertical integration lessens the impact of a country's ability to enforce contracts on the comparative advantage of complex goods. We also find that countries with good financial institutions export disproportionately more in sectors that produce complex goods and that have a high propensity for vertical integration. Our results confirm thus the role of institutions as source of comparative advantage and suggest that this depends not only on the technological characteristics of the goods produced but also on the way firms are able to organize the production process.

3. Heterogeneous Firms and Credit Frictions: a General Equilibrium Analysis of Market Entry Decisions

The third chapter studies the interaction between financial constraints and the market entry behavior of firms. It also analyzes whether the impact of trade liberalization on average firm productivity and on individual welfare is affected by the presence of credit frictions. The rapid and sharp decline in economic activity and international trade that followed the 2007-2009 global financial crisis has boosted interest in these topics and brought a fast growing body of literature to study the linkages between credit constraints, firms activity and international trade. Four year after the first events that triggered the global downturn, financial markets are still very unstable and, according to all indicators, the credit to private sectors is still contracted, with firms facing substantial limitations to credit access. The persistence of this condition makes it crucial to understand the role that financial frictions can have on firms' long-run entry opportunities and to study the effects of worsening credit market conditions that go beyond the short run responses to a negative shock in the credit supply. This chapter focuses on the steady state effects of financial frictions in a setting where firms' domestic and foreign market entry decisions combine in shaping the allocation of resources within an open economy. Differently from previous contributions, I develop a full-fledged general equilibrium model that fully describes how firms' choices, average productivity and the number of producers interact in general equilibrium and how credit frictions affect via these channels consumers welfare and the role of played by trade costs.

I introduce credit market frictions in a heterogeneous firm model in the spirit of Melitz (2003). The model features two symmetric economies where monopolistically competitive firms differ in their productivity levels. Before knowing their productivity, liquidity constrained firms must raise capital to pay for the sunk costs needed for innovation and market entry, both in the domestic and in the foreign market. Capital is provided by a competitive credit market where lenders face imperfect protections in the form of a positive probability that the borrower can avoid the per-period debt reimbursement without incurring in any sanction. First, I solve for the optimal debit contract that maximizes the firm's expected value from variety introduction and satisfies the incentive compatibility constraint of the firm and the participation constraint of the lender. Then, I solve for the aggregate variables that define the general equilibrium characterized by an infinite mass of potential entrants.

A main result of my analysis is that financial frictions create rents that divert resources away from innovation activities, limit the access of firms to credit and constrain entry decisions. Moreover, I find that exporting firms, because bigger in term of total sales and profits, can have an advantage in terms of access to credit and this shifts resources from innovation to foreign market entry. I thus show that a main effect of credit frictions is a too low number of entrants. This implies a lack of competitiveness in the market that allows low-productivity firms to survive. The market can thus be characterized by a low number of big and inefficient firm and this has negative effects on welfare. I also show that credit frictions interact with trade costs in such a way that trade liberalization does not necessarily lead to higher average productivity and higher individual welfare.

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PAPER 1

Firm Heterogeneity in a North-South Trade Model

Abstract

This paper presents a model of North-South trade with firm-level productivity differences and R&D driven growth. Compared to manufacturing, R&D activities are intensive in skilled labor and the two regions differ in the relative endowment of skilled and unskilled workers. The relative market size, via its effect on firm selection, magnifies the North's factor-based cost advantage in variety introduction with respect to good production. Trade liberalization causes reallocation of resources from the least to the most productive firms but the long run welfare effects can be ambiguous. The Southern region is more likely to gain from a reduction in trade costs and the model thus predicts welfare convergence following trade liberalization.

1. Introduction

Empirical research has established that there are large and persistent productivity differences among firms in narrowly defined industries (Bartelsman and Doms, 2000). Other studies, in particular Bernard and Jensen (1999), have shown that firm productivity and export status are positively and strongly correlated and that causality seems to run from ex-ante productivity to export decisions. Furthermore, exposure to trade has been found to enhance growth opportunities only for some firms, reallocating market shares and resources toward the more productive ones and contributing thus to aggregate productivity growth (Clerides, Lach and Tybout, 1998; Bernard and Jensen, 2004; Aw, Chung and Roberts, 2000).

Neither the old trade theory (the Heckscher-Ohlin and Ricardian trade models) nor the so-called new trade theory (the Krugman's increasing returns trade model) can account for any of these empirical patterns. Both classes of models have representative firms and assume away any firm level differences within sectors.

Empirical challenges to old and new trade theory have led to the development of richer theoretical models emphasizing the importance of firm heterogeneity in generating international trade and inducing aggregate productivity growth. In his seminal contribution, Melitz (2003) builds a model with heterogeneous firms operating in monopolistically competitive industries. Firms incur a fixed cost to export but before doing so each firm has to make a productivity draw from an exogenous distribution.

Sunk costs and firm heterogeneity interact and only the most productive firms self-select into export markets.

The Melitz framework has proved to be particularly tractable and has stimulated a great deal of analysis into the implications of firm heterogeneity for a wide range of issues in international trade. Helpman, Melitz and Yeaple (2004) extend it to consider the decision to set up an overseas affiliate. Ghironi and Melitz (2005) and Chaney (2005) analyze the short run effects of trade liberalization and study the transitional dynamic. Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2010, GS hereafter) add steady state productivity growth. Assuming the same R&D technology that drives productivity growth in the first generation endogenous growth model by Grossman and Helpman (1993), Baldwin and Robert-Nicoud (2008) find that trade liberalization permanently retards productivity growth and makes consumers worse off in the long run. Assuming the same R&D technology that drives productivity growth in the second generation endogenous growth model by Jones (1995a), GS find that, for a wide range of parameter values, trade liberalization promotes productivity growth in the short run and makes consumers better off in the long run. However all these papers limit their analysis to the case of symmetric countries and they all assume that the production of goods and the production of knowledge require similar technologies or the same kind of skills. In these papers there is only one factor of production that is perfectly mobile between manufacturing and R&D and country characteristics play no role in the analysis.

A heterogeneous firm model with country asymmetries has recently been developed by Bernard, Redding and Schott (2007, BRS hereafter). In this influential contribution, the authors embed heterogeneous firms and scale economies in a model with endowment-based comparative advantage. They consider a two-country, two-factor, two-sector world and they find that reducing trade costs raises productivity proportionally more in comparative advantage industries. This differential productivity growth across industries widens pre-liberalization differences in the opportunity costs of production and amplifies the ex ante comparative advantage from factor composition. This magnification of countries' original heterogeneity boosts the welfare gains from trade.

Although countries asymmetry is a crucial assumption of the BRS model, the analysis is mainly concerned with how the effects of trade liberalization differ across sectors and it is almost silent on how the effects differ across countries. Another drawback of the BRS model is that it assumes different factor intensities only across production sectors while, within the same industry, production and R&D share the same factor intensity.

In this paper I present a model of North-South trade where firm self-selection into exporting and resources reallocation following trade liberalization vary with the characteristics of the trading partners. I develop an improved version of Melitz (2003) with productivity growth and with the more realistic assumption that R&D activities are skill intensive compared to manufacturing activities. I use the model to study both the equilibrium effects and the welfare effects of changing trade costs, being careful to distinguish between the different roles played by relative market sizes and relative skill abundance in driving the results.

I consider a world consisting of two regions, North and South, which have different endowments of the two production factors, skilled and unskilled labor. In both regions, firms engage in innovative R&D to develop new product varieties and then learn their productivity. Once productivity is known, firms decide whether or not to incur the one-time investment to adapt the product for the local and foreign markets. I assume that manufacturing employs both types of labor. Innovative R&D and market entry investments require the production of knowledge which I assume employs only skilled workers. Knowledge creation is thus the skilled labor intensive activity. The model, differently from BRS, exhibits positive productivity growth. The source of growth is given by the introduction of new products which, in order to avoid the "strong scale effect", is modeled in the same way as GS.

In the steady state equilibrium, firms that develop new products with low productivity immediately exit. Firms that develop new products with intermediate productivity incur the fixed cost of entering the local market and only firms with sufficiently high productivity choose to also incur the fixed cost of entering the foreign market. Consistent with the empirical evidence, only the most productive firms export. I also find that the productivity thresholds that defines firms' entry strategies varies between the two regions and in a way that depends only on the relative size of the two markets. The bigger the domestic market relative to the foreign one, the lower the productivity needed to successfully produce for the domestic market and the higher the productivity needed to become an exporter.[1] The exporter productivity premium is thus higher in the larger economy.

[1] Another paper that studies the effect of market size on the productivity cut-offs is Melitz and Ottaviano (2008). Departing from the conventional CES preference assumption, they construct a model of international trade with variable markups, in which market size and trade affect the toughness of competition in a market, which then feeds backs into the selection of heterogeneous firms even in the absence of any fixed entry cost. Some of their findings are very different from mine. Their main selection mechanism works through the competition on the product market and not through the competition on the factor market, as in my framework or in Melitz (2003). As a result they predict that bigger markets, attracting more firms, are characterized by a higher domestic productivity thresholds and these do not depend on the size of the trading partners.

In equilibrium, the difference in relative factor endowments will determine the relative wage and this will affect the opportunity cost of R&D activity with respect to good production. As in BRS, this factor-based difference in relative costs is magnified by the endogenous firm selection described above and driven by relative market sizes. Since the expected innovation cost is decreasing in the exporter productivity premium, I find that relative market sizes and factor abundance work in the same direction to make innovation relatively cheaper in the North. As a result, the Southern economy will be populated by a smaller number of bigger firm and by a larger fraction of exporters. These results offers a new and richer interpretation of the "market effects" introduced by Krugman (1980) in his homogeneous firm setting.

I find that trade liberalization reallocates resources and market shares toward the more productive firms, that the number of exporters increases and that the total number of firms sustainable in equilibrium decreases. The magnitude of these effects differs substantially across regions. Firm selection is tougher in the South. The cost of R&D relative to manufacturing diverges further in the two regions and, as a result, both the North to South relative number of firms and the North to South relative number of exporters increases. This has important welfare effects. Aggregate welfare is always converging between the two regions because the South benefits more from the increase in the number of imported varieties. On the other hand, due to the reduction in the total number of varieties, it is not necessarily the case that welfare is increasing in falling trade costs and it can move in opposite directions in the two regions, with the South being more likely to gain. As in GS, this depends on the intertemporal spillovers that characterize the knowledge production function.

Another interesting finding is that, for plausible parameter values, the model predicts surprisingly small effects of trade liberalization on aggregate welfare. This is particularly true for the North where, for parameters that match realistic values of TFP growth, a shift from autarky to free trade causes gains in welfare of only 2 percentage points.

The rest of the paper is organized as follows: in Section 2, I provide a full description of the model and of the conditions that describe the equilibrium. In section 3, I define the balanced growth equilibrium that I solve for, and I derive closed form solutions for some of the endogenous variables. The combination of multiple factors, country asymmetries and firm heterogeneity means that I cannot derive closed form solutions for all the endogenous variables. Thus in Section 4, I solve the model numerically and I discuss its equilibrium properties. In Section 5, I discuss the effects of trade liberalization on both industry structure and on consumer welfare of the two regions.

Section 6 illustrates the difference between the short run and long run effects of trade liberalization. Finally, section 7 concludes.

2. The Model

2.1. Overview of the Model. I consider a world of two asymmetric regions, two factors (skilled and unskilled labor), one consumption-good sector where there is Dixit-Stiglitz monopolistic competition and one innovation sector that produces knowledge through R&D activity. The analysis focuses on asymmetries in factor endowments, so I assume that the two regions are identical in all other respects. The two sectors differ in factor intensities. The production sector employs both skilled and unskilled labor, whereas the R&D-sector has skilled workers as the only input. R&D activity is the engine of growth in this model and in contrast with Melitz (2003) and BRS, there is steady-state productivity growth.

Each firm's cost involves one-time fixed costs and variable Cobb-Douglas production costs. To produce, a firm must first develop a new variety and this is done in the innovation sector. This involves a variety-development fixed cost which is thereafter sunk. After having incurred this fixed cost, the firm receives a patent to exclusively produce the new variety and learns its productivity. The productivity parameter is drawn from a probability distribution, so firms have different unit costs of production. In addition to the development fixed cost, there are also sunk market-entry costs.[2] After having learned its productivity, each firm decides whether or not to incur the additional fixed costs and enter only the domestic market or both the domestic and the foreign market. Firms need to draw a sufficiently high productivity to enter the domestic market and an even more favorable draw to enter the export market. For firms that incur the one-time foreign-market entry cost and learn how to export, there are iceberg trade costs associated with shipping their products to the foreign market. The main focus of this paper is on analyzing the steady state equilibrium and welfare implications of changes in trade costs. In particular I want to explore how economies with different characteristics react to trade liberalization, in a framework where the R&D activity plays a crucial role in both skill-abundant and skill-scarce regions.

Whenever a different exposition is not needed, I fully describe the model's equations for only one of the two economies, denoting foreign variables by an asterisk.

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[2] The empirical evidence supports the choice of sunk market-entry costs over period-by period continuation costs. See for example Roberts and Tybout (1997), G.Alessandria and H. Choi (2007) and Bernard and Jensen (2004)). From a theoretical perspective, see Baldwin (1988), Baldwin and Krugman (1989), and Dixit (1989a,b) for the analysis of what is defined as “exporter hysteresis”.

2.2. Consumption. In each region there is a fixed measure of two types of households, those endowed with skilled labor and those endowed with unskilled labor. Each individual member of a household lives forever and in each period inelastically supplies a unit of labor according to her skills. The size of each household, measured by the number of its members, grows exponentially at a fixed rate $n > 0$. The supply of skilled labor at time t is $H_t = H_0 e^{nt}$ and the supply of unskilled labor at time t is $L_t = L_0 e^{nt}$, where H_0 and L_0 represent the initial size of skilled and unskilled households respectively. In addition to wage income, households also receive asset income from their ownership of firms. For simplicity, I assume that firms in each region are owned by households in the same region.

All households share identical preferences. Each household is modeled as a dynastic family that maximizes discounted lifetime utility

$$U_l = \int_0^\infty e^{-(\rho-n)t} \ln(u_{lt})\, dt \qquad l = H, L \tag{2.1}$$

where $\rho > n$ is the subjective discount rate and u_{lt} is the static utility of the representative household member. The static CES utility function is given by

$$u_{lt} = \left[\int_0^{\hat{m}_t} x_{lt}^{\alpha}(\varpi)\, d\varpi \right]^{\frac{1}{\alpha}} \qquad l = H, L \tag{2.2}$$

where $x_{lt}(\varpi)$ is the quantity consumed for each product ϖ, $\hat{m}_t$ is the number of varieties available for consumers in the economy and $\alpha \in (0, 1)$ measures the degree of product differentiation. The elasticity of substitution between product is given by $\varepsilon \equiv \frac{1}{1-\alpha} > 1$. Solving the static consumer optimization problem yields the familiar demand function

$$x_{lt}(\varpi) = \frac{p_t(\varpi)^{-\varepsilon} c_{lt}}{P_t^{1-\varepsilon}} \qquad l = H, L \tag{2.3}$$

where $p_t(\varpi)$ is the price of variety ϖ, $P_t \equiv \left[\int_0^{\hat{m}_t} p_t(\varpi)^{1-\epsilon} d\varpi \right]^{\frac{1}{1-\varepsilon}}$ is the price index and c_{lt} is the individual expenditure. Maximizing (2.1) subject to (2.2) where (2.3) has been used to substitute for $x_{lt}(\varpi)$ yields the Euler condition

$$\frac{\dot{c}_{lt}}{c_{lt}} = r_t - \rho \qquad l = H, L \tag{2.4}$$

implying that individual expenditure for both types of consumers grows over time only if the market interest rate r_t exceeds the subjective discount rate ρ.

I solve the model for a steady state equilibrium where consumer expenditure is constant over time. From (2.4), it follows that, for c_{lt} to be constant, the market interest rate must also be constant over time and given by

$$r = \rho. \tag{2.5}$$

2.3. R&D and Production. Firms create knowledge in the R&D sector employing only skilled labor. This is an important assumption. Given that the production of goods uses both inputs according to a Cobb-Douglas production function, R&D is the skill intensive activity. It follows that for any factor prices, a higher ratio of skilled to unskilled labor is used in R&D than in production. This is a major departure from the BRS (2007) framework where production and R&D are assumed to share the same input intensity.

I assume that the unit cost for knowledge production at time t is given by

$$b_{It} = \frac{w_{Ht}}{(m_t + \lambda m_t^*)^{\phi}} \tag{2.6}$$

where $\phi < 1$ and $\lambda \in [0, 1]$ are given R&D parameters, w_{Ht} is the skilled wage and m_t and m_t^* are the number of varieties successfully introduced by firms in the home and foreign region. The term $(m_t + \lambda m_t^*)$ can be interpreted as the stock of knowledge in the economy. As in GS (2008), ϕ measures the strength of intertemporal knowledge spillovers and λ measures the international dimension of spillovers. The only restriction I impose is $\phi < 1$ to rule out explosive growth. For $0 < \phi < 1$, researchers become more productive in creating new knowledge as the stock of knowledge increases. For $\phi < 0$, the opposite is true and researchers experience a "fishing out" effect in the innovation activity.

To develop a new variety, firms need to create F_I units of knowledge. The fixed cost associated with variety introduction is thus given by $F_I b_{It}$. Knowledge creation is required also to tailor the new product to market-specific tastes, standards and regulation. To do so for the domestic market, firms must create F_D units of knowledge and the associated fixed entry cost is $F_D b_{It}$. Selling in the foreign market requires F_E additional units of knowledge and the fixed entry cost is $F_E b_{It}$.

Once a firm has developed a new variety, it learns its productivity parameter φ, which is drawn from a probability density function $g(\varphi)$ with support $[\bar{a}, \infty]$. Let $G(\varphi)$ be the corresponding cumulative distribution function. I assume that the probability distribution is Pareto, that is

$$G(\varphi) = 1 - \left(\frac{\bar{a}}{\varphi}\right)^k \tag{2.7}$$

where k and $\bar{a}$ are the shape and scale parameters of the distribution. I assume $k > \varepsilon - 1$, which ensures that the expected discounted profits from innovating are finite. The assumption that productivity levels are drawn from a Pareto distribution was first explored by Helpman et al. (2004). This assumption is made for its analytical tractability and because it provides a good approximation of the distribution of firm productivity in several countries. For instance, Del Gatto, Mion and Ottaviano (2007), using a panel for 11 EU countries, show that the Pareto is a reasonable approximation of the underlying productivity distributions and the average k is estimated to be close to 2. Eaton et al. (2009) show that a Pareto distribution with $k = 1.5$ is a good fit for the productivity distribution of French exporters.

After having observed its productivity parameter φ, if the firm starts production it has a Cobb-Douglas unit cost function given by

$$C_t(\varphi) = \frac{w_{Ht}^{\beta} w_{Lt}^{1-\beta}}{\varphi} = \frac{\kappa_t}{\varphi}$$

where $0 < \beta < 1$, w_{Lt} is the unskilled wage and $\kappa_t \equiv w_{Ht}^{\beta} w_{Lt}^{1-\beta}$. Domestic market profits are given by

$$\pi_{Dt}(\varphi) = \left[p_{Dt}(\varphi) - \frac{\kappa_t}{\varphi} \right] \frac{E_t p_{Dt}(\varphi)^{-\varepsilon}}{P_t^{1-\varepsilon}}$$

where the subscript D is used to indicate the domestic market and $E_t \equiv H_t c_{Ht} + L_t c_{Lt}$ is the aggregate expenditure in the economy. Since there exists a continuum of firms, each firm chooses a profit-maximizing price taking aggregate expenditure and other firms' prices as given. This yields

$$p_{Dt} = \frac{\kappa_t}{\alpha \varphi}.$$

By substituting for the price, profits of a firm selling domestically can be written as

$$\pi_{Dt}(\varphi) = E_t \varsigma \left(\frac{\kappa_t}{\varphi P_t} \right)^{1-\varepsilon} \tag{2.8}$$

where $\varsigma \equiv (\varepsilon - 1)^{\varepsilon-1} \varepsilon^{-\varepsilon}$. From selling in the foreign region (subscript E), the firm makes profits

$$\pi_{Et}(\varphi) = \left[p_{Et}(\varphi) - \frac{\tau \kappa_t}{\varphi} \right] \frac{E_t^* p_{Et}(\varphi)^{-\varepsilon}}{P_t^{*1-\varepsilon}}$$

where $\tau > 1$ is the iceberg trade cost, such that τ units must be shipped for one unit to reach its destination. The corresponding profit-maximizing price is given by

$$p_{Et}(\varphi) = \frac{\tau \kappa_t}{\alpha \varphi}$$

with maximized profits equal to

$$\pi_{Et}(\varphi) = E_t^* \varsigma \left(\frac{\tau \kappa_t}{\varphi P_t^*} \right)^{1-\varepsilon}. \tag{2.9}$$

2.4. Market Entry. I define $V_{zt}(\varphi)$ as the value at time t of a firm with productivity φ that sells on the domestic market ($z = D$) or on the foreign market ($z = E$). As there is no risk for the owners of a firm once its productivity is known, the total return on equity claims must equal the risk-free interest rate r_t, that is

$$\pi_{zt}(\varphi)dt + \dot{V}_{zt}(\varphi)dt = r_t V_{zt}(\varphi)dt \qquad z = D, E$$

where $\pi_{zt}(\varphi)dt$ and $\dot{V}_{zt}(\varphi)dt$ are the profits and the capital gains earned during the time interval dt. Solving for $V_{zt}(\varphi_j)$ yields:

$$V_{zt}(\varphi) = \frac{\pi_{zt}(\varphi)}{r_t - \frac{\dot{V}_{zt}(\varphi)}{V_{zt}(\varphi)}} \qquad z = D, E. \tag{2.10}$$

Let φ_D and φ_E denote the productivity levels that make firms indifferent between entering and not entering the local and foreign markets respectively. I solve the model for a steady state equilibrium where φ_D and φ_E are constant over time. These two thresholds are defined by the two conditions

$$V_{zt}(\varphi_z) = F_z b_{It} \qquad z = D, E \tag{2.11}$$

such that the value of the firm is equal to the entry fixed cost. Since profits are strictly increasing in productivity, any firm with $\varphi > \varphi_D$ finds it profitable to enter the domestic market and any firm with $\varphi > \varphi_E$ finds it profitable to enter the foreign market. Firms with productivity draws below these two thresholds will exit and never start production. Substituting (2.10) into (2.11) using the profit flows (2.8) and (2.9) yields the steady-state local market entry condition

$$\frac{E_t \varsigma \left(\frac{\kappa_t}{\varphi_D P_t}\right)^{1-\varepsilon}}{r - \frac{\dot{b}_{It}}{b_{It}}} = F_D b_{It} \tag{2.12}$$

and the steady-state foreign market entry condition

$$\frac{E_t^* \varsigma \left(\frac{\tau\kappa_t}{\varphi_E P_t^*}\right)^{1-\varepsilon}}{r - \frac{\dot{b}_{It}}{b_{It}}} = F_E b_{It}. \tag{2.13}$$

Combining the two gives a relation between the two cut-off values, φ_{Dt} and φ_{Et}

$$\frac{\varphi_E}{\varphi_D} = \frac{P_t}{P_t^*}\tau \left(\frac{F_E}{F_D}\frac{E_t}{E_t^*}\right)^{\frac{1}{\varepsilon-1}}. \tag{2.14}$$

For the model to be consistent with the evidence that not all firms export, I will focus on parameters values such that in equilibrium $\varphi_E > \varphi_D$. The productivity premium of exporting firms $\frac{\varphi_E}{\varphi_D}$ is high when the iceberg trade cost τ is high and when the knowledge requirement to enter the foreign market F_E is large compared to the knowledge requirement for the domestic market F_D. The premium is also high when the domestic market is large relatively to the foreign market (E_t/E_t^* high) and when domestic prices are high relative to foreign prices (P_t/P_t^* high).

Having solved for when firms choose to sell locally and export, it is now possible to work backwards and determine the incentives to develop new varieties. Since I assume that there is free entry by firms into variety innovation, the ex-ante expected benefit of developing a new variety must equal the cost of variety innovation. This can be stated as

$$\int_{\varphi_D}^{\infty} \left(V_{Dt}(\varphi) - F_D b_{It}\right) g(\varphi) d\varphi + \int_{\varphi_E}^{\infty} \left(V_{Et}(\varphi) - F_E b_{It}\right) g(\varphi) d\varphi = F_I b_{It} \tag{2.15}$$

where $g(\varphi)$ is the Pareto probability density function from which a potential market entrant draws its productivity level. In the appendix I show that this condition is equivalent to

$$\frac{\varsigma \kappa_t^{1-\varepsilon}}{r - \frac{\dot{b}_{It}}{b_{It}}} \cdot \Delta_t = \bar{F} b_{It} \tag{2.16}$$

where

$$\bar{F} \equiv \left[F_I \left(\frac{\varphi_D}{\bar{a}}\right)^k + F_D + F_E \left(\frac{\varphi_D}{\varphi_E}\right)^k \right] \tag{2.17}$$

is the expected knowledge requirement for product introduction and

$$\Delta_t \equiv \frac{k}{k - \varepsilon + 1} \left[\frac{E_t}{P_t^{1-\varepsilon}} \frac{1}{\varphi_D^{1-\varepsilon}} + \frac{E_t^*}{P_t^{*1-\varepsilon}} \left(\frac{\tau}{\varphi_E}\right)^{1-\varepsilon} \left(\frac{\varphi_D}{\varphi_E}\right)^k \right]. \tag{2.18}$$

Free entry ensures that ex-ante expected discounted profits must equal ex-ante expected fixed costs of developing a profitable variety. The right hand side of (2.16) represents in fact the full expected cost of successfully introducing a new variety: $\left(\frac{\varphi_D}{\bar{a}}\right)^k = \frac{1}{1-G(\varphi_D)}$ is the number of attempts needed before a profitable variety is discovered, then any successful firm has to adapt the good for the domestic market, and with probability $\left(\frac{\varphi_D}{\varphi_E}\right)^k = \frac{1-G(\varphi_E)}{1-G(\varphi_D)}$ it is profitable to do so even for the foreign market.

In the appendix I show how, combining (2.12), (2.13) and (2.15), it is possible to get

$$\frac{F_D}{\varphi_D^k} + \frac{F_E}{\varphi_E^k} = \frac{k - \varepsilon + 1}{\varepsilon - 1} \frac{F_I}{\bar{a}^k}. \tag{2.19}$$

This condition describes a negative relation between the two productivity thresholds. If the domestic market productivity threshold φ_D increases, introducing a successful variety becomes more difficult. For the ex-ante expected benefit of developing a new variety to equal the unchanged cost of variety innovation $F_I b_{It}$, the probability of earning profits also from the foreign market must increase. This implies that φ_E has to go down. Using (2.19), $\bar{F}$ becomes:

$$\bar{F} = F_I \left(\frac{\varphi_D}{\bar{a}}\right)^k \frac{k}{\varepsilon - 1}. \tag{2.20}$$

This tells us that the expected knowledge requirement for variety introduction $\bar{F}$ is increasing in the domestic market threshold φ_D. When the productivity needed to serve the domestic market increases, more trials are needed in order to develop a successful product. Moreover, because of (2.19), the successful firm is more likely to become an exporter and so to incur the foreign market entry cost $F_E b_{It}$.

The flow of new varieties introduced in the economy at time t is given by the skilled labor devoted to R&D, divided by the labor units required for a successful innovation. A successful innovation requires on average $\bar{F}$ units of knowledge and to create each of them, according to (2.6), $\frac{1}{(m_t+\lambda m_t^*)^\phi}$ unit of skilled labor must be employed. That is,

$$\dot{m}_t = \frac{H_{It}}{\bar{F}/\left(m_t + \lambda m_t^*\right)^\phi} = \frac{H_{It}\left(m_t + \lambda m_t^*\right)^\phi}{\bar{F}} \tag{2.21}$$

where H_{It} is the total amount of skilled labor employed in the innovation sector.

2.5. Goods and Labor Markets. Given the productivity thresholds and other previously derived results, it is possible to complete the description of all variables and relations that define the equilibrium.

The domestic price index P_t satisfies

$$\begin{aligned} P_t^{1-\varepsilon} &= \int_0^{\hat{m}_t} p_t(\varpi)^{1-\varepsilon} d\varpi \\ &= \int_{\varphi_D}^{\infty} p_{Dt}(\varphi)^{1-\varepsilon} m_t \frac{g(\varphi)}{1-G(\varphi_D)} d\varphi + \int_{\varphi_E^*}^{\infty} p_{Et}^*(\varphi)^{1-\varepsilon} m_t^* \frac{g(\varphi)}{1-G(\varphi_D^*)} d\varphi \\ &= \frac{k\alpha^{\varepsilon-1}}{k-\varepsilon+1}\left[m_t\left(\frac{\kappa_t}{\varphi_D}\right)^{1-\varepsilon} + m_t^*\left(\frac{\tau\kappa_t^*}{\varphi_E^*}\right)^{1-\varepsilon}\left(\frac{\varphi_D^*}{\varphi_E^*}\right)^k\right]. \end{aligned} \tag{2.22}$$

Analogously, in the foreign region

$$P_t^{*1-\varepsilon} = \frac{k\alpha^{\varepsilon-1}}{k-\varepsilon+1}\left[m_t^*\left(\frac{\kappa_t^*}{\varphi_D^*}\right)^{1-\varepsilon} + m_t\left(\frac{\tau\kappa_t}{\varphi_E}\right)^{1-\varepsilon}\left(\frac{\varphi_D}{\varphi_E}\right)^k\right].$$

A trade balance condition has to hold in equilibrium. The value of imports has to equal the value of exports. Integrating the revenues over exporting firms in both regions, the condition becomes

$$m_t^* \frac{E_t}{P_t^{1-\varepsilon}}\left(\frac{\kappa_t^*}{\varphi_E^*}\right)^{1-\varepsilon}\left(\frac{\varphi_D^*}{\varphi_E^*}\right)^k = m_t \frac{E_t^*}{P_t^{*1-\varepsilon}}\left(\frac{\kappa_t}{\varphi_E}\right)^{1-\varepsilon}\left(\frac{\varphi_D}{\varphi_E}\right)^k. \tag{2.23}$$

Combining (2.18), (2.22) and (2.23), the aggregate expenditure can be solved as

$$E_t = m_t \left(\frac{\kappa_t}{\alpha}\right)^{1-\varepsilon} \Delta_t. \tag{2.24}$$

This result, together with (2.6), can be used to rewrite (2.16) as

$$\frac{\frac{E_t}{\varepsilon m_t}}{r - \frac{\dot{b}_{It}}{b_{It}}} = \frac{\bar{F} w_{Ht}}{(m_t + \lambda m_t^*)^\phi}. \tag{2.25}$$

It is worth noticing that variable trade costs do not directly affect expected profits from innovation (left hand side of (2.25)). This is a consequence of the firm pricing rule. When τ decreases, the price-cost margin is unaffected and expected profits from exporting increase only because each firm increases its share in the foreign market. Due to the symmetry in trade costs, this is true also for foreign exporters that gain shares in the home market. Given the trade balance condition, the two effects cancel out and τ disappears from the free entry condition (2.25), where the extent of expected profits is simply captured by the domestic market share $\frac{E_t}{m_t}$.[3]

Labor market clearing requires the demand for labor in the production and innovation sectors to equal the labor supply as given by the region's factor endowment. From (2.21), the skilled labor used in R&D is given by

$$H_{It} = \frac{\dot{m}_t \bar{F}}{(m_t + \lambda m_t^*)^\phi}. \tag{2.26}$$

In the appendix, from the firm's cost minimization problem and by summing over firms, I derive

$$H_{Pt} = E_t \frac{\beta\alpha}{w_{Ht}} \tag{2.27}$$

and

$$L_{Pt} = E_t \frac{(1-\beta)\alpha}{w_{Lt}} \tag{2.28}$$

where H_{Pt} and L_{Pt} are the total quantity of skilled and unskilled labor used by firms in the production sector. The labor market clearing conditions are thus given by:

$$H_t = H_{Pt} + H_{It} = \beta E_t \frac{\alpha}{w_{Ht}} + \frac{\dot{m}_t \bar{F}}{(m_t + \lambda m_t^*)^\phi} \tag{2.29}$$

$$L_t = L_{Pt} = (1-\beta) E_t \frac{\alpha}{w_{Lt}}. \tag{2.30}$$

[3] See Migueles (2010) for a formal and detailed analysis of this issue.

3. The Steady State Equilibrium

By the definition of a balanced growth path, in equilibrium all the endogenous variables have to grow at constant (not necessarily identical) rates over time. In particular, as mentioned before, I am solving for a steady state equilibrium where φ_D, φ_E, c_H and c_L are constant over time. It immediately follows that the aggregate expenditure E_t must grow at the same rate as population growth n. If the stock of knowledge $M_t \equiv m_t + \lambda m_t^*$ grows at a constant rate, this has to be the case also for m_t and m_t^* and

$$g \equiv \frac{\dot{M}_t}{M_t} = \frac{\dot{m}_t}{m_t} = \frac{\dot{m}_t^*}{m_t^*}.$$

From (2.21)

$$\frac{\dot{m}_t}{m_t} = \frac{H_{It}\left(m_t + \lambda m_t^*\right)^{\phi}}{m_t \bar{F}}.$$

Since $\frac{\dot{H}_{It}}{H_{It}} = \frac{\dot{H}_t}{H_t} = n$, as I show in the appendix, g can only be constant over time if

$$g = \frac{n}{1-\phi}. \tag{3.31}$$

Equation (3.31) establishes that the steady state rate of innovation g is proportional to the population growth rate n and does not depend on either trade costs or the population size (no "strong scale effect"). This is true also for the per capita GDP growth rate, defined as the growth rate of the per capita real expenditure $E_t/P_t(L_t + H_t)$. As I show in the appendix, this is given by

$$g_{GDP} = \frac{g}{\varepsilon - 1}. \tag{3.32}$$

I view these as virtues of the model because they are supported by the empirical observation that total factor productivity and per capita GDP growth rates have been remarkably stable over time in spite of many public policy changes that one might think would be growth-promoting. For example, plotting data on per capita GDP (in logs) for the US from 1880 to 1987, Jones (1995a) shows that a simple linear trend fits the data extremely well. Moreover, (3.32) implies that the long run level of per capita income, and not its growth rate, is an increasing function of the size of the economy. Jones (2005) has a lengthy discussion of this "weak scale effect" property and cites Alcala and Ciccone (2004) as providing the best empirical support. Controlling for both trade and institutional quality, they find that a 10% increase in the size of the workforce in the long run is associated with 2.5% higher GDP per worker.

Taking logs and differentiating (2.27) and (2.30) and combining them with $\frac{\dot{E}_t}{E_t} = n$, I can show that nominal wages are constant:

$$\frac{\dot{w}_{Lt}}{w_{Lt}} = \frac{\dot{w}_{Ht}}{w_{Ht}} = 0,$$

and I will henceforth denote the unskilled and skilled wage rate as w_L and w_H. Using this result and (3.31), the growth rate of the unit cost of knowledge production is found to be

$$\frac{\dot{b}_{It}}{b_{It}} = -\phi g.$$

I will treat the Northern skill wage rate as the numeraire ($w_H = 1$), so all prices are measured relative to the price of Northern skilled labor.

The equilibrium is referenced by the pair of vectors $[\varphi_D, \varphi_E, E_t, P_t, m_t, w_L, w_H, r]$ and $[\varphi_D^*, \varphi_E^*, E_t^*, P_t^*, m_t^*, w_L^*, w_H^*, r^*]$. All the other endogenous variables can be written as functions of these quantities and model parameters. The numeraire and (2.4), which implies $r = r^* = \rho$, reduce the number of unknowns to 13. The equilibrium is then determined by the following equations: for each region, the zero profit condition for the domestic market (2.12), the relation between the two cutoff values (2.14), the free entry condition (2.25), the expressions for the price indexes (2.22) and the two labor market condition (2.29) and (2.30). The last equation is given by the trade balance condition (2.23).

The combination of multiple factors, region asymmetries, firm heterogeneity and trade costs means that there are no closed form solutions for many endogenous variables of the model. Nonetheless it is possible to derive some analytical results that help to shed light on the numerical solutions described in the following sections. As shown in the appendix, by combining (2.25), (2.26) and (2.29), I derive

$$E_t = \frac{w_H H_t \varepsilon \left(\rho + \phi g\right)}{g + \varepsilon\alpha\beta\left(\rho + \phi g\right)}. \tag{3.33}$$

Thus the aggregate expenditure E_t is pinned down by the total skilled workers remuneration $w_H H_t$. Note that I am not substituting for $w_H = 1$. The reason is that in this way it is easier to generalize the results and the expressions found for the Southern economy, where the skilled wage rate w_H^* is allowed to be different from 1. Plugging (3.33) into (2.30) I can solve for the skill premium as

$$\frac{w_H}{w_L} = \frac{L_t}{H_t}\frac{g + \beta(\varepsilon - 1)(\rho + \phi g)}{(1 - \beta)(\varepsilon - 1)(\rho + \phi g)}. \tag{3.34}$$

In steady state, the skill premium is increasing in the relative factor endowment L_t/H_t and in the production skill intensity β. More interestingly, it is increasing in the innovation rate g and decreasing in the elasticity of substitution ε.[4] A faster variety introduction (higher g) increases the demand of skilled labor from the R&D sector (see (3.36)) and the competition for this factor in the production sector pushes its price up. On the other hand, a more competitive industry (higher ε) reduces the expected profits from entry and so the demand for skilled workers driven by the R&D sector. It is interesting to notice that, if production and R&D shared the same factor intensity, then the innovation rate g and the elasticity ε would have no effect on the skill premium. As I show in the appendix, in that case the relative wage would be a function of the factor endowments and the skill-intensity β only. Its value would be lower than the one given by (3.34) for any $g > 0$.

The absence of any steady state effect of trade costs on the wage ratio is surprising. This result follows directly from the fact that the allocation of skilled labor between the two sectors is also invariant with respect to trade costs. In fact, from (2.27) and (2.28), it follows that the skill premium is an increasing function of the ratio of unskilled and skilled workers employed in production, L_{Pt}/H_{Pt}. Since $L_{Pt} = L_t$ and $H_{Pt} = H_t - H_{It}$, the wage rate changes in response to trade liberalization only through changes in H_{It}. To see why trade costs have no effects on the allocation of skilled workers across sectors, it is helpful to refer back to the free entry condition (2.25) and to the demand of R&D workers (2.26). Combining the two, I obtain

$$\frac{\frac{E_t}{\varepsilon m_t}}{\rho + \phi g} = w_H \frac{H_{It}}{\dot{m}_t}$$

which requires the average firm profit stream, the left hand side, to equal the amount of skilled workers employed by each entrant ($H_{It}/\dot{m}_t$) multiplied by the unit cost of skilled work, the right hand side. Rearranging the terms, I obtain

$$H_{It} = \dot{m}_t \frac{\frac{E_t}{\varepsilon m_t}}{\rho + \phi g} \frac{1}{w_H}. \tag{3.35}$$

In equilibrium the amount of skilled workers employed in R&D is thus equal to the aggregate benefit from innovating adjusted for the unit labor cost. As I have shown above, $\frac{\dot{m}_t}{m_t} = g$ is invariant with respect to trade costs and, according to (3.33), any change in E_t is perfectly compensated by the variation in the wage rate w_H. In other words, the cost-adjusted benefit from innovating is unaffected by trade costs and so it is the allocation of skilled workers between production and R&D. Substituting $\frac{\dot{m}_t}{m_t} = g$ and (3.33) in (3.35) and dividing both sides by H_t, I obtain the steady state equilibrium

[4] The skill premium is always increasing in g, whether g increases because of n or because of ϕ.

value for the proportion of R&D workers

$$h_I \equiv \frac{H_{It}}{H_t} = \frac{g}{g + \beta(\varepsilon - 1)(\rho + \phi g)}. \tag{3.36}$$

In steady state, the share of skilled workers employed in R&D does not depend on τ, is decreasing in the skill intensity of the manufacturing sector β and, as I show in the appendix, is increasing in the intertemporal spillovers ϕ.

Although the standard prediction from endowment based theories of comparative advantage (Heckscher-Ohlin) is that trade liberalization has clear distributional effects between production factors, empirical research has offered no conclusive evidence on the effects of trade liberalization on employment and wages.[5] In particular, studies that seek to decompose the sources of employment and wage inequality changes generally find that trade factors play only a minor role and that dominant factors are productivity growth (a function of g in the model) and technological change.

4. Numerical Solutions

In this section, I report the results I obtained from solving the model numerically: first, I show and explain the choices for the parameter values I used; second, I discuss the equilibrium properties of the model, being careful to distinguish among the different channels that drive my results. In the next section, I will use the same numerical methods to study the impact of trade liberalization.

4.1. Parameters Choice. In the computer simulations, I used the following parameters values: $\rho = 0.07$, $\varepsilon = 3$, $k = 2.1$, $\bar{a} = 0.2$, $n = 0.014$, $g = 0.025$, $\phi = 0.44$, $F_D = 1$, $F_E = 1.7$, $F_I = 2.5$, $\lambda = 0.7$, $\beta = 0.5$, $\tau = 1.3$, $H_T = 1000$, $L_T = 1500$, $H_T^* = 700$, and $L_T^* = 1800$, where $t = T$ stands for an arbitrary moment in time on the balanced growth path. I will refer to this set of parameter values as the benchmark case (BMC hereafter).

The subjective discount rate ρ was set at 0.07 to reflect the real interest rate of 7 percent, consistent with the average real return of the US stock market over the past century as calculated by Mehra and Prescott (1985). The elasticity of substitution parameter ε was chosen in order for $\frac{\varepsilon}{\varepsilon-1}$ to match empirical estimates of firms markups. These vary widely from 3 to 70 percent. I chose a value of $\varepsilon = 3$ that corresponds to a markup of 50 percent and that is consistent with the choice of the shape parameter

[5] See B. Hoekman and A. L. Winters (2005) for a survey of the impact of international trade and trade reform on employment outcomes.

k.[6] In fact, for the expected discounted profits from innovating to be finite, it must be the case that $k > \varepsilon - 1$. Del Gatto, Minon and Ottaviano (2006), using firm level European data, provide the benchmark value $k = 2.1$. The minimum productivity level $\bar{a}$ is just a scale parameter and I set it equal to 0.2. The population growth rate $n = 0.014$ equals the annual rate of world population growth between 1991 and 2000 according to the World Development Indicators (World Bank, 2003). From Jones and Williams (2000) I took the benchmark TFP growth rate $g_{TFP} = 0.0125$ which is the average growth rate in the US private business sector over the period 1948-97. Given the TFP growth rate, defined as the growth rate of real output not explained by the growth of the inputs used in production, I can pin down the value for the innovation rate g. In the appendix I show that

$$g_{TFP} = \frac{g}{\varepsilon - 1}$$

which implies $g = g_{TFP}(\varepsilon - 1) = 0.0125 \times 2 = 0.025$. The intertemporal spillover parameter is based on the equilibrium identity (3.31) and $\phi = \frac{g-n}{g} = 0.44$. In the benchmark case intertemporal spillovers are thus assumed to be positive. The fixed costs, $F_D = 1$, $F_E = 1.7$ and $F_I = 2.5$, were chosen in order to satisfy

$$F_I > F_E > F_D$$

and such that $\frac{\varphi_E}{\varphi_D} < 1$ (see (2.14) above). International spillovers are assumed to be smaller than 1 ($\lambda = 0.7$) and the Cobb-Douglas parameter β is set equal to 0.5 because this is the value such that the skill premium is equal to 1 in case there is no productivity growth ($g = 0$) and $L_t = H_t$. The labor endowment parameters $H_T = 1000$, $L_T = 1500$, $H_T^* = 700$, and $L_T^* = 1800$ were chosen to capture a North-South trade relationship between the two regions. Relative to the foreign region (henceforth, the South), the domestic region (henceforth, the North) is skilled-labor abundant ($H_T/L_T = 0.67 > H_T^*/L_T^* = 0.39$), has more high-skilled labor ($H_T > H_T^*$) and has less low-skill labor ($L_T < L_T^*$).

The analysis focuses on how the exposure to trade differently affects economies that are dissimilar only in terms of their labor endowments. In equilibrium, differences in factor endowments make the two regions heterogeneous along two major dimensions: the factor abundance, measured by H_t/L_t, and the market size or GDP, measured by E_t. In order to disentangle the separate roles of factor abundance and market size in driving the results, I compare in Table 1 the equilibrium for the BMC (first column) with

[6] 50% represents the markup over marginal costs that, in a model with entry costs, differ from average costs. Thus, although $\varepsilon = 3$ implies a fairly high markup over marginal cost, my parametrization delivers reasonable markups over average costs, which take into account the per-period, amortized flow value of the sunk entry costs.

those associated with other choices of factor endowments for the Southern economy. The second column reports the results in case the two economies have different market sizes but equal relative factor endowments. To obtain these values, I imposed the expenditures in the two regions to be the same as in the first column (E_t=2292 and E_t^*=1869) and then I solved for the equilibrium such that the two economies share the same skill abundance given by $H_T/L_T = 1000/1500 = 0.67$. The Southern factor endowments such that this constraint is met in equilibrium are given by $H_T^* = 834$ and $L_T^* = 1251$. The third column displays the results for the equilibrium where the two regions have the same market size but different factor abundance. To solve for it, I imposed Southern aggregate expenditure to be the same as in the North ($E_T^* = E_T = 2292$) and Southern skill abundance to be the same as in the benchmark case ($H_T^*/L_T^* = 700/1800 = 0.39$). The Southern factor endowments consistent with such an equilibrium are given by $H_T^* = 840$ and $L_T^* = 2159$. The last column shows the equilibrium values for the symmetric-region case, when the two region have the same absolute factor endowments, $H_T = H_T^* = 1000$ and $L_T = L_T^* = 1500$, and as a consequence, the same factor abundance and market size. In other words, the second column of Table 1 represents a scenario where the South is a smaller economy compared to the North but it is not less abundant in skilled workers. The comparison of this column with the last one will help to highlight the contribution of differences in market size in driving the results. The third column corresponds to a case where the South is as big as the North but its labor force is less skilled. Contrasting the entries in this column with those in the last one will stress the role played by the difference in skill abundance.

To complete the description of Table 1, I define the relative innovation cost RIC_t as the ratio between the average cost of introducing a new variety and the average unit production cost. This gives the cost of introducing a new good in terms of foregone units of existing varieties. Formally it is given by

$$RIC_t = \frac{\varphi_D \bar{F}(\varphi_D)}{(m_t + \lambda m_t^*)^{\phi}} \frac{k+1}{k} \left(\frac{w_H}{w_L}\right)^{1-\beta}. \tag{4.37}$$

It is increasing in the skill premium w_H/w_L and in the domestic market productivity cut-off φ_D.

4.2. Equilibrium Properties. Consistent with the empirical evidence, the model is calibrated in order to satisfy the selection into export hypothesis.[7] The productivity threshold is always higher for the foreign market than for the domestic market. For

[7] See for example Bernard and Jensen (2004) or Wagner (2007) for an extensive review of the related literature.

Market Size	$E_t > E_t^*$				$E_t = E_t^*$			
Factor Abundance	$H_t/L_t > H_t^*/L_t^*$		$H_t/L_t = H_t^*/L_t^*$		$H_t/L_t > H_t^*/L_t^*$		$H_t = H_t^*; L_t = L_t^*$	
	North	*South*	*North*	*South*	*North*	*South*	*North*	*South*
E_T	2292	1869	2292	1869	2292	2292	2292	2292
H_T	1000	700	1000	834	1000	840	1000	1000
L_T	1500	1800	1500	1251	1500	2159	1500	1500
H_T/L_T	0.67	0.39	0.67	0.67	0.67	0.39	0.67	0.67
φ_D	0.65	0.68	0.65	0.68	0.67	0.67	0.67	0.67
φ_E	1.19	1.08	1.19	1.08	1.13	1.13	1.13	1.13
φ_E/φ_D	1.83	1.57	1.83	1.57	1.69	1.69	1.69	1.69
w_H/w_L	1.96	3.36	1.96	1.96	1.96	3.36	1.96	1.96
κ	0.71	0.63	0.71	0.70	0.71	0.65	0.71	0.71
m_T/m_T^*	1.66		1.38		1.21		1.00	
m_T^{exp}/m_T^{*exp}	1.21		1.00		1.21		1.00	
RIC_T	0.11	0.18	0.10	0.13	0.12	0.16	0.11	0.11

TABLE 1. Steady State Equilibrium when $\tau = 1.3$

instance, $\varphi_E = 1.19 > \varphi_D = 0.65$ and $\varphi_E^* = 1.08 > \varphi_D* = 0.68$ in the BMC (first column in Table 1). In other words, the model shares with previous papers the prediction that exporting firms are more productive than non-exporting ones. Exporting firms are also bigger (see Roberts and Tybout (1997) and Bernard et al. (2005) for empirical evidence). In the appendix, I show that firm size, measured in terms of both employment and sales, is an increasing function of the productivity level φ. It follows that if exporting firms are more productive, then they are also larger than non-exporting firms.

Another important and surprising result from Table 1 is that only the relative size of the two regions matters for the equilibrium values of the productivity thresholds. The productivity cut-offs are in fact the same between the first and the second columns and between the third and fourth columns. Moreover the productivity cut-offs are identical across regions when they have the same GDP ($\varphi_D = \varphi_D^* = 0.67$ and $\varphi_E = \varphi_E^* = 1.13$ in both the third and the fourth column). In other words, for given market sizes, relative factor endowments have no effect on the exporter productivity premium and on the productivity thresholds. The intuition behind this result is the following: any change in relative factor endowments that does not affect the aggregate expenditure leaves the relative profitability between the domestic and the foreign market unchanged. If relative profitability is constant, then the exporter productivity premium is also constant. Analytically, a variation in the relative factor endowment affects wages and production costs (see κ in the table), but when I divide side by side (2.12) and (2.13) to derive (2.14), κ on the left hand sides and w_H that enters b_I on the right hand sides, both cancel out since they affect the profits and the costs on the two markets in the same way. As I show in the appendix, for a given exporter productivity premium φ_E/φ_D, there exists a unique pair (φ_E, φ_D) that satisfies condition (2.19).

The model suggests that the exporter productivity premium φ_E/φ_D is increasing in domestic expenditure E_t and decreasing in foreign expenditure E_t^*. When the South's GDP decreases (from $E_T^* = 2292$ in the last two columns to $E_T^* = 1869$ in the first two columns of Table 1), the exporter productivity premium increases in the North (φ_E/φ_D rises from 1.69 to 1.83) and decreases in the South (φ_E^*/φ_D^* falls from 1.69 to 1.57). The difference in exporter productivity premia between the two regions is associated with a lower domestic market productivity cut-off in the North and a lower foreign market productivity cut-off in the South ($\varphi_D < \varphi_D^*$ and $\varphi_E > \varphi_E^*$ in the first and second columns).[8] The intuition is that the profitability of entering a market is proportional to its size and the higher the profitability the lower the productivity needed to successfully introduce a new product.

The model has interesting implications concerning the size of exporters. The larger is the trading partner, the less tough is the selection into exporting (as Southern income rises from 1869 to 2292, φ_E falls from 1.19 to 1.13). As firm productivity falls, firm employment decreases (this is shown in Appendix 8.13). It follows that average exporter employment in the North falls when the South becomes larger. This result is in line with some recent evidence about U.S. trading firms in Bernard et al. (2005). They found that the average exporter size, measured in terms of firm employment, falls systematically as the income of the firm's trading partners increases.

Market size also has implications for the number of firms active in the two economies. The model offers a sophisticated version of the "home market effect", described by Krugman (1980) as the tendency of firms to locate where the market is larger. In Krugman's homogeneous firms setting, all entrants serve both markets so the larger markets attracts both more firms and more exporters. With firm heterogeneity, the bigger market still attracts more firms (going from the fourth to the second column of Table 1 , m_T/m_T^* increases from 1 to 1.38 when E_T/E_T^* increases from 1 to 2292/1869=1.23), but this is not true for the number of exporters (m_T^{exp}/m_T^{exp*} is almost unchanged).[9] This is due to the self-selection of firms into export and to what could be defined as

[8] In the appendix I show that, from (2.19), it follows that a higher exporter productivity premium φ_E/φ_D always implies a higher foreign market productivity threshold and a lower domestic market productivity threshold.

[9] What Table 1 does not show is that when Southern market size drops by 18 percent (moving from the fourth to the second column), the number of firms in the South decreases by more than 27

a"foreign market effect": facing a bigger export market, Southern exporters can more easily exploit the economies of scale given by the entry cost F_E. As a result m_T^{exp}/m_T^{exp*} is always less than or equal to m_T/m_T^*.

The difference in relative factor endowments explains the difference in the skill premium between the two regions: according to (3.34) the skill premium is higher in the skill scarce country ($w_H^*/w_L^* = 3.36$ is higher than $w_H/w_L = 1.96$ when $H_T^*/L_T^* = 0.39$ is less than $H_T/L_T = 0.67$). This has an important effect on the difference in the relative innovation cost between the two economies, and explain why the South produces less varieties regardless of its market size. Comparing the last two columns of Table 1, when the South becomes the skill-scarce country (H_T^*/L_T^* falls from 0.67 to 0.39), its opportunity cost of introducing a new variety in terms of units produced increases and becomes bigger than the opportunity cost in the North (RIC_T^* increases from 0.11 to 0.16, where as RIC_T increases from 0.11 to 0.12). As a result, the relative number of varieties produced in the South falls (m_T/m_T^* increases from 1 to 1.21), even though the income is unchanged ($E_T^* = E_T = 2292$).[10]

The results reported in the first column (BMC) are thus a combination of differences in market size and labor force composition. In industries where product development and market entry require sunk investments in skill intensive activities, the model predicts that the Southern economy produces (and to a lower extent exports) a smaller number of varieties ($m_T/m_T^* = 1.66$ and $m_T^{exp}/m_T^{*exp} = 1.21$). This is the result of both its smaller market and its less skilled work force. The South's smaller market is responsible for the higher domestic productivity threshold ($\varphi_D^* = 0.68 > 0.65 = \varphi_D$) and, according to (2.20), for the higher expected knowledge requirement $\bar{F}$. The South's less skilled work force is responsible for the lower production cost ($\kappa^* = 0.63 < \kappa = 0.71$) and the higher skill premium in the South. As a result, R&D activities are more expensive in the South and the average unit production cost is more expensive in the North. In terms of (4.37), the opportunity cost of innovation in terms of units produced is higher in the South ($RIC_T^* = 0.18 > 0.11 = RIC_T$), with ultimate effects on trade

percent while the number of firms in the North increases slightly. This is the same result as derived in Feenstra (2004, p.165) for the Krugman (1980) model.

[10] This result follows from the assumption of R&D being more skill-intensive than production. If the two activities shared the same factor intensity, as in BRS (2007), then the only difference between two regions of the same size but different relative factor endowments would be the skill premium. Wages would adjust such that both production and innovation costs would be the same across the two regions.

composition: the South exports bigger quantities of fewer varieties while the North exports smaller quantities of more varieties. In other words, Southern exports are more developed on the intensive margin while Northern exports are more developed on the extensive margin.

5. Comparative Statics

In this section I analyze the effects of lowering variable trade costs on the steady state equilibrium of the model. To do so I consider symmetric reductions in variable trade costs between the two regions, from autarky ($\tau \to \infty$) to free trade ($\tau = 1$). I concentrate on the long run effect of trade liberalization, comparing the equilibria across settings that differ in the level of the variable trade costs τ. A more complete analysis would require a detailed study of the transition dynamics. Although this goes much beyond the purpose of this paper, I will discuss some important differences between the short run and the long run effects of trade liberalization.

5.1. Industry structure. The first important thing to notice from Table 2 is that, despite the new features of asymmetric regions, multiple factors of production and positive productivity growth, the model still delivers the main predictions of Melitz (2003): lowering trade barriers ($\tau \downarrow$) makes it more difficult for firms to enter the domestic market ($\varphi_D \uparrow$) and induces more firms to become exporters ($\varphi_E \downarrow$). For instance, when τ decreases from 1.8 to 1.3, the domestic market productivity cut-off φ_D increases from 0.60 to 0.65 in the North and from 0.61 to 0.68 in the South, while the export market productivity cut-off φ_E decreases from 1.50 to 1.19 in the North and from 1.35 to 1.08 in the South. When trade costs decrease, profitability from exporting increases so ex-ante expected profits from innovation go up and more firms are willing to invest in R&D. This, together with the increased demand for factors needed to produce for export, pushes up the demand for both skilled and unskilled labor. As a result, the higher wages reduce the ex post profits of domestic firms and the least productive ones will not find market entry attractive anymore. The resulting higher competition on a limited amount of resources implies that less firms can be sustained in equilibrium and lower trade costs are thus associated with a lower number of firms in both regions. On the other hand, the fraction of exporting firms, given by the share of successful entrants with productivity $\varphi > \varphi_E$, i.e. $\frac{1-G(\varphi_E)}{1-G(\varphi_D)} = \left(\frac{\varphi_D}{\varphi_E}\right)^k$, is increasing with trade liberalization and the net effect of lower trade barriers is an increase in the total number of exporters.

Trade Costs	Country	φ_D	φ_E	m_t	m_t^{exp}	w_H	w_L	RIC_t	P_t
$\tau \to \infty$	North	0.54	∞	7.4e+04	0	1.000	0.509	0.05	0.00159
	South	0.54	∞	5.0e+04	0	1.144	0.340	0.06	0.00169
$\tau = 1.8$	North	0.60	1.50	4.9e+04	0.7e+04	1.000	0.509	0.08	0.00158
	South	0.61	1.35	3.1e+04	0.6e+04	1.155	0.343	0.11	0.00164
$\tau = 1.5$	North	0.62	1.30	4.2e+04	0.9e+04	1.000	0.509	0.09	0.00157
	South	0.65	1.19	2.6e+04	0.7e+04	1.160	0.345	0.14	0.00162
$\tau = 1.3$	North	0.65	1.19	3.6e+04	1.0e+04	1.000	0.509	0.11	0.00156
	South	0.68	1.08	2.1e+04	0.8e+04	1.165	0.346	0.18	0.00161
$\tau = 1$	North	0.71	1.00	2.4e+04	1.2e+04	1.000	0.509	0.18	0.00155
	South	0.78	0.93	1.4e+04	0.9e+04	1.178	0.350	0.33	0.00156

TABLE 2. Steady State Effects of Trade Liberalization ($\tau \downarrow$)

Trade liberalization has thus three major effects: it raises average productivity, increases the number of firms that export and reduces the total mass of firms. Even though these effects hold in both regions, Table 2 shows that their magnitude can differ substantially. Since the Northern market is bigger, the increase in the expected profits from innovating and exporting is higher for Southern firms. As a result, the pressure on wages is stronger there. Table 2 shows that when trade costs fall, the wages in the North are unaffected while they increase in the South. For instance, going from autarky ($\tau \to \infty$) to free trade (τ=1), w_H^* increases from 1.144 to 1.179 and w_L^* from 0.340 to 0.350. Given the choice of the numeraire ($w_H = 1$), this means that trade liberalization increases the relative wages of both skilled and unskilled workers in the South.

The rise in labor costs makes the selection among the least productive firms particularly tough in the South. For instance, when τ decreases from 1.8 to 1.3, φ_D increases by 8 percent while φ_D^* by more than 11 percent. As a result there is divergence in average firm productivity between the two regions following trade liberalization, and the same holds for the opportunity cost of innovation in terms of units produced. When τ falls from 1.8 to 1.3, RIC_T increases by 63 percent in the South (from 0.11 to 0.18) and by only 37 percent in the North (from 0.08 to 0.11). Trade liberalization, even without affecting the skill premia w_H/w_L, alters the relative innovation cost and magnify the North's ex-ante comparative advantage in R&D. As a result, even though the number of firms decreases in both regions, the Northern relative number of varieties m_t/m_t^* increases (m_t falls by almost 27 percent while m_t^* by almost 32 percent) and the number of exporters, increasing in both regions, rises more in the North (by 41 percent against the 39 percent in the South).

The last column in Table 2 shows the effect of trade liberalization on the price index. This will play a crucial role in the welfare analysis. According to (2.22), P_t depends on production costs, in terms of wages, average productivity and trade barriers, and on the number of varieties available on the market, varieties either imported or domestically produced. These components push the index in different directions. With the benchmark parametrization of the model, the decrease in transport costs, the increase in the number of exported varieties and the increase in average productivity are the dominant forces. The price index is thus decreasing with trade liberalization and, by reason of the bigger increase in the number of Northern exported varieties, it falls faster in the smaller Southern economy (with τ falling from 1.8 to 1.3, P_t drops by 2 percent in the South and by 1 percent in the North).

Trade Costs	Country	E_T	u_H	u_L	W_T
$\tau \to \infty$	North	2292	759	451	1.44e+06
	South	1835	776	301	1.08e+06
$\tau = 1.8$	North	2292	767	456	1.45e+06
	South	1853	805	312	1.12e+06
$\tau = 1.5$	North	2292	770	458	1.46e+06
	South	1861	818	317	1.14e+06
$\tau = 1.3$	North	2292	773	460	1.46e+06
	South	1869	831	322	1.16e+06
$\tau = 1$	North	2292	779	463	1.47e+06
	South	1890	864	335	1.21e+06

Table 3. More Steady State Effects of Trade Liberalization ($\tau \downarrow$)

5.2. Consumer Welfare. According to (3.34) and (3.36), the skill premium w_H/w_L and skilled labor allocation h_I do not vary with trade costs τ. This means that Northern wages do not change with trade liberalization while, as shown before, Southern wages increase both for skilled and unskilled workers. The model thus predicts income convergence between North and South due to trade liberalization. As Table 3 shows, this is true also for nominal GDP: going from autarky ($\tau \to \infty$) to free trade (τ=1), E_T^* increases from 1835 to 1890, while E_T remains constant at 2292.

Individual welfare is given by consumer utility u_{lt}. Plugging (2.3) into (2.2) I obtain:

$$u_{lt} = \frac{c_l}{P_t} \qquad l = H, L.$$

From the individual budget constraint

$$c_l = w_l + a_{lt}(\rho - n) - \dot{a}_{lt} \quad l = H, L.$$

I assume that the asset endowment at time $t = T$ is the same for skilled and unskilled workers and equal to

$$a_{lT} \equiv \frac{A_T}{H_T + L_T} \qquad l = H, L.$$

Aggregate assets in the economy equal the aggregate value of active firms, given by

$$A_t = m_t \left[\int_{\varphi_D}^{\infty} V_{Dt}(\varphi) \frac{g(\varphi)}{1 - G(\varphi_D)} d\varphi + \int_{\varphi_E}^{\infty} V_{Et}(\varphi) \frac{g(\varphi)}{1 - G(\varphi_D)} d\varphi \right].$$

Using (2.15), I can rewrite it as

$$A_t = m_t b_{It} \bar{F}.$$

Combining the above expression with (2.6) and (2.26), in the appendix I solve for individual asset holdings as

$$a_{lt} = \frac{H_t}{H_t + L_t} \frac{h_I w_H}{g} \qquad l = H, L.$$

Since w_H and h_I are constant in steady state, it immediately follows that $\dot{a}_{lt} = 0$. Consequently, I can solve for the individual expenditure as:

$$c_l = w_l + a_l(\rho - n) \quad l = H, L.$$

On the balanced growth path c_H and c_L are constant because wages and asset holdings are constant. Consumer utility grows because the price index falls over time due to variety introduction.

Table 3 reports some important results. First, individual welfare is higher in the North for unskilled workers ($u_L = 460 > u_L^* = 322$ in the BMC) but higher in the South for skilled workers ($u_H^* = 831 > u_H = 773$ in the BMC).

Second, the big economy has the highest aggregate welfare, defined as $W_T \equiv E_T/P_T$. In the benchmark case (τ=1.3) the aggregate welfare is 26 percent higher in the North than in the South ($W_T/W_T^* = 1.46/1.16 = 1.26$). This is the result of both the higher expenditure E_T and the lower price index P_T.

Third, aggregate welfare is converging with trade liberalization between the two regions. Under free trade (τ=1), welfare in the North is 21 percent higher than welfare in the South ($W_T/W_T^* = 1.47/1.21 = 1.21$). With respect to the benchmark case, in fact, aggregate welfare has increased by more than 4 percent in the South and by only 0.6 percent in the North. When variable trade costs decline, welfare improves in both regions because price indexes fall but, as shown in the previous section, Southern consumers benefit more since the increases in the average productivity and in the number of imported varieties are bigger. Furthermore, Southern workers benefit from the rise in wages which is the cause of the convergence in the nominal GDP described above.

While welfare convergence is robust to the choice of parameters value, the welfare gains from trade liberalization are not. The next section discusses this issue.

5.3. The Role of Intertemporal Knowledge Spillovers. The parameter ϕ is a key parameter of the model.[11] It captures the extent of inter-temporal knowledge spillovers which can be either positive or negative. For $\phi > 0$ (< 0) knowledge creation becomes easier (tougher) as the stock of knowledge increases over time. According to (3.31), for given population growth rate n, the choice of ϕ uniquely pins down the growth rate g. When ϕ increases, the skilled labor employed in R&D becomes more productive and this implies an increase in h_I and, as a consequence, in g.

What follows is a replication of the previous comparative static exercises for different values of ϕ in order to assess the role of the spillovers in driving the results. Table 4 shows the equilibrium values for the price indexes and the aggregate welfare. Each column refers to a different choice of ϕ and to the corresponding values for the innovation rate g and the productivity growth rate g_{TFP}. The second column of numbers reports the results for the benchmark parameter choice ϕ=0.44.

For negative or low enough values of the spillovers (ϕ=-0.12 and ϕ=0.44 in the table) both price indexes are decreasing as trade costs fall. For high values of ϕ (ϕ=0.60, in the table) they are instead both increasing. Positive and high intertemporal spillovers magnify the variety loss due to trade liberalization and this becomes so severe that, despite the lower trade cost, the higher average productivity and the increased number of imported goods, the price index goes up. For intermediate values of ϕ (ϕ=0.50, in the table), the price index exhibits different patterns in the two regions ($P_T \uparrow$ and

[11] Also in the symmetric country model developed by Gustafsson and Segerstrom (2010), the welfare effects of trade liberalization depend on the strength of the intertemporal spillovers ϕ. The authors solve algebraically for the critical value $\bar{\phi}$, such that welfare effects are positive for $\phi < \bar{\phi}$ and negative otherwise. On the other hand, they do not solve the model numerically and they focus on the case of negative spillovers which corresponds to implausible values for the TFP growth rate.

		$\phi = -0.12$ $g = 0.012$ $g_{TFP} = 0.006$	$\phi = 0.44$ $g = 0.025$ $g_{TFP} = 0.0125$	$\phi = 0.50$ $g = 0.028$ $g_{TFP} = 0.014$	$\phi = 0.60$ $g = 0.035$ $g_{TFP} = 0.017$
P_T	$\tau = 1.5$	0.022946	0.001573	0.000852	0.000211
	$\tau = 1.2$	0.021274	0.001564	0.000861	0.000222
P_T^*	$\tau = 1.5$	0.023674	0.001627	0.000881	0.000218
	$\tau = 1.2$	0.021683	0.001627	0.000879	0.000227
W_T	$\tau = 1.5$	0.11e+06	1.46e+06	2.64e+06	10.0e+06
	$\tau = 1.2$	0.12e+06	1.47e+06	2.61e+06	9.77e+06
W_T^*	$\tau = 1.5$	0.09e+06	1.14e+06	2.07e+06	8.01e+06
	$\tau = 1.2$	0.10e+06	1.17e+06	2.09e+06	7.77e+06

Table 4. Effects of Varying ϕ and τ on Price Indexes and Welfare

$P_T^* \downarrow$ when $\tau \downarrow$). In the South, the positive effect of trade liberalization on the range of imported varieties is always higher and this compensates for the fall in the total number of active firms for a wider range of parameter values.

The response of welfare to trade liberalization depends on ϕ through the effect on the price index just described. In the previous section, I have shown the effect on welfare for the BMC. The same conclusions hold for lower values of ϕ. When the spillovers are higher (ϕ=0.60 in the table), aggregate welfare decreases with trade liberalization in both regions ($W_T \downarrow$ and $W_T^* \downarrow$ when $\tau \downarrow$). Differently from BRS (2007), the model predicts that trade liberalization is not necessarily welfare improving. Although trade liberalization ($\tau \downarrow$) always leads to higher average productivity ($\varphi_D \uparrow$), the competition that it induces among producers, reducing the range of products available to consumers ($m_T \downarrow$), can have detrimental effects on welfare. For ϕ=0.50, a reduction in trade costs causes Northern welfare to decrease and Southern welfare to increases ($W_T \downarrow$ by roughly 1 percent and $W_T^* \uparrow$ by roughly 1 percent). The model thus predicts that the welfare effects of decreasing trade barriers can differ between the two regions not only in the magnitude, but also in the direction, the South being more likely to gain.

As a more general result, Table 4 suggests that both the welfare and the welfare effects of trade liberalization are decreasing in ϕ (for instance, when τ falls from 1.5 to 1.2, the welfare in the South changes by +11% when $\phi = -0.12$, by +3% when $\phi = 0.44$, by +1% when $\phi = 0.50$ and by -3% when $\phi = 0.6$). It is also interesting to notice how small the welfare effects of trade liberalization are in the North when considering plausible parameter values. In the BMC, when the region goes from autarky to free trade, the welfare increases by only 2 percentage points (see Table 3). These results underline the importance of allowing for productivity growth and the relevance of the assumptions behind the R&D technology when studying the welfare implication of trade liberalization.

6. The short run effects of trade liberalization

In the sections above I have shown that trade liberalization has no steady state effect on the rate of product innovation g. This is true also for the aggregate welfare growth rate. In the appendix, I show that $\dot{W}_t/W_t = n + \frac{g}{\varepsilon-1}$, independent of τ. On the other hand, the model has clear predictions for what concerns the short-run effects of changing trade costs. One of the main long-run effects of trade liberalization is to reduce the number of active firms. In a framework with sunk entry costs and productivity growth, this means that no firm will actually exit the market but that the economy, once having reached the new steady state, will have less firms than it would have had at the same moment in time without the policy intervention. This outcome can be achieved only if the entry of new firms slows down for a while. In other words, trade liberalization temporary decreases the rate of variety innovation, such that in the new steady state m_t will be growing at the same rate g, but on a lower path. The out-of-steady-state expression for the innovation rate is given by

$$g_t \equiv \frac{\dot{m}_t}{m_t} = \frac{H_{It}\left(m_t + \lambda m_t^*\right)^{\phi}}{m_t \bar{F}_t}$$

where $\bar{F}_t$ stands for the out-of steady-state average knowledge requirement for product introduction. As I show in the appendix, this is given by

$$\bar{F}_t = \left[F_I\left(\frac{\varphi_{Dt}}{\bar{a}}\right)^k + F_D + F_E\left(\frac{\varphi_{Dt}}{\varphi_{Et}}\right)^k\right] \tag{6.38}$$

and it is simply the time varying version of $\bar{F}$ as defined by (2.17). When trade liberalization occurs, according to (2.9), the profitability from exporting increases immediately and, as a consequence, the foreign market productivity cut-off goes down. The increased competition for production factors pushes up the domestic market productivity threshold. As a result the expected cost of entry as defined by (6.38) increases. In other words, $\bar{F}_t$ is a jump variable while the number of firms adjusts slowly during the transition to the new steady state, up to the point where $\frac{(m_t+\lambda m_t^*)^{\phi}}{m_t}$ exactly compensate for the rise in $\bar{F}_t$, such that g_t returns to the unchanged steady state value.[12]

[12] This does not necessarily mean that $\bar{F}_t$ immediately jumps to its new steady state value. Right after trade liberalization, when competition is fierce, φ_{Dt} and $\bar{F}_t$ could jump above the new steady

The amount of skilled workers employed in R&D H_{It} could potentially move in either direction but in the long run it has to be back at the old steady state value (remember that h_I does not depend on τ).[13] Panel (a) of Figure 1 shows the effect on the dynamic of innovation rate g: it immediately jumps down in response to the increase in the expected knowledge requirement $\bar{F}_t$, and then slowly rises back to its long run level as m_t converge to the new steady state value. Panel (b) illustrates the corresponding dynamic for the number of firms: this is always increasing but at a lower rate during the transition. Since the jump of $\bar{F}_t$ is bigger in the South $\left(\frac{\varphi_D}{\varphi_D^*}\downarrow\right)$, this region experiences a more severe slump in the variety growth rate following trade liberalization.

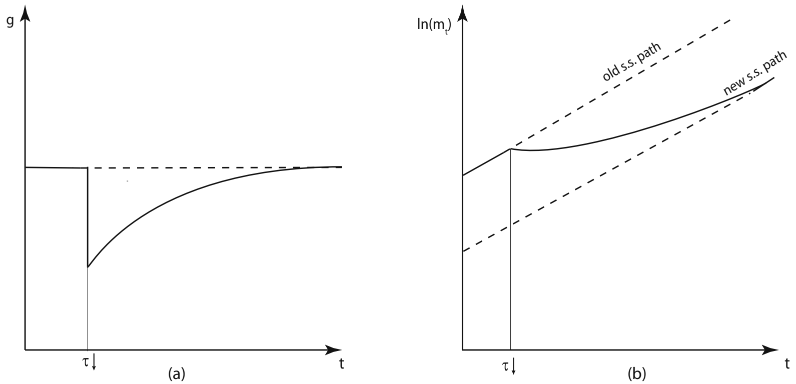

FIGURE 1. Effects of Trade Liberalization on Variety Innovation

The dynamics just described hold for any value of the parameter ϕ, since the number of firms is always decreasing with trade liberalization. On the other hand, ϕ plays a role in determining the speed along the transition path. The intuition is the following: when $\phi < 0$, the decrease in the number of firms exerts a positive externality on new

state values. Along the transition, with the number of firms growing below the potential, competition weakens and they slowly decrease. What matters for analysis is that $\bar{F}_t$ is not a state variable and it experiences a positive initial jump. See Chaney (2005) for a detailed analysis of the transition dynamics of the Melitz (2003) model.

13 H_{It} most likely jumps down as liberalization occurs and then slowly converges to its steady state value. The immediate effect of lower trade costs is an increase in production for the foreign market. This would reallocate resources from R&D to manufacturing. During the transition, firms enter at a lower rate while the labor force keeps growing at the rate n. This raises the labor supply and allows skilled worker to return to innovation activities.

R&D activities and makes g_t rise faster back to its steady state value. When $\phi > 0$, the lower number of firms further depresses the introduction of new varieties.

All I have just said has important implications for how welfare adjusts to lower trade costs. As mentioned before, the main effects on consumer welfare come from the increase in the number of foreign varieties, the reduction in prices due to lower trade costs, the rise of average productivity and the decline in the number of domestic firms. The first two immediately follow trade liberalization and they lead to a reduction of the price index and to a discrete positive jump of consumers welfare (see Figure 2). The number of firms and average productivity instead adjust slowly over time and they have opposite effects on the welfare transition to the new steady state path. If the gradual increase in average productivity more than compensates for the slow-down in variety introduction, welfare grows above its potential and ends up on a higher steady state path (dashed line in panel (a)). If the opposite is true, along the transition consumers welfare grows below its potential and ends up on a either higher or lower path, depending on the size of the initial jump (full lines in panels (a) and (b)).

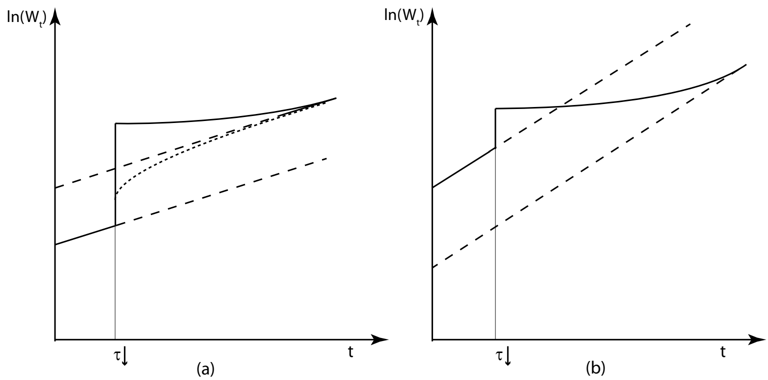

FIGURE 2. Effects of Trade Liberalization on Welfare

The model thus predicts that the short and long run effects of trade liberalization can be very different. In the short run, trade liberalization is always welfare improving because it expands the range of foreign varieties immediately available for consumption. During the transition to the new steady state, the growth of consumer welfare can either slow down or accelerate and the net effect on the long run welfare level is thus ambiguous. This ambiguity is absent in Melitz (2003) and BRS (2007) because, without productivity growth ($g = 0$), the positive effects always dominate. Here the lower is ϕ, the more likely it is that the positive effect on average productivity prevails during the transition and, as shown in the previous section, the higher is the long run effect on welfare. In fact, as discussed above, the lower is ϕ, the faster is the convergence of g back to the steady state value and, as a consequence, the faster is the increase of the average productivity. The discrete initial jump, instead, is bigger the larger is the increase in the number of foreign exporters. Compared to the North, the short run welfare adjustment in the South is thus characterized by a bigger initial jump and, because of the larger increase in φ_D, by a more severe slowdown in variety innovation and a greater improvement in average productivity. The sum of these effects is such that the welfare gap between the two regions is reduced in the long run.

7. Concluding Remarks

In heterogeneous-firm trade models, it is typically assumed that the production of goods and the production of knowledge required for variety introduction share the same factor intensity. In other words, R&D activities are not assumed to be skill intensive relative to the other stages of the production process.

In this paper, I present a North-South trade model with firm-level productivity differences and productivity growth that is consistent with the idea that product innovation and marketing are skill intensive activities. Firms first engage in R&D activities which employ skilled workers only, then they eventually start production which requires both skilled and unskilled workers. The main focus of the paper is on studying how trade openness differently affects economies that differ only in their endowments of skilled and unskilled labor.

I find that firm selection into the domestic and foreign markets varies systematically between the two regions in a way that depends only on the relative size of the two markets. The bigger the domestic market relative to the foreign one, the lower the productivity needed to successfully produce for the local market and the higher the productivity needed to become an exporter. The exporter productivity premium is thus higher in the larger economy. The market size-driven firm selection leads to a lower knowledge requirement for variety introduction in the North relative to the South. This magnifies the North's factor-based cost advantage in variety innovation with respect to unit production. As a result, Northern exports are more developed along the extensive margin while Southern exports are more developed along the intensive margin.

I also study the effects of trade liberalization on both firm selection and consumer welfare. When variable trade costs fall, the number of exporters increases and market shares reallocate from less to more productive firms. This leads to an increase in average productivity but long run welfare effects can be ambiguous and differ substantially between the two regions. I find that the South is more likely to gain from trade liberalization. The model thus predicts welfare convergence between the two regions.

A huge body of literature has already shown that domestic and exporting firms differ systematically along several dimensions, such as productivity, revenues and sizes. But these results most likely hide a lot of heterogeneity depending, as suggested by the analysis in this paper, on country characteristics. The test of this and related hypotheses is left to future empirical research.

8. Technical Appendix

8.1. The Free Entry Condition. Equation (2.11) implies that for the firm with productivity $\varphi = \varphi_z$

$$\frac{\dot{V}_{zt}(\varphi_z)}{V_{zt}(\varphi_z)} = \frac{\dot{b}_{It}}{b_{It}} \qquad z = D, E.$$

Consider now the ratio between the value of a generic firm with productivity $\varphi \geq \varphi_z$ and the value of the firm with productivity equal to the threshold φ_z. According to (2.10) and (2.11) this is given by

$$\frac{V_{zt}(\varphi)}{V_{zt}(\varphi_z)} = \frac{\pi_{zt}(\varphi)}{\pi_{zt}(\varphi_z)} \frac{r - \frac{\dot{b}_{It}}{b_{It}}}{r - \frac{\dot{V}_{zt}(\varphi)}{V_{zt}(\varphi)}} \qquad z = D, E.$$

Exploiting the fact that the ratio between the profits of any two firms can be expressed as a function of their relative productivity (see equations (2.8) and (2.9)), this can be rewritten as

$$\frac{V_{zt}(\varphi)}{V_{zt}(\varphi_z)} = \left(\frac{\varphi_z}{\varphi}\right)^{1-\varepsilon} \frac{r - \frac{\dot{b}_{It}}{b_{It}}}{r - \frac{\dot{V}_{zt}(\varphi)}{V_{zt}(\varphi)}} \qquad z = D, E. \tag{8.39}$$

or

$$V_{zt}(\varphi) = V_{zt}(\varphi_z) \left(\frac{\varphi_z}{\varphi}\right)^{1-\varepsilon} \frac{r - \frac{\dot{b}_{It}}{b_{It}}}{r - \frac{\dot{V}_{zt}(\varphi)}{V_{zt}(\varphi)}} \; z = D, E.$$

By definition of balanced growth path, $\frac{\dot{b}_{It}}{b_{It}}$ and $\frac{\dot{V}_{zt}(\varphi)}{V_{zt}(\varphi)}$ must be constant in equilibrium. This implies that

$$\frac{\dot{V}_{zt}(\varphi)}{V_{zt}(\varphi)} = \frac{\dot{V}_{zt}(\varphi_z)}{V_{zt}(\varphi_z)} = \frac{\dot{b}_{It}}{b_{It}} \quad \forall \varphi > \varphi_z \quad z = D, E. \tag{8.40}$$

Now, free entry in variety introduction implies that the ex-ante expected benefit of developing a new variety equals the cost of variety innovation. Formally:

$$\int_{\varphi_{Dt}}^{\infty} \left(V_{Dt}(\varphi) - F_D b_{It}\right) g(\varphi) d\varphi + \int_{\varphi_{Et}}^{\infty} \left(V_{Et}(\varphi) - F_E b_{It}\right) g(\varphi) d\varphi = F_I b_{It}.$$

Using (2.10) and the steady state result in (8.40), the above becomes

$$\frac{\int_{\varphi_D}^{\infty} \pi_{Dt}(\varphi) g(\varphi) d\varphi + \int_{\varphi_E}^{\infty} \pi_{Et}(\varphi) g(\varphi) d\varphi}{r - \frac{\dot{b}_{It}}{b_{It}}} = b_{It}\left[F_I + F_D(1 - G(\varphi_D)) + F_E(1 - G(\varphi_E))\right]. \tag{8.41}$$

Using (2.8), (2.9) and the probability density function $g(\varphi) = G'(\varphi) = k\bar{a}^k \varphi^{-(1+k)}$, the integrals can be solved as

$$\begin{aligned}
&\int_{\varphi_D}^{\infty} \pi_{Dt}(\varphi) g(\varphi) d\varphi + \int_{\varphi_E}^{\infty} \pi_{Et}(\varphi) g(\varphi) d\varphi = \\
&= \varsigma \kappa_t^{1-\varepsilon} \left[\frac{E_t}{P_t^{1-\varepsilon}} \int_{\varphi_D}^{\infty} \varphi^{\varepsilon-1} g(\varphi) d\varphi + \frac{E_t^* \tau^{1-\varepsilon}}{P_t^{*1-\varepsilon}} \int_{\varphi_E}^{\infty} \varphi^{\varepsilon-1} g(\varphi) d\varphi \right] \\
&= \bar{a}^k k \varsigma \kappa_t^{1-\varepsilon} \left[\frac{E_t}{P_t^{1-\varepsilon}} \int_{\varphi_D}^{\infty} \varphi^{\varepsilon-1} \varphi^{-(k+1)} d\varphi + \frac{E_t^* \tau^{1-\varepsilon}}{P_t^{*1-\varepsilon}} \int_{\varphi_E}^{\infty} \varphi^{\varepsilon-1} \varphi^{-(k+1)} d\varphi \right] \\
&= \bar{a}^k k \varsigma \kappa_t^{1-\varepsilon} \left[\frac{E_t}{P_t^{1-\varepsilon}} \int_{\varphi_D}^{\infty} \varphi^{\varepsilon-2-k} d\varphi + \frac{E_t^* \tau^{1-\varepsilon}}{P_t^{*1-\varepsilon}} \int_{\varphi_E}^{\infty} \varphi^{\varepsilon-2-k} d\varphi \right] \\
&= \frac{\bar{a}^k k}{\varepsilon - 1 - k} \varsigma \kappa_t^{1-\varepsilon} \left[\frac{E_t}{P_t^{1-\varepsilon}} \varphi^{\varepsilon-1-k} \Big|_{\varphi_D}^{\infty} + \frac{E_t^* \tau^{1-\varepsilon}}{P_t^{*1-\varepsilon}} \varphi^{\varepsilon-1-k} \Big|_{\varphi_E}^{\infty} \right] \\
&= \frac{\bar{a}^k k}{k - \varepsilon + 1} \varsigma \kappa_t^{1-\varepsilon} \left[\frac{E_t}{P_t^{1-\varepsilon}} \varphi_D^{\varepsilon-1-k} + \frac{E_t^* \tau^{1-\varepsilon}}{P_t^{*1-\varepsilon}} \varphi_E^{\varepsilon-1-k} \right]
\end{aligned}$$

where the assumption $k > \varepsilon - 1$ guarantees that the expected discounted profits from entering are finite. Using (2.7), the *r.h.s.* of (3.20) can be rewritten as

$$b_{It} \left[F_I + F_D(1 - G(\varphi_D)) + F_E(1 - G(\varphi_E)) \right] = b_{It} \left[F_I + F_D \left(\frac{\bar{a}}{\varphi_D} \right)^k + F_E \left(\frac{\bar{a}}{\varphi_E} \right)^k \right].$$

Now, plugging these last expressions back into (3.20) and multiplying both sides by $\left(\frac{\varphi_D}{\bar{a}}\right)^k$, I obtain

$$\begin{aligned}
&\frac{k}{k - \varepsilon + 1} \frac{\varsigma \kappa_t^{1-\varepsilon}}{r - \frac{\dot{b}_{It}}{b_{It}}} \left[\frac{E_t}{P_t^{1-\varepsilon}} \left(\frac{1}{\varphi_D} \right)^{1-\varepsilon} + \frac{E_t^*}{P_t^{*1-\varepsilon}} \left(\frac{\tau}{\varphi_E} \right)^{1-\varepsilon} \left(\frac{\varphi_D}{\varphi_E} \right)^k \right] = \\
&= b_{It} \left[F_I \left(\frac{\varphi_D}{\bar{a}} \right)^k + F_D + F_E \left(\frac{\varphi_D}{\varphi_E} \right)^k \right].
\end{aligned}$$

8.2. The Productivity Thresholds. Using (8.39) and the steady state result in (8.40), I have

$$\frac{V_{zt}(\varphi)}{V_{zt}(\varphi_z)} = \left(\frac{\varphi_z}{\varphi} \right)^{1-\varepsilon} \qquad z = D, E$$

where $V_{zt}(\varphi)$ is the value of a generic firm with productivity $\varphi \geq \varphi_z$ from selling on market $z = D, E$ and $V_{zt}(\varphi_z)$ the same value for a firm with productivity equal to the threshold φ_z. From (2.11) the equation above becomes

$$V_{Dt}(\varphi) = \left(\frac{\varphi_D}{\varphi} \right)^{1-\varepsilon} F_D b_{It}$$

for the domestic market and

$$V_{Et}(\varphi)=\left(\frac{\varphi_E}{\varphi}\right)^{1-\varepsilon}F_E b_{It}$$

for the export market. Substituting these expressions into (2.15), I obtain:

$$F_I b_{It}=\int_{\varphi_D}^{\infty}\left(\left(\frac{\varphi_D}{\varphi}\right)^{1-\varepsilon}F_D b_{It}-F_D b_{It}\right)g(\varphi)d\varphi+\int_{\varphi_E}^{\infty}\left(\left(\frac{\varphi_E}{\varphi}\right)^{1-\varepsilon}F_E b_{It}-F_E b_{It}\right)g(\varphi)d\varphi.$$

Dividing both sides by b_{It}, I obtain

$$F_I=F_D\int_{\varphi_D}^{\infty}\left(\left(\frac{\varphi_D}{\varphi}\right)^{1-\varepsilon}-1\right)g(\varphi)d\varphi+F_E\int_{\varphi_E}^{\infty}\left(\left(\frac{\varphi_E}{\varphi}\right)^{1-\varepsilon}-1\right)g(\varphi)d\varphi$$

$$F_I=F_D\left[\varphi_D^{1-\varepsilon}\int_{\varphi_D}^{\infty}\varphi^{\varepsilon-1}k\bar{a}^k\varphi^{-k-1}d\varphi-(1-G(\varphi_D))\right]$$
$$+F_E\left[\varphi_E^{1-\varepsilon}\int_{\varphi_E}^{\infty}\varphi^{\varepsilon-1}k\bar{a}^k\varphi^{-k-1}d\varphi-(1-G(\varphi_E))\right]$$

$$F_I=F_D\left[\bar{a}^k k\varphi_D^{1-\varepsilon}\int_{\varphi_D}^{\infty}\varphi^{\varepsilon-2-k}d\varphi-(1-G(\varphi_D))\right]+F_E\left[\bar{a}^k k\varphi_E^{1-\varepsilon}\int_{\varphi_E}^{\infty}\varphi^{\varepsilon-2-k}d\varphi-(1-G(\varphi_E))\right]$$

$$F_I=F_D\left[\bar{a}^k\frac{k}{\varepsilon-1-k}\varphi_D^{1-\varepsilon}\ \varphi^{\varepsilon-1-k}\Big|_{\varphi_D}^{\infty}-\frac{\bar{a}^k}{\varphi_D^k}\right]+F_E\left[\bar{a}^k\frac{k}{\varepsilon-1-k}\varphi_E^{1-\varepsilon}\ \varphi^{\varepsilon-1-k}\Big|_{\varphi_E}^{\infty}-\frac{\bar{a}^k}{\varphi_E^k}\right]$$

$$F_I=F_D\left[\bar{a}^k\frac{k}{k-\varepsilon+1}\frac{1}{\varphi_D^k}-\frac{\bar{a}^k}{\varphi_D^k}\right]+F_E\left[\bar{a}^k\frac{k}{k-\varepsilon+1}\frac{1}{\varphi_E^k}-\frac{\bar{a}^k}{\varphi_E^k}\right]$$

$$F_I=F_D\frac{\bar{a}^k}{\varphi_D^k}\frac{\varepsilon-1}{k-\varepsilon+1}+F_E\frac{\bar{a}^k}{\varphi_E^k}\frac{\varepsilon-1}{k-\varepsilon+1}$$

Rearranging I obtain equation (2.19) in the text:

$$\frac{F_D}{\varphi_D^k}+\frac{F_E}{\varphi_E^k}=\frac{k-\varepsilon+1}{\varepsilon-1}\frac{F_I}{\bar{a}^k}$$

Multiplying both sides by φ_E^k and rearranging, I obtain

$$F_D\left(\frac{\varphi_E}{\varphi_D}\right)^k=\frac{k-\varepsilon+1}{\varepsilon-1}\frac{F_I}{\bar{a}^k}\varphi_E^k-F_E$$

while multiplying both sides by φ_D^k and rearranging, I obtain

$$F_E\left(\frac{\varphi_D}{\varphi_E}\right)^k=\frac{k-\varepsilon+1}{\varepsilon-1}\frac{F_I}{\bar{a}^k}\varphi_D^k-F_D.$$

Thus an increase in the export premium $\frac{\varphi_E}{\varphi_D}$ implies an increase in the export productivity cut off φ_E and a decrease in the domestic productivity cut off φ_D. Furthermore, the above conditions, for a given value for $\frac{\varphi_E}{\varphi_D}$, can be solved for unique values of φ_D and φ_E.

8.3. The expected knowledge requirement $\bar{\mathbf{F}}$. Multiplying both sides of (2.19) by φ_D^k I get

$$\begin{aligned}F_D + F_E\left(\frac{\varphi_D}{\varphi_E}\right)^k &= \frac{k-\varepsilon+1}{\varepsilon-1}F_I\left(\frac{\varphi_D}{\bar{a}}\right)^k\\ F_E\left(\frac{\varphi_D}{\varphi_E}\right)^k &= \frac{k-\varepsilon+1}{\varepsilon-1}F_I\left(\frac{\varphi_D}{\bar{a}}\right)^k - F_D.\end{aligned}$$

Substituting this expression into (2.17), I obtain (2.20) in the text

$$\begin{aligned}\left[F_I\left(\frac{\varphi_D}{\bar{a}}\right)^k + F_D + F_E\left(\frac{\varphi_D}{\varphi_E}\right)^k\right] &= \left[F_I\left(\frac{\varphi_D}{\bar{a}}\right)^k + F_D + \frac{k-\varepsilon+1}{\varepsilon-1}F_I\left(\frac{\varphi_D}{\bar{a}}\right)^k - F_D\right]\\ &= \left[F_I\left(\frac{\varphi_D}{\bar{a}}\right)^k + \frac{k-\varepsilon+1}{\varepsilon-1}F_I\left(\frac{\varphi_D}{\bar{a}}\right)^k\right]\\ &= F_I\left(\frac{\varphi_D}{\bar{a}}\right)^k\frac{\varepsilon-1+k-\varepsilon+1}{\varepsilon-1}\\ &= F_I\left(\frac{\varphi_D}{\bar{a}}\right)^k\frac{k}{\varepsilon-1} = \bar{F}.\end{aligned}$$

8.4. The Price Index. The price index satisfies:

$$\begin{aligned}P_t^{1-\varepsilon} &= \int_0^{\hat{m}_t} p_t(\varpi)^{1-\varepsilon}d\varpi\\ &= \int_{\varphi_D}^{\infty} p_{Dt}(\varphi)^{1-\varepsilon}m_t\frac{g(\varphi)}{1-G(\varphi_D)}d\varphi + \int_{\varphi_E^*}^{\infty} p_{Et}^*(\varphi)^{1-\varepsilon}m_t^*\frac{g(\varphi)}{1-G(\varphi_D^*)}d\varphi\end{aligned}$$

where $\frac{g(\varphi)}{1-G(\varphi_D)}$ is the probability density function conditional on entry. Using (2.7) and substituting $g(\varphi) = G'(\varphi) = k\bar{a}^k\varphi^{-(1+k)}$, $p_{Dt} = \frac{\kappa_t}{\alpha\varphi}$ and $p_{Et}^* = \frac{\kappa_t^*\tau}{\alpha\varphi}$ the above can be solved as

$$
\begin{aligned}
P_t^{1-\varepsilon} &= \alpha^{\varepsilon-1}\left[m_t\kappa_t^{1-\varepsilon}\int_{\varphi_D}^{\infty}\varphi^{\varepsilon-1}k\bar{a}^k\frac{\varphi^{-(1+k)}}{\bar{a}^k/\varphi_D^k}d\varphi + m_t^*\left(\tau\kappa_t^*\right)^{1-\varepsilon}\int_{\varphi_E^*}^{\infty}\varphi^{\varepsilon-1}k\bar{a}^k\frac{\varphi^{-(1+k)}}{\bar{a}^k/\varphi_D^{*k}}d\varphi\right]\\
&= k\alpha^{\varepsilon-1}\left[m_t\kappa_t^{1-\varepsilon}\varphi_D^k\int_{\varphi_D}^{\infty}\varphi^{\varepsilon-2+k}d\varphi + m_t^*\left(\tau\kappa_t^*\right)^{1-\varepsilon}\varphi_D^{*k}\int_{\varphi_E^*}^{\infty}\varphi^{\varepsilon-2+k}d\varphi\right]\\
&= \frac{k\alpha^{\varepsilon-1}}{\varepsilon-k-1}\left[m_t\kappa_t^{1-\varepsilon}\varphi_D^k\left.\varphi^{\varepsilon-1-k}\right|_{\varphi_D}^{\infty} + m_t^*\left(\tau\kappa_t^*\right)^{1-\varepsilon}\varphi_D^{*k}\left.\varphi^{\varepsilon-1-k}\right|_{\varphi_E^*}^{\infty}\right]\\
&= \frac{k\alpha^{\varepsilon-1}}{k-\varepsilon+1}\left[m_t\kappa_t^{1-\varepsilon}\varphi_D^k\varphi_D^{\varepsilon-1-k} + m_t^*\left(\tau\kappa_t^*\right)^{1-\varepsilon}\varphi_D^{*k}\varphi_E^{*\varepsilon-1-k}\right]\\
&= \frac{k\alpha^{\varepsilon-1}}{k-\varepsilon+1}\left[m_t\left(\frac{\kappa_t}{\varphi_D}\right)^{1-\varepsilon} + m_t^*\left(\frac{\tau\kappa_t^*}{\varphi_E^*}\right)^{1-\varepsilon}\left(\frac{\varphi_D^*}{\varphi_E^*}\right)^k\right].
\end{aligned}
$$

8.5. The Trade Balance Condition. The trade balance condition requires the total value of exports of a country to equal the total value of its imports. For the North (South) the total value of exports (imports) is given by the total value of the goods shipped by Northern firms to the Southern market. Formally:

$$
\begin{aligned}
\int_{\varphi_E}^{\infty}p_{Et}(\varphi)\frac{E_t^*p_{Et}(\varphi)^{-\varepsilon}}{P_t^{*1-\varepsilon}}m_t\frac{g(\varphi)}{1-G(\varphi_D)}d\varphi &= \frac{m_tE_t^*}{P_t^{*1-\varepsilon}}\int_{\varphi_E}^{\infty}p_{Et}^{1-\varepsilon}(\varphi)\frac{g(\varphi)}{1-G(\varphi_D)}d\varphi\\
&= \frac{m_tE_t^*}{P_t^{*1-\varepsilon}}\int_{\varphi_E}^{\infty}\left(\frac{\tau\kappa_t}{\varphi\alpha}\right)^{1-\varepsilon}k\bar{a}^k\frac{\varphi^{-(1+k)}}{\bar{a}^k/\varphi_D^k}d\varphi\\
&= \left(\frac{\tau\kappa_t}{\alpha}\right)^{1-\varepsilon}\frac{m_tE_t^*}{P_t^{*1-\varepsilon}}k\varphi_D^k\int_{\varphi_E}^{\infty}\varphi^{\varepsilon-2-k}d\varphi\\
&= \left(\frac{\tau\kappa_t}{\alpha}\right)^{1-\varepsilon}\frac{m_tE_t^*}{P_t^{*1-\varepsilon}}\frac{k\varphi_D^k}{\varepsilon-k-1}\left.\varphi^{\varepsilon-1-k}\right|_{\varphi_E}^{\infty}\\
&= \left(\frac{\tau\kappa_t}{\alpha}\right)^{1-\varepsilon}\frac{m_tE_t^*}{P_t^{*1-\varepsilon}}\frac{k\varphi_D^k}{k-\varepsilon+1}\varphi_E^{\varepsilon-1-k}\\
&= \left(\frac{\tau\kappa_t}{\alpha}\right)^{1-\varepsilon}\frac{m_tE_t^*}{P_t^{*1-\varepsilon}}\frac{k\varphi_E^{\varepsilon-1}}{k-\varepsilon+1}\left(\frac{\varphi_D}{\varphi_E}\right)^k. \qquad (8.42)
\end{aligned}
$$

For the North (South) the total value of imports (exports) is given by the total value of the goods shipped by Southern firms to the Northern market:

$$\begin{aligned}
\int_{\varphi_E^*}^{\infty} p_{Et}^*(\varphi)\frac{E_t p_{Et}^*(\varphi)^{-\varepsilon}}{P_t^{1-\varepsilon}} m_t^* \frac{g(\varphi)}{1-G(\varphi_D^*)} d\varphi &= \frac{m_t^* E_t}{P_t^{1-\varepsilon}} \int_{\varphi_E^*}^{\infty} p_{Et}^{*1-\varepsilon}(\varphi)\frac{g(\varphi)}{1-G(\varphi_D^*)} d\varphi \\
&= \frac{m_t^* E_t}{P_t^{1-\varepsilon}} \int_{\varphi_E^*}^{\infty} \left(\frac{\tau\kappa_t^*}{\varphi\alpha}\right)^{1-\varepsilon} k\bar{a}^k \frac{\varphi^{-(1+k)}}{\bar{a}^k/\varphi_D^{*k}} d\varphi \\
&= \left(\frac{\tau\kappa_t^*}{\alpha}\right)^{1-\varepsilon} \frac{m_t^* E_t}{P_t^{1-\varepsilon}} k\varphi_D^{*k} \int_{\varphi_E^*}^{\infty} \varphi^{\varepsilon-2-k} d\varphi \\
&= \left(\frac{\tau\kappa_t^*}{\alpha}\right)^{1-\varepsilon} \frac{m_t^* E_t}{P_t^{1-\varepsilon}} \frac{k\varphi_D^{*k}}{\varepsilon-k-1} \left.\varphi^{\varepsilon-1-k}\right|_{\varphi_E^*}^{\infty} \\
&= \left(\frac{\tau\kappa_t^*}{\alpha}\right)^{1-\varepsilon} \frac{m_t^* E_t}{P_t^{1-\varepsilon}} \frac{k\varphi_D^{*k}}{k-\varepsilon+1} \varphi_E^{*\varepsilon-1-k} \\
&= \left(\frac{\tau\kappa_t^*}{\alpha}\right)^{1-\varepsilon} \frac{m_t^* E_t}{P_t^{1-\varepsilon}} \frac{k\varphi_E^{*\varepsilon-1}}{k-\varepsilon+1} \left(\frac{\varphi_D^*}{\varphi_E^*}\right)^{k}. \quad (8.43)
\end{aligned}$$

Equating (8.43) and (8.42) I get

$$\left(\frac{\tau\kappa_t^*}{\alpha}\right)^{1-\varepsilon} \frac{m_t^* E_t}{P_t^{1-\varepsilon}} \frac{k}{k-\varepsilon+1} \left(\frac{\varphi_D^*}{\varphi_E^*}\right)^{k} \varphi_E^{*\varepsilon-1} = \left(\frac{\tau\kappa_t}{\alpha}\right)^{1-\varepsilon} \frac{m_t E_t^*}{P_t^{*1-\varepsilon}} \frac{k}{k-\varepsilon+1} \left(\frac{\varphi_D}{\varphi_E}\right)^{k} \varphi_E^{\varepsilon-1}$$

$$m_t^* \frac{E_t}{P_t^{1-\varepsilon}} \left(\frac{\kappa_t^*}{\varphi_E^*}\right)^{1-\varepsilon} \left(\frac{\varphi_D^*}{\varphi_E^*}\right)^{k} = m_t \frac{E_t^*}{P_t^{*1-\varepsilon}} \left(\frac{\kappa_t}{\varphi_E}\right)^{1-\varepsilon} \left(\frac{\varphi_D}{\varphi_E}\right)^{k}$$

where the term $\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon} \frac{k}{k-\varepsilon+1}$ has canceled out.

8.6. The Aggregate Expenditure. From (2.23) I can solve for

$$\frac{E_t^*}{P_t^{*1-\varepsilon}}\frac{1}{\varphi_E^{1-\varepsilon}}\left(\frac{\varphi_D}{\varphi_E}\right)^k = \frac{m_t^*}{m_t}\frac{E_t}{P_t^{-\varepsilon}}\left(\frac{\kappa_t^*}{\kappa_t\varphi_E^*}\right)^{1-\varepsilon}\left(\frac{\varphi_D^*}{\varphi_E^*}\right)^k.$$

Plugging this expression into definition (2.18), I get

$$\begin{aligned}
\Delta_t &= \frac{k}{k-\varepsilon+1}\left[\frac{E_t}{P_t^{1-\varepsilon}}\frac{1}{\varphi_D^{1-\varepsilon}} + \frac{m_t^*}{m_t}\frac{E_t}{P_t^{-\varepsilon}}\left(\frac{\tau\kappa_t^*}{\kappa_t\varphi_E^*}\right)^{1-\varepsilon}\left(\frac{\varphi_D^*}{\varphi_E^*}\right)^k\right] \\
&= \frac{k}{k-\varepsilon+1}\kappa_t^{\varepsilon-1}\left[\frac{E_t}{P_t^{1-\varepsilon}}\left(\frac{\kappa_t}{\varphi_D}\right)^{1-\varepsilon} + \frac{m_t^*}{m_t}\frac{E_t}{P_t^{-\varepsilon}}\left(\frac{\tau\kappa_t^*}{\varphi_E^*}\right)^{1-\varepsilon}\left(\frac{\varphi_D^*}{\varphi_E^*}\right)^k\right] \\
&= \frac{k}{k-\varepsilon+1}\frac{\kappa_t^{\varepsilon-1}}{m_t}\frac{E_t}{P_t^{1-\varepsilon}}\left[m_t\left(\frac{\kappa_t}{\varphi_D}\right)^{1-\varepsilon} + m_t^*\left(\frac{\tau\kappa_t^*}{\varphi_E^*}\right)^{1-\varepsilon}\left(\frac{\varphi_D^*}{\varphi_E^*}\right)^k\right] \\
&= E_t\frac{\kappa_t^{\varepsilon-1}}{m_t}\frac{1}{P_t^{1-\varepsilon}}\frac{k}{k-\varepsilon+1}\left[m_t\left(\frac{\kappa_t}{\varphi_D}\right)^{1-\varepsilon} + m_t^*\left(\frac{\tau\kappa_t^*}{\varphi_E^*}\right)^{1-\varepsilon}\left(\frac{\varphi_D^*}{\varphi_E^*}\right)^k\right] \\
&= E_t\frac{\kappa_t^{\varepsilon-1}}{m_t}\alpha^{1-\varepsilon}
\end{aligned}$$

where the last step follows from (2.22). Rearranging the terms I get

$$E_t = m_t\left(\frac{\kappa_t}{\alpha}\right)^{1-\varepsilon}\Delta_t.$$

8.7. Aggregate Labor Demands. The Cobb-Douglas unit cost function

$$C_t(\varphi) = \frac{w_{Ht}^{\beta}w_{Lt}^{1-\beta}}{\varphi} = \frac{\kappa_t}{\varphi}$$

is the result of the optimization problem

$$\min_{H_t(\varphi),L_t(\varphi)} \quad w_{Ht}H_t(\varphi) + w_{Lt}L_t(\varphi)$$
$$s.t.\ y_t(\varphi) \geq 1$$

where $H_t(\varphi)$ and $L_t(\varphi)$ are the amount of the skilled and unskilled labor employed by the generic firm with productivity φ and $y_t(\varphi)$ is the production function given by the Cobb-Douglas

$$y_t(\varphi) = B\varphi H_t^{\beta}(\varphi)L_t^{1-\beta}(\varphi)$$

where $B \equiv (1-\beta)^{\beta-1}\beta^{-\beta}$. At the optimum the constraint is binding so I can use it to solve for $H_t(\varphi)$ as:

$$H_t(\varphi) = \left[\frac{L_t^{\beta-1}(\varphi)}{B\varphi}\right]^{\frac{1}{\beta}}. \tag{8.44}$$

Next, I plug this into the objective function in order to express it as a function of $L_t(\varphi)$ only:

$$w_{Ht}\left[\frac{L_t^{\beta-1}(\varphi)}{B\varphi}\right]^{\frac{1}{\beta}} + w_{Lt}L_t(\varphi).$$

Taking the derivative with respect to $L_t(\varphi)$ and setting it equal to zero, I obtain

$$\frac{\beta-1}{\beta}w_{Ht}\left[\frac{1}{B\varphi}\right]^{\frac{1}{\beta}} L_{\varphi t}^{-\frac{1}{\beta}} + w_{Lt} = 0$$

where I have switched to the notation $L_{\varphi t} \equiv L_t(\varphi)$ in order to make the equations more readable. From the first oder condition, I can solve for the optimal value of $L_{\varphi t}$

$$\begin{aligned}
\frac{1-\beta}{\beta}w_{Ht}\left(\frac{1}{B\varphi}\right)^{\frac{1}{\beta}} L_{\varphi t}^{-\frac{1}{\beta}} &= w_{Lt} \\
\frac{1-\beta}{\beta}\frac{w_{Ht}}{w_{Lt}}\left(\frac{1}{B\varphi}\right)^{\frac{1}{\beta}} &= L_{\varphi t}^{\frac{1}{\beta}} \\
\left(\frac{1-\beta}{\beta}\right)^{\beta}\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}\frac{1}{B\varphi} &= L_{\varphi t}.
\end{aligned}$$

From the definition of B and back to the original notation, I get the unit unskilled labor requirement

$$L_t(\varphi) = \frac{1-\beta}{\varphi}\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}. \tag{8.45}$$

Substituting this last expression into (8.44), I obtain the optimal demand of skilled labor

$$
\begin{aligned}
H_t(\varphi) &= \left[\frac{\left(\frac{1-\beta}{\varphi}\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}\right)^{\beta-1}}{B\varphi}\right]^{\frac{1}{\beta}} \\
&= \left[\frac{\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta(\beta-1)}(1-\beta)^{\beta-1}\varphi^{1-\beta}}{B\varphi}\right]^{\frac{1}{\beta}} \\
&= \left[\frac{\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta(\beta-1)}\beta^{\beta}}{\varphi^{\beta}}\right]^{\frac{1}{\beta}} \\
&= \frac{\beta}{\varphi}\left(\frac{w_{Lt}}{w_{Ht}}\right)^{1-\beta}. \qquad (8.46)
\end{aligned}
$$

A firm's demand of unskilled labor is equal to the unit unskilled labor requirement (8.45) times the quantity produced. The quantity produced by each firm is given by the domestic and foreign aggregate demand for its variety, given by $\frac{E_t p_{Dt}^{-\varepsilon}(\varphi)}{P_t^{1-\varepsilon}}$ and $\frac{E_t^* p_{Et}^{-\varepsilon}(\varphi)}{P_t^{*1-\varepsilon}}$ respectively. Summing over all firms in the economy I obtain the aggregate demand of unskilled labor:

$$
L_{Pt} = m_t\left[\int_{\varphi_D}^{\infty} L_t(\varphi)\frac{E_t p_{Dt}^{-\varepsilon}(\varphi)}{P_t^{1-\varepsilon}}\frac{g(\varphi)}{1-G(\varphi_D)}d\varphi + \int_{\varphi_E}^{\infty} L_t(\varphi)\frac{E_t^*\tau p_{Et}^{-\varepsilon}(\varphi)}{P_t^{*1-\varepsilon}}\frac{g(\varphi)}{1-G(\varphi_D)}d\varphi\right]
$$

where $\frac{g(\varphi)}{1-G(\varphi_D)}$ is the probability density function conditional on entry. The foreign demand is multiplied by the iceberg trade cost τ. Plugging in the expressions for $L_t(\varphi)$, $p_{Dt}(\varphi)$ and $p_{Et}(\varphi)$ yields

$$
L_{Pt} = m_t(1-\beta)\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}\left[\int_{\varphi_D}^{\infty}\frac{1}{\varphi}\frac{E_t\left(\frac{\kappa_t}{\alpha\varphi}\right)^{-\varepsilon}}{P_t^{1-\varepsilon}}\frac{g(\varphi)}{1-G(\varphi_D)}d\varphi + \int_{\varphi_E}^{\infty}\frac{1}{\varphi}\frac{E_t^*\tau\left(\frac{\tau\kappa_t}{\alpha\varphi}\right)^{-\varepsilon}}{P_t^{*1-\varepsilon}}\frac{g(\varphi)}{1-G(\varphi_D)}d\varphi\right].
$$

Given (2.7) and $g(\varphi) = G'(\varphi) = k\bar{a}^k\varphi^{-(1+k)}$, I can solve the integrals:

$$
\begin{aligned}
L_{Pt} &= m_t (1-\beta)\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta} \\
&\left[\frac{E_t\left(\frac{\kappa_t}{\alpha}\right)^{-\varepsilon}}{P_t^{1-\varepsilon}}\int_{\varphi_D}^{\infty}\varphi^{\varepsilon-1}k\bar{a}^k\frac{\varphi^{-(1+k)}}{\bar{a}^k/\varphi_D^k}d\varphi+\frac{\tau E_t^*\left(\frac{\tau\kappa_t}{\alpha}\right)^{-\varepsilon}}{P_t^{*1-\varepsilon}}\int_{\varphi_E}^{\infty}\varphi^{\varepsilon-1}k\bar{a}^k\frac{\varphi^{-(1+k)}}{\bar{a}^k/\varphi_D^k}d\varphi\right] \\
&= m_t (1-\beta)\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}\left(\frac{\kappa_t}{\alpha}\right)^{-\varepsilon}k\varphi_D^k\left[\frac{E_t}{P_t^{1-\varepsilon}}\int_{\varphi_D}^{\infty}\varphi^{\varepsilon-2-k}d\varphi+\frac{\tau^{1-\varepsilon}E_t^*}{P_t^{*1-\varepsilon}}\int_{\varphi_E}^{\infty}\varphi^{\varepsilon-2-k}d\varphi\right] \\
&= m_t (1-\beta)\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}\left(\frac{\kappa_t}{\alpha}\right)^{-\varepsilon}\frac{k\varphi_D^k}{\varepsilon-1-k}\left[\frac{E_t}{P_t^{1-\varepsilon}}\,\varphi^{\varepsilon-1-k}\Big|_{\varphi_D}^{\infty}+\frac{\tau^{1-\varepsilon}E_t^*}{P_t^{*1-\varepsilon}}\,\varphi^{\varepsilon-1-k}\Big|_{\varphi_E}^{\infty}\right] \\
&= m_t (1-\beta)\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}\left(\frac{\kappa_t}{\alpha}\right)^{-\varepsilon}\frac{k\varphi_D^k}{k-\varepsilon+1}\left[\frac{E_t}{P_t^{1-\varepsilon}}\varphi_D^{\varepsilon-1-k}+\frac{\tau^{1-\varepsilon}E_t^*}{P_t^{*1-\varepsilon}}\varphi_E^{\varepsilon-1-k}\right] \\
&= m_t (1-\beta)\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}\left(\frac{\kappa_t}{\alpha}\right)^{-\varepsilon}\frac{k}{k-\varepsilon+1}\left[\frac{E_t}{P_t^{1-\varepsilon}}\varphi_D^{\varepsilon-1}+\frac{\tau^{1-\varepsilon}E_t^*}{P_t^{*1-\varepsilon}}\varphi_E^{\varepsilon-1}\left(\frac{\varphi_D}{\varphi_E}\right)^{k}\right] \\
&= m_t (1-\beta)\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}\left(\frac{\kappa_t}{\alpha}\right)^{-\varepsilon}\frac{k}{k-\varepsilon+1}\left[\frac{E_t}{P_t^{1-\varepsilon}}\left(\frac{1}{\varphi_D}\right)^{1-\varepsilon}+\frac{E_t^*}{P_t^{*1-\varepsilon}}\left(\frac{\tau}{\varphi_E}\right)^{1-\varepsilon}\left(\frac{\varphi_D}{\varphi_E}\right)^{k}\right].
\end{aligned}
$$

From (2.18) and (2.24) the expression above further simplifies as

$$
\begin{aligned}
L_{Pt} &= m_t (1-\beta)\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}\left(\frac{\kappa_t}{\alpha}\right)^{-\varepsilon}\Delta_t \\
&= (1-\beta)\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}\alpha\kappa_t^{-1}E_t \\
&= (1-\beta)\,\alpha\left(\frac{w_{Ht}}{w_{Lt}}\right)^{\beta}w_{Ht}^{-\beta}w_{Lt}^{\beta-1}E_t \\
&= E_t\frac{(1-\beta)\alpha}{w_{Lt}}.
\end{aligned}
$$

Following the same steps, it is easy to show that the aggregate demand of skilled labor in production is given by:

$$
H_{Pt} = E_t\frac{\beta\alpha}{w_{Ht}}.
$$

8.8. The Growth Rates g and g_{GDP}. From (2.21) it follows

$$
g \equiv \frac{\dot{m}_t}{m_t} = \frac{H_{It}\left(m_t+\lambda m_t^*\right)^{\phi}}{m_t\bar{F}}.
$$

Taking logs and differentiating, I get

$$\begin{aligned}\frac{\dot{g}}{g} &= \frac{\dot{H}_{It}}{H_{It}} + \phi\frac{\dot{m}_t + \lambda\dot{m}_t^*}{(m_t + \lambda m_t^*)} - \frac{\dot{m}_t}{m_t} - \frac{\dot{\bar{F}}}{\bar{F}} \\ &= n + \phi g - g - 0 \\ &= n - g(1-\phi).\end{aligned}$$

Since in steady state $\frac{\dot{g}}{g} = 0$, the above equation implies

$$g = \frac{n}{1-\phi}.$$

The real GDP per capita is defined as the real expenditure per capita E_t/P_tN_t, where $N_t = L_t + H_t$ is the total population. Taking the log and then the derivative with respect to time, I obtain

$$\begin{aligned}g_{GDP} &= \frac{\dot{E}_t}{E_t} - \frac{\dot{P}_t}{P_t} - \frac{\dot{N}_t}{N_t} \\ &= n - \frac{\dot{P}_t}{P_t} - n\end{aligned}$$

According to (2.22), for the growth rate of $P_t^{1-\varepsilon}$ to be constant it must equal the growth rate of variety introduction g. This implies that

$$\frac{\dot{P}_t}{P_t} = \frac{g}{1-\varepsilon} = -\frac{g}{\varepsilon - 1}.$$

Using this result, I derive

$$g_{GDP} = \frac{g}{\varepsilon - 1}.$$

8.9. The Skill Premium. Multiplying both sides of (2.26) by $\frac{w_H}{m_t}$ and rearranging, I get

$$\begin{aligned}\frac{w_H \frac{\dot{m}_t}{m_t}\bar{F}}{(m_t + \lambda m_t^*)^\phi} &= \frac{w_H H_{It}}{m_t} \\ \frac{w_H g \bar{F}}{(m_t + \lambda m_t^*)^\phi} &= \frac{w_H H_{It}}{m_t} \\ \bar{F} b_{It} &= \frac{w_H H_{It}}{m_t g}.\end{aligned} \tag{8.47}$$

I can now use this expression into (2.25) once I have substituted for the equilibrium value of the discounting factor $r - \frac{\dot{b}_{It}}{b_{It}} = \rho + \phi g$ and I get

$$\begin{aligned}
\frac{\frac{E_t}{\varepsilon m_t}}{\rho + \phi g} &= \frac{w_H H_{It}}{m_t g} \\
\frac{E_t}{\varepsilon} &= \frac{w_H H_{It} \left(\rho + \phi g\right)}{g}.
\end{aligned} \tag{8.48}$$

From (2.29) I can solve for H_{It} as

$$\begin{aligned}
H_{It} &= H_t - H_{Pt} \\
&= H_t - \frac{\alpha \beta E_t}{w_H}.
\end{aligned} \tag{8.49}$$

I can now substitute this expression into (8.54) and I obtain (3.33) in the main text

$$\begin{aligned}
\frac{E_t}{\varepsilon} &= \frac{w_H \left(\rho + \phi g\right)}{g} \left(H_t - \beta E_t \frac{\alpha}{w_H}\right) \\
\frac{E_t}{\varepsilon} &= \frac{w_H H_t \left(\rho + \phi g\right)}{g} - \frac{\beta \alpha E_t \left(\rho + \phi g\right)}{g} \\
E_t \left(1 + \frac{\varepsilon \beta \alpha \left(\rho + \phi g\right)}{g}\right) &= \frac{w_H H_t \varepsilon \left(\rho + \phi g\right)}{g} \\
E_t \left(\frac{g + \varepsilon \beta \alpha \left(\rho + \phi g\right)}{g}\right) &= \frac{w_H H_t \varepsilon \left(\rho + \phi g\right)}{g} \\
E_t &= \frac{w_H H_t \varepsilon \left(\rho + \phi g\right)}{g + \varepsilon \alpha \beta \left(\rho + \phi g\right)}.
\end{aligned}$$

Plugging this into (8.49) and dividing both sides by H_t, I can solve for the fraction of skilled workers employed in the R&D sector as

$$\begin{aligned}
h_I &= 1 - \frac{\varepsilon \alpha \beta \left(\rho + \phi g\right)}{g + \varepsilon \alpha \beta \left(\rho + \phi g\right)} \\
&= \frac{g + \varepsilon \alpha \beta \left(\rho + \phi g\right) - \varepsilon \alpha \beta \left(\rho + \phi g\right)}{g + \varepsilon \alpha \beta \left(\rho + \phi g\right)} \\
&= \frac{g}{g + \varepsilon \alpha \beta \left(\rho + \phi g\right)} \\
&= \frac{g}{g + \beta(\varepsilon - 1) \left(\rho + \phi g\right)}
\end{aligned}$$

where the last step follows from $\alpha = \frac{\varepsilon-1}{\varepsilon}$. Dividing both sides by g and using $g = \frac{n}{1-\phi}$, the above expression becomes

$$\begin{aligned} h_I &= \frac{1}{1+\beta(\varepsilon-1)\left(\frac{\rho}{g}+\phi\right)} \\ &= \frac{1}{1+\beta(\varepsilon-1)\left(\frac{\rho(1-\phi)}{n}+\phi\right)} \\ &= \frac{n}{n+\beta(\varepsilon-1)(\rho-\rho\phi+n\phi)} \\ &= \frac{n}{n+\beta(\varepsilon-1)[\rho-\phi(\rho-n)]}. \end{aligned}$$

Since $\rho > n$, it follows that h_I is an increasing function of the spillovers ϕ.

Combining (2.30) and (3.33), I can solve for the skill premium:

$$\begin{aligned} L_t &= (1-\beta)E_t\frac{\alpha}{w_L} \\ L_t &= \frac{(1-\beta)w_H H_t\varepsilon\left(\rho+\phi g\right)}{g+\varepsilon\alpha\beta\left(\rho+\phi g\right)}\frac{\alpha}{w_L} \\ L_t &= H_t\frac{(1-\beta)\alpha\varepsilon\left(\rho+\phi g\right)}{g+\varepsilon\alpha\beta\left(\rho+\phi g\right)}\frac{w_H}{w_L} \\ \frac{w_H}{w_L} &= \frac{L_t}{H_t}\frac{g+\varepsilon\alpha\beta\left(\rho+\phi g\right)}{(1-\beta)\alpha\varepsilon\left(\rho+\phi g\right)} \\ \frac{w_H}{w_L} &= \frac{L_t}{H_t}\frac{g+(\varepsilon-1)\beta\left(\rho+\phi g\right)}{(1-\beta)(\varepsilon-1)\left(\rho+\phi g\right)}. \end{aligned}$$

8.10. The Skill Premium when R&D and production share the same factor intensity. In this section I derive the equilibrium skill premium of a model that is in all respects identical to one described in the paper, except that production of knowledge and production goods are assumed to have the same skill intensity β. This means that the expression for the unit cost of knowledge production (2.6) is replaced by:

$$b_{It} = \frac{w_H^{\beta}w_L^{1-\beta}}{(m_t+\lambda m_t^*)^{\phi}} = \frac{\kappa}{\gamma}. \tag{8.50}$$

where $\gamma \equiv (m_t+\lambda m_t^*)^{\phi}$ to save on notation. This cost function is the result of the following optimization problem:

$$\min_{H_{\varphi It}, L_{\varphi It}} w_H H_{\varphi It} + w_L L_{\varphi It}$$
$$s.t.\ x_t \geq 1$$

where $H_{\varphi It}$ and $L_{\varphi It}$ are the amount of the skilled and unskilled labor employed by a generic firm in knowledge production and x_t is the knowledge production function given by the Cobb-Douglas

$$x_t = B\gamma H_{\varphi It}^{\beta} L_{\varphi It}^{1-\beta}$$

where $B \equiv (1-\beta)^{\beta-1}\beta^{-\beta}$. The problem is thus identical to the one that firms face in good production and that I have solved for in section 8.7 above. Following exactly the same steps, I can solve for the optimal unit requirements of skilled and unskilled labor for knowledge production. They are given by:

$$L_{\varphi It} = \frac{1-\beta}{\gamma}\left(\frac{w_H}{w_L}\right)^{\beta}$$
$$H_{\varphi It} = \frac{\beta}{\gamma}\left(\frac{w_L}{w_H}\right)^{1-\beta}.$$

These quantities apply to all firms engaged in knowledge creation, since firm heterogeneity concerns only good production. As a consequence, to derive the aggregate amount of skilled and unskilled workers employed in R&D in the economy, I just need to multiply the above unit requirements by the aggregate quantity of knowledge produced. This is given by the number of entrants $\dot{m}_t$ multiplied by the average knowledge requirement for product introduction $\bar{F}$. I obtain:

$$L_{It} = L_{\varphi It}\dot{m}_t\bar{F} = \frac{1-\beta}{\gamma}\left(\frac{w_H}{w_L}\right)^{\beta}\dot{m}_t\bar{F}$$
$$H_{It} = H_{\varphi It}\dot{m}_t\bar{F} = \frac{\beta}{\gamma}\left(\frac{w_L}{w_H}\right)^{1-\beta}\dot{m}_t\bar{F}.$$

Using the unchanged demands of labor from the production sector (2.28) and (2.27), the labor market clearing conditions (2.30) and (2.29) become

$$L_t = L_{Pt} + L_{It} = (1-\beta)E_t\frac{\alpha}{w_L} + \frac{1-\beta}{\gamma}\left(\frac{w_H}{w_L}\right)^{\beta}\dot{m}_t\bar{F}. \tag{8.51}$$

$$H_t = H_{Pt} + H_{It} = \beta E_t\frac{\alpha}{w_H} + \frac{\beta}{\gamma}\left(\frac{w_L}{w_H}\right)^{1-\beta}\dot{m}_t\bar{F}. \tag{8.52}$$

Given (8.50), the steady state free entry condition (2.25) is replaced by

$$\frac{\frac{E_t}{\varepsilon m_t}}{\rho + \phi g} = \frac{\bar{F} w_H^{\beta} w_L^{1-\beta}}{\gamma}. \tag{8.53}$$

I can solve for the left hand side from (8.52) as

$$\frac{\bar{F} w_H^{\beta} w_L^{1-\beta}}{\gamma} = \frac{w_H H_t - \beta E_t \alpha}{\dot{m}_t \beta}.$$

Plugging this result into (3.20) and using $g = \frac{\dot{m}_t}{m_t}$, I obtain

$$\begin{aligned} \frac{\frac{E_t}{\varepsilon m_t}}{\rho + \phi g} &= \frac{w_H H_t - \beta E_t \alpha}{\dot{m}_t \beta} \\ \frac{E_t g \beta}{\varepsilon(\rho + \phi g)} &= w_H H_t - \beta E_t \alpha \end{aligned}$$

Using $\alpha = \frac{\varepsilon - 1}{\varepsilon}$ and isolating $\frac{E_t \beta}{\varepsilon}$, I obtain

$$\frac{E_t \beta}{\varepsilon} \left[\frac{g}{(\rho + \phi g)} + (\varepsilon - 1) \right] = w_H H_t$$

which I can solve for E_t:

$$E_t = H_t \frac{\varepsilon w_H}{\beta} \frac{\rho + \phi g}{g + (\rho + \phi g)(\varepsilon - 1)}. \tag{8.54}$$

Now, using (3.20) I can solve for:

$$\begin{aligned} L_{It} &= \frac{1 - \beta}{\gamma} \left(\frac{w_H}{w_L} \right)^{\beta} \dot{m}_t \bar{F} \\ &= \frac{\bar{F} w_H^{\beta} w_L^{1-\beta}}{\gamma} (1 - \beta) \frac{\dot{m}_t}{w_L} \\ &= \frac{\frac{E_t}{\varepsilon m_t}}{\rho + \phi g} (1 - \beta) \frac{\dot{m}_t}{w_L} \\ &= \frac{(1 - \beta) E_t g}{\varepsilon w_L (\rho + \phi g)}. \end{aligned}$$

Using this expression into (8.51), together with $\alpha = \frac{\varepsilon-1}{\varepsilon}$ and (8.54), I obtain

$$
\begin{aligned}
L_t &= (1-\beta)E_t\frac{\alpha}{w_L} + \frac{(1-\beta)E_t g}{\varepsilon w_L(\rho+\phi g)} \\
&= \frac{(1-\beta)E_t(\varepsilon-1)}{\varepsilon w_L} + \frac{(1-\beta)E_t g}{\varepsilon w_L(\rho+\phi g)} \\
&= \frac{(1-\beta)E_t}{\varepsilon w_L}\left[(\varepsilon-1) + \frac{g}{(\rho+\phi g)}\right] \\
&= \frac{(1-\beta)}{\varepsilon w_L}H_t\frac{\varepsilon w_H}{\beta}\frac{\rho+\phi g}{g+(\rho+\phi g)(\varepsilon-1)}\frac{(\rho+\phi g)(\varepsilon-1)+g}{\rho+\phi g} \\
&= H_t\frac{(1-\beta)}{\beta}\frac{w_H}{w_L}.
\end{aligned}
$$

I can now finally solve for the skill premium:

$$
\frac{w_H}{w_L} = \frac{L_t}{H_t}\frac{\beta}{1-\beta}. \tag{8.55}
$$

Comparing (8.55) with (3.34), I can find the condition such that the first is smaller than the second. This is given by:

$$
\begin{aligned}
\frac{\beta}{1-\beta} &< \frac{g+\beta(\varepsilon-1)(\rho+\phi g)}{(1-\beta)(\varepsilon-1)(\rho+\phi g)} \\
\beta(\varepsilon-1)(\rho+\phi g) &< g+\beta(\varepsilon-1)(\rho+\phi g) \\
0 &< g.
\end{aligned}
$$

8.11. The TFP Growth Rate. The growth in total factor productivity (TFP) represents output growth not accounted for by the growth in inputs. Given the generic Cobb-Douglas production function

$$
Y_t = A_t K_t^{\beta} L_t^{1-\beta}
$$

the TFP, or Solow Residual, is given by

$$
A_t = \frac{Y_t}{K_t^{\beta} L_t^{1-\beta}}
$$

where Y_t is the real output and K_t and L_t are the factors employed in production. In my model the real aggregate output is given by E_t/P_t and the factors employed in production are $L_{Pt} = L_t$ and H_{Pt}. Applying the above definition to the model setting, I derive

$$
TFP_t = \frac{E_t}{P_t(H_{Pt}^{\beta} L_t^{1-\beta})}.
$$

Taking logs, differentiating with respect to time and using some of the steady state results found previously, I obtain

$$\begin{aligned} g_{TFP} &= \frac{\dot{E}_t}{E_t} - \frac{\dot{P}_t}{P_t} - \beta\frac{\dot{H}_{Pt}}{H_{Pt}} - (1-\beta)\frac{\dot{L}_t}{L_t} \\ &= n - \frac{g}{1-\varepsilon} - \beta n - (1-\beta)n \\ &= \frac{g}{\varepsilon - 1}. \end{aligned}$$

8.12. The Relative Innovation Cost. The relative innovation cost is defined as the ratio between the average cost of innovation and the average unit cost of production. The numerator is given by the expected knowledge requirement for variety introduction $\bar{F}$ multiplied by the unit cost of knowledge b_{It}. The denominator is given by the manufacturing unit cost $\frac{\kappa_t}{\varphi}$ averaged among active firms. Formally:

$$RIC_t \equiv \frac{\bar{F}(\varphi_D) b_{It}}{\int_{\varphi_D}^{\infty} \frac{\kappa_t}{\varphi} \frac{g(\varphi)}{1-G(\varphi_D)} d\varphi}$$

where, according to (2.20), I am stressing that $\bar{F}$ is a function of the domestic market productivity cut-off φ_D. Plugging in the expression for b_{It} and solving the integral using $g(\varphi) = k\bar{a}^k \varphi^{-(1+k)}$ and $1 - G(\varphi) = \left(\frac{\bar{a}}{\varphi_D}\right)^k$, I derive:

$$\begin{aligned} RIC_t &= \frac{\frac{\bar{F}(\varphi_D) w_H}{(m_t + \lambda m_t^*)^{\phi}}}{\kappa_t \varphi_D^k k \int_{\varphi_D}^{\infty} \frac{\varphi^{-(1+k)}}{\varphi} d\varphi} \\ &= \frac{\frac{\bar{F}(\varphi_D) w_H}{(m_t + \lambda m_t^*)^{\phi}}}{\kappa_t \varphi_D^k k \int_{\varphi_D}^{\infty} \varphi^{-(2+k)} d\varphi} \\ &= \frac{\frac{\bar{F}(\varphi_D) w_H}{(m_t + \lambda m_t^*)^{\phi}}}{-\kappa_t \varphi_D^k \frac{k}{k+1} \varphi^{-(1+k)}\big|_{\varphi_D}^{\infty}} \\ &= \frac{\frac{\bar{F}(\varphi_D) w_H}{(m_t + \lambda m_t^*)^{\phi}}}{\kappa_t \varphi_D^k \frac{k}{k+1} \varphi_D^{-(1+k)}} \\ &= \frac{\frac{\bar{F}(\varphi_D) w_H}{(m_t + \lambda m_t^*)^{\phi}}}{\frac{k}{k+1} \frac{\kappa_t}{\varphi_D}}. \end{aligned}$$

Now using $\kappa_t = w_H^\beta w_L^{1-\beta}$ and simplifying, I obtain

$$RIC_t = \frac{\varphi_D \bar{F}(\varphi_D)}{(m_t + \lambda m_t^*)^\phi} \frac{k+1}{k} \left(\frac{w_H}{w_L}\right)^{1-\beta}.$$

8.13. Firm Size. Firm size can be measured either in terms of sales or in terms of employment.

Firm sales are given by the demand the firm faces for its variety, multiplied by the price charged for each unit. Sales on the domestic market are:

$$\begin{aligned} s_{Dt}(\varphi) &= p_D \frac{E_t p_D(\varphi)^{-\varepsilon}}{P_t^{1-\varepsilon}} \\ &= \frac{E_t}{P_t^{1-\varepsilon}} p_D(\varphi)^{1-\varepsilon} \\ &= \frac{E_t}{P_t^{1-\varepsilon}} \left(\frac{\kappa}{\alpha\varphi}\right)^{1-\varepsilon} \end{aligned}$$

where I used $p_D = \frac{\kappa}{\alpha\varphi}$. With $p_E = \frac{\tau\kappa}{\alpha\varphi}$, sales on the foreign market are:

$$\begin{aligned} s_{Et}(\varphi) &= p_E \frac{E_t^* p_E(\varphi)^{-\varepsilon}}{P_t^{*1-\varepsilon}} \\ &= \frac{E_t^*}{P_t^{*1-\varepsilon}} \left(\frac{\tau\kappa}{\alpha\varphi}\right)^{1-\varepsilon}. \end{aligned}$$

For firms with $\varphi < \varphi_E$, $s_{Dt}(\varphi)$ is also the size measured in terms of total sales $S_t(\varphi)$. For firms with $\varphi > \varphi_E$, the size is given by the sum of the sales on the two markets. In other words:

$$S_t(\varphi) = \begin{cases} \left(\frac{\kappa}{\alpha\varphi}\right)^{1-\varepsilon} \frac{E_t}{P_t^{1-\varepsilon}} & \text{if } \varphi < \varphi_E \\ \left(\frac{\kappa}{\alpha\varphi}\right)^{1-\varepsilon} \left[\frac{E_t}{P_t^{1-\varepsilon}} + \frac{\tau^{1-\varepsilon} E_t^*}{P_t^{*1-\varepsilon}}\right] & \text{if } \varphi \geq \varphi_E. \end{cases}$$

Since $\varepsilon > 1$, $S_t(\varphi)$ is clearly an increasing function of the productivity level φ.

Firm employment is given by the sum of skilled and unskilled workers employed in production. Using the expressions for the unit labor requirements given by (8.45) and (8.46), together with the expressions for the domestic price and demand used above, I obtain the total number of workers employed to produce for the domestic market:

$$\begin{aligned} e_{Dt}(\varphi) &= [L_t(\varphi) + H_t(\varphi)] \frac{E_t p_D(\varphi)^{-\varepsilon}}{P_t^{1-\varepsilon}} \\ &= \left[\frac{1-\beta}{\varphi} \left(\frac{w_H}{w_L} \right)^{\beta} + \frac{\beta}{\varphi} \left(\frac{w_L}{w_H} \right)^{1-\beta} \right] \frac{E_t}{P_t^{1-\varepsilon}} \left(\frac{\kappa}{\alpha\varphi} \right)^{-\varepsilon} \\ &= \left[(1-\beta) \frac{w_H^{\beta} w_L^{1-\beta}}{w_L} + \beta \frac{w_H^{\beta} w_L^{1-\beta}}{w_H} \right] \frac{1}{\varphi} \frac{E_t}{P_t^{1-\varepsilon}} \left(\frac{\kappa}{\alpha\varphi} \right)^{-\varepsilon} \\ &= \frac{\alpha E_t}{P_t^{1-\varepsilon}} \left(\frac{\kappa}{\alpha\varphi} \right)^{1-\varepsilon} \left(\frac{1-\beta}{w_L} + \frac{\beta}{w_H} \right). \end{aligned}$$

Multiplying the foreign demand by the iceberg trade cost τ, the total number of workers employed to produce for the export market is analogously derived as

$$\begin{aligned} e_{Et}(\varphi) &= [L_t(\varphi) + H_t(\varphi)] \frac{\tau E_t^* p_E(\varphi)^{-\varepsilon}}{P_t^{*1-\varepsilon}} \\ &= \frac{\alpha E_t^*}{P_t^{*1-\varepsilon}} \left(\frac{\tau\kappa}{\alpha\varphi} \right)^{1-\varepsilon} \left(\frac{1-\beta}{w_L} + \frac{\beta}{w_H} \right). \end{aligned}$$

As for firm sales, firm employment in both markets is given by

$$e_{Dt}(\varphi) + e_{Et}(\varphi) = \begin{cases} \left(\frac{\kappa}{\alpha\varphi} \right)^{1-\varepsilon} \left(\frac{1-\beta}{w_L} + \frac{\beta}{w_H} \right) \frac{\alpha E_t}{P_t^{1-\varepsilon}} & \text{if } \varphi < \varphi_E \\ \left(\frac{\kappa}{\alpha\varphi} \right)^{1-\varepsilon} \left(\frac{1-\beta}{w_L} + \frac{\beta}{w_H} \right) \left[\frac{\alpha E_t}{P_t^{1-\varepsilon}} + \frac{\alpha \tau^{1-\varepsilon} E_t^*}{P_t^{*1-\varepsilon}} \right] & \text{if } \varphi \geq \varphi_E \end{cases}$$

and is clearly an increasing function of firm productivity φ.

8.14. Consumers Welfare. Plugging (2.3) into (2.2) the individual instantaneous utility becomes

$$\begin{aligned}
u_{lt} &= \left[\int_0^{\hat{m}_t} x_{lt}^{\alpha}(\varpi)\,d\varpi\right]^{\frac{1}{\alpha}} \\
&= \left[\int_0^{\hat{m}_t} \left(\frac{c_l p_t(\varpi)^{-\varepsilon}}{P_t^{1-\varepsilon}}\right)^{\alpha} d\varpi\right]^{\frac{1}{\alpha}} \\
&= \frac{c_l}{P_t^{1-\varepsilon}}\left[\int_0^{\hat{m}_t} p_t(\varpi)^{-\varepsilon\alpha} d\varpi\right]^{\frac{1}{\alpha}} \\
&= \frac{c_l}{P_t^{1-\varepsilon}}\left[\int_0^{\hat{m}_t} p_t(\varpi)^{-\varepsilon\frac{\varepsilon-1}{\varepsilon}} d\varpi\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \frac{c_l}{P_t^{1-\varepsilon}}\left[\int_0^{\hat{m}_t} p_t(\varpi)^{1-\varepsilon} d\varpi\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \frac{c_l}{P_t^{1-\varepsilon}} P_t^{-\varepsilon} \\
&= \frac{c_l}{P_t} \qquad\qquad l = H, L.
\end{aligned}$$

The growth rate of consumers welfare is given by:

$$\frac{\dot{u}_{lt}}{u_{lt}} = \frac{\dot{c}_l}{c_l} - \frac{\dot{P}_t}{P_t} \qquad\qquad l = H, L.$$

In steady state $\frac{\dot{c}_l}{c_l} = 0$ and, according to (2.22), for the growth rate of $P_t^{1-\varepsilon}$ to be constant it must equal the growth rate of variety introduction, g. This implies that

$$\frac{\dot{P}_t}{P_t} = \frac{g}{1-\varepsilon}$$

and

$$\frac{\dot{u}_{lt}}{u_{lt}} = \frac{g}{\varepsilon - 1}.$$

Aggregate welfare is given by

$$\begin{aligned}
W_t &\equiv H_t u_{Ht} + L_t u_{Lt} \\
&= \frac{H_t c_H + L_t c_L}{P_t} \\
&= \frac{E_t}{P_t}.
\end{aligned}$$

Its growth rate is then given by

$$\frac{\dot{W}_t}{W_t} = \frac{\dot{E}_t}{E_t} - \frac{\dot{P}_t}{P_t}$$
$$= n + \frac{g}{\varepsilon - 1}.$$

8.15. Asset Holdings. Aggregate assets in the Northern economy equal the aggregate value of active firms, given by

$$A_t = m_t \left[\int_{\varphi_D}^{\infty} V_{Dt}(\varphi) \frac{g(\varphi)}{1 - G(\varphi_D)} d\varphi + \int_{\varphi_E}^{\infty} V_{Et}(\varphi) \frac{g(\varphi)}{1 - G(\varphi_D)} d\varphi \right]$$
$$= \frac{m_t}{1 - G(\varphi_D)} \left[\int_{\varphi_D}^{\infty} \left(V_{Dt}(\varphi) - F_D b_{It} + F_D b_{It} \right) g(\varphi) d\varphi + \int_{\varphi_E}^{\infty} \left(V_{Et}(\varphi) - F_E b_{It} + F_E b_{It} \right) g(\varphi) d\varphi \right].$$

From (2.15)

$$\int_{\varphi_D}^{\infty} \left(V_{Dt}(\varphi) - F_D b_{It} \right) g(\varphi) d\varphi + \int_{\varphi_E}^{\infty} \left(V_{Et}(\varphi) - F_E b_{It} \right) g(\varphi) d\varphi = F_I b_{It}$$

so aggregate assets can be rewritten as

$$A_t = \frac{m_t}{1 - G(\varphi_D)} \left[F_I b_{It} + \int_{\varphi_D}^{\infty} F_D b_{It} g(\varphi) d\varphi + \int_{\varphi_E}^{\infty} F_E b_{It} g(\varphi) d\varphi \right]$$
$$= \frac{m_t}{1 - G(\varphi_D)} \left[F_I b_{It} + \left(1 - G(\varphi_D)\right) F_D b_{It} + \left(1 - G(\varphi_E)\right) F_E b_{It} \right]$$
$$= b_{It} m_t \left[\frac{F_I}{1 - G(\varphi_D)} + F_D + \frac{1 - G(\varphi_E)}{1 - G(\varphi_D)} F_E \right]$$
$$= b_{It} m_t \bar{F}.$$

Using (8.47) and the definition $h_I \equiv \frac{H_{It}}{H_t}$

$$A_t = m_t \frac{w_H H_{It}}{m_t g}$$
$$= \frac{H_t h_I w_H}{g}.$$

Dividing by the total population, I get the individual assets holdings as

$$a_{lt} = \frac{H_t}{H_t + L_t} \frac{h_I w_H}{g} \qquad l = H, L.$$

8.16. Out-of-Steady-State Growth Rate g_t. The flow of new varieties introduced in the market at *any* time t is given by the total amount of skilled labor devoted to R&D, H_{It}, divided by the labor units required for successful innovation. Denote with $\bar{F}_t$ the out-of-steady-state equivalent of the steady state average knowledge requirement for product introduction $\bar{F}$. By assumption (see (2.6) in the text) the units of skilled labor needed to produce one unit of knowledge are given by $\frac{1}{(m_t+m_t^*)^\phi}$. This implies that the units of labor required to invent a successful variety are given by $\frac{\bar{F}_t}{(m_t+m_t^*)^\phi}$ and the out-of-steady-state flow of new varieties is equal to

$$\dot{m}_t = \frac{H_{It}\left(m_t + \lambda m_t^*\right)^\phi}{\bar{F}_t}$$

which is very similar to the steady state flow given by (2.21). Dividing both sides by m_t I get the expression for the rate of product introduction:

$$g_t = \frac{\dot{m}_t}{m_t} = \frac{H_{It}\left(m_t + \lambda m_t^*\right)^\phi}{m_t \bar{F}_t}.$$

$\bar{F}_t$ is conceptually identical to its steady state equivalent $\bar{F}$ with the only difference that the productivity thresholds are allowed to vary over time. In fact at any time t, a firm that invent a new variety expect to invest F_I unit of knowledge to come up with a new idea, F_D units with probability $1 - G(\varphi_{Dt})$ to enter the domestic market, and F_E units with the even lower probability $1 - G(\varphi_{Et})$ to become an exporter. Since I am looking at the average knowledge requirement for a successful innovation, I must divide this expected sunk cost by the probability of actually being successful, which is $1 - G(\varphi_{Dt})$. Formally this is given by:

$$\begin{aligned}
\bar{F}_t &= \frac{F_I + F_D(1 - G(\varphi_{Dt})) + F_E(1 - G(\varphi_{Et}))}{1 - G(\varphi_{Dt})} \\
&= \frac{F_I}{(1 - G(\varphi_{Dt}))} + F_D + F_E \frac{1 - G(\varphi_{Et})}{1 - G(\varphi_{Dt})} \\
&= \left[F_I \left(\frac{\varphi_{Dt}}{\bar{a}} \right)^k + F_D + F_E \left(\frac{\varphi_{Dt}}{\varphi_{Et}} \right)^k \right].
\end{aligned}$$

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PAPER 2

Institution-Driven Comparative Advantage, Complex Goods and Organizational Choice

with Shon Ferguson

Abstract

The theory of the firm suggests that firms can respond to poor contract enforcement by vertically integrating their production process. The purpose of this paper is to examine whether firms' integration opportunities affect the way institutions determine international trade patterns. We find that vertical integration lessens the impact of a country's ability to enforce contracts on the comparative advantage of complex goods. We also find that countries with good financial institutions export disproportionately more in sectors that produce complex goods and that have a high propensity for vertical integration. In doing so we use a new outcome-based measure of vertical integration propensity and we employ several empirical strategies: cross section, panel and event study analysis. Our results confirm the role of institutions as source of comparative advantage and suggest that this depends not only on the technological characteristics of the goods produced but also on the way firms are able to organize the production process.

1. Introduction

A substantial body of empirical work has established that the quality of a country's institutions has a profound effect on its economic performance. The impact of institutions on economic outcomes was first successfully estimated by Acemoglu, Johnson and Robinson (2001, 2002), who showed that differences in institutions have a large effect on income per capita across countries. Rodrik et al (2004) showed that institutions are more important than geography and trade in explaining differences in income per capita. Many authors pursued this topic further by focusing on the role played by specific types of institutions in explaining cross-country differences in economic performance. The effect of financial institutions was pioneered by King and Levine (1993), who showed that a country's level of financial development is a significant predictor of its future rate of economic growth. Knack and Keefer (1995) and Mauro (1995) are among the first who looked at the impact of specific measures of property rights protection on investment and growth.

More recently increasing attention has been devoted to examine the impact of institutions on trade volumes and trade composition. Anderson and Marcouiller (2002) and Ranjan and Lee (2007) show that poor institutions, in the form of corruption and imperfect contract enforcement, dramatically reduce international trade. Several influential works have studied and explored the idea that legal, financial and other types of institutions are indeed “inputs” to the production process and give a nation a comparative advantage in industries relatively intensive in the use of the services provided by these institutions. These papers show that institutional quality contributes to a country’s comparative advantage in the same way as the more traditional sources such as factor endowments and technology.

Evidence for the effect of legal institutions on comparative advantage is given by Nunn (2007) and Levchenko (2007) who show that countries with better legal systems export relatively more of “complex goods” that are more sensitive to poor contract enforcement.[1] The effect of financial development on comparative advantage was first explored by Svaleryd and Vlachos (2005) who showed that financial development favours the specialization in sectors that are more dependent on external financing. Manova (2008) showed that equity market liberalizations increase exports disproportionately more in sectors that are more dependent on external finance and employ fewer collateralizable assets.

One important matter that the above mentioned empirical contributions do not account for, however, is that firms may adapt their organizational form in order to cope with the limitations of the institutional environment. Namely, firms can respond to poor contract enforcement by vertically integrating their production process. We thus test the hypothesis that vertical integration is a substitute for good legal institutions when producing contract-intense goods. By accounting for endogenous organizational form this allows us to better understand the effect of legal institutions on the composition of exports.

The opportunity and the feasibility of vertical integration may rely on the quality of financial institutions too. A large body of work emphasizes the importance of financial institutions but it offers ambiguous predictions on how financial development should affect internal organization of the firm in general, and vertical integration in

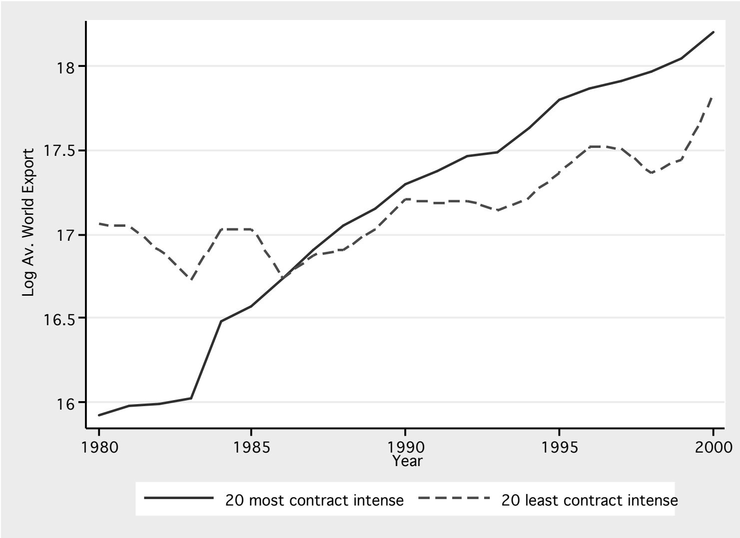
[1] See Levchenko(2007) and Acemoglu et al. (2007) for a theoretical analysis.

particular. One one hand, the lack of financial development could reduce the pool of potential entrepreneurs, limit firm entry and encourage the formation of large and vertically integrated firms (Rajan and Zingales (1998), Kumar, Rajan, and Zingales (1999)). On the other hand, it may be the case that credit market imperfections limit incumbents' investment opportunities and prevent firms that would otherwise like to vertically integrate from doing so (see, for example, McMillan and Woodruff (1999)). We weigh into this debate and provide evidence suggesting that credit market imperfections adversely affect vertically integrated industries only if they are contract-intense as well.

The interactions between financial development, contract intensity, and the extent of vertical integration have been recently explored by Acemoglu et al (2009). They find greater vertical integration in countries that have both higher contracting costs and more developed financial markets. They also find that countries with higher contracting costs are more vertically integrated in more capital-intensive industries, arguing that capital-intensive industries are more susceptible to hold-up problems. They do not investigate the consequences of this mechanism on trade, however, which is the goal of the this paper.

In this paper, we investigate the effect of legal and financial institutional quality on comparative advantage across industries that vary in their complexity and their propensity to vertically integrate. A complex good is defined as a good whose production process is intensive in the use of highly specialized and customized inputs. We measure industry complexity using Nunn's (2007) measure of contract intensity. The trade of complex goods has grown substantially over the past three decades, making its study all the more relevant for the modern economy. Figure 1 shows that the export growth for the 20 most contract intense industries has outpaced the export growth of the 20 least contract intense industries over the period 1980-2000. The main methodological contribution of this paper is that we use a new measure of industry-level "vertical integration propensity" based on the observed vertical integration outcomes from U.S. firm-level data. This measure has the advantage that it is a direct measure of vertical integration based solely on sector characteristics. In contrast, previous literature has used proxy measures such as the number-of-inputs.

We test two ways that vertical organization choice affects institution-driven comparative advantage in producing complex goods. First, we test if the beneficial effect of a country's legal institutional quality on its comparative advantage in complex goods



Source: Feenstra et al (2005), Nunn (2007)

FIGURE 1. World Exports of the 20 highest and 20 lowest contract-intense industries

industries is diminished for industries that also have a high propensity to vertically integrate. This should hold if firms are vertically integrating around the problem of contract incompleteness resulting from poor legal institutions. Second, we test whether or not financial development within a country enhances the comparative advantage of complex goods industries that are more inclined to vertically integrate. This should depend on whether good financial institutions enable firms to finance vertical integration and alleviate thus the hold up problem, more severe in complex goods industries. These hypotheses thus test the role of incomplete contract theory in explaining trade flows.

Our results show that there is a statistically significant interaction between institution-driven comparative advantage in complex goods and propensity to vertically integrate. We first test our hypotheses with a cross-section, which exploits cross-country variation in institutional quality and cross-industry variation in complexity and vertical integration propensity. We then test our hypotheses with panel and event study analyses, exploiting the available time variation in financial development provided by capital account liberalizations that occurred in several countries during the years 1984-2000. The cross-section is the ideal setting to examine the effect of legal institutions, which vary very little over time, while the panel and event study analyses lend themselves well to investigating the effect of financial development. In all our specifications we control for other potential sources of comparative advantage, such as factor endowments and the possibility that countries specialize in different goods according to their level of development.

Our work relates to a recent paper that studies the interactions between financial constraints and contract incompleteness by Carluccio and Fally (2008). Using import data of French multinational firms, they find that financial development generates a comparative advantage in the supply of complex goods and that imports of complex inputs are more likely to occur within the bounds of the firm when the exporter's level of financial development is lower. The purpose of Carluccio and Fally's (2008) paper is to analyze intra-firm trade and the decision of firm to vertically integrate only in relation with the institutional characteristics of the host country. An implicit assumption is that firms face no financial constraints coming from the domestic institutional environment. In contrast, we concentrate on the effect of domestic institutional quality on vertical integration regardless of whether the vertical integration occurs across borders or not.

The paper is organized as follows. The theoretical background is described in section 2. Variable descriptions and data sources are discussed in section 3. The methodology and results for the cross-section analysis, panel analysis and event study are given in sections 4, 5 and 6 respectively. Conclusions follow in section 7.

2. Theoretical Background

The idea that countries with better legal institutions have a comparative advantage in complex goods finds theoretical support in the incomplete contract literature. The argument, pioneered by Williamson (1979) and further developed by Grossman and Hart (1986), is the following: when contracts are not fully enforceable ex post, the contracting parties tend to under-invest ex ante and this problem, the " hold-up problem", is bigger the more the investment is relationship-specific. Consider the case of an up-stream firm (U) and a down-stream firm (D) that transact a customized intermediate good. U's investments in customization and D's effort in adapting its production process to use that specific input are both relationship-specific because their value is higher within this buyer-seller relationship than outside it. If the contract is not enforced and the trade agreement falls apart then U is left with a good that has a lower value for any other buyer, while D will find it difficult to procure a good substitute from another supplier. Given such a risk both parties in the transaction will underinvest in the relationship and the production of the final good will be inefficient. The better legal institutions are the higher the probability for the contract to be enforced and the lower the efficiency loss due to underinvestment. The resulting cost advantage will be greater the more important relation-specific inputs are in the production of the final good. From this it follows that countries with better legal institutions have a comparative advantage in the production of those goods intensive in relationship-specific inputs. Although this hypothesis has found strong empirical support, it takes into account only part of the theoretical predictions. The hold-up problem entails a transaction cost associated with market exchanges and, as Coase (1937) suggested, the transaction cost may be avoided or reduced by choosing the optimal organizational structure. This idea is fully developed by Williamson (1971,1979) who suggested vertical integration as an organizational response to the hold-up problem.[2] Williamson posits that moving the transactions of the specific inputs inside the firm's boundaries should alleviate the dependence on contract enforceability. If this is true then legal institutions should have a lower effect in driving comparative advantage of complex goods when the firms producing them can more easily vertical integrate. This is the the first hypothesis we test.

Given the propensity of firms belonging to a given industry to vertical integrate, one may ask which country-specific characteristics actually make this a viable option. Acemoglu et al. (2009) argue and show that a stronger financial development is a prerequisite for firms to efficiently integrate in response to high contracting cost. Vertical integration, either if achieved via the acquisition of an existing supplier or through the establishment of a new production plant, is a costly option and may require access to external finance.[3] If this argument is correct, good financial institutions should drive

[2] The more sophisticated approach developed by Grossman and Hart (1986) and Hart and Moore (1990) and known as the Property Rights Theory (PRT) emphasizes that transaction costs can also be present in a vertical integrated structure. As a consequence, according to the PRT it is not entirely clear whether relationship-specific investments should induce more or less vertical integration. As noted by Lafontaine and Slade (2007), Williamson's transaction costs approach to vertical integration, perhaps because of its more testable predictions, has stimulated much more empirical work and has found considerable support in the data.

[3] See also McMillan and Woodruff (1999) for evidence on firms in Vietnam.

comparative advantage in those contract-dependent industries where it is easy to vertically integrate around the problem of weak contract enforcement. This is the second hypothesis we test.

3. The Data

To examine the effect of legal and financial institutions on comparative advantage we combine data on countries' characteristics, industries' characteristics and countries' exports by industry. We employ different sources depending on the type of analysis and the time span we consider. For instance, the cross section analysis, mainly based on the data set from Nunn (2007), uses observations for 1997 while the panel and episode analysis use data for the period 1984-2000. This section illustrates the sources and the definitions of our main variables. We refer the reader to the appendix for a more complete description of the entire data set.

3.1. Trade Flows and Institution Quality. Industry level data on trade flows are from Feenstra et al. (2005). We converted the original data, classified by 4-digit SITC Rev.2 code, to the BEA's 1997 I-O industry classification. For the cross section we have trade data for 222 industries and 159 countries, for the panel we have trade data for 206 industries and 176 countries over the period 1984-2000.

The quality of legal institutions is measured by different variables according to data availability. For the cross section, in line with Nunn (2007), we use the "rule of law" from Kaufmann et al. (2008). This variable measures for each country the extent to which agents have confidence in the judiciary system and in law enforcement. In the panel analysis we use a similar index, the "law and order", collected by the International Country Risk Guide (ICRG) and available for more years.

We define the "quality of financial institutions" as the ease for firms to obtain external financing. To capture this idea we use one continuous and two discrete measures. The continuous measure, which we use in the cross section analysis, is the amount of credit by banks and other financial intermediaries to the private sector as a share of GDP. This variable has been extensively used in the literature since it represents an objective measure of the actual use of external funds and is therefore an appropriate proxy for the economy potential to support financial relationships.[4]

[4] See for example Rajan and Zingales (1998), Svaleryd and Vlachos (2005), Acemoglu et al. (2009), Beck (2002, 2003)

Table 1 shows that the "rule of law" measure and the ratio of credit to GDP are positively correlated with countries' GDP per capita and their endowments of physical and human capital. This highlights the importance of controlling for GDP per capita and factor endowments in our analysis.

The discrete measures of financial development are time-varying dummy variables that indicate the removal of equity market restrictions and are taken from Bekaert et al. (2005). Removing equity market restrictions increases the availability of external finance to firms (Mitton (2005) and has similar effects on the sectoral composition of exports as a rise in domestic credit availability. Moreover, as Bekaert et al. (2005) and others have argued, the exact timing of an equity market liberalization is usually the outcome of complex political processes and is therefore exogenous from the perspective of individual producers and potential exporters. We extended the dataset on equity market liberalizations used by Manova (2008) using the updated version of the data described in Bekaert et al. (2005). Our dataset lists 112 countries distinguishing among those that liberalized to foreign equity flows before, during or after the period 1980-2004. For each reforming country we consider both the official year of equity market reform and the "first sign" of liberalization.[5] Our measures of financial development are given by two dummy variables that are equal to 1 in the year of and all the years after an official or first sign of financial liberalization.

3.2. Contract Intensity. According to the theoretical framework we have in mind, the sensitivity of a given industry to the quality of legal institutions is an exogenous industry characteristic and it derives from the relative importance in the production process of those inputs that, due to some specificity, suffer from hold-up problems. A direct measure of such a variable does not exist and we use the proxy constructed and employed by Nunn (2007). As an indicator of whether an input requires or not relation-specific investments he considers Rauch's (1999) commodity classification. This consists of three groups: goods traded on organized exchanges, goods not traded on organized exchanges but nevertheless possessing a reference price in trade publications, and all other goods. Nunn defines an input as being relationship-specific if it is neither purchased on an organized exchange nor reference-priced. Using this information, together with information from the 1997 U.S. I-O Table on input use, Nunn constructs for each final good the following measures of the proportion of its

[5] The first-sign year is the earliest of three dates: official liberalization, American Depository Recipt (ADR) announcement or first country fund launch.

intermediate inputs that are relationship-specific:

$$z_i = \sum_j \theta_{ij} R_j^{neither}$$

where θ_{ij} is the weight of input j in the production of the final good i and $R_j^{neither}$ is the proportion of input j that is neither sold on an organized exchange nor reference priced.[6] A ranking of the five least and five highest contract intense industries is given in Table 2.

Although there are several alternative measures of contract intensity in the literature, we choose Nunn's measure because it most clearly captures the problem of asset specificity with upstream suppliers. Levchenko (2007), for example, uses the Herfindahl index of intermediate input use as an inverse measure of product complexity. The motivation for using the Herfindahl index is that the more suppliers a firm has and the less they are concentrated, the more the firm depends on legal institutions because it has to deal with a higher number of equally important contracts. Costinot (2009) instead bases its measure of complexity on survey data on the length of time needed to become fully trained and qualified in a given industry. Berkowitz et al. (2008) and Ranjan and Lee (2007) also use the data from Rauch (1999) but they do so to classify the downstream industries according to their own good's complexity, without looking at the type of intermediate inputs employed.

The correlation of the contract intensity measure (z_i) with other industry variables is reported in Table 3. Contract intensity is positively correlated with human capital intensity and, more surprisingly, negatively correlated with physical capital intensity.[7]

3.3. Vertical Integration Propensity. In order to test whether firms' organization choice has an impact on the way institutions drive comparative advantage in complex goods we need an industry-specific measure of the ease for firms to vertical integrate. Our measure of industries' propensity to vertically integrate is taken from Acemoglu et al. (2009). As mentioned earlier, they study the institutional determinants of vertical integration and in doing so they use a large and detailed firm level data set from WorldBase. Combining individual firms information with the U.S. I-O Table,

[6] Rauch's original classification groups goods into 1,189 industries according to the 4-digit SITC Rev.2 Classification. Nunn aggregates these data into 342 industries following the BEA's I-O industry classification. This explain why $R_j^{neither}$ is a proportion and not simply a 0/1. We refer to Nunn (2007) for a detailed description of the indicator and its construction.

[7] The negative correlation between Nunn's measure of contract dependence and physical capital intensity is particularly interesting. In fact, Acemoglu et al. (2009) used the industry's capital intensity as proxy for the extent of the hold up problem.

they compute for each firm in the sample the dollar value of inputs from industries in which the firm operates that is required to produce one dollars worth of the firms primary output. They then create a similar index also for secondary industries in which a firm is active. Each firm's vertical integration index is then the average of these indices. For U.S. firms only, they then regress this variable on a set of industry dummies and the resulting estimates are direct measures of vertical integration propensity. These dummy coefficients represent the average level of vertical integration in each industry in the U.S., where institutional constraints are likely to be slacker than everywhere else. They thus devise an industry ranking of the average propensity of firms to vertically integrate based solely on sector characteristics and derived from actual and observed vertical integration outcomes.

To the best of our knowledge there is no variable in the literature that has extensively served as a measure of industry-level vertical integration propensity. Nunn (2007) uses the number of inputs employed in the production process as a measure of the difficulty of vertical integration. The idea behind his choice is the following: if there are fixed costs in producing each single input, the total cost of integrate the entire production chain in-house is increasing in the number of inputs required. According to Lafontaine and Slade (2007), however, the empirical literature has identified plenty of factors as possible determinants of vertical integration.[8] Moreover, Nunn's argument views the decision to vertical integrate as a 0/1 choice: if a firm vertically integrates it does it with all its suppliers. We argue that the Acemoglu et al. (2009) outcome-based measure captures a wider range of factors that determine vertical integration and is thus most suitable for our study.

The only assumption we have to make, as for any other industry-specific variable, is that our index is consistent across countries. It is the external validity of the ranking that matters though, and not its absolute values.[9] Our measure of vertical integration propensity is thus given by 72 dummies that we match with the 222 Input-Output industries for which we have trade data.[10].

Although our variable is a direct measure of vertical integration derived from firm-level data, it could still be the case that it captures some other sector characteristics.

[8] Lafontaine and Slade (2007) mention, for example, the presence of economies of scale or of scope, the existence of uncertainty, monitoring costs or repeated interaction and the importance of relationship-specific investments itself.

[9] See Rajan and Zingales (1998).

[10] Acemoglu et al (2009) estimate a total of 77 industry dummies based on the BEA's 1992 I-O classification. See the appendix for more details.

This is why in the empirical specifications we control for many industry-specific variables. The correlations between our vertical integration propensity measure (vi_i) and some of these variables is reported in Table 3. Vertical integration propensity is positively correlated with physical capital intensity and negative correlated with industry value added. It's interesting to notice that the correlation with Nunn's proxy for vertical integration (In_i) is not significantly different from zero. A ranking of the five least and five most vertically integrated industries in the U.S. in 2003 is given in Table 4. It is interesting to note in Table 3 that the correlation coefficient between vertical integration propensity and Nunn's number-of-inputs variable is very low (0.10) and not statistically significant at the 1 percent level. A ranking of industries with a combined low contract intensity and a low propensity to vertically integrate is given in Table 5.

4. Cross-section Analysis

4.1. Empirical Specification. We take three different approaches to measuring the effect of legal and financial institutions on trade: cross-section analysis, panel analysis and event study. We begin our analysis using a cross-section methodology. The goal of the cross-section analysis is to exploit the variation in institutional quality across countries. This is particularly useful for the case of legal institutions since there is very little time variation in the measures of legal institutional quality that we employ.

We test our hypotheses by estimating the following equation:

$$\begin{aligned} T_{ci} &= \beta_0 + \beta_1 (z_i Q_c) + \beta_2 (z_i Q_c vi_i) + \beta_3 (z_i CR_c vi_i) + \beta_4 (z_i CR_c) \\ &\quad + \beta_5 (Q_c vi_i) + \beta_6 (CR_c vi_i) + \mathbf{X}_{ci} + \alpha_c + \alpha_i + \varepsilon_{ci}. \end{aligned} \tag{4.1}$$

T_{ci} is the log value of country c's exports to the rest of the world in industry i. Q_c is legal institutional quality, proxied by the "Rule of Law" index from Kaufmann et al. (2000). CR_c is financial institutional quality, which is proxied by the log of credit by banks and other financial institutions to the private sector as a share of GDP. z_i is Nunn's (2007) industry-specific measure of contract intensity, while vi_i is Acemoglu et al.'s (2009) measure of vertical integration propensity. $\mathbf{X}_{ci}$ is a vector of country-industry interaction controls, while α_c and α_i denote country fixed effects and industry fixed effects respectively. In this equation exports are explained by interactions of an industry characteristic with a country characteristic. This specification was first used by Rajan and Zingales (1998) to test whether industries that are more dependent on external financing are growing faster in countries with better developed financial markets.

Note that this specification measures the effect of country characteristics and industry characteristics on the composition of trade, not the total volume of trade. The effect of country characteristics such as institutional quality Q_c on the volume of trade is captured here by the country fixed effects. This formulation is thus conceptually distinct from studies such as Anderson and Marcouiller (2002) that use a gravity model to measure the effect of institutional quality on the total volume of trade in all sectors of an economy.

Nunn's (2007) hypothesis was that countries with better contract enforcement have a comparative advantage in producing final goods that use intensively inputs requiring relationship-specific investments. This is indicated by a positive coefficient for β_1, and means that countries with better contract enforcements will specialize in contract-intensive industries. The vi_i variable is standardized with a mean of zero, so we can interpret β_1 as the effect of judicial quality on comparative advantage for an industry with the mean level of vertical integration propensity.

Our analysis focuses on the triple-interactions in equation (4.1), since we are interested in how institution-driven comparative advantage in complex goods interacts with an industry's propensity to vertically integrate. Consider the first triple interaction term, $z_iQ_cvi_i$. A negative coefficient for β_2 implies that the effect of contract enforcement on comparative advantage in contract-intensive industries is diminished when the industry can easily vertically integrate. Vertically integrating around the problem of contract incompleteness thus reduces the necessity of good judicial institutions for producing complex goods. Consider now the second triple interaction, $z_iCR_cvi_i$. A positive coefficient for β_3 means that a country with efficient financial institutions will have a comparative advantage in producing contract-intense goods whose production process can profitably be vertical integrated. In other words, good financial institutions are important for firms producing complex goods and belonging to industries characterized by a high degree of vertical integration. Other interaction terms are also included, such as z_iCR_c, Q_cvi_i and CR_cvi_i. These control interactions are not the focus of the analysis but we report them in all regressions nonetheless. Additional control variables include the typical sources of comparative advantage, physical capital and human capital.

All industry-specific variables in the analysis are taken from U.S. data. Identification thus requires that the ranking of sectors in terms of contract intensity, vertical integration propensity, and other industry-specific controls remains relatively stable across countries.

4.2. Cross-Section Results. The results of the cross-section are presented in Tables 6 and 7. We estimate equation (4.1) using Nunn's (2007) dataset of 70 countries and 182 industries in the year 1997. Using Nunn's data allows us to directly compare our results with his original results.

Table 6 focuses on legal institutions only. As in Nunn (2007), we find that the coefficient for judicial quality interaction, z_iQ_c, is positive and statistically significant across all columns of Table 6. We also observe that the coefficient for the triple interaction, $z_iQ_cvi_i$, is negative and statistically significant across all columns. These results support our hypothesis that legal institutions are less important for comparative advantage within industries for which vertical integration is relatively easy. While Nunn (2007) tested this hypothesis using the number of inputs as an inverse measure of the ease of vertical integration, we use use Acemoglu et al.'s (2009) observed industry-level vertical integration outcomes in the U.S. as our measure of vertical integration propensity.

Columns (1) and (2) of Table 6 do not include any controls for alternative sources of comparative advantage or industry characteristics. The only difference between columns (1) and (2) is the number of observations. Column (1) uses the unrestricted sample, while column (2) is restricted to using the same observations as column (4), which is lower due to limitations in data availability for the control variables. Restricting the sample only affects the coefficients slightly.

Controlling for traditional sources of comparative advantage in column (3) does not change the main results. We report standardized beta coefficients in all specifications, which allows us to directly compare the relative size of the coefficients. We observe that the effect of judicial quality has a greater impact on comparative advantage than human or physical capital. According to the estimate in column (3), a one standard deviation increase in the judicial quality interaction increases exports by .28 standard deviations. In contrast, a simultaneous one standard deviation increase in the physical capital and human capital interactions increases log exports by a combined .17 standard deviations. The judicial quality-vertical integration triple interaction also has a large coefficient, with a one standard deviation increase in vertical integration propensity reduces the effect of the judicial quality interaction by .09 standard deviations.

We control for other determinants of trade flows in column (4) of Table 6. Log income per capita is interacted with industry measures for share of value-added in shipment, intra-industry trade and TFP growth in the previous twenty years. These interactions control for the possibility that, for reasons unrelated to contract enforcement, high-income countries have a comparative advantage in high value-added industries, industries with a high degree of fragmentation of the production process or a rapid rate of technological progress. The final control in column (4) interacts log income with one minus the Herfindahl index of input concentration. Clague (1991a,b) argues that the Herfindahl index measures how "self contained" an industry is, and that less developed countries tend to specialize in industries that are relatively more "self contained". This interaction thus controls for the possibility that high-income countries will specialize in industries that are less "self contained". All control interactions are statistically significant with the expected sign.[11]

The judicial quality interactions and controls in Table 7 are the same as Table 6, but financial institution quality interactions and additional controls are also included. All columns of Table 7 are restricted to the same set of observations. The original judicial quality interaction and the judicial quality-vertical integration interaction continue to be significant with the expected sign across all columns. This implies that legal and financial institutions, although their measures are highly correlated, have separate roles in affecting international trade patterns.

The coefficient of the financial quality-vertical integration triple interaction, $z_i CR_c vi_i$, is significant with a positive sign across all columns of Table 7. This result supports the hypothesis that good financial institutions are relatively more important for industries that produce complex goods and where firms tend to be vertically integrated. At the same time, our control interaction between product complexity and financial institution quality, $z_i CR_c$, is positive and significant. This result is robust across all the specifications we will consider in our study. It is also worth noticing that the effect of good financial institutions is increasing with the industry's vertical integration propensity ($CR_c vi_i$ positive and significant in all the specifications). This seems to

[11] As an additional robustness check, available upon request, we also added to all the specifications of Table 6 an interaction of our measure of good complexity with the index of patent protection available in Park (2008). Our result are robust to the inclusion of this variable whose coefficient is positive and significant.

suggest that financial development is particularly beneficial to industries where firms tend to be large and integrated. This result won't be robust to the more demanding econometric strategies employed in the next sections, proving to be just a spurious correlation.

No controls are included in column (1), while control interactions are successively added in columns (2), (3) and (4). Column (2) includes controls for traditional sources of comparative advantage and industry characteristics interacted with log income. All of the controls in column (2) are significant with the exception of the TFP interaction.

Two additional control interactions are included in column (3). The first controls for the importance of financial development in capital-intense industries, k_iCR_c. The second controls the importance of financial development in industries that are growing quickly, tfp_iCR_c. The inclusion of these controls is motivated by previous studies on financial development and export composition and the possibility that z_ivi_i captures some other source of financial dependence that has nothing to do with contract intensity and organizational choice. Both of these controls are statistically significant with the expected sign. Nonetheless, our triple interactions are robust to these controls.

Two more control interactions are included in column (4) to test whether other country characteristics, rather than judicial quality, cause countries to specialize in the production of complex goods. We do this by interacting z_i with the country-level characteristics of physical and human capital abundance. The coefficient for the z_iK_c control is positive and significant at the 5% level, but the coefficient for the z_iH_c control is negative and significant only at the 10% level. Overall, our significant and economically meaningful results for the triple interactions, $z_iQ_cvi_i$ and $z_iCR_cvi_i$, are robust to a wide array of controls in the cross-section data.[12] [13]

The effect of vertical integration on the response of complex goods to institutional quality is economically significant. Take the hypothetical case of Cambodia improving its Rule of Law Ranking to that of South Korea, which would entail moving from the 25th percentile of the Rule of Law country ranking to the 75th percentile. The point estimates in column 4, Table 7 indicate that complex goods exports (75th percentile of the complexity ranking) would rise by 39 percentage points in Cambodia's low-VI

[12] We also find significant results when using the "net interest margin" from Beck et al. (2000) as our proxy for financial development and when we substitute the "rule of law" with alternative measures of contract enforcement. See the data appendix for more details.

[13] In an additional robustness check available upon request, we confirm in in the cross-section specification that the vi_i variable is not simply a proxy for labor-intensity, measured as the ratio of total wages to value-added.

industries but only 5 percentage points in its high-VI industries (comparing the 25th vs. 75th percentile of the VI ranking). Similarly, if Burundi (25th percentile) improved its credit/GDP ratio to that of South Korea (75th percentile), the point estimates suggest that Burundi's exports of complex goods would increase by 64 percentage points for high-VI industries versus 44 percentage points for low-VI industries.

We complement our regression results with a graphical analysis of how the marginal effects of legal and financial institutions on trade vary with industry characteristics. Derivation of (1) illustrates that these marginal effects are a function of industry-level contract intensity and vertical integration propensity:

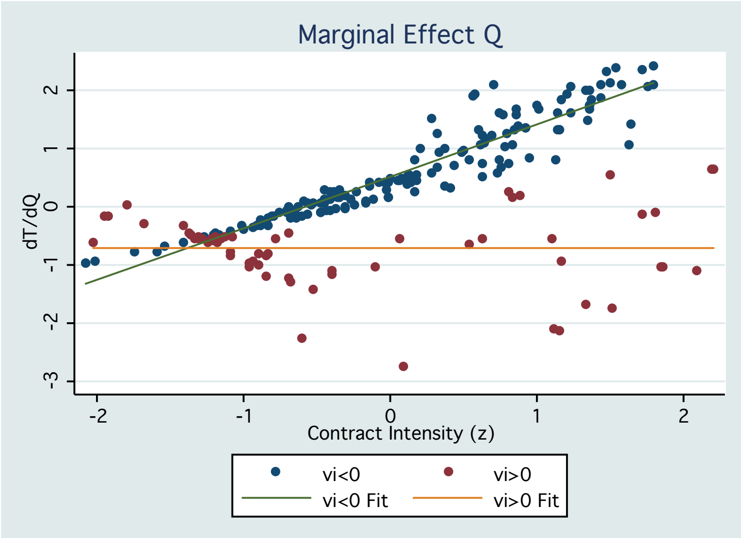
$$\frac{\partial T_{ci}}{\partial Q_c} = z_i(\beta_1 + \beta_2 vi_i) + \beta_5 vi_i$$

$$\frac{\partial T_{ci}}{\partial CR_c} = z_i(\beta_4 + \beta_3 vi_i) + \beta_6 vi_i + \chi_1 k_i + \chi_2 \Delta tfp_i.$$

We cannot capture the true marginal effects because some of the effects of institutions are absorbed by the country dummies. This is not a problem though since we are interested in knowing how the marginal effects differ across industries that vary in contract intensity and vertical integration propensity. The connection between these industry characteristics and the marginal effects is illustrated in Figures 2 and 3 respectively. Figure 2 shows that the marginal effect of legal institutions is increasing with contract intensity for industries that have a low propensity to vertically integrate. However, there is no relationship between the marginal effect and contract intensity for industries that have a high propensity to vertically integrate. A similar pattern is found for the marginal effect of financial institutions in Figure 3. The marginal effect of financial institutions is increasing with contract intensity for industries with both low and high propensity to vertically integrate, but the effect is larger for high-VI sectors.

The cross-section approach is appropriate for analyzing the effect of judicial quality on comparative advantage since there is so little time variation in the available proxies of countries' judicial institution quality. Reverse causality is still an issue though, since it may be that countries that already export contract-intense goods have an incentive to improve their contract enforcement or financial institutions.[14] As for our measure of financial institutional quality, it goes without saying that the ratio of private bank credit to GDP is an outcome variable. Our results thus can only be interpreted as

[14] See Do and Levchenko (2007) for the causal effect of comparative advantage on financial development.



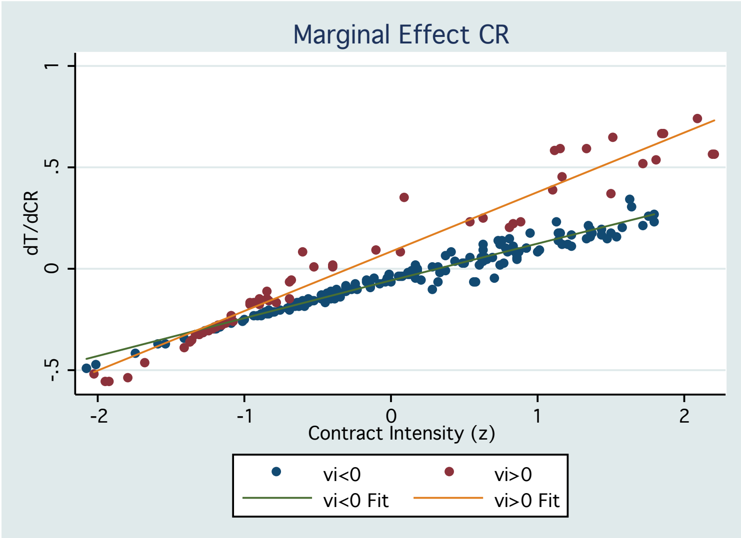
Notes: Marginal effects derived from the regression provided in column (4) of Table 7

FIGURE 2. Marginal effect of legal institutions on exports

interesting correlations but to not indicate a causal relationship between institutional quality and comparative advantage. We address these concerns in the panel analysis and event study by following Manova (2008) and using episodes of equity market liberalization as a source of exogenous variation in the supply of outside finance.

Several authors have attempted to use an instrument for institutional quality in order to isolate the causal impact of institutions on comparative advantage. Nunn (2007), for instance, attempts to use countries' legal origins as an instrument for legal institutions. As our analysis examines two different types of institutions, it requires separate instruments for legal institutional quality and financial development. Since legal origin likely affects both contract enforcement and financial development it is not a suitable instrument for either type of institution. Given the lack of good instruments we elect to exploit the shocks in financial liberalization instead.

Another issue with the cross-section is that it may suffer from the problem of missing variables. Although we include several country-industry controls, there may be unobserved country-industry interaction terms that bias our results. The panel



Notes: Marginal effects derived from the regression provided in column (4) of Table 7

FIGURE 3. Marginal effect of financial institutions on exports

analysis in the next section addresses this problem by employing country-industry fixed effects.

5. Panel Analysis

5.1. Empirical Specification. The goal of the panel analysis is to exploit the sudden shocks to financial development that occurred in several countries between 1984 and 2000 in order to help alleviate the problem of reverse causality that we have in our cross-section analysis. Data on financial market liberalizations from Bekaert et al. (2005) provides us with a source of variation in financial development that we exploit in both the panel data and later in the event study approach. We use a generalized difference in difference methodology similar to Manova (2008) and estimate the following equation:

$$\begin{aligned} T_{cit} &= \beta_0 + \beta_1 \left(z_i L\&O_{ct}\right) + \beta_2 \left(z_i L\&O_{ct} vi_i\right) + \beta_3 \left(z_i Lib\_dum_{ct} vi_i\right) \\ &\quad + \beta_4 \left(z_i Lib\_dum_{ct}\right) + \mathbf{X}_{cit} + \alpha_{ci} + \alpha_t + \varepsilon_{cit}. \end{aligned} \tag{5.2}$$

Here the dependent variable, T_{cit}, is the log value of country c's exports to the rest of the world in industry i in year t. The proxy for legal institutional quality, $L\&O_{ct}$, is the "Law and Order" indicator from the ICRG. We use this measure of legal institutional quality because it is available for more years than the "Rule of Law" measure. Lib_dum_{ct} is the financial liberalization dummy variable, which takes a value of 1 the year of and all years after a financial liberalization in country c and 0 otherwise. $\mathbf{X}_{cit}$ is a vector of controls, while α_{ci} and α_t denote country-industry and time fixed effects respectively. By using country-industry fixed effects we control for all time-constant factors that are related to a particular industry in a particular country. Together with the time fixed effects this means that we are left with industry-year, country-year, and industry-country-year interaction terms. Identification is thus made using purely the time variation in institutional quality.

We are interested in the same triple-interactions in equation (5.2) as we were in the cross-section approach. The interpretation of the triple interaction coefficients is identical to the cross-section case. A negative coefficient for β_2 implies that legal institutional quality is not as important for specialization in contract-intensive industries when it is relatively easy for these industries to vertically integrate around the problem of contract incompleteness. A positive coefficient for β_3 means that a country will have a comparative advantage in producing contract-intense goods in a vertically-integrated production process if financial institutions are strong.

The panel analysis includes many of the same control interaction terms as the cross-section, plus all the variables that vary over time since they are not subsumed by the country-industry fixed effects. This includes country-specific legal and financial institutional quality, physical and human capital, and log income. Almost all industry-specific variables in the panel analysis are time-constant. The only time-varying industry variable is total factor productivity growth, which is both interacted with log income and included on its own.

As Manova (2008) states, the estimates in equation (5.2) may be an underestimation of the true effect if trade increases in the anticipation of a financial or legal reform. Anticipation of the reforms may be occurring, but this downward bias serves to strengthen our results since we find large and statistically significant effects.

5.2. Panel Results. Our panel incorporates data from 76 countries and 153 industries over the years 1984-2000. An advantage with the panel approach is that it allows us to combine data from reforming countries with data on non-reformers. 39

countries in the panel that undertake a reform of their capital account during the time period we study. Of the remaining countries in our sample, 20 have closed capital accounts over the entire timespan of our panel and 19 have fully liberalized capital accounts prior to 1984.

The results of the panel analysis are presented in Table 8. We use two different ways of defining the timing of the financial liberalizations. Columns (1) and (2) use the official year of financial liberalization, while columns (3) and (4) use the year of the first sign of liberalization. Both of these interpretations of the timing of the financial liberalization are taken from Bekaert et al. (2005). All columns include all possible interactions of country-specific legal and financial development with industry-specific complexity and vertical integration propensity. The only control included in columns (1) and (3) is real GDP per capita, while several more controls are added in columns (2) and (4). We observe that the judicial quality interaction, $z_i L\&O_{ct}$, is now weakly significant. Given the lack of time variation in the Law and Order variable, it's remarkable that $z_i L\&O_{ct}$ is still significant at the 10 percent level in 2 out of 4 our very demanding specifications.[15] The judicial quality-vertical integration triple interaction, $z_i L\&O_{ct} vi_i$, is insignificant in all columns of Table 8. The lack of significance may be a symptom of a lack of time variation in the Rule of Law variable. However, the insignificant coefficient for $z_i L\&O_{ct} vi_i$ may instead suggest that vertical integration propensity does not reduce the problem of contract incompleteness when financial development is also poor ($Lib\_dum_{ct} = 0$).

The coefficient attached to the financial quality-vertical integration triple interaction, $z_i Lib\_dum_{ct} vi_i$, is positive and significant across all columns of Table 8. The strongest results are found using the first sign of liberalization, which is probably better in capturing the effect of liberalizations in case the actual reforms have either delayed or anticipated effects. The statistically significant coefficient for $z_i Lib\_dum_{ct} vi_i$ lends support for the hypothesis that good financial institutions are required in complex industries that have a higher propensity to vertically integrate.

As mentioned already, the vertical integration-financial development interaction term, $Lib\_dum_{ct} vi_i$ in Table 8, is now insignificant. This result confirms the ambiguity suggested by the theoretical literature: better financial institutions can both foster entry and the development of small firms (low-vi_i sectors) but also boost investments and the growth of big and integrated firms (high-vi_i sectors). What matters in our

[15] The lack of time variation is a problem common to many measures of governance or institution quality, especially if based on survey data. See Kaufmann et al. (2008).

analysis is that, when we restrict the attention to high-z sectors, the effect becomes positive and significant, because we isolate only the second of the two mechanisms. Complex good industries thus benefit the most from better financial institutions when they facilitate vertical integration required to avoid the hold-up problem.

The panel results complement the cross-section analysis by illustrating that financial development effects comparative advantage not only across countries but also within the same country over time.

6. Event Study

6.1. Empirical Specification. While the panel approach succeeds in measuring the effect of changes in financial development within countries over time, it does not take a firm stand on the number of years it takes for a financial liberalization to affect exports. On the one hand this allows for flexibility but on the other hand it prohibits us from measuring how quickly the financial reforms show up in the export data. We thus complement the panel analysis with an event study approach following Trefler (2004) and Manova (2008). Let $t = 0$ the time period before a liberalization event and $t = 1$ the time period after a liberalization event. We obtain the event study regression equation by first-differencing equation (5.2):

$$\begin{aligned}\Delta T_{ci} &= T_{ci1} - T_{ci0} \\ &= \beta_1 \left(z_i \Delta L\&O_{ct}\right) + \beta_2 \left(z_i \Delta L\&O_{ct} vi_i\right) + \beta_3 \left(z_i vi_i\right) \\ &\quad + \beta_4 \left(z_i\right) + \Delta X_{cit} + \Delta \varepsilon_{cit}.\end{aligned} \tag{6.3}$$

Note that the constant term β_0 and the country-industry and time fixed effects have dropped out of the regression equation. We include the first-differenced judicial quality interactions because we want to control for changes in legal institution quality that occur at the same time as the financial reforms. Note that the the effects may be underestimated since T_{ci0} includes any response in exports to an anticipated reform.

6.2. Event Study Results. We estimate (6.3) using the same set of financial reforms as the panel analysis. The event study only uses the 39 reforming countries since the other 35 non-reformers drop out due to first-differencing. Note that there is only one observation for every country-industry combination. All regressions include liberalization year fixed effects in order to control for changes in exports that may result from macroeconomic fluctuations. We first measure ΔT_{ci} as the difference in the log of average exports between $(t+1, t+3)$ and $(t-1, t-3)$. All time-varying independent variables are differenced in the same manner, taking the difference between the three year average before and after the year of the financial liberalization event.

The results of the event study are presented in Table 9. We find significant effects even when using this econometrically demanding setup. The statistical significance of the coefficient for the financial quality-vertical integration triple interaction, $z_i \Delta L\&O_{ct} vi_i$ depends on the controls used and the definition of the financial reform. Without controls the coefficient is the expected sign and statistically significant at the 5% and 10% levels using the first sign of liberalization and official liberalization respectively. Adding controls and country dummes in columns (3) and (6) does not affect significance levels.

As a robustness check we used an alternative time period measure and defined ΔT_{ci} as the difference in the log of average exports between $t+4$ and $t-1$. The results using this alternative time horizon are also statistically significant. The coefficients are also similar in size, which suggests that the composition of exports did not change in anticipation of the reform.

7. Conclusion

The purpose of this paper was to show that organizational form matters when measuring the effect of institutional quality on comparative advantage. We argue that firms can circumvent the hold-up problem by vertically integrating with suppliers, and that vertical integration requires well-functioning financial markets. These effects will be most pronounced in complex industries that are most susceptible to the hold-up problem. We tested these hypotheses using data for several countries that differ in institutional quality and several industries that differ in their complexity and propensity to vertically integrate.

Overall, the three different empirical strategies that we employ indicate a significant relationship between institutional quality, organizational choice and the exports of complex goods. The cross-section was the most ideal way to measure the effect of judicial quality since the variation in contract enforcement exists across countries but not within countries over the time period of our sample. Financial development varies both across countries and within countries over time, allowing us to find significant results in all three specifications.

Our results suggest that organizational form matters when measuring the effect of institutions on comparative advantage. Our results confirm the role of institutions as source of comparative advantage and suggest that this depends not only on the technological characteristics of the goods produced but also on the way firms are capable to organize the production process.

8. Technical Appendix

8.1. Detailed Data Description. Industry level data on trade flows are from the World Trade database [Feenstra et al. (2005)]. The data are measured in thousands of U.S. dollars and are originally classified by the 4-digit SITC Rev. 2 system. We map the data to the BEA 1997 I-O classification system using the SITC to HS10 concordance tables by Jon Haveman and the concordance from HS10 to the I-O system available from the BEA. When an SITC category maps into multiple I-O categories we pick the more frequent match in terms of the number of HS10 categories linking each SITC and I-O category. When an SITC category maps equally into two or more I-O categories, then the choice of I-O category was made manually.

Our first measure of judicial quality is the "rule of law" and it is from Kaufmann et al. (2008). The variable, using surveys data collected in 1997 and 1998, measures the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, property rights, the police, and the courts, as well as the likelihood of crime and violence. The original variable ranges from -2.5 to +2.5 but we use the variable as rescaled from 0 to 1 by Nunn (2007). The other variable we use is the "law and order" from the International Country Risk Guide. Law and Order are assessed separately, with each sub-component comprising zero to three points. The Law sub-component is an assessment of the strength and impartiality of the legal system, while the Order sub-component is an assessment of popular observance of the law. For this variable we have data from 1984 to 2000. As robustness checks, in the cross section analysis we replace the "rule of law" with other measures of contract enforcement. "Legal quality" is from Gwartney and Lawson (2003). It is an index from 1 to 10 that measures the legal structure and the security of property rights. Data on the "number of procedures", "official costs", and "time" required to collect an overdue debt are from the World Bank (2009).

Our first measure for capital market development is the commonly used amount of credit by banks and other financial institutions to the private sector as a share of GDP. The source is the "World Development Indicators". The second data set for financial development comes from Bekaert et al. (2005). In an ongoing project Bekaert and Hervey are collecting data for "A Chronology of Important Financial, Economic and Political Events in Emerging Markets".[16] For the countries surveyed the authors date both the official year of financial market reforms and the "first sign" of liberalization.

[16] See http://web.duke.edu/ charvey/Country_risk/chronology/chronology_index.htm

This first sign year is the earliest of three dates: official liberalization, first American Depository Receipt (ADR) announcement or first country fund launch. We construct post-liberalization dummies that equal 1 in the year of and all years after an official or first-sign liberalization. The data as used by Manova (2008) are available for 91 countries between 1980 and 1997. We extended this list up to 112 countries according to the most updated information made available by Bekaert and Hervey on their web site. As a robustness check to the cross section analysis we substitute the private credit over GDP with the net interest margin. This is a proxy for the wedge between prices faced by the parties on either side of a loan transaction. The source is Beck et al. (2000).

Annual real GDP is from the Penn World Tables. The stock of physical capital per capita is constructed according to the perpetual inventory method using data on population, investment share and real GDP from the Penn World Tables. Human capital per worker is calculated from the average years of schooling in a country with Mincerian non-linear returns to education. Average years of schooling come from Barro and Lee (2001).

Contract intensity comes from Nunn (2007). It measures the proportion of an industry's inputs, weighted by value, that require relationship-specific investments in their production. More details on the construction of this variable are in the text and in Nunn (2007).

Vertical integration propensity comes from Acemoglu et al. (2009). For each firm in their data-set they know up to five sectors j in which the firm operates and which one is the primary sector of activity, i. The vertical integration index of firm f from country c, whose primary sector is i, is then given by:

$$v_{cif} = \frac{1}{|N_f|} \sum_{j \in N_f} \sum_{h \in N_f} VI_{hj}$$

where N_f is the set of industries in which firm is active, $|N_f|$ denotes the number of these industries and VI_{hj} the entry of he I-O table for input h in producing 1\$ of output j. As explained in the text, the index is the average among the $|N_f|$ sectors of activity of the input requirements produced in-house. Looking only at US firms (i.e. $c = USA$) they run the following regression

$$v_{USAif} = d_i + \epsilon_{if}$$

where the d_is are 72 industry dummies and their estimate our measure for vertical integration propensity. Acemoglu et al.(2009) use the BEA's 1992 I-O Table classification at a 2-digit level of aggregation. We matched their data with our 4-digit 1997 I-O Table classification using the concordances I-O 92-SIC 87-HS10-I-O 97. The sources for the concordance tables are again Jon Haveman's and BEA's web sites.

All the other industry-specific data are from Nunn (2007). Data on factor intensities of production, industry value added and TFP growth were originally from Bartelsman and Gray (1996) and are all based on U.S. data. The TFP growth data is converted from NAICS to the 1997 I-O industry classification using the BEA concordance. Capital intensity is measured as the total real capital stock in each industry divided by the value added and skill intensity as the ratio of non-production worker wages to total wages. Value added is given by total value added of each sector divided by the total value of shipments. TFP growth is averaged over the period 1976 and 1996. Intra industry trade and Herfindahl index of input concentration are constructed by Nunn. Intra-industry trade is the amount of intra-industry trade in each industry according to the Grubel-Lloyd index for the United States in 1997. The Herfindahl index of input concentration is constructed from the 1997 U.S. I-O Use Table.

	Q_c	CR_c	H_c	K_c	Y_c
Q_c	1.00				
CR_c	0.75*	1.00			
H_c	0.68*	0.63*	1.00		
K_c	0.73*	0.69*	0.84*	1.00	
Y_c	0.83*	0.75*	0.84*	0.92*	1.00

Notes: Correlation coefficients are reported.
* indicates significance at the 1 percent level.

TABLE 1. Correlations of Country-Level Variables

	Least contract intensive		Most contract intensive
1.	Poultry processing	1.	Automobile & light truck manuf.
2.	Flour milling	2.	Heavy duty truck manuf.
3.	Wet corn milling	3.	Electronic computer manuf.
4.	Aluminum sheet, plate & foil manuf.	4.	Audio & video equipment manuf.
5.	Primary aluminum production	5.	Other computer peripheral equip. manuf.

Notes: Industry description are based on BEA 1997 6-digit I-O classifications

TABLE 2. The Five Least and Five Most Contract-Intense Industries

	z_i	vi_i	In_i	h_i	k_i	va_i
z_i	1.00					
vi_i	-0.35*	1.00				
In_i	0.16	0.10	1.00			
h_i	0.44*	-0.08	0.23*	1.00		
k_i	-0.49*	0.33*	0.02	-0.23*	1.00	
va_i	0.32*	-0.32*	-0.07	0.26*	-0.45*	1.00

Notes: "In" is the inverse measure of vertical integration used by Nunn (2007). Correlation coefficients are reported.
* indicates significance at the 1 percent level.

TABLE 3. Correlations of Industry-Level Variables

	Least vertically integrated		Most vertically integrated
1.	Health/education services	1.	Mining, nonferrous
2.	Maintenance construction	2.	Petroleum & gas
3.	Furniture, household	3.	Leather
4.	Household appliances	4.	Livestock
5.	Automotive service	5.	Amusement

Notes: Industry description are based on BEA 1992 2-digit I-O classifications

Table 4. The Five Least and Five Most Vertically Integrated Industries, U.S., 2003

	Combined Lowest		Combined Highest
1.	Poultry processing	1.	Electronic computer manuf.
2.	Flour milling	2.	Other electronic component manuf.
3.	Wet corn milling	3.	Cut & sew apparel manuf.
4.	Petroleum refineries	4.	Accessories & other apparel manuf.
5.	Rice milling	5.	Accessories & Audio & video equip. manuf.

Notes: Industry description are based on BEA 1997 6-digit I-O classifications

Table 5. Industries With Lowest and Highest Combined Contract Intensity and Vertical Integration Propensity

VARIABLES	(1)	(2)	(3)	(4)
z_iQ_c	0.252	0.282	0.284	0.241
	(0.011)***	(0.019)***	(0.021)***	(0.022)***
$z_iQ_cvi_i$	-0.047	-0.098	-0.088	-0.074
	(0.014)***	(0.019)***	(0.019)***	(0.019)***
Q_cvi_i	-0.009	-0.063	-0.072	-0.081
	(0.013)	(0.019)***	(0.019)***	(0.019)***
h_iH_c			0.071	0.054
			(0.018)***	(0.018)***
k_iK_c			0.098	0.079
			(0.033)***	(0.035)**
va_iY_c				-0.167
				(0.070)**
iit_iY_c				0.476
				(0.059)***
Δtfp_iY_c				0.043
				(0.050)
$(1-hf_i)Y_c$				0.544
				(0.106)***
Observations	20352	9776	9776	9776
R^2	0.718	0.753	0.754	0.758

Notes: Standardized beta coefficients are reported with robust standard errors in brackets Dependent Variable: Industry-Level Exports. Legal Institution Measure: Rule of Law. All regressions include a constant term, exporter and industry fixed effects.
***, ** and * indicate significance at 0.01, 0.05 and 0.1 levels.

TABLE 6. Cross-Section, Legal Institution Only

VARIABLES	(1)	(2)	(3)	(4)
z_iQ_c	0.145 (0.029)***	0.112 (0.029)***	0.102 (0.030)***	0.086 (0.034)**
$z_iQ_cvi_i$	-0.134 (0.027)***	-0.127 (0.027)***	-0.123 (0.027)***	-0.121 (0.027)***
zi_iCR_c	0.094 (0.013)***	0.090 (0.013)***	0.107 (0.014)***	0.096 (0.015)***
$zi_iCR_cvi_i$	0.031 (0.013)**	0.032 (0.013)**	0.030 (0.013)**	0.029 (0.013)**
Q_cvi_i	-0.146 (0.027)***	-0.159 (0.027)***	-0.144 (0.027)***	-0.146 (0.027)***
CR_cvi_i	0.049 (0.012)***	0.047 (0.012)***	0.033 (0.012)***	0.033 (0.012)***
h_iH_c	0.064 (0.018)***	0.053 (0.018)***	0.048 (0.018)***	0.052 (0.020)***
k_iK_c	0.122 (0.034)***	0.097 (0.036)***	0.021 (0.043)	0.054 (0.048)
va_iY_c		-0.200 (0.072)***	-0.183 (0.072)**	-0.185 (0.072)**
iit_iY_c		0.471 (0.061)***	0.466 (0.061)***	0.463 (0.061)***
tfp_iY_c		0.032 (0.053)	-0.129 (0.076)*	-0.136 (0.077)*
$(1-hf_i)Y_c$		0.512 (0.109)***	0.499 (0.109)***	0.485 (0.110)***
k_iCR_c			0.078 (0.021)***	0.066 (0.023)***
Δtfp_iCR_c			0.037 (0.012)***	0.037 (0.012)***
z_iK_c				0.140 (0.057)**
z_iH_c				-0.052 (0.027)*
Observations	9762	9700	9700	9700
R^2	0.755	0.758	0.759	0.759

Notes: Standardized beta coefficients are reported with robust standard errors in brackets Dependent Variable: Industry-Level Exports. Legal Institution Measure: Rule of Law. Credit Measure: Private Credit/GDP. All regressions include a constant term, exporter and industry fixed effects. ***, ** and * indicate significance at 0.01, 0.05 and 0.1 levels.

TABLE 7. Cross Section, Legal Institution & Credit

VARIABLES	(1) Official Lib. Date	(2) Offical Lib. Date	(3) First Sign of Lib.	(4) First Sign of Lib.
$z_i L\&O_{ct}$	0.039	0.025	0.055	0.036
	(0.023)*	(0.021)	(0.023)**	(0.021)*
$z_i L\&O_{ct} vi_i$	-0.000	0.004	-0.003	0.002
	(0.012)	(0.012)	(0.012)	(0.012)
$z_i Lib\_dum_{ct} vi_i$	0.011	0.014	0.019	0.021
	(0.006)*	(0.006)**	(0.006)***	(0.006)***
$z_i Lib\_dum_{ct}$	0.040	0.029	0.034	0.024
	(0.009)***	(0.008)***	(0.010)***	(0.009)***
$L\&O_{ct} vi_i$	0.018	0.020	0.017	0.019
	(0.011)	(0.012)*	(0.010)	(0.011)*
$Lib\_dum_{ct} vi_i$	0.003	0.004	0.004	0.006
	(0.005)	(0.005)	(0.007)	(0.007)
$L\&O_{ct}$	-0.013	-0.015	-0.008	-0.010
	(0.023)	(0.021)	(0.023)	(0.021)
Lib_dum_{ct}	0.040	0.036	0.034	0.029
	(0.012)***	(0.012)***	(0.013)**	(0.013)**
$RGDP_{ct}$	0.361	0.083	0.373	0.090
	(0.072)***	(0.108)	(0.072)***	(0.108)
K_{ct}		0.041		0.057
		(0.114)		(0.115)
H_{ct}		0.053		0.049
		(0.092)		(0.093)
$k_i K_{ct}$		-0.561		-0.607
		(0.273)**		(0.268)**
$h_i H_{ct}$		0.117		0.126
		(0.055)**		(0.055)**
$iit_i Y_{ct}$		0.804		0.812
		(0.142)***		(0.143)***
$(1 - hf_i) Y_{ct}$		-0.410		-0.436
		(0.162)**		(0.161)***
$va_i Y_{ct}$		0.422		0.420
		(0.171)**		(0.171)**
Δtfp_{it}		-0.047		-0.048
		(0.014)***		(0.014)***
$\Delta tfp_{it} Y_{ct}$		0.046		0.047
		(0.014)***		(0.014)***
Observations	126505	126505	126505	126505
R^2	0.321	0.328	0.319	0.326

Notes: Standardized beta coefficients are reported with robust standard errors in brackets.
Dependent Variable: Industry-Level Exports. Legal Institution Measure: Law and Order.
All regressions include a constant term, exporter-industry fixed effects,year fixed effects and cluster errors at the exporter level. ***, ** and * indicate significance at 0.01, 0.05 and 0.1 levels.

TABLE 8. Panel Regression, Country-Industry and Year Fixed Effects

VARIABLES	(1) Official Liberalization	(2) Official Liberalization	(3) Official Liberalization	(4) First Sign of Liberalization	(5) First Sign of Liberalization	(6) First Sign of Liberalization
$z_i \Delta L\&O_c$			-0.010			0.005
			(0.023)			(0.026)
$z_i \Delta L\&O_c vi_i$			-0.011			0.012
			(0.024)			(0.025)
$z_i vi_i$	0.032	0.034	0.037	0.042	0.060	0.049
	(0.017)*	(0.020)*	(0.022)*	(0.019)**	(0.023)**	(0.024)**
z_i	0.101	0.093	0.096	0.085	0.096	0.096
	(0.017)***	(0.021)***	(0.024)***	(0.018)***	(0.024)***	(0.028)***
$\Delta L\&O_c vi_i$			-0.006			-0.011
			(0.022)			(0.025)
vi_i	-0.012	-0.010	0.001	-0.000	0.018	0.030
	(0.018)	(0.020)	(0.022)	(0.019)	(0.024)	(0.025)
$h_i \Delta H_c$			-0.019			-0.032
			(0.039)			(0.041)
$k_i \Delta K_c$			-0.057			-0.078
			(0.039)			(0.037)**
$va_i \Delta Y_c$			0.023			0.071
			(0.068)			(0.077)
$iit_i \Delta Y_c$			0.114			0.028
			(0.042)***			(0.049)
$\Delta tfp_i \Delta Y_c$			-0.003			0.009
			(0.020)			(0.021)
$(1 - hf_i)\Delta Y_c$			-0.063			-0.046
			(0.032)**			(0.036)
Country Dummies	no	no	yes	no	no	yes
Observations	4576	3014	3014	4083	2325	2325
R^2	0.054	0.064	0.161	0.047	0.049	0.145

Notes: Standardized beta coefficients are reported, with robust standard errors in brackets. Dependent variable: Change in average industry-level exports to the world between $(t+1, t+3)$ and $(t-1, t-3)$. All regressions include liberalization-year fixed effects. *, ** and *** indicate significance at the 10, 5 and 1 percent levels.

TABLE 9. Event Study Regressions

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PAPER 3

Heterogeneous Firms and Credit Frictions: a General Equilibrium Analysis of Market Entry Decisions

Abstract

This paper develops a general equilibrium trade model with heterogeneous firms and imperfect credit markets. In the model, firms must raise external capital in order to finance the costs for product innovation and for domestic and foreign market entry. The model shows the importance of considering a general equilibrium setting in order to fully characterize the misallocations of resources that derive from the existence of credit frictions. These have important implications for firms entry decisions in the different markets and for the welfare effects of imperfect financial institutions. The paper also shows that allowing for liquidity constrained firms and imperfect credit markets changes, and in some cases reverses, some of main results from the heterogeneous firms literature. In particular the model predicts that trade liberalization does not necessarily lead to an increase of average productivity and consumers' welfare.

1. Introduction

This paper studies the interaction between financial constraints and the market entry behavior of firms. It also analyzes whether the impact of trade liberalization on average firm productivity and on individual welfare is affected by the presence of credit frictions. The rapid and sharp decline in economic activity and international trade that followed the 2007-2009 global financial crisis has boosted interest in these topics and brought a fast growing body of literature to study the linkages between credit constraints, firms activity and international trade. Four year after the first events that triggered the global downturn, financial markets are still very unstable and, according to all indicators, the credit to private sectors is still contracted, with firms facing substantial limitations to credit access. The persistence of this condition makes it crucial to understand the role that financial frictions can have on firms' long-run entry opportunities and to study the effects of worsening credit market conditions that go beyond the short run responses to a negative shock in the credit supply.

This paper focuses on the steady state effects of financial frictions in a setting where firms' domestic and foreign market entry decisions combine in shaping the allocation of resources within an open economy. It fully describes how firms' choices, average productivity and the number of producers interact in general equilibrium and how credit frictions affect via these channels consumers welfare and the role of played by trade costs.

Establishing a business entails substantial ex-ante sunk costs in both the initial stage of product development and in the following stages of domestic and foreign markets entry. It is natural to think that the ability of a financial system to provide firms with these innovation and entry investments may play an important role in determining the number and the type of firms active on the markets and, ultimately, consumer welfare. A main result of my analysis is that financial frictions create rents that divert resources away from innovation activities, limit the access of firms to credit and constrain entry decisions. Moreover, I find that exporting firms, because bigger in term of total sales and profits, can have an advantage in terms of access to credit and this shifts resources from innovation to foreign market entry. I thus show that a main effect of credit frictions is a too low number of entrants. This implies a lack of competitiveness in the market that allows low-productivity firms to survive. The market can thus be characterized by a low number of big and inefficient firm and this has negative effects on welfare. I also show that credit frictions interact with trade costs in such a way that trade liberalization does not necessarily lead to higher average productivity and higher individual welfare.

Formally, I introduce credit market frictions in a heterogeneous firm model in the spirit of Melitz (2003). The model features two symmetric economies where monopolistically competitive firms differ in their productivity levels. Before knowing their productivity, liquidity constrained firms must raise capital to pay for the sunk costs needed for innovation and market entry, both in the domestic and in the foreign market. Capital is provided by a competitive credit market where lenders face imperfect protections. In other words, I assume that there is a probability $1 - \lambda > 0$ that the borrower can avoid paying the per-period debt reimbursement without incurring in any sanction. First, I solve for the optimal debit contract that maximizes the firm's expected value from variety introduction and satisfies the incentive compatibility constraint of the firm and the participation constraint of the lender. Then, I solve for the aggregate variables that define the general equilibrium characterized by an infinite mass of potential entrants. In particular, I solve for the minimum productivity levels that grant access to the domestic and the foreign market, for the number of active firms in the economy and for the individual welfare.

I am particularly interested in the general equilibrium effects of the credit market frictions, inversely measured by the parameter λ. I show that, when both domestic firms and exporters face imperfect credit markets and when the number of firms is endogenously derived from a free entry condition, credit frictions imply at least two forms of resources misallocation. First of all, because of the moral hazard problem, firms must be granted resources that would be otherwise allocated to innovation and entry investments. This rent for the firm implies that the creditor's pledgeable profits are only a fraction λ of the return of the investment in fixed costs. As a result, the total credit awarded in the economy is suboptimal, and so the total number of entrants. This reduces competition among incumbents allowing low efficiency firms to survive. Credit frictions thus imply a lower average productivity. A lower number of firms and a lower average productivity have a negative effects on individual utility: consumer welfare is decreasing in the level of credit frictions. The second form of resources misallocation goes from domestic to foreign firms. The fact that domestic profits can be de facto used to back-up the loan to pay for foreign market entry, makes exporters less likely to be credit constrained, at least for not too low levels of financial development.

The result that credit frictions can make foreign market entry relatively easier than domestic market entry can seem in contradiction with what found previously in the literature (see Manova (2008), Muûls (2008) and Wang (2011)). As I illustrate in more detail below, these contributions stress the constraints that credit frictions pose to exporting activities by assuming that domestic entry can be freely financed. In other words, credit frictions apply only to exporting activities by assumption. This approach is justified by the idea that exporting activities entail a higher degree of risk and uncertainty that causes the financing to be more difficult than for domestic activities. My setup could be easily extended to account for exogenously higher frictions to fund foreign market entry. Everything else equal, this would reduce the probability of becoming an exporter. On the other hand, the result that domestic profits can reduce the liquidity constraints faced by exporter would not disappear and this is an interesting general equilibrium effect that act in favor of the number of exporting firms and, once more, against the resources available for a higher number of active firms.

A second important set of results that stems from the general equilibrium properties of the model concerns the effects of decreasing trade costs, modeled as iceberg shipping costs to the foreign market. When these costs fall, export activities becomes more profitable and this tends to increase the number of exporting firms. On the other hand, the increased demand for limited resources lowers the number of domestic firms that can be sustained in equilibrium. Depending on which effect dominates in equilibrium, trade liberalization can either increase or decrease average productivity and consumer welfare. In particular, I show that the lower is the substitutability among varieties and the more firms are concentrated in the lower tail of the productivity distribution, the more likely it is that a reduction in trade costs decreases the total number of varieties available and lowers the competition among firms. As a result, the minimum and the average productivity of domestic firms decrease, and so does the welfare of consumers that have a smaller number of more expensive varieties.

The main contributions of this paper are to the recent but fast growing literature on financial institutions and trade. First of all, it provides this literature with a tractable full-fledged general equilibrium model. This allows taking into account interactions among variables that have been mostly neglected by previous contributions but that have important implications for firms selection and consumers welfare. For instance, Manova (2008) develops a model with credit constrained heterogeneous firms and imperfect financial institutions. The model present several nice features: many countries with different levels of financial development and many sectors with different levels of financial needs and collateralizable assets. It allows to rationalize in a parsimonious way some empirical findings over a panel of country and industry level data. Using credit to the private sector over GDP as a measure for financial development, the author finds that once controlled for sector and country fixed effects, financially developed countries export relatively higher volumes in sectors that requires more outside capital and in sector with fewer collateralizable assets. On the other hand, the model presented in Manova (2008) is solved in a partial equilibrium setting and credit frictions by assumption affect only foreign market investments. Credit constraints interact with firm productivity simply by reinforcing the way in which firms with higher productivity select into exporting. The distortions on the number of firms and on average productivity are completely neglected in this setting and there is no link between domestic and foreign market entry decisions, neither at the aggregate level nor at the firm level.

Two general equilibrium trade models with heterogeneous firms and liquidity constraints to entry decisions are developed in Chaney (2005) and Muûls (2008). In the first one, developed in order to explain the lack of sensitivity of exports to exchange rates fluctuations, only firms that have enough liquidity, either as endowment or as profits from domestic activities, are able to exports. As in the present setting, this allows for the possibility that, thanks to domestic market profits, foreign market entry in not necessarily constrained by the lack of liquidity. On the other hand, Chaney (2005) does not take into account the effects that imperfect credit markets have on the number of firms which is taken exogenously in his analysis. Moreover Chaney does not explicitly model credit market frictions. He assumes that firms inherit an exogenous amount of liquidity and there is no credit market where to collect further resources. Muûls (2008) combines the general equilibrium approach of Chaney (2005) with the more structural specification of financial constraints of Manova (2008), yielding thus a richer framework of analysis. Still, the fixed number of potential entrants does not allow to take into account the mechanism linking firms competition and firms selection that is key in my analysis. Other interesting general equilibrium contributions are Suwantaradon (2008) and Wang (2011). These papers consider a dynamic framework in which some firms can accumulate liquidity over time and partially overcome the credit constraints. The first of the two again takes as given the mass of entrants while the second restricts the effects of imperfect financial markets on exporting activities only. Moreover, none of these papers studies the interaction between trade costs and financial frictions.

Another important difference between this paper and the majority of the contributions in the literature on credit constraint and trade is that my analysis focuses on the role of financial institutions, meant as an exogenous characteristic of the environment in which all firms operate. Many of the papers in this literature, including those listed above, focus more on the role of idiosyncratic firms characteristics in shaping the credit constraints, such as firms' own liquidity (Chaney (20005) and Muûls (2008)), net worth (Wang (2011) and Suwantaradon (2008)) and tangibility of the collaterals (Manova (2008) and Muûls (2008)). These choices are motivated by the increasing availability of firms level data on both exporting status and financial health and the evidence that the two tend to be positively correlated (see Manova (2008), Muûls (2008), Wang (2011) and Suwantaradon (2008), Berman and Héricourt (2010), Bellone et al. (2010)). In this respect, my work can be considered as complementary and opens a new venue for empirical investigation.

This paper also add an important contribution to the more mature literature on trade and heterogeneous firms first introduced by Melitz (2003). My analysis shows that the conclusion that trade liberalization increases average productivity and consumer welfare can be reversed when other frictions, besides trade frictions, are introduced in the model. In Melitz (2003), as well as in many of the following contributions (for instance Bernard et al. (2007)), trade liberalization has three main effects: it increases the number of foreign varieties, it increases average productivity and it reduces the number of domestic firms. The first two effects increase consumers welfare and always dominate. [1]In my analysis, the increase in the number of exported varieties and the fall in the number of domestic varieties can be such that the second dominates and the total number of varieties decreases allowing for less productive firms to enter the domestic market. When this happens consumers welfare drops. When trade cost falls, the profits from exports increase so the minimum productivity level required for foreign market entry goes down. In Melitz (2003), this unambiguosly increases the expected profits from exports and decreases the expected profits from domestic sales. This result is achieved by pushing up the minimum productivity required for domestic market entry. In the present setting, this mechanism can be totally reversed. This happens because, from the lender point of view, the expected net return from foreign market entry can be negative, if covered by positive expected net profits on the domestic market. When this is the case and the minimum productivity for foreign market entry goes down, the expected lender's expected profits from financing export activities can decrease, requiring the expected profits from domestic activities to increase. This happens if the minimum productivity level for domestic market entry goes down.

Finally, given the high tractability of the model, several venues of further analysis can be easily exploited in the future. A first natural extension would be to allow for non-symmetric countries. This would entail some costs in terms of analytical tractability but would allow to address interesting questions as: what are the effects of trade

[1] This prediction has been already questioned for instance in Baldwin and Robert-Nicoud (2008), Gustafsson and Segertrom (2010) and Formai (2010). They show that introducing an engine of growth into the Melitz (2003) model, trade liberalization can permanently retards productivity growth in the short run and and make consumers worse off in the long run. In other words, they add a fourth effect from lower trade costs that also reduces the number of varieties in steady states and that can offset the positive effects driving Melitz's result.

frictions on comparative advantage and what are the effects of unilateral financial markets reforms on trade patterns? Another interesting extension would be, as mentioned above, to allow for the foreign market investments to be more risky and so more exposed to financial frictions. This would enrich the model in such a way that would make it easier to bring it to the data.

The rest of the paper proceeds as follows. In Section 2 I describe the general setting of the model. Section 3 solves the model for the standard case without credit frictions and briefly review the properties of its equilibrium. In section 4 I introduce credit frictions, describe the optimal contract between the firm and the creditor and solve the model for both a closed and an open economy. Section 5 presents the equilibrium properties and the main results of the paper. Section 6 concludes.

2. Set Up of the Model

The economy consists of two symmetric countries, Home and Foreign. The only factor of production is labor, and population is of size L. There are two sectors. One sector provides a single homogeneous good which is freely traded. This good is used as numeraire, and its price is therefore equal to 1 in both countries. Production in this sector is characterized by perfect competition and constant return to scale with $Q_0 = w \times L_0$, where L_0 is the amount of labor used to produce the quantity Q_0 of the homogeneous good and w is the constant productivity of labor employed in this sector. If, as shall be assumed, both countries produce the homogeneous good, then wages will be fixed by this sector's labor productivity w.

The second sector of this economy produces a continuum of differentiated goods. Each firm operating in this sector supplies one of these goods under Dixit-Stiglitz monopolistic competition. Production is characterized by increasing returns to scale. To produce, a firm must first incur a sunk innovation investment. Only after having incurred this fixed cost, the firm receives a patent to exclusively produce the new variety and learns its productivity. The productivity parameter is drawn from a probability distribution, so firms have different variable production costs. After having learned its productivity, each firm decides whether or not to incur the additional fixed costs needed to start selling its variety. The domestic and the foreign market requires separate entry costs. Firms need to draw a sufficiently high productivity to enter the domestic market and an even more favorable draw to enter the export market.

The main assumption in this paper is that firms face liquidity constraints and need to borrow in order to finance both the innovation and the market entry sunk costs. Credit markets do not provide perfect protection to lenders. I assume that in each period the firm can avoid the repayment specified by the debt contract. In this case, there is only a probability less than one that a court will enforce the payment and punish the insolvent firm.

2.1. Demand. The representative consumer is endowed with one unit of labor and her preferences are given by a Cobb-Douglas utility function in the homogeneous good q_0 and in the C.E.S. consumption bundle q_1 :

$$U = q_0^{1-\alpha} q_1^{\alpha}. \tag{2.1}$$

The bundle q_1 is defined over the continuum of differentiated varieties $\omega \in \Omega$ and is given by:

$$q_1 = \left[\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right]^{\frac{1}{\rho}}$$

The constant elasticity of substitution between any two varieties is given by $\sigma \equiv 1/(1-\rho) > 1$. If each variety ω is available domestically at the price $p(\omega)$, the aggregate price index associated with the bundle q_1 is given by

$$P \equiv \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}.$$

Given the individual expenditure e, this can be used to derive the optimal consumption decisions of the representative consumer:

$$\begin{aligned} q_0 &= (1-\alpha)e \\ q_1 &= \alpha e / P \\ q(\omega) &= \alpha e \frac{p(\omega)^{-\sigma}}{P^{1-\sigma}}, \forall \omega \in \Omega. \end{aligned} \tag{2.2}$$

2.2. Production. In the differentiated good sector, there is a continuum of firms, each producing a different variety ω. Firm technology is characterized by a cost function that exhibits constant marginal costs. Labour used in production is thus a linear function of output $Q : l = Q/\varphi$, where φ is the productivity level and l is the labor employed by the firm. Firms have different productivity levels φ, which they draw from a known and common distribution right after variety innovation. All producers face a residual demand with constant elasticity σ, and thus choose the same profit maximizing markup $\sigma/(\sigma-1)=1/\rho$. This yields to the pricing rule

$$p(\varphi)=\frac{w}{\rho\varphi}. \tag{2.3}$$

The firm variable profit, gross of any fixed cost, is

$$\pi(\varphi)=p(\varphi)Q(\varphi)-l(\varphi)w=r(\varphi)/\sigma$$

where $r(\varphi)$ is firm revenue and $Q(\varphi)$ is the demand for variety φ and is given by (2.2), where the individual consumer expenditure e is replaced by the economy-wide consumer expenditure $E=eL$. Using the expressions for the demand and the price (2.3), I can rewrite

$$r(\varphi)=\alpha E\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1} \tag{2.4}$$

$$\pi(\varphi)=\frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1}. \tag{2.5}$$

Notice that the ratio of any two firms' revenues and profits can be written as a function of their productivities only:

$$\frac{r(\varphi_1)}{r(\varphi_2)}=\frac{\pi(\varphi_1)}{\pi(\varphi_2)}=\left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}. \tag{2.6}$$

2.3. Variety introduction and Market Entry. In each economy there is an unbounded pool of prospective entrants into the differentiated good industry. Before entry each potential firm is identical. In order to enter, each firm must first discover a new variety and obtain a patent to exclusively produce it. This requires an investment of F_E units of domestic labor. Only when this investment is sunk, the firm gets to know its productivity level φ. This is drawn from a common probability density function $g(\varphi)$ with support $(0,\infty)$ and corresponding cumulative distribution function $G(\varphi)$. Melitz (2003) worked with a general probability distribution of firm productivity φ, but the model becomes considerably more tractable analytically if a Pareto distribution is assumed. Moreover, the empirical literature on firm productivity distribution suggests that a Pareto distribution is a reasonable approximation (Del Gatto, Mion and Ottaviano (2006), Eaton et al. (2008)). In what follows I thus assume that $G(\varphi)=1-\left(\frac{\varphi_m}{\varphi}\right)^a$ for all $\varphi\geq\varphi_m$, where $\varphi_m>0$ is the minimum productivity value and a is a shape parameter that indexes the dispersion of productivity draws. As a increases, the relative number of low-productivity firms increases, and the productivity distribution is more concentrated at these low productivity levels. In order for the average productivity to be finite, I must assume $a>\sigma-1$.

Once productivity is known, firms decide whether to exit immediately or to start production for either the domestic market or for both the domestic and the foreign market. Entry in the domestic market requires an up-front investment of F_D units of domestic labor and entry in the foreign market an additional investment of F_X units of domestic labor. These costs can be thought as the investments needed to promote the new variety and customize it according to the taste of domestic and foreign consumers respectively. Given these fixed cost, only if the productivity level φ is high enough, it is profitable to enter a market and produce for it. If the firm does produce, it then faces a constant probability δ in every period of a bad shock that would force it to exit.

The equilibrium will be characterized by a mass M of firms in the differentiated good sector and a probability density function $\mu(\varphi)$ of the productivity levels of the active firms defined over a subset of $(0, \infty)$.

3. The Model without Credit Frictions

I will first present and review the standard frictionless setup and then I will introduce the credit frictions and show how they affect the equilibrium properties. In both cases, I will first describe the closed economy equilibrium and then introduce the exporting decisions in the open economy.

3.1. The Closed Economy. In a world where firms have deep pockets or where credit is frictionless, a firm with known productivity φ would decide whether or not to enter the domestic market by comparing the value generated by the investment with its cost $F_D w$. In this paper I consider only steady state equilibria in which aggregate variables remain constant over time. Since upon entry each firm's productivity level φ does not change over time, this implies that the per period profit will also remain constant. Since the only source of uncertainty is the exogenous death rate δ, and assuming that there is no time discounting, the investment value for the domestic market is given by

$$v_D(\varphi) = \sum_{t=0}^{\infty} (1-\delta)^t \pi(\varphi) = \frac{\pi(\varphi)}{\delta}. \tag{3.7}$$

Thus, $\varphi^* = \inf\{\varphi : v_D(\varphi) > F_D w\}$ identifies the lowest productivity level of firms producing for the domestic market. Hereafter φ^* will be referred as the domestic cutoff level and $v_D(\varphi^*) = F_D w$ as the zero profit condition for the domestic market. Together with (3.7), this can be rewritten as

$$\pi(\varphi^*) = \delta F_D w. \tag{3.8}$$

Given that any firm with $\varphi < \varphi^*$ immediately exits, and given that the death rate δ is exogenous and uncorrelated with productivity, $\mu(\varphi)$ is the distribution of $g(\varphi)$ conditional on entry, defined as

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)} & \text{if } \varphi \geq \varphi^*, \\ 0 & \text{otherwise} \end{cases} \tag{3.9}$$

where $1-G(\varphi^*)$ is the ex-ante probability of successful entry. I now define the weighted average productivity

$$\begin{aligned} \tilde{\varphi} &\equiv \left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \\ &= \left[\frac{1}{1-G(\varphi^*)}\int_{\varphi^*}^\infty \varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}. \end{aligned} \tag{3.10}$$

As shown by Melitz (2003), this measure is extremely useful, since it summarizes the information in the density $\mu(\varphi)$ that is relevant for all aggregate variables. I can use $\tilde{\varphi}$ to write aggregate revenues and variable profits as $R = Mr(\tilde{\varphi})$ and $\Pi = M\pi(\tilde{\varphi})$ respectively. Note that

$$\begin{aligned} \bar{r} &= \frac{R}{M} = r(\tilde{\varphi}) \\ \bar{\pi} &= \frac{\Pi}{M} = \pi(\tilde{\varphi}) \end{aligned} \tag{3.11}$$

represent both the average revenue and profit, as well as the revenue and the profit of the firm with productivity level equal to the average productivity $\tilde{\varphi}$. Using the above expression together with the property described in (2.6), the profits of the marginal firm $\pi(\varphi^*)$ can be expressed in terms of the average profits as $\pi(\varphi^*) = \pi(\tilde{\varphi}(\varphi^*))\left[\frac{\varphi^*}{\tilde{\varphi}(\varphi^*)}\right]^{\sigma-1}$. This allows me to rewrite the zero profit condition (3.8) as

$$\bar{\pi} = \pi(\tilde{\varphi}(\varphi^*)) = \delta F_D w \left[\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right]^{\sigma-1}. \tag{3.12}$$

Before knowing its productivity level, each firm investing in the development of a new variety expects to earn a stream of the average profit $\bar{\pi}$ net of the entry fixed cost $F_D w$, but only with probability $1 - G(\varphi^*)$. I thus define v_E as the net value of a new variety:

$$v_E \equiv (1 - G(\varphi^*))\left[\sum_{t=0}^{\infty}(1-\delta)^t\bar{\pi} - F_D w\right] - F_E w. \tag{3.13}$$

If this value is negative, no firm would invest in variety introduction. With unrestricted entry, this value cannot be positive in equilibrium. In other words $v_E = 0$. Using (3.10) and (3.11), the free entry condition for the closed-economy equilibrium without credit constraints is thus given by:

$$\bar{\pi} = \pi(\tilde{\varphi}(\varphi^*)) = \delta w \left(\frac{F_E}{1 - G(\varphi^*)} + F_D \right). \tag{3.14}$$

The expected future profits, given by the flow of profits for the average firms, must equal the expected total cost for variety introduction. This is given by the fixed cost of variety innovation $F_E w$, multiplied by the number of attempts needed to develop a successful variety, $\frac{1}{1-G(\varphi^*_{cc})}$, plus the fixed cost for entry on the domestic market, $F_D w$.

3.2. Equilibrium in a Closed Economy. The equilibrium values for the average profits $\bar{\pi}$ and for the productivity cutoff φ^* are pinned down by the system given by the free entry condition (3.14) and the zero profit condition (3.8). Under the Pareto distribution assumption, the two conditions can be re-written as

$$\begin{aligned} (FEC) \qquad \bar{\pi} =& \delta w \left(F_E \left(\frac{\varphi^*}{\varphi_m} \right)^a + F_D \right) \\ (ZPC) \qquad \bar{\pi} =& \frac{\delta F_D w a}{a - \sigma + 1}. \end{aligned}$$

The FEC describes a positive relationship between $\bar{\pi}$ and φ^*: the higher the average profits the higher the productivity required for a successful entry. The ZPC defines, for a given market entry fixed cost $F_D w$, a constant value of average profits. In a stationary equilibrium, the aggregate variables must also remain constant over time. This requires the number of total entrants M_e in each period to be such that the number of successful entrants, $M_e(1 - G(\varphi^*))$, exactly replaces the mass δM of dying firm: $[1 - G(\varphi^*)]M_e = \delta M$. New entrants employ a total amount of labor equal to the number of workers needed for product innovation plus the number of workers used to pay for domestic market entry: $L_I \equiv M_e F_E + M_e[1 - G(\varphi^*)]F_D$. Production workers in the differentiated good sector, L_P, are such that $\Pi = R - wL_P$. In the homogeneous good sector employed workers L_0 are such that $wL_0 = R_0$, where R_0 is the total revenue for this sector. The labor market clearing condition is then given by $L = L_I + L_P + L_0$. In the closed economy, the optimal consumption rule implies $R_0 = (1 - \alpha)E$ and $R = \alpha E$, where E is the economy aggregate expenditure. Finally, $\Pi = M\bar{\pi}$ (see (3.11)), $\Pi = R/\sigma$ (see appendix) and the expression for the price index $P = M^{1/(1-\sigma)}p(\tilde{\varphi})$ complete the characterization of the equilibrium.

Proposition 1. *The equilibrium exists and is unique.*

Proof: See Appendix □

The average profits are given by $\bar{\pi} = \frac{\delta F_D wa}{a-\sigma+1}$ and the productivity cut-off by $\varphi^* = \varphi_m \left[\frac{\sigma-1}{a-\sigma+1}\frac{F_D}{F_E}\right]^{\frac{1}{a}}$. Combining the labor market clearing conditions and the FEC, I can solve for the aggregate expenditure as a function of the the total labor income, $E = wL$. This implies $\Pi = \frac{\alpha wL}{\sigma}$. Given the identity $\bar{\pi} = \frac{\Pi}{M}$, the number of firms in equilibrium is given by $M = \frac{\alpha wL}{\sigma\bar{\pi}}$.

As a measure of per-capita welfare, I consider the utility U in (2.1). Using the expressions for q_0 and q_1, this is given by

$$U = \frac{e(1-\alpha)^{1-\alpha}\alpha^{\alpha}}{P^{\alpha}}. \tag{3.15}$$

Now, using the closed form solutions derived in the appendix for $e = E/L$ and P, the above can be rewritten as:

$$U = w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi^*)^{\alpha}\left[\frac{\alpha L}{\sigma\delta F_D}\right]^{\frac{\alpha}{\sigma-1}}.$$

3.3. The Open Economy. I now assume that, once the productivity level φ in known, firms can also decide to enter the foreign market. To do so they must make the initial sunk investment of F_X units of labor. International trade also requires additional variable costs, modeled in the standard iceberg formulation: $\tau > 1$ unit of a good must be shipped in order for 1 unit to reach the destination.

Domestic prices, revenues and profits are still given by (2.3), (2.4) and (2.5) and will henceforth indexed by the subscript "D". The subscript "X" will instead refer to the foreign market. Exporting firms will set higher prices on the foreign market that reflect the higher marginal cost due to τ: $p_X(\varphi) = w\tau/\varphi\rho$. This price implies revenues $r_X(\varphi) = \alpha E\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1}$ and profits $\pi_X(\varphi) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1}$, where E and P are not indexed by country due to the symmetry assumption.

Once productivity is revealed, there is no more uncertainty and each firm decides whether to immediately exit, whether to produce for one market only or to produce for both markets. Given the specificity of the sunk costs $F_D w$ and $F_X w$, the decisions to enter the two markets are taken separately, looking at the net values of the two investments. These are given by $v_D(\varphi) - F_D w$ for the domestic market and by $v_X(\varphi) -$

$F_X w$ for the export market, where $v_D(\varphi) = \frac{\pi_D(\varphi)}{\delta}$ (see(3.7)), and, analogously

$$v_X(\varphi) = \frac{\pi_X(\varphi)}{\delta}.$$

As for the closed economy, $\varphi^* = \inf\{\varphi : v_D(\varphi) > F_D w\}$ identifies the lowest productivity level of firms finding it profitable to enter the domestic market. Analogously, $\varphi_x^* = \inf\{\varphi : v_X(\varphi) > F_X w\}$ identifies the lowest productivity level of firms finding it profitable to enter the foreign market (export cutoff level). In line with the empirical evidence, I will restrict the analysis to the case in which no firm becomes an exporter without also serving the domestic market, in other words I will restrict to case $\varphi_x^* > \varphi^*$. This occurs if and only if $\tau^{\sigma-1} > \frac{F_D}{F_X}$.[2] By their definitions, these cutoff levels must satisfy $\pi_D(\varphi^*) = \delta F_D w$ and $\pi_X(\varphi_x^*) = \delta F_X w$. These two conditions imply the following relationship between φ^* and φ_x^*:

$$\varphi_x^* = \tau\varphi^* \left(\frac{F_X}{F_D}\right)^{\frac{1}{\sigma-1}}. \tag{3.16}$$

As before, the density function of productivity levels for incumbent firms, $\mu(\varphi)$, is determined by the ex-ante distribution of productivity levels, conditional on a successful entry (see (3.9)). Furthermore, $p_x = [1 - G(\varphi_x^*)]/[1 - G(\varphi^*)]$ now represents the probability of becoming an exporter and $M_X = p_x M$ represents the mass of exporting firms. Using the same weighted average function defined in (3.10), let $\tilde{\varphi} = \tilde{\varphi}(\varphi^*)$ and $\tilde{\varphi}_x = \tilde{\varphi}(\varphi_x^*)$ denote the average productivity of, respectively, all firms and exporting firms only. By construction, the productivity averages $\tilde{\varphi}$ and $\tilde{\varphi}_x$ can be used to express the average profits earned on the domestic market, $\pi_D(\tilde{\varphi})$, and the average profits from exports, $\pi_X(\tilde{\varphi}_x)$. The overall average of combined revenues and profits earned on both market are given by:

$$\bar{r} = r_D(\tilde{\varphi}) + p_x r_X(\tilde{\varphi}_x)$$

and

$$\bar{\pi} = \pi_D(\tilde{\varphi}) + p_x \pi_X(\tilde{\varphi}_x). \tag{3.17}$$

I can now exploit (2.6), to express the profits of the marginal firms in terms of the average profits: $\pi_D(\varphi^*) = \pi_D(\tilde{\varphi}) \left[\frac{\varphi^*}{\tilde{\varphi}}\right]^{\sigma-1}$ and $\pi_X(\varphi_x^*) = \pi_X(\tilde{\varphi}_x) \left[\frac{\varphi_x^*}{\tilde{\varphi}_x}\right]^{\sigma-1}$. It then

[2] See appendix.

follows:

$$\pi_D(\varphi^*) = \delta F_D w \Leftrightarrow \pi_D(\tilde{\varphi}) = \delta F_D w \left[\frac{\tilde{\varphi}}{\varphi^*}\right]^{\sigma-1}$$
$$\pi_X(\varphi_x^*) = \delta F_X w \Leftrightarrow \pi_X(\tilde{\varphi}_x) = \delta F_X w \left[\frac{\tilde{\varphi}_x}{\varphi_x^*}\right]^{\sigma-1}.$$

Together with (3.17), the above conditions allow me to express the zero profit condition in terms of the overall average profits:

$$\bar{\pi} = \delta F_D w \left[\frac{\tilde{\varphi}}{\varphi^*}\right]^{\sigma-1} + p_x \delta F_X w \left[\frac{\tilde{\varphi}_x}{\varphi_x^*}\right]^{\sigma-1}. \tag{3.18}$$

From the product innovation investment $F_E w$, each potential entrant expects to earn, with probability $1 - G(\varphi^*)$, a stream of average profits net of expected fixed cost of entry. The value of entry is now given by:

$$v_E \equiv (1 - G(\varphi^*)) \left[\sum_{t=0}^{\infty} (1-\delta)^t \bar{\pi} - F_D w - p_x F_X w\right] - F_E w. \tag{3.19}$$

Free entry implies that in equilibrium $v_E = 0$ and the free entry condition for the open economy becomes:

$$\frac{\bar{\pi}}{\delta} = w \left(\frac{F_E}{1 - G(\varphi^*)} + F_D + F_X \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)}\right). \tag{3.20}$$

3.4. Equilibrium in an Open Economy. The Pareto distribution assumption, together with (3.16), allows me to re-write the ZPC and the FEC again as a system in φ^* and $\bar{\pi}$:

$$\begin{aligned} (FEC) \quad \bar{\pi} =& \delta w F_D \left[\frac{F_E}{F_D}\left(\frac{\varphi^*}{\varphi_m}\right)^a + 1 + \tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right] \\ (ZPC) \quad \bar{\pi} =& \frac{\delta w a F_D}{a - \sigma + 1}\left[1 + \tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]. \end{aligned} \tag{3.21}$$

Aggregation follows the same logic as in the closed economy: $\Pi = M\bar{\pi}$ and $R = M\bar{r}$, with $\Pi = R/\sigma$. Stationarity again requires $\delta M = [1 - G(\varphi^*)]M_e$. An important difference with respect to the closed economy case is that L_I now includes also the workers employed for foreign market entry, i.e. $L_I = M_e F_E + M_e[1 - G(\varphi^*)]F_D + M_e[1 - G(\varphi_x^*)]F_x$. The labor market clearing condition is still given by $L = L_I + L_P + L_0$, where production workers in both sectors are determined as in autarky. In an open economy, balanced trade ensures $R_0 = (1-\alpha)E$ and $R = \alpha E$. Finally, I denote with

$\bar{M} \equiv M + M_X$ the total number of varieties available in each market and I define

$$\bar{\varphi} \equiv \left\{ \frac{1}{\bar{M}} [M \tilde{\varphi}^{\sigma-1} + M_x (\tau^{-1} \tilde{\varphi}_x)^{\sigma-1}] \right\}^{\frac{1}{\sigma-1}}$$

as the weighted productivity average that takes into account the different market shares of domestic and exporting firms, as well as the loss of productivity implied by variable transport costs. Given the country-symmetry assumption, $\bar{\varphi}$ also represents the weighted average productivity of all firms, both domestic and foreign, that compete on the same market, again adjusted by the trade cost τ. I thus write the price index as:

$$P = \bar{M}^{\frac{1}{1-\sigma}} p_D(\bar{\varphi}) = \bar{M}^{\frac{1}{1-\sigma}} \frac{w}{\rho \bar{\varphi}}.$$

PROPOSITION 2. *The equilibrium exists and is unique.*

Proof: See Appendix □

In equilibrium, $E = wL$, $\Pi = \alpha w L / \sigma$ and $M = \alpha w L / \sigma \bar{\pi}$, where $\bar{\pi}$ is given by the ZPC and the equilibrium value of φ^* is:

$$\varphi^* = \varphi_m \left\{ \left[\tau^{-a} \left(\frac{F_D}{F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 \right] \frac{\sigma - 1}{a - \sigma + 1} \frac{F_D}{F_E} \right\}^{\frac{1}{a}} \tag{3.22}$$

which is strictly decreasing in τ and higher than in the closed economy case. Welfare is again given by

$$U = w^{1-\alpha} (1-\alpha)^{1-\alpha} (\alpha \rho \varphi^*)^{\alpha} \left(\frac{\alpha L}{\sigma \delta F_D} \right)^{\frac{\alpha}{\sigma-1}}$$

which is increasing in φ^* and, as a consequence, decreasing in τ.

4. Liquidity Constraints and Credit Market Frictions

I now introduce two crucial assumptions. First, I assume that potential entrepreneurs face liquidity constraints. This means that, in order to incur any investment, they need to borrow on the financial market. For simplicity, I assume that entrepreneurs are ex-ante identical and they all have zero initial wealth. As a result, before knowing their productivity level, firms need to borrow the entire sum required for product development and variety introduction. I thus introduce a competitive credit market, where firms can borrow the sum needed to pay for the fixed cost $F_E w$, $F_D w$ and $F_X w$. In order to do that, each firm must sign a debt contract with a deep-pocket investor. At the end of any period of time, once revenues are realized and workers have been paid, the borrower is expected to pay the lender the sum established by the debt contract they had previously signed.

The second assumption is that the credit market is not perfect. I assume that parties can sign full contingent contracts but that there is imperfect lender protection. In other words, when the parties meet, firm's productivity is still unknown but they can sign a contract that establishes the future conduct, contingent on any possible realization of φ. Nevertheless, at the end of each period, the borrower can choose to hide profits and avoid the per period repayment due to the investor. If this happens, there is only a probability $\lambda \in (0, 1)$ that a court enforces the payment and punishes the borrower with a fine that exhausts all its per period profits. In other words, λ measures the efficiency of the financial market. If $\lambda = 1$, then there are no frictions on the credit market, and the entrepreneur's liquidity constraints become irrelevant.

There is another assumption that is standard in the definition of a private ownership equilibrium (see Mas-Colell et al. (1995)) and that is worth to state explicitly here. That is consumers ultimately own the firms and their initial endowment constitutes, not only of labour, but also of a claim to firms' profits. As a result, their total wealth derives from wages and firm's profits, if any. Ownership shares are assumed to be equal across all consumers.

4.1. Timing. Consider the case of a firm that decides to invent a new variety of the differentiated good. This requires the up-front innovation cost $F_E w$ and, eventually, the further entry costs $F_D w$ and $F_X w$ for the domestic and foreign markets respectively. Since the firm is liquidity constrained, it needs to find an investor. Lenders behave competitively, in the sense that the loan, if any, makes zero profits. That is, I assume that several prospective lenders compete for issuing the loans to the firms. I also assume that the contract is renegation-proof, in the sense that no one can be forced to stay in the relationship if in any period his expected profits are negative (see Laffont and Martimont (2002)).[3] In exchange for the amount $F_E w$ that the lender finances immediately, the debt contract sets a fixed ex ante transfer $K \geq 0$ from the lender to the borrower and a plan of action for each possible realization of the productivity φ. The plan of action establishes the entry rules and, in case of entry, the per-period (incentive compatible) repayment $f(\varphi)$ due by the firm as a per-period debt reimbursement. The entry rules are given by $i(\varphi) \in \{0, 1\}$ and $i_x(\varphi) \in \{0, 1\}$ for the domestic market and

[3] Do not confuse renegation-proofness with renegotiation-proofness. The first concept, used in this paper, requires that no one can commit himself to stay in the relationship if, at any stage, his individual continuation value is lower than his outside option. The renegotiation-proofness refers to imposing that the contract is such that the coalition of the firm and the creditor cannot improve the joint pay-off by renegotiating the contract.

foreign market respectively. If $i(\varphi) = 1$ then the firm enters the domestic market, if $i(\varphi) = 0$ then there is no entry. The same is true for foreign market entry rule: if $i_x(\varphi) = 1$ then the firm enters the foreign market, otherwise it stays out. As the contract has been signed, the firm incurs the cost $F_E w$ and discovers its productivity level: if the productivity is such that entry costs are not financed, the firm is forced to exit immediately, and the lender loses the initial investment $F_E w$. If entry is financed, in either one or both markets, the firm starts production and revenues are realized. Once workers have been paid the wage w, the firm decides whether or not to pay the per-period debt reimbursements $f(\varphi)$. In case of no repayment, with probability λ a court will enforce the reimbursement and will impose a fee equal to the remaining firm's profits.[4] At the beginning of the next period, the firm faces the exogenous shock that leads to exit with probability δ. If the firm survives, the game is repeated from the production stage. Figure 1 summarizes the timing of the events.

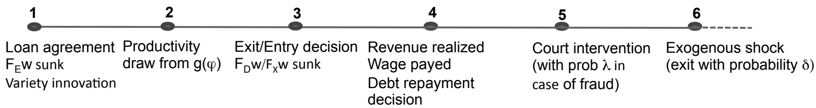

FIGURE 1. Timing

4.2. The Optimal Debt Contract in a Closed Economy. Starting from the closed economy case, where the only entry decision concerns the domestic market and the fixed cost $F_D w$, I will solve for an equilibrium characterized by unrestricted entry of profit maximizing firms and by perfect competition on the credit market. The optimal consumption decisions and period profits are the same as derived in sections 2.1 and 2.2. I split the rest of the problem in two steps. The first step is to derive the optimal debt contract that the firm will offer as a take-it or leave-it offer to the potential investor. The second step is to derive the general equilibrium conditions and to solve for all the endogenous aggregate variables of the model.

[4] The fee can be interpreted as a reimbursement of the trial expenses and of the cost of the court's work that would otherwise be borne by the society.

Each firm will chose the optimal contract by solving:

$$\max_{i(\varphi)\in\{0,1\},f(\varphi),K} v_E \equiv \int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t(\pi(\varphi)-f(\varphi))\right] i(\varphi)g(\varphi)d\varphi + K \tag{P}$$

subject to

$$K \geq 0, \tag{LC}$$

$$\pi(\varphi) - f(\varphi) \geq (1-\lambda)\pi(\varphi) \text{ for all } \varphi \text{ such that } i(\varphi)=1, \tag{IC}$$

$$v_L \equiv \int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi) - F_D w\right] i(\varphi)g(\varphi)d\varphi - F_E w - K \geq 0, \tag{PC}$$

$$\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi) \geq F_D w \text{ for all } \varphi \text{ such that } i(\varphi)=1. \tag{RP}$$

The objective function of the firm is given by the present discounted value of future profits, as defined by (2.5), net of the per period repayment $f(\varphi)$, and plus the ex ante transfer K. The profits are earned only when entry occurs, and this happens when $i(\varphi) = 1$. The requirement that K is non-negative derives from the liquidity constraint of the borrower (LC), which is a key assumption in the analysis. At stage 4 in Figure 1, φ is known, the firm has received the second loan $F_D w$, and it has entered the market and started production. Once revenues are realized and wages paid, the firm is left with profits $\pi(\varphi)$. The firm will behave if the profits $\pi(\varphi)$, net of the per period repayment $f(\varphi)$, exceed the expected gain from avoiding the payment and risking, with probability λ, being fined by a court. This implies the incentive compatibility constraint (IC). The lender's participation constraint (PC) requires the expected value of the contract in period 1 v_L to be non-negative. This happens when the present discounted value of the per period repayments, net of the entry cost $F_D w$, exceeds the the initial investment $F_E w$ and the transfer K. Entry occurs according to the entry rule, when $i(\varphi) = 1$. The last constraint is the renegation-proof condition (RP). It requires the lender's expected payoff to be positive also at stage 3 in Figure 1, once the productivity is known and the initial investment $F_E w$ is sunk. The lender finds it profitable to invest the further amount $F_D w$, if this does not exceed the present discounted value of the flow of future incentive compatible repayments.

In the appendix I spell out all the steps that allow me to solve for the optimal contract. First notice that the firm can increase v_E by raising the term $T \equiv K - \int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi)\right] i(\varphi)g(\varphi)d\varphi$ until the (PC) binds ($v_L = 0$). As a result in equilibrium it must hold that

$$K = \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty} (1-\delta)^t f(\varphi) \right] i(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} F_D w i(\varphi) g(\varphi) d\varphi - F_E w \geq 0. \quad (4.23)$$

It follows that $v_E = \int_{\varphi_m}^{\infty} \frac{\pi(\varphi)}{\delta} i(\varphi) g(\varphi) d\varphi - \int_{\varphi_m}^{\infty} F_D w i(\varphi) g(\varphi) d\varphi - F_E w$. This implies that the optimal contract is not unique. Any pair (K, $f(\varphi)$) such that (4.23) holds together with the (IC) and (RP) requirements that $\delta F_D w \leq f(\varphi) \leq \lambda \pi(\varphi)$ is a valid solution to (P). Nevertheless, the way the transfer T is split between the ex-ante transfer K and the ex-post per period repayments $f(\varphi)$ does not affect the optimal value v_E. If for the generic firm with productivity φ (IC) and (RP) can be slack, this is not the case for the marginal firm. In order to meet the lender's (RP) condition, the marginal firm has the incentive to pay a per period repayment $f(\varphi)$ as high as the (IC) allows for. In other words, a binding (IC) and a binding (RP) solve for the entry rule $i(\varphi) = 1$ if and only if $\varphi \geq \varphi_{cc}^*$, where φ_{cc}^* represents the entry-productivity cut-off such that:

$$\lambda \pi(\varphi_{cc}^*) = \delta F_D w. \quad (4.24)$$

The condition above is thus the equivalent of the zero profit condition (3.8) for the frictionless set-up. Any firm with productivity $\varphi \geq \varphi_{cc}^*$ will obtain the financing needed to enter the market. If $\varphi < \varphi_{cc}^*$, then the firm is forced to exit immediately. Since $\lambda < 1$, the profitability of the marginal firm is higher than the minimum productivity levels that would satisfy the efficiency requirement $\pi(\varphi) \geq \delta F_D w$. The intuition is that higher profits are needed to give firms the right incentives to comply with the debt contract. To summarize:

PROPOSITION 3. *Any optimal contract satisfies:*

I.

$$i(\varphi) = \begin{cases} 1 & \textit{if } \varphi \geq \varphi_{cc}^*, \\ 0 & \textit{otherwise} \end{cases}$$

where φ_{cc}^* *is such that* $\pi(\varphi_{cc}^*) = \frac{\delta F_D w}{\lambda}$*;*

II. $(K, f(\varphi))$ *are such that*

$$K = \int_{\varphi_{cc}^*}^{\infty} \frac{f(\varphi)}{\delta} g(\varphi) d\varphi - (1 - G(\varphi_{cc}^*)) F_D w - F_E w \geq 0 \quad (4.25)$$

and

$$\begin{cases} f(\varphi) = \lambda \pi(\varphi) & \textit{if } \varphi = \varphi_{cc}^*, \\ \delta F_D w < f(\varphi) \leq \lambda \pi(\varphi) & \textit{if } \varphi > \varphi_{cc}^*. \end{cases}$$

Moreover

$$v_E = \int_{\varphi^*_{cc}}^{\infty} \frac{\pi(\varphi)}{\delta} g(\varphi)d\varphi - (1 - G(\varphi^*_{cc}))F_D w - F_E w.$$

Proof: See Appendix □

4.2.1. *Equilibrium in a Closed Economy.* In the frictionless set-up with unrestricted entry, the notion of equilibrium requires that v_E cannot be positive since the mass of prospective entrants is unbounded (see also Melitz (2003)). Since no firms would want to enter the market when $v_E < 0$, $v_E = 0$ represents what was defined as the free entry condition. With liquidity constrained firms and credit frictions, things becomes more complicated. The equilibrium notion provided wants to capture the idea (much as in any other paper) that no firm left outside of the market should find entry strictly profitable ($v_E > 0$) and strictly feasible ($K > 0$) at the same time. Differently from the frictionless set up where profitability and feasibility coincide, here I need to introduce the second condition that refers to the ability of firms to find a creditor. I will require that in equilibrium at least one of the above conditions fails. First of all, as in the frictionless set-up, $v_E < 0$ is ruled out as an equilibrium outcome. Now consider $v_E = 0$. Notice that the maximum value that can be set for K is the one for $f(\varphi) = \lambda\pi(\varphi)$ for all φ such that $\varphi \geq \varphi^*_{cc}$. I thus define $K_{max} \equiv \int_{\varphi^*_{cc}}^{\infty} \frac{\lambda\pi(\varphi)}{\delta} g(\varphi)d\varphi - (1-G(\varphi^*_{cc}))F_D w - F_E w < v_E$. It follows that $v_E = 0$ implies $K_{max} < 0$, meaning that even if the firm were to pledge as much future income as possible (credit markets are imperfect so the firm cannot pledge all future income), the creditors would still require an upfront payment to end up even. Given the liquidity constrained (4.25), $K \geq 0$ is required. By definition, also K_{max} must be non-negative and $v_E = 0$ is thus ruled out as an equilibrium outcome. I conclude that in equilibrium it must be that $v_E > 0$. It follows that the unbounded mass of prospective entrants must find entry strictly not feasible in equilibrium, that is that the maximum possible K must be equal to zero. $K_{max} = 0$ is thus the new FEC and it can be simplified to obtain

$$\lambda\bar{\pi}_{cc} = \delta w \left(\frac{F_E}{1 - G(\varphi^*_{cc})} + F_D \right). \tag{4.26}$$

where $\bar{\pi}_{cc} \equiv [1 - G(\varphi^*_{cc})]^{-1} \int_{\varphi^*_{cc}}^{\infty} \pi(\varphi)g(\varphi)d\varphi$ are the average profits. Condition (4.26) has an intuitive economic interpretation, analogous to the FEC in the frictionless setup (see (3.14)). Firms will be able to obtain credit and enter the market until the maximum lender's expected pledgeable profits, given by λ times the flow of profits for the average firm, equal the expected total cost for variety introduction. This is given by the fixed cost of variety innovation $F_E w$, multiplied by the number of attempts needed to develop a successful variety, $\frac{1}{1-G(\varphi^*_{cc})}$, plus the fixed cost for entry on the domestic market, $F_D w$. The existence of credit market frictions creates thus positive rents equal to $v_E = (1 - G(\varphi^*_{cc}))\frac{(1-\lambda)\bar{\pi}_{cc}}{\delta}$ which reduce the resources invested in innovation and entry. By assumptions, these rents are split uniformly among consumers, according to their ownership shares.

The distribution of firms conditional on entry is now given by

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*_{cc})} & \text{if } \varphi \geq \varphi^*_{cc}, \\ 0 & \text{otherwise} \end{cases}$$

and the weighted average productivity by $\tilde{\varphi}_{cc} \equiv \tilde{\varphi}(\varphi^*_{cc}) = \left[\frac{1}{1-G(\varphi^*_{cc})}\int_{\varphi^*_{cc}}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}$. It is convenient to rewrite the profits of the marginal entry firm $\pi(\varphi^*_{cc})$ in terms of the average profits, which in the appendix are shown to equal the profits of the firm with the weighted average productivity ($\bar{\pi}_{cc} = \pi(\tilde{\varphi}_{cc})$). According to (2.6), $\pi(\varphi^*_{cc}) = \bar{\pi}_{cc}\left[\frac{\varphi^*_{cc}}{\tilde{\varphi}_{cc}}\right]^{\sigma-1}$ and (4.24), the new zero profit condition (ZPC) is

$$\lambda\bar{\pi}_{cc} = \delta F_D w \left[\frac{\tilde{\varphi}_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1}. \tag{4.27}$$

The equilibrium values for the average profits $\bar{\pi}_{cc}$ and for the productivity cutoff φ^*_{cc} are pinned down by the system given by (4.27) and (4.26). Notice that the two equations can also be interpreted as a system in the productivity cutoff φ^*_{cc} and in the new variable $\bar{z} \equiv \lambda\bar{\pi}_{cc}$, that is the maximum pledgeable profits. This is the average revenue of the agent, now the lender, who commits to the innovation and entry investments. For $\lambda < 1$, clearly $\bar{z} < \bar{\pi}_{cc}$. When φ is Pareto distributed on $[\varphi_m, \infty)$ with shape parameter a, the system can be re-written as:

$$\begin{aligned} (ZPC) \quad \lambda\bar{\pi}_{cc} &= \frac{\delta F_D w a}{a - \sigma + 1} \\ (FEC) \quad \lambda\bar{\pi}_{cc} &= \delta w \left(F_E \left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + F_D\right). \end{aligned} \tag{4.28}$$

Compared to the frictionless setup, with $\lambda < 1$ the ZPC and the FEC must associate to each cutoff value a higher average profit for the maximum pledgeable income to be in line with the ante and the ex post investment costs.

Given the equilibrium values $\bar{\pi}_{cc}$ and φ^*_{cc}, the equilibrium values for the other endogenous variables are pinned down by the exact same relationships described in the no-credit friction setup. The number of total entrants M_e in each period is such that the number of successful entrants exactly replace the mass of dying firm:

$[1-G(\varphi^*_{cc})]M_e = \delta M$. The labor market clearing condition implies $L = L_0 + L_P + L_I$, where $wL_0 = R_0$, $wL_P = R - \Pi$, and $L_I = M_e F_E + M_e[1-G(\varphi^*_{cc})]F_D$. The optimal consumption rule implies $R_0 = (1-\alpha)E$ and $R = \alpha E$. Finally, $\Pi = M\bar{\pi}$, $\Pi = R/\sigma$ and the expression for the price index $P = M^{1/(1-\sigma)}p(\tilde{\varphi}_{cc})$ complete the characterization of the equilibrium.

PROPOSITION 4. *The equilibrium exists and is unique. Furthermore some firms are credit constrained ($\pi(\varphi^*_{cc}) > \delta F_D w$) and φ^*_{cc}, $\bar{\pi}_{cc}$ and M satisfy:*

$$\frac{\partial \bar{\pi}_{cc}}{\partial \lambda} < 0,\ \frac{\partial \varphi^*_{cc}}{\partial \lambda} = 0,\ \frac{M}{\partial \lambda} > 0.$$

Proof: See Appendix □

The proposition above states that credit frictions prevent some firms from entering the domestic market, although they would be profitable enough to do so. When firms can avoid repaying the debt, profits must be high enough for the risk of court's intervention to give the right incentive to behave. Knowing that, lenders will finance the entry investment $F_D w$ only to firms with profits $\pi(\varphi) \geq \frac{\delta F_D w}{\lambda} > \delta F_D w$. As a result, average profits are higher than in a frictionless setup ($\bar{\pi}_{cc} > \bar{\pi}$) and they are increasing in credit frictions ($\lambda \downarrow$). Higher equilibrium average profits are not due to a higher average productivity $\tilde{\varphi}(\varphi^*_{cc})$, but to a lower number of active firms M. According to Proposition 4, $\varphi^*_{cc} = \varphi^*$ and is constant in λ, while M is increasing in λ and takes on a maximum value when there are no credit frictions ($\lambda = 1$). The intuition for this result is the following: when credit frictions increase ($\lambda \downarrow$), the profits of the marginal firm, and so the average profits $\bar{\pi}_{cc}$, have to increase for the entry investment $F_D w$ to be profitable from the lender's point of view (the ZPC moves upwards). $\bar{\pi}_{cc}$ can increase either via an increase of the productivity cutoff φ^*_{cc} or via a decrease in the number of firms which leads to an increase in $\pi(\varphi)$ for all $\varphi \geq \varphi^*_{cc}$. In the first case ($\varphi^*_{cc} \uparrow$), the chance to extract a good productivity level would decrease and the expected cost of entry would go up ($\varphi^*_{cc} \uparrow \Rightarrow$ *r.h.s.* of FEC $\uparrow$). According to the FEC, this would require the average profits $\bar{\pi}_{cc}$ to increase even further (*l.h.s.* of FEC). With $\bar{\pi}_{cc}$ increased more than in proportion to the initial change in λ, the *ZPC* would be violated and this cannot represent the new equilibrium. This implies that $\bar{\pi}_{cc}$ can increase only as a result of a lower mass M of firms. The FEC moves upward but in such a way that in equilibrium φ^*_{cc} is unchanged. Another way to interpret this result is to look at the system (4.28) as a system in φ^*_{cc} and in the pledgeable income $\bar{z}$. Then, λ does not directly affect neither the investment for domestic market entry (*r.h.s.* of ZPC) nor the expected cost of innovation ($r.h.s.$ of FEC). As a result both $\bar{z}$ and φ^*_{cc} are constant and only $\bar{\pi}_{cc}$ varies according to $\bar{\pi}_{cc} = \bar{z}/\lambda$.

To summarize, the tighter credit market restricts the number of firms obtaining the loan and this increases the marginal and the average profits. Everything else equal, a less efficient credit market does not affect the average productivity $\tilde{\varphi}_{cc}$, but allows only a smaller number of bigger firms (in terms of average sale $\bar{r}$) to get access to credit ($\lambda \downarrow \Rightarrow \bar{\pi}_{cc} \uparrow \Rightarrow M \downarrow$and $\bar{r} = \bar{\pi}_{cc}/\sigma \uparrow$).

LEMMA 5. *In the closed economy equilibrium with credit frictions, per-capita welfare U is given by*

$$U = \sigma w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi^*_{cc})^{\alpha}\left[\frac{\alpha L}{\delta F_D}\right]^{\frac{\alpha}{\sigma-1}}\lambda^{\frac{\alpha}{\sigma-1}}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}$$

and $\frac{\partial U}{\partial \lambda} > 0$.

Proof: See Appendix □
According the the lemma above, when credit frictions become less severe ($\lambda \uparrow$), per-capita welfare increases. This effect is the result of the increase in the number of varieties that, due to the love for variety assumption, pushes down the price index P and increases real expenditure for the differentiated goods q_1 ($\lambda \uparrow \Rightarrow M \uparrow \Rightarrow M^{1/(\sigma-1)} \uparrow \Rightarrow P \downarrow \Rightarrow U \uparrow$). Welfare reaches the same level as in the frictionless set-up when $\lambda = 1$.

4.3. The Optimal Debt Contract in an Open Economy. I now introduce the possibility of becoming an exporter. This means that the debt contract stipulates, conditionally on the realization of φ, whether the firm should start production and, in this case, whether entry in the domestic market only or entry in both the domestic and the foreign market is financed.[5] Accordingly, the contract specifies the level of the debt per-period repayments. I now denote with $i(\varphi) \in \{0,1\}$ and with $i_x(\varphi) \in \{0,1\}$ the entry rule for the domestic and the foreign market respectively, with $f(\varphi)$ the reimbursement for firms engaged in the domestic market only and with $f'(\varphi)$ the reimbursement for firms engaged in both the domestic and the foreign market. In this second case, the reimbursement also covers the additional investment $F_X w$. Each firm

[5] As for the no-credit friction case, I focus only on the case where exporting firms also serve the domestic market.

will chose the optimal contract by solving:

$$\max_{i(\varphi),i_x(\varphi),f(\varphi),f'(\varphi),K} v_E \equiv \int_{\varphi_m}^{\infty} \left[\sum_{t=0}^{\infty}(1-\delta)^t(\pi_D(\varphi)-f(\varphi))\right](i(\varphi)-i_x(\varphi))g(\varphi)d\varphi$$
$$+\int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t(\pi_D(\varphi)+\pi_X(\varphi)-f'(\varphi))\right] i_x(\varphi)g(\varphi)d\varphi+K \tag{P'}$$

subject to

$$K \geq 0, \tag{LC}$$

$$\pi_D(\varphi)-f(\varphi) \geq (1-\lambda)\pi_D(\varphi), \text{ for all } \varphi \text{ such that } i(\varphi)=1 \text{ and } i_x(\varphi)=0 \tag{IC}$$

$$\pi_D(\varphi)+\pi_X(\varphi)-f'(\varphi) \geq (1-\lambda)(\pi_D(\varphi)+\pi_X(\varphi)), \tag{IC'}$$

for all φ such that $i(\varphi)=1$ and $i_x(\varphi)=1$

$$v_L \equiv \int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi)-F_D w\right](i(\varphi)-i_x(\varphi))g(\varphi)d\varphi$$
$$+\int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t f'(\varphi)-(F_D+F_X)w\right] i_x(\varphi)g(\varphi)d\varphi - F_E w - K \geq 0, \tag{PC}$$

$$\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi) \geq F_D w, \text{ for all } \varphi \text{ such that } i(\varphi)=1 \text{ and } i_x(\varphi)=0 \tag{RP}$$

$$\sum_{t=0}^{\infty}(1-\delta)^t f'(\varphi) \geq (F_D+F_X)w, \tag{RP'}$$

for all φ such that $i(\varphi)=1$ and $i_x(\varphi)=1$

The objective function of the firm is given by the expected discounted value of net future profits plus the ex ante transfer K. When only domestic entry occurs, net profits are given by $\pi_D(\varphi)-f(\varphi)$, and, given the assumption that no firm exports without also serving the domestic market, this happens when $i(\varphi)-i_x(\varphi)=1$. When entry in both markets occurs, then net profits are given by $\pi_D+\pi_X(\varphi)-f'(\varphi)$, and this happens when $i_x(\varphi)=1$. Again, the requirement that K is non-negative derives from the liquidity constraint of the borrower (LC). At stage 4 in Figure 1, the domestic firm with known productivity φ faces the same incentive compatible constraint (IC) described above. The firm with known productivity φ that produces for both markets pays the per-period reimbursement $f'(\varphi)$ if the domestic and foreign market profits, net of the payment $f'(\varphi)$, exceed the expected profits from cheating. This condition is expressed by the second incentive compatibility constraint (IC'). The lender's participation constraint (PC) requires the expected value of the contract in period 1 to be non-negative. This happens when the present discounted value of per period repayments, net of the entry costs, exceeds the the initial investment $F_E w$ and the transfer K. Entry occurs in either one or both markets according to the entry rules $i(\varphi)$ and $i_x(\varphi)$. The last two constraints are the lender's renegation-proof conditions, requiring the lender's continuation payoff at stage 3 to be positive. The lender finds it profitable to invest further in market entry, if the fixed costs do not exceed the present discounted value of the flow of future incentive compatible repayments. Again, I distinguish between the entry in the domestic market only (RP) and the entry in both the domestic and foreign market (RP').

As before, the firm will increase the term $T' \equiv K - \int_{\varphi_m}^{\infty} \frac{f(\varphi))}{\delta}(i(\varphi) - i_x(\varphi))g(\varphi)d\varphi - \int_{\varphi_m}^{\infty} \frac{f'(\varphi))}{\delta} i_x(\varphi)g(\varphi)d\varphi$ until (PC) becomes binding. As a result, in equilibrium

$$K = \int_{\varphi_m}^{\infty} \left[\frac{f(\varphi)}{\delta} - F_D w\right] (i(\varphi) - i_x(\varphi))g(\varphi)d\varphi + \int_{\varphi_m}^{\infty} \left[\frac{f'(\varphi)}{\delta} - F_D w - F_X w\right] i_x(\varphi)g(\varphi)d\varphi \\ - F_E w \geq 0. \tag{4.29}$$

As for the closed economy, the optimal contract is not unique. Any triplet (K, $f(\varphi)$, $f'(\varphi)$) such that (4.29) holds together with (IC), (RP), (IC') and (RP') is a valid solution to (P). Nevertheless, the way the transfer T is split between the ex-ante transfer K and the ex-post per period repayments does not affect the optimal value v_E. Although the (IC) and the (IC') do not have to be binding for the generic firm with productivity φ, they will be for the marginal firms entering the domestic and the foreign market respectively. Setting $f(\varphi)$ and $f'(\varphi)$ as high as the (IC) and the (IC') allow for, makes it easier to meet the lender renegation proof constraint. For the marginal firms it must be that $f(\varphi) = \lambda\pi(\varphi)$ and that $f'(\varphi) = \lambda[\pi_D(\varphi) + \pi_X(\varphi)]$. Plugging these values into the renegation-proof conditions, they become

$$\pi_D(\varphi) \geq \frac{\delta F_D w}{\lambda}$$
$$\pi_D(\varphi) + \pi_X(\varphi) \geq \frac{\delta[F_D + F_X]w}{\lambda}. \tag{4.30}$$

$F_D/F_X < \tau^{\sigma-1}$ still ensures that no firm becomes an exporter without also serving the domestic market. Condition (4.30) implicitly states that firms can use the profits gained on the domestic market to finance the repayment of the exporting fixed costs.[6]

[6] To see this, condition (4.30) can be rewritten as $\pi_X(\varphi) + \left[\pi_D(\varphi) - \frac{\delta F_D w}{\lambda}\right] \geq \frac{\delta F_X w}{\lambda}$.

On the other hand, no firms will be willing to enter the foreign market if the present discounted profits from exporting do not exceed the entry cost $F_X w$. It follows that $\varphi^*_{cc} = \{\varphi : \pi_D(\varphi) = \frac{\delta F_D w}{\lambda}\}$ always identifies the domestic market productivity cutoff, while the lowest productivity level of exporting firms $\varphi^*_{cc\_x}$ is given by the largest between $\inf\{\varphi : \pi_D(\varphi) + \pi_X(\varphi) > \frac{\delta[F_D + F_X]w}{\lambda}\}$ and the cutoff defined, as in the no credit friction case, by $\inf\{\varphi : \pi_X(\varphi) > \delta F_X w\}$. Which of the two cases prevails in defining the equilibrium cutoff $\varphi^*_{cc\_x}$ depends on λ.

PROPOSITION 6. *The foreign market cutoff is defined by* $\varphi^*_{cc\_x} = \{\varphi : \pi_X(\varphi) = \delta F_X w\}$ *if* $\lambda \geq \hat{\lambda}$ *and by* $\varphi^*_{cc\_x} = \{\varphi : \pi_X(\varphi) = \frac{w\delta(F_D + F_X)}{\lambda(1+\tau^{\sigma-1})}\}$ *if* $\lambda < \hat{\lambda}$*, where* $\hat{\lambda} \equiv \frac{F_D/F_X + 1}{1+\tau^{\sigma-1}}$.

Proof: See Appendix □

When $\lambda < 1$, domestic firms are always credit-constrained, meaning that there are firms that would find it profitable to enter the domestic market, meaning $\frac{\pi_D(\varphi)}{\delta} \geq F_D w$, but, due to imperfect creditor protection, are not able to borrow the sum $F_D w$. Conversely, Proposition 6 implies that exporters are credit-constrained only for low values of λ. A firm could obtain the credit needed for both domestic and foreign market entry even when foreign market entry itself is not optimal, meaning $\frac{\pi_X(\varphi)}{\delta} < F_X w$. Entry in the foreign market would then implies net losses and this possibility is excluded by the contract; entry in the export market is determined by the first best condition instead. This happens because when the repayment of the two investments is joint the risk of loosing both $\pi_D(\varphi)$ and $\pi_X(\varphi)$ can make it easier to meet (4.30) than to meet the first best condition $\frac{\pi_X(\varphi)}{\delta} \geq F_X w$. Based on proposition 6, the worse financial institutions are (low λ), the higher the cost of domestic market entry is (high F_D), the lower the cost of foreign market entry is (low F_X) and the lower the variable trade costs are (low τ), the more likely it is for exporting decisions to be constrained. The effect of λ is clear: when credit frictions are higher, the incentive to misbehave is also higher. Higher profits are thus needed to ensure the repayment. The effect of the other three parameters derives from the fact that the net domestic profits $\pi_D(\varphi) - F_D w$ can be seen as extra resources to finance the foreign market entry. The higher F_D, the lower these net profits and so the higher $\pi_X(\varphi)$ must be. The lower τ, the lower the value of $\pi_D(\varphi)$ associated with a given value of $\pi_X(\varphi)$.[7] If domestic net profits are lower, then higher foreign market profits are needed to compensate. A higher F_X always increases the productivity needed for foreign market entry. On the other hand, this effect could

[7] Remember that $\pi_D(\varphi) = \tau^{\sigma-1}\pi_X(\varphi)$.

be stronger without credit frictions than with credit frictions. The reason is that in the first case the fixed cost is covered by foreign market profits only, while in the second case it is covered by both the foreign market profits and the net domestic profits.

The entry rules are thus given by $i(\varphi) = 1$ if and only if $\varphi \geq \varphi^*_{cc}$ and by $i_x(\varphi) = 1$ if and only if $\varphi \geq \varphi^*_{cc\_x}$. The following proposition summarizes the results just described.

Proposition 7. *Any optimal contract satisfies:*

I.
$$i(\varphi) = \begin{cases} 1 & \textit{if } \varphi \geq \varphi^*_{cc}, \\ 0 & \textit{otherwise} \end{cases}$$
where $\varphi^*_{cc} = \{\varphi : \pi_D(\varphi) = \frac{\delta F_D w}{\lambda}\}$*;*

II.
$$i_x(\varphi) = \begin{cases} 1 & \textit{if } \varphi \geq \varphi^*_{cc\_x}, \\ 0 & \textit{otherwise} \end{cases}$$
where $\varphi^*_{cc\_x} = \{\varphi : \pi_X(\varphi) = \delta F_X w\}$ *if* $\lambda \geq \hat{\lambda}$ *and by* $\varphi^*_{cc\_x} = \left\{\varphi : \pi_X(\varphi) = \frac{w\delta(F_D+F_X)}{\lambda(1+\tau^{\sigma-1})}\right\}$ *if* $\lambda < \hat{\lambda}$*;*

III. $(K, f(\varphi), f'(\varphi))$ *are such that*

$$K = \int_{\varphi^*_{cc}}^{\varphi^*_{cc\_x}} \left[\frac{f(\varphi)}{\delta} - F_D w\right] g(\varphi)d\varphi + \int_{\varphi_{cc\_x^*}}^{\infty} \left[\frac{f'(\varphi)}{\delta} - (F_D + F_X)w\right] g(\varphi)d\varphi - F_E w \geq 0, \tag{4.31}$$

$$\begin{cases} f(\varphi) = \lambda\pi_D(\varphi) & \textit{if } \varphi = \varphi^*_{cc} \\ \delta F_D w \leq f(\varphi) \leq \lambda\pi_D(\varphi) & \textit{if } \varphi^*_{cc} < \varphi < \varphi^*_{cc\_x}. \end{cases}$$

and

$$\begin{cases} f'(\varphi) = \lambda(\pi_D(\varphi) + \pi_X(\varphi)) & \textit{if } \varphi = \varphi^*_{cc\_x}, \\ \delta w(F_D + F_X) \leq f'(\varphi) \leq \lambda(\pi_D(\varphi) + \pi_X(\varphi)) & \textit{if } \varphi > \varphi^*_{cc\_x}. \end{cases}$$

Moreover

$$v_E = \int_{\varphi^*_{cc}}^{\infty} \left[\frac{\pi_D(\varphi)}{\delta} - F_D w\right] g(\varphi)d\varphi + \int_{\varphi^*_{cc\_x}}^{\infty} \left[\frac{\pi_X(\varphi)}{\delta} - F_X w\right] g(\varphi)d\varphi - F_E w.$$

Proof: See Appendix □

4.3.1. *Equilibrium in an Open Economy.* In equilibrium no firm left outside of the market should find entry strictly profitable ($v_E > 0$) and strictly feasible ($K > 0$) at the same time. I will require that in equilibrium at least one of the above conditions fails. The maximum value for K is the one for $f(\varphi) = \lambda\pi_D(\varphi)$ for all $\varphi \in [\varphi^*_{cc}, \varphi^*_{cc\_x})$ and $f'(\varphi) = \lambda(\pi_D(\varphi) + \pi_X(\varphi))$ for all $\varphi \geq \varphi^*_{cc\_x}$. Plugging this value into (4.31), I obtain

$$K_{max} = \int_{\varphi^*_{cc}}^{\infty} \left[\frac{\lambda \pi_D(\varphi)}{\delta} - F_D w \right] g(\varphi) d\varphi + \int_{\varphi^*_{cc\_x}}^{\infty} \left[\frac{\lambda \pi_X(\varphi)}{\delta} - F_X w \right] g(\varphi) d\varphi - F_E w < v_E. \tag{4.32}$$

As for the closed economy case, free entry cannot lead neither to $v_E = 0$ nor to $v_E < 0$. The unbounded mass of prospective entrant together with the firms liquidity constraint require that in equilibrium $K_{max} = 0$.[8] This is the FEC, and it can be simplified to obtain:

$$\lambda \bar{\pi}_{cc} = \delta w \left[\frac{F_E}{1 - G(\varphi^*_{cc})} + F_D + F_X \frac{1 - G(\varphi^*_{cc\_x})}{1 - G(\varphi^*_{cc})} \right]. \tag{4.33}$$

In equilibrium firms will be able to obtain credit and enter the market until the maximum lender's expected pledgeable profits $\lambda \bar{\pi}_{cc}$ equal the expected total cost for variety introduction. This is given by the fixed cost of variety innovation $F_E w$, multiplied by the number of attempts needed to develop a successful variety, $\frac{1}{1-G(\varphi^*_{cc})}$, plus the fixed cost for entry on the domestic market, $F_D w$, plus the fixed cost for entry on the foreign market $F_X w$, multiplied by the probability $[1 - G(\varphi^*_{cc\_x})]/[1 - G(\varphi^*_{cc})]$. The existence of credit market friction creates positive rents equal to $v_E = (1 - G(\varphi^*_{cc})) \frac{(1-\lambda)\bar{\pi}_{cc}}{\delta}$ which reduce the resources invested in innovation and entry. These rents are split uniformly among consumers, according to their ownership shares.

The aggregate variables and the equilibrium conditions are derived as in the case for $\lambda = 1$. The density function of productivity levels for incumbent firms, $\mu(\varphi)$, is determined by the ex-ante distribution of productivity levels, conditional on a successful entry (see (3.9)). The ex-ante probability of becoming an exporter is given by $p_x = [1 - G(\varphi^*_{cc\_x})]/[1 - G(\varphi^*_{cc})]$ and $M_X = p_x M$ is the mass of exporting firms. Using the same weighted average function defined in (3.10), let $\tilde{\varphi}_{cc} = \tilde{\varphi}(\varphi^*_{cc})$ and $\tilde{\varphi}_{cc\_x} = \tilde{\varphi}(\varphi^*_{cc\_x})$ denote the average productivity of, respectively, all firms and exporting firms only. The overall average of combined revenues and profits earned on both markets is given by:

$$\bar{r}_{cc} = r_D(\tilde{\varphi}_{cc}) + p_x r_X(\tilde{\varphi}_{cc\_x})$$

and

$$\bar{\pi}_{cc} = \pi_D(\tilde{\varphi}_{cc}) + p_x \pi_X(\tilde{\varphi}_{cc\_x}) \tag{4.34}$$

where $r_D(\tilde{\varphi}_{cc})$ and $r_X(\tilde{\varphi}_{cc\_x})$, $\pi_D(\tilde{\varphi}_{cc})$ and $\pi_X(\tilde{\varphi}_{cc\_x})$ are respectively the average revenue and profits from domestic sales and exports. Using (2.6) to express the profits of the marginal firms in terms of the average profits together with (4.34), I can write the zero

[8] See the section for the closed economy case for a more complete argument.

profit condition in terms of the overall average profits as

$$\bar{\pi}_{cc} = \begin{cases} \frac{\delta F_D w}{\lambda}\left[\frac{\tilde{\varphi}_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1} + p_x \frac{\delta(F_D+F_X)w}{\lambda(1+\tau^{\sigma-1})}\left[\frac{\tilde{\varphi}_{cc\_x}}{\varphi^*_{cc\_x}}\right]^{\sigma-1} & \text{if } 0<\lambda<\hat{\lambda} \\ \frac{\delta F_D w}{\lambda}\left[\frac{\tilde{\varphi}_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1} + p_x \delta F_X w \left[\frac{\tilde{\varphi}_{cc\_x}}{\varphi^*_{cc\_x}}\right]^{\sigma-1} & \text{if } \hat{\lambda}\le\lambda\le 1. \end{cases} \tag{4.35}$$

It is also useful to derive the ratio between the two productivity cutoffs:

$$\frac{\varphi^*_{cc}}{\varphi^*_{cc\_x}} = \begin{cases} \tau^{-1}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{1}{\sigma-1}} & \text{if } 0<\lambda<\hat{\lambda}, \\ \tau^{-1}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{1}{\sigma-1}} & \text{if } \hat{\lambda}\le\lambda\le 1. \end{cases} \tag{4.36}$$

The equilibrium values for the average profits $\bar{\pi}_{cc}$ and for the productivity cutoff φ^*_{cc} are pinned down by the system given by the conditions (4.33) and (4.35). When φ is Pareto distributed on $[\varphi_m, \infty)$ with shape parameter a, the system becomes

$$\begin{aligned} (FEC) \quad \lambda\bar{\pi}_{cc} =& \delta w F_D \left[\frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + \tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}} + 1\right] \\ (ZPC) \quad \lambda\bar{\pi}_{cc} =& \frac{\delta w a F_D}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right] \end{aligned} \tag{4.37}$$

when $0<\lambda<\hat{\lambda}$ and

$$\begin{aligned} (FEC) \quad \lambda\bar{\pi}_{cc} =& \delta w F_D \left[\frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + \tau^{a}\lambda^{-1}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}} + 1\right] \\ (ZPC) \quad \lambda\bar{\pi}_{cc} =& \frac{\delta w a F_D}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right] \end{aligned} \tag{4.38}$$

when $\hat{\lambda}\le\lambda\le 1$. As for the closed economy case, the conditions above can also be interpreted as a system in the cutoff φ^*_{cc} and in the maximum expected pledgeable income of the investor $\lambda\bar{\pi}_{cc}$, which is ultimately the base to evaluate the profitability of the overall entry cost.

Given the equilibrium values $\bar{\pi}_{cc}$ and φ^*_{cc}, the equilibrium values for the other endogenous variables are pinned down by the exact same relationships described in the no-credit friction setup. From (4.36) I can solve for $\varphi^*_{cc\_x}$. Aggregation implies $\Pi = M\bar{\pi}_{cc}$ and $R = M\bar{r}_{cc}$, with $\Pi = R/\sigma$. Stationarity requires $\delta M = [1-G(\varphi^*_{cc})]M_e$. The demands for labor satisfies $\Pi = R - wL_P$ for production workers in the differentiated good sector, $wL_0 = R_0$ for workers employed in the homogeneous good sector, and

$L_I = M_e F_E + M_e[1 - G(\varphi^*_{cc})]F_D + M_e[1 - G(\varphi^*_{cc\_x})]F_X$ for workers employed in product innovation and market entry. The labor market clearing condition is thus given by $L = L_I + L_P + L_0$. In an open economy, balanced trade ensures $R_0 = (1-\alpha)E$ and $R = \alpha E$. M is given by $\Pi/\bar{\pi}_{cc}$ and M_X by $M\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}$. Given the total number of varieties available in each market $\bar{M} \equiv M + M_X$, the overall productivity average weighted by the market shares of domestic and exporting firms is now:

$$\bar{\varphi}_{cc} \equiv \left\{ \frac{1}{\bar{M}} [M\tilde{\varphi}_{cc}^{\sigma-1} + M_x(\tau^{-1}\tilde{\varphi}_{cc\_x})^{\sigma-1}] \right\}^{\frac{1}{\sigma-1}}.$$

Since $\bar{\varphi}_{cc}$ also represents the weighted average productivity of all domestic and foreign firms competing on the same market, the price index is given by $P = \bar{M}^{\frac{1}{1-\sigma}} p_D(\bar{\varphi}_{cc}) = \bar{M}^{\frac{1}{1-\sigma}} \frac{w}{\rho\bar{\varphi}_{cc}}$. In the appendix I also derive the total value of firms rents $V_E \equiv M_e v_E = (1-\lambda)\Pi$. These are resources that are misallocated as a result of credit market frictions. Because of the moral hazard problem, firms must be granted resources that would be otherwise allocated to innovation and entry investments.

PROPOSITION 8. *The equilibrium exists and is unique.*

Proof: See Appendix □

5. Steady-State Properties of the Model

In this section I describe the properties of the open economy equilibrium with respect to the credit market frictions λ and the trade frictions τ. In particular, I study how the endogenous variables of the model change when λ and τ vary symmetrically in the two countries. All of the following analyses rely on comparison of steady state equilibria and should therefore be interpreted as capturing the long run consequences of changes in the economic environment.

PROPOSITION 9. *φ^*_{cc} is continuous and not decreasing in $\lambda \in (0,1]$. In particular, $\varphi^*_{cc} =$*

$$\begin{cases} \varphi_m \left\{ \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{1+\tau^{\sigma-1}}{1+F_D/F_X} \right)^{\frac{a}{\sigma-1}} \left(\frac{F_D}{F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left[\frac{a}{a-\sigma+1} \left(\frac{1+\tau^{\sigma-1}}{1+F_D/F_X} \right)^{-1} - 1 \right] \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} & \text{if } 0 < \lambda < \hat{\lambda}, \\ \varphi_m \left\{ \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} & \text{if } \hat{\lambda} \le \lambda < 1, \\ \varphi_m \left\{ \left[\tau^{-a} \left(\frac{F_D}{F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 \right] \frac{\sigma-1}{a-\sigma+1} \frac{F_D}{F_E} \right\}^{\frac{1}{a}} = \varphi^* & \text{if } \lambda = 1. \end{cases}$$

and

$$\frac{\partial \varphi^*_{cc}}{\partial \lambda}\begin{cases} = 0 & \text{if } 0 < \lambda < \hat{\lambda}, \\ > 0 & \text{if } \hat{\lambda} \leq \lambda < 1. \end{cases}$$

Moreover, the mass of firms M is continuous and increasing in $\lambda \in (0,1]$ and the relative number of exporting firms $M_X/M = \left(\frac{\varphi^*_{cc}}{\varphi^*_{cc\_x}}\right)^a$ is continuous and non-increasing in $\lambda \in (0,1]$.

Proof: See Appendix □

Proposition 9 contains one of the main results and it is surprising. It states that the productivity threshold, and so the average productivity $\tilde{\varphi}_{cc} = \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}} \varphi^*_{cc}$, does not depend on credit market frictions when frictions are high ($\lambda < \hat{\lambda}$), while it increases with better financial institutions when credit market frictions are low ($\lambda \geq \hat{\lambda}$). In other words, because of general equilibrium effects and because of the interaction between domestic and foreign market entry, the minimum and average firm productivity does not increase when credit frictions increase and obtaining credit becomes more difficult. The best way to see this is to look at the FEC and at the conditions defining φ^*_{cc} and $\varphi^*_{cc\_x}$, expressed in terms of the pledgeable profits $z_D(\varphi) \equiv \lambda\pi_D(\varphi)$ and $z_X(\varphi) \equiv \lambda\pi_X(\varphi)$. Setting (4.32) equal to zero, I obtain:

$$\int_{\varphi^*_{cc}}^{\infty}\left[\frac{z_D(\varphi)}{\delta} - F_D w\right] g(\varphi)d\varphi + \int_{\varphi_{cc\_x}}^{\infty}\left[\frac{z_X(\varphi)}{\delta} - F_X w\right] g(\varphi)d\varphi = F_E w \tag{5.39}$$

while the productivity cutoffs are given by:

$$z_D(\varphi^*_{cc}) = \delta F_D w \tag{5.40}$$

and, from Proposition 6, by

$$z_X(\varphi^*_{cc\_x}) = \begin{cases} \frac{w\delta(F_D+F_X)}{1+\tau^{\sigma-1}} & \text{if } 0 < \lambda < \hat{\lambda}, \\ \lambda\delta F_X w & \text{if } \hat{\lambda} \leq \lambda \leq 1 \end{cases} \tag{5.41}$$

Let's first consider the case $0 < \lambda < \hat{\lambda}$. None of the three conditions depends on λ. It follows that φ^*_{cc}, $\varphi^*_{cc\_x}$, $z_D(\varphi^*_{cc})$ and $z_X(\varphi^*_{cc\_x})$ are constant in λ. Given the definitions of $z_D(\varphi)$ and $z_X(\varphi)$, when λ varies, $\pi_D(\varphi)$ and $\pi_X(\varphi)$ have to vary accordingly for all $\varphi \geq \varphi^*_{cc}$ in order for $z_D(\varphi^*_{cc})$ and $z_X(\varphi^*_{cc\_x})$ to be unchanged. For instance, when credit frictions increase ($\lambda \downarrow$), it must be the case that for every firm profits go up ($\pi_D(\varphi) \uparrow$ and $\pi_X(\varphi) \uparrow$). This is possible only if the mass of firms becomes smaller ($M \downarrow$). When $\hat{\lambda} \leq \lambda \leq 1$, a change in λ directly affects (5.41). For instance when λ goes down, then also $z_X(\varphi^*_{cc\_x})$ goes down. Since $z_X(\varphi^*_{cc\_x}) = \lambda\delta F_X w < \delta F_X w$, this means that the value of second integral in (5.39) becomes smaller. If the creditor's net expected pledgeable profit from foreign market entry decreases, then net expected pledgeable profits from domestic market entry must increase for the total value of the investment to be equal to the initial cost $F_E w$. The term $\int_{\varphi^*_{cc}}^{\infty} \left[\frac{z_D(\varphi)}{\delta} - F_D w\right] g(\varphi) d\varphi$ can increase either if φ^*_{cc} goes down or if $z_D(\varphi)$ goes up for all $\varphi \geq \varphi^*_{cc}$. Since from (5.40) $z_D(\varphi^*_{cc})$ is constant in λ, they will both move. If $z_D(\varphi)$ goes up for all $\varphi \geq \varphi^*_{cc}$, then also $z_X(\varphi)$ increase for all $\varphi \geq \varphi^*_{cc\_x}$. To bring this result together with the fact that $z_X(\varphi^*_{cc\_x})$ decreases, $\varphi^*_{cc\_x}$ must also decrease. If $z_D(\varphi)$ and $z_X(\varphi)$ increase for all active firms when λ goes down, then also the profits $\pi_D(\varphi)$ and $\pi_X(\varphi)$ must increase. As a result the mass M of firms must become smaller. ($\lambda \downarrow \Rightarrow \varphi^*_{cc} \downarrow$, $\varphi^*_{cc\_x} \downarrow$ and $M \downarrow$).

The economic intuition behind this result is the following: when credit frictions increase ($\lambda \downarrow$), so does the misallocation of resources from innovation to firms rents V_E. It follows that there are less firms in equilibrium and this lack of competition keeps profits high, allowing also inefficient firms to enter the two markets. As a result φ^*_{cc} falls below φ^*. When credit frictions are high enough ($\lambda < \hat{\lambda}$), both domestic and foreign market entry are restricted and $\frac{\varphi^*_{cc}}{\varphi^*_{cc\_x}}$ does not depend on λ (see (4.36)). In this case the increase in average profits fully compensate for the decrease in λ and the two cutoff are constant in λ. When instead credit fractions are lower ($\lambda \geq \hat{\lambda}$), domestic profits are high enough to facilitate the entry on the foreign market and only domestic market entry is restricted. It follows that, when λ decreases there is an additional effect: $p_x = \frac{M_X}{M} = \left(\frac{\varphi^*_{cc}}{\varphi^*_{cc\_x}}\right)^a$ increases, meaning that entry in the foreign market becomes relatively easier (see (4.36)). This implies that for the creditor the expected investment increases and, for the FEC to keep holding, the probability of a successful productivity draw must go up ($\varphi^*_{cc} \downarrow$).

The bottom line of proposition 9 is that credit frictions imply a misallocation of resources that reduces the number of incumbents, the competition and the average productivity. The resource misallocation is given by the resources diverted from R&D activities and by the too high number of exporters relatively to the total number of firms.

PROPOSITION 10. *For $\lambda \in (0,1)$, $\partial\varphi^*_{cc}/\partial\tau < 0$ if a is sufficiently small (close to $\sigma - 1$) and $\partial\varphi^*_{cc}/\partial\tau > 0$ otherwise (if a is sufficiently large). The mass of firms M is always increasing in τ while M_X/M is decreasing in τ.*

Proof: See Appendix □

The equilibrium properties for M and M_X/M are the same as in Melitz (2003) and in the related heterogeneous firms literature. Lower trade costs ($\tau \downarrow$) induce more firms to become exporters ($M_X/M \uparrow$) and, given the higher expected profits that this implies, push more firms to invest in introducing new products. This increases the demand for both production and innovation workers and, given the limited number of resources L in the economy, less firms can be sustained in equilibrium ($M \downarrow$). In Melitz (2003), this increased competition among firms forces the least productive firms to exit and the domestic productivity cut-off is always increasing as τ decreases. When there are credit markets frictions, this is no longer always true and the effect of τ on φ^*_{cc} depends on the productivity distribution of firms in the differentiated goods sector. According to Proposition 10, trade liberalization induces exit of the least productive firms ($\partial\varphi^*_{cc}/\partial\tau < 0$) only when the parameter a is low enough. This parameter characterizes the shape of the Pareto distribution of the productivity levels. In particular, the smaller is a, the larger the proportion of very-high productivity firms. The explanation for this result is in the wedge that the credit frictions create between the total expected profits from market entry and the maximum expected profits that can be pledged to the lender. For instance, when τ decreases, the expected profits $\bar{\pi}_{cc}$ increase because the expected profits from exports increase. This is due to the effect on $\pi_X(\tilde{\varphi}_{cc\_x})$ and to the effect on p_x (see both (3.17) and (4.34)). On the other hand, a lower τ also affects the expected cost of entry, but only by increasing the probability p_x of incurring the fixed cost $F_X w$. In the frictionless set up, this means that expected profits increase more than the expected cost for foreign market entry and the other components of the expected cost have to decrease according to the FEC. This can happen only if φ^* increases (see (3.20)). In the setting with credit frictions, for each increase in the expected profits, the increase in the fixed costs is proportionally higher because the lender is entitled only up to a fraction λ of those profits (see (4.33)). As a result the increase in the fixed costs for foreign market entry can be higher than the corresponding increase in profits and, for the free entry condition to hold, φ^*_{cc} has to fall. Whether the increase in expected profits is higher or not than the increase in the expected cost depends on the parameter a. In particular, when a is small, the distribution of productivity is more concentrated at high levels and the average productivity $\tilde{\varphi}_{cc\_x}$ tends to be higher. Since the effect of τ on $\pi_X(\tilde{\varphi}_{cc\_x})$ is proportional to $\tilde{\varphi}_{cc\_x}$, when a is small enough this effect is higher than the effect on the expected cost and φ^*_{cc} is decreasing in τ. Conversely, when a is large enough, the average productivity is small and so is the effect of τ on $\pi_X(\tilde{\varphi}_{cc\_x})$. It follows that $\partial\varphi^*_{cc}/\partial\tau > 0$.

Lemma 11. *In the open economy equilibrium with credit frictions, per-capita welfare U is given by*

$$U = \sigma w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi_{cc}^{*})^{\alpha}\left[\frac{\alpha L}{\delta F_D}\right]^{\frac{\alpha}{\sigma-1}}\lambda^{\frac{\alpha}{\sigma-1}}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}$$

Moreover $\frac{\partial U}{\partial \lambda} > 0$, $\partial U/\partial\tau < 0$ *when a is sufficiently small and* $\partial U/\partial\tau > 0$ *otherwise (when a is sufficiently large).*

Proof: See Appendix □

According to Proposition 9, the number of domestic varieties M is increasing in λ, and the domestic productivity φ_{cc}^{*}, which determines the level of average productivity, is non-decreasing in λ. Because of these effects, Lemma 11 claims that consumer welfare, which is increasing in the average productivity and in the number of varieties available to consumers, is also increasing in λ. The Lemma also claims that the effect of trade liberalization on consumers welfare depends on the exogenous parameter a. The effect of τ on U ultimately depends on the domestic productivity cut-off φ_{cc}^{*} and, according to Proposition 10, trade liberalization induces the productivity cut-off to decrease when a is large enough and to increase when a is small enough. This result is in contrast with Melitz (2003) which predicts that trade liberalization has unambiguously positive effects on consumers welfare.

6. Concluding Remarks

This paper presents a general equilibrium trade model with heterogenous firms and imperfect credit markets. Before knowing their productivity level, firms must find a creditor willing to finance the cost for product innovation and the eventual costs associated with domestic and foreign market entry. If a firm obtains the loan and starts production, then in every period there is a probability $1-\lambda > 0$ that it can avoid the debt reimbursement established by the debt contract without incurring in any sanction. To solve for the model equilibrium, I first solve for the optimal contract that maximizes the firm's expected profits, given the firm's liquidity and incentive constraint and the lender's participation constraint. I thus find the entry rules for the domestic and the foreign market. I then solve for the productivity cutoffs for domestic firms and exporters and, given the market clearing conditions, I can find a closed form solution for all the aggregate variables that define the general equilibrium of the model.

The main focus of the paper is on studying the steady-state equilibrium effects of financial frictions in a symmetric countries case. I show the importance of considering a general equilibrium setting in order to fully characterize the misallocations of resources that derive from the existence of credit frictions. First of all, the moral hazard problem generates rents for the firm that reduce the resources employed in product innovation and market entry. As a result the total number of firms is suboptimal; this reduces competition among incumbents allowing low efficiency firms to survive. Credit frictions thus imply a lower average productivity. A lower number of firms and a lower average productivity have a negative effects on individual utility: consumer welfare is decreasing in the level of credit frictions. The second form of resources misallocation goes from domestic to foreign firms. The fact that domestic profits can be de facto used to back-up the loan to pay for foreign market entry, makes exporters less likely to be credit constrained, at least for not too low levels of financial development.

The paper also shows that allowing for liquidity constrained firms and imperfect credit markets changes, and in some cases reverses, some of main results from the heterogeneous firms literature. The model predicts that trade liberalization does not necessarily lead to an increase of average productivity and consumers' welfare. In particular, I show that trade liberalization increases consumer welfare only when the distribution of firm productivity is more concentrated towards high values.

Considering a general equilibrium setting with an endogenous mass of firms, this model fills an important gap in the recent literature on trade and financial development. Moreover, given its tractability, the model could be easily extended in some relevant directions. First of all, allowing for higher frictions of export financing would reconcile the model with the prediction present in other contributions that export activities are more dependent on external capital. Second, allowing for asymmetric credit frictions between countries, it would be possible to study the effects of financial developments on the patterns of trade. These are possible directions for future research.

7. Technical Appendix: the perfect credit market setting

7.1. Aggregate quantities in the closed economy. Given there is a mass M of firms in equilibrium and that $\mu(\varphi)$ is the distribution of productivity of active firms defined on a subset of $(0,\infty)$, from $r(\varphi) = \alpha E \left(P\frac{\rho\varphi}{\omega}\right)^{\sigma-1}$ I can derive the aggregate revenue as

$$\begin{aligned}
R &= \int_0^\infty r(\varphi)M\mu(\varphi)d\varphi \\
&= M\int_0^\infty \alpha E\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1}\mu(\varphi)d\varphi \\
&= M\alpha E\left(P\frac{\rho}{w}\right)^{\sigma-1}\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right] \\
&= M\alpha E\left(P\frac{\rho}{w}\right)^{\sigma-1}\tilde{\varphi}^{\sigma-1} \\
&= M\alpha E\left(P\frac{\rho\tilde{\varphi}}{w}\right)^{\sigma-1} \\
&= Mr(\tilde{\varphi}).
\end{aligned}$$

where $\tilde{\varphi} \equiv \left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$. Using $\pi(\varphi) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{\omega}\right)^{\sigma-1}$, I obtain

$$\begin{aligned}
\Pi &= \int_0^\infty \pi(\varphi)M\mu(\varphi)d\varphi \\
&= M\int_0^\infty \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1}\mu(\varphi)d\varphi \\
&= M\frac{\alpha E}{\sigma}\left(P\frac{\rho}{w}\right)^{\sigma-1}\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right] \\
&= M\frac{\alpha E}{\sigma}\left(P\frac{\rho}{w}\right)^{\sigma-1}\tilde{\varphi}^{\sigma-1} \\
&= M\frac{\alpha E}{\sigma}\left(P\frac{\rho\tilde{\varphi}}{w}\right)^{\sigma-1} \\
&= M\pi(\tilde{\varphi}).
\end{aligned}$$

Moreover,

$$\Pi = M\frac{\alpha E}{\sigma}\left(P\frac{\rho\tilde{\varphi}}{w}\right)^{\sigma-1} = \frac{R}{\sigma}.$$

Analogously, the price index for the consumption bundle can be written as

$$\begin{aligned} P &= \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \\ &= \left[\int_0^\infty p(\varphi)^{1-\sigma} M\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}}. \end{aligned}$$

Now, using $p(\varphi) = w/\rho\varphi$, I obtain

$$\begin{aligned} P &= \left[\int_0^\infty \left(\frac{w}{\rho\varphi}\right)^{1-\sigma} M\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} \\ &= M^{\frac{1}{1-\sigma}}\frac{w}{\rho}\left[\int_0^\infty \left(\frac{1}{\varphi}\right)^{1-\sigma} \mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} \\ &= M^{\frac{1}{1-\sigma}}\frac{w}{\rho}\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{-\frac{1}{\sigma-1}} \\ &= M^{\frac{1}{1-\sigma}}\frac{w}{\rho\tilde{\varphi}} \\ &= M^{\frac{1}{1-\sigma}}p(\tilde{\varphi}). \end{aligned}$$

7.2. FEC in the closed economy. The free entry condition requires that in equilibrium $v_E = 0$. Plugging in (3.13), this is equivalent to

$$\begin{aligned} (1-G(\varphi^*))\left[\sum_{t=0}^{\infty}(1-\delta)^t\bar{\pi} - F_D w\right] - F_E w &= 0 \\ (1-G(\varphi^*))\left[\frac{\bar{\pi}}{\delta} - F_D w\right] &= F_E w \\ \frac{\bar{\pi}}{\delta} - F_D w &= \frac{F_E w}{1-G(\varphi^*)} \end{aligned}$$

Rearranging, I obtain

$$\frac{\bar{\pi}}{\delta} = w\left(\frac{F_E}{1-G(\varphi^*)} + F_D\right)$$

where (3.10) and (3.11) imply that the left hand side is a function of φ^*, since $\bar{\pi} = \pi(\tilde{\varphi})$ and $\tilde{\varphi} = \tilde{\varphi}(\varphi^*)$.

7.3. The FEC and the ZPC under the Pareto distribution assumption-closed economy. Given the Pareto cumulative distribution function $G(\varphi) = 1 - \left(\frac{\varphi_m}{\varphi}\right)^a$, the FEC can be rewritten as:

$$
\begin{aligned}
\bar{\pi} &= \delta w \left(\frac{F_E}{1 - G(\varphi^*)} + F_D \right) \\
&= \delta w \left(\frac{F_E}{1 - 1 + \left(\frac{\varphi_m}{\varphi^*}\right)^a} + F_D \right) \\
&= \delta w \left(F_E \left(\frac{\varphi^*}{\varphi_m} \right)^a + F_D \right).
\end{aligned}
$$

The probability density function of the Pareto distribution is given by:

$$
\begin{aligned}
g(\varphi) &= G'(\varphi) \\
&= -(-a)(\varphi)^{-a-1}(\varphi_m)^a \\
&= \frac{a}{\varphi} \left(\frac{\varphi_m}{\varphi} \right)^a.
\end{aligned}
$$

Given $a > \sigma - 1$, I can now re-write the average productivity as:

$$
\begin{aligned}
\tilde{\varphi} \equiv \tilde{\varphi}(\varphi^*) &= \left[\frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{1 - 1 + \left(\frac{\varphi_m}{\varphi^*}\right)^a} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{a}{\varphi} \left(\frac{\varphi_m}{\varphi} \right)^a d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{\varphi^*}{\varphi_m} \right)^a a \varphi_m^a \int_{\varphi^*}^{\infty} \varphi^{\sigma-1-1-a} d\varphi \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi^*)^a}{\sigma - 1 - a} \varphi^{\sigma-1-a} |_{\varphi^*}^{\infty} \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi^*)^a}{\sigma - 1 - a} \left(0 - (\varphi^*)^{\sigma-1-a} \right) \right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a}{a - \sigma + 1} \right]^{\frac{1}{\sigma-1}} \varphi^*.
\end{aligned}
$$

Using this expression in (3.12), the ZPC becomes:

$$\begin{aligned}\bar{\pi} =& \delta F_D w \left[\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right]^{\sigma-1} \\ =& \delta F_D w \left[\frac{\left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}} \varphi^*}{\varphi^*}\right]^{\sigma-1} \\ =& \frac{\delta F_D w a}{a-\sigma+1}.\end{aligned}$$

7.4. Proof of proposition 1: existence and uniqueness of equilibrium in a closed economy. In the FEC and ZPC, if φ^* is replaced by φ and $\bar{\pi}$ is replaced by π, then these equations can be graphed in (φ, π) space, as illustrated in Figure 2. The FEC is increasing in φ, with $\pi = \delta w(F_E + F_D)$ at $\varphi = \varphi_m$ and the ZPC is constant in φ for all $\varphi \geq \varphi_m$. The FEC cuts the ZPC line only once from below (see Figure 2) when:

$$\begin{aligned}\frac{\delta F_D w a}{a-\sigma+1} >& \delta w(F_E + F_D) \\ \frac{F_D a}{a-\sigma+1} >& F_E + F_D \\ \frac{a}{a-\sigma+1} >& \frac{F_E}{F_D} + 1 \\ \frac{a-(a-\sigma+1)}{a-\sigma+1} >& \frac{F_E}{F_D} \\ \frac{\sigma-1}{a-\sigma+1} >& \frac{F_E}{F_D}.\end{aligned}$$

Under this condition, which holds when F_E is small enough, not all firms are productive enough to enter the market. In this case $\varphi^* > \varphi_m$ and $\bar{\pi} = \frac{\delta F_D w a}{a-\sigma+1}$. The solution for φ^*

is given by:

$$\begin{aligned}
\delta w\left(F_E\left(\frac{\varphi^*}{\varphi_m}\right)^a+F_D\right) =& \frac{\delta F_D w a}{a-\sigma+1}\\
F_E\left(\frac{\varphi^*}{\varphi_m}\right)^a+F_D =& \frac{F_D a}{a-\sigma+1}\\
\frac{F_E}{F_D}\left(\frac{\varphi^*}{\varphi_m}\right)^a+1 =& \frac{a}{a-\sigma+1}\\
\left(\frac{\varphi^*}{\varphi_m}\right)^a =& \frac{a-a+\sigma-1}{a-\sigma+1}\frac{F_D}{F_E}\\
\varphi^* =& \varphi_m\left[\frac{\sigma-1}{a-\sigma+1}\frac{F_D}{F_E}\right]^{\frac{1}{a}}.
\end{aligned}$$

As shown above in the technical appendix, φ^* uniquely determines $\tilde{\varphi}(\varphi^*) = \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi^*$.

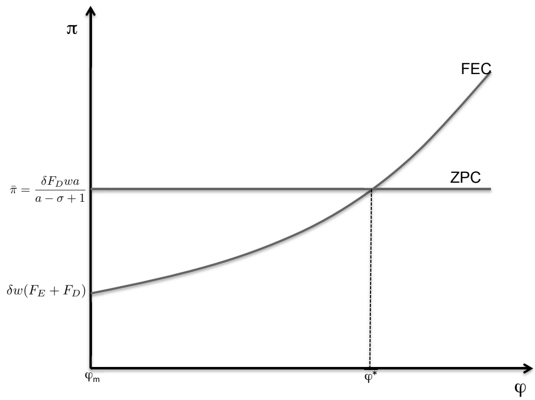

FIGURE 2. Determination of Equilibrium φ^* and $\bar{\pi}$

The FEC also implies that

$$\begin{aligned}
\bar{\pi} &= \delta w\left(\frac{F_E}{1-G(\varphi^*)}+F_D\right)\\
M\bar{\pi} &= M\delta w\left(\frac{F_E}{1-G(\varphi^*)}+F_D\right)\\
\Pi &= M\delta w\left(\frac{F_E}{1-G(\varphi^*)}+F_D\right).
\end{aligned}$$

Given $[1-G(\varphi^*)]M_e = \delta M$, I can derive the total wages of innovation workers wL_I as follows:

$$\begin{aligned}
L_I =& M_e F_E + M_e[1-G(\varphi^*)]F_D \\
L_I =& \frac{\delta M}{1-G(\varphi^*)}[F_E + (1-G(\varphi^*))F_D] \\
L_I =& \delta M \left(\frac{F_E}{1-G(\varphi^*)} + F_D \right) \\
L_I w =& \delta M w \left(\frac{F_E}{1-G(\varphi^*)} + F_D \right).
\end{aligned}$$

Combining the expressions for Π and for $L_I w$ with $R = \sigma\Pi$, I obtain $L_I w = \Pi = \frac{R}{\sigma}$. The wages to production workers in the differentiated sector are given by $wL_P = R - \Pi = R(\sigma-1)/\sigma$ and wages to workers in the homogeneous good sector are given by $L_0 w = R_0$. Now, using $R_0 = (1-\alpha)E$ and $R = \alpha E$, I can rearrange the labor market clearing condition to solve for the aggregate equilibrium expenditure E:

$$\begin{aligned}
L =& L_I + L_P + L_0 \\
wL =& wL_I + wL_P + wL_0 \\
wL =& \frac{R}{\sigma} + \frac{R(\sigma-1)}{\sigma} + R_0 \\
wL =& \frac{\alpha E}{\sigma} + \frac{\alpha E(\sigma-1)}{\sigma} + (1-\alpha)E \\
\sigma wL =& \alpha E + \alpha E\sigma - \alpha E + \sigma E - \sigma\alpha E \\
wL =& E
\end{aligned}$$

This implies $\Pi = \frac{R}{\sigma} = \frac{\alpha E}{\sigma} = \frac{\alpha wL}{\sigma}$. Given the identity $\bar{\pi} = \frac{\Pi}{M}$, the number of firms is pinned down by $M = \frac{\alpha wL}{\sigma\bar{\pi}}$. Given the expression for $\bar{\pi}$ defined by the ZPC, this is equivalent to:

$$M = \frac{\alpha wL}{\sigma}\frac{a-\sigma+1}{\delta F_D wa} = \frac{\alpha L}{\sigma}\frac{a-\sigma+1}{\delta F_D a}$$

Once M is known, also the equilibrium price index is known and given by $P = M^{1/(1-\sigma)}p(\tilde{\varphi})$. Given $p(\tilde{\varphi}) = \frac{w}{\rho\tilde{\varphi}} = \frac{w}{\rho\varphi^*}\left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{1-\sigma}}$, this is equal to:

$$P = \left[\frac{\alpha L}{\sigma}\frac{a-\sigma+1}{\delta F_D a}\right]^{\frac{1}{1-\sigma}} \frac{w}{\rho\varphi^*}\left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{1-\sigma}} = \left[\frac{\alpha L}{\sigma\delta F_D}\right]^{\frac{1}{1-\sigma}} \frac{w}{\rho\varphi^*}.$$

As a measure of per-capita welfare, I consider the individual utility U in (2.1). Given the expressions for q_0 and q_1, this is given by

$$U = q_0^{1-\alpha} q_1^{\alpha} = ((1-\alpha)e)^{1-\alpha} \left(\frac{\alpha e}{P}\right)^{\alpha} = \frac{e(1-\alpha)^{1-\alpha}\alpha^{\alpha}}{P^{\alpha}}.$$

Using the closed form solutions for $e = E/L = w$ and P, welfare becomes:

$$U = \frac{w(1-\alpha)^{1-\alpha}\alpha^{\alpha}}{w^{\alpha}} (\rho\varphi^*)^{\alpha} \left[\frac{\alpha L}{\sigma\delta F_D}\right]^{\frac{\alpha}{\sigma-1}} = w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi^*)^{\alpha} \left[\frac{\alpha L}{\sigma\delta F_D}\right]^{\frac{\alpha}{\sigma-1}}.$$

7.5. Sorting condition between domestic firms and exporters. I need to find the condition such that $\varphi^* < \varphi_x^*$, where φ^* is defined by $\pi_D(\varphi^*) = \delta F_D w$ and φ_x^* by $\pi_X(\varphi_x^*) = \delta F_X w$. Given $\pi_D(\varphi^*) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi^*}{w}\right)^{\sigma-1}$ and $\pi_X(\varphi_x^*) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi_x^*}{w\tau}\right)^{\sigma-1}$, I can solve for the two cutoffs as:

$$\begin{aligned}
\delta F_D w =& \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi^*}{w}\right)^{\sigma-1} \\
\delta\frac{F_D w\sigma}{\alpha E} =& \left(P\frac{\rho\varphi^*}{w}\right)^{\sigma-1} \\
P\frac{\rho\varphi^*}{w} =& \left(\frac{\delta F_D w\sigma}{\alpha E}\right)^{\frac{1}{\sigma-1}} \\
\varphi^* =& \frac{w}{P\rho}\left(\frac{\delta F_D w\sigma}{\alpha E}\right)^{\frac{1}{\sigma-1}}
\end{aligned}$$

and

$$\begin{aligned}
\delta F_X w =& \frac{\alpha E}{\sigma}\left(\frac{P\rho\varphi_x^*}{\tau w}\right)^{\sigma-1} \\
\frac{\delta F_X w\sigma}{\alpha E} =& \left(\frac{P\rho\varphi_x^*}{w\tau}\right)^{\sigma-1} \\
\frac{P\rho\varphi_x^*}{w\tau} =& \left(\frac{\delta F_X w\sigma}{\alpha E}\right)^{\frac{1}{\sigma-1}} \\
\varphi_x^* =& \frac{w\tau}{P\rho}\left(\frac{\delta F_X w\sigma}{\alpha E}\right)^{\frac{1}{\sigma-1}}.
\end{aligned}$$

Dividing side by side the expressions for φ^* and φ_x^*, I can show that they are linked by the relationship:

$$\begin{aligned}\frac{\varphi_x^*}{\varphi^*} =& \frac{\frac{w\tau}{P\rho}\left(\frac{\delta F_X w\sigma}{\alpha E}\right)^{\frac{1}{\sigma-1}}}{\frac{w}{P\rho}\left(\frac{\delta F_D w\sigma}{\alpha E}\right)^{\frac{1}{\sigma-1}}}\\ \varphi_x^* =& \tau\varphi^*\left(\frac{F_X}{F_D}\right)^{\frac{1}{\sigma-1}}.\end{aligned}$$

Furthermore, it follows:

$$\begin{aligned}\varphi^* < \varphi_x^* \Leftrightarrow & \frac{w}{P\rho}\left(\frac{\delta F_D w\sigma}{\alpha E}\right)^{\frac{1}{\sigma-1}} < \frac{w\tau}{P\rho}\left(\frac{\delta F_X w\sigma}{\alpha E}\right)^{\frac{1}{\sigma-1}}\\ \Leftrightarrow & F_D^{\frac{1}{\sigma-1}} < \tau F_X^{\frac{1}{\sigma-1}}\\ \Leftrightarrow & \frac{F_D}{F_X} < \tau^{\sigma-1}.\end{aligned}$$

7.6. Aggregate quantities in the open economy. In the open economy, the price index for the consumption bundle includes all domestic varieties plus the varieties exported by the foreign country. The probability density function of productivity levels on the subset $[\varphi^*, \infty)$ is given by $g(\varphi)/[1 - G(\varphi^*)]$. Because of symmetry I can write P as:

$$\begin{aligned}P =& \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}\\ =& \left[\int_{\varphi^*}^{\infty} p_D(\varphi)^{1-\sigma} M\frac{g(\varphi)}{1-G(\varphi^*)}d\varphi + \int_{\varphi_x^*}^{\infty} p_X(\varphi)^{1-\sigma} M\frac{g(\varphi)}{1-G(\varphi^*)}d\varphi\right]^{\frac{1}{1-\sigma}}.\end{aligned}$$

Now, using $M_X = p_x M$, $p_x = \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}$, $p_D(\varphi) = w/\rho\varphi$ and $p_X(\varphi) = w\tau/\rho\varphi$, I obtain

$$
\begin{aligned}
P &= \left[\int_{\varphi^*}^{\infty} p_D(\varphi)^{1-\sigma} M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} p_X(\varphi)^{1-\sigma} M_X \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi\right]^{\frac{1}{1-\sigma}} \\
&= \frac{w}{\rho}\left[M \int_{\varphi^*}^{\infty} \left(\frac{1}{\varphi}\right)^{1-\sigma} \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + M_X \int_{\varphi_x^*}^{\infty} \left(\frac{\tau}{\varphi}\right)^{1-\sigma} \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi\right]^{\frac{1}{1-\sigma}} \\
&= \frac{w}{\rho}\left[M \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + M_X \tau^{1-\sigma} \int_{\varphi_x^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi\right]^{\frac{1}{1-\sigma}} \\
&= \frac{w}{\rho}\left[M\tilde{\varphi}^{\sigma-1} + M_X \tau^{1-\sigma} \tilde{\varphi}_x^{\sigma-1}\right]^{\frac{1}{1-\sigma}} \\
&= \frac{w}{\rho}\left[\frac{\bar{M}}{\bar{M}}\left(M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}\right)\right]^{\frac{1}{1-\sigma}} \\
&= \bar{M}^{\frac{1}{1-\sigma}} \frac{w}{\rho\bar{\varphi}} \\
&= \bar{M}^{\frac{1}{1-\sigma}} p_D(\bar{\varphi})
\end{aligned}
$$

where $\bar{M} \equiv M + M_X$ and $\bar{\varphi} \equiv \left\{\frac{1}{\bar{M}}[M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}]\right\}^{\frac{1}{\sigma-1}}$. Analogously, using $r_D(\varphi) = \alpha E\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1}$ and $r_X(\varphi) = \alpha E\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1}$, the aggregate revenue is given by

$$
\begin{aligned}
R &= \int_{\varphi^*}^{\infty} r_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} r_X(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi \\
&= \int_{\varphi^*}^{\infty} r_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} r_X(\varphi) M_X \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi \\
&= \int_{\varphi^*}^{\infty} \alpha E\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1} M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} \alpha E\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1} M_X \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi \\
&= \alpha E\left(P\frac{\rho}{w}\right)^{\sigma-1}\left[M \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + M_X \int_{\varphi_x^*}^{\infty} \tau^{1-\sigma}\varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi\right] \\
&= \alpha E\left(P\frac{\rho}{w}\right)^{\sigma-1}\left[M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}\right] \\
&= M\alpha E\left(P\frac{\rho\tilde{\varphi}}{\omega}\right)^{\sigma-1} + M_X\alpha E\left(P\frac{\rho\tilde{\varphi}_x}{\omega\tau}\right)^{\sigma-1} \\
&= M r_D(\tilde{\varphi}) + M_X r_X(\tilde{\varphi}_x).
\end{aligned}
$$

Dividing by M, the average revenue $\bar{r}$ is given by:

$$
\begin{aligned}
\bar{r} = \frac{R}{M} &= r_D(\tilde{\varphi}) + \frac{M_X}{M} r_X(\tilde{\varphi}_x) \\
&= r_D(\tilde{\varphi}) + p_x r_X(\tilde{\varphi}_x).
\end{aligned}
$$

Alternatively, using the definition of $\bar{\varphi}$,

$$\begin{aligned}
R &= \alpha E \left(P\frac{\rho}{w}\right)^{\sigma-1} \left[M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}\right] \\
&= \bar{M}\alpha E \left(P\frac{\rho}{w}\right)^{\sigma-1} \left[\frac{1}{\bar{M}}\left(M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}\right)\right] \\
&= \bar{M}\alpha E \left(P\frac{\rho\bar{\varphi}}{\omega}\right)^{\sigma-1} \\
&= \bar{M} r_D(\bar{\varphi}).
\end{aligned}$$

Given $\pi_D(\varphi) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1}$ and $\pi_X(\varphi) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1}$, following the same steps as above

$$\begin{aligned}
\Pi &= \int_{\varphi^*}^{\infty} \pi_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} \pi_X(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi \\
&= \int_{\varphi^*}^{\infty} \pi_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} \pi_X(\varphi) M_X \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi \\
&= \int_{\varphi^*}^{\infty} \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1} M \frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + \int_{\varphi_x^*}^{\infty} \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1} M_X \frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi \\
&= \frac{\alpha E}{\sigma}\left(P\frac{\rho}{w}\right)^{\sigma-1} \left[M\int_{\varphi^*}^{\infty} \varphi^{\sigma-1}\frac{g(\varphi)}{1-G(\varphi^*)} d\varphi + M_X \int_{\varphi_x^*}^{\infty} \tau^{1-\sigma}\varphi^{\sigma-1}\frac{g(\varphi)}{1-G(\varphi_x^*)} d\varphi\right] \\
&= \frac{\alpha E}{\sigma}\left(P\frac{\rho}{w}\right)^{\sigma-1} \left[M\tilde{\varphi}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}\right] \\
&= M\frac{\alpha E}{\sigma}\left(P\frac{\rho\tilde{\varphi}}{w}\right)^{\sigma-1} + M_X\frac{\alpha E}{\sigma}\left(P\frac{\rho\tilde{\varphi}_x}{w\tau}\right)^{\sigma-1} \\
&= M\pi_D(\tilde{\varphi}) + M_X\pi_X(\tilde{\varphi}_x).
\end{aligned}$$

Dividing by M, the average profits are given by:

$$\bar{\pi} = \frac{\Pi}{M} = \pi_D(\tilde{\varphi}) + p_x\pi_X(\tilde{\varphi}_x).$$

Finally, since $\pi_D(\varphi) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1} = \frac{r_D(\varphi)}{\sigma}$ and $\pi_X(\varphi) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1} = \frac{r_X(\varphi)}{\sigma}$ for all φ, as in the closed economy case, $\Pi = M\pi_D(\tilde{\varphi}) + M_X\pi_X(\tilde{\varphi}_x) = M\frac{r_D(\tilde{\varphi})}{\sigma} + M_X\frac{r_X(\tilde{\varphi}_x)}{\sigma} = \frac{R}{\sigma}$.

7.7. FEC in the open economy. Free entry condition requires that in equilibrium $v_E = 0$. Plugging in (3.19), this is equivalent to

$$\begin{aligned}(1-G(\varphi^*))\left[\sum_{t=0}^{\infty}(1-\delta)^t\bar{\pi} - F_D w - p_x F_X w\right] - F_E w =&0\\ (1-G(\varphi^*))\left[\frac{\bar{\pi}}{\delta} - F_D w - p_x F_X w\right] =&F_E w\\ \frac{\bar{\pi}}{\delta} - F_D w - p_x F_X w =&\frac{F_E w}{1-G(\varphi^*)}.\end{aligned}$$

Now, using $p_x = \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}$ and rearranging, I obtain

$$\frac{\bar{\pi}}{\delta} = w\left(\frac{F_E}{1-G(\varphi^*)} + F_D + F_X\frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}\right).$$

7.8. The FEC and the ZPC under the Pareto distribution assumption-open economy. Using the same steps shown above for the closed economy case, I can derive the expressions for $\tilde{\varphi}(\varphi^*)$ and $\tilde{\varphi}(\varphi_x^*)$ when φ is Pareto distributed. These are given by:

$$\begin{aligned}\tilde{\varphi} \equiv \tilde{\varphi}(\varphi^*) &= \left[\frac{1}{1-G(\varphi^*)}\int_{\varphi^*}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}\\ &= \left[\frac{1}{1-1+\left(\frac{\varphi_m}{\varphi^*}\right)^a}\int_{\varphi^*}^{\infty}\varphi^{\sigma-1}\frac{a}{\varphi}\left(\frac{\varphi_m}{\varphi}\right)^a d\varphi\right]^{\frac{1}{\sigma-1}}\\ &= \left[\left(\frac{\varphi^*}{\varphi_m}\right)^a a\varphi_m^a\int_{\varphi^*}^{\infty}\varphi^{\sigma-1-1-a}d\varphi\right]^{\frac{1}{\sigma-1}}\\ &= \left[\frac{a(\varphi^*)^a}{\sigma-1-a}\varphi^{\sigma-1-a}|_{\varphi^*}^{\infty}\right]^{\frac{1}{\sigma-1}}\\ &= \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi^*,\end{aligned}$$

and

$$\begin{aligned}
\tilde{\varphi}_x \equiv \tilde{\varphi}(\varphi_x^*) &= \left[\frac{1}{1-G(\varphi_x^*)}\int_{\varphi_x^*}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{1-1+\left(\frac{\varphi_m}{\varphi_x^*}\right)^a}\int_{\varphi_x^*}^{\infty}\varphi^{\sigma-1}\frac{a}{\varphi}\left(\frac{\varphi_m}{\varphi}\right)^a d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{\varphi_x^*}{\varphi_m}\right)^a a\varphi_m^a\int_{\varphi_x^*}^{\infty}\varphi^{\sigma-1-1-a}d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi_x^*)^a}{\sigma-1-a}\varphi^{\sigma-1-a}|_{\varphi_x^*}^{\infty}\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi_x^*.
\end{aligned}$$

Substituting these expressions into (3.18), I obtain

$$\begin{aligned}
\bar{\pi} =& \delta F_D w\left[\frac{\tilde{\varphi}}{\varphi^*}\right]^{\sigma-1} + p_x\delta F_X w\left[\frac{\tilde{\varphi}_x}{\varphi_x^*}\right]^{\sigma-1} \\
=& \delta F_D w\left[\frac{\left(\frac{a}{a-\sigma+1}\right)^{\frac{1}{\sigma-1}}\varphi^*}{\varphi^*}\right]^{\sigma-1} + \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}\delta F_X w\left[\frac{\left(\frac{a}{a-\sigma+1}\right)^{\frac{1}{\sigma-1}}\varphi_x^*}{\varphi_x^*}\right]^{\sigma-1} \\
=& \frac{\delta F_D wa}{a-\sigma+1} + \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}\frac{\delta F_X wa}{a-\sigma+1} \\
=& \frac{\delta F_D wa}{a-\sigma+1} + \frac{\left(\frac{\varphi_m}{\varphi_x^*}\right)^a}{\left(\frac{\varphi_m}{\varphi^*}\right)^a}\frac{\delta F_X wa}{a-\sigma+1} \\
=& \frac{\delta wa}{a-\sigma+1}\left[F_D + \left(\frac{\varphi^*}{\varphi_x^*}\right)^a F_X\right].
\end{aligned}$$

Now given $\varphi_x^* = \tau\varphi^* \left(\frac{F_X}{F_D}\right)^{\frac{1}{\sigma-1}}$, the above expression further simplifies:

$$\begin{aligned}
\bar{\pi} =& \frac{\delta w a}{a-\sigma+1}\left\{F_D + \left[\tau\left(\frac{F_X}{F_D}\right)^{\frac{1}{\sigma-1}}\right]^{-a} F_X\right\} \\
=& \frac{\delta w a F_D}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a}{\sigma-1}}\frac{F_X}{F_D}\right] \\
=& \frac{\delta w a F_D}{a-\sigma+1}\left[1+\tau^{-a}\frac{F_D^{\frac{a}{\sigma-1}-1}}{F_X^{\frac{a}{\sigma-1}-1}}\right] \\
=& \frac{\delta w a F_D}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right].
\end{aligned}$$

As for the FEC

$$\begin{aligned}
\bar{\pi} =& \delta w\left[\frac{F_E}{1-G(\varphi^*)} + F_D + F_X\frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}\right] \\
=& \delta w\left[\frac{F_E}{\left(\frac{\varphi_m}{\varphi^*}\right)^a} + F_D + F_X\frac{\left(\frac{\varphi_m}{\varphi_x^*}\right)^a}{\left(\frac{\varphi_m}{\varphi^*}\right)^a}\right] \\
=& \delta w\left[F_E\left(\frac{\varphi^*}{\varphi_m}\right)^a + F_D + F_X\left(\frac{\varphi^*}{\varphi_x^*}\right)^a\right] \\
=& \delta w\left[F_E\left(\frac{\varphi^*}{\varphi_m}\right)^a + F_D + F_X\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a}{\sigma-1}}\right] \\
=& \delta w F_D\left[\frac{F_E}{F_D}\left(\frac{\varphi^*}{\varphi_m}\right)^a + 1 + \tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right].
\end{aligned}$$

7.9. Proof of proposition 2: existence and uniqueness of equilibrium in an open economy. In FEC and ZPC, if φ^* is replaced by φ and $\bar{\pi}$ is replaced by π , then these two equations can be graphed in (φ, π) space. The FEC is increasing in φ with $\pi = \delta w F_D\left[\frac{F_E}{F_D} + 1 + \tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]$ at $\varphi = \varphi_m$ and the ZPC is constant in φ for all $\varphi \geq \varphi_m$. As for the closed economy case, I will consider only the case where not all firms enter the domestic market, $\varphi^* > \varphi_m$. This happens when

$$\begin{aligned}
\frac{\delta w a F_D}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right] &> \delta w F_D\left[\frac{F_E}{F_D}+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}+1\right]\\
\frac{a}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right] &> \frac{F_E}{F_D}+\left[1+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]\\
\left(\frac{a}{a-\sigma+1}-1\right)\left[1+\tau^{\sigma-1}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right] &> \frac{F_E}{F_D}.
\end{aligned}$$

Under this assumption, which holds when F_E is small enough, the FEC cuts the ZPC line only once from below, so the ZPC uniquely determines $\bar{\pi}$ and, given the value for $\bar{\pi}$, the FEC uniquely determines $\varphi^* > \varphi_m$. The closed form solution for φ^* is given by:

$$\begin{aligned}
\delta w F_D\left[\frac{F_E}{F_D}\left(\frac{\varphi^*}{\varphi_m}\right)^a+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}+1\right] &= \frac{\delta w a F_D}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]\\
\frac{F_E}{F_D}\left(\frac{\varphi^*}{\varphi_m}\right)^a+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}+1 &= \frac{a}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]\\
\frac{F_E}{F_D}\left(\frac{\varphi^*}{\varphi_m}\right)^a &= \tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\left(\frac{a}{a-\sigma+1}-1\right)\\
&\quad+\frac{a}{a-\sigma+1}-1\\
\left(\frac{\varphi^*}{\varphi_m}\right)^a &= \left[\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}+1\right]\frac{\sigma-1}{a-\sigma+1}\frac{F_D}{F_E}\\
\varphi^* &= \varphi_m\left\{\left[\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}+1\right]\frac{\sigma-1}{a-\sigma+1}\frac{F_D}{F_E}\right\}^{\frac{1}{a}}.
\end{aligned}$$

Given φ^*, (3.16) uniquely pins down $\varphi_x^* = \tau\varphi^*\left(\frac{F_X}{F_D}\right)^{\frac{1}{\sigma-1}}$. I can then solve for $\tilde{\varphi} = \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi^*$ and for $\tilde{\varphi}_x = \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi_x^*$.

The FEC (3.20) also implies

$$\begin{aligned}
\bar{\pi} &= \delta w\left(\frac{F_E}{1-G(\varphi^*)}+F_D+F_X\frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}\right)\\
\bar{\pi}M &= \delta M w\left(\frac{F_E}{1-G(\varphi^*)}+F_D+F_X\frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}\right)\\
\Pi &= \delta M w\left(\frac{F_E}{1-G(\varphi^*)}+F_D+F_X\frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}\right).
\end{aligned}$$

Given $[1-G(\varphi^*)]M_e = \delta M$ and $M_X = \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}M$, I can derive the total wages of innovation workers wL_I as follows:

$$\begin{aligned}
L_I =& M_eF_E + M_e[1-G(\varphi^*)]F_D + M_e[1-G(\varphi_x^*)]F_X \\
L_I =& \frac{\delta M}{1-G(\varphi^*)}[F_E + (1-G(\varphi^*))F_D + (1-G(\varphi_x^*))F_X] \\
L_I =& \delta M\left(\frac{F_E}{1-G(\varphi^*)} + F_D + \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}F_X\right) \\
L_Iw =& \delta Mw\left(\frac{F_E}{1-G(\varphi^*)} + F_D + \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}F_X\right).
\end{aligned}$$

Combining the expressions for Π and L_Iw with $R = \sigma\Pi$, I obtain $L_Iw = \Pi = \frac{R}{\sigma}$. The wages to production workers in the differentiated sector are given by $wL_P = R - \Pi = R(\sigma-1)/\sigma$ and the wages to workers in the homogeneous good sector are given by $L_0w = R_0$. Now, using $R_0 = (1-\alpha)E$ and $R = \alpha E$, I can rearrange the labor market clearing condition to solve for the aggregate equilibrium expenditure E:

$$\begin{aligned}
L =& L_I + L_P + L_0 \\
wL =& wL_I + wL_P + wL_0 \\
wL =& \frac{R}{\sigma} + \frac{R(\sigma-1)}{\sigma} + R_0 \\
wL =& \frac{\alpha E}{\sigma} + \frac{\alpha E(\sigma-1)}{\sigma} + (1-\alpha)E \\
\sigma wL =& \alpha E + \alpha E\sigma - \alpha E + \sigma E - \sigma\alpha E \\
wL =& E.
\end{aligned}$$

This implies $\Pi = \frac{R}{\sigma} = \frac{\alpha E}{\sigma} = \frac{\alpha wL}{\sigma}$. Given the identity $\bar{\pi} = \frac{\Pi}{M}$, the number of firms is pinned down by $M = \frac{\alpha wL}{\sigma\bar{\pi}}$. Once M is known, I can solve for $M_X = \frac{1-G(\varphi_x^*)}{1-G(\varphi^*)}M$ and then for $\bar{M} \equiv M + M_X$. I can thus pin down the equilibrium values for $\bar{\varphi} \equiv \left\{1/\bar{M}[M\tilde{\varphi}^{\sigma-1} + M_x(\tau^{-1}\tilde{\varphi}_x)^{\sigma-1}]\right\}^{\frac{1}{\sigma-1}}$ and for $P = \bar{M}^{1/(1-\sigma)}p_D(\bar{\varphi}) = \bar{M}^{1/(1-\sigma)}\frac{w}{\rho\bar{\varphi}}$. To compute the welfare $U = q_0^{1-\alpha}q_1^{\alpha}$, I start considering the term

$$q_1 = \frac{\alpha e}{P} = \frac{\alpha E}{LP} = \frac{R}{LP} = \frac{R\rho\bar{\varphi}}{L\bar{M}^{1/(1-\sigma)}w}.$$

Remember that I can always write $\frac{r_D(\bar{\varphi})}{r_D(\varphi^*)} = \left(\frac{\bar{\varphi}}{\varphi^*}\right)^{\sigma-1}$, where $r_D(\bar{\varphi}) = R/\bar{M}$ and, from the zero profit condition for the domestic market, $r_D(\varphi^*) = \sigma\pi_D(\varphi^*) = \sigma\delta F_Dw$. It follows

$$\left(\frac{\bar{\varphi}}{\varphi^*}\right)^{\sigma-1} = \frac{R}{\bar{M}\sigma\delta F_D w}$$
$$\bar{\varphi} = \varphi^* \left(\frac{R}{\bar{M}\sigma\delta F_D w}\right)^{\frac{1}{\sigma-1}}.$$

Using this in the expression for q_1, I obtain

$$\begin{aligned} q_1 =& \frac{R\rho\varphi^*}{L\bar{M}^{1/(1-\sigma)}w}\left(\frac{R}{\bar{M}\sigma\delta F_D w}\right)^{\frac{1}{\sigma-1}} \\ =& \frac{\alpha E\rho\varphi^*}{Lw}\left(\frac{\alpha E}{\sigma\delta F_D w}\right)^{\frac{1}{\sigma-1}} \\ =& \frac{\alpha wL\rho\varphi^*}{Lw}\left(\frac{\alpha wL}{\sigma\delta F_D w}\right)^{\frac{1}{\sigma-1}} \\ =& \alpha\rho\varphi^*\left(\frac{\alpha L}{\sigma\delta F_D}\right)^{\frac{1}{\sigma-1}}. \end{aligned}$$

Given $q_0 = (1-\alpha)e = (1-\alpha)E/L = (1-\alpha)wL/L = (1-\alpha)w$, the individual welfare can be rewritten as

$$U = w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi^*)^{\alpha}\left(\frac{\alpha L}{\sigma\delta F_D}\right)^{\frac{\alpha}{\sigma-1}}.$$

8. Technical Appendix: the imperfect credit market setting

8.1. Proof of proposition 3: the optimal contract in the closed economy.

To solve for the optimal contract, I start from the fact that the firm value

$$\begin{aligned} v_E =& \int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t(\pi(\varphi)-f(\varphi))\right]i(\varphi)g(\varphi)d\varphi + K \\ =& \int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t\pi(\varphi)\right]i(\varphi)g(\varphi)d\varphi - \int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi)\right]i(\varphi)g(\varphi)d\varphi + K \\ =& \int_{\varphi_m}^{\infty}\frac{\pi(\varphi)}{\delta}i(\varphi)g(\varphi)d\varphi - \int_{\varphi_m}^{\infty}\frac{f(\varphi)}{\delta}i(\varphi)g(\varphi)d\varphi + K \end{aligned}$$

is increasing in the term $T \equiv K - \int_{\varphi_m}^{\infty}\frac{f(\varphi)}{\delta}i(\varphi)g(\varphi)d\varphi$. Since I am solving for an equilibrium with perfect competition among potential lenders, the firm will increase T

till the (PC) binds. $v_L = 0$ implies:

$$\int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi) - F_D w\right] i(\varphi)g(\varphi)d\varphi - F_E w - K = 0$$

$$\int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi) - F_D w\right] i(\varphi)g(\varphi)d\varphi - F_E w = K.$$

Using the definition of T, the above condition can also be written as:

$$-\int_{\varphi_m}^{\infty} F_D w i(\varphi)g(\varphi)d\varphi - F_E w = K - \int_{\varphi_m}^{\infty}\frac{f(\varphi)}{\delta}i(\varphi)g(\varphi)d\varphi$$

$$-\int_{\varphi_m}^{\infty} F_D w i(\varphi)g(\varphi)d\varphi - F_E w = T. \tag{8.42}$$

Plugging this value in v_E, I obtain

$$\begin{aligned} v_E &= \int_{\varphi_m}^{\infty}\frac{\pi(\varphi)}{\delta}i(\varphi)g(\varphi)d\varphi + T \\ &= \int_{\varphi_m}^{\infty}\frac{\pi(\varphi)}{\delta}i(\varphi)g(\varphi)d\varphi - \int_{\varphi_m}^{\infty} F_D w i(\varphi)g(\varphi)d\varphi - F_E w. \end{aligned}$$

Notice that for $v_L = 0$ the firm's objective function does not depend on K and $f(\varphi)$ separately, but on T only. It follows that, so far (LC), (IC) and (RP) hold, (8.42) can be achieved by arbitrary varying both K and $f(\varphi)$. This introduce a degree of freedom in determining the optimal contract. On the other hand, this is not the case for the firm with the minimum productivity level needed to obtain the loan $F_D w$ and start production. In fact, in order to meet the lender's (RP) condition, this firm has the incentive to pay a per period repayment $f(\varphi)$ as high as the (IC) allows for. In other word, a binding (IC) and a binding (RP) solve for the entry rule: $i(\varphi) = 1$ if and only if $\varphi \geq \varphi_{cc}^*$, where φ_{cc}^* represents the entry-productivity cut-off. A binding (IC) implies:

$$\begin{aligned} \pi(\varphi) - f(\varphi) &= (1-\lambda)\pi(\varphi) \\ \pi(\varphi) - f(\varphi) &= \pi(\varphi) - \lambda\pi(\varphi) \\ f(\varphi) &= \lambda\pi(\varphi). \end{aligned}$$

Using this in the (RP) condition, φ^*_{cc} is such that

$$\begin{aligned}\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi^*_{cc}) &= F_D w\\ \frac{\lambda\pi(\varphi^*_{cc})}{\delta} &= F_D w\\ \lambda\pi(\varphi^*_{cc}) &= \delta F_D w.\end{aligned}$$

Once the entry rule has been solved for, the firm's problem reduces to:

$$\max_{f(\varphi),K} v_E = \int_{\varphi^*_{cc}}^{\infty}\frac{\pi(\varphi)}{\delta}g(\varphi)d\varphi - (1-G(\varphi^*_{cc}))F_D w - F_E w \tag{P}$$

subject to

$$K = \int_{\varphi^*_{cc}}^{\infty}\frac{f(\varphi)}{\delta}g(\varphi)d\varphi - (1-G(\varphi^*_{cc}))F_D w - F_E w \geq 0 \tag{LC+PC}$$

$$\delta F_D w \leq f(\varphi) \leq \lambda\pi(\varphi) \text{ for all } \varphi \text{ such that } \varphi \geq \varphi^*_{cc} \tag{IC+RP}$$

that, as argued above, does not have a unique solution for the pair $(K, f(\varphi))$.

8.2. The FEC and the ZPC in the closed economy.

FEC: In the text I argue that the FEC is given by

$$K_{max} \equiv \int_{\varphi^*_{cc}}^{\infty}\frac{\lambda\pi(\varphi)}{\delta}g(\varphi)d\varphi - (1-G(\varphi^*_{cc}))F_D w - F_E w = 0.$$

Given $\bar{\pi}_{cc} \equiv [1-G(\varphi^*_{cc})]^{-1}\int_{\varphi^*_{cc}}^{\infty}\pi(\varphi)g(\varphi)d\varphi$, I can rewrite it as

$$\begin{aligned}\frac{\lambda(1-G(\varphi^*_{cc}))}{\delta(1-G(\varphi^*_{cc}))}\int_{\varphi^*_{cc}}^{\infty}\pi(\varphi)g(\varphi)d\varphi =& w[(1-G(\varphi^*_{cc}))F_D + F_E]\\ \frac{\lambda(1-G(\varphi^*_{cc}))\bar{\pi}_{cc}}{\delta} =& w[(1-G(\varphi^*_{cc}))F_D + F_E]\\ \lambda\bar{\pi}_{cc} =& \delta w\left[\frac{F_E}{1-G(\varphi^*_{cc})} + F_D\right]\end{aligned}$$

In the same way, I can rewrite v_E as

$$\begin{aligned} v_E =& \int_{\varphi^*_{cc}}^{\infty} \frac{\pi(\varphi)}{\delta} g(\varphi)d\varphi - (1-G(\varphi^*_{cc}))F_D w - F_E w \\ =& \frac{(1-G(\varphi^*_{cc}))\bar{\pi}_{cc}}{\delta} - (1-G(\varphi^*_{cc}))F_D w - F_E w \\ =& \frac{(1-G(\varphi^*_{cc}))\bar{\pi}_{cc}}{\delta} - \frac{\lambda(1-G(\varphi^*_{cc}))\bar{\pi}_{cc}}{\delta} \\ =&(1-G(\varphi^*_{cc}))\frac{(1-\lambda)\bar{\pi}_{cc}}{\delta} > 0. \end{aligned}$$

ZPC: First of all, define $\tilde{\varphi}_{cc} \equiv \tilde{\varphi}(\varphi^*_{cc}) = \left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$ as the weighted average productivity, and notice that the average profits can be rewritten as the profits of the firm with productivity $\tilde{\varphi}_{cc}$:

$$\begin{aligned} \bar{\pi}_{cc} \equiv& [1-G(\varphi^*_{cc})]^{-1} \int_{\varphi^*_{cc}}^{\infty} \pi(\varphi)g(\varphi)d\varphi \\ =& [1-G(\varphi^*_{cc})]^{-1} \int_{\varphi^*_{cc}}^{\infty} \frac{\alpha E}{\sigma} \left(P\frac{\rho\varphi}{w}\right)^{\sigma-1} g(\varphi)d\varphi \\ =& \alpha E \left(P\frac{\rho}{w}\right)^{\sigma-1} [1-G(\varphi^*_{cc})]^{-1} \int_{\varphi^*_{cc}}^{\infty} \varphi^{\sigma-1} g(\varphi)d\varphi \\ =& \frac{\alpha E}{\sigma}\left(P\frac{\rho}{w}\right)^{\sigma-1} \int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi \\ =& \frac{\alpha E}{\sigma}\left(P\frac{\rho}{w}\right)^{\sigma-1} \tilde{\varphi}_{cc}^{\sigma-1} \\ =& \frac{\alpha E}{\sigma}\left(P\frac{\rho\tilde{\varphi}_{cc}}{w}\right)^{\sigma-1} \\ =& \pi(\tilde{\varphi}_{cc}). \end{aligned}$$

Given (2.6), $\pi(\varphi^*_{cc}) = \pi(\tilde{\varphi}_{cc})\left[\frac{\varphi^*_{cc}}{\tilde{\varphi}_{cc}}\right]^{\sigma-1} = \bar{\pi}_{cc}\left[\frac{\varphi^*_{cc}}{\tilde{\varphi}_{cc}}\right]^{\sigma-1}$. This, together with the condition for the productivity cutoff $\varphi^*_{cc} = \{\varphi : \pi(\varphi^*_{cc}) = \frac{\delta F_D w}{\lambda}\}$, I can derive the zero profit condition as

$$\begin{aligned} \lambda\bar{\pi}_{cc}\left[\frac{\varphi^*_{cc}}{\tilde{\varphi}_{cc}}\right]^{\sigma-1} =& \delta F_D w \\ \lambda\bar{\pi}_{cc} =& \delta F_D w \left[\frac{\tilde{\varphi}_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1}. \end{aligned}$$

8.3. Aggregate quantities in the closed economy. Given there is a mass M of firms in equilibrium, $\mu(\varphi)$ is the distribution of productivity of active firms defined on a subset of $(0,\infty)$ and using $r(\varphi)=\alpha E\left(P\frac{\rho\varphi}{\omega}\right)^{\sigma-1}$ the aggregate revenue is given by

$$\begin{aligned}
R &= \int_0^\infty r(\varphi)M\mu(\varphi)d\varphi \\
&= M\int_0^\infty \alpha E\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1}\mu(\varphi)d\varphi \\
&= M\alpha E\left(P\frac{\rho}{w}\right)^{\sigma-1}\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right] \\
&= M\alpha E\left(P\frac{\rho}{w}\right)^{\sigma-1}\tilde{\varphi}_{cc}^{\sigma-1} \\
&= M\alpha E\left(P\frac{\rho\tilde{\varphi}_{cc}}{w}\right)^{\sigma-1} \\
&= Mr(\tilde{\varphi}_{cc})
\end{aligned}$$

where $\tilde{\varphi}_{cc}\equiv\tilde{\varphi}(\varphi^*_{cc})=\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}$. Analogously, using $\pi(\varphi)=\frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{\omega}\right)^{\sigma-1}$, I obtain

$$\begin{aligned}
\Pi &= \int_0^\infty \pi(\varphi)M\mu(\varphi)d\varphi \\
&= M\int_0^\infty \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1}\mu(\varphi)d\varphi \\
&= M\frac{\alpha E}{\sigma}\left(P\frac{\rho}{w}\right)^{\sigma-1}\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right] \\
&= M\frac{\alpha E}{\sigma}\left(P\frac{\rho}{w}\right)^{\sigma-1}\tilde{\varphi}_{cc}^{\sigma-1} \\
&= M\frac{\alpha E}{\sigma}\left(P\frac{\rho\tilde{\varphi}_{cc}}{w}\right)^{\sigma-1} \\
&= M\pi(\tilde{\varphi}_{cc}) \\
&= M\bar{\pi}_{cc}.
\end{aligned}$$

Moreover,

$$\Pi = M\frac{\alpha E}{\sigma}\left(P\frac{\rho\tilde{\varphi}_{cc}}{w}\right)^{\sigma-1} = \frac{R}{\sigma}.$$

The price index for the consumption bundle can be written as

$$\begin{aligned}
P &= \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}} \\
&= \left[\int_0^\infty p(\varphi)^{1-\sigma}M\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}}.
\end{aligned}$$

Now, using $p(\varphi) = w/\rho\varphi$, I obtain

$$\begin{aligned}
P &= \left[\int_0^\infty \left(\frac{w}{\rho\varphi}\right)^{1-\sigma} M\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} \\
&= M^{\frac{1}{1-\sigma}}\frac{w}{\rho}\left[\int_0^\infty \left(\frac{1}{\varphi}\right)^{1-\sigma} \mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} \\
&= M^{\frac{1}{1-\sigma}}\frac{w}{\rho}\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{-\frac{1}{\sigma-1}} \\
&= M^{\frac{1}{1-\sigma}}\frac{w}{\rho\tilde{\varphi}_{cc}} \\
&= M^{\frac{1}{1-\sigma}}p(\tilde{\varphi}_{cc}).
\end{aligned}$$

8.4. The FEC and the ZPC under the Pareto distribution assumption-closed economy. Given the Pareto cumulative distribution function $G(\varphi) = 1 - \left(\frac{\varphi_m}{\varphi}\right)^a$, the FEC can be rewritten as:

$$\begin{aligned}
\lambda\bar{\pi}_{cc} &= \delta w\left(\frac{F_E}{1-G(\varphi^*_{cc})} + F_D\right) \\
\lambda\bar{\pi}_{cc} &= \delta w\left(\frac{F_E}{1-1+\left(\frac{\varphi_m}{\varphi^*_{cc}}\right)^a} + F_D\right) \\
\lambda\bar{\pi}_{cc} &= \delta w\left(F_E\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + F_D\right).
\end{aligned}$$

As shown already, the probability density function of the Pareto distribution is given by $g(\varphi) = \frac{a}{\varphi}\left(\frac{\varphi_m}{\varphi}\right)^a$. Given $a > \sigma - 1$, in analogy with the frictionless set-up, I can re-write the average productivity as:

$$\begin{aligned}
\tilde{\varphi}_{cc} \equiv \tilde{\varphi}(\varphi^*_{cc}) &= \left[\frac{1}{1-G(\varphi^*_{cc})}\int_{\varphi^*_{cc}}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{1-1+\left(\frac{\varphi_m}{\varphi^*_{cc}}\right)^a}\int_{\varphi^*_{cc}}^{\infty}\varphi^{\sigma-1}\frac{a}{\varphi}\left(\frac{\varphi_m}{\varphi}\right)^a d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a a\varphi_m^a \int_{\varphi^*_{cc}}^{\infty}\varphi^{\sigma-1-1-a}d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi^*_{cc})^a}{\sigma-1-a}\varphi^{\sigma-1-a}|_{\varphi^*_{cc}}^{\infty}\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi^*_{cc})^a}{\sigma-1-a}\left(0-(\varphi^*_{cc})^{\sigma-1-a}\right)\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi^*_{cc}.
\end{aligned}$$

Using this expression in (4.27), the ZPC becomes:

$$\begin{aligned}
\lambda\bar{\pi}_{cc} =& \delta F_D w\left[\frac{\tilde{\varphi}(\varphi^*_{cc})}{\varphi^*_{cc}}\right]^{\sigma-1} \\
\lambda\bar{\pi}_{cc} =& \delta F_D w\left[\frac{\left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi^*_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1} \\
\lambda\bar{\pi}_{cc} =& \frac{\delta F_D w a}{a-\sigma+1}.
\end{aligned}$$

8.5. Proof of proposition 4: existence and uniqueness of equilibrium in a closed economy with imperfect creditor protection. Given the system

$$\begin{aligned}
(ZPC) \qquad \bar{\pi}_{cc} =& \frac{\delta F_D w a}{\lambda(a-\sigma+1)} \\
(FEC) \qquad \bar{\pi}_{cc} =& \frac{\delta w}{\lambda}\left(F_E\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + F_D\right)
\end{aligned}$$

the proof of existence and uniqueness of the equilibrium follows exactly the same line as for the proof of Proposition 1. The only difference is given by the presence of $\lambda < 1$ in the denominator of the l.h.s. of both the the ZPC and the FEC. As a result, the ZPC curve is higher than in the frictionless case and the FEC both higher and steeper (see Figure 3). In FEC and ZPC, if φ^*_{cc} is replaced by φ and $\bar{\pi}_{cc}$ is replaced by π, then these equations can be graphed in (φ, π) space with FEC increasing in φ with

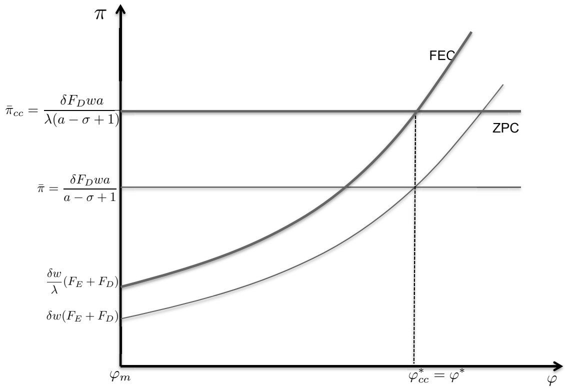

FIGURE 3. Determination of Equilibrium φ^*_{cc} and $\bar{\pi}$

$\pi = \frac{\delta w}{\lambda}(F_E + F_D)$ at $\varphi = \varphi_m$ and ZPC constant for all $\varphi \geq \varphi_m$. The FEC cuts the ZPC line only once from below (see Figure 3) when F_E is sufficiently small:

$$\begin{aligned}
\frac{\delta F_D wa}{\lambda(a-\sigma+1)} &> \frac{\delta w}{\lambda}(F_E + F_D) \\
\frac{F_D a}{a-\sigma+1} &> F_E + F_D \\
\frac{a}{a-\sigma+1} &> \frac{F_E}{F_D} + 1 \\
\frac{a-a+\sigma-1}{a-\sigma+1} &> \frac{F_E}{F_D} \\
\frac{\sigma-1}{a-\sigma+1} &> \frac{F_E}{F_D}.
\end{aligned}$$

In this case $\bar{\pi}_{cc} = \frac{\delta F_D w a}{\lambda(a-\sigma+1)} > \bar{\pi}$ and $\varphi^*_{cc} > \varphi_m$, and I can solve for φ^*_{cc}:

$$\begin{aligned}
\frac{\delta F_D w a}{\lambda(a-\sigma+1)} =& \frac{\delta w}{\lambda}\left(F_E\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + F_D\right) \\
\frac{F_D a}{a-\sigma+1} =& F_E\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + F_D \\
\frac{a}{a-\sigma+1} =& \frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + 1 \\
\frac{a-a+\sigma-1}{a-\sigma+1}\frac{F_D}{F_E} =& \left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a \\
\varphi_m\left[\frac{\sigma-1}{a-\sigma+1}\frac{F_D}{F_E}\right]^{\frac{1}{a}} =& \varphi^*_{cc} = \varphi^*.
\end{aligned}$$

As shown above, φ^*_{cc} uniquely determines $\tilde{\varphi}(\varphi^*_{cc}) = \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}} \varphi^*_{cc}$. The FEC also implies

$$\begin{aligned}
\bar{\pi}_{cc} &= \frac{\delta w}{\lambda}\left(\frac{F_E}{1-G(\varphi^*_{cc})} + F_D\right) \\
M\bar{\pi}_{cc} &= \frac{M\delta w}{\lambda}\left(\frac{F_E}{1-G(\varphi^*_{cc})} + F_D\right) \\
\Pi &= \frac{M\delta w}{\lambda}\left(\frac{F_E}{1-G(\varphi^*_{cc})} + F_D\right).
\end{aligned}$$

Given $[1-G(\varphi^*_{cc})]M_e = \delta M$, I can derive the total wages of innovation workers wL_I as follows:

$$\begin{aligned}
L_I =& M_e F_E + M_e[1-G(\varphi^*_{cc})]F_D \\
L_I =& \frac{\delta M}{1-G(\varphi^*_{cc})}[F_E + (1-G(\varphi^*_{cc}))F_D] \\
L_I =& \delta M\left(\frac{F_E}{1-G(\varphi^*_{cc})} + F_D\right) \\
L_I w =& \delta M w\left(\frac{F_E}{1-G(\varphi^*_{cc})} + F_D\right).
\end{aligned}$$

Combining the expressions for Π and $L_I w$ with $R = \sigma\Pi$, I obtain $L_I w = \lambda\Pi = \frac{\lambda R}{\sigma}$. The wages to production workers in the differentiated sector are given by $wL_P = R - \Pi = R(\sigma-1)/\sigma$ and wages to workers in the homogeneous good sector are given by $L_0 w = R_0$. Now, using $R_0 = (1-\alpha)E$ and $R = \alpha E$, I can rearrange the labor market clearing condition to solve for the aggregate equilibrium expenditure E:

$$\begin{aligned}
L =& L_I + L_P + L_0 \\
wL =& wL_I + wL_P + wL_0 \\
wL =& \frac{\lambda R}{\sigma} + \frac{R(\sigma-1)}{\sigma} + R_0 \\
wL =& \frac{\lambda \alpha E}{\sigma} + \frac{\alpha E(\sigma-1)}{\sigma} + (1-\alpha)E \\
\sigma wL =& \lambda\alpha E + \alpha E\sigma - \alpha E + \sigma E - \sigma\alpha E \\
wL =& \frac{E(\sigma-\alpha(1-\lambda))}{\sigma} \\
E =& \frac{\sigma wL}{\sigma-\alpha(1-\lambda)}
\end{aligned}$$

where $\frac{\sigma}{\sigma-\alpha(1-\lambda)} > 1$. The aggregate expenditure exceeds thus the labor income wL because this is augmented by the extra rents $V_E \equiv M_e v_E$ that belongs to consumers as owners of the firms. Using $M_e = \delta M/(1-G(\varphi^*_{cc}))$, $\Pi = M\bar{\pi}$ and the expression derived above for v_E, I obtain:

$$\begin{aligned}
V_E =& M_e(1-G(\varphi^*_{cc}))\frac{(1-\lambda)\bar{\pi}_{cc}}{\delta} \\
=& \frac{\delta M}{1-G(\varphi^*_{cc})}(1-G(\varphi^*_{cc}))\frac{(1-\lambda)\bar{\pi}_{cc}}{\delta} \\
=& M(1-\lambda)\bar{\pi}_{cc} \\
=& (1-\lambda)\Pi.
\end{aligned}$$

Given $\Pi = \frac{R}{\sigma} = \frac{\alpha E}{\sigma}$, the expression above becomes $V_E = \frac{(1-\lambda)\alpha E}{\sigma}$. Adding this to $wL = \frac{E(\sigma-\alpha(1-\lambda))}{\sigma}$, I get

$$V_E + wL = \frac{(1-\lambda)\alpha E}{\sigma} + \frac{E(\sigma-\alpha(1-\lambda))}{\sigma} = \frac{(1-\lambda)\alpha E}{\sigma} + \frac{E\sigma}{\sigma} - \frac{(1-\lambda)\alpha E}{\sigma} = E.$$

Now, given the identity $\bar{\pi}_{cc} \equiv \frac{\Pi}{M}$ and the equilibrium values for E and $\bar{\pi}_{cc}$, the number of firms is pinned down by

$$\begin{aligned} M =& \frac{\Pi}{\bar{\pi}_{cc}} \\ =& \frac{\alpha E}{\sigma \bar{\pi}_{cc}} \\ =& \frac{\alpha w L}{(\sigma - \alpha(1-\lambda))\bar{\pi}_{cc}} \\ =& \frac{\alpha w L}{(\sigma - \alpha(1-\lambda))} \left[\frac{\delta F_D w a}{\lambda(a-\sigma+1)}\right]^{-1} \\ =& \frac{\alpha L \lambda (a-\sigma+1)}{(\sigma - \alpha(1-\lambda))\delta F_D a}. \end{aligned}$$

Taking the derivative with respect to λ, I obtain

$$\begin{aligned} \frac{\partial M}{\partial \lambda} =& \frac{\alpha L}{\delta F_D a} \frac{(a-\sigma+1)(\sigma - \alpha(1-\lambda)) - \alpha\lambda(a-\sigma+1)}{(\sigma-\alpha(1-\lambda))^2} \\ =& \frac{\alpha L}{\delta F_D a} \frac{(a-\sigma+1)(\sigma-\alpha) + \alpha\lambda(a-\sigma+1) - \alpha\lambda(a-\sigma+1)}{(\sigma-\alpha(1-\lambda))^2} \\ =& \frac{\alpha L}{\delta F_D a} \frac{(a-\sigma+1)(\sigma-\alpha)}{(\sigma-\alpha(1-\lambda))^2} > 0. \end{aligned}$$

Once M is known, also the equilibrium price index is known and given by $P = M^{1/(1-\sigma)} p(\tilde{\varphi}_{cc})$. Given $p(\tilde{\varphi}_{cc}) = \frac{w}{\rho \tilde{\varphi}_{cc}} = \frac{w}{\rho \varphi^*_{cc}} \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{1-\sigma}}$, this is equal to:

$$P = \left[\frac{\alpha L \lambda (a-\sigma+1)}{(\sigma-\alpha(1-\lambda))\delta F_D a}\right]^{\frac{1}{1-\sigma}} \frac{w}{\rho \varphi^*_{cc}} \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{1-\sigma}} = \left[\frac{\alpha L \lambda}{(\sigma-\alpha(1-\lambda))\delta F_D}\right]^{\frac{1}{1-\sigma}} \frac{w}{\rho \varphi^*_{cc}}.$$

8.6. Proof of Lemma 5: Individual welfare U. As a measure of per-capita welfare, I consider the individual utility U in (2.1). Given the expressions for q_0 and q_1, this is given by

$$U = q_0^{1-\alpha} q_1^{\alpha} = ((1-\alpha)e)^{1-\alpha} \left(\frac{\alpha e}{P}\right)^{\alpha} = \frac{e(1-\alpha)^{1-\alpha}\alpha^{\alpha}}{P^{\alpha}}.$$

Using the closed form solutions for $e = E/L = \frac{\sigma w}{\sigma-\alpha(1-\lambda)}$ and P, welfare becomes:

$$\begin{aligned}
U =& \frac{\sigma w(1-\alpha)^{1-\alpha}\alpha^{\alpha}}{(\sigma-\alpha(1-\lambda))w^{\alpha}}(\rho\varphi_{cc}^{*})^{\alpha}\left[\frac{\alpha L\lambda}{(\sigma-\alpha(1-\lambda))\delta F_D}\right]^{\frac{\alpha}{\sigma-1}} \\
=& \frac{\sigma w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi_{cc}^{*})^{\alpha}}{\sigma-\alpha(1-\lambda)}\left[\frac{\alpha L\lambda}{(\sigma-\alpha(1-\lambda))\delta F_D}\right]^{\frac{\alpha}{\sigma-1}} \\
=& \sigma w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi_{cc}^{*})^{\alpha}\left[\frac{\alpha L}{\delta F_D}\right]^{\frac{\alpha}{\sigma-1}}\lambda^{\frac{\alpha}{\sigma-1}}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}.
\end{aligned}$$

Notice that for $\lambda = 1$ welfare is the same as in the frictionless set-up. In fact:

$$\begin{aligned}
U =& \sigma w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi_{cc}^{*})^{\alpha}\left[\frac{\alpha L}{\delta F_D}\right]^{\frac{\alpha}{\sigma-1}}\left[\frac{1}{\sigma}\right]^{\frac{\alpha}{\sigma-1}+1} \\
=& w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi_{cc}^{*})^{\alpha}\left[\frac{\alpha L}{\delta F_D}\right]^{\frac{\alpha}{\sigma-1}}\left[\frac{1}{\sigma}\right]^{\frac{\alpha}{\sigma-1}} \\
=& w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi_{cc}^{*})^{\alpha}\left[\frac{\alpha L}{\sigma\delta F_D}\right]^{\frac{\alpha}{\sigma-1}}.
\end{aligned}$$

Given that φ_{cc}^{*} is constant in λ, in order to study the sign of $\frac{\partial U}{\partial \lambda}$ I can focus on the term $\Lambda \equiv \lambda^{\frac{\alpha}{\sigma-1}}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}$:

$$\begin{aligned}
\frac{\partial \Lambda}{\partial \lambda} =& \frac{\alpha}{\sigma-1}\lambda^{\frac{\alpha}{\sigma-1}-1}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1} + \lambda^{\frac{\alpha}{\sigma-1}}\frac{\alpha+\sigma-1}{\sigma-1}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}}\frac{-\alpha}{[\sigma-\alpha(1-\lambda)]^2} \\
=& \frac{\alpha}{\sigma-1}\lambda^{\frac{\alpha}{\sigma-1}-1}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1} \\
&+\lambda^{\frac{\alpha}{\sigma-1}-1}\frac{\lambda(\alpha+\sigma-1)}{\sigma-1}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}\frac{-\alpha[\sigma-\alpha(1-\lambda)]}{[\sigma-\alpha(1-\lambda)]^2} \\
=& \frac{\alpha}{\sigma-1}\lambda^{\frac{\alpha}{\sigma-1}-1}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}\left[1-\frac{\lambda(\alpha+\sigma-1)}{\sigma-\alpha(1-\lambda)}\right] \\
=& \frac{\alpha}{\sigma-1}\lambda^{\frac{\alpha}{\sigma-1}-1}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}\frac{\sigma-\alpha(1-\lambda)-\lambda(\alpha+\sigma-1)}{\sigma-\alpha(1-\lambda)} \\
=& \frac{\alpha}{\sigma-1}\lambda^{\frac{\alpha}{\sigma-1}-1}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}\frac{\sigma-\alpha+\alpha\lambda-\lambda\alpha-\lambda\sigma+\lambda}{\sigma-\alpha(1-\lambda)} \\
=& \frac{\alpha}{\sigma-1}\lambda^{\frac{\alpha}{\sigma-1}-1}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}\frac{\sigma-\alpha-\lambda\sigma+\lambda}{\sigma-\alpha(1-\lambda)} \\
=& \frac{\alpha}{\sigma-1}\lambda^{\frac{\alpha}{\sigma-1}-1}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+2}[\sigma(1-\lambda)-(\alpha-\lambda)].
\end{aligned}$$

Since $\lambda < 1$, $\sigma > 1 > \alpha$, both $\frac{1}{\sigma-\alpha(1-\lambda)} > 0$ and $\sigma(1-\lambda)-(\alpha-\lambda) > 0$ are true and $\frac{\partial U}{\partial \lambda} > 0$.

8.7. Sorting condition between domestic firms and exporters. Consider the marginal firms, so those firms whose (IC) and (IC') are binding. In case of a firm that produces only for the domestic market $f(\varphi) = \lambda\pi_D(\varphi)$ and the lender renegation proof condition requires

$$\pi_D(\varphi) \geq \frac{\delta F_D w}{\lambda}.$$

Analogously, if there were firms producing for the foreign market only, it should hold that

$$\pi_X(\varphi) \geq \frac{\delta F_X w}{\lambda}. \tag{8.43}$$

I now derive the condition under which, whenever the above condition is met, the firm always finds a creditor willing to finance also the entry on the domestic market. As shown in the text, with $f'(\varphi) = \lambda(\pi_D(\varphi) + \pi_X(\varphi))$ this requires

$$\pi_D(\varphi) + \pi_X(\varphi) \geq \frac{\delta[F_D + F_X]w}{\lambda}.$$

First, using $\pi_D(\varphi) = \tau^{\sigma-1}\pi_X(\varphi)$, I can rewrite the above condition as:

$$\begin{aligned} \pi_X(\varphi)(1+\tau^{\sigma-1}) &\geq \frac{\delta[F_D + F_X]w}{\lambda} \\ \pi_X(\varphi) &\geq \frac{\delta[F_D + F_X]w}{\lambda(1+\tau^{\sigma-1})}. \end{aligned} \tag{8.44}$$

Then, I find the condition such that, whenever (8.43) is met, also (8.44) is met. In other words, I find the condition for $\frac{\delta F_X w}{\lambda} > \frac{\delta[F_D+F_X]w}{\lambda(1+\tau^{\sigma-1})}$:

$$\begin{aligned} F_X &> \frac{F_D + F_X}{1+\tau^{\sigma-1}} \\ F_X + F_X\tau^{\sigma-1} &> F_D + F_X \\ F_D &< F_X\tau^{\sigma-1} \\ \frac{F_D}{F_X} &< \tau^{\sigma-1}. \end{aligned}$$

8.8. Proof of proposition 6: the foreign market cutoff $\varphi^*_{cc\_x}$. In a frictionless world, it is optimal for a firm to enter a market whenever the flow of future profits exceeds the fixed entry cost, meaning when $\frac{\pi_D(\varphi)}{\delta} \geq F_D w$ and when $\frac{\pi_X(\varphi)}{\delta} \geq F_X w$, for the domestic and foreign market respectively. As shown in the text, when there is imperfect creditor protection ($\lambda < 1$), the firm incentive compatibility constraint and the lender incentive rationality constraint imply that firms will be able to borrow $F_D w$ only when $\frac{\pi_D(\varphi)}{\delta} > \frac{F_D w}{\lambda}$. Since $\frac{F_D w}{\lambda} > F_D w$, this means that there are firms that are productive enough to enter in the perfect credit market case, but that are not productive enough to borrow $F_D w$ in the imperfect credit market case. For the export decision, the incentive compatibility constraint and the incentive rationality constraint imply that a firm will be able to borrow both $F_D w$ and $F_E w$ when $\pi_D(\varphi) + \pi_X(\varphi) \geq \frac{\delta[F_D+F_X]w}{\lambda}$. Using $\pi_D(\varphi) = \tau^{\sigma-1}\pi_X(\varphi)$, I can rewrite it as $\frac{\pi_X(\varphi)}{\delta} \geq \frac{[F_D+F_X]w}{\lambda(1+\tau^{\sigma-1})}$. This condition can be weaker than the one under the assumption of no-credit market frictions. This happens when:

$$\begin{aligned}
\frac{[F_D + F_X]w}{\lambda(1+\tau^{\sigma-1})} &< F_X w \\
\frac{F_D + F_X}{\lambda(1+\tau^{\sigma-1})} &< F_X \\
\frac{F_D + F_X}{F_X} &< \lambda(1+\tau^{\sigma-1}) \\
\lambda > \hat{\lambda} &\equiv \frac{F_D/F_X + 1}{1+\tau^{\sigma-1}}. \qquad (8.45)
\end{aligned}$$

Under this condition, creditors would be willing to finance foreign market entry even when this is unprofitable from the firm's point of view. As a result, only firms with productivity φ such that $\frac{\pi_X(\varphi)}{\delta} \geq F_X w$ will indeed enter. When $\lambda \geq \hat{\lambda}$ the foreign market cutoff is thus given by $\varphi^*_{cc\_x} = \{\varphi : \pi_X(\varphi) = \delta F_X w\}$. If instead $\lambda < \hat{\lambda}$, then there are firms that would enter the foreign market in the absence of credit frictions but are prevented to do so and $\varphi^*_{cc\_x} = \{\varphi : \pi_X(\varphi) = \frac{\delta(F_D+F_X)w}{\lambda(1+\tau^{\sigma-1})}\}$. Looking at (8.45), exporters are more likely to be credit constrained the higher the right hand side ($F_D \uparrow$, $F_X \downarrow$ and $\tau \downarrow$) and the lower the left hand side ($\lambda \downarrow$).

8.9. Proof of proposition 7: the optimal contract in the open economy. To solve for the optimal contract, I start from the fact that the firm value

$$\begin{aligned}
v_E =& \int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t(\pi_D(\varphi)-f(\varphi))\right](i(\varphi)-i_x(\varphi))g(\varphi)d\varphi \\
&+ \int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t(\pi_D(\varphi)+\pi_X(\varphi)-f'(\varphi))\right]i_x(\varphi)g(\varphi)d\varphi + K \\
=& \int_{\varphi_m}^{\infty}\left[\frac{\pi_D(\varphi)}{\delta}-\frac{f(\varphi))}{\delta}\right](i(\varphi)-i_x(\varphi))g(\varphi)d\varphi \\
&+ \int_{\varphi_m}^{\infty}\left[\frac{\pi_D(\varphi)+\pi_X(\varphi)}{\delta}-\frac{f'(\varphi))}{\delta}\right]i_x(\varphi)g(\varphi)d\varphi + K \\
=& \int_{\varphi_m}^{\infty}\frac{\pi_D(\varphi)}{\delta}(i(\varphi)-i_x(\varphi))g(\varphi)d\varphi + \int_{\varphi_m}^{\infty}\frac{\pi_D(\varphi)+\pi_X(\varphi)}{\delta}i_x(\varphi)g(\varphi)d\varphi \\
&+ K - \int_{\varphi_m}^{\infty}\frac{f(\varphi))}{\delta}(i(\varphi)-i_x(\varphi))g(\varphi)d\varphi - \int_{\varphi_m}^{\infty}\frac{f'(\varphi))}{\delta}i_x(\varphi)g(\varphi)d\varphi
\end{aligned}$$

is increasing in the term $T' \equiv K - \int_{\varphi_m}^{\infty}\frac{f(\varphi)}{\delta}(i(\varphi)-i_x(\varphi))g(\varphi)d\varphi - \int_{\varphi_m}^{\infty}\frac{f'(\varphi))}{\delta}i_x(\varphi)g(\varphi)d\varphi$. Since I am solving for an equilibrium with perfect competition among potential lenders, the firm will increase T' till the (PC) binds. $v_L = 0$ implies that

$$\begin{aligned}
&\int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi) - F_D w\right](i(\varphi)-i_x(\varphi))g(\varphi)d\varphi \\
&+ \int_{\varphi_m}^{\infty}\left[\sum_{t=0}^{\infty}(1-\delta)^t f'(\varphi) - (F_D+F_X)w\right]i_x(\varphi)g(\varphi)d\varphi - F_E w - K = 0 \\
&\int_{\varphi_m}^{\infty}\left[\frac{f(\varphi)}{\delta} - F_D w\right](i(\varphi)-i_x(\varphi))g(\varphi)d\varphi \\
&+ \int_{\varphi_m}^{\infty}\left[\frac{f'(\varphi)}{\delta} - (F_D+F_X)w\right]i_x(\varphi)g(\varphi)d\varphi - F_E w = K.
\end{aligned}$$

Rearranging the above condition, I get

$$\begin{aligned}
&-\int_{\varphi_m}^{\infty}F_D w(i(\varphi)-i_x(\varphi))g(\varphi)d\varphi - \int_{\varphi_m}^{\infty}(F_D+F_X)w i_x(\varphi)g(\varphi)d\varphi - F_E w = \\
&K - \int_{\varphi_m}^{\infty}\frac{f(\varphi)}{\delta}(i(\varphi)-i_x(\varphi))g(\varphi)d\varphi - \int_{\varphi_m}^{\infty}\frac{f'(\varphi))}{\delta}i_x(\varphi)g(\varphi)d\varphi.
\end{aligned}$$

Using the definition of T', I obtain

$$-\int_{\varphi_m}^{\infty} F_D w(i(\varphi) - i_x(\varphi))g(\varphi)d\varphi - \int_{\varphi_m}^{\infty}(F_D + F_X)wi_x(\varphi)g(\varphi)d\varphi - F_E w = T'$$

$$-\int_{\varphi_m}^{\infty} F_D wi(\varphi)g(\varphi)d\varphi - \int_{\varphi_m}^{\infty} F_X wi_x(\varphi)g(\varphi)d\varphi - F_E w = T'. \tag{8.46}$$

Plugging this value into v_E, I obtain

$$\begin{aligned}
v_E &= \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta}(i(\varphi) - i_x(\varphi))g(\varphi)d\varphi + \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi) + \pi_X(\varphi)}{\delta} i_x(\varphi)g(\varphi)d\varphi + T' \\
&= \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta} i(\varphi)g(\varphi)d\varphi - \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta} i_x(\varphi)g(\varphi)d\varphi + \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta} i_x(\varphi)g(\varphi)d\varphi \\
&+ \int_{\varphi_m}^{\infty} \frac{\pi_X(\varphi)}{\delta} i_x(\varphi)g(\varphi)d\varphi - \int_{\varphi_m}^{\infty} F_D wi(\varphi)g(\varphi)d\varphi - \int_{\varphi_m}^{\infty} F_X wi_x(\varphi)g(\varphi)d\varphi - F_E w \\
&= \int_{\varphi_m}^{\infty} \frac{\pi_D(\varphi)}{\delta} i(\varphi)g(\varphi)d\varphi + \int_{\varphi_m}^{\infty} \frac{\pi_X(\varphi)}{\delta} i_x(\varphi)g(\varphi)d\varphi - \int_{\varphi_m}^{\infty} F_D wi(\varphi)g(\varphi)d\varphi \\
&- \int_{\varphi_m}^{\infty} F_X wi_x(\varphi)g(\varphi)d\varphi - F_E w \\
&= \int_{\varphi_m}^{\infty} \left[\frac{\pi_D(\varphi)}{\delta} - F_D w\right] i(\varphi)g(\varphi)d\varphi + \int_{\varphi_m}^{\infty} \left[\frac{\pi_X(\varphi)}{\delta} - F_X w\right] i_x(\varphi)g(\varphi)d\varphi - F_E w.
\end{aligned}$$

Notice that for $v_L = 0$ the firm's objective function does not depend on K, $f(\varphi)$ and $f'(\varphi)$ separately, but on T' only. It follows that, so far as (LC), (IC), (RP), (IC') and (RP') hold, the condition (8.46) on T' can be achieved by arbitrary varying K, $f(\varphi)$ and $f'(\varphi)$. This introduces a degree of freedom in determining the optimal contract and the (IC) and the (IC') do not have to be binding. On the other hand, the firms with the minimum productivity level needed to obtain the loans $F_D w$ and $F_X w$ have the incentive to make per period repayments as high as the (IC) and the (IC') allow for. This makes it easier to meet the lender's (RP) and (RP') conditions. In other words, the (IC) and the (IC') will be binding for the marginal firms:

$$\begin{aligned}
\pi_D(\varphi) - f(\varphi) &= (1 - \lambda)\pi_D(\varphi) \\
\pi_D(\varphi) - f(\varphi) &= \pi_D(\varphi) - \lambda\pi_D(\varphi) \\
f(\varphi) &= \lambda\pi_D(\varphi)
\end{aligned}$$

for the domestic firm and

$$\begin{aligned}\pi_D(\varphi) + \pi_X(\varphi) - f'(\varphi) &= (1-\lambda)(\pi_D(\varphi) + \pi_X(\varphi)) \\ \pi_D(\varphi) + \pi_X(\varphi) - f'(\varphi) &= \pi_D(\varphi) + \pi_X(\varphi) - \lambda(\pi_D(\varphi) + \pi_X(\varphi)) \\ f'(\varphi) &= \lambda(\pi_D(\varphi) + \pi_X(\varphi))\end{aligned}$$

for the exporting firm. Using these values in (RP) and (RP'), I obtain

$$\begin{aligned}\sum_{t=0}^{\infty}(1-\delta)^t f(\varphi) &\geq F_D w \\ \frac{\lambda \pi_D(\varphi)}{\delta} &\geq F_D w \\ \pi_D(\varphi) &\geq \frac{\delta F_D w}{\lambda}.\end{aligned}$$

$$\begin{aligned}\sum_{t=0}^{\infty}(1-\delta)^t f'(\varphi) &\geq (F_D + F_X)w \\ \frac{\lambda(\pi_D(\varphi) + \pi_X(\varphi))}{\delta} &\geq (F_D + F_X)w \\ \pi_D(\varphi) + \pi_X(\varphi) &\geq \frac{\delta(F_D + F_X)w}{\lambda}.\end{aligned} \tag{8.47}$$

The entry rule for the domestic market is given by $i(\varphi) = 1$ if and only if $\varphi \geq \varphi^*_{cc}$, where φ^*_{cc} is such that $\pi_D(\varphi^*_{cc}) = \frac{\delta F_D w}{\lambda}$. To solve for the entry rule on the export market, condition (8.47) must be compared with the first best condition $\pi_X(\varphi) \geq \delta F_X w$. If the later is not met, exporting induces net losses and no firms will be willing to enter the foreign market although the (RP') is satisfied. It follows that $i_x(\varphi) = 1$ if and only if $\varphi \geq \varphi^*_{cc\_x}$, where $\varphi^*_{cc\_x}$ is given by the larger between $\inf\{\varphi : \pi_D(\varphi) + \pi_X(\varphi) > \frac{\delta[F_D + F_X]w}{\lambda}\}$ and the frictionless case cutoff $\inf\{\varphi : \pi_X(\varphi) > \delta F_X w\}$ (see Proposition 6).

Given the optimal entry rules, the firm's problem reduces thus to:

$$\max_{K,f(\varphi),f'(\varphi)} v_E = \int_{\varphi_{cc}^*}^{\infty}\left[\frac{\pi_D(\varphi)}{\delta} - F_D w\right] g(\varphi)d\varphi + \int_{\varphi_{cc\_x}^*}^{\infty}\left[\frac{\pi_X(\varphi)}{\delta} - F_X w\right] g(\varphi)d\varphi - F_E w \tag{P}$$

subject to

$$K = \int_{\varphi_{cc}^*}^{\varphi_{cc\_x}^*}\left[\frac{f(\varphi)}{\delta} - F_D w\right] g(\varphi)d\varphi + \int_{\varphi_{cc\_x}^*}^{\infty}\left[\frac{f'(\varphi)}{\delta} - (F_D + F_X)w\right] g(\varphi)d\varphi - F_E w \geq 0 \tag{LC+PC}$$

$$\delta F_D w \leq f(\varphi) \leq \lambda\pi_D(\varphi) \text{ for all } \varphi \in [\varphi_{cc}^*, \varphi_{cc\_x}^*] \tag{IC+RP}$$

$$\delta(F_D + F_X)w \leq f'(\varphi) \leq \lambda(\pi_D(\varphi) + \pi_X(\varphi)) \text{ for all } \varphi \geq \varphi_{cc\_x}^* \tag{IC'+RP'}$$

that, as argued above, does not have a unique solution for the triplet $(K, f(\varphi), f'(\varphi))$.

8.10. Aggregate quantities in the open economy. In the open economy, the price index for the consumption bundle includes all domestic varieties plus the varieties exported by the foreign country. The probability density function of productivity levels on the subset $[\varphi^*, \infty)$ is given by $g(\varphi)/[1 - G(\varphi_{cc}^*)]$. Because of symmetry, I can write P as:

$$\begin{aligned} P &= \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \\ &= \left[\int_{\varphi_{cc}^*}^{\infty} p_D(\varphi)^{1-\sigma} M \frac{g(\varphi)}{1 - G(\varphi_{cc}^*)} d\varphi + \int_{\varphi_{cc\_x}^*}^{\infty} p_X(\varphi)^{1-\sigma} M \frac{g(\varphi)}{1 - G(\varphi_{cc}^*)} d\varphi\right]^{\frac{1}{1-\sigma}}. \end{aligned}$$

Now, using $M_X = p_x M$, $p_x = \frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}$, $p_D(\varphi) = w/\rho\varphi$ and $p_X(\varphi) = w\tau/\rho\varphi$, I obtain

$$\begin{aligned}
P &= \left[\int_{\varphi^*_{cc}}^{\infty} p_D(\varphi)^{1-\sigma} M \frac{g(\varphi)}{1-G(\varphi^*_{cc})} d\varphi + \int_{\varphi^*_{cc_x}}^{\infty} p_X(\varphi)^{1-\sigma} M_X \frac{g(\varphi)}{1-G(\varphi^*_{cc_x})} d\varphi\right]^{\frac{1}{1-\sigma}} \\
&= \frac{w}{\rho}\left[M \int_{\varphi^*_{cc}}^{\infty} \left(\frac{1}{\varphi}\right)^{1-\sigma} \frac{g(\varphi)}{1-G(\varphi^*_{cc})} d\varphi + M_X \int_{\varphi^*_{cc_x}}^{\infty} \left(\frac{\tau}{\varphi}\right)^{1-\sigma} \frac{g(\varphi)}{1-G(\varphi^*_{cc_x})} d\varphi\right]^{\frac{1}{1-\sigma}} \\
&= \frac{w}{\rho}\left[M \int_{\varphi^*_{cc}}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*_{cc})} d\varphi + M_X \tau^{1-\sigma} \int_{\varphi^*_{cc_x}}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*_{cc_x})} d\varphi\right]^{\frac{1}{1-\sigma}} \\
&= \frac{w}{\rho}\left[M \tilde{\varphi}_{cc}^{\sigma-1} + M_X \tau^{1-\sigma} \tilde{\varphi}_{cc_x}^{\sigma-1}\right]^{\frac{1}{1-\sigma}} \\
&= \frac{w}{\rho}\left[\frac{\bar{M}}{\bar{M}}\left(M \tilde{\varphi}_{cc}^{\sigma-1} + M_X (\tau^{-1}\tilde{\varphi}_{cc_x})^{\sigma-1}\right)\right]^{\frac{1}{1-\sigma}} \\
&= \bar{M}^{\frac{1}{1-\sigma}} \frac{w}{\rho\bar{\varphi}_{cc}} \\
&= \bar{M}^{\frac{1}{1-\sigma}} p_D(\bar{\varphi}_{cc})
\end{aligned}$$

where $\bar{M} \equiv M + M_X$, $\tilde{\varphi}_{cc} \equiv \left[\frac{1}{1-G(\varphi^*_{cc})}\int_{\varphi^*_{cc}}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}$, $\tilde{\varphi}_{cc\_x} \equiv \left[\frac{1}{1-G(\varphi^*_{cc\_x})}\int_{\varphi^*_{cc\_x}}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}}$ and $\bar{\varphi}_{cc} \equiv \left\{\frac{1}{\bar{M}}[M\tilde{\varphi}_{cc}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_{cc\_x})^{\sigma-1}]\right\}^{\frac{1}{\sigma-1}}$. Analogously, using $r_D(\varphi) = \alpha E\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1}$ and $r_X(\varphi) = \alpha E\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1}$, the aggregate revenue is given by

$$\begin{aligned}
R &= \int_{\varphi^*_{cc}}^{\infty} r_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*_{cc})} d\varphi + \int_{\varphi^*_{cc_x}}^{\infty} r_X(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*_{cc})} d\varphi \\
&= \int_{\varphi^*_{cc}}^{\infty} r_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi^*_{cc})} d\varphi + \int_{\varphi^*_{cc_x}}^{\infty} r_X(\varphi) M_X \frac{g(\varphi)}{1-G(\varphi^*_{cc_x})} d\varphi \\
&= \int_{\varphi^*_{cc}}^{\infty} \alpha E\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1} M \frac{g(\varphi)}{1-G(\varphi^*_{cc})} d\varphi + \int_{\varphi^*_{cc_x}}^{\infty} \alpha E\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1} M_X \frac{g(\varphi)}{1-G(\varphi^*_{cc_x})} d\varphi \\
&= \alpha E\left(P\frac{\rho}{w}\right)^{\sigma-1}\left[M \int_{\varphi^*_{cc}}^{\infty} \varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*_{cc})} d\varphi + M_X \int_{\varphi^*_{cc_x}}^{\infty} \tau^{1-\sigma}\varphi^{\sigma-1} \frac{g(\varphi)}{1-G(\varphi^*_{cc_x})} d\varphi\right] \\
&= \alpha E\left(P\frac{\rho}{w}\right)^{\sigma-1}\left[M\tilde{\varphi}_{cc}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_{cc_x})^{\sigma-1}\right] \\
&= M\alpha E\left(P\frac{\rho\tilde{\varphi}_{cc}}{w}\right)^{\sigma-1} + M_X \alpha E\left(P\frac{\rho\tilde{\varphi}_{cc_x}}{w\tau}\right)^{\sigma-1} \\
&= M r_D(\tilde{\varphi}_{cc}) + M_X r_X(\tilde{\varphi}_{cc_x}).
\end{aligned}$$

Dividing by M, the average revenue $\bar{r}$ is given by:

$$\begin{aligned}\bar{r}_{cc} = \frac{R}{M} =& r_D(\tilde{\varphi}_{cc}) + \frac{M_X}{M} r_X(\tilde{\varphi}_{cc\_x}) \\ =& r_D(\tilde{\varphi}_{cc}) + p_x r_X(\tilde{\varphi}_{cc\_x}).\end{aligned}$$

Alternatively, using the definition of $\bar{\varphi}_{cc}$, I obtain

$$\begin{aligned}R &= \alpha E \left(P\frac{\rho}{w}\right)^{\sigma-1} \left[M\tilde{\varphi}_{cc}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_{cc\_x})^{\sigma-1}\right] \\ &= \bar{M}\alpha E \left(P\frac{\rho}{w}\right)^{\sigma-1} \left[\frac{1}{\bar{M}}\left(M\tilde{\varphi}_{cc}{}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_{cc\_x})^{\sigma-1}\right)\right] \\ &= \bar{M}\alpha E \left(P\frac{\rho\bar{\varphi}_{cc}}{w}\right)^{\sigma-1} \\ &= \bar{M} r_D(\bar{\varphi}_{cc}).\end{aligned}$$

Given $\pi_D(\varphi) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1}$ and $\pi_X(\varphi) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1}$, following the same steps as above I can derive

$$\begin{aligned}\Pi &= \int_{\varphi_{cc}^*}^{\infty} \pi_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi_{cc}^*)} d\varphi + \int_{\varphi_{cc\_x}^*}^{\infty} \pi_X(\varphi) M \frac{g(\varphi)}{1-G(\varphi_{cc}^*)} d\varphi \\ &= \int_{\varphi_{cc}^*}^{\infty} \pi_D(\varphi) M \frac{g(\varphi)}{1-G(\varphi_{cc}^*)} d\varphi + \int_{\varphi_{cc\_x}^*}^{\infty} \pi_X(\varphi) M_X \frac{g(\varphi)}{1-G(\varphi_{cc\_x}^*)} d\varphi \\ &= \int_{\varphi_{cc}^*}^{\infty} \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1} M \frac{g(\varphi)}{1-G(\varphi_{cc}^*)} d\varphi + \int_{\varphi_{cc\_x}^*}^{\infty} \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1} M_X \frac{g(\varphi)}{1-G(\varphi_{cc\_x}^*)} d\varphi \\ &= \frac{\alpha E}{\sigma}\left(P\frac{\rho}{w}\right)^{\sigma-1}\left[M\int_{\varphi_{cc}^*}^{\infty} \varphi^{\sigma-1}\frac{g(\varphi)}{1-G(\varphi_{cc}^*)} d\varphi + M_X \int_{\varphi_{cc\_x}^*}^{\infty} \tau^{1-\sigma}\varphi^{\sigma-1}\frac{g(\varphi)}{1-G(\varphi_{cc\_x}^*)} d\varphi\right] \\ &= \frac{\alpha E}{\sigma}\left(P\frac{\rho}{w}\right)^{\sigma-1}\left[M\tilde{\varphi}_{cc}^{\sigma-1} + M_X(\tau^{-1}\tilde{\varphi}_{cc\_x})^{\sigma-1}\right] \\ &= M\frac{\alpha E}{\sigma}\left(P\frac{\rho\tilde{\varphi}_{cc}}{w}\right)^{\sigma-1} + M_X\frac{\alpha E}{\sigma}\left(P\frac{\rho\tilde{\varphi}_{cc\_x}}{w\tau}\right)^{\sigma-1} \\ &= M\pi_D(\tilde{\varphi}_{cc}) + M_X\pi_X(\tilde{\varphi}_{cc\_x}).\end{aligned}$$

Dividing by M, the average profits are given by:

$$\bar{\pi}_{cc} = \frac{\Pi}{M} = \pi_D(\tilde{\varphi}_{cc}) + p_x\pi_X(\tilde{\varphi}_{cc\_x}).$$

Finally, since $\pi_D(\varphi) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w}\right)^{\sigma-1} = \frac{r_D(\varphi)}{\sigma}$ and $\pi_X(\varphi) = \frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi}{w\tau}\right)^{\sigma-1} = \frac{r_X(\varphi)}{\sigma}$ for all φ, $\Pi = M\pi_D(\tilde{\varphi}_{cc}) + M_X\pi_X(\tilde{\varphi}_{cc\_x}) = M\frac{r_D(\tilde{\varphi}_{cc})}{\sigma} + M_X\frac{r_X(\tilde{\varphi}_{cc\_x})}{\sigma} = \frac{R}{\sigma}$.

8.11. The ZPC in the open economy. Using (2.6), I can express the profits of the marginal firms in terms of the average profits: $\pi_D(\varphi_{cc}^*) = \pi_D(\tilde{\varphi}_{cc})\left[\frac{\varphi_{cc}^*}{\tilde{\varphi}_{cc}}\right]^{\sigma-1}$ and

$\pi_X(\varphi^*_{cc\_x}) = \pi_X(\tilde{\varphi}_{cc\_x})\left[\frac{\varphi^*_{cc\_x}}{\tilde{\varphi}_{cc\_x}}\right]^{\sigma-1}$. It then follows:

$$\pi_D(\varphi^*_{cc}) = \frac{\delta F_D w}{\lambda} \Leftrightarrow \pi_D(\tilde{\varphi}_{cc}) = \frac{\delta F_D w}{\lambda}\left[\frac{\tilde{\varphi}_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1}$$

$$\pi_X(\varphi^*_{cc\_x}) = \delta F_X w \Leftrightarrow \pi_X(\tilde{\varphi}_{cc\_x}) = \delta F_X w\left[\frac{\tilde{\varphi}_{cc\_x}}{\varphi^*_{cc\_x}}\right]^{\sigma-1}$$

$$\pi_X(\varphi^*_{cc\_x}) = \frac{\delta(F_D+F_X)w}{\lambda(1+\tau^{\sigma-1})} \Leftrightarrow \pi_X(\tilde{\varphi}_{cc\_x}) = \frac{\delta(F_D+F_X)w}{\lambda(1+\tau^{\sigma-1})}\left[\frac{\tilde{\varphi}_{cc\_x}}{\varphi^*_{cc\_x}}\right]^{\sigma-1}$$

where the second row refers to the case $\lambda \geq \hat{\lambda}$ and the third row to the case $\lambda < \hat{\lambda}$. Together with (4.34), the above conditions allow me to express the zero profit condition in terms of the overall average profits:

$$\bar{\pi}_{cc} = \begin{cases} \frac{\delta F_D w}{\lambda}\left[\frac{\tilde{\varphi}_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1} + p_x \delta F_X w\left[\frac{\tilde{\varphi}_{cc\_x}}{\varphi^*_{cc\_x}}\right]^{\sigma-1} & \text{if } \lambda \geq \hat{\lambda}, \\ \frac{\delta F_D w}{\lambda}\left[\frac{\tilde{\varphi}_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1} + p_x \frac{\delta(F_D+F_X)w}{\lambda(1+\tau^{\sigma-1})}\left[\frac{\tilde{\varphi}_{cc\_x}}{\varphi^*_{cc\_x}}\right]^{\sigma-1} & \text{if } \lambda < \hat{\lambda}. \end{cases}$$

8.12. The ratio $\varphi^*_{cc}/\varphi^*_{cc\_x}$. When $\lambda < \hat{\lambda}$, the domestic market cutoff is defined by $\pi_D(\varphi^*_{cc}) = \frac{\delta F_D w}{\lambda}$ and the foreign market cutoff by $\pi_X(\varphi^*_{cc\_x}) = \frac{\delta(F_D+F_X)w}{\lambda(1+\tau^{\sigma-1})}$. Combining the two conditions, I get:

$$\begin{aligned} \frac{\pi_D(\varphi^*_{cc})}{\pi_X(\varphi^*_{cc\_x})} &= \frac{\frac{\delta F_D w}{\lambda}}{\frac{\delta(F_D+F_X)w}{\lambda(1+\tau^{\sigma-1})}} \\ \frac{\frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi^*_{cc}}{w}\right)^{\sigma-1}}{\frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi^*_{cc\_x}}{w\tau}\right)^{\sigma-1}} &= \frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X} \\ \left(\frac{\varphi^*_{cc}\tau}{\varphi^*_{cc\_x}}\right)^{\sigma-1} &= \frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X} \\ \frac{\varphi^*_{cc}}{\varphi^*_{cc\_x}} &= \tau^{-1}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{1}{\sigma-1}}, \end{aligned}$$

which does not depend on λ. Notice that the term $\tau^{-1}(1+\tau^{\sigma-1})^{\frac{1}{\sigma-1}}$, can be rewritten as

$$\tau^{-\frac{\sigma-1}{\sigma-1}}(1+\tau^{\sigma-1})^{\frac{1}{\sigma-1}} = (\tau^{-(\sigma-1)}+1)^{\frac{1}{\sigma-1}}$$

which is decreasing in τ. It follows that also $\varphi^*_{cc}/\varphi^*_{cc\_x}$ is also decreasing in τ.

When $\lambda \geq \hat{\lambda}$, the domestic market cutoff is again defined by $\pi_D(\varphi^*_{cc}) = \frac{\delta F_D w}{\lambda}$ and the foreign market cutoff by $\pi_X(\varphi^*_{cc\_x}) = \delta F_X w$. Combining the two conditions I get:

$$\begin{aligned}
\frac{\pi_D(\varphi^*_{cc})}{\pi_X(\varphi^*_{cc_x})} =& \frac{\frac{\delta F_D w}{\lambda}}{\delta F_X w}\\
\frac{\frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi^*_{cc}}{w}\right)^{\sigma-1}}{\frac{\alpha E}{\sigma}\left(P\frac{\rho\varphi^*_{cc_x}}{w\tau}\right)^{\sigma-1}} =& \frac{F_D}{\lambda F_X}\\
\left(\frac{\varphi^*_{cc}\tau}{\varphi^*_{cc_x}}\right)^{\sigma-1} =& \frac{F_D}{\lambda F_X}\\
\frac{\varphi^*_{cc}}{\varphi^*_{cc_x}} =& \tau^{-1}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{1}{\sigma-1}}.
\end{aligned}$$

It follows immediately that $\varphi^*_{cc}/\varphi^*_{cc\_x}$ is now decreasing in both λ and τ. Moreover when $\lambda = 1$ then the value is the same as in the frictionless setup, while when $\lambda = \hat{\lambda} \equiv \frac{F_D/F_X+1}{1+\tau^{\sigma-1}}$ the two expressions computed above have the same value.

8.13. The FEC in the open economy. In the text, I claim that entry will continue till $K_{max} = 0$. K_{max} is achieved when $f(\varphi) = \lambda\pi_D(\varphi)$ for all $\varphi^*_{cc} \leq \varphi < \varphi^*_{cc\_x}$ and $f'(\varphi) = \lambda(\pi_D(\varphi) + \pi_X(\varphi))$ for all $\varphi \geq \varphi^*_{cc\_x}$. Plugging these values into (4.31), I obtain:

$$\begin{aligned}
K_{max} =& \int_{\varphi^*_{cc}}^{\varphi^*_{cc_x}} \left[\frac{\lambda\pi_D(\varphi)}{\delta} - F_D w\right] g(\varphi)d\varphi\\
&+ \int_{\varphi_{cc_x^*}}^{\infty} \left[\frac{\lambda(\pi_D(\varphi) + \pi_X(\varphi))}{\delta} - (F_D + F_X)w\right] g(\varphi)d\varphi - F_E w\\
=& \int_{\varphi^*_{cc}}^{\infty} \left[\frac{\lambda\pi_D(\varphi)}{\delta} - F_D w\right] g(\varphi)d\varphi + \int_{\varphi^*_{cc_x}}^{\infty} \left[\frac{\lambda\pi_X(\varphi)}{\delta} - F_X w\right] g(\varphi)d\varphi - F_E w.
\end{aligned}$$

The FEC is thus given by

$$\begin{aligned}
\int_{\varphi^*_{cc}}^{\infty} \left[\frac{\lambda\pi_D(\varphi)}{\delta} - F_D w\right] g(\varphi)d\varphi + \int_{\varphi^*_{cc_x}}^{\infty} \left[\frac{\lambda\pi_X(\varphi)}{\delta} - F_X w\right] g(\varphi)d\varphi =& F_E w\\
\int_{\varphi^*_{cc}}^{\infty} \frac{\lambda\pi_D(\varphi)}{\delta} g(\varphi)d\varphi - (1 - G(\varphi^*_{cc}))F_D w + \int_{\varphi^*_{cc_x}}^{\infty} \frac{\lambda\pi_X(\varphi)}{\delta} g(\varphi)d\varphi - (1 - G(\varphi^*_{cc_x}))F_X w =& F_E w.
\end{aligned}$$

Using the definitions $\pi_D(\tilde{\varphi}_{cc}) \equiv [1 - G(\varphi^*_{cc})]^{-1}\int_{\varphi^*_{cc}}^{\infty} \pi_D(\varphi)g(\varphi)d\varphi$, $\pi_X(\tilde{\varphi}_{cc\_x}) \equiv [1 - G(\varphi^*_{cc\_x})]^{-1}\int_{\varphi^*_{cc\_x}}^{\infty} \pi_X(\varphi)g(\varphi)d\varphi$ and $\bar{\pi}_{cc} \equiv \pi_D(\tilde{\varphi}_{cc}) + \frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}\pi_X(\tilde{\varphi}_{cc\_x})$, I can rewrite the above condition as:

$$\begin{aligned}\frac{\lambda\pi_D(\tilde{\varphi}_{cc})}{\delta}(1-G(\varphi^*_{cc}))+\frac{\lambda\pi_X(\tilde{\varphi}_{cc\_x})}{\delta}(1-G(\varphi^*_{cc\_x})) =&w\left[F_D(1-G(\varphi^*_{cc}))+F_X(1-G(\varphi^*_{cc\_x}))+F_E\right]\\ \frac{\lambda\pi_D(\tilde{\varphi}_{cc})}{\delta}+\frac{\lambda\pi_X(\tilde{\varphi}_{cc\_x})}{\delta}\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})} =&w\left[F_D+F_X\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}+\frac{F_E}{1-G(\varphi^*_{cc})}\right]\\ \lambda\bar{\pi}_{cc} =&\delta w\left[F_D+F_X\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}+\frac{F_E}{1-G(\varphi^*_{cc})}\right].\end{aligned}$$

The same definitions used above can be used to find the equilibrium value of

$$\begin{aligned}v_E &= \int_{\varphi^*_{cc}}^{\infty}\left[\frac{\pi_D(\varphi)}{\delta}-F_Dw\right]g(\varphi)d\varphi+\int_{\varphi^*_{cc\_x}}^{\infty}\left[\frac{\pi_X(\varphi)}{\delta}-F_Xw\right]g(\varphi)d\varphi-F_Ew\\ &= \frac{\pi_D(\tilde{\varphi}_{cc})}{\delta}(1-G(\varphi^*_{cc}))+\frac{\pi_X(\tilde{\varphi}_{cc\_x})}{\delta}(1-G(\varphi^*_{cc\_x}))\\ &- w\left[F_D(1-G(\varphi^*_{cc}))+F_X(1-G(\varphi^*_{cc\_x}))+F_E\right]\\ &= (1-G(\varphi^*_{cc}))\left[\frac{\pi_D(\tilde{\varphi}_{cc})}{\delta}+\frac{\pi_X(\tilde{\varphi}_{cc\_x})}{\delta}\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}\right]\\ &- w(1-G(\varphi^*_{cc}))\left[F_D+F_X\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}+\frac{F_E}{1-G(\varphi^*_{cc})}\right]\\ &= \frac{1-G(\varphi^*_{cc})}{\delta}\left\{\bar{\pi}_{cc}-\delta w\left[F_D+F_X\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}+\frac{F_E}{1-G(\varphi^*_{cc})}\right]\right\}.\end{aligned}$$

Given the FEC, this becomes

$$\begin{aligned}v_E &= \frac{1-G(\varphi^*_{cc})}{\delta}[\bar{\pi}_{cc}-\lambda\bar{\pi}_{cc}]\\ &= (1-G(\varphi^*_{cc}))\frac{(1-\lambda)\bar{\pi}_{cc}}{\delta}.\end{aligned}$$

8.14. The FEC and the ZPC under the Pareto distribution assumption-open economy. As for the other cases, the expressions for $\tilde{\varphi}_{cc}$ and $\tilde{\varphi}_{cc\_x}$ under the Pareto distribution assumption are given by:

$$\begin{aligned}
\tilde{\varphi}_{cc} \equiv \tilde{\varphi}(\varphi^*_{cc}) &= \left[\frac{1}{1-G(\varphi^*_{cc})}\int_{\varphi^*_{cc}}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{1-1+\left(\frac{\varphi_m}{\varphi^*_{cc}}\right)^a}\int_{\varphi^*_{cc}}^{\infty}\varphi^{\sigma-1}\frac{a}{\varphi}\left(\frac{\varphi_m}{\varphi}\right)^a d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a a\varphi_m^a\int_{\varphi^*_{cc}}^{\infty}\varphi^{\sigma-1-1-a}d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi^*_{cc})^a}{\sigma-1-a}\varphi^{\sigma-1-a}|^{\infty}_{\varphi^*_{cc}}\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi^*_{cc}
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\varphi}_{cc_x} \equiv \tilde{\varphi}(\varphi^*_{cc_x}) &= \left[\frac{1}{1-G(\varphi^*_{cc_x})}\int_{\varphi^*_{cc_x}}^{\infty}\varphi^{\sigma-1}g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{1}{1-1+\left(\frac{\varphi_m}{\varphi^*_{cc_x}}\right)^a}\int_{\varphi^*_{cc_x}}^{\infty}\varphi^{\sigma-1}\frac{a}{\varphi}\left(\frac{\varphi_m}{\varphi}\right)^a d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\left(\frac{\varphi^*_{cc_x}}{\varphi_m}\right)^a a\varphi_m^a\int_{\varphi^*_{cc_x}}^{\infty}\varphi^{\sigma-1-1-a}d\varphi\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a(\varphi^*_{cc_x})^a}{\sigma-1-a}\varphi^{\sigma-1-a}|^{\infty}_{\varphi^*_{cc_x}}\right]^{\frac{1}{\sigma-1}} \\
&= \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi^*_{cc_x}.
\end{aligned}$$

Plugging these equations into (4.35), together with $G(\varphi) = 1 - \left(\frac{\varphi_m}{\varphi}\right)^a$, I obtain

$$\begin{aligned}
\bar{\pi}_{cc} =& \frac{\delta F_D w}{\lambda}\left[\frac{\tilde{\varphi}_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1} + p_x \delta F_X w \left[\frac{\tilde{\varphi}_{cc_x}}{\varphi^*_{cc_x}}\right]^{\sigma-1} \\
=& \frac{\delta F_D w}{\lambda}\left[\frac{\left(\frac{a}{a-\sigma+1}\right)^{\frac{1}{\sigma-1}}\varphi^*_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1} + \frac{1-G(\varphi^*_{cc_x})}{1-G(\varphi^*_{cc})}\delta F_X w \left[\frac{\left(\frac{a}{a-\sigma+1}\right)^{\frac{1}{\sigma-1}}\varphi^*_{cc_x}}{\varphi^*_{cc_x}}\right]^{\sigma-1} \\
=& \frac{\delta F_D w a}{\lambda(a-\sigma+1)} + \frac{1-G(\varphi^*_{cc_x})}{1-G(\varphi^*_{cc})}\frac{\delta F_X w a}{a-\sigma+1} \\
=& \frac{\delta F_D w a}{\lambda(a-\sigma+1)} + \frac{\left(\frac{\varphi_m}{\varphi^*_{cc_x}}\right)^a}{\left(\frac{\varphi_m}{\varphi^*_{cc}}\right)^a}\frac{\delta F_X w a}{a-\sigma+1} \\
=& \frac{\delta w a}{a-\sigma+1}\left[\frac{F_D}{\lambda} + \left(\frac{\varphi^*_{cc}}{\varphi^*_{cc_x}}\right)^a F_X\right]
\end{aligned}$$

when $\lambda \geq \hat{\lambda}$ and

$$\begin{aligned}
\bar{\pi}_{cc} =& \frac{\delta F_D w}{\lambda}\left[\frac{\tilde{\varphi}_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1} + p_x \frac{\delta(F_D+F_X)w}{\lambda(1+\tau^{\sigma-1})}\left[\frac{\tilde{\varphi}_{cc_x}}{\varphi^*_{cc_x}}\right]^{\sigma-1} \\
=& \frac{\delta F_D w}{\lambda}\left[\frac{\left(\frac{a}{a-\sigma+1}\right)^{\frac{1}{\sigma-1}}\varphi^*_{cc}}{\varphi^*_{cc}}\right]^{\sigma-1} + \frac{1-G(\varphi^*_{cc_x})}{1-G(\varphi^*_{cc})}\frac{\delta(F_D+F_X)w}{\lambda(1+\tau^{\sigma-1})}\left[\frac{\left(\frac{a}{a-\sigma+1}\right)^{\frac{1}{\sigma-1}}\varphi^*_{cc_x}}{\varphi^*_{cc_x}}\right]^{\sigma-1} \\
=& \frac{\delta F_D w a}{\lambda(a-\sigma+1)} + \frac{1-G(\varphi^*_{cc_x})}{1-G(\varphi^*_{cc})}\frac{\delta(F_D+F_X)wa}{\lambda(1+\tau^{\sigma-1})(a-\sigma+1)} \\
=& \frac{\delta F_D w a}{\lambda(a-\sigma+1)} + \frac{\left(\frac{\varphi_m}{\varphi^*_{cc_x}}\right)^a}{\left(\frac{\varphi_m}{\varphi^*_{cc}}\right)^a}\frac{\delta(F_D+F_X)wa}{\lambda(1+\tau^{\sigma-1})(a-\sigma+1)} \\
=& \frac{\delta w a}{\lambda(a-\sigma+1)}\left[F_D + \left(\frac{\varphi^*_{cc}}{\varphi^*_{cc_x}}\right)^a \frac{F_D+F_X}{1+\tau^{\sigma-1}}\right]
\end{aligned}$$

when $\lambda < \hat{\lambda}$. Now given (4.36), the above conditions further simplify to:

$$\begin{aligned}
\bar{\pi}_{cc} =& \frac{\delta w a}{a-\sigma+1}\left\{\frac{F_D}{\lambda} + \left[\tau\left(\frac{\lambda F_X}{F_D}\right)^{\frac{1}{\sigma-1}}\right]^{-a} F_X\right\} \\
=& \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left[1+\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a}{\sigma-1}} \frac{\lambda F_X}{F_D}\right] \\
=& \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left[1+\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a}{\sigma-1}-1}\right] \\
=& \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left[1+\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]
\end{aligned}$$

and

$$\begin{aligned}
\bar{\pi}_{cc} =& \frac{\delta w a}{\lambda(a-\sigma+1)}\left\{F_D + \left[\tau\left(\frac{F_D+F_X}{F_D(1+\tau^{\sigma-1})}\right)^{\frac{1}{\sigma-1}}\right]^{-a} \frac{F_D+F_X}{1+\tau^{1-\sigma}}\right\} \\
=& \frac{\delta w a}{\lambda(a-\sigma+1)}\left\{F_D + \tau^{-a}\left(\frac{F_D+F_X}{F_D(1+\tau^{\sigma-1})}\right)^{\frac{-a}{\sigma-1}} F_D \frac{F_D+F_X}{F_D(1+\tau^{1-\sigma})}\right\} \\
=& \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left\{1 + \tau^{-a}\left(\frac{F_D+F_X}{F_D(1+\tau^{\sigma-1})}\right)^{\frac{-a}{\sigma-1}} \frac{F_D+F_X}{F_D(1+\tau^{1-\sigma})}\right\} \\
=& \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left\{1 + \tau^{-a}\left(\frac{F_D+F_X}{F_D(1+\tau^{\sigma-1})}\right)^{\frac{-a}{\sigma-1}+1}\right\} \\
=& \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left\{1 + \tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right\}
\end{aligned}$$

respectively.

As for the FEC

$$\begin{aligned}
\bar{\pi}_{cc} =& \frac{\delta w}{\lambda}\left[\frac{F_E}{1-G(\varphi^*_{cc})} + F_X\frac{1-G(\varphi^*_{cc_x})}{1-G(\varphi^*_{cc})} + F_D\right] \\
=& \frac{\delta w}{\lambda}\left[\frac{F_E}{\left(\frac{\varphi_m}{\varphi^*_{cc}}\right)^a} + F_X\frac{\left(\frac{\varphi_m}{\varphi^*_{cc_x}}\right)^a}{\left(\frac{\varphi_m}{\varphi^*_{cc}}\right)^a} + F_D\right] \\
=& \frac{\delta w}{\lambda}\left[F_E\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + F_X\left(\frac{\varphi^*_{cc}}{\varphi^*_{cc_x}}\right)^a + F_D\right].
\end{aligned}$$

Again, from (4.36) I obtain:

$$\begin{aligned}
\bar{\pi}_{cc} =& \frac{\delta w}{\lambda}\left[F_E\left(\frac{\varphi_{cc}^*}{\varphi_m}\right)^a + F_X\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a}{\sigma-1}} + F_D\right] \\
=& \frac{\delta w F_D}{\lambda}\left[\frac{F_E}{F_D}\left(\frac{\varphi_{cc}^*}{\varphi_m}\right)^a + \tau^{-a}\lambda^{\frac{-a}{\sigma-1}}\frac{F_X}{F_D}\left(\frac{F_D}{F_X}\right)^{\frac{a}{\sigma-1}} + 1\right] \\
=& \frac{\delta w F_D}{\lambda}\left[\frac{F_E}{F_D}\left(\frac{\varphi_{cc}^*}{\varphi_m}\right)^a + \tau^{-a}\lambda^{\frac{-a}{\sigma-1}-1+1}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}} + 1\right] \\
=& \frac{\delta w F_D}{\lambda}\left[\frac{F_E}{F_D}\left(\frac{\varphi_{cc}^*}{\varphi_m}\right)^a + \tau^{-a}\lambda^{-1}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}} + 1\right]
\end{aligned}$$

when $\lambda \geq \hat{\lambda}$ and

$$\begin{aligned}
\bar{\pi}_{cc} =& \frac{\delta w}{\lambda}\left[F_E\left(\frac{\varphi_{cc}^*}{\varphi_m}\right)^a + F_X\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}} + F_D\right] \\
=& \frac{\delta w F_D}{\lambda}\left[\frac{F_E}{F_D}\left(\frac{\varphi_{cc}^*}{\varphi_m}\right)^a + \tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}} + 1\right]
\end{aligned}$$

when $\lambda < \hat{\lambda}$.

8.15. Proof of proposition 8: existence and uniqueness of equilibrium in an open economy with imperfect creditor protection. First, I consider (4.37) when $0 < \lambda < \hat{\lambda}$:

$$\begin{aligned}
(FEC) \quad \bar{\pi}_{cc} =& \frac{\delta w F_D}{\lambda}\left[\frac{F_E}{F_D}\left(\frac{\varphi_{cc}^*}{\varphi_m}\right)^a + \tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}} + 1\right] \\
(ZPC) \quad \bar{\pi}_{cc} =& \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left[1 + \tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right].
\end{aligned}$$

If φ_{cc}^* is replaced by φ and $\bar{\pi}_{cc}$ is replaced by π, I can graph FEC and ZPC in (φ, π) space. The FEC is increasing in φ with $\pi = \frac{\delta w a F_D}{\lambda}\left[\frac{F_E}{F_D} + \tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}} + 1\right]$ at $\varphi = \varphi_m$ and the ZPC is constant in φ for all $\varphi \geq \varphi_m$. Considering only the case where not all firms start production, implies:

$$\frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left[1+\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right] > \frac{\delta w F_D}{\lambda}\left[\frac{F_E}{F_D}+\tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}}+1\right]$$

$$\frac{a}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right] > \frac{F_E}{F_D}+\left[1+\tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}}\right].$$

Rearranging, I obtain

$$\frac{F_E}{F_D} < \frac{a}{a-\sigma+1}-1+\frac{a\tau^{-a}}{a-\sigma+1}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}-\tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}}$$

$$\frac{F_E}{F_D} < \frac{\sigma-1}{a-\sigma+1}+\frac{a\tau^{-a}}{a-\sigma+1}\left(\frac{F_D}{F_X}\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{\frac{a-\sigma+1}{\sigma-1}}-\tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D}{F_X}\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{\frac{a}{\sigma-1}}$$

$$\frac{F_E}{F_D} < \frac{\sigma-1}{a-\sigma+1}+\frac{a\tau^{-a}}{a-\sigma+1}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\left(\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{\frac{a-\sigma+1}{\sigma-1}}$$
$$-\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\left(\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{\frac{a}{\sigma-1}}$$

$$\frac{F_E}{F_D} < \frac{\sigma-1}{a-\sigma+1}+\tau^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\left(\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{\frac{a}{\sigma-1}}\left[\frac{a}{a-\sigma+1}\left(\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{-1}-1\right] \tag{8.48}$$

Under this assumption, which holds when F_E is sufficiently small, the FEC cuts the ZPC line only once from below and $\varphi^*_{cc} > \varphi_m$ and $\bar{\pi}_{cc} = \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left[1+\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]$.

Next, I consider (4.38) when $\hat{\lambda} \leq \lambda < 1$:

$$(FEC) \qquad \bar{\pi}_{cc} = \frac{\delta w F_D}{\lambda}\left[\frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a+\tau^{-a}\lambda^{-1}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}+1\right]$$

$$(ZPC) \qquad \bar{\pi}_{cc} = \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left[1+\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right].$$

If φ^*_{cc} is replaced by φ and $\bar{\pi}_{cc}$ is replaced by π, I can graph FEC and ZPC in (φ, π) space. FEC is increasing in φ with $\pi = \frac{\delta w F_D}{\lambda}\left[\frac{F_E}{F_D}+\left(\tau\lambda^{\frac{1}{\sigma-1}}\right)^{-a}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}+1\right]$ at $\varphi = \varphi_m$ and ZPC is constant in φ for all $\varphi \geq \varphi_m$. Since $\lambda < 1$ always appears at the denominator, both curves are shifted upwards with respect to the case $\lambda = 1$ (compare with (3.21)). As before, I will consider only the case in which not all firms are efficient enough to start production. This happens when:

$$
\begin{aligned}
\frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left[1+\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right] &> \frac{\delta w F_D}{\lambda}\left[\frac{F_E}{F_D}+\tau^{-a}\lambda^{-1}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}+1\right] \\
\frac{a}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right] &> \frac{F_E}{F_D}+\tau^{-a}\lambda^{-1}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}+1 \\
\frac{a}{a-\sigma+1}-1+\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\left(\frac{a}{a-\sigma+1}-\frac{1}{\lambda}\right) &> \frac{F_E}{F_D} \\
\frac{\sigma-1}{a-\sigma+1}+\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\left(\frac{a}{a-\sigma+1}-\frac{1}{\lambda}\right) &> \frac{F_E}{F_D}. \qquad (8.49)
\end{aligned}
$$

Under this assumption, which holds when F_E is sufficiently small, the FEC cuts the ZPC only once from below and $\varphi^*_{cc} > \varphi_m$ and $\bar{\pi}_{cc} = \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left[1+\tau^{-a}\left(\frac{F_D}{\lambda F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]$.

Once φ^*_{cc} is known, (4.36) uniquely pins down $\varphi^*_{cc\_x}$. I can then solve for $\tilde{\varphi}_{cc} = \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi^*_{cc}$ and for $\tilde{\varphi}_{cc\_x} = \left[\frac{a}{a-\sigma+1}\right]^{\frac{1}{\sigma-1}}\varphi^*_{cc\_x}$.

The FEC (4.33) also implies that

$$
\begin{aligned}
\lambda\bar{\pi}_{cc} &= \delta w\left(F_D + F_X\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}+\frac{F_E}{1-G(\varphi^*_{cc})}\right) \\
\bar{\pi}_{cc}M &= \frac{\delta M w}{\lambda}\left(F_D + F_X\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}+\frac{F_E}{1-G(\varphi^*_{cc})}\right) \\
\Pi &= \frac{\delta M w}{\lambda}\left(F_D + F_X\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}+\frac{F_E}{1-G(\varphi^*_{cc})}\right).
\end{aligned}
$$

Given $[1-G(\varphi^*_{cc})]M_e = \delta M$ and $M_X = \frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}M$, I can derive the total wages of innovation workers wL_I as follows:

$$
\begin{aligned}
L_I =& M_e[1-G(\varphi^*_{cc})]F_D + M_e[1-G(\varphi^*_{cc\_x})]F_X + M_e F_E \\
L_I =& \frac{\delta M}{1-G(\varphi^*_{cc})}[(1-G(\varphi^*_{cc}))F_D + (1-G(\varphi^*_{cc\_x}))F_X + F_E] \\
L_I =& \delta M\left(F_D + \frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}F_X + \frac{F_E}{1-G(\varphi^*_{cc})}\right) \\
L_I w =& \delta M w\left(F_D + F_X\frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}+\frac{F_E}{1-G(\varphi^*_{cc})}\right).
\end{aligned}
$$

Combining the expressions derived above together with $R = \sigma\Pi$, I obtain $L_I w = \lambda\Pi = \frac{\lambda R}{\sigma}$. The wages to production workers in the differentiated sector are given by $wL_P = R - \Pi = R(\sigma-1)/\sigma$ and the wages to workers in the homogeneous good sector are given by $L_0 w = R_0$. Now, using $R_0 = (1-\alpha)E$ and $R = \alpha E$, I can rearrange the labor market clearing condition to solve for the aggregate equilibrium expenditure E:

$$\begin{aligned}
L =& L_I + L_P + L_0 \\
wL =& wL_I + wL_P + wL_0 \\
wL =& \frac{\lambda R}{\sigma} + \frac{R(\sigma-1)}{\sigma} + R_0 \\
wL =& \frac{\alpha\lambda E}{\sigma} + \frac{\alpha E(\sigma-1)}{\sigma} + (1-\alpha)E \\
\sigma wL =& \alpha\lambda E + \alpha E\sigma - \alpha E + \sigma E - \sigma\alpha E \\
\sigma wL =& E(\sigma - \alpha(1-\lambda)) \\
\frac{\sigma wL}{\sigma-\alpha(1-\lambda)} =& E > wL.
\end{aligned}$$

The difference between the aggregate expenditure E and the labor income wL is given by the rents gained by new entrants as a result of credit market frictions. The rents $V_E \equiv M_e v_E$ are divided among the consumers who own shares is firms profits. Using $M_e = \delta M/(1-G(\varphi^*_{cc}))$, $\Pi = M\bar{\pi}_{cc}$ and the expression derived above for v_E, I obtain:

$$\begin{aligned}
V_E =& M_e(1-G(\varphi^*_{cc}))\frac{(1-\lambda)\bar{\pi}_{cc}}{\delta} \\
=& \frac{\delta M}{1-G(\varphi^*_{cc})}(1-G(\varphi^*_{cc}))\frac{(1-\lambda)\bar{\pi}_{cc}}{\delta} \\
=& M(1-\lambda)\bar{\pi}_{cc} \\
=& (1-\lambda)\Pi.
\end{aligned}$$

Given $\Pi = \frac{R}{\sigma} = \frac{\alpha E}{\sigma}$, the expression above becomes $V_E = \frac{(1-\lambda)\alpha E}{\sigma}$. Adding this to $wL = \frac{E(\sigma-\alpha(1-\lambda))}{\sigma}$, I get

$$\frac{(1-\lambda)\alpha E}{\sigma} + \frac{E(\sigma-\alpha(1-\lambda))}{\sigma} = \frac{(1-\lambda)\alpha E}{\sigma} + \frac{E\sigma}{\sigma} - \frac{(1-\lambda)\alpha E}{\sigma} = E.$$

Given the identity $\bar{\pi}_{cc} \equiv \frac{\Pi}{M}$, the number of firms in equilibrium is given by $M = \frac{\Pi}{\bar{\pi}_{cc}} = \frac{\alpha wL}{(\sigma-\alpha(1-\lambda))\bar{\pi}_{cc}}$. Once M is known, I can solve for $M_X = \frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})}M$ and then for $\bar{M} \equiv M + M_X$. I can thus pin down the equilibrium values for $\bar{\varphi}_{cc} \equiv \left\{\frac{1}{\bar{M}}[M\tilde{\varphi}_{cc}^{\sigma-1} + M_x(\tau^{-1}\tilde{\varphi}_{cc\_x})^{\sigma-1}]\right\}^{\frac{1}{\sigma-1}}$ and for $P = \bar{M}^{1/(1-\sigma)}p_D(\bar{\varphi}_{cc}) = \bar{M}^{1/(1-\sigma)}\frac{w}{\rho\bar{\varphi}_{cc}}$.

8.16. Proof of proposition 9: steady-state analysis with respect to λ. From (4.37), when $0 < \lambda < \hat{\lambda}$, the closed form solution for φ^*_{cc} is given by:

$$\frac{\delta w F_D}{\lambda}\left[\frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + \tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}} + 1\right] =$$
$$= \frac{\delta w a F_D}{\lambda(a-\sigma+1)}\left[1+\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]$$

Rearranging, I obtain

$$\frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a + \tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}} + 1 = \frac{a}{a-\sigma+1}\left[1+\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}-1}\right]$$

$$\frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a = \frac{a-a+\sigma-1}{a-\sigma+1} + \frac{a}{a-\sigma+1}\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}-1}$$
$$-\tau^{-a}\frac{F_X}{F_D}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}}$$

$$\frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a = \frac{\sigma-1}{a-\sigma+1} + \tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a}{\sigma-1}}\left[\frac{a}{a-\sigma+1}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{-1} - \frac{F_X}{F_D}\right]$$

$$\frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a = \frac{\sigma-1}{a-\sigma+1} + \tau^{-a}\left(\frac{F_D}{F_X}\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{\frac{a}{\sigma-1}}\left[\frac{a}{a-\sigma+1}\left(\frac{F_D}{F_X}\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{-1} - \frac{F_X}{F_D}\right]$$

$$\frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a = \frac{\sigma-1}{a-\sigma+1} + \tau^{-a}\left(\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{\frac{a}{\sigma-1}}\left(\frac{F_D}{F_X}\right)^{\frac{a}{\sigma-1}}$$
$$\left[\frac{a}{a-\sigma+1}\frac{F_X}{F_D}\left(\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{-1} - \frac{F_X}{F_D}\right]$$

$$\frac{F_E}{F_D}\left(\frac{\varphi^*_{cc}}{\varphi_m}\right)^a = \frac{\sigma-1}{a-\sigma+1} + \tau^{-a}\left(\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{\frac{a}{\sigma-1}}\left(\frac{F_D}{F_X}\right)^{\frac{a}{\sigma-1}-1}\left[\frac{a}{a-\sigma+1}\left(\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{-1} - 1\right]$$

$$\varphi^*_{cc} = \varphi_m\left\{\left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a}\left(\frac{1+\tau^{\sigma-1}}{F_D/F_X+1}\right)^{\frac{a}{\sigma-1}}\left(\frac{F_D}{F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\left(\frac{a}{a-\sigma+1}\frac{F_D/F_X+1}{1+\tau^{\sigma-1}} - 1\right)\right]\frac{F_D}{F_E}\right\}^{\frac{1}{a}}$$

The threshold is thus constant in λ. Notice that, because of condition (8.48), $\varphi^*_{cc} > \varphi_m > 0$. Moreover, given $\hat{\lambda} \equiv \frac{F_D/F_X+1}{1+\tau^{\sigma-1}}$, the above can be rewritten as

$$\begin{aligned}
\varphi_{cc}^* &= \varphi_m \left\{ \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \hat{\lambda}^{\frac{-a}{\sigma-1}} \left(\frac{F_D}{F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a\hat{\lambda}}{a-\sigma+1} - 1 \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} \\
&= \varphi_m \left\{ \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \hat{\lambda}^{\frac{-a}{\sigma-1}+1} \left(\frac{F_D}{F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\hat{\lambda}} \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} \\
&= \varphi_m \left\{ \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{F_D}{\hat{\lambda} F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\hat{\lambda}} \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} \qquad (8.50)
\end{aligned}$$

When $\hat{\lambda} \leq \lambda < 1$, the closed form solution for φ_{cc}^* is given by solving (4.38):

$$\frac{\delta w F_D}{\lambda} \left[\frac{F_E}{F_D} \left(\frac{\varphi_{cc}^*}{\varphi_m} \right)^a + \tau^{-a} \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 \right] = \frac{\delta w a F_D}{\lambda(a-\sigma+1)} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]$$

$$\frac{F_E}{F_D} \left(\frac{\varphi_{cc}^*}{\varphi_m} \right)^a + \tau^{-a} \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 = \frac{a}{a-\sigma+1} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]$$

$$\frac{F_E}{F_D} \left(\frac{\varphi_{cc}^*}{\varphi_m} \right)^a + \tau^{-a} \lambda^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} + 1 = \frac{a}{a-\sigma+1} + \frac{a\tau^{-a}}{a-\sigma+1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}}$$

Rearranging, I obtain

$$\begin{aligned}
\frac{F_E}{F_D} \left(\frac{\varphi_{cc}^*}{\varphi_m} \right)^a &= \frac{a-a+\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) \\
\left(\frac{\varphi_{cc}^*}{\varphi_m} \right)^a &= \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) \right] \frac{F_D}{F_E} \\
\varphi_{cc}^* &= \varphi_m \left\{ \left[\frac{\sigma-1}{a-\sigma+1} + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \left(\frac{a}{a-\sigma+1} - \frac{1}{\lambda} \right) \right] \frac{F_D}{F_E} \right\}^{\frac{1}{a}} .
\end{aligned}$$

Again, from (8.49), it follows $\varphi_{cc}^* > \varphi_m > 0$. Moreover, notice that when $\lambda = \hat{\lambda}$, φ_{cc}^* has the same value as in (8.50). To study the sign of $\partial \varphi_{cc}^* / \partial \lambda$, I can focus on the partial derivative of the term $\lambda^{\frac{-a+\sigma-1}{\sigma-1}}\left(\frac{a}{a-\sigma+1}-\frac{1}{\lambda}\right)=\lambda^{\frac{-a}{\sigma-1}}\left(\frac{a\lambda}{a-\sigma+1}-1\right)$. This is equal to:

$$
\begin{aligned}
&-\frac{a}{\sigma-1}\lambda^{\frac{-a}{\sigma-1}-1}\left(\frac{a\lambda}{a-\sigma+1}-1\right)+\lambda^{\frac{-a}{\sigma-1}}\frac{a}{a-\sigma+1}=\\
&-\frac{a}{\sigma-1}\lambda^{\frac{-a}{\sigma-1}}\frac{a}{a-\sigma+1}+\frac{a}{\sigma-1}\lambda^{\frac{-a}{\sigma-1}-1}+\lambda^{\frac{-a}{\sigma-1}}\frac{a}{a-\sigma+1}=\\
&\lambda^{\frac{-a}{\sigma-1}}\frac{a}{a-\sigma+1}\left(1-\frac{a}{\sigma-1}\right)+\frac{a}{\sigma-1}\lambda^{\frac{-a}{\sigma-1}-1}=\\
&\lambda^{\frac{-a}{\sigma-1}}\frac{a}{a-\sigma+1}\left(-\frac{a-\sigma+1}{\sigma-1}\right)+\frac{a}{\sigma-1}\lambda^{\frac{-a}{\sigma-1}-1}=\\
&-\lambda^{\frac{-a}{\sigma-1}}\frac{a}{\sigma-1}+\frac{a}{\sigma-1}\lambda^{\frac{-a}{\sigma-1}-1}=\\
&\lambda^{\frac{-a}{\sigma-1}}\frac{a}{\sigma-1}\left(\frac{1}{\lambda}-1\right)>0.
\end{aligned}
$$

In the proof of Proposition 8, I derived $M=\frac{\Pi}{\bar{\pi}_{cc}}=\frac{\alpha wL}{(\sigma-\alpha(1-\lambda))\bar{\pi}_{cc}}$. Given the equilibrium value of $\bar{\pi}_{cc}$, I can solve for the closed form solution. When $0<\lambda<\hat{\lambda}$, $\bar{\pi}_{cc}=\frac{\delta waF_D}{\lambda(a-\sigma+1)}\left[1+\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]$ and

$$
\begin{aligned}
M=&\frac{\alpha wL}{(\sigma-\alpha(1-\lambda))\bar{\pi}_{cc}}\\
=&\frac{\alpha wL}{(\sigma-\alpha(1-\lambda))}\frac{\lambda(a-\sigma+1)}{\delta waF_D\left[1+\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]}\\
=&\frac{\alpha L\lambda(a-\sigma+1)}{\delta aF_D(\sigma-\alpha(1-\lambda))\left[1+\tau^{-a}\left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X}\right)^{\frac{a-\sigma+1}{\sigma-1}}\right]}. \qquad (8.51)
\end{aligned}
$$

The sign of $\partial M/\partial\lambda$ is pinned down by the sign of the partial derivative of the term $\frac{\lambda}{\sigma-\alpha(1-\lambda)}$ with respect to λ. The derivative is given by

$$
\frac{\sigma-\alpha(1-\lambda)-\alpha\lambda}{[\sigma-\alpha(1-\lambda)]^2}=\frac{\sigma-\alpha+\alpha\lambda-\alpha\lambda}{[\sigma-\alpha(1-\lambda)]^2}=\frac{\sigma-\alpha}{[\sigma-\alpha(1-\lambda)]^2}>0.
$$

When $\hat{\lambda} \leq \lambda < 1$, $\bar{\pi}_{cc} = \frac{\delta w a F_D}{\lambda(a-\sigma+1)} \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]$ and

$$\begin{aligned} M =& \frac{\alpha w L}{(\sigma - \alpha(1-\lambda))\bar{\pi}_{cc}} \\ =& \frac{\alpha w L}{(\sigma - \alpha(1-\lambda))} \frac{\lambda(a-\sigma+1)}{\delta w a F_D \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]} \\ =& \frac{\alpha L \lambda (a-\sigma+1)}{\delta a F_D (\sigma - \alpha(1-\lambda)) \left[1 + \tau^{-a} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]}. \end{aligned} \tag{8.52}$$

As before, when λ increases, M increases because of the effect of the term $\frac{\lambda}{\sigma-\alpha(1-\lambda)}$. Moreover, M increases also because of the additional lambda within the squared brackets ($\lambda \uparrow \Rightarrow$ denominator $\downarrow \Rightarrow M \uparrow$, given the assumption $a > \sigma - 1$). As a result $\partial M / \partial \lambda > 0$. Finally, notice that setting $\lambda = \hat{\lambda}$ in (8.51) yields

$$\begin{aligned} M =& \frac{\alpha L \hat{\lambda} (a-\sigma+1)}{\delta a F_D (\sigma - \alpha(1-\hat{\lambda})) \left[1 + \tau^{-a} \left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]} \\ =& \frac{\alpha L \hat{\lambda} (a-\sigma+1)}{\delta a F_D (\sigma - \alpha(1-\hat{\lambda})) \left[1 + \tau^{-a} \left(\frac{F_D}{F_X} \frac{1+\tau^{\sigma-1}}{F_D/F_X+1} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]} \\ =& \frac{\alpha L \hat{\lambda} (a-\sigma+1)}{\delta a F_D (\sigma - \alpha(1-\hat{\lambda})) \left[1 + \tau^{-a} \left(\frac{F_D}{\hat{\lambda} F_X} \right)^{\frac{a-\sigma+1}{\sigma-1}} \right]} \end{aligned}$$

which is the same value as in (8.52) for $\lambda = \hat{\lambda}$.

Given $M_X = \frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})} M$, I can solve for the share of exporting firms as:

$$\frac{M_x}{M} = \frac{1-G(\varphi^*_{cc\_x})}{1-G(\varphi^*_{cc})} = \frac{\left(\frac{\varphi_m}{\varphi^*_{cc\_x}} \right)^a}{\left(\frac{\varphi_m}{\varphi^*_{cc}} \right)^a} = \left(\frac{\varphi^*_{cc}}{\varphi^*_{cc\_x}} \right)^a.$$

From

$$\frac{\varphi^*_{cc}}{\varphi^*_{cc\_x}} = \begin{cases} \tau^{-1} \left(\frac{F_D(1+\tau^{\sigma-1})}{F_D+F_X} \right)^{\frac{1}{\sigma-1}} & \text{if } 0 < \lambda < \hat{\lambda}, \\ \tau^{-1} \left(\frac{F_D}{\lambda F_X} \right)^{\frac{1}{\sigma-1}} & \text{if } \hat{\lambda} \leq \lambda \leq 1. \end{cases}$$

it immediately follows that $\frac{\partial (M_X/M)}{\partial \lambda} \leq 0$.

8.17. Proof of proposition 10: Steady-state analysis with respect to τ. Given Proposition 9, to study the sign of $\partial\varphi^*_{cc}/\partial\tau$ when $0<\lambda<\hat{\lambda}$, I can focus on the term

$$\begin{aligned}\Gamma \equiv&\tau^{-a}(1+\tau^{\sigma-1})^{\frac{a}{\sigma-1}}\left[\frac{a}{a-\sigma+1}\left(\frac{1+\tau^{\sigma-1}}{1+F_D/F_X}\right)^{-1}-1\right]\\ =&\tau^{-(\sigma-1)\frac{a}{\sigma-1}}(1+\tau^{\sigma-1})^{\frac{a}{\sigma-1}}\left[\frac{a}{a-\sigma+1}\left(\frac{1+\tau^{\sigma-1}}{1+F_D/F_X}\right)^{-1}-1\right]\\ =&\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a}{\sigma-1}}\left[\frac{a}{a-\sigma+1}\frac{1+F_D/F_X}{1+\tau^{\sigma-1}}-1\right]\\ =&\left(\tau^{-(\sigma-1)}+1\right)^{\frac{a}{\sigma-1}}\left[\frac{a}{a-\sigma+1}\frac{1+F_D/F_X}{1+\tau^{\sigma-1}}-1\right].\end{aligned}$$

Taking the partial derivative with respect to τ, I obtain:

$$\begin{aligned}\frac{\partial\Gamma}{\partial\tau}=&-\frac{a}{\sigma-1}\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a}{\sigma-1}-1}(\sigma-1)\tau^{-(\sigma-1)-1}\left[\frac{a}{a-\sigma+1}\frac{1+F_D/F_X}{1+\tau^{\sigma-1}}-1\right]\\ &+\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a}{\sigma-1}}\frac{a(1+F_D/F_X)}{a-\sigma+1}\frac{-(\sigma-1)\tau^{\sigma-2}}{(1+\tau^{\sigma-1})^2}\\ =&-a\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a}{\sigma-1}}\frac{\tau^{\sigma-1}\tau^{-\sigma}}{1+\tau^{\sigma-1}}\left(\frac{a}{a-\sigma+1}\frac{1+F_D/F_X}{1+\tau^{\sigma-1}}-1\right)\\ &-a\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a}{\sigma-1}}\left[\frac{1+F_D/F_X}{a-\sigma+1}\frac{(\sigma-1)\tau^{\sigma-1}\tau^{-1}}{(1+\tau^{\sigma-1})(1+\tau^{\sigma-1})}\right]\\ =&-a\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a}{\sigma-1}}\frac{\tau^{\sigma-1}\tau^{-\sigma+1-1}}{1+\tau^{\sigma-1}}\left(\frac{a}{a-\sigma+1}\frac{1+F_D/F_X}{1+\tau^{\sigma-1}}-1\right)\\ &-a\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a}{\sigma-1}}\left[\frac{1+F_D/F_X}{a-\sigma+1}\frac{(\sigma-1)\tau^{-1}}{1+\tau^{\sigma-1}}\frac{\tau^{\sigma-1}}{1+\tau^{\sigma-1}}\right]\\ =&-a\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a}{\sigma-1}-1}\tau^{-1}\left[\tau^{-(\sigma-1)}\left(\frac{a}{a-\sigma+1}\frac{1+F_D/F_X}{1+\tau^{\sigma-1}}-1\right)+\frac{1+F_D/F_X}{a-\sigma+1}\frac{(\sigma-1)}{1+\tau^{\sigma-1}}\right]\\ =&-a\tau^{-1}\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a-\sigma+1}{\sigma-1}}\left[\frac{a(1+F_D/F_X)}{(a-\sigma+1)(1+\tau^{\sigma-1})\tau^{\sigma-1}}-\frac{1}{\tau^{\sigma-1}}+\frac{(1+F_D/F_X)(\sigma-1)}{(a-\sigma+1)(1+\tau^{\sigma-1})}\right]\end{aligned}$$

Simplifying further, the expression for $\frac{\partial\Gamma}{\partial\tau}$ becomes

$$\begin{aligned}&-a\tau^{-1}\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a-\sigma+1}{\sigma-1}}\frac{a+aF_D/F_X-(a-\sigma+1)(1+\tau^{\sigma-1})+(1+F_D/F_X)(\sigma-1)\tau^{\sigma-1}}{(a-\sigma+1)(1+\tau^{\sigma-1})\tau^{\sigma-1}}=\\ &-a\tau^{-1}\left(\frac{1+\tau^{\sigma-1}}{\tau^{\sigma-1}}\right)^{\frac{a}{\sigma-1}-1}\frac{-a(\tau^{\sigma-1}-F_D/F_X)+(\sigma-1)(1+\tau^{\sigma-1})+(1+F_D/F_X)(\sigma-1)\tau^{\sigma-1}}{(a-\sigma+1)(1+\tau^{\sigma-1})\tau^{\sigma-1}}.\end{aligned}$$

Since by assumption $\tau^{\sigma-1} > F_D/F_X$, the sign of the term in squared brackets is ambiguous and depends on the values taken by the exogenous parameters. In particular it is positive when a is sufficiently small and it is negative when a is sufficiently large. First of all, notice that the term is continuous and decreasing in $a \in (\sigma-1, \infty)$. For $a = \sigma - 1$ the numerator becomes

$$\begin{aligned} -a(\tau^{\sigma-1} - F_D/F_X) + (\sigma-1)(1+\tau^{\sigma-1}) + (1+F_D/F_X)(\sigma-1)\tau^{\sigma-1} &= \\ -(\sigma-1)(\tau^{\sigma-1} - F_D/F_X) + (\sigma-1)(1+\tau^{\sigma-1}) + (1+F_D/F_X)(\sigma-1)\tau^{\sigma-1} &= \\ (\sigma-1)[-\tau^{\sigma-1} + F_D/F_X + 1 + \tau^{\sigma-1} + (1+F_D/F_X)\tau^{\sigma-1}] &= \\ (\sigma-1)[F_D/F_X + 1 + (1+F_D/F_X)\tau^{\sigma-1}) &> 0 \end{aligned}$$

which implies that for a sufficiently close to $\sigma - 1$, the sign of the term in square brackets is positive. It follows that for a sufficiently small $\partial\varphi^*_{cc}/\partial\tau < 0$. On the other hand, whenever

$$\begin{aligned} a(\tau^{\sigma-1} - F_D/F_X) &> (\sigma-1)(1+\tau^{\sigma-1}) + (1+F_D/F_X)(\sigma-1)\tau^{\sigma-1} \\ a(\tau^{\sigma-1} - F_D/F_X) &> (\sigma-1)[1+\tau^{\sigma-1} + (1+F_D/F_X)\tau^{\sigma-1}] \\ a &> (\sigma-1)\frac{1+\tau^{\sigma-1} + (1+F_D/F_X)\tau^{\sigma-1}}{\tau^{\sigma-1} - F_D/F_X} > \sigma - 1 \end{aligned}$$

the numerator is negative. This implies that for a sufficiently large the term in square brackets is also negative and $\partial\varphi^*_{cc}/\partial\tau > 0$.

Given (8.51), the sign of $\partial M/\partial\tau$ is determined by the sign of the partial derivative with respect to τ of the term $\tau^{-a}(1+\tau^{\sigma-1})^{\frac{a-\sigma+1}{\sigma-1}}$. This can be re-written as:

$$\tau^{-(\sigma-1)\frac{a}{\sigma-1}}(1+\tau^{\sigma-1})^{\frac{a}{\sigma-1}-1} = (\tau^{-(\sigma-1)}+1)^{\frac{a}{\sigma-1}}(1+\tau^{\sigma-1})^{-1}.$$

Notice that τ appears only with a negative exponent, meaning that the term above is decreasing in τ. Since the term is in the denominator of M, it follows that $\partial M/\partial\tau > 0$.

When $\hat{\lambda} \leq \lambda < 1$, the sign of $\partial\varphi^*_{cc}/\partial\tau$ depends on the sign of the term in round brackets $\frac{a}{a-\sigma+1} - \frac{1}{\lambda}$ (see the expression in proposition 9), which is multiplied by τ^{-a}.

When

$$\begin{aligned}\frac{a}{a-\sigma+1}-\frac{1}{\lambda}&>0\\ a\lambda-a+\sigma-1&>0\\ -a(1-\lambda)&>1-\sigma\\ a&<\frac{\sigma-1}{1-\lambda}\end{aligned}$$

then $\partial\varphi^*_{cc}/\partial\tau<0$. For a sufficiently small, $\partial\varphi^*_{cc}/\partial\tau<0$, while for a sufficiently high ($a>(\sigma-1)/(1-\lambda)$), $\partial\varphi^*_{cc}/\partial\tau>0$. Since τ appears only at the denominator of (8.52) with a negative exponent, it follows immediately that $\partial M/\partial\tau>0$.

Given $M_X/M=\left(\frac{\varphi^*_{cc}}{\varphi^*_{cc\_x}}\right)^a$, the proof that $\frac{\partial(M_X/M)}{\partial\tau}<0$ follows from the results derived above for $\frac{\varphi^*_{cc}}{\varphi^*_{cc\_x}}$ (see 8.12).

8.18. Proof of Lemma 11: Individual welfare U. To compute the welfare $U=q_0^{1-\alpha}q_1^{\alpha}$, I start considering the term

$$q_1=\frac{\alpha e}{P}=\frac{\alpha E}{LP}=\frac{R}{LP}=\frac{R}{L\bar{M}^{1/(1-\sigma)}p_D(\bar{\varphi}_{cc})}=\frac{R\rho\bar{\varphi}_{cc}}{L\bar{M}^{1/(1-\sigma)}w}.$$

Remember that I can always write $\frac{r_D(\bar{\varphi}_{cc})}{r_D(\varphi^*_{cc})}=\left(\frac{\bar{\varphi}_{cc}}{\varphi^*_{cc}}\right)^{\sigma-1}$, where $r_D(\bar{\varphi}_{cc})=R/\bar{M}$ and, from the renegation proof condition for the domestic investment F_Dw, $r_D(\varphi^*_{cc})=\sigma\pi_D(\varphi^*_{cc})=\frac{\sigma\delta F_D w}{\lambda}$. It follows that

$$\begin{aligned}\left(\frac{\bar{\varphi}_{cc}}{\varphi^*_{cc}}\right)^{\sigma-1}=&\frac{R\lambda}{\bar{M}\sigma\delta F_D w}\\ \bar{\varphi}_{cc}=&\varphi^*_{cc}\left(\frac{R\lambda}{\bar{M}\sigma\delta F_D w}\right)^{\frac{1}{\sigma-1}}.\end{aligned}$$

Using this in the expression for q_1, I obtain

$$\begin{aligned}q_1=&\frac{R\rho\varphi^*_{cc}}{L\bar{M}^{1/(1-\sigma)}w}\left(\frac{R\lambda}{\bar{M}\sigma\delta F_D w}\right)^{\frac{1}{\sigma-1}}\\ =&\frac{\alpha E\rho\varphi^*_{cc}}{Lw}\left(\frac{\alpha E\lambda}{\sigma\delta F_D w}\right)^{\frac{1}{\sigma-1}}\\ =&\frac{\alpha\frac{\sigma wL}{\sigma-\alpha(1-\lambda)}\rho\varphi^*_{cc}}{Lw}\left(\frac{\alpha\lambda\frac{\sigma wL}{\sigma-\alpha(1-\lambda)}}{\sigma\delta F_D w}\right)^{\frac{1}{\sigma-1}}\\ =&\frac{\sigma\alpha\rho\varphi^*_{cc}}{\sigma-\alpha(1-\lambda)}\left(\frac{\alpha\lambda L}{\delta F_D(\sigma-\alpha(1-\lambda))}\right)^{\frac{1}{\sigma-1}}.\end{aligned}$$

Given $q_0 = (1-\alpha)e = (1-\alpha)E/L = (1-\alpha)\frac{\sigma w L}{\sigma-\alpha(1-\lambda)}/L = \frac{\sigma w(1-\alpha)}{\sigma-\alpha(1-\lambda)}$, the individual welfare can be rewritten as

$$\begin{aligned}U =& \left[\frac{\sigma w(1-\alpha)}{\sigma-\alpha(1-\lambda)}\right]^{1-\alpha}\left[\frac{\sigma\alpha\rho\varphi_{cc}^*}{\sigma-\alpha(1-\lambda)}\left(\frac{\alpha\lambda L}{\delta F_D(\sigma-\alpha(1-\lambda))}\right)^{\frac{1}{\sigma-1}}\right]^{\alpha}\\ =&\sigma w^{1-\alpha}(1-\alpha)^{1-\alpha}(\alpha\rho\varphi_{cc}^*)^{\alpha}\left[\frac{\alpha L}{\delta F_D}\right]^{\frac{\alpha}{\sigma-1}}\lambda^{\frac{\alpha}{\sigma-1}}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}.\end{aligned}$$

When $0<\lambda<\hat{\lambda}$, φ_{cc}^* is constant in λ and the sign of $\frac{\partial U}{\partial \lambda}$ is determined by the sign of the partial derivative of the term $\lambda^{\frac{\alpha}{\sigma-1}}\left[\frac{1}{\sigma-\alpha(1-\lambda)}\right]^{\frac{\alpha}{\sigma-1}+1}$. As I shown in the proof of Lemma 5, this is always positive, implying $\frac{\partial U}{\partial \lambda}>0$. When $\hat{\lambda}\leq\lambda<1$, then also $\frac{\partial \varphi_{cc}^*}{\partial \lambda}>0$ and, given $\frac{\partial U}{\partial \varphi_{cc}^*}>0$, this implies again $\frac{\partial U}{\partial \lambda}>0$. The welfare depends on variable trade costs τ only through φ_{cc}^*. According to Proposition 10, $\partial\varphi_{cc}^*/\partial\tau<0$ when a is sufficiently small and $\partial\varphi_{cc}^*/\partial\tau>0$ otherwise. It follows that $\partial U/\partial\tau<0$ when a is sufficiently small and $\partial U/\partial\tau>0$ otherwise.

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