A STUDY OF GEOMETRY CONTENT KNOWLEDGE OF ELEMENTARY PRESERVICE TEACHERS: THE CASE OF QUADRILATERALS

By

FATMA ASLAN TUTAK

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To Bilge and Erdem

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TABLE OF CONTENTS

page

APPENDIX

LIST OF TABLES

LIST OF FIGURES

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

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PRESERVICE TEACHERS: THE CASE OFQUADRILATERALS

By

Fatma Aslan Tutak

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Teacher quality is the most influential factor in student learning (Ferguson, 1991). However, quality teaching requires a unique body of knowledge for teaching. Among the different types of teacher knowledge, the content knowledge of a teacher plays a crucial role (Brown & Borko, 1992). In particular, beginning teachers are not equipped with necessary content for teaching geometry (Jones, 2000, Swafford, Jones & Thornton, 1997). On the other hand, research on teachers' geometry content knowledge is limited.

The purpose of this research is to understand preservice teachers' geometry learning as investigated by qualitative methods which to inform the following investigation to study preservice teachers' geometry content knowledge. This study took place in an elementary methods course. For the qualitative investigation, narrative analysis (Labov, 1972) and thematic analysis (Coffey & Atkinson, 1996) methods were used. As a result of narrative analysis two main kinds of stories emerged: as a learner and as a beginning teacher. The thematic analysis results yield to three themes: history of learning geometry, perceptions about geometry, effective geometry instructional practices. The results informed the following study on geometry content knowledge for the case of quadrilaterals.

During the second phase of the study, 102 participants who enrolled in the methods course completed pre and post test of teachers' geometry content knowledge, measured by Content Knowledge for Teaching Mathematics Measures (CKT-M). Treatment group participants (n=54) received intervention, a protocol focusing on quadrilaterals which was developed as a result of the qualitative investigation, and control group participants (n=48) received traditional instruction. Repeated measures ANOVA results showed a significant change in treatment group participants' geometry content knowledge $F(1, 49) = 16.08$, $p<.001$, $R^2 = .25$, $\text{eta}^2 = .25$. The mixed ANOVA results indicated a significant main effect of knowledge $F(1, 91) = 28.38$, $p<.001$ but no significant interaction between geometry content knowledge and grouping (treatment/control), $F(1, 91) = .21$, $p=.646$. Even though treatment group participants' geometry content knowledge growth was significant, the difference between treatment group and control group participants' growth in geometry content knowledge was not significant.

CHAPTER 1
INTRODUCTION

"The desired learning environments can result only from knowledgeable teachers"

(Putnam, Heaton, Prawat, & Remillard, 1992, p. 225)

Teachers, the agency of change in education, bear an important role for quality learning in the classrooms. One of the imperative elements of effective teaching is the content knowledge of teachers (Ferguson, 1991). Teachers' knowledge should be addressed in preservice teacher education and in professional development for in-service teachers. Teachers tend to teach the way they were taught (Schoenfeld, 1988). They begin learning to teach from their experiences of as learners. Teachers begin their formal education on learning to teach in teacher education programs in which their previous conceptions about teaching may change. As they begin their profession in classrooms, they are shaped by their experiences in the classrooms. Therefore, teacher education programs play a very important role in this process of learning to teach as the first, and sometimes the last, formal steps of preparation other than occasional professional development workshops.

Statement of the Problem

The most recent and comprehensive national report for government on mathematics education, Foundation for Success: The final report of the National Mathematics Advisory Panel, addressed teachers' mathematics knowledge in great detail (The National Mathematics Advisory Panel, 2008). After summarizing several research studies, the report stressed that:

> Overall, across the studies reviewed by the panel, it is clear that teachers' knowledge of mathematics is positively related to student achievement. In the context of a body of literature as inexact as this one, the positive trends we

> identified do support the importance of teachers' knowledge of mathematics as a factor in students' achievement (p. 37).

Furthermore, the report highlighted the lack of rigorous research to show the importance and complexity of teachers' content knowledge. There were recommendations given such as developing a reliable and valid measure for teachers' mathematics knowledge, addressing the mathematics content knowledge preparation of teachers with emphasis on in-depth understanding of school mathematics, and high-quality research projects to develop understanding of teachers' mathematics knowledge.

In efforts to study teachers' content knowledge, the purpose of the early studies was to develop a definition for the concept. The most commonly used and widely accepted definition of teacher's knowledge was given by Shulman (1986, 1987). Shulman's model was highly accepted because it was the manifestation of the paradigm shift in teacher education from a separate focus on only content and only pedagogical skills to combination of these two elements. "How might we think about the knowledge that grows in the minds of teachers, with special emphasis on content? I suggest we distinguish among three categories of content knowledge: (a) subject matter knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge" (Shulman, 1986, p. 9).

Subject matter knowledge which Shulman also referred to as content knowledge (CK) was defined as "the amount and organization of knowledge per se in the mind of teacher" (p. 9). At first glance, CK does not much differ from knowing facts. However, Shulman emphasized the important role of CK as providing explanations and definitions for students. Therefore, CK also addresses explanations for "why a particular propositions is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within the discipline and without, both in theory and practice" (p. 9).

On the other hand, pedagogical content knowledge (PCK) "goes beyond knowledge of subject matter per se to the dimensions of subject matter knowledge for teaching" (p. 9). Shulman warned his readers that PCK was part of teachers' content knowledge, and not just pedagogical skills. PCK is the type of content knowledge that distinguishes a teacher from a scientist. It addresses the topic's "teachability" (Shulman, 1986, p. 9). A scientist does not have to think about effective teaching strategies of the subject while a teacher cannot depend only on subject matter knowledge. PCK is necessary for effective teaching practice because it's the knowledge to choose most appropriate examples, representations, illustrations, and analogies. Therefore, teachers should possess not only subject matter knowledge for teaching but also pedagogical skills specific to the subject.

The third knowledge, curriculum knowledge addresses three features of content knowledge related to curriculum: alternative curriculum materials, lateral and vertical aspects of curriculum. Shulman first stressed the importance of teachers' repertoire of instruction materials such as texts, software, visual displays for effective teaching. In addition to this knowledge of instructional material, Shulman defined lateral curriculum knowledge as a teacher's knowledge of the connections between other subjects and the subject of teaching for the given grade level. Vertical curriculum knowledge was defined as "familiarity with the topics and issues that have been and will be taught in the same subject area during the proceeding and later years in school, and the materials embody them" (p. 10).

Among these knowledge types, content knowledge stands out as a point of focus for teacher education. Brown and Borko (1992) asserted that preservice teachers' limited mathematics content knowledge is an obstacle for their training on pedagogical knowledge. Also many studies have shown that lack of content knowledge affects teacher's methods of teaching (Carpenter, Fennema, Peterson & Carey, 1988; Leinhardt & Smith, 1985). In the project of Cognitively Guided Instruction (CGI), one of the teachers, Ms. Jackson, was identified as expert teacher by researchers (Carpenter et al., 1988). She had extensive background on addition and subtraction but her knowledge of fractions was limited. Observations of her teaching yielded important differences between instruction practices of these two topics. There were less discussion and less mathematics in the classroom when she was teaching fractions than when she was teaching addition and subtraction. Carpenter and his colleagues (1988) emphasized that the content knowledge of a teacher heavily affects the teachers' use of the pedagogical tools.

Mathematical Knowledge for Teaching

Ball and a team of researchers developed mathematical knowledge for teaching (MKT) which addresses how a teacher uses mathematics for teaching while emphasizing the importance of mathematics knowledge in the teaching settings (Ball, 2000). The position of the research team which developed MKT was that the purpose of MKT was not to replace Shulman's model but to provide deeper understanding of teachers' knowledge by building on it (Ball, Thames & Phelps, 2008). As they worked more on the concept of teachers' content knowledge, the researchers developed domains of teacher knowledge under Shulman's knowledge types.

According to MKT model, there are six domains of teacher's content knowledge which can be categorized under Shulman's different types of knowledge as illustrated in Table 1-1 (Ball et al., 2008). There are three domains under content knowledge: common content knowledge (CCK, mathematics knowledge not unique to teaching), specialized content knowledge (SCK, mathematics knowledge unique to teaching), and horizon content knowledge (knowledge of mathematics throughout the curriculum). Also, there are three domains under pedagogical content knowledge: knowledge of content and students (KCS, interaction of knowledge of mathematics and students' mathematical conceptions), knowledge of content and teaching (KCT, interaction of knowledge of mathematics and teaching methods), and knowledge of content and curriculum (interaction of knowledge of mathematics and mathematics curriculum).

Table 1-1. MKT model comparison to Shulman's model

Shulman's Model (1986)	Ball et al. MKT Model (2008)		
Content Knowledge	Common Content Knowledge	Specialized Content Knowledge	Horizon Content Knowledge
Pedagogical Content Knowledge	Knowledge of Content and Students	Knowledge of Content and Teaching	Knowledge of Content and Curriculum

Among the domains of content knowledge of teachers, SCK took attention of the researchers. The important feature of SCK is that this domain of knowledge requires only mathematics but not knowledge of students or teaching. "What caught us [authors] by surprise, however, was how much special mathematical knowledge was required, even in many everyday tasks of teaching – assigning student work, listening to student talk, grading or commenting on student work" (p. 398). In spite of the heavy use of SCK, researchers proposed that this domain of teachers' knowledge needs to be studied further in order to understand the concept of teachers' knowledge. Ball et al. (2008)

suggested addressing this type of knowledge in teacher education in order to improve teachers' mathematics content knowledge for teaching.

Geometry Content Knowledge

In research of teachers' mathematics content knowledge, researchers has been addressed the concept for several mathematics topics. In a study of Borko, Eisenhart, Brown, Underhill, Jones and Agard (1992), the team of researchers studied middle school preservice teachers' content knowledge. The authors reported results from one student teacher, Ms. Daniels, in fraction division. The researchers interviewed the participant, and studied the methods course she completed. Ms. Daniels could not answer questions her students' questions about fraction division even though, she had taken advanced mathematics courses in college and the mathematics methods course before she entered the classroom as a student teacher. She was not able to explain why the fraction division algorithm works.

There are several studies on teachers' knowledge of mathematics focused on topics such as fractions (Carpenter, Fennema, & Franke, 1996; Carpenter et al., 1988) or numbers and operations (Ball, 1990a; Ma, 1999). For example, the comparative study of Chinese and the U.S. elementary school teachers' understanding of three topics in mathematics (division, place value and area-perimeter relationship) conducted by Ma (1999) garnered attention from mathematics teacher educators. The results were groundbreaking because in spite of the advantage of higher education and advanced mathematics courses, American teachers did not have the deep mathematical understanding that Chinese teachers had. Chinese teachers did not have the same level of higher education yet they had more experience with mathematics learning practices in the classroom. Their learning was tailored for teaching rather than advanced degrees in mathematics. The results revealed that higher education mathematics courses were not enough to make sure that teachers have quality mathematics knowledge for teaching.

In spite of the general interest in teachers' mathematics content knowledge in topics such as fractions or place value, there is limited research on knowledge of geometry for teaching. The results of those studies reflect that especially beginning teachers are not equipped with necessary content and pedagogical content knowledge of geometry, and it is important to address this issue in teacher education (Jones, 2000; Swafford, Jones, & Thornton, 1997). On the other hand, the focus of teacher education research in geometry is on middle and high school grades such as Driscoll, Egan, Dimatteo, & Nikula (2009), and Swafford et al. (1997). In the study of Swafford et al. (1997), the researchers investigated middle school teachers' geometry content knowledge and their van Hiele levels of geometry thinking. The results of the study showed improvement in teachers both knowledge and geometric thinking levels after the summer workshops on geometry.

However, closer analysis of geometry topics in the Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence (NCTM, 2006) reveals emphasis on the importance of students' experiences with geometry in early grades on to their preparedness for secondary level geometry learning. Furthermore, "Students should enter high school understanding the properties of, and relationship among, basic geometric shapes" (NCTM, 2000, p. 310). Therefore, elementary school teachers should possess required geometry knowledge to prepare students for more advanced geometric thinking while the role of middle and secondary school teachers is to facilitate learning experiences for higher geometric thinking levels. If the elementary school teachers lack the necessary knowledge of geometry to prepare students for higher level of geometry thinking, students will enter secondary level grades with a limited geometry knowledge which results into rote memorization of geometry without meaningful learning (van Hiele, 1999). Even though the emphasis in geometry education is on upper level grades, these findings suggest that researchers need to study teachers' geometry content knowledge in elementary school level as well.

Many other research projects which on geometry content knowledge of teachers emphasized the lack of teachers' knowledge, especially the beginning teachers (Barrantes & Blanco, 2006; Chinnappan, Nason, & Lawson, 1996; Jacobson & Lehrer, 2000; Lampert, 1988; Leikin, Berman, & Zaslavsky, 2000). "Teachers are expected to teach geometry when they are likely to have done little geometry themselves since they were in secondary school, and possible little even then." (Jones, 2000, p. 110). Therefore, with new understandings in teachers' mathematics content knowledge the mathematics teacher education community needs to study not only middle and high school teachers' geometry content knowledge but also elementary school teachers'.

Analyzing Student Work to Study MKT

One of the core elements of MKT model was that it was developed from practice of teaching. Ball (2000) discussed interweaving content and pedagogy for mathematics teaching. She posited that it is crucial for teacher education to explore ways to provide learning experiences for teachers to know the subject in varied teaching contexts. Her suggestion to approach this challenge was to design learning opportunities for teachers to experience content knowledge in the context of teaching. One of the possible designs is using student work to analyze what students know and what they are learning. Using student work has been widely accepted by teacher educators to improve teacher learning and instructional practices (Lampert & Ball, 1998; Little, 2004; Smith 2003). Using student work to facilitate teacher learning may result in teachers' deeper content knowledge (Franke & Kazemi, 2001; Kazemi & Franke, 2004). The authors discussed that by analyzing student work, teachers may be forced to think deeply and elaborate on mathematics knowledge while they are trying to understand what students did. "Making sense of children's strategies could be an indirect way for teachers to wrestle with the mathematical issues themselves" (p. 7).

Some studies showed that using student work to facilitate teacher learning resulted in teachers' deeper subject matter knowledge and classroom practice (Franke & Kazemi, 2001; Kazemi & Franke, 2004). In the line of CGI research, Franke and Kazemi reported use of student work with elementary school teachers. The researchers conducted a four-year professional development workshop series with teachers from one school. The professional development was designed to promote teachers' understanding of student thinking. The researchers reported that as a result of attending professional development workshops, teachers' both content knowledge and pedagogical content knowledge was improved. "Thus, in detailing student thinking for the group, teachers included rich descriptions of the questions asked to elicit that thinking, the responses of other students, and the work that came before the shared interaction" (p. 107).

However Kazemi and Franke (2004) also stressed the importance of the selection of the student work in order to involve teachers in meaningful content discussions on students' use of algorithms and procedures (Kazemi & Franke, 2003). One important feature of a learning activity for teachers would be providing student work which demonstrates uncommon algorithms or methods of mathematics thinking.

Besides the effectiveness of using student work with teachers, the disadvantages might be the restricted flexibility and limited transformation of content knowledge for different settings (Little, 2004). Teachers face classrooms where they have many different learners which may not be discussed any of the tasks used. Therefore, teacher learning tasks should also provide flexibility of mathematics knowledge for different types of classrooms and for different challenges of teachers. In a professional development setting for in-service teachers, when a facilitator has concern about flexibility, the facilitator may ask teachers to bring their own student work in order to address their own setting. On the other hand, for preservice teacher education courses, the facilitator may provide student work from local schools, potential student population for preservice teachers.

Purpose of the Study

The purpose of this study is to improve our understanding of teachers' geometry content knowledge. As the leading organization of mathematics education in the United States, the National Council of Teachers of Mathematics (NCTM) published two influential documents, Curriculum and Evaluation Standards for School Mathematics (1989) and Principles and Standards for School Mathematics (2000) to support the reform in mathematics teaching and learning. Both documents represented the important influence of teachers on students' learning by addressing the principle of teaching and teacher knowledge in order to reach the goals of reform.

> Teachers must know and understand deeply the mathematics they are teaching and understand and be committed to their students as learners of mathematics and as human beings (NCTM, 2000, p.17).

As NCTM's teaching principle guides this research, preservice teachers were the focus of this study. There are two related yet distinct purposes of this research, (i) preservice teachers' geometry learning and effective instructional strategies for their learning, (ii) possible effects of using geometry learning protocol (developed as a result of the investigation on the first purpose) on their geometry content knowledge. These two purposes of the study were addressed by two connected investigations. The first investigation was designed to understand preservice teachers' geometry learning and effective instructional strategies for their learning. As a result of this qualitative investigation a protocol consisted of geometry activities (focused on quadrilaterals) for teachers and analysis of student works was developed. For the second investigation, quantitative approach was used to study possible effects of using the protocol (on quadrilaterals with analysis of student work) on preservice teachers' geometry content knowledge. Therefore, the research questions for these two purposes of were:

1. What is preservice elementary teachers' understanding of geometry in elementary school?
2. What are the perceptions of preservice elementary teachers on effective instructional strategies to promote their knowledge of geometry in mathematics methods courses?
3. Does use of geometry activities focused on quadrilaterals with analysis of student work influence preservice elementary teachers' geometry content knowledge?
4. Is there a difference in geometry content knowledge between preservice teachers who are in a traditional mathematics methods course and preservice teachers who are in experimental mathematics methods course?

Significance of the Study

One of the most important issues in mathematics teacher education is to understand the teachers' perspective of their learning and to develop practices to increase teachers' knowledge with the blend of their needs and theory. This study will enrich the understanding of how preservice teachers learn geometry during their teacher education program. Also, the results of this study provide further discussion on how to improve geometry content knowledge of teachers. This study serves as an example for research in practice because the geometry protocol was developed as a result of this research and it was tested with preservice teachers. The activities in the protocol are a resource for teacher educators to facilitate geometry learning experiences on quadrilaterals for preservice elementary teachers and professional development experiences for in-service teachers.

Overview of the Dissertation

The document is composed of introduction, literature review, methodology, two journal articles written from the dissertation research, and conclusions. The introduction (Chapter 1) provides an overview of main concepts and the significance of the study. The literature review (Chapter 2) consists of discussions on teachers' content knowledge and gaps in the literature for further investigation. The methodology (Chapter 3) addresses the details of the study in terms of the development of the study in addition to the information regarding data collection and data analysis methods.

There are two journal articles written from this study. The first article (Chapter 4) provides discussion on qualitative investigation which addressed the first two research questions. The goal of the qualitative investigation was to understand preservice elementary teachers' geometry learning. The second article (Chapter 5) was written to report the quantitative investigation of the dissertation study which addressed the last two research questions. The purpose of the quantitative investigation was to study possible effects of using geometry protocol on preservice teachers' geometry content knowledge. The last chapter (Chapter 6) reports the conclusions of the study and the implications for mathematics teacher education in addition to limitations and suggestions for further research.

CHAPTER 2
REVIEW OF THE LITERATURE

With the new developments in understanding of learning, reforms were developed to improve the quality of teaching. Teacher quality is the most influential factor in student learning (Ferguson, 1991). The results from the study of Ferguson showed that for more than 1000 school districts, spending additional dollars on more highly qualified teachers resulted in greater improvements in student achievement than did any other use of school resources. However, quality teaching requires professionalism in a unique body of knowledge for teaching. “Policymakers can change textbooks, they can change tests, and they can recommend classroom activities or approaches, but changing mathematics teaching must involve teachers in fundamental ways…The desired learning environments can result only from knowledgeable teachers” (Putnam, Heaton, Prawat & Remillard, 1990, pp. 225-226). Teacher knowledge is one of the most important components of teacher quality and among the different types of teacher knowledge the content knowledge of a teacher strongly impacts the enactment of pedagogical tools of the teacher. Brown and Borko (1992) asserted that preservice teachers’ limited mathematics content knowledge is an obstacle for their training on pedagogical knowledge.

The aim of this chapter is to provide review of the literature on the theoretical background of teachers’ content knowledge and subsequently on teachers’ mathematics content knowledge. After discussions on theoretical approaches to teachers’ mathematics content knowledge, approaches to measure teachers’ content knowledge will be discussed. The concluding section of the literature review will include selected research projects focused on teachers’ mathematics content knowledge for different mathematics topics, and two suggested research designs; using videos of classrooms and using student work for studying teachers' mathematics knowledge.

Defining Teacher Knowledge

The knowledge and professional preparation of teachers have been an interest for policymakers since the beginning of the professional teaching practice in the 1800s (Hill, Sleep, Lewis, & Ball, 2008). However, it is only in the last 30 years, the study of teachers' knowledge gained more attention. One such study of teacher knowledge which received great attention was by Begle (1979). This meta-analysis of studies from 1960 to 1976 revealed teacher characteristics that yield student achievement. Begle found negative effects of teachers' content knowledge on student achievement. Teachers' content knowledge was measured by number of college mathematics credits taken by the teachers. Therefore, Begle concluded that common conceptions about good teaching are inaccurate or questionable. Further, the researcher proposed that teacher educators should direct their attention on other areas of education because the results of the study could not show the relation between teachers' content knowledge and student achievement. Results of this meta-analysis indeed resulted in increase of interest in the study of teachers' content knowledge to be approached from different research perspectives.

One of the challenges of the field in studying teachers' content knowledge was the lack of common consensus on the definition of teachers' content knowledge in spite of commonly held perceptions of importance of teacher' knowledge (Ball, Lubienski & Mewborn, 2001). In their literature review on teachers' mathematics content knowledge, Ball, Lubienski and Mewborn addressed the efforts of defining teachers' content knowledge in three groups, policy response, characteristics of teachers and teachers' knowledge. These three categories will be used to present the literature review in this chapter.

Policy Response Approach to Define Teachers' Content Knowledge

Some of the examples for a policy response approach involve the National Commission on Teaching and America's Future (1996). This commission provided general suggestions to improve instruction in classrooms such as knowing the topic in-depth and the epistemology of the topic. As a national leading organization, National Council of Teachers of Mathematics (NCTM) provided guidelines and standards in teaching mathematics including the teachers' knowledge of mathematics (NCTM, 1989, 2000).NCTM published two influential documents, Curriculum and Evaluation Standards for School Mathematics (1989) and Principles and Standards for School Mathematics (2000) to support mathematics teaching and learning. Both documents represent the important influence of teachers on students' learning by the principle of teaching.

> Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well…Teachers must know and understand deeply the mathematics they are teaching and understand and be committed to their students as learners of mathematics and as human beings (NCTM, 2000, p.17).

An early attempt to describe teachers' content knowledge by policy response was the analysis of Guyton and Antonelli (1987) on four reports on teaching and teacher education: A Call for Change in Teacher Education (National Commission on Excellence in Teacher Education, 1985), Improving Teacher Education (Southern Regional Education Board, 1985), Tomorrow's Teachers (Homes Group, 1986) and A Nation Prepared (Carnegie Forum on Education and Economy, 1986). The authors highlighted five recommendations:

- Raise standards for admission to teacher education and the teaching profession.

• Move professional education of teachers to the post baccalaureate level.

• Revise the teacher education curriculum, particularly to incorporate research findings.

• Make efforts to enhance the prestige of and respect for teachers and teaching profession.

• Engage Arts and Sciences faculty in the teacher education program. (p. 45)

Ball et al. (2001) stressed that these documents are not based on research results but on “policy deliberations” (p. 441). In spite of policy deliberations of this approach, the following two approaches in studying teachers’ content knowledge were based on research to define and measure teachers’ content knowledge.

Characteristics of Teachers Approach to Define Teachers’ Content Knowledge

The teacher characteristics approach investigates attributes of teachers in order to produce a list of teacher characteristics needed to ensure student achievement. This approach defines teachers’ content knowledge by variables such as certification of teachers, number of college mathematics credits taken, having minor or major degree in mathematics, or degree of the teachers (Begle, 1979; Monk, 1994, Monk & King, 1994). The study of Longitudinal Study of American Youth consists of 2,829 students with 608 mathematics teachers and 483 science teachers from 51 randomly selected school sites (Monk, 1994). The researchers administered a survey about teachers’ completed undergraduate and graduate coursework. In order to measure student achievement, researchers used selected National Assessment of Educational Progress (NAEP) items with 1,492 students from 10th and 11th grade. Correlation research methods were used to study the relationship between teachers’ mathematics content knowledge and student achievement. One of the most interesting results was that being mathematics major did not contribute to student achievement even though number of mathematics courses had a positive effect on student achievement for both 10th and 11th grade students. In terms of mathematics education courses, both undergraduate and graduate courses were positively related to student achievement. Furthermore, undergraduate mathematics education courses affected student achievement more than undergraduate mathematics courses.

The results from Monk (1994) and other studies such as Begle (1979) promoted the necessity to find other approaches to study teachers' mathematics knowledge. Furthermore, "as the paradigm for research on teaching shifted from behavioral psychology to cognitive psychology, researchers shifted the focus from teachers' behaviors to teachers' thinking and knowledge" (Grossman, 2005, pp. 438-439).

Teachers' Knowledge Approach to Define Teachers' Content Knowledge

Even though the second (the teacher characteristics) and third (teachers' knowledge) approaches addressed the need to study teachers' content knowledge through research, the main differences between them are their purpose and their inquiry methods. While the teachers' characteristics approach was heavily quantitative with the primary interest in teachers, the teachers' knowledge approach implemented more qualitative research methods in order to draw attention more on knowledge itself instead of the teacher.

Though the importance of teachers' content knowledge is evident in the literature (The National Mathematics Advisory Panel, 2008), some of the challenges in studying teachers knowledge were limited definitions and lack on common consensus on names for the different knowledge types for teaching. One of the early studies to address types of teacher knowledge was by Fenstermacher (1994) who provided a list of nine different types of teacher knowledge such as strategic knowledge, craft knowledge and propositional knowledge. He emphasized that having such a long list was not helpful to understand teacher knowledge because the fact that all of these names do not address different types of knowledge. Fenstermacher used an effective analogy to understand teacher knowledge. A person has a first name, a nickname, a family favorite name. All of these names are used for same person even though their context of use is different. In the same sense, he proposed that all of the different names to define teacher knowledge actually refer to the same, one body of knowledge. Therefore, Fenstermacher's approach emphasized that teacher knowledge has a complex, interweaved, interdependent structure.

One of the most influential research examples for the teachers' knowledge approach would be the Knowledge Growth in Teaching Project (Shulman, 1986, 1987; Wilson, Shulman, Richert, 1987) conducted by a group of researchers to explore how novice high school teachers use content knowledge (for mathematics, science, history and literature) in the classroom. The purpose of this research project was to study the transfer of subject matter knowledge into knowledge for teaching. The importance of this research project was providing the shift from teachers to teaching itself. Shulman (1986, 1987), one of the leading researchers in the Knowledge Growth in Teaching Project, presented discussions from this research project, and he stressed that the main goal of the project was to study "transition from expert student to novice teacher" (1986, p.8). According to Shulman, other driving questions for this research were "Where do teacher explanations come from? How do teachers decide what to teach, how to represent it, how to question students about it, and how to deal with problems of misunderstanding?" (p. 8). Furthermore, it is important to note that the intention of this study was not to provide a list of what teachers should know but to understand the nature of teachers' knowledge. In the line of this project there were other models presented for teacher knowledge (Grossman, Wilson, & Shulman, 1989) yet Shulman's model that was proposed in 1986 was widely accepted.

Shulman's model for teachers' knowledge

The most commonly used and widely accepted definition of teacher's knowledge was given by Shulman (1986, 1987). Shulman's model was highly accepted because it was the manifestation of the paradigm shift in teacher education from separate focuses on only content and only pedagogical skills to a combination of these two elements. "How might we think about the knowledge that grows in the minds of teachers, with special emphasis on content? I suggest we distinguish among three categories of content knowledge: (a) subject matter knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge" (Shulman, 1986, p. 9).

Subject matter knowledge which Shulman also referred to as content knowledge (CK) was defined as "the amount and organization of knowledge per se in the mind of teacher" (p. 9). At first glance, CK does not much differ from knowing facts. However, Shulman emphasized the important role of CK as providing explanations and definitions for students. Therefore, CK also addresses explanations for "why a particular propositions is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within the discipline and without, both in theory and practice" (p. 9).

On the other hand, pedagogical content knowledge (PCK) "goes beyond knowledge of subject matter per se to the dimensions of subject matter knowledge for teaching" (p. 9). Shulman warned his readers that PCK was part of content knowledge, but not just pedagogical skills. PCK is the type of content knowledge that distinguishes a teacher from a scientist. It addresses the topic's "teachability" (Shulman, 1986, p. 9). A scientist does not have to think about effective teaching strategies of the subject while a teacher cannot depend only on subject matter knowledge. PCK is necessary for effective teaching practice because it is the knowledge to choose most appropriate examples, representations, illustrations, and analogies. Therefore, teachers should possess not only subject matter knowledge for teaching but also pedagogical skills specific to the subject.

The third knowledge, curriculum knowledge addresses three features of content knowledge related to curriculum: "alternative curriculum materials" (p. 10), lateral and vertical aspects of curriculum. Shulman first stressed the importance of teachers' repertoire of instruction materials such as texts, software, visual displays for effective teaching. In addition to this knowledge of instructional material, Shulman defined lateral curriculum knowledge as a teacher's knowledge of the connections between other subjects and the subject of teaching for the given grade level. Vertical curriculum knowledge was defined as "familiarity with the topics and issues that have been and will be taught in the same subject area during the proceeding and later years in school, and the materials embody them" (p. 10). Figure 1 shows the summary of these three types of teachers' content knowledge from Shulman's work (1986, 1987).

This model garnered attention because it provided categories to explore teachers' content knowledge further. For example, PCK was the missing element of the studies like Begle (1979) because only mathematics knowledge does not account for student gain. Therefore, PCK was the category of teachers' knowledge that caught wide-spread attention. It was accepted very fast, and used in several research studies. With attempts to understand PCK, still there is no single one extensive definition for PCK, after Shulman's model (Ball, Thames & Phelps, 2008).

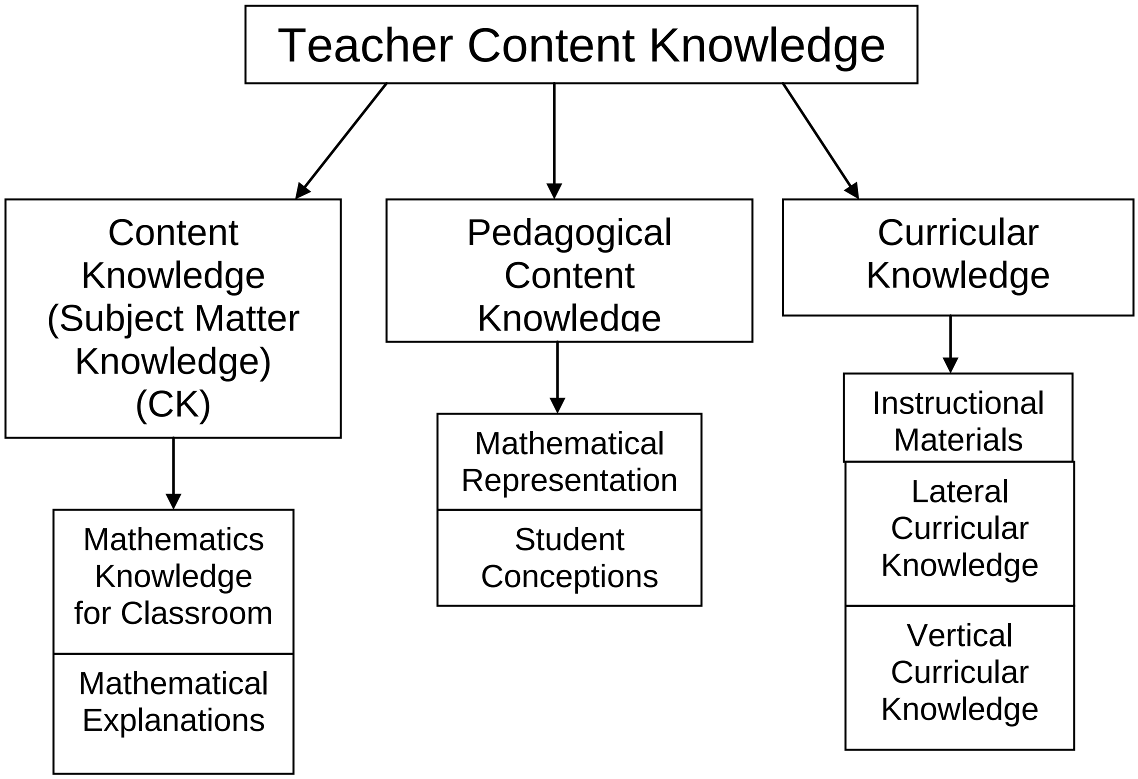

Figure 2-1. Shulman's teachers' content knowledge model

The purpose of this chapter is to discuss teachers' subject matter knowledge (which is also called as CK by Shulman). Several research studies showed that among knowledge types of Shulman's model, CK becomes prominent among other types of knowledge. A teacher needs to know the subject well in order to apply effective pedagogical methods. Brown and Borko (1992) asserted that preservice teachers' limited mathematical content knowledge is an obstacle for their training on pedagogical knowledge. Other research studies showed that lack of content knowledge affects teacher's teaching methods. In the project of Cognitively Guided Instruction (CGI), one of the teachers, Ms. Jackson, was identified as an expert teacher by researchers (Carpenter et al., 1988). She had extensive background knowledge on addition and subtraction but her knowledge of fractions was limited. The observations of her teaching yielded important differences between instruction practices of these two topics. There were less discussion and less mathematics in the classroom when she was teaching fractions than when she was teaching addition and subtraction. Carpenter and his colleagues (1988) emphasized that the content knowledge of a teacher heavily affects the teachers' use of the pedagogical tools. Therefore, it is crucial to study and examine teacher content knowledge in depth in order to improve quality of mathematics instruction in the classrooms (Putnam, Heaton, Prawat, & Remillard, 1992).

Mathematical knowledge for teaching

As mathematics education field was affected from Shulman's findings, there were examples of research which followed the inquiry of Shulman's with the primary interest on teachers' perceptions and understanding for particular mathematics topics (Ball, 1988, Ball 1990a, 1990b; Leinhardt & Smith, 1985; Owens, 1987; Post, Harel, Behr & Lesh, 1988; Steinberg, Haymore, and Marks, 1985). With this growing interest, teacher educators realized that there was no common terminology or definition to address this important concept in mathematics education (Ball et al., 2001). Therefore, the early studies on teachers' mathematics content knowledge focused on improving the understanding of the concept.

One of the research groups who has been studying Shulman's model for teachers' knowledge in mathematics education context was the team of Ball, Bass, Cohen, Hill and others. In her early works, Ball emphasized that teachers' need to "unlearn" and "unpack" their mathematics knowledge (Ball, 1988, 1990a, 1990b). For example, Ball

(1990a) studied mathematics content knowledge of elementary and secondary preservice teachers through interviews. She asked participants to answer division problems (division by fractions, division by zero, division in the context of algebraic equations) for both correct answers and mathematical explanations for their answers. Even though all secondary preservice teachers answered problems by giving correct rules or explanations, only 3 out of 10 elementary preservice teachers were able to give the correct rule or explanation. Results such as these revealed that knowing the subject was not same as knowing mathematics in teaching settings Ball (1990a). The mathematics content courses might provide mathematics knowledge (rules and procedures) yet they might not give required knowledge for core understanding of K-12 mathematics topics. Therefore, the interest in teachers' content knowledge has been increased in a new perspective: knowing mathematics for teaching.

Ball was one of the leading researchers in two projects; Mathematics Teaching and Learning to Teach (MTLT), and Learning Mathematics for Teaching (LMT), to study teachers' mathematics content knowledge. The first project was designed to investigate the work of teachers in order to develop testable hypotheses. The purpose of the second project was to develop a survey measurement to study the framework of teachers' mathematics content knowledge (Ball et al., 2008).

For the first research project, MTLT, the researchers focused on job analysis of teaching mathematics. Ball (2000) stated that job analysis approach is a common practice to study mathematics related professions like engineering and nursing. She suggested that this approach would provide insight for understanding teaching and specifically the knowledge of teachers for teaching in the classrooms (Ball, 2000). The premise was that the definition of teacher knowledge should not depend on only the school curriculum because the curriculum approach may overlook the core activities of teaching, the practice perspective of teacher knowledge, such as decisions on students' knowledge, teaching ideas, choosing between teaching choices, and use of textbooks. Therefore, she stressed the importance of studying the content knowledge along with subtle practices embedded in these activities instead of depending on what teachers need to teach.

The researchers in this project collected longitudinal data from elementary school mathematics classrooms. The data included videotapes, audiotapes and transcripts of classroom lessons, copies of students' written class work, homework, and quizzes in addition to the teacher's plans, notes, and reflections (Ball, 2000). The inspiring questions of this qualitative study were "1. What are the recurrent tasks and problems of teaching mathematics? What do teachers do as they teach mathematics? 2. What mathematical knowledge, skills, and sensibilities are required to manage these tasks?" (Ball et al., 2008, p. 395). As a result of this qualitative study (experiences of research group members and analytical tools developed for this research) the group developed the framework mathematical knowledge for teaching (MKT) (Ball & Bass, 2000a, 2000b, 2003).

Furthermore, the researchers made hypotheses about multidimensional aspect of mathematics knowledge for teaching which was tested in the second project, The Learning Mathematics for Teaching (LMT). The purpose of this second project was the measurement aspect of MKT in order to study it further. The research group developed a survey measurement to study MKT (Hill & Ball, 2004; Hill, Schilling & Ball, 2004; Hill, Rowan & Ball, 2005) which will be discussed further in the next section on measurement aspect of teachers' mathematics content knowledge.

The most important result from these two projects was that mathematics content knowledge of teachers is a complex concept such that general mathematics understanding is not enough to explain the relationship between teachers' content knowledge and student achievement. These two projects resulted in deeper understanding about teachers' knowledge that teacher content knowledge has multidimensional structures. Therefore, Ball and her colleagues (2008) stated that "we [authors] hypothesize that teachers' opportunities to learn mathematics for teaching could be better tuned if we could identify those types more clearly" (p. 403).

The position of this research group was that their purpose in developing MKT was not to replace Shulman's model but to provide further understanding of teachers' knowledge by building on it (Ball et al., 2008). In their own words, the researchers stated that "we are able to fill in some of the rudimentary 'periodic table' of teachers' knowledge" (p. 396). As they worked more on Shulman's model, the researchers developed domains of teacher knowledge under Shulman's knowledge types. According to the MKT model, there are six domains of teacher's mathematics content knowledge which can be categorized under Shulman's two types of knowledge, CK and PCK: common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge being under Shulman's knowledge type of CK; and knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum being under Shulman's category of pedagogical content knowledge.

Table 2-1. MKT model comparison to Shulman's model

Shulman's Model (1986)	Ball et al. MKT Model (2008)		
Content Knowledge	Common Content Knowledge	Specialized Content Knowledge	Horizon Content Knowledge
Pedagogical Content Knowledge	Knowledge of Content and Students	Knowledge of Content and Teaching	Knowledge of Content and Curriculum

Common content knowledge (CCK). This knowledge of a teacher is solving mathematics problems correctly and being able to do mathematics that students need to do. The word common in the name can be misleading. CCK is not defined as knowledge for everyone in the sense of common knowledge, but it is defined as "knowledge of a kind used in wide variety of settings – in other words, not unique to teaching" (Ball et al., 2008, p. 399). CCK in the classroom is consists of knowing the mathematics, recognizing wrong answers and being able to notice inaccurate definitions. For example, when a teacher computes division of two fractions correctly, the teacher uses common content knowledge.

Specialized content knowledge (SCK). Specialized content knowledge of a teacher represents the unique knowledge of mathematics and skills for teaching that someone other than a teacher does not need to possess. As someone's mathematics knowledge gets more advanced, one needs to pack them together and make it denser. On the other hand, a teacher needs to unpack the mathematical knowledge in order to make it easy to be understood by students. Ball et al. (2008) call this practice of teachers as "uncanny kind of unpacking" (p. 400) that people other than teachers do not need to do. For example, when someone is doing fraction division, one does not need to know why the invert and multiply algorithm works. However, a teacher needs to know why it works and to be able to recognize different student answers for fraction division.

Ball et al. (2008) listed some of the mathematical tasks of teaching such as “presenting mathematical ideas, responding to students’ ‘why’ questions, finding an example to make a specific mathematical point…evaluating the plausibility of students’ claims (often quickly)” (p. 400).

The important feature of SCK is that even though it is the practice of teaching, this domain of knowledge requires only mathematics knowledge but not knowledge of students or teaching. “What caught us [authors] by surprise, however, was how much special mathematical knowledge was required, even in many everyday tasks of teaching – assigning student work, listening to student talk, grading or commenting on student work” (p. 398). In spite of the vast amount of use of SCK in the classroom, researchers proposed that this domain of teachers’ knowledge needs to be studied more in order to understand teachers’ mathematics content knowledge.

Horizon content knowledge. This is a domain of teachers’ knowledge that is related to mathematics understanding throughout the curriculum. The concept of horizon content knowledge was developed by Ball (as cited in Ball et al., 2008) to represent the knowledge of teachers as “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (p. 403). For example, a teacher needs to know how fractions are used in higher grades in order to prepare students mathematically. Even though the researchers called it a third category (in addition to SMK, PCK), horizon content knowledge is parallel to Shulman’s vertical curriculum knowledge because it’s teachers’ knowledge of mathematics across the curriculum and the mathematical connections between different grade levels. On the other hand, the researchers stated that they were not sure about placing this domain under subject matter knowledge because it may be addressed across other domains of knowledge. They stressed that this domain of knowledge requires more theoretical, empirical, and pragmatic explorations.

Knowledge of content and students (KCS). This domain of MKT represents the interaction between knowledge of content and knowledge of students. In other words, it is “an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking” (p. 401). Teachers need to know common student conceptions and misconceptions and what students think. For example, during the instruction of fraction division, knowing the example fractions which would cause less confusion among students, or selecting fractions to address common student misconceptions is a kind of task that teachers do. Indeed, the domain of KCS is parallel to Shulman’s one aspect of PCK. According to Shulman there were two main practices in PCK, student conceptions and representations. In MKT model knowledge of content and students is similar to Shulman’s student conceptions aspect of PCK.

Knowledge of content and teaching (KCT). Knowledge of content and teaching represents the interaction between teachers’ mathematics content knowledge and their knowledge of teaching (pedagogical knowledge). A teacher uses KCT to decide which models to represent a topic or to choose appropriate language and metaphors to develop the mathematical concepts. “Knowledge of teaching and content is an amalgam, involving a particular mathematical idea or procedure and familiarity with pedagogical principles for teaching that particular content” (Ball et al., 2008, p. 402). In the same example of fraction divisions, when a teacher realizes that students are having difficulties with the invert and multiply algorithm, and decides what to do about it, the teacher is using the interaction between knowledge of fractions and knowledge of teaching. The domain of KCT is parallel to Shulman's explanation of the representations aspect of PCK.

Knowledge of content and curriculum. This last domain of knowledge placed under PCK is the interaction of knowledge of curriculum with the knowledge of content. Having this domain of knowledge under PCK is different than Shulman's model where curriculum knowledge was separate from both content and pedagogical content knowledge. Ball et al. (2008) proposed that this reduction was consistent with later works of Shulman. On the other hand, the researchers stated that they could not conclude about the final place to put this domain. "We are not yet sure whether this may be a part of our category of knowledge of content and teaching or whether it may run across several categories or be a category in its own right" (p. 403). It is important to note here that the MKT model places Shulman's curriculum knowledge under subject matter knowledge and pedagogical content knowledge in two different pieces as horizon content knowledge and knowledge of content and curriculum.

Final Remarks on MKT. The researchers focused on especially four domains (CCK, SCK, KCS and KCT) of teacher knowledge while building them on Shulman's two categories (CK and PCK) of teacher knowledge. Even though MKT model does not differ much from Shulman's model, it provides deeper understanding of teachers' knowledge by providing domains to explain each category. For example, Ma (1999) identified the difference between Chinese and the U.S. teachers in terms of fundamental and in depth understanding of elementary mathematics. Although Ma stressed the difference between two groups of teachers as mathematical understanding of school mathematics, she identified it as pedagogical content knowledge. However, analysis of the results of the study reveals that the difference found by Ma was what Ball et al. (2008) calls SCK, mathematics knowledge that is special to teachers yet different than knowledge of students and teaching. For example, Ma (1999) used an example of recognizing uncommon student inventions for the concepts of area and perimeter of a rectangle. The teachers were asked if the students' ideas were mathematically correct and if they could show the mathematical reasons for their answers. These kinds of tasks are classified as specialized content knowledge in the MKT model, under content knowledge not under pedagogical content knowledge. Therefore, having these domains of teacher knowledge helps us to study teachers' knowledge further.

Even though these domains help us to grasp teacher knowledge in various ways, still they do not provide a clear cut distinction between them. "The shifts that occur across the four domains…are important yet subtle" (Ball et al., 2008, p. 404). For example, for one teacher recognizing an uncommon student answer might be SCK because the teacher may use mathematics knowledge to understand the student's approach. However, an experienced teacher who is familiar with that kind of student error might use only the interaction of knowledge of mathematics and students rather than special mathematics knowledge.

Furthermore, this model changes considerations for curriculum knowledge within the picture of teacher knowledge. The authors claimed that other researchers from the Knowledge Growth in Teaching Project placed curriculum knowledge with PCK. However, Ball et al. (2008) did not provide much information about curriculum knowledge while they recommended studying curriculum knowledge further. More research on curriculum knowledge of teachers and how it interacts with other domains of knowledge is needed in the field. With more research studies to understand teachers' mathematics knowledge for teaching, the model they developed to show domains of MKT and how they relate to Shulman's model will emerge into a more rigorous model showing interactions between the domains of MKT. On the other hand, it should be stressed that today any model for teachers' content knowledge is not complete because our understanding of teachers' content knowledge is developing with further research in this field. In other words, the MKT model should be studied in different contexts, discussed from different perspectives in order to improve it. Even though there are some studies conducted to test this model by the developers (Hill & Ball, 2004; Hill, Rowan & Ball, 2005), other studies investigating MKT model in various contexts would be beneficial.

Measurement Aspect of Teachers' Mathematics Knowledge

Policymakers had to answer the questions of who can teach or who should teach by certification exams for teachers for more than a century in the United States. "The history of assessing teachers' mathematical knowledge begins with the history of teacher examinations for certification" (Hill et al., 2008, p. 112). In addition to certification exams for teachers, other methods of measurement have emerged as research in teachers' mathematics knowledge has been informing the understanding in teacher knowledge. The efforts to measure teacher knowledge can be grouped in two categories: certification tests and research based measurement tools. Some of the examples for certification tests are Praxis Series (Praxis I, II and III) by the Educational Testing Service (ETS), Interstate New Teacher Assessment and Support Consortium

(INTASC), a portfolio assessment by the Council of Chief State School Officers. There is also National Board for Professional Teaching Standards (NBPTS) certification for teachers who are already certified to teach and have at least three years of experience. Further discussion on these and other certification tests in mathematics education can be found in literature review of Hill et al. (2008). Because the focus of this literature review is the research perspective for teachers' mathematics knowledge, the second assessment category, research based measurement tools, will be discussed.

While the mathematics education researchers have been studying teachers' mathematics content knowledge, one limitation emerged: the difficulty of measuring teacher knowledge. The researchers have been studying the measurement aspect of the concept in order to demonstrate the effect of teacher education programs, to address the importance of teacher knowledge especially for student achievement, and most importantly to understand teachers' content knowledge. There have been several studies which used observations (Leinhardt & Smith, 1985; Borko, Eisenhart, Brown, Underhill, Jones and Agard, 1992), questionnaires and interviews (Ma, 1999, Tirosh & Graeber, 1990) to study teachers' mathematics knowledge as situated in teaching. However, those measurement tools could not generalized to other research settings. The National Mathematics Advisory Panel (2008) suggested the mathematics education field to develop large scale teacher knowledge measurement instruments with established reliability and validity. The research studies were limited by the lack of valid and reliable large scale measures for teachers' mathematics content knowledge. Therefore, this gap in the mathematics education was addressed by two approaches: education production function and instruments for teachers' mathematics knowledge

(Hill et al., 2008). These two approaches can be seen as parallel to last two approaches of the define teacher knowledge (teacher characteristic approach and teachers' knowledge approach), respectively.

Education Production Function for Measuring Teachers' Content Knowledge

In this approach teachers' knowledge was measured through certification or number of courses in order to investigate its effect on student achievement (Monk, 1994; Monk & King, 1994; Rowan, Chiang & Miller, 1997; Rowan, Correnti, & Miller, 2002) In these studies teachers' content knowledge was defined by teacher characteristics. In addition to study of Longitudinal Study of American Youth, which was discussed earlier (Monk, 1994), Rowan et al. (2002), studied teacher knowledge through teacher certification too. The researchers used 3-level linear hierarchical model, new developments in data analysis method. This study focused on both mathematics and reading. For the mathematics component, the student achievement measured in two cohorts, students beginning at 1st grade (n=5,454) who were tested for 3 year, and the students at third grade (n=5926) from 138 schools. The results were parallel to earlier studies; teacher certification had no effect on student achievement. Teacher experience status showed positive effect for only later grades. The most interesting result of this study was that students of teachers with advanced mathematics degree did worse than students of teachers with no advanced mathematics degree. This result showed that advanced degree in mathematics might not result in higher student achievement because teachers' mathematics content knowledge is not only the mathematics knowledge but the knowledge of mathematics for teaching.

Several attempts to the show effect of teacher certification and college mathematics coursework has failed. This yielded into the paradigm shift to study teachers' mathematics knowledge in teaching settings. Both Shulman's (1986, 1987) model and MKT (Ball et al., 2008) model strongly suggest that mathematics content knowledge of a teacher is different than mathematics knowledge of a person from another profession such as an engineer. Therefore, teachers' mathematics content knowledge should be measured by tools and methods designed specifically for teachers' mathematics content knowledge which is the next approach, instruments for teachers' mathematics knowledge approach.

Instruments for Teachers' Mathematics Knowledge for Measuring Teachers' Content Knowledge

The need for instruments to study teachers' mathematics knowledge led researchers to develop mathematics knowledge tests for teachers which can be used with a large number of participants. Two measurement instruments, Study of Instructional Improvement/Learning Mathematics for Teaching (SII/LMT) (LMT, 2009) and Diagnostic Teacher Assessment in Mathematics and Science (DTAMS, 2009) are highlighted in the mathematics education community due to extensive work of these two projects on reliability and validity in addition to covering majority of the mathematics topics. There are some other instruments which have high quality of validity and reliability but they are only for specific mathematics topics. For example, SimCalc (as cited in Hill et al., 2008) is an instrument to measure teachers' content knowledge only in rate and proportionality. The second example is Knowledge for Algebra Teaching (KAT), developed by researchers in Michigan State University to measure teachers' knowledge of algebra (KAT, 2009). The SII/LMT and DTMAS will be discussed further in this chapter. In spite of the common purpose of these two instruments they have different uses in measuring teachers' content knowledge and providing different means to measure it. These two instruments will be discussed in terms of their theoretical approach to the concept of teachers' mathematics content knowledge and their different purposes of use.

SII/LMT. This instrument, also called Content Knowledge for Teaching Mathematics Measures (CKT-M), can be seen as a continuum of research on MKT at the University of Michigan. The researchers were studying theory of mathematics knowledge for teaching. They developed some survey items to study California reform programs to implement and test the MKT model (Hill, Rowan, & Ball, 2005). When the researchers received additional funding, they developed the SII/LMT instrument to measure mathematics knowledge for teaching. This instrument was developed by group of mathematicians, mathematics teachers, mathematics educators, and the members of the research team that developed the MKT (Hill, et al., 2004, Hill et al., 2008). The validity of the items was studied by experts from different backgrounds (Ball et al., 2008; Hill et al., 2004)

Three domains of teacher knowledge from the MKT model are targeted in this instrument: content knowledge (common content knowledge and specialized content knowledge), knowledge of content and students, and knowledge of content and teaching. The instrument addresses the majority of mathematics topics under three categories: number and operations (K-6 and 6-8), patterns functions and algebra (K-6 and 6-8), and geometry (3-8). The research group first developed elementary school items then middle school items.

The purpose of this instrument is to "discriminate accurately among teachers, in essence ordering them as correctly as possible relative to one another and to the underlying trait being assessed, mathematical knowledge for teaching" (Hill et al., 2008, p. 131). Another use of this instrument is to measure change in teachers' knowledge as they learn over time. An important characteristic of this instrument is that it does not provide raw scores. In other words, a teacher's score cannot be interpreted as how much the teacher knows. The instrument developers strongly warn that the instrument is not suitable for the purpose of individual teacher accountability such as certification or qualification (Hill et al., 2008).

DTMAS. In another attempt to measure teachers' mathematics content knowledge, a team of researchers from the University of Louisville developed DTAMS for both elementary and middle school teachers. There are both open-ended questions and multiple choice questions. Even though SII/LMT emerged from previous studies, DTAMS was a project to develop measurement in other words the goal of the research project was to develop an instrument to measure teachers' mathematics content knowledge. The team of mathematicians, mathematics educators, and teachers developed the first items to be tested for validity (Bush, Ronau, Moody & McGatha, 2006; DTAMS, 2009). The items are categorized under four groups of mathematics topics: number/computation, geometry/measurement, probability/statistics, and algebraic ideas. The researchers used three strategies to ensure validity: developing mathematics topics to be measured from national standards and recommendations, item development by experts in the field (mathematicians, mathematics educators and teachers) and sending out the items to the experts to test if the items address what is intended.

For this instrument, the researchers identified teachers' mathematics content knowledge under four types. Type I knowledge (Memorized Knowledge) is rote memorization of mathematics knowledge such as "memorized definitions, procedures, or rules" (DTAMS, 2009). Type II knowledge (Conceptual Understanding) is deep understanding of mathematics and knowledge of multiple ways to solve and explain a mathematics topic in addition to knowledge of relationship between mathematics topics. Teachers who possess Type III knowledge (Problem Solving/ Reasoning) are in higher order of mathematical thinking because they can "reason informally, and formally, conjecture, validate, analyze, and justify" (DTAMS, 2009). The last type of knowledge, Type IV (Pedagogical Content Knowledge) is the only knowledge type which is unique to teaching mathematics. With this type of knowledge, the teacher can recognize student misconceptions and provide instructional methods to correct them. This knowledge type "includes knowledge of the most regularly taught topics in mathematics, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations" (DTAMS, 2009). The research team reported that they reached these four types of knowledge from an analysis of literature on "what mathematics elementary students and teachers should know. They used national recommendations, national and international test objects, and research to determine appropriate mathematics content" (DTAMS, 2009).

Table 2-2. Knowledge types and definitions in DTMAS

Knowledge Type	Definition
Type I (Memorized Knowledge)	Rote memorization of mathematics knowledge
Type II (Conceptual Understanding)	Deep understanding of mathematics
Type III (Problem Solving/ Reasoning)	Mathematical reasoning and problem solving skills
Type IV (Pedagogical Content Knowledge)	Unique knowledge of teachers to teach mathematics

There are two goals of this instrument: to diagnose elementary and middle school teachers' knowledge of mathematics and to study teacher knowledge with teacher learning and student achievement. Since DTAMS was developed to diagnose teachers' knowledge, results of this test can be interpreted as how much mathematics a teacher knows or lacks. This is one of the most important differences of DTAMS from SII/LMT in which the results cannot be interpreted for individual teachers. Besides the difference in theoretical perspective for teachers' mathematics content knowledge, another difference between these two instruments is the source of items. The research team in SII/LMT used classroom observation and classroom teaching in addition to expert suggestions while DTAMS items are based national standards and curriculum.

Teachers' Mathematics Knowledge in Research

The questions of whether and how teachers' mathematical knowledge affects student achievement was the central question of the research of Hill, Rowan, and Ball (2005). The researchers studied teachers from 89 schools which were participants of one of the three reform programs in California (America's Choice, Success for All, and Accelerated School Projects) during 2000-2001 through 2003-2004. The results of the study showed that teachers' content knowledge was a significant predictor of student gain in elementary school. So Hill and others concluded that "efforts to improve teachers' mathematical knowledge through content-focused professional development and preservice programs will improve student achievement, as intended" (p. 400). Most importantly, the results showed that teachers of the lowest third of the distribution of knowledge may benefit most from content based professional development. The teachers of struggling students and schools need content based professional development more than any other group of teachers.

One of the possible solutions for providing quality education for all students might be using content focused professional development opportunities for teachers. However, the underlying question is what quality content learning experiences for teachers is. For the following sections of this chapter, the example research studies will be discussed to demonstrate the efforts to provide content focused research on teachers' content knowledge. First, research studies for various mathematics topics will be discussed. Later, two possible designs to study teachers' mathematics content knowledge will be addressed in order to guide further research studies.

Teachers' Mathematics Knowledge for Mathematics Topics

As many studies expressed the importance of teachers' content knowledge in teaching, there is a need to study teacher knowledge in the context of mathematics topics. Cohen and Hill (2001) found that professional development was most likely to have an affect on teachers' practice when it was focused on particular content as represented by curriculum as well as focused on mathematical ideas, and on students' thinking about that same mathematics topic. In order to improve our understanding of teacher knowledge, we need to study this concept in various contexts and especially for various topics of mathematics. For the purpose of organization of this chapter and to be able to address several research projects, the studies will be considered in three groups according to their mathematics topics: algebraic ideas (numbers, computations, patterns, functions, and algebra), statistics/probability, and geometry/measurement. For the purpose of this literature review, a selection of studies will be discussed in detail. These studies are selected because of their scope and their contributions in understanding of teachers' mathematics content knowledge.

Algebraic ideas

Most of the research studies in teachers' mathematics knowledge are in mathematics topics which can be categorized as algebraic ideas. The first example is one of the series of studies by Ball (1990a, 1990b). These studies were from Teacher Education and Learning to Teach Study (TELT) via the National Center for Research on Teacher Education (NCRTE). Ball (1990a) studied elementary (n=10) and secondary (n=9) preservice teachers to explore their knowledge of mathematics in division concepts (division by fractions, division by zero, and division in the context of algebraic equations). The interview questions were based on both correct answers and conceptual understanding of the algorithms. All but two of the elementary and secondary preservice teachers were able to answer division by fraction correctly with mathematically correct explanation. Many of the participants had difficulty in providing mathematical reasons for the algorithm. In general, while most of the secondary preservice teachers were able to answer mathematics problems correctly, they could not present mathematical reasons beyond rules. In the case of elementary preservice teachers, most of them could not answer the problems correctly and the majority of them could not give mathematical explanations. Therefore, the results of this study showed that most of the preservice teachers in secondary and especially in elementary mathematics teacher education programs lacked mathematics knowledge for concepts for divisions, and they could not choose appropriate representations. Also, the participants who majored in mathematics showed dependency on algorithms and rules in great extend rather than mathematical explanations.

In another research project to investigate teachers' knowledge of mathematics, Ma (1999) conducted a comparative research project on elementary school teachers' understanding of three topics in mathematics: division, place value and area-perimeter relationship. She did a comparative study between Chinese and American in-service teachers. The results were groundbreaking because the results revealed that in spite of having the advantage of higher education and advanced mathematics courses, American teachers did not have the deep mathematical understanding that Chinese teachers had. Chinese teachers did not have the same level of higher education yet they had more experience with mathematics teaching practices in the classroom. Their learning was tailored for teaching rather than advanced degrees in mathematics. The results revealed that higher education mathematics courses were not enough to make sure that teachers have quality mathematics knowledge for teaching.

In an earlier study, Borko et al. (1992) addressed the gap between teacher education programs and expectations of teachers in terms of their knowledge. As part of a research project, Learning to Teach, in which the team of researchers studied middle school preservice teachers, the authors reported the results of one student teacher in fraction division. The researchers interviewed the participant, Ms. Daniels, several times during the last year of her teacher education and during her student teaching. The research group investigated the methods course Ms. Daniels completed. There were interviews conducted with methods course instructors and other preservice teachers. The aim of this study was to investigate student teachers understanding of mathematics concepts and the connection between concepts and the procedures. Ms. Daniels took advanced mathematics courses in college the mathematics methods course before she taught as a student teacher. However, she could not answer students' questions regarding explanation of fraction division algorithm. In the search of answering questions about Mrs. Daniels' weak mathematics understanding, the authors concluded that university mathematics courses "do not stress meaningful learning of mathematics" (p.217). Furthermore, Borko et al. (1992) suggested that Ms. Daniels' knowledge of mathematics was only rote knowledge. "If this was the case, then, her own previous successes may have contributed to a sense that rote knowledge is, at least in cases like division of fractions, enough to know. If so, the conceptual emphasis on the topic in the methods course would have seemed irrelevant" (p. 218).

These example studies and many others (Lamon, 1999; Leinhardt & Smith, 1985; Simon, 1993; Simon & Blume, 1994; Stein, Baxter, Leinhardt, 1990; Stoddart, Connell, Stofflett, & Peck, 1993; Tirosh & Graeber, 1990) showed preservice and in-service teachers lack of fundamental understanding of algebraic ideas of mathematics. Even though teachers may solve a mathematics problem correctly, they may have difficulty in providing mathematical explanations for many algorithms. One of the most important conclusions from these studies was the crucial role of specialized content knowledge for teachers, and the challenge to develop this knowledge in university mathematics courses. Therefore, as these topics are building blocks for mathematics learning, teacher educators should address SCK for arithmetic in mathematics education courses and in professional development workshops. Furthermore, the effective means to address arithmetic topics such as fraction and place value in teacher education should be explored more. There is no common consensus on learning experiences for teachers which may results in deeper understanding for these topics.

Statistics and probability

Stohl (2005) stated that the mathematics education field has been giving emphasis on stochastic (statistics and probability) in school curriculum only for the last 10-15 years. As the field of statistics education being newly developing, Garfield and Ben-Zvi (2007) call research on teachers' statistics knowledge as a new line of research in statistics education. In an earlier work of Garfield (2002) he addressed the challenges of statistical reasoning and its role in statistics education. Garfield developed the Statistical Reasoning Assessment and concluded that in spite of students' good grades and good performances on exams and projects for statistics; they were not showing statistical reasoning. Garfield stressed the role of teachers and their knowledge and conceptions on statistics. Garfield concludes that teachers tend to teach statistics procedurally without addressing statistical reasoning and expecting students to develop statistical reasoning from procedural experiences. "However, it appears that reasoning does not actually develop this way" (p. 3). Therefore, he stressed that teachers' knowledge of statistics should not be rote knowledge but meaningful knowledge with statistical reasoning.

As statistics and probability education gathered more attention from researchers, the interest in teachers' statistics and probability knowledge increased. Some of the statistics topics that have been studied in preservice and in-service teacher settings were sampling, distribution and probability. To study preservice teachers' knowledge of statistical sampling, Groth and Bergner (2005) asked 54 preservice teachers to give idiosyncratic metaphors for the concept of sample. The purpose of the study was to investigate the nature of preservice teachers' knowledge of statistics. The qualitative analysis resulted in seven categories of participants' statistical thinking for sample: (i) no metaphor given, (ii) sample as a collection of objects, (iii) sample as a part of whole, (iv) sample as a representative part of a whole, (iv) metaphors for place of sample within field of statistics, (vi) metaphors describing actions to be taken upon samples, and (vii) pedagogically awkward characterizations for sample. The results revealed that the majority of the preservice teachers, more than 80%, failed to mention representativeness of samples. Also, some preservice teachers could not provide logical metaphors (category seven). For example, one of the participants used “one in a million” as a metaphor for sample. Later, the participant explained that this metaphor shows that sample does not represent everyone. Therefore, the researchers concluded that “the categories of participants’ responses also suggest areas for teacher educators to focus upon in developing preservice teachers’ statistical content knowledge” (p. 38).

In another research on preservice teachers’ statistics knowledge, Leavy (2006) studied the concept of distribution. It was one-group, pre and post-test exploratory study in a methods course for a semester, Leavy asked 23 preservice teachers to complete two statistical inquiry projects, the bean investigation and the popcorn experiment. The bean experiment was used as pre-assessment, and the popcorn experiment was used as post-assessment in addition to learning experiences between these two inquiry activities. The desired outcome of these projects was preservice teachers to compare distributions of data while they are working on collection, representation, analysis and reporting of data. The initial understanding of the participants showed great tendency to use descriptive statistics especially mean with limited graphical representation to show distributional features in analyzing data. After the statistics instruction, the post-assessment activity showed improvement in preservice teachers’ use of visual representations for distribution, however only the participants who were showing interest and knowledge to use them in pre-assessment. “Thus we were successful in helping participants understand the differential limitations and advantages of particular graphical representations … however the stability in strategy use for those who used descriptive statistics indicates that for these participants we were less successful” (p.102). Leavy concluded that even though emphasis on concept of distribution and use of visual representations improve statistics knowledge of preservice teachers the positive outcome was limited.

In addition to statistics, probability is another topic of stochastic which shares the same concerns with statistics education in the area of teacher knowledge. Haller (1997) facilitated the summer institute, Rational Number Project Middle Grades Teacher Enhancement, for middle school teachers to prepare them for the newly adapted curriculum. Part of the institute was focused on probability, and the researcher conducted pre-assessment and post-assessment for probability knowledge of teachers. In spite of the evidence of teachers lack of probability knowledge and misconceptions in pre-assessment, post-assessment results showed improved teacher knowledge and increased confidence of teachers’ knowledge of probability. However, as a follow-up study during the school year, Haller interviewed with four teachers from the summer institute. Some teachers expressed their perspective in teaching probability such that they would teach probability if they would have enough time at the end of the semester. Therefore, even though the summer institute yielded positive results in the short term, the knowledge and beliefs of teachers in probability education was limited in the longer time period.

The research studies in statistics knowledge of preservice and in-service teachers are limited and most of them are exploratory. There is a need to conduct more research to investigate ways to improve teachers' statistics and probability knowledge and effective tools to improve teachers' statistics understanding (Garfield, 1995; Garfield & Ben-Zvi, 2007; Konold & Higgins, 2003; Shaughnessy, 1992, 2003). Even though statistics education programs and materials have been developed for teachers, the field could not answer the effectiveness of these programs (Stohl, 2005). After having exploratory studies, the field would need quantitative studies to measure teachers' knowledge of statistics and probability. Most of the studies in this field were conducted with self-developed assessment tools for teacher knowledge. With more elaborative and quality measurement tools, researchers can also show the efficiency of the programs.

Geometry and measurement

Being one of the most important topic of mathematics curricula (both K-12 and college), geometry is also one of the first branch of mathematics tracing its history from 3000 BCE Mesopotamians and Egyptians. In ancient Greek, geometry (geometria) can be defined as geo- for earth and -metria for measure. The work of Euclid, Elements, is the foundation of the study of geometry and it is dated as far back as 300 BCE. Today, college geometry curriculum includes non-Euclidean geometry, but K-12 curriculum is based on Euclidean geometry (plane geometry). Especially in K-12 curriculum, the Euclidean geometry can be defined as the study of figures of two and three dimensional objects in addition to study of their shapes, sizes and positions.

> School geometry is the study of those spatial objects, relationships, and transformations that have been formalized (or mathematized) and the axiomatic mathematical systems that have been constructed to represent them. Spatial reasoning, on the other hand, consists of the set of cognitive processes by which mental representations for spatial objects, relationships, and transformations are constructed and manipulated. (Clements & Battista, 1992, p. 420)

Especially in secondary school geometry education, the instruction based on teaching and learning geometry is through extensive use of two-column proofs due to the emphasis on using Euclidean axioms and proofs. However, as the theory van Hiele levels for geometric thinking portrays it clearly, students have to go through several other experiences in order to reach the stage where they can do proofs. “they[students] lack prerequisite understandings about geometry. This lack creates a gap between their level of thinking and that required for the geometry that they are expected to learn” (van Hiele, 1999, p. 310). It is necessary to understand the van Hiele Levels before studying geometry learning of students and teachers.

The theory of van Hiele geometric levels of thinking was the results of a research by two Dutch educators, Pierre van Hiele and Dina van Hiele in 1959 who were sharing similar education views with Piaget (Clements & Batitista, 1992). Pierre van Hiele continued this research on geometric thinking, and he was accepted as the father of geometric thinking theory of five levels. In brief, these five levels of geometric thought as van Hiele (1999) addressed are:

- the visual level is the nonverbal level in which “figures are judged by their appearance” (p. 311),
- the descriptive level is observed when students identify figures as certain way due to their properties but not just the way look,
- the informal deduction level is the beginning of deductive reasoning without formal proof skills, “However, at this level [informal deduction level], the intrinsic meaning of deduction, that is, the role of axioms, definitions, theorem and converses, is not understood” (p. 311),

- the deduction level is the last level to be expected to be seen in K-12 curriculum in which students perform deductive proofs by using Euclidean geometry axioms without going beyond Euclidean geometry,
- the rigor level is where students are able to develop theorems in different postulational systems and move their thinking in non-Euclidean geometry and can be evidenced mostly in college level courses.

In development of geometric thinking and geometry knowledge, the role of learning experiences is the key element rather than age or maturation of the learners. Another feature of this theory is the sequential order of levels; one cannot reach a higher level without mastering the previous levels. The last but not least characteristic of the levels is that one may be at different levels for different geometry topics because of various experiences with different topics. For example, one may show higher level of geometry thinking for two-dimensional shapes than three-dimensional shapes. In addition to studies on K-12 students geometric thinking levels (e.g., Senk, 1989) there have been several research studies to investigate preservice and in-service teachers van Hiele levels and the effect of certain learning experiences on their geometric thinking (e.g., Usiskin, 1982).

It is possible to categorize research with teachers in geometry education under three categories: teachers' van Hiele levels of geometric thinking (e.g., Usiskin, 1982), professional development studies to improve geometry teaching in the classroom (e.g., Paniati, 2009) and studies to investigate teachers content and pedagogical content knowledge of geometry. Because the scope of this review of literature is teachers' content knowledge, the last two categories will be discussed further.

An example for professional development projects for geometry teaching is Fostering Geometric Thinking (FGT), a project of Educational Development Center (EDC). This research consisted of two phases: development of professional development materials and field test of the materials. Before developing the professional development materials, a team of researchers first developed the geometric habit of minds (GHM) framework so the professional development activities could foster those geometric habits of minds. There were four components to GHM: reasoning with relationship (understanding “always” and “every” cases in geometry), generalizing geometric ideas (using geometric relationships between shapes), investigating invariants (studying features that stays same or changes) and balancing exploration and reflection (trying various methods to solve and to reflect on those methods) (Driscoll, Egan, Dimatteo, & Nikula, 2009). These four components were built on work of Driscoll (1999) on mathematical habits of mind. To foster GHM, the research team developed series of professional development activities.

The professional development activities were designed to be used by a facilitator with a group of teachers of grades 5-10. There were 20 sessions of two hours each. Each session was focused on a geometry problem to solve. For example, one of the geometry problems is the perimeter problem: “Two vertices of a triangle are located at (4,0) and (8,0). The perimeter of the triangle is 12 units. What are all possible positions for the third vertex? How do you know you have them all?” (Driscoll et al., 2009, p. 166). The design of the professional development had three sections. First, the participating teachers engaged in the given geometry problem during the session, later they used that problem in their classroom. After they collected work of their students on the problem, the professional development participants met again to analyze student work and to reflect on them.

The second phase of the research was to conduct a field test for the professional development activities. For this field testing, there were three research interests: the increase in teachers' mathematics content knowledge, the increase in teachers' attention to students thinking and mathematical communication, and change in teachers' pedagogy towards focusing on productive ways of thinking in geometry and knowing to benefit English Language Learners (J. Nikula, personal communication, May 21, 2009). In the field test, the research team studied treatment and control groups. In treatment groups, there were 15 facilitator and 117 teachers. In control groups, there were 13 facilitator and 104 teachers. The geometry content knowledge of teachers was measured by a geometry survey which was consisted of multiple choice geometry problems, open-ended questions on problem solving strategies, and analysis of transcribed lessons. This geometry survey was used as pre and post test. After the completion of professional development and post-test, the research team made classroom observations the following school year. The results of the field test showed increase in both teachers' content knowledge and attention to students' thinking (J. Nikula, personal communication, May 21, 2009). At the time of my personal communication, the researchers reported that they did not finish the analysis of classroom observations so they could not study the third research question. The purpose of this project was to develop a professional development series to foster middle and secondary school teachers' geometry knowledge. Studies similar to FGT provide valuable resources to enhance teacher geometry knowledge yet they do not provide strong research perspective in the topic of teachers' geometry content knowledge.

In a study with 49 in-service teachers for grades 4-9 and in the follow up study with 8 of the participants with observations and stimulated recall interviews, the researchers looked for change in the geometry content knowledge and van Hiele cognitive levels of teachers after a summer program for 4 weeks (Swafford, Jones & Thornton, 1997). The aim of this research study was to combine the Cognitively Guided Instruction approach (knowledge of student thinking) and teacher content knowledge efforts for geometry teaching. During the intervention program, the participants engaged in problem solving and hands on geometry activities for two and three dimensional explorations. In order to measure teachers' content knowledge, the researchers adapted a test from the Cognitive Development and Achievement in Secondary School Geometry Project (CDASSG) from Usiskin (1982). After administering the pre-test, the researchers decided that the test was too easy for their participants, so they chose only 10 items to be used for this project. Furthermore, in order to measure the van Hiele levels of participants, the researchers used a combination of two instruments, interview instrument from Mayberry (1981) and a multiple choice instrument from Usiskin (1982). The results showed increase in both teachers' content knowledge and the van Hiele levels of teachers. The researchers reported a significant gain in geometry knowledge for teachers especially 4th and 5th grade teachers. Furthermore, according to the pre and post-tests for van Hiele levels, 72% of the teachers had at least one level increase while 50% of them increased for two levels.

As these two examples reflect, the focus of teacher education research in geometry is on middle and high school grades. However, closer analysis of geometry topics in Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A

Quest for Coherence (NCTM, 2006) stresses the importance of students' experiences with geometry in early grades on to their preparedness for secondary level geometry learning. Furthermore, "Students should enter high school understanding the properties of, and relationship among, basic geometric shapes" (NCTM, 2000, p. 310). In elementary school level, two-dimensional geometry is addressed more than other topics. Students should know definitions of the two-dimensional shapes and know the relationship and differences between them. Quadrilaterals are the cornerstone of geometry topics for elementary school grades (NCTM, 2006). Students and their teachers should know the hierarchical nature of the quadrilaterals in addition to the definitions of the quadrilaterals (Fujita & Jones, 2006, 2007)

Therefore, the teachers of elementary school should possess required geometry knowledge to prepare students for more advanced geometric thinking. If the elementary school teachers lack the necessary knowledge of geometry to prepare students for higher level of geometry thinking, students would enter secondary level grades with a limited geometry knowledge which would result in to rote memorization of geometry without meaningful learning (van Hiele, 1999). Even though the emphasis in geometry education is on upper level grades, the mathematics education community needs to study teachers' geometry content knowledge in elementary school level too.

However, many other research projects focused on knowledge of geometry for teaching reached the conclusion that especially beginning teachers are not equipped with necessary subject matter knowledge and pedagogical content knowledge of geometry, and it is important to address this issue in teacher education programs (Barrantes & Blanco, 2006; Chinnappan, Nason, & Lawson, 1996; Jacobson & Lehrer, 2000; Lampert, 1988; Leikin, Berman, & Zaslavsky, 2000). “Teachers are expected to teach geometry when they are likely to have done little geometry themselves since they were in secondary school, and possible little even then” (Jones, 2000, p. 110). Therefore, under the considerations of the new understandings in teachers’ mathematics content knowledge the mathematics teacher education community need to study not only middle and high school teachers’ geometry content knowledge but also elementary school teachers’ too.

In terms of investigating teachers’ measurement content knowledge, in some studies, measurement learning was studied with geometry (e.g., Clements, Battista, Sarama, Swaminathan, & McMillen, 1997). There are some studies on students’ learning measurement (Hiebert, 1981, 1984; Miller, 1994; Stephen & Clements, 2003) but studies in the area of teachers’ knowledge of measurement concepts is limited (Baturo & Nason, 1996; Heaton, 1992; Simon & Blume, 1994).

In one of the studies for teachers’ content knowledge in measurement, Baturo and Nason (1996) interviewed 13 first-year prospective primary teachers. Seven of the participants were identified as high achievers and six of them were identified as low achievers. There were eight tasks on area measurement asked in each interview. With these interviews, the researchers studied not only the content knowledge of participants but also their dispositions towards measurement and mathematics. The results of this study reflected the limited knowledge of prospective primary teachers in the area measurement. “Much of their substantive knowledge was incorrect, and/or incomplete, and often unconnected. The ability of the students to transfer from one form of representation to other forms of representations thus was very limited” (Baturo & Nason, 1996, p. 261). The researchers addressed their concern that these participants might transfer their lack of knowledge into the classroom that they would be teaching. “the impoverished nature of the students' area measurement subject matter knowledge would extremely limit their ability to help their learners develop integrated and meaningful understandings of mathematical concepts and processes” (p. 263).

Efforts in understanding teachers’ content knowledge for geometry and measurement are similar to the efforts in statistics education. There are more exploratory studies and professional development projects than large scale research studies. With the new developments in teachers’ content knowledge the field should study teachers’ content knowledge in all mathematics topics further. Also, the with new measurement instruments, researchers should study to illuminate the effects of professional development projects on teachers’ content knowledge in various mathematics topics.

Designs to Study Teacher Knowledge in the Context of Teaching

In order to improve our understanding of teachers’ mathematics content knowledge, Ball (2000) proposed three topics to be studied. The first two topics were (i) development of a definition of the teacher knowledge, (ii)“how such knowledge needs to be held” (p. 244). In their quest to answer these two topics, the field witnessed the development of the MKT model and extensions of Shulman’s model, in addition to development of measurement instruments for teachers’ mathematics content knowledge.

The third topic was “how to create opportunities for learning subject matter that would enable teachers not only to know but to learn to use what they know in the varied contexts of practice." (p. 246). Many studies showed the importance of addressing content specific teacher knowledge in professional development and preservice teacher education, yet the one of the obstacles is the difficulty in studying teacher knowledge in the context of teaching.

When the goal is to interweave teacher learning experiences with the classroom teaching context, two possible study designs are suggested: (i) using student work as a tool to interpret students' knowledge and their learning, thus, working on the content itself and (ii) using examples of classroom video episodes (Ball, 2000). The core of both of these designs was to study a task of mathematics teaching which would address content knowledge while providing connection to the classroom teaching.

Using video clips from classrooms

Use of video in teacher education began in 1960s and some of the early uses of video recordings were microteaching sessions and using video recordings of field observations instead of observing live classrooms (Sherin & Han, 2004). In the 1990s, teacher educators began using video for video study groups (i.e. video clubs) in in-service teacher education settings (Frederiksen, Sipusic, Sherin, & Wolfe, 1998; Gwyn-Paquette, 2001; Seago, 2004; Sherin 2003, 2004; Tochon 1999) and for methods courses in preservice teacher education (Ball & Lampert, 1998; Copeland & Decker, 1996; Daniel, 1996; Friel& Carboni, 2000; Goldman & Barron, 1990; Lambdin, Duffy, & Moore, 1997; Winitzky & Arends, 1991).

Sherin and Han (2004) defined video clubs as "meetings in which groups of teachers watch and discuss excerpts of videotapes from their classrooms" (p. 163). The most common design of video study groups is where a researcher or teacher educator is the facilitator instead of the less popular design where participating teachers take turns as facilitators. Furthermore, the discussion topics can be chosen together by facilitator and the presenting teacher (Tochon, 1999), or by the presenting teacher only (Sipustic, 1994). Regardless of the design, the purposes of video study groups are to record classroom teaching, and choose a short piece of it to share within a community of teachers in order to generate discussions around classroom teaching.

Using video study groups and the discussions during the meetings provide additional resources for teachers in learning to teach. First of all, it gives teachers more time to reflect on their practice. “Unlike teaching, viewing classroom interactions via video can be a time for reflection rather than action” (Sherin & Han, 2004, p. 165). By having a chance to reflect on their teaching, teachers develop new skills which might be transferred from video study groups to their classrooms (Gamoran, 1994). Furthermore, teachers may develop a new perspective in their knowledge of teaching (Gwyn-Paquette, 2001).

One of the most important features of video study groups is that teachers are able to create a collaborative professional group in which they should be able to discuss teaching openly without judgments. These collaborative groups may even develop their norms such as the language for studying videos (Sipustic, 19994). Due to these valuable opportunities served to teachers by video study groups, Sherin and Han (2004) call them as “an important catalyst for learning” (p. 180).

On the other hand, some the challenges to be considered when using video study groups may be building collaborative learning environment, especially when participating teachers are not open to peer reflection. Some teachers may not prefer to be video recorded even though they are part of the group. In Sherin and Han’s study, two of the teacher preferred not to be recorded yet they participated in discussions in the group (Sherin & Han, 2004). Because of the difficulty of forming a collaborative learning environment, the researcher should be careful in establishing trust and norms within the group. A video study group may benefit from some introductory activities that build professional trust before any discussion on teaching.

In the preservice teacher education setting, researchers used video clips to investigate various aspects of teacher development such as preservice teachers' meaning making (Copeland & Decker, 1996), their beliefs and attitudes (Daniel, 1996; Friel & Carboni, 2000), their learning, teaching and assessment visions (Lambdin, Duffy & Moore, 1997), their knowledge of cooperative learning (Winitzky & Arends, 1991), and their knowledge of classroom management (Overbaugh, 1995).

In the qualitative study of Daniel (1996), videotapes of experienced mathematics teachers were used with preservice teachers. Daniel studied preservice teachers' perceptions of using videotapes to understand constructivist approach to teaching. Majority of the preservice teachers expressed their satisfaction of seeing videotapes and shared its affect on understanding learning theories that they had been studying. Furthermore, interviews revealed that after preservice teachers had seen videotapes of classrooms in which the teachers were using reform-based instruction, they reported that videotapes helped them to have understanding and image of a reform-based classroom while they paid more attention on student-centered instruction ideas (Friel & Carboni, 2000).

In another example, Ball and Lampert (1998) used hypermedia materials in methods course for elementary mathematics teachers. The materials included videotapes of their own third and fifth grade teaching for a year, transcription of the videos, student work, and other classroom artifacts such as teachers' journals. Ball and Lampert used these materials to guide preservice teachers in their investigations. The researchers focused on content perspective of the material and they tried to encourage participants to investigate content of the materials. Even though, the researchers did not provide evidence based results about participants learning, they reported results about the participant investigations. From their analysis of 68 investigations, the results showed that while seven of the investigations were about students and seven of them about policy issues, two third of all investigations were about teaching.

One of the characteristics of video discussions with in-service and preservice teachers is the focus of the participants' on topics related to the teaching rather than content. When teachers view a video of a classroom, the most important feature of teaching to receive their attention is classroom interactions and general pedagogical techniques such as classroom management (Friel, 1997; Richardson & Kile, 1999). In a particular study, researchers focused on discussion topics that emerged in a seven-week video study group (Sherin & Han, 2004). The qualitative analysis of discussions revealed four types of discussion topics: pedagogy, student conceptions, discourse, and mathematics, in the order of higher occurrence. Therefore, teachers tend to pay more attention on pedagogical aspects of teaching with less discussion on student conceptions and very little discussion on mathematics. According to Sherin and Han, by the end of the seven-week study, the teachers began to pay more attention on student conceptions with more complex issues while their pedagogical discussions shifted from discussions of alternative pedagogies to analyzing the teaching method.

In spite of value of using videotapes on teacher learning, researchers should make sure that the research purposes aligns with using videotapes. In other words, in the teacher knowledge models that were previously discussed, video study groups mostly address pedagogical content knowledge in Shulman's model. With a greater focus on student thinking and pedagogy, video discussions may be beneficial to use if a researcher prefers to address knowledge of content and students or knowledge of content and teaching in MKT model. Therefore video discussions may not be the most suitable research tool to use to address mathematics content, especially specific content knowledge in MKT model.

Using student work

When the purpose of the teacher learning activities is to address teachers' mathematics content knowledge in the context of teaching, then analyzing student work may offer an effective tool without observing a classroom. Using student work has been widely accepted by teacher educators as a way to improve teacher learning and instructional practices (Lampert & Ball, 1998; Little, 2004; Smith 2003). In spite of video clips, this practice makes it more relevant to specialized content knowledge by taking focus away from the teaching practices and interactions in the classroom (e.g., questioning, classroom management), to mathematics in the classroom while it still combines instructional elements (Kazemi & Franke, 2003). The authors discussed that by analyzing student work, teachers may be forced to think deeply and elaborate on mathematics knowledge while they are trying to understand what students did. "Making sense of children's strategies could be an indirect way for teachers to wrestle with the mathematical issues themselves" (p. 7).

Some studies showed that using student work to facilitate teacher learning resulted in teachers' deeper subject matter knowledge and classroom practice (Franke & Kazemi, 2001; Kazemi & Franke, 2004). In the line of Cognitively Guided Instruction research, Franke and Kazemi reported use of student work with elementary school teachers. The researchers conducted a four-year professional development workshop series with teachers from one school. The workshops were designed to promote teachers' understanding of student thinking. The researchers reported that as a result of attending the workshops, both teachers' content knowledge and pedagogical content knowledge were improved. "Thus, in detailing student thinking for the group, teachers included rich descriptions of the questions asked to elicit that thinking, the responses of other students, and the work that came before the shared interaction" (p. 107). However the researchers also stressed the importance of the selection of the student work in order to involve teachers in meaningful content discussions about students' use of algorithms and procedures (Kazemi & Franke, 2003). Therefore, an one important feature of a learning activity for teachers would be providing student work which demonstrates uncommon algorithms or methods of mathematics thinking.

There are two possible designs to use student work with teachers as professional development activities. In the first design, teachers bring their own students' work from their classroom, and for the second design the facilitator provides the student work from another source.

Little, Gearhart, Curry, and Kafka (2003) conducted a meta-analysis of three projects, Project Zero, the Coalition of Essential Schools, and the Academy for Educational Development, to study common elements of these three projects for examining student work. The goal of these projects was not teachers' content knowledge but to improve instruction in the classrooms. Even though this meta-analysis was not focused on teachers' knowledge, it provided important features of a professional development activities based on analyzing student work. The researchers found three common elements among the mentioned projects: providing opportunities for teachers to focus on student learning, supporting teachers to use student work part of their discussion, and use of protocols to structure conversations. On the other hand, the researchers expressed three concerns in implementing professional development activities with teachers: "(i) concern for personal comfort and collegial relationship, (ii) scarce time, many interests, and (iii) uncertainty about what to highlight in 'looking at student work'" (p. 191). In order to resolve the last two concerns of using student work, one may use long-term professional development workshops with focused and structured protocols instead of general topics of interest. However, the first concern is hard to overcome yet it is very important to address in any professional development activity with student work from teachers' own classrooms.

As addressed by Little et al. (2003), one of the difficulties of using teachers' own students' work for professional development is that teachers may be unwilling to share or discuss their students' work. Teachers' lack of trust to the professional development community might be due to instructional context that teachers are into rather than the professional development environment (Cobb, McClain, Lamberg & Dean, 2003; Zhao, McClain, & Visnovska, 2007). For example, Zhao et al. (2007) reported results from two different school districts which used analyzing student work for professional development for middle school mathematics teachers. The researchers found that how teachers experience the professional development workshops of analyzing student work was highly affected by the evaluation process of their districts. In one district, teachers were assessed by their students' correct work. Those teachers were timid and uncooperative to bring incorrect student work which was hindering the professional development workshop and their leaning. On the other hand, in another district, when teachers were able to perceive student work as "records of students' diverse ways of reasoning" (Zhao et al., 2007, p. 139), teachers had rich professional development experiences. "They saw a PD [professional development] activity built around student work as aligned with this instructional orientation and thus useful not only in helping them interpret students solutions but also in deepening their own statistical reasoning" (Zhao et al., 2007, p. 139).

The second kind of design for using student work in teacher learning is when facilitators provide student work from resources other than participating teachers' classroom. One of the disadvantages of the first design becomes an advantage of the second design, accessibility of student work. In a similar case like the study of Zhao et al. (2007), teachers may be reluctant to share their student work. For similar cases, using student work from other sources might provide rich discussions during the professional development activity while it relives trust concerns of teachers. Furthermore, especially when the participants do not have access to student work, such as preservice teachers, using other resources for student work may be the best option. The key point for this design is the selection of the student work to use. The student work should be still connected to the local classrooms and be similar to classroom settings that teachers would work.

Especially in preservice teacher education courses such as methods courses, preservice teachers' learning might suffer from their limited connection with actual classroom settings and schools. This limited connection may interfere with their learning especially when the purpose of the courses is to teach SCK and PCK. For example, Nugent and Grant (2009) used NAEP materials with preservice teachers and reached positive results. The researchers used a resource book to facilitate learning sessions with student works from NAEP data pool.

As in the line of using student work for professional development for teachers, NCTM published a book, Learning from NAEP: Professional Development Materials for Teachers of Mathematics (2006), to guide teacher educators who prefer to use student work for professional development. The book used data from NAEP. There are five workshops presented in the book and each workshop address different topics in teacher learning such as studying student understanding, developing mathematical content knowledge, improving classroom assessment practices, exploring states, and addressing issues of equity. For example, for the workshop on teachers' content knowledge, the book provides easy to follow steps for facilitator and materials to use during the professional development workshop. After leading participants to become familiar with NAEP tasks, the facilitator asks them to complete or solve the particular NAEP task. Upon completing the NAEP task, participants discuss different solutions for the task, and then the group works on examining student work samples. The focus of the task of looking at student solutions is to studying mathematics itself from students' solutions.

One advantage of using NAEP data would be availability. This data is always available and one does not need to collect student work to facilitate workshop. On the other hand, the data provided might not respond well to the purpose of the workshop. For example, the NAEP data for geometry, provided by the book, is very limited. There is no student work for quadrilaterals, thus a professional development workshop on quadrilaterals cannot be conducted from NAEP data. Another consideration that a researcher has to pay attention is the relevance of such data. The student work that would be analyzed by teachers should be relevant to their settings and their local or state standards. NAEP data may not be suitable for participants who are teaching in schools, or districts that are not similar to NAEP data pool.

Beside the effectiveness of videotape studies or using student work with teachers, the disadvantages of these kinds of activities might be the restricted flexibility and limited transformation of content knowledge for different settings (Ball, 2000). Teachers face classrooms where they have many different learners which may not be discussed any of the tasks used during the above designs. Therefore, teacher learning tasks should also provide flexibility of mathematics knowledge for different types of classrooms and for different challenges of teachers. If the researcher has concern about flexibility, they may ask teachers to bring videos of their classroom teaching or their own student work in order to address their own setting. On the other hand, for preservice teacher education courses, the researcher may videotape teachers or collect student work from local schools, potential student population for preservice teachers.

Even though both of these designs, videotapes or student works, have potential for studying teacher knowledge, there are some differences between these two designs that a researcher has to be aware of in order to address the research interest properly. One of the differences between these two tools is the focus of the discussions during the workshops. With the technology of video recording, it is possible to observe a classroom without going to an actual classroom. Also, participants can watch same part of teaching more than once which provides deeper discussions about classroom teaching. However, when teachers watch video tapes of classroom, their discussions mostly focus on pedagogical topics (Sherin & Han, 2004) even though the facilitators try to change the focus of the discussion on content (Ball & Lampert, 1998). On the other hand, the task of analyzing student work provides materials for discussions without bringing the classroom interactions. Although, it might look like lack of classroom videos as disadvantage for teacher discussions, indeed this lack creates opportunities to stress other elements of teaching such as content. Therefore, using student work would be suitable for research on teachers' subject matter knowledge, especially specialized content knowledge because this tool helps to connect content and classroom teaching while eliminating pedagogical discussions.

Conclusion

The conclusions derived from this analysis of literature on teachers' content knowledge with a special focus on teachers' mathematics content knowledge brought up several questions to study in addition to providing understanding of the concept. Teacher education and concurrently mathematics teacher education have witnessed tremendous change and development of the understanding of teacher knowledge in past 30 years. The field moved from policy statements about what to do for effective teachers to uncovering the complex nature of teachers' content knowledge. A mathematics teacher does not use only mathematics knowledge as a mathematician but also the mathematics knowledge specific to teaching, teaching strategies for mathematics topics and curricular issues related to mathematics teaching. However, as it may seem simplistic to categorize them under three types of knowledge, indeed teachers' content knowledge is very complex concept that the field has not reached final conclusions. There are several areas to be studied in the concept of teacher knowledge.

First of all, there is a need to conduct more explanatory research. Even though the field has moved from giving so many names for teacher knowledge and identifying every single characteristic separately (Fenstermacher, 1994) to more condensed and more theoretical models (Ball, et al., 2008; Shulman, 1986, 1987), there is still missing information about the understanding of teacher knowledge. For example, more research about teachers' mathematics knowledge that is just for teaching (SCK) should be examined further. When content preparation of teachers does not address SCK but only common mathematics knowledge, teachers do not have chance to develop SCK understanding yet they enter the profession to develop it by practice.

Indeed, as a natural consequence of deeper understanding, development of valid and reliable measurement tools to study teacher knowledge are expected to be emerged. Without valid measurement tools, our conclusions about teacher knowledge would be limited. Even though Ball and the group of researchers developed MKT and measurement tool for teachers' knowledge, there are still some areas of their model that require further study. For example, the relationship between the different domains of knowledge needs to be addressed in order to improve our understanding of teachers' mathematics knowledge for teaching.

One of the contributions of the MKT model to understanding mathematics teachers' content knowledge was revealing the role and importance of SCK. As discussed before, because earlier studies could not address the mathematical knowledge which is different than mathematicians yet still different than PCK, it was very difficult to detect effects of teacher knowledge on student learning. The final report of the National Mathematics Advisory Panel (2008) stated that the problem especially for elementary school teachers is to know mathematics for classroom to be taught rather than knowledge from mathematics courses. Indeed, this idea of knowing and using mathematics brings the question of teacher education programs and teachers' opportunities to learn mathematics deeper. "Mathematics teacher education programs should reconsider how they provide subject matter knowledge and opportunities to teach it" (Borko et al., 1992, p. 194). Furthermore, Ball et al. (2008) stressed the role of teacher education in terms of content preparation of teachers.

> In our research we began to notice how rarely these mathematical demand could be addressed with a mathematical knowledge learned in university mathematics courses. We began to hypothesize that there were aspects of subject matter knowledge – in addition to pedagogical content knowledge- that need to be uncovered, mapped, organized, and included in mathematics courses for teachers (p. 398).

One of the promising tools for teacher education is using student work with teachers (including the preservice teachers). Using student work to discuss mathematics that students did may encourage discussions among teachers to address mathematics knowledge that specific to them (Kazemi & Franke, 2004). However, things to be cautious about are to choose the appropriate student work and to develop collaborative learning community with teachers. The student work should be puzzling and relevant to the teachers' setting. Furthermore, as for any teacher learning environment, the researcher or the teacher educator should be careful in planning of the learning communities.

In spite of mentioned missing parts in general understanding of teachers' mathematics content knowledge, there have been several studies addressing specific topics of mathematics to study teachers' content knowledge. Those studies were effective and helpful to understand teachers' knowledge and as Cohen and Hill (2004) suggested using content specific professional development activities might help better to teachers in order to make impact and change in their teaching. However, the field has emphasized mathematics topics such as numbers, and algebra with limited attention on other topics such as geometry and statistics. By having more studies addressing various topics of mathematics would benefit the whole field resulting in a better understanding of the teachers' content knowledge.

CHAPTER 3
METHODOLOGY

Our knowledge of learning and teaching has been changed tremendously with the new theories on learning. For example, the series of research projects of Cognitively Guided Instruction showed the important role of teachers' knowledge of student thinking. Teachers' who know and address student thinking in the classroom were more effective teachers. On the other hand, good intentions are not enough to be good teachers (Borko, Eisenhart, Brown, Underhill, Jones and Agard 1992). Teachers tend to teach the way they were taught (Schoenfeld, 1988). They begin learning to teach from their experiences as learners and their formal education on teaching begins in teacher education programs in which they may experience changes in their perceptions. As they begin their profession in classrooms, they continue learning through their experiences in the classroom. Teacher education programs play a very important role in this process of learning to teach as it is the first formal step of preparation and in some cases the last formal education besides any possible professional development workshops. Often teacher education programs do not support preservice teachers in their learning in order to transform them to knowledgeable teachers (Borko, et al., 1992).

As the research in teacher education reveals more about teachers' knowledge the practice of teacher education evolves too. Shulman's categories of teachers' content knowledge were influential on studies about teachers' knowledge, yet with new developments such as mathematical knowledge for teaching model brought different aspects of teacher knowledge to be studied. In order to understand teachers' content knowledge for teaching it should be studied in the context of teaching. Furthermore, studies on teachers’ content knowledge should start with understanding their perspectives and their needs.

The theoretical perspective of this study is constructivism. Hatch (2002) addressed the quest of a constructivist researcher as “individual constructions of reality compose the knowledge of interest to constructivist researcher” (p.15). In this dissertation study, the goal of the researcher was to study preservice teachers’ geometry knowledge. Therefore, first, the researcher began by listening to preservice teachers to understand their construction of geometry learning and practices to improve their geometry content knowledge. It was necessary to address preservice teachers’ constructions of geometry learning in order to be able to be able to develop a protocol to improve their geometry content knowledge.

Participating preservice teachers told stories which were expressing their perspectives. The analysis of the participants’ stories informed the researcher in terms of the geometric topic to be studied (quadrilaterals) and effective instructional practices to use with preservice teachers in the methods course. The synthesis of results from preservice teachers’ stories and literature developed into a geometry learning protocol for methods course. In order to study effectiveness of this protocol, the researcher investigated teachers’ geometry content knowledge of preservice teachers who received traditional instruction and those teachers who received the protocol for geometry instruction.

Therefore, the goal of this study is two-fold: (i) preservice teachers’ understanding of geometry learning and effective instructional strategies for their learning, (ii) possible effects of using a geometry learning protocol (developed as a result of the investigation on the first purpose) on quadrilaterals on their geometry content knowledge Therefore, the research questions for these two purposes of were:

1. What is preservice elementary teachers' understanding of geometry in elementary school?
2. What are the perceptions of preservice elementary teachers on effective instructional strategies to promote their knowledge of geometry in mathematics methods courses?
3. Does use of geometry activities focused on quadrilaterals with analysis of student work influence preservice elementary teachers' geometry content knowledge?
4. Is there a difference in geometry content knowledge between preservice teachers who are in a traditional mathematics methods course and preservice teachers who are in experimental mathematics methods course?

The components of the research to address multiple research questions required intensive planning. The study was completed in two years, including the pilots and preparation process. The pilot study for qualitative investigation (understanding preservice teachers' geometry learning) was conducted during the spring semester of the first year. Then during the summer term, the researcher attended the workshop for the instrument (Content Knowledge for Teaching-Mathematics Measures, CKT-M Measures) to measure teachers' mathematic content knowledge for teaching.

In the second year, local elementary schools were contacted to work with 4th and 5th grade teachers to collect student work. A geometry worksheet to use in classrooms was developed as a result of collaboration between the researcher and teachers. The teachers implemented the worksheet with students in April. At the end of the spring semester, the researcher conducted the qualitative investigation on preservice teachers' geometry learning. The methods course was not offered for the summer term. Meanwhile, the qualitative data were analyzed, and from the results of that investigation the protocol to use in the second phase of the research was developed.

During the fall semester of the second year, first the researcher piloted the protocol with preservice teachers who would take the methods course the following semester. Also, the protocol was piloted in a state conference for mathematics teachers and teacher educators (Florida Council of Teachers of Mathematics, FCTM) in order to improve validity of the protocols. After the completion of suggested changes, the protocol was used in treatment group sections of the methods course.

The participants in both treatment and control groups were tested for their geometry content knowledge (measured by CKT-M Measures) one week before their geometry instruction and one week after the completion of the geometry instruction. The test results were used to measure progress in their geometry content knowledge and to study possible difference between geometry content knowledge of the treatment and control groups. The methodology of this study will be discussed under three components: the qualitative investigation, development of the protocol, and the quantitative investigation.

Settings and Participants

This study was conducted in a mathematics methods course at a large southeastern research university for predominantly middle-class, white, female elementary school preservice teachers. Students complete approximately two years of education before starting in college of education for the teacher education program. Students begin their unified elementary education program in their junior year and usually they take the methods course in their senior year. This course plays an important role in preservice teachers’ education because it is the only mathematics methods course. Preservice teachers may prefer to continue their education for master’s degree for which they choose a major (e.g., mathematics/science or special education). Only the preservice teachers who choose mathematics/science as a major would have to take more mathematics methods courses. If a preservice teacher chooses not to complete master's degree or chooses to study a major other than mathematics/science, the preservice teacher would not take any other mathematics methods course. Therefore, the methods course of the interest of this study plays a crucial role in future teachers' education. This course is the last and the only mathematics methods course for many of the preservice teachers.

During the teacher education program, elementary school preservice teachers are required to take three mathematics courses, two general mathematics courses (e.g. calculus) and one content course for elementary teachers, before the mathematics methods course. The mathematics content course addresses mathematics concepts for elementary school level whereas the mathematics methods course is designed to build the future teachers' pedagogical tools for teaching mathematics. Even though the recommendation of this order is given, some students take methods course and the content course at the same time or some take methods course before the content course. Therefore, the methods course instructors are concerned about mathematics content knowledge readiness of their students.

One semester of the mathematics methods course in this university was thirteen or fourteen weeks. The textbook to be used for this course was chosen in advance. Indeed, the textbook, Elementary and Middle School Mathematics: Teaching Developmentally (Van de Walle, 2007) has been used as the major textbook of this course for more than ten years. Also, the students in the course have access to various manipulatives for elementary school classrooms to practice using the manipulatives for teaching mathematics. During the semester, the common practice among instructors is to address problem solving, assessment and technological aspects of mathematics teaching. In addition to these general goals, the instructors address mathematics topics for elementary school for one or two week long instruction. For example, while the instructor discusses development of number sense and operations in two weeks, the instructor may discuss topics of measurement in one week. Even though there is a common consensus about duration of instruction for a topic, the instructors have flexibility to change it. For geometry topics, generally instructors spend two weeks to address learning and teaching geometry.

Qualitative Investigation

One year before the qualitative investigation, a pilot study was conducted for in which individual interviews were used. The purpose of this exploratory study was to understand geometry knowledge of preservice teachers. Two participants from an elementary mathematics methods course were interviewed and artifacts from their class (e.g. geometry related parts of weekly journals) were collected. The planned analysis method was thematic analysis but the analysis of interviews showed great use of stories of preservice teachers. Therefore, both narrative analysis and thematic analysis were used. Most importantly, those stories were focused on participants’ learning experiences rather than their knowledge. Also, the participants addressed their lack of geometry knowledge and their limited experiences with geometry in both mathematics content course and methods course. Therefore, the pilot study informed the actual study in two areas: revision of research questions (to address learning experiences rather than their knowledge) and revision of research methodology (use of narrative analysis and thematic analysis).

Settings and Participants

The goal of the qualitative investigation was to understand preservice teachers' geometry learning especially in methods courses. The focus was the methods course because the results would inform the researcher to develop geometry activities for methods course to be used in the second phase of the study. This study took place in an elementary mathematics methods course which was described above. At the time of the qualitative investigation, the spring semester (January-May), there were three sections of the course being taught by three different instructors. Three participants, Christiana, Emma and Liz (pseudonyms), volunteered to participate in this study. There was one participant from each section in order to capture geometry learning in various classrooms of different instructors.

Data Sources

The data collection methods included individual interviews with one participant from each of the three sections of the course (n=3), observations of geometry instruction in each section (two weeks), and the collection of materials used during the geometry instructions. All three sections of the course were observed for three hours for each two weeks of the geometry instruction. Field notes were taken during the observations. Also, copies of the instruction materials (handouts and transparencies) and student presentations were collected. The primary purpose of the observations and the artifact collection was to capture the process of preservice elementary teachers' geometry learning in order to support the interview data. These data sources were not used in the data analysis process yet the observation and artifact collection improved the validity of the study by providing triangulation for the main data source, the interviews.

The primary data source for this study was individual interviews. The purpose of the interviews was to understand preservice elementary teachers' stories of learning geometry (before college, in mathematics courses in the college and especially in the methods course). The interviews were conducted after the participants had instruction on geometry in methods course. The interviews were at the end of the semester because geometry was one of the last topics to be covered. The interview participation request was sent to students through e-mail. There were five volunteers from three sections. Due to scheduling issues at the end of the semester, two of the volunteers could not participate in interviews. Therefore, there was one volunteer from each section for a total of three. The interviews were 45-60 minutes long and video recorded. The video recordings were used only for audio purposes. The reason to choose video recording the interviews was to make it easier to transcribe interviews (especially because the researcher was a second language user of English).

The interviews were semi-structured. The interview protocol (Appendix A) was not followed strictly because the purpose of it was to guide the researcher during the interviews. The narrative interviews are focused on intriguing story telling from participants through open-ended questions or probes (Reissman, 1993, 2000). The suggested narrative interview probes are "Tell me about" (Reissman, 1993, 2000). For this study, some of the interview questions were "Tell me about your geometry learning before college" or "Tell me about geometry instruction in methods course". As the participants talk about their experiences the researcher asked further questions to elaborate the topics. Another important feature of narrative interviews is that the researcher has to accept the role of the participant as the leading person in the interviews because the participant is the knowledge holder (Bruner, 1990; Reissman, 2000). These features of narratives interviews were explained to the participants at the beginning of the interview along with the confidentiality of the interviews.

Data Analysis

The data analysis in this study was focused on participants' experiences of geometry learning. The interviews, the source of the data analysis, were analyzed for both narratives and non-narrative forms. The stories of preservice teachers analyzed structurally (Labov, 1972) in addition to thematic analysis of both narrative and non-narrative data (Coffey & Atkinson, 1996).

Individuals may use narratives for meaning making in addition to using them for sharing their experiences in stories (McAdams, 1993; Reissman, 1993). Grbich (2007) identified research settings which might be addressed by narrative analysis "those that explore either the structure of narratives or the specific experiences of particular events, e.g. marriage breakdown; finding out information which is life changing; undergoing social/medical procedures; or participating in particular programmes" (p. 124). In the case of teacher learning, narrative analysis may be used to study professional development experiences of in-service teachers or preservice teachers in teacher education programs. Also, literature suggests that teachers may prefer to discuss their learning and their knowledge through stories (Cortazzi, 1993). Teachers' narratives have been used in teacher education and teacher development in various context such as Carter (1993), Clandinin and Connelly (1996), Cortazzi (1993), Doyle and Carter (2003), Elbaz (1991). "Researchers have come to appreciate that teachers' stories offer a wealth of information about their individual identities and classroom experiences" (Lloyd, 2006, p. 58).

The stories told by participants during the interviews were analyzed by using narrative analysis method of Labov (1972). According to Labov (1972, 1982) a narrative has a structure and a sequence. If a narrative is fully formed, it has six components; abstract (AB; summary of the narrative), orientation (OR; time, place people etc.), complicating action (CA; sequence, turning points, crisis, content), resolution (RE; resolution of events, crisis), evaluation (EV; interpretation), and coda (CO; narrative ends and turn back to listener). The structure of the narratives, produced by participants, gives insights about how they perceive their experiences in methods course. The order of the components may change, while some of the components may be absent from some stories. The following table provides a summary of components and their definitions.

Table 3-1. Narrative components

Narrative component	Definition
Abstract (AB) (optional)	Summary of the narrative
Orientation (OR)	Time, place, people etc.
Complicating Action (CA)	Sequence, turning points, crisis, content
Evaluation (EV)	Interpretation
Resolution (RE)	Resolution of events, crisis
Coda (CO) (optional)	Narrative ends and turn back to listener

Also, the narrative analysis was an appropriate choice in terms of the theoretical perspective. The constructivist perspective requires reconstructing the participants' own construction of the topic of interest. In this study, narrative analysis was propitious in the pursuit of participants' constructions of geometry learning. Hatch (2002) also addressed that while interviews and observation of participants' natural settings are primary source of data, narratives are one form of the product of a constructivist research. "Knowledge produced within the constructivist paradigm is often presented in the form of case studies or rich narratives that describe the interpretations constructed as part of the research process" (pp. 15-16).

After the completion of interviews, the videotapes of the interviews were converted to digital video segments. The researcher had a chance to listen to the entire interviews while converting them. During the process of conversion, some of the participants' responses took researchers attention and some analysis notes were taken. Transcriptions of the interviews were completed by using the video program, QuickTime with 1/2 x play option. Playing the videos slower was helpful for the researcher who was a second language user of English person.

The process of coding began with reading the transcripts without any coding. The researcher concentrated to understand overall stories of participants. Then, the researcher completed the open coding by taking notes next to the transcriptions. The codes were either the exact phrases of the participants or words to summarize what the participants told.

For analysis of narratives, first the stories told in each interview were marked. Then, the stories were re-transcribed without non-narrative parts of the interviews. Eliminating non-narrative data helped to concentrate on narratives which were then organized as scenes (K-12, college mathematics courses and methods course experiences). Narrative segments were parsed into numbered lines which were divided according to clauses as units of ideas (Reissman, 2000). Discourse markers such as 'and', 'so', 'then', 'if' were helpful in dividing the stories in clauses. Building scenes was followed by coding according to Labov's structure. The three segments of stories were in focus, complicating action, evaluation and resolution because these segments revealed the participants understanding about geometry learning.

In addition to structural analysis of narratives, thematic analysis (Coffey & Atkinson, 1996) was used and the interviews were coded. Literature supports using other analysis methods in addition to narrative analysis, deepened the analysis of the rich data (Lloyd, 2005, 2006; Reissman, 1993, Robichaux, 2002). In addition to the narratives in the interviews, participants provided information about geometry learning and teaching in non-narrative form. The open codes from interviews provided themes to inform the researcher about effective geometry learning experiences for the participants. It is important to note that, particularly in this study, narrative analysis and thematic analysis were complementing each other rather than one of them being the primary analysis method. In order to address the constructivist perspective, focusing on participants' meaning making and their perceptions, both analysis methods have been used.

Development of the Protocol

The protocol of the geometry learning activities for the methods course consists of two parts: geometry activities for the methods course and analyzing student work activities. The geometry activities were developed from the results of the qualitative investigation and resources for methods courses. In addition to content focus with geometry activities, literature supports using activities in which preservice or in-service teachers would practice mathematics in the context of teaching were also used (Ball, 2000). As discussed in the literature review, one of the promising practices is using student work to analyze in teacher education (e.g. Little, 2004). Literature suggestions (E. Kazemi, personal communication, August 17, 2008; Little, 2004) were used to develop activities to analyze student work. Therefore, this section of methodology will provide information on development of the geometry activities, and the process of collection of student work to be used and development of activities for analyzing student work.

Analysis of the textbooks which were often used for mathematics methods courses, the state standards for elementary school geometry, and literature (Fujita & Jones, 2006; Jones 2000) showed the importance of geometry topics for teachers and students. In addition to some discussions on 3-D geometry topics, elementary school geometry education focuses on introduction and mastery of 2-D topics. In 2-D geometry topics polygons especially the quadrilaterals play an important role in developing geometric thinking in elementary school (NCTM, 2006). While in early grades of elementary school the goal is recognition of the shapes (e.g. square and rectangle), the later grades of elementary school encourage students to identify characteristics of the shapes (e.g. square has four congruent sides) and the relationship between them (e.g. square is a special case of rectangle). Another goal of elementary school geometry education is to prepare students for more advanced geometry thinking needed in middle school. The hierarchical relationship within the quadrilaterals prepares elementary school students for the nature of geometric thinking. Therefore, elementary school teachers are expected to possess required knowledge of 2-D shapes, especially quadrilaterals and the relationship between the quadrilaterals (Fujita & Jones, 2006; 2007). In addition to suggestions from literature to study teachers' knowledge of quadrilaterals (Fujita & Jones, 2007), the results of the qualitative investigation indicated preservice teachers' expectancy of studying quadrilaterals in depth during the content and methods courses. Therefore, the mathematical topic of this study is polygons with a focus on quadrilaterals and the relationship between the quadrilaterals.

Geometry Activities

During the development of protocol (Appendix B), the researcher used the results from the qualitative investigation for the first phase, geometry activities on quadrilaterals. The summary of qualitative results which informed the process of development of the protocol would be necessary to report. The results which informed the development of geometry activities are as follows:

- There is a need to address content in addition to pedagogical practices in the methods course.
- Preservice teachers' reported their lack of knowledge in 2-D geometry topics especially in quadrilaterals.
- Preservice teachers stressed that, in methods course, discussion of content before the discussions of pedagogical practices would improve their learning.
- Preservice teachers expressed the importance of the flow of instruction from easier topics to more advanced topics due to various backgrounds among them.
- Preservice teachers addressed the effectiveness of using visual aids such as drawings for their geometry learning.
- Preservice teachers explained that various forms of activities such as small group works in addition to individual work were helpful in their learning.

The synthesis of the results from the qualitative investigation, methods course resources such as Van de Walle (2007), and the literature on preservice teacher education yielded geometry activities on quadrilaterals as an intervention for this study. The activity types were adapted from the methods course textbooks (Van de Walle, 2007) and resource books for teaching geometry (Muschla, 2002). In order to make the activities suitable for using with preservice teachers, they needed to be revised or reformed because most of the activities were from resources for elementary school classrooms. For example, the first activity, sorting shapes was adapted from Van de Walle (2007) with revised questions to answer while sorting shapes and new set of shapes.

There were three groups of activities: sorting shapes, attributes of shapes, and classification of polygons. The first activity was a sorting activity in which the participants (in pairs) sorted 33 cut-out shapes in groups according to their properties (Appendix B). The groups of shapes were concave, convex, hexagons, pentagons, triangles, quadrilateral, kite, trapezoid, parallelogram, rectangle, rhombus, and square (at least three of each category). When the participants were sorting shapes they experienced defining characteristics of the shapes and the relationships between them. As a result of this activity, the participants developed definitions of those shapes, individually.

For the second group of activity, attributes of shapes, participants worked in pairs to study 10 groups of figures (4 figures in each group, Appendix B). The participants were asked to determine which figure in a group did not belong to others. In other words, the participants had to find a figure which did not share the common characteristics with other three figures. Participants were encouraged to find more than one answer for each group. For example, in a group of four figures, figure B did not belong to others because it was concave while figure D did not belong to others because it was not a quadrilateral. The goal of this activity was for preservice teachers to practice the characteristics of shapes in an open-ended problem solving activity while discussing the relationship between the shapes.

For the last group of activities, classification of polygons, the participants worked in small groups to develop a visual representation (Venn diagram) demonstrating the relationships between the polygons especially the quadrilaterals (Appendix B). Participants were given vocabulary (in alphabetical order) to fill the empty spots in the visual representation. The vocabulary were concave, convex, hexagon, kite, parallelogram, pentagon, polygon, quadrilateral, rectangle, rhombus, square, trapezoid and triangle. After the completion of the diagram, participants answered a set of true-false questions based on the Venn diagram (Appendix B). Some of the examples of true-false questions were “All pentagons are regular” and “Only some trapezoids are parallelograms”.

In addition to individual characteristics of the activities, the combination of them provided coherence. Participants worked individually, in pairs and small groups. At the end of the each activity, the facilitator led whole class discussions on the topics while providing the right answers. The participants experienced geometry topics with visual representations such as cut-out shapes. Also, the activities progressed through van Hiele geometric thinking levels (see literature review for van Hiele levels). Participants began with level 0 and level 1 activities (e.g. sorting) and finished with a level 2 activities (e.g. true-false statements). Therefore, the activities reflected suggestions from both literature and qualitative results.

Analyzing Student Work

Kazemi and Franke (2004) suggested that the student work to be used in professional development to improve teachers’ content knowledge should be challenging. In other words, the student work should show wrong student answers and misconceptions in order to intrigue teachers’ discussions on mathematics topics. With this purpose, the researcher collected student work from elementary schools with mathematically struggling students. After the completion of permissions from county school board, principals of three schools were contacted to reach 4th and 5th grade teachers in their schools. Then, the researcher visited one voluntarily participant teacher from each school for this study. Two of the teachers agreed to participate in this study while one opted out. Therefore, the student work for this study was collected from two low-income, mathematically struggling elementary schools in the same city of the university. Another important decision in selection of schools was to choose local schools. The reason to choose student work from local schools was to provide teaching context similar to participants might be teaching.

The researcher visited the participating teachers, Mrs. Brown and Mrs. Smith (pseudonyms), in their classrooms. The teachers requested to conduct this study after the state standardized achievement tests. As a result of collaboration between the researcher and teachers, the geometry worksheet to use in the classrooms was designed. The worksheet consists of open-ended questions for definitions of some geometry shapes and 10 figures to be determined if they are certain quadrilaterals with mathematical explanations (Appendix C). During the process of worksheet development, the researcher also received consultation from an experienced mathematics teacher educator, an experienced mathematics teacher, and a graduate student who was a methods course instructor.

The teachers used the worksheets with their students. The teachers were requested not to interfere with the students thinking by correcting their mistakes. The researcher explained to the teachers that it would be helpful for preservice teachers to study student misconceptions. Student work copies from each classroom were collected and students' names were removed from the worksheets. First the worksheets with unanswered questions were eliminated. Majority of the student worksheets were eliminated because most of the students left unanswered questions. Then, among the fully answered student worksheets, only the ones with explanation for answers were chosen because explanations provide information about students' understanding of geometry topics. To be used in the research, six students' worksheets which were providing most challenging geometry ideas were selected. One of them can be seen in Appendix D as a sample. For example, for definition of a trapezoid, one student wrote "like skirt". The goal of the selection was to choose worksheets of six different students because there were six small groups (4-5 people) of preservice teachers in each class. Therefore, each group would have one student's worksheet and groups would have different students' work. The purpose of providing the same worksheet in a group was to support small group discussions which would improve participants' learning as suggested by the qualitative investigation results. On the other hand, the purpose of providing different student worksheet was to introduce participants to different student thinking and ideas.

The participants were given a protocol to study student work (Appendix E). The protocol was developed by suggestions from several resources (E. Kazemi, personal communication, August 17, 2008; NCTM, 2006). First in pairs, the participants discussed what the student did, what the student knew (and misconceptions), what they would ask the student in order to learn more about the student's knowledge of geometry. Then, in small groups (two pairs), participants discussed what they would do to teach these concepts to the student and how they would address the student misconceptions. The participants recorded their discussions. For the whole class discussion, the facilitator asked participants to share the student work and their discussions on the given questions.

Pilot of activities

The activities for the intervention were piloted in two settings. For the first pilot study, preservice teachers who would take the mathematics methods course the following semester were contacted. This preservice teachers group was chosen because they did not have any mathematics methods course experience, and they were taking the mathematics content course. Four preservice teachers participated in the pilot study. They did geometry activities and analyzed work of a student. The participants provided some feedback which was taken into consideration to make necessary revisions.

After the revisions from the first pilot, the second pilot was conducted. The activities were presented during a state conference for mathematics teachers and mathematics teacher educators (Florida Council of Mathematics Teachers, FCTM). FCTM is affiliated with the leading national organization for mathematics teaching, National Council of Teachers of Mathematics (NCTM). The audience of the presentation consisted of mathematics teachers of various levels and mathematics educators. The audience was informed about the purpose of the presentation; to receive feedback on the protocols for the intervention. The audience experienced the geometry activities and analyzed student work. The feedback from the audience was used to revise the protocol.

Quantitative Investigation

This third section of the methodology chapter will provide information regarding the last phase, the quantitative investigation of this research. Because the intervention was described in detail in previous section, instrumentation, data collection and data analysis will be reported in this section. This quantitative investigation addressed the last two research question:

- Does use of geometry activities focused on quadrilaterals with analysis of student work influence preservice elementary teachers' geometry content knowledge?
- Is there a difference in geometry content knowledge between preservice teachers who are in a traditional mathematics methods course and preservice teachers who are in experimental mathematics methods course?

Settings and Participants

The quantitative investigation was conducted during the fall semester (August-December) of the second year. The settings for the methods course were described above which were similar to settings of the quantitative investigation. It is important to note some differences such as the instructors and the number of sections. There were three instructors for four sections of the methods course in which one hundred and seven students were enrolled and 102 of them volunteered to participate in the study. All the participants were female. Two of the sections were selected as treatment and other two were selected as control groups. Students took courses as cohorts which were decided, without any criteria, by the department. One of the main differences between sections was the time of day the classes met. There were two morning sections and two afternoon sections. In order to eliminate this factor, the treatment and control groups assigned in order to have one morning and one afternoon section in each group. Moreover, the researcher was one of the instructors of an afternoon section which was chosen to be a treatment group. Also, one of the instructors was a faculty member who taught two sections, and his sections had to be assigned for the same group which had to be the control group. The fourth section, second treatment section, was taught by another graduate student. Therefore, in treatment group there were two sections, one morning and one afternoon, which were taught by two graduate students while in control group there were two sections, one morning and one afternoon which were taught by a faculty member.

All the instructors were teaching geometry for two weeks during the last third of the semester. Because the focus of this research was geometry, the intervention had to be conducted in two weeks during the time of geometry instruction of each section. The intervention took 90 minutes (half of one class time) of each geometry week. The first week, the activities were for geometry more specifically quadrilaterals, and for the second week, the activities were analyzing student work.

A precaution to avoid researcher bias was to train another instructor to deliver intervention activities. Emily Peterek, a graduate student who had taught the course for two years had volunteered to assisst. She also received an award from the university for her excellence in teaching. She was not teaching at the time of this study. She had valuable experience with the student population of this course. She was trained for teaching the intervention activities. The researcher was also present in the class during the intervention in case of consultation with activities.

Instrumentation

The instrument was developed by a research group, Learning Mathematics for Teaching (LMT) at University of Michigan. This project can be seen as continuum of research on mathematics knowledge for teaching (MKT) which was discussed in literature review. This instrument, Content Knowledge for Teaching Mathematics Measures (CKT-M Measures)[1], was developed by group of mathematicians, mathematics teachers and mathematics educators in addition the members of the research team that developed MKT (Hill, et al., 2004, Hill et al., 2008). The validity of the items was studied by experts from different backgrounds (Ball et al., 2008; Hill et al., 2004). The test developers designed a workshop addressing use of the test, and the workshop was required to be able to use this test. The researcher attended the workshop to be able to use this instrument. The test developers warned participants not to publish the test items. In order to honor test developers request the instrument could not be added as an appendix to this dissertation, yet released items can be found in Appendix F.

The purpose of this instrument is to "discriminate accurately among teachers, in essence ordering them as correctly as possible relative to one another and to the underlying trait being assessed, mathematical knowledge for teaching" (Hill et al., 2008, p. 131). Another use of this instrument is to measure change in teachers' knowledge as they learn over time. An important characteristic of this instrument is that it does not provide raw scores. In other words, a teacher's score cannot be interpreted as how much the teacher knows. Therefore, the instrument developers strongly warn that this instrument is not suitable for the purpose of individual teacher accountability such as certification or qualification (Hill et al., 2008).

 Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

The intent of using this instrument in this present study was to compare mathematical knowledge of groups (control and treatment) of preservice elementary school teachers and detect geometry knowledge growth of the preservice teachers of the experimental group. The instrument addresses the majority of mathematics topics under three categories: number and operations (K-6 and 6-8), patterns functions and algebra (K-6 and 6-8), and geometry (3-8). For this current study, I used only geometry section of the instrument. Two parallel forms of the geometry section of the test were administered as pre and post test. The pre-test consisted of 19 multiple choice questions in 8 stems. The post-test consisted of 23 multiple choice questions in 8 stems.

The test developers used the item response theory (IRT) to calculate the internal consistency and equivalency of various forms of the measurement. First, ORDFAC, the factor analysis methods was used which was specifically designed for this instrument (Hill et al., 2004). Three-factor solution composed of algebra, numbers and operation, and geometry was reached. Furthermore, the researchers used BILOG (a program for the estimation and testing of IRT models) to fit initial item response theory one-parameter and two-parameter models to the data (Hill, et al., 2004). Rausch model for item response theory was used to study reliability and equate the different forms of the instrument. Also, the test developers provided item characteristics curve for each item in the test.

The test developers conducted pilot testing in California's Mathematics Professional Development Institute during 2001-2003. They calculated reliability separately for three different sections of the test: numbers and operations; patterns, functions, and algebra; and geometry. There were 18 items in numbers and operations and the calculated reliability coefficients were .76 and .79 for one parameter and two parameter respectively (Hill et al., 2004). Patterns, functions, and algebra section had 15 items and it had one parameter reliability of .84, and two-parameter reliability of .87. Finally, geometry section consisted 43 items and this section showed highest reliability by .91 for one-parameter and .92 for two-parameter.

Data Collection and Analysis

Participants completed the CKT-M Measures geometry test one week before their geometry instruction. For next two weeks they received the geometry instruction and the following week they completed the post-test. Both pre and post tests were administered at the beginning of the class. The course instructors were not present during testing or informed consent agreement by participants. In order to protect students' privacy, instructors were not informed about participation of any student from their classroom. Therefore, as it was stressed to students, their participation in this study did not affect their grade in this course.

Table 3-2. The timeline of the study

First Week	Second Week	Third Week	Fourth Week
Pre-test	Geometry Instruction	Geometry Instruction	Post-test

In order to address two research questions, geometry knowledge growth of treatment group and any difference of knowledge growth between treatment and control group, two different analysis methods, repeated measures ANOVA and mixed ANOVA, were used, respectively.

CHAPTER 4
AN ANALYSIS OF PRESERVICE ELEMENTARY TEACHERS' STORIES OF LEARNING GEOMETRY

This chapter is a journal article to be submitted. It encourages further investigations of teachers' mathematics content knowledge through qualitative investigations to deepen our understanding of the concept. Also, this chapter aims to inform teacher educators of preservice teachers' perspectives on their geometry learning and means to improve their content knowledge in teacher education programs.

Christiana was excited to go to her first class in university after transferring from the community college of the same city. She was hopeful in getting the education to be a good teacher. On the way to her mathematics course, she remembered her mathematics teachers throughout her education. She regretted that none of them had inspired her to learn during her high school years. She wanted to have a new start with this university because she cared about her future students from then. She wanted to learn mathematics that she previously avoided, and she wanted to know everything about teaching mathematics to be the good teacher that she never had. She wanted to be a teacher who would catch her students' interests.

Christiana is one of the participants who told her story of learning geometry for the study discussed in this chapter. This chapter is a component of a broader research study which integrated qualitative and quantitative research methods to study preservice elementary teachers' geometry learning and their content knowledge of geometry. The first phase of the study was the qualitative investigation to understand preservice teachers' geometry learning. By studying effective geometry learning experiences of preservice teachers in the qualitative phase, the researcher created a series of activities for a mathematics methods course. The goal of those activities was to improve the geometry content knowledge of preservice teachers. The geometry activities were used as the intervention for the quasi-experimental quantitative phase. In this chapter, the author will report the qualitative investigation on preservice elementary teachers' geometry learning with a focus on their experiences in mathematics methods course.

The significance of this study is its approach to study preservice teachers' geometry content knowledge. In order to develop effective strategies to enhance preservice elementary teachers' geometry content knowledge, first their learning has been investigated. For this constructivist study with the purpose of understanding participants' perspective and needs, the research questions were about preservice elementary teachers' geometry learning in the mathematics methods course. Without proper knowledge of preservice teachers' learning, the efforts in improving the instruction in methods course would not be helpful. Therefore, the goal of this research is to provide policy implications in mathematics teacher education and to inform mathematics teacher education practice.

Review of the Literature

The most commonly accepted definition of teacher knowledge was given by Shulman (1986, 1987), who developed a cognitive model of teacher knowledge. His definition is consisted of three types of teacher knowledge: content knowledge (CK), pedagogical content knowledge (PCK) and curriculum knowledge. CK refers to knowledge base of the content one is teaching, such as mathematics. PCK "goes beyond knowledge of subject matter per se to the dimensions of subject matter knowledge for teaching" (Shulman, 1986, p. 9). PCK is the type of knowledge that distinguishes the work of a teacher from the work of a scientist. A scientist does not have to think about effective teaching strategies of the subject while a teacher cannot depend only on content knowledge. The third knowledge type curriculum knowledge addresses effective use of curriculum materials and teachers' familiarity with other subjects that students study.

Among these knowledge types, content knowledge stands out as a point of interest for teacher education. Brown and Borko (1992) asserted that preservice teachers' limited mathematics content knowledge is an obstacle for their pedagogical training. Also, several studies have shown that lack of content knowledge affects teacher's methods of teaching (e.g. Carpenter, Fennema, Peterson & Carey, 1988; Leinhardt & Smith, 1985). In the project of Cognitively Guided Instruction (CGI), one of the teachers, Ms. Jackson, was identified as expert teacher by researchers (Carpenter et al., 1988). She had extensive background on addition and subtraction but her knowledge of fractions was limited. The observations of her teaching yielded important differences between instruction practices of these two topics. There were less discussion and less mathematics in the classroom when she was teaching fractions than when she was teaching addition and subtraction. Carpenter and his colleagues (1988) emphasized that the content knowledge of a teacher heavily affects the teachers' use of the pedagogical tools.

As the field of mathematics education was affected from Shulman's findings, there were examples of research which followed the inquiry of Shulman's work. The foci were on teachers' understanding rather than their ability to solve problems correctly for particular mathematics topics (Ball, 1988, 1990a, 1990b; Leinhardt and Smith, 1985; Owens, 1987; Post, Harel, Behr, & Lesh, 1988; Steinberg, Haymore, and Marks, 1985).

One of the research groups who has been studying Shulman's model for teachers' knowledge in mathematics is Ball, Bass, Cohen, Hill and others. The early works of Ball focused on studying teachers' mathematics content knowledge from a different approach. She emphasized that teachers need to "unlearn" and "unpack" their mathematics knowledge (Ball, 1988, 1990a, 1990b).

This research team focused on the concept of job analysis for teaching mathematics. The data collection took place in elementary school classrooms, and the researchers collected audiotapes of lessons, students' work, teachers' plans, and teachers' reflections for a year from elementary school classrooms (Ball, 2000). Results of this study informed about how a teacher uses mathematics for teaching, and the researchers developed the conceptual framework of mathematical knowledge for teaching (MKT) (Ball & Bass, 2000a, 2000b, 2003). Ball, Thames and Phelps (2008) defined MKT as mathematical knowledge that teachers need for teaching as this knowledge being different than mathematical knowledge of other professionals such as engineers. "To avoid a strictly reductionist and utilitarian perspective, however, we seek a generous conception of 'need' that allows for the perspective, habits of mind, and appreciation that matter for effective teaching of the discipline" (Ball et al., 2008, p. 399).

According to MKT model, there are six domains of teacher's content knowledge which can be categorized under Shulman's two types of knowledge. There are three domains under subject matter knowledge: common content knowledge (CCK, mathematics knowledge not unique to teaching), specialized content knowledge (SCK, mathematics knowledge unique to teaching), and horizon content knowledge (knowledge of mathematics throughout the curriculum). Also, there are three domains under pedagogical content knowledge: knowledge of content and students (KCS, interaction of knowledge of mathematics and students' mathematical conceptions), knowledge of content and teaching (KCT, interaction of knowledge of mathematics and teaching methods), and knowledge of content and curriculum (interaction of knowledge of mathematics and mathematics curriculum). The relationship between the Shulman's (1986) model for teachers' content knowledge and the MKT model of Ball et al. (2008) can be summarized as in the following table.

Table 4-1. MKT model comparison to Shulman's model

Shulman's Model (1986)	Ball et al. MKT Model (2008)		
Content Knowledge	Common Content Knowledge	Specialized Content Knowledge	Horizon Content Knowledge
Pedagogical Content Knowledge	Knowledge of Content and Students	Knowledge of Content and Teaching	Knowledge of Content and Curriculum

In the research of teachers' mathematics content knowledge, several mathematics topics has been addressed. In a study of Borko, Eisenhart, Brown, Underhill, Jones and Agard (1992), the team of researchers studied middle school preservice teachers' content knowledge. The authors reported results from one student teacher, Ms. Daniels, in fraction division. The researchers interviewed the participant, investigated the methods course that she had completed. Ms. Daniels who had taken advanced mathematics courses in college and the mathematics methods course before being a student teacher could not answer her students' questions about fraction division. She was not able to provide an explanation for fraction division algorithm.

There are several studies on teachers' knowledge of mathematics focused on topics such as fractions (Carpenter, Fennema, & Franke, 1996; Carpenter et al., 1988) or numbers and operations (Ball, 1990; Ma, 1999). For example, the comparative study

(Ma, 1999) of Chinese and the U.S. on elementary school teachers' understanding of three topics in mathematics (division, place value and area-perimeter relationship) garnered attention from mathematics teacher educators. The results were groundbreaking because in spite of advantage of higher education and advanced mathematics courses, American teachers did not have the deep mathematical understanding that Chinese teachers had. Chinese teachers did not have the same level of higher education yet they had more experience with mathematics learning practices in the classroom. Their learning was tailored for teaching rather than advanced degrees in mathematics. The results revealed that higher education mathematics courses were not enough to make sure that teachers have quality mathematics knowledge for teaching.

In spite of the general interest in teachers' mathematics content knowledge in topics such as fractions or place value, there is a limited number of research projects on knowledge of geometry for teaching. The results of those studies reflect that especially beginning teachers are not equipped with necessary content and pedagogical content knowledge of geometry, and it is important to address it in teacher education (Jones, 2000; Swafford, Jones, & Thornton, 1997).

Therefore, this study is an effort to improve the mathematics teacher educators' understanding of preservice teachers' perspective in geometry learning and teaching. This study's most important characteristic is to understand preservice teachers' needs and strengths from their perspective in order to address their geometry learning needs to enhance their geometry content knowledge. Therefore, this study strives to investigate the following research questions by addressing preservice teachers' stories in elementary mathematics methods course from the constructivist perspective:

- What is preservice elementary teachers' understanding of geometry in elementary school?
- What are the perceptions of preservice elementary teachers on effective instructional strategies to promote their knowledge of geometry in mathematics methods courses?

Research Methods

The theoretical perspective of this study is constructivism. Hatch (2002) addressed the quest of a constructivist researcher as "individual constructions of reality compose the knowledge of interest to constructivist researcher" (p.15). In this dissertation study, the goal of the researcher was to study preservice teachers' geometry knowledge. Therefore, first, the researcher began by listening preservice teachers to understand their construction of geometry learning and means to improve their geometry content knowledge. It was necessary to address preservice teachers' constructions of geometry learning in order to be able to be able to develop a protocol to improve their geometry content knowledge.

One year before the current study, a qualitative pilot study was conducted for which individual interviews were used. The purpose of this exploratory study was geometry knowledge of preservice teachers. Two participants from an elementary mathematics methods course were interviewed and artifacts from their classrooms (such as geometry related parts of weekly journals) were collected. The planned analysis method was thematic analysis but the analysis of interviews showed great use of stories of preservice teachers. Most importantly, those stories were focused on participants' learning experiences rather than their knowledge. Therefore, the pilot study informed the actual study in terms of two topics: revision of research questions (focus on learning experiences rather than their knowledge) and revision of research methodology (narrative analysis and thematic analysis).

Settings and Participants

This study was conducted in mathematics methods course at a large southeastern research university for elementary school teachers who were predominantly middle-class, white, female students. Students complete approximately two years of education before starting in college of education for the teacher education program. Students begin their unified elementary education program in their junior year and usually they take the methods course in their senior year. This course plays an important role in preservice teachers' education because it is the only mathematics methods course in the program. Preservice teachers may prefer to continue their education for master's degree for which they choose a major (e.g. mathematics/science, special education). Only the preservice teachers who choose mathematics/science as a major would have to take more mathematics methods courses. If a preservice teacher chooses not to complete masters or chooses to study a major other than mathematics/science for master's degree, the preservice teacher would not take any other mathematics methods course. The methods course, the interest of this research, is the last and the only mathematics methods course for many of the preservice elementary teachers.

During the teacher education program, elementary preservice teachers are required to take three mathematics courses, two elective (e.g. calculus) and one content course, before the mathematics methods course. The mathematics content course addresses mathematics concepts for elementary school level whereas the mathematics methods course is designed to build the future teachers' pedagogical tools for teaching mathematics. Even though the recommendation of this order is given, some students take methods course and the content course at the same time or some students take methods course before the content course. Therefore, the methods course instructors are concerned about mathematics content knowledge readiness of their students.

The goal of the qualitative component was to understand preservice teachers' geometry learning especially in methods courses. The focus was the methods course because of its importance. Also, the results of this study would inform the researcher to develop geometry activities for methods course to be used in the second phase of the study. Three participants (volunteers), Christiana, Emma and Liz (pseudonyms), were preservice elementary school teachers who were enrolled in the methods course. There was one participant from each section of the course. In this study, only Liz took the content course before methods course. The other two participants, Christiana and Emma were planning to take it the following semester.

Data Sources

The data collection methods included individual interviews with one participant from each of the three sections of the course, observations of geometry instruction in each section (two weeks), and the collection of materials used during the geometry instructions. All three sections of the course were observed during the geometry instruction. Field notes were taken during the observations. Also, copies of the instruction materials (handouts and transparencies) and student presentations were collected. The observation and artifact data were not used for the data analysis purpose. The primary purpose of the observations and the artifact collection was to capture content preparation for the geometry learning process of preservice elementary teachers in order to provide triangulation for the interview data and to support the interview data.

The primary data source for this study was individual interviews. The purpose of the interviews was to understand preservice elementary teachers' stories of learning geometry (before college, in mathematics courses in the college and especially in the methods course). The interviews were conducted after the participants received geometry instruction in methods course. There was one volunteer from each section for a total of three. The interviews were 45-60 minutes long and video recorded.

The interview protocol was designed for semi-structured and open-ended interviews. The narrative interviews are tailored to intrigue story telling from participants through open-ended questions or probes (Reissman, 1993, 2000). The mostly suggested narrative interview probes are "Tell me about" (Reissman, 1993, 2000). For this study, some of the interview questions were "Tell me about your geometry learning before college" or "Tell me about geometry instruction in methods course". Another important feature of narrative interviews is that the researcher accepts the role of the participant as the leading role in the interviews because the participant is the knowledge holder (Bruner, 1990; Reissman, 2000). Therefore, narrative analysis and especially the narrative interviews align with the constructivist research design in which participants are the meaning makers.

Data Analysis

The data analysis in this study was focused on participants' experiences of geometry learning. The interviews, the source of the data analysis, were analyzed for both narratives and non-narrative forms. The stories of preservice teachers analyzed structurally (Labov, 1972) in addition to thematic analysis of both narrative and non-narrative data (Coffey & Atkinson, 1996).

Individuals may use narratives for meaning making in addition to using them for sharing their experiences in stories (McAdams, 1993; Riessman, 1993). Grbich (2007) identified research settings which might be addressed by narrative analysis “those that explore either the structure of narratives or the specific experiences of particular events, e.g. marriage breakdown; finding out information which is life changing; undergoing social/medical procedures; or participating in particular programmes” (p. 124). In the case of teacher learning, narrative analysis may be used to study professional development experiences of in-service teachers or preservice teachers in teacher education programs. Also, literature suggests that teachers may prefer to discuss their learning and their knowledge through stories (Cortazzi, 1993). Teachers’ narratives have been used in teacher education and teacher development in various context such as Carter (1993), Clandinin and Connelly (1996), Cortazzi (1993), Doyle and Carter (2003), Elbaz (1991). “Researchers have come to appreciate that teachers’ stories offer a wealth of information about their individual identities and classroom experiences” (Lloyd, 2006, p. 58).

The stories told by participants during the interviews were analyzed by using narrative analysis method of Labov (1972). According to Labov (1972, 1982) a narrative has a structure and a sequence. If a narrative is fully formed, it has six components; abstract (AB; summary of the narrative), orientation (OR; time, place people etc.), complicating action (CA; sequence, turning points, crisis, content), resolution (RE; resolution of events, crisis), evaluation (EV; interpretation), and coda (CO; narrative ends and turn back to listener). The structure of the narratives, produced by participants, gives insights about how they perceive their experiences in methods course. The order of the components may change, while some of the components may be absent from some stories. Table 4.2 provides a summary of components and their definitions.

Table 4-2. An example of narrative coding

this is really where it gets tricky I did not like the teacher (.) I don't think she (.) taught the class very well (.)	AB
she already had a notebook of notes you have for the rest of the year and she followed it very strictly and	OR
if you would ask a question she would just say either come and see me after class or she would like no its right there you are supposed to get it and	CA
she kept going on so our questions were unanswered and	RE
I really didn't like that and she just she just didn't have a lot of patience and	EV
also the class was at 7: 25 in the morning so students are already kind of have to get up early and	OR
that kind of attitude did not help a lot and (.)	EV
she didn't answer e-mail	AB
she said she didn't have any e-mail so	OR
if we had a question we had to go to her office hour but	CA
I go to school and I have a job so	OR
I could not get out to go to her office hour so e-mail is good	CA
because she can answer questions in e-mail and satisfy and I really really didn't like that class and	EV
I feel like that's the general consensus	RE

AB: abstract, OR: orientation, CA: complicating action, RE: resolution, EV: evaluation, CO: coda

Also, the narrative analysis was an appropriate choice in terms of the theoretical perspective. The constructivist perspective requires reconstructing the participants' own construction of the topic of interest. In this study, narrative analysis was propitious in the pursuit of participants' constructions of geometry learning. Hatch (2002) also addressed that while interviews and observation of participants' natural settings are primary source of data, narratives are one form of the product of a constructivist research. "Knowledge produced within the constructivist paradigm is often presented in the form of case studies or rich narratives that describe the interpretations constructed as part of the research process" (pp. 15-16).

In addition to structural analysis of narratives, thematic analysis (Coffey & Atkinson, 1996) was used and the whole interviews were coded. Literature supports using other analysis methods in addition to narrative analysis in order to deepen the analysis of the rich data similar to the data of this study (Lloyd, 2005, 2006; Reissman, 1993, Robichaux, 2002). In addition to the narratives in the interviews, participants talked about geometry learning and teaching in non-narrative form. The open codes from interviews yielded in to themes to inform the researcher about effective geometry learning experiences for the participants.

Findings

The findings section is organized as narrative analysis findings and thematic analysis findings. There were two main kinds of stories with sub headings emerged from participants' narratives: stories as a learner and stories as a beginning teacher. The thematic analysis yielded three themes from preservice teachers' geometry learning: history of learning geometry, perceptions about geometry, effective geometry instruction approaches.

Narrative Analysis

The participants told stories about their learning experiences of geometry from two different perspectives, as a learner (K-12 and college mathematics courses) and as a beginning teacher (college mathematics courses and mathematics methods course).

Even though participants experienced the methods course as beginning teachers, all three of the participants emphasized the role of their history of learning geometry as a student on their experiences in the methods course as beginning teachers. Therefore, the stories from both perspectives (learner and beginning teacher) are important to study in order to understand preservice elementary mathematics teachers' geometry learning in mathematics methods course.

The resolution (RE) and evaluation (EV) components of the narratives reflected the focus of the participants as a learner or as a beginning teacher in addition to participants' perceptions about geometry learning. In addition to RE and EV components, the OR component informed the researcher about the settings, time and characteristics of the instructors in the narratives. One interesting result from orientation competent of narratives from all three participants was that all of the narratives were about courses that participants took. The participants did not tell any story outside the formal education environment, even though geometry has strong connection with real life applications. The stories were K-12 education courses, college mathematics courses or mathematics methods course related.

Stories as a learner

The stories of learning geometry with an emphasis as a learner were stressed usually in K-12 education and in college mathematics courses. It is not surprising that their stories as learners from K-12 education because the participants did not know that they would be teachers. For example, Emma mentioned about the geometry course that she took in 9th grade and her perceptions about that class. "*we did I remember making bridges and to see how much weight popsicles sticks with different shapes and angles* *how to build together stuff and I didn't love it (.) I didn't really take another I don't think we really did a lot of geometry*".

On the other hand, for college mathematics courses participants told stories from both perspectives, as a learner and as a beginning teacher. In this section the stories as a learner will be reported and in the next section stories as beginning teacher will be reported. The participants had to take three mathematics courses before taking the mathematics methods course due the requirement of the program. Two of the courses were general mathematics course while the third one was the content course. All three participants told stories from the mathematics courses they took and they expressed that those courses were as a review of their high school knowledge. Only Christiana expressed that the college mathematics course was effective in her learning. Due to her weak mathematics background from high school and community college, she expressed that she learned more mathematics in that college mathematics course than in high school mathematics courses. "*in topics of mathematics it went through everything it went through like statistics geometry algebra stuff that I never herd of truth tables*".

The stories told about the mathematics content course for elementary school teachers is limited because only one participant, Liz, took the course before the methods course. The stories of Liz from that course reflected her concerns about the limited mathematics learning and through the absence of the connection of that course to her teaching career. Liz was concerned that she could not learn enough, and her story of geometry learning in that class expressed that the content was confusing for her. "*we reviewed the properties of parallelograms what makes them rhombus and stuff a drawing of each of these things but she really lightly touched on them like on their* *characteristics she did not spend a lot of time on talking about distinctions so sometimes we would be confused wait so is this this (emphasized) or is this that (emphasized) she goes like that its that and just keep going and so its never stop I didn't get it*".

In spite of her focus in methods course as a beginning teacher Liz expressed that her experiences as a student in the methods course was more effective than the content course for learning mathematics. "*even if math was challenging she [methods course instructor] makes it so that get it and she would go back and explain it in other way…what I like this class a lot better than 3811 [the content course] I like concrete models and I like different ways of looking at the same thing*".

Stories as a beginning teacher

Since the participants took their college mathematics courses after they decided to be teacher, they had the consciousness about learning mathematics in those courses as a teacher. The beginning teacher aspect, being able to relate college education into elementary classroom teaching, was briefly expressed in the narratives from mathematics courses. An example of the beginning teacher aspect is Liz's perspective on mathematics content course. Even though her priority in that course was to learn mathematics as a student, she had thoughts about ways to transfer the presented knowledge into her teaching. This was another frustration for her. "*we would do a lattice addition and multiplication and to me that was confusing I don't know if I would wanna go teach the kids that specific method so it was hard*".

Most of the stories as a beginning teacher took place in methods course. Only one participant (Liz) was satisfied with her learning in the methods course. The other two participants expressed their frustration as the lack of the mathematical discussions and connection between content and the teaching methods (Emma), and the misguided flow of the course by moving to the more difficult topics before discussing easier topics (Christiana).

Liz expressed that she could transfer the knowledge that she learned in methods course into the teaching mathematics for elementary school students. She was very impressed by the structural flow of the course as being able to move from mathematics activities to discussion of how to incorporate those activities into the classroom. One of the activities that the instructor used was warm-up mathematics activities. Liz told the story of reading a children's book as a warm up activity for geometry class. The book, Sir Cumference (Neuschwander, 1997), was to teach concept of circle and vocabulary related to circle. "*she read the story even if we were not kids we could still relate to it so that was kind of the warm up she did*". The flow of the content during a class was easily transitioned for students. The instructor first discussed geometry content through use of several activities such as making shapes with geoboards and doing shape-sorting activity. As the class progresses, the instructor provided discussions on how to implement activities in the elementary school classroom. Moreover, the order of the activities was important as they got more advanced in terms of the content knowledge.

On the other hand, Christiana stressed her difficulty in the class due to lack of discussion on easier geometry topics (shapes and simpler vocabulary) before doing activities with more advanced topics (3-D shapes such as polyhedra and related vocabulary). Even though 3-D shapes are not thought as advanced topics in geometry, Christiana had difficulty understanding those concepts. "*I think more complex level of geometry is definitely good to teach in college courses but I think you have to start at* *the basics because not everybody is on the same page*”. As the order of topics discussed was a concern for Christiana, Emma’s concern was the lack of connection between mathematics topics and teaching methods. She expressed that she gathered valuable activities to use in the classroom however she never experienced discussions on those activities. “*I prefer to like do some of the mathematics problems and then learn hands on kind of things and have her explain like why she taught us that way or why she did certain things specific*”.

Thematic Analysis

The narrative analysis was not enough to address the richness of the data in order to investigate preservice teachers’ geometry learning in mathematics methods course. From the thematic analyses, three themes, *history of geometry learning, perceptions about geometry, effective geometry instructional approaches* were emerged. It is important to note that, even though narrative analysis and thematic analysis results are reported separately, they are embedded in each other. Two different kinds of narratives are present in all three themes partly which may be partly because of using both narrative and non-narrative data for thematic analysis. For example, there are both

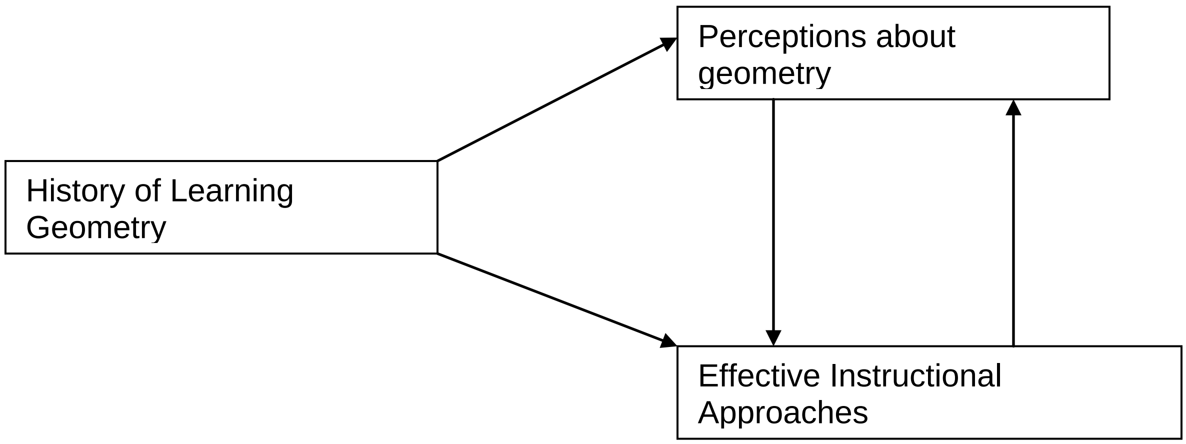

Figure 4-1. Thematic analysis results

stories as learners and as beginning teacher for participants' perceptions about geometry.

History of geometry learning

Preservice teachers bring their perceptions, beliefs and learning experiences into the teacher education programs. All three of the participants mentioned how they learned geometry and their teachers especially before college. Their background in geometry played very important role in their learning in college courses especially the methods course. All of them stressed the emphasis on algebraic topics in K-12 education with limited learning of geometry. They took one geometry course in high school, and they all expressed being dissatisfied with it. Emma expressed that even though her teacher was "*the easy teacher*" and the teacher did "*fun activities*" she did not like the course. When she was asked about the reasons that why she did not like class, she expressed that there are more characteristics of a course than having fun to make it effective. "*I think there is more if you like the teacher the courses is better but if the course is good on its own you don't really have to have a good teacher but if you don't like the teacher and the course was bad that's bad all around*". According to Emma, neither the geometry course nor her teacher was effective. Emma brought her geometry perceptions into the methods course, and she expected the instructor to be able to provide content discussions in addition to pedagogical preparation.

Another aspect of participants' history of learning geometry is the focus on algebraic topics in K-12 education. They all perceive geometry as being different than mathematics because they have the perception of mathematics as algebraic topics. Christiana stated that "*I didn't have any clue about geometry [in high school] and then I went to community college and I had to take intro to algebra and then college algebra* *so it was back to algebra again which algebra is pretty easy I started doing algebra 7th grade middle school so I didn't even think I had to touch*". Furthermore, when participants were asked about effective geometry instruction methods they expressed that effective practices for geometry were different than for the ones for other topics of mathematics. Participants perceived geometry learning different than learning algebraic topics. They preferred to have more real life examples and visual representations for geometry while for other topics learning the formula through direct instruction would be enough.

Perceptions about geometry

All the participants recognized the importance of visualization in geometry. Participants expressed geometry as a study of shapes and measurement features related to the shapes (such as area). Indeed, the participants gave only 2-D shapes rather than 3-D shapes as examples in. For example Christiana thought 3-D geometry as an advanced topic. Some other important topics of geometry such as transformation were not mentioned by any of the participants. Their limited experiences with geometry resulted in distorted perception of geometry. "*for me geometry is basically studying shapes and dimensions and how things fit in things that what I think about geometry*" (Liz).

Effective geometry instructional approaches

The participants addressed the practices and activities which helped their understanding and learning of geometry especially in methods course. The mostly emphasized instruction approach was addressing geometry topics for elementary school (content) before studying instructional aspects of those topics (pedagogical content). Participants stressed their need to study the concepts first in order to be able to understand pedagogical aspects of the topics. Even tough, participants perceived college mathematics courses as reviews before the methods course, because those reviews did not provide desired in-depth geometry understanding for elementary school, they were expecting more content preparation from methods course. As addressed before, only Liz was satisfied from the methods course in terms of experiencing both content and pedagogical content preparation. She experienced "*understanding how a child would see it a child cannot grasp this way but he can understand that way*".

All three of the participants addressed practicing content before the pedagogical aspects of geometry. Especially Emma emphasized content preparation because she thought the pedagogical preparation effective yet she had difficulty to grasp the ideas. Emma stated that she could not relate to the activities for elementary school classroom because they discussed only the pedagogical aspect of the activities. "*she [the instructor] gave us a lot of tricks and fun activities and then she actually taught well but she is still I guess like besides that it was more like stuff to do in your class we never actually did mathematics problems I prefer to like do some of the mathematics problems and then learn hands on kind of things and have her explain like why she taught us that way or why she did certain things specific*". She wanted to be able experience the activities as her students in order to be able to understand students process of learning. Even though Christiana experienced content discussions she could not relate the geometry activities to the pedagogical skills. "*we went through a lot of example we used a lot of manipulatives but I don't know a lot of time that's like how to use that in classroom how is this gonna help for future instruction*".

The second aspect of content preparation in the methods course was to progress from easier to more difficult topics in geometry. Christiana's instructor was providing content preparation before the pedagogical discussions, yet she stated that the instruction was not effective in her learning because the discussed geometry topics were advanced for her. All three participants expressed the need to study basic geometry topics (such as 2-D shapes) before advanced geometry topics (such as 3-D shapes) because they were aware of their limited knowledge of geometry. Christiana especially felt the disproportion because of her limited geometry background. *"[talking about polyhedra and vocabulary for 3-D shapes] I think this is what we went over and that's things I never heard before … I learned new words like I never heard hexahedrons stuff and I didn't even know what was it six sides 3-D shape never heard some of this stuff in my other geometry class*". Then she stressed the importance of starting from basic in order to address students from different background.

In addition to content preparation in the methods course, the participants addressed some instructional practices that were helpful in their geometry learning. The highly stressed feature of an effective geometry instruction was the use of visual aids such as drawing on the board or on the overhead projector, using of manipulatives such as geoboard. All three of the participants mentioned how visual drawings helped them in their geometry learning before the methods class. In the methods course, they experienced geometry manipulatives more than drawings. Especially Liz was very glad to be introduced to the manipulatives in teaching geometry. "*she [the instructor] had the geoboards with rubber band those are really good way of thinking of simpler shapes*".

Another effective instructional practice emphasized by all three of the participants was working in groups. They addressed the supportive feature of group work in classroom activities. Students in groups would explain some topics to each other without asking the instructor. Due to her difficulties with content, Christiana was receiving help from her group members. She could not direct her questions to the instructor so she expressed that "*we do a lot of group work and so there is a lot of interaction going on and that's really helpful*".

Discussion and Implications

The findings of this research inform mathematics teacher educators on several important issues in preservice teacher education. The most important result of this study is preservice elementary teachers' lack of geometry knowledge as reported by them. All the participants were very enthusiastic in teaching in elementary school. They all stressed the professionalism in teaching that one needs teacher education to be able to an effective teacher. They all favor hands-on and meaningful teaching in mathematics. However, they still felt that they were not ready to teach mathematics in elementary school. They expressed that they need to learn more before they began teaching. Good intentions are not enough to be good teachers (Borko et al., 1992). Often teacher education programs do not support preservice teachers in their learning in order to transform them to knowledgeable teachers.

Preservice teachers were aware of their lack of content knowledge which in turn affected their learning pedagogical aspects of teaching (Fennema & Franke, 1992). Even though preservice teachers should have been prepared content wise before the methods course, many of them were not equipped with enough content knowledge to focus on pedagogical content preparation. They stressed that content prepation before the methods course was not addressing in-depth understanding for elementary mathematics (Ball et al., 2008). As preservice teachers come from different backgrounds (e.g. Christina from community college) they may require further content preparation.

The methods course for elementary preservice teachers should provide content knowledge in addition to the pedagogical content knowledge. Even though methods course instructors addressed content, they used different instructional approaches. Among the three participants only one of them reported receiving an effective integration of content and pedagogy preparation in the methods course. The results of this study stress two important characteristics of studying mathematics content in methods course. First, the mathematics topics should be accessible to the preservice teachers. The difficulty of mathematics topics should be from easier to the more advanced topics. The teacher educators should and should aim to address the diverse mathematical background that the preservice teachers bring in the classroom. The second characteristic of an effective content preparation in a methods course is to provide the content with the pedagogical aspects. Participants were aware of that the primary purpose of methods course is not to teach mathematics, but to concentrate on teaching. However, without any content discussion the preservice teachers were having trouble relating to the pedagogical examples.

It is important to note that the type of content knowledge that has been asked by preservice teachers was not college level mathematics, but mathematics that they would be teaching. They did not feel confident about knowing elementary school mathematics for teaching it meaningfully. This type of knowledge is the type of content knowledge that Ball et al. (2008) called as specialized content knowledge (SCK). In studying SCK, Ball et al. (2008) stressed the importance of using mathematics in the context of teaching because SCK is the mathematics knowledge for only teachers to use it in teaching.

In terms of geometry content knowledge, the results showed preservice teachers' limited experiences with geometry in both K-12 education and in college education. Jones (2000) stressed that teachers' lack of geometry knowledge is partly because of their limited experiences in high school. Their understanding of geometry was limited with shapes and measurement aspects of shapes. Preservice teachers could not identify other important topics of geometry such as transformation, and symmetry. Even in the study of shapes, they were having trouble to learn classification of quadrilaterals. Preservice teachers recognized geometry as an important topic of mathematics to teach in elementary school, yet they were afraid of teaching it due to their lack of knowledge. They were planning to learn geometry and teaching geometry from experienced teachers in their schools after they would begin teaching.

Teacher educators should address the content needs of preservice teachers not only in content courses, but also in methods courses too. In methods courses, it is important to discuss content in the context of teaching (Ball et al., 2008). In terms of geometry knowledge, preservice teachers' understanding differs from their understanding of algebraic topics. Compared to their algebra experiences, they have very limited experiences with geometry which results in limited geometry knowledge. Not only content courses but also the methods courses in teacher preparation programs should address geometry content knowledge.

CHAPTER 5
PRESERVICE ELEMENTARY TEACHERS' GEOMETRY CONTENT KNOWLEDGE: IMPACT OF USING GEOMETRY LEARNING ACTIVITIES FOCUSED ON QUADRILATERALS WITH ANALYSIS OF STUDENT WORK

This chapter is a journal article to be submitted. It reports the second phase of the research, impact of using the protocol developed as a synthesis of qualitative results and the literature. This chapter informs the mathematics teacher education community in transferring research into practice as the previous qualitative research informed the researcher into development of the protocol to be used in the quantitative investigation, the object of this chapter.

Teachers tend to teach the way they were taught (Schoenfeld, 1988). Their knowledge of learning to teach starts from their experiences as learners. However, teachers begin their formal education on learning to teach in teacher education programs in those programs they may experience changes in the way they learned about teaching. As they begin their profession in classrooms, they continue learning through their experiences in the classroom. The teacher education programs plays very important role in this process of learning to teach due to being the first formal step of preparation and in some cases the last formal education besides any possible professional development workshops.

The study of Ferguson (1991) showed that for more than 1000 school districts, spending additional dollars on more highly qualified teachers resulted in greater improvements in student achievement than did any other use of school resources. However quality teaching requires professionalism in a unique body of knowledge for teaching. Teacher knowledge is one of the most important components of teacher quality. The content knowledge of a teacher strongly impacts the enactment of pedagogical tools of the teacher. Brown and Borko (1992) asserted that preservice teachers' limited mathematics content knowledge is an obstacle for their training on pedagogical knowledge. Preservice teachers usually have good intentions such as applying meaningful learning in their class. However, good intentions are not enough to be good teachers (Borko, Eisenhart, Brown, Underhill, Jones and Agard, 1992). Often teacher education programs do not support preservice teachers in their learning in order to transform them to knowledgeable teachers.

The goal of this chapter is to present a quasi-experimental study to address geometry content knowledge of preservice elementary school teachers. The researcher developed a protocol to use in elementary mathematics methods course. The protocol was consisted of two components: geometry activities for teachers and set of activities to analyze student work. An earlier research, a qualitative investigation on preservice teachers' geometry learning in methods course (Aslan-Tutak, 2009) has been used to develop the protocol. The protocol was administered to the treatment groups of a methods course at a large south eastern public university. The knowledge growth of treatment group participants and the difference between knowledge growth of control and treatment group participants were studied. This chapter will provide discussions on the development of the protocol and research results of using the protocol in order to provide policy implications in mathematics teacher education and to inform mathematics teacher education practice.

Review of the Literature

The most recent and comprehensive national report for government on mathematics education, Foundation for Success, addressed teachers' mathematics knowledge in great detail (The National Mathematics Advisory Panel, 2008). The report highlighted the lack of rigorous research to show the importance and complexity of teachers’ content knowledge. There were recommendations given such as developing a reliable and valid measure for teachers’ mathematics knowledge, addressing the mathematics content knowledge preparation of teachers with emphasis on in-depth understanding of school mathematics, and high-quality research projects to develop understanding of teachers’ mathematics knowledge.

In efforts to study teachers’ content knowledge, the purpose of the early studies was to develop a definition for the concept. Shulman (1986, 1987) developed a model for teacher knowledge. This model has influenced mathematics education as it affected research in the teacher education. Shulman (1986) proposed three types of teachers’ content knowledge: subject matter knowledge, which he called as content knowledge (CK), pedagogical content knowledge (PCK) and curriculum knowledge. Shulman’s work garnered attention because he suggesting studying teachers’ content knowledge and pedagogical knowledge together. In the case of mathematics, CK refers to mathematics knowledge of the teacher. He also stated that CK is not mere knowledge of mathematics but also knowledge of mathematics of the classrooms. On the other hand, PCK is unique knowledge of mathematics for teaching that a scientist does not need to possess. Effective teaching strategies of the subject are not concerns of a scientist, but a teacher needs to know how to choose helpful examples to discuss a topic in addition to knowledge of the topic. The curriculum knowledge addresses effective use of curriculum materials and teachers’ familiarity with other subjects that students study.

Among these knowledge types, content knowledge stands out as a point of interest for teacher education. In a study of Borko et al. (1992), the team of researchers studied middle school preservice teachers' content knowledge. The authors reported results from one student teacher, Ms. Daniels, in fraction division. Ms. Daniels who had taken advanced mathematics courses in college, could not answer her students' questions about fraction division. Several studies have shown that lack of content knowledge affects teacher's methods of teaching (Carpenter, Fennema, Peterson & Carey, 1988; Leinhardt & Smith, 1985). Brown and Borko (1992) asserted that preservice teachers' limited mathematical content knowledge is an obstacle for their training on pedagogical knowledge.

The foci of research in mathematics teacher education were on teachers' understanding instead of their ability to respond correctly to mathematics questions for particular mathematics topics (Ball, 1988, 1990a, 1990b; Leinhardt and Smith, 1985; Owens, 1987; Post, Harel, Behr, & Lesh, 1988; Steinberg, Haymore, and Marks, 1985). One of the research groups in mathematics education which were influenced by the Shulman's work on teachers' content knowledge was the group of Ball, Bass, Cohen, Hill and others. Ball stressed the importance of studying teachers' mathematics content knowledge from a different approach that teachers needed to "unlearn" and "unpack" their mathematics knowledge (Ball, 1988, 1990a, 1990b). The research team focused on the concept of job analysis for teaching mathematics. The job analysis of mathematics teaching yielded into the conceptual framework of mathematical knowledge for teaching (MKT) (Ball & Bass, 2000a, 2000b, 2003). Ball, Thames and Phelps (2008) defined MKT as mathematical knowledge that teachers need for teaching as this knowledge being different than mathematical knowledge of other professionals such as engineers. “To avoid a strictly reductionist and utilitarian perspective, however, we seek a generous conception of ‘need’ that allows for the perspective, habits of mind, and appreciation that matter for effective teaching of the discipline” (Ball et al., 2008, p. 399).

Ball et al. (2008) identified six domains of teacher’s content knowledge. The researchers organized the domains according to their relationship to the Shulman’s model of teachers’ content knowledge. Common content knowledge (CCK, mathematics knowledge not unique to teaching), specialized content knowledge (SCK, mathematics knowledge unique to teaching), and horizon content knowledge (knowledge of mathematics throughout the curriculum) were listed as three domains of the CK, and knowledge of content and students (KCS, interaction of knowledge of mathematics and students’ mathematical conceptions), knowledge of content and teaching (KCT, interaction of knowledge of mathematics and teaching methods), and knowledge of content and curriculum (interaction of knowledge of mathematics and mathematics curriculum) were listed as three domains of the PCK.

Table 5-1. MKT model comparison to Shulman’s model

Shulman’s Model (1986)	Ball et al. MKT Model (2008)		
Content Knowledge	Common Content Knowledge	Specialized Content Knowledge	Horizon Content Knowledge
Pedagogical Content Knowledge	Knowledge of Content and Students	Knowledge of Content and Teaching	Knowledge of Content and Curriculum

In content knowledge of teachers, SCK took attention of the researchers because this domain of knowledge requires only mathematics knowledge but not knowledge of students or teaching. “What caught us[authors] by surprise, however, was how much special mathematical knowledge was required, even in many everyday tasks of teaching – assigning student work, listening to student talk, grading or commenting on student work" (p. 398). In spite of the heavy use of SCK in teaching settings, researchers proposed that this domain of teachers' knowledge needs to be studied further in order to understand the concept of teachers' knowledge. Ball and others suggested addressing this type of knowledge in teacher education in order to improve teachers' mathematics content knowledge for teaching.

There are several studies on teachers' knowledge of mathematics focused on topics such as fractions (Carpenter, Fennema, & Franke, 1996; Carpenter et al., 1988) or numbers and operations (Ball, 1990; Ma, 1999). For example, the comparative study of Chinese and the U.S. elementary school teachers' understanding of three topics in mathematics: division, place value and area-perimeter relationship (Ma, 1999) garnered attention from mathematics teacher educators. The results were groundbreaking because in spite of advantage of higher education and advanced mathematics courses, American teachers did not have the deep mathematical understanding that Chinese teachers possessed. Chinese teachers did not receive the same level of higher education yet they had more experience with mathematics teaching practices in the classroom. Their learning was tailored for teaching rather than advanced degrees in mathematics. The results revealed that higher education mathematics courses were not enough to make sure that teachers have quality mathematics knowledge for teaching.

Geometry Content Knowledge of Teachers

In spite of the general interest in teachers' mathematics content knowledge in topics such as fractions or place value, there is a limited number of research projects on knowledge of geometry for teaching. The results of those studies reflect that especially beginning teachers are not equipped with necessary content and pedagogical content knowledge of geometry, and it is important to address it in teacher education (Jones, 2000; Swafford, Jones, & Thornton, 1997).

An example for professional development projects for geometry teaching is Fostering Geometric Thinking (FGT) (Driscoll, Egan, Dimatteo, & Nikula, 2009). In this project, the researchers first developed 20 professional development sessions for middle school and high school geometry topics. The design of the professional development had three sections. First, the participating teachers engage in the given geometry problem, later they use that problem in their classroom. After they collect their students' work on the problem, the professional development participants would meet again to analyze student work and to reflect on them. After the completion of professional development, in the field test, the research team studied treatment (15 facilitators and 117 teachers) and control groups (13 facilitator and 104 teachers) (J. Nikula, personal communication, May 21, 2009). The geometry content knowledge of teachers was measured by a geometry survey which was consisted of multiple choice geometry problems, open-ended questions on problem solving strategies and analysis of transcribed lessons. This geometry survey was used as pre and post test and the results showed significant improvement in content knowledge of participating teachers.

In another study, with 49 in-service teachers for grades 4-9, a group of researchers looked for change into the geometry content knowledge and van Hiele cognitive levels of teachers after a summer program for 4 weeks (Swafford, Jones & Thornton, 1997). The researchers continued to investigate participating teachers' geometry knowledge as a follow up study with 8 of the teachers by observations and stimulated recall interviews. The aim of this research study was to combine the Cognitively Guided Instruction approach (knowledge of student thinking) and teachers' content knowledge for geometry teaching. During the intervention program, the participants engaged in problem solving and hands on geometry activities for two and three dimensional explorations. The results showed increase in both teachers' content knowledge and their van Hiele geometric thinking levels. The researchers reported a significant gain in geometry knowledge for teachers especially 4th and 5th grade teachers. Furthermore, according to the pre and post test results for van Hiele levels, 72% of the teachers had at least one level increase while 50% of them increased for two levels.

As these two examples reflect, the focus of teacher education research in geometry is on middle and high school grades. However, closer analysis of geometry topics in Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence (NCTM, 2006) stresses the importance of students' experiences with geometry in early grades on to their preparedness for secondary level geometry learning. Furthermore, "students should enter high school understanding the properties of, and relationship among, basic geometric shapes" (NCTM, 2000, p. 310). Therefore, the elementary school teachers should possess required geometry knowledge to prepare students for more advanced geometric thinking. If elementary school teachers lack the necessary knowledge of geometry to prepare students for higher level of geometry thinking, students would enter secondary level grades with a limited geometry knowledge that would result in rote memorization of geometry without meaningful learning (van Hiele, 1999). Even though the emphasis in geometry education is on upper level grades, the mathematics education community needs to study teachers' geometry content knowledge at the elementary level too.

Many other research projects that were focused on knowledge of geometry for teaching emphasized the lack of geometry content knowledge of teachers, especially for beginning teachers (Barrantes & Blanco, 2006; Chinnappan, Nason, & Lawson, 1996; Jacobson & Lehrer, 2000; Lampert, 1988; Leikin, Berman, & Zaslavsky, 2000). "Teachers are expected to teach geometry when they are likely to have done little geometry themselves since they were in secondary school, and possible little even then" (Jones, 2000, p. 110). Therefore, with consideration of new understanding in teachers' mathematics content knowledge, the mathematics teacher education community needs to study not only middle and high school teachers' geometry content knowledge but also elementary school teachers too.

Using Student Work to Study Teachers' Content Knowledge

Using student work has been widely accepted by teacher educators to improve teacher learning and instructional practices (Lampert & Ball, 1998; Little, 2004; Smith 2003). Analysis of student work addresses SCK by making participants to focus on mathematics in the classroom while it still combines instructional elements (Kazemi & Franke, 2003). Kazemi and Franke discussed that by analyzing student work, teachers may be forced to think deeply and elaborate on mathematics knowledge while they are trying to understand what students did. "Making sense of children's strategies could be an indirect way for teachers to wrestle with the mathematical issues themselves" (p. 7).

Studies showed that using student work to facilitate teacher learning resulted in teachers' deeper subject matter knowledge and classroom practice (Franke & Kazemi, 2001; Kazemi & Franke, 2004). In the line of Cognitively Guided Instruction research, Franke and Kazemi reported use of student work with elementary school teachers. The researchers conducted a four-year professional development workshop series with teachers from one school. The professional developments were designed to promote teachers' understanding of student thinking. The researchers reported that as a result of attending professional development workshops, teachers' both content knowledge and pedagogical content knowledge was improved. "Thus, in detailing student thinking for the group, teachers included rich descriptions of the questions asked to elicit that thinking, the responses of other students, and the work that came before the shared interaction" (p. 107). However the researchers also stressed the importance of the selection of the student work in order to involve teachers in meaningful content discussions about students' use of algorithms and procedures (Kazemi & Franke, 2003). Therefore, one important feature of a learning activity for teachers would be providing student work which demonstrates uncommon algorithms or methods of mathematics thinking.

Especially in preservice teacher education courses such as methods courses, preservice teachers' learning might suffer from their limited experience with classrooms. This limitation may interfere with their learning especially when the purpose of the courses is to teach SCK and PCK. For example, Nugent and Grant (2009) used National Assessment of Educational Progress (NAEP) materials, student work on NAEP questions with preservice teachers to study their content knowledge. This study with student work on NAEP questions resulted in increase in preservice teachers' mathematics content knowledge. The research team used Learning from NAEP: Professional Development Materials for Teachers of Mathematics (NCTM, 2006). This book was published to guide teacher educators who prefer to use student work for professional development.

Besides the effectiveness of using student work with teachers, the disadvantages of using it might be the restricted flexibility and limited transformation of content knowledge for different settings (Ball, 2000). Teachers face classrooms where they have many different learners which may not be discussed any of the tasks used during the above designs. Therefore, teacher learning tasks should also provide flexibility of mathematics knowledge for different types of classrooms and for different challenges of teachers. If the facilitator has concern about flexibility, they may ask teachers to bring their own student work in order to address their own setting. On the other hand, for preservice teacher education courses, the facilitator may collect student work from local schools, potential student population for preservice teachers.

Research Questions

This study is an effort to improve preservice elementary teachers' geometry content knowledge in methods course. The goal of this study is to address content knowledge of preservice teachers rather than PCK in methods course. The protocol used as the intervention in this study addresses only content knowledge but not pedagogical knowledge. Therefore, this study strives to investigate the following research questions:

- Does use of geometry activities focused on quadrilaterals with analysis of student work influence preservice elementary teachers' geometry content knowledge?
- Is there a difference in geometry content knowledge between preservice teachers who are in a traditional mathematics methods course and preservice teachers who are in experimental mathematics methods course?

Methods

Settings and Participants

This study was conducted in mathematics methods course at a large southeastern research university for elementary school teachers who were predominantly middle-class, white, female students. Students complete approximately two years of education before starting their courses in the teacher education program. Students begin their unified elementary education program in their junior year and usually they take the methods course in their senior year. This course plays an important role in preservice teachers' education because it is the only mathematics methods course for most of them. Preservice teachers may prefer to continue their education in master's degree for which they choose a major (e.g. mathematics/science or special education). Only the preservice teachers who choose mathematics/science as a major would have to take more mathematics methods courses. If a preservice teacher chooses not to complete masters or chooses to study a major other than mathematics/science for master's degree, the student does not take any other mathematics methods course. Therefore, the methods course of the interest of this study plays a crucial role in future teachers' education. This course is the last and the only mathematics methods course for most of the preservice teachers.

During the teacher education program, elementary school preservice teachers are required to take three mathematics courses (two elective, one content course) before the mathematics methods course. The mathematics content course addresses mathematics concepts for elementary school level whereas the mathematics methods course is designed to build the future teachers' pedagogical tools for teaching mathematics. Even though, this order of the courses is recommended, some students take methods course and the content course at the same time or some students may take methods course before the content course. Therefore, mathematics content knowledge readiness of the students is a concern for the methods course instructors.

One semester of the mathematics methods course in this university was thirteen or fourteen weeks. The textbook to be used for this course was chosen in advance. Indeed, the textbook, Elementary and Middle School Mathematics: Teaching Developmentally (Van de Walle, 2007) has been used as the major textbook of this course for more than ten years. Also, the students in the course have access to various manipulatives for elementary school classrooms to practice incorporating the manipulatives for teaching mathematics. During the semester, the common practice among instructors is to address problem solving, assessment and technological aspects of mathematics teaching. In addition to these general goals, the instructors address mathematics topics for elementary school for one or two week long instruction. For example, while the instructor discusses development of number sense and operations in two weeks, the instructor may discuss topics of measurement in one week. Even though there is a common consensus about duration of instruction for a topic, the instructors have flexibility to change it. For geometry topics, generally instructors spent two weeks to address learning and teaching geometry.

At the time of the study, there were four sections of the course, two treatment sections and two control sections. The instructors of the course were a faculty member (two control sections), the researcher (one treatment section) and one graduate student (one treatment section). Students were assigned to cohorts by the department and then they were assigned to sections as cohorts. One of the main differences between sections was the time of the classes during the day, two morning sections and two afternoon sections. The treatment and control groups assigned in order to have one morning and one afternoon section in each group. The faculty member classes assigned to control groups because he could not be assigned to both treatment and control groups. Therefore, in treatment group there were two sections, one morning and one afternoon, which were taught by two graduate students while in control group there were two sections, one morning and one afternoon which were taught by a faculty member.

At the beginning of the pre-test, participants answered some survey questions addressing demographics of the participants. There were one hundred and seven students registered to the course and one hundred and two of them participated in the study. There were forty-eight participants in control group and fifty-four participants in the treatment group. The average of the participants' age was 21.16 and majority of the participants, 67%, were 21 years old. Also the average number of mathematics credits taken by the participants was 9.11. The percentage of participants who took more than 10 mathematics credits was 36.8 (n=36) while 63.2% (n= 66) of them took less than 10 mathematics credits in college. Also, only sixty-one students (59.8%) took required mathematics content course before the methods course and thirty-seven of them (36.3%) were taking it at the same time with the methods course. The participants were also asked about their possible choice of major for their master's degree. Only two participants reported that they would not continue their education in master's degree. Among one hundred participants, only twenty of them (19.6%) would study mathematics and science for their master's degree while the majority of them (27.5%) would study special education. The following table illustrates the descriptive data for the control, treatment groups and whole group of participants.

Table 5-2. Demographics of the participants

	Mathematics Credits (average)	Less than 10 Credits (percent)	More than 10 Credits (percent)	Content Course-Before (percent)	Content Course-Together (percent)	Math/Science Major for Master's (percent)
Control	8.41	69.6	30.4	56.3	37.5	16.7
Treatment	9.73	57.7	42.3	63.0	35.2	22.2
Whole	9.11	63.2	36.8	59.8	36.3	19.6

Intervention

The geometry instruction was for two weeks in all four sections of the course. The control group instruction stayed as the traditional instruction without any changes. All four sections of the course were observed in order to be informed about the instructional practices in those classes.

The treatment group received ninety minutes interventions for each of the two weeks for geometry instruction. In order to avoid researcher bias, another mathematics educator instead of the researcher facilitated the intervention activities. The facilitator had taught the course for two years and had received award from the university for her excellence in teaching. She was not teaching at the time of the study. She was trained to facilitate the intervention activities. The researcher was present during the intervention only to observe the instruction and to provide consultation in any case of need.

The synthesis of results from the qualitative investigation, methods course resources (Van de Walle, 2007), and the literature on preservice teacher education; yielded geometry activities as an intervention for this study. The activity types were adapted from the methods course textbook (Van de Walle, 2007) and resource books for teaching geometry (Muschla, 2002). The activities were revised to adapt for teacher learning activities. For example, the first activity, sorting shapes is adapted from Van de Walle (2007) with self-developed shapes to sort out to focus on quadrilaterals and questions to answer in order to produce definition of various shapes.

The results from the earlier study which informed the development of the intervention protocol can be summarized as the following list:

- There is a need to address content in addition to pedagogical practices in the methods course.
- Preservice teachers' reported their lack of knowledge in 2-D geometry topics especially in quadrilaterals.
- Preservice teachers stressed that, in methods course, discussion of content before the discussions of pedagogical practices would improve their learning.
- Preservice teachers expressed the importance of the flow of instruction from easier topics to more advanced topics due to their various backgrounds.
- Preservice teachers addressed the effectiveness of using visual aids such as drawings for their geometry learning.
- Preservice teachers explained that various forms of activities such as small group works in addition to individual work were helpful in their learning.

There were three groups of activities: sorting shapes, attributes of shapes, and classification of polygons. The first activity was a sorting activity in which the participants (in pairs) sorted 33 cut-out shapes in groups according to their properties (Appendix A). The groups of shapes were concave shapes, convex shapes, hexagons, pentagons, triangles, quadrilateral, kite, trapezoid, parallelogram, rectangle, rhombus, and square. When the participants were sorting shapes they experienced defining characteristics of the shapes and the relationships between them. As a result of this activity, the participants developed definitions of those shapes, individually.

For the second group of activity, attributes of shapes, participants worked in pairs to study 10 groups of figures (4 figures in each group, Appendix A). The participants were asked to determine which figure in a group did not belong to others. In other words, the participants had to find a figure which did not share the common characteristics with other three figures. Participants were encouraged to find more than one answer for each group. For example, in a group of four figures, figure B did not belong to others because it was concave while figure D did not belong to others because it was not a quadrilateral. The goal of this activity was preservice teachers to practice the characteristics of shapes in an open-ended problem solving activity while discussing the relationship between the shapes.

For the last group of activities, classification of polygons, the participants worked in small groups to develop a visual representation (Venn diagram) demonstrating the relationships between the polygons especially the quadrilaterals (Appendix B). Participants were given vocabulary (in alphabetical order) to fill the empty spots in the visual representation. The vocabulary were concave, convex, hexagon, kite, parallelogram, pentagon, polygon, quadrilateral, rectangle, rhombus, square, trapezoid and triangle. After the completion of the diagram, participants answered a set of true-false questions based on the Venn diagram (Appendix B). Some of the examples for true-false questions were “All pentagons are regular” and “Only some trapezoids are parallelograms”.

In addition to individual characteristics of the activities, the combination of them provided a coherent set. Participants worked in pairs or small groups in addition to individual work. At the end of the each activity, the facilitator led whole class discussions on the topics while providing the right answers. The participants experienced geometry topics with visual representations such as cut-out shapes. Also, the activities progressed through van Hiele geometric thinking levels. Participants began with level 0 and level 1 activities (e.g. sorting) and finished with a level 2 activities (e.g. true-false statements). Therefore, the activities reflected suggestions from both literature and qualitative results.

For the second week, participants analyzed geometry work of 4th and 5th grade students from two local public schools (Appendix C). Among the collected student work, only the fully completed ones selected. Then, the ones with wrong answers and especially with student misconceptions were selected in order to intrigue content discussions among preservice teachers, as suggested by the literature on using student works with teachers (Kazemi, 2004). For example, one of the selected student work showed a misconception for definition of a trapezoid. The student defined trapezoid as "like skirt". The students' names were removed before using the student works with the participants.

First, the participants worked individually on the worksheet developed for elementary school students. After the completion of the worksheet, they analyzed elementary school students' worksheets. The participants were given protocol to study student work (Appendix E). In pairs, they discussed what the student did, what the student knew (and student misconceptions), what they would ask the student in order to learn more about the student's knowledge of geometry. Then, in small groups (two pairs), participants discussed what they would do to teach these concepts to the student and how they would address the student misconceptions. The participants recorded their discussions. There were six small groups and each group received a different student work. For the whole class discussion, the facilitator asked participants to share their student work and their discussions on the given questions.

Instrumentation: Teacher Knowledge Measurement

The test, Content Knowledge for Teaching Mathematics Measures (CKT-M Measures)[1], was developed by a research group at the University of Michigan. The purpose of this instrument is to "discriminate accurately among teachers, in essence ordering them as correctly as possible relative to one another and to the underlying trait being assessed, mathematical knowledge for teaching" (Hill et al., 2008, p. 131). Another use of this instrument is to measure change in teachers' knowledge as they learn over time. An important characteristic of this instrument is that it does not provide raw scores. In other words, a teacher's score cannot be interpreted as how much the teacher knows. Therefore, the instrument developers strongly warn that this instrument is not suitable for the purpose of individual teacher accountability such as certification or qualification (Hill, Schilling, & Ball, 2004; Hill, Sleep, Lewis, & Ball, 2008). The intent of using this instrument in this present study was to compare mathematical knowledge of groups (control and treatment) of preservice elementary school teachers and detect any growth of the preservice teachers of the experimental group.

The instrument is a multiple choice test. There are three sections of the test; numbers and operations; patterns, functions, and algebra; and geometry. For the

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1 Copyright © 2006 The Regents of the University of Michigan. For information, questions, or permission requests please contact Merrie Blunk, Learning Mathematics for Teaching, 734-615-7632. Not for reproduction or use without written consent of LMT. Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

reliability, the test developers studied reliability separately for three different sections of the test: numbers and operations; patterns, functions, and algebra; and geometry (Hill, et al., 2004). For this study only the geometry section questions were used. The geometry section showed highest reliability from item response theory study of reliability by .91 for one-parameter and .92 for two-parameter. Furthermore, for the validity study, cognitive interviews were conducted in addition to evaluation of mathematicians and mathematics educators (Hill, et al., 2004).

Data Collection and Analysis

Participants completed the CKT-M Measures geometry test one week before their geometry instruction. For next two weeks they received the geometry instruction and the following week they completed the post-test. Both pre and post tests were administered at the beginning of the classes. The course instructors were not present during testing or informed consent agreement of participants. In order to protect students' privacy, instructors were not informed about participation of any of the students from their classroom. Therefore, as it was stressed to students, their participation in this study did not affect their grade in this course. In order to address two research questions, geometry knowledge growth of treatment group and difference of knowledge growth between treatment and control group, two different analysis methods, repeated measures ANOVA and mixed ANOVA, were used, respectively.

Results

In order to study the first research question, geometry knowledge growth of treatment group, repeated measures ANOVA was used. Results showed a significant change in participants' geometry content knowledge, $F(1, 49) = 16.08$, $p<.001$, R2 = .25, eta2 = .25. This indicates statistically significant positive change in treatment group participants' geometry content knowledge. A mixed ANOVA method of analysis was conducted to study whether there was difference of knowledge growth between treatment and control groups. Results indicated a significant main effect of time $F(1, 91) = 28.38$, $p<.001$ but there was no significant interaction between time and grouping (treatment/control), $F(1, 91) = .21$, $p=.646$. The results showed that geometry knowledge of participants was increased significantly, however the grouping did not have any affect on participants' knowledge growth. It can be concluded that even though treatment group participants' geometry content knowledge growth was significant, the difference between treatment group and control group participants' growth in geometry content knowledge was not significant.

Discussion and Conclusions

The demographics of the participants show that control and treatment groups reflect similar trends in terms of participants' age, number of mathematics credits taken and taking content course before the methods course. The majority of the students took less than 10 credits of college mathematics. This also shows that they only took required courses (one or two) before the methods course. This result is parallel with the small number of participants who wanted to study mathematics and science for their master's degree. Therefore, the demographics of the participants is parallel with the results from the previous investigation (Aslan-Tutak, 2009), the limited mathematics experience of preservice teachers, and the small number of preservice teachers with interest in mathematics.

The analysis of growth in treatment group can be interpreted as that use of the protocol developed from the previous studies resulted in significant increase in preservice teachers' geometry content knowledge. However, the control group results showed increase in preservice teachers who received regular instruction too. Even though treatment group participants' increase was more than the increase of control group participants, the difference was not statistically significant. Therefore, it would be difficult to reach the conclusion on the protocol as being effective for preservice teachers in this study. Control group instruction was not entirely controlled. The control group instructor decided on practices to be used in control group sections. The researcher did not have authority on the control group instruction which may be the main limitation of this study. For further research, in a similar setting in which the control group instruction designed not to address geometry in the context of teaching could provide more insight on affect of using the protocol with preservice teachers.

Closer look at both of the protocol and the traditional instruction in control groups may reveal common characteristics to inform future research. The most important feature of the protocol was that it was developed with the insights from the previous research conducted by the author from similar settings, same course with different sample and instructors. Mathematics teacher educators should consider examining the settings especially the participants and their needs before developing a learning tool for them. For example, one of the highlighted characteristics of the preservice teachers in this setting was limited experience with mathematics and different levels of mathematics preparation among them. The protocol provided content discussions before the pedagogical discussions. Also, the activities in the protocol were in an order to prepare participants to higher thinking levels and more complex parts of the topics.

During the geometry instruction of the control group, the researcher observed the control group instruction. The observations revealed that the control group participants also received similar geometry activities. The control group instructor who has certain experience with preservice elementary teachers used an instruction based on elementary school curriculum. The activities addressed how geometry was taught from 3rd to 5th grade. The instructor used activities from elementary school curriculum materials in order to provide geometry experiences in the context of teaching. The focus of the instruction was the topic of quadrilaterals. Therefore, some common characteristics of these two instructions can be identified as use of learning activities in the context of teaching especially closely linked to the classroom and use of the topic of quadrilaterals. Even though, it is not possible to drive conclusions on these two instructions from this study without further qualitative investigation, it is helpful to recognize possible affects of providing content knowledge activities in the context of teaching in the methods course.

The limitations of this study would provide not only the explanations for the non-significant difference between the groups but also ideas for future research in mathematics teacher education. The time limitation for the intervention activities is one of the highest risks for this study. Due to nature of the methods course, there were only two weeks for geometry instruction. Two weeks of intervention may not be long enough to provide detectable change. For a small extend change, the time period for the intervention should have been longer than two weeks to be able to result in changes. Because it is not feasible to spend more time for geometry in this course, this research can be expended with a similar design for a longer period of time in a different setting.

In a study of middle and secondary school teachers' geometry content knowledge, Fostering Geometric Thinking (FGT), Driscoll and his colleagues used content activities and analysis of student work with in-service teachers (Driscoll et al., 2009). This study showed significant difference between control group teachers who did not receive any professional development and treatment group teachers who received 20-week long intervention. The intervention was designed to provide geometry content experiences for teachers and analysis of student work from teachers own classroom.

Comparison of FGT study and this study reveals other limitations such as selection of the student work. Using student work with preservice teachers might not be as effective as using them with in-service teachers. This study provides a new direction in using student work with teachers. The effects of using student work might vary in the context of preservice or in-service teacher education. In the case of in-service teachers, participants first experience teaching the materials and then analyze student work. On the other hand, in the case of preservice teachers, participants only experience the materials as a student without teaching them. Therefore, this study might start the discussions such that the role of actual teaching of the materials before analyzing student work might have a crucial influence on teachers' learning.

Therefore, as this study provides further understanding on teacher' geometry content knowledge, it also stresses the necessity to study teachers' mathematics content knowledge especially geometry knowledge. This study informs mathematics teacher education in three important points. Preservice teachers' have limited geometry knowledge as previous research have showed (Jones, 2000; Swafford et al., 1997). Our understanding of preservice teachers' geometry content knowledge needs to be improved for many geometry topics. It also provides another dimension of discussions on using student works with teachers. Using student works in the context of preservice and in-service teacher settings might result in different outcomes. Using student works with preservice teachers should be studied further.

CHAPTER 6
CONCLUSIONS AND IMPLICATION

The purpose of this dissertation study was to provide policy implications in mathematics teacher education, and to inform the practice to improve preservice teachers' geometry content knowledge. The design of this research was two fold: to investigate preservice teachers' geometry learning and their geometry content knowledge. For this chapter, the discussion on conclusions and implication on these two investigations will be addressed in two sections: geometry learning and geometry knowledge. In the section of geometry learning, the qualitative investigation implications will be addressed while the following section, geometry knowledge, will provide discussion on quantitative investigation results. This chapter will be concluded with the discussion on the limitations of the study and future research suggestions.

Geometry Learning

The qualitative investigation informed not only the following quantitative investigation but also the teacher education practices especially for the mathematics methods courses for elementary school teachers. All three participants from the qualitative investigation were very enthusiastic about teaching in elementary school. They all stressed the professionalism in teaching that one needed teacher education to be able to an effective teacher. They all favored hands-on and meaningful teaching in mathematics. However, they still felt that they were not ready to teach mathematics in elementary school. Good intentions are not enough to be good teachers (Borko et al., 1992). Borko and others asserted that often teacher education programs do not support preservice teachers in their learning in order to transform them to knowledgeable teachers. In the case of geometry, the investigation on preservice teachers' geometry learning revealed that preservice elementary teachers' perceived their geometry knowledge limited. The participants of this investigation expressed their need to study geometry further before beginning to teaching.

Preservice teachers were aware of their lack of geometry content knowledge and its possible effect on their learning pedagogical aspects of teaching (Fennema & Franke, 1992). Even though preservice teachers should have been prepared content wise before the methods course, many of them were not equipped with enough content knowledge to focus on pedagogical content preparation. Also, preservice teachers came from different backgrounds (e.g. community college). Therefore, the preservice teachers' quest for content knowledge preparation continued in methods courses too.

The methods course for elementary preservice teachers should provide content knowledge in addition to the pedagogical content knowledge. Even though some methods course instructors addressed content, they were not effective. The results of the qualitative investigation reflect two important characteristics of studying mathematics content in methods course. First, the mathematics topics should be accessible to the preservice teachers. The difficulty of mathematics topics should be from easier to the more advanced. The teacher educators should remember the diverse mathematical background of the preservice teachers and should aim to address them. The second characteristic of an effective content preparation in a methods course is integrating content with pedagogy. Participants were aware of that the primary purpose of methods course was not to teach mathematics content, but to address how to teach mathematics. However, in the case of absence of content discussion, the preservice teachers were having trouble relating to the pedagogical examples.

It is important to note that the type of content knowledge that has been asked by preservice teachers was not college level mathematics, but mathematics that they would be teaching. They were confident about doing elementary school mathematics but they were not confident about knowing elementary school mathematics for teaching it meaningfully. This type of knowledge is the type of content knowledge that (Ball, Thames & Phelps, 2008) called as specialized content knowledge (SCK). Preservice teachers need to unpack their mathematics knowledge in order to be able to learn how to teach mathematics. In studying SCK, Ball et al. (2008) stress the importance of using mathematics in the context of teaching because SCK is the mathematics knowledge for only teachers to use it in teaching.

In terms of geometry content knowledge, the results showed preservice teachers' limited experiences with geometry in both K-12 education and in college education. Jones (2000) stressed that teachers' lack of geometry knowledge is partly because of their limited experiences in high school. The qualitative investigation showed that preservice teachers' understanding of geometry was limited with shapes and measurement aspects of shapes. Preservice teachers could not identify other important topics of geometry such as transformation, and symmetry. Even in the study of shapes, they were having trouble to understand classification of quadrilaterals. Preservice teachers recognized geometry as an important topic of mathematics to teach in elementary school, yet they were afraid of teaching it due to their lack of knowledge. They were planning to learn geometry and teaching geometry from experienced teachers in their schools after they begin teaching in the classroom.

Teacher educators should address the content need of preservice teachers not only in content courses, but also in methods courses too (Ball et al., 2008). In methods courses, it is important to discuss content in the context of teaching. In terms of geometry knowledge, preservice teachers' understanding differed from their understanding of algebraic topics. Compared to their algebra experiences, they expressed that they had limited experiences with geometry which resulted in limited geometry knowledge. Not only content courses but also the methods courses in teacher preparation programs should address geometry content knowledge.

Geometry Knowledge

The second phase of this dissertation study, the quantitative investigation, addressed preservice teachers' geometry content knowledge. The analysis of growth in the treatment group can be interpreted as the use of the protocol developed as a result from previous studies resulted in significant increase in preservice teachers' geometry content knowledge. However, the control group results showed increase in preservice teachers who received regular instruction, too. Even though treatment group participants' increase was more than the increase of control group participants, the difference was not statistically significant. Therefore, it would be difficult to reach the conclusion on the protocol as being effective for preservice teachers in this study. Control group instruction was not entirely controlled. The control group instructor decided on practices to be used in control group sections. The researcher did not have authority on the control group instruction which may be the main limitation of this study. For further research, in a similar setting in which the control group instruction designed not to address geometry in the context of teaching could provide more insight on affect of using the protocol with preservice teachers.

Closer look at both of the protocol and the traditional instruction in control groups may reveal common characteristics to inform future research. The most important feature of the protocol was that it was developed with the insights from the previous research conducted by the author from similar settings, same course with different sample and instructors. Mathematics teacher educators should consider examining the settings especially the participants and their needs before developing a learning tool for them. For example, one of the highlighted characteristics of the preservice teachers in this setting was limited experience with mathematics and different levels of mathematics preparation among them. The protocol provided content discussions before the pedagogical discussions. Also, the activities in the protocol were in an order to prepare participants to higher thinking levels and more complex parts of the topics.

During the geometry instruction of the control group, the researcher observed the control group the instruction. The observations revealed that the control group participants also received similar geometry activities. The control group instructor who has certain experience with preservice elementary teachers used an instruction based on elementary school curriculum. The activities addressed how geometry was taught from 3^{rd} to 5^{th} grade. The instructor used activities from elementary school curriculum materials in order to provide geometry experiences in the context of teaching. The focus of the instruction was the topic of quadrilaterals. Therefore, some common characteristics of these two instructions can be identified as use of learning activities in the context of teaching especially closely linked to the classroom and use of the topic of quadrilaterals. Even though, it is not possible to drive conclusions on these two instructions from this study without further qualitative investigation, it is helpful to recognize possible affects of providing content knowledge activities in the context of teaching in the methods course.

Limitations and Future Research

Examination of this study and its design would highlight the limitations of it and provide suggestions for further research. The limitations of this study would provide not only the explanations for the non significant difference between the groups but also ideas for future research in mathematics teacher education. Some of the limitations which could be addressed by further research were small number of participants, limited control on control group, duration of the study, use of student work, and instrumentation. The limitation of the control group was discussed above. In this part of the chapter discussion on other limitation factors and suggestions for further research will be provided.

Number of Participants

There were only three participants for the first phase of this study. The qualitative results were limited by only one participant from each section of the course. Because the selection of the participant depended on participants' volunteering, the participant might not provide rich information from the classroom. In order to be able to capture various perspectives from methods course, it might be needed to have two or three participants from each section. In her study of critical care nurses' stories, Robichaux (2002) interviewed twenty-one participants. In narrative analysis it is a common practice to interview larger number of participants. A future study could be narrative analysis of at least three participants from each section of the methods course to investigate their stories of geometry learning.

Duration of the Study

The time limitation for the intervention activities is one of the highest risks for this study. Due to nature of the methods course, there were only two weeks for geometry instruction. Two weeks of intervention may not be long enough to provide detectable change. For a change of a small extend, the time period for the intervention should have been longer than two weeks to be able to result in detectable changes. A geometry intervention longer than 2 weeks in this methods course is not feasible. Therefore, a similar intervention (content in the context of teaching) should be implemented in another setting which would allow for a longer period of study.

Use of Student Work

Another limitation of this study might be the selection of the student work. In order to provide relative context, for this study, the student work were chosen from local schools. However, teachers' learning might be improved by using the student work form their own classrooms. Using student work with preservice teachers might not be as affective as using them with in-service teachers. Therefore, this study provides a new direction in using student works with teacher. The affects of using student work might vary in the context of preservice and in-service teacher education. In the case of in-service teachers, participants first experience teaching the materials and then analyze student work. On the other hand, in the case of preservice teachers, participants do not experience the teaching component. Therefore, this study might start discussion on the role of actual teaching of the materials before analyzing student work as a crucial influence on teachers' learning. For a further study, the preservice teacher may be asked to develop lesson plan and implement it in the classroom as a part of the methods course. Then they would be asked to collect student work from classrooms and to discuss them in the methods course with other preservice teachers. A research project with this kind of design would be able to inform teacher educators further in use of student work with preservice teachers.

Instrumentation

The last but not the least limitation of this study was the instrumentation. Indeed, limitation of instrument to measure mathematics teachers' content knowledge is one of the concerns of the mathematics teacher education field. There is no clearinghouse for instruments for teachers' mathematics knowledge (Hill et al., 2008). "Research projects interested in studying the development of teacher knowledge design their own instruments, tailored to their own program or purpose" (pp. 135-136). In this study, the researcher chose not to develop an instrument in order to avoid validity and reliability concerns. In their literature review, Hill and others, reported four nationally recognized research-based instruments. CKT-M Measures, the instrument of this study, was one of them. CKT-M Measures is the only instrument addressing goals of this study, as discussed in methods section. However, this instrument is not perfect (Hill et al., 2008). Therefore, a general geometry knowledge test for teachers might not detect small changes of teacher knowledge especially when the intervention focuses on a topic of geometry. For further research, the investigation may also address the development of a geometry knowledge survey to measure preservice teachers' geometry content knowledge in the case of quadrilaterals. On the other hand, Hill and others addressed the difficulty of producing an instrument for a concept, teachers' mathematics content knowledge, which is still developing. "Without better theoretical mapping of this domain [mathematical knowledge for teaching], no instrument can hope to fully capture the knowledge and reasoning skills teachers possess" (p. 136).

Geometry Topic

In an effort to improve our understanding of teachers' geometry content knowledge, further qualitative investigations is necessary to study teachers' perception of geometry and their learning in not only the topics of polygons but also other topics of geometry such as transformation. For example, the results of the first phase of this study showed that preservice teachers understanding of geometry was limited by the 2-D topics. Teachers', especially the preservice teachers' understanding and knowledge of other geometry topics (e.g. symmetry) should be addressed by further study.

As this study provides further understanding on teachers' geometry content knowledge, it also stresses the necessity to study teacher mathematics content knowledge especially geometry knowledge in other aspects. This study informs mathematics teacher education field in terms of three important issues: preservice teachers' lack of geometry experience and knowledge, need to improve our understanding of preservice teachers' geometry content knowledge, and possible difference between using student work in the context of preservice and in-service teacher settings.

APPENDIX A
INTERVIEW PROTOCOL

Interview Guide
I want to talk to you about your geometry learning during the mathematics methods course (MAE 4310), and your geometry knowledge for teaching. I would like to ask you a few questions.

1. Could you tell me your story of learning geometry?
2. What are your perceptions about geometry learning?
3. What are your perceptions about geometry teaching?
4. Could you tell me your story of your geometry learning during this course?
5. What is geometry?
6. Which concepts in geometry is most important than others in mathematics curriculum?
7. How important in your teaching is the content knowledge?
8. How would you describe your geometry content knowledge?
9. How was effective the geometry instruction during this course?
10. What kind of learning experiences would promote your geometry instruction during your teacher education program?

Is there anything that you would like to add? Do you have any questions or comments?

Thank you for your participation.

APPENDIX B
GEOMETRY PROTOCOL

Names: MAE 4310
Fall 2008

Sorting Polygons and Quadrilaterals

In this activity you will work in pairs to sort polygons into different categories, polygons in the same category share certain characteristics. Please follow the instructions below.

Materials: Set of 33 polygons (each numbered).

1. Which polygons are *concave polygons*, those with an *interior angle greater than 180⁰*?
   Please write their numbers. ________________. The remaining polygons are convex
2. Now put all the polygons back together. Select those with *six sides*. These are *hexagons*. Please write their numbers ________________.
3. From the remaining polygons, select those with *five sides*. These are *pentagons*.
   Please write their numbers ________________.
4. From the remaining polygons select those with *three sides*. These are *triangles*.
   Please write their numbers ________________.
5. All the remaining polygons are *quadrilaterals*, four sided polygons. Select *convex* quadrilaterals with *no parallel sides*. Please write their numbers ________________.
6. In the group of convex quadrilaterals with no parallel sides, select those with two opposing pairs of *congruent adjacent sides*. These are *kites*. Please write their numbers ________________.
7. From the remaining convex quadrilaterals select those with *at least one pair* of parallel sides. These are *trapezoids*. Please write their numbers ________________
8. Now in the group of trapezoids select those with exactly *two pairs of* parallel sides. These are *parallelograms*. Please write their numbers ________________.
9. From the group of parallelograms now select those with *four right angles*. These are *rectangles*.
   Please write their numbers ________________
10. From the group of rectangles now select those with *four congruent sides*. These are *squares*.
    Please write their numbers ________________
11. Now put all the parallelogram cards back together and select those with *four congruent* sides. These are *rhombi* (pl. of *rhombus*). Please write their numbers ________________

Names: MAE 4310

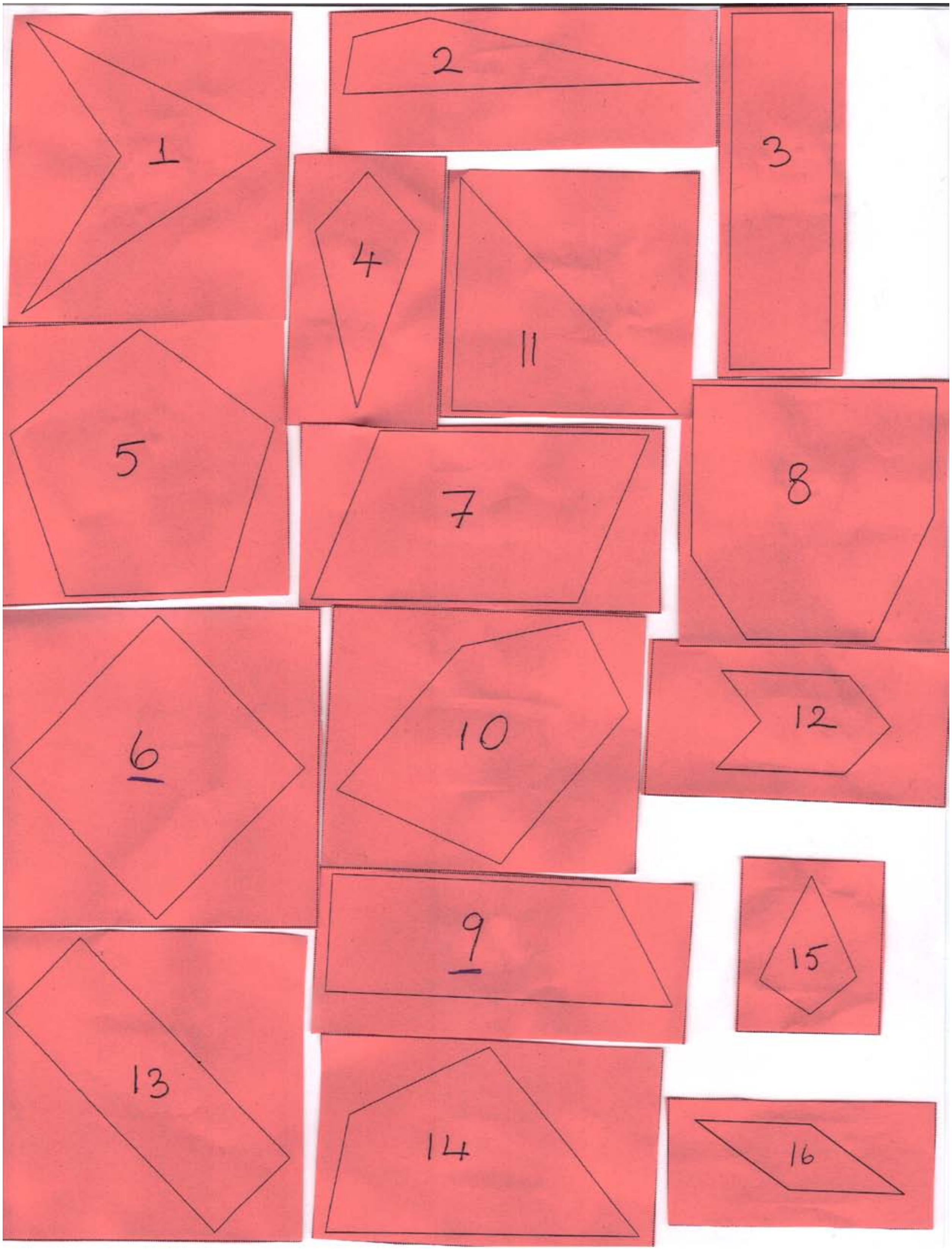
Fall 2008

Sorting Polygons and Quadrilaterals

In this activity you will work in pairs to sort polygons into different categories, polygons in the same category share certain characteristics. Please follow the instructions below.

Materials: Set of 33 polygons (each numbered).

1. Which polygons are *concave polygons*, those with an *interior angle greater than 180^0*? Please write their numbers. _1, 12, 31______. The remaining polygons are convex
2. Now put all the polygons back together. Select those with *six sides*. These are *hexagons*. Please write their numbers __8, 12, 23, 24____.
3. From the remaining polygons, select those with *five sides*. These are *pentagons*. Please write their numbers _5, 10, 18, 31_____.
4. From the remaining polygons select those with *three sides*. These are *triangles*. Please write their numbers _11, 17, 19_____.
5. All the remaining polygons are *quadrilaterals*, four sided polygons. Select *convex* quadrilaterals with *no parallel sides*. Please write their numbers _2,4,14,15 30,32_______________.
6. In the group of convex quadrilaterals with no parallel sides, select those with two opposing pairs of *congruent adjacent sides*. These are *kites*. Please write their numbers _4,15,32_______________.
7. From the remaining convex quadrilaterals select those with *at least one pair* of parallel sides. These are *trapezoids*. Please write their numbers 3,6,7,8,10,13,15,16,20,21,22,25,26,27,28,29,33_______________
8. Now in the group of trapezoids select those with exactly *two pairs of* parallel sides. These are *parallelograms*. Please write their numbers _6,7,13,16,21,22,25,26,27,28,33_______________.
9. From the group of parallelograms now select those with *four right angles*. These are *rectangles*. Please write their numbers _3,6,13,21,26,33_______________
10. From the group of rectangles now select those with *four congruent sides*. These are *squares*. Please write their numbers _3,13,33_______________
11. Now put all the parallelogram cards back together and select those with *four congruent* sides. These are *rhombi* (pl. of *rhombus*). Please write their numbers _6,16,21,25,26,28_______________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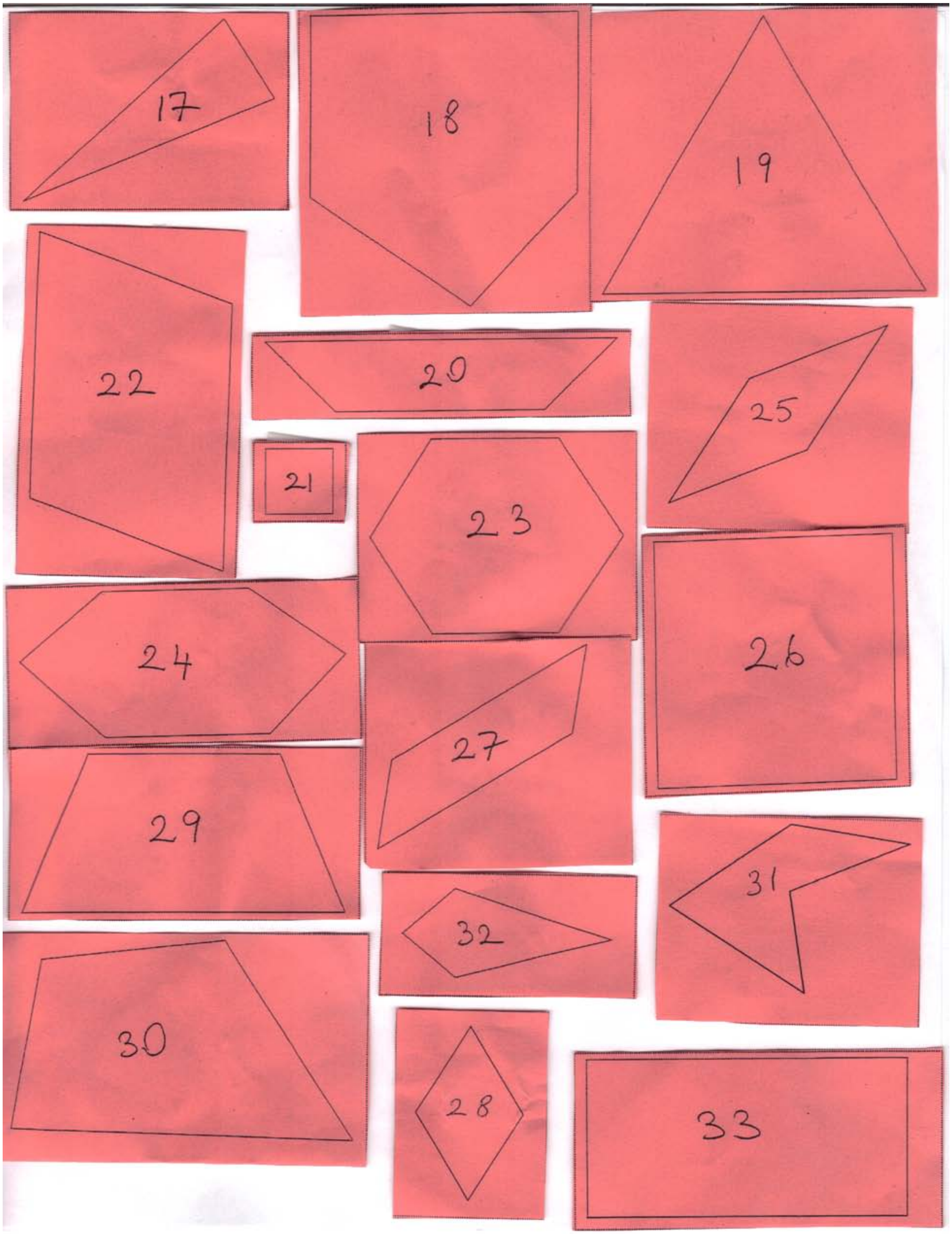
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Names: MAE 4310
Fall 2008

Vocabulary of Polygons

Please work individually and write the definitions of the following terms.

Concave	**Convex**
Hexagon	**Pentagon**
Triangle	**Quadrilaterals**
Kite	**Trapezoid**
Parallelogram	**Rectangle**
Square	**Rhombus**

Names: MAE 4310
Fall 2008

Vocabulary of Polygons

Polygon: A <u>closed</u> <u>plane</u> figure bounded by three or more <u>line segments</u>

Concave A polygon with at least one angle of more than 180 OR Non-convex, simple (the boundary of polygon does not intersect itself) polygon	**Convex** A polygon with all the angles of less than 180. OR Any line drawn through the polygon meets its boundary exactly twice.
Hexagon A polygon with six sides and six angles. It can be concave or convex.	**Pentagon** A polygon with five sides and five angles. It can be concave or convex.
Triangle A polygon with three sides and three angles.	**Quadrilaterals** A polygon with four sides, and four angles. It can be concave or convex
Kite A convex quadrilateral with two pairs of equal length adjacent sides but not all sides are of equal length.	**Trapezoid** A convex quadrilateral with at least one pair of opposite sides parallel.
Parallelogram A convex quadrilateral with two opposite sides parallel. OR A trapezoid with two pairs of parallel sides	**Rectangle** A convex quadrilateral with two pairs of parallel sides and four right angles. OR A parallelogram with four right angles.
Square A convex quadrilateral with two pairs of parallel sides whose all sides are equal and all angles are equal.	**Rhombus** A convex quadrilateral with all equal sides. OR A parallelogram with all equal sides with opposite angles are equal.

Names: Attributes of Polygons MAE 4310

In the following table, there are sets of four figures given in each row. One of them does not belong to other three. Determine which figure does not share the common characteristic with other three. There maybe more than one right answer, try to find as many as possible. Please work in <u>pairs</u> and provide explanation for your answer(s).

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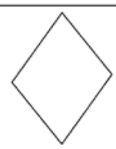
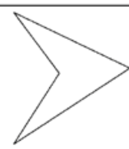
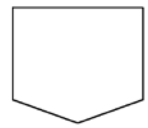
1					
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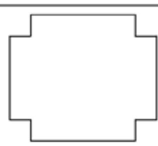
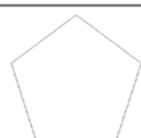
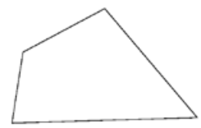
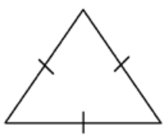
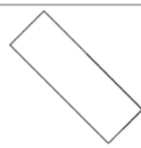
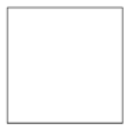
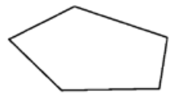
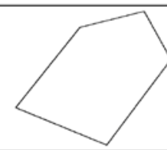
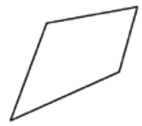
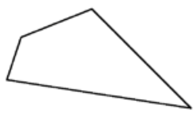
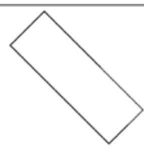
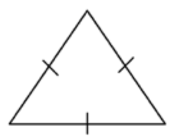
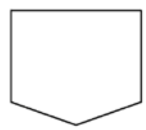
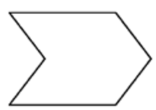
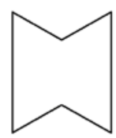
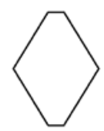
3					
4					
5					

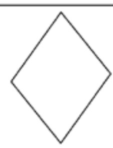
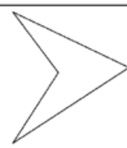
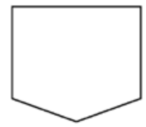
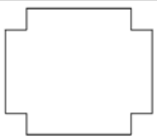
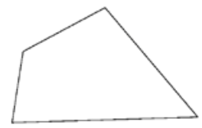
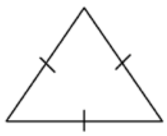
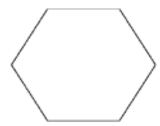
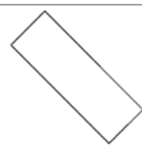
Names: Attributes of Polygons MAE 4310

In the following table, there are sets of four figures given in each row. One of them does not belong to other three. Determine which figure does not share the common characteristic with other three. There maybe more than one right answer, try to find as many as possible. Please work in pairs and provide explanation for your answer(s).

	Figure A	Figure B	Figure C	Figure D	Explanation
6					
7					
8					
9					
10					

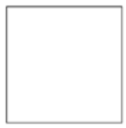
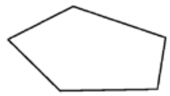
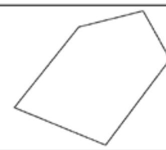
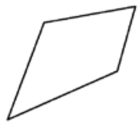
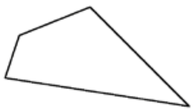
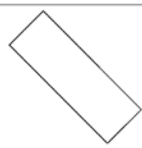
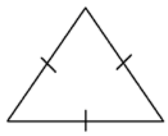
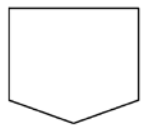
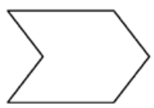
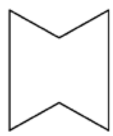
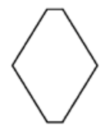
Names: Attributes of Polygons MAE 4310

In the following table, there are sets of four figures given in each row. One of them does not belong to other three. Determine which figure does not share the common characteristic with other three. There maybe more than one right answer, try to find as many as possible. Please work in <u>pairs</u> and provide explanation for your answer(s).

	Figure A	Figure B	Figure C	Figure D	Explanation
1					**B: concave D: not a quadrilateral**
2					**D: not a parallelogram**
3					**C: Not a polygon**
4					**B: Not regular**
5					**A: not a rectangle A: not 90 degrees**

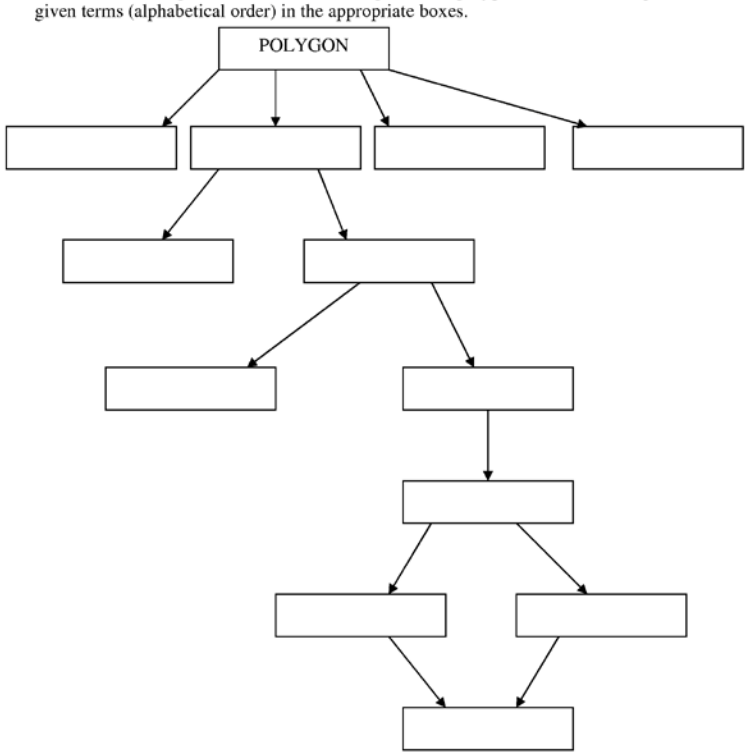
Names: Attributes of Polygons MAE 4310

In the following table, there are sets of four figures given in each row. One of them does not belong to other three. Determine which figure does not share the common characteristic with other three. There maybe more than one right answer, try to find as many as possible. Please work in pairs and provide explanation for your answer(s).

	Figure A	Figure B	Figure C	Figure D	Explanation
6					**A: not a pentagon**
7					**D: not quadrilateral**
8					**B: not trapezoid**
9					**C: Not 90 degrees**
10					**A: not hexagon**

Names:

There is a visual representation of relationship between polygons with focus on quadrilaterals. Please work in pairs, and place the given terms (alphabetical order) in the appropriate boxes.



CONCAVE

CONVEX

HEXAGON

KITE

PARALLELOGRAM

PENTAGON

POLYGON

QUADRILATERAL

REACTANGLE

RHOMBUS

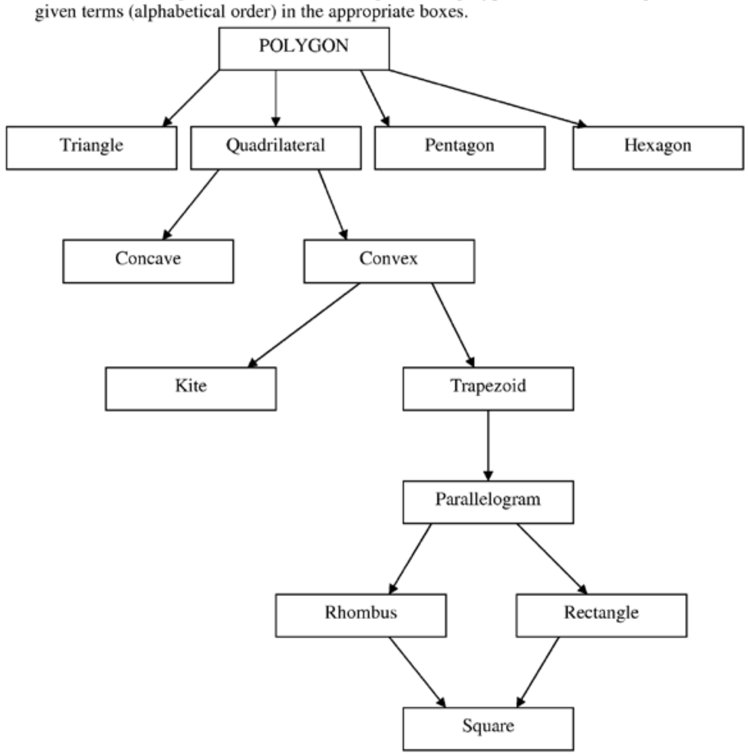
SQUARE

TRAPEZOID

TRIANGLE

Names:

There is a visual representation of relationship between polygons with focus on quadrilaterals. Please work in pairs, and place the given terms (alphabetical order) in the appropriate boxes.



CONCAVE

CONVEX

HEXAGON

KITE

PARALLELOGRAM

PENTAGON

POLYGON

QUADRILATERAL

REACTANGLE

RHOMBUS

SQUARE

TRAPEZOID

TRIANGLE

Name: MAE 4310
Fall 2008

Classifying Polygons

By using the diagram you have produced, decide if the given statements are TRUE or FALSE. Please, justify your answer by referring back to the diagram.

1. ___All pentagons are regular.

2. ___Only some rhombi are parallelograms.

3. ___All squares are rectangles.

4. ___Only some trapezoids are parallelograms.

5. ___All quadrilaterals are convex.

6. ___Only some squares are rhombi.

7. ___Only some parallelograms are rectangles.

8. ___All hexagons have five sides.

9. ___All parallelograms are trapezoids.

10. ___Only some rectangles are squares.

11. ___All rhombi are squares.

12. ___Only some parallelograms are rhombi.

13. ___Only some rectangles are parallelograms.

Name: MAE 4310
Fall 2008

Classifying Polygons

By using the diagram you have produced, decide if the given statements are TRUE or FALSE. Please, justify your answer by referring back to the diagram.

1. _F__All pentagons are regular.

2. _F__Only some rhombi are parallelograms.

3. _T__All squares are rectangles.

4. _T__Only some trapezoids are parallelograms.

5. _F__All quadrilaterals are convex.

6. _F__Only some squares are rhombi.

7. _T__Only some parallelograms are rectangles.

8. _F__All hexagons have five sides.

9. _T__All parallelograms are trapezoids.

10. _T__Only some rectangles are squares.

11. _F__All rhombi are squares.

12. _T__Only some parallelograms are rhombi.

13. _F__Only some rectangles are parallelograms.

APPENDIX C
ELEMENTARY SCHOOL STUDENTS WORKSHEET

Name:
Teacher:

Part 1:
Define the following geometric shapes:

1. Parallelogram

2. Quadrilateral

3. Rectangle

4. Square

5. Trapezoid

6. Triangle

Name:
Teacher:
Part II
For each given figure decide if it is a parallelogram, quadrilateral, rectangle, square, trapezoid, and/or triangle.

Figure 1

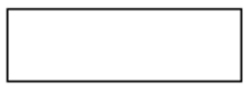


Figure 1	Yes	No	Not Sure	Explanation
Parallelogram				
Quadrilateral				
Rectangle				
Square				
Trapezoid				
Triangle				

Figure 2

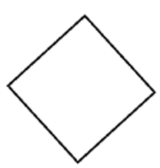


Figure 2	Yes	No	Not Sure	Explanation
Parallelogram				
Quadrilateral				
Rectangle				
Square				
Trapezoid				
Triangle				

Name:
Teacher:

Figure 3

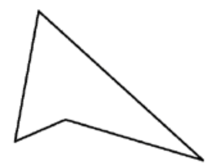


Figure 3	Yes	No	Not Sure	Explanation
Parallelogram				
Quadrilateral				
Rectangle				
Square				
Trapezoid				
Triangle				

Figure 4

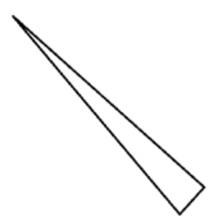


Figure 4	Yes	No	Not Sure	Explanation
Parallelogram				
Quadrilateral				
Rectangle				
Square				
Trapezoid				
Triangle				

Name:
Teacher:

Figure 5

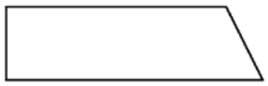


Figure 5	Yes	No	Not Sure	Explanation
Parallelogram				
Quadrilateral				
Rectangle				
Square				
Trapezoid				
Triangle				

Figure 6

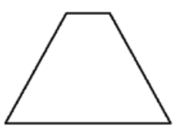


Figure 6	Yes	No	Not Sure	Explanation
Parallelogram				
Quadrilateral				
Rectangle				
Square				
Trapezoid				
Triangle				

Name:
Teacher:

Figure7

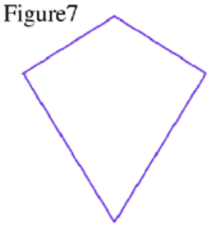


Figure 7	Yes	No	Not Sure	Explanation
Parallelogram				
Quadrilateral				
Rectangle				
Square				
Trapezoid				
Triangle				

Figure 8

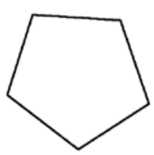


Figure 8	Yes	No	Not Sure	Explanation
Parallelogram				
Quadrilateral				
Rectangle				
Square				
Trapezoid				
Triangle				

- 5 -

Name:
Teacher:

Figure 9

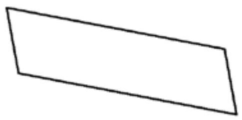


Figure 9	Yes	No	Not Sure	Explanation
Parallelogram				
Quadrilateral				
Rectangle				
Square				
Trapezoid				
Triangle				

Figure 10

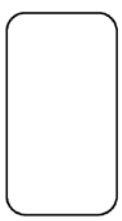
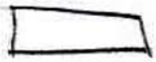


Figure 10	Yes	No	Not Sure	Explanation
Parallelogram				
Quadrilateral				
Rectangle				
Square				
Trapezoid				
Triangle				

APPENDIX D
SAMPLE STUDENT WORK

Part 1:
Define the following geometric shapes: ADAM

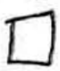
1. Parallelogram
HaS four congrent Side and on Side is parrlel



2. Quadrilateral
Has four Side and 4 angles

3. Rectangle
HaS four parrlel Sides

4. Square
HaS four Side & four angles



5. Trapezoid
HaS two parrlel Sides

6. Triangle
HaS three Sides & parrlel Sides

-1-

For each given figure [illegible] square, trapezoid, and/or triangle.

Figure 1

Figure 1	Yes	No	Not Sure	Explanation
Parallelogram	✓			It has 2 sides that equal
Quadrilateral	✓			It has 4 Sides that equal
Rectangle	✓			It is Shaped like a rectangle
Square	✓			It has 4 Side like a Square does
Trapezoid		✓		It dosen't look like a trapezoid
Triangle		✓		It dosen't have three Sides

Figure 2

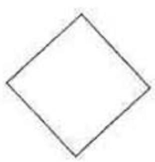


Figure 2	Yes	No	Not Sure	Explanation
Parallelogram	✓			It has four equal sizes
Quadrilateral	✓			It has four Sides
Rectangle		✓		It is not long and the Side are Short
Square	✓			It look like a Square but tilted
Trapezoid		✓		I dosen't look like a skirt
Triangle		✓		It dosen't have three Sides

Figure 3

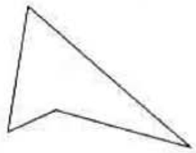


Figure 3	Yes	No	Not Sure	Explanation
Parallelogram		✓		It dosen't have parall-l lines
Quadrilateral	✓			It has four sides
Rectangle		✓		It is not long and have Short Sides
Square	✓			I has four Sides
Trapezoid		✓		It donsen't look like a Skirt
Triangle		✓		It dosen't have four Side that equal

Figure 4

Figure 4	Yes	No	Not Sure	Explanation
Parallelogram			✓	
Quadrilateral		✓		It dosen't have four eqaul Sides
Rectangle		✓		It is not long and have Short Sides
Square		✓		It dosen't have four Side that equal
Trapezoid		✓		It dosent look like Skirt
Triangle	✓			It haves three Side that looks like a triangle

Figure 5

Figure 5	Yes	No	Not Sure	Explanation
Parallelogram			✓	
Quadrilateral	✓			It have's four sides
Rectangle		✓		It not long and have short sides
Square	✓			It have's four sides
Trapezoid		✓		It's not long and have short sides
Triangle		✓		It dosen't have three sides

Figure 6

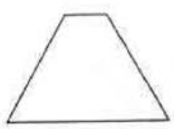


Figure 6	Yes	No	Not Sure	Explanation
Parallelogram	✓			It has two sides that are equal
Quadrilateral	✓			It have's four sides
Rectangle		✓		It dosen't have a long side and a short sides
Square	✓			It had four sides
Trapezoid	✓			It looks like a skirt
Triangle		✓		It dosen't have three sides

Figure7

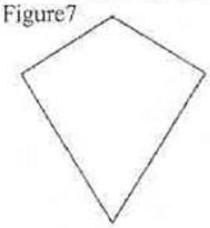


Figure 7	Yes	No	Not Sure	Explanation
Parallelogram	✓			It haves two equal sides
Quadrilateral	✓			It has four sides
Rectangle		✓		It is not long and have short sides
Square	✓			It haves four sides
Trapezoid		✓		It donsen't look like a skirt
Triangle		✓		It dosen't have three sides

Figure 8

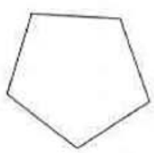


Figure 8	Yes	No	Not Sure	Explanation
Parallelogram	✓			It haves two side that are equal
Quadrilateral		✓		It haves four sides
Rectangle		✓		It is not long and haves thort sides
Square		✓		I have four sides
Trapezoid		✓		It dosent look like a skirt
Triangle		✓		I dosen't hav three sides

Figure 9

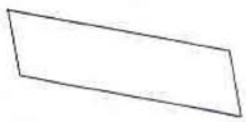


Figure 9	Yes	No	Not Sure	Explanation
Parallelogram	✓			It haves four equal parallel sides
Quadrilateral	✓			It haves four equal sides
Rectangle	✓			It has long side and short ends
Square	✓			It haves four equal sides
Trapezoid		✓		It dosen't look like a skirt
Triangle		✓		

Figure 10

Figure 10	Yes	No	Not Sure	Explanation
Parallelogram	✓			It haves foor equal sides
Quadrilateral	✓			It haves equal sides
Rectangle		✓		It dosen't have long sides and short sides
Square	✓			It haves four equal sides
Trapezoid		✓		It dosen't look like a skirt
Triangle		✓		It dosen't have three sides

- 6 -

APPENDIX E
ANALYZING STUDENT WORK PROTOCOL

Names: MAE 4310
Fall 2008

Analyzing Student Work

You will be given a copy of geometry worksheet completed by an elementary school student. Please discuss the following questions for the given student work.

a. Please work in pairs and discuss what the student did. (Please identify what the student did mathematically)

b. With your pair now discuss what the student knows. (Which geometry concepts does the student know and how well does the student know them? Does this work show any misconceptions on the part of the student?)

c. With your pair now discuss what you would ask the student in order to learn more about the student's knowledge of geometry. (Try to understand how the student thinks and what the student knows.)

Aslan-Tutak, F.

Names: MAE 4310
Fall 2008

d. In your table, as a small group discuss student approaches to the geometry concepts in the worksheet, then discuss the following questions:
   i. What would you do to help this student in learning geometry concepts?

   ii. How would you teach these geometry concepts to be sure your students know them?

e. Each table has different students' work. Please report your small group discussions to the whole class: different student approaches to geometry concepts, what you would do help the student, and how you would teach these concepts.

Aslan-Tutak, F.

APPENDIX F
CKT-M MEASURES RELEASED ITEMS

**STUDY OF INSTRUCTIONAL IMPROVEMENT/
LEARNING MATHEMATICS FOR TEACHING**

CONTENT KNOWLEDGE FOR TEACHING MATHEMATICS MEASURES (CKT-M MEASURES)

MATHEMATICS RELEASED ITEMS
2005

University of Michigan, Ann Arbor
610 E. University #1600
Ann Arbor, MI 48109-1259
(734) 647-5233
www.soe.umich.edu/lmt

 Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

April, 2005

Dear Colleague:

Thank you for your interest in our survey items measuring teachers' knowledge for teaching mathematics. Because of the expense of developing and piloting items, we do not release items from our general pool over the web. Instead, we provide here a small set of items that illustrate our larger item pool; those interested in using this larger item pool can contact Geoffrey Phelps (gphelps@umich.edu) for information about training sessions and permissions.

These released items may be useful as open-ended prompts which allow for the exploration of teachers' reasoning, as materials for professional development or teacher education, or as exemplars of the kinds of mathematics teachers must know to teach. We encourage their use in such contexts. However, this particular set of items is, as a group, NOT appropriate for use as an overall measure, or scale, representing teacher knowledge. In other words, one cannot calculate a teacher score that reliably indicates either level of content knowledge or growth over time.

We ask users to keep in mind that these items represent steps in the process of developing measures. Many of these released items failed, statistically speaking, in our piloting; in these cases, items may contain small mathematical ambiguities or other imperfections. If you have comments or ideas about these items, please feel free to contact one of us by email at the addresses below.

These items are the result of years of thought and development, including both qualitative investigations of the content teachers use to teach elementary mathematics, and quantitative field trials with large numbers of survey items and participating teachers. Because of the intellectual effort put into these items by SII investigators, we ask that *all* users of these items satisfy the following requirements:

1) Please request permission from SII/LMT for any use of these items. To do so, contact Geoffrey Phelps at gphelps@umich.edu. Include a brief description of how you plan to use the items, and if applicable, what written products might result.

2) In any publications, grant proposals, or other written work which results from use of these items, please cite the development efforts which took place at SII/LMT by referencing this document:

> Hill, H.C., Schilling, S.G., & Ball, D.L. (2004) Developing measures of teachers' mathematics knowledge for teaching. Elementary School Journal 105, 11-30.

3) Refrain from using these items in multiple choice format to evaluate teacher content knowledge in any way (e.g., by calculating number correct for any individual teacher, or gauging growth over time). Use in professional development, as open-ended prompts, or as examples of the kinds of knowledge teachers might need to know is permissible.

You can also check the SII website (http://www.sii.soe.umich.edu/) or LMT website (http://www.soe.umich.edu/lmt) for more information about this effort.

Below, we present three types of released item – elementary content knowledge, elementary knowledge of students and content, and middle school content knowledge. Again, thank you for your interest in these items.

Sincerely,

Deborah Loewenberg Ball
Study of Instructional Improvement
University of Michigan
dball@umich.edu

Heather Hill
Learning Mathematics for Teaching
University of Michigan
hhill@umich.edu

Study of Instructional Improvement/Learning Mathematics for Teawching
Content Knowledge for Teaching Mathematics Measures (CKT-M measures)
Released Items, 2005
ELEMENTARY CONTENT KNOWLEDGE ITEMS

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

2. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x 25 125 +75 875	35 x25 175 +700 875	35 x 25 25 150 100 +600 875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

3. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

4. Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

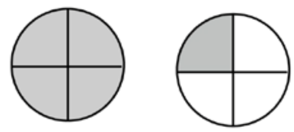
a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

c) Check to see whether 371 is divisible by any prime number less than 20.

d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



a) 5/4

b) 5/3

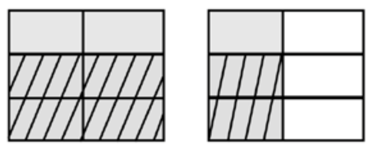
c) 5/8

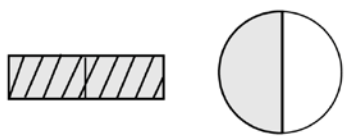
d) 1/4

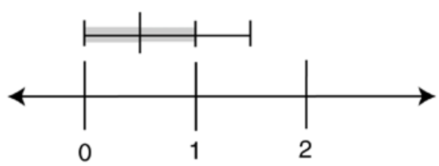
6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)

A) 

B) 

C) 

D) 

7. Which of the following story problems could be used to illustrate $1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have $1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3

8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

```
  983
 x  6
  488
+5410
 5898
```

What is Todd doing here? (Mark ONE answer.)

a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.

b) Todd is using the traditional multiplication algorithm but working from left to right.

c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.

d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.

ELEMENTARY KNOWLEDGE OF STUDENTS AND CONTENT ITEMS

9. Mr. Garrett's students were working on strategies for finding the answers to multiplication problems. Which of the following strategies would you expect to see some elementary school students using to find the answer to 8 x 8? (Mark YES, NO, or I'M NOT SURE for each strategy.)

	Yes	No	I'm not sure
a) They might multiply 8 x 4 = 32 and then double that by doing 32 x 2 = 64.	1	2	3
b) They might multiply 10 x 10 = 100 and then subtract 36 to get 64.	1	2	3
c) They might multiply 8 x 10 = 80 and then subtract 8 x 2 from 80: 80 – 16 = 64.	1	2	3
d) They might multiply 8 x 5 = 40 and then count up by 8's: 48, 56, 64.	1	2	3

10. Students in Mr. Hayes' class have been working on putting decimals in order. Three students — Andy, Clara, and Keisha — presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. What error are these students making? (Mark ONE answer.)

a) They are ignoring place value.

b) They are ignoring the decimal point.

c) They are guessing.

d) They have forgotten their numbers between 0 and 1.

e) They are making all of the above errors.

11. You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here? (Mark ONE answer.)

a) Bonny doesn't know how large 23 is.

b) Bonny thinks that 2 and 20 are the same.

c) Bonny doesn't understand the meaning of the places in the numeral 23.

d) All of the above.

12. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I)	II)	III)
$\begin{array}{r} {}^{1} \\ 38 \\ 49 \\ +\ 65 \\ \hline 142 \end{array}$	$\begin{array}{r} {}^{1} \\ 45 \\ 37 \\ +\ 29 \\ \hline 101 \end{array}$	$\begin{array}{r} {}^{1} \\ 32 \\ 14 \\ +\ 19 \\ \hline 64 \end{array}$

Which have the same kind of error? (Mark ONE answer.)

a) I and II

b) I and III

c) II and III

d) I, II, and III

13. Ms. Walker's class was working on finding patterns on the 100's chart. A student, LaShantee, noticed an interesting pattern. She said that if you draw a plus sign like the one shown below, the sum of the numbers in the vertical line of the plus sign equals the sum of the numbers in the horizontal line of the plus sign (i.e., 22 + 32 + 42 = 31 + 32 + 33). Which of the following student explanations shows sufficient understanding of <u>why this is true</u> for all similar plus signs? (Mark YES, NO or I'M NOT SURE for each one.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

	Yes	No	I'm not sure
a) The average of the three vertical numbers equals the average of the three horizontal numbers.	1	2	3
b) Both pieces of the plus sign add up to 96.	1	2	3
c) No matter where the plus sign is, both pieces of the plus sign add up to three times the middle number.	1	2	3
d) The vertical numbers are 10 less and 10 more than the middle number.	1	2	3

14. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students around particular difficulties that they are having with subtracting from large whole numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I	II	III
4 12	4 15	6 9 8 15
~~5~~0~~2~~	3~~5~~00~~5~~	~~7~~~~0~~~~0~~~~5~~
− 6	− 6	− 7
406	34009	6988

Which have the same kind of error? (Mark ONE answer.)

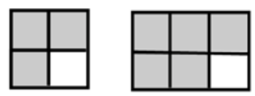
a) I and II

b) I and III

c) II and III

d) I, II, and III

15. Takeem's teacher asks him to make a drawing to compare $\frac{3}{4}$ and $\frac{5}{6}$. He draws the following:



and claims that $\frac{3}{4}$ and $\frac{5}{6}$ are the same amount. What is the most likely explanation for Takeem's answer? (Mark ONE answer.)

a) Takeem is noticing that each figure leaves one square unshaded.

b) Takeem has not yet learned the procedure for finding common denominators.

c) Takeem is adding 2 to both the numerator and denominator of $\frac{3}{4}$, and he sees that that equals $\frac{5}{6}$.

d) All of the above are equally likely.

16. A number is called "abundant" if the sum of its proper factors exceeds the number. For example, 12 is abundant because 1 + 2 + 3 + 4 + 6 > 12. On a homework assignment, a student incorrectly recorded that the numbers 9 and 25 were abundant. What are the most likely reason(s) for this student's confusion? (Mark YES, NO or I'M NOT SURE for each.)

	Yes	No	I'm not sure
a) The student may be adding incorrectly.	1	2	3
b) The student may be reversing the definition, thinking that a number is "abundant" if the number exceeds the sum of its proper factors.	1	2	3
c) The student may be including the number itself in the list of factors, confusing proper factors with factors.	1	2	3
d) The student may think that "abundant" is another name for square numbers.	1	2	3

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BIOGRAPHICAL SKETCH

Fatma Aslan Tutak was born in Antalya, Turkey, where she completed her K-12 education. She was fortunate to have great mathematics teachers who would inspire her to study mathematics. She decided to be mathematics teacher. She entered Secondary School Mathematics Education Program in Bogazici University, Istanbul, Turkey as third rank. At a point in her education, she could not decide if she wanted to study mathematics or teaching mathematics. She chose to study teaching because she believed that she could make a difference in other people's lives through education. As a result of her enthusiasm in mathematics education, she finished her program in first rank.

The first time Fatma taught, she knew that she chose the right profession. She was teaching geometry and algebra courses in a private institution which prepared students to the national university entrance exam. She was able to communicate with her students who were mathematically struggling with mathematics anxiety. Fatma realized that she enjoyed assisting students into realizing that mathematics was not to fear but to enjoy.

Fatma pursued her interest in mathematics education further and started to study in the doctoral program of University of Florida for mathematics education on January 2005. The Higher Education Institute of Turkish government awarded her for doctoral fellowship. She experienced several valuable learning opportunities during her education in University of Florida. She has participated in research projects with in-service teachers, Project TALL; attended or presented in many conferences such as American Educational Research Association, Association of Mathematics Teacher Educators, National Council of Teachers of Mathematics, Psychology of Mathematics

Education North America Chapter; published scholar articles; taught elementary mathematics methods course; and taught mathematics remediation courses for College-Level Academics Test (CLAST). Her contributions to the University of Florida and the community were recognized by the Outstanding International Student Award.

Fatma is planning to return back to Turkey and continue her academic career. Her interests include studying preservice mathematics teacher education, and developing professional development workshops for in-service teachers in addition to critical mathematics education.