Nadiia Lazhevska

ESSAYS ON TRADE AND CONSUMPTION

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This doctoral thesis in Economics consists of three self-contained chapters.

"How large are the dynamic gains from trade?" uses a theoretical trade model to study the welfare gains from trade in a dynamic setting. I show that trade liberalization leads to a higher productivity growth and higher aggregate consumer welfare even when the returns to firm entry are decreasing.

"Localized effects of the China trade shock: Is there an effect on consumer expenditure?" applies empirical methods to study the distributional consequences of increased trade competition. I examine the effect of the China trade shock on local labor market outcomes and local consumer expenditure in the U.S.

"The effect of the fracking boom on non-durable consumer expenditure: evidence from the consumer scanner data" continues to investigate the changes in nominal and real consumer expenditure following localized economic shocks. In particular, I show that the fracking boom in the U.S. had a positive and persistent effect on local consumer expenditure.

NADIIA LAZHEVSKA holds a B.Sc. in Economics from Kyiv Taras Shevchenko National University and M.A. in Economics from Central European University. Her main research fields are International Trade, Regional Economics, and Applied Microeconomics.

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To my family

Foreword

This volume is the result of a research project carried out at the Department of Economics at the Stockholm School of Economics (SSE).

This volume is submitted as a doctoral thesis at SSE. In keeping with the policies of SSE, the author has been entirely free to conduct and present her research in the manner of her choosing as an expression of her own ideas.

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Stockholm, October 1, 2018

Nadiia Lazhevska

Contents

Introduction

This doctoral thesis in Economics consists of three self-contained chapters. The first two chapters address the economic benefits and costs of trade integration. In the first chapter, I solve a dynamic theoretical trade model to study how an assumption about the shape of firm entry technology affects the aggregate welfare gains from trade. The aggregate gains from trade are shown to be large, even in a setting where the returns to innovation are decreasing. In the second chapter, I apply empirical methods to study the distributional consequences of increased trade competition with China for the U.S. consumers. Being motivated by the prior findings about the negative impact of the localized China trade shock on local labor market outcomes, I examine how this shock affects an important measure of consumer welfare, non-durable consumer expenditure. The third chapter of this thesis explores the adjustments in consumer expenditure following another localized economic shock, the fracking boom. I examine how this positive local productivity shock has affected the consumer expenditure in terms of quantity and composition of consumer basket. Abstracts for each chapter follow bellow.

"How large are the dynamic gains from trade?"

The recent dynamic trade literature has established that the dynamic gains are an additional and quantitatively large source of welfare gains from trade. However, this literature does not account for the decreasing returns to scale in R&D. In this paper, I extend a dynamic trade model with heterogeneous firms and knowledge spillovers to allow for decreasing returns to scale in R&D. By calibrating the model to match the U.S. economy, I quantify the dynamic gains from trade under alternative assumptions about the returns to scale in R&D. I find that the dynamic gains from trade are still quantitatively important when a realistic degree of decreasing returns is assumed. In particular, the model is able to generate total gains from trade that are 2.95 times larger than the static gains. Further, I explore numerically the interaction between firm entry, productivity growth, and welfare gains from trade, and conduct counterfactual exercises by varying trade costs and the R&D subsidy rate.

"Localized effects of the China trade shock: Is there an effect on consumer expenditure?"

The paper contributes to a vast literature on the effects of recent rise in Chinese import competition on the U.S. local labor markets. The previous literature has shown that higher imports cause higher unemployment and reduced wages in local labor markets that house import-competing manufacturing industries. This paper revisits these findings and examines whether the exposure of local labor markets to increased import competition has an impact on local consumer expenditure. Using household scanner data, I show that the effect of the China trade shock on changes in local non-durable consumer expenditure in nominal and real terms are not distinguishable from zero. Moreover, I show that, in the period of 2000 to 2007, the localized China trade shock had a weak effect on average wages and median household income at the commuting zone level, which may explain why I observe no effect on household non-durable expenditure.

"The effect of the fracking boom on non-durable consumer expenditure: evidence from the consumer scanner data."

I use consumer scanner data to study the response of non-durable consumer expenditure to a localized economic shock induced by the fracking boom in the U.S. The identification strategy utilizes the spatial variation in the location of geological resources and the variation in the timing of the fracking boom across the U.S. Using difference-in-differences and event study approaches, I find on average 4.4% quarterly increase in the household expenditure on non-durable goods in areas affected by fracking. I find no difference in the effects on nominal expenditure and real consumption. I also examine changes in composition of consumer baskets, and heterogeneous effects by age, income, education, and occupation.

Chapter 1

How large are the dynamic gains from trade?[1]

[1]I am thankful to my advisor Paul Segerstrom for invaluable guidance, encouragement and support. I benefited greatly from discussions with Kerem Coşar, Eleonora Freddi, Marta Giagheddu, Gene Grossman, Mathias Iwanowsky, Arieda Muço, Bengt Söderlund, Mark Voorneveld, Jörgen Weibull, and seminar participants at SSE, IFN, and NOITS 2017 International Trade Workshop. Financial support from the Jan Wallander and Tom Hedelius Foundation is gratefully acknowledged. All remaining errors are my own.

1.1 Introduction

An influential finding by Arkolakis, Costinot and Rodríguez-Clare (2012) suggests that a large set of static trade models with or without heterogeneous firms predict identical gains from trade. The recent dynamic trade literature has established that the dynamic gains are an additional and quantitatively large source of gains from trade. Sampson (2016) distinguishes between static and dynamic gains, where the static gains correspond to the ones described in Arkolakis, Costinot and Rodríguez-Clare (2012), and the dynamic gains are a result of the interplay between knowledge spillovers and the Melitz-type firm selection mechanism. When calibrating the model to match the U.S. economy, he finds that the welfare gains from trade are 3.17 times as large as the gains implied by static trade models, such as the Melitz (2003) model.

To study the dynamic gains from trade, Sampson (2016) develops an endogenous growth model with heterogeneous firms and knowledge spillovers from incumbent firms to entrants, where new firms enter by conducting R&D activity. The baseline model assumes constant returns to scale in R&D, which implies that doubling the labor employed in R&D doubles the flow of emerging firms. However, the empirical literature on patents and R&D finds decreasing returns to scale in R&D to be more realistic. Blundell, Griffith and Windmeijer (2002) estimate the long run elasticity of patents with respect to R&D to be approximately 0.5, which is also in line with the survey by Kortum (1993). The shape of the R&D technology is important in this model as it directly affects firm entry. Depending on the functional form of the R&D technology, the model can generate quantitatively different results about the dynamic gains from trade.

In this paper I revisit the Sampson (2016) model and derive a version of the model with decreasing returns to scale in R&D. To understand why the form of the R&D technology matters for the dynamic gains from trade, the nature of these gains needs to be explained. Similar to the Melitz (2003) model, after the R&D phase is done, each entering firm draws its productivity. The knowledge spillovers are modeled such that the entrant productivity distribution depends on the average productivity of existing producers. Through the firm selection mechanism, entry of new firms leads to an exit of the least productive incumbent firms, and the average productivity of existing producers increases. Due to knowledge spillovers, entrants also become on average more productive, which induces further firm selection. As a result, on the balanced growth path, the productivity cutoff for domestic firms grows at a constant rate and the productivity distributions of incumbent firms and entrants constantly shift to the right. When trade liberalization occurs, it leads to an upward shift in the productivity distribution of domestic producers, as in the Melitz model. This also improves the entrant productivity distribution, which leads to a higher growth rate of productivity cutoff and a higher economic growth rate in the domestic economy. Thus, the dynamic gains from trade are a result of an interplay between knowledge spillovers from incumbent firms to entrants and the Melitz-style firm selection mechanism. By introducing decreasing returns to scale in R&D, I effectively impose a higher entry cost for an individual firm, which in equilibrium should result in a lower firm entry response following trade liberalization and lower dynamic gains from trade.

I prove analytically that, under the functional forms for the R&D technology considered in this paper, trade liberalization unambiguously leads to an increase in economic growth and an increase in aggregate consumer welfare. I calibrate the model to match the U.S. economy under alternative assumptions about the functional form of the R&D technology. In the numerical exercise I show that the dynamic gains from trade are in general smaller with decreasing returns to R&D. However, the extent to which the dynamic gains deteriorate depends crucially on the assumption about the functional form of the R&D technology. I compare two alternative assumptions about the functional form of the R&D technology, and find that, depending on the functional form, the dynamic gains from trade are 1.97 or 2.95 times larger than the static gains[2]. This suggests that the dynamic gains from trade remain quantitatively important when a realistic degree of decreasing returns is assumed.

The model discussed in this paper is an extension of the static Melitz model to a dynamic setting, and, in addition to generating novel welfare implications, this model provides a useful framework for analysis of questions related to trade, firm entry and economic growth. In the static Melitz model, entry leads to firm selection and in equilibrium increases the average productivity of domestic producers. In the Sampson (2016) model, entry affects the growth rate of productivity cutoff and leads to higher economic growth and higher consumer welfare. It has been shown in Melitz and Redding (2015) that the

[2]This result is obtained when calibrating elasticity of patents with respect to R&D expenditure to the Blundell, Griffith and Windmeijer (2002) estimate of 0.5.

equilibrium allocation in the Melitz model is efficient. This is not the case in the Sampson (2016) model, as here entrant firms do not take into account the positive externalities generated by knowledge spillovers, and hence there is not sufficient entry in equilibrium. Inefficiency of the decentralized equilibrium due to insufficient entry provides intuition for the existence of dynamic gains from trade in this model. Higher openness leads to more entry to the domestic market, which helps to cover the gap between the inefficient amount of entry and the amount of entry which would have been chosen by a social planner.

The above reasoning also suggests that subsidizing entry would be welfare improving in this model. In particular, one of the extensions briefly analyzed in the original paper deals with the analysis of the optimal R&D subsidy. It is shown that under particular assumptions, an R&D subsidy can indeed be welfare improving. However, the solution for the level of the optimal subsidy is shown to be heavily affected by the form of the R&D technology, and the analytical results presented in Sampson (2016) are too general to provide clear implications about the optimal subsidy level. In this paper, I show analytically that an increase in the R&D subsidy rate unambiguously leads to an increase in the rate of economic growth. I then present a numerical analysis of the optimal R&D subsidy rate under differing assumptions on the returns to scale in R&D. I numerically compare the welfare effect of an increase in the R&D subsidy rate to the effect from trade liberalization. The properties of the Sampson (2016) model change substantially when decreasing returns to scale is assumed. I document that, under constant returns to scale in R&D, moving from autarky to the current U.S. level of trade has a much smaller effect on consumer welfare than switching to the optimal R&D subsidy rate, and this optimal R&D subsidy rate is huge, around 90%. Introducing decreasing returns to scale in R&D mitigates the mentioned effect and leads to a lower optimal R&D subsidy rate.

Related literature

There are a number of papers that incorporate dynamics in heterogeneous firm trade settings to study the implications of trade liberalization on firm entry-exit decisions, as well as to study the welfare consequences of trade liberalization. Atkeson and Burstein (2010) present a dynamic general equilibrium model with heterogeneous firms and both product and process innovation to study the response of firms' decisions to operate and innovate to a change in the marginal cost of international trade. Schröder and Sørensen (2012) introduce exogenous economy-wide technological progress into the Melitz model to study the firm exit decisions in relation to technological advancement and trade liberalization. The above papers, however, abstract from knowledge spillover effects that might lead to endogenous growth. Baldwin and Robert-Nicoud (2008), and the revision of this paper by Ourens (2016), combine the Melitz model with the standard model of endogenous growth with expanding product varieties. They consider different types of spillovers in the product innovation phase, but do not model knowledge spillovers in the process innovation phase as in Sampson (2016) and the current paper. Segerstrom and Stepanok (2018) quantify welfare gains from trade in a standard quality ladders endogenous growth model with heterogeneous firms, where it takes time for firms to learn how to export.

The current model is also related to a set of models that have an idea flow mechanism similar to Sampson (2016). Alvarez, Buera and Lucas (2013) introduce the idea diffusion mechanism into the Eaton and Kortum (2002) model, where domestic producers learn from both domestic and foreign producers through random meetings, and trade costs affect these learning possibilities. They show that high trade costs have large long-run effects on productivity and consumer welfare. Buera and Oberfield (2016) augment the Eaton and Kortum (2002) model by allowing for trade linkages and FDI as further sources of knowledge diffusion. They separate the gains from trade into static and dynamic components, where the static component consists of the gains from increased specialization and comparative advantage, whereas the dynamic component are the gains that operate through the flow of ideas between trading partners.

The paper also contributes to the literature emphasizing the importance of the assumption of decreasing returns to scale in R&D. This assumption has been widely discussed in the literature on endogenous growth (Kortum (1993), Davidson and Segerstrom (1998)). Within the trade literature this assumption has been under-used. An exception is the recent paper by Segerstrom and Sugita (2016), who suggest that decreasing returns to scale in R&D appears to be a crucial assumption when studying unilateral and non-uniform trade liberalization in the Melitz (2003) setting. They present a static model which matches the empirical finding by Trefler (2004) that industrial productivity increases more strongly in liberalized industries than in non-liberalized industries. Segerstrom and Sugita (2016) show that a sufficient degree of decreasing returns to scale in R&D helps to obtain results consistent with the Trefler (2004) evidence. In turn, I present an analysis of the effect of the decreasing returns to scale in R&D assumption on predictions about the gains from trade in an extension of a symmetric Melitz model to a dynamic setting.

The rest of the paper is organized as follows. Section 2 provides a brief description of the model and equilibrium. Section 3 describes the balanced growth path and defines the gains from trade. Section 4 explains the calibration and presents the numerical results. The final section concludes. A full solution to the model can be found in the Theoretical Appendix.

1.2 The model

In this section I lay out an open economy endogenous growth model based on Sampson (2016). Presentation of the model closely follows the text of the original paper.

1.2.1 Consumers

The world is comprised of $J + 1$ symmetric economies, each consisting of a set of identical households with dynastic preferences and discount rate ρ. The population $L_t = e^{nt}$ at time t grows at the exogenous rate $n > 0$. Households can lend or borrow at the interest rate r_t. Each household has constant intertemporal elasticity of substitution preferences over the final consumption good and maximizes its lifetime utility:

$$U = \int_{t=0}^{\infty} e^{-\rho t} e^{nt} \frac{c_t^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} dt \tag{1.1}$$

subject to the budget constraint

$$\dot{a}_t = w_t + r_t a_t - c_t - n a_t - b_t, \tag{1.2}$$

where c_t is consumption per capita of the final consumption good, a_t and b_t denote assets and lump-sum tax per capita, w_t denotes the wage, and $\gamma \in (0, 1)$ is the intertemporal elasticity of substitution.

Solving the intertemporal consumer optimization problem under a no Ponzi game condition yields the standard Euler equation

$$\frac{\dot{c}_t}{c_t} = \gamma(r_t - \rho) \tag{1.3}$$

and the transversality condition

$$\lim_{t\to\infty}\left\{a_t \exp\left[-\int_0^t (r_s - n)ds\right]\right\} = 0. \tag{1.4}$$

The final good is a composite of a continuum of intermediate varieties produced by the monopolistically competitive sector. At every point in time, conditional on individual expenditure on the final consumption good, each consumer decides how much of her expenditure is spent on each variety ω belonging to the set of varieties Σ available in the economy. Let $C_t = c_t L_t$ denote the total amount of final good consumed in the economy. Consumer preferences over varieties are constant elasticity of substitution (CES) and can be written as:

$$C_t = \left(\int_{\omega\in\Sigma} y_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}}, \tag{1.5}$$

where $\sigma > 1$ is the elasticity of substitution, and $y_t(\omega)$ is total quantity of variety ω consumed in the economy at time t.

The intratemporal consumer maximization problem yields the standard total demand for individual variety:

$$y_t(\omega) = \frac{p_t(\omega)^{-\sigma} E_t}{P_t^{1-\sigma}}, \tag{1.6}$$

where E_t is the total consumer expenditure in the economy, $p_t(\omega)$ is the price of individual variety ω, and P_t is the price index, or the price of the final consumption good:

$$P_t \equiv \left[\int_{\omega\in\Sigma} p_t(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}. \tag{1.7}$$

The final consumption good is assumed to be the numeraire, and its price, P_t, is normalized to unity. So all prices are measured relative to the price of the final consumption good.

1.2.2 Product markets

Workers inelastically supply one unit of labor at every moment in time. Labor can be used either for production of the final good, which is a composite of intermediate varieties produced by the monopolistically competitive sector modeled similar to Melitz (2003), or in the creation of new intermediate varieties through research and development (R&D). Each innovation produced by R&D generates a product and a process innovation. Product ownership for intermediate varieties is protected by an infinitely lived patent, but when it comes to the process innovation the R&D technology allows for knowledge spillovers from existing firms to potential entrants.

Intra-temporal problem of a firm

The final good is non-tradable and is produced under perfect competition using the CES production function. Production in the intermediate sector is performed by monopolistically competitive firms producing differentiated varieties. The only factor of production is labor, and firms are heterogeneous in their labor productivity θ, which is constant over time for each firm. The labor needed to produce quantity y_t of a differentiated good by a firm with productivity θ is

$$l(y_t) = f + \frac{y_t}{\theta}, \tag{1.8}$$

where f is the fixed overhead production cost, denominated in units of labor. Similar to Melitz (2003), firms can sell their intermediate varieties both at home and abroad. Firms selecting into exporting face the additional fixed cost f_x per market and variable iceberg trade cost $\tau > 1$. Both fixed and variable trade costs are denominated in units of labor. As it is costless for producers to differentiate their product, and because all varieties enter symmetrically into consumer preferences, each firm produces a unique variety, and in what follows I will index firms by their productivity θ instead of variety ω.

The firm's static optimization problem is equivalent to Melitz (2003), and the solution is standard to the literature. At every point in time the combined profits from domestic and export sales for a firm with productivity θ are given by

$$\pi_t(\theta) = \pi_t^d(\theta) + \max[0, J\pi_t^x(\theta)], \tag{1.9}$$

where $\pi_t^d(\theta)$ are the profits from domestic sales, and $\pi_t^x(\theta)$ are the profits from exporting to a single foreign destination. I assume that if a firm exports, it exports to all J foreign countries.

Let p_t^d denote the price a firm with productivity θ charges domestic consumers. Then, due to the iceberg trade cost, $\tau > 1$, firms charge the export price $p_t^x(\theta) = \tau p_t^d(\theta)$ to foreign consumers. Maximizing its combined profits subject to demand for its intermediate variety, each firm sets the domestic price equal to a constant markup over its marginal cost:

$$p_t^d(\theta) = \frac{\sigma}{\sigma - 1} \frac{w_t}{\theta}. \tag{1.10}$$

Similar to Melitz (2003), I define θ_t^* to be a cutoff level of productivity such that firms with productivity below this level choose not to produce: $\pi_t^d(\theta_t^*) = 0$. I also introduce a productivity cutoff for exporting: firms with productivity lower than θ_t^x find it not optimal to export: $\pi_t^x(\theta_t^x) = 0$. Following Melitz (2003), it is possible to use the zero-cutoff profit conditions to derive the relationship between domestic and export cutoff productivities:

$$\theta_t^x = \theta_t^* \tau \left(\frac{f_x}{f} \right)^{1/(\sigma - 1)}. \tag{1.11}$$

Notice that the usual assumption about fixed costs and variable trade costs $\tau^{\sigma-1} f_x > f$ implies that not all firms choose to export ($\theta_t^x > \theta_t^*$).

By choosing the final consumption good as a numeraire and setting its price to unity, $P_t = 1$, I solve for the domestic productivity cutoff as a function of the wage rate, total consumer expenditure, fixed production costs, and elasticity of substitution between intermediate varieties:

$$\theta_t^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1} \left(\frac{w_t^\sigma f}{c_t L_t} \right)^{\frac{1}{\sigma-1}}. \tag{1.12}$$

In the remaining part of the paper, it will be useful to use relative productivity notation. For a firm with productivity θ, let ϕ_t denote the firm's productivity relative to the domestic productivity cutoff:

$$\phi_t \equiv \frac{\theta}{\theta_t^*}. \tag{1.13}$$

Then the exporter productivity cutoff relative to the domestic productivity cutoff is a constant:

$$\tilde{\phi} \equiv \frac{\theta_t^x}{\theta_t^*} = \tau \left(\frac{f_x}{f} \right)^{1/(\sigma - 1)}. \tag{1.14}$$

Note that if $\phi_t \geq 1$, a firm chooses to produce, and if $\phi_t \geq \tilde{\phi}$, a firm chooses to export.

Using the zero profit cutoff conditions, equilibrium profits and labor demanded for production for the home market and for export can be rewritten as functions of the relative productivity cutoff:

$$\pi_t^d(\phi_t) = fw_t(\phi_t^{\sigma-1} - 1) \qquad \pi_t^x(\phi_t) = fw_t\tau^{1-\sigma}(\phi_t^{\sigma-1} - \tilde{\phi}_t^{\sigma-1}) \tag{1.15}$$

$$l_t^d(\phi_t) = f[(\sigma-1)\phi_t^{\sigma-1} + 1] \qquad l_t^x(\phi_t) = f\tau^{1-\sigma}[(\sigma-1)\phi_t^{\sigma-1} + \tilde{\phi}^{\sigma-1}]. \tag{1.16}$$

Let $I_t[\phi_t \geq \tilde{\phi}]$ be an indicator function which takes value of one if the firm is exporting at time t, and zero otherwise. Since there are J export markets, total firm employment is given by $l_t(\phi_t) = l_t^d(\phi_t) + Jl_t^x(\phi_t) \cdot I_t[\phi_t \geq \tilde{\phi}]$ and combined firm profits are $\pi_t(\phi_t) = \pi_t^d(\phi_t) + J\pi_t^x(\phi_t) \cdot I_t[\phi_t \geq \tilde{\phi}]$.

Firm entry and decreasing returns to scale in R&D

A firm in the intermediate sector can lend or borrow at interest rate r_t. Let $W_t(\phi_t)$ be the value of a firm with relative productivity ϕ_t at time t, given by the present discounted value of the firm's future profits:

$$W_t(\phi_t) = \int_t^\infty \pi_\nu(\phi_\nu) \exp\left[-\int_t^\nu r_s ds\right] d\nu. \tag{1.17}$$

In each economy firm entry takes place via a research and development (R&D) activity, financed through a costless intermediation sector, which owns existing firms and pools the risk faced by innovators. Let N_t denote the aggregate labor employed in the R&D sector at time t, which produces a flow Ω_t of innovations, where each innovation represents an emerging firm. It follows that the R&D cost in terms of units of labor per individual entering firm is given by

$$F_t \equiv N_t/\Omega_t. \tag{1.18}$$

The R&D cost F_t potentially depends on the aggregate mass of entrants Ω_t. However each individual firm treats this cost as given.

The baseline model in Sampson (2016) features a constant returns to scale in R&D assumption. In particular, it is assumed that the flow of innovations Ω_t is linear in the labor employed in R&D:

$$\Omega_t = N_t/f_e, \tag{1.19}$$

where f_e is the entry cost parameter. Intuitively, this means that doubling the aggregate R&D labor would lead to twice as many innovations. Also, it means that the entry cost for an individual firm is constant and is equal to the entry cost parameter $F_t = f_e$.

The assumption of constant returns to scale in R&D does not find support in the empirical literature on patents and R&D, and, instead, the decreasing returns to scale in R&D assumption is found to be more realistic. In particular, Blundell, Griffith and Windmeijer (2002) use data on R&D expenditures and patents of large US firms to show that the long run elasticity of patents with respect to R&D is approximately 0.5. Moreover, Kortum (1993) surveys the literature and finds that the estimates for this elasticity are in range of 0.1 to 0.6. To accommodate this evidence and to model congestion in entry, I consider functional forms for the R&D technology that allow for decreasing returns to scale in R&D.

In their recent paper, Segerstrom and Sugita (2016) suggest the following functional form for the flow of innovations to model decreasing returns to scale in R&D:

$$\Omega_t = \left(\frac{N_t}{f_e}\right)^{\beta}, \tag{1.20}$$

where $\beta \in (0,1)$ measures the degree of decreasing returns to scale in R&D.[3] Setting $\beta = 1$ yields the functional form for constant returns given by (1.19). With $\beta < 1$, doubling the R&D labor N_t would less than double the flow of innovations Ω_t. From (1.18) and (1.20), the R&D cost in units of labor for an individual firm can be expressed as $F_t = f_e \Omega_t^{(1-\beta)/\beta}$, which is not a constant anymore and is strictly increasing in the mass of emerging firms Ω_t. In this model the R&D activity results in creation of new varieties, hence the competition in innovation is about coming up with an original idea. When more firms enter the chance of duplicating increases and entry becomes more expensive for each individual firm.

An alternative way to model decreasing returns to scale in R&D was suggested by Sampson (2016) as one of the robustness checks in his appendix.[4] In

[3]The parameter β corresponds to $1/(\zeta+1)$ with $\zeta > 0$ in Segerstrom and Sugita (2016) notation.

[4] Sampson imposes additional restrictions on the function Ω: it is homogeneous of degree one, strictly increasing in the R&D labor N_t, weakly increasing in the mass of incumbent firms

the numerical exercise Sampson (2016) uses the following functional form for the flow of innovations:

$$\Omega_t = \left(\frac{N_t}{f_e}\right)^{\alpha} M_t^{1-\alpha} \tag{1.21}$$

where M_t is the mass of incumbent firms, $\alpha \in (0,1)$ measures the degree of decreasing returns to scale in R&D, and $\alpha = 1$ corresponds to constant returns. The entry cost in units of labor encountered by an individual firm can be expressed as $F_t = f_e \left(\Omega_t / M_t\right)^{(1-\alpha)/\alpha}$, which is strictly increasing in the mass of entrants Ω_t, similar to the Segerstrom and Sugita (2016) functional form, but is also strictly decreasing in the mass of incumbent producers M_t. The assumption that an individual entry cost increases in the aggregate mass of entrants Ω_t helps to model congestion in entry. However, the assumption that the entry cost is decreasing in the mass of incumbent producers M_t is less obvious. Sampson (2016) suggests that the R&D is more productive when there are more incumbent firms to learn from. One more possible explanation is that an increase in M_t expands the variety space and provides additional opportunities for creation of new products. On the other hand, one could argue that the R&D cost is increasing in the mass of incumbents M_t due to the fishing out effect: it becomes more difficult for researchers to come up with new inventions because the easiest discoveries have already been made.

In Section 4.2.1, I perform numerical simulation of the model using both functional forms and I show that the Segerstrom and Sugita (2016) functional form generates larger dynamic gains from trade compared to the Sampson (2016) specification.

Knowledge spillovers

After the R&D cost has been paid, every newly emerging firm draws its productivity. The draw θ depends on the average productivity of incumbent firms x_t at time t and on a stochastic component ψ with cumulative distribution function $F(\psi)$:

$$\theta = x_t \psi. \tag{1.22}$$

M_t, and satisfies $\Omega(0,0) = 0$. The homogeneity of degree one restriction on the functional form of the R&D technology is not necessary to model decreasing returns to scale in R&D, and it is possible to solve for the balanced growth equilibrium path using the simpler functional form given by (1.20).

Variation in x_t captures knowledge spillovers from existing firms to entrants. As suggested by Sampson (2016), when assuming such a form of knowledge spillovers between existing producers and entrants, the model is able to explain the substantial heterogeneity of entrants' productivity and the co-movement of productivity distributions of entrants and incumbents over time observed in the data. Note that the knowledge spillovers are assumed to be intra-national in nature.[5]

Let $G_t(\theta)$ be the cumulative productivity distribution of firms that produce at time t. The distribution of entrants' productivity is given by $\tilde{G}_t(\theta) = F(\theta/x_t)$. Denote the cumulative distribution functions of relative productivity ϕ for existing firms and entrants as $H_t(\phi_t)$ and $\tilde{H}_t(\phi_t)$ respectively. The free entry condition implies that in equilibrium the expected cost of innovating equals the expected value of creating a new firm:

$$F_t w_t(1 - v_e) = \int_\phi W_t(\phi) d\tilde{H}_t(\phi), \tag{1.23}$$

where F_t is the labor cost of generating a new firm, w_t is the wage, v_e is the share of R&D costs covered by the government (the R&D subsidy rate), and the integral represents the expected present discounted value of a firm entering at time t, which is itself affected by the productivity distribution of entrants.

Let M_t denote the mass of producers in the economy at time t. I assume that the domestic productivity cutoff θ_t^* is strictly increasing over time. Then at time $t + \Delta$ the mass of producing firms with relative productivity below ϕ is approximated[6] by

$$\begin{aligned} M_{t+\Delta} H_{t+\Delta}(\phi) \approx & M_t \left[H_t \left(\frac{\theta_{t+\Delta}^*}{\theta_t^*} \phi \right) - H_t \left(\frac{\theta_{t+\Delta}^*}{\theta_t^*} \right) \right] + \\ & + \Delta \Omega_t \left[F \left(\frac{\phi \theta_{t+\Delta}^*}{x_t} \right) - F \left(\frac{\theta_{t+\Delta}^*}{x_t} \right) \right], \end{aligned} \tag{1.24}$$

where the first term on the right hand side is the mass of time t incumbents that still produce but have relative productivity less than ϕ at time $t + \Delta$, and

[5] Sampson (2016) also studies international knowledge spillovers, which is not a focus of this paper.

[6] This is a minor correction to Sampson (2016). where the equation corresponding to (1.24) is written with equality. However, the approximation is only exact in the limit as Δ converges to zero.

the second term on the right hand side is the approximate mass of firms that entered between t and $t+\Delta$ with relative productivity between 1 and ϕ as of time $t+\Delta$. I use (1.24) to derive the laws of motion for the mass of incumbent firms and their productivity distribution. In particular, the growth rate of the mass of incumbents at time t is given by

$$\frac{\dot{M}_t}{M_t} = -H_t'(1)\frac{\dot{\theta}_t^*}{\theta_t^*} + \frac{\Omega_t}{M_t}\left[1 - F\left(\frac{\theta_t^*}{x_t}\right)\right], \tag{1.25}$$

which means that the growth rate of the mass of producers is affected by the rate at which the productivity cutoff grows leading to some firms exiting when their relative productivity falls below 1, and by the rate of successful entry. The law of motion for the relative productivity distribution is

$$\begin{aligned}\dot{H}_t(\phi) = \left(H_t'(\phi)\phi - H_t'(1)\left[1 - H_t(\phi)\right]\right)\frac{\dot{\theta}_t^*}{\theta_t^*} + \\ + \frac{\Omega_t}{M_t}\left[F\left(\frac{\phi\theta_t^*}{x_t}\right) - F\left(\frac{\theta_t^*}{x_t}\right) - H_t(\phi)\left[1 - F\left(\frac{\theta_t^*}{x_t}\right)\right]\right],\end{aligned} \tag{1.26}$$

indicating that the relative productivity distribution evolves due to growth in the productivity cutoff and to new entry.

1.2.3 Equilibrium

Three more conditions are needed in order to fully characterize the equilibrium in the economy. The first one is the labor market clearing condition:

$$L_t = M_t \int_\phi l_t(\phi)dH_t(\phi) + N_t, \tag{1.27}$$

which means that the sum of labor used in production and in the innovation activities should sum up to the total supply of labor in the economy.

The asset market clearing condition implies that the aggregate household assets should be equal to the total present discounted value of all firms operating in the economy:

$$a_tL_t = M_t \int_\phi W_t(\phi)dH_t(\phi), \tag{1.28}$$

Each household holds a balanced portfolio of all firms and R&D projects in the economy.

An additional equilibrium condition is the balanced budget of the government, i.e. the R&D subsidy is financed by an aggregate lump-sum tax on consumers:

$$b_t L_t = w_t N_t v_e, \tag{1.29}$$

where b_t is the lump-sum tax per capita, so $b_t L_t$ is the total tax payment, $w_t N_t$ is the total cost of R&D, and v_e is the share of R&D costs covered by the government.

The equilibrium in the world economy is defined by time paths $t \in [0, \infty)$ for

$$c_t, a_t, b_t, w_t, r_t, \theta_t^*, \theta_t^x, W_t(\phi), M_t, N_t, \Omega_t, \text{ and } H_t(\phi),$$

such that the following conditions hold:

- consumers maximize (1.1) subject to (1.2), which gives the Euler Equation (1.3) and the transversality condition (1.4);
- producers maximize profits, which gives the export productivity cutoff (1.11), the domestic productivity cutoff (1.12), and the firm value (1.17);
- the free entry into R&D condition (1.23);
- the domestic productivity cutoff is strictly increasing over time ($\dot{\theta}_t^* > 0$), and the laws of motion for M_t and $H_t(\phi)$ are given by (1.25) and (1.26);
- the labor market clearing condition (1.27);
- the asset market clearing condition (1.28);
- the balanced budget of the government condition (1.29);
- the initial mass of potential producers at time zero is given by $\hat{M}_0$ with productivity distribution $\hat{G}_0(\theta)$.

1.3 Balanced growth path

On a balanced growth equilibrium path $c_t, a_t, w_t, r_t, \theta_t^*, \theta_t^x, W_t(\phi), M_t, N_t,$ and Ω_t grow at constant rates and the distribution of relative productivity ϕ is stationary, meaning $\dot{H}_t(\phi) = 0$ for all t and ϕ.

1.3.1 Stationary relative productivity distribution

As in Sampson (2016), I make the following assumption about the sampling productivity distribution[7]:

Assumption 1. *The sampling productivity distribution F is Pareto:* $F(\psi) = 1 - (\psi/\psi_{min})^{-k}$ *for* $\psi \geq \psi_{min}$*, where* $k > \max\{1, \sigma - 1\}$. *Moreover, the lower bound of the sampling productivity distribution satisfies* $x_t \psi_{min} < \theta_t^*$.

The second part of the assumption means that not all entrants draw productivity that is above the exit cutoff θ_t^*.

By substituting for F in (1.26), and by focusing on a stationary distribution by setting $\dot{H}_t(\phi) = 0$, I obtain the following differential equation:

$$0 = \left\{H'(\phi)\phi - H'(1)\left[1 - H(\phi)\right]\right\} \frac{\dot{\theta}_t^*}{\theta_t^*} + \frac{\Omega_t}{M_t}\left(\frac{\theta_t^*}{x_t}\right)^{-k} \psi_{min}^k \left[1 - \phi^{-k} - H(\phi)\right]. \tag{1.30}$$

This differential equation has to be solved for a stationary relative productivity distribution $H(\phi)$. It is easy to see that the Pareto distribution $H(\phi) = 1 - \phi^{-k}$ is a solution independently of the functional form of Ω (use $H'(\phi) = k\phi^{-k-1}$ and $H'(1) = k$). Lemma 1 states this result[8].

Lemma 1. *Given Assumption 1, there exists a stationary relative productivity distribution:* $H(\phi) = 1 - \phi^{-k}$.

In solving for the balanced growth path, I will focus on the stationary relative productivity distribution given by Lemma 1. Then on the balanced growth path the productivity distribution of incumbents $G_t(\theta)$ is Pareto with shape parameter k and scale parameter θ_t^*. Using properties of the Pareto distribution, the average productivity of incumbents is $x_t = \frac{k}{k-1}\theta_t^*$. It is useful to

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[7] Sampson (2016) shows that it is also possible to solve for the balanced growth path without restricting the functional form of the entrants' productivity distribution.

[8] In the Theoretical Appendix I show that this is not the only stationary relative productivity distribution satisfying (1.30). This constitutes a correction to Sampson (2016), where the uniqueness of the stationary relative productivity distribution is stated in Lemma 1. However the proof to the original lemma misses the fact that the Picard-Lindelöf theorem can not be applied due to presence of the term $H'(1)$ in (1.30). Ignoring the $H'(1)$ term in Sampson's proof leads to the loss of an infinite number of solutions to the differential equation. See the Theoretical Appendix for details.

define a measure of the strength of knowledge spillovers:

$$\lambda \equiv \frac{x_t \psi_{min}}{\theta_t^*} = \frac{k}{k-1}\psi_{min} < 1. \tag{1.31}$$

Then on the balanced growth path the relative productivity distribution of entrants is

$$\tilde{H}_t(\phi) = H\left(\frac{\phi}{\lambda}\right) \tag{1.32}$$

and the fraction of entrants that draw productivity below the productivity cutoff θ_t^* and exit immediately is $\tilde{H}(1) = 1 - \lambda^k$. So, the productivity distribution of entrants is of the same functional form as the productivity distribution of incumbents, but shifted to the left.

1.3.2 Dynamic selection

Let $q \equiv \dot{c}_t/c_t$ be the growth rate of consumption per capita. Then from the budget constraint (1.2), wages, assets and tax per capita all grow at the same constant rate:

$$\frac{\dot{a}_t}{a_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{b}_t}{b_t} = q. \tag{1.33}$$

Substituting for the consumption growth rate in the Euler equation (1.3), I obtain the relationship between the growth rate q and the interest rate r_t. It follows that the interest rate is constant $r_t = r$:

$$q = \gamma(r - \rho). \tag{1.34}$$

Finally, for the transversality condition (1.4) to hold with $r_t = r$, I need $r - n > q$, which implies that $q\frac{1-\gamma}{\gamma} + \rho - n > 0$.

Let g be the growth rate of the domestic productivity cutoff θ_t^*. As a firm's actual productivity θ remains constant over time, g is also the rate at which the firm's relative productivity ϕ_t decreases. Log-differentiating (1.12) I obtain

$$g \equiv \frac{\dot{\theta}_t^*}{\theta_t^*} = \frac{1}{\sigma - 1}\left(\sigma\frac{\dot{w}_t}{w_t} - \frac{\dot{c}_t}{c_t} - \frac{\dot{L}_t}{L_t}\right).$$

Using this expression, I can express the growth rate of consumption $q \equiv \dot{c}_t/c_t$ as a function of the productivity cutoff growth rate $g \equiv \dot{\theta}_t^*/\theta_t^*$ and the population growth rate $n \equiv \dot{L}_t/L_t$:

$$q = g + \frac{n}{\sigma - 1}. \tag{1.35}$$

Equation (1.35) shows that the growth in the productivity cutoff g contributes to the economic growth rate q. The combination of the Melitz-style firm selection with the knowledge spillovers generates a long-run growth in the productivity cutoff and continuously shifts the productivity distribution of producing firms upwards. This channel for economic growth is absent from most endogenous growth models.

Moreover, as is standard in most endogenous growth models, the growth rate of the economy $q \equiv \dot{c}_t/c_t$ depends on the population growth rate $n \equiv \dot{L}_t/L_t$. To see why this is the case, consider the labor market clearing condition (1.27). Substituting for labor demand from (1.16) and for the relative productivity distribution given by Lemma 1, the labor market clearing condition implies that

$$L_t = M_t f \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} \left[1 + J \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k} \right] + N_t. \tag{1.36}$$

The population L_t, the mass of producers M_t, and the labor involved in R&D N_t should grow at the same constant rate on the balanced growth path:

$$\frac{\dot{L}_t}{L_t} = \frac{\dot{M}_t}{M_t} = \frac{\dot{N}_t}{N_t} = n. \tag{1.37}$$

With growth of the population the mass of produced varieties M_t also grows, and the more love for variety consumers exhibit (the lower the elasticity of subsitution σ is in (1.35)), the higher is the consumption growth associated with an increase in the number of produced varieties.

I have established that the mass of producing firms grows at a constant rate n and the productivity cutoff grows at rate g. Applying the functional form of the stationary relative productivity distribution given by Lemma 1, and the fraction of successful entry given by λ^k, the law of motion for the mass of producing firms (1.25) can be rewritten as follows:

$$\frac{\dot{M}_t}{M_t} = -k \frac{\dot{\theta}_t^*}{\theta_t^*} + \frac{\Omega_t}{M_t} \lambda^k. \tag{1.38}$$

The fraction of entrants that draw productivity below the exit cutoff is $\tilde{H}(1) = 1 - \lambda^k$, so the fraction of entrants that draw productivity above the exit cutoff is λ^k and the successful entry rate is $\frac{\Omega_t}{M_t}\lambda^k$. The rate at which existing producers exit is proportional to the growth rate of the productivity cutoff and is equal to kg. Then, the successful entry rate is equal to the population growth rate plus the exit rate for existing firms:

$$\frac{\Omega_t}{M_t}\lambda^k = n + kg. \tag{1.39}$$

Finally, substituting (1.39) into the labor market clearing condition (1.36), I can solve for the mass of producing firms:

$$M_t = \left[f \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} \left[1 + J \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma - 1}} \tau^{-k} \right] + \frac{n + kg}{\lambda^k} F_t \right]^{-1} L_t. \tag{1.40}$$

1.3.3 Solving for balanced growth path

Because the relative productivity of existing firms is decreasing over time as the productivity cutoff grows, it is easy to show that, conditional on existing at time t, a firm will exit at time $t + \frac{\ln \phi_t}{g}$ and, conditional on exporting at time t, a firm will stop exporting at time $t + \frac{\ln(\phi_t/\tilde{\phi})}{g}$. Then the present discounted value of future profits of a firm with relative productivity ϕ_t at time t is given by:

$$\begin{aligned} W_t(\phi_t) = &\int_t^{t+\frac{\ln \phi_t}{g}} \pi_v^d(\phi_v) e^{-r(v-t)} dv \\ &+ J \int_t^{t+\frac{\ln(\phi_t/\tilde{\phi})}{g}} \pi_v^x(\phi_v) e^{-r(v-t)} \cdot I[\phi_t \geq \tilde{\phi}] dv. \end{aligned} \tag{1.41}$$

By substituting for profits from (1.15) and for the growth rates of wages and relative productivity, I obtain the following expression for the present discounted value of a firm:

$$\begin{aligned} W_t(\phi_t) = &f w_t \left[\frac{\phi_t^{\sigma-1} - \phi_t^{-\frac{r-q}{g}}}{g(\sigma-1) + r - q} + \frac{\phi_t^{-\frac{r-q}{g}} - 1}{r - q} \right] \\ &+ f w_t I[\phi_t \geq \tilde{\phi}] J \frac{f_x}{f} \left[\frac{\left(\phi_t/\tilde{\phi}\right)^{\sigma-1} - \left(\phi_t/\tilde{\phi}\right)^{-\frac{r-q}{g}}}{g(\sigma-1) + r - q} + \frac{\left(\phi_t/\tilde{\phi}\right)^{-\frac{r-q}{g}} - 1}{r - q} \right]. \end{aligned} \tag{1.42}$$

Substituting (1.42) into the free entry condition (1.23), while accounting for (1.32), the free entry condition can be rewritten as:

$$F_t w_t(1-v_e) = \lambda^k f w_t \left(1 + J\frac{f_x}{f}\tilde{\phi}^{-k}\right)\frac{\sigma-1}{k+1-\sigma}\frac{1}{r+kg-q}, \tag{1.43}$$

where $\sigma - 1$ and $k+1-\sigma$ are positive constants. The left-hand side of (1.43) represents the entry cost for an individual firm, whereas the right-hand side represents the expected present discounted value of profits from firm entry at time t. The firm profits are appropriately discounted using the market interest rate r and the rate kg at which existing producers exit. Also taken into account are the capital gains q that capture the fact that firm profits grow over time.

To further explain the intuition behind the free entry condition, it is useful to rewrite it using (1.34) and (1.35):

$$\begin{aligned} F_t(1-v_e) =& \lambda^k f \left(1 + J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}\right) \cdot \\ & \cdot \frac{\sigma-1}{k+1-\sigma}\frac{1}{g\frac{1+(k-1)\gamma}{\gamma} + \frac{1-\gamma}{\gamma}\frac{n}{\sigma-1} + \rho}. \end{aligned} \tag{1.44}$$

Consider first the case of constant returns to scale in R&D given by (1.19). In this case the entry cost for an individual firm is constant, $F_t = f_e$. Trade liberalization, which takes the form of a decrease in variable trade cost τ, raises the term in large brackets on the right hand side of the free entry condition (1.44), leading to an increase in the profits from exporting. The expected value of entry goes up, which encourages more firms to enter. By (1.39), more firms entering leads to an increase in the growth rate of the productivity cutoff g. A higher growth rate of the productivity cutoff means that each firm's relative productivity declines faster and the expected lifetime of a firm decreases, leading to a decrease in the expected value of entry.

Thus, the free entry condition implies that in equilibrium an increase in the expected profits from exporting is offset by a decrease in the expected lifetime of a firm due to a higher dynamic selection. The free entry condition does not help to pin down productivity cutoffs, as it does in the Melitz (2003) model, however it does help to pin down the dynamic selection rate g $[\tau \downarrow \Longrightarrow g \uparrow]$.

When there are decreasing returns to scale in R&D, an increase in the mass of entrants also affects the individual entry cost F_t. Then, by the free entry condition (1.44), an increase in the expected profit from exporting can be offset by a decrease in the expected lifetime of a firm or by an increase in the individual entry cost F_t. Proposition 1 shows that, for particular forms of the R&D technology, the effects of trade liberalization are qualitatively the same as when there is constant returns. Moreover, it also shows that, increasing the R&D subsidy rate has the same qualitative effects as trade liberalization.

Proposition 1. *When the R&D technology is given by the functional forms* (1.19), (1.20) *or* (1.21), *a decrease in the variable trade cost τ unambiguously leads to an increase in the rate of dynamic selection g, an increase in the successful entry rate $\lambda^k \Omega_t / M_t$, and a decrease in the equilibrium mass of producers M_t.* $[\tau \downarrow \Longrightarrow g \equiv \dot{\theta}_t^* / \theta_t^* \uparrow, \lambda^k \Omega / M_t \uparrow,$ *and* $M_t \downarrow]$ *An increase in the R&D subsidy rate v_e has the same effect.* $[v_e \uparrow \Longrightarrow g \equiv \dot{\theta}_t^* / \theta_t^* \uparrow, \lambda^k \Omega / M_t \uparrow,$ *and* $M_t \downarrow]$

Using (1.34), (1.35), and (1.44), I can solve for the growth rate of consumption per capita:

$$q = \frac{\gamma}{1 + (k-1)\gamma} \cdot \left(\frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{F_t (1 - v_e)} \left(1 + J \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma - 1}} \tau^{-k} \right) + k \frac{n}{\sigma - 1} - \rho \right). \tag{1.45}$$

Under constant returns to scale in R&D, (1.45) provides a closed form solution for the growth rate of consumption, (1.39) solves for the rate of successful entry, and (1.40) defines the equilibrium mass of producers in the economy. Under decreasing returns to scale in R&D, equations (1.45), (1.39), and (1.40) jointly determine equilibrium values of q, Ω_t, and M_t.

To complete the solution for the balanced growth path, I solve for the initial consumption level.

Assumption 2. *Assume that the initial mass of potential entrants is exogenously given by $\hat{M}_0$, and their productivity distribution $\hat{G}_0(\theta) = 1 - \left(\theta / \hat{\theta}_0^*\right)^{-k}$ is Pareto with scale parameter $\hat{\theta}_0^*$ and shape parameter k.*

Let θ_0^* be the equilibrium domestic productivity cutoff at time $t = 0$. Then the equilibrium mass of incumbents at time $t = 0$ is given by $M_0 = \hat{M}_0 (1 - \hat{G}_0(\theta_0^*))$. Given Assumption 2, the equilibrium domestic productivity cutoff

θ_0^* at time $t = 0$ is given by:

$$\theta_0^* = \hat{\theta}_0^* \left(\frac{\hat{M}_0}{M_0} \right)^{1/k}. \tag{1.46}$$

Substituting for the initial domestic productivity cutoff in (1.12), I obtain an expression for the wage rate at time $t = 0$:

$$w_0 = \left(\hat{\theta}_0^*\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\hat{M}_0}{M_0} \right)^{\frac{1}{k}\frac{\sigma-1}{\sigma}} c_0^{\frac{1}{\sigma}} L_0^{\frac{1}{\sigma}} f_0^{-\frac{1}{\sigma}} \frac{(\sigma-1)^{\frac{\sigma-1}{\sigma}}}{\sigma}. \tag{1.47}$$

Further, I can use $W_0(\phi_0)$ given by (1.42) in the asset market clearing condition (1.28) to obtain

$$a_0 = \frac{M_0}{L_0} w_0 \frac{F_0}{\lambda^k} (1 - v_e), \tag{1.48}$$

and from the government balanced budget condition (1.29), I obtain

$$b_0 = w_0 \frac{N_0}{L_0} v_e. \tag{1.49}$$

From the budget constraint (1.2), I can derive the following condition for the value of initial consumption:

$$c_0 = w_0 + \left(q\frac{1-\gamma}{\gamma} + \rho - n \right) a_0 - b_0, \tag{1.50}$$

where $q\frac{1-\gamma}{\gamma} + \rho - n$ is the marginal propensity to consume from wealth and is positive by the transversality condition. By substituting into (1.50) for M_0, w_0, a_0, and for b_0 from (1.40), (1.47), (1.48), and (1.49), I obtain the final expression for initial consumption:

$$\begin{aligned} c_0 =& A f^{-\frac{k+1-\sigma}{k(\sigma-1)}} \left[1 + J \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k} \right]^{\frac{1}{k}} \cdot \\ & \cdot \left[k\sigma + 1 - \sigma + \frac{n + kg}{q\frac{1-\gamma}{\gamma} + kg + \rho} \frac{\sigma - 1}{1 - v_e} \right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}, \end{aligned} \tag{1.51}$$

where A is a positive constant:

$$A \equiv (\sigma - 1)k^{\frac{\sigma}{\sigma-1}} (k + 1 - \sigma)^{-\frac{1}{k}} \hat{\theta}_0^* \hat{M}_0^{\frac{1}{k}} L_0^{\frac{k+1-\sigma}{k(\sigma-1)}}.$$

This completes the characterization of the balanced growth path.

Assumption 1 ensures that in equilibrium the relative productivity distribution $H_t(\phi)$ jumps immediately at $t = 0$ to the stationary relative productivity distribution given by Lemma 1. Moreover, if the economy is on the balanced growth path, there is no transitional dynamics following trade liberalization. To see this notice that the state variables in this model are the relative productivity distribution $H(\phi)$, which is independent of time, and the mass of producing firms. When trade liberalization occurs, the mass of producing firms declines instantaneously through (1.40) following the jump in the domestic productivity cutoff θ_t^*. This ensures that the economy jumps from one balanced growth path to another without any transitional dynamics.

1.3.4 Gains from trade

Consumption per capita c_0 and its growth rate q affect consumer welfare U through the following relationship, obtained by substituting $c_t = c_0 e^{qt}$ into the consumer welfare function (1.1):

$$U = \frac{\gamma}{1-\gamma} \left(\frac{1}{\rho - n} - \frac{\gamma c_0^{-\frac{1-\gamma}{\gamma}}}{(1-\gamma)q + \gamma(\rho - n)} \right). \tag{1.52}$$

Trade liberalization affects consumer welfare through both c_0 and q. First, from Proposition 1, trade liberalization leads to an increase in the rate of dynamic selection g and, consequently, to an increase in the consumption growth rate q. As $0 < \gamma < 1$, q has a direct positive effect on consumer welfare in (1.52). Second, trade liberalization has a direct positive effect on the consumption level c_0 in (1.51) through the term $J\tau^{-k} (f/f_x)^{(k+1-\sigma)/(\sigma-1)}$. In addition, consumption level c_0 is affected by trade liberalization indirectly through an increase in q, and the total effect of higher q on c_0 can be either positive or negative. For the model without an R&D subsidy and with a general form of R&D technology, it can be shown that the total effect of trade liberalization on the consumer welfare is positive regardless of the effect of the growth rate

q on consumption c_0. Proposition 2 states this results formally. For the model with a positive R&D subsidy rate, the effect is ambiguous.

Proposition 2. *A decrease in the variable trade cost τ unambiguously leads to an increase in welfare U in the model without an R&D subsidy and with a general form of R&D technology.* [$\tau \downarrow \implies U \uparrow$, *when* $v_e = 0$.]

The total gains from trade z are defined as the proportional increase in the autarky level of consumption required to obtain the open economy welfare level:

$$U(zc_0^A, q^A) = U(c_0, q).$$

Using (1.52), I can solve for the total gains from trade z:

$$z = \frac{c_0^A}{c_0}\left[\frac{(1-\gamma)q + \gamma(\rho - n)}{(1-\gamma)q^A + \gamma(\rho - n)}\right]^{\frac{\gamma}{1-\gamma}}.$$

The total gains from trade can be further decomposed into the static gains and the dynamic gains:

$$z = z_s \cdot z_d.$$

From (1.51), trade raises welfare by increasing c_0 for any given growth rate. If the growth rate was not affected by trade integration (the case where $q = q^A$), then the total gains from trade would be equal to the static gains:

$$z_s \equiv \frac{c_0}{c_0^A} = \left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]^{\frac{1}{k}}. \tag{1.53}$$

Thus the dynamic gains from trade are defined by $z_d \equiv z/z_s$, capturing the fact that trade integration also affects welfare by increasing the consumption growth rate ($q > q^A$).

The static gains from trade are the only gains from trade in the model without knowledge spillovers and these gains from trade correspond to the ones obtained in Arkolakis, Costinot and Rodríguez-Clare (2012) for a wide range of static trade models. The dynamic gains from trade represent a new source for welfare gains from trade.

1.4 Calibration and numerical results

1.4.1 Calibration

While in the baseline case of constant returns to scale in R&D it is enough for Sampson (2016) to calibrate only a subset of parameters of the model to obtain a numerical estimate of the dynamic gains from trade, in the version of the model with decreasing returns to scale in R&D, this task requires calibration of a wider set of parameters.

As in Arkolakis, Costinot and Rodríguez-Clare (2012), the static gains from trade can be expressed as a function of the import penetration ratio (IPR) and the trade elasticity. To see this, first calculate the import expenditure (IMP) in each country:

$$IMP_t = JM_t \int_{\tilde{\phi}}^{\infty} e_t^x(\phi) dH_t(\phi) = \frac{k\sigma}{k+1-\sigma} M_t w_t f J \tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}. \quad (1.54)$$

Thus k is the trade elasticity, the elasticity of imports with respect to variable trade costs. To derive the import penetration ratio (IPR_t), divide (1.54) by total domestic sales $c_t L_t$ and use (1.50):

$$IPR_t = \frac{IMP_t}{c_t L_t} = \frac{J\tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}}{1 + J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}}. \quad (1.55)$$

By applying notation from (1.53), I obtain the expression for static gains from trade:

$$z_s = \left[\frac{1}{1 - IPR_t}\right]^{1/k}. \quad (1.56)$$

Sampson (2016) computes the U.S. import penetration ratio for year 2000 as imports of goods and services divided by gross output, and reports it to be equal to 0.081. As I would like to follow the original calibration as closely as possible, I will set $IPR = 0.081$. For the value of the trade elasticity, I follow Sampson (2016) and set $k = 7.5$. I also follow Sampson (2016) in setting parameters σ, γ, ρ, and n. The elasticity of substitution is calibrated to $\sigma = 8.1$, the intertemporal elasticity of substitution is set to $\gamma = 0.33$ and the discount rate to $\rho = 0.04$. Based on the average annual U.S. population growth rate from 1980 to 2000, Sampson (2016) sets $n = 0.011$.

The key to calibrate the dynamic gains from trade, as well as to solve numerically for endogenous variables, is to calibrate the expression λ^k. Notice that λ^k is the fraction of innovations that lead to the creation of new firms. Let NF be the rate of successful entry, whereas the flow of all innovations is Ω_t. Then, I have

$$NF = \lambda^k \frac{\Omega_t}{M_t}, \tag{1.57}$$

and using (1.39), I obtain:

$$g = \left(\lambda^k \frac{\Omega_t}{M_t} - n\right)\frac{1}{k} = \frac{NF - n}{k}. \tag{1.58}$$

Using this result in (1.45) gives:

$$\begin{aligned}\lambda^k =& \frac{k+1-\sigma}{\gamma k(\sigma-1)}(1-IPR)\frac{F_t}{f}(1-v_e)\cdot \\ &\cdot\left[(1+(k-1)\gamma)(NF-n) + \frac{k(1-\gamma)}{\sigma-1}n + \gamma k\rho\right].\end{aligned} \tag{1.59}$$

In order to calibrate λ^k, I follow Sampson (2016) and match the U.S. firm entry rate. As reported by Luttmer (2007), the U.S. entry rate per annum in 2002 was 11.6%, thus making it reasonable to set $NF = 0.116$.[9]

In calibrating the model to match the U.S. economy, Sampson (2016) assumes constant returns to scale in R&D ($\beta = 1$) and a zero R&D subsidy rate ($v_e = 0$). I search for more realistic values to match these parameters to.

Blundell, Griffith and Windmeijer (2002) estimate the long run elasticity of patents with respect to R&D to be approximately 0.5, which is in line with the survey by Kortum (1993). Applying this evidence to the context of the current model suggests that a reasonable value for the degree of decreasing returns to

[9] An alternative way to calibrate λ^k is to match the U.S. growth rate q. Jones (2015) suggests that GDP per person in the U.S. economy has grown at a steady average rate of around 2% per year. Due to equations (1.35) and (1.58), the firm entry rate NF is linearly related to the growth rate of consumption q, hence I can target only one of these observables at a time.

scale in R&D is $\beta = 0.5$ for the Segerstrom and Sugita (2016) functional form[10] and $\alpha = 0.5$ for the Sampson (2016) functional form.

According to OECD (2014), as of 2012 around 23% of business R&D expenditure in the U.S. was financed via direct public funding, including grants and subsidies. Another 10% of innovation costs were covered via indirect public funding, such as tax incentives. On average across OECD countries 10-20% of business R&D costs are funded by public money. Thus, I set the level of the R&D subsidy in the U.S. to $v_e = 0.25$.

Equation (1.45) implies that it is not the magnitude of the entry cost F_0, but the ratio of this cost to fixed cost of production f that is relevant for assessing the impact of entry costs on the growth rate of consumption. I follow Barseghyan and DiCecio (2011) in targeting an average ratio of entry to fixed production costs across several U.S. industries of $F_0/f = 0.82$. Equations (1.39), (1.40), and (1.45) jointly determine values of Ω_0, M_0, and q in trade equilibrium.

I also need to assume the initial level of labor in the economy. Notice that equations (1.40), (1.47), (1.48), (1.49), and (1.51) all depend on the ratio L_0/f, and never on levels of these parameters. As I am only interested in relative values of endogenous variables, I set $L_0/f = 1$ since this normalization does not affect the results.

To solve for the level of consumption and the productivity cutoffs under trade and autarky, I will need to assume the values of initial parameters $\hat{\theta}_0^*$ and $\hat{M}_0$. Notice that $\hat{\theta}_0^*$ is the scale parameter for the Pareto distribution, thus normalizing this to $\hat{\theta}_0^* = 1$ only affects the scale on which I compute productivities. $\hat{M}_0$ is a potential mass of entrants at time $t = 0$, and can also be safely normalized to $\hat{M}_0 = 1$. Using (1.12), (1.47), (1.48), (1.49), and (1.51), I can solve for $\theta_0^*, w_0, a_0, b_0$ and c_0.

Finally, I have calibrated the model to match the U.S. economy, which corresponds to a trade equilibrium with some level of trade costs. In order to quantify the gains from trade for different levels of trade costs, I calibrate parameters τ and the ratio f_x/f to match the two moments from U.S. data: the import penetration ratio IPR, given by (2.2.4), and the share of exporting

[10] The elasticity of patents with respect to R&D $\beta = 0.5$ corresponds to $\zeta = 1$ in the original Segerstrom and Sugita (2016) notation.

firms in manufacturing. The share of exporting firms in manufacturing is

$$SEF = \tilde{\phi}^{-k} = \left[\left(\frac{f_x}{f} \right)^{1/(\sigma-1)} \tau \right]^{-k}, \tag{1.60}$$

which is reported to be equal to 18% for the U.S. by Melitz and Redding (2015). Hence I set $SEF = 0.18$. To calibrate J, the number of foreign economies, I compute the share of U.S. GDP in world GDP using data for 2012 from the World Bank, and it is around 21.7%. Thus, it is reasonable to assume that the U.S. is one of five hypothetical equal-sized countries in the world, i.e. $J+1 = 5$, or $J = 4$. Using (1.60) together with (2.2.4) and the calibrated value of J, I can solve for implied values of τ and f_x/f.

Tables 1.1 and 1.2 summarize the values of observables and parameters used to calibrate the model.

Table 1.1: Calibration: Observables

Observable		Value	Source
Import penetration ratio	IPR	0.081	Sampson (2016)
Firm entry rate	NF	0.116	Sampson (2016), Luttmer (2007)
Share of exporting firms in manufacturing	SEF	0.18	Melitz and Redding (2015)
Population growth rate	n	0.011	Sampson (2016)
R&D subsidy rate	v_e	0.25	OECD (2014)
Ratio of entry cost per firm to fixed production cost	F_0/f	0.82	Barseghyan and DiCecio (2011)
Share of U.S. GDP in the world GDP		0.217	World Bank (2012)

Table 1.2: Calibration: Parameters

Parameter		Value	Source
Trade elasticity	k	7.5	Sampson (2016)
Elasticity of substitution across goods	σ	8.1	Sampson (2016)
Intertemporal elasticity of substitution	γ	0.33	Sampson (2016)
Discount rate	ρ	0.04	Sampson (2016)
Elasticity of patents with respect to R&D	α, β	0.5	Blundell, Griffith and Windmeijer (2002)
Population to fixed cost of production	L_0/f	1	normalization
Initial productivity cutoff	$\hat{\theta}_0^*$	1	normalization
Potential mass of entrants	$\hat{M}_0$	1	normalization

Comparing the current calibration with Melitz and Redding (2015), I notice that they calibrate the fixed entry costs by matching the U.S. firm exit rate. From (1.38), the rate at which producing firms exit is given by kg. This rate is endogenously defined by values of population growth n and firm entry rate NF through (1.57). The implied firm exit rate using the current calibration is $kg = 0.105$, which is much bigger than the level 0.005 targeted in Melitz and Redding (2015). However, it is easy to rationalize such a difference. Melitz and Redding (2015) use the exit rate for firms with more than 500 employees, which corresponds to a stochastic exit rate (δ in the Melitz (2003) model). This underestimates the real exit rate, as exit of small, low productivity firms is not considered. In the Sampson (2016) model, firm exit is endogenous and depends on each firm's relative productivity and its size, which decline as firms age. This implies that exit in the model is generated by small firms, and the endogenous exit rate of 0.105 represents the exit of less productive firms.

1.4.2 Numerical results

In this section I replicate numerical results for the baseline model of Sampson (2016) and present the numerical estimation of the version of the model with decreasing returns to scale in R&D. I compare the model's predictions about the dynamic gains from trade under two assumptions on the functional form of the R&D technology, the Segerstrom and Sugita (2016) assumption given by (1.20) and the Sampson (2016) assumption given by (1.21). In addition, I present a numerical analysis of the effect of subsidizing R&D on the gains from trade and consumer welfare.

Gains from trade

Table 1.3 shows the implied values of parameters for several alternative calibrations and Table 1.4 presents the numerical solution under these alternative calibrations for two scenarios: for the observed U.S. trade level and for autarky. The first column in both Table 1.3 and Table 1.4 corresponds to the baseline calibration in Sampson (2016), which sets the R&D subsidy in the U.S. to zero and assumes constant returns to scale in R&D. The second column takes into account that the observed R&D subsidy in the U.S. is around 25%. The third and the fourth columns present the calibration of parameters assuming decreasing returns to scale in R&D, with the elasticity of patents with respect to R&D expenditure of 0.5. In particular, the third column uses the functional form for the R&D technology by Segerstrom and Sugita (2016) and the fourth column uses the functional form by Sampson (2016).

From Table 1.3, the implied probability of successful entry is roughly the same across different calibrations and is around 0.6%. The relative fixed trade cost f_x/f and the variable trade cost τ are constant across calibrations.[11]

Table 1.3: Alternative calibrations: Implied values of parameters

		(1)	(2)	(3)	(4)
		$v_e = 0$ $\beta = 1$	$v_e = 0.25$ $\beta = 1$	$v_e = 0.25$ $\beta = 0.5$	$v_e = 0.25$ $\alpha = 0.5$
Probability of successful entry	λ^k	0.007	0.006	0.006	0.006
Fixed exporting cost to fixed production cost	f_x/f	0.122	0.122	0.122	0.122
Variable trade cost	τ	1.69	1.69	1.69	1.69

In Table 1.4 the rate of successful entry NF in trade equilibrium is calibrated to match the U.S. firm entry rate, and the growth rate q is a function of NF and parameters. Therefore both of these variables are not affected by the

[11] It is easy to check that the transversality condition and the assumption $g \equiv \dot{\theta}_t^*/\theta_t^* > 0$ are satisfied across alternative calibrations. The condition $\tau^{\sigma-1} f_x/f > 1$, required for not all firms choosing to export, also holds.

R&D technology assumption. Static gains from trade are calibrated using the U.S. import penetration ratio and the value of the trade elasticity. Hence these variables also do not change across different simulations.[12]

Column (1) in Table 1.4 replicates numerical results presented in Sampson (2016). In addition, having assigned values to all parameters in the model, I am able to compute the firm entry rate and consumer welfare under trade and under autarky. Results in the first column indicate that the growth rate of consumption q is 10.7% higher at the observed U.S. trade level than in autarky. This leads to the total gains from trade being 3.17 times larger than the static gains from trade, confirming that the dynamic selection provides a quantitatively important source of welfare gains from trade. This replicates the main result in Sampson (2016).

Comparison of columns (2), (3), and (4) in Table 1.4, where the R&D subsidy rate is set to $v_e = 0.25$, shows that the dynamic gains from trade are lower under decreasing returns to scale in R&D (columns (3) and (4)) than under constant returns (column (2)). However, when the elasticity of patents with respect to R&D is calibrated to 0.5, the dynamic gains from trade under the Segerstrom and Sugita (2016) functional form are twice as large as under the Sampson (2016) functional form. The model with the Segerstrom and Sugita (2016) functional form generates total gains from trade that are 2.95 larger than the static gains, which is only slightly lower than in the baseline case of constant returns to R&D. This suggests that the dynamic gains from trade remain quantitatively important even with a realistic assumption about the returns to scale in R&D.

[12] The growth rate of the U.S. economy generated by the numerical simulation in Table 1.4 is 1.55%, which is lower than the long term growth rate of the U.S. GDP per capita. Table 1.A.2 in the Appendix shows results when targeting the 2% GDP per capita growth rate. The dynamic gains from trade are slightly higher than those presented in Table 1.4. To obtain results comparable to Sampson (2016), I continue with calibrations that match the firm creation rate NF.

Table 1.4: Alternative calibrations: Results

		(1)	(2)	(3)	(4)
		$v_e = 0$	$v_e = 0.25$	$v_e = 0.25$	$v_e = 0.25$
		$\beta = 1$	$\beta = 1$	$\beta = 0.5$	$\alpha = 0.5$
Trade growth rate	q	0.0155	0.0155	0.0155	0.0155
Autarky growth rate	q^A	0.0140	0.0140	0.0142	0.0149
Relative growth rate	q/q^A	1.107	1.107	1.097	1.047
Relative consumption level	c_0/c_0^A	1.010	1.009	1.010	1.011
Static gains from trade	$z_s - 1$	0.011	0.011	0.011	0.011
Dynamic gains from trade	$z_d - 1$	0.024	0.024	0.022	0.011
Total gains from trade	$z - 1$	0.036	0.035	0.033	0.022
Relative gains from trade	$\frac{z-1}{z_s-1}$	3.17	3.13	2.95	1.97
Trade entry rate	NF	0.116	0.116	0.116	0.116
Autarky entry rate	NF^A	0.105	0.105	0.106	0.111
Trade welfare	U	12.524	12.277	12.277	12.277
Autarky welfare	U^A	12.193	11.932	11.952	12.061

In Figures 1.1 and 1.2, I depict the ratios of main endogenous variables in trade versus autarky equilibria as functions of the degree of the decreasing returns to scale in R&D, by assuming the Segerstrom and Sugita (2016) functional form in Figure 1.1 and the Sampson (2016) functional form in Figure 1.2.[13] The numerical results for $\beta < 0.2$ and $\alpha < 0.2$ can not be presented, as very low values of this elasticity imply very large values of labor employed in R&D N_t, and solving the model in such cases would require excessive computing power.

From Figure 1.1a, it follows that under the Segerstrom and Sugita (2016) functional form, the ratio of total to static gains from trade slightly decline as I allow for congestion in entry. Figure 1.2a shows that the dynamic gains from trade deteriorate much more with the introduction of decreasing returns to scale in R&D under the Sampson (2016) functional form compared to the previous case. For low values of the elasticity α, the total gains from trade are getting close to the static gains.

To understand the lower dynamic gains from trade under the Sampson (2016) functional form compared to the Segerstrom and Sugita (2016) func-

[13]Figure 1.2a is similar to the one presented in Sampson (2016) with the difference that I re-calibrate the model for each value of parameter α and calibrate the R&D subsidy rate to $v_e^{US} = 0.25$.

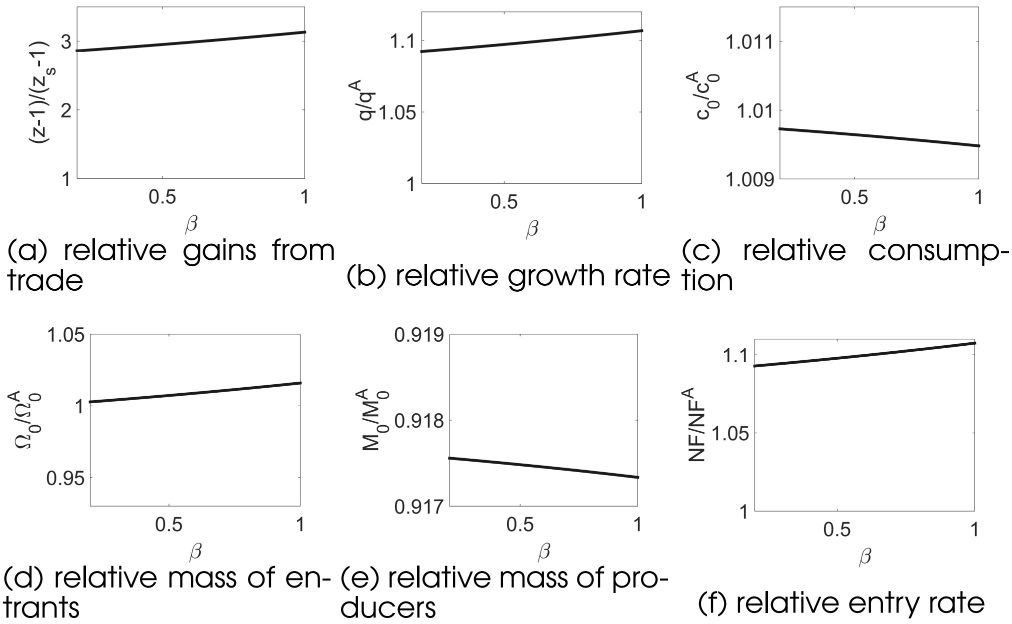
tional form, I analyse the behaviour of other endogenous variables. First, consider the constant returns to R&D case, which corresponds to $\beta = 1$ in Figure 1.1. By (1.44), trade liberalization leads to an increase in the dynamic selection rate, $g/g^A > 1$, and to the higher economic growth rate, $q/q^A > 1$. By (1.39) and (1.57), the successful entry rate is higher in trade equilibrium, $NF/NF^A > 1$, and by (1.40) the equilibrium mass of producers declines in trade equilibrium, $M_0/M_0^A < 1$.

Second, assume that there are decreasing returns to R&D. When the flow of innovations is only affected by the labor employed in R&D, as in (1.20), introducing the decreasing returns to scale in R&D makes entry more expensive for each individual firm.[14] As a consequence, from (1.44), less dynamic selection takes place following trade liberalization compared to the constant returns case, and the rate of economic growth in trade relative to autarky is also lower (Figure 1.1b). By (1.39), this leads to a smaller increase in the successful entry rate compared to the constant returns case (Figure 1.1f), and to a smaller decline in the mass of producers after trade liberalization (Figure 1.1e).

Now, assume that in addition to this, the flow of innovations is also increasing in the mass of producing firms as in (1.21). As before, trade liberalization would encourage more entry, leading to a higher dynamic selection and a reduction in the mass of producers. However, by (1.21), any reduction in the mass of producers affects negatively the flow of innovations. To offset this negative effect, in equilibrium the mass of producing firms in trade relative to autarky (Figure 1.2e) will be higher under the Sampson (2016) assumption than under the Segerstrom and Sugita (2016) or constant returns to scale in R&D case. With introduction of decreasing returns to R&D as in Sampson (2016), the relative successful entry rate in Figure 1.2f moves closer to unity. This, by (1.39), means that the dynamic selection in trade equilibrium is not much higher than the dynamic selection in autarky, which explains low dynamic gains from trade.

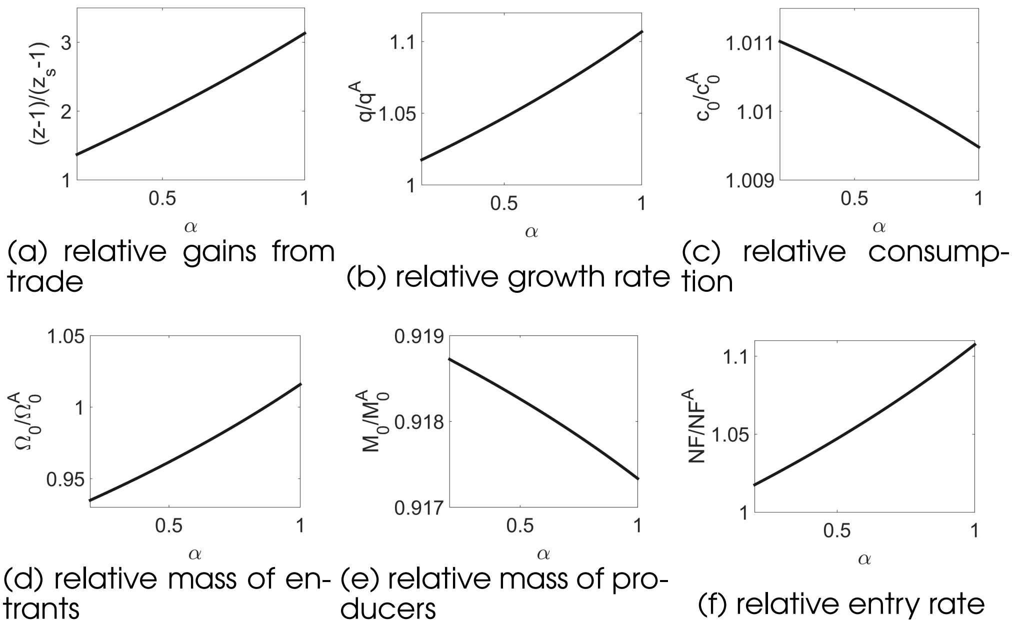
[14] This follows from the functional form for the individual entry cost $F_t = f_e \Omega_t^{1/\beta - 1}$, which is increasing in Ω_t when $\beta \in (0, 1)$.

Figure 1.1: Segerstrom and Sugita (2016) functional form, $\beta = 1$ corresponds to constant returns.



(a) relative gains from trade
(b) relative growth rate
(c) relative consumption
(d) relative mass of entrants
(e) relative mass of producers
(f) relative entry rate

Figure 1.2: Sampson (2016) functional form, $\alpha = 1$ corresponds to constant returns.



(a) relative gains from trade
(b) relative growth rate
(c) relative consumption
(d) relative mass of entrants
(e) relative mass of producers
(f) relative entry rate

Hence, it is possible to get large gains from trade while assuming a realistic degree of decreasing returns to R&D. In particular, with Segerstrom and Sugita (2016) functional form for R&D technology and the value of parameter $\beta = 0.5$, the ratio of total to static gains from trade is about 2.95. In the rest of this section I will use this functional form when comparing models with constant and decreasing returns to scale in R&D. In the appendix, I also present the results for the Sampson (2016) functional form.

Counterfactual: variable trade costs and import penetration ratio

In this subsection I examine how the numerical estimates of the gains from trade are changing with the variable trade costs. Table 1.5 shows a counterfactual exercise of looking at moving from autarky to a given variable trade cost τ. Column (1) corresponds to autarky, column (3) restores the variable trade cost implied by the U.S. economy, and column (6) corresponds to costless trade. Moving to lower variable trade cost increases economic growth and consumer welfare. Interestingly, the model predicts that a decrease in variable trade cost τ from the implied U.S. level of 1.69 to 1.5 leads a to higher increase in consumer welfare than moving from autarky to $\tau = 1.69$. As the variable trade cost τ decreases, the dynamics gains from trade become relatively more important than the static gains.

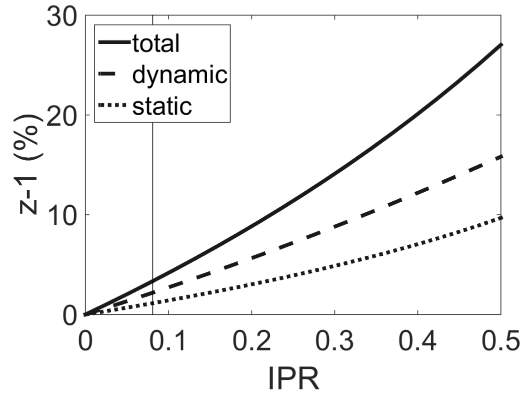
Table 1.5: Gains from trade when moving from autarky to trade with variable trade cost τ: decreasing returns to scale in R&D $\beta = 0.5$.

		(1)	(2)	(3)*	(4)	(5)	(6)
Variable trade cost	τ	∞	2.00	1.69	1.50	1.25	1.00
Growth rate	q	0.0142	0.0146	0.0155	0.0175	0.0274	0.0849
Welfare	U	11.95	12.05	12.28	12.68	14.02	16.04
Total gains	$z-1$	0	0.010	0.033	0.080	0.297	1.280
Static gains	z_s-1	0	0.003	0.011	0.026	0.085	0.255
Dynamic gains	z_d-1	0	0.006	0.022	0.053	0.195	0.817
Relative gains	$\frac{z-1}{z_s-1}$	na	2.90	2.95	3.06	3.49	5.02

*level of variable trade cost implied for the U.S. economy

The previous counterfactual exercise examined a change in variable trade costs τ while keeping other parameters of the model unchanged. Next, I examine an increase in the import penetration ratio IPR, which by (2.2.4) can be caused by a decrease in variable trade costs τ, fixed trade costs f_x, or an increase in the number of trading partners J. Figure 1.3 shows that the total gains from trade are increasing in the import penetration ratio. Increasing the import penetration ratio from 0.051 (Japan) to 0.36 (Belgium) raises total gains from trade from around 2% to around 17%. The dynamic gains are always larger than the static gains, and it can be computed that the ratio of total to static gains decreases from 3 to 2.8 times when increasing IPR in interval from 0 to 0.5 (see Figure 1.A.6 in the appendix).

Figure 1.3: Gains from trade ($z-1$) in percents and import penetration ratio. Decreasing returns to scale in R&D, $\beta = 0.5$. Vertical line corresponds to the U.S. import penetration ratio of 0.081.



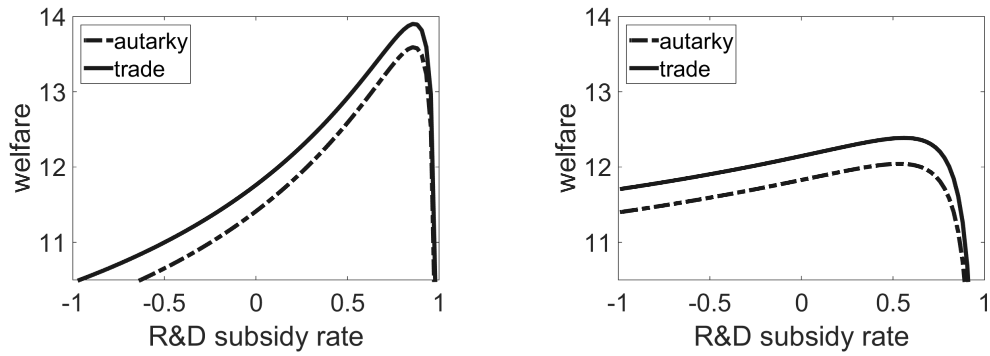
Counterfactual: the R&D subsidy rate

The equilibrium allocation in the Sampson (2016) model is not efficient, as entering firms do not take into account the positive externalities generated by knowledge spillovers, and hence there is not sufficient entry in equilibrium. Inefficiency of the decentralized equilibrium due to insufficient entry provides intuition for the existence of dynamic gains from trade, and also suggests that subsidizing entry would be welfare improving in this model. In particular, one of the extensions briefly analyzed in Sampson (2016) deals with the analysis of the optimal R&D subsidy. It is shown that under particular assumptions, an R&D subsidy can indeed be welfare improving. However, the solution for the level of the optimal subsidy is shown to be heavily affected by the form of the R&D technology, and the analytical results presented in Sampson (2016) are too general to provide clear implications about the optimal subsidy level.

In this subsection I address this issue and present numerical analysis of the optimal R&D subsidy rate under differing assumptions about the returns to scale in R&D.

Figure 1.4 shows a counterfactual analysis of the effect of the R&D subsidy rate on consumer welfare in trade and autarky. Figure 1.4a presents the calibration with constant returns to scale in R&D[15]. Under such assumptions, an increase in the R&D subsidy rate generates an enormous welfare increase. In particular, moving from autarky to a trade equilibrium (calibrated to match current U.S. trade status) would increase consumer welfare by 3.1% (from 11.93 to 12.30), whereas increasing the R&D subsidy rate from 25% to 50% would increase welfare by 5.6% (from 11.93 to 12.60). Thus, under constant returns to scale in R&D, the model predicts that the welfare gains from international trade are low compared to the welfare gains from subsidizing R&D.

Figure 1.4: Consumer welfare as a function of the R&D subsidy rate in trade and autarky.



(a) Constant returns to R&D, $\beta = 1$ (b) Decreasing returns to R&D, $\beta = 0.5$

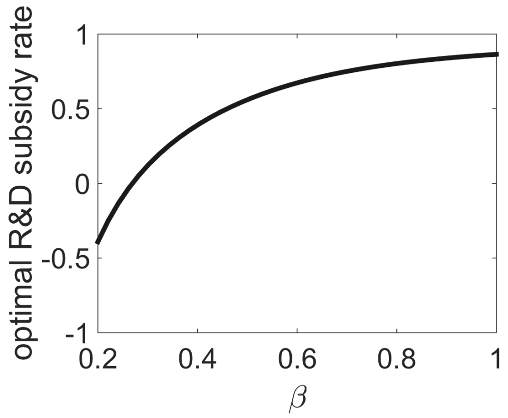
In Figure 1.4b I present the same counterfactual analysis as before, but now I am allowing for decreasing returns to scale in R&D. The striking difference with Figure 1.4a is that now the welfare curve is flatter and the implied optimal R&D subsidy rate is much lower. Moreover, the welfare effect of introducing the optimal R&D subsidy is always smaller than the welfare effect of trade liberalization. Figures 1.A.1 and 1.A.2 in the appendix show similar graphs for different values of β and the alternative Sampson (2016) functional form.

Figure 1.5 presents the numerical solution for the optimal R&D subsidy rate, corresponding to different degrees of decreasing returns to scale in R&D. Table 1.A.3 in the appendix provides actual values of the optimal subsidy rate for several selected values of parameters β. The solution for the optimal subsidy rate varies tremendously with the assumption on degree of decreasing returns

[15]The difference to the original calibration by Sampson (2016) is that here I set $v_e = 25\%$ to match the U.S. level of R&D subsidy.

to scale in R&D: the more congestion there is in entry, the more expensive it is for individual firms to enter and the costlier it is to finance the R&D subsidy. It is even optimal for the government to tax R&D when entry becomes too costly relative to production. On the other hand, with constant returns to scale in R&D, the model predicts an optimal subsidy of 86%. Assuming a reasonable level of β around 0.5 yields an optimal subsidy around 56%, which suggests that the existing level of government financing of R&D in the U.S. (23% in direct funding and around 10% in indirect funding) could be too low.

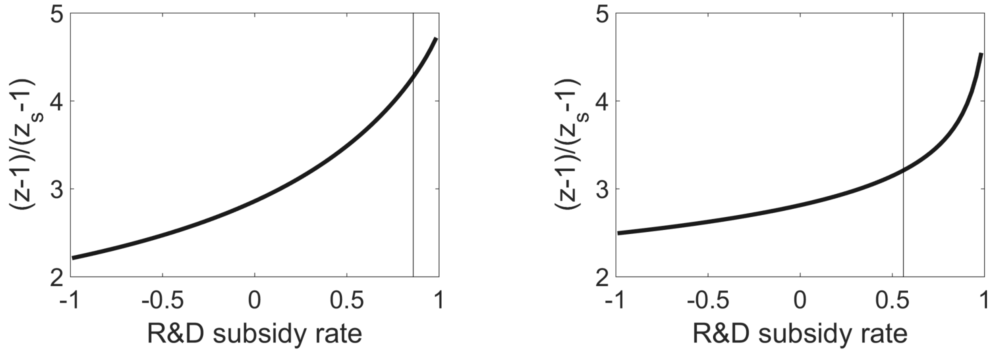
Figure 1.5: The optimal R&D subsidy rate as a function of decreasing returns to scale in R&D



Finally, I examine the relationship between the dynamic gains from trade and the R&D subsidy rate. Figure 1.6 plots the ratio of total to static gains from trade against the R&D subsidy rate when $\beta = 1$ and $\beta = 0.5$.[16] In both cases the ratio of total to static gains from trade is strictly increasing in the R&D subsidy rate. However, in the constant returns case, the relative gains from trade are much more sensitive to a change in the R&D subsidy rate. In particular, moving from a 25% to 50% R&D subsidy rate increases the relative gains from trade from 3.13 to 3.49 in case of constant returns (Figure 1.6a) and from 2.95 to 3.15 in case of decreasing returns (Figure 1.6b). When R&D is subsidized at the optimal rate, the gains from trade are much larger in the case of constant returns to scale in R&D than in case of decreasing returns (4.27 versus 3.21).

[16]Results for the Sampson (2016) functional form of decreasing returns to R&D are presented in the Appendix.

Figure 1.6: Relative gains from trade for different R&D subsidy rates. Constant returns to scale in R&D with $\beta = 1$ and decreasing returns with $\beta = 0.5$. Vertical lines correspond to the optimal R&D subsidy rate.



(a) $\beta = 1$ (b) $\beta = 0.5$

The intuition for the above results is the following. First, consider the case of constant returns to R&D. Trade liberalization raises the expected value of future profits for firms and encourages entry. Due to knowledge spillovers from incumbents to entrants, a rise in firm entry leads to an increase in the dynamic selection rate and boosts economic growth. In the presence of the R&D subsidy, the entry cost for an individual firm is reduced and even more firms enter, hence, the effect of trade liberalization on economic growth is more pronounced than under the zero-subsidy rate, and the dynamic gains from trade are higher. In the model with decreasing returns to scale in R&D, the congestion in entry deters some part of entry, leading to a smaller dynamic selection rate and lower dynamic gains from trade compared to the constant returns to scale case.

1.5 Conclusions

In this paper I focus on the implications of the functional form of the R&D technology for the predictions about welfare gains from trade in the Sampson (2016) model. The baseline model assumes constant returns to scale, however the empirical literature on patents and R&D suggests that decreasing returns to scale in R&D are more realistic. I show that depending on the form of the R&D technology, the model generates quantitatively different results on the dynamic gains from trade.

I confirm the finding by Sampson (2016) that under constant returns to scale in R&D, the ratio of total to static gains from trade is around 3.17, when the model is calibrated to match the U.S. observables. Second, when the R&D costs of each individual firm are increasing in a mass of entrants, as in Segerstrom and Sugita (2016), the ratio of total to static gains decreases. Nevertheless, it still remains a little lower than 3 even when a realistic degree of decreasing returns to scale in R&D is assumed. However, when an alternative functional form is considered, namely if the mass of incumbent firms positively affects the flow of innovations as in Sampson (2016), the dynamic gains from trade are found to deteriorate rapidly. This happens because, under Sampson's (2016) assumption, a decrease in the equilibrium mass of producing firms following trade liberalization affects negatively a flow of innovations (due to the form of the R&D technology), which in turn leads to less entry, lower dynamic selection, and lower dynamic gains from trade in equilibrium. As the effect of the mass of existing firms on the flow of innovations is ambiguous, and due to the large impact the particular shape of the R&D technology has on the dynamic gains from trade, it is important to take this result into consideration when choosing the exact functional form of the R&D technology in similar models.

Further, I conduct a counterfactual exercise and predict welfare gains from trade for alternative values of the import penetration ratio and variable trade cost. Intuitively, both static and dynamic gains from trade are increasing with the import penetration ratio and decreasing with variable trade cost. Interestingly, the model predicts larger welfare effect of reducing variable trade cost when it is already low.

Finally, I analyze the effect of the R&D subsidy rate on consumer welfare and gains from trade. I show that the dynamic gains from trade are strictly increasing in the R&D subsidy rate. Moreover, I predict the optimal level of this subsidy under different assumptions on the R&D technology. Assuming constant returns to scale in R&D in this model leads to a large predicted level of optimal subsidy, whereas introducing decreasing returns leads to a lower level of optimal subsidy.

1.A Appendix

1.A.1 Tables and Figures

Table 1.A.1: Implied values of parameters: matching the U.S. growth rate

		(1)	(2)	(3)	(4)
		$v_e = 0$ $\beta = 1$	$v_e = 0.25$ $\beta = 1$	$v_e = 0.25$ $\beta = 0.5$	$v_e = 0.25$ $\alpha = 0.5$
Probability of successful entry	λ^k	0.009	0.007	0.007	0.007
Fixed exporting cost to fixed production cost	f_x/f	0.122	0.122	0.122	0.122
Variable trade cost	τ	1.69	1.69	1.69	1.69

Table 1.A.2: Results: matching the U.S. growth rate

		(1)	(2)	(3)	(4)
		$v_e = 0$ $\beta = 1$	$v_e = 0.25$ $\beta = 1$	$v_e = 0.25$ $\beta = 0.5$	$v_e = 0.25$ $\alpha = 0.5$
Trade growth rate	q	0.0200	0.0200	0.0200	0.0200
Autarky growth rate	q^A	0.0181	0.0181	0.0183	0.0191
Relative growth rate	q/q^A	1.103	1.103	1.095	1.046
Relative consumption level	c_0/c_0^A	1.010	1.010	1.010	1.011
Static gains from trade	$z_s - 1$	0.011	0.011	0.011	0.011
Dynamic gains from trade	$z_d - 1$	0.027	0.026	0.025	0.012
Total gains from trade	$z - 1$	0.038	0.038	0.036	0.024
Relative gains from trade	$\frac{z-1}{z_s-1}$	3.39	3.35	3.20	2.09
Trade entry rate	NF	0.149	0.149	0.149	0.149
Autarky entry rate	NF^A	0.135	0.135	0.136	0.143
Trade welfare	U	13.079	12.855	12.855	12.855
Autarky welfare	U^A	12.769	12.530	12.546	12.654

Table 1.A.3: Optimal R&D subsidy as a function of decreasing returns to scale in R&D. $\beta = 1$ corresponds to constant returns in R&D.

Degree of decreasing returns to scale in R&D	β	0.25	0.50	0.75	1.00
Optimal R&D subsidy	v_e^*	-0.09	0.56	0.78	0.86
Maximized welfare under trade	U^*	12.31	12.39	12.99	13.90
Maximized welfare under autarky	U_A^*	12.01	12.04	12.63	13.59

Table 1.A.4: Optimal R&D subsidy as a function of decreasing returns to scale in R&D. $\alpha = 1$ corresponds to constant returns in R&D.

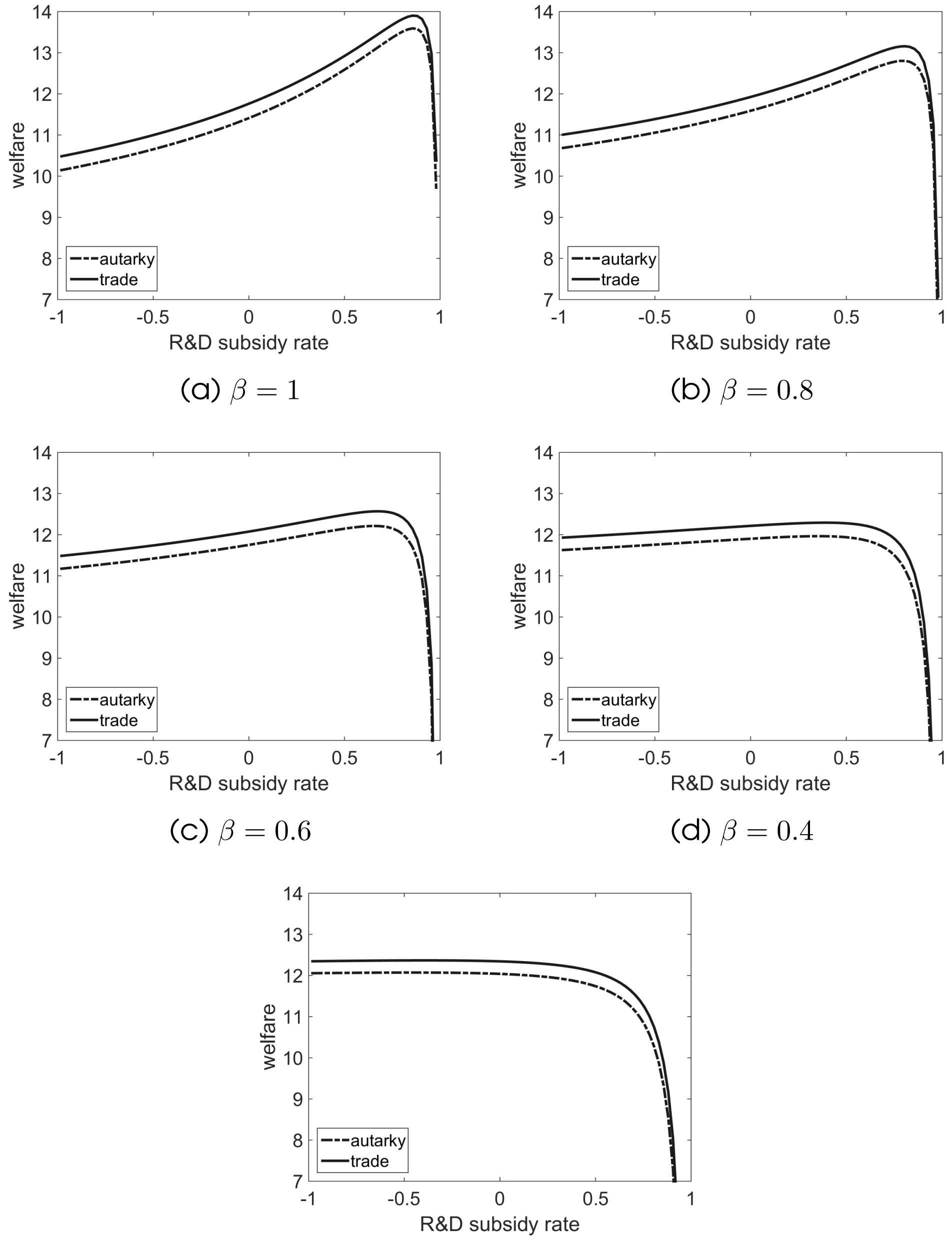
Degree of decreasing returns to scale in R&D	α	0.25	0.50	0.75	1.00
Optimal R&D subsidy	v_e^*	-0.36	0.43	0.73	0.86
Maximized welfare under trade	U^*	12.39	12.31	12.82	13.90
Maximized welfare under autarky	U_A^*	12.23	12.09	12.53	13.59

Table 1.A.5: Gains from trade when moving from autarky to trade with variable trade cost τ: decreasing returns to scale in R&D $\alpha = 0.5$.

		(1)	(2)	(3)*	(4)	(5)	(6)
Variable trade cost	τ	∞	2.00	1.69	1.50	1.25	1.00
Growth rate	q	0.0149	0.0151	0.0155	0.0165	0.0206	0.0366
Welfare	U	12.06	12.12	12.28	12.55	13.47	15.17
Total gains	$z - 1$	0	0.006	0.022	0.053	0.180	0.633
Static gains	$z_s - 1$	0	0.003	0.011	0.026	0.085	0.255
Dynamic gains	$z_d - 1$	0	0.003	0.011	0.026	0.088	0.301
Relative gains	$\frac{z-1}{z_s-1}$	na	1.96	1.97	2.00	2.12	2.48

*level of variable trade cost implied for the U.S. economy

Figure 1.A.1: Consumer welfare as a function of the R&D subsidy rate v_e, for different values of β ($\beta = 1$ means constant returns to scale in R&D).



(a) $\beta = 1$

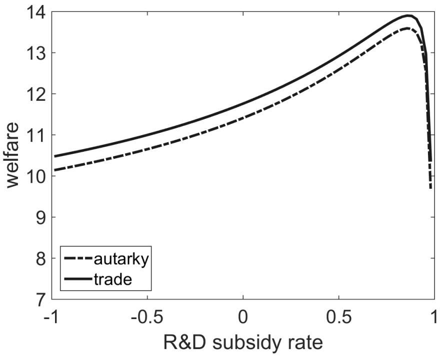
(b) $\beta = 0.8$

(c) $\beta = 0.6$

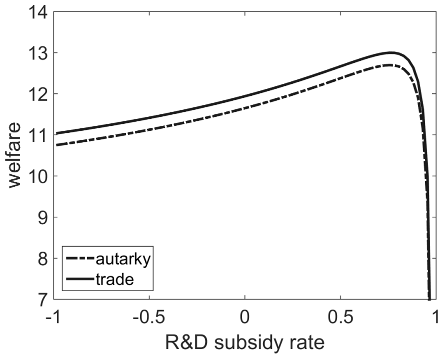
(d) $\beta = 0.4$

(e) $\beta = 0.2$

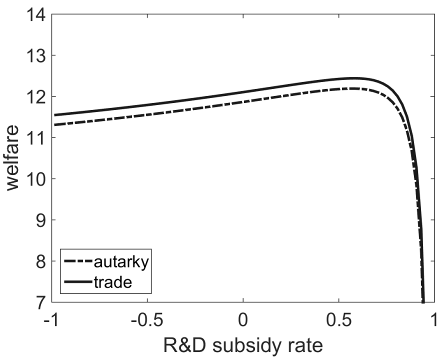
Figure 1.A.2: Consumer welfare as a function of the R&D subsidy rate v_e, for different values of α ($\alpha = 1$ means constant returns to scale in R&D).



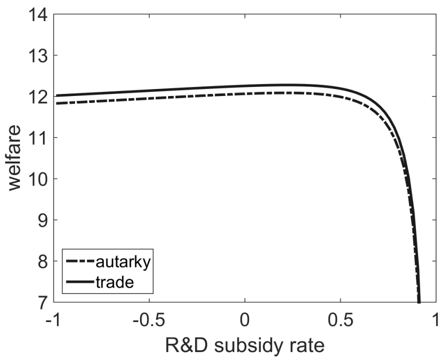
(a) $\alpha = 1$



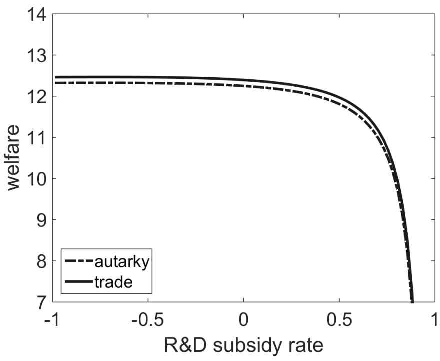
(b) $\alpha = 0.8$



(c) $\alpha = 0.6$



(d) $\alpha = 0.4$



(e) $\alpha = 0.2$

Figure 1.A.3: Optimal R&D subsidy rate as a function of the degree of decreasing returns to scale in R&D, Sampson (2016) functional form

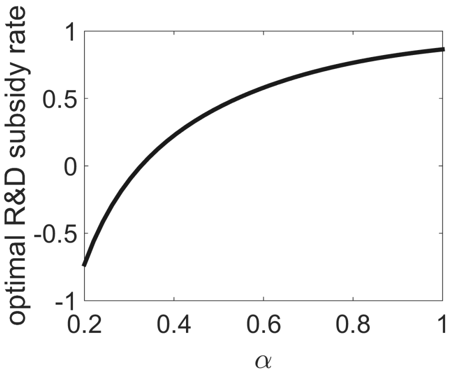
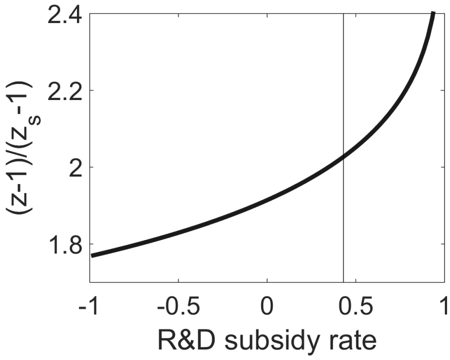


Figure 1.A.4: Relative gains from trade. Decreasing returns to scale in R&D, Sampson (2016) functional form with $\alpha = 0.5$. Vertical line correspond to the optimal R&D subsidy rate of 43%.



(a) $\alpha = 0.5$

Figure 1.A.5: Counterfactual: gains from trade $z - 1$ and import penetration ratio IPR. Decreasing returns to scale in R&D, Sampson (2016) functional form with $\beta = 0.5$. Vertical line corresponds to the U.S. import penetration ratio of 0.081.

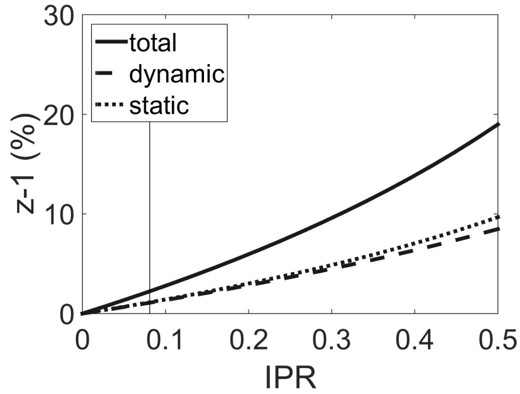
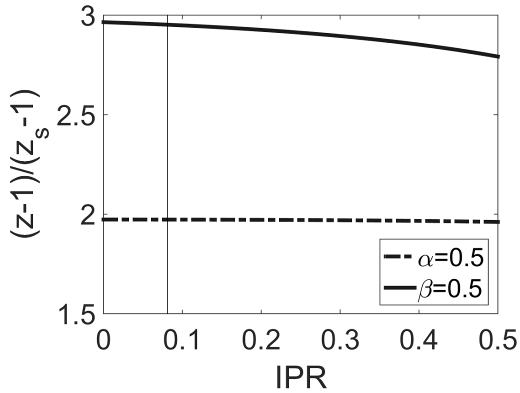


Figure 1.A.6: Counterfactual: relative gains from trade and import penetration ratio. Decreasing returns to scale in R&D, Sampson (2016) vs Segerstrom and Sugita (2016) functional form. Vertical line corresponds to the U.S. import penetration ratio of 0.081.



1.B Theoretical Appendix

This theoretical appendix contains the full solution to the model and is self-contained. The text in this appendix closely follows Sampson (2016) as well as the material in the main part of this paper. The numbering of equations corresponds to the main text.

1.B.1 Consumers

Intertemporal consumer's problem Each household has constant intertemporal elasticity of substitution preferences over the final consumption good:

$$U = \int_{t=0}^{\infty} e^{-\rho t} e^{nt} \frac{c_t^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} dt, \tag{1.1}$$

where c_t denotes consumption per capita, $0 < \gamma < 1$ is the intertemporal elasticity of substitution, n is the population growth rate, and ρ is the discount rate. The law of motion of assets in the economy is

$$\dot{A}_t = r_t A_t + w_t L_t - C_t - B_t, \tag{1.B.1}$$

where A_t denotes total assets in the economy, L_t is labor, C_t is total consumption of the final good, B_t is a lump-sum tax, and r_t and w_t denote the interest rate and the wage rate. Let $a_t = A_t/L_t$, $b_t = B_t/L_t$, and $c_t = C_t/L_t$ denote per capita variables. Note that $\dot{L}_t = nL_t$, as $L_t = e^{nt}$, hence $\dot{A}_t = \dot{a}_t L_t + \dot{L}_t a_t = \dot{a}_t L_t + nL_t a_t$. Plug this and $A_t = a_t L_t$ into (1.B.1):

$$\dot{a}_t L_t + nL_t a_t = r_t a_t L_t + w_t L_t - c_t L_t - b_t L_t$$

and divide through by L_t to obtain:

$$\dot{a}_t = w_t + r_t a_t - c_t - n a_t - b_t. \tag{1.2}$$

Each household maximizes its welfare subject to the budget constraint (1.2).

Let $u(c_t) \equiv (c_t^{1-\frac{1}{\gamma}} - 1)/(1 - \frac{1}{\gamma})$ denote the consumer's utility at consumption level c_t. The Hamiltonian and two first order conditions are

$$\mathbb{H}_t = u(c_t)e^{-(\rho-n)t} + \nu_t(w_t + (r_t - n)a_t - b_t - c_t)$$

$$\frac{\partial \mathbb{H}_t}{\partial c_t} = u'(c_t)e^{-(\rho-n)t} - \nu_t = 0$$

$$\dot{\nu}_t = -\frac{\partial \mathbb{H}_t}{\partial a_t} = -\nu_t(r_t - n),$$

where v_t is a co-state variable. So, the system becomes

$$\nu_t = u'(c_t)e^{-(\rho-n)t}$$

$$\dot{\nu}_t = -\nu_t(r_t - n).$$

Differentiate the first equation:

$$\dot{\nu}_t = u''(c_t)\dot{c}_t e^{-(\rho-n)t} - (\rho - n)e^{-(\rho-n)t}u'(c_t).$$

Plug it into the second equation:

$$u''(c_t)\dot{c}_t e^{-(\rho-n)t} - (\rho - n)e^{-(\rho-n)t}u'(c_t) = -u'(c_t)e^{-(\rho-n)t}(r_t - n),$$

so,

$$u''(c_t)\dot{c}_t = u'(c_t)(\rho - n - r_t + n)$$

$$\frac{u''(c_t)}{u'(c_t)}\dot{c}_t = \rho - r_t.$$

The Euler Equation then is:

$$\frac{u''(c_t)c_t}{u'(c_t)}\frac{\dot{c}_t}{c_t} = \rho - r_t.$$

And with $u(c_t) \equiv (c_t^{1-\frac{1}{\gamma}} - 1)/(1 - \frac{1}{\gamma})$ it becomes

$$\frac{-\frac{1}{\gamma}c_t^{-\frac{1}{\gamma}-1}c_t}{c_t^{-\frac{1}{\gamma}}}\frac{\dot{c}_t}{c_t} = \rho - r_t,$$

resulting in the usual Euler Equation:

$$\frac{\dot{c}_t}{c_t} = \gamma(r_t - \rho). \tag{1.3}$$

The transversality condition for our optimization program is written as

$$\lim_{t\to\infty} \nu_t a_t = 0.$$

Solving the differential equation $\dot{\nu}_t = -\nu_t(r_t - n)$ for ν_t:

$$\begin{aligned}
\frac{\dot{\nu}_t}{\nu_t} &= -(r_t - n) \\
\int_0^t \frac{\dot{\nu}_s}{\nu_s} ds &= -\int_0^t (r_s - n) ds \\
\ln \nu_s|_0^t &= -\int_0^t (r_s - n) ds \\
\ln \frac{\nu_t}{\nu_0} &= -\int_0^t (r_s - n) ds \\
\nu_t &= \nu_0 \exp\left\{-\int_0^t (r_s - n) ds\right\}.
\end{aligned}$$

So, the transversality condition is

$$\lim_{t\to\infty} a_t \nu_0 \exp\left\{-\int_0^t (r_s - n) ds\right\} = 0.$$

Since $\nu_t = u'(c_t)e^{-(\rho-n)t}$, it follows that $\nu_0 = u'(c_0) > 0$ is a constant. Therefore, I obtain:

$$\lim_{t\to\infty} a_t \exp\left\{-\int_0^t (r_s - n) ds\right\} = 0. \tag{1.4}$$

Intratemporal consumer's problem The final good is a composite of a continuum of intermediate varieties produced by the monopolistically competitive sector. At every point in time, conditional on individual expenditure on a final consumption good, each consumer decides how much of her expenditure is spent on each variety ω belonging to the set of varieties Σ available in the economy. Let $C_t = c_t L_t$ denote the total amount of final good consumed in the economy. Consumers' preferences over varieties are constant elasticity of substitution (CES) and can be written as:

$$C_t = \left(\int_{\omega\in\Sigma} y_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}}, \tag{1.5}$$

where $\sigma > 1$ is the elasticity of substitution, and $y_t(\omega)$ is total quantity of variety ω consumed in the economy at time t.

Let $E_t \equiv C_t P_t$ denote total consumer expenditure, where P_t is the price of the final consumption good. Then the budget constraint for consumption of individual varieties at each point in time is as follows:

$$\int_{\omega \in \Sigma} p_t(\omega) y_t(\omega) d\omega = E_t,$$

where $p_t(\omega)$ is the price of variety ω.

Rewrite the intratemporal consumer's problem as an optimal control problem, taking the total expenditure as given:

$$\max_{y_t(\omega)} \int_{\omega \in \Sigma} y_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega,$$

subject to the state equation

$$\dot{e}_t(\omega) = p_t(\omega) y_t(\omega), \qquad e_t(+\infty) = E_t, \quad \text{and} \quad e_t(0) = 0.$$

Note that the budget constraint can be rewritten as $\int_{\omega \in \Sigma} \dot{e}_t(\omega) d\omega = e_t(+\infty) - e_t(0) = E_t$.

Below is the Hamiltonian, where $\lambda(\omega)$ is the co-state variable:

$$\mathbb{H}_t = y_t(\omega)^{\frac{\sigma-1}{\sigma}} + \lambda(\omega) p_t(\omega) y_t(\omega).$$

Then, from the first order conditions, it follows that

$$\begin{aligned} \dot{\lambda}(\omega) &= -\frac{\partial \mathbb{H}_t}{\partial e_t} = 0 \\ \lambda(\omega) &= \lambda \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \mathbb{H}_t}{\partial y_t} &= \frac{\sigma-1}{\sigma} y_t(\omega)^{\frac{\sigma-1}{\sigma}-1} + \lambda p_t(\omega) = 0 \\ y_t(\omega) &= \left(-\frac{\sigma}{\sigma-1} \lambda p_t(\omega)\right)^{-\sigma}. \end{aligned}$$

Plug this back into the budget constraint:

$$\begin{aligned}\int_{\omega\in\Sigma} p_t(\omega)y_t(\omega)d\omega = \int_{\omega\in\Sigma} p_t(\omega)\left(-\frac{\sigma}{\sigma-1}\lambda p_t(\omega)\right)^{-\sigma} d\omega = E_t \\ \left(-\frac{\sigma}{\sigma-1}\lambda\right)^{-\sigma}\int_{\omega\in\Sigma} p_t(\omega)p_t(\omega)^{-\sigma}d\omega = E_t \\ \left(-\frac{\sigma}{\sigma-1}\lambda\right)^{-\sigma} = \frac{E_t}{\int_{\omega\in\Sigma} p_t(\omega)^{1-\sigma}d\omega}.\end{aligned}$$

Then

$$y_t(\omega) = \left(-\frac{\sigma}{\sigma-1}\lambda p_t(\omega)\right)^{-\sigma} = \frac{p_t(\omega)^{-\sigma}E_t}{\int_{\omega\in\Sigma} p_t(\omega)^{1-\sigma}d\omega}.$$

Thus, maximization of static utility taking the total expenditure E_t as given yields the following time t demand for individual variety:

$$y_t(\omega) = \frac{p_t(\omega)^{-\sigma}E_t}{P_t^{1-\sigma}}, \tag{1.6}$$

where P_t is the price index, or the price of the final consumption good:

$$P_t \equiv \left[\int_{\omega\in\Sigma} p_t(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}}. \tag{1.7}$$

Define expenditure on individual variety ω as:

$$e_t(\omega) \equiv p_t(\omega)y_t(\omega) = p_t(\omega)\frac{p_t(\omega)^{-\sigma}E_t}{P_t^{1-\sigma}} = E_t\left(\frac{p_t(\omega)}{P_t}\right)^{1-\sigma}. \tag{1.B.2}$$

Now, I am left to show that the aggregation is correct:

$$
\begin{aligned}
E_t \equiv C_t P_t &= \left[\int_{\omega\in\Sigma} y_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \left[\int_{\omega\in\Sigma} p_t(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \\
&= \left[\int_{\omega\in\Sigma} \left(\frac{p_t(\omega)^{-\sigma} E_t}{P_t^{1-\sigma}}\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \left[\int_{\omega\in\Sigma} p_t(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \\
&= \frac{E_t}{P_t^{1-\sigma}} \left[\int_{\omega\in\Sigma} p_t(\omega)^{1-\sigma} d\omega\right]^{\frac{\sigma}{\sigma-1}} \left[\int_{\omega\in\Sigma} p_t(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \\
&= \frac{E_t}{P_t^{1-\sigma}} \int_{\omega\in\Sigma} p_t(\omega)^{1-\sigma} d\omega = \int_{\omega\in\Sigma} E_t \left(\frac{p_t(\omega)}{P_t}\right)^{1-\sigma} d\omega \\
&= \int_{\omega\in\Sigma} p_t(\omega) y_t(\omega) d\omega.
\end{aligned}
$$

1.B.2 Product markets

Intra-temporal problem of a firm

The final good is non-tradable and is produced under perfect competition using the CES production function. The production in the intermediate sector is done by the monopolistically competitive firms producing differentiated varieties. The only factor of production is labor. Each worker inelastically supplies one unit of labor. Firms are heterogeneous in their labor productivity θ, which is constant over time for each firm. The distribution of productivities across firms that produce at time t in each economy is $G_t(\theta)$.

The labor demand for producing quantity y_t of a differentiated good for a firm with productivity θ is:

$$
l(y_t) = f + \frac{y_t}{\theta}, \tag{1.8}
$$

where $1/\theta$ is the marginal cost of production and f is the fixed overhead cost.

Firms can sell at home or abroad. Firms selecting into exporting encounter additional fixed costs f_x per market and variable iceberg trade costs τ.

At each point in time an intermediate variety producer with productivity θ seeks to maximize combined profits from domestic sales ($\pi_t^d(\theta)$) and exporting ($\pi_t^x(\theta)$)

$$
\pi_t(\theta) = \pi_t^d(\theta) + \max[0, J\pi_t^x(\theta)]. \tag{1.9}
$$

I assume that if a firm exports, it exports to all J foreign countries.

Using (1.6), let $p_t^d(\theta)$ and $p_t^x(\theta)$ denote price for domestic and imported goods. Let $y_t^d(\theta)$ and $y_t^x(\theta)$ denote demand for domestic and imported goods:

$$y_t^d(\theta) = \frac{p_t^d(\theta)^{-\sigma} E_t}{P_t^{1-\sigma}}$$
$$y_t^x(\theta) = \frac{p_t^x(\theta)^{-\sigma} E_t}{P_t^{1-\sigma}}.$$

Let $e_t^d(\theta)$ and $e_t^x(\theta)$ denote consumer expenditure on domestic and imported varieties:

$$e_t^d(\theta) = y_t^d(\theta) p_t^d(\theta) \qquad e_t^x(\theta) = y_t^x(\theta) p_t^x(\theta).$$

Moreover, these also correspond to the firm's revenues from domestic sales and from sales in a foreign country.

As τ is an iceberg trade cost, the labor demand to serve the domestic market and a single foreign country is as follows:

$$l^d(y_t^d) = f + \frac{y_t^d(\theta)}{\theta} \qquad l^x(y_t^x) = f_x + \frac{\tau y_t^x(\theta)}{\theta}.$$

Then, profits from domestic sales and from exporting to a single foreign country are

$$\pi_t^d(\theta) = e_t^d(\theta) - w_t l^d(y_t^d) \qquad \pi_t^x(\theta) = e_t^x(\theta) - w_t l^x(y_t^x).$$

Substituting for firm revenues and labor demand, I obtain

$$\begin{aligned}\pi_t^d(\theta) &= y_t^d(\theta) p_t^d(\theta) - w_t \left[f + \frac{y_t^d(\theta)}{\theta} \right] \\ &= E_t \left(\frac{p_t^d(\theta)}{P_t} \right)^{1-\sigma} - w_t \left[f + \frac{1}{\theta} \left(\frac{p_t^d(\theta)}{P_t} \right)^{-\sigma} \frac{E_t}{P_t} \right]\end{aligned}$$

$$\begin{aligned}\pi_t^x(\theta) &= y_t^x(\theta) p_t^x(\theta) - w_t \left[f_x + \tau \frac{y_t^x(\theta)}{\theta} \right] \\ &= E_t \left(\frac{p_t^x(\theta)}{P_t} \right)^{1-\sigma} - w_t \left[f_x + \tau \frac{1}{\theta} \left(\frac{p_t^x(\theta)}{P_t} \right)^{-\sigma} \frac{E_t}{P_t} \right]\end{aligned}$$

Taking the FOC of domestic profits with respect to the domestic price, I obtain:

$$\begin{aligned}\frac{\partial \pi_t^d(\theta)}{\partial p_t^d(\theta)} &= \frac{E_t}{P_t^{1-\sigma}}(1-\sigma)p_t^d(\theta)^{-\sigma} - w_t\frac{E_t(-\sigma)p_t^d(\theta)^{-\sigma-1}}{\theta P_t^{1-\sigma}} = 0\\ &= \frac{E_t p_t^d(\theta)^{-\sigma}}{P_t^{1-\sigma}}\left(1-\sigma+w_t\frac{\sigma}{\theta p_t^d(\theta)}\right) = 0.\end{aligned}$$

So, $\sigma - 1 = w_t\frac{\sigma}{\theta p_t^d(\theta)}$, and the price is a constant markup over the marginal cost:

$$p_t^d(\theta) = \frac{\sigma}{\sigma-1}\frac{w_t}{\theta}. \tag{1.10}$$

Taking the FOC of foreign profits with respect to the foreign price, I obtain:

$$\begin{aligned}\frac{\partial \pi_t^x(\theta)}{\partial p_t^x(\theta)} &= \frac{E_t}{P_t^{1-\sigma}}(1-\sigma)p_t^x(\theta)^{-\sigma} - w_t\tau\frac{E_t(-\sigma)p_t^x(\theta)^{-\sigma-1}}{\theta P_t^{1-\sigma}} = 0\\ &= \frac{E_t p_t^x(\theta)^{-\sigma}}{P_t^{1-\sigma}}\left(1-\sigma+w_t\tau\frac{\sigma}{\theta p_t^x(\theta)}\right) = 0,\end{aligned}$$

and it follows that

$$p_t^x(\theta) = \frac{\sigma}{\sigma-1}\frac{w_t\tau}{\theta}. \tag{1.B.3}$$

Hence, firms charge the export price $p_t^x(\theta) = \tau p_t^d(\theta)$ to foreign consumers.

Now use (1.10) and (1.6) to rewrite revenue and profits from domestic and exporting production. Revenue from domestic and export sales:

$$\begin{aligned}e_t^d(\theta) &= E_t\left(\frac{p_t^d(\theta)}{P_t}\right)^{1-\sigma} = E_t\left(\frac{\sigma}{\sigma-1}\frac{w_t}{\theta P_t}\right)^{1-\sigma}\\ e_t^x(\theta) &= E_t\left(\frac{p_t^x(\theta)}{P_t}\right)^{1-\sigma} = E_t\left(\frac{\sigma}{\sigma-1}\frac{w_t\tau}{\theta P_t}\right)^{1-\sigma} = \tau^{1-\sigma}e_t^d(\theta).\end{aligned}$$

Profits from domestic sales:

$$\begin{aligned}
\pi_t^d(\theta) &= e_t^d(\theta) - w_t l^d(\theta) \\
&= e_t^d(\theta) - w_t \frac{y_t^d(\theta)}{\theta} - w_t f \\
&= e_t^d(\theta) - \frac{w_t}{\theta}\frac{E_t}{P_t}\left(\frac{p_t^d(\theta)}{P_t}\right)^{-\sigma} - w_t f \\
&= e_t^d(\theta) - \frac{w_t}{\theta}\frac{E_t}{P_t^{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\frac{w_t}{\theta}\right)^{-\sigma} - w_t f \\
&= e_t^d(\theta) - \frac{\sigma-1}{\sigma}E_t\left(\frac{\sigma}{\sigma-1}\frac{w_t}{\theta P_t}\right)^{1-\sigma} - w_t f \\
&= e_t^d(\theta) - \frac{\sigma-1}{\sigma}e_t^d(\theta) - w_t f \\
&= \frac{e_t^d(\theta)}{\sigma} - w_t f.
\end{aligned}$$

Similarly, profits from exporting:

$$\begin{aligned}
\pi_t^x(\theta) &= e_t^x(\theta) - w_t l^x(\theta) \\
&= e_t^x(\theta) - w_t \tau \frac{y_t^x(\theta)}{\theta} - w_t f_x \\
&= e_t^x(\theta) - w_t\frac{\tau}{\theta}\frac{E_t}{P_t}\left(\frac{p_t^x(\theta)}{P_t}\right)^{-\sigma} - w_t f_x \\
&= e_t^x(\theta) - w_t\frac{\tau}{\theta}\frac{E_t}{P_t^{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\frac{w_t\tau}{\theta}\right)^{-\sigma} - w_t f_x \\
&= e_t^x(\theta) - \frac{\sigma-1}{\sigma}E_t\left(\frac{\sigma}{\sigma-1}\frac{w_t\tau}{\theta P_t}\right)^{1-\sigma} - w_t f_x \\
&= e_t^x(\theta) - \frac{\sigma-1}{\sigma}e_t^x(\theta) - w_t f_x \\
&= \frac{e_t^x(\theta)}{\sigma} - w_t f_x.
\end{aligned}$$

Zero cutoff profit conditions Let θ_t^* be a cutoff level of productivity such that firms with productivity below this level choose not to produce:

$$\pi_t^d(\theta_t^*) = \frac{e_t^d(\theta_t^*)}{\sigma} - w_t f = 0,$$

hence, the ZCP condition for domestic producers is

$$\frac{e_t^d(\theta_t^*)}{\sigma} = w_t f. \tag{1.B.4}$$

I also introduce a productivity cutoff for exporting, θ_t^x. Firms with productivity lower than this level find it not optimal to export:

$$\pi_t^x(\theta_t^x) = \frac{e_t^x(\theta_t^x)}{\sigma} - w_t f_x = 0,$$

which implies the ZCP condition for exporting:

$$\frac{e_t^x(\theta_t^x)}{\sigma} = w_t f_x. \tag{1.B.5}$$

Next, use the ZCP conditions to derive the relationship between domestic and export cutoff productivities:

$$\frac{e_t^x(\theta_t^x)}{e_t^d(\theta_t^*)} = \frac{E_t\left(\frac{\sigma}{\sigma-1}\frac{w_t\tau}{\theta_t^x P_t}\right)^{1-\sigma}}{E_t\left(\frac{\sigma}{\sigma-1}\frac{w_t}{\theta_t^* P_t}\right)^{1-\sigma}} = \tau^{1-\sigma}\left(\frac{\theta_t^x}{\theta_t^*}\right)^{\sigma-1} = \frac{\sigma w_t f_x}{\sigma w_t f} = \frac{f_x}{f}.$$

Thus

$$\begin{aligned}\left(\frac{\theta_t^x}{\theta_t^*}\right)^{\sigma-1} &= \tau^{\sigma-1}\frac{f_x}{f} \\ \theta_t^x &= \theta_t^*\tau\left(\frac{f_x}{f}\right)^{1/(\sigma-1)}.\end{aligned} \tag{1.11}$$

Assume $\tau^{\sigma-1}f_x > f$, which implies that $\theta_t^x > \theta_t^*$.

Next I derive two more useful results:

$$\frac{e_t^d(\theta)}{e_t^d(\theta_t^*)} = \frac{E_t\left(\frac{\sigma}{\sigma-1}\frac{w_t}{\theta P_t}\right)^{1-\sigma}}{E_t\left(\frac{\sigma}{\sigma-1}\frac{w_t}{\theta_t^* P_t}\right)^{1-\sigma}} = \left(\frac{\theta}{\theta_t^*}\right)^{\sigma-1}. \tag{1.B.6}$$

$$\frac{e_t^x(\theta)}{e_t^x(\theta_t^x)} = \frac{E_t\left(\frac{\sigma}{\sigma-1}\frac{w_t\tau}{\theta P_t}\right)^{1-\sigma}}{E_t\left(\frac{\sigma}{\sigma-1}\frac{w_t\tau}{\theta_t^x P_t}\right)^{1-\sigma}} = \left(\frac{\theta}{\theta_t^x}\right)^{\sigma-1}. \tag{1.B.7}$$

Price index Write down the expression for the price index using (1.7) and taking into account that there are domestic and imported goods available for domestic consumers:

$$P_t^{1-\sigma} = \int_{\theta_t^*}^{\infty} p_t^d(\theta)^{1-\sigma} M_t dG_t(\theta) + J \int_{\theta_t^x}^{\infty} p_t^x(\theta)^{1-\sigma} M_t dG_t(\theta),$$

where M_t is the mass of all firms operating in the domestic economy (and by symmetry also in each foreign economy), $G_t(\theta)$ is the productivity distribution of firms that produce in each economy, and J is the number of foreign countries.

Using (1.10) and (1.B.3), rewrite the price index:

$$\begin{aligned} P_t^{1-\sigma} &= M_t \left[\int_{\theta_t^*}^{\infty} \left(\frac{\sigma}{\sigma-1} \frac{w_t}{\theta} \right)^{1-\sigma} dG_t(\theta) + J \int_{\theta_t^x}^{\infty} \left(\frac{\sigma}{\sigma-1} \frac{\tau w_t}{\theta} \right)^{1-\sigma} dG_t(\theta) \right] \\ &= M_t \left(\frac{\sigma}{\sigma-1} w_t \right)^{1-\sigma} \left[\int_{\theta_t^*}^{\infty} \theta^{\sigma-1} dG_t(\theta) + J\tau^{1-\sigma} \int_{\theta_t^x}^{\infty} \theta^{\sigma-1} dG_t(\theta) \right]. \end{aligned}$$

Express the mass of producing firms as a function of the price index, the wage, and the productivity cut-offs:

$$M_t = P_t^{1-\sigma} \left(\frac{\sigma}{\sigma-1} w_t \right)^{\sigma-1} \left[\int_{\theta_t^*}^{\infty} \theta^{\sigma-1} dG_t(\theta) + J\tau^{1-\sigma} \int_{\theta_t^x}^{\infty} \theta^{\sigma-1} dG_t(\theta) \right]^{-1}.$$

Use the final good as the numeraire, thus normalize the price index to $P_t = 1$. This results in

$$M_t = \left(\frac{\sigma}{\sigma-1} w_t \right)^{\sigma-1} \left[\int_{\theta_t^*}^{\infty} \theta^{\sigma-1} dG_t(\theta) + J\tau^{1-\sigma} \int_{\theta_t^x}^{\infty} \theta^{\sigma-1} dG_t(\theta) \right]^{-1}. \quad (1.B.8)$$

Total expenditure The total consumer expenditure on domestic and imported intermediate goods is

$$E_t = \int_{\theta_t^*}^{\infty} e_t^d(\theta) M_t dG_t(\theta) + J \int_{\theta_t^x}^{\infty} e_t^x(\theta) M_t dG_t(\theta).$$

From (1.B.6) and (1.B.7), and the zero cutoff profit conditions (1.B.4) and (1.B.5):

$$
\begin{aligned}
E_t &= \int_{\theta_t^*}^{\infty} e_t^d(\theta_t^*)\left(\frac{\theta}{\theta_t^*}\right)^{\sigma-1} M_t dG_t(\theta) + J\int_{\theta_t^x}^{\infty} e_t^x(\theta_t^x)\left(\frac{\theta}{\theta_t^x}\right)^{\sigma-1} M_t dG_t(\theta) \\
&= \int_{\theta_t^*}^{\infty} \sigma w_t f\left(\frac{\theta}{\theta_t^*}\right)^{\sigma-1} M_t dG_t(\theta) + J\int_{\theta_t^x}^{\infty} \sigma w_t f_x\left(\frac{\theta}{\theta_t^x}\right)^{\sigma-1} M_t dG_t(\theta) \\
&= M_t\sigma w_t\left[f(\theta_t^*)^{1-\sigma}\int_{\theta_t^*}^{\infty}\theta^{\sigma-1}dG_t(\theta) + Jf_x(\theta_t^x)^{1-\sigma}\int_{\theta_t^x}^{\infty}\theta^{\sigma-1}dG_t(\theta)\right].
\end{aligned}
$$

Use (1.11):

$$
\begin{aligned}
E_t &= M_t\sigma w_t\left[f(\theta_t^*)^{1-\sigma}\int_{\theta_t^*}^{\infty}\theta^{\sigma-1}dG_t(\theta) + Jf_x(\theta_t^*)^{1-\sigma}\tau^{1-\sigma}\frac{f}{f_x}\int_{\theta_t^x}^{\infty}\theta^{\sigma-1}dG_t(\theta)\right] \\
&= M_t\sigma w_t f(\theta_t^*)^{1-\sigma}\left[\int_{\theta_t^*}^{\infty}\theta^{\sigma-1}dG_t(\theta) + J\tau^{1-\sigma}\int_{\theta_t^x}^{\infty}\theta^{\sigma-1}dG_t(\theta)\right].
\end{aligned}
$$

Substitute for M_t from (1.B.8):

$$
\begin{aligned}
E_t =& \frac{\left(\frac{\sigma}{\sigma-1}w_t\right)^{\sigma-1}}{\left[\int_{\theta_t^*}^{\infty}\theta^{\sigma-1}dG_t(\theta) + J\tau^{1-\sigma}\int_{\theta_t^x}^{\infty}\theta^{\sigma-1}dG_t(\theta)\right]}\sigma w_t f(\theta_t^*)^{1-\sigma}. \\
&\cdot\left[\int_{\theta_t^*}^{\infty}\theta^{\sigma-1}dG_t(\theta) + J\tau^{1-\sigma}\int_{\theta_t^x}^{\infty}\theta^{\sigma-1}dG_t(\theta)\right].
\end{aligned}
$$

The terms in square brackets cancels out:

$$
E_t = \left(\frac{\sigma}{\sigma-1}w_t\right)^{\sigma-1}\sigma w_t f(\theta_t^*)^{1-\sigma}.
$$

Productivity cutoff Rearrange the previous equation:

$$
\begin{aligned}
(\theta_t^*)^{\sigma-1} &= \left(\frac{\sigma}{\sigma-1}w_t\right)^{\sigma-1}\frac{\sigma w_t f}{E_t} \\
\theta_t^* &= \frac{\sigma}{\sigma-1}\sigma^{\frac{1}{\sigma-1}}\left(\frac{w_t^\sigma f}{E_t}\right)^{\frac{1}{\sigma-1}} \\
\theta_t^* &= \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left(\frac{w_t^\sigma f}{E_t}\right)^{\frac{1}{\sigma-1}},
\end{aligned}
$$

and using $E_t = C_t P_t$, $C_t = c_t L_t$ and $P_t = 1$, I obtain the expression for θ_t^*:

$$\theta_t^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1} \left(\frac{w_t^\sigma f}{c_t L_t} \right)^{\frac{1}{\sigma-1}}. \tag{1.12}$$

Introducing relative productivities Define ϕ_t to be a firm's productivity relative to the exit cut-off, or a firm's relative productivity:

$$\phi_t \equiv \frac{\theta}{\theta_t^*}. \tag{1.13}$$

Then the exporter threshold relative to the exit cut-off is:

$$\tilde{\phi}_t \equiv \frac{\theta_t^x}{\theta_t^*} = \tau \left(\frac{f_x}{f} \right)^{1/(\sigma-1)}. \tag{1.14}$$

Note that if $\phi_t \geq 1$ a firm chooses to produce and if $\phi_t \geq \tilde{\phi}$ a firm chooses to export.

In what follows, for convenience, I will use relative productivity notation. I will rewrite prices, employment and profits as functions of a firm's relative productivity. The prices are

$$\begin{aligned}
p_t^d(\theta_t) &= \frac{\sigma}{\sigma - 1} \frac{w_t}{\theta_t} = \frac{\sigma}{\sigma - 1} \frac{w_t}{\frac{\theta_t}{\theta_t^*} \theta_t^*} \\
&= \frac{\sigma}{\sigma - 1} \frac{w_t}{\phi_t \theta_t^*} \equiv p_t^d(\phi_t) \\
p_t^x(\phi_t) &= \tau p_t^d(\phi_t).
\end{aligned}$$

Using the zero profit cut-off condition $\pi^d(\theta_t^*) = 0$, the profits can be rewritten as a function of the relative productivity cutoff:

$$\begin{aligned}
\pi_t^d(\theta_t) &= \frac{e_t^d(\theta_t)}{\sigma} - w_t f = \left(\frac{\sigma}{\sigma - 1} \frac{w_t}{\theta_t} \right)^{1-\sigma} \frac{E_t}{\sigma} - w_t f \\
&= \left(\frac{\sigma}{\sigma - 1} \frac{w_t}{\frac{\theta_t}{\theta_t^*} \theta_t^*} \right)^{1-\sigma} \frac{E_t}{\sigma} - w_t f \\
&= \left(\frac{\sigma}{\sigma - 1} \frac{w_t}{\theta_t^*} \right)^{1-\sigma} \frac{E_t}{\sigma} \phi_t^{\sigma-1} - w_t f \\
&= \frac{e_t^d(\theta_t^*)}{\sigma} \phi_t^{\sigma-1} - w_t f = w_t f \phi_t^{\sigma-1} - w_t f \\
&= w_t f (\phi_t^{\sigma-1} - 1).
\end{aligned}$$

Similarly,

$$\begin{aligned}
\pi_t^x(\theta_t) &= \frac{e_t^x(\theta_t)}{\sigma} - w_t f_x = \frac{\tau^{1-\sigma} e_t^d(\theta_t)}{\sigma} - w_t f_x \\
&= \left(\frac{\sigma}{\sigma-1} \frac{w_t}{\frac{\theta_t}{\theta_t^*}\theta_t^*} \right)^{1-\sigma} \frac{E_t}{\sigma} \tau^{1-\sigma} - w_t f_x \\
&= \left(\frac{\sigma}{\sigma-1} \frac{w_t}{\theta_t^*} \right)^{1-\sigma} \frac{E_t}{\sigma} \phi_t^{\sigma-1} \tau^{1-\sigma} - w_t f_x \\
&= \frac{e_t^d(\theta_t^*)}{\sigma} \phi_t^{\sigma-1} \tau^{1-\sigma} - w_t f_x = w_t f \phi_t^{\sigma-1} \tau^{1-\sigma} - w_t f_x \\
&= w_t f \tau^{1-\sigma} (\phi_t^{\sigma-1} - \frac{f_x}{f} \tau^{\sigma-1}) \\
&= w_t f \tau^{1-\sigma} (\phi_t^{\sigma-1} - \tilde{\phi}_t^{\sigma-1}).
\end{aligned}$$

Thus,

$$\pi_t^d(\phi_t) = w_t f(\phi_t^{\sigma-1} - 1) \qquad \pi_t^x(\phi_t) = w_t f \tau^{1-\sigma} (\phi_t^{\sigma-1} - \tilde{\phi}_t^{\sigma-1}). \tag{1.15}$$

The zero cutoff profit condition $\pi_t^d(\theta_t^*) = 0$ implies that

$$\begin{aligned}
\frac{e_t^d(\theta_t^*)}{\sigma} \frac{1}{w_t} &= f \\
\left(\frac{\sigma}{\sigma-1} \frac{w_t}{\theta_t^*} \right)^{1-\sigma} \frac{E_t}{\sigma w_t} &= f \\
\frac{\sigma}{\sigma-1} \frac{w_t}{\theta_t^*} \left(\frac{\sigma}{\sigma-1} \frac{w_t}{\theta_t^*} \right)^{-\sigma} \frac{E_t}{\sigma w_t} &= f \\
\left(\frac{\sigma}{\sigma-1} \frac{w_t}{\theta_t^*} \right)^{-\sigma} \frac{E_t}{\theta_t^*} &= f(\sigma - 1).
\end{aligned} \tag{1.B.9}$$

The use of labor in production for domestic consumption and for export as a function of relative productivity cutoff is:

$$\begin{aligned}
l^d(\theta_t) &= f + \frac{y_t^d(\theta_t)}{\theta_t} = f + \frac{p_t^d(\theta_t)^{-\sigma} E_t}{P_t^{1-\sigma} \theta_t} \\
l^x(\theta_t) &= f_x + \tau \frac{y_t^x(\theta_t)}{\theta_t} = f_x + \tau \frac{p_t^x(\theta_t)^{-\sigma} E_t}{P_t^{1-\sigma} \theta_t}.
\end{aligned}$$

Using (1.B.9), (1.10), (1.B.3), and $P_t = 1$, I obtain:

$$\begin{aligned} l^d(\theta_t) &= \left(\frac{\sigma}{\sigma-1}\frac{w_t}{\theta_t}\right)^{-\sigma}\frac{E_t}{\theta_t} + f \\ &= \left(\frac{\sigma}{\sigma-1}\frac{w_t}{\frac{\theta_t}{\theta_t^*}\theta_t^*}\right)^{-\sigma}\frac{E_t}{\frac{\theta_t}{\theta_t^*}\theta_t^*} + f \\ &= \left(\frac{\sigma}{\sigma-1}\frac{w_t}{\theta_t^*}\right)^{-\sigma}\frac{E_t}{\theta_t^*}\phi_t^{\sigma-1} + f \\ &= f(\sigma-1)\phi_t^{\sigma-1} + f = f[(\sigma-1)\phi_t^{\sigma-1}+1] \end{aligned}$$

$$\begin{aligned} l^x(\theta_t) &= \tau\left(\frac{\sigma}{\sigma-1}\frac{w_t\tau}{\theta_t}\right)^{-\sigma}\frac{E_t}{\theta_t} + f_x \\ &= \tau\left(\frac{\sigma}{\sigma-1}\frac{w_t}{\theta_t^*}\right)^{-\sigma}\frac{E_t}{\theta_t^*}\tau^{-\sigma}\phi_t^{\sigma-1} + f_x \\ &= f\tau^{1-\sigma}[(\sigma-1)\phi_t^{\sigma-1} + \frac{f_x}{f}\tau^{\sigma-1}] = f\tau^{1-\sigma}[(\sigma-1)\phi_t^{\sigma-1} + \tilde{\phi}^{\sigma-1}]. \end{aligned}$$

Thus,

$$l^d(\phi_t) = f[(\sigma-1)\phi_t^{\sigma-1}+1] \qquad l^x(\phi_t) = f\tau^{1-\sigma}[(\sigma-1)\phi_t^{\sigma-1} + \tilde{\phi}^{\sigma-1}]. \tag{1.16}$$

Let $I_t[\phi_t \geq \tilde{\phi}]$ be an indicator function which takes value one if the firm is exporting at time t and zero otherwise. Since there are J export markets, total firm employment is given by $l_t(\phi_t) = l_t^d(\phi_t) + Jl_t^x(\phi_t)\cdot I_t[\phi_t \geq \tilde{\phi}]$ and combined firm profits are $\pi_t(\phi_t) = \pi_t^d(\phi_t) + J\pi_t^x(\phi_t)\cdot I_t[\phi_t \geq \tilde{\phi}]$.

Firm entry and decreasing returns to scale in R&D

A firm in the intermediate sector can lend or borrow at interest rate r_t. Let $W_t(\phi_t)$ be the value of a firm with relative productivity ϕ_t at time t, given by the present discounted value of the firm's future profits:

$$W_t(\phi_t) = \int_t^\infty \pi_\nu(\phi_\nu)\exp\left[-\int_t^\nu r_s ds\right]d\nu. \tag{1.17}$$

In each economy firm entry takes place via the research and development (R&D) activity, financed through a costless intermediation sector, which owns existing firms and pools the risk faced by innovators. Let N_t denote the aggregate labor employed in the R&D sector at time t, which produces a flow Ω_t of innovations, where each innovation represents an emerging firm. It follows that the R&D cost in terms of units of labor per individual entering firm is given by

$$F_t \equiv N_t/\Omega_t. \tag{1.18}$$

The R&D cost F_t potentially depends on the aggregate mass of entrants Ω_t. However each individual firm treats this cost as given.

The baseline model in Sampson (2016) features a constant returns to scale in R&D assumption. In particular, it is assumed that the flow of innovations Ω_t is linear in the labor employed in R&D:

$$\Omega_t = N_t/f_e, \tag{1.19}$$

where f_e is the entry cost parameter. Intuitively, this means that doubling the aggregate R&D labor would lead to twice as many innovations. Also, it means that the entry cost for an individual firm is constant and is equal to the entry cost parameter $F_t = f_e$.

Two alternative functional forms for R&D technology are considered. The first one is used in Segerstrom and Sugita (2016) and is given by

$$\Omega_t = \left(\frac{N_t}{f_e}\right)^{\beta}, \tag{1.20}$$

where $\beta \in (0,1)$ measures the degree of decreasing returns to scale in R&D. Notice that setting $\beta = 1$ yields the functional form for constant returns given by (1.19). From (1.20) one can express the labor employed in R&D for producing Ω_t innovations:

$$N_t = f_e \Omega_t^{1/\beta}.$$

Substituting this into an expression for the R&D cost of an individual firm (1.18), I obtain:

$$F_t \equiv \frac{N_t}{\Omega_t} = f_e \frac{\Omega_t^{1/\beta}}{\Omega_t} = f_e \Omega_t^{(1-\beta)/\beta}. \tag{1.B.10}$$

An alternative functional form for Ω_t is suggested by Sampson (2016) as one of the robustness checks in his appendix:

$$\Omega_t = \left(\frac{N_t}{f_e}\right)^{\alpha} M_t^{1-\alpha} \tag{1.21}$$

where M_t is the mass of existing producers in each economy, and $\alpha \in (0, 1)$ measures the degree of decreasing returns to scale in R&D. Again, setting $\alpha = 1$ yields the functional form for constant returns given by (1.19). The labor employed in R&D for producing Ω_t innovations is

$$N_t = f_e \Omega_t^{1/\alpha} M_t^{(\alpha-1)/\alpha}.$$

Then the entry cost per firm can be rewritten as:

$$F_t \equiv \frac{N_t}{\Omega_t} = f_e \frac{\Omega_t^{1/\alpha} M_t^{(\alpha-1)/\alpha}}{\Omega_t} = f_e \left(\frac{\Omega_t}{M_t}\right)^{(1-\alpha)/\alpha}. \tag{1.B.11}$$

Knowledge spillovers

After the R&D cost has been paid, every newly emerging firm draws its productivity. The productivity of entrants is given by

$$\boldsymbol{\theta} = x_t \boldsymbol{\psi},$$

where x_t is the mean of the productivity distribution of incumbents, and $\boldsymbol{\psi}$ is a stochastic component distributed with a cumulative distribution function $F(\psi)$. Let θ and ψ denote particular realizations of the random variables $\boldsymbol{\theta}$ and $\boldsymbol{\psi}$. Then the realization of an entrant's productivity is given by:

$$\theta = x_t \psi. \tag{1.22}$$

The cumulative productivity distribution of firms that produce at time t is $G_t(\theta)$, and the cumulative distribution of entrants' productivity is given by

$$\tilde{G}_t(\theta) = \Pr\left[\boldsymbol{\theta}_t < \theta\right] = \Pr\left[x_t \boldsymbol{\psi} < \theta\right] = \Pr\left[\boldsymbol{\psi} < \frac{\theta}{x_t}\right] = F\left(\frac{\theta}{x_t}\right).$$

Let H_t and $\tilde{H}_t$ be the cumulative distribution functions of relative productivity ϕ for existing firms and entrants, respectively. Then, using $\boldsymbol{\phi}_t = \frac{\boldsymbol{\theta}}{\theta_t^*}$ and $\boldsymbol{\theta} =$

$x_t\boldsymbol{\psi}$, for a particular realization of a random variable $\phi = \frac{\theta}{\theta_t^*}$ and $\theta = x_t\psi$, I can write:

$$\tilde{G}_t(\theta) = F\left(\frac{\theta}{x_t}\right) = F\left(\frac{\phi\theta_t^*}{x_t}\right) = \Pr\left[\boldsymbol{\psi} < \frac{\phi\theta_t^*}{x_t}\right] = \Pr\left[\frac{\boldsymbol{\psi}x_t}{\theta_t^*} < \phi\right] = \Pr\left[\frac{\boldsymbol{\theta}}{\theta_t^*} < \phi\right]$$
$$= \Pr\left[\boldsymbol{\phi}_t < \phi\right] = \tilde{H}_t(\phi).$$

The free entry condition implies that in equilibrium the expected cost of innovating equals the expected value of creating a new firm:

$$F_t w_t(1 - v_e) = \int_\phi W_t(\phi) d\tilde{H}_t(\phi), \tag{1.23}$$

where F_t is the labor cost of generating a new firm from (1.18), w_t is the wage, v_e is the share of R&D costs covered by the government, and the integral represents the expected present discounted value of a firm entering at time t, which is itself affected by the productivity distribution of entrants.

Assuming that the exit cut-off θ_t^* is strictly increasing over time, the probability that the incumbent from time t has productivity less than ϕ at time $t+\Delta$ can be written as follows

$$\Pr\left[\boldsymbol{\phi}_{t+\Delta} \leq \phi\right] = \Pr\left[\frac{\boldsymbol{\theta}}{\theta_{t+\Delta}^*} \leq \phi\right] = \Pr\left[\frac{\boldsymbol{\theta}}{\theta_t^*}\frac{\theta_t^*}{\theta_{t+\Delta}^*} \leq \phi\right]$$
$$= \Pr\left[\boldsymbol{\phi}_t\frac{\theta_t^*}{\theta_{t+\Delta}^*} \leq \phi\right] = \Pr\left[\boldsymbol{\phi}_t \leq \frac{\theta_{t+\Delta}^*}{\theta_t^*}\phi\right] = H_t\left(\frac{\theta_{t+\Delta}^*}{\theta_t^*}\phi\right),$$

as $\boldsymbol{\phi}_t = \frac{\boldsymbol{\theta}}{\theta_t^*}$ and $\boldsymbol{\phi}_{t+\Delta} = \frac{\boldsymbol{\theta}}{\theta_{t+\Delta}^*}$. The probability that the incumbent at time t has productivity less than 1 at time $t+\Delta$ is similarly:

$$\Pr\left[\boldsymbol{\phi}_{t+\Delta} \leq 1\right] = \Pr\left[\boldsymbol{\phi}_t \leq \frac{\theta_{t+\Delta}^*}{\theta_t^*}\right] = H_t\left(\frac{\theta_{t+\Delta}^*}{\theta_t^*}\right).$$

M_t is the mass of producers in each economy at time t. Then the mass of time t incumbents that have relative productivity less than ϕ, but greater than 1, at time $t+\Delta$, is given by

$$M_t\left[H_t\left(\frac{\theta_{t+\Delta}^*}{\theta_t^*}\phi\right) - H_t\left(\frac{\theta_{t+\Delta}^*}{\theta_t^*}\right)\right].$$

Now consider entrants. The probability that the entrant who enters at time t has relative productivity less than ϕ at time $t+\Delta$ is

$$\begin{aligned}\Pr\left[\boldsymbol{\phi}_{t+\Delta} < \phi\right] &= \Pr\left[\frac{\boldsymbol{\theta}}{\theta^*_{t+\Delta}} < \phi\right] = \Pr\left[\frac{x_t \boldsymbol{\psi}}{\theta^*_{t+\Delta}} < \phi\right] = \\ &= \Pr\left[\boldsymbol{\psi} < \phi\frac{\theta^*_{t+\Delta}}{x_t}\right] = F\left(\frac{\phi\theta^*_{t+\Delta}}{x_t}\right),\end{aligned}$$

and the probability that the entrant who enters at time t has relative productivity less than 1 at time $t+\Delta$ is

$$\begin{aligned}\Pr\left[\boldsymbol{\phi}_{t+\Delta} < 1\right] &= \Pr\left[\frac{\boldsymbol{\theta}}{\theta^*_{t+\Delta}} < 1\right] = \Pr\left[\frac{x_t \boldsymbol{\psi}}{\theta^*_{t+\Delta}} < 1\right] = \\ &= \Pr\left[\boldsymbol{\psi} < \frac{\theta^*_{t+\Delta}}{x_t}\right] = F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right).\end{aligned}$$

The flow of innovations at time t is Ω_t. Then the mass of entrants between t and $t+\Delta$ with relative productivity between 1 and ϕ at time $t+\Delta$ is approximately

$$\Delta\Omega_t\left[F\left(\frac{\phi\theta^*_{t+\Delta}}{x_t}\right) - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right)\right].$$

Finally, the mass of firms with relative productivity less than ϕ at time $t+\Delta$ is approximated by:

$$\begin{aligned}M_{t+\Delta}H_{t+\Delta}(\phi) \approx{}& M_t\left[H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\phi\right) - H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right)\right] + \\ &+ \Delta\Omega_t\left[F\left(\frac{\phi\theta^*_{t+\Delta}}{x_t}\right) - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right)\right].\end{aligned} \tag{1.24}$$

Solving for the law of motion of $H_t(\phi)$. Take the limit of (1.24) as $\phi \to \infty$ (in order to get the mass of firms with all possible relative productivities above 1). Note that as $\phi \to \infty$, $H_{t+\Delta}(\phi) \to 1$:

$$M_{t+\Delta} \approx M_t\left[1 - H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right)\right] + \Delta\Omega_t\left[1 - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right)\right]. \tag{1.B.12}$$

Rearrange, divide through by M_t and Δ, and take the limit as $\Delta \to 0$. Note that $H_t(1) = 0$, as H_t is a cdf of relative productivities of existing firms.

$$M_{t+\Delta} - M_t \approx -M_t \left[H_t \left(\frac{\theta^*_{t+\Delta}}{\theta^*_t} \right) - H_t \left(\frac{\theta^*_t}{\theta^*_t} \right) \right] + \Delta\Omega_t \left[1 - F \left(\frac{\theta^*_{t+\Delta}}{x_t} \right) \right]$$
$$\frac{1}{M_t} \frac{M_{t+\Delta} - M_t}{\Delta} \approx -\frac{H_t \left(\theta^*_{t+\Delta}/\theta^*_t \right) - H_t \left(\theta^*_t/\theta^*_t \right)}{\Delta} + \frac{\Omega_t}{M_t} \left[1 - F \left(\frac{\theta^*_{t+\Delta}}{x_t} \right) \right]$$

$$\frac{1}{M_t} \lim_{\Delta \to 0} \frac{M_{t+\Delta} - M_t}{\Delta} = - \lim_{\Delta \to 0} \frac{H_t \left(\theta^*_{t+\Delta}/\theta^*_t \right) - H_t \left(\theta^*_t/\theta^*_t \right)}{\Delta} + \\ + \lim_{\Delta \to 0} \frac{\Omega_t}{M_t} \left[1 - F \left(\frac{\theta^*_{t+\Delta}}{x_t} \right) \right].$$

Consider the first term on the RHS. Let $k_t(v) \equiv H_t \left(\theta^*_v/\theta^*_t \right)$. Then

$$\dot{k}_t(v) \equiv \frac{d}{dv} k_t(v) = H'_t \left(\frac{\theta^*_v}{\theta^*_t} \right) \frac{\dot{\theta}^*_v}{\theta^*_t}$$
$$\dot{k}_t(t) = H'_t \left(\frac{\theta^*_t}{\theta^*_t} \right) \frac{\dot{\theta}^*_t}{\theta^*_t} = H'_t (1) \frac{\dot{\theta}^*_t}{\theta^*_t}.$$

Rewriting the limit:

$$\lim_{\Delta \to 0} \frac{H_t \left(\theta^*_{t+\Delta}/\theta^*_t \right) - H_t \left(\theta^*_t/\theta^*_t \right)}{\Delta} = \lim_{\Delta \to 0} \frac{k_t(t+\Delta) - k_t(t)}{\Delta} = \dot{k}_t(t) = H'_t (1) \frac{\dot{\theta}^*_t}{\theta^*_t}. \tag{1.B.13}$$

Thus, I obtain the law of motion for the mass of producing firms in the economy:

$$\frac{\dot{M}_t}{M_t} = -H'_t(1) \frac{\dot{\theta}^*_t}{\theta^*_t} + \frac{\Omega_t}{M_t} \left[1 - F \left(\frac{\theta^*_t}{x_t} \right) \right]. \tag{1.25}$$

Now I derive a law of motion for $H_t(\phi)$. Plug (1.B.12) into (1.24):

$$\left[M_t \left(1 - H_t \left(\frac{\theta^*_{t+\Delta}}{\theta^*_t} \right) \right) + \Delta\Omega_t \left(1 - F \left(\frac{\theta^*_{t+\Delta}}{x_t} \right) \right) \right] H_{t+\Delta}(\phi) \approx \\ M_t \left[H_t \left(\frac{\theta^*_{t+\Delta}}{\theta^*_t} \phi \right) - H_t \left(\frac{\theta^*_{t+\Delta}}{\theta^*_t} \right) \right] + \Delta\Omega_t \left[F \left(\frac{\phi \theta^*_{t+\Delta}}{x_t} \right) - F \left(\frac{\theta^*_{t+\Delta}}{x_t} \right) \right].$$

Dividing through by M_t gives:

$$\left(1 - H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right)\right) H_{t+\Delta}(\phi) + \Delta\frac{\Omega_t}{M_t}\left(1 - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right)\right) H_{t+\Delta}(\phi) \approx$$
$$H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\phi\right) - H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right) + \Delta\frac{\Omega_t}{M_t}\left[F\left(\frac{\phi\theta^*_{t+\Delta}}{x_t}\right) - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right)\right].$$

Rearrange and use that $H_t(1) = 0$:

$$H_{t+\Delta}(\phi) - H_t(\phi) \approx \left[H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\phi\right) - H_t\left(\frac{\theta^*_t}{\theta^*_t}\phi\right)\right]$$
$$- \left[H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right) - H_t\left(\frac{\theta^*_t}{\theta^*_t}\right)\right] + \left[H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right) - 0\right] H_{t+\Delta}(\phi)$$
$$+ \Delta\frac{\Omega_t}{M_t}\left[F\left(\frac{\phi\theta^*_{t+\Delta}}{x_t}\right) - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right) - \left(1 - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right)\right) H_{t+\Delta}(\phi)\right].$$

Divide through by Δ:

$$\frac{H_{t+\Delta}(\phi) - H_t(\phi)}{\Delta} \approx \frac{H_t\left(\theta^*_{t+\Delta}\phi/\theta^*_t\right) - H_t\left(\theta^*_t\phi/\theta^*_t\right)}{\Delta}$$
$$- \frac{H_t\left(\theta^*_{t+\Delta}/\theta^*_t\right) - H_t\left(\theta^*_t/\theta^*_t\right)}{\Delta}\left[1 - H_{t+\Delta}(\phi)\right]$$
$$+ \frac{\Omega_t}{M_t}\left[F\left(\frac{\phi\theta^*_{t+\Delta}}{x_t}\right) - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right) - \left(1 - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right)\right) H_{t+\Delta}(\phi)\right].$$

Take the limit as $\Delta \to 0$:

$$\lim_{\Delta\to 0}\frac{H_{t+\Delta}(\phi) - H_t(\phi)}{\Delta} = \lim_{\Delta\to 0}\frac{H_t\left(\theta^*_{t+\Delta}\phi/\theta^*_t\right) - H_t\left(\theta^*_t\phi/\theta^*_t\right)}{\Delta}$$
$$- \lim_{\Delta\to 0}\frac{H_t\left(\theta^*_{t+\Delta}/\theta^*_t\right) - H_t\left(\theta^*_t/\theta^*_t\right)}{\Delta}\left[1 - H_{t+\Delta}(\phi)\right]$$
$$+ \lim_{\Delta\to 0}\frac{\Omega_t}{M_t}\left[F\left(\frac{\phi\theta^*_{t+\Delta}}{x_t}\right) - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right) - \left(1 - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right)\right) H_{t+\Delta}(\phi)\right].$$

Consider the first term on the RHS. Let $h_t(v) \equiv H_t\left(\theta^*_v\phi/\theta^*_t\right)$ and

$$\dot{h}_t(v) \equiv \frac{d}{dv}h_t(v) = H'_t\left(\frac{\theta^*_v}{\theta^*_t}\phi\right)\frac{\dot{\theta}^*_v}{\theta^*_t}\phi$$
$$\dot{h}_t(t) = H'_t\left(\frac{\theta^*_t}{\theta^*_t}\phi\right)\frac{\dot{\theta}^*_t}{\theta^*_t}\phi = H'_t\left(\phi\right)\frac{\dot{\theta}^*_t}{\theta^*_t}\phi.$$

Then,

$$\lim_{\Delta\to 0} \frac{H_t\left(\theta^*_{t+\Delta}\phi/\theta^*_t\right) - H_t\left(\theta^*_t\phi/\theta^*_t\right)}{\Delta} = \lim_{\Delta\to 0} \frac{h_t(t+\Delta) - h_t(t)}{\Delta} =$$
$$= \dot{h}_t(t) = H'_t(\phi)\frac{\dot{\theta}^*_t}{\theta^*_t}\phi.$$

Use (1.B.13) to solve for the limit of the second term on the RHS:

$$\lim_{\Delta\to 0} \frac{H_t\left(\theta^*_{t+\Delta}/\theta^*_t\right) - H_t\left(\theta^*_t/\theta^*_t\right)}{\Delta}\left[1 - H_{t+\Delta}(\phi)\right] =$$
$$= H'_t(1)\frac{\dot{\theta}^*_t}{\theta^*_t} \cdot \lim_{\Delta\to 0}\left[1 - H_{t+\Delta}(\phi)\right]$$
$$= H'_t(1)\frac{\dot{\theta}^*_t}{\theta^*_t} \cdot \left[1 - H_t(\phi)\right].$$

Thus I obtain the law of motion for $H_t(\phi)$:

$$\begin{aligned}\dot{H}_t(\phi) = &\left\{H'_t(\phi)\phi - H'_t(1)\left[1 - H_t(\phi)\right]\right\}\frac{\dot{\theta}^*_t}{\theta^*_t} + \\ &+ \frac{\Omega_t}{M_t}\left[F\left(\frac{\phi\theta^*_t}{x_t}\right) - F\left(\frac{\theta^*_t}{x_t}\right) - H_t(\phi)\left[1 - F\left(\frac{\theta^*_t}{x_t}\right)\right]\right].\end{aligned} \tag{1.26}$$

1.B.3 Equilibrium

The labor market clearing condition is:

$$L_t = M_t \int_\phi l_t(\phi) dH_t(\phi) + N_t. \tag{1.27}$$

The asset market clearing condition is:

$$a_t L_t = M_t \int_\phi W_t(\phi) dH_t(\phi). \tag{1.28}$$

An additional equilibrium condition is the balanced budget of the government, i.e. the R&D subsidy is financed by an aggregate lump-sum tax on consumers:

$$b_t L_t = w_t N_t v_e. \tag{1.29}$$

In this equation, b_t is the lump-sum tax per capita, so $b_t L_t$ is the total tax payment, $w_t N_t$ is the total cost of R&D, and v_e is the share of R&D costs covered by the government.

The equilibrium in the world economy is defined by time paths $t \in [0, \infty)$ for

$$c_t, a_t, b_t, w_t, r_t, \theta_t^*, \theta_t^x, W_t(\phi), M_t, N_t, \Omega_t, H_t(\phi),$$

such that the following conditions hold:

- consumers maximize (1.1) subject to (1.2), which gives the Euler Equation (1.3) and the transversality condition (1.4);
- producers maximize profits, which gives the domestic productivity cut-off (1.12), the export productivity cut-off (1.11), and the firm value (1.17);
- the free entry into R&D condition (1.23);
- the domestic productivity cut-off is strictly increasing over time ($\dot{\theta}_t^* > 0$), and the laws of motion for M_t and $H_t(\phi)$ are given by (1.25) and (1.26);
- the labor market clearing condition (1.27);
- the asset market clearing condition (1.28);
- the balanced budget of the government condition (1.29);
- the initial mass of potential producers at time zero is given by $\hat{M}_0$ with productivity distribution $\hat{G}_0(\theta)$.

1.B.4 Balanced growth path

I solve for a balanced growth equilibrium path of the world economy. On a balanced growth equilibrium path $c_t, a_t, w_t, r_t, \theta_t^*, \theta_t^x, W_t(\phi), M_t, N_t, \Omega_t$ grow at constant rates and the distribution of relative productivity ϕ is stationary, meaning $\dot{H}_t(\phi) = 0 \ \forall t, \phi$.

Stationary relative productivity distribution

Assumption 1. *The sampling productivity distribution F is Pareto:* $F(\psi) = 1 - (\psi/\psi_{min})^{-k}$ *for* $\psi \geq \psi_{min}$*, where* $k > \max\{1, \sigma - 1\}$*. Moreover, the lower bound of the sampling productivity distribution satisfies* $x_t \psi_{min} < \theta_t^*$*.*

Substitute for F in (1.26) and set $\dot{H}_t(\phi) = 0$:

$$0 = \dot{H}_t(\phi) = \left\{H_t'(\phi)\phi - H_t'(1)\left[1 - H_t(\phi)\right]\right\} \frac{\dot{\theta}_t^*}{\theta_t^*}$$
$$+ \frac{\Omega_t}{M_t}\left[1 - \left(\frac{\phi\theta_t^*/x_t}{\psi_{min}}\right)^{-k} - 1 + \left(\frac{\theta_t^*/x_t}{\psi_{min}}\right)^{-k} - H_t(\phi)\left[1 - 1 + \left(\frac{\theta_t^*/x_t}{\psi_{min}}\right)^{-k}\right]\right]$$
$$0 = \left\{H_t'(\phi)\phi - H_t'(1)\left[1 - H_t(\phi)\right]\right\} \frac{\dot{\theta}_t^*}{\theta_t^*}$$
$$+ \frac{\Omega_t}{M_t}\left[-\left(\frac{\phi\theta_t^*}{x_t}\right)^{-k}\psi_{min}^k + \left(\frac{\theta_t^*}{x_t}\right)^{-k}\psi_{min}^k - H_t(\phi)\left(\frac{\theta_t^*}{x_t}\right)^{-k}\psi_{min}^k\right],$$

which gives the following differential equation (as I am solving for a stationary productivity distribution, I can omit the time subscript on $H(\phi)$):

$$0 = \left\{H'(\phi)\phi - H'(1)\left[1 - H(\phi)\right]\right\} \frac{\dot{\theta}_t^*}{\theta_t^*} + \frac{\Omega_t}{M_t}\left(\frac{\theta_t^*}{x_t}\right)^{-k}\psi_{min}^k\left[1 - \phi^{-k} - H(\phi)\right]. \tag{1.30}$$

Lemma 1. *Given Assumption 1, there exists a stationary relative productivity distribution:*

$$H(\phi) = 1 - \phi^{-k}. \tag{1.B.14}$$

Proof. Substitute into (1.30) for $H(\phi) = 1 - \phi^{-k}$, $H'(\phi) = k\phi^{-k-1}$, and $H'(1) = k$:

$$0 = \left\{k\phi^{-k-1}\phi - k\phi^{-k}\right\}\frac{\dot{\theta}_t^*}{\theta_t^*} + \frac{\Omega_t}{M_t}\left(\frac{\theta_t^*}{x_t}\right)^{-k}\psi_{min}^k\left[1 - \phi^{-k} - 1 + \phi^{-k}\right]$$
$$0 = \left\{k\phi^{-k} - k\phi^{-k}\right\}\frac{\dot{\theta}_t^*}{\theta_t^*} + 0.$$

Thus, (1.30) holds when $H(\phi) = 1 - \phi^{-k}$. □

On a balanced growth path, the growth rate $g \equiv \dot{\theta}_t^*/\theta_t^* > 0$ is constant over time. As the stationary relative productivity distribution $H(\phi)$ is not a function of time and (1.30) should be satisfied for some stationary $H(\phi)$ for all ϕ, it follows that the term $\frac{\Omega_t}{M_t}\left(\frac{\theta_t^*}{x_t}\right)^{-k}\psi_{min}^k$ is constant over time. Denote

$$\mu \equiv \frac{\Omega_t}{M_t}\left(\frac{\theta_t^*}{x_t}\right)^{-k}\psi_{min}^k \Big/ \left(\frac{\dot{\theta}_t^*}{\theta_t^*}\right)$$

and rewrite the differential equation in a simplified form (for $\phi \geq 1$):

$$0 = \left\{H'(\phi)\phi - H'(1)\left[1 - H(\phi)\right]\right\} + \mu\left[1 - \phi^{-k} - H(\phi)\right].$$

Rearranging the terms, the differential equation can be rewritten as

$$H'(\phi) = \frac{H'(1)\left[1 - H(\phi)\right]}{\phi} - \frac{\mu}{\phi}\left[1 - \phi^{-k} - H(\phi)\right]. \tag{1.B.15}$$

The initial condition is

$$H(1) = 0. \tag{1.B.16}$$

Lemma B.1. *There are infinitely many solutions to the differential equation* (1.B.15) *together with the initial condition* $H(1) = 0$, *one for each* $H'(1) > 0$.

Proof. To prove this result, I will use the following definition and theorems from Appendix A.1 and A.4 in Edwards and Penney (2008):

Definition. *If* R *is a region in* $(m+1)$*-dimensional* $(\mathbf{x}, t)$*-space, then the function* $\mathbf{f}(\mathbf{x}, t)$ *is said to be* ***Lipschitz continuous*** *on* R *if there exists a constant* $k > 0$ *such that*

$$|\mathbf{f}(\mathbf{x}_1, t) - \mathbf{f}(\mathbf{x}_2, t)| \leq k|\mathbf{x}_1 - \mathbf{x}_2| \tag{1.B.17}$$

if $(\mathbf{x}_1, t)$ *and* $(\mathbf{x}_2, t)$ *are points of* R.

Theorem. ***Global Existence of Solutions.*** *Let* $\mathbf{f}$ *be a vector-valued function (with* m *components) of* $m+1$ *real variables, and let* I *be a [bounded or unbounded] open interval containing* $t = a$. *If* $\mathbf{f}(\mathbf{x}, t)$ *is continuous and satisfies the Lipschitz condition* (1.B.17) *for all t in I and for all* $\mathbf{x}_1$ *and* $\mathbf{x}_2$, *then the initial value problem*

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t), \qquad \mathbf{x}(a) = \mathbf{b} \tag{1.B.18}$$

has a solution on the [entire] interval I.

Theorem. ***Uniqueness of Solutions.*** *Suppose that on some region* R *in* $(m+1)$*-space, the function* $\mathbf{f}$ *in* (1.B.18) *is continuous and satisfies the Lipschitz condition* (1.B.17). *If* $\mathbf{x}_1(t)$ *and* $\mathbf{x}_2(t)$ *are two solutions to the initial problem in* (1.B.18) *on some open interval I containing* $x = a$, *such that the solution curves* $(\mathbf{x}_1(t), t)$ *and* $(\mathbf{x}_2(t), t)$ *both lie in* R *for all t in* I, *then* $\mathbf{x}_1(t) = \mathbf{x}_2(t)$ *for all t in* I.

Now, consider the initial value problem (1.B.15) and (1.B.16) in light of the above stated theorems. Let $I = (1-\epsilon, \infty)$ be an open interval containing initial value $\phi = 1$, with ϵ arbitrarily small. Let R be an infinite strip - a set of all points (H, ϕ) where $\phi \in I$. For $H'(1)$ taking on any finite positive real value η, and for an arbitrarily small ϵ, the RHS of (1.B.15) is a Lipschitz continuous function $f(H,\phi)$ on R:

$$\begin{aligned}
f(H,\phi) \equiv& \frac{\eta\left[1-H\right]}{\phi} - \frac{\mu}{\phi}\left[1-\phi^{-k}-H\right] \\
|f(H_1,\phi) - f(H_2,\phi)| =& \left| \frac{\eta\left[1-H_1\right]}{\phi} - \frac{\mu}{\phi}\left[1-\phi^{-k}-H_1\right] \right. \\
& \left. -\frac{\eta\left[1-H_2\right]}{\phi} + \frac{\mu}{\phi}\left[1-\phi^{-k}-H_2\right] \right| \\
=& \left| \frac{\eta}{\phi}(H_2-H_1) + \frac{\mu}{\phi}(H_1-H_2) \right| = \left| \frac{\eta-\mu}{\phi}(H_2-H_1) \right| \\
=& \left| \frac{\eta-\mu}{\phi} \right| |H_2-H_1| \leq \left| \frac{\eta-\mu}{1-\epsilon} \right| |H_2-H_1|,
\end{aligned}$$

where the last inequality is satisfied because $1-\epsilon < \phi$ for all $\phi \in I$ and for all H_1 and H_2.

Then the initial value problem (1.B.15) and (1.B.16) has a unique solution on the entire interval I. □

This constitutes a correction to Sampson (2016), where the uniqueness of the stationary relative productivity distribution is stated in Lemma 1. However the proof to the original lemma misses the fact that the Picard-Lindelöf theorem can not be applied due to presence of the term $H'(1)$ in (1.B.15). Ignoring the $H'(1)$ term in Sampson's proof leads to the loss of an infinite number of solutions to the differential equation.

Further, Lemma B.2 specifies the solution for each value of the initial slope $H'(1)$. However, not each of these solutions is a productivity distribution. For a solution H to be a productivity distribution function, I need to impose certain restrictions on parameters.

Lemma B.2. *For any positive* μ, k *and* η*, such that* $k-\eta+\mu \neq 0$*, the following function* $H(\phi)$ *is a solution to* (1.B.15) *together with the initial condition* $H(1) = 0$*:*

$$H(\phi) = 1 - \frac{\mu}{k-\eta+\mu}\phi^{-k} - \frac{k-\eta}{k-\eta+\mu}\phi^{-(\eta-\mu)}, \tag{1.B.19}$$

which corresponds to $H'(1) = \eta$. *Moreover, if* $\eta = k$ *or condition* $\mu < \eta < k$ *holds, then* $H(\phi)$ *is a cumulative distribution function.*

Proof. The first order derivative of (1.B.19) is:

$$H'(\phi) = \frac{\mu k}{k-\eta+\mu}\phi^{-k-1} - \frac{(k-\eta)(\mu-\eta)}{k-\eta+\mu}\phi^{-(\eta-\mu)-1},$$

which gives the initial slope

$$\begin{aligned} H'(1) &= \frac{\mu k}{k-\eta+\mu} - \frac{(k-\eta)(\mu-\eta)}{k-\eta+\mu} = \frac{\mu k - k\mu + \eta\mu + k\eta - \eta^2}{k-\eta+\mu} \\ &= \frac{\eta(k-\eta+\mu)}{k-\eta+\mu} = \eta. \end{aligned}$$

Substitute these and (1.B.19) into (1.B.15) and rearrange the terms:

$$\begin{aligned} H'(\phi) &= \frac{H'(1)\left[1-H(\phi)\right]}{\phi} - \frac{\mu}{\phi}\left[1-\phi^{-k}-H(\phi)\right] \\ H'(\phi) &= \left[H'(1)-\mu\right]\frac{1-H(\phi)}{\phi} + \mu\phi^{-k-1} \end{aligned}$$

$$\begin{aligned} &\frac{\mu k}{k-\eta+\mu}\phi^{-k-1} - \frac{(k-\eta)(\mu-\eta)}{k-\eta+\mu}\phi^{-(\eta-\mu)-1} = \\ &\qquad = \frac{\eta-\mu}{\phi}\left(\frac{\mu}{k-\eta+\mu}\phi^{-k} + \frac{k-\eta}{k-\eta+\mu}\phi^{-(\eta-\mu)}\right) + \mu\phi^{-k-1} \end{aligned}$$

$$\begin{aligned} &\left(\frac{\mu k}{k-\eta+\mu} - \frac{(\eta-\mu)\mu}{k-\eta+\mu} - \mu\right)\phi^{-k-1} = \\ &\qquad = \left(\frac{(\eta-\mu)(k-\eta)}{k-\eta+\mu} + \frac{(k-\eta)(\mu-\eta)}{k-\eta+\mu}\right)\phi^{-(\eta-\mu)-1} \end{aligned}$$

$$\begin{aligned} \left(\frac{\mu(k-\eta+\mu)}{k-\eta+\mu} - \mu\right)\phi^{-k-1} &= \left(\frac{(\eta-\mu)(k-\eta)}{k-\eta+\mu} - \frac{(k-\eta)(\eta-\mu)}{k-\eta+\mu}\right)\phi^{-(\eta-\mu)-1} \\ 0 &= 0 \end{aligned}$$

Hence, I have shown that for any given initial slope $H'(1) = \eta > 0$, (1.B.19) is a solution to (1.B.15) and (1.B.16) for any values of $\mu, k > 0$, such that $k-\eta+\mu \neq 0$.

For H to be a cumulative distribution function, in addition to the initial condition $H(1) = 0$, two conditions must be satisfied. First, the limit of the function as ϕ goes to infinity must be $\lim_{\phi\to\infty} H(\phi) = 1$. This holds if and only if either $\eta = k$, or $\mu < \eta$.

Second, the function should be increasing on $\phi \geq 1$. The first order condition is:

$$H'(\phi) = \frac{\mu k}{k-\eta+\mu}\phi^{-k-1} - \frac{(k-\eta)(\mu-\eta)}{k-\eta+\mu}\phi^{-(\eta-\mu)-1} \geq 0.$$

Assume $\eta = k$. The first order condition simplifies to

$$H'(\phi) = k\phi^{-k-1} \geq 0,$$

which holds for all $\phi \geq 1$.

Assume $\eta \neq k$ and $k-\eta+\mu > 0$. Noting that $\phi \geq 1$, the first order condition simplifies to

$$H'(\phi) = \frac{\phi^{-k-1}}{k-\eta+\mu}\left[\mu k - (k-\eta)(\mu-\eta)\phi^{k-\eta+\mu}\right] \geq 0$$

from where it follows that

$$\mu k - (k-\eta)(\mu-\eta)\phi^{k-\eta+\mu} \geq 0.$$

Since this condition has to be satisfied for all $\phi \geq 1$, including very large values of ϕ, it has to be that $(k-\eta)(\mu-\eta) \leq 0$. I have already established the condition $\mu < \eta$ for the function to converge to one, hence it has to be that $\eta < k$.

Assume $\eta \neq k$ and $k-\eta+\mu < 0$. The first order condition simplifies to

$$H'(\phi) = \frac{\phi^{-1-\eta+\mu}}{k-\eta+\mu}\left[\mu k\phi^{-k+\eta-\mu} - (k-\eta)(\mu-\eta)\right] \geq 0,$$

from where it follows that

$$\mu k\phi^{-k+\eta-\mu} \leq (k-\eta)(\mu-\eta).$$

As $-k+\eta-\mu > 0$ and the inequality has to hold as $\phi \to \infty$, it has to be that $(k-\eta)(\mu-\eta)$ is infinitely large. For the function to converge to one the condition $\mu < \eta$ must hold. Then for the RHS to be infinitely large, it has to be that $\eta \to \infty$, which would mean that the initial slope of H is infinite, and H is not a productivity distribution if $k - \eta + \mu < 0$.

Thus, I have shown that $\lim_{\phi\to\infty} H(\phi) = 1$ and H is increasing if $0 < \mu < \eta < k$ or when $\eta = k$. Together with initial condition (1.B.16) these conditions also ensure that the function satisfies $0 \leq H(\phi) \leq 1$ for all $\phi \geq 1$. Hence, I established that if $0 < \mu < \eta < k$ or $\eta = k$, then H given by (1.B.19) is a cumulative distribution function. If $\eta = k$, then $H(\phi) = 1 - \phi^{-k}$ is the Pareto distribution.

□

It is important to notice that the constant μ contains endogenous variables. The only productivity distribution function that solves (1.B.15) for any μ is the Pareto distribution given in Lemma 1, which can be obtained by setting $\eta = k$. This solution is the only solution found by Sampson (2016).

In the rest of the paper I am interested in the balanced growth path where the relative productivity distribution is Pareto and is given by Lemma 1. Then, on the balanced growth path the productivity distribution of incumbents $G_t(\theta)$ is Pareto with shape parameter k and scale parameter θ_t^*:

$$\begin{aligned}
H_t(\phi) &= \Pr[\boldsymbol{\phi} < \phi] = 1 - \phi^{-k} \\
&= \Pr\left[\frac{\boldsymbol{\theta}}{\theta_t^*} < \frac{\theta}{\theta_t^*}\right] = 1 - \left(\frac{\theta}{\theta_t^*}\right)^{-k} \\
G_t(\theta) &= \Pr\left[\boldsymbol{\theta} < \theta\right] = 1 - \left(\frac{\theta}{\theta_t^*}\right)^{-k}.
\end{aligned}$$

By the properties of the Pareto distribution, $G_t'(\theta) = k\theta^{-k-1}(\theta_t^*)^k$. By Assumption 1, $k > 1$, and it follows that $-k + 1 < 0$. Then the average productivity of incumbents is

$$\begin{aligned}
x_t &= \int_{\theta_t^*}^{\infty} \theta dG_t(\theta) = \int_{\theta_t^*}^{\infty} \theta k \left(\theta_t^*\right)^k \theta^{-k-1} d\theta \\
&= k\left(\theta_t^*\right)^k \int_{\theta_t^*}^{\infty} \theta^{-k} d\theta = k\left(\theta_t^*\right)^k \left.\frac{\theta^{-k+1}}{-k+1}\right|_{\theta_t^*}^{\infty} \\
&= k\left(\theta_t^*\right)^k \frac{0 - \left(\theta_t^*\right)^{-k+1}}{-k+1} = \frac{k}{k-1}\left(\theta_t^*\right)^k \left(\theta_t^*\right)^{-k+1} \\
x_t &= \frac{k}{k-1}\theta_t^*. \qquad (1.B.20)
\end{aligned}$$

Define a measure of the strength of knowledge spillovers:

$$\lambda \equiv \frac{x_t \psi_{min}}{\theta_t^*}.$$

Then, by (1.B.20) and Assumption 1:

$$\lambda = \frac{k}{k-1}\psi_{min} < 1. \tag{1.31}$$

The relative productivity distribution of entrants is

$$\tilde{H}_t(\phi) = F\left(\frac{\phi\theta_t^*}{x_t}\right) = F\left(\frac{\phi\psi_{min}}{\lambda}\right) = 1 - \left(\frac{\phi\psi_{min}}{\lambda\psi_{min}}\right)^{-k} = 1 - \left(\frac{\phi}{\lambda}\right)^{-k},$$

from where it follows that

$$\tilde{H}_t(\phi) = H\left(\frac{\phi}{\lambda}\right). \tag{1.32}$$

The entrant relative productivity distribution is Pareto distributed with shape parameter k and scale parameter $\lambda < 1$. So, the entrant relative productivity distribution has similar functional form as the relative productivity distribution of incumbents, but shifted to the left.

Then, on the balanced growth path, the fraction of entrants that draw productivity below the exit cut-off is

$$\tilde{H}(1) = F\left(\frac{\theta_t^*}{x_t}\right) = F\left(\frac{\psi_{min}}{\lambda}\right) = 1 - \left(\frac{\psi_{min}}{\lambda\psi_{min}}\right)^{-k} = 1 - \lambda^k. \tag{1.B.21}$$

Dynamic selection

Let $q \equiv \dot{c}_t/c_t$ be the growth rate of consumption per capita. Then from the budget constraint (1.2):

$$\frac{\dot{a}_t}{a_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{b}_t}{b_t} = q. \tag{1.33}$$

From the Euler Equation (1.3):

$$q \equiv \frac{\dot{c}_t}{c_t} = \gamma(r_t - \rho).$$

As consumption grows at the constant rate q, it follows that the interest rate $r_t = r$ is constant over time:

$$q = \gamma(r - \rho), \qquad (1.34)$$
$$r = q/\gamma + \rho.$$

As $r_t = r$ and assets per capita a_t grow at the rate q, the transversality condition (1.4) is:

$$\lim_{t\to\infty} a_t \exp\left\{-\int_0^t (r_s - n)ds\right\} = \lim_{t\to\infty} a_t e^{-(r-n)t} = \lim_{t\to\infty} a_0 e^{qt} e^{-(r-n)t} = 0.$$

For the above condition to hold it has to be that $q - (r - n) < 0$, or $r - n > q$. Then, using (1.34) I obtain:

$$\begin{aligned} r - n - q &> 0 \\ \frac{q}{\gamma} + \rho - n - q &> 0 \\ q\frac{1-\gamma}{\gamma} + \rho - n &> 0. \end{aligned} \qquad (1.B.22)$$

Next, log-differentiate (1.12)

$$\begin{aligned} \theta_t^* &= \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1}\left(\frac{w_t^\sigma f}{c_t L_t}\right)^{1/(\sigma-1)} \\ \ln\theta_t^* &= \ln\frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1} + \frac{1}{\sigma-1}\left(\ln w_t^\sigma + \ln f - \ln c_t - \ln L_t\right) \\ d\ln\theta_t^* &= \frac{1}{\sigma-1}\left(\sigma \cdot d\ln w_t - d\ln c_t - d\ln L_t\right) \\ g \equiv \frac{\dot{\theta}_t^*}{\theta_t^*} &= \frac{1}{\sigma-1}\left(\sigma\frac{\dot{w}_t}{w_t} - \frac{\dot{c}_t}{c_t} - \frac{\dot{L}_t}{L_t}\right) \\ g &= \frac{1}{\sigma-1}\left(\sigma q - q - n\right). \end{aligned}$$

Rearrange to get:

$$q = g + \frac{n}{\sigma - 1}, \qquad (1.35)$$

where g is the growth rate of both the exit cut-off and the export cut-off, as well as the rate at which the firm's relative productivity ϕ_t decreases.

Labor market clearing condition The labor market clearing condition is:

$$L_t = M_t \int_\phi l(\phi_t) dH_t(\phi) + N_t,$$

where $l(\phi_t) = l^d(\phi_t) + Jl^x(\phi_t) \cdot I_t[\phi_t \geq \tilde{\phi}]$, and $l^d(\phi_t)$ and $l^x(\phi_t)$ are given by (1.16).

Plug these into L_t:

$$\begin{aligned} L_t =& M_t \int_\phi \left(l^d(\phi_t) + Jl^x(\phi_t) \cdot I[\phi_t \geq \tilde{\phi}] \right) dH_t(\phi) + N_t \\ =& M_t \left[\int_1^\infty l^d(\phi_t) dH_t(\phi) + J \int_{\tilde{\phi}}^\infty l^x(\phi_t) dH_t(\phi) \right] + N_t \\ =& M_t \left[\int_1^\infty f[(\sigma-1)\phi_t^{\sigma-1} + 1] dH_t(\phi) \right. \\ & \left. + J \int_{\tilde{\phi}}^\infty f\tau^{1-\sigma}[(\sigma-1)\phi_t^{\sigma-1} + \tilde{\phi}^{\sigma-1}] dH_t(\phi) \right] + N_t \end{aligned}$$

Using $H_t(\phi) = 1 - \phi^{-k}$ and $dH_t(\phi) = k\phi^{-k-1} d\phi$:

$$\begin{aligned} L_t = & M_t f \left[\int_1^\infty (\sigma-1)\phi_t^{\sigma-1} k\phi_t^{-k-1} d\phi_t + \int_1^\infty dH_t(\phi) \right. \\ & + J\tau^{1-\sigma} \int_{\tilde{\phi}}^\infty (\sigma-1)\phi_t^{\sigma-1} k\phi_t^{-k-1} d\phi_t + J\tau^{1-\sigma} \left. \int_{\tilde{\phi}}^\infty \tilde{\phi}^{\sigma-1} dH_t(\phi) \right] + N_t \\ = & M_t f \left[(\sigma-1)k \frac{\phi_t^{\sigma-1-k}}{\sigma-1-k} \Bigg|_1^\infty - \phi^{-k} \Big|_1^\infty \right. \\ & + J\tau^{1-\sigma}(\sigma-1)k \frac{\phi_t^{\sigma-1-k}}{\sigma-1-k} \Bigg|_{\tilde{\phi}}^\infty \left. - J\tau^{1-\sigma}\tilde{\phi}^{\sigma-1}\phi^{-k} \Big|_{\tilde{\phi}}^\infty \right] + N_t \end{aligned}$$

Remember that, by Assumption 1, $k > \max\{1, \sigma - 1\}$. Then $\sigma - 1 - k < 0$, and the above expression becomes:

$$\begin{aligned}
L_t &= M_t f \Big[(\sigma-1)k\cdot\Big(-\frac{1}{\sigma-1-k}\Big)-(0-1) \\
&\quad - J\tau^{1-\sigma}(\sigma-1)k\frac{\tilde{\phi}^{\sigma-1-k}}{\sigma-1-k}+J\tau^{1-\sigma}\tilde{\phi}^{\sigma-1}\tilde{\phi}^{-k}\Big]+N_t \\
&= \frac{M_t f}{k+1-\sigma}\Big[(\sigma-1)k+(k+1-\sigma)+J\tau^{1-\sigma}(\sigma-1)k\tilde{\phi}^{\sigma-1-k} \\
&\quad +J\tau^{1-\sigma}\tilde{\phi}^{\sigma-1-k}(k+1-\sigma)\Big]+N_t \\
&= \frac{M_t f}{k+1-\sigma}\left[(\sigma-1)k+(k+1-\sigma)\right. \\
&\quad \left.+J\tau^{1-\sigma}\tilde{\phi}^{\sigma-1-k}\left((\sigma-1)k+(k+1-\sigma)\right)\right]+N_t \\
&= M_t f\frac{k\sigma+1-\sigma}{k+1-\sigma}\Big[1+J\tau^{1-\sigma}\tilde{\phi}^{\sigma-1-k}\Big]+N_t.
\end{aligned}$$

Substitute $\tilde{\phi}=(f_x/f)^{1/(\sigma-1)}\tau$ to obtain:

$$L_t = M_t f\frac{k\sigma+1-\sigma}{k+1-\sigma}\left[1+J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right]+N_t. \tag{1.36}$$

Thus, on the balanced growth path I have that

$$\frac{\dot{L}_t}{L_t}=\frac{\dot{M}_t}{M_t}=\frac{\dot{N}_t}{N_t}=n. \tag{1.37}$$

Now use (1.25) together with the fact that M_t grows at the rate n, the exit cutoff θ_t^* grows at the rate g, $H_t'(\phi)=k\phi^{-k-1}$, $H_t'(1)=k$, and $F(\theta_t^*/x_t)=\tilde{H}(1)=H(1/\lambda)=1-\lambda^k$:

$$\begin{aligned}
\frac{\dot{M}_t}{M_t} &= -H_t'(1)\frac{\dot{\theta}_t^*}{\theta_t^*}+\frac{\Omega_t}{M_t}\left[1-F\left(\frac{\theta_t^*}{x_t}\right)\right], \\
\frac{\dot{M}_t}{M_t} &= -k\frac{\dot{\theta}_t^*}{\theta_t^*}+\frac{\Omega_t}{M_t}\lambda^k, \\
n &= -kg+\frac{\Omega_t}{M_t}\lambda^k.
\end{aligned} \tag{1.38}$$

From (1.B.21), the fraction of entrants that draw productivity below the exit cutoff is $\tilde{H}(1)=1-\lambda^k$, so the fraction of entrants that draw productivity above the exit cutoff is λ^k and the successful entry rate is $\frac{\Omega_t}{M_t}\lambda^k$. The rate at which existing producers exit is proportional to the growth rate of the productivity cutoff and is equal to kg. Then, the successful entry rate is equal to the population growth rate plus the exit rate for existing firms:

$$\frac{\Omega_t}{M_t}\lambda^k = n + kg. \tag{1.39}$$

Divide the labor market clearing condition (1.36) by M_t:

$$L_t = M_t f \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma}\left[1 + J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right] + N_t$$
$$\frac{L_t}{M_t} = f \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma}\left[1 + J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right] + \frac{N_t}{M_t}.$$

Substitute (1.39):

$$M_t = \left[f \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma}\left[1 + J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right] + \frac{n + kg}{\lambda^k}\frac{N_t}{\Omega_t}\right]^{-1} L_t$$

Substitute (1.18):

$$M_t = \left[f \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma}\left[1 + J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right] + \frac{n + kg}{\lambda^k}F_t\right]^{-1} L_t. \tag{1.40}$$

Solving for the balanced growth path

Firstly, show that relative productivity declines at rate g:

$$\phi_t \equiv \frac{\theta}{\theta_t^*}$$
$$\frac{\dot{\phi}_t}{\phi_t} = -\frac{\dot{\theta}_t^*}{\theta_t^*} = -g.$$

Then, at time s the relative productivity is

$$\phi_s = e^{-g(s-t)}\phi_t.$$

Now solve for the time when the firm exits, which happens once its relative productivity becomes $\phi_s = 1$:

$$\begin{aligned} e^{-g(s-t)}\phi_t &= 1 \\ g(s-t) &= \ln \phi_t \\ s &= t + \frac{\ln \phi_t}{g}. \end{aligned}$$

So, the firm will exit at time $t + \ln \phi_t / g$.

Further, assume the firm exports at time t, i.e. $\phi_t > \tilde{\phi}$. Solve for the time when the firm will stop exporting, or when $\phi_s = \tilde{\phi}$:

$$\begin{aligned} e^{-g(s-t)}\phi_t &= \tilde{\phi} \\ g(s-t) &= \ln \phi_t - \ln \tilde{\phi} \\ s &= t + \frac{\ln \phi_t - \ln \tilde{\phi}}{g} = t + \frac{\ln(\phi_t/\tilde{\phi})}{g}. \end{aligned}$$

So, conditional on exporting at time t, the firm stops exporting at time $t + \ln(\phi_t/\tilde{\phi})/g$.

Substitute (1.15) into (1.17), taking into account $r_t = r$, the time when firms stop exporting, and the time when firms exit:

$$\begin{aligned} W_t(\phi_t) &= \int_t^{\infty} \pi_v(\phi_v) \exp\left[-\int_t^v r_s ds\right] dv \\ &= \int_t^{t+\frac{\ln \phi_t}{g}} \pi_v^d(\phi_v) e^{-r(v-t)} dv \\ &\quad + J \int_t^{t+\frac{\ln(\phi_t/\tilde{\phi})}{g}} \pi_v^x(\phi_v) e^{-r(v-t)} \cdot I[\phi_t \geq \tilde{\phi}] dv \\ &= \int_t^{t+\frac{\ln \phi_t}{g}} f w_v (\phi_v^{\sigma-1} - 1) e^{-r(v-t)} dv \\ &\quad + J \int_t^{t+\frac{\ln(\phi_t/\tilde{\phi})}{g}} f w_v \tau^{1-\sigma} (\phi_v^{\sigma-1} - \tilde{\phi}^{\sigma-1}) e^{-r(v-t)} \cdot I[\phi_t \geq \tilde{\phi}] dv. \end{aligned} \tag{1.41}$$

Note that the wage and the relative productivity grow at rates q and $-g$, substitute $w_v = w_t e^{q(v-t)}$ and $\phi_v = \phi_t e^{-g(v-t)}$:

$$W_t(\phi_t) = f \int_t^{t+\frac{\ln \phi_t}{g}} w_t e^{q(v-t)} [\phi_t^{\sigma-1} e^{-g(v-t)(\sigma-1)} - 1] e^{-r(v-t)} dv$$
$$+ fJ\tau^{1-\sigma} \cdot$$
$$\cdot \int_t^{t+\frac{\ln(\phi_t/\tilde{\phi})}{g}} w_t e^{q(v-t)} [\phi_t^{\sigma-1} e^{-g(v-t)(\sigma-1)} - \tilde{\phi}^{\sigma-1}] e^{-r(v-t)} \cdot I[\phi_t \geq \tilde{\phi}] dv. \tag{1.B.23}$$

Work separately on the two terms. The first one is

$$f \int_t^{t+\frac{\ln \phi_t}{g}} w_t e^{q(v-t)} [\phi_t^{\sigma-1} e^{-g(v-t)(\sigma-1)} - 1] e^{-r(v-t)} dv =$$
$$= f w_t \left[\phi_t^{\sigma-1} \int_t^{t+\frac{\ln \phi_t}{g}} e^{-(r-q)(v-t)} e^{-g(v-t)(\sigma-1)} dv \right.$$
$$\left. - \int_t^{t+\frac{\ln \phi_t}{g}} e^{-(r-q)(v-t)} dv \right]$$
$$= f w_t \left[- \frac{\phi_t^{\sigma-1}}{g(\sigma-1)+r-q} \, e^{-(v-t)[g(\sigma-1)+r-q]} \Big|_t^{t+\frac{\ln \phi_t}{g}} \right.$$
$$\left. + \frac{1}{r-q} \, e^{-(r-q)(v-t)} \Big|_t^{t+\frac{\ln \phi_t}{g}} \right]$$
$$= f w_t \left[- \frac{\phi_t^{\sigma-1}}{g(\sigma-1)+r-q} \left(e^{-\frac{\ln \phi_t}{g}[g(\sigma-1)+r-q]} - e^0 \right) \right.$$
$$\left. + \frac{1}{r-q} \left(e^{-\frac{\ln \phi_t}{g}(r-q)} - e^0 \right) \right]$$
$$= f w_t \left[- \frac{\phi_t^{\sigma-1}}{g(\sigma-1)+r-q} \left(\phi_t^{\frac{-[g(\sigma-1)+r-q]}{g}} - 1 \right) + \frac{1}{r-q} \left(\phi_t^{\frac{-(r-q)}{g}} - 1 \right) \right]$$
$$= f w_t \left[\frac{\phi_t^{\sigma-1} - \phi_t^{-\frac{r-q}{g}}}{g(\sigma-1)+r-q} + \frac{\phi_t^{-\frac{r-q}{g}} - 1}{r-q} \right]. \tag{1.B.24}$$

This term is unambiguously positive since it equals present discounted value of profits from domestic sales, and the profit flow is positive for active firms.

Furthermore, the transversality condition implies $r - n - q > 0$, from where it follows that $r - q > n > 0$ and $g(\sigma - 1) + r - q > 0$.

The second term is:

$$f J \tau^{1-\sigma} \int_t^{t+\frac{\ln(\phi_t/\tilde{\phi})}{g}} w_t e^{q(v-t)} [\phi_t^{\sigma-1} e^{-g(v-t)(\sigma-1)} - \tilde{\phi}^{\sigma-1}] e^{-r(v-t)} \cdot I[\phi_t \geq \tilde{\phi}] dv$$

$$= f w_t I[\phi_t \geq \tilde{\phi}] J \tau^{1-\sigma} \left[\phi_t^{\sigma-1} \int_t^{t+\frac{\ln(\phi_t/\tilde{\phi})}{g}} e^{-(r-q)(v-t)} e^{-g(v-t)(\sigma-1)} dv \right.$$

$$\left. - \tilde{\phi}^{\sigma-1} \int_t^{t+\frac{\ln(\phi_t/\tilde{\phi})}{g}} e^{-(r-q)(v-t)} dv \right]$$

$$= f w_t I[\phi_t \geq \tilde{\phi}] J \tau^{1-\sigma} \left[- \frac{\phi_t^{\sigma-1}}{g(\sigma-1) + r - q} e^{-(v-t)[g(\sigma-1)+r-q]} \Big|_t^{t+\frac{\ln(\phi_t/\tilde{\phi})}{g}} \right.$$

$$\left. + \frac{\tilde{\phi}^{\sigma-1}}{r-q} e^{-(r-q)(v-t)} \Big|_t^{t+\frac{\ln(\phi_t/\tilde{\phi})}{g}} \right]$$

$$= f w_t I[\phi_t \geq \tilde{\phi}] J \tau^{1-\sigma} \left[- \frac{\phi_t^{\sigma-1}}{g(\sigma-1) + r - q} \left(e^{-\frac{\ln(\phi_t/\tilde{\phi})}{g}[g(\sigma-1)+r-q]} - e^0 \right) \right.$$

$$\left. + \frac{\tilde{\phi}^{\sigma-1}}{r-q} \left(e^{-\frac{\ln(\phi_t/\tilde{\phi})}{g}(r-q)} - e^0 \right) \right]$$

$$= f w_t I[\phi_t \geq \tilde{\phi}] J \tau^{1-\sigma} \left[- \frac{\phi_t^{\sigma-1}}{g(\sigma-1) + r - q} \left(\left[\frac{\phi_t}{\tilde{\phi}} \right]^{\frac{-[g(\sigma-1)+r-q]}{g}} - 1 \right) \right.$$

$$\left. + \frac{\tilde{\phi}^{\sigma-1}}{r-q} \left(\left[\frac{\phi_t}{\tilde{\phi}} \right]^{\frac{-(r-q)}{g}} - 1 \right) \right]$$

$$= fw_t I[\phi_t \geq \tilde{\phi}] J\tau^{1-\sigma} \left[-\frac{\phi_t^{\frac{-(r-q)}{g}} \tilde{\phi}^{\frac{g(\sigma-1)+r-q}{g}}}{g(\sigma-1)+r-q} + \frac{\phi_t^{\sigma-1}}{g(\sigma-1)+r-q} \right.$$

$$\left. + \frac{\phi_t^{\frac{-(r-q)}{g}} \tilde{\phi}^{\sigma-1+\frac{r-q}{g}}}{r-q} - \frac{\tilde{\phi}^{\sigma-1}}{r-q} \right]$$

$$= fw_t I[\phi_t \geq \tilde{\phi}] J\tau^{1-\sigma} \left[\frac{\phi_t^{\sigma-1} - \left(\tilde{\phi}/\phi_t\right)^{\frac{r-q}{g}} \tilde{\phi}^{\sigma-1}}{g(\sigma-1)+r-q} + \frac{\left(\tilde{\phi}/\phi_t\right)^{\frac{r-q}{g}} \tilde{\phi}^{\sigma-1} - \tilde{\phi}^{\sigma-1}}{r-q} \right]. \tag{1.B.25}$$

The second term is also unambiguously positive since it equals present discounted value of profits from sales to foreign consumers, and the profit flow is positive for active exporters.

Now, substitute (1.B.24) and (1.B.25) back into (1.B.23):

$$W_t(\phi_t) = fw_t \left[\frac{\phi_t^{\sigma-1} - \phi_t^{-\frac{r-q}{g}}}{g(\sigma-1)+r-q} + \frac{\phi_t^{-\frac{r-q}{g}} - 1}{r-q} \right]$$

$$+ fw_t I[\phi_t \geq \tilde{\phi}] J\tau^{1-\sigma} \left[\frac{\phi_t^{\sigma-1} - \left(\tilde{\phi}/\phi_t\right)^{\frac{r-q}{g}} \tilde{\phi}^{\sigma-1}}{g(\sigma-1)+r-q} + \frac{\left(\tilde{\phi}/\phi_t\right)^{\frac{r-q}{g}} \tilde{\phi}^{\sigma-1} - \tilde{\phi}^{\sigma-1}}{r-q} \right].$$

From (1.14) it follows that $\tau^{1-\sigma} = \tilde{\phi}^{1-\sigma} f_x/f$. Use this to rewrite the above expression:

$$W_t(\phi_t) = fw_t \left[\frac{\phi_t^{\sigma-1} - \phi_t^{-\frac{r-q}{g}}}{g(\sigma-1)+r-q} + \frac{\phi_t^{-\frac{r-q}{g}} - 1}{r-q} \right]$$

$$+ fw_t I[\phi_t \geq \tilde{\phi}] J \frac{f_x}{f} \left[\frac{\left(\phi_t/\tilde{\phi}\right)^{\sigma-1} - \left(\tilde{\phi}/\phi_t\right)^{\frac{r-q}{g}}}{g(\sigma-1)+r-q} + \frac{\left(\tilde{\phi}/\phi_t\right)^{\frac{r-q}{g}} - 1}{r-q} \right].$$

Finally,

$$W_t(\phi_t) = fw_t \left[\frac{\phi_t^{\sigma-1} - \phi_t^{-\frac{r-q}{g}}}{g(\sigma-1)+r-q} + \frac{\phi_t^{-\frac{r-q}{g}} - 1}{r-q} \right]$$
$$+ fw_t I[\phi_t \geq \tilde{\phi}] J \frac{f_x}{f} \left[\frac{\left(\phi_t/\tilde{\phi}\right)^{\sigma-1} - \left(\phi_t/\tilde{\phi}\right)^{-\frac{r-q}{g}}}{g(\sigma-1)+r-q} + \frac{\left(\phi_t/\tilde{\phi}\right)^{-\frac{r-q}{g}} - 1}{r-q} \right]. \tag{1.42}$$

Remember that the free entry condition (1.23) is:

$$F_t w_t(1-v_e) = \int_\phi W_t(\phi_t) d\tilde{H}_t(\phi_t),$$
$$F_t w_t(1-v_e) = \int_1^\infty W_t(\phi_t) dH\left(\frac{\phi_t}{\lambda}\right).$$

Use (1.32), $H(\phi/\lambda) = 1-\lambda^k\phi^{-k}$, and $dH(\phi/\lambda) = k\lambda^k\phi^{-k-1}d\phi$. Next use (1.42) to solve the integral on the right hand side:

$$\int_1^\infty W_t(\phi_t) dH\left(\frac{\phi_t}{\lambda}\right) = \int_1^\infty W_t(\phi_t) k\lambda^k \phi_t^{-k-1} d\phi_t$$
$$= k\lambda^k f w_t \left(\int_1^\infty \left[\frac{\phi_t^{\sigma-1} - \phi_t^{-\frac{r-q}{g}}}{g(\sigma-1)+r-q} + \frac{\phi_t^{-\frac{r-q}{g}} - 1}{r-q} \right] \phi_t^{-k-1} d\phi_t \right.$$
$$\left. + J\frac{f_x}{f} \int_{\tilde{\phi}}^\infty \left[\frac{\left(\phi_t/\tilde{\phi}\right)^{\sigma-1} - \left(\phi_t/\tilde{\phi}\right)^{-\frac{r-q}{g}}}{g(\sigma-1)+r-q} + \frac{\left(\phi_t/\tilde{\phi}\right)^{-\frac{r-q}{g}} - 1}{r-q} \right] \phi_t^{-k-1} d\phi_t \right). \tag{1.B.26}$$

By the transversality condition $q < r-n$. Then $q-r < -n < 0$ and $\frac{q-r}{g} - k < 0$.

Moreover, by Assumption 1, $\sigma - 1 - k < 0$. Then

$$\int_1^\infty \phi_t^{\sigma-1-k-1} d\phi_t = \left.\frac{\phi_t^{\sigma-1-k}}{\sigma-1-k}\right|_1^\infty = \frac{1}{k+1-\sigma},$$

$$\int_1^\infty \phi_t^{-\frac{r-q}{g}-k-1} d\phi_t = \left.\frac{\phi_t^{-\frac{r-q}{g}-k}}{-\frac{r-q}{g}-k}\right|_1^\infty = \frac{1}{\frac{r-q}{g}+k},$$

$$\int_1^\infty \phi_t^{-k-1} d\phi_t = \left.\frac{\phi_t^{-k}}{-k}\right|_1^\infty = \frac{1}{k}.$$

Similarly,

$$\begin{aligned}
\int_{\tilde{\phi}}^\infty (\phi_t/\tilde{\phi})^{\sigma-1} \phi_t^{-k-1} d\phi_t &= \tilde{\phi}^{1-\sigma} \int_{\tilde{\phi}}^\infty \phi_t^{\sigma-1-k-1} d\phi_t \\
&= \tilde{\phi}^{1-\sigma} \left.\frac{\phi_t^{\sigma-1-k}}{\sigma-1-k}\right|_{\tilde{\phi}}^\infty = \tilde{\phi}^{1-\sigma} \frac{\tilde{\phi}^{\sigma-1-k}}{k+1-\sigma} \\
&= \frac{\tilde{\phi}^{-k}}{k+1-\sigma}, \\
\int_{\tilde{\phi}}^\infty (\phi_t/\tilde{\phi})^{-\frac{r-q}{g}} \phi_t^{-k-1} d\phi_t &= \tilde{\phi}^{\frac{r-q}{g}} \int_{\tilde{\phi}}^\infty \phi_t^{-\frac{r-q}{g}-k-1} d\phi_t \\
&= \tilde{\phi}^{\frac{r-q}{g}} \left.\frac{\phi_t^{-\frac{r-q}{g}-k}}{-\frac{r-q}{g}-k}\right|_{\tilde{\phi}}^\infty = \tilde{\phi}^{\frac{r-q}{g}} \frac{\tilde{\phi}^{-\frac{r-q}{g}-k}}{\frac{r-q}{g}+k} \\
&= \frac{\tilde{\phi}^{-k}}{\frac{r-q}{g}+k}, \\
\int_{\tilde{\phi}}^\infty \phi_t^{-k-1} d\phi_t &= \left.\frac{\phi_t^{-k}}{-k}\right|_{\tilde{\phi}}^\infty = \frac{\tilde{\phi}^{-k}}{k}.
\end{aligned}$$

Then the first term in (1.B.26) becomes

$$\begin{aligned}
&\int_1^\infty \left[\frac{\phi_t^{\sigma-1} - \phi_t^{-\frac{r-q}{g}}}{g(\sigma-1)+r-q} + \frac{\phi_t^{-\frac{r-q}{g}} - 1}{r-q}\right] \phi_t^{-k-1} d\phi_t \\
&= \frac{1}{g(\sigma-1)+r-q}\left(\frac{1}{k+1-\sigma} - \frac{1}{\frac{r-q}{g}+k}\right) + \frac{1}{r-q}\left(\frac{1}{\frac{r-q}{g}+k} - \frac{1}{k}\right),
\end{aligned}$$

and the second term in (1.B.26) becomes

$$\int_{\tilde{\phi}}^{\infty}\left[\frac{\left(\phi_t/\tilde{\phi}\right)^{\sigma-1}-\left(\phi_t/\tilde{\phi}\right)^{-\frac{r-q}{g}}}{g(\sigma-1)+r-q}+\frac{\left(\phi_t/\tilde{\phi}\right)^{-\frac{r-q}{g}}-1}{r-q}\right]\phi_t^{-k-1}d\phi_t$$

$$=\frac{1}{g(\sigma-1)+r-q}\left(\frac{\tilde{\phi}^{-k}}{k+1-\sigma}-\frac{\tilde{\phi}^{-k}}{\frac{r-q}{g}+k}\right)+\frac{1}{r-q}\left(\frac{\tilde{\phi}^{-k}}{\frac{r-q}{g}+k}-\frac{\tilde{\phi}^{-k}}{k}\right)$$

$$=\tilde{\phi}^{-k}\left[\frac{1}{g(\sigma-1)+r-q}\left(\frac{1}{k+1-\sigma}-\frac{1}{\frac{r-q}{g}+k}\right)+\frac{1}{r-q}\left(\frac{1}{\frac{r-q}{g}+k}-\frac{1}{k}\right)\right].$$

Then,

$$\int_{1}^{\infty}W_t(\phi_t)dH\left(\frac{\phi_t}{\lambda}\right)=k\lambda^k f w_t\left(1+J\frac{f_x}{f}\tilde{\phi}^{-k}\right)\cdot$$
$$\cdot\left[\frac{1}{g(\sigma-1)+r-q}\left(\frac{1}{k+1-\sigma}-\frac{1}{\frac{r-q}{g}+k}\right)+\frac{1}{r-q}\left(\frac{1}{\frac{r-q}{g}+k}-\frac{1}{k}\right)\right].$$

The expression in the brackets can be simplified:

$$\frac{1}{g(\sigma-1)+r-q}\left(\frac{1}{k+1-\sigma}-\frac{1}{\frac{r-q}{g}+k}\right)+\frac{1}{r-q}\left(\frac{1}{\frac{r-q}{g}+k}-\frac{1}{k}\right)=$$
$$=\frac{\sigma-1}{(r+kg-q)(k+1-\sigma)k}.$$

To show this, rewrite the above expression by opening the brackets:

$$\begin{aligned}
&\frac{1}{g(\sigma-1)+r-q}\left(\frac{1}{k+1-\sigma}-\frac{1}{\frac{r-q}{g}+k}\right)+\frac{1}{r-q}\left(\frac{1}{\frac{r-q}{g}+k}-\frac{1}{k}\right)=\\
&\quad=\frac{1}{[g(\sigma-1)+r-q]}\frac{1}{(k+1-\sigma)}-\frac{1}{[g(\sigma-1)+r-q]}\frac{g}{(r+kg-q)}\\
&\quad\quad+\frac{1}{[r-q]}\frac{g}{(r+kg-q)}-\frac{1}{[r-q]}\frac{1}{k}\\
&\quad=\frac{1}{[g(\sigma-1)+r-q](k+1-\sigma)}-\frac{1}{[r-q]k}\\
&\quad\quad+\frac{-g(r-q)+g[g(\sigma-1)+r-q]}{[g(\sigma-1)+r-q](r+kg-q)[r-q]}\\
&\quad=\frac{1}{[g(\sigma-1)+r-q](k+1-\sigma)}-\frac{1}{[r-q]k}\\
&\quad\quad+\frac{g^2(\sigma-1)}{[g(\sigma-1)+r-q](r+kg-q)[r-q]}.
\end{aligned} \tag{1.B.27}$$

Consider the first two terms on the LHS in (1.B.27):

$$\begin{aligned}
&\frac{1}{[g(\sigma-1)+r-q]}\frac{1}{(k+1-\sigma)}-\frac{1}{(r-q)k}= \\
&\quad=\frac{(r-q)k-[g(\sigma-1)+r-q](k+1-\sigma)}{[g(\sigma-1)+r-q](k+1-\sigma)(r-q)k} \\
&\quad=\frac{(r-q)k-g(\sigma-1)(k+1-\sigma)-(r-q)k-(r-q)(1-\sigma)}{[g(\sigma-1)+r-q](k+1-\sigma)(r-q)k} \\
&\quad=\frac{-g(\sigma-1)(k+1-\sigma)-(r-q)(1-\sigma)}{[g(\sigma-1)+r-q](k+1-\sigma)(r-q)k} \\
&\quad=\frac{(\sigma-1)[-g(k+1-\sigma)+(r-q)]}{[g(\sigma-1)+r-q](k+1-\sigma)(r-q)k} \\
&\quad=\frac{(\sigma-1)[-g(k+1-\sigma)+(r-q)](r+kg-q)}{k[g(\sigma-1)+r-q](k+1-\sigma)(r-q)(r+kg-q)}. \qquad (1.B.28)
\end{aligned}$$

Multiply and divide by $k(k+1-\sigma)$ the last term from (1.B.27):

$$\begin{aligned}
&\frac{g^2(\sigma-1)}{[g(\sigma-1)+r-q](r+kg-q)[r-q]}= \\
&\quad=\frac{g^2(\sigma-1)k(k+1-\sigma)}{k[g(\sigma-1)+r-q](k+1-\sigma)(r-q)(r+kg-q)}. \qquad (1.B.29)
\end{aligned}$$

Now, as the denominators of expressions (1.B.28) and (1.B.29) are the same, add the expressions in their numerators:

$$\begin{aligned}
&(\sigma-1)[-g(k+1-\sigma)+(r-q)](r+kg-q)+g^2(\sigma-1)k(k+1-\sigma)= \\
&\quad=(\sigma-1)\left[-g(k+1-\sigma)kg+(r-q)kg+g(k+1-\sigma)(q-r)\right. \\
&\qquad\left.+(r-q)^2+g^2k(k+1-\sigma)\right] \\
&\quad=(\sigma-1)\left[(r-q)kg+g(k+1-\sigma)(q-r)+(r-q)^2\right] \\
&\quad=(\sigma-1)(r-q)\left[kg-g(k+1-\sigma)+(r-q)\right] \\
&\quad=(\sigma-1)(r-q)\left[kg-kg+g(\sigma-1)+r-q\right] \\
&\quad=(\sigma-1)(r-q)[g(\sigma-1)+r-q].
\end{aligned}$$

Using (1.B.27) I obtain:

$$\begin{aligned}
&\frac{1}{[g(\sigma-1)+r-q]}\frac{1}{(k+1-\sigma)}-\frac{1}{(r-q)k}+ \\
&\quad+\frac{g(\sigma-1)}{r-q}\frac{1}{[g(\sigma-1)+r-q]}\frac{g}{(r+kg-q)}= \\
&=\frac{(\sigma-1)(r-q)[g(\sigma-1)+r-q]}{k[g(\sigma-1)+r-q](k+1-\sigma)(r-q)(r+kg-q)} \\
&=\frac{\sigma-1}{(r+kg-q)(k+1-\sigma)k}.
\end{aligned}$$

Thus, the expected value of creating a new firm is:

$$\int_1^\infty W_t(\phi_t)dH\left(\frac{\phi_t}{\lambda}\right)=\lambda^k f w_t\left(1+J\frac{f_x}{f}\tilde{\phi}^{-k}\right)\frac{\sigma-1}{(r+kg-q)(k+1-\sigma)}. \tag{1.B.30}$$

The free entry condition becomes:

$$F_t w_t(1-v_e)=\lambda^k f w_t\left(1+J\frac{f_x}{f}\tilde{\phi}^{-k}\right)\frac{\sigma-1}{k+1-\sigma}\frac{1}{r+kg-q}. \tag{1.43}$$

Substitute for $\tilde{\phi}$ from (1.14):

$$F_t w_t(1-v_e)=\lambda^k f w_t\left(1+J\frac{f_x}{f}\left(\frac{f_x}{f}\right)^{-k/(\sigma-1)}\tau^{-k}\right)\frac{\sigma-1}{k+1-\sigma}\frac{1}{r+kg-q}.$$

$$F_t w_t(1-v_e)=\lambda^k f w_t\left(1+J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right)\frac{\sigma-1}{k+1-\sigma}\frac{1}{r+kg-q}.$$

Divide through by w_t:

$$F_t(1-v_e)=\lambda^k f\left(1+J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right)\frac{\sigma-1}{k+1-\sigma}\frac{1}{r+kg-q}.$$

Substitute for r and q into the term $r + kg - q$ from (1.35) and (1.34):

$$
\begin{aligned}
r + kg - q &= kg + \frac{q}{\gamma} + \rho - q \\
&= kg + \frac{1-\gamma}{\gamma} q + \rho \\
&= kg + \frac{1-\gamma}{\gamma}\left(g + \frac{n}{\sigma - 1}\right) + \rho \\
&= g\left(k + \frac{1-\gamma}{\gamma}\right) + \frac{1-\gamma}{\gamma}\frac{n}{\sigma-1} + \rho \\
&= g\frac{1+(k-1)\gamma}{\gamma} + \frac{1-\gamma}{\gamma}\frac{n}{\sigma-1} + \rho.
\end{aligned}
$$

Substitute this back into the free entry condition:

$$
\begin{aligned}
F_t(1-v_e) = \lambda^k f \left(1 + J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}\right) \frac{\sigma-1}{k+1-\sigma} \cdot \\
\cdot \frac{1}{g\frac{1+(k-1)\gamma}{\gamma} + \frac{1-\gamma}{\gamma}\frac{n}{\sigma-1} + \rho}.
\end{aligned}
\tag{1.44}
$$

Proposition 1. *When the R&D technology is given by the functional forms* (1.19), (1.20) *or* (1.21), *a decrease in the variable trade cost* τ *unambiguously leads to an increase in the rate of dynamic selection* g, *an increase in the successful entry rate* $\lambda^k\Omega_t/M_t$, *and a decrease in the equilibrium mass of producers* M_t. $[\tau \downarrow \implies g \equiv \dot{\theta}_t^*/\theta_t^* \uparrow,\ \lambda^k\Omega/M_t \uparrow,$ *and* $M_t \downarrow]$ *An increase in the R&D subsidy rate* v_e *has the same effects.* $[v_e \uparrow \implies g \equiv \dot{\theta}_t^*/\theta_t^* \uparrow,\ \lambda^k\Omega/M_t \uparrow,$ *and* $M_t \downarrow]$

Proof. Let $Z \equiv 1 + J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}$ denote the term in large brackets in (1.44). An increase in the number of trading partners J, as well as a decrease in the variable trade cost τ or the fixed trade cost f_x always lead to an increase in Z. An increase in the R&D subsidy v_e leads to a decline in term $(1 - v_e)$. Throughout the proof I use that $\frac{1+(k-1)\gamma}{\gamma}$, $\frac{\sigma-1}{k+1-\sigma}$, and $\frac{k\sigma+1-\sigma}{k+1-\sigma}$ are positive constants.

Consider alternative functional forms for R&D technology each in turn.

First, consider the case (1.19) of constant returns to scale in R&D, where $F_t = f_e$ is a positive constant. Then, by the free entry condition (1.44), an increase in Z or an increase in v_e, holding all other parameters fixed, must be offset by an increase in g. Then, from (1.40) the equilibrium mass of producers M_t declines, and from (1.39) the successful entry rate $\lambda^k\Omega_t/M_t$ increases.

Second, consider the Sampson (2016) functional form for R&D technology in (1.21). The individual entry cost can be rewritten using (1.B.11) and (1.39) as

$$F_t = f_e \left(\frac{\Omega_t}{M_t} \right)^{(1-\alpha)/\alpha} = f_e \left(\frac{n + gk}{\lambda^k} \right)^{(1-\alpha)/\alpha},$$

so F_t is strictly increasing in g for $0 < \alpha < 1$. Then, by the free entry condition (1.44), an increase in Z or an increase in v_e lead to only one possible outcome, where the dynamic selection rate g and the individual fixed entry cost F_t increase simultaneously. In such case, by (1.39) the successful entry rate increases, and by (1.40) the equilibrium mass of entrants M_t declines.

Third, consider the Segerstrom and Sugita (2016) functional form for R&D technology in (1.20). By (1.B.10), the individual entry cost is $F_t = f_e \Omega_t^{(1-\beta)/\beta}$. Equation (1.44) implies that an increase in Z or an increase in v_e can potentially lead to the following outcomes:

1. The dynamic selection rate g decreases. Then, the right-hand side of (1.44) increases and the fixed individual entry cost F_t increases. By (1.B.10) the mass of entrants Ω_t also increases. Use (1.40) to substitute for M_t into equation (1.39):

$$\frac{\Omega_t}{M_t} \lambda^k = n + gk$$

$$\frac{\Omega_t}{L_t} \lambda^k \left[f \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} \left(1 + J \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k} \right) + \frac{n + kg}{\lambda^k} F_t \right] = n + gk$$

Divide through by the expression on the right-hand side and use notation for Z:

$$\frac{\Omega_t}{L_t} \left[f \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} Z \frac{\lambda^k}{n + gk} + F_t \right] = 1 \qquad (1.B.31)$$

In case Z increases, simultaneous increases in Z, F_t, and Ω_t and a decrease in g contradicts the above equation, as it would lead to an unambiguous increase in the term on the left-hand side. In case v_e increases holding Z fixed, a simultaneous increases in F_t and Ω_t and a decrease in g contradicts (1.B.31). Thus, such an outcome is not possible.

2. The dynamic selection rate g is not affected. Then, the right-hand side of (1.44) increases, and the fixed individual entry cost F_t must increase. By (1.39) the successful entry rate $\lambda^k\Omega_t/M_t$ stays constant, whereas by (1.B.10) the mass of entrants Ω_t has to increase. So the equilibrium mass of producers M_t has to increase, which contradicts (1.40). Thus, such an outcome is not possible.

3. The dynamic selection rate g increases. Given the first two outcomes are not possible, this outcome has to occur, that is, an increase in Z or an increase in v_e has to lead to g increasing. Then, by (1.39) the successful entry rate $\lambda^k\Omega_t/M_t$ increases. Consider the alternative outcomes for F_t.

   (a) The fixed individual entry cost F_t is not affected. Then, by (1.B.10) the mass of entrants Ω_t is not affected. This is possible, as by (1.40) the equilibrium mass of producers M_t unambiguously declines.

   (b) The fixed individual entry cost F_t increases. Then, by (1.B.10) the mass of entrants Ω_t must increase. This is possible, as by (1.40) the equilibrium mass of producers M_t declines.

   (c) The fixed individual entry cost F_t decreases. Then, by (1.B.10) the mass of entrants Ω_t decreases. In case v_e increases holding Z fixed, a simultaneous decrease in F_t and Ω_t and an increase in g contradicts (1.B.31). Thus, such an outcome is not possible when v_e increases. Consider the case when Z increases holding v_e fixed. Rewrite the free entry condition (1.44):

$$\frac{F_t(1-v_e)}{\lambda^k f}\frac{k+1-\sigma}{\sigma-1}\left(g\frac{1+(k-1)\gamma}{\gamma}+\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}+\rho\right)=Z.$$

   Use the above expression to substitute for Z into (1.B.31):

$$\frac{\Omega_t}{L_t}\left[f\frac{k\sigma+1-\sigma}{k+1-\sigma}\frac{F_t(1-v_e)}{\lambda^k f}\frac{k+1-\sigma}{\sigma-1}\cdot\right.$$
$$\left.\cdot\left(g\frac{1+(k-1)\gamma}{\gamma}+\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}+\rho\right)\frac{\lambda^k}{n+gk}+F_t\right]=1.$$

Simplify the expression:

$$\frac{\Omega_t}{L_t}\left[\frac{k\sigma+1-\sigma}{\sigma-1}F_t(1-v_e)\cdot\right.$$
$$\left.\cdot\left(g\frac{1+(k-1)\gamma}{\gamma}+\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}+\rho\right)\frac{1}{n+gk}+F_t\right]=1$$

$$\frac{\Omega_t}{L_t}F_t\left[\frac{k\sigma+1-\sigma}{\sigma-1}(1-v_e)\frac{g\frac{1+(k-1)\gamma}{\gamma}+\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}+\rho}{n+gk}+1\right]=1.$$

Define the following function:

$$h(g)\equiv\frac{g\frac{1+(k-1)\gamma}{\gamma}+\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}+\rho}{n+gk}.$$

Given that an increase in Z leads to decreases in F_t and Ω_t and an increase in g, this outcome is possible only if h is an increasing function in g. To investigate the behavior of this function, take its first order derivative with respect to g:

$$\begin{aligned}\frac{\partial h(g)}{\partial g}&=\frac{\frac{1+(k-1)\gamma}{\gamma}(n+gk)-\left(g\frac{1+(k-1)\gamma}{\gamma}+\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}+\rho\right)k}{[n+gk]^2}\\&=\frac{\frac{1+(k-1)\gamma}{\gamma}n+\frac{1+(k-1)\gamma}{\gamma}gk-gk\frac{1+(k-1)\gamma}{\gamma}-\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}k-\rho k}{[n+gk]^2}\\&=\frac{\frac{1+(k-1)\gamma}{\gamma}n-\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}k-\rho k}{[n+gk]^2}.\end{aligned}$$

The first order derivative of h with respect to g is positive when the following condition holds:

$$\frac{1+(k-1)\gamma}{\gamma}n-\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}k-\rho k>0.$$

When parameters satisfy the above condition, it is possible that the mass of entrants Ω_t decreases following trade liberalization. In such case, from (1.39) the equilibrium mass of producers M_t must decrease even more.[17]

[17] In the calibration and numerical exercise I use values of parameters that do satisfy this condition.

To conclude, I have shown that, under functional forms for R&D technology considered in this paper, the only possible outcomes given an increase in Z or an increase in v_e are where the dynamic selection rate g increases. The effect of trade liberalization on the successful entry rate $\lambda^k\Omega_t/M_t$ is always positive, and the effect of trade liberalization on the equilibrium mass of producers M_t is always negative. The effects on the mass of entrants Ω_t and on the fixed individual entry cost F_t is unclear and depends on specific values of parameters. □

Rearrange the terms in (1.44):

$$g\frac{1+(k-1)\gamma}{\gamma}+\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}+\rho=\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{F_t(1-v_e)}\cdot \cdot\left(1+J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right).$$

Solve for the dynamic selection rate:

$$g=\frac{\gamma}{1+(k-1)\gamma}\left(\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{F_t(1-v_e)}\left(1+J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right) -\frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}-\rho\right). \tag{1.B.32}$$

This equation solves for the equilibrium dynamic selection rate as a function of parameters and endogenous R&D cost F_t. Using (1.35) I can solve for an equilibrium growth rate of consumption:

$$\begin{aligned} q &= g+\frac{n}{\sigma-1} \\ &= \frac{\gamma}{1+(k-1)\gamma}\left(\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{F_t(1-v_e)}\left(1+J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right) +k\frac{n}{\sigma-1}-\rho\right), \end{aligned} \tag{1.45}$$

where I used

$$\begin{aligned} 1-\frac{\gamma}{1+(k-1)\gamma}\frac{1-\gamma}{\gamma} = 1-\frac{1-\gamma}{1+(k-1)\gamma} &= \frac{1+(k-1)\gamma-1+\gamma}{1+(k-1)\gamma} \\ &= \frac{\gamma}{1+(k-1)\gamma}k. \end{aligned}$$

Finally, I can solve for the interest rate r using (1.45) and $r = q/\gamma + \rho$.

To ensure that the growth rate of productivity cutoff g (the dynamic selection rate) is positive the following condition must be satisfied:

$$\frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{F_t(1 - v_e)} > \frac{1 - \gamma}{\gamma} \frac{n}{\sigma - 1} + \rho.$$

This inequality ensures that the smallest possible g is positive, i.e. for $J = 0$. For $J > 0$ the term $\left(1 + J\left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}\right)$ is positive, and as $\sigma > 1$ and $k > \max\{1, \sigma - 1\}$, then $g > 0$.

For the transversality condition (1.B.22) to hold the following inequality must be satisfied:

$$q\frac{1 - \gamma}{\gamma} + \rho - n > 0.$$

Substitute for q from (1.45):

$$\frac{1 - \gamma}{\gamma} \frac{\gamma}{1 + (k - 1)\gamma} \cdot \left(\frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{F_t(1 - v_e)} \left(1 + J\left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}\right) + k\frac{n}{\sigma - 1} - \rho\right) > n - \rho.$$

Rearrange the terms:

$$\begin{aligned}
\frac{(1 - \gamma)(\sigma - 1)}{k + 1 - \sigma} \frac{\lambda^k f}{F_t(1 - v_e)} \left(1 + J\left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}\right) > \\
&> (n - \rho)(1 + (k - 1)\gamma) + (1 - \gamma)(\rho - k\frac{n}{\sigma - 1}) \\
&= n - \rho + (n - \rho)k\gamma - (n - \rho)\gamma + (1 - \gamma)\rho - \frac{(1 - \gamma)kn}{\sigma - 1} \\
&= (n - \rho)k\gamma + (1 - \gamma)\left(n - \frac{kn}{\sigma - 1}\right) \\
&= (n - \rho)k\gamma - (1 - \gamma)\frac{k + 1 - \sigma}{\sigma - 1}n.
\end{aligned}$$

In case of constant returns to scale in R&D, when $F_t = f_e$, the above conditions are the functions of parameters only and can easily be checked. For the case when $F_t \neq f_e$, the numerical solution to the model ensures that the dynamic selection is positive and the transversality condition is satisfied.

Asset market clearing condition Substitute (1.42) into (1.28):

$$a_t L_t = M_t \int_\phi W_t(\phi) dH_t(\phi)$$

By setting $\lambda = 1$ in (1.B.30), we obtain that

$$\int_1^\infty W_t(\phi_t) dH(\phi) = f w_t \left(1 + J \frac{f_x}{f} \tilde{\phi}^{-k}\right) \frac{\sigma - 1}{(r + kg - q)(k + 1 - \sigma)}.$$

Then the asset market clearing condition becomes

$$a_t L_t = M_t f w_t \left(1 + J \frac{f_x}{f} \tilde{\phi}^{-k}\right) \frac{\sigma - 1}{(r + kg - q)(k + 1 - \sigma)}.$$

Firstly, solve for the term $r + kg - q$ from (**??**):

$$r + kg - q = \frac{\sigma - 1}{(k + 1 - \sigma)} \frac{\lambda^k f}{F_t(1 - v_e)} \left(1 + J \frac{f_x}{f} \tilde{\phi}^{-k}\right).$$

Substitute this into the asset market clearing condition:

$$a_t L_t = M_t f w_t \left(1 + J \frac{f_x}{f} \tilde{\phi}^{-k}\right) \frac{\sigma - 1}{(k + 1 - \sigma)} \frac{k + 1 - \sigma}{\sigma - 1} \frac{F_t(1 - v_e)}{\lambda^k f} \frac{1}{\left(1 + J \frac{f_x}{f} \tilde{\phi}^{-k}\right)}$$

$$a_t L_t = M_t w_t \frac{F_t}{\lambda^k} (1 - v_e).$$

Solving for initial consumption

Assumption 2. *Assume that the initial mass of potential entrants is exogenously given by $\hat{M}_0$, and their productivity distribution $\hat{G}_0(\theta) = 1 - \left(\theta / \hat{\theta}_0^*\right)^{-k}$ is Pareto with scale parameter $\hat{\theta}_0^*$ and shape parameter k.*

Let θ_0^* be an equilibrium domestic productivity cutoff at time $t = 0$. Then equilibrium mass of incumbents at time $t = 0$ is given by $M_0 = \hat{M}_0(1 - \hat{G}_0(\theta_0^*))$. Given Assumption 2, it can be rewritten as

$$M_0 = \hat{M}_0 \left(\frac{\theta_0^*}{\hat{\theta}_0^*}\right)^{-k}$$

and the equilibrium domestic productivity cutoff θ_0^* at time $t = 0$ is given by:

$$\theta_0^* = \hat{\theta}_0^* \left(\frac{\hat{M}_0}{M_0} \right)^{1/k}. \tag{1.46}$$

Moreover, it was shown previously in (1.12) that the productivity cutoff can be computed as:

$$\theta_0^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1} \left(\frac{f w_0^\sigma}{c_0 L_0} \right)^{1/(\sigma-1)} = \hat{\theta}_0^* \left(\frac{\hat{M}_0}{M_0} \right)^{1/k},$$

which I can use to solve for the wage w_0 as a function of c_0 and M_0:

$$w_0 = \left(\hat{\theta}_0^*\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\hat{M}_0}{M_0} \right)^{\frac{1}{k}\frac{\sigma-1}{\sigma}} c_0^{\frac{1}{\sigma}} L_0^{\frac{1}{\sigma}} f^{-\frac{1}{\sigma}} \frac{(\sigma-1)^{\frac{\sigma-1}{\sigma}}}{\sigma}. \tag{1.47}$$

From the asset market clearing I have:

$$a_0 = \frac{M_0}{L_0} w_0 \frac{F_0}{\lambda^k} (1 - v_e), \tag{1.48}$$

and from the government balanced budget condition (1.29) I have

$$\begin{aligned} b_0 &= w_0 \frac{N_0}{L_0} v_e \\ &= \frac{M_0}{L_0} w_0 \frac{N_0}{M_0} v_e. \end{aligned} \tag{1.49}$$

Substitute for $1/M_0$ from (1.39):

$$\begin{aligned} b_0 &= \frac{M_0}{L_0} w_0 \frac{n + kg}{\lambda^k} \frac{N_0}{\Omega_0} v_e \\ &= \frac{M_0}{L_0} w_0 \frac{n + kg}{\lambda^k} F_0 v_e. \end{aligned}$$

From the budget constraint (1.2) using $r_t = r$ I obtain

$$\dot{a}_t - a_t(r - n) = w_t - c_t - b_t.$$

Multiply both sides of the budget constraint by $e^{-(r-n)t}$ and integrate over time:

$$\begin{aligned}
e^{-(r-n)t}(\dot{a}_t - a_t(r-n)) &= e^{-(r-n)t}(w_t - c_t - b_t) \\
\int_0^T e^{-(r-n)t}(\dot{a}_t - a_t(r-n))dt &= \int_0^T e^{-(r-n)t}(w_t - c_t - b_t)dt \\
e^{-(r-n)t}a_t|_0^T &= \int_0^T e^{-(r-n)t}(w_t - c_t - b_t)dt \\
a_T e^{-(r-n)T} - a_0 &= \int_0^T e^{-(r-n)t}(w_t - c_t - b_t)dt.
\end{aligned}$$

Take the limit as $T \to \infty$ and use the transversality condition $\lim_{t\to\infty} a_t e^{-(r-n)t} = 0$ to obtain

$$-a_0 = \int_0^\infty e^{-(r-n)t}(w_t - c_t - b_t)dt.$$

Now use that $c_t = c_0 e^{qt}$ and $w_t = w_0 e^{qt}$, and also $b_t = b_0 e^{qt}$, and $q - (r-n) < 0$ by the transversality condition:

$$\begin{aligned}
-a_0 &= \int_0^\infty e^{-(r-n)t}(w_0 e^{qt} - c_0 e^{qt} - b_0 e^{qt})dt \\
&= (w_0 - c_0 - b_0)\int_0^\infty e^{-(r-n)t}e^{qt}dt \\
&= (w_0 - c_0 - b_0)\int_0^\infty e^{(q-r+n)t}dt \\
&= (w_0 - c_0 - b_0)\left.\frac{e^{(q-r+n)t}}{q-r+n}\right|_0^\infty \\
&= (w_0 - c_0 - b_0)\left(0 - \frac{1}{q-r+n}\right) \\
-a_0 &= \frac{w_0 - c_0 - b_0}{r-n-q} \\
c_0 &= w_0 + (r-n-q)a_0 - b_0.
\end{aligned}$$

Using (1.34), rewrite $r - n - q = (q/\gamma + \rho) - n - q = q\,(1/\gamma - 1) + \rho - n$. Then,

$$c_0 = w_0 + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right)a_0 - b_0. \tag{1.50}$$

Substitute for a_0 and b_0 into consumption equation (1.50):

$$c_0 = w_0 \left[1 + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right) \frac{M_0}{L_0}\frac{F_0}{\lambda^k}(1-v_e) - \frac{M_0}{L_0}\frac{n+kg}{\lambda^k}F_0 v_e\right] \quad (1.B.33)$$

Substitute for w_0:

$$c_0 = \left(\hat{\theta}_0^*\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\hat{M}_0}{M_0}\right)^{\frac{1}{k}\frac{\sigma-1}{\sigma}} c_0^{\frac{1}{\sigma}} L_0^{\frac{1}{\sigma}} f^{-\frac{1}{\sigma}} \frac{(\sigma-1)^{\frac{\sigma-1}{\sigma}}}{\sigma} \cdot$$
$$\cdot \left[1 + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right) \frac{M_0}{L_0}\frac{F_0}{\lambda^k}(1-v_e) - \frac{M_0}{L_0}\frac{n+kg}{\lambda^k}F_0 v_e\right]$$

Solve for c_0:

$$c_0^{\frac{\sigma-1}{\sigma}} = \left(\hat{\theta}_0^*\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\hat{M}_0}{M_0}\right)^{\frac{1}{k}\frac{\sigma-1}{\sigma}} L_0^{\frac{1}{\sigma}} f^{-\frac{1}{\sigma}} \frac{(\sigma-1)^{\frac{\sigma-1}{\sigma}}}{\sigma} \cdot$$
$$\cdot \left[1 + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right) \frac{M_0}{L_0}\frac{F_0}{\lambda^k}(1-v_e) - \frac{M_0}{L_0}\frac{n+kg}{\lambda^k}F_0 v_e\right]$$

$$c_0 = (\sigma-1)\hat{\theta}_0^* \left(\frac{\hat{M}_0}{M_0}\right)^{\frac{1}{k}} L_0^{\frac{1}{\sigma-1}} f^{-\frac{1}{\sigma-1}} \sigma^{-\frac{\sigma}{\sigma-1}} \cdot$$
$$\cdot \left[1 + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right) \frac{M_0}{L_0}\frac{F_0}{\lambda^k}(1-v_e) - \frac{M_0}{L_0}\frac{n+kg}{\lambda^k}F_0 v_e\right]^{\frac{\sigma}{\sigma-1}}$$

$$c_0 = (\sigma-1)\hat{\theta}_0^* \left(\frac{\hat{M}_0}{M_0}\right)^{\frac{1}{k}} L_0^{\frac{1}{\sigma-1}} f^{-\frac{1}{\sigma-1}} \sigma^{-\frac{\sigma}{\sigma-1}} \left(\frac{M_0}{L_0}\right)^{\frac{\sigma}{\sigma-1}} \cdot$$
$$\cdot \left[\frac{L_0}{M_0} + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right) \frac{F_0}{\lambda^k}(1-v_e) - \frac{n+kg}{\lambda^k}F_0 v_e\right]^{\frac{\sigma}{\sigma-1}} .$$

Collect the M_0, L_0 terms:

$$c_0 = (\sigma-1)\hat{\theta}_0^* \hat{M}_0^{\frac{1}{k}} L_0^{-1} f^{-\frac{1}{\sigma-1}} \sigma^{-\frac{\sigma}{\sigma-1}} M_0^{\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \cdot$$
$$\cdot \left[\frac{L_0}{M_0} + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right) \frac{F_0}{\lambda^k}(1-v_e) - \frac{n+kg}{\lambda^k}F_0 v_e\right]^{\frac{\sigma}{\sigma-1}} .$$

Now substitute for M_0 using (1.40):

$$M_0 = \left[f\frac{k\sigma+1-\sigma}{k+1-\sigma}\left[1 + J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}\right] + \frac{n+kg}{\lambda^k}F_0\right]^{-1} L_0, \quad (1.B.34)$$

and denoting $Z \equiv \left[1 + J\left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}\right]$:

$$c_0 = \frac{(\sigma-1)\hat{\theta}_0^* \hat{M}_0^{\frac{1}{k}} L_0^{-1}}{f^{\frac{1}{\sigma-1}} \sigma^{\frac{\sigma}{\sigma-1}}} L_0^{\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \left[f \frac{k\sigma+1-\sigma}{k+1-\sigma} Z + \frac{n+kg}{\lambda^k} F_0\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \cdot$$

$$\cdot \left[\frac{L_0}{M_0} + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right) \frac{F_0}{\lambda^k}(1-v_e) - \frac{n+kg}{\lambda^k} F_0 v_e\right]^{\frac{\sigma}{\sigma-1}}.$$

Work separately on the two expressions in brackets. Let X_1 denote the first one:

$$\begin{aligned} X_1 &\equiv \left[f \frac{k\sigma+1-\sigma}{k+1-\sigma} Z + \frac{n+kg}{\lambda^k} F_0\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \\ &= \left(\frac{fZ}{k+1-\sigma}\right)^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \left[k\sigma + 1 - \sigma + \frac{n+kg}{\lambda^k} F_0 \frac{k+1-\sigma}{fZ}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}. \end{aligned}$$

From (1.45):

$$q = \frac{\gamma}{1+(k-1)\gamma}\left(\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{F_0}\frac{1}{1-v_e}Z + k\frac{n}{\sigma-1} - \rho\right)$$

$$\begin{aligned} \frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{F_0}\frac{1}{1-v_e}Z + k\frac{n}{\sigma-1} - \rho &= \frac{q\,(1+(k-1)\gamma)}{\gamma} \\ \frac{1}{k+1-\sigma}\frac{\lambda^k f}{F_0}\frac{\sigma-1}{1-v_e}Z &= \frac{q}{\gamma} + kq - q - k\frac{n}{\sigma-1} + \rho \\ &= q\frac{1-\gamma}{\gamma} + k\left(q - \frac{n}{\sigma-1}\right) + \rho \\ &= q\frac{1-\gamma}{\gamma} + kg + \rho \\ \frac{1}{k+1-\sigma}\frac{\lambda^k f}{F_0}Z &= \left(q\frac{1-\gamma}{\gamma} + kg + \rho\right)\frac{1-v_e}{\sigma-1}. \end{aligned}$$

Then X_1 becomes:

$$X_1 = \left(\frac{fZ}{k+1-\sigma}\right)^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \left[k\sigma + 1 - \sigma + \frac{n+kg}{q\frac{1-\gamma}{\gamma} + kg + \rho}\frac{\sigma-1}{1-v_e}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}.$$

Work on the expression in second brackets, which I denote as X_2. Firstly substitute for $\frac{L_0}{M_0}$:

$$\begin{aligned}
X_2 &\equiv \left[\frac{L_0}{M_0} + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right)\frac{F_0}{\lambda^k}(1-v_e) - \frac{n+kg}{\lambda^k}F_0 v_e\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[f\frac{k\sigma+1-\sigma}{k+1-\sigma}Z + \frac{n+kg}{\lambda^k}F_0\right. \\
&+ \left.\left(q\frac{1-\gamma}{\gamma} + \rho - n\right)\frac{F_0}{\lambda^k}(1-v_e) - \frac{n+kg}{\lambda^k}F_0 v_e\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{fZ}{k+1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}\left[k\sigma+1-\sigma+\frac{n+kg}{\lambda^k}F_0\frac{k+1-\sigma}{fZ}\right. \\
&+ \left.\left(q\frac{1-\gamma}{\gamma} + \rho - n\right)\frac{F_0}{\lambda^k}(1-v_e)\frac{k+1-\sigma}{fZ} - \frac{n+kg}{\lambda^k}F_0 v_e\frac{k+1-\sigma}{fZ}\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{fZ}{k+1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}\left[k\sigma+1-\sigma\right. \\
&\quad \left.+\frac{F_0}{\lambda^k}\frac{k+1-\sigma}{fZ}(1-v_e)\left[n+kg+q\frac{1-\gamma}{\gamma}+\rho-n\right]\right]^{\frac{\sigma}{\sigma-1}}.
\end{aligned}$$

Substitute for

$$\frac{1}{k+1-\sigma}\frac{\lambda^k f}{F_0}Z = (1-v_e)\left(q\frac{1-\gamma}{\gamma}+kg+\rho\right)\frac{1}{\sigma-1}$$

to obtain:

$$\begin{aligned}
X_2 &= \left(\frac{fZ}{k+1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}\left[k\sigma+1-\sigma\right. \\
&\quad \left.+\frac{\sigma-1}{q\frac{1-\gamma}{\gamma}+kg+\rho}\frac{1}{1-v_e}(1-v_e)\left[q\frac{1-\gamma}{\gamma}+kg+\rho\right]\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{fZ}{k+1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}\left[k\sigma+1-\sigma+\sigma-1\right]^{\frac{\sigma}{\sigma-1}} \\
&= \left(\frac{fZ}{k+1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}\left[k\sigma\right]^{\frac{\sigma}{\sigma-1}}.
\end{aligned}$$

Now collect all the parts given by X_1 and X_2 and substitute back into the expression for c_0:

$$c_0 = \frac{(\sigma-1)\hat{\theta}_0^* \hat{M}_0^{\frac{1}{k}} L_0^{-1}}{f^{\frac{1}{\sigma-1}} \sigma^{\frac{\sigma}{\sigma-1}}} L_0^{\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \cdot$$
$$\cdot \left(\frac{fZ}{k+1-\sigma}\right)^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \left[k\sigma+1-\sigma+\frac{n+kg}{q\frac{1-\gamma}{\gamma}+kg+\rho}\frac{\sigma-1}{1-v_e}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \cdot$$
$$\cdot \left(\frac{fZ}{k+1-\sigma}\right)^{\frac{\sigma}{\sigma-1}} [k\sigma]^{\frac{\sigma}{\sigma-1}}.$$

Collect the terms and cancel what can be canceled:

$$c_0 = (\sigma-1)\hat{\theta}_0^* \hat{M}_0^{\frac{1}{k}} L_0^{\frac{k+1-\sigma}{k(\sigma-1)}} f^{-\frac{k+1-\sigma}{k(\sigma-1)}} Z^{\frac{1}{k}} \cdot$$
$$\cdot \left(\frac{1}{k+1-\sigma}\right)^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \left[k\sigma+1-\sigma+\frac{n+kg}{q\frac{1-\gamma}{\gamma}+kg+\rho}\frac{\sigma-1}{1-v_e}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \cdot$$
$$\cdot \left(\frac{1}{k+1-\sigma}\right)^{\frac{\sigma}{\sigma-1}} k^{\frac{\sigma}{\sigma-1}}.$$

Now I have obtained:

$$c_0 = A f^{-\frac{k+1-\sigma}{k(\sigma-1)}} \left[1+J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}\right]^{\frac{1}{k}} \cdot \left[k\sigma+1-\sigma+\frac{n+kg}{q\frac{1-\gamma}{\gamma}+kg+\rho}\frac{\sigma-1}{1-v_e}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}, \tag{1.51}$$

where

$$A \equiv (\sigma-1)k^{\frac{\sigma}{\sigma-1}} (k+1-\sigma)^{-\frac{1}{k}} \hat{\theta}_0^* \hat{M}_0^{\frac{1}{k}} L_0^{\frac{k+1-\sigma}{k(\sigma-1)}}.$$

Note that the following terms are positive: $k+1-\sigma > 0$, $k\sigma+1-\sigma > 0$, $g > 0$, $n+kg > 0$, $q\frac{1-\gamma}{\gamma}+\rho-n > 0$, thus $A > 0$ and the final term in (1.51) is positive.

1.B.5 Gains from trade

Substitute $c_t = c_0 e^{qt}$ into (1.1), integrate, and use the transversality condition (1.B.22) to obtain:

$$\begin{aligned}
U &= \int_{t=0}^{\infty} e^{-\rho t} e^{nt} \frac{(c_0 e^{qt})^{1-1/\gamma} - 1}{1 - \frac{1}{\gamma}} dt \\
&= \frac{\gamma}{\gamma - 1} \left(c_0^{1-1/\gamma} \int_{t=0}^{\infty} e^{\left(-\rho+n+q\frac{\gamma-1}{\gamma}\right)t} dt - \int_{t=0}^{\infty} e^{(-\rho+n)t} dt \right) \\
&= \frac{\gamma}{\gamma - 1} \left(c_0^{1-1/\gamma} \frac{e^{(-\rho+n+q\frac{\gamma-1}{\gamma})t}}{-\rho + n + q\frac{\gamma-1}{\gamma}} |_0^{\infty} - \frac{e^{(-\rho+n)t}}{-\rho + n} |_0^{\infty} \right) \\
&= \frac{\gamma}{\gamma - 1} \left(-\frac{c_0^{\frac{\gamma-1}{\gamma}}}{-\rho + n + q\frac{\gamma-1}{\gamma}} - \frac{1}{\rho - n} \right) \\
&= \frac{\gamma}{\gamma - 1} \left(\frac{\gamma c_0^{\frac{\gamma-1}{\gamma}}}{(1-\gamma)q + \gamma(\rho - n)} - \frac{1}{\rho - n} \right) \\
&= \frac{\gamma}{1 - \gamma} \left(\frac{1}{\rho - n} - \frac{\gamma c_0^{-\frac{1-\gamma}{\gamma}}}{(1-\gamma)q + \gamma(\rho - n)} \right). \qquad (1.52)
\end{aligned}$$

Trade affects growth q and the consumption level c_0 through the value of the term $J\,(f/f_x)^{\frac{k+1-\sigma}{\sigma-1}}\,\tau^{-k}$, which is strictly increasing in J and fixed cost f, and strictly decreasing in τ and f_x.

Proposition 2. *A decrease in the variable trade cost τ unambiguously leads to an increase in welfare U in the model without an R&D subsidy and with a general form of R&D technology.* $[\tau \downarrow \Longrightarrow\ U \uparrow,$ *when* $v_e = 0.]$

Proof. Differentiate (1.52) with respect to q, remembering that c_0 is a function of q given by (1.51):

$$\begin{aligned}\frac{\partial U}{\partial q} &= -\frac{\gamma}{1-\gamma}\left(\frac{-\gamma\frac{1-\gamma}{\gamma}c_0^{-\frac{1-\gamma}{\gamma}-1}\frac{\partial c_0}{\partial q}[(1-\gamma)q+\gamma(\rho-n)]-(1-\gamma)\gamma c_0^{-\frac{1-\gamma}{\gamma}}}{[(1-\gamma)q+\gamma(\rho-n)]^2}\right)\\ &= \gamma c_0^{-\frac{1-\gamma}{\gamma}}\left(\frac{c_0^{-1}\frac{\partial c_0}{\partial q}[(1-\gamma)q+\gamma(\rho-n)]+\gamma}{[(1-\gamma)q+\gamma(\rho-n)]^2}\right)\\ \frac{\partial U}{\partial q} &= \gamma c_0^{-\frac{1-\gamma}{\gamma}}\frac{\gamma}{[(1-\gamma)q+\gamma(\rho-n)]^2}\left(c_0^{-1}\frac{\partial c_0}{\partial q}\frac{(1-\gamma)q+\gamma(\rho-n)}{\gamma}+1\right).\end{aligned} \tag{1.B.35}$$

As $(1-\gamma)q+\gamma(\rho-n)>0$, the sign of the above derivative is determined by the sign of the expression in brackets:

$$\frac{\partial U}{\partial q} \propto c_0^{-1}\frac{\partial c_0}{\partial q}\left(\frac{1-\gamma}{\gamma}q+\rho-n\right)+1.$$

Differentiate (1.51) with respect to q, noting from (1.35) that $\partial g/\partial q = 1$, and $Z \equiv 1 + J\left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}$:

$$c_0 = Af^{-\frac{k+1-\sigma}{k(\sigma-1)}}Z^{\frac{1}{k}}\left[k\sigma+1-\sigma+\frac{n+kg}{q\frac{1-\gamma}{\gamma}+kg+\rho}\frac{\sigma-1}{1-v_e}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}$$

$$\begin{aligned}\frac{\partial c_0}{\partial q} = -\frac{k\sigma+1-\sigma}{k(\sigma-1)}&Af^{-\frac{k+1-\sigma}{k(\sigma-1)}}Z^{\frac{1}{k}}\cdot\\ &\cdot\left[k\sigma+1-\sigma+\frac{n+kg}{q\frac{1-\gamma}{\gamma}+kg+\rho}\frac{\sigma-1}{1-v_e}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}-1}\cdot\\ &\cdot\left[\frac{\sigma-1}{1-v_e}\frac{k\left[q\frac{1-\gamma}{\gamma}+kg+\rho\right]-(n+kg)\left[\frac{1-\gamma}{\gamma}+k\right]}{\left[q\frac{1-\gamma}{\gamma}+kg+\rho\right]^2}\right]\end{aligned}$$

$$\frac{\partial c_0}{\partial q} = -\frac{k\sigma+1-\sigma}{k(\sigma-1)}\left[k\sigma+1-\sigma+\frac{n+kg}{q^{\frac{1-\gamma}{\gamma}}+kg+\rho}\frac{\sigma-1}{1-v_e}\right]^{-1} c_0 \cdot$$

$$\cdot\left[\frac{\sigma-1}{1-v_e}\frac{k\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right]-(n+kg)\left[\frac{1-\gamma}{\gamma}+k\right]}{\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right]^2}\right]$$

The following term becomes:

$$c_0^{-1}\frac{\partial c_0}{\partial q} = -\frac{k\sigma+1-\sigma}{k(\sigma-1)}\left[k\sigma+1-\sigma+\frac{n+kg}{q^{\frac{1-\gamma}{\gamma}}+kg+\rho}\frac{\sigma-1}{1-v_e}\right]^{-1}\cdot$$

$$\cdot\frac{\sigma-1}{1-v_e}\frac{k\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right]-(n+kg)\left[\frac{1-\gamma}{\gamma}+k\right]}{\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right]^2}$$

$$= -\frac{k\sigma+1-\sigma}{k(\sigma-1)}\cdot$$

$$\cdot\left[\frac{[k\sigma+1-\sigma]\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right](1-v_e)+(n+kg)(\sigma-1)}{q^{\frac{1-\gamma}{\gamma}}+kg+\rho}\frac{1}{1-v_e}\right]^{-1}\cdot$$

$$\cdot\frac{\sigma-1}{1-v_e}\frac{k\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right]-(n+kg)\left[\frac{1-\gamma}{\gamma}+k\right]}{\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right]^2}$$

$$= -\frac{k\sigma+1-\sigma}{k(\sigma-1)}\frac{1}{[k\sigma+1-\sigma]\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right](1-v_e)+(n+kg)(\sigma-1)}\cdot$$

$$\cdot(\sigma-1)\frac{k\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right]-(n+kg)\left[\frac{1-\gamma}{\gamma}+k\right]}{q^{\frac{1-\gamma}{\gamma}}+kg+\rho}$$

$$= -\frac{k\sigma+1-\sigma}{k[k\sigma+1-\sigma]\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right](1-v_e)+k(n+kg)(\sigma-1)}\cdot$$

$$\cdot\frac{k\left[q^{\frac{1-\gamma}{\gamma}}+kg+\rho\right]-(n+kg)\left[\frac{1-\gamma}{\gamma}+k\right]}{q^{\frac{1-\gamma}{\gamma}}+kg+\rho}.$$

Welfare is proportionate to

$$\frac{\partial U}{\partial q} \propto c_0^{-1} \frac{\partial c_0}{\partial q} \left(\frac{1-\gamma}{\gamma} q + \rho - n \right) + 1$$

$$= - \frac{k\sigma + 1 - \sigma}{k[k\sigma + 1 - \sigma] \left[q \frac{1-\gamma}{\gamma} + kg + \rho \right] (1 - v_e) + k(n + kg)(\sigma - 1)} \cdot$$

$$\cdot \frac{k \left[q \frac{1-\gamma}{\gamma} + kg + \rho \right] - (n + kg) \left[\frac{1-\gamma}{\gamma} + k \right]}{q \frac{1-\gamma}{\gamma} + kg + \rho} \cdot \left(\frac{1-\gamma}{\gamma} q + \rho - n \right) + 1$$

Denote $D_1 \equiv \frac{1-\gamma}{\gamma} q + \rho - n > 0$ and $D_2 \equiv n + gk > 0$. Then $D_1 + D_2 = q \frac{1-\gamma}{\gamma} + kg + \rho$, and

$$\frac{\partial U}{\partial q} \propto 1 - \frac{k\sigma + 1 - \sigma}{k[k\sigma + 1 - \sigma] [D_1 + D_2] (1 - v_e) + kD_2(\sigma - 1)} \cdot$$

$$\cdot \frac{k [D_1 + D_2] - D_2 \left[\frac{1-\gamma}{\gamma} + k \right]}{D_1 + D_2} D_1$$

$$= \frac{h(v_e)}{(k[k\sigma + 1 - \sigma] [D_1 + D_2] (1 - v_e) + kD_2(\sigma - 1)) (D_1 + D_2)},$$

where the denominator is positive and the numerator is a function h of the

R&D subsidy rate v_e:

$$
\begin{aligned}
h(v_e) \equiv & \left(k[k\sigma+1-\sigma]\left[D_1+D_2\right](1-v_e)+kD_2(\sigma-1)\right)(D_1+D_2) \\
& -(k\sigma+1-\sigma)\left(k\left[D_1+D_2\right]-D_2\left[\frac{1-\gamma}{\gamma}+k\right]\right)D_1 \\
= & k(k\sigma+1-\sigma)(1-v_e)(D_1+D_2)^2+kD_2(\sigma-1)(D_1+D_2) \\
& -(k\sigma+1-\sigma)D_1\left(kD_1-\frac{1-\gamma}{\gamma}D_2\right) \\
= & k(D_1+D_2)\left[(k\sigma+1-\sigma)(1-v_e)(D_1+D_2)+D_2(\sigma-1)\right] \\
& -k(k\sigma+1-\sigma)D_1^2+(k\sigma+1-\sigma)\frac{1-\gamma}{\gamma}D_2D_1 \\
= & k(D_1+D_2)\left[(k\sigma+1-\sigma)(1-v_e)D_1\right. \\
& \left.+((k\sigma+1-\sigma)(1-v_e)+(\sigma-1))D_2\right] \\
& -k(k\sigma+1-\sigma)D_1^2+(k\sigma+1-\sigma)\frac{1-\gamma}{\gamma}D_2D_1 \\
= & -k(k\sigma+1-\sigma)v_eD_1^2+k(k\sigma+1-\sigma)(1-v_e)D_1D_2 \\
& +k(D_1+D_2)((k\sigma+1-\sigma)(1-v_e)+(\sigma-1))D_2 \\
& +(k\sigma+1-\sigma)\frac{1-\gamma}{\gamma}D_2D_1,
\end{aligned}
$$

where $0<\gamma<1$. When $v_e=0$, the above term is positive, and hence the derivative satisfies $\frac{\partial U}{\partial q}>0$. In this case, consumer welfare is strictly increasing in the growth rate of consumption q. Then, using Proposition 1, trade liberalization leads to an increase in consumer welfare. This proves the proposition. □

Define the gains from trade z in equivalent variation terms as the proportional increase in the autarky level of consumption required to obtain the open economy welfare level:

$$U(zc_0^A, q^A) = U(c_0, q).$$

Use (1.52) to solve for z:

$$\frac{\gamma}{1-\gamma}\left[\frac{1}{\rho-n}-\frac{\gamma(zc_0^A)^{-\frac{1-\gamma}{\gamma}}}{(1-\gamma)q^A+\gamma(\rho-n)}\right]=\frac{\gamma}{1-\gamma}\left[\frac{1}{\rho-n}-\frac{\gamma c_0^{-\frac{1-\gamma}{\gamma}}}{(1-\gamma)q+\gamma(\rho-n)}\right]$$

$$\frac{(zc_0^A)^{-\frac{1-\gamma}{\gamma}}}{(1-\gamma)q^A+\gamma(\rho-n)}=\frac{c_0^{-\frac{1-\gamma}{\gamma}}}{(1-\gamma)q+\gamma(\rho-n)}$$

$$\left(\frac{zc_0^A}{c_0}\right)^{-\frac{1-\gamma}{\gamma}}=\frac{(1-\gamma)q^A+\gamma(\rho-n)}{(1-\gamma)q+\gamma(\rho-n)}$$

$$z=\frac{c_0}{c_0^A}\left[\frac{(1-\gamma)q^A+\gamma(\rho-n)}{(1-\gamma)q+\gamma(\rho-n)}\right]^{-\frac{\gamma}{1-\gamma}}$$

$$z=\frac{c_0^A}{c_0}\left[\frac{(1-\gamma)q+\gamma(\rho-n)}{(1-\gamma)q^A+\gamma(\rho-n)}\right]^{\frac{\gamma}{1-\gamma}}.$$

The total gains from trade z can be further decomposed into static gains and dynamic gains:

$$z=z_s\cdot z_d.$$

From (1.51), trade raises welfare by increasing c_0 for any given growth rate. In the case $q=q^A$, all the gains from trade are static:

$$z_s\equiv\frac{c_0}{c_0^A}=\left[1+J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right]^{\frac{1}{k}}. \tag{1.53}$$

Then the dynamic gains from trade are defined as $z_d\equiv z/z_s$, capturing the welfare effect from the direct effect of trade on the consumption growth rate q and the indirect effect on the consumption level c_0.

1.B.6 Calibration

The static gains from trade depend only on the import penetration ratio (IPR) and the trade elasticity. To see this first calculate the import expenditure (IMP) in each country. By symmetry, total expenditures on imports in each country should be equal the total value of exports from each country:

$$IMP_t = JM_t \int_{\tilde{\phi}}^{\infty} e_t^x(\phi) dH_t(\phi),$$

where $e_t^x(\phi)$ is the value of exports by the firm with relative productivity ϕ_t. When deriving (1.15) it was shown that $e_t^x(\phi)/\sigma = w_t f \tau^{1-\sigma} \phi_t^{\sigma-1}$. Then the IMP at time t becomes:

$$IMP_t = JM_t \int_{\tilde{\phi}}^{\infty} \sigma w_t f \tau^{1-\sigma} \phi_t^{\sigma-1} dH_t(\phi).$$

Using $H_t(\phi) = 1 - \phi_t^{-k}$, $\sigma - 1 - k < 0$, and that $\tilde{\phi} = (f_x/f)^{1/(\sigma-1)}\tau$:

$$\begin{aligned}
IMP_t &= JM_t \sigma w_t f \tau^{1-\sigma} \int_{\tilde{\phi}}^{\infty} \phi_t^{\sigma-1} k \phi_t^{-k-1} d\phi_t \\
&= JM_t \sigma w_t f \tau^{1-\sigma} k \left. \frac{\phi_t^{\sigma-1-k}}{\sigma - 1 - k} \right|_{\tilde{\phi}}^{\infty} \\
&= JM_t \sigma w_t f \tau^{1-\sigma} k \frac{\tilde{\phi}^{\sigma-1-k}}{k + 1 - \sigma} \\
&= \frac{k\sigma}{k + 1 - \sigma} M_t w_t f J \tau^{-k} \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}}. \qquad (1.54)
\end{aligned}$$

So, k is the trade elasticity (elasticity of imports with respect to variable trade costs).

Divide (1.54) by total domestic sales $c_t L_t$ to write down the import penetration ratio (IPR) at time t:

$$IPR_t \equiv \frac{IMP_t}{c_t L_t} = \frac{\frac{k\sigma}{k+1-\sigma} M_t w_t f J \tau^{-k} \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}}}{c_t L_t}.$$

Use $\dot{w}_t/w_t = \dot{c}_t/c_t = q$, $\dot{M}_t/M_t = \dot{L}_t/L_t = n$, and (1.B.33):

$$\begin{aligned} IPR_t &= \frac{\frac{k\sigma}{k+1-\sigma} M_o e^{nt} w_0 e^{qt} f J \tau^{-k} \left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}}}{c_0 e^{qt} L_0 e^{nt}} \\ &= \frac{\frac{k\sigma}{k+1-\sigma} w_0 f J \tau^{-k} \left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}}}{\frac{L_0}{M_0} w_0 \left[1 + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right) \frac{M_0}{L_0} \frac{1}{\lambda^k} F_0(1-v_e) - \frac{M_0}{L_0} \frac{n+kg}{\lambda^k} F_0 v_e\right]} \\ &= \frac{\frac{k\sigma}{k+1-\sigma} f J \tau^{-k} \left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}}}{\left[\frac{L_0}{M_0} + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right) \frac{1}{\lambda^k} F_0(1-v_e) - \frac{n+kg}{\lambda^k} F_0 v_e\right]}. \end{aligned}$$

Notice that the expression in the denominator is equal to $X_2^{\frac{\sigma-1}{\sigma}}$, where

$$\begin{aligned} X_2 &\equiv \left[\frac{L_0}{M_0} + \left(q\frac{1-\gamma}{\gamma} + \rho - n\right) \frac{1}{\lambda^k} F_0(1-v_e) - \frac{n+kg}{\lambda^k} F_0 v_e\right]^{\frac{\sigma}{\sigma-1}} \\ X_2 &= \left(\frac{f}{k+1-\sigma}\right)^{\frac{\sigma}{\sigma-1}} \left[1 + J\left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}\right]^{\frac{\sigma}{\sigma-1}} [k\sigma]^{\frac{\sigma}{\sigma-1}}. \end{aligned}$$

Then:

$$\begin{aligned} IPR_t &= \frac{\frac{k\sigma}{k+1-\sigma} f J \tau^{-k} \left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}}}{\frac{f}{k+1-\sigma}\left[1 + J\left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}\right] k\sigma} = \frac{J\tau^{-k}\left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}}}{1 + J\left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}} \\ &= 1 - \frac{1}{1 + J\left(\frac{f}{fx}\right)^{\frac{k+1-\sigma}{\sigma-1}} \tau^{-k}} = 1 - \frac{1}{(z_s)^k}. \end{aligned} \tag{2.2.4}$$

Thus, I obtain the expression for the static gains from trade:

$$z_s = \left[\frac{1}{1 - IPR_t}\right]^{1/k}. \tag{1.56}$$

Let NF be the rate of successful entry. From (1.B.21) the fraction of entrants that draw productivity below the exit cut-off is $\tilde{H}(1) = 1 - \lambda^k$, so the fraction of innovations leading to the creation of new firms is λ^k. The mass of entrants is Ω_t. Then

$$NF = \lambda^k \frac{\Omega_t}{M_t}. \tag{1.57}$$

Combine this with (1.39) to obtain:

$$\frac{\Omega_t}{M_t} = \frac{n+kg}{\lambda^k}$$
$$g = \left(\lambda^k \frac{\Omega_t}{M_t} - n\right)\frac{1}{k} = \frac{NF-n}{k}. \tag{1.58}$$

Substitute for q from (1.35) into (1.45):

$$\begin{aligned} q =& g + \frac{n}{\sigma-1} \\ =& \frac{\gamma}{1+(k-1)\gamma} \cdot \\ & \cdot \left(\frac{\sigma-1}{k+1-\sigma}\lambda^k f \frac{1}{F_t}\frac{1}{1-v_e}\left(1+J\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\tau^{-k}\right) + k\frac{n}{\sigma-1} - \rho\right) \end{aligned}$$

Now, use (1.58):

$$\frac{NF-n}{k} + \frac{n}{\sigma-1} = \frac{\gamma}{1+(k-1)\gamma}\left(\frac{\sigma-1}{k+1-\sigma}\lambda^k f \frac{1}{F_t}\frac{1}{1-v_e}(z_s)^k + k\frac{n}{\sigma-1} - \rho\right)$$

$$\begin{aligned} \frac{(NF-n)(\sigma-1)+nk}{k(\sigma-1)} \cdot \frac{1+(k-1)\gamma}{\gamma} =& \frac{\sigma-1}{k+1-\sigma}\lambda^k f \frac{1}{F_t}\frac{1}{1-v_e}(z_s)^k \\ & + k\frac{n}{\sigma-1} - \rho \end{aligned}$$

Rearrange the terms to express λ^k:

$$\begin{aligned} \lambda^k =& \left[\frac{(NF-n)(\sigma-1)+nk}{k(\sigma-1)} \cdot \frac{1+(k-1)\gamma}{\gamma} - k\frac{n}{\sigma-1} + \rho\right] \cdot \\ & \cdot \frac{k+1-\sigma}{\sigma-1} F_t \frac{1}{f}(1-v_e)(z_s)^{-k} \end{aligned}$$

$$\begin{aligned} \lambda^k =& \frac{k+1-\sigma}{\gamma k(\sigma-1)}(z_s)^{-k}\frac{F_t}{f}(1-v_e) \cdot \\ & \cdot \left[(1+(k-1)\gamma)(NF-n) + \frac{nk(1+(k-1)\gamma)}{\sigma-1} - k\frac{n\gamma k}{\sigma-1} + \gamma k \rho\right] \end{aligned}$$

Substitute for $(z_s)^{-k}$ using the import penetration ratio from (2.2.4):

$$\lambda^k = \frac{k+1-\sigma}{\gamma k(\sigma-1)}(1-IPR)\frac{F_t}{f}(1-v_e)\cdot \\ \cdot\left[(1+(k-1)\gamma)(NF-n)+\frac{k(1-\gamma)}{\sigma-1}n+\gamma k\rho\right]. \tag{1.59}$$

Finally, to solve for the share of exporting firms, use that the probability that a firm with relative productivity ϕ_t is an exporter is given by

$$\Pr(\phi_t > \tilde{\phi}) = 1 - H(\tilde{\phi}) = \tilde{\phi}^{-k}. \tag{1.B.36}$$

Then the share of exporting firms SEF is constant over time:

$$SEF = \tilde{\phi}^{-k} = \left[\left(\frac{f_x}{f}\right)^{1/(\sigma-1)}\tau\right]^{-k}. \tag{1.60}$$

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Chapter 2

Localized effects of the China trade shock: Is there an effect on consumer expenditure?[1]

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2.1 Introduction

Since China's entry to the WTO in 2001, its export growth has represented a large positive net global supply shock for manufacturing. Improvements in China's productive capabilities and reductions in its trade costs have changed the intensity of competition for the U.S. manufacturing sector, leading to a contraction in the U.S. industries subject to greater import exposure. In particular, industries more exposed to import competition from China are shown to exhibit higher rates of plant exit (Bernard, Jensen and Schott (2006)), larger contraction in employment (Pierce and Schott (2016), Acemoglu et al. (2016)), and lower incomes for affected workers (Autor et al. (2014)).

Moreover, given the spatial concentration of manufacturing in the U.S., such industry shocks have translated into localized employment shocks. In particular, Autor, Dorn and Hanson (2013) analyze the effect of rising Chinese import competition on the U.S. local labor markets. They document the substantial job loss, increase in unemployment, and decline in average wages and median household income in local labor markets that host more import-competing industries.

In this paper, I further assess the distributional consequences of rising trade with China in the U.S. by studying the effect of local import exposure on consumer expenditure. The aggregate economic benefits of trade integration, resulting in lower consumer prices and higher number of available varieties of consumer goods, have been well recognized by the trade literature (Broda and Weinstein (2006)). However, as the China trade shock has differential impact on employment and household income across local labor markets within the U.S., the implications for the local consumer expenditure are less clear. The previous literature has documented that local non-durable consumer expenditure is responsive to a variety of local economic shocks: a decline in housing values (Mian, Rao and Sufi (2013) and Kaplan, Mitman and Violante (2016)), a change in the minimum wage rate (Alonso (2016)), a local productivity shock (Lazhevska (2018)), and local extreme weather conditions (Kozlova (2016)).[2]

[2] Mian, Rao and Sufi (2013) find a negative effect of a decline in household net worth on both durable and non-durable consumer expenditure. Kaplan, Mitman and Violante (2016) replicate Mian, Rao and Sufi (2013) for non-durable expenditure using retailer scanner data and find a similar negative effect. Alonso (2016) uses retailer scanner data to find that non-durable expenditure increases in the minimum wage rate. Lazhevska (2018) uses consumer scanner

The China trade shock can be considered a negative local productivity shock with economically significant impact on local labor markets, hence, it is reasonable to expect a negative effect of this shock on local consumer expenditure.

The previous literature on the effects of local economic shocks on consumer expenditure has focused mostly on non-durable consumer goods. This literature makes use of a detailed scanner data from several sources that record non-durable expenditure.[3] Similar to the previous literature, I also focus on non-durable consumer goods and use data on consumer expenditure from the Kilts Nielsen Consumer Panel (KNCP). The KNCP is a consumer scanner dataset containing price and quantity data on purchases of consumer packaged goods by a large panel of U.S. households starting from 2004. The detailed nature of the data allows me to compute total household expenditure as well as expenditure by product category, and to control for a number of initial household level characteristics.

To describe the local labor market exposure to import competition from China, I rely on the measure constructed by Autor, Dorn and Hanson (2013) and available in their replication package. It exploits the differences in initial industrial composition across local labor markets, and variation in Chinese import competition across manufacturing industries in the U.S. The local labor market is defined as a commuting zone, which is a cluster of counties with strong internal commuting ties. Autor, Dorn and Hanson (2013) compute changes in the commuting zone import exposure for periods 1990-1999 and 2000-2007. As consumer expenditure data is only available after 2004, I use the measure of import exposure computed for the 2000-2007 period. This period also captures the largest increase in Chinese exports since its entry to the WTO. As a robustness check, I show the results using an alternative measure of exposure, the change in the import penetration ratio as in Acemoglu et al. (2016).

To identify the effect of import competition from China on local consumer expenditure, I need to take into account several important issues. First, as an increase in Chinese imports directly affects prices of non-durables in the

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data to show a positive effect of a local productivity shock, the fracking boom, on local non-durable expenditure. Finally, Kozlova (2016) utilizes the regional variation in the number of heating days to estimate a negative effect of a decline in household disposable income on quantity and quality of household food consumption using consumer scanner data.

[3]The U.S. marketing datasets available for researchers are Kilts Nielsen Consumer Panel dataset, Kilts Nielsen Retail Scanner dataset, and retail scanner dataset from IRi Worldwide.

U.S., one needs to isolate the shift in local demand for non-durable goods from the shift in local supply of these goods. An important identifying assumption that makes this possible is that the China trade shock is not likely to cause a differential shift in local supply of non-durable goods, to the extent that these goods are traded on a national market. Even though the trade shock can affect labor costs for retailers, hence shifting the supply curve at the retailer level, labor cost is a rather small component of retailer marginal cost and such an effect should be of a second order (as discussed in Stroebel and Vavra (2014)). To account for a possibility of differential price changes across commuting zones due to potential changes in either marginal costs or markups, I construct a measure of real consumer expenditure by fixing prices for each product at the national average in a given year.

Second, an increase in the Chinese imports can be considered an exogenous shock only if it was not driven by a decline in local productivity and changes in local demand in the U.S. I follow Autor, Dorn and Hanson (2013) in using an instrument which captures the supply-driven increase in the imports from China. It is computed using the data on industry exports from China to other developed countries, weighted by a lagged industrial composition for each commuting zone in the U.S.

As a preview of my results, I do not find either statistically or economically significant effects of changes in commuting zone trade exposure on household non-durable expenditure. The null result is precise and suggests that locations with increased import competition are not likely to face a decline in demand for non-durable products. The effect on the real non-durable expenditure, as measured at the average national prices, is also close to zero. When examining different product categories separately, I find that households in negatively affected commuting zones on average significantly increase their alcohol consumption. This result provides evidence for a positive effect of an adverse local economic shock on alcohol consumption. I find no significant effect on any other food or non-food expenditure category. Importantly, the results about non-durable expenditure should not be generalised to all consumer expenditure, as the effects on durable consumption might differ.[4]

There are several potential explanations for the estimated effect of the China trade shock on household non-durable expenditure to be close to zero. One

[4]Expenditures on durables are known to be more cyclical than expenditures on non-durables, as is stated, for example, in Bils and Klenow (1998).

possibility is that, the sample size used in the analysis is rather small, with on average 40 households per commuting zone, which might be not representative of the commuting zone population. Another potential concern is related to using the data on the China trade shock from 2000 to 2007 while observing household expenditure only from 2004 to 2007. It is possible that the main adjustment to the China trade shock in terms of household expenditure had happened before 2004, which would explain no significant effect after 2004. I address these issues to the extent that the data allows: by computing the average commuting zone expenditures for 2004 and 2007 using a larger unbalanced sample of households, or by computing an alternative measure of import penetration ratio that matches the periods of 2004-2007 or 2004-2011. Even though the main result is robust to these alternative measures, it is still not possible to fully rule out the above explanations for the absence of the effect.

An alternative explanation for why household non-durable expenditure does not respond to the local China trade shock may be related to the nature of the underlying shock in the period of 2000-2007. To better understand this shock, I explore its effect on commuting zone income and wages during the period 2000-2007, using data from the replication package for Autor, Dorn and Hanson (2013). In the original study, Autor, Dorn and Hanson (2013) analyze the differences in outcomes across the periods 1990-1999 and 2000-2007 by stacking observations from the two periods. Among other results, they find a substantial reduction in manufacturing employment, an increase in the unemployment rate, a reduction in average wage and wage in non-manufacturing, and a decline in median household income in local labor markets that experience stronger import competition from China. By examining the period 2000-2007 separately, I confirm that an increase in import exposure per worker during this period had a significant and negative effect on commuting zone manufacturing employment. Other results, however, do not appear as robust: I find a positive and statistically significant effect of the China trade shock on local non-manufacturing employment, and no significant effect on the unemployment rate, average wage per worker, or median household income.[5] These

[5]These findings add to a recent discussion about the robustness of the results in Autor, Dorn and Hanson (2013) beyond the changes in manufacturing employment. While originally, Autor, Dorn and Hanson (2013) suggest a significant negative effect on both manufacturing employment and household income, later the authors emphasise that the effects on median income and average wages should indeed be considered weaker than the effect on manufacturing employment. This discussion consists of a series of comments by Roth-

findings can explain the absence of the effect of the China trade shock on total household non-durable expenditure in the period of interest. The effect of the China trade shock on household alcohol consumption is then likely driven by the decline in manufacturing employment rather than an income shock.

The paper is organised as follows. Section 2 describes data and defines measures of household expenditure and commuting zone import exposure. Section 3 presents the empirical approach. Section 4 provides descriptive statistics and main results. Section 5 discusses the effect of the trade shock on local labor market outcomes. Section 6 concludes.

2.2 Data and construction of key variables

2.2.1 Consumer expenditure

The data on consumer expenditure at the household level is obtained from the Kilts-Nielsen Consumer Panel (KNCP) dataset.[6] This dataset consists of an unbalanced panel of more than 150,000 households residing across all the U.S., and covers the period from 2004 to 2015. Households use a portable scanner to register every transaction made in supermarket, grocery, convenience store, or pharmacy. The price and quantity purchased is recorded for about 2 million universal product codes (UPC), covering General Merchandise, Non-Food Grocery, Health and Beauty Aids, as well as food categories such as Dry Grocery, Frozen Foods, Dairy, Deli, Packaged Meat, Fresh Produce, and Alcohol. The household information includes a wide range of demographic characteristics.

Nielsen aims at constructing a geographically dispersed and demographically balanced panel of households. In addition to demographic representation, the KNCP sample is also balanced on county size dispersion as well as key county population totals. To my knowledge, there is no better data source available that would allow to measure household expenditure (either durable or non-durable) at the level of a U.S. commuting zone for the period of inter-

well (2017), Feenstra, Ma and Xu (2017), Autor, Dorn and Hanson (2017*b*), and Autor, Dorn and Hanson (2017*c*), and it has not reached a consensus yet. Comments by Autor, Dorn and Hanson (2017*b*) and Autor, Dorn and Hanson (2017*c*) can be accessed via http://chinashock.info/papers/.

[6]The dataset is provided through the Nielsen Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business.

est.[7]

As I am interested in long differences in household expenditure, I focus on a subsample of households who report purchases in years 2004 and 2007 (or 2011), which leaves me with about 25,000 (or 15,000) households. Households can decide to terminate their participation in the Nielsen panel at any time, which explains the unbalanced nature of the panel. For each year, only households who use scanners continuously within a year are included in the Nielsen sample.

From the high-frequency transaction data I construct household annual expenditure as a sum of expenditure over UPC products across all transactions during the year:

$$C_{ict} = \sum_{u \in U} \sum_{w \in W} P_{uwict} Q_{uwict}, \tag{2.2.1}$$

where C_{ict} is a total annual expenditure of household i, located in commuting zone c, in year t, and P_{uwict} and Q_{uwict} are price and quantity corresponding to UPC code u in transaction w. Further, to account for the possibility that prices for consumer goods change differentially across locations and to measure household expenditure in real terms, I construct a volume-based measure of expenditure similar to the one used in Kaplan, Mitman and Violante (2016) and Alonso (2016):

$$RC_{ict} = \sum_{u \in U} \sum_{w \in W} P_{u\bar{y}} Q_{uwict}, \tag{2.2.2}$$

where $P_{u\bar{y}}$ is a national average price of UPC-code u in year $\bar{y}$, where $\bar{y}$ is the last year the good is available in the sample.[8]

2.2.2 Local trade exposure

The main measure

The measure of commuting zone import exposure per worker ΔIPW_c^{US} is obtained from Autor, Dorn and Hanson (2013) and corresponds to the period

[7]For example, the Consumer Expenditure Survey from the U.S. Bureau of Labor Statistics reports consumer expenditure only for the following geographic areas: national, census regions, census divisions, selected states, and selected metropolitan statistical areas.

[8]Due to the high turnover of products in the KNCP dataset, it is not feasible to fix the price of all products in one specific year.

2000-2007. It is defined as the change in import exposure per worker in commuting zone c given the initial industry employment structure in this commuting zone and the national change in industry imports:

$$\Delta IPW_c^{US} = \sum_j \frac{L_{cj0}}{L_{j0}} \frac{\Delta M_j^{CHUS}}{L_{c0}}, \tag{2.2.3}$$

where c stands for commuting zone and j is industry; ΔIPW_c^{US} is a change in import exposure per worker in commuting zone c from 2000 to 2007, L_{cj0}/L_{j0} is commuting zone's c initial fraction in industry j national employment, L_{c0} is initial total manufacturing employment in commuting zone c, and ΔM_j^{CHUS} is a change in imports in industry j from China to the U.S. from 2000 to 2007.

The measure of import exposure per worker is directly taken from the replication package by Autor, Dorn and Hanson (2013) and is used in all baseline regressions.[9] Data on commuting zone demographics and labor market outcomes also come from this replication package.

An alternative measure

For robustness, I compute an alternative measure of the commuting zone import exposure, an import penetration ratio described in Acemoglu et al. (2016). A change in import penetration ratio for commuting zone c ΔIPR_c^{US} is defined as a ratio of industry imports from China to industry initial absorption, weighted by the share of each industry in commuting zone employment:

$$\Delta IPR_c^{US} = \sum_j \frac{L_{cj0}}{L_{c0}} \frac{\Delta M_j^{CHUS}}{Y_{j0} + M_{j0} - E_{j0}}, \tag{2.2.4}$$

where L_{cj0}/L_{c0} is industry j's initial share of total employment in commuting zone c, ΔM_j^{CHUS} is a change in imports from China to the U.S., Y_{j0} is initial U.S. industry shipments, M_{j0} and E_{j0} are initial U.S. industry imports and exports respectively, and the term $Y_{j0} + M_{j0} - E_{j0}$ stands for the initial U.S. industry absorption.

I compute the changes in the import penetration ratio for alternative time periods (2000-2007, 2000-2011, 2004-2007, and 2004-2011) by combining the

[9]The replication package for Autor, Dorn and Hanson (2013) is available for download at the AER webpage (https://www.aeaweb.org/articles?id=10.1257/aer.103.6.2121) or at David Dorn's personal webpage (https://www.ddorn.net/data.htm).

data on international trade flows from the UN Comtrade Database with the local industry employment structure by commuting zone from the County Business Patterns (CBP), and using a crosswalk from HS to SIC to NAICS industrial classifications provided by David Dorn.[10]

2.3 Empirical strategy and identification

To estimate the effect of the increased import exposure from China on household non-durable expenditure across the U.S. commuting zones, I estimate the following equation:

$$\Delta logC_{ic} = \alpha + \beta \Delta IPW_c^{US} + X_i'\gamma + Z_c'\nu + \epsilon_{ic}, \qquad (2.3.5)$$

where $\Delta logC_{ic}$ is a change in the logarithm of the annual expenditure of household i living in commuting zone c between 2004 and 2007, ΔIPW_c^{US} is the change in import exposure per worker for commuting zone c from 2000 to 2007, X_i is a vector of household-level controls corresponding to 2004, and Z_c is a vector of controls at the commuting zone level corresponding to 2000. Household-level controls include household size, race, income, and education and average age of household heads. Controls at the commuting zone level include a full set of controls from Autor, Dorn and Hanson (2013): initial percentage of employment in manufacturing, percentage of college-educated population, percentage of foreign-born population, percentage of employment among women, percentage of employment in routine occupations, average offshorability index of occupations.[11] The regression also includes dummies for nine census divisions in the U.S. to absorb region-specific trends.

The equation (2.3.5) describes a reduced form relationship between commuting zone import exposure and household consumer expenditure. The underlying first stage in this relationship is the adverse effect of import exposure on household employment and income. A possible threat to identification, however, is that the Chinese imports can directly affect prices and supply of consumer packaged goods. Thus, it is important to isolate the shift in local demand for non-durable goods caused by an economic shock from the shift

[10] The crosswalk from HS to SIC to NAICS industrial classifications provided by David Dorn can be found at https://www.ddorn.net/data.htm.

[11] The last two variables capture the susceptibility of a commuting zone occupations to substitution by technology or task offshoring.

in local supply of these goods caused by increase in imports. This appears to be possible because the China trade shock is not likely to cause a differential shift in local supply of non-durable goods across commuting zones, as imported goods are traded on a national market. A potential threat to identification is that the local labor demand shock caused by the import exposure can affect labor costs for retailers, hence shift the supply curve at the retailer level. However, as discussed in Stroebel and Vavra (2014), labor costs are a rather small component of retailer marginal costs and such an effect on local supply should be of a second order.[12] There is a possibility of differential price changes across commuting zones due to local markups responding to changes in local demand for non-durables. Using a volume-based measure of consumer expenditure as in (2.2.2) helps to account for potential changes in consumer prices due to changes in marginal costs or markups.

In addition, there is a concern regarding the endogeneity of the measure of trade exposure. In particular, both regional economic performance, such as employment, income, and consumption, and measures of trade exposure might be simultaneously affected by domestic demand and supply shocks. To identify the effect of trade shocks driven only by the Chinese supply, I use an instrument for import exposure ΔIPW_c^{OT}, as described in Autor, Dorn and Hanson (2013):

$$\Delta IPW_c^{OT} = \sum_j \frac{L_{cj,-1}}{L_{j,-1}} \frac{\Delta M_j^{CHOT}}{L_{c,-1}}, \tag{2.3.6}$$

where ΔM_j^{CHOT} is a change in industry j import from China to a set of other developed countries, and $\frac{L_{cj,-1}}{L_{j,-1}}$ is a commuting zone's c fraction in industry j's national employment from the prior decade. Similar to Autor, Dorn and Hanson (2013), other developed countries used for construction of the IV are Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland. The identifying assumptions are that the lagged industrial composition in the commuting zone does not cause the future changes in commuting zone outcome variables, and that the China industry exports to other developed countries are not correlated to the U.S. demand and supply shocks.[13]

[12]Stroebel and Vavra (2014) decompose retail prices into components and show that labor costs and retail rents account for less than 25% of retailers' marginal costs, whereas wholesale costs constitute more than 75% and vary little across geographies.

[13]The instrument in Autor, Dorn and Hanson (2013) is a shift-share or Bartik instrument.

Autor, Dorn and Hanson (2013) discuss potential threats to identification. First, the product demand shocks may be correlated across high-income countries, including the U.S. They rule out this possibility by estimating a gravity-based model, in which they isolate supply- and trade-cost-driven changes in China's export performance, and show that the gravity and the IV estimates are very similar. Another potential threat to identification is possibility of an increase in high-income country imports from China due to a negative productivity shock in the U.S., or common technological developments in high-income countries (e.g. automation). While not being able to rule out this possibility, the authors argue that productivity growth in China is likely to be an important driver of China's export surge.

The instrument for an alternative measure of import exposure, the import penetration ratio from Acemoglu et al. (2016), follows a similar logic and is defined as follows:

$$\Delta IPR_c^{OT} = \sum_j \frac{L_{cj,-1}}{L_{c,-1}} \frac{\Delta M_j^{CHOT}}{Y_{j,-1} + M_{j,-1} - E_{j,-1}}, \tag{2.3.7}$$

where ΔM_j^{CHOT} is the growth in industry j imports from China to a set of other developed countries, and $Y_{j,-1} + M_{j,-1} - E_{j,-1}$ is a lagged U.S. industry absorption.

2.4 Results

2.4.1 Descriptives

Table 2.6.1 presents summary statistics for main variables in the analysis. Panel A summaries KNCP dataset for year 2004. Changes in variables correspond to 2004-2007. There are 25331 households in the KNCP dataset, for whom purchases are observable in both 2004 and 2007. Income for the year 2004 is only observabe for 23010 of these households, with the average reported annual income of 59370$.[14] The average household size in the sample is 2.28, and an

More on Bartik instruments and their validity in the context of Autor, Dorn and Hanson (2013) implementation can be found in the recent paper by Borusyak, Hull and Jaravel (2018).

[14] An income variable in the KNCP dataset is self-reported on an annual basis with a two-year lag. Therefore, income for year 2004 is reported by a household in 2006.

average age of a household head is 55. The share of black and Hispanic households are 10% and 6% respectively. 46% of households in the sample have at least one head of household with a college degree. In 2004 households registered on average 3355$ in annual expenditure on groceries, out of which around two thirds was spent on food.[15] Between 2004 and 2007 the average household expenditure declined by 13%, and this change varies significantly across households. A possible reason for the decline in average expenditure registered by households in the KNCP is a recent shift towards online purchases, which are not in the KNCP dataset.

Figure 2.6.1 shows the number of households in the KNCP per commuting zone. Of the 722 commuting zones in the U.S., 618 commuting zones are covered with at least one household in the sample. There are on average 40 households per commuting zone, as is shown in Table 2.6.1.

Panel B of Table 2.6.1 summarizes data from Autor, Dorn and Hanson (2013) on the change in commuting zone import exposure per worker from 2000 to 2007, and commuting zone characteristics corresponding to the beginning of this period. Figure 2.6.2 shows a spatial distribution of the change in import exposure per worker from 2000 to 2007 across commuting zones. The average change in value of Chinese imports per worker from 2000 to 2007 is 2800$, and it varies substantially across commuting zones, with the most affected regions being located in the South East and Midwest.

Table 2.6.2 presents descriptive statistics for commuting zones experiencing the smallest (bottom quartile) and the largest (top quartile) changes in Chinese import exposure per worker between 2000 and 2007. The average change in the import exposure per worker in the top quartile was about 6420$, and only 560$ in the lower quartile. Panel A describes households residing in the most and least affected commuting zones. As of 2004, households in the top quartile of import exposure were on average larger, younger, had lower income, and were less likely to be black or Hispanic. However, these households did not differ significantly in their grocery expenditure. Also, over the period of 2004 to 2007 households in top affected commuting zones didn't see a significantly different change in expenditure compared to households in the least affected commuting zones. Panel B shows that commuting zones in the top quartile of import exposure per worker had significantly higher initial share of manufacturing employment, lower education, and lower share of foreign born

[15]All expenditure values are brought to 2011 prices using the CPI Food at Home index.

population.

2.4.2 Consumer expenditure

To assess the effect of import exposure per worker on household consumer expenditure I estimate equation (2.3.5) at the household level and present results in Table 2.6.3. Column (1) shows results for OLS and column (2) for IV estimation of equation (2.3.5) without controls. Both OLS and IV regressions suggest that an increase in commuting zone import exposure per worker does not have either an economically or statistically significant effect on household expenditure. Adding a full set of household level controls (column (3)) and commuting zone level controls (column (4)) does not change the results significantly. Following Autor, Dorn and Hanson (2013), all regressions include census division dummies, and state-clustered standard errors. The first stage for the 2SLS estimation in columns (2)-(4) is presented in Panel B of Table 2.6.3.

Table 2.6.4 presents the IV estimation of equation (2.3.5) with the fullest set of controls for alternative outcome variables. Column (1) repeats the baseline result for nominal household expenditure. Column (2) estimates the effect on real consumer expenditure, which is larger in magnitude, but is still not significantly different from zero. Columns (3) and (4) present results separately for food and non-food expenditure, with effects not significantly different from zero.

Table 2.6.5 divides food expenditure into further categories: dry produce, frozen produce, dairy, deli, packaged meat, fresh produce, and alcohol. Out of all food categories, only alcohol shows both an economically and statistically significant effect. In particular, an increase in the value of Chinese imports per worker in the commuting zone by 1000$ over the period of 2000-2007 causes on average a 3.9% increase in household grocery expenditure on alcohol from 2004 to 2007. It can be further computed that for households at the 25th and 75th percentiles of a change in import exposure the difference in the 2004-2007 change in average alcohol consumption is about 9.7 percentage points. Such an increase in alcohol consumption in areas more affected by the trade competition shock can be attributed to a substantial loss of jobs in the manufacturing sector following the shock.[16]

[16]The KNCP provides no information about purchases of alcohol in specialized alcohol stores, as well as consumption of alcohol out of home, such as in restaurants and bars. There-

2.4.3 Robustness

There are several potential explanations for the estimated effect of the China trade shock on local household non-durable expenditure to be close to zero. In this section I discuss some of them, as well as further examine the robustness of the baseline result.

Sample size

There is a concern that the average number of households per commuting zone in the baseline analysis is rather small, and that the sample may be not representative of the commuting zone population. Even though, the KNCP is a geographically dispersed and demographically balanced panel of households, it is important to keep in mind that the sample in my baseline analysis is not the original Nielsen sample, as I exclude households who do not report consumption in either 2004 or 2007. To address this concern, I use a full unbalanced sample of households from the KNCP and compute the average expenditure per commuting zone for 2004 and 2007. I then estimate the equation (2.3.5) using changes in the average expenditure at the commuting zone level. The summary statistics for the unbalanced sample are presented in Table 2.6.6, and the estimation of the effect for average total expenditure at the commuting zone level is in Table 2.6.7 (without household level controls). The sample of households used for the analysis is now almost twice as large, with on average 62 households per commuting zone in 2004 and 98 households in 2007. The number of commuting zones in the sample is now 642. The estimated effect of the China trade shock on average total expenditure at the commuting zone level is shown to be not statistically different from zero. Still, there is a remaining concern, that in some commuting zones the number of households is particularly low, and it is difficult to known whether the households in the sample were directly affected by the local trade shock, as the KNCP provides no information about the industry of employment of a household head.

Alternative import exposure measures

Previous literature has used two alternative measures of commuting zone import exposure: the import exposure per worker by Autor, Dorn and Hanson

fore, the overall effect on consumption of alcohol in affected commuting zones can not be assessed.

(2013) and the import penetration ratio by Acemoglu et al. (2016). To examine robustness of the main results to the choice of import exposure measure, I estimate equation (2.3.5) using changes in the import penetration ratio derived as in Acemoglu et al. (2016).

Moreover, there is a possibility that some of the expenditure adjustment to the China trade shock at the household level has already happened before 2004. As the KNCP data is only available from 2004, I can not directly test or rule out this possibility. However, I can estimate the effect of the change in import penetration ratio that corresponds to the timing of the expenditure data in the KNCP, i.e. derived for the period of 2004-2007.

Another potential issue is that the time period 2004-2007 for household expenditure in the baseline estimation is rather short. Looking at a longer period of 2004-2011 may provide additional evidence about the adjustment of household expenditure to the local China trade shock. The reasons for not considering this period in the main specification are that this period includes the years of the Great Recession, and that there are few households in the sample who report expenditure both in 2004 and 2011.

In Table 2.6.8, I estimate the effect using the changes in import penetration ratio computed for alternative time periods. The dependent variable is the change in log household annual expenditure from 2004 to either 2007 or 2011. Panel A presents results where the change in the import penetration ratio is computed starting from 2000, and Panel B - from 2004 in order to match the expenditure data. Such measure of import exposure has different scale and units compared to the Autor, Dorn and Hanson (2013) measure used in the baseline analysis, hence the magnitude of estimated coefficients differs from those in Table 2.6.3.[17] All coefficients in Table 2.6.8 are close to zero and are not statistically significant. Thus, regardless of the measure used, there is no evidence of a significant effect of the change in commuting zone import exposure on consumer expenditure.

Change in household size

The baseline estimation of the effect of the China trade shock on household expenditure does not control for a change in the marital status of a household head or a change in household size in general. Autor, Dorn and Han-

[17] The average value of ΔIPR from 2000 to 2007 is 1.24 and standard deviation is 1.26.

son (2017*a*) suggest that an increase in the commuting zone import exposure per worker causes shifts in the relative economic stature of young men versus young women, which in turn leads to a decline in marriage rates and fertility. As the consumer expenditure used for analysis in this paper is at the household level, it is possible that the relative reduction in marriage rates and fertility rates in areas more affected by the trade shock would mechanically lead to a relative decline in the total household expenditure. Thus, not taking into account possible changes in household composition following the China trade shock would lead to overestimation of the effect of the shock on consumer expenditure. As the estimated effect of the China trade shock on household expenditure is in fact close to zero, controlling for a change in household expenditure should not substantially affect the result.

Indeed, Table 2.6.9 presents estimation of equation (2.3.5) taking into account a potential change in the household composition. In the first three columns the dependent variable is the 2004-2007 change in annual household expenditure. Column (1) presents the baseline estimation. In column (2), I control for a change in household marital status (either into or out of marriage) and a change in household size. Column (3) excludes households who report either a change in the marital status or in the number of household members. Column (4) estimates equation (2.3.5) for a change in annual expenditure per household member. Neither results are statistically or economically significant or different from the baseline estimate.

2.5 Reassessing the effect of the trade shock on local labor market outcomes

Another potential reason for the absence of the effect of the China trade shock on household expenditure is that this shock may not have an expected effect on local labor market outcomes in the sample of commuting zones, that I use for analysis. While a list of studies (Autor, Dorn and Hanson (2013), Acemoglu et al. (2016), Autor, Dorn and Hanson (2017*a*), Feler and Senses (2017) etc.) have emphasized the negative effect of the China trade shock on commuting zone manufacturing employment and household income, it is important to investigate whether this shock affects commuting zones in my sample. Moreover, the majority of the literature on the China trade shock have been following Autor, Dorn and Hanson (2013) by pooling observations from two periods, 1990-1999 and 2000-2007.[18] In my estimation, I am confined to the second period 2000-2007, as I only observe outcome variables in the KNCP after 2004.

The original analysis in Autor, Dorn and Hanson (2013) estimates the following equation at the commuting-zone level:

$$\Delta Y_{ct} = \alpha_t + \beta \Delta IPW_{ct}^{US} + Z_{ct}' \nu + \epsilon_{ct}, \tag{2.5.8}$$

where ΔY_{ct} is the change in the outcome variable for commuting zone c for the period 1990-1999 or 2000-2007, ΔIPW_{ct}^{US} is the change in regional import exposure for commuting zone c for corresponding period, α_t is a dummy variable indicating that data comes from the second period, and Z_{ct} is a vector of start of the period controls at the commuting zone level. The regression is estimated using 2SLS, includes census division dummies, and observations are weighted by the share of each commuting zone in the U.S. population.

I begin by replicating the original results from Autor, Dorn and Hanson (2013) in Panel A of Table 2.6.10. This panel shows the estimation of equation (2.5.8) for a wide range of commuting zone outcome variables with a full set of commuting zone controls. All data in this analysis comes from the replication package provided by Autor, Dorn and Hanson (2013). The original results show that commuting zones facing a higher increase in the China import exposure per worker experience a significant reduction in manufacturing employment, an increase in unemployment rate, a reduction in average wage, in average wage in non-manufacturing, and in median household income. The results also suggest that commuting zone population does not respond to the shock.

In my analysis I am mainly interested in the second period, therefore I replicate the baseline results for the period of 2000-2007 only. The estimation is presented in Panel B of Table 2.6.10, where equation (2.5.8) is estimated with a full set of commuting zone controls. First, an increase in import exposure per worker has no statistically significant effect on commuting zone population, which confirms the lack of mobility in response to this negative local economic shock. This finding confirms Autor, Dorn and Hanson (2013), and is in line

[18]The original regression in Autor, Dorn and Hanson (2013), and consequent literature (e.g. Acemoglu et al. (2016), Autor, Dorn and Hanson (2017*a*), Feler and Senses (2017)), is estimated on data pooled over two sub-periods 1990-1999 and 2000-2007, with the dummy variable for the second sub-period.

with Glaeser and Gyourko (2005) who find mobility responses to negative local productivity shocks in the U.S. to be slow. Second, the effect on the local manufacturing employment is negative and statistically significant. In particular, an increase in Chinese imports per worker by 1000$ leads to a decline in commuting zone share of employed in manufacturing by 0.47%.[19] This effect is significant at the 1% significance level.

The results for other variables from the original study are less robust to estimating equation (2.5.8) for the period 2000-2007 alone. In particular, Panel B shows that local non-manufacturing employment increases by 0.23% for every 1000$ increase in Chinese imports per worker, and this effect is statistically significant at 10% significance level. The effect on unemployment rate is not statistically significant, as well as the effect on log average wages. The estimated effect on average wages in manufacturing is positive and statistically significant, and average wages in non-manufacturing seem to not respond to the shock. Panel B shows that the estimated effect of the China trade shock on median household income is small in magnitude and is not statistically significant: while Autor, Dorn and Hanson (2013) suggest an average decline in median household income of 1.7% for 1000$ increase in import exposure per worker, for the period 2000-2007 I find an average effect of 0.6% and this effect is not statistically distinguishable from zero.

Panel C further explores the robustness of results in Autor, Dorn and Hanson (2013) by estimating the effect separately for the period 1990-1999. Panel D tests whether the differences in the results across all three estimations are significant. The effects of the shock on most outcome variables are not significantly different between 1990-1999 and 2000-2007. However, when comparing both these results to the baseline estimation in Autor, Dorn and Hanson (2013), the effects of the China trade shock on most outcome variables differ substantially. A further research is needed in order to explain why the effects for a set of outcome variables in Autor, Dorn and Hanson (2013) are sensitive to splitting the sample into two periods.[20]

[19]In Autor, Dorn and Hanson (2013), the effects are computed for a 10-year change in the outcome variable. For the period 2000-2007, they scale (i.e. multiply by 10/7) the changes in outcome variables and trade exposure to be comparable to the changes for 1990-1999 period. As I use replication data from Autor, Dorn and Hanson (2013), the effects should also be interpreted as equivalent to a 10-year change in outcome variable.

[20] The regression in Panel A of Table 2.6.10 assumes that the coefficients on control variables are constant across the two periods, whereas separate estimation in Panels B and C allow for

The results presented in Panel B of Table 2.6.10 carry over to a restricted sample of commuting zones, that are used in the analysis of household expenditure. In particular, Table 2.6.11 estimates equation (2.5.8) using 618 commuting zones for the period of 2000-2007 and finds very similar results to those for all commuting zones. In particular, the local China trade shock had a weak effect on average wages and median household income in 2000-2007. This may help to explain the insignificant effect of this shock on household expenditure in this period. The positive effect of the China trade shock on household alcohol consumption is then likely driven by the decline in local manufacturing employment.

The above findings speak to several papers in trade and labor economics, which use the China trade shock as a source of variation in earnings and household income across regions in the U.S. (for example, Autor, Dorn and Hanson (2017*a*) and Feler and Senses (2017)). The results in this section suggest that the China trade shock should be considered primarily as a local manufacturing employment shock rather than a local income shock.

2.6 Conclusion

Motivated by previous findings in the literature about the adverse effects of increased import competition with China on the U.S. local labor markets, I further study the distributional consequences of the China trade shock by measuring its impact on local household expenditure. Using the detailed household scanner data, I show that the effect of the China trade shock on changes in local non-durable consumer expenditure in nominal and real terms are not distinguishable from zero. When decomposing household expenditure into product categories, I find a positive and significant effect on alcohol consumption, however no effect on other food and non-food categories. Among potential explanations for these findings is that the China trade shock in 2000-2007 was mainly a local manufacturing employment shock and less so a local income shock.

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coefficients on control variables to vary. To further understand the results in Table 2.6.10 it is important to investigate the relationship between the China shock and controls in Autor, Dorn and Hanson (2013), however such analysis is beyond the scope of this paper.

Tables and Figures

Table 2.6.1: Summary statistics

	Count	Mean	SD	p25	p75
Household-level variables					
Δ Log(total exp)	25331	-0.13	0.35	-0.29	0.07
Δ Log(food exp)	25331	-0.11	0.39	-0.28	0.10
Δ Log(non-food exp)	25331	-0.17	0.47	-0.42	0.10
Log(total exp)	25331	3355.09	1726.90	2101.14	4276.90
Log(food exp)	25331	2244.23	1197.93	1370.94	2897.56
Log(non-food exp)	25331	1103.36	728.78	606.28	1422.59
Household size	25331	2.28	1.26	1.00	3.00
Black	25331	0.10	0.30	0.00	0.00
Hispanic	25331	0.06	0.23	0.00	0.00
Average head age	25331	55.21	11.49	47.00	64.75
Income	23010	59.37	36.55	30.68	72.52
College	25331	0.46	0.50	0.00	1.00
Commuting zone-level variables					
KNCP households per czone	618	40.99	105.60	4.00	28.00
ΔIPW	618	2.86	3.09	1.13	3.59
Share of emp in mfg	618	20.11	10.56	11.80	27.53
Share of college-educated pop	618	48.14	8.62	41.78	54.28
Share of foreign-born pop	618	6.06	6.54	2.24	6.86
Share of emp among women	618	64.06	6.97	59.63	68.65
Share of emp in routine occ	618	29.26	2.76	27.56	31.20
Avg offshorability index of occ	618	-0.57	0.43	-0.88	-0.34

Notes: Panel A describes household-level outcomes and controls from the Kilts Nielsen Consumer Panel corresponding to 2004. Changes correspond to the period 2004-2007. Panel B describes variables at the commuting zone level corresponding to 2000. ΔIPW is a 2000-2007 change in commuting zone import exposure per worker (in 1000$). Data from the replication package by Autor, Dorn and Hanson (2013).

Table 2.6.2: Descriptives: Commuting Zones with Largest and Smallest Trade Shocks, 2000-2007

	(1) Mean Top quartile of ΔIPW	(2) Mean Bottom quartile of ΔIPW	(3) Difference
ΔIPW	6.42	0.56	5.86***
Household-level variables			
Δ Log(total exp)	-0.14	-0.13	-0.01
Δ Log(food exp)	-0.13	-0.12	-0.01
Δ Log(non-food exp)	-0.18	-0.16	-0.02
Log(total exp)	3348.88	3401.44	-52.57
Log(food exp)	2243.24	2259.37	-16.14
Log(non-food exp)	1098.40	1133.51	-35.11
Income	53.95	56.68	-2.73**
Household size	2.38	2.22	0.16***
Black	0.07	0.10	-0.03***
Hispanic	0.04	0.06	-0.02***
Average head age	54.88	56.61	-1.72***
College	0.42	0.45	-0.03*
Commuting zone-level variables			
KNCP households per czone	23.37	15.23	8.14*
Share of emp in mfg	30.20	10.09	20.11***
Share of college-educated pop	44.16	50.62	-6.46***
Share of foreign-born pop	4.07	7.02	-2.95***
Share of emp among women	63.94	63.20	0.74
Share of emp in routine occ	30.44	27.35	3.09***
Avg offshorability index of occ	-0.50	-0.76	0.26***

Notes: This table compares the means of outcome variables and controls across commuting zones with high and low changes in import exposure per worker. Panel A shows means of household-level outcomes and characteristics from the Kilts Nielsen Consumer Panel. Changes in household expenditure correspond to the period 2004-2007. Levels of household expenditure and household characteristics correspond to 2004. Panel B describes variables at the commuting zone level. ΔIPW is a 2000-2007 change in commuting zone import exposure per worker (in 1000$). All commuting zone characteristics correspond to 2000. Data on import exposure and initial commuting zone characteristics come from the replication package by Autor, Dorn and Hanson (2013). * $p < .1$, ** $p < .05$, *** $p < .01$

Table 2.6.3: The effect of Chinese import exposure on changes in household expenditure: 2004-2007.

	(1) OLS	(2) IV	(3) IV	(4) IV
	Panel A: Δ Log(household annual expenditure)			
ΔIPW	-0.000933	-0.000633	0.0000811	-0.000734
	[0.00105]	[0.00170]	[0.00180]	[0.00266]
Household size			-0.0257***	-0.0259***
			[0.00147]	[0.00145]
Black			-0.00802	-0.00457
			[0.00826]	[0.00803]
Hispanic			-0.00323	-0.00236
			[0.00783]	[0.00869]
Average head age			-0.000787***	-0.000790***
			[0.000242]	[0.000242]
Log Income			0.00152	0.00228
			[0.00373]	[0.00379]
College			0.00164	0.00213
			[0.00471]	[0.00481]
Share of emp in mfg				-0.000128
				[0.000704]
Share of college-educated pop				-0.000930
				[0.000674]
Share of foreign-born pop				0.000937***
				[0.000324]
Share of emp among women				0.00324***
				[0.000948]
Share of emp in routine occ				0.000957
				[0.00146]
Avg offshorability index of occ				-0.0221**
				[0.00907]
Census division dummies	✓	✓	✓	✓
	Panel B: 2SLS First stage estimates			
ΔIPW^{CHOT}		0.849***	0.848***	0.636***
		[0.101]	[0.0998]	[0.111]
Observations	25331	25331	23010	23010

Notes: The dependent variable is the change in log annual household expenditure on groceries from 2004 to 2007. ΔIPW is a 2000-2007 change in commuting zone import exposure per worker (in 1000$). Column (1) is estimated with OLS. Columns (2)-(4) are estimated with 2SLS, using the instrument defined in equation (2.3.6). Household-level controls correspond to 2004, commuting zone-level controls correspond to 2000. First stage estimates in Panel B include the control variables that are indicated in the corresponding columns of Panel A. All regressions include a constant and a set of census division dummies. Standard errors are clustered at the state level. * $p < .1$, ** $p < .05$, *** $p < .01$

Table 2.6.4: The effect of Chinese import exposure on changes in household real expenditure, and log expenditure on food and non-food: 2004-2007.

Dependent variable:	Δ Log(household annual expenditure)			
	(1) Nominal	(2) Real	(3) Food	(4) Non food
ΔIPW	-0.000734	-0.00165	-0.000131	-0.00448
	[0.00266]	[0.00270]	[0.00268]	[0.00472]
Observations	23010	23010	23010	23010
Census division dummies	✓	✓	✓	✓
CZ controls	✓	✓	✓	✓
HH controls	✓	✓	✓	✓

Notes: This table estimates equation (2.3.5), where the dependent variables are the 2004-2007 change in log annual household expenditure (column (1)), in log annual household real expenditure (column (2)), and in log annual household expenditure on food and non-food grocery (columns (3)-(4)). ΔIPW is a 2000-2007 change in commuting zone import exposure per worker (in 1000$). All regressions are estimated with 2SLS, using the instrument defined in equation (2.3.6), include a constant, a set of census division dummies, a set of household-level controls, and a set of commuting-zone level controls. Data on household level outcomes and controls is derived from the Kilts Nielsen Consumer Panel. Data on trade exposure and commuting zone controls comes from the replication package by Autor, Dorn and Hanson (2013). Standard errors clustered at the state level. * $p < .1$, ** $p < .05$, *** $p < .01$

Table 2.6.5: The effect of Chinese import exposure on changes in household expenditure by food category: 2004-2007.

Dependent variable:	Δ Log(household annual expenditure)						
	(1) Dry	(2) Frozen	(3) Dairy	(4) Deli	(5) Packaged meat	(6) Fresh	(7) Alcohol
ΔIPW	-0.00419	0.00584	-0.00404	-0.00574	-0.00499	-0.00293	0.0396***
	[0.00375]	[0.00542]	[0.00364]	[0.00836]	[0.00786]	[0.00753]	[0.0152]
Observations	23010	23010	23010	23010	23010	23010	23010
Census division dummies	✓	✓	✓	✓	✓	✓	✓
CZ controls	✓	✓	✓	✓	✓	✓	✓
HH controls	✓	✓	✓	✓	✓	✓	✓

Notes: This table estimates equation (2.3.5), where the dependent variables are the 2004-2007 change in log annual household expenditure by food category. ΔIPW is a 2000-2007 change in commuting zone import exposure per worker (in 1000$). All regressions are estimated with 2SLS, using the instrument defined in equation (2.3.6), include a constant, a set of census division dummies, a set of household-level controls, and a set of commuting-zone level controls. Data on household level outcomes and controls is derived from the Kilts Nielsen Consumer Panel. Data on trade exposure and commuting zone controls comes from the replication package by Autor, Dorn and Hanson (2013). Standard errors are clustered at the state level. * $p < .1$, ** $p < .05$, *** $p < .01$

Table 2.6.6: Robustness: Summary statistics for the average household expenditure in commuting zone using the full KNCP sample.

	Count	Mean	SD	p25	p75
Δ Log(avg total exp)	642	0.10	0.23	-0.00	0.18
KNCP households per czone in 2004	642	61.63	157.01	5.00	42.00
KNCP households per czone in 2007	642	98.48	207.09	12.00	87.00

Notes: Summary statistics for the change in log average annual expenditure on groceries across households in commuting zone, computed using the full sample of households available in the KNCP in years 2004 or 2007. The table also shows summary statistics for the number of households per commuting zone in 2004 and 2007.

Table 2.6.7: Robustness: Average household expenditure in commuting zone using the full KNCP sample.

Dependent variable:	Δ Log(average total expenditure)	
	(1)	(2)
ΔIPW	0.00415	0.00507
	[0.00340]	[0.00468]
Observations	642	642
Census division dummies	✓	✓
CZ controls		✓

Notes: This table estimates equation (2.3.5) at the commuting zone level. The dependent variable is the 2004-2007 change in log average annual expenditure on groceries per commuting zone, computed using the full sample of households available in the KNCP in years 2004 or 2007. ΔIPW is a 2000-2007 change in commuting zone import exposure per worker (in 1000$). All regressions are estimated with 2SLS, using the instrument defined in equation (2.3.6), include a constant and a set of census division dummies. Standard errors are clustered at the state level. * $p < .1$, ** $p < .05$, *** $p < .01$

Table 2.6.8: Robustness: Change in import penetration ratio.

Dependent variable:	Δ Log(household annual expenditure)	
	(1)	(2)
	2004-2007	2004-2011
Panel A:		
ΔIPR 2000-2007	-0.00126	
	[0.00486]	
ΔIPR 2000-2011		-0.00211
		[0.00642]
Panel B:		
ΔIPR 2004-2007	0.00182	
	[0.00897]	
ΔIPR 2004-2011		0.00172
		[0.00961]
Observations	23010	15041
Census dividision dummies	✓	✓
CZ controls	✓	✓
HH controls	✓	✓

Notes: This table estimates equation (2.3.5), where the dependent variable is the change in log annual household expenditure on groceries either from 2004 to 2007 (column (1)) or from 2004 to 2011 (column (2)). ΔIPR is the change in commuting zone import penetration ratio as in Acemoglu et al. (2016), computed for alternative periods. Data on household level outcomes and controls is derived from the Kilts Nielsen Consumer Panel. All regressions are estimated with 2SLS, using the instrument defined in equation (2.3.7), include a constant, a set of census division dummies, a set of household-level controls, and a set of commuting-zone level controls. Standard errors are clustered at the state level. * $p < .1$, ** $p < .05$, *** $p < .01$

Table 2.6.9: Robustness: Controlling for the change in household size.

Dependent variable:	Δ Log(household annual expenditure)			
	(1) Baseline	(2) Control for Δ hh size	(3) Exclude Δ hh size	(4) Per hh member
ΔIPW	-0.000734	-0.000947	0.000733	-0.000943
	[0.00266]	[0.00266]	[0.00282]	[0.00291]
Change in hh size		0.0351***		
		[0.00341]		
Change in marital status to not married		-0.164***		
		[0.0171]		
Change in marital status to married		0.156***		
		[0.0207]		
Observations	23010	23010	17692	23010
Census division dummies	✓	✓	✓	✓
CZ controls	✓	✓	✓	✓
HH controls	✓	✓	✓	✓

Notes: This table estimates equation (2.3.5), where the dependent variables are the 2004-2007 change in log annual household expenditure. ΔIPW is a 2000-2007 change in commuting zone import exposure per worker (in 1000$). Column (1) present the baseline estimation. Column (2) controls for a change in household marital status and a change in household size. Column (3) excludes households who reported either a change in the marital status or in the number of household members. Column (4) estimates equation (2.3.5) for a change in annual expenditure per household member. All regressions are estimated with 2SLS, using the instrument defined in equation (2.3.6), include a constant, a set of census division dummies, a set of household-level controls, and a set of commuting-zone level controls. Data on household level outcomes and controls is derived from the Kilts Nielsen Consumer Panel. Data on trade exposure and commuting zone controls comes from the replication package by Autor, Dorn and Hanson (2013). Standard errors are clustered at the state level. * $p < .1$, ** $p < .05$, *** $p < .01$

Table 2.6.10: Replicating Autor, Dorn and Hanson (2013): Stacked differences vs single periods.

	Mfg emp/pop	Nonmfg emp/pop	Unemp rate	Log avg wage	Log mfg wage	Log nonmfg wage	Log med income	Log population
Panel A: Estimated on stacked differences from periods 1990-1999 and 2000-2007								
ΔIPW	-0.596***	-0.178	0.221***	-0.759***	0.151	-0.761***	-1.732***	-0.0502
	[0.0988]	[0.137]	[0.0576]	[0.253]	[0.482]	[0.261]	[0.381]	[0.746]
Dummy for 2000-2007	✓	✓	✓	✓	✓	✓	✓	✓
Observations	1444	1444	1444	1444	1444	1444	1444	1444
Panel B: Estimated for period 2000-2007								
ΔIPW	-0.469***	0.230*	0.109	-0.135	1.214**	-0.0847	-0.575	0.178
	[0.123]	[0.118]	[0.0988]	[0.359]	[0.572]	[0.366]	[0.442]	[0.960]
Observations	722	722	722	722	722	722	722	722
Panel C: Estimated for period 1990-1999								
ΔIPW	-0.222	0.195	-0.0524	0.966**	1.600**	0.493	0.864	1.080
	[0.169]	[0.202]	[0.0875]	[0.478]	[0.694]	[0.358]	[0.639]	[0.743]
Observations	722	722	722	722	722	722	722	722
Panel D: Testing differences in coefficients								
P-value: $\beta^{1990-2007} = \beta^{2000-2007}$	0.0520	0.00120	0.0962	0.00135	0.0326	0.000233	0.00125	0.482
P-value: $\beta^{1990-2007} = \beta^{1990-1999}$	0.0372	0.0535	0.0234	0.0000871	0.00610	0.00122	0.0000376	0.0839
P-value: $\beta^{1990-1999} = \beta^{2000-2007}$	0.250	0.865	0.343	0.0289	0.644	0.207	0.0688	0.321

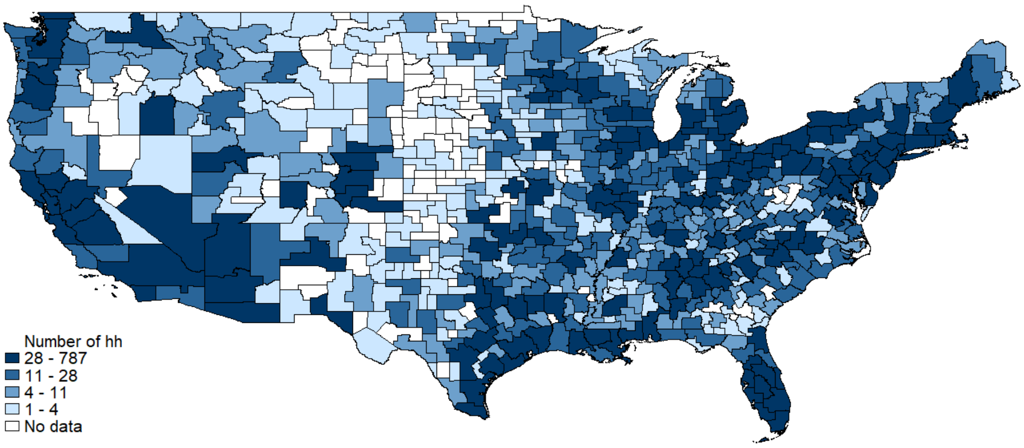
Notes: Dependent variables: Δ share of manufacturing and non-manufacturing employment, Δ unemployment rate, Δ log average wage, Δ log average wage in manufacturing and non-manufacturing, Δ log median household income, Δ log commuting zone population. Outcome variables are multiplied by 100 and should be interpreted as changes in % points. ΔIPW is a change in commuting zone import exposure per worker (in 1000$). Data from the replication package by Autor, Dorn and Hanson (2013). All regressions include constant, census division dummies, controls at the commuting zone level, and weighted by the share of commuting zone in national population. SE clustered at the state level. * $p < .1$, ** $p < .05$, *** $p < .01$

Table 2.6.11: The effect of a 2000-2007 change in commuting zone import exposure per worker on labor market outcomes: reduced sample of commuting zones.

	(1) Mfg emp/pop	(2) Nonmfg emp/pop	(3) Unemp rate	(4) Log avg wage	(5) Log mfg wage	(6) Log nonmfg wage	(7) Log med income	(8) Log population
ΔIPW	-0.478***	0.244**	0.109	-0.112	1.284**	-0.0642	-0.559	0.155
	[0.127]	[0.120]	[0.103]	[0.371]	[0.584]	[0.378]	[0.458]	[0.987]
Observations	618	618	618	618	618	618	618	618
Controls	✓	✓	✓	✓	✓	✓	✓	✓
Census division dummies	✓	✓	✓	✓	✓	✓	✓	✓

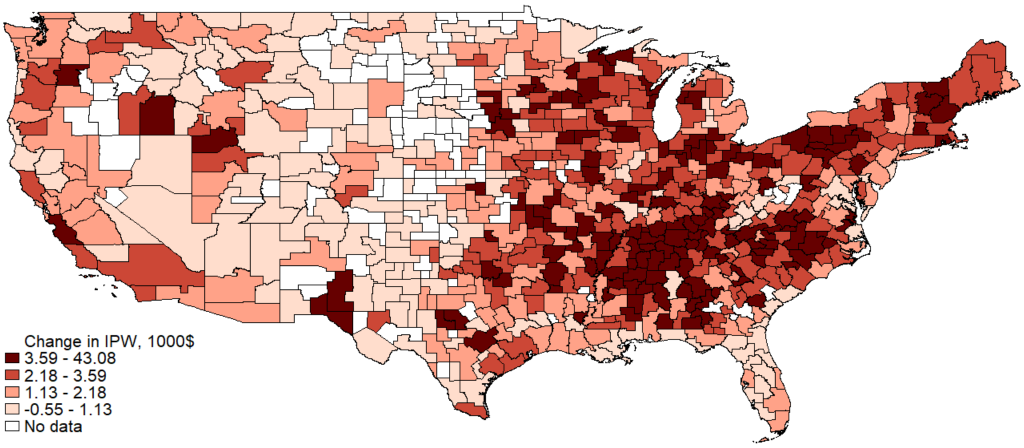
Notes: This table estimates equation (2.5.8) for 618 commuting zones which are included in the sample in this paper. Dependent variables: Δ share of manufacturing and non-manufacturing employment, Δ unemployment rate, Δ logarithm of average wage, Δ logarithm of average wage in manufacturing and non-manufacturing, Δ logarithm of median household income, Δ logarithm of commuting zone population. All outcome variables are multiplied by 100 and should be interpreted as changes in percentage points. ΔIPW is a 2000-2007 change in commuting zone import exposure per worker (in 1000$). Data from the replication package by Autor, Dorn and Hanson (2013). Regressions include a constant, a set of census division dummies, and a full set of controls at the commuting zone level. Regressions are weighted by the share of each commuting zone in national population. Standard errors are clustered at the state level. * $p < .1$, ** $p < .05$, *** $p < .01$

Figure 2.6.1: Average number of households per commuting zone.



Notes: This map depicts the average number of households in the sample per commuting zone. Data comes from the Kilts Nielsen Consumer Panel. Only households who report purchases in 2004 and 2007 are included.

Figure 2.6.2: Distribution of the change in import exposure per worker from 2000 to 2007 across U.S. commuting zones.



Notes: This map depicts the change in import exposure per worker, measured in 1000$, from 2000 to 2007. Data comes from the replication package by Autor, Dorn and Hanson (2013). Only commuting zones with households available in the Kilts Nielsen Consumer Panel are presented.

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Chapter 3

The effect of the fracking boom on non-durable consumer expenditure: evidence from the consumer scanner data.[1]

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3.1 Introduction

The term "fracking boom" refers to a sharp increase in production of shale gas and shale oil in the U.S. in the recent two decades due to a successful combination of a newly available technique of horizontal drilling and improvements in hydraulic fracturing. The combination of these techniques, further referred to as "fracking", made it possible to extract oil and gas resources trapped in geological formations called "shale plays". With introduction of fracking, locations with shale resources have experienced an enormous increase in oil and gas extraction.[2] This sudden boom has led to the creation of thousands of new jobs in mining, construction, transportation, and other sectors. In addition to the rise in employment, the fracking regions have seen an increase in wage income, dividend income, housing values, and rental prices.[3]

This paper studies the effect of the localized economic shock induced by the fracking boom in the U.S. on consumer expenditure, with a focus on non-durables. Non-durables are a large component of household expenditure, with groceries alone accounting on average for about 10.3%.[4] The previous literature has emphasized that the non-durable expenditure reacts significantly to a variety of economic shocks: changes in the housing prices, changes in the minimum wage rate, and changes in household disposable income (Mian, Rao and Sufi (2013), Kaplan, Mitman and Violante (2016), Alonso (2016), Kozlova (2016)).

I use the Kilts-Nielsen Consumer Panel (KNCP) dataset, which provides detailed price and quantity information on household purchases for a universe of grocery products, to estimate the effect of the fracking boom on household non-durable expenditure. Moreover, as it is real income and real consumption that directly matter for household welfare, I distinguish between the effect of the fracking boom on nominal expenditure and real consumption. I also use the detailed household demographic characteristics available in the KNCP

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[2] According to the U.S. Energy Information Administration, EIA (2011), the production of natural gas from hydraulically fractured wells grew from 7% to 67% of the total natural gas output of the United States, and the production of oil grew from 2% to about 50% between 2000 and 2015.

[3] See, for example, Bartik et al. (2017), Feyrer, Mansur and Sacerdote (2017) for estimates of the effect of fracking on local economic outcomes.

[4] The share of groceries in the total household expenditure is even larger for poorer households. Derived from the Consumer Expenditure Survey.

dataset to address the heterogeneous impacts of the fracking boom.

I estimate the effect of residing in the area with fracking potential on household non-durable expenditure. The identification strategy used in this paper utilizes the spatial variation in the location of shale plays and the variation in the timing of the fracking boom in each fracking region. The main identifying assumption is that neither the availability of the resources nor the timing of the fracking boom are correlated to the local economic conditions. While the location of geological shale formations is predetermined, the timing of the development of each shale play can still be correlated to local economic outcomes. I follow Bartik et al. (2017) and focus on the dates when the fracking potential of each shale play became public for the first time. This timing is mostly determined by the local geology and the availability of a fracking technique suited for each particular shale play.

Using difference-in-differences and event study approaches, I find a positive, significant, and persistent effect of fracking on household total grocery expenditure. Households residing in counties above shale plays have on average 4.4% higher expenditure on groceries after the beginning of fracking, compared to households in the control group.[5] The persistence of this result for at least six years after the beginning of fracking can be an evidence of the long-lasting effect of the natural resource boom on local communities.

Next, this paper gives an insight on the effect of fracking on real consumption. The changes in nominal expenditure can be driven by changes in quantity or quality of goods purchased, or changes in local consumer prices. Local consumer prices can be affected by fracking through several channels: marginal costs can rise as a result of a positive local labor demand shock, local markups can increase in response to increase in local demand for non-durables, or markups can fall as a result of increased competition in retail (in case economic development of fracking regions results in entry of new retailers and vendors). To distinguish between the effect of fracking on prices and real consumption, I construct a volume measure of expenditure by using fixed prices for all products. I show that there is no significant difference between the effect of fracking on nominal expenditure and real consumption. This finding is in particular important as it suggests that the increase in expenditure is mainly driven by the changes in quantity of goods purchased, or by changes in composition of consumer baskets. I analyze changes in the composition of consumer

[5]See Section 3.3 for the details about construction of treatment and control groups.

baskets following the fracking boom, and find that the share of food in grocery expenditure slightly decreases, and within food households tend to shift towards processed or ready made food.

Further, the detailed household-level data allows to estimate the effect of the fracking boom on consumption of households with different background. In particular, I estimate the heterogeneous effects of the fracking boom by initial household composition, household head age, education, and occupation. The results suggest that older households, males living alone, and households with at least one head with college degree tend to increase their expenditure more after the beginning of fracking.

The empirical strategy utilized in this paper only allows to estimate the "intent to treat" effect of being potentially affected by fracking on household economic outcomes. In the robustness section I discuss the results for counties that are more likely to be directly affected by fracking, such as those with lower density of population or counties fully located within shale play boundaries. I also discuss whether the results change when the treatment is assigned at the commuting zone rather than at the county level.

Finally, to assess the magnitude of the effect of the fracking boom on household grocery expenditure, I compare this effect to the effect on wages per capita at the county level. I find that, at the average levels of expenditure per household member and per capita wages, an increase in grocery expenditure after the beginning of fracking corresponds to one tenth of the increase in wage income per capita.

This paper is related to a growing body of literature focusing on the effect of the fracking boom on local economic outcomes. A number of papers explore whether there is evidence for the natural resource curse following the development of shale gas and oil in the U.S. Maniloff and Mastromonaco (2014) and Fetzer (2014) study the spillover effects of the fracking boom in the U.S. to other sectors, focusing on local job growth and average earnings, and find no evidence for the resource curse. Allcott and Keniston (2017) study oil and gas booms in the U.S. from 1960 to 2014, including the most recent fracking revolution, and similarly find no evidence for the resource curse. Their findings suggest that local manufacturers benefit from productivity spillovers during the natural resource booms. Feyrer, Mansur and Sacerdote (2017) study geographic propagation of local economic shocks from shale oil and gas production in the U.S., and find substantial increases in regional employment due to fracking. They conclude that new extraction between 2005 and 2012 increased aggregate U.S. employment by 640,000, where one-half of the increase is in sectors not directly related to extraction. Bartik et al. (2017) analyze the effect of the fracking boom in the U.S. on a wide range of economic indicators, such as total county employment, income, housing prices, rental prices, public expenditure, and find positive effects on all these outcomes. The authors conduct a welfare analysis where they account for changes in real wages by analyzing housing prices and rents. However, they do not consider changes in local consumer prices, which would also affect real wages and consumer welfare. Among other recent papers on local economic consequences of the fracking boom are Weber (2012), Weinstein (2014), who focus on labor market outcomes, and Muehlenbachs, Spiller and Timmins (2014), Gopalakrishnan and Klaiber (2014), who focus on the housing market. The effect of the fracking boom on non-economic outcomes, such as crime rates (Bartik et al. (2017)), high-school dropout rates (Cascio and Narayan (2015)), and fertility rates (Kearney and Wilson (2017)) have also been studied.

This paper contributes to the literature by being the first to use the detailed micro-data on a large number of U.S. households to estimate the effect of fracking on household non-durable expenditure and to address the effect of fracking on the real consumption, which is important for consumer welfare. Another important contribution to the literature is the analysis of the distributional impacts of the fracking boom via estimating the effects for different types of households. Moreover, the panel structure of the KNCP data with the detailed information on household location of residence allows to estimate the effect of fracking on local population, as opposed to the previous literature that used county-level averages and couldn't distinguish between the effects of the fracking boom on local population and in-migrants.

This paper is also closely related to the recent literature using consumer and retailer scanner data to study consumer expenditure and prices of non-durables. Mian, Rao and Sufi (2013) estimate the elasticity of consumer expenditure to housing share of household net worth for both durable and non-durable sectors, whereas Kaplan, Mitman and Violante (2016) estimate this elasticity for non-durable consumer expenditure only using retailer scanner data. Stroebel and Vavra (2014) use the retailer scanner data to study the effect of changes in local housing prices on local retail prices. The response of non-durable consumer expenditure to changes in the minimum wage rate was studied in Alonso (2016), and the pass-through from the minimum wage rate to local prices of non-durables is studied in Renkin, Montialoux and Siegenthaler (2017). Kozlova (2016) addresses the effect of changes in disposable income due to weather conditions on the quantity and quality of household food purchases. My paper sheds light on how both local non-durable expenditure and real consumption respond to the local productivity shock, namely the fracking boom.

The rest of this paper is structured as follows. Section 2 introduces the data used in the paper. Section 3 discusses the identification and empirical strategy. Section 4 presents descriptive statistics and Section 5 presents the results. I discuss the magnitude of the effect and the potential mechanism in Section 6. Section 7 concludes.

3.2 Data

This paper combines data from several sources. The data on non-durable consumer expenditure at the household level is obtained from the Kilts-Nielsen Consumer Panel (KNCP) dataset.[6] This dataset consists of an unbalanced panel of more than 150,000 households residing across all the U.S., and covers the period from 2004 to 2015. Households in the sample are given portable scanners to register every purchase made in supermarket, grocery, convenience store, or pharmacy. The price and quantity is recorded for about 2 million universal product codes (UPC) with detailed product characteristics. The product departments covered include General Merchandise, Non-Food Grocery, Health and Beauty Aids, as well as food categories such as Dry Grocery, Frozen Foods, Dairy, Deli, Packaged Meat, Fresh Produce, and Alcohol.

I construct the household quarterly expenditure as a sum of expenditures over all UPC products across transactions during the quarter:

$$C_{icst} = \sum_{j \in J} \sum_{w \in W} P_{jwicst} Q_{jwicst},$$

where C_{icst} is a total quarterly expenditure of household i, located in county c, state s in quarter t, and P_{jwicst} and Q_{jwicst} are price and quantity corresponding to product j in transaction w.

[6]This dataset is provided through the Nielsen Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business.

To distinguish between real and nominal effects, I construct a measure of real quarterly household expenditure by aggregating the quantities of each UPC code bought using the fixed price for each UPC:

$$RC_{icst} = \sum_{j \in J} \sum_{w \in W} P_{j\bar{y}} Q_{jwicst},$$

where $P_{j\bar{y}}$ is a national average price of good j in year $\bar{y}$, where $\bar{y} = 2004$ or the first year good is available in the sample.[7] I also define an alternative measure, where $\bar{y} = 2015$ or the last year good is available in the sample. This measure is similar to the one described in Kaplan, Mitman and Violante (2016) and Alonso (2016), and is meant to abstract from price changes and give a sense of the change in quantities purchased.

The household information includes a wide range of demographic characteristics, which are self-reported on an annual basis, including the location of residence at the 5 digit zip-code level. Households report their total annual income with a two-year lag, therefore, to construct the contemporaneous income I use records from two years in the future. Moreover, household income and employment hours of each household head are reported as categorical variables with wide bins, from which I construct continuous variables by using a middle point of each bin.[8] I then add the employment hours for household heads to obtain a total number of employment hours for each household. These approximations lead to both household income and household employment hours variables to be rather noisy, and relying on them in the estimation can result in a measurement error. I use these variables for descriptives and for robustness checks.

To estimate the effect of the fracking boom on local economic outcomes, I use data on county-level total employment and wages from the Quarterly Census of Employment and Wages (the U.S. Bureau of Labor Statistics). I complement this data with county population estimates from Regional Economic Accounts (the U.S. Bureau of Economic Analysis). All monetary variables, including household-level outcomes, are inflation-adjusted using the Consumer Price Index for All Urban Consumers: Food at home, available from the U.S. Bureau of Labor Statistics.

[7]Due to the high turnover of products in the KNCP dataset, it is not feasible to fix the price of all products in 2004.

[8]For households reporting annual income in the bin ">100,000$", I impute income of 125,000$. This choice is rather arbitrary.

To identify whether the county is located above a shale play, I overlay shale play boundary shapefiles from the U.S. Energy Information Administration with county boundary shapefiles from the U.S. Census Bureau, and assign a county to a shale play if there is an intersection between county and shale play polygons. If the county lies above more than one shale play, then the shale play with the largest area of intersection is used, and if there is a tie, then the shale play with the latest date of beginning of fracking is used. In the robustness check, I exclude the counties on the intersection of shale plays.

3.3 Identification and empirical strategy

3.3.1 Evolution of fracking

The technique of hydraulic fracturing has been in use for more than six decades, but only after its combination with horizontal drilling of wells did it become the technique by which most natural gas and oil are produced in the United States. According to EIA estimates, natural gas production from hydraulically fractured wells constitutes about two-thirds of total U.S. marketed gas production, and the share of crude oil produced using this method is about one half of current U.S. crude oil production.[9]

The implementation of horizontal drilling in conjunction with hydraulic fracturing in the U.S. has been spread over time and space. Historically, the majority of wells in the U.S. were drilled vertically. In 1990s the mining company operating above the Barnet shale play in Texas introduced an improved technique of hydraulic fracturing combined with horizontal drilling and proved that shale plays had a great potential for oil and gas extraction. Due to a high cost of construction and operation of horizontal wells, and due to uncertainty about applicability of hydraulic fracturing to other shale plays, it took more than a decade for the new technology to spread throughout the country.[10] Over time, however, the enormous potential of fracking across shale plays has become public, leading to a widespread implementation of this technique. Figure 3.7.1 shows the evolution of the number of horizontal wells per county

[9] https://www.eia.gov/todayinenergy/detail.php?id=26112

[10] The fracking activity in each geological region could be performed on a full scale only after the technology was proven to be effective in this region, and it took mining companies certain engineering effort to establish the potential of each geological formation.

between 2000 and 2010 using data from the U.S. Geological Survey.

3.3.2 Identification

The location of fracking sites and the timing of when each particular well is built depends on several factors: availability and quality of geological resources, timing of the technological progress which makes these resources accessible for extraction, a prior history of oil and gas development in the region, and the potential cost of extraction. The potential cost of extraction is defined by conditions of the local labor market, local land ownership structure, rental and housing prices, and general willingness of local communities to allow fracking. This paper's empirical analysis aims to determine the effect of the fracking boom on household expenditure on non-durables. However, the areas which are chosen for fracking may differ from the rest of the country due to factors that are correlated to local economic outcomes. To overcome this endogeneity problem, I exploit exogenous variation in the availability of the shale resources and variation in the timing of the fracking boom across different regions.

The availability of shale oil and gas is predetermined by the location of multiple shale plays in the U.S. Figure 3.7.2 maps the geographical boundaries for the U.S. shale gas and shale oil plays. I follow the majority of papers in the literature and compare areas over shale plays to areas without shale plays.[11] Most of these papers use availability of a shale play as an instrument for the production of gas and oil or for the number of productive wells. My identification, however, differs from these papers, as I do not use the data on actual production or number of wells.[12] Instead, I estimate the intention to treat effect, i.e. the effect of living in the area with fracking potential.

The second source of variation is the timing of the fracking boom. Ideally, I would like to account for differences in the timing of fracking which are determined by factors other than local economic conditions. Therefore, I follow Bartik et al. (2017) and use information on the first date when the fracking

[11] Papers which compare areas over shale plays to areas without shale plays are Feyrer, Mansur and Sacerdote (2017), Maniloff and Mastromonaco (2014), Fetzer (2014), Cascio and Narayan (2015), Kearney and Wilson (2017), and Weber (2012). In contrast, Bartik et al. (2017) use within-shale play variation in the quality of underlying resources as an indicator for potential fracking activity, however the data on quality of underlying resources is not freely accessible.

[12] The data on actual production or the number of wells drilled is proprietary.

potential of each shale play became public knowledge. By analyzing a large amount of information from sources like news, business and industry reports, Bartik et al. (2017) collected evidence on dates when the fracking potential of a number of the biggest shale plays in the U.S. became public. This timing was mainly determined by specifics of a local geology and the time it took to develop a technique suited for each particular shale play. Table 3.7.1 summarizes this data.

3.3.3 Empirical strategy

The main empirical analysis in this paper is at the household level. I consider a household being potentially affected by fracking if its reported county of residence is located above a shale play. The reason to assign treatment at the county level is related to the potential for a regional propagation of the effect. Using a finer geographical definition, such as a zip-code, would not take into account that some individuals commute to work and are still affected by fracking even if their zip-code is not within a shale play boundaries. As an alternative, I could assign treatment at the level of a commuting zone. However, a commuting zone on average is much bigger than a county and usually includes densely populated areas along with low-density areas, which makes it difficult to distinguish where the most of the effect takes place. I present the results using the assignment based on the commuting zone as a robustness check.

Bartik et al. (2017) raise an important issue of differences in the underlying economic characteristics of counties that are located above shale plays compared to the rest of the U.S. I address this problem by narrowing down the control group to households living in counties that are located within same states as shale plays, and controlling for state-time fixed effects in all regressions. Figure 3.7.3 shows the main sample for analysis in this paper. The counties which are located above shale plays are coded as treated, whereas counties that are not lying above any shale play but are still located within shale-play-states are coded as a control group. I code the county as treated or control if at least one household in the KNCP dataset reports it as a county of residence. The household-level data is only available for years 2004-2015, therefore I analyze shale plays which experienced fracking boom in 2007 or later, in order to observe at least three years prior to the beginning of fracking.

Most of the previous literature has studied the effect of the fracking boom on county-level outcomes, which are derived using data on all available population in the county at a given time, including households who have migrated to the region following the fracking boom.[13] The data aggregated at a county level doesn't allow to estimate the effect for households who were living in the regions prior to fracking and were directly affected by it. In the KNCP data I observe individual households and their geographical location of residence. Thus, I am able to estimate the effect of fracking on local population by restricting my analysis to those households which report to never have changed the county of residence.[14]

Difference-in-differences

The unexpected nature of the fracking boom and the exogenous variation in the location of shale resources lends itself to a difference-in-difference technique. To estimate the effect of the fracking boom on household-level outcomes, I estimate the following model:

$$Y_{icst} = \alpha_{st} + \gamma_i + \beta \text{Above play}_c \times \text{Post fracking}_{ct} + \epsilon_{icst} \qquad (3.3.1)$$

where Y_{icst} is the value of an outcome variable for household i, located in county c, state s in quarter t. Above play$_c$ is an indicator for whether county c is located above any shale play, and Post fracking$_{ct}$ is an indicator that equals 1 in every quarter after the potential of the shale play corresponding to county c became public. α_{st} denotes state-quarter fixed effect, and γ_i is a household fixed effect.

The parameter of interest β corresponds to a difference between the average outcome of a household, who lives above shale play, before and after the fracking potential of this play became public, relative to the average outcome of households in the control group.

Event-study specification

The main identifying assumption behind difference-in-difference estimation is absence of differential trends in the outcome variable across treated and control households prior to the beginning of fracking. To visually examine any

[13]Bartik et al. (2017) shows that fracking regions experience a significant increase in in-migration.

[14]Availability of detailed geographical location data for each household allows to study the moving patterns of households following the fracking boom, which I will address in my future work.

differential pre-trends in the data, as well as to study the propagation of the effect over time, I use an event-study approach. I normalize the time around the quarter when the fracking potential of each shale play became public, and estimate the following equation:

$$Y_{icst} = \alpha_{st} + \gamma_i + \sum_{k=-30}^{30} \beta_k \text{Above play}_c \times I[\tau_{ct} = k] + \epsilon_{icst} \qquad (3.3.2)$$

where $\tau_{ct} = t - \text{First post-fracking quarter}_c$ measures time relative to the event in each particular county. As before, Above play$_c$ is an indicator for whether county c is located above any shale play, α_{st} denotes state-quarter fixed effect, γ_i is a household fixed effect.

3.4 Descriptives

Table 3.7.2 reports the distribution of households, counties, and states in the sample of analysis by shale play. There are in total 14 states in the analysis, located above at least one of the following nine shale plays: Marcellus and Utica in the Northeast, Niobrara-Denver Basin, Niobrara-Powder River and Bakken in the Midwest, and Haynesville, Eagle Ford, Woodford-Anadarko and Woodford-Ardmore in the South. Each state contains both counties above shale play and counties in the control group. There are in total 823 counties, out of which 227 are located above one of the above-mentioned shale plays. There are 40,569 households in the sample, of which 12,670 live in counties above shale plays. The largest number of households in the sample reside above Utica shale play. The reason for the large number of households residing above Utica is explained by high population density and the large area covered by Utica shale play. Moreover, Marcellus and Utica shale plays partly overlap, and I assign counties located on this overlap to the Utica shale play. In the robustness section I present results for the sample where counties located on the overlap of two shale plays are excluded. Finally, I only consider households who report expenditure for all quarters within each year they are in the sample, in order to remove any households who are not consistent reporters.

Table 3.7.3 shows the descriptive statistics for key variables. Panel A shows descriptives for main household characteristics and Panel B focuses on household outcome variables. Column (1) reports household-level means across all households in the sample. The majority of households in the sample are white (89%) and non-hispanic (93%), with an average age of a household head about 49.5 years. About two thirds of households in the sample report to be married and one third of households report to have kids. About half of all households have at least one head with college degree. An average household stays in the Nielsen sample for about four years. On average a household has 2.65 members. The average total number of employment hours per week across all household heads is a bit more than 40, which captures the fact that a large share of households report at least one household head not in the labor force. Total annual income per household is on average 64,780$. The total quarterly expenditure on grocery amounts on average to 1,071$, where food accounts for more than 70%. The largest expenditure category within food is Dry Grocery, which accounts for about one third of all grocery expenditure of households. The smallest category is Alcohol. The KNCP doesn't include purchases of alcohol in special liquor stores or bars and restaurants.

Columns (2)-(5) in Table 3.7.3 compare pre-treatment characteristics of households living above shale plays with those of households in the control group. The sample is balanced on several characteristics, such as household size, marital status, and presence of kids. However, prior to the fracking boom, households in the regions above shale plays were more likely to be white, less likely to have at least one household head with college degree, they had significantly lower income, and worked fewer hours per week.

Despite the income and other differences, there were no significant differences in average pre-treatment total consumer expenditure, which is the main outcome variable in the analysis. Figure 3.7.4 suggests that the distribution of total grocery expenditure for households residing above shale play is very similar to the control group. Table 3.7.3 also suggests that there are no significant differences in pre-treatment expenditure on most food categories, with exception of deli and alcohol, which account for higher expenditure in the control group.

Comparing KNCP to Consumer Expenditure Survey

It is interesting to compare the reported household expenditure in the Kilts Nielsen Consumer Panel dataset with the Consumer Expenditure Survey (CEX) estimates. Table 3.7.4 presents average income and expenditure by consumer unit from the CEX[15]. Panel A presents descriptives on selected consumer unit characteristics. The characteristics of households in the KNCP sample, presented in Table 3.7.3, are very similar to those of the CEX consumer units. The average size of a household in the KNCP is 2.65 compared to 2.5 in the CEX, and the average household head age is about 50 in both samples. The shares of White and Not Hispanic are only slightly higher in the KNCP. The average reported household income before tax is lower in the KNCP dataset, 64,780$ compared to 74,664$ in the CEX.[16] The comparison suggests that the KNCP sample in my analysis matches well most of the observable characteristics of a more general population, except for the household income.

Panel B presents average expenditure in levels and shares for the largest expenditure categories in the CEX survey. Categories in italics correspond to those represented in the KNCP dataset. *Housekeeping supplies* and *Personal care products and services* correspond to Nielsen categories that can be found in non-food grocery, whereas *Food at home* and *Alcoholic beverages* correspond to food grocery. The sum of annual expenditure for these categories is 5,900$, which corresponds to on average 1,475$ per quarter. This is significantly higher than the reported 1,071$ of quarterly expenditure in the KNCP, suggesting that households in the KNCP on average under-report their expenditure on groceries.

3.5 Results

3.5.1 Consumer expenditure

I begin my empirical analysis by estimating the effect of a potential fracking activity on household total quarterly expenditure on groceries. Table 3.7.5 reports the results of estimating difference-in-difference specification (3.3.1) for the logarithm of household total quarterly expenditure. Column (1) shows estimation with county fixed effects and state-quarter fixed effects. The average effect of living in the fracking area on total consumer expenditure is 3%.

[15]Consumer Expenditure Survey, U.S. Bureau of Labor Statistics, August, 2017 (https://www.bls.gov/cex/2016/combined/decile.pdf)

[16] Households in the KNCP sample report the total annualized income of all members of the household, which refers to income before tax. The continuous income variable used for descriptives is derived from the KNCP dataset and is prone to a measurement error.

There is a substantial heterogeneity among households within counties, and controlling for household fixed effects should help to capture this heterogeneity. When including household fixed effects as in column (2), the effect on expenditure increases, but the difference between the two estimates is not statistically significant. The estimation with household fixed effects implies that after the fracking becomes possible, households above shale plays spend on groceries on average 4.4% more compared to households in the control group. At the average quarterly expenditure of about 1,071$, this implies an increase of 47$ per quarter.

Figure 3.7.5 is an event study plot of the logarithm of total grocery expenditure, after adjustment for household and state-quarter fixed effects, as in specification (3.3.2). The effect of residing above shale play has a persistent effect on household grocery expenditure, which lasts for at least six years. This can indicate that the natural resource boom has a potential for a long-lasting effect on the local economies.

There is little evidence of a trend in total household expenditure in advance of the fracking boom in the shale play counties, however, the effect seems to take off around $\tau = -3$ quarter relative to the first quarter the potential of the shale play became public. This might happen, for example, if in the prospective fracking locations households become employed in the exploratory drilling and construction activities. Alternatively, households might observe the exploratory activity in their area and expect an increase in the future profits.

3.5.2 Real consumption

Table 3.7.6 presents the results for the log of total quarterly expenditure in nominal and real terms. Column (1) repeats the estimation in Column (3) of Table 3.7.14. Columns (2)-(3) show the effect of fracking on real expenditure, by using the average national price available for the first and the last years for each UPC. Even though the estimate is higher in Column (2), the difference between estimates in all three columns are not significantly different from one another.

Figure 3.7.6 shows the event-study estimation of the effect of fracking on household-level log real total quarterly expenditure, where the real expenditure is computed using average national prices in the first year for each UPC. The plot looks very similar to the nominal expenditure in Figure 3.7.5, both in terms of the timing of when the effect kicks in, the magnitude of the effect, and its persistence.

These results suggest that the changes in the nominal household expenditure on groceries following the fracking boom are mostly due to the changes in quantities of goods purchased (if households buy more of the same products), or due to the changes in the composition of consumer baskets (if households shift their consumption to more expensive products).

3.5.3 Composition of consumer basket

Share of food expenditure

To better understand the effect of the fracking boom on household expenditure on groceries, I analyze the compositional changes in consumer baskets.

Columns (1)-(2) of Table 3.7.7 show the estimates of equation 3.3.1 for log quarterly expenditure. The effect on non-food expenditure is almost twice as large as the effect on food expenditure, but this difference is not statistically significant even at 10% significance level. Figure 3.7.7 suggests that both effects on food and non-food expenditure are statistically significant and persist for all 6 years after the beginning of fracking boom in each respective shale region.

Column (3) of Table 3.7.7 presents the effect on the share of food expenditure in total household expenditure on groceries. The average share of food falls by 0.5 percentage points following the fracking boom, which is a rather small effect compared to the mean share of food expenditure of 68%. Even though the effect on the share of food expenditure is small, it is negative and in line with the Engel's Law, stating that the share of consumer expenditure on food is decreasing in consumer income (Hamilton (2001)).

Expenditure by food category

Further, Table 3.7.8 presents more detailed results on the effect of the fracking boom on household food purchases. Panel A reports estimates for log quarterly expenditure by food category, and Panel B presents results for shares of each food category in a household total expenditure on food. The expenditure on almost all food categories increase significantly for households potentially affected by fracking, with an exception of alcohol and dairy products. The largest increase in percentage terms is for deli (9.1%), packaged meat (8.1%), followed by fresh (6.3%) and frozen produce (5.9%). The highest increase in absolute terms happens for dry grocery, which at the average level of expenditure of 356$ per quarter sees an increase of 11.4$ or 3.2%.

What is more interesting is to analyze the effect of fracking on shares of each category in the total food expenditure. Panel B shows a significant decline in the share of dairy products, and a significant increase in the share of frozen and deli departments (although the last results are significant only at 5% and 10% significance levels). These effects are rather small quantitatively, however they indicate that households in the areas affected by fracking are shifting their food consumption towards processed or ready made food.[17]

3.5.4 Heterogeneous effects

The available data on household demographics in the KNCP makes it possible to study the heterogeneous effects of fracking on households along several dimensions: household composition, age, education and occupation of household head. Tables 3.7.10-3.7.12 show the results of estimating difference-in-difference specifications, where the Above play$\times$Post fracking indicator is interacted with household characteristics in the first quarter a household is observed.

Household composition

Table 3.7.9 shows the results for households with different composition. The effect for males living alone is the highest: these households increase their expenditure by 11% after the fracking boom. This effect is significantly different from the effect for males or females living with others. The possible explanation for the large effect on households who reported to be single males in the initial period is that these individuals are more likely to become employed in the fracking sector when it booms. Moreover, if males and females living alone were more likely to get married after the fracking boom compared to the control group, the expenditure of such households would also increase.[18]

[17]The data used in this paper does not say anything about the changes in household food consumption in restaurants or their purchases of take away.

[18]However, Kearney and Wilson (2017) did not find evidence for a significant increase in marriage rates in the regions affected by fracking.

Household head age

Table 3.7.10 shows a heterogeneous effect by the average age of a household head. I split households into three groups, where the first one is young (those aged less than 35), and the last one is old (those aged 65 or older, which is a retirement age). The effect of fracking on consumer expenditure of households with the average age from 35 to 65 is the highest: they spend on average 4.8% more on groceries after the fracking boom takes place. However, this effect is not significantly different from the effect for households of other ages. There seems to be no statistically significant effect on expenditure of younger households (despite the anecdotal evidence that members of such households are more likely to be employed in low-skilled jobs in mining and construction), and of households aged more than 65 (which are likely to be retired).

Education

Table 3.7.11 compares the effect of fracking for households with different initial levels of education. Households report education for each household head, and I distinguish between households where no one has high school degree, at least one head of household has high school degree and no college degree, and at least one head of household has college degree. Households where at least one head has college degree prior to the fracking show on average larger increase in consumer expenditure following the fracking boom, compared to those who have only high school degree or did not finish high school.

There can be several reasons for why older and more educated households increase their grocery expenditure more. These households are more likely to be landowners or homeowners, hence in addition to increased labor income they might gain from increase in housing values and rents. Another possible explanation is that fracking is not a trivial technique and values skills and competence beyond the level of high school degree. Finally, it is possible that, due to in-migration to fracking regions, the younger and low-skilled local workers face an increased labor market competition, which adversely affects their wages in the fracking sector.

Occupation

Table 3.7.12 compares the effect of fracking for households with different initial occupations. I group all occupations in the KCNP into three groups: General occupation (which includes for example such occupations as economist, architect, system developer, teacher, lawyer, landscaper, buyer, clerk, postal worker, salesman, barber, farmer, etc.), Construction (which includes such occupations as construction and road machine operator, mechanic, technician, factory or transportation worker, etc.), and Not in labor force (which includes students, housewives, retired, and unemployed). Interestingly, there are no significant differences in the effect of fracking boom on households with different occupations. One of the explanations for this is that the spillovers from fracking to other sectors can lead to a significant increase in expenditure across other occupations and those not in labor force. Also, as was discussed previously, labor income is not the only source of income that is affected by fracking. Finally, it is possible that individuals switch occupations following the local labor demand shock.

3.5.5 Robustness

This subsection tests the robustness of the baseline specification.

Sample composition

Table 3.7.13 shows robustness checks related to the sample composition. There is a concern about spillovers of the effect from fracking areas to nearby locations (Feyrer, Mansur and Sacerdote (2017)). First, it is possible that members of households commute to work to nearby fracking locations. I take this into account by assigning treatment at the commuting zone level, instead of the county level. Column (2) in Table 3.7.13 presents estimation of the effect on households where treatment is defined at the commuting zone level. The effect is smaller in magnitude, however not statistically different from the baseline result.

Second, the reverse flow of economic activity is possible. If financial revenues from fracking flow to nearby cities (which, for example, host headquarters of mining companies), these cities can observe a boost in economic activity. By including these cities in the control group, I underestimate the effect of fracking on local economic outcomes. Column (3) in Table 3.7.13 presents the results for the sample restricted to households residing in counties with low density of population. The effect is slightly higher if I only focus on counties with less than 500 persons per square mile, however the difference compared to the baseline is not statistically significant.

Counties with low population density are also more likely to be directly affected by fracking activity, compared to cities and dense residential areas which are unlikely to allow fracking. Other counties that are less likely to be affected by fracking are those where fracking activity is limited by law. Fracking has been limited in the New York State due to a temporary moratorium being enacted in 2010. Column (4) in table 3.7.13 takes this into account by excluding counties that belong to the New York state from treated and control groups. The effect is lower than the baseline effect, but the difference is not statistically significant.

Further, I examine whether the results are robust to the definition of the shale-play county. Previously, I defined the county to be located above shale play if it had at least some overlap with the shale play. In column (5) in table 3.7.13, I estimate the effect of fracking on counties that have 100% overlap with a shale play. The effect is higher now, but the difference with the baseline effect is not statistically significant.

The last column of table 3.7.13 tests the robustness of main result to excluding the counties located above more than one shale play. The biggest concern is associated with the overlap between Marcellus and Utica shale plays, which became known for their fracking potential in 2007 and 2011 respectively. Column (6) estimates the effect of fracking on total expenditure when excluding households residing at the overlap. The effect is smaller, but not significantly different from the baseline estimate.

Time-varying controls

As a further robustness check, I examine whether the effect of fracking on household expenditure changes when adding household-level time-varying controls. Column (1) in Table 3.7.14 shows the baseline estimate with household fixed effects. In Column (2), I control for annual household income and employment hours.[19] The sample size is smaller in Column (2) because household income in the KNCP is reported with a 2-year lag, hence it is not observed for the last two years each household is in the sample. From the prior literature on fracking (e.g. Bartik et al. (2017), Feyrer, Mansur and Sacerdote (2017)), we know that both local income and employment are affected by the fracking

[19]See Data section for description of how these variables were constructed.

boom, and I expect this effect to further propagate into non-durable expenditure. Thus, including these controls in the baseline specification should result in a biased estimate of the effect of fracking on household expenditure. However, what I observe is that including these controls doesn't change the effect of the shock on expenditure. I believe that the explanation for why adding time-varying household income and employment hours doesn't change the baseline effect is that both these controls, being approximated from the KNCP data, are rather noisy measures. As will be shown later in Table 3.7.18 and Figure 3.7.11, the effect of the fracking boom on the measure of annual household income from the KNCP is very weak.

In column (3), I control for household size, marital status and whether there are children in the household. An economic shock related to the fracking boom can potentially affect individual decisions for getting married or have children, which would in turn mechanically affect the total household expenditure. Thus, it is important to check whether an increase in total household expenditure is not driven by changes in household composition. Column (3) shows that effect doesn't change when controlling for household composition. This may imply that there is little effect of the fracking boom on these controls. The prior research by Kearney and Wilson (2017) finds positive effect of the fracking boom on birth rates, but no evidence of the increase in marriage rates, which is partly in line with what I observe. Finally, in column (4) I estimate the effect of fracking on logarithm of total expenditure per household member to be around 5%.

Standard errors

In the baseline specification the reported standard errors are clustered at the county level to allow for serial correlation in residuals from the same county. Although treatment is assigned at the level of a county in my analysis, there still may be a concern about serial correlation in residuals from same states. Table 3.7.15 presents estimation of the baseline specification with standard errors clustered at the state level. As there are just 14 states in the analysis, this results in having too few clusters. I also present the standard errors clustered at the interaction of state and the treatment status, which increases the number of clusters to 28. These standard errors should be conservative enough to allow for correlation in errors across households living within same states and with the same treatment status.

3.6 Discussion

Magnitude of the effect

The previous section has established that households residing above shale plays increase their expenditure on groceries by on average 4.4% after the beginning of the fracking boom, compared to households who do not reside above shale plays. This effect is both statistically and economically significant. At the average quarterly expenditure on groceries of 1,071$, an increase of 4.4% means an increase of 47$ each quarter, and this effect persists for at least six years observed in the data. To put this number in context, it amounts to 1.5 times the household quarterly expenditure on fresh grocery, deli or packaged meat, or to more than a half of household quarterly expenditure on all frozen produce registered in the KNCP purchases.

It is also interesting to compare the magnitude of the effect with results in the literature on the effect of different shocks on consumer expenditure. In particular, Kaplan, Mitman and Violante (2016) find that a 30% percent decline in house values over the period from 2007 to 2011 can account for a 6.2% fall in non-durable consumer expenditure. Alonso (2016) finds that an increase in the minimum wage by 41% from 2006 to 2014 (the federal increment during this period) leads to an average increase in non-durable consumer expenditure by about 4.5%. My estimate is quantitatively comparable to these effects, although the nature of the shock that I study is much different.

Mechanism

In this section I draw a link between the estimated effect of fracking on consumer expenditure to the effect of fracking on local labor market outcomes. There is an abundant literature estimating the effect of fracking on county-level economic outcomes. However, in this paper I focus on a specific sample of counties and employ a slightly different identification strategy. It is important to show that the effect of fracking on county-level employment and income for the sample of counties covered by the KNCP sample is in line with the previous literature.

Table 3.7.17 shows the effect of the fracking boom on county-level quarterly wages and employment per capita from Quarterly Census of Employment and Wages (BLS). The sample for analysis includes only those counties where the households from the KNCP sample reside. Columns (1)-(2) present the effect of fracking boom on quarterly total and per capita employment, and columns (3)-(4) – on quarterly total and per capita wages at the county level. Counties affected by fracking experience an increase in total employment by 3.6% and in total wages by 6.4%. The effect on per capita variables is very similar. Column (5) shows the effect on county population, and it is not significantly different from zero. Figure 3.7.10 presents event-study plots for county-level employment and wages per capita. There is little evidence for differential pre-trends between shale play counties and control group, and the effect of fracking is significant and persistent. The estimated effects of fracking on county employment and wages are comparable in magnitude to estimates by Bartik et al. (2017). However, Bartik et al. (2017) find a weakly significant positive effect of fracking on county population, which is not supported by my results.[20]

The county-level outcomes are based on individuals who were initially living in future fracking areas, and those who in-migrated following the beginning of fracking. This makes the estimated effects not directly comparable to those at the household level. Moreover, since household expenditure decisions are more likely to be made at the household rather than at the individual level, analyzing changes in household income rather than in county-level income per capita would be more relevant.

Unfortunately, the categorical household income variable provided by the KNCP can not capture the changes in household income that are happening within the bins. This limitation also holds for the continuous measure of household income constructed from the KNCP, as described in section 2. Table 3.7.18 estimates equation (3.3.1) using this continuous household income variable. The unit of observation is household-year, and the specification includes household and state-year fixed effects. The result in Table 3.7.18 implies that the household income increases by 1.69% after the beginning of fracking. This effect is significant at the 10% significance level. However, figure 3.7.11 shows the evidence of differential pre-trends prior to the beginning of fracking, and the confidence intervals are very wide. Large standard errors might be a result of the artificial construction of income variable from the categorical variable, which introduces noise into estimation. The low point estimate com-

[20]Bartik et al. (2017) estimate effects on in- and out-migration separately, and find that in-migration responds by about 4%, whereas out-migration doesn't seem to respond.

pared to the county-level results might be due to inability of household income measure in the KNCP to capture changes in income that are not large enough. Also, the highest category for household income in the KNCP is 100k$, which leads to a loss of information about income changes in the upper end of income distribution.

To assess whether the estimates of the effect on household grocery expenditure are plausible, I compute the average increase in county-level wages per capita induced by the fracking boom and compare it with the average increase in household-level expenditure per capita. The average county quarterly wage per capita is 3,085$, as is reported in Table 3.7.16. An increase by 6.27% implies an increase by 199$ per quarter. It can be computed that fracking leads to an average increase in quarterly household grocery expenditure per capita of 21$, which constitutes about 1/10 of the implied increase in wage income following the fracking boom. I can not observe other types of expenditure for households in the KNCP sample, however from the CEX survey, I know the fraction of total household expenditure covered by departments in the KNCP dataset, which is roughly 1/10. One can infer that, following the shock, households were spending their additional income in proportion to their usual expenditure shares.

3.7 Conclusions

The recent fracking boom in the U.S. had been previously shown to have a significant effect on local labor market outcomes, such as wages and employment. It has also affected local housing markets, leading to an increase in housing values and rental prices. This paper addresses the effect of the fracking boom on an important indicator of consumer welfare, the non-durable consumer expenditure. By making use of the consumer scanner dataset, I am able to examine in detail the changes in household expenditure on a universe of grocery items. The main finding of the paper is that households potentially affected by fracking increase their total grocery expenditure on average by 4.4%. This effect is both statistically and economically significant, and persists for several years after the beginning of fracking. I also find that this increase in the household expenditure on groceries following the fracking boom is mostly due to the changes in quantities of goods purchased or in the composition of consumer baskets. I show that the share of food in the total grocery expenditure decreases by about 0.5 percentage points, and within the food categories households shift expenditure towards frozen produce and deli. To assess whether the estimates of the effect on household grocery expenditure are plausible, I compare this effect to the effect on county-level per capita wages. At the average levels of per capita household expenditure and per capita county wage income, the implied increase in household grocery expenditure after the beginning of fracking corresponds to one tenth of the increase in wage income.

Tables and Figures

Table 3.7.1: The first date the fracking potential of the shale play became public

Shale play	Basin	First Publicity Date
Woodford-Ardmore	Ardmore	1/10/2007
Bakken	Williston	2/1/2007
Marcellus	Appalachian	12/9/2007
Haynesville-Bossier	TX-LA-MS Salt	3/24/2008
Woodford-Anadarko	Anadarko	2/28/2008
Eagle Ford	Western Gulf	10/21/2008
Niobrara-Denver	Denver	4/7/2010
Niobrara-Powder River	Powder River	4/7/2010
Utica	Appalachian	7/28/2011

Notes: The table shows the dates when the fracking potential of each shale play became public from Bartik et al. (2017). Shale plays, for which Bartik et al. (2017) do not present an estimate of when their fracking potential became public, are not used in the analysis. Shale plays with the fracking potential becoming public before 2007 are not used in the analysis. Shale plays for which there are no corresponding household observations in the Kilts-Nielsen Consumer Panel are not used in the analysis.

Table 3.7.2: Households by shale play

Shale play	Households	Counties	States
No shale play	27899	596	all states below
Marcellus	905	43	NY, OH, VA, WV
Utica	6714	93	MD, NY, OH, PA, WV
Niobrara-Denver Basin	2047	14	CO, NE, WY
Niobrara-Powder River	51	5	MT, WY
Bakken	63	9	MT, ND
Haynesville	739	25	LA, TX
Eagle Ford	1270	19	TX
Woodford-Ardmore	113	8	OK, TX
Woodford-Anadarko	768	11	OK
Total	40569	823	14

Notes: The distribution of households, counties, and states in the sample of analysis by shale play. *Households* and their county of residence are from Kilts-Nielsen Consumer Panel. Only households who do not change county of residence during the sample period are included. Column *Counties* shows the number of counties, with any households, located above each play. If county is located above more than one play, then it is assigned to the shale play which covers the largest area within a county, and with the latest known year of its fracking potential becoming public. The column *States* presents shale-play-states, i.e. states which contain counties located above a shale play and for which there is a household in Kilts-Nielsen Consumer Panel. The first row summarizes data on all counties that are located within the shale-play-states, but are not above any specific shale play. These observations serve as controls in the analysis.

Table 3.7.3: Consumer Panel: Descriptive statistics

	Total	Balance test			
		No shale play	Above shale play	Difference	t-test
	(1)	(2)	(3)	(4)	(5)
Panel A: Household characteristics					
Household size	2.65	2.66	2.64	0.02	(0.60)
Married	0.63	0.63	0.65	-0.02	(-1.20)
Have kids	0.34	0.34	0.33	0.01	(1.37)
Average head age	49.46	49.31	49.80	-0.49**	(-2.14)
Black	0.11	0.13	0.06	0.07***	(4.46)
White, Asian, other	0.89	0.87	0.94	-0.07***	(-4.46)
Hispanic	0.07	0.08	0.07	0.01	(0.51)
Not hispanic	0.93	0.92	0.93	-0.01	(-0.51)
High school	0.47	0.46	0.52	-0.06***	(-4.67)
College	0.51	0.53	0.47	0.06***	(4.43)
Total annual income, $	64,780	67,560	58,850	8,710***	(5.92)
Employment hours	41.86	42.25	41.00	1.25**	(2.54)
Years in sample	4.03	3.98	4.13	-0.14*	(-1.76)
Panel B: Household-level outcomes					
Total quarterly expenditure, $	1,071.21	1,069.71	1,074.53	-4.82	(-0.41)
Food expenditure, $	715.05	716.62	711.59	5.02	(0.55)
Dry Grocery, $	356.86	355.66	359.52	-3.86	(-0.60)
Frozen, $	91.06	91.59	89.89	1.70	(0.93)
Dairy, $	81.46	81.03	82.40	-1.37	(-0.80)
Deli, $	28.11	29.08	25.98	3.10***	(3.64)
Packaged Meat, $	28.78	29.00	28.31	0.69	(0.81)
Fresh Grocery, $	29.65	30.07	28.72	1.35	(1.62)
Alcohol, $	26.02	27.62	22.51	5.11***	(3.60)
Non-food expenditure, $	354.55	351.51	361.26	-9.75*	(-1.82)
Number of Households	40,569	27,899	12,670		

Notes: The table presents descriptive statistics for households residing in counties with and without shale play. Data is derived from from Kilts-Nielsen Consumer Panel dataset and corresponds to the first year a household appears in the sample. *High school* and *College* is an indicator variable, which equals to one if at least one household head has high school or college degree. *Total annual income* is self-reported with a two-year lag. *Employment* refers to a sum of weekly hours of employment by each head of a household. Household expenditure by product categories corresponds to standard Nielsen product department definitions. t-test uses standard errors clustered at the county level. * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3.7.4: Consumer Expenditure Survey: Annual expenditure means

Panel A: Consumer unit characteristics		
Number of consumer units	129,549,000	
People in consumer unit	2.5	
Age of reference person	50.9	
Black	0.13	
White, Asian, other	0.87	
Not hispanic	0.87	
Hispanic	0.13	
Panel B: Income and Expenditure	**Average, $**	**Share**
Income before taxes	74,664	
Income after taxes	64,175	
Average annual expenditures	57,311	100.0
Housing	18,886	33.0
Housekeeping supplies	*660*	*1.2*
Personal care products and services	*707*	*1.2*
Transportation	9,049	15.8
Food	7,203	12.6
Food at home	*4,049*	*7.1*
Cereals and bakery products	524	.9
Meats, poultry, fish, and eggs	890	1.6
Dairy products	410	.7
Fresh fruits and vegetables	542	0.9
Processed fruits and vegetables	242	0.4
Other food at home	1,442	2.5
Food away from home	3,154	5.5
Healthcare	4,612	8.0
...	...	...
Alcoholic beverages	*484*	*0.8*
...	...	...

Notes: This table presents average income and expenditure by consumer unit from Consumer Expenditure Survey, U.S. Bureau of Labor Statistics, August, 2017 (https://www.bls.gov/cex/2016/combined/decile.pdf). Panel A presents descriptives on selected consumer unit characteristics. Panel B presents average expenditures in USD and shares for the largest expenditure categories. In addition, expenditures for alcoholic beverages presented. Categories in italics correspond to those represented in the Kilts Nielsen Consumer Panel dataset. Housekeeping supplies and Personal care products and services correspond to non-food grocery, whereas Food at home and Alcoholic beverages correspond to food grocery.

Table 3.7.5: Impact of fracking on household total quarterly expenditure on groceries

Dependent variable:	Log of total quarterly expenditure	
	(1)	(2)
Above play×Post fracking	0.0300***	0.0438***
	[0.0108]	[0.00894]
Observations	653576	653576
R^2	0.052	0.694
County FE	✓	
Household FE		✓
State#Quarter FE	✓	✓

Standard errors in brackets

* $p < .1$, ** $p < .05$, *** $p < .01$

Notes: This table presents estimation of equation (3.3.1), where the outcome variable is natural logarithm of household total quarterly expenditure on all groceries registered in Kilts-Nielsen Consumer Panel dataset. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. Column (1) includes county FE and state×quarter FE, column (2) includes household FE instead of county FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3.7.6: Impact of fracking on nominal and real household quarterly expenditure

Dependent variable:	Log of total quarterly expenditure		
	Nominal	Real	Real
		$\bar{y} = 2004$	$\bar{y} = 2015$
	(1)	(2)	(3)
Above play×Post fracking	0.0438***	0.0496***	0.0432***
	[0.00894]	[0.00978]	[0.00903]
Observations	653576	653576	653576
R^2	0.694	0.687	0.688
Household FE	✓	✓	✓
State×Quarter FE	✓	✓	✓

Notes: This table presents estimations of equation (3.3.1). The outcome variable in column (1) is natural logarithm of household total quarterly expenditure on groceries. The outcome variable in columns (2)-(3) is logarithm of household real quarterly expenditure, which is computed using the average national price for each UPC for the first (column (2)) or the last (column (3)) year the UPC is available. The data is derived from Kilts-Nielsen Consumer Panel dataset. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. All columns include household FE and state×quarter FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3.7.7: Impact of fracking on household Food and Non-Food expenditure

Dependent variable:	Log quarterly expenditure		Share of food
	Food	Non-Food	in total
	(1)	(2)	(3)
Above play×Post fracking	0.0358***	0.0603***	-0.00504**
	[0.00824]	[0.0152]	[0.00232]
Observations	653576	653576	653576
R^2	0.692	0.631	0.576
Average	715.05$	354.55$	0.68
Household FE	✓	✓	✓
State×Quarter FE	✓	✓	✓

Notes: This table presents estimations of equation (3.3.1). The outcome variables in columns (1) and (2) are natural logarithm of household quarterly expenditure on food and non-food items. The outcome variable in column (3) is the share of food expenditure in total expenditure on groceries. Data is derived from Kilts-Nielsen Consumer Panel dataset. Food includes the following standard Nielsen product departments: Dry Grocery, Frozen Foods, Dairy, Deli, Packaged Meat, Fresh Produce, Alcohol. Non-food includes the following product departments: General Merchandise, Non-Food Grocery, Health and Beauty Aids. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. All columns include household FE and state×quarter FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3.7.8: Impact of fracking on household expenditure by food category

	Dry	Frozen	Dairy	Deli	Meat	Fresh	Alcohol
Panel A: Log quarterly expenditure							
Above play×	0.0319***	0.0597***	0.0182*	0.0908***	0.0807***	0.0629***	0.00263
×Post fracking	[0.00797]	[0.0166]	[0.00973]	[0.0218]	[0.0175]	[0.0220]	[0.0183]
Observations	653576	653576	653576	653576	653576	653576	653576
R^2	0.682	0.613	0.683	0.555	0.596	0.668	0.669
Average expenditure, $	356.86	91.06	81.46	28.11	28.78	29.65	26.02
Panel B: Share of quarterly food expenditure							
Above play×	-0.00281	0.00303**	-0.00242***	0.00180*	0.000773	0.000694	-0.00107
×Post fracking	[0.00214]	[0.00138]	[0.000735]	[0.000982]	[0.000537]	[0.000879]	[0.00101]
Observations	653068	653068	653068	653068	653068	653068	653068
R^2	0.623	0.532	0.580	0.555	0.503	0.623	0.724
Average share	0.57	0.14	0.13	0.04	0.04	0.04	0.04
Household FE	✓	✓	✓	✓	✓	✓	✓
State×Quarter FE	✓	✓	✓	✓	✓	✓	✓

Notes: This table presents estimations of equation (3.3.1). Panel A shows the results for the natural logarithm of household quarterly expenditure by food category. Panel B shows resukts for the share of food category in the total household expenditure on food. Data is derived from Kilts-Nielsen Consumer Panel dataset. Food categories correspond to the standard Nielsen product departments. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. All columns include household FE and state×quarter FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3.7.9: Heterogeneous effect of fracking on household expenditure by initial household composition

Dependent variable:	Log quarterly expenditure	
	Nominal	Real
Above play×Post fracking × Married	0.0332***	0.0355***
	[0.00967]	[0.0103]
Above play×Post fracking × Male alone	0.110***	0.120***
	[0.0277]	[0.0311]
Above play×Post fracking × Female alone	0.0687***	0.0839***
	[0.0155]	[0.0176]
Above play×Post fracking × Male/Female with others	0.0234	0.0319*
	[0.0172]	[0.0175]
Observations	653576	653576
Household FE	✓	✓
State×Quarter FE	✓	✓
Test: $\beta1 = \beta2$	0.00436	0.00391
Test: $\beta1 = \beta3$	0.0106	0.00103
Test: $\beta1 = \beta4$	0.590	0.853
Test: $\beta2 = \beta3$	0.116	0.208
Test: $\beta2 = \beta4$	0.0145	0.0240
Test: $\beta3 = \beta4$	0.0700	0.0530

Notes: This table presents estimations of equation (3.3.1) for the natural logarithm of household quarterly expenditure by initial household composition. Data is derived from Kilts-Nielsen Consumer Panel dataset. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. All columns include household FE and state×quarter FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$. The p-value is shown for the tests of whether coefficients are equal.

Table 3.7.10: Heterogeneous effect of fracking on household expenditure by initial average household head age

Dependent variable:	Log of quarterly expenditure	
	Nominal	Real
Above play×Post fracking × <35 y.o.	0.0209	0.0175
	[0.0255]	[0.0278]
Above play×Post fracking × 35-65 y.o.	0.0485***	0.0566***
	[0.00884]	[0.00956]
Above play×Post fracking × >65 y.o.	0.0269	0.0241
	[0.0171]	[0.0185]
Observations	653576	653576
Household FE	✓	✓
State×Quarter FE	✓	✓
Test: $\beta 1 = \beta 2$	0.261	0.147
Test: $\beta 1 = \beta 3$	0.834	0.836
Test: $\beta 2 = \beta 3$	0.169	0.0453

Notes: This table presents estimations of equation (3.3.1) for the natural logarithm of household quarterly expenditure by households with different initial average of head of household. Data is derived from Kilts-Nielsen Consumer Panel dataset. The average age of head of household corresponds to the first year the household is present in the sample. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. All columns include household FE and state×quarter FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$. The p-value is shown for the tests of whether coefficients are equal.

Table 3.7.11: Heterogeneous effect of fracking on household expenditure by initial education

Dependent variable:	Log of quarterly expenditure	
	Nominal	Real
Above play×Post fracking × Some school	-0.0201	-0.00523
	[0.0631]	[0.0741]
Above play×Post fracking × High School	0.0319***	0.0387***
	[0.0106]	[0.0113]
Above play×Post fracking × College	0.0590***	0.0636***
	[0.0111]	[0.0116]
Observations	653576	653576
Household FE	✓	✓
State×Quarter FE	✓	✓
Test: $\beta 1 = \beta 2$	0.411	0.555
Test: $\beta 1 = \beta 3$	0.222	0.365
Test: $\beta 2 = \beta 3$	0.0256	0.0330

Notes: This table presents estimations of equation (3.3.1) for the natural logarithm of household quarterly expenditure by initial education of household head. Data is derived from Kilts-Nielsen Consumer Panel dataset. Households are classified into those where no head of household has high school degree, at least one head of household has high school degree and no college degree, and at least one head of household has college degree. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. All columns include household FE and state×quarter FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$. The p-value is shown for the tests of whether coefficients are equal.

Table 3.7.12: Heterogeneous effect of fracking on household expenditure by initial occupation

Dependent variable:	Log of quarterly expenditure	
	Nominal	Real
Above play×Post fracking × General occupation	0.0456***	0.0490***
	[0.00894]	[0.00947]
Above play×Post fracking × Construction	0.0347**	0.0465***
	[0.0172]	[0.0180]
Above play×Post fracking × Not in labor force	0.0480***	0.0538***
	[0.0137]	[0.0144]
Observations	653576	653576
Household FE	✓	✓
State×Quarter FE	✓	✓
Test: $\beta 1 = \beta 2$	0.491	0.875
Test: $\beta 1 = \beta 3$	0.869	0.741
Test: $\beta 2 = \beta 3$	0.465	0.698

Notes: This table presents estimations of equation (3.3.1) for the natural logarithm of household quarterly expenditure by initial occupation of household head. Data is derived from Kilts-Nielsen Consumer Panel dataset. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. All columns include household FE and state×quarter FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$. The p-value is shown for the tests of whether coefficients are equal.

Table 3.7.13: Robustness checks: Sample composition

Dependent variable:	Log of total quarterly expenditure					
	Baseline	CZ	Density<500	Exclude NY	Inner county	No overlap
	(1)	(2)	(3)	(4)	(5)	(6)
Above play×Post fracking	0.0438***	0.0294***	0.0508***	0.0365***	0.0523***	0.0278***
	[0.00894]	[0.00910]	[0.0111]	[0.00934]	[0.0126]	[0.00969]
Observations	653576	593064	319572	534728	545812	578280
R^2	0.694	0.693	0.689	0.693	0.692	0.693
Household FE	✓	✓	✓	✓	✓	✓
State×Quarter FE	✓	✓	✓	✓	✓	✓

Notes: This table presents robustness checks for estimations of equation (3.3.1) for the natural logarithm of household quarterly expenditure. Data is derived from Kilts-Nielsen Consumer Panel dataset. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. Column (1) reports the baseline results. Column (2) uses definition of treatment at the commuting zone level. Column (3) exclude counties with population density above 500 persons per square mile. Column (4) excludes counties in New York state. Column (5) defines county above play only if it has 100% intersection with a play. Column (6) excludes counties above overlapping shale plays. All columns include household FE and state×quarter FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3.7.14: Robustness: Impact of fracking on household total quarterly expenditure on groceries

Dependent variable:	Log total expenditure			Log expend per capita
	(1)	(2)	(3)	(4)
Above play×Post fracking	0.0438***	0.0431***	0.0432***	0.0505***
	[0.00894]	[0.00860]	[0.00889]	[0.00990]
Log annual income		0.0360***		
		[0.00524]		
Employment hours		-0.0000925		
		[0.000125]		
Household size			0.0327***	
			[0.00259]	
Married			0.129***	
			[0.00840]	
Have kids			0.0267***	
			[0.00699]	
Observations	653576	381156	653576	653576
R^2	0.694	0.730	0.695	0.686
Household FE	✓	✓	✓	✓
State#Quarter FE	✓	✓	✓	✓

Standard errors in brackets

* $p < .1$, ** $p < .05$, *** $p < .01$

Notes: This table presents estimation of equation (3.3.1). In columns (1)-(3) the outcome variable is a natural logarithm of household total quarterly expenditure on all groceries registered in Kilts-Nielsen Consumer Panel dataset. In column (4) the outcome variable is a logarithm of expenditure per household member. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. All columns include the household FE and state×quarter FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3.7.15: Robustness checks: Standard errors

Dependent variable:	Log of total quarterly expenditure		
	Baseline	State-cluster	State-treated cluster
	(1)	(2)	(3)
Above play×Post fracking	0.0438***	0.0438***	0.0438***
	[0.00894]	[0.0118]	[0.00949]
Observations	653576	653576	653576
R^2	0.694	0.694	0.694
Household FE	✓	✓	✓
State×Quarter FE	✓	✓	✓

Notes: This table presents robustness checks for estimations of equation (3.3.1) for the natural logarithm of household quarterly expenditure. Data is derived from Kilts-Nielsen Consumer Panel dataset. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. Column (1): standard errors clustered at the county level, column (2): standard errors clustered at the state level, column (3): standard errors clustered at the state×above play level. All columns include household FE and state×quarter FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3.7.16: Descriptives: County-level outcome variables

	Total	Balance test			
		No shale play	Above shale play	Difference	t-test
	(1)	(2)	(3)	(4)	(5)
Total employment	40058.49	40776.63	38260.10	2516.53	(0.25)
Employment per ca	0.34	0.35	0.33	0.02	(1.56)
Total wages, m$	507	544	415	130	(0.61)
Wages per ca, $	3085.74	3135.45	2970.60	164.84	(0.94)
Population	94320.72	96673.52	88870.56	7802.95	(0.41)
Observations	834	596	238	834	

Notes: The table presents descriptive statistics for counties with and without shale play. Employment and Wages are from Quarterly Census of Employment and Wages, BLS. Quarterly Total employment represent the number of covered workers who worked or received pay at the beginning of the quarter. County population is from BEA. Only counties from the household-level analysis included. * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3.7.17: Impact of fracking on county-level outcomes

Dependent variable:	Log employment		Log wages		Log population
	Total	Per ca	Total	Per ca	
	(1)	(2)	(3)	(4)	(5)
Above play×Post fracking	0.0363***	0.0344***	0.0645***	0.0627***	0.00255
	[0.0110]	[0.00909]	[0.0157]	[0.0142]	[0.00625]
Observations	53365	50301	53365	50301	50304
R^2	0.998	0.963	0.996	0.955	0.999
County FE	✓	✓	✓	✓	✓
State×Quarter FE	✓	✓	✓	✓	✓

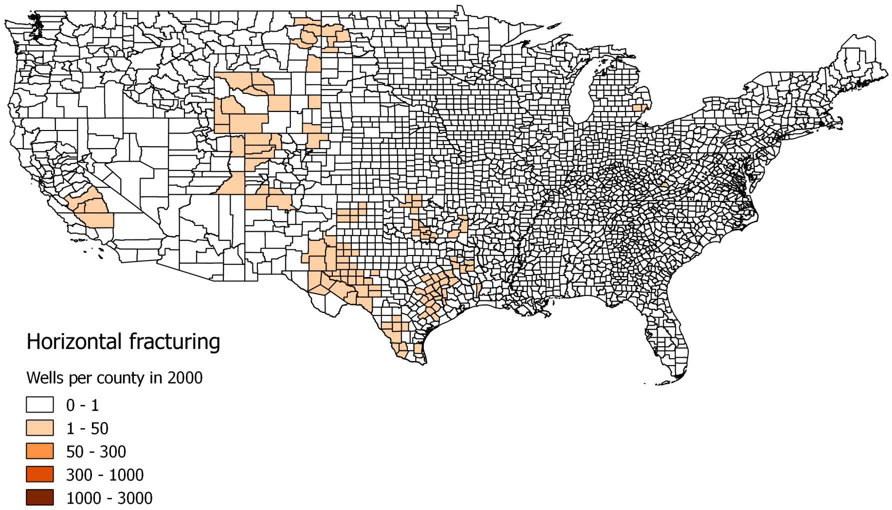
Notes: The effect of fracking on county-level outcomes. Only counties from the household-level analysis included. Employment and Wages are from Quarterly Census of Employment and Wages, BLS. County population is from BEA. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. All columns include county fixed effects and state×quarter fixed effects. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3.7.18: Impact of fracking on household-level income

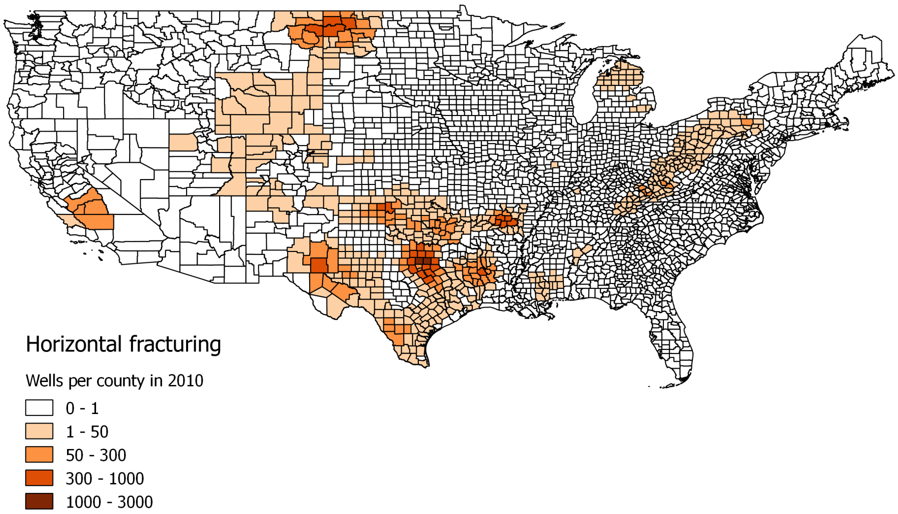
Dependent variable:	Log of annual household income
	(1)
Above play×Post fracking	0.0169*
	[0.00931]
Observations	90555
R^2	0.876
Household FE	✓
State×Year FE	✓

Notes: This table presents estimation of equation (3.3.1) for the natural logarithm of household annual income. Data is derived from Kilts-Nielsen Consumer Panel dataset. *Above play×Post fracking* is an indicator for county being located above a shale play interacted with an indicator for a year after the fracking potential of a respective shale play became public. All columns include household FE and state×year FE. Standard errors clustered at the county level in parentheses. * $p < .1$, ** $p < .05$, *** $p < .01$.

Figure 3.7.1: Evolution of hydraulic fracturing in the U.S.



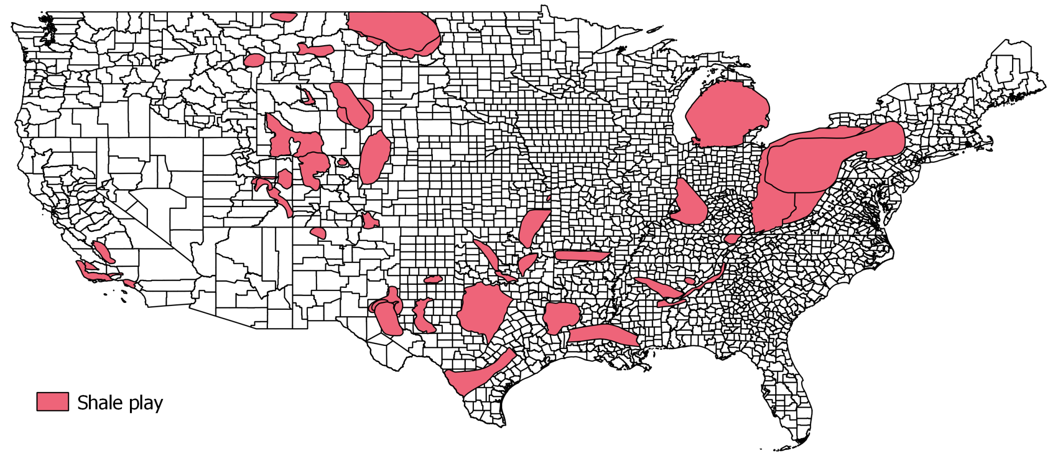
(a) 2000



(b) 2010

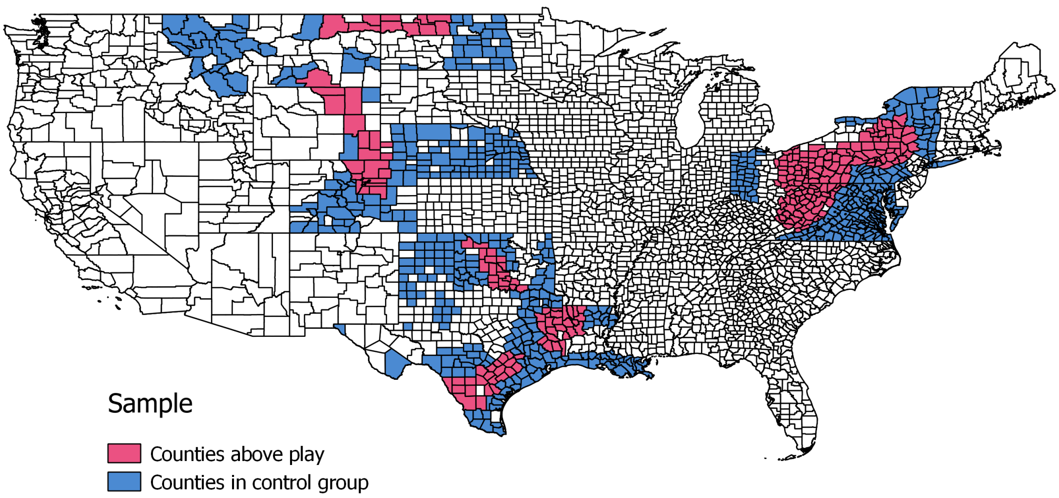
Notes: The number of wells drilled using horizontal fracturing technique per county. The data was constructed from the number of wells per hydrologic unit available from Gallegos and Varela (2015), the U.S. Geological Survey.

Figure 3.7.2: U.S. Shale Gas and Shale Oil Plays.



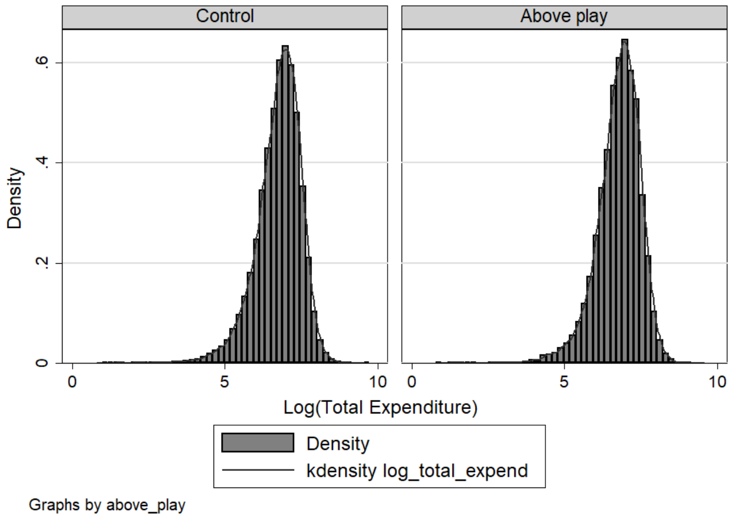
Notes: Data: U.S. Energy Information Administration (EIA).
https://www.eia.gov/maps/maps.htm×geodata

Figure 3.7.3: Counties above shale play and control group.



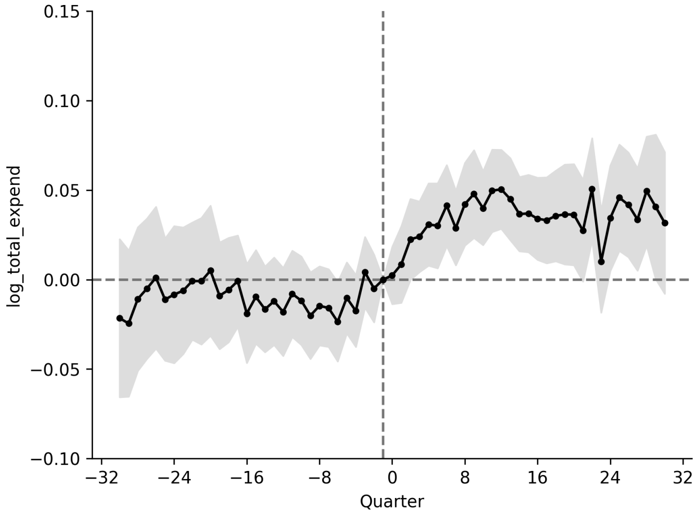
Notes: Counties located above any shale play from Table 3.7.1 are colored with red color. Counties that are located within the shale-play-states, but are not above any specific shale play are colored with blue color and serve as controls in the analysis. Only counties with at least one household in the KNCP sample are included in the analysis.

Figure 3.7.4: Distribution of household total expenditure on groceries.



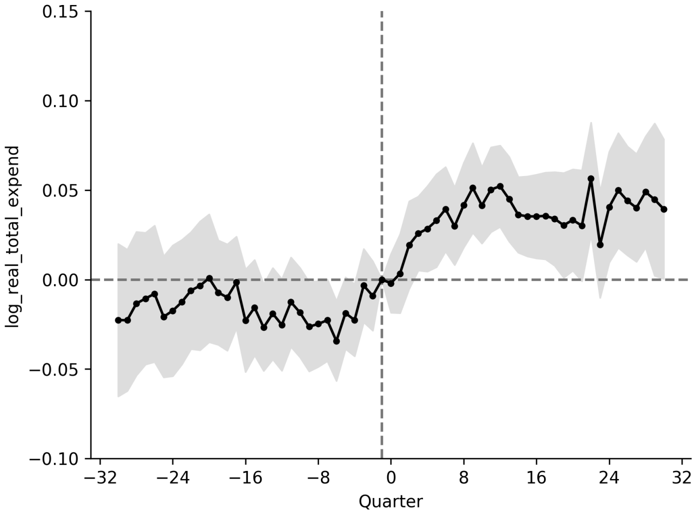
Notes: The histogram of the logarithm of household quarterly expenditure on groceries for households residing in counties above shale play and in control counties. The expenditure variable is derived from Kilts-Nielsen Consumer Panel dataset, and corresponds to the first quarter each household is present in the sample.

Figure 3.7.5: Event study analysis: household quarterly expenditure.



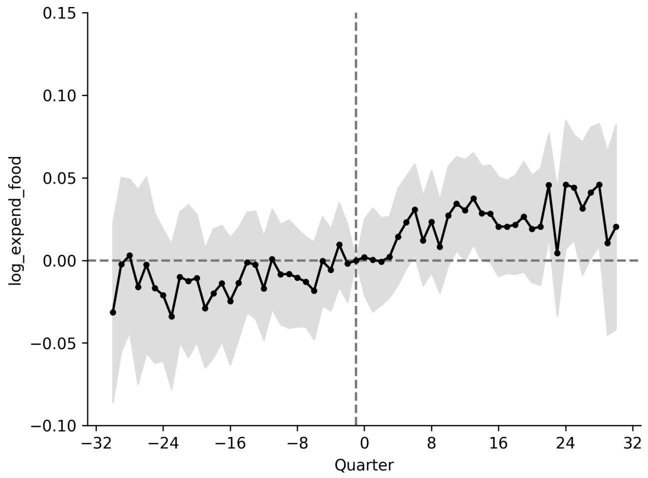
Notes: This plot depicts estimation of equation (3.3.2), where the outcome variable is natural logarithm of household total quarterly expenditure on groceries. The expenditure variable is derived from Kilts-Nielsen Consumer Panel dataset. On the x-axis is the calendar quarter relative to the first quarter when the fracking potential of the corresponding shale play became public: $\tau = t -$ *First post-fracking quarter*. The effect in quarter $\tau = -1$ relative to the event is normalized to zero. Specification includes household FE and state×quarter FE. Standard errors are clustered at the county-level. Shaded area corresponds to 95% confidence interval.

Figure 3.7.6: Event study analysis: household real quarterly expenditure.

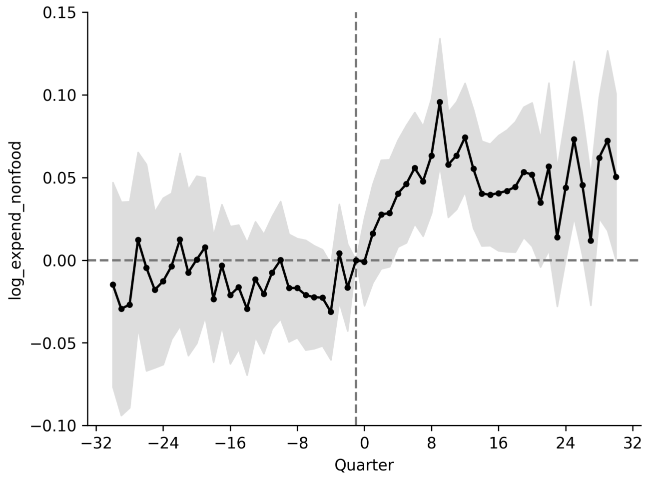


Notes: This plot depicts estimation of equation (3.3.2), where the outcome variable is natural logarithm of household *real* total quarterly expenditure on groceries. The real expenditure variable is derived from Kilts-Nielsen Consumer Panel dataset, and is computed using average national prices for each UPC code in the first year it is available in the sample. On the x-axis is the calendar quarter relative to the first quarter when the fracking potential of the corresponding shale play became public: $\tau = t - \textit{First post-fracking quarter}$. The effect in quarter $\tau = -1$ relative to the event is normalized to zero. Specification includes household FE and state×quarter FE. Standard errors are clustered at the county-level. Shaded area corresponds to 95% confidence interval.

Figure 3.7.7: Event study analysis: household quarterly expenditure on Food and Non-Food.



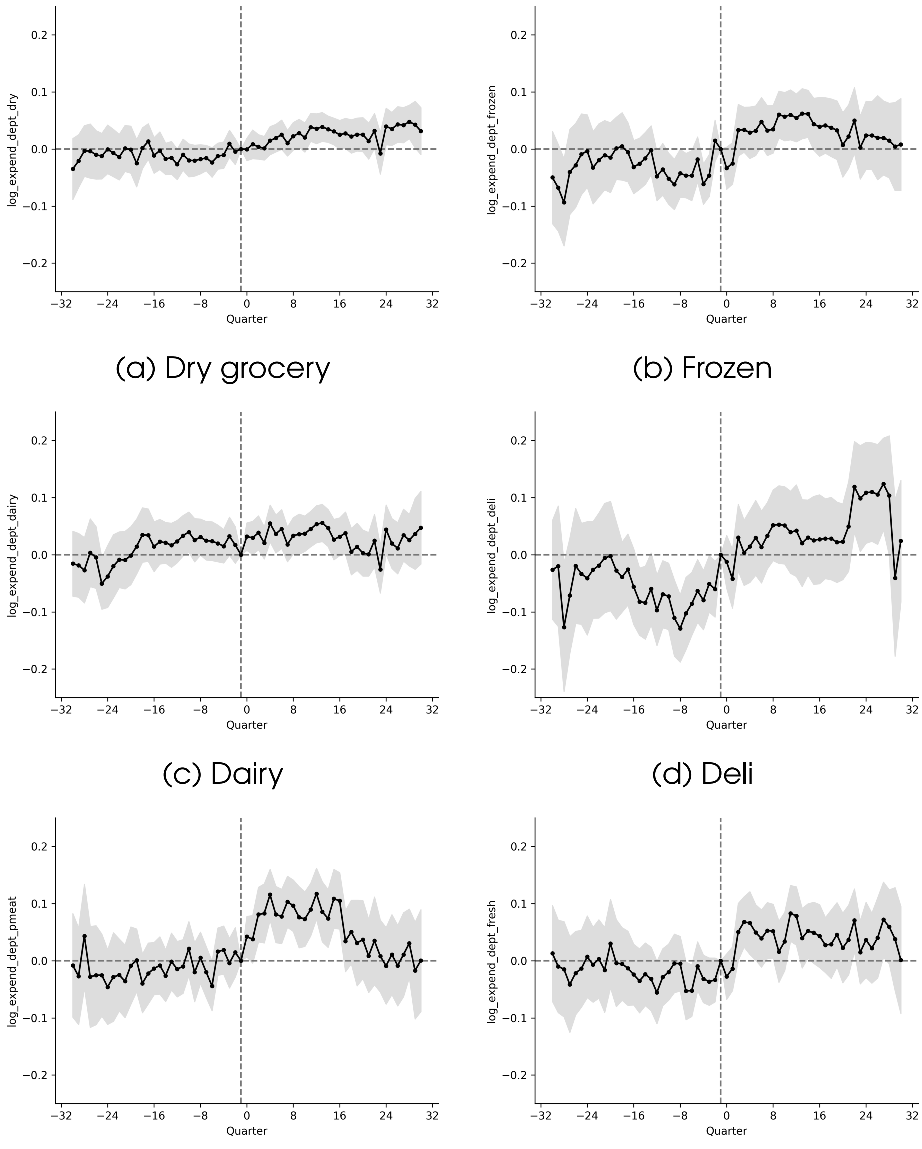
(a) Food



(b) Non-Food

Notes: This plot depicts estimation of equation (3.3.2), where the outcome variables are natural logarithm of household quarterly expenditure on Food and Non-Food. The expenditure variables are derived from Kilts-Nielsen Consumer Panel dataset. Food includes the following standard Nielsen product departments: Dry Grocery, Frozen Foods, Dairy, Deli, Packaged Meat, Fresh Produce, Alcohol. Non-Food includes the following product departments: General Merchandise, Non-Food Grocery, Health and Beauty Aids. On the x-axis is the calendar quarter relative to the first quarter when the fracking potential of the corresponding shale play became public: $\tau = t - \textit{First post-fracking quarter}$. The effect in quarter $\tau = -1$ relative to the event is normalized to zero. Specification includes household FE and state$\times$quarter FE. Standard errors are clustered at the county-level. Shaded area corresponds to 95% confidence interval.

Figure 3.7.8: Event study analysis: household quarterly expenditure by food category.



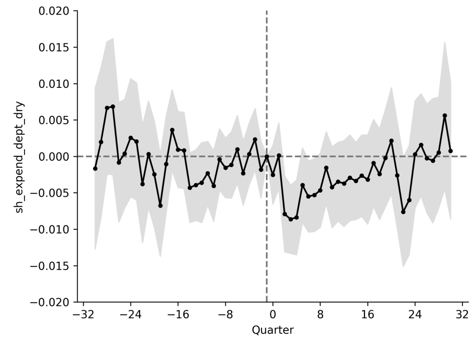
(a) Dry grocery (b) Frozen

(c) Dairy (d) Deli

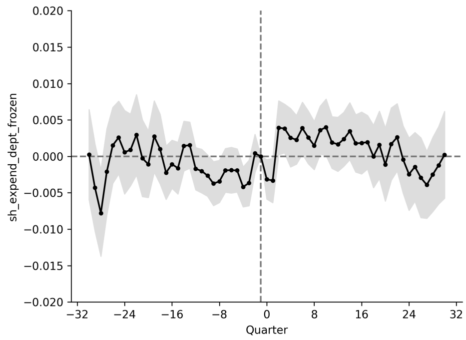
(e) Packaged meat (f) Fresh produce

Notes: Estimation of equation (3.3.2), where the outcome variables are natural logarithm of household quarterly expenditure by food category. The expenditure variables are derived from Kilts-Nielsen Consumer Panel dataset, and food categories correspond to standard Nielsen product department definitions. On the x-axis is the calendar quarter relative to the first quarter when the fracking potential of the corresponding shale play became public: $\tau = t - \textit{First post-fracking quarter}$. The effect in quarter $\tau = -1$ relative to the event is normalized to zero. Specification includes household FE and state×quarter FE. Standard errors are clustered at the county-level. Shaded area corresponds to 95% confidence interval.

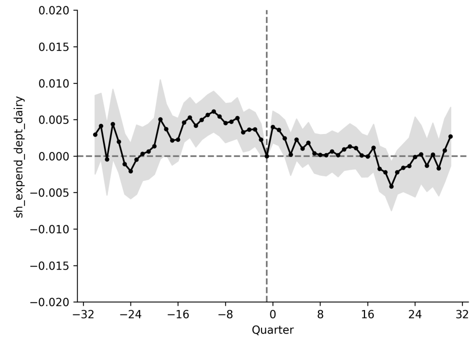
Figure 3.7.9: Event study analysis: share of household quarterly expenditure by food category.



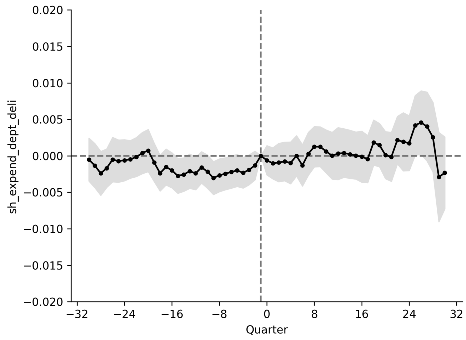
(a) Dry grocery



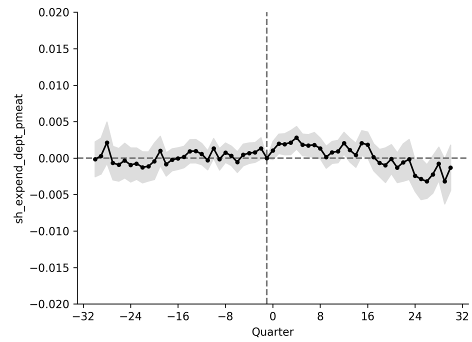
(b) Frozen



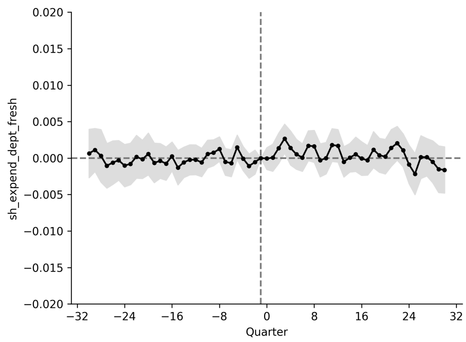
(c) Dairy



(d) Deli



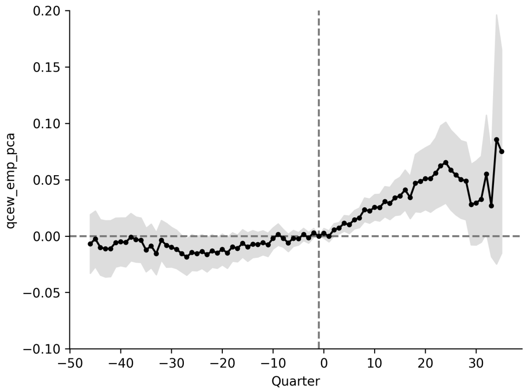
(e) Packaged meat



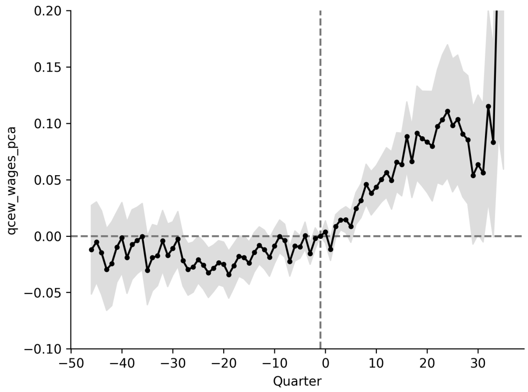
(f) Fresh produce

Notes: Estimation of equation (3.3.2), where the outcome variables are shares of household quarterly expenditure on food category relative to the total food expenditure. Food categories correspond to standard product department definitions in the Kilts-Nielsen Consumer Panel dataset. On the x-axis is the calendar quarter relative to the first quarter when the fracking potential of the corresponding shale play became public: $\tau = t -$ *First post-fracking quarter*. The effect in quarter $\tau = -1$ relative to the event is normalized to zero. Specification includes household FE and state$\times$quarter FE. Standard errors are clustered at the county-level. Shaded area corresponds to 95% confidence interval.

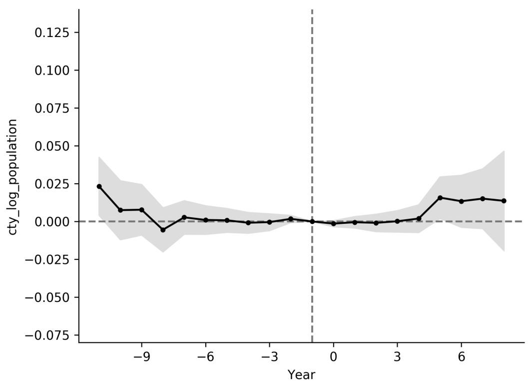
Figure 3.7.10: Event study analysis: county-level quarterly employment and wages per capita and yearly population.



(a) Employment per ca



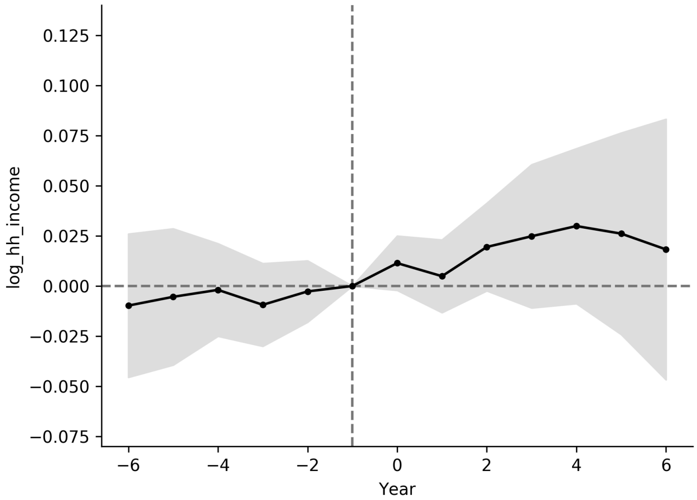
(b) Wages per ca



(c) Population

Notes: This plot depicts estimation of equation (3.3.2), where the outcome variables are natural logarithm of county-level per capita employment and wages or county-level population. Data on total county-level employment and wages comes from from Quarterly Census of Employment and Wages (U.S. Bureau of Labor Statistics), and per capita variables constructed by dividing with a county population estimate from the U.S. Bureau of Economic Analysis. On the x-axis is the calendar quarter relative to the first quarter when the fracking potential of the corresponding shale play became public: $\tau = t - \textit{First post-fracking quarter}$. The effect in quarter $\tau = -1$ relative to the event is normalized to zero. Specification includes county FE and state$\times$quarter FE. Standard errors are clustered at the county-level. Shaded area corresponds to 95% confidence interval.

Figure 3.7.11: Event study analysis: household annual income.



Notes: This plot depicts estimation of equation (3.3.2), where the outcome variable is natural logarithm of household annual income. Household income is derived from the categorical variable reported as a part of household demographics in the Kilts-Nielsen Consumer Panel dataset. On the x-axis is the calendar quarter relative to the first quarter when the fracking potential of the corresponding shale play became public: $\tau = t -$ *First post-fracking quarter*. The effect in quarter $\tau = -1$ relative to the event is normalized to zero. Specification includes household FE and state$\times$year FE. Standard errors are clustered at the county-level. Shaded area corresponds to 95% confidence interval.

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