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Dempster-Shafer Theory Applications in Structural Damage Assessment and Social Vulnerability Ranking

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DEMPSTER-SHAFER THEORY APPLICATIONS IN STRUCTURAL DAMAGE ASSESSMENT AND SOCIAL VULNERABILITY RANKING

By

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Abstract

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Dempster-Shafer Theory Applications in Structural Damage Assessment and Social Vulnerability Ranking

Thesis directed by Professor Ross B. Corotis

This thesis explores the different mathematical frameworks that have been used in risk assessment, with an emphasis on Dempster-Shafer Evidence Theory and the applicability in cases of post-seismic structural damage assessments and social vulnerability ranking. Evidence Theory allows the combination of multiple expert beliefs while considering uncertainties that are often inherent in such evaluations. In cases such as seismic hazards, for which structural vulnerability and structural damage are evaluated in a case-by-case scenario, subjective assessments are not only useful but necessary. The results of experimentation and survey distribution suggest that probability may not be the most natural framework in which to quantitatively incorporate the involved uncertainty. Ignorance and evidence-based assessments may be better represented using Evidence Theory.

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1. Introduction

Risk assessment is an integral component of modern engineering and hazard mitigation, but presents a mathematical obstacle due to the inherent uncertainties involved in such evaluations. The field of civil engineering often requires assessments that are inherently subjective in nature, as no structure or location is exactly the same. At any given project, a limited number of experts may be available to provide their risk evaluations using varying amounts of evidence and information. Similarly, the study of social vulnerability deals with great amounts of uncertainty and yet is often determined using rigid frameworks and objective census data. A variety of frameworks should be considered when handling such uncertainties. Probability often provides a reliable structure in such situations, where an educated guess can be made based on the outcome of previous similar occurrences and professional judgment. There are many circumstances, however, in which probability is not optimal. Such circumstances include ignorance (when there is limited amount of data), varying degrees and sources of confidence, and situations that are not repetitive enough to use previous data or frequentist probability.

The "subjectivists" offered another interpretation of probabilities as a "degree of belief;" the probability of an uncertain event as a measure of one's belief about its occurrence (Vick 2002). As stated in *Structural Reliability Analysis and Prediction,* "a subjective probability estimate reflects the degree of ignorance about the phenomenon under consideration" (Melchers 1999). The class of subjective probabilities, or Bayesian degrees of belief, allows for a broader context of probability theory, justified not necessarily by the objective or frequentist basis but to single occurrence events in the form of a measure of one's uncertainty about a particular event. As such, judgments manifested in the form of subjective probabilities can be manipulated with the axioms of probability theory. Although this offers a powerful framework for systematically incorporating uncertainty into almost any problem, subjective probabilities cannot distinguish between known equal outcomes and complete ignorance. Further, in cases with little or partial knowledge, judgments treated with a probabilistic model suggest there is precise information not only about the event itself, but also its contrary. These realizations inspired research into a broader conception of uncertainty, exploring important facets of uncertainty that are not probabilistic in nature. These other forms of characterizing uncertainty have received very limited attention in the area of structural risk and vulnerability, and it is now apparent that a complete paradigm shift in embodying uncertainty is needed for more robust and resilient theories of structural and community vulnerability (Corotis 2015).

There has been a significant amount of research that explores the relevance and applicability of other mathematical theories dedicated to the treatment of uncertainty, but many of these methods remain only partially developed and not investigated in terms of their applicability to engineering and social studies. The research presented in this thesis examines the characteristics of uncertainty beyond traditional probabilistic modeling, and is motivated by these primary objectives: (i) introducing appropriate roles for uncertainty theories beyond probability theory and their associated relevance, (ii) developing a deeper awareness and understanding of uncertainty's role within civil engineering, and (iii) creating a new comprehensive uncertainty model for future research in this field. The motivation for new approaches is not intended to challenge the fundamentals of probability theory, but to present different mathematical models, which may be relevant in a variety of contexts.

2. Literature Review

2.1 Uncertainties in Risk Assessment

In 1976, Glenn Shafer presented his work and the work of his mentor, Arthur Dempster, in "A Mathematical Theory of Evidence" (Beynon, Curry, and Morgan 2000). This work features a *theory of evidence* in which *belief functions* can be formalized from a degree of belief based on available evidence, termed *beliefs* and *plausibilities* (Yager and Liu 2008). As the works became known to the artificial intelligence community, the theory fell under the name of the Dempster-Shafer Theory (DST) of evidence, or commonly, *Evidence Theory* (Shafer 1976). Since Evidence Theory's origination, it has been evaluated as a potential alternative to classical, frequentist, and subjective probability. Classical and frequentist probabilities are the number of outcomes resulting in the specified event over the total number of outcomes and the number of times the specified event occurs if the situation were repeated, respectively, while subjective is entirely on the assigner's degree of belief (Aven et al. 2014). As Aven et al. (2014) state in *Uncertainty in Risk Assessment*, "in looking for a general framework for treating uncertainties in risk assessment, we started with the probabilistic treatment of uncertainties, recognizing its merits and limitations, and thus ventured beyond probability to describe uncertainties in a risk assessment context whose setting demands an extension of concepts and methods". There is a clear demand in the world of science and engineering for a method of risk assessment that addresses the inevitable uncertainties of the field (Cooke 1991).

Ang and Tang (2007) write in *Probability Concepts in Engineering* that "sources of uncertainty may be classified into two broad types: (1) those that are associated with natural randomness; and (2) those that are associated with inaccuracies in our prediction and estimation of reality. The former may be called the aleatory type, whereas the latter the epistemic type." (Ang and Tang 2007). Aleatory represents that the randomness of the circumstance – an uncertainty that cannot be eliminated but can be reduced with more information. Epistemic, on the other hand, is the uncertainty within the model itself. While the distinction between aleatory and epistemic uncertainty brings our understanding of sources of uncertainty into clearer focus, they still are subject to the axioms of probability theory. Aven et al (2014) introduce hybrid approaches for propagating uncertainty that combine probabilistic and possibilistic theories.

Recent theories that extend beyond probability include imprecise probabilities, probability-bound analysis, Possibility Theory, and Evidence Theory. Motivated by the emergence of various mathematical models for handling uncertainty and partial information of different types, a new area of study termed *Generalized Information Theory* (GIT) was formally introduced in the early 1990s (Ayyub 1998; Klir 2006; Ross 2010). This area of study is aimed at formally recognizing and systematically dealing with the nature and scope of uncertainty and its association with partial knowledge. In other words, GIT is concerned with the development of uncertainty theories.

2.2 Monotone Measures

Generalized Information Theory (GIT) expands probability theory in two dimensions by including non-additive probability measures and fuzzy sets (rather than classical set theory) (Klir 2006). This section focuses on the latter, specifically the generalization of the uncertainty associated with the assignment of an element. This area of study falls under the *theory of monotone measures* (Klir and Smith 2001; Wang and Klir 2009).

Monotone measures broaden the mathematical framework of probability theory. There are several classes of monotone measures that generalize the notion of uncertainty in the assignment of an element (x), to a particular set (A). Measures include possibility/necessity measures, Sugeno λ-measures, belief/plausibility measures, interval-valued probability distributions and imprecise probabilities (general lower and upper probabilities). Of these, possibility/necessity measures, belief/plausibility measures, and imprecise probabilities are among the most promising for the evaluation of uncertainty in a structural or community risk, reliability, vulnerability, and resilience context. In the context of classical probability, the assignment of an element x, to the set A is typically interpreted as a matter of likelihood or chance, or in the context of subjective probabilities, as a degree of certainty. Monotone measures generalize this interpretation, typically associating notions of incomplete information with ‘evidence’ pertaining to x.

From this perspective, likelihood in the context of monotone measures can be viewed as a specific form of evidence.

Mathematically, a monotone measure---denoted $g(\mathrm{A})$---is a mapping to the power set (a set of beliefs on any available event or event combination) on the unit interval. The value assigned to $g(\mathrm{A})$ is an expression of the degree of evidence that supports the belief that an element x belongs to a given crisp subset A (Ross 2010). The two axioms for monotone measures are:

$$
\begin{aligned}
&(g1)\ g(\varnothing) = 0,\ g(\mathrm{X}) = 1 \\
&(g2)\ g(\mathrm{A}) \leq g(\mathrm{B}),\ \text{for all A, B} \in \mathrm{P(X)}\ \text{if A} \subseteq \mathrm{B}
\end{aligned}
\tag{1}
$$

The first requirement ($g1$) establishes the boundary conditions for any monotone measure: where $g(\varnothing) = 0$ signifies no evidence or degree of support in the null set and $g(\mathrm{X}) = 1$ indicates complete evidence for the entire universe. The second requirement ($g2$), states that the evidence supporting B must be at least as great as the evidence assigned to A, when A is completely contained in B, the statement of *monotonicity*. This requires that all monotone measures satisfy the inequalities in Equation (2) for A, B, and $\mathrm{A} \cup \mathrm{B}$.

$$
\begin{aligned}
g(\mathrm{A} \cap \mathrm{B}) &\leq \min[g(\mathrm{A}), g(\mathrm{B})] \\
g(\mathrm{A} \cup \mathrm{B}) &\geq \max[g(\mathrm{A}), g(\mathrm{B})]
\end{aligned}
\tag{2}
$$

Probability theory satisfies the axioms of monotone measures, but in addition must satisfy the *additivity requirement*), which is a critical restriction for the use of expert opinions. As demonstrated in *Uncertainty and Information* (Klir 2006), additivity describes the circumstance in which probability measures can be obtained from subsets of X if bound within the disjoint set as shown below:

$$P(A \cup B) = P(A) + P(B) \tag{3}$$

2.3 Possibility/Necessity Measures

Possibility theory differs from probability theory in that it explicitly recognizes the case when evidence or judgments support the possibility of one event, but does not necessarily implicate evidence regarding the contrary event (Dubois 2006; Dubois and Prade 1988). In probability theory, uncertainty is represented by a single probability measure. If either the probability of an event or the probability of its negation (or complement) is known, the additivity requirement guarantees that the probabilities of both are known. In possibility theory, by comparison, to characterize fully the uncertainty of an event A, uncertainty is represented by dual measures, termed *possibility* and *necessity measures* as shown below (Ayyub and Klir 2006):

$$Pos_E(\{x\} = \begin{cases} 1 \; when \; x \; \in E \\ 0 \; when \; x \in \; \bar{E} \end{cases} \quad for \; all \; x \in X \tag{4}$$

$$Nec(E_i) = 1 - Pos(\overline{E_\iota}) \tag{5}$$

where all alternatives in set E are possible, and where $\overline{E_\iota}$ is the complement of E. As shown, the Possibility measure is 1 when x is within E, and 0 when x is within anything other than E. The Necessity measure is then calculated by subtracting the possibility measure for anything other than E from 1.

These measures are founded on the basic concepts of *possibility theory*. Possibility theory provides a mathematical framework to represent ignorance explicitly (Ross 2010). In this context, pairs of necessity and possibility measures are linked to the mathematical framework of Evidence Theory.

2.4 Belief/Plausibility Measures in Evidence Theory

Evidence Theory is based on a measure of degree of belief, called a *belief measure*, Bel(A), which expresses a degree of belief that the correct or true alternative belongs to the set A, from which a basic assignment or *Mobius Measure,* m(x), can be calculated. Mobius Measures are related to the previously discussed belief and plausibility measures, and provide "an assessment of the likelihood of each set in a family of sets identified by the analyst" (Ayyub and Klir 2006). In other words, Mobius Measures are the evidence that is compiled for each event. Belief and plausibility measures are calculated as follows (Aven et al. 2014):

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{6}$$

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B) \tag{7}$$

in which the belief in A is the sum of all Mobius measures relating to B in which B is fully contained within or equal to A. The plausibility measure is then the sum of all Mobius measures relating to B in which A and B have no commonality. The plausibility measure Pl(A) represents not only the evidence represented by the belief Bel(A), but also the evidence associated with any sets which overlap with A. Hence, at a minimum, the plausibility will be as strong as indicated by a belief. From these equations, it is clear that the relationship between plausibilities and belief measures are related through the following (Ayyub and Klir 2006):

$$Pl(\bar{A}) = 1 - Bel(A) \tag{8}$$

$$Pl(A) \geq Bel(A) \tag{9}$$

A degree of belief or evidential support of A, Bel(A), does not implicate disbelief of $\bar{A}$. For this reason, Evidence Theory differs from classical probability theory in that it provides a natural framework for modeling ignorance (Shafer 1976), which is the difference between one and the sum of the belief and the belief of the complement (Ross 2010):

$$\text{Ignorance} = 1 - [\text{Bel}(A) + \text{Bel}(\bar{A})] \tag{10}$$

2.5 Belief Combination using Evidence Theory

Another facet of Evidence Theory is the ability to combine information from multiple sources, which can be thought of as a joint message, or a *joint evidence assignment* of the two pieces of evidence (Shafer 1987). Combining beliefs using the Dempster-Shafer theory (or Dempster's Rule of Combination) can be done by using Eq.11 below:

$$m_{1,2}(A) = \frac{\sum_{B\cap C=A} m_1(B)\cdot m_2(C)}{1-c} \tag{11}$$

Where the denominator is calculated using Eq.12 below:

$$c = \sum_{B\cap C=\emptyset} m_1(B) \cdot m_2(C) \tag{12}$$

The numerator is determined by multiplying the belief in every event or event combination in which the only commonality is the event in question. Every combination is summed. The denominator is then determined by multiplying the belief in every event or event combination that has nothing in common, and summing the results. The results vary based on the evidence provided for the other events, as well as the amount of belief in a combination of events as opposed to single events (i.e., the belief in the occurrence of either A and/or B versus the belief in A or B singly). As stated in *Elicitation of Expert Opinions for Uncertainty and Risks* (Ayyub 2001), "Probability Theory can be treated as a special case of the Theory of Evidence. For cases in which all focal elements for a given basic assignment, *m* are singletons, the associated belief measure and plausibility measure collapse into a single measure, a classical probability measure".

The concepts of combining judgment from multiple experts in a mathematically-founded framework could be very powerful in combining engineering judgment with quantitatively- and qualitatively-based risk calculations. Field judgment in damage assessment and building vulnerability has great potential to take advantage of Evidence Theory combinations of belief and necessity, as is demonstrated with a damage assessment survey presented in chapter 4.

2.6 Conflict among Expert Beliefs

Conflicting belief has been a noted weak point of Evidence Theory (Xin, Xiao, and You 2005). Ayyub and Klir (2006) discuss Yager's Rule of Combination which introduces a *ground probability mass function* ($q_{1,2}$) to assign contradiction to the universal 'X'. The term *c*, the conflict variable, is removed from the denominator in Eq. 11 (making the denominator unity), and each term in Eq. 12 is added to the corresponding numerator term in Eq. 11 (where the conflict exists). This correctly renormalizes the sum of all terms in Eq. 11, and places the conflict within each term. This process is detailed below in equations 13-15.

$$q_{1,2}(A_i) = \sum_{all\ A_j \cap A_k = A_i} m_1(A_j) m_2(A_k) \tag{13}$$

$$m_{1,2}(A_i) = q_{1,2}(A_i) \quad for\ A_i \neq \emptyset\ and\ A_i \neq X \tag{14}$$

$$m_{1,2}(X) = q_{1,2}(X) + q_{1,2}(\emptyset) \tag{15}$$

Uncertainty Modeling and Analysis in Engineering and the Sciences also presents Inagaki's Rule of Combination (Ayyub and Klir 2006), which is primarily based on Dempster's rule of Combination but allows for Yager's rule in such cases when a combination parameter, *k,* is set equal to the values below (Ayyub and Klir 2006):

$$k = 0 \qquad \text{[use Yager's rule of Combination]}$$

$$k = \frac{1}{1-q(\emptyset)} \qquad \text{[use Dempster's rule of Combination]} \tag{16}$$

where q is the ground probability mass function defined in equation 13. This k value is then integrated into Yager's Rule of Combination by the following (Ayyub and Klir 2006):

$$m_{1,2}(A_i) = \left[1 + kq_{1,2}(\emptyset)\right]q_{1,2}(A_i) \quad for\ A_i \neq \emptyset \ \ and\ A_i \neq X \tag{17}$$

$$m_{1,2}(X) = \left[1 + kq_{1,2}(\emptyset)\right]q_{1,2}(X) + [1 + kq_{1,2}(\emptyset) - k]q_{1,2}(\emptyset) \tag{18}$$

$$m_{1,2}(\emptyset) = 0 \tag{19}$$

While these approaches have been shown to produce reasonable results, the differences are minor unless there is significant conflict among the experts, and it lacks the straightforward interpretation behind Eqs. 11 and 12. Therefore, it has not been adopted in the seismic damage data analysis in this thesis.

2.7 Imprecise Probabilities

Walley (1991) introduced the idea of imprecise probabilities as a generalization of probability theory which follows the principles of probability theory but does not require precise probability assignments (Ayyub and Klir 2006). As discussed previously, a main goal of this theory analysis is to determine how to best represent uncertainty in risk assessments. Providing upper and lower bound probabilities is one way to demonstrate some uncertainty of a model output in a quantifiable way. The lower bound probabilities are related to the previously discussed Mobius measures by the following (Ayyub and Klir 2006):

$$m(A) = \textstyle\sum_{all\ B\ such\ that\ B \subseteq A} (-1)^{|A-B|} \underline{P}(B) \tag{20}$$

$$\underline{P}(B) = \ \textstyle\sum_{all\ B\ such\ that\ B \subseteq A} m(A) \tag{21}$$

where $\underline{P}$ is lower bound probability. Upper bound probabilities can be calculated from lower bound probabilities with the following (Ayyub and Klir 2006):

$$\overline{P}(A) = 1 - \underline{P}(\overline{A}) \tag{22}$$

where $\overline{P}$ is upper bound probability of A and $\overline{A}$ is the complement of A. Probability bounds are used in line with Dempster Shafer Theory to analyze data in Chapter 4 of this thesis.

2.8 Social Vulnerability

As Dempster-Shafer Theory is evaluated in multiple capacities in this thesis, including as an analysis tool for social vulnerability, a short review of this topic is necessary as well. A community's vulnerability to a hazard is often thought of in physical terms; their infrastructure, environmental surroundings/global location, etc. Social vulnerability is the inter-personal counterpart – the vulnerability one might experience due to factors such as income disparity, class, gender, age, disability, health, living situation, income, or race/ethnicity (Thomas et al. 2013). One current method of determining a community's social vulnerability is called the Social Vulnerability Index (SVI), and is dependent on 15 census data variables: percent of people below poverty, unemployment, per capita income, possession of a high school degree, age (above 65 or below 17), disability, single parenthood, minority status, speaking English "well", vehicle access, and housing (multi-unit structure, mobile home, group quarters, over-crowding) (Flanagan et al. 2011). These variables are ranked among all census tracts in the United States and the top 10% most vulnerable receive a "flag" that indicates a vulnerability (Flanagan et al. 2011). These variables are grouped into four themes: socioeconomic status, household composition/disability, minority status/language, and housing/transportation (Flanagan et al. 2011). If the census tract is in the top 10% within any of these themes, that indicates another flag. Finally, all 15 variables are summed and if that value is in the top 10% when compared to all other tracts, another flag is recorded (Flanagan et al. 2011). Due to the fairly limited scope, this method might not reflect all the factors that influence one's true social vulnerability. More in-depth methods have been proposed, such as the one presented in *Social Vulnerability to Environmental Hazards* in which 42 independent variables were used to compile 11 factors: personal wealth, age, density of built environment, single-sector economic dependence, housing stock and tenancy, race – African American, race – Hispanic, race – Native American, race – Asian, occupation, and infrastructure dependence (Cutter, Boruff, and Shirley 2003). Another proposed method involves combining the SVI with hazard event frequency and economic loss data to determine what factors influence large dollar losses (Cutter, Boruff, and Shirley 2003). As Cutter et al. state, “there is no consensus within the social science community about social vulnerability or it correlates” (Cutter, Boruff, and Shirley 2003). Given the many uncertainties and nuances involved in social vulnerability, using a belief-based analysis tool like Dempster-Shafer Theory has the potential to provide a more comprehensive evaluation of social vulnerability. While the variables above may indicate that vulnerability is more or less likely to be present, the ability of a community to recover after a disaster is influenced by much more. For example, *Community Resilience as a Metaphor, Theory, Set of Capacities, and Strategy for Disaster Readiness* discusses the importance of community bonds such as those established by town-wide participation in common activities: church, school, self-help groups, or neighborhood watch committees (Pfefferbaum et al. 2007). These types of values cannot be quantified in the current indexing method of social vulnerability, but may be represented by a more subjective framework like Dempster-Shafer Theory.

2.9 Literature Review Conclusion

Aven et al. (2014) write that "Evidence Theory provides an alternative to the traditional manner in which probability theory is used to represent uncertainty by means of the specification of two degrees of likelihood, belief and plausibility, for each event under consideration." By using a model that accounts for the uncertainty in the incoming data, it is possible to achieve a more reliable output. While the work above reinforces the idea that uncertainty is, perhaps, not being given the appropriate consideration in risk assessment, the alternatives to frequentist probability need to now be evaluated in authentic scenarios. Thus, this work aims to analyze the applications of Evidence Theory in the field of civil engineering and hazard mitigation, specifically in post-seismic structural damage assessments and social vulnerability, to determine how such uncertainties can be acknowledged.

3. Program Testing

Since its origination, Dempster-Shafer theory has undergone a fairly small amount of exploration. Much of the available literature provides Equations 11 and 12 along with a short example using two or three power sets. As a main goal of this thesis was to determine how DST could practically be put to use, a comprehensive set of tests was performed to determine trends and behavior. These tests include combining:

- different sets of beliefs
- identical sets of beliefs
- power sets that having missing information
- power sets with extra confidence in combined events
- power sets that have varying amounts of ignorance
- power sets with strongly conflicting beliefs.

3.1 Combining Beliefs

Dempster-Shafter Evidence Theory can be used to combine the beliefs of multiple experts to achieve a combined Mobius measure, which can then be used to calculate a combined belief value. When experts provide a full power set of information, or their belief for any single event and any combination of events, then the belief in single events along with any extra belief they have in combinations of events is redistributed to the event with the most information. As an example, five different expert opinions are shown below in Table 1.

Table 1. Combined Power Sets

Event	**m1**	***bel1***	**m2**	***bel2***	**m3**	***bel3***	**m4**	***bel4***	**m5**	***bel5***	**m combined**	**bel combined**
A	0.05	0.05	0.15	0.15	0	0	0.3	0.3	0.1	0.1	0.40	0.40
B	0	0	0	0	0	0	0	0	0.2	0.2	0.17	0.17
C	0.05	0.05	0.05	0.05	0	0	0.1	0.1	0.3	0.3	0.38	0.38
AB	0.15	0.2	0.05	0.2	0.4	0.4	0.2	0.5	0	0.3	0.02	0.59
AC	0.1	0.2	0.2	0.4	0.1	0.1	0.2	0.6	0.2	0.6	0.01	0.79
BC	0.05	0.1	0.05	0.1	0.5	0.5	0.2	0.3	0	0.5	0.02	0.57
ABC	0.6	1	0.5	1	0	1	0	1	0.2	1	0.00	1.00

Note: m is Mobius measure, bel is belief

Events “AB”, “AC”, “BC”, and “ABC” signify an either/or relationship. All beliefs in “ABC”, then, are equal to 1 because that is the belief that either A, B, or C will happen. As these are the only options, one of them must occur. The Mobius measure and belief value are shown for each expert, denoted as “m1” and “bel1” for expert 1 and so on. Recall that the belief value is what is provided by the expert, and the m value, or Mobius Measure/evidence, is what is calculated from the belief. The combined values of m and belief can then be seen on the right side of the table. By looking at the initial beliefs for each event and then the combined belief, it is evident that the combined belief and Mobius measures are dependent on several factors. It is important to look at the starting individual event values, but also the amount of extra information provided with the belief in combined events. The added certainty that one may have in a combination of events without having to associate it with any individual event is filtered back to the single events. For example, examine event A. Each expert provides a belief value for the single event of A, but their beliefs for any combined event involving A (“AB” or “AC”) is almost always higher than simply the combination of those individual event beliefs. When these beliefs are combined, a higher combined belief and Mobius measure for the single event of A are produced. Through this process, experts are allowed to express uncertainty or ignorance on their belief of any single event without ignoring evidence they may have on combined events.

3.2 Combining Identical Beliefs

An interesting result of this calculation occurs when people with identical beliefs are combined. Rather than resulting in the combined expert belief equaling the identical individual beliefs, the combined belief is redistributed based on the strength of the original beliefs. The belief is distributed with priority on the events with the strongest starting belief and with the most amount of extra certainty from joint events. To illustrate this point, one set of beliefs was chosen and then duplicated to calculate the combined belief as if several

Table 2. Sample Power Set

Event	Mobius Measure	Belief
A	0.15	0.15
B	0.00	0.00
C	0.05	0.05
AB	0.05	0.20
AC	0.20	0.40
BC	0.05	0.10
ABC	0.50	1.00

experts had the exact same belief. The starting beliefs are shown in Table 2.

This test was performed with the use of a computer program written in Matlab that combines expert beliefs using Dempster-Shafter Evidence Theory based on the number of experts. It should be noted that the program written for this purpose allows the combination of up to five experts. However, the results are continuous in that combining two sets of two experts will yield the same result as combining 4 experts of the same beliefs (Note: combining one set of two experts with one set of three experts does not yield the same result as combining five experts, as this weights the beliefs differently). This was verified via preliminary testing on the program. Using this, up to 20 experts were combined to analyze the trends. There are gaps at 7, 11, 13, 14, 17, and 19 experts, as these numbers are not divisible by the available 1-5 expert combinations. The resulting Mobius measures and belief values of combining several experts with the identical belief shown above can be seen in Figure 1.

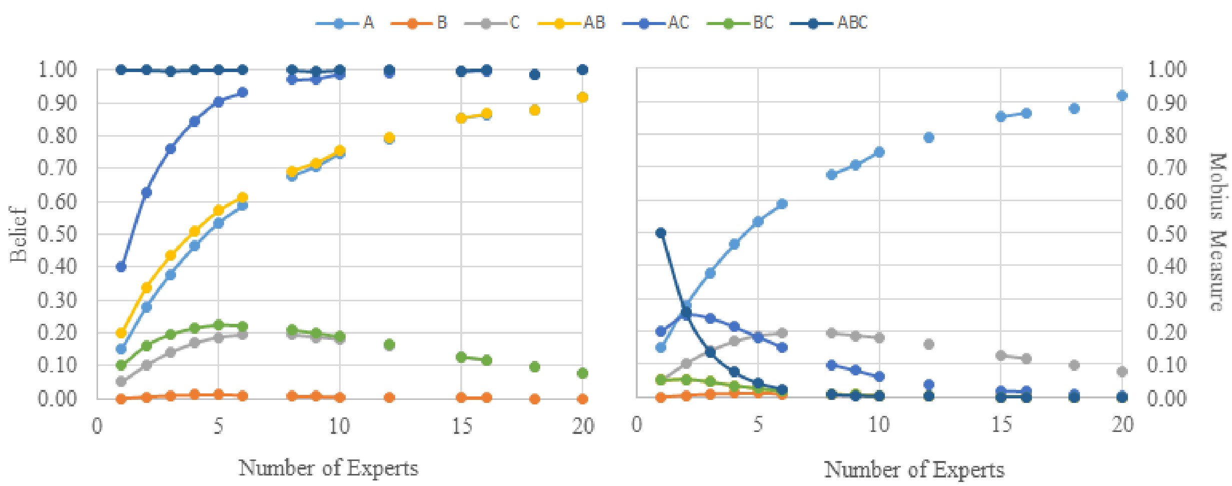

Figure 1. Belief and Mobius Measure of Combined Experts with Identical Beliefs

Although the original power set of beliefs provides extra confidence in every double event beyond the confidence of any single event, the A event acquires the most combined evidence (Mobius Measure) with every added expert. While A starts out with the highest single event belief in the power set in Table 2, it also gains the most from the extra belief associated with the combined events of AB and AC. Since every expert has more overall belief in event A than he or she does in B or C, the evidence for A will accumulate with each added expert, therefore the combined evidence value for A will continue to approach 1 and all other values will approach 0. The belief plot on the left is slightly different in that the belief in any event involving A will approach 1, while any event absent of A will approach 0. This is expected, as the more evidence there is for A, the higher the belief is that any event where A is an option will occur.

3.3 Dealing with "Missing Information"

The reason to consider how missing information is treated is that asking an expert for his or her belief in any combination of events might not be practical. For example, if an expert is assessing a structure and A is light damage, B is moderate damage, and C is extreme damage, asking for one's belief in light or extreme damage but not moderate damage does not make sense. This leaves the question of how to fill in the missing information of one's belief in A or C. Several ways of handling this missing information were considered. A sample data set of five different experts' beliefs was used, with the value of the belief in A or C being calculated differently each time. These beliefs were then combined to see how the different treatments of AC affected the trend of combined Mobius measures and beliefs.

The initial test was done with full power sets to provide base combined values for comparison. That is, the belief values of AC were provided by the experts. For this particular data set, every expert's belief in AC is more than just the sum of the beliefs in A and C, signifying that each expert has increased confidence in either one of those events occurring.

The first attempt at filling the missing AC information was simply setting the belief in A or C equal to 0. The idea behind this theory was that, as asking for one's belief in either A or C is not a logical question to ask, the expert's belief would be 0. The results yielded several negative values. It can be concluded that since the beliefs in A and C are not 0 individually, their combined belief cannot be less than the sum.

Based on the results of the previous test, the next attempt was to set the combined belief of A and C equal to the sum of the individual beliefs in A and C. Using the idea of common probability, the belief in A or C should simply be the sum of the beliefs in A and C. However, since Evidence Theory allows for extra belief in combined events rather than just the combination of single events, we need to consider that the combined belief in A and C might be more than just the combined belief in the individual events. If extra information is provided for AB and BC, but AC is simply the sum of the individual beliefs in A and C, the combined beliefs for A and C will be negatively affected even if that is not the true belief of the experts. Since the provided beliefs for AC in the original power set did have extra information, the combined Mobius measures and beliefs were strongly influenced by the lack of extra information in this treatment.

The next test was setting the combined belief of A and C equal to the belief in A, B, or C minus the belief of B alone (theoretically leaving behind the belief of A or C). Again, this follows the general rule of probability in that one's beliefs must add to 1. Therefore, the belief in 2 of 3 events should be 1 minus the belief of the third event. Since evidence theory allows ignorance on the part of the single events and does not require these beliefs to sum to 1 (but can be no larger than 1), the calculated values of the belief in AC turned out much higher than the expert-provided beliefs. This led to overly inflated values of combined Mobius measures and beliefs.

The final test was calculating a value of the belief in A or C based on the provided individual values of A and C while also taking into account the provided extra information in AB and BC. The belief value of AC was calculated by summing the individual A and C belief values, then adding half of the extra belief assigned to AB and BC, with the assumption that half of the extra belief for AB was for A, and half of the extra belief for BC was for C. This is shown below in Eq. 23.

$$Bel(A \cap C) = Bel(A) + Bel(C) + \frac{[Bel(A \cap B) - Bel(A) - Bel(B)] + [Bel(B \cap C) - Bel(B) - Bel(C)]}{2} \tag{23}$$

While the true amount of expert belief associated with the individual events is dependent on each unique assessment, this method produced combined Mobius measures and beliefs that were the closest to the values calculated with the full expert-provided power set. A potential problem arises when the individual A and C beliefs combined with the calculated added information are large enough that this calculation leads to an AC belief greater than 100. If this is the case, the calculated belief in AC should logically be capped at 100.

3.4 The Effects of Extra Confidence

As stated, Evidence Theory allows experts to acknowledge that they have ignorance about individual events, but be more confident in combined events. The effects of this extra confidence was tested to determine how having low individual beliefs but significant extra beliefs in combined events might weigh against having high individual beliefs with no extra confidence in combined events. To test this, three different power sets of information were evaluated by repeatedly combining them to analyze trends.

The first power set gives event A a starting belief of 40%, and B and C comparatively low beliefs of 10%. In this first set, the belief in the double events is the sum of the single events; there is no added confidence. As expected, the combined belief in A, AB, and AC continued to grow while the belief in events B, C, and BC trended towards 0. The results of this test can be seen below in Table 3 and Figure 2.

Table 3. No Extra Confidence

# of Experts	***1***	***2***	***3***	***4***	***5***	***6***	***8***	***9***	***10***
	Combined Beliefs								
A	0.4	0.585	0.707	0.794	0.859	0.905	0.959	0.974	0.983
B	0.1	0.110	0.096	0.076	0.057	0.040	0.019	0.012	0.008
C	0.1	0.110	0.096	0.076	0.057	0.040	0.019	0.012	0.008
AB	0.5	0.695	0.803	0.871	0.916	0.945	0.978	0.986	0.991
AC	0.5	0.695	0.803	0.871	0.916	0.945	0.978	0.986	0.991
BC	0.2	0.220	0.192	0.153	0.114	0.081	0.037	0.025	0.016
ABC	1	1	1	1	1	1	1	1	1
	Combined Mobius Measures								
A	0.4	0.585	0.707	0.794	0.859	0.905	0.959	0.974	0.983
B	0.1	0.110	0.096	0.076	0.057	0.040	0.019	0.012	0.008
C	0.1	0.110	0.096	0.076	0.057	0.040	0.019	0.012	0.008
AB	0	0	0	0	0	0	0	0	0
AC	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0
ABC	0.4	0.195	0.101	0.053	0.028	0.014	0.004	0.002	0.001

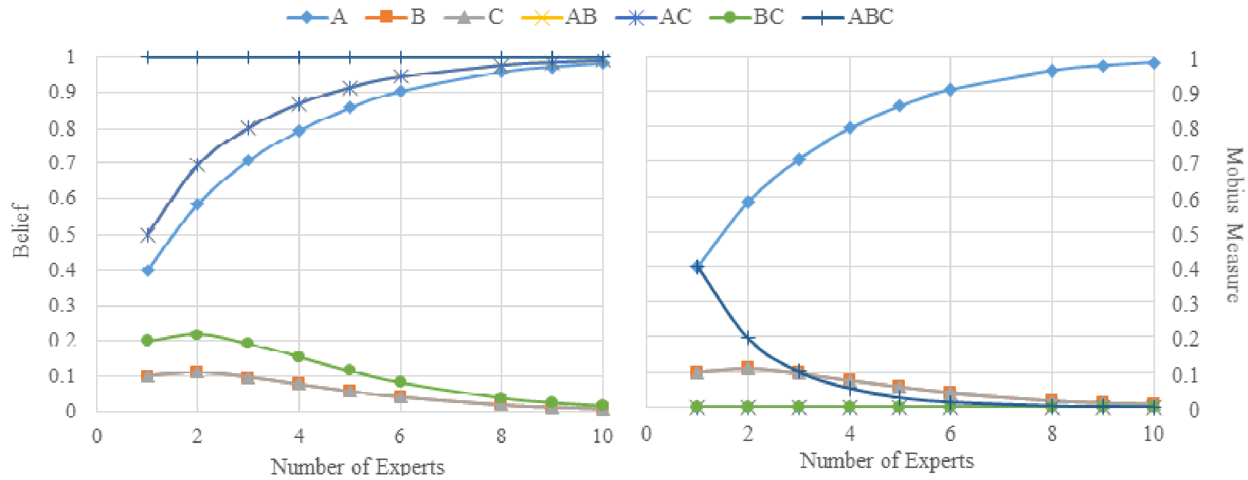

Figure 2. No Extra Confidence in Combined Events

The second power set had the same individual event beliefs, but the extra confidence in the combined event of BC was increased by 10%. The results are similar to the original test, suggesting that 10% in added belief in BC was not significant enough to diminish the higher starting belief in event A. The results of this test can be seen below in Table 4 and Figure 3.

Table 4. Extra Confidence of 10%

# of Experts	*1*	*2*	*3*	*4*	*5*	*6*	*8*	*9*	*10*
	Combined Beliefs								
A	0.4	0.541	0.630	0.700	0.760	0.812	0.890	0.917	0.939
B	0.1	0.122	0.122	0.111	0.096	0.080	0.050	0.038	0.029
C	0.1	0.122	0.122	0.111	0.096	0.080	0.050	0.038	0.029
AB	0.5	0.662	0.751	0.811	0.857	0.892	0.940	0.956	0.968
AC	0.5	0.662	0.751	0.811	0.857	0.892	0.940	0.956	0.968
BC	0.3	0.338	0.317	0.276	0.229	0.183	0.110	0.082	0.061
ABC	1	1	1	1	1	1	1	1	1
	Combined Mobius Measures								
A	0.4	0.541	0.630	0.700	0.760	0.812	0.890	0.917	0.939
B	0.1	0.122	0.122	0.111	0.096	0.080	0.050	0.038	0.029
C	0.1	0.122	0.122	0.111	0.096	0.080	0.050	0.038	0.029
AB	0	0	0	0	0	0	0	0	0
AC	0	0	0	0	0	0	0	0	0
BC	0.1	0.095	0.074	0.053	0.036	0.023	0.009	0.006	0.003
ABC	0.3	0.122	0.054	0.024	0.011	0.005	0.001	0.000	0.000

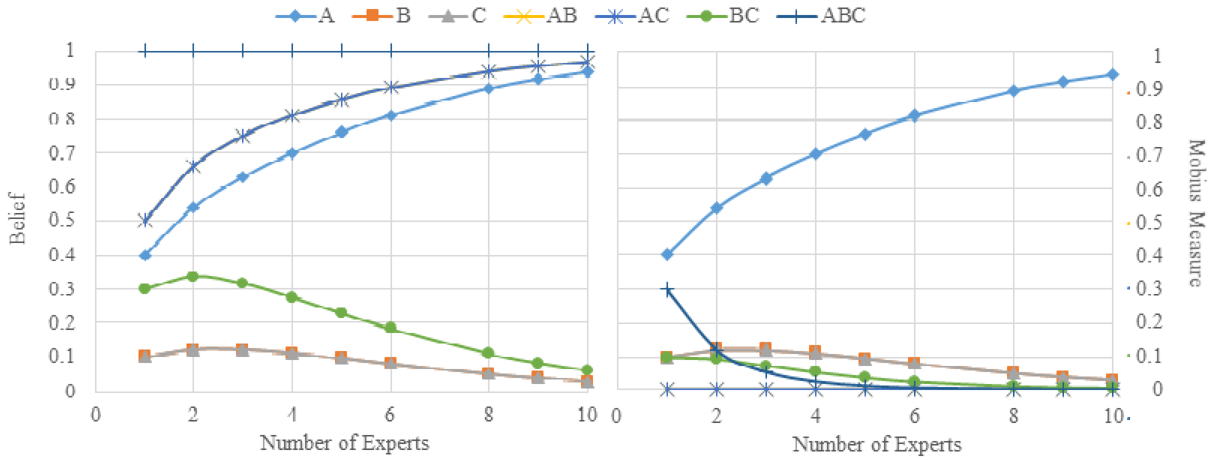

Figure 3. 10% Extra Confidence in Combined Events

The third power set added another increment of 10% belief to BC, making the belief in event A and the belief in BC are equal. However, the combined belief in A still trended toward 1, while the belief in B and C briefly increased and then trended back down towards 0. This is still expected, as the beginning beliefs in AB and AC are still greater than the belief in BC. The results of this test can be seen below in Table 5 and Figure 4.

Table 5. Extra Confidence of 20%

# of Experts	*1*	*2*	*3*	*4*	*5*	*6*	*8*	*9*	*10*
	Combined Beliefs								
A	0.4	0.485	0.528	0.563	0.597	0.632	0.701	0.734	0.766
B	0.1	0.136	0.155	0.162	0.162	0.156	0.136	0.123	0.110
C	0.1	0.136	0.155	0.162	0.162	0.156	0.136	0.123	0.110
AB	0.5	0.621	0.683	0.725	0.759	0.788	0.837	0.858	0.876
AC	0.5	0.621	0.683	0.725	0.759	0.788	0.837	0.858	0.876
BC	0.4	0.455	0.452	0.430	0.401	0.367	0.299	0.266	0.234
ABC	1	1	1	1	1	1	1	1	1
	Combined Mobius Measures								
A	0.4	0.485	0.528	0.563	0.597	0.632	0.701	0.734	0.766
B	0.1	0.136	0.155	0.162	0.162	0.156	0.136	0.123	0.110
C	0.1	0.136	0.155	0.162	0.162	0.156	0.136	0.123	0.110
AB	0	0	0	0	0	0	0	0	0
AC	0	0	0	0	0	0	0	0	0
BC	0.2	0.182	0.142	0.106	0.077	0.055	0.027	0.019	0.013
ABC	0.2	0.061	0.020	0.007	0.003	0.001	0.000	0	0

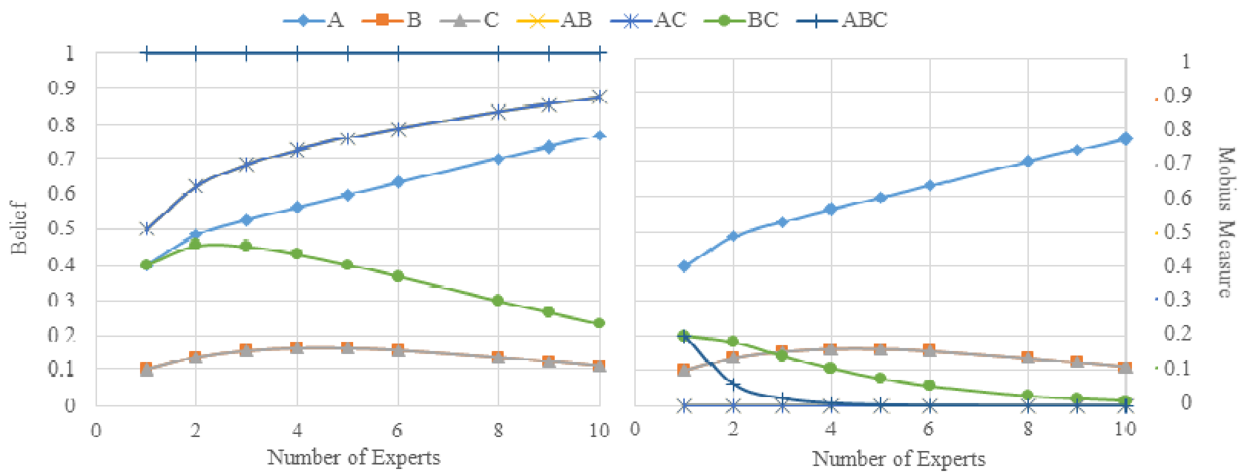

Figure 4. 20% Extra Confidence in Combined Events

The fourth power set increased the belief in BC by another 10% to a total of 50%. While the belief in B and C remain at 10%, the belief in BC now matched the beliefs in AB and AC. The trends were not clear after combining 10 experts with identical beliefs, so the test was extended through 20 experts. The beliefs in A, B, and C all trend towards 33%, with the belief in double events trending towards 67%. The results of this extra belief are significant because even though A had a significantly higher starting belief than B and C, and the beliefs in AB, AC, and BC were all identical, B and C trended upwards while A trended down. This proved that this extra belief in BC is the turning point in extra confidence overtaking individual starting belief. However, these results are interesting because the added belief in BC does not necessarily give enough belief to B and C individually to have this effect. Since the belief in AB and AC are solely the sum of the individual beliefs in A and B, and A and C respectively, it makes sense to split the extra belief in B and C and add it back to the starting beliefs of B and C. Following this theory, the starting beliefs of B and C would each gain 15%, putting them both at 25% and still less than the starting belief of A. This suggests that extra confidence in combined events is handled differently, and potentially more seriously, than starting beliefs in individual events. These results can be seen below in Table 6 and Figure 5.

Table 6. Extra Confidence of 30%

# of Experts	***1***	***2***	***3***	***4***	***5***	***6***	***8***	***9***	***10***	***12***	***15***	***16***	***18***	***20***
	Combined Beliefs													
A	0.4	0.41	0.40	0.39	0.37	0.37	0.35	0.35	0.35	0.34	0.34	0.34	0.34	0.33
B	0.1	0.16	0.20	0.23	0.25	0.27	0.29	0.30	0.31	0.32	0.33	0.33	0.33	0.33
C	0.1	0.16	0.20	0.23	0.25	0.27	0.29	0.30	0.31	0.32	0.33	0.33	0.33	0.33
AB	0.5	0.57	0.60	0.61	0.63	0.63	0.65	0.65	0.65	0.66	0.66	0.66	0.66	0.67
AC	0.5	0.57	0.60	0.61	0.63	0.63	0.65	0.65	0.65	0.66	0.66	0.66	0.66	0.67
BC	0.5	0.57	0.60	0.61	0.63	0.63	0.65	0.65	0.65	0.66	0.66	0.66	0.66	0.67
ABC	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Combined Mobius Measures													
A	0.4	0.41	0.40	0.39	0.37	0.37	0.35	0.35	0.35	0.34	0.34	0.34	0.34	0.33
B	0.1	0.16	0.20	0.23	0.25	0.27	0.29	0.30	0.31	0.32	0.33	0.33	0.33	0.33
C	0.1	0.16	0.20	0.23	0.25	0.27	0.29	0.30	0.31	0.32	0.33	0.33	0.33	0.33
AB	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AC	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BC	0.3	0.26	0.20	0.16	0.12	0.10	0.06	0.05	0.04	0.02	0.01	0.01	0.01	0
ABC	0.1	0.02	0	0	0	0	0	0	0	0	0	0	0	0

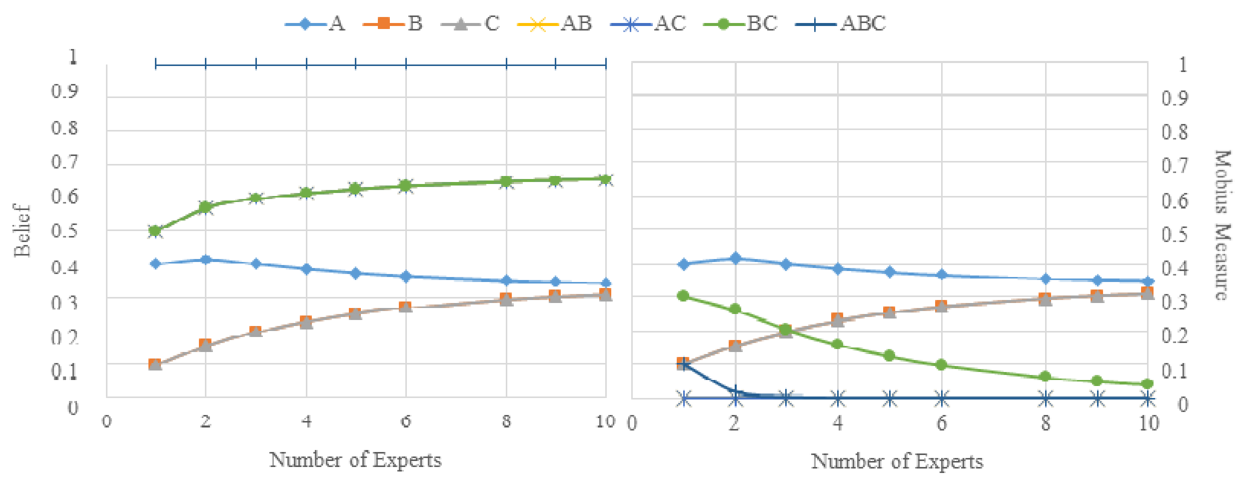

Figure 5. 30% Extra Confidence in Combined Events

The final power set increased the starting belief in BC to 60%. Again, the individual belief in B and C remain low, but the belief in BC is now higher than AB or AC. The combined belief in A trended quickly towards 0, while the beliefs in B, C, and BC trended significantly upwards. The belief in AB and AC stayed at 50%. Based on the previous test results, it was expected that B and C would outweigh the belief in A with this extra belief in BC. The results of this test can be seen below in Table 7 and Figure 6.

Table 7. Extra Confidence of 40%

# of Experts	*1*	*2*	*3*	*4*	*5*	*6*	*8*	*9*	*10*
	Combined Beliefs								
A	0.4	0.320	0.256	0.205	0.164	0.131	0.084	0.067	0.054
B	0.1	0.180	0.244	0.295	0.336	0.369	0.416	0.433	0.446
C	0.1	0.180	0.244	0.295	0.336	0.369	0.416	0.433	0.446
AB	0.5	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
AC	0.5	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
BC	0.6	0.680	0.744	0.795	0.836	0.869	0.916	0.933	0.946
ABC	1	1	1	1	1	1	1	1	1
	Combined Mobius Measures								
A	0.4	0.320	0.250	0.205	0.164	0.131	0.084	0.067	0.054
B	0.1	0.180	0.244	0.295	0.336	0.369	0.416	0.433	0.446
C	0.1	0.180	0.244	0.295	0.336	0.369	0.416	0.433	0.446
AB	0	0	0	0	0	0	0	0	0
AC	0	0	0	0	0	0	0	0	0
BC	0.4	0.320	0.256	0.205	0.164	0.131	0.084	0.067	0.054
ABC	0	0	0	0	0	0	0	0	0

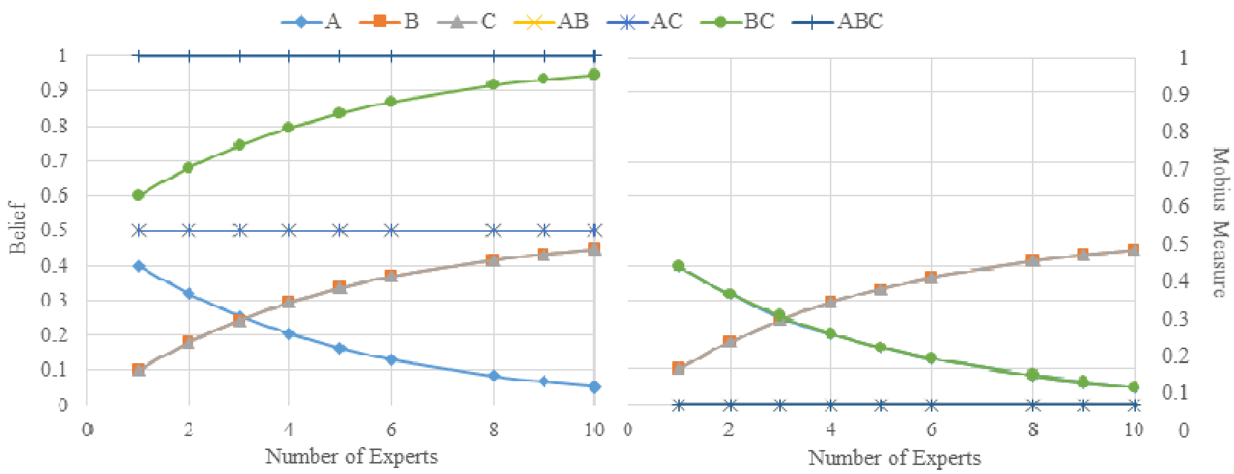

Figure 6. 40% Extra Confidence in Combined Events

The purpose of this test was to examine how Evidence Theory handles the extra confidence that one may have in the belief of either one of two options, rather than choosing between the two. The results are interesting in that the extra confidence in combined events appears to be more heavily weighted than the belief in single events. On one hand, this seems counterintuitive since the expert was not confident enough to assign any of the extra belief to the individual events, only to the chance of their either/or occurrence. On the other hand, this makes sense in that the expert is confident enough that one of the two will occur, but might be equally split between the individual events and is uncomfortable choosing to assign more belief to either of the two.

3.5 Varying Amounts of Ignorance

A key difference between Evidence Theory and Probability is that Evidence Theory does not require one to assign all of their belief to any individual or combination of events. Ignorance is allowed and, accordingly, the sum of one's beliefs is often less than unity. The impact of the amount of ignorance one can have was tested to determine how combining experts with relatively low beliefs might differ from combining experts with higher, though proportional, beliefs. For the purposes of identifying trends, one set of beliefs was repeatedly combined until the combined expert belief no longer had any ignorance, or all 100% of the belief was accounted for between the 3 single events.

The first test uses a set of beliefs that has a large amount of ignorance. There was only 10% belief for the single events and 20% for the double events (the sum of their respective single events with no extra confidence). This leaves 70% of one's belief unassigned. As stated previously, the belief in A, B, or C must always be 100% as there are no other events that may occur. This set of beliefs was combined 20 times to reach a point when the belief of the individual events did sum to 100% (with a belief of ~33% for each individual event) and there was no longer any ignorance, as probability would require. The combined beliefs can be seen below in Table 8.

Table 8. Ignorance of 70%

# of Experts	*1*	*2*	*3*	*4*	*5*	*6*	*8*	*9*	*10*	*12*	*15*	*16*	*18*	*20*
	Combined Beliefs													
A	0.1	0.16	0.20	0.23	0.25	0.26	0.28	0.29	0.30	0.31	0.32	0.32	0.32	0.33
B	0.1	0.16	0.20	0.23	0.25	0.26	0.28	0.29	0.30	0.31	0.32	0.32	0.32	0.33
C	0.1	0.16	0.20	0.23	0.25	0.26	0.28	0.29	0.30	0.31	0.32	0.32	0.32	0.33
AB	0.2	0.32	0.40	0.45	0.49	0.52	0.57	0.58	0.60	0.61	0.63	0.63	0.65	0.65
AC	0.2	0.32	0.40	0.45	0.49	0.52	0.57	0.58	0.60	0.61	0.63	0.63	0.65	0.65
BC	0.2	0.32	0.40	0.45	0.49	0.52	0.57	0.58	0.60	0.61	0.63	0.63	0.65	0.65
ABC	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Combined Mobius Measures													
A	0.1	0.16	0.20	0.23	0.25	0.26	0.28	0.29	0.30	0.31	0.32	0.32	0.32	0.33
B	0.1	0.16	0.20	0.23	0.25	0.26	0.28	0.29	0.30	0.31	0.32	0.32	0.32	0.33
C	0.1	0.16	0.20	0.23	0.25	0.26	0.28	0.29	0.30	0.31	0.32	0.32	0.32	0.33
AB	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AC	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ABC	0.7	0.52	0.40	0.32	0.26	0.21	0.15	0.13	0.11	0.08	0.05	0.05	0.03	0.02

The next tested power set has proportional starting values, but with less ignorance. Each individual event was assigned a belief of 20% with the double events again being the sum of their respective individual events (40%), leaving 40% of the belief unassigned. This set of beliefs was combined 8 times to reach a combined belief with no ignorance. This can be seen below in Table 9.

Table 9. Ignorance of 40%

# of Experts	*1*	*2*	*3*	*4*	*5*	*6*	*8*
Combined Beliefs							
A	0.2	0.263	0.292	0.308	0.317	0.323	0.329
B	0.2	0.263	0.292	0.308	0.317	0.323	0.329
C	0.2	0.263	0.292	0.308	0.317	0.323	0.329
AB	0.4	0.526	0.585	0.616	0.635	0.646	0.658
AC	0.4	0.526	0.585	0.616	0.635	0.646	0.658
BC	0.4	0.526	0.585	0.616	0.635	0.646	0.658
ABC	1	1	1	1	1	1	1
	Combined Mobius Measures						
A	0.2	0.263	0.292	0.308	0.317	0.323	0.329
B	0.2	0.263	0.292	0.308	0.317	0.323	0.329
C	0.2	0.263	0.292	0.308	0.317	0.323	0.329
AB	0	0	0	0	0	0	0
AC	0	0	0	0	0	0	0
BC	0	0	0	0	0	0	0
ABC	0.4	0.211	0.123	0.076	0.048	0.031	0.013

The final test had starting beliefs with almost no ignorance. Each starting individual event belief was 30%, and each double event was assigned a belief of 60%. Only 10% of the belief was left unassigned. This relatively high power set only had to be combined 2 times to reach a combined belief with no remaining ignorance. These test results can be seen below in Table 10.

Table 10. Ignorance of 10%

# of Experts	*1*	*2*
	Combined Beliefs	
A	0.3	0.326
B	0.3	0.326
C	0.3	0.326
AB	0.6	0.652
AC	0.6	0.652
BC	0.6	0.652
ABC	1	1
	Combined Mobius Measures	
A	0.3	0.326
B	0.3	0.326
C	0.3	0.326
AB	0	0
AC	0	0
BC	0	0
ABC	0.1	0.022

Understanding how this theory handles ignorance is important due to the fact that allowing ignorance is a key component of Evidence Theory that makes it a contending alternative to probability. The tests above suggest that the ignorance provided by experts is retained in their combined belief until enough evidence is provided to allow otherwise. When there is substantial ignorance in the starting beliefs, many experts are required to contribute their belief to reach a combined belief that no longer has ignorance. As shown in Figures 7 and 8 below, the higher the starting beliefs, the lower the number of experts are required to reach a full combined belief power set with no ignorance.

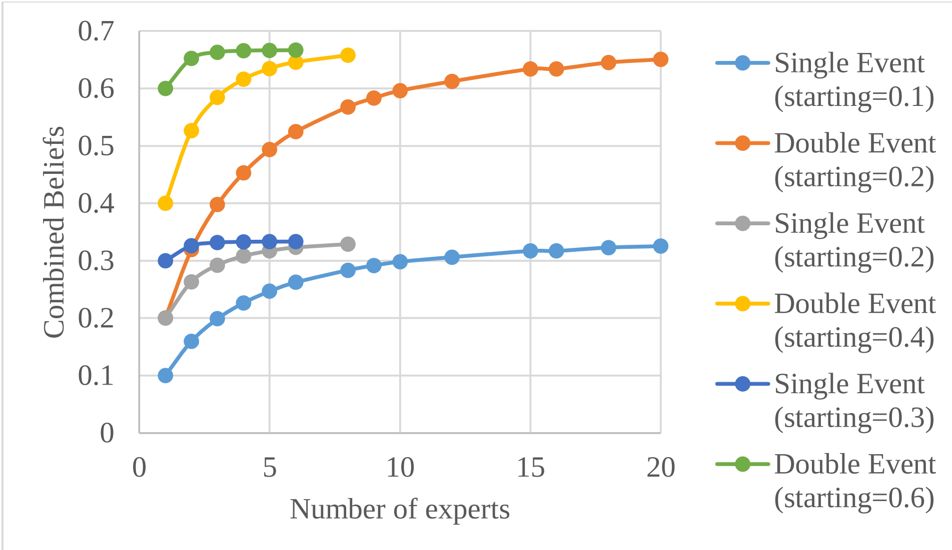

Figure 7. Number of Experts Required to Reach Stability

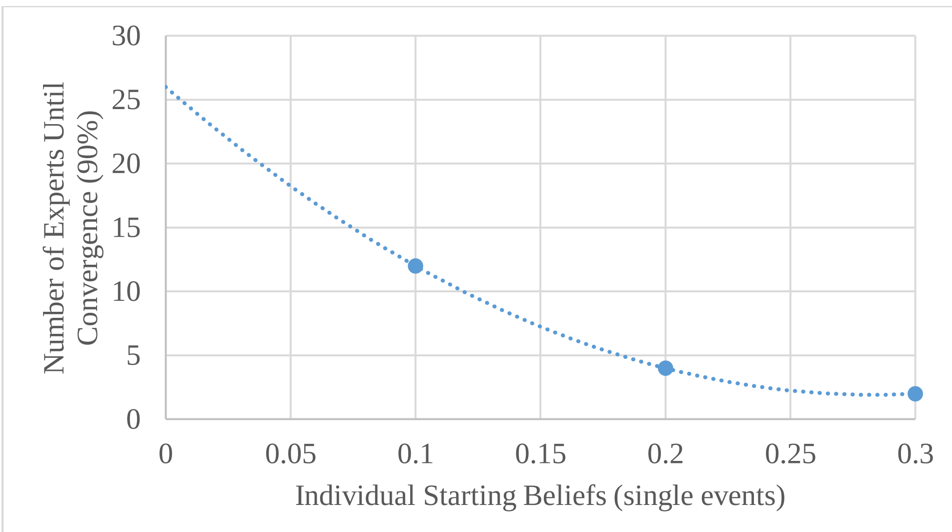

Figure 8. Effects of Varying Starting Beliefs

3.6 Conflicting Expert Opinion

As discussed in Chapter 2.6: Conflict among Expert Beliefs, Dempster Shafer Theory might be inadequate when strongly conflicting beliefs are present. To determine how the equations outlined in this paper handle conflicting opinion, a series of basic tests were performed.

The first test assigned absolute belief in event A to one expert and absolute belief in event B to a second. The belief in combined events is the sum of the individuals. In such cases of 100% conflicting belief, this theory is not able to compute a joint belief as there is no commonality between the two experts, from which the numerator in Eq. 11 originates.

The second test assigns near absolute belief to the same events as the first, but with 1% ignorance. In this case, Evidence Theory essentially takes the average of the provided beliefs as there is nearly no commonality between the experts, but both admit some small ignorance. Since probability would handle these beliefs in a similar method, this seems like a natural result. The results are outlined in Table 11 below.

Table 11. 1% Ignorance in Combined Beliefs

Event	Expert 1 Belief	Expert 2 Belief	Combined Belief
A	99.0%	0.0%	49.8%
B	0.0%	99.0%	49.8%
C	0.0%	0.0%	0.0%
AB	99.0%	99.0%	99.5%
AC	99.0%	0.0%	49.8%
BC	0.0%	99.0%	49.8%
ABC	100.0%	100.0%	100.0%

A third test was carried out by assigning three experts 50% belief in different events. Rather than producing an average combined belief, the result was a 25% joint belief in each individual event (larger than the 16.67% that an average would yield) and a 50% belief in each combined event, as seen below in Table 12. This result reflects a notable difference in how probability and Evidence Theory deal with multiple inputs and significant ignorance.

Table 12. 50% Ignorance in Combined Beliefs

Event	Expert 1 Belief	Expert 2 Belief	Expert 3 Belief	Combined Belief
A	50%	0%	0%	25%
B	0%	50%	0%	25%
C	0%	0%	50%	25%
AB	50%	50%	0%	50%
AC	50%	0%	50%	50%
BC	0%	50%	50%	50%
ABC	100%	100%	100%	100%

In order to analyze how larger quantities of conflicting beliefs are handled, one expert (Expert 4) was assigned absolute belief in event C, which was combined with multiple experts that had strong belief in A. This test is of significant interest because even though it seems there is more belief in event A, the fourth expert's absolute belief in C allows no ignorance or commonality, and therefore trumps the less-than-absolute that the other three experts had in event A. Again, the idea of ignorance plays a significant role in Evidence Theory that probability would ignore. The results of this test can be seen below in Table 13.

Table 13. Ignorance Effect on Combined Belief

Event	Expert 1 Belief	Expert 2 Belief	Expert 3 Belief	Expert 4 Belief	Combined Belief
A	90%	90%	90%	0%	0%
B	0%	0%	0%	0%	0%
C	0%	0%	0%	100%	100%
AB	90%	90%	90%	0%	0%
AC	90%	90%	90%	100%	100%
BC	0%	0%	0%	100%	100%
ABC	100%	100%	100%	100%	100%

A fourth test was performed to determine at what level of confidence expert 4's belief in event C gives way to the other three experts' belief in event A. Keeping the first three experts' belief in event A at 90%, the confidence that expert 4 has in C was varied from 90% to 100%. As shown in Figure 9 below, when expert 4's confidence drops below around 99.5%, the combined belief shifts towards event A. Any belief above 99.5% does not leave enough ignorance to allow for the first three expert's beliefs to influence the combined belief.

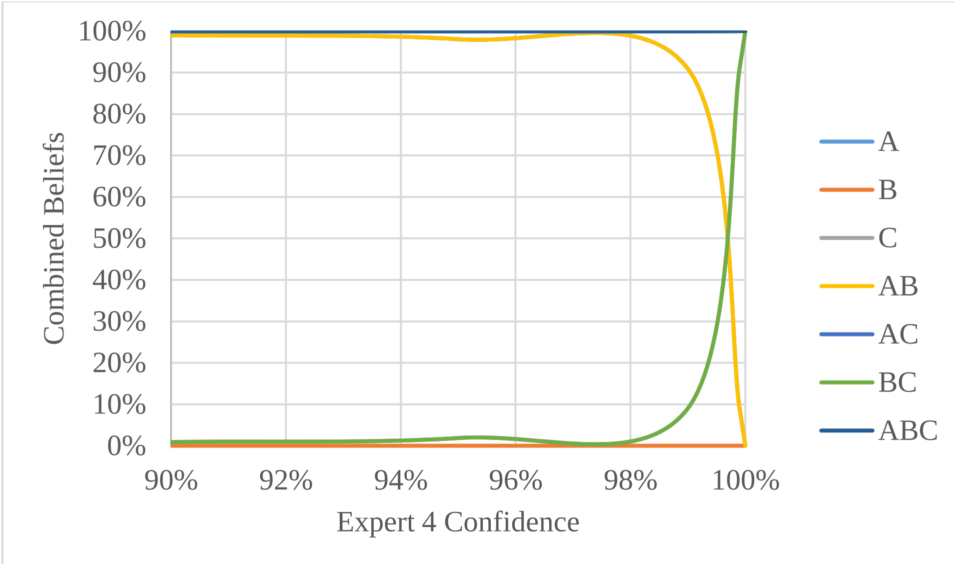

Figure 9. Effects of Varying Amounts of Confidence

This last test demonstrates how different belief measures are from probability. Belief measures represent one's confidence in a certain outcome. When this confidence is near 100%, this is taken almost as fact of what will happen, rather than just an estimate. The remaining ignorance is so small that other beliefs less than 100% are considered negligible or not likely enough to occur.

3.7 Summary of Program Testing Results

The series of tests performed on the MatLab programs helped determine how Evidence Theory behaves in specified situations and in which scenarios it might be applicable. Several key outcomes were determined throughout this testing, many of which highlighted the contrast between this theory and basic probability. Combining expert beliefs using Evidence Theory yields a significantly different result than simple averaging; as more and more beliefs are contributed, one event will eventually reach a joint belief of 100%, while all other single events will have 0%. Several other aspects influence the joint belief, including any amount of ignorance the experts may have (a total belief less than 100%), how much extra belief he or she has in the joint events versus the single events, and how conflicting the contributing beliefs are. The ignorance allowed in beliefs less than 100% act as a sort of weighting measure – experts with more ignorance do not influence the joint belief as strongly as experts who assign 100% of their belief. The confidence level of the contributing experts influences how many experts are required to reach total belief in a definitive answer. For example, a small number of very confident experts with similar beliefs might have a joint belief of 100% in one event, while a larger group of less certain experts may yield a more ambivalent result. When contributing beliefs are strongly conflicting, the amount of ignorance present plays a key role. One expert that very strongly believes in event A (little to no ignorance) combined with another expert who has a moderately high belief in event B (slightly more ignorance), will yield a joint belief that strongly backs event A, even though the contributing beliefs in A and B are both high. Another key outcome of conflicting belief testing is that when there is no possible overlap, for example if one expert has 100% belief in A and one has 100% belief in B, Evidence Theory is not capable of calculating a joint belief.

Overall, these results were able to provide a general foundation for the behavior and trends of Dempster Shafer Theory under several conditions. This basis allows for further real-world testing in practical risk situations, such as the survey instrument in the next chapter.

4. Survey

A structural damage assessment survey was constructed to test one of the real-life applications of this program. The survey includes 5 different aerial images of Port-au-Prince, Haiti, taken shortly after the 2010 earthquake there. A damage scale is provided giving examples of images that have damage ranges of 0 - 20%, 20 - 40%, 40 - 60%, 60 - 80%, and 80 - 100%. The participants are asked to evaluate each image and assign their belief that the image has a damage in the ranges of 0 - 33%, 34 - 66%, and 67 - 100%. They are also asked to assign their belief that the damage is within the ranges of 0-66% and 34 – 100%, with the provided explanation that they may have more confidence in the larger ranges than simply the sum of the smaller ranges. The participants were not asked for their belief in [0 - 33% + 67 - 100%] (the combination of the two outer ranges), as this is not a commonsense question in terms of damage assessment. Since the belief in this combined event is necessary to calculate the total combined belief, the missing value was calculated using the provided beliefs. The individual beliefs in 0 - 33% and 67 - 100% were summed along with half of the extra belief in 0 - 66% and 34 - 100%, as this method proved to be the most apt at approximating the missing information that the damage might be 0 - 33% + 67 - 100% (see Chapter 3.3). The full survey is provided in Appendix A.

4.1 Survey Delivery

The survey was delivered by hand to selected structural-focused engineering classes at both the undergraduate and graduate level. Professors with a similar focus were also offered the survey either in person or via email. The survey results were recorded and each participants’ beliefs were combined to achieve combined damage beliefs in each of the five images.

4.2 Survey Results

A total of 46 surveys were filled out and returned. If any questions were filled out not in accordance to the specified rules, namely if the provided belief exceeded 100%, the individual question was not considered in the results. Each of the five survey questions produced at least 40 correctly filled out beliefs.

4.2.1 Evaluating Survey Results using Evidence Theory

The results for each question were calculated in two different ways. The first combines five groups of eight experts each, with the idea that many real life damage assessments will not have 40 available experts to provide their beliefs. The second combines all 40 available surveys to analyze the result of large quantities of opinions. The results can be seen below in Table 14.

Table 14. Combined Belief Results Using Evidence Theory

	Damage Range (%)	*Group 1*	*Group 2*	*Group 3*	*Group 4*	*Group 5*	*All 40*
Question 1	0 - 33	98.5%	97.4%	54.7%	98.2%	100.0%	100.0%
	34 - 66	0.4%	0.3%	12.9%	0.5%	0.0%	0.0%
	67 - 100	0.7%	1.9%	29.7%	0.7%	0.0%	0.0%
	0 - 66	99.0%	97.7%	67.6%	98.7%	100.0%	100.0%
	0-33 + 67-100	99.6%	99.7%	87.1%	99.5%	100.0%	100.0%
	34-100	1.1%	2.1%	42.6%	1.2%	0.0%	0.0%
	0 - 100	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Question 2	0 - 33	100.0%	79.2%	58.4%	99.2%	100.0%	100.0%
	34 - 66	0.0%	2.4%	4.7%	0.2%	0.0%	0.0%
	67 - 100	0.0%	17.1%	35.6%	0.6%	0.0%	0.0%
	0 - 66	100.0%	81.5%	63.1%	99.3%	100.0%	100.0%
	0-33 + 67-100	100.0%	97.6%	95.2%	99.8%	100.0%	100.0%
	34-100	0.0%	19.5%	40.3%	0.7%	0.0%	0.0%
	0 - 100	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Question 3	0 - 33	1.0%	2.4%	0.0%	2.8%	5.2%	0.0%
	34 - 66	0.5%	0.2%	1.0%	1.6%	1.0%	0.0%
	67 - 100	98.0%	97.1%	97.9%	94.9%	92.1%	100.0%
	0 - 66	1.5%	2.5%	1.1%	4.4%	6.2%	0.0%
	0-33 + 67-100	99.6%	99.8%	99.0%	98.4%	99.0%	100.0%
	34-100	98.5%	97.3%	99.9%	96.5%	93.0%	100.0%
	0 - 100	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Question 4	0 - 33	91.3%	89.5%	92.9%	99.4%	92.4%	100.0%
	34 - 66	3.0%	0.4%	1.1%	0.1%	2.0%	0.0%
	67 - 100	3.6%	9.1%	5.2%	0.4%	4.6%	0.0%
	0 - 66	94.3%	89.8%	94.0%	99.6%	94.4%	100.0%
	0-33 + 67-100	97.0%	99.6%	98.8%	99.9%	98.0%	100.0%
	34-100	6.5%	9.5%	6.3%	0.5%	6.6%	0.0%
	0 - 100	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Question 5	0 - 33	23.4%	2.4%	0.0%	0.0%	0.2%	0.0%
	34 - 66	1.2%	0.2%	0.0%	0.0%	0.0%	0.0%
	67 - 100	73.6%	97.0%	100.0%	100.0%	99.7%	100.0%
	0 - 66	24.6%	2.6%	0.0%	0.0%	0.2%	0.0%
	0-33 + 67-100	97.7%	99.8%	100.0%	100.0%	100.0%	100.0%
	34-100	74.8%	97.1%	100.0%	10.0%	99.7%	100.0%
	0 - 100	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Two observations are evident when examining the results above. One; when all 40 experts are combined, the beliefs either reach 0% or 100%. Regardless of the individual beliefs, there is enough provided evidence among the 40 experts to fully support one single event. Second, the smaller groups of experts provide combined beliefs that vary heavily based on the individual beliefs. For example, consider the results of Question 1. Groups 1, 2, 4, and 5 strongly back 0-33% damage. The group 5 experts supported this damage range so strongly that just combining those 8 experts' beliefs produced 100% belief in 0-33% damage. Group 3, on the other hand, had enough varied individual beliefs that the combined belief was still fairly scattered. Regardless, when combining all 5 groups of experts, Group 3's split belief became negligible and the total combined belief supported 0-33%.

4.2.1.1 Secondary Survey Results

A second version of the survey was constructed and distributed to a separate civil engineering undergraduate class. In this version, the same images were used but the participants were asked to assign their beliefs to larger damage groups first, and then based on how confident they were on the damage rating, to assign their beliefs to the smaller ranges. One question from this survey is included in Appendix B for reference on how the question was asked differently. The purpose of this version was to see if participants responded better to this question progression, as this concept might be difficult to understand outside of the axioms of probability. The survey was distributed to approximately 130 students, 25 of whom had taken the first survey. Only about 35 responses were filled out correctly for each question (30 new participants, five repeat participants). The most common reason that a question was thrown out was due to a participant assigning more belief to single events than to the combined event. For example, assigning 35% belief to light damage and 25% belief to moderate damage, but only assigning 50% to belief and/or moderate damage. This suggests that asking the participant to assign whatever belief they are confident in to smaller ranges, and then asking if they have any extra belief in a wider range makes sense to a larger population than asking for any belief in a wide range and then asking to narrow it down into small ranges. For the correctly filled-out surveys, the value of light and/or severe damage (0%-33% +

Table 15. Secondary Survey Results

Combined Expert Damage Beliefs			
	Damage Range (%)	***30 New Participants***	***5 Repeat Participants***
Question 1	0 - 33	100.0%	79.7%
	34 - 66	0.0%	3.0%
	67 - 100	0.0%	13.7%
	0 - 66	100.0%	82.7%
	0-33 + 67-100	100.0%	97.0%
	34-100	0.0%	16.7%
	0 - 100	100.0%	100.0%
Question 2	0 - 33	89.0%	74.0%
	34 - 66	0.0%	2.6%
	67 - 100	11.0%	20.0%
	0 - 66	89.0%	76.6%
	0-33 + 67-100	100.0%	97.4%
	34-100	11.0%	22.6%
	0 - 100	100.0%	100.0%
Question 3	0 - 33	0.1%	36.8%
	34 - 66	0.0%	8.2%
	67 - 100	100.0%	53.9%
	0 - 66	0.1%	45.0%
	0-33 + 67-100	100.0%	91.9%
	34-100	100.0%	62.1%
	0 - 100	100.0%	100.0%
Question 4	0 - 33	94.0%	83.9%
	34 - 66	0.0%	1.3%
	67 - 100	6.1%	12.8%
	0 - 66	94.0%	85.2%
	0-33 + 67-100	100.0%	98.8%
	34-100	6.1%	14.1%
	0 - 100	100.0%	100.0%
Question 5	0 - 33	0.0%	21.6%
	34 - 66	0.0%	6.4%
	67 - 100	100.0%	69.1%
	0 - 66	0.0%	28.0%
	0-33 + 67-100	100.0%	93.6%
	34-100	100.0%	75.5%
	0 - 100	100.0%	100.0%

67%-100%) was calculated the same way as the original survey. Again, the results were scaled so that the sum of all belief (the belief in the individual events plus the extra belief in the combined events) did not exceed 100%. The 30 new participants were combined as one group and the five repeat participants were combined in one group. The results of this secondary survey can be seen in Table 15 to the left.

While this survey did not return as many successful responses as the first round, these results do support the initial survey results. The combined belief between the 30 new participants and the five repeat participants both support the same damage range for all five questions, which also agree with the original survey responses. The comparison between the group of 30 new participants and five repeat participants also highlights some of the conclusions made from the first round; combining 30 results yielded enough evidence to completely or nearly completely support one damage range, while combining just five results did not have enough evidence for any one damage range to produce 100% joint belief in any one range.

A primary purpose of this second survey was to understand how Evidence Theory can best be conveyed to potential users in terms of using one’s belief outside the axioms of probability, while still understanding the mathematical restrictions within Evidence Theory. Since the first round of surveys returned many more usable results, those results are further analyzed in the rest of this thesis.

4.2.2 Computing Probabilities from Combined Beliefs

The combined beliefs from the original survey instrument were used to compute probabilities, including upper and lower bound probabilities. Only the smaller groups of experts were used to calculate probabilities, as the full groups of 40 experts produced "all or nothing" results that would produce identical probabilities. It is also important to note the difference in how probability and Evidence Theory handle multiple events. In Evidence Theory, experts are able to have belief in combined events without being forced to allocate all of that belief to the individual events comprising it. Probability simply sums the individual event probabilities to obtain the probability in disjoint combined events. The results for the first survey question are outlined in Table 16 below, and the results for all five questions are available in Appendix B.

Table 16. Probabilities Computed from Combined Beliefs

Damage Range (%)	***Group 1***			***Group 2***			***Group 3***			***Group 4***			***Group 5***		
	LB (%)	P_1 (%)	UP (%)	LB (%)	P_2 (%)	UB (%)	LB (%)	P_3 (%)	UB (%)	LB (%)	P_4 (%)	UB (%)	LB (%)	P_5 (%)	UB (%)
0 - 33	98.5	98.5	98.9	97.4	97.4	97.9	54.7	54.7	57.4	98.2	98.2	98.8	100	100	100
34 - 66	0.4	0.4	0.4	0.3	0.3	0.3	12.9	12.9	12.9	0.5	0.5	0.5	0	0	0
67 - 100	0.7	1.1	1	1.9	2.3	2.3	29.7	32.4	32.4	0.7	1.3	1.3	0	0	0
0 - 66	99	98.9	99.3	97.7	97.7	98.1	67.6	67.6	70.3	97.8	98.7	99.3	100	100	100
0-33 + 67-100	99.6	99.6	99.6	99.7	99.7	99.7	87.1	87.1	87.1	99.5	99.5	99.5	100	100	100
34-100	1.1	1.5	1.5	2.1	2.6	2.6	42.6	45.3	45.3	1.2	1.8	1.8	0	0	0
0 - 100	100%	100	100	100	100	100	100	100	100	100	100	100	100	100	100

Note: "LB" = Lower Bound, "UP" = Upper Bound, "P" = Probability

There are several notable observations to be made from the results above. First, examine Group 5. As expected, the calculated probabilities are identical to the combined beliefs since there was enough combined belief from the experts in Group 5 to produce either 100% belief or none at all. Groups 1, 2, and 4 all produced very similar results; the calculated probability for each event in these groups was nearly identical to the calculated beliefs. Since almost all of the combined belief was in the 0% - 33% damage range, the probability bounds are very tight. There is less than one percent difference between the upper and lower bounds for all damage ranges in these three groups. Group 3 probabilities display slightly larger upper and lower bounds of up to 3% in range. Larger bounds are expected with this group as the combined belief is more diverse.

To further examine how combined belief affects probability bounds, the 20 surveys with the largest extra belief in combined events were analyzed and the probability bounds were determined. As expected, the bounds for this group of respondents were significantly wider with an average bounds range of 3.8%, compared to an average range of 0.5% for the results in Table 14. Since more belief was allocated to a range of events, rather than one single event, the probability of single events is less absolute. In some cases for the results above, the lower bounds exceed the middle probability or the upper bounds undercut the middle probability by a very small percent. Since the bounds in these cases are so tight, this is apparently due to rounding error.

4.2.3 Further Evaluation of Survey Results by Averaging

For further comparison, the individual beliefs provided by the surveys were combined to approximate probability, rather than using Eqs. 11-12. For each of the small damage ranges seen in Table 14, the additional belief from the associated larger range(s) was allocated according to the fraction of belief for the smaller ranges. The results were normalized to ensure the results summed to unity. Although the survey respondents were not asked to think in terms of probability, this method provides a way to logically convert their individual beliefs to the laws of probability. The results of this can be seen below in Table 17.

Table 17. Combined Belief Results Using Averaging

Survey Results: Probability					
Damage Rating (%)	***Question 1***	***Question 2***	***Question 3***	***Question 4***	***Question 5***
0 - 33	41.6%	39.1%	4.0%	37.2%	5.9%
34 - 66	48.5%	42.7%	33.7%	44.0%	29.6%
67 - 100	10.0%	18.1%	62.3%	18.8%	64.5%

The difference between these results and the results using Evidence Theory are significant. Most noticeable is the effect of combining many experts. When combining more and more expert opinions, Evidence Theory will continuously weight single events until they reach 100% or 0%, as shown by the results of combining all 40 expert beliefs using Evidence Theory in Table 14. Outliers and small amounts of contradicting opinion are eventually considered negligible as more and more beliefs are combined. Averaging, however, will continue to incorporate all individual responses. Outliers here will have a more significant impact on the combined belief. While the results above do have favored damage ranges for each question, the highest combined belief is 64.5%. Even though 40 individual beliefs were combined to obtain these values, the results are varied and arguably inconclusive. The most strongly supported damage range in questions 1, 2, and 4 is 34-66% damage, while the results for those three questions in Table 14 strongly support the 0-33% range. Although these results are not strictly probability-based, it is clear that the influence of ignorance and conflicting opinion in Evidence Theory is significant.

4.2.4 Actual Damage Results

The actual damage state for each image on the survey is provided in the table below. Whichever range the majority of buildings in that image fell into is the range shown. The most strongly supported damage range calculated from the survey results using both Dempster Shafer Theory and averaging/Probability are also shown for comparison.

Table 18. Actual Damage Results - Comparison

Survey Question	Actual Damage	DST Results	Averaging Results
1	0-33	0-33	34-66
2	0-33	0-33	34-66
3	33-66	67-100	67-100
4	0-33	0-33	34-66
5	67-100	67-100	67-100

This table provides valuable insight on how a different framework can produce significantly different results from the same data. The Dempster-Shafer results correctly matched four of the five actual damage ranges, while the results from averaging correctly matched one. This not only reinforces the idea that a framework that allows the user to have some uncertainties changes the output, but it suggests that this framework could produce a more accurate output.

5. Dempster Shafer Theory Applications in Social Vulnerability

5.1 Introduction to Social Vulnerability

Social vulnerability is an aspect of hazard management that is often hard to quantify. This type of vulnerability might be defined as how well someone is able to recover after experiencing a disaster, such as an earthquake, hurricane, or heat wave. One's ability to respond in such a scenario could depend on such factors as whether they own a vehicle, if they have children/dependents, if they are native in the local language, their income, etc. These aspects are all founded in the "differential social relations among groups in a given society" (Thomas et al. 2013). The current method of analyzing social vulnerability relies on percentiles in 15 different census variables that are separated into 4 themes as shown below (Flanagan et al. 2011):

Socioeconomic Status

- Percent individuals below poverty
- Percent civilians unemployed
- Per capita income
- Percent persons with no high school diploma

Household Composition/Disability

- Percent persons 65 years of age or older
- Percent persons 17 years of age or younger
- Percent persons more than 5 years old with a disability
- Percent single parent with child under 18 years old

Minority Status/Language

- Percent minority
- Percent persons 5 years or old who speak English less than "well"

Housing/Transportation

- Percent multi-unit structure (10 or more units in structure)

- Percent mobile homes
- Crowding (more people than rooms at household level)
- No vehicle available
- Percent of persons in group quarters (nursing homes, dorms, military quarters)

A Social Vulnerability Index for Disaster Management outlines how these variables are analyzed; "To construct the SVI, each of the 15 census variables, except per capita income, was ranked from highest to lowest across all census tracts in the United States with a non-zero population. Per capita income was ranked from lowest to highest because, unlike the other variables, a higher value indicates less vulnerability" (Flanagan et al. 2011). If the percentile is 90% or higher (inverse for "Per capita income"), the town is flagged. These percentiles are also summed within each of the four themes. If the group percentile is 90% or higher, this symbolizes another flag. Finally, the percentiles for all 15 variables are summed, and if the overall percentile is 90% or higher, the town receives another flag. The number of total flags is reviewed on a variable level, theme level, and overall level. The number of received flags indicates the level of social vulnerability within the town (Flanagan et al. 2011).

5.2 Rethinking Social Vulnerability Indexing

There are several potential issues with the current method of ranking. The most simplistic one is that being in or out of the 90+ percentile does not necessarily mean this group is or is not socially vulnerable. It also assigns an "all or nothing" ranking – those with an 89th percentile ranking would not receive a flag but are nearly just as vulnerable as those in the 90th percentile. Along the same lines, ranking in the 90th percentile or above may not actually indicate a vulnerability. For example, those living in group quarters such as a dorm might experience a benefit of having close-knit groups, or they may have predetermined recovery plans laid out by the school. Another aspect to consider is that the 15 variables are not split evenly among the four themes. Since the groups have the potential to be assigned one flag, and there are differing numbers of variables within each group, then each variable does not carry equal weight. For example, not having access to a vehicle is in a group with four other variables, while being under the age of 17 is in a group with three other variables. This means that being under the age of 17 carries more weight than not having a vehicle. The ability to escape or recover from a disaster may or may not rely more on a method of travel than age. Finally, consider the result if one group is in the 90^{th}+ percentile in a few variables but ranked very low in all other categories. This group would register as two or three flags. Compare this to a group that is ranked in the 60^{th} or 70^{th} percentile in nearly every category. They would receive zero flags, and be ranked as less vulnerable than the first group.

5.3 Dempster-Shafer Theory in Social Vulnerability

Using Dempster-Shafer Theory offers a new perspective on social vulnerability. Two potential methods of using DST are presented here:

1) If, instead of assigning percentiles to these categories, analysts are asked to rank towns on their likelihood of being vulnerable for each of the 15 variables. For example, one is asked to assign their belief that a town is either A) not vulnerable, B) moderately vulnerable, or C) very vulnerable in each of the 15 categories. The axioms of DST do not require that these add to 100%, so uncertainty of the level of social vulnerability does not count for or against the group. DST also allows extra belief to be allotted to combined groups, so that an analyst may say they are 100% certain the group is either moderately or very vulnerable, but does not have to distribute all 100% to the moderately and very vulnerable ranges. These beliefs can then be combined using Dempster-Shafer Theory to determine a social vulnerability “likelihood” that the town is either not vulnerable, moderately vulnerable, or very vulnerable. This can be evaluated within each of the 4 groups, or as an overall combined percentage.

   This option has many advantages. As discussed previously, qualifying in the 90^{th} percentile or higher in any one of the 15 variables does not necessary signify vulnerability. While the analysts will have access to this census data, they can consider a wider range of contributing factors, such as surrounding resources and the type of hazard that is likely in that area. Further, if this analysis is performed by local analysts, the type of subjective data that could be used is invaluable. A main issue in social vulnerability is the quantification of the valuable contributions made by communities despite their measured vulnerabilities (Thomas et al. 2013). Local analysts would be able to use this intimate knowledge of the community in their analysis when assigning their beliefs to each vulnerability ranking, rather than being restricted to the census data.

2) The second proposed method involves combining the provided census data using DST rather than summing and ranking. The census percentage for each variable is assigned to C) very vulnerable. The rest of the population is assigned to A) not vulnerable and B) moderately vulnerable. For example, if 30% of a town is below poverty, that category is marked as 30% very vulnerable, and the remaining 70% is assigned to not vulnerable and/or moderately vulnerable. How this 70% is split between A and B can be determined by a standard rule or by more subjective means depending on the analyst. These values can then be combined with the other variables within their theme, and with the other 14 categories to determine a combined percentage for “not vulnerable”, “moderately vulnerable”, and “very vulnerable”.

5.3.1 Testing Dempster-Shafer Theory

The second method was tested using the available 2016 census data for three census tracts of varying social vulnerability (1, 4, and 9 flags using the existing methodology) in Denver, Colorado. The census percentage for each of the 15 variables was assigned to C (very vulnerable). The rest of the population was assign to AB (not vulnerable and/or moderately vulnerable). The individual values for A and B were varied between 0% and 25% (at 5% increments) of the remaining population that was not assigned to C. The combined values for AC and BC were the sum of the individual percentages for A and C, and B and C, respectively. The results using values of A and B as 15% of the population that was not assigned to C are outlined below in Table 19. The 15% value was chosen for 2 reasons. First, since the census data pertain to the “very vulnerable” population of each variable, it is difficult to confidently assign the rest of the population to one or the other. Second, this method was tried with values of 0%, 5%, 10%, 15%, 20%, and 25%. The effect on the percentage of “very vulnerable” from changing the percentage assigned to A and B was small or nonexistent. Since the 15% value is somewhat arbitrary at this point (ideally it would be based off of more in-depth census data), the observations made from these results should only be considered satisfactory for this level of analysis. Note that the "very vulnerable" percentages highlighted in red indicate that this variable was flagged using the 90th percentile rule. Also note that the Per Capita Income was calculated based on inverse percentile (the lowest per capita income was in the highest percentile so that it counted as the most vulnerable). See Table 20 for reference on the Vulnerability Ranking.

Table 19. Social Vulnerability Calculated Using Dempster-Shafer Theory

Social Vulnerability Calculated Using Dempser-Shafer Theory																						
TRACT NO.	VULNERABILITY RANKING	Socioeconomic Status					Household Composition/Disability					Minority Status/Language			Housing/Transportation						TOTAL COMBINED VULNERABILITY	TOTAL FLAGS
		BELOW POVERTY	UNEMPLOYED	PER CAPITA INCOME	NO HS DIPLOMA	**COMBINED THEME**	OVER 65	UNDER 17	DISABLED	SINGLE PARENT	**COMBINED THEME**	MINORITY	ENGLISH SECOND LANGUAGE	**COMBINED THEME**	MULTIPLE UNIT HOUSING	MOBILE HOME	CROWDING	NO VEHICLE	GROUP QUARTERS	**COMBINED THEME**		
1	A	12.6	13.8	4.5	14.8	**35**	14.7	13.7	14.4	14	**35.1**	12.2	14.9	**24.3**	9.5	15	15	13.8	9.7	**38.3**	48.6	1
	B	12.6	13.8	4.5	14.8	**35**	14.7	13.7	14.4	14	**35.1**	12.2	14.9	**24.3**	9.5	15	15	13.8	9.7	**38.3**	48.6	
	C	15.8	8.3	70	1.1	**0.05**	1.9	8.6	4	6.9	**0**	18.8	0.5	**0.12**	36.5	0	0	8	35.4	**0**	0	
	AB	84.2	91.7	30	98.9	**100**	98.1	91.4	96	93.1	**100**	81.2	99.5	**99.9**	63.5	100	100	92	64.6	**100**	100	
	AC	28.4	22.1	74.5	15.9	**35.1**	16.6	22.3	18.4	20.9	**35.1**	31	15.4	**24.4**	46	15	15	21.8	45.1	**38.3**	48.6	
	BC	28.4	22.1	74.5	15.9	**35.1**	16.6	22.3	18.4	20.9	**35.1**	31	15.4	**24.4**	46	15	15	21.8	45.1	**38.3**	48.6	
	ABC	100	100	100	100	**100**	100	100	100	100	**100**	100	100	**100**	100	100	100	100	100	**100**	100	
2	A	11.5	14.3	3	10.5	**34**	13.8	11.2	13	12.3	**35**	4.8	13.8	**20.3**	14.9	15	14.4	13.5	14.4	**38.3**	48.6	4
	B	11.5	14.3	3	10.5	**34**	13.8	11.2	13	12.3	**35**	4.8	13.8	**20.3**	14.9	15	14.4	13.5	14.4	**38.3**	48.6	
	C	23.3	4.5	79.7	29.8	**2.9**	8	25.3	13	17.8	**0.1**	68	8.3	**16.8**	0.8	0	3.8	10.1	4.1	**0**	0	
	AB	76.7	95.5	20.3	70.2	**97.1**	92	74.7	87	82.2	**99.9**	32	91.7	**83.2**	99.2	100	96.2	89.9	95.9	**100**	100	
	AC	34.8	18.8	82.7	40.3	**36.8**	21.8	36.5	26.1	30.1	**35.1**	72.8	22.1	**37.1**	15.7	15	18.2	23.6	18.5	**38.3**	48.6	
	BC	34.8	18.8	82.7	40.3	**36.8**	21.8	36.5	26.1	30.1	**35.1**	72.8	22.1	**37.1**	15.7	15	18.2	23.6	18.5	**38.3**	48.6	
	ABC	100	100	100	100	**100**	100	100	100	100	**100**	100	100	**100**	100	100	100	100	100	**100**	100	
3	A	9.6	13.6	0.41	10.3	**16**	14.2	11.4	12.8	12.5	**35**	5	12.8	**17.8**	12.5	15	13	12.6	13	**38.3**	48.6	9
	B	9.6	13.6	0.41	10.3	**16**	14.2	11.4	12.8	12.5	**35**	5	12.8	**17.8**	12.5	15	13	12.6	13	**38.3**	48.6	
	C	35.7	9.6	97.3	31.1	**54.4**	5.3	24.3	14.6	16.6	**0.1**	67	14.8	**27**	17	0	13.2	16.3	13.1	**0**	0	
	AB	64.3	90.4	2.7	68.9	**45.6**	94.7	75.7	85.4	8..4	**99.9**	33	85.2	**73**	83	100	86.8	83.7	86.9	**100**	100	
	AC	45.3	23.2	97.7	41.4	**70.3**	19.5	35.7	27.4	29.1	**35.1**	72	27.6	**44.8**	29.5	15	26.2	28.9	26.1	**38.3**	48.6	
	BC	45.3	23.2	97.7	41.4	**70.3**	19.5	35.7	27.4	29.1	**35.1**	72	27.6	**44.8**	29.5	15	26.2	28.9	26.1	**38.3**	48.6	
	ABC	100	100	100	100	**100**	100	100	100	100	**100**	100	100	**100**	100	100	100	100	100	**100**	100	

Table 20. Legend for Table 19

Legend	
A	Not Vulnerable
B	Moderately Vulnerable
C	Very Vulnerable
AB	Not or Moderately Vulnerable
AC	Not or Very Vulnerable
BC	Moderately or Very Vulnerable
ABC	Not, Moderately, or Very Vulnerable

There are several observations to be made about the results above:

- The total vulnerability of all 15 variables is identical for each tract; nearly 50% not vulnerable, 50% moderately vulnerable, and 0% very vulnerable. There are several reasons for this. First, consider that the most number of flags a tract could receive is 15 (one for each variable). The most vulnerable tract observed here has 9, which is 60% of the maximum. The overall low vulnerability of these tracts correlates to relatively low percentages of "very vulnerable" population. As the variable percentages are combined using DTS, the low percentage of vulnerable population is damped out by the overwhelming evidence that most of the population is "not/moderately vulnerable". Second, since the values for A and B were determined by taking an identical specified percentage of the population (15% of the remaining population that wasn't assigned to C), it makes sense that their values are equal, and nearly exhaustive. Note that they do not add to 100%, which DST allows as the percentages assigned to A, B, and C for each variable do not include the entire tract population.

  The practical implications of this result deserve some consideration. On one hand, the third tract evaluated in this thesis is one of the most vulnerable tracts in the Denver area even though it is only 60% of the maximum vulnerability on the currently used scale, so if this small group is surrounded by largely "not vulnerable" tracts with resources, then perhaps it really is not very vulnerable. On the other hand, it is unreasonable to look at the total combined vulnerability and consider all three tracts to have the same amount of vulnerability. While this method provides valuable insight within the four themes, it loses this subtlety when all 15 variables are combined into a final vulnerability ranking. Using more in-depth census data to appropriately assign values to A and B may affect the final combined results, but in general it is recommended that further studies be done to evaluate how analyzing these 15 individual variables as a whole relates to the actual vulnerability experienced by a census tract, and how the surrounding tracts may influence this value.

- Compare the number of overall flags each tract received to the vulnerability ranking for each theme. The sum of the “very vulnerable” percentage for the 4 themes is 0.17%, 19.8%, and 81.5% for tracts 1, 2, and 3, respectively. This corresponds positively with the number of flags (being 1, 4, and 9 for tracts 1, 2, and 3, respectively). While the ranking is slightly different (81.5% very vulnerable compared to 60% of the possible flags), it might provide different insight on the type and range of vulnerability experienced by each tract, as explained in the next bullet point.
- Note which variables were flagged compared to which ones were not, and which of those contributed to the high vulnerability calculated by DST. For example, look at the Socioeconomic Status Theme for the second tract. 29.8% of this tract does not have a high school diploma, which received a flag for ranking in the 90th percentile or higher of all tracts in this variable. However, this tract is also in the 79.9th percentile for low per capita income, and did not receive a flag for this variable. This means that this tract was flagged as “vulnerable” for about 30% of the population not having a high school diploma but was not flagged for having a lower per capita income than almost 80% of all other tracts, and since these variables are in the same theme, they count for the same weight. By using DST, both of these factors influence the vulnerability output of this tract with the amount of influence being respective of their weight, not as a yes-or-no flag.
- The Housing/Transportation theme (blue) has a combined percent of 0 for “very vulnerable” for each tract. Note that none of the population in any of these tracts resides in mobile homes. Due to this factor, and the low vulnerable percentages in the other variables in this theme, there is a 0% very vulnerable ranking for this group in all 3 tracts.
- The most vulnerable output is the Minority/Language Theme (purple) for the third tract. This makes sense, as there are only 2 variables in this theme, and the minority percentage is comparatively very high (67%). Even though the minority percentage for the second tract is even

higher at 68%, the percentage of people who speak English “less than well” is only at 8.3% for the second tract, compared to 14.8% for the third.

5.4 Social Vulnerability Conclusion

It is clear that using Dempster Shafer Theory to analyze social vulnerability produces different results when compared to the method of indexing, and also compared to standard probability. Rather than ranking each group and taking the top 10%, or averaging each variable, DST analyzes the evidence for or against each vulnerability ranking. If not all of the population has been assigned to a specific vulnerability ranking, then the output will have some ignorance factor. As stated above, ideally the vulnerability ranking would be based off more in-depth census data rather than just the percentage of the population under or over a certain standard. For example, the per capita income, age, disability type, and language variables could all be ranked on a scale rather than as a “yes or no” output. These types of data would provide a far more comprehensive look at the vulnerability of individual tracts and their overall communities/towns. The first method presented in this chapter has the potential to offer an even more comprehensive analysis of social vulnerability, captivating the subjective nuances present in this type of vulnerability. Since this method requires analysts familiar or local to every area being analyzed, there are potential obstacles such as availability and making sure every analyst is operating on the same scale. Both methods offer solutions to some of the problems with the current indexing method, but may have their own shortcomings. The results of combining all 15 variables using DST clearly presented an issue when the results put all three tracts at the same vulnerability ranking. On a theme level, however, the results offer more detail and insight into the type and range of vulnerability when compared to the current indexing method. By gaining a more comprehensive understanding of the vulnerability scale, it is possible to prepare or mitigate hazards in areas that currently do not register as vulnerable.

6. Conclusion

A primary goal of this thesis was to further understand the role of uncertainty in civil engineering and analyze potential frameworks with which this uncertainty might be captured. While a variety of frameworks have been presented in previous publications, the testing program that was executed and analyzed in this thesis offers a more comprehensive understanding of Dempster Shafer Theory and how such a theory would react if used in damage assessments. Evidence Theory provides significantly different results in subjective cases when compared to the frequentist alternative of probability. Such results often provide a much more definitive and involved joint belief that takes into account aspects such as what confidence levels the experts have, any extra belief there may be in a wider range of events, and how conflicting the contributing beliefs are. Using a method that contains these nuances could yield significantly different results in damage assessments when compared to probability. In such cases of post-seismic structural analysis, a limited number of experts may be available to visit the site and provide an evaluation. Combining these valuable subjective individual beliefs to obtain a result requires consideration of those factors that make this assessment subjective, such as ignorance or confidence in a wider range of damage as opposed to a more specific range. Further, as each expert provides more or less evidence (or their belief) of an event, the combined belief will increase or decrease support for one event, rather than averaging each added belief. The rules of probability, namely additivity, handle a potential doubt, or lack of belief, in an event as evidence to its contrary. Many of the tests performed in this thesis involved some level of ignorance, and its influence was not insignificant. By using a framework that acknowledges such a lack of belief as ignorance, rather than belief of the contrary, it is possible to achieve more meaningful results. The results discussed in this thesis suggest that Evidence Theory is a viable, if not preferential, treatment of post-seismic structural assessments.

A secondary goal of this thesis was to investigate the applications of Dempster-Shafer Theory in social vulnerability ranking. While the current indexing method provides an objective analysis using readily available data for every census tract in the United States, the output ignores many key factors that play into a community's ability to deal with a disaster. Dempster-Shafer theory was tested using the same census data to determine of this framework would provide a more in-depth analysis. The theme-level output offered a more detailed analysis of a tract's vulnerability, but combining all 15 variables produced an illogical result. Another method for using DST in this capacity was presented (ranking tracts or communities on a belief basis, rather than relying solely on census data), and further testing is recommended to pursue this possibility.

This program was put through several trials in this thesis to determine general trends and behaviors, but many unknowns remain. The particular equation analyzed here showed some limitations, such as with strongly conflicting beliefs that have no commonality. The adjustment to the equation presented in the conflicting beliefs section might provide an ideal solution to such situations, and should be investigated further. In addition, while the survey employed in this thesis yielded a variety of results, the participants were mostly students. Structural engineers in real post-seismic analysis scenarios might have different confidence trends when evaluating damage. Due to these reasons, further testing is recommended.

The main applications of Evidence Theory explored in this thesis are post-seismic structural damage analysis and social vulnerability indexing, but the possibilities extend far beyond that. Subjective investigations and assessments are unavoidable in many civil engineering and social science operations, as no two locations, projects, communities, and environments are exactly the same. The fields of civil engineering and social science are challenged with recognizing and accounting for these uncertainties. A mathematical framework such as Evidence Theory that allows for this uncertainty has the potential to change the outcome of infrastructure and hazard management decisions on a large scale.

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Appendix A: Original Damage Assessment Survey

Damage Assessment Survey

This survey provides 5 different images of Port-au-Prince, Haiti after an earthquake. You are asked to evaluate the percent of damage in each image. A damage scale is provided for your reference in evaluating the pictures.

After each image, you are asked to indicate your belief that the damage in the image falls within certain ranges. The first 3 ranges are 0%-33%, 34%-66%, and 67%-100%. The latter 2 ranges are 0%-66%, and 34%-100%. The images are fuzzy, and you will likely not be confident enough about the amount of damage to assign all 100% of your belief to the first 3 smaller, more specific ranges. The larger ranges are provided in case you feel more confident in assigning your belief to a larger range. <u>The beliefs you assign for all 5 ranges do not have to sum to 100%, but cannot exceed it.</u>

An example is provided below:

What is your belief that the damage in the image above is in the following ranges?

0% to 33%	34% to 66%	67%-100%
0	40	20

If you are not confident in assigning all your belief to the smaller ranges above, you can put your remaining belief into the larger ranges below. **Remember that the sum of these 5 boxes cannot exceed 100%.**

0%-66%?	34%-100%?
10	15

Please use the damage scale below as a reference for evaluating further pictures.

QUESTION 1 of 5

What is your belief that the damage in the image above is in the following ranges?

0% to 33%	34% to 66%	67%-100%

If you are not confident in assigning all your belief to the smaller ranges above, you can put your remaining belief into the larger ranges below. **Remember that the sum of these 5 boxes cannot exceed 100%.**

0%-66%?	34%-100%?

QUESTION 2 of 5

What is your belief that the damage in the image above is in the following ranges?

0% to 33%	34% to 66%	67%-100%

If you are not confident in assigning all your belief to the smaller ranges above, you can put your remaining belief into the larger ranges below. **Remember that the sum of these 5 boxes cannot exceed 100%.**

0%-66%?	34%-100%?

QUESTION 3 of 5

What is your belief that the damage in the image above is in the following ranges?

0% to 33% | 34% to 66% | 67%-100%

If you are not confident in assigning all your belief to the smaller ranges above, you can put your remaining belief into the larger ranges below. **Remember that the sum of these 5 boxes cannot exceed 100%.**

0%-66%? | 34%-100%?

QUESTION 4 of 5

What is your belief that the damage in the image above is in the following ranges?

0% to 33%	34% to 66%	67%-100%

If you are not confident in assigning all your belief to the smaller ranges above, you can put your remaining belief into the larger ranges below. **Remember that the sum of these 5 boxes cannot exceed 100%.**

0%-66%?	34%-100%?

QUESTION 5 of 5

What is your belief that the damage in the image above is in the following ranges?

0% to 33%	34% to 66%	67%-100%

If you are not confident in assigning all your belief to the smaller ranges above, you can put your remaining belief into the larger ranges below. **Remember that the sum of these 5 boxes cannot exceed 100%.**

0%-66%?	34%-100%?

Appendix B: Secondary Damage Assessment Survey

QUESTION 1 of 5

What is your belief that the damage in the image above is in the following ranges?
You can have up to 100% in either range.

0%-66%?

34%-100%?

Using the amount of belief you assigned to the large ranges above, split that into the smaller ranges below. If you are not as confident in the smaller ranges, you can assign less belief, but you cannot have more confidence in the smaller ranges than you do in the larger ones (e.g., belief for 0%-33% plus belief for 34%-66% cannot be greater than the belief you assigned above for 0%-66%). **The sum of these 3 boxes cannot exceed 100%.**

0% to 33%

34% to 66%

67%-100%

Appendix C: Probability Bounds for Original Survey Groups

Combined Expert Damage Probabilities

	Damage Range (%)	***Group 1***			***Group 2***			***Group 3***			***Group 4***			***Group 5***		
		Lower Bound	P_1	Upper Bound	Lower Bound	P_2	Upper Bound	Lower Bound	P_3	Upper Bound	Lower Bound	P_4	Upper Bound	Lower Bound	P_5	Upper Bound
Question 1	0 - 33	98.5%	98.5%	98.9%	97.4%	97.4%	97.9%	54.7%	54.7%	57.4%	98.2%	98.2%	98.8%	100%	100%	100%
	34 - 66	0.4%	0.4%	0.4%	0.3%	0.3%	0.3%	12.9%	12.9%	12.9%	0.5%	0.5%	0.5%	0.0%	0.0%	0.0%
	67 - 100	0.7%	1.1%	1.0%	1.9%	2.3%	2.3%	29.7%	32.4%	32.4%	0.7%	1.3%	1.3%	0.0%	0.0%	0.0%
	0 - 66	99.0%	98.9%	99.3%	97.7%	97.7%	98.1%	67.6%	67.6%	70.3%	97.8%	98.7%	99.3%	100%	100%	100%
	0-33 + 67-100	99.6%	99.6%	99.6%	99.7%	99.7%	99.7%	87.1%	87.1%	87.1%	99.5%	99.5%	99.5%	100%	100%	100%
	34-100	1.1%	1.5%	1.5%	2.1%	2.6%	2.6%	42.6%	45.3%	45.3%	1.2%	1.8%	1.8%	0.0%	0.0%	0.0%
	0 - 100	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Question 2	0 - 33	100%	100%	100%	79.2%	79.2%	80.5%	58.4%	58.4%	59.7%	99.2%	99.2%	99.3%	100%	100%	100%
	34 - 66	0.0%	0.0%	0.0%	2.4%	2.4%	2.4%	4.7%	4.8%	4.8%	0.2%	0.2%	0.2%	0.0%	0.0%	0.0%
	67 - 100	0.0%	0.0%	0.0%	17.1%	18.4%	18.5%	35.6%	36.8%	36.9%	0.6%	0.6%	0.7%	0.0%	0.0%	0.0%
	0 - 66	100%	100%	100%	81.5%	81.6%	82.9%	63.1%	63.2%	64.4%	99.3%	99.4%	99.4%	100%	100%	100%
	0-33 + 67-100	100%	100%	100%	97.6%	97.6%	97.6%	95.2%	95.2%	95.3%	99.8%	99.8%	99.8%	100%	100%	100%
	34-100	0.0%	0.0%	0.0%	19.5%	20.8%	20.8%	40.3%	41.6%	41.6%	0.7%	0.8%	0.8%	0.0%	0.0%	0.0%
	0 - 100	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Question 3	0 - 33	1.0%	1.6%	1.5%	2.4%	2.7%	2.7%	0.0%	0.1%	0.1%	2.8%	3.5%	3.5%	5.2%	6.9%	7.0%
	34 - 66	0.5%	0.4%	0.4%	0.2%	0.2%	0.2%	1.0%	2.0%	1.0%	1.6%	1.6%	1.6%	1.0%	1.0%	1.0%
	67 - 100	98.0%	98.0%	98.5%	97.1%	97.1%	97.5%	97.9%	97.9%	98.9%	94.9%	94.9%	95.6%	92.1%	92.1%	93.8%
	0 - 66	1.5%	2.0%	2.0%	2.5%	2.9%	2.9%	1.1%	2.1%	2.1%	4.4%	5.1%	5.1%	6.2%	7.9%	7.9%
	0-33 + 67-100	99.6%	99.6%	99.5%	99.8%	99.8%	99.8%	99.0%	98.0%	99.0%	98.4%	98.4%	98.4%	99.0%	99.0%	99.0%
	34-100	98.5%	98.4%	99.0%	97.3%	97.3%	97.6%	99.9%	99.9%	100%	96.5%	96.5%	97.2%	93.0%	93.1%	94.8%
	0 - 100	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Question 4	0 - 33	91.3%	91.3%	93.5%	89.5%	89.5%	90.5%	92.9%	92.9%	93.7%	99.4%	99.4%	99.5%	92.4%	92.4%	93.4%
	34 - 66	3.0%	3.0%	3.0%	0.4%	0.4%	0.4%	1.1%	1.2%	1.2%	0.1%	0.1%	0.1%	2.0%	2.0%	2.0%
	67 - 100	3.6%	5.7%	5.7%	9.1%	10.1%	10.2%	5.2%	5.9%	6.0%	0.5%	0.5%	0.4%	4.6%	5.6%	5.6%
	0 - 66	94.3%	94.3%	96.4%	89.8%	89.9%	90.9%	94.0%	94.1%	94.8%	99.6%	99.5%	99.5%	94.4%	94.4%	95.4%
	0-33 + 67-100	97.0%	97.0%	97.0%	99.6%	99.6%	99.6%	98.8%	98.8%	98.9%	99.9%	99.9%	99.9%	98.0%	98.0%	98.0%
	34-100	6.5%	8.7%	8.7%	9.5%	10.5%	10.5%	6.3%	7.1%	7.1%	0.5%	0.6%	0.6%	6.6%	7.6%	7.6%
	0 - 100	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Question 5	0 - 33	23.4%	24.1%	25.2%	2.4%	2.8%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.3%	0.3%
	34 - 66	1.2%	2.3%	2.3%	0.2%	0.2%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	67 - 100	73.6%	73.6%	75.4%	97.0%	97.0%	97.4%	100%	100%	100%	100%	100%	100%	99.7%	99.7%	99.8%
	0 - 66	24.6%	26.4%	26.4%	2.6%	3.0%	3.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.3%	0.3%
	0-33 + 67-100	97.7%	97.7%	98.8%	99.8%	99.8%	99.8%	100%	100%	100%	100%	100%	100%	100%	100%	100%
	34-100	74.8%	75.9%	76.6%	97.1%	97.2%	97.6%	100%	100%	100%	100%	100%	100%	99.7%	99.7%	99.8%
	0 - 100	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%