# VALIDATING A BAYESIAN MODEL FOR LINKING SERIAL CRIMES THROUGH SIMULATION 

by<br>Jonathan Allen Kringen, M.S.C.J.<br>A dissertation submitted to the Graduate Council of<br>Texas State University in partial fulfillment of the requirements for the degree of<br>Doctor of Philosophy with a Major in Criminal Justice<br>May 2014

Committee Members:

Kim Rossmo, Chair

Pete Blair

Paul Brantingham

Marcus Felson

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## DEDICATION

This work is dedicated to Anne Li Kringen; my life, my love, and my intellectual better half.

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#### Abstract

Crime linkage analysis tries to determine which crimes were committed by the same offender. This is an important police investigative function, as research has shown that a significant proportion of most crime types are committed by a small number of prolific offenders. Case clearance is related to the amount of information available to investigators, and a series of linked crimes provides more information than individual cases examined alone. However, there are only a few available methods for crime linkage. These methods commonly utilize information provided by physical evidence, offender description, and crime scene behavior (i.e., proximity in time and space, modus operandi, and signature). Recognizing that very few factors definitively link crimes, researchers have demonstrated the utility of probabilistically linking crimes using less than definitive information. Bayesian methods provide a promising method of analyzing these links. While some research has demonstrated the efficacy of these methods, the initial work validating the models has relied on limited samples. As such, the generalizability of this research is unknown. This study assesses the validity of a Bayesian crime linkage method using computational methods.

Using empirical observations for both serial murder and commercial robbery as the basis of offender behavior, simulated observations were generated for distance, time difference, and 12 modus operandi (M.O.) factors for serial and non-serial offenses for each crime type. In total 3,500,000 linkage analyses were generated for each crime type


using the Bayesian crime linkage method. Receiver operating curve (ROC) analysis was utilized to assess the predictive capacity of the method on the simulated data. The mean area under the curve (AUC) for the entire set of linkage analyses was 0.81 for serial murder and 0.80 for commercial robbery indicating that the model represents a "good" predictor of serial linkage.

The Bayesian hypothesis test was applied to the likelihood ratio, and results indicated that the extreme level of evidence utilized in the test was a good indicator of linkage (exhibiting a median hit rate of $90 \%$ and a mean percent of series identified of $43.22 \%$ ) for the commercial robbery data using spatial and time difference in conjunction with all 12 M.O. factors. For the murder data using the same set of factors, the extreme level of evidence was less effective as a predictor (exhibiting a median hit rate of 54.55\% and a mean percent of series identified of $56.38 \%$ ).

The inclusion of additional information was shown to increase the predictive capacity of both models using AUCs as a measure of predictive validity. Using the levels of evidence from the Bayesian hypothesis test as decision thresholds, the inclusion of additional information increased both the percent of true positives and the percent of a series identified for all levels of evidence for the commercial robbery data. Adding additional information had little effect on the percent of true positives for the murder data at the highest and lowest levels of evidence and a negative effect at the two middle levels of evidence. In contrast, adding additional information had no effect on the hit rate using
the murder data at the three lower levels of evidence but increased the percent of a series identified at the highest level of evidence. The difference in performance between the commercial robbery and murder data was ascribed to the lower base rate of serial murder and the higher predictive capacity of distance for serial murder data. The higher predictive capacity of distance for serial murders resulted in overall higher likelihood ratios than those observed for the commercial robbery data. Greater performance capacity was found to be associated with longer serial distances and time differences, shorter nonserial distances and time differences, and greater offender consistency and uniqueness for M.O. factors in both the murder and commercial robbery data. Distance and time measures were more important for serial murder linkage, though they were still strong factors for commercial robbery linkage. Consistency and uniqueness were found to have equal value in serial murder linkage, but uniqueness had twice the impact on commercial robbery linkage performance. The greater impact of uniqueness in serial commercial robbery linkage than in serial murder linkage was attributed to higher average levels of consistency in the commercial robbery M.O. data and the reduced ability of distance and time to predict commercial robbery linkage.

## CHAPTER I

## STATEMENT OF PROBLEM

Over 100 years ago, the idea that crime scene behavior could be used to link serial crimes emerged (Gross, 1906). Following this, elaborate schemes for recording crime scene data were developed (Fosdick, 1915). In the modern era, technological advances allowed for the adoption of sophisticated computer systems that could be used to manage recorded data. In addition to storing data, these systems were utilized in attempts to identify patterns that indicated the presence of crime series (Howlett, Hanfland, \& Ressler, 1986). However, the early techniques employed to link crimes were based upon little empirical evidence, and few efforts were made to demonstrate their validity (Bennell \& Woodhams, 2012).

Recognizing these limitations, researchers began to study serial crimes and the methods used for linking them. The purpose of these studies was twofold. From a theoretical perspective, the studies were believed to illuminate whether a similar underlying psychological process influenced how different types of criminals committed their crimes (Bennell \& Jones, 2005). At the extreme end of this perspective, researchers asserted that the characteristics of offenders were related to their crime scene actions and that the more similar two offenders were, the more similar their crime scene behavior would be (Mokros \& Alison, 2002). Based upon this reasoning, it was believed that the features of a crime would provide insight into the type of individual that may have perpetrated the crime. However, research has demonstrated that this homology
assumption is too simplistic and may not provide a strong basis for offender profiling (Mokros \& Alison, 2002).

From a practical perspective, the studies were designed to determine whether investigative strategies previously employed to link serial crimes were sufficient, or whether other methods might prove superior. Several pertinent questions arose such as whether crime linkage methods should be developed for specific geo-demographic situations and/or crime types, or whether more general techniques would suffice (Bennell \& Jones, 2005). Despite calls for linkage procedures to be compared to other approaches and to be vigorously tested to demonstrate their predictive validity (Funder \& Colvin, 1991), little research has focused on comparing linkage analysis methods (Dowden, Bennell, \& Bloomfield, 2007). The failure to demonstrate robust predictive validity for such methods represents a substantial limitation in the crime linkage analysis literature.

## Linking Serial Crimes

Crime linkage is an important part of a serial crime investigation (Bennell, Jones, \& Melnyk, 2009; Burrell, Bull, \& Bond, 2012). By pooling information from each of the individual crimes in a series, investigators receive more information than they would receive from a single isolated crime. This information gain carries several practical advantages (Woodhams, Hollin, \& Bull, 2007). It can help police departments to allocate their resources in a more efficient manner (Woodhams et al., 2007) resulting in greater productivity during investigative efforts (Bennell et al., 2009; Grubin, Kelly, \& Brundson, 2001; Labuschagne, 2012). It can increase the speed and likelihood of identifying and apprehending an offender (Burrell et al., 2012), as well as allow the use
of additional investigative tools such as geographic profiling (Rossmo, 2000). Furthermore, the additional information can sometimes strengthen the evidence in the case (Woodhams, Bull, \& Hollin, 2007), thereby yielding more successful trial outcomes (Labuschagne, 2006).

Despite the benefits of linking serial crimes, research has demonstrated that informal linkage decisions made by investigators are often based on limited, subjective impressions (Canter, 2000). These impressions frequently differ between officers (Maltz, Gordon, \& Friedman, 1990), and consequently investigators often poorly perform when assessing linkages (Wilson, Canter, \& Butterworth, 1996). To overcome these shortcomings, researchers have attempted to develop objective analytic models to establish crime linkages (Green, Booth, \& Biderman, 1976; Grubin et al., 2001).

Establishing a crime series is straightforward when certain types of evidence are present. Confessions, eyewitness testimony, physical and/or forensic evidence such as fibers, fingerprints, or DNA may be used to establish linkages (Grubin et al., 2001). However, these types of evidence may be rare or not available (Ewart, Oatley, \& Burn, 2005; Hazelwood \& Warren, 2003). Even when such evidence is present at a crime scene, it is not always collected (Davies, 1991).

In the absence of these types of physical evidence, behavioral indicators may be the only information available to investigators for a linkage analysis (Bennell \& Woodhams, 2012). Thus, some type of behavioral analysis is often necessary to link serial crimes (Burrell et al., 2012; Mokros \& Alison, 2002; Rossmo, 2000). This type of process relies on analyzing the patterns of behavior associated with the crime scenes themselves as similarities between crime scene factors can help determine the probable
extent of a crime series (Rossmo, 2000). Some of these factors include location in time and space, modus operandi (MO), and signature (Rossmo, 2000; Rossmo, Kringen, \& Allen, 2012).

Whereas position in time and space are straightforward ideas, M.O. and signature are more difficult concepts. M.O. itself is a vague term. M.O. stands for the method of operation, and it refers to the actions that an offender undertakes that are necessary for the completion the crime (Bennett, 1989; Rossmo, 2000; Rossmo et al., 2012). Additionally, M.O. is often conceived to include characteristics of the target (McCarthy, 2007). In contrast, signature typically refers to actions undertaken by a criminal that are unnecessary to the commission of the crime (Douglas \& Munn, 1992; Rossmo, 2000). While signatures are often thought of as behaviors that would be particularly useful in linking crimes, their rarity generally makes them unavailable for linking most serial crimes (Rossmo et al., 2012). Although some researchers have claimed that criminal signatures exist (Keppel, 2000; Keppel \& Birnes, 1997), others have argued that they are likely rare and unlikely to be identifiable for more common crimes such as burglary (Canter, 2000).

In the absence of physical evidence and signature, the only information available for linking crimes may be crime scene characteristics that are known, such as location, time, and the more common M.O. factors typically recorded by police. Research has demonstrated that crime location, temporal factors, and M.O. factors all have potential in classifying crimes as linked or unlinked (Davies, Tonkin, Bull, \& Bond, 2012), with most research in the area focusing on the value of space and time. Despite the demonstrated value of spatial and temporal factors in linking crimes, M.O. characteristics
may be useful as well. The very concept of M.O. implies that there are behavioral similarities between linked crimes, that these are recognizable, and that they could be useful in the classification of serial offenses (McCarthy, 2007, Rossmo, 2000). However, M.O. factors have been the subject of less empirical study (Bennell, \& Jones, 2005).

The best way to accurately classify crimes as linked or unlinked using M.O. factors would be to identify factors that reliably correspond to linked crimes but not to unlinked crimes. Unfortunately, it has proven difficult to identify such factors. Criminal behavior is complex and the identification of a set of actions that perfectly discriminate between linked and unlinked crimes is challenging. Empirical research suggests that perfect discriminators are unlikely to exist in the criminal context, making the value of this approach questionable (Bennell \& Canter, 2002).

However, the process of linking crimes using M.O. factors may still be effective if it can be shown that identifying factors occur at different rates for linked versus unlinked crimes. M.O. factors such as crime selection choices, entry behaviors, characteristics of the targeted properties, and items stolen may vary between offenders in patterns similar to the way spatial factors and temporal factors vary (Davies et al., 2012). Based upon this understanding, researchers have attempted to develop analytic linkage methods based on this variation (Green et al., 1976; Grubin et al., 2001).

The use of this probabilistic approach in linking crimes requires identifying a linking feature or set of features reliably associated with crimes committed by the same offender that are not as commonly associated with crimes committed by different offenders (McCarthy, 2007; Rossmo, 2000). This implies there is value in determining which aspects of offenders' crime scene actions are most often repeated across crimes.

Identified areas of behavioral repeatability may also have practical value as a basis for decision support tools in crime linkage (Rossmo, 2000). Variation in the frequency of some M.O. factors has been shown to have discrimination accuracy establishing crime linkages when used in conjunction with spatial and temporal factors (Davies et al., 2012).

## The Present Study

Recently, a Bayesian model for linking serial crimes has been proposed (Rossmo et al., 2012). The method involves computing a likelihood ratio that indicates the likelihood of linkage for each crime in a potential series. Initial investigation has demonstrated support for the predictive validity of the method (Rossmo et al., 2012). However, the initial analyses were limited to spatial and temporal factors. Therefore, the impact and value of M.O. factors remains untested.

Building upon these prior analyses, the present study seeks to establish additional evidence of the predictive validity of the linkage method. Simulation models are employed, and the inclusion of M.O. factors is tested to determine the extent to which they increase the predictive validity of the linkage method. Finally, the study determines the value of information to the linkage method by establishing the characteristics of information related to model improvement.

## CHAPTER II

## REVIEW OF THE LITERATURE

Two general areas of literature are important for crime linkage analysis. The first body consists of theoretical literature that describes offender behavior. Crime pattern theory, the journey to crime, routine activity theory, and social-cognitive theory provide insight into the ways in which offenders operate in space, in time, and behaviorally. Empirical support has been demonstrated for each of these four, and this understanding of offender behavior provides a theoretical basis for a crime linkage system.

The second body of literature consists of work focused specifically on crime linkage analysis. This work empirically tests the utility of various factors and different systems that can be utilized to predict linkage. Through a variety of techniques and multiple types of analysis, support has been demonstrated for the use of distance, time difference, and M.O. for crime linkage.

The first section of this chapter covers the four theoretical foundations. Each is explained as it applies to crime linkage analysis, and empirical evidence resulting from tests of each theory is presented. The second section of this chapter covers issues that are important for understanding crime linkage analysis. The third section outlines previous research on crime linkage methods. The fourth section presents an introduction to Bayesian probability and the proposed crime linkage method. The final section discusses the research questions addressed by the present study.

## Theoretical Foundations

Four main theoretical areas have substantial importance for crime linkage analysis. The first, crime pattern theory, arose from the work of Brantingham and Brantingham (1981, 1984, 1993a, 1993b). Crime pattern theory focuses on spatial patterns in offending that arise from an offender's awareness space which defines areas where offenders tend to commit crimes. As a result of regularities in individual offender's awareness space, the theory suggests that geographic patterns in an individual offender's crimes will emerge.

The second, journey to crime, focuses on the idea that offenders, like other hunting and foraging species, minimize effort when seeking criminal opportunities. Thus, with the exception of possibly avoiding criminal opportunities immediately near their homes or offices where they may be recognized, offenders will tend to commit crimes closer to these locations rather than farther away. As a result, the spatial location of an individual offender's crimes will demonstrate distinct patterns.

The third, routine activity theory, arose out of the work of Cohen and Felson (1979). Routine activities focuses on the spatial and temporal patterns that arise from the sustenance activities of everyday life. Because these activities disperse individuals in patterned ways, offenders and targets intersect in distinct patterns. Thus, the underlying opportunity structure for crime is non-random, and spatial and temporal locations of crime exhibit distinct patterns.

The final theory, social-cognitive theory, asserts that individuals have inherent behavioral tendencies that they express when engaging in certain behaviors, and that the
expression of these behaviors represents an interaction between the individual's underlying tendencies and the environment. This theory is particularly important for crime linkage systems that utilize M.O. factors, because M.O. factors are likely more useful for crime linkage analysis if an offender more commonly expresses them when committing their crimes. The following sections present each of these four areas in greater detail.

## Crime Pattern Theory

Crime pattern theory arose from the work of Brantingham and Brantingham (1981; 1984; 1993a, 1994b). Brantingham and Brantingham noted that crime is not uniformly distributed across space. They argued that, "criminal behavior can be viewed as a complex form of subjective spatial behavior in which movement patterns depend on underlying spatial mobility biases, knowledge, and experience" (1984, p. 332). This view relies on the idea of a behavioral environment in which crime occurs. The behavioral environment can be broken down into the physical setting (buildings, roads, climate), the social setting (social and economic conditions as well as group and friend networks), the legal setting (laws and law enforcement behavior), and the cultural setting (the beliefs that influence actions). Brantingham and Brantingham (1984) view the first setting as primarily related to crime whereas the remaining three form a backcloth that exerts an indirect influence on crime.

Crime pattern theory is based on the idea that offenders use a spatially structured, hierarchical decision process when navigating the physical environment in search of criminal opportunity (McCarthy, 2007). The process consists of two distinct stages. In the
first stage, criminals identify a suitable area to commit an offense. In the second stage, criminals identify targets within the selected area. Crime pattern theory focuses on the first step, how criminals identify suitable areas. The theory is based on the idea that each offender has an awareness space or areas of familiarity, and that offenders exhibit a preference for committing their crimes within this space (Brantingham \& Brantingham, 1993a).

Criminals prefer to commit their crimes in familiar locations for several reasons. First, familiar surroundings allow offenders to identify environmental cues more easily. Second, offenders are more able to identify getaway routes in familiar places. Third, offenders blend in more easily in familiar surroundings. Finally, operating in familiar areas that are part of an offender's everyday activities requires less effort than operating in other areas.

According to the theory, three important features define an offender's awareness space. These three factors are nodes, paths, and edges (Brantingham \& Brantingham, 1993b). Nodes are the locations or the centers of activity where an offender engages in non-criminal acts. These include locations such as an offender's home, an offender's work or school, locations where an offender shops, and locations where an offender regularly goes for entertainment. Because an offender spends time at each of these locations, they develop familiarity with the areas. As a result, these areas serve as anchor points for offending. Of the locations mentioned, the offender's home has been shown to be the most influential on location of an offender's offenses (Rengert \& Wasilchick, 2000).

Paths are the routes that connect nodes. Together, nodes and paths form an activity space wherein individuals are most likely to commit crimes (Brantingham \& Brantingham, 1993b). The greater activity space is influenced by the physical environment and the way in which an individual travels. For example, an individual's activity space may include a path from home to work, but if the travel between the two locations is undertaken primarily by train, the individual may have limited awareness of locations along the path (Brantingham \& Brantingham, 1984). Edges are the meeting zone of two or more distinct areas. The concept of edges extends the basic understanding of the importance of environment. Brantingham and Brantingham (1993b) contend that greater opportunity for crime exists near edges.

Brantingham and Brantingham (1984) argue that it is possible to describe the process of target selection as spatial behavior. Specifically, they argue that an individual begins their search at or near a node that composes part of their activity space. The individual covers areas that they know before moving further away. Because potential targets and victims are unevenly distributed across space, an opportunity space exists separate from the offender's activity space. Crimes occur at the intersection of both spaces. While criminals may encounter targets outside of their awareness space, offenders are more likely to encounter targets within or near their awareness space. Offenders assess the risk of individual targets seeking characteristics which suggest some targets are better than others. While offenders will vary in the factors that lead them to conclude that a target is suitable, the relationship between awareness space, opportunity space, and areas of crime occurrence are present in all criminal acts (Brantingham \& Brantingham, 1984).

## Journey to Crime

The journey to crime is a logical extension of crime pattern theory. Journey-tocrime focuses on the distances between an offender's residence and their crime. While each offender is unique, at an aggregate level basic similarities regarding offenders' search processes emerge. These similarities lead to general principles regarding criminal search patterns which aid in understanding their search behavior. As suggested by crime pattern theory, most offenders commit offenses near an anchor point from which they begin their search. While the anchor point is commonly the offender's home, it may represent another location such as their place of work.

The typical distance between an offender's anchor point and the locations of their targets varies by type of crime and type of offender. However, the basic principle that offenders target opportunities near their anchor point rather than far away remains consistent. Although the search for criminal opportunities may not originate from the offender's home (Pettiway, 1995), many studies support the contention that offenders generally live within a short distance from the location where they commit crimes (Canter \& Larkin, 1993; Rengert, Piquero \& Jones, 1999, Wiles \& Costello, 2000; Ratcliffe, 2001; Bernasco \& Lux, 2003; Bernasco, 2009; Sarangi \& Youngs, 2006). While these studies have demonstrated variation in the actual distances travelled, some of the variation may be attributed to situational factors, sampling issues, and the particular distance metric used (McCarthy, 2007).

Importantly, the type of crime committed influences the actual distance travelled.
For example, property crime trips tend to be longer than other crime types (Levine,
2004). This may vary as a function of the value of the property stolen. Bernasco (2009) suggests that optimal foraging strategy explains that when a higher reward is perceived, offenders will travel greater distances to commit crimes. Other crime types have different impacts on the journey to crime. Rossmo (1993) indicates that violent serial offenders typically do not offend within a buffer zone around their home because they perceive targets in close proximity to be too risky.

Journey-to-crime research has utilized a variety of different distance measures. The majority of research uses either Euclidean or Manhattan distances. The Euclidean distance represents the shortest distance between two points and can be visualized as the hypotenuse of a triangle. The Manhattan distance represents the length of travel between two points along a right angle street network and can be visualized as the two right-angle sides of a triangle. While which distance measure is appropriate is the subject of an ongoing debate, both measurements provide similar results because they essentially serve as proxies for each other (O’Leary, 2009).

Other researchers theorize that different distance measures might prove superior. However, shortest actual travel path and the quickest temporal path have both been found inferior to either the Euclidean distance or the Manhattan distance (Kent, Leitner, and Curtis, 2006). While some view such tests as overly technical, modeling different distance metrics provides insight into the nature of the criminal search. For example, Canter (2003) theorizes that actual travel paths might not completely define an offender's awareness space. Instead, his research suggested that individuals may choose routes based upon their mental maps. Because an individual's mental map is limited, the additional information required to find the shortest travel and quickest temporal path may
be far beyond the reach of an offender. Further, the observation that the distance an offender actually travels to commit a crime may be much longer than the measured distance from his or her home to the crime site (Rossmo, Lu, \& Fang, 2011).

## Routine Activities

Routine activities is an approach for explaining how crime rates vary over space and time. While the theory is essentially a macro-level explanation of crime, it involves several micro-level assumptions about individual behavior and the nature of criminal opportunity. The approach states that three minimal elements must converge for a crime to occur: a motivated offender, a suitable target, and the absence of a capable guardian (Cohen \& Felson, 1979). Routine activities themselves are the basic sustenance activities that individuals engage in as part of their daily lives. While these activities are noncriminal, they distribute offenders, targets, and guardians over space and time thus affecting the spatial and temporal locations of crime.

The routine activities approach was based largely on Amos Hawley's (1950) theory of community life. Hawley (1950) treated communities as a set of symbiotic and competitive relationships. These relationships vary as human activities are performed. Three important temporal aspects of Hawley's work were essential for the development of the routine activities perspective. These temporal aspects include tempo, rhythm, and timing. Tempo is the number of events in a time period. Rhythm is the regular periodicity at which an event occurs. Timing concerns the coincidence of different, unrelated activities.

Applying these temporal ideas to crime considering the three minimal elements (i.e., a motivated offender, a suitable target, and the absence of a capable guardian) is straightforward. Cohen and Felson (1979) contend that the organization of non-criminal activities affect the spatio-temporal convergence of motivated offenders, suitable targets, and capable offenders. These convergences have rhythms driven by the organization of everyday activities. For example, the incidence of certain crimes increases in the afternoons on weekdays as school children are let out because when children are released from school, the number of potential offenders and targets within areas around schools becomes larger.

The inclusion of the temporal aspects of daily life in the routine activity perspective provides a means to understand temporal patterns in crime. In conjunction with the ideas of awareness space and opportunity space, routine activities incorporates the idea of temporal convergence. Because individual offenders have non-criminal routines, their opportunities to offend also exhibit temporal regularity.

## Social-cognitive Theory

Social-cognitive theory is a behavioral theory that addresses whether individual behavior varies between individuals as the result of momentary situational influences or as a result of enduring differences in their personality (Schoda, Mischel, \& Wright, 1994). The theory asserts that individual differences in patterns of behavior result from underlying personal characteristics such as individual experiences, expectations, values, goals, and self-regulating strategies (Mischel, 1999). Importantly, the theory specifically
considers situations where individuals are similar in their behavior on average, but differ in the specific situations where they exhibit the behaviors.

According to the theory, the tendency to exhibit a behavior in some situations but not others is itself a distinct aspect of individuals. Enduring personality traits are viewed as interacting with situational characteristics to result in generally stable behavior. Thus, individuals exhibit sets of "temporally stable prototypic behaviors" (Mischel \& Peake, 1982, p. 754) that produce discriminative patterns. These patterns of behavior can be conceptualized as unique behavioral indicators of personality (Schoda, Mischel, \& Wright, 1994).

Situational theories of behavior have long stood in opposition to dispositional theories. Essentially, dispositional theories of behavior assert that there are fixed dispositions or traits that define personalities (Casta \& McCrae, 1997; Wiggins \& Trapnell, 1997). These dispositions are believed to be invariant across situations and distinctive to the individual (Funder, 1937; Goldberg, 1993). In contrast, situational theories of behavior posit that intra-individual variation across situations results, in part, due to individual differences in their reactions to situations (Higgins, 1996). Essentially, the two perspectives differ in that dispositional theories focus on broad stable characteristics that differentiate between individuals and situational theories focus on the effect of the environment on behavior (Mischel \& Shoda, 1995).

Social-cognitive theory represents a union of these two extremes, overcoming some of the problems inherent in viewing behavior under only one of the models. Whereas a dispositional approach would provide a theoretical basis for behavioral profiling of individuals, its inability to integrate variability in responses due to between-
situation variation represents a substantial limitation. In contrast, social-cognitive theory provides a theoretical basis for explaining individuals that may be likely to exhibit behaviors yet do so only on certain occasions.

## Other Issues in Crime Linkage Analysis

Beyond these theoretical foundations, certain other ideas are important for understanding crime linkage analysis. These include the difference between signature and thematic models of behavior, the ideas of behavioral consistency and distinctiveness, and the issues related to classification instruments and prediction error. These ideas are explained in the following sections.

## Consistency and Distinctiveness

Identifying the degree to which features of an offense help link it to others by the same offender is important for crime linkage methodologies (McCarthy, 2007). Two central assumptions about behavior are important considerations when assessing the value of behavior for linkage. The first assumption is known as the behavioral consistency hypothesis. This hypothesis asserts that an individual offender's behavior is relatively consistent from crime to crime (Canter, 1995). The second assumption is the behavioral distinctiveness hypothesis. This hypothesis asserts that offenders' behaviors are heterogeneous and vary largely between individual offenders (Goodwill \& Allison, 2006; Salfati \& Bateman, 2005).

Taken together, these two assumptions suggest there should be at least some differences in the behavioral characteristics associated with crimes committed by
different offenders and some similarities in the behavioral characteristics of crimes committed by the same individual offender. This consistency within offenders and the variation between offenders should help link an individual offender's crimes to a series while distinguishing it from crimes committed by other offenders (Burrell et al., 2012). Empirical research has demonstrated support for the behavioral consistency hypothesis. Grubin et al. (2001) showed evidence of behavioral consistency among serial sexual assaults. Salfati and Bateman (2005) showed evidence of behavioral consistency among serial murders. Bennell and Canter (2002) showed evidence of behavioral consistency among serial burglaries, and Burrell et al. (2012) showed evidence of behavioral consistency among serial robberies. Other research has echoed these findings (e.g., Bateman \& Salfati, 2007; Bennell \& Jones, 2005; Markson, Woodhams, \& Bond, 2010; Canter \& Youngs, 2003; Santtila, Fritzon, \& Tamelander, 2004; Sorochiniski \& Salfati, 2010; Tonkin, Grant, \& Bond, 2008; Woodhams \& Toye, 2007). While these findings provide strong evidence that serial offenders are largely consistent in their behavior, some studies suggest behavioral similarity alone is insufficient to demonstrate a high likelihood of linkage, because other offenders may exhibit the same behaviors consistently as well (Klein, 1984).

Highly consistent behaviors that also exhibit high levels of distinctiveness between offenders can overcome this limitation. Substantial research has demonstrated support for the behavioral distinctiveness hypothesis across serial sexual assault, serial murder, serial burglary, and serial robbery as well (Bateman \& Salfati, 2007; Bennell \& Canter, 2002; Bennell \& Jones, 2005, Bennell, Gauthier, Gauthier, Melnyk, \& Musolino,

2010; Burrell et al., 2012; Grubin et al., 2001; Santtila et al., 2004; Woodhams \& Toye, 2007).

Empirical evidence for both behavioral consistency and behavioral distinctiveness has been found for spatial location (Markson et al., 2010; Lundrigan, Czarnomski, \& Wilson, 2010; Santilla, Laukkanen, \& Zappala, 2007, \& Tonkin et al., 2008) and temporal proximity (Goodwill \& Alison, 2006; Markson et al., 2010). Since location of target may be the most crucial decision, and is the one that an offender has the most control over, it follows that this aspect of behavior will be more consistent than other, context-dependent behaviors (Bennell, \& Jones, 2005; Harbers, Deslauriers-Varin, Beauregard, \& Van Der Kemp, 2012). Likewise, consistency and distinctiveness of spatial behavior is important for crime linkage because location can be recorded in a reliable fashion by the police (Bennell \& Jones, 2005). This allows for the detection of consistent patterns of spatial behavior. To a lesser extent, temporal aspects of crimes exhibit this same benefit. However, certain crime types, such as burglary, are less likely to result in accurate temporal data. Factors other than time and location may be even less reliable.

Beyond the issue of reliability is the problem that the consistency assumptions underlying M.O. may be incorrect (Douglas \& Munn, 1992). Serial offenders may modify aspects of their criminal activity over the course of their crime series due to a range of situational and learning factors. Due to a lack of empirical support for the presence of enduring personality traits (Shoda, 1999), it is believed that offender behavior changes depending upon the situation. However, researchers argue that these changes themselves are consistently manifested when an individual faces similar situations
(Mischel, 1999). Therefore, one individual's reactions to a set of situations will be consistent and may also be distinct from another individual's behaviors in the same set of situations (Mischel \& Schoda, 1995). This implies that M.O. behavior can be both relatively consistent and distinctive.

The claim that time moderates consistency is problematic for the use of crime linkage. If this phenomenon occurs, events that happen closer in time will show a greater behavioral consistency than those that happen further apart (Pervin, 2010). It has been suggested that experience will aid behavioral control as individuals become more familiar with the situations they face (Hettema \& Van Bakel, 1997), or it could be that as criminals become more experienced in offending, they specialize and refine their M.O. Another theory suggests that offenders may simply mature and change their offending patterns over time (Davies, 1992). However, research fails to find evidence that time moderates consistency (Markson et al., 2010; Tonkin et al., 2008; Woodhams, Hollin, \& Bull, 2008; Woodhams \& Labuschagne, 2011). Likewise, other empirical work suggests that there is little relationship between behavioral consistency, distinctiveness, and expertise (Snook, 2004; Tonkin et al., 2008).

In addition to the research contradicting claims that M.O. changes over time, other research has shown that M.O. factors demonstrate statistically significant levels of behavioral consistency and distinctiveness (Grubin et al., 2001; Woodhams \& Toye, 2007). The empirical evidence contradicts the assertion that M.O. is too dynamic to be of practical value in linking serial crimes (Davies, 1992, Douglas \& Munn, 1992). Although the use of some M.O. indicators may result in lower levels of discriminatory accuracy, it
is possible to identify certain elements that are relatively stable and distinct across a crime series (Bennell, \& Jones, 2005).

## Signature and Thematic Models

Two distinct approaches to classifying offender behavior have emerged. The first of these is the thematic model. The thematic model asserts that comparison of behavioral characteristics of crime scenes should focus on themes of behavior, or that individual behaviors should be grouped together when the behaviors are suggestive of a general domain (Bateman \& Salfati, 2007). This approach suggests that crime scene actions corresponding to the same domain of behavior should be considered the same actual behavior when attempting to link crimes (Salfati \& Bateman, 2005). Thus, the thematic model implies that the consistency and distinctiveness of the presence of themes of behavior are the important considerations for crime linkage analysis.

Early crime linkage methodologies relied on this thematic approach using generalist typologies to link crimes (Clinard \& Quinney, 1986). These methods attempted to differentiate between offenders using thematic categories such as skills and outlook (Osterburg \& Ward, 1992) and degree of planning or typologies of items targeted (Waller \& Okihiro, 1978). Recent attempts to develop crime linkage methods have used the thematic approach as well. For example Merry and Harsent (2000) used degree of professionalism, and Goodwin and Canter (1997) developed a typology of offenders on the basis of their script.

The second approach is the signature model. Whereas signature is often used when speaking about criminal behavior to refer to an offender's unique behavior or
calling card (Douglas \& Munn, 1992), the signature model of behavior for crime linkage is a different concept. The signature model for crime linkage uses specific behaviors rather than domains of behavior when attempting to link crimes (Bateman \& Salfati, 2007). Unlike the thematic approach, the signature approach makes no assumptions by grouping behaviors that may be unrelated. Under the signature model, the likelihood that two crimes are part of a series should be determined by comparing the specific behavioral characteristics present at the crime scene.

Researchers have argued that the thematic approach should be more effective at linking serial offenses compared to the signature approach as the use of themes is less sensitive to variations in consistency than the use of the key individual behaviors (Bateman \& Salfati, 2007). Thus, comparing behavior thematically should result in greater behavioral consistency than would be found when comparing individual behaviors.

However, the superiority of one approach over the other remains undetermined. Studies have found mixed support when analyzing both individual behaviors and behaviors corresponding to domains or themes (Bateman \& Salfati, 2007; Salfati \& Bateman, 2005; Woodhams et al., 2008). Although the assertion that the thematic approach yields greater consistency persists, some assert that individual behaviors may have more relevance for practice (Harbers et al., 2012).

Due to the assumption-free nature of crime linkage using individual behaviors, there is less chance of linking crimes where different offenders exhibit different yet thematically similar behaviors. However, to utilize individual behaviors for crime linkage analysis it becomes important to clearly establish which features are the most consistent
and which are the least consistent from crime to crime (Harbers et al., 2012). Further, each individual behavior present at a crime that meets the consistency and distinctiveness requirements for linkage should be considered in conjunction with the other factors of the scene (Rossmo et al., 2012). This approach yields a set of individual behaviors exhibiting greater consistency overall than any individual behavior while avoiding the problematic assumptions of the thematic model.

Importantly, some crime linkage methods incorporate M.O. factors that can be viewed as either signature or thematic behaviors. For example, behavioral profilers with the Federal Bureau of Investigation (FBI) rely on type of crime, style of crime, primary intent, victim risk, offender risk, and escalation in addition to the time and the location of the crime (Douglas, Ressler, Burgess \& Hartman, 1986). Several of the M.O. factors included by the FBI could be classified as thematic or signature depending on how broadly the categories are defined.

## Diagnostic Tests and Decision Outcomes

The goal of crime linkage analysis methods is to successfully classify linked and unlinked crimes using some set of information as the basis for the classification decisions. This process can be conceptualized as a diagnostic task (Bennell \& Canter, 2002). Diagnostic tests where there are two possible outcomes to be classified are called two alternative, yes-no tests (Swets, 1988). Crime linkage analysis is this type of test. For any set of crimes, there are two possibilities (i.e., the crimes are either linked or unlinked). Thus, there are two possible predictions. When tests result in the two possible
predictions and there are only two possible outcomes, there are four possible decision outcomes for each observation/prediction pair. These outcomes are presented in Table 1.

Table 1: Decision Outcomes

|  | Actually Linked | Actually Unlinked |
| :---: | :---: | :---: |
| Predicted Linked | Hit | False alarm |
| Predicted Unlinked | Miss | Correct rejection |

The four decision outcomes are known as hits, misses, correct rejections, and false alarms. Hits occur when the prediction that two crimes are linked is correct. Misses occur when the prediction indicates that two linked crimes are unlinked. Correct rejections occur when the prediction that two crimes are unlinked is correct, and false alarms refer to predictions that indicate that two unlinked crimes are linked. While hits are also known as true positive, misses are often referred to as false negatives. Similarly, correct rejections are also known as true negatives, whereas false alarms are sometimes called false positives.

The probability of certain types of linkage decisions are used to measure the accuracy of a diagnostic test, and these probabilities are calculated using the frequencies of the four decision outcomes (Swets, 1988). The calculation of the probabilities for each decision outcome are presented in Table 2 with the letters $A, B, C$ and $D$ indicating the frequency within each cell.

Table 2: Decision Outcome Probabilities

|  | Actually Linked | Actually Unlinked |
| :---: | :---: | :---: |
| Predicted Linked | $A$ | $B$ |
| Predicted Unlinked | $P(A)=\frac{A}{A+C}$ | $P(B)=\frac{B}{B+D}$ |
| $C(C)=\frac{C}{A+C}$ | $P(D)=\frac{D}{B+D}$ |  |

The hit rate, or true positive rate, is known as the sensitivity of the test. The correct rejection rate, or true negative rate, is known as the specificity of the test. Whereas the sensitivity of the test indicates the probability that a crime classified as linked is actually linked, the specificity of the test indicates the probability that a crime classified as unlinked is actually unlinked. Since the probabilities in each column of Table 2 sum to one, two pieces of information can be used to summarize all the information indicated in Table 2; therefore, sensitivity and specificity are commonly used as measures of the performance of classification systems (Swets, 1988).

## Research on Crime Linkage Analysis

While several studies have addressed the behavioral assumptions of crime linkage systems, less empirical research has focused specifically on crime linkage analysis itself. Within crime linkage analysis research, some work has focused primarily on the value of spatial and temporal factors to discern linkage. Other research has used M.O. factors. The majority of crime linkage analysis research using M.O. factors has been based on thematic models of behavior with less research focusing on the signature approach.

Previous research on crime linkage analysis including M.O. factors is presented in this section.

Early efforts at MO-based linkage analysis relied on thematic models with a limited number of categories for each type of behavior analyzed (Clinnard \& Quinney, 1986; Waller \& Okihiro, 1978). Because of the generalist typologies that were used, a substantial number of offenders were grouped in each class. As a result, the early studies failed to find variables that could classify linkage (McCarthy, 2007).

One early study conducted by Green et al. (1976) demonstrated predictive validity ${ }^{1}$ for crime linkage methods. Using seven M.O. factors for serial burglars (location of entry, side of entry, location on block, method of opening, day of week, value of property stolen, and type of property stolen), Green et al. (1976) demonstrated that measures of similarity calculated for pairs of crimes could predict linkage. The measures of similarity were plotted, and both objective and subjective methods were found to be capable of determining clusters of linkage. While the study showed that $93 \%$ of the offenses analyzed were correctly linked, there were two issues with the study. First, the study was based on simulated data generated without an empirical basis. Second, the cases selected for validation were chosen because they exhibited distinctive M.O. factors. Thus, the validation occurred on a biased sample of data. Without testing on data that corresponded to the indistinct nature of the majority of M.O. factors, the findings provide little insight into the utility of crime linkage analysis in most real-world applications.

[^0]Bennell and Canter (2002) used logistic regression models to determine M.O. factors that distinguished between linked and unlinked crimes. Bennell and Canter (2002) analyzed M.O. factors in commercial burglary that included entry behavior, target characteristics, property stolen, and inter-crime distance. The study demonstrated that inter-crime distance, target characteristics, entry behaviors, and property stolen all had predictive validity. Of the factors, inter-crime distance was the best predictor of serial linkage accurately predicting $80 \%$ of serial burglary. The study defined an approach to evaluating crime linkage analyses that later research largely followed. However, the study had an important limitation; it did not actually mimic the real-world task of crime linkage. Instead of testing the ability of factors to correctly classify linked and unlinked crimes from all crimes that occurred within a jurisdiction during a period of time, the study analyzed the ability of factors to correctly classify a subset of pairs of crimes (some pairs committed by the same offender and others committed by different offenders) that removed many offenses committed by the same offender. While the authors argued that this approach increased the confidence in their results because the effect of very prolific criminals had been removed, the subset approach substantially altered the data that would have been considered by an actual crime analyst.

Bennell and Jones (2005) used the same techniques as Bennell and Canter (2002) on another set of burglary data. The data included both commercial and residential burglary. Consistent with the earlier study, the research showed that inter-crime distance, target characteristics, items stolen and entry behaviors were all predictive for serial commercial burglary. The same M.O. factors were predictive for serial residential robbery, and, of the factors studied, inter-crime distance demonstrated the greatest
predictive validity for both crimes. Interestingly, inter-crime distance was a better predictor (85-94\% accurate) in the residential model than in the commercial model (76$88 \%$ accurate). However, due to the use of the same techniques as the previous study, Bennell and Jones' (2005) study suffered from the same limitations.

Woodhams and Toye (2007) studied crime linkage analysis for serial commercial robberies. Data on 71 offender behaviors were coded into four behavioral domains. These included target selection, planning, control, and property. Together with inter-crime distance, these factors were analyzed for predictive validity. The study showed evidence that all five factors were predictive of serial linkage. However, unlike previous research, Woodhams and Toye (2007) found the control domain to be the best predictor of linkage. The authors theorize that, while inter-crime distance is the best predictor for other types of crime such as burglary, the inter-personal nature of robbery lends itself to more controlling behavior. They assert that features of control such as use of weapons, language used, level of violence, and so on reflect an individual's demeanor more accurately than other behaviors. This suggests that there should be both greater consistency and uniqueness exhibited in controlling behaviors.

McCarthy (2007) studied the effectiveness of linkage analysis on serial burglaries. Using data on how, why, when, and where each burglary was committed, McCarthy (2007) demonstrated the predictive validity of an optimal model based on 21 characteristics of burgled premises, 34 behaviors of burglars during the commission of the crimes, and the Euclidian distance between crime sites. The model correctly classified $98 \%$ of linked and unlinked burglaries. Further, McCarthy demonstrated that $94 \%$ accuracy could be achieved using only nine of the predictors. These predictors consisted
of three characteristics of the target (premises type, whether premises was in a cul-de-sac, and the presence of deadlocks), four behaviors of offenders (entry using a tool to disable a lock, use of a cutting tool, and not stealing CDs even though they were present), and distance between crime sites. Absolute distance between crime sights was found to be the best predictor of linkage. McCarthy (2007) found that the use of both conjoint presence (i.e., similarity due to the same factors being present in two crimes) and conjoint absence (i.e., similarities due to the same factors not being present in two crimes) of behavior resulted in greater predictive validity than conjoint presence alone.

Santilla et al. (2008) demonstrated the ability of a thematic linkage model to correctly distinguish between homicides that were part of different series. Based on pathologist findings, witness statements, and interrogation information, in conjunction with other data collected by the police, Santilla et al. (2008) derived a set of variables for offense information, victim characteristics, and situational factors. Using a nonparametric alternative to factor analysis, seven dimensions were found in the data. Five of the dimensions related to motivation, one related to level of planning, and the final related to crime scene behavior. In conjunction, these seven dimensions were capable of correctly classifying $63 \%$ of the crimes to the correct criminal. The authors acknowledged a noteworthy limitation; the data used for the analysis consisted of only solved homicides. As pointed out by Bennell and Jones (2005), solved cases may represent a biased sample as solved cases may exhibit higher levels of behavioral constancy and distinctiveness than unsolved cases.

Burrell, Bull, and Bond (2012) tested behavioral similarity as the basis for crime linkage of personal robbery. The data for analysis included 48 behavioral factors. These
factors comprised three domains of behavior (target selection, control, and property) which were analyzed in conjunction with temporal proximity and inter-crime distance. The analysis demonstrated that inter-crime distance was the best factor for crime linkage. While control was able to discern linkage, the optimal model in the study included both inter-crime distance and target selection. Property was not a valid predictor of serial linkage. Importantly, inter-crime distance was found to be a better predictor of serial linkage when analyzing larger areas. When the analysis was limited to smaller localities, while inter-crime distance was still predictive, it was less accurate. As a result, Burrell et al. (2012) conclude that distance alone should not be used to link serial crimes.

Davies, Tonkin, Bull, and Bond (2012) studied crime linkage analysis for serial auto theft. They coded their data into the domains of target selection, target acquisition, target disposal behaviors as well as inter-spatial distances for location of theft and location of dump and time differences between thefts. Both measures of distance, the measure of time difference, and the domain of target selection behaviors were found to have predictive validity. Importantly, Davies et al. (2012) addressed whether adding other behaviors would improve the predictive validity of the model. The study showed that adding time differences and additional measures of target selection increased the ability to identify linked crimes.

## Bayesian Probability

One of the primary tasks of detectives is the interpretation of available information (Kuykendall, 1982; Rossmo, 2004; Sanders, 1977), and investigators must decide what the information tells them about the possibility that a particular suspect
committed the crime in question (Blair \& Rossmo, 2010). Similarly, evaluating what a given piece of evidence implies about the likelihood that two crimes are linked is the essence of crime linkage. However, researchers have demonstrated that, when presented with a specific piece of information, individuals are generally unable to correctly assess what it tells them about the likelihood of a particular outcome (Blair \& Rossmo, 2010). Thus, one of the goals of a valid crime linkage method is to correctly quantify the likelihood of linkage (i.e., the certainty that the same offender committed the crime).

Probability is the mathematical system for quantifying chance; precisely, probability is the systematic and rigorous process for dealing with uncertainty (Gill, 2006). Although humans think in probabilistic terms daily, issues, such as cognitive biases, often result in incorrect characterizations of probability (Gigerenzer \& Murray, 1987). However, a detailed application of the formal rules of probability yields the correct probability of an outcome. This has particular value for a crime linkage system that assesses the likelihood that two crimes are linked.

The correct assessment of conditional probabilities is an important consideration when applying probability to crime linkage analysis. Conditional probability allows additional information to affect the calculation of a probability (Gill, 2006). For example the probability that a male individual is guilty of some crime is different than the probability that he is guilty of the crime in light of evidence that the offender was female. Having the additional information (the evidence that the offender was female) changes the probability that the male suspect is guilty. Formally, probability statements that acknowledge this additional information are known as conditional probability statements.

Simple probability statements, such as the probability of event $A$ occurring, are written as $P(A)$. Conditional probability statements such as the probability of event $A$ occurring given that event $B$ has occurred are written as $P(A \mid B)$ (Good, 2005). Applying this method of writing probabilities to crime linkage, an analyst might be interested in the simple probability of linkage, or $P(L)$ where $L$ implies linkage. More likely, however, an analyst would be interested in $P(L \mid E)$, the probability of linkage given $E$, the presence of an item of evidence.

Bayesian probability is the field of probability that relates conditional probabilities (Gill, 2009). Bayesian probability follows from the work of Thomas Bayes, published posthumously in 1763. Bayes' contribution to the field of probability is known as Bayes' rule or Bayes' theorem. The theorem provides a way of inverting conditional probabilities (Gelman, Carlin, Stern, \& Rubin, 2004, Gill, 2009).

Bayes' theorem is:

$$
P(A \mid B)=\frac{P(B \mid A) P(A)}{P(B)}
$$

where:
$P(A \mid B)$ is the probability of $A$ given $B$
$P(B \mid A)$ is the probability of $B$ given $A$
$P(A)$ is the unconditional probability of $A$
$P(B)$ is the unconditional probability of $B$

Bayes' theorem could be applied to the probability of crime linkage as follows:

$$
P(L \mid E)=\frac{P(E \mid L) P(L)}{P(E)}
$$

where:
$P(L \mid E)$ is the probability of linkage given the evidence $P(E \mid L)$ is the probability of the evidence given linkage $P(L)$ is the unconditional probability of linkage $P(E)$ is the unconditional probability of the evidence

Bayes' theorem is often expressed using odds rather than proportions. Odds are ratios of probabilities and are often considered more intuitive than probabilities (Gill, 2006). When considering a sample space with only two possible outcomes, $P$ and $Q$, by definition, $Q=1-P$. The odds are the ratio of $P$ and $Q$, or the ratio of $P$ and $(1-P)$. In other words, the odds are the probability of one outcome divided by the probability of the other outcome. Expressing Bayes' theorem in this way removes the denominator (in this example the unconditional probability of the evidence) which can be difficult to determine. When using odds, Bayes' theorem is expressed as:

```
posterior odds = prior odds x likelihood ratio
```

Bayes' theorem expressed in odds follows algebraically:

$$
\frac{P(L \mid E)}{P(U \mid E)}=\frac{P(L)}{P(U)} x \frac{P(E \mid L)}{P(E \mid U)}
$$

where:

$$
\begin{aligned}
& \frac{P(L \mid E)}{P(U \mid E)}=\text { the posterior odds } \\
& \frac{P(L)}{P(U)}=\text { the prior odds } \\
& \frac{P(E \mid L)}{P(E \mid U)}=\text { the likelihood ratio (Bayes' factor) }
\end{aligned}
$$

Ideally, a crime linkage method would be most concerned with the posterior odds since the posterior odds summarize the probability of linkage given an item of evidence to the probability of non-linkage given an item of evidence. Ultimately, crime linkage analysis is interested in the probability of linkage, not the probability of evidence given linkage. However, Bayes' theorem can only yield these probabilities with the inclusion of a prior which, for crime linkage, is usually unknown.

Bayesian methods have been criticized because of their use of priors (Kruschke, 2011). Because there may be little prior information available, some methodologists claim this element biases the posterior odds. If an incorrect prior was included in the crime linkage method, it could fundamentally alter the predictive validity of the method. There is, however, a way out of this conundrum. Some Bayesian methods, such as the

Bayesian hypothesis test, do not rely on the use of a prior. This allows a Bayesian evaluation without any bias due to the prior. In the Bayesian hypothesis test, both the numerator and the denominator of the Bayes' factor can be viewed as hypotheses (Hoijtink, Klugkist \& Boelen, 2008). Importantly, these are competing hypotheses that are mutually exclusive and fully exhaustive (i.e., a crime is either linked or not, and it cannot be both).

The Bayes' factor is the value interpreted in the Bayesian hypothesis test. The Bayes' factor is a ratio of the support for the one hypothesis (the numerator) to the support for the alternate hypothesis (the denominator). The Bayesian hypothesis test is viewed as quantifying the odds of the hypothesis expressed in the numerator. Because the Bayes' factor consists of probabilities for each hypothesis, it can demonstrate evidence for either. A Bayes' factor less than one indicates support for the hypothesis in the denominator while a Bayes' factors greater than one indicates support for the hypothesis in the numerator (Wagenmakers, Wetzels, Borsboom, \& Van der Maas, 2010).

For example, a Bayes' factor of two would indicate that the numerator hypothesis of crime linkage is twice as likely as the denominator hypothesis of no linkage. A Bayes’ factor of one indicates no support for either possibility. Finally, a Bayes' factor of onehalf indicates that the unlinked hypothesis is twice as likely as the linked hypothesis. The Bayes' factor can be interpreted as the weight of the evidence provided by the data (Good, 1985). Table 3 presents a classification system that summarizes the weight of evidence for various levels of Bayes' factors:

Table 3: Strength of Evidence using Bayes' Factor

| Bayes' factor |  |  | Interpretation |
| :---: | :---: | :---: | :---: |
| 30 | - | 100 | Extreme evidence for $H_{2}$ |
| 10 | - | 30 | Very strong evidence for $H_{2}$ |
| 3 | - | 10 | Strong evidence for $H_{2}$ |
| 1 | - | 3 | Moderate evidence for $H_{2}$ |
|  | 1 |  | Weak evidence for $H_{2}$ |
| $1 / 3$ | - | 1 | No evidence |
| $1 / 10$ | - | $1 / 3$ | Weak evidence for $H_{1}$ |
| $1 / 30$ | - | $1 / 10$ | Moderate evidence for $H_{1}$ |
| $1 / 100$ | - | $1 / 30$ | Strong evidence for $H_{1}$ |
|  | $<$ | $1 / 100$ | Very strong evidence for $H_{1}$ |
|  |  |  | Extreme evidence for $H_{1}$ |

## Bayes' Factor for Crime Linkage

Rossmo et al. (2012) proposed a Bayesian method for linking serial crimes involving estimating a likelihood ratio of the probability of linkage to the probability of non-linkage $\left(L R_{\text {final }}\right)$. Because the joint probability of multiple events is equal to the product of the probabilities of the individual events ${ }^{2}$, a final likelihood ratio can be computed by multiplying the likelihood ratios for each factor as follows:

[^1]$$
L R_{\text {final }}=L R_{\text {distance }} \times L R_{\text {time }} \times L R_{M O(1)} \times L R_{M O(2)} \times L R_{M O(3)} x \ldots L R_{M O(i)}
$$
where:
\[

$$
\begin{aligned}
& L R_{\text {distance }}=\frac{P(\text { distance } \mid \text { linked })}{P(\text { distance } \mid \text { unlinked })} \\
& L R_{\text {time }}=\frac{P(\text { time difference } \mid \text { linked })}{P(\text { time difference } \mid \text { unlinked })} \\
& L R_{M O(i)}=\frac{P(M O(\text { i }) \mid \text { linked })}{P(M O(i) \mid \text { unlinked })}
\end{aligned}
$$
\]

The numerators in each of the likelihood ratios for distance, time, and various M.O. factors can be viewed as measures of consistency of behavior within a crime series with a high probability indicating a serial offender exhibits a given behavior with greater frequency. In contrast, the denominators in each of the likelihood ratios can be viewed as measures of the distinctiveness of the behavior. High probability in the uniqueness measure indicates the behavior is common for the crime type, whereas a low probability indicates the behavior is rare. As the consistency measure of the serial behavior (the numerator) increases, holding uniqueness (the denominator) constant results in a larger likelihood ratio. However, as uniqueness (the denominator) increases, holding consistency (the numerator) constant, the likelihood ratio decreases. In this way, larger likelihood ratios indicate that either the behavior being measured is consistent within a series or generally unique for the type of crime being analyzed.

Importantly, likelihood ratios themselves are distinct from probabilities. As such, they are interpreted differently. Because probabilities express a precise estimate of the chance of an event, they have an absolute interpretation. For example, a probability of 1.0
(the upper bound of probability) expresses certainty. In contrast, likelihood ratios have no upper bound. Thus, they are not situated along a scale that allows for absolute interpretation. However, likelihood ratios are useful for making comparisons, and relative judgments about chance can be made using them. This characteristic poses no limitation for crime linkage systems that use likelihood ratios, because the task of crime analysis is largely related to prioritization of unsolved crimes. Likelihood ratios inform this task, as crimes exhibiting higher total likelihood have a greater chance of being linked than those with lower likelihood ratios.

Initial empirical work involving a sample of 162 cases consisting of 4,192 crimes showed support for the likelihood ratio (Bayes' factor) method for crime linkage (Rossmo et al., 2012). The data consisted of a variety of crime types with robbery, sexual assault, burglary, and serial murder forming the majority (76.5\%) of the crimes. As an important note, Rossmo et al.'s method (2012) graphically analyzed the log of the likelihood ratio. This study proceeds numerically analyzing the Bayes' factor, which is the simple (unlogged) likelihood ratio. This is done for two reasons. First, the Bayes' factor is easily interpretable. Second, numerically analyzing Bayes' factor allows the analysis to be placed in the context of the Bayesian hypothesis test classification system.

## Research Questions

The present research employed Monte Carlo simulation methods to assess the predictive validity of the proposed linkage method and to assess the utility of the Bayesian hypothesis test to crime linkage analysis using the likelihood ratio approach. Ultimately the analysis in this research addressed the following specific research questions:

1) Does the proposed linkage method demonstrate predictive validity?
2) Does the Bayesian hypothesis test provide a useful framework for classifying the likelihood ratio generated from the method?
3) What is the contribution of additional information?
4) What are the characteristics of information that affect model performance?

## CHAPTER III

## METHODOLOGY

The present study tested the predictive validity of the linkage method proposed by Rossmo et al. (2012). The analysis involved validation using simulated data generated under assumptions that were consistent with empirical analysis of serial murder and serial commercial robbery data. These assumptions were used to generate a pseudo-population of data for each crime type, and samples from this pseudo-population were drawn. Each sample included both serial and non-serial crimes as well as the data on a combination of factors (e.g., measures of distance, measures of time, and some number of M.O. factors) for each simulated crime. A linkage analysis was performed on each sample. Linkage likelihood ratios were calculated for these simulated factors following Rossmo et al.'s (2012) method. Decision thresholds suggested by the Bayesian hypothesis classification system were applied to these likelihood ratios yielding predictions indicating which crimes were believed to be part of a series and which crimes were not. These predictions were then compared to the data to determine predictive validity for the individual linkage analysis.

Monte Carlo methods were employed to perform this analysis on multiple samples, thus determining the overall predictive validity of the method given the information state used to generate the data. The procedure was repeated under different information states (e.g., inclusion of additional M.O. factors). This process generalized the predictive validity of the method across models and provided insight into the value of
different types of information (e.g., the number of variables, the consistency of variables, and the uniqueness of variables). The following section presents an introduction to Monte Carlo simulation techniques and explains the application of the technique to the validation of the crime linkage model. The second section explains the generation of the simulated data. The third section presents a description of the simulated data, and the final section presents the analyses performed to address the specific research questions.

## Monte Carlo Simulations

Monte Carlo techniques are computational techniques that rely on repeated sampling to obtain results. By sampling from a large dataset repeatedly, the method can provide information on the overall performance of a model as well as provide information on the variability in performance between samples. As the number of samples tested grows larger, the information provided by the simulation becomes more precise.

There are essentially four steps in general Monte Carlo simulations. The first step involves defining the information that will go into the simulation. The second step involves generating data randomly from probability distributions. The third involves sampling from this data and performing computations on the samples. The fourth involves aggregating the results. The following sections explain these steps.

## Data Generating Process

The first step in a Monte Carlo simulation involves defining the information that will go into the simulation. This process is based on the assumption that there is some
data generating process (DGP) that models measureable observations (Mooney, 1997). In a traditional analysis designed to estimate parameters, the DGP is unknown. If it were known, then there would be no need for estimation. However, in Monte Carlo simulation, the DGP is known because it is set up by the researcher. Once this DGP has been setup, simulated data are generated that are known a pseudo-population (Mooney, 1997).

Defining the DGP for a simulation involves assigning probability distributions that are used to randomly draw values from. Observations are then generated by sampling from these probability distributions. These observations form the pseudo-population of data. Samples are then drawn from this pseudo-population, and the measure of interest is calculated. This process is repeated many times (e.g., 100,000 times) drawing new samples each time. The measures generated from each round can be compared to evaluate the behavior of the estimates. Because of the importance of probability distributions to the DGP, an explanation of the probability distributions used in the present study follows.

## Distributions

To simulate criminal behavior, distributions were defined that represent spatial, temporal, and M.O. behavior. Observations for individual offenders were drawn from these distributions. Three specific distributions were utilized to generate the data: (1) the beta distribution; (2) the uniform distribution; and (3) the Bernoulli distribution. Beta distributions were utilized to generate distance and time difference observations for linked crimes, while the uniform distribution was used to generate these observations for unlinked crimes. The Bernoulli distribution was used to generate dichotomous
presence/absence characteristics for all M.O. factors for both linked and unlinked crimes. An explanation of each distribution follows.

The beta distribution is a very flexible distribution that is bounded between zero and one. It is particularly useful for social science simulation due to this flexibility (Mooney, 1997). The distribution is defined by two parameters, alpha ( $\alpha$ ) and beta ( $\beta$ ). The parameters alpha and beta have no direct interpretation independently, but together they define the moments of the distribution as follows:
notation and parameterization:

$$
\operatorname{Beta}(\alpha, \beta) \text { where } \alpha>0 \& \beta>0
$$

mean:

$$
\frac{\alpha}{\alpha+\beta}
$$

median (approximate):

$$
\frac{\alpha-\frac{1}{3}}{\alpha+\beta-\frac{2}{3}}
$$

mode:

$$
\frac{\alpha-1}{\alpha+\beta-2}
$$

variance:

$$
\frac{\alpha \beta}{(\alpha+\beta)^{2}(\alpha+\beta+1)}
$$

The parameterization of the beta distribution allows for straightforward generation of probability distributions with known means, medians, modes, and
variances. Distributions can therefore be generated that define proportional distance and time differences. Observations can then be drawn from the distributions to represent values associated with linked crimes. Figures 1 to 3 demonstrate the versatility of the beta distribution using different parameters.


Figure 1: Beta(2,7) Probability Density


Figure 2: Beta(5,5) Probability Density


Figure 3: Beta(9,3) Probability Density

The uniform distribution is another useful distribution for social science simulation (Mooney, 1997). All outcomes supported on a uniform distribution's interval are equally probable. The distribution is defined by two parameters, $a$ and $b$. The two parameters define the interval of support for the distribution, where $a$ is the minimum and $b$ is the maximum value of the $x$-variable. As with the beta distribution, the moments and functions for the uniform distribution are well known. They are defined as follows:
notation and parameterization:

$$
\mathrm{U}(a, b) \text { where }-\infty<a<b<\infty
$$

mean:

$$
\frac{1}{2}(a+b)
$$

median (approximate):

$$
\frac{1}{2}(a+b)
$$

variance:

$$
\frac{1}{12}(b-a)^{2}
$$

The uniform distribution is defined on the interval $a, b$. Thus, a $\mathrm{U}(0,1)$ distribution can be viewed as randomness on the interval from zero to one. Draws from the uniform distribution were used to represent proportional distance and time differences for unlinked crimes. The probability density for a uniform $(0,1)$ distribution is shown in Figure 4.


Figure 4: Uniform(0,1) Probability Density

The Bernoulli distribution is the final distribution that was used to simulate data. The Bernoulli distribution is used to model a process that results in two possible outcomes with constant probabilities (Mooney, 1997). The distribution has a range of two values, zero and one, and is parameterized by a single parameter, $p$, where $p$ is the probability of a getting a one on from a single random draw. The moments and functions for the Bernoulli distribution (Ber) are as follows:
notation and parameterization:

$$
\operatorname{Ber}(p) \text { where } 0<p<1
$$

mean:

## $p$

variance:

$$
p(1-p)
$$

Since draws from the Bernoulli distribution result in zeros and ones, draws from the Bernoulli distribution were used to simulate the presence or absence of M.O. characteristics. For linked crimes, the probability used represented the consistency of behavior for the serial offender. For unlinked crimes, the probability represented the uniqueness of the M.O. characteristic in general.

## Model Parameterization

The parameters utilized for each of the distributions used in the simulations defined the behavior of the simulated offenders. Thus, the selection of these parameters was fundamental to the validity of the subsequent analysis. Therefore, the parameters that defined the distributions used for the DGP were determined based on empirical assessment of actual offender behavior. Four sets of data were used for this process.

The first data set provided observations on serial murderers' spatial and temporal behavior. The second data set provided observations on serial murderers' behavioral
consistency. The third data set provided observations on commercial robbers' spatial and temporal behavior, and the fourth data set provided observations on commercial robbers' behavioral consistency. Parameters were selected for the distributions used in the DGP that were consistent with these observations. The process is presented in Chapter 4.

## Generation of Simulated Data

The data were generated on a sample-by-sample basis by generating all observations for an individual sample for linkage analysis then proceeding to the next sample. Each sample consisted of both serial and non-serial crimes. The following two sections explain the data generating process that was used. The first section presents the method used to simulate serial crimes, and the second presents the method used to simulate non-serial crimes. Importantly, this process of generating simulated data was performed twice. The first time used assumptions based on serial murder, and the second time used assumptions based on serial commercial robbery.

## Serial Crime Data Generation

Serial crime data were generated for individual offenses within each case and included the sample number, a code indicating serial, a distance measurement, a time difference measurement, and measurements for each of the $12 \mathrm{M} . \mathrm{O}$. factors for each serial crime. The number of serial crimes simulated for each case was randomly selected from a distribution that matched empirical observations of the number of serial offenses for the crime type being simulated.

The distance measurements for each of the crimes within a case were randomly drawn from a beta distribution that closely resembled the empirical data on serial distances for the crime type being simulated. Because draws from the beta distribution range from zero to one, these distances were conceptually defined as proportional distances. The time difference measurements for each of the crimes within a case were randomly drawn from a beta distribution that closely resembled the empirical data on serial crime time differences related to the crime type being simulated. These measurements were bounded between zero and one, and were conceptually defined as proportional time differences.

Simulation of the serial M.O. data involved a two-step sampling process. First, a probability representing an offender's consistency for a particular behavior was randomly drawn for each M.O. factor from a beta distribution that closely resembled an empirical distribution of offender consistencies generated from the observational data. In total, 12 probabilities drawn for each sample, with each probability assigned to a single M.O. factor. The second step involved sampling observations for the presence or absence of a characteristic for each of the M.O. factors. These data were randomly drawn from Bernoulli distributions defined by the probabilities that were assigned to each factor in the first step. These draws resulted in either zeros or ones which were interpreted as the presence (1) or the absence (0) of the behavior.

## Non-serial Crime Data Generation

The non-serial data were generated using a similar process. The non-serial data also included an identifying number for the sample, a code indicating non-serial, a
distance measurement, a time difference measurement, and measurements for each of the 12 M.O. factors for non-serial crimes. The number of non-serial crimes for each sample was randomly selected from a range of values such that the mean proportion of serial crimes to total crimes corresponded to a realistic expectation of the true proportion. It also provided for adequate difference between the murder and commercial robbery data to allow for comparison.

Generation of the non-serial distance data was accomplished by randomly sampling from a uniform( 0,1 ) distribution. Because this distribution represents randomness on the interval from zero to one, these data were conceptually defined as proportional distances for non-serial crime. The time difference data were also simulated using random draws from a uniform( 0,1 ) distribution.

Simulation of the non-serial M.O. data involved the same two-step sampling process used to simulate the serial M.O. data. First, probabilities for non-serial M.O. were sampled. Unlike the serial M.O. data simulation, the non-serial M.O. data simulation drew these probabilities from a uniform( 0,1 ) distribution. Thus, these probabilities were random. The second step proceeded in precisely the same manner as the second step for the serial M.O. data. The sampled probabilities were assigned to individual M.O. factors, and the M.O. data were sampled from a Bernoulli distribution based on the assigned probabilities.

## Description of Simulated Data

In total, 3.5 million samples were generated for each of the crime types. A detailed description of the simulated data for each crime type follows.

The 3.5 million samples for serial murder linkage analysis included 3,574,400,080 individual simulated offenses. Of those, 52,571,380 (1.5\%) were simulated serial offenses, and the remaining 3,521,828,700 (98.5\%) were simulated non-serial offenses. Descriptive statistics for the individual samples drawn for serial murder linkage analysis are presented in Table 4.

## Table 4: Descriptive Statistics for Simulated Murder Data Samples

|  | Mean | Median | Std. Dev. | Min | Max |
| :--- | ---: | :---: | ---: | ---: | ---: |
| Total crimes | $1,021.26$ | 1,022 | 574.43 | 12 | 2,039 |
| Serial crimes | 15 | 14 | 7.83 | 2 | 40 |
| Non-serial crimes | $1,006.40$ | 1,007 | 574.38 | 10 | 2,000 |
| Proportion of serial crimes | 0.03 | 0.01 | 0.06 | 0.0005 | 0.80 |

While the maximum number of total crimes within a linkage analysis $(2,039)$ for the murder data may seem high $^{3}$, the number was chosen to yield an average proportion of serial murders around 0.02 . Because the number of serial murders was modeled based on the empirical serial murder data, the distribution of the total number of murders was fixed to render this proportion. Thus, in this research, linkage analyses conducted on samples with high proportions of serial murders were exceedingly rare. The distribution of the proportion of serial offenses is presented in Figure 5.

[^2]

Figure 5: Distribution of Proportion of Serial Murder Offenses

The 3.5 million samples for the serial commercial robbery linkage analysis included 297,503,803 individual simulated offenses. Of those, $87,516,746$ (29.4\%) were simulated serial offenses, and the remaining 209,987,057 (70.6\%) were simulated nonserial offenses. Descriptive statistics for the individual samples drawn for serial commercial robbery linkage analysis are presented in Table 5.

Table 5: Descriptive Statistics for Simulated Commercial Robbery Data Samples

|  | Mean | Median | Std. Dev. | Min | Max |
| :--- | ---: | :---: | :---: | ---: | ---: |
| Total crimes | 85.07 | 85 | 26.22 | 25 | 145 |
| Serial crimes | 25 | 25 | 11.84 | 5 | 45 |
| Non-serial crimes | 60 | 60 | 23.38 | 20 | 100 |
| Proportion of serial crimes | 0.30 | 0.29 | 0.14 | 0.05 | 0.69 |

The simulated serial commercial robbery crimes were modeled based on the empirical serial commercial robbery data. In contrast to the murder data, the commercial robbery data was modeled to provide a higher proportion of serial crimes. This was done to provide a comparison between the effects of different proportions of serial offenses on the linkage method. The distribution of the proportion of serial commercial robbery offenses is presented in Figure 6.


Figure 6: Distribution of Proportion of Serial Commercial Robbery Offenses

## Analysis of Simulated Data

Analysis of the simulated data involved three steps. The first step was evaluation and aggregation. In this step, measures of predictive validity were generated for each linkage analysis performed on a sample of data. These measures were then aggregated across all samples to determine the overall performance for that model. The second step involved analyzing simulated data based on a different number of factors. In this step, factors that define the model were changed (e.g., additional M.O. factors were included) yielding a different model for comparison. New data were sampled, and the evaluation and aggregation was repeated on the new samples. Finally, results were generated for the new model. The third step involved evaluating the overall performance of the linkage method considering the factors that were used to generate the data. This step consisted of comparisons between models from the same DGP using different amounts of information (e.g., more M.O. factors). This final step identified the value of additional information and the differences in linkage performance due to the characteristics of the data used (e.g., distance, time difference, consistency, and uniqueness). A detailed description of the different types of analysis conducted to answer each research question is presented below.

## Evaluation and Aggregation

Each simulation followed a process where a sample of data representing a single set of crimes for linkage analysis was drawn from the pseudo-population. Crimes generated from the linked DGP were notated as linked, and crimes generated from the
non-linked DGP were notated as unlinked. The $L R_{\text {total }}$ was calculated for each of the observations in the sample. Measures of predictive validity were generated for the individual linkage analysis. The process was repeated using a different sample, and the performance measures were summarized for the entire set of linkage analyses conducted yielding a general conclusion about the linkage method's behavior.

## Research Question One

The first research question asked, "Does the proposed linkage method demonstrate predictive validity?" This was a general question about the potential of the linkage method and was addressed prior to consideration of the Bayesian hypothesis test. To answer this question, receiver operating curve (ROC) analysis was utilized.

ROC analysis is a technique used to assess the ability of a diagnostic tool to correctly distinguish between two outcomes. Importantly, the sensitivity of a test (i.e., the true positive rate) and the specificity of a test (i.e., the true negative rate of a test) are both functions of the decision threshold used to classify observations. Thus, the decision threshold (also known as a cut score) plays an important role in the overall performance of a test. For example, a linkage model where the cut score is higher than any observation will classify all observations as unlinked. The test will therefore have a sensitivity of zero but a specificity of one. This implies that the probability of a false positive is 0.0 , but the probability of a true positive is 0.0 as well. In contrast, a model where the cut score is lower than any observation will predict all crimes as linked. Thus, this model will have a sensitivity of one but will also have a specificity of zero. This means that the probability of a true positive is 1.0 , but the probability of a false positive is 1.0 as well.

Because the probabilities of true and false positives are a function of the cut score, analyses of classification performance based on the use of any particular cut score determine the impact of the cut score rather than the overall capacity of the classification system. ROC analysis overcomes this limitation and determines the predictive capacity of a model without consideration of a cut score. The ROC curve is the curve that emerges by plotting the specificity along the x -axis of a graph in decreasing order ${ }^{4}$ and the sensitivity along the $y$-axis in increasing order. A diagonal line splitting the graph from $(0,0)$ to $(1,1)$ (i.e., $y=x$ ) indicates that both true positives and false positives increase at an equal rate across possible cut scores. Therefore, this line represents a test that is uninformative. ROC curves are indicative of predictive capacity when they increase more rapidly along the $y$-axis than the $x$-axis. An example of a ROC curve generated from a crime linkage analysis is presented in Figure 7.

[^3]

Figure 7: Example ROC curve

ROC curves provide an intuitive method for assessing the predictive capacity of tests. However, ROC curves themselves are graphical methods that correspond to analysis of a single sample of data. Therefore, they are not well-suited to aggregation across multiple samples as employed in Monte Carlo simulation. However, the area under the ROC curve (AUC) is appropriate for aggregation. Because the line $y=x$ is uninformative, an AUC of 0.5 or less indicates that the model performs no better than chance. However, AUCs greater than 0.5 indicate predictive capacity. An AUC between 0.50 and 0.70 is considered a predictive yet "poor" model, 0.70 to 0.80 indicates a "fair" model, 0.80 to 0.90 indicates a "good" model, and 0.90 to 1.0 indicates an "excellent" model.

To demonstrate general predictive validity of the linkage model, the AUCs for each sample drawn for linkage analysis generated under a given DGP were calculated. The descriptive statistics for the AUCs for the set of samples were then calculated to provide a description of the overall performance of the model given the information used to calculate the $L R_{\text {final }}$.

## Research Question Two

The second research question asked, "Does the Bayesian hypothesis test provides a useful framework for classifying the likelihood ratio generated from the linkage method?" Essentially, the Bayesian hypothesis test provides a set of cut scores to apply to the likelihood ratio to classify linked and unlinked crimes. Because this method incorporates cut scores, a different type of analysis was required.

To analyze the behavior of the linkage method under the Bayesian hypothesis test, the likelihood ratios generated for each set of data were used to predict whether each crime was linked or unlinked using decision thresholds from the Bayesian hypothesis test. For each linkage analysis, the number of actual hits was recorded along with the hit rate. The number of actual hits can be viewed as a measure of information gain, and the hit rate can be viewed as a measure of confidence in this information. Likewise, the proportion of a series detected was recorded. Along with the actual number of crimes, these data summarized all of the information necessary to evaluate the predictive validity of the method.

Analysis consisted of calculating descriptive statistics for the measures recorded. Additionally, histograms were generated for each of the measures recorded at each level
of evidence, and performance measures were plotted against performance percentile for the set of linkage analyses. This additional step provided greater detail into the performance of the linkage method at the different levels of evidence.

## Research Question Three

The third research question asked, "What is the value of including additional information in the linkage model?" Analysis for this question involved using Monte Carlo methods to run simulated experiments. Monte Carlo experiments follow the same logic as a laboratory experiment (Carsey \& Harden, 2011). The researcher draws samples from the data calculating the measure of interest from each sample. Next, a single factor is varied and new samples are drawn. For each of these samples, the measure of interest is calculated. The measures generated from the samples obtained from the first model are then compared to the measures generated from the second model, and the effect of the varied factor is observed (Carsey \& Harden. 2011).

Analysis of the previous research questions involved generation of performance measures for each DGP at each number of included factors. The analyses for research question three utilized graphical methods to interpret changes in the performance measures between models. Specifically, the measures of central tendency for the AUCs calculated previously for model (e.g., distance only, distance and time difference, distance and time difference in conjunction with a single M.O. factor, etc.) were plotted in order of ascending information. This analysis provided a description of the value of increasing information in general.

A second set of analyses followed a similar strategy. Specifically, the performance measures for each of the models were plotted in order of ascending information at each level of evidence under the Bayesian hypothesis test. This process yielded a description of the effect of additional information on the linkage method when evaluated as a Bayesian hypothesis.

## Research Question Four

The fourth research question asked, "What are the characteristics of information that impact model performance?" To address this question, linear regression was utilized. Two regression models were estimated. In the first model, the AUCs for each serial murder linkage analysis were defined as the measure of performance, and this measure was regressed onto seven characteristics of the information used in each linkage analysis. These characteristics included median distance and time differences for the serial and non-serial crimes, serial offender mean behavioral consistency, mean behavioral uniqueness, and the proportion of serial offenses in the linkage analysis. This regression model estimated the effect of the information on the overall predictive capacity of the linkage method for serial murder. The second model was estimated by regressing the AUCs for the commercial robbery linkage analyses on the same set of independent variables. The estimates were then compared between the serial murder linkage analysis and the commercial robbery linkage analysis models.

## CHAPTER IV

## LIKELIHOOD CALCULATIONS AND EMPIRICAL FOUNDATIONS

The first step in analyzing the linkage methodology involved a detailed consideration of the specific calculations used in generating the final likelihood ratio used to predict linkage. The following three sections present the calculation of the distance, time difference, and behavior probabilities used in the linkage method. Within each section, the empirical analyses conducted to provide a basis for the are explained. Finally, the processes used to generate probability distributions and define the DGPs for each simulation are discussed.

## Distance for Crime Linkage Analysis

Previous research has found geography to provide important information for crime linkage analysis. Crime pattern theory suggests that offenders operate from various nodes or anchor points, and that offenders develop an awareness space around the paths travelled between these nodes. This results in a mental map that the offender uses when seeking targets. Because offenders are most familiar with their nodes and the paths that they use in inter-nodal travel, they tend to have greater awareness in areas around these nodes and the travel paths that link them.

Journey-to-crime research has demonstrated that offenders tend to commit crimes that exhibit predictable spatial relationships in relation to their anchor points. Some crime types exhibit a distinct pattern where an offender is less likely to commit an offense near
their anchor point. As they move away, the likelihood of offending increases until they reach a certain distance. The area inscribed by this distance is known as a buffer zone. After passing beyond the buffer zone, the likelihood of offending decreases as suggested by the least effort principle and foraging theory. The crime locations that emerge for an individual offender are based on these behavioral consistencies and should result in distinctive patterns.

Interspatial distances provide one method for measuring the relationships between these locations. Several studies have demonstrated distinct patterns in interspatial distances for linked crimes. This consistency has been used in a variety of linkage tests, the majority of which have shown the utility of interspatial distance in differentiating between linked and unlinked crimes.

The proposed Bayesian linkage method includes distance as a component. The final likelihood ratio evaluated in the method is the product of the distance likelihood multiplied by the time difference likelihood and the M.O. likelihoods. A description of the distance likelihood calculation and a discussion of the effects of the underlying assumptions follows.

The distance likelihood is given by:

$$
L R_{\text {distance }}=\frac{P(\text { distance } \mid \text { linked })}{P(\text { distance } \mid \text { unlinked })}
$$

The numerator, the probability of a distance given that the crime is linked, is estimated by assuming a probability density function for distances between linked crimes. The cumulative density function is evaluated at the observed distance, and the
corresponding probability is used. The following example assumes a $\operatorname{Beta}(2,5)$ distribution. The $\operatorname{Beta}(2,5)$ probability density is shown in Figure 8, and the cumulative density is shown in Figure 9.


Figure 8: Beta(2,5) Probability Density


Figure 9: Beta(2,5) Cumulative Density

Because the beta distribution is defined only on the interval from zero to one, distances must be converted into proportional distances where the longest possible distance equals one. The conversion is as follows:

$$
\text { Proportional distance }=\frac{\text { Observed distance }}{\text { Maximum possible distance }}
$$

The denominator of the distance likelihood ratio, the probability of a distance given that the crime is unlinked, is estimated by calculating the probability of observing a distance up to the observed distance assuming that all possible distances within the
analysis area are equally probable. Ignoring edge effects ${ }^{5}$, this assumption allows the probability to be estimated by calculating the proportion of the area of a circle with a radius equal to the observed distance to the area of the entire analysis space. This proportion is given by:

$$
P(\text { spatial distance } \mid \text { unlinked })=\frac{\pi * \text { distance }^{2}}{\text { Total area of analysis space }}
$$

Figure 10 provides a graphic illustration.


Figure 10: Areas Utilized in the Calculation of $\boldsymbol{P}$ (distance|unlinked)

[^4]An example of the distance likelihood ratio calculated for the observations in Table 6 follows:

Table 6: Sample Data for $L R_{\text {distance }}$ Calculation

| Factor | Observation |
| :--- | :---: |
| Distance | 1.3 mi. |
| Longest possible distance | 7.2 mi. |
| Area of analysis space | $16.1 \mathrm{mi.}^{2}$ |

Using the data from Table 6, the probability of the distance given that the crime is linked is estimated by first converting the observed distance of 1.3 miles into a proportional distance.

$$
\text { Proportional distance }=\frac{1.3 \mathrm{mi} .}{7.2 \mathrm{mi} .}=0.18
$$

The assumed $\operatorname{Beta}(2,5)$ cumulative density function is then evaluated at 0.18 which yields a probability of 0.30 . Therefore the probability of the distance given the crime is linked is equal to 0.30 , and this value is $P($ distance $\mid$ linked $)$, the numerator in the calculation of the distance likelihood ratio (i.e., $L R_{\text {distance }}$ ).

The probability of the distance given that the crime is unlinked is calculated by assuming that the observed distance occurred randomly. This allows the $P($ distance|unlinked $)$ to be estimated by calculating the area of a circle with a radius equal
to the observed distance ( 1.3 miles) and then dividing this area by the total area of the analysis space ( 16.1 miles $^{2}$ ).

$$
\begin{gathered}
\text { Numerator of } P(\text { distance } \mid \text { unlinked })=\pi * 1.3^{2}=5.31 \mathrm{mi}^{2} \\
P(\text { distance } \mid \text { unlinked })=\frac{5.31 m i^{2}}{16.1 m i^{2}}=0.33
\end{gathered}
$$

Thus, 0.33 is the denominator of the likelihood ratio, and the distance likelihood ratio is given by:

$$
L R_{\text {distance }}=\frac{0.30}{0.33}=0.91
$$

The distance likelihood ratio of 0.91 indicates that it is slightly more likely that the crime is unlinked than that it is linked. However, this conclusion can be altered drastically given minor changes to the assumptions that underlie the calculations.

Because the numerator of the likelihood ratio, the probability of the distance given that the crime is linked, is estimated using the assumed probability distribution, using a different probability distribution will change the likelihood ratio. Consider instead that the distance measures were assumed to follow a distribution with a lower median. A Beta $(1,15)$ distribution is such a distribution. The probability density for a Beta $(1,15)$ is plotted in Figure 11, and the cumulative density is potted in Figure 12.


Figure 11: Beta(1,15) Probability Density


Figure 12: Beta(1,15) Cumulative Density

Because the Beta $(1,15)$ distribution demonstrates a lower median than a $\operatorname{Beta}(2,5)$ distribution, the cumulative density function results in a greater probability when evaluated at the same distance used in the previous example. The cumulative density of a $\operatorname{Beta}(1,15)$ when evaluated at 0.18 is 0.95 . This replaces 0.30 in the previous example as $P($ distance $\mid$ linked $)$. Because nothing else in the calculation has changed, this results is a greater distance likelihood ratio.

$$
L R_{\text {distance }}=\frac{0.95}{0.33}=2.88
$$

The conclusion that results from this higher likelihood ratio is that the hypothesis that the crime is linked is almost three times as likely as the hypothesis that the crime is unlinked, which is significantly different than under the previous assumption.

A similar change in the conclusion can be caused by a change in the area of analysis. Because the denominator, the probability of the distance given that the crime is unlinked, is a function of the total area of analysis, the likelihood ratio changes greatly when the area analyzed changes. Beginning with the previous example that resulted in a likelihood ratio of 2.88 , consider the effect of changing the total area of analysis from 16.1 square miles to 36.2 square miles. This changes the calculation of the denominator.

$$
P(\text { spatial distance } \mid \text { unlinked })=\frac{5.31 m i^{2}}{36.2 m i^{2}}=0.15
$$

This change in the denominator results in a greater likelihood ratio.

$$
L R_{\text {distance }}=\frac{0.95}{0.15}=6.33
$$

Whereas the previous calculation based on 16.1 square miles rendered the conclusion that the hypothesis that the crime is linked is almost three times as likely as the hypothesis that the crime is unlinked, the calculation based on 36.2 square miles yields the conclusion that linkage is over six times as likely. However, this is an overestimate because there is another way in which a larger analysis area changes the likelihood.

A larger analysis area also affects the likelihood because as the analysis area increases, the longest possible distance increases. Because the numerator, the probability of a distance given that a crime is linked, is based on a proportional distance, increasing the longest possible distance reduces this probability. For example, assume that increasing the total analysis area from 16.1 square miles increases the longest possible distance from 7.2 miles to 12.1 miles. The calculation of the proportional distance now changes.

$$
\text { Proportional distance }=\frac{1.3 \mathrm{mi} .}{12.1 \mathrm{mi} .}=0.11
$$

Thus, the proportional distance is reduced from 0.18 to 0.11 . Evaluating the $\operatorname{Beta}(1,15)$ cumulative density function at 0.11 yields a probability of 0.83 which is lower than the previous probability of 0.95 . This lowers the likelihood ratio.

$$
L R_{\text {distance }}=\frac{0.83}{0.15}=5.53
$$

Although increasing the longest possible distance lowers the likelihood ratio, this reduction does not offset the increase caused by changes in the denominator. Thus, the resulting likelihood ratio experiences a net increase, and the conclusion tilts toward greater belief in the hypothesis that linkage exists.

In general, the changes in the conclusions that result from using a different distribution or from altering the analysis area correspond to an underlying issue. Specifically, crime linkage systems that utilize distances are essentially measuring clustering, and clustering varies based on the scale of analysis. Figure 13 illustrates the locations of several linked and unlinked crimes observed within a small area. Plainly, there is little evidence of clustering that might differentiate the linked crimes from the unlinked crimes based on their locations.


Figure 13: Locations of Linked Crimes Appear Random

However, when the same data, are evaluated at a different scale, the clustering of the serial crime locations becomes apparent. Figure 14 illustrates the same observations within a larger sample space.


Figure 14: Locations of Linked Crimes Appear Clustered

This implies that, for crime linkage analysis, the assumptions about the spatial distribution of crimes are inherently tied to the size of the area of analysis. Thus, a linkage model using proportional distance measures that analyzes the entire United States would generate high distance likelihood ratios for all the crimes in a given city despite the fact that it is unreasonable to conclude that all the crimes in a city are actually linked. The distance likelihood ratios would change drastically if the analysis were conducted only the city, and the substantive conclusions of the linkage analysis would differ markedly.

For this reason, it is important to consider the scale of the area of analysis and to select a probability distribution that generates proportional distance measures that are consistent with reasonable assumptions about offenders' spatial behavior. These
assumptions should be rooted in empirical observations about offender behavior.
Therefore, an empirical evaluation of offenders' spatial behavior follows.

## Empirical Observations on Distance

To provide an empirical basis for offenders' spatial behavior two datasets were analyzed. The first dataset consisted of 27 serial murders and included information on 461 geographic locations. The second dataset consisted of 31 serial commercial robberies and included information on 519 geographic locations. The individual analysis for each crime type follows.

## Serial Murder

The serial murder dataset included 27 unique murder series with a total of 461 sites. The 461 sites included both primary and secondary locations including where murderers encountered their victims, where they committed the actual murders, and where they dumped the bodies ${ }^{6}$. The mean number of sites per series was 17.07 with a median of 15 . The maximum number of sites within a single series was 40 , and the minimum was 4 . The standard deviation was 10.21 . The 461 sites corresponded to 10,939 distance measurements. Analysis of the serial murder geographic data began with calculating descriptive statistics for the entire distribution of unstandardized distances. The results are presented in Table 7.

[^5]
# Table 7: Descriptive Statistics for Unstandardized Serial Murder Distances 

| Statistic | Observation |
| :--- | :---: |
| Mean | 48.96 mi. |
| Median | 7.63 mi. |
| Standard deviation | 128.31 mi. |
| Maximum | 1033.75 mi. |
| Minimum | 0.00 mi. |

The descriptive statistics in Table 7 indicate a highly skewed distribution, as can be seen in Figure 15.


Figure 15: Distribution of Unstandardized Serial Murder Distances

The next step involved standardizing the distances by dividing each distance within a single series by the longest distance within the series. Because the beta distributions used for the simulations are bounded from zero to one, the observed distances needed to be placed on this scale for analysis. While standardizing by the mean or median is a more common approach to standardization in general, this process would not have placed all distances on the interval from zero to one. Standardizing by the longest distance resulted in data that were measured as proportional distances, and the resulting scale ranged from zero to one. Since standardization using the longest distance was a linear transformation, the relative distances within a single series remained
unchanged. Descriptive statistics were calculated for the standardized data. The results are presented in Table 8.

# Table 8: Descriptive Statistics for Standardized ${ }^{7}$ <br> Serial Murder Distances 

| Statistic | Observation |
| :--- | :---: |
| Mean | 0.29 |
| Median | 0.19 |
| Standard deviation | 0.31 |
| Maximum | 1.00 |
| Minimum | 0.00 |

Standardizing the distances allowed the distance distribution to be approximated using a beta distribution. Additionally, standardization reduced the skew present in the total set of unstandardized data. The distribution of the standardized data is presented in Figure 16.

[^6]

Figure 16: Distribution of Standardized Serial Murder Distances

The method used to approximate a distribution from sample data involved the moment matching approach (AbouRizk, Halpin, \& Wilson, 1994). This approach takes advantage of the fact that the equations for the first two moments, the mean and the variance, are parameterized in terms of $\alpha$ and $\beta$. The mean of the beta distribution is defined as:

$$
\mu=\frac{\alpha}{\alpha+\beta}
$$

Rearranging this equation algebraically yields:

$$
\beta=\alpha\left(\frac{1}{\mu}-1\right)
$$

The variance of the beta distribution is defined as:

$$
\sigma^{2}=\frac{\alpha \beta}{(\alpha+\beta)^{2}(\alpha+\beta+1)}
$$

Substituting the previous solution for $\beta$, the variance equation, rearranged algebraically, yields:

$$
\alpha=\left(\frac{1-\mu}{\sigma^{2}}-\frac{1}{\mu}\right) \mu^{2}
$$

The equations for $\alpha$ and $\beta$ are then solved using the mean and variance from the sample data to complete the moment matching. Using the mean and variance observed for the standardized serial murder distance data indicated that a $\operatorname{Beta}(0.34,0.84)$ distribution was a relatively close approximation for the observed data. The summary statistics for the $\operatorname{Beta}(0.34,0.84)$ distribution are compared with the summary statistics from the serial murder distance data in Table 9:

Table 9: Comparison of Standardized Data to Proposed Beta Distribution

| Statistic | Data | Beta(0.34, 0.84) |
| :--- | :---: | :---: |
| Mean | 0.29 | 0.29 |
| Median | 0.19 | 0.17 |
| Standard deviation | 0.31 | 0.30 |

Figure 17 presents the $\operatorname{Beta}(0.34,0.84)$ distribution's probability density.


Figure 17: Beta(0.34, 0.84) Probability Density

Graphical comparison of the cumulative density for a $\operatorname{Beta}(0.34,0.84)$ distribution and the empirical cumulative density (ECD) of the serial murder spatial data is presented in Figure 18.


Figure 18: Comparison of ECD for Serial Murder Data to CDF for Proposed Distribution

Figure 18 demonstrates that the $\operatorname{Beta}(0.34,0.84)$ distribution approximated the serial murder data. As such, distances for serial murder simulations were drawn from a Beta $(0.34,0.84)$ distribution. As this distribution likewise represents an empirical understanding for distances in serial murder cases, $P($ distance $\mid$ linked $)$ was estimated from the $\operatorname{Beta}(0.34,0.84)$ cumulative density function.

## Serial Commercial Robbery

The serial commercial robbery dataset included 31 unique commercial robbery series with a total of 519 sites. Unlike the serial murder data, the serial commercial robbery data only included primary locations. Therefore, the locations represent the sites of the actual robberies. For the serial commercial robbery data, the mean number of sites per series was 16.74 with a median of 15 . The maximum number of sites within a single series was 45 , and the minimum was 3 . The standard deviation was 10.99 . The 519 sites correspond to 11,792 distance measurements. Analysis of the serial commercial robbery data began with calculating descriptive statistics for these unstandardized distances. The results are presented in Table 10:

# Table 10: Descriptive Statistics for Unstandardized Serial Commercial Robbery Distances 

| Statistic | Observation |
| :--- | :---: |
| Mean | 16.39 mi. |
| Median | 7.99 mi. |
| Standard deviation | 27.32 mi. |
| Maximum | 277.23 mi. |
| Minimum | 0.00 mi. |

As with serial murder, the descriptive statistics in Table 7 indicated a highly skewed distribution, as can be seen in Figure 19.


Figure 19: Distribution of Unstandardized Commercial Robbery Distances

Next, the distances within each commercial robbery series were standardized by dividing each by the longest distance within the series. This placed all of the distances on the interval from zero to one allowing the observations to be referenced against a beta distribution. Descriptive statistics were calculated for the standardized data.

# Table 11: Descriptive Statistics for Standardized ${ }^{8}$ Serial Commercial Robbery Distances 

| Statistic | Observation |
| :--- | :---: |
| Mean | 0.33 |
| Median | 0.27 |
| Standard deviation | 0.24 |
| Maximum | 1.00 |
| Minimum | 0.00 |

As with standardizing the serial murder distance data, standardization reduced skew for the aggregated distribution of the serial commercial robbery data. This is illustrated in Figure 20:

[^7]

Figure 20: Distribution of Standardized Commercial Robbery Distances

The equations for the parameters of a beta distribution with moments equal to the statistics observed for the standardized serial commercial robbery data suggested that a Beta $(0.94,1.96)$ distribution was a close approximation for the observed data. The summary statistics for the $\operatorname{Beta}(0.94,1.96)$ distribution are compared with the summary statistics for the serial commercial robbery data in Table 12.

Table 12: Comparison of Standardized Data to Proposed Beta Distribution

| Statistic | Data | Beta(0.94, 1.96) |
| :--- | :---: | :---: |
| Mean | 0.33 | 0.32 |
| Median | 0.27 | 0.28 |
| Standard deviation | 0.24 | 0.24 |

Figure 21 presents the probability density for the Beta( $0.94,1.96$ ) distribution.


Figure 21: Beta(0.94, 1.96) Probability Density

Graphical comparison of the cumulative density function for the $\operatorname{Beta}(0.94,1.96)$ distribution and the empirical cumulative density of the commercial robbery spatial data is presented in Figure 22.


Figure 22: Comparison of ECD for Commercial Robbery Data to CDF for Proposed Distribution

Figure 22 shows that the $\operatorname{Beta}(0.94,1.96)$ distribution approximated the commercial robbery data. Thus, distances for serial commercial robbery simulations were drawn from a $\operatorname{Beta}(0.94,1.96)$ distribution. Similar to the serial murder models, $P($ distance $\mid$ linked $)$ was estimated from the $\operatorname{Beta}(0.94,1.96)$ cumulative density function for the commercial robbery data.

## Time Difference for Crime Linkage Analysis

Criminological research also suggests that temporal information about crimes should provide information useful for crime linkage analysis. One method of measuring the relationship of events in time is to measure time differences between crimes. Studies have demonstrated distinct patterns in time differences for linked crimes. This consistency has been used in a variety of linkage tests. While generally less effective than distance, multiple studies have shown that time difference has utility in differentiating between linked and unlinked crimes. The proposed Bayesian linkage method includes time difference as a component. Therefore, a description of the time difference likelihood calculation and a discussion of the effects of the underlying assumptions follows.

The time difference likelihood is given by:

$$
L R_{\text {time }}=\frac{P(\text { time difference } \mid \text { linked })}{P(\text { time difference } \mid \text { unlinked })}
$$

As with distance, the numerator, the probability of a time difference given that the crime is linked, is estimated by assuming a probability density function for distances of linked crimes. The cumulative density function is evaluated at the observed distance, and the corresponding probability is used. The assumed probability density function for this example is a $\operatorname{Beta}(1,5)$ distribution. The $\operatorname{Beta}(1.5)$ probability density is plotted in Figure 23, and the cumulative density is plotted in Figure 24.


Figure 23: Beta(1,5) Probability Density


Figure 24: Beta(1,5) Cumulative Density

As with the distance measures, the time differences must be converted to proportional differences to fit a beta distribution. The conversion is similar to the distance conversion. However, since time is unidirectional within a single dimension, the denominator, the longest possible time difference, is simply the length of time searched by the analyst.

$$
\text { Proportional difference }=\frac{\text { Observed time difference }}{\text { Time period searched }}
$$

The denominator of the time difference likelihood ratio, the probability of the time difference given that the crime is unlinked, is estimated by calculating the probability of observing a time difference up to the observed time difference assuming that all possible time differences are equally likely. This assumption allows the probability to be estimated by calculating a simple proportion. This proportion is the same as the proportional time difference and is given by:

$$
P(\text { time difference } \mid \text { unlinked })=\frac{\text { Observed time difference }}{\text { Time period searched }}
$$

An example of the time difference likelihood ratio calculated for the observations in Table 13 follows:

Table 13: Sample Data for $L R_{\text {time }}$ Calculation

| Factor | Observation |
| :---: | :---: |
| Time difference | 3 days |
| Length of time searched | 62 days |

Using the data from Table 13, the probability of the time difference given that the crime is linked would be estimated by first converting the observed time difference of three days into a proportional difference.

$$
\text { Proportional difference }=\frac{3 \text { days }}{62 \text { days }}=0.05
$$

The assumed $\operatorname{Beta}(1,5)$ cumulative density function is then evaluated at 0.05 which yields a probability of 0.23 . Therefore, the probability of the time difference given that the crime is linked is equal to 0.23 , and this value is $P$ (time difference|linked). Because the denominator of the time difference likelihood ratio is simply the proportional difference, the time difference likelihood ratio is given by:

$$
L R_{\text {time }}=\frac{0.23}{0.05}=4.60
$$

The time difference likelihood ratio of 4.60 indicates that the hypothesis that the crime is linked is 4.6 times as likely as the hypothesis that the crime is unlinked. As with the distance likelihood ratio, this conclusion rests heavily on the assumptions used in the calculation. Again, changes in these assumptions can substantially alter the conclusions drawn.

Because the numerator of the time difference likelihood ratio, the probability of the time difference given that the crime is linked, is estimated with an assumed probability distribution, using a different probability distribution will change the likelihood ratio. If an offender were believed to exhibit a larger gap between crimes, then a distribution with a higher median, such as $\operatorname{Beta}(3,5)$ distribution, would be appropriate. The probability density for a $\operatorname{Beta}(3,5)$ distribution is plotted in Figure 25, and the cumulative density is plotted in Figure 26.


Figure 25: Beta(3,5) Probability Density


Figure 26: Beta(3,5) Cumulative Density

Because the $\operatorname{Beta}(3,5)$ distribution demonstrates a higher median than a $\operatorname{Beta}(1,5)$ distribution, the cumulative density function results in a smaller probability when evaluated at the same time difference used in the previous example. The cumulative density of a $\operatorname{Beta}(3,5)$ distribution when evaluated at 0.05 equals 0.004 . This replaces 0.23 in the previous example as $P$ (time differencelinked). Because nothing else in the calculation has changed, this results in a lower time difference likelihood ratio.

$$
L R_{\text {time }}=\frac{0.004}{0.05}=0.08
$$

Therefore, the conclusion changes from moderate evidence in support of the linked hypothesis to strong evidence in favor of the hypothesis against linkage.

The same phenomena observed for the denominator in the distance likelihood ratio calculation is observed for the time difference likelihood ratio. Specifically, searching data over longer periods of time will raise the likelihood ratio. This is because the proportional difference which equals the probability of the time difference given that the crime is unlinked is inversely related to the length of the search. Consider the search period changing from 62 to 124 days. The calculation of the proportional distance now changes.

$$
\text { Proportional difference }=\frac{3 \text { days }}{124 \text { days }}=0.024
$$

Thus the denominator in the time difference likelihood ratio is reduced from 0.05 to 0.024 raising the ratio.

$$
L R_{\text {time }}=\frac{0.004}{0.024}=0.167
$$

Whereas the previous calculation based on searching 62 days rendered strong evidence in favor of the crime not being linked, the calculation based on searching 124 days represents only moderate evidence of the conclusion.

As with the distance likelihood ratio, the effects of the change in the period of analysis on the numerator must be considered as well. Because the proportional
difference decreased from 0.05 to 0.024 , the cumulative density must be reevaluated at the new proportional difference to yield the proper probability of the time difference given that the crime is linked. Evaluating a $\operatorname{Beta}(3,5)$ distribution at 0.024 yields a probability of 0.0005 . This lowers the likelihood ratio.

$$
L R_{\text {time }}=\frac{0.0005}{0.024}=0.021
$$

As has been shown, changes in the distribution used in the calculation of the likelihood ratio alter the substantive conclusions. As with distance, this issue results from changes in the ability to observe temporal clustering that varies based on the length of time searched. Figure 27 illustrates the timeline for a series of linked crimes alongside the timeline for the unlinked crimes in the same time period. There is little evidence of clustering that might help differentiate the linked crimes from the unlinked crimes.


Figure 27: Temporal Locations of Linked Crimes Appear Random

Again, when evaluating the same data over a longer search period, the linked crimes appear clustered rather than random. Figure 28 illustrates the same observations over a longer search period.


Figure 28: Temporal Locations of Linked Crimes Appear Clustered

Figures 27 and 28 imply that, for crime linkage analysis using time differences, the assumptions about the temporal distribution of crimes are inherently tied to the length of the time period searched. A linkage model using time differences when searching a longer period of time (e.g., five years) would generate higher time difference likelihood ratios for all the crimes in a given month, than the same linkage analysis conducted when searching a shorter period (e.g., six months) using the same distribution.

For this reason, it is important to consider the length of time searched and to select a probability distribution that generates proportional time difference measures that are consistent with reasonable assumptions about offenders' temporal behavior. These assumptions should be rooted in empirical observations about offender behavior. Therefore, an empirical evaluation of offenders' temporal behavior follows.

## Empirical Observations on Time Difference

The analyses undertaken to provide an empirical basis for offenders' temporal behavior utilized the same two datasets that were analyzed for spatial behavior. In addition to information on the geographic locations associated with each incident, both datasets included the dates that each incident took place. This allowed for temporal analyses to be conducted on both the serial murder and the serial commercial robbery data, and the individual analysis for each crime type follows.

## Serial Murder

All 27 series in the serial murder dataset included temporal information. However, date information was missing for 17 incidents. This left a total of 444 incidents with known dates. The mean number of incidents with date information per series was 16.44 with a median of 15 . The maximum number was 40 , and the minimum was 4 . The standard deviation was 9.22 . The 444 incidents with date information yielded 9,885 time difference measures. Analysis of the serial murder temporal data began with calculating descriptive statistics for the unstandardized time differences. The results are presented in Table 14.

Table 14: Descriptive Statistics for Unstandardized Serial Murder Time Differences

| Statistic | Observation |
| :--- | ---: |
| Mean | 775.73 days |
| Median | 281.00 days |
| Standard deviation | $1,483.43$ days |
| Maximum | $16,480.43$ days |
| Minimum | 0.00 days |

The descriptive statistics in Table 14 indicate a highly skewed distribution. This is further demonstrated in Figure 29.


Figure 29: Distribution of Unstandardized Serial Murder Time Differences

The next step involved standardizing the time differences by dividing each difference within a single series by the longest difference within the series. Descriptive statistics were calculated for the standardized data. The results are presented in Table 15.

## Table 15: Descriptive Statistics for Standardized ${ }^{9}$ <br> Serial Murder Time Differences

| Statistic | Observation |
| :--- | :---: |
| Mean | 0.29 |
| Median | 0.19 |
| Standard deviation | 0.30 |
| Maximum | 1.00 |
| Minimum | 0.00 |

As with the serial murder distance data, standardization reduced the skew present in the unstandardized data. The distribution for the standardized data is presented in Figure 30.

[^8]

Figure 30: Distribution of Standardized Serial Murder Time Differences

Following the method of moment matching previously presented indicated that a $\operatorname{Beta}(0.39,0.96)$ distribution is a close approximation to the observed data. The summary statistics for a $\operatorname{Beta}(0.39,0.96)$ distribution are compared with the summary statistics from the serial murder time difference data in Table 16.

Table 16: Comparison of Standardized Data to Proposed Beta Distribution.

| Statistic | Data | Beta(0.39, 0.96) |
| :--- | :---: | :---: |
| Mean | 0.29 | 0.29 |
| Median | 0.19 | 0.18 |
| Standard deviation | 0.30 | 0.30 |

Figure 31 presents the $\operatorname{Beta}(0.39,0.96)$ distribution's probability density.


Figure 31: Beta(0.39, 0.96) Probability Density

Graphical comparison of the cumulative density for a $\operatorname{Beta}(0.39,0.96)$ distribution and the empirical cumulative density (ECD) for the serial murder time difference data is presented in Figure 32.


Figure 32: Comparison of ECD for Serial Murder Temporal Data to CDF for Proposed Distribution

Figure 32 demonstrates that the $\operatorname{Beta}(0.39,0.96)$ distribution approximated the serial murder temporal data. As such, time difference for serial murder simulations were drawn from a $\operatorname{Beta}(0.39,0.96)$ distribution. Likewise, $P($ time differencellinked $)$ was estimated using a $\operatorname{Beta}(0.39,0.96)$ cumulative density function.

## Serial Commercial Robbery

All 31 series in the commercial robbery dataset included temporal information. However, date information was missing for one incident. This left a total of 518 incidents with known dates. The mean number of incidents with date information per series was 16.71 with a median of 15 . The maximum number was 45 , and the minimum was 3 . The standard deviation was 11.0. The 518 incidents with date information yielded 11,772 time difference measures. Analysis of the commercial robbery temporal data analysis began with calculating descriptive statistics for the unstandardized time differences. The results are presented in Table 17.

## Table 17: Descriptive Statistics for Unstandardized <br> Commercial Robbery Time Differences

| Statistic | Observation |
| :--- | ---: |
| Mean | 89.63 days |
| Median | 40.00 days |
| Standard deviation | 168.75 days |
| Maximum | 1788.0 days |
| Minimum | 0.00 days |

The descriptive statistics in Table 17 indicated a highly skewed distribution. This is further demonstrated in Figure 33.


Figure 33: Distribution for Unstandardized Commercial Robbery Time Differences

The next step involved standardizing the time differences by dividing each difference within a single series by the longest difference within the series. Descriptive statistics were calculated for the standardized data. The results are presented in Table 18.

# Table 18: Descriptive Statistics for Standardized ${ }^{10}$ <br> Commercial Robbery Time Differences 

| Statistic | Observation |
| :--- | :---: |
| Mean | 0.33 |
| Median | 0.28 |
| Standard deviation | 0.25 |
| Maximum | 1.00 |
| Minimum | 0.00 |

As with the commercial robbery distance data, standardization reduced the skew present in the aggregated unstandardized time difference data. The distribution for the standardized data is presented in Figure 34.

[^9]

Figure 34: Distribution of Standardized Commercial Robbery Time Differences

Following the method of moment matching previously used indicated that a $\operatorname{Beta}(0.87,1.73)$ distribution was a close approximation to the observed data. The summary statistics for a $\operatorname{Beta}(0.87,1.73)$ distribution are compared with the summary statistics from the serial murder time difference data in Table 19.

Table 19: Comparison of Standardized Data to Proposed Beta Distribution

| Statistic | Data | Beta(0.87, 1.73) |
| :--- | :---: | :---: |
| Mean | 0.33 | 0.33 |
| Median | 0.28 | 0.29 |
| Standard deviation | 0.25 | 0.24 |

Figure 35 presents the $\operatorname{Beta}(0.87,1.73)$ distribution's probability density.


Figure 35: Beta(0.87, 1.73) Probability Density

Graphical comparison of the cumulative density for a $\operatorname{Beta}(0.87,1.73)$ distribution and the empirical cumulative density (ECD) for the commercial robbery time difference data is presented in Figure 36.


Figure 36: Comparison of ECD for Commercial Robbery Temporal Data to CDF for Proposed Distribution

Figure 36 demonstrates that the $\operatorname{Beta}(0.87,1.73)$ distribution approximated the commercial robbery temporal data. As such, time distance for commercial robbery simulations were drawn from a $\operatorname{Beta}(0.87,1.73)$ distribution. Likewise, $P$ (time differencelinked) was estimated using a $\operatorname{Beta}(0.87,1.73)$ cumulative density function.

## Modus Operandi for Crime Linkage Analysis

Social-cognitive theory suggests that behavioral information about crimes should provide information that is useful for crime linkage analysis, and criminological research has supported this suggestion by demonstrating both offender consistency and distinctiveness. Likewise, research has shown that patterns of behavior can be used to establish crime linkage. The proposed Bayesian linkage method includes M.O. factors as a component of the linkage analysis. Therefore, a description of the time difference likelihood ratio calculation follows.

The M.O. likelihood ratio is given by:

$$
L R_{M O(i)}=\frac{P(\text { behavior } \mid \text { linked })}{P(\text { behavior } \mid \text { unlinked })}
$$

The numerator, the probability of observing a behavior given that the crime is linked, is estimated by assuming a probability for the individual offender's consistency. The denominator, the probability of observing a behavior given that a crime is unlinked, is estimated using the base rate of the behavior in the data being analyzed for linkage. Therefore, under the assumption that an offender is $75 \%$ consistent in exhibiting a certain behavior and observing this behavior in $25 \%$ of the population of linkage analyses, the M.O. likelihood ratio is:

$$
L R_{M O(i)}=\frac{0.75}{0.25}=3.5
$$

The M.O. likelihood ratio of 3.5 indicates that the hypothesis that the crime is linked is 3.5 times as likely as the hypothesis that the crime is unlinked. As with the previous likelihood ratios, this conclusion rests heavily on the assumption used in the calculation, specifically the assumption that the offender in question is $75 \%$ consistent in exhibiting the behavior. Changing this assumption drastically alters the conclusion. For example, assuming that the offender is $50 \%$ consistent lowers the likelihood ratio.

$$
L R_{M O(i)}=\frac{0.50}{0.25}=2.0
$$

Lowering the consistency reduces the M.O. likelihood ratio to 2.0 thus reducing the evidence of linkage from moderate to weak evidence. Similarly, assuming that an offender is more consistent is his or her behavior raises the likelihood ratio. For example, assume that the offender is perfectly consistent exhibiting a M.O. behavior in $100 \%$ of linkage analyses. The likelihood ratio becomes:

$$
L R_{M O(i)}=\frac{1.00}{0.25}=4.0
$$

Altering the assumption about the offender's consistency therefore changes the conclusion. It is important to note that, while changes in assumptions concerning consistency alter the conclusions, the denominator ultimately has a greater effect on the likelihood ratio. Considering that the likelihood ratio is a ratio of probabilities and that probabilities are bounded between zero and one, extremely high likelihood ratios are only
possible with very small denominators. As the denominator approaches zero, the likelihood ratio approaches infinity regardless of the value of the numerator. Thus, uniqueness of behavior has a greater overall effect on the likelihood ratio and subsequent conclusions than consistency. Figure 37 demonstrates the effects of consistency and uniqueness on the likelihood ratio.


Figure 37: Effects of Consistency and Uniqueness on Likelihood Ratio ${ }^{11}$

[^10]DNA evidence is an example of this phenomenon. Because the probability that two crimes are unlinked given matching DNA evidence is extremely low (i.e., DNA is very unique), the likelihood ratio associated with this evidence would be very high. This results in the substantive conclusion that there is little possibility that two crimes with the same DNA evidence present are unlinked.

Because the numerator is estimated using the base rate of a behavior in the data analyzed for possible linkages, there is potentially less assumptive error in this calculation. In contrast, assumptive error concerning offender consistency can still substantially alter conclusions resulting from M.O. likelihood ratios. Therefore, an empirical basis for consistency assumptions is necessary.

## Empirical Observations on Modus Operandi

To provide a glimpse on offender consistency for M.O. factors associated with various types of serial crime, two datasets were used. The first dataset included M.O. factors for a variety of serial murders. Because serial murder is a rare occurrence, the data included closed serial murders that occurred between 1963 and 1993 in the United States, Canada, and the United Kingdom. The second dataset included M.O. factors for closed serial commercial robberies that occurred between 2009 and 2012 in San Antonio, Texas.

## Serial Murder

The serial murder dataset consisted of 18 serial murderers. The mean number of crimes committed by each offender was $13.83(\mathrm{SD}=7.27)$ with individual series ranging
from 5 to 33 crimes. The data set included information on 12 separate M.O. characteristics. The M.O. characteristics included: (1) gender of victim; (2) victim/killer relationship; (3) selection method; (4) victim traits; (5) victim activity; (6) hunting style; (7) attack style; (8) approach; (9) control method; (10) murder method; (11) crime location set; and (12) attempt to hide body.

Gender of victim was coded as either male or female. Victim/killer relationship was coded as either stranger or acquaintance. Killer selection method was coded as either random or patterned, with patterned representing selection of victims based on their membership in a specific group (e.g., child), their unique characteristics (e.g., living near a freeway ramp), or their unique actions that are not part of typical routine activities (e.g., hitchhiking). Victim traits were coded as either specific or non-specific with specific indicating victim selection based on individual traits (i.e., a particular appearance, action, response, etc.). Victim activity was coded as either home, work, commuting, walking or jogging, hitchhiking, other travel, visiting friend, outdoor recreation, bar, other social event, or prostitution. Hunting style and attack style were coded as either hunter, poacher, stalker, troller, or trapper. A hunter sets out in search of a victim from their home. A poacher sets out in search of a victim from a site other than their home. A troller opportunistically encounters victims while involved in other, nonpredatory activities. A trapper is an individual that assumes a position or creates a situation that allows him to encounter victims in a location under their control. Killer approach was coded as either confidence approach, surprise, or blitz. Control method was coded as either firearm, knife, blunt instrument, strangulation, physical force, intoxicant, threat, or blitz (i.e., where the victim was immediately killed). Murder method
was coded as either firearm, knife, blunt instrument, strangulation, physical force, or poison. Crime location set describes which actions (encounter, attack, murder, disposal) were undertaken at each of the known sites involved in the crime and was coded as either $E \rightarrow A \rightarrow M \rightarrow D$ (all actions occurring at different sites), $E \rightarrow A \rightarrow M D$ (encounter at site one, attack at site two, and murder and disposal at site three, etc.), $E \rightarrow A M \rightarrow D, E A$ $\rightarrow M \rightarrow D, E A \rightarrow M D, E \rightarrow A M D, E A M \rightarrow D, E A M D$. Body disposal was coded as either displayed, dumped, other-not hidden, casually hidden, or well hidden.

The data were recoded dichotomously for each of the factors corresponding to a given M.O. characteristic. For example, victim gender was coded in the variables female and male. Each of these variables was dichotomously coded indicating that the victim was female with female equal to one and male equal to zero. This allowed the offender's consistency for a given M.O. characteristic to be calculated using the proportion of crimes exhibiting a specific M.O. factor. From these consistency measures for each offender, descriptive statistics were calculated for each behavior. The means, standard deviations, the minimums and maximums, and the number of offenders exhibiting each characteristic are presented in Table 20.

Table 20: Descriptive Statistics for Serial Murder Consistency

| MO Characteristic | Mean | Std. Dev. | Min | Max | N |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Victim gender |  |  |  |  |  |
| Male | 0.53 | 0.36 | 0.10 | 1.00 | 13 |
| Female | 0.83 | 0.20 | 0.40 | 1.00 | 14 |
| Victim/killer relationship |  |  |  |  |  |
| $\quad$ Stranger | 0.90 | 0.14 | 0.58 | 1.00 | 18 |
| $\quad$ Acquaintance | 0.29 | 0.16 | 0.10 | 0.50 | 9 |
| Selection method |  |  |  |  |  |
| $\quad$ Patterned | 0.73 | 0.26 | 0.20 | 1.00 | 15 |
| Random | 0.57 | 0.30 | 0.13 | 1.00 | 13 |
| Victim traits |  |  |  |  |  |
| Specific | 0.72 | 0.26 | 0.10 | 1.00 | 14 |
| Non-specific | 0.58 | 0.36 | 0.10 | 1.00 | 13 |
| Victim activity |  |  |  |  |  |
| Home | 0.60 | 0.33 | 0.13 | 1.00 | 8 |
| Work | 0.26 | 0.12 | 0.10 | 0.39 | 5 |
| Commuting | 0.28 | 0.17 | 0.17 | 0.59 | 5 |
| Walking or jogging | 0.43 | 0.26 | 0.10 | 0.82 | 10 |
| Hitchhiking | 0.29 | 0.19 | 0.10 | 0.59 | 7 |
| Other travel | 0.42 | 0.37 | 0.12 | 1.00 | 8 |
| Visiting friends | 0.40 | 0.16 | 0.29 | 0.59 | 3 |
| Other outdoor | $0.82^{*}$ |  |  |  | 1 |
| Bar | 0.26 | 0.16 | 0.10 | .50 | 7 |
| Other social activity | 0.68 | 0.11 | 0.59 | 0.80 | 3 |
| Prostitution | 0.48 | 0.31 | .010 | 1.00 | 6 |
| Hunting style | 0.58 | 0.34 | 0.12 | 1.00 | 11 |
| Hunter | 0.75 | 0.29 | 0.24 | 1.00 | 13 |
| Poacher | $0.82^{*}$ |  |  |  | 1 |
| Stalker | 0.47 | 0.36 | 0.10 | 1.00 | 6 |
| Troller | 0.19 | 0.06 | 0.13 | 0.25 | 3 |
| Trapper |  |  |  |  |  |
|  |  |  |  |  |  |

Table 20-Continued

| MO Characteristic | Mean | Std. Dev. | Min | Max | N |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Approach |  |  |  |  |  |
| $\quad$ Surprise | 0.60 | 0.33 | 0.25 | 1.00 | 6 |
| Biltz | 0.63 | 0.39 | 0.12 | 1.00 | 5 |
| Attack style |  |  |  |  |  |
| $\quad$ Hunter | 0.88 | 0.22 | 0.20 | 1.00 | 17 |
| $\quad$ Stalker | 0.51 | 0.31 | 0.25 | 1.00 | 6 |
| Control method |  |  |  |  |  |
| $\quad$ Firearm | 0.67 | 0.38 | 0.21 | 1.00 | 6 |
| Knife | 0.73 | 0.29 | 0.36 | 1.00 | 4 |
| Blunt | 0.15 | 0.01 | 0.14 | 0.15 | 2 |
| Strangle | 0.68 | 0.16 | 0.50 | 0.80 | 3 |
| Physical force | 0.45 | 0.28 | 0.12 | 0.85 | 9 |
| Intoxicant | 0.75 | 0.38 | 0.17 | 0.95 | 4 |
| Threat | 0.50 | 0.47 | 0.17 | 0.83 | 2 |
| Blitz | 0.65 | 0.33 | 0.17 | 1.00 | 10 |
| Murder method |  |  |  |  |  |
| Firearm | 0.74 | 0.30 | 0.25 | 1.00 | 9 |
| Knife | 0.53 | 0.34 | 0.12 | 1.00 | 9 |
| Blunt Instrument | 0.45 | 0.26 | 0.12 | 0.90 | 9 |
| Strangle | 0.62 | 0.31 | 0.10 | 1.00 | 10 |
| Physical Force | 0.46 | 0.33 | 0.10 | 0.95 | 7 |
| Poison | $0.83^{*}$ |  |  |  | 1 |
| Crime location set | 0.24 | 0.19 | 0.10 | 0.38 | 2 |
| E $\rightarrow$ A $\rightarrow$ M $\rightarrow$ D | 0.27 | 0.14 | 0.13 | 0.40 | 3 |
| E $\rightarrow$ A $\rightarrow$ MD | 0.54 | 0.32 | 0.20 | 1.00 | 8 |
| E $\rightarrow$ AM $\rightarrow$ D | 0.10 | 0.13 | 0.33 | 3 |  |
| EA $\rightarrow$ M $\rightarrow$ D | 0.19 | 0.13 | 0.10 | 0.33 | 3 |
| EA $\rightarrow$ MD | 0.27 | 0.18 | 1.00 | 8 |  |
| E $\rightarrow$ AMD | 0.06 | 0.14 | 0.25 | 3 |  |
| EAM $\rightarrow$ D | 0.18 | 0.50 | 1.00 | 7 |  |
| EAMD |  |  |  |  |  |
|  |  |  |  |  |  |

Table 20-Continued

| MO Characteristic | Mean | Std. Dev. | Min | Max | N |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Hidden body |  |  |  |  |  |
| $\quad$ Display | $0.93^{*}$ |  |  |  | 1 |
| Dumped | 0.38 | 0.29 | 0.14 | 0.75 | 6 |
| Other - not hidden | 0.64 | 0.28 | 0.10 | 1.00 | 9 |
| Casually hidden | 0.31 | 0.23 | 0.10 | 0.78 | 9 |
| Well hidden | 0.58 | 0.30 | 0.13 | 1.00 | 11 |

* As only one offender exhibited the behavior; standard deviations, minimums, and maximums are not presented.

Table 20 demonstrates that offenders were very consistent for some M.O. factors and inconsistent for others. The mean of the mean consistencies reported in Table 20 was $0.54(\mathrm{SD}=0.22)$, and the median was slightly higher at 0.57 . This indicated that offenders were, on average, slightly more than 50 percent consistent. For linkage analysis, an important issue concerns M.O. characteristics for which offenders are inconsistent. The first quartile for the mean of mean consistencies reported in Table 20 was 0.40 and the third quartile was 0.73 , supporting the contention that offenders were generally more consistent than inconsistent across this set of M.O. factors. This phenomenon is illustrated in Figure 38.


Figure 38: Histogram of Mean Consistencies for Serial Murder M.O. Behaviors

Another important consideration is the frequency at which each individual M.O. consistency is observed. Calculating a distribution consisting of all observed individual offender M.O. consistencies provided a way to establish the relative chance that an observed behavior occurred with a given consistency. When considering consistency in this way, the mean consistency was $0.59(\mathrm{SD}=0.32)$, and the median was 0.59 as well. The first and third quartiles were 0.28 and 0.91 respectively. Figure 39 presents the distribution of individual offender M.O. consistencies.


Figure 39: Histogram of Individual Offender M.O. Consistencies for Serial Murder

This illustrates the important consideration that, for serial murder, offenders exhibited very high levels of consistency (over 0.90) for more than $25 \%$ of M.O. factors.

The previously used method of moment matching indicated that a $\operatorname{Beta}(0.79$, 0.55 ) distribution was a close approximation to the observed serial murder M.O. data. The summary statistics for a $\operatorname{Beta}(0.79,0.55)$ distribution are compared with the summary statistics from the serial murder M.O. data in Table 21.

Table 21: Comparison of Observed Data
to Proposed Beta Distribution.

| Statistic | Data | Beta(0.79, 0.55) |
| :--- | :---: | :---: |
| Mean | 0.59 | 0.59 |
| Median | 0.59 | 0.62 |
| Standard deviation | 0.32 | 0.32 |

Figure 40 presents the $\operatorname{Beta}(0.79,0.55)$ distribution's probability density.


Figure 40: Beta(0.79, 0.55) Probability Density

Graphical comparison of the cumulative density for a $\operatorname{Beta}(0.79,0.55)$ distribution and the empirical cumulative density (ECD) for the serial murder temporal distance data is presented in Figure 41.


Figure 41: Comparison of ECD for Serial Murder M.O. Data to CDF for Proposed Distribution

Figure 41 demonstrates that the $\operatorname{Beta}(0.79,0.55)$ distribution approximated the serial murder M.O. data. As such, M.O. probabilities for serial murder simulations were drawn from a $\operatorname{Beta}(0.79,0.55)$ distribution. $P($ behavior $\mid$ linked $)$ was estimated at 0.59 , the mean of the $\operatorname{Beta}(0.79,0.55)$ distribution.

## Serial Commercial Robbery

The dataset for serial commercial robbery consisted of crimes from 15 serial commercial robbers. The mean number of crimes committed by each offender in the set was $12.29(\mathrm{SD}=6.30)$ with individual series ranging from 3 to 22 crimes. The dataset included information on 12 M.O. characteristics. The M.O. characteristics included: (1) target; (2) time of day: (3) day of week; (4) attempt to hide identity; (5) use of gloves; (6) use of a bag; (7) transportation;
(8) presence of a co-offender; (9) weapon used; (10) use of physical violence; (11) mannerism; and (12) items stolen.

Target was coded as a bank, government building, automotive supply store, gas station or convenience store, grocery store, fast food establishment, restaurant, retail store, or hotel or motel. Time of day was coded into one of four categories morning (6:00 am to 1:59 pm), afternoon (2:00 pm to 5:59 pm) evening (6:00 pm to $10: 59 \mathrm{pm}$ ), and night (11:00 pm to 5:59 am). Day of week was coded by day (Sunday to Saturday). Attempt to hide identity was coded as mask, bandanna, hat, or sunglasses. The use of gloves was dichotomously coded as gloves (yes $=1$ or no $=0$ ). Bag was coded as either improvised or brought. Transportation was coded as either vehicle or on foot. The presence of a co-offender was coded as absent or present. The use of a weapon was coded as a hand gun, shotgun, knife, or hidden weapon. Physical violence was coded as implied or used. Mannerism was coded as calm and confident, loud and aggressive, reassuring, instructional, or restrained. Items stolen were coded as cash, cellphonelelectronics, jewelry, purse, store products, or items from individuals.

As with serial murder, the data were recoded dichotomously for each of the variables that form a given M.O. characteristic. For example, time of day was recoded into four new variables: (1) morning; (2) afternoon; (3) evening; (4) and night. Each of these variables was coded as either yes (1) or no (0). For example, a morning robbery was coded as morning $=1$, afternoon $=0$, evening $=0$, and night $=0$. As before, an offender's consistency for a given M.O. characteristic was calculated using the proportion of crimes exhibiting a specific M.O. factor. From these consistency measures for each offender, descriptive statistics were calculated for each of behaviors. The means, standard deviations, the minimums and maximums, and the number of offenders exhibiting each of the characteristics are presented in Table 22.

Table 22: Descriptive Statistics for Serial Commercial Robbery Consistency

| MO Characteristic | Mean | Std. Dev. | Min | Max | N |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Target |  |  |  |  |  |
| Automotive supply store | 0.41 | 0.43 | 0.11 | 0.71 | 2 |
| Gas station | 0.60 | 0.38 | 0.33 | 0.88 | 2 |
| Grocery store | 0.63* |  |  |  | 1 |
| Fast food | 0.64 | 0.28 | 0.16 | 1.00 | 9 |
| Restaurant | 0.73 | 0.25 | 0.50 | 1.00 | 3 |
| Retail store | 0.52 | 0.33 | 0.11 | 1.00 | 10 |
| Time of day |  |  |  |  |  |
| Morning | 0.62 | 0.21 | 0.25 | 0.77 | 5 |
| Afternoon | 0.53 | 0.32 | 0.15 | 1.00 | 7 |
| Evening | 0.51 | 0.28 | 0.25 | 1.00 | 7 |
| Night | 0.51 | 0.28 | 0.22 | 1.00 | 6 |
| Day of week |  |  |  |  |  |
| Sunday | 0.47 | 0.23 | 0.15 | 0.71 | 5 |
| Monday | 0.24 | 0.08 | 0.14 | 0.33 | 7 |
| Tuesday | 0.34 | 0.23 | 0.11 | 0.71 | 8 |
| Wednesday | 0.47 | 0.33 | 0.14 | 1.00 | 7 |
| Thursday | 0.25 | 0.15 | 0.11 | 0.50 | 7 |
| Friday | 0.20 | 0.06 | 0.11 | 0.25 | 4 |
| Saturday | 0.32 | 0.18 | 0.11 | 0.67 | 10 |
| Hide identity |  |  |  |  |  |
| Mask | 0.84 | 0.33 | 0.25 | 1.00 | 5 |
| Bandanna | 1.00** |  |  |  | 3 |
| Hat | 0.82 | 0.20 | 0.50 | 1.00 | 8 |
| Sunglasses | 0.89 | 0.18 | 0.68 | 1.00 | 3 |
| Hoodie | 0.57 | 0.39 | 0.22 | 1.00 | 4 |
| Gloves |  |  |  |  |  |
| Used | 0.76 | 0.35 | 0.25 | 1.00 | 4 |
| Bag used |  |  |  |  |  |
| Improvised | 0.71* |  |  |  | 1 |
| Brought | 0.72 | 0.26 | 0.25 | 1.00 | 7 |

## Table 22-Continued

| MO Characteristic | Mean | Std. Dev. | Min | Max | N |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Transportation |  |  |  |  |  |
| Vehicle | 1.00** |  |  |  | 9 |
| On foot | 1.00 |  |  |  | 3 |
| Co-offender |  |  |  |  |  |
| Absent | 1.00** |  |  |  | 11 |
| Present | 1.00** |  |  |  | 4 |
| Weapon |  |  |  |  |  |
| Knife | 0.33* |  |  |  | 1 |
| Hand gun | 0.94 | 0.19 | 0.33 | 1.00 | 12 |
| Shotgun | 1.00* |  |  |  | 1 |
| Hidden weapon | 0.33* |  |  |  | 1 |
| Physical violence |  |  |  |  |  |
| Implied | 0.99 | 0.03 | 0.89 | 1.00 | 12 |
| Used | 0.39 | 0.34 | 0.11 | 0.77 | 3 |
| Mannerism |  |  |  |  |  |
| Calm and confident | 0.78 | 0.31 | 0.29 | 1.00 | 6 |
| Loud and aggressive | 0.89 | 0.18 | 0.50 | 1.00 | 9 |
| Reassuring | 1.00* |  |  |  | 1 |
| Instructional | 0.67 | 0.34 | 0.13 | 1.00 | 10 |
| Restrained | 0.77 | 0.24 | 0.53 | 1.00 | 3 |
| Items stolen |  |  |  |  |  |
| Cash | 0.98 | 0.04 | 0.89 | 1.00 | 12 |
| Cell phone/electronics | 0.31 | 0.03 | 0.29 | 0.33 | 2 |
| Jewelry | 0.56 | 0.38 | 0.33 | 1.00 | 3 |
| Purse | 0.50 | 0.24 | 0.33 | 0.67 | 2 |
| Store products | 0.19 | 0.07 | 0.14 | 0.24 | 2 |
| Items from individuals | 0.53 | 0.27 | 0.14 | 0.77 | 5 |

* As only one offender exhibited the behavior; standard deviations, minimums, and maximums are not presented. ** All offenders exhibiting the behavior were $100 \%$ consistent. Standard deviations, minimums, and maximums are not presented.

Table 22 demonstrates that serial commercial robbers varied in their M.O. consistency across factors. However, serial commercial robbers were slightly more
consistent than serial murderers. The mean of the mean consistencies reported in Table 19 was $0.61(\mathrm{SD}=0.27)$, and the median was slightly higher at 0.62 . Thus, on average, offenders were more than $50 \%$ consistent. The first quartile for the means reported in Table 19 was 0.33 , and the third quartile was 1.0 . This indicated that the consistency of serial commercial robbers was more evenly distributed across lower ranges than the M.O. consistencies for serial murderers. This is illustrated in Figure 42.


Figure 42: Histogram of Mean Consistencies for Serial Commercial Robbery M.O. Behaviors

Analyzing the frequencies of individual offender M.O. consistencies yielded a higher mean consistency of $0.67(\mathrm{SD}=0.33)$ and a median of 0.73 . The first and third quartiles were 0.33 and 1.0 respectively. Figure 43 presents the distribution of individual offender M.O. consistencies.


Figure 43: Histogram of Individual Offender M.O. Consistencies for Serial Commercial Robbery

As with serial murder, the general conclusion was that serial commercial robbers were highly consistent with more than $25 \%$ of the distribution demonstrating perfect consistency (i.e., consistency $=1.0$ ), across many M.O. factors.

Moment matching indicated that a $\operatorname{Beta}(0.67,0.33)$ distribution was a close approximation to the observed commercial robbery M.O. data. The summary statistics for a $\operatorname{Beta}(0.67,0.33)$ distribution are compared with the summary statistics from the commercial robbery M.O. date in Table 23.

Table 22: Comparison of Observed Data to Proposed Beta Distribution.

| Statistic | Data | Beta(0.67, 0.33) |
| :--- | :---: | :---: |
| Mean | 0.67 | 0.67 |
| Median | 0.73 | 0.76 |
| Standard deviation | 0.33 | 0.33 |

Figure 44 presents the $\operatorname{Beta}(0.67,0.33)$ distribution's probability density.


Figure 44: Beta(0.67, 0.33) Probability Density

Graphical comparison of the cumulative density for a $\operatorname{Beta}(0.67,0.33)$ distribution and the empirical cumulative density (ECD) for the commercial robbery M.O. data is presented in Figure 45.


Figure 45: Comparison of ECD for Commercial Robbery M.O. Data to CDF for Proposed Distribution

Figure 45 demonstrates that the $\operatorname{Beta}(0.67,0.33)$ distribution approximated the commercial robbery M.O. data. As such, M.O. probabilities for commercial robbery simulations were drawn from a $\operatorname{Beta}(0.67,0.33)$ distribution. $P($ behavior|linked $)$ was estimated using 0.67 , the mean of the $\operatorname{Beta}(0.67,0.33)$ distribution.

## CHAPTER V

## PREDICTIVE VALIDITY OF THE LINKAGE MODEL

To address the four research questions, a variety of separate analyses are presented. The first section addresses the first research question, whether the proposed linkage method using the likelihood ratio demonstrates predictive validity, using ROC analysis. The second section addresses whether the Bayesian hypothesis test provides a useful framework for classifying the likelihood ratio using descriptive statistics and graphical methods to analyze various measures of performance at the different levels of evidence. The third section assesses the value of adding additional information to the model using graphical methods to analyze the results of both the ROC analysis and the Bayesian hypothesis analysis. The last section addresses the final research question, identifying the characteristics of information that impact model performance, using linear regression models to assess the relationships between characteristics of the data and measures of predictive validity from each linkage analysis.

## Validation of the Likelihood Ratio for Linkage Analysis

Validation of the likelihood ratio for crime linkage involved calculating the AUC for each of the 3.5 million sets of crimes for both the murder and commercial robbery simulations. Descriptive statistics were calculated for the AUCs from each set of crimes that corresponded to the same crime type. The analysis of each set of AUCs is presented in the following two sections.

## Serial Murder AUC Analysis

AUCs were calculated for each of the 3.5 million simulated murder linkage analyses. Descriptive statistics were then generated for the entire set of AUCs. The descriptive statistics for the murder AUCs are presented in Table 24, and the distribution of AUCs is presented in Figure 46.

Table 24: Descriptive Statistics for Murder AUCs

| Statistic | Observation |
| :--- | :---: |
| Mean | 0.81 |
| Median | 0.83 |
| Standard deviation | 0.10 |
| Maximum | 1.00 |
| Minimum | 0.34 |



Figure 46: Histogram of Murder AUCs

The descriptive statistics indicated that across all models (i.e., from the distance only model to the model that included distance, time, and 12 M.O. factors) the likelihood ratio exhibited "good" predictive capacity on average (mean AUC $=0.81$ ) for serial murder linkage analyses. Further the median of 0.83 indicated that $50 \%$ of serial murder linkage analyses exhibited either "good" or "excellent" predictive capacity. Table 25 presents the proportion of linkage analyses exhibiting AUCs at different levels of predictive capacity for the murder models.

Table 25: Serial Murder AUC Performance Levels

| Level of Performance | Proportion |
| :--- | :---: |
| Uninformative $(\mathrm{AUC}<0.5)$ | 0.01 |
| Poor $(0.5 \leq \mathrm{AUC}<0.7)$ | 0.12 |
| Fair $(0.7 \leq \mathrm{AUC}<0.8)$ | 0.27 |
| Good $(0.8 \leq \mathrm{AUC}<0.9)$ | 0.40 |
| Excellent $(\mathrm{AUC} \geq 0.9)$ | 0.20 |

## Serial Commercial Robbery AUC Analysis

AUCs were calculated for each of the 3.5 million simulated commercial robbery linkage analyses. Descriptive statistics were then generated for the entire set of AUCs. The descriptive statistics for the commercial robbery AUCs are presented in Table 26, and the distribution of AUCs is presented in Figure 47.

Table 26: Descriptive Statistics for Commercial Robbery AUCs

| Statistic | Observation |
| :--- | :---: |
| Mean | 0.80 |
| Median | 0.81 |
| Standard deviation | 0.14 |
| Maximum | 1.00 |
| Minimum | 0.32 |



Figure 47: Histogram of Commercial Robbery AUCs

The descriptive statistics indicated that across all models (i.e., from the distance only model to the model that included distance, time, and 12 M.O. factors) the likelihood ratio exhibited "good" predictive capacity on average (mean AUC $=0.80$ ) for commercial robbery linkage analyses. Further the median of 0.81 indicated that $50 \%$ of the linkage analyses exhibited either "good" or "excellent" predictive capacity. Table 27 presents the proportion of cases exhibiting AUCs at different levels of predictive capacity for the commercial robbery models.

Table 27: Commercial Robbery AUC Performance Levels

| Level of Performance | Proportion |
| :--- | :---: |
| Uninformative $(\mathrm{AUC}<0.5)$ | 0.02 |
| Poor $(0.5 \leq \mathrm{AUC}<0.7)$ | 0.26 |
| Fair $(0.7 \leq \mathrm{AUC}<0.8)$ | 0.21 |
| Good $(0.8 \leq \mathrm{AUC}<0.9)$ | 0.20 |
| Excellent $(\mathrm{AUC} \geq 0.9)$ | 0.30 |

## Crime Linkage Using the Bayesian Hypothesis Test

Analyzing the utility of the Bayesian hypothesis test as a method to predict serial crime linkage consisted of calculating descriptive statistics for the percent of a series identified, the number of actual serial crimes identified, and the percent of true positives for each level of evidence for each set of crimes of a given type using the same number of factors to predict linkage. Histograms of each measure as well as performance plots were also generated. The descriptive statistics, histograms, and performance plots are presented in the following two sections.

## Serial Murder Linkage at Different Levels of Evidence

To assess the utility of the Bayesian hypothesis test for murder linkage analysis, three measures of predictive performance were assessed using the murder data. The first measure, the percent of a series identified, provided a comparative measure of the method's ability to predict linkages. The number of hits provided insight into the actual
information gain yielded by the model, and the percent of true positives provided an estimate of the confidence in the estimates. Means and medians for each of these measures at each level of evidence are presented in Table 28 for the full information model (i.e., serial murder predictions based on distance and time difference in conjunction with 12 M.O. factors ${ }^{12}$ ). Detailed distributional information as well as performance plots are presented in Figures 48 to 51.

Table 28: Serial Murder Linkage for Full Information Model

|  | Level of Evidence |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 199,048 | 214,634 | 221,099 | 242,946 |
| Percent of samples with predictions | 79.62 | 85.85 | 88.44 | 97.18 |
|  |  |  |  |  |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 18.57 | 22.44 | 26.22 | 56.38 |
| $\quad$ Median | 13.33 | 17.65 | 21.74 | 52.94 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 2.91 | 3.44 | 3.92 | 8.04 |
| $\quad$ Median | 2.00 | 2.00 | 3.00 | 7.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 12.31 | 19.13 | 29.17 | 54.98 |
| Median | 4.69 | 7.96 | 14.81 | 54.55 |

[^11]

Figure 48: Serial Murder Full Information Model at the Substantial Level of Evidence


Figure 49: Serial Murder Full Information Model at the Strong Level of Evidence


Figure 50: Serial Murder Full Information Model at the Very Strong Level of Evidence


Figure 51: Serial Murder Full Information Model at the Extreme Level of Evidence

The analysis indicated that, for the serial murder data, the substantial, strong, and very strong levels of evidence resulted in poor predictions (i.e., median true positive percentages of $4.69,7.96$, and 14.81 ). In $50 \%$ of the linkage analyses at these levels of evidence, approximately 85 to $95 \%$ of the predictions were false positives. This indicated that the substantial, strong, and very strong levels of evidence represent decision thresholds that do not discriminate between linked and unlinked murders when using all available information.

At the extreme level of evidence, the performance improved. The median true positive percentage of $54.55 \%$ indicated that in $50 \%$ of the linkage analyses, more predictions were correct than incorrect. As indicated in Figure 49, at the extreme level of evidence, the model was $100 \%$ accurate in approximately $10 \%$ of linkage analyses. Thus, the extreme level of evidence demonstrated utility in discerning between linked and unlinked murders.

## Serial Commercial Robbery at Different Levels of Evidence

To assess the utility of the Bayesian hypothesis test for commercial robbery linkage, the same three measures of predictive performance were examined using the commercial robbery data. Means and medians for each of these measures at each level of evidence are presented in Table 29 for the commercial robbery full information model (i.e., serial murder predictions based on distance and time difference in conjunction with

12 M.O. factors ${ }^{13}$ ). Detailed distributional information as well as performance plots are presented in Figures 52 to 55 .

Table 29: Commercial Robbery Linkage for Full Information Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 233,534 | 200,981 | 146,436 | 88,035 |
| Percent of samples with predictions | 93.41 | 80.39 | 58.57 | 35.21 |
|  |  |  |  |  |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 34.04 | 34.49 | 35.36 | 43.22 |
| $\quad$ Median | 27.59 | 26.32 | 25.00 | 31.25 |
|  |  |  |  |  |
| Number of hits | 8.72 | 8.12 | 7.53 | 7.91 |
| $\quad$ Mean | 6.00 | 5.00 | 5.00 | 5.00 |
| $\quad$ Median |  |  |  |  |
|  |  |  |  |  |
| Percent of true positives | 43.76 | 55.62 | 64.20 | 71.54 |
| $\quad$ Mean | 38.46 | 58.33 | 75.00 | 90.00 |
| Median |  |  |  |  |

[^12]

Figure 52: Commercial Robbery Full Information Model at the Substantial Level of

## Evidence



Figure 53: Commercial Robbery Full Information Model at the Strong Level of Evidence


Figure 54: Commercial Robbery Full Information Model at the Very Strong Level of Evidence


Figure 55: Commercial Robbery Full Information Model at the Extreme Level of Evidence

The analysis indicated that, for the commercial robbery data, the substantial level of evidence resulted in poor predictions (i.e., a median true positive percentage of $38.46 \%$ ). In $50 \%$ of the linkage analyses, approximately $60 \%$ of the predictions were inaccurate at the substantial level of evidence.

At the strong, very strong, and extreme levels of evidence, the performance improved. The median true positive percentages of $58.33 \%, 75 \%$, and $90 \%$ indicated that in $50 \%$ of the linkage analyses, more predictions were correct than incorrect for the strong level of evidence. Using the very strong level of evidence, more than 3 out of 4 predictions were correct in more than $50 \%$ of linkage analyses, and, at the extreme level of evidence, more than 9 out of 10 predictions were correct in $50 \%$ of the linkage analyses. Figure 52 indicates that, at the strong level of evidence, the model was $100 \%$ accurate in approximately $25 \%$ of linkage analyses. Similarly, Figure 53 indicates that the model was $100 \%$ accurate in approximately $40 \%$ of linkage analyses using the very strong level of evidence. Finally, Figure 54 indicates that the model was $100 \%$ accurate in approximately $45 \%$ of linkage analyses using the extreme level of evidence.

## The Value of Additional Information

For any particular sample, the inclusion of additional information (e.g., adding additional M.O. factors) altered the predictive capacity of the linkage method. These changes sometimes improved and, other times, degraded the method's ability to accurately predict crime linkage. Figures 56 through 62 illustrate the changes that occurred in a single linkage analysis as the amount of information used in the calculation of $L R_{\text {final }}$ was increased. As these ROC curves represent the linkage method's
performance for a single linkage analysis, the figures are illustrative rather than informative about performance.


Figure 56: Example ROC Curve Based on Distance Only


Figure 57: Example ROC Curve Based on Distance and Time


Figure 58: Example ROC Curve Based on Distance, Time, and One M.O. Factor


Figure 59: Example ROC Curve Based on Distance, Time, and Two M.O. Factors


Figure 60: Example ROC Curve Based on Distance, Time, and Three M.O. Factors


Figure 61: Example ROC Curve Based on Distance, Time, and Four M.O. Factors


Figure 62: Example ROC Curve Based on Distance, Time, and Five M.O. Factors

Figures 56 to 62 demonstrate the fact that including additional information has the potential to increase predictive capacity as well as reduce it. Because, it is important to understand how the inclusion of additional information generally affected the linkage model across a large number of cases, the following analyses were conducted.

Determining the impact of adding additional information involved two separate sets of analyses for each crime type. The first set of analyses involved comparing ROC curves generated using a different number of factors in the calculation of $L R_{\text {final }}$. The second set of analyses involved comparing changes in performance measures resulting from different numbers of factors in the calculation of $L R_{\text {final }}$ for predictions rendered using the Bayesian hypothesis test. The following two sections present these analyses for both the serial murder and commercial robbery data.

## Serial Murder Linkage and Additional Information

Analysis of the value of additional information in the serial murder linkage tests began with a comparison of AUC values that emerged when including different numbers of factors. The changes in the mean and median AUCs for each model are presented in Figure 63.


Figure 63: Changes in Serial Murder AUC with Additional Information ${ }^{14}$

Figure 63 indicates that inclusion of additional information increased the model's predictive capacity using the serial murder data. Importantly, including the time difference measurements substantially increased the model's predictive capacity. Adding additional M.O. factors resulted in less substantial gains.

The next set of analyses assessed the value of additional information for serial murder linkage under the Bayesian hypothesis test. The analyses involved plotting the means and medians of two measures of model predictive performance, the hit rate (i.e., the percent of crimes classified as linked that are actually linked) and the percent of a

[^13]series identified against the information used in the model to generate $L R_{\text {final }}$. This process was performed at each of the four levels of evidence.


Figure 64: Changes in Serial Murder Hit Rate at Substantial Evidence


Figure 65: Changes in Serial Murder Percent Identified at Substantial Evidence

Figures 64 and 65 indicate that the including additional information had a slight negative impact on the percent of true positives for the serial murder data at the substantial level of evidence. However, the inclusion of the time difference measurement substantially increased the percent of a series identified without altering the hit rate. The additional M.O. data had little impact on the percent of a series identified. Given that the individual M.O. likelihood ratios were calculated assuming a consistency (the numerator of $L R_{M O}$ ) of 0.59 , including additional M.O. factors raised the total likelihood ratio whenever the behavior exhibited a uniqueness (the denominator of $L R_{M O}$ ) less than 0.59 . This implied that, on average, including additional M.O. factors raised the total likelihood ratio for crimes that were correctly predicted as linked by a model based on less
information. In turn, this resulted in these crimes being correctly classified but at a higher level of evidence.


Figure 66: Changes in Serial Murder Hit Rate at Strong Evidence


Figure 67: Changes in Serial Murder Percent Identified at Strong Evidence

Figures 66 and 67 indicate that the inclusion of the additional information had a negative impact on the percent of true positives for the serial murder data at the strong level of evidence as well. As with the substantial level of evidence, the inclusion of the time difference measurement substantially increased the percent of a series identified. However, unlike the substantial level of evidence, the inclusion of the time difference data had a negative effect on the hit rate. The inclusion of additional M.O. data increased the percent identified when eight or more factors were included.


Figure 68: Changes in Serial Murder Hit Rate at Very Strong Evidence


Figure 69: Changes in Serial Murder Percent Identified at Very Strong Evidence

Figures 68 and 68 indicate that the inclusion of the additional information had a negative impact on the percent of true positives for the serial murder data at the very strong level of evidence as well. The negative effect of including the time difference data was more extreme than observed at the strong level of evidence. However, the inclusion of the time difference measurement substantially increased the percent of a series identified. The additional M.O. data increased the percent identified when eight or more M.O. factors were included. The additional M.O. data reduced the hit rate as more information was included. However, the effect was less substantial than the effect of including time difference.


Figure 70: Changes in Serial Murder Hit Rate at Extreme Evidence


Figure 71: Changes in Serial Murder Percent Identified at Extreme Evidence

Figures 70 and 71 indicate that including additional information had a negative impact on the percent of true positives for the serial murder data at the extreme level of evidence as well. As before, the inclusion of the time difference measurement substantially increased the percent of a series identified. The additional M.O. data increased the percent identified regardless of the number of M.O. factors included. Thus, including additional M.O. information resulted in a greater number of total true positives while increasing the number of false positives. This indicated that the decision threshold for the extreme level of evidence may have simply been too low for the serial murder data.

## Serial Commercial Robbery Linkage and Additional Information

Analysis of the value of additional information in the serial commercial robbery linkage tests began with a comparison of AUC values between the predictions that resulted from inclusion of different numbers of factors. The changes in the mean AUC for each model is presented in Figure 72.


Figure 72: Changes in Commercial Robbery AUC with Additional Information

Figure 72 indicates that inclusion of additional information increased the model's overall predictive capacity using the commercial robbery data. While the inclusion of the time difference measurements resulted in a greater increase to the model's predictive capacity than the inclusion of M.O. factors, the inclusion of additional M.O. factors resulted in greater gains than observed for the serial murder models.

The next set of analyses to determine the value of additional information for serial commercial robbery involved plotting the means and medians of two measures of model predictive performance, the hit rate and the percent of a series identified, against the information used to generate $L R_{\text {final }}$. As with the serial murder analysis, this process was performed at each of the four levels of evidence.


Figure 73: Changes in Commercial Robbery Hit Rate at Substantial Evidence


Figure 74: Changes in Commercial Robbery Percent Identified at Substantial Evidence

Figures 73 and 74 indicate that the inclusion of the time difference measurement had a positive effect on both the hit rate and the percent of a series identified at the substantial level of evidence for the commercial robbery data. The inclusion of M.O. data had little effect on either performance measure.


Figure 75: Changes in Commercial Robbery Hit Rate at Strong Evidence ${ }^{15}$

[^14]

Figure 76: Changes in Commercial Robbery Percent Identified at Strong Evidence

Figures 75 and 76 indicate that the additional information had a negative impact on the percent of true positives for the commercial robbery data at the strong level of evidence as well. However, the inclusion of additional data (temporal distance and M.O. information) increased the percent of a series identified.


Figure 77: Changes in Commercial Robbery Hit Rate at Very Strong Evidence


Figure 77: Changes in Commercial Robbery Percent Identified at Very Strong Evidence

Figures 77 and 78 indicate that the adding M.O. information had an initial negative impact on the percent of true positives for the commercial robbery data at the very strong level of evidence. However, the additional information from the next five M.O. factors increased the hit rate. After six M.O. factors were included, additional M.O. information had a negligible impact on the hit rate. Including additional M.O. information had a positive effect on the percent identified. The first M.O. factor included had the greatest impact with each additional M.O. factor having a lower but consistent effect.


Figure 79: Changes in Commercial Robbery Hit Rate at Extreme Evidence


Figure 80: Changes in Commercial Robbery Percent Identified at Extreme Evidence

Figures 79 and 80 indicate that the inclusion of the additional information had a positive impact on model performance at the extreme level of evidence. Adding M.O. information had a positive impact on the hit rate for the first six M.O. factors. After that, the impact of additional M.O. information was negligible. More M.O. information generally improved the percent of a series identified, with the first M.O. factor having the greatest impact.

## Characteristics of Information that Impact Model Performance

To assess the characteristics of information that impacted model performance, linear regression analysis was utilized. Two linear regression models were estimated. Because AUC is a measure of the overall predictive capacity of a test, each regression
model was estimated using the AUC for the type of linkage analysis (i.e., serial murder or commercial robbery) as the dependent variable. This analysis informed about the effect of the characteristics of the information on the overall ability of the proposed linkage method to correctly classify linked and unlinked crimes.

The first model involved regressing the AUCs for the serial murder linkage analyses on seven characteristics of the information used to perform the linkage analyses. These characteristics included the proportion of serial offenses, the median serial distance, the median non-serial distance, the median serial time difference, the median non-serial time difference, the mean consistency, and the mean uniqueness. The second model involved regressing the AUCs for the commercial robbery linkage analyses on the same set of information characteristics. All seven characteristics were independent, with each one exhibiting a variance inflation factor less than 1.02. The regression models are presented in the following sections.

## AUC Regression for Serial Murder Linkage

The first model was based on the serial murder linkage analyses. The model was significant ( $\mathrm{p}<0.001$ ), and all of the individual coefficients were significant ( $\mathrm{p}<0.001$ ) as well. The model explained $54.8 \%$ of the variation in the predictive capacity of the linkage method as measured by the AUCs. On average, greater predictive capacity was related to lower serial crime proportions, lower serial distances, lower serial time differences, and lower M.O. uniqueness. Additionally, predictive capacity was related to greater non-serial distances, greater non-serial time differences, and greater offender M.O. consistency. The regression estimates are presented in Table 30.

Table 30: Serial Murder Linear Regression for AUC

|  | Coefficient |
| :--- | :---: |
| Serial proportion | -0.006 |
| Serial distance | -0.454 |
| Non-serial distance | 0.329 |
| Serial time | -0.427 |
| Non-serial time | 0.309 |
| Consistency | 0.230 |
| Uniqueness | -0.230 |
| Intercept | 0.732 |
| $R^{2}$ | 0.548 |

Serial distance was the strongest predictor followed by serial time difference. Behavioral consistency and uniqueness had equally strong effects albeit in differing directions. Because uniqueness is defined as $P$ (behavior|linkage), higher values of the uniqueness measure represent behaviors that are less unique. Therefore the differing directions of these two effects are consistent with the expected behavior outlined in Chapter 4. With the exception of the proportion of serial offenses, the other effects were all in the anticipated directions.

Because two-outcome classification systems typically perform better when the base rate of the outcome being classified is closer to 0.5 , the proportion of serial crimes was expected to have a positive relationship to predictive validity which suggested that the linkage method's performance should improve as the proportion of serial murders increased. However, the observed negative effect indicated the opposite, that the performance of the model degraded as the proportion of linked murders increased. However, it is important to note that this effect
(-0.006) was negligible. While the effect was statistically significant, the finding of statistical significance was driven largely by the sample size, and the effect of the proportion of serial crimes on AUC was substantially lower that the effects of the other information.

## AUC Regression for Commercial Robbery Linkage

The second linear regression model was estimated using the commercial robbery data. Again, the AUC for each linkage analysis was used as the dependent variable. The model was significant ( $\mathrm{p}<0.001$ ). As in the serial murder model, all of the coefficients were likewise significant ( $\mathrm{p}<0.001$ ), and the model explained $54.7 \%$ of the variation in performance as measured by the AUCs. On average, greater predictive capacity was related to lower serial crime proportions, lower serial distances, lower serial time differences, and lower M.O. uniqueness. Additionally, predictive capacity was related to greater non-serial distances, greater non-serial time differences, and greater offender M.O. consistency. The regression estimates for the commercial robbery analysis are presented in Table 31.

Table 31: Commercial Robbery Linear Regression for AUC

|  | Coefficient |
| :--- | :---: |
| Serial proportion | -0.008 |
| Serial distance | -0.158 |
| Non-serial distance | 0.218 |
| Serial time | -0.110 |
| Non-serial time | 0.068 |
| Consistency | 0.116 |
| Uniqueness | -0.239 |
| Intercept | 0.886 |
| $R^{2}$ | 0.547 |

Unlike the serial murder model, behavioral uniqueness was the strongest predictor followed by non-serial distance and serial distance for the commercial robbery data. The reduced effect for distance was attributed to the lower spatial clustering observed in the serial commercial robbery crimes. Behavioral consistency was more strongly related to linkage performance for two reasons. First, because distance was less able to predict commercial robbery linkage than serial murder linkage, the M.O. information had more potential to impact the performance. Second, because greater M.O. consistency was present in the commercial robbery data, the M.O. likelihood ratios were higher for the same level of uniqueness than in the serial murder linkage analyses. This allowed for a greater overall effect for uniqueness in the commercial robbery data. As before, the proportion of serial crimes was negatively related to linkage performance, but the effect was again negligible as in the serial murder regression model.

## CHAPTER VII

## CONCLUSION

This study attempted to provide evidence of the predictive validity of the Bayesian crime linkage method proposed by Rossmo et al. (2012). The analysis attempted to answer four specific research questions: (1) does the proposed linkage method demonstrate predictive validity; (2) does the Bayesian hypothesis test provide a useful framework for classifying the likelihood ratio generated from the method; (3) what is the value of additional information; and (4) what are the characteristics of information that impact model performance? This chapter presents the interpretation of the findings related to each question. Three sections are presented in this chapter. The first section discusses the findings placing them into the context using information presented in the study as well as previous research on crime linkage methods. The second section discusses limitations of the study, and the third section discusses directions for future research on crime linkage analysis.

## Discussion

The analysis indicated that the proposed linkage method exhibited predictive validity. Evidence of the utility of the Bayesian hypothesis test for crime linkage was split, with some evidence indicating that the levels used for the Bayesian decision thresholds worked well for commercial robbery but poorly for serial murder. Evidence indicated that the inclusion of additional information improved the predictive capacity of
the test, but that the inclusion of additional information was problematic for the Bayesian hypothesis test applied to the serial murder data at lower levels of evidence. Finally, multiple analyses indicated that several of the characteristics of the data were related to greater predictive ability. The following section of this chapter is divided into four distinct parts, each part addressing the interpretation of a single research question.

## Research Question One

The analysis directed at the first research question was the most fundamental in demonstrating the predictive validity of the linkage method. Using AUCs to quantify predictive capacity, the results demonstrated that the model exhibited predictive validity for both the murder and commercial robbery data. This was an important finding because it established the model's capacity to differentiate between linked and unlinked crimes. The analyses conducted thereafter would have been meaningless if the AUC analysis had failed to demonstrate predictive capacity for the likelihood ratio.

The findings of the AUC analysis were consistent with previous research that demonstrated the predictive capacity of distance, time difference, and M.O. factors using other linkage methods. Incorporation of this information through the proposed likelihood ratio generated valid predictions as suggested by the previous research. The evidence indicated that the proposed likelihood ratio is a viable way of incorporating this information for making classifying crimes as linked.

Importantly, the AUC analysis indicated that the model's predictive capacity was relatively similar for both the serial murder and commercial robbery data (serial murder mean $\mathrm{AUC}=0.81$, commercial robbery mean $\mathrm{AUC}=0.80$ ). This was an interesting
finding considering that the two models varied substantially in the base rate of the occurrence of serial crime, the average distance (i.e., serial murder exhibiting shorter distances than serial commercial robbery), the average time difference (i.e., serial murder exhibiting shorter time differences than serial commercial robbery), and the average behavioral consistency (i.e., serial murder mean consistency $=0.59$, serial commercial robbery mean consistency 0.67 ). However, this finding provides additional support for the linkage method in general, as the similar performance observed likely resulted from both the serial murder and commercial robbery simulations referencing the same distributions for data generation and likelihood ratio estimation. This indicates that the linkage method results in valid predictions when the underlying assumptions are correct.

## Research Question Two

The analysis addressing the second research question provided additional insight into the predictive ability of the linkage method and provided information concerning the behavior of the method when evaluated at the levels of evidence suggested by the Bayesian hypothesis test. In general, the Bayesian hypothesis test was a useful tool for predicting linkages from the commercial robbery data, but was not as useful for serial murder predictions. Predictions for both crime types made at the extreme level of evidence were the most accurate.

The substantial level of evidence was determined to exhibit poor performance overall (e.g., median $95.31 \%$ false positive rate for the full information serial murder data, and median $61.54 \%$ false positive rate for the full information commercial robbery data). The strong and very strong levels of evidence resulted in poor predictions for the
serial murder data (e.g., median false positive rates for the full information model of $92.04 \%$ and $85.19 \%$, respectively). However, these levels of evidence performed better for the commercial robbery data (e.g., median false positive rates for the full information model of $41.67 \%$ and $25 \%$, respectively). Finally, the extreme level of evidence performed best for both sets of data. Predictions made for the murder data at the extreme level of evidence resulted in a median false positive rate of $44.45 \%$. Predictions made for the commercial robbery data at the extreme level of evidence were substantially better with a median false positive rate of $10 \%$. Importantly, at the extreme level of evidence, predictions were $100 \%$ accurate for approximately $25 \%$ of serial murder linkage analyses and $45 \%$ of commercial robbery linkage analyses. The greater performance for the extreme level of evidence suggests that this level may be most appropriate for conducting linkage analysis.

The lower performance of the Bayesian hypothesis test for classifying the serial murder data in general likely resulted in part from the lower base rates of serial crime in the serial murder dataset. The serial murder data exhibited very low rates of linked crimes $($ mean proportion of linked crimes $=0.03$, median proportion of linked crimes $=0.01)$ compared to the commercial robbery data (mean proportion of linked crimes $=0.30$, median proportion of linked crimes $=0.29$ ). Thus, although the true positive rates observed for serial murder predictions were low, they represented substantial gains over chance predictions. Additionally, the lower performance resulted from overall higher likelihood ratios generated by the serial murder data. This indicated that the decision thresholds in the Bayesian hypothesis tests were simply too low for accurate serial murder linkage.

Although this finding indicated that the Bayesian hypothesis test may be problematic for use in serial murder linkage analysis, these results did not indicate that the linkage method itself does not have potential to be used in linking serial murder. The results of the AUC analysis indicated that the proposed linkage method has the ability to correctly classify linked murders. In contrast, the results of the Bayesian hypothesis test analyses only indicated that the Bayesian levels of evidence were improper decision thresholds for classifying the likelihood ratio. The performance of the Bayesian hypothesis test varied between the crime types, indicating that a fixed numeric threshold may be inappropriate for the linkage method. Instead, the graphical method of analyzing the $\log$ of the likelihood ratio advanced by Rossmo et al. (2012) may be superior. Because the graphical method does not rely on a fixed decision threshold, it may be potentially more useful across a wide range of likelihood ratios. Additionally, since graphical analysis orders the crimes analyzed and prioritization is a fundamental task of crime analysis, graphical analysis may be a more intuitive approach to accomplishing the goals of crime linkage analysis.

## Research Question Three

The analysis addressing the third research question provided insight into the value of adding additional information to the model. The first set of analyses addressing this issue involved calculating the mean AUCs for each crime type for each model (i.e., the distance only model, the distance and time model, the distance, time, and one M.O. factor model, etc.). These AUCs were then compared as the models included more information.

For both crime types, the inclusion of additional information consistently increased the predictive capacity of the linkage method. This finding was consistent with Davies et al.'s (2012) conclusion that more information increases the predictive capacity of linkage analysis. For both types of crime, distance had the greatest predictive capacity, inclusion of time difference increased predictive capacity, and inclusion of M.O. factors had less substantial effects. Whereas the distance only model for the serial murder data had a mean AUC of 0.71, the distance and time model had a mean AUC of 0.81 . Each additional M.O. factor increased the AUC for the murder data by approximately 0.4 . The same phenomenon was observed for the commercial robbery data although the effect was smaller. Whereas the distance only model for the commercial robbery data had a mean AUC of 0.72 , the distance and time model had a mean AUC of 0.74 . Each M.O. factor increased the mean AUC for the commercial robbery data, but the effect of each additional M.O. factor was less than the effect of the previous.

The greater increase observed when incorporating the temporal information in the serial murder predictions was likely due to the fact that serial murder observations were more closely clustered in time when time was measured proportionally. The mean proportional serial murder time difference was 0.29 , whereas the mean proportional serial commercial robbery time difference was 0.33 . While this difference may seem trivial, considering the rapid increase of the cumulative density functions used to generate the time difference linkage probabilities, this difference had a large effect on $L R_{\text {time }}$ which translated into a large effect on $L R_{\text {final }}$.

The lesser apparent value of the M.O. information and the diminishing effect of including more information was expected. Considering that distance alone was a fair
predictor for both sets of data, there was less left to predict after the distance information had already been incorporated. Additionally, the inclusion of M.O. information for the serial murder data may have added less to the predictive capacity of the method due to the base rate of serial offending. Individual samples in the serial murder data had as many as 2,000 non-serial crimes. Due to the large number of non-serial crimes, several individual non-serial crimes likely had $100 \%$ accordance for all M.O. factors despite the low probability of this occurring. Because the mean consistency for non-serial offenders was 0.5 , the probability of observing all 12 factors was $0.5^{12}(0.0002)$. With 1,000 crimes in a sample, the expectation of a non-serial crime exhibiting this perfect set of factors was 0.2. For high, although not perfect, levels of M.O. match (e.g., matches on 9 of the 12 factors), this issue was more problematic. The probability of observing a match on 9 of the 12 factors was 0.002 . Thus, in a sample of 1,000 cases, two crimes would have been expected to exhibit these high levels of concordance.

This also explains the observation that, for both types of crime, inclusion of additional M.O. information tended to lower the true positive rate. However, because of higher consistencies for serial crimes, adding this information tended to increase the percent of a series identified for both crime types.

Importantly, the lower levels of evidence (i.e., the substantial and strong levels) both showed initial gains in the percent of a series identified as additional information was included. However, these gains quickly reached a point of diminishing returns. The same phenomenon was observed for the commercial robbery data at the substantial level of evidence. This is likely due to the fact that, for crimes correctly predicted as linked, the inclusion of additional information, on average, elevated the likelihood ratio. Thus, the
predictions made at lower levels of evidence were reclassified as predictions at higher levels of evidence. For the murder data at the extreme level of evidence adding additional information resulted in a consistent gain in the percent of a series identified. For the commercial robbery data, the increases in the percent of series identified were seen at the three higher levels of evidence (i.e., the strong, the very strong, and the extreme levels of evidence).

## Research Question Four

The analysis used to address the fourth research question provided insight into the types of information that were related to better predictive performance. The linear regression analysis indicated that the relationships between characteristics of spatial and temporal information and predictive performance were consistent with the prior assumptions. Specifically, the linkage method performed better when serial crimes were closer together in space and time and when non-serial crimes were further apart in space and time. This is consistent with the theoretical analysis in Chapter 4 that indicated predictions based on spatial and time difference are essentially based on the clustering of the data. It is likewise supportive of Burrell et al.'s (2012) conclusion that distance is better for determining linkages when analyzing larger areas.

Overall, the relationships between M.O. behavioral consistency and uniqueness were in the directions expected. As serial offenders exhibited greater consistency, the predictive performance of the method improved. Likewise, as behavior became more unique, the performance of the method improved. The magnitudes of the effects for consistency and uniqueness were equal for the serial murder linkage analysis, but the
effect for uniqueness was almost twice the magnitude than the effect for consistency in the commercial robbery data. Because this increased effect for uniqueness is related to greater consistency in the commercial robbery data, this finding underscores both the value of relying on distinct evidence when it is available and the value of incorporating behaviors that are most consistent into crime linkage analysis models.

## Study Limitations

While the results of this study provide evidence that the proposed method has utility linking crimes, there were several important limitations that should be acknowledged. Some of these limitations were related to the use of simulated data, while others were related to the processes used to estimate the probabilities and to calculate the likelihood ratios.

## Limitations of Simulated Data

Analysis of simulated data provides useful conclusions when the assumptions used to generate the data closely mirror reality. In this study, simulated data for serial crimes were generated based on empirical assessments of serial offender behavior. Specifically, empirical data on spatial and temporal distances between serial murders and between serial commercial robberies were analyzed. The observations made from these data were then generalized to distributions that were reasonable approximations about serial offenders' spatial and temporal behavior. These distributions, in turn, were used to simulate serial spatial and temporal observations. Thus, the assumptions used to generate the serial spatial and temporal data had empirical support.

The same process was used to provide an empirical basis for M.O. observations. Empirical data on offender M.O. behavior was analyzed, and observations from these data were generalized to distributions designed to approximate offenders' M.O. behavior in serial crimes. However, two important issues concerning this process are noteworthy. First, the data used to estimate distributions for offender M.O. behavior had fewer observations than the data used to estimate distributions for spatial and temporal behavior. Likewise, the data used for the M.O. behavior analysis included few M.O. factors. In reality, offenders exhibit many behaviors in the commission of their crimes. Considering this reality, generalizing about an offenders' average behavioral consistency from analyzing limited data with information on only a few M.O. factors is problematic. Thus, while the analysis involving M.O. factors presented is informative, it should be viewed as exploratory.

The second limitation related to the assumptions used to generate the data results from the simulation of non-serial offender behavior. Unlike the serial behavior, the nonserial behavior lacked an empirical basis. Instead, non-serial spatial, temporal, and M.O. data were generated randomly. Because research indicates that criminal opportunity is non-random, generating the non-serial data in this way was a significant limitation. However, the nature of the simulations was designed to limit the effect of this problem. The power of Monte Carlo simulation for testing the linkage method came, in part, from the sheer number of linkage analyses performed. While the random nature of the nonserial data was problematic, the size of the simulation somewhat offset this issue. Importantly, the data generation for non-serial offenses resulted in observations that were
random on average when all samples were viewed at the same time but exhibited nonrandom pattering within samples.

## Limitations of Probability and Likelihood Ratio Estimation

The most important limitation related to the estimation of the probabilities used to calculate the likelihood ratios concerns the underlying assumptions. Analysis of empirical data was used to generalize probability distributions. Because this analysis of empirical data represented an understanding of behavior, these same probability distributions were used to estimate probabilities for distance and time difference observations as well as for estimating probabilities for M.O. consistency. Therefore, there was no average assumptive error in this study. If there were substantial variations between the actual distributions of offender behavior and the distributions used to estimate the probabilities employed to calculate the likelihood ratios, then the error between these two sets of distributions would have an impact on the predictive validity of the method. The effect of assumptive error on linkage prediction is an important consideration for crime linkage analysis; however, that question was beyond the scope of this study.

Another limitation of the study relates to the estimation of serial consistency. Analysis of empirical data rendered a set of M.O. consistencies, and the mean of all consistencies was used as the estimate of $P$ (behaviorlinked) for all M.O. factors. While this measure of central tendency represents the best guess of this probability for any unknown factor, this could be a biased estimate. Because offenders may be much more consistent in certain behaviors than others, estimating this probability using the mean empirical consistency for the actual behavior observed may result in greater predictive
validity. As addressed earlier, the limited empirical data used to understand offender M.O. behavior made it difficult to pursue this approach. The mean of all consistencies is a viable estimate of the probability but represents a limitation if better estimates could be found.

The final limitation concerns the number of M.O. factors used. AUC analysis of both the serial murder and commercial robbery data indicated that the overall predictive capacity of the method improved as more information was added. Neither model reached a point where the AUC failed to increase as additional information was added. Therefore, it is possible that inclusion of additional M.O. information may improve model performance. The decision to include only 12 M.O. factors was based on preliminary analyses that indicated performance maximization at 8 to $10 \mathrm{M} . \mathrm{O}$. factors and on certain computational limitations. However, the full scale simulation presented in this research failed to demonstrate this phenomenon. Thus, limiting the analysis to 12 M.O. factors was another important limitation of the study.

## Directions for Future Research

While this research provided additional support for the validity of the proposed Bayesian method for linking serial crimes, the results are best viewed as exploratory. However, the findings and limitations of the study provide guidance for future research both on this linkage method as well as on crime linkage analysis in general. Several of these directions for future research are presented in this section.

To improve understanding and development of crime linkage systems, several issues should be addressed. Importantly, some of these issues involve additional studies
on offender behavior rather than studies directed at crime linkage. Specifically, a greater understanding of behavioral consistency among offenders is necessary for developing improved crime linkage tools.

As noted in the limitations, different offenders likely vary in their consistency between behaviors. Identification of the behaviors that offenders are most consistent at exhibiting is fundamental to using M.O. to predict crime linkage. The analysis for the fourth research question demonstrated that offender consistency was a strong predictor of predictive performance. Therefore, using a set of behaviors that are believed to be more consistent overall should render better predictions. Additionally, identifying a set of behaviors that exhibit utility for crime linkage has important implications. Once these behaviors are identified, procedures can be implemented to insure that data are collected for these behaviors from crime sites. Current analysis assumes that the absence of a record of a behavior is an indication that the offender did not exhibit the behavior. As it is possible that the behavior was simply not noted in the police report, the effect of measurement error on crime linkage is unknown. Developing protocols to improve the collection of data to be used for linkage analysis would help address this issue.

In addition to this general direction, understanding the behavior of the proposed linkage method requires additional research. Primarily, the method needs to be subjected to a test of validity using empirical rather than simulated data. Because the simulated data are based on empirical assumptions, it is possible that empirical data collected for different types of serial crimes, from different areas and at different times, may be unlike the assumptions applied in this study. Testing the linkage method using empirical data for
a variety of crime types from different areas and at different times will add to the generalizability of the method.

Finally, the effect of assumptive error on the linkage method needs to be assessed.
Analyzing the predictive validity of the crime linkage method using incorrect assumptions about offenders' spatial, temporal, and M.O. behaviors will provide insight into whether general assumptions are sufficient for the method, or whether assumptions about offender behavior should be generated specifically for different jurisdictions.

## APPENDIX SECTION

## APPENDIX A


#### Abstract

A brief discussion of edge effects and an alternate method for estimating $L R_{\text {distance }}$ is presented in this Appendix.


The calculation used to generate $L R_{\text {distance }}$ presented in Chapter 4 has two specific limitations. First, the calculation of $P($ distance $\mid$ unlinked $)$ ignores the location of the crimes within the analysis area. Thus, the method does not control for the changes in probability that occur when observations are situated near the edge of the analysis space. This is not particularly problematic when locations are situated well within the boundaries of the analysis area and the area is large relative to the distances being analyzed. However, it may become problematic when the distances become longer. The following figures demonstrate the issue.


Locations of Crimes Do Not Affect Calculation of $\boldsymbol{P}$ (distance|unlinked)


Area of circle with radius equal to distance
$\square$ Total area of analysis space

Both figures present possible observed distances within the same area. In the first figure, all of the possible points contained within the circle inscribed by the observed distance fall within the greater area. In this case, the calculation presented in Chapter 4 yields the correct probability. However, in the second figure, the calculation is incorrect. This is due to the fact that some of the points inscribed by the circle lie outside the area of analysis. This is an edge effect where the location of the crime site within the analysis space affects the probability of distances related to that site. In this case, the probability would be correctly calculated by dividing the area of the circle that falls within the analysis area by the total area. This may be an issue that affects the performance of $L R_{\text {distance }}$ as distances becomes much longer.

The issue is particularly problematic when the distance becomes so long that the area of the circle that it inscribes becomes greater that the area being analyzed. When this occurs, the calculation of $P($ distance $\mid$ unlinked $)$ using this method results in a value over one. The following figure demonstrates the issue.


## Area of Circle is Greater than Area Analyzed

The second potential issue results from both this approach to calculating $P($ distance $\mid u n l i n k e d)$ and the use of cumulative densities to calculate $P($ distance $\mid$ linked $)$. Both of these methods utilize probabilities that represent the chance of observing a distance equal to or less than the distance observed. Estimating the probability of observing a specific distance may result in better performance of the linkage method. The alternate technique presented herein addresses both of these issues.

The empirical serial murder distances were found to correspond to a $\operatorname{Beta}(0.34,0.84)$ distribution, and cumulative density function for this distribution was used to estimate the probability of observing a distance equal to or less than the observed distance. $\operatorname{Beta}(0.34,0.84)$ is a probability density function and is given by:

$$
\beta_{(0.34,0.84)}(x)=\frac{x^{(0.34-1)}(1-x)^{(0.84-1)}}{\int_{0}^{1} u^{(0.34-1)}(1-u)^{(0.84-1)} d u}
$$

Estimating probabilities for specific observations $(x)$ from a probability density function is accomplished by calculating the definite integral of the function. Because definite integrals must range over some values of $x$, some small quantity $(\delta)$ is added and subtracted from $x$ to create the range. This value can be infinitesimally small and can be thought of as measurement error. Using the $\operatorname{Beta}(0.34,0.84)$ probability density function, the calculation for the numerator of the likelihood ratio is:

$$
P(\text { distance } \mid \text { linked })=\int_{x-\delta}^{x+\delta} \beta_{(0.34,0.84)}(x) d x
$$

To generate the denominator of the likelihood ratio, a probability density function representing the distribution of all possible distances within an analysis area must be derived. This distribution can be derived for any area using a Monte Carlo approach. Generating this distribution involves the following steps:
(1) Randomly sample two points within the analysis area.
(2) Calculate the distance between these points.
(3) Add this distance to the distribution for the area.
(4) Repeat steps 1 to 3 until the moments of the distribution converge.
(5) Fit a probability density function that corresponds to the derived distribution.

The fit probability density function summarizes the possible distances within the analysis area assuming that they are random. For example, assume that the analysis area is one square mile. Sampling 10,000 pairs of points and recording 10,000 distances
renders a distribution with a mean of 0.518 miles and a variance of 0.062 . Standardizing these values renders a mean proportional distance of 0.408 and a variance of 0.196 . The derived distribution of random distances within the area is presented in the following figure.


Distribution of Randomly Occurring Distances in One Square Mile

Moment matching indicates that a $\operatorname{Beta}(2.37,3.87)$ distribution is a close approximation to this distribution. The $\operatorname{Beta}(2.37,3.87)$ probability density is presented in the following figure:


Beta(2.37, 3.87) Probability Density Function

Using the $\operatorname{Beta}(2.37,3.87)$ as the probability density function, the probability for the denominator, $P($ distance $\mid$ unlinked $)$, can be estimated as:

$$
P(\text { distance } \mid \text { unlinked })=\int_{x-\delta}^{x+\delta} \beta_{(2.37,3.87)}(x) d x
$$

The distance likelihood ratio can now be calculated based on the probabilities for the observed distance rather than the cumulative probabilities, and this estimate now controls for edge effects. The distance likelihood ratio is ultimately given by:

$$
L R_{\text {distance }}=\frac{\int_{x-\delta}^{x+\delta} \beta_{(0.34,0.84)}(x) d x}{\int_{x-\delta}^{x+\delta} \beta_{(2.37,3.87)}(x) d x}
$$

The extent to which calculating the distance likelihood ratio in this way affects the ability of the linkage method to accurately classify linked crimes is undetermined. However, estimating the probabilities in this way would allow for specific distributions to be generated for any area analyzed regardless of shape or size. Empirical data on serial offending within that area alone could be used to generate a distinct distribution of serial crime distances specifically related to the area as well. This would result in direct correspondence between the two distributions that might prove superior to using general distributions that incorporate data from multiple areas.

## APPENDIX B

Analysis of the performance of the Bayesian hypothesis test for serial murder linkage for models with incomplete information (i.e., models with less than distance, time, and 12 M.O. factors) is presented in this Appendix. The complete information models (i.e., those using distance, time, and all 12 M.O. factors) are presented in Chapter 5.

## Serial Murder Linkage Performance for Distance Only Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with <br> predictions | 194,992 | 171,488 | 138,293 | 46,817 |
| Percent of samples with <br> predictions | 77.96 | 68.56 | 55.28 | 81.72 |
|  |  |  |  |  |
| Percent of series identified | 7.11 | 3.94 | 2.97 | 10.06 |
| $\quad$ Mean | 0.00 | 0.00 | 0.00 | 9.09 |
| $\quad$ Median |  |  |  |  |
|  | 1.12 | 0.62 | 0.48 | 1.51 |
| Number of hits | 0.00 | 0.00 | 0.00 | 1.00 |
| $\quad$ Mean |  |  |  |  |
| $\quad$ Median | 17.64 | 42.33 | 73.67 | 94.88 |
| Percent of true positives | 11.11 | 40.00 | 100.00 | 100.00 |
| Mean |  |  |  |  |



Serial Murder Distance Only Model Performance at the Substantial Level of Evidence


Serial Murder Distance Only Model Performance at the Strong Level of Evidence


Serial Murder Distance Only Model Performance at the Very Strong Level of Evidence


Serial Murder Distance Only Model Performance at the Extreme Level of Evidence

## Serial Murder Linkage Performance for Distance and Time Model

|  | Level of Evidence |  |  |  |
| :--- | ---: | ---: | :---: | ---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 166,765 | 148,224 | 132,149 | 239,245 |
| Percent of samples with predictions | 66.68 | 59.25 | 52.82 | 95.67 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean |  |  |  |  |
| $\quad$ Median | 21.07 | 17.14 | 14.46 | 24.43 |
|  | 16.67 | 14.29 | 12.50 | 23.08 |
| Number of hits |  |  |  |  |
| $\quad$ Mean |  |  |  |  |
| $\quad$ Median | 3.35 | 2.74 | 2.30 | 3.67 |
|  | 2.00 | 2.00 | 2.00 | 3.00 |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 12.74 | 27.83 | 49.88 | 85.17 |
| Median | 7.69 | 22.22 | 50.00 | 100.00 |



Serial Murder Distance and Time Model Performance at the Substantial Level of Evidence


Serial Murder Distance and Time Model Performance at the Strong Level of Evidence


Serial Murder Distance and Time Model Performance at the Very Strong Level of Evidence


Serial Murder Distance and Time Model Performance at the Extreme Level of Evidence

## Serial Murder Linkage Performance for Distance, Time, and One-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 169,812 | 155,283 | 140,807 | 239,172 |
| Percent of samples with predictions | 67.90 | 62.08 | 56.29 | 95.65 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 19.84 | 17.36 | 15.11 | 27.49 |
| $\quad$ Median | 15.00 | 14.29 | 12.50 | 25.00 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 3.18 | 2.75 | 2.38 | 4.09 |
| Median | 2.00 | 2.00 | 2.00 | 3.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| Mean | 12.66 | 26.33 | 46.20 | 81.84 |
| Median | 6.90 | 19.05 | 50.00 | 100.00 |



Serial Murder Distance, Time, and One-M.O.-Factor Model Performance at the Substantial Level of Evidence


Serial Murder Distance, Time, and One-M.O.-Factor Model Performance at the Strong Level of Evidence


Serial Murder Distance, Time, and One-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Serial Murder Distance, Time, and One-M.O.-Factor Model Performance at the Extreme Level of Evidence

## Serial Murder Linkage Performance for Distance, Time, and Two-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | ---: | ---: | ---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 173,188 | 164,841 | 151,199 | 239,746 |
| Percent of samples with predictions | 69.26 | 65.93 | 60.44 | 95.86 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 19.31 | 17.43 | 15.84 | 30.71 |
| $\quad$ Median | 14.29 | 14.29 | 12.50 | 27.27 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 3.08 | 2.76 | 2.48 | 4.51 |
| $\quad$ Median | 2.00 | 2.00 | 2.00 | 4.00 |
|  |  |  |  |  |
| Percent of true positives | 12.66 | 25.01 | 43.65 | 78.69 |
| $\quad$ Mean | 6.47 | 16.67 | 40.00 | 88.89 |
| Median |  |  |  |  |



Serial Murder Distance, Time, and Two-M.O.-Factor Model Performance at the Substantial Level of Evidence


Serial Murder Distance, Time, and Two-M.O.-Factor Model Performance at the Strong Level of Evidence


Serial Murder Distance, Time, and Two-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Serial Murder Distance, Time, and Two-M.O.-Factor Model Performance at the Extreme Level of Evidence

## Serial Murder Linkage Performance for Distance, Time, and Three-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | ---: | ---: | ---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 175,726 | 171,015 | 161,149 | 240,116 |
| Percent of samples with predictions | 70.29 | 68.37 | 64.42 | 96.02 |
|  |  |  |  |  |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 18.40 | 17.18 | 16.02 | 33.78 |
| $\quad$ Median | 14.29 | 13.79 | 13.04 | 29.17 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 2.93 | 2.70 | 2.50 | 4.93 |
| $\quad$ Median | 2.00 | 2.00 | 2.00 | 4.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 12.64 | 24.23 | 40.84 | 75.44 |
| Median | 6.06 | 14.29 | 33.33 | 83.33 |



Serial Murder Distance, Time, and Three-M.O.-Factor Model Performance at the Substantial Level of Evidence


Serial Murder Distance, Time, and Three-M.O.-Factor Model Performance at the Strong Level of Evidence


Serial Murder Distance, Time, and Three-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Serial Murder Distance, Time, and Three-M.O.-Factor Model Performance at the Extreme Level of Evidence

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | :---: | ---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 179,099 | 177,893 | 171,597 | 240,316 |
| Percent of samples with predictions | 71.60 | 71.14 | 68.63 | 96.11 |
|  |  |  |  |  |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 17.81 | 17.19 | 16.54 | 36.56 |
| $\quad$ Median | 13.64 | 14.29 | 13.64 | 31.25 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 2.84 | 2.69 | 2.56 | 5.27 |
| $\quad$ Median | 2.00 | 2.00 | 2.00 | 4.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 12.65 | 23.60 | 39.08 | 72.53 |
| Median | 5.88 | 12.73 | 33.33 | 80.00 |



Serial Murder Distance, Time, and Four-M.O.-Factor Model Performance
at the Substantial Level of Evidence


Serial Murder Distance, Time, and Four-M.O.-Factor Model Performance at the Strong Level of Evidence


Serial Murder Distance, Time, and Four-M.O.-Factor Model Performance
at the Very Strong Level of Evidence


Serial Murder Distance, Time, and Four-M.O.-Factor Model Performance
at the Extreme Level of Evidence

# Serial Murder Linkage Performance for Distance, Time, and Five-M.O.-Factor Model 

|  | Level of Evidence |  |  |  |
| :--- | ---: | ---: | :---: | ---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 185,155 | 188,798 | 184,815 | 240,622 |
| Percent of samples with predictions | 74.05 | 75.49 | 73.91 | 96.23 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 17.28 | 16.94 | 16.72 | 39.61 |
| $\quad$ Median | 13.33 | 14.29 | 14.29 | 33.33 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 2.75 | 2.65 | 2.57 | 5.73 |
| $\quad$ Median | 2.00 | 2.00 | 2.00 | 5.00 |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 12.83 | 22.98 | 37.63 | 70.09 |
| Median | 5.71 | 12.00 | 28.13 | 77.78 |



Serial Murder Distance, Time, and Five-M.O.-Factor Model Performance at the Substantial Level of Evidence


Serial Murder Distance, Time, and Five-M.O.-Factor Model Performance at the Strong Level of Evidence


Serial Murder Distance, Time, and Five-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Serial Murder Distance, Time, and Five-M.O.-Factor Model Performance at the Extreme Level of Evidence

## Serial Murder Linkage Performance for Distance, Time, and Six-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | :---: | ---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 189,645 | 197,608 | 197,389 | 241,171 |
| Percent of samples with predictions | 75.83 | 79.03 | 78.92 | 96.44 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 16.68 | 16.57 | 16.92 | 42.37 |
| $\quad$ Median | 13.33 | 14.29 | 14.29 | 36.00 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 2.65 | 2.61 | 2.61 | 6.08 |
| $\quad$ Median | 2.00 | 2.00 | 2.00 | 5.00 |
|  |  |  |  |  |
| Percent of true positives | 12.92 | 22.45 | 36.18 | 67.38 |
| $\quad$ Mean | 5.77 | 11.11 | 25.00 | 75.00 |
| Median |  |  |  |  |



Serial Murder Distance, Time, and Six-M.O.-Factor Model Performance at the Substantial Level of Evidence


Serial Murder Distance, Time, and Six-M.O.-Factor Model Performance at the Strong Level of Evidence


Serial Murder Distance, Time, and Six-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Serial Murder Distance, Time, and Six-M.O.-Factor Model Performance at the Extreme Level of Evidence

## Serial Murder Linkage Performance for Distance, Time, and Seven-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | :---: | ---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 195,454 | 208,831 | 211,084 | 241,382 |
| Percent of samples with predictions | 78.17 | 83.50 | 84.83 | 96.52 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 16.11 | 16.39 | 17.14 | 44.86 |
| $\quad$ Median | 14.29 | 14.29 | 15.00 | 38.46 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 2.57 | 2.57 | 2.63 | 6.48 |
| $\quad$ Median | 2.00 | 2.00 | 2.00 | 5.00 |
|  |  |  |  |  |
| Percent of true positives | 12.99 | 22.46 | 35.46 | 65.14 |
| $\quad$ Mean | 5.68 | 11.11 | 25.00 | 71.43 |
| Median |  |  |  |  |



Serial Murder Distance, Time, and Seven-M.O.-Factor Model Performance
at the Substantial Level of Evidence


Serial Murder Distance, Time, and Seven-M.O.-Factor Model Performance at the Strong Level of Evidence


Serial Murder Distance, Time, and Seven-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Serial Murder Distance, Time, and Seven-M.O.-Factor Model Performance at the Extreme Level of Evidence

# Serial Murder Linkage Performance for Distance, Time, and Eight-M.O.-Factor Model 

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 204,965 | 226,073 | 231,239 | 241,708 |
| Percent of samples with predictions | 81.97 | 90.39 | 92.47 | 96.67 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 15.50 | 15.88 | 17.15 | 47.57 |
| $\quad$ Median | 14.29 | 15.00 | 15.79 | 40.91 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 2.48 | 2.49 | 2.63 | 6.85 |
| $\quad$ Median | 2.00 | 2.00 | 2.00 | 5.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 13.34 | 22.48 | 34.88 | 62.89 |
| Median | 5.88 | 11.11 | 23.08 | 66.67 |



## Serial Murder Distance, Time, and Eight-M.O.-Factor Model Performance at the Substantial Level of Evidence



Serial Murder Distance, Time, and Eight-M.O.-Factor Model Performance at the Strong Level of Evidence


## Serial Murder Distance, Time, and Eight-M.O.-Factor Model Performance at the Very Strong Level of Evidence



Serial Murder Distance, Time, and Eight-M.O.-Factor Model Performance at the Extreme Level of Evidence

# Serial Murder Linkage Performance for Distance, Time, and Nine-M.O.-Factor Model 

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 204,947 | 224,987 | 229,297 | 242,099 |
| Percent of samples with predictions | 81.94 | 89.97 | 91.71 | 96.81 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 16.10 | 17.40 | 19.48 | 50.11 |
| $\quad$ Median | 13.04 | 14.29 | 16.67 | 44.44 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 2.54 | 2.69 | 2.95 | 7.13 |
| $\quad$ Median | 2.00 | 2.00 | 2.00 | 6.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 12.97 | 21.35 | 32.98 | 60.48 |
| Median | 5.41 | 9.74 | 20.00 | 66.67 |



Serial Murder Distance, Time, and Nine-M.O.-Factor Model Performance at the Substantial Level of Evidence


Serial Murder Distance, Time, and Nine-M.O.-Factor Model Performance at the Strong Level of Evidence


Serial Murder Distance, Time, and Nine-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Serial Murder Distance, Time, and Nine-M.O.-Factor Model Performance at the Extreme Level of Evidence

## Serial Murder Linkage Performance for Distance, Time, and 10-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 203,505 | 221,925 | 226,444 | 242,464 |
| Percent of samples with predictions | 81.37 | 88.77 | 90.56 | 96.96 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 16.70 | 19.18 | 21.91 | 52.59 |
| $\quad$ Median | 12.50 | 15.00 | 17.65 | 48.28 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 2.63 | 2.95 | 3.29 | 7.47 |
| $\quad$ Median | 2.00 | 2.00 | 2.00 | 6.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 12.45 | 19.97 | 30.76 | 58.01 |
| Median | 4.88 | 8.33 | 16.67 | 60.00 |



Serial Murder Distance, Time, and 10-M.O.-Factor Model Performance at the Substantial Level of Evidence


Serial Murder Distance, Time, and 10-M.O.-Factor Model Performance at the Strong Level of Evidence


Serial Murder Distance, Time, and 10-M.O.-Factor Model Performance
at the Very Strong Level of Evidence


Serial Murder Distance, Time, and 10-M.O.-Factor Model Performance at the Extreme Level of Evidence

## Serial Murder Linkage Performance for Distance, Time, and 11-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 201,575 | 217,510 | 223,520 | 242,569 |
| Percent of samples with predictions | 80.63 | 87.00 | 89.39 | 97.01 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 17.61 | 20.90 | 24.29 | 54.53 |
| $\quad$ Median | 12.5 | 16.67 | 20.00 | 50.00 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 2.76 | 3.21 | 3.65 | 7.82 |
| $\quad$ Median | 2.00 | 2.00 | 3.00 | 6.00 |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 12.38 | 19.39 | 30.26 | 56.73 |
| Median | 4.76 | 7.89 | 16.22 | 58.06 |



Serial Murder Distance, Time, and 11-M.O.-Factor Model Performance at the Substantial Level of Evidence


Serial Murder Distance, Time, and 11-M.O.-Factor Model Performance at the Strong Level of Evidence


Serial Murder Distance, Time, and 11-M.O.-Factor Model Performance
at the Very Strong Level of Evidence


Serial Murder Distance, Time, and 11-M.O.-Factor Model Performance at the Extreme Level of Evidence

## APPENDIX C

Analysis of the performance of the Bayesian hypothesis test for serial commercial robbery linkage for models with incomplete information (i.e., models with less than distance, time, and 12 M.O. factors) is presented in this appendix. The complete information models (i.e., those using distance, time, and all 12 M.O. factors) are presented in Chapter 5.

Commercial Robbery Linkage Performance for Distance Only Model

|  | Level of Evidence |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 10,091 | n/a | n/a | n/a |
| Percent of samples with predictions | 4.04 | n/a | n/a | n/a |
| Percent of series identified |  |  |  |  |
| Mean | 1.84 | n/a | n/a | n/a |
| Median | 0.00 | n/a | n/a | n/a |
| Number of hits |  |  |  |  |
| Mean | 0.51 | n/a | n/a | n/a |
| Median | 0.00 | n/a | n/a | n/a |
| Percent of true positives |  |  |  |  |
| Mean | 49.23 | n/a | n/a | $\mathrm{n} / \mathrm{a}$ |
| Median | 0.00 | n/a | n/a | n/a |

Distance alone failed to yield any Bayes' factors at the strong, very strong, or extreme level of evidence.


# Commercial Robbery Distance Only Model Performance at the Substantial Level of Evidence 

## Commercial Robbery Linkage Performance for Distance and Time Model

|  | Level of Evidence |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 233,695 | 4,500 | n/a | n/a |
| Percent of samples with predictions | 93.48 | 1.80 | n/a | n/a |
| Percent of series identified |  |  |  |  |
| Mean | 32.26 | 2.84 | n/a | n/a |
| Median | 36.36 | 2.63 | $\mathrm{n} / \mathrm{a}$ | n/a |
| Number of hits |  |  |  |  |
| Mean | 8.74 | 0.77 | n/a | n/a |
| Median | 7.00 | 1.00 | $\mathrm{n} / \mathrm{a}$ | n/a |
| Percent of true positives |  |  |  |  |
| Mean | 38.08 | 75.47 | n/a | n/a |
| Median | 40.00 | 100.00 | $\mathrm{n} / \mathrm{a}$ | n/a |

Distance and time together failed to yield any predictions at the very strong or extreme levels of evidence.


Commercial Robbery Distance and Time Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance and Time Model Performance at the Strong Level of Evidence

## Commercial Robbery Linkage Performance for Distance, Time, and One-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 231,459 | 54,289 | 5,406 | 874 |
| Percent of samples with predictions | 92.58 | 21.72 | 2.16 | 0.35 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean |  |  |  |  |
| $\quad$ Median | 30.30 | 13.71 | 14.76 | 39.73 |
|  | 23.08 | 5.55 | 5.65 | 16.67 |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 8.15 | 3.11 | 3.41 | 8.31 |
| $\quad$ Median | 5.00 | 1.00 | 1.00 | 3.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 38.16 | 56.52 | 53.31 | 51.81 |
| Median | 35.71 | 62.07 | 50.00 | 50.00 |



Commercial Robbery Distance, Time, and One-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and One-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and One-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and One-M.O.-Factor Model Performance at the Extreme Level of Evidence

## Commercial Robbery Linkage Performance for Distance, Time, and Two-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | ---: | ---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 227,975 | 88,994 | 16,953 | 3,383 |
| Percent of samples with predictions | 91.19 | 35.60 | 6.78 | 1.35 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 30.18 | 17.15 | 16.74 | 37.78 |
| $\quad$ Median | 21.74 | 7.69 | 7.14 | 37.25 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 8.08 | 3.99 | 3.51 | 6.81 |
| $\quad$ Median | 5.00 | 2.00 | 1.00 | 3.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 38.16 | 53.82 | 57.49 | 61.73 |
| Median | 33.33 | 54.55 | 66.67 | 62.50 |



Commercial Robbery Distance, Time, and Two-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and Two-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and Two-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and Two-M.O.-Factor Model Performance at the Extreme Level of Evidence

Commercial Robbery Linkage Performance for Distance, Time, and Three-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 226,058 | 112,466 | 31,351 | 8,206 |
| Percent of samples with predictions | 90.42 | 44.99 | 12.54 | 3.28 |
|  |  |  |  |  |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 30.06 | 19.29 | 18.12 | 34.66 |
| $\quad$ Median | 22.22 | 9.52 | 8.57 | 28.18 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 8.02 | 4.56 | 3.8 | 6.25 |
| $\quad$ Median | 5.00 | 2.00 | 2.00 | 3.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 38.55 | 52.85 | 59.40 | 67.21 |
| $\quad$ Median | 33.33 | 40.06 | 66.67 | 81.82 |



Commercial Robbery Distance, Time, and Three-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and Three-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and Three-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and Three-M.O.-Factor Model Performance at the Extreme Level of Evidence

| Model |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Level of Evidence |  |  |  |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 225,019 | 131,515 | 47,313 | 14,880 |
| Percent of samples with predictions | 90.00 | 52.61 | 18.93 | 5.95 |
| Percent of series identified |  |  |  |  |
| Mean | 29.93 | 20.70 | 19.29 | 35.24 |
| Median | 22.22 | 11.11 | 10.00 | 20.00 |
| Number of hits |  |  |  |  |
| Mean | 7.95 | 4.92 | 4.02 | 6.08 |
| Median | 5.00 | 2.00 | 2.00 | 3.00 |
| Percent of true positives |  |  |  |  |
| Mean | 38.96 | 52.39 | 59.93 | 69.07 |
| Median | 33.33 | 50.00 | 70.00 | 87.50 |



Commercial Robbery Distance, Time, and Four-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and Four-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and Four-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and Four-M.O.-Factor Model Performance at the Extreme Level of Evidence

Commercial Robbery Linkage Performance for Distance, Time, and Five-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 225,680 | 146,636 | 63,358 | 22,869 |
| Percent of samples with predictions | 90.26 | 58.65 | 25.46 | 9.15 |
|  |  |  |  |  |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 29.71 | 21.96 | 20.46 | 35.74 |
| $\quad$ Median | 23.08 | 12.82 | 11.11 | 20.00 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 7.88 | 5.24 | 4.32 | 6.09 |
| $\quad$ Median | 5.00 | 3.00 | 2.00 | 3.00 |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 39.60 | 52.65 | 60.67 | 69.87 |
| Median | 33.33 | 50.00 | 71.43 | 88.89 |



Commercial Robbery Distance, Time, and Five-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and Five-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and Five-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and Five-M.O.-Factor Model Performance at the Extreme Level of Evidence

Commercial Robbery Linkage Performance for Distance, Time, and Six-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | ---: | ---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 226,876 | 159,328 | 79,506 | 32,248 |
| Percent of samples with predictions | 90.75 | 63.73 | 31.80 | 12.90 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean |  |  |  |  |
| $\quad$ Median | 29.58 | 22.93 | 21.80 | 36.59 |
|  | 23.14 | 14.63 | 13.04 | 20.00 |
| Number of hits |  |  |  |  |
| $\quad$ Mean |  |  |  |  |
| $\quad$ Median | 7.85 | 5.53 | 4.64 | 6.32 |
|  | 5.00 | 3.00 | 2.00 | 3.00 |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 40.50 | 53.21 | 61.83 | 70.89 |
| Median | 34.21 | 53.15 | 75.00 | 91.67 |



Commercial Robbery Distance, Time, and Six-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and Six-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and Six-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and Six-M.O.-Factor Model Performance at the Extreme Level of Evidence

# Commercial Robbery Linkage Performance for Distance, Time, and Seven-M.O.-Factor Model 

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 229,361 | 172,349 | 95,751 | 41,955 |
| Percent of samples with predictions | 91.74 | 68.94 | 38.80 | 16.78 |
|  |  |  |  |  |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 29.59 | 23.95 | 23.00 | 37.52 |
| $\quad$ Median | 25.71 | 16.67 | 15.00 | 22.22 |
|  |  |  |  |  |
| Number of hits | 7.80 | 5.75 | 4.92 | 6.54 |
| $\quad$ Mean | 6.00 | 4.00 | 3.00 | 4.00 |
| $\quad$ Median |  |  |  |  |
|  |  |  |  |  |
| Percent of true positives | 41.37 | 54.11 | 62.84 | 71.11 |
| $\quad$ Mean | 35.71 | 55.10 | 75.00 | 91.67 |
| $\quad$ Median |  |  |  |  |



Commercial Robbery Distance, Time, and Seven-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and Seven-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and Seven-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and Seven-M.O.-Factor Model Performance at the Extreme Level of Evidence

# Commercial Robbery Linkage Performance for Distance, Time, and Eight-M.O.-Factor 

 Model|  | Level of Evidence |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 235,078 | 185,817 | 111,843 | 51,671 |
| Percent of samples with predictions | 94.03 | 74.33 | 44.74 | 20.67 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 29.63 | 25.04 | 24.57 | 38.72 |
| $\quad$ Median | 28.57 | 20.93 | 18.18 | 23.53 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 7.76 | 6.02 | 5.25 | 6.84 |
| $\quad$ Median | 6.00 | 4.00 | 3.00 | 4.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 42.14 | 54.90 | 63.67 | 71.44 |
| Median | 36.84 | 57.14 | 75.00 | 91.67 |



Commercial Robbery Distance, Time, and Eight-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and Eight-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and Eight-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and Eight-M.O.-Factor Model Performance at the Extreme Level of Evidence

| Model |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Level of Evidence |  |  |  |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 234,900 | 198,861 | 122,316 | 61,594 |
| Percent of samples with predictions | 93.36 | 76.34 | 48.93 | 34.64 |
| Percent of series identified |  |  |  |  |
| Mean | 30.59 | 27.70 | 27.84 | 39.68 |
| Median | 27.27 | 21.43 | 20.00 | 25.00 |
| Number of hits |  |  |  |  |
| Mean | 7.96 | 6.60 | 5.93 | 7.04 |
| Median | 6.00 | 4.00 | 4.00 | 4.00 |
| Percent of true positives |  |  |  |  |
| Mean | 42.45 | 55.07 | 63.90 | 71.57 |
| Median | 37.21 | 57.14 | 75.00 | 91.67 |



Commercial Robbery Distance, Time, and Nine-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and Nine-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and Nine-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and Nine-M.O.-Factor Model Performance at the Extreme Level of Evidence

Commercial Robbery Linkage Performance for Distance, Time, and 10-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | ---: | ---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 234,620 | 194,769 | 131,497 | 70,608 |
| Percent of samples with predictions | 93.85 | 77.91 | 52.60 | 28.24 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean |  |  |  |  |
| $\quad$ Median | 31.78 | 30.23 | 30.60 | 41.12 |
|  | 26.67 | 22.50 | 21.05 | 27.28 |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 8.21 | 7.16 | 6.50 | 7.34 |
| $\quad$ Median | 6.00 | 5.00 | 4.00 | 5.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 42.94 | 55.12 | 63.94 | 71.60 |
| Median | 37.50 | 57.14 | 75.00 | 90.47 |



Commercial Robbery Distance, Time, and 10-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and 10-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and 10-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and 10-M.O.-Factor Model Performance at the Extreme Level of Evidence

Commercial Robbery Linkage Performance for Distance, Time, and 11-M.O.-Factor Model

|  | Level of Evidence |  |  |  |
| :--- | :---: | ---: | :---: | :---: |
|  | Substantial | Strong | Very Strong | Extreme |
| Number of samples with predictions | 234,088 | 197,831 | 139,242 | 79,816 |
| Percent of samples with predictions | 93.64 | 79.13 | 55.70 | 31.93 |
| Percent of series identified |  |  |  |  |
| $\quad$ Mean | 32.78 | 32.44 | 33.20 | 41.87 |
| $\quad$ Median | 26.67 | 24.44 | 23.08 | 28.89 |
|  |  |  |  |  |
| Number of hits |  |  |  |  |
| $\quad$ Mean | 8.43 | 8.05 | 7.06 | 7.55 |
| $\quad$ Median | 6.00 | 5.00 | 4.00 | 5.00 |
|  |  |  |  |  |
| Percent of true positives |  |  |  |  |
| $\quad$ Mean | 43.28 | 55.37 | 64.21 | 71.7 |
| Median | 38.08 | 57.89 | 75.00 | 90.00 |



Commercial Robbery Distance, Time, and 11-M.O.-Factor Model Performance at the Substantial Level of Evidence


Commercial Robbery Distance, Time, and 11-M.O.-Factor Model Performance at the Strong Level of Evidence


Commercial Robbery Distance, Time, and 11-M.O.-Factor Model Performance at the Very Strong Level of Evidence


Commercial Robbery Distance, Time, and 11-M.O.-Factor Model Performance at the Extreme Level of Evidence

## APPENDIX D

An example of R code to generate simulated data and to calculate the likelihood ratios is presented in this Appendix.

```
# ---------------------------------------------------------------------------
# Simulation data generation (distance, time, and 12 M.O. factors) and
LR Calculation
# -----------------------------------------------------------------------------
# Load required R package
require(car)
# Generate an object to store the entire set of summary data
(analysis.data is a list of 14 objects with each object being a matrix
of 250,000 performance measure summaries from 250,000 simulated cases)
analysis.data <- list()
for(j in 1:14){
    # Set the number of cases for each run of the simulation
    n <- 250000
    # Generate an object to store the distance, time, and M.O.
observations (runs is a list of 3,500,000 objects with each object
being a matrix of observations and calculated likelihood ratios for an
individual case)
    runs <- list()
    # Generate an object to store the performance measure summaries for
250,000 cases (this object is posted to analysis.data each iteration of
the outer j loop)
    summaries <- matrix(NA, ncol = 53, nrow = n)
    colnames(summaries) <- c("per.hit.substantial", "per.hit.strong",
"per.hit.verystrong", "per.hit.extreme", "per.series.substantia7",
"per.series.strong", "per.series.verystrong", "per.series.extreme",
"count.hit.substantial", "count.hit.strong","count.hit.verystrong",
"count.hit.extreme", "n.ser", "n.non", "n.case", "count.hit.tota\eta",
"per.hit.tota1", "count.fp.substantia1", "count.fp.strong",
"count.fp.verystrong", "count.fp.extreme", "per.fp.substantial",
""per.fp.strong", "per.fp.verystrong", "per.fp.extreme",
"mean.dist.ser", "median.dist.ser", "sd.dist.ser", "mean.dist.non",
"median.dist.non", "sd.dist.non", "mean.time.ser", "median.time.ser",
"sd.time.ser", "mean.time.non", "median.time.non", "sd.time.non",
"mean.mo.ser", "median.mo.ser", "sd.mo.ser", "mean.mo.non",
"median.mo.non", "sd.mo.non", "mean.uniq", "median.uniq", "sd.uniq",
"min.ser.MO",' "max.ser.MO", "min.non.MO",' "max.non.MO", "min.uniq",
"max.uniq", "case #")
```

for (i in 1:n) \{
\# Sample number of serial crimes within the case
\# Alter this code to change the number of serial crimes
ser.n <- round (rgamma(1, 3.24, 0.21))
while(ser.n > 40 | ser.n < 1) \{
ser.n <- round(rgamma(1, 3.24, 0.21))
\}
\# Sample the number of non-serial crimes within the case
\# Alter this code to change the number of non-serial crimes
non.n <- sample(100:2000, 1)
\# Generate serial observations
alpha.dist <- 0.34
beta.dist <- 0.84
\# Sample serial distance observations
ser.dist <- matrix(rbeta(ser.n, alpha.dist, beta.dist), nrow = ser.n, ncol = 1)
a1pha.time <- 0.39
beta.time <- 0.96
\# Sample serial time observations
ser.time <- matrix(rbeta(ser.n, alpha.time, beta.time), nrow = ser.n, ncol = 1)

```
ser.p1 <- rbeta(1, .79, .55)
ser.p2 <- rbeta(1, .79, .55)
ser.p3 <- rbeta(1, .79, .55)
ser.p4 <- rbeta(1, .79, .55)
ser.p5 <- rbeta(1, .79, .55)
ser.p6 <- rbeta(1, .79, .55)
ser.p7 <- rbeta(1, .79, .55)
ser.p8 <- rbeta(1, .79, .55)
ser.p9 <- rbeta(1, .79, .55)
ser.p10 <- rbeta(1, .79, .55)
ser.p11 <- rbeta(1, .79, .55)
ser.p12 <- rbeta(1, .79, .55)
```

\# Sample for each M.O. factor using the sampled M.O. probabilities to generate M.O. observations.

```
ser.mo1 <- matrix(rbinom(ser.n, 1, ser.p1))
ser.mo2 <- matrix(rbinom(ser.n, 1, ser.p2))
ser.mo3 <- matrix(rbinom(ser.n, 1, ser.p3))
ser.mo4 <- matrix(rbinom(ser.n, 1, ser.p4))
ser.mo5 <- matrix(rbinom(ser.n, 1, ser.p5))
ser.mo6 <- matrix(rbinom(ser.n, 1, ser.p6))
ser.mo7 <- matrix(rbinom(ser.n, 1, ser.p7))
ser.mo8 <- matrix(rbinom(ser.n, 1, ser.p8))
ser.mo9 <- matrix(rbinom(ser.n, 1, ser.p9))
ser.mo10 <- matrix(rbinom(ser.n, 1, ser.p10))
ser.mo11 <- matrix(rbinom(ser.n, 1, ser.p11))
```

```
ser.mo12 <- matrix(rbinom(ser.n, 1, ser.p12))
```

\# Bind all serial M.O. data together
ser.mo.data <- cbind(ser.mo1, ser.mo2, ser.mo3, ser.mo4, ser.mo5,
ser.mo6, ser.mo7, ser.mo8, ser.mo9, ser.mo10, ser.mo11, ser.mo12)
\# Generate a serial code
ser.dich <- matrix (1, nrow = ser.n, ncol = 1)
\# Bind all serial data together (i.e, distance, time, and M.O.
observations with serial code)
ser.data <- cbind(ser.dist, ser.time, ser.mo.data, ser.dich)
\# Generate non-serial observations
\# Sample non-serial distance observations
non.dist <- matrix(runif(non.n, 0, 1), nrow = non.n, ncol = 1)
\# Sample non-serial time observations
non.time <- matrix (runif(non.n, 0,1$)$, nrow $=$ non. $n$, ncol $=1$ )
non.p1 <- runif(1, 0, 1)
non.p2 <- runif(1, 0, 1)
non.p3 <- runif(1, 0, 1)
non.p4 <- runif(1, 0, 1)
non.p5 <- runif(1, 0, 1)
non.p6 <- runif(1, 0, 1)
non.p7 <- runif(1, 0, 1)
non.p8 <- runif(1, 0, 1)
non.p9 <- runif(1, 0, 1)
non.p10 <- runif(1, 0, 1)
non.p11 <- runif(1, 0, 1)
non.p12 <- runif(1, 0, 1)
\# Sample for each M.O. factor using the sampled M.O. probabilities to generate M.O. observations

```
non.mo1 <- matrix(rbinom(non.n, 1, non.p1))
non.mo2 <- matrix(rbinom(non.n, 1, non.p2))
non.mo3 <- matrix(rbinom(non.n, 1, non.p3))
non.mo4 <- matrix(rbinom(non.n, 1, non.p4))
non.mo5 <- matrix(rbinom(non.n, 1, non.p5))
non.mo6 <- matrix(rbinom(non.n, 1, non.p6))
non.mo7 <- matrix(rbinom(non.n, 1, non.p7))
non.mo8 <- matrix(rbinom(non.n, 1, non.p8))
non.mo9 <- matrix(rbinom(non.n, 1, non.p9))
non.mo10 <- matrix(rbinom(non.n, 1,non.p10))
non.mo11 <- matrix(rbinom(non.n, 1,non.p11))
non.mo12 <- matrix(rbinom(non.n, 1,non.p12))
```

\# Bind all non-serial M.O. data together
non.mo.data <- cbind(non.mo1, non.mo2, non.mo3, non.mo4, non.mo5, non.mo6, non.mo7, non.mo8, non.mo9, non.mo10, non.mo11, non.mo12)

```
# Generate a non-serial code
```

non.dich <- matrix $(0$, nrow $=$ non. $n, \operatorname{ncol}=1)$
\# Bind all non-serial data together (i.e., distance, time, and M.o. observations with non-serial code)
non.data <- cbind(non.dist, non.time, non.mo.data, non.dich)
\# Bind both the serial and non-serial data together to generate the total set of data for the case

```
data <- rbind(ser.data, non.data)
```

\# Write the probabilities for each observation

```
const.dist <- pbeta(data[,1], alpha.dist, beta.dist)
```

unique.dist <- data[,1]
1r.dist <- const.dist / unique.dist

```
const.time <- pbeta(data[,2], alpha.time, beta.time)
```

unique.time <- data[,2]
$1 r$.time <- const.time / unique.time
holder <- data[,3]
const.mo.1 <- recode(holder, "1 = . 59; else = .41")
unique.mo. 1 <- sum(data[,3]) / nrow(data)
1r.mo. 1 <- const.mo. 1 unique.mo. 1
holder <- data[,4]
const.mo.2 <- recode(holder, "1 = .59; e1se = .41")
unique.mo. 2 <- sum(data[,4]) / nrow(data)
$1 \mathrm{r} . \mathrm{mo} .2$ <- const.mo. 2 / unique.mo. 2
holder <- data[,5]
const.mo. 3 <- recode(holder, "1 = . 59; else = .41")
unique.mo. 3 <- sum(data[,5]) / nrow(data)
$1 r . m o .3$ <- const.mo. 3 / unique.mo. 3
holder <- data[,6]
const.mo. 4 <- recode(holder, "1 = .59; else = .41")
unique.mo. 4 <- sum(data[,6]) / nrow(data)
$1 r . m o .4$ <- const.mo. 4 / unique.mo. 4
holder <- data[,7]
const.mo. 5 <- recode(holder, "1 = .59; else = .41")
unique.mo. 5 <- sum(data[,7]) / nrow(data)
$1 \mathrm{r} . \mathrm{mo}$.5 <- const.mo. 5 / unique.mo. 5
holder <- data[,8]
const.mo.6 <- recode(holder, "1 = .59; else = .41")
unique.mo. 6 <- sum(data[,8]) / nrow(data)
$1 r . m o .6$ <- const.mo. 6 / unique.mo. 6
holder <- data[,9]
const.mo.7 <- recode(holder, "1 = .59; else = .41")
unique.mo. 7 <- sum(data[,9]) / nrow(data)
$1 \mathrm{r} . \mathrm{mo} .7$ <- const.mo. 7 / unique.mo. 7
holder <- data[,10]
const.mo.8 <- recode(holder, "1 = .59; else = .41")
unique.mo.8 <- sum(data[,10]) / nrow(data)
1 r .mo. 8 <- const.mo. 8 / unique.mo. 8

```
holder <- data[,11]
const.mo.9 <- recode(holder, "1 = .59; e1se = .41")
unique.mo.9 <- sum(data[,11]) / nrow(data)
1r.mo.9 <- const.mo.9 / unique.mo.9
holder <- data[,12]
const.mo.10 <- recode(holder, "1 =.59; e1se = .41")
unique.mo.10 <- sum(data[,12]) / nrow(data)
1r.mo.10 <- const.mo.10 / unique.mo.10
holder <- data[,13]
const.mo.11 <- recode(holder, "1 =.59; else = .41")
unique.mo.11 <- sum(data[,13]) / nrow(data)
1r.mo.11 <- const.mo.11 / unique.mo.11
holder <- data[,14]
const.mo.12 <- recode(holder, "1 =.59; e1se = .41")
unique.mo.12 <- sum(data[,14]) / nrow(data)
1r.mo.12 <- const.mo.12 / unique.mo.12
```

\# Generate the likelihood ratios

```
1r.dist.time <- 1r.dist * 1r.time
1r.mo1 <- 1r.dist * 1r.time * 1r.mo.1
1r.mo2 <- 1r.dist * 1r.time * 1r.mo.1 * 1r.mo.2
1r.mo3 <- 1r.dist * 1r.time * 1r.mo.1 * 1r.mo.2 * 1r.mo.3
1r.mo4 <- 1r.dist * 1r.time * 1r.mo.1 * 1r.mo.2 * 1r.mo.3 * 1r.mo.4
1r.mo5 <- 1r.dist * 1r.time * 1r.mo.1 * 1r.mo.2 * 1r.mo.3 * 1r.mo.4
    * 7r.mo.5
1r.mo6 <- 1r.dist * 1r.time * 1r.mo.1 * 1r.mo.2 * 1r.mo.3 * 1r.mo.4
    * 1r.mo.5 * 1r.mo.6
1r.mo7 <- 1r.dist * 1r.time * 1r.mo.1 * 1r.mo.2 * 1r.mo.3 * 1r.mo.4
    * 1r.mo.5 * 7r.mo.6 * 7r.mo.7
1r.mo8 <- 1r.dist * 7r.time * 1r.mo.1 * 1r.mo.2 * 1r.mo.3 * 1r.mo.4
    * 1r.mo.5 * 1r.mo.6 * 1r.mo.7 * 1r.mo.8
7r.mo9 <- 7r.dist * 1r.time * 7r.mo.1 * 7r.mo.2 * 7r.mo. 3 * 1r.mo.4
    * 7r.mo.5 * 7r.mo.6 * 1r.mo.7 * 1r.mo.8 * 7r.mo.9
1r.mo10 <- 1r.dist * 1r.time * 1r.mo.1 * 1r.mo.2 * 1r.mo.3 *
    7r.mo.4 * 1r.mo.5 * 7r.mo.6 * 7r.mo.7 * 1r.mo.8 * 1r.mo.9 *
        1r.mo.10
1r.mol1<- 1r.dist * 1r.time * 1r.mo.1 * 1r.mo.2 * 7r.mo.3 *
    1r.mo.4 * 1r.mo.5 * 1r.mo.6 * 1r.mo.7 * 1r.mo.8 * 1r.mo.9 *
    1r.mo.10 * 1r.mo.11
1r.mol2 <- 1r.dist * 1r.time * 1r.mo.1 * 1r.mo.2 * 1r.mo.3 *
        1r.mo.4 * 1r.mo.5 * 1r.mo.6 * 1r.mo.7 * 1r.mo.8 * 1r.mo.9 *
        1r.mo.10 * 7r.mo.11 * 7r.mo.12
```

\# Bind the observed data and the likelihood ratios together
data. $1 r<-$ cbind(data, 1r.dist, 1r.dist.time, 1r.mo1, 1r.mo2, 1r.mo3, $1 r . m o 4,1 r . m o 5,1 r . m o 6,1 r . m o 7,1 r . m o 8,1 r . m o 9,1 r . m o 10$, 1r.mo11, 1r.mo12)
\# Name the columns for the observed data and likelihood ratio object
colnames(data.1r) <- с("dist", "тime", "мо.1", "мо.2", "мо.3",
"мо.4", "мо.5", "мо.6", "мо.7", "мо.8", "мо.9", "мо10", "мо11", "мо12",
"Seria1", "1r.dist", "1r.dist-time", "1r.mo1", "1r.mo2", "1r.mo3", "1r.mo4"', "1r.mo5",’"1r.mo6", "1r.mo7", "1r.mo8", "1r.mo9", "1r.mo10", "1r.mo11", "1r.mo12" )

```
\# Post the observed data and likelihood ratio object to the runs object
```

```
runs[[i]] <- data.1r
```

runs[[i]] <- data.1r
\# Generate performance summaries
\# Isolate the data where predictions were made at each level of evidence

```
```

    test.substantial <- subset(data.1r, data.1r[,15+j] >= 3 &
    ```
    test.substantial <- subset(data.1r, data.1r[,15+j] >= 3 &
data.1r[,25] < 10)
    test.strong <- subset(data.1r, data.1r[,15+j] >= 10 & data.1r[,25]
< 30)
    test.verystrong <- subset(data.1r, data.1r[,15+j] > 30 &
data.1r[,25] > 100)
    test.extreme <- subset(data.1r, data.1r[,15+j] > 100)
    # Generate performance measures for each level of evidence
hit.test1 <- sum(test.substantia1[,15]) / nrow(test.substantial)
```

    \# Post the performance measures to the summaries object
    \(\begin{array}{ll}\text { summaries }[i, 1] & <- \text { hit.test1 } \\ \text { summaries }[1,2] & <- \text { hit.test2 } \\ \text { summaries }[1,3] & <- \text { hit.test3 } \\ \text { summaries }[i, 4] & <- \text { hit.test4 } \\ \text { summaries }[i, 5] & <- \text { per.test1 } \\ \text { summaries }[1,6] & <- \text { per.test2 } \\ \text { summaries }[1,7] & <- \text { per.test3 } \\ \text { summaries }[1,8] & <- \text { per.test4 } \\ \text { summaries }[1,9] & <- \text { count.test1 }\end{array}\)
    ```
    summaries[i,10] <- count.test2
    summaries[i,11] <- count.test3
    summaries[i,12] <- count.test4
    summaries[i,13] <- ser.n
    summaries[i,14] <- non.n
    summaries[i,15] <- ser.n + non.n
    summaries[i,16] <- count.test1 + count.test2 + count.test3 +
count.test4
    summaries[i,17] <- summaries[i,16] / ser.n
summaries[i,18] <- nrow(test.substantial) - count.test1
summaries[i,19] <- nrow(test.strong) - count.test2
summaries[i,20] <- nrow(test.verystrong) - count.test3
summaries[i,21] <- nrow(test.extreme) - count.test4
summaries[i,22] <- summaries[i,18] / nrow(test.substantial)
summaries[i,23] <- summaries[i,19] / nrow(test.strong)
summaries[i,24] <- summaries[i,20] / nrow(test.verystrong)
summaries[i,25] <- summaries[i,21] / nrow(test.extreme)
summaries[i,26] <- mean(ser.dist)
summaries[i,27] <- median(ser.dist)
summaries[i,28] <- sd(as.numeric(ser.dist))
summaries[i,29] <- mean(non.dist)
summaries[i,30] <- median(non.dist)
summaries[i,31] <- sd(as.numeric(non.dist))
summaries[i,32] <- mean(ser.time)
summaries[i,33] <- median(ser.time)
summaries[i,34] <- sd(as.numeric(ser.time))
summaries[i,35] <- mean(non.time)
summaries[i,36] <- median(non.time)
summaries[i,37] <- sd(as.numeric(non.time))
summaries[i,38] <- mean(c(ser.p1, ser.p2, ser.p3, ser.p4, ser.p5,
    ser.p6, ser.p7, ser.p8, ser.p9, ser.p10, ser.p11, ser.p12))
summaries[i,39] <- median(c(ser.p1, ser.p2, ser.p3, ser.p4, ser.p5,
    ser.p6, ser.p7, ser.p8, ser.p9, ser.p10, ser.p11, ser.p12))
summaries[i,40] <- sd(c(ser.p1, ser.p2, ser.p3, ser.p4, ser.p5,
    ser.p6, ser.p7, ser.p8, ser.p9, ser.p10, ser.p11, ser.p12))
summaries[i,41] <- mean(c(non.p1, non.p2, non.p3, non.p4, non.p5,
    non.p6, non.p7, non.p8, non.p9, non.p10, non.p11, non.p12))
summaries[i,42] <- median(c(non.p1, non.p2, non.p3, non.p4, non.p5,
    non.p6, non.p7, non.p8, non.p9, non.p10, non.p11, non.p12))
summaries[i,43]<- sd(c(non.p1, non.p2, non.p3, non.p4, non.p5,
    non.p6, non.p7, non.p8, non.p9, non.p10, non.p11, non.p12))
summaries[i,44] <- mean(c(unique.mo.1, unique.mo.2, unique.mo.3,
    unique.mo.4, unique.mo.5, unique.mo.6, unique.mo.7, unique.mo.8,
    unique.mo.9, unique.mo.10, unique.mo.11, unique.mo.12))
summaries[i,45] <- median(c(unique.mo.1, unique.mo.2, unique.mo.3,
        unique.mo.4, unique.mo.5, unique.mo.6, unique.mo.7, unique.mo.8,
    unique.mo.9, unique.mo.10, unique.mo.11, unique.mo.12))
summaries[i,46] <- sd(c(unique.mo.1, unique.mo.2, unique.mo.3,
        unique.mo.4, unique.mo.5, unique.mo.6, unique.mo.7, unique.mo.8,
        unique.mo.9, unique.mo.10, unique.mo.11, unique.mo.12))
summaries[i,47] <- min(c(ser.p1, ser.p2, ser.p3, ser.p4, ser.p5,
    ser.p6, ser.p7, ser.p8, ser.p9, ser.p10, ser.p11, ser.p12))
summaries[i,48] <- max(c(ser.p1, ser.p2, ser.p3, ser.p4, ser.p5,
    ser.p6, ser.p7, ser.p8, ser.p9, ser.p10, ser.p11, ser.p12))
summaries[i,49] <- min(c(non.p1, non.p2, non.p3, non.p4, non.p5,
```

```
            non.p6, non.p7, non.p8, non.p9, non.p10, non.p11, non.p12))
        summaries[i,50] <- max(c(non.p1, non.p2, non.p3, non.p4, non.p5,
            non.p6, non.p7, non.p8, non.p9, non.p10, non.p11, non.p12))
        summaries[i,51] <- min(c(unique.mo.1, unique.mo.2, unique.mo.3,
        uniqque.mo.4, unique.mo.5, unique.mo.6, unique.mo.7, unique.mo.8,
        unique.mo.9, unique.mo.10, unique.mo.11, unique.mo.12))
        summaries[i,52] <- max(c(unique.mo.1, unique.mo.2, unique.mo.3,
            unique.mo.4, unique.mo.5, unique.mo.6, unique.mo.7, unique.mo.8,
            unique.mo.9, unique.mo.10, unique.mo.11, unique.mo.12))
        summaries[i,53] <- i
    # Print counter to screen to monitor progress
    print(j)
    print(i)
    # Repeat inner (i) loop
    }
    # Drop unnecessary objects from RAM
    rm(1ist = 1s()[!(1s() %in% c('data.1r', 'summaries', 'runs',
    # Post summaries object to the analysis.data object
    analysis.data[[j]] <- summaries
# Repeat outer (j) 1oop
}
```


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[^0]:    ${ }^{1}$ Predictive validity is defined as the extent to which a test correctly classifies some criterion (Cronbach \& Meehl, 1955). Consistent with the idea of predictive validity, the word "predict" is used in crime linkage research when one factor has been determined be useful in correctly classifying serial crime. For example, distance measures have been shown to be able to correctly classify linked crimes; therefore, distance can be said to "predict" linkage. This should not be confused with the idea of future event prediction, as none of the studies referenced herein made predictions about future linked crimes.

[^1]:    ${ }^{2}$ Calculating $L R_{\text {final }}$ this way assumes independence.

[^2]:    ${ }^{3}$ Serial murders may take place over large areas and/or over long periods of time, thus resulting in a large number of potential crimes for analysis. For example, in 2012 the Northeastern United States experienced 2,106 murders and non-negligent manslaughters. The state of Texas alone experienced 1,144, and the city of Los Angeles experienced 299.

[^3]:    ${ }^{4}$ This is equivalent to plotting $P($ false positive $)$ along the x -axis in increasing order.

[^4]:    ${ }^{5}$ An alternate method for estimating $L R_{\text {dist }}$ that addresses edge effects is presented in Appendix A.

[^5]:    ${ }^{6}$ Additional analyses were conducted using only the primary sites. These analyses resulted in no changes to the substantive conclusions about the distribution of distances for serial murders. Thus, these analyses did not significantly alter the distribution used for serial murder distance simulations and had no effect on the performance of the proposed linkage method.

[^6]:    ${ }^{7}$ The standardized distance measures are proportional distances and have no units of measure.

[^7]:    ${ }^{8}$ The standardized distance measures are proportional distances and have no units of measure.

[^8]:    ${ }^{9}$ The standardized time difference measures are proportional differences and have no units of measure.

[^9]:    ${ }^{10}$ The standardized time difference measures are proportional differences and have no units of measure.

[^10]:    ${ }^{11}$ As uniqueness is defined as $P$ (behavior|unlinked), low probabilities on this measure indicate rare behaviors. As the uniqueness measure increases, behavior becomes more common.

[^11]:    ${ }^{12}$ A detailed descriptive analysis of serial murder linkage performance using incomplete information (e.g., distance only, distance and time, or distance, time, and one MO factor, etc.) is presented in Appendix B.

[^12]:    ${ }^{13}$ A detailed descriptive analysis of serial commercial robbery linkage performance using incomplete information (e.g., distance only, distance and time, or distance, time, and one MO factor, etc.) is presented in Appendix C.

[^13]:    ${ }^{14}$ The x-axes for Figures 62 to 79 are cumulative (i.e., 'Dist' represents the distance only model, 'Time' represents the model based on distance and temporal difference, ' 1 MO ' represents the model based on distance and time difference in conjunction with one MO factor, etc.).

[^14]:    ${ }^{15}$ No predictions were made at the strong level of evidence or above for the distance-only model. For comparability with other graphs, models where no predictions were made are still included on the x -axis in figures where this occurs.

