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Quark-gluon correlation functions relevant to single transverse spin asymmetry

Hong Zhang
Iowa State University

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Hong Zhang

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Jianwei Qiu, Major Professor
John Lajoie
Kerry Whisnant
Jim W. Evans

Iowa State University

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ABSTRACT

This thesis presents the evaluation of the relative contributions of various twist-3 quark-gluon correlation functions relevant to single transverse spin asymmetry (SSAs) in a quark-diquark model of the nucleons. The twist-3 quark-gluon correlation functions responsible for gluonic pole and fermionic pole contributions are calculated and compared. We find that at the leading nontrivial order, only gluonic pole contribution is finite and all others are zero for both scalar diquark and axial-vector diquark. We also evalute the symmetries of the these twist-3 quark-gluon correlation functions explicitely

1. INTRODUCTION

The measurement of single transverse-spin asymmetry (SSA) provides an excellent opportunity to evaluate our understanding of strong interaction and hadron structure, and thus has attracted wide insterest in both experimental and theoretical sides in recent years. Two types of single transverse-spin asymmetry experiments have been conducted so far. One is the single particle inclusive production in a high energy collision: $A^{\uparrow} + B \longrightarrow C + X$, where A and B are the initial particles with the spin of A perpendicular to its momentum direction, C is the observed particles (such as pions) of momentum l, X represents all other particles in the final state. In such a process, SSA is defined as

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{d\Delta\sigma}{2d\sigma^{unp}}, \tag{1.1}$$

where $d\sigma^{\uparrow}$ and $d\sigma^{\downarrow}$ represent the invariant differential cross section $E_C d\sigma^{\uparrow}/d^3\mathbf{p_C}$ and $E_C d\sigma^{\downarrow}/d^3\mathbf{p_C}$, respectively, for the production of C with momentum $p_C^{\mu} = (E_C, \mathbf{p}_{\perp}, p_L)$. $d\sigma^{unp}$ in Eq. (1.1) is the differential cross section in unpolarized scattering $A+B \rightarrow C+X$. A_N is also refered to as "left-right" asymmetry because, by rotational invariance, the spatial distribution of produced particle C on the left with A spin-up, is the same as the spatial distribution of produced C on the right with A spin-down. The other type of experiments of SSA is $A + B \longrightarrow C^{\uparrow} + X$, where C is transversely polarized while A and B are unpolarized. In this case, SSA can also be defined as Eq. (1.1) with $d\sigma^{\uparrow(\downarrow)}$ stands for the differential cross section for produced spin-up (spin-down) particle C with respect to the reaction plane, which is defined by the incoming and outgoing particle momenta: p_A, p_B, and p_C.

Single transverse-spin asymmetry was first observed in 1976 by Bunce et al in the process $p + Be \rightarrow \Lambda^{0\uparrow} + X$ [1]. In their experiment, SSA was significantly nonzero at relatively small transverse momentum $p_{\perp}$ and thus was once interpreted as non-perturbative effects. However, during the 90s, the E581/E704 Collaborations at Fermilab reported SSA up to 30%-40% for π polarizaiton in the forward region of the process $p^{\uparrow}(\bar{p}^{\uparrow}) + p \to \pi + X$ with collision energy $\sqrt{s} \simeq 20\ GeV$. They also observed a strong rise of SSA with x_F for all pion charges [2]. Later, the E925 Collaboration conducted the same experiment in BNL independently and confirmed the results obtained in Fermilab [3]. From the end of the 90s, a series of experiments in BNL have shown large SSA in $p^{\uparrow}p$ process with collision energy as large as $\sqrt{s} = 200GeV$ [4]. At the same time, the measurements in HERMES, COMPASS and JLAB-CLAS Collaboration have also shown azimuthal single spin asymmetry in semi-inclusive particle production in the deeply inelastic collisions (DIS) of longitudinally polarized leptons off either transversely or longitudinally polarized proton and deuteron targets [4].

Theoretically, two years after the experiment of Bunce et al., Kane, Pumplin and Repko concluded that SSA should be negligible at high-energy scales [5]. However, several years later, Efremov and Teryaev pointed out that a non-vanishing single transverse-spin asymmetry can exist in pQCD if one goes beyond the leading term [6].In 2002, Brodsky et al. demonstrated that large SSA could be generated in perturbative QCD by calculating SSA of single particle production in a semi-inclusive lepton hadron deep inelastic scattering [9]. Their explicit calculations show that the SSA is not suppressed by Q, the large scale of the scattering. Now it is widely accepted that SSAs in high energy collisions are directly connected to the transverse motion of quarks and gluons inside the transversely polarized hadron. It is the left-right asymmetry of such internal motion of partons that results in SSA.

Two complementary QCD-based approaches have been proposed to analyze single transverse-spin asymmetry theoretically: the transverse momentum dependent (TMD) factorization approach [7, 8, 9, 10, 11, 12, 13] and the collinear factorization approach [6, 14, 15, 16, 17, 18]. Both approaches treat SSA perturbatively and consistent with each other in the region where they both apply [19, 20]. The TMD factorization approach is more suitable when two very different momentum transfers exist, i.e. $Q_1 \gg Q_2 \gtrsim \Lambda_{QCD}$. It directly probes the spin dependence of the parton's transverse motion at the momentum scale Q_2, while the larger scale Q_1 defines the hard collision. The collinear factorization approach is valid when all observed momentum transfers $Q \gtrsim \Lambda_{QCD}$. In this approach, the leading power term in the $1/Q$ expansion, which treats partons inside hadrons as free particles with certain momentum distributon, does not contribute to SSA because of parity and time-reversal invariance for strong interaction. Therefore in the collinear factorization approach, SSA originates from the correlation of quarks and gluons inside a polarized nucleon in the form of the twist-3 quark-gluon and tri-gluon correlation functions [14, 15, 16].

Both TMD factorization approach and collinear factorization approach exploit the basic factorization theorem and inherit, to some extent, the pattern of cross section in a unpolarized hadron collsion, which is

$$d\sigma_{A+B\to C} \propto \sum_{abc} \phi_{a/A}(x_a) \otimes \phi_{b/B}(x_b) \otimes \hat{\sigma}_{a+b\to c} \otimes D_{c\to C}(z), \tag{1.2}$$

where the summations run over parton flavors: quark, antiquark and gluon. $\phi_{a/A}(x_a)$ and $\phi_{b/B}(x_b)$ are parton distribution functions, which represent the probability of finding parton a of momentum $x_a P_A$ in hadron A of momentum P_A, and parton b of momentum $x_b P_B$ in hadron B of momentum P_B, respectively. $D_{c\to C}(z)$ is the fragmentation function for a parton c of momentum $p_c = l/z$ to fragment into a hadron C of momentum l. $\hat{\sigma}_{a+b\to c}$ is the semi-inclusive partonic hard-scattering cross section of the process $a + b \to c$. In Eq. (1.2), only the paronic had-scattering cross section can be calculated in pQCD while both the parton distribution function and fragmentation function are non-perturbative. Likewise, TMD factorization and collinear factorization approaches rely on some non-perturbative functions: the TMD parton distribution functions (PDFs) and the twist-3 three-parton correlation functions, respectively. Our understanding of these non-perturbative functions determines the predictive power of both approaches [21, 22]. Although the quantum evolution of these functions from one perturbative scale to another where these functions are probed can be evaluated in pQCD [23], the absolute normalization of these functions or the boundary conditions are non-perturbative and can only be extracted from experimental data. However, before the precise data are obtained from experiments, a calculation based on a good model could provide important insight in the mechanism for generating single transverse-spin asymmetry, as well as valuable knowledge of the relative importance of various functions. In this thesis, we present our calculations of all twist-3 quark-gluon correlation functions relevant to the SSAs in the collinear factorization approach based on quark-diquark model of the nucleon [9, 25]. We find, in both scalar diquark and axial-vector diquark cases, that at the first non-trivial order all quark-gluon correlation functions corresponding to the fermionic pole contribution, $T_{q,F}(0, x), T_{\Delta q,F}(0, x), T_{q,F}(x, 0), T_{\Delta q,F}(x, 0)$ vanish. For those functions corresponding to the gluonic pole contribution, $T_{q,F}(x, x)$ is finite while $T_{\Delta q,F}(x, x) = 0$. Our results, though model dependent, indicate that the gluonic pole contribution is more important than that of fermionic pole contribution. The result seems to agree with a general expectation that a quark-gluon state with the quark carries all of its momentum is more likely than a quark-gluon state with the gluon carries all of its momentum to interfere with a quark state of the same momentum [15].

2. COLLINEAR FACTORIZATION APPROACH TO SSAS

Before we go into more details of Collinear factorization approach, a physical picture of the hadron-hadron scattering process is of great help. It is accepted that in a hadron-hadron hard collision, two partons (quark, antiquark or gluon) from different hadrons collide to produce a leading parton c, while other partons are spectators of the process in the leading power term. Then c fragments into hadron C which is detected. With this whole process in mind, the physical interpretation of each term in Eq. (1.2) is obvious. To study polarization phenomena of the process $A^{\uparrow} + B \to C + X$ (C unpolarized), one has to generalize Eq. (1.2) to include spin

$$d\sigma_{A+B\to C+X} \propto \sum_{abc} \phi_{a/A}(x_a, \vec{s_\perp}) \otimes \phi_{b/B}(x_b) \otimes \hat{\sigma}_{a+b\to c}(x_a, x_b) \otimes D_{c\to C}(z). \tag{2.1}$$

This leading term keeps unchanged if spin of hadron A is flipped because of parity and time-reversal invariance for strong interaction. Therefore it has no contribution to the numerator in Eq. (1.1) and higher twist correlation must be evaluated. The first consistent calculation in collinear pQCD of a considerable SSA in the region with large x_F was given by Qiu and Sterman [14, 15]. With neglectable quark mass, they introduced new twist-3 quark-gluon correlator functions convoluted with ordinary twist-two parton distribution functions in the light-cone coordinates. The new twist-3 functions, unlike those twist-2 functions, do not have a simple partonic distribution interpretation. More precisely, they obtain the numerator in

Eq. (1.1) in the process $A^{\uparrow} + B \to C + X$ (C unpolarized) [15]

$$\begin{aligned}
\Delta\sigma_{A+B\to C+X}(l_\perp, \vec{s}_\perp) = &\sum_{abc} \phi^{(3)}_{a/A}(x_1, x_2, \vec{s}_\perp) \otimes \phi_{b/B}(x') \otimes \hat{\sigma}_{ab\to c}(\vec{s}_\perp) \otimes D_{c\to C}(z) \\
&+ \sum_{abc} \delta q_{a/A}(x, \vec{s}_\perp) \otimes \phi^{(3)}_{b/B}(x'_1, x'_2) \otimes \hat{\sigma}'_{ab\to c}(\vec{s}_\perp) \otimes D_{c\to C}(z) \\
&+ \sum_{abc} \delta q_{a/A}(x, \vec{s}_\perp) \otimes \phi_{b/B}(x') \otimes \hat{\sigma}''_{ab\to c}(\vec{s}_\perp) \otimes D^{(3)}_{c\to C}(z_1, z_2) \\
&+ \textit{higher power corrections}. \qquad (2.2)
\end{aligned}$$

Additional arguments such as the factorization/renormalization scale have been suppressed. In Eq. (2.2). The superscript "(3)" indicates the corresponding function is a twist-3 function. $\phi_{b/B}(x')$ and $\delta q_{a/A}(x)$ are the standard twist-2 unpolarized and transversity parton distribution functions. $D_{c\to C}(z)$ is the standard fragmentation funtion. The symbol $\otimes$ denotes an appropriate convolution of these functions in partonic light-cone momentum fraction. The twist-3 terms in Eq. (2.2) represent the twist-3 contributions from the polarized hadron A (first row), the unpolarized hadron B (second row) and the fragmentation process $c \to C$ (third row). For each of these contribution, the partonic hard-scattering cross sections functions $\hat{\sigma}_{ab\to c}(\vec{s}_\perp)$,$\hat{\sigma}'_{ab\to c}(\vec{s}_\perp)$ and $\hat{\sigma}''_{ab\to c}(\vec{s}_\perp)$ are different. Because the contribution from the first line in Eq. (2.2) to the SSA is proportional to the derivative of the twist-3 correlation functions, which leads to a characteristic growth of SSA when x_F is large [16, 17]. Therefore, in the forward region, Eq. (2.2) could be simplified as

$$\Delta\sigma_{A+B\to C+X}(l_\perp, \vec{s}_\perp) = \sum_{abc} \phi^{(3)}_{a/A}(x_1, x_2, \vec{s}_\perp) \otimes \phi_{b/B}(x') \otimes \hat{\sigma}_{ab\to c}(\vec{s}_\perp) \otimes D_{c\to C}(z). \qquad (2.3)$$

It is important to note that in Eq. (2.3), only the partonic hard-scattering cross section $\hat{\sigma}_{ab\to c}(\vec{s}_\perp)$ is calculatble in pQCD while the parton distribution functions and fragmentation functions both have non-perturbative nature. Compared Eq. (2.3) with the leading term of unpolarized hadron-hadron hard scattering cross section in Eq. (2.1), one could assert that the twist-3 correlation function includes all information of single transverse-spin asymmetry phenomena in the leading nontrivial term.

The twist-3 quark gluon correlation functions were first introduced in Ref. [14] and the complete set of quark gluon correlation functions relevant to the SSA were constructed by

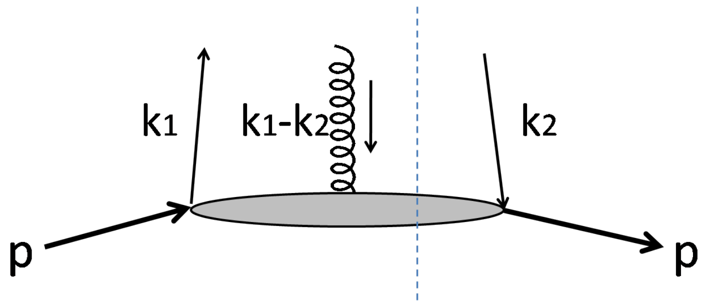

Figure 2.1: The Feynman diagram representation for the twist-3 quark-gluon correlation functions, where $k_i \approx x_i p$ with $i = 1, 2$ [23].

Kang and Qiu in Ref. [23] as

$$T_{q,F}(x_1, x_2) = \int \frac{dy_1^- dy_2^-}{4\pi} e^{ix_1 p^+ y_1^- + i(x_2 - x_1)p^+ y_2} \langle p, s_\perp | \bar{\psi}_q(0) \gamma^+ [\epsilon^{s_\perp \sigma n \bar{n}} g F_\sigma^+(y_2^-)] \psi_q(y_1^-) | p, s_\perp \rangle, \tag{2.4}$$

$$T_{\Delta q,F}(x_1, x_2) = \int \frac{dy_1^- dy_2^-}{4\pi} e^{ix_1 p^+ y_1^- + i(x_2 - x_1)p^+ y_2} \langle p, s_\perp | \bar{\psi}_q(0) \gamma^+ \gamma^5 [i s_\perp^\sigma g F_\sigma^+(y_2^-)] \psi_q(y_1^-) | p, s_\perp \rangle, \tag{2.5}$$

where $x_i = (k_i \cdot n)/(p \cdot n)$ with $i = 1, 2$ are momentum fractions for two independent partons. $\bar{n}^\mu = [1^+, 0^-, 0_\perp]$ and $n^\mu = [0^+, 1^-, 0_\perp]$ are two light-like vectors and $\bar{n} \cdot n = 1$. Because of parity and time-reversal invariance, one could derive the following symmetry property under the exchange of the two arguments $x_1 \leftrightarrow x_2$ [14, 15, 16, 23]

$$T_{q,F}(x_1, x_2) = T_{q,F}(x_2, x_1), \qquad T_{\Delta q,F}(x_1, x_2) = -T_{\Delta q,F}(x_2, x_1). \tag{2.6}$$

the diagonal quark-gluon correlation functions $T_{q,F}(x, x)$ and $T_{\Delta q,F}(x, x)$ is responsible for the leading order gluonic pole contribution to the SSAs [6, 14, 15, 16], while the leading order fermionic pole contribution to the SSAs is given by the off-diagonal quark-gluon correlation functions, $T_{q,F}(0, x)$ and $T_{\Delta q,F}(0, x)$, or $T_{q,F}(x, 0)$ and $T_{\Delta q,F}(x, 0)$ [6, 23]. From Eq. (2.6), It is straightforward to find $T_{\Delta q,F}(x, x) = 0$.

In the same paper, Kang and Qiu also demonstrated that these twist-3 correlation functions can be represented by the Feynman diagram as shown in Fig. 2.1 with proper cut vertices. In the light-cone gauge, the cut vertices for these two quark-gluon correlation functions are given by [23]

$$\mathcal{V}^{\rm LC}_{q,F} = \frac{\gamma^+}{2p^+}\, 2\pi g \delta\left(x - \frac{k^+}{p^+}\right) y\, \delta\left(y - \frac{q^+}{p^+}\right) \left(i\, \epsilon^{s_\perp \mu n \bar{n}}\right) [-g_{\mu\sigma}]\, \mathcal{C}_q\,, \tag{2.7}$$

$$\mathcal{V}^{\rm LC}_{\Delta q,F} = \frac{\gamma^+\gamma^5}{2p^+}\, 2\pi g \delta\left(x - \frac{k^+}{p^+}\right) y\, \delta\left(y - \frac{q^+}{p^+}\right) \left(-s^\mu_\perp\right)\, [-g_{\mu\sigma}]\, \mathcal{C}_q, \tag{2.8}$$

where the color contraction factor $\mathcal{C}_q$ is

$$(\mathcal{C}_q)^c_{ij} = (t^c)_{ij}. \tag{2.9}$$

$i, j = 1, 2, 3 = N_c$ and $c = 1, 2, \ldots, 8 = N_c^2 - 1$, respectively. t^c are the generators of the fundamental representation of color SU(3) group. To evaluate Fig. 2.1, one still need to know how the quark and gluon are connected to the physical proton state, which depends on the model of the nucleon. In the next section, the calculation is presented in quark-diquark model of the nucleon for both scalar diquark and axial-vector diquark cases [24].

3. CALCULATION OF TWIST-3 QUARK CORRELATION FUNCTIONS IN THE QUARK-DIQUARK MODEL

In this section, the twist-3 quark-gluon correlation functions relevant to the gluonic and fermionic pole are calculated based on quark-diquark model of the nucleon [9, 25]. Both scalar and axial-vector diquark cases are evaluated. The section is organized as follows. In the first subsection, the Feynman rules and form factor used in the calculation are introduced. In subsection B, the calculations in scalar diquark case for twist-3 quark-gluon correlation functions relevant to both gluonic and fermionic pole contributions are presented. Finally, a similar calculation with an axial diquark is conduced in the last subsection.

3.1 The quark-diquark model of the nucleon

In the quark-diquark model, two of the three quarks in a hadron are much closer to each other than to the third one and form a diquark [9, 25]. The diquark could be either a scalar diquark or an axial-vector diquark. The single quark of mass m and the diquark of mass M_s compose the nucleon of mass M. The Feynman rules are as follows. The interaction between

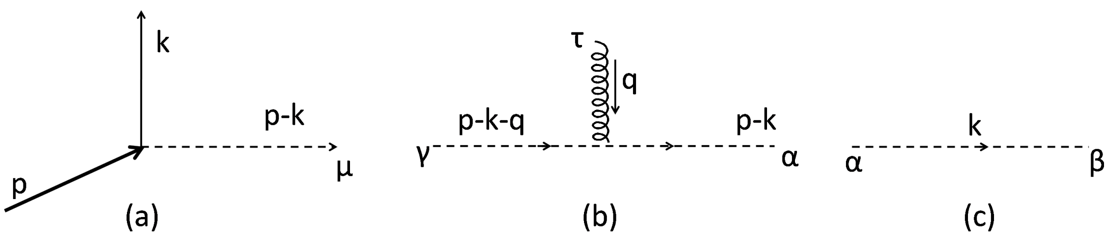

Figure 3.1: Feynman rules in the quark-diquark model of the nucleon: (a) vertex links the nucleon, the quark, and the diquark, (b) interaction vertex between the gluon and the diquark, and (c) the diquark propagator. The diquark could be a scalar particle or an axial-vector particle. The Lorentz indices are for the gluon and the axial-vector diquark.

the nucleon, the quark and the diquark (see Fig. 3.1(a)) is given by

$$i\lambda_s F_s(k^2) \qquad \text{scalar diquark,} \tag{3.1}$$

$$i\frac{\lambda_v}{\sqrt{2}}\gamma^\mu\gamma^5 F_v(k^2) \qquad \text{axial-vector diquark,} \tag{3.2}$$

with λ_s and λ_v represent the point-like interaction strength for a scalar and an axial-vector diquark respectively. $F_s(k^2)$ and $F_v(k^2)$ are the form factors which are used to remove the unphysical ultraviolet divergence in $k_\perp$ integration, with k the four momentum of the constituent quark. Several choices for the form factor were introduced and discussed in Ref.[25]. In the following discussion, We choose the same form factor for both the scalar and the axial-vector diquark as

$$F(k^2) = F_s(k^2) = F_v(k^2) = \frac{k^2 - m^2}{(k^2 - \Lambda_s^2)^2}\Lambda_s^2, \tag{3.3}$$

where $\Lambda_s^2 \gtrsim M^2$ is the ultraviolet cutoff. We will demonstrate below that the introduction of the form factor smoothly suppresses the influence of the ultraviolet region of $k_\perp^2$ or k^2 without affecting the pole structure. The Feynman rule for the coupling between the gluon and the diquark in Fig. 3.1(b) is

$$ig_s(2p - 2k - q)^\tau \qquad \text{scalar diquark,} \tag{3.4}$$

$$ig_v V^{\tau\gamma\alpha}(q, p-k-q, k-p) \qquad \text{axial-vector diquark,} \tag{3.5}$$

g_s and g_v are the coupling constant for scalar diquark and axial-vector diquark respectively. $V^{\tau\gamma\alpha}(q, p-k-q, k-p)$ is given by

$$V^{\tau\gamma\alpha}(q, p-k-q, k-p) = g^{\tau\gamma}(2q - p + k)^\alpha + g^{\gamma\alpha}(2p - 2k - q)^\tau + g^{\alpha\tau}(k - p - q)^\gamma. \tag{3.6}$$

The Feynman rule for the point-like diquark propagator with momentum k (as in Fig. 3.1(c)) is

$$\frac{i}{k^2 - M_s^2} \qquad \text{scalar diquark,} \tag{3.7}$$

$$\frac{i}{k^2 - M_s^2} d^{\alpha\beta}(k, M_s) \qquad \text{axial-vector diquark,} \tag{3.8}$$

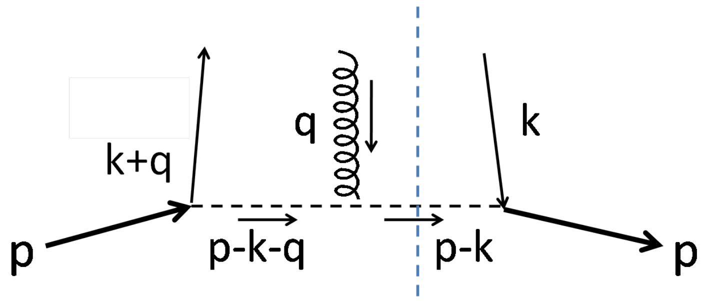

Figure 3.2: The lowest order Feynman diagram for twist-3 quark-gluon correlation functions in the quark-diquark model.

$d^{\alpha\beta}$ is the summation of different polarizations of the spin-1 axial-vector diquark. Several forms of $d^{\alpha\beta}$ are discussed in Ref. [25]. In the following calculation, We choose [25]

$$d^{\alpha\beta}(k, M_s) = -g^{\alpha\beta} + \frac{k^\alpha n^\beta + k^\beta n^\alpha}{n \cdot k} - \frac{M_s^2 n^\alpha n^\beta}{(n \cdot k)^2}. \tag{3.9}$$

It has the following properties

$$d^{\alpha\beta}(k, M_s) = d^{\beta\alpha}(k, M_s), \tag{3.10}$$

$$n_\alpha \cdot d^{\alpha\beta}(k, M_s) = 0, \tag{3.11}$$

$$k_\alpha \cdot d^{\alpha\beta}(k, M_s) = 0 \qquad \text{for } k^2 = M_s^2. \tag{3.12}$$

3.2 Calculation with a scalar diquark

In the leading power term of Fig. 2.1, The diquark can be treated as a point-like particle. Therefore, the diquark can be expressed as the propagator shown in Eq. (3.7). The lowest order Feynman diagram for $T_{q,F}$ and $T_{\Delta q,F}$ is shown in Fig. 3.2.

First study the quark-gluon correlation function with vertex in Eq. (2.7). For simplicity, the beam direction is along z-axis. With this consideration, the initial hadrons have no transverse momentum $p_\perp$ and $M^2 = p^2 = 2p^+p^-$. Apply Eq. (2.7), (3.1), (3.4) and (3.7) to the diagram in Fig. 3.2, one could obtain

$$T^{(s)}_{q,F}(x+y,x) = -N_c C_F \frac{g\,\lambda_s^2\, g_s\, \pi^2}{p^+} \int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\,\delta\left(x-\frac{k^+}{p^+}\right) y\,\delta\left(y-\frac{q^+}{p^+}\right)\delta\left((p-k)^2-M_s^2\right)$$
$$\times \epsilon^{s_\perp \sigma n \bar{n}}(2p-2k-q)^\tau \left(-g_{\sigma\tau}+\frac{q_\sigma n_\tau + q_\tau n_\sigma}{q\cdot n}\right)\mathrm{Tr}\left[\gamma^+(\not{k}+\not{q}+m)(\not{p}+M)\gamma^5 \not{s}_\perp(\not{k}+m)\right]$$
$$\times \frac{1}{k^2-m^2-i\epsilon}\frac{1}{q^2+i\epsilon}\frac{1}{(k+q)^2-m^2+i\epsilon}\frac{1}{(p-k-q)^2-M_s^2+i\epsilon} F(k^2)F((k+q)^2), \tag{3.13}$$

where ϵ is a small positive parameter, $F(k^2)$ and $F((k+q)^2)$ are the form factors contained in the Feynman rule as in Eq. (3.3), the subscript (s) indicates the scalar diquark. Integrating over k^+,k^-, and q^+ using the three δ-functions in Eq. (3.13),

$$\delta(x-\frac{k^+}{p^+}) = p^+\delta(k^+-xp^+)\ , \tag{3.14}$$

$$\delta(y-\frac{q^+}{p^+}) = p^+\delta(q^+-yp^+)\ , \tag{3.15}$$

$$\delta((p-k)^2-M_s^2) = \frac{1}{2(1-x)p^+}\delta\left(k^- - \frac{(1-x)M^2-k_\perp^2-M_s^2}{2(1-x)p^+}\right), \tag{3.16}$$

one could obtain

$$T^{(s)}_{q,F}(x+y,x) = N_c C_F \frac{g\,\lambda_s^2\, g_s}{16\pi p^+(1-x)} \int \frac{d^2q_\perp}{(2\pi)^2}\frac{d^2k_\perp}{(2\pi)^2}\frac{1}{k^2-m^2}\int\frac{dq^-}{2\pi}$$
$$\times \epsilon^{s_\perp \sigma n \bar{n}}(2p-2k-q)^\tau (q^+ g_{\sigma\tau} - q_\sigma n_\tau)\mathrm{Tr}\left[\gamma^+(\not{k}+\not{q}+m)(\not{p}+M)\gamma^5 \not{s}_\perp(\not{k}+m)\right]$$
$$\times \frac{1}{q^2+i\epsilon}\frac{1}{(k+q)^2-m^2+i\epsilon}\frac{1}{(p-k-q)^2-M_s^2+i\epsilon}\, F(k^2)\, F((k+q)^2), \tag{3.17}$$

with

$$q^+ = yp^+, \tag{3.18}$$

$$k^+ = xp^+, \tag{3.19}$$

$$k^- = \frac{(1-x)M^2-k_\perp^2-M_s^2}{2(1-x)p^+}. \tag{3.20}$$

The integration over q^- is crucial and is evaluated by taking the residue of relevant pole(s) of the integrand in Eq. (3.17), which provides the necessary phase for a real quark-gluon correlation function $T_{q,F}(x+y,x)$. For the leading nontrivial gluonic and fermionic pole contribution to the SSAs, one can only analyze the pole structure in Eq. (3.17) at $y=0$ (gluonic pole) and $x+y=0$ (fermionic pole) while $x>0$. From

$$(p-k-q)^2 - M_s^2 + i\epsilon = -2(1-x-y)p^+q^- - \frac{y(k_\perp^2+M_s^2)}{1-x} - 2k_\perp\cdot q_\perp - q_\perp^2 + i\epsilon = 0, \tag{3.21}$$

one obtains the location of the pole

$$q^- = -\frac{1}{2(1-x-y)p^+}\left[\frac{y(k_\perp^2+M_s^2)}{1-x} + 2k_\perp\cdot q_\perp + q_\perp^2 - i\epsilon\right]. \tag{3.22}$$

Since $x+y<1$, the pole is in the upper half plane of the q^- whenever $y=0$ or $x+y=0$. However, the potential poles from $q^2+i\epsilon=0$ and $(k+q)^2-m^2+i\epsilon=0$ are sensitive to these two limits. For the fermionic pole case, $x+y=0$ while $y<0$ since $x>0$, $q^2+i\epsilon=0$ provides a pole at

$$q^- = -\frac{q_\perp^2}{2|y|p^+} + i\epsilon, \tag{3.23}$$

which is in the upper half plane of the q^-, while

$$(k+q)^2 - m^2 + i\epsilon = 2(x+y)p^+(k+q)^- - (k_\perp+q_\perp)^2 - m^2 + i\epsilon = -(k_\perp+q_\perp)^2 - m^2 + i\epsilon \tag{3.24}$$

does not provide any pole in the q^- integration. Therefore, when $x+y=0$ and $x>0$, the integrand of q^- integration in Eq. (3.17) has only two poles from $(p-k-q)^2-M_s^2+i\epsilon=0$ and $q^2+i\epsilon=0$. Because both of the poles are in the upper half plane of q^- and the integration dq^- in Eq. (3.17) is sufficiently converging when $|q^-|\to\infty$, the q^- integration vanishes by closing the q^--contour through the lower half plane. In conclusion, $T_{q,F}^{(s)}(0,x)=0$ from this leading order calculation with a scalar diquark, so as $T_{q,F}^{(s)}(x,0)=0$, which can be derived by an explicit calculation or the symmetry property $T_{q,F}(x,0)=T_{q,F}(0,x)$.

In the gluonic pole case, $y=0$

$$(k+q)^2 - m^2 + i\epsilon = 2xp^+(k^-+q^-) - (k_\perp+q_\perp)^2 - m^2 + i\epsilon = 0, \tag{3.25}$$

contributes to a pole at

$$q^- = \frac{(k_\perp + q_\perp)^2 + m^2 - i\epsilon}{2xp^+} - k^-, \tag{3.26}$$

which is in the lower half plane of q^- since $x > 0$, while

$$q^2 + i\epsilon = -q_\perp^2 + i\epsilon \tag{3.27}$$

is independent of q^-. In conclusion, for the quark-gluon correlation functions relevant to the leading gluonic pole contribution to the SSAs, the q^--integration in Eq. (3.17) has two poles from $(p-k-q)^2 - M_s^2 + i\epsilon = 0$ and $(k+q)^2 - m^2 + i\epsilon = 0$ with the former in upper and the latter in lower half plane of q^-. In principle, one can close the contour in either the upper or the lower half plane and obtain the same result. Here we choose the contour in the upper half plane and thus

$$(p-k-q)^2 - M_s^2 + i\epsilon = 0, \tag{3.28}$$

from which we can get

$$q^- = \frac{2q_\perp \cdot k_\perp + q_\perp^2}{2(x-1)p^+}. \tag{3.29}$$

Use has been made of $y = 0$ and ϵ is ignored. After q^- integration, one could obtain

$$\begin{aligned} T_{q,F}^{(s)}(x+y,x) = & -i\, N_c C_F \frac{g\, \lambda_s^2\, g_s}{32\pi {p^+}^2 (1-x)^2} \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{d^2 k_\perp}{(2\pi)^2} \frac{1}{k^2 - m^2} \\ & \times \epsilon^{s_\perp \sigma n \bar{n}} (2p - 2k - q)^\tau (q^+ g_{\sigma\tau} - q_\sigma n_\tau) \mathrm{Tr}\left[\gamma^+ (\not{k} + \not{q} + m)(\not{p} + M)\gamma^5 \not{s}_\perp (\not{k} + m)\right] \\ & \times \frac{1}{q^2} \frac{1}{(k+q)^2 - m^2} F(k^2) F((k+q)^2). \end{aligned} \tag{3.30}$$

Further simplification could be achieved by introducing $L_s^2(m^2)$ as

$$L_s^2(m^2) = x M_s^2 + (1-x) m^2 - x(1-x) M^2, \tag{3.31}$$

with which one could derive

$$\begin{aligned} k^2 &= 2k^+ k^- - k_\perp^2 \\ &= 2xp^+ \frac{(1-x)M^2 - k_\perp^2 - M_s^2}{2(1-x)p^+} - k_\perp^2 \\ &= m^2 - \frac{1}{1-x}\left[k_\perp^2 + L_s^2(m^2)\right], \end{aligned} \tag{3.32}$$

and similarly

$$(k+q)^2 \;=\; m^2 - \frac{1}{1-x}\left[(k_\perp+q_\perp)^2 + L_s^2(m^2)\right], \tag{3.33}$$

where Eq. (3.20) , (3.29) and $y=0$ are used in the derivation. Eq. (3.30) is simplified as

$$\begin{aligned} T_{q,F}^{(s)}(x,x) = i\; N_c C_F \frac{g\,\lambda_s^2\, g_s}{32\pi p^{+2}} &\int \frac{d^2q_\perp}{(2\pi)^2}\frac{d^2k_\perp}{(2\pi)^2}\frac{1}{k_\perp^2 + L_s^2(m^2)} \\ &\times \epsilon^{s_\perp \sigma n\bar{n}}(2p-2k-q)^\tau (q^+ g_{\sigma\tau} - q_\sigma n_\tau)\mathrm{Tr}\left[\gamma^+(\not{k}+\not{q}+m)(\not{p}+M)\gamma^5 \not{s}_\perp(\not{k}+m)\right] \\ &\times \frac{1}{q_\perp^2}\frac{1}{(k_\perp+q_\perp)^2 + L_s^2(m^2)} F(k^2)F((k+q)^2). \end{aligned} \tag{3.34}$$

In evaluating the second row of Eq. (3.34), one could take advange of $y=0$ and simplify the calculation

$$\begin{aligned} &\epsilon^{s_\perp \sigma n\bar{n}}(2p-2k-q)^\tau (q^+ g_{\sigma\tau} - q_\sigma n_\tau)\mathrm{Tr}\left[\gamma^+(\not{k}+\not{q}+m)(\not{p}+M)\gamma^5 \not{s}_\perp(\not{k}+m)\right] \\ &= -2(1-x)p^+ \epsilon^{s_\perp q n\bar{n}}\mathrm{Tr}\left[\gamma^+(\not{k}+\not{q}+m)(\not{p}+M)\gamma^5 \not{s}_\perp(\not{k}+m)\right] \\ &= -2(1-x)p^+ \epsilon^{s_\perp q n\bar{n}}\left(4im\epsilon^{nqps_\perp} - 4iM\epsilon^{nqs_\perp k}\right) \\ &= -8i(1-x)p^{+2}(m+xM)\left[q_\perp^2 - (q_\perp\cdot s_\perp)^2\right]. \end{aligned} \tag{3.35}$$

With this result, Eq. (3.34) could be shown as

$$T_{q,F}^{(s)}(x,x) = \frac{N_c C_F g\lambda_s^2 g_s}{4\pi}(1-x)(m+xM)\int\frac{d^2k_\perp}{(2\pi)^2}\frac{d^2q_\perp}{(2\pi)^2}\frac{\left[q_\perp^2 - (q_\perp\cdot s_\perp)^2\right]F(k^2)\,F((k+q)^2)}{q_\perp^2\left[k_\perp^2 + L_s^2(m^2)\right]\left[(k_\perp+q_\perp)^2 + L_s^2(m^2)\right]}. \tag{3.36}$$

Choose both form factors $F(k^2)$ and $F((k+q)^2)$ as in Eq. (3.3), which remove the ultraviolet divergence of the integration, use Eq. (3.31) to rewrite the form factors as

$$F(k^2) = (1-x)\frac{k_\perp^2 + L_s^2(m^2)}{(k_\perp^2 + L_s^2(\Lambda_s^2))^2}\,\Lambda_s^2, \tag{3.37}$$

and similarly

$$F((k+q)^2) = (1-x)\frac{(k_\perp+q_\perp)^2 + L_s^2(m^2)}{((k_\perp+q_\perp)^2 + L_s^2(\Lambda_s^2))^2}\,\Lambda_s^2, \tag{3.38}$$

where $L_s^2(\Lambda_s^2)$ is given in Eq. (3.31) with m^2 replaced by the cutoff scale Λ_s^2. Thus one can obtain

$$\begin{aligned} T_{q,F}^{(s)}(x,x)\Big|_{\text{dipolar}} &= \frac{N_c C_F g\lambda_s^2 g_s}{4\pi}(1-x)^3(m+xM)\int\frac{d^2k_\perp}{(2\pi)^2}\frac{d^2q_\perp}{(2\pi)^2}\frac{\left[q_\perp^2 - (q_\perp\cdot s_\perp)^2\right](\Lambda_s^2)^2}{q_\perp^2\left[k_\perp^2 + L_s^2(\Lambda_s^2)\right]^2\left[(k_\perp+q_\perp)^2 + L_s^2(\Lambda_s^2)\right]^2} \\ &= \frac{N_c C_F g\lambda_s^2 g_s}{4\pi}(1-x)^3(m+xM)(\Lambda_s^2)^2\int\frac{d^2k_\perp}{(2\pi)^2}\frac{I}{\left[k_\perp^2 + L_s^2(\Lambda_s^2)\right]^2}, \end{aligned} \tag{3.39}$$

I could be calculated using Feynman parameter,

$$
\begin{aligned}
I &\equiv \int \frac{d^2q_\perp}{(2\pi)^2} \frac{q_\perp^2 - (q_\perp \cdot s_\perp)^2}{q_\perp^2 [(k_\perp + q_\perp)^2 + L_s^2(\Lambda_s^2)]^2} \\
&= \int \frac{d^2q_\perp}{(2\pi)^2} [q_\perp^2 - (q_\perp \cdot s_\perp)^2] \int_0^1 d\alpha \frac{2\alpha}{[(1-\alpha)q_\perp^2 + \alpha\left((k_\perp + q_\perp)^2 + \alpha L_s^2(\Lambda_s^2)\right)]^3} \\
&= \int \frac{d^2q_\perp}{(2\pi)^2} [q_\perp^2 - (q_\perp \cdot s_\perp)^2] \int_0^1 d\alpha \frac{2\alpha}{[(q_\perp + \alpha k_\perp)^2 + \alpha(1-\alpha)k_\perp^2 + \alpha L_s^2(\Lambda_s^2)]^3} \\
&= \int \frac{d^2q_\perp}{(2\pi)^2} \int_0^1 d\alpha \frac{2\alpha[q_\perp^2 + \alpha^2 k_\perp^2 - (q_\perp \cdot s_\perp)^2 - \alpha^2 (k_\perp \cdot s_\perp)]}{[q_\perp^2 + \alpha(1-\alpha)k_\perp^2 + \alpha L_s^2(\Lambda_s^2)]^3} \\
&= \pi \int \frac{dq_\perp^2}{(2\pi)^2} \int_0^1 d\alpha \frac{\alpha(q_\perp^2 + \alpha^2 k_\perp^2)}{[q_\perp^2 + \alpha(1-\alpha)k_\perp^2 + \alpha L_s^2(\Lambda_s^2)]^3} \\
&= \frac{1}{8\pi} \frac{1}{L_s^2(\Lambda_s^2)}.
\end{aligned}
\tag{3.40}
$$

Therefore

$$
\begin{aligned}
T_{q,F}^{(s)}(x,x)\Big|_{\text{dipolar}} &= \frac{N_c C_F g \lambda_s^2 g_s}{8(2\pi)^2} (1-x)^3 (m + xM) \frac{(\Lambda_s^2)^2}{L_s^2(\Lambda_s^2)} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{1}{\left[k_\perp^2 + L_s^2(\Lambda_s^2)\right]^2} \\
&= \frac{N_c C_F g \lambda_s^2 g_s}{16(2\pi)^3} (1-x)^3 (m + xM) \left(\frac{\Lambda_s^2}{L_s^2(\Lambda_s^2)}\right)^2.
\end{aligned}
\tag{3.41}
$$

Because the form factor in Eq. (3.3) suppresses the ultraviolet divergence without altering the pole structure of the original diagram, Eq. (3.36) holds independent of the existence of the form factors. Choose $F(k^2) = F((k+q)^2) = 1$, which is the same with no form factors. We present the calculation below. After integration over k^-

$$
\begin{aligned}
T_{q,F}^{(s)}(x,x)\Big|_{\text{point-like}} &= \frac{N_c C_F g \lambda_s^2 g_s}{4\pi} (1-x)(m+xM) \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2q_\perp}{(2\pi)^2} \frac{[q_\perp^2 - (q_\perp \cdot s_\perp)^2]}{q_\perp^2 \left[k_\perp^2 + L_s^2(m^2)\right]\left[(k_\perp + q_\perp)^2 + L_s^2(m^2)\right]} \\
&= \frac{N_c C_F g \lambda_s^2 g_s}{16\pi^2} (1-x)(m + xM) \int_0^1 d\alpha \int \frac{d^2q_\perp}{(2\pi)^2} \frac{q_\perp^2 - (q_\perp \cdot s_\perp)^2}{q_\perp^2 \left[\alpha(1-\alpha)q_\perp^2 + L_s^2(m^2)\right]}. \\
&= \frac{N_c C_F g \lambda_s^2 g_s}{16\pi^2} (1-x)(m+xM) \int \frac{d^2q_\perp}{(2\pi)^2} \frac{1}{q_\perp \sqrt{q_\perp^2 + 4L_s^2(m^2)}} \ln \frac{\sqrt{q_\perp^2 + 4L_s^2(m^2)} + q_\perp}{\sqrt{q_\perp^2 + 4L_s^2(m^2)} - q_\perp},
\end{aligned}
\tag{3.42}
$$

The integration over $q_\perp$ is ultraviolet divergent as expected.

The calculation for $T_{\Delta q,F}(x+y,x)$ is almost the same with that of $T_{q,F}(x+y,x)$ except the cut vertex is replaced by the one in Eq. (2.8). One can obtain the expression for diagram in Fig. 3.2 as follows

$$T^{(s)}_{\Delta q,F}(x+y,x) = -i\, N_c C_F \frac{g\,\lambda_s^2\, g_s\, \pi^2}{p^+}\int\frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\,\delta\left(x-\frac{k^+}{p^+}\right) y\,\delta\left(y-\frac{q^+}{p^+}\right)\delta\left((p-k)^2-M_s^2\right)$$
$$\times s_\perp^\sigma(2p-2k-q)^\tau\left(-g_{\sigma\tau}+\frac{q_\sigma n_\tau+q_\tau n_\sigma}{q\cdot n}\right)\mathrm{Tr}\left[\gamma^+\gamma^5(\not k+\not q+m)(\not p+M)\gamma^5\not s_\perp(\not k+m)\right]$$
$$\times\frac{1}{k^2-m^2-i\epsilon}\frac{1}{q^2+i\epsilon}\frac{1}{(k+q)^2-m^2+i\epsilon}\frac{1}{(p-k-q)^2-M_s^2+i\epsilon}F(k^2)F((k+q)^2) \tag{3.43}$$

As one can find from the the pole analysis for $T_{q,F}(x+y,x)$, that the contraction, trace and the form factors do not contribute poles, all poles come from the denominators of the quark and gluon propagators. Since these denominators are the same in Eq. (3.13) and Eq. (3.43), for the fermionic pole with $x+y=0$, both poles are in the upper half plane of q^- and thus

$$T^{(s)}_{\Delta q,F}(0,x) = T^{(s)}_{\Delta q,F}(x,0) = 0. \tag{3.44}$$

Because of the symmetry property in Eq. (2.6), one can expect the diagonal correlation function $T^{(s)}_{\Delta q,F}(x,x)$ relevant to the gluonic pole contrubution is zero. As a consistent test of our model calculation, we verify this result explicitly as follows. In the same procedure as we analyze $T^{(s)}_{q,F}(x,x)$, after integration over k^+,q^+,k^- and q^-, we obtain

$$T^{(s)}_{\Delta q,F}(x,x) = -\frac{N_c C_F g \lambda_s^2 g_s}{32\pi p^{+2}}\int\frac{d^2k_\perp}{(2\pi)^2}\frac{d^2q_\perp}{(2\pi)^2}\frac{F(k^2)\,F((k+q)^2)}{q_\perp^2[k_\perp^2+L_s^2(m^2)][(k_\perp+q_\perp)^2+L_s^2(m^2)]}$$
$$\times s_\perp^\sigma(2p-2k-q)^\tau(q^+g_{\sigma\tau}-q_\sigma n_\tau)\mathrm{Tr}\left[\gamma^+\gamma^5(\not k+\not q+m)(\not p+M)\gamma^5\not s_\perp(\not k+m)\right], \tag{3.45}$$

where use is made of Eq. (3.32) and (3.33) for simplication. The contraction part and the trace part in Eq. (3.45) can be calculated in a straightforward way,

$$\begin{aligned}&s_\perp^\sigma(2p-2k-q)^\tau\left(q^+g_{\sigma\tau}-q_\sigma n_\tau\right)\\ =\;& 2(1-x)p^+ s_\perp\cdot q_\perp\,,\end{aligned} \tag{3.46}$$

where we have used y=0. The trace part is

$$
\begin{aligned}
& \mathrm{Tr}\left[\gamma^+\gamma^5(\not{k}+\not{q}+m)(\not{p}+M)\gamma^5\not{s}_\perp(\not{k}+m)\right] \\
= & \ \mathrm{Tr}\left[\gamma^+(\not{k}+\not{q}-m)(\not{p}-M)\not{s}_\perp(\not{k}+m)\right] \\
= & \ -m\mathrm{Tr}\left[\gamma^+\not{p}\not{s}_\perp\not{k}\right] - M\mathrm{Tr}\left[\gamma^+(\not{k}+\not{q})\not{s}_\perp\not{k}\right] + m\mathrm{Tr}\left[\gamma^+(\not{k}+\not{q})\not{p}\not{s}_\perp\right] \\
= & \ -4mp^+(s_\perp\cdot k_\perp) - 4xMp^+(2s_\perp\cdot k_\perp + s_\perp\cdot q_\perp) - 4mp^+(s_\perp\cdot k_\perp + s_\perp\cdot q_\perp) \\
= & \ -4(xM+m)(2s_\perp\cdot k_\perp + s_\perp\cdot q_\perp).
\end{aligned} \tag{3.47}
$$

Therefore,

$$
T^{(s)}_{\Delta q,F}(x,x) = \frac{N_c C_F g \lambda_s^2 g_s}{4\pi}(1-x)(m+xM)\int\frac{d^2k_\perp}{(2\pi)^2}\frac{d^2q_\perp}{(2\pi)^2}\frac{q_\perp\cdot s_\perp[2k_\perp\cdot s_\perp + q_\perp\cdot s_\perp]F(k^2)F((k+q)^2)}{q_\perp^2[k_\perp^2+L_s^2(m^2)][(k_\perp+q_\perp)^2+L_s^2(m^2)]}\ . \tag{3.48}
$$

After utilization of the form factor as in Eq. (3.37) and (3.38), one could apply Feynman parameter

$$
T^{(s)}_{\Delta q,F}(x,x)\Big|_{\rm dipolar} = \frac{N_c C_F g \lambda_s^2 g_s}{4\pi}(1-x)^3(m+xM)\int\frac{d^2q_\perp}{(2\pi)^2}\frac{d^2\ell_\perp}{(2\pi)^2}\int_0^1 d\alpha\, \frac{3!\,\alpha(1-\alpha)(1-2\alpha)(q_\perp\cdot s_\perp)^2}{q_\perp^2\left[\ell_\perp^2+\alpha(1-\alpha)q_\perp^2+L_s^2(m^2)\right]^4}, \tag{3.49}
$$

where $l_\perp = k_\perp + (1-\alpha)q_\perp$. The numerator of the α integral is antisymmetric under $\alpha \leftrightarrow 1-\alpha$ while the denominator and the integration limits are symmetric. Therefore, Eq. (3.49) is zero as expected. To demonstrate the zero result is independent of the specific form of the form factor, we evaluate Eq. (3.48) for point-like interaction by setting $F(k^2) = F((k+q)^2) = 1$

$$
T^{(s)}_{\Delta q,F}(x,x)\Big|_{\rm point-like} = \frac{N_c C_F g \lambda_s^2 g_s}{4\pi}(1-x)(m+xM)\int\frac{d^2q_\perp}{(2\pi)^2}\frac{d^2\ell_\perp}{(2\pi)^2}\int_0^1 d\alpha\frac{(1-2\alpha)(q_\perp\cdot s_\perp)^2}{q_\perp^2\left[\ell_\perp^2+\alpha(1-\alpha)q_\perp^2+L_s^2(m^2)\right]^2}. \tag{3.50}
$$

The numerator in Eq. (3.50) is still antisymmetric while the denominator is symmetric under $\alpha \leftrightarrow 1-\alpha$. Therefore $T^{(s)}_{\Delta q,F}(x,x)$ vanishes independent of the specific form of the form factor.

3.3 Calculation with an axial-vector diquark

Because proton and quark are both spin-$\frac{1}{2}$ particles, diquark can be spin-0 or spin-1, which correspond to a scalar diquark or an axial-vector diquark, respectively. To test the sensitivity of our results derived above on the choice of the scalar diquark, we present here a similar calculation with an axial-vector diquark.

The Feynman diagram is the same as Fig. 3.2 but with an axial-vector diquark. Using Eq. (3.2), (3.5), (3.6), (3.8) and (3.9), one can obtain the expression

$$\begin{aligned} T^{(v)}_{q,F}(x+y,x) =& -N_c C_F \frac{g\,\lambda_v^2\, g_v\, \pi^2}{p^+} \int \frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\,\delta\left(x-\frac{k^+}{p^+}\right) y\,\delta\left(y-\frac{q^+}{p^+}\right)\delta\left((p-k)^2-M_s^2\right) \\ &\times \epsilon^{s_\perp \mu n\bar{n}} V^{\mu'\alpha'\omega'}(q,p-k-q,k-q)\left(-g_{\mu'\mu}+\frac{q_{\mu'}n_\mu+q_\mu n_{\mu'}}{q\cdot n}\right) d_{\alpha\alpha'}(p-k) d_{\omega\omega'}(p-k-q) \\ &\times \mathrm{Tr}\left[\gamma^+(\not{k}+\not{q}+m)\gamma^\alpha\gamma^5(\not{p}+M)\gamma^5\not{s}_\perp\gamma^\omega\gamma^5(\not{k}+m)\right] \\ &\times \frac{1}{k^2-m^2-i\epsilon}\frac{1}{q^2+i\epsilon}\frac{1}{(k+q)^2-m^2+i\epsilon}\frac{1}{(p-k-q)^2-M_s^2+i\epsilon} F(k^2)F((k+q)^2)\,. \end{aligned} \tag{3.51}$$

We still choose the form factors $F(k^2)$ and $F((k+q)^2)$ as in Eq. (3.3), or in Eq. (3.37) and (3.38). Following the same procedure, one could make use of the three δ-functions in the integration over k^+, k^- and q^+. In the integration over q^-, after analysing the denominators in Eq. (3.51), one could find that all poles are from $1/q^2$, $1/((k+q)^2-m^2)$ and $1/((p-k-q)^2-M_s^2)$, which are exactly the same with Eq. (3.13) . In other words, the pole structure of the Feynman diagram in Fig. 3.2 is insensitive to whether the spectator diquark is a scalar or an axial-vector. Therefore, as in the case of a scalar diquark, that all off-diagonal quark-gluon correlation functions relevant to the leading fermionic pole contribution vanish immediately,

$$T^{(v)}_{q,F}(0,x) = T^{(v)}_{\Delta q,F}(0,x) = 0. \tag{3.52}$$

Calculation of the gluonic pole $T_{q,F}(x,x)$ is not straightforward. After the integration over k^+, k^- and q^+ using the three δ-functions, one could abtain

$$\begin{aligned} T^{(v)}_{q,F}(x+y,x) =& N_c C_F \frac{g\lambda_v^2\, g_v}{16\pi p^+(1-x)} \int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2q_\perp dq^-}{(2\pi)^3} \\ &\times \epsilon^{s_\perp \mu n\bar{n}} V^{\mu'\alpha'\omega'}(q,p-k-q,k-q)\left[q^+ g_{\mu'\mu}-(q_{\mu'}n_\mu+q_\mu n_{\mu'})\right] d_{\alpha\alpha'}(p-k) d_{\omega\omega'}(p-k-q) \\ &\times \mathrm{Tr}\left[\gamma^+(\not{k}+\not{q}+m)\gamma^\alpha\gamma^5(\not{p}+M)\gamma^5\not{s}_\perp\gamma^\omega\gamma^5(\not{k}+m)\right] \\ &\times \frac{1}{k^2-m^2-i\epsilon}\frac{1}{q^2+i\epsilon}\frac{1}{(k+q)^2-m^2+i\epsilon}\frac{1}{(p-k-q)^2-M_s^2+i\epsilon} F(k^2)F((k+q)^2)\,. \end{aligned} \tag{3.53}$$

For the q^- integration, here we still choose the pole $(p-k-q)^2+M_s^2=0$ as in the scalar diquark case. After q^- integration and the simplification with $L_s^2(m^2)$, we obtain

$$
\begin{aligned}
T_{q,F}^{(v)}(x+y,x) = i\, N_c C_F \frac{g\lambda_v^2\, g_v}{32\pi {p^+}^2} \int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2q_\perp}{(2\pi)^2} \\
\times \epsilon^{s_\perp \mu n\bar{n}} V^{\mu'\alpha'\omega'}(q,p-k-q,k-q)\left[q^+ g_{\mu'\mu} - (q_{\mu'}n_\mu + q_\mu n_{\mu'})\right] d_{\alpha\alpha'}(p-k) d_{\omega\omega'}(p-k-q) \\
\times \mathrm{Tr}\left[\gamma^+(\not{k}+\not{q}+m)\gamma^\alpha\gamma^5(\not{p}+M)\gamma^5 \not{s}_\perp \gamma^\omega\gamma^5(\not{k}+m)\right] \\
\times \frac{1}{k_\perp^2+L_s^2(m^2)}\frac{1}{q_\perp^2}\frac{1}{(k_\perp+q_\perp)^2+L_s^2(m^2)+i\epsilon} F(k^2)F((k+q)^2)\,.
\end{aligned}
\tag{3.54}
$$

For clarity, we list Eq. (3.18), (3.19), (3.20) and (3.29) again as follows.

$$q^+ = yp^+, \tag{3.55}$$

$$k^+ = xp^+, \tag{3.56}$$

$$k^- = \frac{(1-x)M^2 - k_\perp^2 - M_s^2}{2(1-x)p^+}, \tag{3.57}$$

$$q^- = \frac{2q_\perp\cdot k_\perp + q_\perp^2}{2(x-1)p^+}. \tag{3.58}$$

The contraction and trace are long and tedious. Eq.(3.11), (3.12), (3.12) and $y=0$ should be exploited for simplification.

$$
\begin{aligned}
&\epsilon^{s_\perp \mu n\bar{n}} V^{\mu'\alpha'\omega'}(q,p-k-q,k-q)\left[q^+ g_{\mu'\mu} - (q_{\mu'}n_\mu + q_\mu n_{\mu'})\right] d_{\alpha\alpha'}(p-k) d_{\omega\omega'}(p-k-q) \\
\times\ & \mathrm{Tr}\left[\gamma^+(\not{k}+\not{q}+m)\gamma^\alpha\gamma^5(\not{p}+M)\gamma^5 \not{s}_\perp \gamma^\omega\gamma^5(\not{k}+m)\right]. \\
=\ & -8ix(m+Mx){p^+}^2[q^2-(S_\perp\cdot q)^2]
\end{aligned}
\tag{3.59}
$$

After all these calculation, we obtain

$$T_{q,F}^{(v)}(x,x) = \frac{N_c C_F g\lambda_v^2 g_v}{4\pi}\, x(m+xM) \int \frac{d^2k_\perp}{(2\pi)^2}\frac{d^2q_\perp}{(2\pi)^2} \frac{[q_\perp^2 - (q_\perp\cdot s_\perp)^2]\, F(k^2)F((k+q)^2)}{q_\perp^2\left[k_\perp^2+L_s^2(m^2)\right]\left[(k_\perp+q_\perp)^2+L_s^2(m^2)\right]}\,, \tag{3.60}$$

which are the same with in Eq. (3.36) except the overall $(1-x)$ factor is replaced by x due to the difference in diquark spin. Therefore, the rest of evaluation and discussion following

Eq. (3.36) in last subsection should be the same. We just show the results as follows,

$$T_{q,F}^{(v)}(x,x)\Big|_{\text{point-like}} = \frac{N_c C_F g \lambda_v^2 g_v}{16\pi^2} x(m+xM) \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{1}{q_\perp \sqrt{q_\perp^2 + 4L_s^2(m^2)}} \ln \frac{\sqrt{q_\perp^2 + 4L_s^2(m^2)} + q_\perp}{\sqrt{q_\perp^2 + 4L_s^2(m^2)} - q_\perp}, \tag{3.61}$$

$$T_{q,F}^{(v)}(x,x)\Big|_{\text{dipolar}} = \frac{N_c C_F g \lambda_v^2 g_v}{16(2\pi)^3} x(1-x)^2(m+xM) \left(\frac{\Lambda_s^2}{L_s^2(\Lambda_s^2)}\right)^2, \tag{3.62}$$

which are the same with Eq. (3.41) and (3.42) except that one factor of $(1-x)$ is replaced by x.

For fermionic pole $T_{\Delta q,F}(0,x)$, one starts with

$$\begin{aligned} T_{\Delta q,F}^{(v)}(x+y,x) = & -iN_c C_F \frac{g\,\lambda_v^2\, g_v\, \pi^2}{p^+} \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4}\, \delta\left(x - \frac{k^+}{p^+}\right) y\, \delta\left(y - \frac{q^+}{p^+}\right) \delta\left((p-k)^2 - M_s^2\right) \\ & \times s_\perp^\mu V^{\mu'\alpha'\omega'}(q, p-k-q, k-q) \left(-g_{\mu'\mu} + \frac{q_{\mu'} n_\mu + q_\mu n_{\mu'}}{q\cdot n}\right) d_{\alpha\alpha'}(p-k) d_{\omega\omega'}(p-k-q) \\ & \times \mathrm{Tr}\left[\gamma^+\gamma^5(\not{k} + \not{q} + m)\gamma^\alpha\gamma^5(\not{p} + M)\gamma^5 \not{s}_\perp \gamma^\omega \gamma^5 (\not{k} + m)\right] \\ & \times \frac{1}{k^2 - m^2 - i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{1}{(k+q)^2 - m^2 + i\epsilon} \frac{1}{(p-k-q)^2 - M_s^2 + i\epsilon} F(k^2) F((k+q)^2). \end{aligned} \tag{3.63}$$

After the integration over k^+, k^- and q^+ using the three δ-function, and the integration over q^- by the pole structure which is the same with $T_{\Delta q,F}(x,x)$ for scalar diquark case, one obtains

$$\begin{aligned} T_{\Delta q,F}^{(v)}(x+y,x) = & -N_c C_F \frac{g \lambda_v^2\, g_v}{32\pi p^{+2}} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 q_\perp}{(2\pi)^2} \\ & \times s_\perp^\mu V^{\mu'\alpha'\omega'}(q, p-k-q, k-q) [q^+ g_{\mu'\mu} - (q_{\mu'} n_\mu + q_\mu n_{\mu'})] d_{\alpha\alpha'}(p-k) d_{\omega\omega'}(p-k-q) \\ & \times \mathrm{Tr}\left[\gamma^+\gamma^5(\not{k} + \not{q} + m)\gamma^\alpha\gamma^5(\not{p} + M)\gamma^5 \not{s}_\perp \gamma^\omega \gamma^5 (\not{k} + m)\right] \\ & \times \frac{1}{k_\perp^2 + L_s^2(m^2)} \frac{1}{q_\perp^2} \frac{1}{(k_\perp + q_\perp)^2 + L_s^2(m^2) + i\epsilon} F(k^2) F((k+q)^2), \end{aligned} \tag{3.64}$$

The contraction (second row) and trace (third row) can be calculated as

$$\begin{aligned} & s_\perp^\mu V^{\mu'\alpha'\omega'}(q, p-k-q, k-q) \left[q^+ g_{\mu'\mu} - (q_{\mu'} n_\mu + q_\mu n_{\mu'})\right] d_{\alpha\alpha'}(p-k) d_{\omega\omega'}(p-k-q) \\ & \times \mathrm{Tr}\left[\gamma^+\gamma^5(\not{k} + \not{q} + m)\gamma^\alpha\gamma^5(\not{p} + M)\gamma^5 \not{s}_\perp \gamma^\omega \gamma^5 (\not{k} + m)\right] \\ = \; & 8p^{+2}(m+xM)x(q\cdot s_\perp)(q\cdot s_\perp + 2k\cdot s_\perp). \end{aligned} \tag{3.65}$$

Therefore

$$T_{\Delta q,F}(x,x) = -N_c C_F \frac{g\lambda_v^2 g_v}{4\pi} x(m+xM) \int \frac{d^2k_\perp}{(2\pi)^4} \int \frac{d^2q_\perp}{(2\pi)^4} \frac{(q_\perp \cdot s_\perp)(q_\perp \cdot s_\perp + 2k_\perp \cdot s_\perp) F(k^2) F((k+q)^2)}{[(k_\perp + q_\perp)^2 + L_s^2(m^2)][k_\perp^2 + L_s^2(m^2)] q_\perp^2}, \tag{3.66}$$

which is different from Eq. (3.48) only by a constant factor. So we also explicitly verify that $T_{\Delta q,F}(x,x) = 0$ when it is calculated with an axial-vector diquark.

As a conclusion, we summarize our key results as follows. We find, in terms of an explicit calculation in the quark-diquark model of the nucleon, that at the leading non-trivial order all quark-gluon correlation functions relevant to the leading fermionic pole contribution to the SSAs vanish,

$$T_{q,F}(0,x) = T_{q,F}(x,0) = 0, \qquad T_{\Delta q,F}(0,x) = -T_{\Delta q,F}(x,0) = 0\,. \tag{3.67}$$

We also verify that $T_{\Delta q,F}(x,x) = 0$, and find that only the diagonal quark-gluon correlation function, $T_{q,F}(x,x)$, is finite.

4. SUMMARY AND CONCLUSIONS

Based on the quark-diquark model of the nucleon, We calculated various twist-3 quark-gluon correlation functions of a transversely polarized nucleon relevant to the leading soft pole and compared their contributions to the SSAs. We found in both scalar diquark and axial vector diquark cases, the leading fermionic pole contribution, $T_{q,F}(0,x)$,$T_{\Delta q,F}(0,x)$,$T_{q,F}(x,0)$ and $T_{\Delta q,F}(x,0)$, vanish. Only one of the diagonal quark-gluon correlation functions relevant to the leading gluonic pole contribution, $T_{q,F}(x,x)$, is finite. The other diagonal quark-gluon correlation function, $T_{\Delta q,F}(x,x)$, also vanishes from both the symmetry argument and explicit calculation. Our conclusions are independent of the diquark being a scalar or an axial-vector.

Although our calculations is based on certain model which does not include all information of a nucleon, the features of the results should allow us to conclude with confidence that the diagonal quark-gluon correlation function $T_{q,F}(x,x)$ is much larger than all other quark-gluon correlation functions that are relevant to the leading soft pole contribution to the SSAs. This conclusion is significant and important for phenomenological study of the SSAs. It enables us to study the physics of SSAs without including too many unknown correlation functions at the early stage of probing this new domain of QCD dynamics.

To have a complete understanding of twist-3 correlation functions, one still need to calculate the tri-gluon correlation functions [23]. However, it is impossible in quark-diquark model because of the limitation of this model.

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