An Optimal Selection of Induction Heater Capacitance Considering Dissipation Loss Caused by ESR

Jung-gi Lee, Sun-kyoung Lim, Kwang-hee Nam and and Dong-ik Choi*
Department of Electrical Engineering, POSTECH University,
Hyoja San-31, Pohang, 790-784 Republic of Korea.
Tel:(82)54-279-2218, Fax:(82)54-279-5699,
E-mail:kwnam@postech.ac.kr
*POSCO Gwangyang Works, 700, Gumho-dong,
Gwangyang-si, Jeonnam, 541-711, Korea.

***Abstract*—In the design of a parallel resonant induction heating system, choosing a proper capacitance for the resonant circuit is quite important. The capacitance affects the resonant frequency, output power, Q-factor, heating efficiency and power factor. In this paper, the role of equivalent series resistance (ESR) in the choice of capacitance is significantly recognized. Without the effort of reducing temperature rise of the capacitor, the life time of capacitor tends to decrease rapidly. This paper, therefore, presents a method of finding an optimal value of the capacitor under voltage constraint for maximizing the output power of an induction heater, while minimizing the power loss of the capacitor at the same time. Based on the equivalent circuit model of an induction heating system, the output power, and the capacitor losses are calculated. The voltage constraint comes from the voltage ratings of the capacitor bank and the switching devices of the inverter. The effectiveness of the proposed method is verified by simulations and experiments.**

I. INTRODUCTION

Induction heating is widely used in metal industry for melting or heating of thin slab in a continuous casting plant because of good heating efficiency, high production rate, and clean working environments. A typical parallel resonant inverter circuit for induction heater is shown in Fig. 1. The phase controlled rectifier provides a constant DC current source. The H-bridge inverter consists of four thyristors and a parallel resonant circuit comprised capacitor bank and heating coil. Thyristors are naturally commutated by the ac current flowing through the resonant circuit. Therefore, this type of inverter called as a load commutated inverter [1], [2], [3].

In the parallel resonant inverter, if the switching frequency is closer to the resonant frequency, higher voltage is generated at capacitor bank [2]. However, due to the limit in the voltage tolerance of the capacitor bank, the inverter output voltage v_C needs to be limited below the rated voltage V_{max}. One method of limiting v_C is to reduce the DC-link current I_{dc} by increasing the firing angle of the rectifier. However, it results in the decrease of I_{dc}, thereby the decrease of the output power.

At the mini-mill in a POSCO steel plant, eight induction heaters of load commutated type were installed to heat thin slabs for next milling process in the continuous casting plant. The rated output power of each induction heater is 1.5 MW. However, there were two problems: One was the insufficiency in the output power, and the other was the frequent damages of the capacitor bank. Insufficiency in output power was caused by a poor power factor of the inverter. On the other hand, the damage to the capacitor bank was due to a little voltage margin between v_C and V_{max}, and it resulted in a large power dissipation in the capacitor causing a high temperature rise.

The capacitance of the capacitor bank affects the overall operating factors of induction heater such as resonant frequency, Q-factor, efficiency, and power factor [6], [11], [12]. Hence, in this work, we propose a method of choosing an optimal capacitance value C_{opt}, which maximizes the output power, and at the same time, minimizes the capacitor loss. Firstly, an equivalent model of the induction heater is developed based on previous works [1], [4], [11], [12]. The heating coil and slab is modelled as an inductance plus a series resistance, and the capacitor bank is modelled as a pure capacitance with an equivalent series resistance(ESR) [14]. Utilizing this model, the output power of the induction heater and the power loss in the capacitor are derived as functions of capacitance. An optimal value of the capacitance under the voltage constraint is found with the help of Lagrange multiplier [13].

This procedure looks very useful in the design of the induction heating system, specifically, in determining the size of capacitor bank. All the parameters used in this work come from the induction heater operating in POSCO.

II. SYSTEM ANALYSIS

A. Equivalent Circuit

In general, the heating coil and the load are modelled as a transformer with a single turn secondary winding as shown in Fig. 3(a). Almost all magnetic flux generated by the induction coil (primary winding) penetrates into the slab (secondary winding). Hence, in the secondary circuit no leakage inductance appears and the coupling coefficient is equal to one. The secondary circuit can be moved to the primary part as shown in Fig. 3(b)[11]. Denoting the mutual inductance by L_M, it follows that

$$L_1 = L_l + NL_M, \quad (1)$$

$$L_2 = \frac{L_M}{N}, \quad (2)$$

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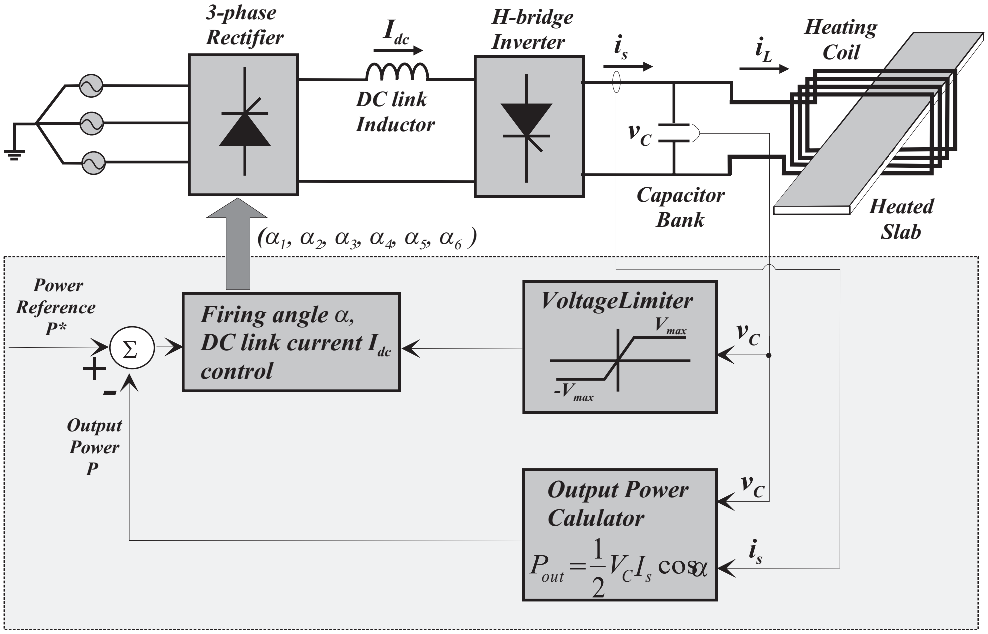

Fig. 1. Block diagram of induction heating system.

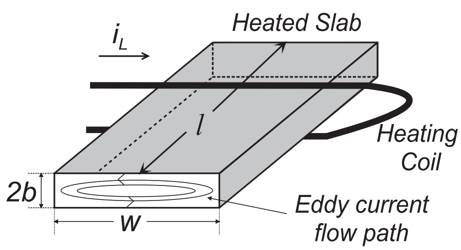

Fig. 2. Model of slab for one turn coil.

where L_1 and L_l denote the inductance of the heating coil and the leakage inductance, respectively. Using Wheeler's formula [9], the inductance of the heating coil is calculated as follows:

$$L_1(\mu H) = \frac{r^2 \times N^2}{9r + 10h}, \tag{3}$$

where r is the radius, N is the turn number and h is the height of the coil. The slab resistance R_L for one turn coil is given by

$$R_L = \rho \frac{L}{A} = \rho \frac{2(w + 2b)}{l\delta}, \tag{4}$$

where L and A are length and area of eddy current, l is the effective length of the slab occupied by one turn coil and b and w are defined in Fig. 2 [10], δ and ρ are skin depth almost distributed over the surface of slab and electrical resistivity of the material. Simplified equivalent model for a transformer can be represented in Fig. 3(c) by an equivalent inductance L_{eq} and resistance R_{eq} [1], [11]. These equivalent parameters depend on several variables including the shape of the heating coil, the spacing between the coil and slab, the electrical conductivity and magnetic permeability of the slab, and the angular frequency of the varying current ω_s [11].

$$L_{eq} = L_1 - A^2 L_2, \tag{5}$$

$$R_{eq} = R_1 + A^2 R_L, \tag{6}$$

where R_1 denotes the resistance of the heating coil, R_L denotes the resistance of the heated slab, and $A = \omega_s L_M / \sqrt{\omega_s{}^2 L_2{}^2 + R_L{}^2}$. It is noted that the inductance of heating coil L_1 is not affected by the existence of the slab in the heating coil, since at about $1100^o C$ temperature the permeability of the iron slab is equal to that of air, i.e., $\mu = 4\pi \times 10^{-7}$ (H/m) [11].

To represent the power dissipation in the capacitor bank, it is modelled by a pure capacitance C and an equivalent series resistance (ESR) R_{ESR}. It is noted that R_{ESR} is inversely proportional to the capacitance, hence, it is modelled as $R_{ESR} = k/C$, where k is a coefficient of ESR ranged from $1.2 \times 10^{-4}\ \Omega \cdot F$ to $1.5 \times 10^{-4}\ \Omega \cdot F$ [14].

B. Power Equation

A useful variable to calculate the power is a total impedance seen from the impedance of equivalent circuit in Fig. 3(c) [8]. The total impedance is given by

$$\mathbf{Z}_t = \frac{(\mathbf{Z}_C + \mathbf{Z}_{ESR}) \cdot (\mathbf{Z}_L + \mathbf{Z}_R)}{\mathbf{Z}_L + \mathbf{Z}_R + \mathbf{Z}_C + \mathbf{Z}_{ESR}} = \frac{(\omega_s k R_{eq} + \omega_s L_{eq}) + j(\omega_s^2 L_{eq} k - R_{eq})}{\omega_s (C R_{eq} + k) + j(\omega_s L_{eq} C - 1)}, \tag{7}$$

where $\mathbf{Z}_L = j\omega_s L_{eq}$, $\mathbf{Z}_C = 1/j\omega_s C$, $\mathbf{Z}_R = R_{eq}$, and $\mathbf{Z}_{ESR} = R_{ESR} = k/C$.

The rectifier and H-bridge inverter of the induction heater are represented by a square waved current source whose magnitude is equal to the DC-link current I_{dc}. Therefore, the

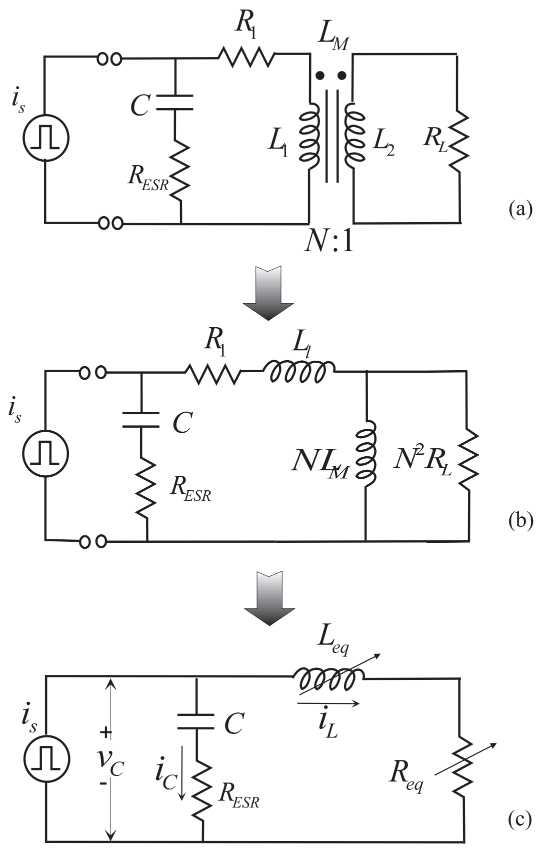

Fig. 3. (a) Based circuit of the induction heater. (b) Equivalent circuit of the induction heater based on a transformer concept. (c) Simplified equivalent circuit of the induction heater.

current source expanded in a fourier series is described as follows:

$$i_s(t) = \sum_{n-1}^{\infty} \frac{4I_{dc}}{n\pi} \sin n\omega_s t \qquad n = 1, 3, 5, \ldots \quad . \tag{8}$$

The fundamental component of the square waved current source is given by

$$i_s(t) = \frac{4}{\pi} I_{dc} \sin \omega_s t = I_s \sin \omega_s t, \tag{9}$$

where $I_s = 4I_{dc}/\pi$ is a peak of i_s. The current through R_{eq} and R_{ESR} are represented by i_L and i_C, respectively. The phasor expression of i_L and i_C are described as follows:

$$\mathbf{I}_L = \frac{\mathbf{V}_C}{\mathbf{Z}_L + \mathbf{Z}_R} = \frac{\mathbf{Z}_C + \mathbf{Z}_{ESR}}{\mathbf{Z}_L + \mathbf{Z}_R + \mathbf{Z}_C + \mathbf{Z}_{ESR}} \mathbf{I}_s, \tag{10}$$

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C + \mathbf{Z}_{ESR}} = \frac{\mathbf{Z}_L + \mathbf{Z}_R}{\mathbf{Z}_L + \mathbf{Z}_R + \mathbf{Z}_C + \mathbf{Z}_{ESR}} \mathbf{I}_s, \tag{11}$$

where $\mathbf{V}_C$ and $\mathbf{I}_s$ are phasors of v_C and i_s and $\mathbf{V}_C = \mathbf{Z}_t \cdot \mathbf{I}_s$.

In Fig. 3(b), the power consumption generate equivalent resistor R_{eq} and ESR R_{ESR} of the capacitor bank. therefore, the output power of the induction heater P_{out} and the capacitor loss P_{loss} are given by

$$\begin{aligned} P_{out}(C) &= \left(\frac{I_L}{\sqrt{2}}\right)^2 \mathbf{Z}_{eq} \\ &= \frac{1}{2} \left| \frac{\mathbf{Z}_C + \mathbf{Z}_{ESR}}{\mathbf{Z}_L + \mathbf{Z}_R + \mathbf{Z}_C + \mathbf{Z}_{ESR}} \right|^2 I_s^2 \mathbf{Z}_{eq} \\ &= \frac{8I_{dc}^2}{\pi^2} \left| \frac{\omega_s k - j}{\omega_s C \left(R_{eq} + \frac{k}{C}\right) + j(\omega_s^2 C L_{eq} - 1)} \right|^2 R_{eq}, \end{aligned} \tag{12}$$

$$\begin{aligned} P_{loss}(C) &= \left(\frac{I_C}{\sqrt{2}}\right)^2 \mathbf{Z}_{ESR} \\ &= \frac{1}{2} \left| \frac{\mathbf{Z}_L + \mathbf{Z}_R}{\mathbf{Z}_L + \mathbf{Z}_R + \mathbf{Z}_C + \mathbf{Z}_{ESR}} \right|^2 I_s^2 \mathbf{Z}_{ESR} \\ &= \frac{8I_{dc}^2}{\pi^2} \left| \frac{\omega_s C(R_{eq} + j\omega_s L_{eq})}{\omega_s C \left(R_{eq} + \frac{k}{C}\right) + j(\omega_s^2 C L_{eq} - 1)} \right|^2 \frac{k}{C}, \end{aligned} \tag{13}$$

where I_L and I_C denote the peak of i_L and i_C, respectively [7]. It is noted that P_{out} and P_{loss} are function of capacitance C, since all the parameters except capacitance are known values in (12) and (13).

III. Optimal Capacitance for Maximizing Output Power and Minimizing Capacitor Loss

In the load commutated inverter, the switching frequency of the inverter must be higher than the resonant frequency of the L-C load to guarantee commutation of the thyristors [3]. Hence, for more suitable value for the inverter while working close to the resonant frequency, we let $\omega_s = 1.1\omega_0$, then the voltage constraint is given by

$$V_C = |\mathbf{Z}_t(j\omega_s)| \cdot I_s = |\mathbf{Z}_t(j1.1\omega_0)| \cdot \frac{\pi}{4} I_{dc} \leq V_{max}, \tag{14}$$

where V_C is the peak of v_C, V_{max} is rated voltage of the capacitor bank, and $\mathbf{Z}_t$ is total impedance of capacitor bank and heating parts. One can see that $|\mathbf{Z}_t(j1.1\omega_0)|$ is also a function of capacitance, since $\omega_0 = 1/\sqrt{L_{eq}C}$.

The aim is to find an optimal capacitance value that maximize the output power of the induction heater and minimize the capacitor losses simultaneously under the voltage constraint (14). If the cost function is selected as $P_{out} - P_{loss}$, maximizing $P_{out} - P_{loss}$ is equivalent to maximizing P_{out} and at the same time minimizing P_{loss}.

$$\begin{aligned} &\text{maximizing} \quad P_{out} - P_{loss} \\ &\text{subject to} \quad |\mathbf{Z}_t(j1.1\omega_0)| \cdot (\tfrac{\pi}{4} I_{dc}) \leq V_{max}. \end{aligned}$$

It leads to apply Kuhn-Tucker theorem [13]. Firstly, the cost function is defined by

$$\begin{aligned} J(C) &= P_{out}(C) - P_{loss}(C) \\ &+ \lambda \cdot \left(V_{max} - |\mathbf{Z}_t(j1.1\omega_0)| \cdot \frac{\pi}{4} I_{dc}\right), \end{aligned} \tag{15}$$

where $\lambda \geq 0$ is the Lagrange multiplier. The maximum point is found under the following conditions,

$$\frac{\partial J}{\partial C} = 0, \tag{16}$$

$$\lambda \cdot \left(V_{max} - |\mathbf{Z}_t(j1.1\omega_0)| \cdot \frac{\pi}{4} I_{dc} \right) = 0. \tag{17}$$

The optimal capacitance value was found by using MATLAB. But, the resulting equation is quite long, hence it is described in Appendix.

IV. Simulation and Experimental Results

Simulation was performed with SIMPLORER circuit simulator. The parameters of POSCO induction heater were utilized in this simulation: L_{eq}=8.3 μH, R_{eq}=0.053 Ω, V_{max} =1700 V, I_s=1300 A, and $k = 1.35 \times 10^{-4}$ $\Omega \cdot F$. With the help of MATLAB, the optimal capacitance value is found to be $C_{opt} = 126\mu$F and $\lambda = 0$ by (19) and (20) in Appendix. Fig. 4 shows the output power P_{out}, the capacitor loss P_{loss}, and the power difference $P_{out} - P_{loss}$ versus capacitance value. MATLAB is used for deriving various powers in functional forms, and SIMPLORER is used for simulating powers versus time. One can see that the cost function is maximized at the optimal capacitance $C = 126\mu$F.

Fig. 5 shows circuit simulation results of the source current i_s, the capacitor voltage v_C, the output power P_{out}, and the capacitor loss P_{loss} when C = 90 μF, C = 126 μF, and $C = 170$ μF, respectively.

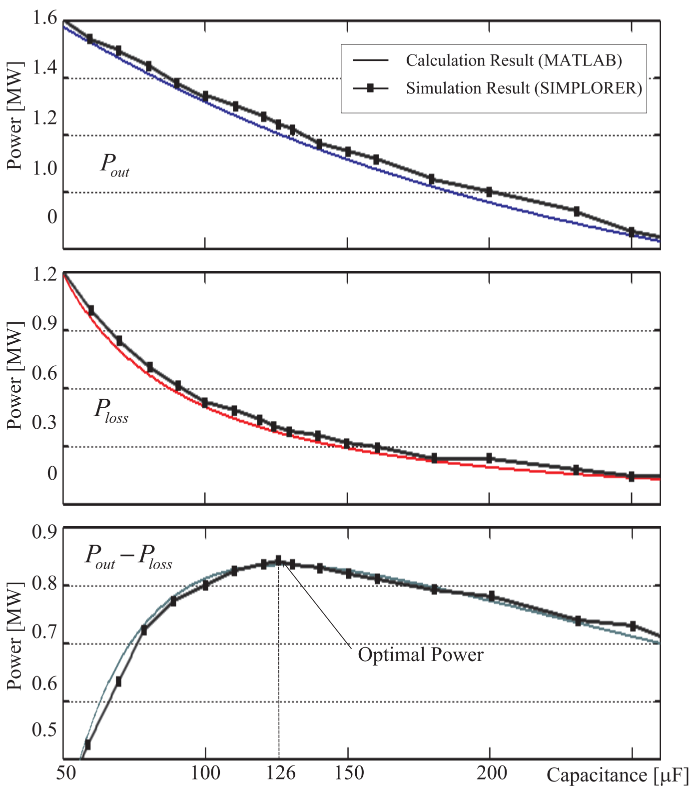

Fig. 4. The output power P_{out}, the capacitor loss P_{loss}, and the power difference (cost function) $P_{out} - P_{loss}$ versus capacitance value.

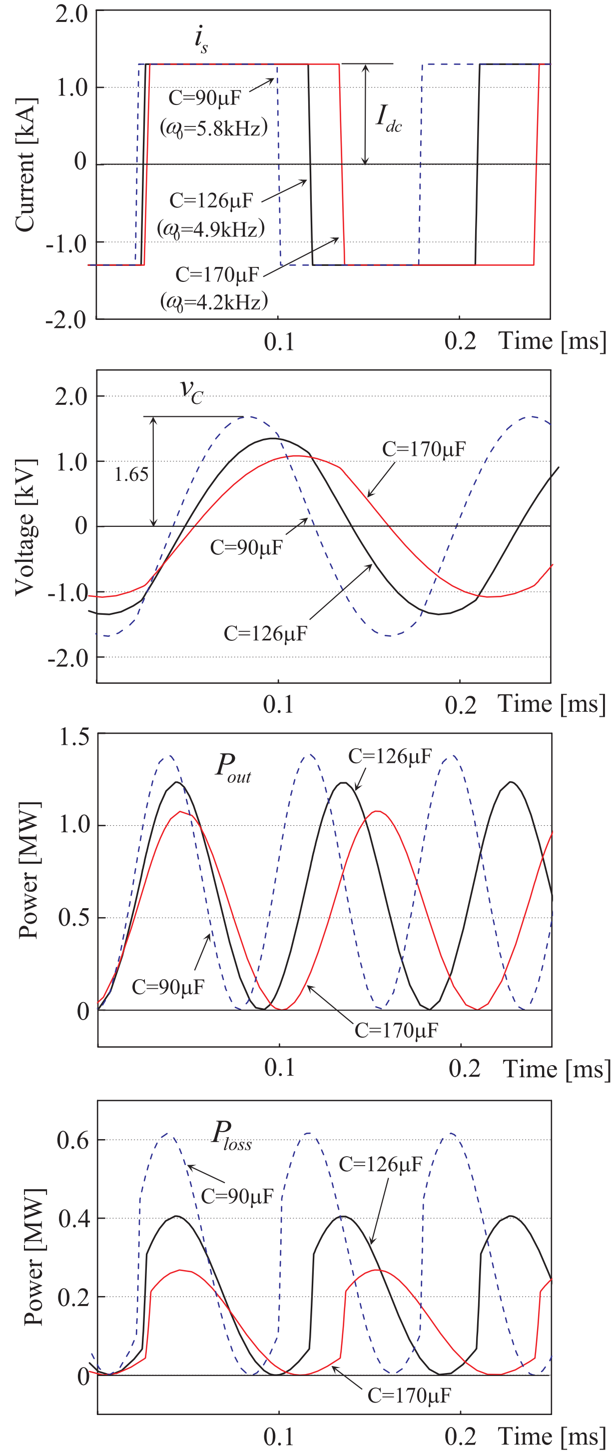

Fig. 5. The source current i_s, the capacitor voltage v_C, the output power P_{out}, and the capacitor loss P_{loss} when C = 90 μF, C = 126 μF, and $C = 170$ μF, respectively.

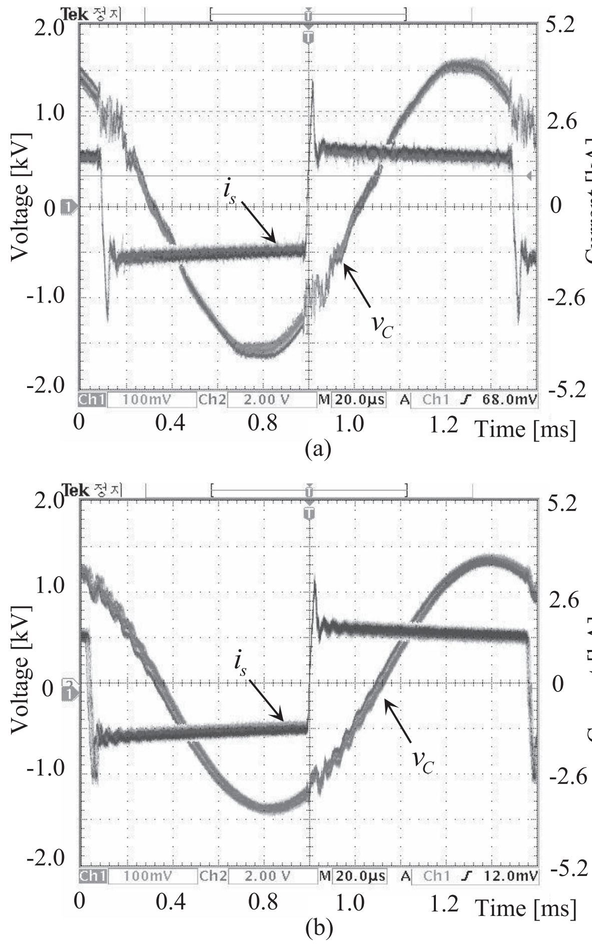

Fig. 6. The source current i_s and the capacitor voltage v_C when (a) $C = 115$ μF and (b) $C = 126$ μF, respectively.

With $C = 90$ μF, the largest output power (1.38 MW) is achieved. However, the capacitor loss (0.62 MW) is impractically large. With the optimal value $C = 126$ μF, the capacitor loss is reduced to a reasonable value, 0.4 MW, while the output power is 1.27 MW. Fig. 6 shows experimental results of the source current i_s and capacitor voltage v_C when $C = 115$ μF and $C = 126$ μF, respectively. Note that the source current i_s is constant at 1300A and the peak of capacitor voltage v_C falls to 1250V at the optimal operating point($C = 126$ μF). Note also that the experimental results are identical with the simulation results.

V. Conclusion

This paper suggests a new method in the choice of capacitance and operating frequency with the considerations on voltage tolerance and ESR of the capacitor, maximum output power, and high power factor. The optimal solution is found with Lagrange multiplier. In the derivation of the desired solution, MATLAB is utilized. This optimal choice is thought to contribute to increasing the life time of the capacitor bank and generating a maximum output power.

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VI. Appendix

Based on these parameters, the cost function and the condition equations are calculated as follows:

$$
\begin{aligned}
J(C) &= 8\,\frac{{I_{dc}}^2\left(10000\,L_{eq}C+12100\,k^2\right)R_{eq}}{\pi^2\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)} \\
&- 968\,\frac{{I_{dc}}^2\left(121\,L_{eq}+100\,{R_{eq}}^2C\right)k}{\pi^2\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)} \\
&+ \lambda\left(\frac{V_{max}}{I_{dc}}-\sqrt{\frac{\left(121\,L_{eq}+100\,{R_{eq}}^2C\right)\left(100\,L_{eq}C+121\,k^2\right)}{C\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)}}\right),
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\frac{\partial J(C)}{\partial C} &= 80000\,\frac{{I_{dc}}^2L_{eq}R_{eq}}{\pi^2\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)} \\
&- 8\,\frac{{I_{dc}}^2\left(10000\,L_{eq}C+12100\,k^2\right)R_{eq}\left(24200\,{R_{eq}}^2C+24200\,kR_{eq}+441\,L_{eq}\right)}{\pi^2\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)^2} \\
&- 96800\,\frac{{I_{dc}}^2{R_{eq}}^2k}{\pi^2\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)} \\
&+ 968\,\frac{{I_{dc}}^2\left(121\,L_{eq}+100\,{R_{eq}}^2C\right)k\left(24200\,{R_{eq}}^2C+24200\,kR_{eq}+441\,L_{eq}\right)}{\pi^2\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)^2} \\
&+ \lambda\Bigg\{-\frac{L_{eq}C}{2}\times\Bigg\{100\,\frac{{R_{eq}}^2\left(100\,L_{eq}C+121\,k^2\right)\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)}{{L_{eq}}^2C^3} \\
&+ 100\,\frac{\left(121\,L_{eq}+100\,{R_{eq}}^2C\right)\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)}{L_{eq}C^3} \\
&+ \frac{\left(121\,L_{eq}+100\,{R_{eq}}^2C\right)\left(100\,L_{eq}C+121\,k^2\right)\left(24200\,{R_{eq}}^2C+24200\,kR_{eq}+441\,L_{eq}\right)}{{L_{eq}}^2C^3} \\
&- 3\,\frac{\left(121\,L_{eq}+100\,{R_{eq}}^2C\right)\left(100\,L_{eq}C+121\,k^2\right)\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)}{{L_{eq}}^2C^4}\Bigg\} \\
&\times \sqrt{\frac{{L_{eq}}^2C^3}{\left(121\,L_{eq}+100\,{R_{eq}}^2C\right)\left(100\,L_{eq}C+121\,k^2\right)\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)^3}} \\
&- \sqrt{\frac{\left(121\,L_{eq}+100\,{R_{eq}}^2C\right)\left(100\,L_{eq}C+121\,k^2\right)}{C^3\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)}} \\
&+ \sqrt{\frac{\left(121\,L_{eq}+100\,{R_{eq}}^2C\right)\left(100\,L_{eq}C+121\,k^2\right)\left(24200\,{R_{eq}}^2C+24200\,kR_{eq}+441\,L_{eq}\right)^2}{C\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)^3}}\Bigg\}=0,
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\lambda\Bigg(\frac{V_{max}}{I_{dc}} &- \left|Z\left(j\frac{1.1}{\sqrt{L_{eq}C}}\right)\right|\Bigg) \\
&= \lambda\left(\frac{V_{max}}{I_{dc}}-\sqrt{\frac{\left(121\,L_{eq}+100\,{R_{eq}}^2C\right)\left(100\,L_{eq}C+121\,k^2\right)}{C\left(12100\,{R_{eq}}^2C^2+24200\,CkR_{eq}+12100\,k^2+441\,L_{eq}C\right)}}\right)=0.
\end{aligned} \tag{20}
$$