

# Introduction to Solid State Power Electronics



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## Acknowledgement

The material for this updated text was originally written by Dr. William E. Newell, a noted authority on power electronics with the aid of his associates at the Westinghouse Research and Development Center. Until his untimely death in 1976, Dr. Newell had devoted much of his life to helping practicing engineers and students alike better understand the field of power electronics. As a “teacher” within Westinghouse, as well as at Carnegie Mellon University, Dr. Newell tested the materials in this text well, with over five years of use and refinement.

We dedicate this text to the memory of Dr. Newell for his unselfish contributions to the field of power electronics. In order that future generations of engineers and students can profit from the efforts of Dr. Newell, The Westinghouse Electric Corporation has engaged his friends and colleague John W. Motto, Jr., to edit this work. John brings to this task over 17 years of applications and ratings experience on power semiconductor devices.

We hope you find this text a worthwhile addition to your reference library. We appreciate any comments you might have so that your ideas can be reflected in succeeding editions of this text.

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# Foreword

Although power electronics has a considerably longer history than many better known areas of electrical engineering, its widespread significance is only now beginning to gain recognition. In a sense, power electronics is the marriage of techniques which are characteristically power. But the incompatibilities which have traditionally separated these two specialties makes the consummation of this marriage far from simple. Power engineers find it difficult to adapt their intuitive thought patterns to a microsecond time scale, and electronics engineers have similar problems in adjusting to the demands of a megawatt power scale. Thus there is a growing need for electrical engineers whose capabilities transcend the two fields.

At least three current trends dictate that in spite of its slow rise to prominence, power electronics will soon emerge as a discipline and profession of major importance:

1. The growing shortage of oil and gas and growing environmental concern will stimulate the utilization of clean electrical power in countless new areas now predominantly served by other forms of energy.
2. Efficiency in the manipulation and control of electrical power will have increasing priority as the rising cost of power forces the abandonment of techniques which are short-term cheap but long-term wasteful.
3. The evolution of present applications and the creation of new applications will cause increasing demands for speed, precision, and reliability in power control that can be satisfied by no other technology. This text seeks to introduce power electronics in a way which emphasizes both the shared and the unique aspects of a wide variety of circuits for power conversion and control. It also seeks to build on elementary fundamentals with which all electrical engineers should be familiar. A review of these fundamental principals needed to establish a basic understanding of power electronics is concisely stated in Appendix I as the "Ten Cornerstones of Power Electronics."

This text is divided into the following major chapters:

- I. Introduction
- II. Diodes and Uncontrolled Rectifiers
- III. Thyristors, AC/DC Converters, and Other Naturally Commutated Circuits
- IV. Turn-off Devices and Self-commutated Circuits
- V. Power Semiconductor Device Protection

Each of these chapters is further divided into concise discussions of the topics and ideas that are felt to be of greatest importance in appreciating both the capabilities and limitations of power electronics. This format should allow the reader to quickly and selectively locate the section(s) of greatest interest to him.

Although a familiarity with the vocabulary of power electronics may result from reading or listening, experience has shown that relatively little useful learning (defined as the acquisition of new skills) occurs in this way. The problems at the end of each chapter are the "meat" of the text. These problems seek to illustrate important results while encouraging the reader to think and analyze in terms of broadly applicable physical principles. They seek to avoid both routine "formula plugging" and excessive mathematical grinding.

Once power electronics has penetrated your "awareness barrier," hopefully you will be aroused to continue your involvement with the field, becoming a contributor to the profession as well as a benefactor from the technology.

# Table of Contents

## CHAPTER 1

### INTRODUCTION TO POWER ELECTRONICS

#### Section

1. What is power electronics?
2. Power Converters as Switching Matrices
3. The Role of Power Filters
4. Types of Solid-State Switches
5. Commutation
6. The Internal Functional Sections of a Generalized Power Converter
7. Power/Frequency Domains of Power Electronics
8. Specialized Technologies of Power Electronics
9. Converter Terminology Defined
10. Operation of an AC/DC Converter
11. Functional Categories of Applications
12. Power Electronics – Lab.. Curiosity or Competitive Necessity?
13. Power Electronics Books
14. Problems

## CHAPTER 2

### DIODES AND UNCONTROLLED RECTIFIERS

#### Section

1. Type 1 Switch or Diode
2. Steady-State V-I Characteristic of a Diode
3. Transient V-I Characteristic of a Diode
4. Pulse Number
5. Simplifying Assumptions
6. Single-Phase Half-Wave Diode Rectifier
7. Single-Phase Half-Wave Diode Rectifier – R/L Loads
8. Single-Phase Half-Wave Diode Rectifier – Other Loads
9. Single-Phase Full-Wave Diode Rectifiers and Continuous Current
10. Average Voltage Across an Inductance
11. Two-Pulse Rectifier with Inductance Filter
12. Power Relationships
13. Three-Phase Diode Rectifiers
14. Generalized Center-Tap Rectifier
15. Generalized Bridge Rectifier
16. Applications of Diode Rectifiers
17. Transformers and Nonsinusoidal Waveforms
18. Problems

## CHAPTER 3

### THYRISTORS, AC/DC CONVERTERS AND OTHER NATURALLY COMMUTATED CIRCUITS

#### Section

1. Type 2 Switch Thyristor
2. Steady-State V-I Characteristic of a Thyristor
3. Two-Transistor Analog for Explaining Thyristor Turn On
4. Gate Characteristics
5. Transient V-I Characteristic of a Thyristor
6. Thyristor Ratings – Design Tradeoffs and Commercial Availability
7. Circuit Functions Performed by Type 2 Switches
8. Two-Pulse AC/DC Converter Operating as a Controlled Rectifier
9. Two-Pulse AC/DC Converter Operating as a Synchronous Inverter
10. AC/DC Converter as a linear Amplifier
11. AC/DC Converter as a DC Motor Drive
12. Two-Pulse Semiconverter
13. Two-Pulse Dual Converter
14. Waveform Derivation for a 3-Phase Bridge AC/DC Converter
15. Naturally Commutated Cycloconverter
16. Bilateral Solid-State Switches
17. Triac
18. AC Switches and Regulators
19. Vargulators
20. Brushless Machines
21. Summary
22. Problems

## CHAPTER 4

### TURN-OFF DEVICES AND SELF COMMUTATED CIRCUITS

#### Section

1. Power Transistor
2. Gate Controlled Switch
3. DC Switching and Regulation
4. Chopper Using Thyristors
5. Fundamentals of Inverters
6. Full-Bridge Inverter
7. Half-Bridge Inverter with an Inductive Load
8. Thyristor Inverters with Forced Commutation
9. Types of Forced Commutation
10. Series Inverter
11. Parallel Inverter
12. Complementary Impulse – Commutated Inverter
13. Inverter Output Voltage Control
14. Inverter Output Waveform Improvement
15. Frequency and Power Factor Changers
16. Problems

## **CHAPTER 5**

### **POWER SEMICONDUCTOR DEVICE PROTECTION**

#### **Section**

1. *The Power Module*
2. *Importance of Junction Temperature*
3. *Components of Dissipation*
4. *Average Dissipation at 60 Hz Switching Rate*
5. *Average Dissipation at 40 KHz Switching Rate*
6. *Simplified Geometry Assumed for the “Typical” Device*
7. *Junction-to-Case Thermal Resistance*
8. *Heat Sink Calculation*
9. *Selection of Device Package and Type of Heat Sink*
10. *Natural Convection Air Cooling*
11. *Forced Air Cooling*
12. *Water Cooling*
13. *Using the Data Sheet for Heat Sink Calculations*
14. *Thermal Capacity*
15. *Transmission Line for Transient Thermal Analysis*
16. *Initial Temperature Rise*
17. *Transient Thermal Impedance of Each Section*
18. *Calculation of Thermal Time Constant*
19. *Junction-to-Case Transient Thermal Impedance*
20. *Using Transient Thermal Impedance*
21. *Calculation of Pulse and Surge Current Capabilities*
22. *Turn-On  $di/dt$*
23. *Transient Overvoltage*
24. *Off-State  $dv/dt$*
25. *Series/Parallel Arrays for High-Power Applications*
26. *Summary*
27. *Problems*

## **APPENDIX I**

*Ten Cornerstones of Power Electronics*

## **APPENDIX II**

*Selected Symbol List*

## **APPENDIX III**

*Power Electronic Circuit Configurations — Voltage and  
Current Relationships*

# Chapter 1

## INTRODUCTION TO POWER ELECTRONICS

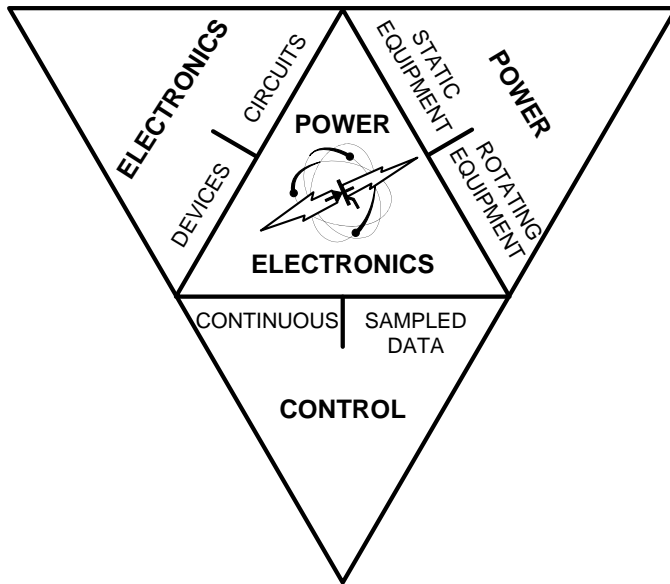
### Section

1. What is power electronics? . . . . .	1-2
2. Power Converters as Switching Matrices . . . . .	1-3
3. The Role of Power Filters . . . . .	1-3
4. Types of Solid-State Switches . . . . .	1-4
5. Commutation . . . . .	1-5
6. The Internal Functional Sections of a Generalized Power Converter . . . . .	1-5
7. Power/Frequency Domains of Power Electronics . . . . .	1-6
8. Specialized Technologies of Power Electronics . . . . .	1-7
9. Converter Terminology Defined . . . . .	1-7
10. Operation of an AC/DC Converter . . . . .	1-8
11. Functional Categories of Applications . . . . .	1-8
12. Power Electronics – Lab. Curiosity or Competitive Necessity? . . . . .	1-9
13. Power Electronics Books . . . . .	1-10
14. Problems . . . . .	1-12

### Introduction

The objectives of this chapter are to establish a perspective for the entire text which will enable the reader:

- To relate and distinguish power electronics from other fields of electronics and electrical engineering;
- To recognize the importance of power efficiency and to understand how it is achieved;
- To distinguish between the basic types of solid-state switching devices;
- To define the basic functional sections within a power converter, the specialized technologies involved in converter design, and the functional applications which converters serve.



**Figure 1.1**  
*Power Electronics...*  
*Interstitial to all of the major disciplines of electrical engineering.*

## 1. What is Power Electronics? (Figure 1.1)

For descriptive purposes, it is frequently useful to divide the overall field of electrical engineering into three areas of specialization: electronics, power, and control. The *electronics* area deals primarily with devices and circuits for the processing of information; the *power* area deals with both rotating and static equipment for the generation, transmission, distribution, and utilization of vast quantities of electrical power; and the *control* area deals with the stability and response characteristics of closed-loop systems using feedback on either a continuous or sampled-data basis.

Interstitial to all three of these areas is *power electronics*, which deals with the use of *electronics* for the *control* and conversion of large amounts of electrical *power*.

The origins of power electronics can be traced back many years, at which time mercury-arc devices were utilized for the rectification of AC to DC or the inversion of DC to AC. However, today's rapidly growing usage of power electronics has resulted from the development of solid-state power devices.

Specifically, then, we will limit the use of the name power electronics to those applications in which

electrical power flows through and is controlled by one or more solid-state power devices. (Note that this definition excludes such applications as those where a low-level electronic control circuit actuates an electromechanical relay in the power circuit.)

All of the important parameters of the electrical waveform are subject to regulation or conversion by solid-state power devices, including effective voltage, effective current, frequency, and/or power factor. Often the control of electrical power is desired simply as a means for controlling some non-electrical parameter. For example, drives for controlling the speed of a motor. In other applications, power electronics is used to control the temperature of an oven, the rate of an electrochemical refining process, the intensity of lighting, etc.

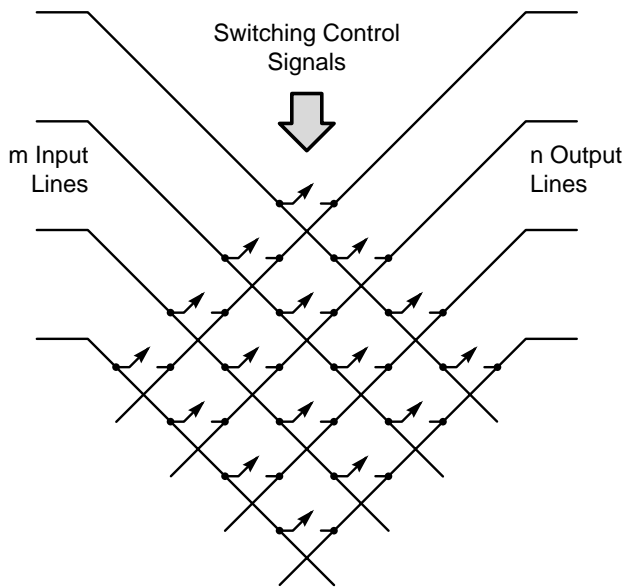
The design of power electronics equipment involves interactions with the source and the load, and utilizes small-signal electronic control circuits as well as power devices. Therefore power electronics draws upon, and indeed depends upon all of the other areas of electrical engineering. Obviously, the potential scope of this field is quite vast!

Despite the growing importance of power electronics, very few universities presently offer courses devoted to this area, and few courses in the other areas deals significantly with it either. Therein lies the motivation for this text. To keep the text within bounds, we will assume that you have a basic knowledge of the other areas of electrical engineering. We will concentrate most heavily on those important topics which are either ignored or given inadequate emphasis in available texts in the other areas.

Probably the single most important distinction between power electronics and small-signal electronics is the importance attached to overall power efficiency. In most cases, other factors outweigh power efficiency in small-signal electronic circuits, and power efficiency is calculated as an interesting after thought, if at all. But in power electronics, power efficiency is critical because both the cost of dissipating the heat and the cost of the wasted power are significant compared to the total cost of the equipment.

The importance of power efficiency dictates that the basic control element in power electronics must be a switch, NOT a continuously variable element.





**Figure 1.2**  
*m x n Switching Matrix*

## 2. Power Converters as Switching Matrices (Figure 1.2)

Newcomers to power electronics are frequently bewildered by the hundreds of different circuits which confront them when they begin to search typical references in the field. The essential differences between the various circuits are seldom clearly distinguished from their basic similarities. And if you attempt to understand each of these circuits individually, you have a lifetime project before you.

The first fundamental principle which you should remember forever is that power efficiency dictates that switches be used as control devices. The second fundamental principle, which is of equal importance, is that all high-power controls or converters are simply switching matrices. Like a telephone exchange, a general switching matrix permits any incoming line to be connected for a specified period of time to any outgoing line.

If there are  $m$  incoming lines and  $n$  outgoing lines, the matrix has  $m \times n$  switches. Because of the fact that most solid-state switches are unilateral, the general solid-state switching matrix has  $2 \times m \times n$  switches, one switch of each polarity for each intersection.

Specific converter circuits may have one line common to the input and output, in which case the 2 switches associated with the corresponding

intersection degenerate to a short circuit. Other circuits require only one-way conduction so that 1 of the 2 switches at each intersection can be discarded. In summary, the number of solid-state switches in a particular converter circuit may be less than or equal to  $2 \times m \times n$ , but any converter circuit can be derived from the general switching matrix.

A power switching matrix synthesizes the desired voltages on the output lines from selected “chunks” of the input voltages as dictated by the switching control signals.\* The functions which a given converter circuit can perform are dictated by

- (a) which of the switches are omitted from the general matrix,
- (b) the switching control signals, and
- (c) the types of solid-state switches used.

\*In a sense, a power converter is a combination analog/digital circuit. It uses switches to perform the required processing, like a digital circuit but unlike an analog circuit. On the other hand, the power “signal” that is processed is continuous as in an analog circuit, not quantized as in a digital circuit.

## 3. The Role of Power Filters

The voltage waveforms which are synthesized by the switching matrix are usually only an approximation to the desired output waveforms. Although the approximation could be improved by using continuously variable devices in place of the switches, this would cause an intolerable loss in power efficiency. Because of the repetitive nature of the switching action, the discrepancy in the waveforms appears as spurious frequencies which can be removed by a filter in the output lines. In other words, the output filter absorbs the unwanted ripple voltages from the output of the matrix.

A secondary effect is that the input current waveforms to the matrix contain spurious frequencies which may be undesirable in the supply lines or the source. Therefore another filter may be needed on the input lines to provide a local path for the unwanted ripple currents required by the matrix.

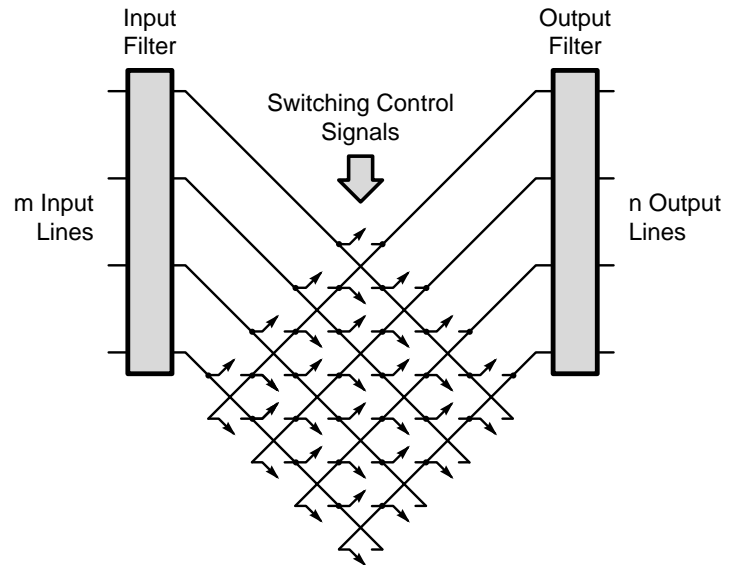
These discrepancies may be viewed as an undesired ripple in the instantaneous power flow through the matrix. The filters reduce this ripple in the input and output lines by storing energy during intervals of



excess power flow and returning energy to the circuit during intervals of insufficient power flow.

Filters contribute significantly to the cost, size, and weight of power electronics equipment, as well as to its performance. The cost and weight of the filters depend on the maximum energy storage which is required, which depends in turn on both the power rating of the circuit and the frequency of the undesired ripple. The lower the ripple frequency, the longer the interval during which the filter must be able to absorb or supply a significant portion of the rated power. Conversely, if the ripple frequency can be increased, comparable performance can be obtained from smaller and less costly filters.

These considerations, together with the related savings on transformers, motivate the selection of a power frequency greater than 60Hz in some applications. In other applications which are limited to 60Hz, the ripple frequency can be increased by the selection of a circuit configuration with a higher “pulse number”, as we shall see later.



**Figure 1.3**  
*Power Electronic Converter as a Switching Matrix*

#### 4. Types of Solid-State Switches (Figure 1.3)

Types of Switches:

- Type 1 DIODES** - Conduct Automatically when Forward Polarity is Applied.
- Type 2 THYRISTORS or SCR'S** - Begin to Conduct in the Forward Direction Upon Command of a Control Signal and Continue to Conduct Until the Next Current Zero Crossing.
- Type 3 TRANSISTORS, GATE CONTROLLED SWITCHES, FORCE-COMMUTATED THYRISTORS** - Forward Conduction can be Initiated and Interrupted by Control Signals

As stated previously, one of the factors which determine the performance of a power switching matrix is the type of switch that is used. For present purposes we don't need to get into the physics of solid-state switches, but we should distinguish between three main types of switches that are available to us. All three types are UNILATERAL. They can conduct current in one direction, known as the forward direction, but they very rapidly cease conduction when the reverse polarity is applied. The other characteristics of these switches are as follows:

- Type 1** Switches always conduct whenever a forward polarity of voltage is applied to their terminals. Type 1 switches are known as rectifiers or diodes.
- Type 2** Switches do not conduct forward current until commanded to do so by a control signal. Therefore Type 2 switches behave as Type 1 switches and continue to conduct as long as forward current flows. Type 2 switches are known as thyristors (the full official name is reverse blocking triode thyristor) or silicon controlled rectifiers (SCR's).
- Type 3** Switches can not only turn on forward conduction when commanded by the control signal, but can also interrupt conduction upon command without waiting for reverse polarity to be applied. Power transistors and gate controlled switches exhibit Type 3 behavior. By artificial circuit means, thyristors can also be “force commutated” to operate as Type 3 switches.

It is probably obvious that the 3 types of switches have been listed in the order of increasing generality. That is, with appropriate control signals a Type 3

switch can replace either a Type 2 or a Type 1 switch, and a Type 2 switch can replace a Type 1 switch. But they are not interchangeable in the opposite order.

## 5. Commutation

Definitions Regarding Commutation:

**COMMUTATION** – Current Interruption or Transfer to an Alternate Path.

**NATURAL OR LINE COMMUTATION** – Occurs Because of the Changing AC Line Voltages.

**SELF OR FORCED COMMUTATION** – Occurs Because of Inherent or Artificial Turn-off Ability.

**LOAD COMMUTATION** – Depends on Inherent or Artificially Produced Load Characteristics.

In the discussion of switches, we have just noted that it is the ability to interrupt conduction which distinguishes the third type from the first two types. The process by which current in a switch is caused to cease is called “commutation”, and the available means of commutation play an important role in determining the capabilities of any given circuit.

Classical rectifier or inverter circuits based on mercury-arc devices utilize what is known as “natural” or “line” commutation. Commutation occurs “naturally” because of the changing AC line voltages, either:

- (a) because the circuit current goes to zero, or
- (b) because the circuit current is transferred to another switch which is connected to a higher potential. The analysis of these circuits dates back more than 30 years, although solid-state power devices have made many new applications technically and economically feasible.

In a turn-off circuit, commutation can be caused to occur at arbitrary times in one of two ways. The most obvious way is to use a Type 3 switch having inherent turn-off ability. The second way is to “force commute” a Type 2 switch, and we will subsequently explore various methods of doing this. Either of these ways is known as “self commutation”.

When commutation depends on the load having certain characteristics, the process is known as “load commutation”. Load commutation may be similar to either line or forced commutation according to

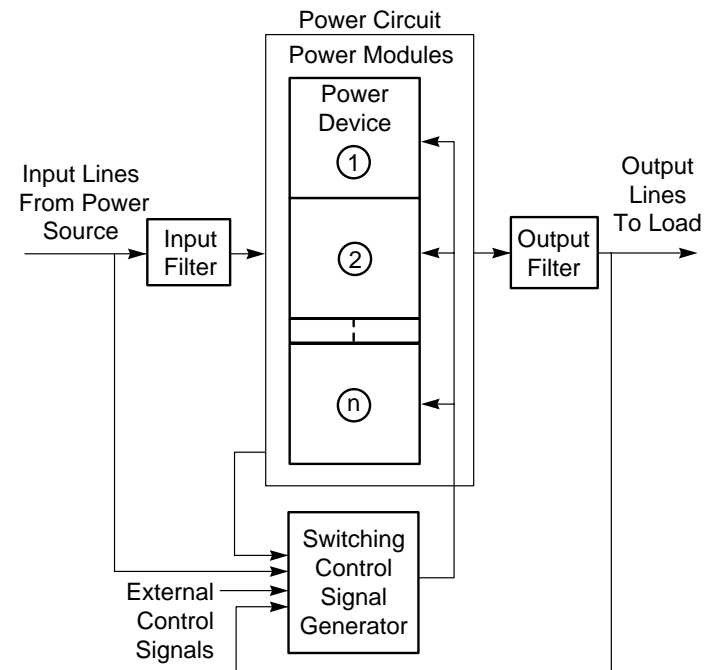
whether the required characteristics are inherent in the load or must be produced artificially.

Most applications of turn-off circuits were not feasible until the advent of solid-state power devices. This technology is developing rapidly and is providing many new opportunities for applying solid-state power electronics.

## 6. The Internal Functional Sections of a Generalized Power Converter (Figure 1.4)

We stated previously that power electronics encompasses a vast amount of technology from both the electronics and power fields, plus considerable technology not ordinarily covered by either of those fields. In the time available, this text will focus on those topics which will help you to understand how typical converter circuits operate, what limitations are encountered in designing such circuits, and how these limitations are dealt with.

To clarify the emphasis of this text, the components of a generalized power converter can be grouped into six main internal functional sections: power devices, power modules, the power circuit, the switching control signal generator, and input and output filters.



**Figure 1.4**  
*Internal Functional Sections of a Generalized Power Converter*

The *power devices* are the solid-state devices which have made modern power electronics possible. We will emphasize their important terminal characteristics, but the details of semiconductor physics and device fabrication are outside the scope of this text.

Each power device or group of devices is embedded within a *power module* which interfaces it with the surrounding power circuit. The power module implements the function of a single switch in the switching matrix. In addition to one or more power devices, each power module contains one or more heat sinks, fuses and other components to protect the power devices, and sometimes local circuitry for gating and forced commutation.

The *power circuit* interconnects the power modules into the appropriate switching matrix, and includes the transformers needed for isolation, changing voltage levels, etc. Starting from simple bridge and center-tap arrangements and wye or delta transformer connections, the number of power circuit configurations is seemingly endless. For example, the ANSI Standard C34.2-1968, "Practices and Requirements for Semiconductor Power Rectifiers," designates some 70 standard circuit configurations for rectifiers alone.

The *output filter* smooths the discrepancies between the voltage waveform synthesized by the power circuit and the desired output voltage waveform.

The *input filter* provides a path for harmonic currents which are required by the power circuit but which are undesirable in the input supply lines.

The *switching control signal generator* receives signals from the input lines, the output lines, from within the power circuit, and/or from external control lines and processes them to generate appropriate gating pulses to actuate each Type 2 or Type 3 switch. These gating pulses usually have a periodicity which is determined in one of three ways:

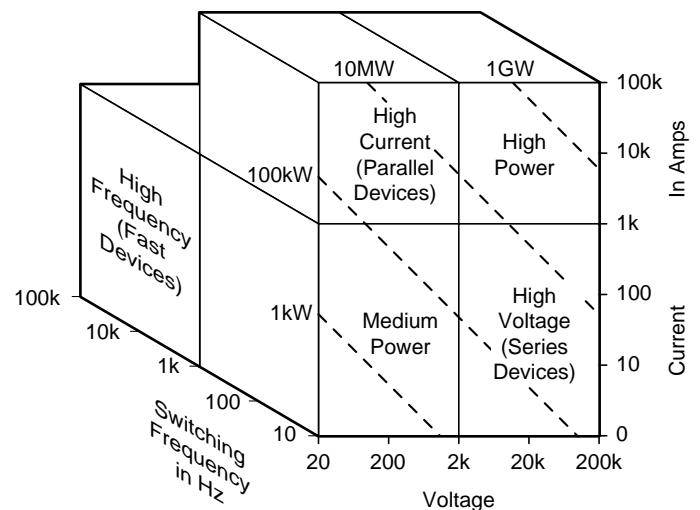
- (a) In the simplest case, the gating signals are synchronized in frequency with the AC supply, but with adjustable relative phase.
- (b) When the converter is powered from a DC source, internal time constants or an internal reference oscillator determine the periodicity of switching.
- (c) In a frequency changer, the gating signals must be properly synchronized with both the AC source frequency and the desired output frequency. In effect, the smaller of these two frequencies modulates the larger frequency.

The design of the switching control signal generator utilizes all of the applicable circuit technology from both analog and digital electronics.

This text will concentrate most heavily on various common configurations for the power circuit and on some aspects of the power module. Appropriate timing of the switching control signals will be discussed, but the design of the control circuits which generate these signals is outside the scope of the text.

## 7. Power/Frequency Domains of Power Electronics (Figure 1.5)

The three-dimensional space of voltage-current-frequency provides another interesting way to categorize power electronics. Within this space, the first and by far the most widely useful domain is defined by voltage and current limits which are within the capability of a single solid-state device per power module. These limits are rather arbitrary, and change with time as new solid-state devices are developed, but for present purposes they will be shown at 2kV and 1000A. This domain can be called the "Medium Power Domain", although at its upper limit it extends to a power of 2 MW!



**Figure 1.5**  
Power Frequency Domains of Power Electronics

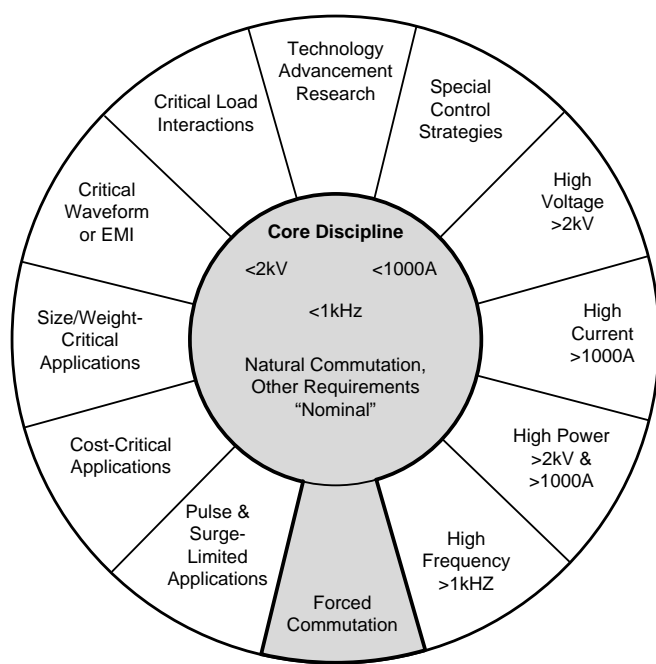
Power devices can be cascaded in series to increase the voltage limit. When this is necessary, the associated technology falls within the “High Voltage Domain”. Similarly power devices can be paralleled to increase the current limit, thereby defining a “High Current Domain”. For some applications, series/parallel arrays are required, leading to a “High Power Domain”.

Conventional solid-state power devices are capable of much faster switching action than a mechanical switch, but the switching time is not zero. Consequently there is an upper limit on the switching frequency of conventional power devices, which for present purposes will be arbitrarily assumed to be 1 kHz. However, power devices especially designed for fast switching action are available. The technology associated with the use of these fast devices defines a “High Frequency Domain”.

## 8. Specialized Technologies of Power Electronics

(Figure 1.6)

A pinwheel can be used to further extend the specialized technological categories of power electronics without trying to show an n-dimensional picture. The “Core Discipline” incorporates the “Medium Power Domain”, with the additional requirements of natural commutation and other



**Figure 1.6**  
*Specialized Technologies of Power Electronics*

requirements being “nominal”. Power electronics equipment for a particular application incorporates technology from the Core Discipline plus technology from one or more of the specialized categories.

The High Voltage, High Current, High Power, and High Frequency Domains just discussed constitute some of these specialized technologies. Forced Commutation is another category of increasing importance. Numerous other categories of specialized technology could be noted. For example, in some applications the power devices must be specified on the basis of their transient surge ratings rather than on their steady-state ratings. In some applications, such as those for the consumer market, cost reduction is a critical consideration. In other applications, such as those for the consumer market, cost reduction is a critical consideration. In other applications, such as aerospace, size and weight are critical considerations. And so on and on.

This introductory text will touch lightly upon a number of these categories, but the Core Discipline and Forced Commutation form the “keyhole” which we hope will unlock a basic understanding of power electronics for you.

## 9. Converter Terminology Defined

Power Converter Terminology:

**CONVERTER** – General Term

**SWITCH** – Full Off or Full On

**REGULATOR** – Intermediate Control of AC or DC

**RECTIFIER** – Converts AC to DC

**INVERTER** – Converts DC to AC

**FREQUENCY CHANGER** – Changes Frequency of AC Power

**POWER FACTOR CHANGER** – Changes Power Factor of an AC Load

**AC/DC CONVERTER** – Operates as Rectifier or Inverter

Unfortunately the field of power electronics abounds with inconsistent and ambiguous usage of terminology. To maintain internal consistency, we will adopt the following definitions:

**CONVERTER** – A general term which can be used to describe any one of the following circuit types, a combination of several types, or all types as a group.

**SWITCH** – A device or circuit which has two extreme states - full off or full on.

**REGULATOR** – A circuit which permits intermediate control of AC or DC power. The desired output frequency is the same as the input frequency. If implemented by switching devices, regulation is achieved by time - averaging with a suitable time constant. A DC regulator is often called a chopper.

**RECTIFIER** – Converts AC power to DC power.

**INVERTER** – Converts DC power to AC power.

**FREQUENCY CHANGER** – A converter in which the desired AC output frequency is generally different from the AC input frequency.

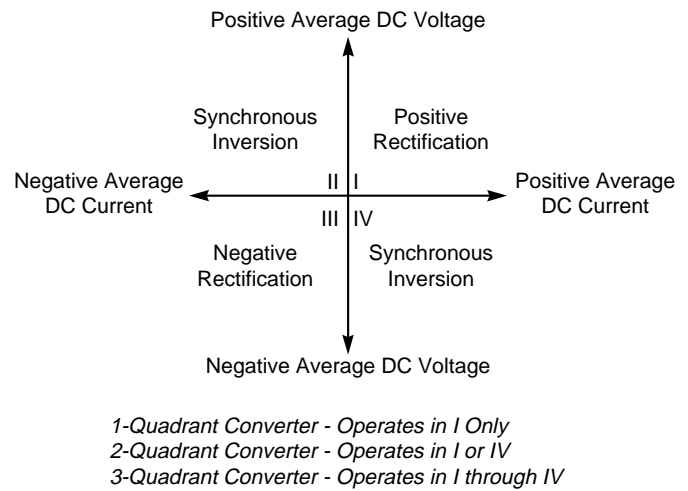
**POWER FACTOR CHANGER** – A converter which manipulates the power factor of an AC load without changing frequency.

In contrast to the previous discussion of the internal functions and the specialized technologies of power electronics, these terms form the basis for categorizing power electronics according to external or application-oriented functions. However, as will be seen, rectifiers and inverters are not exclusive types of circuits. Certain circuits can perform either function. Therefore we will define another intermediate category:

**AC/DC CONVERTER** – A circuit which may operate either as a rectifier or as an inverter.

## 10. Operation of an AC/DC Converter (Figure 1.7)

As the name implies, an AC voltage appears at one set of terminals of an AC/DC converter, and a DC voltage appears at the other set of terminals. The operation of the converter can be described in terms of 4 quadrants as viewed from the DC terminals. The horizontal axis represents the average DC current, and the vertical axis represents the average DC voltage. In quadrant I, where average voltage and average current are both “positive”, net power flow is from the AC terminals to the DC terminals; the process is known as positive rectification.



**Figure 1.7**

*Regions of Operation of an AC/DC Converter*

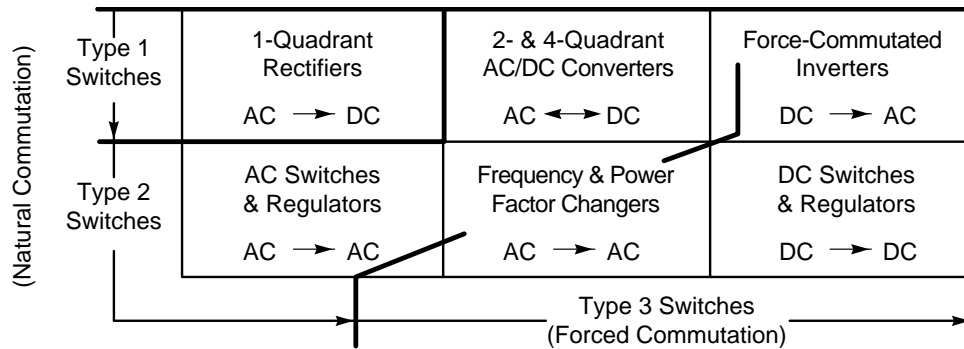
Similarly, if both the average voltage and average current are negative, net power flow is still from the AC terminals to the DC terminals and the process is known as negative rectification.

If either the average voltage or the average current reverses (but not both), power flow also reverses. That is, in Quadrants II and IV, power flows from the DC terminals to the AC terminals. In AC/DC converters, the AC line provides the current zeros needed to naturally commute Type 2 switches. In quadrants II and IV the process is known as synchronous inversion to distinguish it from the type of inversion which requires forced commutation.

As will be seen later, some AC/DC converters can operate only in certain quadrants. If operation is confined to Quadrant I, the circuit is called a 1-Quadrant Converter. A 2-Quadrant Converter can operate in either Quadrant I or Quadrant IV. And a 4-Quadrant Converter can operate in any of the 4 quadrants.

## 11. Functional Categories of Applications (Figure 1.8)

Prior to the advent of solid-state power devices, power converters were limited almost exclusively to rectifiers and synchronous inverters. We will now attempt to meaningfully categorize the variety of functional applications which are possible with solid-state power devices.



**Figure 1.8**  
*Types of Power converters*

As we have discussed, 1-Quadrant Rectifiers are limited to net power flow from the AC to DC terminals. These rectifiers can be implemented using only Type 1 switches (diodes).

2- and 4-Quadrant AC/DC Converters permit power flow in either direction, but require Type 2 switches. Since power can flow either way, AC/DC converters bridge the gap between the traditional “Rectifier” and “Inverter” categories.

It is also possible to implement AC Switches and Regulators and certain types of Frequency Changers with naturally commutated Type 2 switches.

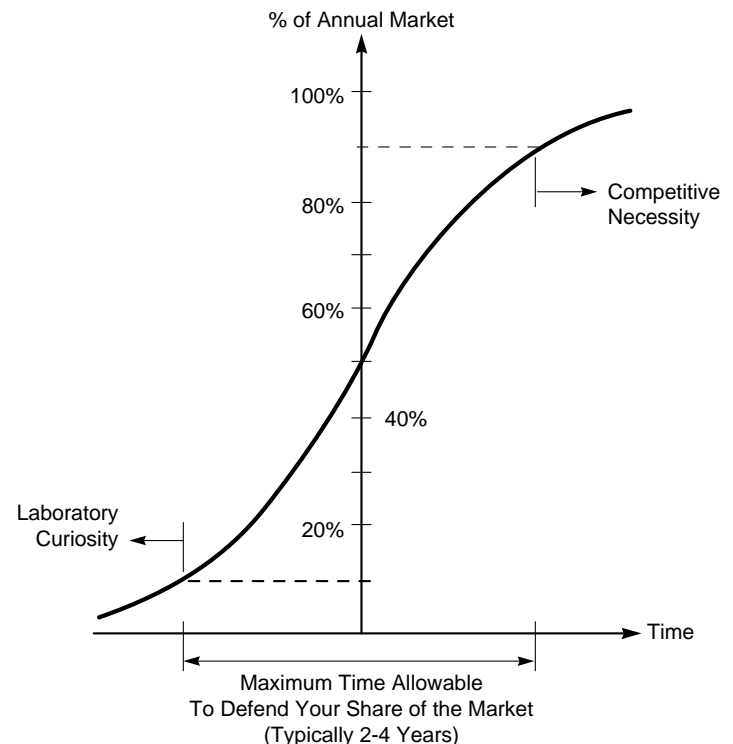
With Type 3 switches (or force-commutated Type 2 switches), other types of inverters, DC Switches and Regulators (i.e. choppers), and Frequency and Power Factor Changers become feasible.

The text will explain the basic principles of operation of circuits which serve each of these functions.

## 12. Power Electronics - Laboratory Curiosity or Competitive Necessity? (Figure 1.9)

The potential advantages of solid-state power electronics over electromechanical equipment are well known: increased reliability and service life, and reduced size and maintenance. In some cases, there is a sizable cost advantage, while in other cases the performance capabilities can be achieved in no other way.

For these reasons, power electronics is penetrating many new application areas and outmoding many traditional design approaches. Product engineering



**Figure 1.9**  
*Universal S-Curve Showing the Capture of a Market by a Product Which Takes Advantage of Power Electronics*

departments accustomed to well established design procedures involving Ward-Leonard drives, or relay logic, or selenium rectifiers are faced with extinction overnight if they cannot adapt to the higher levels of complexity and analytical sophistication both made possible and made necessary by solid-state electronics.

In assessing business risks, a company must try to guard against squandering development funds on a premature attempt to market a laboratory curiosity.

It must also guard against being caught without adequate competence in a rapidly evolving technology when that technology becomes a competitive necessity. Many technologies evolve over a period of decades, leaving adequate reaction time for business planning.

One aspect of power electronics which is not widely appreciated is that when it captures a market, it frequently does so with a bang. Solid-state typically moves from an insignificant share of the market to a dominate share in a period of only 2 to 4 years. Thus a company which does not maintain a nucleus of competence in this new technology can easily lose a secure market position because of inadequate reaction time.

Power electronics has already become a competitive necessity in many applications, and will soon emerge from its laboratory curiosity status in many others. Therefore this text will not dwell on the details of existing applications, but will seek to convey an understanding of basic principals and techniques which are equally applicable to countless new applications.

---

*To supplement this brief introduction to power electronics, the following review articles are recommended:*

A. Kusko, "Solid-State Motor-Speed Controls," IEEE Spectrum, pp. 50-55; October 1972.

H.F. Storm, "Solid-State Power Electronics in the U.S.A.," IEEE Spectrum, pp. 49-59; October 1969.

F.W. Gutzwiller, "Thyristors and Rectifier Diodes — The Semiconductor Workhorses," IEEE Spectrum, pp. 102-111; August 1967.

R.G. Hoft, "Static Power Converters in the U.S.A.," IEEE Intl. Semiconductor Power Converter Conference Record, pp. 2-8-1 to 8; May 1972.

J.J. Bates and R.E. Colyer, "The Impact of Semiconductor Devices on Electrical Power Engineering," Radio & Electronic Engineer, vol. 43, pp. 115-124; Jan./Feb. 1973.

W.E. Newell, "Power Electronics - Emerging from Limbo," IEEE Trans., vol. IA-10, pp. 7-11; Jan./Feb. 1974.

A.M. Curry and R.V. Wachter, "A User Looks at the Semiconductor Converter," IEEE Trans., vol. IGA-4, pp. 41-46; Jan./Feb. 1968.

A. Ludbrook, "Basic Economics of Thyristor Adjustable Speed Drive Systems," IEEE Industry and General Applications 4th Annual Meeting Record, pp. 591-596; Oct. 1969.

D.W. Borst, et. al., "Voltage Control by Means of Power Thyristors," IEEE Trans., vol. IGA-2, pp. 102-124; Mar./Apr. 1966.

### 13. Power Electronics Books

#### A. Power Device Manufacturer's Handbooks

1. SCR Designers Handbook (Second Edition, 1970) Westinghouse Semiconductor Div., Youngwood, PA.
2. SCR Manual (Fifth Edition, 1972) General Electric Co., Syracuse, NY.
3. Solid-State Power Circuits (1971) RCA Technical Series SP-52. RCA Solid-State Division, Somerville, NJ.
4. SCR Applications Handbook (First Printing) International Rectifier Corporation; September 1974.
5. Semiconductor Power Circuits Handbook (1968) Motorola Semiconductor Products Div., Phoenix, Ariz.
6. *Power Semiconductor DATAbook D.A.T.A., Inc., Orange, NJ (A worldwide tabulation of the ratings of commercial power diodes, transistors, and thyristors, revised twice a year.)*

#### B. Textbooks and Special Issues on Power Devices

1. *Semiconductor Controlled Rectifiers: Principals and Applications of PNP Devices* (1964) by F.E. Gentry, et. al. (General Electric) Prentice-Hall, Englewood Cliffs, NJ.
2. *Special Issue on High-Power Semiconductor Devices* IEEE Transactions on Electron Devices, vol. ED-17, no. 9; September 1970.
3. *Special Issue on High-Power Semiconductor Devices* Proceedings of the IEEE, vol. 55, no. 8; August 1967.

#### C. Textbooks on Power Circuit Analysis

1. *The Fundamental Theory of Arc Convertors* (1939) by H. Rissik. Chapman & Hall, London. Facsimile reprint available from University Microfilms, Ann Arbor, Mich. (A classic book, equally applicable today if "thyristor converter" is substituted for "arc convertor".)
2. *Power Diode and Thyristor Circuits* (1971) by R.M. Davis. Cambridge University Press, London.
3. *Rectifier Circuits: Theory and Design* (1965) by J. Schaefer. John Wiley, New York.
4. *Principles of Inverter Circuits* (1964) by B.D. Bedford and R.G. Hoft (General Electric) John Wiley, New York.
5. *Thyristor Phase-Controlled Converters and Cycloconverters* (1971) by B.R. Pelly (Westinghouse) Wiley-Interscience, New York.
6. *The Theory and Design of Cycloconverters* (1972) by W. McMurray (General Electric) MIT Press, Cambridge, Mass.
7. *Line-Commuted Thyristor Converters* (Second Edition, 1972) by G. Molgen (Siemens) Pitman Publishing, London.



8. *Semiconductor Devices in Power Engineering* (1968) Edited by J. Seymour. Sir Isaac Pitman, London.
9. *Power Engineering Using Thyristors: Vol. 1 - Techniques of Thyristor Power Control* (1970) Edited by M.J. Rose (Mullard) Mullard Ltd., London.
10. *Thyristor Control* (1973) by F.F. Mazda (ITT). John Wiley, New York.
11. *Power Electronics: Thyristor Controlled Power for Electric Motors* (1973) by R. Ramshaw (Univ. Waterloo, Ontario) Chapman and Hall, London. (Distributed in USA by John Wiley, NY.)
12. *Power Semiconductor Circuits* (1975) by S.B. Dewan and A. Straughen Wiley-Interscience, New York.
13. *Theory of SCR Circuit and Application of Motor Control* (1968) by T.J. Takeuchi (Tokyo EE College) Tokyo Electrical Engineering College Press, Tokyo. (This book is unique in its application of Laplace transforms to the analysis of power control circuits.)
14. *Rectifier Circuits* (1972) Edited by W.F. Waller. Macmillan Press, Basingstoke, Hampshire, U.K.
15. *Transistor Inverters and Converters* (1963) by T. Roddam. Van Nostrand, Princeton, NJ.

#### **D. Textbooks on Applications**

1. *Solid-State DC Motor Drives* (1969) by A. Kusko, MIT Press, Cambridge, Mass.
2. *Design of Solid-State Power Supplies* (1971) by E.R. Hnatek (National Semiconductor) Van Nostrand Reinhold, New York.
3. *Direct Current Transmission - Vol. I* (1971) by E.W. Kimbark (Bonneville Power Admin.) Wiley-Interscience, New York.
4. *High Voltage Direct Current Convertors and Systems* (1965) Edited by B.J. Cory, Macdonald, London.
5. *Electric Power Utilization* by N.N. Hancock (Univ. Manchester) Sir Isaac Pitman, London.
6. *Power Semiconductor Applications: Vol. I - General Considerations; Vol. II - Equipment and Systems* (1972) Edited by J.D. Harnden and F.B. Golden IEEE Press, New York. (An extensive and inexpensive collection of selected papers reprinted from literature.)
7. *Thyristor Control of AC Motors* (1973) by J.M.D. Murphy, Pergamon Press, Oxford, England.

#### **E. Elementary Books and Experimenters' Guides**

1. *Understanding Silicon Controlled Rectifiers* (1968) by S. Heller (Voorhees Technical Institute) Hayden, New York.
2. *RCA SCR Experimenter's Manual* (1967) RCA Technical Series KM-71 (Experimenters Kit KD2105 also available.) RCA Distributor Products, Harrison, NJ.
3. *110 Thyristor Projects Using SCR's and Triacs* (1972) by R.M. Marston, Iliffe, London.
4. *Thyristors* (1971) by L.W. Owens (Mullard) Mullard, Mitcham, Surrey, U.K.
5. *Thyristors and Their Applications* (1972) by P. Atkinson (University of Reading) Mills & Boon, London.

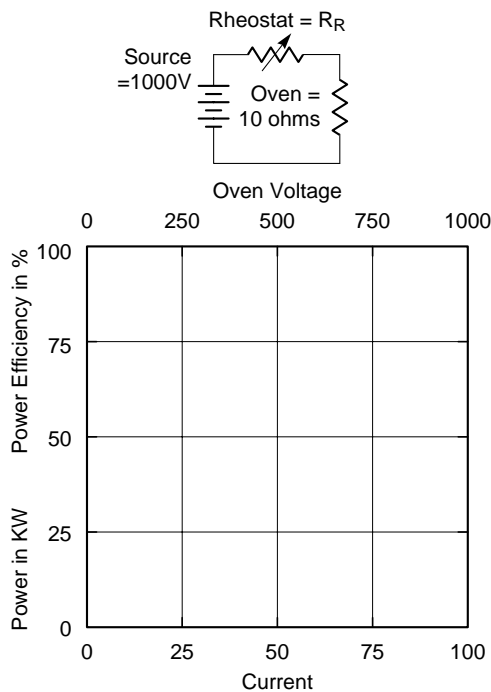
# 14. Problems

## Problem 1.1 Power Efficiency of a Series Rheostat as a Power Control

Assume that a series rheostat is used to control the flow of power to a 100-kW oven as shown by the circuit below. When  $R_R = 0$ , full power is delivered to the oven. As  $R_R$  is increased, power to the oven decreases and ultimately approaches zero as  $R_R$  becomes infinite.

- (a) On the axes provided, plot the source power, the oven power, and the power dissipated in the rheostat as functions of the oven voltage.
- (b) Plot the power efficiency, defined as the ratio of oven power to source power.
- (c) How much power must the heat sink attached to the rheostat be capable of dissipating? \_\_\_\_\_ watts.
- (d) Making whatever assumptions seem reasonable to you, estimate the value of the power wasted by this control over its useful life, neglecting the additional cost of dissipating the heat. If a simple but more efficient control can be purchased for \$15 per kW of controlled power, how long will it take for the better control to completely pay for itself by reducing wasted power? (Assume this control has a power efficiency of 95%.)

## Problem 1.1 Power Efficiency

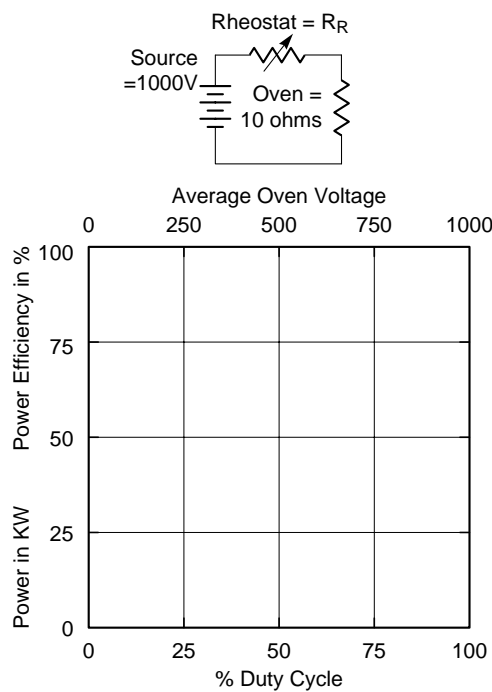


## Problem 1.2 Switching Power Efficiency

After working Problem 1.1 and being impressed by the poor efficiency of a rheostat as a power control, an engineer invents a way to improve the efficiency. He notes that the rheostat dissipates no power when it is in either of its limiting positions, zero or infinite resistance. Therefore he devises a mechanism which causes the rheostat to spend nearly all of the time in one or the other of these limiting positions, with negligible time at intermediate positions. Average power to the oven is controlled by varying the relative time (duty cycle) spent in the two positions. For instance, if half of the time is spent in each position, the average oven voltage is 500 V and the average oven power is 50 kW. Repeat (a), (b), and (c) of Problem 1.1 for this modified control, and plot results on the axes below. (d) Is oven power proportional to the square of average oven voltage? Explain.

*You have probably observed that the “rheostat” is no longer a rheostat but a switch, and you have just established for yourself why high-power controls use a switching mode of operation.*

## Problem 1.2 Switching Efficiency



# Chapter 2

## DIODES AND UNCONTROLLED RECTIFIERS

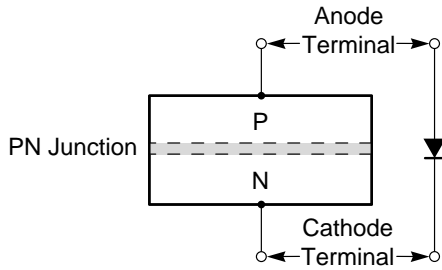
### Section

1. Type 1 Switch or Diode . . . . .	2-2
2. Steady-State V-I Characteristic of a Diode . . . . .	2-2
3. Transient V-I Characteristic of a Diode . . . . .	2-2
4. Pulse Number . . . . .	2-3
5. Simplifying Assumptions . . . . .	2-3
6. Single-Phase Half-Wave Diode Rectifier . . . . .	2-4
7. Single-Phase Half-Wave Diode Rectifier – R/L Load . . .	2-5
8. Single-Phase Half-Wave Diode Rectifier – Other Loads . .	2-6
9. Single-Phase Full-Wave Diode Rectifiers and Continuous Current . . . . .	2-7
10. Average Voltage Across an Inductance . . . . .	2-8
11. Two-Pulse Rectifier with Inductance Filter . . . . .	2-8
12. Power Relationships . . . . .	2-9
13. Three-Phase Diode Rectifiers . . . . .	2-10
14. Generalized Center-Tap Rectifier . . . . .	2-12
15. Generalized Bridge Rectifier . . . . .	2-12
16. Applications of Diode Rectifiers . . . . .	2-13
17. Transformers and Nonsinusoidal Waveforms . . . . .	2-13
18. Problems . . . . .	2-15

## Introduction

The objectives of this chapter are:

- To name and describe the important steady-state and transient terminal characteristics of solid-state diodes.
- To list the simplifying assumptions usually made in rectifier analysis.
- To explain the operation of a one-pulse rectifier and sketch the voltage and current waveforms for various types of loads.
- To work simple problems related to voltage-averaging by an inductor in the DC circuit.
- To explain the real, reactive, and distortion components of total apparent power for the non-sinusoidal AC line current drawn by a rectifier.
- To describe the operation of single-phase and three-phase center-tap and bridge rectifiers.
- To list several common applications of uncontrolled rectifiers.



**Figure 2.1**  
*Type 1 Switch or Diode*

### 1. Type 1 Switch or Diode (Figure 2.1)

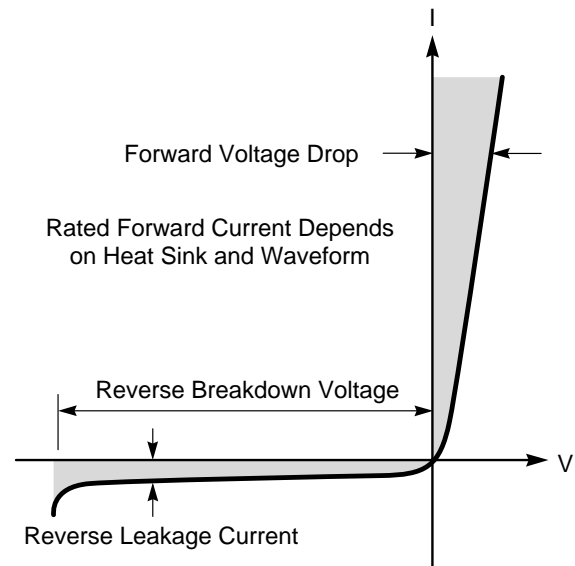
A conventional solid-state diode is a Type 1 switch. It has two terminals, known as the anode and the cathode. Internally, a simple diode consists of a single PN junction within a crystal of silicon. The anode terminal connects to the P side of the junction, and the cathode connects to the N side. When the anode or P side is positive with respect to the cathode or N side, the diode conducts forward current with a relatively low voltage drop. The triangle in the schematic symbol for a diode indicates the polarity of forward current flow.

When the polarity is reversed, a large reverse voltage can be applied but only a small reverse leakage current will flow.

### 2. Steady-State V-I Characteristic of a Diode (Figure 2.2)

The highly nonlinear behavior of a diode which has just been discussed can best be illustrated by the steady-state V-I characteristic. The primary steady-state parameters which describe a diode are shown. The *forward voltage drop* is determined primarily by the semiconductor material from which the diode is fabricated, independent of the current rating of the diode. The material is nearly always silicon, and the voltage drop is about one volt.

The internal or junction temperature is an important factor in the operation of all solid-state devices. Power diodes are usually rated for operation at junction temperatures up to 200°C. The forward power dissipation of a device is proportional to the forward voltage drop and the forward current. Since the voltage drop is nearly constant, it is the ability to dissipate the heat without exceeding the maximum



**Figure 2.2**  
*Steady State V-I Characteristic of a Diode*

*junction temperature* which determines the *forward current* rating of a device. Larger currents cause more dissipation, leading to physically larger devices to get the heat out.

The *reverse leakage current* of a diode depends on its size and internal design, and also depends on leakage at the junction surfaces. Reverse leakage current increases rapidly with junction temperature.

In addition to the forward current rating of a diode, another parameter of extreme practical importance is the *reverse breakdown voltage*. Together these two parameters determine the maximum load power which the diode can control. Although the reverse breakdown voltage depends on the internal design (but not the physical size) of the diode, practical fabrication processes lead to a spread in each batch of devices. Consequently this rating is determined by testing each individual diode.

### 3. Transient V-I Characteristic of a Diode (Figure 2.3)

The switching action of a solid-state device is much faster than that of a mechanical switch of the same power capacity. Nevertheless it is not instantaneous.

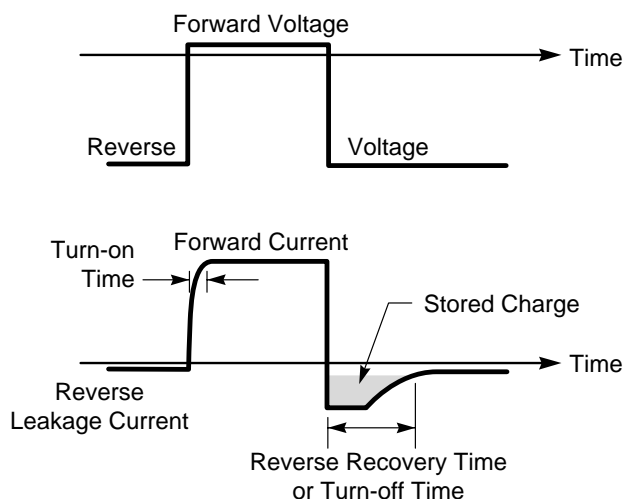
When forward voltage is applied to a diode, a short *turn-on time* elapses before the charges at the PN junction reach equilibrium to carry full forward current. This time is measured in nanoseconds.

Of greater practical importance is the transient which occurs when reverse voltage is applied to turn off a diode which has been conducting forward current. The charges which had been carrying this current must be “swept out” from the junction region before the reverse non-conducting state can be established. These charges are called “*stored charge*”, and depend on the internal design of the diode and on the forward current which has been flowing. The instantaneous reverse current which flows is usually limited by the circuit external to the diode, but the amp-seconds of the reverse current transient depend on the stored charge. The duration of this transient is called the reverse recovery time or *turn-off time*. This time is also nanoseconds for fast low voltage computer diodes, but is measured in tenths of microseconds and microseconds for high power diodes.

Elementary descriptions of the physics of solid-state diodes are contained in many textbooks on electronics. More advanced information on power diodes is contained in:

F. E. Gentry, et al, *Semiconductor Controlled Rectifiers: Principles and Applications of PNP Devices*, Chapter 1; Prentice-Hall; 1964.

Y. C. Kao, “The Design of High-Voltage, High-Power Silicon Junction Rectifiers,” *IEEE Trans.*, vol. ED-17, pp. 657-660; September 1970.



**Figure 2.3**  
*Transient V-I Characteristic of a Diode*

#### 4. Pulse Number

Among the important factors which influence the choice and design of rectifiers are the magnitude and frequency of the ripple voltage at the DC terminals. The smaller the magnitude and the higher the frequency, the cheaper it is to filter the ripple to specific tolerances.

The ratio of the fundamental frequency of the DC ripple to the AC supply frequency is commonly called the pulse number. Most single-phase rectifiers are 2-pulse, and most three-phase rectifiers are 6-pulse.

$$\text{Pulse Number} = \frac{\text{Fundamental Ripple Frequency}}{\text{AC Supply Frequency}}$$

#### 5. Simplifying Assumptions

The analysis of a rectifier circuit usually begins with the study of the idealized version of the circuit. In most cases the characteristics of the idealized circuit can be read directly from design tables as contained in Appendix III. Practical deviations from the idealized circuit can then be added as needed.

The simplifying assumptions involved in the idealized circuits are as follows:

- (a) The voltage drop across switching devices is neglected while they are conducting, and the leakage current is neglected while they are blocking.
- (b) The turn-on and turn-off times of the switching devices are negligible.
- (c) The AC line voltage is sinusoidal and there is no stray impedance (line impedance, transformer reactance, ect.).
- (d) The DC terminals are connected to an ideal filter (an infinite inductance) which maintains the DC current constant over each cycle at its average value. We will look at what is involved in this assumption shortly.

In review the assumptions are:

**Simplifying Assumptions**

- (a) **Switches have no voltage drop or leakage current.**
- (b) **Instantaneous switching.**
- (c) **Sinusoidal voltage source.**
- (d) **Constant DC current over each cycle.**

It was stated in the Introduction that the ideal voltage waveforms at the output of the switching matrix consist of segments selected from the input voltage waveforms as dictated by the switching control signals. If the input voltages and the switching control signals are specified, it would seem that the output voltage waveform from the matrix could easily be determined. Once the voltage waveform is known, the current waveform in a specified load can be determined by conventional analysis.

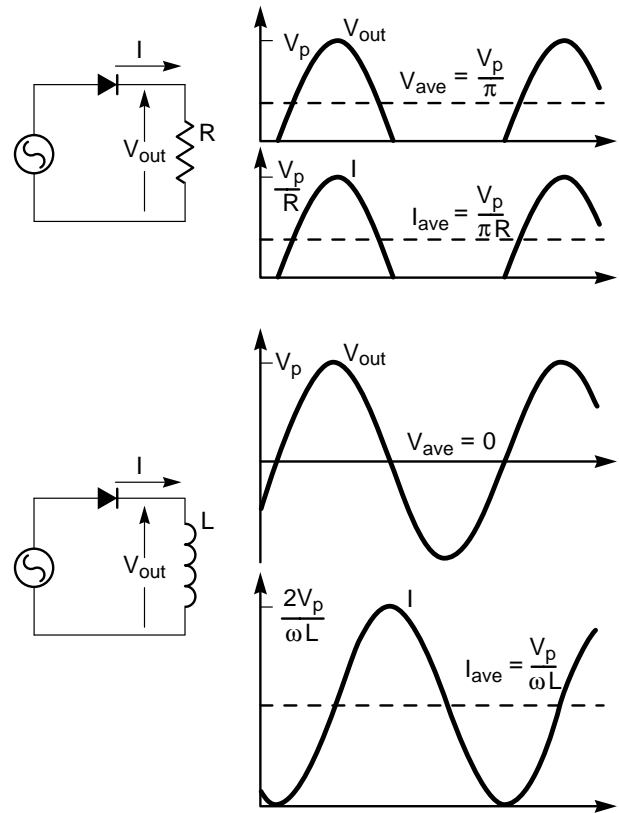
Fortunately an adequate analysis is sometimes as straight-forward as the above discussion implies. However, subtle and unexpected factors can often have significant effects. For example, the load current waveform may influence the commutation points, which in turn modify the voltage waveform. This interaction can lead to an analysis that is somewhat less straightforward. Also, the commutation of current from one switch to another requires a finite time, even if the dynamic response of the switching devices is instantaneous. This “commutation overlap” further complicates the ideal voltage waveforms.

Therefore the best approach to an analytical understanding of the operation of switched power circuits is to proceed slowly from basic fundamentals, being careful not to be misled by implicit assumptions which are invalid.

The problems encountered in converter analysis are discussed in:

S.B. Dewan and G. R. Slemon, “Analysis Techniques for Thyristor Converters,” IEEE Power Electronics Specialists Conference Record, pp. 141-148; June 1973.

S. B. Dewan and J. B. Forsythe, “Techniques of Analysis for SCR Converters,” IEEE Industry & General Applications 4th Annual Meeting Record, pp. 453-468; Oct. 1969.



**Figure 2.4**  
*Single-Phase Half-Wave Diode Rectifier*

## 6. Single-Phase Half-Wave Diode Rectifier

(Figure 2.4)

Although the half-wave diode rectifier is not a useful circuit for high power applications, it nevertheless permits a number of useful principles to be explained in their simplest terms. For the moment we will adopt simplifying assumptions (a), (b), and (c), but assumption (d) is not valid for this one-pulse circuit.\* We will use this circuit to introduce the study of rectifier waveforms and the effects which the load has on these waveforms.

If the load is purely resistive, the output voltage waveform consists of half-cycles of a sine wave separated by half-cycles of zero output voltage, for which the average value is:

$$V_{ave} = \frac{\omega}{2\pi} \int_0^{\frac{\pi}{\omega}} V_p \sin \omega t dt = \frac{V_p}{\pi}$$

The current waveform is identical in shape to the voltage waveform, and the average current is:

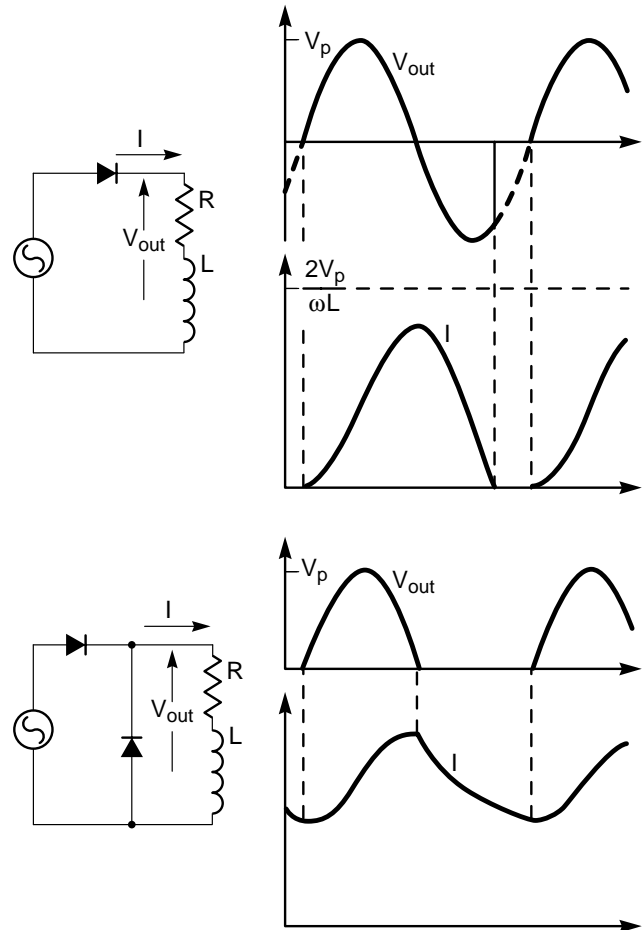
$$I_{ave} = \frac{V_p}{\pi R}$$

On the other hand, if the load is purely inductive, the waveforms change considerably. During the first half cycle, the current builds up from zero to a peak value,  $I_p = 2V_p/\omega L$ . During the half cycle, energy has been transferred from the AC source to the inductor, and  $(1/2) LI_p^2$  watt-seconds are stored in the magnetic field. However, the diode cannot interrupt the non-zero current which exists when the source voltage reverses polarity. The diode must wait for the current to go to zero by itself before conduction ceases. In fact, the diode will continue to conduct throughout the second half-cycle, during which the output voltage is negative and the total energy stored in the inductor is returned to the AC line. If the inductor is lossless, the diode will conduct continuously, and at the end of each full cycle the total net energy transfer will be zero. Note that although the instantaneous output voltage goes negative, the current never does. If the current did go negative, it would violate the assumed behavior of the ideal diode.

Since the diode conducts continuously, the load voltage is identical to the AC source voltage, and has an average value of *zero*. However, the average value of the current is  $V_p/\omega L$ . This example illustrates that average voltages and currents must be manipulated with care. In general, average current does *NOT* equal average voltage divided by impedance!

*Thought Question:* In steady state, the diode in this ideal circuit never becomes reverse biased, and the current is the same as if the diode were shorted. Would the circuit operate in the same way if the diode were not present?

\*The terminology "one-pulse circuit" indicates that the circuit has a pulse number of one.



**Figure 2.5**

*Single-Phase Half-Wave Diode Rectifier – R/L Load*

## 7. Single-Phase Half-Wave Diode Rectifier – R/L Load (Figure 2.5)

In practice, the inductor is not lossless, so we must consider at least some series resistance. At the end of the first half cycle, the current will be less than the previously calculated  $I_p$  because of the voltage drop and consequent power loss in R. During the second half cycle, R will continue to dissipate power as long as current flows. Since there is less total energy to return to the AC line, current will always cease before the second half cycle is completed. That is, commutation does occur, but is delayed until after the zero crossing of the AC source voltage. Net power flow over the full cycle will be from the AC line to the load, as it must be to account for the loss in R.

During the time when the instantaneous voltage and current are both of the same polarity, power flow is



from the AC line to the load, and the circuit acts as a rectifier. During the time when the instantaneous voltage and current are not of the same polarity, power flow is from the load to the AC line, and the circuit acts as a synchronous inverter. Since the load is passive, the net power flow over a full cycle must be from the AC line to the load. Since the diode must conduct for the full half cycle of rectification, but ceases conduction part way through the inversion half-cycle, the circuit has positive average voltage and cannot invert net power from its DC terminals to its AC terminals.

If the  $L/R$  time constant of the load is small compared to a half-period of the AC source, commutation is delayed very little, and the operation of the circuit approaches that of a circuit with a resistive load. As  $L/R$  increases, the delay in commutation increases and the current waveform approaches a sine wave (plus a DC component). However, since the peak current cannot exceed  $2V_p/\omega L$ , the inductance cannot be increased sufficiently to hold the current constant as required by simplifying assumption (d).

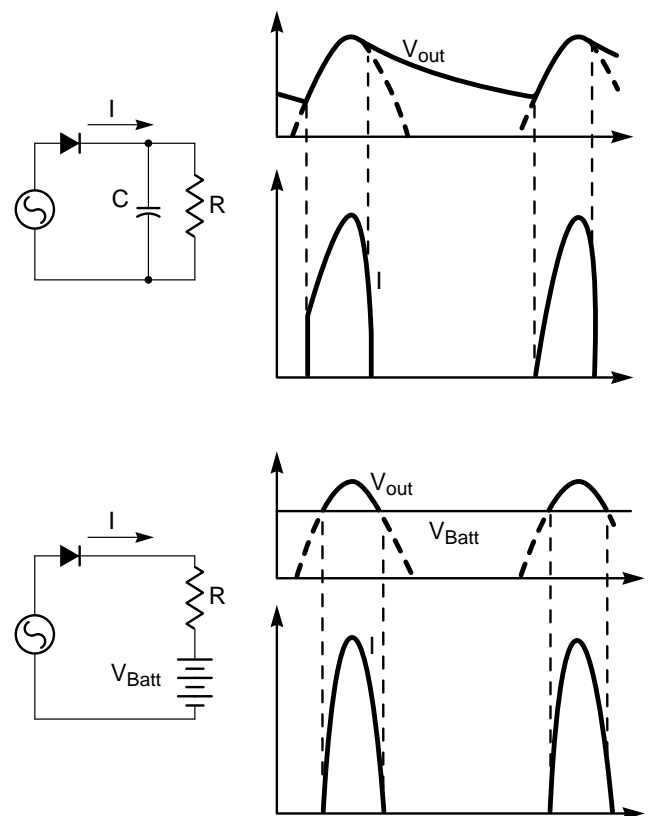
If the intent of the rectifier is to get maximum power transfer into the load, the flow of power back into the AC line on alternate half cycles is undesirable. This reverse flow occurs because of the reversal of the polarity of the instantaneous output voltage. Both can be prevented by connecting a “freewheeling” or “bypass” diode across the output to conduct whenever the output voltage tries to go negative. The rectifier diode then commutates off at the zero-crossing of the AC source voltage. The load current continues to flow, but is transferred to the loop formed by the freewheeling diode. This diode is forward biased by the “kickback” voltage of the inductance. The energy stored in  $L$  then discharges into  $R$  rather than being returned to the AC line. The freewheeling diode helps to prevent the load current from ever going to zero, and thereby reduces the ripple.

## 8. Single-Phase Half-Wave Diode Rectifier – Other Loads (Figure 2.6)

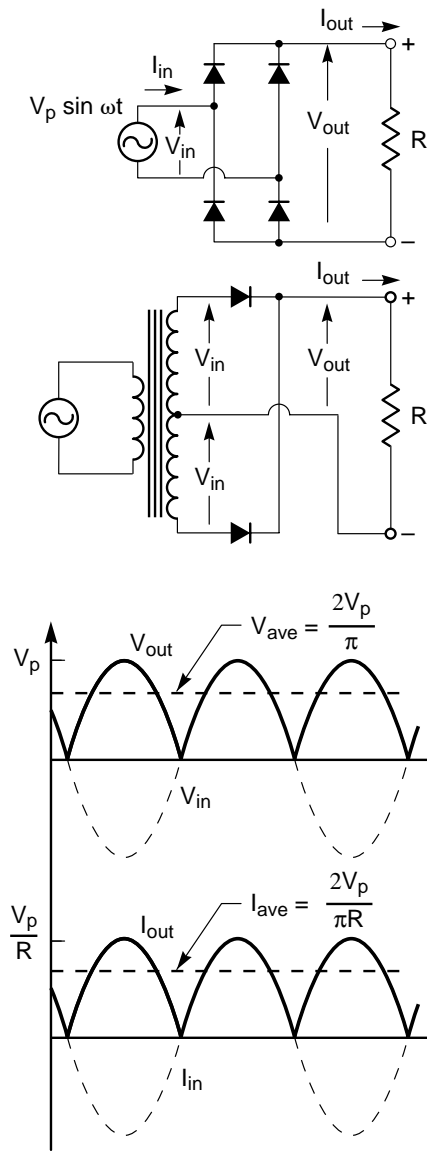
Two other types of loads should also be noted briefly – capacitive loads and back-emf loads. If a capacitive load can be represented by  $R$  and  $C$  in series, the steady-state load voltage charges up to  $V_p$  and thereafter current ceases.

However, if the equivalent circuit is a parallel combination of  $R$  and  $C$ , the capacitor tends to smooth the ripple in output voltage, but the current (in the diode) occurs in brief spikes. Note that in contrast to an inductive load which tends to postpone commutation after the voltage zero crossing, a capacitive load causes commutation to occur before the voltage zero crossing.

Similarly, a back-emf load of the polarity shown, such as a battery or a DC motor, also causes current spikes and early commutation. No current flows until the AC line voltage exceeds the back-emf. If the polarity of this load emf is reversed, conduction begins before the first zero crossing of the AC source voltage and commutation is delayed until after the second zero crossing. As in the case of the inductive load, the AC source receives power during part of each cycle. But the net power flow over a full cycle in this and all other diode rectifier circuits is always from the AC to the DC terminals. Hence they are 1-Quadrant rectifiers.



**Figure 2.6**  
Single-Phase Half-Wave Diode Rectifier - Other Loads



**Figure 2.7**  
*Two-Pulse Diode Rectifier Circuits and Waveforms*

### 9. Single-Phase Full-Wave Diode Rectifiers and Continuous Current (Figure 2.7)

Single-phase full-wave diode rectifiers are two-pulse circuits since the fundamental ripple frequency is twice the AC supply frequency. With a resistive or partially inductive load, the current in the DC circuit is continuous,\* and the diodes always commute at the zero crossings of the AC source voltage. Hence the DC voltage waveform consists of successive half-cycles of a sine-wave, independent of the load. In effect, the two diodes that are tied to the positive DC

terminal connect it to the AC terminal which is instantaneously more positive. Similarly, the two diodes that are tied to the negative DC terminal (in the bridge circuit) connect it to the more negative AC terminal.

If the load is purely resistive, the DC current has the same waveform as the voltage except that the amplitude is multiplied by  $1/R$ . The average value of these waveforms constitutes the DC or zero-frequency component, and is given by:

$$RI_{ave} = V_{ave} = \frac{\omega}{\pi} \int_0^{\pi} V_p \sin \omega t dt = \frac{2}{\pi} V_p$$

The AC line current is the same as the DC current except that alternate half cycles are reversed in polarity. Thus the AC line current is purely sinusoidal with no harmonic content.

Two-pulse circuits have limited practical value for high-power applications because of the large ripple which is contained in the DC output voltage. It is readily seen that the ripple has a peak-to-peak value of  $V_p$  and contains all even harmonics of the AC source frequency.

The ripple current is ordinarily reduced by adding series inductance in the DC circuit to pass the DC component of current while attenuating the harmonics. In practice, part or all of this inductance is frequently in the load itself, as in the case of a motor armature or field. However for the sake of discussion, the inductance will be considered as a filter which is part of the rectifier, and the resistance will be considered as the load.

The presence of this inductance will change the current waveform and consequently the voltage waveform across the resistance. However, as long as the current is continuous, the presence or the value of the inductance does not influence the commutation points of the diodes, and hence does not affect the output voltage waveform from the diodes. In particular, the inductance does not change the average value of this waveform calculated above. This average voltage must be equal to the sum of the average voltages across the inductance and the resistance. As we shall see in the next section, the average voltage across the inductance must be zero. Therefore, the average voltage across the resistance is  $(2/\pi)V_p$  and the average current is  $(2/\pi)(V_p/R)$

independent of the inductance.

It should also be noted that rectifier analysis normally assumes steady-state conditions. Obviously there is a transient with an  $L/R$  time constant which can alter the waveforms when a rectifier is first started.

*\*The DC current is said to be discontinuous if it contains finite intervals of zero current. With a capacitive or back-emf load, the current will be discontinuous unless the series inductance exceeds some minimum critical value. (See R.J. Distler and S.G. Munshi, "Critical Inductance and Controlled Rectifiers," IEEE Trans., vol. IECI-12, pp. 34-37; March 1965.) With discontinuous current, the commutation points and the DC voltage waveform from the diodes are load-dependent.*

## 10. Average Voltage Across an Inductance

A fundamental principle which should be remembered is that in steady-state, the voltage across an inductance, averaged over a full cycle, is always zero.

$$V_{Lave} = \frac{\omega}{2\pi} \int_0^{2\pi} V_L(t) dt = \frac{\omega L}{2\pi} \int_{i_0}^{i_{2\pi}} di = 0$$

This principle may be verified as follows. The voltage across an inductance averaged over a cycle is defined as:

$$V_{Lave} = \frac{\omega}{2\pi} \int_0^{2\pi} V_L(t) dt$$

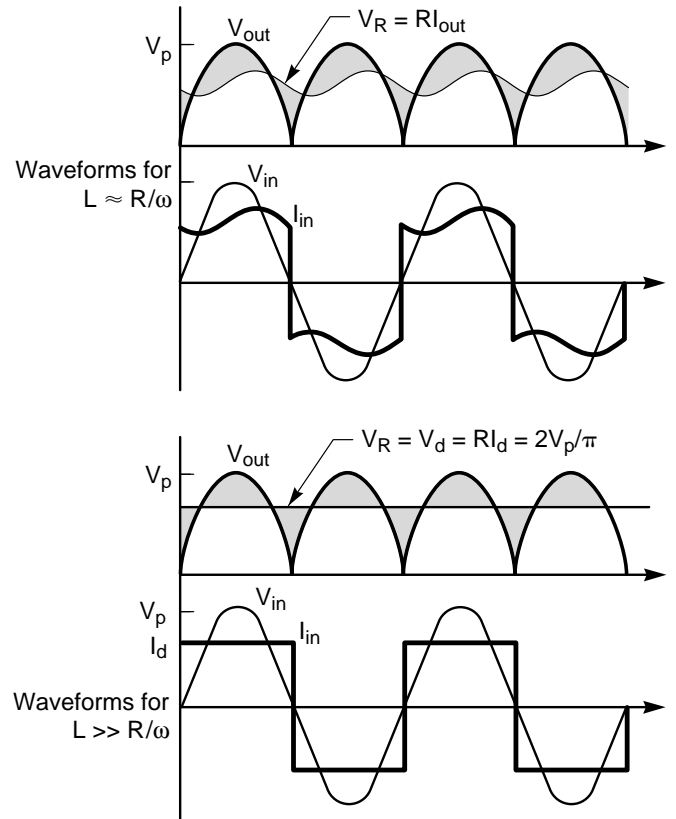
We also know that the defining relationship between voltage and current for an inductance is:

$$V_L(t) = L \frac{di}{dt} \text{ or } di = \frac{1}{L} V_L(t) dt$$

Integrating this equation over a full cycle gives the change in current:

$$\Delta i = \int_{i_0}^{i_{2\pi}} di = \frac{1}{L} \int_0^{2\pi} V_L(t) dt = \frac{2\pi}{\omega L} V_{Lave}$$

But, again by definition, the change in current over a full cycle in steady-state is zero. Hence  $V_{Lave}$  must also be zero.



**Figure 2.8**  
Effects of Inductance on the Waveforms of a Two-Pulse Diode Rectifier  
(Voltage Across Inductance Shown Shaded)

## 11. Two-Pulse Rectifier with Inductance Filter (Figure 2.8)

Returning to the two-pulse diode rectifier, we can consider the effects of inductance in series with the load in two cases. In the first case, the inductance is "nominal". That is, it is big enough to have an appreciable smoothing effect but small enough that the ripple is still significant. An approximate value for this inductance is  $R/\omega$ .

The load current waveform no longer consists of half sine waves,\* but the average current is still the same as before,  $2V_p/\pi R$ . The AC line current is no longer sinusoidal, but approximates a poor square wave with superimposed ripple. The inductance has reduced the harmonic content of the load current by increasing the harmonic content of the AC source current.

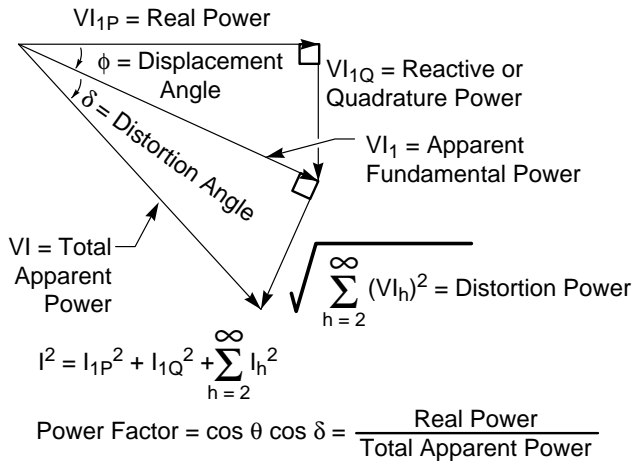
The voltage which appears across the inductance is shown shaded. In order for the average voltage across the inductance to be zero, the positive shaded areas must be equal to the negative shaded areas.

In the second case, the series inductance is increased until it is much larger than  $R/\omega$ . The ripple across the load is now insignificant, and simplifying assumption (d) is valid. The load current is constant at its average value. These constant values of load voltage and current are designated  $V_d$  and  $I_d$ :

$$V = RI_d = V_{ave} = \frac{1V_p}{\pi}$$

The inductance has totally removed the harmonic content of the load voltage and current, but the AC source current waveform has become a square wave of value  $I_d$ : Unless otherwise stated, most rectifier analysis assumes these idealized conditions.

*\*In the one-pulse circuit, the single diode could not commute until the load current went to zero. Note that in this two-pulse circuit the load current never goes to zero. Commutation occurs, not because the load current goes to zero, but because this current is "captured" when the voltage in some other diode loop exceeds the voltage in the loop of the diode previously conducting.*



**Figure 2.9**  
Power Relationships

## 12. Power Relationships (Figure 2.9)

The purpose of a rectifier is to convert AC power into DC power. The fundamental nature of power transfer in power electronics makes it essential for us to understand the relationships between the various components of power. Although these relationships are probably quite familiar when the waveforms are purely sinusoidal, they must be modified to include the nonsinusoidal waveforms encountered in rectifiers and other types of converters.

The two-pulse rectifier which we have just discussed presents an opportunity to explain these modified relationships. Under simplifying assumptions (a), there is no power dissipation in the switching devices. Therefore the net power flow at the AC terminals averaged over a full cycle must be exactly equal to the net power flow at the DC terminals.

With a large inductance in the DC circuit, the DC current is constant at  $I_d$ , and the voltage across the load resistance is constant at  $V_d$ . Therefore the real DC power is easily calculated as:

$$P_d = V_d I_d = (2/\pi) V_p I_d$$

According to simplifying assumptions (c), the AC source voltage is sinusoidal. Therefore its RMS value,  $V$ , is related to its peak value by:

$$V = V_p / \sqrt{2}$$

However, as we have seen, the AC current waveform is a square wave of amplitude  $I_d$ , in phase with the AC voltage. The RMS value of this square wave is  $I_d$ , and there is no apparent power factor phase angle. We might be led to conclude that the AC power is the product of the RMS AC voltage,  $V$ , and the RMS AC current,  $I = I_d$ :

$$P_T = VI = (V_p / \sqrt{2}) I_d$$

It is apparent that this "total apparent power" is *not* equal to the real DC power calculated above. We have been misled by applying relationships derived for sinusoidal waves to a case where the current is nonsinusoidal.

To reconcile this situation, expand the current waveform into a Fourier series where the RMS value of harmonic “h” is  $I_h$ . The RMS value of the waveform is then:

$$I = \sqrt{I_1^2 + I_2^2 + I_3^2 + \dots}$$

or

$$I^2 = I_1^2 + \sum_{h=2}^{\infty} I_h^2$$

In general, the fundamental current,  $I_1$ , will consist of an in-phase component,  $I_{1p}$ , and a quadrature component,  $I_{1Q}$  (if the fundamental current is not in phase with the voltage). Then:

$$I^2 = I_{1p}^2 + I_{1Q}^2 + \sum_{h=2}^{\infty} I_h^2$$

Multiplying both sides of this equation by the square of the RMS voltage gives:

$$(VI)^2 = (VI_{1p})^2 + (VI_{1Q})^2 + \sum_{h=2}^{\infty} (VI_h)^2$$

The left side of the equation is the square of the total apparent power, but of the three terms on the right side, only the *in-phase* component of the *same* frequency as the voltage represents *real* power (watts). The second term represents the square of the reactive power (VAR's) which results if the fundamental current is out of phase with the voltage, and the third term is a “wattless” power caused by the harmonic components (i.e., distortion) of the nonsinusoidal waveform.

For sinusoidal waves, a right triangle displays geometrically the mathematical relationship between real power, reactive power, and apparent power. The phase angle,  $\emptyset$ , is one of the angles of this triangle, and the power factor (ratio of real power to apparent power) is equal to  $\cos \emptyset$ .

Similarly two right triangles can be used to give a geometric representation of the equation above. The phase angle between the voltage and the fundamental component of current is known as the “displacement angle,”  $\phi$ , and  $\cos \phi$  is called the “displacement factor”. The harmonic term is associated with a second right triangle and an

angle,  $\delta$ , which has little significance. However,  $\cos \delta$  is known as the “distortion factor”, and is equal to the ratio of the apparent fundamental power to the total apparent power. The two triangles may be cascaded as shown in the figure, and it is apparent that the power factor is now equal to the product of the displacement factor and the distortion factor.

For the two-pulse diode rectifier, the fundamental component of current is in phase with the voltage. Therefore,  $\phi = 0$ , and the displacement factor is unity. However, the RMS value of the fundamental component of a square wave is equal to  $4/\pi\sqrt{2}$  times the RMS value of the square wave. Therefore the distortion factor is  $4/\pi\sqrt{2}$ .

The real AC power is then the product of the displacement factor, the distortion factor, and the total apparent power:

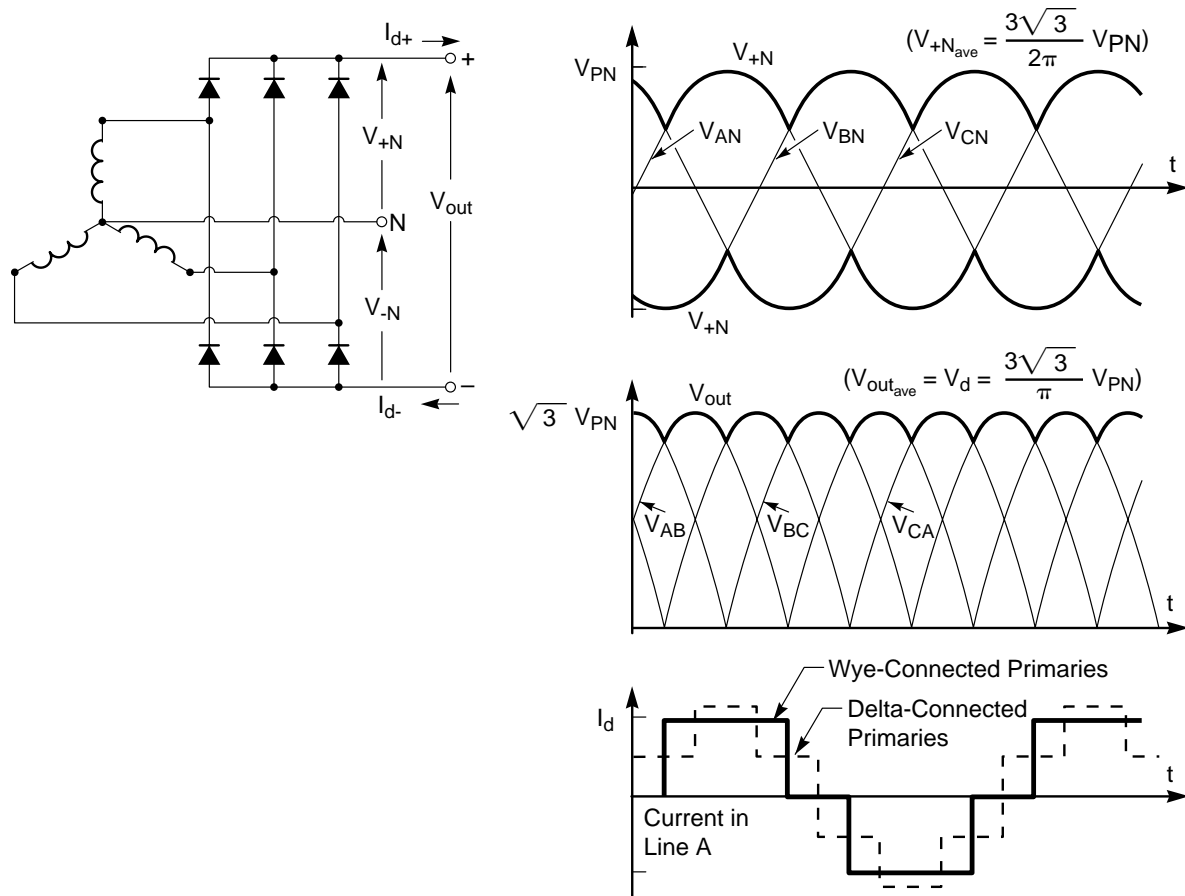
$$VI_{1p} + (1)\left(\frac{4}{\pi\sqrt{2}}\right)\left(\frac{V_p I_d}{\sqrt{2}}\right) = \frac{2}{\pi} V_p I_d$$

The real AC power is seen to be equal to the real DC power calculated previously, and the desired power balance has been verified.

### 13. Three-Phase Diode Rectifiers (Figure 2.10)

The pulse number of a single-phase rectifier cannot be increased beyond two unless some artificial means is available for generating out-of-phase voltages. On the other hand, the pulse number of a polyphase rectifier may be made arbitrarily high by various interconnections of transformer windings. The most common high-power rectifiers are supplied from a three-phase source and use a three-pulse or six-pulse circuit.

The easiest way to derive the waveforms of a three-phase bridge rectifier is to consider each half separately with respect to the neutral. The upper three diodes constitute a three-pulse center-tap circuit, and each diode conducts for  $120^\circ$ . The voltage from the positive DC terminal to the neutral,  $V_{+N}$ , has a positive average value of  $(3\sqrt{3}/2\pi) V_{pN}$ , where  $V_{pN}$  is the peak phase voltage to the neutral. The waveform is as shown. If a load is connected between the positive DC terminal and the neutral (including enough inductance to maintain the current constant), a current  $I_{d+}$  will flow.



**Figure 2.10**  
*Waveforms for a Three-Phase Bridge Rectifier*

Similarly the lower group of three diodes generates a negative three-pulse waveform between the negative DC terminal and the neutral, and a load will cause a current  $I_{d-}$  to flow. As long as the neutral is connected, the two groups of diodes operate independently as three-pulse rectifiers, and the currents  $I_{d+}$  and  $I_{d-}$  need not be equal. However there will be a net DC component of current in each winding, tending to cause DC magnetization of the cores, if these currents are unequal.

If the load is connected directly between the positive and negative DC terminals without a neutral connection, the average DC voltage doubles and  $I_{d+}$  and  $I_{d-}$  must be equal because there is no return path for a difference current. Therefore core magnetization is no longer a problem. However, another significant advantage also occurs. Note that commutation occurs alternately, not simultaneously, in the two groups of diodes. When the two three-pulse groups operate together, a six-pulse output waveform results.

The current in each wye-connected secondary winding supply AC to the bridge is a square wave of amplitude  $I_d$ , consisting of a  $120^\circ$  positive pulse, a  $60^\circ$  period to zero current, a  $120^\circ$  negative pulse, and another  $60^\circ$  period of zero current. If the primaries of the transformer are also wye-connected, the AC line currents will have the same waveform. On the other hand, if the primaries are delta-connected, the AC line current becomes different in waveform although the harmonic content remains the same.

Circuits having higher pulse numbers further increase the ripple frequency, making filtering easier and reducing the harmonic content of the AC current. However, these circuits may also reduce the conduction angle of each diode, thereby reducing the utilization of its capacity. Therefore, circuits having pulse numbers higher than six are usually used only for special purposes.

We will now state, without proof, several general laws

concerning the harmonics generated by ideal diode rectifiers:

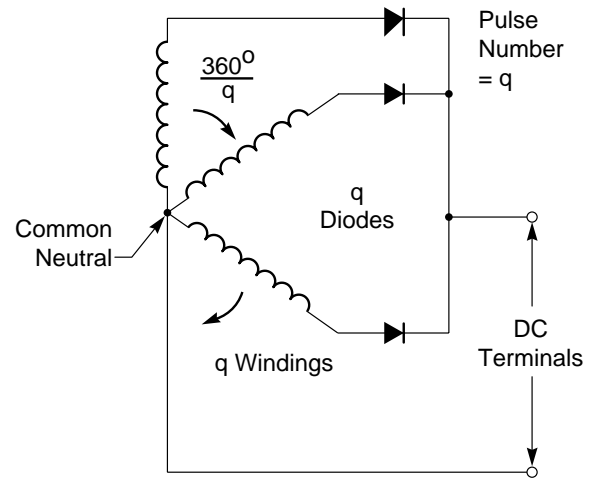
- (1) The ripple voltage at the output has a fundamental of  $pf_s$ , where  $p$  is the pulse number and  $f_s$  is the source frequency. Because of the lack of symmetry of the positive and negative parts of the ripple waveform, *all* harmonics of this fundamental ripple frequency will be present:  $npf_s$ ,  $n = 1, 2, 3, \dots$ . The peak amplitude of any harmonic of the source frequency which is present, relative to the uncontrolled DC voltage, is  $2/(n^2p^2 - 1)$ . That is, the pulse number determines which harmonics are present in the output, but *does not* change the relative amplitude of a given harmonic if it is present.
- (2) The only harmonics present in the input current (in addition to the fundamental current at  $f_s$ ) occur at  $(np \pm 1) f_s$ ,  $n = 1, 2, 3, \dots$ . The amplitude of each harmonic current, relative to the fundamental current, is inversely proportional to its order (i.e., to  $np \pm 1$ ).

#### References:

- J. Schaefer, *Rectifier Circuits: Theory and Design*, p. 257 and p. 298, John Wiley, New York; 1965.
- B. R. Pelly, *Thyristor Phase-Controlled Converters and Cycloconverters*, pp. 94-107, John Wiley, New York; 1971.
- C. J. Amato, "A Simple and Speedy Method for Determining the Fourier Coefficients of Power Converter Waveforms," IEEE Industry & General Applications 4th Annual Meeting Record, pp. 477-483; Oct. 1969.
- I. K. Dortort, "Phase Shifting of Harmonics in AC Circuits of Rectifiers," IEEE Trans., vol. IGA-4, pp. 655-658; Nov./Dec. 1968.

#### 14. Generalized Center-Tap Rectifier (Figure 2.11)

The two-pulse and three-pulse center tap circuits which we have discussed are characterized by the fact that one terminal of the DC circuit is connected directly to a center-tap or neutral point of the transformer windings which couple the rectifier to the AC supply. The generalized center-tap rectifier has  $q$  secondaries, with adjacent secondaries separated by  $360^\circ/q$ . The required number of diodes is  $q$ , one for each secondary. The pulse number is also equal to  $q$ .



**Figure 2.11**  
*Generalized Center-Tap Rectifier*

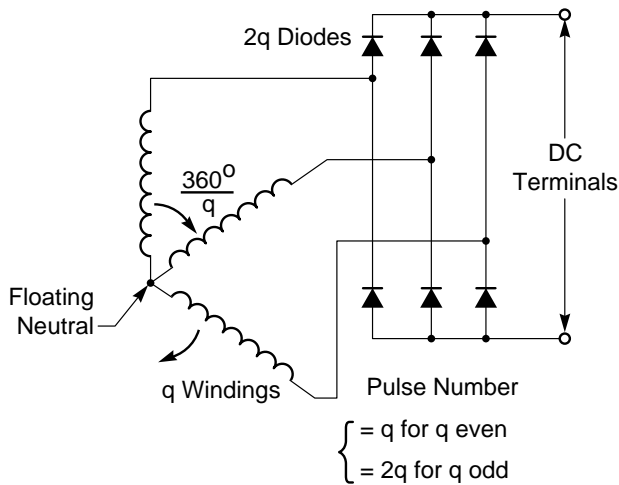
For  $q = 1$ , the circuit is a single-phase half-wave rectifier. Although widely used for low-current power supplies, this circuit is not useful at high power levels because of its large ripple voltage and because of the DC current which it draws from the AC line. Circuits for which  $q = 2, 3$ , or  $6$  are of greater practical importance.

Center-tap rectifiers are also known as "single-way" circuits because the current in each transformer winding is unidirectional. If  $q$  is even, currents in each pair of windings flow in opposite directions so that the net DC magnetization of transformer cores is zero. However, if  $q$  is odd, there is no cancellation and DC magnetization of transformer cores is a problem. In this case, magnetization is usually avoided by a "zig-zag" series connection of pairs of transformer secondaries, arranged so that the effect of each winding carrying unidirectional current is cancelled by another winding which carries current in the opposite direction.

#### 15. Generalized Bridge Rectifier (Figure 2.12)

Bridge rectifiers do not ordinarily have a direct connection between the DC terminals and the transformer secondaries, but each winding is connected to two oppositely polarized diodes so that both polarities of current flow in each winding. Hence DC magnetization of the transformer cores is not a problem. Another name for bridge circuits is "double-way" circuits.





**Figure 2.12**  
*Generalized Bridge Rectifier*

The generalized bridge circuit also contains  $q$  secondaries separated by  $360^\circ/q$  in phase, but requires  $2q$  diodes. Because of the lack of a common connection between input and output terminals,  $q$  must be greater than one. For even values of  $q$ , the pulse number is equal to  $q$ . However, for odd values of  $q$ , diodes on opposite sides of the bridge commute alternately (not simultaneously), and the ripple frequency is doubled. Thus for odd  $q$ , the pulse number is  $2q$ .

The most useful values for  $q$  are 2 and 3, for single-phase and three-phase AC supplies respectively.

Since the advent of solid-state rectifiers, bridge circuits are much more widely used than center-tap circuits because of their better utilization of the transformer windings. Previously, this advantage had to be balanced against the disadvantage of the larger forward voltage drop of two mercury-arc rectifiers in series.

## 16. Applications of Diode Rectifiers

Diode rectifiers are most commonly used in those applications where control of the DC voltage is unnecessary or can be achieved by other means. These rectifiers are used in the largest quantities in low-power DC supplies for radios, television sets, and other electronic equipment. Solid-state diodes also led to the changeover from DC generators to alternators in automotive electrical systems. These

diodes permitted the elimination of the commutator, and the lack of rectifier control is no problem because regulation is achieved in the field circuit.

Other applications of diode rectifiers include battery chargers, power supplies for DC welding, and DC traction drives. Important applications for very high-power diode rectifiers are brushless exciters for large turbine-generators (to 5 MW or more) and DC power supplies for electrochemical process lines (to 100 MW or more). A summary of diode application is:

### *Applications of Diode Rectifiers*

- **Radio, TV, & other Electronic Equipment**
- **Automotive Electrical Systems**
- **Battery Chargers & DC Welding Supplies**
- **DC Traction Drives**
- **Brushless Exciters for Turbine-Generators**
- **DC Supplies for Electrochemical Process Lines**

Diode rectifiers are seldom analyzed separately but are usually treated as a special case of the more general AC/DC converters which use thyristors. Separate consideration has been given here in order to introduce some of the basic concepts of rectifiers in the simplest possible context.

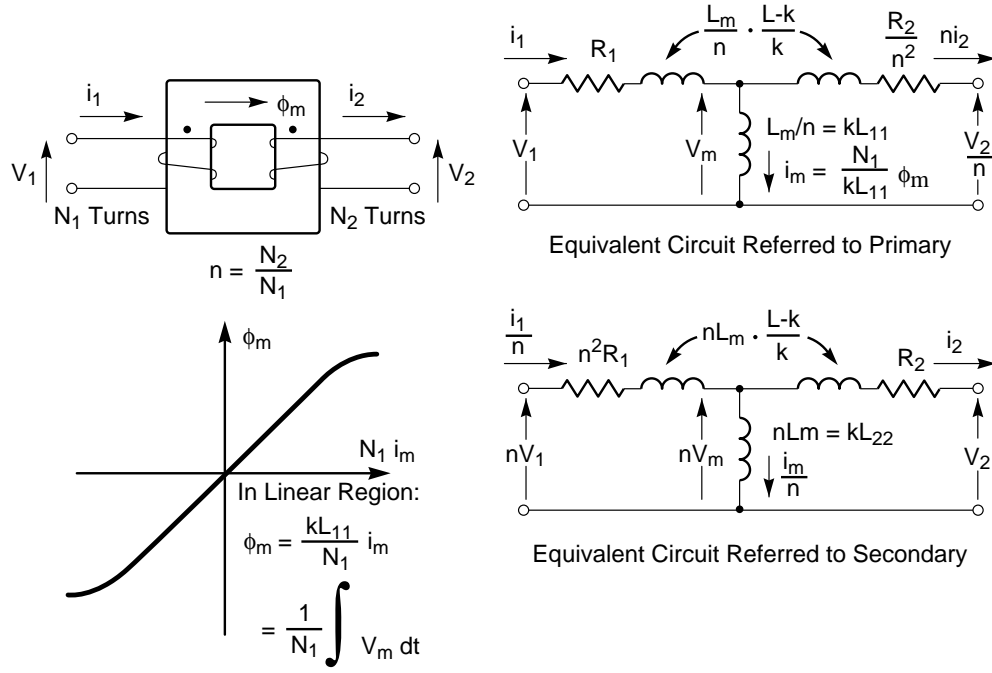
An excellent reference which uses a similar approach, but goes into all aspects of the subject much more thoroughly than is possible here, is:

J. Schaefer, *Rectifier Circuits: Theory and Design*, Wiley (1965).

## 17. Transformers and Nonsinusoidal Waveforms (Figure 2.13)

In elementary electrical courses, transformers are usually studied in the context of sinusoidal waveforms. Therefore questions frequently arise concerning the behavior of transformers (and other magnetic components) when exposed to the nonsinusoidal current and/or voltage waveforms in power electronics circuits. These questions are best answered by a brief review of fundamentals.

Consider a two-winding transformer having  $N_1$  turns on the primary and  $N_2$  turns on the secondary, as shown. Assume a magnetic core with no hysteresis, no eddy-current losses, and constant permeability. Let  $L_{11}$  be the inductance of the primary and  $L_{22}$  be



**Figure 2.13**  
Basic Transformer Relationships

the inductance of the secondary. Then:

$$n = \frac{N_2}{N_1} \approx \sqrt{\frac{L_{22}}{L_{11}}}$$

and

$$L_m = k \sqrt{L_{11} L_{22}}$$

where  $L_m$  is the “mutual inductance” and  $k$  is known as the coefficient of coupling.

When a “magnetizing” current  $i_m$  is applied to the primary, the flux generated in the core is given by:

$$\Phi_m = \frac{kL_{11}}{N_1} i_m = \frac{L_m}{nN_1} i_m$$

The relationship between  $\Phi_m$  and  $i_m$  is linear as long as  $\Phi_m$  is below the value where saturation begins. From the differential relationship for an inductance, the induced voltage is:

$$V_m = kL_{11} \frac{di_m}{dt} = \frac{L_m}{n} \bullet \frac{di_m}{dt} = N_1 \frac{d\Phi_m}{dt}$$

Integration gives the inverse relationship:

$$\Phi_m = \int \frac{V_m}{N_1} dt = \frac{kL_{11}}{N_1} i_m$$

In general, the coefficient of coupling is less than one, and the response of the transformer is determined by solving the loop or node equations involving one of the equivalent circuits shown. (Winding resistances  $R_1$  and  $R_2$  are indicated, but winding capacitance is neglected.) However, most power transformers are designed with a coefficient of coupling near unity. Thus the “leakage inductances” in the windings are usually small compared to the mutual inductance. Frequently both the leakage inductances and the winding resistances are neglected in power circuit analysis.

To the extent that these assumptions are valid, the primary voltage must be equal to the induced (or “mutual” or “magnetizing”) voltage (i.e.,  $V_1 = V_m$ ). Under the same assumptions:

$$V_2 = nV_m = nV_1$$

independent of waveforms. If the transformer is

driven from a voltage source, the flux and the magnetizing current are proportional to  $\int V_1 dt$ , or “volt-second area”. When a load is connected across the secondary, the secondary AC ampere-turns  $N_2 i_2$  must be compensated by an equal number of primary ampere-turns to maintain the flux change and the induced voltage constant. (DC ampere-turns induce no voltage and therefore cannot be compensated by primary current. Instead they offset the flux swing.) Hence:

$$N_1 i_1 = N_1 i_m + N_2 i_2$$

or

$$i_1 = i_m + N i_2$$

Stated in another way, the applied voltage determines  $N_1 i_m$  which is the net difference in primary and secondary ampere-turns.

Under these idealized assumptions, it is seen that there is no difficulty in principle when nonsinusoidal voltage and/or current waveforms are applied to a transformer. Specifying the ratings of the transformer must, however, take the waveforms into account.

First, if the secondary current contains a DC component, the transformer core must be designed to accommodate the flux offset and prevent excessive saturation. Otherwise, the current rating of a transformer depends on the heating in the winding resistances, which is proportional to the RMS value of the current waveforms.

If the voltage waveform is nonsinusoidal, the insulation must be specified according to the peak voltage, and the core area must be specified to handle the peak flux without excessive saturation. The flux depends on the volt-second area of the voltage waveform which is influenced by voltage amplitude, waveform, and frequency.

Sometimes a transformer is deliberately designed with appreciable leakage inductance (i.e., with  $k < 1$ ) to limit peak currents into a capacitive or back-emf load. In this case the validity of the previous assumptions should be examined, and the full equivalent circuit used if necessary. If core losses are appreciable, these may be represented by adding resistance in parallel with  $L_m$ .

## 18. Problems

### Problem 2.1 Comparison of Diode Rectifier Circuits

Diodes are available which have a reverse breakdown voltage rating of  $V_{RRM} = 1000V$  and an average forward current rating of  $I_T = 100A$ . Therefore they each have the capacity to supply a nominal DC load of  $V_{RRM} I_T = 100kW$ . Their actual load capacity depends on the rectifier circuit which is used, derating factors for overload capacity requirements, ect. The purpose of this problem is to compare the various rectifier circuits with respect to their utilization of diode capacity and transformer capacity.

The attached table lists five commonly used rectifier circuit configurations.

- Sketch each circuit in the form of a switching matrix to verify the generality of this concept.
- Perform the calculations necessary to design each configurations to utilize the diodes specified above as fully as possible, and thereby obtain the information needed to complete the table. Assume that the four simplifying assumptions discussed in the text are valid, and also assume that the diodes will not overheat as long as their *average* current does not exceed their rated value. Transformer ratings are based on *RMS* voltage  $v_s$  and current  $I_L$ .\* Ignore the effect of the DC magnetizing current on the transformer core of the center-tapped three-pulse circuit.
- Based on the data in the table alone, is any one rectifier circuit clearly superior?

\*The following information is useful in calculating RMS values:

$$f_{RMS} = \left[ \frac{1}{(T_2 - T_1)} \int_{T_1}^{T_2} f^2(t) dt \right]^{1/2}$$

### Problem 2.1

*Comparison of Diode Utilization and Transformer Utilization for Various Rectifier Circuit Configurations.*

$V_d, I_d$ = Smooth DC Load Voltage, Current		$V_s, I_s$ = RMS Winding Voltage, Current							
Configuration	Pulse No.	Diode Factors				Transformer Secondary Factors			
		No. of Diodes $n$	Voltage Ratio $\frac{V_d}{V_{RRM}}$	Current Ratio $\frac{I_d}{nI_T}$	Diode Utilization $\frac{V_d I_d}{n V_{RRM} I_T}$	No. of Windings $q$	Secondary Voltage Ratio $\frac{V_d}{V_s}$	Secondary Current Ratio $\frac{I_d}{q I_L}$	Transformer Utilization $\frac{V_d I_d}{q V_s I_L}$
Center-Tap	2	2				2			
	3	3				3			
	6	6				6			
Bridge	2	4				2 (Floating Neutral)			
	6	6				3			

Comparison of Diode Utilization and Transformer Utilization for Various Rectifier Circuit Configurations.

### Problem 2.2 Average vs. RMS Current Ratings for Diodes

In the previous problem, the diodes were rated in terms of average current while transformer windings were rated in terms of RMS current. The data sheets on solid-state power devices frequently contain both average and RMS current ratings. The purpose of this problem is to explore how the two ratings differ and why both are used.

The steady-state current rating of a diode depends on the average amount of heat which the diode can dissipate. For the purposes of this problem, assume that dissipation occurs only when the diode is conducting forward current. Also assume that the load current,  $I_d$ , is smooth, and that each diode carries this current for a time  $\eta T$  during each cycle, where  $\eta$  is the duty cycle and  $T$  is the period of the AC source. For a specified circuit configuration, the duty cycle decreases as the pulse number is increased.

The diode to be considered has an average current rating of 100A at a 50% duty cycle, an RMS current rating of 140A, and its V-I characteristic is as shown in the graph.

First, calculate the average diode current,  $I_{AVE}$  and the RMS diode current,  $I_{RMS}$ , as functions of the load current,  $I_d$ , and the duty cycle,  $\eta$ .

Second, assume that  $I_d$  varies with duty cycle in such a way that  $I_{ave}$  is held constant at 100A. Use the V-I characteristic to calculate and plot the actual average diode dissipation vs. duty cycle. Is the average current rating valid for low duty cycles?

Third, assume that  $I_d$  varies with duty cycle in such a way that  $I_{RMS}$  is held constant at 140A, and repeat the calculation and plot of dissipation vs. duty cycle. Is the RMS current rating valid for low duty cycles?

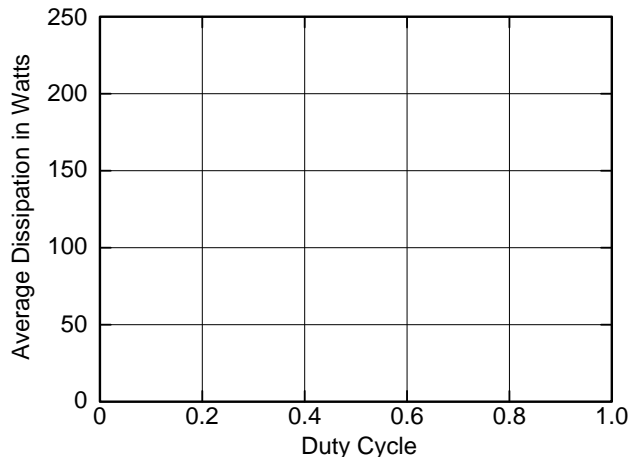
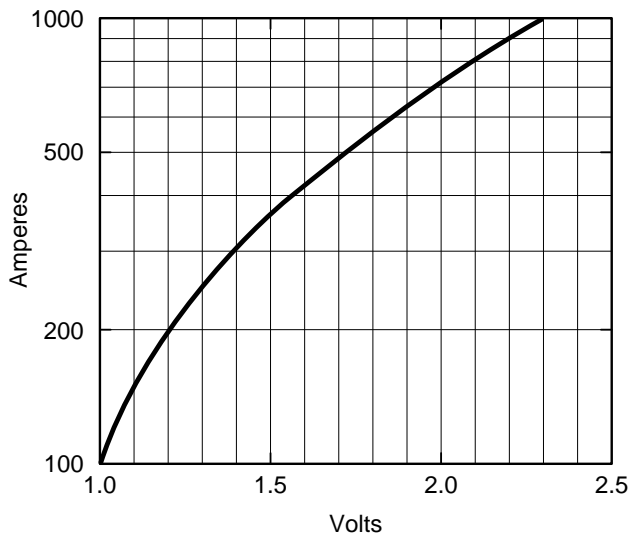
And last, attempt to interpret the two plots so as to reach a general conclusion concerning the usefulness and limitations of average and RMS current ratings for the more complicated current waveforms found in other types of converter circuits. Can you give a simple physical explanation of why solid-state devices are more difficult to rate than such components as transformers?

(A rating which attempts to avoid the shortcomings of both average and RMS current ratings for diodes and thyristors is proposed in:

W.E. Newell, "Dissipation in Solid-State Devices — The Magic of  $I^{1+N}$ ", IEEE Power Electronics Specialists Conference Record, pp. 162-173; June 1974.)

### Problem 2.2

#### Average vs. RMS Current Ratings for Diodes



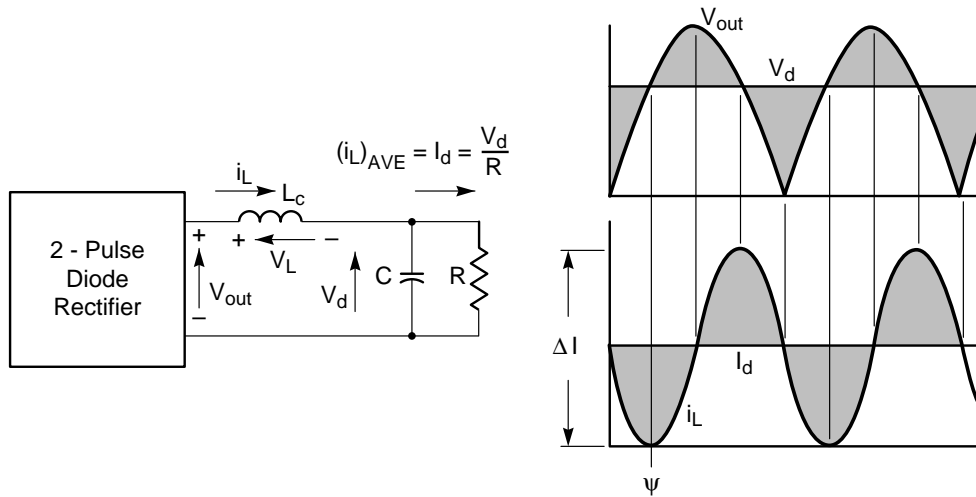
### Problem 2.3 Commutation Overlap

Inductance in the DC circuit of a rectifier is an asset in reducing the ripple in the DC load current. The purpose of this problem is to investigate the very different effect of inductance in the AC circuit. Although the AC inductance is frequently ignored in rectifier analysis, in practice it is always present in the form of AC-line impedance, transformer leakage reactance, ect., and as will be seen, it causes significant changes in the idealized waveforms.

Consider a two-pulse center-tap diode rectifier having an inductor  $L_s$  in series with each of the two diodes and constant current  $I_d$  in the load. Each time the load current commutates from one diode to the other, the current in one of these inductors must decrease from  $I_d$  to zero while the current in the other increases from zero to  $I_d$ . Since the current in an inductor cannot change instantaneously, there is a time during which both diodes are conducting. This time is known as the "commutation overlap".

- Show that the output voltage during commutation overlap is equal to the average of the two phase voltages to which the diodes are connected. Sketch the actual waveforms of output voltage and diode currents, and compare them with the ideal waveforms obtained for  $L_s = 0$ .
- The effect of  $L_s$  is to decrease the average DC output voltage by an amount which is proportional to  $I_d$ . Therefore, its effect on the regulation of the rectifier is equivalent to a (lossless) resistor in the DC circuit. Determine the value of this resistor.
- Discuss the generalization of the above results to rectifiers of higher pulse numbers and controlled rectifiers in which the diodes are replaced by thyristors.

**Problem 2.4**  
Critical Inductance



**Problem 2.4 Critical Inductance**

The current in a rectifier without a filter inductance becomes discontinuous as the AC source voltage drops below the DC load voltage. This will happen with a capacitor filter or a back-emf load such as a battery or a motor. Usually it is assumed that the filter inductance is large enough that the DC current is not only continuous but constant. The purpose of this problem is to determine the minimum or “critical” inductance,  $L_C$ , required to prevent the DC current from becoming discontinuous. In particular, calculate  $L_C$  in terms of load resistance  $R$  and source frequency  $\omega$  for a 2-pulse diode rectifier.  $C$  in the following circuit may be assumed to be so large that there is negligible ripple voltage across  $R$ .

(Hint: Determine and integrate  $V_L(t)$  to obtain  $\Delta I$ . For  $L = L_C$ ,  $i_L$  will be zero at  $\varphi$  as indicated. Furthermore, it can be shown from the symmetry of the waveforms (for  $\alpha = 0$ ) that  $(i_L)_{ave} = 0.5 \Delta I$ .)

# Chapter 3

## THYRISTORS

### AC/DC CONVERTERS

### AND OTHER

### NATURALLY

### COMMUTATED

### CIRCUITS

#### Section

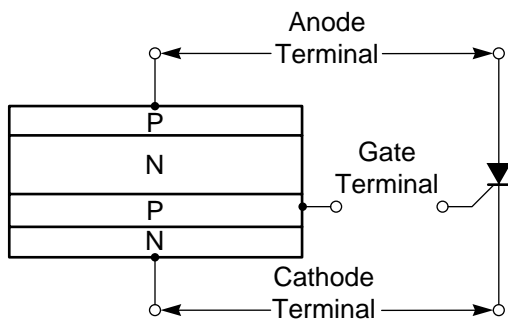
1. Type 2 Switch Thyristor . . . . .	3-2
2. Steady-State V-I Characteristic of a Thyristor . . . . .	3-2
3. Two-Transistor Analog for Explaining Thyristor Turn On . . . . .	3-3
4. Gate Characteristics . . . . .	3-4
5. Transient V-I Characteristic of a Thyristor . . . . .	3-5
6. Thyristor Ratings – Design Tradeoffs and Commercial Availability . . . . .	3-6
7. Circuit Functions Performed by Type 2 Switches . . . . .	3-7
8. Two-Pulse AC/DC Converter Operating as a Controlled Rectifier . . . . .	3-7
9. Two-Pulse AC/DC Converter Operating as a Synchronous Inverter . . . . .	3-9
10. AC/DC Converter as a Linear Amplifier . . . . .	3-10
11. AC/DC Converter as a DC Motor Drive . . . . .	3-11
12. Two-Pulse Semiconverter . . . . .	3-13
13. Two-Pulse Dual Converter . . . . .	3-14
14. Waveform Derivation for a Three-Phase Bridge AC/DC Converter . . . . .	3-14
15. Naturally Commutated Cycloconverter . . . . .	3-18
16. Bilateral Solid-State Switches . . . . .	3-19
17. Triac . . . . .	3-19
18. AC Switches and Regulators . . . . .	3-21
19. Varying Regulators . . . . .	3-22
20. Brushless Machines . . . . .	3-23
21. Summary . . . . .	3-25
22. Problems . . . . .	3-25

## Introduction

The objectives of this chapter are:

- To name and describe the important steady-state and transient terminal characteristics of thyristors and triacs.
- To analyze the effect of phase delay on average DC current for an AC/DC converter in both the rectifier and synchronous inverter modes of operation.
- To explain and distinguish between 1-, 2-, and 4-quadrant converters.
- To compare two-step (rectifier/inverter) and one-step (cycloconverter) AC frequency changers.
- To show several ways of implementing a bilateral AC switch.
- To describe and compare the four modes of operation of an AC regulator.





**Figure 3.1**  
*Type 2 Switch or Thyristor*

### 1. Type 2 Switch or Thyristor (Figure 3.1)

The thyristor is a Type 2 switch containing three internal PN junctions in series. When the anode is negative with respect to the cathode, the center junction is forward biased, but the two outer junctions are reversed biased. Therefore a thyristor blocks the flow of reverse current until the breakdown voltage of the two other junctions is exceeded.

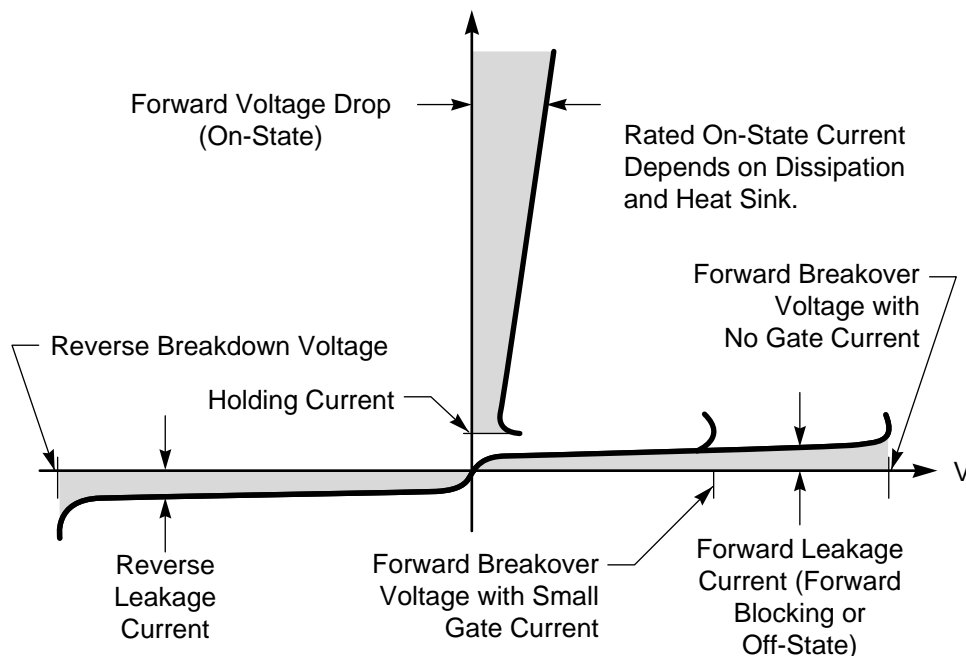
If the anode is positive with respect to the cathode, the two outer junctions are forward biased but the center junction is reverse biased. Therefore forward current is also blocked until the breakdown voltage of the center junction is exceeded.

However, the third terminal of a thyristor is connected to the P layer adjoining the cathode N layer. If this terminal is made positive with respect to the cathode, forward current flows across the cathode PN junction. If the anode terminal is also positive, this forward current across the cathode junction initiates a regenerative internal action which then permits current flow across the reverse biased center junction. In other words, this third terminal allows the thyristor to be turned on. Because of the control which this terminal exercises over current flow, it is known as the “gate”. Once forward current flow is initiated, the thyristor latches in its on state, the gate loses its control, and current continues until it is commutated by some external means.

### 2. Steady-State V-I Characteristic of a Thyristor (Figure 3.2)

The V-I characteristic of a thyristor and its primary parameters are shown below. The reverse blocking state is similar to that of a diode, as is the forward blocking state, and each is characterized by a leakage current and a breakdown voltage.

When a small current is applied to the gate, the forward breakover voltage decreases. If the applied forward voltage exceeds this decreased breakover voltage, the thyristor switches to its on-state. This state is characterized by a forward voltage drop,



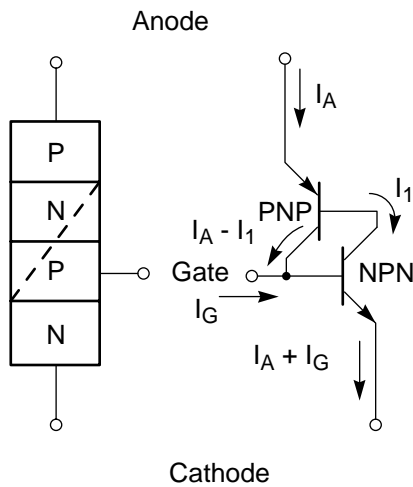
**Figure 3.2**  
*Steady-State V-I Characteristic of a Thyristor*

typically about 1.4 volts. The forward current rating of a thyristor in its on-state is determined primarily by the internal dissipation and the thermal efficiency involved in getting the heat out.

Internal junction temperature is even more critical in a thyristor than it is in a diode. Leakage current increases rapidly with temperature. Since leakage across the center junction into the gate layer is indistinguishable from external gate current, the forward blocking state will disappear if the junction becomes overheated, and irreversible damage may occur. Most thyristors cannot be relied upon to block their rated maximum forward blocking voltage if the junction temperature exceeds 125°C.

Once the thyristor is in its on-state, it will remain there until the current is reduced (by external means) to a value which is less than a minimum value known as the holding current. It then reverts to the blocking state.

Since the forward voltage drop of a thyristor is higher and the maximum allowable junction temperature is lower than for a diode, a thyristor has a lower current rating than a diode of the same physical size.



**Figure 3.3**  
*Two-Transistor Analog of a Thyristor*

### 3. Two-Transistor Analog for Explaining Thyristor Turn-On (Figure 3.3)

The regenerative turn-on behavior of a thyristor may be explained qualitatively by splitting the four internal layers into two equivalent transistors as shown above. With positive bias between the anode and cathode, the anode junction acts as the emitter

of a PNP transistor and the cathode acts as the emitter of an NPN transistor. The collector of each transistor feeds current to the base of the other transistor.

The analysis of turn-on follows from the gain equation for each transistor, which states that:

$$I_C = \alpha I_E + I_{CBO}$$

where  $I_C$  and  $I_E$  are the collector and emitter currents,  $I_{CBO}$  is the collector-to-base leakage current, and  $\alpha$  is common-base current gain. Applying this relationship to each transistor in terms of the currents labeled in the Figure:

$$\text{PNP: } I_A - I_1 = \alpha_p I_A + I_{COBP}$$

$$\text{NPN: } I_1 = \alpha_N (I_A + I_G) + I_{CBON}$$

Substituting the second equation into the first to eliminate  $I_1$ , and then solving for anode current  $I_A$  gives:

$$I_A = \frac{\alpha_N I_G + I_{CBON} + I_{COBP}}{1 - (\alpha_N + \alpha_p)}$$

To interpret this equation, it should first be remembered that  $\alpha$  is much less than one for very small collector currents, but increases as the current increases. In forward blocking state with zero gate current  $I_G$ , the leakage currents are so small that  $\alpha_N$  and  $\alpha_p$  are both small. Therefore their sum is less than one, the denominator in the above equation is positive, and the anode current is stable at a small value.

The stable forward blocking state can be destroyed in a number of ways. The normal way is to apply gate current, which increases the base current to the NPN transistor and thereby increases  $\alpha_N$ . The increased NPN collector current increases the PNP base current, thereby also increasing  $\alpha_p$ . As  $\alpha_N + \alpha_p$  approaches one, the denominator of the anode current equation approaches zero. The transistors are regeneratively driven into saturation, and the anode current rises to a value determined by the external load circuit. The load current maintains  $\alpha_N + \alpha_p > 1$ , and the thyristor is latched into its on state even if the gate current is removed.

Turn-on can also be initiated by other effects which increase the off-state current to the point where  $\alpha_N + \alpha_p > 1$ . If the forward voltage is increased above its maximum rating, avalanche multiplication increases the leakage current. This mode of turn-on will destroy some devices. Increasing the temperature increases the

leakage current, and above 125° C turn-on can occur at less than the rated blocking voltage. If the anode voltage is positive and increases rapidly, the  $C dv/dt$  current through the capacitance of the center PN junction can cause unexpected turn-on. In place of gate turn-on, some thyristors are designed to be fired by a pulse of light which generates electrons and holes in the depletion region of the blocking junction, thereby increasing the leakage current.

Once latched, the thyristor remains in its on-state until something occurs which causes  $\alpha_N + \alpha_P$  to decrease below unity. The anode current below which this occurs is known as the “holding current”.

Although the simple two-transistor analog is quite useful explaining many aspects of turn-on, others require that the thyristor be viewed as a two-dimensional array of filamentary devices. The external gate lead does not connect directly to each of these devices. Gate current must pass through the lateral resistance of the gate layer, so the gate potential changes as the distance from the gate contact increases.

One result of this lateral gate resistance is that the gate current does not turn the entire thyristor on simultaneously. Only the area immediately surrounding the gate contact conducts initially, and the size of this initial area depends on the amplitude and rise time of the gate pulse.

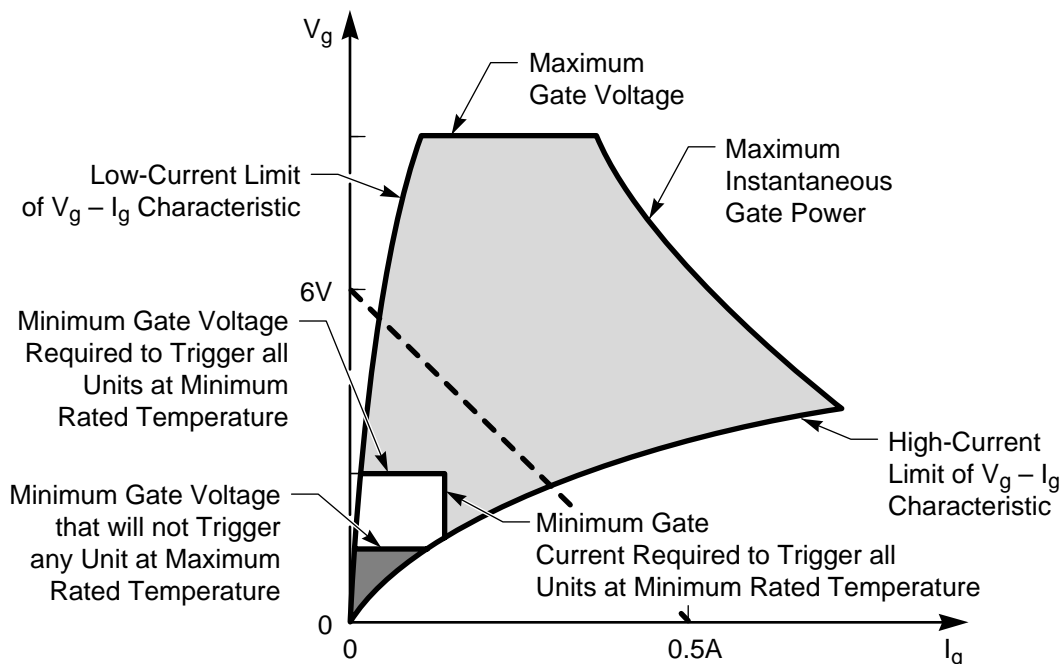
Another result is that the gate of an ordinary thyristor cannot cause it to turn-off. In the two-transistor analog, shorting the gate to the cathode or reverse biasing it would decrease the current, thereby decreasing  $\alpha_N$  and  $\alpha_P$  and causing turn-off to occur. In an actual thyristor, the lateral gate resistance prevents an appreciable part of the load current from being withdrawn through the gate lead, and the  $\alpha$ 's are not significantly affected.

#### 4. Gate Characteristics (Figure 3.4)

The V-I characteristic of the gate is similar to a diode, but varies considerably between units. The circuit which supplies firing signals to the gate must be designed:

- (1) to accommodate these variations,
- (2) not to exceed the maximum voltage and power capabilities of the gate,
- (3) to assure that triggering does not occur from spurious signals or noise, and
- (4) to assure that triggering does occur when desired.

These requirements are all summarized in a design chart which is usually provided by the device manufacturer to characterize the gates of a specified thyristor type.



**Figure 3.4**  
*Design Chart for the Gate Triggering Circuit of a Thyristor*

All possible safe operating points for the gate are bounded by the low and high current limits for the V-I characteristic, the maximum gate voltage, and the hyperbola representing maximum gate power. Within these boundaries there are three regions of importance.

The first region lies near the origin (shown shaded) and is defined by the maximum gate voltage which will *not* trigger any device. This value is obtained at the maximum rated junction temperature (usually 125° C). The gate must be operated in this region whenever forward bias is applied across the thyristor and triggering is *not* wanted. In other words, this region sets a limit on the maximum spurious signals that can be tolerated in the gate firing circuit.

The second region is further defined by the minimum values of gate voltage and current required to trigger *all* devices at the minimum rated junction temperature. This region contains the *actual* minimum firing points of all devices. In a sense, it is a forbidden region for the firing circuit because a signal in this region cannot be depended upon either to always fire all devices or to never fire any devices.

The third region is the largest and shows the limits on the gate signal for reliable firing. Ordinarily a firing signal in the lower-left part of this region is adequate. For applications where fast turn-on is required, a “hard” firing signal in the upper right part of the region may be needed.

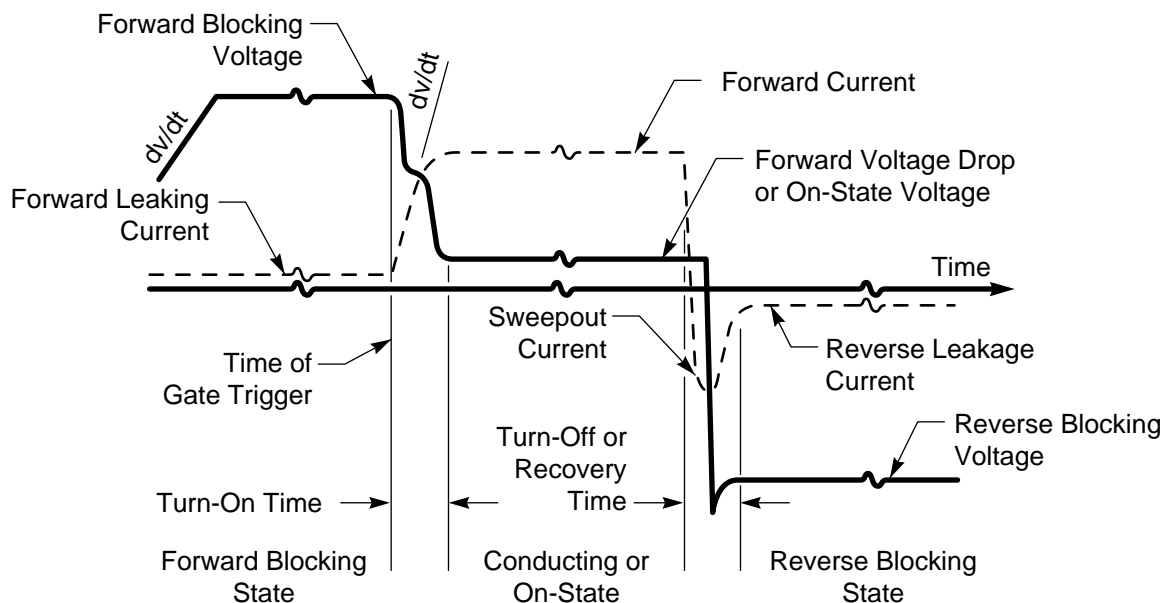
A typical firing circuit for a 100A thyristor should deliver about 6V (no load) with a source impedance of 12 ohms.

## 5. Transient V-I Characteristic of a Thyristor

(Figure 3.5)

The transient characteristics of a thyristor are similar to those of a diode in that turn-on and turn-off are not instantaneous, and a pulse of reverse current flows when the device is reverse biased immediately after conducting forward current. However, the fact that a thyristor has a forward blocking state not possessed by a diode causes several additional parameters to be significant.

First consider the turn-on transient. Since a diode turns on automatically as soon as it becomes forward biased, its entire area turns on simultaneously. On the other hand, the turn-on of a thyristor begins at a very small area adjacent to the gate electrode, and conduction must then propagate laterally from this initial area. The propagation velocity is approximately 0.1mm/μ sec. Therefore a device 2 cm. in diameter with the gate electrode in the center will require some 100μ sec. before the full area is conducting, although the forward voltage will approach its steady-state value much sooner than this. Because the area which is conducting is so small during the early part of this transient, the current



**Figure 3.5**  
*Transient V-I Characteristics of a Thyristor*

density can become extremely high if the rate of increase of current is not limited. Excessive current density causes a hot spot which can destroy the device. Consequently, high power thyristors have a maximum  $di/dt$  rating at turn-on.

Another parameter of great importance is  $dv/dt$ . As we have seen, the center junction of a thyristor is reverse biased when the device is in its forward blocking state, thereby limiting the steady-state current to a small leakage value. However, this junction is shunted by a junction capacitance which passes appreciable current into the gate layer if the device is subjected to a rapidly changing voltage while it is in its blocking state. If the device is forward biased and  $dv/dt$  is positive, this current can turn the device on when it is supposed to remain in its blocking state. Therefore a thyristor also has a maximum  $dv/dt$  rating, and the circuit may fail to operate properly if this rating is exceeded.

If a thyristor is naturally commutated, the primary factors in the turn-off transient are the peak value and duration of the reverse sweepout current, as in the case of a diode. Naturally commutated thyristors are normally reverse biased for a time which is considerably greater than the recovery time. For some types of forced commutation, the thyristor is reverse biased for a specified time, and must then block forward voltage. Under these circumstances, other factors which will be discussed under forced commutation must be taken into account.

The literature on the physics, characterization, rating, and application of thyristors is voluminous. The most comprehensive reference is:

F. E. Gentry, et al., *Semiconductor Controlled Rectifiers: Principles and Applications of PNP Devices*, Prentice-Hall; 1964.

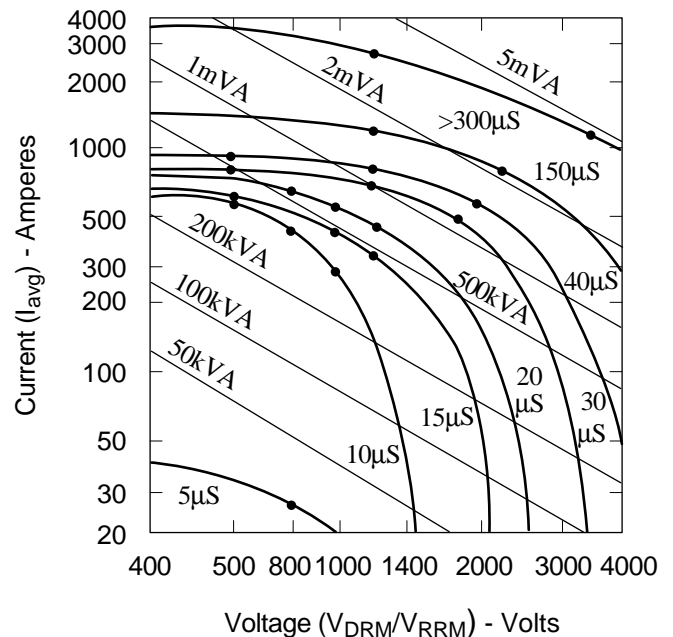
Manufacturers' handbooks also contain much useful information:

*General Electric SCR Manual* (Fifth Edition), General Electric Semiconductor Products Dept., Syracuse, N.Y.; 1972.

*Westinghouse SCR Designers Handbook* (Second Edition), Westinghouse Semiconductor Div., Youngwood, Pa.; 1970.

The history of thyristor development up to 1965 is interestingly summarized in:

J. D. Harnden, Jr., "Present-Day Solid-State Power Switches," *IEEE Spectrum*, vol. 2, no. 9, pp. 107-112; 1965.

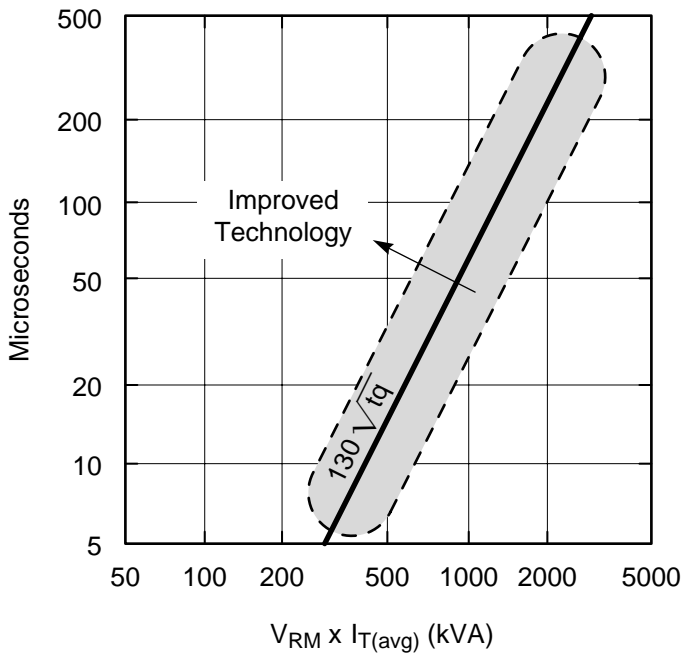


**Figure 3.6**  
*Average Current Ratings vs. Blocking Voltage Ratings for Commercial Thyristors. (Fastest Turn-Off Time, in  $\mu s$  Also Shown for Each Curve.)*

## 6. Thyristor Ratings – Design Tradeoffs and Commercial Availability (Figures 3.6 & 3.7)

The design of commercial thyristors involves tradeoffs between power controlling capability and speed. In an optimum device, both can be increased simultaneously only by increasing the silicon area. The development of larger and larger devices accounts for much of the progress that has been made in thyristors, but the largest area that is feasible at any specified time is limited by the technology of growing silicon crystals of adequate quality, by the differential expansion between silicon and package materials, and by the heat density which can be withdrawn through the package. Thyristors having a silicon diameter of 50mm are now readily available, and 75-100mm devices are being developed.

Although the subject of thyristor design tradeoffs and their interactions with circuit design tradeoffs is quite complex, some simple relationships between the voltage, current, and turn-off time ratings can be illustrated. A survey of high-power commercially available thyristors is summarized in Figure 3.6 and Figure 3.7. The interaction between the ratings of state-of-the-art thyristors is plotted in Figure 3.6  $I_{Tave}$  vs.  $V_{RM}$ , with  $t_q$  shown as a running parameter.



**Figure 3.7**  
*Turn-Off Time Ratings Vs. Rated Power Capability for Commercial Thyristors.*

Figure 3.7 shows  $t_q$  plotted vs. the product of  $V_{RM}$  and  $I_{Tave}$ . Most devices cluster about the line whose equation is:

$$V_{RM} I_{Tave} \text{ (kVA)} = 130 \sqrt{t_q \text{ (mS)}}$$

Thus we have deduced an empirical relationship between the power controlling capability and the speed of commercially available thyristors.

#### Converters Using Type 2 Switches

- Utilize an AC Source for Natural Commutation
- Can Perform
  - Controlled Rectification
  - Synchronous Inversion
  - Frequency Changing
  - AC Switching and Regulation

#### 7. Circuit Functions Performed by Type 2 Switches

Type 2 switches allow forward current to be turned on at arbitrary times but do not permit arbitrary

turn-off or commutation. Consequently, almost all circuits using Type 2 switches must be connected to an AC source which periodically reverses polarity and causes commutation to occur “naturally”.

The functions which these circuits can perform include:

**CONTROLLED RECTIFICATION** – the controllable conversion of AC power to DC power.

**SYNCHRONOUS INVERSION** – supplying power from a DC source into an AC line.

**FREQUENCY CHANGING** – Conversion of AC power at one frequency into AC power of a somewhat lower frequency.

**AC SWITCHING AND REGULATION** – on/off switching and “continuous” regulation of the effective value of AC power.

The basic circuits for all of these functions are the same, since all are based on the switching matrix. The circuits are distinguished primarily by which switches are omitted from the general matrix and by the timing of the switching control signals, and secondarily by the presence of freewheeling diodes, various interconnections of transformer windings, ect.

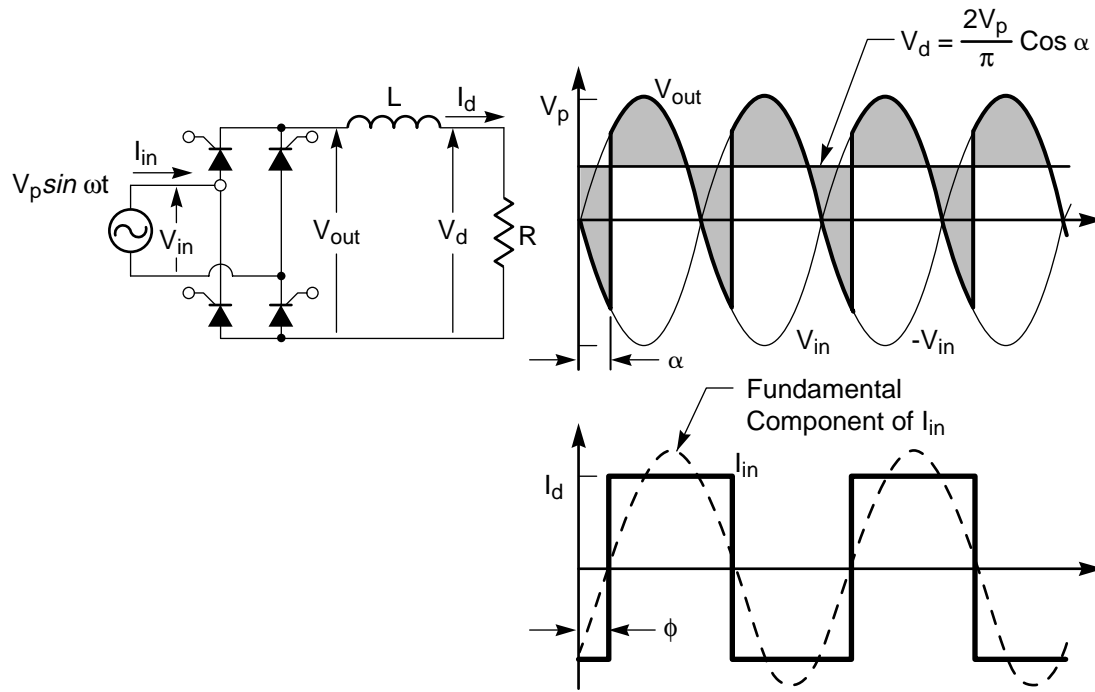
The circuits for performing controlled rectification and/or synchronous inversion are called AC/DC converters. These circuits have an obvious relationship to diode rectifier circuits and will be discussed first.

#### 8. Two-Pulse AC/DC Converter Operating as a Controlled Rectifier (Figure 3.9)

Consider what can be done if all of the diodes in a two-pulse bridge (or center-tap) rectifier are replaced by thyristors. If each thyristor receives a firing pulse at the instant when its anode becomes positive with respect to its cathode, no change occurs and the circuit operates exactly as it did as a diode rectifier. But if these firing pulses are uniformly delayed by an angle  $\alpha$ , somewhat different results occur. This change is easily implemented by building a controllable phase delay into the switching control signal generator.

Remember that all four simplifying assumptions are imposed on the analysis of this circuit. In particular, the DC current is assumed to be constant over each





**Figure 3.9**  
Two-Pulse AC/DC Converter  
(Waveforms shown for  $\alpha=45^\circ$ )

cycle.\* Therefore a continuous current path must exist at all times between AC and the DC terminals. If firing of one set of thyristors is delayed, commutation is also delayed and the other set must continue conduction in the meantime. If the AC source reverses polarity, the forward bias on the thyristors to maintain this conduction will be supplied by the kickback voltage of the filter inductance.

As a typical example, let  $\alpha = 45^\circ$ . The voltage and current waveforms are shown in the figure for this case. The ripple voltage appearing across the filter inductance is shaded, and as before its average value over a full cycle must be zero. Therefore the average load voltage is:

$$V_d(45^\circ) = \frac{\omega}{\pi} \int_{\frac{\pi}{4\omega}}^{\frac{5\pi}{4\omega}} V_p \sin \omega t dt = \frac{V_p}{\pi} [-\cos \omega t]_{\frac{\pi}{4\omega}}^{\frac{5\pi}{4\omega}} = \frac{\sqrt{2}V_p}{\pi}$$

It is seen that the phase delay has reduced the average load voltage from the corresponding value for a diode rectifier. In general,  $V_d$  may be set to any intermediate value by proper choice of  $\alpha$ , according to the relationship:

$$V_d(\alpha) = \frac{2V_p}{\pi} \cos \alpha$$

Note also that the AC source current waveform remains a square wave of amplitude  $I_d$ , but it is now displaced from the voltage sine wave by an angle  $\phi = \alpha$ . Therefore the converter has an increasingly lagging power factor at the AC terminals as the output DC voltage is reduced.

Since the DC current is constant, the instantaneous power dissipated in the load resistor is the same throughout each half cycle. However, the power interchange between the other parts of the circuit changes continually. When  $V_{out}$  exceeds  $V_d$ , the voltage across the inductance is positive, and both the inductance and the load resistor are absorbing power from the AC source. When  $V_{out}$  drops below  $V_d$ , the inductance starts to supply part of the power to the resistor. Then when the source voltage reverses polarity, the inductance supplies instantaneous power to the load resistor *and* to the AC source. As with the one-pulse diode rectifier having an inductive load, the AC source receives instantaneous power during part of each cycle even though the net power flow over a cycle is from the AC source to the load resistance.

*\*Note that this assumption does not prevent  $I_d$  from changing as a function of  $\alpha$ .*

## 9. Two-Pulse AC/DC Converter Operating as a Synchronous Inverter (Figure 3.10)

The previous discussion is valid as long as the phase delay is less than  $90^\circ$ . When  $\alpha = 90^\circ$ , the average DC voltage as given by the previous equation drops to zero. Therefore the current in the load resistor also drops to a very low value, ideally zero. The circuit is “coasting” with the entire output voltage appearing across the filter inductance and with no net power flow.

Actually this case is only of hypothetical interest with a passive load because as soon as the current drops below the holding current, the thyristors refuse to fire. But now assume that the passive load is replaced by an active load which is independently capable of maintaining a constant flow of current greater than the holding current. This current must be in the positive direction or the thyristors will not be able to carry it. For the sake of discussion, assume that the load is a DC current source of value  $I_d$ .

The AC source current is still a square wave of amplitude  $I_d$ , but now it is displaced  $90^\circ$  from the AC voltage. Therefore, both the average DC voltage and the net power are zero. The instantaneous power flow from the AC voltage source to the DC current source during one part of the cycle is just balanced by the instantaneous power flow in the opposite direction during another part of the cycle.

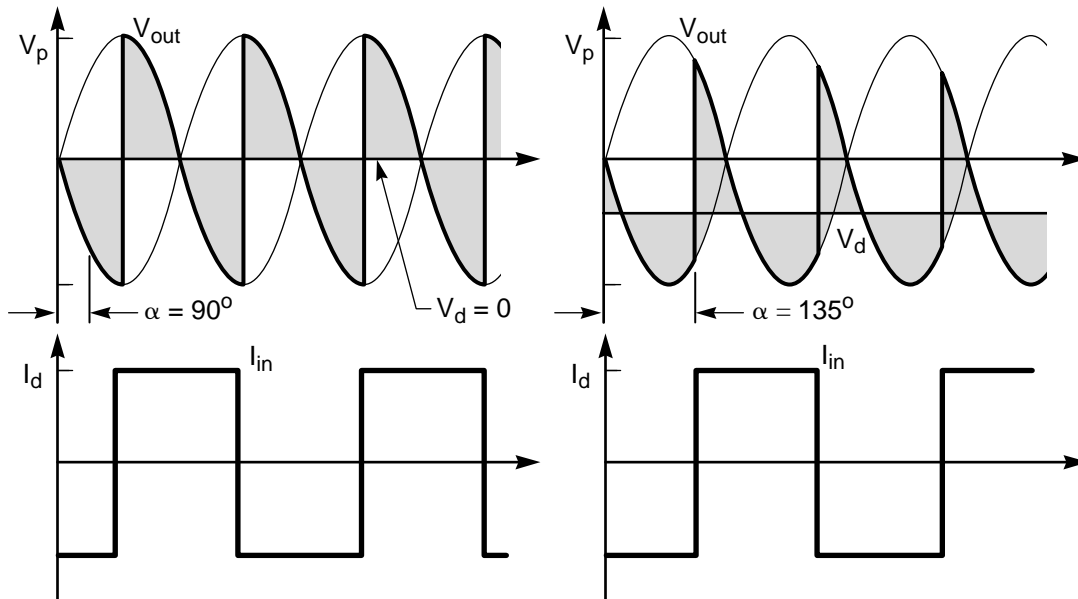
However, we have now established a set of conditions under which  $\alpha$  can be allowed to exceed  $90^\circ$ . If, for

example,  $\alpha = 135^\circ$ , the waveform are as shown. The average DC is negative:

$$V_d(135^\circ) = \frac{2V_p}{\pi} \cos 135^\circ = -\frac{\sqrt{2}V_p}{\pi}$$

and net power flow is from the DC current source into the AC voltage source. The AC/DC converter is performing synchronous inversion. In other words, since the average DC voltage has reserved polarity but the average DC current has not, operation has entered Quadrant IV. The net power flow in this direction may be controlled by changing  $\alpha$  just as it was in the other direction when  $\alpha$  was less than  $90^\circ$ . One difference, however, is that there is a practical upper limit on  $\alpha$  at about  $165^\circ$ . If this limit is exceeded, the thyristor which is commutating off does not have sufficient time to regain its blocking ability before it is subjected to forward voltage. Therefore it continues to conduct and a “commutation failure” occurs.

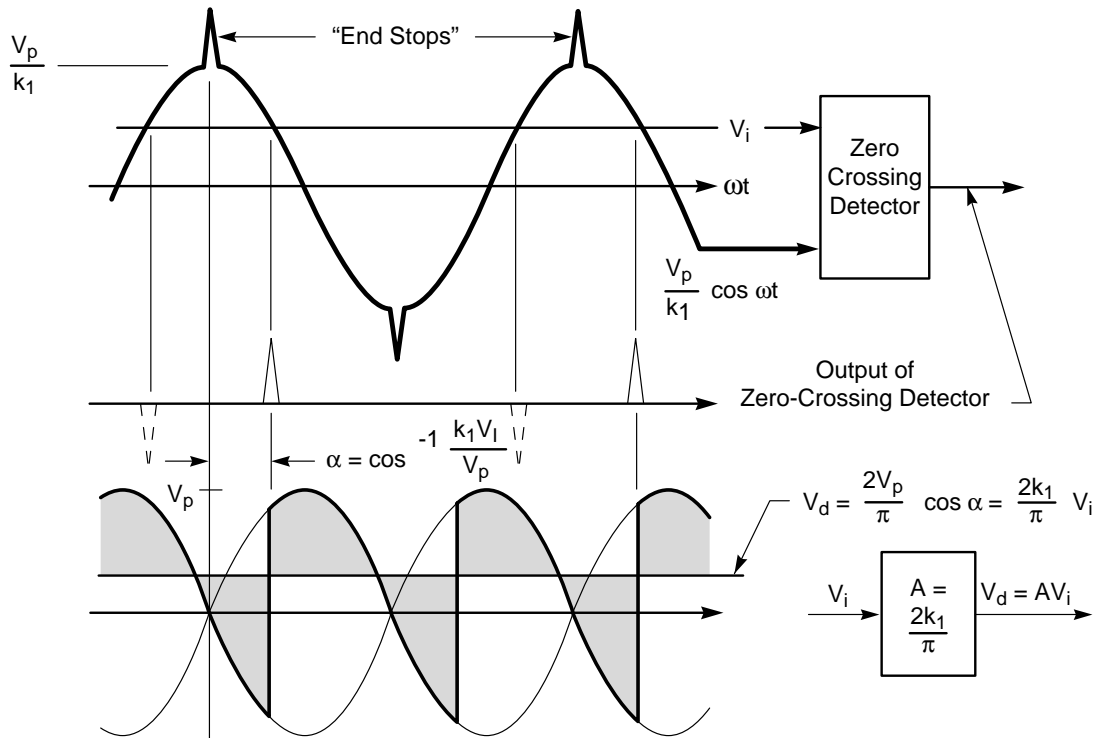
The operation of the AC/DC converter which we have been discussing depends on the ability of a Type 2 switch to block forward voltage. As a result, the converter permits the average DC voltage to be controlled and the net power flow to be reversed, neither of which is possible in a rectifier using only Type 1 switches. These converters, in which all of the Type 1 switches have been replaced by Type 2 switches, are known as 2-quadrant or fully-controlled AC/DC converters.



**Figure 3.10**

*Waveforms for a Two-Pulse AC/DC Converter for  $\alpha = 90^\circ$  and  $135^\circ$*





**Figure 3.11**

*Two-Pulse AC/DC Converter Using Cosine-Crossing Method to Achieve Linear Amplification*

An application for this type of converter which is destined to become very important in the future is high-voltage DC (HVDC) transmission. A converter at each end of a DC transmission line links it to two AC systems. The AC systems need not be synchronized (in fact, they can operate at different frequencies), and power can be caused to flow in either direction.

Much literature exists on HVDC converters and system considerations. Introductory articles include:

- F. D. Kaiser, "Solid-State HVDC Transmission Technology," IEEE Spectrum, vol. 3, no. 11, pp. 48-51; 1966.
- J. Kauferle, "Using DC Links to Enhance AC System Performance," IEEE Spectrum, vol. 9, no. 6, pp. 31-37; 1972.

Textbooks devoted to the subject include:

- E. W. Kimbark, *Direct Current Transmission*, vol. I, Wiley-Interscience; 1971.
- B. J. Cory (Editor), *High Voltage Direct Current Convertors and Systems*, MacDonald (London); 1965.

## 10. AC/DC Converter as a Linear Amplifier (Figure 3.11)

We have seen that the output voltage of an AC/DC converter varies according to the phase delay angle. The general form of this relationship for a fully controlled AC/DC converter is:

$$V_d(\alpha) = \left( \frac{P}{\pi} \sin \frac{\pi}{p} \right) V_p \cos \alpha$$

where  $P$  is the pulse number and  $V_p$  is the peak of the unfiltered output voltage. Obviously  $V_d$  is a nonlinear function of  $\alpha$ . However, in many applications it is desirable for the output to be a linear function of an input control signal. One way of implementing this linear relationship in the firing control circuit is known as the "cosine crossing" method.

At the heart of this method is a circuit known as a zero crossing detector. This circuit compares two input signals and emits a pulse when they cross each other (that is, when they are instantaneously equal). One input to the zero crossing detector is the input control signal,  $V_i$ . The other input is an internally generated sinusoidal signal having the same frequency as the power frequency but displaced in phase so that it peaks at the  $\alpha = 0$  commutation

point. Assume that the peak value of this signal is  $V_p/k_1$ , where  $k_1$  is a constant. Then pulses will appear at the output of the zero crossing detector at:

$$\alpha = \cos^{-1} \frac{V_i}{V_p/k_1}$$

The pulses which occur when the sinusoidal signal crosses  $V_i$  in a negative-going direction can then be used to initiate one set of firing pulses.\* The output voltage from the AC/DC converter is then:

$$V_d = \left( \frac{P}{\pi} \sin \frac{\pi}{P} \right) V_p \cos \alpha = \left( \frac{P}{\pi} \sin \frac{\pi}{P} \right) V_p \frac{V_i}{V_p/k_1} = A V_i$$

where:

$$A = \frac{k_1 P}{\pi} \sin \frac{\pi}{P}$$

Thus the AC/DC converter operates as a linear voltage amplifier with a gain of  $A$ .

In this analysis we have assumed that the input voltage,  $V_i$ , is a constant. The analysis remains valid for a time-varying input,  $V_i(t)$ , as long as its frequency spectrum lies well below the fundamental output ripple frequency,  $P\omega$ . Because of the nonlinearities inherent in the switching action, a more exact analysis which is valid for rapidly varying inputs requires advanced techniques such as describing functions or z-transforms. These techniques are described in the following references:

G. De and A. K. Mandal, "Incremental Describing Function Analysis of Subharmonic Oscillations in Control Systems with Thyristor Converters," IEEE Trans., vol. IECI-20, pp. 229-235; Nov. 1973.

G. De and A. K. Mandal, "Impulse Analysis of Subharmonic Oscillations in Control Systems with Thyristor Converters," Proc. IEE, vol. 120, pp. 1030-1037; Sept. 1973.

J. P. Sucena-Paiva, R. Hernandez and L. L. Freris, "Stability Study of Controlled Rectifiers Using a New Discrete Model," Proc. IEE, vol. 119, pp. 1285-1293; Sept. 1972.

P. A. Hazell and J. O. Flower, "Stability Properties of Certain Thyristor-Bridge Control Systems: Part 1-The Thyristor Bridge as a Discrete Control System; Part 2-The Inter-relationships of Discrete-and Continuous-Design Methods," Proc. IEE, vol. 117, pp. 1405-1420; July 1970.

\*When the sinusoidal signal crosses  $V_i$  in a positive-going direction, the output pulses have the opposite polarity and should be ignored. This scheme is repeated  $P$  times with the sinusoidal signal shifted by  $2\pi/P$  to generate the firing pulses required by other thyristors.

## 11. AC/DC Converter as a DC Motor Drive

(Figure 3.12)

AC/DC converters are widely used in closed-loop DC motor drives, supplying controllable power either to the armature or to the field. Usually the other time constants in the loop are much greater than the time constant of the AC/DC converter. Hence the linear analysis given in Section 10 is frequently valid.

The basic elements of a closed-loop speed control supplying the armature of a DC motor are shown above. The AC/DC converter is represented by a linear voltage amplifier with a gain  $A$ . The inside current-limiting loop serves to override the outside speed-control loop if necessary to prevent the current from exceeding a safe value. Otherwise this inside loop may be ignored.

The analysis of the dynamics of the speed-control loop can be performed with standard linear techniques, such as the Laplace transform. The electrical equation of the armature is:

$$A V_i = V_d = (L_a s + R_a) I_d = E_a$$

where  $s = d/dt$  is the complex frequency operator,  $L_a$  and  $R_a$  are the armature circuit inductance and resistance, and  $E_a$  is the back emf generated in the armature. Assuming a constant field flux,  $E_a$  is proportional to the angular speed of the armature,  $\omega_a$ :

$$E_a = k_a \omega_a \quad \text{or} \quad \omega_a = \frac{1}{k_a} E_a$$

where  $k_a$  is the same constant which enters the torque equation:

$$T = k_a I_d$$

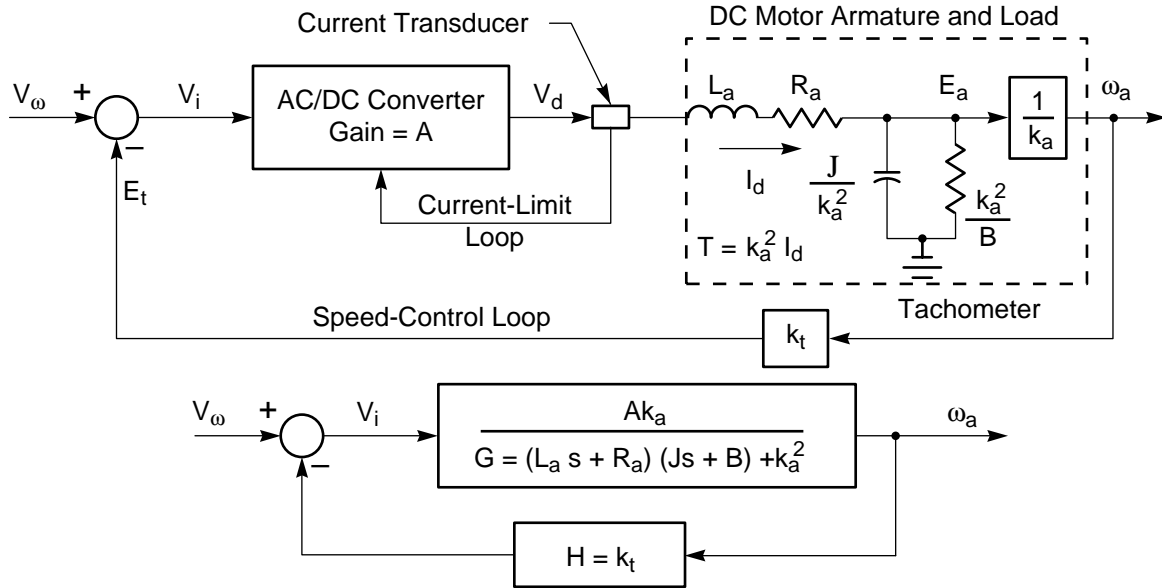
The electrical analog of the mechanical "circuit" of the armature may be derived as follows. If  $J$  the moment of inertia and  $B$  is a linear friction load (torque proportional to  $\omega_a$ ), then:

$$T = (Js + B) \omega_a = (Js + B) \frac{E_a}{k_a} = k_a I_d$$

or

$$\frac{I_d}{E_a} = \frac{J}{k_a^2} s + \frac{B}{k_a^2} = C_m s + \frac{1}{R_m}$$

where  $C_m$  is an electrical capacitance analogous to moment of inertia, and  $R_m$  is a parallel resistance analogous to the dissipative load.



**Figure 3.12**

*AC/DC Converter in a DC Motor Drive Using Type O or Proportional Speed Control in the Armature Circuit*

These equations can now be used to eliminate  $I_d$  and  $E_a$ , yielding the overall forward transfer function:

$$G = \frac{\omega_a}{V_i} = \frac{A k_a}{(L_a s + R_a)(Js + B) + K_a^2}$$

The back emf of the armature,  $E_a$ , could be used as an accurate indicator of armature speed except that it is physically inaccessible. The applied armature voltage,  $V_d$ , is sometimes used as an approximate indicator of speed where precise speed control is unnecessary. Better regulation is achieved by means of a tachometer which generates a feedback voltage proportional to armature speed. In this case, the transfer function of the feedback path is a constant:

$$H = \frac{E_t}{\omega_a} = k_t$$

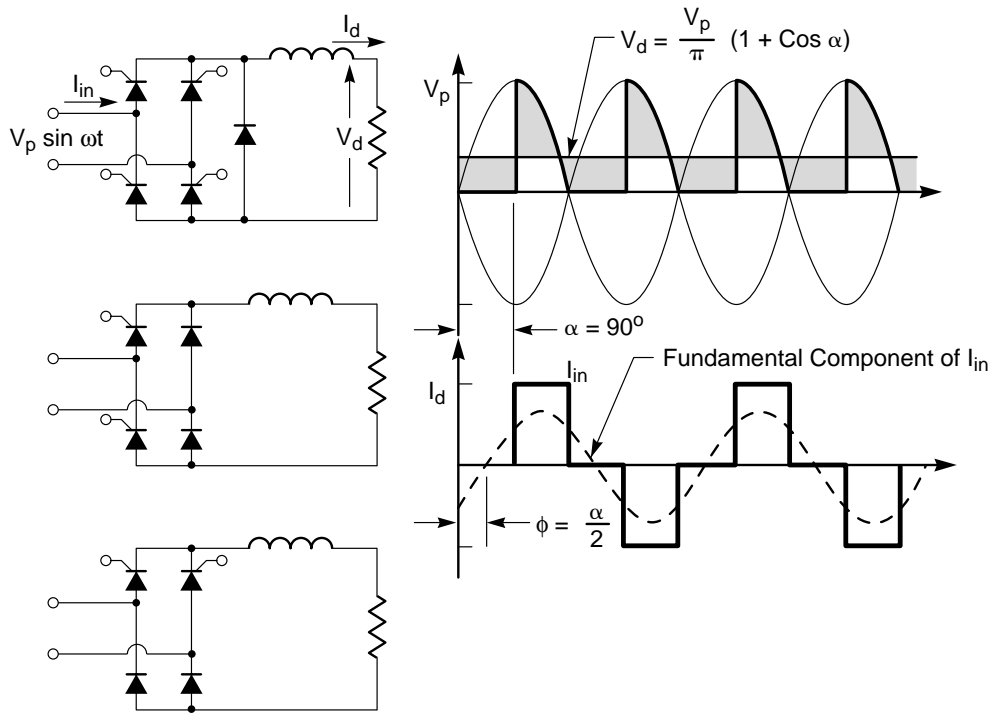
Examination of the loop gain,  $GH$ , reveals that this is a second-order Type 0 or proportional control system. Such systems are analyzed in any textbook on feedback control. The steady-state armature speed is directly proportional to the input variable,  $V_\omega$ , and the speed of response is related to the loop gain and the armature time constants,  $L_a/R_a$  and  $J/B$ .

Since the motor torque is proportional to  $I_d$ , and since  $I_d$  cannot be reversed in a two-quadrant AC/DC converter, this type of motor drive cannot regeneratively brake or reverse the direction of rotation unless a polarity reversing switch is incorporated into either the armature or the field circuit. We will soon see

that this limitation can be removed through the use of a four-quadrant AC/DC converter.

#### References:

1. Solid-State DC Motor Drives (Chapter 2), A. Kusko; MIT Press, Cambridge, Mass.; 1969.
2. Feedback Control System Analysis and Synthesis, J. J. D'Azzo and C. H. Houppis; McGraw-Hill, New York.
3. R. Cushman, "Powerful SCR's, Connected in Parallel, Control Industry's Biggest Machinery," Electronics, pp. 110-120; Oct. 4, 1965.
4. R. H. Neer and R. M. Brenza, "SCR's in Large Motor Drives," Control Engineering, vol. 20, pp. 42-45; Feb. 1973.
5. A.S.A. Farag, et al, "Studies on an SCR Controlled Variable Speed DC Shunt Motor," IEEE Trans., vol. PAS-93, pp. 785-792; May/June 1974.
6. K. H. Bezoid, et al., "Thyristor Converters for Traction DC Motor Drives," IEEE Trans., vol. IA-9, pp. 612-617; Sept./Oct. 1973.
7. C. E. Robinson, "Redesigning of DC Motors for Applications with Thyristor Power Supplies," IEEE Trans., vol. IGA-4, pp. 508-514; Sept./Oct. 1968.
8. N.A.O. Demerdash, "Effect of Complex Forms on Copper Losses in Large DC Motors," IEEE Industry & General Applications 5th Annual Meeting Record, pp. 77-81; Oct. 1970.
9. W. C. Carter, "Mechanical Factors Affecting Electrical Drive Performance," IEEE Trans., vol. IGA-5, pp. 282-290; May/June 1969.



**Figure 3.13**

*Two-Pulse Semiconverter Circuits and Waveforms for  $\alpha = 90^\circ$*

## 12. Two-Pulse Semiconverters (Figure 3.13)

In the previous two-pulse 2-quadrant converter, all of the diodes in the diode bridge were replaced by thyristors.

$$V_d = \frac{V_p}{\pi} (1 + \cos \alpha)$$

There are a number of similar two-pulse converters which have controllable DC output voltages, but are confined to 1-quadrant operation. That is, they can operate as controlled rectifiers, but since the output voltage cannot reverse polarity, they cannot operate as inverters. These circuits are known as semiconverters or half-controlled converters.

The most obvious way to change a full converter into a semiconverter is to connect a freewheeling diode across the DC output terminals of the bridge. During the interval from  $180^\circ$  to  $(180 + \alpha)^\circ$ , load current circulates through the freewheeling diode rather than through the bridge. But the same effect may be achieved more inexpensively by replacing only two of the four diodes in the original diode bridge by thyristors. Two possible configurations are shown. In each case, there is always a freewheeling path which prevents the output voltage from going negative.

Voltage and current waveforms are shown for  $\alpha = 90^\circ$ . Note that  $V_d$  is controllable from  $2V_p/\pi$  to zero, but the corresponding range of  $\alpha$  is from zero to  $180^\circ$  rather than zero to  $90^\circ$  as in the fully controlled converter. In general, the relationship for a two-pulse semiconverter is:

Note also that the displacement angle of the AC source current is now  $\alpha/2$ , giving an improved power factor.

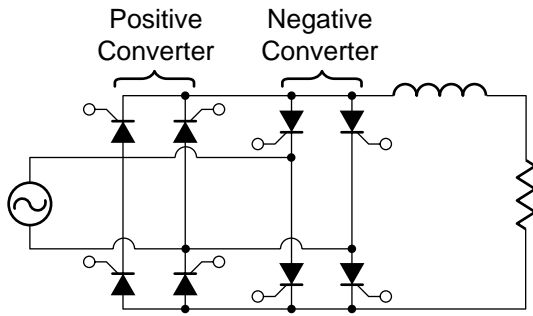
Semiconverters are widely used for DC motor drives where reversing is not necessary or can be provided by a reversing contactor. For these applications, semiconverters are preferred to fully controlled converters because:

- (a) they require fewer thyristors which are more expensive than diodes and
- (b) the input displacement factor is increased.

Depending on other factors, the harmonic content of the input current and output voltage may be reduced or increased.

### Reference:

D. L. Duff and A. Ludbrook, "Semiconverter Rectifiers Go High Power," IEEE Trans., vol. IGA-4, pp. 185-192; Mar./Apr. 1968.



**Figure 3.14**  
*Two-Pulse Dual Converter*

### 13. Two-Pulse Dual Converter (Figure 3.14)

Some types of DC motor drives require that the motor be accelerated, controlled, and regeneratively braked in both directions. This complete 4-quadrant operation is readily provided by a pair of fully controlled AC/DC converters connected in parallel so as to be capable of either controlled rectification or synchronous inversion with either polarity at the DC terminals. One of these converters is called the “positive converter” and the other the “negative converter,” and together they constitute a “dual converter”. Note that the bridge form of dual converter implements the complete general switching matrix.

Usually firing pulses are supplied to only one of the two converters at a time. If the motor load is to be soft-started in the forward direction, firing pulses must be supplied to the positive converter. Initially,  $\alpha$  should be nearly  $90^\circ$  to limit the in-rush current, but then decrease to nearly zero as the motor approaches full speed.

When the motor is to be reversed, its current must be reversed. The positive converter cannot conduct this reverse current, so the firing pulses are transferred to the negative converter. However, the motor will continue to supply positive back-emf until its direction of rotation actually reverses. In the meantime,  $\alpha$  can be greater than  $90^\circ$  so that the negative converter acts as a synchronous inverter. The motor acts as a generator during regenerative braking, and the energy stored in its inertia is returned to the AC line. Then as  $\alpha$  is reduced below  $90^\circ$ , the motor accelerates in the reverse direction.

Note that although the inductance in the DC circuit helps to smooth the current waveform, it also hinders rapid current reversal. Therefore its value

must be a compromise between the large value required for smoothing and a small value for minimum response time.

### 14. Waveform Derivation for a Three-Phase Bridge AC/DC Converter (Figures 3.15 & 3.16)

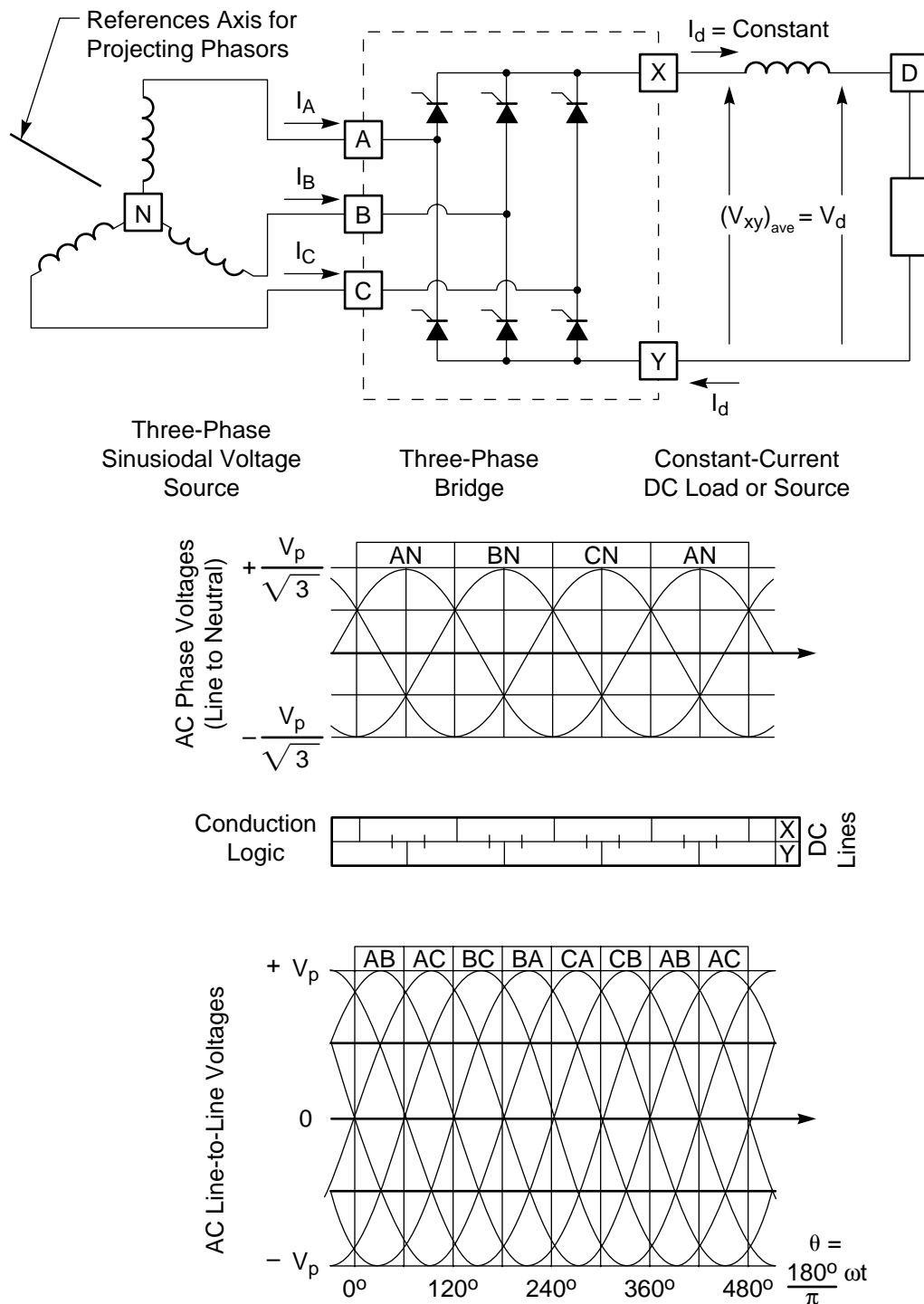
The Three-Phase bridge is the most widely used high-power AC/DC converter connection. Hence a thorough understanding of the operation of this circuit is a fundamental part of power electronics. The derivation of the voltage and current waveforms which occur throughout the bridge under a specified mode of operation quickly tests whether or not that understanding has been achieved. Intuitive attempts to derive these waveforms usually lead to utter confusion, but a methodical analysis can be performed very easily once a few simple procedures have been learned.

The following analysis assumes that the three-phase AC source voltage is balanced and sinusoidal, the DC current is ideally filtered so that  $I_d$  is constant, the switching devices are ideal (zero on-state voltage and off-state current, and instantaneous switching), and commutation overlap is negligible. If these assumptions are not valid, corrections can usually be made in an obvious way. We will also assume a fully controlled (two-quadrant), six-pulse converter, although the method is equally applicable to a variety of semiconductor, dual converter, and cycloconverter circuits.

The symmetries which are evident in the operation of a six-pulse bridge tempt us to neglect the complete labeling of the circuit and to skip apparently obvious steps. Yielding to this temptation is a sure route to frustration.

The bridge itself has a total of 5 nodes or terminals: 3 AC terminals labeled A, B, C, and 2 DC terminals labeled X, Y. In addition, the filtered DC node has been labeled D, and the AC neutral has been labeled N. (If the windings are delta-connected, a “virtual neutral” can be defined as a reference even though it doesn’t exist physically.) Using these 7 labels, we can precisely specify any voltage that can exist anywhere in the converter. For instance,  $V_{AN}$  is the voltage drop from A to N, and is positive when A is positive with respect to N.

Furthermore, the AC phase windings have been drawn so as to represent the phasors of the phase

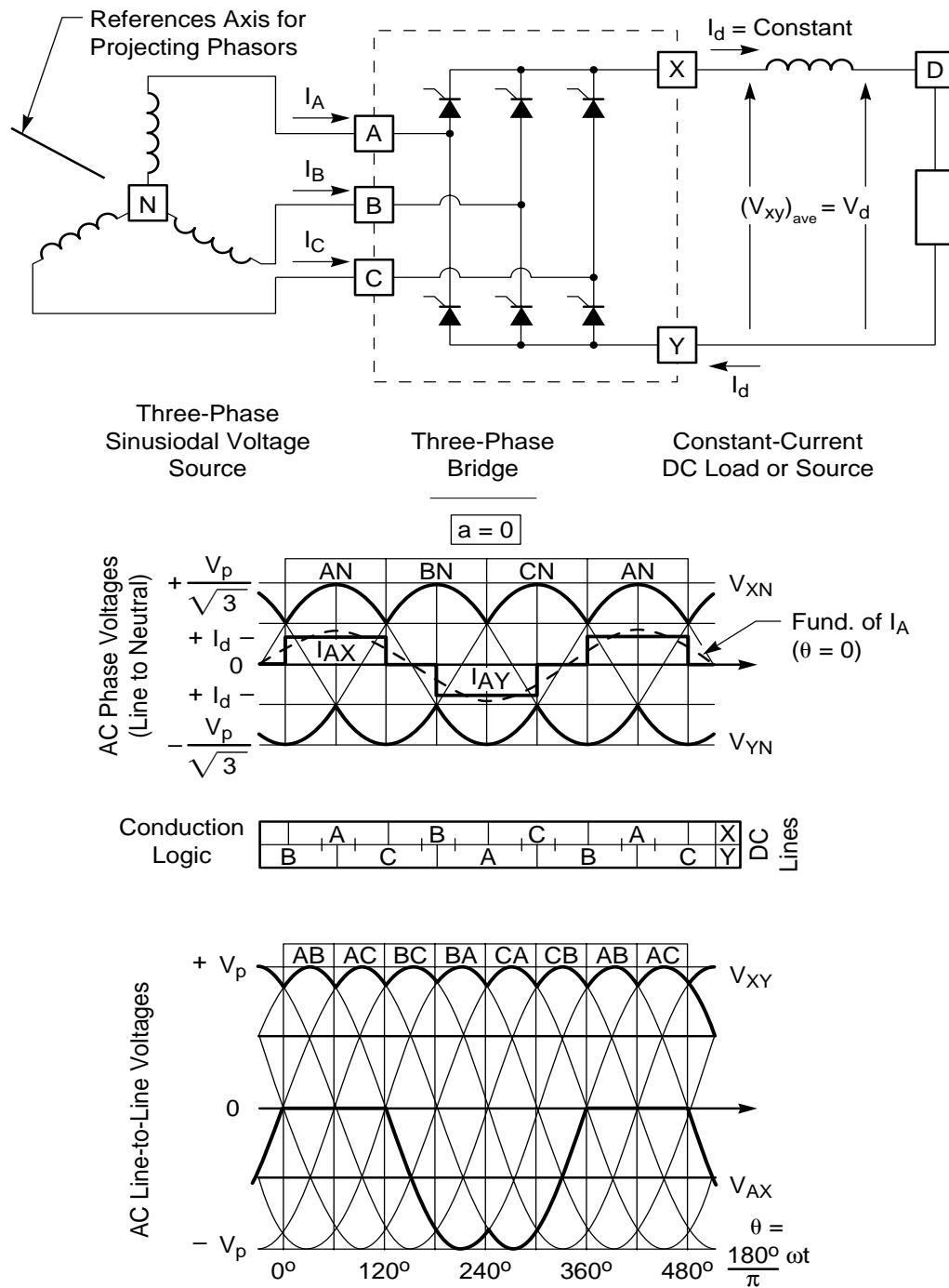


**Figure 3.15**  
Guide for Drawing 3-Phase AC/DC Bridge Waveforms

voltages. A reference axis is indicated which is consistent with the electrical angle  $\theta = (180^\circ/\pi)\omega t$  defined at the bottom of the figure. As time passes from  $t = 0$ , the phase windings may be pictured as rotating counterclockwise with angular velocity  $\omega$ . The instantaneous phase voltage is proportional to the projection of the length of the winding onto the

reference axis. Alternately, the windings can be held fixed if the reference axis is rotated clockwise with angular velocity  $\omega$ .

The two three-pulse halves of the bridge should always be analyzed separately before combining them to determine the overall six-pulse waveforms. It is



**Figure 3.16**  
Three-Phase AC/DC Bridge Waveform Example

obvious that each thyristor in each three-pulse group must conduct for  $120^\circ$  of each cycle under normal operating conditions. As the conduction state of each phase is determined as a function of time, this is recorded on the “conduction logic” diagram. The three-pulse voltage waveforms are then easily derived from the AC phase (or line-to-neutral) voltages, and finally the six-pulse waveforms from the AC line-to-line voltages. The figure can be used repeatedly as a

waveform sketching guide if the waveforms are actually drawn on tracing paper.

The process is most easily described in terms of an example (Figure 3.16). The firing delay angle of  $\alpha$  of each thyristor is measured from the point where a diode (substituted for the thyristor) would begin to conduct. Let us begin by determining the *waveforms* for  $\alpha = 0$ .

In the three-pulse group of thyristors connected to X, thyristor AX is connected to the most positive phase voltage over the interval  $0^\circ < \theta < 120^\circ$ . A diode in this position would conduct over this interval, and with  $\alpha = 0$  a thyristor will do the same. This fact is recorded on the conduction logic diagram by showing that X is tied to A over  $0^\circ < \theta < 120^\circ$ . Similarly it can be argued that X is tied to B by the conduction of thyristor BX over the interval  $120^\circ < \theta < 240^\circ$ , and X is tied to C over the interval  $240^\circ < \theta < 360^\circ$ . Thereafter the cycle repeats. The  $V_{XN}$  waveform should now be obvious.

In the three-pulse group connected to Y, phase A is most negative and Y is tied to A over the interval  $180^\circ < \theta < 300^\circ$ . Similarly Y is tied to B over  $300^\circ < \theta < 420^\circ$  (or  $-60^\circ < \theta < 60^\circ$ ), and to C over  $60^\circ < \theta < 180^\circ$ . In this way, the conduction logic diagram is completed and the  $V_{YN}$  waveform is derived.

The unfiltered  $V_{XY}$  waveform can now be derived from the relationship:

$$V_{XY} = V_{XN} - V_{YN}$$

since the  $V_{XN}$  and  $V_{YN}$  waveforms are both known. Alternately,  $V_{XY}$  can be derived from the conduction logic diagram by taking one interval of  $\theta$  at a time. Thus for  $0 < \theta < 60^\circ$ , X is tied to A, Y is tied to B, and  $V_{XY} = V_{AB}$ . For  $60^\circ < \theta < 120^\circ$ ,  $V_{XY} = V_{AC}$ ; ect. In each case,  $V_{XY}$  is equal to one of the AC line-to-line voltages, and the corresponding segment may be sketched.

The average of the  $V_{XY}$  waveform is equal to the filtered DC voltage  $V_{DY} = V_d$ . The instantaneous voltage across the filter inductor is the difference between these two voltages:

$$V_{XD} = V_{XY} - V_d = L \frac{dl_d}{dt}$$

The peak-to-peak ripple current in the DC circuit can be evaluated from the volt-second area imposed on the inductor:

$$\Delta I_d = \frac{1}{L} \int V_{XD} dt$$

where the limits on the integral are the points where  $V_{XY}$  crosses  $V_d$  (i.e. where  $V_{XD} = 0$ ).

The only remaining voltage waveforms are those which appear across the various thyristors. These, too, may be derived directly from the conduction logic diagram and the AC line-to-line voltages. The

voltage across thyristor AX, for instance, is simply  $V_{AX}$ . Thus:

$$0^\circ < \theta < 120^\circ \quad V_{AX} = 0 \quad (\text{Since this thyristor is conducting and the conduction logic diagram shows that X is tied to A})$$

$$120^\circ < \theta < 240^\circ \quad V_{AX} = V_{AB} \quad (\text{Since X is tied to B})$$

$$240^\circ < \theta < 360^\circ \quad V_{AX} = V_{AC} \quad (\text{Since X is tied to C})$$

Having completed the analysis of the voltage waveforms, we can turn our attention to the current waveforms. Under the assumptions of continuous, constant DC current  $I_d$  and no commutation overlap, at any instant the current in any part of the converter must be either  $+I_d$  or zero. The conduction logic diagram tells the interval during which each thyristor conducts  $I_d$ . The AC line currents are then obtained by adding the two thyristor currents in each phase:

$$I_A = I_{AB} + I_{AY} = I_{AX} - I_{YA}$$

From symmetry, the angular position of the fundamental component of AC line current can usually be determined and compared with the corresponding AC phase voltage to establish the displacement angle  $\phi$ .

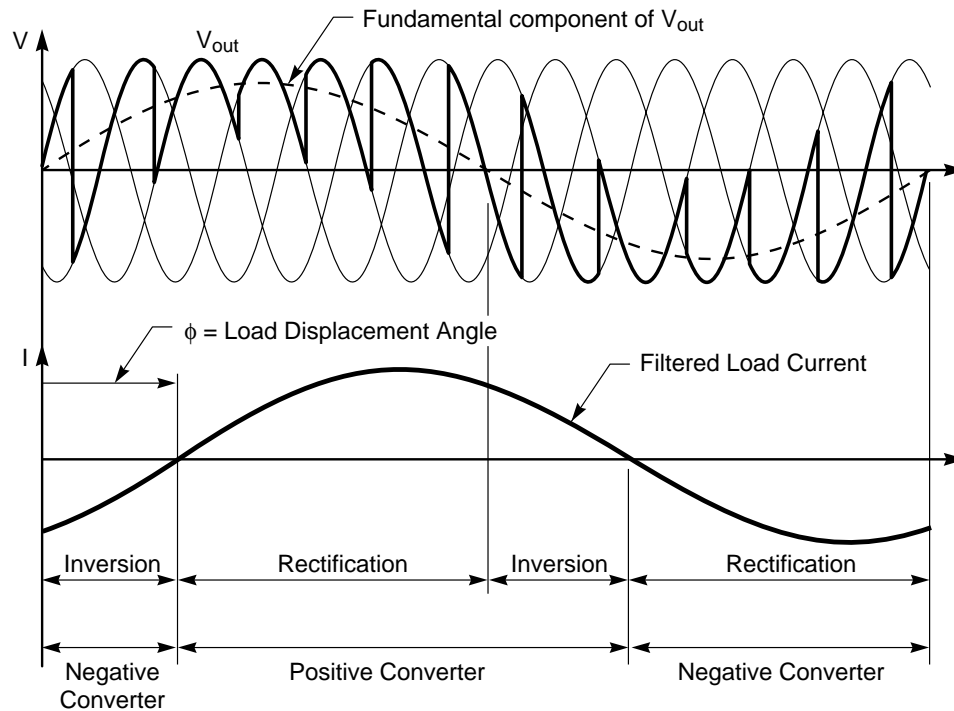
This complete analysis of the converter for  $\alpha = 0$  probably did not reveal any unexpected waveforms because of the relative ease with which a six-pulse diode rectifier bridge can be analyzed. However, the beauty of the analytical process that has been described is that it gives the waveforms for other values of  $\alpha$  with equal ease. The entire conduction logic is simply shifted to the right by angle  $\alpha$ . The corresponding changes in the voltage waveforms may surprise you, but these are left as an exercise.

The graphical approach to converter analysis has two major advantages:

- (1) it is easy to perform, and
- (2) it leads to a good physical understanding of what is happening at any time at any point in the circuit.

Once the graphical analysis has been completed, appropriate mathematical expressions can be derived, if necessary, for each segment of each waveform. However, an attempt to derive these expressions without first thinking through the details





**Figure 3.17**

*Voltage and Current Waveforms for a Three-Pulse Cycloconverter with a 5:1 Frequency Reduction Ratio*

of the physical behavior of the converter is guaranteed to produce false results.

Various aspects of this method of analysis have been adapted from a number of sources. The following references are recommended for further study:

"A Simplified Technique for Analyzing the three-Phase Bridge Rectifier Circuit,"  
A. Ludbrook and R. M. Murray, IEEE Trans. on Industry & General Applications, vol. IGA-1, pp. 182-187; May/June 1965.

*Solid-State DC Motor Drives* (Chapters 3 and 5), A. Kuskov; MIT Press, Cambridge, Mass.; 1969.

*Rectifier Circuits: Theory and Design*,  
J. Schaefer, John Wiley, New York; 1965.

*Principles of Inverter Circuits* (Section 3.4),  
B. D. Bedford and R. G. Hoft, John Wiley, New York; 1964.

*Thyristor Phase-Controlled Converters and Cycloconverters*, B. R. Pelly, John Wiley, New York; 1971.

*The Fundamental Theory of Arc Converters*,  
H. Rissik, Chapman & Hall, London; 1939. Facsimile reprints available from University Microfilms, Ann Arbor, Michigan.

"The Rectifier Calculus," W. M. Goodhue, AIEE Trans., vol. 59, pp. 687-691; 1940. (Discussion on pp. 1073-1076.)

For historical research on the early development of the power electronics field, an invaluable aid is the

"Bibliography on Electric Power Converters," AIEE Publication No. S-35, February 1950. Nearly 1300 publications between 1903 and 1947 are listed.

## 15. Naturally Commutated Cycloconverter (Figure 3.17)

We have examined the two-pulse dual converter and shown that it is capable of supplying a DC output voltage of either polarity, and can also support current of either polarity in the DC circuit. If the timing of the firing control signals is modified, this same circuit can operate as a one-step frequency changer. In this mode of operation it is known as a naturally commutated cycloconverter, and does not require an intermediate DC link such as that involved when a controlled rectifier and a synchronous inverter are used together as a frequency changer.

In essence, a cycloconverter synthesizes the desired low-frequency AC waveform as a succession of periodically varying "DC" values. Since these "DC" values are functions of the corresponding firing delays, this synthesis is achieved by periodically

modulating  $\alpha$  for each thyristor. But the derivation of the firing control signals is not as straightforward as it might sound from this elementary description.

Two facts should be emphasized:

- (1) It is the desired output *voltage* waveform which determines the proper value of  $\alpha$  each time a thyristor is fired, but
- (2) it is the *current* waveform which determines whether the positive or negative converter should be activated.

Of course the phase and amplitude relationships between the voltage and current waveforms are load-dependent. Therefore the firing control logic usually includes a feedback loop which maintains proper synchronization in spite of varying load conditions.

Specifically, the firing signals cause the cycloconverter to operate as follows: If the current is positive, the positive converter is activated. If the voltage is also positive, then  $\alpha < 90^\circ$  and the positive converter acts as a rectifier. But if the voltage is negative, then  $\alpha > 90^\circ$  and the positive converter acts as a synchronous inverter. Similarly, when the current is negative the negative converter rectifies when the voltage is negative and inverts when the voltage is positive.

Naturally commutated signal-phase cycloconverters are limited to output frequencies less than about one third of the source frequency. Because of the greater choice of source voltages from which to synthesize the desired output waveform, three-phase cycloconverters have less output ripple and operate at somewhat higher output frequencies, although still generally confined to frequencies below the source frequency.\*

The inherent constraints of natural commutation cause this type of cycloconverter to draw a lagging source current even for resistive load. The spurious frequencies present in the voltage and current waveforms are also usually not integer harmonics or subharmonics of source frequency.

The primary application for naturally commutated cycloconverters is for variable speed AC motor drives, especially at low speeds where the low output frequency which is obtainable helps to eliminate gearing. A cycloconverter can also be used in conjunction with a wound-rotor induction motor to vary the rotor frequency and thereby provide speed variation about the nominal speed of the motor.

The most comprehensive reference on cycloconverters is:

B.R. Pelly, *Thyristor Phase-Controlled Converters and Cycloconverters*, Wiley-Interscience; 1971.

Other interesting references include:

L. J. Lawson, "The Practical Cycloconverter," *IEEE Trans. on Industry & General Applications*, vol. IGA-4, pp. 141-144; 1968.

T. M. Hamblin and T. H. Barton, "Cycloconverter Control Circuits," *IEEE Trans. on Industry Applications*, vol. IA-8, pp. 443-453; 1972.

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\*A novel cycloconverter which operates at 3 times the source frequency is described by J. E. Jenkins in "A Frequency Tripling Cycloconverter," *IEEE Industry Applications 7th Annual Meeting Record*, pp. 81-83; Oct. 1972.

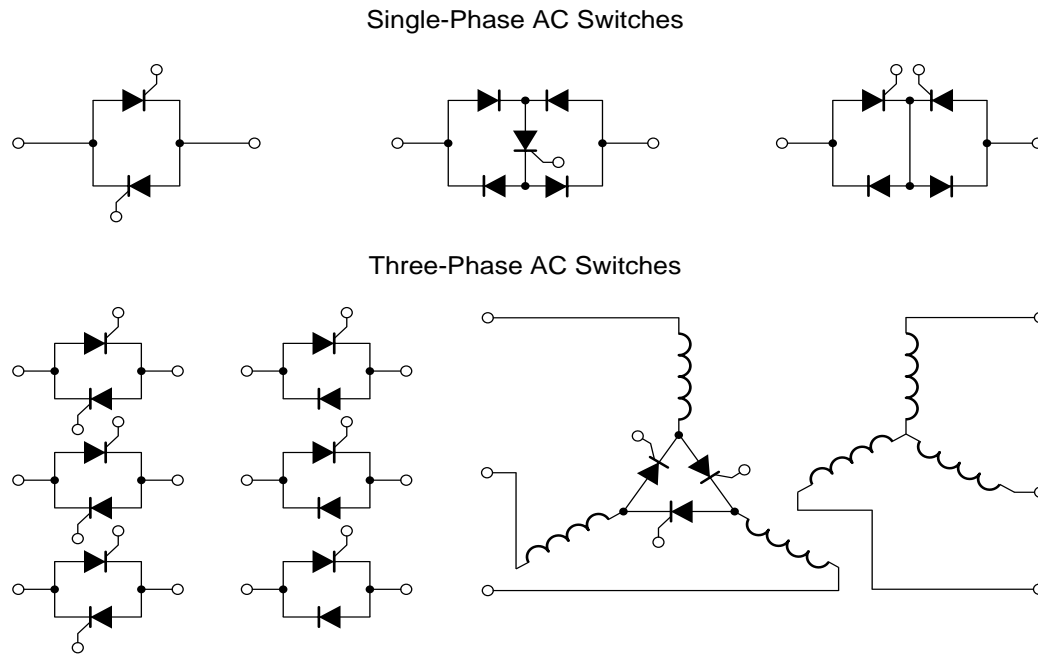
## 16. Bilateral Solid-State Switches (Figure 3.18)

The converter circuits which have been considered utilize unilateral solid-state switches. However, there are many AC applications where bilateral conduction is required. A bilateral or AC switch can be implemented in a variety of ways. The most obvious way is to use a pair of unilateral devices connected in parallel with opposite polarity as was done in the general switching matrix. In fact, this approach has already been used in the dual converter and cycloconverter circuit, as examination of the circuit will reveal. Each thyristor in the positive converter is paralleled by one of the opposite polarity in the negative converter.

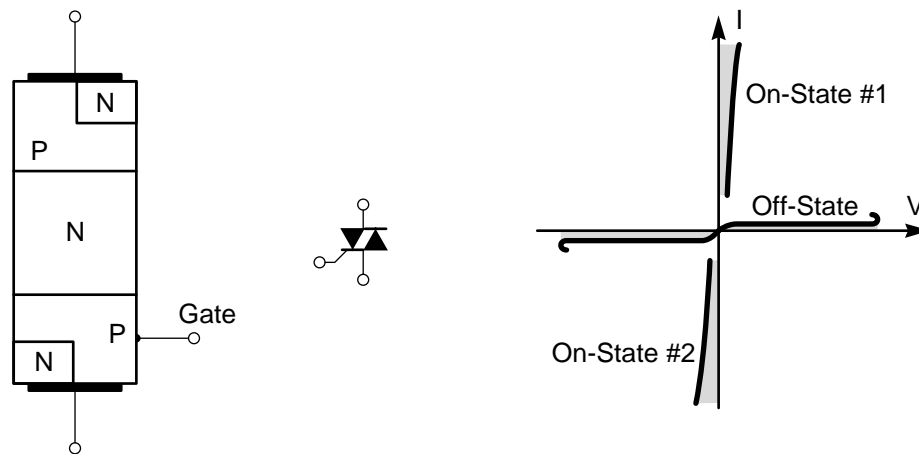
Several other ways of implementing single-phase and three-phase AC switches are also shown below. In some cases it is feasible to replace some of the thyristors by diodes which are less expensive, and do not require a firing circuit.

## 17. Triac (Figure 3.19)

The two antiparallel thyristors may also be integrated into a single device structure, commonly known as a triac. Triacs are widely used in AC switches for moderate power levels because they tend to cost less than a pair of thyristors, and also eliminate the need for the second firing circuit. However, in most applications, one half of the device must block forward current immediately after the other half has ceased conduction, a very difficult requirement. Because of the interaction of the two halves, it has



**Figure 3.18**  
*Bilateral Solid-State Switch Configurations*



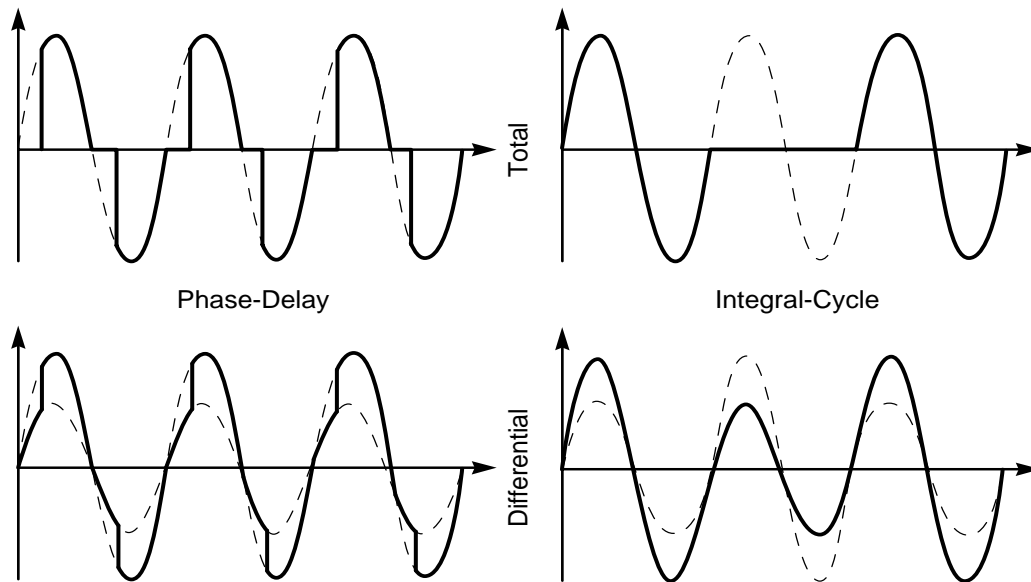
**Figure 3.19**  
*Triac Junction Geometry, Symbol, and V-I Characteristic*

not been possible to build triacs with voltage, current, or frequency ratings as high as those readily obtainable in thyristors. Therefore, high power AC switches must be implemented with thyristors.

**References on triacs include:**

General Electric SCR Manual (Fifth Edition), Section 7, General Electric Semiconductor Products Dept., Syracuse, N. Y.: 1972.

F. E. Gentry, et al, "Bidirectional Triode PNP Switches," Proc. IEEE, vol. 53, pp. 355-369; April 1965.



**Figure 3.20**  
*Four Modes of Operation for an AC Regulator*

### 18. AC Switches and Regulators (Figure 3.20)

The deceptive simplicity of AC switching and regulation makes it easy to underestimate its usefulness and overlook potentially valuable applications. In fact, solid-state AC switches offer many advantages over electromechanical switches. For example:

- a. The elimination of moving parts and arcing leads to quiet operation, longer life without wear, greater reliability, and safety in hazardous environments (such as explosive atmospheres).
- b. A solid-state switch is easy to actuate remotely by a logic-level signal, or it can be interfaced with other electronic circuits and transducers for sensing and control. Photo-isolation of the input, by means of a light-emitting diode and a photodetector, eliminates the possibility of surges in the power circuit finding their way back into sensitive control circuits.
- c. The speed of a solid-state switch is nearly instantaneous compared to the period of 60 Hz power.

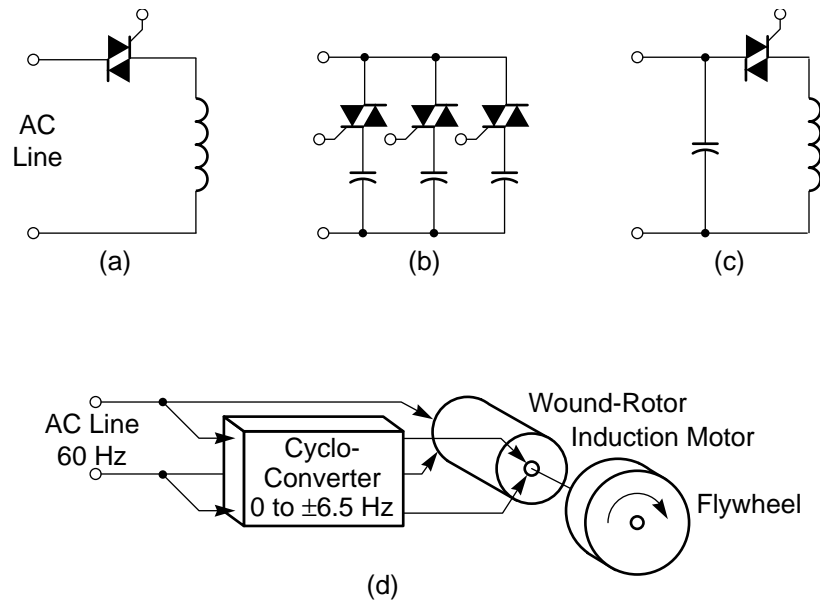
A number of manufacturers offer general-purpose “solid-state relay” modules which are rated for standard AC line voltages to 480V and currents to 200A.

Obviously these solid-state switches can be used for occasionally turning an AC load off or on. But

because of their speed and their capability for a nearly infinite number of switching operations, they have much greater flexibility to achieve control on a cycle-by-cycle basis that is impossible with an electromechanical switch. This control permits efficient regulation of the effective AC power to intermediate values as well as full off or full on.

Solid-state AC regulators can be designed to operate in various modes. If the initiation of conduction is phase delayed each half cycle, the regulation is equivalent to the phase-delay control of a rectifier. This mode is easy to implement and is widely used in lamp dimmers and speed controls for small tools and appliances. The fundamental power frequency remains 60 Hz so that flicker or pulsations are not a problem. However, the fact that turn-on does not occur at zero voltage causes significant high-frequency interference to be generated. Appropriate filters are therefore needed.

If the load time constant is somewhat longer than a cycle of the power frequency, an alternate mode of regulation which eliminates high-frequency interference can be used. In this mode, turn-on always occurs at zero voltage. Of course, with line commutation, turn-off always occurs at zero current. Regulation is achieved by simply omitting one or more complete cycles of conduction as appropriate, and is therefore known as integral-cycle control. (If the existence of an unbalanced DC component is not a problem, half-cycle control can be used.) This



**Figure 3.21**  
*Types of Varegulators*

mode of control is feasible, for example, with water heaters, room heaters, ovens, and the like.

Phase control and integral-cycle control as described above permit “total” control of the effective output voltage from full line voltage to zero. Both generate transients which can be troublesome in some applications. These transients can be minimized by “differential” control if a reduced range of control is adequate. The minimum required voltage is obtained from a tapped transformer winding, and switching occurs from this minimum value to full value rather than from zero. Intermediate values may be obtained either by phase delay or by integral-cycle switching.

The voltage waveforms resulting from these four modes of AC regulation are compared above for a resistive load. Inductive loads cause commutation to be delayed after the zero crossing of voltage, and the design of the control circuits must take this fact into account.

#### References:

W. Shepherd, “Steady-State Analysis of the Series Resistance-Inductance Circuit Controlled by Silicon Controlled Rectifiers,” *IEEE Trans.*, vol. IGA-1, pp.259-265; July/Aug. 1965.

R. Thompson, “A Thyristor Alternating Voltage Regulator,” *IEEE Trans.*, vol. IGA-4, pp. 162-166; Mar./Apr. 1968.

#### 19. Varegulators (Figure 3.21)

The preceding power circuits have been “two-port” circuits which accept real power from a source and convert or control it as they transmit it to a load. Recently another “one-port” class of power circuits is beginning to appear. These circuits incorporate an internal “load” which acts as a reservoir to controllably store and return energy. If the input power is averaged over an appropriate interval, there is ideally no net flow of real power or watts but only an interchange of reactive power or vars. Hence we will call these circuits “varegulators”.

An obvious type of varegulator is shown in Figure 3.21 (a). An AC switch controls the portion of each voltage half-cycle which is applied to an inductor. If the firing of the AC switch is phase delayed until after the peak of the source voltage, the conduction interval will be less than  $180^\circ$ , and the effective current will be a function of the delay angle. Hence, the reactive power drawn from the source can be continuously controlled by varying the delay angle. Unfortunately, this circuit can supply only inductive or “lagging” vars, while most practical applications require capacitive or “leading” vars to compensate for a lagging power factor elsewhere in the system.

Circuit (b) illustrates the use of a capacitive load to generate leading vars. However, if the firing angle is arbitrarily delayed to control the vars, the current through the AC switch occurs in huge pulses as the

capacitor voltage equalizes with the source voltage. These pulses destroy the switching devices. Hence firing must be synchronized to occur when the instantaneous source voltage is equal to the capacitor voltage. Var control is achieved by splitting the capacitor into sections and controlling each section by an independent AC switch. Only enough sections are fired to obtain the desired vars. This circuit has the disadvantages of requiring numerous AC switches and their associated firing circuits while achieving only stepwise rather than continuous var control.

Circuit (c) combines circuits (a) and (b). An unswitched capacitor provides the maximum leading vars that will be required, while phase delay of the AC switch in series with the inductor subtracts a continuously controllable number of lagging vars.

These static varegulators are beginning to replace rotary synchronous condensers for preventing lighting flicker on power systems which supply highly pulsating loads, such as industrial arc furnaces.

To appreciate the versatility of the varegulator concept, consider circuit (d) which was developed to meet the needs of a rather unique application. Huge electrically powered draglines were to be operated at a remote mining site supplied only by a relatively small-capacity transmission line. Normal operation of the draglines involves large cyclic pulsations during which the peak power can reach several times the average power level for up to 30 sec. During the remainder of the cycle, comparable peaks of power can be regenerated back into the power system. Obviously the system voltage will swing widely and cannot be stabilized from the generating station because of the impedance of the transmission line. Not only the draglines but all other loads on the power system would be severely affected.

The problem has been solved by installing a varegulator near the draglines. Because of the large amount of energy storage needed to smooth the power pulsations (some 600 MW-seconds), mechanical storage in a large flywheel was chosen over electrical storage. Rapid control of the coupling between the flywheel and the power system was achieved by means of a wound-rotor induction machine. The rotor is fed from a cycloconverter whose frequency and phasing relative to the system voltage determine the magnitude and direction of power flow.

Much of the peak power to the draglines is drawn from the flywheel, and regenerated power is returned

to the flywheel. The power system must supply only nominal variations about the average power level.

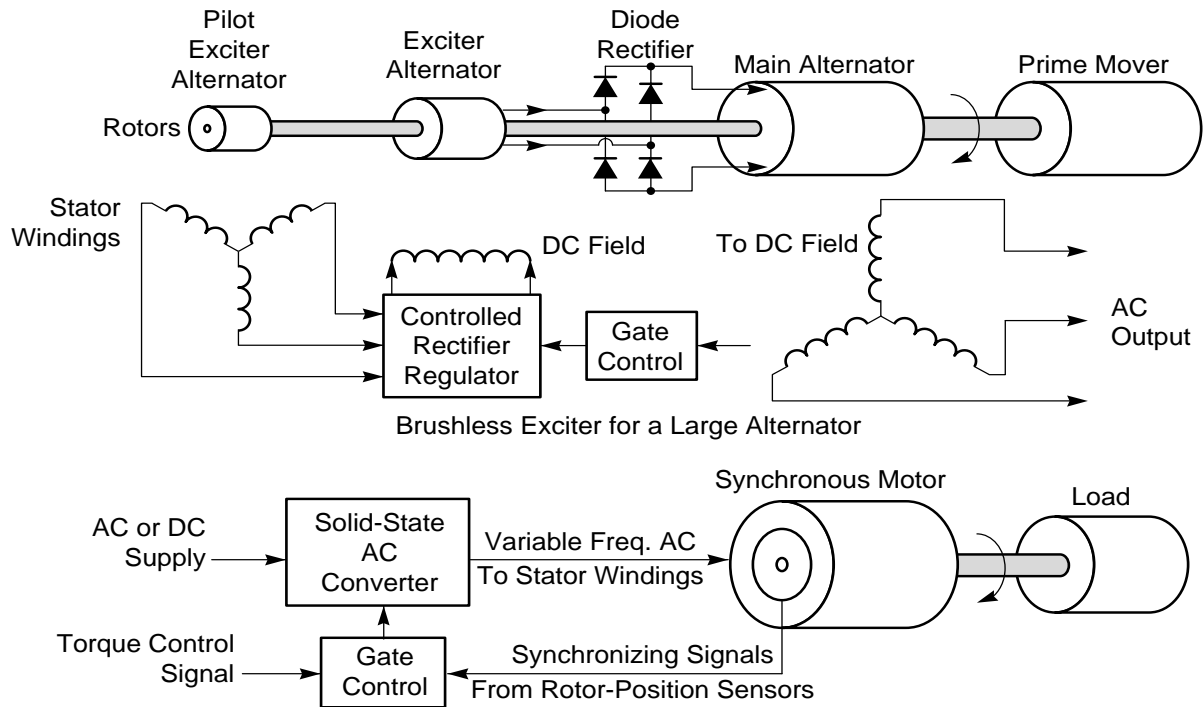
#### References:

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## 20. Brushless Machines (*Figure 3.22*)

Increasing attention is being given to the use of solid-state switches, not only as drives and exciters for conventional DC and AC machines, but also as integral parts of these machines to improve their overall performance in various ways. In particular, solid-state devices make it possible to eliminate mechanical commutators and/or slip rings together with their well-known limitations and maintenance problems. Brushless excitation of large alternators and brushless motors are two typical examples of brushless machines.

The DC field excitation of today's largest power station alternators may require 600V at 10,000A or more. Traditionally this excitation power was



**Figure 3.22**  
*Brushless Machines*

supplied through slip rings to the rotor field windings from a DC generator with a commutator. Solid-state has now done away with both the slip rings and the commutator. The principles of a brushless excitation system are shown above. The rotors of all machines are mounted on a single shaft driven by a turbine or other prime mover. Rotating windings are interconnected along the shaft, but require no electrical connections to the stator.

The smallest machine is a pilot alternator which generates the excitation for the exciter alternator. The pilot alternator has a permanent magnet rotor. The AC output from its stator windings is coupled to the stator field windings of the exciter alternator through a controlled rectifier. Gate control of this rectifier is used to provide regulation of the main AC output.

The rotor of the exciter alternator generate AC which is rectified by diodes mounted on the shaft. The components of this rotating rectifier must be designed to withstand high centrifugal forces (up to 8000g). The DC output supplies the rotor field windings of the main alternator. The main AC output is obtained from the stator windings. (See Reference 1 for further information.)

Various modifications of this basic approach utilize thyristor control on the shaft (2,3,4), or rectification of the voltages induced in the field windings themselves (5).

The development of brushless motors is making it possible to achieve the many advantages of DC motors without the disadvantages of a commutator. Such motors are being used in applications ranging from miniature instrument drives to hundreds of horsepower. Most motors of this type resemble a synchronous motor in that the main polyphase AC windings are on the stator and the DC field windings or permanent magnets for small sizes are on the rotor, as shown above. To avoid the loss of torque at speeds well below synchronous speed (which is characteristic of AC motors), the stator windings are supplied by a solid-state AC converter which switches in synchronism with the actual rotor position. That is, the AC frequency is determined by rotor-position sensors (photoelectric, magnetic, etc.), while the phasing, is determined by the torque required.

If the power source is DC, the AC converter is an inverter. For an AC power source, the AC converter

may be a cycloconverter, a self-commutated frequency changer, or a rectifier/inverter.

The DC rotor field is usually supplied through slip rings. However, the use of a solid “claw-pole” rotor allows the field windings (as well as the AC windings) to be placed on the stator. Further information is contained in References 6, 7, and 8.

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1. T. L. Dillman, et al., “Brushless Excitation,” IEEE Spectrum, pp. 58-66; March 1972.
2. W. F. Wright, et al., “Brushless Thyristor Excitation Systems,” IEEE Trans., vol. PAS-91, pp. 1848-1854; Sept./Oct. 1972.
3. E. C. Hartung, et al., “A Rotating Thyristor Excitation System for Hydroelectric Generators,” IEEE Trans., vol. PAS-91, pp. 2171-2178; Sept./Oct. 1972.
4. I. R. Smith and C. D. Manning, “Brushless Synchronous Generator with Rotating Thyristor Excitation,” Proc. IEE, vol. 120, pp. 901-902; Aug. 1973.
5. B. J. Chalmers, et al., “General Principle for Brushless Synchronous Machines and Its Application in an Inverter-Fed Drive,” Proc. IEE, vol. 119, pp. 1641-1642; Nov. 1972.
6. J. Inagaki, et al., “Commutators Get the Brushoff,” IEEE Spectrum, pp. 52-58; June 1973.
7. T. Maeno and M. Kobata, “AC Commutatorless and Brushless Motor,” IEEE Trans., vol. PAS-91, pp. 1476-1484; July/Aug. 1972.
8. N. Sato, “A Brushless DC Motor with Armature Induced Voltage Commutation,” IEEE Trans., vol. PAS-91, pp. 1485-1492; July/Aug. 1972.
9. T. Tsuchiya, “Basic Characteristics of Cycloconverter-Type Commutatorless Motors,” IEEE Trans., vol. IGA-6, pp. 349-356; July/Aug. 1970.
10. A. H. Hoffmann, “Brushless Synchronous Motors for Large Industrial Drives,” IEEE Trans., vol. IGA-5, pp. 158-162; Mar./Apr. 1969.
11. J. M. D. Murphy, *Thyristor Control of AC Motors*, Pergamon Press, Oxford; 1973. See Chapter 8, Commutatorless DC Motors, pp. 140-149.

## SUMMARY

- *Most established applications utilize Type 2 Switches and Natural Commutation: AC/DC Converters, Cycloconverters, and AC Switches and Regulators.*
- *Many newer applications utilize Type 3 Switches with Turn-Off Capability or Forced Commutation: DC Switches and Regulators, Inverters, and Generalized Frequency Changers.*

## 21. Summary

In summary, we have seen that naturally commutated circuits can perform a variety of power conversion/control functions in many applications. The number of naturally commutated circuits in use far exceeds the number of circuits which can interrupt current. Part of the reason for this fact is historical; many present applications of solid-state power electronics have evolved from mercury-arc converters where the cost of forced commutation was prohibitive. Nevertheless, there is an increasing number of applications which cannot be served by naturally commutated circuits but where turn-off solid-state switching devices is feasible. Most of these applications involve a DC power source and lack a connection to an AC source which could cause commutation to occur naturally. In the next section we will examine the technology and applications of circuits using Type 3 or turn-off switches.

## 22. Problems

### *Problem 3.1 Inductive Loading of a Controlled Rectifier*

To test your understanding of the effects of inductance and delayed conduction, consider a one-pulse controlled rectifier which is loaded by a pure inductance. The waveforms for this circuit with no phase delay ( $\alpha = 0$ ) were discussed in the course notes under uncontrolled rectifiers. If conduction is delayed ( $\alpha > 0$ ), current flow will not be continuous but will cease to  $360^\circ$ .

- (a) If the extinction angle is  $\delta (\alpha < \delta < 360^\circ)$ , sketch the voltage and current waveforms for some arbitrary value of  $\alpha$  and calculate  $\delta$  and the average load current as functions of  $\alpha$ .
- (b) Repeat (a) if the circuit is modified by connecting a free-wheeling diode across the load inductance.



- (c) Describe what would happen if a two-pulse rectifier were loaded by a pure inductance. First consider the case when  $\alpha < 90^\circ$  and then the case for  $\alpha > 90^\circ$ .

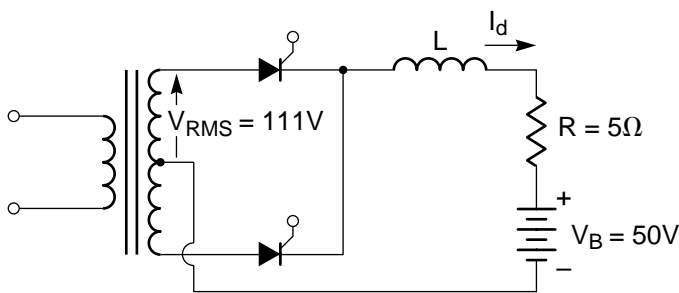
### Problem 3.2 Capacitor Switching

Capacitors are frequently used to correct the power factor of lagging loads. An engineer proposes using a solid-state AC switch in series with such a capacitor to permit rapid compensation for changes in power factor. Furthermore, control of the phase delay of the AC switch allows continuous variation of the effective leading KVAR's rather than the usual stepped variation as individual capacitors are connected or disconnected by mechanical switches.

When the engineer tries the circuit in the lab, he finds that the AC switches constantly burn out, although the average current calculated from the KVA rating of the capacitor is well within the current rating of the solid-state devices.

- Examine the operation of a phase-delayed AC switch loaded by a capacitor and explain the cause of this engineers problem.
- Are solid-state devices totally unsuited for switching capacitors or can they be used with certain restrictions? What restrictions do you think are necessary?

### Problem 3.3 Two-Pulse Battery Charger



The two-pulse AC/DC converter shown here is used as a battery charger.

- Calculate and plot the charging current  $I_d$  as a function of the phase delay angle  $\alpha$ . First assume that  $L$  is zero, and then assume  $L$  is large enough that the ripple current is negligible. Does  $L$  influence the average value of  $I_d$ ? The waveform of  $I_d$ ?

- If the battery is inadvertently connected into the circuit with the opposite polarity to that shown, calculate and plot  $I_d$  as a function of  $\alpha$ .
- Discuss what would happen in the circuit if  $R$  were negligibly small.

### Problem 3.4 three-Phase Bridge AC/DC Converter

- The results of a complete analysis of a three-phase bridge AC/DC converter are shown in the text for  $\alpha = 0$ . Repeat the analysis, showing the waveforms for  $V_{XN}$ ,  $V_{YN}$ ,  $I_{AX}$ ,  $I_{AY}$ ,  $I_A$ ,  $V_{XY}$ ,  $V_d$ ,  $V_{AX}$ , and  $V_{YA}$ , and showing the conduction logic diagram for  $\alpha = 45^\circ$ . What is the displacement angle,  $\phi$ ? What is the pulse number?
- Assuming that there is a "load" capable of maintaining continuous current, repeat (a) for  $\alpha = 90^\circ$ .
- Repeat (b) for  $\alpha = 135^\circ$ .
- Assume that the three thyristors whose anodes are tied to Y are replaced by diodes so the circuit becomes a *semiconverter*. Repeat the analysis for  $\alpha = 90^\circ$ , and compare the results with those of (b). Have the pulse number and displacement angle changed?
- Assume that the arc is supplied through a transformer having its secondaries connected delta and its primaries connected wye (with neutral floating). Determine the current waveforms in the AC lines for  $\alpha = 45^\circ$ , and compare them with (a).
- Repeat (c) if there is a commutation overlap angle of  $15^\circ$ . Show where  $dv/dt$  firing might be expected to occur.

### Problem 3.5 Motor Drive Dynamics

*Note: This problem assumes previous knowledge of the theory of simple feedback systems.*

A six-pulse bridge dual converter draws power from a 60-Hz, 240V RMS line-to-line three-phase AC supply and drives the armature of a 240V DC motor rated 100 hp at 1750 rpm. Constant field excitation is provided by another power supply. The moment of inertia of the armature and load is  $J = 25 \text{ Nms}^2$ , and the armature circuit constants are  $R\alpha = 0.07 \text{ ohms}$

and  $L\alpha = 5\text{mH}$ . Assume that the load can be represented by constant  $B$ , and that the friction and windage losses are 10 hp at 1750 rpm. The peak value of the cosinusoidal reference signal into the zero-crossing detector is 5V. The current-limiting loop is set to operate at  $\pm 400\text{A}$ . (The footnote\* contains reference information that may be useful.)

- (a) Assume  $k_t = 0$  (i.e., no feedback) and that the motor is operating at 1750 rpm with full load. What voltage  $V_\omega$  is needed in steady state? Find  $\alpha$  and the AC source power factor. Calculate  $R_m$  and  $C_m$  (the electrical analogs of the mechanical circuit), and compare the electrical ( $L_\alpha/R_\alpha$ ) and mechanical ( $J/B$ ) time constants. Determine the natural frequency  $\omega_o$ , damping factor  $\delta$ , and speed regulation  $\eta$  of the motor.

$$*1 \text{ hp} = 745.7\text{W}$$

Sinusoidal coefficients of a Fourier series expansion are given by:

$$b_k = \frac{1}{\pi} \int_0^{2\pi} f(t) \sin ktdt$$

For a quadratic characteristic equation:

$$A_2 s^2 + A_1 s + A_0 = 0,$$

$$\omega_0 = \sqrt{A_0/A_2},$$

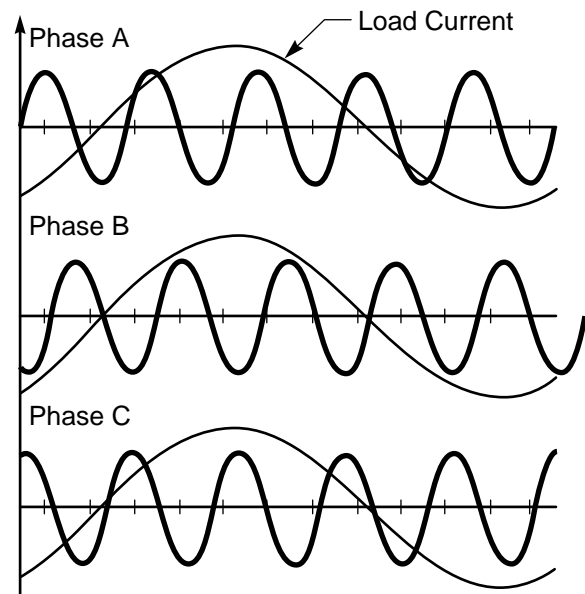
$$\delta = A_1/2\sqrt{A_2 A_0}$$

Speed regulation  $= \eta = (\text{no-load speed} - \text{full-load speed}) / (\text{full-load speed})$ .

- (b) Assume  $k_t = 0.15\text{Vs/rad}$  and re-evaluate those factors in (a) that are affected.
- (c) The motor is operating in steady-state as in (b) when  $V_\omega$  is suddenly changed to  $-25\text{V}$ . Determine in detail how the system responds. In particular, plot  $I_d$  and  $\omega_\alpha$  as functions of time. (You are sure to run into trouble if your analysis isn't guided by physical reasoning.)
- (d) If the motor drive contained a 2-quadrant converter instead of a dual converter, explain how an armature reversing switch could be used to duplicate the results of (c).
- (e) Would your explanation in (d) have to be changed to apply to a semiconverter? If so, how?

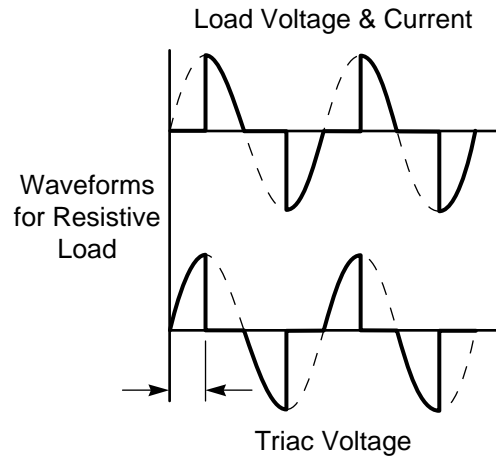
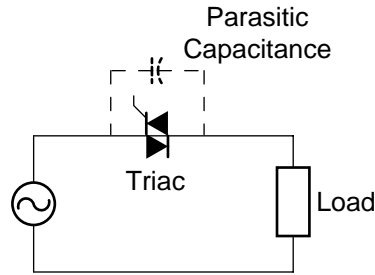
### Problem 3.6 Operation of a Three-Pulse Cycloconverter

#### Load Current Waveforms



The waveforms for a three-pulse cycloconverter operating with a 5:1 frequency reduction ratio are shown in the course notes. (a) Draw the circuit for this converter, and also show it in the form of a switching matrix. (b) The filtered sinusoidal load current waveform is reproduced three times below with the times indicated at which switching occurs. Show how the load current is divided between the three-phase of the AC supply by sketching the three-phase-current waveform. (c) Compare the phase-current waveforms with the corresponding phase voltages and discuss how it could be established that the AC source sees a lagging displacement factor. (d) Use the switching matrix concept to determine how many thyristors are required to implement a  $3\text{-}\phi$  to  $3\text{-}\phi$  cycloconverter without neutral connection.

**Problem 3.7 Qualitative Analysis of an AC Regulator**



The circuit above shows a triac used in a simple phase-delay AC regulator intended for resistive loads. The purpose of this program is to qualitatively explore some of the effects of inductive loads and parasitic parameters. Seek to understand in detail what happens in the circuit with the goal of setting up a solution procedure, but an actual analytical solution is not required.

- (a) Idealized waveforms of triac voltage and load voltage and current are shown for a resistive load and  $\alpha = 90^\circ$ . How would these waveforms change if the load had a lagging power factor of 70%? Would load power vary as  $(\cos \alpha)^2$ ? What aspect of the modified waveform might cause the triac to fail to commutate at all?
- (b) Assume that the triac turns off when the current becomes less than a small (but non-zero) holding current, and that there is a small (but non-zero) capacitance across the triac. How would these parasitic parameters further modify the waveforms during the turn-off and turn-on transients? Would the previous commutation problem be more or less severe?
- (c) What corrective measures might be designed into the circuit to assure proper commutation?

# Chapter 4

## TURN-OFF DEVICES AND SELF-COMMUTATED CIRCUITS

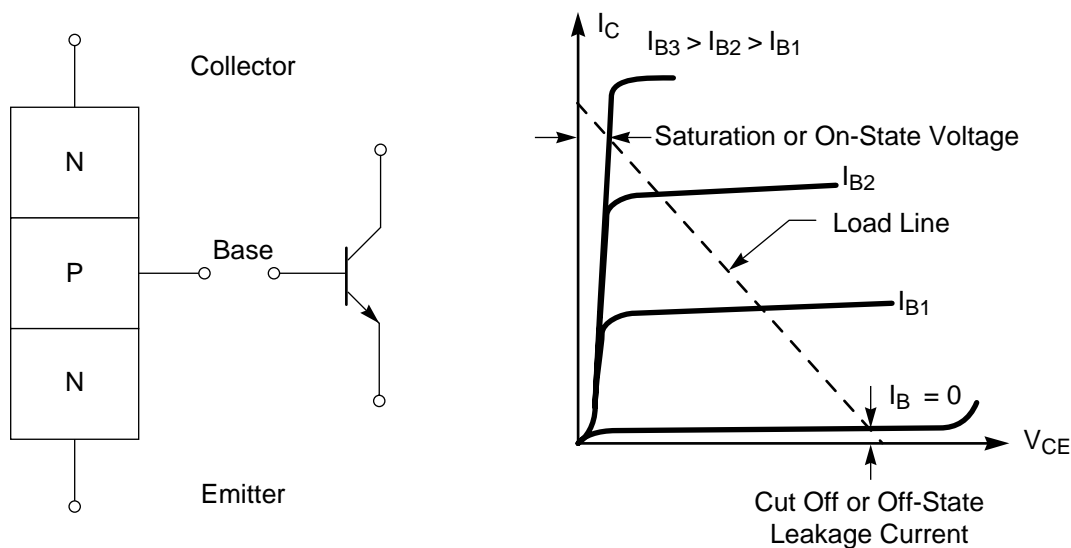
### Section

1. Power Transistor . . . . .	4-2
2. Gate Controlled Switch . . . . .	4-3
3. DC Switching and Regulation . . . . .	4-3
4. Chopper Using Thyristors . . . . .	4-4
5. Fundamentals of Inverters . . . . .	4-5
6. Full-Bridge Inverter . . . . .	4-7
7. Half-Bridge Inverter with an Inductive Load . . . . .	4-7
8. Thyristor Inverters with Forced Commutation . . . . .	4-8
9. Types of Forced Commutation . . . . .	4-9
10. Series Inverter . . . . .	4-10
11. Parallel Inverter . . . . .	4-11
12. Complementary Impulse – Commutated Inverter . . . . .	4-12
13. Inverter Output Voltage Control . . . . .	4-12
14. Inverter Output Waveform Improvement . . . . .	4-14
15. Frequency and Power Factor Changers . . . . .	4-15
16. Problems . . . . .	4-16

### Introduction

The objectives of this chapter are:

- To describe and compare the three ways of implementing turn-off switches: transistors, gate controlled switches, and force-commutated thyristors.
- To explain the basic ways in which a charged capacitor can be used to turn a thyristor off and the factors which determine the size of capacitor required in each case.
- To analyze the operation of a chopper in decreasing or increasing a DC voltage.
- To describe and compare the basic circuit configurations for transistor and thyristor inverters, and discuss ways of controlling output voltage and waveform.
- To discuss the functions that can be performed by a force-commutated frequency changer.



**Figure 4.1**

*NPN Transistor Junction Geometry, Symbol, and V-I Characteristic (A Non-Latching Type 3 Switch)*

### 1. Power Transistor (Figure 4.1)

In contrast to a thyristor or Type 2 switch which can be turned on by a short gate pulse and “latches” into conduction until current is commutated externally, a power transistor is a non-latching Type 3 switch. Operated as a switch, a transistor must be held in its on-state by a continuous control signal in the base circuit. When this signal is removed, the switch automatically reverts to its off-state.

The junction structure, circuit symbol, and V-I characteristic of an NPN transistor are shown above. In the forward direction (collector positive with respect to the emitter for an NPN transistor), the limiting values of the V-I characteristic are similar to those of a thyristor, with the cut-off and saturation regions corresponding to the off and on states respectively. Two important differences should be noted, however.

First, since the transistor is not self-latching, the transition between the two limiting states can be stopped at any intermediate value of collector current. If the base current is inadequate to carry the collector into saturation for a specified load line, the operating point remains in a region of high dissipation, and the temperature increase may damage the device.

Second, after the transistor reaches its saturation state, it can easily drop out of saturation (with the same destructive effects) if the collector current

increases above  $\beta I_B$  or if the base current decreases below  $I_C/\beta$ , where  $\beta$  is the common-emitter current gain. Therefore, the current overload capacity of a transistor is very limited.

Also contrary to a thyristor, a power transistor is not designed to operate in the third quadrant of its V-I characteristics. Normally, if the transistor is exposed to reverse currents, a shunting diode between collector and emitter is used to bypass these currents.

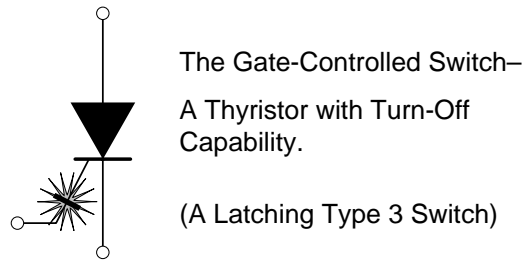
At present, transistors are usually the most economical Type 3 switches for relatively low power levels, and because of the slower response of thyristors, transistors must be used for operating frequencies above about 10kHz. However, transistors are not available with power capacities greater than about 10kVA.

The following references may be consulted for further information on power transistor design, ratings, and circuit applications:

*Solid-State Power Circuits*, (Designer's Handbook SP-52), RCA Solid State Div., Somerville, N.J.; 1971.

*Westinghouse Silicon Power Transistor Handbook*, Westinghouse Semiconductor Div., Youngwood, Pa.; 1967.

*Power Transistors*, (Technical Series PM-81), RCA Solid State Div., Somerville, N.J.; 1971.



**Figure 4.2**  
*The Gate Controlled Switch*

## 2. Gate Controlled Switch (Figure 4.2)

A conventional thyristor switches from its forward conducting state to its forward blocking state only when its anode current is reduced by external means to a value less than its required holding current. Since the holding current is normally a very weak function of gate current, the gate cannot interrupt appreciable anode current. However, thyristors can be specially designed to increase the dependence of the holding current on gate current. If the gate current can cause the holding current to exceed the load current which is flowing, then the device is capable of gate controlled turn-off. This type of device is known as a turn-off thyristor or a gate controlled switch. The ratio of load current which can be interrupted to the gate current needed is typically 3 to 10.

Although the gate controlled switch offers attractive possibilities for the future, it is presently commercially available only in current ratings up to about 10A.

### References:

- J. M. Goldey, et al., "Turn-Off Gain in PNP Triodes," *Solid-State Electronics*, vol. 3, pp. 119-122; 1961.
- T. C. New, et al., "High-Power Gate Controlled Switch," *IEEE Trans.*, vol. ED-17, pp. 706-710; Sept. 1970.
- E. D. Wolley, "Gate Turn-Off in PNP Devices," *IEEE Trans.*, vol. ED-13, pp. 590-597; July 1966.
- H. F. Storm, "Introduction to Turn-Off Silicon-Controlled Rectifiers," *IEEE Trans. Comm. & Elect.*, pp. 375-383; July 1963.
- E. D. Wolley, et al., "Characteristics of a 200-Amp Gate Turn-Off Thyristor," *IEEE Industry Applications Soc. 8th Annual Meeting Record*, pp. 251-257; Oct. 1973.
- P. S. Raderecht, "The Development of a Gate Assisted Turn-Off Thyristor for Use in High Frequency Applications," *Intnl. Jour. Electronics*, vol. 36, pp. 399-416; 1974.

## 3. DC Switching and Regulation (Figure 4.3)

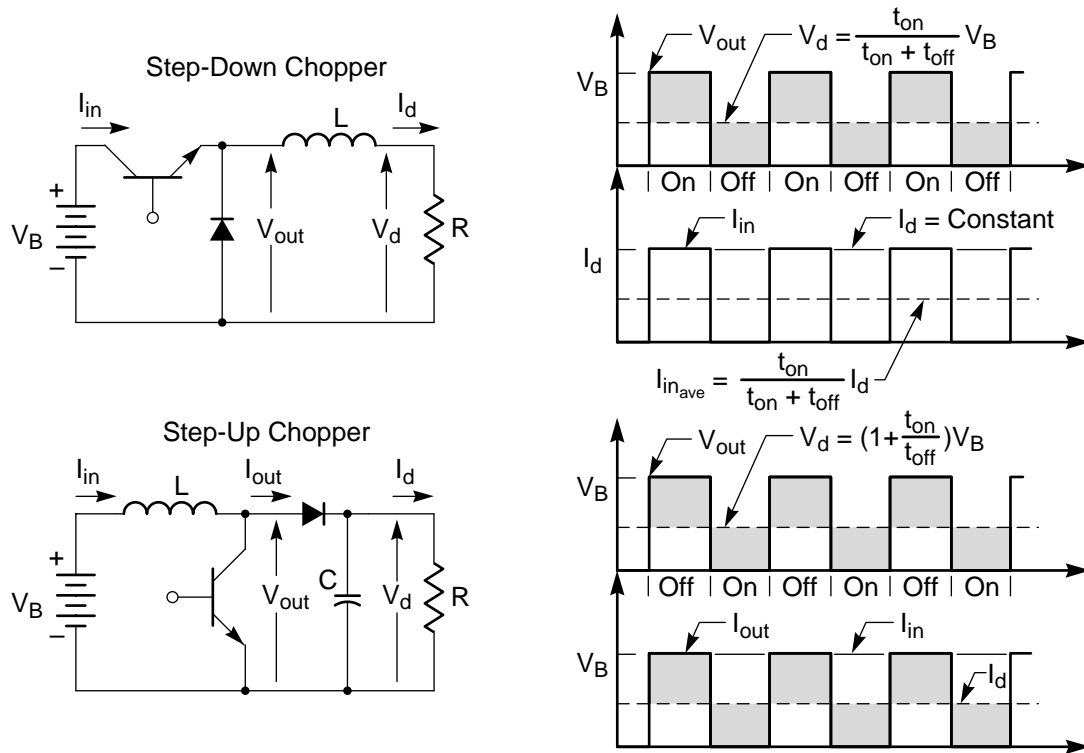
Since a Type 3 switch can both initiate and interrupt conduction, it can act as a DC switch if connected in series with a DC source and load.

The same circuit, with appropriate repetitive switching control signals, can also be extended to change or regulate the average value of a DC source. The average value is determined by the relative on-time or duty cycle of the "chopped" output waveform. The duty cycle can be adjusted by varying the on-time, the off-time, or both, and all three modes of control are used in practical choppers.

Step-down choppers, in which the average output voltage is less than the input voltage, are most commonly used. A circuit of this type is shown in the example, including an inductance to smooth the output waveform and a freewheeling diode to provide a path for the load current while the Type 3 switch is off. Although the average voltage is decreased by this circuit, the peak source current is equal to the load current, so the average source current is less than the load current. Therefore this chopper steps up the average current.

With some rearrangement, the circuit can increase the average voltage and decrease the average current. As shown in the second circuit, L is placed in series with the source and the Type 3 is moved to a shunt position. When the switch is closed, L stores volt-seconds. When the switch opens, the inductive kickback voltage adds to the source voltage, causing the diode to conduct, and raising the load voltage above the source voltage. L performs voltage averaging, while C performs current averaging. Both L and C are assumed large, yielding waveforms with perfect smoothing. (In fact, except for the lack of an AC source, all the assumptions made in rectifier analysis have also been made here.)

From this description, it is seen that choppers are the functional equivalent of a step-down or a step-up DC transformer, with the additional advantage that the effective turns ratio can easily be changed electronically by changing the duty cycle of the switching control signals. Choppers are widely used as speed controls in DC-powered vehicles such as industrial fork lifts, golf carts, electric cars, etc. The size of the smoothing inductors and capacitors is reduced if the frequency of operation of the chopper is increased. But then switching losses increase, so the frequency chosen is a compromise.



**Figure 4.3**  
DC Regulators or Choppers

#### References:

- E. Reimers, "Design Analysis of Multiphase DC Chopper Motor Drive," IEEE Trans., vol. IA-8 pp. 136-144; Mar./Apr. 1972.
- D. Y. Chen, et al., "Computer-Aided Design and Graphics Applied to the Study of Inductor-Energy-Storage DC-to-DC Electronic Power Converters," IEEE Trans., vol. AES-9, pp. 585-597; July 1973.
- G. W. Wester and R. D. Middlebrook, "Low-Frequency Characterization of Switched DC-DC Converters," IEEE Trans., vol. AES-9, pp. 376-385; May 1973.
- R. V. Langmuir, "Repetitively Switched Circuits," IEEE Trans., vol. AES-9, pp. 59-64; Jan. 1973.
- P. W. Franklin, "Theory of the DC Motor Controlled by Power Pulses: Part I-Motor Operation; Part II-Braking Methods, Commutation, and Additional Losses," IEEE Trans., vol. PAS-91, pp. 249-262; 1972.

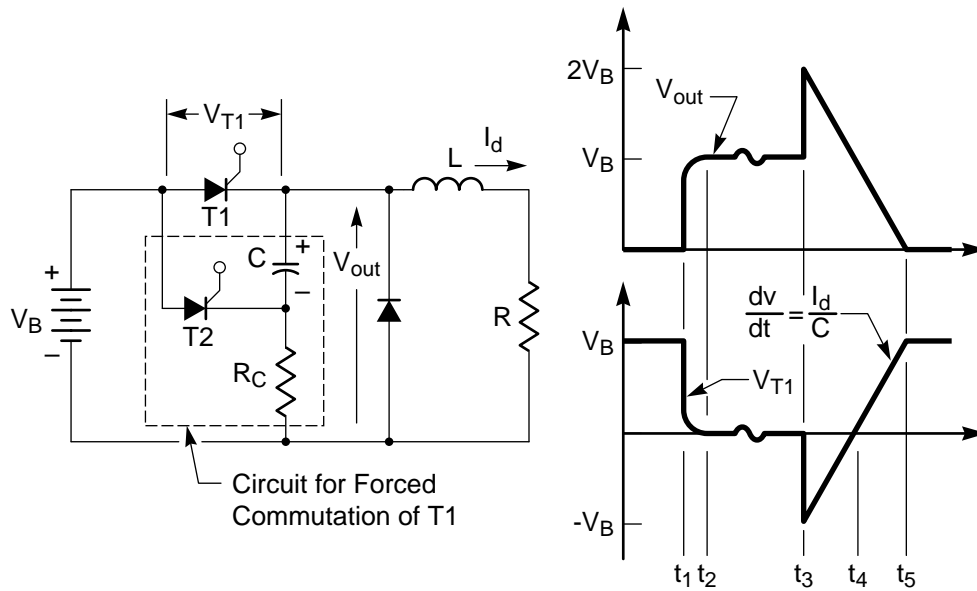
#### 4. Chopper Using Thyristors (Figure 4.4)

The choppers which were just considered used power transistors as Type 3 switches. For larger power ratings, a transistor can be replaced by a thyristor

with an associated circuit for forced commutation, as shown above. Assume initially that thyristor T1 is in its blocking state and that the circuit in the dashed box is absent.

At time  $t_1$  a firing pulse is applied to the gate of T1. By  $t_2$ , following a brief turn-on transient, the voltage  $V_{out}$  has reached its steady-state value equal to the source voltage,  $V_B$ . Since T1 cannot interrupt current, conduction will continue forever. The circuit does not have an AC source to cause natural commutation and to restore control to the gate of T1.

Now insert the circuit shown in the dashed box. Capacitor C will charge through  $R_C$  to voltage  $V_B$  with the polarity indicated. When it is desired to turn T1 off, say at time  $t_3$ , a firing pulse is applied to the gate of auxiliary thyristor T2, thereby connecting C across T1. The capacitor voltage adds to the battery voltage, increasing  $V_{out}$  to  $2V_B$ . But more importantly, it reverse biases T1 and diverts the load current. As C recharges, the polarity on T1 again becomes positive at  $t_4$ . If the interval from  $t_3$  to  $t_4$  equals or exceeds the turn-off time of T1 and if  $dv/dt$  is not excessive, then T1 will be able to block forward voltage and gate control is reestablished. By  $t_5$ , C has charged to



**Figure 4.4**  
*Chopper Using a Thyristor with Forced Commutation*

$V_B$  with polarity opposite to that shown, and the load current begins to flow in the holding current of T2 and T2 will turn off. The thyristor circuit has now returned to its fully off state and will remain off until another firing pulse is applied to T1.

The circuit in the dashed box is one of many possible circuits for the forced commutation of regular thyristors to give them Type 3 turn-off capability. Other types of circuits for forced commutation will be discussed in later sections. The critical component in these circuits is the commutation capacitor which must store enough energy to support load current for some minimum turn-off and must prevent excessive  $dv/dt$  being applied to the main thyristor. Note that because of the freewheeling diode, the energy stored in the DC inductance does not influence the design of the commutating circuit.

The minimum turn-off time ( $t_4 - t_3$ ) is an important rating of the thyristor and is somewhat longer than the recovery time when the device is not immediately expected to block forward voltage. Because of the interdependence of this turn-off time and the preceding forward current, junction temperature,  $dv/dt$ , etc., it must be determined in a tester which is capable of dynamically simulating the conditions encountered in the actual circuit under consideration.

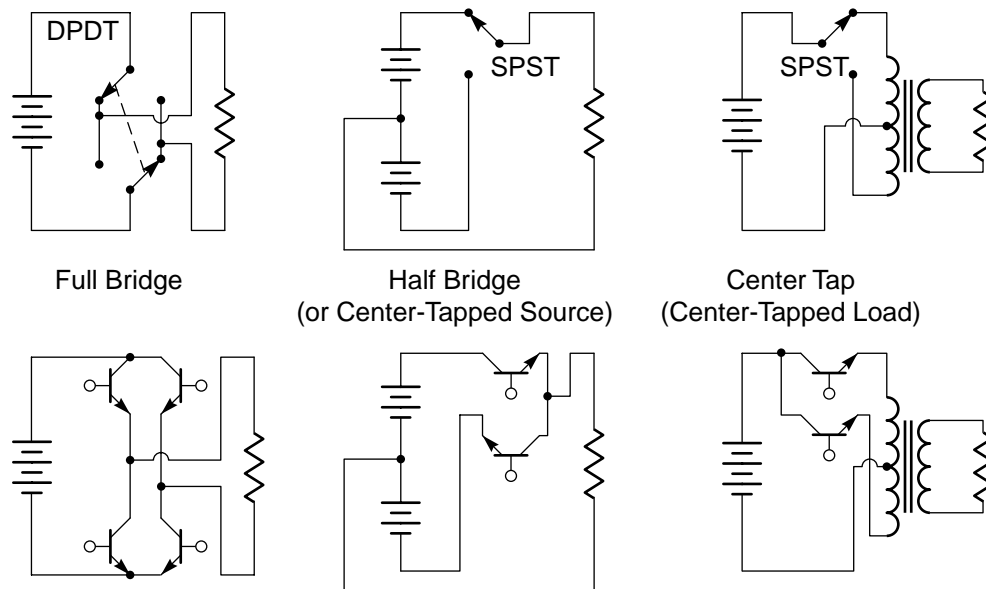
#### References:

- B. D. Bedford and R. G. Hoft, *Principals of Inverter Circuits* (Chapter 10), John Wiley, New York; 1964.
- R. M. Davis, *Power Diode and Thyristor Circuits* (Chapter 7), Cambridge Univ. Press, London; 1971.
- K. Heumann, "Pulse Control of DC and AC Motors by Silicon-Controlled Rectifiers," *IEEE Trans. Comm. & Elect.*, vol. 83, pt. 2, pp. 390-399; July 1964
- R. Cushman, "Powerful SCR's, Connected in Parallel, Control Industry's Biggest Machinery," *Electronics*, pp. 110-120; Oct. 4, 1965.
- I. I. Ponner, "Design Criteria for DC Switch with Resonant Circuit Connected Between Thyristors," *IEEE Trans.*, vol. IECI-17, pp. 394-395; Aug. 1970.
- B. H. Smith, "A simple Bilateral Variable-Ratio DC Pulse Converter," *IEEE Trans.*, vol. IECI-15, pp. 1-11; Nov. 1968.

## 5. Fundamentals of Inverters (Figure 4.5)

A chopper converts DC power to DC power and therefore the output voltage maintains a constant polarity. An inverter, on the other hand, supplies AC power and must be able to cyclically reverse the





**Figure 4.5**  
*Basic Inverter Circuit Configurations*

output polarity. Three basic single-phase circuit configurations are used to achieve reversal.:

- (a) A full bridge is equivalent to a DPDT reversing switch which allows the full source to be connected to the full load with either polarity. Four Type 3 switches are needed to implement the DPDT switch.
- (b) A center tap on the source allows the load polarity to be reversed when the load is switched from one half of the source to the other. Only half of the source is used at a time, but a half bridge (or SPDT switch) containing only two Type 3 switches is adequate.
- (c) A center tap on the load (which is usually achieved by a center tapped transformer winding) requires only two Type 3 switches and also allows the full source voltage to be used.

If the load on any of these inverter circuits is resistive, the output voltage and current waveforms are square waves. Under these idealized conditions, the Type 3 switches must each block forward voltage during the half cycle when they are turned off, but they are never exposed to reverse voltage or current.

However, an inductive load is more typical. In this case, the inductance stores appreciable energy during each half cycle. If the Type 3 switches turn off

rapidly, thereby interrupting the load current before it can reverse and be picked up by the complementary switches which are turning on, a huge reverse inductive voltage is induced across these switches and may destroy them. In a rectifier or a chopper, which have a DC output, this reverse voltage could be avoided by shunting the output by a freewheeling diode. But with an AC output from an inverter, the “reverse” voltage is of opposite polarity on alternate half cycles. Two freewheeling diodes of opposite polarity would simply short the output.

The solution is to transfer the freewheeling diodes from the AC output circuit to the DC circuit where the reverse transients are always of the same polarity. These diodes are in fact connected either directly or effectively in parallel with the Type 3 switches. The parallel combination of a Type 3 switch and a diode can support (conduct) current in both directions but can support (block) voltage in only one direction. Note that the switch required by an inverter is the dual switch required by an AC/DC converter which can support (block) voltage in both directions but can support (conduct) current in only one direction.

Other important aspects of transistor inverters are analyzed in the following references:

F. C. Y. Lee, et al., “Analysis of Limit Cycles in a Two-Transistor Saturable-Core Parallel Inverter,” IEEE Trans., vol. AES-9, pp. 571-584; July 1973.

F. C. Y. Lee and T. G. Wilson, "Analysis and Modeling of a Family of Two-Transistor Parallel Inverters," IEEE Trans., vol. MAG, pp. 414-418; Sept. 1973.

F. C. Y. Lee and T. G. Wilson, "Analysis of Starting Circuits for a Class of Hard Oscillators: Two-Transistor Saturable-Core Parallel Inverters," IEEE Trans., vol. AES-10, pp. 100-112; Jan. 1974.

## 6. Full-Bridge Inverter (Figure 4.6)

The full-bridge inverter circuit has an apparent disadvantage in that it requires four switches rather than only two for the other two configurations. However, it avoids the need for a center-tapped source or a transformer coupling to the load. With the addition of one more leg containing two more switches, it is also capable of supplying a three-phase AC output.

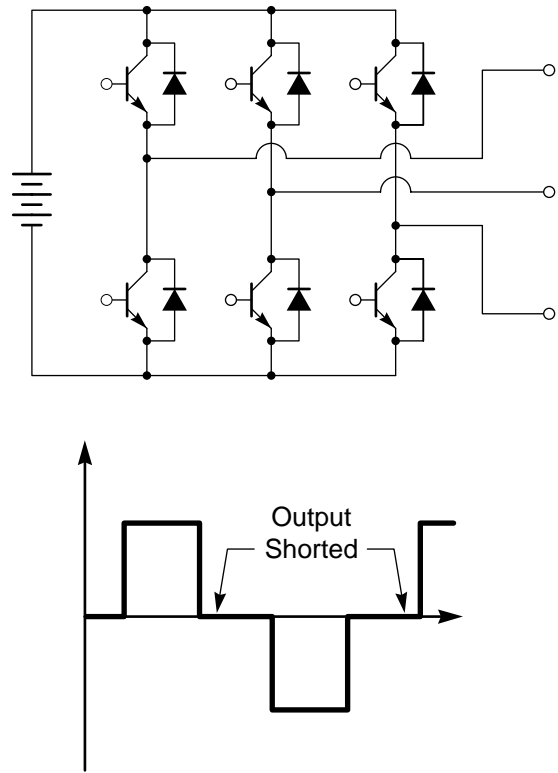
Another advantage is less obvious. The other configurations connect the load to the source with either positive or negative polarity, but they cannot short the load to insert segments of zero voltage into the output waveform while continuing to carry inductive load current. However, a full bridge with shunting diodes can supply load current in either direction at zero voltage if all of the switches on one side of the bridge are turned on simultaneously. As will be discussed subsequently, this capability is useful in permitting regulation of the output voltage.

Shown in Figure 4.6 is a full-bridge, three-phase inverter with shunting diodes. Also shown is an output voltage waveform having segments of zero voltage.

An interesting theoretical technique using "Park vectors" for the analysis of three-phase inverters is discussed in:

R. K. Jordan, et al., "General Analysis of Three-Phase Inverters," IEEE Trans., vol. IGA-5, pp. 672-679; Nov./Dec. 1969.

R. K. Jordan, "Modes of Operation of Three-Phase Inverters," IEEE Trans., vol. IGA-5, pp. 680-685; Nov./Dec. 1969.



**Figure 4.6**

*Full-Bridge Three-Phase Inverter with Shunting Diodes*

## 7. Half-Bridge Inverter with an Inductive Load (Figure 4.7)

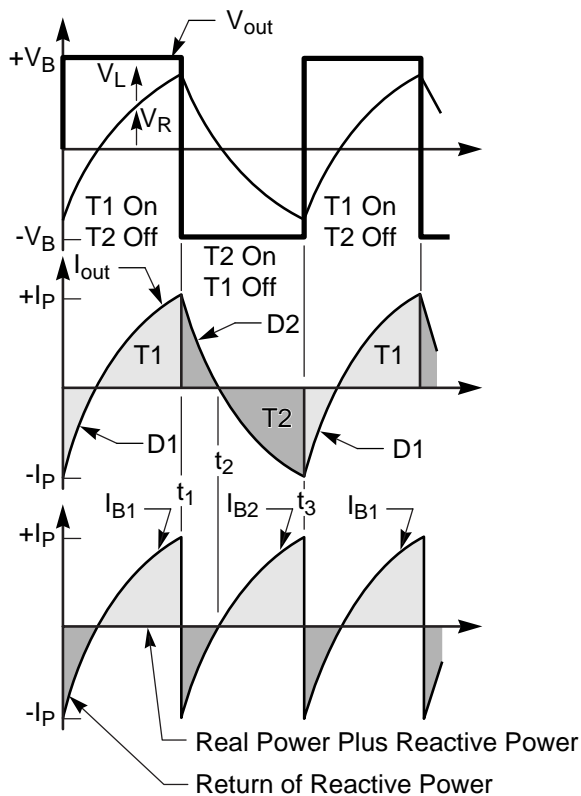
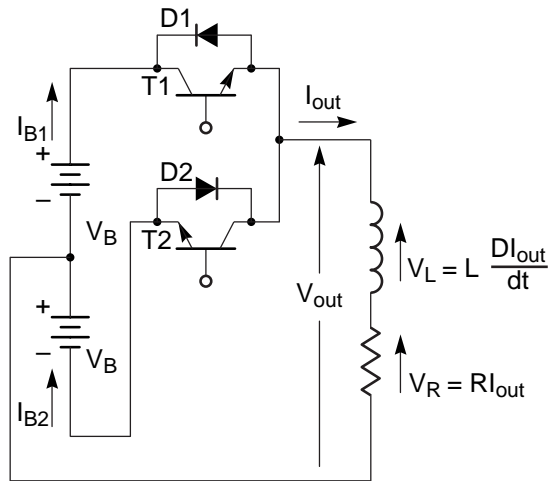
To be certain that the operation of an inverter with an inductive load is clear, let us examine the half-bridge circuit shown in more detail. Similar conclusions are also valid for the other circuit configurations.

Immediately prior to time  $t_1$ , Type 3 switch T1 (shown as a transistor) has been conducting and the load current  $I_{out}$  has reached a value  $+I_p$ . At  $t_1$ , base current is removed from T1 to turn it off, and base current is supplied to T2 to turn it on. However, current in the inductance cannot change instantly; it must be a continuous function of time. Since  $I_{out}$  at  $t_1$  is positive, it cannot flow through T2 but transfers from T1 to the parallel diode D2 instead. But since D2 is connected to the other half of the DC source,  $V_{out}$  reverses polarity immediately.

D2 continues to conduct until  $t_2$  when the current reverses. During the interval from  $t_1$  to  $t_2$ , the energy stored in the load inductance is being discharged,

partly into the load resistance and partly back into the DC source. This flow of power back to the DC source represents the return of the reactive component of power.

At  $t_2$ ,  $I_{out}$  becomes negative and transfers from D2 to T2.  $I_{out}$  increases in the negative direction (asymptotically approaching  $V_B/R$ ) until  $t_3$  when T2 is turned off and T1 is turned on. This completes the



**Figure 4.7**  
*Circuit and Waveforms for a Half-Bridge Inverter with an Inductive Load*

half cycle, and the next half cycle is the same except that T1 and T2 interchange roles.

The waveform of current in the DC source is the same as that in the load, except that the relative polarity reverses each half cycle, as shown. When the source current waveform is multiplied by the constant source voltage, the result is the source power waveform. Positive power represents the flow of real and reactive power from the source to the load, and negative power represents the return of the reactive component of power.

Note that the switching control signals applied to T1 and T2 determine the frequency but not the effective value of  $V_{out}$ , which is just opposite to the effect of the control signals in an AC/DC converter.

**Note: The Commutation Circuit in a Thyristor Inverter Is Ordinarily Not Separable from the Rest of the Circuit.**

## 8. Thyristor Inverters with Forced Commutation

In principle, a thyristor and its associated circuit for forced commutation can be substituted directly into the previous inverter circuits in place of each transistor. In practice, a number of difficulties arise when this is done, including the following:

- The shunting diodes prevent the application of an appreciable reverse voltage to achieve minimum turn-off time.
- Application of a brief firing pulse at  $t_1$  is ineffective because the thyristor is not forward biased until after  $t_2$ , and the delay between  $t_1$  and  $t_2$  is load dependent.
- Inefficient charging of the commutating capacitor can significantly reduce the overall efficiency of the inverter.

Therefore, in most practical inverters, the means for forced commutation become an integral part of the circuit rather than a subcircuit which can be isolated and studied separately. Because of this intimate involvement between the commutation circuit and the operation of the inverter, we will next distinguish between various types of forced commutation.

### References:

A. J. Humphrey, "Inverter Commutation Circuits," IEEE Trans., vol. IGA-4, pp. 104-110; Jan./Feb. 1968.

S. P. Jackson, "Three-Phase Static Inverters – A Survey," IEEE Industry & General Applications 3rd Annual Meeting Record, pp. 797-810; Oct. 1968.

A. Verhoef, "Basic Forced-Commutated Inverters and Their Characteristics," IEEE Trans., vol. IA-9, pp. 601-606; Sept./Oct. 1973.

S. B. Dewan and D. L. Duff, "Practical Considerations in the Design of Commutation Circuits for Choppers and Inverters," IEEE Industry & General Applications 4th Annual Meeting Record, pp. 469-475; Oct. 1969.

N. W. Mapham, "The Classification of SCR Inverter Circuits," IEEE Intl. Convention Record, pt. 4, pp. 99-105; 1964.

## 9. Types of Forced Commutation (Table 4.1)

Circuits using a capacitor to force-commutate a thyristor may be classified on the basis of

- (a) the energy storage requirement on the commutating capacitor, and
- (b) the way in which turn-off occurs.

Because of the variety of intermediate and special cases, the boundaries of the classes are not distinct and clear-cut. Compounding the difficulty is the lack of standard terminology: the literature freely uses different names to mean the same thing and the same names to mean different things. Nevertheless, an attempt at classification is believed to be worthwhile because of the emphasis which it places on those aspects which are of fundamental importance. But you should be cautioned that the literature is not universally consistent with the terminology adopted here.

The size of the commutating capacity is determined by:

- (a) the current which flows through it during the commutation interval,
- (b) the duration of that interval, and
- (c) the change in voltage which occurs.

If the duration of the commutation interval is held to the minimum which will assure turn-off of the thyristor, the size of the required capacitor is also minimized, and "*Impulse commutation*" occurs. If there is inductive energy stored in the load, the inductive current must be routed elsewhere by bypass diodes. In this type of commutation, the capacitor size depends on the turn-off time of the thyristor but tends to be independent of the load phase angle.

Otherwise, the commutating capacitor must be capable of absorbing the inductive energy, and commutation is said to be "*load dependent*". In most cases, the mode of commutation requires a much larger capacitor and is more difficult to design to accommodate wide load variations. However, if the output waveform is important, the commutating capacitor may replace an equivalent amount of capacitance in the output filter.

Some circuits for forced commutation, such as the example discussed in connection with the chopper, utilize an "*auxiliary*" thyristor to turn off the main thyristor which carries load current. This auxiliary thyristor is not located in the main load circuit, and is provided with some means of self-extinction.

		Means		
		Auxiliary	Complementary	Inherent
Type	Load Dependent (Commutating Capacitor Absorbs Energy from Load Inductance)		Parallel Inverter	Series Inverter
	Impulse (Dependent on Device Recovery Time; Uses Bypass Diodes)	Auxiliary Impulse Chopper	McMurray-Bedford Inverter	

**Table 4-1**  
*Classes of Forced Commutation*

In another class of forced commutation, the thyristors which carry load current occur in “complementary” groups, and the circuit is so arranged that the presently conducting thyristor is automatically commutated off when (but not until) the next load carrying thyristor is fired. This class is also known as auto or self commutation. Although this type of circuit avoids the need for auxiliary commutating thyristors, it has the disadvantage that external means must be provided to turn the equipment off. If control signals are simply removed from the thyristor gates, thyristors which were then conducting will continue to conduct.

With both auxiliary and complementary commutation, turn-off of a conducting thyristor is initiated by firing another thyristor. We will lump the remaining circuits into the category of “inherent” commutation, although this is not a term which is used in the literature. Load current in these circuits is made self-extinguishing, as for instance by means of a series capacitor.

Each feasible combination of these types of forced commutation leads to a corresponding family of thyristor inverters. We have discussed impulse commutation using an auxiliary thyristor in connection with the thyristor chopper, and the same technique can also be used in an inverter. \*We will now examine the

series, parallel, and McMurray-Bedford inverter configurations as typical examples of the countless possible inverter circuits using the other types of forced commutation.

#### References:

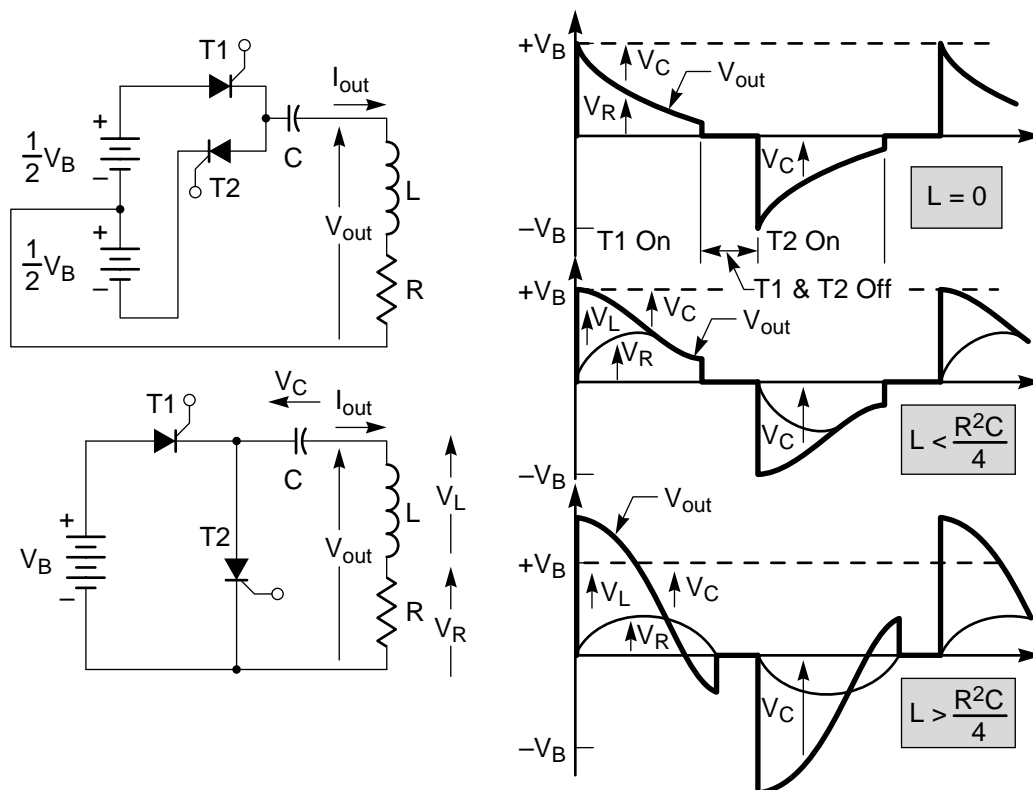
\*W. McMurray, “SCR Inverter Commutated by an Auxiliary Impulse,” IEEE Trans. Comm. & Elect., vol. 83, pt. I, pp. 824-829; Nov. 1964.

J. B. Klaassens, “Analysis of a Forced Commutation Circuit for Design of a Class of Thyristor Inverters,” IEEE Trans., vol. IECI-20, pp. 125-129; Aug. 1973.

S. B. Dewan and D. L. Duff, “Optimum Design of an Input-Commutated Inverter for AC Motor Control,” IEEE Trans., vol. IGA-5, pp. 699-705; Nov./Dec. 1969.

### 10. Series Inverter (Figure 4.8)

The first inverter circuit shown illustrates one form of load-dependent, inherent commutation used in a half-bridge (i.e., center tapped source) configuration. The capacitor in series with the load causes inherent commutation to occur, and also forces the *average* (not the effective or RMS) load current to be zero. Because of this fact, the load current  $I_{out}$  and voltage  $V_{out}$  are



**Figure 4.8**  
Series Inverter and Waveforms for the Lower Circuit for Various Values of  $L$

independent of the location of the return lead from the load. It can be returned to either end of the DC source instead of to the center tap. Therefore the second configuration is more commonly used, and is known simply as a series inverter.

First consider the operation of this inverter with a resistive load (i.e.,  $L = 0$ ). When T1 is fired,  $V_{out}$  jumps discontinuously to  $+V_B$  and an initial current  $V_B/R$  begins to flow in the load. As the capacitor charges, the current decreases exponentially. Eventually the capacitor voltage will approach  $V_B$  with the polarity indicated, the current will decrease to the holding current, and T1 will turn off. Note that turn-off is “inherent” in the behavior of this circuit; a turn-off signal is not required as it is with an auxiliary or complementary type of turn-off circuit.

The circuit will now remain inactive, with a voltage of nearly  $+V_B$  appearing across the capacitor, until T2 is fired. This connects C across R, causing  $V_R$  to jump discontinuously to  $-V_B$ , and causing C to discharge into R. Again, when the current decreases to the holding current, T2 turns off, and C is left with nearly zero voltage across its terminals. The entire cycle is ready to repeat when T1 is fired again.

Note that the frequency of the circuit is limited by the RC time constant which determines how rapidly the current decreases. If, in attempt to increase the frequency, one thyristor is fired before the previous one turns off, both thyristors will conduct and form a short circuit across the DC source.

Consider next the case where the load circuit contains inductance, but it is small enough that the circuit is overdamped (i.e.,  $L < R^2 C/4$ ). The current  $I_{out}$ , and therefore voltage  $V_R$  can no longer change discontinuously, but the voltage waveform  $V_{out}$  is changed relatively little, although it now has zero initial slope.

The final case to be considered is when L is large enough that the circuit is underdamped oscillatory (i.e.,  $L > R^2 C/4$ ). The output voltage (across L and R) now goes negative before the current decreases to the holding current and turn-off occurs. Consequently, the capacitor is charged to a voltage in excess of  $V_B$ . In contrast to the exponential current waveforms when L was small, the current waveform now closely approximates half sine waves. It is clear that in this resonant mode the commutating capacitor must be capable of absorbing most of the energy stored by the load inductance each half cycle

(bypass diodes are not needed), and that the operation of the circuit is highly load dependent.

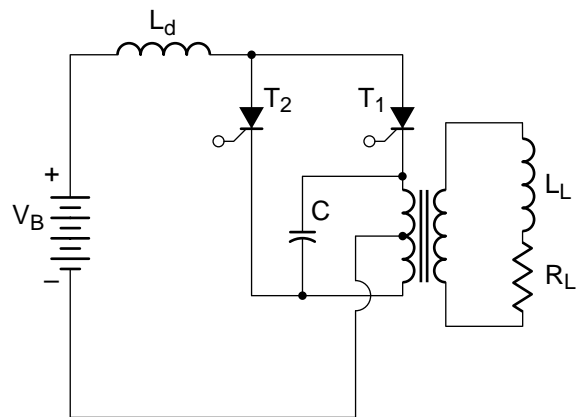
To summarize the characteristics of series inverters, they have a fixed pulse width which is determined by the time constant of the load and the commutating capacitor, and are limited to operation at relatively low frequencies. However, they have the advantages of being self-starting when firing pulses are applied and self-stopping when they are removed.

For an analysis of the series inverter, see chapter 5 of B. D. Bedford and R. G. Hoft, *Principles of Inverter Circuits*, John Wiley, New York; 1964.

## 11. Parallel Inverter (Figure 4.9)

The inverter shown here uses a center-tapped load, and the commutation capacitor is effectively in parallel with the load. Hence, this circuit is commonly called a parallel inverter. The inductor,  $L_D$ , in the DC circuits is required to prevent excessive spikes of current from being drawn by C when switching occurs.

The commutating capacitor effectively resonates with the load, thereby absorbing the energy stored by  $L_L$  each half cycle. As in the series inverter, operation of the circuit is highly load dependent, although this circuit uses complementary commutation rather than inherent commutation. Current does not automatically decrease below the holding current to achieve turn-off. However, when the next load-carrying thyristor is fired, the commutation capacitor is connected so that its voltage reverse biases the thyristor which had been conducting, causing it to turn-off.



**Figure 4.9**  
Parallel Inverter

Complementary commutation permits the parallel inverter to be operated with variable pulse widths and at somewhat higher frequencies than the corresponding series inverter. However, the circuit is not self-starting. Special arrangements must be made to establish an appropriate initial charge on C so that the first thyristor can be turned off after the first half cycle. The circuit also cannot be turned off by simply removing the firing pulses.

Detailed steady-state analysis of the parallel inverter and the load dependence of its waveforms involves solving a third-order differential equation and matching initial and final conditions on a half cycle. Therefore general analytical solutions are not feasible, but the waveforms can be generated by analog or digital simulation. These are presented in:

B. D. Bedford and R. G. Hoft, *Principles of Inverter Circuits* (Chapter 4), John Wiley, New York; 1964.

and in:

C. F. Wagner, "Parallel Inverter with Resistive Load," AIEE Trans., vol. 54, pp. 1227-1235; Nov. 1935.

C. F. Wagner, "Parallel Inverter with Inductive Load," AIEE Trans., vol. 55, pp. 970-980; Sept. 1936.

Another inverter, known as a current-source inverter, is related to the parallel inverter in that there is inductance in series with the DC source and the commutating capacitor parallels the load. The current-source inverter is described in the following references:

B. R. Pelly, "Latest Developments in Static High-Frequency Power Sources for Induction Heating," IEEE Trans., vol. IECI-17, pp. 297-312; June 1970.

K. P. Phillips, "Current-Source Converter for AC Motor Drives," IEEE Trans., vol. IA-8, pp. 679-683; Nov./Dec. 1972.

W. Farrer and J. D. Miskin, "Quasi-Sine-Wave Fully Regenerative Inverter," Proc. IEE, vol. 120, pp. 969-976; Sept. 1973.

R. B. Maag, "Characteristics and Application of Current Source/Slip Regulated AC Induction Motor Drives," IEEE Industry & General Applications 6th Annual Meeting Record, pp. 411-416; Oct. 1971.

G. R. Slemon, et al., "Synchronous Motor Drive with Current-Source Inverter," IEEE Trans., vol. IA-10, pp. 412-416; May/June 1974.

## 12. Complementary Impulse-Commutated Inverter (Figure 4.10)

Impulse-commutated inverters require the commutating capacitor to reverse bias the thyristor which is being turned off only for a minimum time. Therefore the capacitor can be of minimum size. It does not have to absorb the energy stored in the load inductance, and therefore circuit operation is also more nearly independent of load variations.

Since the commutating capacitor does not absorb the inductive load energy, provision must be made to route this energy elsewhere. Bypass diodes directly across the thyristors form a freewheeling path which serves this purpose, but because of the long time constant of the low impedance path, the energy does not usually have a chance to diminish very far in the available time of a half cycle. The energy becomes "trapped" in a circulating current which hinders subsequent commutations. The circuits shown above, using tapped windings and slightly modified diode connections, couple this energy more quickly and efficiently back to the DC source, thereby avoiding unnecessary dissipation. The first circuit above was derived from the parallel inverter which has a center tapped load winding. The second circuit is equivalent, but was derived from the half bridge or center-tapped source configuration. These circuits, together with similar complementary impulse-commutated circuits having other minor modifications, and known as "McMurray-Bedford inverters" after their inventors.

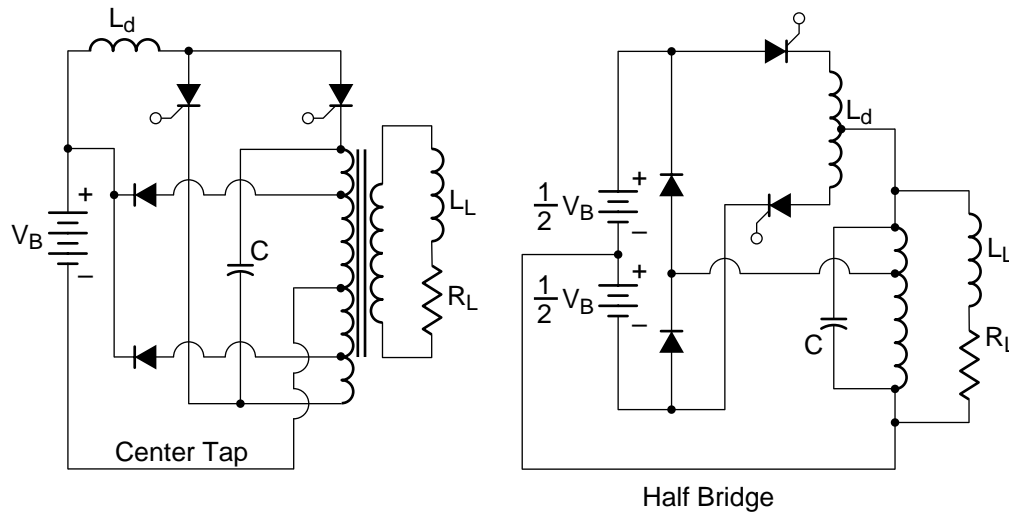
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B. D. Bedford and R. G. Hoft, *Principles of Inverter Circuits* (Chapter 7 by W. McMurray), John Wiley, New York; 1964.

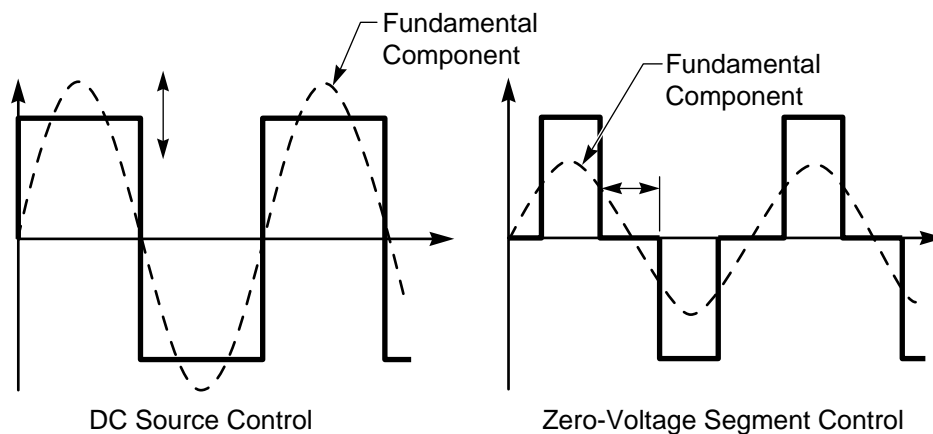
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## 13. Inverter Output Voltage Control (Figure 4.11)

Inverters are used primarily for variable-speed AC motor drives, for induction heating, and for reliable sources of AC power to critical loads such as computers (uninterruptible power supplies). They may be supplied either from a battery or other DC source, or from rectified AC.



**Figure 4.10**  
*McMurray-Bedford Inverters*



**Figure 4.11**  
*Ways of Controlling the Effective Output Voltage of an Inverter*

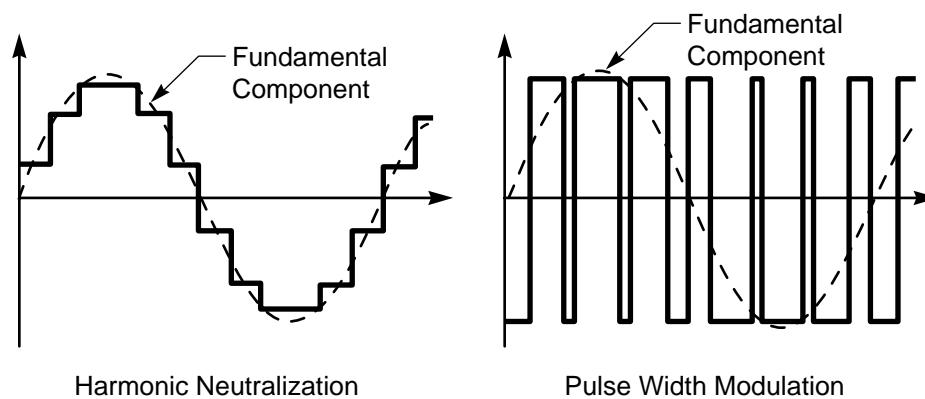
Two practical problem areas which have not been discussed in connection with inverters are:

- (a) control of the output, and
- (b) achieving a sinusoidal output voltage waveform.

Simple “phase control” of the firing signals changes the frequency but does not appreciably affect the output voltage of an inverter as it did in an AC/DC converter. One way of controlling the AC output voltage is to control the DC input voltage. If the DC originates from rectified AC, phase control of the

rectifier will serve this purpose. If the DC source is fixed, such as a battery, a chopper can be used ahead of the inverter. Although control of the DC input voltage is a useful technique where only a limited control range of the AC output is desired, wide variations lead to problems in achieving satisfactory commutation. An alternate approach is to use a full bridge configuration and insert controllable segments of zero voltage into the output waveform. The effective value of the waveform will depend on the relative length of these zero segments.





**Figure 4.12**

*Ways of Eliminating Low-Order Spurious Frequencies to Improve the Output Voltage Waveform of an Inverter*

#### 14. Inverter Output Waveform Improvement

(Figure 4.12)

Improvement of the basic square-wave voltage output of an inverter may be sought in three different ways. The first, brute-force way is to filter the output so as to adequately attenuate the harmonic frequencies, leaving only the sinusoidal fundamental frequency. This approach is used where the requirements on harmonic output are not too severe, but the filter soon becomes both large and expensive for tighter requirements.

The other two ways seek to minimize the size of the filter by modifying the output waveform so as to cancel the lower order harmonics which otherwise are both largest in amplitude and most difficult to attenuate. One approach, known as pulse width modulation (PWM), involves repetitive switching of a single inverter during each half cycle. The other, known as harmonic neutralization, involves a number of series inverters staggered in phase but each switching at the fundamental rate.

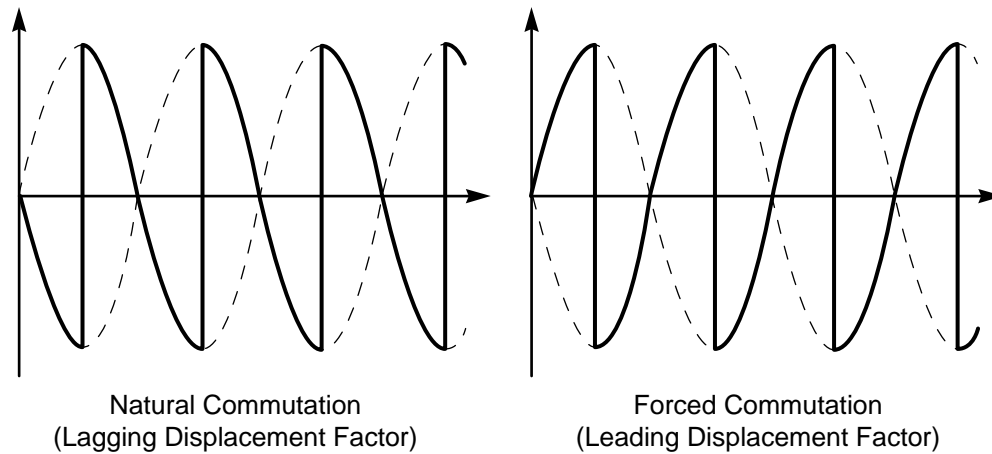
One form of PWM inverter derives its firing control signals by comparing a sinusoidal reference signal at the desired frequency with a sawtooth carrier signal of a considerably higher frequency. The firing signals are generated at the instants when the other two signals are equal. With this type of inverter, spurious frequencies in the output waveform cluster around the carrier frequency and its harmonics rather than at harmonics of the desired power frequency.

In a harmonic neutralized inverter,  $N$  conventional inverter stages which are staggered in phase by  $180^\circ/N$  are connected in parallel across the DC

source but in series across the AC output terminals. The amplitude of each stage is adjusted so as to cancel all harmonics less than  $(2N-1)$  in the  $2N$ -stepped approximation to a sine wave. For the waveform shown,  $N=6$  and the inverter stages are staggered by  $180^\circ/6=30^\circ$ . The  $90^\circ$  stage is degenerate (i.e., has zero amplitude) so that only five stages are required. The lowest order harmonic appearing in the output is the eleventh.

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**Figure 4.13**

*The Two Complementary Waveforms of a Two-Pulse Frequency Changer Without Phase Modulation*

T. M. Corry, "A New Concept for Generating Three-Phase Sine Wave Voltages with Semiconductor Power Switches," IEEE Power Electronics Spec. Conf. Rec., pp. 230-236; June 1973.

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H. S. Patel and R. G. Hoft, "Generalized Techniques of Harmonic Elimination and Voltage Control in Thyristor Inverters: Part I - Harmonic Elimination," IEEE Trans., vol. IA-9, pp. 310-317; May/June 1973.

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J. J. Pollack, "Advanced Pulsewidth Modulated Inverter Techniques," IEEE Trans., vol. IA-8, pp. 145-154; Mar./Apr. 1972.

R. D. Adams and R. S. Fox, "Several Modulation Techniques for a Pulsewidth Modulated Inverter," IEEE Trans., vol. IA-8, pp. 636-643; Sept./Oct. 1972.

## 15. Frequency and Power Factor Changers

(Figure 4.13)

Frequency changers are basically dual converters which allow periodic modulation of the switching control signals rather than simple phase delay. Because of its dependence on natural commutation, the cycloconverter is a special case which is limited to output frequencies which are less than about 1/3 of the source frequency and to a lagging input displacement factor. Converters with self commutation have neither of these restrictions.

If the switches can interrupt current, it is obvious that the source waveform(s) can be chopped into brief segments to synthesize output frequencies which are higher than the source frequency. The fact that independent control can be exercised over the displacement factor (and thereby the power factor) is not as obvious. The control logic which determines when each input is connected to the output and for how long is not uniquely specified by the output frequency. In the cycloconverter, the control logic is constrained by the requirements of natural commutation. With self commutation it can be chosen to implement other features. For example, it can be chosen to minimize the ripple or deviation from the desired output waveform. Or by tolerating more ripple, the converter can control the displacement angle seen by the source.

The mathematical theory of these generalized converters has only recently been explored and is beyond the scope of this text. However, their operation can be explained qualitatively in terms of a single-phase two-pulse example. Imagine a dual converter operating as a naturally commutated cycloconverter. The output waveforms shown above occur when  $\alpha = 90^\circ$  and the average output is zero. The waveform on the left occurs if the output current is positive, and the one on the right if the output current is negative. Natural commutation must occur to a phase which is more positive when the current is positive, and to a phase which is more negative when the current is negative. The desired output frequency is generated by periodically modulating  $\alpha$  about  $90^\circ$  at this frequency, thereby modulating the average value of each switching cycle about zero. Inherent in

this naturally commutated mode of operation is a lagging displacement factor at the AC input.

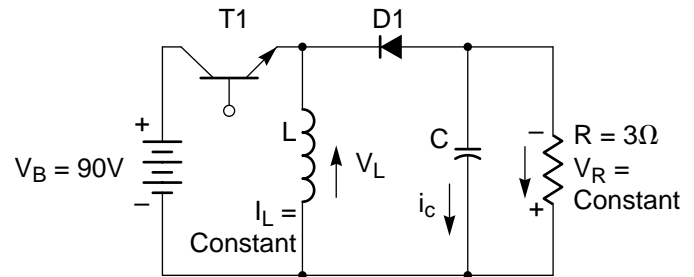
If the dual converter is self commutated, it can be made to commute at any time. It can, for instance, generate the “naturally commutated” waveforms just described. It can also generate waveforms which are exactly complementary to these. That is, it can commute to a phase which is more negative when the current is positive, and to a phase which is more positive when the current is negative. In this complementary mode, it would generate an equivalent output except that the displacement factor at the input would change sign and be leading.

Since the converter can generate equivalent outputs with either a lagging or leading input displacement factor depending on which mode is selected, it can also generate controllable intermediate displacement factors by interleaving the two modes. The details of the interleaving process are determined by the control logic which characterizes the various types of frequency changers. It can, for instance, maintain the displacement factor constant at unity for varying load power factors. With different control logic, the converter can be made to change the sign of the load displacement factor. This converter is called an Unrestricted Frequency Changer (UFC), and causes an inductive load to appear capacitive to the source, or a capacitive load to appear inductive.

Admittedly this explanation of frequency and power factor changers is brief. Hopefully it has been suggestive of the very general functional capabilities of self-commutated power converters which incorporate the tremendous computational versatility of today's small-signal electronic circuits. The full implications of being able to reliably and repeatedly switch megawatts in microseconds are not widely appreciated, even among electrical engineers. Yet the theoretical and practical possibilities offer a wealth of challenge to the imaginative designer.

## 16. Problems

### Problem 4.1 Operation of a Thyristor Chopper



The circuit shown above is used as a DC regulator or chopper. Assume that the reactive components,  $L$  and  $C$ , are so large that  $I_L$  and  $V_C (= V_R)$  are constant over a cycle.

- If transistor T1 is on for 25% of each cycle, calculate and plot the waveforms for  $V_L$  and  $i_C$ . Also calculate the load voltage, current, and power. Show that the average power drawn from the battery is equal to the load power.
- Repeat (a) for 75% duty cycle on T1.
- What can this chopper circuit do which the two step-down and step-up chopper circuits shown in the course notes cannot do?

### Problem 4.2 Impulse Commutation of a Thyristor Chopper

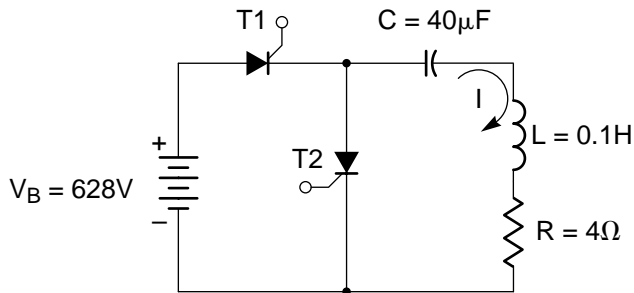
The purpose of this problem is to investigate some of the basic relationships and constraints involved in the design of a thyristor chopper using auxiliary impulse commutation, the circuit for which is shown in the course notes. Assume that the chopper is to operate at a fixed frequency of 400 Hz and that the nominal duty cycle is to be variable from 20% to 100%. For proper turn-off, the main thyristor T1 must be reverse biased for at least 50  $\mu\text{sec}$ . The filter inductor is 0.2 H, and it may be assumed initially that  $I_d$  is constant over each cycle. The DC source voltage is 1000V, and the load resistor is 10 ohms.

- Calculate the minimum value of  $C$  which is required to give satisfactory commutation over the reduced range of duty cycle. What is the minimum  $dv/dt$  rating for thyristor T1? Sketch the waveform for the voltage  $V_{out}$  for the limiting values of the duty cycle.

- (b) Calculate the maximum energy stored in L and C and compare the two values. Note that impulse commutation allows the ratio of these two energies to be quite high.
- (c) Check the assumption that  $I_d$  is constant over a cycle by calculating the worst-case peak-to-peak ripple as a percentage of  $I_d$ .
- (d) Investigate the influence of the value of the charging resistor  $R_C$  on circuit operation and attempt to specify a suitable value.

#### Problem 4.3 Operation of a Series Inverter

The purpose of this problem is to explore some of the important aspects of the operation of the series inverter which was discussed in the course notes. The circuit is as follows:



In order to produce a reasonably sinusoidal output, this type of inverter is frequently designed to be considerably underdamped, and is driven with a very small conduction gap (assumed negligible) between the extinction of one thyristor and the firing of the other one. Assuming sinusoidal waveforms and that the circuit operates at its resonant frequency ( $f = 1/2\pi\sqrt{LC}$ ):

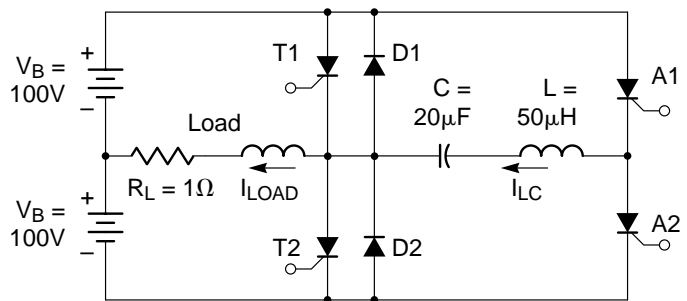
- (a) Sketch the waveforms of voltage and current for each component in the circuit. Show by appropriate shading those areas which must be equal in order that the average capacitor current and inductor voltage are zero.
- (b) The average power supplied by the battery must be equal to the average power dissipated by the load resistor. Use this fact to calculate the current I. Then use I to calculate the voltage across each component, and appropriately label the waveforms in (a).
- (c) Calculate the peak energy which the commutating capacitor must be able to store. Calculate the peak

energy in L. How do these two values compare? Is the comparison consistent with the discussion of "load-dependent" commutation?

- (d) Compare the KVA ratings required for L and C with the KW rating of the load. What KVA rating (peak blocking voltage x average current) is required for thyristors T1 and T2? Are the components utilized efficiently?
- (e) Would you expect this circuit to have good voltage regulation for changing load resistance? Explain.
- (f) Aside from the small conduction gap which has been assumed to simplify the analysis, is there a danger that turning on one thyristor will cause the other one to turn on also? Explain. Discuss how the waveforms and analysis of the circuit would change if the switching frequency were decreased to permit a longer conduction gap.

#### Problem 4.4 Resonant Impulse Commutation

Shown below is a half-bridge auxiliary impulse commutated inverter. Commutation of main thyristors T1 and T2 is achieved by the resonant charge reversal on C via L and auxiliary thyristors A1 and A2. The initial voltage on C depends on Q and on the load, but for the purposes of this problem assume that  $V_C(0) = 250V$  with the polarity shown, and that T1 is initially conducting. Assume that the load current has reached equilibrium at  $I_{LOAD} = V_B / R_L = 100A$  and that the load inductance is large enough to hold  $I_{LOAD}$  constant during the commutation interval.



To commutate T1 off, A1 is fired. As  $I_{LC}$  increases,  $I_{T1}$  decreases by a corresponding amount. When  $I_{LC}$  exceeds  $I_{LOAD}$ ,  $I_{T1}$  becomes zero and the excess current flows through D1. When  $I_{LC}$  decreases below  $I_{LOAD}$  again, T1 has regained its blocking ability,  $I_{LOAD}$  transfers to D2, and the polarity of the load voltage reverses. C recharges to  $-V_C(0)$  before A1

ceases conduction, and is therefore ready to commute T2 off one-half cycle later.

Determine the maximum rated turn-off time ( $t_q$ ) for T1 and T2.

#### **Problem 4.5 Harmonic Neutralization**

The amplitudes of the square-wave components of a harmonic neutralized inverter are given by:

$$V_n = \frac{\pi V_{RMS}}{\sqrt{2N}} \cos \frac{n\pi}{N}$$

where  $V_{RMS}$  is the desired RMS value of the fundamental output frequency and  $N$  is the number of inverter stages. (a) Calculate the amplitudes of the components for a six-stage inverter used to synthesize a sine wave having  $V_{RMS} = 120$  V. (b) Plot the desired sinusoidal output waveform, and superimpose upon it the actual stepped output waveform. (c) Sketch the difference between these two waveforms which constitutes the remaining ripple. What is its peak amplitude as a percentage of  $V_{RMS}$ ? What is the fundamental frequency of the ripple relative to the desired output?

#### **Problem 4.6 Operation of a Frequency Changer**

The Unrestricted Frequency Changer (UFC) is a particular type of force-commutated frequency changer which has the following desirable properties:

1. The time between successive commutations is constant and is given by:

$$T_c = \frac{1}{P(F_{in} + F_{out})}$$

where  $f_{in}$  and  $f_{out}$  are the input and output frequencies and  $P$  is the pulse number.

2. The phases always conduct in the same sequence and for the same length of time.
  - (a) Using these properties of the UFC, construct the output waveform for a three-pulse UFC having  $f_{out}/f_{in} = 1/5$ .
  - (b) If the load draws a  $30^\circ$  lagging current, indicate which of the commutations occur “naturally”, and which must be “forced”.
  - (c) Using the same commutation frequency as before, construct an output waveform which could be achieved with a naturally commutated cycloconverter. Compare the conduction sequence with that in (a).
  - (d) Repeat (a) for  $f_{out}/f_{in} = 5/1$ .

# Chapter 5

## POWER SEMICONDUCTOR DEVICE PROTECTION

### Section

1. The Power Module	5-2
2. Importance of Junction Temperature	5-2
3. Components of Dissipation	5-3
4. Average Dissipation at 60 Hz Switching Rate	5-4
5. Average Dissipation at 40 KHz Switching Rate	5-4
6. Simplified Geometry Assumed for the "Typical" Device	5-4
7. Junction-to-Case Thermal Resistance	5-5
8. Heat Sink Calculation	5-5
9. Selection of Device Package and Type of Heat Sink	5-5
10. Natural Convection Air Cooling	5-6
11. Forced Air Cooling	5-6
12. Water Cooling	5-7
13. Using the Data Sheet for Heat Sink Calculations	5-7
14. Thermal Capacity	5-14
15. Transmission Line for Transient Thermal Analysis	5-14
16. Initial Temperature Rise	5-15
17. Transient Thermal Impedance of Each Section	5-15
18. Calculation of Thermal Time Constant	5-16
19. Junction-to-Case Transient Thermal Impedance	5-16
20. Using Transient Thermal Impedance	5-17
21. Calculation of Pulse and Surge Current Capabilities	5-17
22. Turn-On $di/dt$	5-19
23. Transient Overvoltage	5-20
24. Off-State $dv/dt$	5-21
25. Series/Parallel Arrays for High-Power Applications	5-22
26. Summary	5-23
27. Problems	5-23

### Introduction

The objectives of this chapter are to provide information which will enable the reader:

- To evaluate and compare the various components of dissipation within a solid-state device in a variety of applications.
- To calculate the requirements and select a heat sink for a specified device dissipation.
- To estimate the steady-state and transient thermal impedance and the thermal time constant of a simple device geometry.
- To calculate the rise in junction temperature caused by a current pulse or surge.
- To explain the physical basis for voltage, current,  $dv/dt$  and  $di/dt$  ratings on devices and the approaches that are used to protect solid-state devices from stresses in excess of these ratings.
- To discuss the maximum voltage and current capabilities of presently available power devices and the precautions that are required in the design of series/parallel arrays of devices to extend the maximum power capabilities of solid-state equipment.

Understanding of these topics is vital in the design of solid-state power equipment if its full potential reliability is to be achieved. It is in this area that intuition developed from design experience in traditional power or electronic equipment is most misleading and most likely to cause unforeseen problems in the design.

## 1. Power Module

The Power Module Protects the Power Device from:

- Junction Temperature
- Voltage
- Current
- $dv/dt$
- $di/dt$

in Excess of its Ratings.

The economic feasibility of a power converter may be determined either by the transformers, inductors and capacitors or by the solid-state components. On the other hand, technical feasibility is nearly always limited by the current state of the art of available solid-state devices. The function of the power module is to interface the solid-state power device and its limitations with the requirements and peculiarities of the power circuit so as to achieve technical feasibility, reliability, and overall minimum cost. In particular, the purpose of the power module is to protect the power device from junction temperature, voltage, current,  $dv/dt$ , and  $di/dt$  in excess of its ratings. Each of these factors is associated with a corresponding potential failure mechanism.

Unfortunately, many factors interact in determining whether or not a solid-state power device will fail to operate properly or be damaged under a particular set of conditions. In arriving at the ratings shown on the specification sheet, a device manufacturer tests his devices under set of conditions which may or may not be representative of a particular application. (And since the test conditions are not uniform between manufacturers, comparisons are not always easy to make.) One manufacturer may be conservative and rate his devices under worst-case conditions, but this puts him in an apparent price disadvantage compared to another manufacturer who may for example specify typical values. Typical values can not be used for design purposes because processes do change and the typical values may be altered significantly. It is not enough to simply compare device data sheets the actual capability of the device should be carefully evaluated. The power semiconductor device selected should consider the trade-offs of price, performance, and availability of a continuing supply of reliable devices. Frequently one rating is of crucial concern to a designer and he would like to know whether he can safely exceed that rating if he stays well below other ratings, or whether he must buy a premium device with apparent excess

capacity on most of its ratings. There are no infallible rules, but skepticism is advisable. The neophyte designer is cautioned that the reliability of solid-state power devices tends to be a precipice function. Within their capabilities, they can last much longer than the equipment they are controlling. But if their capabilities are exceeded, their lifetime may literally be measured in microseconds! All other avenues should be exhausted before trying to squeeze the last cent out of the cost of the solid-state power devices which are selected.

With these forewarnings, we will proceed to explore the fundamentals of device protection, beginning with thermal considerations. Device protection principles will be described primarily using thyristors characteristics and ratings; however, most of these principles apply equally well to all power semiconductor devices.

Additional information on device rating and procurement can be found in:

J. S. Read and R. F. Dyer, "Power Thyristor Rating Practices," Proc. IEEE, vol. 55, pp. 1288-1301; 1967.

M. C. Sardi, "Semiconductor Yield and Life Cycles," Electro Procurement, November 1973.

## 2. Importance of Junction Temperature (*Table 5.1*)

Nearly all of the characteristics of solid-state devices are sensitive to the internal junction temperature. Since ratings are merely design limits on characteristics, they too are temperature dependent. Therefore thermal protection will be considered first.

Although the dissipation in switching devices can often be ignored in the analysis of the power circuit (as we have done), it is a vital factor in the choice of the device itself and its heat sink. Since the dissipation is highly concentrated at the junction of a device where temperature is critical, and since we do not have access to the junction to measure its temperature directly, thermal design must be performed analytically. Both the average junction temperature when the device is nearly in thermal equilibrium and the instantaneous peak junction temperature resulting from a transient are important, but each involves a different analytical approach.

In order to illustrate what is involved in thermal calculations, we will analyze a “typical” thyristor which, although grossly oversimplified, nevertheless yields values which are representative of actual devices. The assumed characteristics of this thyristor are listed at the beginning of this section. The important characteristics include the forward voltage drop, the leakage current, and the switching times, all of which have previously been neglected.

**Table 5.1**

*Assumed Characteristics of a “Typical” Thyristor*

$I_{T(RMS)}$	Current Rating	100A
$V_{RRM}, V_{DRM}$	Blocking Voltage Rating	600V
$V_T$	Forward Voltage Drop at 100A	1.4V
$I_{DRM}, I_{RRM}$	Leakage Current at 600V	10mA
$t_{ON}$	Turn-On Time	3ms
$t_{OFF}$	Turn-Off Time	20ms

### 3. Components of Dissipation (Figure 5.1)

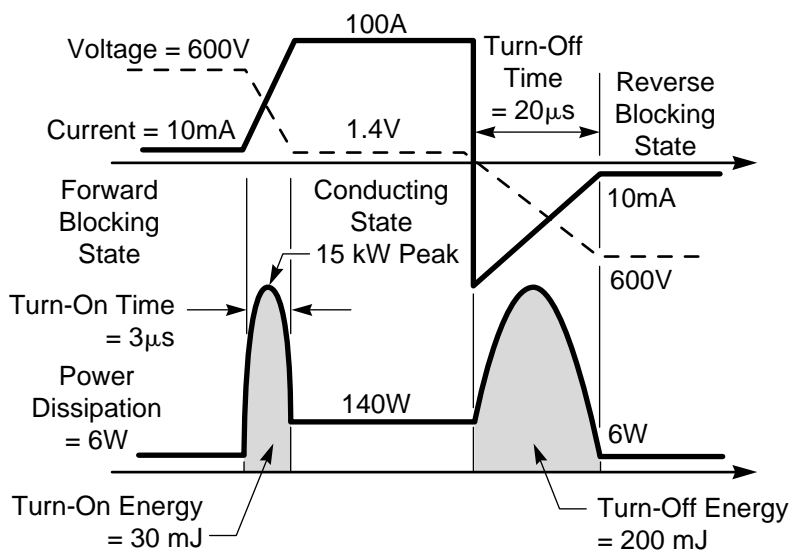
Junction temperature depends both on the dissipation or heat input to the device and on the thermal resistance encountered by the heat in flowing to a cooling medium. In normal cyclic operation, the dissipation is not constant but varies as a function of time, depending on the state of the switch. There are five distinct components of dissipation corresponding to the forward blocking state, the reverse blocking state, the conducting state, the turn-on interval, and the turn-off interval. (Power

dissipated by gating signals will be neglected here, but may be significant in some applications.) For present purposes, the two blocking states are assumed to have equal dissipations, and it is assumed that the device is loaded to its full ratings, 600V and 100A, thereby controlling a 60 KW load.

In either blocking state, the device dissipates  $6000V \times 10 \text{ mA} = 6W$ , while in the conducting state it dissipates  $1.4V \times 100A = 140W$ . For simplicity we will assume that these values are constant rather than variable with time, as they are with sinusoidal waveforms.

During the switching transitions we cannot avoid the fact that the dissipation changes rapidly. During the  $3\mu s$  turn-on interval, the voltage across the device must decrease from 600V to 1.4V, while the current is increasing from 10 mA to 100A. The instantaneous power is the instantaneous product of voltage and current, and hence it depends on the dynamic curves followed by the voltage and current. Again for simplicity we will assume that these transistors are linear, as shown in figure 5-1. The power waveform during turn-on is then an inverter parabola having a peak value of 15 kW! Obviously, when subjected to this rate of dissipation, the junction temperature rises very rapidly. The energy dissipated in the switch during the turn-on transient is the area under the power parabola,  $(2/3) \times 15 \text{ kW} \times 3\mu s = 30 \text{ mJ}$ .

Assuming the same linear transitions during the turn-off interval, the dissipation again peaks at 15 kW, although the energy increases to 200 mJ because of the longer turn-off time.



**Figure 5.1**

*Assumes Linear Switching Transitions Showing Components of Dissipation in Switching Device*



**Table 5-2**

*Comparison of the Components of Dissipation for a 60Hz Switching Rate with a 50% Duty Cycle*

**On-Time = Off-Time = 8.3ms**

Conducting Dissipation	140W x 8.3ms = 1.17J
Blocking Dissipation	6W x 8.3ms = 0.05J
Turn-On Dissipation	0.03J
Turn-Off Dissipation	0.20J
Total Dissipation per Cycle	1.45J

$$\text{Average Power Dissipation} = \frac{1.45\text{J}}{16.7\text{ms}} = 87\text{W}$$

#### 4. Average Dissipation at 60 Hz Switching Rate

(Table 5.2)

To calculate the average junction temperature, we must first calculate the average dissipation under specified conditions. For example, with equal on-and-off-times at 60 Hz, the various components of dissipation per cycle are calculated above, and the average dissipation is 87W. For low switching rates, the switching times are very short compared to the total period, and the average dissipation is determined primarily by the dissipation in the conducting state.

#### 5. Average Dissipation at 40 KHz Switching Rate

(Table 5.3)

To emphasize what happens at high switching rates, let us consider an extreme case. Judging by the switching times alone, it should be possible to complete a full cycle in 25μs, corresponding to a switching rate of 40 kHz. Again the individual components of dissipation and the average dissipation are calculated above. The switching components of dissipation are now dominant, and it is seen that the average dissipation has risen to a phenomenal 9.2 kW! When the small physical size of a solid-state device with a 100A rating is considered, it is obviously impossible to dissipate this much heat at a tolerable temperature.

Therefore the maximum frequency at which switching is feasible depends on switching dissipation as well as on switching speed, and the current rating of a device must be decreased at high frequencies. Since the switching dissipation tends to be a constant

**Table 5-3**

*Comparison of the Components of Dissipation for a 40Hz Switching Rate with a 50% Duty Cycle*

**On-Time = Off-Time = 0.5 (25 – 3 – 20) = 1μs**

Conducting Dissipation	140.W x 1μs =	0.140mJ
Blocking Dissipation	6.W x 1μs =	0.006mJ
Turn-On Dissipation		30. mJ
Turn-Off Dissipation		200. mJ
Total Dissipation per Cycle		230. mJ

$$\text{Average Power Dissipation} = \frac{230\text{mJ}}{25\mu\text{s}} = 9.2\text{kW}$$

amount of energy per cycle while the conducting and blocking components tend to be constant power, the “corner frequency” above which switching dissipation is a dominant factor can be estimated from:

$$f_c \approx \frac{\text{Conducting \& Blocking Power Averaged Over a Cycle}}{\text{Switching Energy Per Cycle}}$$

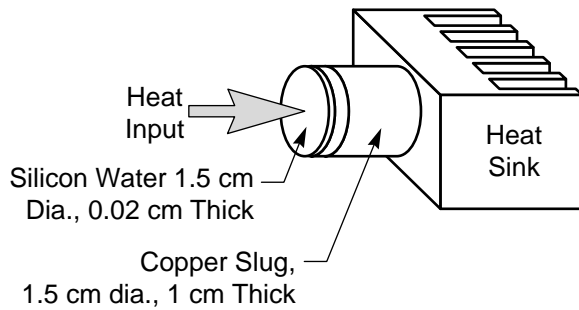
For our “typical” thyristor operating at 100A with a 50% duty cycle, the conducting and blocking power averaged over a cycle is (140 + 6)/2 = 73W, and the switching energy per cycle is 0.23 J. Thus  $f_c \approx 73/0.23$  or about 300 Hz.

**Note:** Although this simplified analysis seems to indicate that the dissipation during turn-off is somewhat more important than during turn-on, the opposite is usually true in practice. The turn-on dissipation is concentrated near the periphery of the gate electrode of a thyristor, causing a hot spot whose average and peak temperatures can greatly exceed those elsewhere. In addition, the turn-off waveforms are highly oversimplified.

#### 6. Simplified Geometry Assumed for the “Typical” Device

(Figure 5.2)

In addition to average dissipation, the other factor which determines average junction temperature is the thermal resistance between the junction and the cooling medium. Part of this resistance is internal to the packaged device as purchased, and part is determined by the heat sink and the flow rate of the coolant. To facilitate calculations, we will assume the



**Figure 5.2**

*Simplified geometry assumed for a silicon switching device mounted on a heat sink*

simplified device geometry shown above. The dissipation is assumed to occur at one surface of the silicon wafer than at the junction within the silicon, and the device package is assumed to be equivalent to a cylindrical copper slug. We would like to calculate the heat sink which is required for a specified average junction temperature and a specified average dissipation. First we must calculate the “junction-to-case” thermal resistance.

## 7. Junction-to-Case Thermal Resistance

*Calculation of Steady-State Thermal Resistance for the Silicon Wafer and Copper Slug*

$$R_{\theta}(\text{Silicon}) = \frac{d}{kA} = \frac{2 \times 10^{-4} \text{ m}}{(84 \text{ W/m}^{\circ}\text{C})(1.5^2 \times 10^{-4})(\pi/4) \text{ m}^2} = 0.0135^{\circ}\text{C/W}$$

$$R_{\theta}(\text{Copper}) = \frac{d}{kA} = \frac{1 \times 10^{-2} \text{ m}}{(388 \text{ W/m}^{\circ}\text{C})(1.5^2 \times 10^{-4})(\pi/4) \text{ m}^2} = 0.146^{\circ}\text{C/W}$$

The thermal resistance of the cylindrical silicon wafer is equal to the thickness (d) divided by the product of the thermal conductivity (k) and the cross-sectional area (A)- (in a consistent set of units). For the dimensions assumed previously, this resistance is calculated above and is found to be 0.0135°C/W. That is, in steady state, 100W of dissipation will cause a temperature differential across the silicon of only 1.35°C.

Similarly the thermal resistance of the copper package is calculated to be 0.146°C/W. The sum of these two

thermal resistances gives the “junction-to-case” thermal resistance. This value is listed by the device manufacturer on the data sheet, so that it need not be calculated. We have gone through the calculation simply to show what is involved.

## 8. Heat Sink Calculation

*Heat Sink Calculation Assuming Switch Must Be Capable of Carrying 100A Continuously (140W) in a 35°C Ambient with a 120°C Junction Temperature*

$$R_{\theta}(\text{Total}) = \frac{T_J - T_A}{\text{Ave. Dissipation}} = \frac{(120 - 35)^{\circ}\text{C}}{140 \text{ W}} = 0.607^{\circ}\text{C/W}$$

$$R_{\theta}(\text{Heat Sink}) = R_{\theta}(\text{Total}) - R_{\theta}(\text{Silicon}) - R_{\theta}(\text{Copper}) = 0.607 - 0.014 - 0.146 = 0.447^{\circ}\text{C/W}$$

Assume that the switch must be capable of carrying 100A continuously, that the maximum temperature of the cooling ambient is 35°C, and that the average junction temperature is to be 120°C. The total “junction-to-ambient” thermal resistance cannot exceed 0.607°C/W, as calculated above. Subtracting the junction-to-case thermal resistance gives the maximum thermal resistance of the heat sink (and the interface between the device and the heat sink) as 0.447°C/W.

Obviously, this calculation may be reversed to obtain the average junction temperature for a specified heat sink and average dissipation.

## 9. Selection of Device Package and Type of Heat Sink

Early semiconductor rectifiers and thyristors were stud-mount devices. They were easy to mount and could be installed by almost any manufacturer. This package design was suitable for many years, since the current rating/device never exceeded 200 to 300A. As the demand for higher current devices evolved, the *disc* package design was developed. (Trade names for disc-type power semiconductors include “Pow-R-Disc”, “Hockey Puck” and “Press Pak”.)

Unlike the stud-mount device example we have been considering, where all of the heat is dissipated through the stud base of the package to a single heat sink, the disc-packaged device dissipates heat through both upper and lower pole faces to two separate heat sinks.

This effectively cuts junction-to-case thermal impedance in half. Thus, a disc rectifier or SCR with double-sided cooling offers higher output current ratings than conventional stud-mount rectifiers or SCR's of equivalent element sizes. For example a disc unit can offer up to 80% more current capability and cost less than the same size silicon element in a stud package.

The selection of the best type of heat sink for a given application depends upon many factors such as volume & weight restrictions, economics, and the capacity required compared to available devices. The main categories of heat sinks are natural (convection) cooling, and forced convection cooling. For natural convection, air is normally the medium although oil may be used in high voltage applications. Forced convection air cooling is most widely used for power semiconductor applications although there has been an increased use of water cooling in recent years.

Another technique which is being developed but is not yet commercially available is the heat-pipe device package. This package places a cooling liquid in intimate contact with the silicon wafer. As the liquid absorbs its heat of vaporization, it becomes a gas which flows to another part of the package where the heat is exchanged with an outside cooling medium. The very high thermal efficiency of this type of cooling results from the fact that the vapor transport carries heat away with a very small temperature gradient compared to normal conduction (i.e., diffusion) of heat.

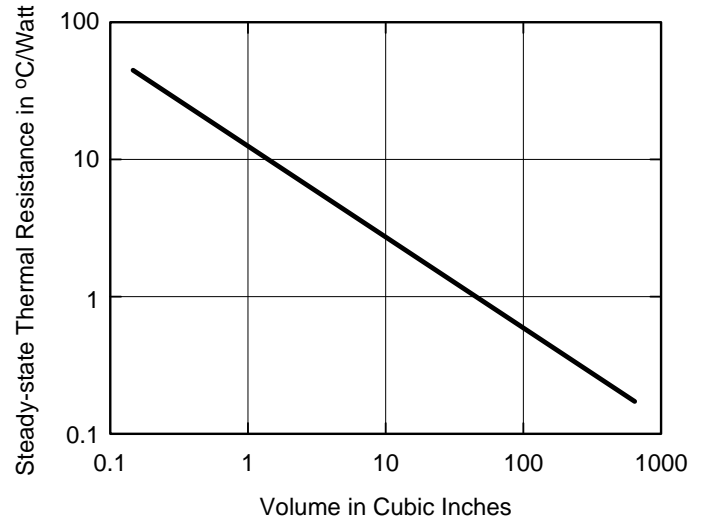
#### 10. Natural Convection Air Cooling (Figure 5.3)

The graph in Figure 5.3 shows the thermal resistance of one manufacturer's optimized line of natural convection cooled, finned heat sinks plotted against volume. For a specified thermal resistance, the closest larger standard heat sink should be selected. The thermal resistance of  $0.447^{\circ}\text{C}/\text{W}$  resulting from the previous example requires a heat sink having a volume of about 150 in.<sup>3</sup>.

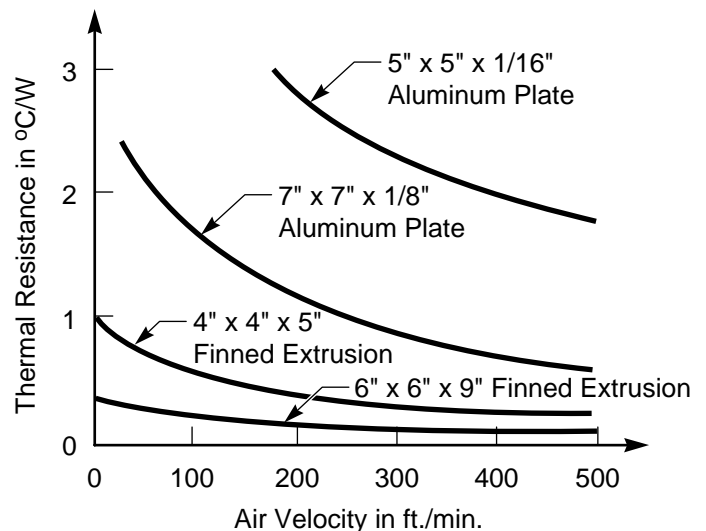
#### 11. Forced Air Cooling (Figures 5.4 & 5.5)

The thermal resistance of a heat sink may be decreased below its ratings with natural convection by using forced air cooling. Figure 5.4 illustrates the decrease that can be expected for several typical heat

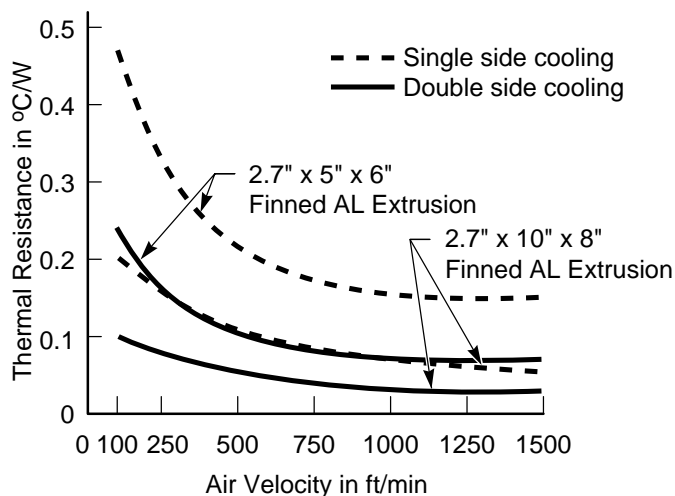
sinks. Figure 5.5 illustrates the low heat sink thermal resistance that can be realized for disc type devices and forced air cooling. Note that two of the specified heat sinks are used for double size cooling (solid line).



**Figure 5-3**  
*Thermal Resistance vs. Volume for an Optimized Line of Natural Convection Cooled, Finned Heat Sinks (at 50° Rise Above Ambient)*



**Figure 5-4**  
*Forced Air Cooling of Stud Type Devices*



**Figure 5-5**  
*Forced Air Cooling of Stud Type Devices*

## 12. Water Cooling

Water cooling is the most efficient type of cooling in general use today. While it may not be a practical solution to every design problem, more and more systems are being converted to water cooling each year. Inherent advantages in size, weight and cost simply cannot be overlooked.

A typical example of a water-cooled assembly is shown at the beginning of this chapter.

## 13. Using the Data Sheet for Heat Sink Calculations (Figure 5.6)

Figure 5.6 the data sheet for the Powerex Type T500 thyristor will be used to illustrate how the device manufacturer attempts to ease standard thermal calculations. For low frequencies (400 Hz or less) where switching losses are small, and for standard current waveforms such as square waves and phase-delayed sine waves, curves are given which show the maximum average power which will be dissipated and the maximum allowable case temperature (for 125°C junction temperature) versus the average current and the conduction angle. Another curve gives the forward voltage drop as a function of current.

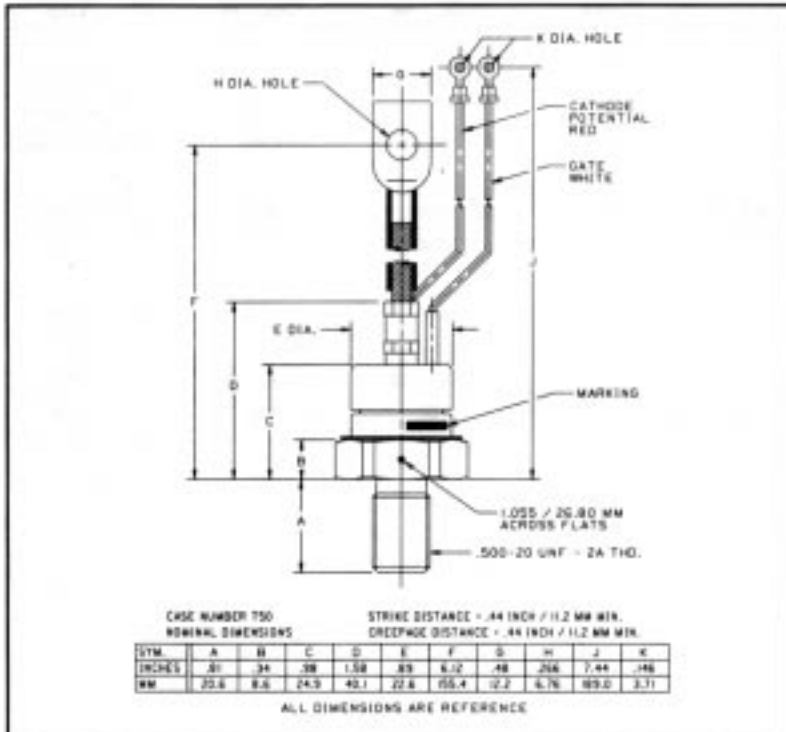
Using the T500–80 (Figure 5.6) with a square wave of current having an amplitude of 120A and a duty cycle of 50%. (The average current is then 60A and the conduction angle is 180°.) Figure 5.6-A shows that the

voltage drop at 120A is 1.5 volts. The dissipation during conduction is  $120A \times 1.5V = 180W$ , but since there is little dissipation during the other half cycle, the average dissipation is approximately  $180/2 = 90W$ . Figure 5.6-I shows the average dissipation for this square wave as 94W. Under “Thermal and Mechanical Characteristics,” we find the junction-to-case thermal resistance listed as  $0.28^{\circ}C/W$ . Therefore, the average junction temperature is  $94W \times 0.28^{\circ}C/W = 26^{\circ}C$  above the case temperature. For an average junction temperature of  $125^{\circ}C$ , the corresponding case temperature would be  $99^{\circ}C$ . Figure 5.6-J shows the maximum allowable case temperature for 60A average and 180° conduction as  $92^{\circ}C$ . This curve has a  $7^{\circ}C$  margin to allow for the fact that the peak junction temperature will exceed the average temperature.

Figures 5.6-E and 5.6-F give dissipation and maximum case temperature for phase-delayed half-wave sinusoidal waveforms.

Note that although this thyristor is rated 125A RMS or 80A half-wave average, its average current capacity must be derated if the conduction angle is less than 180° or if the heat sink is inadequate to maintain the case temperature at or below  $76^{\circ}C$ .

### Phase Control SCR 40-80 Amperes (63-125 RMS) 1600 Volts



T500 Phase Control SCR  
40-80 Amperes (63-125 RMS),  
1600 Volts

T500, TO-94 (Outline Drawing) Also Available with Flag Lead, TO-83 Package

#### Ordering Information:

Select the complete part number you desire from the following table:

Type	Voltage *		Current		Turn-off		Gate Current		Leads	
	V <sub>DRM</sub> & V <sub>RRM</sub> (Volts)	Code	I <sub>T(av)</sub> (A)	Code	I <sub>g</sub> (μsec)	Code	I <sub>GT</sub> (mA)	Code	Case	Code
T500	700	07	40	40	100	0	100	5	TO-94	AQ
	800	08			(Typ.)		150	4		
	900	09					**			
	1000	10	80	80					TO-83	AB
	1100	11								
	1200	12								
	1300	13								
	1400	14								
	1500	15								
	1600	16								

\* For 600V and Below, see T510

\*\* For Lower I<sub>GT</sub> Consult Factory

Example: Type T500 rated at 80A average with V<sub>DRM</sub> = 1600V, I<sub>GT</sub> = 150MA, and standard flexible lead, order as:

Type	Voltage	Current	Turn-off	Gate Current	Leads
T 5 0 0	1 6	8 0	0	4	A Q

#### Features:

- ☐ Center Fired, di/namic Gate
- ☐ All Diffused Design
- ☐ Low V<sub>TM</sub>
- ☐ Compression Bonded Encapsulation
- ☐ Low Thermal Impedance
- ☐ High Surge Current Capability
- ☐ Low Gate Current

#### Applications:

- ☐ Phase control
- ☐ Motor Control
- ☐ Power Supplies

Figure 5-6  
T500 Data Sheet



Powerex, Inc., 200 Hillis Street, Youngwood, Pennsylvania 15697-1600 (724) 925-7272

T500 Phase Control SCR  
40-80 Amperes (63-125 RMS),  
1600 Volts

#### Absolute Maximum Ratings

Characteristics	Symbol	T500 _ 40	T500 _ 80	Units
RMS Forward Current	$I_{T(rms)}$	63	125	Amperes
Average Forward Current	$I_{T(av)}$	40	80	Amperes
One-half Cycle Surge Current	$I_{TSM}$	1200	1800	Amperes
3 Cycle Surge Current	$I_{TSM}$	950	1300	Amperes
10 Cycle Surge Current	$I_{TSM}$	800	1170	Amperes
Minimum Rate of Rise of On-State Current (Non-repetitive)	$di/dt$	800	800	Amperes/ $\mu s$
$I^2t$ (for Fusing), $\geq 8.3$ milliseconds	$I^2t$	6000	13500	$A^2sec$
Peak Gate Power Dissipation	$P_{GM}$	16	16	Watts
Average Gate Power Dissipation	$P_{G(av)}$	3	3	Watts
Storage Temperature	$T_{stg}$	-40 to +150	-40 to +150	$^{\circ}C$
Operating Temperature	$T_j$	-40 to +125	-40 to +125	$^{\circ}C$
Mounting Torque (Lubricated)		130	130	in-lb

Figure 5-6 (Continued)  
T500 Data Sheet

**T500 Phase Control SCR**  
40-80 Amperes (63-125 RMS),  
1600 Volts

## Electrical and Thermal Characteristics

Characteristics	Symbol	Test Conditions	T500 __40	T500 __80	Units
<b>Current - Conducting State Maximums</b>					
Forward Voltage Drop	$V_{TM}$	$T_J = 25^\circ\text{C}$ , $I_{TM} = 500\text{A}$	3.7	2.2	Volts
<b>Voltage - Blocking State Maximums</b>					
Rep. Peak Forward Blocking Voltage (Rated Limit)	$V_{DRM}$		1600	1600	Volts
Repetitive Peak Reverse Voltage (Rated Limit)	$V_{RRM}$		1600	1600	Volts
Non-Rep. Trans. Peak Rev. Voltage (Rated Limit)	$V_{RSM}$	$t_p \leq 5.0 \text{ msec}$	1800	1800	Volts
Forward Leakage Current	$I_{DRM}$	$T_J = 125^\circ\text{C}$ , $V_{DRM} = \text{Rated}$	10	10	mA
Reverse Leakage Current	$I_{RRM}$	$T_J = 125^\circ\text{C}$ , $V_{RRM} = \text{Rated}$	10	10	mA
<b>Switching</b>					
Typical Turn-off Time	$t_q$	$I_T = 50\text{A}$ , $di_T/dt = 5 \text{ A}/\mu\text{sec}$ , reapplied $dv/dt = 20\text{V}/\mu\text{sec}$ linear to $0.8 V_{DRM}$ , $T_J = 125^\circ\text{C}$	100	100	$\mu\text{sec}$
Typical Turn-on Time	$t_{on}$	$I_T = 100\text{A}$ , $V_D = 100\text{V}$	4	4	$\mu\text{sec}$
Minimum Critical $dv/dt$ Exponential to $V_{DRM}$	$dv/dt$	$T_J = 125^\circ\text{C}$	300	300	$\text{V}/\mu\text{sec}$
<b>Thermal</b>					
Maximum Resistance, Junction to Case	$R_{\theta(j-c)}$		0.28	0.28	$^\circ\text{C}/\text{Watt}$
Maximum Resistance, Case to Sink (Lubricated)	$R_{\theta(c-s)}$		0.12	0.12	$^\circ\text{C}/\text{Watt}$
<b>Gate - Maximum Parameters</b>					
Gate Current to Trigger	$I_{GT}$	$T_J = 25^\circ\text{C}$ , $V_D = 12\text{V}$	(See Ordering Information)		mA
Gate Voltage to Trigger	$V_{GT}$	$T_J = 25^\circ\text{C}$ , $V_D = 12\text{V}$	3	3	Volts
Non-Triggering Gate Voltage	$V_{GDM}$	$T_J = 125^\circ\text{C}$ , $V_{DRM} = \text{Rated}$	0.15	0.15	Volts
Peak Forward Gate Current	$I_{GTM}$		4	4	Amperes
Peak Reverse Gate Voltage	$V_{GRM}$		5	5	Volts

Figure 5-6 (Continued)  
T500 Data Sheet

**T500 Phase Control SCR**  
40-80 Amperes (63-125 RMS),  
1600 Volts

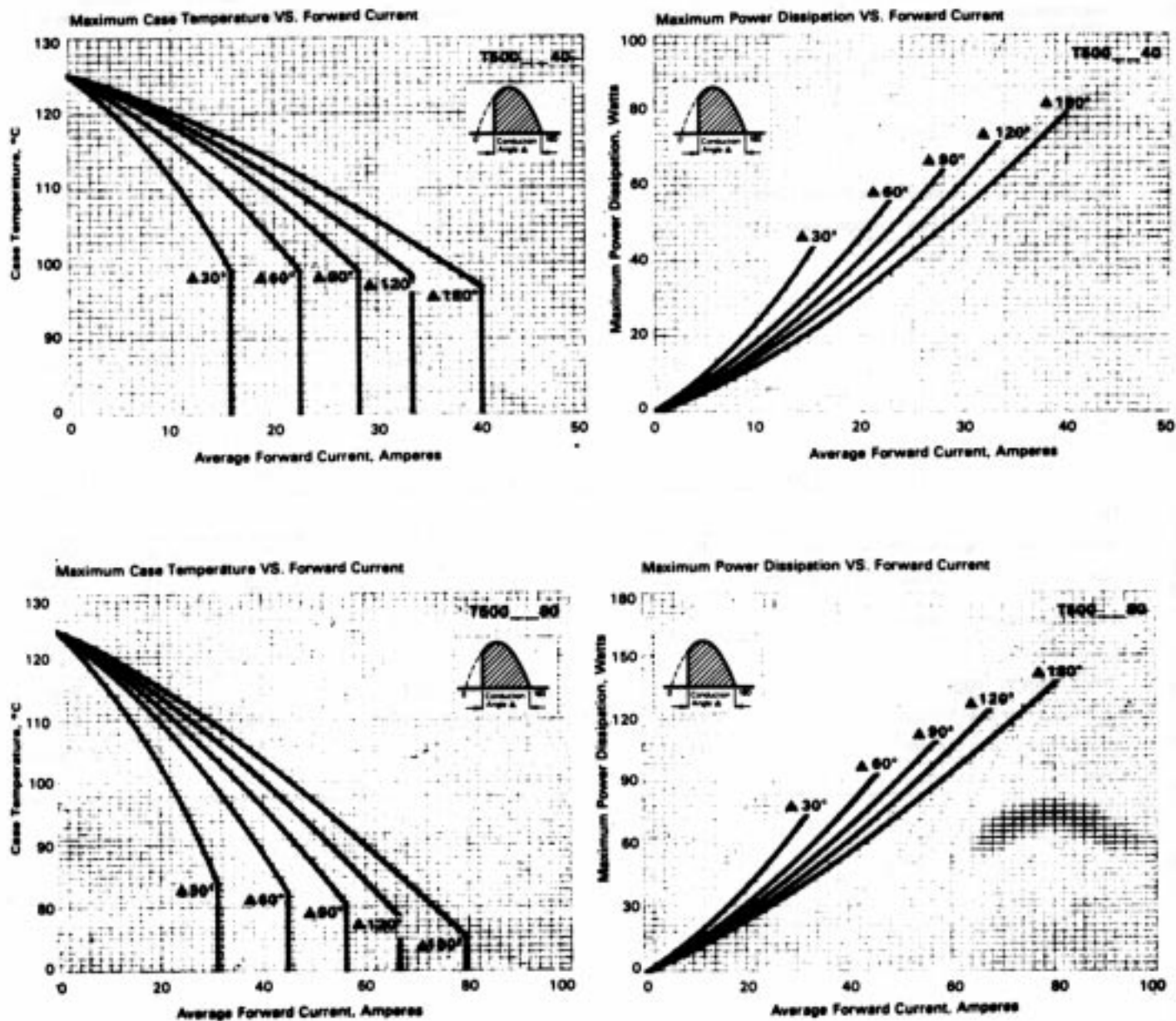


Figure 5-6 (Continued)  
T500 Data Sheet



**T500 Phase Control SCR**  
40-80 Amperes (63-125 RMS),  
1600 Volts

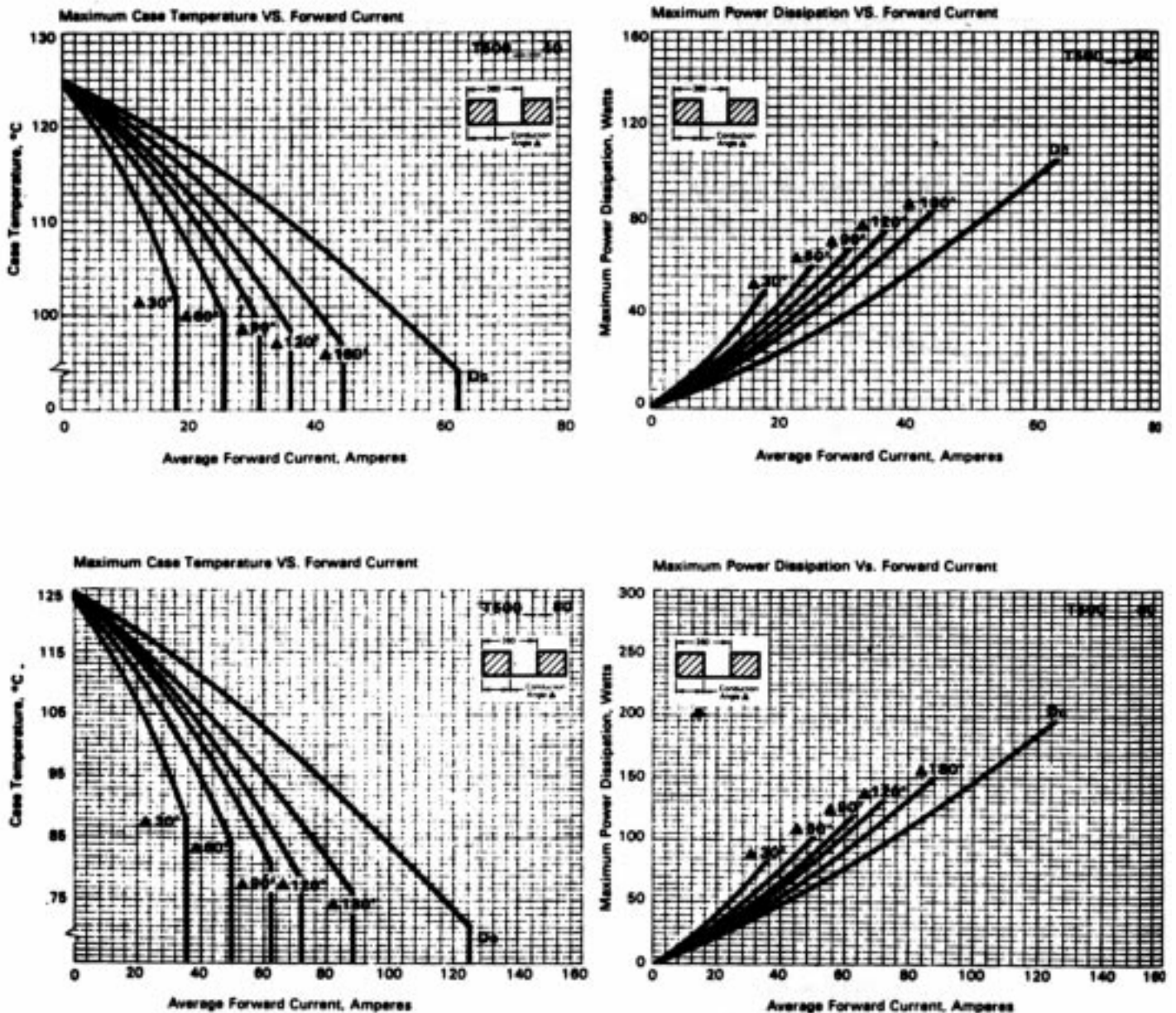


Figure 5-6 (Continued)  
T500 Data Sheet

**T500 Phase Control SCR**  
40-80 Amperes (63-125 RMS),  
1600 Volts

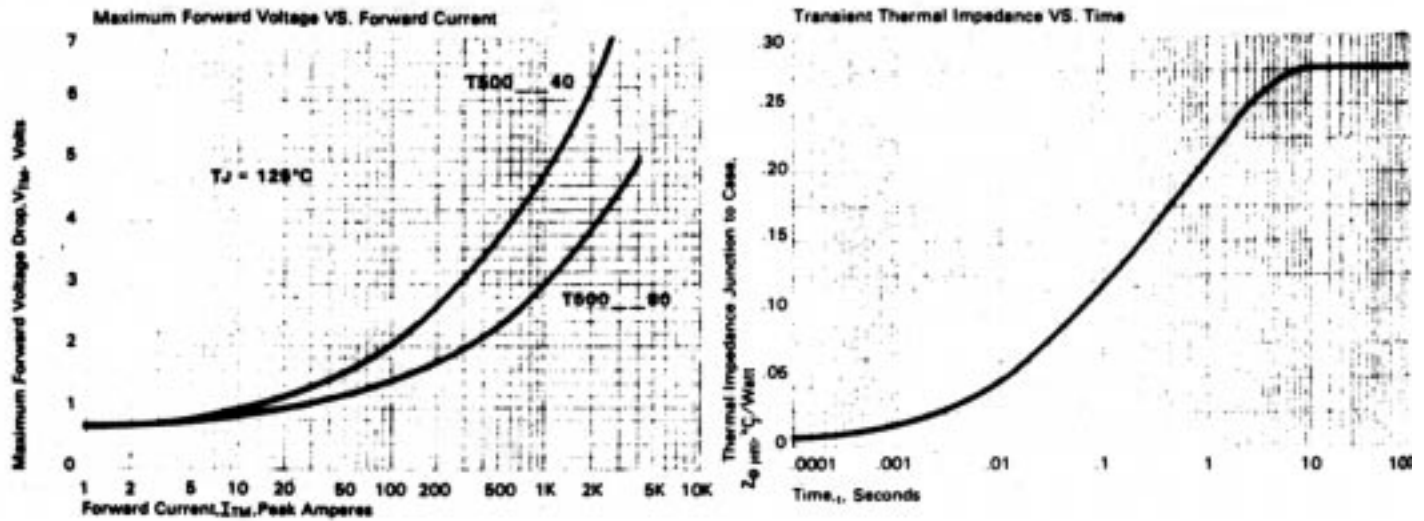


Figure 5-6 (Continued)  
T500 Data Sheet

## 14. Thermal Capacity

### *Calculation of Thermal Capacity for the Silicon Wafer and Copper Slug*

$$C_{\theta}(\text{Silicon}) = \rho c A d$$

$$\begin{aligned} &= (2.34 \times 10^3 \text{ kg/m}^3)(735 \text{ J/kgC}^\circ)(1.77 \times 10^{-4} \text{ m}^2)(2 \times 10^{-4} \text{ m}) \\ &= 6.1 \times 10^{-2} \text{ J/C}^\circ \end{aligned}$$

$$C_{\theta}(\text{Copper}) = \rho c A d$$

$$\begin{aligned} &= (8.9 \times 10^3 \text{ kg/m}^3)(393 \text{ J/kgC}^\circ)(1.77 \times 10^{-4} \text{ m}^2)(10^{-2} \text{ m}) \\ &= 6.2 \text{ J/C}^\circ \end{aligned}$$

To this point, the discussion on thermal considerations has dealt only with average junction temperature and steady-state thermal resistance. For normal, low frequency waveforms, this type of analysis is adequate. But for calculating peak junction temperatures resulting from low duty cycle (pulsed) waveforms or from transient overloads, steady-state analysis is not adequate.

We are all familiar with the fact that electrical equipment can be temporarily overloaded without permanent damage, sometimes to hundreds of times its steady-state ratings. The reason is that in a piece of equipment such as a motor or a transformer, there is a considerable mass of copper and iron which must rise in temperature before damage from overheating occurs. The energy of the transient overload is absorbed by the large thermal capacity of the equipment without excessive temperature rise. (Of course different considerations prevail if the heating is localized, as when an arc develops.)

The same generalization holds true for a solid-state power device except that the maximum temperature (at which control can be maintained) is more critical and the thermal capacity is orders of magnitude smaller (than for a motor or transformer of comparable power rating). Therefore an engineer who is accustomed to traditional electrical equipment must be very cautious until he retrains his intuition on solid-state devices. Although the excess thermal capacity of these devices is small, it must be fully utilized if reliability is to be achieved without the prohibitive cost of an overly conservative design.

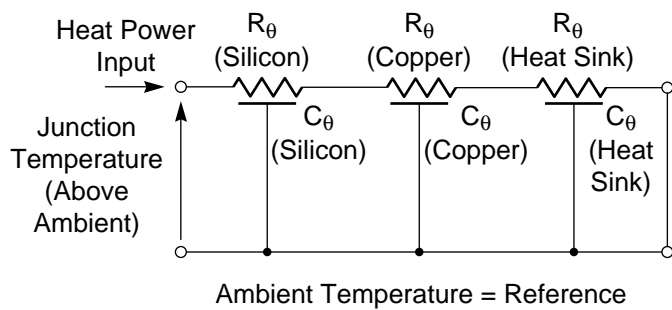
Let us return to the simple device geometry which was previously defined and calculate its thermal capacity. Thermal resistance is the temperature rise

per watt of heat flow. Thermal capacity is the heat energy which can be stored per degree of temperature rise. Consequently thermal capacity is equal to the product of the specific heat of the material ( $c$ ) and its mass (density ( $\rho$ ) times volume ( $Ad$ )). The thermal capacities of the silicon wafer and the copper slug are calculated above. Note that together they can absorb only a little more than 6 watt-seconds (i.e., joules) per  $^\circ\text{C}$  of temperature rise. Remember that the peak dissipation during switching was 15kW. Obviously the junction temperature will change very rapidly under these circumstances.

## 15. Transmission Line for Transient Thermal Analysis (Figure 5-7)

Thermal resistance and thermal capacity are now “lumped” elements but are both distributed throughout the device being considered. The device is, in fact, a thermal transmission line in which heat flow is governed by the diffusion equation. For the cylindrical geometry we have assumed, the one-dimensional diffusion equation applies, and there is an exact analogy with a corresponding RC electrical transmission line. Heat power is then analogous to current, and temperature difference is analogous to voltage. The complete transmission line between the junction and the cooling ambient is shown above. All temperatures (voltages) are measured relative to the ambient. Since the heat is ultimately dissipated at ambient temperature, the output of the transmission line is “short circuited”. Each of the three sections of the line, representing the silicon wafer, the copper slug, and the heat sink, respectively, may be assumed to be uniform transmission lines.

The analytical solution for the junction temperature for a specified power input (as a function of time) is extremely tedious, even for this simplified geometry, and involves solving the diffusion equation for each of the three sections and matching the boundary conditions between sections. Fortunately the “time constants” of the three sections differ widely so that they can be analyzed separately. Both the individual analysis and the combination of the separate results lend themselves to a very simple graphical approach.



**Figure 5-7**

*Thermal Transmission Line for Analyzing Temperature Rise of Junction*

## 16. Initial Temperature Rise

*Initial Increase in Junction Temperature as a Function of Time for a Homogeneous Cylinder*

$$\frac{\Delta T_j}{P} = \frac{2}{A\sqrt{\pi k \rho c}} \sqrt{t} = R_\theta \sqrt{\frac{t}{\frac{\pi}{4} R_\theta C_\theta}}$$

Consider one section of a uniform RC transmission line (thermal or electrical). Assume initially that the section is infinite in length so that we do not need to worry about the boundary condition at the output end. Also assume that the heat power input is zero prior to  $t = 0$  and constant at a value  $P$  thereafter. That is, the power input is assumed to be a step function.

At  $t = 0$ , the junction temperature,  $T_j$ , will begin to rise above its previous value. We will not solve the diffusion equation, but it can be shown that the solution for the junction temperature rise as a function of time takes the simple form shown above. The left side of this equation is the temperature rise per watt of power input, and therefore has the same units as thermal resistance. This quality is known as the “transient thermal impedance.”

The right side of the equation consists of a constant times the square root of time. Note first that the constant depends only on the thermal conductivity, density, and specific heat of the material and its cross-sectional area. With a little algebraic manipulation, this constant can be shown to be equal to  $R_\theta/\sqrt{\pi/4} R_\theta C_\theta$ .

Next, note that temperature rises in proportion to  $\sqrt{t}$ , not  $t$  itself. As the heat propagates down the transmission line, more and more material must be heated for each degree of temperature rise at the junction. Since the power input is constant, the rate of temperature rise decreases. If this relationship is plotted on log-log graph paper, the graph will be a straight line with a slope of one half.

This solution for an infinite transmission line accurately represents the initial temperature rise of a finite line (that is, a finite thickness of material). But when the heat penetrates to the other face of the material and the temperature at the face begins to increase, the solution rapidly becomes inaccurate. Stated concisely, this is the asymptotic solution for very small values of time.

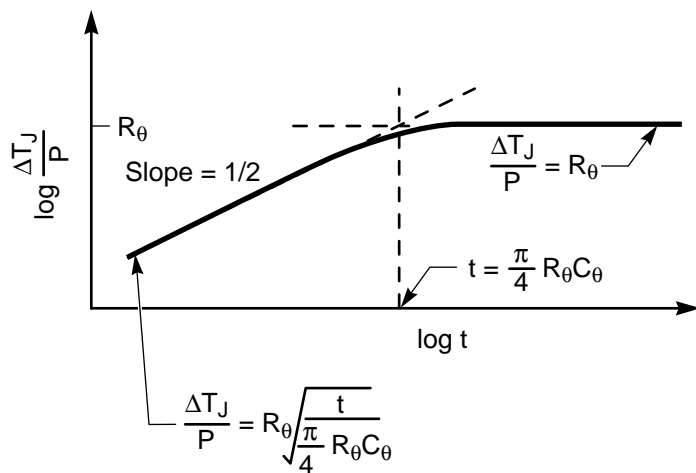
## 17. Transient Thermal Impedance of Each Section (Figure 5-8)

Next consider that the section of transmission line is terminated in a short circuit, corresponding to the actual piece of material being mounted on an infinite heat sink which can absorb any amount of heat with no temperature increase. Steady-state will have been reached when the input temperature rise reaches  $\Delta T_j = PR_\theta$ , or when the transient thermal impedance reaches  $\Delta T_j/P = R_\theta$ . In other words, the steady-state thermal resistance forms the asymptote of transient thermal impedance for very long times.

The two asymptotic solutions are shown above. It is seen that they intersect at  $t = (\pi/4) R_\theta C_\theta$ . This is the effective time for the heat to penetrate a specified thickness of material, and is therefore an effective thermal “time constant”. For values of time near the time constant, neither asymptotic solution is accurate, and the actual solution “rounds the corner” between them.\*

The asymptotic solutions for the individual sections of the overall thermal transmission line are uniquely determined by the steady-state thermal resistance and the time constant for each section.

\*At  $t = (\pi/4) R_\theta C_\theta$ , the actual value of  $\Delta T_j/PR_\theta$  is 0.883. Note the similarity between this asymptotic approach to transient thermal impedance and the well-known Bode plot of frequency response which has an actual value of  $1/2 = 0.707$  at the corner frequency of a first-order network.



**Figure 5-8**

*Transient Junction Temperature Rise as a Function of Time Following the Application of Constant Power Input*

## 18. Calculation of Thermal Time Constant

*Calculation of Thermal Time Constants for the Silicon Wafer and Copper Slug*

$$\frac{\pi}{4} R_{\theta} C_{\theta} \text{ (Silicon)} = \frac{\pi}{4} (0.0135 \text{ C}^{\circ}/\text{W}) (0.061 \text{ J/C}^{\circ}) = 0.64 \text{ ms}$$

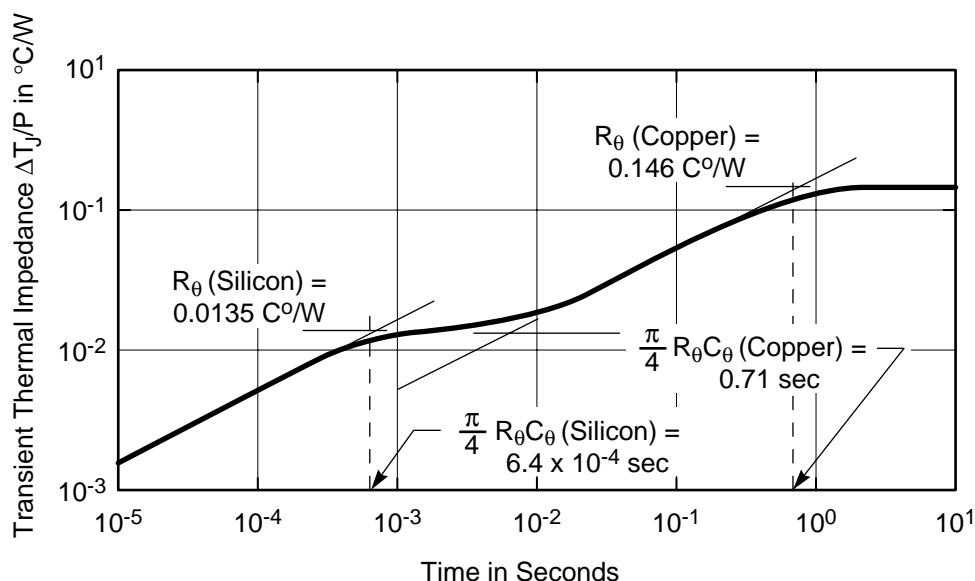
$$\frac{\pi}{4} R_{\theta} C_{\theta} \text{ (Copper)} = \frac{\pi}{4} (0.146 \text{ C}^{\circ}/\text{W}) (6.2 \text{ J/C}^{\circ}) = 0.71 \text{ s}$$

The thermal time constants of the silicon wafer and the copper slug are calculated above from the previously calculated values of  $R_{\theta}$  and  $C_{\theta}$ . It is seen that the time constant of the package is more than three orders of magnitude larger than that of the silicon wafer.

## 19. Junction-to-Case Transient Thermal Impedance (Figure 5-9)

If the asymptotic solutions for the silicon wafer and the copper slug are each plotted on the same log-log graph, and the curve rounded at the intersections as shown in Figure 5-9, the junction-to-case transient thermal impedance plot is the result. The asymptote on the right is actually the junction-to-case thermal resistance, but on a logarithmic scale the difference from the thermal resistance of the copper alone is not perceptible. When the packaged device is mounted onto a heat sink, the transient thermal impedance of the heat sink is added in the same way. Since the heat sink will have an even longer time constant, only the right portion of the curve will be changed.

Note that when the transient thermal impedance is plotted on log-log paper, the time constants of the constituent parts are recognizable from the bumps in the curve. For transients which are short compared to the time constant of the silicon wafer, the



**Figure 5-9**

*Log-log Plot of the Transient Thermal Impedance of the “Packaged” Silicon Wafer (Not Including the Heat Sink)*

transient thermal impedance is determined solely by the characteristics of the silicon, independent of whether the silicon is mounted on anything. For longer transients which are still shorter than the package time constant, the transient thermal impedance is independent of the heat sink. But for transients having a duration greater than the package time constant, the heat sink does play a role.

This analysis has assumed uniform distribution of the heat over the cross-sectional area of the device. This assumption is not valid during the turn-on transient when the conducting area is less than the total area. Therefore transient thermal impedance, as presented here, cannot be used for transients which are shorter than 10 to 100 $\mu$ s, depending on the size of the device.

In spite of the fact that the curve of transient thermal impedance is easier to plot and interpret on log-log scales, it is the universal practice of device manufacturers to give this curve on log-linear scales (see Figure 5.6-B of the Type T500 data sheet).

## 20. Using Transient Thermal Impedance (Table 5.4)

We are now in a position to use the transient thermal impedance curve to calculate instantaneous junction temperature or peak temperature for specified current surges, or to calculate the allowable current surges for a specified peak junction temperature. To illustrate, refer to Figures 5.6-A and 5.6-B on the data sheet for the Powerex T500 thyristor.

First, let us calculate the fluctuation in junction temperature about its average value when the current is a 120A square wave with a 50% duty cycle. This corresponds to the example previously considered, where we found the dissipation during conduction to be 180W. If the frequency of the square wave is 60 Hz, the duration of each half cycle is 8.3 ms. From Figure 5.6-B we see that the transient thermal impedance for  $t = 8.3$  ms is  $0.04^{\circ}\text{C}/\text{W}$ . Therefore, the temperature rise during each half cycle of conduction is  $180\text{W} \times 0.04^{\circ}\text{C}/\text{W} = 7^{\circ}\text{C}$ . Of course the junction must cool by an equal amount during the half cycle of nonconduction. But it may be surprising to you to find that there is an appreciable temperature fluctuation of the junction even during normal 60 Hz operation. Since the duration is less than the time constant of the device, this fluctuation cannot be reduced by an improved heat sink.

**Table 5.4**

*Fluctuation of Junction Temperature*

### **Junction Temperature Fluctuation at 60 Hz (Square Wave with 50% Duty Cycle Assumed)**

Dissipation During Half-Cycle of Conduction	$P = 180\text{W}$
Duration of Half-Cycle at 60 Hz	8.3 ms
Transient Thermal Impedance at 8.3 ms	$Z_{\theta} = 0.04^{\circ}\text{C}/\text{W}$

$$\Delta T_j = P Z_{\theta} = 180\text{W} \times 0.04^{\circ}\text{C}/\text{W} = 7^{\circ}\text{C}$$

## 21. Calculation of Pulse and Surge Current Capabilities (Figure 5-10)

As another example, let us determine the rise in junction temperature when the Type T500 is subjected to a 1000A pulse lasting 1.5 ms. From Figure 5.6-A on the data sheet, the forward voltage drop is 3.0V, and from Figure 5.6-B the transient thermal impedance is about  $0.016^{\circ}\text{C}/\text{W}$ . Therefore:

$$\Delta T_j = 3.0\text{V} \times 1000\text{A} \times 0.016^{\circ}\text{C}/\text{W} = 48^{\circ}\text{C}$$

If the device is operated with an average junction temperature of  $77^{\circ}\text{C}$ , this pulse will not cause the peak junction temperature to exceed  $125^{\circ}\text{C}$ . Stated in another way, when operating with a junction temperature of  $77^{\circ}\text{C}$ , a Type T500 can carry a 1.5 ms current pulse equal to 8 times ( $1000/125 = 8$ ) its RMS current rating without losing its blocking ability.

Repetition of this calculation for a constant  $\Delta T_j = 50^{\circ}\text{C}$  yields the normalized graph shown above. It is evident that a thyristor can tolerate substantial overload currents without hindering its operation if:

- (a) the duration of the overload is sufficiently short,
- and
- (b) the junction is operating somewhat below its rated maximum temperature.

The current capacity calculated above is a "repetitive" value for an arbitrarily selected temperature rise in that the thyristor can be subjected to such currents an unlimited number of



times without shortening its life expectancy. Current ratings such as the surge current are “non-repetitive” in the sense that a thyristor should not be exposed to more than 100 of these overloads during its entire useful life. Under these overloads, the junction temperature will exceed 125°C and the thyristor may not immediately block its rated peak forward voltage.

The surge current rating gives the *peak* half-wave sinusoidal current 60 Hz which the thyristor can withstand for the specified number of cycles. If gating pulses are removed immediately when a fault occurs, the Type T500 can withstand 1800A peak for one-half cycle (8.3 ms) until the current passes through zero. If the fault is cleared by a circuit breaker which acts more slowly, the allowable peak fault current drops to 1300A for 3 cycles (i.e., 3 half-cycles of conduction separated by 3 half-cycles of non-conduction) or 1170A for 10 cycles. These peak values have been converted to RMS values and are shown for comparison on the following graph.

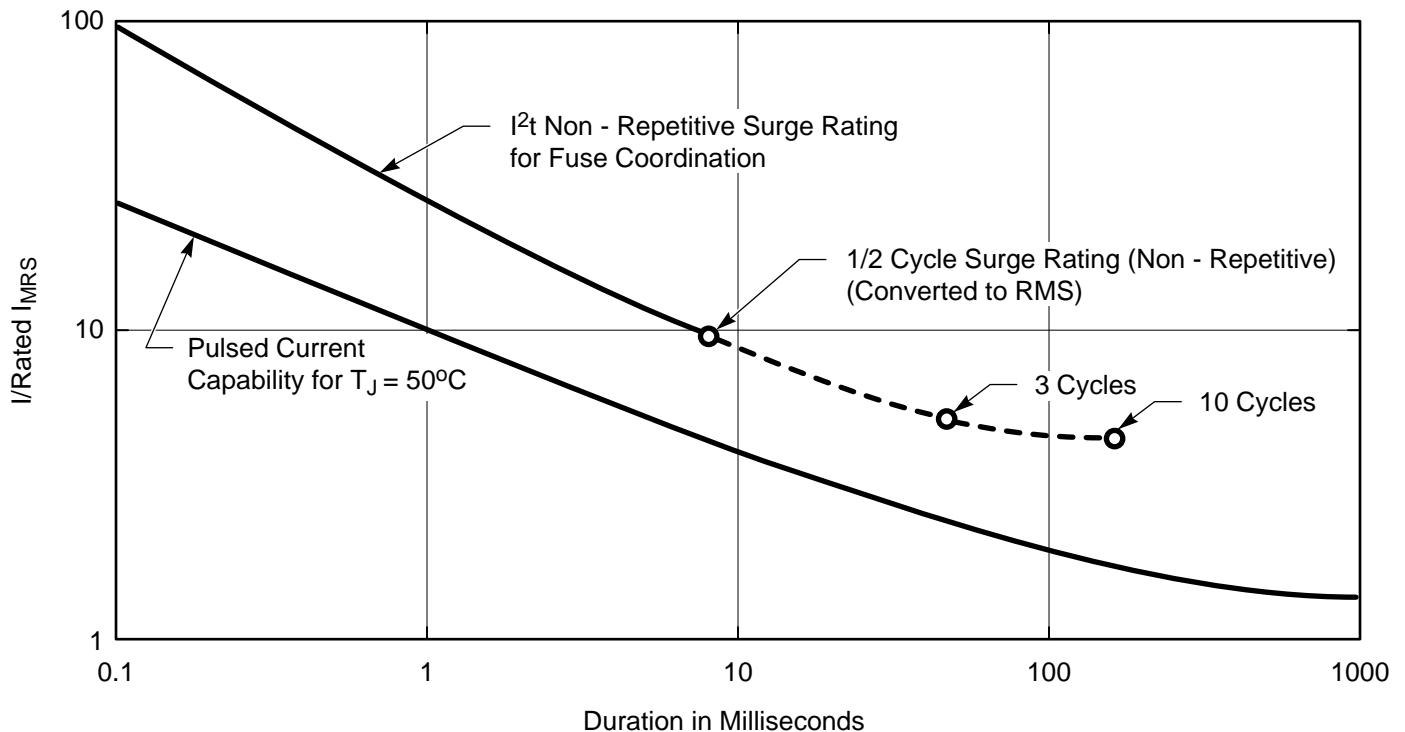
In many cases, the circuit in which a thyristor is used is so “stiff” that a short-circuit fault can cause currents much higher than these values. If the thyristor is to be protected, either additional impedance must be designed into the circuit to limit the current to a safe value until a circuit breaker can

open, or a special fast-acting fuse must be placed in series with the device. The “ $I^2t$ ” rating of a thyristor is provided as a guideline in selecting an appropriate fuse which will prevent permanent damage but will not blow unnecessarily. This is a non-repetitive rating, and is equal to the square of the *RMS* current capacity times the duration of the surge. The  $I^2$  rating may be used to calculate the maximum non-repetitive surge current for less than one-half cycle (i.e., 8.3 ms). The corresponding curve is shown above for the Type T500 which has an  $I^2t$  rating of 13,495 A<sup>2</sup>s.

In most applications, the power device is selected on the basis of its steady-state current ratings and protected so that its surge ratings are not exceeded. However, in other applications where shutdown of the equipment cannot be tolerated, oversize devices with spare steady-state capacity must be used to achieve the required surge capacity.

References on Pulse and Surge Current Capacity include:

William E. Newell, “Dissipation In Solid State Devices - The Magic of  $I^1 + N$ ,” IEEE Transactions on Industry Application, July/Aug. 1976.



**Figure 5-10**

*Current Carrying Capability (Repetitive and Non-Repetitive) for a Westinghouse Type T500 Thyristor (Rated  $I_{RMS} = 125\text{A}$ ; Rated  $Pt = 13,495\text{ A}^2\text{s}$ )*

John W. Motto, "A New Quality to Describe Power Semiconductor Subcycle Current Ratings," IEEE Transaction on Industry Applications, July/Aug. 1971.

William E. Newell, "Transient Thermal Analysis of Solid-State Power Devices-Making a Dreaded Process Easy," IEEE Transactions on Industry Applications, July/Aug. 1976.

John W. Motto, "Computer Aided Thyristor Overload Current Ratings for Motor Drive and Other Applications Requiring Repetitive Overloads," IEEE Industry and General Applications, Sept./Oct. 1969.

Additional information on thermal analysis, heat sink selection, and current ratings may be found in:

Westinghouse SCR Designers Handbook (Second Edition), Sections 4.3 to 4.6 and 10, Westinghouse Semiconductor Division, Youngwood, PA; 1970.

General Electric SCR Manual (Fifth Edition), Section 3.1 to 3.6, 15, and 18, General Electric Semiconductor Products Dept., Syracuse, New York; 1972.

Wakefield Heat Sink Product Bulletin; Wakefield Eng. Inc., Wakefield, Mass; 1966.

E. T. Schonhoizer, "Fuse Protection for Power Thyristors," IEEE Trans. on Industry Applications, vol. IA-8, pp. 301-309; 1972.

W. E. Newell, "Dissipation in Solid-State Devices - The Magic of  $I^{1+N}$ ," IEEE Power Electronics Specialists Conference Record, pp. 162-173; June 1974.

Westinghouse Thyristors Surge Suppression Ratings AD 54-550; Oct. 1976.

W. Savage, "How to Get the Most Out of High-Power Rectifiers and SCR's," EDN; August 1974.

The effects of maximum temperature and temperature cycling on the life expectancy of power devices are discussed in:

D. E. Piccone and I. Somos, "Are You Confused by High di/dt SCR Ratings?," Electronic Engineer, pp. 89-92; Jan. 1969.

D. E. Piccone and I. Somos, "Accelerated Life Tests for Determining the Life Expectancy of a Thyristor Due to di/dt Failure Modes," IEEE Industry Applications 7th Annual Meeting Record, pp. 469-476; Oct. 1972.

R. E. Locher, "Large Diameter Rectifier Diode and Thyristor In-Service Reliability," IEEE Industry Applications 9th Annual Meeting Record, pp.463-465; Oct. 1974.

M. Morozowich, "Are Semiconductors Reliable?," Electronic Products Magazine; July 1976.

## 22. Turn-On di/dt (Figures 5-11 & 12)

### Turn-On di/dt

- Is Determined by the Lateral Rate of Spreading of the Turned-On Area,
- Can be limited by increasing Circuit Inductance,
- Capability is Increased by Special Device Geometries and by "Hard" Gate Drive.

The preceding discussion has emphasized the intimate relationship between the thermal characteristics of a thyristor and its steady-state and transient current-carrying ability. The corresponding data sheet ratings are a concise statement of the basic information needed to choose a heat sink and to coordinate fuses and circuit breakers with thyristor capabilities. We will now examine some of the other factors which influence the design of the power module to protect the power device.

When the transient characteristics of thyristors were discussed earlier in the text, it was stated that the gate turns on only a small part of the total thyristor area. The time required for the conducting area to propagate laterally, typically 0.1mm per microsecond, across the full device necessitates a corresponding limitation on the rate of increase of load current at turn on. This limitation is expressed as a maximum di/dt rating. Exceeding this rating causes excessive current density at turn-on, and the resulting hot spot will degrade or destroy the thyristor.

In some applications, high di/dt occurs at turn-on but is not essential. For this case, the solution is usually to design additional inductance (either linear or saturating) into the circuit to limit di/dt to a safe value.

The di/dt capability of any thyristor is decreased if minimum gate drive is used and increased if the gate is driven "hard", thereby increasing the initially conducting area. Most thyristors today incorporate an integrated amplifier termed an amplifying or di/namic gate design which includes a small pilot thyristor in the gate circuit which generates a very strong pulse to turn on the main thyristor rapidly. This is illustrated in Figure 5.11.

In other applications, high di/dt is essential and must be accommodated by the power devices. For this reason, fast turn-on devices with increased di/dt ratings are available and development is continuing. Originally, the gate contact on thyristors was located at one edge, and conduction had to propagate across the entire diameter. Later turn-on time was reduced



by moving the gate contact to the center, cutting the maximum distance for propagation in half. Still newer thyristor geometries use interdigitated gate electrodes to initiate conduction over a large percentage of the total area. This is illustrated in Figure 5.12.

References on  $di/dt$  and gating considerations for fast turn-on include:

Westinghouse SCR Gate Turn-on Characteristics, A.D. 54-540, pp. 1-4; December 1976.

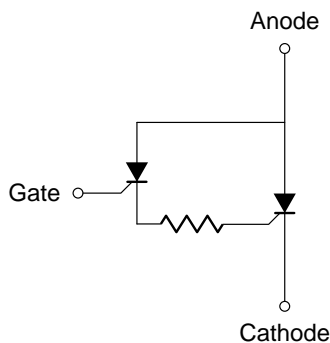
General Electric SCR Manual (Fifth Edition), Section 3.7 and 5.5, General Electric Semiconductor Products Dept., Syracuse, N.Y.; 1972.

H. F. Storm and J. G. St. Clair, "An Involute Gate-Emitter Configuration for Thyristors," IEEE Trans., vol. ED-21, pp. 520-522; Aug. 1974.

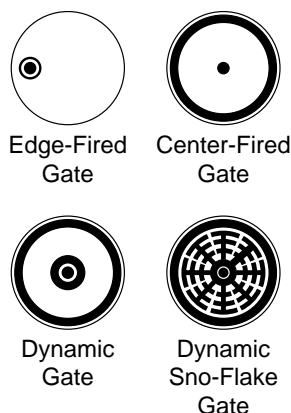
N. Mapham, "The Rating of Silicon Controlled Rectifiers when Switching into High Currents," IEEE Trans. on Communications and Electronics, vol. 83, pp. 515-519; September 1964.

D. A. Paice and P. Wood, "Nonlinear Reactors as Protective Elements for Thyristor Circuits," IEEE Trans. on Magnetics, vol. MAG-3, pp. 228-232; September 1967.

S. Ikeda, et al., "The Current Pulse Ratings of Thyristors," IEEE Trans. on Electron Devices, vol. ED-17, pp. 690-693; September 1970.



**Figure 5-11**  
*Equivalent Circuit of an Integrated Gate Amplifier*



**Figure 5-12**  
*Various Gate Cathode Geometries*

## 23. Transient Overvoltage

### Transient Overvoltage

- Is Usually Caused by Inductive Kickback.

### Damage Can Be Prevented By

- Circuit Redesign,
- Devices Having Adequate Voltage Ratings,
- RC Suppressor Circuits,
- Nonlinear Suppressor Devices,
- Solid-State "Crowbar" Which Rapidly Shorts the Voltage Transient.

Unanticipated voltage transients which exceed the rated blocking voltages of solid-state power devices are probably the most frequent single cause of unreliability in these devices. Depending on the severity of the overvoltage, the energy which it represents, and its frequency of repetition, the device may fail immediately or its characteristics may progressively deteriorate. Because voltage breakdown tends to occur at the surface of the device rather than within the silicon, the energy required to cause permanent damage may be relatively small.

Except for random disturbances such as lightning, most voltage transients can be traced to an inductor, either within or external to the equipment, in which current has been initiated or interrupted abruptly. Obvious cases include removing the load on a rectifier having a large smoothing inductor in the DC circuit, for example, by blowing a fuse. Cases which are much more subtle involve hidden inductance such as the leakage reactance of a transformer, or unintentional current such as the rapid cessation of reverse sweepout current in a diode or thyristor (which occurs each cycle). Other causes of transient overvoltage are the capacitive coupling from a high voltage circuit to a low voltage circuit and the energizing of a RLC circuit where the capacitor will charge to two times the peak line voltage.

Protection against destructive overvoltage transients can be achieved in four general ways:

- Redesign the circuit operation or physical location to remove or minimize the cause of the transient.
- Suppress the transient by absorbing its energy in an appropriately designed RC circuit located across the source of the transient.
- Shunt the power devices by nonlinear resistive elements which clip the voltage transient at a safe

level. Selenium transient suppressors (known by the tradenames Voltrap, Thyrector, ect.) and zener diodes have been used for this purpose, and ceramic suppressors (known as Zinc Nonlinear Resistors, ZNR's and Metal Oxide Varistors MOV's) have recently become available. However currently available ceramic suppressors devices are relatively low in energy absorbing capability compared to selenium transient suppressors.

- d. Occasional severe transients can sometimes best be limited by a solid-state "crowbar" circuit which shorts the line and absorbs the transient energy, at least until an auxiliary circuit breaker can open.

References on overvoltage protection of devices include:

Westinghouse SCR Designers Handbook (Second Edition), Sections 9.3 to 9.5, Westinghouse Semiconductor Div., Youngwood, Pa.; 1970.

General Electric SCR Manual (Fifth Edition), Section 16, General Electric Semiconductor Products Dept., Syracuse, N.Y.; 1972.

F. D. Martzoff, "Surge Voltages in Residential and Industrial Power Circuits," IEEE Trans. on Power Apparatus and Systems, vol. PAS-89, pp. 1049-1055; July/Aug. 1970.

T. Liao and T. H. Lee, "Surge Suppressors for the Protection of Solid-State Devices," IEEE Transactions on Industry and General Applications, vol. IGA-2, pp. 44-52; Jan./Feb. 1966.

D. C. Kay, et al., Transient Voltage Suppression Manual General Electric Semiconductor Products Dept., Syracuse, N.Y.; 1976.

Voltrap Surge Suppressor Westinghouse  
TD-16435 Westinghouse Electric Corporation, Buffalo, N.Y.  
14240.

## 24. Off-State $dv/dt$

Off-State  $dv/dt$  Capability

- Is Determined by Capacitive Current Into Gate Layer,
- Can Be Limited to a Safe Value by an RC Snubber,
- Is Increased by Shorted Emitter Design.

If a thyristor is in its forward blocking state, it is the reverse-biased center junction which prevents the flow of appreciable current. As we have seen, gate current which is greater than some critical minimum value causes regenerative action which turns the device on and thereafter prevents this junction from blocking current.

However, this center junction is shunted by capacitance. If the device is subjected to a rapidly changing voltage (anode increasingly positive with respect to the cathode), this capacitance causes a current

$$i = C_j \frac{dv}{dt}$$

to flow across the blocking junction into the gate layer. This current is equivalent to externally supplied gate current, and will turn the device on if it exceeds the critical value. Thus, if the device is to remain in its blocking state, it cannot be subjected to  $dv/dt$  in excess of a critical value.

The  $dv/dt$  seen by a device can be reduced by connecting a capacitor across the device. Used alone, this capacitor will cause excessive  $di/dt$  when the device turns on and shorts the capacitor. Therefore a resistor is needed in series with the capacitor to limit the current transient at turn-on. The RC  $dv/dt$  limiter is known as a "snubber", and suitable values for R and C are frequently recommended by the thyristor manufacturer. The values chosen are a compromise between  $dv/dt$  reduction, tolerable  $di/dt$ , and minimum added dissipation.

Some applications, such as high-frequency inverters, have an inherently high  $dv/dt$ . The cost of snubbers can be eliminated in other applications if the devices have adequate  $dv/dt$  capability. Therefore there are continuing efforts to develop devices having even higher  $dv/dt$  ratings. One noteworthy approach to the design of such devices involves the use of a "shorted emitter". In this type of device, the cathode electrode partially shorts the gate P-layer to the cathode N-layer, providing an alternate path for the  $dv/dt$  current. Although this design modification increases the gate drive needed to turn the device on, it permits substantial increases in  $dv/dt$  ratings.

References on  $dv/dt$  effects, snubber design, and shorted emitter devices include:

Westinghouse SCR Designers Handbook (Second Edition), Sections 4.7, 5.3.1 and 7.2.1, Westinghouse Semiconductor Div., Youngwood, Pa.; 1970.

General Electric SCR Manual (Fifth Edition), Sections 1.5, 3.10, 5.4, and 16.3.2, General Electric Semiconductor Products Dept., Syracuse, N.Y.; 1972.

International Rectifier SCR Application Handbook, P. G. Hoff et al., International Rectifier Corp., Semiconductor Div., El Segundo, Ca.; 1974.

J. B. Rice and L. E. Nickles, "Commutation  $dv/dt$  Effects in Thyristor Three-Phase Bridge Converters," IEEE Trans. on Industry and General Applications, vol. IGA-4, pp. 665-672; Nov./Dec. 1968.

J. B. Rice, "Design of Snubber Circuits for Thyristor Converters," Conference Record of the 4th Annual Meeting of the IEEE Industry and General Applications Group, pp. 485-489; Oct. 1969.

W. McMurray, "Optimum Snubbers for Power Semiconductors," IEEE Trans., vol. IA-8, pp. 593-600; Sept./Oct. 1972.

C. K. Chu, "Geometry of Thyristor Cathode Shunts," IEEE Trans. on Electron Devices, vol. ED-17, pp. 687-690; Sept. 1970.

P. S. Raderacht, "A Review of the 'Shorted-Emitter' Principle as Applied to PNP Silicon Controlled Rectifiers," Intl. Jour. of Electronics, vol. 31, pp. 541-564; 1971.

## 25. Series/Parallel Arrays for High-Power Applications (*Figures 5-13*)

The maximum power which can be controlled by a single solid-state power device is determined by its rated blocking voltage and by its rated forward current. To maximize either of these ratings requires some compromise of the other rating. Also laboratory devices can always be cited with capabilities considerably beyond those of devices which are commercially available at any particular time.

Present limits on diode ratings are about 5 kV and 3000A individually, with a maximum product of about 5 MW. Similar limiting ratings for thyristors are about 4 kV, 2500A, and 4 MW.

Applications requiring voltages which are higher than the capabilities of individual devices can be served by series strings of devices, and higher currents can be achieved by devices connected in parallel. For example, electrochemical process lines may require 100,000A or more, necessitating many parallel devices. High-voltage DC converters require high voltage and high current, leading to series/parallel arrays of devices.

If all of the characteristics of solid-state devices of a given type were closely matched, series/parallel arrays of devices could be routinely assembled. In some cases, screening of devices to select matched sets is an economical alternative. However, in most cases the spread of characteristics of production devices necessitates an equalizing network to assure that no individual device in the array is overstressed.

Because of the rapid change in on-state current with forward voltage, parallel devices tend to share current unequally. The best device with the lowest forward voltage monopolizes the load current, becomes overheated, and is destroyed. Similarly, when parallel thyristors are first turned on, the device with the shortest turn-on time is subjected to a very high  $di/dt$  which may destroy it.

Analogous problems occur in series strings of devices. Because of the wide variation in off-state leakage currents between production devices of a type, those devices having the lowest leakage currents are subjected to excessive blocking voltages when the devices are in equilibrium. Also, mismatches in junction capacitances and stored charge cause unequal distribution of transient voltage stresses.

The design of series/parallel equalization networks to achieve maximum overall power rating, reliability, minimum dissipation, and minimum cost is beyond this text. Suffice it to say that parallel devices can be forced to share current by a resistor in series with each device or by equalizing reactors (which are more costly but also dissipate less power). Series devices can be forced to share static blocking voltages by means of a resistor in parallel with each device. Transient blocking voltages are equalized by a series RC circuit in parallel with each device. Series and parallel assemblies are available from some manufacturers.

The problem of equalizing network design can often be avoided by purchasing standard multiple-device assemblies complete with heat sinks. In this case, the device manufacturer assumes responsibility for the selection of properly matched devices or for the adequacy of the equalization network.

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Further information on series/parallel arrays of devices and on the design of equalization networks can be found in:

Westinghouse SCR Designers Handbook (Second Edition), Section 9, Westinghouse Semiconductor Div., Youngwood, Pa.; 1970.

General Electric SCR Manual (Fifth Edition), Section 6, General Electric Semiconductor Products Dept., Syracuse, N.Y.; 1972.

High Voltage Rectifier Stacks TD 54-200 12-76, Powerex, Inc., Youngwood, Pa.

Air & Water Cooled Pow-R-Disc Assemblies Data Book, Powerex, Inc., Youngwood, Pa.

A. R. Mulica, "How to Use Silicon Controlled Rectifiers in Series or Parallel," Control Engineering, vol. 11, pp. 95-99; May 1964.

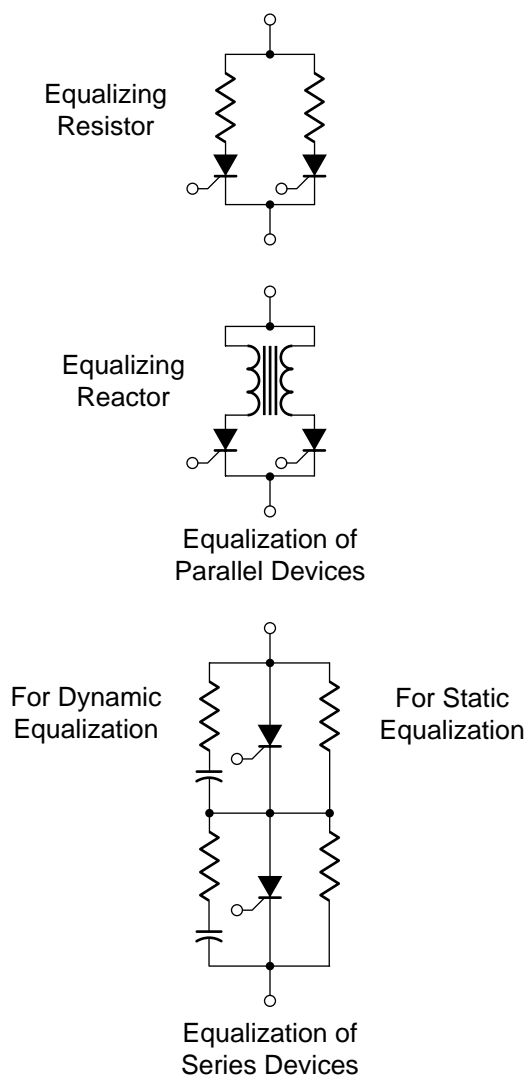
G. Karady and T. Gilsig, "The Calculation of Transient Voltage Distribution in a High Voltage DC Thyristor Valve," IEEE Trans., vol. PAS-92, pp. 893-899; May/June 1973.

G. Karady and T. Gilsig, "The Calculation of the Turn-Off Overvoltages in a High Voltage DC Thyristor Valve," IEEE Trans., vol. PAS-91, pp. 565-574; Mar./Apr. 1972.

G. Karady and T. Gilsig, "The Calculation of Turn-On Overvoltages in a High Voltage DC Thyristor Valve," IEEE Trans., vol. PAS-90, pp. 2802-2811; Nov./Dec. 1971.

J. Reeve and R. Marttila, "Calculation of Turn-On Stresses in a HVDC Valve Having Parallel Thyristor Strings," IEEE Trans., vol. PAS-92, pp. 864-870; May/June 1973.

G. Sherbondy A., Graphical Approach to Paralleling Power Semiconductors Tech-Tip 5-6, Westinghouse Semiconductor Division, Youngwood, Pa.; Dec. 1976.

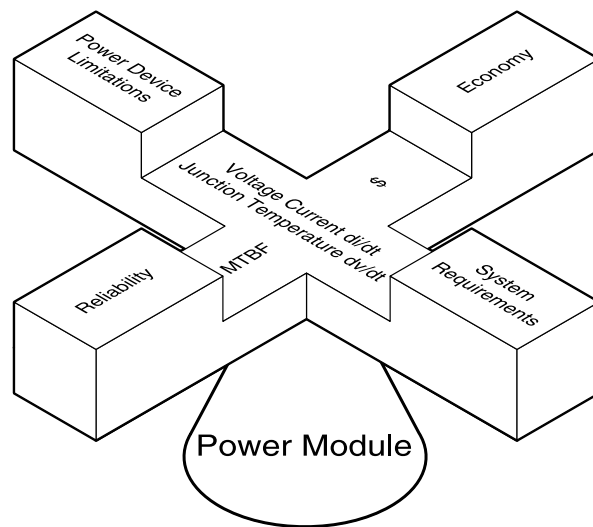


**Figure 5-13**  
*Voltage and Current Balancing Techniques*

## 26. Summary

The technical feasibility of a new application of power electronics technology is nearly always determined by the state of the art in solid-state power devices. Many potential applications are well within the direct capabilities of existing devices without extensive interfacing, and the design of the associated circuitry is simple and routine. In other cases, peculiar requirements or economic trade-offs may dictate that very careful attention be given to the design of the power module to protect the power device from excessive stresses and to squeeze out the full performance of which it is capable.

Each aspect of the design of the power module to interface device limitations with system requirements is the subject of considerable specialized technology. Hopefully this text has helped you to appreciate both the immense capabilities of the devices that are available and the general approaches that are available when needed to overcome particular device limitations.



**Figure 5-14**  
*The Power Module is the Pivot of Balanced Power Electronics Design*

## 27. Problems

### *Problem 5.1 Switching Dissipation*

The "typical" thyristor considered in the text was assumed to have a  $3\mu\text{s}$  turn-on time during which both the voltage and the current changed from their blocking values to their conducting values. Under

these conditions, the turn-on energy for 100A of forward current was calculated to be 30 mJ.

Assume that the voltage across the thyristor drops linearly in 3μs as before, but that inductance is added to the circuit so that the current requires twice as long (i.e., 6μs) to build up to 100A. How does this change the energy dissipated in the switch during the turn-on interval?

---

*The peak dissipation in the “typical” thyristor during the switching interval was found to be 15 KW. Because this dissipation is concentrated in a very small volume surrounding the junction, the corresponding power density is of the order of 1 MW/cm<sup>3</sup>, and the energy flux is about the same as that at the surface of the sun, 6 KW/cm<sup>2</sup>! Why doesn’t the thyristor light up like the sun (or does it)?*

### **Problem 5.2 Steady-State Thermal Resistance and Average Current Rating**

Six Type T500 thyristors are used in a 6-pulse bridge AC/DC converter. If they are to operate with a case temperature of 90°C and an average junction temperature of 120°C, how much DC load current can the converter tolerate? If they are cooled by ambient air at 40°C, what is the maximum thermal resistance of each heat sink?

### **Problem 5.3 Transient Thermal Impedance**

The I<sup>2</sup>t rating of a thyristor is a non-repetitive rating. Therefore it can be expected that a thyristor which is stressed to the limit allowed by this rating will have a peak junction temperature considerably higher than 125°C and that an appreciable delay is required before forward blocking ability is regained.

Use the plot of transient thermal impedance for the Type T500 to estimate the increase in junction temperature which occurs when a matched fuse (I<sup>2</sup>t = 13,495 A<sup>2</sup>s) blows. Assume that the clearing time for the fuse is 2 ms and that the current is constant over this interval.

### **Problem 5.4 Transient Thermal Impedance**

A graphical process is determined in the text for combining the transient thermal impedances of two dissimilar cylinders, namely silicon and copper. Test your understanding of this process by using it to determine the increase in thermal time constant which takes place when the length of a cylinder is doubled. In other words, use the process to combine the thermal impedance graphs of two identical cylinders.

Verify your result by showing that it is consistent with the analytical expression for the time constant.

---

*The plot of junction-to-case thermal impedance shows the rise in junction temperature which would occur (as a function of time) if the device were mounted on an infinite heat sink. Use physical reasoning to predict how the plot would change for a device which is insulated from thermal contact with any other body.*

### **Problem 5.5 Using Superposition with Transient Thermal Impedance**

Idealized heat conduction is a linear process. Therefore the rise in junction temperature for an arbitrary dissipation waveform can be obtained by the superposition of step-function responses. Determine the peak junction temperature rise of a Type T500 thyristor caused by a 700A, 20 ms pulse followed after an interval of 40 ms by a 500A, 40 ms pulse.

### **Problem 5.6 Current Sharing of Parallel Thyristors**

Type T500 thyristors have an RMS current rating of 125A. Assume that 2 of these thyristors are to be paralleled in order to control a 250A load current. The forward V-I characteristic may be approximately by a linear relationship of the form:

where the nominal values are  $E_0 = 1.2\text{v}$  and  $R = 0.003$  ohms. Assume that production devices have a  $\pm 10\%$

$$V = E_0 + R_0 I$$

spread in both  $E_0$  and  $R_0$ .

- Calculate the worst case distribution of currents if two production devices are paralleled to carry 250A. Compared to the  $\pm 10\%$  tolerance on  $E_0$  and  $R_0$ , what is the corresponding percentage tolerance on current?
- Calculate the value of the equalizing resistor which must be connected in series with each thyristor to hold the current tolerance within  $\pm 10\%$ . Calculate the power dissipated in the equalizing resistors and compare it with the power dissipated in the thyristors.

# APPENDICES

## APPENDIX I

### Ten Cornerstones of Power Electronics

The fundamental principles needed to understand the repetitively switched power circuits used in power electronics are not new to electrical engineers. Once you gain self confidence in applying these old principles, mixed with patience and preserverance, you will find that the mysteries of the new field of power electronics disappear.

In power electronics, concise conclusions which are both useful and general are as rare as the circuit configurations are diverse. Therefore, an understanding of how the fundamentals can be applied to reach conclusions which are both specific and correct is the only safe foundation. If conclusions are remembered, the necessary restrictions and qualifications on their validity are soon forgotten. And intuitive guessing is even riskier.

The ten cornerstones needed to establish a basic understanding of power electronics can be stated in simple terms:

1. Kirchhoff's Voltage Law: The sum of the instantaneous voltages around a closed loop is zero.  $\sum e_N = 0$
2. Kirchhoff's Current Law: The sum of the instantaneous currents into a junction is zero.  $\sum i_N = 0$
3. Ohm's Law:  $e_R(t) = R i_R(t)$

$$4. \quad e_L(t) = L \frac{di_L(t)}{dt} \quad \text{or} \quad \Delta i_L = \frac{1}{L} \int e_L(t) dt$$

$$5. \quad i_C(t) = C \frac{de_C(t)}{dt} \quad \text{or} \quad \Delta e_C = \frac{1}{C} \int i_C(t) dt$$

(Note: Because of the nonsinusoidal waveforms in repetitively switched circuits, the relationships in (4) and (5) must be used in place of impedance relationships based on  $Z_L = j \omega L$  and  $Z_C = 1/j \omega C$ . In general, instantaneous and average values are more useful in switched circuits than the RMS values which are used almost exclusively in sinusoidal analysis.)

6. Instantaneous Power:  $P(t) = e(t) \times i(t)$

$$7. \quad [f(t)]_{\text{AVE}} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f(t) dt$$

$$8. \quad [f(t)]_{\text{RMS}} = \left[ \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f^2(t) dt \right]^{\frac{1}{2}}$$

By definition, there is no net change in a periodically varying function in steady-state over a full cycle. From (4) and (7):

$$[e_L(t)]_{\text{AVE}} = \frac{1}{T} \int_T e_L(t) dt = \frac{L}{T} \Delta i_L \Big|_T = 0$$

Therefore:

9. The average voltage across an inductor over a full cycle in steady-state is zero.

Similarly from (5) and (7):

10. The average current through a capacitor over a full cycle in steady-state is zero.

$$[i_C(t)]_{\text{AVE}} = \frac{1}{T} \int_T i_C(t) dt = \frac{C}{T} \Delta e_C \Big|_T = 0$$

Regardless of the apparent simplicity of these principles and the apparent complexity of many power circuits, lack of understanding of the circuit operation can nearly always be traced to a lack of understanding of the implications of one or more of these principles.

## APPENDIX II

### Selected Symbol List

#### Standard Circuit Symbols

$V_{do}$ =	DC or average voltage with no phase control and no load.
$V_d$ =	DC or average voltage with phase control and/or load.
$I_d$ =	DC or average current.
$P_d = V_d I_d$ =	Real Power (W).
$P_a = V_{do} I_d$ =	Apparent fundamental power (VA) neglecting harmonics.
$f_s$ =	Source frequency.
$\alpha$ =	Phase control angle.
$\phi, \cos \phi$ =	Displacement angle, displacement factor.
$\mu$ =	Commutation overlap angle.

#### Other Circuit Symbols

$V_s$ =	RMS line-to-line sinusoidal source voltage (secondary).
$V_P = \sqrt{2}V_S$ =	Peak line-to-line sinusoidal source voltage (secondary).
$I_L$ =	RMS AC line current (secondary).
$I_{L1}$ =	RMS fundamental AC line current (secondary).
$P$ =	Pulse number.

#### Standard Device Rating Symbols

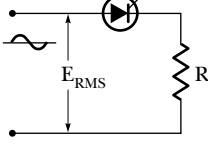
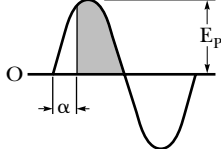
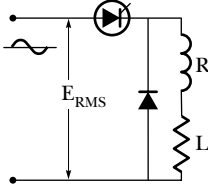
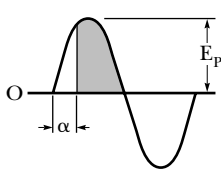
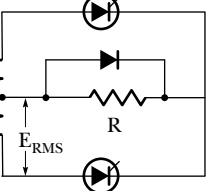
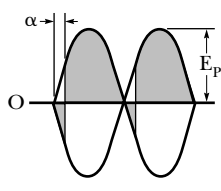
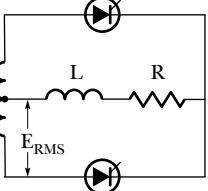
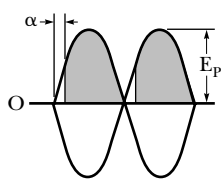
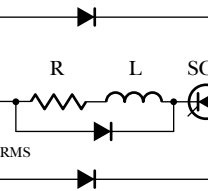
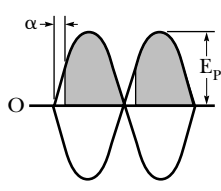
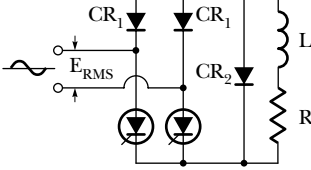
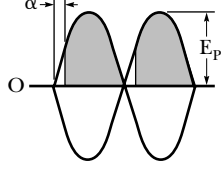
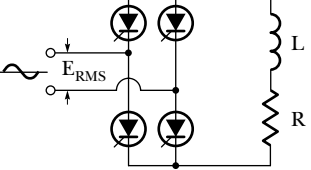
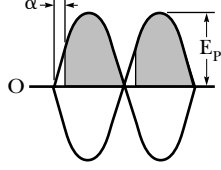
$V_{RRM}, V_{DRM}$ =	Thyristor reverse and forward off-state blocking voltages (repetitive).
$V_T$ =	On-state voltage.
$I_T$ =	On-state current.
$I_{TSM}$ =	Maximum on-state thyristor surge current (non-repetitive).
$I_{RRM}, I_{DRM}$ =	Reverse and forward off-state thyristor blocking (leakage) currents.
$t_q$ =	Circuit-commutated turn-off time.
$t_{rr}$ =	Junction reverse recovery time.
$T_J$ =	Junction temperature. (Subscripts c = Case A = Ambient).
$R_{\theta JC}$ =	Junction-to-case thermal resistance.
$Z_{\theta JC}(t)$ =	“Transient thermal impedance” Unit step function thermal response with device mounted on fixed-temperature heat sink.

**Characteristic** – An inherent and measurable property of a device, or a set of related values, for stated or recognized conditions.

**Rating** – A maximum or minimum limiting capability or condition for a device determined for specified operating and environmental conditions.

### APPENDIX III

#### Power Electronic Circuit Configurations — Voltage and Current Relationships

Name	Circuit		Waveforms	Max. Thyristor Voltage	PRV	
	Notation	Schematic			SCR	Diode
1. Half-Wave Resistive Load	1-1-1-H			1.4 $E_{RMS}$	$E_P$	...
2. Half-Wave Inductive Load with Free Wheeling Diode	1-1-1-H with Free Wheeling Diode			1.4 $E_{RMS}$	$E_P$	$E_P$
3. Center Tap Full Wave Resistive Load or Inductive Load with FWD	2-1-1-C			2.8 $E_{RMS}$	$2E_P$	...
4. Center Tap Inductive Load	2-1-1-C			2.8 $E_{RMS}$	$2E_P$	...
5. Center Tap Inductive Load with FWD Thyristor in DC Circuit	2-1-1-C with Thyristor in DC Circuit			1.4 $E_{RMS}$	0	2.8 $E_{RMS}$
6. Single Phase Bridge Half Control Resistive or Inductive FWD Load	Half Control 4-1-1-B and FWD			1.4 $E_{RMS}$	$E_P$	$E_P$
7. Single Phase Bridge Full Control Resistive or Inductive FWD Load	Full Control 4-1-1-B			1.4 $E_{RMS}$	$E_P$	...



Maximum Load Voltage $E_d = \text{Average}$ $E_a = \text{RMS}$	Load Voltage with Delayed Firing	Maximum Average Thyristor Current		Maximum Average Diode Current	
		Average Amps.	Cond. Period	Average Amps.	Cond. Period
$E_d = \frac{E_p}{\pi}$ $E_a = \frac{E_p}{2}$	$E_d = \frac{E_p}{2\pi} (1 + \cos\alpha)$ $E_a = \frac{E_p}{2\sqrt{\pi}} \sqrt{(\pi - \alpha + \frac{1}{2}\sin 2\alpha)}$	$\frac{E_p}{\pi R}$	180°	...	...
$E_d = \frac{E_p}{\pi}$	$E_d = \frac{E_p}{2\pi} (1 + \cos\alpha)$	<i>Highly Ind. Load</i> $\frac{E_p}{2\pi R}$	180°	$.54 \frac{E_p}{\pi R}$	210°
		<i>Conventional</i> $\frac{E_p}{\pi R}$			
$E_d = \frac{2E_p}{\pi}$	$E_d = \frac{E_p}{\pi} (1 + \cos\alpha)$	$\frac{E_p}{\pi R}$	180°	$.26 \frac{2E_p}{\pi R}$	<180°
$E_d = \frac{2E_p}{\pi}$	$E_d = \frac{2E_p}{\pi} (\cos\alpha)$ $(I_{dc} = K)$	$\frac{E_p}{\pi R}$	180°	...	...
$E_d = \frac{2E_p}{\pi}$	$E_d = \frac{E_p}{\pi} (1 + \cos\alpha)$	$\frac{2E_p}{\pi R}$	360°	$\frac{E_p}{\pi R}$	180°
				$\frac{.5(E_p)}{\pi R}$	<180° FWD
$E_d = \frac{2E_p}{\pi}$	$E_d = \frac{E_p}{\pi} (1 + \cos\alpha)$	$\frac{E_p}{\pi R}$	180°	$\frac{E_p}{\pi R}$	180°
				$\frac{.26(2E_p)}{\pi R}$	<180° FWD
$E_d = \frac{2E_p}{\pi}$	$E_d = \frac{2E_p}{\pi} (\cos\alpha)$ $(I_{dc} = K)$	$\frac{E_p}{\pi R}$	180°	...	...

Name	Circuit		Waveforms	Max. Thyristor Voltage	PRV	
	Notation	Schematic			SCR	Diode
8. Single Phase Bridge with Thyristor in DC and Free Wheeling Diode	4-1-1-B with Thyristor Control and FWD			1.4 $E_{RMS}$	$E_p$	$E_p$
9. Three Phase Half Wave Inductive Load	3-1-1-Y			2.45 $E_{RMS}$	2.45 $E_{RMS}$	...
10. Three Phase Half Wave Resistive Load or Inductive Load with Free Wheeling Diode	3-1-1-Y with Free Wheeling Diode			2.45 $E_{RMS}$	2.45 $E_{RMS}$	$E_p$
11. Three Phase Bridge Full Control Inductive Load	6-1-1-B with Full Control			2.45 $E_{RMS}$	2.45 $E_{RMS}$	...
12. Three Phase Bridge Full Control Resistive Load or Inductive Load with Free Wheeling Diode	6-1-1-B Full Control and FWD			2.45 $E_{RMS}$	2.45 $E_{RMS}$	2.45 $E_{RMS}$
13. Three Phase Bridge Half Control Resistive Load or Inductive Load with Free Wheeling Diode	6-1-1-B Half Control and FWD			2.45 $E_{RMS}$	2.45 $E_{RMS}$	2.45 $E_{RMS}$
14. Inverse Parallel Thyristors Resistive Load	AC Switch			1.4 $E_{RMS}$	$E_p$	...

Maximum Load Voltage $E_d = \text{Average}$ $E_a = \text{RMS}$	Load Voltage with Delayed Firing	Maximum Average Thyristor Current		Maximum Average Diode Current	
		Average Amps.	Cond. Period	Average Amps.	Cond. Period
$E_d = \frac{2E_p}{\pi}$	$E_d = \frac{E_p}{\pi} (1 + \cos \alpha)$	$\frac{2E_p}{\pi R}$	360°	$\frac{E_p}{\pi R}$	180°
				$\frac{5E_p}{\pi R}$	<180°
$E_d = \frac{3\sqrt{3}E_p}{2\pi}$	$E_d = \frac{3\sqrt{3}E_p}{2\pi} (\cos \alpha)$ $(I_{dc} = K)$	.33 $I_D$	120	...	...
$E_d = \frac{3\sqrt{3}E_p}{2\pi}$	$E_d = \frac{3\sqrt{3}E_p}{2\pi} (\cos \alpha)$ $(0 < \alpha < 30^\circ)$	.33 $I_D$	120°	Size to .33 $I_D$	<120° FWD
	$E_d = \frac{3E_p}{2\pi} [i + \cos(\alpha + 30)]$ $(30^\circ < \alpha < 150^\circ)$				
$E_d = \frac{3\sqrt{3}E_p}{\pi}$	$E_d = \frac{3\sqrt{3}E_p}{\pi} (\cos \alpha)$	.33 $I_D$	120°	...	...
$E_d = \frac{3\sqrt{3}E_p}{\pi}$	$E_d = \frac{3\sqrt{3}E_p}{\pi} (\cos \alpha)$ $(0 < \alpha < 60^\circ)$	.33 $I_D$	120°	Size to .33 $I_D$	<120° FWD
	$E_d = \frac{3\sqrt{3}E_p}{\pi} \left( 1 + \frac{\cos \alpha}{2} - \frac{\sqrt{3}}{2} \sin \alpha \right)$ $(60^\circ < \alpha < 120^\circ)$				
$E_d = \frac{3\sqrt{3}E_p}{\pi}$	$E_d = \frac{3\sqrt{3}E_p}{2\pi} (i + \cos \alpha)$	.33 $I_D$	120°	.33 $I_D$	120°
				Size to .33 $I_D$	<120° FWD
$E_a = \frac{E_p}{1.4}$	$E_a = \frac{E_p}{\sqrt{2\pi}} \sqrt{(\pi - \alpha + \frac{1}{2} \sin 2\alpha)}$	$\frac{E_p}{\pi R}$	180°	...	...