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Development of Educational Software for the Design of Slender Reinforced Concrete Columns

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DEVELOPMENT OF EDUCATIONAL SOFTWARE
FOR THE DESIGN OF
SLENDER REINFORCED CONCRETE COLUMNS

By

KHALID BOUDAOU

Directed by

Dr. GEORGE HEARN

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in
Structural Engineering and Structural Mechanics

University of Colorado – Boulder, 2013
Boulder, CO

SIGNATURE PAGE

This thesis entitled:

Development of Educational Software for the
Design of Slender Reinforced Concrete Columns

written by Khalid Boudaoui

has been approved for the Department of Civil, Architectural and Environmental
Engineering

Dr. George Hearn

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The final copy of this thesis has been examined by the signatories, and we find that both the content and the form meet acceptable presentation standards of scholarly work in the above mentioned discipline

ABSTRACT

Boudaoui, Khalid (M.S., Civil, Environmental and Architectural Engineering)

Development of Educational Software for the Design of Slender Reinforced Concrete Columns

Thesis directed by Professor George Hearn

Education of undergraduate students in first class of reinforced concrete design must include design of slender compression members since they are common components in structures such as buildings and bridges. Education must engage students to function as designers, that is, to determine the geometry of the member as well as the layout of the steel reinforcement to withstand a set of loadings. Educational software application, CU Slender Column, is developed to support the education of students in reinforced concrete design. CU Slender Column studies single slender reinforced concrete columns with circular cross section. Designs account for slenderness effects by applying the American Concrete Institute (ACI) 318 moment magnification method for both non-sway and sway frames. To help students in the design, CU Slender Column provides feedbacks relative to the accuracy of numerical values of strengths and loads as well as the satisfaction of ACI provisions. Also, it is demonstrated that CU Slender Column is an efficient tool able to deal with a wide range of design situations. The demonstration of the performance of CU Slender Column is done by developing 38 examples exploring different column designs passing or failing in strength or in ACI provisions.

DEDICATION

My research at the University of Colorado – Boulder on the development of education software for the design of slender columns has been valuable on many points. Being part of the University of Colorado – Boulder has been possible thanks to the unlimited help of my family especially my mother. Thus, I would like to dedicate my work to my loved ones.

Besides, working on a topic touching the education of students in structural engineering taught me the importance of the education of those who will build the world of tomorrow. I hope that my work will bring a humble contribution to enhance the education of future structural engineers. Thus, I would also like to dedicate my work to this large and wonderful community devoted to design and construct sustainable and reliable structures.

ACKNOWLEDGMENTS

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Chapter 1 - Introduction

Compression members are common components in reinforced concrete structures such as buildings and bridges. The structural engineering design of reinforced concrete columns must furnish members with adequate strength under simultaneous effects of axial force and end moments. In addition, for slender columns, the design must deal with stability issue by accounting for the slenderness effects.

Thus, engineering courses in design of reinforced concrete structures must include design of compression members and should engage students in design situations that include combined axial force and end moments as well as the effects of member slenderness. Education in design should allow students to function as designers, that is, to select the dimensions of the concrete member and the details of the steel reinforcement.

Efficiency in engineering courses requires to evaluate in detail the designs prepared by students and to check the designs for adequate strengths and for compliance with all provisions of design code.

In classes with large enrollments, checking and evaluation must be automated if each design is to be checked in detail. Evaluation is more than grading. Options for structural systems are considered as preliminary designs evolve with revision into final designs. In coursework, there is an additional level of iteration as students encounter, recognize and correct errors in their works as well as revise and refine their designs.

Evaluation of students' works must support and advance the process of design iterations. Evaluation must notify students of errors, provide some guidance on the nature of errors and the likely remedies. The educational goal is to guide and sometimes prod the student to a study of design requirements and discovers of appropriate design solutions.

For this purpose, it is developed educational software: CU Slender Column. CU Slender Column checks, evaluates and guides students in the design of a single slender reinforced concrete column.

CU Slender Column operates in two levels. First, given the student's column design and the set of loadings, CU Slender Column executes a benchmark study to compute the axial and the bending strengths of the column as well as the factored demands. Second, CU Slender Column provides feedbacks on whether the column design satisfies strength requirements, design code provisions as well as if the student's numerical values of strengths and loads are accurate compared to values computed by the benchmark study.

The Design Code

CU Slender Column implements the provisions of Chapter 10 for flexure and axial loads of the latest version of the ACI building code: ACI 318 for structural concrete [*Building*, 2011].

The Design Method

Slenderness effects refer to the phenomenon when the action of the axial force combined with the column deflection leads to additional moments along the column. ACI 318 proposes two methods to account for slenderness effects in the design of slender reinforced concrete columns. First, the moment magnification method which accounts for slenderness effects by magnifying the larger column end moments. Second, the P- Δ or 2nd order analysis method which discretizes the column to plot the bending moment along the member and catches the maximum moment [*Building*, 2011]. CU Slender Column uses the traditional moment magnification method which is reliable, less difficult to implement and appropriate for students in first class of reinforced concrete design.

Limitation on Structural Members

The study considers single slender reinforced concrete column with circular cross section and spiral ties reinforcement. Other structural members, cross-sectional geometries and type of ties are beyond the scope of the study.

Organization of the Study

The study is organized in Chapters.

Chapter 1 is an introduction to the study. It highlights the interest of developing educational software for the design of slender reinforced concrete columns. Also, it presents the building code the study uses as well as the design method to account for slenderness effects.

Chapter 2 focuses on the theory and the application of the moment magnification method. The theory is explained through an elastic analysis of slender columns subjected to axial forces and end moments in non-sway frames. The purpose is to show that ACI 318 formula for the moment magnification approaches the elastic solution. Besides, Chapter 2 provides a detailed method for computing axial and bending strengths for a circular column. Chapter 2 ends with two detailed procedures showing how to apply the moment magnification method in the non-sway and sway cases.

Chapter 3 inventories some professional software available in the market as well as educational tools developed for the design of structural members. Also, Chapter 3 presents the graphical user interface and the organization of the program running CU Slender Column.

Chapter 4 starts by presenting the three demonstrations that aim at validating CU Slender Column. Then, it is detailed two examples of the application of ACI 318 moment magnification method for non-sway and sway frames with references to program routines that perform the computations. Chapter 4 ends with the presentation of the 38 examples developed for the validation of CU Slender Column.

Chapter 5 summarizes the study for educational software for the design of slender reinforced concrete columns and opens to possible improvements.

Chapter 2 - Theory and Application of ACI 318 Moment Magnification Method

To begin, the Chapter presents the elastic solution for secondary moments in compression members and the ACI 318 method for moment magnification for slender compression members within a non-sway frame. Then, results are analyzed to show the consistency between both methods.

Besides, the Chapter presents a detailed method for the computations of both axial and bending strengths for reinforced concrete column with circular cross section.

Finally, the Chapter presents step by step the application of the ACI 318 moment magnification method for both non-sway and sway frames.

It is noted that throughout the study, the same ACI 318 label formats are used in reference of ACI 318 sections and equations. For example, ACI 318 sections are referred using the format 10.13.2 and ACI 318 equations are referred using the format (10-7).

ACI 318 Loading Combinations

ACI 318 has developed a set of seven loading combinations (Eq. 1 to Eq. 7) to assess the loading demand for the design of concrete structures. The loading demand accounts for all loading contributions and has to affect to each contribution a factor reflecting the accuracy the contribution is computed and the variation of the contribution during the lifetime of the structure. Also, different combinations are made to reflect the probability of simultaneous occurrence of contributions [Building, 2011]. The load computed from a loading combination is the factored load.

ACI (9-1)	$U = 1.4(D + F)$	Eq. 1
ACI (9-2)	$U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$	Eq. 2
ACI (9-3)	$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W)$	Eq. 3
ACI (9-4)	$U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$	Eq. 4
ACI (9-5)	$U = 1.2D + 1.0E + 1.0L + 0.2S$	Eq. 5
ACI (9-6)	$U = 0.9D + 1.6W + 1.6H$	Eq. 6
ACI (9-7)	$U = 0.9D + 1.0E + 1.6H$	Eq. 7

Contributions used in the ACI 318 loading combinations are defined in Table 1.

Table 1 – Contributions in ACI 318 Loading Combinations

Contribution	Signification
U	Factored load
D	Dead load
E	Load effects of earthquake
F	Loads due to weight and pressure of fluids with well-defined densities
H	Loads due to weight and pressure or soil, water in soil, or other materials
L	Live loads
L _r	Roof live loads
R	Rain load
S	Snow load
T	Cumulative effect of temperature, creep, shrinkage, differential settlement
W	Wind load

Elastic Theory of the Moment Magnification Method for Non-Sway Frame

This section aims at presenting the elastic theory behind the moment magnification method. The Figure 1 shows on the left a non-sway frame structure and on the right a detail of the studied column. The column is modeled as a 1-Dimensional (1D) member and has 3 external factored loads at each end. The end i has a factored bending moment M_i , a factored axial force P_u and a factored shear force V . The end j has a factored bending moment M_j , a factored axial force P_u and a factored shear force V .

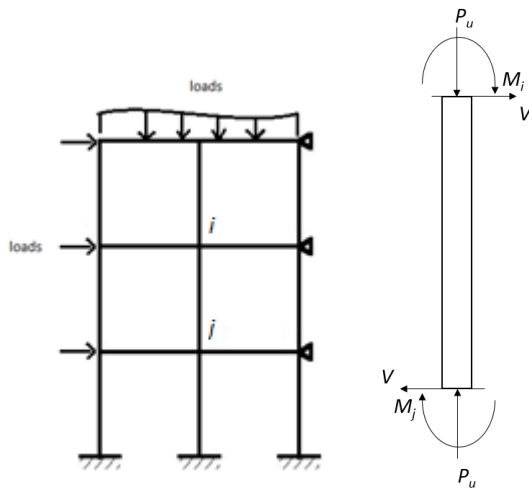


Figure 1 – Non-sway frame structure

Sign Convention

The study uses the frame convention. It defines positive axial forces, positive shear forces and positive moments as oriented in the Figure 2. Clockwise acting end moments are counted positive. By convention the larger of the two factored end moments, either M_i or M_j , is noted M_2 and the smaller of the two factored end moments is noted M_1 .

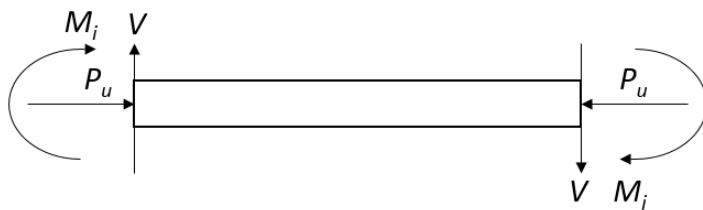


Figure 2 – Frame sign convention

Throughout the study, the column is laid out horizontally for a better reading.

Expression of the Moment $M(x)$ along the column

The Figure 3 shows in dotted-line the deformed shape of the column under the external loads. Also, a free body cut into the column is made at the abscissa x to determine through an equilibrium analysis the expression of the moment $M(x)$ at the abscissa x . The deflection of the column at the abscissa x is noted $y(x)$.

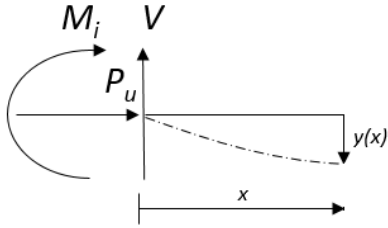


Figure 3 – Free Body Diagram

Governing Equation and Solution

The primary moment noted $M_p(x)$ corresponds to the moment along the column when there is no axial force P_u . The secondary moment noted $P_u * y$ is the moment created by the coupling of the column deflection $y(x)$ and P_u .

A derivation of the linear differential equation from the equilibrium analysis of the Figure 3 is provided by [Salmon and Johnson, 1990] in case of a column with unequal end moments without transverse loadings.

$$M''(x) + \omega^2 M(x) = M''_p(x) \quad \text{Eq. 8}$$

where

$$\omega = \sqrt{\frac{P_u}{EI}} \quad \text{Eq. 9}$$

EI is the reduced stiffness. E is the elastic modulus and I is the second moment of area.

$M_p(x)$ refers to the primary moment varying along the column as:

$$M_p(x) = M_i + \frac{-M_j - M_i}{L} x \quad \text{Eq. 10}$$

The resolution of the linear differential equation gives a solution valid for any x . The maximum moment in the column which accounts for the primary moment and the second moment is determined as [Salmon and Johnson, 1990]:

$$M_{max} = \sqrt{M_i^2 + \left(\frac{-M_j - M_i \cos(\omega L)}{\sin(\omega L)} \right)^2} \quad \text{Eq. 11}$$

It is noted that the length of the column $[0, L]$ constitutes the domain of interest. Any maximum moment M_{max} greater than the larger end moments for an abscissa outside the domain does not refer to the moment magnification. In fact, the curve of the moment along the column continues away from the domain and reaches larger values.

The maximum moment is reached at the abscissa x_0 when:

$$\tan(\omega x_0) = \frac{-M_j - M_i \cos(\omega L)}{M_i \sin(\omega L)} \quad \text{Eq. 12}$$

$$x_0 = \frac{1}{\omega} \operatorname{atan}\left(\frac{-M_j - M_i \cos(\omega L)}{M_i \sin(\omega L)}\right) \quad \text{Eq. 13}$$

If x_0 is outside the domain of interest $[0, L]$, then the maximum moment in the column is the larger factored end moments, M_2 .

If x_0 is inside the domain of interest $[0, L]$, then the maximum moment in the column is the computed M_{max} using (Eq. 11).

In the case of a column without transverse loading, the maximum moment along the column before counting for the secondary moment is M_2 .

The moment magnification occurs by increasing the axial force P_u , that is, by compressing axially the column harder, the deflection of the column increases, then the second moment $P_u * y$ increases until the maximum moment M_{max} becomes bigger than M_2 .

The elastic magnification factor, $\delta_{ns,elast}$, is defined as:

$$\delta_{ns,elast} = \left| \frac{M_{max}}{M_2} \right| \quad \text{Eq. 14}$$

ACI 318 Moment Magnification Method for Non-Sway Frame

ACI 318 provides an approximate method to estimate the magnified moment.

The buckling load P_c is by definition the axial load needed to buckle a column. It depends only on the column characteristics: The length L , the effective length factor k and EI .

$$\text{ACI (10-10)} \quad P_c = \frac{\pi^2 EI}{(kL)^2} \quad \text{Eq. 15}$$

The Figure 4 shows values of k for different end conditions.









		k			
TOP	Hinged 	0.81	0.91	0.95	1.00
	Elastic 	0.77	0.86	0.90	0.95
	Elastic 	0.74	0.83	0.86	0.91
	Stiff 	0.67	0.74	0.77	0.81
					
		Stiff	Elastic	Elastic	Hinged
		BOTTOM			

Figure 4 – Values of k for different column configuration [Saatcioglu, 2012]

ACI 318 constrains the axial load P_u to not more than 75% of the buckling load P_c . This is partially justified by the approximation of the buckling load formula which is derived using a linear model. However, while buckling, the column tends to respond in a nonlinear fashion due to large displacements and the nonlinear behavior of concrete.

For non-sway frames, ACI 318 introduces C_m , the factor relating actual moment diagram to an equivalent uniform moment diagram, to account for the curvature of the column. Indeed, in case of identical end moments magnitudes, a double-curved column is less deflected than a single-curved column. Thus, C_m is used to mitigate the effect of magnification when a column is double-curved.

For sway frames, conditions of single or double curvature do not affect the values of magnification factor.

ACI 318 defines a magnification factor for non-sway frame, δ_{ns} , as:

$$\text{ACI (10-9)} \quad \delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0 \quad \text{Eq. 16}$$

$$C_m = 0.6 - 0.4 \frac{M_1}{M_2} \geq 0.4 \quad \text{Eq. 17}$$

It is noted that ACI 318 uses a different expression for C_m with a positive sign instead of a negative sign because ACI 318 relies on the single beam convention where moments sagging the beam are positive. The upper bound for C_m is 1.0.

The magnified moment, M_c , for which the column is designed is computed using δ_{ns} and M_2 .

$$\text{ACI (10-8)} \quad M_c = \delta_{ns} * M_2 \quad \text{Eq. 18}$$

Comparison between elastic analysis and ACI 318 expressions for magnification factor

A set of two examples aim at showing the consistency between the magnification factor derived from elastic analysis and the magnification factor defined by ACI 318.

Columns in both examples are identical. The first example studies a single-curved column and the second example studies a double-curved column.

Example 1 – Non-Sway / Single Curvature

Consider a 20 ft-long pin supported circular reinforced concrete column in a non-sway frame. The reduced stiffness of the column, EI , is equal to $20 * 10^6 \text{ kin}^{-2}$. Consider the column is subjected to positive clockwise M_i , negative counterclockwise M_j and axial force P_u .

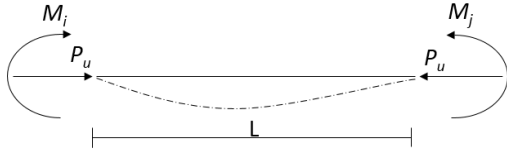


Figure 5 – Single-curved column in non-sway frame

For a pin supported column, the effective length factor k is 1.0.

The buckling load P_c is:

$$\text{ACI (10-10)} \quad P_c = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 20 * 10^6}{(1.0 * 20 * 12)^2} \quad \text{Eq. 19}$$

$$= 3427 \text{ k}$$

The end moments are:

$$M_i = 100 \text{ kft} = M_1 \quad \text{Eq. 20}$$

$$M_j = -100 \text{ kft} = M_2 \quad \text{Eq. 21}$$

The factor C_m is:

$$C_m = 0.6 - 0.4 \frac{M_1}{M_2} = 0.6 - 0.4 \frac{100}{-100} \quad \text{Eq. 22}$$

$$= 1.0$$

The Figure 6 shows the moment along the column while P_u increases. The moment magnification occurs as soon as P_u is applied.

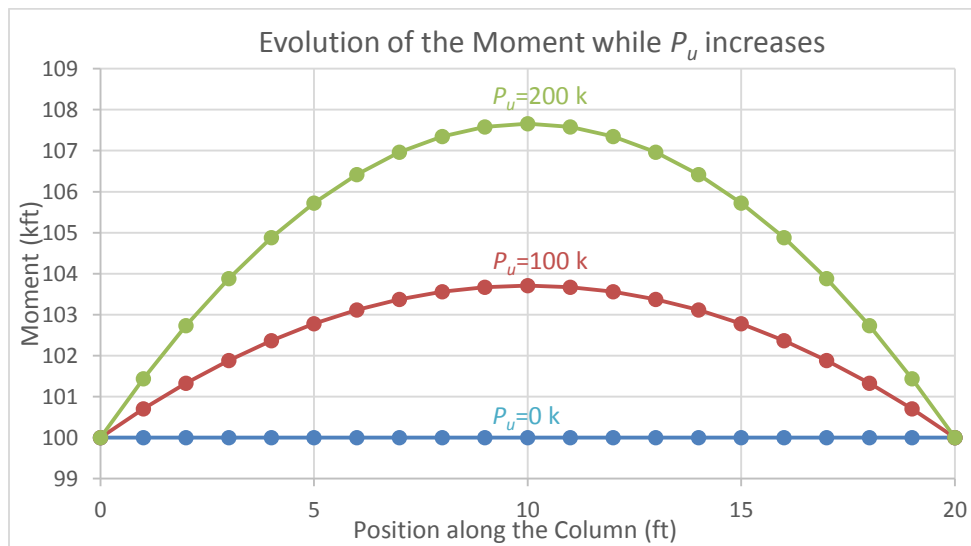


Figure 6 – Magnification from Elastic analysis

The Table 2 lists the magnification factor from the elastic analysis $\delta_{ns,elast}$ and from the ACI 318 method δ_{ns} for increasing P_u .

Table 2 – Moment magnification factor from elastic analysis and ACI 318 for example 1

L	ft	20		
EI	kin^{-2}	$20 * 10^6$		
C_m		1.0		
P_c	k	3427		
M_i	kft	100		
M_j	kft	-100		

P_u	ω	M_{max}	Elastic Analysis	ACI 318
			$\delta_{ns,elast}$	δ_{ns}
k	in^{-1}	kft		
0.1	0.00007	100	1.00	1.00
100	0.00224	104	1.04	1.04
200	0.00316	108	1.08	1.08
400	0.00447	116	1.16	1.18
400	0.00447	116	1.16	1.18
800	0.00632	138	1.38	1.45
1600	0.00894	210	2.10	2.65

The Figure 7 shows the evolution of the magnification factor from the ACI 318 and the elastic analysis.

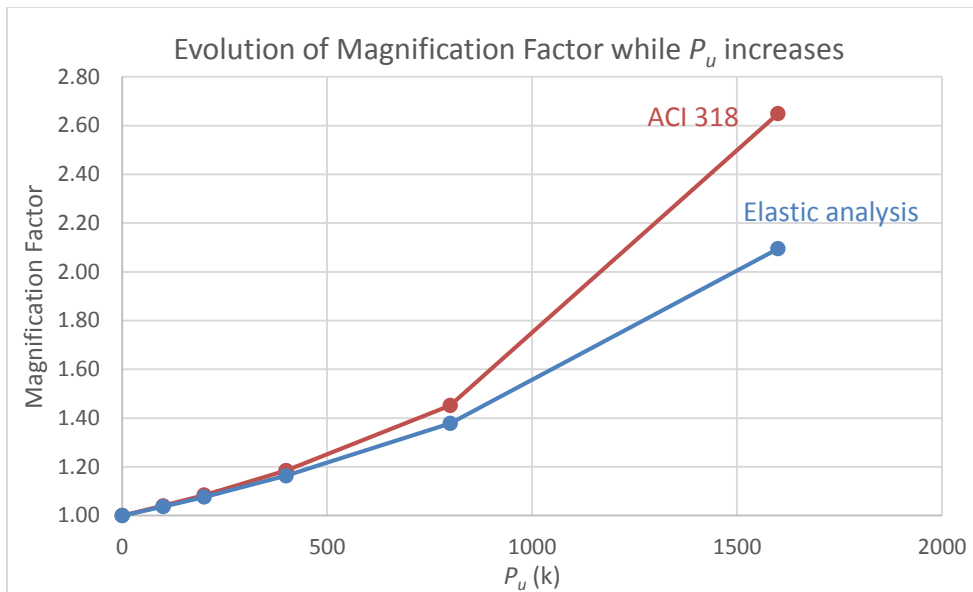


Figure 7 – Moment Magnification factor for Elastic Analysis and ACI 318

By increasing the magnitude of the axial force, the moment magnification factor curve becomes convex and shows the magnification effect.

It is noted that the curves fit well for an axial load ranging from 0 to 1000 k and that the ACI 318 curve is above the elastic curve to provide a more conservative design.

Beyond an axial force magnitude of 800 k and values of magnification factor larger than 1.40, the two curves diverge and highlights a limitation in the ACI 318 method. In this range, the linear model is no longer valid due to large displacements and the non-linear behavior of the concrete.

Example 2 – Non-Sway / Double Curvature

Consider a 20 ft-long pin supported circular reinforced concrete column in a non-sway frame. The reduced stiffness of the column, EI , is equal to $20 * 10^6 \text{ kin}^{-2}$. Consider the column is subjected to positive clockwise M_i , positive clockwise M_j and axial force P_u .

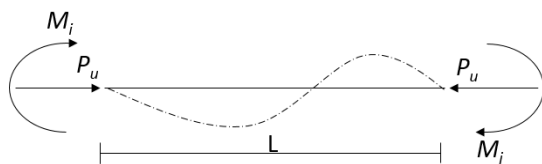


Figure 8 – Double-curved column in non-sway frame

The buckling load P_c remains at 3427 k.

The end moments are:

$$M_i = 50 \text{ kft} = M_1 \quad \text{Eq. 23}$$

$$M_j = 100 \text{ kft} = M_2 \quad \text{Eq. 24}$$

The factor C_m is:

$$\begin{aligned} C_m &= 0.6 - 0.4 \frac{M_1}{M_2} = 0.6 - 0.4 \frac{50}{100} \\ &= 0.4 \end{aligned} \quad \text{Eq. 25}$$

The Figure 9 shows the moment along the column while P_u increases. The moment magnification occurs when P_u is larger than 1600 k.

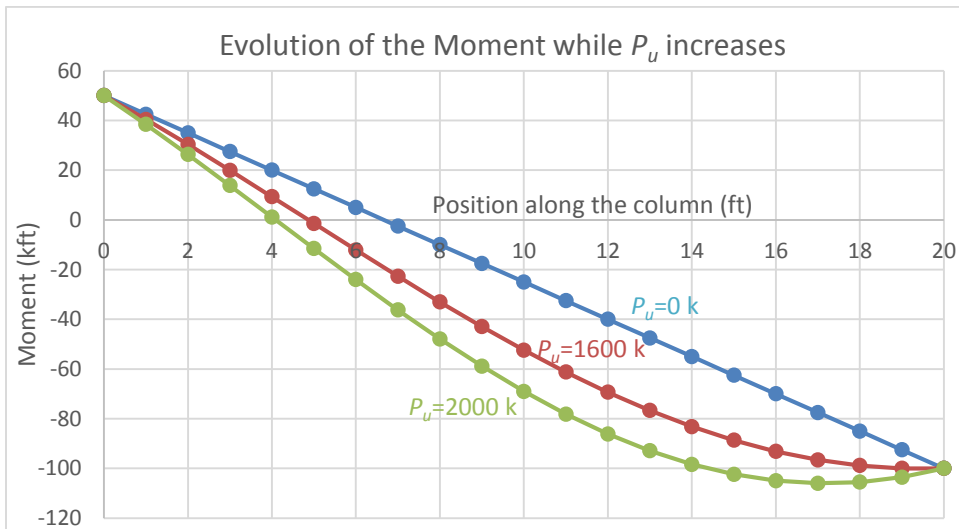


Figure 9 – Magnification from Elastic analysis

The Table 3 lists the magnification factor from the elastic analysis $\delta_{ns,elast}$ and from the ACI 318 δ_{ns} for increasing P_u .

Table 3 – Moment magnification factor from elastic analysis and ACI 318 for example 2

L	ft	20
EI	kin^{-2}	$20 * 10^6$
C_m		0.4
P_c	k	3427
M_i	kft	50
M_j	kft	100

		Elastic Analysis		ACI 318
P_u	ω	M_{max}	$\delta_{ns,elast}$	δ_{ns}
k	in^{-1}	kft		
0.1	0.00007	100	1.00	1.00
100	0.00224	100	1.00	1.00
200	0.00316	100	1.00	1.00
400	0.00447	100	1.00	1.00
800	0.00632	100	1.00	1.00
1600	0.00894	100	1.00	1.06
2000	0.01000	106	1.06	1.06

The Figure 10 shows the evolution of the magnification factor from the ACI 318 and the elastic analysis.

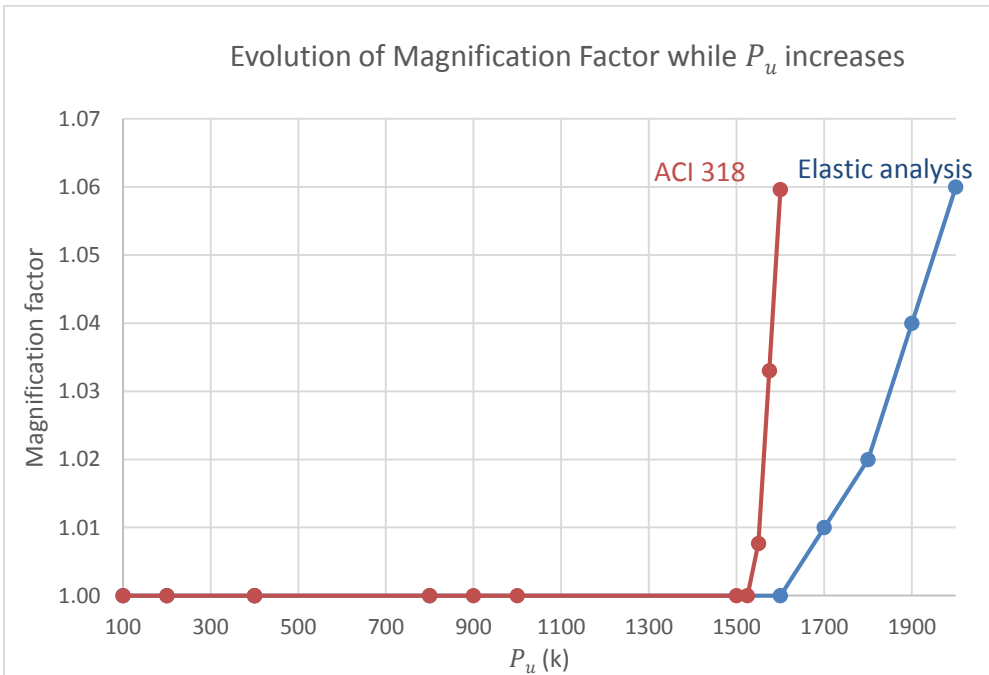


Figure 10 – Moment Magnification factor for Elastic Analysis and ACI 318

The double curvature of the column decreases the deflection y and the secondary moment $P_u * y$. The Figure 10 shows that M_2 remains the maximum moment in the column for the axial force ranging from 0 to 1600 k. For this range, there is no magnification and the magnification factor is equal to 1.0. When applying an axial force larger than 1600 k, the magnification starts, the magnification factor takes values greater than 1.0 and the maximum moment is located within the column.

Here is illustrated the role of C_m as a factor mitigating the magnification factor because of the double curvature of the column.

As in the non-sway case, the ACI 318 curve is beyond the elastic curve in order to provide a more conservative design.

ACI 318 Moment Magnification Method for Sway Frame

In the application of ACI 318 moment magnification method, ACI 318 notations are used: M_{1ns} is the smaller factored end moments in non-sway case, M_{2ns} the larger factored end moments in non-sway case, M_{1s} is the smaller factored end moments in sway case and M_{2s} is the larger factored end moments in sway case.

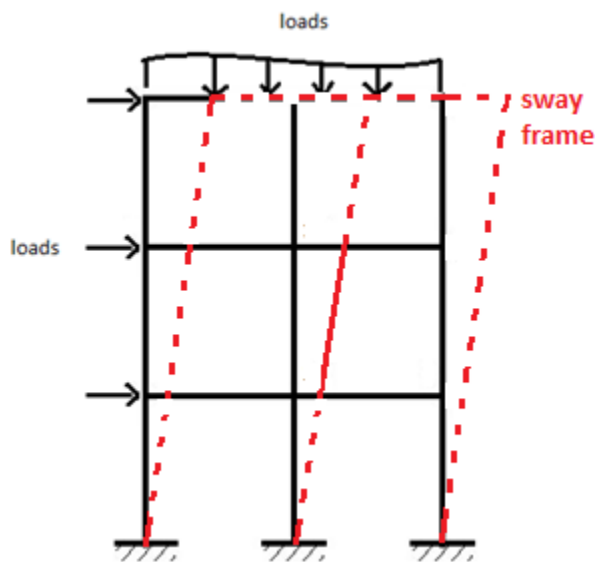


Figure 11 – Sway frame structure in dotted line

ACI 318 computes the magnified sway moment, $\delta_s M_s$, as:

$$\text{ACI (10-18)} \quad \delta_s M_s = \frac{M_s}{1 - \frac{P_u}{0.75P_c}} \geq M_s \quad \text{Eq. 26}$$

The magnification factor, δ_s , in case of sway frame is defined as:

$$\delta_s = \frac{1.0}{1 - \frac{P_u}{0.75P_c}} \geq 1.0 \quad \text{Eq. 27}$$

For sway frame, the upper bound for δ_s is 2.5 [*Building*, 2011]. For values larger than 2.5, the frame might be subjected to foundation rotations and stiffness variations along the column. It is noted that the factor C_m is equal to 1.0 for a sway frame, because as stated earlier, the curvature of the column does not influence the magnification factor.

ACI 318 method estimates total magnified moment, M_c , as the sum of two contributions: Non-sway moment M_{2ns} and magnified sway moment $\delta_s M_{2s}$.

$$M_c = M_{2ns} + \delta_s M_{2s} \quad \text{Eq. 28}$$

Strength Computations

This section describes the method to compute the axial strength ϕP_n and the bending strength ϕM_n for columns with circular cross section and subjected to combined axial force and end moments. The method applies for both sway and non-sway frames.

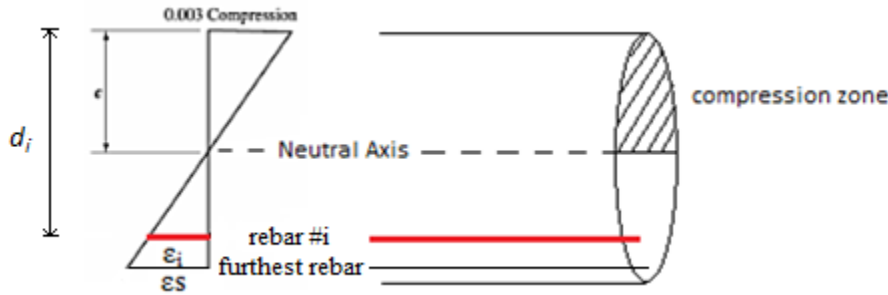


Figure 12 – Strain diagram of column cross section

To begin, the depth of the concrete compression zone, c , is determined by equalizing the strength eccentricity, e_n and the demand eccentricity, e_u .

$$e_u = \frac{M_c}{P_u} \quad \text{Eq. 29}$$

$$e_n = \frac{\phi M_n}{\phi P_n} \quad \text{Eq. 30}$$

$$e_n = e_u \quad \text{Eq. 31}$$

The strain of the i^{th} longitudinal rebar, ε_i , is computed using c and the distance between the i^{th} longitudinal rebar and the most compressed concrete fiber, d_i .

$$\varepsilon_i = 0.003 \frac{d_i - c}{c} \quad \text{Eq. 32}$$

The stress in the i^{th} longitudinal rebar, f_i , is computed using ε_i and the steel Young's modulus E_s .

$$f_i = \varepsilon_i * E_s \quad \text{Eq. 33}$$

By convention, the rebar is in compression when f_i is negative and the rebar is in tension when f_i is positive.

A parabolic stress distribution is used to describe the actual stresses in concrete (Figure 13). However, calculating internal forces with the parabolic stress distribution involves integration over a parabolic curve. To make the computations easier, an equivalent rectangular stress distribution is used (Figure 13).

Table 4 – Notations used in Figure 13

a (in)	equivalent depth of the concrete compression zone in rectangular distribution
c (in)	depth of the concrete compression zone in parabolic distribution
f'_c (ksi)	concrete compression strength
NA	Neutral Axis

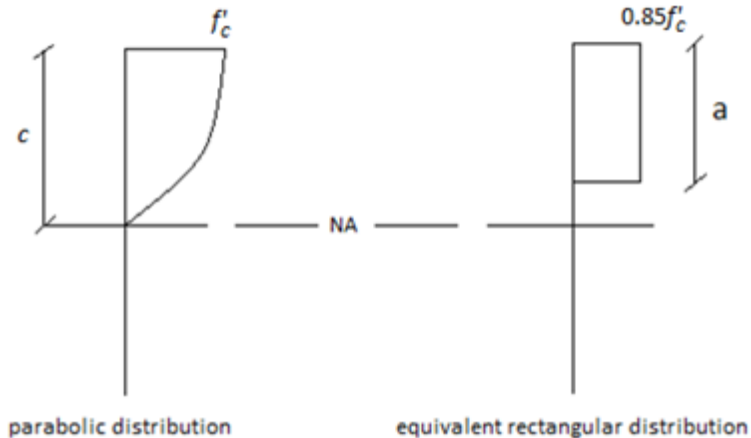


Figure 13 – Concrete Stress Diagram

ACI 318 computes a based on c as:

$$a = \beta_1 c \quad \text{Eq. 34}$$

The parameter β_1 is equal to 0.85 for concrete with f'_c less than 4 ksi and 0.05 less for each 1 ksi of f'_c in excess of 4 ksi [Building, 2011].

For a circular column, the compression zone is a segment of a circle. Two cases are studied: The compression zone is either smaller or larger than half of the column cross-sectional area.

Compression zone smaller than half the column cross-sectional area

The Figure 14 shows the cross section of a circular reinforced concrete column with a compression zone smaller than half the column cross-sectional area.

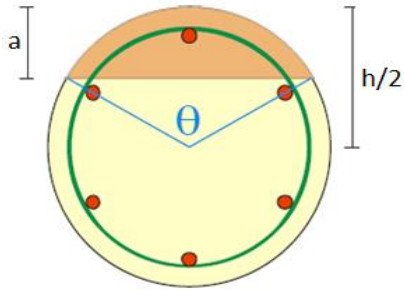


Figure 14 – Compression zone smaller than half the column cross-sectional area

For known column diameter h and depth of compression zone a , the central angle of the compression zone, θ , is:

$$\theta = 2 * \arccos\left(1 - \frac{2a}{h}\right) \quad \text{Eq. 35}$$

The area of concrete compression zone, A_c , is:

$$A_c = \frac{1}{8} * h^2(\theta - \sin(\theta)) \quad \text{Eq. 36}$$

The distance from the center of the column to the line of action of the compression force, \bar{y} , is:

$$\bar{y} = \frac{2 * h * \sin^3\left(\frac{\theta}{2}\right)}{3(\theta - \sin(\theta))} \quad \text{Eq. 37}$$

P_n is computed using the equilibrium of forces at the section, that is, by summing the compressive force in concrete section and the force in steel rebars where A_i is the area of a steel rebar. In (Eq. 38), the negative sign comes from the convention counting positive the tension of a steel rebar and positive the compression in concrete. M_n is computed by summing the product of compressive force in concrete and distance \bar{y} and the product of force in steel rebars and distance from rebars to column center.

$$P_n = 0.85 f'_c A_c - \sum A_i f_i \quad \text{Eq. 38}$$

$$M_n = 0.85 f'_c A_c \bar{y} + \sum A_i f_i \left(d_i - \frac{h}{2}\right) \quad \text{Eq. 39}$$

Compression zone larger than half the column cross-sectional area

The Figure 15 shows the cross section of a circular reinforced concrete column with a compression zone larger than half of the column cross-sectional area.

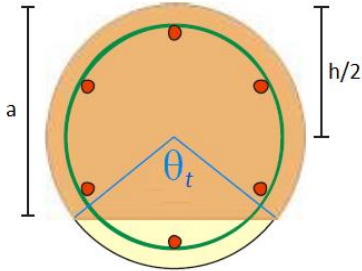


Figure 15 – Compression zone larger than half the column cross-sectional area

The central angle of the concrete zone not in compression, θ_t , is:

$$\theta_t = 2 * \arccos\left(\frac{2a}{h} - 1\right) \quad \text{Eq. 40}$$

The area of the concrete zone not in compression, A_t , is:

$$A_t = \frac{1}{8} * h^2 (\theta_t - \sin(\theta_t)) \quad \text{Eq. 41}$$

The distance from the center of the column to the line of action of the tension force acting in the concrete zone not in compression, \bar{y}_t , is:

$$\bar{y}_t = \frac{2 * h * \sin^3\left(\frac{\theta_t}{2}\right)}{3(\theta_t - \sin(\theta_t))} \quad \text{Eq. 42}$$

The strength computations are similar to the previous case. P_n is computed by summing the compressive force in concrete section and the force in steel rebars. M_n is computed by summing the product of compressive force in concrete and distance \bar{y} and the product of force in steel rebars and distance from rebars to column center.

$$P_n = 0.85 f_c \left(\frac{\pi h^2}{4} - A_t \right) - \sum A_i f_i \quad \text{Eq. 43}$$

$$M_n = 0.85 f_c A_t \bar{y} + \sum A_i f_i \left(d_i - \frac{h}{2} \right) \quad \text{Eq. 44}$$

The reduction factor ϕ lowers the column strength to make the design more conservative.

$$\phi = 0.70 + 0.20 \frac{(\varepsilon_t - 0.002)}{0.003} \quad \text{Eq. 45}$$

$$0.70 \leq \phi \leq 0.90$$

where ε_t is the strain in the furthest rebar from the most compressed concrete fiber.

By convention, ε_t is positive in tension and negative in compression.

The notations used in Figure 16 are defined in Table 5:

Table 5 – Notations used in steel linear stress-strain diagram

σ (ksi)	Stress in steel rebar
ε	Strain of steel rebar

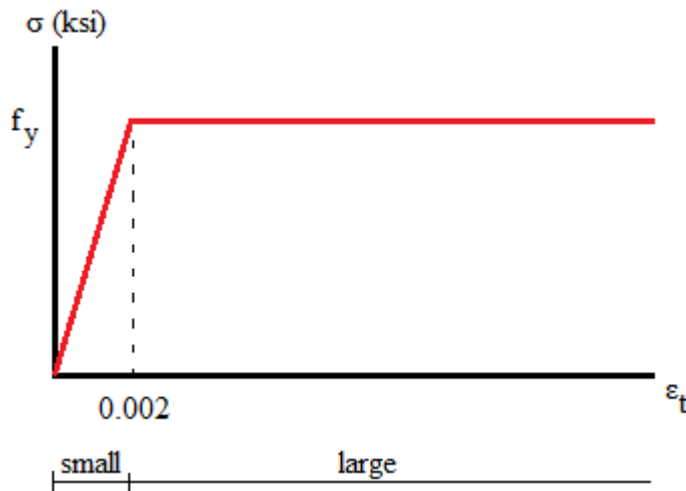


Figure 16 – Steel linear stress-strain diagram

It is noted that the expression of ϕ is proportional to ε_t . The linear stress-strain steel diagram shows that variations in small values of strain influence the values of stresses hence the strengths and variations in large values of strain do not influence stresses. Thus, when ε_t is small, the design must be more conservative by lowering ϕ to account for uncertainty on stresses in steel rebars.

ACI 318 Moment Magnification Method and Checks of Strength Requirements for Non-Sway and Sway Frames

Two procedures for both non-sway and sway frames show step by step the ACI 318 moment magnification method and the checks of the column design. It is noted that ACI 318 notations are used.

Non-sway frame

The following procedure shows in detail the sequence of computations for the application of the ACI 318 moment magnification method in non-sway case and the checks of strength requirements.

Step #1:

Compute the factored loads.

The student gets the values of end moments contributions using a structural analysis software. Besides, the factored end moments are computed using ACI 318 loading combinations. For columns subjected to little or no moments, a minimum moment (Eq. 47) is computed using the minimum eccentricity (Eq. 46) because the axial loads are never rigorously at the center of the member and little unpredictable moments may occur at both ends [Building, 2011]. Accidental bending effects are taken proportional to the column diameter.

$$e_{min} = 0.6 + 0.03h \quad \text{Eq. 46}$$

$$\text{ACI (10-14)} \quad M_{2,min} = P_u (0.6 + 0.03h) \quad \text{Eq. 47}$$

Step #2:

Compute the slenderness of the column.

For non-sway frames, ACI 318 allows to ignore the slenderness effects, that is, not applying the moment magnification method, if the slenderness of the column is less than:

$$\frac{kL}{r} \leq 34 + 12 \frac{M_1}{M_2} \quad \text{Eq. 48}$$

where r is the radius of gyration of the concrete cross-section defined as a function of I_g , the second moment of area of the concrete gross area and A_g , the concrete gross area.

$$r = \sqrt{\frac{I_g}{A_g}} \quad \text{Eq. 49}$$

Step #3:

Compute C_m .

$$C_m = 0.6 - 0.4 \frac{M_1}{M_2} \geq 0.4 \quad \text{Eq. 17}$$

Step #4:

Apply the ACI 318 moment magnification method.

In the ACI 318 moment magnification method, several intermediate parameters are needed.

The parameter βd accounts for the concrete creep. The creep is caused by the action of long-term loads or sustained loads such as the dead load. If the contribution of the sustained loads with respect to the global factored load P_u increases, the creep will increase as well [Building, 2011].

$$\beta d = \frac{1.2 * P_d}{P_u} \quad \text{Eq. 50}$$

The creep leads to the shortening of the column length, then increases the lateral deflection and creates an additional eccentricity which increases the secondary moment. Thus, the column reduced stiffness, EI , must be lowered to reduce P_c hence to increase δ_{ns} [Building, 2011]. The factor 0.4 accounts for the stiffness reduction due to cracks in concrete. E_c is the concrete Young's modulus.

$$\text{ACI (10-12)} \quad EI = \frac{0.4 * E_c * I_g}{1 + \beta d} \quad \text{Eq. 51}$$

The buckling load P_c is computed as (Eq. 19).

$$\text{ACI (10-10)} \quad P_c = \frac{\pi^2 * EI}{(kL)^2} \quad \text{Eq. 19}$$

The magnification factor for non-sway frame is computed as (Eq. 16).

$$\text{ACI (10-9)} \quad \delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 * P_c}} \quad \text{Eq. 16}$$

The magnified moment M_c for which the column is designed is computed as (Eq. 18).

$$\text{ACI (10-8)} \quad M_c = \delta_{ns} * M_2 \quad \text{Eq. 18}$$

Step #5:

Compute strengths ϕM_n and ϕP_n .

ϕM_n and ϕP_n are computed using the method detailed in the section “Strength Computations” (page 18). The strength computations are based on the matching of the demand eccentricity and the strength eccentricity.

Step #6:

Pass/Fail in strength.

The column satisfies strength requirements if ϕP_n is larger than P_u and ϕM_n is larger than M_c .

Table 6 – Checks on strength requirements in non-sway frame

$\phi P_n \backslash \phi M_n$	$\geq M_c$	$< M_c$
$\geq P_u$	Pass	Fail
$< P_u$	Fail	Fail

Sway frame

The following procedure shows in detail the sequence of computations for the application of the ACI 318 moment magnification method in sway case and the checks of strength requirements.

Step #1:

Compute the slenderness of the column.

For sway frames, ACI 318 allows to ignore the slenderness effects, that is, not applying the moment magnification method, if the slenderness of the column is less than:

$$\frac{kL}{r} \leq 22 \quad \text{Eq. 52}$$

Step #2:

Compute the factored loads.

The student gets the values of end moments contributions using a structural analysis software. Besides, the factored end moments are computed using ACI 318 loading combinations. In ACI 318 moment magnification method for sway frames, it is required to provide the end moments contributions in sway case and the end moments contributions in non-sway case to compute M_{2s} and M_{2ns} .

Step #3:

Determine if the frame can be considered as non-sway.

The stability index for a story Q represents the importance of moments induced by the sway with respect to the moments induced by external lateral loads. If Q is smaller than 0.05, the sway contribution is considered as small and the frame is considered as non-sway [Building, 2011]. The story drift Δ_0 is obtained through a structural analysis software.

$$\text{ACI (10-6)} \quad Q = \frac{P_u * \Delta_0}{V_u * L} \quad \text{Eq. 53}$$

Step #4:

Apply the ACI 318 moment magnification method.

The computation of P_c , βd and EI are similar to the non-sway case. The magnification factor for sway frame is computed as (Eq. 27).

$$\delta_s = \frac{1}{1 - \frac{P_u}{0.75 * P_c}} \geq 1.0 \quad \text{Eq. 27}$$

The magnified moment M_c for which the column is designed is computed as (Eq. 28).

$$M_c = M_{2ns} + \delta_s M_{2s} \quad \text{Eq. 28}$$

Step #5:

Compute strengths ϕM_n and ϕP_n .

ϕM_n and ϕP_n are computed using the method detailed in the section “Strength Computations” (page 18). As the non-sway case, the strength computations are based on equalizing the demand eccentricity and the strength eccentricity.

Step #6:

Pass/Fail in strength.

The column satisfies strength requirements if ϕP_n is larger than P_u and ϕM_n is larger than M_c .

Table 7 – Checks on strength requirements in sway frame

$\phi P_n \backslash \phi M_n$	$\geq M_c$	$< M_c$
$\geq P_u$	Pass	Fail
$< P_u$	Fail	Fail

Flowchart of ACI 318 Moment Magnification Method

The Figure 17 presents a flowchart of the ACI 318 moment magnification method.

The sequence is as follows:

- The student chooses whether the column is in a sway or a non-sway frame.
- The student determines whether the column is slender using (Eq. 52) for a sway frame or (Eq. 48) for a non-sway frame. A non-slender column is beyond the scope of the study.
- To account for slenderness effects, the student uses ACI 318 moment magnification method. The use of the 2nd order analysis is beyond the scope of the study.
- For each type of frame (sway or non-sway), a sequence of computations leads to the magnified moment for which the column is designed.
- The software checks whether the column satisfies strength requirements.

The notations used in Figure 17 are defined in Table 8

Table 8 – Notations used in Figure 17

C_m	Factor relating actual moment diagram to an equivalent uniform moment diagram
δ_{ns}	Moment magnification factor for non-sway case
δ_s	Moment magnification factor for sway case
M_{ij}	Moment due to load contribution j at end i
M_c	Magnified moment for which the column is designed
P_c	The buckling load
P_u	Factored axial demand

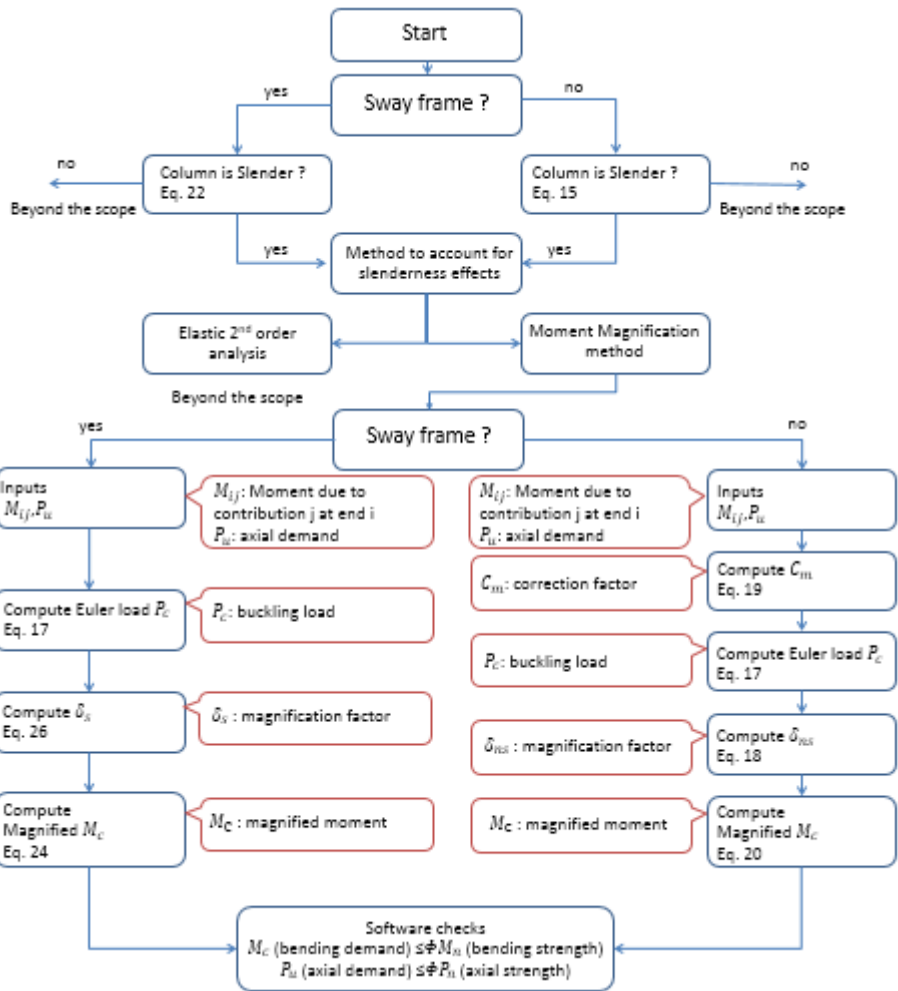


Figure 17 – Flowchart for ACI 318 moment magnification method

Chapter 3 - Educational Software for the Design of Slender Reinforced Concrete Columns

Available Professional Software

Civil engineers use structural engineering software to design a structure. Several structural engineering software are available in the market. Many deal with slender reinforced concrete columns with circular cross section. The slenderness effects are taking into account through different methods: 2nd order analysis, moment magnification and the nominal curvature method. The latter is used in the European Building Code Eurocode 2 (EC 2, 2002) in the section (Cl 5.8.8). The 2nd order analysis and the nominal curvature method are not discussed in the study.

In the list of presented professional software, two use the nominal curvature method to account for slenderness effects. Three software use the simplified moment magnification method and four more advanced software deal with both 2nd order analysis and moment magnification method.

SAP2000 [*SAP2000*, 2013] uses in the analysis of slender reinforced concrete column the 2nd order analysis and the moment magnification method. Irregular column cross sections can be input as well as biaxial bending moments. The main building codes SAP2000 relies on are ACI 318 and EC 2.

Dlubal [*Concrete Column 8.xx*, 2013] uses in the analysis of slender reinforced concrete column the nominal curvature method. Rectangular and circular column cross sections can be input as well as biaxial bending moments. The main building codes Dlubal relies on are ACI 318 and the Chinese Building Code GB 50010.

ASDIP Concrete [*Concrete Column Design*, 2013] uses in the analysis of slender reinforced concrete column the moment magnification method. Rectangular and circular column cross sections can be input as well as uniaxial bending moments. The main building code ASDIP relies on is ACI 318.

Digital Canal [*Technical Specification*, 2013] uses in the analysis of slender reinforced concrete column the 2nd order analysis and the moment magnification method. Rectangular

and circular column cross sections can be input as well as biaxial bending moments. The main building code Digital Canal relies on is ACI 318.

StructurePoint Column [*Reinforced Concrete Column*, 2013] uses in the analysis of slender reinforced concrete column the moment magnification method. Irregular column cross sections can be input as well as biaxial bending moments. The main building code StructurePoint Column relies on is ACI 318.

Graitec [*Advance Concrete*, 2013] uses in the analysis of slender reinforced concrete column the 2nd order analysis. Irregular column cross sections can be input as well as biaxial bending moments. The main building codes Graitec relies on are ACI 318 and EC 2.

Real3D Analysis [*cColumn*, 2013] uses in the analysis of slender reinforced concrete column the 2nd order analysis and the moment magnification method. Rectangular and circular column cross sections can be input as well as biaxial bending moments. The main building code Real3D Analysis relies on is ACI 318.

Oasys [*GSA Building*, 2013] uses in the analysis of slender reinforced concrete column the moment magnification method. Rectangular and circular column cross sections can be input as well as uniaxial bending moments. The main building codes Oasys relies on are ACI 318 and EC 2.

BuildSoft ConCrete [*ConCrete (Plus)*, 2013] uses in the analysis of slender reinforced concrete column the nominal curvature method. Irregular column cross sections can be input as well as biaxial bending moments. The main building code BuildSoft ConCrete relies on is EC 2.

Educational modules

As stated in the introduction, education in design of reinforced concrete structures should allow students to function as designers, that is, to select the dimensions of concrete members and the details of steel reinforcement to satisfy strength requirements as well as ACI provisions. Some software applications have been developed to support the education of students in design of concrete structural members.

Jiang H. developed virtual laboratories for education in design of reinforced concrete [Virtual, 2013]. Virtual laboratories are written in Java and provide students the complete reinforcement given a set of loads, member geometry and material properties. By performing the design for the student, the module does not educate the way of designing a structural member through the application of ACI provisions and the satisfaction of strength requirements.

Urgessa [2011] developed Matlab modules for the design in reinforced concrete. Using student's initial estimates of geometry and reinforcement ratio, the tool leads the student to the optimum values of geometry and reinforcement that withstand the demand. The module does not check for ACI provisions for a design entirely developed by the student.

Al-Ansari and Senouci [1999] developed worksheets in Mathcad supporting in class students' education in reinforced concrete design. The teacher designs the structure using the application while the classroom learns the steps of the design process. The teacher can alter the design and obtain immediate outcomes from the application. This tool is useful during students' learning but is not used to check students' works for a term project.

Gambatese J. wrote a module using Microsoft PowerPoint for education in reinforced concrete [Reinforced, 2013]. The module provides practical information about the construction of structural members rather than details of the design process. For instance, it is described the sequence of building a structural member from the placing of reinforcement to the curing of the structural member.

Kastanos et al. [2011] developed a software in Matlab environment about the influence of soil-structure interaction on the seismic design of reinforced concrete buildings. Based on

student's design of the building (general dimensions and shear walls geometry), the software provides outcomes on the influence of considering soil-structure interaction over fundamental period and base shear force. The software was not developed to provide feedbacks to students on whether the design satisfies the Eurocode 8 provisions.

Canakci [2007] developed an educational module called Pile-D using Microsoft Excel. The software supports design of pile foundation in clay. The module executes a benchmark study based on the student's design and displays outcomes on many geotechnical parameters. Then, the student inputs its values and can check whether they are correct or not. However, no feedbacks are given to students regarding the design and its compliance with building codes.

Adeli and Kim [2000] developed a web-based interactive courseware written in Java and supporting structural steel design. The process is as follows: The student fills the Input panel with design characteristics on loadings, supports, steel type and section type. The student can impose a nominal depth for the steel member. Then, the applet executes the design and provides under the Results panel internal loads diagrams as well as the detail of the cross section. The applet performs the design for the student based on the AISC code. The applet was not developed to let the student determine the complete design of steel beam and then to provide feedbacks on whether the design satisfies AISC provisions.

Wang and Adeli [2012] developed a web-based module in Java supporting educational in steel design for beams, columns, beam-columns, connections and girders. The paper presents a specific applet called *Beam Tutor* which focuses on design of steel beams. Given a set of loadings, the beam supports and student's choice of cross section shape, the module selects the lightest shape that withstands the maximum moment and shear force. But, the student can choose a nominal depth for the selected cross section shape. The user is informed if no shape verifies AISC requirements. The module performs almost the entire design for the student, and then does not provide feedbacks on whether the design satisfies AISC provisions. The paper points that comments on errors are displayed if inputs such as tensile strength and nominal depth are out of range.

Many educational tools supporting education in structural design have been developed. However, none of them was developed to provide feedbacks on the satisfaction of building codes provisions for a design entirely performed by the student. Regarding the evaluation of the student's numerical values, one tool [Canakci, 2007] compares the student's outcomes with that of the software.

Operating System

CU Slender Column is developed in C++ using Microsoft's Windows Forms framework and targets personal computers running Windows operating systems. The integrated development environment is Microsoft Visual Studio Express.

Organization CU Slender Column Program

CU Slender Column is developed using Microsoft Visual Studio Express which organizes the program into interactive classes. A class is a substructure able to interact with other classes by calling them. This organization is useful in the development of CU Slender Column because a class is used to support a grid.

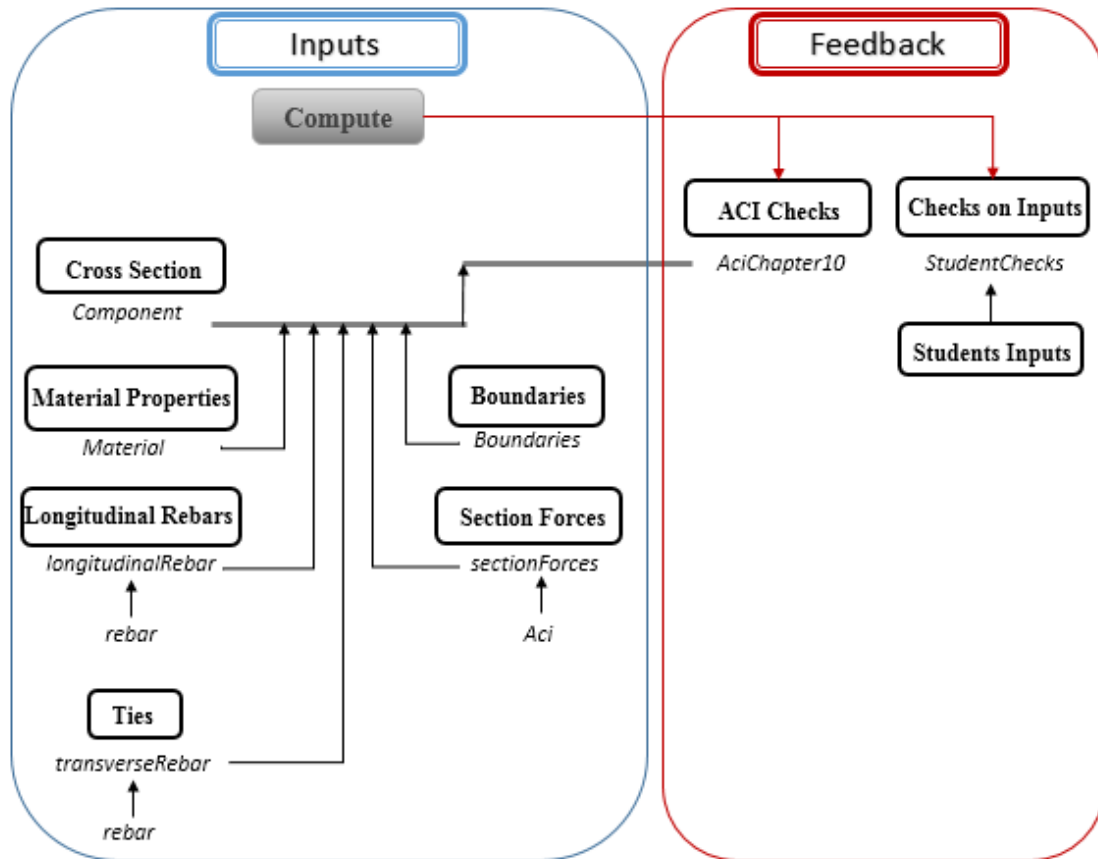


Figure 18 – Flowchart for the organization of CU Slender Column program

The tab Inputs is composed of six grids: Cross Section, Material Properties, Longitudinal Rebars, Ties, Boundaries and Section Forces.

Under the tab Inputs, the *rebar* class is used in the classes *transverseRebar* and *longitudinalRebar*. The *Aci* class is used in the *sectionForces* class. Then, five classes: *Material*, *longitudinalRebar*, *transverseRebar*, *sectionForces* and *boundaries* are used in the *component* class which supports the *AciChapter10* class.

The tab Feedback is composed of three grids: Checks on Inputs, Students Inputs and ACI Checks.

Under the tab Feedback, the Students Inputs are used in the *StudentsChecks* class.

While clicking on the button Compute located under the tab Inputs, feedbacks are generated and checks on accuracy of student's inputs are performed.

Graphical User Interface of CU Slender Column

The graphical user interface is constituted of two tabs: The tab Inputs where the student inputs the design of the column and the tab Feedback where the student sees the feedbacks generated by the software regarding the design.

Tab Inputs

The tab Inputs is made of six fillable grids.

Table 9 presents the student's inputs needed to describe the column design and the supporting grids.

Table 9 – Student's Inputs in tab Inputs

Student's Inputs	Supporting grid
Column dimensions	Cross-Section
Material properties	Material Properties
Boundary conditions	Boundaries
Sizes and placement of longitudinal rebars	Longitudinal Rebars
Size and placement of spiral ties	Ties
Values of end moments contributions	Section Forces

Figure 19 presents the graphical user interface of the tab Inputs.

The button Save allows the student to save the inputs into a .txt file.

The button Load allows the student to upload the inputs previously saved on a .txt file.

The button Compute triggers all the computations and generates the feedbacks to the student.

Also the graphic of the column cross-section with longitudinal rebars and ties appears by clicking on the Graphic button.

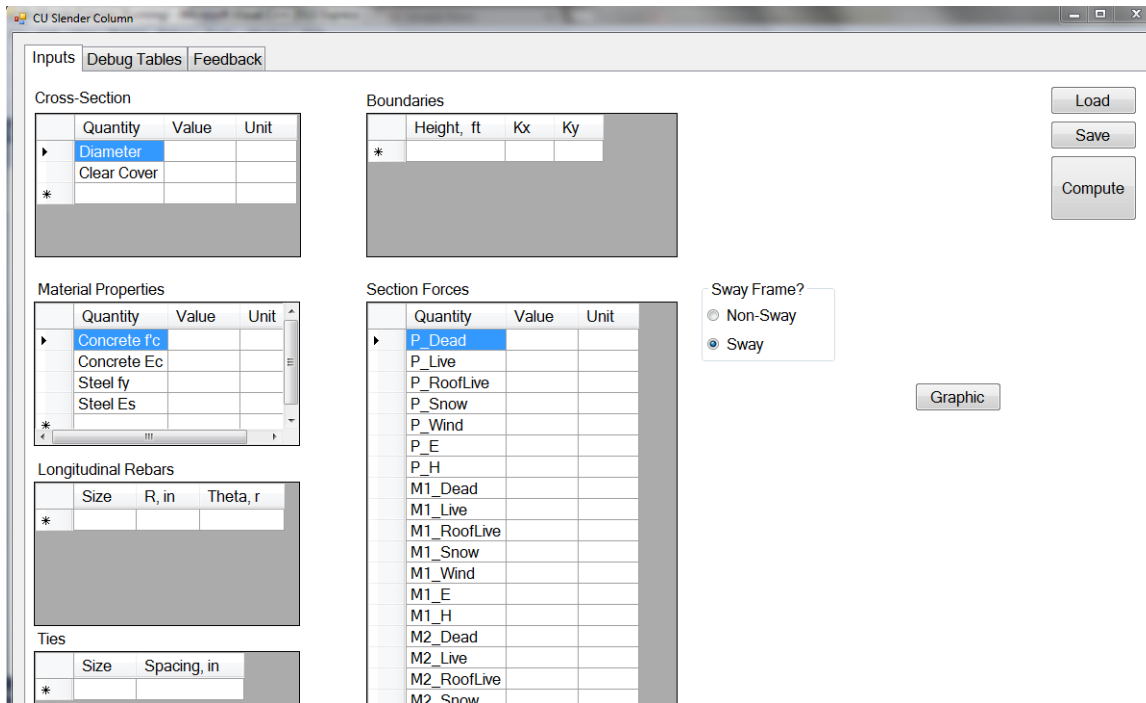


Figure 19 – Graphical user interface of the tab Inputs

Tab Feedback

The tab Feedback is made of three grids.

The ACI Checks grid lists the ACI provisions used in the design process and tells the student whether the provisions are satisfied or not. If a provision is not satisfied, a feedback helping to improve the design appears on the right of the provision reference.

The Student Inputs grid gets the student’s values for demands, strengths and intermediate parameters as presented in Table 10.

Table 10 – Student’s inputs in tab Feedback

Student’s Inputs	Supporting grid
Values of factored axial force P_u	Student Inputs
Values of magnified moment M_c	
Values of eccentricity demand e	
Values of depth of compression zone c	
Values of reduction factor ϕ	
Values of axial strength ϕP_n	
Values of moment strength ϕM_n	

The Checks on Inputs grid tells the student whether the values of demands, strengths and intermediate parameters are accurate or not.

Figure 20 presents the graphical user interface of the tab Feedback.

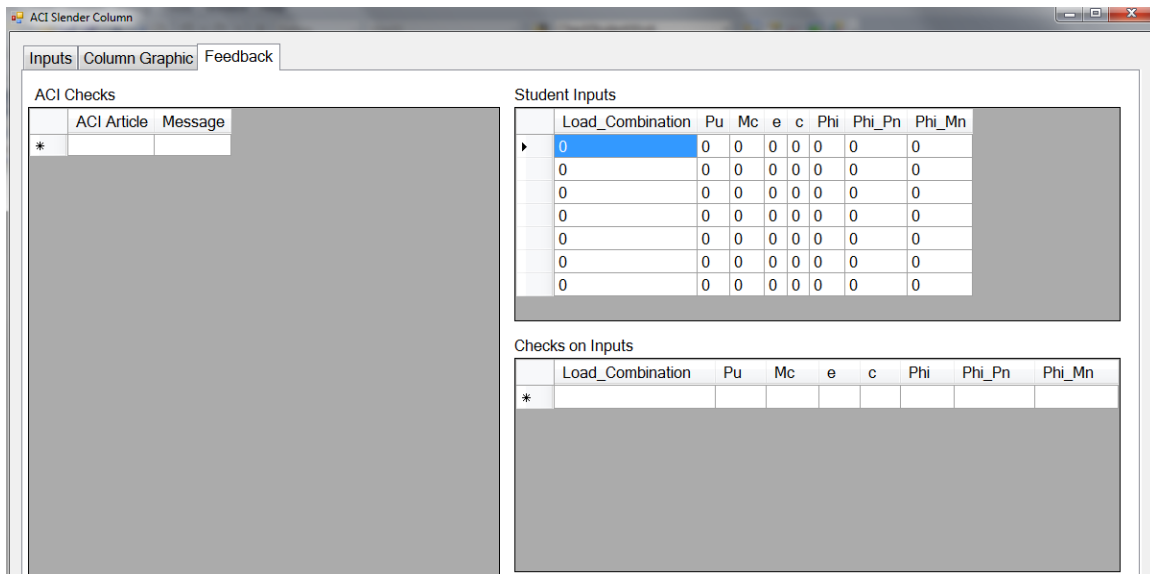


Figure 20 – Graphical user interface of the Tab Feedback

Grid construction

Two types of grids are constructed in the program: The static grid whose dimensions are fixed and the dynamic grid whose dimensions depend on students' designs.

The static grid is made by giving a fixed number of rows and columns. For instance, the Cross Section grid, the Material Properties grid, the Ties grid, the Boundaries grid and the Section Forces grid are static. The number of inputs for these grids is fixed and does not depend on students' designs.

The dynamic grid is made by adding a new row when the previous one has been filled. For instance, the Longitudinal Rebars grid is dynamic because the number of rebars depends on students' designs.

CU Slender Column Classes

In this section are provided the descriptions of the classes used in the development of CU Slender Column.

Aci Chapter 10 class

AciChapter10 class runs all ACI 318 Chapter 10 provisions for compression members. When a provision is called in the program, it unveils a feedback to the student about whether the provision is satisfied or not. When a provision is not satisfied, the key parameter and some explanations related to the error are given.

Aci class

Aci class runs required ACI provisions located in other ACI Chapters such as the loading combinations in Chapter 9 section 9.2.1 and the reduction factor ϕ in Chapter 9 section 9.3.2.2.

Section forces class

SectionForces class runs student's values of each contribution for axial force and end moments (Dead, Live, Live Roof, Snow, Wind, Earthquake, Lateral Earth Pressure). This class supports the static grid Section Forces.

Rebar class

Rebar class interacts with classes that use rebars such as *Material* to inventory the characteristics of a steel rebar, *longitudinalRebar* for the circular layout of the longitudinal rebars and *transverseRebar* for the spiral ties reinforcement.

Material class

Material class runs concrete properties such as f'_c and E_c as well as steel properties such as the yield stress f_y and E_s . This class supports the static grid Material Properties.

Transverse rebar class

TransverseRebar class runs spiral ties reinforcement characteristics such as the rebar size and the spacing. Ties reinforcement are made of one spiral rebar. This class supports the static grid Ties.

Longitudinal rebar class

LongitudinalRebar class runs longitudinal rebars characteristics such as the rebar size and the polar coordinates of the rebar: The distance from the column center and the angle from the horizontal axis. This class supports the dynamic grid Longitudinal Rebars.

Boundaries class

Boundaries class takes into account the column height and the effective length factor. This class supports the static grid Boundaries.

Component class

Component class is supported by all classes under the tab Inputs as shown in Figure 18. It contains design information for columns as well as the loadings. Also, it checks the unit given by the student and generates a “bad unit” message if the inputs differs from the United States customary units. Also, routines to build static and dynamic grids are written under this class.

StudentChecks class

StudentChecks contains a program which compares the student’s values to the CU Slender Column benchmark values. An arbitrary tolerance of 2% is considered. If the relative error is greater than 2%, a “ng” message appears, if the error is within 2%, a “ok” message appears and if only the sign is different, a “bad sign” message appears.

TabInput

TabInput contains ACI 318 moment magnification method calculations and strength calculations.

columnGraphic

columnGraphic is used to sketch the column cross-section. The size of a cross-section is scaled as follows: 216 pixels is equivalent to a 32 in diameter.

CU Slender Column computations of ϕP_n and ϕM_n

Column strength computations require an accurate assessment of c .

The Figure 21 shows in red line the variation of the strength eccentricity e_n in function of the depth of the concrete compression zone c . The blue line refers to the target eccentricity demand e_u that has to be approached. The procedure to approach the eccentricity demand and then to obtain the corresponding c value is as follows:

The interpolation of c is linear as shown in Figure 21. The program creates integer values of c (index i), then detects the point i when e_u exceeds e_n . Besides, the program splits the segment $[i, i + 1]$ into 20 segments (index j). If more precision is expected, the segment $[i, i + 1]$ can be split into a larger number. The same detection process is done and gets the point j . Eventually, the linear interpolation is computed between the point j and the point $j+1/20$.

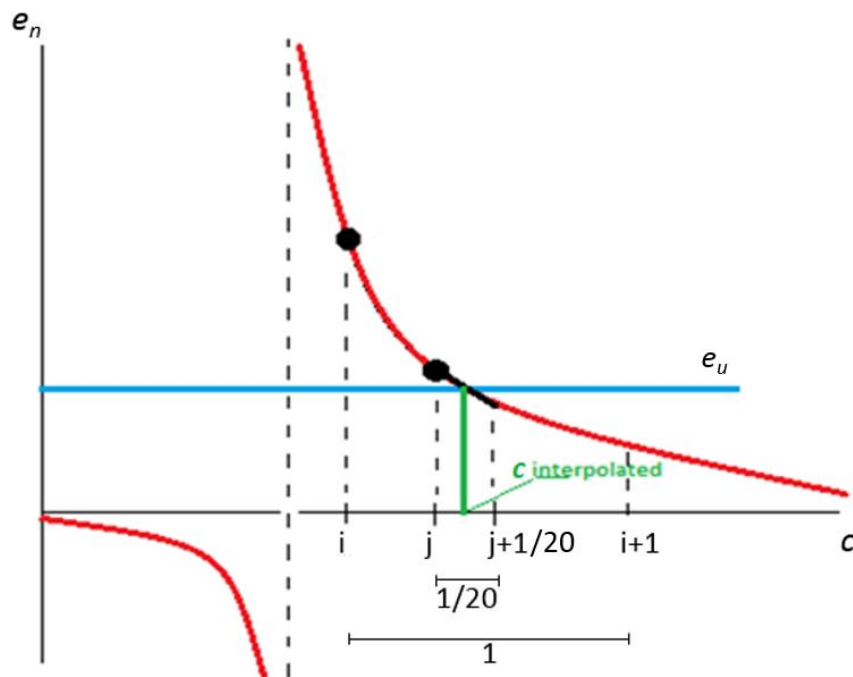


Figure 21 – Variation of eccentricity e_n in function of the depth of concrete compression zone c

Chapter 4 - Demonstration of Educational Software

Presentation of Examples

The validation of CU Slender Column is done by three demonstrations aiming at covering many design situations. These demonstrate that CU Slender Column detects whether a column passes or fails in strength through 24 examples (Demo #1), CU Slender Column detects whether a design does not satisfy ACI provisions through 10 examples (Demo #2), and CU Slender Column detects whether a student values are not accurate through 4 examples (Demo #3).

Demo #1 - Evaluation of column strength in non-sway and sway frames

A column either in a non-sway or a sway frame passes in strength if the strengths exceed the demands for every single load combination:

$$\begin{aligned} \phi P_n &\geq P_u && \text{Eq. 54} \\ \phi M_n &\geq M_u && \text{Eq. 55} \\ e_n &= e_u && \text{Eq. 31} \end{aligned}$$

Demo #1 is made of 24 examples covering a wide range of column designs either passing or failing in strength. The variety of examples is obtained by varying parameters such as eccentricity demand e_u through values of factored axial force and end moments, the type of frame: Sway or non-sway frame, C_m through the value of the ratio of factored end moments and the slenderness. To explore both single curvature and double curvature cases, columns in the first half of examples are single-curved and columns in the second half are double-curved.

Table 11 – Examples for Demo #1

# Example	Slenderness	Sway/Non Sway	C_m	Eccentricity	Strength
1	Low Value	N-S	Low Value	Diameter	Pass
2	Low Value	N-S	Low Value	Diameter/10	Pass
3	Low Value	N-S	1.0	Diameter	Pass
4	Low Value	N-S	1.0	Diameter/10	Pass
5	Low Value	S	1.0	Diameter	Pass
6	Low Value	S	1.0	Diameter/10	Pass
7	High Value	S	1.0	Diameter	Pass
8	High Value	S	1.0	Diameter/2	Pass
9	High Value	N-S	Low Value	Diameter	Pass
10	High Value	N-S	Low Value	Diameter/2	Pass
11	High Value	N-S	1.0	Diameter	Pass
12	High Value	N-S	1.0	Diameter/2	Pass
13	Low Value	N-S	Low Value	Diameter	Fail
14	Low Value	N-S	Low Value	Diameter/10	Fail
15	Low Value	N-S	1.0	Diameter	Fail
16	Low Value	N-S	1.0	Diameter/10	Fail
17	Low Value	S	1.0	Diameter	Fail
18	Low Value	S	1.0	Diameter/10	Fail
19	High Value	S	1.0	Diameter	Fail
20	High Value	S	1.0	Diameter/2	Fail
21	High Value	N-S	Low Value	Diameter	Fail
22	High Value	N-S	Low Value	Diameter/2	Fail
23	High Value	N-S	1.0	Diameter	Fail
24	High Value	N-S	1.0	Diameter/2	Fail

The Figure 22 presents the scattering of geometry of columns presented in this section and shows that examples in Demo #1 deal with many column dimensions. Columns diameters range from 14 in to 32 in and columns heights range from 10 ft to 32 ft.

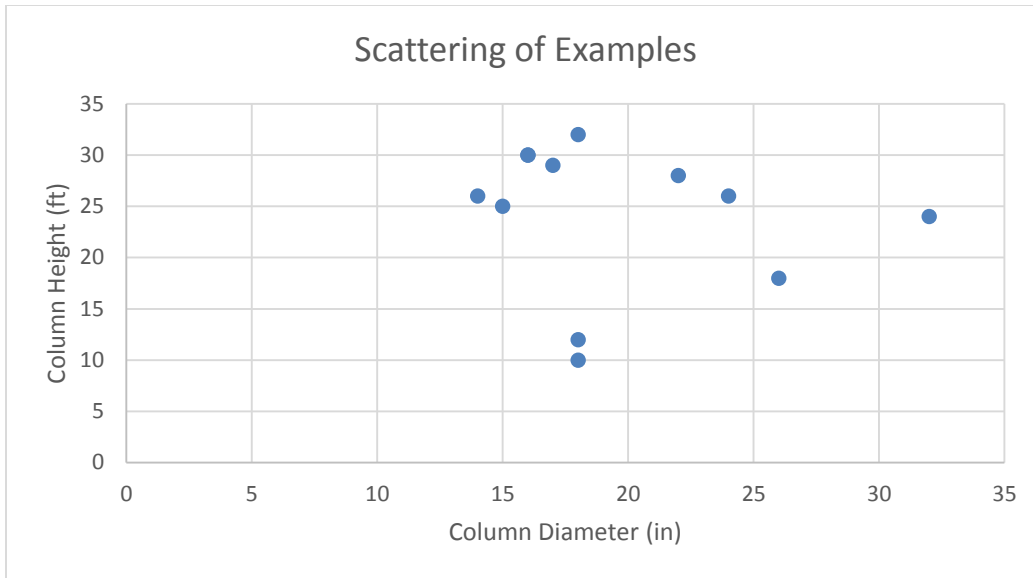


Figure 22 – Scattering of columns geometry for Demo #1

The Figure 23 shows the elevated view of the columns used in Demo #1.

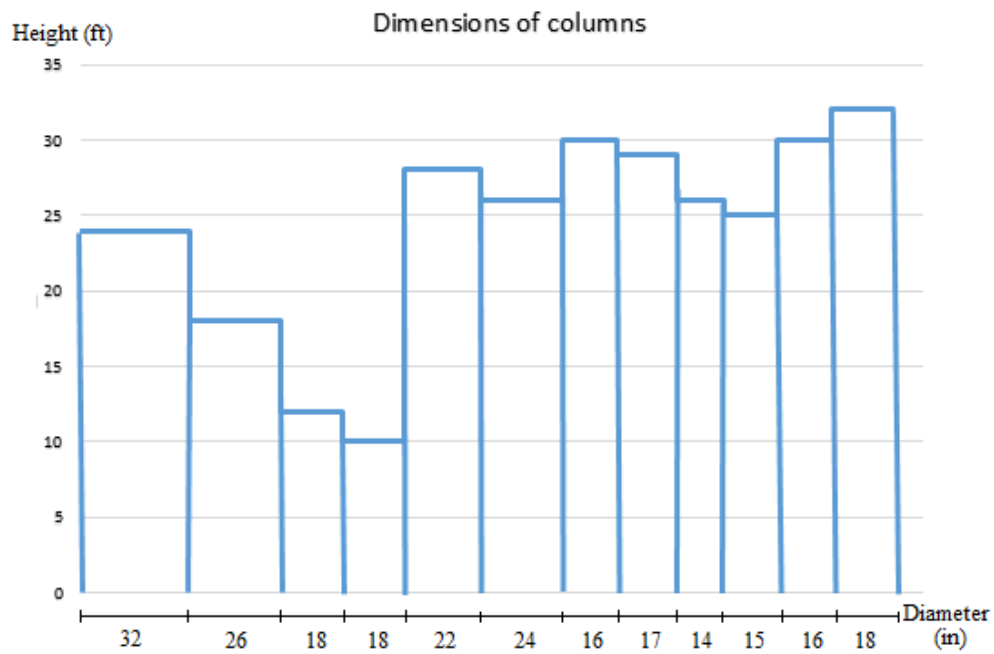


Figure 23 – Dimensions of columns for Demo #1

Figure 24 shows the cross sections of columns developed in Demo #1.

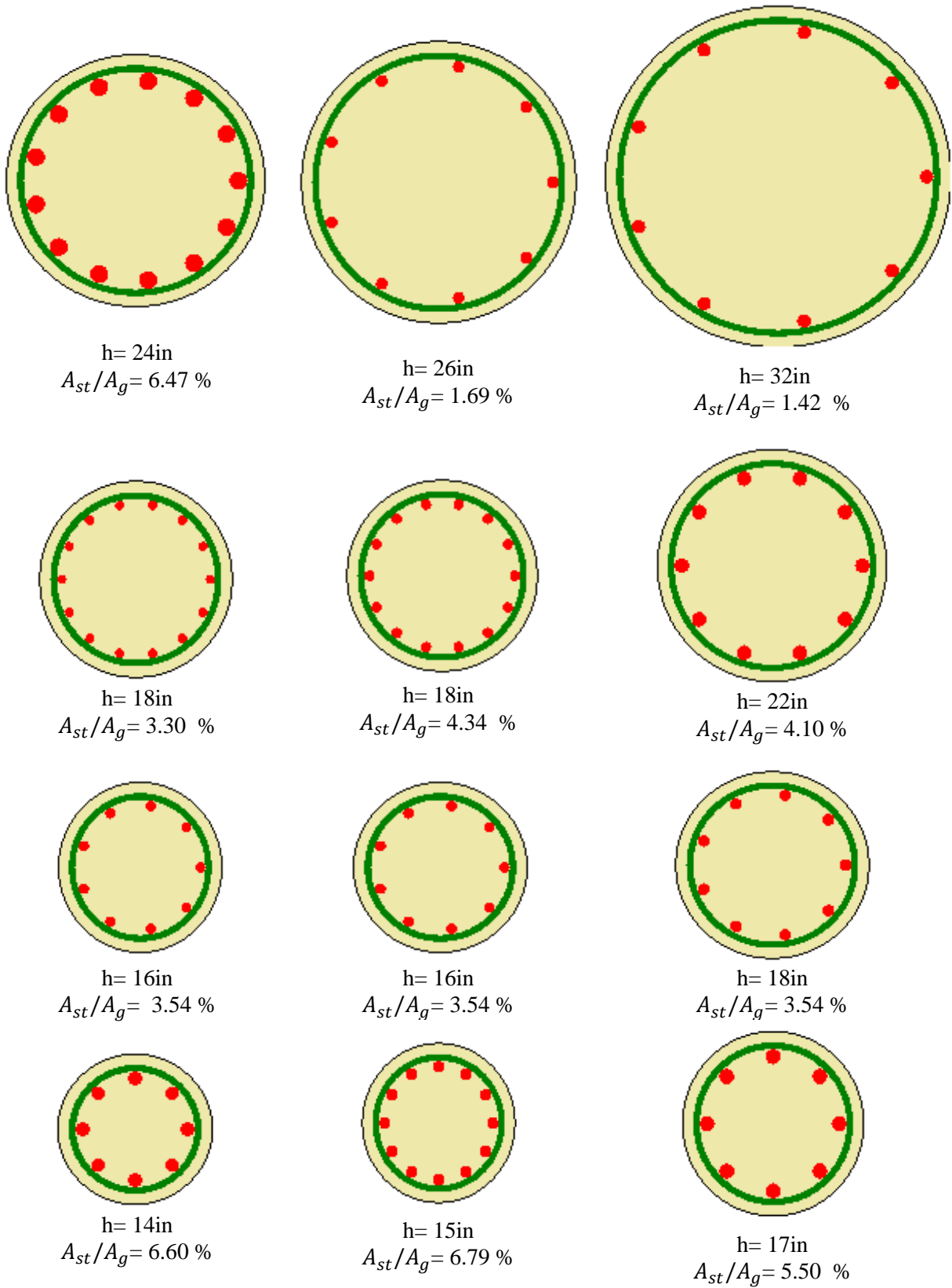


Figure 24 – Cross sections of columns for Demo #1

Demo #2 – Non satisfaction of ACI provisions

A column passes in ACI provisions if all ACI provisions are satisfied.

Demo #2 is made of 10 examples failing in ACI provisions. Provisions that are considered are the confinement spiral spacing, the spiral reinforcement ratio, the longitudinal rebars ratio, the longitudinal rebars count and the applicability of the moment magnification method.

Table 12 – Examples for Demo #2

# Example	ACI provisions	Content	Strength
#25	ACI 7.10.4.3 - FAIL	Confinement Spiral Spacing	PASS
#26	ACI 10.9.3 - FAIL	Spiral Reinforcement Ratio	PASS
#27	ACI 10.9.1 - FAIL	Longitudinal Rebars Ratio	PASS
#28	ACI 10.9.2 - FAIL	Longitudinal Rebars Count	PASS
#29	ACI 10.11.5 - FAIL	Application of Moment Magnification	PASS
#30	ACI 7.10.4.3 - FAIL	Confinement Spiral Spacing	FAIL
#31	ACI 10.9.3 - FAIL	Spiral Reinforcement Ratio	FAIL
#32	ACI 10.9.1 - FAIL	Longitudinal Rebars Ratio	FAIL
#33	ACI 10.9.2 - FAIL	Longitudinal Rebars Count	FAIL
#34	ACI 10.11.5 - FAIL	Application of Moment Magnification	FAIL

Demo #3 – Accuracy of student's inputs

Student's inputs are accurate if they match CU Slender Column outcomes within a tolerance set at 2% in the study. A table entitled Checks on Inputs under the tab Feedback allows the student to check whether the inputs are accurate with those computed by CU Slender Column. As stated earlier, if the values are not accurate, a message “ng” appears, if the values are accurate, a message “ok” appears then, if the sign is wrong, a message “bad sign” appears.

Demo #3 is made of 4 examples where student's inputs are not accurate with CU Slender Column outcomes.

Table 13 – Examples for Demo #3

# Example	2% Accuracy	ACI provisions	Strength
#35	FAIL	PASS	PASS
#36	FAIL	PASS	FAIL
#37	FAIL	FAIL	PASS
#38	FAIL	FAIL	FAIL

Comprehensiveness of examples for validation of CU Slender Column

The Table 14 shows the strong demonstration of CU Slender Column through the validation of examples covering a wide range of design situations. It is shown that all possibilities under the three demonstrations are covered.

Table 14 – Comprehensiveness of examples for validation of CU Slender Column

Accuracy	ACI Provisions	Strength	Demo	Examples
PASS	PASS	PASS	#1	#1 to #12
		FAIL		#13 to #24
	FAIL	PASS	#2	#25 to #29
		FAIL		#30 to #34
FAIL	PASS	PASS	#3	#35
		FAIL		#36
	FAIL	PASS		#37
		FAIL		#38

To generate examples for validation more rapidly, Dead, Live and Wind contributions are considered equal. Other contributions are not considered for those examples.

$$P_d = P_l = P_w = P \quad \text{Eq. 56}$$

$$M_{id} = M_{il} = M_{iw} = M_{ei} \quad \text{Eq. 57}$$

$$M_{jd} = M_{jl} = M_{jw} = M_{ej} \quad \text{Eq. 58}$$

It is noted that CU Slender Column program through the routine Aci9_2_1 under the *Aci* class considers all load contributions and loading combinations.

Detailed Examples for Moment Magnification Method in Non-Sway Frame

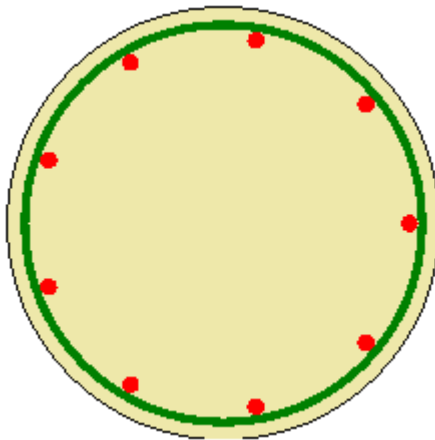
Examples #1 and #13 composing the Example Pair #1 and presented in Table 11 are detailed to show step by step the ACI 318 moment magnification method for a non-sway frame as described in the section “ACI 318 Moment Magnification Method and Checks of Strength Requirements for Non-Sway and Sway Frames” (page 23). References to the routines written in the program performing the computations are provided.

Examples #1 and #13 use the same single-curved column. The column is in a non-sway frame with a diameter of 32 in and a height of 24 ft. The concrete Young’s modulus E_c is equal to 4287 psi. Examples target a low value of slenderness, a low factored end moments ratio and a high eccentricity.

Table 15 – Column geometry for detailed examples in non-sway frame

Column Geometry

Height (ft)	24
Diameter (in)	32



SCALE 1:14

Longitudinal Rebars, 9 –No 10

Spiral Ties, No 5 @ 2 in

Figure 25 – Column cross section for detailed examples in non-sway frame

Step #1:

Compute the factored loads.

The student gets the values of load contributions using a structural analysis software. In the detailed examples, the Dead, Live and Wind contributions for the axial force are identical and equal to P (Eq. 56). The Dead, Live and Wind contributions at the end i are identical and equal to M_{ei} (Eq. 57). The Dead, Live and Wind contribution at the end j are identical and equal to M_{ej} (Eq. 58).

Table 16 – Student’s loading inputs for detailed examples in non-sway frame

Loading	Ex #1	Ex #13
P (k)	50	700
M_{ei} (kft)	10	5
M_{ej} (kft)	-200	-1500

The routine `Aci9_2_1` under the `Aci` class computes the factored loads and the routine `Aci_10_12_3_1` defines M_1 as the smaller factored end moments and M_2 as the larger factored end moments.

For example #1, the factored axial force P_u is equal to 190 k and the larger factored end moment M_2 is equal to -760 kft. For example #13, the factored axial force P_u is equal to 2660 k and the larger factored end moment M_2 is equal to -5700 kft.

Table 17 – Factored loads for detailed examples in non-sway frame

Factored Load	Ex #1	Ex #13
P_u (k)	190	2660
M_1 (kft)	38	19
M_2 (kft)	-760	-5700

Step #2:

Compute the slenderness of the column.

In the program, the routine `Aci_10_12_2` under the `AciChapter10` class computes the slenderness of the column. The effective length factor k , equal to 1.0, and the column length L , equal to 24 ft, are given by the student in the grid Boundaries. The routine `SectionProperties` under the `component` class computes the radius of gyration r .

Columns in both examples have identical geometrical features, the slenderness is the same:

$$\frac{kL}{r} = \frac{1.0 * 24 * 12}{8} = 36 \quad \text{Eq. 59}$$

Table 18 – Column slenderness for detailed examples in non-sway frame

	Ex #1	Ex #13
Slenderness	36	36

The routine `Aci_10_12_2` under the `AciChapter10` class computes the slenderness lower bound for the application of the ACI 318 moment magnification method as defined in (Eq. 46). The slenderness lower bound for both examples is 34. The column used in both examples is slender and the ACI 318 moment magnification method applies.

Step #3:

Compute C_m .

The routine `Aci_10_12_3_1` under the `AciChapter10` class computes C_m from the factored end moments.

C_m is identical for both examples and equal to 0.6.

Table 19 – C_m for detailed examples in non-sway frame

Correction factor	Ex #1	Ex #13
C_m	0.6	0.6

Step #4:

Apply the ACI 318 moment magnification method.

The routine `Aci_10_13_6` under the `AciChapter10` class computes the parameter βd . The factored sustained loads are computed using the routine `Aci9_2_1_Sus` under the `Aci` class and the factored loads are computed using the routine `Aci9_2_1` under the `Aci` class.

Parameter βd is identical for both examples and equal to:

$$\beta d = \frac{1.2P}{P_u} = \frac{1.2 * 50}{190} = 0.316 \quad \text{Eq. 60}$$

The routine `Aci_10_12_3` under the `AciChapter10` class computes the reduced stiffness EI .

EI is identical for both examples and equal to:

$$EI = \frac{0.4 * E_c * I_g}{1 + \beta d} = \frac{0.4 * 4287 * 51472}{1 + 0.316} = 6.7 * 10^7 \text{ kin}^{-2} \quad \text{Eq. 61}$$

The routine `Aci_10_12_3` under the `AciChapter10` class computes the buckling load P_c .

P_c is identical for both examples and equal to:

$$P_c = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 * 6.7 * 10^7}{(1.0 * 24 * 12)^2} = 7982 \text{ k} \quad \text{Eq. 62}$$

The magnification factor δ_{ns} is calculated in the routine `Compute` under `tabInput`.

For example #1, δ_{ns} is equal to:

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 * P_c}} = \frac{0.6}{1 - \frac{190}{0.75 * 7972}} = 0.64 \Rightarrow \delta_{ns} = 1.0 \quad \text{Eq. 63}$$

The magnification factor is taken as 1.0 because the column has to withstand at least the existing larger factored end moment.

For example #13, δ_{ns} is equal to:

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 * P_c}} = \frac{0.6}{1 - \frac{2660}{0.75 * 7972}} = 1.08 \quad \text{Eq. 64}$$

Table 20 – δ_{ns} for detailed examples in non-sway frame

Magnification factor	Ex #1	Ex #13
δ_{ns}	1.0	1.08

The magnified moment M_c is calculated in the routine Compute under *tabInput*.

For example #1, M_c is equal to:

$$M_c = \delta_{ns} * M_2 = 1.0 * 760 = 760 \text{ kft} \quad \text{Eq. 65}$$

For example #13, M_c is equal to:

$$M_c = \delta_{ns} * M_2 = 1.08 * 5700 = 6170 \text{ kft} \quad \text{Eq. 66}$$

Table 21 – M_c for detailed examples in non-sway frame

Magnified moment	Ex #1	Ex #13
M_c (kft)	760	6170

Step #5:

Compute the strengths.

The strengths are computed when the student presses the button Compute under the tab Inputs. The program interpolates c by equalizing the eccentricities. The strengths are computed by the method described in the section “Strength Computations” (page 18).

Table 22 – Strengths for detailed examples in non-sway frame

Strength	Ex #1	Ex #13
$\phi P_n(k)$	202	400
$\phi M_n(kft)$	806	826

Step #6:

The column satisfies strength requirements if the load demands are smaller than the column strengths.

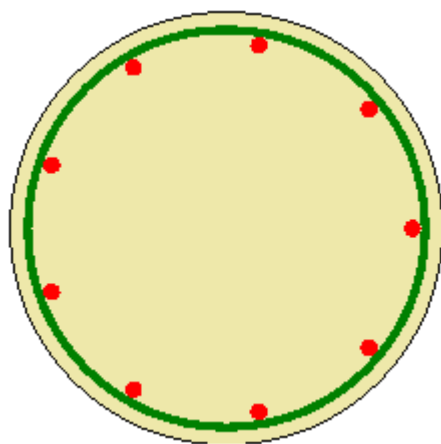
Table 23 – Findings for detailed examples in non-sway frame

Finding	Ex #1	Ex #13
Strength	PASS	FAIL
ACI Provisions	PASS	PASS

The complete detailed examples #1 and #13 for non-sway frame are summarized in Table 24.

Table 24 – Example Pair #1 – Low Slenderness / Non-Sway / Low C_m / Low Eccentricity / Single Curvature

Column Geometry		
Height (ft)	24	
Diameter (in)	32	
Column Configuration		
Slenderness	36	~Low Value
Sway/Non-Sway?	Non-Sway	
C_m	0.60	~Low Value
Eccentricity (in)	33	~diameter
Loading	Ex #1	Ex #13
P (k)	50	700
M_{ei} (kft)	10	5
M_{ej} (kft)	-200	-1500
δ_{ns}	1	1.08
Results		
P_u (k)	190	2660
ϕP_n (k)	202	400
M_2 (kft)	760	5700
M_c (kft)	760	6170
ϕM_n (kft)	806	826
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



SCALE 1:14
Longitudinal Rebars, 9 – No 10
Spiral Ties, No 5 @ 2in

Detailed Examples for Moment Magnification Method in Sway Frame

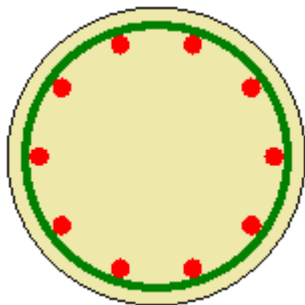
Examples #5 and #17 composing the Example Pair #5 and presented in Table 11 are detailed to show step by step the ACI 318 moment magnification method for a sway frame as described in the section “ACI 318 Moment Magnification Method and Checks of Strength Requirements for Non-Sway and Sway Frames” (page 23). References to the routines written in the program performing the computations are provided.

Examples #5 and #17 use the same single-curved column. The column is in a sway frame with a diameter of 22 in and a height of 28 ft. Examples target a low value of slenderness, and a high eccentricity.

Table 25 – Column geometry for detailed examples in sway frame

Column Geometry

Height (ft)	28
Diameter (in)	22



SCALE 1:14

Longitudinal Rebars, 10 –No 11

Spiral Ties, No 5 @ 2 in

Figure 26 – Column cross section for detailed examples in sway frame

Step #1:

Compute the slenderness of the column.

In the program, the routine `Aci_10_12_2` under the `AciChapter10` class computes the slenderness of the column. The effective length factor k , equal to 1.0, and the column length L , equal to 28 ft, are given by the student in the grid Boundaries. The routine `SectionProperties` under the `component` class computes the radius of gyration r .

Since columns in both examples have identical geometrical features, the slenderness is identical and equal to:

$$\frac{kL}{r} = \frac{1.0 * 28 * 12}{5.5} = 61 \quad \text{Eq. 67}$$

Table 26 – Slenderness for detailed examples in sway frame

	Ex #5	Ex #17
Slenderness	61	61

In sway frame, the slenderness lower bound for the application of the ACI 318 moment magnification method is equal to 22.

Thus, the ACI 318 moment magnification method is applicable in examples #5 and #17.

Step #2:

Compute the factored loads.

Structural analysis software provides load contributions as well as contributions due to non-sway and sway case. Likewise, in the detailed examples, the Dead, Live and Wind contribution for the axial force are identical and equal to P . The Dead, Live and Wind contribution at the end i are identical and equal to M_{ei} . The Dead, Live and Wind contribution at the end j are identical and equal to M_{ej} .

Table 27 – Student's loading inputs for detailed examples in sway frame

Loading	Ex #5	Ex #17
P (k)	80	75
M_{ei} (kft)	100	120
M_{ej} (kft)	-100	-120

The routine `Aci9_2_1` under the `Aci` class computes the factored loads and the routine `Aci_10_12_3_1` computes M_2 .

For example #5, the factored axial force P_u is equal to 304 k and the larger factored end moment M_2 is equal to -380 kft. For example #17, the factored axial force P_u is equal to 285 k and the larger factored end moment M_2 is equal to -456 kft.

Table 28 – Factored loads for detailed examples in sway frame

Factored Load	Ex #5	Ex #17
P_u (k)	304	285
M_1 (kft)	380	456
M_2 (kft)	-380	-456

Step #3:

Determine if the frame can be considered as non-sway.

In the examples, it is considered that a structural software analysis gives a story drift Δ_0 of 1.0 in, a factored shear force V_5 equal to 7k for example #5 and a factored shear force V_{17} equal to 8.5k for example #17. The routine `Aci_10_11_4_2` under the `AciChapter10` class computes the story index Q .

For example #5, Q is equal to:

$$Q = \frac{P_u * \Delta_0}{V_5 * L} = \frac{304 * 1.0}{7 * 28 * 12} = 0.13 \quad \text{Eq. 68}$$

For example #17, Q is equal to:

$$Q = \frac{P_u * \Delta_0}{V_{17} * L} = \frac{285 * 1.0}{8.5 * 28 * 12} = 0.10 \quad \text{Eq. 69}$$

For both examples, as Q is larger than 0.05, the frame is considered as sway and the ACI 318 moment magnification method for sway frames applies.

Step #4:

Apply the ACI 318 moment magnification method.

The routine `Aci_10_13_6` under the `AciChapter10` class computes the parameter βd . The factored sustained loads are computed using the routine `Aci9_2_1_Sus` under the `Aci` class and the factored loads are computed using the routine `Aci9_2_1` under the `Aci` class.

Parameter βd is identical for both examples and equal to:

$$\beta d = \frac{1.2P}{P_u} = \frac{1.2 * 80}{304} = 0.316 \quad \text{Eq. 70}$$

The routine `Aci_10_12_3` under the `AciChapter10` class computes the reduced stiffness EI .

EI is identical for both examples and equal to:

$$EI = \frac{0.4 * E_c * I_g}{1 + \beta d} = \frac{0.4 * 4287 * 11499}{1 + 0.316} = 1.5 * 10^7 \text{kin}^{-2} \quad \text{Eq. 71}$$

The routine `Aci_10_12_3` under the `AciChapter10` class computes the buckling load P_c .

P_c is identical for both examples and equal to:

$$P_c = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 * 1.5 * 10^7}{(1.0 * 28 * 12)^2} = 1310 \text{ k} \quad \text{Eq. 72}$$

The magnification factor δ_s is calculated in the routine `Compute` under `tabInput`.

For example #5, δ_s is equal to:

$$\delta_s = \frac{1.0}{1 - \frac{P_u}{0.75 * P_c}} = \frac{1.0}{1 - \frac{304}{0.75 * 1310}} = 1.45 \quad \text{Eq. 73}$$

For example #17, δ_s is equal to:

$$\delta_s = \frac{1.0}{1 - \frac{P_u}{0.75 * P_c}} = \frac{1.0}{1 - \frac{285}{0.75 * 1310}} = 1.41 \quad \text{Eq. 74}$$

Table 29 – δ_s for detailed examples in sway frame

Magnification factor	Ex #5	Ex #17
δ_{ns}	1.45	1.41

The magnified moment M_c is calculated in the routine Compute under *tabInput*.

For example #5, M_c is equal to:

$$M_c = 452 \text{ kft} \quad \text{Eq. 75}$$

For example #17, M_c is equal to:

$$M_c = 534 \text{ kft} \quad \text{Eq. 76}$$

Table 30 – M_c for detailed examples in sway frame

Magnified moment	Ex #5	Ex #17
$M_c(\text{kft})$	452	534

Step #5:

Compute the strengths. The strengths are computed when the student presses the button Compute under the tab Inputs. The program interpolates c by equalizing the eccentricities. The strengths are computed by the method described in the section “Strength Computations” (page 18).

Table 31 – Strengths for detailed examples for sway frame

Strength	Ex #5	Ex #17
$\phi P_n(\text{k})$	314	251
$\phi M_n(\text{kft})$	466	471

Step #6:

The column satisfies the strength requirements if the load demands are smaller than the column strengths.

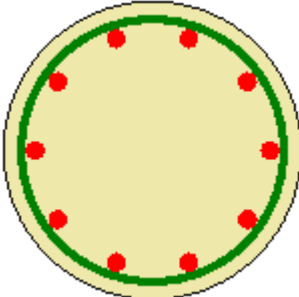
Table 32 – Findings for detailed examples in sway frame

Finding	Ex #5	Ex #17
Strength	PASS	FAIL
ACI Provisions	PASS	PASS

The complete detailed examples #5 and #17 for sway frame are summarized in Table 33.

Table 33 – Example Pair #5 – Low Slenderness / Sway / High Eccentricity / Single Curvature

Column Geometry		
Height (ft)	28	
Diameter (in)	22	
Column Configuration		
Slenderness	61	~Low Value
Sway/Non-Sway?	Non-Sway	
C_m	1.0	~High Value
Eccentricity (in)	33	~diameter
Loading	Ex #5	Ex #17
P (k)	80	75
M_{ei} (kft)	100	120
M_{ej} (kft)	-100	-120
δ_{ns}	1.45	1.41
Results		
P_u (k)	304	285
ϕP_n (k)	314	251
M_2 (kft)	380	456
M_c (kft)	452	534
ϕM_n (kft)	466	471
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



SCALE 1:14
Longitudinal Rebars, 10 – No 11
Spiral Ties, No 5 @ 2in

CU Slender Column Feedbacks

CU Slender Column provides useful feedbacks about satisfaction of ACI provisions and accuracy of numerical values of loads and strengths. The feedbacks aim at helping students in the design process. To show that CU Slender Column generates correct feedbacks, two examples are developed for a passing column and a failing column.

Passing Column

The following example targets a column design satisfying strength requirements as well as ACI provisions. The purpose is to show that CU Slender Column detects that the design satisfies ACI provisions as well as strength requirements and provides the correct feedbacks. The Figure 27 shows the tab Inputs with the student's inputs and the Figure 28 shows the tab Feedback with the feedbacks to the student.

The screenshot shows the 'Inputs' tab of the CU Slender Column software. The interface is organized into several sections, each with a table of input parameters:

- Cross-Section:**

Quantity	Value	Unit
Diameter	32	in
Clear Cover	1	in
- Boundaries:**

Height, ft	Kx	Ky
24	1	1
- Material Properties:**

Quantity	Value	Unit
Concrete f_c	5	ksi
Concrete E_c	4287	ksi
Steel f_y	60	ksi
Steel E_s	29000	ksi
- Section Forces:**

Quantity	Value	Unit
P_Dead	50	k
P_Live	50	k
P_RoofLive	0	
P_Snow	0	
P_Wind	50	k
P_E	0	
P_H	0	
M1_Dead	10	kft
M1_Live	10	kft
M1_RoofLive	0	
M1_Snow	0	
M1_Wind	10	kft
M1_E	0	
M1_H	0	
M2_Dead	-200	kft
M2_Live	-200	kft
M2_RoofLive	0	
- Longitudinal Rebars:**

Size	R, in	Theta, r
10	13.74	0
10	13.74	0.698
10	13.74	1.396
10	13.74	2.094
10	13.74	2.793
- Ties:**

Size	Spacing, in
5	2

Figure 27 – Student's inputs for passing column

The screenshot shows the 'CU Slender Column' software window with three tabs: 'Inputs', 'Debug Tables', and 'Feedback'. The 'Feedback' tab is active, displaying a table titled 'ACI Checks'. The table has two columns: 'ACI Article' and 'Message'. The 'Column Design' row is highlighted in blue and shows a 'Pass' message. Other rows include 'Sway/Non-Sway?' (Non-Sway), 'ACI Design Provision(s)' (Pass), 'Design Strengths' (Pass), and various ACI codes (Aci 5.1.1, Aci 7.10.4.2, Aci 7.10.4.3, Aci 10.1, Aci 10.2.1-10.2.7, Aci 10.2.7.1-10.2.7.3, Aci 10.3.3-10.3.6, Aci 10.9.1-10.9.3, Aci 10.11.4.2) with 'ok' or empty message fields.

ACI Article	Message
Column Design	Pass
Sway/Non-Sway?	Non-Sway
ACI Design Provision(s)	Pass
Design Strengths	Pass
Aci 5.1.1	ok
Aci 7.10.4.2	ok
Aci 7.10.4.3	ok
Aci 10.1	
Aci 10.2.1	
Aci 10.2.2	
Aci 10.2.3	
Aci 10.2.4	
Aci 10.2.5	
Aci 10.2.6	
Aci 10.2.7	
Aci 10.2.7.1	ok
Aci 10.2.7.2	ok
Aci 10.2.7.3	ok
Aci 10.3.3	
Aci 10.3.4	
Aci 10.3.5	
Aci 10.3.6	ok
Aci 10.9.1	ok
Aci 10.9.2	ok
Aci 10.9.3	ok
Aci 10.11.4.2	

Figure 28 – CU Slender Column feedbacks for student in case of passing column

The Figure 28 shows that CU Slender Column detects that the column passes in strength by providing the feedback “Pass” in front of “Design Strengths”. Also CU Slender Column detects that ACI 318 provisions are satisfied by providing the feedback “Pass” in front of “ACI Design Provision(s)”.

Failing Column

The following example aims at showing that CU Slender Column is able to detect non satisfied strength requirements as well as ACI provisions. The Figure 29 shows the tab Inputs with the student's inputs and the Figure 30 shows the tab Feedback with the useful feedbacks helping student to improve the design.

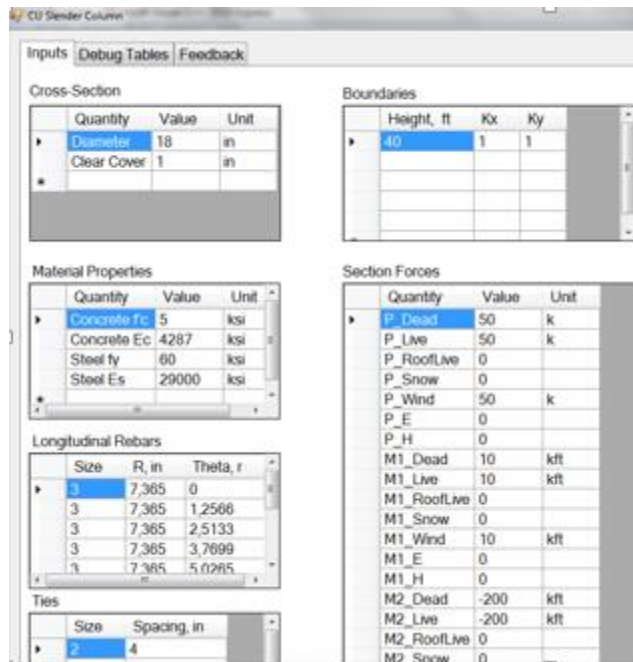
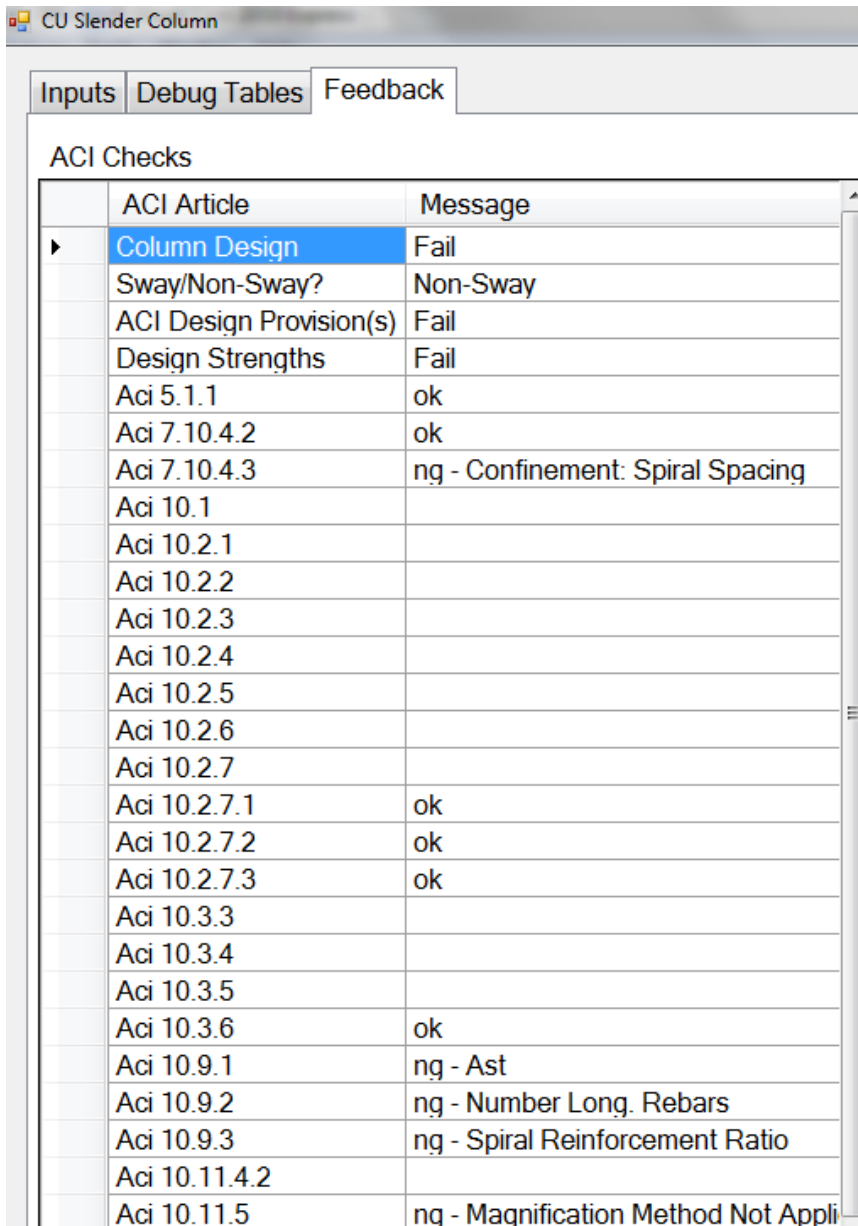


Figure 29 – Student's inputs for failing column

The Table 34 shows design parameters that fail ACI provisions. The values of those parameters that satisfy ACI provisions are described in the section “Demo #2 – Non Satisfaction of ACI Design Provisions” (page 83).

Table 34 – Design features of failing column

ACI Provision	Value	Content
7.10.4.3	4 in	Confinement Spiral Spacing
10.9.3	0.00313	Spiral Reinforcement Ratio
10.9.1	0.55%	Longitudinal Rebars Ratio
10.9.2	5	Longitudinal Rebars Count
10.11.5	107	Slenderness to Apply Moment Magnification Method



ACI Article	Message
Column Design	Fail
Sway/Non-Sway?	Non-Sway
ACI Design Provision(s)	Fail
Design Strengths	Fail
Aci 5.1.1	ok
Aci 7.10.4.2	ok
Aci 7.10.4.3	ng - Confinement: Spiral Spacing
Aci 10.1	
Aci 10.2.1	
Aci 10.2.2	
Aci 10.2.3	
Aci 10.2.4	
Aci 10.2.5	
Aci 10.2.6	
Aci 10.2.7	
Aci 10.2.7.1	ok
Aci 10.2.7.2	ok
Aci 10.2.7.3	ok
Aci 10.3.3	
Aci 10.3.4	
Aci 10.3.5	
Aci 10.3.6	ok
Aci 10.9.1	ng - Ast
Aci 10.9.2	ng - Number Long. Rebars
Aci 10.9.3	ng - Spiral Reinforcement Ratio
Aci 10.11.4.2	
Aci 10.11.5	ng - Magnification Method Not Appli

Figure 30 – CU Slender Column feedbacks for student in case of failing column

The Figure 30 shows that CU Slender Column detects that the column fails in strength by providing the feedback “Fail” in front of “Design Strengths”. Also CU Slender Column detects that ACI 318 provisions are not satisfied by providing the feedback “Fail” in front of “ACI Design Provision(s)”. For non-satisfied provisions, CU Slender Column identifies the parameters that have to be corrected.

Evaluation of Accuracy of CU Slender Column

As part of the development of CU Slender Column, it is developed an independent study with Microsoft Excel 2013. This independent study is used as the benchmark for the evaluation of CU Slender Column accuracy. All outcomes are computed with a precision of three significant figures.

Accuracy of the approach determining c

The accuracy of the approach to determine c is illustrated by the Figure 31. It is shown that with a precision of three significant figures, CU Slender Column outcomes match those of the benchmark study. Thus, the linear interpolation implemented in CU Slender Column provides a good accuracy in the assessment of c .

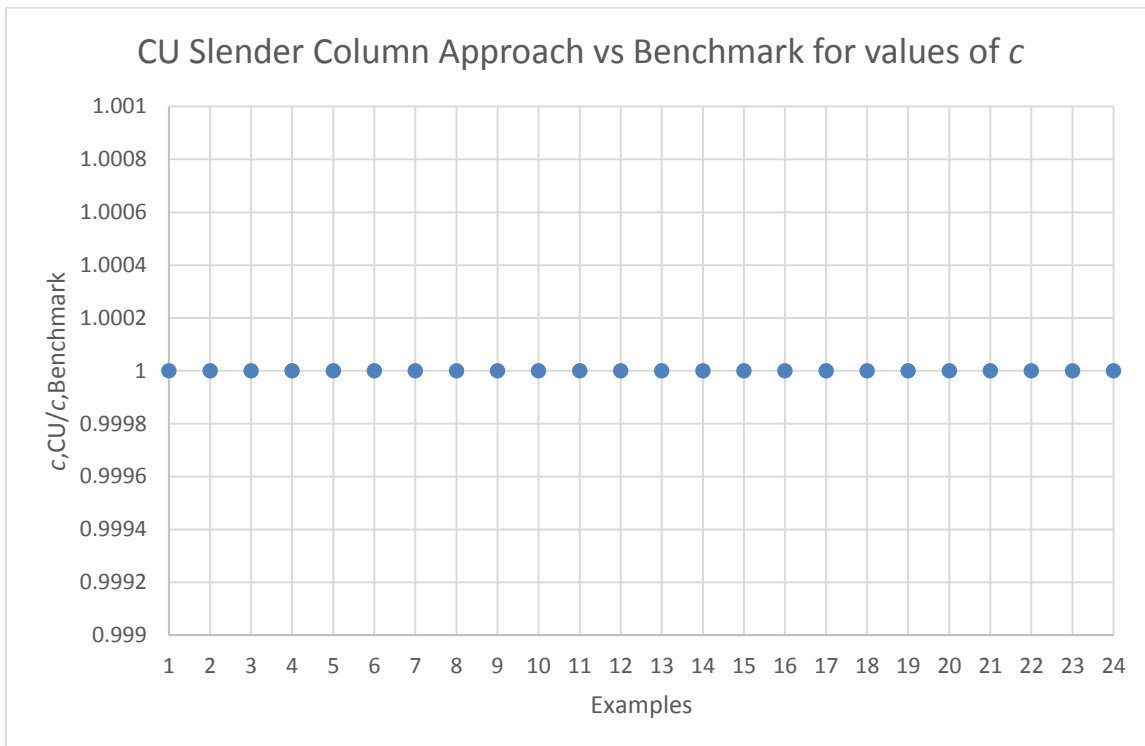


Figure 31 – CU Slender Column Approach vs Benchmark for values of c .

Accuracy of Strengths

The accuracy of CU Slender Column to determine the axial strength is illustrated in the Figure 32 and the accuracy for the moment strength is illustrated in the Figure 33. It is shown that with a precision of three significant figures, CU Slender Column outcomes match those of the benchmark study.

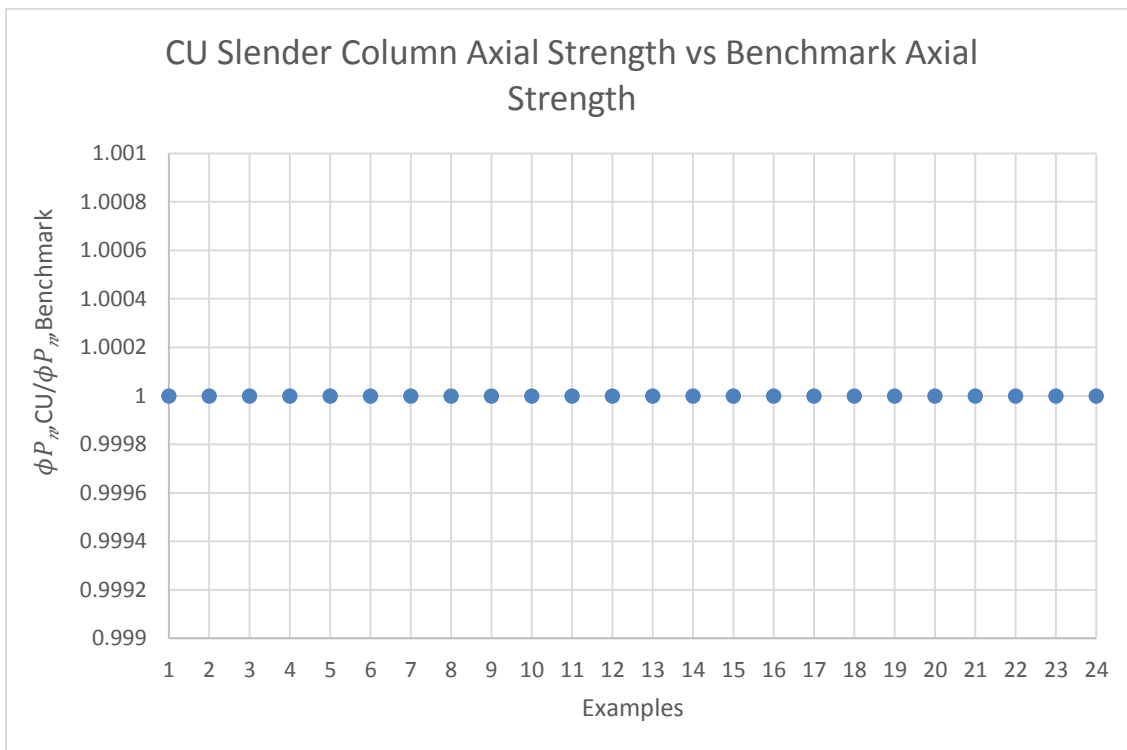


Figure 32 – CU Slender Column Axial Strength vs Benchmark Axial Strength

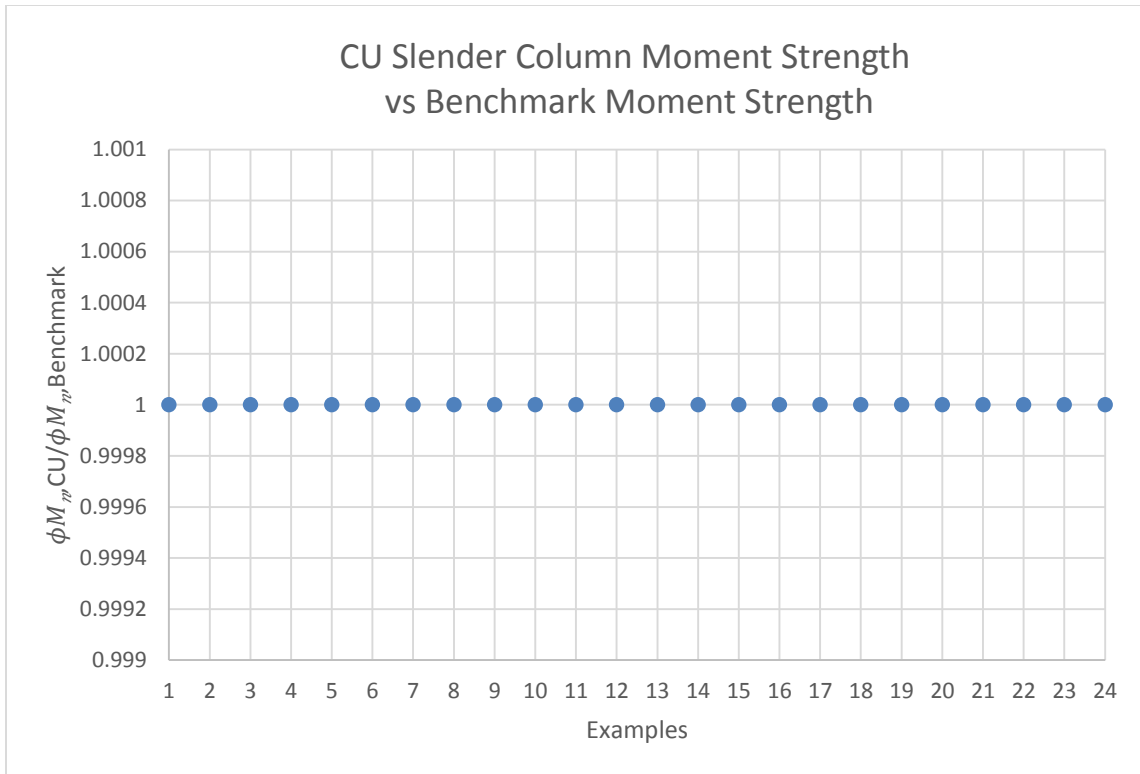


Figure 33 – CU Slender Colum Moment Strength vs Benchmark Moment Strength

Accuracy of Demands and Moment Magnification Factor

The accuracy of CU Slender Column to determine the axial demand P_u is illustrated in Figure 34, the accuracy for the magnified moment demand M_c is illustrated in Figure 35, and the accuracy for the moment magnification factor is illustrated in Figure 36. It is shown that with a precision of three significant figures, CU Slender Column outcomes match those of the benchmark study.

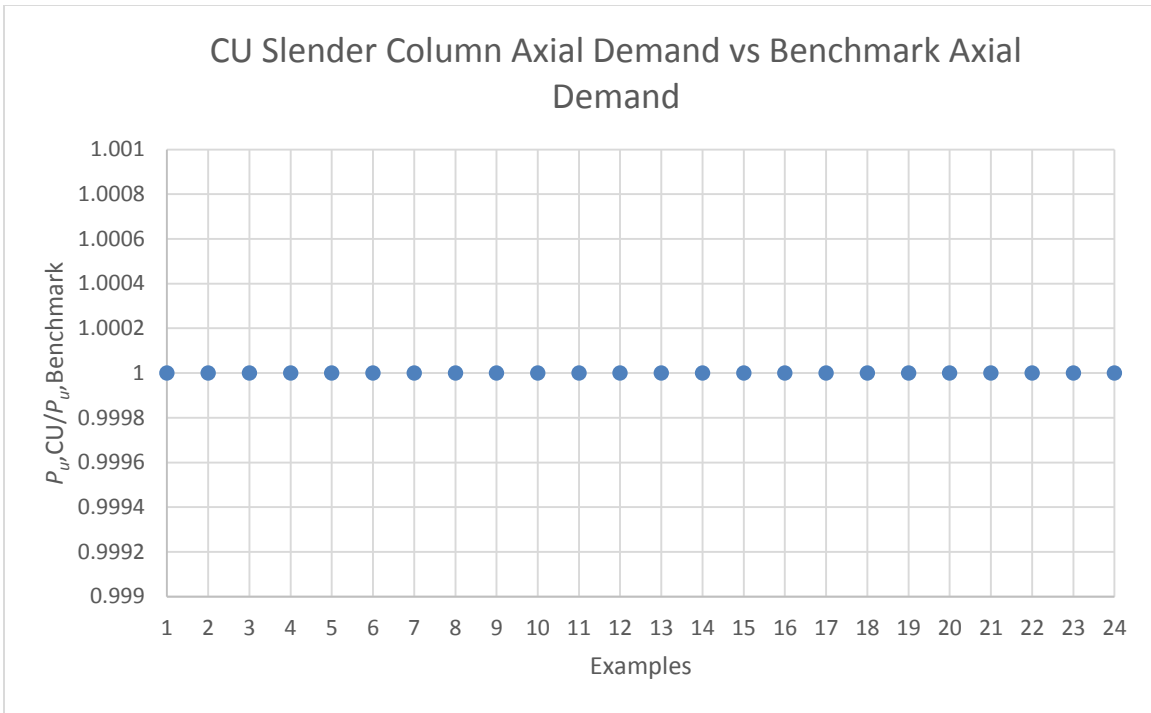


Figure 34 – CU Slender Column Axial Demand vs Benchmark Axial Demand

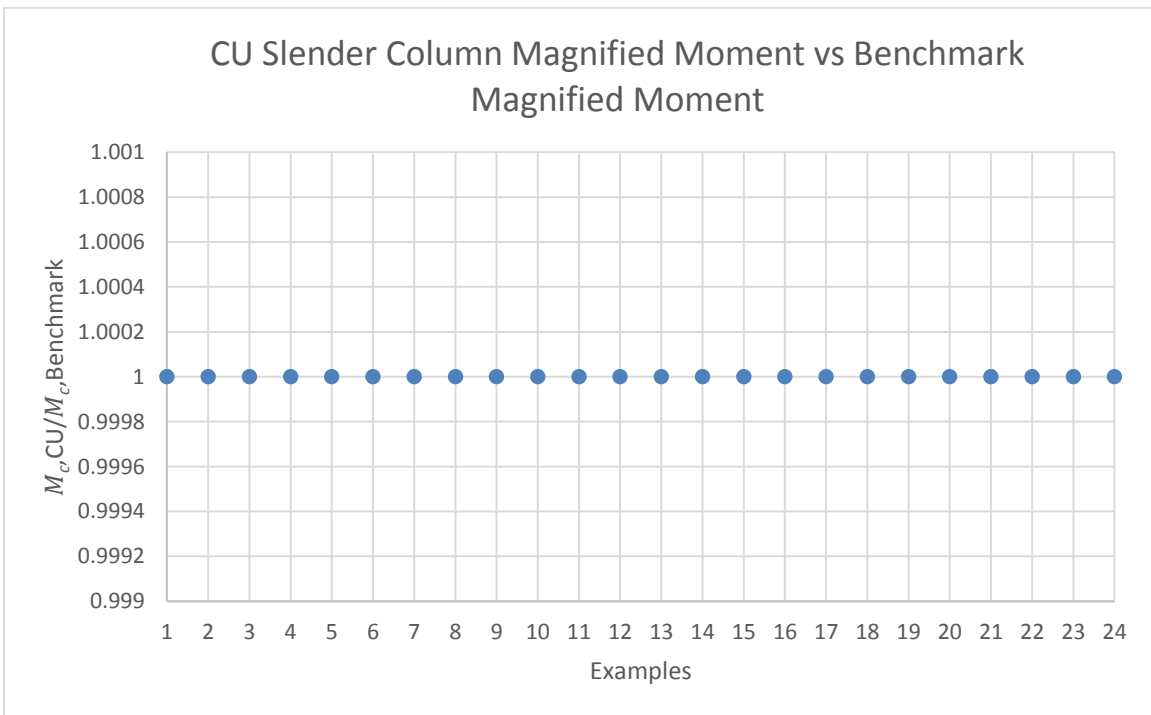


Figure 35 – CU Slender Column Magnified Moment vs Benchmark Magnified Moment

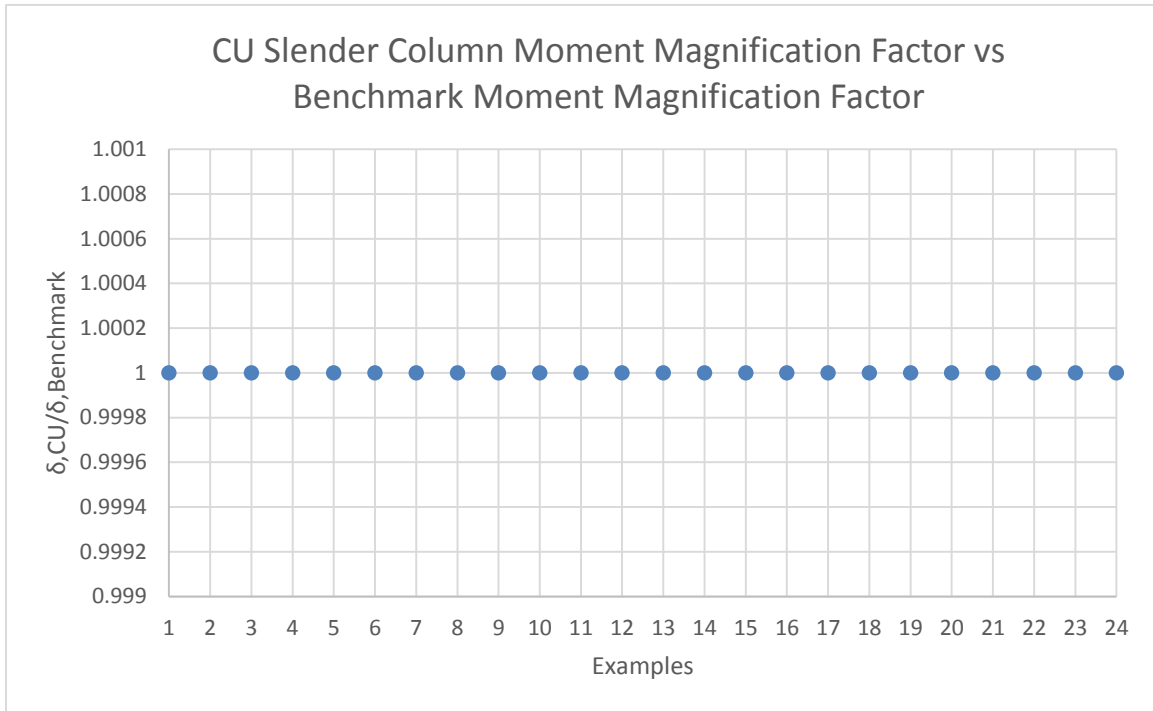


Figure 36 – CU Slender Column Moment Magnification Factor vs Benchmark Moment Magnification Factor

The evaluation of the accuracy of CU Slender Column is based on the study of the accuracy of the software application approach to determine c , the study of the accuracy of the axial strength ϕP_n , the study of the accuracy of the moment strength ϕM_n as well as the study of the accuracy of the axial demand P_u , the magnified moment demand M_c and the moment magnification factor. It is shown that the outcomes generated by CU Slender Column are accurate with a three significant figures precision and can be used as a benchmark in the evaluation of students' numerical values.

Demo #1 – Variations on Evaluation of Columns in Non-Sway Frame

Columns in examples for validation in non-sway frame have different geometry and subjected to different loadings to explore low and high values of design parameters such as slenderness, eccentricity and ratio of factored end moments. Those variations show that CU Slender Column detects whether a column passes or fails in strength for a wide range of design situations.

Example Pairs #1 to #4 target a low slenderness. As shown in Table 35, variations touch on low and high values of C_m and eccentricity. It is noted that examples #1 and #13 which compose the Example Pair #1 are the detailed examples for non-sway frame presented earlier (page 48).

Table 35 – Examples varying C_m and eccentricity for Examples Pairs #1 to #4

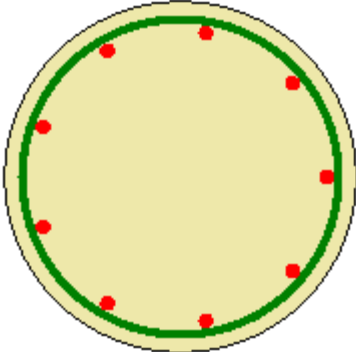
Eccentricity \ C_m	Low	High
	Low	#2 #14
High	#1 #13	#3 #17

Example Pair #2 – Low Slenderness / Non-Sway / Low C_m / Low Eccentricity

The column in Example Pair #2 is in a non-sway frame and single-curved. Compared to Example Pair #1, Example Pair #2 explores a longer column with a smaller diameter and targets a low slenderness. Also, the axial force is larger and the end moments smaller to target a low eccentricity. In Example Pair #2, longitudinal rebars have a smaller size.

Table 36 – Example Pair #2 – Low Slenderness / Non-Sway / Low C_m / Low Eccentricity / Single Curvature

Column Geometry		
Height (ft)	18	
Diameter (in)	26	
Column Configuration		
Slenderness	33	~Low Value
Sway/Non-Sway?	Non-Sway	
C_m	0.63	~Low Value
Eccentricity (in)	3.5	~diameter/10
Loading	Ex #2	Ex #14
P (k)	275	350
M_{ei} (kft)	10	20
M_{ej} (kft)	-120	-140
δ_{ns}	1.0	1.0
Results		
P_u (k)	1045	1330
ϕP_n (k)	1093	1153
M_2 (kft)	456	532
M_c (kft)	456	532
ϕM_n (kft)	477	461
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



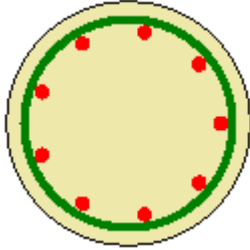
SCALE 1:14
Longitudinal Rebars, 9 – No 9
Spiral Ties, No 5 @ 2in

Example Pair #3 – Low Slenderness / Non-Sway / High C_m / High Eccentricity

The column in Example Pair #3 is in a non-sway frame and single-curved. Compared to Example Pair #2, Example Pair #3 explores a shorter column with a smaller diameter and targets a low slenderness. Also, the axial force is smaller and the end moments bigger to target a high eccentricity. The ratio of factored end moments is bigger.

Table 37 – Example Pair #3 – Low Slenderness / Non-Sway / High C_m / High Eccentricity / Single Curvature

Column Geometry		
Height (ft)	12	
Diameter (in)	18	
Column Configuration		
Slenderness	32	~Low Value
Sway/Non-Sway?	Non-Sway	
C_m	1.0	~High Value
Eccentricity (in)	24	~diameter
Loading	Ex #3	Ex #15
P (k)	20	100
M_{ei} (kft)	50	180
M_{ej} (kft)	-50	-180
δ_{ns}	1.03	1.19
Results		
P_u (k)	76	380
ϕP_n (k)	87	106
M_2 (kft)	190	684
M_c (kft)	196	813
ϕM_n (kft)	226	227
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



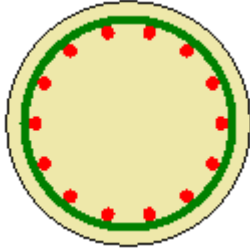
SCALE 1:14
Longitudinal Rebars, 9 – No 9
Spiral Ties, No 5 @ 2in

Example Pair #4 – Low Slenderness / Non-Sway / High C_m / Low Eccentricity

The column in Example Pair #4 is in a non-sway frame and single-curved. Compared to Example Pair #3, Example Pair #4 explores a shorter column and targets a low slenderness. Also, the axial force is larger and the end moments smaller to target a low eccentricity. In Example Pair #4, more longitudinal rebars are used with a smaller size.

Table 38 – Example Pair #4 – Low Slenderness / Non-Sway / High C_m / Low Eccentricity / Single Curvature

Column Geometry		
Height (ft)	10	
Diameter (in)	18	
Column Configuration		
Slenderness	27	~Low Value
Sway/Non-Sway?	Non-Sway	
C_m	1.0	~High Value
Eccentricity (in)	1.4	~diameter/10
Loading		
	Ex #4	Ex #16
P (k)	240	350
M_{ei} (kft)	20	50
M_{ej} (kft)	-20	-50
δ_{ns}	1.36	1.63
Results		
P_u (k)	912	1330
ϕP_n (k)	952	778
M_2 (kft)	76	190
M_c (kft)	118	309
ϕM_n (kft)	123	181
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



SCALE 1:14
Longitudinal Rebars, 14 – No 8
Spiral Ties, No 5 @ 2in

Example Pairs #9 to #12 target a high slenderness. As shown in Table 39, variations touch on low and high values of C_m and eccentricity.

Table 39 – Examples varying C_m and eccentricity for Example Pairs #9 to #12

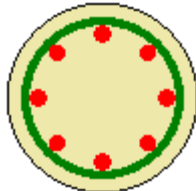
Eccentricity \ C_m	Low	High
	Low	#10 #22
High	#9 #21	#11 #23

Example Pair #9 - High Slenderness / Non-Sway / Low C_m / High Eccentricity

The column in Example Pair #9 is in a non-sway frame and double-curved. Compared to Example Pair #4, Example Pair #9 explores a longer column with a smaller diameter and targets a high slenderness. Also, the axial force is smaller to target a high eccentricity. The ratio of factored end moments is smaller. In Example Pair #9, less longitudinal rebars are used with a bigger size.

Table 40 – Example Pair #9 – High Slenderness / Non-Sway / Low C_m / High Eccentricity / Double Curvature

Column Geometry		
Height (ft)	26	
Diameter (in)	14	
Column Configuration		
Slenderness	89	~High Value
Sway/Non-Sway?	Non-Sway	
C_m	0.4	~Low Value
Eccentricity (in)	10	~diameter
Loading	Ex #9	Ex #21
P (k)	15	40
M_{ei} (kft)	10	20
M_{ej} (kft)	20	25
δ_{ns}	1.00	2.14
Results		
P_u (k)	57	152
ϕP_n (k)	103	102
M_2 (kft)	76	95
M_c (kft)	76	204
ϕM_n (kft)	137	137
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



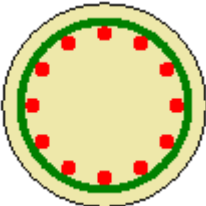
SCALE 1:14
Longitudinal Rebars, 8 – No 10
Spiral Ties, No 5 @ 2in

Example Pair #10 – High Slenderness / Non-Sway / Low C_m / Low Eccentricity

The column in Example Pair #10 is in a non-sway frame and double-curved. Compared to Example Pair #9, Example Pair #10 explores a slightly shorter column with a slightly smaller diameter and targets a high slenderness. Also, the axial force is bigger to target a low eccentricity. The ratio of factored end moments is slightly bigger. In Example Pair #10, more longitudinal rebars are used with a smaller size.

Table 41 – Example Pair #10 – High Slenderness / Non-Sway / Low C_m / Low Eccentricity / Double Curvature

Column Geometry		
Height (ft)	25	
Diameter (in)	15	
Column Configuration		
Slenderness	80	~High Value
Sway/Non-Sway?	Non-Sway	
C_m	0.47	~Low Value
Eccentricity (in)	10	~diameter/2
Loading	Ex #10	Ex #22
P (k)	40	55
M_{ei} (kft)	10	20
M_{ej} (kft)	30	27
δ_{ns}	1.09	1.86
Results		
P_u (k)	152	209
ϕP_n (k)	218	196
M_2 (kft)	114	102
M_c (kft)	124	191
ϕM_n (kft)	179	179
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



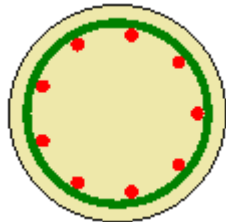
SCALE 1:14
Longitudinal Rebars, 12 – No 9
Spiral Ties, No 5 @ 2in

Example Pair #11 – High Slenderness / Non-Sway / High C_m / High Eccentricity

The column in Example Pair #11 is in a non-sway frame and double-curved. Compared to Example Pair #10, Example Pair #11 explores a longer column and targets a high slenderness. Also, the axial force is smaller and end moments bigger to target a high eccentricity. The ratio of factored end moments is lower. In Example Pair #11, less longitudinal rebars are used with a smaller size.

Table 42 – Example Pair #11 – High Slenderness / Non-Sway / High C_m / High Eccentricity / Double Curvature

Column Geometry		
Height (ft)	30	
Diameter (in)	16	
Column Configuration		
Slenderness	90	~High Value
Sway/Non-Sway?	Non-Sway	
C_m	0.55	~High Value
Eccentricity (in)	17	~diameter
Loading	Ex #11	Ex #23
P (k)	30	30
M_{ei} (kft)	5	5
M_{ej} (kft)	37	40
δ_{ns}	1.04	1.05
Results		
P_u (k)	114	114
ϕP_n (k)	120	110
M_2 (kft)	141	152
M_c (kft)	147	160
ϕM_n (kft)	154	154
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



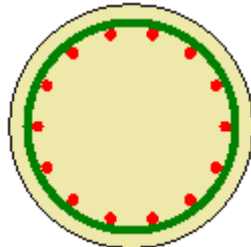
SCALE 1:14
Longitudinal Rebars, 9 – No 8
Spiral Ties, No 5 @ 2in

Example Pair #12 – High Slenderness / Non-Sway / High C_m / Low Eccentricity

The column in Example Pair #12 is in a non-sway frame and double-curved. Compared to Example Pair #11, Example Pair #12 explores a longer column with a larger diameter and targets a high slenderness. Also, the axial force is smaller to target a low eccentricity. The ratio of factored end moments is lower. In Example Pair #12, more longitudinal rebars are used with a smaller size.

Table 43 – Example Pair #12 – High Slenderness / Non-Sway / High C_m / Low Eccentricity / Double Curvature

Column Geometry		
Height (ft)	32	
Diameter (in)	18	
Column Configuration		
Slenderness	85	~High Value
Sway/Non-Sway?	Non-Sway	
C_m	0.53	~High Value
Eccentricity (in)	11	~diameter/2
Loading	Ex #12	Ex #24
P (k)	60	65
M_{ei} (kft)	5	5
M_{ej} (kft)	30	30
δ_{ns}	1.65	2.00
Results		
P_u (k)	228	247
ϕP_n (k)	271	240
M_2 (kft)	114	114
M_c (kft)	188	227
ϕM_n (kft)	223	221
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



SCALE 1:14
Longitudinal Rebars, 14 – No 7
Spiral Ties, No 5 @ 2in

Demo #1 – Variation on Evaluation of Column in Sway Frame

Similarly to the evaluation of columns in non-sway frame, columns in examples for validation in sway frame have different geometry and subjected to different loadings to explore low and high values of design parameters such as slenderness and eccentricity. Those variations show that CU Slender Column detects whether a column passes or fails in strength for a wide range of design situations.

Example Pairs #5 to #8 target a high value of C_m . As shown in Table 44, variations touch on low and high values of slenderness and eccentricity. It is noted that examples #5 and #17 are the detailed examples for sway frame presented earlier (page 55).

Table 44 – Examples varying slenderness and eccentricity for Example Pairs #5 to #8

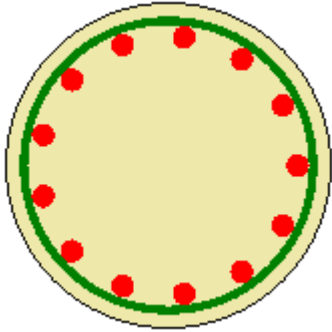
Slenderness \ Eccentricity	Low	High
	Low	#6 #18
High	#5 #17	#7 #19

Example Pair #6 – Low Slenderness / Sway / Low Eccentricity

The column in Example Pair #6 is in a sway frame and single-curved. Compared to Example Pair #5, Example Pair #6 explores a shorter column with a larger diameter and targets a low slenderness. Also, the axial force is larger to target a low eccentricity. In Example Pair #6, less longitudinal rebars are used with a bigger size.

Table 45 – Example Pair #6 – Low Slenderness / Sway / Low Eccentricity / Single Curvature

Column Geometry		
Height (ft)	26	
Diameter (in)	24	
Column Configuration		
Slenderness	52	~Low Value
Sway/Non-Sway?	Sway	
C_m	1.0	~High Value
Eccentricity (in)	3	~diameter/10
Loading	Ex #6	Ex #18
P (k)	120	200
M_{ei} (kft)	30	200
M_{ej} (kft)	-30	-200
δ_{ns}	1.39	1.89
Results		
P_u (k)	456	760
ϕP_n (k)	1699	577
M_2 (kft)	114	760
M_c (kft)	133	1045
ϕM_n (kft)	495	794
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



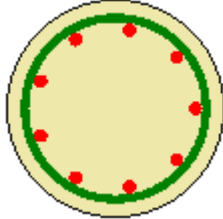
SCALE 1:14
Longitudinal Rebars, 13 – No 14
Spiral Ties, No 5 @ 2in

Example Pair #7 – High Slenderness / Sway / High Eccentricity

The column in Example Pair #7 is in a sway frame and double-curved. Compared to Example Pair #6, Example Pair #7 explores a longer column with a smaller diameter and targets a high slenderness. Also, the axial force is smaller to target a high eccentricity. In Example Pair #7, less longitudinal rebars are used with a smaller size.

Table 46 – Example Pair #7 – High Slenderness / Sway / High Eccentricity / Double Curvature

Column Geometry		
Height (ft)	30	
Diameter (in)	16	
Column Configuration		
Slenderness	90	~High Value
Sway/Non-Sway?	Sway	
C_m	1.0	~High Value
Eccentricity (in)	16	~diameter
Loading	Ex #7	Ex #19
P (k)	25	35
M_{ei} (kft)	30	40
M_{ej} (kft)	30	40
δ_{ns}	1.66	2.25
Results		
P_u (k)	95	133
ϕP_n (k)	100	88
M_2 (kft)	114	152
M_c (kft)	146	232
ϕM_n (kft)	154	153
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



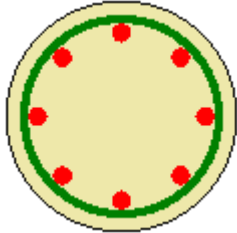
SCALE 1:14
Longitudinal Rebars, 9 – No 8
Spiral Ties, No 5 @ 2in

Example Pair #8 – High Slenderness / Sway / Low Eccentricity

The column in Example Pair #8 is in a sway frame and double-curved. Compared to Example Pair #7, Example Pair #8 explores a shorter column with a shorter diameter and targets a high slenderness. Also, the axial force is larger to target a low eccentricity. In Example Pair #8, less longitudinal rebars are used with a bigger size.

Table 47 – Example Pair #8 – High Slenderness / Sway / Low Eccentricity / Double Curvature

Column Geometry		
Height (ft)	29	
Diameter (in)	17	
Column Configuration		
Slenderness	82	~High Value
Sway/Non-Sway?	Sway	
C_m	1.0	~High Value
Eccentricity (in)	8	~diameter/2
Loading	Ex #8	Ex #20
P (k)	50	50
M_{ei} (kft)	10	30
M_{ej} (kft)	30	50
δ_{ns}	2.39	2.39
Results		
P_u (k)	190	190
ϕP_n (k)	246	149
M_2 (kft)	114	190
M_c (kft)	181	301
ϕM_n (kft)	234	236
Finding		
Strength	PASS	FAIL
ACI Provisions	PASS	PASS



SCALE 1:14
Longitudinal Rebars, 8 – No 11
Spiral Ties, No 5 @ 2in

Demo #2 – Non Satisfaction of ACI Design Provisions

In addition to satisfy strength requirements, a column design must satisfy ACI provisions. Demo #2 aims at showing that CU Slender Column detects whether a column does not satisfy ACI provisions. Examples present different columns designs that violate a provision. Among the violations are: Inadmissible steel spiral ties spacing, inadmissible steel spiral ties ratio, inadmissible amount of longitudinal rebars, inadmissible number of longitudinal rebars, and inadmissible slenderness for the application of the moment magnification method.

Example Pair #1 – ACI 318 7.10.4.3: Confinement Spiral Spacing

The provision ACI 318 7.10.4.3 requires that the clear spacing between spirals does not exceed 3 in or is less than 1 in. The routine `Aci_7_10_4_3` under the `AciChapter10` class gives the following message if the provision is not satisfied: “ng – Confinement: Spiral Spacing”. The message appears in the ACI Checks grid under the tab Feedback in front of the corresponding ACI 318 provision 7.10.4.3.

Table 48 – Example Pair #1 – Confinement Spiral Spacing

Loading	Ex #25	Ex #30
P_u (k)	950	1330
ϕP_n (k)	974	1100
M_c (kft)	456	532
ϕM_n (kft)	467	444
<i>ACI 7.10.4.3</i>		
Spiral spacing (in)	0.5	3.5
<i>Finding</i>		
Strength	PASS	FAIL
ACI provisions	ng	ng

Figure 37 and Figure 38 are obtained from the ACI Checks grid under the tab Feedback and show that CU Slender Column detects non-satisfied provision ACI 7.10.4.3.

CU Slender Column		
Inputs Debug Tables Feedback		
ACI Checks		
	ACI Article	Message
▶	Column Design	Fail
	Sway/Non-Sway?	Non-Sway
	ACI Design Provision(s)	Fail
	Design Strengths	Pass
	Aci 5.1.1	ok
	Aci 7.10.4.2	ok
	Aci 7.10.4.3	ng - Confinement: Spiral Spacing

Figure 37 – Ex 25: Detection of failed provision ACI 7.10.4.3

CU Slender Column		
Inputs Debug Tables Feedback		
ACI Checks		
	ACI Article	Message
▶	Column Design	Fail
	Sway/Non-Sway?	Non-Sway
	ACI Design Provision(s)	Fail
	Design Strengths	Fail
	Aci 5.1.1	ok
	Aci 7.10.4.2	ok
	Aci 7.10.4.3	ng - Confinement: Spiral Spacing

Figure 38 – Ex 30: Detection of failed provision ACI 7.10.4.3

Example Pair #2 – ACI 318 10.9.3: Spiral Reinforcement Ratio

Figure 39 shows a spiral reinforcement in a circular concrete column.

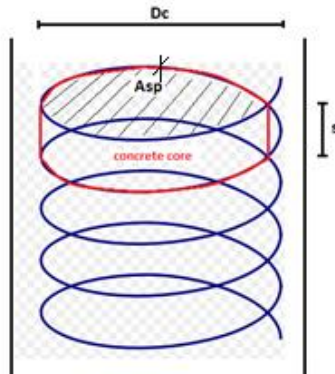


Figure 39 – Spiral reinforcement in circular column

The spiral volumetric ratio ρ is defined as the volume of spirals over the volume of concrete core. For the computation of ρ , the curvature of the spiral is neglected, one circle is considered.

$$\rho = \frac{V_{spiral}}{V_{core}} = \frac{Asp * \pi * Dc}{s * \pi * Dc^2 / 4} = \frac{4 * Asp}{s * Dc} \quad \text{Eq. 77}$$

where:

A_{sp} is the cross sectional area of the spiral

s the spacing between spirals

D_c the diameter of the concrete core

The provision ACI 318 10.9.3 requires that ρ is larger than ρ_s , a minimum spiral volumetric ratio specified by ACI 318.

$$\rho \geq \rho_s \quad \text{Eq. 78}$$

$$\rho_s = 0.45 \left(\frac{A_g}{A_{ch}} - 1 \right) \frac{f'_c}{f_{yt}} \quad \text{Eq. 79}$$

where:

A_g is the concrete gross area

A_{ch} is the concrete core area

f'_c and f_{yt} are respectively the concrete compression strength and the transverse reinforcement yield stress

The routine `Aci_10_9_3` under the `AciChapter10` class gives the following message if the provision is not satisfied: “ng – Spiral Reinforcement Ratio”. The message appears in the ACI Checks grid under the tab Feedback in front of the corresponding ACI 318 provision 10.9.3.

Table 49 – Example Pair #2 – ACI 318 10.9.3: Spiral Reinforcement Ratio

Loading	Ex #26	Ex #31
P_u (k)	1045	1330
ϕP_n (k)	1049	1108
M_c (kft)	456	532
ϕM_n (kft)	458	443
<i>ACI 10.9.3</i>		
ρ_s	0.0065	0.0065
ρ	0.0061	0.0061
<i>Finding</i>		
Strength	PASS	FAIL
ACI provisions	ng	ng

Figure 40 and Figure 41 are obtained from the ACI Checks grid under the tab Feedback and show that CU Slender Column detects non-satisfied provision ACI 10.9.3.

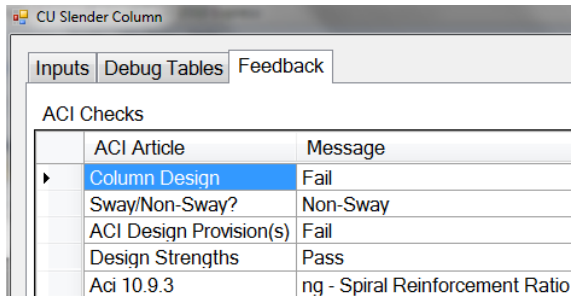


Figure 40 – Ex 26: Detection of failed provision ACI 10.9.3

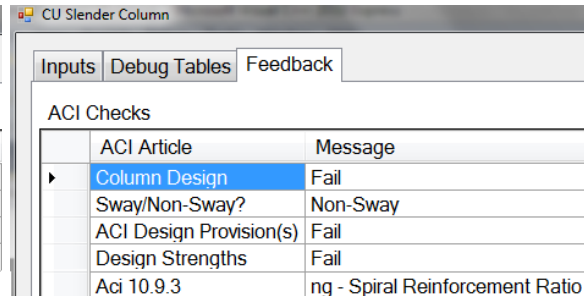


Figure 41 – Ex 31: Detection of failed provision ACI 10.9.3

Example Pair #3 – ACI 318 10.9.1: Longitudinal Rebars Ratio

The provision ACI 318 10.9.1 requires that the ratio of longitudinal rebars area over concrete gross area does not exceed 8% or is less than 1%. The routine *Aci_10_9_1* under the *AciChapter10* class gives the following message if the provision is not satisfied: “ng – Ast”. The message appears in the ACI Checks grid under the tab Feedback in front of the corresponding ACI 318 provision 10.9.1.

Table 50 – Example Pair #3 – Longitudinal Rebars Ratio

Loading	Ex #27	Ex #32
P_u (k)	1045	1330
ϕP_n (k)	1116	1146
M_c (kft)	380	1332
ϕM_n (kft)	406	1148
<i>ACI 10.9.1</i>		
A_{st}/A_g	0.99%	9.04%
<i>Finding</i>		
Strength	PASS	FAIL
ACI provisions	ng	ng

Figure 42 and Figure 43 are obtained from the ACI Checks grid under the tab Feedback and show that CU Slender Column detects non-satisfied provision ACI 10.9.1.

The screenshot shows the 'ACI Checks' grid in the 'Feedback' tab. The grid has three columns: 'ACI Article' and 'Message'. The 'Column Design' row is highlighted in blue and shows a 'Fail' message. Other rows include 'Sway/Non-Sway?' (Non-Sway), 'ACI Design Provision(s)' (Fail), 'Design Strengths' (Pass), and 'Aci 10.9.1' (ng - Ast).

ACI Article	Message
Column Design	Fail
Sway/Non-Sway?	Non-Sway
ACI Design Provision(s)	Fail
Design Strengths	Pass
Aci 10.9.1	ng - Ast

Figure 42 – Ex 27: Detection of failed provision ACI 10.9.1

The screenshot shows the 'ACI Checks' grid in the 'Feedback' tab. The grid has three columns: 'ACI Article' and 'Message'. The 'Design Strengths' row is highlighted in blue and shows a 'Fail' message. Other rows include 'Column Design' (Fail), 'Sway/Non-Sway?' (Non-Sway), 'ACI Design Provision(s)' (Fail), and 'Aci 10.9.1' (ng - Ast).

ACI Article	Message
Column Design	Fail
Sway/Non-Sway?	Non-Sway
ACI Design Provision(s)	Fail
Design Strengths	Fail
Aci 10.9.1	ng - Ast

Figure 43 – Ex 32: Detection of failed provision ACI 10.9.1

Example Pair #4 – ACI 318 10.9.2: Longitudinal Rebars Count

The provision ACI 318 10.9.2 requires that at least 6 longitudinal rebars have to be provided when spiral ties are used. The routine *Aci_10_9_2* under the *AciChapter10* class gives the following message if the provision is not satisfied: “ng – Number Long. Rebars”. The message appears in the ACI Checks grid under the tab Feedback in front of the corresponding ACI 318 provision 10.9.2.

Table 51 – Example Pair #4 – ACI 318 10.9.2: Longitudinal Rebars Count

Loading	Ex #28	Ex #33
P_u (k)	1045	1330
ϕP_n (k)	1447	1199
M_c (kft)	245	532
ϕM_n (kft)	340	480
<i>ACI 10.9.2</i>		
Rebars count	4	5
<i>Finding</i>		
Strength	PASS	FAIL
ACI provisions	ng	ng

Figure 44 and Figure 45 are obtained from the ACI Checks grid under the tab Feedback and show that CU Slender Column detects non-satisfied provision ACI 10.9.2.

ACI Article	Message
Column Design	Fail
Sway/Non-Sway?	Non-Sway
ACI Design Provision(s)	Fail
Design Strengths	Pass
Aci 10.9.2	ng - Number Long. Rebars

Figure 44 – Ex 28: Detection of failed provision ACI 10.9.2

ACI Article	Message
Column Design	Fail
Sway/Non-Sway?	Non-Sway
ACI Design Provision(s)	Fail
Design Strengths	Fail
Aci 10.9.2	ng - Number Long. Rebars

Figure 45 – Ex 33: Detection of failed provision ACI 10.9.2

Example Pair #5 – ACI 318 10.11.5: Applicability of Moment Magnification Method

The moment magnification method is used within a specific range of slenderness. For non-sway frames, the provision ACI 318 10.12.2 allows to ignore slenderness effects, that is not to apply the moment magnification method if:

$$\frac{kL}{r} \leq 34 + 12 \frac{M_1}{M_2} \quad \text{Eq. 48}$$

For sway frames, the provision ACI 318 10.13.2 allows to ignore slenderness effects if:

$$\frac{kL}{r} \leq 22 \quad \text{Eq. 52}$$

Also, the provision ACI 318 10.11.5 specifies that a 2nd order analysis must be applied instead of the moment magnification method if:

$$\frac{kL}{r} \geq 100 \quad \text{Eq. 80}$$

The routine `Aci_10_11_5` under the `AciChapter10` class gives the following message if the provision is not satisfied: “ng – Moment Magnification Not Applicable”. The message appears in the ACI Checks grid under the tab Feedback in front of the corresponding ACI 318 provision 10.11.5.

Table 52 – Example Pair #5 – Application of Moment Magnification

Loading	Ex #29 / Non-Sway	Ex #34 / Sway
P_u (k)	380	1330
ϕP_n (k)	581	189
M_c (kft)	38	532
ϕM_n (kft)	58	76
<i>ACI 10.11.5</i>		
Slenderness	110	17
Limit	100	22
<i>Finding</i>		
Strength	PASS	FAIL
ACI provisions	Method N/A	Method N/A

Note: N/A stands for Not Applicable

Figure 46 and Figure 47 are obtained from the ACI Checks grid under the tab Feedback and show that CU Slender Column detects non-satisfied provision ACI 10.11.5.

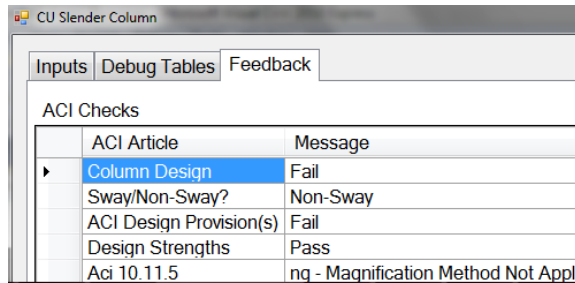


Figure 46 – Ex 29: Detection of failed provision ACI 10.11.5

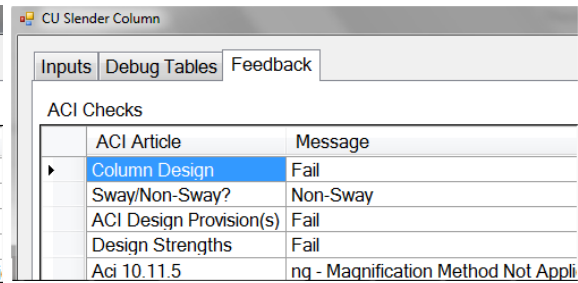


Figure 47 – Ex 34: Detection of failed provision ACI 10.13.2

Demo #3 – Accuracy of Student’s Inputs

Demo #3 aims at showing that CU Slender Column detects whether student’s inputs are accurate. A relative error between the student’s inputs and CU Slender Column outcomes of less than 2% is tolerated. In the program, the routine CheckStudentWork under the StudentChecks class compares the student’s inputs to outcomes computed by CU Slender Column and generates the messages reporting to the student whether the inputs are accurate or not.

Designs Admissible in ACI Provisions

The examples #35 and #36 explore student’s numerical values of loads and strengths that are not accurate with satisfied ACI provisions. Examples aim at demonstrating that the application detects a wrong value or a wrong sign and provides the correct feedbacks, either “ng” or “bad sign”.

The numerical error is computed as a percentage of the CU Slender Column outcome:

$$\varepsilon = \frac{|Va - Vs|}{|Va|} * 100 \tag{Eq. 81}$$

where:

Va is the software outcome

Vs is the student’s input

Va and Vs are any of the following values: P_u , M_c , e , c , ϕ , ϕP_n , ϕM_n

Table 53 – Examples #37 and #38 with design admissible in ACI Provisions

Software Loading		Ex #35	Ex #36	
P_u (k)		190		228
ϕP_n (k)		241		145
M_c (kft)		174		352
ϕM_n (kft)		221		224

Student’s Loading		error	error	
P_u (k)	260	36.9% - ng	-228	bad sign
ϕP_n (k)	200	17.0% - ng	-145	bad sign
M_c (kft)	-174	bad sign	344	2.27% - ng
ϕM_n (kft)	-221	bad sign	231	3.13% - ng

Finding			
ACI Provisions	PASS		PASS
Strength	PASS		FAIL

Figure 48 shows feedbacks to the student relatively to the accuracy of numerical values of loads and strengths for Example 35.

Checks on Inputs

	Load_Combination	Pu	Mc	Phi_Pn	Phi_Mn
	LC #4	ng	bad sign	ng	bad sign

Figure 48 – Ex 35: detection of inaccuracy of student's numerical values of loads and strengths

Figure 49 shows feedbacks to the student relatively to the accuracy of numerical values of loads and strengths for Example 36.

Checks on Inputs

	Load_Combination	Pu	Mc	Phi_Pn	Phi_Mn
	LC #4	bad sign	ng	bad sign	ng

Figure 49 – Ex 36: detection of inaccuracy of student's numerical values of loads and strengths

Designs Inadmissible in ACI Provisions

The examples #37 and #38 explore student’s numerical values of loads and strengths that are not accurate with non-satisfied ACI provisions

Table 54 – Examples #39 and #40 with designs inadmissible in ACI Provisions

Software Loading		Ex #37		Ex #38
P_u (k)		190		228
ϕP_n (k)		241		145
M_c (kft)		174		352
ϕM_n (kft)		221		224

Student’s Loading		error		error
P_u (k)	-190	bad sign	234	2.63% - ng
ϕP_n (k)	-241	bad sign	159	9.66% - ng
M_c (kft)	260	49.4% - ng	-352	bad sign
ϕM_n (kft)	215	2.71% - ng	-224	bad sign

Finding	
ACI Provision	FAIL
Strength	PASS

	FAIL
	FAIL

Figure 50 shows feedbacks to the student relatively to the accuracy of numerical values of loads and strengths for Example 37.

Checks on Inputs

	Load_Combination	Pu	Mc	Phi_Pn	Phi_Mn
	LC #4	bad sign	ng	bad sign	ng

Figure 50 – Ex 37: detection of inaccuracy of student’s numerical values of loads and strengths

Figure 51 shows feedbacks to the student relatively to the accuracy of numerical values of loads and strengths for Example 38.

Checks on Inputs

	Load_Combination	Pu	Mc	Phi_Pn	Phi_Mn
	LC #4	ng	bad sign	ng	bad sign

Figure 51 – Ex 38: detection of inaccuracy of student’s numerical values of loads and strengths

Chapter 5 - Summary. Further Study

Summary

First class in design of reinforced concrete structures must include design of slender compression members since they are common components in many structures such as buildings and bridges. Education must engage students to consider basic loads such as axial forces and end moments as well as teach the importance of stability effects through the consideration of slenderness effects. Education in design must allow students to function as designers, that is, to determine the geometry of the member as well as the complete steel reinforcement that verify strength requirements as well as design code provisions. In design of reinforced concrete compression members, students use ACI 318 [*Building*, 2011] and implement the Chapter 10 for flexure and axial loads. To support the education of students in reinforced concrete design, educational software application, CU Slender Column, is developed. CU Slender Column considers members that are single slender reinforced concrete columns with circular cross section. CU Slender Column uses ACI 318 moment magnification method to account for slenderness effects for both non-sway and sway frames.

CU Slender Column evaluates students' designs in two levels. First, it realizes a benchmark study based on the students' designs. Second, it provides feedbacks on satisfaction of ACI provisions and on accuracy of numerical values of loads and strengths. Strength requirements are satisfied if the column strengths exceed both axial and bending demands and ACI provisions are satisfied if the design respects all design provisions written under the Chapter 10 of ACI 318.

The graphical user interface of CU Slender Column is composed of two tabs. The tab Inputs collects the student's design as well as the values of contributions of axial force and end moments. Three buttons, Save and Load and Compute allow the student to save the work, to load it and to perform computations that evaluate the work and generates the feedbacks on the design. The tab Feedback displays the feedbacks to the student on ACI provisions and provides messages on accuracy of student's numerical values of strengths and loads.

CU Slender Column is validated with a set of 38 examples. Those examples aim at exploring a wide range of design situations to provide a strong and comprehensive demonstration of the software performance. The first set of examples is made of 24 examples with columns either passing or failing strength requirements. To show that CU Slender Column is able to consider a wide range of column features, the examples explore variations in column geometry, reinforcement ratio, loadings, curvature and type of frame. The second set of examples is made of 10 examples with columns failing in ACI provisions. Five ACI design provisions are explored. The third set of examples is made of 4 examples touching on inaccurate student's numerical values of strengths and loads.

CU Slender Column is used as a benchmark in the evaluation of students' designs. It is paramount to demonstrate that the outcomes computed by CU Slender Column are accurate. An independent study using Microsoft Excel 2013 is performed to evaluate the accuracy of the computations of the software. Evaluations on several outcomes such as demands, magnification factor and strengths show that CU Slender Column provides accurate outcomes and can be used as a benchmark for evaluation of students' designs.

Future Work

CU Slender Column aims at improving the evaluation of students' projects in design of slender reinforced concrete columns. By using this application, the teacher can quickly evaluate in detail a large number of projects, which is more efficient than the method of evaluation by hand. This traditional method is time consuming and more likely to miss some errors. CU Slender Column achieves the goal of providing students a complete check of the design by pointing out the errors and providing the remedies. The report on the design is done by the use of feedbacks which are efficient because they tend to prod the students in the discovery of a design passing strength requirements and complying with ACI provisions. However, the application is restrained in several levels and it is possible to enhance the educational performance of the software to allow students to work on more complex and more practical designs. Further studies are possible on inputs and feedbacks.

The variables input by the student to characterize the design and the loadings can be increased to consider more elaborate designs.

For column geometry, CU Slender Column only deals with circular cross sections but can expand to rectangular and irregular cross sections. It can propose a library of three or four predefined cross-sectional shapes usually used for compression members in bridges and buildings. To find those shapes, a large panel of recently built concrete bridges and buildings across the United States can be studied and then the most common shapes are selected.

For spiral ties reinforcement, CU Slender column only considers a single spiral but it can consider circular reinforcement as an alternative. The transverse reinforcement does not influence the axial and bending strengths of the column but does play a role in case of shear design. The application should allow the student to create a pattern for the transverse reinforcement along the member so that it provides just enough steel to withstand the shear demand along the column.

For types of end conditions, CU Slender Column can either let the student input the value of effective length factor beforehand computed using the alignment charts in case of a

column within a frame, or propose several combinations whether the column ends are fixed, hinged, rollers or free in case of a single column.

For the loadings, CU Slender Column only considers axial forces and end moments. The software can expand to biaxial bending moments to allow the student to consider a column within a 3-Dimensional (3D) frame. If biaxial bending moments are considered, two different effective length factors, one for each bending direction, have to be defined. Also, it would be interesting to take into account distributed lateral loads such as wind and ground excitation due to earthquake for design in areas with high seismicity.

Finally, since students may use a structural engineering software to get values of loads contribution in case of sway and non-sway frames, it would be useful to enable CU Slender Column to accept outputs files generated by structural analysis software. This compatibility would save time to students because they would not type the values of loads contributions in the Section Forces grid.

The feedbacks aim at reporting accuracy of the student's work as well as detecting design flaws. Improvements are possible to make those feedbacks more efficient.

Design error can be ranked according to its importance in the context of education. A design error can either be evaluated as critical, weak or inaccurate. A critical error signifies a design that does not satisfy basic design rules affecting the column strengths and the disposition of the steel reinforcement. Thus, the critical parameters the student has to work on are the minimum number of reinforcement rebars, a feasible layout of reinforcement rebars and the dimensions of the cross section to withstand the loads. A weak design signifies a design that does not satisfy strength requirements. Thus, the parameter the student must review is the area of steel reinforcement. An inaccurate design signifies a design which values of factored loads and strengths are not accurate. Thus, the student must review the computations of loads and strengths.

Evaluations of the design can be performed throughout the student's work and detects design flaws early in the design process to avoid waste of effort by the student. For instance, the design process can be split into several important steps: Computation of factored loads, layout of steel reinforcement, computation of magnified moments ... Then, when the

student achieves a step, he can ask for a check and the software will provide feedbacks on what he has achieved.

Students must be educated regarding efficiency of material use. The student's design can be checked for material use by performing an optimal design and comparing the quantities of steel and concrete required in both optimal and student's designs.

Finally, for evaluation of intermediate outcomes obtained by the student, it is helpful to tell the student whether those outcomes are "consistent" with the starting inputs. For instance, an error in the computation of the eccentricity demand e_u leads to an error for c , then an error for strengths. The student must be able to see that even though e_u is wrong, the computation of strengths, for example, are "consistent". Thus, the student can better point the flaws in the computations and spend time in computing only the wrong values.

To evaluate the educational efficiency of the software application, students' evaluations are collected by the professor at the term of each semester. Students will be asked to give their impressions regarding the use of the software and the relevance of the feedbacks. Those impressions will be used to adjust the software objectives and features in order to become a valuable educational tool in design of reinforced concrete structures.

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