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Real options modeling and valuation of price adjustment flexibility with an application to the leasing industry

Ahmed Al sharif

Department: Engineering Management and Systems Engineering

REAL OPTIONS MODELING AND VALUATION OF PRICE ADJUSTMENT FLEXIBILITY WITH AN APPLICATION TO THE LEASING INDUSTRY

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AHMED A.A. AL SHARIF

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Approved by

Dr. Ruwen Qin, Advisor
Dr. Xiaoping Du
Dr. David Enke
Dr. Suzanna Long
Dr. Stephen A. Raper

ABSTRACT

Uncertainty poses not only threats but also opportunities. This study sought to build the scientific foundation for introducing a real options (ROs) methodology for price risk management to the leasing industry. A price risk management that allows for both coping with threats and taking advantage of opportunities. In the leasing industry, fixed rate long-term lease contracts help contract parties stabilize cash flows within volatile markets. The contract's term, however, may be extended long enough that prevent capturing the opportunities of gaining greater profits or reducing expenses. Therefore, the flexibility that enables participants to take advantage of favorable market price is desirable.

This discussion is dedicated to the study of three different forms of price adjustments flexibility: 1) single-sided price adjustment flexibility (SSPAF). 2) double-sided price adjustment flexibility (DSPAF) with the preemptive right to exercise. 3) DSPAF with the non-preemptive right to exercise. Each was designed to meet various participants flexibility requirements and budgets. An ROs methodology was developed to model, price, and optimize these flexibility clauses. The proposed approach was then tested in the example of Time Charter (TC) rate contracts from the maritime transport industry. Both the metric and the process for quantifying the benefit of the proposed flexibility clauses are provided.

This work provides an alternative approach to the price risk management, which is accessible to all participants in the leasing industry. It is also the starting point in studying the multiple-party, multiple-exercisable price adjustment flexibility. Moreover, both the flexibility designs and the proposed ROs methodology for price risk management are applicable to not only other forms of lease contracts but also to other forms of contract relationships.

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1. INTRODUCTION

1.1. BACKGROUND

Uncertainty and ambiguity are key challenges faced by today's business leaders and decision makers. Uncertainty is a state of having limited knowledge, lack of certainty or being not precisely determined about a future outcome. Uncertainty comes from a great variety of sources including changes in economic and financial policies, demand and market conditions, and the competitive environment. Uncertainty that have some possible undesired outcomes inheres risk. The greater the uncertainty the greater the risk. The recent economic difficulties and global crises have created more uncertainties and risks, while also increasing the challenges to todays businesses. For instance, the uncertainty in the spot market price creates market price risk, and requires appropriate management to control or reduce the risk of adverse price changes in the spot market. Uncertainty, however, encompasses both risks and opportunities. For example, the market price uncertainty presents risks to sellers as the market price may decrease. However, the uncertain environment also presents opportunities, such as the market price increase for sellers. Uncertainty can be managed with flexibility. Flexibility is the ability to change with relative ease [1].

Increasing competition leads to more stringent business environments. Companies and market participants in highly competitive environments find themselves under the pressure to respond to ever-faster changes in market conditions. Companies are forced to look for more flexible tools and forms in their businesses. Flexibility in economic activities and "tailor-made" relations among business participants are necessary to correspond to the specific requirements and adjust to changing market conditions. Companies also need flexibility to both hedge risks and capture opportunities.

Traditional long-term fixed price agreements historically dominated in the business environment. Long-term agreements enable economic actors to coordinate behavior. It also provides participants with a hedge against market price risk. Moreover, in a long-term agreement the seller offers better prices, because seller will have a long period of stable cash flows and less market price risk. Short-term agreements bringes more cost to all participants including re-negotiation cost. Long-term agreements, however, come with some shortcomings. Long-term agreements lock participants in for a the agreement time, and participants are unable to capture opportunities when market prices move in their favor. This inefficiency could be of high price to companies and market participants. For a long time now, companies have been looking for ways to build flexibilities into their contracts. Participants of long-term fixed price agreements such as long-term procurement contracts and lease contracts loaded contracts with different clauses that allows for flexible relationships between contract participants while also transferring different types of risk between the concerned parties. Historically, risk sharing has been one of the main factors influencing contract choice. Loading long-term contracts with different types of flexibility clauses has become an unavoidable trend in today's competitive business environment. Examples of important flexibilities embedded in long-term contracts include the flexibility to renew and to terminate the contract. Such flexibilities create additional value to contracts and allow contract parties to optimize contracts and allocate the risks. According to the pricing theory this flexibility should not be provided for free. Participants are willing to pay for these flexibilities if they were properly priced.

Besides long-term fixed rate agreements, participants in some markets may use financial derivatives to lock the price at a desired level and, thus, hedge the price risk. For instance, in the maritime transport industry participants use Freight

Forward Agreement (FFA) and options on FFAs. However, not all markets have a relevant derivative market. Even if one is available, not all participants are able to use derivatives for various reasons. For example, a desired derivative instrument may not be available when it is needed; not all participants have sufficient knowledge on derivatives or enough cash reserve to participate in the derivative market.

Participants may still have to enter long-term contracts even when market conditions are not favorable. In a such case, without the help of financial derivatives, participants entering long-term contracts could incure considerable losses. Under such circumstances, the flexibility that allows participants to adjust the lease price is attractive to some practitioners. For example, a lessor in a lease contract may gain greater profit if she can either exit the active lease contract or renegotiate the lease price when the spot price rises dramatically. Such flexibility can also help lessees save operating costs in case that the spot price decreases dramatically. Consequently, long-term contracts with the price adjustment flexibility allow the flexibility owner to take advantage of the favorable movement of market price. The price adjustment flexibility provides another alternative for managing the price risk, and is more accessible to participants than financial derivatives. The flexibility become even more valuable in markets that are not associated with financial derivatives.

Leasing is an important type of long-term agreement. The leasing industry is very large. Various sorts of assets can be leased, such as cars and trucks, commercial aircraft and ships, production machinery, industrial equipment (e.g., construction and medical), plants, offshore drilling and satellites. "Lease" is a term that refers to several different kinds of contractual relationships between a lessee and a lessor. Operating leasing separates property ownership from property use where the lessor receives lease payments and the residual property value and the lessee receives the use of the property over the lease term [2]. Under operating lease contracts, lessees require services that are in short term relative to the life of the asset and may be repeated many times, possibly at different locations [3].

Lease contract valuation, and valuation in general focus on determining the price that an investor should pay today in order to obtain the right to receive a specific set of cash flows through a period of time in the future. Particularly, the valuation translates a sequence of risky cash flows into a Net Present Value (NPV). Algebraic models of contract equivalents and Discounted Cash Flow (DCF) methods were traditionally used to value lease contracts. The difficulty in implementing DCF methods in valuing flexible lease contracts is that the anticipated future cash flows to be estimated typically do not properly reflect the flexibility that exists in the lease. In DCF methods the uncertainty is incorporated by discounting risky cash flows at a higher rate. The used discount rate depends on the risk level. The estimated discount rate cannot appropriately represent the risk and may lead to significant error in valuing future cash flows (e.g., [4]). All the above DCF methods are inefficient methods to model and value the price adjustment flexibility for lease contracts that are associated with highly volatile cash flows.

1.2. PROBLEM STATEMENT

This dissertation work aims at addressing the problem of long-term contract price inflexibility, which prevents contract parties from taking advantage of favorable market price movement. Missing this opportunity could deprive the participants of long-term contracts from a substantial increase in income as the price changes dramatically in participants interest. Missing this opportunity could affect the participants competitive position.

The general objective of this work is to create the required flexibility for long-term contractual relationships. A flexibility that enables participants to manage the market price risks that arise from the economic uncertainty while simultaneously enabling the pursuit of opportunities that may arise in the market. Having flexibility in operations compared to that with no flexibility clearly reduces risk. The hypothesis for this work is that price adjustment flexibility embedded in long-term lease contracts would allow the flexibility owner to take advantage of favorable market price changes, thus complementing the functionality of traditional long-term lease contracts.

This dissertation research has the following three specific objectives:

1. modeling the price adjustment flexibility appropriately to obtain in-depth insights into its role;
2. pricing the price adjustment flexibility precisely and quantifying the return from investing in it; and
3. providing a user-friendly tool for optimizing the use of price adjustment flexibility for the flexibility owner.

1.3. POTENTIAL SOLUTIONS

Real options (ROs) are a well-recognized method for modeling and pricing flexibility under highly uncertain conditions (e.g., [5]; [6]; [7] ; [8]). An RO gives the option holder (decision maker) the right, but not the obligation, to take adaptive actions when the future condition is changed or to postpone decisions until more information is available, thereby capturing the essence of flexibility [1]. ROs valuation has traditionally been applied to valuing business investment decisions under uncertainty by taking into account the managerial flexibility. [9, 10] Applications of ROs to the investment and operations have been the focus of much academic research [11]. ROs are a well recognized method for modeling and valuing flexibility (e.g., 5, 6, 7, 8). ROs analysis, in recent years, has gained considerable attention from researchers and been considered in new areas far beyond valuation of projects. After identifying the flexibility and modeling it as an RO, the ROs valuation can be utilized to value the flexibilities beneficial outcome to identify whether the flexibility is worthwhile.

This dissertation, therefore, is motivated to develop a ROs methodology to model, price and optimize the price adjustment flexibility for long-term lease contracts.

1.4. CHALLENGES IN THE REAL OPTIONS APPROACH

ROs "translate" the basic pricing theories of financial options into decision making methods for highly uncertain non-financial domains; therefore, theories, models, and methods of financial options may not be directly used for ROs. Specifically, there are three major challenges presenting in this dissertation research: model complexity, computational issue, and exercise strategy specification.

- *Model Complexity*: Existing models of financial options may not be able to capture the characteristics of ROs. The context of ROs are often more complex than that of the financial options. Modification of existing models of financial options or developing new models is often needed.

- *Computational Issue*: Estimating one participant's decision involves estimating the other participant's decisions. This interdependence between participants' decisions escalates the computational complexity. The price adjustment can be exercised at any time during the contract life. The dynamic nature of the decision further increases the computational complexity.

- *Exercise Strategy Specification*: Much of the ROs literature focuses on the ROs valuation. The way to implement the optimal exercise strategy is often ignored or simplified. Specifically, the process for developing a tool that optimizes the use of ROs is often missing.

Therefore, the development of a ROs methodology for the price risk management requires creative efforts of identifying the flexibility or opportunity for uncertain conditions, building RO models, and probing solution approaches.

1.5. AN OVERVIEW OF THE DISSERTATION RESEARCH

The dissertation is composed of three research essay where the price adjustment flexibility is developed in stages. In the first essay "Valuation of Lease Contracts with a Price Adjustment Option: An Application to The Maritime Transport Industry", the single-sided price adjustment flexibility (SSPAF) for the lessor is modeled as an American call option and that for the lessee is modeled as an American put option. The effect of two variables of flexibility design are studied, including the lock-up period that prohibits the price adjustment very early in the contract life and the time dependent exercise price that diminishes as time passes. The finite difference method (FDM) is developed for pricing the SSPAF and visualizing the optimal exercise strategy. This work has built the fundamentals of modeling, pricing and optimizing the price adjustment flexibility for long-term fixed rate lease contracts.

The second essay "A Real Options Approach to the Modeling and Valuation of Double-Sided Price Adjustment Flexibility with the Preemptive Right to Exercise" is the first attempt to introduce double-sided price adjustment flexibility (DSPAF) to the leasing industry. The DSPAF aims at addressing the dilemma of allowing only one party to have the price adjustment flexibility while both parties of a lease contract want it. Yet one party may enjoy superior flexibility over the counterparty through buying a preemptive right of the flexibility. The essay models the double-sided flexibility as sequentially compounded ROs and provides both contract party the optimal exercise strategy. The proposed flexibility is embedded in Time Charter (TC) contracts from the maritime transport industry to illustrate the effectiveness of it in helping manage the price risk of lease contracts. Numerical experiments are analyzed to determine the best tradeoff between computational complexity and result accuracy.

The third essay "A Real Options Approach to the Modeling and Valuation of Double-Sided Price Adjustment Flexibility with the Non-preemptive Right to Exercise" is dedicated to the study of the non-preemptive right to exercise the price adjustment. The non-preemptive right is defined as an equal, parallel right for both contract parties to adjust the lease price. The DSPAF with the non-preemptive right is expected to meet the flexibility requirements in more cooperative relationships, in situations where both parties have concerns with the random movement of future price yet are unable to predict its trend, and in cases where both contract parties agree on sharing the price adjustment flexibility at the same level. This work provides the mathematical insight into the complexity in modeling and valuation.

The three developed forms of price adjustment flexibility are valuable options for negotiating flexibility clauses for long-term lease contracts. These are designed to meet different participants' price risk management requirements. The suitability depends on the participants' expectation on the market prices, the goal off risk management, the participants' budget for the risk management purpose, and their attitude towards cooperation.

1.6. ANTICIPATED CONTRIBUTION

Expected contributions of this dissertation research are four-fold:

- The dissertation provides an alternative risk management tool that is more accessible to all participants in the leasing industry than financial derivatives.

- It provides a variety of price adjustment flexibility clauses to meet different needs for the flexibility and budget constraints.

- It builds the scientific foundation for studying the multiple-sided, multiple-exercisable price adjustment flexibility (MMPAF) for general contract relationships.

- It contributes to the literature on ROs by pushing the boundaries of RO applications.

2. VALUATION OF LEASE CONTRACTS WITH A PRICE ADJUSTMENT OPTION: AN APPLICATION TO THE MARITIME TRANSPORT INDUSTRY

2.1. INTRODUCTION

Leasing an asset corresponds to purchasing the use of the asset over a fixed period of time. The term “lease” is a generic term that refers to different kinds of contractual relationships between a lessee and a lessor. Lessees under lease contracts require services that are in short term relative to the asset life and may be repeated multiple times, possibly at different locations [3]. The leasing industry is very wide and leasing is an attractive option for many businesses and services that involve capital-intensive assets performing specific functions. Examples of important leasing markets include the real estate market, shipping chartering market, and vehicle leasing market. Each lease market has specific features and dynamics. This essay is presented in the context of maritime transport.

The primary task of the international maritime transport industry is to provide the service of delivering certain cargo from one port to another. The freight rate is the price of shipping service. The service is provided under specific contractual agreements which are the shipping freight contracts. Freight markets in the international bulk shipping industry include the spot market for single voyages and the auxiliary market for period time charters [12].

Freight rates volatility denotes the variability or the dispersion of the freight rates. Volatility is a well-known characteristic of freight markets. Ocean freight rates may change substantially over a short time span. The volatility has not dampened over the years. Since year 2000 ship owners and cargo owners have faced both very high and rather low freight rates [13]. The volatility in freight markets may come from seasonal, cyclical reasons or random shocks [14]. The larger the volatility, the greater chance by which freight rates may deviate from the expectation [15].

The basic theoretical concept in the maritime economic literature states that the shipping price in a competitive freight market is determined by the demand and supply relationship (see, for example, [16].However, the volatility of freight rates observed constitutes a major source of market risk for ship owners, charterers, and other parties involved in the maritime transport industry including shipping hedge funds and shipping banks. For a commodity or energy producer (e.g., a refiner) hiring in vessels, high freight rates increase the production cost and, thus, affect the product price or the production profit. For a ship owner, low freight rates yield less income from hiring out vessels [17, 18]. The combination of huge investments and high uncertainty in freight rates creates a substantial demand for hedging and management of freight rate risk.

Revenues of maritime transport industry followed the booming world trade fairly closely until mid-2008, with the Clarksea index of freight rates reaching a peak of 47,567 at the end of year 2007 (The Clarksea index is a weighted average index of freight earnings for the three major vessel types: bulker, tanker, and container vessels). However, as the global financial crisis spread and deepened in 2008, the index dropped almost 85%, from its peak to a low of 8,025 in April 2009. Extreme changes in revenues, operating cash flows, and asset values during the recent financial crisis have upset the usual means of financing shipping companies. Risk management has become a critical task for shipping companies in this new environment [19].

The objective of hedging is to control or reduce the risk of adverse price changes in the spot market [20]. One available tool of hedging freight rate risk is freight derivatives, including Freight Forward Agreement (FFA) and options on FFAs. However, the use of FFAs is not always coming flawless. The effectiveness of FFAs hedge depends on the liquidity of specific routes and the accuracy of forward assessments. The use of freight derivatives requires qualified and well-educated personnel who are familiar with the derivative market. Besides, a large amount of cash should be sufficiently reserved in case of loss in the FFA market. Freight derivatives have become a strong interest of sophisticated investors who are not active in physical shipping markets [19]. However, the use of freight derivatives by ship owners and charterers is limited according to different surveys (see, for example, 21, 22, 23).

The freight rate risk has traditionally been managed by Time Charter (TC) contracts [24]. TC contracts reduce the exposure to the spot freight market during the entire life of the contract. Furthermore, it is well known that industrial charterers use TC contracts to meet most of their long-term transportation requirements and use spot contracts for extra needs, which might be seasonal or cyclical [25]. The TC rate is less volatile than the spot rate as the latter is more exposed to the day-to-day market condition than the former. Under a TC contract, the charterer hires in a vessel for a specified time period at a pre-determined rate (i.e. a TC rate) and locks in the future rate, accordingly. The hire period ranges from several weeks to 15 years, during which the charterer has the operational control of the ship and the ship owner receives fixed income from hiring out the ship. The TC market tends to be peripheral to the spot freight market; it also share many similarities with futures markets in that the TC rate reflects the expectation of future spot rate [26].

TC contracts, however, present certain limitations. A TC contract commits the two parties of the contract to a fixed rate for the contract term. The committed TC rate after some time could be far from the prevailing TC rate as the spot market moves up or down, squeezing one party of the contract in an unfavorable contractual situation. Consider a ship owner hiring out a vessel for a period of time under certain TC contract rate. If the freight rate in the spot market rises while the ship owner is still paid at the low TC rate, she losses the opportunity of gaining greater income if the ship were working in the spot market or hired out at the higher current TC rate. Similarly, if the spot rate goes down, the charterer, under the TC contract, pays more than she would if she were working in the spot market or at a more recent TC rate. [27, 28]. [29] argued that a major contention of lease contracts in the maritime transport market is the penalty for pre-terminating the lease or higher price potentially paid by the lessee.

In practice, as an attempt to resolve the above-mentioned problem, a clause may be included in a TC contract, which gives one party the right to extend the TC contract term or the right to renegotiate the TC rate. In some cases one party of a shipping contract, in order to get out of unfavorable situations, would put pressure on the other party to change the contract rate although a relevant clause is not included in the original contract. These extension and renegotiation flexibilities if included in the contract are practically given for free, without a fair compensation to the flexibility maker. Consequently, this could let one party benefit from the clause while hurts the other party. If these flexibilities were included in a TC contract at certain costs, an appropriate valuation of the flexibilities is an issue for the party paying for these flexibilities. The limitation of TC contracts critically affects the viability of TC contracts as a risk management tool and explains why many charterers and ship owners avoid entering TC contracts for extended periods. This necessitates the needs for quantifying the benefits of rate adjustment flexibility and determining the best way to use it.

It is well known that real options can be best used to build flexibility arrangement into systems under uncertain environments. The concept of real options is originated from financial options. A real option gives the option holder a right, yet not an obligation, to undertake an action in the future at a predetermined price, which is termed the "exercise price" (e.g. 9, 10). The highly volatile environment of maritime transport industry is a suitable context for real options applications. Moreover, real options share similarities with the flexibility in TC rate adjustment in volatile freight markets. Therefore, the author is motivated to model the flexibility in TC rate adjustment as a real option embedded in the TC contract system. Real options valuation is used to help determine the fair price of the flexibility and provide the best strategy of utilizing the flexibility.

The proposed approach improves the hedging effectiveness of traditional TC contracts in an unprecedented manner. It is not envisioned to let TC contracts substitute other risk management tools such as freight derivatives. Instead, it is proposed as another alternative tool that practitioners can directly use and customize for various conditions.

The rest of this essay is organized as follows. The following section summarizes the relevant literature. Then, TC contracts with a flexibility to adjust the contract rate are modeled and valued, followed by numerical examples and result analysis. Findings from this research and potential future research are summarized at the end.

2.2. LITERATURE REVIEW

The breadth of relevant literature for the current problem includes theoretical studies on the freight rate dynamics and the applications of real options valuation to the shipping industry, which are summarized as follows.

2.2.1. Models of Freight Rates Dynamics. Modeling the dynamics of spot freight rate is the basic building block of any pricing applications in shipping economics. Starting with [30], the functioning of freight markets and modeling of spot freight rate for bulk shipping have been the topic of much research in maritime economics [31]. The classical research in maritime economics attempts to model the spot rate in a structural model setting (e.g., 16, 30, 32) where the rate is determined by the demand-supply equilibrium. However, recent research, inspired by advancements in financial economics, suggests the use of some well-known stochastic models. Major models of freight rate processes used in the literature are briefly summarized as follows.

In early stages, [33], [34], and [9] suggested that the spot freight rate follows a Geometric Brownian Motion (GBM) process. However, [35] argued that there is no evidence supporting the use of GBM model for freight rates. [36] proposed to model the spot freight rate as an Ornstein-Uhlenbeck (OU) mean reverting process, identical to the famous interest rate dynamic process in [37]. This process has been widely used in the commodity market [38]. In the shipping freight literature, the OU model has been applied and/or supported by many studies including [33], [36], [39], [40], [41], [42], [43], and [44]. [35] used a different form of mean reverting process named Geometric Mean Reversion process (GMR). [39] used a Mean Reversion with Absorption level (MRA) process to model the freight rate dynamics.

Lately, new trends of freight rate modeling are emerging. For example, non-parametric specification models that take account of non-linearity in freight rates [42, 45], and the model of spot freight rate in a stochastic partial equilibrium framework [31]. Moreover, there exist a substantive number of other useful models capable of formulating the volatility and variance of time series data, such as the Autoregressive Conditional Heteroskedasticity (ARCH), Generalized ARCH (GARCH), and Exponential GARCH (EGARCH) (e.g., 18, 46, 47). Empirical research into freight rate dynamics, however, has found strong evidence to support mean reverting process models [44].

2.2.2. Real Options Valuation in Shipping. In the shipping literature, the applications of real options to the investment and operations have been the focus of much academic research [11]. For instance, [35] valued Very Large Crude Carrier (VLCC) and focused on lay-up and scrapping problems using two alternative spot freight rate processes; [48] developed a real options valuation model to value the flexibility in switching between wet and dry bulk shipping markets; [49] built an entry-exit model for the switchover between dry bulk and tanker markets (see [50] for more examples); and [51] and [52] analyzed strategies for managing general lease contracts with purchase and exit options. [53] discussed real options applications to logistics and transportation; and [54] valued a truck transportation option. These two studies are also relevant to the research of this essay.

Although there exist a considerable number of studies applying real options to the shipping investment and operations, applications of real options to the design and management of shipping contracts are very limited. [36] is a pioneer in applying real options to the valuation of TC contracts. The authors modeled the freight rate as a mean reverting process and valued a European option to extend a TC contract with contingent claim analysis. In a more advanced work, [43] proposed a valuation method for TC contracts with built-in Bermudan purchase options on chartered ships and developed a new two-factor stochastic model. [44] analyzed and priced TC contracts with extension and purchase options. They formulated the stochastic spot freight rate as the single-factor model presented in [36].

The literature review reveals that real option applications to TC contracts were mainly focused on the valuation of TC contracts with an option to purchase the ship or to extend the contract term. To the best of our knowledge there has been no research that attempted to solve the genuine problem of TC contracts, that is the risk associated with the long term commitment to a fixed rate in very volatile shipping markets.

2.3. THE MODEL

This essay assumes that the spot rate market is arbitrage-free and the term structure of interest rate is flat. The model of real options valuation is built on the general option pricing theory. Major symbols used in the modeling are listed in the Nomenclature in Appendices.

2.3.1. Valuation of Time Charter Contracts. For simplicity and tractability this research has chosen to adopt the one-factor model in [36], which assumes the spot freight rate follows an OU process. The model is commonly used in the literature and receives support from empirical studies. The dynamics of the spot freight rate, $X(t)$, is modeled by the following stochastic differential equation (SDE):

$$dX(t) = k(\alpha - X(t))dt + \sigma dZ(t), \tag{2.1}$$

Where k is the reverting speed, α is the constant long term rate, σ is the instantaneous volatility of the spot freight rate, and $Z(t)$ is a one-dimensional standard Wiener process. Given a time series data of $X(t)$, the model parameters can be fitted through, for example, the regression analysis in the Appendices. The stochastic process followed by the spot freight rate is transformed from the actual probability measure to an equivalent Martingale measure ($*$), becoming

$$dX(t) = k(\alpha^* - X(t))dt + \sigma dZ(t)^*, \tag{2.2}$$

where Z^* is the standard Wiener process under the equivalent Martingale measure and the long term rate under the Martingale measure, α^*, is given by

$$\alpha^* = \alpha - \frac{\sigma\lambda}{k}. \tag{2.3}$$

λ in Eqn. (2.3) is the market price of risk. A method of estimating λ has been discussed in [55] and applied by [56]. The instantaneous cash flow generated by a ship can be calculated as

$$D(t)dt = (aX(t) - b)dt, \tag{2.4}$$

where a is the size of the cargo, which is equal to one whenever the freight rate is quoted for the whole ship. b is the total cost flow rate. The spot freight rate in this section is modeled as the Time Charter Equivalent (TCE) spot rate (dollar/day) rather than the spot freight rate (Worldscale or dollar/ton) itself. The TCE rate is the income of a ship on a daily basis less voyage related costs with bunkers, harbor and channel charges deducted from the transport income [57]. Since the TCE spot rate already includes voyage expenses, b only accounts for operating expenses and capital costs. b is considered as a constant in this section because operating expenses and capital costs are fairly stable in the maritime transport industry [44].

The risk-neutral expected value of the continuous cash flows from a spot rate contract that starts at time t and will mature at time T is defined as

$$\hat{E}_t\left[\int_t^T e^{-rs}D(s)ds\right] = aA(T-t, r+k)X(t) - B(T-t, r, k), \tag{2.5}$$

where A is the annuity value factor defined as

$$A(t, r) = \left[1 - \exp\left(-rt\right)\right]/r, \tag{2.6}$$

and the term B is calculated as

$$B(t, r, k) = a\alpha^* A(t, r+k) - (a\alpha^* - b)A(t, r). \tag{2.7}$$

r in Eqns. (2.5,2.7) is the risk-free interest rate, assumed constant.

Now consider a TC contract written at time t, where the charterer has the right to operate the ship from time t through T. The charterer pays a fixed rate, $F(X(t), t)$, and the value of the lease cost is calculated as

$$\hat{E}_t\left[\int_t^T e^{-rs}F(t, T)ds\right] = F(X(t), t)A(T-t, r). \tag{2.8}$$

According to the non-arbitrage assumption, the value of the TC contract is equal to that of the spot rate contract within the same time frame. Therefore, the TC rate, $F(X(t), t)$, is

$$F(t,T) = \frac{1}{A(T-t,r)}\left[aA(T-t,r+k)X(t) - B(T-t,r,k)\right]. \qquad (2.9)$$

As the TC contract is originally written at time 0, the initial TC rate, $F(X(0), 0)$, is

$$F(0,T) = \frac{1}{A(T,r)}\left[aA(T,r+k)X(0) - B(T,r,k)\right]. \qquad (2.10)$$

Equation (2.9) shows that $F(X(t), t)$ is positively correlated to $X(t)$, meaning that an increase of the spot freight rate would lead to an increase of the TC rate and vice versa. The essay exploits this feature to design and value the suggested real options.

2.3.2. Design of TC Contracts With Rate Adjustment Flexibility.

This essay models a TC contract that allows the ship owner or the charterer to adjust the TC rate for once during the contract life. The new rate is the prevailing TC rate at the time of rate adjustment, which would be applied to the remaining life of the contract. The major design variables for TC contracts with rate adjustment flexibility are the following:

- A lock-up period, starting at time zero and ending at t_L, during which the original freight rate is not adjustable.
- A predetermined cost, K, that the option holder pays at the the time of adjusting the contracted TC rate to the prevailing TC rate.

A TC contract with such a structure provides the rate adjustment flexibility at a reasonable cost (i.e., buying an insurance on the risks she has concerns) and mimics the actual practice in shipping contracts. The choice of these two design variables impacts the option price (i.e., the premium of the insurance).

The TC contract structure is further illustrated in Fig. 2.1. The lock-up period starts at time zero and ends at time t_L ($t_L < T$). A reasonable lock-up period may help reduce the cost of flexibility without giving up major benefits of flexibility. This is because the evolution of spot rate takes time and, therefore, the chance of using the flexibility early in the contract life is relatively small. The longer the lock-up period, the lower the flexibility and option price. The adjustment cost, K, can be used to control the difficulty in option exercise, so the option price. We choose K to be equal to a constant percentage of the remaining cash flows of the original contract:

$$K(t) = hA(T - t, r)F(X(0), 0), \tag{2.11}$$

where h ($h \geq 0$) is the proportional factor for determining the adjustment cost. Equation (2.11) indicates that the option holder will pay a higher adjustment cost early in the contract life because the price adjustment impacts a large portion of cash flows of the original TC contract. With time elapses, less cash flows are affected by the price adjustment; therefore, the option holder will pay a lower adjustment cost if she exercises the option late in the contract life. Moreover, the formula of $K(t)$ in Eqn. (2.11) indicates that the time value of money is taken care of too when determining the adjustment cost . We further explain practical aspects of call and put options as follows.

In oreder to illustrates the call option modeling. Consider a ship owner who enters a TC contract at a certain rate, $F(X(0), 0)$, and yet has concerns with the increase in freight rates in the future. The ship owner does not like to be locked in a TC contract that may produce a significantly lower income than that she would get if she were working in the spot market or entering a more recent TC contract. Ship owner can request to embed a call option to the TC contract that gives her the right but not the obligation to adjust the freight rate for once at any time t ($t_L < t \leq T$)

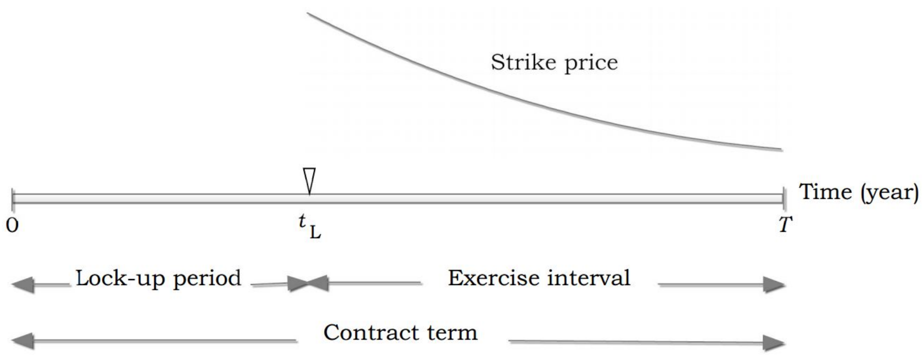


Figure 2.1: Contract structure

to the prevailing TC rate, $F(X(t),t)$, at the predetermined adjustment cost, $K(t)$. This case is analogous to buying back the original freight service from the charterer (i.e., terminating the contract) and then selling a new service at the prevailing price (i.e., entering a new contract). This option also shares similarity with the financial exchange option with floating exercise price. The intrinsic value of exercising the option at time t is $A(T-t,r)[F(X(t),t)-(1+h)F(X(0),0)]$. The exercise decision is an optimal stopping time problem where the option holder will continually compare the TC rate she is already receiving to the prevailing TC rate to determine the best time to adjust the TC rate (i.e., exercise the call option). Like in any American option valuation, the optimal exercise decision at any point of time is determined by the maximum between the intrinsic value of immediate exercise and the expected continuation value (i.e., the value of the risk-free portfolio that contains the option and the underlying asset). Therefore, the value of the American call option embedded in this TC contract, $C(X(t),t)$, satisfies

$$C(X(t),t) \geq A(T-t,r)\max\{F(X(t),t)-(1+h)F(X(0),0),0\}, \qquad (2.12)$$

where the prevailing TC rate for the remaining life of the contract, $F(X(t), t)$, is a function of the spot rate, $X(t)$, and time, t, which is defined in Eqn. (2.9). The adjustment cost factor, h, effectively controls the exercise price of the call option. The greater the adjustment factor, the more difficult the option exercise and the lower the option price.

In oreder to illustrates the put option modeling. Consider a charterer who has concerns with over-payment of freight cost if the freight rate decreases in the future. Charterer can request to embed a put option to the TC contact, which gives her the right to adjust the TC rate for once at any time t ($t_L < t \leq T$) to the prevailing TC rate, $F(X(t), t)$, at the predetermined adjustment cost, K. This case is analogous to selling the original freight service back to the ship owner (i.e, terminating the contract) and then buying a new service from the ship owner at the prevailing price (i.e., entering a new contract). The intrinsic value of exercising the put option at time t is $A(T-t, r)[(1-h)F(X(0), 0) - F(X(t), t)]$. The value of the put option, $P(X(t), t)$, satisfies

$$P(X(t), t) \geq A(T-t, r) \max\{(1-h)F(X(0), 0) - F(X(t), t), 0\}. \tag{2.13}$$

The adjustment cost factor, h, effectively controls the exercise price of the put option. The greater the adjustment factor, the more difficult the option exercise and the lower the option price.

2.3.3. The Options Valuation. The inequality for valuing the options is given by the following:

$$\frac{\partial V}{\partial t} + k(\alpha^* - X)\frac{\partial V}{\partial X} + \frac{1}{2}\sigma^2 \frac{\partial^2 V}{\partial X^2} - rV \leq 0, \tag{2.14}$$

where $V \triangleq V(X(t), t)$ is the contingent claim value, that is, $C(X(t), t)$ defined in Eqn. (2.12) or $P(X(t), t)$ in Eqn. (2.13). If the option exercise is optimal at time t,

$V(X(t), t)$ is equal to the intrinsic value of exercising the option and the inequality is strict. Otherwise, $V(X(t), t)$ is greater than the intrinsic value of option exercise and the inequality is an equality. The option value at maturity is equal to zero because the value of TC contract diminishes over and, finally, reduces to zero at the end of the contract term,

$$V(X(T), T) = 0. \tag{2.15}$$

While there is no closed form solution to the above derived inequality (very few optimal stopping problems allow for the derivation of closed form solutions), it is solved numerically. This section uses the finite difference method (FDM) where a numerical solution to the the partial differential equation (PDE) in (2.14) can be found by converting the PDE to a set of finite difference equations [58].

Numerical procedures of the FDM are discussed below and the pseudo code in Appendices further delineates the procedures. The FDM starts with building a two-dimensional (2D) grid for approximating the spaces of spot rate, $X(t)$, and time, t. The grid is determined by choosing proper values for the minimum value of $X(t)$, X_{min}, the maximum value of $X(t)$, X_{max}, the step size of $X(t)$, ΔX, and the step size of time t, Δt. M denotes the number of steps on the dimension of $X(t)$ and it is equal to $(X_{max} - X_{min})/\Delta X$; therefore, $X(t)$ on the grid is approximated by

$$X_j = X_{min} + j\Delta X, \quad \text{for } j = 0, 1, \ldots, M. \tag{2.16}$$

N is the number of steps on the dimension of t, and it is equal to $T/\Delta t$. t on the grid is approximated by

$$t_i = i\Delta t, \quad \text{for } i = 0, 1, \ldots, N. \tag{2.17}$$

t, $X(t)$ and $V(X(t),t)$ in the PDE of (2.14) are replaced by t_i, X_j and $V_{i,j}$, respectively. Then, the portfolio value, $V_{i,j}$, is determined backwards (i.e., $i = N-1, N-2, \ldots, 0$). Boundary conditions, which need to be first determined at each time step, require special attention. The section has adopted the method introduced by [59] and implemented by [44] in order to overcome the problem of PDE being "convection dominated" for mean reverting processes. The method uses an explicit approximation for the option value at the boundaries of $X(t)$ instead of directly defining it. On the lower boundary of $X(t)$ (i.e., when $j = 0$), the portfolio value, $V_{i,0}$, is determined by

$$V_{i,0} = \gamma_{0,0}V_{i+1,0} + \gamma_{0,1}V_{i+1,1} + \gamma_{0,2}V_{i+1,2}, \tag{2.18}$$

where

$$\begin{aligned}
\gamma_{0,0} &= 1 + r\Delta t - \frac{1}{2}\left[\frac{\sigma^2\Delta t}{(\Delta X)^2} - \frac{3k(\alpha^* - X_{min})\Delta t}{\Delta X}\right], \\
\gamma_{0,1} &= \frac{\sigma^2\Delta t}{(\Delta X)^2} - \frac{2k(\alpha^* - X_{min})\Delta t}{\Delta X}, \\
\gamma_{0,2} &= -\frac{1}{2}\left[\frac{\sigma^2\Delta t}{(\Delta X)^2} + \frac{k(\alpha^* - X_{min})\Delta t}{\Delta X}\right].
\end{aligned} \tag{2.19}$$

On the upper boundary of $X(t)$ (i.e., when $j = M$), the portfolio value, $V_{i,M}$, is determined by

$$V_{i,M} = \gamma_{M,M-2}V_{i+1,M-2} + \gamma_{M,M-1}V_{i+1,M-1} + \gamma_{M,M}V_{i+1,M}, \tag{2.20}$$

where

$$\begin{aligned}
\gamma_{M,M-2} &= -\frac{1}{2}\left[\frac{\sigma^2\Delta t}{(\Delta X)^2} + \frac{k(\alpha^* - X_{max})\Delta t}{\Delta X}\right], \\
\gamma_{M,M-1} &= \frac{\sigma^2\Delta t}{(\Delta X)^2} + \frac{2k(\alpha^* - X_{max})\Delta t}{\Delta X}, \\
\gamma_{M,M} &= 1 + r\Delta t - \frac{1}{2}\left[\frac{\sigma^2\Delta t}{(\Delta X)^2} - \frac{3k(\alpha^* - X_{max})\Delta t}{\Delta X}\right].
\end{aligned} \tag{2.21}$$

For interior points of the grid (i.e., $j = 1, 2, \ldots, M-1$), an implicit method is used to determine $V_{i,j}$, which involves solving a system of linear equations:

$$\gamma_{j,j-1}V_{i,j-1} + \gamma_{j,j}V_{i,j} + \gamma_{j,j+1}V_{i,j+1} = V_{i+1,j}, \tag{2.22}$$

where

$$\begin{aligned}
\gamma_{j,j-1} &= -\frac{1}{2}\left[\frac{\sigma^2\Delta t}{(\Delta X)^2} - \frac{k(\alpha^* - X_j)\Delta t}{\Delta X}\right], \\
\gamma_{j,j} &= 1 + \frac{\sigma^2\Delta t}{(\Delta X)^2} + r\Delta t, \\
\gamma_{j,j+1} &= -\frac{1}{2}\left[\frac{\sigma^2\Delta t}{(\Delta X)^2} + \frac{k(\alpha^* - X_j)\Delta t}{\Delta X}\right].
\end{aligned} \tag{2.23}$$

If t_i is greater than t_L, V_{ij} obtained from Eqns. (2.18), (2.20), and (2.22) is compared to the intrinsic value of option, $G_{i,j}$, to examine the optimality of immediate exercise. Finally, the option price is obtained, equal to $V_{0,\tilde{j}}$ where $\tilde{j}$ is the index of $X(0)$ (i.e., $\tilde{j} = (X(0) - X_{min})/\Delta X$).

2.4. RESULT ANALYSIS

This section presents numerical examples for analyzing the rate adjustment flexibility for TC contracts, the design of flexibility, and the value the flexibility adds to TC contracts. In all the examples, the market price of risk, λ, is assumed zero. The assumption is supported by the work of [56] who found that the market price of risk is close to zero for most levels of freight rate, indicating that ship owners are not compensated for the risk associated with trading in the spot market. A risk free interest rate of 5% per year is used for discounting future cash flows (converted to $\frac{5}{360}$% per day in the valuation). For simplicity, the freight rate is quoted for the whole ship (i.e., $a = 1$) and net of all costs (i.e., $b = 0$) such that $X(t)$ is the instantaneous net cash flow from an operating vessel. In the FDM, the 2D grid is built by choosing

$X_{max} = 130 \times 10^3$ dollar/day, $X_{min} = -60 \times 10^3$ dollar/day, $\Delta X = 500$ dollar/day, and $\Delta t = 30$ days.

2.4.1. Benchmark Case. A benchmark case is first defined, where the TC contract includes an embedded American option to adjust the contracted TC rate to the prevailing TC rate for once at any time during the contract life (i.e., $t_L = 0$) with no adjustment cost (i.e., $h = 0$). The term of contract, T, is equal to five years (converted to 5×360 days in the valuation) in the benchmark case. The long term rate under the equivalent Martingale measure, α^*, is 40×10^3 dollar/day. Volatility, σ, is 3×10^3 dollar/day$^{3/2}$. The reverting speed, k, is 1 per year (converted to $\frac{1}{360}$ per day in the valuation).

Figure 2.2 illustrates results from the benchmark case. The plot on the upper left of the figure displays the call option value, $C(X(t), t)$. The initial spot rate, $X(0)$, is equal to 20×10^3 dollar/day, and the original TC rate, $F(X(0), 0)$, is equal to 35.717×10^3 dollar/day. Accordingly, the value of the 5-year TC contract without the rate adjustment flexibility is $\$56.884 \times 10^6$. The price of this call option, $C(X(0), 0)$, which measures the expected value that the rate adjustment flexibility adds to the TC contract, is found to be $\$13.810 \times 10^6$. That is, the flexibility is expected to add 24.28% of total value to the TC contract in the benchmark case. The plot on the upper right of Fig. 2.2 illustrates the free boundary of option exercise along with a random sample path of spot rate. The first time the sample path crosses the free boundary is 2.62 years. The ship owner (who is also the owner of the call option) exercises the option at that time and receives the prevailing TC rate, $F(2.62, 5)$, equal to 55.372×10^3 dollar/day for the remaining life of the contract.

Similarly, the plot on the bottom left of Fig. 2.2 displays the put option value, $P(X(t), t)$. The original TC rate, $X(0)$, is equal to 60×10^3 dollar/day, and the original TC rate, $F(0, 5)$, is equal to 44.283×10^3 dollar/day. Therefore, the value of the TC contract without the rate adjustment flexibility is $\$70.527 \times 10^6$. The price of the put option embedded in the contract is found to be $\$13.814 \times 10^6$, indicating that the flexibility is expected to increase the contract value by 19.59%. The plot on the bottom right shows the free boundary of this put option and a random sample path of spot rate. The charterer exercises the option when the sample path crosses the free boundary the first time, at 2.58 years, and pays the prevailing TC rate, $F(2.58, 5)$, equal to 25.178×10^3 dollar/day for the remaining life of the contract.

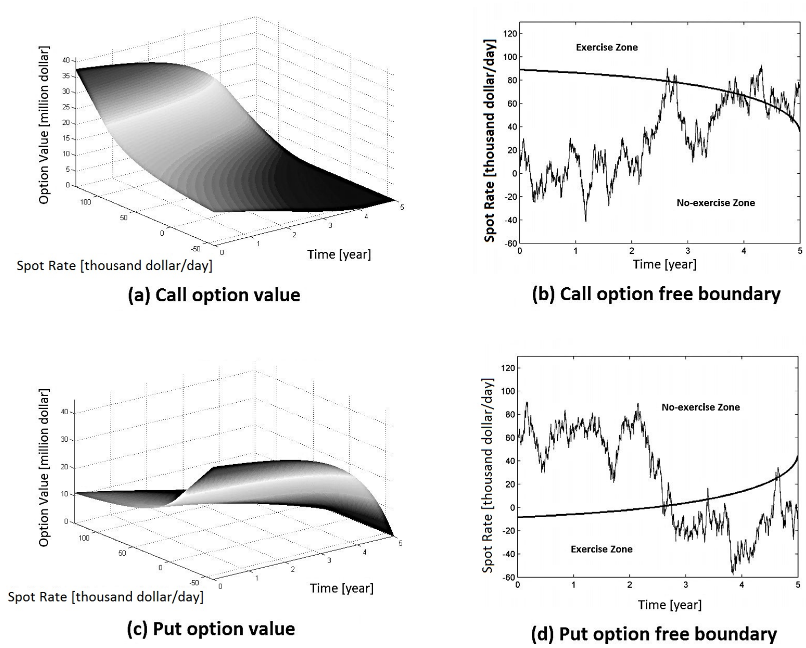
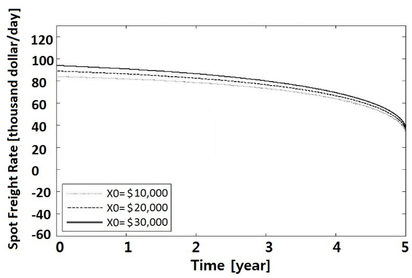


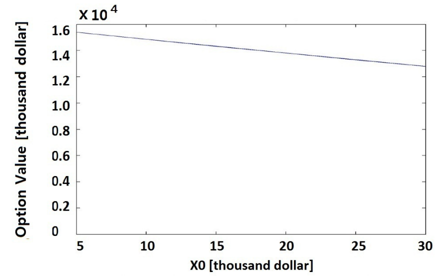
Figure 2.2: Option prices and free boundaries in the benchmark case

The sensitivity analysis of the call and put options to the spot rate model examines the reliability of conclusions from the benchmark case across a broader range of scenarios. Figures 2.4 and 2.3 illustrate how the option prices and free boundaries react to changes in model parameters ($X(0)$, k, σ, α^* and T). Results of the sensitivity analysis are also summarized in Tables 2.1 and 2.2 in rough approximation in that the level of impact is indicated by a number of plus and minus signs.

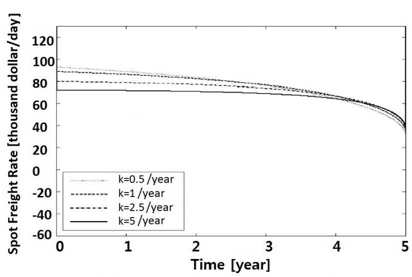
Main observations concluded from the sensitivity analysis are as follows:



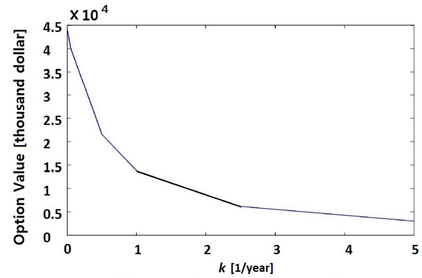
(a) Free Boundary vs $X(0)$



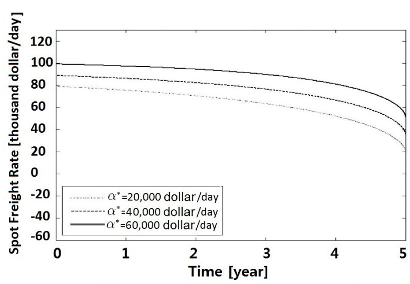
(b) Option Value vs $X(0)$



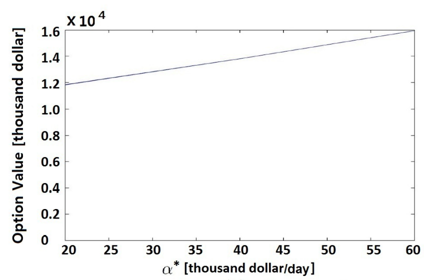
(c) Free Boundary vs k



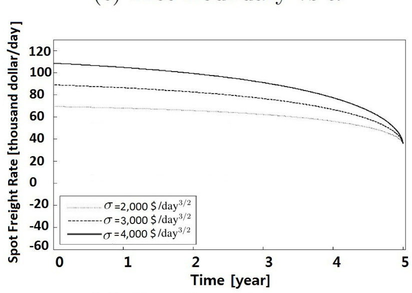
(d) Option Value vs k



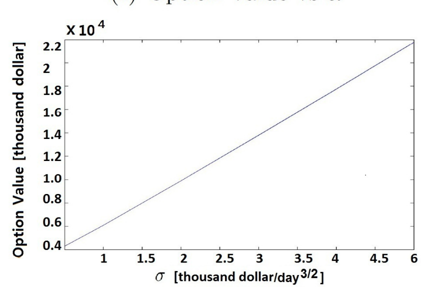
(e) Free Boundary vs α^*



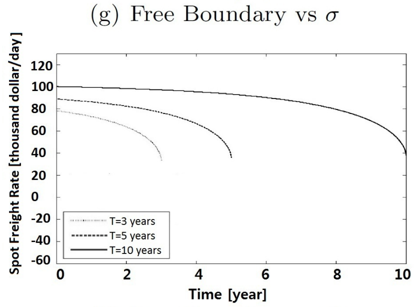
(f) Option Value vs α^*



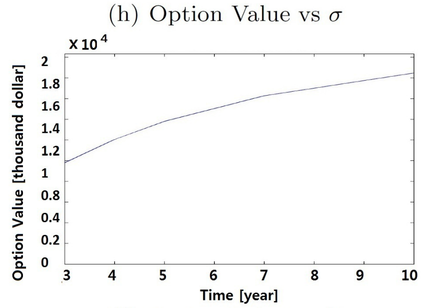
(g) Free Boundary vs σ



(h) Option Value vs σ

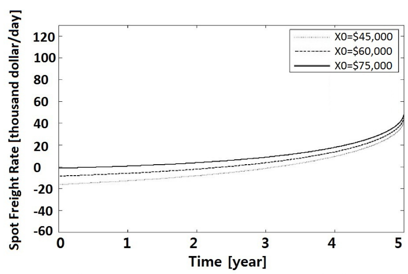


(i) Free Boundary vs T

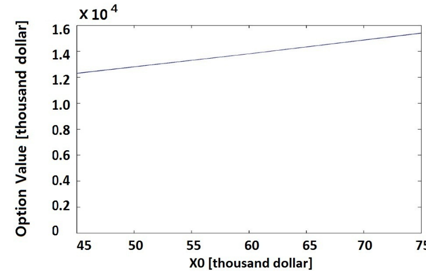


(j) Option Value vs T

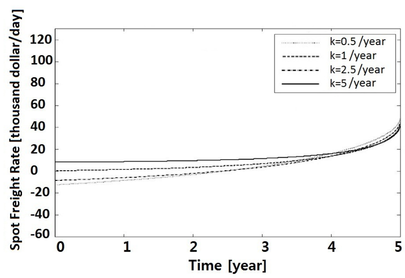
Figure 2.3: Sensitivity of call option price and free boundary to the parameters of spot rate model in the benchmark case



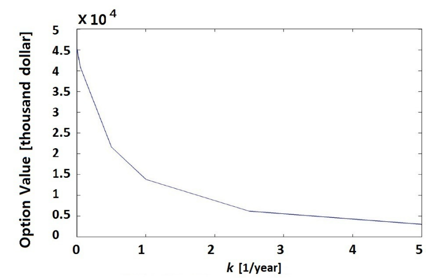
(a) Free Boundary vs $X(0)$



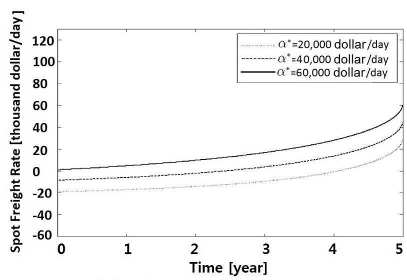
(b) Option Value vs $X(0)$



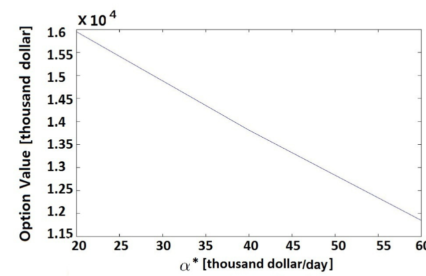
(c) Free Boundary vs k



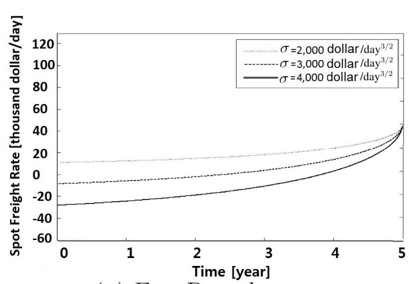
(d) Option Value vs k



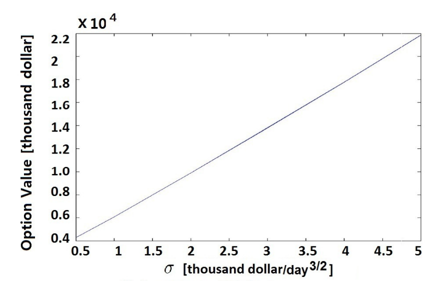
(e) Free Boundary vs α^*



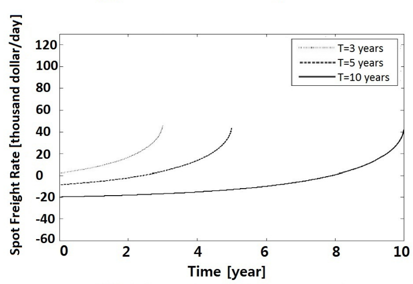
(f) Option Value vs α^*



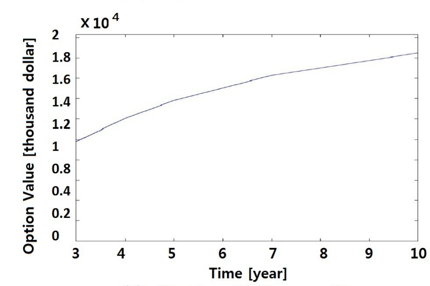
(g) Free Boundary vs σ



(h) Option Value vs σ



(i) Free Boundary vs T



(j) Option Value vs T

Figure 2.4: The sensitivity of put option price and free boundary to the parameters of spot rate model in the benchmark case

Table 2.1: Sensitivity of option prices to the spot rate model

Basic Parameter	Call	Put
k	- - - -	- - - -
σ	+++	+++
T	++	++
$X(0)$	-	+
α^*	+	-

- The increase in the reverting speed, k, has a significant, negative effect on both the call and put option prices. This is understandable because a high value of k restricts the variability of the spot rate and thus lowers the price of options.

- The effects of σ and T come next in importance, and both positively impact the call and put prices. This means the greater the volatility and the longer the contract life, the higher the option prices. These are typical features of options, generally.

- $X(0)$ and α^* have less impacts on option prices. The effect of $X(0)$ is obvious. The increase in $X(0)$ increases $F(X(0), 0)$, which increases the put option value and decreases the call option value (see Eqns. (2.12) and (2.13)). The effect of α^* is not obvious. The increase in α^* reduces the negative value of $B(t, r, k)$, which increases $F(X(0), 0)$ and $F(X(t), t)$ in different ways. In conclusion α^* has a positive impact on the call option price and yet a negative impact on the put option price.

Meanwhile, main observations regarding the sensitivity of exercise zone to the spot rate model in the benchmark case are as follows:

- The increase of σ or T value has a significant, negative impact on the exercise zone. A larger volatility or a longer maturity is associated with a greater chance for the freight rate to deviate from the original expectation. Therefore, an

Table 2.2: Sensitivity of exercise zone to the spot rate model

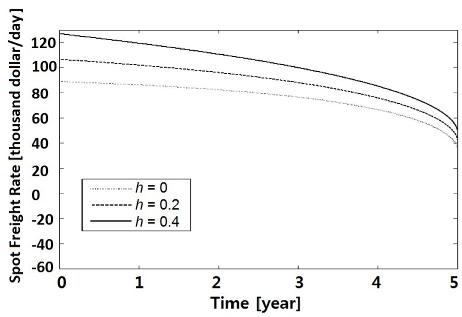
Basic Parameter	Call	Put
k	+	+
σ	- -	- -
T	- -	- -
$X(0)$	-	+
α^*	-	+

increase of σ or T value moves the free boundaries outwards and decreases the exercise zone.

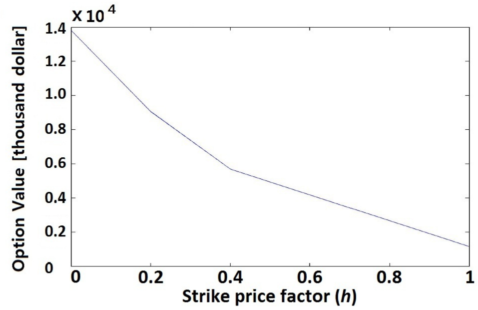
- $X(0)$ and α^* work the same way and have less effect on the exercise zone. A large initial spot rate or long term rate reduces the exercise zone for the call option and yet increases the exercise zone for the put option. This is due to the fact that a larger value of $X(0)$ or α^* makes it easier and more difficult to exercise the put and call option, respectively.

- the increase of k value expands the exercise zone due to the fact that the variability of spot rate is reduced by larger k.

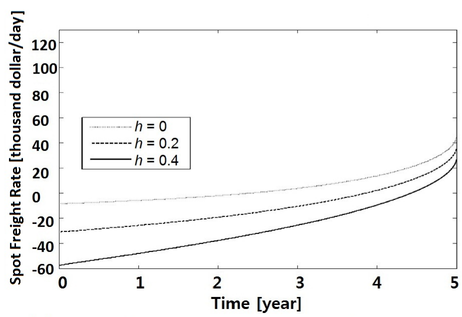
2.4.2. Effectiveness of the Option Design for TC Contracts. A different case is consider further to examine the effectiveness of the option design for TC contacts. In this new case, the option holder has the right to adjust the contracted TC rate to the prevailing TC rate for once after an agreed lock-up period, $[0, t_L]$, at a predetermined adjustment cost, $K(t)$. The option holder pays a premium (i.e., the option price) to obtain the rate adjustment flexibility at the beginning of the contract; option owner will pay the adjustment cost if she exercises the option during the contract term. Some parties favor a low premium because it is all they would lose if they do not exercise the option. The lock-up period and adjustment cost can be tailored to offer desired levels of flexibility and prices.



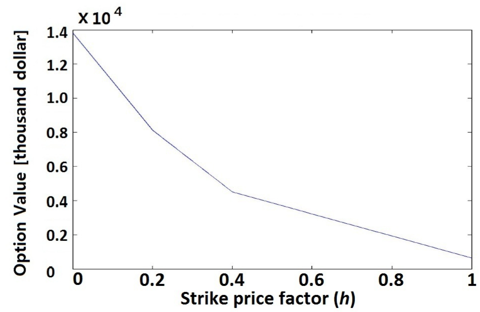
(a) Call Option Free Boundary vs K



(b) Call Option Value vs K

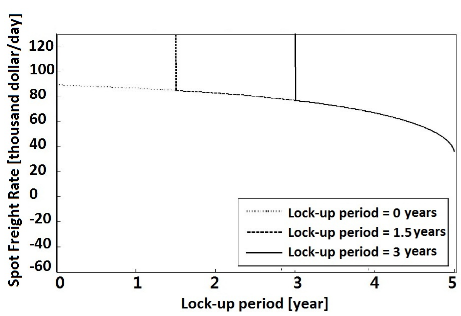


(c) Put Option Free Boundary vs K

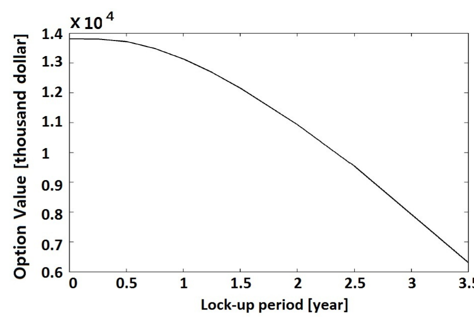


(d) Put Option Value vs K

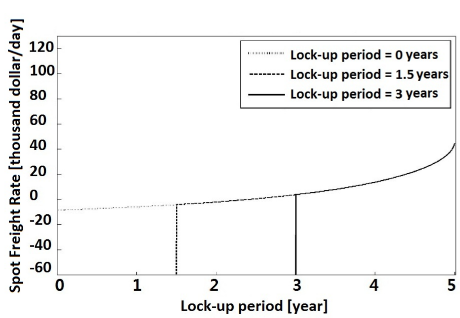
Figure 2.5: Sensitivity of option prices and free boundaries to the adjustment cost, k



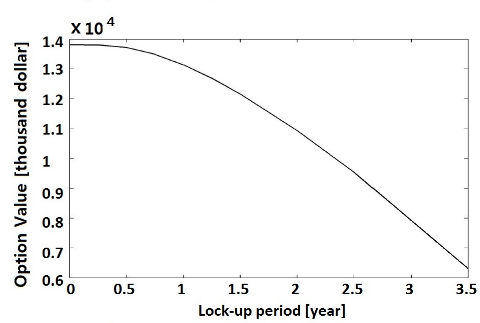
(a) Call Option Free Boundary vs t_L



(b) Call Option Value vs t_L



(c) Put Option Free Boundary vs t_L



(d) Put Option Value vs t_L

Figure 2.6: Sensitivity of option prices and free boundaries to the lock-up period, t_L

Figure 2.5 illustrates the option prices and free boundaries at different levels of adjustment cost, and Fig. 2.6 illustrates how the option prices and free boundaries are changed by the lock-up period. Major observations concluded from the sensitivity analysis are as follows:

- The lock-up period reduces the flexibility simply by prohibiting the option exercise early in the contract term, whereas the adjustment cost reduces the flexibility by pushing the free boundaries away from the long term rate (i.e., making it more difficult to exercise the options).
- Including a lock-up period and/or an adjustment cost can reduce the option price to be paid upfront, making the embedded options more attractive to entities with a limited budget for the adjustment flexibility.
- The adjustment cost is more effective than the lock-up period in adjusting option prices.

2.4.3. Discounted and Premium TC Contracts. The option price, instead of being paid upfront, can also be in the form of periodic payments equivalent to a freight rate discount given to a charterer (in case of the call option) or a freight rate premium paid to a ship owner (in case of the put option). The percentage of discount or premium also measures the expected contract added value from the rate adjustment flexibility. Tables 2.3 and 2.4 calculate the percentage of TC rate discount and the percentage of premium, respectively.

Table 2.3 shows that the maximum expected value that the rate adjustment flexibility (for the ship owner) can add to the TC contract for is 24.28% (the benchmark case). The increases in the lock-up period and the adjustment cost reduce the flexibility, and the discount percentage. For example, the ship owner, who buys a call option with a lock-up period of 1.5 years and an adjustment cost equal to 20% of the value of remaining contract cash flows, will give the charterer a 14.62% discount on the regular TC rate throughout the 5-year contract term. If the ship owner feels that the discount given to the charterer is too large, ship owner may choose to increase the adjustment cost, for example, to 40% of the value of remaining contract cash flows, which reduces the discount percentage to 9.56%. Similarly, Table 2.4 indicates the maximum expected value that the rate adjustment flexibility (for the charterer) can add to the TC contract is 19.59% (the benchmark case). By increasing the lock-up period and the adjustment cost, the value added by the flexibility and the premium paid to the ship owner is reduced. For example, the charterer, who buys a put option with the same lock-up period ($t_L = 1.5$ years) and adjustment cost ($h = 0.2$), will pay the ship owner a 10.96% premium over the regular TC rate throughout the 5-year contract term. The premium would be reduced to 6.20% if the charterer accepts an adjustment cost equal to 40% of the value of remaining contract cash flows.

Tables 2.3 and 2.4 both confirm that the lock-up period is less capable of adjusting option price than the adjustment cost would do, particularly when h is large. For instance, the discount and premium rates are not affected by the change in lock-up period when h exceeds 0.8. The reason is revealed by looking at Figs. 2.5 and 2.6. The plots on the left of Fig. 2.5 illustrates that the increase in h quickly eliminates the possibility of exercising the options early in the contract life, which invalids the adjustment capability of lock-up period illustrated in the left plots of Fig. 2.6. Therefore, the lock-up period is more effective in controlling option prices for scenarios of lower adjustment cost.

Tables 2.3 and 2.4 provide a convenient tool of contract negotiation to both the ship owner and charterer in that it facilitates the accomplishment of various levels of risk management targets and accommodates different budgets for the flexibility. Moreover, after all terms of the TC contract are negotiated and determined by the two parties of a TC contract, the corresponding free boundary of option exercise can be used by the option holder as a tool of contract risk management.

Table 2.3: The call option price as a TC rate discount (%) to the charterer

	t_L								
h	0	0.25	0.5	0.75	1.0	1.25	1.5	2.0	2.5
0	24.28	24.27	24.11	23.72	23.10	22.30	21.38	19.22	16.77
0.1	19.76	19.76	19.68	19.43	19.02	18.46	17.79	16.17	14.25
0.2	15.90	15.90	15.87	15.73	15.46	15.09	14.62	13.45	12.00
0.3	12.67	12.67	12.65	12.58	12.43	12.20	11.88	11.06	10.00
0.4	10.01	10.01	10.01	09.97	09.89	09.75	09.56	09.01	08.25
0.6	06.08	06.08	06.08	06.08	06.08	06.05	06.01	05.81	05.47
0.8	03.52	03.52	03.52	03.52	03.52	03.52	03.52	03.52	03.48
1.0	02.03	02.03	02.03	02.03	02.03	02.03	02.03	02.03	02.03

Table 2.4: The put option price as a TC rate premium (%) paid to the ship owner

	t_L								
h	0	0.25	0.5	0.75	1.0	1.25	1.5	2.0	2.5
0	19.59	19.58	19.45	19.13	18.63	17.99	17.25	15.51	13.53
0.1	15.15	15.15	15.09	14.92	14.62	14.21	13.70	12.49	11.04
0.2	11.52	11.52	11.50	11.41	11.25	11.01	10.96	09.89	08.87
0.3	08.63	08.63	08.63	08.59	08.51	08.38	08.20	07.70	07.03
0.4	06.40	06.40	06.40	06.39	06.36	06.29	06.20	05.91	05.48
0.6	03.42	03.42	03.42	03.42	03.42	03.42	03.41	03.35	03.21
0.8	01.77	01.77	01.77	01.77	01.77	01.77	01.77	01.77	01.77
1.0	00.91	00.91	00.91	00.91	00.91	00.91	00.91	00.91	00.91

2.4.4. Illustrative Examples. Consider an example in the same setting as the benchmark case except that it has a lock-up period of 1.5 year and an adjustment cost equal to 20% of the remaining contracted cash flows. A ship owner, who expects the TC rate to increase in the future and worries about a possible loss from that price change, buys the rate adjustment flexibility (i.e., a call option). Consequently, the ship owner has the right to adjust the original TC rate to the prevailing TC rate for once any time after the lock-up period at the agreed adjustment cost. The call option price is $\$8.318 \times 10^6$, equivalent to a 14.62% discount of the TC contract rate to the charterer throughout the 5-year contract life. The free boundary of option exercise and two random sample paths of spot rate are displayed in Fig. 2.7. The first random path of spot rate crosses the free boundary the first time at 1.592 years when the rate increases to about 99×10^3 dollar/day. The option holder (i.e., the ship owner) exercises the option at that point of time. The option holder, who used to receive the contracted TC rate equal to 35.717×10^3 dollar/day, pays an adjustment cost of $\$8.059 \times 10^6$ to exercise the option and starts receiving a new TC rate, $F(1.592, 5)$, equal to 57.419×10^3 dollar/day. The gross gain the option holder receives from exercising the option and adjusting the freight rate for the remaining contract life is $\$15.168 \times 10^6$. The option holder makes a net gain (after deducting the call option price) from the call option, equal to $\$6.849 \times 10^6$ (all the gain values are discounted to time zero at the risk-free rate). In case that the spot rate develops in contrary to the ship owner's expectation and never increases high enough to go across the free boundary during the contract life, just like the second random sample path of spot rate, the call option will expire and never be exercised. The option holder then loses the amount of $\$8.318 \times 10^6$ she paid for the flexibility of rate adjustment (or equivalently the 14.62% discount she gave to the charterer). The TC contract, however, guarantees the ship owner the contracted TC rate regardless of how low the spot freight rate goes.

Similarly, consider a charterer who enters a similar TC contract when the spot rate is 60×10^3 dollar/day and expects the TC rate to decrease in the future. The charterer would like to hedge against the risk of decreasing TC rate; therefore, she buys the flexibility (i.e., a put option) of adjusting the contracted TC rate to the prevailing TC rate any time after the lock-up period of 1.5 years at the adjustment cost equal to 20% of the remaining contract cash flows. Fig. 2.8 illustrates the free boundary of this put option and two random sample paths of spot rate. In this example, the put option price is $\$7.541 \times 10^6$, equivalent to paying a premium of 10.96% over the TC rate. The option then is exercised at 2.344 years when the spot rate of the first random sample drops to -16.500×10^3 dollar/day and crosses the

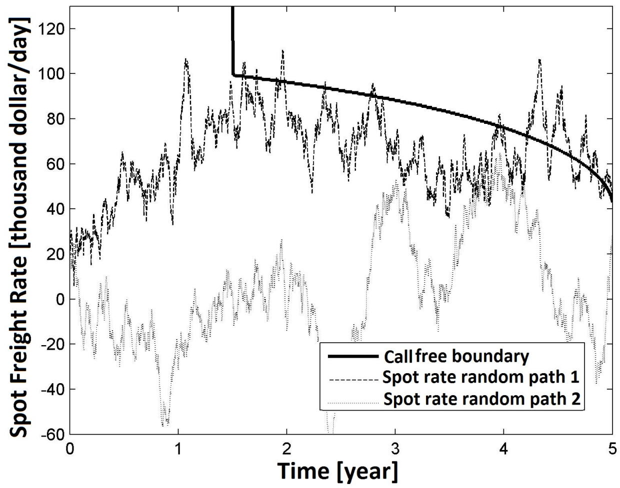


Figure 2.7: The free boundary of call option and two random sample paths of spot rate

free boundary the first time. The freight rate here can be a negative value as it is represented by instantaneous net cash flow. The negative value implies that the spot rate drops below costs. The adjustment cost is $\$7.937 \times 10^6$. Upon option exercise, the TC rate is adjusted from the original TC rate of 44.283×10^3 dollar/day to the new TC rate of 19.709×10^3 dollar/day. The net gain the put option holder (the charterer) receives is $\$4.973 \times 10^6$. Again, in case that the spot rate never drops low enough to cross the free boundary of option exercise, just like the second random sample path, the option holder then loses the amount of $\$7.541 \times 10^6$ she paid for the put option. Anyhow, the TC contract guarantees the charterer the contracted TC rate when the rate is high.

2.5. CONCLUSIONS AND FUTURE RESEARCH

A fixed rate contact may lock one party of the contract in an unfavorable condition for a long period of time because the market rate may change dramatically.

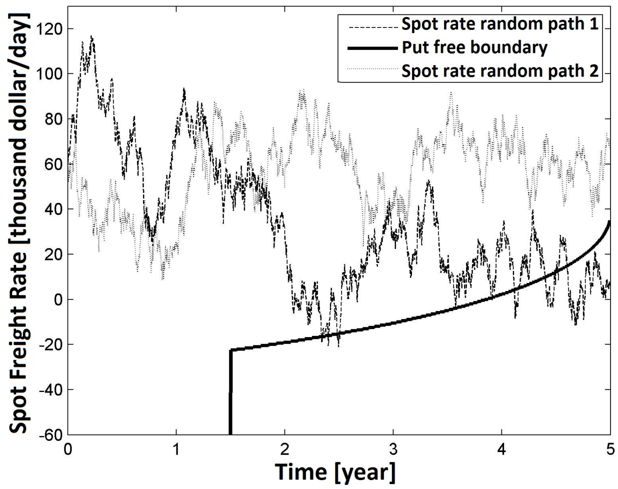


Figure 2.8: The free boundary of put option and two random sample paths of spot rate

The use of freight derivatives to hedge the freight rate risk requires special knowledge and experiences with derivatives and inherits certain limitations that affect the hedging effectiveness. This essay exploited an alternative approach - embedding a flexibility to lease contracts, which allows for adjusting the contracted fixed rate when it is far away from the prevailing rate. The essay modeled the flexibility as real options, quantified the price of the flexibility (i.e., the expected value of the flexibility), and provided a decision management tool. The way of tailoring the flexibility level and the price of it was also discussed in the contract design.

Contributions of this essay are two-fold. Firstly, the real options model serves as a tool of lease contract management. The model can be easily set up as a simple, user friendly tool with GUI (e.g., a web based tool). Ship owners, charterers, and brokers can directly use the tool for contracting and risk management without going to the hassle of financial options. The option model and valuation can also be implemented by any other type of leases if the spot leasing rate follows an OU process. Secondly, the study presented in this essay enriches real options theories and applications. Real options are not about simply applying the financial derivative theories and methods to non-financial domains. Great efforts of real options lie in a "domain translation". This work serves as a good starting point of modeling flexibilities needed by fixed rate lease contracts in a volatile market environment.

More importantly, this work initiates the discussion of a series of topics that are interesting to both researchers in the area of real options and contract managers. For instance, multiple factor models may be implemented to include other risk components such as uncertainties in the interest rate (r). Geometric mean reverting (GMR) process is an alternative model for freight rate dynamics, and the performance of it is worth of further assessment. One possible extension of current work may involve in the study of futures options on lease contracts. Other forms of flexibility are more practical and attractive, which call for further analysis. For instance, while this essay has revealed that the flexibility in rate adjustment should not be given for free, ship owners and charterers are still reluctant to pay for the flexibility, particularly in cash. One possibility is to pay an in-kind adjustment cost, for example, the option holder when exercising her option provides a rate adjustment option to the other party. A more attractive practice is to let both parties of a lease contract have equal, parallel flexibility. The pricing and exercising strategies of these flexibilities are challenging and subject to further research.

3. REAL OPTIONS MODELING AND VALUATION OF DOUBLE-SIDED PRICE ADJUSTMENT FLEXIBILITY WITH THE PREEMPTIVE RIGHT TO EXERCISE

3.1. INTRODUCTION

The leasing industry is very wide. Various sorts of equipment can be leased, such as cars and trucks, commercial aircraft and ships, production machinery, industrial equipment (e.g., construction and medical), plants, offshore drilling and satellites. “Lease” is a term referring to several different kinds of contractual relationships between a lessee and a lessor. Under lease contracts, operators (lessees) require services that are in short term relative to the life of the asset and may be repeated many times, possibly at different locations [3]. Some leasing markets are highly volatile. The world recent economic crisis created even more sources of uncertainty for the leasing industry. In today’s uncertain business environment, participants of the leasing industry show growing interests in flexibilities that can help improve their competences.

The flexibility in adjusting the lease price complements the risk management capability of fixed rate contracts. Traditional lease contracts began with a simple format of contractual relationships between a lessee and a lessor. This simple format allows the lessor to lock the leasing income over the lease period and the lessee to fix the operating cost related to the leased equipment during that lease period, thus reduce price risk. However, uncertainty not only poses risks but also offer opportunities. While fixed rate lease contracts help stabilize cash flows in volatile markets, lease periods can extend long enough to make the lease contract parties miss opportunities of gaining greater profits or saving costs. Clearly, flexibility is valuable to fixed rate contracts. For example, a lessor may gain greater profit if she can either exit the active lease contract or renegotiate the lease price when the spot price rises. Such flexibilities can also help lessees save operating costs in case that the spot price decreases dramatically.

The flexibility in adjusting the lease price is an alternative tool for managing the price risk, and it is more accessible to participants than derivatives. Some leasing markets use financial derivatives to lock the lease price at a desired level. For instance, the maritime transport industry can use Freight Forward Agreement (FFA) and options on FFAs. However, not all leasing markets have a relevant derivative market. Even if a derivative market is associated with the leasing market, not all participants are able to use derivatives for various reasons. For example, a desired derivative instrument may not be available when it is in need; not all participants have sufficient knowledge on derivatives or enough cash reserve to participate in the derivative market. Without the help of derivatives, participants may have to enter lease contracts when lease market prices are not favorable. For example, lessors usually hesitate to idle the leasing assets because they need lease incomes to pay down the bank loan or the capital investment in leased assets. Therefore, they may still enter a lease contract although the market condition is unfavorable. Under such circumstances, the flexibility that allows them to adjust the lease price when the market returns favorable is attractive.

Unsurprisingly, loading lease contracts with flexibility clauses has become an unavoidable trend in today's competitive business environment. This essay proposes to provide both parties of a lease contract the flexibility in adjusting the lease price, termed the *double-sided flexibility*. It is an indispensable option for the negotiation of flexibility clauses because both parties of a lease contract may all want to have the flexibility under some circumstances. For example, if the lease price at the contract underwriting can go either way with nontrivial chances, both parties may want to have the flexibility. The double-sided flexibility can address the dilemma of allowing only one party to have the flexibility yet both parties want it. The two parties of a lease contract may have either different budgets on acquiring the flexibility or different expectations on the market. This requires offering the flexibility at unequal levels. The party who wants superior flexibility over the counterparty can purchase the *preemptive right* of the flexibility. Preemptive right flexibility defines the relationship in which a flexibility belongs solely to, and is used by, one party before it is granted to the other. In contrast, the *non-preemptive right* defines the relationship as one in which both parties of a contract have an equal, parallel right in adjusting the lease price.

Through modeling, pricing and optimizing the double-sided flexibility constrained by the preemptive right, this work builds a theoretic foundation for promoting the use of price adjustment flexibility as an alternative tool of price risk management. This essay models the proposed flexibility as sequentially compounded real options. The flexibility held by the owner of the preemptive right is defined as the *primary option*; that held by the counterparty is defined as the *secondary option*. These options allow both parties to take advantage of favorable market price changes yet at different levels of superiority. Real Options (ROs) are a well recognized method for modeling and pricing flexibility (e.g., 5, 6, 7, 8). Both the reliability and applicability of the proposed model are tested using Time Charter (TC) contracts, a type of fixed rate lease agreements commonly used by the maritime transport market where the price volatility and the peer competition are notable.

The remainder of this essay is organized in the following manner. Section 3.2 summarizes relevant literature to identify the gap existent in current studies. Section 3.3 both models and values the double-sided price adjustment flexibility constrained by the preemptive right with application to TC contracts. Numerical examples and result analysis are presented in Section 3.4. Findings from this research and future research are summarized at the end in Section 3.5.

3.2. LITERATURE REVIEW

The valuation of lease contracts with flexibility clauses has been a subject drawing significant interest in economic circles for many years [60]. Numerous models proposed in the literature have approached the subject from a variety of perspectives. The literature relevant to this essay is presented in two streams. The first stream includes both theoretical and empirical studies on embedding real options in different lease contracts. The second stream focuses on modeling lease price flexibilities as real options.

3.2.1. Real Options Embedded in Lease Contracts. Leases, particularly real estate leases, have been examined in different approaches and frameworks from the perspective of asset users. The seminal work of [61] viewed leasing as purchasing the right to use an asset over a specific period and suggested an equilibrium model for the valuation of lease contracts. This idea was applied by [62] and [63]. [62] provided an option-like characterization for leasing and introduced a methodology for modeling leases in an option approach. Continuing that work, [63] established a no-arbitrage framework of leasing assets' pricing. Their framework includes a variety of lease options, such as a cancellation option and an option to buy the leased asset at a fixed price or at the market price. [64] theoretically developed leasing flexibility for real estate leases. Grenadier built a continuous-time model and proposed a unified option pricing approach that can be used to price a variety of leasing contracts with different embedded options, such as forward leases, leases with options to either renew or cancel contracts and lease insurance contracts. [65] derived a model that provides a unified framework to the equilibrium valuation of leases subject to default risk. [52] valued complex leasing contracts with a variety of embedded operating options and discussed the interactions among the combined options. [66] used the same equilibrium approach used by [64], however, instead of the monopolistic assumption in a game theoretical approach, his work was built on the competitive market assumption. [67] developed a no-arbitrage based valuation model that calculates the Net Present Value (NPV) of each lease contract taking into account both the contractual payment amounts and any embedded options. [68] proposed an endogenous structural model for the term structure for real estate lease rates. [69] discussed four typical lease contracts and proposed a decomposition and diagram method. They expressed the complex real options as a portfolio of both vanilla options and simple exotic options.

Some research with a focus on specific lease markets have been conducted. [70] developed a game theoretical framework to price lease contracts with options in imperfect leasing markets for durable goods. [71] priced standard automobile leases with both cancellation and purchase options. They also discussed the penalty for early termination of a lease. [72] developed a method for valuing claims on offshore petroleum lease. [3] valued different options on short-term leases for capital-intensive equipment. The asset utilization and idle time were key factors in the proposed methodology. The methodology was illustrated by pricing options for oil-drilling services.

A significant amount of research has been done on applying ROs to lease contracts in the maritime transport industry. [36] applied ROs to the valuation of TC contracts with embedded options. They modeled the spot freight rate as mean reverting process and used the contingent claim analysis to value a European option to extend the TC contract. [43] proposed a valuation method for TC contracts with built-in Bermudan options to purchase chartered ships. [44] analyzed and priced TC contracts with extension and purchase options. [54] attempted to hedge the uncertainties in transportation capacity by creating truckload options.

3.2.2. Options to Adjust Leasing Prices. Several studies have specifically attempted to analyze leasing price flexibilities in the real estate market. The adjustable-rate lease is one type of researched lease contracts. In an adjustable-rate lease the rental rate of real estate is fixed at the lease commencement. The rate can be either periodically reset to the market rate or adjusted according to some pre-specified reference index, such as the Consumer Price Index (CPI) or a real estate index. [73] discussed options embedded in adjustable-rate real estate lease contracts. These contracts give the lessee a right to either renew the lease or purchase the rental property at a certain price. The price is tied to the cumulative change in some index, such as the CPI. [64] also discussed adjustable-rate lease contracts, advocating that adjustable-rate leasing may provide the lessor with a hedge against both unexpected inflations and cost fluctuations.

A special form of adjustable-rate leases is the upward-only adjustable leasing, a common feature of UK commercial leases. [2] presented a stochastic pricing model of upward-only adjustabal leases. As defined in his study, an upward-only adjustable lease includes a fixed rental rate for an extended term at the lease commencement. In addition, however, it gives the lessor the option to periodically adjust the rent to the market rate (every five years). The lessor will only raise the initial rate to the prevailing market level if the market rent the increases. The contract rent remains unchanged, however, if the market rent declines. This upward-only review reduces both the risk and volatility of cash flows from property investments by setting a floor for the investment return. [2] indicated that the initial rent for an upward-only adjustabal lease should be significantly lower than that for a corresponding lease with both upward and downward rent reviews.

Overage is one universal common feature of retail leases in multi-tenanted shopping centers. It is one kind of the Percentage Lease Agreements (PLAs), paying a flat base rent plus a turnover-related income (i.e., the overage). [74] treated the overage rent as a call option on the tenant's sale turnover. They applied the binomial tree model to pricing the option, finding that the option-like feature of the overage rent adds value to retail leases when the sale volatility is high.

All the lease price flexibilities found in the literature are *single-sided flexibility* that gives only one party (usually the lessor) the right to adjust the price at predetermined times, normally every five years with the contract life spanning 15 years or longer. To the best of our knowledge no previous research attempted to price the double-sided flexibility.

3.3. THE MODEL

The modeling of a lease contract depends on the type of the contract being studied, variables underlying the contract, and features of the lease market. This section illustrates both the modeling and the valuation methods for the TC contract in the maritime transport industry. The same methodology can be easily modified and applied to other lease contracts in different lease markets.

3.3.1. Valuation of Operating Lease Contracts. This section models the spot market rate, $X(t)$, as a mean reverting process, a widely applied model in the maritime transportation literature (e.g., 36). In a risk-neutral world, the dynamics of $X(t)$ is defined as

$$dX(t) = k(\alpha^* - X(t))dt + \sigma dZ^*(t), \tag{3.1}$$

where $X(t)$ is represented by the time charter equivalent (TCE) spot freight rate [57], k is the reverting speed, α^* is the long-term steady rate under the risk-neutral measurement, σ is the instantaneous volatility of the spot rate, and $Z^*(t)$ is the one-dimensional standard Wiener process under the risk-neutral measurement. The closed form solution to Eqn.(3.1) is

$$X(t+dt) = (1 - e^{-kdt})\alpha^* + e^{-kdt}X(t) + \sigma\sqrt{\frac{1 - e^{-2kdt}}{2k}}\epsilon, \tag{3.2}$$

where ϵ follows a standard normal distribution.

The instantaneous cash flow generated by an operating vessel is

$$D(t)dt = (aX(t) - b)dt, \tag{3.3}$$

where a is the size of the cargo (which is equal to 1 when a freight is quoted for the entire ship) and b denotes the rate of total cost (including both operating costs and capital expenses).

The value of spot rate contracts during the time period $[t, T]$ is the risk-neutral expected value of the cash flow stream, $\{D(s)|t \leq s \leq T\}$:

$$\begin{aligned} \hat{E}_t \left[\int_t^T e^{-rs} D(s) ds \right] =& A(T-t, r+k) a X(t) \\ & + \left[A(T-t, r) - A(T-t, r+k) \right] \alpha^* - A(T-t, r) b, \end{aligned} \tag{3.4}$$

where $\hat{E}_t$ is the risk-neutral expected value at time t, r is the risk-free interest rate (assumed constant), and A is the annuity value factor.

In a TC contract starting at time t and ending at time T, the lessee of the contract has the right to operate the ship during the time period $[t, T]$ by paying continuously at a fixed rate, $F(X(t), t)$. The equivalent value of the total leasing cost evaluated at time t (i.e., the value of the TC contract) is

$$\hat{E}_t \left[\int_t^T e^{-rs} F(X(t), t) ds \right] = F(X(t), t) A(T-t, r), \tag{3.5}$$

According to the non-arbitrage assumption, the value of the TC contract is equal to that of the spot rate contracts within the same time frame. Therefore, the TC rate, $F(X(t), t)$, is calculated with

$$F(X(t), t) = a \frac{A(T-t, r+k)}{A(T-t, r)} X(t) + \left[1 - \frac{A(T-t, r+k)}{A(T-t, r)} \right] \alpha^* - b. \tag{3.6}$$

Eqn. (3.6) determines that the TC rate for the remaining life of the contract, $F(X(t),t)$, is a function of the spot rate at time t, $X(t)$. This is an important feature we will exploit to design and value the price adjustment flexibilities.

3.3.2. The Real Options Model of Double-Sided Flexibility. This section models the double-sided flexibility in adjusting the contracted TC rate to the prevailing rate as sequentially compounded real options. The primary option held by the owner of the preemptive right can be exercised any time during the contract's life, whereas the secondary option is not allowed to be exercised until the primary option has been exercised. Therefore, giving the secondary option to the counterparty can be seen as paying an in-kind exercise cost (i.e., non-cash cost) for exercising the primary option. The practicalities of the options are explained below.

The following discusses the DSPAF case when the preemptive right is held by the lessor. Consider a lessor in a low market negotiating a new T-year lease contract for the rate $F(X(0),0)$. The lessor expects the market rate to go up in the future during the contract term and is concerned over being locked in an underpaid lease. She wants to hedge against this price risk and, thus, asks to load the lease contract with a preemptive right to adjust the contracted TC rate once during the contract term. This preemptive right is equivalent to an *American call option* that gives her the right, but not the obligation, to exchange the original TC rate, $F(X(0),0)$, for the prevailing rate, $F(X(t),t)$, at any time t ($0 \leq t \leq T$). The exercise cost of this primary call is the right granted to the lessee to re-adjust the TC rate once during the remaining life of the contract. The right of the lessee is equivalent to an *American put option*. The value of the primary call option at any time t is designated by $V^{PC}(X(t),t)$, while $V^{SP}(X(\tau),\tau)$ designates the value of the secondary put option at any time τ ($t \leq \tau \leq T$).

The primary option generates value if the spot rate, $X(t)$, rises high enough. Specifically, the lessor may consider exercising the call option at time t if it has not been exercised prior to t and the payoff from exercising the primary call option at t, $A(T-t,r)[F(X(t),t)-F(X(0),0)]$, is higher than the value of the secondary put option at that time, $V^{SP}(X(t),t)$. The intrinsic value from exercising the primary call at time t, $G^{PC}(X(t),t)$, is defined as

$$G^{PC}(X(t),t)=A(T-t,r)[F(X(t),t)-F(X(0),0)]-V^{SP}(X(t),t). \tag{3.7}$$

Moreover, because the primary call option can be exercised at any time during the contract's life, the value of it must satisfy

$$V^{PC}(X(t),t)\geq\max\{G^{PC}(X(t),t),0\}, \tag{3.8}$$

at any time t $(0\leq t\leq T)$. Since the secondary put is also an American option, its value must satisfy the following inequality during the remaining life of the contract (i.e., $t\leq\tau\leq T$):

$$V^{SP}(X(\tau),\tau)\geq\max\{G^{SP}(X(\tau),\tau),0\} \tag{3.9}$$

where $G^{SP}(X(\tau),\tau)$ is the intrinsic value of exercising the secondary put option at any time τ $(t\leq\tau\leq T)$,

$$G^{SP}(X(\tau),\tau)=A(T-\tau,r)[F(X(t),t)-F(X(\tau),\tau)]. \tag{3.10}$$

Unlike the primary call (which has an in-kind exercise price), the secondary put option is assumed to have a zero exercise price, thus simplifying contract management. The secondary put option is valued only if the spot rate first rises high enough to make the secondary put option come into being and then falls to a low level to make the exercise of the secondary put option valuable.

The following discusses the DSPAF case when the preemptive right is held by the lessee. A lessee who expects the market rate to go down and wants to protect against overpaying for the lease would buy a preemptive right to adjust the contracted TC rate $F(X(0), 0)$. The lessee gets the right to adjust the contract rate at any time t ($0 \leq t \leq T$) to the prevailing rate $F(X(t), t)$ once. This flexibility with the preemptive right is a primary put option for her, and its value at time t is denoted by $V^{PP}(X(t), t)$. The exercise cost of the primary put is a right allowing the lessor to re-adjust the TC rate once during the remaining life of the contract. The right of the lessor is the secondary call option, and the value of it is designated by $V^{SC}(X(\tau), \tau)$ ($t \leq \tau \leq T$). If the primary put option was not exercised prior to time t, the intrinsic value of exercising it at time t is

$$G^{PP}(X(t), t) = A(T - t, r)[F(X(0), 0) - F(X(t), t)] - V^{SC}(X(t), t). \qquad (3.11)$$

Because the value of the primary put option can be exercised at any time, the value of it must satisfy

$$V^{PP}(X(t), t) \geq \max\{G^{PP}(X(t), t), 0\}, \qquad (3.12)$$

at any time t ($0 \leq t \leq T$). The secondary call option must satisfy the following inequality during the remaining life of the contract (i.e., $t \leq \tau \leq T$),

$$V^{SC}(X(\tau), \tau) \geq \max\{G^{SC}(X(\tau), \tau), 0\} \qquad (3.13)$$

where $G^{SC}(X(\tau), \tau)$ is the intrinsic value of exercising the secondary call option at time τ,

$$G^{SC}(X(\tau), \tau) = A(T - \tau, r)[F(X(\tau), \tau) - F(X(t), t)]. \qquad (3.14)$$

Again, the secondary call option has zero exercise cost.

3.3.3. Valuation of Options. The general inequality for valuing the options was derived using the Ito's lemma and standard no-arbitrage argument [58],

$$\frac{\partial V}{\partial s} + k(\alpha^* - X)\frac{\partial V}{\partial X} + \frac{1}{2}\sigma^2\frac{\partial^2 V}{\partial X^2} - rV \leq 0, \tag{3.15}$$

where $V \triangleq V(X(s), s)$ is the contingent claim value of any of the options in Eqns. (3.8), (3.9),(3.12), and(3.13) (i.e., V can be V^{PC}, V^{SP}, V^{PP}, or V^{SC}). Accordingly, s is used to designate either the time variable in valuing the exercise of a primary option, t, or the time variable in valuing the corresponding secondary option, τ. When exercise is optimal, $V(X(s), s)$ is equal to the intrinsic value of exercising the option, $G(X(s), s)$, and the inequality in (3.15) is strict. Otherwise, $V(X(s), s)$ is greater than $G(X(s), s)$ and the inequality becomes an equality.

The derived inequality in (3.15) was solved numerically. This section applies the Finite Difference Method (FDM) to find the solution. With this method the solution can be found by converting the partial differential equation (PDF) in (3.15) to a set of finite difference equations [58].

The FDM procedures begin with building a two-dimensional grid on the spaces of both the spot rate, $X(s)$, and the time, s. This grid is built by appropriately choosing a range of the spot rate, $[X_{min}, X_{max}]$, the step of spot rate, ΔX, and the step of time, Δs. $X(s)$ is approximated on the grid as $X_j = X_{min} + j\Delta X$ for $j = 0, 1, \ldots, M$, where M is equal to $(X_{max} - X_{min})/\Delta X$. t on the grid is represented by $t_i = i\Delta s$ for $i = 0, 1, \ldots, N$, where $N = T/\Delta s$. τ on the grid is $\tau_{i'} = i'\Delta s$ for $i' = i, \ldots, N$. To derive the finite difference equations for determining the option value on the grid, s_i in this section is used to designate both t_i and $\tau_{i'}$. We also replace s, $X(s)$ and $V(X(s), s)$ with s_i, X_j and $V_{i,j}$ in (3.15), respectively. The option value at maturity, $V_{N,j}$, is equal to zero because the contract value is reduced to zero at the end of the contract term. The portfolio value determined by the PDE,

$V_{i,j}$, is then found on the grid backwards (i.e., $i = N-1, N-2, \ldots, 0$). Boundary conditions must first be determined at each time step. This section has adopted an improved FDM introduced by [59] and implemented by [44] to overcome the problem of the PDE being convection-dominated for mean reverting processes. This method uses an explicit approximation for the option value on the boundaries of X_j instead of directly defining it. On the lower boundary of X_j (i.e., $j = 0$), the portfolio value, $V_{i,0}$, is determined by

$$V_{i,0} = \gamma_{0,0} V_{i+1,0} + \gamma_{0,1} V_{i+1,1} + \gamma_{0,2} V_{i+1,2}, \tag{3.16}$$

where

$$\begin{aligned}
\gamma_{0,0} &= 1 + r\Delta s - \frac{1}{2}\left[\frac{\sigma^2 \Delta s}{(\Delta X)^2} - \frac{3k(\alpha^* - X_{min})\Delta s}{\Delta X}\right], \\
\gamma_{0,1} &= \frac{\sigma^2 \Delta s}{(\Delta X)^2} - \frac{2k(\alpha^* - X_{min})\Delta s}{\Delta X}, \\
\gamma_{0,2} &= -\frac{1}{2}\left[\frac{\sigma^2 \Delta s}{(\Delta X)^2} + \frac{k(\alpha^* - X_{min})\Delta s}{\Delta X}\right].
\end{aligned} \tag{3.17}$$

On the upper boundary of X_j (i.e., $j = M$), the portfolio value, $V_{i,M}$, is determined by

$$V_{i,M} = \gamma_{M,M-2} V_{i+1,M-2} + \gamma_{M,M-1} V_{i+1,M-1} + \gamma_{M,M} V_{i+1,M}, \tag{3.18}$$

where

$$\begin{aligned}
\gamma_{M,M-2} &= -\frac{1}{2}\left[\frac{\sigma^2 \Delta s}{(\Delta X)^2} + \frac{k(\alpha^* - X_{max})\Delta s}{\Delta X}\right], \\
\gamma_{M,M-1} &= \frac{\sigma^2 \Delta s}{(\Delta X)^2} + \frac{2k(\alpha^* - X_{max})\Delta s}{\Delta X}, \\
\gamma_{M,M} &= 1 + r\Delta s - \frac{1}{2}\left[\frac{\sigma^2 \Delta s}{(\Delta X)^2} - \frac{3k(\alpha^* - X_{max})\Delta s}{\Delta X}\right].
\end{aligned} \tag{3.19}$$

An implicit FDM is used to determine the option value at interior points of the grid (i.e., $j = 1, 2, \ldots, M-1$). This method involves solving a system of linear equations:

$$\gamma_{j,j-1} V_{i,j-1} + \gamma_{i,j} V_{i,j} + \gamma_{j,j+1} V_{i,j+1} = V_{i+1,j}, \text{ for } j = 1, 2, \ldots, M-1 \tag{3.20}$$

where

$$\gamma_{j,j-1} = -\frac{1}{2}\left[\frac{\sigma^2\Delta s}{(\Delta X)^2} - \frac{k(\alpha^* - X_j)\Delta s}{\Delta X}\right],$$
$$\gamma_{j,j} = 1 + \frac{\sigma^2\Delta s}{(\Delta X)^2} + r\Delta s, \tag{3.21}$$
$$\gamma_{j,j+1} = -\frac{1}{2}\left[\frac{\sigma^2\Delta s}{(\Delta X)^2} + \frac{k(\alpha^* - X_j)\Delta s}{\Delta X}\right].$$

$V_{i,j}$ obtained from Eqns.(3.16), (3.18), and (3.20) is compared to the intrinsic value of the option exercise, $G_{i,j}$, to examine the optimality of immediate exercise. The option value is determined, accordingly, by

$$V_{i,j} = \max\{V_{i,j}, G_{i,j}\}. \tag{3.22}$$

Thus, the option price is finally obtained. For a primary option, the option price is equal to $V_{0,j'}$, where j' is the index of $X(0)$ (i.e., $j' = (X(0) - X_{min})/\Delta X$). For a secondary option, the option price at any time t is equal to $V_{i,\tilde{j}'}$ where $i = t\Delta s$ is the index of time t and $\tilde{j}' = (X(t) - X_{min})/\Delta X$ is the index of $X(t)$.

The pseudo code of the algorithm for valuing a primary option is listed in Table 3.1, and that for valuing a secondary option is listed in Table 3.2. The pseudo codes illustrate that the exercise decision for the primary option at any time t and any possible spot rate at that time $X(t)$ involves valuing the corresponding secondary option at that state. Consequently, the computational time is driven up substantially.

3.4. RESULT ANALYSIS

3.4.1. Numerical Examples. A numerical example of TC contracts is discussed here to illustrate both the cost and benefit of the preemptive right of price adjustment flexibility. The cost paid by the holder of preemptive right is determined

Table 3.1: Pseudo Code of the Algorithm for Valuing a Primary Option V^P

0. Build a 2D grid of size $N \times M$:
   $N \leftarrow T/\Delta s$,
   $M \leftarrow (X_{max} - X_{min})/\Delta X$.
1. Define terminal condition: $V^P_{i,j} \leftarrow 0$ for $i = N$ and any j.
2. Value the option backward:
   for $i = N-1, N-2, \ldots, 0$
      Determine the expected value of continuation, $V^P_{i,j}$:
         for boundary points ($j = 0, M$), use the explicit method in Eqns. (3.16-3.19);
         for interior points ($j = 1, \ldots, M-1$), use the implicit method in Eqns. (3.20-3.21).
      Determine the intrinsic value of exercising the option, $G_{i,j}$:
         if $V^P_{i,j} = V^{PC}_{i,j}$
            $G_{i,j} \leftarrow G^{PC}(j\Delta X, i\Delta s)$ defined in Eqn. (3.7),
         else
            $G_{i,j} \leftarrow G^{PP}(j\Delta X, i\Delta s)$ defined in Eqn. (3.11).
         end.
      Determine the value of primary option:
         $V^P_{i,j} \leftarrow \max\{V^P_{i,j}, G_{i,j}\}$.
   end.
3. The value of the primary option at time zero is $V^P_{0,\tilde{j}}$, where $\tilde{j} \leftarrow (X(0) - X_{min})/\Delta X$.

Table 3.2: Pseudo Code of the Algorithm for Valuing a Seconary Option V^S

0. Build a 2D grid of size $(N-i) \times M$:
   $N \leftarrow T/\Delta s$,
   $M \leftarrow (X_{max} - X_{min})/\Delta X$,
   $i \leftarrow t/\Delta s$.
1. Define terminal condition: $V^S_{i',j} \leftarrow 0$ for $i' = N$ and any j.
2. Value the option backward:
   for $i' = N-1, N-2, \ldots, i$
      Determine the expected value of continuation, $V^S_{i',j}$:
         for boundary points ($j = 0, M$), use the explicit method in Eqns. (3.16-3.19);
         for interior points ($j = 1, \ldots, M-1$), use the implicit method in Eqns. (3.20-3.21).
      Determine the intrinsic value of exercising the option, $G_{i',j}$:
         if $V^S_{i',j} = V^{SC}_{i',j}$
            $G_{i',j} \leftarrow G^{SC}(j\Delta X, i'\Delta s)$ defined in Eqn. (3.14),
         else
            $G_{i',j} \leftarrow G^{SP}(j\Delta X, i'\Delta s)$ defined in Eqn. (3.10).
         end.
      Determine the value of the secondary option:
         $V^S_{i',j} \leftarrow \max\{V^S_{i',j}, G_{i',j}\}$.
   end.
3. The value of secondary option at time t is $V^S_{i,\tilde{j}'}$, where $\tilde{j}' \leftarrow (X(t) - X_{min})/\Delta x$.

by the primary option's value; the return from the preemptive right is random, depending on the evolution of spot rate in the market. There are three possible scenarios of option exercise:

- *Double Exercises (2E) Scenario*: The market rate is favorable for the primary option holder and, thus, the option holder exercises the option. Later, the market rate reverses enough so that the secondary option is exercised too.

- *Single Exercise (1E) Scenario*: The market rate is favorable for the primary option holder and, thus, the holder exercises the option. The market rate, however, never reverses enough for the secondary option holder to exercise her option.

- *No Exercise (0E) Scenario*: The market rate is unfavorable for the primary option holder and, thus, she does not exercise her option, killing the secondary option (although the market rate may go in favor of the secondary option holder).

Consider a 5-year TC contract (one year = 360 days) written in the maritime transportation market. The risk-neutral, long-term steady rate, α^*, is 40×10^3 dollar/day, the spot rate volatility, σ, is 3×10^3 dollar/day$^{3/2}$, and the reverting speed of the spot rate, k, is 1 per year. Values of spot rate model parameters were determined based on practical data available in the literature [44]. The continuously compounded risk-free rate, r, was assumed fixed at 5% per year. We built an appropriate grid for the FDM using $X_{max} = \$130 \times 10^3$ dollar/day, $X_{min} = -\$60 \times 10^3$ dollar/day, $\Delta X = 1 \times 10^3$ dollar/day, and $\Delta s = 10$ days.

The following is an examples of the preemptive right held by the lessor. At an initial spot rate of 20×10^3 dollar/day, the original TC contract rate, $F(20 \times 10^3, 0)$, is 35.717×10^3 dollar/day. The lessor (ship owner) in a TC contract expects the spot rate to go up because the current rate is significantly lower than the long-term steady rate of 40×10^3 dollar/day. This owner wants to protect against a potential future earnings loss (from having hired out the ship for a price lower than the market price) and thus buys a preemptive right to adjust the contracted TC rate to the prevailing rate once during the contract's life. The cost to obtain this right is the value of the primary call option, $V^{PC}(X(0), 0)$, which is equal to $\$6.337 \times 10^6$. Because the value of the lease contract is $\$56.884 \times 10^6$, the preemptive right of the price adjustment flexibility is expected to add 11.1% value to the contract. Table 3.3 summarizes the example results.

The free boundary of an option is an optimized tool for supporting the exercise decision. It divides the state space into two zones: "exercising the option" zone and "holding the option" zone. The option is exercised when the spot rate crosses the free boundary the first time. Fig. 3.1 illustrates the free boundary of primary call option, three random sample paths of the spot rate (with each corresponding to one of the three scenarios), and two free boundaries of the secondary put option (with one for the 2E scenario and the other for the 1E scenario).

Table 3.3: Examples of The Preemptive Right Held by the Lessor

$X(0)$, [thousand dollar/day]	20.000		
$dX(t) = (40000 - X(t))dt + 3000dZ^*(t)$			
$F(X(0), 0)$, [thousand dollar/day]	35.717		
Value of TC contract w/o options, [million dollar]	56.884		
Price of the preemptive right, [million dollar]	6.337		
	Examples		
	2E	1E	0E
Exercise time of the primary call, t_P, [year]	0.555	2.744	N/A
Prevailing rate,$F(X(t_P), t_P)$, [thousand dollar/day]	50.243	53.665	N/A
Gain from the option exercise [million dollar]	20.843	13.782	N/A
Exercise time of the secondary put, t_S, [year]	3.194	N/A	N/A
Prevailing rate,$F(X(t_S), t_S)$, [thousand dollar/day]	29.088	N/A	N/A
Loss from the option exercise [million dollar]	13.151	N/A	N/A
Net gain (loss) [million dollar]	2.725	5.678	-6.337
Net gain (loss)/Original contract value	4.8%	10.0%	-11.1%
Net return from preemptive right	43.0%	89.6%	-100%

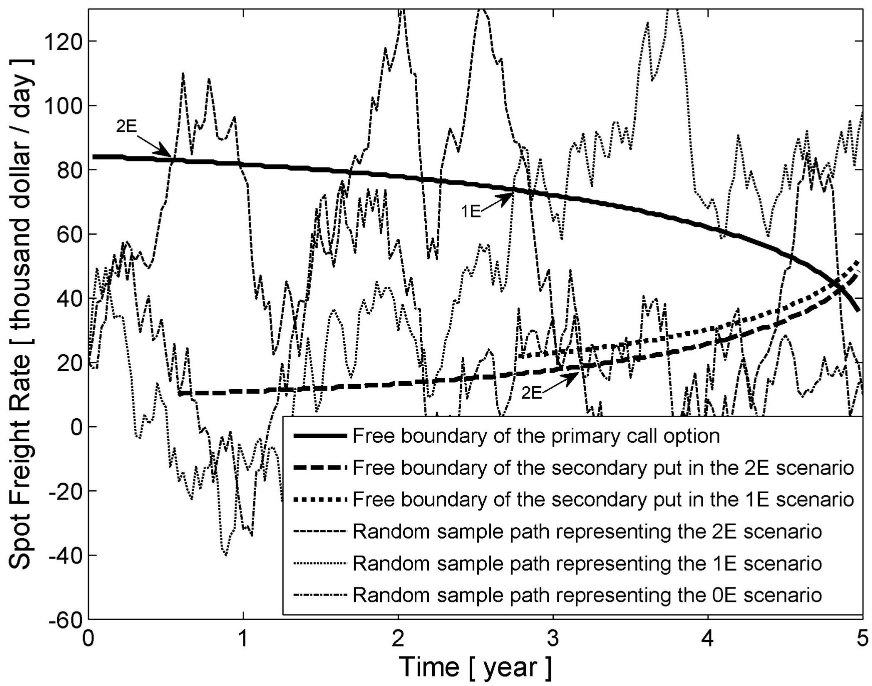


Figure 3.1: Primary Call - Secondary Put: An Illustrative Example

The random path displayed as dash curve in Fig. 3.1 illustrates the 2E scenario. The spot rate goes up to 83.500×10^3 dollar/day and crosses the free boundary of the primary call option (the bold solid curve) at time 0.555 years. The call option is then exercised and the TC rate is adjusted to the prevailing rate, $F(83.500 \times 10^3, 0.555)$. This rate is equal to 50.243×10^3 dollar/day. The exercise of the primary call option increases the income cash flows for the lessor, equivalent to $\$20.743 \times 10^6$ at time 0.555 years. The exercise of the primary call option provides a secondary put option to the lessee. The value of the secondary put option is worth $\$15.074 \times 10^6$ (assessed at time 0.555 years). The corresponding free boundary of the secondary option is the bold dash curve illustrated in Fig. 3.1. The spot rate reverses down to 16.500×10^3 dollar/day and crosses the free boundary of the secondary put option at time 3.194 years. The lessee, as the holder of the secondary put option, exercises the option and the TC rate is re-adjusted to the prevailing rate, $F(16.500 \times 10^3, 3.194)$, equal to 29.088×10^3 dollar/day. The exercise of the secondary put reduces the incomes cash flows to the lessor, equivalent to $\$13.151 \times 10^6$ at time 3.194 years. The net gain for the preemptive right holder is $\$2.725 \times 10^6$ (the net gain is evaluated at time zero and after the option price deducted). The preemptive right increases the lease value by 4.8%. The net return from investing in the flexibility with the preemptive right is 43.0%.

The random path displayed as dotted curve in Fig. 3.1 illustrates the 1E scenario. The spot rate goes up to 74.000×10^3 dollar/day and crosses the free boundary of the primary call option at time 2.744 years. Consequently, the primary option's holder exercises the option at that point in time. The TC rate is then adjusted to the prevailing TC freight rate, $F(74.000 \times 10^3, 2.744)$, equal to 53.665×10^3 dollar/day. Exercising the primary call option increases the income cash flows to the lessor, equivalent to $\$13.782 \times 10^6$ at time 2.744 years, as well as grants a secondary put option to the counterparty worth of $\$8.3754 \times 10^6$ at then. The corresponding free boundary of the secondary put option is illustrated by the solid dotted curve. The spot rate, however, never reverses strongly enough to trigger an exercise of the secondary put. Consequently, the rate adjustment made a net gain of $\$5.678 \times 10^6$ or a 10.0% increase in the lease value to the lessor. The net return from investing in the preemptive right is 89.6%.

The random path displayed as dash-dotted curve in Fig. 3.1 illustrates the 0E scenario. The spot rate in this scenario is never high enough to cross the free boundary of the primary call option. Therefore, the primary call option expires and is worthless. Although the spot rate has gone very low during the contract life, the lessee (the secondary option's holder) is not allowed to adjust the contracted TC rate because the preemptive right of adjusting the lease rate belongs to the lessor. In this scenario the lessor loses the amount of money she paid for the flexibility, equal to

11.1% of lease value; however, the TC contract has guaranteed her the contracted TC rate regardless of how low the market prices go during the contract life. The following is an examples of the preemptive right held by the lessee. At an initial spot rate of 60×10^3 dollar/day, the original TC contract rate, $F(60 \times 10^3, 0)$, is 44.283×10^3 dollar/day. A lessee (charterer) who expects the spot price to fall and wants to protect against a potential future loss (from having hired in a ship at a price higher than the market rate), buys a preemptive right to adjust the TC rate once during the contract's life. This right is equivalent to a primary put option. The put option price, $V^{PP}(X(0), 0)$, is equal to $\$6.345 \times 10^6$. Because the value of the TC contract is $\$70.527 \times 10^6$, the preemptive right is expected to increase the contract value by 9.0%. Table 3.4 summarizes the example results.

Figure 3.2 illustrates the free boundary of primary put option, three random sample paths of spot rate (with each corresponding to one of the three representative scenarios), and two free boundaries of the two secondary call options (with one associated with the 2E scenario and the other associated with the 1E scenario).

The random sample path displayed as dash curve in Fig. 3.2 illustrates the 2E scenario. The spot rate falls to -22.500×10^3 dollar/day (the negative value is acceptable as the rate is represented by the net cash flow) and crosses the free boundary of primary put option at time 1.833 years. Consequently, the primary put option is exercised, and the TC rate is adjusted to the prevailing rate, $F(-22.500 \times 10^3, 1.833)$, which is equal to 20.407×10^3 dollar/day. The exercise of the primary put option reduces the cash flows the lessee will pay during the contract's remaining life, equivalent to $\$25.173 \times 10^6$ at time 1.833 years. The contract grants the lessor a secondary call option worth $\$8.330 \times 10^6$ for the contract's remaining life once the primary put option is exercised. The free boundary of the secondary call option is displayed as the solid dash curve in Fig. 3.2. The spot rate then reverses up to 48.500×10^3 dollar/day, crossing the free boundary of the secondary call option at

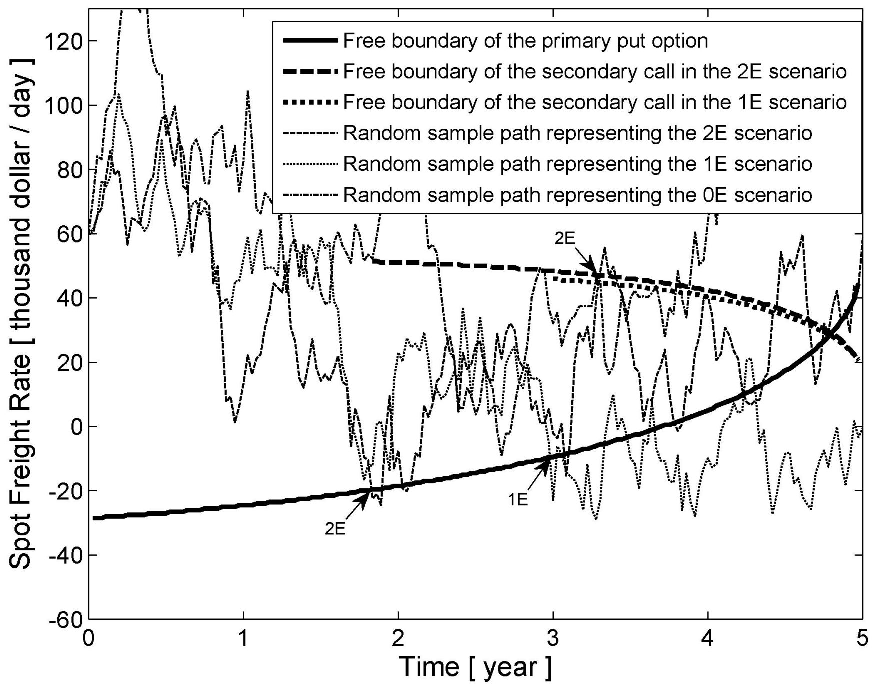


Figure 3.2: Put option Illustrative Example

Table 3.4: Examples of Preemptive Right Held by the Lessee

$X(0)$, [thousand dollar/day]	60.000		
$dX(t) = (40000 - X(t))dt + 3000dZ^*(t)$			
$F(X(0), 0)$, [thousand dollar/day]	44.283		
Value of TC contract w/o options, [million dollar]	70.527		
Price of the preemptive right, [million dollar]	6.345		
	Examples		
	2E	1E	0E
Exercise time of the primary call, t_P, [year]	1.833	2.979	N/A
Prevailing rate,$F((t_P), T_P)$, [thousand dollar/day]	20.407	18.655	N/A
Gain from the option exercise [million dollar]	25.173	17.731	N/A
Exercise time of the secondary put, t_S, [year]	3.305	N/A	N/A
Prevailing rate,$F(X(t_S), t_S)$, [thousand dollar/day]	44.142	N/A	N/A
Loss from the option exercise [million dollar]	13.886	N/A	N/A
Net gain (loss) [million dollar]	4.852	8.932	-6.345
Net gain (loss)/Original contract value	6.9%	12.7%	-9.0%
Net return from the preemptive right	76.5%	140.8%	-100%

3.305 years. At this time the lessor exercises the secondary call option, and the TC rate is re-adjusted to the prevailing TC rate, $F(48.500 \times 10^3, 3.305)$. The rate is equal to 44.142×10^3 dollar/day. The exercise of the secondary call option increases the cash flows the lessee will pay during the remaining life of the contract, equivalent to $\$13.886 \times 10^6$ at the time of 3.305 years. The net gain from this case is $\$4.852 \times 10^6$, which increases the lease value by 6.9%. The net return from investing in the preemptive right is 76.5% in this case.

The random path, displayed as dotted curve, illustrates the 1E scenario. The spot rate falls to -9.500×10^3 dollar/day and crosses the free boundary of the primary put option at 2.979 years. Consequently, the primary put option is exercised, and the TC rate is adjusted to the prevailing rate, $F(-9.500 \times 10^3, 2.979)$, which is equal to 18.655×10^3 dollar/day. The exercise of the primary put option saves a value of $\$17.731 \times 10^6$ for the lessee starting at 2.979 years. The free boundary of the secondary call option is illustrated in Fig. 3.2 in solid dotted curve; the spot rate never reverses strongly enough to cross that boundary. The net gain the lessee made is $\$8.932 \times 10^6$, equivalent to 12.7% net increase in the lease value. The net return from investing in the preemptive right is 140.8%.

The random path in dash-dotted curve is a case of the 0E scenario. The spot rate never falls low enough to cross the free boundary of the primary put option. The primary put option thus expires and is worthless. Although the spot rate has gone very high during the contract life, the lessor is not allowed to request for adjusting the spot rate because the lessee has the preemptive right. In this case, the lessee loses the option price she paid, approximately 9.0% of the lease value. Again, the TC lease contract has guaranteed the lessee the contracted TC rate regardless how high the market price climbs during the contract life.

The following discusses the preemptive right holder's decision behavior. To understand the impact of double-sided flexibility to the decision behavior of the primary option holder, this work analyzes the secondary option's value. This is because the secondary option is the in-kind exercise cost for the primary option in the ROs model.

Figure 3.3 displays the value of the secondary put option on the state space, and that of the secondary call option is illustrated on Fig. 3.4. The figures suggest that the in-kind exercise price decreases as time passes. This indicates that the double-sided flexibility may motivate the owner of the primary option to delay the option exercise to limit the chance of exercising the secondary option. This explains the reason for the primary options being exercised relatively early in the contract life in the 2E scenario and yet relatively late in the 1E scenario. While delaying the exercise of the primary option limits the chance of exercising the secondary option, it also reduces the benefit of cash flow increase. Therefore, the owner of the preemptive right have to consider the trade-off between the exercise cost and the profit of cash flow increase when deciding on the timing of option exercise.

The figures also show that the in-kind exercise cost for the primary put option, $V^{SC}(X(t), t)$, decreases as the spot rate rises. In contrast, the in-kind exercise cost for the primary call option, $V^{SP}(X(t), t)$, decreases as the spot rate drops. Therefore, the double-sided flexibility motivates the owner of the preemptive right to move the free boundary of the primary option towards the steady rate α^* in order to reduce the value of the secondary option to the counterparty. This, however, is at the cost of reducing the benefit of cash flow increase. The double-sided flexibility requires participants, particularly the owner of the preemptive right, to assess their decisions from a game perspective.

3.4.2. Sensitivity Analysis. Sensitivity analysis can test both the explanatory validity as well as the consistency of the ROs model with classic option theories.

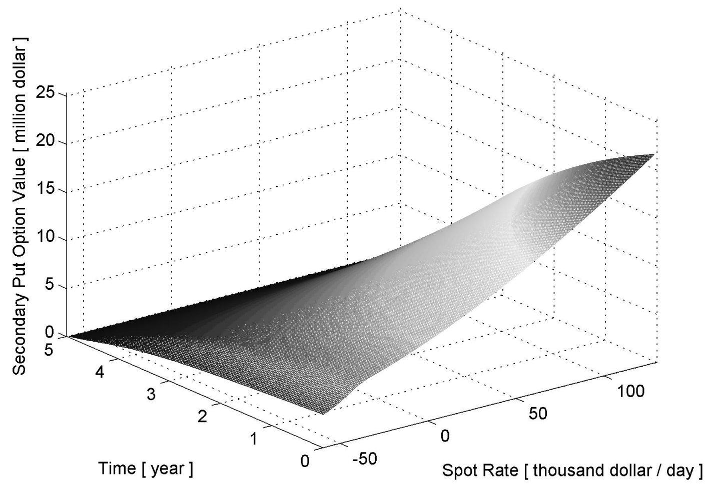


Figure 3.3: In-kind exercise price for the primary call option held by the lessor

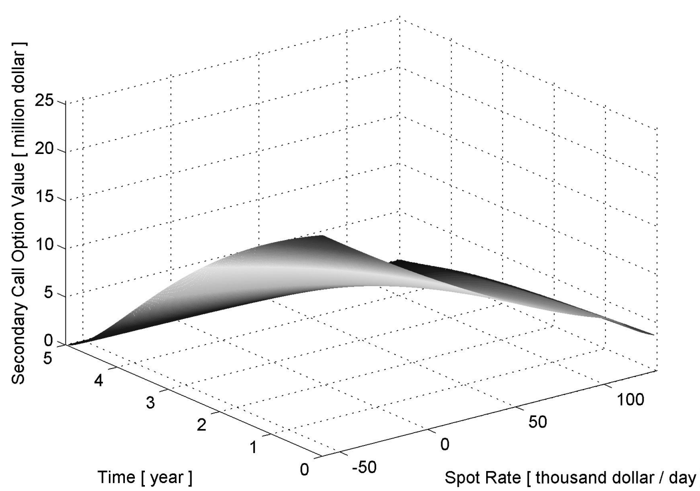


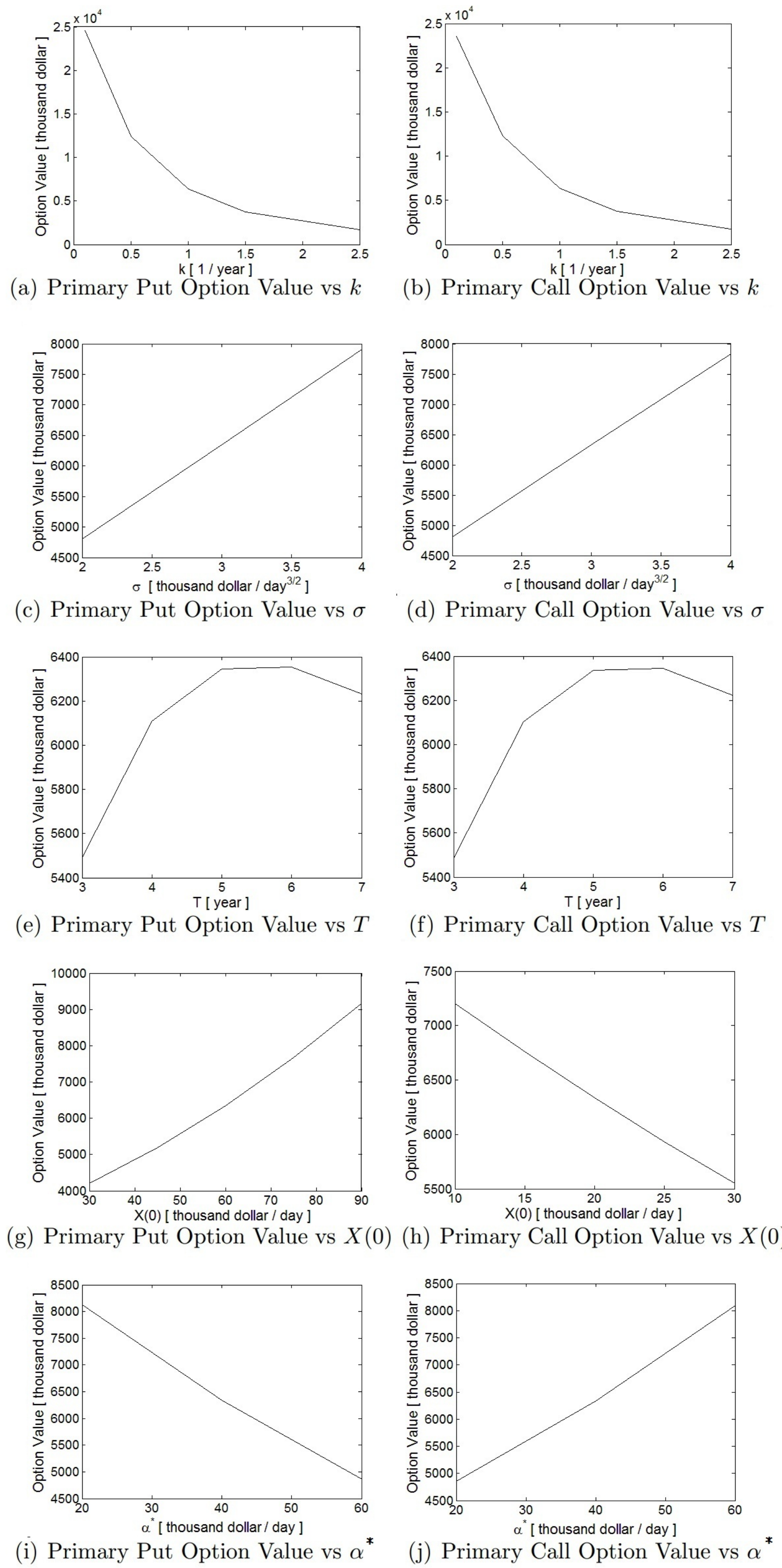
Figure 3.4: In-kind exercise price for the primary put option held by the lessee

This analysis also helps clarify how the value of a preemptive right is impacted by the spot rate dynamics. The sensitivity of the value of primary options to the changes in spot rate model parameters is given in Fig. 3.5. Major findings include the following:

- **Reverting speed,** k: The reverting speed is a decisive factor in determining the value of the rate adjustment flexibility with the preemptive right. Figs. 3.5(a) and 3.5(b) illustrate that an increase in k value quickly reduces the value of the primary option. This observation can be explained by the dynamic process of spot rate. Eqn. (3.2) indicates that the spot rate in a time interval dt, $X(t+dt)$, follows a normal distribution. The mean of $X(t+dt)$ is equal to the weighted average of the risk-neutral long-term rate, α^*, and the current spot rate, $X(t)$; the standard deviation is $\sigma\sqrt{\frac{1-e^{-2kdt}}{2k}}$. An increase in the value of k quickly reduces both the standard deviation of $X(t+dt)$ and the weight on $X(t)$ (i.e., e^{-kdt}). The dominating factor for determining the future spot rate then becomes α^*. In extreme situations the spot rate follows a Wiener process when k is equal to zero; it follows a deterministic process (i.e., $X(t)=\alpha^*$) when k goes to infinity. In summary, a fast reverting speed restricts the variability of the spot rate, lowering the primary option' price accordingly.

- **Volatility,** σ: Figs. 3.5(c) and 3.5(d) show that the value of primary options grows as the volatility increases. This observation is consistent with classic option theories and can be explained by Eqn. (3.2). The standard deviation of $X(t+dt)$ is proportional to σ. Therefore, a large value of volatility enlarges future spot rate's distribution and, thus, increases the value of primary options.

- **Contract term,** T: The manner in which the contract term impacts the preemptive rights' value is quite interesting. As T is prolonged, the preemptive right's value first increases and then decreases as illustrated in Figs. 3.5(e)



(a) Primary Put Option Value vs k (b) Primary Call Option Value vs k

(c) Primary Put Option Value vs σ (d) Primary Call Option Value vs σ

(e) Primary Put Option Value vs T (f) Primary Call Option Value vs T

(g) Primary Put Option Value vs $X(0)$ (h) Primary Call Option Value vs $X(0)$

(i) Primary Put Option Value vs α^* (j) Primary Call Option Value vs α^*

Figure 3.5: Sensitivity of the Preemptive Right's Value to the Spot Rate Model

and 3.5(f). This observation conflicts with classic option theories. The non-monotonic change of options value is related to the in-kind exercise cost for primary options. A lengthy contract term results in a high exercise cost that may outweigh the benefit the lengthy contract term can bring to the primary option. Therefore, the preemptive right's value may be decreased by a very long contract term.

- **Initial spot rate,** $X(0)$: Figs. 3.5(g) and 3.5(h) suggest that an increase in the initial spot rate increases and decreases the value of the primary put and call options, respectively. The underlying reason for this observation is that the original TC rate, $F(X(0), 0)$, is positively related to $X(0)$, and the value of the primary put and call options are increased and decreases, respectively, by an increase in $F(X(0), 0)$.

- **Risk-neutral long-term rate,** α^*: Figs. 3.5(i) and 3.5(j) show that a decrease in the primary put option value, and an increase in the primary call option value, are associated with an increase in the risk-neutral long-term rate. This observation can be explained by Eqn. (3.2), which indicates that a higher spot rate is expected if the value of α^* increases. Moreover, the value of the primary put and call option is negatively and positively, respectively, impacted by an increase in $X(t)$.

3.4.3. Discounted and Premium TC Contracts. Embedding the double-sided price adjustment flexibility in a lease contract results in either a discount on the TC rate (when the preemptive right is held by the lessor) or a premium above the TC rate (when the preemptive right is held by the lessee). The level of discount/premium is dependent on both the contract term and the spot rate at the time of the contract underwriting (according to the sensitivity analysis discussed in the previous section).

The ROs model presented in this section can help determine both the discount and the premium levels.

Table 3.5: The price of the preemptive right as a contract discount (%) to the lessee

		Spot Rate, $X(0)$ [thousand dollar/day]				
		10	15	20	25	30
Contract Term, T [year]	3	16.65	13.91	11.76	10.04	8.64
	4	14.11	12.04	10.35	8.95	7.78
	5	11.94	10.33	8.99	7.84	6.87
	6	10.18	8.89	7.78	6.84	6.03
	7	8.75	7.68	6.77	5.98	5.30

Table 3.6: The price of preemptive right as a contract premium (%) to the lessor

		Spot Rate, $X(0)$ [thousand dollar/day]				
		30	45	60	75	90
Contract Term, T [year]	3	10.32	11.02	11.77	12.55	13.35
	4	8.43	9.36	10.36	11.42	12.50
	5	6.99	7.94	9.00	10.14	11.34
	6	5.89	6.77	7.80	8.94	10.18
	7	5.01	5.83	6.78	7.88	9.12

Tables 3.5 and 3.6 respectively illustrate the discount and premium for TC contracts with different contract terms and spot rates at underwriting. For example, a lessor who enters a 4-year TC contract when the spot rate is 15×10^3 dollar/day would provide a discount of 12.04% to the lessee to obtain the preemptive right. In negotiating a longer life contract, for example a 6-year contract, the lessor would only give an 8.89% discount to the lessee for the preemptive right. Similarly, a lessee entering a 4-year TC contract when the spot rate is 75×10^3 dollar/day would pay a premium of 11.42% to the lessor to buy the preemptive right. If the spot rate at contract underwriting is 45×10^3 dollar/day, the premium is reduced to 9.36%.

3.4.4. Computational Complexity. The interdependence between option holders' decisions increases the computational complexity. The valuation of the primary option at any point (except for $i = N$) on the grid requires the valuation of the secondary option at the same point. The valuation of the secondary option at any point (i, j) involves solving the system of $M - 1$ linear equations defined in Eqn. (3.20) for $N + 1 - i$ times. Consequently, the linear equation system needs to be solved for $0.5(N + 1)N(M + 1) + N$ times to determine the primary option value at time zero.

The values of Δt and ΔX that define M and N, respectively, substantially affect the computational time and result accuracy. Valuation experiments are coded using Matlab to illustrate how the selections of Δt and ΔX impact both the result accuracy and the computational time. The experiments are performed on a workstation (Dell T7500: 2 quad-core Intel®Xeon®processors CPU, 1.4 GHz and 2.39 GHz, 48GB RAM). Results are listed in Tables 3.7 and 3.8.

The computational time increases as both Δt and ΔX decrease. According to the complexity formula (derived above), reducing the time step Δt to one-third should increase the computational time for approximately nine times. The results of the average computational times listed in Tables 3.7 and 3.8 are relatively consistent with the analytical results of computational complexity. Reduction of ΔX not only increases the number of times the linear equation system is valued but also the size of the linear equation system. For example, when ΔX is reduced by half, the number of times the linear equation system solved doubles and the computational time should increase by more than two times. Tables 3.7 and 3.8 suggest that a decrease in ΔX either from 2.0×10^3 dollar/day to 1.0×10^3 dollar/day or from 1.0×10^3 dollar/day to 0.5×10^3 dollar/day prolongs the computational time for approximately eight times.

As both Δt and ΔX become smaller, the result accuracy becomes better. Tables 3.7 and 3.8 indicate that the result accuracy is more sensitive to a change in

Table 3.7: Result accuracy and average computational time: Primary call option

Primary call value[1] / Avg. comp. time[2]		ΔX [thousand dollar/day] 0.5	1.0	2.0	5.0	10.0
Δt [day]	10	6.337 2.934e+04	6.337 3.833e+03	6.336 4.871e+02	6.333 3.644e+01	6.324 8.244e+00
	30	6.241 3.636e+03	6.241 4.333e+02	6.240 5.636e+01	6.238 4.243e+00	6.231 9.575e-01
	90	6.078 4.193e+02	6.078 5.034e+01	6.078 6.696e+00	6.076 5.355e-01	6.074 1.340e-01

1. Option values are in millions of dollars;
2. Average computational times are in seconds and based on five replications.

Table 3.8: Result accuracy and average computational time: Primary put option

Primary put value[1] / Avg. comp. time[2]		ΔX [thousand dollar/day] 0.5	1.0	2.0	5.0	10.0
Δt [day]	10	6.345 2.870e+04	6.345 3.725e+03	6.345 4.651e+02	6.342 3.646e+01	6.336 7.601e+00
	30	6.249 3.471e+03	6.249 4.204e+02	6.248 5.266e+01	6.247 4.135e+00	6.241 9.304e-01
	90	6.084 4.112e+02	6.084 4.817e+01	6.084 6.103e+00	6.083 5.186e-01	6.082 1.278e-01

1. Option values are in millions of dollars;
2. Average computational times are in seconds and based on five replications.

Δt than it is to the change in ΔX within the range of study. Moreover, a good trade-off between the computational time and the results accuracy seems to be obtained at $\Delta t = 10$ days and $\Delta X = 1 \times 10^3$ dollar/day.

3.5. CONCLUSION AND FUTURE WORK

This essay developed the double-sided price adjustment flexibility for the lease industry that is practicing in highly volatile markets. The double-sided flexibility adds a valuable option to the negotiation of flexibility clauses. The preemptive right of the price adjustment flexibility can be obtained by any party at a certain cost in order to enjoy superior flexibility over the counterparty. The double-sided flexibility complements the risk management capability of fixed rate lease contracts, and it is a tool more accessible to participants than derivatives. A ROs methodology was developed to model, value, and optimize the use of the flexibility held by each party of a lease contract. The developed methodology also helped obtain an insightful understanding of the decision behavior of participants who have options. The model was applied to TC contract examples from the volatile maritime lease market, which proved both the reliability and the applicability of the developed ROs framework. The same flexibility design is also applicable to other forms of lease contracts as well as to other contract relationships such as supply chain contracts. This work also contributes to the ROs literature by pushing the boundary of ROs applications.

The proposed model can be extended in a number of directions. One important direction of future studies is to model other attractive forms of flexibility, such as, the non-preemptive right of the rate adjustment flexibility, where both parties of the contract have an equal, parallel flexibility. The double-sided multiple-exercisable flexibility is also an important flexibility design worth of studying. Another study direction is to relax the model's assumptions and use more realistic spot price dynamics models (e.g., the Geometric Mean Reverting process). Finally this work can be improved by implementing multiple factor models that include other risk factors, such as the stochastic interest rate, in the options valuation. Such extensions would provide a richer picture of both the design and pricing of lease contract flexibilities.

4. REAL OPTIONS MODELING AND VALUATION OF DOUBLE-SIDED PRICE ADJUSTMENT FLEXIBILITY WITH THE NON-PREEMPTIVE RIGHT TO EXERCISE

4.1. INTRODUCTION

Much economic activity takes place within a framework of long-term contracts. Long-term contracts enable economic actors to coordinate their behavior[75]. Long-term contracts are popular in property lease in many domains, such as real estate, heavy equipment and transportation industry. Operating leasing separates property ownership from property use. The lessor receives the lease payments and the residual property value while the lessee receives the right to use the property over the lease term [65]. Lessors offer better pricing for long-term lease, because they have a longer stream of cash flow that minimizes their risk. Short term contracts incur more cost to all parties including re-negotiation cost. Long-term leases locks contract parties in for the contract term. This is an important shortcoming to the traditional long-term leases. This is an inherent shortcoming to the traditional long-term contracts. Obviously, contract parties would look for different kinds of appropriate flexibility in long-term contracts.

The flexibility in adjusting the lease price is an alternative tool for managing the price risk. The price uncertainty in the volatile business environment of the 21st century makes price flexibility even more valuable to both contract parties. In general price flexibility lessens price risk by limiting losses in downturn economic conditions and/or taking advantage of upturn economic conditions. However, flexibilities don't go without a price, flexibility seeker agrees to pay a premium to a lease contract counterparty to incorporate a specific flexibility in its lease.

Traditional long-term contracts do not allow participants to take advantages of favorable price movement, which is a limitation to long-term contracts. Some flexible rate leases have been practiced in leasing industry to overcome this limitation. Example of rate flexible leases is the Up-ward Only Rate Review (UORR) clause in the real estate industry. A lease in which landlords have the option to review the initial rental figure in line with market conditions at pre-determined intervals [2]. In the UORR, the possible adjustments (typically every five years) are known as rent reviews. If rents in the market have increased over the interim period, the rent will be adjusted upwards. However, if market rents have decreased, landlords will choose not to invoke the rent review clause and the existing rent will continue [76]. The UORR flexibility was considered fair to both lessee and lessor. It was fair to the lessor because it enabled the lessor to obtain a fair rent instead of a rent far below that which reflects the value of the property, and both inflationary and real increases in rents. It was fair to the lessee because, without it, under inflationary conditions, it would not be possible for a lessee to obtain a long-term lease [77].

Turnover (or overage or percentage) rent contracts is another flexible rate lease exists in some retail leases. Turnover contract specify that lessee will pay a base rent and a turnover rent equal to a percentage of the difference between sales in the current period and threshold sales, if the difference is positive [78]. In down markets lessee pays base rent. In high markets lessee's sales increases and lessor's gets the base plus the overage. Leasing, however, is not limited to real estates. Almost all expensive equipment and assets can be leased and lease industry is growing in the modern economy. For example, more than 33% of the worlds aviation fleet is rented and the proportion is likely to keep growing [79].

The lease rate flexibility in practice and in the literature are generally one-sided flexibility. However, a double-sided flexible rate lease is potentially more fair to both the lessor and the lessee. In their study [80] proposed a DSPAF in operating leases.

They suggested a rate flexibility in the form of an embedded real option in the lease. This flexibility gives the option holder the right but not the obligation to exercise the option and adjust the contract rate to the prevailing rate, when it is financially sound. The exercise right, however, is preemptive to the primary option holder. Only when primary option holder exercise her option the counterparty is granted a secondary option to re-adjust the rate. This was the first work that introduces the DSPAF in lease contracts in the literature. The preemptive right flexibility to primary option holder is superior over the other party right. The preemptive right suits certain rate flexibility requirements, paricularly, when contract parties have different market price expectations or when one party is more concerned about the price risk and willing to pay for the flexibility.

This study sought to develop a DSPAF in the form of non-preemptive right that provides equal, parallel right to both contract parties. The non-preemptive right flexibility is expected to come for a price that is less than the preemptive right flexibility as both parties enjoys similar and equal flexibility. The non-preemptive right is expected to meets contracts parties flexibility requirements in more cooperative relationship context. When both parties have concerns about the future price dramatic movements, yet unable to predict the price trend. The non-preemptive right flexibility is expexted to suit situations when both parties agree on equally sharing the flexibility to the reduce flexibility cost. Moreover, the preemptive right along with the non-preemptive right could make different lease flexibility alternatives for participants with different flexibility requirement and budgets. The non-preemptive right can be more attractive to participants working in lease markets where derivatives are not available. The valuation of the non-preemptive right, however, could be challenging considering the interaction between the contract parties exercise decisions.

The remainder of this essay is organized in the following manner. Section 2 summarizes relevant literature. Section 3 both models and values lease contracts with an embedded non-preemptive right with application to TC contracts. Section 4 present numerical examples and result analysis. Findings from this research and discussions of future research are summarized in Section 5.

4.2. LITERATURE REVIEW

In general, valuing lease contracts from a real options perspective is already well developed [2]. For example, used an endogenously derived term-structure for lease rates. Grenadier, determined the equilibrium lease rates for many different types of leases under various economic assumptions [64]. Lease prices flexibility has also been an area of interest for many researchers. The primary focus, however, has been on both analyzing and valuing commonly practiced forms of price adjustable contracts in real estate commercial lease. The practiced forms of adjustable price contracts are one-sided flexibility models. In essence they are agreements that allow the lessor additional rent over a minimum base. Contracts are often fashioned in discrete time when the price is reviewed periodically and then reset according to certain market conditions, often either inflation-indexing or market reviews. Examples of the adjustable lease contracts include the UORR.

The up-ward only price adjustment in a real estate market has been the focus of many studies. Ward et al. (1988) concluded that the UORR option premium is significant. they suggested the removal of the UORR clause from a lease contract would lead to a significant increase in the initial rent [76]. Ward and French (1997) used the Black-Scholes option-pricing equation and determined that an UORR has value to landlords. They demonstrated that approximately 17% of a lease's value is attributed to "upward only" constraints [81].

Ward et al. (1998) simulated a stochastic rent generating process, concluding that, for their UK lease example, a rental uplift of between 5 % and 16 % should apply to a lease with a UORR clause versus one without [76]. Booth and Walsh (2001) applied an adjusted standard option-pricing technique to the valuation of UORR properties [82]. Ambrose et al. (2002) presented a stochastic pricing model of upward-only adjusting leases. They developed an implicit equation for securities with path dependent cash flows and then applied it to the upward-only adjusting lease [2].

Another example of lease rate adjustment flexibility is the Percentage Lease Agreement (PLA). PLA or turnover rent) used in retail leases for multi-tenanted shopping centers. In PLA the lessee pays a flat base rent plus a turnover-related income. Hendershott and Ward (2000) treated the overage rent as a call option on the tenant's sale turnover. They applied the binomial option pricing approach to pricing the option. Hendershott and Ward (2000) demonstrated that ignoring the impact of future uncertainties on overage rents may underestimate the lease value by more than 10% [74].

Chiang et al.(1986) treated the tenant's obligation to either pay a percentage or turnover rent as if the landlord had a call option contingent on gross sales [83]. In contrast, Lee (1995) demonstrated that a percentage rent in a retail lease shares the risks of the variations in the success of the tenant's business. Therefore the expected rent should be higher than the fixed rent [84]. These studies revealed that practiced adjustable rate options significantly affect the contract's value. Provided flexibility, however, is one-sided and does not meet recent economic changes.

The maritime transportation market (the application of this essay), known for it's price volatility and high competition, is another important lease market. In this market both ships and tankers can be leased under Time Charter (TC) contracts for only a few months, or up to several years. [85] introduced and valued a new option to adjust price in the TC contract. The option also offers one-sided flexibility. However, unlike former options, this option is an American option and exercise is allowed any time during the contract's term. Al sharif and Qin modeled both the ship owners call option and the charterers put option. This put option allows the option owner, at a predefined exercise price, the right to adjust the lease price once. [80] added a DSPAF to the TC lease contract, allowing one party to enjoy superior flexibility by buying a preemptive right on that flexibility. They utilized the in-kind exercise feature to create DSPAF. The level of flexibility, however, is unequal. The primary option owner is given the preemptive right to adjust the price. The other party, however, is given a secondary option to re-adjust the price only after the primary option has been exercised.

A non-preemptive DSPAF equally serves both contract parties. It is also expected to be cheaper than preemptive flexibility and is thus more attractive to participants with certain flexibility requirements.

4.3. THE MODEL

This section illustrates both the modeling and the valuation methods for TC contracts from the maritime transport industry. The modeling of a lease contract depends on the contract type, the variables underlying the contract, and features of the lease market. The same methodology, however, can be adopted by other lease contracts in different lease markets.

4.3.1. Valuing Lease Contracts without the DSPAF. This study modeled the time charter equivalent (TCE) spot market freight rate, $X(t)$, as an Ornstein-Uhlenbeck (OU) mean-reverting process, a widely applied stochastic model of the spot freight rate discussed in the maritime transportation literature (e.g., [36]). It is also a model of the real estate lease price (e.g., [82]). The dynamics of $X(t)$ in a risk-neutral world is defined as

$$dX(t) = k(\alpha^* - X(t))dt + \sigma dZ^*(t), \tag{4.1}$$

where k is the reverting speed, α^* is the long-term steady rate under the risk-neutral measurement, σ is the instantaneous volatility of the spot rate, and $Z^*(t)$ is the one-dimensional standard Wiener process under the risk-neutral measurement.

The instantaneous cash flow generated by an operating vessel is

$$D(t)dt = (aX(t) - b)dt, \tag{4.2}$$

where a is the size of the cargo (which is equal to 1 when a freight is quoted for the entire ship) and b denotes the rate of total cost (including both operating costs and capital expenses).

According to the non-arbitrage assumption, the value of the TC contract is equal to that of the spot rate contracts within the same time frame. Therefore, the prevailing TC rate beginning at time t and ending at time T, $F(X(t), t)$, is calculated with

$$F(X(t), t) = a\frac{A(T-t, r+k)}{A(T-t, r)}X(t) + \left[1 - \frac{A(T-t, r+k)}{A(T-t, r)}\right]\alpha^* - b. \tag{4.3}$$

The $A(r, t)$ in Eqn. (4.3) is the annuity value factor equal to $(1 - e^{-rt})/r$. Equation (4.3) indicates that the TC rate is a linear function of the spot rate. Both the slope and the intercept, however, vary as time passes. Therefore, the stochastic movement of the spot rate may take the prevailing fixed rate, $F(X(t), t)$, away from the initial fixed rate, $F(X(0), 0)$, in an unpredictable manner.

4.3.2. Modeling the DSPAF with the Non-Preemptive Right. The DSPAF, as described in Fig. 4.1, are two real options written on the same underlying asset. Each contract party keeps one of the options and gives the other to the counterparty. For example, the lessor holds a call option that allows her to receive additional cash flows when the spot rate rises high enough to trigger the exercise of the call option. Meanwhile, she provides to the lessee a put option that requires herself to reduce the lease rate if the lessee exercises the put option. Thus, the two options cannot be exercised simultaneously. That is,

Lemma: *If both contract parties exercise their options, these two options must be exercised sequentially.*

Proof: Assume that both contract parties exercise their options at time t. Without loss of generality, assume the exercise of the call option is associated with cash inflows for the lessor during the remaining contract life: $\{\Delta CF^{call}(\tau) \geq 0 | t \leq \tau \leq T\}$. The cash flows to the lessee associated with the exercise of the put option at the same time are $\{\Delta CF^{put}(\tau) | t \leq \tau \leq T\}$, and $\Delta CF^{put}(\tau) = -\Delta CF^{call}(\tau)$ at any time τ. Therefore, the assumption contradicts to the fact that the lessee would not exercise her option if the payoff from exercising the option is negative. $\square$

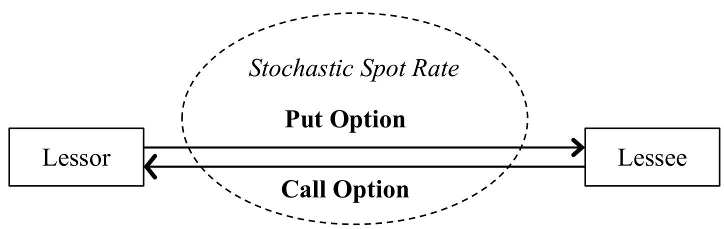


Figure 4.1: Schematic Diagram of the Double-Sided Price Adjustment Flexibility

A*secondary option* is defined here as the option in which the counterparty's option has already been exercised. Otherwise, the option is a *primary option.* In the remaining of the essay, PC, PP, SC, SP stand for the primary call option, primary put option, secondary call option, and secondary put option, respectively.

An optimal exercise policy for an option specifying that the *critical spot rate* triggers an exercise of the option. This critical rate may vary as time. consequently, the optimal exercise policy is represented by a trajectory of the critical spot rate on the time horizon. This is termed as *free boundary* of option exercise. The optimal exercise policy for the DSPAF with the non-preemptive right to exercise consists of four boundaries of option exercise. For a non-preemptive right to exercise case. Both contract parties hold a primary option until their counterparty exercises her option. Therefore, the optimal exercise policy is first specified for the primary options, and then for the secondary options.

Under the constraint of a non-preemptive right, either the lessor or the lessee can be the first to exercise her option. Therefore, the state space, $\{(X(t), t)|0 \leq t \leq T\}$, is divided into three zones: 1) "no options have been exercised yet (NE)", 2) the "primary call option has been exercised (PCE)", and 3) "the primary put option has been exercised (PPE)". these three zones are separated by two boundaries, (see Fig. 4.2) (a), the three zones are separated by two boundaries. The upper boundary is the free boundary of the primary call option that specifies the trajectory of the critical spot rate, triggering the exercise of the primary call option. The lower boundary is the free boundary of the primary put option. These two boundaries neither cross noor meet each other before the contract ends, according to the Lemma.

If the primary call option is exercised, the put option held by the lessee immediately becomes the secondary put option. The exercise policy for the secondary put option is the trajectory of the critical spot rate, triggering the exercise of this option. Figure 4.2 (b) illustrates that the state space for the remaining life of the contract (the shaded area) is separated by the free boundary of the secondary put option into two zones: 1) "secondary put option exercised (SPE)" and 2) "secondary put option not exercised (SPN)". The optimal exercise policy for the secondary call option, presented as the free boundary of it, is illustrated in Fig. 4.2 (c).

4.3.3. The Optimal Policy of Option Exercise. Determination of the optimal exercise policy for an option involves defining the trajectory of the critical spot rate for the option (the free boundary for exercising the option). To find the critical spot rate for an option at any time t requires determining both the option value at t, $V(X(t), t)$ and the intrinsic value of exercising the option, $G(X(t), t)$. This essay also uses superscripts to indicate different types of options. For example,

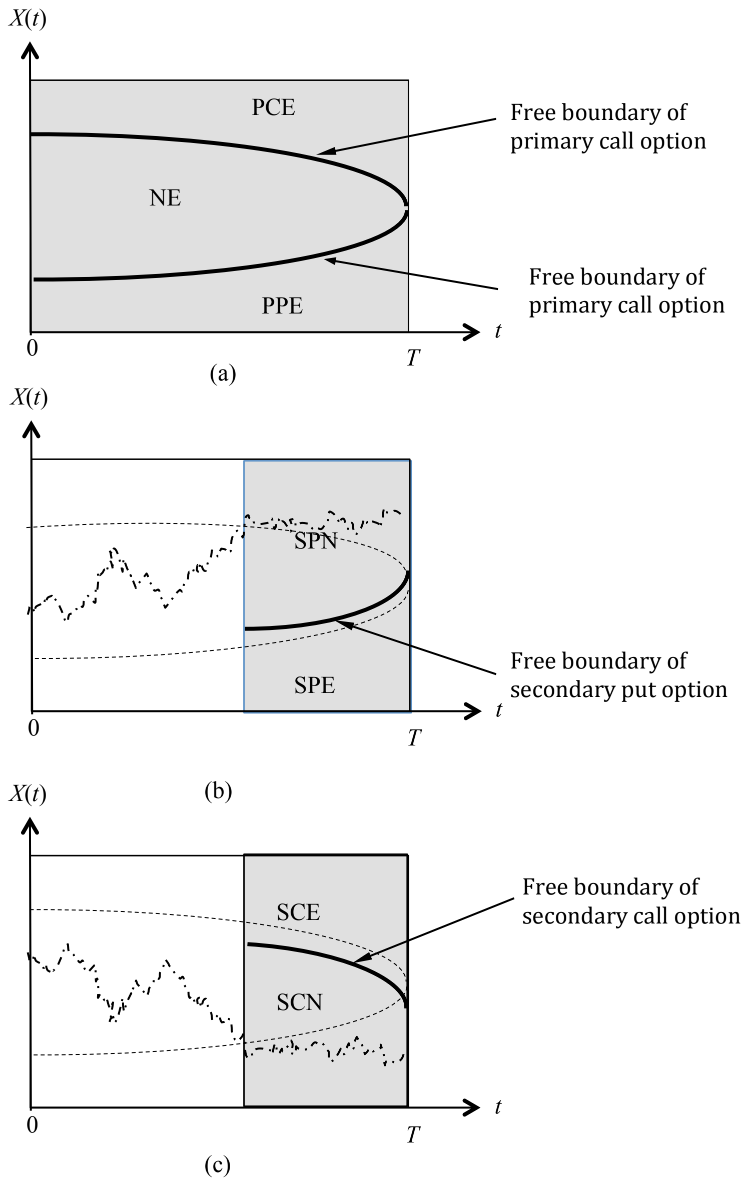


Figure 4.2: Schematic Diagram of the Optimal Exercise Policy as Four Free Boundaries of Option Exercise

$V^{PC}(X(t), t)$ designates the value of the primary call option at time t. Moreover, τ $(t \leq \tau \leq T)$ is used to designate the time during the remaining contract life if a primary option is exercised at time t $(0 \leq t \leq T)$.

The following discusses the intrinsic value of exercising an option. Assuming that a primary option is exercised at time t $(0 \leq t \leq T)$, the option held by the counterparty becomes the secondary option (which is the only option left for the remaining contract life). The secondary option can be exercised at any time during the remaining contract life, τ $(t \leq \tau \leq T)$. The intrinsic value of exercising the secondary option is the payoff from adjusting the lease rate:

$$G(X(\tau), \tau) = \begin{cases} G^{SC}(X(\tau), \tau) = A(T-\tau, r)[F(X(\tau), \tau) - F(X(t), t)] \\ G^{SP}(X(\tau), \tau) = A(T-\tau, r)[F(F(X(t), t) - X(\tau), \tau)] \end{cases} \tag{4.4}$$

If one contractparty becomes the first to exercise her option by time t, the payoff from exercising the primary option may be reduced if the secondary option will be exercised later. This cost is not deterministic. Therefore, the primary option's holder uses the value of the secondary option as an estimated cost for exercising her primary option. She subtracts this cost from the payoff of exercising the primary option in the calculation of the intrinsic value:

$$G(X(t), t) = \begin{cases} G^{PC}(X(t), t) = A(T-t, r)[F(X(t), t) - F(X(0), 0)] - V^{SP}(X(t), t), \\ G^{PP}(X(t), t) = A(T-t, r)[F(X(0), 0) - F(X(t), t)] - V^{SC}(X(t), t). \end{cases} \tag{4.5}$$

The Value of the Option: All four of the options discussed here are American options because their holders can exercise the price adjustment at any time during the contract's life. If immediately exercising the option is not optimal, the option holder will wait for a better opportunity. Determination of the primary options' value is challenging. This study approximates the value of the primary option with the non-preemptive right with that with the preemptive right. Accordingly, the following inequality must be satisfied:

$$\begin{cases} V(X(t),t) \geq \max\{G(X(t),t),0\}, 0 \leq t \leq T, & \text{for primary options;} \\ V(X(\tau),\tau) \geq \max\{G(X(\tau),\tau),0\}, t \leq \tau \leq T, & \text{for secondary options.} \end{cases} \tag{4.6}$$

The inequality can be solved with an appropriate numerical method such as the finite different method. For a more detaild discussion on solving theses equations and valuing the options, please review the work written by [80].

Option Free Boundary: A call (or a put) option's critical spot rate at any time is either the smallest (or the greatest) spot rate at which the intrinsic value of exercising the option equals the option's value:

$$\tilde{X}(t) = \begin{cases} \tilde{X}^{PC}(t) = \min\{X(t) : V^{PC}(X(t),t) = G^{PC}(X(t),t)\}, \\ \tilde{X}^{SC}(t) = \min\{X(\tau) : V^{SC}(X(\tau),\tau) = G^{SC}(X(\tau),\tau)\}, \\ \tilde{X}^{PP}(t) = \max\{X(t) : V^{PP}(X(t),t) = G^{PP}(X(t),t)\}, \\ \tilde{X}^{SP}(t) = \max\{X(\tau) : V^{SP}(X(\tau),\tau) = G^{SP}(X(\tau),\tau)\}. \end{cases} \tag{4.7}$$

The free boundary of an option is the trajectory of its critical spot rate on the decision time horizon:

$$\begin{cases} \{\tilde{X}(t) | 0 \leq t \leq T\}, & \text{for primary options;} \\ \{\tilde{X}(\tau) | t \leq \tau \leq T\}, & \text{for secondary options.} \end{cases} \tag{4.8}$$

The relationship between the two free boundaries of primary options can be determined, which is stated as the following Proposition.

Proposition I: $\tilde{X}^{PC}(t) > \tilde{X}^{PP}(t)$ *for* $0 < t < T$.

Proposition I means that the free boundary of the primary call option is above that of the primary put option.

Proof: For a primary option to be exercised, the payoff from exercising the option must be greater than zero. Therefore, $F(\tilde{X}^{PC}(t), t) > F(X(0), 0) > F(\tilde{X}^{PP}(t), t)$ for $0 < t < T$ as determained by Eqn. (4.5) and Eqn. (4.7). Moreover, at any time before an option expires, $X(t) > X'(t)$ if $F(X(t), t) > F(X'(t), t)$ as determained by Eqn. (4.3). Therefore, $\tilde{X}^{PC}(t) > \tilde{X}^{PP}(t)$ for $0 < t < T$. □

This study remarks that the free boundaries of the primary options are not the true boundaries. They are approximation due to the fact that the values of the primary options are approximation.

4.3.4. Pricing the DSPAF with the Non-Preemptive Right. Facilitated by the free boundaries of primary options, the Monte Carlo simulation (MCS) method can be applied to estimate the value of DSPAF with the non-preemptive right.

The following discusses Monte Carlo Simulation method. The state space is divided into three zones: 1) "PCE", 2) "PPE" and 3) "NE" - according to Proposition I. Any random trajectory of the spot rate originated from the zone 'NE". If the trajectory leaves the zone "NE", either the primary call or the primary put option is exercised. If the trajectory first enters the zone "PCE", the option exercised is the primary call option; otherwise it is the primary put option. Without loss of generality, Table 4.1 illustrates the pseudo code of the Monte Carlo simulation method for estimating the DSPAF value for the lessor.

The following discusses boundaries of the flexibility value. The value of the DSPAF with the non-preemptive right is bounded. These boundaries can be estimated using the value of options with the preemptive right. The relationship between the value of DSPAF with the non-preemptive right and that with the preemptive right, V_{pre}, is summarized as the following Proposition:

Table 4.1: Pseudo Code of the MSC for Estimating the DSPAF Value for the Lessor

```
0: Generate N_MC samples of spot rate trajectory:
   {X^i(t)|0 ≤ t ≤ T} for i = 1, 2, ..., N_MC.
1: Determine the flexibility value for each sample path
   for i = 1, 2, ..., N_MC
       if ∃ X^i(t) ≥ X^PC(t) for 0 < t < T
           t^PCE ← min{t|X^i(t) ≥ X^PC(t)}
           if ∃ X^i(t) ≤ X^PP(t) for 0 < t < T
               t^PPE ← min{t|X^i(t) ≤ X^PP(t)}
               if t^PCE < t^PPE
                   V^i ← exp(−r t^PCE) G^PC(X(t^PCE), t^PCE)
               else
                   V^i ← exp(−r t^PPE) G^PP(X(t^PPE), t^PPE)
               end
           else
               V^i ← exp(−r t^PCE) G^PC(X(t^PCE), t^PCE)
           end
       else
           if ∃ X^i(t) ≤ X^PP(t) for 0 < t < T
               t^PPE ← min{t|X^i(t) ≤ X^PP(t)}
               V^i ← exp(−r t^PPE) G^PP(X(t^PPE), t^PPE)
           else
               V^i ← 0
           end
       end
   end
2: Estimate the value of DSPAF with a non-preemptive right
   V(X(0), 0)_non−preemptive ← (Σ_{i=1}^{N_MN} V^i)/N^MN.
```

Proposition II: $V_{pre}^{PC} \geq V_{lessor} \geq -V_{pre}^{PP}$, *and* $V_{pre}^{PP} \geq V_{lessee} \geq -V_{pre}^{PC}$.

For the lessor, the value of the DSPAF with the non-preemptive right, V_{lessor}, is bounded from above by the value of the primary call option with the preemptive right, V_{pre}^{PC}, and from below by the negative value of the primary put option with the preemptive right, $-V_{pre}^{PP}$. For the lessee, the value of the DSPAF with the non-preemptive right, V_{lessee}, is bounded from above by the value of the primary put option with the preemptive right, V_{pre}^{PP}, and from below by the negative value of the primary call option with the preemptive right, $-V_{pre}^{PC}$.

Proof: The following lists contains five mutually exclusive scenarios of the spot rate trajectory relative to the free boundaries of primary options:

I. The spot rate trajectory crosses the free boundary of the primary call option before it crosses the free boundary of the primary put option.

II. The spot rate trajectory crosses only the free boundary of the primary call option.

III. The spot rate trajectory never crosses any free boundaries of primary options.

IV. The spot rate trajectory crosses only the free boundary of the primary put option.

V. The spot rate trajectory crosses the free boundary of the primary put option before it crosses the free boundary of the primary put option.

In Table 4.2 the realized value (on any random sample of spot rate trajectory) of DSPAF with the non-preemptive right is compared to those with the preemptive right (held by the lessor and the lessee, respectively), for each scenario. On any scenario i, the realized option value, $V_i(X(t), t)$, is equal to either $G(\tilde{X}(t), t)$ (if the option is exercised at time t) or 0 (if the option expired). Again, without loss of generality, the viewpoint of the lessor is used as an illustration.

Table 4.2: Realized Value of DSPAF for the Lessor: Non-Preemptive vs. Preemptive

Scenario	Preemptive Right (held by the lessor), $V^{PC}_{pre_i}(X(t), t)$	Non-preemptive Right (equally held by both), $V_{lessor_i}(X(t), t)$	Preemptive Right (held by the lessee), $-V^{PP}_{pre_i}(X(t), t)$
I	$G^{PC}(\tilde{X}^{PC}(t), t)$	$G^{PC}(\tilde{X}^{PC}(t), t)$	$-G^{PP}(\tilde{X}^{PP}(t), t)$
II	$G^{PC}(\tilde{X}^{PC}(t), t)$	$G^{PC}(\tilde{X}^{PC}(t), t)$	0
III	0	0	0
IV	0	$-G^{PP}(\tilde{X}^{PP}(t), t)$	$-G^{PP}(\tilde{X}^{PP}(t), t)$
V	$G^{PC}(\tilde{X}^{PC}(t), t)$	$-G^{PP}(\tilde{X}^{PP}(t), t)$	$-G^{PP}(\tilde{X}^{PP}(t), t)$

A DSPAF value is the weighted average of the expected present values on the five scenarios:

$$\sum_{i=I}^{i=V} Prob\{i\}E\{\exp(-rt)V_i(X(t),t)\}. \tag{4.9}$$

Table 4.2 shows that

$$V^{PC}_{pre_i}(X(t),t) \geq V_{lessor_i}(X(t),t) \geq -V^{PP}_{pre_i}(X(t),t). \tag{4.10}$$

That is, within each scenario the realized option value for the DSPAF with the non-preemptive right is bounded from above by that with the preemptive right held by the lessor and from below by the negative value of that with the preemptive right held by the lessee. According to Eqn. (4.10, this is a sufficient condition for the Proposition II.)

4.4. NUMERICAL EXAMPLES

Consider a ship owner (i.e., the lessor) and a charterer (i.e., the lessee) working in the maritime transportation TC market. Both expected dramatic movements in the market price and wanted to protect against the market price risk. They agreed on loading the TC contract with a DSPAF with the non-preemptive right to exercise the adjustment. This clause gives the contract parties an equal right to adjust the contracted TC rate to the prevailing rate once during the contract's life.

The contract has a term of five years. The initial spot rate is equal to the long-term steady rate, α^*, which equals 40×10^3 dollar/day. Other parameters of the spot rate model are as follows: the spot rate volatility, σ, equals 3×10^3 dollar/day$^{3/2}$; the reverting speed of the spot rate, k, is 1 per year; the market price of risk, λ, is equal to zero. a is equal to 1, (indicating that the price is quoted for the entire ship); and b is equal to zero because the cash flows are net cash flows. The continuously compounded risk-free rate, r, is 5% per year (assumed fixed). The TC contract rate, under these conditions, is 40×10^3 dollar/day, and the value of the 5-year TC contract is $\$63.705 \times 10^6$.

The values of the primary and the corresponding secondary options were determined throughout this example using the FDM in [80]. The value of the primary call option is $\$4.848 \times 10^6$ and the primary put option value is $\$4.849 \times 10^6$. Consequently, the expected value of DSPAF with the non-preemptive right to adjust the TC rate is within $\pm 7.61\%$ of the contract value. With the MCS method, the expected value of the flexibility is found to be close to zero; that is, the cost for embedding the DSPAF with the non-preemptive right is close to zero. It is remarked that the cost to the lessor (or the lessee) will increase if the initial spot rate decreases (or increases), departing away from the steady rate.

The following list contains five scenarios of TC rate adjustments when the contract parties have the non-preemptive right to exercise the adjustment: 1) "the primary call option exercised only", 2) "both the primary call option and the secondary put option exercised", 3) "the primary put option exercised only", 4) "both the primary put option and the secondary call option exercised", 5) and "no option exercised." Figure 4.3 illustrates four examples with each representing one of the four scenarios. The free boundaries of the primary call option and the primary put option are displayed in Fig. 4.3 with the bold solid curve and the bold dash curve, respectively. Random samples of the spot rate trajectory are in dash curve and TC rates are in solid horizontal lines. Numerical results from the examples are further summarized in Table 4.3.

Each free boundary divides the state space into the "holding the option" zone and the "exercising the option" zone. The spot rate is always originated from the "holding the option" zone. The price adjustment is exercised when the spot rate

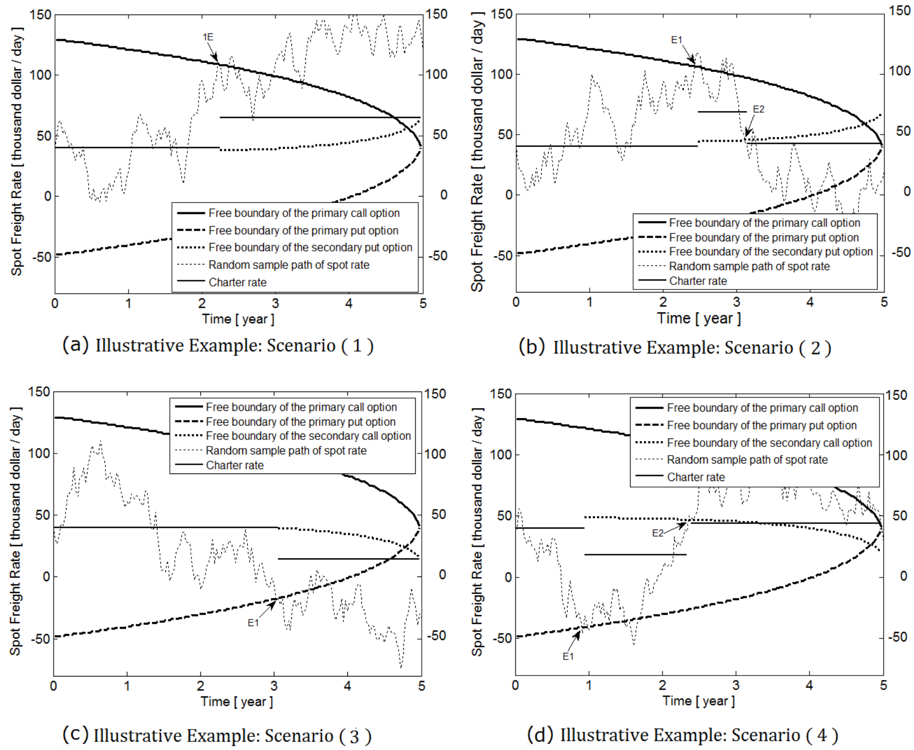


Figure 4.3: Illustrative Examples of TC Rate Adjustments

trajectory crosses the free boundary, transferring from “holding the option” zone to “exercising the option” zone.

In scenario 1 (illustrated in Fig. 4.3(a)) the random sample path of the spot rate rises up. The price becomes favorable for the lessor. The sample path crosses the primary call option’s free boundary at time 2.222 years when the spot rate is equal to 110.900×10^3 dollar/day. The call option (held by lessor) is then exercised and the contracted TC rate is adjusted to the prevailing TC rate, F(110.900×10^3,2.222), equal to \$$64.474 \times 10^3$ dollar/day. This adjustment increases the lessor’s income cash flows equivalent to \$$22.851 \times 10^6$ (assessed at the exercise time). At the exercise of the primary call option, the put option held by the lessee becomes the secondary option. The lessee is then allowed to re-adjust the rate, in case the price falls low enough in her favor. The bold dotted curve demonstrates the secondary option’s free boundary. The spot rate random path, however, keeps rising and never crosses the free boundary of the corresponding secondary put option. Consequently, the lessee let her option expired. In this particular scenario the spot rate and TC rate become highly favorable for the lessor. The net increase in the lessor's income from exercising flexibility is $\$20.449 \times 10^6$ (assessed at time zero). The flexibility increased the contract value by 32.1% for the lessor, and increased the lessee's leasing cost by 32.1%. The lessee partially compensated the lessor. However, the lessee did not pay more than the adjusted price for the rest of the contract term even when TC prices kept rising after the first adjustment and considerably limited the risk of price increase.

In scenario 2 (illustrated in Fig. 4.3(b)) the random path of the spot rate also rises and crosses the free boundary of the primary call option at time 2.444 years.The primary call option is exercised at that point and the TC rate adjusted to the prevailing rate, $F(117.800 \times 10^3, 2.444)$, equal to $\$68.618 \times 10^3$ dollar/day. The increased income cash flows to the lessor is equivalent to $\$24.717 \times 10^6$. The trend of spot rate movement, in this scenario, reverses after the exercise of the primary call option. The spot rate crosss the secondary put option's free boundary at time 3.111 years. Secondary put option' holder exercises her option to re-adjust the TC rate to the more favorable prevailing TC rate $\$42.257 \times 10^3$ dollar/day. The option exercise increases her income cash flows by $\$17.105 \times 10^3$. The net gain or loss from of flexibility in this case is 7.233×10^6, in favor of the lessor. The change in the contract value is equivalent to 11.35% .

In scenario 3 (illustrated in Fig. 4.3(c)) the random path of spot rate falls against the interest of the lessor and crosses the primary put option's free boundary at time 3.028 years when the spot rate is equal to -19.150×10^3 dollar/day (negative rate is acceptable as the rate is modeled by the net ship income, TCE). The put option is then exercised and the contracted TC rate is adjusted to the prevailing TC rate at this point, equal to $\$14.092 \times 10^3$ dollar/day. The adjustment reduces the lessee's cash outflows equivalent to $\$17.5167 \times 10^6$. The random path of the spot rate keeps falling and never crosses the secondary call option's free boundary. In this scenario the market TC rates dramatically decreased against the lessor interest. The net gain to the lessee was a sum of $\$17.517 \times 10^6$, reducing the lessee' leasing cost by 23.64%.

In scenario 4 (illustrated in Fig. 4.3(d)) the random path of the spot rate also falls and crosses the free boundary of the primary put option at time 0.917 years when the spot rate is $\$-46.060 \times 10^3$. The primary put option is then exercised and the TC rate is adjusted to a prevailing rate equal to $\$18.254 \times 10^3$ dollar/day. The adjustment reduces the lessee's cash outflows by $\$33.680 \times 10^6$. The trend of the spot rate in this scenario reverses and rises up after the exercise of the primary put option. The spot rate crosses the secondary call option's free boundary at the time of 2.361 years when the spot rate is 50.040×10^3 dollar/day. The TC rate is re-adjusted to $\$43.583 \times 10^3$ dollar/day. By exercising her option and re-adjusting the TC rate back to a more favorable prevailing rate, the lessor increases her income cash flows by $\$22.543 \times 10^6$. This case is favorable for the lessee whose net gain from the flexibility (discounted to time zero) is 12.138×10^6, about a 19.05% saving in the leasing cost.

Figure 4.3 illustrates that the free boundaries of the two primary options meet at the end of the contract period, which means that at least one contract party will exercise the price adjustment.

4.5. CONCLUSION AND FUTURE WORK

This study not only models but also values a double-sided price adjustment flexibility (DSPAF) of a non-preemptive right to exercise the adjustment within the context of the leasing industry. The DSPAF with non-preemptive right in the form of

Table 4.3: Examples of the DSPAF with the non-preemptive right to exercise the adjustment

$X(0)$, [dollar/day]									40,000	
$dX(t) = (40000 - X(t))dt + 3000dZ^*(t)$										
Contract life, [year]									5	
$F(X(0), 0)$, [dollar/day]									40,000	
Value of TC contract w/o options, [million dollar]									63.705	
The Value of the DSPAF with the non-preemptive right [million dollar]							For lessor		-0.001332	
							For lessee		0.001332	
Example Scenarios	Scenario 1		Scenario 2		Scenario 3		Scenario 4		Scenario 5	
Option type	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put
Exercise type	Pri.	Sec.	Pri.	Sec.	Sec.	Pri.	Sec.	Pri.	Pri.	Pri.
Exercise time, [year]	2.222	N/A	2.444	3.111	N/A	3.028	2.361	0.917	N/A	N/A
Rate after exercise, [thousand dollar/day]	46.474	N/A	68.618	42.257	N/A	14.092	43.583	18.254	N/A	N/A
Viewpoint of the call option's holder (the lessor)										
Gain/loss at exercise [million dollar]	22.851	N/A	24.717	-17.105	N/A	-17.517	22.543	-33.680	0.000	0.000
Net gain (loss) [million dollar]	20.449		7.233		-15.057		-12.138		0.000	
Change in contract value	32.10%		11.35%		-23.64%		-19.05%		0.000%	
Viewpoint of the put option's holder (the lessee)										
Gain/loss at exercise [million dollar]	-22.851	N/A	-24.717	17.105	N/A	17.517	-22.543	33.680	0.000	0.000
Net gain (loss) [million dollar]	-20.449		-7.233		15.057		12.138		0.000	
Change in contract value	-32.10%		-11.35%		23.64%		19.05%		0.000%	

real options provides a price flexibility for both contract parties at an equal level. This non-preemptive right also allows both contract parties to take advantage of favorable price changes, thus may adding value to the lease contract and making the long-term contract a more fair contractual relationship. The price of the DSPAF with the non-preemptive right depends heavily on the initial contract rate. The flexibility's price for any contract parties found to be less than that proposed by [80].

This non-preemptive right is expected to meet some participants flexibility requirements and budgets. As a result it can better serves both parties in certain market conditions, particularly, when the trend of future price movement in a volatile market is difficult to predict. It also serves in cooperative situations where the lessee and the lessor would like to have longer fair relationships.

The DSPAF with the non-preemptive right is also expected to complement that with the preemptive right. It can do so by providing the lease parties with another form of flexibility during the contracting process.

The work of this essay can be extended in multiple directions. A similar model can be developed to manage the price risk in other types of contracts, such as a long-term supply contracts on commodity and services. Another direction of future work is to relax some model assumptions and examine the effect of the relaxation. Other stochastic specifications of the spot price process such as the Geometric Mean Reverting process may be more appropriate. Additionally, multiple risk factors may be included in the model so that the price adjustment decision can comprehensively consider different sources of uncertainty. For example, the stochasticity in the interest rate. This work also builds the foundation for modeling multiple-exercisable price adjustment flexibility and that with more than multiple (more than two) participants.

5. CONCLUSIONS

5.1. SUMMARY

This dissertation research utilized the real options theory to exploit an alternative approach to managing the price risk for long-term lease contracts in volatile markets. It proposed to embed the price adjustment flexibility to long-term lease contracts. The proposed flexibility allows contract parties to adjust the contracted lease rate when it is far from the prevailing rate. The work modeled the flexibility as real options, quantified the price of the flexibility (i.e., the expected value of the flexibility), and tested the proposed approach with examples of Time Charter contracts from the maritime transport industry.

Three forms of price adjustment flexibility that can be embedded in the long-term lease contracts were developed. The first form of developed flexibility is the single-sided price adjustment flexibility (SSPAF) in the form of an American call option to the lessor and an American put option to the lessee. In this form of flexibility only one contract party is allowed to adjust the contract rate to the prevailing rate.

The second form of developed flexibility is a double-sided price adjustment flexibility (DSPAF) with the preemptive right to exercise. That is, both contract parties are allowed to adjust the lease rate to take advantage of favorable price changes, yet at different levels. One party can enjoy superior level of flexibility over the counterparty through purchasing the preemptive right to adjust the price.

The third form of developed flexibility is the DSPAF with the non-preemptive right to exercise the price adjustment. That is, both contract parties have equal, parallel flexibility in the price adjustment, making the lease contact more fair to both sides.

5.2. FINDINGS

Insightful findings from the research build our knowledge and experience with the practice of price adjustment flexibility. Particularly,

- This work shows that price adjustment flexibility can be provided to both contract parties when they want it. However, a DSPAF is not a linear combination of the lessor SSPAF and the lessees SSPAF. Additionally, the DSPAF with the non-preemptive right is not a linear combination of the DSPAF with the preemptive right belonging to the lessor and that belongs to the lessee.

- The price adjustment flexibility is a value added in that it allows contract parties to take advantages of favorable market conditions. The price of the flexibility can be quantified with the Real Options (ROs) valuation. The flexibility's price is dependent on multiple factors, including the lease rate dynamics, contract term, and constraints on the flexibility.

- Each form of the price adjustment flexibility meets specific purpose of risk management and budget requirement, and is suitable for specific market condition. Thus, it is important to have a variety of flexibility options in the negotiation of flexibility clauses for long-term contracts.

- A straightforward tool for implementing the price adjustment flexibility is the trajectory of critical spot rate, termed the free boundary for exercising the price adjustment. At any time the boundary determines whether the adjustment should be made by comparing the realized market rate to the critical rate. When both contract parties have the price adjustment flexibility, the free boundary of the secondary option is not fixed. It depends on the time the primary option was exercised and the spot rate at the option exercise.

5.3. CONTRIBUTIONS

This research advances our knowledge on modeling and valuation of flexibility, real options, and risk management. Particularly,

- This work develops a ROs methodology for managing the price risk with flexibility, which complements the risk management capability of fixed rate long-term lease contracts. With the price adjustment flexibility, participants of long-term contracts can take advantages of favorable price movement. This helps maintain a good long-term relationship between contract parties.

- This work proposes to embed the price adjustment flexibility in long-term fixed rate lease contracts, which provides an alternative tool of price risk management to participants in a more natural way. It is a tool more accessible to participants than derivatives. Market participants can directly use the tool for contracting and risk management without going to the hassle of financial derivatives. Because financial depravities are more complex and require special knowledge and experiences with derivatives and inheres certain limitations. The model can be easily set up as a simple, user-friendly tool with GUI (e.g., a web based tool). The price adjustment flexibility becomes even more important in lease markets where derivatives are not available.

- This work contributes to the ROs literature by pushing the boundary of real options applications to further spaces. ROs are not about simply applying the financial derivative theories and methods to non-financial domains. Great efforts of ROs methods lie in a "domain translation" in that ROs translate the basic options pricing theories into insightful decision methods. The valuation and modeling methodology developed in this work enrich ROs theories and applications.

- This work builds a solid mathematical foundation for designing and implementing other forms of flexibility, such as multiple-sided multiple-exercisable flexibility, for general contract relationships.

5.4. FUTURE RESEARCH

The completion of this dissertation is not the end, rather, the starting point of inspired future exploration. The dissertation work initiates the discussion of a series of topics that are interesting to both industrial practitioners and academic researchers. A few thoughts of the future research are the following:

- *Broader Applications*: This dissertation serves as a good starting point of modeling various price flexibilities needed in different market environments. For instance, a similar model can be developed to manage price risk for other types of contracts such as long-term procurement and supply contracts on commodity and services.

- *Model Generalization*: The developed models of price adjustment flexibility can be extended in a number of directions that call for further studies. One important direction of future studies is to model other attractive forms of flexibility. For example, the double-sided, multiple-exercisable flexibility with the preemptive right, where both contract parties have an equal, parallel price adjustment flexibility that can be exercised more than once during the contract life. Multiple-sided, multiple-exercisable price adjustment flexibility is another important flexibility design worth of studying.

- *Quality Improvement of Stochastic Models*: Another direction of future work is to relax some model assumptions and examine the effect of the relaxation. The stochastic model of the underlying variable plays a critical role in dynamic decision. The quality of the stochastic model directly impacts the effectiveness of the decision outcomes. Other stochastic specifications of the spot price process may be more appropriate such as the Geometric Mean Reverting process and worth testing. Moreover, multiple risk factors may be included in the model so that the price adjustment decision can comprehensively consider different sources of uncertainty, for example the stochasticity in the interest rate.

These extensions would provide a richer picture of both the design and pricing of price adjustment flexibility for long-term contracts.

APPENDIX A

LIST OF NOMENCLATURE

Table A.1: List of Nomenclature

Term	Unit	Definition
α	[dollar/day]	the long term rate of $X(t)$
α^*	[dollar/day]	α under the equivalent Martingale measure (*)
γ		parameters of finite difference methods
λ	[day$^{-1/2}$]	the market price of risk
σ	[dollar/day$^{3/2}$]	the volatility of $X(t)$
ΔX	[dollar/day]	the step size of spot freight rate on the grid (t_i, X_j)
Δt	[day]	the step size of time on the grid (t_i, X_j)
$A(t, r)$	[day]	annuity value factor
$B(t, r, k)$	[dollar]	a term of cash for determining contract values
$C(X(t), t)$	[dollar]	the call option value at time t
$\hat{E}_t$		the risk-neural expected value assessed at time t
$F(X(t), t)$	[dollar/day]	the TC rate realized at time t and effective until T
$G(X(t), t)$	[dollar]	the intrinsic value of exercising option at time t
$K(t)$	[dollar]	the adjustment cost
M		the number of steps of spot rate on the grid (t_i, X_j)
N		the number of time steps on the grid (t_i, X_j)
$P(X(t), t)$	[dollar]	the put option value at time t
T	[day]	the term of contract
$V(X(t), t)$	[dollar]	the (general) option value at time t
$X(t)$	[dollar/day]	the spot freight rate at time t
X_j	[dollar/day]	the (discrete)spot freight rate
$Z(t)$	[day$^{1/2}$]	standard Wiener process
$Z^*(t)$	[day$^{1/2}$]	$Z(t)$ under the equivalent Martingale measure (*)
a		the size of cargo
b	[dollar/day]	the cost flow rate
h		the proportional factor for determining $K(t)$
i		the index of (discrete) time on the grid (t_i, X_j)
j		the index of (discrete) spot rate on the grid (t_i, X_j)
k	[year^{-1}]	the reverting speed of spot rate
r	[year^{-1}]	the risk-free rate
t	[day]	(continuous) time
t_i	[day]	(discrete) time
t_L	[day]	lock-up period

Terms showing no unit are dimensionless.

APPENDIX B

THE CALIBRATION OF OU PROCESS MODEL

According to Eqn. (2.1), the OU mean-reverting process followed by the spot rate, $X(t)$, is

$$X(t+dt) = \alpha(1-e^{-kdt}) + e^{-kdt}X(t) + \sigma\sqrt{\frac{1-e^{-2kdt}}{2k}}\epsilon, \tag{B.1}$$

where ϵ follows the standard normal distribution. We rewrite Eqn. (B.1) as a regression function,

$$X(t+dt) = c_1 + c_2 X(t) + e_t, \tag{B.2}$$

where

$$c_1 = \alpha(1-e^{-kdt}), \tag{B.3}$$

$$c_2 = e^{-kdt}, \tag{B.4}$$

and e_t follows a normal distribution with a mean of zero and a standard deviation of σ_e:

$$\sigma_e = \sigma\sqrt{\frac{1-e^{-2kdt}}{2k}}. \tag{B.5}$$

Given a time series data of $X(t)$, the regression model in Eqn. (B.2) can be fitted. Let $\hat{c}_1$, $\hat{c}_2$, and $\hat{\sigma}_e$ denote the estimated values for c_1, c_2, and σ_e, respectively, obtained from the regression analysis. The estimates of parameters k, α, and σ are

$$\hat{k} = -\ln(\hat{c}_2)/dt, \tag{B.6}$$

$$\hat{\alpha} = \hat{c}_1/(1-\hat{c}_2), \tag{B.7}$$

$$\hat{\sigma} = \hat{\sigma}_e\sqrt{-2\ln(\hat{c}_2)/[dt(1-\hat{c}_2^2)]}. \tag{B.8}$$

Other techniques can also be used to calibrate the model parameters, for example the Maximum Likelihood Estimation.

APPENDIX C

PSEUDO CODE OF THE OPTION VALUATION ALGORITHM

The pseudo code of the algorithm for valuing the options are shown in Table C.1. Inputs of the valuation include:

- parameters for setting up the 2D grid, (t_i, X_j): X_{max}, X_{min}, ΔX, T, and ΔT;
- parameters of the spot rate model: $X(0)$, α, λ, k, and σ;
- contract design parameters: h, and t_L; and
- parameters of contract valuation model: a, b, and r.

Table C.1: Pseudo Code of the Algorithm for Valuing the Options

0. Build a 2D grid of size $N \times M$:
   $N \leftarrow T/\Delta t$,
   $M \leftarrow (X_{max} - X_{min})/\Delta X$.
1. Define the terminal condition: $V_{i,j} \leftarrow 0$ for $i = N$ and any j.
2. Value the option backward:
   for $i = N-1, N-2, \ldots, 0$
      Determine the expected value of continuation, $V_{i,j}$:
         for boundary points ($j = 0, M$), use the explicit method in Eqns. (2.18-2.21);
         for interior points ($j = 1, \ldots, M-1$), use the implicit method in Eqns. (2.22-2.23).
      Determine the intrinsic value of option exercise, $G_{i,j}$, if $i\Delta t > t_L$:
         if $V_{i,j} = C(X_{min} + j\Delta X, i\Delta t)$
            $G_{i,j} \leftarrow A(T - i\Delta t, r)[F(i\Delta t, T) - (1+h)F(X(0), 0)]$,
         else
            $G_{i,j} \leftarrow A(T - i\Delta t, r)[(1-h)F(X(0), 0) - F(i\Delta t, T)]$.
         end;
         $V_{i,j} \leftarrow \max\{G_{i,j}, V_{i,j}\}$.
3. The option value is determined at at time zero, equal to $V_{0,\tilde{j}}$ where $\tilde{j} = (X(0) - X_{min})/\Delta X$.

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VITA

Ahmed Al sharif was born on January 22nd, 1967 in Tripoli, Libya. In 1989 received his bachelors degree in mechanical engineering from Al-Fatah University in Tripoli. In 2006, he received his MBA from Higher Studies Academy also in Tripoli.

Ahmeds work experiences in Libya include the Engineering Industries Board from 1989 to 1992 and the Industrial Research Center from 1992 to 2008.

In 2009, Ahmed began attending Missouri University of Science and Technology (Missouri S&T) in Rolla, USA. While there, he received his masters degree in engineering management. He also earned both the Project Management and Financial Engineering Certificates and the 2012/2013 Outstanding PhD Graduate Student Research Award. During the Fall semester of 2013, he worked as a graduate teaching assistant with the Engineering Management Department, teaching a graduate course on economic decision analysis.

In December, 2013, Ahmed received his doctoral degree in engineering management with a specialization in financial engineering from Missouri S&T.