Modelling of Inventory Management in Humanitarian Logistics

by

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This dissertation is dedicated to my parents,

Rian and Estie van Wyk

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"If you think about disaster, you will get it. Brood about death and you hasten your demise. Think positively and masterfully, with confidence and faith, and life becomes more secure, more fraught with action, richer in achievement and experience."

- Swami Vivekanada

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Executive Summary

Title:	Modelling of Inventory Management in Humanitarian Logistics
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Natural and man-made disasters are becoming more frequent in many countries throughout the world. Countries with inadequate infrastructure and poorly planned emergency logistics are subject to such events which may lead to the destruction of a community and/or may prevent efficient and successful recovery. Despite the progress that disaster planning, mitigation and new management systems have made, the need for disaster relief continues everlasting. Extensive research is on-going to improve the various phases in the disaster operations life cycle. However, the impact of disaster will not diminish and improved disaster relief planning and management should be addressed intensely. This dissertation addresses various possible mathematical models comprising stochastic and deterministic models, to provide generic means to address the damage and consequences associated with disaster events. The models are applied to countries such as Somalia and the Southern African Development Community (SADC), which have been prone to catastrophic events and poverty consequences.

Keywords: Inventory management, humanitarian logistics, operational research modelling

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List of Acronyms

AFP	Agency France-Presse
CBA	Cost-benefit Analysis
DIM	Deterministic Inventory Model
DRC	Democratic Republic of the Congo
EDA	Euclidean Distance Algorithm
GIS	Geographic Information Systems
IDNDR	International Decade for Natural Disaster Reduction
IT	Information Technology
OR	Operations Research
PPF	Pre-positioning Facility
PSA	Present-state-of-art
SA	Simulated Annealing
SADC	Southern African Development Community
SMIP	Stochastic Mixed Integer Program
SIM	Stochastic Inventory Management
SOS	Save Our Souls

UN United Nations

UNISDR United Nations International Strategy for Disaster Reduction

Chapter 1

Introduction

1.1 The Importance of Disaster Management

The severe consequences of natural and man-made disasters are evident from media reporting. On 13 November 2011, Agence France-Presse (AFP) reported that United Nations (UN) climate scientists forecast the likelihood of increasing heat waves in Southern Europe (News 24, 2011). In addition, North Africa will be more susceptible to droughts, and rising seas will cause storm surges in small island states. According to the AFP, peer reviewed scientific journals are claiming that the impact of disasters have a 90% probability of becoming unbearable over time (News 24, 2011). A summary for policy makers drafted by the AFP claims:

"Global warming will create weather on steroids."

To mention a few more disastrous events: On 11 March 2011, a 8.9 magnitude earthquake struck Japan triggering a tsunami alert along Japan's Pacific Coast and to at least 20 countries. This majestic tsunami catastrophe was the worst historic event in Japan. A tornado struck the East Rand, Johannesburg in South Africa on February 2011, causing extensive damage at Duduza township. This was shortly after a storm wind, suspected also to have been a tornado, flattened houses at Ficksburg, in the Free State province of South Africa, causing death and destruction.

Concern exists that in the future, entire communities could be obliterated by a single disaster. The living conditions of communities will degrade as disasters increase in frequency and/or severity, which in turn will cause an increase in permanent migration and present more pressures in areas of relocation, leading to a greater need for disaster management.

Tomasini and Van Wassenhove (2009) define disaster management as: "…the result of a long and structured process of strategic process design, ultimately resulting in successful execution". Disaster management can be divided into four phases: mitigation, preparedness, response and recovery. These phases are known collectively as the disaster operations life cycle. Mitigation comprises the application of measures that either prevent the onset of a disaster or reduce the impact should a disaster occur. Preparedness relates to the community's ability to respond when a disaster occurs; response refers to the employment of resources and emergency procedures as guided by plans to preserve life, property, and the governing structure of the community. Finally, recovery involves actions taken to stabilize the community subsequent to the immediate impact of a disaster (Altay and Green, 2006). The disaster cycle is illustrated in Figure 1.1[1].

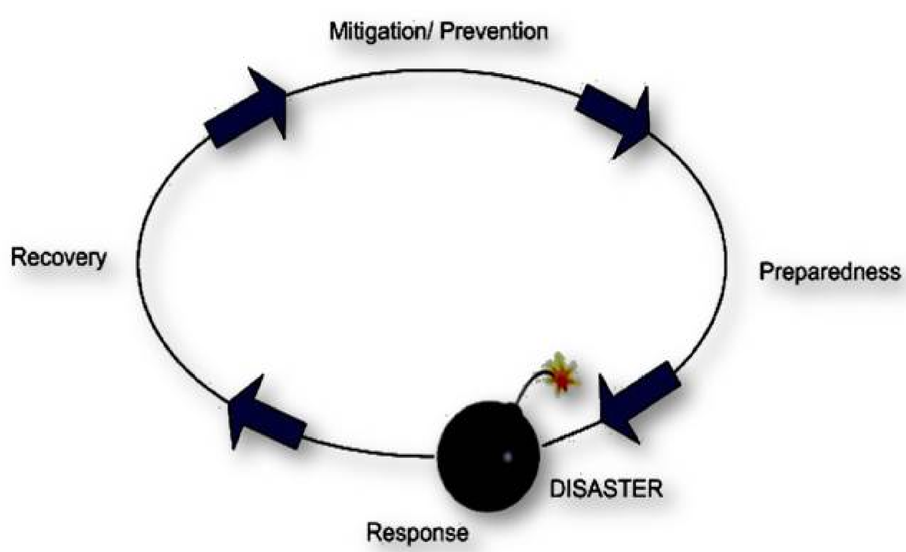

Figure 1.1: The disaster life cycle

[1]Source: Ciottone (2006)

Humanitarian logistics form part of disaster management, specifically focusing on the preparedness and response phases. Kovacs and Spens (2007) define the characteristics of humanitarian logistics as the unpredictability of demand, suddenness of its occurrence, the high stakes associated with the timeliness of deliveries, and the lack of resources. Evidently, humanitarian logistics can be summed up into the following factors, namely, time, capacity, resources and location.

Tomasini and Van Wassenhove (2009) emphasize that the first 72 hours after a disaster has occurred are crucial to save the maximum human lives. Saving lives, however, relies on the correct quantity and types of aid supplies, which would be a fairly effortless exercise, if all disaster incidents were predictable. Arminas (2005) suitably describes this predicament as follows:

> *"...purchasing and logistics for major disaster relief is like having a client from hell: You never know beforehand what they want, when they want it, how much they want and even where they want it sent."*

To address this complexity, it is vital that relief supplies be pre-positioned, managed and effectively transported to disaster affected areas to improve emergency response times. This will ensure a capable and practical disaster operations life cycle for any disaster-prone country. Demand for aid supplies vary in type and quantity depending on a specific disaster, the level of destruction caused and the available infrastructure of the particular country. These supplies must meet the immediate needs of those affected and will include the supply of items such as food, medicine, tents, sanitation equipment, tools and related necessities (Whybark, 2007).

1.2 Research Methodology

1.2.1 Managerial Implications

The aid sector lacks operational knowledge and is not equipped with the latest methods and techniques available to solve disaster relief complexities (Kovacs and Spens, 2007). Even though appropriate methods are presented for disaster preparedness, its full potential will not be achieved until the other phases in the disaster operations life cycle are properly planned and managed. All four phases need to be well collaborated and co-ordinated to complete a successful life cycle. A humanitarian organisation may be well prepared for a disaster, but may lack effective re-action if the response and recovery phases are not properly managed.

A further consideration is that, although a pre-positioned facility can be effectively stocked with the correct amounts and types of aid supplies, the logistics involved about receiving the items from suppliers and transporting these supplies to disaster affected areas involve vital managerial responsibilities. The inventory for disaster relief must be kept up to date, with reference to the quantity and types of aid supplies. Certain countries have poor infrastructure and therefore transport and prepositioning need to be well pre-prepared. A final consideration is the prevention of crime, such as theft, which is of great concern in certain countries. Consequently a prepositioned facility should be fully equipped with security measures.

Analysis of humanitarian logistics thus shows that risk management, crisis management, continuity planning and project management form an important part of the logistics process. For successful recovery from a disaster, organisations need to incorporate these managerial processes into the disaster operations life cycle, which will lead to increased assurance that lives will be saved.

1.2.2 Questions to be addressed

In order to evaluate all aspects of adequate disaster management, various factors need to be addressed:

- The nature of the inventory to be introduced during and after a disaster has occurred, taking into consideration that some products may be perishable;
- the likely occurrence of man-made or natural disasters with reference to certain countries;
- location of the pre-positioned facilities and how many facilities are required;
- the practical and cost-effective transport required to correspond with a region's infrastructure;
- the quantity of aid supplies required;

- the anticipated consequences of any disaster and how measured;
- the optimal man-power required to maximise recovery;
- dependency of disasters on each other and how this should be approached;
- identification of risks and management thereof, whether general or country specific;
- the impact of disasters different for each country, requiring the management thereof to be personalised to suit the conditions of each country.

All these problem areas should be approached in a generic manner so that the solutions may apply to any affected country.

1.2.3 Research Objectives

By addressing the above factors, it becomes apparent that the main problem areas of immediate response lie in the pre-positioning and the distribution of aid supplies influenced by the unpredictability of demand (Tomasini and Van Wassenhove, 2009). The serious consequences of inadequate management of disasters raises the following question:

> *How can the logistics of disaster management in a disaster response network within a country be determined?*

This research seeks to resolve the issues by encouraging the formulation of mathematical models as a means of anticipating the types and quantities of aid supplies to be kept in a pre-positioned facility, and the transportation of these supplies to disaster affected areas. Eccles and Groth (2007) describe the reason for addressing problem solving with technologies such as mathematical modelling as follows:

> *"Problems often impose demands that cannot be met given the natural human cognitive and physical limitations. One solution is to adapt oneself to the problem through training and practice, but when humans are unable or unwilling to adapt themselves, they often turn to adapting the environment, by creating technologies, to augment their problem-solving capabilities."*

For the above reason, this research is based on the formulation of mathematical models. These models provide a generic approach that can be applied to any pre-positioned facility for disaster preparedness within a specified country, by simply entering relevant input values, such as disaster types, their estimated impact and the specified locations.

The proposed models aim to maximise the recovery capability of disaster victims. The formulation encompasses likely disasters that may occur, taking into account the possible effect and impact of any such disaster. The output of the models provide the amount and types of aid supplies required subsequent to a disaster. These supplies will be retained in inventory at prepositioned facilities. It must however be anticipated that supplies may be damaged during a disaster event and that perishables will have to be replaced if not used within a certain time period. This will evidently affect the types and quantities of aid supplies to be stocked. For this reason, risk management forms part of the formulation of the models. Appropriate transport is selected and consequently the most suitable route available within the designated region should be determined.

1.2.4 Importance of the Research Problem

Countries around the world do suffer from the effects of disasters which occur most frequently. Improving the logistics of disaster management will benefit communities around the world. Somalia, for example, has been without an effective central government since 1991. According to BBC News, there have been years of conflict between rival warlords causing an inability to deal with famine and disease which have led to the deaths of up to one million people (BBC, 2011). In Southern Africa a research gap has been identified in the pre-positioning of aid supplies. A country such as Somalia and the countries of the Southern African Development Community (SADC) will benefit substantially from proper logistics for disaster relief.

1.2.5 Limitations and Assumptions of the Study

It is known that natural and man-made disasters remain unpredictable. A disaster may or may not occur and the effects may be worse or less than expected. The planning, response and recovery phases may be over-planned or insufficient to meet the needs of disaster victims.

Further, should a disaster fail to occur within a specific region, a loss will be made if all the supplies are not used. However, supplies should be sold to defray losses if not used within a certain time frame. Mitigation involves the implementation of laws and mechanisms to reduce the vulnerability of a population. Recovery comes after the response phase and is executed when surviving institutions and infrastructure seek to restore some form of normality to disaster victims. This is usually considered an improvement to the disaster area. Since the focus of this research is on humanitarian logistics, the mitigation and recovery phases fall outside the scope of this dissertation.

1.3 Concluding Remarks and Scope of the Work

This research comprises a comprehensive literature review of existing methods used to resolve the logistics of disaster management. In addition, data is gathered from various disasters, countries and in general the provision of aid supplies. The aim of this research is to fully understand the impact of disasters in a selected region and to provide sufficient relief to victims in the process of disaster management. An extensive literature review is undertaken to analyse previous models developed for the purpose of stock pre-positioning and distribution for disaster relief. The gathered information is used as a framework to serve as a basis to develop generic models for the specific countries examined.

In order to obtain an indication of the nature and quantity of aid supplies required, further research is done on the impact of possible disasters in a country in terms of the number of people affected. The variety of supplies varies with time, during and after a disaster. These varying needs are addressed with "probability distributions". Finally, optimal locations will be considered to ensure effective coverage for disaster victims. Various methods are utilised to illustrate the functionality of the models, as well as case studies applied to Somalia and the SADC countries.

In summary, humanitarian logistics forms part of disaster management, specifically focusing on the preparedness and response phases. Due to the suddenness of the occurrence of the disasters, governments must prepare for strategic inventory management as one of their important strategies. Several models of disaster inventory management are studied in this dissertation:

Chapter 1 is introductory in nature and gives a brief description on the importance of disaster management, humanitarian logistics and different issues to be addressed when disasters occur.

The literature review addresses the problem variants, the methodology and the various techniques used in the models of the dissertation. This is discussed in Chapter 2.

In Chapter 3, four theoretical models, in particular stochastic models have been studied to analytically pre-position humanitarian facilities and determine the capacity levels of the inventory required during a disaster event.

The anticipated quantities and types of aid supplies required during a disaster event is studied in Chapter 4, with the aid of a Deterministic Inventory Model (DIM), Stochastic Inventory Model (SIM) and an Euclidean Distance Algorithm (EDA). The results of the DIM, SIM and EDA are analyzed in a comparative study. It is justified that these models may serve as an effective decision-tool for emergency organizations in Somalia and the SADC region. Somalia is introduced as a case study due to the various man-made disasters taking place within the country.

In Chapter 5, a pre-emptive multi-objective optimization inventory model for pre-positioning facilities is developed for Somalia and the SADC region.

A Simulated Annealing (SA) heuristic is developed in Chapter 6, to determine the locations of pre-positioning facilities for the SADC region. This model is likewise justified as advantageous to decision-makers.

In Chapter 7, the concluding remarks of the dissertation are presented. The conclusion places emphasis on why "Industrial Engineering" expertise is required to solve the problems addressed in this dissertation.

Chapter 2

Literature Review

2.1 Introduction

This chapter focuses on obtaining an overview of inventory management in humanitarian logistics. Section 2.2 gives a background of the countries of interest for this dissertation; the SADC region and Somalia. Section 2.3 addresses the challenges present in humanitarian logistics. Section 2.4 highlights potential solutions. Section 2.5 explains techniques used in the analysis of the disaster relief models in Chapter 3. Section 2.6 summarizes this chapter.

2.2 The Countries of Interest

The regions addressed in this dissertation are specifically chosen due to the absence of proper inventory management methods and techniques in these disaster-prone areas. These countries are categorized as developing countries in which some of the areas have dilapidated infrastructures. Even a small disaster will thus demolish these regions if proper precautions are not introduced. This study will contribute prolonged value to inhabitants residing in these areas.

2.2.1 An Overview of Disasters in the SADC

The Southern African Development Community (SADC) was established in 1980 by the majority ruled countries in Southern Africa. The SADC currently comprises 15 member states, namely; Angola, Botswana, Democratic Republic of Congo (DRC), Lesotho, Madagascar, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Swaziland, United Republic of Tanzania, Zambia and Zimbabwe. These countries are shown in Figure 2.1[2]. Currently, the total population size of the SADC community is 257.7 million inhabitants (SADC, 2012).

Figure 2.1: The SADC countries

The United Nations International Strategy for Disaster Reduction (UNISDR) Secretariat maintains that there has been remarkable progress in climate science and technology. The increase in climate forecasting has allowed disaster managers to improve prevention, mitigation, preparedness, early response and recovery programs. Past events, however, have shown that the challenge is to integrate climate science into disaster risk reduction policy and operations. There is a lack of organized, pro-active and sustained partnerships, which means that even though reliable disaster data exists, there is little or no action taken to concretely reduce related risks (UNISDR, 2011). To bridge these gaps, it is important to utilize reliable

[2]Source: ACTSA (2012)

updated data correctly and frequently and to maintain the disaster management phases.

The UNISDR has revealed that Africa is one of the continents mostly affected by climate change and variability, which conditions are further aggravated by existing socio-economic challenges. In the SADC, 90 percent of all disasters that occur in the region are climate related, causing food insecurity and water stress, which are critical in these countries due to their reliance on agricultural products (UNISDR, 2011). The SADC region has a disaster profile characterized by drought, floods and cyclones, mostly attributed to the Zambezi River and tropical cyclones in Madagascar and Mozambique. Floods and drought have been the main focus of early warning and disaster preparedness strategies in the region over the past 20 years.

2.2.2 Somalia as a Case Study

Somalia, a country situated in Eastern Africa (Figure 2.2[3]) is currently characterized as a suffering and failed state. Since the spring in 2011, Somalia experienced a drought, which is considered to be the worst in 60 years (Bureau of African Affairs Somalia, 2012).

The country has suffered from crop failure, an extreme rise in food prices, as well as the grip of "Al-Shabaab" on central and south Somalia. These factors have forced the United Nations (UN) to declare famine in six areas of Somalia. These areas have currently been reduced to three areas due to the involvement of humanitarian organizations over the past few years. This increased involvement, however, has been insufficient as there are still 3.7 million victims in need of emergency assistance and 250,000 in danger of dying. The famine has compelled thousands of victims to move to overfull refugee camps in Ethiopia, Kenya, and Djibouti, while other victims have fled to Internally Displaced Persons camps in Mogadishu (Bureau of African Affairs Somalia, 2012). The United States, UN, and international humanitarian organizations have ongoing endeavours to address the immediate needs of the victims in Somalia (Bureau of African Affairs Somalia, 2012). These unfortunate circumstances leave Somalia in need of a more permanent solution, i.e. more research should be done so that adequate solutions are obtained and facilities should be pre-positioned in disaster areas with a sufficient supply of relief aid.

[3]Source: (Somalia Pirates, 2012)

Figure 2.2: Somalia

2.3 Humanitarian Logistics Challenges

The onset of disaster provokes cries for help requiring immediate response. Tomasini and Van Wassenhove (2009) identify this immediate response as: **"...the right goods, at the right time, to the right place, and distributed to the right people."** This statement proposes that the following factors should be addressed namely, time, resources, capacity and location, which forms the framework of this research. The logistics strategy is to implement these factors thereby creating a humanitarian supply chain which is agile, adaptable and aligned, producing an effective disaster cycle (Figure 1.1).

Considerable literature has addressed the management of disaster relief organizations. Much of this deals with the social and organizational implications of responding to disasters in many parts of the world, including countries with poor infrastructure that may be involved in hostilities. Blecken et al. (2010) state that even though research contributions to supply chain management in the context of humanitarian operations have increased, a serious gap remains when considering humanitarian logistics in countries such as the SADC and Somalia. Being prepared for a disaster requires the knowledge of knowing when or where an event is likely to take place, how many people will be affected and what supplies will be required. Despite the progress that disaster planning, mitigation and new management systems have made, the need for relief, specifically in underdeveloped countries, still remains (Whybark, 2007). Improving disaster relief planning and management is a continuous process.

Due to the unpredictable nature of a disaster, disaster management is a process that cannot be comprehensively controlled. Altay and Green (2006) explain that even though it is known that response to disasters requires effective planning, it is essential to leave room for improvisation to deal with the unusual challenges that manifest. Hills (1998) approvingly states that the phrase "disaster management" implies a degree of control, which rarely exists in disaster cases. It is for this reason that Standard Management Methods used in industry may not always apply directly to disaster situations (Hills, 1998).

Rawls and Turnquist (2009) raise an added concern, namely that the capacities of resource providers are the key components in managing response efforts subsequent to disaster events, but that very little research has been conducted on the planning and distribution of aid supplies kept in inventory at prepositioned facilities. In addition, Duran et al. (2009) maintain that an important element to take into account when considering stock pre-positioning is that facilities should always have sufficient inventory to satisfy demand. It should also be considered that stored aid supplies may be destroyed during a disaster event (Duran et al., 2009). The pre-positioned stock should thus meet the needs of a disrupted region by taking the effect of the disaster into consideration (Bryson et al., 2002). Any shortcomings may result in serious consequences for victims of disasters and could mean the difference between life and death (Tomasini and Van Wassenhove, 2009). Public demand therefore expects accurate orders and that humanitarian supply chains need to be more adaptable and agile towards the changing needs of disaster victims (Tomasini and Van Wassenhove, 2009). This need requires effective methods to address the uncertainty of a disaster and possible solutions.

2.4 Existing Solutions

On 22 December 1989, the General Assembly of the UN unanimously resolved to make prevention and preparedness against disasters caused by natural extreme events its task and declared the 1990s as the International Decade for Natural Disaster Reduction (IDNDR) (Plate and Kron, 1994). The purpose was to inform people in disaster prone countries about protective measures against natural disasters. Plate and Kron (1994) review the history of disaster management which has existed since the beginning of time. Consequently, the need to prepare, respond and recover from disasters will continue till the end of time. This highlights the importance of further research is this field, specifically to achieve more viable solutions.

The majority of practical solutions to disaster management problems are supported by mathematical methods and operations research techniques (Van Wyk et al., 2011b). These approaches provide effective tools for planning the preparedness, response and recovery phases of disaster management, as they address uncertainty by means of probabilistic scenarios. These scenarios represent disasters and their outcome [(Mete and Zabinsky, 2009), (Snyder, 2006), (Beraldi and Bruni, 2009), (Beamon and Kotleba, 2006)].

In order to obtain an accurate view of what research has been conducted in the area of disaster management, it is necessary to understand where disaster relief models originated. De la Torre et al. (2012) compiled a summary of characteristics in disaster relief distribution methods. The earliest publications depicted in this summary is by Knott (1987). The author addresses the logistics of bulk relief supplies by focusing on minimising unsatisfied demand, single depots and heterogeneous vehicles. The author limits unsatisfied demand, whilst introducing vehicle routes which begin and end at a single depot. The vehicles introduced vary in transportation capacity, speed and fuel consumption. Knott (1988) extends this research by introducing multiple types of goods, each with different application and demand. More recent publications have evolved and improved immensely. Some of these valuable contributions are discussed in the "Present-State-of-Art" (PSA).

The PSA of the sourced literature is compiled for a period of 10 years, from 2002 to 2012. The PSA is portrayed in Table 2.1, Table 2.2 and Table 2.3. Table 2.1 addresses the literature associated with the preparedness phase. Table 2.2 discusses the literature associated with the response phase and finally, Table 2.3 addresses the literature associated with both the preparedness and response phases. This approach is acquired due to the scope of the dissertation which comprises the humanitarian logistics phases, i.e. preparedness and response as explained in Section 1.2.5. The tables are divided into the following headings: year, author/s, "method and approach" and "contribution and/or comments". The "method and approach" outlines the main methodology of the paper, whereas the "contribution" emphasizes the significance of the results obtained and finally "comments" addresses any future work or limitations of the work done. The 10 year survey gives an appropriate overview of current analytical modelling and techniques applicable to humanitarian logistics. The researched literature specifically highlights relevant publications to supporting the methods and techniques presented in this dissertation during the past 10 years.

Table 2.1: **PSA (Preparedness Phase)**

Preparedness Phase Literature			
Year	**Author/s**	**Method and Approach**	**Contribution and/or Comments**
2002	Barbarosoğlu, Özdamar & Cevik	•Hierarchical multi-criteria methodology	•Helicopter mission planning •Procedure to establish a framework for revealing different aspiration levels for various objectives
	Bryson, Millar, Joseph & Mobulurin	•Development of DRPs for organizational preparedness •Mathematical model maximizing total value of coverage	•Reasonable solutions in short time •Forward step in providing MS/OR literature for disaster relief
2003	Morris & Wodon	•Conceptual model suggesting funding allocation	•Higher weight placed on losses than on pre-disaster wealth when targeting relief efforts
	Zerger & Smith	•Evaluation of GIS for cyclone disaster risk management	•Model scenarios and interact with the spatial dimension of disasters

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Year	**Author/s**	**Method and Approach**	**Contribution and/or Comments**
		•Observing real-time emergency management disaster scenarios	•Failure exists due to implementation, user access and knowledge impediments against the availability of spatial data and models
2005	Shao	•Discrete optimization model •Maximize the overall survival capability of an organization's IT functions	•Allocation of redundancy to critical IT functions for disaster recovery planning •Extended from Bryson et al. (2002) •Incorporate redundancy into critical IT functions and maximize survival against disasters
2007	Jia, Ordonez & Dessouky	•Facility location model as covering model: P-median and P-center	•Reducing loss of life and economic losses
	Chang, Tseng & Chen	•Flood emergency logistics problem formulated as Stochastic programming model	•Decision-making tool to be used by government agencies •Flood emergency logistics

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Year	Author/s	Method and Approach	Contribution and/or Comments
2008	Balcik & Beamon	•Facility location decisions for humanitarian relief chains •Maximal covering location model •Integrates facility location and inventory decisions	•Highlights important implications for decision-makers •Number of scenarios, items and pre-positioning facilities must be increased
2009	Lodree Jr. & Taskin	•Stochastic inventory control problem •Inventory control problem formulated as optimal stopping problem with Bayesian updates	•Proactive disaster recovery planning for potential hurricane activity •Solution appropriate to manage hurricanes predictions •Only considers two demand classes per hurricane category
2010	Wu, Ru & Li	•Normalizing Euclidean Distance algorithm to forecast demand	•Accurate prediction of emergency supplies
	Rawls & Turnquist	•SMIP	•Emergency response pre-positioning strategy for hurricanes
	Salmerón & Apte	•Two-stage stochastic optimization model	•Allocation of budget to acquire and position relief assets

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Year	Author/s	Method and Approach	Contribution and/or Comments
2011	Campbell & Jones	•Development of equations for determining optimal stocking quantities •Equations to derive total costs when delivering to a demand point from a supply point	•First publication to consider both risk and inventory levels without the use of scenarios

Table 2.2: **PSA (Response Phase)**

Response Phase Literature			
Year	**Author/s**	**Method and Approach**	**Contribution and/or Comments**
2004	Barbarosoğlu & Arda	•Two-stage stochastic programming model, planning the transportation of vital first-aid supplies to disaster-affected areas •Multi-commodity, multi-modal network flow formulation for flow of material	•Reveals value of information when uncertainty exists •Enhances early warning and quick response performance
2006	Mendonça, Beroggi, van Gent & Wallace	•Gaming simulation to assess decision support	•Recognize that crises events are influenced by judgements and behaviour
2007	Clark & Culkin	•Mathematical transshipment multi-commodity supply chain network model	•Applied to earthquakes •Better understanding of humanitarian supply chains

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Year	**Author/s**	**Method and Approach**	**Contribution and/or Comments**
	Sheu	•Hybrid fuzzy clustering-optimization approach •Numerical study: Real large-scale Taiwan earthquake data	•Method for the operation of emergency logistics co-distribution responding to relief demands •Ensures response during the three-day crucial rescue period
	Yi & Özdamar	•Mixed integer multi-commodity network flow model •Fast relief access to affected areas •Locating temporary emergency units in appropriate sites	•Coordinating logistics support and evacuation operations •Maximizing response service levels •Can handle large numbers of vehicles
	Gong & Batta	•Ambulance allocation and reallocation methods •Deterministic model to depicts cluster growing after disaster strikes	•Focus: One disaster •Limitation: Balance between long waiting times and frequent re-allocations
2008	Balcik, Beamon & Smilowitz	•Mixed integer programming model determining the delivery schedules/routes for each vehicle	•Minimizing transportation costs and maximizing benefits to disaster victims

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Year	Author/s	Method and Approach	Contribution and/or Comments
		•Allocation of supply, vehicle capacity and delivery time restrictions	•Faster algorithms required to enable testing of more complex problems
2009	Yuan & Wang	•Single objective path selection model taking into account that travel speed on each arc will be affected by disaster extension •Minimize total travel time along path •Dijkstra algorithm used to solve single-objective model •Multi-objective path selection model minimizes total travel time along path and path complexity •Ant colony optimization algorithm used to solve multi-objective model	•Results show correctness and effectiveness and feasibility •Only path selection problem considered
2010	Sheu	•Relief-demand management model	•Approximating relief demands under information uncertainty •Allocating relief demand

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Year	Author/s	Method and Approach	Contribution and/or Comments
			•Relief-demand model rarely found in literature
2011	Lin, Batta, Rogerson, Blatt & Flanigan	•Delivery of prioritized items: multi-items, multi-vehicles, multi-periods, soft time windows, split delivery strategy scenario •Multi-objective integer programming model •Two heuristic approaches: genetic algorithm and decomposition of original problem	•Improvement of Balcik et al. (2008): Demand is served immediately when it occurs •Need to consider temporary depots •Long travel distances
2012	Rottkemper, Fisher & Belcken	•Mixed-integer programming model •Minimization of unsatisfied demand •Minimization of operational costs •Model solved by rolling horizon solution method	•Study of trade-ff between demand satisfaction and logistical costs •Model applicable with single relief item •Need to introduce capacity of humanitarian staff

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Year	Author/s	Method and Approach	Contribution and/or Comments
	Lin, Batta, Rogerson, Blatt & Flanigan	•Location of temporary depots around disaster affected areas •Two phase heuristic approach	•Focus: Earthquake •Need to account for depots being destroyed

Table 2.3: **PSA (Combination of phases)**

Combination of Humanitarian Logistics Phases			
Year	**Author/s**	**Method and Approach**	**Contribution and/or Comments**
2006	Altay & Green III	•Survey literature to identify potential research directions in disaster operations	•Literature survey 1984 - 2004 •Literature categorized: mitigation, preparedness, response, recovery •Starting point for further research •Encourage research collaboration
	Van Wassenhove	•Survey to highlight complexities of managing supply chains	•Emphasizes that private sector logistics should be used to improve disaster logistics Initiates the need for collaboration between humanitarians, businesses and academics

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Year	Author/s	Method and Approach	Contribution and/or Comments
2007	Whybark	•Analysis of: nature of disaster relief, research on disaster relief, disaster relief inventories from storage to distribution	•Induce research agendas •Recognition of a lack of research for disaster relief inventories •Recommendations to develop better theories, systems and management guidelines
	Currion, De Silva & Van De Walle	•Open-access approach	•Innovation to save more lives
2010	Balcik, Beamon, Krejci, Muramatsu & Ramirez	•Review of challenges in coordinating humanitarian relief chains •Description of current coordination practices in disaster relief	•Emphasize importance of coordination •Emphasizes challenge to create integrated global relief chain
	Horner & Downs	•Warehouse location model •GIS-based spatial model	•Focus: Hurricane disaster relief •Limitations: Treat demand as a stochastic problem

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Year	Author/s	Method and Approach	Contribution and/or Comments
2011	White, Smith & Currie	•Review of the overall picture of OR in developing countries	•Instigate contributions towards poverty
2012	De la Torre, Dolinskaya & Smilowitz	•Analysis of the use of operations research models from both practitioners and academics perspectives •Interviews with aid organizations •Reviews of publications •Literature review of operations research models	•Enhanced methods to transport relief goods •Area with potential to help relief organizations •Opportunity for growth in OR
	Noyan	•Two-stage stochastic programming •Uncertainty in demand and damage level of the disaster network	•Determining response facility locations and inventory level
	Caunhye, Nie & Pokharel	•Review of optimization models utilized in emergency logistics	•Identify gaps: Facility location, Linkage between shelters and stores, Capacity planning and inventory planning

2.4.1 Salient features of the Present-State-of-Art

The primary focus of the PSA is to accentuate relevant research conducted within the past 10 years. This time period permits a reasonable overview of the comprehensive research being done in the field of inventory management for humanitarian logistics as follows:

(i) **Preparedness Phase:** Literature addressing this phase during earlier years, emphasize the inadequate research done and the importance to stimulate this area of research to save lives. The general contribution of the earlier mathematical models are directed towards single disasters. More recent papers focus on the combination of inventory quantities required and optimal facility locations. In addition, as research progressed, more consideration was given to the unpredictability of demand for disaster relief. The current models developed therefore focus specifically on stochastic modelling. An added concern for future research is to solve more complex problems, i.e. consider various aid supplies, various scenarios and more than one disaster.

(ii) **Response Phase:** The earlier stages of the research done focused on acquiring accurate information to compliment the results of the models. It is further revealed that allocation of relief-demand has only been addressed recently. It is also noted that single facilities were utilized for distribution purposes, which were inadequate. Researchers have now incorporated the need to consider multiple facilities (Lin et al., 2011). Finally, future research considerations should also focus on the possible destruction of facilities during the event of a disaster.

(iii) **Combination of the Humanitarian Logistics Phases**: Most of the studies comprising the preparedness and response phases are publications in the form of literature surveys. The concerns in these papers are aimed at the necessity to incorporate private sector logistics and humanitarian logistics. This is supported by the need emphasized to initiate collaboration between businesses, humanitarians and academics (Whybark, 2007).

Even though a vast volume of literature exists, new directions for operations research and other analytical methods should remain a continuous process. There are numerous factors which still need to be considered and various innovative methods and techniques to be adopted and/or improved. According to Caunhye et al. (2012), the research gaps in humanitarian logistics exist within capacity planning, facility location and inventory planning. The purpose of this dissertation is therefore to address some of these gaps. Such techniques used are discussed in the next section.

2.5 Techniques used in the Stochastic Models of Disaster Relief

This section analyses various stochastic processes including Poisson processes, stochastic point processes and stochastic differential equations, for the analysis of various stochastic disaster relief models in Chapter 3.

2.5.1 Stochastic Processes

Definition: A stochastic process is a family of random variables $\{X(t) \mid t \in T\}$, defined on a given probability space, indexed by the parameter t, where t varies over an index set $T \subset R$. More explicitly, the family $\{X(t), t \in T\}$ is called a stochastic process for every $n \in N$ and $t_1, t_2, \ldots, t_n \in T$, the set of random variables $X(t_1), X(t_2), \ldots, X(t_n)$ have the joint probability distribution.

2.5.2 Process with Independent Increments

Definition: A stochastic process $\{X(t); t \in T\}$ has independent increments if for all $n = 3, 4, \ldots$ and for all n time points $\{t_1, t_2, \ldots, t_n\}$ with $t_1 < t_2 < \ldots < t_n$ and $t_i \in T$, the increments $X(t_2) - X(t_1), X(t_3) - X(t_2), \ldots, X(t_n) - X(t_{n-1})$ are independent random variables.

2.5.3 Stochastic Point Processes

Stochastic point processes are from a class of processes which are more general. This is due to the point processes having several definitions, each appearing quite natural from the view point of the particular problem under study [(Bartlett, 1966), (Harris, 1971), (Khintchine, 1955), (Moyal, 1962)].

Stochastic point processes is a mathematical abstraction which arises from considering such phenomenon as a randomly located population, or a sequence of events in time. Generally, a stochastic point process can be defined as a continuous time parameter discrete state space stochastic process. Typically there is a state space X, and a set of points x_n from X representing the locations of the different members of the population or the times at which the events occur. Because a realization of these phenomena is just a set of points in time or space, a family of such realizations is defined as a point process.

It is now proposed that X is the real line. One of the ways to characterize such a stochastic point process is through product densities in the case of non-renewal processes (Srinivasan, 1974) . Let $N(t,\tau)$ denote the random variable representing the number of occurrence of events in the interval $(t, t+\tau)$. Then the product density of order n is defined as:

$$h_n(t_1, t_2, \dots t_n) = \lim_{\Delta_i \to 0, i=1,2,\dots n} \frac{P[N(t_i, \Delta_i) > 1]}{\prod_{i=1}^{n} \Delta_i}; t_1 \neq t_2 \neq \dots \neq t_n \tag{2.1}$$

A point process defined on the real line is said to be regular or orderly if the probability of occurrence of more than one event in $(t, t+\Delta)$, where Δ is small, is of order $\circ(\Delta)$. For a regular point process the n^{th} order product density is equivalent given by:

$$h_n(t_1, t_2, \dots, t_n) = \lim_{\Delta_i \to 0, i=1,2,\dots n} \frac{E[\prod_{i=1}^{n} N(t_i, \Delta_i)]}{\prod_{i=1}^{n} \Delta_i}; t_1 \neq t_2 \neq \dots t_n \tag{2.2}$$

These densities represent the probability of an event in each of these intervals $(t_1, t_1+\Delta_1)$, $(t_2, t_2+\Delta_2)$,..., $(t_n, t_n+\Delta_n)$. Even though the functions $h_n(t_1, t_2, \dots, t_n)$ are called densities, it is important to note that their integration will not give probabilities but will yield the factorial moments. The ordinary moments can be obtained by relaxing the condition that the x_i's are distinct. For instance the mean number of events in the interval $[0, t]$ is given by:

$$E[N[0,t]] = \int_0^t h_1(t)dt \quad (2.3)$$

A point process is said to be completely stationary, if:

$$P[N(t_i,\tau_i) = n_i; i = 1,2,\ldots,n] = P[N(t_i + h,\tau_i) = n_i; i = 1,2,\ldots,n] \quad (2.4)$$

For all $t_i, \tau_i, n_i \geq 0, h \geq 0$ and for all positive integral values of n. Passivity of a point process implies that $\lim_{t\to\infty} h_1(t) = \mu$, (a constant); (Khintchine, 1955). In addition, by Korolyuk's theorem (Khintchine, 1955), a stationary process is regular if and only if:

$$\mu = \lim_{\Delta\to 0} \frac{P[N(t,\Delta) \geq 1]}{\Delta} \quad (2.5)$$

2.5.4 Homogeneous Poisson Process

Definition: A stochastic point process $\{X(t), t \geq 0\}$ is a homogeneous poisson process with intensity $\lambda, \lambda > 0$, if it has the following properties:

(i) $X(0) = 0$

(ii) $\{X(t), t \geq 0\}$ is a stochastic process with independent increments.

(iii) The increments of the process in any interval $[s,t], s < t$, are Poisson distributed with parameter $\lambda(t-s)$:

$$P[X(t) - X(s) = i] = \frac{[\lambda(t-s)]^i}{i!} e^{-\lambda(t-s)}; i = 0,1,\ldots \quad (2.6)$$

If $\tau = t - s$, then:

$$P[X(s+\tau) - X(s) = i] = \frac{(\lambda\tau)^i}{i!} e^{-\lambda\tau}; i = 0, 1, \ldots \tag{2.7}$$

2.5.5 Stochastic Differential Equations

The study of several physical and biological phenomena requires the applications of stochastic processes to model the quantities which describe the phenomena. The dynamics of these quantities are modelled by differential equations involving stochastic processes. Such differential equations are referred to as "stochastic differential equations". Because of the nature of the stochastic processes (Brownian motion or Weiner Process and Poisson Process) which are used to model the random phenomena, the rules of ordinary calculus can not be applied. Instead, a separate calculus (termed as stochastic calculus) is developed to analyze the stochastic differential equations. Almost all dynamical systems are modelled by the stochastic differential equations:

$$dX(t) = f[X(t), \zeta(t)]dt; X(0) = x(0) \tag{2.8}$$

Where, $X(t)$ represents the state of the system at time t, $\zeta(t)$ is some kind of stochastic noise process affecting the deterministic behaviour of the system and $f[X(t), \zeta(t)]$ is the rate of growth of the state variable of the system. The stochastic noise process $\zeta(t)$ is usually taken as the white-noise or the shot-noise. Even in simple cases, the presence of such noises makes the analysis quite complex. The monograph of Srinivasan and Vasudevan (1971) brings out several applications of stochastic differentials equations in modelling several response phenomena arising in physical and engineering sciences. The excellent treatise of Arnold (2012) gives the theory and applications of differential equations in several control problems. To highlight the difficulty in handling stochastic differential equations, one should consider a dynamical system which is presented by:

$$dX(t) = rX(t)dt + \sigma X(t)dN(t) \tag{2.9}$$

Where $N(t)$ is a Poisson process with rate λ, it is necessary to find $E[X(t)]$. For this, one has to get $X(t)$ from equation 2.9. Setting $Y(t) = \ln X(t)$, thus:

$$\begin{aligned}
dY(t) =& Y(t+dt) - Y(t) \\
&= \ln X(t+dt) - \ln X(t) \\
&= \ln\left(\frac{X(t+dt)}{X(t)}\right) \\
&= \ln\left(1 + \frac{X(t+dt) - X(t)}{X(t)}\right) \\
&= \ln\left(1 + \frac{dX(t)}{X(t)}\right) \\
&= \frac{dX(t)}{X(t)} - \frac{1}{2}\left(\frac{dX(t}{X(t)}\right)^2 + \frac{1}{3}\left(\frac{dX(t)}{X(t)}\right)^3 \dots \\
&= rdt + \sigma dN(t) - \frac{1}{2}(rdt + \sigma dN(t))^2 + \frac{1}{3}(rdt + \sigma dN(t))^3 \dots \\
&= rdt + [\sigma - \frac{1}{2}\sigma^2 + \frac{1}{3}\sigma^3 \dots]dN(t) \\
&= rdt + \ln(1+\sigma)dN(t)
\end{aligned} \tag{2.10}$$

and so the result is:

$$Y(t) = Y(0) + rt + \ln(1+\sigma)N(t) \tag{2.11}$$

from which:

$$X(t) = X(0)e^{rt + \ln(1+\sigma)N(t)} \tag{2.12}$$

From the equation 2.12, the following is obtained:

$$\begin{aligned}
E[X(t)] =& X(0)e^{rt}\sum_{n=0}^{\infty} e^{\ln(1+\sigma)}\frac{e^{-\lambda t}(\lambda t)^n}{n!} \\
=& X(0)e^{rt}e^{-\lambda t}\sum_{n=0}^{\infty}\frac{[\lambda(1+\sigma)t]^n}{n!} \\
=& X(0)e^{rt}e^{-\lambda t}e^{\lambda(1+\sigma)t} \\
=& X(0)e^{(r+\lambda\sigma)t}
\end{aligned} \tag{2.13}$$

In achieving the above result, a more sophisticated approach was used, namely the "Itö-differentiation rule", for shot-noise process. In the same way, the Itö-differentiation rule for white-noise process manifests. This approach of stochastic differential equations is used in Chapter 3 for the disaster models.

2.6 Conclusions

In this chapter, the focus was on literature specifically addressing important humanitarian logistic factors such as time, resources, capacity and location. These factors are fundamental to improve disaster logistics. Altay and Green (2006) state that disasters have hit, and will continue hitting our communities, businesses, and economies. It is therefore important to continuously develop and build on previous research done. This chapter provides the building blocks of the dissertation to construct various models in order to facilitate relief logistics decisions. These models are addressed in the subsequent chapters.

Chapter 3

Stochastic Models of Humanitarian Inventories for Disaster Relief Management

3.1 Introduction

Disaster relief management has become a challenging issue to governing organisations, due to the unpredictable nature of various events related to disasters. Natural disasters such as hurricanes, tsunamis and earthquakes occur in various parts of the world. These disasters either exterminate numerous people, leave many homeless or render people struggling to survive. Accordingly, governments plan to prepare communities to manage the extreme events by setting up pre-positioning facilities across vulnerable locations. These facilities store adequate medicines and food supplies for distribution in the event of disasters [(Kovacs and Spens, 2007),(Özdamar et al., 2004)].

In addition to these, man-power and transportation should also be made available at these locations. In some cases, the inventory locations themselves are prone to catastrophic events. Hence, strategic planning should be made to position the storage locations and the quantities to be stored (Beamon and Kotleba, 2006). Furthermore, in keeping such inventories, the main objective is purely humanitarian in the sense that the measure of benefit is calculated based upon the quantum of relief rendered and lives saved from disasters. In this chapter, a mathematical model is proposed to determine the storage locations and optimum storage levels of humanitarian inventories.

The remainder of this chapter is structured as follows: In Section 3.2, a non-spatial model is formulated to study the total relief measures offered by the government as a function of time. A spatial and time dependant model, which obtains the optimum positioning of inventory storage in a disaster-prone area, is developed in Section 3.3. In Section 3.4, the optimal positioning of humanitarian aid-supplies is studied, with the assumption of a Poisson process for the number of disasters. This is achieved with the help of utility maximization principle.

3.2 A Non-spatial Model

3.2.1 Analysis of the Model

In this section, a non-spatial model is formulated to address time as a crucial factor. Let there be N emergency inventories which are stationed at different N locations. Let the disasters/catastrophes (extreme events) occur as point events on the time axis. Whenever an extreme event occurs, a "Save Our Souls" (SOS) is recorded at all the N locations and actions are initiated from each inventory for providing relief at the disaster location. It is assumed that the i^{th} inventory is responding to the j^{th} disaster with probability p_{ij}. If an action taken by a pre-positioning facility for the n^{th} disaster is not realised before a fixed time T_n, then it is rendered useless without making any contribution to the disaster management activities.

It is assumed that the time taken by a relief action initiated from the i^{th} pre-positioning facility to reach the n^{th} disaster is a random variable having the probability density function $\mu e^{-\mu t}$; $t > 0$, $\mu > 0$. Not all the actions are realized at the disaster location due to problems of logistics. The number of successful relief actions realised at the j^{th} disaster location is denoted by X_j. It is assumed that X_j is binomially distributed with parameters N, p_j; where p_j stands for the probability that a relief is realized at the place of the j^{th} catastrophe.

Let the disasters occur at random time points t_1, t_2,... t_n on the time axis. Let $N(t)$ be the number of disasters that have occurred up to time t. Let λ be the rate of occurrence of disasters in a location. Then X_i's are independent and identically distributed binomial random variable with probability function:

$$P(X_i = r) = \binom{N}{r} p_i^r (1 - p_i)^{N-r}; r = 1, 2, \ldots, N \tag{3.1}$$

Let q_{ij} be the probability that the relief measure from the i^{th} location is successful for the j^{th} catastrophe. Then it is evident that:

$$q_{ij} = p_{ij}(1 - e^{-\mu T_j}); \quad i = 1, 2, \ldots N, \quad j = 1, 2, \ldots N(t) \tag{3.2}$$

Defining:

$$X_{ij} = \begin{cases} 1, & \text{if the contribution of the jth inventory for the ith disaster is successful} \\ 0, & \text{otherwise.} \end{cases} \tag{3.3}$$

Then it is noted that:

$$P(X_{ij} = 1) = q_{ij} = 1 - P(X_{ij} = 0) \tag{3.4}$$

This leads to the result that $E(X_{ij}) = q_{ij}$. Now $X_i = \sum_{j=1}^{N} X_{ij}$, then:

$$E(X_i) = \sum_{j=1}^{N} E(X_{ij}) = \sum_{j=1}^{N} q_{ij} = \sum_{j=1}^{N} p_{ij}(1 - e^{-\mu T_j}) \tag{3.5}$$

Alternatively, $E(X_i) = Np_i$. Consequently, then:

$$Np_i = \sum_{j=1}^{N} p_{ij}(1 - e^{-\mu T_j}) \tag{3.6}$$

$$p_i = \frac{1}{N}\sum_{j=1}^{N} p_{ij}(1 - e^{-\mu T_j}); \quad i = 1, 2, \ldots, N(t) \tag{3.7}$$

If the relief operation X_{ij} is successful, then it would have contributed a relief quantity Y_{ij} to the disaster management. Then the total contribution up to time t is given by:

$$\Phi(t) = \sum_{i=1}^{N(t)}\sum_{j=1}^{N} Y_{ij}X_{ij} \tag{3.8}$$

Clearly $\Phi(t)$ is a random quantity for which the following is found:

$$\begin{aligned} E[\Phi(t)] &= \sum_{n=1}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}\sum_{i=1}^{n}\sum_{j=1}^{N} Y_{ij}E(X_{ij}) \\ &= \sum_{n=1}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}\sum_{i=1}^{n}\sum_{j=1}^{N} Y_{ij}q_{ij} \\ &= \sum_{n=1}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}\sum_{i=1}^{n}\sum_{j=1}^{N} Y_{ij}p_{ij}(1 - e^{-\mu T_j}) \end{aligned} \tag{3.9}$$

For simplicity, the case $N = 2$ is considered, then:

$$\begin{aligned} E\left[\Phi(t)\right] &= \sum_{n=1}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}\sum_{i=1}^{n}\left[Y_{i1}p_{i1}(1 - e^{-\mu T_1}) + Y_{i2}p_{i2}(1 - e^{-\mu T_2})\right] \\ &= \sum_{n=1}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}\left[(1 - e^{-\mu T_1})\sum_{i=1}^{n} Y_{i1}p_{i1} + (1 - e^{-\mu T_2})\sum_{i=1}^{n} Y_{i2}p_{i2}\right] \end{aligned} \tag{3.10}$$

Hypothesis 1

Let the quantity Y_{ij} increase linearly with respect to the number of occurrences of disasters. To be specific, the assumption is made that:

$$Y_{ij} = a_j + b_j i; \quad j = 1, 2 \tag{3.11}$$

It is also assumed that each inventory is responding to an SOS call with equal probability. Then $p_{ij} = p$, where $0 < p < 1$. Consequently:

$$\begin{aligned}
E\left[\Phi(t)\right] =& \sum_{n=1}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}\left[(1-e^{-\mu T_1})\sum_{i=1}^{n}(a_1+b_1 i)p \right.\\
&\left. +(1-e^{-\mu T_2})\sum_{i=1}^{n}(a_2+b_2 i)p\right] && (3.12)\\
=& \sum_{n=1}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}\left[p(1-e^{-\mu T_1})\left\{na_1+b_1\frac{n(n+1)}{2}\right\}\right.\\
&\left. +p(1-e^{-\mu T_2})\left\{na_2+b_2\frac{n(n+1)}{2}\right\}\right] && (3.13)\\
=& p(1-e^{-\mu T_1})e^{-\lambda t}\left[a_1\sum_{n=1}^{\infty}\frac{(\lambda t)^n}{n-1!}+b_1\sum_{n=1}^{\infty}\frac{(n+1)(\lambda t)^n}{(n-1)!}\right]\\
&+p(1-e^{-\mu T_2})e^{-\lambda t}\left[a_2\sum_{n=1}^{\infty}\frac{(\lambda t)^n}{(n-1)!}+b_2\sum_{n=1}^{\infty}\frac{(n+1)(\lambda t)^n}{(n-1)!}\right] && (3.14)\\
=& p(1-e^{-\mu T_1})e^{-\lambda t}\left[a_1\lambda t\sum_{n=1}^{\infty}\frac{(\lambda t)^{n-1}}{(n-1)!}+b_1\sum_{n=1}^{\infty}\frac{(n-1+2)(\lambda t)^n}{(n-1)!}\right]\\
&+p(1-e^{-\mu T_2})e^{-\lambda t}\left[a_2\lambda t\sum_{n=1}^{\infty}\frac{(\lambda t)^{n-1}}{(n-1)!}+b_2\sum_{n=1}^{\infty}\frac{(n-1+2)(\lambda t)^n}{(n-1)!}\right] && (3.15)\\
=& p(1-e^{-\mu T_1})e^{-\lambda t}\left[a_1\lambda t e^{-\lambda t}+b_1\left\{\sum_{n=1}^{\infty}\frac{(\lambda t)^n}{(n-2)!}+2\sum_{n=1}^{\infty}\frac{(\lambda t)^n}{(n-1)!}\right\}\right]\\
&+p(1-e^{-\mu T_2})e^{-\lambda t}\left[a_2\lambda t e^{-\lambda t}+b_2\left\{\sum_{n=1}^{\infty}\frac{(\lambda t)^n}{(n-2)!}+2\sum_{n=1}^{\infty}\frac{(\lambda t)^n}{(n-1)!}\right\}\right] && (3.16)\\
=& p(1-e^{-\mu T_1})e^{-\lambda t}\left[a_1\lambda t e^{-\lambda t}+b_1\left\{(\lambda t)^2\sum_{n=2}^{\infty}\frac{(\lambda t)^{n-2}}{(n-2)!}+2\lambda t\sum_{n=1}^{\infty}\frac{(\lambda t)^{n-1}}{(n-1)!}\right\}\right]
\end{aligned}$$

$$+ p(1 - e^{-\lambda T_2})e^{-\lambda t}\left[a_2\lambda te^{-\lambda t} + b_2\left\{\sum_{n=1}^{\infty}\frac{(\lambda t)^n}{(n-2)!} + 2\lambda t\sum_{n=1}^{\infty}\frac{(\lambda t)^{n-1}}{(n-1)!}\right\}\right] \tag{3.17}$$

$$= p(1 - e^{-\mu T_1})e^{-\lambda t}\left\{a_1\lambda te^{\lambda t} + b_1\{(\lambda t)^2 e^{\lambda t} + 2\lambda te^{\lambda t}\right\}$$

$$+ p(1 - e^{-\mu T_2})e^{-\lambda t}\left[\left\{a_2\lambda te^{\lambda t} + b_2\left\{(\lambda t)^2 e^{\lambda t} + 2\lambda te^{\lambda t}\right\}\right] \tag{3.18}$$

$$= p(1 - e^{-\mu T_1})\left[a_1\lambda t + b_1\left\{(\lambda t)^2 + 2\lambda t\right\}\right] + p(1 - e^{-\mu T_2})\left[a_2\lambda t + b_2\left\{(\lambda t)^2 + 2\lambda t\right\}\right] \tag{3.19}$$

The above equation predicts that the quantum of relief measures provided by the pre-positioning facilities increases quadratically with respect to time.

Hypothesis 2

It is now assumed that the quantity Y_{ij} increases geometrically with respect to the number of disasters. To be specific, it is deduced that:

$$Y_{ij} = a_j + b_j x^i; 0 < x < 1; j = 1, 2 \tag{3.20}$$

It is also assumed that each inventory is responding to a SOS call with equal probability. Then $p_{ij} = p$; $0 < p < 1$. Consequently:

$$E[\Phi(t)] = \sum_{n=1}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}[(1 - e^{-\mu T_1})\sum_{i=1}^{n}(a_1 + b_1 x^i)p$$

$$+ (1 - e^{-\mu T_2})\sum_{i=1}^{n}(a_2 + b_2 x^i)p] \tag{3.21}$$

$$= \sum_{n=1}^{\infty} e^{-\lambda t}\frac{(\lambda t)^n}{n!}[p(1 - e^{-\mu T_1})\{na_1 + b_1(\frac{x - x^{n+1}}{1 - x})\}$$

$$+ p(1 - e^{-\mu T_2})\{na_2 + b_2(\frac{x - x^{n+1}}{1 - x})\}] \tag{3.22}$$

$$= p(1 - e^{-\mu T_1})e^{-\lambda t}[a_1\lambda te^{\lambda t} + \frac{b_1 x}{1 - x}(e^{\lambda t} - e^{\lambda xt})]$$

$$+ p(1 - e^{-\mu T_2})e^{-\lambda t}[a_2 \lambda t e^{\lambda t} + \frac{b_2 x}{1-x}(e^{\lambda t} - e^{\lambda x t})] \tag{3.23}$$

$$= \lambda p t[a_1(1 - e^{-\mu T_1}) + a_2(1 - e^{-\mu T_2})]$$
$$+ \frac{px}{1-x}(1 - e^{-\lambda(1-x)t})[b_1(1 - e^{-\mu T_1}) + b_2(1 - e^{-\mu T_2})] \tag{3.24}$$

3.2.2 Numerical Work

Using the method of least squares, Table 3.1 gives: $a_1 = 74.667, b_1 = 0.0788, a_2 = 87.429, b_2 = -0.6429$. Let the other parameters be: $p = 0.5, T_1 = 5, T_2 = 10, \mu = 3.3, \lambda = 0.01$. Then the equation for the quantum of relief is presented as:

$$E[\Phi(t)] = 0.00007t^2 + 1.6593t + 0.000002 \tag{3.25}$$

Table 3.1: Results of PPF I and PPF II

PPF I		PPF II	
Disaster no i	Amount of relief (Y_{i1})	Disaster no i	Amount of relief (Y_{i2})
1	51	1	87
2	86	2	77
3	98	3	105
4	63	4	71
5	91	5	93
6	49	6	74
7	88	7	87
8	90	-	-
9	67	-	-
10	68	-	-

To conclude, in the instance of disaster management, even if all pre-positioning facilities are connected for relief and if all initiate their respective options, the total cost appears to vary quadratically in relation to time. This is because of the realistic assumption that the quantity of relief realized from a pre-positioning facility at the disaster site increases linearly in relation to the number of occurrences of disasters. The total cost varies exponentially as the quantity increases geometrically.

3.3 A Temporo-spatial Stochastic Model[1]

3.3.1 Introduction

In this section, a one-dimensional temporo-spatial stochastic model is proposed to analyse the problem of positioning humanitarian pre-positioning facilities for optimum disaster-relief management. To be specific, it is assumed that disasters occur in a line segment region of a real line according to a Poisson Process. It is further assumed that two pre-positioning facilities are positioned at two different points of the line segment for the provision of humanitarian relief to the disaster locations. The total relief rendered is now quantified up to any time t. By optimizing the expected value of the total relief, the optimum positions of the two relief facilities are obtained. A numerical example is illustrated in the results.

Brotcorne et al. (2003) have investigated the problem of identifying appropriate locations and positions of disaster relief measures. In addition, Chang et al. (2007) have addressed the problem of locating and distributing relief supplies to disaster victims and obtaining optimal decisions for effective relief operations.

The problem of locating the emergency inventories has also been studied by Jia et al. (2007). These authors gave consideration to the fact that fluctuating demands occur at disaster sites. Rajagopalan et al. (2008) have addressed the issue of locating the minimum quantity of relief measures to be kept at humanitarian pre-positioning facilities. The above models however, do not consider the theory of point processes, which is inherent in the occurrences of catastrophic events.

Vere-Jones (1970) addresses models and procedures required to analyze the sequence of energies and origin times of earthquakes from a given region. Hence, it is necessary to formulate a stochastic point process model which will equate to an earthquake model of Vere-Jones (1970) in the problem of positioning humanitarian pre-positioning facilities for disaster relief management.

This section proposes a one dimensional spatial model. It is assumed that the disasters occur randomly as a stochastic point process in the segment $[0, L]$ of the real line. The assumption is also made that disasters are independent of each other.

[1] A modified version of this section has been submitted to the European Journal of Operations Research

3.3.2 Analysis of a Temporo-spatial Model for Disaster Relief

Let $f_n(x_n, t_n)dt_n dx_n$ be the probability that a disaster occurs in the region $(x_n, x + dx_n)$ and in the time interval $(t_n, t_n + dt_n)$ given that $n-1$ disasters have occurred before time t_n. Let there be two pre-positioning facilities situated in the region $[0, L]$. Let l_1 and l_2 be the locations of the facilities. For avoiding ambiguity, it is assumed that $0 \leq l_1 < l_2 \leq 1$. Let $N(t)$ be the total number of disasters that have occurred up to time t in the region $[0, L]$. Then the n^{th} order product density of the point process (Srinivasan, 1974) $N(t)$ is:

$$p_n(t_1, t_2, \ldots, t_n) = \int_0^L \ldots \int_0^L \prod_{j=1}^{n} f_n(x_j, t_j)dx_j; \quad 0 < t_1 < t_2 < \ldots < t_n \tag{3.26}$$

It is clear that $p_n(t_1, t_2, \ldots, t_n)dt_1 dt_2 \ldots dt_n$ has the probabilistic interpretation that it represents the probability that the first disaster occurs in the time interval $(t_1, t_1 + \Delta)$, the second disaster occurs in the time interval $(t_2, t_2 + \Delta)$, and the n^{th} disaster occurs in the time interval $(t_n, t_n + \Delta)$. It is assumed that the probability that the relief from site l_i reaches the position x_n of the n^{th} disaster is given by:

$$1 - e^{-\alpha|x_n - l_i|}; \quad i = 1, 2 \tag{3.27}$$

Suppose that the amount of relief realised at the position x_n of a disaster due to the inventory at the location l_i is given by $\mid l_i - x_n \mid^n$, then the amount of relief realised due to the n^{th} disaster at time t_n is given by:

$$R_n(l_1, l_2, t_n) = \int_0^L f_n(x_n, t_n) \sum_{i=1}^{2} \mid l_i - x_n \mid^n (1 - e^{-\alpha|x_n - l_i|})dx_n \tag{3.28}$$

For simplicity, the following is assumed:

$$f_n(x_n, t_n)dx_n dt_n = \frac{6\lambda x(L-x)}{L^3}dx_n dt_n \tag{3.29}$$

The above equation is plausible in the sense that the disasters occur as a Poisson process and the amount of relief will be further away from the disaster point. Then $R_n(l_1, l_2, t_n)$ is independent of t_n, which presents the following:

$$R_n(l_1, l_2) = \int_0^L \frac{6\lambda x_n(L-x_n)}{L^3} \sum_{i=1}^{2} \mid l_i - x_n \mid^n (1 - e^{-\alpha|x_n - l_i|})dx_n \tag{3.30}$$

$$= \frac{6\lambda}{L^3} \sum_{i=1}^{2} \int_0^L x_n(L-x_n) \mid l_i - x_n \mid^n (1 - e^{-\alpha|x_n - l_i|})dx_n \tag{3.31}$$

Splitting the interval $[0, L]$ into $[0, l_i]$ and $[l_i, L]$, the following is obtained:

$$\begin{aligned} R_n(l_1, l_2) \quad = \quad & \frac{6\lambda}{L^3} \sum_{i=1}^{2} \left[\int_0^{l_i} x_n(L-x_n) \mid l_i - x_n \mid^n (1 - e^{-\alpha|x_n - l_i|})dx_n \right. \\ & \left. + \int_{l_i}^{L} x_n(L-x_n) \mid l_i - x_n \mid^n (1 - e^{-\alpha|x_n - l_i|})dx_n \right] \end{aligned} \tag{3.32}$$

By noting the fact that:

$$\mid x_n - l_i \mid = \begin{cases} l_i - x_n \text{ if } x_n \leq l_i \\ x_n - l_i \text{ if } x_n \geq l_i \end{cases}$$

Therefore:

$$\begin{aligned} R_n(l_1, l_2) = & \frac{6\lambda}{L^3} \sum_{i=1}^{2} \left[\int_0^{l_i} x_n(L-x_n)(l_i - x_n)^n (1 - e^{-\alpha(l_i - x_n)})dx_n \right. \\ & \left. + \int_{l_i}^{L} x_n(L-x_n)(x_n - l_i)^n (1 - e^{-\alpha(x_n - l_i)})dx_n \right] \end{aligned} \tag{3.34}$$

$$=\frac{6\lambda}{L^3}\sum_{i=1}^{2}\left[\int_0^{l_i}\{l(L-l)u^n-(L-2l)u^{n+1}-u^{n+2}\}(1-e^{-\alpha u})du\right.$$
$$\left.+\int_0^{L-l_i}\{l(L-l)v^n+(L-2l)v^{n+1}-v^{n+2}\}(1-e^{-\alpha v})dv\right] \tag{3.35}$$

Using the identity:

$$\int_0^t e^{-\alpha u}u^m du=\frac{1}{\alpha^{m+1}}-m!\sum_{j=0}^{m}\frac{l^{m-j}}{(m-j)!}\left(\frac{e^{-\alpha t}}{\alpha^{j+1}}\right) \tag{3.36}$$

Hence:

$$R_n(l_1,l_2)=\frac{6\lambda}{L^3}\sum_{i=1}^{2}\left[\left\{\frac{L(l_i^{n+2}+(L-l_i)^{n+2})}{(n+1)(n+2)}-\frac{2(l_i^{n+3}+(L-l_i)^{n+3})}{(n+1)(n+2)(n+3)}\right\}\right.$$
$$-l_i(L-l_i)\left\{\frac{1}{\alpha^{n+1}}-n!e^{-\alpha l_i}\sum_{j=0}^{n}\frac{(l_i^{n-j})}{(n-j)!}\left(\frac{1}{\alpha^{j+1}}\right)\right\}$$
$$+(L-2l_i)\left\{\frac{1}{\alpha^{n+2}}-(n+1)!e^{-\alpha l_i}\sum_{j=0}^{n+1}\frac{l_i^{n+1-j}}{(n+1-j)!}\left(\frac{1}{\alpha^{j+1}}\right)\right\}$$
$$+\left\{\frac{1}{\alpha^{n+3}}-(n+2)!e^{-\alpha l_i}\sum_{j=0}^{n+2}\frac{l_i^{n+2-j}}{(n+2-j)!}\left(\frac{1}{\alpha^{j+1}}\right)\right\}$$
$$-l_i(L-l_i)\left\{\frac{1}{\alpha^{n+1}}-n!e^{-\alpha(L-l_i)}\sum_{j=0}^{n}\frac{(L-l_i)^{n-j}}{(n-j)!}\left(\frac{1}{\alpha^{j+1}}\right)\right\}$$
$$-(L-2l_i)\left\{\frac{1}{\alpha^{n+2}}-(n+1)!e^{-\alpha(L-l_i)}\sum_{j=0}^{n+1}\frac{(L-l_i)^{n+1-j}}{(n+1-j)!}\left(\frac{1}{\alpha^{j+1}}\right)\right\}$$
$$\left.+\left\{\frac{1}{\alpha^{n+3}}-(n+2)!e^{-\alpha(L-l_i)}\sum_{j=0}^{n+2}\frac{(L-l_i)^{n+2-j}}{(n+2-j)!}\left(\frac{1}{\alpha^{j+1}}\right)\right\}\right] \tag{3.37}$$

Let $\alpha=1$, then:

$$R_n(l_1,l_2)=\frac{6\lambda}{L^3}\sum_{i=1}^{2}\left[\left\{\frac{L(l_i^{n+2}+(L-l_i)^{n+2})}{(n+1)(n+2)}\right]-\frac{2(l_i^{n+3}+(L-l_i)^{n+3})}{(n+1)(n+2)(n+3)}\right\}$$
$$-l_i(L-li)\left\{1-n!e^{-l_i}\sum_{j=0}^{n}\frac{l_i^{n-j}}{(n-j)!}\right\}$$

$$+ (L - 2l_i)\left\{1 - (n+1)! e^{-l_i} \sum_{j=0}^{n+1} \frac{l_i^{n+1-j}}{(n+1-j)!}\right\}$$
$$+ \left\{1 - (n+2)! e^{-l_i} \sum_{j=0}^{n+2} \frac{l_i^{n+2-j}}{(n+2-j)!}\right\}$$
$$- l_i(L - l_i)\left\{1 - n! e^{-(L-l_i)} \sum_{j=0}^{n} \frac{(L-l_i)^{n-j}}{(n-j)!}\right\}$$
$$- (L - 2l_i)\left\{1 - (n+1)! e^{-(L-l_i)} \sum_{j=0}^{n+1} \frac{(L-l_i)^{n+1-j}}{(n+1-j)!}\right\}$$
$$+ \left\{1 - (n+2)! e^{-(L-l_i)} \sum_{j=0}^{n+2} \frac{(L-l_i)^{n+2-j}}{(n+2-j)!}\right\} \tag{3.38}$$

The above equation simplifies to:

$$\begin{aligned} R_n(l_1, l_2) =& \frac{6\lambda}{L^3} \sum_{i=1}^{2} \Bigg[\left\{\frac{L(l_i^{n+2} + (L-l_i)^{n+2}}{(n+1)(n+2)} - \frac{2(l_i^{n+3} + (L-l_i)^{n+3})}{(n+1)(n+2)(n+3)}\right\} \\ &+ 2 + l_i(L-l_i) n! \left\{\sum_{j=0}^{n} \frac{e^{-l_i} l_i^{n-j} + e^{-(L-l_i)}(L-l_i)^{n-j}}{(n-j)!}\right\} \\ &+ (L-2l_i)(n+1)! \left\{\sum_{j=0}^{n+1} \frac{e^{-(L-l_i)}(L-l_i)^{n+1-j} - e^{-l_i} l_i^{n+1-j}}{(n+1-j)!}\right\} \\ &-(n+2)! \left\{\sum_{j=0}^{n+2} \frac{e^{-l_i} l_i^{n+2-j} + e^{-(L-l_i)}(L-l_i)^{n+2-j}}{(n+2-j)!}\right\}\Bigg] \end{aligned} \tag{3.39}$$

If $R(l_1, l_2, t)$ denote the total relief rendered up to time t, then:

$$R(l_1, l_2, t) = \sum_{j=0}^{\infty} e^{-\lambda t} \frac{(\lambda t)^j}{j!} \sum_{n=0}^{j} R_n(l_1, l_2) \tag{3.40}$$

Where, $R_0(l_1, l_2) = 0$. For the optimum location, it is noted that the condition $R(l_1, l_2, t)$ is maximum. Accordingly, the following necessary conditions are achieved:

$$\frac{\partial R(l_1, l_2, t)}{\partial l_1} = 0 \tag{3.41}$$

$$\frac{\partial R(l_1, l_2, t)}{\partial l_2} = 0 \tag{3.42}$$

For further simplicity:

$$\frac{L-l_1}{l_1}=\frac{l_2}{L-l_2}=\theta \tag{3.43}$$

where $\theta > 0$, consequently:

$$\begin{aligned}R(l_1,l_2) =&\frac{12\lambda L^{n-1}}{(1+\theta)^{n+2}}\left[\frac{L(1+\theta^{n+2})}{(n+1)(n+2)}-\frac{\frac{2L}{1+\theta}(1+\theta^{n+3})}{(n+1)(n+2)(n+3)}\right]\\&+2\frac{(1+\theta)^{n+2}}{L^{n+2}}+\theta n!\sum_{j=0}^{n}\frac{L^{-j}}{(1+\theta)^{-j}}\left\{\frac{e^{\frac{-L}{1+\theta}}+e^{\frac{-\theta L}{1+\theta}}\theta^{n-j}}{(n-j)!}\right\}\\&-L(1-\theta)(n+1)!\sum_{j=0}^{n+1}\frac{L^{-j}}{(1+\theta)^{-j}}\left\{\frac{e^{\frac{-\theta L}{1+\theta}}\theta^{n+1-j}-e^{\frac{-L}{1+\theta}}}{(n+1-j)!}\right\}\\&-(n+2)!\sum_{j=0}^{n+2}\frac{L^{-j}}{(1+\theta)^{-j}}\left\{\frac{e^{\frac{-L}{1+\theta}}+e^{\frac{-\lambda L}{1+\theta}}\theta^{n+2-j}}{(n+2-j)!}\right\}\end{aligned} \tag{3.44}$$

The above function is a function of θ only. Hence, the condition for optimality becomes:

$$\frac{\partial R(l_1,l_2,t)}{\partial\theta}=0 \tag{3.45}$$

3.3.3 Numerical Illustration

Equation 3.45 is quite tedious to solve and hence a search technique is adopted to obtain the optimum value of θ. These results are obtained using *Matlab*, the model is depicted in Appendix A. For this, the following values are utilised: $\lambda = 0.1, L = 100, t = 100$. Then:

$$R(l_1,l_2,t)=\sum_{j=0}^{k}\frac{e^{-\lambda t}(\lambda t)^j}{j!}\sum_{n=0}^{j}R_n(l_1,l_2) \tag{3.46}$$

Where k is sufficiently large. For θ ranging from 0.01 to 0.99, the value of $R(l_1,l_2,t)$ is presented in Table 3.2.

The above table illustrates that the relief is maximum when $\theta = 5.22$ to 5.52. Taking the

Table 3.2: Value of $R(l_1, l_2, t)$

θ	$R(\theta)$	θ	$R(\theta)$	θ	$R(\theta)$	θ	$R(\theta)$	θ	$R(\theta)$	θ	$R(\theta)$
0.1000	0.7819	1.1000	0.0209	2.1000	0.0402	3.1000	0.0641	4.1000	0.0789	5.1000	0.0844
0.2000	0.3673	**1.2000**	**0.0212**	2.2000	0.0428	3.2000	0.0661	4.2000	0.0798	5.2000	0.0845
0.3000	0.1941	1.3000	0.0221	2.3000	0.0455	3.3000	0.0679	4.3000	0.0806	**5.3000**	**0.0846**
0.4000	0.1128	1.4000	0.0236	2.4000	0.0481	3.4000	0.0696	4.4000	0.0814	**5.4000**	**0.0846**
0.5000	0.0711	1.5000	0.0255	2.5000	0.0506	3.5000	0.0712	4.5000	0.0820	**5.5000**	**0.0846**
0.6000	0.0483	1.6000	0.0276	2.6000	0.0531	3.6000	0.0728	4.6000	0.0826	5.6000	0.0845
0.7000	0.0353	1.7000	0.0299	2.7000	0.0555	3.7000	0.0742	4.7000	0.0831	5.7000	0.0843
0.8000	0.0278	1.8000	0.0324	2.8000	0.0578	3.8000	0.0755	4.8000	0.0836	5.8000	0.0841
0.9000	0.0236	1.9000	0.0349	2.9000	0.0600	3.9000	0.0767	4.9000	0.0839	5.9000	0.0838
1.0000	0.0216	2.0000	0.0376	3.0000	0.0621	4.0000	0.0778	5.0000	0.0842	6.0000	0.0835

average 5.37 and using the condition:

$$\frac{L - l_1}{l_1} = \frac{l_2}{L - l_2} = 0 \tag{3.47}$$

the result, $\frac{100-l_1}{l_1} = \frac{l_2}{100-l_2} = 5.37$, and so:

$$l_1 = \frac{100}{6.37} = 15.6986, l_2 = \frac{100 \times 5.37}{6.37} = 84.3014 \tag{3.48}$$

It can be concluded that the two inventories should be positioned at distances 15.6986 meters and 84.3014 meters away from the origin, in order to derive the maximum relief from the centers up to time $t = 100$ days.

3.4 Stochastic Control Model

As in Section 3.3, it is assumed that disasters occur in the region $[0, L]$ and the time points of the disasters occur from a Poisson Process with rate λ. Let $N(t)$ be the number of disasters that have occurred up to time t. Let there be a single pre-positioning facility of relief measures for the disaster management. Let it be situated at ζ in $[0, L]$. If a disaster occurs at point α in $[0, L]$, then the amount of relief rendered instantly from the inventory to the disaster site is a random variable J of which the probability density function $f(x)$ is given by:

$$f(x; \alpha, \zeta) = k \mid \alpha - \zeta \mid e^{-|\alpha-\zeta|x}; \quad 0 < x < L \tag{3.49}$$

Where K is the scaling factor given by $\int_0^L f(x; \alpha, \zeta)dx = 1$. Let r be the constant rate of humanitarian relief rendered by the inventory to the society. Let $X(t)$ be the total amount of relief rendered by the inventory up to time t. Then $X(t)$ satisfies the following stochastic deferential equation:

$$dX(t) = rX(t)dt + JdN(t) \tag{3.50}$$

If $Y(t) = e^{-rt}X(t)$, then:

$$\begin{aligned} dY(t) &= e^{-rt}dX(t) - re^{-rt}X(t)dt \\ &= re^{-rt}X(t)dt + e^{-rt}JdN(t) - rY(t)dt \\ &= rY(t)dt + e^{-rt}JdN(t) - rY(t)dt \\ &= e^{-rt}JdN(t) \end{aligned} \tag{3.51}$$

Consequently, the strong solution is derived:

$$X(t) = X(0)e^{rt} + J\int_0^t e^{-r(t-u)}dN(u) \tag{3.52}$$

Now one can find the optimal positioning of the humanitarian inventory. One of the several methods is to apply the utility maximization principle. Accordingly, an exponential utility function is introduced:

$$U(x) = \frac{1}{\zeta}(1 - e^{-\zeta x}); \quad x > 0 \tag{3.53}$$

Suppose that a finite time horizon T exists. Let V_T be the cumulative utility up to time T. Then the stochastic integral is obtained:

$$\begin{aligned} V_t &= \int_0^T U[X(t)]dt \\ &= \int_0^T \frac{1}{\zeta}(1 - e^{-\xi X(t)})dt \\ &= \int_0^T \frac{1}{\zeta}\left[1 - e^{-\xi\{X(0)e^{rt} + J\int_0^t e^{r(t-u)}dN(u)\}}dt\right] \\ &= \frac{T}{\zeta} - \frac{1}{\zeta}\left[\int_0^T e^{-\xi\{X(0)e^{rt} + J\int_0^t e^{r(t-u)}dN(u)\}}dt\right] \\ &= \frac{T}{\zeta} - \frac{1}{\zeta}\int_0^T e^{-\xi\{X(0)e^{rt}\}}dt - \frac{1}{\zeta}\int_0^T e^{-\xi J\int_0^t e^{r(t-u)}dN(u)}dt \end{aligned} \tag{3.54}$$

Let $M(t) = \int_0^t e^{r(t-u)}dN(u)$, then:

$$V_T = \frac{T}{\zeta} - \frac{1}{\zeta}\int_0^T e^{-\zeta X(0)e^{rt}}dt - \frac{1}{\zeta}\int_0^T e^{-\zeta JM(t)}dt \tag{3.55}$$

Then the expected value of the stochastic integral V_T is given by:

$$E(V_T) = \frac{T}{\zeta} - \frac{1}{\zeta}\int_0^T e^{-\zeta x(0)e^{rt}}dt - \frac{1}{\zeta}\int_0^T E(e^{-\zeta JM(t)})dt \tag{3.56}$$

For this, the moment generating function of the random variable is required.

$$M(t) = \int_0^t e^{r(t-u)}dN(u) \tag{3.57}$$

Let $m(s,t)$ be the moment generating function of $M(t)$, then:

$$m(s,t) = E\left[exp\left\{s\int_0^t e^{r(t-u)}dN(u)\right\}\right] \tag{3.58}$$

To find $m(s,t)$, the following function is introduced:

$$g(s,t,\tau) = E\left[exp\left\{s\int_\tau^t e^{r(t-u)}dN(u)\right\}\right] \tag{3.59}$$

Subsequently, $g(s,t,\tau) = 0$ and $g(s,t,0) = m(s,t)$. It is effortless to write a differential equation for $g(s,t,\tau)$ as a function of τ. Now:

$$\begin{aligned} g(s,t,\tau) &= E\left[exp\left\{s\int_\tau^t e^{r(t-u)}dN(u\Big)\}\right]\\ &= E\left[exp\left\{s\int_\tau^{\tau+\Delta} e^{r(t-u)}dN(u)\right\} + \left\{s\int_{\tau+\Delta}^t e^{r(t-u)}dN(u)\right\}\right]\\ &= E\left[exp\left\{s\int_\tau^{\tau+\Delta} e^{r(t-u)}dN(u)\right\}\right]E\left[exp\left\{s\int_{t+\Delta}^t e^{r(t-u)}dN(u)\right\}\right]\\ &= [(1-\lambda\Delta) + \lambda\Delta exp\{se^{r(t-u)}\}]g(s,t,\tau+\Delta) \end{aligned} \tag{3.60}$$

because of independent increments of N(t).

Expanding $g(s,t,\tau+\Delta)$ by Taylor's series:

$$g(s,t,\tau) = [(1-\lambda\Delta) + \lambda\Delta exp\{se^{r(t-u)}\}][g(s,t,\tau) + \Delta\frac{\partial g}{\partial\tau} + \circ(\Delta)] \tag{3.61}$$

Simplifying and taking $\Delta \to 0$:

$$0 = \lambda[e^{se^{r(t-u)}} - 1]g + \frac{\partial g}{\partial\tau} \tag{3.62}$$

Integrating this equation:

$$\log g(s,t,t) - \log g(s,t,0) = -\lambda \int_0^t (e^{se^{r(t-\tau)}} - 1)d\tau \tag{3.63}$$

Applying the boundary conditions $g(s,t,t) = 0$ and $g(s,t,0) = m(s,t)$ in (3.63):

$$\log m(s,t) = \lambda \int_0^t (e^{se^{r(t-\tau)}} - 1)d\tau \tag{3.64}$$

Hence, the moment generating function is derived:

$$m(s,t) = exp\{\lambda \int_0^t (e^{se^{r(t-\tau)}} - 1)d\tau\} \tag{3.65}$$

Then:

$$E[e^{-\zeta JM(t)}] = \int_0^\infty m(-\zeta s,t) f(s;a,\zeta) ds \tag{3.66}$$

Consequently:

$$E(V_T) = \frac{T}{\zeta} - \frac{1}{\zeta}\int_0^T e^{-\zeta X(0)e^{rt}} dt - \frac{1}{\zeta}\int_0^T \int_0^\infty m(-\zeta s;t) f(s;\alpha,\zeta) ds dt \tag{3.67}$$

Subsequently $E(V_t)$ is maximized with respect to the positioning of the inventory. For this, the necessary condition is:

$$\frac{\partial E(V_T)}{\partial \zeta} = 0 \tag{3.68}$$

Then:

$$-\frac{T}{\zeta^2}+\frac{1}{\zeta^2}\int_0^T e^{-\zeta X(0)e^{rt}}dt+\frac{X(0)}{\zeta}\int_0^T e^{-\zeta X(0)e^{rt}}e^{rt}dt+\frac{1}{\zeta^2}\int_0^T\int_0^\infty m(-\zeta s,t)f(s;\alpha,\zeta)dsdt \tag{3.69}$$

$$-\frac{1}{\zeta}\int_0^T\int_0^\infty\{\frac{\partial m(-s\zeta,t)}{\partial\zeta}f(s;\alpha,\zeta)+m(-\zeta s,t)\frac{\partial f}{\partial\zeta}(s;\alpha,\zeta)dsdt=0 \tag{3.70}$$

Solving this equation, the optimal position ζ^* of the inventory is obtained.

3.5 Fluid Queue Model of Humanitarian Inventory for Disaster Relief Management

3.5.1 Introduction

In this section, a single relief item in inventory for disaster relief management is addressed. Besides the purpose of managing disaster situations, this inventory also encompasses location regional requirements. The time intervals of disaster relief operations and that of no disaster relief operations alternate on time axis; these intervals are assumed to be independent. The assumption is made that a disaster occurs according to a Poisson process with rate $\lambda > 0$ during the time interval in which a disaster is managed by the pre-positioning facility, no other disaster, if any occurs, is taken care of by the pre-positioning facility. Further the pre-positioning facility, if it is free, goes immediately for the relief of the disaster that occurs after the time it has become free. The time interval during which a disaster relief is recorded is an exponential random variable having the probability density function $\mu e^{-\mu t}$; $t > 0, \mu > 0$. The rate of catering during no disaster relief period (normal period) is a positive constant c_o and the rate of catering during the disaster relief period (critical period) is a positive constant c_1. The maximum storage capacity of the inventory is assumed as S, where $S < \infty$ and when the inventory level comes down to the level $s(< S)$, it is immediately re-equipped to the level S. Let $D(t)$ denote the state of the disaster region at time t. Then it is noted that:

$$D(t) = \begin{cases} 0, & \text{if the region is in a normal state} \\ 1, & \text{if the region is in a critical state} \end{cases}$$

The $D(t)$ is a two-state Markov-process which is well documented (Karlin and Taylor, 1975). If $P_{ij}(t)$ denote its transition probabilities, and defined as:

$$P_{ij}(t) = P[D(t+s) = j \mid D(s) = i]; i, j = 0, 1 \tag{3.71}$$

It is known that:

$$P_{00}(t) = e^{-\alpha t} + \alpha e^{-\alpha t} \copyright P_{10}(t) \tag{3.72}$$

$$P_{01}(t) = \alpha e^{-\alpha t} \copyright P_{11}(t) \tag{3.73}$$

$$P_{10} = \beta e^{-\beta t} \copyright P_{00}(t) \tag{3.74}$$

$$P_{11} = e^{-\beta t} + \beta e^{-\beta t} \copyright P_{01}(t) \tag{3.75}$$

Where © is the convolution symbol. Solving these equations:

$$P_{oo}(t) = \frac{\beta}{\alpha + \beta} + \frac{\alpha}{\alpha + \beta} e^{-(\alpha+\beta)t} \tag{3.76}$$

$$P_{01}(t) = \frac{\alpha}{\alpha+\beta} - \frac{\alpha}{\alpha+\beta}e^{-(\alpha+\beta)t} \tag{3.77}$$

$$P_{10}(t) = \frac{\beta}{\alpha+\beta} - \frac{\beta}{\alpha+\beta}e^{-(\alpha+\beta)t} \tag{3.78}$$

$$P_{11}(t) = \frac{\alpha}{\alpha+\beta} + \frac{\beta}{\alpha+\beta}e^{-(\alpha+\beta)t} \tag{3.79}$$

Let $X(t)$ be the level of the humanitarian inventory at time t. Then the pair $(X(t), D(t))$ is a Markov process. It is assumed that at time $t = 0$ the inventory level S and the state of the region is 0. Then, $X(0) = S$ and $D(0) = 0$. The time points at which the inventory is replenished is considered and $N(t)$ is the random number of time points at which inventory is replenished in the interval $(0, t]$. Just after the time of replenishment the state of the region may be normal or critical and the inventory level is S. Let $c(t)$ be the cost of equipping one unit of relief item to the inventory at time t. Let δ be the discounting factor of the monetary value. Then the total expenditure $Z(t)$ incurred up to time t is given by the stochastic integral:

$$Z(t) = \int_0^t e^{-\delta u} c(u) dN(u) \tag{3.80}$$

Next, the expected value $E[Z(t)]$ is obtained. For this, the probability density function of the interval is derived between successive time points of replenishment of the inventory. In this interval, the following is defined:

$$f_{ij}(x;t;y) = \lim_{\Delta \to 0} \frac{P[x < X(t) < x + \Delta, D(t+\tau) = j \mid X(\tau) = y, D(\tau) = i]}{\Delta}; \quad i, j = 0, 1$$

(3.81)

Where $S \geq y > x \geq 0$. Then:

$$f_{oj}(x,t;y) = e^{-\alpha t}\delta\left(t - \frac{y-x}{c_0}\right) + \alpha e^{-\alpha t} \copyright f_{ij}(x,t;y-c_{0t}); j = 0,1 \tag{3.82}$$

$$f_{1j}(x,t;y) = e^{-\beta t}\delta\left(t - \frac{y-x}{c_1}\right) + \beta e^{-\beta t} \copyright f_{0j}(x,t_i;y-c_{1t}); j = 0,1 \tag{3.83}$$

Putting $y = S$ and $x = 0$ in the above equations:

$$f_{0j}(0,t;S) = e^{-\alpha t}\delta\left(t - \frac{S}{c_0}\right) + \alpha e^{-\alpha t} \copyright f_{ij}(x,t;y-c_{0t}); j = 0,1 \tag{3.84}$$

$$f_{1j}(0,t;S) = e^{-\beta t}\delta\left(t - \frac{S}{c_1}\right) + \beta e^{-\beta t} \copyright f_{0j}(x,t;y-c_{1t}); j = 0,1 \tag{3.85}$$

Where:

$$0 < \frac{S}{c_1} \leq t \leq \frac{S}{c_0} \tag{3.86}$$

The above equations can be solved by a random motion approach:

$$\begin{aligned} f_{00}(0;t;S) =& e^{-\alpha t}\delta\left(t - \frac{S}{c_0}\right) + e^{-\alpha t}\sum_{n=1}^{\infty}\alpha^n\beta^n\int_0^t\int_0^{t_1}\dots\int_{t_{2n-1}}^t e^{-(\alpha-\beta)\sum_{j=1}^{2n}(-1)^j t_j}\times \\ & \delta\left(t - \frac{S+(c_0-c_1)\sum_{j=1}^{2n}(-1)^j t_j}{c_0}\right)dt_{2n}\dots dt_2 dt_1 \end{aligned} \tag{3.87}$$

$$f_{01}(0,t;S) = \alpha e^{-\beta t}\int_0^t e^{-(\alpha-\beta)t_1}\delta\left(t-\frac{S-(c_0-c_1)t_1}{c_1}\right)dt_1 + \alpha e^{-\beta t}\sum_{n=1}^{\infty}\alpha^n\beta^n\int_0^t\int_0^{t_1}\dots\int_{t_{2n}}^{t} e^{-(\alpha-\beta)\sum_{j=1}^{2n+1}(-1)^j t_j}\times \delta\left(t-\frac{(S-s)(c_0-c_1)\sum_{j=1}^{2n+1}(-1)^j t_j}{c_1}\right)dt_{2n+1}dt_{2n}\dots dt_2 dt_1 \quad (3.88)$$

$$f_{10}(0,t;S) = \beta e^{-\alpha t}\int_0^t e^{-(\alpha-\beta)t_1}\delta\left(t-\frac{S+(c_0-c_1)t}{c_0}\right)dt_1 + \beta e^{-\alpha t}\sum_{n=1}^{\infty}\alpha^n\beta^n\int_0^t\int_0^{t_1}\dots\int_{t_{2n}}^{t} e^{(\alpha-\beta)\sum_{j=1}^{2n+1}(-1)^{j-1} t_j}\times \delta\left(t-\frac{S+(c_0-c_1)\sum_{j=1}^{2n+1}(-1)^{j-1} t_j}{c_0}\right)dt_{2n+1}dt_{2n}\dots dt_2 dt_1 \quad (3.89)$$

$$f_{11}(0,t;S) = e^{-\beta t}\delta\left(t-\frac{S}{c_1}\right) + e^{-\beta t}\sum_{n=1}^{\infty}\alpha^n\beta^n\int_0^t\int_0^{t_1}\dots\int_{t_{2n-1}}^{t} e^{-(\alpha-\beta)\sum_{j=1}^{2n}(-1)^{j-1} t_j}\times \delta\left(t-\frac{S+(c_0-c_1)\sum_{j=1}^{2n}(-1)^{j-1} t_j}{c_1}\right)dt_{2n}\dots dt_2 dt_1 \quad (3.90)$$

Where $\delta(.)$ is a Dirac delta function. Let $g_0(t)$ be the probability density of inter-replenishment times with the inventory replenished just at time $t = 0$ and the region is in state $t = 0$. Then it is clear that:

$$g_0(t) = f_{00}(0,t;S) + f_{01}(0,t;S) \quad (3.91)$$

Now the process $N(t)$ is studied. The joint process $N(t)$ is considered the following equation is defined:

$$P_n(t) = P[N(t) = n] \tag{3.92}$$

Then:

$$P_n(t) = g_0(t) \copyright P_{n-1}(t); \quad n = 1, 2, \ldots \tag{3.93}$$

$$P_0(t) = \left[1 - \int_0^t g_0(\tau)d\tau\right]\left[1 - H(t - \frac{S}{c_0})\right] \tag{3.94}$$

Where $H(.)$ is a Heaviside function. Taking Laplace transforms on both sides of the above equations:

$$P_n^*(\theta) = g_0^*(\theta)P_{n-1}^*(\theta); \quad n = 1, 2, \ldots \tag{3.95}$$

$$P_0^*(\theta) = \frac{1 - e^{\frac{-\theta S}{c_0}}}{\theta} + e^{\frac{-\theta S}{c_0}}\theta \int_0^{\frac{S}{c_0}} g_0(\tau)d\tau \tag{3.96}$$

The above equation precedes to:

$$\begin{aligned} P_n^*(\theta) &= [g_0^*]^n P_0^*(\theta) \\ &= \{g_0^*(\theta)\}^n \left[\frac{1 - e^{\frac{-\theta S}{c_0}}}{\theta} + e^{\frac{-\theta S}{c_0}} \int_0^{\frac{S}{c_0}} g_0(\tau)d\tau\right] \end{aligned} \tag{3.97}$$

Now the conditional first order product density $h_j(t); j = 0, 1$ (Srinivasan, 1974) of the point process $N(t)$ of epochs of replenishment is obtained. It is known that $h_j(t)\Delta$ has the interpretation for the probability of occurrence of a replenishment in a small interval $(t, t+\Delta)$ given that a replenishment has occurred and the region is in state j at time $t = 0$. It is now noted that:

$$h_j(t)dt = E[dN(t) \mid X(0) = S, E(0) = j]; j = 0, 1 \tag{3.98}$$

From renewal theoretical arguments (Cox, 1962), it is known that the replenishment that occurs in the interval $(t, t + \Delta)$ may be the first one after time 0 or it may be a subsequent one. Consequently:

$$h_j(t) = g_j(t) + f_{j0}(t)\copyright h_0(t) + f_{j1}(t)\copyright h_1(t); j = 0, 1 \tag{3.99}$$

From the above equation, the Laplace transform is derived:

$$h_0^*(\theta) = g_0^*(\theta) + f_{00}^*(\theta)h_0^*(\theta) + f_{01}^*(\theta)h_1^*(\theta) \tag{3.100}$$

$$h_1^*(\theta) = g_1^*(\theta) + f_{10}^*(\theta)h_0^*(\theta) + f_{11}^*(\theta)h_1^*(\theta) \tag{3.101}$$

Solving the above system of equations, the following is derived:

$$h_0^*(\theta) = \frac{g_0^*(\theta)[1 - f_{11}^*(\theta)] + f_{01}^*(\theta)g_1^*(\theta)}{[1 - f_{00}^*(\theta)][1 - f_{11}^*(\theta)] - f_{10}^*(\theta)f_{01}^*(\theta)} \tag{3.102}$$

$$h_1^*(\theta) = \frac{g_1^*(\theta)[1 - f_{00}^*(\theta)] + f_{10}^*(\theta)g_0^*(\theta)}{[1 - f_{00}^*(\theta)][1 - f_{11}^*(\theta)] - f_{10}^*(\theta)f_{01}^*(\theta)} \tag{3.103}$$

Inverting the above equations, the densities $h_j(t); j = 0, 1$ are obtained. Taking expectation on both sides of the above equation, the expected value of the total quantity of replenishment made up to time t is given by:

$$\begin{aligned} E[Z(t)] &= \int_0^t c(u)E[dN(u)] \\ &= \int_0^t c(u)h_0(u)du \end{aligned} \tag{3.104}$$

3.6 Conclusions

In this chapter, the objective was to formulate stochastic models which incorporate the various factors of disaster logistics: time, resources, capacity and location. The non-spatial model incorporates time and capacity by determining the required capacity at a centre point in time, once a disaster has occurred. The one-dimensional temporo-spatial stochastic model analyzes the problem of positioning humanitarian pre-positioning facilities. This model is developed to solve the location problem of the pre-positioning facilities. The stochastic control model and the fluid queue model, both are developed to regulate the capacity in pre-positioning facilities. Due to the complexity of the constraints of these models, the validation will be incorporated in future research.

Chapter 4

Applications of DIM, SIM and EDA for Pre-positioning Facilities

4.1 Introduction

This chapter addresses the application of three different models providing suitable solutions to determine the quantity and types of aid supplies required if a disaster occurs. These supplies should then be kept in inventory. All three models are validated by applying the models to disaster-prone regions. This chapter is structured as follows. Section 4.2 presents an overview of the Deterministic Inventory Model (DIM). Section 4.3 explains the formulation and results of the Stochastic Inventory Model (SIM). Section 4.4 discusses the Euclidean Distance Algorithm (EDA). Section 4.5 provides a comparison of the three models to illustrate the significance of each. Finally, Section 4.6 summarizes the chapter.

4.2 Deterministic Inventory Model (DIM)[1]

4.2.1 The Disaster Recovery Plan (DRP) Concept

Organizations are susceptible to various random events, such as man-made or natural disasters, leading to further internal dilemmas within the organization. It is therefore imper-

[1] A modified version of this section has been published in Management Dynamics

ative to ensure that recovery strategies are in place to survive and recover from any such event. Bryson et al. (2002) define such strategies as Disaster Recovery Plans (DRPs) that aim to ensure that organizations function effectively during and after the occurrence of a disaster. Bryson et al. (2002) state that there has been little modelling of disaster recovery issues in operations research literature and that the development of hardware and software tools for addressing specific aspects of disasters have been absent.

An effective DRP will integrate the following properties in the results of the model: feasibility, completeness, consistency, and reliability. These properties establish an adept framework when considering different types of resources required to satisfy demand induced by any relevant disaster. The purpose of the model is to determine the resources that are required so that the total expected value of the recovery capability is maximised. This model assumes that for each disaster effect, there is a set of Disaster Recovery Sub-plans (DRSP) in place to provide protection against such effect. A set of DRSPs therefore encompasses the entire DRP. Each resource will provide different utilisation levels, and some resources may be used for more than one disaster effect. A DRP aims to minimize potential loss by identifying, prioritizing and safeguarding those organizational assets that are most valuable and require maximum protection (Bryson et al., 2002).

This section therefore addresses the application of a DIM to assist the decision maker to select the most appropriate sub-plan to maximize the recovery capability of a recovery strategy. A deterministic model is one in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables. Therefore, deterministic models perform the same way for a given set of initial conditions.

The DRP principle as explained by Bryson et al. (2002) is adopted to develop a mixed integer model to protect a series of countries against a defined set of possible disasters. Each DRSP has an associated cost, which remains within a budget to simultaneously maximise the recovery capability within the relevant budget. The DRP model provides a generic approach which addresses different types of resources required to satisfy demand induced by any given disaster. Testing of the DIM for the SADC countries is conducted by applying case studies of the region to the model. The parameters are kept constant while the budget is altered to show how the quantities of each aid supply varies accordingly. Next, a Cost-benefit Analysis

(CBA) is performed to clearly indicate the functionality of the model, and whether any further modifications are required. The solving of the model is accomplished by using an optimisation software such as *LINGO*.

4.2.2 Mathematical Model for DIM

Assuming a country in the SADC region will be severely affected by a disaster, the primary goal of a humanitarian relief organisation will be to provide relief to as many victims as possible. To limit these severe effects, the supplies should be cautiously planned and pre-positioned beforehand. In the first instance the organisation, ("the decision-maker"), will determine a budget limit. Thereafter, the model will maximise the total recovery capability of the defined sub-plans, in respect of which each sub-plan represents the amount and type of aid supplies required, without exceeding the budget limit. The following notation and constraints are used in the DIM model:

Sets

$K \triangleq$ set of SADC countries

$R \triangleq$ set of aid supplies that occur in integer quantities

$J \triangleq$ set of disaster effects in terms of population affected

$I \triangleq$ set of disaster types

$S \triangleq$ set of recovery sub-plans

$S_j \triangleq$ set of sub-plans that can protect against disaster effect j, where $j \in J$

$S_r \triangleq$ set of sub-plans that use supply r, where $r \in R$

Decision Variables

$$Y_S \triangleq \begin{cases} 1 \text{ if sub-plan } S \text{ is selected for recovery, where } s \in S \\ 0 \text{ otherwise} \end{cases}$$

$Z_r \triangleq$ the amount of aid supply r that is acquired, where $r \in R$

Utility Variables

$p_j \triangleq$ given that a disaster has occurred, the likelihood of experiencing effect j, where $j \in J$

$g_j \triangleq$ the relative importance of a disaster effect j, based on its potential impact on a country, where $j \in J$

Parameters

$f_{ij} \triangleq$ the likelihood of a disaster type i having disaster effect j, where $i \in I, j \in J$

$a_k \triangleq$ the relative importance of SADC country k, where $k \in K$

$h_{jk} \triangleq$ the likelihood that effect j would affect country k, where $j \in J, k \in K$

$U_{rs} \triangleq$ the quantity of aid supply r required by sub-plan s where $r \in R, s \in S$

$c_r \triangleq$ the unit cost for aid supply r, where $r \in R$

$B \triangleq$ budget limit

$$w_{sj} \triangleq \begin{cases} 1 \text{ if sub-plan } S \text{ provides recovery capability for effect } j \\ 0 \text{ otherwise} \end{cases}$$

The total recovery capability of a set of sub-plans can be formulated as follows:

$$\max Z = \sum_{s \in S} \sum_{j \in J} w_{sj} g_j p_j y_s \tag{4.1}$$

s.t.

$$\sum_{r \in R} c_r z_r \leq B \tag{4.2}$$

$$\sum_{S \in S_j} y_s \leq 1 \;\forall\; j \in J \tag{4.3}$$

$$\sum_{S \in S_r} u_{rs} y_s - z_r \leq 0 \;\forall\; r \in R \tag{4.4}$$

$$\sum_{i \in I} f_{ij} = p_j \ \forall \ j \in J \tag{4.5}$$

$$\sum_{k \in K} a_k h_{jk} = g_j \ \forall \ j \in J \tag{4.6}$$

$$Z_r \geq 0 \text{ and integer } \forall \ r \in R \tag{4.7}$$

The objective function (4.1) of the linear programming model is used to maximise the total recovery capability of a set of sub-plans that have been chosen. The units of the objective function are calculated as a percentage which represents the reliability of the set of sub-plans. Constraint (4.2) provides a budget limit which is identified by the decision-maker. This constraint ensures that the resources selected for the sub-plans do not exceed the specified budget. Constraint (4.3) ensures that only one sub-plan is selected for a given effect. Constraint (4.4) allows for the possibility that a resource is obtained for a selected sub-plan. Constraint (4.6) is used to determine the probability of experiencing a certain effect, given that a disaster has occurred. Finally, Constraint (4.7) determines the relative importance of a disaster effect based on its potential impact in a country. The overall objective of this model is to support the decision-maker with inventory decisions for disaster relief in the SADC region.

The model is developed by incorporating a few assumptions:

(i) Certain sub-plans provide relief for more than one disaster;

(ii) the budget limit is defined by the decision-maker based on the desired investment, therefore the model output determines the optimal implementation of sub-plans which will not exceed the budget limit;

(iii) each sub-plan provides a comprehensive recovery reliability; and

(iv) the model provides the user with an integer value z_r which specifies the necessary quantities of aid supplies to be kept in the pre-positioned facility.

These assumptions allow the model to be converted into a possible solution for disaster preparedness when applied to a country in crisis.

4.2.3 DIM: Data Analysis

This section describes the computational results obtained via the model. The generic inventory model was coded in *LINGO*, version 8.0, on a standard personal computer. A short description of how the relevant sets and parameter values were obtained is addressed, followed by the results of the model in terms of the number of aid supplies required according to a selected budget. Finally, a cost-benefit analysis is done to test the functionality of the model. The model input was determined by systematically defining each set, namely, the SADC countries, disaster effects, disaster types, disaster recovery sub-plans and the aid supplies required. From these defined sets, the parameter values were determined. Table 4.1 illustrates the manner in which the data of each disaster was gathered for Swaziland and has been obtained in the same manner for the other 14 countries.

Table 4.1: Summary of disasters in Swaziland

Country	Disaster	Type	No. of events	Total no. killed per event	Total no. affected per event
Swaziland	Drought	Drought	5	100	326 000
	Epidemic	Bacterial infectious diseases	2	31	1 830
		Parasitic infectious diseases	1	80	-
	Flood	General Flood	2	-	137 250
	Storm	Unspecified	2	1	3 843
		Tropical cyclone	1	53	623 500
	Wildfire	Forest fire	1	2	1 500
	Transport	Road	2	26	30

The sets K and I, used in the utility variables p_j and g_j, are simply defined by listing the SADC countries K and the relevant disaster types I, that are associated with these countries. The disaster types are identified by listing all the disasters that have occurred in the SADC over the last 30 years as well as the required aid supplies R. The set of disaster effects J, was identified in ten different ranges of populations affected. These ranges were estimated by considering all the data of the disasters that have affected population groups in the SADC during the selected time period. The percentiles of the list of values were determined by computing the 10^{th} percentile, 20^{th} percentile up to the 100^{th} percentile and from these the ranges were developed. This method was used to anticipate that a country with a smaller population is also incorporated when affected by the worst possible eventuality. The ranges are depicted in Table 4.2.3.

Table 4.2: Summary of disasters in the SADC

Effect (J)	Range(No of people affected)
1	1-39
2	40 - 99
3	100 - 299
4	300 - 899
5	900 - 2 999
6	3 000 - 9 999
7	10 000 - 23 999
8	24 000 - 101 999
9	102 000 - 504 999
10	505 000 - 15 000 000

The set of sub-plans S, was determined by cross referencing aid supplies with disaster types. As presented in Table 4.3, different supplies are required for different disasters. Some disasters do however require the same aid supplies, namely, an epidemic, flood, miscellaneous accident, transport accident, insect infestation, storm and earthquake. This approach was followed to ensure that no unnecessary inventory is kept in a pre-positioned warehouse. Subsequently, there exists 5 different aid supply combinations. Taking into consideration that there are 10 effects, each possible combination thus provides 10 different options. This means that 50 sub-plans are identified to provide relief. The first ten sub-plans provide relief for the first combination of supplies, the next 10, for the second combination, and in the same manner for all 50 sub-plans. With reference to the parameters, the probability f_{ij} was determined by analysing disaster types i in relation to disaster effects j. This analysis provides a method to determine the frequency of a disaster type in relation to its unique consequence. This is shown in Table 4.4.

From this table it was possible to determine f_{ij}, by dividing each frequency value by the total of 475 disasters that have occurred in the SADC in the past 30 years. The result is seen in Table 4.5 which summarizes the likelihood of disaster type i having disaster effect j.

Similarly, h_{jk} was developed by cross referencing disaster effect j with country k; the same steps were followed with the parameter f_{ij}. This data was obtained from the CRED (2009).

Table 4.3: Aid supply R required by disaster type I

	R																
I	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Drought					x	x	x			x	x	x	x	x	x		
Epidemic	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Flood	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Mass movement wet		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Idustrial accident		x	x	x	x	x	x		x			x	x	x	x		x
Miscellaneous accident	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Transport accident	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Insect infestation	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Storm	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Earthquake	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Wildfire		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Extreme temperatures		x	x	x	x	x	x	x	x	x	x	x	x	x	x		

Table 4.4: Frequency of disaster type I having disaster effect J

J/I	1	2	3	4	5	6	7	8	9	10	11	12	Total
1	0	2	5	0	0	1	0	0	4	1	36	1	50
2	0	2	4	0	0	1	0	0	5	0	33	0	45
3	0	0	12	0	3	4	0	1	4	3	22	0	49
4	0	3	25	0	4	1	0	0	2	7	5	0	47
5	0	3	15	0	11	1	0	1	1	8	0	4	44
6	0	2	13	0	19	0	0	0	1	11	0	2	48
7	1	1	16	0	26	0	0	0	1	3	0	0	48
8	6	1	9	0	25	0	0	0	0	7	1	0	49
9	17	0	2	0	18	0	0	0	0	10	0	0	47
10	31	0	0	0	9	0	0	0	0	8	0	0	48
Total	55	14	101	0	115	8	0	2	18	58	97	7	475

Table 4.5: Probability of disaster type i having disaster effect j (f_{ij})

J/I	1	2	3	4	5	6	7	8	9	10	11	12	Total
1	0	0.0042	0.0105	0	0	0.0021	0	0	0.0084	0.0021	0.0758	0.0021	0.1053
2	0	0.0042	0.0084	0	0	0.0021	0	0	0.0105	0	0.0695	0	0.0947
3	0	0	0.0253	0	0.0063	0.0084	0	0.0021	0.0084	0.0063	0.0463	0	0.1032
4	0	0.0063	0.0526	0	0.0084	0.0021	0	0	0.0042	0.0147	0.0105	0	0.0989
5	0	0.0063	0.0316	0	0.0232	0.0021	0	0.0021	0.0021	0.0168	0	0.0084	0.0926
6	0	0.0042	0.0274	0	0.0400	0	0	0	0.0021	0.0232	0	0.0042	0.1011
7	0.0021	0.0021	0.0337	0	0.0547	0	0	0	0.0021	0.0063	0	0	0.1011
8	0.0126	0.0021	0.0189	0	0.0526	0	0	0	0	0.0147	0.0021	0	0.1032
9	0.0358	0	0.0042	0	0.0379	0	0	0	0	0.0211	0	0	0.0989
10	0.0653	0	0	0	0.0189	0	0	0	0	0.0168	0	0	0.1011
Total	0.1158	0.0295	0.2126	0	0.2421	0.0168	0	0.0042	0.0379	0.1221	0.2042	0.0147	1

Each country k is given a weight a_k according to important objectives: total population, disaster frequency, the Gross Domestic Product (GDP) of each country, the number of people affected, and the number of people extinguished by disasters over the last 30 years. The Analytical Hierarchy Process (AHP) was applied to determine the weight of each country, which is an effective tool, employed to make decisions when multiple objectives are involved (Saaty, 2003). The weight of each country is shown in Table 4.6.

Table 4.6: Country Importance

Country (K)	Weight (a_k)
Angola	0.0379
Botswana	0.0172
Congo	0.0306
Lesotho	0.0960
Madagascar	0.0481
Malawi	0.0809
Mauritius	0.0207
Mozambique	0.1206
Namibia	0.0254
Seychelles	0.1734
South Africa	0.0972
Swaziland	0.0589
Tanzania	0.0681
Zambia	0.0425
Zimbabwe	0.0825

The parameter u_{rs} (the quantity of aid supply r required per sub-plan s), was calculated to supply relief for 30 days which, according to Kovacs and Spens (2007), allows sufficient time for the recovery phase to be planned. The cost of the aid supplies, c_r, was determined per unit in South African Rands (ZAR) and is presented in Table 4.7. The table further specifies the number of persons that can benefit by one unit. An estimate of the delivery charges is included in the unit costs, which makes it possible to apply the model to any SADC country. This data was all obtained from appropriate suppliers.

As mentioned above, all 50 sub-plans provide relief for one or more effect. A matrix is therefore developed, in which each cell indicates that sub-plan s provides recovery capability for disaster effect j. Each cell represents a binary value w_{sj} (Bryson et al., 2002).

The model was tested by keeping the defined parameters constant and varying the budget

Table 4.7: Unit cost of aid supplies (c_r)

Item R	Unit cost (ZAR)	No of people
Mosquito nets	45.76	1
Waterproof ponchos	9.60	1
Waterproof ground mats	45.76	1
Children's activity pack	40	10
Durable plastic box	286.00	10
Collapsible water containers	34.32	1
Water purifying equipment	194.48	10
Ten-person tent	57.20	10
Thermal fleece blankets	228.80	1
Cooking equipment	686.40	10
Gel stove	686.40	10
Toolkit	171.60	10
First aid kit	114.82	10
Food supplies	24.60	1
Water	3594.00	110
Portable toilet	2662.00	20
Waste bin	588.00	120

limit. The decision variable of interest to the decision maker is z_r, giving an indication of the amount and types of aid supplies to keep in a pre-positioning facility. Table 4.8 reflects the outcome of four different budgets. According to Kovacs and Spens (2007) an estimated annual budget for relief agencies amounts to ZAR 7 280 000. The budget parameter was varied between ZAR 1 000 000, ZAR 7 500 000, ZAR 10 000 000 and ZAR 10 500 000, to indicate how the model output changes between two extremes and two realistic estimates.

Table 4.8 displays interesting results, considering that the quantity of each item does not necessarily increase as the budget increases. Furthermore, when observing the values of the water supplies for example, the quantity of units increase as the budget increases, whereas with the food supplies, the quantities initially increase, but thereafter remain constant as the budget increases. The results evidently provide an approach to satisfy total recovery for any SADC country rather than just increasing inventory when the budget increases. This means that the pre-positioned facilities will not be over-stocked with unnecessary types and quantities of aid supplies.

To draw further conclusions from the varying budget a Cost-benefit Analysis (CBA) is performed. This is a standard method utilised to determine and compare the cost and benefits of a potential investment. The measured cost and benefits are weighed up against each other to establish criteria for decision making. The cost of each item identified in Table 4.7 are incorporated in the budget compiled to fund the inventory kept in the pre-positioning facilities, whereas the benefits include the objective values of the model which represents the total recovery capability of the sub-plans chosen. The budgets are selected randomly for testing purposes, from the lowest possible value to the highest possible extreme. Figure 4.1 indicates the results of the CBA.

Table 4.8: Model Results

No	Item	ZAR 1 000 000	ZAR 7 500 000	ZAR 10 000 000	ZAR 10 500 000
1	Mosquito nets	0	1	1	1
2	Waterproof ponchos	2	2	2	2
3	Waterproof ground mats	0	0	0	0
4	Children's activity pack	0	0	0	0
5	Durable plastic box	1	1	1	1
6	Collapsible water containers 182	200	200	200	
7	Water purifying equipment	921	1 010	1 010	1 010
8	Ten-person tent	0	1	1	1
9	Thermal fleece blankets	55	60	60	60
10	Cooking equipment	1 861	2 040	2 040	2 040
11	Gel stove	0	0	0	0
12	Toolkit	1	1	1	1
13	First aid kit	10	10	10	10
14	Food supplies	1 513	263 062	364 688	385 014
15	Water	55	60	60	60
16	Portable toilet	5	6	6	6
17	Waste bin	187	204	204	204

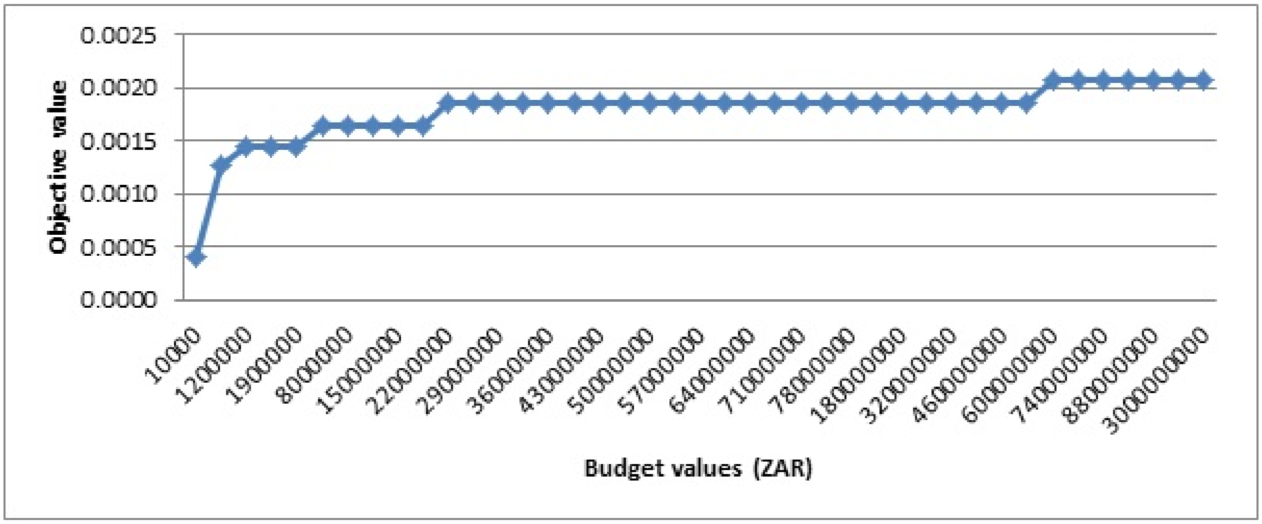

Figure 4.1: Cost-Benefit Analysis

The CBA interpretation indicates that as the budget increases, the objective value also increases. The benefits do, however, reach an optimum, which means that by reaching a certain budget the objective value will no longer increase, thus achieving a saturated limit. In other words, the decision-maker should not have to invest more than ZAR 600 000 000 for the pre-positioning of aid supplies, as any larger investment will become increasingly irrelevant.

4.3 Stochastic Inventory Model (SIM)[2]

4.3.1 Introduction

Humanitarian relief organisations aim to provide relief for as many disaster victims as possible, subject to limited funding. It is therefore appropriate to consider a model that assists the decision-maker with inventory decisions at the lowest possible cost. In this section a Stochastic Inventory Model (SIM) is considered to determine the adequate quantities of supplies to keep. In a stochastic model, randomness is present, and variable states are not described by unique values, but by probability distributions. The notation of the SIM is as follows:

[2] A modified version of this section has been published in the South African Journal of Industrial Engineering

4.3.2 SIM Analysis

$Q_{ik} \triangleq$ the number of aid supplies i required for demand scenario k

$c_i \triangleq$ the unit ordering cost of aid supply i

$x_{ik} \triangleq$ the total expected demand for aid supply i for demand scenario k

$q_k \triangleq$ the probability of a scenario k

$v_{ik} \triangleq$ the excess inventory of aid supply i for scenario k

$h_i \triangleq$ the unit holding cost of aid supply i

$u_{ik} \triangleq$ the number of shortages of aid supply i observed for scenario k

$s_i \triangleq$ the shortage cost of aid supply i

The notation is used to formulate the following objective function:

$$\min Z = \sum_{i \in I} \sum_{k \in K} q_k (c_i Q_{ik} + h_i v_{ik} + s_i u_{ik}) \tag{4.8}$$

s.t.

$$Q_{ik} + u_{ik} - v_{ik} = x_{ik} \tag{4.9}$$

$$Q_{ik}, v_{ik}, u_{ik} \geq 0 \;\; i \in I, k \in K \tag{4.10}$$

The objective function (4.8) selects the appropriate quantities and types of aid supplies to minimise the overall cost of inventory kept. Constraint (4.9) guarantees that the number of aid supplies required for a demand scenario corresponds with the expected demand of scenario, while taking excess inventory and shortages into consideration. Constraint (4.10) ensures that decision variables Q_{ik}, v_{ik} and u_{ik}, remain greater than equal to 0. It is assumed that no excess inventory is present during the first usage of the model.

The model was adapted to apply to the SADC region, and so a few assumptions are changed to convert the model into a more appropriate solution. Taskin and Lodree (2010) simplify the model for the various disaster scenarios and aid supplies. The inventory levels remain constant for a period of one year. It is suggested that the model should be revised annually with updated data, assuming that a disaster has occurred. The updated data will allow pre-positioning facilities to be re-stocked with adequate quantities.

According to Taskin and Lodree (2010), only one type of item is considered; but the model formulated for the SADC will include all the aid supplies selected to provide relief. The final adjustment is made to the unit cost of an aid supply, which is adapted to the shortage cost of an item. It is essential to address shortage cost, considering that human lives are at stake. These adjustments ensure that the model is suitable for the various disasters and their impact in the SADC.

4.3.3 SIM: Data Analysis

This section illustrates the computational results of the model. The generic inventory model was coded in LINGO version 8.0, on a standard personal computer, rendering a result in less than 9 seconds. The model has a total of 561 variables and 749 constraints. A short description of how the relevant parameter values were obtained is addressed, followed by the results of the model and the sensitivity analysis applied to verify the functionality of the model.

For the purpose of this second application, it was necessary to identify disaster scenarios or, more simply stated, disaster impact. To analyze a scenario effectively, all the possible characteristics of a disaster, i.e. disaster types and effects, have to be considered. For each disaster type and each related consequence, a probability is determined, which is multiplied to obtain a disaster scenario. Figure 4.2 illustrates this method.

Predicting a disaster is challenging, and in most cases impossible. However, a probability can be determined to pre-determine the likelihood of such an event. The approach used to determine these probabilities was to observe the number of times the identified disasters have occurred in the SADC in the past 30 years. The total number of occurrences of each disaster is then divided by the overall total of all the SADC disasters, presented in Table 4.9. The stochastic model addresses the probability that a potential disaster has failed to manifest within a given year in the 30 year period. Table 4.9 displays these probabilities.

In addition to identifying the frequency of each disaster occurrence, it is important to understand the impact of all disasters. Therefore the repetition of a disaster and its effect

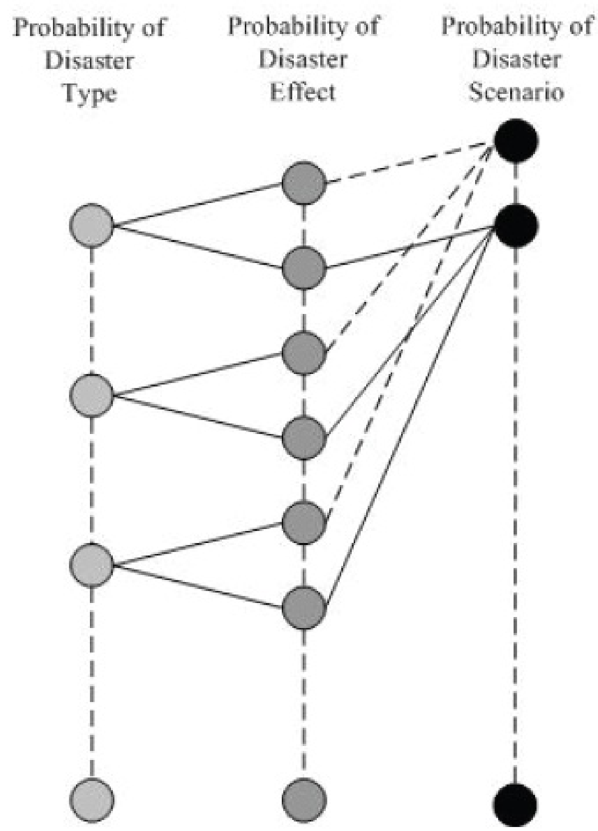

Figure 4.2: Determining scenario probabilities

Table 4.9: Probability of a disaster occurrence

No	Disaster	Total	Probability
1	Drought	70	0.01341
2	Earthquake	17	0.00326
3	Epidemic	154	0.02950
4	Extreme temperature	2	0.00038
5	Flood	168	0.03218
6	Industrial accident	26	0.00498
7	Insect infestation	5	0.00096
8	Mass movement mass	4	0.00077
9	Miscellaneous accident	0.00536	
10	Storm	100	0.01916
11	Transport accident	298	0.05709
12	Wildfire	12	0.00230
13	No disaster	4 336	0.83065
	Total	5 220	1

Table 4.10: Probability of a scenario (q_k)

Scenario k	Probability
1 (No effect)	0.9090
2	0.0096
3	0.0086
4	0.0094
5	0.0090
6	0.0084
7	0.0092
8	0.0092
9	00094
10	0.0090
11	0.0092
Total	1

is determined. Disaster effects were identified in ten different ranges of the total population affected. These ranges were compiled by digesting all the data of disasters that affected population groups in the SADC during the selected period. The percentiles of the list of values were determined by computing the 10^{th} percentile, 20^{th} percentile, and so on up to the 100^{th} percentile; and from these the ranges were developed. This method was used to anticipate that a country with a smaller population is also incorporated when affected by the worst possible eventuality. Effect 1 thus represents no disaster, and effect 11 the worst potential disaster. By identifying each frequency it was possible to compute the probable effect of a defined disaster, determined by dividing each value by the associated totals. Referring back to Figure 4.2, the probability of a disaster scenario is obtained by multiplying the probability of each similar disaster consequence by the probability of every disaster type, and adding these values. Table 4.10 shows the related probabilities, which represent the parameter q_k.

To determine the parameter x_{ik}, the total expected demand is calculated for aid supply i for demand scenario k. The quantities of these supplies, however, are determined by establishing the required demand for every possible disaster scenario. The total supply of all the items will be sufficient for 30 days, which according to Kovacs and Spens (2007) is enough time for the recovery phase to be planned. The unit ordering costs c_i of each item are obtained from appropriate suppliers.

The final parameters to be considered are the holding cost h_i and the shortage cost s_i. To emphasise the significance of these costs, Kovacs and Spens (2007) raise the following question:

How to balance the costs of shortages and/or holding inventory with human suffering, and should they be balanced?

Due to the complications that arise when determining these costs for humanitarian organisations, the assumption is made that when any shortages manifest, it merely implies that there is insufficient quantity of relief supplies for disaster victims, resulting in a possible loss of life. Therefore, the shortage cost is determined as follows:

Shortage cost = (Monetary value of human life × Probability that item i will be required) × Number of persons that can utilise one unit

It may seem insensitive to evaluate human life in monetary terms, but to estimate the cost of a treatment or solution to save a life, it is necessary to determine such value (Card and Mooney, 1977). In addition, by not assigning a selected monetary value to human life in relation to the shortage costs associated with humanitarian organisations, it could be inaccurate if just any random value is selected.

The monetary value of a life for the purpose of this model is determined by using the fatal injury cost per person from the National Department of Transport in South Africa (De Beer and Van Niekerk, 2004). The estimated value is ZAR 529,459.

The probability that an aid supply will be required is simply the sum of the probabilities of the disasters in which the aid supply is demanded. Holding cost comprises the cost of carrying one unit of inventory for one time period, and usually includes storage and insurance cost, taxes on inventory, labour cost, and cost of spoilage, theft, or obsolescence (Winston, 2004). Unlike supply and distribution cost, not all humanitarian relief chains will have substantial inventory cost, in that some relief organisations will maintain and operate their own supply warehouses (Beamon and Kotleba, 2008). The holding costs will therefore depend entirely on the decision-maker's personal preference and demand. Factors such as the size of the warehouse, number of staff, and insurance rates need to be incorporated. The implications involved in determining these factors, will cause the holding costs to be computed as a percentage of the unit cost of each item. The inventory carrying cost will vary according to each individual warehouse, but for testing purposes it is assumed that inventory carrying cost equals 25% of product value per annum (Coyle et al., 2003).

At the outset, the stochastic inventory model is tested by applying the defined parameters addressed above. The graph in Figure 4.3 illustrates the resulting quantities and types of aid supplies required. The results indicate that, as the scenario effects worsen, the quantity of aid supplies do not increase accordingly. The model is concerned with the minimum cost of inventory kept, and so it endeavours to provide sufficient relief whilst avoiding unnecessary costs. Table 4.11 illustrates the individual values of each aid supply.

To test the functionality of the model further, a sensitivity analysis is conducted. The method used is to alternate the holding and shortage cost, while the other parameters are kept constant. The shortage cost is alternated with nine consecutive progressive ranges of values, while the holding cost remains constant. Thereafter, the holding cost is alternated with nine consecutive progressive ranges of values, while the shortage cost remains constant. Figure 4.4 shows how the overall cost given by the objective function remains constant with each progressive range. The graph shows that, when the holding cost is kept constant, the total cost is presented as an acceptable minimum value as long as the shortage cost is kept as low as possible, and the same result is shown when the holding cost is kept at a minimum. From this graph it can be concluded that the model will provide a reasonably low overall cost if holding and shortage costs are kept as low as possible.

The model evidently provides a means to determine the quantities and types of aid supplies to be kept in a pre-positioned facility at the lowest possible cost. The model identifies the required inventory to be kept for one year, and to be revisited annually with updated data to provide relief for the following year.

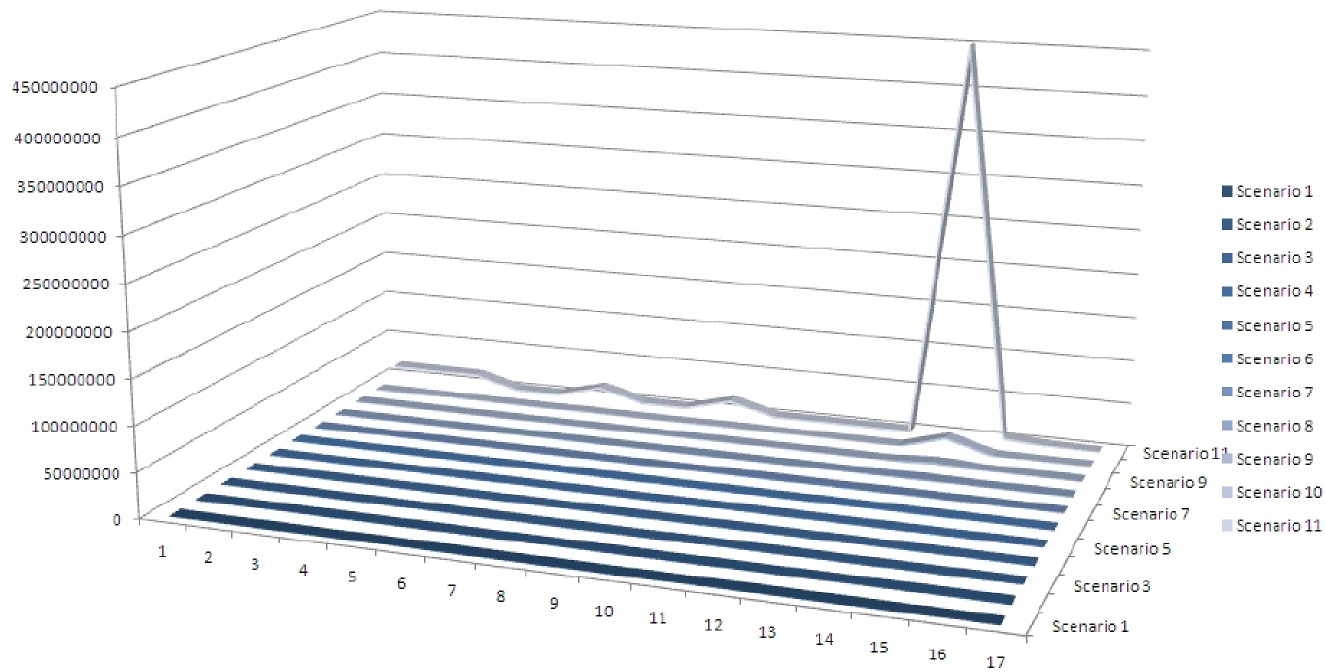

Figure 4.3: Model Results: Quantities and types of aid supplies

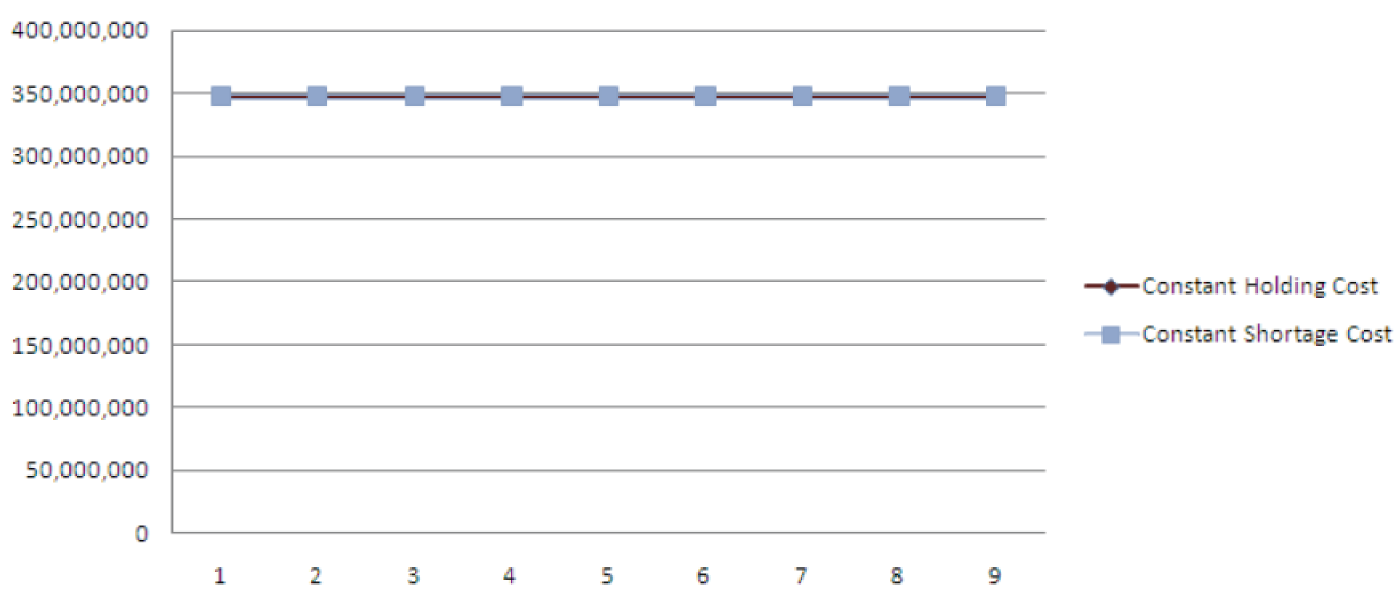

Figure 4.4: Effect of alternating holding and shortage cost

Table 4.11: Model Results

I/K	1	2	3	4	5	6	7	8	9	10	11
1	0	40	100	300	900	3 000	10 000	23 000	102 000	505 000	15 000 000
2	0	40	100	300	900	3 000	10 000	23 000	102 000	505 000	15 000 000
3	0	40	100	300	900	3 000	10 000	23 000	102 000	505 000	15 000 000
4	0	4	10	30	90	300	1 000	2 300	10 200	50 500	1 500 000
5	0	4	10	30	90	300	1 000	2 300	10 200	50 500	1 500 000
6	0	40	100	300	900	3 000	10 000	23 000	102 000	505 000	15 000 000
7	0	4	10	30	90	300	1 000	2 300	10 200	50 500	1 500 000
8	0	4	10	30	90	300	1 000	2 300	10 200	50 500	1 500 000
9	0	40	100	300	900	3 000	10 000	23 000	102 000	505 000	15 000 000
10	0	4	10	30	90	300	1 000	2 300	10 200	50 500	1 500 000
11	0	4	10	30	90	300	1 000	2 300	10 200	50 500	1 500 000
12	0	4	10	30	90	300	1 000	2 300	10 200	50 500	1 500 000
13	0	4	10	30	90	300	1 000	2 300	10 200	50 500	1 500 000
14	0	1 200	3 000	9 000	27 000	90 000	300 000	690 000	3 060 000	15 150 000	450 000 000
15	0	11	27	82	245	818	2 727	6 273	27 818	137 727	4 090 909
16	0	2	5	15	45	150	500	1 150	5 100	25 250	750 000
17	0	1	1	3	8	25	83	192	850	4 208	125 000

4.4 Euclidean Distance Algorithm (EDA)[3]

4.4.1 Introduction

In this section an Euclidean distance algorithm is developed. The method is adopted from Wu et al. (2010) where a similarity calculation method is employed to identify the most similar case. This method allows the prediction of possible disasters to be compared with similar disasters which have occurred in the past, and select the appropriate inventory accordingly.

Dattorro (2005) defines euclidean distance geometry as the determination of point configuration, relative position or location by inference from interpoint distance information.

In the EDA developed in this chapter, an attribute value of the case is normalized to a non-dimensional interval according to a function producing all relevant properties normalized to the same order of magnitude. This method guarantees that the results accurately reflect the matching degree between source case and target case (Wu et al., 2010). Due to the valuable capability of this method, the Euclidean distance algorithm is used to develop an appropriate model applicable to the SADC countries.

4.4.2 Analysis

The notation of the Euclidean distance algorithm for the SADC is addressed below:

$X_{kl} \triangleq$ number of estimated people affected within effect k by disaster l, where $k \in K, l \in L$

$X_{max_{ij}} \triangleq$ maximum number of people affected within effect i by disaster j, where $i \in \{i \dots n\}, j \in \{j \dots m\}$

$X_{min_{ij}} \triangleq$ minimum number of people affected within effect i by disaster j, where $i \in \{i \dots n\}, j \in \{j \dots m\}$

$X^*_{ij} \triangleq$ represents the dimensionless value of effect i by disaster j, where $i \in \{i \dots n\}, j \in \{j \dots m\}$

[3]A modified version of this section has been published in the proceedings of the Computers and Industrial Engineering (CIE) Conference 2011, USA. This paper was selected to be submitted for the Computers and Industrial Engineering Journal

$T_{ij} \triangleq$ represents the dimensionless value of effect i, by disaster j, where $i \in \{i \dots n\}, j \in \{j \dots m\}$

$\bar{A}_j \triangleq$ the attribute value of disaster j, where $j \in \{i \dots m\}$

$w_j \triangleq$ the weight of each attribute value of disaster j, where $j \in \{i \dots m\}$

$d_n \triangleq$ the decision variable d_n representing the smaller, more similar case when compared with the preceding target case

Where:

$$X_{max_{ij}} = \max \begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{nm} \end{pmatrix}$$

$$X_{min_{ij}} = \min \begin{pmatrix} X_{11} & X_{12} & \cdots & X_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{nm} \end{pmatrix}$$

$$X^*_{ij} = \begin{cases} 0 \\ \frac{X_{kl} - X_{min_{ij}}}{X_{max_{ij}} - X_{min_{ij}}} \\ 1 \end{cases}$$

$$\bar{A}_j = \frac{1}{n} \sum_{i=1}^{n} X^*_{ij} \tag{4.11}$$

$$\bar{V}_j = \left[\frac{\sum_{i=1}^{n} (X^*_{ij})^2}{n} \right]^2 \tag{4.12}$$

$$w_j = \frac{V_j}{\sum_{j=1}^{m} V_j} \tag{4.13}$$

$$d_n = \left[\sum_{j=1}^{m} w_j (X_{ij}^* - T_{ij})^2 \right]^{\frac{1}{2}} \tag{4.14}$$

The decision variable d_n represents the smaller, more similar case when compared with the predicted target case.

4.4.3 EDA: Data Analysis

Predicting disasters is challenging and not always possible, but probabilities can be determined to approximate the likelihood of such events. To determine these estimates, the numbers of disasters which have occurred in the SADC in the past 30 years are observed. The data is collected from the Emergency Disaster Database (EM-DAT) as provided by the Centre for Research on the Epidemiology of Disasters (CRED, 2012). In this database, an event qualifies as a disaster if at least one of the following criteria is fulfilled: 10 or more people are reported killed; 100 or more people are reported affected, injured and/or homeless; there has been a declaration of a state of emergency; or there has been a call for international assistance (CRED, 2012). In this analysis, the severity of a disaster in the SADC is measured by the number of people affected by the relevant disasters.

The Euclidean distance algorithm was coded in MATLAB version 7.0, on a standard personal computer. For the purpose of a sensitivity analysis, the X_{ij} values were chosen to represent ten different scenarios. Each scenario represents 10 possible effects, listed in Table 4.12 subsequently increasing in severity. The results of the d_n values are listed in Table 4.13. If for example, a drought is predicted to occur in 2013 with a probability of 15 000 people to be affected, the model will compare this value with a similar drought which has already occurred, providing value d_n. The highlighted values represent the smaller, most similar cases which are identified in Table 4.14 in terms of the relevant d_n value.

Table 4.12: d_n values for 10 different scenarios

d_n/**Scenario**	1	2	3	4	5	6	7	8	9	10
d_1	0.54355	0.4902	0.47815	0.56503	0.58329	0.67311	0.53323	0.64868	0.55391	0.57914
d_2	0.85341	0.83048	0.89806	0.848	0.8643	0.68428	0.87737	0.76135	0.83588	0.9099
d_3	0.45413	**0.25982**	0.53557	0.53586	0.46436	0.45217	0.59627	0.50272	0.46956	0.47077
d_4	0.5328	0.40626	0.54374	0.46426	0.33353	0.39962	0.53224	**0.42417**	0.49982	**0.38242**
d_5	**0.41105**	0.51063	0.49168	0.49579	0.44738	**0.31636**	0.43584	**0.45313**	0.59176	0.46087
d_6	0.46314	0.51456	0.52985	0.54238	0.51474	0.49793	0.44652	0.57017	0.64951	0.53153
d_7	0.41396	0.54371	**0.2841**	0.54543	**0.25237**	0.4984	**0.44752**	0.64957	0.44895	0.4591
d_8	0.56157	0.5161	0.59922	0.56695	0.61099	0.53651	0.4514	0.56326	0.53932	0.55032
d_9	0.48899	0.52963	0.55335	**0.36274**	0.50086	0.42269	0.46606	0.68195	0.58133	0.53677
d_{10}	0.5985	0.57031	0.42461	0.65023	0.51267	0.56066	0.56199	0.54661	0.55842	0.51839

Table 4.13: Selected d_n values

1	2	3	4	5	6	7	8	9	10
d_5	d_3	d_7	d_9	d_7	d_5	d_5	d_4	d_7	d_4

Table 4.14: Inventory required

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
1	40	100	300	900	3000	10000	23000	102000	505000	15000000
2	40	100	300	900	3000	10000	23000	102000	505000	15000000
3	40	100	300	900	3000	10000	23000	102000	505000	15000000
4	4	10	30	90	300	1000	2300	10200	50500	1500000
5	4	10	30	90	300	1000	2300	10200	50500	1500000
6	40	100	300	900	3000	10000	23000	102000	505000	15000000
7	4	10	30	90	300	1000	2300	10200	50500	1500000
8	4	10	30	90	300	1000	2300	10200	50500	1500000
9	40	100	300	900	3000	10000	23000	102000	505000	15000000
10	4	10	30	90	300	1000	2300	10200	50500	1500000
11	4	10	30	90	300	1000	2300	10200	50500	1500000
12	4	10	30	90	300	1000	2300	10200	50500	1500000
13	4	10	30	90	300	1000	2300	10200	50500	1500000
14	1200	3000	9000	27000	90000	300000	690000	3060000	15150000	450000000
15	11	27	82	245	818	2727	6273	27818	137727	4090909
16	2	5	15	45	150	500	1150	5100	25250	750000
17	1	1	3	8	25	83	192	850	4208	125000

For each d_n, appropriate quantities of aid supplies are identified to supply sufficient relief to disaster victims. Table 4.14 outlines the required inventory to keep for each case. Aid supply 1 to 17 represent the following supplies respectively: mosquito nets, waterproof ponchos, waterproof ground mats, childrens activity pack, durable plastic box, collapsible water containers, water purifying equipment, ten-person tent, thermal fleece blankets, cooking equipment, gel stove, toolkit, first aid kit, food supplies, water, portable toilet and finally waste bins. The table evidently shows that the quantity of aid supplies increase with each d_n possibility.

4.5 Comparison of the Models (DIM, SIM and EDA)

A comparison is made amongst the three models discussed in this chapter; the DIM, SIM and EDA. The three models similarly focus on finding the appropriate quantities and types of aid supplies for pre-positioning facilities. The comparison is therefore made by observing the quantities of supplies required for different scenarios. The differences of these models are observed in Figure 4.5, 4.6 and 4.7 in terms of the amount of aid supplies to keep in inventory. The figures each present four different scenarios: Scenario 1 being the smallest effect and increasing in severity with each subsequent scenario.

The DIM model discussed in Section 4.2 is a linear programming model that can be used to effectively identify the type of disaster and determine the quantities of aid supplies required in pre-positioning facilities. The model is formulated as a DRP model to enhance the survival capability of people in the region directly after the occurrence of any of the disasters identified. Figure 4.5 shows that item 14, i.e. food supplies will have the highest demand.

The second model, SIM, which also applied to the SADC region (Section 3.2) selects the appropriate quantities and types of aid supplies to keep so that the overall inventory cost is kept at an acceptable minimum. The model considers excess inventory and shortages. Figure 4.6 also indicates that the food supplies will be high in demand. The final model is the Euclidean distance algorithm, shown in Figure 4.7.

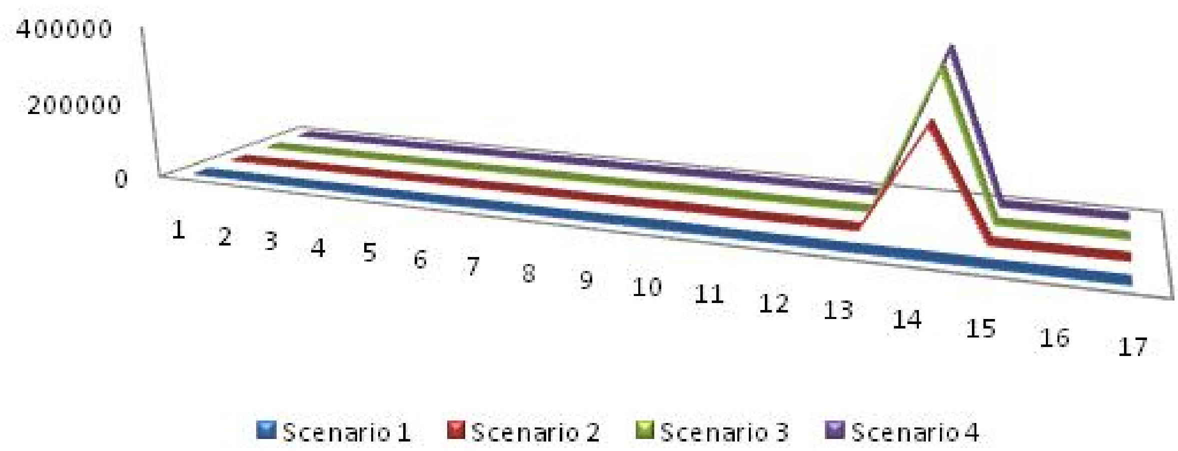

Figure 4.5: DIM Quantities

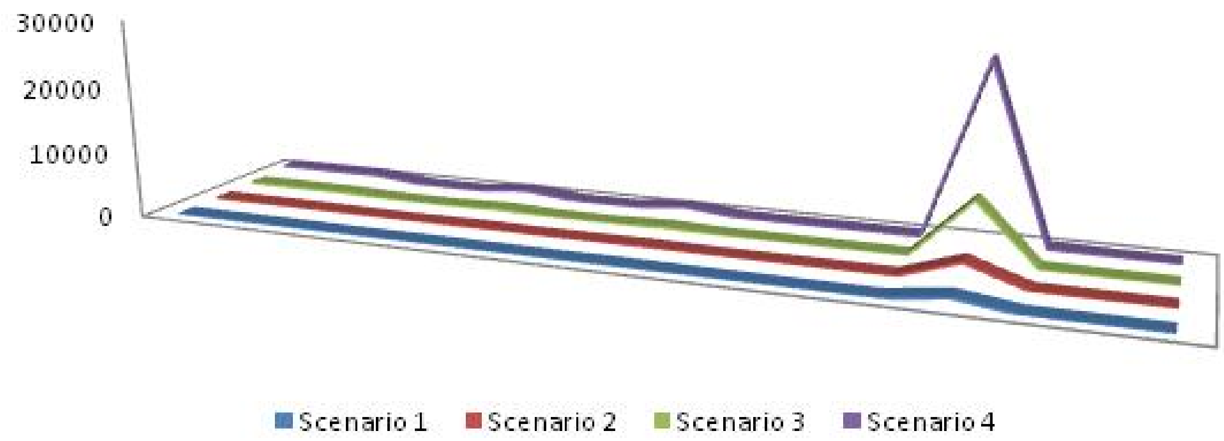

Figure 4.6: SIM Quantities

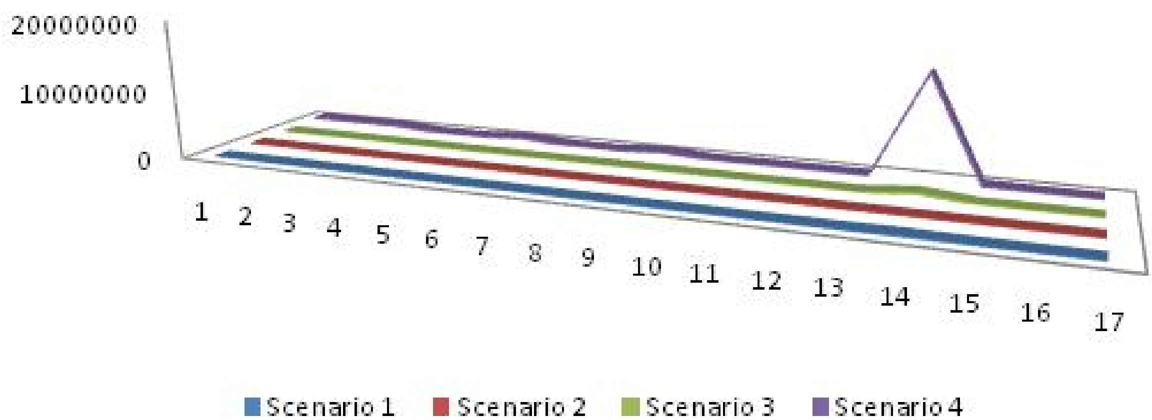

Figure 4.7: EDA Quantities

4.6 Conclusions

The objective of this study was to show the formulation of a mathematical model and to provide strategic decision support, by using this model for selecting the correct quantities and types of aid supplies. All three models provide functional results, given that the quantities increase extensively when food supplies are considered (Number 14 in the figures). The three models provide three different approaches to satisfy a decision-makers need. The deterministic model maximizes the total value of the coverage provided. The stochastic inventory model, however, minimizes the total costs incurred when accumulating and storing relief supplies. The Euclidean distance algorithm aims at selecting the most similar disaster compared to a predicted disaster.

This chapter shows that the models may serve as an effective decision-tool for emergency relief organisations in the SADC countries. However, the models can easily be adapted to be suitable for other countries in the world. The next chapter addresses the application of a pre-emptive multi-objective model using the SADC countries and Somalia as case studies.

Chapter 5

Pre-emptive Multi-objective Inventory Model for Pre-positioning Facilities

5.1 Introduction[1]

Rardin (1998) explains that although practical problems almost always involve more than one measure of solution merit, many can be modelled quite satisfactorily with a single cost or profit objective. Other criteria are either presented as constraints or weighted in a composite objective function to produce a model sufficiently suitable for productive analysis. Many applications such as those in disaster management must be treated as multi-objective. When goals cannot be reduced to a similar scale of cost or benefit, trade-offs need to be addressed. To obtain useful results from such an analysis, the multi-objective model must be reduced to a sequence of single objective optimizations (Rardin, 1998). This leads to pre-emptive multi-objective optimization when considering objectives separately. The most important objective is optimized subject to a requirement that the first has achieved its optimal value; and so on (Rardin, 1998).

The pre-emptive approach to multi-objective optimization is that it results in solutions

[1]A modified version of this chapter has been accepted to be published as a chapter in New Paradigm in Internet Computing, Springer Verlag

that cannot be improved in one objective without degrading another. If each stage of the pre-emptive optimization yields a single-objective optimum, the final solution is an efficient point of the full multi-objective model. The pre-emptive process uses one objective function at a time to improve one without worsening others. At the completion of this process, no further improvement is possible. As usual, infeasible and unbounded cases can produce complications, but the typical outcome is an efficient point (Rardin, 1998).

This chapter presents a pre-emptive multi-objective model, an alternative approach to determine the total capacity of supplies to keep in pre-positioning facilities. The remainder of this chapter is structured as follows Section 5.2 presents the mathematical model. Section 5.3 depicts the data analysis and findings. Section 5.4 concludes the chapter.

5.2 Mathematical Model

$$x_{ik} \triangleq \begin{cases} 1 & \text{if aid supply } i \text{ is required for disaster } k \\ 0 & \text{otherwise.} \end{cases}$$

$q_k \triangleq$ the probability that disaster k will occur

$n_k \triangleq$ the number of people affected by disaster k

$c_i \triangleq$ the unit cost of aid supply i

$h_i \triangleq$ the holding cost of supply i

$s_i \triangleq$ the shortage cost of supply i

$u_i \triangleq$ the number of people affected if supply i is not available

$v_i \triangleq$ the number of aid supply i in excess

$Q_i \triangleq$ the number of aid supply i required

The objective functions have been formulated as follows:

$$min \quad Z_1 = \sum_{i=1}^{I} Q_i c_i + h_i v_i \tag{5.1}$$

$$min \quad Z_2 = \sum_{i=1}^{I} s_i u_i \tag{5.2}$$

$s.t.$

$$Q_i - v_i + u_i = \sum_{k=1}^{K} \frac{x_{ki} n_k q_k}{s_i}, \quad i \in I \tag{5.3}$$

$$Q_i, v_i, u_i \geq 0, \tag{5.4}$$

Objective function 5.1 minimises the overall cost of holding excess aid supplies. The second objective function 5.2, minimises the shortage cost (number of lives affected) of not having an aid supply. Constraint 5.3 guarantees that the number of aid supplies required for a specific disaster corresponds with the expected demand of a given scenario, while taking excess inventory and shortages into consideration. Constraint 5.4 ensures that decision variables Q_i, v_i and u_i remain greater or equal to 0. It is assumed that no excess inventory is present during the first application of the model.

The parameters were obtained in a similar manner to the parameters in Section 4.2 in chapter 4. The approach used to determine the likelihood of a disaster event was to observe the number of times the identified disasters have occurred in the region in the past 30 years. The parameter q_k, was determined by observing the repetition of occurrences of each disaster. This amount is then divided by the overall total of disasters in the region. The parameter, n_k represents the estimated number of victims to be affected by a disaster in its worst magnitude. Therefore, if a drought occurs, it is most likely that the entire population (100%) may be affected. These values are multiplies by the total population of an area to give an indication of the total victims affected. Holding and shortage cost have similar definitions to those of Section 4.2. The shortage cost represents the amount of people who will be affected if an aid supply is not available during and after the disaster event.

The preemptive optimisation model performs multi-objective optimisation by first optimising objective function 5.1 (the cost of holding an aid supply) and then objective function 5.2 (the cost of the total shortages) is optimised subject to the requirement that 5.1 has achieved its optimal value (Rardin, 1998).

5.3 Data Analysis for the Pre-emptive Multi-objective Inventory Model

5.3.1 SADC Region

The preemptive optimisation model was coded in *LINGO* version 12.0. The model was structured to construct four efficient frontier curves, each representing a category. The efficient frontier indicates the efficient points when considering the holding and shortage cost for each category. Category A illustrates the efficient frontier for 0 - 1 million people affected, category B between 1 million - 2.5 million people, category C between 2.5 million - 5 million people and category D between 5 million - 7.7 million people affected. The four categories are given in Figure 5.1, illustrating that with each category the number of aid supplies will increase, increasing the overall costs. The categories can be used as a decision tool to determine the quantities of supplies to be kept within an acceptable budget.

5.3.2 Somalia

The model for Somalia was also formulated to provide four efficient frontier curves, each representing a category. Category A illustrates the efficient frontier for 0 1 million people affected, category B between 1 million 2 million people, category C between 2 million 3 million people and category D between 3 million and 4 million people affected. The four categories are given in Figures 5.2, illustrating that with each category the number of aid supplies will increase, increasing the overall costs.

5.4 Conclusions

The model provides an adequate quantity of the types of aid supplies required in pre-positioning facilities. The pre-emptive model incorporates cost as part of the model and thus identifies a realistic quantity of supplies.

The efficiency of this model is related to the funding associated with disaster preparedness. Finding funds to support disaster preparedness is problematic, according to Bernard

Chomiller, former Head of Logistics at the IFRC (Tomasini and Van Wassenhove, 2009):

> *"It is easy to find resources to respond, it is hard to find resources to be more ready to respond."*

There is an immanent need to attract increased funding for disasters; - a good starting point is to apply more case studies to models to illustrate the urgent demand for preparedness.

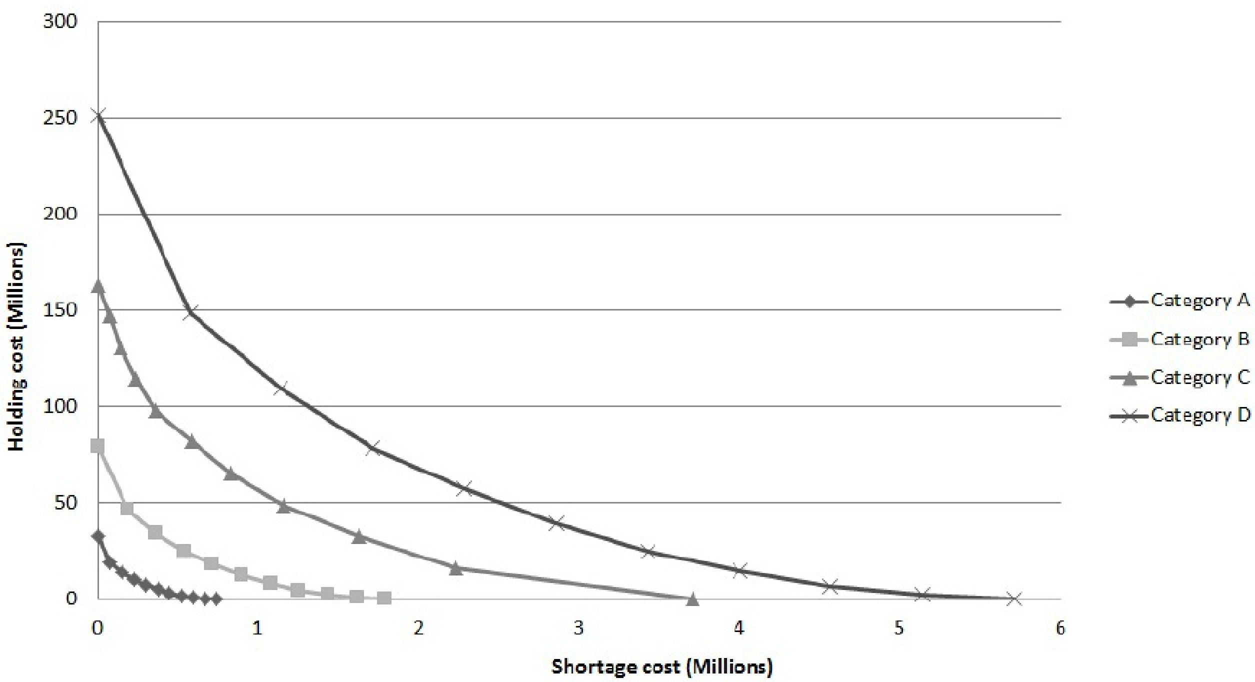

Figure 5.1: Efficient frontier for SADC for each category

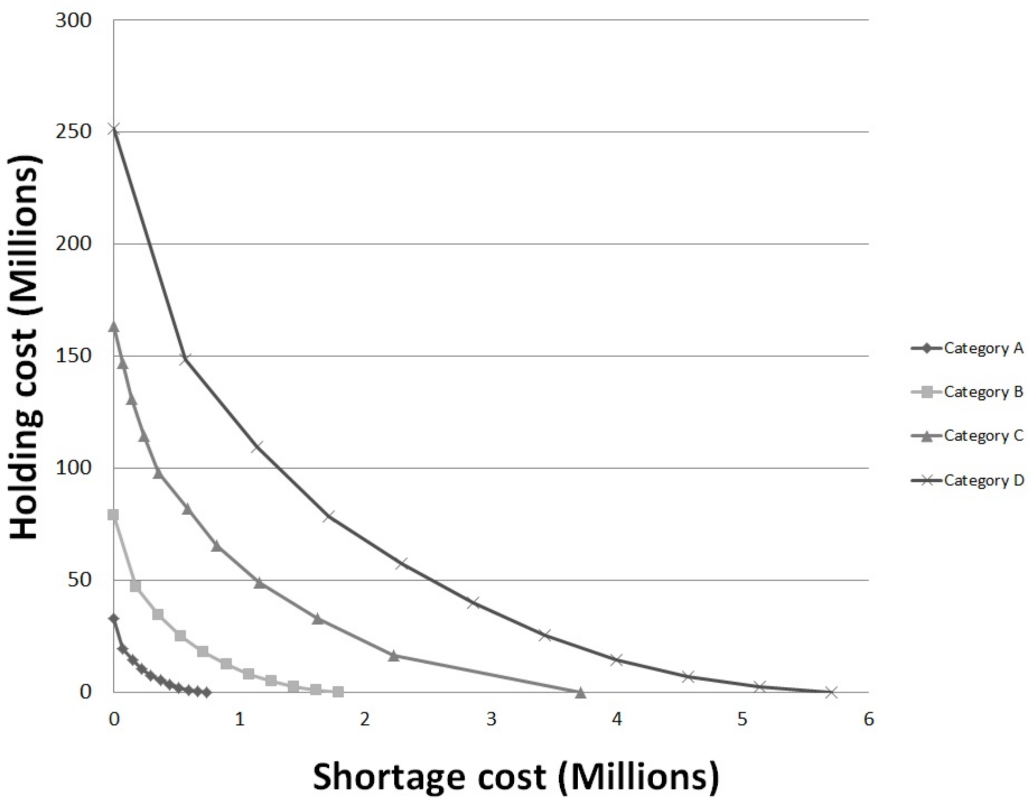

Figure 5.2: Efficient frontier for Somalia for each category

Chapter 6

Application of a Simulated Annealing Heuristic for the Location of Pre-positioning Facilities

6.1 Introduction

In this chapter a solution to the location problem in humanitarian logistics is developed. The solution is the application of a Simulated Annealing (SA) heuristics to determine the optimal locations of pre-positioning facilities in a region such as the SADC. This section introduces the Simulated Annealing method. Section 6.2 addresses the formulated algorithm. Section 6.3 explains the data analysis and finally, Section 6.4 summarizes the chapter.

Emergency logistics support is limited in the initial phases of disaster response. The exact impact of the phenomenon is not known and it takes time to explore the affected regions, communicate the impacts of the disaster and coordinate national and international involvement (Yi and Kumar, 2007). Supply warehouses need to be located at strategic positions, considering a number of factors such as, population size, infrastructure, political views and the probability that a disaster may occur within that region. In addition, if a pre-positioning facility is destroyed through disaster, another should be located within an appropriate range.

According to Balcik (2008), metaheuristics perform a more comprehensive exploration of the solution space and often embed some of the classical route construction and improvement heuristics in searching for good solutions. The main types of metaheuristics that have been applied to location problems are Simulated Annealing, Deterministic Annealing, Tabu Search, Genetic Algorithm, Ant Colony and Neural Networks (Balcik, 2008).

Simulated Annealing was originally introduced by Kirkpatrick et al. (1983)as an optimization technique. It is a stochastic local search algorithm, which is motivated by the annealing process of solids. The method randomly walks within the solution space, gradually adjusting the temperature based on a cooling schedule. Each feasible solution attained during the algorithm is analogous to the state of the solid in the annealing process and objective function at a feasible solution corresponds to the current system energy. Therefore, the optimal solution in an optimization problem is analogous to the minimum energy state in the annealing process. At each iteration of the Simulated Annealing algorithm, a new (candidate) feasible solution is generated in the neighborhood of the current feasible solution using a solution generator, which is chosen depending on the problem characteristics (Balcik et al., 2008).

Simulated Annealing is amongst the most popular iterative methods for solving combinatorial optimization problems. The differentiating characteristic of Simulated Annealing compared to other heuristics is that it is able to improve upon the relatively poor performance of local search by simply replacing the deterministic acceptance criterion by a stochastic criterion. In addition, Balcik (2008) agrees that Simulated Annealing is efficient and has low memory requirements.

In this study, a modified Simulated Annealing Heuristic was selected for the pre-positioning facility placement problem. The 100 largest towns, with the largest population for each country within the SADC are identified. More towns were selected for South Africa, and fewer towns for smaller countries like Lesotho. The Seychelles and Mauritius were excluded from the analysis as they can be regarded as special cases and would bias the algorithm eastward unnecessarily. Town locations were obtained from the GEOnet Names Server, administrated by the National Geospatial-Intelligence Agency (NGA), an agency of the US federal government. The spherical law of cosines was then used to create a large distance table, which in turn was used by the algorithm. It is assumed that an Oryx aircraft is always used for all disasters in all countries, which is the aircraft normally utilised by the South African National

Defence Force for relief operations. Country borders and natural features such as lakes and mountains are not considered by the heuristic. An assumption is made that a disaster type and the proportion of people affected by a specific disaster is equally likely in all towns. The mathematical model has been formulated as recorded hereinafter.

6.2 Mathematical Model

$$x_i \triangleq \begin{cases} 1 & \text{if location } i \text{ has no PPF and has less than 2 PPFs within 300km} \\ 0 & \text{otherwise.} \end{cases}$$

$$b_i \triangleq \begin{cases} 1 & \text{if location i has no PPF and is greater than 300km from nearest PPF} \\ 0 & \text{otherwise.} \end{cases}$$

$$a_i \triangleq \text{number of PPFs } -1 \text{ closer than 300km from PPF at location } i$$

$$c_i \triangleq \text{distance from PPF at } i \text{ to nearest air force base}$$

$$min \quad z = \sum_{n=1}^{I} x_i + a_i + 3b_i + 0.01c_i \tag{6.1}$$

The Simulated Annealing (SA) heuristic has the advantage of being a relatively fast heuristic in terms of processing speed per iteration. Furthermore, constructing the problem as a “heuristic” has the advantage of flexibility in formulation, whilst it may be a Linear Program in its simplest form, adding complexity by taking multiple factors into account could conceivably convert it to a NP-hard problem. Lastly SA is much less sensitive to the starting solution than a Genetic Algorithm, for example.

The standard SA algorithm was modified to converge and intensify on higher quality results by altering the cooling schedule and employing a restart. Whereas the standard SA uses a linear cooling schedule, it was found that an exponential cooling schedule increases the proportion of time spent at lower temperatures. This allows the algorithm to converge to higher quality results, provided adequate diversity on incumbent solutions is obtained early during the run.

The restart adjusts the algorithm to the best known solution once a certain temperature is reached. This ensures that a high quality solution is intensified during the final stages of the algorithm, rather than the incumbent solution the algorithm happens to be working on at that stage.

Due to the unconstrained nature of a heuristic, the problem was formulated as a multiple-objective program. Essentially a set of locations was sought that would minimise the number of pre-positioning facilities whilst ensuring that every town is still within range of an aid drop (300km) from two facilities and not more. Also, minimising the number of facilities that are in range of more than two other facilities. The towns outside range of an aid drop were weighted (penalised) at three times the value of the other numbers in the objective function. A weighting of 0.01 was added to the criterion of a location's distance to the nearest air force base or country capital. This is in order to bias the solution in favour of towns closer to these features, because the aircraft will not be stationed at the pre-positioning facility itself, and will have to fly there first.

The effective radius of aid delivery around a pre-positioning facility was determined by consulting the Flight Manual of an Atlas Oryx (a modified Eurocopter Super Puma). Many variables need to be taken into account. Assuming a warm summers day in the South African highveld, the Oryx can safely transport 1 tonne of relief aid to a location 300 km away in 1.5 hours, and still have sufficient fuel to return to base. The following section describes the computational results of the models.

6.3 Data Analysis for the Location Model

The modified Simulated Annealing (SA) heuristic was programmed in Octave 3.2.4. The algorithm showed satisfactory intensification and repeatability, but not as much diversity as a Hamiltonian cycle normally does. This is because the randomization employed (adding a pre-positioning facility in a random town if there was none, or removing it if there was one), only changes the objective function slightly, whereas interchanging the order of a Hamiltonian cycle affects and alters the objective function significantly. The overall result is, however, acceptable. The progression of the algorithm is presented in Figure 6.1. The quantity and locations of the algorithm is illustrated in Figure 6.2 with 89 pre-positioning facilities, which can be used as a decision tool for humanitarian organizations. The facilities are strategically positioned to accommodate a large population. If however, a facility is demolished, an additional facility will be located within an acceptable radius from the disaster area.

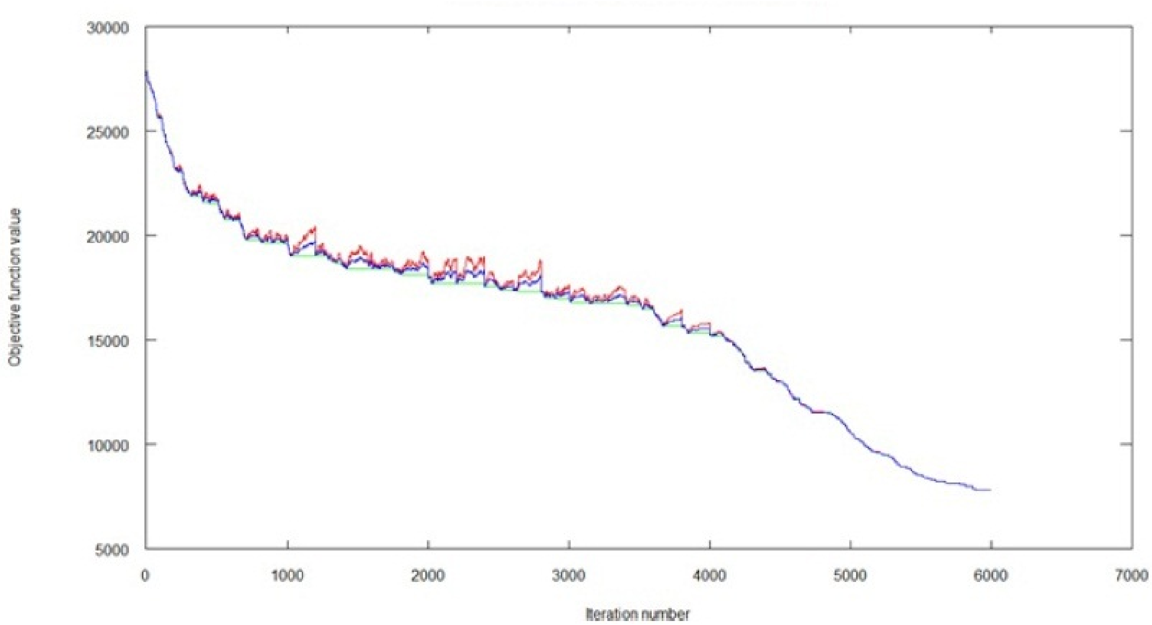

Figure 6.1: Simulated Annealing algorithm progression of SADC PPF

6.4 Conclusions

In this chapter, SA was used to address the location problem for pre-positioning facilities in the SADC region. From the results obtained, the SA algorithm proves to be useful to a decision-maker. The algorithm may be adjusted to comply with another region in the world by adapting the data accordingly.

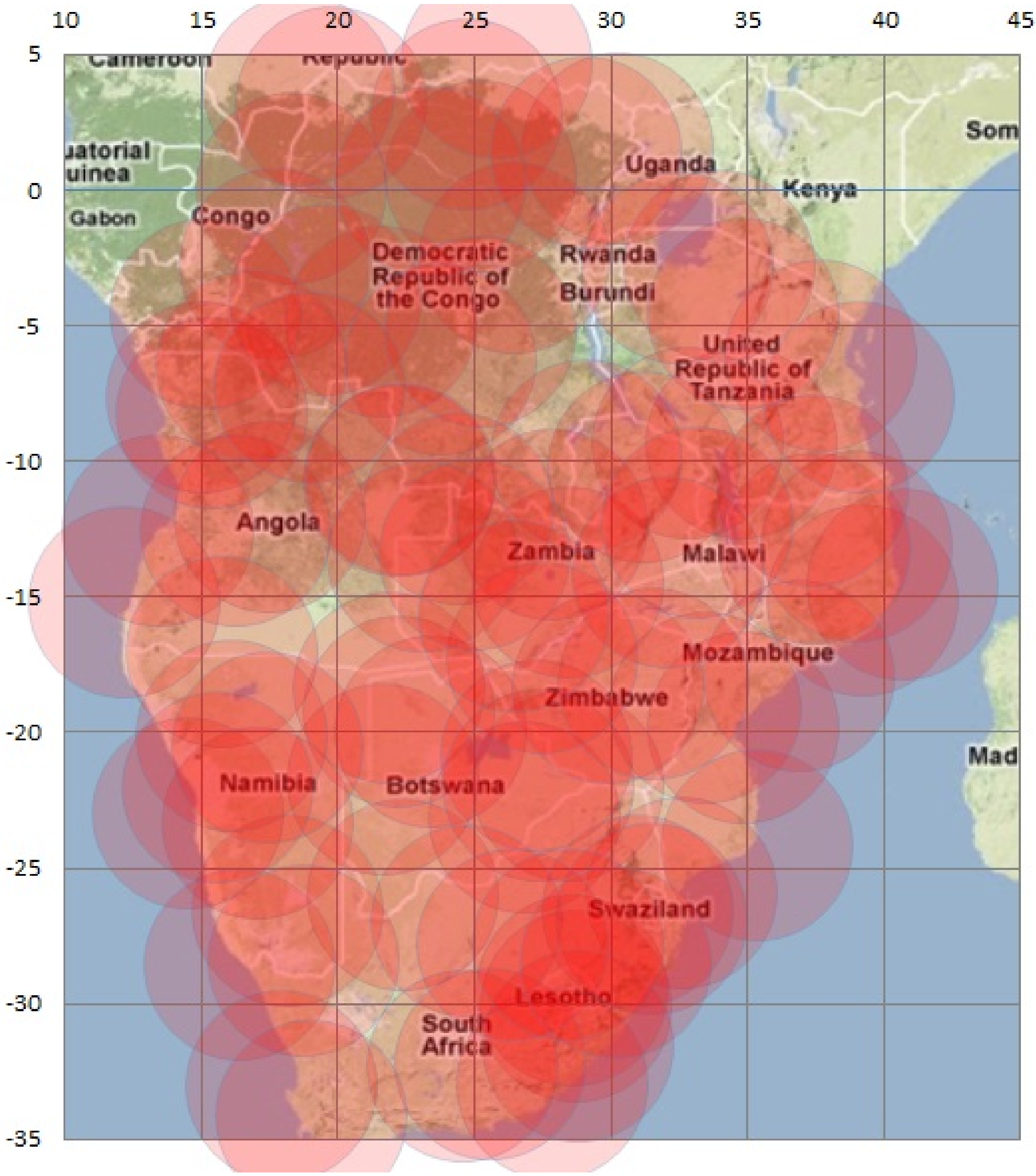

Figure 6.2: Location of pre-positioning facilities

Chapter 7

Conclusions - Modelling of Inventory Management for Humanitarian Logistics

In recent years, there has been an increase in the number and severity of natural disasters. This increase has been accompanied by massive global relief operations involving a large number of relief organizations requiring substantial support. Although relief organizations have important strengths in managing large-scale disaster relief operations in unpredictable and chaotic environments, the scale, magnitude and complexity of recent disaster relief operations have showed that the capabilities of current management approaches may not meet the logistical needs. The trends in the number and impact of disasters and increasing emphasis on accountability have also stressed the need for improving the time, capacity, resource and location factors in humanitarian relief operations. In addition, there has not been enough literature to ensure coordination amongst these factors. The lack of systematic approaches addressing logistics problems, has led relief organizations to use ad-hoc methods in disaster response. This has led to poor performance in preparing and responding to disasters (Balcik, 2008).

This dissertation was motivated by the needs of relief organizations to develop and implement quantitative and qualitative methods that will enable them to manage the logistics operations effectively and efficiently. In addition, there exists a need in developing countries such as the SADC countries and Somalia to implement proper methods for disaster relief. A further motivation is triggered from an industrial engineering perspective. Industrial engineering expertise should be more involved in the interests of communities around the world. The analytical and management techniques developed within an industrial engineer's ability will enhance and improve humanitarian logistics activities. The objectives of the dissertation was to define and characterize critical problems in inventory management for humanitarian logistics. Another objective was to develop analytical models and techniques to support the decision-making process before, during and after a disaster has taken place. Finally, from the results obtained in this dissertation, further research opportunities can be identified.

The focus of the dissertation was mainly on four critical areas: time, resources, capacity and location. Various studies illustrate how mathematical modelling can be utilised to provide strategic decision support for the aftermath of disasters. Countries within SADC, and Somalia are not sufficiently prepared for disaster events. Thousands of people in SADC and Somalia will suffer severe trauma if a storm should strike the country in the same magnitude as Japan was affected in March 2011. In general, this research will be able to give any country the opportunity to be more adequately prepared in the event of disaster occurrences.

The research is gathered in such a way that is accommodates inventory management methods in humanitarian logistics. The literature has addressed the latest methods, techniques, contributions and comments of the publications during the past 10 years. The literature was used to develop theoretical models. The theoretical stochastic models introduced time, capacity and location in the formulation of the models. The ensuing part of the dissertation was the development of models relevant to the countries under consideration. These models considered the capacity of the supplies required in pre-positioning facilities. Next, a pre-emptive multi-objective model was introduced as an improved methodology to determine the types and quantities of aid supplies to be kept. Finally, a Simulated Annealing heuristic was developed to introduce the optimal locations of pre-positioning facilities within the SADC region.

The models address the concerns of feasibility, consistency and completeness. From a decision-maker's point of view, these models can serve as a handy guideline to assist with planning, response and recovery phases in the disaster operations life cycle, as well as incorporating risk where appropriate. Workable solutions have been identified which have unveiled the option to increase the use of operational research methods to enhance disaster relief options. These models are formulated in a generic manner, therefore future research developments can be made to the models by gathering data from the specific country and applying the models to the pre-selected regions. Due to the useful results provided by both models, the implementation thereof will effectively assist and guide decision-makers with humanitarian relief decisions for disaster relief. With adequate marketing, the models can be implemented by humanitarian organizations.

The United Nations (UN) stated that appropriate measures of preparedness and relief may not be able to prevent disasters, but they may significantly reduce the consequences. Although many such measures are known, there exists a demand for additional information, which can only be gained by further research (Plate and Kron, 1994). Natural and man-made disasters are occurring more frequently and are becoming more severe with serious humanitarian consequences. Humanitarian logistics is therefore a research discipline which should never reach an optimum limit.

Future research proposed for this work is to determine the most practical transport and cost-effective routes. The routes may be selected by using Geographic Information Systems (GIS), which is a system designed to capture, store, manipulate, analyze, manage and present all types of geographically referenced data. In addition, greater accuracy of the model validation may be done by gathering data from the UN, and other international organizations.

In conclusion, disasters cause a great deal of suffering, but through careful planning, evaluation and the implementation of the models contained in this research, it is possible to achieve the aim of improving the mitigation, preparedness, response and recovery phases of the disaster cycle, at reasonable and affordable cost structures.

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Appendix A

Matlab Model

```
al=0.001;

cl=100.0;

t=100;

e1=exp(-al*t);

for theta=0.1:0.1:10.0

ck=50;

cr=0;

for j=0:ck

rj=0;

for n=0:j

rn3=0;

for j=0:n+2

    rn3=rn3+((1+theta)^j/cl^j)*((exp(-cl/(1+theta))+exp(-
al*cl/(1+theta))*theta^(n+2-j))/factorial(n+2-j));

end

t6=rn3*factorial(n+2);

rn2=0;

for j=0:n+1

    rn2=rn2+((1+theta)^j/cl^j)*((exp(-theta*cl/(1+theta))*theta^(n+1-j)-exp(-
cl/(1+theta)))/factorial(n+1-j));

end

t5=rn2*cl*(1-theta)*factorial(n+1);

rn1=0;

for j=0:n

    rn1=rn1+((1+theta)^j/cl^j)*((exp(-cl/(1+theta))+exp(-
theta*cl/(1+theta))*theta^(n-j))/factorial(n-j));

end

t4=rn1*theta*factorial(n);

t3=(2*(1+theta)^(n+2))/(cl^(n+2));

t2=(2*cl*(1+theta^(n+2)))/((1+theta)*(n+1)*(n+2)*(n+3));

t1=(cl*(1+theta^(n+2)))/((n+1)*(n+2));

tt=t1-t2+t3+t4-t5-t6;

rn=((12*al*cl^(n-1))/((1+theta)^(n+2)))*tt;

rj=rj+rn;

end
```

```
rj=rj*(exp(-al*t)*(al*t)^j)/(factorial(j));

cr=cr+rj;

end

[theta' cr']

end
```

Appendix B

Profile (Estelle van Wyk)

Education

Highest qualification: BEng(Honours)(Industrial Engineering)

Research interests

- Humanitarian logistics
- Logistics
- Operational research modelling (deterministic and stochastic)
- Operations management
- Reliability engineering

Publications

- Van Wyk, E., Bean, W., and Yadavalli, V.S.S. (2011a). Modelling of uncertainty in minimising the cost of inventory for disaster relief. *South African Journal of Industrial Engineering*, 22(1):1-11.

- Van Wyk, E., Bean, W., and Yadavalli, V.S.S. (2011b). Strategic inventory management for disaster relief. *Management Dynamics*, 20:32-41.

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- Van Wyk, E., and Yadavalli, V.S.S.(2012b). A temporo-spatial stochastic model for optimal positioning of humanitarian inventories for disaster relief management. *European Journal of Operations Research*,(Paper communicated to journal).

Conference Presentations

- Decision support in supply chain management for disaster relief in Somalia; (2012) Published in the proceedings of the International Conference on Mechanical and Industrial Engineering (ICMIE), Singapore. **This paper has been invited and accepted for publication in a book published by SPRINGER-VERLAG, with the book title 'New Paradigm in Internet Computing.**

- Application of an Euclidean Distance Algorithm in inventory management for disaster relief in SADC; (2011) Published in the proceedings of the International conference on Computers and Industrial Engineering (CIE), Los Angeles. **This paper has been invited to submit for the Journal 'Computers and Industrial Engineering'.**

- A Stochastic model towards managing the aftermath of a Tsunami disaster: A case study; (2011) Published in the proceedings of the International conference on Industrial, Systems and Engineering Management (ISEM), Stellenbosch, South Africa.

- A generic inventory model for a disaster relief in the SADC countries; Presented at the Operations Research Society of South Africa (ORSSA) conference, South Africa.

- A comparative study of a Euclidean Distance Algorithm and Pre-emptive Multi-objective modelling for disaster logistics: A case study for the SADC, will be presented at the International conference on Computers and Industrial Engineering (CIE) (2012), Cape Town, Sout Africa.

Awards

- Presenter at the Projects Evening of the Department of Industrial and Systems Engineering (2010) - First prize winner for **project presentation and project document.**

- ICMIE conference paper in Singapore received the best paper presentation.

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