The theory of Homo comperiens, the firm's market price, and the implication for a firm's profitability

Joachim Landström

The theory of Homo comperiens, the firm's market price, and the implication for a firm's profitability

Joachim Landström

ABSTRACT

Landström, J., 2007, The theory of Homo Comperiens, the firm's market price, and the implication for a firm's profitability, *Doctoral thesis / Företagsekonomiska institutionen, Uppsala universitet* 128, 259 pp., Uppsala.

This thesis proposes a theory of inefficient markets that uses limited rational choice as a central trait and I call it the theory of Homo comperiens. The theory limits the alternatives and states that the subjects are aware of and only allow them to have rational preference relations on the limited action set and state set, i.e. limited rationality is introduced. With limited rational choice, I drive a wedge between the market price and the intrinsic value and thus create an arbitrage market.

In the theory, the subjects are allowed to gain knowledge about something that they previously were unaware of. As the discovery proceeds, the arbitrage opportunities disappear, and the market prices regress towards the intrinsic values.

The theory is applied to firms and market-pricing models for a Homo comperiens environment is a result. The application of the theory to firms also leads to testable propositions that I test on a uniquely comprehensive Swedish accounting database that cover the years 1978—1994.

Hypotheses are tested which argues that risk-adjusted residual rates-of-returns exist. The null hypotheses argue that risk-adjusted residual rates-of-returns do not exist (since they assume a no-arbitrage market). The null hypotheses are rejected in favor of their alternatives at a 0.0 percent significance level. The tests use approximately 22,200 observations.

I also test hypotheses which argue that risk-adjusted residual rates-of-returns regress to zero with time. The null hypotheses are randomly walking risk-adjusted residual rates-of-returns, which are rejected in favor of the alternative hypotheses. The hypotheses are tested using panel regression models and goodness-of-fit tests. I reject the null hypotheses of random walk at a 0.0 percent significance level.

Finally, the results are validated using out-of-sample predictions where my models compete with random-walk predictions. It finds that the absolute prediction errors from my models are between 12 to 24 percent less than the errors from the random walk model. These results are significant at a 0.0 percent significance level.

Joachim Landström, Uppsala University, Department of Business Studies, Box 513, SE-751 20 Uppsala, Sweden.

To Maria for being at my side at all times.

TABLE OF CONTENTS

LIST OF TABLES

SYMBOLS AND ABBREVIATIONS

Note that this is selection and not a comprehensive list of symbols and abbreviations.

$\{a,b\}$ a and b are the elements of a set.

$\neg$ Logical not.

$a|b$ a when b.

$a \in A$ a is an element of set A.

$a \notin A$ a is not an element of set A.

$\varnothing$ The empty set.

$\forall$ For all.

$\exists$ There exists.

$\wedge$ Logical and.

$\cap$ Intersection.

$\cup$ Union.

$a \neq b$ a is not equal to b.

$A = \bigcup_i a_i$ A is the set of the union of all a:s.

S_Ω The objective (i.e., the universal) state set. Subscript Ω always denotes the universal set.

S_K The subjective state set. Subscript K is mnemonic for known. $S_K = S_\Omega \setminus I_S$

I_S The ignorance of states set. The states that the subject is unaware of.

A_Ω The objective actions set. I.e., the objective set of alternatives to choose from

A_K The subjective action set. A.k.a. the set of alternatives that is known. $A_K = A_\Omega \setminus I_A$

I_A The ignorance of actions set. The alternative that the subject is unaware of.

C_Ω The objective consequence set.

C_K The subjective consequence set

$A \subset B$ A is a strict subset of B. I.e., At least one element of B is not part of A.

$A \subseteq B$ A is a weak subset of B. I.e., All element of B may be part of A

$A \Rightarrow B$ A implies B but B does not necessarily imply A. A is a sufficient condition for B.

$A \Leftrightarrow B$ A is equivalent to B. I.e., A implies B and B implies A. A is a sufficient and necessary condition for B.

$A \setminus B$ Set theoretical minus. I.e., The part of set A that is not part of set B.

$\complement A$ The complement set to set A.

$a \succsim b$ A weak preference relation. I.e., a is either preferred to b or indifferent to b.

$a \succ b$ A strict preference relation. I.e., a is preferred to b.

$a > b$ Real value a is greater than the real value b.

$a \in [0,1]$ $0 \leq a \leq 1$

$a \in (0,1]$ $0 < a \leq 1$

$a \in [0,1)$ $0 \leq a < 1$

$a \in (0,1)$ $0 < a < 1$

$f : A \rightarrow B$ The real function f maps domain A to its co-domain B.

$f \circ u$ The composition of real functions f and u.

$f(a)$ The real value of the real function f at a.

$\mathbb{R}$ The real line.

$\mathbb{R}_+$ The positive real numbers.

$\mathbb{R}^L$ The real space having L dimensions.

$_{t-1}ROE_t$ The firm's return on equity for the period starting at the end of $t-1$ to the end of t.

$_{t-1}ROE_t^I$ The industry's return on equity for period t.

$_{t-1}RROE_t$ Residual return on equity.

$_{t-1}RNOA_t$ Return on net operating assets.

$_{t-1}RROE_t^*$ The ex post risk-adjusted residual return on equity.

$_{t-1}RRNOA_t^*$ The ex post risk-adjusted residual return on equity.

$\mathrm{E}_0[\cdot]$ The objective expectation at present.

$\mathrm{E}_{K0}[\cdot]$ The subjective expectation at present.

$\mathrm{E}_{K0}^*[\cdot]$ The risk-adjusted subjective expectation at present.

$\mathrm{E}_0\left[_{t-1}r_t\right]$ The objective expected market rate-of-return, also denoted objective MROR.

$_{t-1}r_t$ Simplified notation for the objective market rate-of-return.

$\mathrm{E}_{K0}\left[_{t-1}r_{Kt}\right]$ The subjective expected market rate-of-return, also denoted subjective MROR.

$_{t-1}r_{Kt}$ Simplified notation for the subjective market rate-of-return.

$_{t-1}p_t$ The objective price at time t-1 for delivery at time t. It is also denoted

     p^* when optimization is considered.

$_{t-1}p_{Kt}$ The subjective price.

$\displaystyle\prod_{t=1}^{T}$ The product symbol. E.g., $_0p_T = {}_0p_1 \cdot {}_1p_2 \cdot \ldots \cdot {}_{T-1}p_T = \displaystyle\prod_{t=1}^{T} {}_{t-1}p_t$

V_0 The intrinsic value of a firm at present.

P_0 The market price of a firm at present.

LIST OF CENTRAL DEFINITIONS

LIST OF PROPOSITIONS

RRNOA because of arbitrage opportunities. That is, $E_{K0}^*[{}_{t-1}RROE_t] = E_{K0}[\textit{net arbitrage rate of return}_t] \neq 0$, and

$E_{K0}^*[{}_{t-1}RRNOA_t] = E_{K0}[\textit{operating arbitrage rate of return}_t] \neq 0$

Proposition 4-7: In a market that meets the conjectures of the theory of Homo comperiens (Proposition 2-4, Proposition 3-2) and with unbiased accounting the limit values of risk-adjusted subjective expected RROE and RRNOA are zero. That is: $\lim\limits_{t\to\infty}\left(E_{K0}^*[{}_{t-1}RROE_t]\right) = 0$, and $\lim\limits_{t\to\infty}\left(E_{K0}^*[{}_{t-1}RRNOA_t]\right) = 0$.

LIST OF COROLLARIES

"Ignorance is like subzero weather: by a significant expenditure its effects upon people can be kept within tolerable or even comfortable bounds, but it would be wholly uneconomical entirely to eliminate all its effects. And, just as an analysis of man's shelter and apparel would be somewhat incomplete if cold weather is ignored, so also our understanding of economic life will be incomplete if we do not systematically take account of the cold winds of ignorance." (Stigler 1961, p. 224)

CHAPTER 1—INTRODUCTION

*"Thought is only a flash in the middle of a long night, but the flash that
means everything" Poincaré (1854-1912)*

1.1 Background

At least three areas within market-based accounting research use the efficient market hypothesis
(EMH) as a reference point. Accounting researchers who correlate contemporaneous accounting
information with contemporaneous stock prices (e.g., the earning response coefficient literature)
(e.g., Collins and Kothari, 1989; Easton and Zmijewski, 1989; Liu and Thomas, 2000) need a model
to identify normal earnings to be able to study how actual market prices move regarding unexpected
earnings. The earning response coefficient literature conjectures market efficiency to find normal
earnings.

Researchers that test market efficiency demand models for earnings prediction in order to be
able to form hedge portfolios on this type of information (e.g., Abarbanell and Bushee, 1998; Pio-
troski, 2000). This research also makes heavy use of the EMH.

Positive accounting theorists seek to explain accounting choice based on management's op-
portunistic behavior (e.g., Watts and Zimmerman, 1986). In addition, these researchers need to es-
tablish a level of normal earnings, which leads them to assume market efficiency.

In fact, when reading the market-based accounting review articles by Lev and Ohlson (1982),
Bernard (1989), and Kothari (2001), it appears as if the bulk of market-based accounting research
conjectures the EMH.

Lee (2001, p. 237) argues that research that builds on the efficient market hypothesis "is akin
to believing that the ocean is flat, simply because we have observed the forces of gravity at work on
a glass of water. No one questions the effect of gravity, or the fact that water is always seeking its
own level. But it is a stretch to infer from this observation that oceans should look like millponds on
a still summer night. If oceans were flat, how do we explain predictable patterns, such as tides and
currents? How can we account for the existence of waves, and of surfers?"

Nowadays, there are even calls for a theory of inefficient markets. Kothari (2001, p. 191)
writes, "while much of the research concludes market inefficiency, further progress will be made if
researchers develop a theory that predicts a particular return behavior and based on that theory de-
sign tests that specify market inefficiency as the null hypothesis." Lee (2001, p. 251) writes: "Rather
than conjecturing market efficiency, we should study how, when, and why price becomes efficient
(and why at other times it fails to do so)." Even an adamant proponent of EMH, such as Malkiel

(2003, p. 80), admits that the market has pricing irregularities, predictable patterns, and is not perfectly efficient.

The growing awareness of the problems with the EMH provides an opportunity to develop new theories of inefficient markets. This thesis develops and tests such a theory of market inefficiency, which uses a limited rational choice as a point of departure. I call this theory[1] the theory of Homo comperiens and it allows me — continuing with Lee's metaphor — to account for the existence of waves. It further explains how these waves disappear in the world of markets.

The theory of Homo comperiens is general but thesis applies it to financial economics to maintain a connection to EMH. By applying it to financial economics, I derive firm valuation model assuming ignorance. Some propositions from the theory are also tested on Swedish accounting data that places it in the market-based accounting field.

Before I present and discuss any details of the research presented in this thesis, it is necessary to put the research into its proper context. Therefore, what follows is an initial discussion of EMH and consequences of applying EMH onto market-based accounting research. This chapter closes by fencing this research from other non-equilibrium theories and with the disposition of the thesis.

1.2 The efficient market hypothesis

Fama (1965a, p. 94) proposes the EMH and defines an efficient market as "…a market where prices at every point in time represent best estimates of intrinsic values. This implies in turn that, when intrinsic value changes, the actual price will adjust 'instantaneously'…"

Fama's initial definition of the efficient market is changed in Fama (1970, p. 383) to a market "…in which prices always 'fully reflect' available information."

The second definition presumes the first definition but opens for gradual dissemination of information into the market. Fama (1970) also endows the definition with testable properties.

The first definition is an application of the rational expectations hypothesis onto the financial market in that it refers to best estimates. The rational expectations hypothesis conjectures that the subject makes unbiased forecasts (e.g., Huang & Litzenberger 1988, p. 179, 185). A rational expectation model is Pareto optimal if and only if the forecasts are unbiased (Huang & Litzenberger 1988, p. 191-193). Indeed, the rational expectations hypothesis is flexible enough to ensure that a Pareto optimal equilibrium exists even when information is asymmetrically distributed among the subjects in a market (Grossman 1981). Therefore, it follows that EMH is valid if the hypothesis about rational expectations is valid since it yields a Pareto optimal equilibrium.

---

[1] Indeed, I call it theory in a narrow sense since it purports to be "…a set of statements, organized in a characteristic way, and designed to serve as partial premises for explaining as well as
predicting an indeterminately large (and usually varied) class of economic phenomena (Nagel, 1963, p. 212)."

The unbiasedness comes from the fact that the subject continuously updates his or her expectations based on the realizations from a stochastic process (Grossman 1981, p. 543-544). The updating of expectations follows a Bayesian learning procedure (e.g., Huang & Litzenberger 1988, p. 179-182, Congleton 2001, p. 392), which implies the use of a the standard state-space-and-partition model[2]. Even under information asymmetry, the asymmetry is limited to be on the events within the standard state-space-and-partition model (e.g., Grossman 1981).

The standard state-space-and-partition model allows the subject to learn by introducing a finer and finer partition of the state set until the limit scenario occurs where the partition only holds a unique state. This means that the subject gradually resolves previous uncertainty and traverse from knowing that the subject does not know (as Modica and Rustichini, 1999, p. 266 put it, the subject faces a conscious uncertainty), to knowing that the subject knows (i.e., certainty).

To conclude, for EMH to be valid it follows that the rational expectations hypothesis must be valid, the subjects' forecasts must be unbiased for the rational expectations hypothesis to be valid, and the subjects' knowledge must be able to be captured using the standard state-space-and-partition model for the forecasts to be unbiased. When this chain of conjectures holds, it implies that the market is in a Pareto optimal equilibrium. A Pareto optimal equilibrium is a situation where there does not exist arbitrage opportunities (e.g., Ohlson 1987, p. 17).

1.3 The standard state-space-and-partition model, ignorance, and equilibrium

The use of standard state-space-and-partition model permeates financial economics (e.g., Eichberger & Harper 1997; Debreu 1959; Demski 1972; Dothan 1990; Duffie 1992; Fishburn 1979; Huang & Litzenberger 1988; Ohlson 1987; Pliska 1997; Silberger & Suen 2000). Another way to express this is that financial economics generalizes consumer theory with the use of state-contingent commodities (Silberger & Suen 2000, p. 399).

A state is in this perspective "a description of the world, leaving no relevant aspect undefined" Savage ([1954] 1972, p. 9). The true state is the state that "does in fact obtain" Savage ([1954] 1972, p. 9). The standard state-space-and-partition model envisions the state space as exhaustive and consisting of mutually exclusive states. Whichever state obtains is beyond the control of the subject.

The use of the standard state-space-and-partition model is necessary for a Pareto optimal equilibrium solution in financial economics when uncertainty is present (e.g., Ohlson 1987, p. 8-25).

No arbitrage opportunities are necessary to derive at the Capital Asset Pricing Model (CAPM) (e.g., Ohlson 1987, p. 71-94). Existence of no arbitrage opportunities is necessary to derive the firm valuations models used in financial economics (e.g., Feltham & Ohlson 1999), and the standard state-space-and-partition model is an integral part to the no-arbitrage solution.

---

[2] I borrow the use of the concept "standard state-space-and-partition model" from Dekel, Lipman & Rustichini (1998, p. 164) and Samuelson (2004, p. 399).

The standard state-space-and-partition model assumes that the subject can identify everything that the he or she does not know (Dekel, Lipman & Rustichini 1998, p. 164; Rubinstein 1998, p. 47; Samuelson 2004, p. 372, 398). This is known as the axiom of awareness (Samuelson 2004, p. 372) or the axiom of wisdom (Samuelson 2004, p. 398), or that the subject "knows all tautologies" (Dekel, Lipman & Rustichini 1998, p. 164).

The axiom of wisdom ascertains that the subject can never be surprised and that the arrival of new information allows him or her to reduce his or her uncertainty about what is (Samuelson 2004, p. 398). That is, the subject was aware of this possibility before but with the arrival of new information, the subject knows it with certainty. This means that the subject first knows that he or she does not know but with the arrival of new information, the he or she learns such that he or she knows that he or she knows.

The axiom of wisdom ascertains that the subject is not unaware of anything (Samuelson 2004, p. 399). In fact, the main result of Dekel, Lipman, and Rustichini (1998) is that the standard state-space-and-partition model cannot cope with the subject's ignorance of some state. That is, for the subject to be ignorant of a state requires that the subject cannot identify the state, but the standard state-space-and-partition model conjectures that the subject can identify all states.

This may not be a problem when a unique subject's choice is considered, but considering a market means that the problem expands into a multi-subject setting with the implication that the other subjects' choices become part of the uncertainty of the focal subject's choice-problem (Samuelson 2004, p. 388). This means that the standard state-space-and-partition model's axiom of wisdom requires each subject to have a complete understanding of all other subjects' plans for choices— i.e. perfect knowledge.

The discussion above shows that models in financial economics, such as the CAPM and firm valuation model, require every subject in the market to have perfect knowledge about everyone else's supply and demand plans. Anything short of that implies ignorance of at least a state and the existence of ignorance ascertains arbitrage opportunities, which mean that the market no longer satisfies the requirement for a Pareto optimal equilibrium.

Some may argue that requiring all individuals to be rational with perfect knowledge is overly restrictive. Two alternative explanations are that irrational subjects' trades are random and cancel each other, or even if they do not cancel each other, rational arbitrageurs in the market eliminate their influence on prices (Shleifer 2000).

If irrational subjects' trades are random and cancel each other, it follows that at least two individuals in the market are surprised with their consumption-investment plans. This means that at least one individual continues to jostle to improve his or her well-being, with possible changes to the price as a result, i.e. this is not a Pareto optimal equilibrium. The additional weakness with such a

hypothesis is that it lacks theoretical coherence: Why would irrational subjects trade random so that they cancel each other? The existing no-arbitrage theory does not support such a proposition since it imposes the axiom of wisdom.

The second argument surmises that rational arbitrageurs in the market eliminate irrational subjects' influence on prices. Again, such a proposition lacks credible formal theoretical foundation. Such a proposition requires a choice theory where rational subjects are given certain traits and where irrational subjects are given other traits, and where it leads to a Pareto optimal equilibrium. I still have not seen a formal theory that solves this problem.

1.4 Empirical research

Much empirical research is devoted to investigate different operationalizations of EMH and it appears as if stock markets are efficient in the short run (Kothari 2001, p. 187-204).

Tests for stock market efficiency are mainly conducted on U.S. stock market data: for example, Beaver (1968), Landsman, and Maydew (2002). How smaller markets behave is less clear (e.g., Graflund (2001) showed market inefficiency for the Swedish stock market and Jacobsen and Voetmann (1999) found market inefficiency in the Danish stock market. The efficiency in the market for ownership in non-public firms is even less understood, but there are reasons to expect them to be not as efficient as organized stock markets because of less well-developed market microstructures (e.g., liquidity and lack of transparency).

The efficiency tests of organized stock markets use, e.g., short-window event studies. Event studies having short windows in which they study stock market reactions are studies that test if short-run arbitrage opportunities exist in the stock market and, if present, how fast they disappear.

Short-run market efficiency tests cannot separate between the effect from an inefficient market and the effect from deviations from the no-arbitrage condition in the market. This is due to the joint hypothesis of market efficiency tests: Any test of a market's efficiency has to be jointly tested with a no-arbitrage asset-pricing model (Fama 1970, 1991).

CAPM is applied as the no-arbitrage asset-pricing model when the market's efficiency is tested. If the market is in disequilibrium (e.g., because of less-than perfect knowledge) while CAPM is applied, it means that a miss-specified asset-pricing model contaminates the market efficiency test. This implies that such tests cannot distinguish between effects from a miss-specified asset-pricing model and effects from market inefficiency.

Deficiencies in the tests of short-run market efficiency have led to long-horizon event studies (e.g., DeBond and Thaler 1985; Abarbanell and Bushee 1997, 1998). Long-horizon tests of market efficiency find conflicting results to those in the short-run tests, with the results indicating that the stock market is unable to adjust for long-horizon arbitrage opportunities (Kothari 2001, p. 188-189).

Anecdotic evidence of long-horizon mispricing effects abound: e.g., the 17th century tulip mania in Holland, the stock market crash in the 1930s, the October 1987 stock market crash, the Japanese stock market price bubble from the 1980s, and the Internet stock price bubble from the 1990s.

Low-frequency events as those above create the long-horizon post-event anomalies and are observable in other markets as well (e.g., the real-estate markets' price bubbles in Sweden and the UK in the 1980s).

For several reasons (e.g., the long-run indicators of market inefficiency) there is a long-standing and growing unease among empirical market-based accounting and finance researchers in the EMH (e.g., Lev & Ohlson 1982; Bernard 1989; Daniel et al. 1998; Barberis et al. 1998; Hong & Stein 1999). Moreover, it is this unease that led Kothari (2001) and Lee (2001) to start to ask for a theory of inefficient markets.

1.5 The relation of the thesis to other research

This thesis is a response to Kothari (2001) and Lee (2001). It is an attempt to develop and empirically test a theory of inefficient markets using accounting data. I call the choice theory for Homo comperiens. It uses Dekel, Lipman, and Rustichini's (1998) findings that the standard state-space-and-partition model cannot cope with the subject's ignorance, and Modica and Rustichini's (1999) result that shows that a state-space-and-partition model allowing for ignorance is still partional.

It is well known that there exist innumerable ways to be in non-equilibrium, i.e. that potentially there exist innumerable theories of inefficient markets. Examples of inefficient market theories are theories that posit that the subjects are not price takers (monopoly theory and oligopoly theory). Then there is the tradition in behavioral economics that focuses on bounded rational choices in the form of modeling the choice process in a non-perfect manner. Research that uses a bounded rational framework focus on, e.g., heuristics, framing or other types of market inefficiency (Kahneman 2003; Shefrin 2002; Shleifer 1999).

The theory of Homo comperiens presented in this thesis is part of behavioral economics research in the sense that it focuses on the subject's bounded rationality. It separates itself from the core of behavioral economics (e.g., Simon 1955; Kahneman & Tversky 1979; Tversky and Kahneman 1981) since it does not model the choice process. It positions itself deliberately close to the core financial economics (e.g., Debreu 1959; Demski 1972; Dothan 1990) and empirical market-based accounting research (e.g., Abarbanell & Bushee 1998; Collins & Kothari 1989; Liu & Thomas 1999a, b; Piotroski 2000) by conjecturing rational preference relations. It is also closely related to parts of game theory research (e.g. Halpern 2001; Modica & Rustichini 1999) insofar that it allows for ignorance. Modica and Rustichini (1999) defines ignorance as a situation where the subject only

is aware of a subset of the "grand" set of states. Moreover, these authors show that even such a state-space model is partional.

The only central conjecture that is modified in Homo comperiens as compared with mainstream economics is the completeness axiom. By changing only one central conjecture in mainstream economics, a new world opens, which is a world in which disequilibrium becomes the rule and not the exception. Nevertheless, it is at the same time a world that also exhibits equilibrating forces. Alternatively, the theory predicts the existence of waves and how they disappear.

In this thesis I attempt to falsify the theory by empirically testing two testable hypotheses. Since they are based on the firm valuation models conjecturing ignorance that stems from the theory of Homo comperiens, they conjecture market inefficiency. Therefore, the empirical tests do not focus on tests of stock market efficiency. However, again inspired by Kothari (2001, p. 191), in a test of the theory's validity, I let predictions of return behavior from a rational expectations model compete with predictions of return behavior from the theory of Homo comperiens.

Because the market is theorized as being inefficient, market prices are expected to be the same as intrinsic values, implying that there is nothing to be gained from using market prices as part of the empirical tests, as is the standard operating procedure in market-based accounting research.

1.6 Expected contribution

Both the economist and the accounting researcher should find an interest in this research since it proposes a formal theory of limited rational choice that yields inefficient markets. The empirical section of this thesis can also interest users, producers, auditors, and regulators of accounting.

Users of accounting are the existing and potential equity subjects in firms. They can use the results of this research as they analyze the intrinsic value of Swedish manufacturing firms. They can also apply the same basic models, but with different parameter estimates, to other industries and to other countries.

Producers of accounting variables can use the models from several perspectives. As the models reveal how the accounting rate-of-return is affected by discovery, this can provide ideas of how to use a firm's financial control mechanism. It could also be used as a strategic tool for guiding the choice of top-management.

Regulators have use of the empirical accounting research in this thesis since it sheds light on how acquired goodwill changes over time. Consequently, this research can affect the regulation of goodwill.

The thesis also provides a tool for auditors when analyzing firms' impairment test of goodwill in that it provides clues on how to evaluate a firm's forecasted earnings capacity for the acquired assets.

1.7 Outline of thesis

This thesis consists of eight chapters and appendices. The thesis can be read in many ways depending on interest. I propose three alternative reading strategies.

The first strategy is aimed at theoretically interested readers who wonder what the theory of Homo comperiens is and how it relates to economic theory. Such a reader is primarily interested, after having read Chapter 1, in reading Chapter 2—4 and Appendices A - C.

With such an interest, I would advise the reader to read Appendix A before reading Chapter 2. While reading Chapter 2, it is advisable to also read Appendix C.

Having read Appendix A, Chapter 2, and Appendix C, the reader should proceed to Chapter 3, Appendix B, and Chapter 4, in that order. While reading Appendix B, it is advisable to again skim Appendix A because of their similarities.

Then, as closure, I would take a quick look at the empirical findings and conclusions of the thesis by reading the summaries in Chapters 6 and 7 and the concluding discussion in Chapter 8.

The second strategy is designed for the non-theoretically interested accounting researcher, the non-theoretically interested industrial economics researcher, and accounting practitioners, including users, producers, regulators, and auditors.

Such people should, after having read Chapter 1, read Chapter 5 and 6 and Appendix D—N. Chapter 5 is read with frequent cross-referencing into Appendix D—H. Those appendices provide the operationalizations of the variables to the database names and discuss how the database was set up.

Chapter 6 makes final operationalizations of the testable propositions and reports the tests and test results. While reading the chapter there are plentiful cross-references into Appendix I – N that I urge the reader to follow. Those appendices provide further operationalizations (Appendix I), technical discussion on test procedures (Appendix K, L, and M) and complementary tests in Appendix J and N. I also urge such a reader to read the concluding discussion in Chapter 8.

The third strategy focuses on the reader who has an interest in the whole thesis. I would like to advise such a reader to follow both reading strategies, perhaps after first having read Chapter 1 and the concluding discussion in Chapter 8.

Others, with only a cursory interest in thesis, can focus on the first and last chapters and possibly on each chapter's summary

CHAPTER 2—THE THEORY OF HOMO COMPERIENS: LIMITED RATIONAL CHOICE

> "…there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns -- the ones we don't know we don't know." (Rumsfeld 2002)

2.1 Introduction

This thesis aims to develop and test a theory of market inefficiency. At the heart of the debate regarding market efficiency and market inefficiency, lays the conjecture of the subject's rationality. Chapter 1 argues that the nucleus to a theory of market inefficiency should be based on a limited rational choice and not on other limitations such as non-atomistic actors. The purpose of this chapter is to develop a theory of limited rational choice that can be applied to such areas as financial economics and market-based accounting research.

Before I introduce the details, consider the following story[3]. Before you leave home in the morning, you consider whether to wear a raincoat or not. The choice depends on whether it rains or not. You look at the window and if there are raindrops on the window, you take the raincoat because that signal implies that it is raining. If there are no raindrops on the window, you do not wear the raincoat since it implies that it is not raining. When you look at the window, you see there are no raindrops and consequently you do not wear the raincoat when you open the door.

This short story is a choice problem with a set of alternative actions that contains two elements $\{\text{raincoat}, \neg\text{raincoat}\}$. The action set stands in relation to a set of consequences with elements $\{\text{wet}, \text{dry}\}$ and this relation is intercepted by the following state set $\{\text{rain}|\text{drops}, \neg\text{rain}|\neg\text{drops}\}$. The state set is presented in an event way such that the event with drops on the window allows you to learn whether it is raining or not.

However, when you open the door and leave the house, after having received the signal no drops, without wearing the raincoat, you find yourself walking in a rain shower. Walking wet in the rain you realize that you were unaware of the fact that it could rain even when you receive the signal no drops. Therefore, when you prepare to walk to your job the next morning, you consider the expanded state set $\{\text{rain}|\text{drops}, \text{rain}|\neg\text{drops}, \neg\text{rain}|\neg\text{drops}\}$ when choosing what to wear. This time you receive the signal drops and therefore conclude that it is raining and wear the raincoat. When you leave the home, you realize that it does not rain because the drops on the window come from a neighbor watering his lawn, which is a possibility that you somehow where unaware of. Again, you learn from your experience so that the next day you consider an even more complete state

---

[3] This story originally appeared in Rubinstein (1998, p. 41) but it is here modified to allow for ignorance.

set $\{\text{rain}|\text{drops}, \text{rain}|\neg\text{drops}, \neg\text{rain}|\neg\text{drops}, \neg\text{rain}|\text{drops}\}$ when choosing whether to wear the raincoat.

Making these choices, you are unaware of the possibility that it might or might not rain regardless of the signal, and this leads you to incorrect inferences. However, you are not only unaware of the possibility that the signal may not indicate that it rains, you are also unaware of the fact that you are unaware of this. Consequently, you are completely amiss making your choice, i.e. there are uncertainties that are not anticipated – unknown unknowns.

Through the learning effect, you are able to expand the consequence set such that the unknown unknowns become known unknowns, i.e. you know that you do not know whether it is raining after having received either the no drops or the drops signals and the uncertainties are therefore anticipated.

Not only are there unknown unknowns in this simple problem depicting themselves as limited state sets, but the set of alternatives to choose from is also limited. The choice problem is restricted to choose only whether to wear or not to wear a raincoat based on the drops on the window signal. When you learn that this is not an appropriate signal (i.e., unknown unknowns are now known unknowns), you realize that by opening the front door you can judge whether it is raining or not. The set of choice alternatives is therefore limited and by expanding the action set it becomes possible to convert the known unknowns to known knowns, i.e. to remove the uncertainty altogether by redefining the choice problem to include more alternatives in the action set. The set of alternatives can of course be expanded to cover other options too such as bringing an umbrella in case it begins to rain, etc.

Restricting rationality is not novel. Simon (1955, p. 101, 104; 1990, p. 7) notes that there are basic physiological constraints to a subject's choice making on both the subject's computational ability and the subject's memory that physically prohibits perfect rationality. Rubinstein (1998, p. 3) notes that it is not until recently that bounded rationality has affected economics. However, bounded rationality focuses on the elements of the process of choice and hence not specifically on the type of limitations considered above (Rubinstein 1998, p. 1, 192-193; Simon 1993, p. 156). Lipman (1995, p. 43) notes that there are only a few papers that explore the implications of bounded rationality, but that there are many modeling approaches, which may be due to lack of agreement as to what bounded rationality is.

To the best of my knowledge, there are no formal bounded rationality models in economics that have been applied to financial economics models. There is a vast literature in behavioral economics inspired by bounded rationality arguments (see, e.g., Shiller (2003) for a review of behavioral finance and Conlisk (1996) for a review of bounded rationality research). However, as Kahneman (2003) notes, behavioral economics typically addresses only a particular observed market anomaly at

a time, whereas this thesis aims to be more general in scope with its approach to model choice as choosing from a set of alternatives.

It is possible to argue that theory already exists in Austrian economics since Kirzner (1973) and Congleton (2001) restrict choice due to ignorance. Neither Kirzner nor Congleton treats the ideas in a manner such that it can be transformed into a theory of market inefficiency that is precise enough to allow it to be formally tested. The theory of Homo comperiens builds particularly on Kirzner's ideas but takes the analysis further in such a direction that it becomes a testable theory of human action.

In developing the theory I sought vital inspiration from the work of Hayek (1936, 1945), Kirzner (1973), and Simon (1955, 1956). Since the model is mathematically grounded leading to the subjective utility maximizing behavior (which is only similar to a [objective] utility maximizing behavior that appears in a general equilibrium), it is also inspired by the general equilibrium researchers Debreu (1959) and Demski (1972). Rubinstein (1998) and Samuelson (2004) provide inspiration for the chapter's discourse.

Chapter 2 is organized as follows: After the introduction, a subsection follows in which rational choice and limited rational choices are discussed. This subsection focuses on developing the main body of the theory. The ensuing subsection converts the limited rational choice into a utility-based limited rational choice. Learning is introduced before the closure of the chapter, which is a central trait to Homo comperiens. It leads to the sought-for equilibrating forces in a sequence of choices that can best be described as being in a state of disequilibrium.

2.2 The limited rational choice

This section proposes a model of human choice-making that allows for ignorance on both the action and the state set. It does so using an axiomatic approach, which is suggested by Lipman (1995, p. 64) to be an especially promising route in the bounded rationality research.

Lipman (1995, p. 64) points out that it is important to try to apply these models to empirical data, but that there has been very limited attempts in that direction. The goal with this section, and the rest of the chapter, is to be specific enough such that it is possible to use the results to build tractable firm valuation models in uncertainty conjecturing ignorance that can be empirically tested.

The section retains all the traits of a perfectly rational subject but forces those choices to be made on a limited set of alternatives and by considering a limited state set. Thus, the chapter discusses human action that is rational but where any choice is made on a strict subset of the objective set of available alternatives and on a strict subset of the objective set of available consequences.

An intriguing result with this section is that human action, as described by Homo comperiens, can be thought of as if it follows a principle of maximizing the subjective expected utility. This

is similar yet it differs from human action according to the perfectly rational choice. A perfectly rational choice can be described as if it follows a utility maximization principle, which I here call the objective expected utility. First follows the perfectly rational choice that acts as a frame of reference and then follows the limited rational choice model.

2.2.1 The perfectly rational choice, the starting point

First, as in economics, I conjecture a complete preference relation, which is similar to e.g., Kreps (1988), Mas-Colell et al. (1995, p. 6). That is, the subject knows[4] of all the alternatives that the subject can choose among, and the subject can choose between them. Formally, that means $a \succsim b$, or $b \succsim a$, or both $\forall a, b \in A$.

Second, I conjecture again, as in economics (e.g., Kreps 1988; Mas-Colell et al. 1995, p. 6), that the subject's preference relation is transitive. That is $a \succsim b$, $b \succsim c \Rightarrow a \succsim c$ $\forall a, b, c \in A$.

A preference relation that is complete and transitive is, according to Mas-Colell et al. (1995, p. 6), a rational preference relation.

The set of alternatives above is generic and hence more structure is needed. Therefore, let A_Ω be the subject's nonempty objective[5,6] action set, i.e. the set of all available alternatives for a subject. Since there are no a priori limitations to the objective action set, it can be infinite.

A subject who holds a rational preference relation on the objective action set is a perfectly rational subject.

A perfectly rational subject can make any necessary transformations so that the subject can rank all the alternatives. Since the objective action set is potentially infinite, there can be neither bounds to the subject's physiological storage capacity of information nor any limitations to the subject's computational abilities necessary to achieve transitivity.

2.2.2 Actions, ignorance of available actions, and limited rationality

Suppose that the subject is unaware of at least one action in the objective action set. To be more precise I define this ignorance below.

---

[4] By knowing I mean that when the subject knows, e.g., that it is raining, it means that it is true that it is raining. Knowing is therefore not the same as believing, which may or may not turn out to be true. Knowing is to be aware of, or to perceive. Similarly, not knowing is equivalent to be unaware of, equivalent to be ignorant of, and equivalent to not perceive.

[5] The words objective and subjective sets are used in this thesis. Modica and Rustichini (1999) use them with the same meaning, i.e. when they differentiate between the universal set and a subset of it.

[6] Objective is also used by Hayek (1936, p. 39) in the meaning of objective data, where objective data is exogenous data common to all subjects in the market. This should be understood as if there is an objective reality with "real" alternatives and "real" consequences. That is, a world that is not socially constructed. This departs from sociological research that frequently sees the world as being a social construct.

Definition 2-1: Definition of ignorance of actions: Let the subject be unaware of at least one action in the objective action set, i.e. the subject's ignorance set is nonempty, $I_A \notin \varnothing$, and a strict subset to the objective action set, $I_A \subset A_\Omega$.

With the introduction of the ignorance of actions set, it is also possible to define a subject's limited knowledge of action, as done below.

Definition 2-2: Definition of limited knowledge of actions. A subject's knowledge of alternative actions is limited when the subject has a nonempty ignorance set according to Definition 2-1.

Using Definition 2-1 together with the objective action set, it is also possible to define the subject's subjective action set.

Definition 2-3: Definition of the subjective action set. The subjective action is defined as $A_K = A_\Omega \setminus I_A$.

Definition 2-1 to Definition 2-3 can be verbally explained as follows. When a subject has limited knowledge of actions, the subject does not know of all the alternative actions that exist. All the actions that are available are found in the objective action set. Those actions that exist but that the subject does not know of constitute the ignorance of action set. The actions that the subject knows of make up the subjective action set, which is defined as the difference between the objective action set and the ignorance of actions set.

Note that $A_\Omega, I_A \notin \varnothing \wedge I_A \subset A_\Omega \Rightarrow A_K \notin \varnothing$. The fact that the subjective action set and the ignorance of action set are non-empty implies that $A_K \subset A_\Omega$, and so it follows that any choice that the subject makes is based on an incomplete description of the available alternatives. Since A_K is nonempty, it means the model conjectures that the subject always knows of at least one alternative to choose from.

Conjecture that the subject holds a rational preference relation on A_K. Note that that if $A_K = A_\Omega$ is possible, it implies that the subject makes a perfectly rational choice. The strict subset conjecture therefore ascertains that there is a difference between the two choice models.

Simon (1955, p. 101, 104; 1990, p. 7) notes that there are basic physiological constraints to a subject's potential choice making capacity because of the subject's computational ability and because of the subject's limited short-term memory. If Simon is correct, it is reasonable to conjecture that a subject can be ignorant of some of the elements of the objective action set. A subject's ignorance of alternative actions is also something that both Hayek (1936, 1945) and Kirzner (1973) use.

It is possible to conceive that other factors than the subject's physiological constraints can restrict a subject's subjective action set such that it becomes a strict subset of the objective action set. In particular, legislative restrictions may prohibit some goods and services from being traded (e.g., nuclear weapons). These legislative restrictions create incomplete markets, which is a phenomenon already appreciated in economics (e.g., Pliska 1997, p. 58). The subjective action set dis-

cussed above is not of this kind, however: It is a subset that only exists because of limited knowledge. Had the subject had perfect knowledge, the subject would have had the possibility to evaluate all the possible actions rationally. This is guaranteed by the completeness axiom. From this it also follows that incomplete markets may appear due to limited knowledge and does not only depend on legislative measures.

The definition of ignorance of actions in connection with the definition of the objective and the subjective action set leads to limited knowledge of the available alternatives. Limited knowledge of the available alternatives is used to define limited rationality.

> **Definition 2-4:** Definition of limited rationality: A subject that has a rational preference relation, i.e. a preference relation that is complete and transitive on the subjective action set (Definition 2-3) is a limited rational subject.

2.2.3 Limited rationality versus bounded rationality

The concept of limited rationality can itself be loosely seen as a strict subset to bounded rationality. Consider the definition of bounded rationality by Simon (1993, p. 156):

> "Human beings (and other creatures) do not behave optimally for their fitness, because they are wholly incapable of acquiring the knowledge and making the calculations that would support optimization. They do not know all of the alternatives that are available for action; they have only incomplete and uncertain knowledge about the environmental variables, present and future, that will determine the consequences of their choices; and they would be unable to make the computations required for optimal choice even if they had perfect knowledge."

That is, when Simon (1955) introduces bounded rationality as a critique to perfect rationality, it is clear that the subject not only limits his or her action set but the he or she also limits the process of making a choice. The subject does so by introducing, e.g., a satisfying behavior. In research, the concept of bounded rationality has become a waste basket into which almost any sort of non-perfect rationality is deposited (see, e.g., Conlisk (1996) for a review of the diverse field of bounded rationality research). Rubinstein (1998, p. 1) notes that bounded rationality sometimes is even used to depict situations having incomplete or bad models. Rubinstein challenges Kreps to come up with a definition of bounded knowledge, but Kreps acknowledges his inability to provide a precise definition (Kreps 1990, p. 150-156).

Even though Simon (1955, 1993) includes choices based on limited actions as part of bounded rationality, it is clear from Simon (1955, 1956, 1986, 1993) and from Simon's correspondence with Rubinstein (1998, p. 192-193) that bounded rationality primarily focuses on the process of making a choice and the physiological as well as the psychological limitations that guide the process. In fact Simon (1986) even calls this procedural rationality.

Limited rationality does not consider procedural rationality but retains the economics view that the model (hopefully) describes the subject's choice as if it is limited rational, i.e. as will become

clear as the analysis proceeds, limited rationality focuses on the optimization of given means to given ends, to use Kirzner's (1973) word, but here the means and ends are subsets to the objective sets.

With this definition of limited rationality, it clearly demarcates itself from both perfect rationality and bounded rationality. Perfect rationality never implies limited rationality. Bounded rationality may imply limited rationality. Limited rationality implies bounded rationality. But limited rationality and bounded rationality are not equivalent models of a choice problem.

2.2.4 Consequences, ignorance, and the subjective action function in certainty

So far, nothing has been said about how the subject forms his or her preference relation. It is assumed, as in economics (e.g., Rubinstein 1998, p. 9) that a subject forms his or her preference relation based on the consequences of the choice alternatives. An action can only lead to one possible consequence when certainty is present. This subsection maintains the certainty conjecture. A subsequent subsection (2.2.5) introduces the uncertain choice.

Let C_Ω be the nonempty objective consequence set, which is the set whose elements are the outcomes from choosing among the objective action set. Let the objective action function be $f_\Omega : A_\Omega \to C_\Omega$, which attaches a consequence to each action. The unique element in the consequence set is also written as $f_\Omega(a)$ to indicate that it is the real value of the function f_Ω at a.

A (mathematical) function is a special type of correspondence between two sets of which one set is the domain and the other set is the co-domain. It is a correspondence defined once for each element in the domain. To each element in the domain exactly one element is attached in the co-domain. Every element in the domain must be accounted for only once, but to each element in the co-domain there can be attached more than one element from the domain. It is possible that not all the elements in the co-domain are associated with elements in the domain. The essential feature is that each element in the domain must be used once, and once only, to form an association with a unique element in the co-domain. The association is the function's arguments (see, e.g., Sydsaeter et al. (1999) for such a discussion).

Since $f_\Omega : A_\Omega \to C_\Omega$ is a function, it follows that it is a one-to-one relation, which implies that the subject is a priori certain about the outcome of a particular choice. Hence, it is a certain choice. Since it is a one-to-one relation, it follows that it is a surjective function, i.e. every element of the co-domain is used at least once.

Conjecture that the subject does not know of at least one consequence in the objective consequence set. Let the subject have a subjective consequence set, which is defined as the difference between the objective consequence set and the set of consequences that the subject is unaware of. That is, $C_K = C_\Omega \setminus I_C$, where $I_C \subset A_\Omega$ and $I_C \notin \varnothing$. This is analogous to Definition 2-1 to Definition

2-3, but since this subsection is an expansion of the more general choice structure in the previous subsection, no formal definitions are introduced.

Since the subject then has a subjective consequence set, the subject has a subjective action function $f_K : A_K \rightarrow C_K$.

With these definitions, it is possible to state the following proposition.

> **Proposition 2-1:** When knowledge limits the actions inferred as available by the subject, it can, but must not, reduce the subject's set on subjective consequences. That is, $A_K \subset A_\Omega \Rightarrow C_K \subseteq C_\Omega$.

The intuition underlying this proposition can be seen from this simple example: Conjecture that $A_\Omega = \{a, b\}$ and that $f_\Omega(a) \sim f_\Omega(b)$. Furthermore, conjecture that $A_K = \{a\}$ and $I_A = \{b\}$. From this it follows that $A_K \subset A_\Omega$, which implies that $C_K \subset C_\Omega$, but since $f_\Omega(a) \sim f_\Omega(b)$, it also follows that $C_K = C_\Omega$, and so we have $C_K \subseteq C_\Omega$.

This means that even when the choice is limited because of ignorance of available actions, it must not lead to a subjective consequence set that is a strict subset to the objective consequence set. It also means that even when the number of alternatives to choose from increases, the subject may nevertheless feel no significant difference in the choice situations since the subjective consequences can remain unchanged.

Assume the following situation: $A_\Omega = \{a, b, c, d, e\}$ but the subject's subjective action set is only $A_K = \{a, b, c\}$. The corresponding consequences are $C_\Omega = \{f(a), f(b), f(c), f(d), f(e)\}$ and $C_K = \{f(a), f(b), f(c)\}$. Note that $f_\Omega(\cdot) = f_K(\cdot) = f(\cdot)$, i.e. the outcomes are objectively verifiable such as, e.g., eating a meal. Furthermore, assume that $f(a) \sim f(d)$ and $f(b) \sim f(c)$. If the subject can make choices based on the objective action set, he or she would hold the following preferences $f(e) \succ f(a) \sim f(d) \succ f(b) \sim f(c)$. Such preference relations imply that $e \succ a \sim d \succ c \sim b$. Since the subject's knowledge of the available alternatives is restricted to A_K, it implies that the preference relations on the consequences are $f(a) \succ f(b) \sim f(c)$. This means that when knowledge of the available alternatives and their consequences is limited, it follows that the choice can result in a different choice. This, however, is not always true. Consider if $C_K = \{f(a), f(b), f(c), f(e)\}$; it means that the choice in A_Ω is identical to that in A_K Furthermore, since $f(a) \sim f(d)$, the additional consequence that is attainable by expanding the subjective action set to the objective actions is already attainable in the subjective actions set.

2.2.5 Consequences, ignorance, and the subjective action function in uncertainty

The previous subsection analyzes the certain choice. The subject knows at the point of choice what the consequence is for each subjective action. This subsection focuses on the situation where the

subject, at the point of choice, infers that each subjective action can only lead to one consequence, but where the consequence is uncertain. Uncertainty implies that the subject can specify each of the inferred potential consequences for an action, but the subject cannot control which of the consequences that eventually unfolds[7].

The inference from an action to its consequence is, with the introduction of uncertainty, intercepted by exogenous factors. In the economics, the uncontrollable determinants of an action's consequence are captured by the concept of states (e.g., Debreu 1959; Demski 1972; Fishburn 1979; Kreps 1988). Savage ([1954] 1972, p. 9) calls the state a "description of the world, leaving no relevant aspect undefined."

A state is a complete description of the factors about which the subject is uncertain and which are relevant to the consequences that are associated with the choice of an action. Let S_Ω be the subject's nonempty objective set of states where $s \in S_\Omega$ is a unique state. The set of states are mutually exclusive and the objective set of states is $S_\Omega = \bigcup_i s_i$ which is assumed closed and bounded.

Since an action can lead to only one consequence but where the consequence a priori is known to be unknown because the subject cannot know for sure the state that does in fact obtain, it follows that the objective action function is state-dependent. That is, $f_{\Omega,s} : A_\Omega \to C_{\Omega,s}$ where $f_{\Omega,s}$ denotes the state-dependent objective action function that attaches a unique state-dependent consequence to each action and where $C_{\Omega,s}$ is the objective consequence set for a specific state.

Conjecture that the subject is ignorant of at least one state in the objective state set and use this to define the limited knowledge of states as below.

> **Definition 2-5:** Let the subject be unaware of at least one state in the objective state set, i.e., the subject's ignorance set is nonempty, $I_S \notin \varnothing$, and a strict subset to the objective action set, $I_S \subset S_\Omega$.

> **Definition 2-6:** Definition of limited knowledge of states. A subject's knowledge of potential states is limited when the subject has a nonempty ignorance set of states according to Definition 2-5.

As with Definition 2-3 on page 29 it is possible to use the ignorance of states to define the subject's subjective state set as below.

> **Definition 2-7:** Definition of the subjective state set. Let the subjective state set be $S_K = S_\Omega \setminus I_S$.

Again note that $S_\Omega, I_S \notin \varnothing \land I_S \subset S_\Omega \Rightarrow S_K \notin \varnothing$. Since the subjective state set is nonempty, it is a subset to the objective state set, and the fact that the ignorance of states set is also nonempty assures that the subjective state set always is a strict subset to the objective state set. This

---

[7] This is, as in the case of certainty, completely in line with the use of uncertainty in microeconomics.

means that the subject always makes choices that are to some extent based on erroneous expectations.

When the subject faces the subjective rather than the objective state set, the action function no longer is the objective state-dependent action function. Instead, it means that the subject faces the subjective state-dependent action function, which is: $f_{Ks} : A_K \rightarrow C_{K,s}$. Note that $f_{Ks} \neq f_{\Omega s}$ for two reasons. First, the subject's ignorance of actions leads to this difference. Second, the subject's ignorance of states also contributes to this difference.

The separation of the subjective state set from the objective state set resembles the work of Modica and Rustichini (1999) who use a set-theoretical approach to model ignorance. Modica and Rustichini (1999) see ignorance as non-full awareness and awareness is, according to the authors, the union of certainty and conscious uncertainty. That is, awareness implies that either you know for certain what is (e.g., it is for sure raining) or that you know that you do not know (e.g., that it is raining or not). Ignorance implies that the occurrence of rain, or of its opposite, is a surprise to the subject. In other words, it is an unknown unknown.

The definition of states and the state sets implies that each action has one consequence per state. By gathering the consequences across states per action, all possible consequences for an action are given.

Since the objective set of states is exhaustive and because the states are mutually exclusive, it follows that it is possible to assign probabilities to each of the states. That is, it is possible to depict a human choice under uncertainty as if the subject holds probability beliefs over the states. The objective state probability for a state is denoted $\pi_{\Omega s} \in \Pi_{\Omega}$, and Π_{Ω} is the objective state-probability set (see, e.g., Sydsaeter et al. 1999 for the axioms of probability).

The subjective set of states consists of a subset of the objective state set. This means that the subjective state set also consists of mutually exclusive states. Furthermore, since $I_S \notin \varnothing$, the subjective set of states differs from the objective state set, i.e. it cannot be exhaustive in the objective sense. From the subject's perspective, the subjective state set is exhaustive since the subject believes that this is the objective state set, i.e. the subject believes that the subjective state set is exhaustive. This implies that it is possible to assign state probabilities to the subjective states. For convenience, I call these state probabilities subjective state probabilities.

When the subjective set of states is a strict subset of the objective set of states, it follows that at least one element from the objective set of states is missing in the subjective set of states. Or, to put it differently, $S_K \cup I_S = S_{\Omega}$. $s \in S_K \cap S_{\Omega}$ are all the states that are common to both choice situations. Since the subject can be thought to hold probability beliefs on the states, it follows that at

least one state of the states in $S_K \cap S_\Omega$ must have a different subjective state probability belief, $\pi_{Ks} \in \Pi_K$ than the objective state probability. The symbol Π_K is the subjective state probability set.

> **Proposition 2-2:** There exists at least one subjective state probability that differs from the objective state probability when the subject faces a strict subset of states: That is, $\exists \ \pi_{Ks} \neq \pi_{\Omega s}$ where
>
> $\pi_{Ks}, \pi_{\Omega s} \in \Pi_K \cap \Pi_\Omega$, when $S_K \subset S_\Omega$.

Despite that there is a difference in the probability beliefs when a choice is made on the subjective or on the objective set of states, it follows from the previous definition of states and the set of states that summing all of the subjective state probabilities yields exactly unity, as required by the axioms of probability. The only problem is that at least one of these state probabilities is incorrect and thus the subject makes an erroneous choice if his or her choice is described as if the subject makes use of probability calculus.

With the definitions and the proposition above, it is possible to enlarge the definition of limited rationality as put forward in Definition 2-4. It is expanded here to encompass also ignorance of states.

> **Definition 2-8:** Definition of limited rationality in the uncertain choice. In addition to Definition 2-4, a subject exhibits limited rationality when the subject has a rational preference relation on uncertain consequences that are limited because of limited knowledge of states (Definition 2-6).

With Definition 2-1 to Definition 2-8 , Proposition 2-1**,** and Proposition 2-2, I have set up the nucleus to a choice theory that resembles the perfect rational choice but with significant differences. In my model, the subject makes a rational choice on a strict subset of the objective set of alternatives while only considering a strict subset of the objective set of states. This subject has limited knowledge of the alternatives he or she can choose from and limited knowledge of the states that does obtain, all of which leads to limited rational choices.

To the best of my knowledge, limited rational choice in the way I use it in this thesis is novel. There exists game theoretical research that models unawareness, or the lack thereof (e.g., Dekel, Lipman & Rustichini 1998; Halpern 2001; Modica & Rustichini 1994; Modica & Rustichini 1999), but it is limited to the unawareness of states and not of the available actions. Furthermore, to the best of my knowledge, none of the unawareness research has yet found its way into applied economics in terms of financial economics, and hence not into valuation models in uncertainty, which is what I use here. My proposed theory of limited rational choice allows for unawareness of states and of actions and it is precise enough to allow the posing of testable propositions in the area of financial economics and market-based accounting research. Moreover, my proposed theory allows for learning as becoming aware (more precisely this implies that the ignorance set is reduced) in addition to typical Bayesian learning, but that is the topic of section 2.4. The next section avoids the dynam-

ics that come from learning and focus on the static choice. It introduces utility to the limited rational choice model.

2.3 The limited rational choice's utility representation

Having established the core to my theory of limited rational choice in section 2.2, this section takes the analysis forward to the point where it is possible to speak of a limited rational choice as if the subject chooses an action based on a subjective expected utility maximization behavior. This should not be confused with what is often subjective as an expected utility[8] maximizing behavior because what is then thought of is what I call the expected utility maximizing behavior facing the objective action and state sets. I call such a behavior for objective expected utility maximizing. To emphasize, objective is in this research used to emphasize the special case where human action is based on perfect knowledge and hence where the subjective expectation of a subject is identical to the objective facts of the choice problem.

Human action that can be explained using objective expected utility is contrived by theory of choice that makes use of $A_\Omega, C_\Omega, S_\Omega, \Pi_\Omega$. This can be contrasted with human action that can be explained using subjective expected utility since such human action is contrived by the theory of Homo comperiens that makes use of A_K, C_K, S_K, Π_K.

In deriving the subjective expected utility maximization behavior of Homo comperiens the conjectures of continuous, insatiable preferences are imposed. The preferences are also supposed to be state uniform, independent, and that abide to the Archimedean assumption. These conjectures are usually invoked on the perfectly rational choice (e.g., Kreps 1988), but are equally applicable to the situation at hand. With these additional conjectures, it is possible to apply the theory of Homo comperiens to firm valuation.

To be able to connect the theory of Homo comperiens to firm valuation requires that it can be described using real-valued (real) functions. This makes it necessary to be able to describe choice as if it is based on a real number, which in economics is called utility. Utility is a numerical representation of the consequences, where it can be thought of as an index used to rank the consequences of each action (Mas-Colell et al. 1995).

The reader should think when reading this section on how the choice problem is designed. The subject faces a choice among alternative actions. The subject has to choose an action, and does so, according to the previous section as well as the traditional theory of choice, based on the outcomes of the actions. The subject makes the choice by ranking the outcomes and this is done by the subject no matter how diverse these outcomes may be. In this research the utility is merely an index that conserves the ranking of outcomes performed by the subject. Accordingly, utility is merely an

---

[8] Or, more specifically, von Neumann-Morgenstern's expected utility.

analytical device derived from the ranking of the outcomes and thus has no value or importance in itself. The index is given cardinal properties to ensure compatibility with real mathematics.

To be able to claim that a subject acts as if he or she chooses based on an action's utility, i.e. $a \succ c \Leftrightarrow u(a) > u(c)$, a functional relation must exist between the action set and the consequence set, as well as between the consequence set and the utility set. The previous section described the choice as if the subject chooses an action based on consequences using such a functional relation between the action set and the consequence set. This section focuses on the functional relation between the consequence set and the utility set.

A unique numerical representation is attached to each consequence, whether it is from C_Ω, C_K, $C_{\Omega s}$, or C_{Ks}. This representation is the subject's utility for a particular consequence. Recall from subsection 2.2.1 that the preference relation is conjectured weak and therefore there is the possibility that a subject is indifferent among alternative consequences. This implies that those consequences must have identical utilities; the utility otherwise fails to be a measure of how well off the subject is[9]. Based on this reasoning, it follows that each consequence must have only one corresponding element in the utility set, but that each element in the utility set may have more than one consequence attached to it. This is equivalent to the discussion on the action function.

The preferences must be continuous to guarantee the existence of a functional relation between the consequence set and the utility set (Mas-Colell et al. 1995).

For simplicity, let the utility set span the complete real line, here denoted $\mathbb{R}$. Note that this is unproblematic in that a functional relation does not require that all elements of the co-domain have a corresponding element in the domain (Sydsaeter et al. 1999).

Conjecture that the consequence set is the subjective consequence set. The correspondence between the subjective consequence set and the utility set satisfies all the necessary properties of a functional relation: All the elements in the domain exists only once and to each element of the domain is attached a unique element in the co-domain. This means that the correspondence between the subjective consequence set and the utility set is a functional relation. It further means that it is possible to express this function as $u_K : C_K \rightarrow \mathbb{R}$ for a certain choice. When the choice is uncertain, if follows that there exists a subjective state-dependent utility function that is $u_{Ks} : C_{Ks} \rightarrow \mathbb{R}$.

The utility function would be $u_\Omega : C_\Omega \rightarrow \mathbb{R}$ had the certain choice not been limited since the choice is based on the objective consequence set. Function u_Ω is the utility function normally used in economics and it may be expressed as an objective state-dependent utility function also, which is

---

[9] Utility is broad enough in this research to encompass both the subjective and the objective utility. The well-off-ness should be interpreted relatively: Is the subject better or worse off by choosing an action before another?

$u_{\Omega s} : C_{\Omega s} \to \mathbb{R}$. Derivations of a traditional utility function can be found in many textbooks on microeconomics but also in more formal treatment in (e.g., Fishburn 1979).

The general theory of rational choice describes the subjects as if they rank their actions based on their consequences. The action function makes it possible to describe choice as the choice of an action based on the action's consequence. Since the utility function is a real-valued function from the consequence set to the utility set, it opens the possibility to view the choice of action as the composition of the action and the utility function. That is, in certainty, the subjective action function $f_K : A_K \to C_K$, maps an action into a unique subjective consequence. The utility function $u_K : C_K \to \mathbb{R}$, maps the subjective consequence into a unique real number. Thus, we have their composition $f_K \circ u_K : A_K \to \mathbb{R}$, which maps an action into a unique real number.

This means that when the subject has limited knowledge, he or she exhibits $a \succ b$ if and only if the he or she also exhibits $f_K \circ u_K(a) > f_K \circ u_K(b)$, which occurs if and only if $u_K(a) > u_K(b)$, where $a,b \in A_K$. Consequently, $u_K(a) > u_K(b)$ is a necessary and sufficient condition for $a \succ c$. Limited rational choice is then possible to be described as if a subject's choice is based on the action's subjective utilities.

In uncertainty, there is no clear solution to the choice problem. For instance, it is possible to sum the state-dependent utilities for each action and then compare these sums to each other since the system is of an ordinal scale. A cardinal utility is needed to be able to sum the utilities, and according to Kreps (1988), it is necessary to conjecture that the subject's preference of a consequence is uniform across states to introduce cardinal utility. For example, winning a million is equivalent to a subject whether it is won in a boom or in a recession.

In financial economics von Neumann-Morgenstern's expected utility is used to derive the value of the firm. This utility is called the objective expected utility in this research and is denoted $\mathrm{E}\left[U_\Omega\left(c_1, \ldots c_S ; \pi_{\Omega 1}, \ldots \pi_{\Omega S}\right)\right] = \sum_{s \in S_\Omega} \pi_{\Omega s} \cdot u_\Omega\left(c_s\right)$. The objective expected utility is a cardinal utility that is achieved by focusing on a setting meeting the criteria of the perfectly rational choice. The focus on Homo comperiens in this chapter makes it necessary to derive a cardinal utility for a limited rational choice. According to the present thesis, the cardinal utility in uncertainty for Homo comperiens is as follows:

> **Proposition 2-3:** When the subject has a preference relation on the subjective consequence sets, which are complete, transitive, continuous, state uniform, independent, and that follow the Archimedean assumption, it is possible to express the subject's choice as if he or she makes his or her choice based on an action's subjective expected utility: $\mathrm{E}_{K0}\left[U_K\left(c_1, \ldots c_S ; \pi_{K1}, \ldots \pi_{KS}\right)\right] = \sum_{s \in S_K} \pi_{Ks} \cdot u_K\left(c_s\right)$, where $c_s \in C_{Ks}$, and $\pi_{Ks} \in \Pi_K$.

See Appendix C (p. 191) for proof of Proposition 2-3.

Savage ([1954] 1972) proposes a subjective expected utility. This utility should not be confused with the subjective expected utility I propose since my subjective expected utility is a direct effect of the subject's limited knowledge, which means that even in certainty there is a difference in utility functions. Savage focuses on the existence of the subject's subjective probabilities in an uncertain choice and does not limit the action set as I do. Nor does Savage restrict the state set as I do. Indeed, I suppose that Savage's axioms can be applied to my setting transforming the Savage subjective expected utility function into a subjective subjective expected utility function. Such a development is outside the scope of this thesis and is therefore deferred into the future.

The subjective expected utility in Proposition 2-3 differs from the objective expected utility in that at least one of the subjective state probabilities, π_{Ks}, differs from the objective state probability, and that the subjective Bernoulli utilities, $u_K(c_s)$, differ from the objective Bernoulli utilities. The difference in the Bernoulli utilities is due to the subject's failure to specify correctly the supremum and/or the infimum consequences. That is, the subjective supremum and infimum consequences are not the same as the objective supremum and infimum consequences. The failure of having incorrect subjective state probabilities, having an incorrect supremum consequence, and having an incorrect infimum consequence are due to the subject's limited knowledge of actions and states (Definition 2-2 and Definition 2-6). These failures imply that the subject's choice is erroneous when compared with a choice made on perfect knowledge.

Proposition 2-3 should be interpreted as follows. A subject who faces a choice forms an understanding of the available actions that he or she may take. The subject also forms an understanding on what the ensuing consequences are and acts as if he or she assigns probabilities to the states that he or she infers. The subject then acts as if he or she measures the subjective expected utility for each action. With the construction of the action function, the subjective utility function, and the composition thereof, it also follows that the subject acts as if he or she chooses the action that delivers the subjective expected utility that he or she prefers most,

i.e., $a \succ b \Leftrightarrow f_K \circ U_K(a) > f_K \circ U_K(b) \Leftrightarrow U_K(a) > U_K(b)$.

This proposition rests on the conjecture that the subject has preferences that are complete (on the subjective action sets and their associated states), transitive, state-uniform, continuous, independent, and that follow the Archimedean assumption. The only difference between this limited rational choice and a perfectly rational choice is that in the limited rational choice the choice is restricted to a strict subset of the objective action set and to a strict subset of the objective state set.

Since this thesis is particularly concerned with firm valuation, it makes sense to think of the consequence set as if it consists of monetary consequences. This is identical to the choice set in financial economics. Financial economics posits that the perfectly rational subject is never satiated. By

imposing the insatiability conjecture on the limited rational choice, I have a subject that appears to maximize his or her subjective expected utility.

Financial economics rests on the subject that acts as if he or she maximizes his or her objective expected utility. This chapter alters this by introducing limited knowledge: A subject who acts according to Proposition 2-3, is insatiable, has the ability to learn, and is a limited rational subject.

2.4 Learning and the limited rational choice

The previous sections focused on a single choice and not on a sequence of choices. This section introduces sequential choices allowing for learning. Learning is considered here a two-fold process in which the subject learns by Bayesian learning that allows him or her to resolve uncertainty and by discovery. That is, the subject learns such that the unknown unknowns become known unknowns and perhaps even known knowns.

2.4.1 Learning as a resolution of uncertainty

The standard state-space partition model allows the subject to learn by introducing a gradually refined partition of the state set until the limit scenario occurs where the partition only holds a unique state. This means that the subject gradually resolves previous uncertainty and traverse from knowing that he or she does not know (facing a conscious uncertainty as Modica and Rustichini (1999, p. 266) put it) to the limit knowing that the subject knows (i.e., certainty).

Traversing from uncertainty to certainty uses Bayesian learning. In Bayesian learning, the subject holds a prior probability on the states based on the available information. A search for more information (or the gradual dissemination of more information as time passes) allows the subject to update the probabilities based on the new information, which, in turn, allows the individual to update his or her prior probability to a posterior probability. Another way to explain this is that the information partition above is gradually refined, i.e. it holds fewer groups of states. This process repeats itself and if this search for new information is considered costless, it continues until all information is uncovered. When all information is uncovered, the information partition holds only one state and thus all uncertainty has been resolved using Bayesian learning.

The rational expectations model (which EMH relies on) uses Bayesian learning since it presumes that the subject has perfect knowledge of the standard state-space partition model (Huang & Litzenberger 1988, p. 179-182; Congleton 2001, p. 392). Rational expectations further assume that the subject makes unbiased forecasts (Huang & Litzenberger 1988, p. 179, 185). A rational expectation model is Pareto optimal if and only if the forecasts are unbiased (Huang & Litzenberger 1988, p. 191-193). Thus, it follows that EMH is valid if and only if the hypothesis about rational expectations is valid. The unbiasedness comes from the fact that the subject continuously updates his or her expectations based on the realizations from a stochastic process (Grossman 1981, p. 543-544), which means that Bayesian learning is present using the standard state-space model.

Dekel, Lipman and Rustichini's (1998) central contribution is that they show that the standard state-space partition model cannot accommodate ignorance. The standard state-space partition model requires that the axiom of wisdom is in place (Samuelson 2004, p. 399), which means that the subject must know of every state in the objective state set. That is, the subject's state set state must be a complete description of the world, without leaving any important aspect overlooked.

Another way to put this is that ignorance of a state implies a zero prior probability for the state on which the subject is unaware. A zero prior of a state implies that the posterior probability also is zero for the state, implying that Bayesian learning does not reduce the ignorance of states.

In a multi-subject setting, this has even stronger implications. Since the subject can describe the his or her uncertainty in the form of states and that he or she knows of each and every state that is relevant for his or her choice, the subject must also know of each and every other action and state that he or she makes. With such a perspective, the axiom of wisdom is very strong indeed.

With this perspective, it follows that the economic system is without external shocks since a shock is something that was not anticipated and the standard state-space model, by definition, anticipates everything. Recall Savage's ([1954] 1972, p. 9) words: The state is a "description of the world, leaving no relevant aspect undefined." Congleton (2001, p. 9) expresses this as "what is learned is general, rather than anything truly unanticipated or new."

The important contribution by Modica and Rustichini (1999) is that they show that the standard state-space partition model can be mended such that when the subject no longer is aware of the objective state set, but is aware of a subset of the objective set, this subset is still partitional.

Modica and Rustichini (1999) therefore allow for a relaxation of the axiom of wisdom while still allowing for learning in the Bayesian sense, where more information gives a finer partition of the subjective state set, which allows the subject to update his or her priors and perhaps come to another choice. Note that the process is now restricted to take place on the subjective state set and not on the objective state set. Traversing from the subjective to the objective state set cannot be accommodated through Bayesian learning.

This implies that the subject can engage in an information search, which, if the information search is costless, can continue until everything within the confines of the subjective state set is identified. Indeed, it becomes possible to conceive of a limited rational expectations model in which the subject makes unbiased forecasts about the future. However, since the existing knowledge is confined to be within the subjective state set, it means that it may not be, and probably is not, unbiased in an objective sense. So, considering a rational expectations framework, even if we conjecture homogenous preferences such that all subjects in the market have access to the identical subjective state set, it is not possible to conjecture unbiased forecasts. Thus, conjecturing Homo comperiens

implies conjecturing a limited rational expectations model that provides biased forecasts of the future.

Learning as a resolution of uncertainty therefore logically implies that nothing novel is ever discovered since it is restricted to learn more about what is already known.

2.4.2 Learning as discovery

Suppose that the subject faces a sequence of choices, and at present he or she faces a choice whether or not to invest in a firm. The subject proceeds to investigate the firm and given all his or her current and previous knowledge he or she finds that the firm's current market price is below/above its intrinsic value, so the subject invests in the firm. In this setting, the subject has three alternative strategies to choose from: (1) investing in the firm, which implies buying the stock, (2) not buying the stock, and (3) short selling the stock.

With all of the subject's previous and current knowledge, he or she infers only two choices, namely buying and refraining from buying the stock. This means that the subject's subjective action set is a strict subset of the objective action set. The subject is not aware of the possibility of borrowing the stock from someone else (and promising to return it after some time) and selling it in the stock market. This is short selling and makes it possible for him to make a profit on a firm whose market price is above its intrinsic value.

The firm that the subject considers to invest in is in the business of long-term storage of nuclear waste. Its main storage facility is going to be deep in the Swedish mountains in an area with solid rock, and in it, the firm plans to store highly radioactive waste from nuclear power plants. While waiting for the facilities to be constructed it has a short-term storage facility somewhere in the middle of Sweden.

The company's management has recently reviewed its insurance policy and has decided to change the conditions of the insurance for the firm in order that it is not insured against acts of terrorism. The choice to scrap the insurance against acts of terrorism is not publicly disclosed since management does not consider this important.

The subject has, given all of his or her previous and current knowledge, an understanding of the likely states that can occur. The subject can infer various states that incorporate different levels of recessions, boom periods, earthquakes, and so on. But the subject is not aware that there is an immediate risk of a terrorist attack on the firm's facilities and that such an attack can destroy the storage facility in such a fashion that radioactive dust is sent up into the air where it pollutes the whole Stockholm region to such a degree that it must be evacuated for hundreds of years. Nor is the subject aware that in such an event the firm goes bankrupt. This means that the subject has a limited comprehension of all the available states. Consequently, the subject makes his or her choice based on a subjective state set, which is a strict subset of the objective state space. To be able to span the

objective state set the subject would have to know if there were any terrorist considering committing an act of terror against this facility, that such an act could destroy the building, and that the destruction would pollute the whole Stockholm region. The subject would also have to know that management had ruled out this possibility and discarded the terrorist insurance.

With the entire subject's knowledge, he or she finds that the company is priced below its intrinsic value and thus purchases the stock. The next week a terrorist blows up the facility such that it sends a radioactive dust cloud into the air, which drifts into the Stockholm region where it rains down and pollutes the whole area. Since the firm is missing an insurance that covers such an act of terrorism, it means that the firm is sued for the damages that resulted, and the firm goes bankrupt. Thus, the subject loses his or her entire investment.

Had the subject been aware of the fact that a terrorist was planning an attack on the facility, or aware of the fact that a terrorist could be planning such an attack on the facility, that such an attack would blow up the building, and that the company was uninsured against such an accident, the subject would probably have reached another choice and thus refrained from investing in the firm. However, even refraining from investing is an action that is not optimal since the subject was not aware of the third possibility of going short. Had the subject instead been aware of the objective state and action space, the subject could have short-sold the stock and made a profit on the event of an act of terrorism.

After having made this disastrous investment, the subject has discovered the possibility of having terrorists attacking such facilities, with severe and lethal consequences. This knowledge becomes a part of the subjective state set as the subject makes his or her next choice.

The aim with this story is to exemplify how this thesis views learning as discovery. When a choice is made, the result of this choice is communicated to everyone (they discover) that is affected by the choice. Some people make a good choice, implying that the consequence is matched with its anticipation. Others may find consequences that are even better than expected, while some people become disappointed because they have made an erroneous choice.

The knowledge of the effects of the choice above allows people to alter their expectations over and above the effect from Bayesian learning and maybe their behavior as the next round of choices commence. That means that when a subject makes his or her next choice, he or she has a better appreciation of the subjective state and action sets, i.e. the subjective action and state sets are expanded and approaches the objective sets. In my view, discovery concerns the expansion of the subjective sets towards the objective sets.

Conjecture that the objective action and state sets are fixed. A sequence of choices lead to that the subjective sets in the limit approach the objective sets, but since they are defined as strict subsets, it also follows that despite the fact that the subject discovers more states, there is always

more to become aware of. That is, $\lim_{t \to \infty} (A_K) \approx A_\Omega$ and $\lim_{t \to \infty} (S_K) \approx S_\Omega$. Learning as discovery is therefore defined as follows:

> **Definition 2-9:** Definition of learning as discovery. Discovery takes place when the subject that acts according to Definition 2-4 and Definition 2-8 and that faces the next choice in a sequence of choices expands his or her subjective state set and/or the subjective action set. Discovery takes place because of the subject's experience from previous choices: Formally, learning as discovery means that the previous subjective state and/or action sets are strict subsets to the current subjective state set and/or action sets. With symbols, learning as discovery is defined as $S_{Kt-1} \subset S_{Kt}$, $A_{Kt-1} \subset A_{Kt}$ $A_{Kt-1} \subset A_{Kt}$, or when both situations occur and this is because discovery make certain that $I_{At-1} \subset I_{At}$ and $I_{St-1} \subset I_{St}$.

A subtlety with Definition 2-9 is that it implies that the subject, by design, always learns from a sequential choice and that he or she never forgets. Never forgetting is known as perfect recall in game theory (Rubinstein 1998, p. 68-69). Had the definition been such that the previous subjective state and/or action sets are allowed to be just subsets to the current subjective state set and/or action sets, i.e. $S_{Kt-1} \subseteq S_{Kt}$, $A_{Kt-1} \subseteq A_{Kt}$, learning would also encompass non-learning, i.e. $S_{Kt-1} = S_{Kt}$, and/or $A_{Kt-1} = A_{Kt}$. However, even such a weaker definition of learning excludes de-learning (forgetting) since that, in the strict sense, requires that $S_{Kt-1} \supset S_{Kt}$ and/or $A_{Kt-1} \supset A_{Kt}$.

Central conjectures in the analysis above are that both A_Ω and S_Ω are fixed. Another route to follow can perhaps be to allow the objective sets to be strict subsets of some universal objective sets, e.g., $A_{\Omega t} \subset A_\Omega$ and $S_{\Omega t} \subset S_\Omega$. This implies that when the subject makes a choice he or she faces objective action and state sets that are strict subsets to some universal action and state sets, and the subjective sets are strict subsets to the objective sets that exist at the point of choice. Allowing the objective set to be non-fixed as suggested here can again open the possibility of external shocks since the change $S_{\Omega t-1} \subset S_{\Omega t}$, and $A_{\Omega t-1} \subseteq A_{\Omega t}$ can be allowed to be the defining trait of external shocks.

Conjecturing the objective sets constant allows for treating the optimal solution as a fixed point, and tractable solutions to learning as discovery can be devised. This makes the external shock endogenous to the model in the form of the discovery. I am at this point not certain that tractable solutions can be modeled when the objective sets are allowed to be non-fixed. It can provide an interesting future development for the theory of Homo comperiens.

2.4.3 Human action and Homo comperiens

The ignorance[10] that exhibits itself as a subjective state and action sets diminish from a choice to another because of discovery. That is not to say that all ignorance disappears at once. The speed in

---

[10] Ignorance is borrowed from Austrian Economics. Ignorance is central for Kirzner (1973). The subject uses it, e.g., to discuss disequilibrium (1973, p. 69): "Market participants are unaware of the real opportunities for beneficial exchange

learning may be different among different subjects and the decrease of ignorance is thus unique for each subject. While the subject makes more choices and experiences the consequences, he or she discovers more about the objective action and state sets. The subjective action and state sets are in the limit (when discovery has run its course to the end) nearly identical to the objective sets. In other words, the subject still makes limited rational choices but with only a small amount of ignorance when the discovery process has run its course.

Suppose that the market consists of many subjects. A subject's choice of action then depends on his or her anticipation on how the other subjects plan to act. Since choice is in this thesis assumed to be compatible with Homo comperiens, it follows that any subject has an incomplete understanding of the others' plans. In addition, since the market consists of many subjects, it follows that ignorance permeates the market. Even when some ignorance is discovered and dealt with, there is always more ignorance in the market to be discovered and dealt with. In my view, it is unreasonable to expect that a subject will ever be able to infer the objective action and state sets since it implies that all subjects must do so, and to do so at the same time. Any choice is and will always be limited rational.

As the subject goes through life making choices, he or she gathers knowledge that is specific to the choices that he or she has made. This means that a subject follows his or her own unique choice path. Then, it also follows that any subject must be endowed with his or her own unique set of knowledge. It implies that every subject that faces an identical choice makes that choice based on inferences from his or her own unique subjective action and state sets. The difference can sometimes be minimal, but at other times, it is immense and induces vastly different choices.

Since subjects are likely to interpret a situation in their own unique way, this implies that as a subject scrutinizes another subject's choice, he or she may feel that the choice appears illogical. Nevertheless, it must be remembered that it is only illogical in the eyes of the beholder. Given that the subject's actions can be described as following the theory of Homo comperiens, he or she made a limited rational choice according to his or her own unique knowledge.

The discussion hitherto has added increasing structure to the proposed theory of limited rational choice. Given the definition of learning as discovery and learning as resolution of uncertainty, all the pieces of the theory of Homo comperiens are in place. I therefore propose the theory of Homo comperiens:

> **Proposition 2-4:** Homo comperiens is a subject who is limited rational (Definition 2-4, Definition 2-8), that learns using Bayesian learning and through discovery (Definition 2-9). The subject has a complete, transitive, insatiable, continuous weak preference relation, which under uncertainty also is independent, state-uniform and that follows the Archimedean conjecture.

---

which are available to them in the market. The result of this state of ignorance is that countless opportunities are passed up."

2.5 Summary

This chapter develops a theory of choice that I call Homo comperiens. The theory is suppose to, using Nagel's (1963, p. 212) words, "…serve as partial premises for explaining as well as predicting an indeterminately large (and usually varied) class of economic phenomena." Further, structure is needed to make it applicable to financial economics and market-based accounting research. Such a structure is provided in the next chapter.

The theory of Homo comperiens stems from this proposition:

Proposition 2-4: Homo comperiens is a subject who is limited rational (Definition 2-4, Definition 2-8), that learns using Bayesian learning and through discovery (Definition 2-9). The subject has a complete, transitive, insatiable, continuous weak preference relation, which under uncertainty also is independent, state-uniform and that follows the Archimedean conjecture.

The theory of Homo comperiens uses the following core definitions:

Definition 2-4: Definition of limited rationality: A subject that has a rational preference relation, i.e. a preference relation that is complete and transitive on the subjective action set (Definition 2-3) is a limited rational subject.

Definition 2-8: Definition of limited rationality in the uncertain choice. In addition to Definition 2-4, a subject exhibits limited rationality when the subject has a rational preference relation on uncertain consequences that are limited because of limited knowledge of states (Definition 2-6).

Definition 2-9: Definition of learning as discovery. Discovery takes place when the subject that acts according to Definition 2-4 and Definition 2-8 and that faces the next choice in a sequence of choices expands his or her subjective state set and/or the subjective action set. Discovery takes place because of the subject's experience from previous choices: Formally, learning as discovery means that the previous subjective state and/or action sets are strict subsets to the current subjective state set and/or action sets. With symbols, learning as discovery is defined as $S_{Kt-1} \subset S_{Kt}$, $A_{Kt-1} \subset A_{Kt}$ $A_{Kt-1} \subset A_{Kt}$, or when both situations occur and this is because discovery make certain that $I_{At-1} \subset I_{At}$ and $I_{St-1} \subset I_{St}$.

Definition 2-4 and Definition 2-8 rests on separate definitions of the subjective action and state set that are:

Definition 2-3: Definition of the subjective action set. The subjective action is defined as $A_K = A_\Omega \setminus I_A$.

Definition 2-7: Definition of the subjective state set. Let the subjective state set be $S_K = S_\Omega \setminus I_S$.

And the subjective action and state sets depend on the definitions of the ignorance sets.

Definition 2-1: Definition of ignorance of actions: Let the subject be unaware of at least one action in the objective action set, i.e. the subject's ignorance set is nonempty, $I_A \notin \varnothing$, and a strict subset to the objective action set, $I_A \subset A_\Omega$.

Definition 2-5: Let the subject be unaware of at least one state in the objective state set, i.e., the subject's ignorance set is nonempty, $I_S \notin \varnothing$, and a strict subset to the objective action set, $I_S \subset S_\Omega$.

The ignorance sets are also used to define limited knowledge: The limited knowledge definitions are:

> **Definition 2-2:** Definition of limited knowledge of actions. A subject's knowledge of alternative actions is limited when the subject has a nonempty ignorance set according to Definition 2-1.

> **Definition 2-6:** Definition of limited knowledge of states. A subject's knowledge of potential states is limited when the subject has a nonempty ignorance set of states according to Definition 2-5.

A static limited rational choice is achieved by forcing the subject to make his or her choices based on limited knowledge that lead to a choice based on a limited action set and, where uncertainty is present, also on a limited state set. These limited sets are strict subsets to their universal counterparts. The limited sets are referred to as subjective sets where the universal sets are the objective sets. The axioms of rational choice are imposed on the subjective sets.

This means that choice, as pictured in my theory of Homo comperiens, meets the comparability, transitivity, insatiability, and continuous preference relation conjectures. In the uncertain choice, the conjectures of independent, state-uniform preferences, and the Archimedean conjecture is also added to the conjectures on the preference relation.

With this structure on choice, it is possible to explain human action as if the subject maximizes his or her subjective expected utility. That is,

> **Proposition 2-3:** When the subject has a preference relation on the subjective consequence sets, which are complete, transitive, continuous, state uniform, independent, and that follow the Archimedean assumption, it is possible to express the subject's choice as if he or she makes his or her choice based on an action's subjective expected utility: $\mathrm{E}_{K0}\left[U_K\left(c_1,\ldots c_S; \pi_{K1},\ldots \pi_{KS}\right)\right] = \sum_{s \in S_K} \pi_{Ks} \cdot u_K\left(c_s\right)$, where $c_s \in C_{Ks}$, and $\pi_{Ks} \in \Pi_K$.

The difference between subjective expected utility and von Neumann and Morgenstern's expected utility resides in different probabilities and different Bernoulli utilities. In the theory of Homo comperiens they are referred to as subjective probabilities and subjective Bernoulli utilities.

The subjective probability differs from the objective probability because of the limited state set and the subjective Bernoulli utilities differ from the objective Bernoulli utilities because of erroneous specification of the supremum and the infimum consequences. These errors are due to the limited action set, which is a direct effect of limited knowledge. All the erroneous choices can therefore be traced back to my introduction of limited knowledge as the ignorance of actions and states, which can be thought of as an unawareness conjecture. For proof of Proposition 2-3 see Appendix C (p. 191).

The dynamics in the theory of Homo comperiens comes as a convergence of market prices to the intrinsic values. This is achieved through the introduction of learning using both Bayesian learning and discovery. Discovery is the aftermath of choice: Having made a choice, the effect of it

is experienced and when it yields an unanticipated consequence, it is incorporated into the choice set of the next choice. Alternatively, more formally the subjective action and state sets for the previous choice are strict subsets to the current choice.

This means that the subject initially is unaware that he or she is unaware. As the subjects learn through discovery, they learn that they do not know for sure what the consequences are because of the exogenous factors, which means that the subjects face an uncertain choice. By engaging in an information search, the subjects use Bayesian learning to the point where they know that they know and so face a certain choice.

CHAPTER 3—HOMO COMPERIENS AND PRICE THEORY

> "Only two things are infinite, the universe and human stupidity, and I'm not
> sure about the former." Einstein (1879-1955)

3.1 Introduction

This thesis aims to develop a theory of inefficient markets applicable to financial economics and
market-based accounting research. Chapter 2 served to develop the proposed theory of limited ra-
tional choice while the present chapter applies it to price theory. This is necessary since it creates a
structure precise enough that it allows market-pricing models of firms to be derived. Such models
are necessary for traversing from economics into market-based accounting. The results of this chap-
ter are applied to firm valuation in the next chapter with associated appendices.

When a market consists of subjects who act according to the proposed the theory of Homo
comperiens, it follows that the necessary conditions for a general equilibrium is not met and thus the
market is in disequilibrium: Disequilibrium becomes the rule and equilibrium the exception.

The main interest in this chapter is the market price. What is it and does it differ from the
market price in the perfectly rational theory?

This chapter's applied limited rational choice model is first specified, followed by a microana-
lysis and finally a macroanalysis that allows for discovery and price adaptation.

3.2 A point of departure for the application of Homo comperiens to price theory

The micro analysis restricts itself to a one-period choice with only exchange to avoid too much un-
necessary detail. Thus, the analysis focuses on consumer theory. The reader may think of this chap-
ter as an application of the theory to financial markets. Its application follows the standard applica-
tion of microeconomics on financial markets (Huang & Litzenberger 1988, Silberger & Suen 2000),
with one important exception: This thesis posits that human action meets the conjectures stated in
Chapter 2, i.e. choice is limited rational and not perfectly rational. A similar analysis assuming a per-
fectly rational choice is for reference included in Appendix A (p. 147). Note that Chapter 2 argues
that the rational expectations hypothesis yields biased expectations in a Homo comperiens' setting.
It therefore follows that such an analysis is not implemented in this chapter. This chapter's analysis
mode makes instead use of state-contingent consumption following, e.g., Huang & Litzenberger
(1988) and Silberger & Suen (2000).

More than one period is used in the macroanalysis so that it can incorporate the learning
conjectures in Chapter 2. The one-period context should be interpreted as if the subjects make their
choice today about their consumption plans for today and tomorrow.

The analysis follows standard economics procedures and it assumes that the subject can choose among several subjective consumption bundles (actions) that all delivers consequences, which are measured in a numerarie good. The first good is defined as the numerarie; the price for the numerarie is 1, which, in practical terms, can be thought of as money.

The subject has preferences on the consumption bundles according to the definition of Homo comperiens (Proposition 2-4, p. 45). The choice is bounded from above and below since the Homo comperiens assumes that the subject makes choices that are restricted to span the subjective action and state sets that are themselves by design bounded. The choice variable follows the symbols used in Appendix A. Note that all the sets are the objective sets in Appendix A, but this chapter conjectures Homo comperiens and hence are all the states part of the subjective state set.

This chapter's subject chooses among different portfolios of current consumption and portfolios of future consumption (the subject's subjective action set). The goods are denoted $l \in L_K$ where the superscript is mnemonic for known. The subjective set of goods is a strict subset of the objective set of goods, $L_K \subset L_\Omega$ having L goods for consumption because of Definition 2-3 (p. 29). Chapter 2 also conjectures that the subject's subjective state set is a strict subset to the objective state set (Definition 2-7, p. 33). The subjective state set has S states and a state is designated by $s \in S_K$.

The portfolio of current consumption is symbolized by $\mathbf{c}_{K0} = \left(c_{K01}, \ldots, c_{K0L} \right)$[11] and a portfolio of future consumption is represented by $\mathbf{c}_0^K$. The focus of the analysis is on the subjective consumption bundle, designated $\mathbf{C}_K = \left(\mathbf{c}_{K0}, \mathbf{c}_{K1} \right)$. The subjective consumption bundle differs from the objective consumption bundle in that the subject has limited knowledge of the available consumption alternatives. That is, the subject knows only of L_K and not of L_Ω.

The cost of current consumption of a good is $c_{K0l} = {}_0 p_{K0l} \cdot q_{0l}$ and the cost of the current consumption portfolio is $\mathbf{c}_{K0} = \mathbf{Q}_0^\top \cdot {}_0 \mathbf{P}_{K0}$, where $\mathbf{Q}_0^\top = \left[q_{01}, \ldots, q_{0L} \right]$ is the transpose of the column matrix with the quantities of current consumption goods held by the subject. This is a new meaning of the symbols compared with Appendix A, where $\mathbf{c_0}$ represents the portfolio of quantities of goods for current consumption. The shift in meaning is due to this thesis' focus on the cost of consumption and not on quantities per se. Another difference pertains to the fact that Appendix A uses all goods and services in L_Ω as available consumption alternatives, i.e. knowledge of available alternatives are not restricted.

---

[11] The reader is reminded that bold upper case letters are the general representation of a matrix. The exception is the column matrix that uses lower bold case letters as symbols. To reduce space requirement in the text this thesis writes column matrixes as vectors. The inner product of two vectors, $\boldsymbol{\rho} \cdot \mathbf{c}$, can also be written in matrix notation, $\boldsymbol{\rho}^\top \cdot \mathbf{c}$, where $\boldsymbol{\rho}^\top$ denotes the transpose of column matrix $\boldsymbol{\rho}$.

The subject can hold negative, nil, or positive quantities of goods for current consumption. When the subject holds negative quantities, the subject holds a debt that must be cleared some day. Nil indicates that the subject does not hold this particular good for current consumption, whereas a positive sign on the quantity variable indicates that the subject holds this good for current consumption.

The current unit price for each good is summarized into the column matrix whose transpose is symbolized by ${}_0\mathbf{P}_{K0}^\top = \left[{}_0 p_{K01}, \ldots, {}_0 p_{K0L} \right]$. Note that it retains the superscript K to signify that it really is subjective current unit prices. The current price for a good, ${}_0 p_{K0l}$, is the good's spot price, which can be interpreted as the price paid today for the delivery of a good today, measured in terms of the numerarie.

Today's consumption is certain but tomorrow's consumption is uncertain. The uncertainty is subjective and not objective. Using the state-space-model, makes it possible to describe the uncertain subjective future consumption portfolio as a column matrix of subjective state-dependent future consumption. The transpose of the uncertain subjective future consumption portfolio column matrix can be described as $\mathbf{c}_{K1}^\top = \left(\mathbf{c}_{K11}, \ldots, \mathbf{c}_{K1S} \right)$, where $\mathbf{c}_{K1s}$ designates the subjective cost of a future consumption portfolio for a state. The subjective cost of the future consumption portfolio for each subjective state is a column matrix and consists of its own bundle of state-contingent consumption, $\mathbf{c}_{K1s} = \left(c_{K1s}, \ldots, c_{Kls} \right)$.

Since the model is a one-period model, the subjective cost of the state-contingent consumption of a specific good is equal to the subjective payoff of a unit of this good times the quantity of claims to future consumption of this good, i.e., $c_{Kls} = q_l \cdot r_{Kls}$, where $\mathbf{Q}_1 = \left[q_1, \cdots, q_L \right]$ is all the current holdings of claims to the future consumption of goods. Note that $\mathbf{Q}_1$ is state-independent: It is a legal claim that the holder has today to the consumption of goods in the future. What can vary with the state is the subjective payoff of the claim. The payoff, r_{Kls}, is the subjective payoff (in consumption measured in the numerarie) for a unit of a particular good in a particular state. The total subjective payoff for a particular state is then $\mathbf{R}_{Ks} = \left(r_{K1s}, \cdots, r_{KLs} \right)$

The subjective payoff matrix above is for the future consumption in a state and since the subject knows of S states, if follows that the subjective payoff matrix needs to be enlarged to cover all known states. That is,

$$\mathbf{R}_K^\top = \left[\mathbf{R}_{K1}, \ldots, \mathbf{R}_{KS} \right] = \begin{bmatrix} r_{K11} & \cdots & r_{KL1} \\ \vdots & \ddots & \vdots \\ r_{K1S} & \cdots & r_{KLS} \end{bmatrix}$$

The subjective payoff shown above should be interpreted as all potential payoffs that the subject can perceive in the future given his or her restrictions through the subjective set of states

and the subjective action set. It is therefore designated $\mathbf{R}_K$ in this research, which is the subject's subjective payoff set. Note that the subjective payoff set is a strict subset to the objective payoff set. It is restricted both in the number of columns (goods and services known to be available in the future) and in the number of rows (states that the subject believes that the market can enter into as the future unfolds) since both the action set and the state set are the subjective and not the objective sets.

With the definitions above, it is possible to reconsider the subjective cost of a bundle of claims for consumption of goods delivered tomorrow. It can now be written as:

$$\mathbf{c}_{K1} = \begin{bmatrix} \mathbf{c}_{K1} \\ \vdots \\ \mathbf{c}_{KS} \end{bmatrix} = \begin{bmatrix} r_{K11} & \cdots & r_{KL1} \\ \vdots & \ddots & \vdots \\ r_{K1S} & \cdots & r_{KLS} \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ \vdots \\ q_L \end{bmatrix} = \mathbf{R}_K^\top \cdot \mathbf{Q}_1 \Leftrightarrow \mathbf{c}_{K1}^\top = \mathbf{Q}_1^\top \cdot \mathbf{R}_K$$

I initially propose that the subject chooses among different strategies that deliver different subjective consumption bundles, where the bundle is depicted $\left(\mathbf{c}_{K0}, \mathbf{c}_{K1} \right)$. With the setting above, it can instead be described as $\left(\mathbf{Q}_0^\top \cdot {}_0\mathbf{P}_{K0}, \mathbf{Q}_1^\top \cdot \mathbf{R}_K \right)$.

Chapter 1 makes a critical assumption that thesis rests on. It assumes that the actors in the market are price takers. This implies that the research sets the current price matrix to be exogenously determined. The subjective payoff matrix is also exogenous since production is not considered here. Hence, the subject's choice of strategies only affects the quantity of current consumption and the quantity of claims held on future consumption goods.

The subject can only acquire the claims to future consumption at a subjective price which is to the best of his or her knowledge beyond his or her control (again part of fundamental premises of the thesis). The price today for one claim on a good to be delivered tomorrow is ${}_0 p_{K1l}$ (futures price) and the total futures price that the subject has to pay to get his or her bundle of claims for future consumption is $\sum_{l=1}^{L_K} q_l \cdot {}_0 p_{K1l}$. The column matrix ${}_0\mathbf{P}_{K1} = \begin{bmatrix} {}_0 p_{K11}, \cdots, {}_0 p_{K1L} \end{bmatrix}$ represents all unit futures prices.

The subject has at the outset the choice a limited endowment of goods and of claims to future goods that are exogenous and cannot be affected by any choice. Together with the spot and futures prices they make up in this research exogenous subjective wealth:

$\bar{\mathbf{c}} = \bar{\mathbf{Q}}_0^\top \cdot {}_0\mathbf{P}_{K0} + \bar{\mathbf{Q}}_1^\top \cdot {}_0\mathbf{P}_{K1}$. Any choice must be equal to the subject's subjective wealth. The budget restriction for Homo comperiens is therefore: $\mathbf{Q}_0^\top \cdot {}_0\mathbf{P}_{K0} + \mathbf{Q}_1^\top \cdot {}_0\mathbf{P}_{K1} = \bar{\mathbf{Q}}_0^\top \cdot {}_0\mathbf{P}_{K0} + \bar{\mathbf{Q}}_1^\top \cdot {}_0\mathbf{P}_{K1}$. This system allows the subject to hold negative quantities of claims to future goods as well as positive quantities of claims to future goods.

3.3 A micro analysis of limited rational choice

Faced with having to choose among different consumption bundles, the subject chooses in a manner that may be described as if he or she chooses the consumption bundle that maximizes his or her subjective expected utility (cf., Proposition 2-3, p. 38 and the insatiability discussion). This can be expressed as a mathematical constrained optimization problem.

3.3.1 Optimization of Homo comperiens' subjective expected utility maximization problem

Using Lagrange's technique allows the problem to be posed as:

$$\max_{\mathbf{c}_{K0},\mathbf{c}_{K0},\lambda} L\left(\mathbf{c}_0,\mathbf{c}_1\right) = \max_{\mathbf{Q}_0,\mathbf{Q}_1,\lambda} \mathrm{E}_{K0}\left[U_K\left(\mathbf{Q}_0^\top \cdot {}_0\mathbf{P}_0^K, \mathbf{Q}_1^\top \cdot \mathbf{R}_K\right)\right] - \lambda \cdot \left(\mathbf{Q}_0^\top \cdot {}_0\mathbf{P}_{K0} + \mathbf{Q}_1^\top \cdot {}_0\mathbf{P}_{K1} - \overline{\mathbf{c}}\right) = 0 \qquad \text{[EQ 3-1]}$$

The symbol λ is Lagrange's multiplier. Differentiating L with respects to the quantities of current consumption goods and setting the derivative to zero give:

$$\partial L\big/\partial q_{0l} = v_K^0\left(c_{K0},c_{K1}\right) - \lambda \cdot {}_0 p_{K0l} = 0, \ \forall l \in L_K \qquad \text{[EQ 3-2]}$$

Differentiating L with respects to the claims to future consumption goods and setting the derivative to zero give:

$$\partial L\big/\partial q_{1l} = \sum_{s\in S_K} v_K^1\left(c_{K0},c_{K1s}\right)\cdot r_{Kls} - \lambda \cdot {}_0 p_{K1l} = 0, \ \forall l \in L_K \qquad \text{[EQ 3-3]}$$

Differentiating L with respects to Lagrange's multiplier and setting the derivative to zero give:

$$\partial L\big/\partial \lambda = \mathbf{Q}_0^\top \cdot {}_0\mathbf{P}_{K0} + \mathbf{Q}_1^\top \cdot {}_0\mathbf{P}_{K1} - \overline{\mathbf{c}} = 0 \qquad \text{[EQ 3-4]}$$

Dividing [EQ 3-3] with [EQ 3-2] and solving for the ratio between the subjective futures price and the current price show:

$$\frac{{}_0 p_{K1l}}{{}_0 p_{K0l}} = \sum_{s\in S_K} \frac{v_K^1\left(c_{K0},c_{K1s}\right)\cdot r_{Kls}}{v_K^0\left(c_{K0},c_{K1}\right)}, \ \forall l \in L_K \qquad \text{[EQ 3-5]}$$

Specifically, by focusing the analysis of [EQ 3-5] on the numerarie (money), it is clear that the subject's subjective future's price for the numerarie is the marginal rate of substitution between consumption and saving[12]. In symbols it is:

$$_0 p_{K11} = 1 \cdot \sum_{s\in S_K} \frac{v_K^1\left(c_{K0},c_{K1s}\right)\cdot 1}{v_K^0\left(c_{K0},c_{K1}\right)} \qquad \text{[EQ 3-6]}$$

The subjective marginal utilities, $v_K^0\left(c_0^K,c_{1l}^K\right)$ and $v_K^1\left(c_0^K,c_{1s}^K\right)$ have more meaning because of Proposition 2-4:

$$v_K^0\left(c_{K0},c_{K1}\right) = \frac{\partial \mathrm{E}_{K0}\left[U_K\left(c_{K0},\mathbf{c}_{K1}\right)\right]}{\partial c_{K0l}} = \sum_{s\in S_K} \pi_{Ks} \cdot u_{Ks}\left(c_{K0},c_{K1}\right), \ \forall l \in L_K \qquad \text{[EQ 3-7]}$$

---

[12] Saving is the equivalent to postponed consumption.

$$v_K^1 \left(c_{K0}, c_{K1s}\right) = \frac{\partial \mathrm{E}_{\mathrm{K}0} \left[U_K \left(\mathbf{c}_{K0}, \mathbf{c}_{K1}\right)\right]}{\partial c_{1s}^K} = \pi_{Ks} \cdot u_K \left(c_{K0}, c_{K1}\right), \ \forall s \in S_K \qquad \text{[EQ 3-8]}$$

3.3.2 Interpretation of the optimization

When the subjective price is exogenous to the model, it follows that the subject can only adjust his or her consumption quantities that he or she demands and supplies according to the prevalent subjective price. The particular quantities of consumption that the subject chooses is the quantities that satisfy [EQ 3-4] and [EQ 3-5]. This means that the quantities are chosen so that the subject's subjective marginal rates of substitution equal the subjective prices and that the subject does not waste goods and services.

The subjective marginal rates of substitution are themselves functions, where the arguments are based on the subject's inference on the subjective state probabilities as well as on the subject's assignment of subjective Bernoulli utilities to consumption. This is expressed by [EQ 3-7] and [EQ 3-8]. From this, it follows that as soon as the subjective action set is limited, the subjective state set is limited, or when both occur, the subject chooses to trade erroneous amounts of current and future goods: The subject makes an erroneous choice when compared with the situation where the subject has access to the objective action and state sets. Had the subject made his or her choice based on the objective sets, the choice would be identical to a choice using a von Neumann-Morgenstern utility function as in financial economics.

The erroneous choice has profound implications on the macroanalysis as seen in the next section. But first follows some important reflections on what effects the above analysis has for financial economics and the parts of market-based accounting research that needs valuation models and a capital asset pricing model.

First, assume that the preferences are homogenous in the population and hence that it is possible to analyze the situation using the representative individual conjecture, which is used in financial economics when e.g., deriving the CAPM (e.g., Huang & Litzenberger 1988, or Ohlson 1987, for a discussion of the conjectures underlying CAPM).

Even when there is no exogenous change as time passes, the representative subject's utility function changes since the representative subject learns through discovery (Definition 2-9, p. 44), i.e. since the subjective action and state sets change. When the subjective sets change, it implies that the representative subject's subjective state-probabilities change and that the subjective Bernoulli utilities change as well. The subjective Bernoulli utilities change since the representative subject becomes better at specifying the supremum and infimum consequences.

This means that [EQ 3-7] and [EQ 3-8] change as time passes, even when there is no exogenous change and when a representative subject is assumed. Since they change, it also means that the relative prices change, i.e. [EQ 3-5] and [EQ 3-6].

So, even when conjecturing a representative subject, it is without merit to push the analysis into a subjective CAPM model since it is unique for each time and therefore cannot be given empirical content.

Furthermore, since each subject has his or her own unique understanding of which actions are available and the states that can occur, it is fallacious to assume that all subjects in the market can be assigned identical utility functions so that every subject has the same marginal utilities, which is what is assumed by the representative subject conjecture.

So when the world consists of subjects who are limited rational it follows that, e.g., the conjectures for CAPM are violated and models that posit it, such as tests of EMH, invariably fail to provide valid and reliable results.

3.4 A macroanalysis of limited rational choice

A downside with the microanalysis is that it posits that the subjective price is exogenous. To include the formation of a subjective price into the model it is necessary to stop seeing the subject as a Robinson Crusoe (although with the possibility of trading): When the subject trades, he or she trades with other subjects, and these subjects make up his or her environment in which he or she interacts.

The macroanalysis conjectures that the world is populated by more than one subject, that they trade between each other, and that from the outset there exist an exogenous price, but as trading is allowed between the subjects, it becomes endogenously determined. The exogenous initial subjective price can also be thought of as a subjective price that prevails in an already existing market as a subject enters. All the subjects in the analysis behave according to Homo comperiens (Proposition 2-4, p. 45).

The microanalysis shows that when a subject acts as Homo comperiens he or she, given the opportunity, decides on a subjective price that deviates from the price he or she would have decided on had he or she had access to the objective action and state sets. In Chapter 1 where learning is discussed, it is argued that each subject, which learns by making choices and experiencing their consequences, accumulates a unique composition of knowledge that dictates how he or she perceives the world and its accompanying choices.

3.4.1 The dynamics of learning

The subject's unique composition of knowledge implies that each subject assigns a unique subjective price to a good, regardless if it is consumed today or tomorrow. This is because it is a function of the subject's utility function, which is a function of the subjective state probabilities and the subjective Bernoulli utilities, which are themselves functions of the subject's limited knowledge.

If the subjective price, on the other hand, is exogenous and there are several subjects who make up the market, they can adjust their quantities so that each of them supply and demand goods

that make their subjective marginal rate of substitution equal to the quoted subjective prices. The subjective marginal rate of substitution is a function of the subjective state probabilities and the subjective Bernoulli utilities, which also means that each subject's supplied and offered quantities are not the same as if they had had access to the objective action and state sets. When all subjects' demand and supply plans are aggregated, the planned supply of goods does not match the demanded quantities of goods since the subjects fail to consider the peers' demand and supply plans: The demand and supply plans fail to dovetail (except by chance) and the failure causes erroneous choices that are manifested as excess supply and excess demand of goods and services. Only when all subjects make their choices on the objective action and state sets and when all subjects know each other's choice rules, can we expect to see perfectly dovetailing supply and demand plans.

Since the subjects all have their own unique inference of what the available actions are and what states that can occur, it follows that it is incomprehensible that all subjects, at the same time and by chance, happen to infer the objective action and state sets. However, since Homo comperiens learns from past mistakes, it also follows that there is a sequence of choices of demand and supply plans for the subjects from which the subjects learn.

Consider the following two examples: Each round of demand and offer plans are followed by transactions based on these plans and hence are any failure and success with the plans transmitted among the subjects through the transactions. Some subjects succeed in carrying out their plans while others fail to do so. The order in which the subjects' demand and offer plans are cleared in any given round of transactions is then important for the subjects. For instance, imagine that there is an excess demand of a particular good. This implies that the last subject(s) cannot carry out his or her (their) demand plan(s) as imagined, and end(s) up being dissatisfied. Conversely, in the case of excess supply there is at the end some subject(s) who end(s) up not being able to sell all the goods that he or she (they) planned and he or she (they) will also be dissatisfied. The dissatisfaction among subjects induces them to make revised plans for the next round of transactions. The sequence of choices of demand and supply plans is therefore where the subject gradually learns about other subjects' demand and offer plans, as well as about what actions are available.

Taken together, the dynamics lead to a better and better match among the subjects' aggregated demand and supply plans. When enough transactions have taken place, learning has reduced excess demand and supply to almost nil, and the market approaches perfectly dovetailing demand and offer plans since $\lim_{t \to \infty} (A_K) \approx A_\Omega$ and $\lim_{t \to \infty} (S_K) \approx S_\Omega$. When this has happened, the market nears a general equilibrium in which each subject can carry out his or her demand and supply plans almost exactly according to his or her intentions. Thus, I propose the following:

Proposition 3-1: Learning through discovery (Definition 2-9**)** ascertains that $\lim_{t\to\infty}\left(A_K\right) \approx A_\Omega$ and

$\lim_{t\to\infty}\left(S_K\right) \approx S_\Omega$ since the ignorance sets decrease. This implies that the subjective price approaches the objective price as t goes to infinity. That is $\lim_{t\to\infty}\left(_{t-1}p_{Kt}\right) \approx {}_{t-1}p_t$.

Note that even though we fix the objective action and state sets, which implies a fixed price situation, it is not correct to think of the objective price as a fixed and certain price. All I say is that in the limit the subject knows of (almost) all states and of (almost) all actions that are part of the choice. I do not say that the subject knows exactly which state will come true because that requires me to conjecture that the objective state set is a singleton, and I have not imposed such a conjecture here. Therefore, in the limit it is more correct to think of the objective price as a variable that is approximately randomly walking.

Proposition 3-1 also implies that I do not conjecture hog-tail like situations where the subjective price gradually gravitates away from the objective price in the theory of Homo comperiens. Instead, the subjective price gradually regresses to the objective price.

This can be expressed more precisely as below:

Definition 2-1, Definition 2-5, and Definition 2-9 $\Rightarrow \lim_{t\to\infty}\left(I_A\right) \approx \varnothing$, and $\lim_{t\to\infty}\left(I_S\right) \approx \varnothing$

$$\lim_{t\to\infty}\left(I_A\right) \approx \varnothing \Rightarrow \lim_{t\to\infty}\left(A_K\right) \approx A_\Omega \qquad\qquad\text{[EQ 3.9]}$$

$$\lim_{t\to\infty}\left(I_S\right) \approx \varnothing \Rightarrow \lim_{t\to\infty}\left(S_K\right) \approx S_\Omega \qquad\qquad\text{[EQ 3.10]}$$

$$\text{[EQ 3.9]} \Rightarrow \lim_{t\to\infty}\left(u_K\left(c_{K0}, c_{K1}\right)\right) \approx u\left(c_0, c_1\right) \qquad\qquad\text{[EQ 3.11]}$$

$$\text{[EQ 3.9]} \Rightarrow \lim_{t\to\infty}\left(u_K\left(c_{K0}, c_{K1s}\right)\right) \approx u\left(c_0, c_{1s}\right) \qquad\qquad\text{[EQ 3.12]}$$

$$\text{[EQ 3.10]} \Rightarrow \lim_{t\to\infty}\left(\Pi_K\right) \approx \Pi_\Omega \qquad\qquad\text{[EQ 3.13]}$$

$$\text{[EQ 3.13]} \Rightarrow \lim_{t\to\infty}\left(\pi_{Ks}\right) \approx \pi_s \qquad\qquad\text{[EQ 3.14]}$$

$$\text{[EQ 3.12], and [EQ 3.14]} \Rightarrow \lim_{t\to\infty}\left(\frac{\partial E_{K0}\left[U_K\left(\mathbf{c}_{K0}, \mathbf{c}_{K1}\right)\right]}{\partial c_{K0l}}\right) \approx \frac{\partial E\left[U\left(\mathbf{c}_0, \mathbf{c}_1\right)\right]}{\partial c_{0l}} \qquad\qquad\text{[EQ 3.15]}$$

$$\text{[EQ 3.12], and [EQ 3.14]} \Rightarrow \lim_{t\to\infty}\left(\frac{\partial E_{K0}\left[U_K\left(\mathbf{c}_{K0}, \mathbf{c}_{K1}\right)\right]}{\partial c_{1s}^K}\right) \approx \frac{\partial E\left[U\left(\mathbf{c}_0, \mathbf{c}_1\right)\right]}{\partial c_{1l}} \qquad\qquad\text{[EQ 3.16]}$$

$$\text{[EQ 3.15]} \Leftrightarrow \lim_{t\to\infty}\left(v_K^0\left(c_{K0}, c_{K1}\right)\right) \approx v^0\left(c_0, c_1\right) \qquad\qquad\text{[EQ 3.17]}$$

$$\text{[EQ 3.16]} \Leftrightarrow \lim_{t\to\infty}\left(v_K^1\left(c_{K0}, c_{K1s}\right)\right) \approx v^1\left(c_0, c_1\right) \qquad\qquad\text{[EQ 3.18]}$$

$$\text{[EQ 3.17], and [EQ 3.18]} \Rightarrow \lim_{t\to\infty}\left(\frac{_0p_{K1l}}{_0p_{K0l}}\right) \approx \frac{_0p_{1l}}{_0p_{0l}}$$

$$\text{[EQ 3.18],} \; _0p_{K01}, {}_0p_{01} = 1 \Rightarrow \lim_{t\to\infty}\left(_{t-1}p_{Kt}\right) \approx {}_{t-1}p_t \quad \forall l = 1 \quad \text{Q.E.D.}$$

That is, in the long run the subjective price converges towards the objective price since the subjective state-dependent probabilities and the subjective Bernoulli utilities approach their objective counterparts, all of which because the subject's ignorance decreases because of discovery, i.e. limited knowledge becomes perfect knowledge.

3.4.2 The adaptation processes

It is conceivable to think of the subjective price as the information carrying device that brings supply and demand plans to dovetail. This is because it is the subjective price that decides how much a subject is prepared to demand and how much the subject is prepared to offer (cf. section 3.3). So, as subjects craft and attempt to execute erroneous demand and offer plans, excess demand and supply situations occur. The hitherto erroneous plans are taken into account as the next set of demand and offer plans are crafted. The resulting change reduces the excess supply and demand in the market, with the market price changing accordingly. As the sequence of choices continues, the subjective price changes to accommodate the decreasing excess demand and excess supply plans.

When there is an excess demand, subjects are prepared to buy more of the good than what the current supply plans allow. Since subjects are insatiable, it follows that in the next round the suppliers increase the subjective price to counter the excess demand. It is also conceivable that the offered quantity is increased at the given subjective price, which also delivers a decrease in the excess demand. Any combination thereof can also be concocted.

The increase in offered quantities is an effect of discovery: Because the supplier has learned more about conceivable actions and states, the supplier makes the new choice (of increased/decreased offered quantities) based on a new utility function (it is defined on a new and greater action/consequence set that maps into a greater utility set). The ability to overlook larger action and consequence sets changes the subject's perception (i.e. the marginal utilities and hence the subjective marginal rate of substitution changes) and he or she is willing to supply larger quantities at a given subjective price.

If there is an excess supply, is follows from the non-satiation conjecture that the subjective prices must drop so that the demand increases (and the supply decreases) to a level where the excess supply disappears. It is also conceivable that there is an adjustment of the supply plans so that it, at a given price, better corresponds to the demanded quantities, or there can be a combination of the two alternatives. The quantity adjustment is motivated analogous to the preceding quantity adjustment analysis that can take place when there is excess demand.

From above, it should be clear that the subjective price is not an equilibrium price since subjects are limited rational and the subjective price changes in response to new choices that are made based on more knowledge, i.e. the subjective action set and the subjective state set approach the objective sets.

Proposition 3-1 suggests that $\lim_{t \to \infty} \left({}_{t-1}p_{Kt} \right) \approx {}_{t-1}p_t$. This proposition thus suggests an adaptive expectations model of the price. There are many such suggestions in economics starting with Fisher (1930) through, e.g., Nerlove (1958; 1972), but it can also be found in market-based accounting research (Ohlson 1995). These adaptive expectations models are, to use Grossman's (1981, p. 543) words, ad hoc models. That is, the models lack theoretical support since they do not derive from the basic theory of choice. The theory of Homo comperiens, however, suggests such a behavior based on the subjects' choices since it proposes that $\lim_{t \to \infty} \left({}_{t-1}p_{Kt} \right) \approx {}_{t-1}p_t$.

Economics have instead followed another direction with the introduction of rational expectations (Muth 1960), which implies that the price process follows a stochastic structure such that $p_t = \mathrm{E}\left(p_t \,|\, p_1, p_2, \ldots\right)$ (Grossman 1981, p. 543). The rational expectations imply that prices follow random walk, but at present not even Malkiel (2003, p. 80) believes that prices do so to such an extent that markets are perfectly efficient.

Grossman (1981, p. 543-544) shows that a rational expectations equilibrium can be obtained even when subjects face asymmetric information. This finding critically uses the unbiasedness conjecture by conjecturing that the expected price equals the price that comes true. Grossman calls this perfect foresight (Grossman 1981, p. 543). However, [EQ 3-5] show that prices depend on the subjective sets, i.e. as the subject discovers new actions and/or new states, prices change non-randomly.

Chapter 2 argues that it is conceivable to talk about a limited rational expectation model since the theory of Homo comperiens is general enough to allow for Bayesian learning. This means that it is general enough to allow for random release of new information about the subjective state set, which implies that in the event of no discovery we have a situation where the price follows random walk, but in the event of discovery (of new actions and/or states) prices no longer follow random walk. Perhaps it is conceivable to think of the price movements as following a random walk with drift, where the drift follows in the direction of discovery.

I therefore suggest a non-random adaptation of the subjective price such that it regresses to the equilibrium price, i.e. towards the objective price.

> **Proposition 3-2:** Suppose that the Pareto optimal equilibrium price is fixed, which is reasonable since the objective action and state sets are assumed to be fixed and since inflation is not conjectured. Then, with Proposition 3-1 in mind, I propose that price convergence can be described as follows: Let the subjective price be a function of the objective price (p) and a fraction of the previous period's discrepancy between the subjective price and the objective price. That is, ${}_{t-1}p_t^K = p + \beta \cdot \left({}_{t-2}p_{t-1}^K - p \right) + \varepsilon_t$ where $\beta \in [0,1)$ and where ε_t is a white noise disturbance.

The adaptation process proposed above suggests that if there initially is a price discrepancy, i.e. $p_{t-1}^K - p \neq 0$, there will be an adjustment process since $\beta \in (0,1)$. If $\beta = 1$, the process is a ran-

dom walk that suggests there is no learning by discovery. If $\beta = 0$, it implies that the learning process is very fast and complete after one period. Having $\beta \in (-1,0)$ implies an oscillating process with overreactions but where the subjective price nevertheless regresses to the equilibrium price. Having $\beta = -1$ implies a never-ending oscillating process with constant over- and under-reactions, which, consequently, do not allow the subjective price to regress towards the equilibrium price. Since the theory of Homo comperiens suggests that $\lim_{t \to \infty} \left({}_{t-1}p_{Kt} \right) \approx {}_{t-1}p_t$ it is reasonable to expect to have $\beta \in [0,1)$, i.e. the market gradually discovers such that the subjective price regresses towards the equilibrium price without an oscillating behavior. The pace of learning is kept unspecified from being very quick $\beta = 0$ to being very slow $\beta \approx 1$.

The suggested adaptive model resembles Nerlove's (1958, p. 231) model as well as Ohlson's (1995, p. 667-668) model with some exceptions. Nerlove's model does not focus on an adaptive process that leads to an equilibrium price but it is instead a gradual adjustment of the expected normal price based on the difference between the actual price and what was previously expected to be normal. The model proposed above sets the expectation for $t+1$ to be the equilibrium price plus a fraction of a difference between the actual price and the equilibrium price.

Ohlson's (1995) model is similar to that which is proposed above with an important exception. Ohlson's (equation 2A, p. 668) model focuses on v_t only and it assumes that the autoregressive process is intercepted by other non-random value-relevant effects that have an autoregressive pattern. Furthermore, Ohlson presumes that the range of the two autoregressive parameters is non-negative and less than one.

My proposed model is more tractable insofar that any other information is assumed random and enters through the white noise residuals. I think that it is a reasonable assumption since two types of learning is assumed to be present. First, there is Bayesian learning and the release of such information (i.e. information about something that the subject knows that he or she does no know about) is assumed to be random. If that conjecture is correct, it follows that such learning feeds into the model's white noise residuals. The second type of learning is learning through discovery, which is what ascertains that $\lim_{t \to \infty} \left(A_K \right) \approx A_\Omega$ and that $\lim_{t \to \infty} \left(S_K \right) \approx S_\Omega$, and thus $\lim_{t \to \infty} \left({}_{t-1}p_{Kt} \right) \approx {}_{t-1}p_t$. Accordingly, it cannot be a white noise process. Learning through discovery is therefore captured in the model by having $\beta \in [0,1)$.

3.5 Homo comperiens and Walras' tâtonnement process

Assuming perfect rationality suggests that there is neither excess demand nor any excess supply, but with the introduction of limited rational choice, there exists excess demand and supply in the mar-

ket. Excess demand or supply is a sufficient and necessary condition for an arbitrage opportunity to exist in the market.

This means that the existence of arbitrage opportunities is a defining characteristic of a market when limited rational choice is conjectured. The general equilibrium theory was developed by Walras ([1874] 1954) in which the author argues that the equilibrium price is a price that equates the demanded and the offered quantities of goods (Walras [1874] 1954).

Since a general equilibrium requires no-arbitrage, it follows that existence of arbitrage opportunities invalidates equilibrium analysis, and since general equilibrium analysis lies at the heart of traditional economics, any arbitrage opportunity is assumed nonexistent (e.g., Debreu 1959). Walras ([1874]1954, p. 166-172), e.g., discusses what happens if there is an arbitrage opportunity.

According to Walras, it is possible to describe the market in terms of its subjects, their initial endowments, their utility functions, and the market prices. If there is disequilibrium, the market adjusts itself by increasing the price of the goods that are in excess demand, and by decreasing the goods that are in excess supply until the demand and supply are in balance for all goods. This takes place in what Walras calls a tâtonnement process, and is in the subject's book conceived to occur without any outside interference. Walras, however, does not explain how it can happen.

Walras' tâtonnement process has come to be described as if there is a Walrasian auctioneer responsible for balancing the demand and supply of goods (e.g., Gravelle & Reese 1998). The balancing process is assumed to follow a pattern such as the following: The Walrasian auctioneer calls out to all subjects in the market the price for each current and future good. Every subject takes these prices as given and submits to the auctioneer his or her demand and supply plans for each good. All plans for all subjects in the society are summarized so that the auctioneer can determine for each good if there is excess demand, excess supply or if the demand and supply is in balance. If there is imbalance, the auctioneer calls out a new set of prices and the subjects again submit their supply and demand plans at the given price. If there still is an imbalance in the system, the auctioneer commences another round. This continues until there is a price that equates the demand with the supply for all goods in the market. Once the set of equilibrium prices has been found, all plans are executed and trading takes place.

An interesting feature of this Walrasian auctioneer and the tâtonnement process is that it presumes that there is a subject (or perhaps a machine?) that is smarter than all other subjects and who can act as the auctioneer.[13] Since the subjects in the market fail to make choices that are perfectly dovetailing, there is a need for someone else to step in and correct the erroneous demand and

---

[13] More carefully put, it is argued that the Walrasian auctioneer is an aberration in the general equilibrium analysis. It is inserted ad-hoc to guarantee a solution to the general equilibrium's system of simultaneous linear equations.

supply plans. How is this possible when the general equilibrium theory assumes that the world is populated by subjects who are omniscient (e.g. Mas-Colell et al. 1995, p. 547-548)?

These are in fact mutually contradictory conjectures: In perfect rationality subjects already know everything there is to know about the alternatives and their consequences that they face, although they may not know it for sure since there is uncertainty. This means that subjects cannot make an erroneous choice while disequilibrium is due to such erroneous choices! This fact has indeed been appreciated long before today; e.g., Kirzner (1973, 1992) discussed this and came to the conclusion that a market is in disequilibrium precisely because subjects fail to make demand and supply plans that dovetail. A drawback with Kirzner's analysis is that he does not formally a structure the analysis such that he can explain the existence of ignorance and hence why markets are in disequilibrium. This means that the practical use of the subject's thoughts remains elusive. The theory of Homo comperiens explains why markets are in disequilibrium and what the process is that guides the market process (Chapter 2).

By allowing limited rational choice as defined by Homo comperiens, there is a structure that, by design, creates a market that from the outset is in disequilibrium and where the market process is a gradual adjustment to a general equilibrium while it retains the familiar traits of von Neumann - Morgenstern's utility function. A market that is constituted by subjects who act according to Homo comperiens does also, as seen in section 3.3, neatly fit Walras' tâtonnement process without having to make a cumbersome, contradictory, ad-hoc conjecture about a Walrasian (near omnipresent) auctioneer, which has become a standard operating procedure in the application of general equilibrium analysis.

3.6 Summary

This chapter applies the theory of Homo comperiens to a pure exchange market. It shows that the subjective marginal rate of substitution between current and future consumption is:

$$\frac{_0p_{K1l}}{_0p_{K0l}} = \sum_{s \in S_K} \frac{v_K^1\left(c_{K0}, c_{K1s}\right) \cdot r_{Kls}}{v_K^0\left(c_{K0}, c_{K1}\right)}$$

The subjective marginal rate of substitution between current and future consumption is different to the objective marginal rate of substitution between current and future consumption, which is (e.g. Ohlson 1987, p. 23):

$$\frac{_0p_{1l}}{_0p_{0l}} = \sum_{s \in \mathbb{S}} \frac{v^1\left(c_0, c_{1s}\right) \cdot r_{ls}}{v^0\left(c_0, c_{1l}\right)}$$

The difference resides in the difference between the marginal utilities for current and future consumption. The subjective marginal utilities for current and future consumption are:

$$v_K^0\left(c_{K0}, c_{K1}\right) = \frac{\partial \mathrm{E}_{\mathrm{K}0}\left[U_K\left(\mathbf{c}_{K0}, \mathbf{c}_{K1}\right)\right]}{\partial c_{K0l}} = \sum_{s \in S_K} \pi_{Ks} \cdot u_{Ks}\left(c_{K0}, c_{K1}\right)$$

$$v_K^1\left(c_{K0}, c_{K1s}\right) = \frac{\partial \mathrm{E}_{\mathrm{K}0}\left[U_K\left(\mathbf{c}_{K0}, \mathbf{c}_{K1}\right)\right]}{\partial c_{1s}^K} = \pi_{Ks} \cdot u_K\left(c_{K0}, c_{K1}\right)$$

The reason that subjective marginal utilities for current and future consumption differ from the objective marginal utilities for current and future consumption is because of two reasons. First, there is a difference between the subjective state-dependent probabilities and the objective state-dependent probabilities. Second, the subjective Bernoulli utilities are different. That is,

$$\pi_{K,s} \neq \pi_{\Omega,s}, \; u\left(c_0^K, c_1^K\right) \neq u\left(c_0, c_1\right), \text{ and } \tilde{u}\left(c_0^K, c_{1,s}^K\right) \neq u\left(c_0, c_{1s}\right)$$

See section 2.3 (p.36) and Appendix C (p. 191) for a further discussion on this topic. Chapter 2 shows that the subjective state probabilities and the subjective Bernoulli utilities differ from the objective state probabilities and the Bernoulli utilities. From this, it follows that as soon as the subjective action set is limited, the subjective state set is limited, or when both occur, the subject chooses to trade erroneous amounts of current and future goods.

Since the subjective marginal rate of substitution between current and future consumption differs from the objective marginal rate of substitution between current and future consumption, it follows that the subjective prices (i.e. market prices) differ from the objective prices (i.e. intrinsic values).

The chapter also demonstrates that the quantities demanded and quantities supplied of goods and services are chosen so that the subjective marginal rates of substitutions of the goods and services equal the subjective prices. This is a deviation from microeconomic theory that rests on perfect rationality, where the quantities chosen depend on equality between the objective marginal rates of substitutions and the objective prices.

The chapter then focuses on the endogenous determination of the subjective price and concludes that it is not set independently of historical subjective prices (i.e. subjective prices are not expected to follow a random walk process) but instead proposes:

Proposition 3-1: Learning through discovery (Definition 2-9**)** ascertains that $\lim_{t \to \infty}\left(A_K\right) \approx A_\Omega$ and

$\lim_{t \to \infty}\left(S_K\right) \approx S_\Omega$ since the ignorance sets decrease. This implies that the subjective price approaches the

objective price as t goes to infinity. That is $\lim_{t \to \infty}\left({}_{t-1}p_{Kt}\right) \approx {}_{t-1}p_t$.

Or, to put it another differently: The market prices approach the intrinsic values since the subjects learn through discovery from sequential choices when there is no exogenous change that interferes. This also leads to this chapter's next proposition.

Proposition 3-2: Suppose that the Pareto optimal equilibrium price is fixed, which is reasonable since the objective action and state sets are assumed to be fixed and since inflation is not conjectured. Then, with Proposition 3-1 in mind**,** I propose that price convergence can be described as follows: Let the subjective price be a function of the objective price (p) and a fraction of the previous period's discrepancy between the subjective price and the objective price. That is, $_{t-1}p_t^K = p + \beta \cdot \left(_{t-2}p_{t-1}^K - p \right) + \varepsilon_t$ where $\beta \in [0,1)$ and where ε_t is a white noise disturbance.

The discovery variable β is assumed to be less than one but greater than or equal to zero, which provides for a gradual adaptation process that does not overreact. If the adaptation variable were less than zero, it indicates overreactions; had it been minus one, it implies random walk as it does when it is one.

Another subtle but yet important element of this chapter is that it shows that the market price for money is:

$$_0 p_{K11} = 1 \cdot \sum_{s \in S_K} \frac{v_K^1 \left(c_{K0}, c_{K1s} \right) \cdot 1}{v_K^0 \left(c_{K0}, c_{K1} \right)}$$

This differs from the intrinsic value of money (cf. A.6.1, p. 158, for the intrinsic value of money in certainty). And, as A.8.2 (p. 169) shows, the market rate-of-return (MROR) in equilibrium is a function of the intrinsic value. It follows that MROR is a function of the market price. This has important implications for valuation theory since MROR is at the core of it, as Appendix A shows.

Chapter 3 argues that a market that assumes Homo comperiens is a market with arbitrage opportunities, which is in opposition to a general equilibrium market that, by design, excludes arbitrage. This has an important bearing on the existence of a firm and its profits and also precludes economic analysis based on general equilibrium conditions.

CHAPTER 4—HOMO COMPERIENS AND THE FIRM

Homo comperiens' effect on the firm's market price and accounting
variables

4.1 Introduction

Chapter 2 and Chapter 3 focus on the subjects and on how they create the market. Less is said about the firm, which is the focus of the present chapter. This chapter presents limited rational market pricing models for firms by conjecturing Homo comperiens. It also discusses the effect that the theory of Homo comperiens has on the firm's income and on its accounting rate of return (ARR).

The previous chapters present a theory of limited rational choice that is tested using accounting data in latter chapters. Thus, there is a need to connect theory to accounting data. This chapter provides the formal link and poses operationalizable propositions.

Appendix B provides details to this chapter's market pricing model propositions and corollaries.

4.2 The firm as a choice entity in the theory of Homo comperiens

In the theory of Homo comperiens the micro unit is the subject and not the firm. The firm is only an analytical device that assembles the supply activities in the market.

The implication is that each subject can also be a firm, and when this occurs, the firm meets the assumptions underlying the theory of Homo comperiens. Therefore, when the market consists of one subject, it follows that the firm in the market, i.e. the market's supply activities, also has limited knowledge.

However, firms are in practice more often seen as separate units operating in a market. When this occurs, that unit is created, populated, and enacted by subjects.

Assume that the market has many subjects and all meet assumptions of the theory of Homo comperiens. Assembling all their supply activities into a single firm is the analytical equivalent to a planned market. Will such a firm also have limited knowledge and thus be limited rational?

It is possible that the subjects' knowledge in the union of the market is the equivalent to perfect knowledge. The question can then be formulated as follows: Is it possible to bring together the knowledge of all subjects into a single choice body such that it has perfect knowledge?

If it is possible to assemble the market's subjects' knowledge into a single firm such that it becomes endowed with perfect knowledge, that firm becomes omniscient and can decide the supply of goods and services such that a Pareto optimal equilibrium is obtainable. However, it is enough

that the firm fails to gather some of the complementary knowledge for the firm to have limited knowledge.

This view on the subject and the firm's knowledge leads to a proposition on the firm's action set and a corollary on the firm's state set, but first comes a definition of the firm's action set.

> **Definition 4-1:** The firm's action set is defined as the union of all subjects', who participate in the firm's endeavor, action sets. That is, $A_{Firm} = \bigcup_{i=1}^{I} A_i$, where $i \in I$ is a subject.

Definition 4-1, which should be seen as a general definition, does not define whether the action set is the subjective or the objective action set.

Every subject in the firm faces, according to Definition 2-3 (p. 29), a subjective action set that is a strict subset of the objective action set. Since the firm's action set is the union of its subject's action set, it follows that:

> **Proposition 4-1**: Since the subjects in a firm face subjective action sets according to Definition 2-3, and since the firm's knowledge is the union of its entire subject's knowledge (Definition 4-1), the firm faces a subjective action set that is a weak subset of the objective action set, i.e. $A_K^{Firm} \subseteq A_\Omega^{Firm}$.

Proposition 4-1 implies that the firm knows of a weak subset of the objective action set, i.e. it can be possible to endow it with perfect knowledge of available actions. In practice, it is not likely that the union of all subjects' knowledge becomes perfect knowledge. Even in the event that such a conjecture is incorrect, it is not likely that all such knowledge can be collected without any friction losses.

This means that the weak subset in Proposition 4-1 can be treated is if it is a strict subset. It also means that not even in the boundary case, when all subjects in the population are part of a gigantic firm, the firm is able to make perfectly rational choices. Hence, a firm must make its choices based on limited knowledge of the available actions, i.e. $A_K^{Firm} \subseteq A_\Omega^{Firm}$.

Chapter 2 discusses the uncertain choice using the states-and-partition-model. It finds that the subject's subjective state set is a strict subset of the objective state set because of limited knowledge. This also leads to the following corollary.

> **Corollary 4-1:** The firm, populated by subjects who act according to Definition 2-7, faces a subjective state set that is a weak subset of the objective state set, i.e. $S_K^{Firm} \subseteq S_\Omega^{Firm}$.

Corollary 4-1 in practice is also likely to be even stricter and is it probable that a firm's subjective state set is a strict subset to the objective state set, $S_K^{Firm} \subseteq S_\Omega^{Firm}$ (cf. above).

The latter part of Chapter 3 finds that a market populated by subjects whose action meets the conjectures of Homo comperiens is an arbitrage market. Proposition 4-1 and Corollary 4-1 ascertain that a firm's choice follows the analysis set forth in Chapter 2 and Chapter 3.

The firm is therefore an actor in the market that faces limited knowledge when considering both the action and the state sets. Thus, it is not possible to insert the firm into the market and expect that it can act as an omniscient equilibrating mastermind: the firms and the subjects all face and act on arbitrage opportunities. This has important implications on the risk and returns of firms, which is what is treated in the rest of the chapter.

4.3 Certainty, subjective certainty, and uncertainty

The conjecture of perfect rationality ascertains that the firm makes choices based on the objective action and state sets. Since the firm can take into consideration all possible states that can occur, it follows that the choice is a certain choice when the state set only contains one element. Using standard vocabulary, it is a certain choice.

Another version of a certain choice occurs - assuming a state-independent utility function - when the firm foresees more than one element in the state set, but where the state-dependent consequences are constant.

Homo comperiens conjectures limited knowledge, which means that the firm faces subjective action and state sets. The choice is then seemingly a certain choice when the firm's subjective state set contains one element. However, since some elements are ignored because of the firm's limited knowledge, it follows that the choice no longer meets the criteria for a certain choice and the choice is only anticipated to be certain. Hence, as the consequences unfold, there is room for unanticipated consequences for the firm because of failure to account for all conceivable states. I call such a scenario for a subjectively certain choice.

A subjectively certain choice also occurs when the firm correctly accounts for all states that can occur but fails to realize that the state-dependent consequences are not constant (again conjecturing a state-independent utility function).

Facing an uncertain choice, the difference between the perfect rational choice and the limited rational choice is less clear, but it is useful to distinguish between uncertainty in the objective sense and uncertainty in the subjective sense.

Objective uncertainty is present in a choice when the firm faces the objective state set and when the set contains more than one element. It occurs when the conjectures of perfect rationality are applied. If Homo comperiens is assumed, subjective uncertainty is the choice when the firm faces the subjective state set and when it contains more than one element.

Section 4.4 focuses on presenting the disequilibrium market-pricing models that the present thesis uses. The details of their derivations are found in Appendix B.

4.4 Homo comperiens and firm market-pricing models in subjective certainty

Equation [EQ A-52] shows that the intrinsic value (i.e. the objective price) of a firm equals the product of the numerarie's objective futures price and the objective expected future dividends, $V_{10} = {_0}p_{11}^* \cdot d_{11}$, where ${_0}p_{11}^*$ is the numerarie's objective futures price. Conjecturing Homo comperiens, the market price of the firm is in analogy (cf. Appendix B, p. 179 for details):

$$P_{10} = {_0}p_{K11} \cdot \mathrm{E}_{K0}\left[d_{K11}\right] \qquad \text{[EQ B-23]}$$

That is, the market price of a firm equals the product of the numerarie's subjective futures price and the objective expected future dividends.

The subjective futures price differs from the objective futures price since the subject exhibits a subjective and not an objective utility function (cf. Proposition 2-3, p. 38).

Appendix A shows that the objective futures price of a unit of the numerarie is in equilibrium equal to the objective marginal rate of substitution between the numerarie tomorrow and the numerarie at present, [EQ A-19]. Assuming Homo comperiens, the subjective futures price of the numerarie equals the subjective marginal rate of substitution between the numerarie tomorrow and the numerarie at present (cf. Appendix B, p. 179 for details):

$$\frac{{_0}\boldsymbol{p}_{K1}}{{_0}\boldsymbol{p}_{K0}} = \frac{\nabla U_K\left(\mathbf{c}_{K1}\right)}{\nabla U_K\left(\mathbf{c}_{K0}\right)} \qquad \text{[EQ 4-1]}$$

The subjective marginal rate of substitution is a function of the marginal rates of substitutions, which are functions of the subjective consequence set, $U_K : C_K \to \mathbb{R}$. The subjective consequence set is itself a function of the subjective action set, $f_K : A_K \to C_K$.

The objective action set equals the consumption set $X = \mathbb{R}_0^L \times \mathbb{R}_1^L$ after further restrictions (the objective budget hyperplane and transformation frontier) are added. The subjective action set equals the subjective consumption set X_K, which is a strict subset to the objective consumption set $X_K \subset X_\Omega$ after further restrictions (the subjective budget hyperplane and transformation frontier) are added.

For more detailed information, turn to sections 2.3 and 3.3 for an analysis of the utility function, its relationship to the action set and consequence set and section, and for optimization. See Appendix A for an analysis that assumes perfect rationality and Appendix B for an analysis that assumes limited rationality.

The subjective expected dividends in [EQ 4-1] differ from objective expected dividends because of the subject's subjective action set, i.e. the subject may have false anticipations. This means that the subjective expected dividends need not be the dividends that come true even though the subject perceives the choice as certain. Again, this shows how the rational expectations hypothesis assumption of unbiased forecasts is not possible to apply in the limited rational choice.

Appendix A argues that the firm's intrinsic value in a certain choice is equal to the present value of the firm's expected dividends. It also reasons that it is equal to the firm's current book value of equity and its present value of expected future residual income. Finally, the appendix states that the firm's intrinsic value is equal to the firm's present book value of equity and its present value of expected future residual operating income after it is adjusted for its present value of expected future residual interest expense (cf. [EQ A-58], [EQ A-65], and [EQ A-68]). This is knowledge already known (see Feltham & Ohlson, 1999, for a rather technical yet interesting set of derivations under stochastic interest rates).

When subjects meet the conjectures in the theory of Homo comperiens, there exist equivalent market-pricing models. Assuming homogenous preferences means that the market price of a firm is as follows.

> **Proposition 4-2:** Assuming the theory of Homo comperiens (Proposition 2-4), homogenous preferences and a mild regulatory assumption (cf. Appendix B, p. 186 for details), the market price of a firm is:
>
> $P_0 = \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}[d_{Kt}]$.

As in Appendix A, the subscript identifying the numerarie good is excluded from the subscript in Proposition 4-2 to reduce cluttering. The only difference with a market price according to above and the intrinsic value according to [EQ A-58] is that the subjective futures prices, ${}_0 p_{Kt}$, are different from the objective futures prices (hence, we have biased rational expectations) and that the subjective expected future dividend is different from the objective expected future dividend.

Another way to express the market price of a firm is as a function of the subjective expected residual net income.

> **Proposition 4-3:** Assuming the theory of Homo comperiens (Proposition 2-4), homogenous preferences, the clean surplus relationship, and a mild regulatory assumption (cf. Appendix B, p. 186 for details), the market price of a firm is: $P_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}[RI_t]$, where
>
> $\mathrm{E}_{K0}[RI_t] = \mathrm{E}_{K0}[CNI_t] - \mathrm{E}_{K0}[{}_{t-1} r_t] \cdot EQ_{t-1}$.

It is also possible to express Proposition 4-3 using accounting return on equity.

> **Corollary 4-2:** Assuming the theory of Homo comperiens (Proposition 2-4), homogenous preferences, the clean surplus relationship, and a mild regulatory assumption (cf. Appendix B, p. 188 for details), the market price of a firm is: $P_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}[{}_{t-1} RROE_t] \cdot EQ_{t-1}$, where
>
> $\mathrm{E}_{K0}[{}_{t-1} RROE_t] = \mathrm{E}_{K0}[{}_{t-1} ROE_t] - \mathrm{E}_{K0}[{}_{t-1} r]_t$.

Using the value additivity principle of Modigliani and Miller (1958) allows the market price of equity to be described as a function of residual operating income as shown below.

Proposition 4-4: Conjecturing Assuming the theory of Homo comperiens (Proposition 2-4), homogenous preferences, the clean surplus relationship, and a mild regulatory assumption (cf. Appendix B, p. 186 for details), the market price of a firm is: $P_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot E_{K0}[ROI_t] - \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot E_{K0}[RIE_t]$

where $E_{K0}[ROI_t] = E_{K0}[OI_t] - E_{K0}[{}_{t-1}r_t] \cdot NOA_{t-1}$, and $E_{K0}[RIE_t] = E_{K0}[IE_t] - E_{K0}[{}_{t-1}r_t] \cdot NFL_{t-1}$.

It is also possible to express Proposition 4-4 using accounting return on net operating assets.

Corollary 4-3: Assuming the theory of Homo comperiens (Proposition 2-4), homogenous preferences, the clean surplus relationship, and a mild regulatory assumption (cf. Appendix B, p. 189 for details), the market price of a firm is:

$P_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot E_{K0}[{}_{t-1}RRNOA_t] \cdot NOA_{t-1} - \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot E_{K0}[{}_{t-1}RNBC_t] \cdot NFL_{t-1}$, where

$E_{K0}[{}_{t-1}RRNOA_t] = E_{K0}[{}_{t-1}RNOA_t] - E_{K0}[{}_{t-1}r_t]$ and $E_{K0}[{}_{t-1}RNBC_t] = E_{K0}[{}_{t-1}NBC_t] - E_{K0}[{}_{t-1}r_t]$.

Proposition 4-3 differs from [EQ A-65] and Proposition 4-4 differs from [EQ A-68] in that the expected residual income, $E_{K0}[RI_t]$, the expected residual operating income, $E_{K0}[ROI_t]$, and the expected residual net interest expense, $E_{K0}[RIE_t]$, are the subjective and not objective.

Since this is a subjectively certain choice, it follows that the firm, as in subsection A.8.3's certain choice, uses an identical subjective market rate-of-return ${}_{t-1}r_{Kt}$ for Proposition 4-2 to Proposition 4-4 and that it is equal to the risk-free market rate-of-return.

With Proposition 4-3, Proposition 4-4, Corollary 4-2, and Corollary 4-3, it is possible to discuss what bearing they have on a firm's income, accounting rate-of-return, and risk. This is considered in section 4.5.

4.5 Homo comperiens and the firm's risk and return

This section analyzes what is expected from a firm's accounting rates-of-returns and risk when Homo comperiens is assumed. The section is divided into two subsections where subsection 4.5.1 focuses on a subjectively certain choice and subsection 4.5.2 deals with the subjectively uncertain choice.

Appendix A discusses the intrinsic values, income, and accounting rates-of-returns assuming unbiased accounting. This section and Appendix B retain the conjecture assumption of unbiased accounting.

4.5.1 Homo comperiens and the firm's rate-of-return in the subjectively certain choice

Appendix A argues that no residual income is a defining trait of no-arbitrage. Invoking the theory of Homo comperiens implies that there are innumerable arbitrage opportunities ascertaining the existence of residual income. This means that in such a market there exists subjective expected residual income, residual operating income, and residual interest expense.

In equilibrium the comprehensive net income is equal to the product of the objective market rate-of-return and the beginning-of-period equity. The comprehensive operating income is equal to the product of the objective market rate-of-return and the beginning-of-period net operating assets. Moreover, the comprehensive net interest expense is equal to the product of the objective market rate-of-return and the beginning-of-period net financial liabilities.

In a Homo comperiens setting, they are functions of the subjective market rate-of-return and of arbitrage income. Since whole market is in disequilibrium, arbitrage income can be earned within any part of the firm. Therefore:

$$\mathrm{E}_{K0}\left[CNI_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}r_t\right] \cdot EQ_{t-1} + \mathrm{E}_{K0}\left[\textit{net arbitrage income}_t\right] \qquad \text{[EQ 4-2]}$$

$$\mathrm{E}_{K0}\left[OI_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}r_t\right] \cdot NOA_{t-1} + \mathrm{E}_{K0}\left[\textit{operating arbitrage income}_t\right] \qquad \text{[EQ 4-3]}$$

$$\mathrm{E}_{K0}\left[IE_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}r_t\right] \cdot NFL_{t-1} + \mathrm{E}_{K0}\left[\textit{financial arbitrage income}_t\right] \qquad \text{[EQ 4-4]}$$

The expected net arbitrage income $\mathrm{E}_{K,0}\left[\textit{net arbitrage income}_t\right]$ in [EQ 4-2] is the sum of the arbitrage income expected in [EQ 4-3] to [EQ 4-4].

These equations can be measured using accounting rates-of-returns according to the equations below. The analysis focuses on the return on equity and the return on net operating assets since they provide the accounting explanation of the capital growth of the firm when it is either financially leveraged or financially unleveraged (cf. subsection A.8.4.2, p. 174). To focus the analysis the financial accounting rate-of-return is ignored.

From [EQ 4-2] and [EQ 4-3], it is clear that the subjective expected return on equity and the subjective expected return on net operating assets is, given the definitions in Appendix B, functions of the subjective expected market rate-of-return and of the subjective expected arbitrage rate-of-return:

$$\mathrm{E}_{K0}\left[{}_{t-1}ROE_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}r_t\right] + \mathrm{E}_{K0}\left[\textit{net arbitrage rate of return}_t\right] \qquad \text{[EQ 4-5]}$$

$$\mathrm{E}_{K0}\left[{}_{t-1}RNOA_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}r_t\right] + \mathrm{E}_{K0}\left[\textit{operating arbitrage rate of return}_t\right] \qquad \text{[EQ 4-6]}$$

Appendix B also uses the residual accounting rates-of-returns. The residual accounting rates-of-returns are in the subjectively certain choice:

$$\mathrm{E}_{K0}\left[{}_{t-1}RROE_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}ROE_t\right] - \mathrm{E}_{K0}\left[{}_{t-1}r_t\right] = \mathrm{E}_{K0}\left[\textit{net arbitrage rate of return}_t\right] \qquad \text{[EQ 4-7]}$$

$$\mathrm{E}_{K0}\left[{}_{t-1}RRNOA_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}RNOA_t\right] - \mathrm{E}_{K0}\left[{}_{t-1}r_t\right] = \mathrm{E}_{K0}\left[\textit{operating arbitrage rate of return}_t\right] \qquad \text{[EQ 4-8]}$$

Model [EQ 4-2] to [EQ 4-8] holds in expectations since the subjectively certain choice is a chimera because of limited knowledge. As the future unfolds, there can be a difference between the expectation and the outcome because the subject's expectation is based on the subjective state set and not on the objective state set, or because the subjective state-dependent consequence was not

constant. The ex post variation of [EQ 4-2] to [EQ 4-8] is therefore affected by events that the firm is ignorant of and we thus have accounting affected by unexpected income:

$$CNI_{Kt} = {}_{t-1}r_t \cdot EQ_{t-1} + \textit{net arbitrage income}_t + \textit{unexpected income}_t \qquad \text{[EQ 4-9]}$$

$$OI_t = {}_{t-1}r_t \cdot NOA_{t-1} + \textit{operating arbitrage income}_t + \textit{unexpected income}_t \qquad \text{[EQ 4-10]}$$

$$IE_t = {}_{t-1}r_t \cdot NFL_{t-1} + \textit{financial arbitrage income}_t + \textit{unexpected income}_t \qquad \text{[EQ 4-11]}$$

$${}_{t-1}ROE_t = {}_{t-1}r_t + \textit{net arbitrage rate of return}_t + \textit{unexpected rate of return}_t \qquad \text{[EQ 4-12]}$$

$${}_{t-1}RNOA_t = {}_{t-1}r_t + \textit{operating arbitrage rate of return}_t + \textit{unexpected rate of return}_t \qquad \text{[EQ 4-13]}$$

$${}_{t-1}RROE_t = {}_{t-1}ROE_t - {}_{t-1}r_t = \textit{net arbitrage rate of return}_t + \textit{unexpected rate of return}_t \qquad \text{[EQ 4-14]}$$

$${}_{t-1}RRNOA_t = {}_{t-1}RNOA_t - {}_{t-1}r_t = \textit{operating arbitrage rate of return}_t + \textit{unexpected rate of return}_t \qquad \text{[EQ 4-15]}$$

The residual return on equity and the residual return on net operating assets are in expectations equal to the arbitrage rates-of-returns obtained by the firm. When they are measured ex post, there is noise affecting them since there are occurrences that the firm is unaware of when making its choice.

Homo comperiens is endowed with the capacity to discover (cf. Definition 2-9 and its corresponding section), and it is argued in Chapter 2 that the subject, and hence the firm, through discovery makes choices on subjective action and state sets that are increasingly more similar to their objective counter parts. Given enough transactions, the subjective and the objective sets become almost identical. This implies that the limited rational choice in limit becomes almost as a perfect rational choice, with the market close to equilibrium.

This means that the market prices trend towards the Pareto optimal prices (cf. Proposition 3-2, p. 59), which implies that the subjective expected RROE [EQ 4-7] approaches zero when the numbers of transaction increase since the subjective expected ROE approaches the objective expected ROE. The same applies to the subjective expected RRNOA [EQ 4-8] and since the subjective expected RNOA approach the objective expected RNOA.

> **Proposition 4-5:** In a subjectively certain market that meets the assumptions of Homo comperiens (Proposition 2-4, Proposition 3-2), with unbiased accounting, the subjective expected RROE and RRNOA regress until, in the limit, they are zero. That is, $\lim\limits_{t \to \infty}\left(\mathrm{E}_{K0}\left[{}_{t-1}RROE_t\right]\right) = 0$, and
>
> $\lim\limits_{t \to \infty}\left(\mathrm{E}_{K0}\left[{}_{t-1}RRNOA_t\right]\right) = 0$.

The ex-post RROE and RRNOA are thus only affected by the unexpected rate-of-return. If the subjective state sets meet their objective counterpart, it implies that the unexpected rates-of-returns also diminish as the number of transactions increases. It is consequently also likely that the limit values of ex post RROE and RRNOA approach zero assuming subjective certainty.

This subsection takes as a point of departure the case where the subject and the firm perceive a certain future. That is, when they believe that they know for sure what the future payoffs are going

to be. There is a difference between ex ante and ex post even in this scenario because of limited knowledge. The following section analyses a more complete scenario in which subjectively uncertainty affects the formation of expectations.

4.5.2 The firm's rate-of-return in the objective uncertain choice

This subsection expands the problem from analyzing market pricing of firms in subjective certainty to analyzing market pricing of firms in subjective uncertainty. The analysis takes as its point of departure perfect rationality, as in the previous subsection.

Typical estimation of a firm's intrinsic value in an uncertain choice assumes constant market rate-of-return (MROR) and that it can be estimated using CAPM (e.g., Penman 2004). CAPM is a general equilibrium model using constant MROR (Ohlson 1987). Section 4.4 allows for non-constant MROR, which is similar yet different to Feltham and Ohlson (1999). Feltham and Ohlson (1999) provide intrinsic valuation models for objective uncertainty with stochastic MROR. According to these authors, it is possible to find the firm's intrinsic value using (p. 171 [DVR-f*], 172 [AVR-f*], 176-178):

$$V_0 = \sum_{t=1}^{\infty} {}_0p_t \cdot \mathrm{E}_0^*[d_t] \Leftrightarrow EQ_0 + \sum_{t=1}^{\infty} {}_0p_t \cdot \mathrm{E}_0^*[RI_t] \Leftrightarrow EQ_0 + \sum_{t=1}^{\infty} {}_0p_t \cdot \mathrm{E}_0^*[ROI_t] - \sum_{t=1}^{\infty} {}_0p_t \cdot \mathrm{E}_0^*[RIE_t]$$

The expectations operator $\mathrm{E}_0^*[\cdot]$ is different from $\mathrm{E}_0[\cdot]$ since it is a risk-adjusted expectations operator. For example, the expected dividend is the sum of the products of the state-dividend and the objective state probability. Risk-adjusted expected dividend is the sum of the products of the state-dividend and the risk-adjusted objective state probability.

A subject's risk-adjusted objective state probability for a good in a given period is equal to the product of the subject's state price for this good and period and the risk-free return for the period. The state price of the numerarie equals the marginal rate of substitution between a claim to the numerarie tomorrow and a claim to the numerarie today. The marginal rate of substitution stems from the subject's objective probability belief for the state to come true and on the claim's objective Bernoulli utility (see, e.g., Pliska (1997, p. 33-36) for more on the risk-adjusted probability function).

By using the risk-adjusted objective probabilities, it is possible to express the risk-adjusted expected dividend according to (cf. e.g. Feltham & Ohlson 1999, p. 169, 171):

$$\mathrm{E}_0^*[d_t] = \sum_{s \in \mathbb{S}} d_{st} \cdot \pi_{st}^* = \sum_{s \in \mathbb{S}} d_{st} \cdot {}_0p_{st} \cdot {}_0R_t^F \qquad \text{[EQ 4-16]}$$

where

$$\pi_{st}^* = {}_0p_{st} \cdot {}_0R_t^F$$

$${}_0R_t^F = \left(\sum_{s \in \mathbb{S}} {}_0p_{st} \right)^{-1}$$

In the equation above π^*_{st} is the risk-adjusted state probability, $_0 p_{st}$ is the state price that reflects the subject's risk aversion and beliefs, and $_0 R^F_t$ is the risk-free return on a numerarie invested at present until t.

All subjects have the same marginal rates of substitutions between saving and consumption in general equilibrium and it is therefore no loss in generality to assume homogenous preferences, i.e. the conjecture of the "representative subject".

Assuming homogenous preferences means that the risk-adjusted expected dividend can be expressed as a function of the expected dividend and a covariance term between the expected dividend and a risk adjustment index (Ohlson 1987, p. 85). The covariance term is negative for a risk-averse subject. That is,

$$\mathrm{E}^*_0[d_t] = \mathrm{E}_0[d_t] - \mathrm{cov}\left(\mathrm{E}_0[d_t], Q\right) \qquad \text{[EQ 4-17]}$$

Adjusting for risk in valuation using a negative covariance between the payoff and a risk adjustment index is an accepted part of financial theory and can be found in, e.g., Rubinstein (1976). Feltham & Ohlson (1999, p. 173) use this model for both the dividend valuation model and the residual income valuation model.

Adding additional structure to the subject's utility function can lead to a setting in which the familiar CAPM is found. The reader is referred to Ohlson (1987) for a comprehensive treatment of the theory of security valuation in a no-arbitrage setting.

By using [EQ 4-17] it is possible to express the objective risk-adjusted expected residual income, residual operating income, and residual net interest-bearing expense as follows (cf. Feltham & Ohlson (1999, p. 173) who apply [EQ 4-17] to the residual income valuation model).:

$$\mathrm{E}^*_0[RI_t] = \mathrm{E}_0[RI_t] - \mathrm{cov}\left(\mathrm{E}_0[RI_t], Q_t\right) \qquad \text{[EQ 4-18]}$$

$$\mathrm{E}^*_0[ROI_t] = \mathrm{E}_0[ROI_t] - \mathrm{cov}\left(\mathrm{E}_0[ROI_t], Q_t\right) \qquad \text{[EQ 4-19]}$$

$$\mathrm{E}^*_0[RIE_t] = \mathrm{E}_0[RIE_t] - \mathrm{cov}\left(\mathrm{E}_0[RIE_t], Q_t\right) \qquad \text{[EQ 4-20]}$$

A Pareto optimal equilibrium implies that there is no arbitrage opportunity, which means that there are no strictly positive NPV investments. Unbiased accounting and zero NPV therefore imply that the LHS in [EQ 4-18] to [EQ 4-20] are zero (e.g., Ohlson (2003) for a discussion of residual income and NPV).

Given the previous definitions of the different alternative residual incomes models, this enables the following relations:

$$\mathrm{E}_0[CNI_t] = \mathrm{E}_0\left[_{t-1} r_t\right] \cdot EQ_{t-1} + \mathrm{cov}\left(\mathrm{E}_0[RI_t], Q_t\right) = \mathrm{E}_0\left[_{t-1} r_t\right] \cdot EQ_{t-1} + total\ risk\ premium_t \qquad \text{[EQ 4-21]}$$

$$\mathrm{E}_0[COI_t] = \mathrm{E}_0\left[_{t-1} r_t\right] \cdot NOA_{t-1} + \mathrm{cov}\left(\mathrm{E}_0[ROI_t], Q_t\right) = \mathrm{E}_0\left[_{t-1} r_t\right] \cdot NOA_{t-1} + operating\ risk\ premium_t \qquad \text{[EQ 4-22]}$$

$$\mathrm{E}_0\left[CIE_t\right] = \mathrm{E}_0\left[{}_{t-1}r_t\right] \cdot NIBL_{t-1} + \mathrm{cov}\left(\mathrm{E}_0\left[RIE_t\right], Q_t\right) = \mathrm{E}_0\left[{}_{t-1}r_t\right] \cdot NIBL_{t-1} + \textit{financial risk premium}_t \quad \text{[EQ 4-23]}$$

This means that the subject expects a firm to pay off at par with a risk-free investment for any period, as well as an additional premium to cover for the objective uncertainty. It is possible to rephrase this using objective expected ROE and objective expected RNOA.

$$\mathrm{E}_0\left[{}_{t-1}ROE_t\right] = \mathrm{E}_0\left[{}_{t-1}r_t\right] + \textit{total risk premium}_t \qquad \text{[EQ 4-24]}$$

$$\mathrm{E}_0\left[{}_{t-1}RNOA_t\right] = \mathrm{E}_0\left[{}_{t-1}r_t\right] + \textit{operating risk premium}_t \qquad \text{[EQ 4-25]}$$

The risk premium for [EQ 4-24] is $\mathrm{cov}\left(\mathrm{E}_0\left[RI_t\right], Q_t\right) \cdot EQ_{t-1}^{-1}$ and it is different compared with the risk premium in [EQ 4-25], which is $\mathrm{cov}\left(\mathrm{E}_0\left[ROI_t\right], Q_t\right) \cdot EQ_{t-1}^{-1}$ since the covariances do not use the same residual income variables.

The risk premium in [EQ 4-24] is the compensation that the subject requires because of the objective operating uncertainty, i.e. the objective uncertainty that the firm faces if it is without financial leverage. The risk premium in [EQ 4-24] is a combination of the objective operating risk and the objective financial risk. It thus follows that the objective expected ROE and RNOA differ in a financially leveraged firm. Another subtlety in the risk premiums is that they are non-constant: the risk premiums must be constant if CAPM is applied since it is a one-period equilibrium model.

It is useful to note that in a perfect rational choice, [EQ 4-24] and [EQ 4-25] imply, given the definitions of the objective expected RROE and RRNOA, that they are functions of the objective risk premiums:

$$\mathrm{E}_0\left[{}_{t-1}RROE_t\right] = \mathrm{E}_0\left[{}_{t-1}ROE_t\right] - \mathrm{E}_0\left[{}_{t-1}r_t\right] = \textit{total risk premium}_t \qquad \text{[EQ 4-26]}$$

$$\mathrm{E}_0\left[{}_{t-1}RRNOA_t\right] = \mathrm{E}_0\left[{}_{t-1}RNOA_t\right] - \mathrm{E}_0\left[{}_{t-1}r_t\right] = \textit{operating risk premium}_t \qquad \text{[EQ 4-27]}$$

Appendix A finds that the objective expected RROE and RRNOA must be zero when objective certainty is present, i.e. conjecturing perfect rationality and certainty. This section finds that, in the presence of objective uncertainty, the objective expected RROE and RRNOA are no longer expected to be zero. Rather, they are equal to the corresponding non-constant objective risk premiums.

4.5.3 Homo comperiens and the firm's rate-of-return in the subjectively uncertain choice

A similar analysis to that in 4.5.2 can be performed on choice that meets the assumptions of Homo comperiens. This subsection assumes homogenous preferences, an assumption that enables a discussion of a market price rather than of a unique price of the firm for each subject. However, since the subjects' only have access to their limited action and state set, the market price is a disequilibrium price. Therefore, there is a difference between the market price and the intrinsic value.

The subject, acting according to the conjectures of Homo comperiens, therefore finds the firm's market price with the use of the risk-free market rate-of-return and with the use of risk-adjusted subjective expected payoffs:

$$P_0 = \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}^* [d_t] \Leftrightarrow EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}^* [RI_t] \Leftrightarrow EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}^* [ROI_t] - \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}^* [RIE_t]$$

The key difference between section 4.4 and the perfect rational choice in uncertainty (subsection 4.5.2) centers on the risk-adjusted expectations operator and the subjective price.

The objective risk-adjusted expectations operator $\mathrm{E}_0^* [\cdot]$ i.e., assuming perfect rationality, is based on the objective state prices for the numerarie and on the objective risk-free return. The objective state prices equal the objective marginal rate of substitution between a claim to the numerarie for that period and a claim to the numerarie today. Moreover, equilibrium forces all subjects to have the same objective marginal rate of substitution between savings and consumption.

The risk-adjusted expectations operator $\mathrm{E}_{K0}^* [\cdot]$ is based, instead, on the subjective state prices for the numerarie and on the subjective risk-free return. The subjective state prices equal the subjective marginal rate of substitution between a claim to the numerarie for that period and a claim to the numerarie today (cf. A.6.1 on page 157).

This means that the difference between $\mathrm{E}_0^* [\cdot]$ and $\mathrm{E}_{K0}^* [\cdot]$ rests on the difference between the objective and the subjective marginal rate of substitution between savings and consumption. The objective marginal rate of substitution is the same across the population but the subjective marginal rate of substitution is unique for each subject. It depends on the subject's unique perception of the available actions and on the subject's subjective state set. These, in turn, depend on the subject's limited knowledge, which also depends on the subject's unique transaction pattern.

Further, recall from [EQ 4-16] that the objective risk-free return for a period is a function of the sum of the objective state prices for the numerarie in that period (Feltham & Ohlson 1999, p. 168-169). A corollary in the subjective uncertain choice is that the subjective risk-free return for a period is a function of the subjective state prices for the numerarie in that period. In addition, this implies that the subjective risk-free return is affected by the subjective marginal rate of substitution between savings and consumption, which depends on limited knowledge of actions and states.

It is therefore possible to run a similar analysis to [EQ 4-17]—[EQ 4-25], assuming that the effect of arbitrage opportunities enters in the same way as in subsection 4.5.1. Such an analysis gives that the subjective expected ROE and RNOA are functions of the subjective risk-free rate-of-return, a risk premium, and arbitrage rate-of-return. That is,

$$\mathrm{E}_{K0} \left[{}_{t-1} ROE_t \right] = \mathrm{E}_{K0} \left[{}_{t-1} r_t \right] + \mathrm{E}_{K0} \left[\textit{net arbitrage rate of return}_t \right] + \mathrm{E}_{K0} \left[\textit{total risk premium}_t \right] \qquad \text{[EQ 4-28]}$$

$$\mathrm{E}_{K0}\left[{}_{t-1}RNOA_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}r_t\right] + \mathrm{E}_{K0}\left[\textit{operating arbitrage rate of return}_t\right] + \mathrm{E}_{K0}\left[\textit{operating risk premium}_t\right] \qquad \text{[EQ 4-29]}$$

The subjective expected RROE and RRNOA is a function of the respective arbitrage rates-of-returns and subjective risk premiums. That is, I define subjective expected RROE and RRNOA as:

$$\mathrm{E}_{K0}\left[{}_{t-1}RROE_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}ROE_t\right] - \mathrm{E}_{K0}\left[{}_{t-1}r_t\right] \qquad \text{[EQ 4.30]}$$

$$\mathrm{E}_{K0}\left[{}_{t-1}RRNOA_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}RNOA_t\right] - \mathrm{E}_{K0}\left[{}_{t-1}r_t\right] \qquad \text{[EQ 4.31]}$$

Using [EQ 4.30] in [EQ 4-28] and using [EQ 4.31] in [EQ 4-29] gives the subjective expected RROE and RRNOA to be functions of the risk premiums and arbitrage profits.

$$\mathrm{E}_{K0}\left[{}_{t-1}RROE_t\right] = \mathrm{E}_{K0}\left[\textit{net arbitrage rate of return}_t\right] + \mathrm{E}_{K0}\left[\textit{total risk premium}_t\right] \qquad \text{[EQ 4-32]}$$

$$\mathrm{E}_{K0}\left[{}_{t-1}RRNOA_t\right] = \mathrm{E}_{K0}\left[\textit{operating arbitrage rate of return}_t\right] + \mathrm{E}_{K0}\left[\textit{operating risk premium}_t\right] \qquad \text{[EQ 4-33]}$$

Subsection 4.3 finds that in subjective certainty the subjective expected RROE and RRNOA trend towards zero as the number of transactions mounts. The argument is as follows: The subject learns through discovery, which increases the subjective action and state sets until it in the limit almost reaches the objective action and state sets.

Define the risk-adjusted subjective expected RROE and RRNOA as:

$$\mathrm{E}_{K0}^{*}\left[{}_{t-1}RROE_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}RROE_t\right] - \mathrm{E}_{K0}\left[\textit{total risk premium}_t\right] \qquad \text{[EQ 4.34]}$$

$$\mathrm{E}_{K0}^{*}\left[{}_{t-1}RRNOA_t\right] = \mathrm{E}_{K0}\left[{}_{t-1}RRNOA_t\right] - \mathrm{E}_{K0}\left[\textit{operating risk premium}_t\right] \qquad \text{[EQ 4.35]}$$

Substituting [EQ 4.34] into [EQ 4-32] and substituting [EQ 4.35] into [EQ 4-33], and rearranging show that [EQ 4.34] and [EQ 4.35] are non-zero because of the arbitrage rate-of-return. A key difference between perfect rationality and limited rationality is therefore that the latter produces non-zero arbitrage rates-of-returns that manifest themselves as non-zero risk-adjusted subjective expected RROE and RRNOA.

Allowing the subjects to learn through discovery means that the risk-adjusted subjective expected RROE and RRNOA trend towards zero. They trend toward zero since the limited action and state sets trend towards the objective sets that imply that the market trends towards a Pareto optimal equilibrium. That is,

$$\lim_{t\to\infty}\left(\mathrm{E}_{K0}\left[{}_{t-1}RROE_t\right]\right) = \mathrm{E}_{K0}\left[\textit{total risk premium}_t\right] \qquad \text{[EQ 4-36]}$$

$$\lim_{t\to\infty}\left(\mathrm{E}_{K0}\left[{}_{t-1}RRNOA_t\right]\right) = \mathrm{E}_{K0}\left[\textit{operating risk premium}_t\right] \qquad \text{[EQ 4-37]}$$

Due to the analysis above, I pose these two propositions.

Proposition 4-6: In a market that meets the conjectures of the theory of Homo comperiens (Proposition 2-4), and with unbiased accounting, there exists non-zero risk-adjusted subjective expected RROE and RRNOA because of arbitrage opportunities. That is, $\mathrm{E}_{K0}^{*}\left[{}_{t-1}RROE_t\right] = \mathrm{E}_{K0}\left[\textit{net arbitrage rate of return}_t\right] \neq 0$, and $\mathrm{E}_{K0}^{*}\left[{}_{t-1}RRNOA_t\right] = \mathrm{E}_{K0}\left[\textit{operating arbitrage rate of return}_t\right] \neq 0$.

Proposition 4-7: In a market that meets the conjectures of the theory of Homo comperiens (Proposition 2-4, Proposition 3-2) and with unbiased accounting the limit values of risk-adjusted subjective expected RROE and RRNOA are zero. That is: $\lim_{t\to\infty}\left(\mathrm{E}^*_{K0}\left[_{t-1}RROE_t\right]\right)=0$, and $\lim_{t\to\infty}\left(\mathrm{E}^*_{K0}\left[_{t-1}RRNOA_t\right]\right)=0$.

With the propositions above, I have traversed from proposing a theory of choice in the second chapter all the way into market-based accounting in a way that enables me to test the theory of Homo comperiens. However, such tests need operationalization of the risk-adjusted subjective expected residual accounting rates-of-returns. This is the topic of the next two chapters. They make necessary operationalizations and pose testable hypotheses.

Proposition 4-6 and Proposition 4-7 imply there exists expected arbitrage residual income and that such residual income decreases. A pertinent question is then how my market-pricing models with discovery that use residual income differ from the models proposed by Ohlson (1995) and Feltham and Ohlson (1995).

It should be noted that I argue in A.8.4.3 (p. 174) that Ohlson's model and Ohlson and Feltham's model disallow the existence of expected residual income when the accounting bias is removed, i.e. they measure expected no-arbitrage residual income. This means that empirical investigations that use Ohlson's and Feltham and Ohlson's models critically assume no-arbitrage and zero expected residual income of the sort that I model in here. Indeed, Feltham and Ohlson appreciate this fact since they write (196, p. 209-210) "The resulting book value and accounting earning numbers are such that, for all periods, the book rate of return equals the cost of capital; that is, the firm reports normal profits for all periods," in a situation where we have unbiased accounting.

Since Ohlson's and Feltham and Ohlson's models assume no-arbitrage residual income, the implication is that empirical assessments based on their models can only purport to explain residual income because of accounting earnings only (i.e. when NPV is zero for all firms) and cannot purport to explain residual income based on economic earnings (i.e. when NPV is non-zero for at least one firm). This is something also noted by Lo and Lys (1999, p. 348), Lundholm (1995, p. 761), and Beaver (2002, p. 458).

4.6 Summary

Chapter 4 discusses a firm's market price in a market, assuming Homo comperiens. The firm's market price is then different from the firm's intrinsic value because the market is in disequilibrium. The chapter poses the following propositions and corollaries.

Proposition 4-2: Assuming the theory of Homo comperiens (Proposition 2-4), homogenous preferences and a mild regulatory assumption (cf. Appendix B, p. 186 for details), the market price of a firm is:

$$P_0 = \sum_{t=1}^{\infty} {}_0p_{Kt} \cdot \mathrm{E}_{K0}\left[d_{Kt}\right].$$

Proposition 4-3: Assuming the theory of Homo comperiens (Proposition 2-4), homogenous preferences, the clean surplus relationship, and a mild regulatory assumption (cf. Appendix B, p. 186 for details), the market price of a firm is: $P_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}[RI_t]$, where

$\mathrm{E}_{K0}[RI_t] = \mathrm{E}_{K0}[CNI_t] - \mathrm{E}_{K0}[{}_{t-1}r_t] \cdot EQ_{t-1}$.

Proposition 4-4: Conjecturing Assuming the theory of Homo comperiens (Proposition 2-4), homogenous preferences, the clean surplus relationship, and a mild regulatory assumption (cf. Appendix B, p. 186 for details), the market price of a firm is: $P_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}[ROI_t] - \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}[RIE_t]$ where $\mathrm{E}_{K0}[ROI_t] = \mathrm{E}_{K0}[OI_t] - \mathrm{E}_{K0}[{}_{t-1}r_t] \cdot NOA_{t-1}$, and $\mathrm{E}_{K0}[RIE_t] = \mathrm{E}_{K0}[IE_t] - \mathrm{E}_{K0}[{}_{t-1}r_t] \cdot NFL_{t-1}$.

Corollary 4-2: Assuming the theory of Homo comperiens (Proposition 2-4), homogenous preferences, the clean surplus relationship, and a mild regulatory assumption (cf. Appendix B, p. 188 for details), the market price of a firm is: $P_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}[{}_{t-1}RROE_t] \cdot EQ_{t-1}$, where

$\mathrm{E}_{K0}[{}_{t-1}RROE_t] = \mathrm{E}_{K0}[{}_{t-1}ROE_t] - \mathrm{E}_{K0}[{}_{t-1}r]_t$.

Corollary 4-3: Assuming the theory of Homo comperiens (Proposition 2-4), homogenous preferences, the clean surplus relationship, and a mild regulatory assumption (cf. Appendix B, p. 189 for details), the market price of a firm is:

$P_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}[{}_{t-1}RRNOA_t] \cdot NOA_{t-1} - \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}[{}_{t-1}RNBC_t] \cdot NFL_{t-1}$, where

$\mathrm{E}_{K0}[{}_{t-1}RRNOA_t] = \mathrm{E}_{K0}[{}_{t-1}RNOA_t] - \mathrm{E}_{K0}[{}_{t-1}r_t]$ and $\mathrm{E}_{K0}[{}_{t-1}RNBC_t] = \mathrm{E}_{K0}[{}_{t-1}NBC_t] - \mathrm{E}_{K0}[{}_{t-1}r_t]$.

The corollaries above make it possible to form propositions that, when operationalized, are testable.

In a subjectively certain choice the firm has an opportunity to earn a subjective expected residual rate-of-return on equity (RROE) and a residual rate-of-return on net operating assets (RRNOA). These opportunities exist because the market is an arbitrage market. However, the chapter also proposes that the opportunities dissolve because of the subjects' propensity to discover.

Proposition 4-5: In a subjectively certain market that meets the assumptions of Homo comperiens (Proposition 2-4, Proposition 3-2), with unbiased accounting, the subjective expected RROE and RRNOA regress until, in the limit, they are zero. That is, $\lim_{t \to \infty} \left(\mathrm{E}_{K0}[{}_{t-1}RROE_t] \right) = 0$, and

$\lim_{t \to \infty} \left(\mathrm{E}_{K0}[{}_{t-1}RRNOA_t] \right) = 0$.

The proposition above is not testable since it takes place in subjective certainty and hence its alternative formed in a subjectively uncertain choice. The propositions in subjective uncertainty are:

Proposition 4-6: In a market that meets the conjectures of the theory of Homo comperiens (Proposition 2-4), and with unbiased accounting, there exists non-zero risk-adjusted subjective expected RROE and RRNOA because of arbitrage opportunities. That is, $\mathrm{E}_{K0}^{*}[{}_{t-1}RROE_t] = \mathrm{E}_{K0}[\textit{net arbitrage rate of return}_t] \neq 0$, and $\mathrm{E}_{K0}^{*}[{}_{t-1}RRNOA_t] = \mathrm{E}_{K0}[\textit{operating arbitrage rate of return}_t] \neq 0$.

Proposition 4-7: In a market that meets the conjectures of the theory of Homo comperiens (Proposition 2-4, Proposition 3-2) and with unbiased accounting the limit values of risk-adjusted subjective expected RROE and RRNOA are zero. That is: $\lim_{t\to\infty}\left(\mathrm{E}^{*}_{K0}\left[_{t-1}RROE_{t}\right]\right)=0$, and $\lim_{t\to\infty}\left(\mathrm{E}^{*}_{K0}\left[_{t-1}RRNOA_{t}\right]\right)=0$.

CHAPTER 5—THE EMPIRICAL DATA AND THE FINANCIAL STATEMENTS

An operationalization of the balance sheet, the income statement, and of the clean surplus relation

5.1 Introduction

The previous chapter poses propositions based on Homo comperiens that uses accounting measures. This chapter operationalizes the empirical financial statements such that they can be used to measure the components to ROE and RNOA, which are needed for hypothesis testing of the previous chapter's propositions.

ROE is defined in [EQ A-71] (cf. also [EQ B-39]) and RNOA is defined in [EQ A-76] (cf. also [EQ B-42]). Subsection A.8.3.2 on page 172 provides a further disaggegation and classification of the components to ROE and RNOA. This chapter provides empirical content to these definitions and classifications.

The definitions of the accounting rates-of-returns are directly connected to theory. This chapter's operationalizations, together with Appendix D—Appendix H, attempt to determine if this connection continues to exist. Having operationalized accounting rates-of-returns that maintain their theoretical connections is not part of the mainstream in empirical market-based accounting research. Empirical market-based accounting research using accounting rate-of-return often does not pay close attention to the definitions and to the operationalization of the variables used. The research often misses at least some of the dirty-surplus problem, the effects of extraordinary items, and the effects that are due to discontinued operations. Other problems that often exist are measurement inconsistencies (inconsistent definition of numerators and denominators in the accounting rates-of-returns), and timing inconsistency (uses end-of-period balance sheet values).

The problems seem to very frequent in market-based accounting research (Albrecht et al. 1977; Ball & Watts 1972; Beaver 1970; Brooks & Buckmaster 1976; Fama & French 2000; Lookabill 1976; Ou & Penman 1989a,b; Penman 1991; Watt & Leftwich 1977). Notable exceptions are Penman & Zhang (2000) and Nissim & Penman (2001) who pay close attention to the definitions and the operationalization of the variables.

Empirical industrial economics research also uses accounting rates-of-returns (Baginski et al. 1999; Brown & Ball 1967; Buzzell & Gale 1987; Brozen 1970; Cubbin & Geroski 1987; Jacobsen 1988; Jacobson & Aaker 1985; Geroski & Jacquemin 1988; Lev 1983; Lev & Sougiannis 1996; Mueller 1977; Mueller 1990; Waring 1996). This research has the same problems as the market-based

accounting research although the problems are even worse in empirical industrial economics research.

The chapter first presents the empirical data, followed by the operationalization of the clean surplus relationship, the balance sheet, and finally, the operationalization of the income statement.

5.2 A description of the empirical data

The total sample of the present thesis covers 33,251 firm-year observations of which 25,245 are usable[14]. The empirical data are acquired from Statistics Sweden (SCB). It covers the period 1977—1996 but data only up to 1994 are used in this research. The data consist of active limited companies in Sweden's manufacturing industry.[15, 16]

The study uses Swedish accounting data rather than, e.g., US data for two reasons. Focusing on Swedish accounting data suggests that I will have a better understanding of the accounting principles and standards that allows me to operationalize the variables in a more valid manner. The second reason is that there exist no such study of this scope on Swedish data but there exist quantitative accounting studies on a similar scope using, e.g., US accounting data.

The data cover a historical period. Using more up to date data would compromise the comparability since there was a major change in the collection and classification of the accounting data at the end of the studied period. The fact that the accounting data are old does not materially affect this research since I study a general phenomenon and thus it is not influenced by the choice of time.

To each firm-year observation is attached a balance sheet and an income statement, as well as additional information (e.g., a standard industry classification code specified to the 4th digit). Each firm-year observation has between 80—96 specified line items. See Appendix G for a full specification of the variables in the empirical data.

The data do not cover the consolidated financial statements but focus on the financial statements from the subject legal entities. A focus on legal entities rather than groups opens for the pos-

---

[14] A usable firm-year observation is an observation that is structurally stable and that is not imputed. See Appendix C for a description of the process of finding usable firm data from the total sample.

[15] The focus on the Swedish manufacturing industry serves several purposes. First, it takes heed to Bernard's (1989) call for intra-industry research in accounting, and second, it takes into consideration McDonald and Morris' (1985) caution against inter-industry financial ratio analysis. In addition, because of a lack of comparable accounting standards, an international comparison is avoided.

[16] Data from 1977—1978 use the whole population of limited Swedish manufacturing firms (LIMFs) having more than 50 employees and a sample from those LIMFs having less than 50 employees. The sampling method was designed by SCB to keep the same firms in the sample over the years (Företagen 1978, p. 16-17). Between 1979 and 1994, the whole population of LIMFs having more than 20 employees is used (Företagen 1994, p. 6). During the same period, a sample of LIMFs having less than 20 employees is also used. In the period from 1995 to 1996, all LIMFs having more than 10 employees are in the sample, but LIMFs having less than 10 employees are still sampled (Erikson 2003, p. 2).

In this research the manufacturing industry is defined according to the standard industry classification from 1969 (SNI69) as SNI-code 38. From 1992, the new standard for industry classification, SNI92, is used. The old SNI-code 38 corresponds to the new SNI-codes 28—35. The re-definition of the manufacturing industry was developed in dialogue with Statistics Sweden (SCB), which provided the raw data.

sibility of errors because of consolidation issues[17]. However, these problems are small in comparison with the benefit of having access to such a large database. I conjecture that, e.g., transfer pricing errors even out as the empirical analysis focuses on several firms. Group contributions are operationalized as part of dividends and hence so do not affect the analysis.

SCB's database is designed to meet the need of the national accounts in Sweden. Because of its importance for economic statistics in Sweden, the quality of the information in the database is high. I tested the data supplied by SCB for consistency at the point of delivery and it revealed hardly any errors; errors that were found were not of a material size.

Erikson (2003) describes how SCB screen data collected from 1995. SCB first makes a consistency check that requires that the income statement and the balance sheet must sum up. If the firm does not fulfill the consistency check, the income statement and the balance sheet are either adjusted or considered as a non-response and have their data imputed. All large firms are manually adjusted. Smaller firms are automatically corrected or left as non-response and imputed. The second screening is designed to find extreme values and to validate single indicators. All firms that pass the first step and that have at least one potential error according to the second screening are manually checked. Finally, a model for changes in equity (EQ) is applied, and when its result is not good enough to estimate the changes in EQ, a manual check is introduced.

Firms that SCB classifies as non-response after the first screening procedure are imputed.[18] SCB's imputation either uses the firm's last year data or uses the average values for the relevant SNI-code and size class. Data from the most detailed level of SNI-code are used in the imputation process. I eliminate imputed firms using a method described in Appendix C, which reports the method applied for tracing firms and classifying them as usable.

To enable tracking of firms across years SCB has provided me with an extract from its database that also includes the firms' names, organization numbers, and local unit information. This enables me to detect, e.g., mergers, acquisitions, and divestments of whole or parts of firms, as well as name changes and changes of organization numbers.

---

[17] These issues can emanate from, e.g., transfer pricing, different measurement techniques in different legal entities for assets and liabilities, accounting for foreign subsidiaries, etc.

[18] For 1996 and SNI 10 to 37 and for all types of legal entity (from which the supplied raw data are a strict subset) imputation was carried out on 13.2 percent of all the firms. That accounted for 1.7 percent of the total sales for this group or 2.3 percent of the total employees. (Nv 11 SM9801:5)

In 1994, 16.7 percent of all the firms were imputed, accounting for 1.7 percent of sales and 2.8 percent of the employees. (Företagen, 1994:86)

In 1978 (SNI-code 3), 15 percent of the firms were imputed. Those firms represented 2.6 percent of total sales and 3.6 percent of total number of employees. (Företagen, 1978:19)

The vast majority of the imputations over the years 1977 to 1994 make use of the last year's accounting data of the firm instead of the industry average. E.g., only 1 percent of the imputed 16 percent in 1978 was based on industry averages. For 1996, representing the new method introduced from 1995 the relation was higher (6.3%/13.2%).

In section 5.3 the information from SCB's is operationalized to meet the classifications in subsection A.8.3.2 on page 172. Since the supplied income statements and balance sheets do not meet the clean surplus relationship, the analysis from the previous chapter is also operationalized. This determines that the firms in the database report comprehensive net income (CNI).

5.3 Operationalization of the financial statements

Transcending from theory to practice implies several interpretive problems that go beyond the problems with having dirty-surplus conservative accounting. Since the data span almost 20 years, they are also exposed to introductions, changes, and scraping of GAAP. This is not unique for this thesis since any study that faces time-series of accounting data is inevitably exposed to the problem. Associated to introduction and changes of accounting standards is also the problem of inconsistent implementation of the accounting standards across firms.

The problem with introductions, changes, and scraping of GAAP can be divided into two parts. There is at first the problem of changing recognition and measurement rules. To solve this problem requires individual adjustments of the financial reports for all firm-years, which is clearly unfeasible. This problem is therefore acknowledged but it does not induce action. However, when changing recognition and measurement rules creates dirty-surplus accounting, it is captured by [EQ 5-10].

The second problem focuses on changes in the classification of assets and liabilities in the financial reports and the corresponding revenues and expenses. Those who use, e.g., Compustat database or Datastream do not face this problem since the database designers have already addressed it. This thesis uses information from SCB and designs its own database. The implementations issues are discussed in this section and accompanying appendices. Whenever possible a focus on substance rather than form guides the operationalization.

The operationalization of the financial statements is discussed below in three subsections. First, follows the operationalization of the clean surplus relationship, then the operationalization of the balance sheet, and finally, the operationalization of the income statement. The operationalization of the income statement is to some extent driven by the operationalization of the balance sheet.

5.3.1 Operationalization of the clean surplus relationship, book value of equity, and paid net dividends

The clean surplus relationship is the relationship between the change in the balance sheet and the income statement. Its importance means that it needs to be operationalized with care. This section reports on the operationalization.

Appendix A uses unbiased accounting (i.e. an accounting system that is defined such that the book value of assets and liabilities yields accounting rates-of-returns equal to the internal rates-of-returns) that assumes that the clean surplus relation holds.

When theory meets practice, I therefore expose the models to accounting rules that do not meet the conditions in Appendix A. Comprehensive income thus equals that period's change in the book value of EQ and any net dividend paid by the firm to the owner. This means that an operationalization of the clean surplus relationship implies operationalizations of CNI (CNI) and of net dividends.

Swedish firms do not report CNI so it must be inferred from the clean surplus relationship. By defining, operationalizing, and measuring the net dividends paid to the owner and the change in the book value of EQ, the firm's CNI can be inferred. Using the definition of the clean surplus relationship in [EQ A-59] on page 172, the CNI for a period is defined as a function of the change in EQ and the net dividends (d) that was paid to the owners during that period:

$$CNI_t = [EQ_t - EQ_{t-1}] + d_t \qquad \text{[EQ 5-1]}$$

When the CNI is compared with the adjusted reported net income (NI) dirty-surplus accounting can be identified as the discrepancy between the two income measures[19] (see also [EQ 5-10]).

It is necessary to operationalize the book value of EQ and the paid net dividends to have the complete operationalized clean surplus relationship (this is done below).

The book value of EQ is usually defined unambiguously since it is part of the firm's balance sheet. However, in Sweden a firm can reduce its taxes by making appropriations (APR) that reduce the taxable income, where the cumulative appropriations are reported in the balance sheet as an untaxed reserve.

The use of appropriations to manipulate taxable income implies that the untaxed reserve consists of deferred tax and an EQ proportion of the untaxed reserve (EQUTR). The EQUTR is classified here as part of the firm's EQ. This means that the book value of EQ is in this thesis:

$$EQ_t = eq_t + EQUTR_t \qquad \text{[EQ 5-2]}$$

where eq is the reported book value of EQ and EQUTR is the EQ proportion of the untaxed reserve. The database operationalization of the reported book value of EQ is found in Appendix F (p. 205).

The EQ proportion of the untaxed reserve is identified using the full tax method, which is used by, e.g., Runsten (1998). That is,

$$EQUTR_t = [1 - \text{tax rate}_t] \cdot UTR_t \qquad \text{[EQ 5-3]}$$

---

[19] Adjusted reported NI is equal to reported NI adjusted for the equity proportion of the current period's appropriations. The adjustment follows the full tax method, which is an accepted principle in Sweden. It has been used by, e.g., Runsten (1998).

UTR is the untaxed reserve and the tax rate is marginal tax. For further specification of, e.g., the marginal tax rate, see subsection 5.3.2. Appendix D (p. 195) provides the database operationalizations of the untaxed reserve.

The net dividends paid summarize the transactions between the owners and the firm during the year. This means that it includes dividends paid (PDIV), share issues (ISSUE) during the year. Since the database uses legal entities, it is also affected by group contributions (GCs). GCs are also classified as part of the paid net dividends. That is,

$$d_t = PDIV_t - GC_t - ISSUE_t$$ [EQ 5-4]

Dividends paid are operationalized as variable UTD1 in the database. Operationalization of the group contribution is given in Appendix D. The share issue is operationalized as variable UTD3 in the database. Appendix H (p. 209) provides a specification of all variables in the database.

With these operationalizations in place, it is possible to measure ROE but it is also necessary to be able to measure RNOA, which requires an operationalization of net operating assets (NOA) and of comprehensive operating income (OI). These operationalizations amount to operationalizations of the firm's balance sheet and income statement, which is done next.

5.3.2 Operationalization of the balance sheet

According to [EQ A-67] in subsection A.8.3.2, the firm's balance sheet equation uses EQ, NOA, and net financial liabilities (NFL) as below:

$$EQ_t = NOA_t - NFL_t$$ [EQ A-67]

In this subsection NOA is first disaggregated another step so that it is the sum of operating assets (OA) and of operating liabilities (OL):

$$NOA_t = OA_t - OL_t$$ [EQ 5-5]

An OA is an asset that does not generate explicit or implicit interest income and OL are liabilities that do not carry explicitly or implicitly any interest expenses.

NFL are disaggregated into financial liabilities (FL) and financial assets (FA):

$$NFL_t = FL_t - FA_t$$ [EQ 5-6]

As a corollary to the definitions of OA and OL are the financial assets. These are assets that generate explicit or implicit financial revenue and financial liabilities are those liabilities that carry explicit or implicit interest expenses.

This subsection provides empirical content to the four components of NOA and NFL.

The NOA consist only of the assets that are financed by the investors. The investors contribute with EQ and with NFL. With the classifications above, it is possible to view a firm's net operating asset as financed from two investment sources (the debt market and the EQ market). Conse-

quently, this follows the tradition of Modigliani and Miller (1958) and allows for the application of the value additivity principle.

The empirical balance sheets are converted to fit the classifications above since the existing financial reporting follows the above classification. The operationalization is carried out for each segment of the period between 1977 and 1996 in which the financial reports classification system is fixed. In addition, the operationalization is carried out such that it facilitates longitudinal comparisons.

The conversion of the empirical balance sheets to the classification according to [EQ 5-5] and [EQ 5-6] is made in two steps. This subsection discusses the first step and focuses on identifying stable assets and stable liability classes in the empirical balance sheets, as well as the assignment of those to the classification scheme. The second step concerns the assignment of specific line items in the database to this sections' classification scheme. This step is reported in Appendix E (p. 197).

Below follows a table with a summary of the identification of the stable assets and liability classes in the empirical balance sheets, as well as their assignment to the classification scheme.

THE BALANCE SHEET	
NET OPERATING ASSETS (NOA)	**EQUITY (EQ)**
Operating assets (OA)	Share capital
Inventories and advance payments	Restricted reserves
Prepaid expenses and accrued income	Retained earnings
Intangible assets	Reported net income
Land and buildings	Equity proportion of the untaxed reserve
Plant and equipments	**NET FINANCIAL LIABILITIES (NFL)**
Construction in process and advance payments for tangible assets	*Financial liabilities (FL)*
Shares and participations in group companies	Accounts payable
Operating liabilities (OL)	Advance payments from customers
Deferred tax proportion of the untaxed reserve	Other current liabilites
Provision for taxes	Pension liabilities
	Other long term liabilities
	Financial assets (FA)
	Accounts receivables
	Cash and bank balances
	Other current receivables
	Other long-term receivables
	Bonds and other securities
	Shares and participations in other firms

Table 5-1: A specification of the balance sheet's components.

Liabilities are broadly divided into financial liabilities and OL. A financial liability is a liability to which is attached a legal obligation to pay interest to the lender, whereas operating liability is a liability that is not interest-bearing.

The financial liability can also be a liability to a lender who does not charge an explicit interest rate. This is the case with, e.g., accounts payable, where the supplier lends money to the customer and usually does not charge interest. Such a loan has an opportunity cost whom the lender pre-

sumably adds to the price of the goods. This suggests that the firm pays interest to its supplier. However, it is not accounted for as an interest expense but as part of the cost of goods sold.

To have unbiased rates-of-returns measures require the classification of the accounts payables as financial liabilities and that its implicit interest rate is identified and classified as an interest expense. This is done in subsection 5.3.3 where the income statement is operationalized.

Advance payments from customers are also considered as an implicit loan and thus treated in a similar manner. For further information, see subsection 5.3.3.

The pension liability is also classified as a financial liability despite the fact that SCB assigns the liability's interest expense to operating cost. The reclassification follows the same method as for accounts payable. For further information, see subsection 5.3.3.

This means that among the liabilities only deferred taxes and provision for taxes are classified as non-interest-bearing liabilities for reasons discussed below.

In Sweden, firms are allowed to reduce its taxable income in order to reduce income taxes paid. This is done using appropriations. The firms debit the untaxed reserve account, which is an interest-free liability. Had the appropriation not existed, the firm would have to pay tax equal to the marginal tax on the appropriation. The untaxed reserve is normally dissolved through the income statement. Thus, the firm must pay tax on the reduction of the untaxed reserve. Operationalizations of the appropriations and the untaxed reserve are found in Appendix F.

Using the full-tax method, the untaxed reserve is divided into a deferred tax proportion and an EQ proportion. E.g., Runsten (1998) uses this method, where the deferred tax is estimated as the product of the marginal tax rate and the untaxed reserve. The EQ proportion is what remains of the untaxed reserve after the deferred tax has been removed.

In some research (e.g., Hamberg 2000) it is suggested that the untaxed reserve should be classified as EQ. By using arguments such as in an inflationary market, the untaxed reserve will grow in size. What is typical for an untaxed reserve is, however, that it can only be withheld from taxation for a limited time. That means that income tax is eventually paid on the untaxed reserve, except in those situations where the firm makes a loss, which it cancels against the untaxed reserve. The full-tax method is therefore applied in this research.

To apply the full-tax method it is necessary to estimate the marginal income tax. This research uses Runsten's (1998, p. 117) estimation of the marginal income tax for the period 1977—1993. From 1994— 1996 28 percent is used as a marginal income tax. During 1984 to 1990, there was a profit sharing tax, but this is accounted for as an appropriation and thus is not included in the measurement of the marginal tax. The marginal tax used in this research is found in Table 5-2.

Year	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
Tax rate	56%	57%	57%	57%	58%	58%	58%	53%	52%	52%
Year	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
Tax rate	52%	52%	40%	40%	30%	30%	30%	28%	28%	28%

Table 5-2: Estimated yearly marginal tax in Sweden from 1977—1996.

Deferred tax in Sweden refers to the sum of government-supported postponed income tax. Since the government allows the firms to defer tax payments, it becomes a lender to the firm. However, the government does not require any interest payments from the firm because of the deferred tax, and consequently, deferred tax in this research is treated as a non interest-bearing liability. The treatment of deferred tax is consistent with, e.g., Penman (2004), Levin (1998), and Finansanalytikernas rekommendationer (1994).

Provision for taxes does not normally carry interest. It becomes interest bearing only when its due date has expired. In the present research to pay taxes too late is considered unusual and the provision for taxes is therefore classified as a non interest-bearing liability.

Additional to the treatment of accounts payable, deferred tax, and provision for taxes, there are reasons to discuss the treatment of the shares and participations and the accounts receivable. These are discussed individually below.

Accounts payable and the accounts receivables are often classified as OA and liabilities in financial analysis (cf., Levin 1998). In this research they are treated as part of the NFL. Accounts payable has already been discussed, but discussing accounts receivables remains. It is treated as analogous to the accounts payable, which means that the accounts receivables are seen here as created by the firm when it decides not to require a cash payment from the customer at the point of sale. When this occurs, the selling firm becomes a lender and incurs an opportunity cost on the loan. However, only rarely will the selling firm charge an explicit interest on the loan; instead, part of the total invoiced amount will be an implicit interest on the loan. With a substance-over-form perspective, it follows that the accounts receivables should be accounted for as interest-generating assets, and consequently, the implicit interest should be deducted from the sales and added to the firm's interest revenue. Penman (2004) also discusses this topic and proposes this method for adjusting the accounting.

As with the implicit interest expense on the accounts payable, the implicit interest revenue on the accounts receivables is discussed in subsection 5.3.3, which focuses on income classification.

Other current receivables include other loans and claims that the firm has on its group (when it is part of a corporate group). Loans typically carry interest, the claims on the group can be exchanged for cash, and so has an opportunity cost. It may also be the case that the group controls the liquidity in the firm using these claims. Analogous to the accounts receivables, the other current receivables are therefore classified as interest-generating assets.

Shares and participations are divided into two parts based on whether they are deemed an integral part of the firm's operations or not. Shares and participations in subsidiaries are seen as an integral part of the firm's operations. As a result, those assets are classified as non interest-generating assets.

Shares and participations in other firms are not considered an integral part of the firm's operations. Had the shares and participations been an integral part of the owning firm's operations, it is likely that the owning firm would acquire the other firm, or at least invest in it to the point where it gains a controlling interest. When the firm gains the controlling interest in the other firm, its ownership is reclassified to shares and participations in subsidiaries. Since the owning firm has no controlling interest in the other firm, the shares and participations are classified as interest-generating assets.

Other long-term receivables, bonds and other securities, cash and bank balances are all classified as interest-generating assets, which should not be controversial. Next, follows that section on the operationalization of the income statement.

5.3.3 Operationalization of the income statement

According to [EQ A-66] in subsection A.8.3.2, the firm's CNI consists of the comprehensive OI and the comprehensive net interest expense:

$$CNI_t = COI_t - CNIE_t$$

Applying the clean surplus relationship to the firm's operating activity implies that:

$$\Delta NOA = COI_t - FCF_t \Leftrightarrow \qquad \text{[EQ 5-7]}$$

$$COI_t = \Delta NOA + FCF_t \qquad \text{[EQ 5-8]}$$

Defining the free cash flow, FCF, as did done below, allows COI to be measured according to [EQ 5-8].

$$FCF_t = d_t + CNIE_t - \Delta NFL_t \qquad \text{[EQ 5-9]}$$

The system above is using the same structure as used by Feltham and Ohlson (1995), Feltham and Ohlson (1999), Lundholm and O'Keefe (2001), and Penman (2004).

Net dividend paid is symbolized by d_t. The net dividend is defined in subsection 5.3.1 as the net of gross dividends, group contributions, and any new common stock issue. Gross dividends are operationalized as the proposed dividend reported in the previous year according to Appendix G. Group contributions are operationalized according to Appendix F. The new common stock issue is defined in subsection 5.3.1, where it includes both the stocks' face value and any agio. NFL are operationalized according to 5.3.3 and Appendix E. What remains is the operationalization of $CNIE_t$. This implies an operationalization of the income statement, which is the topic for the rest of this subsection.

A guiding principle in the operationalization of the income statement is to have a consistent treatment of assets, liabilities, revenues, and expenses. That is, the assets that are classified as financial assets must have their revenues classified as interest revenues. As an analogy, it follows that liabilities that are classified as financial liabilities must have their expenses classified as interest expenses. The revenues and expenses that are not identified as interest revenue or as interest expense are classified as operating revenue or as operating expenses.

The transformation of the empirical income statement to an income statement comparable to [EQ A-66] is made in two steps. This subsection discusses the first step and focuses on identifying stable revenue classes and stable expense classes in the empirical income statements, as well as the assignment of those to a theoretical classification scheme. The second step is concerned with the assignment of specific line items in the database to this subsections' operationalized classification scheme. This can be found in Appendix D.

Table 5-3 presents a summary of the identification of the stable revenue and expense classes in the empirical income statements and their assignment to a theoretical classification scheme: after the table follows a discussion on the previously signaled issues.

THE INCOME STATEMENT	
COMPREHENSIVE OPERATING INCOME (COI)	**COMPREHENSIVE NET INTEREST EXPENSE (CNIE)**
Adjusted sales	Adjusted interest revenue (IR)
Sales	Interest revenue
-Implicit interest revenue on AR	+Implicit interest revenue on AR
+Implicit interest expense on APL	
Adjusted cost of material	Adjusted interest expense (IE)
- Cost of material	Interest expense
+Implicit interest expense on AP	-Implicit interest expense on AP
+Implicit interest expense on PL	-Implicit interest expense on APL
	-Implicit interest expense on PL
- Labor cost	Other dividends (ODIV)
- Depreciation	Exchange rate difference (FXRD)
Dividends from the group	Summation errors in income (ERRFI)
Government subsidies (GS)	- Tax on CNIE
Items affecting comparability (IAC)	
Dirty-surplus accounting	
Summation errors in OI (ERROI)	
- Tax on COI	

Table 5-3: A specification of the type of components of the income statement

Credit sales inflate the firm's reported operating revenue with the implicit interest revenue. A credit purchase implies that the reported costs of goods sold are over estimated with the estimated implicit interest expense. These issues were discussed in subsection 5.3.2 in conjunction with the discussion of how to classify accounts receivables and accounts payable. The existence of implicit interest is also acknowledged in national GAAP and in international GAAP (e.g., RR 3 and IAS 18).

This thesis deflates the overestimated operating revenue by decreasing it with the implicit interest revenue. The implicit revenue is thus reclassified as interest revenue. It also deflates the overestimated cost of material with the implicit interest expense, reclassifying it as interest expense.

Following GAAP, the implicit interest rate charged to the accounts receivable would be equal to the firm's short-term lending rate, which equals a risk-free interest rate and a risk premium. However, since the accounts receivable typically have short duration and since conservatism requires that uncertain claims to be immediately expensed, it is assumed that the disclosed value of accounts receivable is risk-free. This means that the correct opportunity cost for such a loan is the risk-free interest rate.

Estimating the implicit interest expense that is an effect of the accounts payable is only slightly different. The implicit interest rate charged to the accounts payable should, in analogy to the above, be equal to the firm's short-term borrowing rate, which would equal the risk-free interest rate and a risk premium. In view of the fact that the accounts payable typically has a short duration and because it is not likely that a firm in deep financial trouble would be allowed to use credit purchases, it is assumed that the risk-free rate-of-return can also be used as a proxy.

Advances from customers are also a loan that neither is interest free. It is assumed that customers do not finance a company's activities free, and so demand a reduced price in compensation for having to pay in advance. This reduced price is estimated similarly to the implicit interest expense above considering that these are rather short-term liabilities. The reduction is reversed and is instead classified as an interest expense.

The cost for pension liabilities is usually according to Swedish GAAP classified as an interest expense but SCB reclassifies the expense as an operating expense. I classify the pension liability as a financial liability and consequently classify the expense as an interest expense. However, I lack detailed information of the size of the expenses and therefore need to assume it. I do this similarly by using the one-year risk-free rate-of-return. This is probably a little bit too low considering that pension liabilities are long-term but reclassifying it in this manner is better than no reclassification at all. Additionally, the induced error is of a minor scale and I estimate that it will not have a significant affect on the results.

Below follows the one-year risk-free interest rate that is used.

Year	Risk-free rate of return	Year	Risk-free rate of return	Year	Risk-free rate of return
1978	8.00%	1985	11.80%	1992	12.55%
1979	6.50%	1986	12.50%	1993	9.49%
1980	9.00%	1987	9.55%	1994	6.41%
1981	10.00%	1988	9.73%	1995	9.35%
1982	11.00%	1989	10.87%	1996	8.20%
1983	10.00%	1990	13.02%		
1984	11.80%	1991	13.85%		

Table 5-4: The one-year risk-free rate-of-return in Sweden (Source: SCB 1979—1984; Svenska Dagbladet, 1984—1996).

In Table 5-4, the Swedish government's "diskonto", as of 1st of January, is applied a risk-free rate-of-return for the period of 1978 —1983. Later periods use the yield-to-maturity on a one-year T-bill as of 1 January.

When there are summation errors in the dataset supplied by SCB, these are part of OI. This ensures that the clean surplus relationship is maintained after the dirty surplus adjustments are operationalized. Summation errors in the income statement occur in the raw data but are not frequent, nor are they of material size.

Items affecting comparability include extra ordinary income, and for the period of 1994 to 1996, it incorporates capital gains from sales of fixed assets and write-downs of those assets. The aggregation of capital gains from sales of fixed assets and write-downs of fixed assets with extra ordinary income into items affecting comparability reflects an effort of keeping a comparable accounting interpretation despite changing GAAP (cf. FAR No. 13 and its successor RR 4 in, e.g., FARs Samlingsvolym 1994). Before 1994, capital gains from sales of fixed assets and write-downs of fixed assets were usually included in extra ordinary income but they are not included in extra ordinary income after 1994.

Items affecting comparability are treated as an OI even though it can contain revenues and expenses that should be classified as financial income. When a firm makes a capital gain on the sale of long-term financial assets, it is reported as extra ordinary income before 1994 and as OI after 1994. Since the data do not specify the components of extra ordinary income, or of capital gains from sales of fixed assets, or of write-downs of fixed assets, they must be uniformly classified. Because the firms are manufacturing firms, it is assumed here that the bulk of items affecting comparability can be traced to operating activities and thus are concurrently classified as part of OI.

The effects of dirty surplus accounting are effects that occur when the accounting is not prepared on a clean surplus basis. In view of [EQ 5-8] with definition [EQ 5-9] and its operationalization and with the operationalized balance sheet it is possible to use [EQ 5-8] to measure comprehensive operation income directly.

Defining OI (i.e. as consisting of all line items in COI according to Table 5-3 except the dirty surplus accounting), it is possible to measure the aggregate effect of dirty surplus accounting as:

$$DTSACTG = COI - OI \qquad \text{[EQ 5-10]}$$

The firm's adjusted total tax is allocated to the OI and the financial income. It follows standard procedures applied in, e.g., Copeland, Koller and Murrin (2000) and Penman (2004), which are amended for effects of dirty surplus accounting. Tax on comprehensive net interest expense is estimated as the product of the marginal income tax rate and the pre-tax comprehensive net interest expense.

The operationalization of income taxes is found in Appendix D.

As discussed in the previous section, Swedish GAAP allows firms to reduce the income taxes paid by making appropriations before the income tax is measured. The appropriation is undone in the present research with the use of the full tax method. That means that the deferred tax is added to the reported tax. The deferred tax is estimated as the product of the marginal income tax rate (see Table 5-2) and the period's appropriation. To get the adjusted total reported income tax the deferred tax component is added to the reported tax.

The dividend from the group is exempted from tax and hence it is excluded from the tax shield calculation.

5.4 Summary

This chapter bridges some of the gap between Appendix B and the empirical data. Bridging the gap entails reclassification of assets into non interest-generating assets (also known as OA) and interest-generating assets. It also entails reclassifying the liabilities into interest-bearing liabilities and non interest-bearing liabilities.

The reclassification of the balance sheet follows standard procedures in financial analysis, with the exception of the treatment of accounts receivables, accounts payable, and advance payments from customers. These are classified as financial assets and financial liabilities. This reclassification also carries over into the income statement that reclassifies some of the firm's operating revenues to interest revenue because of the implicit interest revenue from accounts receivables. It reclassifies some of the firm's cost of goods sold as interest expense because of the implicit interest expense from accounts payable. Further, it reverses the discount in sales from getting advance payments from customers and reclassifies it as an interest expense.

The reclassification of the income statement also entails changes that ascertain that the accounting follows the clean surplus principle. It also identifies summary errors in the raw data and treats it as part of the income.

Since the data are based on legal entities rather than consolidated accounting, a firm reports appropriations and untaxed reserves. These effects are also undone using the full-tax method.

Because of the operationalizations in this chapter and in Appendix D—Appendix G, all of the components to ROE and RNOA are in place such that the ratios are measured on a clean surplus basis and the separation components to ROE and RNOA are done in the spirit of Modigliani & Miller (1958).

CHAPTER 6—HYPOTHESES TESTS AND THE TEST VARIABLES

> "…it must be possible for an empirical scientific system to be refuted by experience" (Popper 1959, p. 41)

6.1 Introduction

I evaluate the proposed theory of Homo comperiens in two ways. I test Proposition 4-6 and Proposition 4-7 using hypotheses tests and I assess (to avoid committing a Type I error such that I reject the null hypotheses when they are true) the theory's predictive ability in out-of-sample tests.

This chapter is devoted to discuss the hypothesis tests and their results. Before I present the testable hypotheses and results from the hypothesis tests, I first discuss the operationalization of the risk-adjusted expected residual rates-of-returns. In Chapter 7, I discuss the predictive tests and their results.

Because of their apparent similarities, in subsection 6.4.2.3 I relate my results with those from empirical research based on Ohlson's (1995) model. However, it should be noted that Ohlson's model only purports to measure accounting residual income since it assumes zero arbitrage residual income. If the markets are inefficient, it implies that empirical assessments based on Ohlson's model, unintentionally, come to measure the combined effect of both arbitrage and accounting residual income.

6.2 Operationalization of the risk-adjusted subjective expected residual accounting rates-of-returns

Chapter 4 finds that if a market meets the assumptions of Homo comperiens, the market exhibits arbitrage profits. The existence of arbitrage opportunities implies the existence of risk-adjusted residual accounting rates-of-returns. The existence of risk-adjusted residual accounting rates-of-returns implies the existence of arbitrage opportunities. That is, they are equivalent expressions.

Two risk-adjusted subjective expected residual accounting rates-of-returns are defined in Chapter 4. These include the risk-adjusted subjective expected residual return on equity, $E_{K0}^{*}[_{t-1}ROE_t]$ and the risk-adjusted subjective expected residual return on net operating assets (RRNOA), $E_{K0}^{*}[_{t-1}RRNOA_t]$:

$$E_{K0}^{*}[_{t-1}RROE_t] = E_{K0}[_{t-1}RROE_t] - E_{K0}[\textit{total risk premium}_t] \qquad \text{[EQ 4.34]}$$

$$E_{K0}^{*}[_{t-1}RRNOA_t] = E_{K0}[_{t-1}RRNOA_t] - E_{K0}[\textit{operating risk premium}_t] \qquad \text{[EQ 4.35]}$$

The risk-adjusted subjective expected residual accounting rates-of-returns, as shown above, are strictly theoretical and need to be attributed testable properties. This process is reported in section 6.2, with the assistance of the operationalization in Chapter 5 and relevant appendices.

Subsection 6.2.1 considers the expectations parameter, subsection 6.2.2 addresses the risk-adjusted residual accounting rate-of-return, and subsection 6.2.3 considers both the risk premiums and biased accounting. Section 6.2 concludes with descriptive statistics for the proxies to [EQ 4.34] and [EQ 4.35].

Appendix I (p. 211) is related to section 6.2 in the sense that it operationalizes the industry-specific accounting rates-of-returns, which are necessary components to the testable proxies for the risk-adjusted subjective expected residual accounting rates-of-returns.

6.2.1 Ex ante and ex post rates-of-returns

Chapter 4 finds that in subjective certainty ex ante and ex post rates-of-returns differ because the subject makes choices on the subjective state set. The state that eventually emerges when the future unfolds may not have been part of the subject's subjective state set.

The distinction between ex ante and ex post rates-of-returns because of the subjective state set's interaction with the unfolding future is not only present in a subjective certain choice but it is also present in a subjective uncertain choice. The effect from when an unnoticed (not part of the subjective state set) state suddenly comes true is in the subjective uncertain choice subsumed by the fact that only one of all states considered in the subjective state set can occur. Hence, there is by design a difference between ex ante and ex post rates-of-returns in the subjective uncertainty. That is, the uncertainty creates a difference between ex ante and ex post rates-of-returns even when the true state is within the subjective state set.

Although ex ante and ex post rates-of-returns most likely differ between firms, it is difficult to use ex post rates-of-returns to draw conclusions on ex ante rates-of-returns. However, the state that comes true is, using the state-and-partition-model, uncontrollable by the subjects, which implies that the state that comes true sometimes is better than expected and sometimes it is worse than expected. Because of uncertainty, ex ante and ex post rates-of-returns are different for individual firms at individual times, but I suggest that the positive and negative effects cancel out as the number of observations increase. That is, I assume that the ex post accounting rates-of-returns are, on average, equal to the ex ante expected accounting rates-of-returns since the unfolding of the states is a random process.

Given that ex post and ex ante rates-of-returns are presumed to be comparable, ex post risk-adjusted residual accounting rates-of-returns are used as proxies for [EQ 4.34] and [EQ 4.35] in the empirical assessment of the theory. The ex post risk-adjusted residual accounting rates-of-returns are:

$$_{t-1}RROE_t^* = {}_{t-1}RROE_t - \textit{total risk premium}_t \qquad \text{[EQ 6-1]}$$

$$_{t-1}RRNOA_t^* = {}_{t-1}RRNOA_t - \textit{operating risk premium}_t \qquad \text{[EQ 6-2]}$$

The use of ex post rates-of-returns to make inferences of ex ante rates-of-returns follows a long tradition in both economics and accounting and is therefore not controversial.

6.2.2 The residual accounting rates-of-returns

Appendix B defines the subjective residual accounting rates-of-returns in [EQ B-40] and in [EQ B-43] as the difference between the subjective accounting rate-of-return and the subjective market rate-of-return:

$$_{t-1}RROE_{Kt} = {}_{t-1}ROE_{Kt} - {}_{t-1}r_{Kt} \qquad \text{[EQ B-40]}$$

$$_{t-1}RRNOA_{Kt} = {}_{t-1}RNOA_{Kt} - {}_{t-1}r_{Kt} \qquad \text{[EQ B-43]}$$

The subjective ROE and the subjective RNOA are defined in Appendix B as:

$$_{t-1}ROE_{Kt} = CNI_{Kt} \cdot EQ_{t-1}^{-1} \qquad \text{[EQ B-39]}$$

$$_{t-1}RNOA_{Kt} = COI_{Kt} \cdot NOA_{t-1}^{-1} \qquad \text{[EQ B-42]}$$

Standard empirical ex post financial analysis (e.g., Penman 2004, Stickney & Brown 1999) use average book values rather than beginning book values when accounting rates-of-returns are operationalized. The present research maintains theoretical connection by keeping theoretical definitions of ROE and RNOA when operationalizing them.

Since the profit that a firm reports is compounded over the year, there may be large differences between these two capital definitions. Capital defined on beginning values implies that the risk of having denominators close to zero increases and hence it is likely that the number of observations having extreme ROE and RNOA increases. It is also likely that asset build-up is going to exacerbate the volatility in the rate-of-return patterns since the small denominator effect quickly disappears. However, the asset build-up problem is very small in this thesis because the empirical assessment is done on a very large number of observations. The extreme effects from the capital definition are subsumed by the effect from more normal behaviors.

The capital definition exacerbates the need to devise statistical measures that are insensitive and to non-normal tails and efficient when the tails are non-normal since the extreme values that the capital definitions likely generate create non-normal tails. Tolerance to non-normal tails is known as robustness of validity, and high efficiency when non-normal tails are present is known as robustness of efficiency (Mosteller & Tukey 1977). Robustness issues are considered in subsection I.2 and in Appendix K.

Chapter 4 argues that a period's subjective market rate-of-return is equal to the subjective risk-free rate-of-return for that period, irrespective if the market rate-of-return is coming from the definition of subjective RROE, [EQ 4.30], or coming from the definition of subjective RRNOA,

[EQ 4.31]. It is therefore possible to rewrite the risk-adjusted residual accounting rates-of-returns in [EQ 6-1] and [EQ 6-2] to:

$$_{t-1}RROE_t^* = {}_{t-1}ROE_t - {}_{t-1}r_t^{risk-free} - total\ risk\ premium_t \qquad \text{[EQ 6-3]}$$

$$_{t-1}RRNOA_t^* = {}_{t-1}RNOA_t - {}_{t-1}r_t^{risk-free} - operating\ risk\ premium_t \qquad \text{[EQ 6-4]}$$

The operationalization of the components to subjective ROE and RNOA is given in Chapter 5 and associated appendices.

6.2.3 The risk-premiums and biased accounting

The risk-adjusted subjective expected residual accounting rates-of-returns contain risk premiums and assume unbiased accounting. This subsection addresses the issue of how the risk premiums and the unbiased accounting can be operationalized such that [EQ 6-3] and [EQ 6-4] measure what they should measure.

Subsection 6.2.3 first addresses the issue of proxies for risk premiums and then follows a discussion on proxies for unbiased accounting.

6.2.3.1   Proxies for the risk premiums

If the market had been in a Pareto optimal equilibrium, it would have been possible to measure the risk-adjusted market rate-of-return by using CAPM and from CAPM it would have been possible to find a firm's total risk premium. Specifying the market so that CAPM can be applied also requires that the market rate-of-return is constant, which implies a constant total risk premium.

When the subject makes limited rational choices according to the assumptions of Homo comperiens, the total risk premium cannot be specified using an asset valuation model such as CAPM, which is based on general-equilibrium. Thus, CAPM is not applicable to this research.

In view of the fact that, in this research, CAPM is not available in this respect, two strategies remain. The first strategy implies the development of an asset-pricing model in a Homo comperiens setting that provides the possibility to price subjective uncertainty. It is likely that such a model needs to define operational proxies for the difference between subjective uncertainty and objective uncertainty. This is clearly outside the scope of this thesis and hence is not considered any further.

Where the first strategy is deductive and uses theory to discern the risk premiums, the second strategy uses empirical proxies for the risk premiums. This thesis uses empirical proxies for assessing the risk premiums.

I assume that firms within the same industry sell similar products and have a similar production function. This assumption is similar to that of, e.g., Lev and Sunder (1979). The fluctuations in the demand of the goods then affect the firms within the industry in a similar fashion because they sell similar products. Having similar production functions means that the fluctuations in the factor markets affect the firms within the same industry similarly. Together, these assumptions imply that

the fluctuations affect the firms' profits similarly and hence also their accounting rates-of returns. Gupta & Huefner (1972) find that cross-sectional variation of financial ratios is primarily related to industry characteristics (see also Fieldsend, Longford & McLeay 1987, Lee 1985, McDonald & Morris 1984, and McLeay & Fieldsend 1987 for similar results).

Suppose that it is possible to divide a firm's operating risk premium into two components, where the first component is the risk premium required by the investor for investing in a particular industry and the second component is the firm-specific risk premium. This means:

$$operating\ risk\ premium_t = firm\text{-}specific\ operating\ risk\ premium_t + industry\text{-}specific\ operating\ risk\ premium_t \qquad \text{[EQ 6-5]}$$

Assume that the firm-specific risk premium consists of two components. The firm-specific risk premium's first component is because the firm's sensitivity to the market fluctuations is different from the industry's sensitivity. The second component of the firm-specific risk premium is because the firm's production function is not identical to the industry's production function. Assume that the firm-specific differences from the industry are small. This implies that the firm-specific risk premium is approximately zero. The firm's operating risk premium is then approximately equal to the industry-specific risk premium.

There are studies showing that there is a relationship between a firm's risk and its accounting rates-of-returns (Beaver, Kettler & Scholes 1970; Beaver & Manegold 1975; Gonedes 1975; Lev 1974). I thus assume that an empirical proxy for industry-specific risk, assuming unbiased accounting and using an accounting-based risk perspective as a point of departure, is:

$$industry\text{-}specific\ operating\ risk\ premium_t =\ _{t-1}RNOA_t^I -\ _{t-1}r_t^{risk-free} \qquad \text{[EQ 6-6]}$$

where $_{t-1}RNOA_t^I$ is the industry's return on net operating assets.

A similar argument can be used on total risk. Here it is assumed that the firm-specific total risk is negligible and the firm's total risk premium can therefore be measured using the industry-specific total risk premium. The assumption is that the industry-specific total risk premium can be measured with the spread between the relevant accounting rate-of-returns and the risk-free rate-of return:

$$industry\text{-}specific\ total\ risk\ premium_t =\ _{t-1}ROE_t^I -\ _{t-1}r_t^{risk-free} \qquad \text{[EQ 6-7]}$$

where $_{t-1}ROE_t^I$ is the industry's return on equity (it is operationalized in Appendix I).

The risk-adjusted residual accounting rates-of-returns in [EQ 6-3] and in [EQ 6-4] can be restated as functions of firm profitability and industry profitability since the firm-specific risk is negligible and because spread between the industry accounting rate-of-return and the risk-free rate-of return is conjectured to be a proxy for industry-specific risk. The ex post risk-adjusted residual accounting rates-of-returns are then:

$$_{t-1}RROE_t^* =\ _{t-1}ROE_t -\ _{t-1}ROE_t^I \qquad \text{[EQ 6-8]}$$

$$_{t-1}RRNOA_t^* = {}_{t-1}RNOA_t - {}_{t-1}RNOA_t^I \qquad \text{[EQ 6-9]}$$

The present study uses industry accounting rates-of-returns as proxies for risk. This treatment presupposes that the accounting is unbiased. Unbiased accounting is unlikely and the subsection 6.2.3.2 addresses the issue of how biased accounting is treated in this study.

6.2.3.2 Biased accounting and ex post risk-adjusted residual accounting rates-of-returns

The accounting bias exists because the firm's financial accounting is supposed to provide conservative estimations of the firm's earnings and financial position. The accounting can be biased in many ways. For instance, it is possible that there are opening (direct expensing of research outlays) and closing (historical cost) valuation errors that affect the balance sheet. There can also be other types of related bias such as late recognition of revenue (not at the point of contract but at the point of delivery) or early recognition of costs (at the point of decision and not at the cash outlay). For a discussion of how conservative accounting enter formal accounting models, see Feltham & Ohlson (1996), Penman & Zhang (2002), and Cheng (2005).

The effects of conservatism are well known yet hard to empirically measure. Nonetheless, there exist attempts exist to model and empirically estimate the bias. Runsten (1998), e.g., develops an empirical permanent measurement bias (PMB) variable that is alleged to measure the accounting bias. Runsten's PMB model assumes no-arbitrage, which means that the model is not useful for my purpose.

Subsection 6.2.3.1 reasons that firms within the same industry have similar products and similar production functions. If that is a valid assumption, it is reasonable to assume that firms within the same industry are exposed to similar accounting rules and use similar accounting procedures (e.g., approximately the same planned depreciation rate for fixed assets and similar operationalization of the revenue recognition principle). McDonald & Morris (1984) show that within-industry, comparisons of accounting ratios are possible, but not intra-industry comparisons. Such a comparison would not be possible with a heterogeneous application of the accounting rules within an industry. Lee (1985) also reports significant industry effects. McLeay & Fieldsend (1987) and Fieldsend, Longford & McLeay (1987) both report a significant industry effect in accounting ratios. On balance, this indicates that firms in an industry produce similar products, have similar production functions and apply the accounting rules in a similar manner. Thus, that kind of comparison of accounting data is valid within an industry.

With the accounting data within the industry being comparable, it should be possible to sweep away the accounting bias from a firm's accounting ratios by reducing them with the industry ratio. This means that the adjustment in the ex post risk-adjusted residual accounting rates-of-returns using [EQ 6-8] and [EQ 6-9] remove not only the risk premiums but also the biased accounting. Therefore, [EQ 6-8] and [EQ 6-9] are used as proxies for the risk-adjusted subjective ex-

pected residual accounting rates-of-returns [EQ 4.34] and [EQ 4.35]. The hypotheses tests are therefore performed using operationalizations [EQ 6-8] and [EQ 6-9].

The last component needed in the operationalization of the ex post risk-adjusted residual accounting rates-of-returns are the industry ex post accounting rate-of-returns, which are operationalized in Appendix I (p. 211).

6.2.4 Descriptive statistics of risk-adjusted RROE and RRNOA from 1978 to 1994

The components to risk-adjusted RROE, [EQ 6-8], and to risk-adjusted RRNOA, [EQ 6-9], are ratios and this implies that as the denominator approaches zero for a component, the residual accounting rate-of-return values goes moves towards infinity. The denominators in the ratios are balance sheet data that can be erroneous and therefore outliers affect the data.

Table 6-1 and Table 6-2 present descriptive statistics for risk-adjusted RROE and for risk-adjusted RRNOA for the whole set of firms in the data set.

Descriptive statistics for risk-adjusted RROE									
Year	1978	1979	1980	1981	1982	1983	1984	1985	1986
n	611	727	1356	1337	1342	1385	1436	1401	1444
Max	6,560%	5,600%	4,340%	2,279%	36,683%	1,998%	3,804%	4,474%	9,924%
Mean	41%	10%	10%	-52%	54%	8%	26%	19%	27%
Median	5.0%	9.4%	13.9%	12.5%	11.4%	13.0%	19.2%	13.2%	11.5%
$Location_{bw}$	5.4%	8.4%	15.8%	14.8%	12.7%	15.9%	21.2%	14.4%	11.7%
Min	-2,562%	-2,620%	-3,357%	-71,700%	-3,268%	-3,273%	-5,066%	-12,996%	-3,624%
StdDev	390%	295%	251%	1,992%	1,095%	216%	269%	378%	362%
$StdDev_{bw}$	34%	44%	33%	36%	36%	38%	40%	25%	25%
QRange	38%	41%	37%	39%	37%	39%	47%	30%	28%
Year	1987	1988	1989	1990	1991	1992	1993	1994	
n	1471	1453	1498	1412	1429	1351	1269	1278	
Max	6,314%	22,664%	1,969%	11,038%	72,357%	83,545%	105,119%	54,232%	
Mean	20%	46%	26%	16%	47%	45%	126%	6%	
Median	9.5%	9.8%	21.8%	6.6%	13.6%	2.1%	11.0%	27.3%	
$Location_{bw}$	11.3%	11.0%	23.3%	6.6%	13.5%	1.7%	12.6%	32.6%	
Min	-7,473%	-1,249%	-6,042%	-8,045%	-23,502%	-13,564%	-18,875%	-108,200%	
StdDev	309%	634%	235%	392%	2,062%	2,379%	3,105%	3,419%	
$StdDev_{bw}$	25%	25%	34%	28%	50%	46%	50%	54%	
QRange	27%	27%	38%	29%	56%	51%	52%	68%	

Table 6-1: Descriptive statistics for risk-adjusted RROE per year, [EQ 6-8].

Table 6-1 shows some descriptive statistics for the whole set of the available set of risk-adjusted RROE on a per-year basis. In the table *n* is the symbol for the number of observations available. $Location_{bw}$ is the biweight location estimate. The mean is the arithmetic mean and so it is the location estimate for a Gaussian-shaped distribution. StdDev is the standard deviation, i.e. the scale measure for Gaussian shaped distributions, while the $StdDev_{bw}$ is the biweight estimation of the scale. QRange is the interquartile range for the empirical distributions and thus describes the spread between the 75th and 25th percentile observation.

From a visual inspection of the data, it is apparent that the yearly empirical distributions of RROE have non-normal and fat tails because the interquartile range is small as compared with the standard deviation and to the spread between the Max and Min observations. The effects of the outliers are also seen in the mean estimates of the distributions that often are very far from the median observation. This means that outliers affect the yearly distributions, which prohibits the use of Mean and StdDev for statistical tests.

Descriptive statistics for risk-adjusted RRNOA									
Year	**1978**	**1979**	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**
n	611	727	1356	1337	1341	1385	1436	1401	1444
Max	4,030%	399%	792%	409%	1,423%	12,090%	3,790%	2,049%	3,667%
Mean	26%	8%	11%	10%	-17%	23%	21%	15%	2%
Median	4.2%	5.7%	7.3%	7.5%	7.3%	7.7%	9.7%	8.1%	7.0%
Location$_{bw}$	4.5%	7.0%	8.3%	8.2%	7.9%	8.8%	12.1%	8.7%	7.6%
Min	-600%	-279%	-868%	-467%	-35,714%	-772%	-1,083%	-734%	-12,524%
StdDev	193%	35%	43%	33%	981%	341%	149%	81%	372%
StdDev$_{bw}$	13%	15%	12%	13%	14%	15%	18%	11%	12%
QRange	16%	16%	14%	15%	15%	17%	21%	13%	13%
Year	**1987**	**1988**	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	
n	1471	1453	1498	1411	1428	1351	1266	1277	
Max	45,212%	7,892%	5,141%	27,220%	2,301%	396,709%	504%	1,161%	
Mean	-64%	9%	16%	-12%	12%	302%	12%	29%	
Median	6.7%	6.6%	12.5%	5.9%	9.8%	5.9%	9.6%	17.6%	
Location$_{bw}$	7.5%	7.0%	13.4%	5.9%	10.6%	5.8%	11.3%	19.4%	
Min	-155,870%	-10,516%	-2,525%	-45,693%	-2,697%	-345%	-1,948%	-938%	
StdDev	4,246%	350%	175%	1,444%	123%	10,793%	83%	78%	
StdDev$_{bw}$	12%	12%	15%	13%	23%	21%	23%	25%	
QRange	14%	13%	18%	14%	25%	22%	25%	30%	

Table 6-2: Descriptive statistics for risk-adjusted RRNOA per year, [EQ 6-9].

Table 6-2 shows some descriptive statistics for risk-adjusted RRNOA. The table shows similar results as the table with descriptive statistics for risk-adjusted RROE. Thus, the yearly distributions of risk-adjusted RRNOA exhibit non-normal tails and thus the Mean and StdDev cannot be used for statistical tests on the variable.

The arithmetic mean can possibly be replaced by the median but Mosteller & Tukey (1977) show that the median's efficiency in large samples is rather poor in comparison with the biweight estimate, which has over 90 percent efficiency. An estimate having efficiency over 90 percent is very difficult to distinguish from an estimate having 100 percent efficiency (Mosteller & Tukey 1977). A 100 percent efficiency is, e.g., the arithmetic mean in a Gaussian distribution (Mosteller & Tukey 1977).

The biweight estimate of location and scale uses c=9, which is equivalent to approximately 6 standard deviations, or 99.99 percent (Mosteller & Tukey 1977). This means that the biweight scale and location estimate use all observations up to six standard deviations in its iterative procedure and assigns a zero weight to the observations beyond the cut-off points. Therefore, by only excluding

the most extreme of the extreme values, the location estimate and the scale estimate change dramatically.

6.3 Do risk-adjusted residual accounting rates-of-returns exist?

Chapter 2 develops the theory of Homo comperiens. The theory of Homo comperiens proposes that the subject is limited rational because he or she possesses limited knowledge of the available action and state sets.

Even when assuming homogenous preferences, the theory of Homo comperiens implies that the subjective expected utility function (Proposition 2-3) is different to the objective utility function. This implies that the subjective marginal rate of substitution between savings and consumption differs from the objective marginal rate of substitution and we thus face an arbitrage market.

Chapter 4 finds that firms in the arbitrage market exhibit risk-adjusted subjective expected RROE and risk-adjusted subjective expected RRNOA that are non-zero (Proposition 4-6). The empirically testable proxy for the risk-adjusted subjective expected RROE is [EQ 6-8] and it is [EQ 6-9] for the risk-adjusted subjective expected RRNOA.

Following this analysis, it is possible to test Proposition 4-6 by analyzing [EQ 6-8] and [EQ 6-9]. If some firms report these proxies as statistically non-zero, Proposition 4-6 is not falsified.

Subsection 6.3.1 operationalizes these alternative hypotheses and subsection 6.3.2 presents the results. Appendix K (p. 219) presents the robust t-test that I use.

6.3.1 The hypotheses to be tested

An alternative hypothesis and its null hypothesis are stated per variable.

$\mathbf{H_{1A}}$. A market that meets the conjectures of Homo comperiens is an arbitrage market. In the arbitrage market at least one firm earns arbitrage profits such that the risk-adjusted RROE is non-zero. This is a testable hypothesis based on Proposition 4-6 (p. 77), which focuses on the overall residual earnings ability.

$$_{t-1}RROE_{it}^{*} \neq 0 + \varepsilon_t \qquad \qquad \exists i \in I \in M$$

where i is the firm, I is the industry, M is the market, t is the year, and ε_t is a white noise residual.

$\mathbf{H_{0A}}$. The market is a no-arbitrage market and all firms thus report zero risk-adjusted RROE.

$$_{t-1}RROE_{it}^{*} = 0 + \varepsilon_t \qquad \qquad \forall i \in I \in M$$

where i is the firm, I is the industry, M is the market, t is the year, and ε_t is a white noise residual.

$\mathbf{H_{1B}}$. A market that meets the assumptions of Homo comperiens is an arbitrage market. In the arbitrage market at least one firm earns arbitrage profits such that the risk-adjusted RRNOA is

non-zero. This is a testable hypothesis based on Proposition 4-6 (p. 77), which focuses on the operating residual earnings ability.

$$_{t-1}RRNOA_{it}^* \neq 0 + \varepsilon_t \qquad\qquad \exists i \in I \in M$$

where i is the firm, I is the industry, M is the market, t is the year, and ε_t is a white noise residual.

H$_{0A}$. The market is a no-arbitrage market and all firms thus report zero risk-adjusted RRNOA.

$$_{t-1}RRNOA_{it}^* = 0 + \varepsilon_t \qquad\qquad \forall i \in I \in M$$

where i is the firm, I is the industry, M is the market, t is the year, and ε_t is a white noise residual.

6.3.2 Results from the robust double-sided t tests

Appendix K discusses the robust double-sided t test that is applied when assessing the hypotheses in subsection 6.3.1. The hypotheses are tested using t tests on [EQ 6-8] and [EQ 6-9].

There is a risk that the operationalization of the financial statements in Chapter 5 can drive the results and the hypotheses tests are thus performed under alternative specifications. The alternative tests are reported in Appendix J (p. 215).

There is a further risk that the results are affected by the measurement of the industry profitability confidence interval and therefore two alternative confidence interval methods are tested. These are (1) the 'best' location estimate using the biweight scale estimate and (2) the biweight location using the biweight scale estimate. See Appendix I (p. 211) for details.

The tests of the hypotheses are with the alternative industry profitability operationalizations:

(i) H0: Risk-adjusted RROE is zero for all firms against the alternative hypothesis that risk-adjusted RROE is non-zero for at least one firm. The industry ROE is estimated using subsection I.2: That is, the 'best' location estimate and the biweight scale estimate are used to form the robust confidence intervals (RCI).

(ii) H0: Risk-adjusted RROE is zero for all firms against the alternative hypothesis that risk-adjusted RROE is non-zero for at least one firm. The industry ROE is estimated with RCI using the biweight location and scale estimates. See Appendix K for details.

(iii) H0: Risk-adjusted RRNOA does not exist for any firm against the alternative hypothesis that risk-adjusted RRNOA exists for at least one firm. The industry RNOA is estimated using subsection I.2: That is, the 'best' location estimate and the biweight scale estimate are applied to form the RCI.

(iv) H0: Risk-adjusted RRNOA does not exist for any firm against the alternative hypothesis that risk-adjusted RRNOA exists for at least one firm. The industry RNOA is estimated with RCI using the biweight location and the biweight scale estimates. See Appendix K for details.

6.3.2.1 The data set for the tests of Proposition 4-6

According to Appendix C, the database has 33,251 firm-year observations when the full period from 1977 to 1996 is considered; of those, only 25,245 observations are structurally stable enough to test.

The tests of the hypotheses are further restricted to cover years up to and including 1994. This is because of comparability problems of the data from 1995 forward with the previous periods.

All firms belonging to industry-years where there are fewer than five firms per year are excluded in order to provide a minimum acceptable level of firms in an industry-year.

The first year, 1977, is excluded from the test since the accounting rates-of-returns require beginning of period capital and no such data are available for 1977.

An accounting rate-of-return is deleted if the data are missing or if the capital measure is zero or negative. Traceable imputations are also deleted from the data set.

With these auxiliary restrictions, there are 22,200 observations available for testing the hypothesis whether $_{t-1}RROE_{it}^*$ is non-zero for at least one firm. There are 22,193 observations available for testing the hypothesis whether $_{t-1}RRNOA_{it}^*$ is non-zero for at least one firm.

6.3.2.2   The results for the double-sided t test on risk-adjusted RROE

The t-statistic is measured for each observation according to [EQ K-2] and it is evaluated against a robust threshold value according to the choice rule in Appendix K (p. 219). If the absolute value of the t-statistic is greater than the threshold value, the null hypothesis is rejected in favor of the alternative hypothesis.

The test is evaluated at both the five and the one percent significance level in Table 6-3. Supplementary results are also reported from tests at the significance level of two and ten percent.

The RCI method uses the biweight location and scale estimate with $c = 9$. RCI is used for the tests of hypotheses that have significance levels set at five, two and, one percent.

The best location estimate (BLE) method is also used in the tests of the hypotheses that have significance levels at five percent. Since the BLE uses the biweight scale estimate with $c = 9$ and a location estimate that is not a biweight estimate, the reliability of the test is not known in advance and thus it might be like comparing apples and oranges. This problem is mended with Table 6-4 where the fit between the BLE and the RCI methods is listed.

The results from the hypotheses tests are presented in Table 6-3.

Risk-adjusted RROE=0 for all firms vs. H_{1A}: Risk-adjusted RROE≠0 for at least one firm									
Estimators Significance	**RCI** α=10%		**RCI** α=5%		**BLE** α=5%		**RCI** α=2%		**RCI** α=1%
	Observations	**%**	**Observations**	**%**	**Observations**	**%**	**Observations**	**%**	**Observations**	**%**
Rejected H_0	17,486	79%	16,710	75%	15,645	70%	15,786	71%	15,171	68%
Not rejected	4,714	21%	5,490	25%	6,555	30%	6,414	29%	7,029	32%
Σ	22,200		22,200		22,200		22,200		22,200	

Table 6-3: Summary of the double-sided t test of H_0: Risk-adjusted RROE=0 for all firms vs. H_{1A}: Risk-adjusted RROE≠0 for at least one firm.

The results from the tests reject the null hypothesis of risk-adjusted RRNOA=0 in favor of the alternative hypothesis at a significance level of one percent for the great majority of observations. Approximately 68 percent of all the risk-adjusted RROE are non-zero at the one percent significance level, suggesting that the hypothesis test fails to reject Proposition 4-6 when both the

firm's operating and financial activities are allowed to impinge on the results. Failure to reject Proposition 4-6 means that it is not possible to exclude the possibility that there exists arbitrage opportunities in the market. These arbitrage opportunities may exist at the operative level, at the financial level, or at both the operative and financial levels of the firm. The additional hypothesis test uses the risk-adjusted RRNOA and tests Proposition 4-6 when only operating activities are allowed to affect the results.

RROE=0 vs RROE≠0			
α=5%			
BLE	RCI	Observations	%
		5,245	23.6%
	Reject H_0	1,310	5.9%
Reject H_0		245	1.1%
Reject H_0	Reject H_0	15,400	69.4%
	Σ	22,200	

Table 6-4: Comparison of the robust confidence interval t-test and the best location estimate double-sided t test of H_0: Risk-adjusted RROE=0 for all firms vs. H_{1A}: Risk-adjusted RROE≠0 for at least one firm.

The method of best robust location uses the best possible robust estimate of location while it retains the biweight scale estimate. This could decrease its reliability, but as Table 6-4 shows, the fit between the two alternative methods of estimating industry ROE is not seriously affecting the outcome of the tests.

The two methods of estimating the RCIs agree on the significantly non-zero risk-adjusted RROE. Both methods find that in 15,400 of 22,200 the observation of risk-adjusted RROE is significantly non-zero. In only about seven percent is there dissimilarity between the methods.

Since both measurement methods arrive at the same conclusion for the vast majority of observations, the results are not materially affected by miss-specification of the RCI estimation method.

6.3.2.3   The results for the double-sided t test of risk-adjusted RRNOA

Risk-adjusted RROE is affected by the firms' financial activities, which can drive the results in subsection 6.3.2.2. This section studies the results for the double-sided t test where the alternative hypothesis argues that risk-adjusted RRNOA is non-zero for at least one firm. This means that the firm's residual rates-of-returns are applied in the tests while excluding the effect of the firm's financial activities.

Since risk-adjusted RRNOA requires NOA rather than EQ, more firms are likely to have zero or negative capital, which implies that they cannot be used in the analysis. This shows up in the analysis as six fewer observations when risk-adjusted RRNOA is investigated than if risk-adjusted RROE is investigated. Table 6-5 and Table 6-6 show the outcomes of these tests.

Risk-adjusted RRNOA=0 for all firms vs. H_{1B}: Risk-adjusted RRNOA≠0 for at least one firm										
Estimators Significance	RCI α=10%		RCI α=5%		BLE α=5%		RCI α=2%		RCI α=1%	
	Observations	%	Observations	%	Observations	%	Observations	%	Observations	%
Rejected H_0	17,676	80%	16,856	76%	15,725	71%	15,783	71%	15,179	68%
Not rejected	4,517	20%	5,337	24%	6,468	29%	6,410	29%	7,014	32%
Σ	22,193		22,193		22,193		22,193		22,193	

Table 6-5: Summary of the double-sided t test of H_0: Risk-adjusted RRNOA=0 for all firms vs H_{1B}: Risk-adjusted RRNOA≠0 for at least one firm.

Table 6-5 corroborates the findings from Table 6-3 in that the vast majority of observations (68 percent) reject the null hypothesis in favor of its alternative at the one percent significance level. This suggests that the results reported in Table 6-3 do not depend on whether the residual accounting rate-of-returns includes or excludes the effect of the firms' financial activities.

From Table 6-3 and Table 6-5, it appears to be rather normal for firms to report risk-adjusted residual accounting rates-of-returns, an observation consistent with Proposition 4-6.

RRNOA=0 vs RRNOA≠0			
α=5%			
BLE	RCI	Observations	%
		5,066	22.8%
	Reject H_0	1,402	6.3%
Reject H_0		271	1.2%
Reject H_0	Reject H_0	15,454	69.6%
	Σ	22,193	

Table 6-6: Comparison of the robust confidence interval t-test and the best location estimate double-sided t test of H_0: Risk-adjusted RRNOA=0 for all firms vs H_{1B}: Risk-adjusted RRNOA≠0 for at least one firm.

The alternative hypothesis of having non-zero risk-adjusted RRNOA for at least one firm is tested using (i) the biweight estimator for location and scale with $c = 9$ and (ii) the BLE with the biweight scale estimator. The difference between the two methods, which is small, is reported in Table 6-6. Alternative (i) rejects about five percent more observations than what alternative (ii) does, which is similar to the difference in methods using the risk-adjusted RROE.

6.3.2.4 Do risk-adjusted residual accounting rates-of-returns exist?

The tests whose results are presented on Table 6-3 to Table 6-6 fail to reject Proposition 4-6 and thus they show that risk-adjusted residual accounting rates-of-returns permeate the firms' financial reporting. They also show that it appears to be the rule rather than the exception for firms to report risk-adjusted residual accounting rates-of-returns. Thus, arbitrage opportunities are abundant and that firms find and use those opportunities to earn arbitrage profits.

It also appears as if the majority of the arbitrage opportunities are found already at the firms' operating level. No study has been carried out on the possibility to earn risk-adjusted residual accounting rates-of-returns from the firms' financial activities and so it cannot be excluded that there are possibilities for firms to earn arbitrage profits in their financial activities as well. Indeed, the re-

sults show that the market is in disequilibrium and thus it must be the case that financial activities can also earn arbitrage profits.

Of 22,200 statistical tests based on the risk-adjusted RROE, 15,171 observations report a risk-adjusted RROE significant non-zero at a significance level of 1 percent. 22,193 statistical tests that use risk-adjusted RRNOA are available for tests of hypotheses. These tests reveal that 15,179 observations are significant non-zero at a one percent significance level, indicating that 68 percent of all observations fail to falsify Proposition 4-6.

Dechow et al. (1999), Meyers (1999), McCrae & Nilsson (2001), Callen & Morel (2001), Gregory, Saleh, & Tucker (2005), and Giner & Iñiguez (2006) all report that residual income exists since they find that residual income regresses. However, these studies operationalized the residual income differently than the way I do in that they fail to distinguish between residual income that is due to the accounting system and residual income that is due to arbitrage opportunities. Indeed, they even assume no residual income because of arbitrage opportunities.

Instead, I remove the residual income pertaining to the accounting generated residual income and focus on residual income whose source is arbitrage opportunities. From section 6.3.2, it is clear that risk-adjusted residual rates-of-returns exist even after the bias from the accounting system is removed and thus arbitrage-based residual income exists. Thus, I fail to reject Proposition 4-6.

6.4 Does the market learn through discovery?

Chapter 2 posits that individuals who are limited rational because of limited knowledge have the capacity to learn from past mistakes in the form of discovery (Definition 2-9, p. 44). With discovery as a learning process, Chapter 3 asserts that prices cannot be randomly walking; rather, they are partly a function of the price history that trends towards no-arbitrage prices (Proposition 3-1, p. 57; Proposition 3-2, p. 59).

Chapter 4 takes the analysis further by proposing that limits values for risk-adjusted subjective expected residual accounting rates-of-returns are zero (Proposition 4-5 (p. 72) states it in a subjective certain choice and Proposition 4-7 (p. 78) states it in a subjective uncertain choice). The empirically testable proxy for the risk-adjusted subjective expected RROE is [EQ 6-8] and [EQ 6-9] for risk-adjusted subjective expected RRNOA.

Proposition 4-7 implies that [EQ 6-8] and [EQ 6-9] should (i) approach zero as time passes and (ii) not be randomly walking variables. Alternative (i) is equivalent to proposing that the firm's profitability approaches the industry profitability as time passes. This is what is tested in this chapter. A goodness-of-fit test assesses alternative (ii) and is reported in Appendix N (p. 241).

Subsection 6.4.1 operationalizes the hypotheses and subsection 6.4.2 presents the results from the tests.

6.4.1 The tested hypotheses

As in section 6.3, one alternative hypothesis is posed per variable.

H$_{1A}$: Proposition 4-7 argues that the risk-adjusted subjective expected residual returns on equity (RROE) regress towards zero with time. Using the operationalized risk-adjusted RROE, [EQ 6-8], it can be expressed as:

$$_t RROE^*_{it+1} = \alpha + \beta \cdot {}_{t-1}RROE^*_{it} + \varepsilon_{t+1} \qquad \forall i \in I \in M \qquad \text{[EQ 6-10]}$$

$$\text{where} \qquad \beta < 1$$

and where $\alpha = 0$, i is the firm, I is the industry, M is the market, t is the year, and ε_t is a white noise disturbance.

H$_{0A}$: The risk-adjusted RROE do not diminish as time passes. That is, using [EQ 6-10] this can be expressed as:

$$\beta \geqslant 1 \qquad \qquad \forall i \in I \in M$$

and where $\alpha = 0$, i is the firm, I is the industry, M is the market, t is the year, and ε_t is a white noise disturbance.

H$_{1B}$: Proposition 4-7 argues that the risk-adjusted subjective expected RRNOA regress towards zero with time. Using the operationalized risk-adjusted RRNOA, [EQ 6-9], it can be expressed as:

$$_t RRNOA^*_{it+1} = \alpha + \beta \cdot {}_{t-1}RRNOA^*_{it} + \varepsilon_{t+1} \qquad \forall i \in I \in M \qquad \text{[EQ 6-11]}$$

$$\text{where} \qquad \beta < 1$$

and where $\alpha = 0$, i is the firm, I is the industry, M is the market, t is the year, and ε_t is a white noise disturbance.

H$_{0B}$: The risk-adjusted RRNOA do not diminish as time passes. Using [EQ 6-11] this can be expressed as:

$$\beta \geqslant 1 \qquad \qquad \forall i \in I \in M$$

and where $\alpha = 0$, i is the firm, I is the industry, M is the market, t is the year, and ε_t is a white noise disturbance.

Having $\alpha \neq 0$ can imply many things. For example, it might imply that firms can earn sustainable risk-adjusted RROE and sustainable risk-adjusted RRNOA. However, it is equally plausible that the test models are incorrectly specified and that there is an omitted variable problem.

6.4.2 Results of the tests of the hypotheses

The hypotheses in subsection 6.4.1 are tested using panel regressions. Many different panel regression models may be applicable. Dielman (1989), e.g., describes seven panel regression models from the pooled regression model to models with time-varying and cross-sectional varying regression pa-

rameters. The choice of a panel regression model depends on the panel's statistical properties or on assumptions about those properties.

Dechow, Hutton & Sloan (1999), Gregory, Saleh, & Tucker (2005), McCrae & Nilsson (2001) all use the pooled regression model. There is no information in their research about specification tests that assess the feasibility of such an assumption. Nor is there any explicit discussion on the choice of panel regression model.

Empirical industrial economics research that applies panel regressions on accounting data (e.g., Jacobsen 1988; Jacobson & Aaker 1985; Mueller 1977; Mueller 1990; Waring 1996) also applies the pooled regression model without presenting support for it in the form of specification tests.

The panel regression model that I use is a fixed-effect panel regression model with panel corrected standard errors (PCSE) having AR(1) errors and not a pooled regression model. The choice of a panel regression model is based on four specification tests whose results are reported in Appendix M (p. 233). The panel regression models considered and the evaluation criteria are discussed in Appendix L (p. 221).

6.4.2.1   Results from panel regressions using risk-adjusted RROE

The results from the panel regressions are organized such that the fit statistics are first presented followed by the results of the parameter estimates.

Table 6-7 reports the fit statistics for the panel regressions on the risk-adjusted RROE. See [EQ L-18] for its definitions. RHOHAT is the estimated first-order serial correlation. DFE is the degrees of freedom, SST is the total sum of squares, SSR is the regression sum of squares, and SSE is the error sum of squares. MSE is the mean squared error, RMSE is the root mean squared error, and RSQ is the coefficient of determination, which shows how large the proportion of the data set's variability can be explained by the econometric model. See subsection L.3.4 for definitions of the fit statistics.

Field	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
							Fit origin						
RHOHAT	-0.141	-0.034	-0.068	-0.057	-0.059	-0.105	-0.009	-0.065	-0.065	-0.070	-0.037	-0.133	-0.114
DFE	899	1,064	1,907	1,928	1,970	2,009	1,925	1,988	1,850	1,826	1,748	1,724	1,856
SST	117.0	126.1	243.0	210.0	191.3	159.8	110.6	136.9	130.5	210.0	241.7	302.4	408.1
SSR	56.6	52.9	97.9	82.5	67.3	52.6	38.0	47.1	48.8	72.4	86.0	139.2	192.6
SSE	60.4	73.2	145.2	127.5	124.0	107.2	72.6	89.8	81.7	137.6	155.7	163.3	215.5
MSE	0.068	0.069	0.076	0.066	0.063	0.054	0.038	0.045	0.044	0.076	0.089	0.096	0.118
RMSE	0.262	0.262	0.276	0.258	0.251	0.232	0.194	0.213	0.211	0.275	0.299	0.310	0.343
RSQ	**48.4%**	**41.9%**	**40.3%**	**39.3%**	**35.2%**	**32.9%**	**34.4%**	**34.4%**	**37.4%**	**34.5%**	**35.6%**	**46.0%**	**47.2%**

Table 6-7: Estimated serial correlation in the fixed-effects panel regressions and fit statistics for the fixed-effects panel regression using the risk-adjusted RROE with PCSE when assuming AR(1) errors.

The most notable result in Table 6-7 is that the model appears to be able to explain rather much of the variability in the data. The coefficients of determination range from 32.9 percent to 48.4 percent (average is 39 percent).

Dechow et al. (1999, p. 17) report a coefficient of determination of 34 percent, Gregory et al. (2005, p. 509) 48 percent, and McCrae & Nilsson (2001, p. 328) 29 percent.

Meyers (1999) initiates another type of evaluation of the Ohlson (1995) and Feltham & Ohlson (1995) models that uses individual time-series regressions for each firm. Some of those articles present coefficient of determination (Meyers and Callen & Morel do not). Gregory et al. (2005, p. 509) report an average coefficient of determination of 49 percent. Giner & Iñiguez (2006, p. 179) report an average coefficient of determination of 51 percent.

It appears that the coefficients of determination that I report is approximately are comparable with previous research.

However, applying a pooled regression model with spherical disturbances to my data using the risk-adjusted RROE only yields coefficients of determination ranging from 0.8 to 7.4 percent. The random effect model on the same variable yields coefficients of determination from 0.3 to 5.3 percent.

Both of the alternative regression models deliver coefficients of determination that are much lower than what the fixed effects model produces. As seen from the results of the specification tests in Appendix M, the pooled regression model is not an applicable panel regression estimation model for the data.

Table 6-8 shows the results from the panel regressions using the risk-adjusted RROE. In the table, ESTIMATES are the estimated regression parameters. STDE is the regression's standard error and TVALUE_1 is the t-statistics according to definition [EQ L-1], i.e. the t-statistics for assessing whether the estimated regression parameters are significantly less than one. PVALUE is the corresponding p-value. TVALUE_0 is the t-statistics for testing if the estimated regression parameter is significantly different from zero and PVALUE_0 is the corresponding p-value assuming the two-sided test.

	Fit origin												
Field	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
ESTIMATES	-0.095	-0.126	-0.108	-0.159	-0.111	-0.065	-0.142	-0.198	-0.181	-0.196	-0.091	-0.051	-0.102
STDE	0.243	0.181	0.238	0.221	0.238	0.221	0.159	0.231	0.239	0.241	0.304	0.244	0.250
TVALUE_1	-4.50	-6.21	-4.66	-5.24	-4.67	-4.82	-7.20	-5.19	-4.94	-4.96	-3.59	-4.31	-4.41
PVALUE_1	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.02%**	**0.00%**	**0.00%**
TVALUE_0	-0.39	-0.69	-0.45	-0.72	-0.47	-0.29	-0.89	-0.86	-0.76	-0.81	-0.30	-0.21	-0.41
PVALUE_0	**69.5%**	**48.8%**	**65.0%**	**47.1%**	**64.1%**	**76.9%**	**37.1%**	**39.1%**	**44.9%**	**41.5%**	**76.6%**	**83.4%**	**68.4%**

Table 6-8: Parameter estimates from the fixed-effects panel regression using the risk-adjusted RROE with PCSE and AR(1) errors.

The estimated regression parameters are presented in Table 6-8. If the alternative hypotheses are correct, they should be less than one and PVALUE_1 should be at least less than five percent. The table reports that the estimated regression parameters are significantly less than one and not

significantly different from zero. Indeed the p-values indicate that the results are significant even at a threshold level less than one percent and are consequently strong results.

Proposition 4-7 cannot be falsified based on the results reported in Table 6-8 since the null hypothesis is rejected. This means that the test cannot falsify the notion that the market learns according to Proposition 3-2.

Proposition 3-2 (p. 59) is silent on how fast the actors in the market learn. All that it proposes is that they learn through discovery, i.e. that $\beta \in [0,1)$. If the process of learning is slow, β is close to 1. If β is close to zero, it means that the firms learn fast.

If we have no-arbitrage, there are random walks in the rates-of-returns (e.g., Fama 1965a; Fama 1965b), which implies that $\beta = 1$. Since the test finds that $\beta = 0$, it rejects randomly walking risk-adjusted RROE.

6.4.2.2   Results from panel regressions using risk-adjusted RRNOA

This subsection focuses on the results from the panel regression using the risk-adjusted RRNOA. Table 6-9 reports the results for the panel regressions using the risk-adjusted RRNOA. The coefficients of determination range from 34.1 to 54.1 percent (average is 42.5 percent)

The coefficients of determination are slightly higher for risk-adjusted RRNOA as compared with their counterparts in subsection 6.4.2.1. A good explanation for this difference is probably that the risk-adjusted RROE allows for financial leverage, which brings up a firm's volatility in ROE as compared with the volatility in RNOA. The high volatility in the underlying ratios therefore cascades into greater problems for the regressions to minimize the squared errors. This is possible to see by studying the SST, the SSR, and the sum of squared errors (SSE). SST and SSE are about 5 times greater for risk-adjusted RROE than they are for risk-adjusted RRNOA, suggesting that the variability is much greater for risk-adjusted RROE than for risk-adjusted RRNOA, which makes it more difficult to fit a straight-line through the cluster of observations while yielding low SSE.

| Field | Fit origin |||||||||||||
	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
RHOHAT	-0.129	-0.084	-0.087	-0.035	0.005	-0.088	-0.067	-0.138	-0.126	-0.024	-0.027	-0.160	-0.187
DFE	869	1,061	1,976	1,919	1,931	1,904	1,844	1,829	1,664	1,736	1,679	1,721	1,880
SST	16.9	21.5	47.0	41.9	39.9	36.1	27.4	31.8	28.4	47.6	57.9	76.6	107.3
SSR	7.9	9.6	20.1	17.1	14.0	13.3	11.2	14.5	12.7	16.2	21.3	38.4	58.0
SSE	9.0	12.0	26.9	24.8	26.0	22.7	16.3	17.2	15.7	31.4	36.6	38.2	49.3
MSE	0.011	0.011	0.014	0.013	0.013	0.012	0.009	0.010	0.010	0.018	0.022	0.023	0.027
RMSE	0.103	0.106	0.117	0.114	0.116	0.110	0.094	0.098	0.098	0.134	0.148	0.151	0.165
RSQ	**46.6%**	**44.5%**	**42.7%**	**40.9%**	**35.0%**	**37.0%**	**40.7%**	**45.7%**	**44.8%**	**34.1%**	**36.8%**	**50.1%**	**54.1%**

Table 6-9: Estimated serial correlation in the fixed-effects panel regressions and fit statistics for the fixed-effects panel regression using the risk-adjusted RRNOA with PCSE and assuming AR(1) errors.

In Table 6-10 the parameter estimates and standard errors are presented for risk-adjusted RRNOA. The t-values and p-values that the tests of the hypotheses use also appear in Table 6-10

The table shows that the null hypotheses $\beta \geq 1$ are rejected for all panels in favor of the alternative hypotheses $\beta < 1$. This means that also when risk-adjusted RRNOA is used to test empirically Proposition 4-7 the tests fail to falsify it. Thus, I cannot reject the possibility that firms learn through discovery.

Field	Fit origin												
	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
ESTIMATES	-0.072	-0.127	-0.122	-0.215	-0.145	-0.057	-0.086	-0.157	-0.183	-0.161	-0.039	0.005	-0.054
STDE	0.251	0.222	0.258	0.245	0.252	0.248	0.160	0.239	0.230	0.246	0.309	0.252	0.254
TVALUE_1	-4.27	-5.08	-4.34	-4.96	-4.55	-4.26	-6.78	-4.85	-5.15	-4.73	-3.36	-3.95	-4.15
PVALUE_1	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.04%**	**0.00%**	**0.00%**
TVALUE_0	-0.29	-0.57	-0.47	-0.88	-0.58	-0.23	-0.54	-0.66	-0.80	-0.65	-0.13	0.02	-0.211
PVALUE_0	**77.5%**	**56.8%**	**63.8%**	**38.0%**	**56.5%**	**82.0%**	**59.1%**	**51.1%**	**42.6%**	**51.3%**	**89.8%**	**98.3%**	**83.3%**

Table 6-10: Parameter estimates from the fixed-effects panel regression using the risk-adjusted RRNOA with PCSE and AR(1) errors.

As with the risk-adjusted RROE, the tests reveal that the parameter estimates are not significantly different from zero. This means that the actors learn fast, and within a year of public disclosure of the discovery of an arbitrage opportunity, the risk-adjusted RROE disappears.

To conclude, the null hypothesis of random walk is again rejected in favor of learning through discovery that leads towards equilibrium. The variable PVALUE_1 shows that the tests are all significant at a significance level less than one percent.

6.4.2.3   Does the market discover? — A discussion

All panel regressions using the risk-adjusted RROE and the risk-adjusted RNOA fail to reject Proposition 4-7. Since the tests fails to reject Proposition 4-7 then Proposition 3-2 cannot be rejected. Because Proposition 3-2 cannot be rejected Proposition 3-1 could not be rejected. They therefore fail to falsify that the market learns through discovery (Definition 2-9).

Another effect of the panel regressions is that they show that the variables do not behave as if they are randomly walking, which is an EMH assumption.

The learning-through-discovery effect is so strong that it is complete within a year after discovery. This is completely at odds with random walk, and has important suggestions for forecasting too: Since β is zero, or very close to zero, the best forecast of next year's risk-adjusted residual accounting rate-of-return is zero.

This means that it is not correct to assume that a firm can make sustainable arbitrage profits. Arbitrage profits are temporary by nature since the actors in the market discover and use the arbitrage opportunities. It implies that there are innumerable arbitrage profit opportunities in the market but that those are fleeting, vanishing swiftly after they have been discovered and acted on.

Although it is difficult to compare research that uses similar yet different models, operationalizations, and estimation techniques, I nevertheless compare my results with those from Dechow, Hutton, & Sloan (1999), Meyers (1999), McCrae & Nilsson (2001), Callen & Morel (2001), Gregory,

Saleh, & Tucker (2005), and Giner & Iñiguez (2006), all of whom perform tests using Ohlson's (1995) linear information dynamics model.

Dechow et al. estimate the parameter ω in Ohlson's model: $RI_{t+1} = \omega RI_t + \upsilon_t + \varepsilon_{1t+1}$ while assuming $\upsilon_t = 0$. Dechow et al. (1999, p. 16-17) appear to be using the pooled regression model in the estimation and find that $\omega = 0.62$. I estimate my $\beta = 0$ (which at face value is similar to ω).

I believe there are at least four reasons for this difference between my results and those reported by Dechow et al. These differences are as follows: (i) I remove both risk and accounting bias in my operationalization of the variables, whereas Dechow et al. only remove risk. (ii) They remove risk naïvely since they assume that the cost of equity is 12 percent for all firms at all times (p. 14), whereas I remove risk at different levels for different industries, even allowing it to be a time variable. (iii) They measure residual income from earnings before extraordinary items, whereas I maintain the clean-surplus relation; (iv) I use a fixed effect regression model, whereas Dechow et al. use the pooled regression model.

Indeed, I believe that the fourth reason is an important reason for the differences in results since Table 7-3 in section 7.3 (p. 127) shows my parameter estimates using the pooled regression model. These parameter estimates indicate that the median β is strictly positive. Indeed, Myers (1999) corrects some of the above-mentioned problems and finds significant different results than what Dechow et al. report.

Meyers (1999, p. 12) notes that the parameters "…must be a function of the firm's economic pressures, production technology, and accounting policies". Meyers solves this problem by performing firm-by-firm time-series regressions. The author (1999, p. 15) also maintains that the naïve risk adjustment is too simple and thus replaces it using an industry cost of equity estimation. Meyers' method is not much unlike my own method since I adjust for these errors by sweeping away differences in economic pressures, production functions, accounting rules, and risks by using industry rates-of-returns defined at the three-digit industry classification level. I addition, I use a more sophisticated panel regression model than both what Dechow et al. and Meyers use since I allow for fixed effects in the panel regressions. I thus also allow for heterogeneity across the firms. Meyers (1999, p. 17) solves this problem by performing individual firm regressions and taking the median ω that he finds to be 0.23, which is much closer to my parameter estimates.

Differences still exist between Meyers' and my results and I believe that part of this difference is possible to trace to Meyers' firm-by-firm regressions, which only have 15—22 observations. It is a problem for regressions to have so few observations and this is something that Meyers notes (p. 26). This may bias Meyers' results even though his median is based on 2,601 regressions. The

bias from such a procedure does not asymptotically regress to zero with an increased number of individual regressions, but regresses to zero with an increased number of observations per firm.

The fixed-effect method that I use is not fault free since bias in dynamic fixed-effect models do not asymptotically regress to zero with increasing number of firms. Its bias also regresses to zero with an increase in the number of years in the panel (cf. the Nickell bias in dynamic fixed-effect regression models).

Thus, I conclude that Meyers' method is a significant improvement to the method employed by Dechow et al. Moreover, we can see that it changes the results from Dechow et al. levels towards my levels. The remaining difference can be attributed to the fact that both Meyers and I have room for technical improvements in our estimation methods.

Research that builds on Dechow et al. and Meyers includes the work of McCrae & Nilsson (2001), Callen & Morel (2001), Gregory, Saleh, &Tucker (2005), and Giner & Iñiguez (2006). McCrae & Nilsson report $\omega = 0.523$, Callen & Morel $\omega = 0.469$, Gregory et al. $\omega = 0.62$, and Giner & Iñiguez $\omega = 0.55$.

These articles either use pooled regression models, which I find are not valid for my data and that probably are not valid for any panel regression models using accounting data because of the problems that I discuss in Appendix M, or they use cross-sectional regression, which still exposes them to Meyers criticism of, e.g., differences that are due to differ accounting rules.

The cross sectional samples that Giner & Iñiguez use are small (ranges between 89 and 573 observations), which may make their results spurious. Callen & Morel use firm-by-firm regressions but do not have more than 27 years of observations, which is very little for regression estimation. Small samples (by cross section or by years) can be a factor that affects the results.

There may be additional factors that affect the difference between the cited research and my findings, which is due to how I operationalize the risk-adjusted residual rates-of-returns. I remove, e.g., accounting bias and risk by using the industry accounting rates-of-returns. It may be that industries also earn risk-adjusted residual rates of returns. If that is the case, I also remove those, whereas Dechow et al., Meyers, McCrae & Nilsson, Callen & Morel, Gregory et al., and Giner & Iñiguez all study the effect of jointly regressing both accounting and arbitrage residual incomes if one assumes that the market is inefficient. This can be another explanation to account for the differences between studies.

6.5 Summary

Chapter 2 and Chapter 3 (with Appendix A—Appendix C) provide the core to the theory of Homo comperiens. is the theory is applied to firms in Chapter 4, and it proposes market-pricing models for firms. These models are not testable, but as the analysis in Chapter 4 shows it is possible to deduce

Proposition 4-6 and Proposition 4-7 from the market-pricing models. Proposition 4-6 and Proposition 4-7 are testable. Chapter 6 presents the test method and the results of the tests.

Proposition 4-6 argues that the theory of Homo comperiens' arbitrage market exhibits a certain trait, i.e. that firms (at least one) report risk-adjusted subjective expected residual accounting rates-of-returns. This proposition is tested using robust double-sided t tests, the results of which are presented in this chapter.

The proposition is evaluated using 22,200 (ex post) risk-adjusted return on equity observations and 15,171 of the observations reject the null hypothesis of significant zero risk-adjusted return on equity at the one percent significance level in favor of the alternative hypothesis of having significant non-zero risk-adjusted return on equity.

The proposition is also evaluated using 22,193 risk-adjusted returns on net operating assets observations and 15,179 of those observations reject the null hypothesis of significant zero risk-adjusted return on net operating assets at the one percent significance level in favor of the alternative hypothesis of having significant non-zero risk-adjusted return on net operating assets.

The robust double-sided t tests therefore fail to falsify Proposition 4-6.

Proposition 4-6 assumes that the market has arbitrage opportunities and this allows for the introduction of Proposition 4-7. Proposition 4-7 posits that the market's actors learn from past mistakes through discovery and that this shows itself as diminishing risk-adjusted subjective expected residual accounting rates-of-returns. This chapter applies a panel regression model to assess the proposition.

Proposition 4-7 is evaluated on 13 panels of risk-adjusted residual return on equity and 13 panels having risk-adjusted RRNOA. This means there are 26 tests of the hypotheses tests that assess Proposition 4-7. In all these tests the null hypothesis of random walking risk-adjusted residual accounting rates-of-returns is rejected in favor of its alternative hypotheses of diminishing risk-adjusted residual accounting rates-of-returns.

The panel regression tests also find that the discovery is so strong that it appears as if firm's risk-adjusted residual rates-of-returns diminish within one year after discovery, which implies that arbitrage opportunities are temporary (however, the results also show that they are abundant).

It is possible to reject the hypotheses without them being false (Type I error). To determine that the results are not false it must be possible to create reproducible effects on other sets of empirical observations before a valid rejection of a theory can take place. That is, it is necessary to assess theory's external validity. Chapter 7 assesses the external validity of the theory of Homo comperiens by testing its predictive accuracy using a fixed-size rolling-window out-of-sample forecasting method.

CHAPTER 7—ASSESSING HOMO COMPERIENS' PREDICTIVE ACCURACY

> "…a few basic statements contradicting a theory will hardly induce us to reject it as false." Popper (1959, p. 86)

7.1 Introduction

The theory of Homo comperiens is a theory that attempts to describe how choice is done and how it affects the market. It is not a theory that seeks to make normative statements of how choice should be made. This means that it is a positive theory that makes an endeavor to provide a system of generalizations that can be used to make predictions about phenomena.

Some theorists propose that the appropriateness of a positive theory should be judged on its internal validity (e.g., Mises 1949, p. 858). That is, the appropriateness of a theory is based on whether it is consistent and exhaustive. Nevertheless, since positive theory tries to explain what will happen as well as the consequences of actions, the meaningfulness of the theory must be primarily based on its predictive ability even though the internal validity is of interest.

Popper (1959) argues that only theory that cannot be falsified using empirical observations can endure. It is not enough to use tests of hypotheses, whose results are presented in Chapter 6 because such tests may lead to the rejection of the hypotheses when they are true. This is equivalent to the Type I error in statistics or as Popper (1959, p. 86) says, "…a few basic statements contradicting a theory will hardly induce us to reject it as false."

Popper (1959, p. 86) submits that a theory is rejected only when a reproducible effect that rebuts theory is revealed. This means that tests that just establish the goodness-of-fit of empirical observations to hypotheses are not enough to discard a theory. According to Popper, the results must be reproducible on other sets of empirical observations before a valid rejection of a theory takes place. Friedman's (1953) view on positive theory resembles Poppers idea in that Friedman calls for predictive tests.

An example of research that commit a Type I error is Ou & Penman (1989a) who reject the null hypothesis of random walk in favor of non-random walk, and where a subsequent attempt by Holthausen & Larcker (1992) to replicate the results on another data set fails.

This chapter presents a method for assessing the predictive accuracy and the results thereof of the theory of Homo comperiens. The validation method is designed such that it assesses whether it is possible to make forecasts on out-of-sample data based on the parameterization reported in Chapter 6 that yield greater predictive ability than the random-walk model's predictive ability.

Proposition 4-6 and Proposition 4-7 withstand the hypotheses tests in Chapter 6 and Appendix N. If the propositions also withstand predictive tests, this provides considerable support for the theory of Homo comperiens.

The chapter is organized such that the method for testing predictive accuracy is presented first, followed by the section containing the results from the tests of predictive accuracy.

7.2 The method for assessing the theory's external validity

Friedman (1953, p. 7) writes: "The ultimate goal of a positive science is the development of a 'theory' or, 'hypothesis' that yields valid and meaningful (i.e. not truistic) predictions about phenomena not yet observed." Friedman appears to be inspired by Popper (1959). Further, it is apparent that Friedman thinks that the development of a theory needs empirical corroboration and that such external validation should be based on predictions.

Despite the need for validation, much research in both economics and market-based accounting often fails to validate their propositions and findings using predictive accuracy. Examples from economics that do not validate the results include Brown & Ball (1967), Brozen (1970), Cubbin & Geroski (1987) Mueller (1977, 1990). Beaver (1970), and Fama & French (2000) are examples from market-based accounting research that do not validate their results.

In the situation when it is used for external validation, the prediction is often done on the data set on which that models parameters where originally estimated on. This is referred to as in-sample validation. Research using this approach includes studies by Dechow et al. (1999), Freeman et al. (1982), Giner & Iñiguez (2006), Gregory, et al. (2005), McCrae & Nilsson (2001), and Watts & Leftwich (1977).

There is presently a consensus among forecasters that predictive models should be assessed on out-of sample tests (Tashman 2000, p. 438). Out-of sample tests of predictive accuracy can, e.g., make predictions on a phenomenon, and then wait to see how the future unfolds. This is neither a convenient nor a time efficient method. Holdout samples are presently a common way in which out-of-sample tests are carried out (Tashman 2000, p. 438). A holdout sample is part of the data set that was not used to estimate the model.

This thesis tests the predictive accuracy of the theory of Homo comperiens by making predictions on holdout samples that are compared with other predictions based on perfect rationality. Next follows the tested models, which is followed by a description of the prediction method and the evaluation criteria.

7.2.1 The prediction models

Proposition 4-7 (p. 78) reasons that the risk-adjusted residual return on equity (risk-adjusted RROE) and the risk-adjusted residual return on net operating assets (risk-adjusted RRNOA) regress towards

zero. This is contrary to general equilibrium theory, which proposes randomly walking rates-of-returns.

Proposition 4-7 does not predict how fast the learning process is and two alternatives for the limited rational choice are offered based on Proposition 3-2 (p. 59). The first alternative suggests that learning is immediate, i.e. within one period (here it is a year after discovery) and the second alternative proposes that learning is a gradual process in which the risk-adjusted residual accounting rates-of-returns regress over more than one period until they are zero.

This implies that I test three predictive models. The results reported in Chapter 6 differ from those reported by Dechow, et al. (1999), McCrae & Nilsson (2001), Callen &Morel (2001), Gregory, et al. (2005), and Giner & Iñiguez (2006).

Thus, I also compare the three predictive models with a fourth model that is based on the results reported by McCrae & Nilsson (2001). I compare my predictive models with McCrae & Nilsson rather than with Dechow et al. because the former authors use Swedish data.

The basic prediction model is:

$$x_{iT+n} = \alpha_T + x_{iT} \cdot \beta_T \qquad \text{[EQ 7.1]}$$

Then we have the four prediction models based on [EQ 7.1]:

$$\alpha_T = 0 \,, \ \beta_T = 1 \qquad \forall i \in M \qquad \text{[EQ 7-2]}$$

$$\alpha_T \in (\infty, \infty) \,, \ \beta_T \in (0,1) \qquad \forall i \in M \qquad \text{[EQ 7-3]}$$

$$\alpha_T = 0 \,, \ \beta_T = 0 \qquad \forall i \in M \qquad \text{[EQ 7-4]}$$

$$\alpha_T = -0.012 \,, \ \beta_T = 0.523 \qquad \forall i \in M \qquad \text{[EQ 7-5]}$$

where x_{iT} is the last observed (in year T) risk-adjusted residual accounting rate-of-return for firm i in the set of firms M. x_{iT+n} is the n-year predicted risk-adjusted residual accounting rate-of-return.

Note that when $\alpha_T \neq 0$, the prediction model becomes more complex than a $\alpha_T = 0$ prediction model as we forecast further into the future. This is because a three-year forecast is $x_{T+3} = \alpha_T \cdot \left(1 + \beta_T + \beta_T^2\right) + x_T \cdot \beta_T^3$ rather than only $x_{T+3} = x_T \cdot \beta_T^3$.

Model [EQ 7-2] is equivalent to the proposition that random walks in the rates-of-returns, which mean that the best prediction of tomorrow's observation is today's observation. It is therefore a permanent earnings model.

A different model is [EQ 7-3] that describes a market with actors who are limited rational and where they gradually learn. The risk-adjusted residual accounting rates-of-returns gradually regress towards zero over more than one year. Because Chapter 6 reports $\beta_T = 0$, I apply the results from the pooled regression model (with spherical disturbances) for the gradual learning process

since this is the regression model that Dechow, et al. (1999), Gregory, et al. (2005), and McCrae & Nilsson (2001) use. Thus, it is easier to compare and contrast my results with previous research. See Table 7-3 (p. 127) for the intercepts and slopes that I use in this model.

The extreme case of discovery occurs when it is complete after one year. This occurs with model [EQ 7-4], where the best prediction of next year's and the future years' risk-adjusted residual accounting rates-of-returns is zero. That is, this is a transitory earnings model: a firm discovers and acts on ignorance in a year, which alerts its peers who start to act. The action eliminates the arbitrage opportunity within the following year. This prediction model is based on my dynamic fixed-effect (with spherical AR(1)-disturbances) panel regression model, and since heterogeneity is swept away from the model through the intercept, the intercept is biased and useless for predictions. Thus, I fix it to zero.

Model [EQ 7.5] is a special case of [EQ 7-3] since it fixes both the intercept and the slope to the values reported by McCrae & Nilsson (2001, p. 328, Table 2). This allows for the assessment of the validity of my models not only compared with my data but also compared with other results. If McCrae & Nilsson's model is more valid than mine, it should outperform models [EQ 7-3] and [EQ 7-4].

Next, follows a description on how the out-of-sample method for assessing validity by predictive accuracy is implemented.

7.2.2 The out-of-sample predictive method

The external validity of the theory of Homo comperiens is tested by making predictions with [EQ 7-3] and [EQ 7-4] on holdout samples. The forecasts are compared with the outcomes through forecast errors. Similar predictions are also made using the random walk model, [EQ 7-2] and McCrae & Nilsson's (2001) model [EQ 7.5]. The resulting forecast errors from model [EQ 7-2]—[EQ 7.5] are compared using the criteria from subsection 7.2.3. Subsection 7.2.2.1 discusses the method used to make the predictions.

There are several methods to perform tests of predictive ability on holdout samples (see, e.g., Tashman, 2000 for a discussion of alternatives). I validate the results using a fixed-size rolling-window forecasting method.

7.2.2.1   Fixed-size rolling-window forecasts

The validation method uses the period from 1978 to 1994, which is divided into fit periods and test periods. The test periods are the holdout samples on which predictions are made. The fit periods are the periods on which the predictive models are calibrated. A fit period directly precedes the test period and the last observation in the fit period is the forecast origin. Predictions are made for the holdout sample from the forecast origin.

A rolling-window prediction occurs when the forecast origin is gradually updated and new forecasts are made on the updated forecast origin. A benefit with a rolling window compared with its alternative (i.e. the fixed window) is that the number of forecasts increases significantly. A rolling-window forecast provides $T \cdot (T + 1) \cdot 2^{-1}$ predictions, where T is the length of the test period (Tashman, 2000, p. 439). A fixed window only provides T forecasts, one per forecast length. I apply the rolling-window method to increase the number of predictions.

The predictions can either be updated using the existing calibrated model when the forecast origin rolls forward or they can be predicted anew from a recalibrated model (Tashman, 2000, p. 440). If the model is recalibrated, the model is re-estimated on the fit period. Recalibrations are used when the fit period changes. Figure 7-1 illustrates two versions of recalibration, namely the fixed-origin and the fixed-size fit period forecasting.
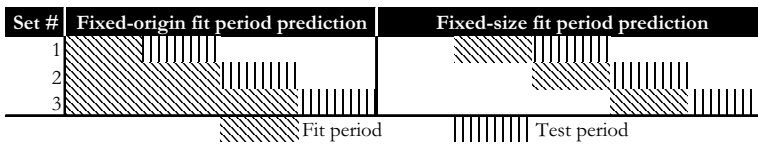
Figure 7-1: The principal difference between having a fixed-origin fit period and having a fixed-size fit period in rolling-window forecasts.

When the forecast origin rolls forward, another year of variables becomes available on to which the model can be fitted. Suppose that the fit period's origin is fixed (thus, the fit period is of a variable size), it implies that another year of variables is added to the fit period as the forecast origin rolls forward. The fit period's length therefore grows as the forecast origin rolls forward. Another option is to have a fixed-size fit period and to discard the oldest observations as a new year of observations becomes available (Figure 7-1).

Having a fixed-origin fit period means that the relative weights of the years change as new years are added to the fit period. This also implies that any already existing outliers or other aberrations in the variable stay in the fit period and where new ones are gradually added to the fit period. This further contaminates the fit period and adversely affects the reliability of the recalibrated model.

Having a fixed-size fit period allows the oldest observations to be discarded. This means that outliers and other anomalies are gradually removed from the fit period while the most current information is simultaneously kept in the fit period. A fixed-size fit period also ascertains that the relative weights of the years in the fit period are constant. I use the fixed-size fit period method.

Because Homo comperiens discovers, a market evolves as time passes and old information is not as important as new information. Removing the oldest observations from the fit period using a fixed-size fit period is therefore reasonable.

There cannot be any discovery using the random walk assumption since it implies that the actors' knowledge already spans the objective state set. Thus, the length of the fit period does not matter for model [EQ 7-2] as long as it is long enough to provide a reasonable number of observations in the fit period.

7.2.2.2   The forecast length

The total period covers 16 years, which is divided into fit and test periods. The panel regressions from subsection 6.4.1 are fitted on periods of four years; the output from the estimations (see section 6.4.2) serves as inputs into the prediction models, which means that the fit periods in the validation procedure are four years long.

A cursory look at the descriptive data indicates, as noted in subsection L.4.1 (p. 229), a strong dynamic process. When the process is strong, there is no need to predict far-off into the future. The forecast length is limited to four years, and thus both the fit and test periods are four years long. Having a four-year test period provides prediction of the most important periods and therefore captures market dynamics.

However, forecasting four years and having a holdout sample of four years implies that there is only one prediction for a single firm's forecast at year $T + 4$. This reduces the assessment accuracy for the relatively long-term forecasts. Tashman (2000, p. 440) uses a minimum number of three maximum length forecasts in the Tashman example. I assume that at least three maximum length forecasts per firm are needed for reasonable forecast accuracy.

Setting the holdout sample's length to six years means that for each firm there will be at least three $T + 4$ forecasts. The relationship between the period of the holdout length (HL), the maximum forecast length (FL), and the minimum number of forecasts at maximum forecast length is (MIN): $HL = FL + MIN - 1$ (Tashman, 2000, p. 440).

The relation between the fit periods and the holdout samples in the data is found in Table 7-1.

The fit periods and their corresponding holdout samples																	

Panel #	Year	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
1	1978																	
2	1979																	
3	1980																	
4	1981																	
5	1982																	
6	1983																	
7	1984																	
8	1985																	
9	1986																	
10	1987																	
11	1988																	
12	1989																	

Fit origin   Fit period

Forecast origin   Forecast period
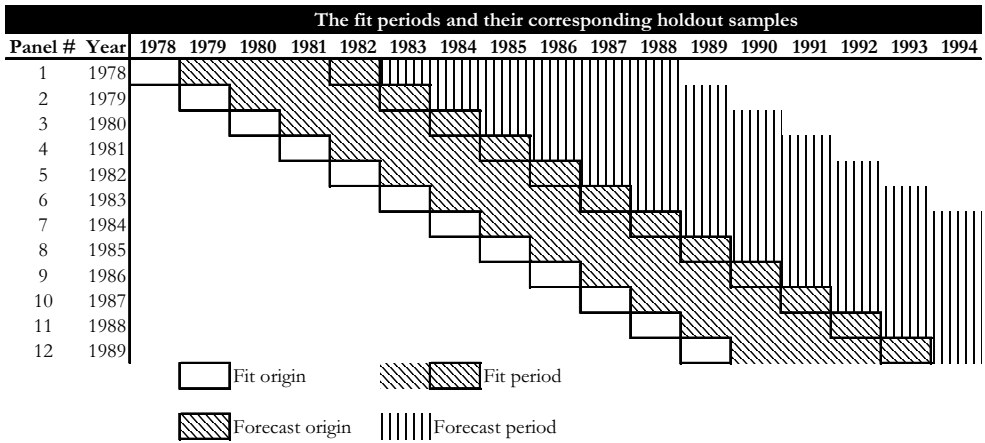
Figure 7-2: Fit periods and test periods for the fixed-size rolling-window validation.

Holdout samples #8 to #12 do not meet the criteria of having a forecast length of four years and a minimum of three forecasts per firm since the holdout period is reduced. The reduction is because the last available year of empirical data for tests is 1994. Figure 7-2 provides a graphical summary of fit periods and holdout samples.

Table 7-1 shows the forecast length, the minimum number of forecasts, and how they are adjusted to allow for external validation using holdout samples #8 to #12.

Panel #	Forecast origin	FL	Min	HL	Panel #	Forecast origin	FL	Min	HL
1	1982	4	3	6	7	1988	4	3	6
2	1983	4	3	6	8	1989	3	3	5
3	1984	4	3	6	9	1990	2	3	4
4	1985	4	3	6	10	1991	1	3	3
5	1986	4	3	6	11	1992	1	2	2
6	1987	4	3	6	12	1993	1	1	1

Table 7-1: A summary of the relationships between holdout sample length, forecast length, and the minimum required predictions per firm at maximum forecast length.

The minimum number of forecasts at maximum forecast length is kept at three for the holdout samples with forecast origins up to 1991; the forecast length is decreased for holdout samples #8 to #10. For holdout samples #11 and #12, the forecast length is only a year. Thus, the minimum number of forecasts at maximum forecast length is reduced to accommodate the data that is available for validation.

The reduction of the minimum forecast per firm reduces the reliability since aberrations are likely to have greater impact on the results. However, it should be noted that the number of firms meeting the criteria increases as the lengths of the holdout period are reduced, which reduces the effect of aberrations in the short-window holdout samples.

7.2.2.3 Outliers

Outliers are present in both the fit and test periods. The fit periods are rid of outliers using the method discussed in subsection L.4.2 because the regression estimates from the tests of the hypotheses are inputs into the prediction models.

Outliers in the holdout samples affect the reliability of the external validity tests using predictive accuracy, too. The influence of the outliers is reduced by using robust forecast error statistics and robust hypothesis tests (see subsection 7.2.3 for more information on the robust forecast error statistic that I use).

7.2.3 The pooling of forecast errors and the forecast error statistic

Assessing predictive accuracy using a single time series with a fixed window method is a rather straightforward exercise. In such a setting focus can be on choosing an appropriate forecast error statistic. If the prediction is for a single time series but with a rolling window, there will be multiple forecasts for the firm (cf. subsection 7.2.2.2). In this case attention is needed to also focus on how the forecast errors are aggregated a cross the predictions. I face forecasts on multiple firms using rolling-window prediction.

To focus the analysis some notation is introduced. In this section a firm is denoted i where M is the set of firms. A holdout sample is symbolized by n, where N is the total number of available holdout samples. See Table 7-2 for a description of the data matrix of available forecast errors and an overview of possible forecast error summary statistics. P-1 signifies panel one, i.e. a firm's holdout sample one.

The forecast error data matrix for two firms						
	Firm i			Firm j			Summary
	P-1	**P-2**	**P-N**	**P-1**	**P-2**	**P-N**	**measure**
T+1	e_{i11}	e_{i12}	e_{i1N}	e_{j11}	e_{j12}	e_{j1N}	Σ_{T+1}
T+2	e_{i21}	e_{i22}	e_{i2N}	e_{j21}	e_{j22}	e_{j2N}	Σ_{T+2}
T+3	e_{i31}	e_{i32}	e_{i3N}	e_{j31}	e_{j32}	e_{j3N}	Σ_{T+3}
T+H	e_{iH1}	e_{iH2}	e_{iHN}	e_{jH1}	e_{jH2}	e_{jHN}	Σ_{T+H}
Σ per t-s	Σ_{i1}	Σ_{i2}	Σ_{iN}	Σ_{j1}	Σ_{j2}	Σ_{jN}	$\Sigma_M\Sigma_H$
Σ per firm		Σ_i			Σ_j		

Table 7-2: The data matrix of forecast errors (e) with N predictions per firm having forecast horizon H with M firms.

An important choice is whether to measure the forecast error statistic over the whole forecast length, H, for a firm's panel time series $\sum_{in}\{e_{i1n},\cdots,e_{iHn}\}$, or if the forecast error statistic should be measured across time series per forecast length, i.e. $\sum_{n\in N}\sum_{i\in M}\{e_{i11},\cdots,e_{M1N}\}$. It is also possible to measure the forecast error statistic per holdout sample and forecast length: e.g., $\sum_{i\in M}\{e_{i1n},\cdots,e_{M1n}\}$, but then there is only one prediction per firm and forecast length, as well as the additional problem of how to interpret that error statistic.

Measuring the forecast error for each time series is not useful since it mixes the forecast error for one-year predictions with the two-year forecast errors, and so on. The interpretation of such a measure does not permit a distinction between short- and long-term predictive accuracy. I measure across the time series per forecast length since such a procedure does not mix short-term predictions with long-term predictions.

Measuring across the time series requires attention to which forecast error statistic is used. Armstrong & Collopy (1992), Fildes (1992), Hyndman & Koehler (2005), and Tashman (2000) discuss this topic.

When the forecast errors are summarized across the times series, Armstrong & Collopy (1992, p. 70), Fildes (1992, p. 85), Hyndman & Koehler (2005, p. 16), and Tashman (2000, p. 445) point out that scale dependent measures, such as the mean square error (MSE) or the mean absolute error (MAE), should be avoided. Ahlburg (1992, p. 99) notes that there is a consensus in the research community that unit free forecast error statistics should be used when comparing forecast methods. The other cited authors argue similarly.

According to Tashman (2000, p. 445), a better measure than scale dependent measures is a scale independent measure such as the median absolute percentage error (MdAPE). MdAPE is robust and can withstand skewed data. However, the MdAPE is not preferable when the time series has different volatility (Tashman 2000, p. 445).

When volatility differs across the time series, Tashman (2000, p. 445) asserts that relative measurement errors are preferable. Tashman maintains that the median relative absolute error (MdRAE) is useful as a summary measure under such premises, or that the geometric mean relative absolute error (GMRAE) is useful. Armstrong & Collopy (1992, p. 71) prefer the MdRAE and GMRAE. Fildes calls for a statistic similar to the GMRAE.

The data in the holdout samples are contaminated by outliers, which mean that the single time series has different volatility. Since this is the case and since the researchers seem to be almost indifferent between choosing MdRAE or GMRAE, I decided to use the MdRAE statistic in that it is computationally simpler than the GMRAE. The MdRAE-statistic is measured as:

$$e_{mhn} = O_{mhn} - F_{mhn} \tag{EQ 7-6}$$

$$RAE_{mhn} = |e_{mhn}| \cdot |e^{*}_{mhn}|^{-1} \tag{EQ 7-7}$$

$$MdRAE_{h} = median\left\{RAE_{1h1}, \cdots, RAE_{MhN}\right\} \tag{EQ 7-8}$$

where O_{mhn} is the acronym for the observation at time $T+h$ for firm m in holdout sample n. F is the acronym for the forecasted value, e is generic for the forecast error, and e^{*} is the forecast error for the benchmark model. $MdRAE_{h}$ is the median relative absolute error across all holdout samples for forecast length h.

When MdRAE<100 percent, the alternative prediction model is more accurate than the benchmark model. I evaluate the significance of the MdRAE statistic using a one-sided robust t test that uses the biweight scale estimate setting its constant to nine. This is equivalent to the 'best' location estimate method discussed in Appendix I (p. 211).

7.3 Results from the predictions

Popper (1959) sees theories as nets cast to catch 'the world'. Section 6.4.2 reports the results from hypothesis tests that fit econometric models to panels. The fit statistics indicates that the theory of Homo comperiens on which the model rests seems capable of rationalizing and explaining 'the world'. However, does the theory of Homo comperiens master 'the world' too? Is it such a valid description of how 'the world' works that the model can predict 'the world'?

This section reports the results from the fix-sized rolling-window forecasts where the theory's predictive ability is tested against the predictive ability of the general equilibrium theory. Homo comperiens predicts that risk-adjusted RROE and risk-adjusted RRNOA regress towards zero. This adjustment process can be so fast that it is complete after one period (i.e. transitory), or it can be slower (semi-transitory). A no-arbitrage theory predicts that the risk-adjusted RROE and the risk-adjusted RRNOA should be randomly walking, which is thus the test's benchmark model. I call it the permanent earnings model.

The predictive ability is measured using MdRAE statistic. The MdRAE-statistic is formed as the median of the ratio of the alternative forecast models' absolute forecast errors and the benchmark model's absolute forecast errors. This section reports the results for the following statistics:

(i) The MdRAE when the transitory earnings model [EQ 7-4] is the alternative forecasting model and where the benchmark model is the permanent earnings model [EQ 7-2].

(ii) The MdRAE when the semi-transitory earnings model [EQ 7-3] is the alternative forecasting model and where the permanent earnings model is the benchmark model.

(iii) The MdRAE when the alternative forecasting model is McCrae & Nilsson's (2001) model [EQ 7.5] and where the benchmark model is the permanent earnings model.

I use the permanent earnings model as the benchmark model with which the transitory earnings and the semi-transitory earnings models are compared. This is similar to the formal hypothesis tests whose results are reported in Chapter 6, since the null hypotheses in that chapter are based on the random-walk model, which, in turn, is an offspring of the perfectly rational choice.

Because my results in subsection 6.4.2 differ from those of McCrae & Nilsson (2001), and therefore need to assess the methods against each other, I perform double-sided paired t tests to assess if the results significantly differ between the models. The tests' null hypothesis is that the difference is zero and the alternative hypothesis is that they are non-zero. Giner & Iñiguez (2006) also use paired t tests to discriminate between forecasting models.

Proposition 3-2 (p. 59) argues when actors discover this implies $\beta < 1$, and if the learning is so fast that the arbitrage opportunity disappears within a period after discovery, it follows that

$\beta = 0$. The opposite to $\beta = 0$ is random walk, i.e. $\beta = 1$, which only occurs when the actors do not discover. Assuming $\beta = 1$ implies a perfectly rational choice since all actors already face the standard state-partition-model and thus there are no more discoveries.

The first test places the transitory earnings model against the permanent earnings model and therefore tests if the actors discover at a very fast rate such that $\beta = 0$ serves as a reasonable approximation as opposed to no discovery in the form of $\beta = 1$.

The second comparison places the semi-transitory earnings model against the permanent earnings model and assesses if the discovery in the form of $\beta = (0,1)$ is a better approximation of the actors' behavior rather than no discovery, which would imply $\beta = 1$.

The loadings for β can possibly come from subsection 6.4.2, but they are non-significantly different from zero and cannot be used for forecasting in model [EQ 7-3] since it requires the loadings to be strictly positive and less than one. Instead, the semi-transitory earnings model uses β from the pooled regression model assuming spherical errors since that is most likely the model that Dechow, et al. (1999), Gregory, et al. (2005), and McCrae & Nilsson (2001) use. These parameter estimates are presented in Table 7-3.

		Parameter estimates based on the pooled regression model with spherical disturbances											
		Fit origin											
Variable	Field	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
RROE	**INTERCEPT**	**0.030**	**0.030**	**0.026**	**0.023**	**0.022**	**0.022**	**0.017**	**0.016**	**0.013**	**0.015**	**0.021**	**0.033**	**0.033**
	STDE	0.009	0.008	0.006	0.006	0.005	0.005	0.004	0.004	0.005	0.006	0.007	0.007	0.008
	TVALUE_0	-113.80	-125.05	-161.78	-175.66	-187.94	-208.71	-238.23	-222.69	-215.06	-166.22	-149.14	-133.83	-125.17
	PVALUE_0	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.0%**	**0.0%**	**0.00%**	**0.00%**	**0.00%**
	SLOPE	**0.279**	**0.191**	**0.222**	**0.168**	**0.169**	**0.133**	**0.074**	**0.100**	**0.113**	**0.128**	**0.223**	**0.262**	**0.311**
	STDE	0.029	0.024	0.020	0.018	0.018	0.017	0.017	0.021	0.020	0.026	0.023	0.023	0.022
	TVALUE_1	-25.28	-33.81	-38.56	-45.88	-46.24	-50.99	-55.38	-43.09	-43.84	-33.94	-33.23	-32.30	-30.94
	PVALUE_1	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**
	TVALUE_0	9.80	7.99	11.00	9.29	9.42	7.83	4.42	4.78	5.58	5.00	9.53	11.45	13.98
	PVALUE_0	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**
RRNOA	**INTERCEPT**	**0.003**	**0.006**	**0.008**	**0.009**	**0.011**	**0.010**	**0.007**	**0.005**	**0.002**	**0.003**	**0.004**	**0.008**	**0.010**
	STDE	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.004	0.004
	TVALUE_0	1.04	1.79	3.12	3.52	4.46	4.31	3.11	2.40	0.92	1.01	1.17	2.23	2.73
	PVALUE_0	**29.9%**	**7.34%**	**0.18%**	**0.04%**	**0.00%**	**0.00%**	**0.19%**	**1.6%**	**35.9%**	**31.5%**	**24.1%**	**2.60%**	**0.6%**
	SLOPE	**0.307**	**0.244**	**0.261**	**0.179**	**0.200**	**0.195**	**0.145**	**0.212**	**0.177**	**0.211**	**0.282**	**0.321**	**0.350**
	STDE	0.029	0.027	0.022	0.019	0.019	0.018	0.017	0.022	0.021	0.026	0.024	0.022	0.021
	TVALUE_1	-23.94	-28.20	-34.33	-42.72	-42.12	-44.20	-49.60	-36.39	-39.20	-29.89	-30.34	-30.39	-30.34
	PVALUE_1	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**
	TVALUE_0	10.62	9.09	12.13	9.33	10.50	10.68	8.40	9.80	8.40	7.99	11.94	14.36	16.362
	PVALUE_0	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**

Table 7-3: The parameter estimates from the pooled regression model for risk-adjusted residual rates-of-returns.

If the prediction tests show that MdRAE is significantly less than 100 percent for both comparison (i) and (ii), the permanent earnings model is rejected in favor of the alternative forecast models proposed by Proposition 3-2.

Section 6.4.2 consistently reports that β is close to zero and formal tests fail to reject that it is non-zero, i.e. the transitory earnings model dominates over the semi-transitory earnings model. However, Chapter 6 notes that the dynamic fixed model may be affected by the "Nickell effect". That is, the parameter estimates may be biased and hence the formal tests may give misleading results. By performing separate predictions based on the estimates from the dynamic fixed model (i.e. the transitory earnings model) and from the pooled regression model, I also evaluate the validity of using the dynamic fixed-effect model.

7.3.1 Results from using the risk-adjusted RROE

Table 7-4 presents the MdRAE statistic for forecast lengths (FL) of one year up to four years. The MdRAE-statistic is measured according to [EQ 7-8]. FM is the alternative forecast model. BM is the benchmark model and FL is the forecast length. Table 7-4 also reports the number of forecasts (NOBS), the test statistics t- and p-value. The tests' null hypotheses are MdRAE=100 percent and the alternative hypotheses are MdRAE<100 percent.

MdRAE for risk-adjusted RROE (one-sided t-test)										
FM		Transitory			Semi-transitory			McCrae & Nilsson		
BM		Permanent			Permanent			Permanent		
FL	NOBS	MdRAE	T-value	P-value	MdRAE	T-value	P-value	MdRAE	T-value	P-value
1	11,185	84.7%	-19.8	0.0%	83.7%	-24.0	0.0%	84.1%	-35.7	0.0%
2	9,574	79.3%	-28.3	0.0%	80.0%	-27.5	0.0%	80.2%	-33.4	0.0%
3	8,087	75.9%	-32.9	0.0%	77.3%	-30.6	0.0%	76.7%	-34.9	0.0%
4	6,704	76.4%	-30.5	0.0%	77.5%	-28.9	0.0%	77.0%	-30.6	0.0%

Table 7-4: The MdRAE-statistic for all holdout samples using the risk-adjusted RROE.

The results in Table 7-4 show that the transitory earnings model, the semi-transitory earnings model, and McCrae & Nilsson's (2001) model all dominate over the permanent earnings model at all forecast lengths since the aggregate MdRAE statistic is significantly less than 100 percent for all models at all forecast lengths.

These results are all significant at the 0.0 percent significance level for all forecast lengths using a one-sided robust significance test.

It is clear from Table 7-4 that the transitory earnings model and the semi-transitory earnings model fair less well for the one-year forecasts than for longer forecasts, which is a result similar to the findings reported from the goodness-of-fit tests.

The goodness-of-fit tests' results in section N.5 cannot rule out that the risk-adjusted RNOA follows a random-walk process between two consecutive years. The results in Table 7-4 indicate that the process does not meet the conjectures of a permanent earning model using risk-adjusted RROE since its MdRAE statistic is significantly less than 100 percent for the one-year forecasts.

For the longer forecasts, the relative absolute forecast errors from the transitory earnings model and from the semi-transitory earnings model are 23—24 percent lower than the relative absolute forecasts errors from the permanent earnings model. This is a considerable improvement.

McCrae & Nilsson's (2001) model performs similarly in the forecasts and I use the paired t test to compare and contrast this model to the other models. I also use this test to discriminate between the transitory earnings model and the semi-transitory earnings model.

Difference in MdRAE for risk-adjusted RROE (Paired two-sided t-test)										
M1		Transitory			Transitory			Semi-Transitory		
M2		Semi-Transitory			McCrae & Nilsson			McCrae & Nilsson		
FL	NOBS	DIFF	T-value	P-value	DIFF	T-value	P-value	DIFF	T-value	P-value
1	11,185	0.38%	1.5	13.0%	-1.05%	-2.0	4.1%	-2.20%	-5.4	0.0%
2	9,574	-1.03%	-5.7	0.0%	-0.90%	-2.9	0.3%	-0.46%	-1.3	19.1%
3	8,087	-1.25%	-6.9	0.0%	-0.90%	-4.5	0.0%	-0.36%	-1.2	24.5%
4	6,704	-1.55%	-7.8	0.0%	-0.49%	-2.8	0.5%	0.17%	0.5	62.5%

Table 7-5: The results from paired t tests used to discriminate between the transitory earnings model, the semi-transitory earnings models, and McCrae & Nilsson's (2001) model.

Table 7-5 shows the results from the paired robust t test having as the null hypothesis that the differences are zero and with the alternative hypothesis that they are non-zero.

Table 7-5 shows that the semi-transitory earnings model is not significantly more accurate than the transitory earnings model in the one-year forecasts. For the forecasts longer than one year, however, the transitory earnings model yields significantly less forecast errors than the semi-transitory earnings model.

In the paired t test McCrae & Nilsson's (2001) model indicates having significantly (at the five percent threshold) larger RAEs than the transitory earnings model in the one-year forecast. However, this result is contradicted by the point estimates in Table 7-4, although the difference is small in Table 7-4. It should also be noted that the difference is significant at a 4.1 percent level, which is rather close to the cut-off level and therefore I judge that result rather weak. Thus, I am inclined to conclude that McCrae & Nilsson's model and the transitory earnings model perform similarly in the one-year predictions.

For the forecasts more than one year into the future, McCrae & Nilsson's model performs significantly less well than the transitory earnings model having significance levels less than or equal to 0.5 percent, indicating rather strong results.

The semi-transitory earnings model is not significantly more accurate than McCrae & Nilsson's (2001) model for forecast lengths more than one year. Indeed, the two models are almost identical in the longer forecasts.

Despite that my estimated models have zero slope coefficients (in the dynamic fixed-effect model), or close to zero (in the pooled regression model), and consequently, are far from those reported by Dechow, et al. (1999), Meyers (1999), McCrae & Nilsson (2001), Callen & Morel (2001),

Gregory, et al. (2005), and Giner & Iñiguez (2006), I must conclude that my models have greater predictive accuracy. This conclusion is based on the above results.

These results imply that the results from section 6.4.2 maintain, i.e. the $\beta = 0$ is not only a statistically significant result but also a valid description of the process that guides the risk-adjusted residual rates-of-returns. The results also imply that the semi-transitory earnings model, using the regression parameter estimates from Table 7-3, is inferior to the transitory earnings model in all but the one-year forecasts where it performs comparably with the semi-transitory earnings model. McCrae & Nilsson's model is inferior to the transitory earnings model for all forecast lengths.

The fact that the results from subsection 6.4.2.1 (p. 110) are asserted be the results presented above indicate that those results are robust and thus not a function of spurious events.

7.3.2 Results from using the risk-adjusted RRNOA

The previous subsection shows how the transitory earnings model significantly outperforms the permanent earnings model, the semi-transitory earnings model, and McCrae & Nilsson's (2001) model in almost all circumstances when risk-adjusted RROE is used as the assessment variable. This section reports the same tests using the risk-adjusted RRNOA as the assessment variable.

MdRAE for risk-adjusted RRNOA (one-sided t-test)									
FM		**Transitory**			**Semi-transitory**			**McCrae & Nilsson**		
BM		**Permanent**			**Permanent**			**Permanent**		
FL	**NOBS**	**MdRAE**	**T-value**	**P-value**	**MdRAE**	**T-value**	**P-value**	**MdRAE**	**T-value**	**P-value**
1	11,176	87.7%	-15.3	0.0%	85.1%	-22.1	0.0%	84.9%	-32.1	0.0%
2	9,568	82.3%	-23.6	0.0%	82.1%	-24.5	0.0%	81.9%	-29.2	0.0%
3	8,082	78.0%	-28.9	0.0%	78.6%	-28.1	0.0%	79.1%	-29.7	0.0%
4	6,701	79.1%	-26.2	0.0%	79.9%	-24.9	0.0%	80.5%	-24.6	0.0%

Table 7-6: The MdRAE statistic for all holdout samples using the risk-adjusted RRNOA.

The results from subsection 6.3.2.3 (p. 106) and subsection 6.4.2.2 (p. 112) show that risk-adjusted RRNOA behaves as risk-adjusted RROE in the tests of the hypotheses on the panel regression estimates.

Table 7-6 reports the MdRAE statistics for the risk-adjusted RRNOA. The results from these predictions are similar to those from the risk-adjusted RROE. That is, the transitory model, the semi-transitory earnings model, and McCrae & Nilsson's model are significantly more accurate than the permanent earnings model. All results are significant at the 0.0 percent significance level for all years.

The RAEs from the all these models are for the four-year forecasts of risk-adjusted RRNOA and are about 20 percent less than the RAEs from the permanent earnings model (i.e. from random-walk predictions).

The results in Table 7-6 show that the forecasting models proposed by the theory of Homo comperiens are superior to the permanent earnings model and thus that the results presented in sub-section 6.4.2.2 are not spurious. The results from the paired t tests are presented in Table 7-7.

Difference in MdRAE for risk-adjusted RRNOA (Paired two-sided t-test)									
M1 **M2**		**Transitory** **Semi-Transitory**			**Transitory** **McCrae & Nilsson**			**Semi-Transitory** **McCrae & Nilsson**		
FL	**NOBS**	**DIFF**	**T-value**	**P-value**	**DIFF**	**T-value**	**P-value**	**DIFF**	**T-value**	**P-value**
1	11,176	2.43%	8.5	0.0%	0.24%	0.4	67.5%	-2.43%	-6.0	0.0%
2	9,568	0.08%	0.6	52.5%	-0.87%	-2.4	1.7%	-1.28%	-3.3	0.1%
3	8,082	-0.25%	-2.2	2.7%	-1.80%	-5.9	0.0%	-1.69%	-4.4	0.0%
4	6,701	-0.34%	-2.8	0.6%	-2.22%	-6.9	0.0%	-2.15%	-5.0	0.0%

Table 7-7: The results from paired t tests used to discriminate between the transitory earnings model, the semi-transitory earnings model, and McCrae & Nilsson's (2001) model.

The results reported in Table 7-7 are somewhat different to the results reported in Table 7-5. Table 7-5 reveals no significant difference between the transitory earnings model and the semi-transitory earnings model for the one-year forecasts. The table further shows that the transitory earnings model is superior for forecast longer than two years.

Table 7-7 shows that the semi-transitory earnings model is significantly more accurate than the transitory earnings model at the one-year forecasts, that the two models are not significantly different at two year forecasts, and that three- and four-year forecasts by the transitory earnings model are superior to the forecasts by the semi-transitory earnings model.

Table 7-5 shows that the transitory earnings model is significantly (though weakly) more accurate than McCrae & Nilsson's (2001) model at one-year forecasts. The difference disappears in Table 7-7 and there is no significant difference between them for that forecast length. For two-year and longer forecasts, the results reported in Table 7-7 show that the transitory earnings model is superior to McCrae & Nilsson's (2001) model, which is similar to the conclusions based on Table 7-5. That is, the transitory earnings model is significantly more accurate than McCrae &Nilsson's (2001) model.

The semi-transitory model outperforms McCrae & Nilsson's model for all forecast lengths when the forecasts are based on risk-adjusted RRNOA.

When considering the results reported in Table 7-5 and Table 7-7 , my conclusion is that the slope coefficient is perhaps not zero but positive (as my theory also proposes), yet significantly less than one.

Furthermore, the fact that I have different results for the two variables risk-adjusted RROE and risk-adjusted RRNOA indicate that the firm's financial activities reduce the residual profitability faster than it would have otherwise. This is perhaps an indication that the financial activities yield negative risk-adjusted rates-of-returns. However, more studies are needed to determine if this hypothesis holds, which is not within the scope of this study.

It is difficult to discriminate between the semi-transitory earnings model and McCrae & Nilsson's model since the former is significantly better than McCrae & Nilsson's model in five out of eight tests and McCrae & Nilsson's model is better than the semi-transitory earnings model in the remaining three tests.

Taking into account both the results presented in Table 7-5 and in Table 7-7, I am confident that my findings are robust. I believe this indicates that the pooled regression method applied by Dechow, et al. (1999), McCrae & Nilsson (2001), Callen & Morel (2001), Gregory, et al. (2005), and Giner & Iñiguez (2006) should be avoided when fitting Ohlson's (1995) and Feltham & Ohlson's (1995) model to data. At the very least, I expect that the choice of which panel regression model is selected should be based on formal specification tests.

7.3.3 Conclusions from the external validation

Section 7.3 focuses on investigating whether the theory of Homo comperiens not only can rationalize and explain 'the world,' but if it can also master it. That is, to study whether the theory has predictive ability over and above the random walk hypothesis. This assessment builds on Proposition 3-2 and the results from section 6.4 (p. 108).

The section uses the MdRAE to assess the predictive accuracy of the models. I measure the statistic across all firms and holdout samples.

The results show that the transitory earnings model and the semi-transitory earnings model are significantly more accurate in predictions than the permanent earnings model. The results are robust and the forecast accuracy using MdRAE is significant at the 0.0 percent significance level. The results thus show that Proposition 3-2 has considerably greater predictive accuracy than the random walk prediction.

The MdRAE statistics for the one-year forecasts indicates an improvement of approximately 12—16 percent when Proposition 3-2 is evaluated against the permanent earnings prediction model. These results are also significant at the 0.0 percent significance level.

The MdRAE statistics at the four-year forecast length show that my models yield an improvement from 19-24 percent when compared with the permanent earnings model.

I also show inconsistent results between the transitory earnings model and the semi-transitory earnings model. Of eight paired t tests, five show that the transitory earnings model yields lower RAEs, one test shows that the semi-transitory earnings model yields lower RAEs, and two tests fail to reject the null hypothesis of no difference. There is a bias for the transitory earnings model and thus I suggest that the slope coefficient is very close to zero, yet positive. The fact that I fail to reject the hypothesis of zero slope coefficient in Chapter 6 may be due to the Nickell bias. If that is the case, it nevertheless implies that its effect is non-material in my research since it does not affect my conclusions.

I also make predictions using McCrae & Nilsson's (2001) model since it reports rather different slope coefficients than what I find. McCrae & Nilsson's model is also significantly more accurate as a forecasting model than the permanent earnings model for risk-adjusted residual accounting rates of returns.

At the same time, I find that McCrae & Nilsson's model is significantly worse at making predictions than my transitory earnings model. The results are weaker when comparing my semi-transitory earnings model with McCrae & Nilsson's model since the tests reveal that the semi-transitory earnings model significantly outperforms McCrae & Nilsson's model in five out of eight tests while there is no significant difference in the remaining three tests. McCrae & Nilsson's model never outperforms my models.

The findings in this chapter thus show the external validity of the theory of Homo comperiens and the appropriateness of use of dynamic fixed-effect panel regressions. This implies that the results reported in section 6.4.2 cannot be easily dismissed by reference to such factors as spurious results or bad specifications.

Indeed, despite a battery of attempts to falsify Proposition 3-2 posed by the theory of Homo comperiens, the proposition nevertheless manages to prevail.

7.4 Summary

Chapter 6 and Appendix N reject the null hypotheses in favor of the alternative hypotheses posed by the theory of Homo comperiens. The null hypotheses are in all but one test (risk-adjusted RRNOA in Appendix N for $t = 2$) rejected. However, it is possible to reject the hypotheses without them being false; to determine if the results are valid it must be possible to create reproducible effects on other sets of empirical observations. That is, it is necessary to assess the theory's external validity.

Chapter 7 assesses the external validity of the theory of Homo comperiens by testing its predictive accuracy using a fixed-size rolling-window out-of-sample forecasting method. More specifically, Proposition 3-2 is validated using this method. The predictive accuracy of Proposition 3-2 is tested against the predictive accuracy of the random walk hypothesis using the MdRAE statistic.

The theory of Homo comperiens' predictive accuracy for the one-year forecasts is from 12—14 percent better than the random walk predictions. The improvement increases as the forecasts length increases up to the forecast horizon at four years. At the four-year forecast length, the errors of the theory of Homo comperiens are 19 to 24 percent less than those of the random walk model.

All these results are significant at the 0.0 percent significance level.

The results further show that McCrae & Nilsson's model, which is similar to mine, is inferior to my transitory earnings model since its predictive accuracy is significantly worse in seven out of

eight paired t tests (in the remaining test no difference was observed between models). When I consider the performance of McCrae & Nilsson's model as compared with the semi-transitory earnings model, I find that my model is significantly more accurate in five out of eight tests, with no significant differences in the remaining three tests.

The chapter also shows that in five out of eight paired t tests the fixed-effect regression model is superior to the pooled regression model. The pooled regression model is superior to the fixed-effect regression model in one test, and the remaining two tests the models perform equally well. Although the results are not unidirectional, it appears as though the fixed-effect model is preferable. I expect, however, that the future use of models similar to those in this chapter will use specification tests to discriminate between the final choice of models. Some of the inconclusiveness in this chapter may stem from a Nickell effect. Thus, it may be of interest to explore other panel regression models that use the dynamics from lagged models in an unbiased fashion.

CHAPTER 8—CONCLUDING DISCUSSION

8.1 Introduction

This chapter summarizes the study's theoretical and empirical findings, directions for future research, and the implications for the role of accounting in our society.

8.2 Research problems and the research design

Kothari (2001, p. 208) observes that evidence against market efficiency has mounted. Lee (2001, p. 234) writes:

> "My thesis is that a naïve view of market efficiency, in which price is assumed to equal fundamental value, is an inadequate conceptual starting point for future market-related research. In my mind, it is an over simplification that fails to capture the richness of market pricing dynamics and the process of price discovery. Prices do not adjust to fundamental value instantly by fiat. Price convergence toward fundamental value is better characterized as a process, which is accomplished through the interplay between noise traders and information arbitrageurs. This process requires time and effort, and is only achieved at substantial cost to society."

Kothari (2001, p. 208) writes that "…advocates of market inefficiency should propose robust hypotheses and empirical tests to differentiate their behavioral-finance theories from the efficient market hypothesis that does not rely on investor irrationality". He further argues (p. 191) that there is a need for a theory of market inefficiency.

In a similar view Lee (2001) reasons that we should assume that the market price and the intrinsic value are two distinct measures. Lee (2001, p. 251) also writes "…we should study how, when, and why price becomes efficient (and why at other times it fails to do so)".

This thesis is a response to Kothari and Lee's calls for a theory of market inefficiency that sees the market price and the intrinsic value as two distinct measures.

Rather than following the current tradition in behavior economics of (Kahneman 2003, p. 1469) "…retaining the basic architecture of the rational model, adding conjectures about cognitive limitations designed to account for specific anomalies," this thesis aims at developing a general theory of market inefficiency that has a potential to be widely applied. I call it the theory of Homo comperiens.

I do not start to develop the theory from a sheet of blank paper. There already exist several important pieces of knowledge in the research community (over and above the standard models in economics) that I synthesize into this theory. Particularly important pieces of existing knowledge come from the work of Hayek (1936, 1945), Kirzner (1973), and Simon (1955, 1956). Of less importance, but still significant are Congleton (2001), who continues in the tradition of Hayek and Kirzner, and game theoretical research from Dekel, Lipman &Rustichini (1998), Modica & Rustichini

(1994), and Modica & Rustichini (1999), all of whom contribute in one way or another to the way that I structure the problem.

Yet, despite the inspiration from the research described above, no one has structured a comprehensive limited rational choice theory in the same (or similar) way as I do here. Furthermore, there is no one that has brought similar theoretical models into empirical financial economics or market-based accounting research and attempted to empirically test them.

I empirically test theory using tests of hypotheses on a database with accounting data from the Swedish manufacturing industry. The present thesis uses accounting data from 1978 to 1994 (approximately 22,000 firm-year observations), which is as far as I know by far the most comprehensive Swedish accounting database applied to this type of research. The external validity of my findings is tested in out-of-sample tests having up to approximately 11,200 predictions.

8.3 The theory of Homo comperiens

Standard research in economics, finance, and market-based accounting research builds largely on the conjecture of the perfect rational choice. This means that two conjectures are invoked. They are the completeness conjecture and the transitivity conjecture. A third conjecture, the non-satiation conjecture, is often invoked to enable the researcher to draw sharper inferences.

On top of these main conjectures are a host of auxiliary conjectures levied onto the perfectly rational choice, such as continuous preferences, state-independent preferences, homogenous preferences, and rational expectations. None of the auxiliary conjectures is normally critical and are used to enable tractable solutions for specific problems.

The perfect rational choice that is derived in this way creates a no-arbitrage market. The perfect rational choice allows for gradual dissemination of information into the market where the subjects respond to the new information and adjust the prices accordingly using Bayesian learning. Bayesian learning is modeled within the framework of the standard state-space-and-partition model.

A crux with the standard state-space-and-partition model is that it requires the subjects to identify everything that they do not know (Dekel, Lipman & Rustichini 1998, p. 164; Rubinstein 1998, p. 47; Samuelson 2004, p. 372, 398). This means that the subjects cannot be unaware of one or more states in a standard state-space-and-partition model. This is known as, e.g., the axiom of awareness (Samuelson 2004, p. 372). In a multi-subject situation the standard state-space-and-partition model means that the other subjects' consumption and investment plans become part of the focal subjects' state set (e.g., Samuelson 2004, p. 388), and therefore it follows that the standard state-space-and-partition model's axiom of awareness requires each subject to have a complete understanding of all other subjects' plans for choices, i.e. perfect knowledge.

Dekel, Lipman, & Rustichini (1998) show that the standard state-space-and-partition model cannot cope with the subject's unawareness of some state. I build on this fact and develop a theory of inefficient market (i.e. an arbitrage market theory) that forces the subjects to make limited rational choices in the sense that the choices are made on both strict subsets of the actions available and on strict subsets of the states that may occur. This is the first main trait that is particular to the theory of Homo comperiens.

I also make an auxiliary conjecture that we have atomistic competition so that I can reduce the problem that is dealt with in this thesis.

By replacing the perfect rational choice with a limited rational choice, the market description is fundamentally changed. It is no longer possible to think of the market as a no-arbitrage market. Rather, the market is in a non-random state of flux, where there are innumerable arbitrage opportunities. Thus, the market price and the intrinsic value are two distinct measures. It is also not possible to argue that the subject holds rational expectation and that price is randomly walking.

I create the market disequilibrium without having to resort to the conjectures about non-atomistic competition. Nor do I have to conjecture a non-rational choice process (as is done in behavioral finance or bounded rationality models in game theory).

Since the subjects have incomplete knowledge because of the strict subset conjecture, there is an opportunity for the subjects to revise their price expectations using not only Bayesian learning. I allow the subjects to also learn about actions and states that they previously were ignorant of. This is what I refer to as discovery. The discovery of states that the subjects were previously ignorant of falls outside of Bayesian learning since the prior probability for a previously ignorant state is zero, and consequently, the posterior probability is also zero. Bayesian learning does not encapsulate the possibility of discovery of previous actions that the subjects previously were ignorant of.

The ability to learn through discovery is the theory of Homo comperiens' second central trait. Endowing the subjects with the capacity to discover induces a price process, much like that called for by Lee (2001), in which the market prices regress towards the intrinsic value.

The theory of Homo comperiens is built upon the following central definitions:

Definition 2-4: Definition of limited rationality: A subject that has a rational preference relation, i.e. a preference relation that is complete and transitive on the subjective action set (Definition 2-3) is a limited rational subject.

Definition 2-8: Definition of limited rationality in the uncertain choice. In addition to Definition 2-4, a subject exhibits limited rationality when the subject has a rational preference relation on uncertain consequences that are limited because of limited knowledge of states (Definition 2-6).

Definition 2-9: Definition of learning as discovery. Discovery takes place when the subject that acts according to Definition 2-4 and Definition 2-8 and that faces the next choice in a sequence of choices expands his or her subjective state set and/or the subjective action set. Discovery takes place because of the subject's experience from previous choices: Formally, learning as discovery means that the previous subjective state and/or action sets are strict subsets to the current subjective state set and/or action sets. With symbols, learning as discovery is defined as $S_{Kt-1} \subset S_{Kt}$, $A_{Kt-1} \subset A_{Kt}$ $A_{Kt-1} \subset A_{Kt}$, or when both situations occur and this is because discovery make certain that $I_{At-1} \subset I_{At}$ and $I_{St-1} \subset I_{St}$.

Definition 2-4 and Definition 2-8 rest on separate definitions of the subjective action and state set that are:

Definition 2-3: Definition of the subjective action set. The subjective action is defined as $A_K = A_\Omega \setminus I_A$.

Definition 2-7: Definition of the subjective state set. Let the subjective state set be $S_K = S_\Omega \setminus I_S$.

And the subjective action and state sets depend on the definitions of the ignorance sets.

Definition 2-1: Definition of ignorance of actions: Let the subject be unaware of at least one action in the objective action set, i.e. the subject's ignorance set is nonempty, $I_A \notin \varnothing$, and a strict subset to the objective action set, $I_A \subset A_\Omega$.

Definition 2-5: Let the subject be unaware of at least one state in the objective state set, i.e., the subject's ignorance set is nonempty, $I_S \notin \varnothing$, and a strict subset to the objective action set, $I_S \subset S_\Omega$.

The ignorance sets are also used to define limited knowledge: The limited knowledge definitions are:

Definition 2-2: Definition of limited knowledge of actions. A subject's knowledge of alternative actions is limited when the subject has a nonempty ignorance set according to Definition 2-1.

Definition 2-6: Definition of limited knowledge of states. A subject's knowledge of potential states is limited when the subject has a nonempty ignorance set of states according to Definition 2-5.

With this structure on the subjects' choice, it is possible to explain choice as if they maximize their subjective expected utility. That is,

Proposition 2-3: When the subject has a preference relation on the subjective consequence sets, which are complete, transitive, continuous, state uniform, independent, and that follow the Archimedean assumption, it is possible to express the subject's choice as if he or she makes his or her choice based on an action's subjective expected utility: $\mathrm{E}_{K0}\left[U_K\left(c_1, \ldots c_S; \pi_{K1}, \ldots \pi_{KS} \right) \right] = \sum_{s \in S_K} \pi_{Ks} \cdot u_K\left(c_s \right)$, where $c_s \in C_{Ks}$, and $\pi_{Ks} \in \Pi_K$.

The difference between subjective expected utility and von Neumann and Morgenstern's expected utility resides in different state probabilities and different Bernoulli utilities. In the theory of Homo comperiens they are referred to as subjective state probabilities and subjective Bernoulli utilities.

The subjective state probability differs from the objective state probability because of the limited state set; the subjective Bernoulli utilities differ from the objective Bernoulli utilities because of erroneous specification of the supremum and the infimum consequences, an error that is due to the limited action set. All erroneous choices can therefore be traced back to my introduction of limited knowledge as the ignorance of actions and states, which can be thought of as an unawareness conjecture.

The price discovery process is covered by the following propositions.

Definition 2-9: Definition of learning as discovery. Discovery takes place when the subject that acts according to Definition 2-4 and Definition 2-8 and that faces the next choice in a sequence of choices expands his or her subjective state set and/or the subjective action set. Discovery takes place because of the subject's experience from previous choices: Formally, learning as discovery means that the previous subjective state and/or action sets are strict subsets to the current subjective state set and/or action sets. With symbols, learning as discovery is defined as $S_{Kt-1} \subset S_{Kt}$, $A_{Kt-1} \subset A_{Kt}$, or when both situations occur and this is because discovery make certain that $I_{At-1} \subset I_{At}$ and $I_{St-1} \subset I_{St}$.

Proposition 3-1: Learning through discovery (Definition 2-9) ascertains that $\lim_{t \to \infty}(A_K) \approx A_\Omega$ and $\lim_{t \to \infty}(S_K) \approx S_\Omega$ since the ignorance sets decrease. This implies that the subjective price approaches the objective price as t goes to infinity. That is $\lim_{t \to \infty}\left({}_{t-1}p_{Kt}\right) \approx {}_{t-1}p_t$.

Proposition 3-2: Suppose that the Pareto optimal equilibrium price is fixed, which is reasonable since the objective action and state sets are assumed to be fixed and since inflation is not conjectured. Then, with Proposition 3-1 in mind, I propose that price convergence can be described as follows: Let the subjective price be a function of the objective price (p) and a fraction of the previous period's discrepancy between the subjective price and the objective price. That is, ${}_{t-1}p_t^K = p + \beta \cdot \left({}_{t-2}p_{t-1}^K - p\right) + \varepsilon_t$ where $\beta \in [0,1)$ and where ε_t is a white noise disturbance.

When the theory of Homo comperiens is applied onto firms, it renders propositions on what a firm's market price is. From the Homo comperiens propositions, I derive the following propositions on how the risk-adjusted subjective expected residual rates-of-returns behave.

Proposition 4-6: In a market that meets the conjectures of the theory of Homo comperiens (Proposition 2-4), and with unbiased accounting, there exists non-zero risk-adjusted subjective expected RROE and RRNOA because of arbitrage opportunities. That is, $\mathrm{E}_{K0}^*\left[{}_{t-1}RROE_t\right] = \mathrm{E}_{K0}\left[\textit{net arbitrage rate of return}_t\right] \neq 0$, and $\mathrm{E}_{K0}^*\left[{}_{t-1}RRNOA_t\right] = \mathrm{E}_{K0}\left[\textit{operating arbitrage rate of return}_t\right] \neq 0$.

Proposition 4-5: In a subjectively certain market that meets the assumptions of Homo comperiens (Proposition 2-4, Proposition 3-2), with unbiased accounting, the subjective expected RROE and RRNOA regress until, in the limit, they are zero. That is, $\lim_{t \to \infty}\left(\mathrm{E}_{K0}\left[{}_{t-1}RROE_t\right]\right) = 0$, and $\lim_{t \to \infty}\left(\mathrm{E}_{K0}\left[{}_{t-1}RRNOA_t\right]\right) = 0$.

Proposition 4-7: In a market that meets the conjectures of the theory of Homo comperiens (Proposition 2-4, Proposition 3-2) and with unbiased accounting the limit values of risk-adjusted subjective expected RROE and RRNOA are zero. That is: $\lim_{t \to \infty} \left(E_{K0}^* \left[{}_{t-1}RROE_t \right] \right) = 0$, and $\lim_{t \to \infty} \left(E_{K0}^* \left[{}_{t-1}RRNOA_t \right] \right) = 0$.

8.4 Summary of the hypotheses tests

I subject the theory of Homo comperiens to a large number of attempts to falsify it. Proposition 4-6 and Proposition 4-7 are exposed to the hypotheses tests. The tests are based on ex post proxies for the risk-adjusted subjective expected residual rates-of-returns. These proxies are:

$$ {}_{t-1}RROE_t^* = {}_{t-1}ROE_t - {}_{t-1}ROE_t^I $$

$$ {}_{t-1}RRNOA_t^* = {}_{t-1}RNOA_t - {}_{t-1}RNOA_t^I $$

where ${}_{t-1}RROE_t^*$ is the ex post risk-adjusted residual return on equity and ${}_{t-1}RRNOA_t^*$ is the ex post risk-adjusted residual return on net operating assets.

Proposition 4-6 proposes the existence of an arbitrage market while Proposition 4-7 proposes that subjects in the market discover previously unknown action and states such that price converges towards the no-arbitrage price.

8.4.1 Does an arbitrage market exist?

The risk-adjusted residual return on equity and the risk-adjusted residual return on net operating assets should be, according to Proposition 4-6, non-zero for at least one firm. This notion is tested against the conjecture of no-arbitrage, i.e. zero risk-adjusted residual return on equity and zero risk-adjusted residual return on net operating assets for all firms.

The t tests reject the null hypotheses of zero risk-adjusted residual return on equity in favor of the alternative hypothesis posed by Proposition 4-6. It finds that in 15,216 double-sided t tests of 22,200 tests the variable is significantly different from zero at the one percent significance level.

The t tests reject the null hypothesis of zero risk-adjusted residual return on net operating assets in favor of the alternative hypothesis posed by Proposition 4-6. It finds that in 15,179 double-sided t tests of 22,193 tests the variable is significantly different from zero at the one percent significance level.

The double-sided t tests apply a robust confidence interval method that uses the biweight location and scale estimates as well as an alternative robust method in which the best possible location estimate is applied together with the biweight scale estimate. The results from the two test methods are nearly identical.

Proposition 4-6 is also tested using goodness-of-fit tests in which the alternative hypotheses state that there is serial dependency in the disturbances. This is posed against the null hypotheses of no serial dependency in the disturbances. The hypotheses tests for $t > 2$ corroborate the findings

above and reject the null hypotheses at the 0.0 percent significance level. The hypotheses tests at $t = 2$ cannot reject the null hypotheses; however, it is argued that the tests' failure to reject the null hypotheses is due to too many bins in the study. The hypotheses tests at $t = 2$ use 56 bins, which is far more than what is used in other research on which accounting rates-of-returns are classified into bins.

8.4.2 Is discovery part of human action?

Proposition 4-6 ascertains that the market has arbitrage opportunities and this allows for Proposition 4-7. Proposition 4-7 argues that there is a price process in which prices are not independent. This dependency is a direct effect of the discovery trait as assumed by the theory of Homo comperiens.

Proposition 4-7 is evaluated on 13 panels of risk-adjusted residual return on equity and 13 panels having risk-adjusted residual return on net operating assets. This means that there are 26 hypotheses tests that assess Proposition 4-7. In all of these hypotheses tests the null hypothesis of random walking risk-adjusted residual accounting rates-of-returns is rejected in favor of its alternative hypothesis of regressing risk-adjusted residual accounting rates-of-returns at the 0.0 percent significance level.

The panel regression hypotheses tests reveal that the learning trait is so strong that it appears as if a firm's risk-adjusted residual accounting rates-of-returns diminish within (or at least almost within) a year after discovery. This implies that the arbitrage opportunities are temporary. Transitory arbitrage opportunities mean that it is almost impossible to attain a sustainable competitive advantage. Attaining a sustainable competitive advantage is probably more by luck than by design.

Six alternative operationalizations of the accounting rates-of-returns that act as inputs into risk-adjusted residual accounting rates-of-returns are also considered. This means that an additional 78 hypotheses tests are evaluated at the 0.0 percent significance level to determine whether the results are sensitive to the operationalization of the risk-adjusted residual accounting rates-of-returns. None of these 78 hypotheses tests reveal conflicting results to those of the original 26 hypotheses tests, which implies robustness to the operationalization of accounting rates-of-returns.

The results from the hypotheses tests reported in Chapter 6 classify observations outside of six standard deviations as outliers and remove them from the analysis. This operationalization may adversely affect the results and an alternative operationalization of outliers is also tested, one in which I classify all observations outside of four standard deviations as outliers. The conclusions do not change because of this change.

Three alternative panel regression models are considered when evaluating Proposition 4-7. These models are the pooled regression model, the random effect model, and the fixed-effect model. Three specification tests carried out on all 26 panels show that the fixed-effect model is the best

panel regression model to be fitted on the data sets. The fixed-effect model is used to evaluate the hypotheses based on Proposition 4-7.

Specification tests also show significant panel heteroscedasticity in all panels. This is incorporated into the panel regression model by allowing for a covariance structure known as panel-corrected standard errors (Beck & Katz 1995). The covariance structures are also corrected for first-order serially correlated disturbances.

8.5 The external validity of the theory of Homo comperiens

In performing hypotheses tests there is the risk for Type I errors, i.e. the risk of rejecting the null hypotheses when they are true. It is therefore necessary to assess the external validity of the theory of Homo comperiens through predictions.

I test the theory's external validity by testing its predictive accuracy using a fixed-size rolling-window out-of-sample forecasting method. The theory's predictive ability is tested against the random-walk model's predictive accuracy over forecasts up to four years into the future.

The research design that I implement is similar to what Kothari (2001, p. 191) proposes. Kothari uses Bernard & Thomas (1990) as an example of how tests of market inefficiency should be carried out. Bernard & Thomas specify stock-price behavior under a random-walk earnings expectation model as well as under another sophisticated earnings expectation model.

When forecast errors are summarized across a times series and where volatility differs, the forecasters (e.g., Tashman 2000) prefer the MdRAE statistic or another similar statistic when evaluating the predictive accuracy. I use the MdRAE statistic to evaluate the theory's external validity.

The external validity of the theory in the short run (defined here as one year) is assessed using approximately 11,187 and 11,176 predictions depending on the tested variable. The four-year forecasts use approximately 6,704 and 6,701 predictions depending on the tested variable.

In the four-year predictions the MdRAE statistic shows that the theory of Homo comperiens generates relative absolute prediction errors that are 19—24 percent less than the random walk model. In the one-year predictions the theory of Homo comperiens generates relative absolute prediction errors that are 12—16 percent less than the random walk model. These results are significant at the 0.0 percent significance level. The two- and three-year predictions exhibit similar results with a sliding trend from the one-year level to the four-year level. All these results are significant at the 0.0 significance percent level.

This implies that the theory of Homo comperiens has significantly greater predictive accuracy than the random walk model and that I thus validate the other findings of the thesis regarding the discovery process and the existence of arbitrage-based residual rates-of-returns.

8.6 Directions for future research

The hypotheses tests and the tests of the external validity of the theory of Homo comperiens confirm my theory. The results are even robust with regard to alternative specifications of operationalizations and test methods. For the most part, the tests are significant at the 0.0 percent level.

However, it is possible to commit a Type II error, i.e. fail to reject an incorrect hypothesis. It may be that one or more of the choices I have made are incorrect.

For instance, the theory of Homo comperiens is still young and thus lacks an asset-pricing model. This makes it necessary to use an empirical proxy to remove the risk effect from the variables. I remove both the risk effect and the effect of conservatism by using the industry accounting rate-of-return. If that method is incorrect, it may adversely affect the reliability of the results in this thesis.

I believe that it is a significant improvement if an asset-pricing model can be developed using the theory of Homo comperiens.

Another way to improve the theory of Homo comperiens is to adapt it to include Savage's ([1954] 1972) subjective utility theory. Another way to improve the theory of Homo comperiens is to allow for imperfect recall, but how this can be done is still an open question.

I also believe that if the theory of Homo comperiens can withstand repeated attempts of falsification it can be attributed a more important status. Such tests should follow at least four avenues.

(i) Testable hypotheses of market prices should be posed and tested against random walk propositions. Such a hypothesis can, e.g., try to connect the firm's market price (assuming no growth) to current industry profitability. This could then be posed against the null hypothesis of the market price being connected to the firm's own current profitability.

(ii) Better panel regression models that, e.g., consider the Nickell bias.

(iii) Exploring other hypotheses based on accounting data. Obvious improvements are based on the hypotheses in section N.1. In these tests the alternative hypotheses state that $corr\left(\varepsilon_{t+1},\varepsilon_t\right) \neq 0$, but stronger tests are, e.g., where the alternative hypotheses are $corr\left(\varepsilon_{t+1},\varepsilon_t\right) < 0$. Indeed, the present thesis finds support for $corr\left(\varepsilon_{t+1},\varepsilon_t\right) = -0.5$, and this can also be tested.

(iv) Using other statistical methods. Other tests than the goodness-of-fit-test that directly targets the implied correlation is also an interesting avenue to pursue.

All these improvements are outside the scope of this thesis but will be evaluated in the near future.

My research also questions the use of the pooled panel regression model in favor of the fixed-effect panel regression model. This, in conjunction with the fact that the within-sample predictions are used for validation, calls for replication studies of Dechow et al. (1999), Gregory, et al. (2005), McCrae & Nilsson (2001) using their databases that perform a specification test to single out the relevant panel regression model and that use out-of-sample predictions. I believe that such research will show results that are substantially different than what we now take for granted.

8.7 The implications for the role of accounting in society

It is assumed that an efficient market ascertains that there is no role for accounting data in the society since the market price always equals the intrinsic value (e.g., Beaver 2002, p. 458; Lundholm 1995, p. 761). One may then wonder why we observe a demand for accounting data in our society. To answer this question we need to turn to agency theory and argue that managers have private information that owners do not have. However, I take another approach in this thesis where I question the efficient market hypothesis altogether.

If the theory of Homo comperiens can function as a first approximation of how people make choices, we have an inefficient market. Based on to this future research can introduce, e.g., asymmetric information.

With the existence of an inefficient market, there exists an inherent demand for accounting data since it is fundamental in distributing information about a firm's profitability, which is a necessary condition for learning by discovery. That is, accounting data are needed to facilitate the discovery process that determines that the market prices regress to the intrinsic values. With no accounting data available, we can expect that this learning process is hampered and will work at a slower speed. Thus, I expect that such a situation will induce considerable societal costs. A practical example of what might happen is when we consider the Kreuger collapse that probably would not have happened if we would have had consolidated accounting back then.

Another aspect of accounting data pertains to IFRS' current introduction of fair value valuation at the expense of historical cost accounting. If the market is inefficient, the fair values (i.e. the market prices) will be different from the intrinsic values. This means that the market exhibits price bubbles that inflate and explode. Indeed, the theory of Homo comperiens suggests that there will be almost an innumerable amount of such bubbles (both positive and negative). The theory also suggests that those bubbles disappear as people discover and act on them. If the balance sheet valuation is dominated by fair value valuation, we can expect to see large and fast changes in the valuation of assets and liabilities. I therefore am concerned that the introduction of fair value valuation will increase a firm's bankruptcy risk.

Related to this fact is the important role that accounting data plays. Central for accounting data is its forecast relevance. With conservative accounting, we (normally) only allow slow changes in the valuation of the assets and liabilities. Slow changes are probably a good thing if we have price bubbles that inflate and explode since the conservatism allows the accounting data to exist relatively unharmed of market price bubblesand therefore I believe that conservative accounting has greater forecast relevance than fair value accounting. Therefore, since an inefficient market that meets the assumptions of the theory of Homo comperiens demands accounting data that has forecast relevance, I am concerned with the current trend of introducing fair value valuation in accounting.

Another effect of my findings pertains to how we ought to account for purchased goodwill. With the introduction of IFRS 3, we have completely abandoned the amortization of goodwill in favor of impairment tests. This signals the end of a long process in which purchased goodwill in Sweden was amortized within five years, which then increased to 10 years and finally up to 20 years. My findings show that this is a dangerous trend. If my parameter estimations are correct, we should immediately expense purchased goodwill.

However, if we also believe that industries can exhibit residual income, we might use Meyers' (1999) results to infer the economic life of purchased goodwill. Interpreting Meyers' results, purchased goodwill implies that we can expect its economic life to be three to four years. Thus, Meyer's results indicate that goodwill should, on average, be amortized within three to four years.

Alternatively, if we choose to go by the results from Dechow, Hutton, & Sloan (1999), McCrae & Nilsson (2001), Callen & Morel (2001), Gregory, Saleh, & Tucker (2005), and Giner & Iñiguez (2006), the purchased goodwill should be depreciated within five to seven years.

Additionally, all the cited research, including my own, indicates that the value of purchased decreases degressively and therefore purchased goodwill should be amortized faster in the beginning than in the end (at least 50 percent should be amortized within the first year). This is very different from today's accounting rule for purchased goodwill.

Returning to accounting research as a subset of the society, I wish to point to the accounting anomaly that is known as post-earnings announcement drift. Research on post-earnings announcement drift has difficulty explaining why such drift occurs.

The theory of Homo comperiens may be able to provide a clue for the post-earnings announcement drift research since the theory suggests that people gradually discover. Gradual discovery is not consistent with random walk, but it is in accordance with a drift in the prices in the direction of discovery towards the intrinsic value. Using accounting data, my findings indicate that the process is finished within a year after discovery, but the post-earnings announcement literature (Kothari 2001, p. 193-196) suggests that the drift continues up to a year after the event. It may be that the drift pattern in the post-earnings announcement drift research can be explained by the proposed theory of limited rational choice.

UPPSALA, July 10, 2007.

APPENDIX A—PERFECT RATIONALITY AND THE INTRINSIC VALUE OF THE FIRM IN AN OBJECTIVE CERTAIN CHOICE

General equilibrium models of the firm's intrinsic value

A.1 Introduction

There exist many different models for valuing firms. Every microeconomic-based model is based on restrictive assumptions. This appendix derives two accounting-based microeconomic valuation models that allow for non-constant rates-of-returns. A discussion on the role and association of economy and accounting rates-of-returns is also part of this appendix.

This appendix's purpose is to stress the connection between the axioms of the perfect rational choice and the firm's intrinsic value, as well as to establish a common language for this thesis. The focus of the analysis is on an individual. The assumption here is that the preferences are homogenous and therefore possible to consider a representative individual for the whole economy. This is not restrictive in this appendix since I assume that theory of the perfect rational choice is applicable and thus all individuals are omniscient.

His or her objective opportunity set (and its components the objective budget set and the objective production set, and the objective prices) are all objective as opposed to where they are the subjective counterparts (as in Appendix B). The difference between objective and subjective is elaborated in Chapter 2 and Chapter 3 and where focus is on limited knowledge.

A reader well versed in economics should have no problem following the discussion in sections A.2—A.7 and can read them in a cursory manner to become familiar with the symbol language and definitions that this thesis applies.

Subsection A.8.1can be skipped if the reader is familiar with how net intrinsic values rules and the dividend valuation models are descendants of optimization problems.

Subsection A.8.3 presents the accounting-based valuation models used in this thesis.

The accounting-based valuation models that use certain non-constant rates-of-returns have not been previously published. Thus, they present a new development in the field of accounting-based firm valuation models. Ohlson (1995) and Feltham & Ohlson (1995) use certain constant rates-of-returns to arrive at accounting-based valuation models while Feltham & Ohlson (1999) use stochastic rates-of-returns to arrive at accounting-based valuation models. My models provide a middle ground between these two extremes.

The first accounting-based valuation models (see subsection A.8.3.1) presents the intrinsic valuate of the firm as a function of comprehensive net income, and the second model (subsection A.8.3.2) presents the intrinsic valuate of the firm as a function of comprehensive operating income.

The latter model uses the value additivity idea that Modigliani & Miller (1958) propose. Having two valuation models serves the purpose of allowing the theory of present thesis to develop propositions on firms with and without financial effects.

Before closing the appendix, subsection A.8.4 is presented, which derives a theoretically consistent return on equity and a theoretically consistent return on net operating assets. These are then operationalized in the chapters and appendices in this thesis.

A.2 A point of departure

The present analysis is based on an individual's choice among desired objects, which in this analysis are portfolios of current and future goods for consumption. A portfolio of consumption should be thought of as the set of goods that is consumed at a particular point in time. Current consumption means the portfolio of all goods available for consumption today. For example, an individual may at present time consume a certain number of apples and oranges. It is these quantities of apples and oranges that constitute his or her current portfolio of consumption.

The subject has preferences that order the available consumption alternatives. All the available alternatives form his or her objective opportunity set, $X = \mathbb{R}_0^1 \times \ldots \times \mathbb{R}_0^\ell \times \mathbb{R}_1^1 \times \ldots \times \mathbb{R}_1^\ell$, where $\mathbb{R}_0^\ell$ is the objective current consumption set of good ℓ, and $\mathbb{R}_1^\ell$ is good ℓ's objective future consumption set.

The set is also abbreviated $X = \mathbb{R}_0^L \times \mathbb{R}_1^L$, which implies that his or her objective opportunity set can be seen as two portfolios: One portfolio of current goods for consumption and another portfolio for the consumption of future goods. To avoid cumbersome notation the time subscripts are sometimes deleted when necessary. Note that the objective set is not restricted a priori in any way except that it must be in the real commodity set since there are no limits to his or her knowledge. Theoretically, the consumption set is therefore unlimited is at this point. Restrictions follow as the analysis proceeds but limiting the consumption set with limited knowledge is done in Appendix B.

His or her behavior is set forth in a certain environment, where he or she chooses among combinations of quantities of current and future consumption. Current consumption is certain.

A.3 Choice of consumption over time in objective certainty

This thesis' basic choice-theoretic structure is introduced in this section and the structure is referred to as the perfect rational choice[20]. It involves choosing one action among a set of alternative actions.

---

[20] The Latin form the perfect rational choice has been used for a very long time in the meaning of a rational self-interested individual. It has been traced back to at least Pareto (1906) by Persky (1995).

The choice of an action in this appendix is made knowing the exact consequence of each permissible action. That is, the state set has only one element, or, if it has more than one element, the consequences are constant between each state.

It is necessary to build a model that guarantees that there is only one solution to the choice problem facing the subject since he or she can only choose one action among several. The appendix uses mathematical optimization to guarantee the solution's uniqueness.

The choice problem has tree bricks to build on: the domain, the function, and the co-domain. With use of the function, the domain is mapped onto the co-domain. The domain consists of choice variables, which are the portfolios of current and future consumption. This is called the objective opportunity set, which is in the spirit of Fisher (1930).

The function is a real function that specifies the relationship that maps the objective opportunity set into the co-domain, $U : X \to \mathbb{R}$. The co-domain, which lies on the real line, is the set of possible utilities. This can be amended and the utility can, e.g., be made to only cover a portion of the real line such as $[0,1]$ [21]. This means that the function is the objective utility function and since the analysis in the appendix focuses on an equilibrium-setting meeting the assumptions of the perfect rational choice, it is the objective utility function. Rather, Appendix B makes use of the subjective objective utility function.

The objective opportunity set changes its properties depending on the situation. There is always the overarching objective opportunity set, which is called the objective consumption set. This latter set covers the real commodity set. However, there is also an objective opportunity set when pure exchange is considered and yet another set when production is available. More will be said about this as the analysis develops.

It is not enough to only establish the components of the choice problem to guarantee an objective optimal solution. The components must be endowed with specific properties.

To guarantee a unique solution it is necessary for the objective opportunity set to be strictly convex, or the objective function must be strictly quasi-concave, or both (e.g., Gravelle & Rees 1998). The objective opportunity set is convex in this analysis and it is thus necessary to endow the objective function with properties that make certain that it is strictly quasi-concave.

The choice variables are defined at the beginning of the analysis. Any individual must choose between different objective portfolios of quantities of a finite number of current goods $(\ell \in L)$, $\mathbf{c_0} = (c_{10}, \ldots, c_{\ell 0}) \subseteq L = \mathbb{R}^L$, and objective portfolios of quantities of a finite number of future

---

[21] A closed interval uses the following notation $[\cdot]$, whereas an open interval is designated $(\cdot)$.

goods, $\mathbf{c_1} = \left(c_{11},...,c_{\ell 1}\right) \subseteq L = \mathbb{R}^L$. The focus is on the objective set, $\mathbf{C} = \left(\mathbf{c_0}, \mathbf{c_1}\right) = \begin{pmatrix} c_{10} & c_{11} \\ \vdots & \vdots \\ c_{\ell 0} & c_{\ell 1} \end{pmatrix}$[22], that

the subject chooses. The objective consumption set must be attainable, i.e. within his or her objective opportunity set, $\mathbf{C} \in X$.

These relations are the basic building blocks from which the analysis proceeds. The next step is to create the objective function. It is necessary to establish a preference order among the various sets that the subject chooses among to create the objective function.

A.4 His or her preference relation and the objective utility function in objective certainty

A relation, $\succ$, on the objective opportunity set is defined as a preference relation if it is asymmetric, i.e. $\mathbf{C} \succ \hat{\mathbf{C}} \Rightarrow \hat{\mathbf{C}} \nsucc \mathbf{C}$, and negatively transitive, i.e. $\mathbf{C} \nsucc \hat{\mathbf{C}}$ and $\hat{\mathbf{C}} \nsucc \tilde{\mathbf{C}} \Rightarrow \mathbf{C} \nsucc \tilde{\mathbf{C}}$ (Kreps 1988). It is assumed that an individual possess such a strict preference relationship on the objective opportunity set, but for the purpose of this paper a weak preference relation is also defined.

A weak preference relationship $\succsim$ on the objective opportunity set is a relation in which the strict relationship is absent, i.e. $\hat{\mathbf{C}} \nsucc \mathbf{C} \Rightarrow \mathbf{C} \succsim \hat{\mathbf{C}}$. When $\mathbf{C} \nsucc \hat{\mathbf{C}}$, and $\hat{\mathbf{C}} \nsucc \mathbf{C} \Rightarrow \mathbf{C} \sim \hat{\mathbf{C}}$, which is called indifference (Kreps 1988). This means that the subject can always say if he or she prefers one set to the other, or if he or she is indifferent. To always be able to choose between two alternatives is a prerequisite for a unique solution to the choice problem.

Since his or her preference satisfies $\mathbf{C} \in X$, $\mathbf{C} \succsim \hat{\mathbf{C}}$, and $\hat{\mathbf{C}} \succsim \mathbf{C}$ for all permissible consumption pairs, it is complete: The subject has an understanding of all the choice opportunities available, makes pair-wise comparisons of them all, and expresses what the subject prefers over the other, or if the subject is indifferent between them.

Not only is his or her preference relation complete but it is also impossible to face the subject with any sequence of pair-wise choices and find inconsistent behavior. That is, it is not possible to find a behavior such as $\mathbf{C} \succsim \hat{\mathbf{C}}$, $\hat{\mathbf{C}} \succsim \tilde{\mathbf{C}}$ and also $\mathbf{C} \precsim \tilde{\mathbf{C}}$. Faced with this pair-wise comparison, the subject exhibits $\mathbf{C} \succsim \tilde{\mathbf{C}}$. The ability to act in this manner is known as the transitivity assumption (Kreps 1988).

---

[22] This text makes use of both vector and matrix algebra. The reader should bear in mind that a vector is equivalent to a column matrix. Standard matrix notation is used in the text with bold uppercase letters signifying a matrix, e.g., $\mathbf{A}$. There is an exception for column matrices. Column matrices are depicted using bold lowercase letters. Thus, we have the following relationship:

$\mathbf{c_0} = \left(c_{10},...,c_{\ell 0}\right) = \begin{pmatrix} c_{10} \\ \vdots \\ c_{\ell 0} \end{pmatrix}$. The inner product of two vectors, $\boldsymbol{\rho} \cdot \mathbf{c}$, can also be written in matrix notation, $\boldsymbol{\rho}^{\mathbf{T}} \cdot \mathbf{c}$, where

$\boldsymbol{\rho}^{\mathbf{T}}$ denotes the transpose of column matrix $\boldsymbol{\rho}$.

Facing a choice between any two sets of consumption, the subject will always choose the set that holds more consumption. Consider two potential object sets of consumption, $\mathbf{C} = (\mathbf{c}_0, \mathbf{c}_1)$, $\hat{\mathbf{C}} = (\hat{\mathbf{c}}_0, \hat{\mathbf{c}}_1)$, where any or both of the following two relationships hold $\mathbf{c}_0 \succsim \hat{\mathbf{c}}_0$, $\mathbf{c}_1 \succsim \hat{\mathbf{c}}_1$ [23], where any or both of $\mathbf{c}_0 \neq \hat{\mathbf{c}}_0$, and $\mathbf{c}_1 \neq \hat{\mathbf{c}}_1$ hold, the subject will always choose objective consumption set $\mathbf{C}$ before $\hat{\mathbf{C}}$, $\mathbf{C} \succ \hat{\mathbf{C}}$. This is the strongly monotone assumption (Mas-Colell et al. 1995).

The assumption of strong monotonicity is also known as the non-satiation assumption (e.g., Fama & Miller 1972); it implies that the objective portfolios of current and future consumption are normal goods: As wealth increases so will consumption. It also ensures that that movement between sets in the indifference set can only be done by substituting current for future consumption, and vice-versa. This amounts to a negatively sloped objective indifference curve between objective current and future consumption. The assumption also makes certain that the indifference set can never be wider than a single point, a set of points, or a curve. If it would be wider (e.g., a band), it would be possible to find the indifference point also in the preferred and the inferior set, which violates the transitivity assumption (Gravelle & Rees 1998).

With this preference relation and a fixed objective consumption set $\mathbf{C}^\circ$, it is possible to identify three consumption subsets to the objective opportunity set. The first set is the preferred set, which contains all available objective consumption sets that are preferred to $\mathbf{C}^\circ$:

$$X_P = \left\{ \bar{\mathbf{C}} \in X : \bar{\mathbf{C}} \succsim \mathbf{C}^\circ \right\} \subset X \qquad \text{[EQ A-1]}$$

The inferior set is the complement set to the preferred set:

$$X_{INF} = \complement X_P = \left\{ \tilde{\mathbf{C}} \in X : \mathbf{C}^\circ \succsim \tilde{\mathbf{C}} \right\} \subset X \qquad \text{[EQ A-2]}$$

It follows that the indifference set equals the intersection between the preferred and the inferior set:

$$X_{IND} = X_P \cap X_{INF} = \left\{ \hat{\mathbf{C}} \in X : \hat{\mathbf{C}} \sim \mathbf{C}^\circ \right\} \subset X \qquad \text{[EQ A-3]}$$

With the assumptions of completeness, transitivity, and strong monotonicity, it is possible to define the indifference set as the intersection between the preferred and the inferior set. This ensures that that the preferences will be continuous (Mas-Colell et al. 1995).

Continuity implies that the preference relation can be represented by a continuous mathematical function that here is called the objective utility function. This means that $U : X \to \mathbb{R}$ is established. Continuity also implies that consumption can be divided into however small bits as necessary, i.e. the goods in the economy must be infinitely divisible.

---

[23] The relation $\mathbf{c}_0 \succsim \hat{\mathbf{c}}_0$ implies that at least one of the elements is strictly preferred, i.e. $c_{\ell 0} \succ \hat{c}_{\ell 0}$.

With the objective utility function available, it is possible rephrase the preferred set to be the set that contains all sets that yield higher utility than set $\mathbf{C}°$:

$$X_P = \left\{ \overline{\mathbf{C}} \in X : U\left(\overline{\mathbf{C}}\right) \succsim U\left(\mathbf{C}°\right) \right\} \subset X \qquad \text{[EQ A-4]}$$

Similarly, the inferior set contains all sets with less utility than $\mathbf{C}°$:

$$X_{INF} = \complement X_P = \left\{ \tilde{\mathbf{C}} \in X : U\left(\mathbf{C}°\right) \succsim U\left(\tilde{\mathbf{C}}\right) \right\} \subset X \qquad \text{[EQ A-5]}$$

The indifference set is then the intersection between the preferred and the not-preferred set. An equivalent expression would be to state that it contains all sets that yield the same level of objective utility as the fixed set $\mathbf{C}°$. It is known as the objective indifference curve:

$$X_{IND} = X_P \cap X_{INF} = \left\{ \hat{\mathbf{C}} \in X : U\left(\hat{\mathbf{C}}\right) = U\left(\mathbf{C}°\right) \right\} \subset X \qquad \text{[EQ A-6]}$$

The constraint $U\left(\hat{\mathbf{C}}\right) = U\left(\mathbf{C}°\right)$ can be thought of as all combinations of objective current and future consumption that yields an identical level of objective utility. This means that the contours of the objective function, $U\left(\hat{\mathbf{C}}\right) = c$, are the objective indifference curves with which c are various constant levels of utilities. Achieving higher levels of objective utility is equivalent to climb to higher levels of contour lines.

Despite continuity, it may still not be possible to differentiate the function, which is necessary for the solution of the choice problem. It is therefore assumed that the objective utility function is differentiable to any necessary degree.

The preference relation is also assumed to exhibit strict convexity. Form an objective consumption set, $\overline{\mathbf{C}}$, to be a linear combination of any two objective consumption sets that the subject is indifferent between, $\mathbf{C} \sim \hat{\mathbf{C}}$, where $\alpha \in [0,1]$. That is, $\overline{\mathbf{C}} = \alpha \cdot \mathbf{C} + (1-\alpha) \cdot \hat{\mathbf{C}}$. The new objective consumption set is then always preferred to any of the original two: $\overline{\mathbf{C}} \succ \mathbf{C} \sim \hat{\mathbf{C}}$. Strict convexity rules out the possibility that the new combined objective consumption set could be indifferent to the initial two sets (Mas-Colell et al. 1995).

The solution could yield a set of objective consumption sets among which the consumer cannot choose if only a convexity assumption is invoked. This is not in line with the intention of having a unique solution. It is therefore necessary to impose strict convexity, which leads to a solution in which the consumer can only choose one set. Strict convexity implies that the objective utility function will be strictly quasi-concave (Gravelle & Rees 1998).

A convex objective indifference curve is the same as diminishing an objective marginal rate of substitution between current and future consumption (Mas-Colell et al. 1995). Convexity guarantees that the subject will never give up all current consumption for future consumption, or the other way around. It is ultimately a matter of survival: the perfect rational choice demands increasing

compensation for each unit of consumption forfeited until, in the limit, the subject demands an infinite compensation for sacrificing consumption. It also implies that the subject has a basic inclination for diversification. The subject always prefers a combination of the two objective consumption sets rather than only one of those (Mas-Colell et al. 1995).

The strongly monotone assumption translates directly into the objective utility function in the sense that it will also be strongly monotone. The increasing function makes it possible to analyze the choice problem as if the subject is a utility maximizer, i.e. the subjects appear to be maximizing the value of their objective utility function over their objective opportunity set.

A.5 His or her objective opportunity set in objective certainty

It was possible from a behavioral perspective to divide the objective opportunity set into three subsets: the preferred, the indifferent, and the inferior set. This operation has nothing to do with what elements the objective opportunity set contain. The choice is constrained to be among the objective consumption sets available within the objective opportunity set.

It is the objective opportunity set that is in focus in this subsection. It can be divided into two subsets, namely the objective opportunity set under pure exchange and the objective opportunity set under pure production. The objective opportunity set is at this point not limited except to the commodity set, i.e. $X = \mathbb{R}_0^L \times \mathbb{R}_1^L$.

Convexity of the objective opportunity set has already been mentioned as a necessary requirement to achieve a mathematically unique solution to the choice problem. This is not the only property an opportunity must possess for it to be useful. The objective opportunity set must have four properties (Gravelle & Rees 1992). It must be:

    (i)   Non-empty. I.e. it must contain at least one objective consumption set: otherwise, there would be nothing to optimize and the problem would be trivial.

    (ii)   Closed. All points on the set's boundary are elements of the set.

    (iii)   Bounded. The objective opportunity set must be limited.

    (iv)   Convex.

An individual must consume at least some good (e.g., oxygen), both today and tomorrow or the subject will die. It could be argued that an individual can have non-positive consumption in the presence of a government that provides social security since the government would step in and help the subject if necessary. This problem is ignored and it is assumed that the subject must at least consume non-negative amounts of goods both in the present and in the future.

Setting the constraint on consumption to be non-negative permits the boundary elements of current and future consumption to be part of the solution. This is one step in the direction of assuring a closed objective opportunity set. The objective opportunity set can now be expressed as

$$X_+ = \mathbb{R}_+^L \times \mathbb{R}_+^L = \left\{ \mathbf{C} \in \mathbb{R}_+^L \times \mathbb{R}_+^L : \mathbf{c}_0 \geq 0, \mathbf{c}_1 \geq 0 \right\} \subset X .$$

The objective opportunity set is still open since the subject can and will consume infinite amounts of consumption. This is due to the non-satiation assumption. The objective opportunity set must be closed and the closure must be bounded.

The closure is achieved with the introduction of further constraints on his or her choice. The final closure looks different depending on whether the subject is only limited by exchange opportunities or whether the subject is also limited by productive opportunities. The two closures are called the opportunity set in a pure exchange economy and the opportunity set in a pure production economy. These are considered next.

A.5.1 The objective opportunity set in a pure exchange economy

In this section the objective opportunity set for the subject in a pure exchange economy is considered. The pure exchange economy has a spot economy and a futures economy. In the spot economy all goods in the commodity set are traded for immediate consumption. Claims to all future consumption commodities, available in the future commodity set, are traded in the futures economy, which is open at present. The spot economy exists both in the present and in the future, which means that the future spot economy opens when the future becomes the present. The claims from the previous futures economy are cleared when the future spot economy opens.

There are two basic types of objective price available in the economy: the objective price paid today for commodities that are consumed today, i.e. the objective spot price, and the objective price paid today for goods that are consumed tomorrow, i.e. the objective futures price. The objective price vector for current consumption is $_0\boldsymbol{p}_0 = \left(_0p_{10},\ldots, _0p_{\ell 0}\right)$ and the objective current price vector for future consumption is $_0\boldsymbol{p}_1 = \left(_0p_{11},\ldots, _0p_{\ell 1}\right)$. Analogous to the objective current spot price vector is the objective future price vector $_1\boldsymbol{p}_1 = \left(_1p_{11},\ldots, _1p_{\ell 1}\right)$. All objective prices are assumed to be on the real line, strictly positive, and finite, i.e. $0 < {}_0p_{\ell 0}, {}_0p_{\ell 1} < \infty \subset \mathbb{R}_+$.

The objective price $_0p_{\ell 0}$ is the objective price paid today for consumption of commodity ℓ delivered today. It is the objective spot price for a particular good. Equivalently is the objective price $_0p_{\ell 1}$, the objective price paid today for commodity ℓ delivered in the future. That is, it is an objective futures price. The first good is set to be the numerarie in this analysis and its objective spot prices are defined to be one, i.e. $_0p_{10} = 1$, and $_1p_{11} = 1$.

It is necessary to make certain that the subject can consume at least one objective consumption set to avoid a trivial solution to the choice problem. In other case, the objective opportunity set would be empty. A non-empty objective opportunity set is achieved by endowing the subject with strictly positive wealth at the same time as the objective prices are finite and the goods are infinitely dividable.

Wealth is defined as the quantities of commodities that the subject already has when entering the choice situation. Some of his or her objective wealth exists today and some is allocated to the future. That is, the subject has an objective current wealth vector $\boldsymbol{\omega}_0 = (\omega_{10}, \ldots, \omega_{\ell 0})$ and an objective future wealth vector $\boldsymbol{\omega}_1 = (\omega_{11}, \ldots, \omega_{\ell 1})$

The subject is limited to consume less than what his or her wealth is. His or her constraint is today $_0 p_{\ell 0} \cdot c_{\ell 0} \leq {_0 p_{\ell 0}} \cdot \omega_{\ell 0}$, $\forall \ell \in L$. The subject can decrease his or her current consumption by purchasing a claim for $\mathbf{s}_1$ units of an asset to be delivered in the future. When the subject buys $\mathbf{s}_1$ units he or she saves. As the future spot economy is opened, the claims are cleared. At the same time as the subject buys a claim for future consumption, someone else is selling the claim for the asset and that person is borrowing. An individual saves when $\mathbf{s}_1 > 0$ and borrows when $\mathbf{s}_1 < 0$.

The restrictions on consumption today and tomorrow follow these definitions:

$$_0 \boldsymbol{p}_0 \cdot \mathbf{c}_0 \leq {_0 \boldsymbol{\rho}_0} \cdot \boldsymbol{\omega}_0 - {_0 \boldsymbol{p}_1} \cdot \mathbf{s}_1 \qquad \text{[EQ A-7]}$$

$$_1 \boldsymbol{p}_1 \cdot \mathbf{c}_1 \leq {_1 \boldsymbol{p}_1} \cdot (\boldsymbol{\omega}_1 + \mathbf{s}_1) \qquad \text{[EQ A-8]}$$

It is possible to substitute [EQ A-8] into [EQ A-7]. The objective budget restriction on today's consumption then becomes:

$$_0 \boldsymbol{p}_0 \cdot \mathbf{c}_0 + {_0 \boldsymbol{p}_1} \cdot \mathbf{c}_1 \leq {_0 \boldsymbol{p}_0} \cdot \boldsymbol{\omega}_0 + {_0 \boldsymbol{p}_1} \cdot \boldsymbol{\omega}_1 \qquad \text{[EQ A-9]}$$

His or her non-empty, closed, bounded, and convex objective opportunity set can be expressed given these assumptions. It is his or her objective budget set:

$$B = \left\{ \mathbf{C} \in X_+ : {_0 \boldsymbol{p}_0} \cdot \mathbf{c}_0 + {_0 \boldsymbol{p}_1} \cdot \mathbf{c}_1 \leq {_0 \boldsymbol{p}_0} \cdot \boldsymbol{\omega}_0 + {_0 \boldsymbol{p}_1} \cdot \boldsymbol{\omega}_1 \right\} \subseteq X_+ \qquad \text{[EQ A-10]}$$

Together with a strictly quasi-concave objective utility function, the objective budget set guarantees that there is a unique and non-trivial solution to his or her choice problem. The next step is the introduction of production and the consideration of the objective opportunity set available through the transactions with nature.

A.5.2 The objective opportunity set in a pure production economy

Production mitigates the problem when no exchange possibilities are available that enables the transfer of current consumption into the future. It is assumed that production takes time: It makes use of current goods in the production of future goods. Production shifts current goods into future goods, a process that is irreversible.

In the pure exchange problem his or her consumption was restricted to be less than or equal to his or her objective wealth while at the same time non-negative. It is possible for his or her to increase his or her objective wealth by investing in production.

The objective opportunity set in a production economy must also be non-empty, closed, bounded, and convex to guarantee the existence of a unique solution to the consumer's choice prob-

lem (Gravelle & Rees 1998). The consumer once again faces the real commodity set $\mathbb{R}_0^L \times \mathbb{R}_1^L$. In the pure exchange situation the objective opportunity set was restricted to be a subset of the real commodity set that was bounded from below with the assumption of consumption being non-negative and from above with the wealth restriction. When only production is available, the objective opportunity set is called the objective production set and is denoted $\mathbb{D}$, which is a mnemonic for dividends.

The objective production set consists of all attainable objective production sets. An objective production set is a combination of the objective vectors of goods used in production. If a vector is negative, it is used as an input whereas a positive vector indicates output.

The objective production set is denoted $\mathbf{D} = (\mathbf{d}_0, \mathbf{d}_1)$ and the objective current production vector is denoted $\mathbf{d}_0 = (d_{10}, \ldots, d_{\ell 0}) = \left\{ d_{\ell 0} \in \mathbb{R}_-^L : d_{\ell 0} \leq 0 \right\}$. Being negative, it signifies that the current goods are used as input into the production. The objective future production vector is $\mathbf{d}_1 = (d_{11}, \ldots, d_{\ell 1}) = \left\{ d_{\ell 1} \in \mathbb{R}_+^L : d_{\ell 1} \geq 0 \right\}$. Since the objective future production vector is positive, the goods are outputs. In summary, this means that the objective production set can be expressed as $\mathbb{D} = \left\{ \mathbf{D} \in \mathbb{R}_-^L \times \mathbb{R}_+^L : \mathbf{d}_0 \leq 0, \mathbf{d}_1 \geq 0 \right\} \subset \mathbb{R}_0^L \times \mathbb{R}_1^L$.

The firm uses a transformation technology to shift current goods into future goods, which can be expressed as a function: $g : \mathbb{R}_-^L \to \mathbb{R}_+^L$. The objective production function can also be expressed as $g(\mathbf{d}_1, \mathbf{d}_0) \leq \varnothing$, where it is understood that the input vector is negative.

Let $\mathbf{d}_0' = \mathbf{d}_0 \setminus \{d_{10}^\circ\}$. For any fixed level of the current numerarie, d_{10}°, we have the objective transformation function $g\!\left(\mathbf{d}_1, \mathbf{d}_0' \middle| d_{10}^\circ\right) \leq \varnothing$. If $g\!\left(\mathbf{d}_1, \mathbf{d}_0' \middle| d_{10}^\circ\right) < \varnothing$. This means that the firm could produce more output without adding more input and that it is output inefficient. With $g\!\left(\mathbf{d}_1, \mathbf{d}_0' \middle| d_{10}^\circ\right) = \varnothing$, it is not possible to produce more with the given input and the firm is output efficient. Every objective production set that satisfies $g\!\left(\mathbf{d}_1, \mathbf{d}_0' \middle| d_{10}^\circ\right) = \varnothing$ lies on the objective transformation frontier. When the objective transformation function has been defined for a firm, it is possible to describe the objective production set as:

$$\mathbb{D} = \left\{ \mathbf{D} \in \mathbb{R}_-^L \times \mathbb{R}_+ : g\!\left(\mathbf{d}_1, \mathbf{d}_0' \middle| d_{10}^\circ\right) \leq \varnothing \right\} \tag{EQ A-11}$$

where, $\mathbf{d}_0 = \left\{ d_{\ell 0} \in \mathbb{R}_-^L : d_{\ell 0} \leq 0 \right\}$, $\mathbf{d}_0' = \mathbf{d}_0 \setminus \{d_{10}^\circ\}$, and $\mathbf{d}_1 = \left\{ d_{\ell 0} \in \mathbb{R}_+^L : d_{\ell 0} \geq 0 \right\}$

The defined objective production set implies that the firm could keep inserting current goods and obtain ever more future goods. This possibility must be restricted with the assumption that the

objective production set exhibits strict convexity. Strict convexity implies that if an objective production set $\overline{\mathbf{D}}$ is formed as a linear combination of two output efficient sets $\mathbf{D}$ and $\hat{\mathbf{D}}$, i.e. $\overline{\mathbf{D}} = \beta \cdot \mathbf{D} + (1 - \beta) \cdot \hat{\mathbf{D}}$, where $\beta = [0,1]$, the new objective production set will always be output inefficient and hence not be on the objective transformation frontier (Mas-Colell et al. 1995). An objective transformation frontier that exhibits decreasing returns to scale corresponds to the assumption of strict convexity (Mas-Colell et al. 1995). This is the same as saying that the objective marginal rate of transformation between current and future goods decreases.

With the assumptions above the objective production set is closed. It is bounded in so far that it is possible to produce on to the objective transformation frontier.

It is now possible to study his or her choice since the objective function, the objective opportunity set in a pure exchange environment, and the objective opportunity set in a pure production environment have been defined. His or her choice follows next.

A.6 His or her optimization problems in objective certainty

The key building blocks are in place that guarantee the uniqueness to his or her choice-problem. The subject acts as if the subject maximizes his or her strictly quasi-concave objective utility function over the available objective opportunities. The available objective opportunities are constrained by non-negative consumption, the exchange opportunities, and by the production opportunities. Corner solution where the subject would consume zero amounts of current or future consumptions is unlikely. The non-negativity constraints are ignored and the focus is on interior solutions to the optimization problem.

Optimization is first studied in a pure exchange situation. This is followed by the situation where the subject faces both exchange and productive opportunities.

A.6.1 Optimization in a pure exchange economy

It has been derived that the subject behaves as if he or she maximizes the objective function subject to a constraint. The objective function is called the objective utility function and the compact and convex set $B = \left\{ \mathbf{C} \in \mathbb{X}_+ : {}_0\boldsymbol{p}_0 \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1 \cdot \mathbf{c}_1 \leq {}_0\boldsymbol{p}_0 \cdot \boldsymbol{\omega}_0 + {}_0\boldsymbol{p}_1 \cdot \boldsymbol{\omega}_1 \right\} \subseteq X_+$ is the constraint in a pure exchange economy.

Since the subject is non-satiated, the subject always consumes as much as possible. This means that the subject never consumes less than what his or her wealth permits his or her, i.e. the subject never accepts ${}_0\boldsymbol{p}_0 \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1 \cdot \mathbf{c}_1 \leq {}_0\boldsymbol{p}_0 \cdot \boldsymbol{\omega}_0 + {}_0\boldsymbol{p}_1 \cdot \boldsymbol{\omega}_1$. The weak inequality in the objective budget constraint is therefore replaced with equality ${}_0\boldsymbol{p}_0 \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1 \cdot \mathbf{c}_1 = {}_0\boldsymbol{p}_0 \cdot \boldsymbol{\omega}_0 + {}_0\boldsymbol{p}_1 \cdot \boldsymbol{\omega}_1$. The objective budget set then becomes the objective budget hyperplane:

$$B' = \left\{ \mathbf{C} \in X_+ : {}_0\boldsymbol{p}_0 \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1 \cdot \mathbf{c}_1 = {}_0\boldsymbol{p}_0 \cdot \boldsymbol{\omega}_0 + {}_0\boldsymbol{p}_1 \cdot \boldsymbol{\omega}_1 \right\} \subseteq B \qquad \text{[EQ A-12]}$$

The objective budget hyperplane implies that the subjects choose their his or her consumption pattern so that the intrinsic value of his or hertheir consumption is equal to their intrinsic value of wealth.

The exact choice of set depends on his or her preferences. The subject chooses the set that gives most objective utility, i.e. the set that lies on the highest possible objective indifference curve. The highest attainable objective indifference curve is the objective indifference curve X_{IND} tangent to his or her objective budget hyperplane B', and his or herhis or her optimal objective consumption set is $\mathbf{C}^* = B' \cap V' X_{IND}$.

There can only be one unique optimal objective consumption set since B' is compact and convex and since X_{IND} is strictly convex. The optimal objective consumption set is found solving the problem:

$$\max_c \ U(\mathbf{C}) \text{ subject to} \qquad \text{[EQ A-13]}$$

$$_0\boldsymbol{p}_0 \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1 \cdot \mathbf{c}_1 - {}_0\boldsymbol{p}_0 \cdot \boldsymbol{\omega}_0 - {}_0\boldsymbol{p}_1 \cdot \boldsymbol{\omega}_1 = 0$$

Lagrange's method is used to solve the problem and Lagrange's equation based on the objective function. Its constraint is:

$$\max_{c,\lambda} \mathcal{L} = U(\mathbf{C}) - \lambda \cdot \left[{}_0\boldsymbol{p}_0 \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1 \cdot \mathbf{c}_1 - {}_0\boldsymbol{p}_0 \cdot \boldsymbol{\omega}_0 - {}_0\boldsymbol{p}_1 \cdot \boldsymbol{\omega}_1 \right] \qquad \text{[EQ A-14]}$$

The partial derivative of Lagrange's equation with respect to current consumption is:

$$\nabla \mathcal{L}(\mathbf{c_0}) = \nabla U(\mathbf{c_0}) - \lambda \cdot {}_0\boldsymbol{p}_0 = 0, \qquad \text{[EQ A-15]}$$

where $\nabla \mathcal{L}(\mathbf{c_0}) = \left(\dfrac{\partial \mathcal{L}}{\partial c_{10}}, \cdots, \dfrac{\partial \mathcal{L}}{\partial c_{\ell 0}} \right)$, and where $\nabla U(\mathbf{c_0}) = \left(\dfrac{\partial U(\mathbf{C})}{\partial c_{10}}, \cdots, \dfrac{\partial U(\mathbf{C})}{\partial c_{\ell 0}} \right)$.

For future consumption the partial derivative is:

$$\nabla \mathcal{L}(\mathbf{c}_1) = \nabla U(\mathbf{c}_1) - \lambda \cdot {}_0\boldsymbol{p}_1 = 0, \qquad \text{[EQ A-16]}$$

where $\nabla \mathcal{L}(\mathbf{c}_1) = \left(\dfrac{\partial \mathcal{L}}{\partial c_{11}}, \cdots, \dfrac{\partial \mathcal{L}}{\partial c_{\ell 1}} \right)$, and where $\nabla U(\mathbf{c_0}) = \left(\dfrac{\partial U(\mathbf{C})}{\partial c_{11}}, \cdots, \dfrac{\partial U(\mathbf{C})}{\partial c_{\ell 1}} \right)$.

The study of the partial derivative with respect to the numerarie good, whose objective spot price is defined to be 1, in equation [EQ A-15] yields the solution for Lagrange's multiplier:

$$\frac{\partial L}{\partial c_{10}} = \frac{\partial U(\mathbf{C})}{\partial c_{10}} = \lambda \cdot {}_0\rho_{10} = \lambda \qquad \text{[EQ A-17]}$$

Dividing equation [EQ A-15] with equation [EQ A-16] gives:

$$\frac{{}_0\boldsymbol{p}_1}{{}_0\boldsymbol{p}_0} = \frac{\nabla U(\mathbf{c}_1)}{\nabla U(\mathbf{c}_0)} \qquad \text{[EQ A-18]}$$

The partial derivative with respect to the numerarie in the future gives the solution for the objective futures price of the numerarie good:

$$_0\boldsymbol{p}_{11} = \frac{\dfrac{\partial U(\mathbf{C})}{\partial c_{11}}}{\dfrac{\partial U(\mathbf{C})}{\partial c_{10}}} \qquad \text{[EQ A-19]}$$

Equation [EQ A-19] says that the objective price paid today for a claim to a unit of the numerarie good to be delivered tomorrow equals the objective marginal rate of substitution between the numerarie tomorrow and the numerarie at present. That is, it is the objective marginal rate of substitution between saving and consuming.

It is also possible to study objective spot prices between different commodities in the same period. In the case of [EQ A-19] the relative objective prices are between the goods equal to the objective marginal rate of substitution between the commodities:

$$\frac{_0\rho_{\ell 0}}{_0\rho_{m0}} = \frac{\dfrac{\partial U(\mathbf{C})}{\partial c_{\ell 0}}}{\dfrac{\partial U(\mathbf{C})}{\partial c_{m0}}}, \quad \frac{_1\rho_{\ell 1}}{_1\rho_{m1}} = \frac{\dfrac{\partial U(\mathbf{C})}{\partial c_{\ell 1}}}{\dfrac{\partial U(\mathbf{C})}{\partial c_{m1}}} \qquad \text{[EQ A-20]}$$

The partial derivative of Lagrange's equation with respect to Lagrange's multiplier gives the constraint that must be satisfied.

$$\nabla\mathcal{L}(\lambda) = {}_0\boldsymbol{p}_0 \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1 \cdot \mathbf{c}_1 - {}_0\boldsymbol{p}_0 \cdot \boldsymbol{\omega}_0 - {}_0\boldsymbol{p}_1 \cdot \boldsymbol{\omega}_1 = 0 \qquad \text{[EQ A-21]}$$

where $\nabla\mathcal{L}(\lambda) = \left(\dfrac{\partial \mathcal{L}}{\partial \lambda}\right)$.

The solution must satisfy the constraint in equation [EQ A-21] and the optimal consumption points then become:

$$\mathbf{c}_0^* = \left(\boldsymbol{\omega}_0 + \frac{_0\boldsymbol{p}_1}{_0\boldsymbol{p}_0} \cdot \boldsymbol{\omega}_1\right) - \frac{_0\boldsymbol{p}_1}{_0\boldsymbol{p}_0} \cdot \mathbf{c}_1^* \qquad \text{[EQ A-22]}$$

$$\mathbf{c}_1^* = \left(\boldsymbol{\omega}_0 \cdot \frac{_0\boldsymbol{p}_0}{_0\boldsymbol{p}_1} + \boldsymbol{\omega}_1\right) - \frac{_0\boldsymbol{p}_0}{_0\boldsymbol{p}_1} \cdot \mathbf{c}_0^* \qquad \text{[EQ A-23]}$$

Due to equation [EQ A-18], it is possible to reformulate the optimal consumption points to be expressed as functions of the marginal rates of substitution between future and current consumption:

$$\mathbf{c}_0^* = \left(\boldsymbol{\omega}_0 + \frac{\nabla U(\mathbf{c}_1)}{\nabla U(\mathbf{c}_0)} \cdot \boldsymbol{\omega}_1\right) - \frac{\nabla U(\mathbf{c}_1)}{\nabla U(\mathbf{c}_0)} \cdot \mathbf{c}_1^* \qquad \text{[EQ A-24]}$$

$$\mathbf{c}_1^* = \left(\boldsymbol{\omega}_0 \cdot \frac{\nabla U(\mathbf{c}_0)}{\nabla U(\mathbf{c}_1)} + \boldsymbol{\omega}_1\right) - \frac{\nabla U(\mathbf{c}_0)}{\nabla U(\mathbf{c}_1)} \cdot \mathbf{c}_0^* \qquad \text{[EQ A-25]}$$

From above ([EQ A-18] and [EQ A-22] to [EQ A-25]), it is possible to conclude that the subject who is a utility maximizer, given an objective budget constraint, chooses the optimal objec-

tive consumption set. And the optimal objective consumption set is at the tangency point where the strictly convex objective indifference curve meets the convex objective budget restriction.

At the tangency point is his or her objective marginal rate of substitution between a future and a current good equal to the current intrinsic value for a unit of the numerarie, adjusted for the relative objective price of the focal good between the future and the present.

A.6.2 Optimization in an exchange and production economy

The subject is not only limited by his or her economy exchange opportunities but also limited by the productive opportunities. This amounts to a joint optimization problem, where the subject must choose an optimal consumption level and at the same time choose an optimal investment level.

In the pure exchange situation the subject was restricted to consume less than his or her exogenous given objective wealth. The exogenous objective wealth is no longer an absolute restriction when production is available. With production available, his or her wealth can change by engaging in exchange with nature.

The subject is assumed to buy shares equal to a portion, θ_i^j, $\left(\sum_i \theta_i^j = 1 \right)$, of the objective future output from the firm $j \in J$. Replacing $\boldsymbol{\omega}_0$ with $\boldsymbol{\omega}_0 + \theta \cdot \mathbf{d}_0$ and $\boldsymbol{\omega}_1$ with $\boldsymbol{\omega}_1 + \theta \cdot \mathbf{d}_1$ in the objective budget restriction [EQ A-9] yields a new objective budget restriction:

$$_0 \boldsymbol{p}_0 \cdot \mathbf{c}_0 + {}_0 \boldsymbol{p}_1 \cdot \mathbf{c}_1 \leq {}_0 \boldsymbol{p}_0 \cdot (\boldsymbol{\omega}_0 + \theta \cdot \mathbf{d}_0) + {}_0 \boldsymbol{p}_1 \cdot (\boldsymbol{\omega}_1 + \theta \cdot \mathbf{d}_1) \qquad \text{[EQ A-26]}$$

The weak inequality is changed to equality with an individual who is always non-satiated, and the objective budget restriction becomes the objective budget hyperplane:

$$B' = \left\{ \begin{array}{l} \mathbf{C} \in \mathbb{X}_+ : {}_0 \boldsymbol{p}_0 \cdot (\mathbf{c}_0 - \boldsymbol{\omega}_0) + {}_0 \boldsymbol{p}_1 \cdot (\mathbf{c}_1 - \boldsymbol{\omega}_1) = \\ {}_0 \boldsymbol{p}_0 \cdot \theta \cdot \mathbf{d}_0 + {}_0 \boldsymbol{p}_1 \cdot \theta \cdot \mathbf{d}_1, \mathbf{D} \in \mathbb{R}_-^L \times \mathbb{R}_+^L \end{array} \right\} \subseteq B \qquad \text{[EQ A-27]}$$

The expanded objective budget restriction says that the subject chooses an objective consumption set that equates the intrinsic value of consumption with the intrinsic value of the payoffs from production and with the intrinsic value of exogenous wealth.

The larger the RHS of the objective budget restriction, the more the subject can consume, i.e. the larger is the LHS. Since the subject is better off with more consumption, he or she will always want to increase the RHS. His or her payoff from the production is within the grasp of his or her control and could be increased.

Any individual, no matter how he or she prefers to balance his or her consumption between the present and the future, will always want the firm to maximize its net present value of the payoffs. With a fixed input numerarie good, d_{10}°, this amounts to maximizing the intrinsic value of the output factor, i.e. $_1 \boldsymbol{p}_1 \cdot \mathbf{d}_1$, which is equal to maximizing the current intrinsic value of the firm.

The objective production set has been specified in equation [EQ A-11] to

$\mathbb{D} = \left\{ \mathbf{D} \in \mathbb{R}^L_- \times \mathbb{R}_+ : g\left(\mathbf{d}_1, \mathbf{d}'_0 \middle| d^\circ_{10}\right) \leq \varnothing \right\}$, but since the subject wants the firm to maximize its intrin-

sic value, this puts an additional constraint on production. Only output efficient production plans,

$g\left(\mathbf{d}_1, \mathbf{d}'_0 \middle| d^\circ_{10}\right) = \varnothing$, are acceptable by the perfect rational choice since it maximizes the output for a

given input. It means that the optimal objective production set must lie on the objective transforma-

tion frontier:

$$\mathbb{D}' = \left\{ \mathbf{D} \in \mathbb{R}^L_- \times \mathbb{R}_+ : g\left(\mathbf{d}_1, \mathbf{d}'_0 \middle| d^\circ_{10}\right) = \varnothing \right\} \qquad \text{[EQ A-28]}$$

The subject maximizes his or her objective utility function $U(\mathbf{C})$, but in the joint optimiza-

tion problem it is subject to the two constraints: the new objective budget hyperplane, [EQ A-27],

and the objective transformation frontier, [EQ A-28].

His or her choice is the optimal objective consumption set that lies on the objective budget

line, satisfying $\mathbf{C}^* = B' \cap X_{IND}$. The optimal choice of production plan satisfies $\mathbf{D}^* = \mathbb{D}' \cap B'$. With

both production and exchange opportunities available, it is not necessary for the optimal objective

production set and the optimal objective consumption sets to be the same and hence $\mathbf{C}^* \neq \mathbf{D}^*$ will

generally be the case. This is also known as Fisher's (1930) separation theorem.

To find the intersection between the objective budget hyperplane, the indifference set, and

the objective production set the problem is formulated as a constrained maximization problem:

$$\max_c U(\mathbf{C}) \text{ s.t.} \qquad \text{[EQ A-29]}$$

$$_0\boldsymbol{p}_0 \cdot (\mathbf{c}_0 - \boldsymbol{\omega}_0) + {}_0\boldsymbol{p}_1 \cdot (\mathbf{c}_1 - \boldsymbol{\omega}_1) - {}_0\boldsymbol{p}_0 \cdot \theta \cdot \mathbf{d}_0 - {}_0\boldsymbol{p}_1 \cdot \theta \cdot \mathbf{d}_1 = 0$$

$$g\left(\mathbf{d}_1, \mathbf{d}'_0 \middle| d^\circ_{10}\right) = 0$$

Using Lagrange's method the constrained maximization problem turned into an uncon-

strained maximization problem:

$$\max_{c,d,\lambda,\mu} \mathcal{L} = \max_{c,d,\lambda,\mu} \left\{ \begin{array}{l} U(\mathbf{C}) \\ -\lambda \cdot \left[\begin{array}{l} {}_0\boldsymbol{p}_0 \cdot (\mathbf{c}_0 - \boldsymbol{\omega}_0) + {}_0\boldsymbol{p}_1 \cdot (\mathbf{c}_1 - \boldsymbol{\omega}_1) \\ -{}_0\boldsymbol{p}_0 \cdot \theta \cdot \mathbf{d}_0 - {}_0\boldsymbol{p}_1 \cdot \theta \cdot \mathbf{d}_1 \end{array} \right] \\ -\mu \cdot g\left(\mathbf{d}_1, \mathbf{d}'_0 \middle| d^\circ_{10}\right) \end{array} \right\} \qquad \text{[EQ A-30]}$$

Solving the unconstrained problem gives the following results.

$$\nabla \mathcal{L}(\mathbf{c_0}) = \nabla U(\mathbf{c_0}) - \lambda \cdot {}_0\boldsymbol{p}_0 = 0 \qquad \text{[EQ A-31]}$$

$$\nabla \mathcal{L}(\mathbf{c_1}) = \nabla U(\mathbf{c_1}) - \lambda \cdot {}_0\boldsymbol{p}_1 = 0 \qquad \text{[EQ A-32]}$$

$$\nabla \mathcal{L}(\mathbf{d_0}) = \lambda \cdot \theta \cdot {}_0\boldsymbol{p}_0 - \mu \cdot \theta \cdot \nabla g(\mathbf{d_0}) = 0 \qquad \text{[EQ A-33]}$$

$$\nabla \mathcal{L}\left(\mathbf{d}_1\right) = \lambda \cdot \theta \cdot {}_0\boldsymbol{p}_1 - \mu \cdot \theta \cdot \nabla g\left(\mathbf{d}_1\right) = 0 \qquad \text{[EQ A-34]}$$

$$\frac{{}_0\boldsymbol{p}_1}{{}_0\boldsymbol{p}_0} = \frac{\nabla U\left(\mathbf{c}_1\right)}{\nabla U\left(\mathbf{c}_0\right)} = \frac{\nabla g\left(\mathbf{d}_1\right)}{\nabla g\left(\mathbf{d}_0\right)} \qquad \text{[EQ A-35]}$$

Equation [EQ A-35] shows that the subject chooses to consume where his or her objective marginal rate of substitution between current and future consumption equals the objective marginal rate of transformation between current and future goods. The objective marginal rate of substitution and the objective marginal rate of transformation intrinsic value equal the objective futures price for the commodity adjusted for the objective spot price for future consumption.

$$\nabla \mathcal{L}\left(\lambda\right) = {}_0\boldsymbol{p}_0 \cdot \left(\mathbf{c}_0 - \boldsymbol{\omega}_0\right) + {}_0\boldsymbol{p}_1 \cdot \left(\mathbf{c}_1 - \boldsymbol{\omega}_1\right) - {}_0\boldsymbol{p}_0 \cdot \theta \cdot \mathbf{d}_0 - {}_0\boldsymbol{p}_1 \cdot \theta \cdot \mathbf{d}_1 = 0 \qquad \text{[EQ A-36]}$$

$$\nabla \mathcal{L}\left(\mu\right) = g\left(\mathbf{d}_1, \mathbf{d}_0' \middle| d_{10}^{\circ}\right) = 0 \qquad \text{[EQ A-37]}$$

The choices made by the subject and by the firm are subject to the constraints expressed as equations [EQ A-36] and [EQ A-37]. With equation [EQ A-35] and [EQ A-36], it is possible to express the optimal consumption that an individual takes to find his or her tangency solution $\mathbf{C}^* = B' \cap X_{IND}$ as:

$$\mathbf{c}_0^* = \left(\boldsymbol{\omega}_0 + \mathbf{d}_0^*\right) + \frac{{}_0\boldsymbol{p}_1}{{}_0\boldsymbol{p}_0} \cdot \theta \cdot \mathbf{d}_1^* - \frac{{}_0\boldsymbol{p}_1}{{}_0\boldsymbol{p}_0} \cdot \left(\boldsymbol{\omega}_1 + \mathbf{c}_1^*\right) \qquad \text{[EQ A-38]}$$

$$\mathbf{c}_1^* = \left(\boldsymbol{\omega}_1 + \theta \cdot \mathbf{d}_1^*\right) + \frac{{}_0\boldsymbol{p}_0}{{}_0\boldsymbol{p}_1} \cdot \theta \cdot \mathbf{d}_0^* - \cdot \frac{{}_0\boldsymbol{p}_0}{{}_0\boldsymbol{p}_1} \cdot \left(\mathbf{c}_0^* + \boldsymbol{\omega}_0\right) \qquad \text{[EQ A-39]}$$

To specify the tangency solution $\mathbf{D}^* = \mathbb{D}' \cap B'$ demands further structure on the production constraint [EQ A-37]. Since the purpose with this appendix is to derive a valuation model, a further specification is not necessary. This means the production equivalent to [EQ A-38] and [EQ A-39] is not derived.

The objective price has until now been exogenous to the model. The subject has to choose an optimal consumption and investment choice for a given objective price structure. The following section lets the objective price be determined from within the model.

A.7 The competitive equilibrium

The present discussion has focused thus far on a single individual who is described as a perfect rational choice. In the pure exchange situation the subject was engaged in exchange with other individuals who acted in the same manner, without further specifying the circumstances. The same applies to the situation where the subject faces the dual problem of exchange and production.

This section considers the full extent of the problem where there are several individuals and where several firms are active in the economy. The expansion makes it possible to let the objective price be established within the model. Previously, the expansion was, together with his or her preferences and the firm's transformation technology, made from outside the model.

The goal of this section is to present a model in which all individuals are satisfied with their consumption and investment plan and where there are no incentives to alter the resource allocation. When everyone is satisfied and there are feasible plans, the supply and demand of the commodities match and the economy is Pareto optimally allocated.

To go from one individual and one firm acting in a economy to consider several individuals and several firms active in the economy calls for further restrictions. Throughout this thesis it is assumed that no individual or productive unit in the economy has the ability to materially affect the prevailing intrinsic values with their transactions. That is, everyone is an objective price taker.

It is also necessary to add constraints that guarantee that the consumption and investment plans are feasible. These are the market clearing constraints and they look different when comparing a pure exchange and a production economy.

A.7.1 Market clearing

Market clearing is a constraint that is imposed to avoid consumption patterns that are not feasible. E.g., it is inconceivable that consumption of fresh water on earth exceeds the total supply of fresh water on earth. The constraint is sometimes also known as a materials balance constraint (Gravelle & Rees 1998) or conservation constraint (Hirshleifer 1970).

Before a formalization of the market clearing constraint is made, it is necessary to define total consumption and total wealth. Let the aggregate current consumption be the sum of all individuals' current consumption: $\sum_i \mathbf{c}_0^i$. Similarly, the aggregate future consumption is equal to the sum of all individuals' future consumption: $\sum_i \mathbf{c}_1^i$. The aggregated current and future endowed wealth are $\sum_i \boldsymbol{\omega}_0^i$, and $\sum_i \boldsymbol{\omega}_1^i$, and the aggregated current and future productions are $\sum_i \mathbf{d}_0^i$, and $\sum_i \mathbf{d}_1^i$.

Feasible consumption patterns assume that the aggregate current consumption must be less than or equal to the sum of all individuals' endowed current exogenous wealth and production. The same restriction applies to future consumption and wealth. That is, $\sum_i \mathbf{c}_0^i \leq \sum_i \boldsymbol{\omega}_0^i + \sum_i \mathbf{d}_0^i$, and $\sum_i \mathbf{c}_1^i \leq \sum_i \boldsymbol{\omega}_1^i + \sum_i \mathbf{d}_1^i$ prohibit unfeasible consumption plans. This means that supply equals demand.

However, since the subjects in the economy are always non-satiated, they will not exhibit wasteful behavior. It means that they do not consume less than what is possible because the remaining resources are lost. This shifts the inequalities to equalities and the supply of resources exactly corresponds to the demands of the resources. The behavior clears the economy and it is symbolized by: $\sum_i c_0^i = \sum_i \omega_0^i$, and $\sum_i c_1^i = \sum_i \omega_1^i$.

A.7.2 Dovetailing consumption and investment choices

Each subject in society is a perfect rational choice. In section A.4 Homo economics' preference relations were defined. The most central assumption concerned completeness, which in words was defined as:

> The subject has an understanding of all the choice opportunities available, makes pair-wise comparison of them all, and expresses whom he or she prefers over the other, or if he or she is indifferent between them.

This means that the subject can assess, fully and correctly, all the consumption and investments opportunities that are available for his or her.

If there are as many individuals in society, it means that their objective opportunity set is affected by what others are willing to trade. If no commodities at all are exogenously inserted into the economy, all the commodities in the economy must be produced, which means that the subject cannot only focus on the consumption choice and the related trade but must also consider the investment choice.

The only way for each subject in an economy to be able to fully and correctly assess his or her objective opportunity set and the related objective production set is to have knowledge of each other's preferences, opportunities, and objective production sets, including budgetary constraints; otherwise, some of his or her consumption/investment plans will be erroneous and thus cannot be executed as expected. This means that some individuals will not be satisfied with the current allocation of goods in the economy and thus will have incentive to change behavior given another chance. When the plans are erroneous, the economy does not clear and there is no match between the supply and demand of commodities.

When the economy clears, there are perfectly dovetailing consumption and investment patterns in society. That is, each subject's consumption and investment choice fit into each other to form a compact and harmonious whole.

The completeness assumption from section A.4, together with this section's assumption on market clearing, means that the subject no longer can be assumed to abide to.

> The subject has an understanding of all the choice opportunities available, makes pair-wise comparisons of them all, and expresses whom he or she prefers over the other, or if he or she is indifferent between them.

Now every individual must behave as below:

> Every individual in society has an understanding of all consumption opportunities, all production opportunities, and all budgetary constraints available for all the subjects in society. Each subject also has knowledge of everyone's preferences. Based on all the opportunities and constraints, the subjects form their expectations on what the aggregate supply and demand will be that clear the economy and what intrinsic values that are needed to clear the economy. Indigenous to this process is also the subjects' individual choice process in which they make up their own consumption and investment plans. When all individuals in society have performed this process, the consumption and investment plans are executed.

This is indeed a very strong assumption. In fact, researchers have tried to avoid it by proposing another process that gives a similar result. Walras ([1874] 1954) devised a strategy that today is

known as the tâtonnement process. In this process a mastermind decides tentative objective prices that are transmitted to the subjects in society, who then hand in their tentative consumption and investments plans to the mastermind. A new set of tentative objective prices is distributed if the economy does not clear based on the initial tentative plans. This process continues until an objective price is found that makes all consumption and investment plans dovetail. The plans are executed when the economy clears.

With the subjects' tâtonnement process, they are no longer able by themselves to discover the feasible consumption and investment patterns in society. There is someone else who is more rational and smarter who acts as a mastermind that mediates to avoid unfeasible plans. This is at odds with the assumptions that each subject in society is a rational economic choice maker. Therefore, these types of ad hoc device are non-void. This thesis solely builds on individual choice making and not on a mastermind.

A.7.3 The market equilibrium

With a dovetailing market, it is possible to focus on the objective price formation process. This process can be described as an aggregate optimization process in which each subject in society participates. Equilibrium is achieved when the aggregate optimization process has run its course, which gives a Pareto optimal allocation of resources. This means that it is not possible to choose a different allocation of resources in society that makes an individual strictly better off without making any other individual worse off.

Recall from section A.4 that each subject has an objective utility function U^i, and from section A.6 how this objective utility function is optimized based on the restrictions from the objective budget set and the objective opportunity set. This can be described on an economy level in which all individuals choose consumption plans that maximize their utilities given their restraints from the budget and objective opportunity set. On top of that, the chosen consumption and production plans must be feasible. The best possible allocation is the Pareto optimal allocation that can be found as the solution to the following constrained maximization problem:

$$\max_c U^1(\mathbf{C}) \text{ s.t.} \qquad \text{[EQ A-40]}$$

$$U^i(\mathbf{C}) \geq U^i(\overline{\mathbf{C}}), \ \forall (2,\ldots,I) \text{ s.t.}$$

$$\sum_i \mathbf{c}_0^i = \sum_i \boldsymbol{\omega}_0^i + \sum_i \mathbf{d}_0^j, \ \forall \ell \in L$$

$$\sum_i \mathbf{c}_1^i = \sum_i \boldsymbol{\omega}_1^i + \sum_i \mathbf{d}_1^j, \ \forall \ell \in L$$

$$\sum_j \left[\mathbf{d}_1^j - g\left(\mathbf{d}_0^{j\prime}, d_{10}^{j\circ}\right) \right] = 0, \ \forall j \in J$$

This maximization gives rise to the following unconstrained optimization:

$$\max_{c,d,\delta,\lambda,\mu} \mathcal{L} = \max_{c,d,\delta,\lambda,\mu} \left\{ \begin{array}{l} U^1(\mathbf{C}) \\ -\boldsymbol{\delta}\left[U^i(\mathbf{C}) - U^i(\overline{\mathbf{C}})\right] \\ -_0\boldsymbol{p}_0 \cdot \left(\sum_i \mathbf{c}_0^i - \sum_i \boldsymbol{\omega}_0^i - \sum_j \mathbf{d}_0^j\right) \\ -_0\boldsymbol{p}_1 \cdot \left(\sum_i \mathbf{c}_1^i - \sum_i \boldsymbol{\omega}_1^i - \sum_j \mathbf{d}_1^j\right) \\ -\boldsymbol{\mu} \cdot \sum_j \left[g\left(\mathbf{d}_1^j, \mathbf{d}_0^{j\prime} \,\big|\, d_{10}^{j\circ}\right)\right] \end{array} \right\}, \begin{array}{l} \forall i \in I \\ \forall \ell \in L, \\ \forall j \in J \end{array} \qquad \text{[EQ A-41]}$$

$$\nabla \mathcal{L}\left(\mathbf{c}_0^i\right) = \boldsymbol{\delta} \cdot \nabla U^i\left(\mathbf{c}_0\right) = {}_0\boldsymbol{p}_0, \; \forall i \in I, \, \ell \in L \qquad \text{[EQ A-42]}$$

$$\nabla \mathcal{L}\left(\mathbf{c}_1^i\right) = \boldsymbol{\delta} \cdot \nabla U^i\left(\mathbf{c}_1\right) = {}_0\boldsymbol{p}_1, \; \forall i \in I, \, \ell \in L \qquad \text{[EQ A-43]}$$

$$\nabla \mathcal{L}\left(\mathbf{d}_0^j\right) = {}_0\boldsymbol{p}_0 - \boldsymbol{\mu} \cdot g\left(\mathbf{d}_0^j\right) = 0, \; \forall j \in J, \, \ell \in L \qquad \text{[EQ A-44]}$$

$$\nabla \mathcal{L}\left(\mathbf{d}_1^j\right) = {}_0\boldsymbol{p}_1 - \boldsymbol{\mu} \cdot g\left(\mathbf{d}_1^j\right) = 0, \; \forall j \in J, \, \ell \in L \qquad \text{[EQ A-45]}$$

$$\frac{{}_0\boldsymbol{p}_1}{{}_0\boldsymbol{p}_0} = \frac{\nabla U^i\left(\mathbf{c}_1\right)}{\nabla U^i\left(\mathbf{c}_0\right)} = \frac{\nabla g\left(\mathbf{d}_1\right)}{\nabla g\left(\mathbf{d}_0\right)} \qquad \text{[EQ A-46]}$$

Equation system [EQ A-46] shows how each subject in society has the same objective marginal rate of substitution between current and future consumption in a Pareto optimal equilibrium. The objective marginal rates of transformation are also equal across firms in the economy in this situation and they are all equal to the prevailing intrinsic value for current consumption scaled by the objective current objective price for future consumption.

The fact that each subject in society has the same objective marginal rate of substitution is conceivable given that conjecture of the theory of the perfect rational choice. The perfect rational choice posits that each subject has complete knowledge, i.e. that he or she can fully specify his or her action and the state set. With the additional assumption of materials balance constraint, each subject must have complete knowledge of every other individual's action set and state set.

Since each subject in society has the same objective marginal rate of substitution, the assumption of homogenous preferences posited by the assumption of the representative individual in financial economics is not restrictive.

A.8 The intrinsic value rule for firms in objective certainty
The subjects' choice when facing both productive and exchange opportunities allowed for the conclusion that they would always be better off with a higher net present payoff from the firm. This conclusion was reached by studying the expanded objective budget set in subsection A.6.2. With the previous optimization analysis, it is possible to derive this relationship formally.

A.8.1 The intrinsic value rule based on dividends

All analyses hitherto have focused on a one-period model. This subsection elaborates on the intrinsic value rule based on net dividends. It continues to use a one-period framework that is expanded into a multi-period context.

A.8.1.1 The one-period dividend valuation model in objective certainty

It was claimed in [EQ A-30] that subjects maximize their utility given the objective budget restriction and the productive opportunities. This is:

$$
\max_{c,d,\lambda,\mu} \mathcal{L} = \max_{c,d,\lambda,\mu} \left\{ \begin{array}{l} U\left(\mathbf{C}\right) \\ -\lambda \cdot \left[{}_0\boldsymbol{p}_0^* \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1^* \cdot \mathbf{c}_0 - {}_0\boldsymbol{p}_0^* \cdot \mathbf{d}_0 - {}_0\boldsymbol{p}_1^* \cdot \mathbf{d}_1' \right] \\ -\mu \cdot g\left(\mathbf{d}_1, \mathbf{d}_0' \Big| d_{10}^\circ \right) \end{array} \right\} \qquad \text{[EQ A-47]}
$$

The maximization problem can be re-written so it focuses on the two separate choices of choosing an optimal objective consumption set and choosing the optimal production plan.

$$
\begin{aligned}
&\max_{c,\lambda} \left\{ U\left(\mathbf{C}\right) - \lambda \cdot \left[{}_0\boldsymbol{p}_0^* \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1^* \cdot \mathbf{c}_1 \right] \right\} + \\
&\max_{d,\lambda,\mu} \left\{ \lambda \cdot \left[{}_0\boldsymbol{p}_0^* \cdot \mathbf{d}_0 + {}_0\boldsymbol{p}_1^* \cdot \mathbf{d}_1' \right] - \mu \cdot g\left(\mathbf{d}_1, \mathbf{d}_0' \Big| d_{10}^\circ \right) \right\}
\end{aligned} \qquad \text{[EQ A-48]}
$$

The maximization problem can be simplified further by defining $\gamma \equiv \mu \cdot \lambda^{-1}$. Focusing on the production maximization problem, it becomes:

$$
\begin{aligned}
&\max_{d,\lambda,\mu} \left\{ \lambda \cdot \left[{}_0\boldsymbol{p}_0^* \cdot \mathbf{d}_0 + {}_0\boldsymbol{p}_1^* \cdot \mathbf{d}_1' \right] - \lambda \cdot \gamma \cdot g\left(\mathbf{d}_1, \mathbf{d}_0' \Big| d_{10}^\circ \right) \right\} = \\
&\lambda \cdot \max_{d,\gamma} \left\{ \left[{}_0\boldsymbol{p}_0^* \cdot \mathbf{d}_0 + {}_0\boldsymbol{p}_1^* \cdot \mathbf{d}_1' \right] - \gamma \cdot g\left(\mathbf{d}_1, \mathbf{d}_0' \Big| d_{10}^\circ \right) \right\}
\end{aligned} \qquad \text{[EQ A-49]}
$$

The subjects' choice can now be interpreted as if they first maximize the production equation [EQ A-49]. This yields the optimal production plan, $\left(\mathbf{d}_0^*, \mathbf{d}_1^* \right)$, that maximizes the net intrinsic value of the firm given the production constraint and equilibrium objective prices. Since the initial investment is assumed fixed (see section A.5.2), what remains is choosing the investment plan that gives the highest attainable intrinsic value (given the restraint that is due to production technology), i.e. $\max \left({}_0\boldsymbol{p}_1^* \cdot \mathbf{d}_1 \Big| \mathbf{d}_{10}^\circ \right)$. Then, with an optimal production plan, the subjects proceed to optimize the remaining problem, i.e.:

$$
\max_{c,d,\lambda,\mu} \left\{ U\left(\mathbf{C}\right) - \lambda \cdot \left[{}_0\boldsymbol{p}_0^* \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1^* \cdot \mathbf{c}_1 - {}_0\boldsymbol{p}_0^* \cdot \mathbf{d}_0^* - {}_0\boldsymbol{p}_1^* \cdot \mathbf{d}_1^* \right] \right\} \qquad \text{[EQ A-50]}
$$

From which the subjects get their optimal consumption plan $\left(\mathbf{c}_0^*, \mathbf{c}_1^* \right)$.

This analysis further highlights the separation of the firm's production choice and his or her consumption choice. Any individual prefers equation [EQ A-49] to be maximized no matter what he

or she plans to consume since this increases the maximum attainable level of well being, as expressed by the objective utility function.

Equation [EQ A-49] establishes how investments into a firm are made in a Pareto optimal setting. For a given level of initial investments (here defined as a fixed portion of the present numerarie) and for Pareto optimal objective current objective prices for future consumption, the investments are made such that they maximize the intrinsic value of the future output.

Since the future output can be traded in the future spot economy at Pareto optimal objective spot prices, it is possible, without loss of generality, to assume that all future commodities are converted into the future numerarie. This allows us to rewrite [EQ A-49] as:

$$\max_{d_{11}} \left[{}_0 p_{10}^* \cdot d_{10}^{\circ} + {}_0 p_{11}^* \cdot d_{11} \right]$$ [EQ A-51]

The technological constraint is ignored in [EQ A-51], but it should not be interpreted as if it is not important. Rather, [EQ A-51] can be interpreted as a maximizing problem given the technological constraints. In everyday language this equation is known as the net intrinsic value rule. It should also be noted that since the future commodity vectors into the future numerarie, it follows that conceptually d_{11} will be the net of all future commodities. In practice this would then be called the net dividends.

The objective spot prices for the numerarie at present and in the future are defined to be 1 in section A.5.1. This allows the numerarie commodity to be called capital, and accordingly, from a firm's perspective, the input and output in [EQ A-51] are equivalent to the firm's dividends.

The Pareto optimal objective price of any investment can thus be defined as its intrinsic value of future payoffs. In algebra it becomes:

$$V_{10} = {}_0 p_{11}^* \cdot d_{11}$$ [EQ A-52]

Equation [EQ A-52] is said to show the Pareto optimal objective price of a firm. In this thesis I call this value the firm's intrinsic value in order to signify that it is the value that the firm should have if the economy is in a Pareto optimal equilibrium.

The subscript that keeps track of the commodity is hereafter dropped to avoid a too cumbersome notation. The stars indicating the Pareto optimal objective prices are also dropped to simplify the notation.

A.8.1.2   A multi-period dividend valuation model in objective certainty

The one period intrinsic value can be expanded to cover more than one period. Assume that [EQ A-52] holds for $T-1$, where $T > 2$. This means that $V_0 = {}_0 p_1 \cdot d_1, \cdots, V_{T-1} = {}_0 p_T \cdot d_T$. This also implies that the Pareto optimal allocation holds across all periods.

As long as $V_1 \neq 0$ it follows that [EQ A-52] needs to be modified to incorporate the remaining value at the end of the period. Hence, $V_0 = {}_0 p_1 \cdot (d_1 + V_1), \cdots, V_{T-1} = {}_{T-1} p_T \cdot (d_T + V_T)$. Substituting this expression into the latter gives the multi-period dividend valuation model:

$$V_0 = \sum_{t=1}^{T} {}_0 p_t \cdot d_t + {}_0 p_T \cdot V_T \qquad \text{[EQ A-53]}$$

Where: ${}_0 p_T = {}_0 p_1 \cdot {}_1 p_2 \cdot \cdots \cdot {}_{T-1} p_T = \prod_{t=1}^{T} {}_{t-1} p_t$.

Equation [EQ A-53] is the multi-period dividend valuation model for a finite period. Before the multi-period dividend valuation model is expanded to cover infinity, the objective market rates-of-return (MROR) is introduced.

A.8.2 The intrinsic value rule with market rates-of-returns and not objective prices

Up to now, the analysis has only made use of spot and objective future prices. The analysis now turns the objective futures price ${}_0 \rho_1$ into an objective MROR.

Let q_0 be the quantity of the current capital and let q_1 be the quantity of the objective future capital. In order to get q_1 units of the capital in the future, the subject will have to forfeit in the present:

$$q_0 = {}_0 p_1 \cdot q_1 \qquad \text{[EQ A-54]}$$

It is also possible to express the quantity of the objective future capital based on the present capital plus the change in the capital, i.e.:

$$q_1 = q_0 + \Delta q_0 \qquad \text{[EQ A-55]}$$

Substituting [EQ A-55] into [EQ A-54] and rearranging gives:

$$1 + \Delta q_0 \cdot q_0^{-1} = {}_0 \rho_0^{-1} \qquad \text{[EQ A-56]}$$

Define the MROR as the rate of growth of capital, i.e. ${}_0 r_1 = \Delta q_0 \cdot q_0^{-1}$. This gives:

$$1 + {}_0 r_1 = {}_0 p_0^{-1} \Leftrightarrow {}_0 p_1 = \left(1 + {}_0 r_1\right)^{-1} \qquad \text{[EQ A-57]}$$

From [EQ A-57], it is apparent that the objective future objective price for capital is equivalent to the commonly used discount factor in investment appraisal. The MROR is equivalent to the risk-free rates-of-return since the model used is based on objective certainty.

With the introduction of the MROR, it is also possible to expand [EQ A-53] to infinity and still have a bounded current intrinsic value. It is assumed that ${}_{t-1} r_t > \Delta V_{t-1} \cdot V_{t-1}^{-1}$, $\forall t$. This implies that $\lim_{t \to \infty} \left(\prod_{t=1}^{\infty} {}_{t-1} p_t \cdot V_\infty \right) = 0$, and then [EQ A-53] collapses to:

$$V_0 = \sum_{t=1}^{\infty} {}_0 p_t \cdot d_t \qquad \text{[EQ A-58]}$$

Where: $_0p_\infty = \,_0p_1 \cdot \,_1p_2 \cdot \cdots \cdot \,_{\infty-1}p_\infty = \prod\limits_{t=1}^{\infty} \,_{t-1}p_t$, and where $_{t-1}p_t = \left(1 + \,_{t-1}r_t\right)^{-1}$.

Equation [EQ A-58] shows the infinite dividend valuation model. It expresses the intrinsic value of, e.g., a firm as a function of the intrinsic value of future dividends with an infinite time horizon. The model allows for non-constant market rates-of-returns, which in practice is often simplified to constant rates-of-returns (Penman 2004; Beaver 1989).

A.8.3 Multi-period, non-constant rates-of-returns, and the intrinsic value rule based on residual income

It is possible to define a firm's intrinsic value as a function of future net dividends based on a Pareto optimal allocation in society. It is also possible to derive accounting valuation models that are equivalent to the dividend valuation model. This section derives the residual income valuation model measured on both comprehensive net income and on comprehensive operating income. First, follows the residual income valuation model measured on comprehensive net income and then the residual income valuation model measured on comprehensive operating income.

A.8.3.1   The residual income valuation model measured on comprehensive net income

The residual income valuation model was originally developed by Preinreich (1936, 1937a, 1937b, 1938), but received scarce attention until 1995 when Ohlson popularized it. Ohlson (1995) developed the model using objective certainty and constant MROR. I present a similar model developed in objective certainty that instead uses non-constant MROR.

Accounting uses several rules. One such rule is the clean surplus relationship, which states that any change in the equity account is a function of net transactions with the owners and of comprehensive net income. Algebraically this implies:

$$\Delta EQ_t = CNI_t - d_t \qquad \text{[EQ A-59]}$$

Where CNI_t is the comprehensive net income for a period, ΔEQ_t is the change in the equity account between two adjacent periods starting at $t-1$, and d_t is the period's net dividends. Substituting [EQ A-59] into [EQ A-58] yields:

$$V_0 = \,_0p_1 \cdot \left(CNI_1 - \Delta EQ_0\right) + \,_0p_1 \cdot \,_1p_2 \left(CNI_2 - \Delta EQ_1\right) + \cdots \qquad \text{[EQ A-60]}$$

Defining residual income as the portion of comprehensive net income that deviates from the expected comprehensive net income

$$RI_t = CNI_t - \,_{t-1}r_t \cdot EQ_{t-1}, \qquad \text{[EQ A-61]}$$

opens the possibility of re-writing [EQ A-60] as a function of residual income, comprehensive net income, and change in equity. Expected comprehensive net income is the comprehensive net income expected by the owners at the inception of the period, i.e. $_{t-1}r_t \cdot EQ_{t-1}$.

$$\begin{aligned} V_0 = \,&_0p_1 \cdot \left(RI_1 + \,_0r_1 \cdot EQ_0 - \Delta EQ_0\right) + \\ &_0p_1 \cdot \,_1p_2 \left(RI_2 + \,_1r_2 \cdot EQ_1 - \Delta EQ_1\right) + \cdots \end{aligned}$$

which is simplified to

$$V_0 = {}_0p_1 \cdot \left(RI_1 + \left(1 + {}_0r_1\right) \cdot EQ_0 - EQ_1\right) + \\ {}_0p_1 \cdot {}_1p_2 \left(RI_2 + \left(1 + {}_1r_2\right) \cdot EQ_1 - EQ_2\right) + \cdots$$ [EQ A-62]

Since [EQ A-57] provides the means for translating the objective futures price (discount factor) into MROR, and vice versa, it follows that ${}_{t-1}p_t = \left(1 + {}_{t-1}r_t\right)^{-1}$. Substituting this into [EQ A-62] gives:

$$V_0 = {}_0p_1 \cdot RI_1 + EQ_0 - {}_0p_1 \cdot EQ_1 + \\ {}_0p_1 \cdot {}_1p_2 \cdot RI_2 + {}_0p_1 \cdot EQ_1 - {}_0p_1 \cdot {}_1p_2 \cdot EQ_2 + \cdots$$ [EQ A-63]

Rearranging and summing [EQ A-63] over a finite period gives:

$$V_0 = EQ_0 + \sum_{t=1}^{T} {}_0p_t \cdot RI_t + {}_0\rho_T \cdot \left(V_T - EQ_T\right)$$ [EQ A-64]

Assuming a mild regulatory condition, ${}_0r_t > \Delta EQ_0 \cdot EQ_0$, implies $\lim_{t \to \infty} \left(\prod_{t=1}^{\infty} {}_{t-1}p_t \cdot EQ_\infty \right) = 0$

and it collapses [EQ A-64] into the multi-period residual income valuation model with non-constant rates-of-returns as the horizon is pushed into infinity:

$$V_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0p_t \cdot RI_t$$ [EQ A-65]

Equation [EQ A-65] is an equivalent expression to [EQ A-58] assuming the clean surplus relationship holds. It shows that the firm's intrinsic value is a function of the present book value of equity and of the intrinsic value of future residual income. The clean surplus relationship makes it possible to solve the dividend conundrum posed by Modigliani & Miller (1958) and analyze a firm's value creation rather than the value distribution, which is the object of the study in model [EQ A-58]. When $V_t > EQ_t$, value has been created. From model [EQ A-65], it is apparent that this can only occur when $RI_{t+1} > 0$. This implies that value is created only when the residual income is positive. Conversely, value has been destroyed when $V_t < EQ_t$, and this can only occur when $RI_{t+1} < 0$.

The model [EQ A-65] shows how the intrinsic value is measured when objective certainty is present and where the rates-of-returns are non-constant. This is a further development compared with Ohlson (1995) who discusses this model with constant rates-of-returns.

Feltham & Ohlson (1999) derive the residual income valuation model with stochastic rates-of-returns. This is a more complex case compared with the case where model [EQ A-65] operates. An important conclusion is nevertheless the same: The normal income is measured as the product of the one-period objective MROR and the beginning of period book value of equity. This is at odds with how it is proposed to be measured among practitioners such as Copeland, Koller & Murrin

(2000), Damodaran (2001), and Stern & Stewart (1999). It is also at odds with how textbooks teach it. E.g., Penman (2004) applies a constant rate-of-return rather than a time-varying one-period rate-of-return when normal income is estimated.

Since firms are not restricted to finance their activities only through equity it is also useful to convert [EQ A-58] into a valuation model that also considers Modigliani & Miller's (1958) value additivity proposition. This is the residual income valuation model measured on comprehensive operating income.

A.8.3.2   <u>The residual income valuation model measured on comprehensive operating income</u>
Using the fact that that the firm's intrinsic value can be expressed as a function of the value of the whole firm and the value of debt (Modigliani & Miller 1958), it is possible to express [EQ A-58] as a function of the intrinsic values of residual operating income, ROI, and the residual net interest expense, RIE. Feltham & Ohlson (1995) derive at a similar model in objective certainty having constant rates-of-returns.

A theoretical income statement and a balance sheet must be introduced in order to derive such a model. First, the comprehensive net income is classified into comprehensive operating income, COI and comprehensive net interest expense, $CNIE$:

$$CNI_t = COI_t - CNIE_t \qquad \text{[EQ A-66]}$$

Second, the balance sheet is classified into net operating assets, NOA, net financial liabilities, NFL, and equity:

$$EQ_t = NOA_t - NFL_t \qquad \text{[EQ A-67]}$$

With these relationships and with the clean surplus relationship [EQ A-59], it is possible to convert [EQ A-58] into another accounting valuation model similar to [EQ A-65] but where residual operating income and residual interest expense are income components rather than residual income.

Using a certain setting having non-constant MROR gives the following valuation model:

$$V_0 = NOA_0 + \sum_{t=1}^{\infty} {}_0 p_t \cdot ROI_t - \left(NFL_0 + \sum_{t=1}^{\infty} {}_0 p_t \cdot RIE_t \right) \qquad \text{[EQ A-68]}$$

where

$$ROI_t = COI_t - {}_{t-1}r_t \cdot NOA_{t-1} \qquad \text{[EQ A-69]}$$
$$RIE_t = CNIE_t - {}_{t-1}r_t \cdot NFL_{t-1} \qquad \text{[EQ A-70]}$$

Since the model is placed in a certain setting, the MROR is the same for the net operating assets and the net financial liabilities. Similar models derived from a setting in uncertainty would use the weighted average cost of capital as MROR for the net operating assets and the after-tax net cost of debt as MROR for the net financial liabilities (e.g., Lundholm & O'Keefe 2001).

See Chapter 5 for a further discussion of classification and operationalization of the components to the balance sheet and income statement.

With the models [EQ A-65] and [EQ A-68], it is possible to define and study accounting rates-of-returns and their relationships to firm valuation. This is done in the next subsection.

A.8.4 The relationship between market rates-of-returns and accounting rates-of-returns

This section defines the accounting rates-of-returns used in this thesis and discusses how they relate to the objective MROR. The accounting rates-of-returns are the return on equity and the return on net operating assets. First follows the return on equity.

A.8.4.1   Return on equity and residual rate-of-return on equity

The most fundamental accounting rate-of-return is the growth rate of the owner's capital. In accounting the growth of the owner's capital is known as the return on equity (ROE). This section defines the ROE and also the residual rate-of-return on equity, which can be used to rewrite [EQ A-65] into a residual rate-of-return on an equity-based valuation model.

In these studies sometimes the net income is divided by beginning of period equity; other times by ending of period equity or the average equity is used to measure the return on equity. This thesis defines return on equity such that it matches the derived valuation models. ROE is thus defined as comprehensive net income divided by beginning of period equity:

$$_{t-1}ROE_t = CNI_t \cdot EQ_{t-1}^{-1}$$

[EQ A-71]

Solving for comprehensive net income in [EQ A-71], substituting this into the clean surplus definition [EQ A-59], and rearranging gives:

$$d_t + EQ_t = \left(1 + {}_{t-1}ROE_t\right) \cdot EQ_{t-1}$$

[EQ A-72]

Suppose that the firm is just formed. This implies that $V_0 = EQ_0$. Moreover, assume that the firm has no value at the end of the period, i.e. $V_1 = EQ_1 = 0$. These two assumptions can be substituted into [EQ A-72], and that gives:

$$V_0 = d_1 \cdot \left(1 + {}_0ROE_1\right)^{-1}$$

[EQ A-73]

Thus, when there is no opening or closing valuation error, the ROE, defined as [EQ A-71], equals the one-period objective MROR, i.e. $_{t-1}ROE_t = {}_{t-1}r_t$. This fundamental relationship can be further utilized.

Define the one-period residual rate-of-return on equity as the difference between the one-period return on equity and its corresponding objective MROR. Algebraically this becomes:

$$_{t-1}RROE_t = {}_{t-1}ROE_t - {}_{t-1}r_t$$

[EQ A-74]

Recall that residual income was defined in [EQ A-61]. Substituting [EQ A-74] into [EQ A-61], rearranging and inserting this result into [EQ A-65] gives a new description of how the value of a firm may be estimated:

$$V_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_t \cdot {}_{t-1}RROE_t \cdot EQ_{t-1} \qquad \text{[EQ A-75]}$$

The following section defines return on net operating assets, residual rate-of-return on net operating assets (RRNOA), and connects them to valuation theory and ROE.

Note that the assumption that there is no opening or closing valuation error for [EQ A-73] implies that for [EQ A-75] there is no intertemporal valuations errors either. That is, the book values are always equal to the intrinsic values and the depreciation equals the change in net book value during a given period. This is unbiased accounting, and in such a setting the accounting rate-of-return always equals MROR, i.e. ${}_{t-1}ROE_t = {}_{t-1}r_t \ \ \forall t$ (e.g., Feltham & Ohlson 1996).

A.8.4.2 <u>Return on net operating assets and residual rate-of-return on net operating assets</u>
Return on equity is related to valuation theory through [EQ A-65]. With a careful definition of return on net operating assets, it can be tied to valuation theory through [EQ A-68]. ROE and RNOA can also be tied to each other and that also requires the definition of net borrowing cost (NBC) and financial leverage (FLEV). The necessary definitions are:

$$_{t-1}RNOA_t = COI_t \cdot NOA_{t-1}^{-1} \qquad \text{[EQ A-76]}$$

$$_{t-1}NBC_t = CNIE_t \cdot NFL_{t-1}^{-1} \qquad \text{[EQ A-77]}$$

$$FLEV_{t-1} = NFL_{t-1} \cdot EQ_{t-1}^{-1} \qquad \text{[EQ A-78]}$$

With these definitions ROE, RNOA and NBC are related to each other through the familiar leverage formula. That is,

$$_{t-1}ROE_t = {}_{t-1}RNOA_t + \left({}_{t-1}RNOA_t - {}_{t-1}NBC_t \right) \cdot FLEV_{t-1} \qquad \text{[EQ A-79]}$$

Also define RRNOA and residual rate-of-return on net financial liabilities (RRIBL) as:

$$_{t-1}RRNOA_t = {}_{t-1}RNOA_t - {}_{t-1}r_t \qquad \text{[EQ A-80]}$$

$$_{t-1}RRIBL_t = {}_{t-1}NBC_t - {}_{t-1}r_t \qquad \text{[EQ A-81]}$$

With [EQ A-76] to [EQ A-81] it is possible to rewrite [EQ A-69] and [EQ A-70] as:

$$ROI_t = {}_{t-1}RRNOA_t \cdot NOA_{t-1}$$

$$RIE_t = {}_{t-1}RRNFL_t \cdot NFL_{t-1}$$

Substituting these intermediate results into [EQ A-68] gives the following description of how the value of a firm can be estimated:

$$V_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_t \cdot {}_{t-1}RRNOA_t \cdot NOA_{t-1} - \sum_{t=1}^{\infty} {}_0 p_t \cdot {}_{t-1}RRNFL_t \cdot NFL_{t-1} \qquad \text{[EQ A-82]}$$

A.8.4.3 <u>Residual accounting rates-of-returns and their relationship to no-arbitrage: A discussion</u>
This appendix is devoted to no-arbitrage valuation models. The models [EQ A-75] and [EQ A-65] are the foundation on which all analyses are related. The rates-of-returns are Pareto optimal when there is no arbitrage and there exists no incentive for individuals to choose other consumption and investment patterns since we have zero NPV. This implies that there are no investments in society

that yield non-zero residual rates and return when there is no opening and closing valuation error since this would provide the subjects to choose other consumption patterns.

E.g., assume that one asset yields a positive RRNOA, the residual rate-of-return on equity is null and there are no valuation errors. This means that the owner of this net operating asset can increase his or her income more than what others can achieve. At the same time, since the residual rate-of-return on equity is null, some net financial liabilities also have a negative residual rate-of-return.

Since we have homogenous beliefs, other investors strive to gain access to the positive residual rate-of-return net operating assets investment in order to be able to increase their consumption. Likewise there will be a shift away from those investments that yield a negative residual rate-of-return on net financial liabilities. The effect will be a rebalancing of the consumption pattern where some will gain and others lose consumption. Striving for increased consumption is always sought for because of the non-satiation assumption.

The incentive to shift consumption and investment patterns is at odds with the Pareto optimal definition. Further, the conditions for models [EQ A-58], [EQ A-65], and [EQ A-68] are violated.

At face value, it appears as though the models above only require that the present value of the sum of future RROE, RRNOA, and RRIBL to be zero when there are no opening or closing valuation errors. However, this restriction is even more austere than is looks when we also consider the assumptions under which the models are derived

Lundholm (1995, p. 761) notes: "Homogenous investors value the firm under a no-arbitrage equilibrium condition; there are no differing intertemporal preferences for consumption, no differing risk preferences and no differing beliefs." Thus, the implication here is that we have a situation where all subjects have perfect knowledge. Hence, even though the models seem to allow for temporary residual rates-of-returns as long as they are reversed in future periods (with appropriate compensation for the time value of money), this goes against the assumptions of the models.

If there exists, even in expectation, some future positive residual income for a particular firm, all subjects hold the same beliefs and thus everyone knows that this firm is expected to be more profitable than other identical firms at that time. Such a situation cannot be Pareto optimal because the assumption implies that all other firms also know this and hence they have a propensity to seek to improve in that they no longer are expected to be maximizing their values.

It is therefore not possible for any firm to be expected to earn any residual rates-of-returns in a no arbitrage situation with homogenous beliefs when we have no opening and closing valuation errors.

Indeed, this fact is recognized by Lo & Lys (1999). Lo & Lys (1999) argue that the unconditional residual income is zero; for this to happen, a firm cannot earn more than the cost-of-capital.

Both Lundholm (1995) and Beaver (2002) are aware of the effect from assuming homogenous beliefs and no arbitrage. Lundholm (1995) even points out that under such a setting there is not even any demand for accounting information since there is no demand for financial assets.

Beaver (2002, p. 458) also points out that these assumptions imply that there is "…no endogenous demand for accounting data" in the valuation model.

Ohlson (2003) argues differently to Lo and Lys (1999) but focuses on the technicalities of the conjectured linear information dynamics. This seems to be off the target since Ohlson does not consider the limitations that the assumptions about homogenous beliefs in no arbitrage impose on the model.

I have to agree with Ohlson (2003) to the extent that, if we remove the assumption of no opening or closing valuation error, we have a setting that allows for non-zero residual income.

It is also plausible that accounting rules that create early (late) recognition of revenues and expenses can create expected residual rates-of-returns that are non-zero for individual periods. But to develop such an accounting system, which should be different to that in the paragraph above and thus have no opening or closing valuation error, requires careful consideration. In my view, it is unclear how to form such an accounting system.

A.9 Summary

This appendix derives two accounting-based valuation models that are equivalent to the dividend valuation model. These two models use certain, but non-constant, rates-of-returns, which separates them from, e.g., the models by Ohlson (1995), Feltham & Ohlson (1995), and Feltham & Ohlson (1999). The valuation models are derived from basic assumptions about his or her preference function that meets a Pareto optimal general equilibrium that allows non-constant rates-of-returns.

The models are:

$$V_0 = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_t \cdot RI_t$$

$$V_0 = NOA_0 + \sum_{t=1}^{\infty} {}_0 p_t \cdot ROI_t - \left[NFL_0 + \sum_{t=1}^{\infty} {}_0 p_t \cdot RIE_t \right]$$

By defining ROE, and RNOA as:

$$_{t-1}ROE_t = CNI_t \cdot EQ_{t-1}^{-1}$$

$$_{t-1}RNOA_t = COI_t \cdot NOA_{t-1}^{-1}$$

and by defining RROE and RRNOA as:

$$_{t-1}RROE_t = {}_{t-1}ROE_t - {}_{t-1}r_t$$

$$_{t-1}RRNOA_t = {}_{t-1}RNOA_t - {}_{t-1}r_t$$

it instead becomes possible to express the derived valuation models using residual rates-of-returns. The residual rates-of-returns equivalents to the models above are found as [EQ A-75] and [EQ A-82] above.

It should be noted that RROE and RRNOA are zero in expectation if the economy meets the no arbitrage conditions (with homogenous beliefs) and if the accounting is unbiased. Thus, they can be also thought of as arbitrage profitability.

APPENDIX B—HOMO COMPERIENS AND THE MARKET PRICE OF THE FIRM IN A SUBJECTIVE CERTAIN CHOICE

Disequilibrium models of the firm's market price

B.1 Introduction

This appendix provides the proof for the Homo comperiens market-pricing models that are used in the analysis in Chapter 2 and Chapter 3. To reduce complexity the analysis in this appendix is limited to deriving the market price of the firm in a subjective certain decision.

The subjective certain decision can be described in two ways: (i) The decision occurs when the subjective state set has only one element; (ii) The decision occurs when the subjective state set has more than one element but where the consequence is constant in all states.

To stress that case (ii) can be present, or that it may be a false hope to believe that the subjective state set has only one element, Chapter 4 uses a subjective expectations operator, $E_{K0}[\cdot]$, when analyzing the subjective certain decision.

Furthermore, this appendix assumes homogenous preferences such that it is possible to analyze the decision and optimize it for a representative individual. This means that the perceive prices in this appendix become the market's prices.

Since the representative individual in this appendix is assumed to behave according to the assumptions of the theory of Homo comperiens, it is not possible here to talk about intrinsic values. Appendix A focuses on the objective prices and those prices are the intrinsic values.

The appendix derives the market price of firms as functions of

   (i)   subjective expected dividends,
   (ii) subjective expected residual income,
   (iii) subjective expected residual operating income,
   (iv) subjective expected residual ROE, and
   (v) subjective expected residual RNOA.

The derivations are analogous to those in Appendix A, but they are more succinct here.

B.2 The subject's opportunity set

Appendix A finds that an individual's decision is limited to a budget set that is

$B = \left\{ \mathbf{C} \in X_+ : {}_0\boldsymbol{p}_0 \cdot \mathbf{c}_0 + {}_0\boldsymbol{p}_1 \cdot \mathbf{c}_1 \leq {}_0\boldsymbol{p}_0 \cdot \boldsymbol{\omega}_0 + {}_0\boldsymbol{p}_1 \cdot \boldsymbol{\omega}_1 \right\} \subseteq X_+$. In this appendix the subject's budget set is also limiting the decision. To distinguish the budget set in Appendix A from this budget set, the budget set from Appendix A is the objective budget set and the budget set in this appendix is the subjective budget set.

In Appendix A the subject is also limited by a production set which is

$$\mathbb{D} = \left\{ \mathbf{D} \in \mathbb{R}^L_- \times \mathbb{R}_+ : g\left(\mathbf{d}_1, \mathbf{d}'_0 \middle| d^\circ_{10}\right) \le \varnothing \right\}.$$ Appendix A's objective production set is the parent to the

subjective production set, i.e. $\mathbb{D}_K \subset \mathbb{D}$ that is used in this appendix.

This means that this appendix uses the subjective budget set:

$$B = \left\{ \tilde{\mathbf{C}} \subset \mathbf{C} \in X_+ : {}_0\boldsymbol{p}_{K0} \cdot \mathbf{c}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \mathbf{c}_{K1} \le {}_0\boldsymbol{p}_{K0} \cdot \boldsymbol{\omega}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \boldsymbol{\omega}_{K1} \right\} \subseteq X_+ \qquad \text{[EQ B-1]}$$

The subjective budget set is assumed to be non-empty, closed, bounded, and convex as the budget set in Appendix A. The subjective current price vector is ${}_0\boldsymbol{p}_{K0} = \left({}_0 p_{K10}, \ldots, {}_0 p_{K\ell 0} \right)$ and the subjective current price vector is ${}_0\boldsymbol{p}_{K1} = \left({}_0 p_{K11}, \ldots, {}_0 p_{K\ell 1} \right)$. The subjective choice portfolio is $\mathbf{C}_K = \left(\mathbf{c}_{K0}, \mathbf{c}_{K1} \right)$ and it is a strict subset to the objective choice portfolio, i.e. $\mathbf{C}_K \subset \mathbf{C}$. The subjective quantity vector for current consumption is $\mathbf{c}_{K0} = \left(c_{K10}, \ldots, c_{K\ell 0} \right) \subset \mathbb{R}^L$ and the subjective quantity vector for future consumption is $\mathbf{c}_{K1} = \left(c_{K11}, \ldots, c_{K\ell 1} \right) \subset \mathbb{R}^L$, where $\ell \in \mathbb{L}$. The subjective current wealth vector is $\boldsymbol{\omega}_{K0} = \left(\omega_{K10}, \ldots, \omega_{K\ell 0} \right)$ and the subjective future wealth vector is $\boldsymbol{\omega}_{K1} = \left(\omega_{K11}, \ldots, \omega_{K\ell 1} \right)$.

To separate the analysis from that in Appendix A it is further assumed that because of limited knowledge $\mathbf{c}_{K0} \subset \mathbf{c}_0$, $\mathbf{c}_{K1} \subset \mathbf{c}_1$. Whether the subjective wealth vectors are subsets to the objective wealth vectors or not is not restrictive and so it is instead assumed that $\boldsymbol{\omega}_{K0} \subseteq \boldsymbol{\omega}_{\Omega 0}$ and $\boldsymbol{\omega}_{K1} \subseteq \boldsymbol{\omega}_{\Omega 1}$.

The first good is set to be the numerarie in this analysis and its subjective spot prices are defined to be one, i.e. ${}_0 \rho_{K10} = 1$, and ${}_1 \rho_{K11} = 1$.

The subject is a subjective expected utility maximizer endowed with a strictly positive subjective wealth. This is so at the same time as the subjective prices are finite and the goods are infinitely dividable. It is also assumed that the subject must at least consume non-negative amounts of goods, both at the present and in the future.

With the setting above, $B_K \ne B$ and the budget set is artificially restricted. Had the subjects in the market been completely rational, it would perceive more combinations and the prices in the market would be different. The subjective budget set is therefore restricted, which leads to inefficient solutions where the subject chooses less than optimal consumption combinations.

This appendix uses the following subjective production set.

$$\mathbb{D}_K = \left\{ \mathbf{D}_K \subset \mathbf{D} \in \mathbb{R}^L_- \times \mathbb{R}_+ : g_K\left(\mathbf{d}_{K1}, \mathbf{d}'_{K0} \middle| d^\circ_{K10}\right) \le \varnothing \right\} \qquad \text{[EQ B-2]}$$

The subjective production vector is $\mathbf{D}_K = (\mathbf{d}_{K0}, \mathbf{d}_{K1})$ and it is a strict subset to the objective production set, i.e. $\mathbf{D}_K \subset \mathbf{D}$. The subjective current production vector is represented as $\mathbf{d}_{K0}$, and $\mathbf{d}'_{K0} = \mathbf{d}_{K0} \setminus \{d^\circ_{K10}\}$. The subjective current production vector is a strict subset to the objective production vector, $\mathbf{d}_{K0} \subset \mathbf{d}_0$, and its elements are zero or negative since there is a consumption of goods in the production process. Similarly, the subjective future production vector is denoted as $\mathbf{d}_{K1}$. It is a strict subset to the objective future production vector, $\mathbf{d}_{K1} \subset \mathbf{d}_1$, and the vector's elements are either zero or positive, indicating that there is some output from the production process.

B.3 The subject's optimization problems

The subject behaves in this setting according to Homo comperiens. Thus, the subject acts as though he or she is striving to maximize his or her strictly quasi-concave subjective utility function over his or her subjective available opportunities, which are constrained by non-negative consumption, the subjective exchange opportunities, and the subjective production opportunities.

The setting in this appendix is similar to that in Appendix A but here the subject is limited to choosing from an inefficient opportunity set. It is bounded when compared with the opportunity set in Appendix A in that the knowledge is limited.

As in Appendix A, the non-negativity constraints are ignored and focus is on interior solutions to the optimization problem.

The optimization is first studied in a pure exchange situation. Next, follows the situation where the subject faces both exchange and productive opportunities.

B.3.1 Optimization in a pure exchange economy

The subject is assumed to be a utility maximizer which means that the weak inequality in [EQ B-1] is replaced by an equality. This means that the subject never chooses to waste resources and therefore the subjective budget restriction is replaced by the hyperplane:

$$\tilde{\mathbb{B}}' = \left\{ \tilde{\mathbf{C}} \in \mathbf{C} \in \mathbb{X}_+ : {}_0\boldsymbol{p}_{K0} \cdot \mathbf{c}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \mathbf{c}_{K1} = {}_0\boldsymbol{p}_{K0} \cdot \boldsymbol{\omega}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \boldsymbol{\omega}_{K1} \right\} \subseteq B_K \qquad \text{[EQ B-3]}$$

The solution to the optimization problem that will let the subject consume according to the subjective budget hyperplane while attaining the highest possible indifference curve is obtained from solving the problem:

$$\max_{c_K} U_K (\mathbf{C}_K) \text{ subject to} \qquad \text{[EQ B-4]}$$

$${}_0\boldsymbol{p}_{K0} \cdot \mathbf{c}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \mathbf{c}_{K1} - {}_0\boldsymbol{p}_{K0} \cdot \boldsymbol{\omega}_{K0} - {}_0\boldsymbol{p}_{K1} \cdot \boldsymbol{\omega}_{K1} = 0$$

That is:

$$\max_{\tilde{c}, \lambda} \mathcal{L} = \tilde{U} (\tilde{\mathbf{C}}) - \lambda \cdot \left[{}_0\boldsymbol{p}_{K0} \cdot \mathbf{c}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \mathbf{c}_{K1} - {}_0\boldsymbol{p}_{K0} \cdot \boldsymbol{\omega}_{K0} - {}_0\boldsymbol{p}_{K1} \cdot \boldsymbol{\omega}_{K1} \right] \qquad \text{[EQ B-5]}$$

Taking the partial derivative of Lagrange's equation with respect to subjective current consumption gives:

$$\nabla \mathcal{L}\left(\mathbf{c}_{K0}\right) = \nabla U_K\left(\mathbf{c}_{K0}\right) - \lambda \cdot {}_0\boldsymbol{p}_{K0} = 0 \,, \qquad \text{[EQ B-6]}$$

where $\nabla \mathcal{L}\left(\mathbf{c}_{K0}\right) = \left(\dfrac{\partial \mathcal{L}}{\partial c_{K10}}, \cdots, \dfrac{\partial \mathcal{L}}{\partial c_{K\ell 0}}\right)$, and where $\nabla U_K\left(\mathbf{c}_{K0}\right) = \left(\dfrac{\partial U_K\left(\mathbf{C}_K\right)}{\partial c_{K10}}, \cdots, \dfrac{\partial U_K\left(\mathbf{C}_K\right)}{\partial c_{K\ell 0}}\right)$.

For the subjective future consumption the partial derivative is:

$$\nabla \mathcal{L}\left(\mathbf{c}_{K1}\right) = \nabla U_K\left(\mathbf{c}_{K1}\right) - \lambda \cdot {}_0\boldsymbol{p}_{K1} = 0 \,, \qquad \text{[EQ B-7]}$$

where $\nabla \mathcal{L}\left(\mathbf{c}_{K1}\right) = \left(\dfrac{\partial \mathcal{L}}{\partial c_{K11}}, \cdots, \dfrac{\partial \mathcal{L}}{\partial c_{K\ell 1}}\right)$, and where $\nabla U\left(\mathbf{c}_{K0}\right) = \left(\dfrac{\partial U_K\left(\mathbf{C}_K\right)}{\partial c_{K11}}, \cdots, \dfrac{\partial U_K\left(\mathbf{C}_K\right)}{\partial c_{K\ell 1}}\right)$.

Dividing equation [EQ B-7] with equation [EQ B-6] gives the subjective marginal rate of substitution between consumption today and consumption in the future:

$$\frac{{}_0\boldsymbol{p}_{K1}}{{}_0\boldsymbol{p}_{K0}} = \frac{\nabla U_K\left(\mathbf{c}_{K1}\right)}{\nabla U_K\left(\mathbf{c}_{K0}\right)} \qquad \text{[EQ B-8]}$$

The subjective marginal rate of substitution between consumption today and consumption in the future is almost identical to the objective marginal rate of substitution between consumption today and consumption in the future, which is found in Appendix A. However, in Appendix A the prices are the objective prices since the solution is Pareto optimal and the marginal utilities are the objective marginal utilities. Since the subject is endowed with limited knowledge, the marginal utilities are the subjective marginal utilities and the prices are the subjective prices. The solution would have been Pareto optimal only if the objective budget set had been used instead of the subjective budget set.

Using the conjecture that ${}_0 p_{K0} = 1$ and taking the partial derivative with respect to the numerarie in the future gives the solution for the subjective futures price of the numerarie good:

$${}_0 p_{11} = \frac{\dfrac{\partial U_K\left(\mathbf{C}_K\right)}{\partial c_{K11}}}{\dfrac{\partial U_K\left(\mathbf{C}_K\right)}{\partial c_{K10}}} \qquad \text{[EQ B-9]}$$

The equation above shows the subjective marginal rate of substitution between saving and consuming, which is very similar to the objective marginal rate of substitution between saving and consuming. Again, the difference resides between them in the fact that it is the subjective marginal utilities that are used in [EQ B-9] and not the objective marginal utilities.

B.3.2 Optimization in an exchange and production economy

In addition to being limited by the subject's market exchange opportunities, this subsection also limits the subject's opportunities by the productive set, which this amounts to a joint optimization problem.

The subject is assumed to buy shares equal to a portion θ_i^j, $\left(\sum_i \theta_i^j = 1\right)$, of the subjective future output from the firm $j \in J$. This means that the subject has $\boldsymbol{\omega}_{K0} + \theta \cdot \mathbf{d}_{K0}$ of subjective current wealth and has $\boldsymbol{\omega}_{K1} + \theta \cdot \mathbf{d}_{K1}$ of subjective future wealth in the budget hyperplane:

$$B' = \begin{Bmatrix} \mathbf{C}_K : {}_0\boldsymbol{p}_0 \cdot \left(\mathbf{c}_{K0} - \boldsymbol{\omega}_{K0}\right) + {}_0\boldsymbol{p}_{K1} \cdot \left(\mathbf{c}_{K1} - \boldsymbol{\omega}_{K1}\right) = \\ {}_0\boldsymbol{p}_{K0} \cdot \theta \cdot \mathbf{d}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \theta \cdot \mathbf{d}_{K1}, \mathbf{D}_K \subset \mathbf{D} \in \mathbb{R}_-^L \times \mathbb{R}_+^L \end{Bmatrix} \subseteq B_K \qquad \text{[EQ B-10]}$$

Since $\mathbf{D}_K \subset \mathbf{D}$, we have a situation where there are more efficient combinations that can either produce more outputs given a certain input, or produce the same outputs with less inputs, or a combination thereof. The efficient transformation frontier is put to use in Appendix A, but here the firm is only producing at its subjective transformation frontier. The use of an inefficient transformation frontier is an effect of the limited knowledge. The transformation frontier is here:

$$\mathbb{D}_K = \left\{ \mathbf{D}_K \subset \mathbf{D} \in \mathbb{R}_-^L \times \mathbb{R}_+ : g_K\left(\mathbf{d}_{K1}, \mathbf{d}'_{K0} \middle| d^\circ_{K10}\right) = \varnothing \right\} \qquad \text{[EQ B-11]}$$

To find the intersection between the budget hyperplane, the indifference set, and the production set is formulated as a constrained maximization problem:

$$\max_{c_K} U_K\left(\mathbf{C}_K\right) \text{ s.t.} \qquad \text{[EQ B-12]}$$

$$_0\tilde{\boldsymbol{\rho}}_0 \cdot \left(\tilde{\mathbf{c}}_0 - \tilde{\boldsymbol{\omega}}_0\right) + {}_0\boldsymbol{p}_{K1} \cdot \left(\mathbf{c}_{K1} - \boldsymbol{\omega}_{K1}\right) - {}_0\boldsymbol{p}_{K0} \cdot \theta \cdot \mathbf{d}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \theta \cdot \mathbf{d}_{K1} = 0$$

$$g_K\left(\mathbf{d}_{K1}, \mathbf{d}'_{K0} \middle| d^\circ_{K10}\right) = \varnothing$$

This can be expressed using Lagrange's method as the following unconstrained maximization problem:

$$\max_{c_K, d_K, \lambda, \mu} \mathcal{L} = \max_{c_K, d_K, \lambda, \mu} \begin{Bmatrix} U_K\left(\mathbf{C}_K\right) \\ -\lambda \cdot \left[{}_0\boldsymbol{p}_{K0} \cdot \left(\mathbf{c}_{K0} - \boldsymbol{\omega}_{K0}\right) + {}_0\boldsymbol{p}_{K1} \cdot \left(\mathbf{c}_{K1} - \boldsymbol{\omega}_{K1}\right) - {}_0\boldsymbol{p}_{K0} \cdot \theta \cdot \mathbf{d}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \theta \cdot \mathbf{d}_{K1}\right] \\ -\mu \cdot g_K\left(\mathbf{d}_{K1}, \mathbf{d}'_{K0} \middle| d^\circ_{K10}\right) \end{Bmatrix} \text{[EQ B-13]}$$

Solving the unconstrained problem gives the following results.

$$\nabla\mathcal{L}\left(\mathbf{c}_{K0}\right) = \nabla U_K\left(\mathbf{c}_{K0}\right) - \lambda \cdot {}_0\boldsymbol{p}_{K0} = 0 \qquad \text{[EQ B-14]}$$

$$\nabla\mathcal{L}\left(\mathbf{c}_{K1}\right) = \nabla U_K\left(\mathbf{c}_{K1}\right) - \lambda \cdot {}_0\boldsymbol{p}_{K1} = 0 \qquad \text{[EQ B-15]}$$

$$\nabla\mathcal{L}\left(\mathbf{d}_{K0}\right) = \lambda \cdot \theta \cdot {}_0\boldsymbol{p}_{K0} - \mu \cdot \theta \cdot \nabla g_K\left(\mathbf{d}_{K0}\right) = 0 \qquad \text{[EQ B-16]}$$

$$\nabla\mathcal{L}\left(\mathbf{d}_{K1}\right) = \lambda \cdot \theta \cdot {}_0\boldsymbol{p}_{K1} - \mu \cdot \theta \cdot \nabla g_K\left(\mathbf{d}_{K1}\right) = 0 \qquad \text{[EQ B-17]}$$

$$\frac{{}_0\boldsymbol{p}_{K1}}{{}_0\boldsymbol{p}_{K0}} = \frac{\nabla U\left(\mathbf{c}_{K1}\right)}{\nabla U\left(\mathbf{c}_{K0}\right)} = \frac{\nabla g_K\left(\mathbf{d}_{K1}\right)}{\nabla g_K\left(\mathbf{d}_{K0}\right)} \qquad \text{[EQ B-18]}$$

Equation [EQ B-18] shows that the subject chooses to consume where his or her subjective marginal rate of substitution between current and future consumption equals the subjective marginal rate of transformation between current and future goods. The subjective marginal rate of substitution and the subjective marginal rate of transformation equal the subjective price for future consumption adjusted for the subjective price for current consumption.

B.4 The market price and the subjective dividends

This subsection elaborates on the market price based on dividends in a one-period model that is expanded into a multi-period setting.

B.4.1 The one-period subjective dividend market-pricing model

Subsection B.3.2 discusses optimization for an individual that maximizes his or her subjective expected utility given the subjective budget restriction and the subjective productive opportunities. That is:

$$\max_{c_K,d_K,\lambda,\mu} \mathcal{L} = \max_{c_K,d_K,\lambda,\mu} \left\{ \begin{array}{l} U_K\left(\mathbf{C}_K\right) \\ -\lambda \cdot \left[{}_0\boldsymbol{p}_{K0} \cdot \left(\mathbf{c}_{K0} - \boldsymbol{\omega}_{K0}\right) + {}_0\boldsymbol{p}_{K1} \cdot \left(\mathbf{c}_{K1} - \boldsymbol{\omega}_{K1}\right) - {}_0\boldsymbol{p}_{K0} \cdot \theta \cdot \mathbf{d}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \theta \cdot \mathbf{d}_{K1}\right] \\ -\mu \cdot g_K\left(\mathbf{d}_{K1}, \mathbf{d}'_{K0}\middle| d^{\circ}_{K10}\right) \end{array} \right\} \text{[EQ B-13]}$$

This maximization problem can be expressed as consisting of two choices, namely that of finding a subjective optimal consumption plan and that of finding a subjective optimal production plan. Hence:

$$\begin{array}{l} \max_{c_K,\lambda} \left\{U_K\left(\mathbf{C}_K\right) - \lambda \cdot \left[{}_0\boldsymbol{p}_{K0} \cdot \mathbf{c}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \mathbf{c}_{K1}\right]\right\} + \\ \max_{d_K,\lambda,\mu} \left\{\lambda \cdot \left[{}_0\boldsymbol{p}_{K0} \cdot \mathbf{d}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \mathbf{d}'_{K1}\right] - \mu \cdot g_K\left(\mathbf{d}_{K1}, \mathbf{d}'_{K0}\middle| d^{\circ}_{K10}\right)\right\} \end{array} \qquad \text{[EQ B-19]}$$

Defining $\gamma = \mu \cdot \lambda^{-1}$ and by focusing on the subjective production maximization problem, it becomes:

$$\lambda \cdot \max_{d_K,\gamma} \left\{\left[{}_0\boldsymbol{p}_{K0} \cdot \mathbf{d}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \mathbf{d}'_{K1}\right] - \gamma \cdot g_K\left(\mathbf{d}_{K1}, \mathbf{d}'_{K0}\middle| d^{\circ}_{K10}\right)\right\} \qquad \text{[EQ B-20]}$$

The subject's choice can now be interpreted as if he or she first maximizes the subjective production equation [EQ B-20]. This yields the subjective optimal production plan, $\left(\mathbf{d}^*_{K0}, \mathbf{d}^*_{K1}\right)$, that maximizes the market value of the firm given the production constraint and equilibrium prices. Since the initial investment is assumed fixed, what remains is choosing the investment plan that gives the highest attainable market price, i.e. $\max\left({}_0\boldsymbol{p}_{K1} \cdot \mathbf{d}_{K1}\middle| d^{\circ}_{K10}\right)$. Then, with a subjective optimal production plan, the subject proceeds to optimize the remaining problem, i.e.:

$$\max_{c_K, d_K, \lambda, \mu} \left\{ U_K \left(\mathbf{C}_K \right) - \lambda \cdot \left[{}_0\boldsymbol{p}_{K0} \cdot \mathbf{c}_{K0} + {}_0\boldsymbol{p}_{K1} \cdot \mathbf{c}_{K1} - {}_0\boldsymbol{p}_{K0} \cdot \mathbf{d}_{K0}^* - {}_0\boldsymbol{p}_{K1} \cdot \mathbf{d}_{K1}^* \right] \right\} \qquad \text{[EQ B-21]}$$

From which the subject gets his or her subjective optimal consumption plan $\left(\tilde{\mathbf{c}}_0^*, \tilde{\mathbf{c}}_1^* \right)$.

Equation [EQ B-20] establishes how investments into a firm are made in a Homo comperiens setting. For a given level of initial investments (here defined as a fixed portion of the present numerarie) and for subjective optimal current prices for future consumption, the investments are made such that they maximize the market price of the subjective future output.

As in Appendix A, I assume that all future commodities are converted into the future numerarie. This allows me to rewrite [EQ B-20] as:

$$\max_{d_{K11}} \left[{}_0 p_{K10} \cdot d_{K10}^{\circ} + {}_0 p_{K11} \cdot d_{K11} \right] \qquad \text{[EQ B-22]}$$

Since the initial investment level is assumed constant it is possible to focus [EQ B-22] further. The market price of any investment is thus according to the theory of Homo comperiens:

$$P_{10} = {}_0 p_{K11} \cdot \mathrm{E}_{K0} \left[d_{K11} \right] \qquad \text{[EQ B-23]}$$

The equation above is the one-period subjective dividend market-pricing model. It is similar to [EQ A-52]. The difference focuses on the fact that (i) the futures price used in the equation above is the subjective futures price of money and not the objective futures price of money and (ii) the dividend considered is the subjective dividend and not the objective dividend.

Compared to [EQ A-52] these differences can appear very small at first glance, but are in fact large. The subjective prices are different from the objective prices because we have (1) limited knowledge of the attainable consumption portfolios, (2) limited knowledge of the attainable production technology that can be put to use, and (3) limited knowledge of how to best use the existing production technology. Consequently, despite their apparent similarity, there is a stormy ocean of ignorance separating the two equations.

The subscript that keeps track of the commodity is dropped to simplify matters.

B.4.2 A multi-period subjective dividend market-pricing model

The one period market price can be expanded to cover more than one period. Assume that [EQ B-23] holds for $T - 1$ where $T > 2$. This means that $P_{K0} = {}_0 p_{K1} \cdot d_{K1}, \cdots, P_{KT-1} = {}_0 p_{KT} \cdot d_{KT}$.

When $P_{K1} \neq 0$ it follows that [EQ B-23] must be modified to fit in the remaining price at the end of the period. Hence, $P_{K0} = {}_0 p_{K1} \cdot \left(d_{K1} + P_{K1} \right), \cdots, P_{KT-1} = {}_{T-1} p_{KT} \cdot \left(d_{KT} + P_{KT} \right)$. Substituting this expression into the latter gives the multi-period subjective dividend market-pricing model:

$$P_{K0} = \sum_{t=1}^{T} {}_0 p_{Kt} \cdot d_{Kt} + {}_0 p_{KT} \cdot P_{KT} \qquad \text{[EQ B-24]}$$

Where ${}_0 p_{KT} = {}_0 p_{K1} \cdot {}_1 p_{K2} \cdot \cdots \cdot {}_{T-1} p_{KT} = \prod_{t=1}^{T} {}_{t-1} p_{Kt}$.

Equation [EQ B-24] is the multi-period dividend market-pricing model for a finite period. This is the Homo comperiens equivalent to [EQ A-58].

Before the multi-period dividend market-pricing model is expanded to cover infinity, the market rate-of-return is introduced.

B.4.3 An infinite subjective dividend market-pricing model with the subjective market rate-of-return

Let q_0 be the quantity of the current capital and let q_{K1} be the quantity of the subjective future capital. To get q_{K1} units of the capital in the future the subject has to forfeit at present:

$$q_0 = {}_0p_{K1} \cdot q_{K1} \qquad \text{[EQ B-25]}$$

It is also possible to express the subjective quantity of the future capital based on the present capital plus the change in the capital, i.e.:

$$q_{K1} = q_0 + \Delta q_{K0} \qquad \text{[EQ B-26]}$$

Substituting [EQ B-25] into [EQ B-26] and rearranging gives:

$$1 + \Delta q_{K0} \cdot q_0^{-1} = {}_0p_{K0}^{-1} \qquad \text{[EQ B-27]}$$

Define the subjective MROR as the subjective rate of growth of capital, i.e. ${}_0r_{K1} = \Delta q_{K0} \cdot q_0^{-1}$. This gives:

$$1 + {}_0r_{K1} = {}_0p_{K0}^{-1} \Leftrightarrow {}_0p_{K1} = \left(1 + {}_0r_{K1}\right)^{-1} \qquad \text{[EQ B-28]}$$

Assume that ${}_{t-1}r_{Kt} > \Delta P_{Kt-1} \cdot P_{Kt-1}^{-1}$, $\forall t$. This implies that $\lim_{t \to \infty}\left(\prod_{t=1}^{\infty} {}_{t-1}\rho_{Kt} \cdot P_{K\infty}\right) = 0$, and so we have the infinite subjective dividend market-pricing model with the subjective market rate-of-return:

$$P_{K0} = \sum_{t=1}^{\infty} {}_0p_{Kt} \cdot \mathrm{E}_{K0}\left[d_{Kt}\right] \qquad \text{[EQ B-29]}$$

Where ${}_0p_{K\infty} = {}_0p_{K1} \cdot {}_1p_{K2} \cdot \ldots \cdot {}_{\infty-1}p_{K\infty} = \prod_{t=1}^{\infty} {}_{t-1}p_{Kt}$, and where ${}_{t-1}p_{Kt} = \left(1 + {}_{t-1}r_{Kt}\right)^t$.

To further highlight that this is only a subjective certain decision, i.e. that the expectations of certainty may be incorrect because of limited knowledge, the subjective expectations operator, $\mathrm{E}_{K0}[\cdot]$, is added.

The infinite subjective dividend market-pricing model with the subjective market rate-of-return is the Homo comperiens equivalent to the equilibrium model in [EQ A-53].

B.5 The market price based on subjective residual income

This section derives the residual income market-pricing model measured on both comprehensive net income and comprehensive operating income.

B.5.1 The residual net income market-pricing model

Assume the clean surplus relationship:

$$\Delta EQ_t = CNI_t - d_{Kt} \qquad \text{[EQ B-30]}$$

Where CNI_t is the comprehensive net income for a period, ΔEQ_t is the change in the equity account between two adjacent periods starting at $t-1$, and $\tilde{d}_t$ is the period's net dividends. Substituting [EQ B-30] into [EQ B-29] yields:

$$P_{K0} = {}_0 p_{K1} \cdot \left(CNI_{K1} - \Delta EQ_0 \right) + {}_0 p_{K1} \cdot {}_1 p_{K2} \left(CNI_{K2} - \Delta EQ_1 \right) + \cdots \qquad \text{[EQ B-31]}$$

Defining the subjective residual income as the portion of comprehensive net income that deviates from the subjective expected comprehensive net income

$$\mathrm{E}_{K0}\left[RI_{Kt} \right] = \mathrm{E}_{K0}\left[CNI_{Kt} \right] - \mathrm{E}_{K0}\left[{}_{t-1} r_{Kt} \right] \cdot EQ_{t-1}, \qquad \text{[EQ B-32]}$$

Substituting [EQ B-32] into [EQ B-31] and simplifying gives:

$$\begin{aligned} P_{K0} = {}_0 p_{K1} \cdot \left(RI_{K1} + \left(1 + {}_0 r_{K1} \right) \cdot EQ_0 - EQ_1 \right) + \\ {}_0 p_{K1} \cdot {}_1 p_{K2} \left(RI_2 + \left(1 + {}_1 r_{K2} \right) \cdot EQ_1 - EQ_2 \right) + \cdots \end{aligned} \qquad \text{[EQ B-33]}$$

Replacing the subjective futures price with the equivalent expression of subjective MROR, substituting this into [EQ B-33], rearranging and simplifying gives:

$$P_{K0} = EQ_0 + \sum_{t=1}^{T} {}_0 p_{Kt} \cdot RI_{Kt} + {}_0 p_{KT} \cdot \left(P_{KT} - EQ_T \right) \qquad \text{[EQ B-34]}$$

Adding the assumption ${}_0 r_{Kt} > \Delta EQ_0 \cdot EQ_0$ implies $\lim_{t \to \infty} \left(\prod_{t=1}^{\infty} {}_{t-1} p_{Kt} \cdot EQ_\infty \right) = 0$. This assumption collapses [EQ B-34] into the multi-period subjective residual net income market-pricing model with non-constant subjective rates-of-returns as the horizon is pushed into infinity:

$$P_{K0} = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}\left[RI_t \right] \qquad \text{[EQ B-35]}$$

The multi-period subjective residual net income market-pricing model is the Homo comperiens equivalent to [EQ A-58].

Since firms are not restricted to finance their activities only through equity, it is also useful to convert [EQ B-35] into a market-pricing model that considers Modigliani & Miller's (1958) value additivity proposition. This is the residual income market-pricing model measured on comprehensive operating income.

B.5.2 The residual operating income market-pricing model

Classify the comprehensive net income into comprehensive operating income, COI, and comprehensive net interest expense, $CNIE$:

$$CNI_t = COI_t - CNIE_t \qquad \text{[EQ A-66]}$$

Classify the balance sheet into net operating assets, NOA, net financial liabilities, $NetIBL$, and equity:

$$EQ_t = NOA_t - NFL_t \qquad \text{[EQ A-67]}$$

By substituting [EQ B-30], [EQ A-66], and [EQ A-67] into [EQ B-29], I get the subjective residual income market-pricing model measured on comprehensive operating income:

$$P_{K0} = NOA_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}\left[ROI_{Kt}\right] - \left(NetIBL_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}\left[RIE_t\right] \right) \qquad \text{[EQ B-36]}$$

where

$$\mathrm{E}_{K0}\left[ROI_t\right] = \mathrm{E}_{K0}\left[COI_t\right] - \mathrm{E}_{K0}\left[{}_{t-1}r_{Kt}\right] \cdot NOA_{t-1} \qquad \text{[EQ B-37]}$$

$$\mathrm{E}_{K0}\left[RIE_t\right] = \mathrm{E}_{K0}\left[CNIE_t\right] - \mathrm{E}_{K0}\left[{}_{t-1}r_{Kt}\right] \cdot NetIBL_{t-1} \qquad \text{[EQ B-38]}$$

Since the model is placed in a subjective certain decision, the subjective MROR is the same for both the net operating assets as for the net financial liabilities.

Similar models derived for a setting in uncertainty would use the weighted average subjective cost of capital as MROR for the net operating assets and the subjective after-tax net cost of debt as MROR for the net financial liabilities.

See Chapter 5 for further discussion of classification and operationalization of the components to the balance sheet and to the income statement.

Model [EQ B-36] is the Homo comperiens equivalent to model [EQ A-68].

B.6 The market price based on accounting rates-of-returns

This section converts [EQ B-35] and [EQ B-36] into market pricing models that use accounting rates-of-returns rather than accounting income.

Define subjective return on equity as:

$$_{t-1}ROE_{Kt} = CNI_{Kt} \cdot EQ_{t-1}^{-1} \qquad \text{[EQ B-39]}$$

Define the one-period subjective residual rate-of-return on equity as the difference between the one-period subjective return on equity and its corresponding subjective market rate-of-return:

$$_{t-1}RROE_{Kt} = {}_{t-1}ROE_{Kt} - {}_{t-1}r_{Kt} \qquad \text{[EQ B-40]}$$

Substituting [EQ B-39] and [EQ B-40] into [EQ B-35] gives the multi-period subjective residual return on equity market-pricing model with non-constant rates-of-returns:

$$P_{K0} = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}\left[RROE_{Kt}\right] \cdot EQ_{t-1} \qquad \text{[EQ B-41]}$$

Define subjective return on net operating assets as:

$$_{t-1}RNOA_{Kt} = COI_{Kt} \cdot NOA_{t-1}^{-1} \qquad \text{[EQ B-42]}$$

Define the one-period subjective residual rate-of-return on net operating assets as the difference between the one-period subjective return on net operating assets and its corresponding subjective market rate-of-return:

$$_{t-1}RRNOA_{Kt} = {}_{t-1}RNOA_{Kt} - {}_{t-1}r_{Kt} \qquad \text{[EQ B-43]}$$

Define the one-period subjective residual rate-of-return on net financial liabilities, RRIBL, as:

$$_{t-1}RRNBC_{Kt} = {}_{t-1}NBC_t - {}_{t-1}r_{Kt} \qquad \text{[EQ B-44]}$$

Substituting [EQ B-43] and [EQ B-44] into [EQ B-36] gives the multi-period subjective residual return on the net operating asset market-pricing model with non-constant subjective rates-of-returns:

$$P_{K0} = EQ_0 + \sum_{t=1}^{\infty} {}_0p_{Kt} \cdot {}_{t-1}RRNOA_{Kt} \cdot NOA_{t-1} - \sum_{t=1}^{\infty} {}_0p_{Kt} \cdot {}_{t-1}RRNBC_{Kt} \cdot NFL_{t-1} \qquad \text{[EQ B-45]}$$

B.7 Summary

This appendix derives market-pricing models for a market meeting the assumptions in the theory of Homo comperiens.

It finds that it is possible to describe individual decision-making in a subjective certain decision as though the subjects are choosing its subjective optimal consumption bundle such that its subjective marginal rate of substitution between future and current consumption equals the ratio between the subjective futures price vector and the subjective current price vector. That is:

$$\frac{_0\boldsymbol{p}_{K1}}{_0\boldsymbol{p}_{K0}} = \frac{\nabla U_K(\mathbf{c}_{K1})}{\nabla U_K(\mathbf{c}_{K0})}$$

It also finds that the firm chooses its production such that its subjective marginal rate of transformation between current and future goods equals the marginal rate of substitution between future and current consumption.

Assuming homogenous preferences opens the door for describing the market price of the firm in a Homo comperiens setting as:

$$P_{K0} = \sum_{t=1}^{\infty} {}_0p_{Kt} \cdot \mathrm{E}_{K0}[d_{Kt}]$$

Where $_{t-1}p_{Kt} = (1 + {}_{t-1}r_{Kt})^t$ and where $_{t-1}r_{Kt}$ is the one-period market rate-of-return. The subjective MROR is in the subjective certain setting equal to the market's risk-free rate of return.

Defining subjective return on equity, ROE, and the subjective return on net operating-assets, RNOA, as:

$$_{t-1}ROE_{Kt} = CNI_{Kt} \cdot EQ_{t-1}^{-1}$$

$$_{t-1}RNOA_{Kt} = COI_{Kt} \cdot NOA_{t-1}^{-1}$$

and by defining the residual rate-of-return on equity, RROE, the subjective residual rate-of-return on net operating assets, RRNOA, and the subjective residual rate-of-return on net financial liabilities, RRIBL as:

$$_{t-1}RROE_{Kt} = {}_{t-1}ROE_{Kt} - {}_{t-1}r_{Kt}$$

$$_{t-1}RRNOA_{Kt} = {}_{t-1}RNOA_{Kt} - {}_{t-1}r_{Kt}$$

$$_{t-1}RRNBC_{Kt} = {}_{t-1}NBC_t - {}_{t-1}r_{Kt}$$

it becomes possible to express the derived market-pricing models using residual rates-of-returns. (Note that subjective MROR is the same for all definitions since this is a subjective certain decision.)

The multi-period subjective residual return on equity market-pricing model with non-constant rates-of-returns is:

$$P_{K0} = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot \mathrm{E}_{K0}\left[RROE_{Kt}\right] \cdot EQ_{t-1}$$

The multi-period subjective residual return on net operating asset market-pricing model with non-constant subjective rates-of-returns is:

$$P_{K0} = EQ_0 + \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot {}_{t-1}RRNOA_{Kt} \cdot NOA_{t-1} - \sum_{t=1}^{\infty} {}_0 p_{Kt} \cdot {}_{t-1}RRNBC_{Kt} \cdot NFL_{t-1}$$

APPENDIX C—PROOF OF PROPOSITION 2-3

Appendix C is the proof of the existence of subjective expected utility in a limited rational choice and mimics the proof of von Neumann-Morgenstern's expected utility function in a perfect rational choice (e.g., Huang & Litzenberger 1988, for a derivation of the vNM utility function).

C.1 Structuring the problem

Assume that the subject has state-independent utility function, i.e. the subject values a consequence uniformly no matter in which state it occurs: $u_a(c) = u_b(c) \quad \forall a, b \in S_K$. State-independent utility makes it possible to form the union of all the state-dependent consequence sets such that it becomes a total subjective consequence set: $C_K = \bigcup_{s \in S_K} C_s$.[24]

Since the subjective state consequence sets are strict subsets to their objective sets because of limited knowledge, it is possible to expect total subjective consequence set to be bounded from above and below. However, when generalizing the concept of the subjective consequence set, it follows from Proposition 2-1 (p. 32) that the total subjective consequence set can become unbounded unless there are further restrictions from the state set. Since the theory of Homo comperiens also restricts the subjective state set to be a strict subset to its objective set, this set is also bounded and therefore the total subjective consequence set also becomes bounded.

Since the total subjective consequence set is bounded, it has a upper bound, $\bar{z}$, and a lower bound, $\underline{z}$.[25]

I assume that all the intermediate consequences between the upper and lower bounds can be measured as a weighted average of the two bounds. This means that the subject is, e.g., indifferent between consequence g and a weighted average, $u_K(\bar{z}) \cdot \bar{z} + (1 - u_K(\bar{z})) \cdot \underline{z}$, of the supremum and infimum consequences, where $u_K(\bar{z}) \in [0,1]$.[26]

---

[24] The total subjective consequence set consists of all consequences that the subject knows. The knowledge of possible consequence is restricted because of the limited knowledge of available alternatives and by the subject's limited knowledge of the possible states.

[25] Baumol (1961) calls these two extreme consequences "eternal bliss" and "damnation." Mathematics calls these extreme consequences supremum (eternal bliss) and infimum (damnation). Upper bound, eternal bliss, and supremum are used interchangeably in this appendix. Damnation, lower bound, and infimum are also used interchangeably in this text For the mathematical definition of a supremum and infimum, as well as the definition of a bounded set, see, e.g., Sydsæter (1999).

Note that I focus on the subjective supremum and infimum consequences and not the objective counterparts.

[26] In this study, a consequence, which is a function of two other consequences, is called a complex consequence.

The symbol $u_K(\cdot)$ is the subjective unique mass assigned by the subject to the upper bound consequence in order to have indifference between consequence g and the complex consequence. This type of mass is interpreted in financial economics as a probability in a lottery where the two potential consequences are the upper and lower bound, but where only the objective mass is considered (Huang & Litzenberger, 1988).

The possibility to have $g \sim u_K(\overline{z}) \cdot \overline{z} + \left(1 - u_K(\overline{z})\right) \cdot \underline{z}$ rests on the existence of a preference relation that is complete, transitive, continuous, state-uniform, independent, and which follows the Archimedean conjecture. For a proof in a perfect rational choice, see, e.g., Kreps (1988).

The completeness, transitivity, continuity, and the state-uniform conjectures have already been assumed in Chapter 2. To these I add the independence and the Archimedean conjectures. These are presented below.

The independence conjecture asserts that for all consequences $\overline{z}, z, \underline{z} \in C_K$, there exists some $u_K(\overline{z}) \in (0,1]$, such that $\overline{z} \succ z \;\Rightarrow\; u_K(\overline{z}) \cdot \overline{z} + \left(1 - u_K(\overline{z})\right) \cdot \underline{z} \succ u_K(\overline{z}) \cdot z + \left(1 - u_K(\overline{z})\right) \cdot \underline{z}$.

Similarly, the Archimedean conjecture asserts that for all consequences $\overline{z}, z, \underline{z} \in C_K$, and $u_K(\overline{z}), u_K(\overline{z}') \in [0,1]$, $\overline{z} \succ z \succ \underline{z} \;\Rightarrow\; u_K(\overline{z}) \cdot \overline{z} + \left(1 - u_K(\overline{z})\right) \cdot \underline{z} \succ z \succ u_K(\overline{z}') \cdot \overline{z} + \left(1 - u_K(\overline{z}')\right) \cdot \underline{z}$. The definition of the independence conjecture and the Archimedean conjecture is found in many financial economics books (e.g., Huang & Litzenberger 1988), although here I apply them in a limited rational choice.

The independence conjecture determines that the subject's preference towards the complex consequence $u_K(\overline{z}) \cdot \overline{z} + \left(1 - u_K(\overline{z})\right) \cdot \underline{z}$ and the complex consequence $u_K(\overline{z}) \cdot z + \left(1 - u_K(\overline{z})\right) \cdot \underline{z}$ depends solely on how he or she feels about $\overline{z}$ and z, given that this is the only thing that differs between the two complex consequences. Consequence $\left(1 - u_K(\overline{z})\right) \cdot \underline{z}$ is irrelevant in the choice between the two consequences because it is part of both complex consequences.

The Archimedean conjecture is best exemplified using a gamble. Suppose $\overline{z}$ is eternal bliss, z is just managing to go around, and $\underline{z}$ is a gruesome death. Then, if the complex consequence $u_K(\overline{z}) \cdot \overline{z} + \left(1 - u_K(\overline{z})\right) \cdot \underline{z}$ is a lottery, there will be an $u_K(\overline{z}) \in [0,1]$ (most likely close to 1) where the subject exhibits a preference such as $u_K(\overline{z}) \cdot \overline{z} + \left(1 - u_K(\overline{z})\right) \cdot \underline{z} \succ z$.

Armed with this structure, I present the derivation of subjective expected utility in a limited rational choice.

C.2 Derivation

Since every consequence can be described as weighted averages of the supremum and infimum consequences, all state consequences for an uncertain choice can be described using the same argu-

ments. Suppose that an action e has three state-dependent consequences denoted p, h, j. Assume for now that the subject is only aware of the two state-dependent consequences p and h. These consequences can, with the additional conjectures of continuous state-uniform preferences that are independent and that follow the Archimedean conjecture, be described as:

$p \sim u_K(\overline{z}) \cdot \overline{z} + (1 - u_K(\overline{z})) \cdot \underline{z}$, and $h \sim u_K(\overline{z}') \cdot \overline{z} + (1 - u_K(\overline{z}')) \cdot \underline{z}$, where $u_K(\overline{z}), u_K(\overline{z}') \in [0,1]$.

Chapter 2 defines states such that it allows probabilities to be assigned to each state. Since the occurrence of a state is beyond the control of the subject, it is also not possible for the subject to affect the probability for a state to occur. The probability-on-states view is useful in the above example. Choice alternative e has the following consequence $(p, h; \pi_{Kp}, \pi_{Kh})$. Since only two consequences can ensue, it can be expressed more succinctly as $(p, h; \pi_{Kp})$. Replacing p with

$u_K(\overline{z}) \cdot \overline{z} + (1 - u_K(\overline{z})) \cdot \underline{z}$ and h with $u_K(\overline{z}') \cdot \overline{z} + (1 - u_K(\overline{z}')) \cdot \underline{z}$ gives the following complex consequence $\left(u_K(\overline{z}) \cdot \overline{z} + (1 - u_K(\overline{z})) \cdot \underline{z}, u_K(\overline{z}') \cdot \overline{z} + (1 - u_K(\overline{z}')) \cdot \underline{z}; \pi_{Kp} \right)$. This means that

$(p, h; \pi_{Kp}) \sim \left(\overline{z}, \underline{z}; \pi_p \cdot u_K(\overline{z}) + (1 - \pi_{Kp}) \cdot u_K(\overline{z}') \right)$ holds.

Precisely as it is possible to describe consequences p and e in terms of a bundle that is a weighted average of the supremum and infimum consequences, it is also possible to describe the uncertain consequence $(p, h; \pi_{Kp})$ in terms of a weighted average of the extreme consequences. This standard bundle is signified by $\left(\overline{z}, \underline{z}; \mathrm{E}_{K0} \left[\mathrm{U}_K(p, h; \pi_{Kp}) \right] \right)$, where $\mathrm{E}_{K0} \left[\mathrm{U}_K(p, h; \pi_{Kp}) \right] \in [0,1]$ is the subjective mass assigned at present to the supremum consequence necessary for $(p, h; \pi_{Kp}) \sim$

$\left(\overline{z}, \underline{z}; \mathrm{E}_{K0} \left[\mathrm{U}_K(p, h; \pi_{Kp}) \right] \right)$.

The existence of the standard bundle means the following relations exist

$(p, h; \pi_{Kp}) \sim \left(\overline{z}, \underline{z}; \mathrm{E}_{K0} \left[\mathrm{U}_K(p, h; \pi_{Kp}) \right] \right) \sim \left(\overline{z}, \underline{z}; \pi_{Kp} \cdot u_K(\overline{z}) + (1 - \pi_{Kp}) \cdot u_K(\overline{z}') \right)$, and from this relation it follows that $\mathrm{E}_{K0} \left[\mathrm{U}_K(p, h; \pi_{Kp}) \right] = \pi_p \cdot u_K(\overline{z}) + (1 - \pi_{Kp}) \cdot u_K(\overline{z}')$.

The subjective expected utility function is cardinal with range $[0,1]$. $u_K(\overline{z})$ and $u_K(\overline{z}')$ are utilities with the same range but on sure-thing consequences. This kind of utility is also known as a Bernoulli utility (Mas-Colell et al. 1995) in the perfect rational choice. Here I call it subjective Bernoulli utility since it is formed, with limited knowledge on supremum and infimum consequences that stem from the subjective consequence set.

$u_K(\overline{z})$ is from now on denoted $u_K(p)$ and $\tilde{u}(\overline{z}')$ is denoted $u_K(h)$. They are the real values of the function $u_K : C_K \rightarrow \mathbb{R}$ at p and h, where $C_K \subset C_\Omega$.

The subjective expected-utility function can be expanded to cover more than two states. When it covers all states in the general description of the state space, it can be written as

$$\mathrm{E}_{K0}\left[\mathrm{U}_K\left(c_1,\ldots c_S;\pi_{K1},\ldots \pi_{KS}\right)\right] = \sum\nolimits_{s \in S_K} \pi_{Ks} \cdot u_K\left(c_s\right), \text{ where } c \in C_K, \text{ and } \pi_{Ks} \in \Pi_K.$$

C.3 Discussion

The difference between the subjective expected-utility of an action and the objective expected-utility of the action rests on (i) the subjective state probability and (ii) the subjective Bernoulli utility.

The subjective states are mutually exclusive and form an exhaustive state set. However, the subject fails to appreciate all the states because of limited knowledge. The subject thus infers that each state has a subjective probability, π_{Ks}. Proposition 2-2 (p. 35) argues that at least one subjective state probability must be different to the objective state probability when the subjective state set is a strict subset of the objective state set. It therefore follows that the subject's limited rational choice is erroneous when compared with the perfect rational choice. This is due to limited knowledge.

The supremum and the infimum consequences in the perfect rational choice are based on the objective consequence set. The subjective consequence set is a strict subset of the objective consequence set and thus the subjective supremum and the infimum consequences are probably different from the objective supremum and the infimum consequences. This means that the subjective Bernoulli utilities change when we go from a perfect rational choice to a limited rational choice; again, this is due to limited knowledge.

In conclusion, both the subjective Bernoulli utilities and the subjective expected utilities retain the range $[0,1]$ in the limited rational choice. However, since the subject only knows of a subset of the objective state set, it is reasonable to expect to see subjective Bernoulli utilities differ from the objective Bernoulli utilities, as well as a difference between the subjective expected utility and the objective expected utility. The difference between the subjective expected utility and the objective expected utility is due to different Bernoulli utilities and different state probabilities and both of these differences are due to limited knowledge (Definition 2-2, p. 29, and Definition 2-6, p. 33).

APPENDIX D—OPERATIONALIZATION OF GROUP CONTRIBUTION AND OF UNTAXED RESERVE

	77NAME	80NAME	85NAME	91NAME	94NAME	95NAME
Group contribution						
Contribution from group companies, net	SPECB4	SPECB6	SPECB6	SPECB7	SPECC6	SPECC5
Shareholders' contribution					SPECC7	
Appropriations						
Change of tax allocation reserve					SPECC1	SPECC1
Change of tax equalization reserve					SPECC2	SPECC2
Change of investment reserve					SPECC3	SPECC3
Dissolving inventory reserve and profit equalization reserve				SPECB1		
Transfer to tax equalization reserve				SPECB2		
Transfer to transitional reserve				SPECB3		
Change of inventory reserve	SPECB1	SPECB1	SPECB1			
Change of profit equalization reserve		SPECB2	SPECB2			
Transfer to investment reserve	SPECB2	SPECB3	SPECB3	SPECB4		
Accelerated depreciation, excl. investment reserve	SPECB3	SPECB4	SPECB4	SPECB5	SPECC4	SPECC4
Reversal of accelerated depreciation for sold fixed assets		SPECB5	SPECB5	SPECB6	SPECC5	
Other changes in the untaxed reserve	SPECB5	SPECB7	SPECB7	SPECB8	SPECC8	
Untaxed reserve						
Total untaxed reserve		BAL40	BAL41	BAL41	BAL40	BAL32
Inventory reserve	BAL35					
Accumulated accelerated depreciation	BAL36					
Investment reserve, et cetera	BAL37					
Other untaxed reserves	BAL38					

Total untaxed reserve for the period 1977 to 1979 is sometimes not equal to the sum of its components, and an analyzis shows that it is the total untaxed reserve calculation that is incorrect and the components are correct.

From 1995 is the data collection method changed so that the firm's group contribution and shareholders' contribution is aggregated with the appropriation called "other changes in the untaxed reserve." To get an estimation of firm's group contributions and shareholders' contributions during 1995 to 1996 is the following method applied.

To get an estimation of firms' group contributions and shareholders' contributions during 1995 to 1996 is the ratio between the group contributions and shareholders' contributions, and the sum of the group contributions, shareholders' contributions, and other changes in the untaxed reserve studied for 1994. The ratio is defined as
(SPECC6+SPECC7)/(SPECC5+SPECC6+SPECC7).

In the full sample of 1511 firms, only two percent (30 firms) reported other changes in the untaxed reserve. Of these firms, 16 reports a ratio between 0.98-1.02, three firms have a ratio higher than 1.02, and 11 firms reports a ratio lower than 0.98.

A total of 780 firms (of 1511) had a sum of group contributions and shareholders' contributions other than zero for 1994. Of these firms, the relation was evaluated to 1.0 for 758 firms, the maximum was 1.6, and the minimum was -1.3. This means that of those firms that has group contributions, or shareholders' contributions, the vast majority reports no "other changes in the untaxed reserve."

Based on these findings is all of other changes in the untaxed reserve for 1995 to 1996 classified as group contribution/shareholders' contribution.

APPENDIX E—TRACING FIRMS

E.1 Introduction

This appendix supplies a description of the method used for following firms across the empirical data's period and for finding the opening balance sheet for the firms.

The financial data set supplied by SCB consists of a table for each year between 1977 and 1996 with financial data for all active[27] limited companies (firms) involved in the Swedish manufacturing industry. In these tables are firms the individual elements and they are identified using their civic registration numbers (Organisationsnummer in Swedish).

Year	No.	Year	No.	Year	No.	Year	No.
1977	893	1982	1,689	1987	1,859	1992	1,586
1978	947	1983	1,706	1988	1,907	1993	1,446
1979	1,643	1984	1,738	1989	1,961	1994	1,511
1980	1,644	1985	1,781	1990	1,889	1995	1,801
1981	1,663	1986	1,836	1991	1,869	1996	1,882

Table E-1: The total number of limited companies (firms) in the empirical data.

SCB also supplied tables for each year with information on local units. A local unit is according to SCB's definition the same as each address, building or group of buildings where the enterprise carries out economic activity. The local unit is identified in the tables using a unique local unit identification number. To each local unit identification number is a firm's civic registration number attached. This means that the local units are unique, and to each local unit is only one civic registration number attached. An active firm has at least one local unit.[28]

E.2 Tracing firms and identifying each firm's opening balance sheet

In the financial tables are, among other things, the firms' current income statement and the closing balance sheet supplied.

To be able to calculate rates-of-returns must the opening balance sheet be identified and therefore must at least two consecutive balance sheets be identified for a particular firm. The change in the equity account is also analyzed. This requires at least two consecutive balance sheets to be identified for a particular firm. When the hypotheses have been posed and are tested is pooled time-series analysis applied to increase the sample size, and this requires that more than two consecutive

---

[27] SCB defines an active firm as a firm that is registered as employer, it is registered for VAT, and it has a certificate for business tax.

[28] An active firm can also have additional local units when the following criteria are met: (i) there are some activity, (ii) there is a place where activity is going on, (iii) the activity is ongoing, and (iv) it has employees

balance sheets can be identified for a particular firm. Because of these reasons is the following method used to identify firm across time.

A firm's civic registration number may exist in a certain year but can appear to have disappeared in the year that follows. The can be interpreted several ways: (i) The firm may continue to exist but has changed civic registration number, (ii) the firms has closed its operations without selling off its production structure, (iii) the firm has merged with another firm. Or (iv) the firm has spun-off all of its production structure to another firms, that may be already existing or where they are newly incepted with the purpose of taking over parts of the old production structure.

Since the purpose is to find consecutive balance sheets are scenario (iii) and (iv) analyzed with the purpose of tracing if any firm can be classified as a surviving firm.

It is also possible that firms only merges parts of its operations with another firm, or it may just spin-off parts of its operations. It is also conceivable that a firm also appears as a seller and buyer of production structure within a financial year. Also these partial transactions are analyzed with the purpose of tracing if any firm can be classified as a surviving firm.

Whether a firm is classified as a survivor depends on the change in the firms production structure is deemed significant. The firm's production structure is in this research operationalized as the local units attached to the firm.

A merger is defined as follows. First are the information on local units used to identify those local units that exist in two consecutive years. Those local units are used to identify the civic registration number for the two years. Then are the number of civic registration number in the preceding year calculated based on each new civic registration number. This procedure provides information where new firms have purchased at least one local unit during the year from another firm.

A complete merger is defined as those mergers where all of the old civic registration numbers (having one or more local units attached to it) is completely subsumed by a new civic registration number. The new civic registration number may have been part of the set of old civic registration numbers participating in the merger, but this is not a requirement. It is conceivable that the new firm changes its civic registration number at the time of the merger. Partial mergers are all those mergers not fulfilling the conditions of a complete merger.

When there is a complete merger between two firms, the size of them can be used as a signal on the structural change. Size can be measured using e.g., market prices, book values of assets, book values of equity, or sales.

In this thesis, market prices are considered disequilibrium prices and hence not good descriptors of the firm's size of its production structure. Book values are subjected to conservativism and are not good descriptors of the firm's size. The sales revenue indicates the firm's market power on its customers, but is silent on the firm's productions structure. Since the focus is on the structural

stability is also sales a bad descriptors of the firm's size of its production structure. This thesis used value added (calculated before depreciation) as a descriptor on the firm's size of its production structure.

The second issue that must be addressed in survival classification is when the change is significant. This thesis makes an analogy to FAR no. 3 that states that a material difference exists between the book value and market price of an asset when it is greater than 20 percent. Thus is a change in the firm's production structure, operationalized as value added, greater than 20 percent, classified as a significant structural change.

When there is a complete merger, is the firms' value added from the year preceding the merger used for estimating the structural change. If a firm has at least 80 percent of the combined value added, it is classified as a survivor, and is deemed structurally stable. This implies that that firm's closing balance sheet, preceding the merger, is used as the opening balance sheet after the merger. In essence, this means that this firm is treated as if there had been no merger. Those firms that do not satisfy the threshold value are treated as if they close their operations at the time of the merger.

When the complete merger implies that all merging firms are structurally instable, all of them are treated as if they close their operations at the time of the merger. Since the merger is complete, the sum of the merging firms' opening balance sheets can function as the opening balance sheet for the new firm. By also using these firms' income statements for the year preceding the merger and that year's opening balance sheet the change in rate-of-returns can be analyzed already from the merging year.

Partial mergers imply that only some of the production structure is sold to another firm. The purchasing firm may or may not have existed before the financial year when the purchase of takes place.

When the firm did not exist in the previous year, i.e., when the new civic registration number did not exist in the previous year, it is classified as structurally instable. When the purchasing firm existed in the preceding year is its change in real value added[29] between the two consecutive years used for measuring structural stability. When the change in the real value added is less than or equal to 20 percent it is treated as a structurally stable firm. A structurally stable firm participating in a partial merger is treated identically as a structurally stable firm participating in a complete merger.

A spin-off is defined as follows. Again, the local units are used to identify the civic registration number for the two years. The number of civic registration numbers in the new year is calculated based on each old civic registration number. This procedure provides information on old firms that have sold at least one local unit during the year to another firm.

---

[29] The nominal value added is deflated using the production price index on SIC-code 38 (SNI62) published by SCB. This ensures that industry-specific inflation does not affect the reliability of the method.

A complete spin-off is defined as a spin-off where all of the new civic registration numbers can be completely related to an old civic registration number. The new civic registration number may have been part of the set of old civic registration numbers participating in the merger, but this is not a requirement. No other old civic registration number is allowed among the new civic registration number in the complete spin-off. This assures that the set all new civic registration numbers, for a particular spin-off, has an identical production set as that of the old civic registration number that spun-off the units. Partial spin-offs are all those sales of local units that do not fulfill the conditions of a complete spin-off.

The evaluation of the structural stability in the complete spin-off is similar to that in a complete merger. In this research is the value added of each new civic registration number, for a given spin-off, compared with the sum of the new civic registration numbers' value added. If a civic registration number has a ratio of at least 80 percent it is evaluated as structurally stable. When this occurs, it is treated identically to a structurally stable complete merger.

A partial spin-off is evaluated similarly to a partial merger. Only a firm that sold part of its local units and that continued to exist is evaluated in this research as a potentially structurally stable firm. Any partial spin-off is treated as structurally instable. The structural stability is evaluated using the change in the real value added, and any change greater than 20 percent implies a structural instable spin-off. A structural stable partial spin-off is treated in the same manner as a structurally stable complete merger.

Sometimes a firm both sells and buys local units in a given financial year. This implies that it appear as both a merger and a spin-off. When this happens is again the change in real value added used as a measure of the structural stability, but since it has made at least two transactions is the threshold value decreased to 10 percent. Any composite change in the real value added less than or equal to 10 percent is classified as a structural stable change.

With the procedure above can most of the firms be analyzed. Those firms that are not considered in the analysis above are compared on their civic registration numbers and names. Where the new civic registration number is identical to an old civic registration number it is classified as a structurally stable firm. That firm that cannot be matched on the civic registration number is compared based on the firm's name. Firms having identical names over two consecutive years are also classified as structurally stable firms.

All civic registration numbers that exists in the data set for the new year and that have not been covered in the analysis above, are classified as completely new firms that lacks an opening balance before the focal year.

The table on the next page provides a summary of the number of structurally stable firm and structurally instable firms.

Number of firms in the sample	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Structurally stable firms	805	859	1,496	1,505	1,534	1,506	1,540	1,587	1,621	1,638	1,668	1,723	1,629	1,639
of which competely stable firms	796	845	1,483	1,486	1,518	1,482	1,511	1,558	1,592	1,620	1,646	1,692	1,609	1,626
of which complete mergers	1	1	1	2	0	2	2	4	3	4	2	4	2	3
of which partial mergers	0	0	0	0	0	0	0	0	0	0	0	0	0	0
of which complete spin-offs	0	2	2	2	3	2	3	2	2	0	6	7	4	5
of which partial spin-offs	0	2	0	6	2	2	3	4	0	1	3	2	2	1
of which composite activity	0	1	0	1	0	0	1	1	2	0	1	1	1	0
of which matching civic registration no.	8	8	10	6	7	15	17	13	18	10	7	12	10	2
of which matching firm name	0	0	0	2	4	3	3	5	4	3	3	5	1	2
Structurally instable firms	88	88	147	139	129	183	166	151	160	198	191	184	332	250
of which complete mergers	4	4	6	2	2	12	9	6	10	22	11	16	26	13
of which partial mergers	2	8	0	10	2	2	4	2	2	4	8	2	2	6
of which complete spin-offs	0	2	1	1	4	4	2	1	3	2	9	5	6	4
of which partial spin-offs	2	4	1	1	0	0	3	1	3	3	4	0	1	2
Total number of firms	893	947	1,643	1,644	1,663	1,689	1,706	1,738	1,781	1,836	1,859	1,907	1,961	1,889

Note that the analysis on which this table rests, focuses on the structural stability/instability for the next following year. That means that the complete mergers for year 1995 should be interpreted as if there was three firms in 1995 that merged in 1996 with other firms, but where enough to be classified as structurally stable, i.e., as survivors in the mergers.

Table E-2: A summary of the number of structurally stable and structurally instable firms for the empirical data

E.3 Finding usable firm-year observations

The table summarizing the effect of the preceding sections method of tracing firms does not suffice for economic analysis. Some firms have been classified as non-response by SCB have their data imputed. Erikson (2003) reports on the procedure used by SCB on non-response firms.

Non-response firms are according to Erikson firms that did not file an income declaration, firms that filed their income declaration too late and was not in the database at the time of delivery to SCB, firms having filed income declaration but is still missing from the database, and the firm may no longer exist but data is still needed. The last point stems from erroneous definition of the population of active firms.

SCB uses an all or nothing method for imputation. This means that either is the firm's data used or it is not used at all. It implies that when some data is still missing after manual editing, and the firm is classified as non-response, it has all of its data imputed. The imputation uses either of two methods. SCB can use the firm's previous period's financial data and then imputes it into the current year. Or SCB uses the average values for the industry and size class as a source in the imputation. The size class is based on average number of employees.

None-response firms have imputed industry average values or imputed one-year old financial data instead of their true values. Making comparison over time for an individual firm, such as calculating its rate-of-return on equity, gives incorrect values for the non-response firms due to the imputation. The structurally stable firm-year observations from the previous section are tainted by non-response firms and are cleansed using the method described below. The remaining firm-years are classified as usable firm-year observations, and on them can economic analysis can be performed.

There is no a prioi way to identify non-response firms. In this research are two avenues pursued to identify non-response firms. First is non-response firms identified that has had its data im-

puted by the industry average. Then is non-response firms identified that have had its data imputed by the previous year's financial data.

Non-response firms that uses imputed industry data is identified by finding those firms, for a given year, that have an identical string of variables. When more than one firm has identical average number of employees, value added, net income, total assets, total untaxed reserve, and total equity, are they classified as imputed firms. It is argued that it is virtually impossible for more than one firm to have so many identical variables.

This method finds the industry average values for those firms where SCB has used the industry average more than once. It fails to detect those situations where the string of variables has been imputed only once.

The table below reports the number of identified non-response firms per year that uses industry averages.

Year	No.	Year	No.	Year	No.	Year	No.
1977	158	1982	100	1987	76	1992	53
1978	100	1983	93	1988	105	1993	28
1979	90	1984	74	1989	110	1994	8
1980	40	1985	80	1990	96	1995	80
1981	35	1986	73	1991	91	1996	15

Table E-3: The number of imputed firm in the data set using industry averages.

Given the analysis in this and the preceding section is the number of firm-year observations reduced from being totally 33,251 to 26,246 firm-years.

Non-response firms that uses imputed previous year's financial data is identified as those firms that have two consecutive years of identical average number of employees, value added, net income, total assets, total untaxed reserve, and total equity. This method uses the same string of variables as that is used to identify imputed industry data.

Year	No.	Year	No.	Year	No.	Year	No.
1978	0	1983	17	1988	123	1993	6
1979	5	1984	16	1989	104	1994	7
1980	0	1985	0	1990	102	1995	8
1981	117	1986	73	1991	123	1996	68
1982	103	1987	89	1992	43		

Table E-4: The number of imputed firm in the data set using previous year's financial data.

Cleaning the data set from the imputed firm-year observations in the table above reduces the number of usable observations to 25,245. The table below summarizes the effects of the tracing and cleansing process.

Total number of firm-year observations 1977-1996   33,251
Potentially usable firm-year observations (1978-1996) 32,358
Structurally stable (or structurally instable w complete merger) 27,864
Usable firm-year observations   25,245

A usable firm-year is a firm's complete set of usable financial data for a particular year where the firm has a preceding year of usable financial data.

For the financial data to be usable must two criteria be satisfied. (i) The firm must be classified as either structurally stable or where it is structurally instable must there be a complete merger. A structurally instable complete merger allows the merging firms' financial data to be merged for the preceding year. This creates a structurally stable opening balance sheet for the merged firm. (ii) No financial data appears to have been imputed by SCB.

Table E-5: Summary of number of firm-year observations.

   The firm-year observations can be matched to form time-series of financial data for each firm. The length of each time series depends on the structural stability of each firm and of any imputation of the firm's financial data. The table below reports the number of usable times-series for various lengths of the time-series.

Yrs	A	B	C	D	Yrs	A	B	C	D
1	876	876	3,971	25,245	11	105	1,155	883	13,283
2	499	998	3,095	24,369	12	109	1,308	778	12,128
3	308	924	2,596	23,371	13	111	1,443	669	10,820
4	289	1,156	2,288	22,447	14	68	952	558	9,377
5	238	1,190	1,999	21,291	15	73	1,095	490	8,425
6	236	1,416	1,761	20,101	16	90	1,440	417	7,330
7	183	1,281	1,525	18,685	17	132	2,244	327	5,890
8	161	1,288	1,342	17,404	18	59	1,062	195	3,646
9	147	1,323	1,181	16,116	19	136	2,584	136	2,584
10	151	1,510	1,034	14,793					

Table E-6: The time-series of structurally stable non-imputed firms.

   The table above should be interpreted as follows. Column A reports the number of firms that have only a given number of years of structurally stability and that are not affected by imputation. In this case, for example, 499 firms have two years of structurally stable non-imputed data. These firms use 998 firm-years observations (column B) of the usable set's 25,245 firm-year observations.

   Column C reports the accumulated number of available time-series: There are 3,095 unique time-series of firms that has at least two years of structurally stable non-imputed data. Column D reports the accumulated number of usable firm-year observations. A pure cross-sectional analysis on

the data set employs 25,245 firm-year observations, where a pooled time-series analysis over five years employs 21,291 firms-year observations (and 1,999 unique time-series).

APPENDIX F—OPERATIONALIZATION OF THE BALANCE SHEET

	77NAME	80NAME	85NAME	91NAME	94NAME	95NAME
NET OPERATING ASSETS (NOA)						
Operating assets (OA)						
Inventories and advance payments	BAL9, BAL10	BAL9, BAL10	BAL9, BAL10	BAL9, BAL10	BAL9, BAL10	BAL7
Prepaid expenses and accrued income	BAL8	BAL8	BAL8	BAL8	BAL8	BAL5
Intangible assets	BAL17	BAL17	BAL17	BAL17	BAL17	BAL11
Land and buildings	BAL20	BAL20	BAL20	BAL20	BAL20	BAL14, BAL15
Plant and equipments	BAL19	BAL19	BAL19	BAL19	BAL19	BAL13
Construction in process and advance payments for tangible assets	BAL18	BAL18	BAL18	BAL18	BAL18	BAL12
Shares and participations in group companies	BAL13	BAL13	BAL13	BAL13	BAL13	Part of BAL9
Operating liabilities (OL)						
Deferred tax proportion of the untaxed reserve	Tax rate*BAL39	Tax rate*BAL40	Tax rate*BAL41	Tax rate*BAL41	Tax rate*BAL40	Tax rate*BAL32
Provision for taxes	BAL26	BAL26	BAL26	BAL26	BAL26	BAL19
EQUITY (EQ)						
Share capital	BAL40	BAL41	BAL42	BAL42	BAL41	EGKAP1
Restricted reserves	BAL41	BAL42	BAL43	BAL43	BAL42	EGKAP2
Retained earnings	BAL42	BAL43	BAL44	BAL44	BAL43	EGKAP3
Reported net income	BAL43	BAL44	BAL45	BAL45	BAL44	EGKAP4
Equity proportion of the untaxed reserve	Rest of BAL39	Rest of BAL40	Rest of BAL41	Rest of BAL41	Rest of BAL40	Rest of BAL32
NET FINANCIAL LIABILITIES (NFL)						
Financial liabilities (FL)						
Accounts payable	BAL24, BAL25	BAL24, BAL25	BAL24, BAL25	BAL25	BAL25	BAL18
Advance payments from customers	BAL29	BAL29	BAL29	BAL29	BAL29	BAL22
Other current liabilites	BAL23, BAL27, BAL28	BAL23, BAL27, BAL28	BAL23, BAL27, BAL28	BAL23, BAL24, BAL27, BAL28	BAL23, BAL24, BAL27, BAL28	BAL20, BAL21
Pension liabilities	BAL32	BAL32	BAL32	BAL32	BAL32	BAL24
Other long term liabilities	BAL31, BAL33	BAL31, BAL33	BAL31, BAL33	BAL31, BAL33	BAL31, BAL33	BAL25
Financial assets (FA)						
Accounts receivables	BAL5, BAL6	BAL5, BAL6	BAL5, BAL6	BAL6	BAL6	BAL4
Cash and bank balances	BAL1, BAL12	BAL1, BAL12	BAL1, BAL12	BAL1, BAL12	BAL1, BAL12	BAL1
Other current receivables	BAL4, BAL7	BAL4, BAL7	BAL4, BAL7	BAL4, BAL5 BAL7	BAL4, BAL5 BAL7	BAL6
Other long-term receivables	BAL15, BAL16	BAL15, BAL16	BAL15, BAL16	BAL15, BAL16	BAL15, BAL16	BAL10
Bonds and other securities	BAL3	BAL3	BAL3	BAL3	BAL3	BAL3
Shares and participations in other firms	BAL2, BAL14	BAL2, BAL14	BAL2, BAL14	BAL2, BAL14	BAL2, BAL14	BAL2, Rest of BAL9

For 1995 and 1996, the shares and participations in group companies are aggregated with shares and participations in other companies. This thesis split, according to the analysis below, the sum of shares and participations into the two components to maintain comparability across all periods.

The data from 1994 is used for identifying firms that should have ownership in group companies. For those firms that exist from 1994 through 1995 and 1996, is the 1994-ratio between shares and participations in group companies and shares and participations in group companies aggregated with shares and participations in other companies [BAL13/(BAL13+BAL14)] assumed to persist through the following years.

Firms that did not exist in 1994 are analyzed based on two cases: (i) the firm report dividends from group companies, (ii) the firm report no dividends from group companies.

In 1994 firms falling into category (i) have a median ratio of 99,5 percent, indicating that virtually all shares and participations can be ascribed to holdings of group firms. Furthermore, 93 percent of the firms report shares and participations in group companies. Category (ii) has a median ratio of 0 percent for the same year, and 70 percent of the firms report no shares and participations in group companies.

It is assumed that firms that did not exist in 1994, but that exist later and that fall into category (i), only have shares and participations in group companies. For firms falling into category (ii) during 1995 and 1996, and that did not exist in 1994, it is assumed that there are no shares and participations in group companies.

	77NAME	80NAME	88NAME	91NAME	94NAME	95NAME
Comprehensive operating income						
Adjusted sales						
Sales	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below
	RES1	RES1	RES1	RES1	RES1	RES1
-Implicit interest revenue on AR	-RFRET(t)*Accounts receivable(t-1)	-RFRET(t)*Accounts receivable(t-1)	-RFRET(t)*Accounts receivable(t-1)	-RFRET(t)*Accounts receivable(t-1)	-RFRET(t)*Accounts receivable(t-1)	-RFRET(t)*Accounts receivable(t-1)
+Implicit interest expense on APL	+RFRET(t)*Adv. paym. fr cust (t-1)	+RFRET(t)*Adv. paym. fr cust (t-1)	+RFRET(t)*Adv. paym. fr cust (t-1)	+RFRET(t)*Adv. paym. fr cust (t-1)	+RFRET(t)*Adv. paym. fr cust (t-1)	+RFRET(t)*Adv. paym. fr cust (t-1)
Adjusted cost of material						
Cost of material	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below
	VA-RES1	VA-RES1	VA-RES1	VA-RES1	VA-RES1	VA-RES1
+Implicit interest expense on AP	+RFRET(t)*Accounts payable (t-1)	+RFRET(t)*Accounts payable (t-1)	+RFRET(t)*Accounts payable (t-1)	+RFRET(t)*Accounts payable (t-1)	+RFRET(t)*Accounts payable (t-1)	+RFRET(t)*Accounts payable (t-1)
+Implicit interest expense on PL	+RFRET(t)*Pension liabilities (t-1)	+RFRET(t)*Pension liabilities (t-1)	+RFRET(t)*Pension liabilities (t-1)	+RFRET(t)*Pension liabilities (t-1)	+RFRET(t)*Pension liabilities (t-1)	+RFRET(t)*Pension liabilities (t-1)
Labor cost	-BAS4	-BAS4	-BAS4	-BAS4	-BAS4	-BAS4
Depreciation	RES4	RES4	RES4	RES4	RES4	RES4
Tax on core operating income						
Reported tax	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below
	RES12	RES12	RES12	RES12	RES12	RES12
Deferred tax	Tax rate*APR(t)	Tax rate*APR(t)	Tax rate*APR(t)	Tax rate*APR(t)	Tax rate*APR(t)	Tax rate*APR(t)
Tax on government subsidies	-TGS	-TGS	-TGS	-TGS	-TGS	-TGS
Tax on items affecting comparability	-TIAC	-TIAC	-TIAC	-TIAC	-TIAC	-TIAC
Tax on ERROI	-TERROI	-TERROI	-TERROI	-TSERROI	-TSERROI	-TSERROI
Tax on core CFI	-TCCMI	-TCCMI	-TCCMI	-TCCMI	-TCCMI	-TCCMI
Tax on peripheral CFI	-TPCMI	-TPCMI	-TPCMI	-TPCMI	-TPCMI	-TPCMI
Dividends from the group	SPECA1	SPECA1	SPECA1	SPECA1	SPECB1	SPECB1
Government subsidies (GS)	BIDRAG1	BIDRAG1+BIDRAG2	BIDRAG1+BIDRAG2	BIDRAG	SPECA4	SPECA4
Items affecting comparability (IAC)	RES9	RES9-BIDRAG2	RES9-BIDRAG2	RES9	RES9+SPECA1+SPECA2	RES10+SPECA1+SPECA2
Dirty-surplus accounting	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below
Unaccounted changes in UTR	UTR(t)-UTR(t-1)+APR(t)	UTR(t)-UTR(t-1)+APR(t)	UTR(t)-UTR(t-1)+APR(t)	UTR(t)-UTR(t-1)+APR(t)	UTR(t)-UTR(t-1)+APR(t)	UTR(t)-UTR(t-1)+APR(t)
GAIN*	GAIN*	GAIN*/(1-Tax rate)	GAIN*/(1-Tax rate)	GAIN*/(1-Tax rate)	GAIN*/(1-Tax rate)	GAIN*/(1-Tax rate)
ACC*	ACC*	ACC*/(1-Tax rate)	ACC*/(1-Tax rate)	ACC*/(1-Tax rate)	ACC*/(1-Tax rate)	ACC*/(1-Tax rate)
Gain or loss due changing tax rate	[Tax rate(t-1)-Tax rate(t)]*UTR(t-1)	[Tax rate(t-1)-Tax rate(t)]*UTR(t-1)	[Tax rate(t-1)-Tax rate(t)]*UTR(t-1)	[Tax rate(t-1)-Tax rate(t)]*UTR(t-1)	[Tax rate(t-1)-Tax rate(t)]*UTR(t-1)	[Tax rate(t-1)-Tax rate(t)]*UTR(t-1)
Summation errors in OI (ERROI)	-(RES8+RES9-RES10) - / (RES10+RES11+RES12-RES13)	-(RES8+RES9-RES10) - / (RES10+RES11+RES12-RES13)	-(RES8+RES9-RES10) - / (RES10+RES11+RES12-RES13)	-(RES8+RES9-RES10) - / (RES10+RES11+RES12-RES13)	-(RES1+RES2+RES4-RES5) -(RES8+RES9-RES10)- / (RES10+RES11+RES12-RES13)	-(RES1+RES2+RES4-RES5)-(RES9+RES10- / RES11) -(RES11+RES12+RES13-RES14)
Tax on peripheral operating income						
Tax on government subsidies (TGS)	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below
	-Tax rate*GS	-Tax rate*GS	-Tax rate*GS	-Tax rate*GS	-Tax rate*GS	-Tax rate*GS
Tax on items affecting comparability (TIAC)	-Tax rate*IAC	-Tax rate*IAC	-Tax rate*IAC	-Tax rate*IAC	-Tax rate*IAC	-Tax rate*IAC
Tax on unaccounted changes in UTR	-Tax rate*[UTR(t)-UTR(t-1)+APR(t)]	-Tax rate*[UTR(t)-UTR(t-1)+APR(t)]	-Tax rate*[UTR(t)-UTR(t-1)+APR(t)]	-Tax rate*[UTR(t)-UTR(t-1)+APR(t)]	-Tax rate*[UTR(t)-UTR(t-1)+APR(t)]	-Tax rate*[UTR(t)-UTR(t-1)+APR(t)]
Tax on ERROI (TERROI)	-Tax rate*ERROI	-Tax rate*ERROI	-Tax rate*ERROI	-Tax rate*ERROI	-Tax rate*ERROI	-Tax rate*ERROI
Comprehensive net interest expense						
Adjusted interest revenue (IR)	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below	Sum of below
	SPECA3, SPECA4	SPECA3, SPECA4	SPECA3, SPECA4	SPECA3, SPECA4	SPECB3, SPECB4, SPECB8	SPECB3
Interest revenue	RFRET(t)*Accounts receivable(t-1)	RFRET(t)*Accounts receivable(t-1)	RFRET(t)*Accounts receivable(t-1)	RFRET(t)*Accounts receivable(t-1)	RFRET(t)*Accounts receivable(t-1)	RFRET(t)*Accounts receivable(t-1)
+Implicit interest revenue on AR						Sum of below
Adjusted interest expense (IE)						SPECB4
-Implicit interest expense on PL	-RFRET(t)*Pension liabilities (t-1)	-RFRET(t)*Pension liabilities (t-1)	-RFRET(t)*Pension liabilities (t-1)	-RFRET(t)*Pension liabilities (t-1)	-RFRET(t)*Pension liabilities (t-1)	-RFRET(t)*Accounts payable (t-1) +RFRET(t)*Pension liabilities (t-1)
-Implicit interest expense on APL	-RFRET(t)*Adv. paym. fr cust (t-1)	-RFRET(t)*Adv. paym. fr cust (t-1)	-RFRET(t)*Adv. paym. fr cust (t-1)	-RFRET(t)*Adv. paym. fr cust (t-1)	-RFRET(t)*Adv. paym. fr cust (t-1)	-RFRET(t)*Adv. paym. fr cust (t-1)
Other dividends (ODIV)	SPECA2	SPECA2	SPECA2	SPECA2	SPECA2	SPECB2
Tax on CFI (TCCFI)	-Tax rate*(IR+IE+ODIV)	-Tax rate*(IR+IE+ODIV)	-Tax rate*(IR+IE+ODIV)	-Tax rate*(IR+IE+ODIV)	-Tax rate*(IR+IE+ODIV)	-Tax rate*(IR+IE+ODIV)
Exchange rate difference (FXRD)	RES7-(SPECA5+SPECA6)	SPECA7	SPECA7	SPECA7	SPECA7	SPECB5, SPECB6
Summation errors in income (ERRIF)	-(RES5+RES6+RES7-RES8)	-(RES5+RES6+RES7-RES8)	-(RES5+RES6+RES7-RES8)	-(RES5+RES6+RES7-RES8)	-(RES5+RES6+RES7-RES8)	-(RES5+RES6+RES7-RES8+RES9)
Tax on peripheral CFI (TPCFI)	-Tax rate*(FXRD+SERR)	-Tax rate*(FXRD+SERR)	-Tax rate*(FXRD+SERR)	-Tax rate*(FXRD+SERR)	-Tax rate*(FXRD+SERR)	-Tax rate*(FXRD+SERR)
Supporting variable						
Value added (VA)	RES3+BAS4-BIDRAG1	RES3+BAS4-BIDRAG1	RES3+BAS4-BIDRAG1	RES3+BAS4-BIDRAG1	RES3+BAS4-SPECA1-SPECA2,SPECA3	RES3+BAS4-SPECA1-SPECA2,SPECA3

English description	Swedish description	77STATUS	77NAME	80STATUS	80NAME	85STATUS	85NAME	88STATUS	88NAME	90STATUS	90NAME	91STATUS	91NAME	94STATUS	94NAME	95STATUS	95NAME
Financial year	Redovisningsår	E	AR	E	AR	E	AR	E	AR	E	AR	E	AR	E	AR	E	AR
Identification number	Organisationsnummer	E	ORGNR	E	ORGNR	E	ORGNR	E	ORGNR	E	ORGNR	E	ORGNR	E	ORGNR	E	ORGNR
Company name	Företagsnamn	E	NAMN	E	NAMN	E	NAMN	E	NAMN	E	NAMN	E	NAMN	E	NAMN	E	NAMN
Standard industry classification code, 1969	Industriklassificering, 1969	E	SNI69	E	SNI69	E	SNI69	E	SNI69	D		N					
Standard industry classification code, 1992	Industriklassificering, 1992									N	SNI92	E	SNI92	E	SNI92	E	SNI92
Number of employees	Antal anställda	E		E	BAS2	E	BAS2	E	BAS2	E	BAS2	E	BAS2	E	BAS2	E	BAS2
Salaries	Löner	E	BAS3	E	BAS3	E	BAS3	E	BAS3	E	BAS3	E	BAS3	E	BAS3	E	BAS3
Labor expenses total	Arbetskraftskostnader, totalt	E	BAS4	E	BAS4	E	BAS4	E	BAS4	E	BAS4	E	BAS4	E	BAS4	E	BAS4
Value added	Förädlingsvärde	E	BAS5	E	BAS5	E	BAS5	E	BAS5	E	BAS5	E	BAS5	E	BAS5	E	BAS5
Sales revenue	Omsättning	E	RES1	E	RES1	E	RES1	E	RES1	E	RES1	E	RES1	E	RES1	E	RES1
Operating expenses	Rörelsens kostnader	E	RES2	E	RES2	E	RES2	E	RES2	E	RES2	E	RES2	E	RES2	E	RES2
Operating income before depreciation (See spec. A)	Resultat före avskrivningar (Se spec. A)	E	RES3	E	RES3	E	RES3	E	RES3	E	RES3	E	RES3	E	RES3	E	RES3
Depreciation according to plan	Avskrivningar	E	RES4	E	RES4	E	RES4	E	RES4	E	RES4	E	RES4	E	RES4	E	RES4
Operating income	*Resultat efter avskrivningar*	E	RES5	E	RES5	E	RES5	E	RES5	E	RES5	E	RES5	E	RES5	E	RES5
Items affecting comparability	Jämförelsestörande poster (Se spec. A)															N	RES6
Financial revenue (See spec. B)	Finansiella intäkter (Se spec. B)	E	RES6	E	RES6	E	RES6	E	RES6	E	RES6	E	RES6	E	RES6	E	RES7
Financial expenses (See spec. B)	Finansiella kostnader (Se spec. B)	E	RES7	E	RES7	E	RES7	E	RES7	E	RES7	E	RES7	E	RES7	E	RES8
Income after financial items	*Resultat efter finansnetto*	E	RES8	E	RES8	E	RES8	E	RES8	E	RES8	E	RES8	E	RES8	E	RES9
Non-recurring items	Extraordinärt netto	E	RES9	E	RES9	E	RES9	E	RES9	E	RES9	E	RES9	E	RES9	E	RES10
Income before changes in untaxed reserves and taxes	*Resultat före bokslutsdispositioner och skatt*	E	RES10	E	RES10	E	RES10	E	RES10	E	RES10	E	RES10	E	RES10	E	RES11
Changes in untaxed reserves (See Spec. C)	Bokslutsdispositioner (Se spec. C)	E	RES11	E	RES11	E	RES11	E	RES11	E	RES11	E	RES11	E	RES11	E	RES12
Reported tax (See spec. C)	Skatt (Se spec. C)	E	RES12	E	RES12	E	RES12	E	RES12	E	RES12	E	RES12	E	RES12	E	RES13
Net income	*Redovisat årsresultat*	E	RES13	E	RES13	E	RES13	E	RES13	E	RES13	E	RES13	E	RES13	E	RES14
Proposed dividend	Föreslagen utdelning	E	UTD1	E	UTD1	E	UTD1	E	UTD1	E	UTD1	E	UTD1	D	UTD1	D	
New common stock issue, including agio	Nyemission inkl. överkurs	E	UTD2	E	UTD2	E	UTD2	E	UTD2	E	UTD2	E	UTD2	D	UTD2	D	
Bonus issue	Fondemission mot uppskrivning av anläggningstillgångar	E	UTD3	E	UTD3	E	UTD3	E	UTD3	E	UTD3	E	UTD3	D	UTD3	D	
Government and municipal subsidies	Statliga och kommunala bidrag	E	BIDRAG1	E	BIDRAG1	E	BIDRAG1	E	BIDRAG1	E	BIDRAG1	D	BIDRAG				
Subsidies to operating expenses	Bidrag till rörelsekostnader			E	BIDRAG2	E	BIDRAG2	E	BIDRAG2	E	BIDRAG2						
Non-recurring subsidies	Extraordinära bidrag			N		N		N		N		D					
Capital gain/loss	Realisationsvinst/förlust	E	SPECA1	E	SPECA1	E	SPECA1	E	SPECA1	E	SPECA1	E	SPECA1	N		E	SPECA1
Write-down of shares, material FA, and restructuring costs	Nedskrivning av aktier o materiella AT samt omstrukt. kostn.	E	SPECA2	E	SPECA2	E	SPECA2	E	SPECA2	E	SPECA2	E	SPECA2	N		E	SPECA2
Write-down of fixed assets	Nedskrivning av AT													N		E	SPECA3
Exchange rate difference, net	Kursdifferenser, netto													N		E	SPECA4
Government and municipal subsidies	Statliga och kommunala bidrag													E	SPECB1	E	SPECB1
Dividends from group companies	Utdelning från koncernföretag	E	SPECA3	E	SPECA3	E	SPECA3	E	SPECA3	E	SPECA3	E	SPECA3	E	SPECB2	E	SPECB3
Dividends from other companies	Utdelning från övriga	E	SPECA4	E	SPECA4	E	SPECA4	E	SPECA4	E	SPECA4	E	SPECA4	E	SPECB3	E	SPECB3
Interest income	Ränteintäkter m.m.	E	SPECA5	E	SPECA5	E	SPECA5	E	SPECA5	E	SPECA5	E	SPECA5	E	SPECB4	E	SPECB3
Interest expense	Räntekostnader m.m.	E	SPECA6	E	SPECA6	E	SPECA6	E	SPECA6	E	SPECA6	E	SPECA6	E	SPECB5	E	SPECB3
Exchange rate gain	Valutakursvinster			N		N		N		N		E	SPECA7	N		E	SPECB5
Exchange rate loss	Valutakursförluster													N		E	SPECB6
Interest revenue from group companies	Räntor från koncernföretag	E	SPECB1	E	SPECB1	E	SPECB1	E	SPECB1	E	SPECB1	E	SPECB1	E	SPECB3	E	SPECB7
Other interest revenue	Övriga ränteintäkter													E	SPECB4	E	SPECB8
Interest expense paid to group companies	Räntor till koncernföretag													E	SPECB5	N	SPECC1
Other interest expense	Övriga räntekostnader													E	SPECB6	N	SPECC2
Exchange rate difference, net	Valutakursdifferenser, netto													E	SPECB7	N	SPECC3
Other financial items, net	Övriga finansiella poster, netto													N		N	
Change of tax allocation reserve	Förändring av periodiseringsfond													N		D	SPECC1
Change of tax equalization reserve	Förändring av skatteutjämningsreserv													N		D	SPECC2
Change of investment reserve	Förändring av investeringsfonder m. m.													N		D	SPECC3
Change of inventory reserve	Förändring av lagerreserv													N		D	
Dissolving inventory reserve and profit equalization reserve	Upplösning av lagerreserv och resultatutjämningsfond	E	SPECB2	E	SPECB2	E	SPECB2	E	SPECB2	E	SPECB2	E	SPECB1	D	SPECC1	D	SPECC4
Transfer to tax equalization reserve	Avsättning till skatteutjämningsreserv	E	SPECB3	E	SPECB3	E	SPECB3	E	SPECB3	E	SPECB3	E	SPECB2	D	SPECC2	D	SPECC5
Transfer to transitional reserve	Avsättning till transitionsreserv	N		N		N		N		N		N	SPECB3	D	SPECC3	D	
Change of profit equalization reserve	Förändring av resultatutjämningsfond	E	SPECB4	E	SPECB4	E	SPECB4	E	SPECB4	E	SPECB4	E	SPECB4	D	SPECB4	D	
Transfer to investment reserve	Avsättning till investeringsfonder	N		N		E	SPECB5	E	SPECB5	E	SPECB5	E	SPECB5	E	SPECB5	E	SPECC4
Accelerated depreciation, excluding investment reserve	Avskrivningar utöver plan, exkl. investeringsfonder	N		N		N		N		E	SPECB6	E	SPECB6	E	SPECB6	E	SPECC5
Reversal of accelerated depreciation for sold fixed assets	Återföring av överavskrivningar för sålda anläggningstillgångar	N		N		N		N		N		E	SPECB7	E	SPECB7	E	SPECC6
Contribution from group companies, net	Koncernbidrag, netto	E	SPECB4	E	SPECB5	E	SPECB6	E	SPECB6	E	SPECB6	N		N		E	SPECC7
Shareholders' contribution	Aktieägartillskott, netto													N		N	
Other changes in the untaxed reserve	Övriga bokslutsdispositioner	E	SPECB5	E	SPECB7	E	SPECB7	E	SPECB7	E	SPECB7	E	SPECB8	E	SPECC8	E	SPECC8
Profit sharing tax	Vinstdelningsskatt							E	SPECB8	E	SPECB8	D		D		D	
Other taxes	Övriga skatter					N		N		N		N		E	SPECC9	E	
Cash and bank	Kassa och bank	E	BAL1	E	BAL1	E	BAL1	E	BAL1	E	BAL1	E	BAL1	E	BAL1	E	BAL1
Shares and participation	Aktier och andelar	E	BAL2	E	BAL2	E	BAL2	E	BAL2	E	BAL2	E	BAL2	E	BAL2	E	BAL2
Bonds and other securities	Obligationer m. m.	E	BAL3	E	BAL3	E	BAL3	E	BAL3	E	BAL3	E	BAL3	E	BAL3	E	BAL3
Liquid assets held by group companies	Likvida medel hos koncernföretag											E	BAL4	E	BAL4	D	

APPENDIX I—THE INDUSTRY-SPECIFIC ACCOUNTING RATES-OF-RETURNS

I.1 Introduction

It is not apparent how an industry's ex post accounting rates-of-returns should be estimated. At least two problems are pertinent when considering the industry profitability. These include the measurement problem and the classification problem. Both issues are addressed in this appendix. First, the measuring problem is discussed in subsection I.2, and second, the classification of the industry is discussed.

I.2 Estimation of the industry year's ex post accounting rates-of-returns

A common method to estimate an industry year's accounting rates-of-returns is to assume that the distribution of the accounting rates-of-returns is Gaussian (Normal). See, e.g., McDonald & Morris (1984) with respect to how this can be done. The assumption that the distribution of the accounting rates-of-returns is Gaussian is often not explicit in published research since no discussion is made on this topic. The tacit assumption of normality shows itself in the choice of location measure since the arithmetic average is applied when estimating the distribution's location. Only the location (center) of a theoretical Gaussian distribution can be measured using the arithmetic average.

Empirical distributions of accounting ratios are not Gaussian for at least two possible reasons: (i) The ratio's limit value is infinite when the denominator approaches zero and (ii) errors in the raw data create outliers.

Suppose that the theoretical distribution of the accounting rates-of-returns is Gaussian, but the empirical distribution is affected by factors (i) and (ii) above. The arithmetic mean is in such a case no longer a valid location estimator since outliers contaminate the Gaussian distributed data and another robust location estimate such as the median is needed.

Given the uncertainty about the empirical distributional characteristics of the accounting rates-of-returns because outliers affect the data because of factors (i) and (ii), it is necessary to apply a robust location estimate. Since the accounting rates-of-returns must be estimated per industry-year, the industry-years have to rely on a relatively small number of accounting rates-of-returns. It is therefore necessary to estimate the industry-year's accounting rates-of-returns using a location estimate that is robust even in small distributions.

The current subsection uses Rosenberger & Gasko (1983) and Goodall (1983) to determine robust location estimates. Rosenberger & Gasko (1983) and Goodall (1983) show that the choice of

robust location estimate varies across empirical distributions and across sample sizes, making it necessary to asses the empirical distribution of the accounting rates-of-returns for each industry-year. Rosenberger and Gasko argue that the thickness of the distribution's tails can be used to classify the empirical distribution.

To asses the tail thickness Rosenberger & Gasko (1983, p. 322) use a tail-weight index. The tail-weight index measures the empirical distribution's ratio between the 99th and 75th percentile from the median. This ratio is standardized against an equivalent ratio of a Gaussian distribution's, which sums to 3.457 to create the tail weight index:

$$\tau(F) = \left(\frac{F^{-1}(0.99) - F^{-1}(0.5)}{F^{-1}(0.75) - F^{-1}(0.5)} \right) \cdot 3.457^{-1} \qquad \text{[EQ I-1]}$$

where F' is the cumulative empirical distribution function, and $F^{-1}(0.99)$ is the 99 percentile's accounting rate-of-return for a industry-year. $F^{-1}(0.5)$ is consequently the distribution's median.

A Gaussian distribution has tail weight index 1.0. Lighter tailed distributions have indices less than 1.0 and heavier tailed distributions such as e.g., Cauchy, has indices above 1.0. Table I-7 is adopted from Rosenberger & Gasko (1983, p. 322) and describes tail weights for different types of empirical distribution.

Tail-weight index					
Distribution	$\tau(F)$	Distribution	$\tau(F)$	Distribution	$\tau(F)$
Uniform	0.57	CN(.05;3)	1.20	CN(.05;10)	3.42
Triangular	0.86	Logistic	1.21	Slash	7.85
Gaussian	1.00	D-exponential	1.63	Cauchy	9.22

Table I-7: Tail-weight indices for different distributions.

In Table I-7 CN(.05;3) is a contaminated Gaussian distribution where 95 percent of the observations come from a Gaussian distribution having one standard deviation while five percent come from a Gaussian distribution having three standard deviations. D-exponential is a double exponential distribution.

Rosenberger & Gasko test eighth location measures of efficiency on different sample sizes ($n = 5$, 10, and 20) for the distributions in Table I-7. Goodall (1983) also tests several M-estimators of location. From Rosenberger & Gasko (1983) and Goodall (1983), a set of robust location measures is chosen for the estimation of the robust industry-year accounting rates-of-returns. The most efficient estimate of location is chosen per sample size and distribution type.

The location estimates used in the present study to estimate the industry-year's accounting rates-of-returns are the arithmetic mean (Mean), the median, trimmed means (TM α), midmean (MIDM), broadened mean (BMEAN), Huber's 1-step M-estimate with $c = 2\pi$, and biweight 1-step

M-estimate with $c = 8.8$. Table I-8 provides an overview of which location estimate is employed in this study per industry size.

Robust location estimator per industry size						
		Industry size				
Empirical distribution		Unique	Micro	Small	SemiSmall	Large
Name	Tail-weight	n=1	1<n<4	4≤n<8	8≤n<15	n>14
Uniform	τ(F)<0.71	i=I	Median	Median	Median	Median
Triangular	0.71≤τ(F)<0.93	i=I	Median	Median	Median	Median
Gaussian	0.93≤τ(F)<1.04	i=I	Mean	TM10	Huber(2)	Huber(2)
	1.04≤τ(F)<1.1	i=I	Median	MIDM	TM10	Biweight(8.8)
One-out	1.1≤τ(F)<1.3	i=I	Median	MIDM	TM10	Biweight(8.8)
	1.3≤τ(F)<1.5	i=I	Median	MIDM	TM10	Biweight(8.8)
D-exp	1.5≤τ(F)<1.8	i=I	Median	MIDM	BMEAN	BMEAN
One-wild	1.8≤τ(F)<5.6	i=I	Median	Median	BMEAN	BMEAN
Slash	5.6≤τ(F)<8	i=I	Median	Median	Median	BMEAN
Cauchy	τ(F)≥8	i=I	Median	Median	Median	BMEAN

Table I-8: Robust estimates for the industry-year's accounting rates-of-returns per industry size.

The estimators in Table I-8 are per tail-weight and per number of observations per industry-year.

The trimmed mean is calculated following Rosenberger and Gasko (1983, p. 308):

$$TM\,(\alpha) = \frac{1}{n \cdot (1 - 2\alpha)} \cdot \sum_{i=g+1}^{n-g} x_i \qquad \text{[EQ I-2]}$$

Where $g = \alpha \cdot n$, α is 10 percent, n is the number of firms per industry-year, and x_i is the accounting rates-of-returns for a specific firm-year.

The midmean is equivalent to an α-trimmed mean where α is 25 percent (Rosenberger & Gasko 1983, p. 311).

The broadened mean is also equivalent to α-trimmed mean but having a variable α. α depends on the number of firms per industry-year and is calculated as (Rosenberger & Gasko 1983, p. 313):

$$\left(.5 - \frac{1.5}{n}\right) \qquad\qquad \forall\ 5 \leq n \leq 12 \qquad \text{[EQ I-3]}$$

$$\left(.5 - \frac{2.5}{n}\right) \qquad\qquad \forall\ n \geq 13$$

Note that the limit value of α is 50 percent. The trimmed mean is equivalent to the median when α is equal to 50 percent. $\alpha = 49$ percent when $n = 250$, but α is already 45 percent at $n = 40$. This means that the trimmed mean asymptotically approaches the median and it is approximately equal to the median already at relatively few observations.

Huber's 1-step M-estimate and the biweight 1-step M-estimate are used for some empirical distributions. See Goodall (1983) for a discussion on how to calculate them.

An alternative method to the method used in this subsection for assessing the accounting rates-of-returns for industry-years is also used when the hypotheses in section 6.4 are assessed. See subsection L.4.2 for more information about the alternative estimation method.

I.3   Classifying the industry

There are considerable problems in defining an industry, but a common practice is to use the standard industry classification system (SIC). The SIC system is a hierarchal classification system in which the first-digit level signifies the economy at large in a country and the fifth-digit level is the most narrow industry definition (Statistics Sweden 2006). A firm is assigned to an industry based on the activity it carries out; a firm can be active in several industries at the same time (Statistics Sweden 2006).

This research has access to each firm-year's main SIC code at the fourth-digit level. Because of a shift in the classification system in 1990 (from classification system SNI69 to SNI92), it is not possible to use a four-digit level industry definition. The most narrow and consistent definition of the industry across the two classification systems is at the three-digit level.

Classifying the industry at a three-digit level serves several purposes: (i) it is practical despite the change in the classification system and (ii) it provides enough firm observations to separate the firm from the industry. Sweden is a rather small market and classifying the industry at a four-digit or at a five-digit level likely renders several industries having only one firm. This also occurs at the three-digit level, but this is a minor problem at this level. (iii) It is narrow enough to provide for a stable industry belonging. Gupta & Huefner (1972) argue that a four-digit level is too narrow since this is such a narrow industry definition that many firms shift industry on a frequent basis.

At least two drawbacks can be found when having an industry definition at a three-digit level and not at a more narrow level. (i) The products produced in the industry are not close substitutes, i.e. the industry is not homogenous. E.g., the industry name for SIC-code 291 is manufacture of machinery for the production and use of mechanical power, except aircraft, vehicle, and cycle engines, suggesting a relatively heterogeneous industry. Relatively homogenous industry classifications are found at a four-digit level. E.g., the SIC code 2912 is the manufacture of pumps and compressors. However, one can argue that the industry classification at a three-digit level can also capture the competitive effect of a broader class of substitute products. (ii) With a relatively heterogeneous classification of the industry, the production functions may differ and this may transfer to a relatively heterogeneous implementation of the accounting standards. E.g., the depreciation policies may vary relatively more within an industry classified at a three-digit level than at a four-digit level.

Despite these problems, it is common in research to have the industry classification even at the two-digit level (cf. Geroski & Jacquemin 1988; Gupta & Huefner 1972).

APPENDIX J— HYPOTHESES TESTS USING ALTERNATIVE OPERATIONALIZATIONS OF INCOME

J.1 Introduction

Return on equity is defined in this thesis as $_{t-1}ROE_t = CNI_t \cdot EQ_{t-1}^{-1}$, and as

$_{t-1}RNOA_t = COI_t \cdot NOA_{t-1}^{-1}$. This chapter explores if some misspecification of comprehensive net income (CNI) and comprehensive operating income (COI) may drive the results. The effects of three alternative specifications of CNI and the effects of three alternative specifications of COI are investigated in this appendix.

This means that this appendix changes the operationalization of

$_{t-1}RROE_t^* = {}_{t-1}ROE_t - {}_{t-1}ROE_t^I$ and $_{t-1}RRNOA_t^* = {}_{t-1}RNOA_t - {}_{t-1}RNOA_t^I$ since the meaning of ROE and RNOA changes.

The changes are then evaluated on new panels using the identical method to that applied in section 6.4 (p. 108) with associated appendices.

The alternative specifications of CNI are:

1. Basic CNI less dividends from the group.

2. As #2 less government subsidies.

3. As #3 less dirty surplus accounting (see [EQ 5-10])

The alternative specifications of COI are:

4. Basic COI less dividends from the group.

5. As #2 less government subsidies.

6. As #3 less dirty surplus accounting (see [EQ 5-10])

J.2 The result from panel regression on alternative specification of CNI

The coefficients of determination for the panels having the basic definition of CNI are found in Table 6-7; they are, on average, 39 percent. This increases to 49 percent when alternative CNI #1 is tested and 51 percent when alternative CNI #2 is tested. However, by also removing dirty surplus accounting from the definition, i.e. using alternative CNI #3, it increases to 52 percent. This shows that dividends from the group have important effects in the variability of the panels despite that outliers have been removed. Government subsidies and dirty surplus accounting do not contribute much to the variability.

Variable	Field	Fit origin												
		1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
RROE_ALT#1	ESTIMATES	-0.028	-0.058	-0.063	-0.083	-0.013	-0.001	-0.013	-0.032	-0.015	-0.018	0.062	0.018	-0.023
	STDE	0.249	0.250	0.240	0.241	0.226	0.216	0.180	0.226	0.247	0.262	0.290	0.250	0.257
	TVALUE_1	-4.12	-4.23	-4.44	-4.49	-4.49	-4.63	-5.64	-4.57	-4.10	-3.88	-3.23	-3.93	-3.98
	PVALUE_1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.06%	0.00%	0.00%
	TVALUE_0	-0.11	-0.23	-0.26	-0.34	-0.06	0.00	-0.07	-0.14	-0.06	-0.07	0.21	0.07	-0.09
	PVALUE_0	91.1%	81.7%	79.2%	73.2%	95.3%	99.6%	94.2%	88.8%	95.2%	94.6%	83.2%	94.4%	93.0%
RROE_ALT#2	ESTIMATES	-0.008	-0.040	-0.048	-0.073	-0.013	0.053	-0.008	0.006	0.038	0.050	0.089	0.084	0.032
	STDE	0.249	0.251	0.236	0.239	0.220	0.221	0.181	0.227	0.233	0.252	0.282	0.252	0.256
	TVALUE_1	-4.05	-4.14	-4.45	-4.50	-4.61	-4.29	-5.57	-4.38	-4.13	-3.77	-3.23	-3.63	-3.79
	PVALUE_1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.06%	0.01%	0.01%
	TVALUE_0	-0.03	-0.16	-0.21	-0.31	-0.06	0.24	-0.04	0.03	0.16	0.20	0.31	0.34	0.13
	PVALUE_0	97.6%	87.3%	83.7%	75.8%	95.3%	81.2%	96.7%	97.8%	87.0%	84.4%	75.3%	73.8%	90.0%
RROE_ALT#3	ESTIMATES	-0.020	-0.040	-0.060	-0.081	0.002	0.067	-0.004	0.005	0.053	0.047	0.096	0.075	0.036
	STDE	0.246	0.251	0.235	0.239	0.218	0.221	0.181	0.230	0.234	0.248	0.281	0.256	0.258
	TVALUE_1	-4.15	-4.15	-4.52	-4.53	-4.57	-4.22	-5.54	-4.33	-4.05	-3.84	-3.22	-3.62	-3.73
	PVALUE_1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.07%	0.02%	0.01%
	TVALUE_0	-0.08	-0.16	-0.26	-0.34	0.01	0.30	-0.02	0.02	0.23	0.19	0.34	0.29	0.14
	PVALUE_0	93.4%	87.4%	79.7%	73.5%	99.1%	76.2%	98.3%	98.3%	82.2%	85.0%	73.2%	76.9%	88.8%

Table J-9: Parameter estimates from the fixed-effects panel regression using the risk-adjusted RROE with PCSE and AR(1) errors for alternative operationalizations of CNI.

Table J-9 reports the parameter estimates with t-values and p-values for the alternatively operationalized variables. The table shows that for all alternative specifications and for all panels, the conclusions from 6.4.2.1 (p. 110) remain unchanged. That is, the parameter estimates are significantly less than one (which therefore rules out random walk) and not significantly different from zero.

J.3 The results from panel regression on alternative specification of COI

The coefficients of determination for the panels having the basic definition of COI are found in Table 6-9; it is, on average, 43 percent. This does not change materially as alternative COI #and alternative CNI #2 are tested. However, as in the previous section, by removing group dividends (alternative COI #1), it increases to 53 percent. Again, this is a sign that the group dividends have important effects in the variability of the panels.

Variable	Field	Fit origin												
		1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
RRNOA_ALT#1	ESTIMATES	-0.021	-0.027	-0.024	-0.085	-0.010	-0.014	-0.008	0.004	-0.006	0.044	0.076	0.032	0.026
	STDE	0.237	0.257	0.255	0.250	0.246	0.229	0.204	0.249	0.239	0.253	0.282	0.241	0.259
	TVALUE_1	-4.30	-3.99	-4.01	-4.35	-4.11	-4.43	-4.94	-4.01	-4.21	-3.77	-3.28	-4.03	-3.76
	PVALUE_1	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.01%**	**0.05%**	**0.00%**	**0.01%**
	TVALUE_0	-0.09	-0.10	-0.09	-0.34	-0.04	-0.06	-0.04	0.01	-0.02	0.17	0.27	0.13	0.10
	PVALUE_0	92.9%	91.7%	92.6%	73.4%	96.8%	95.1%	96.9%	98.8%	98.1%	86.3%	78.7%	89.6%	91.9%
RRNOA_ALT#2	ESTIMATES	-0.071	-0.127	-0.122	-0.215	-0.145	-0.057	-0.086	-0.157	-0.183	-0.161	-0.039	0.005	-0.054
	STDE	0.251	0.222	0.258	0.245	0.252	0.248	0.160	0.239	0.230	0.246	0.309	0.252	0.254
	TVALUE_1	-4.27	-5.08	-4.34	-4.96	-4.55	-4.26	-6.78	-4.85	-5.15	-4.73	-3.36	-3.95	-4.15
	PVALUE_1	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.04%**	**0.00%**	**0.00%**
	TVALUE_0	-0.28	-0.57	-0.47	-0.88	-0.58	-0.23	-0.54	-0.66	-0.80	-0.66	-0.13	0.02	-0.21
	PVALUE_0	77.8%	56.8%	63.8%	38.0%	56.5%	81.9%	59.1%	51.1%	42.6%	51.2%	89.8%	98.3%	83.3%
RRNOA_ALT#3	ESTIMATES	0.018	0.008	0.032	-0.077	0.009	0.022	0.002	0.041	0.045	0.094	0.126	0.077	0.091
	STDE	0.224	0.256	0.258	0.245	0.250	0.232	0.187	0.248	0.239	0.245	0.277	0.241	0.264
	TVALUE_1	-4.39	-3.88	-3.75	-4.39	-3.96	-4.22	-5.33	-3.87	-4.00	-3.69	-3.15	-3.82	-3.44
	PVALUE_1	**0.00%**	**0.01%**	**0.01%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.01%**	**0.00%**	**0.01%**	**0.08%**	**0.01%**	**0.03%**
	TVALUE_0	0.08	0.03	0.12	-0.31	0.04	0.10	0.01	0.17	0.19	0.38	0.46	0.32	0.34
	PVALUE_0	93.6%	97.4%	90.2%	75.3%	97.0%	92.4%	98.9%	86.9%	84.9%	70.1%	64.8%	74.9%	73.2%

Table J-10: Parameter estimates from the fixed-effects panel regression using the risk-adjusted RRNOA with PCSE and AR(1) errors for alternative operationalizations of COI.

Table J-10 shows no material changes when compared with the original operationalization of COI. The table shows that for all alternative specifications and for all panels, the conclusions from 6.4.2.2 (p. 112) remain unchanged. The parameter estimates are significantly less than one and not significantly different from zero.

APPENDIX K—THE ROBUST T-TEST ASSESSING IF NON-ZERO RISK-ADJUSTED RESIDUAL ACCOUNTING RATES-OF-RETURNS EXISTS

The null hypotheses assume that the risk-adjusted residual return on equity (risk-adjusted RROE), [EQ 6-8] and the risk-adjusted residual return on net operating assets (risk-adjusted RRNOA), [EQ 6-4], are zero for all firms. The alternative hypotheses posed by Homo comperiens maintain that that the risk-adjusted RROE and the risk-adjusted RRNOA are non-zero for at least one firm. This is tested with a robust double-sided t test.

A t test means that confidence intervals are implicitly estimated. Subsection I.2 discusses how to measure the industry-year's accounting rates-of-returns and finds that a robust location estimate is needed. When estimating a confidence interval, it is not enough with a robust location estimate: the confidence interval must be robust.

Iglewicz (1983) discusses the robustness of confidence intervals in three empirical distributions (Gaussian, One-wild[30], and Slash). Iglewicz finds that a location estimate based on the wave estimator slightly dominates a location estimate based on the biweight estimator (93 percent triefficiency compared with 91 triefficiency). These two robust methods completely dominate the usual way to create confidence intervals using mean and standard deviation, which have only one percent triefficiency.

However, finding t-values for the biweight estimation method is much easier than for the wave estimation method. Mosteller & Tukey (1977) suggest that using the t-values from the t-distribution with $0.7 \cdot (n\text{-}1)$ degrees of freedom provide a conservative approximation of the cut-off point when both the location and the scale are estimated using the biweight estimation technique. Iglewicz (1983) validates Mosteller and Tukey's approximation. Finding the t-values for the use of the wave estimation method is not as easy.

This thesis measures robust confidence intervals using two methods. The first method uses the best possible robust location estimate based on the method discussed in subsection I.2 in conjunction with a biweight scale estimate. This method is subsequently referred to as the Best possible robust location estimate (BLE). The t-values used for conservative approximation of the cut-off point for BLE are based on the t-values provided by Gross (1976). The second method uses a biweight estimation of location and a biweight estimation of scale. The cut-off point for the second

---

[30] One-wild in this thesis represents a distribution where 19 observations are from a Gaussian distribution that has one standard deviation and one observation from a Gaussian distribution that has 100 standard deviations.

method is based on Mosteller and Tukey's suggestion, and is subsequently referred to as the Bi-weight method (BW).

The biweight estimate of the robust scale estimator (in the research estimated with $c = 9$) is defined as (Iglewicz 1983, p. 416):

$$s^{bw} = \frac{n^{\frac{1}{2}} \cdot \left[\sum_{|u_i|<1}(x_i - Median)^2 \cdot \left(1 - u_i^2\right)^4\right]^{\frac{1}{2}}}{\left[\sum_{|u_i|<1}\left(1 - u_i^2\right) \cdot \left(1 - 5 \cdot u_i^2\right)\right]}$$ [EQ K-1]

where $u_i = (x_i - Median) \cdot (c \cdot MAD)^{-1}$, and x_i is the accounting rate-of-return for a firm-year in an industry.

The t-test statistic is calculated for the years according to the following equation (follows Iglewicz 1983, p. 416):

$$t = (x_{it} - x_{It}) \cdot \left[s_{It} \cdot \left(\sqrt{n_{It}}\right)^{-1}\right]^{-1}$$ [EQ K-2]

where x_{it} is the firm-year accounting rate-of-return, i.e. $_{t-1}ROE_t$ and $_{t-1}RNOA_t$. x_{It} is the industry-year's accounting rate-of-return, i.e. $_{t-1}ROE_t^I$ and $_{t-1}RNOA_t^I$ estimated using method BLE or BW above. s_{It} is the scale estimate per industry-year estimated using either method BLE or BW, and n_{It} is the number of firms in the industry-year.

The decision rule is thus:

$$\text{Reject H}_0 \text{ if } t > \left|t_{0.7 \cdot (n-1), 0.5 \cdot \alpha}\right|$$

where $t_{0.7 \cdot (n-1), 0.5 \cdot \alpha}$ is the cut-off point for a t-distribution having $0.7 \cdot (n-1)$ degrees of freedom and with significance level α. The hypothesis test uses significance levels of 1, 5, and 10 percent.

APPENDIX L—THE PANEL REGRESSION TEST METHOD

L.1 Introduction

This appendix discusses the panel regression method applied for testing the learning hypothesis of the thesis. The appendix is organized by first discussing the three alternative panel regression models that are considered. Next, the appendix discusses alternative covariance structures for the fixed-effect model. The appendix also addresses the question of how the panels are formed. This includes choices of panel length as well as treatment of outliers in the data.

L.2 Three panel regression models

This subsection focuses on panel models for assessing whether learning takes place by focusing on the β factor in [EQ 6-10] and [EQ 6-11].

The null hypotheses are $\beta \geqslant 1$ and the alternative hypotheses are $\beta < 1$. The hypotheses lend themselves to estimation and statistical tests using panel regression analysis[31]. The t-test statistics for assessing the hypotheses is measured for the regressions as:

$$t = \left(\hat{\beta} - 1 \right) \cdot stde^{-1} \qquad \text{[EQ L-1]}$$

The standard error, *stde*, is the estimated standard deviation for the estimated regression parameter (Newbold 1995).

The decision rule is:

Reject H$_0$ if $t < -t_{dfe,\alpha}$

where $t_{dfe,\alpha}$ is the cut-off point for a t-distribution having *dfe* degrees of freedom and with a significance level α and where $\hat{\beta}$ is the panel regression's estimated regression parameter. The hypothesis test employs a five percent significance level.

---

[31] The thesis uses SAS 9.1.3 for its statistical analysis. SAS 9.1.3 is weak on panel regression techniques and on specification tests associated to the panel regressions. SAS 9.1.3 has PROC TCSREG and PROC MIXED that can be used for panel regression but neither is sufficiently versatile to allow for more than cursory panel regression analysis. To compensate for this the panel regression models and all associated specification tests have been hand-written in PROC IML. Since the programs are hand-written, Appendix L and especially L.3 elaborate more than what would have been necessary if standardized programs had been used.

The written programs are tested in most cases on Grunfeld's investment data, where the outputs have been compared with the outputs that Green (2003) provides. Accordingly, this determines the programs' reliability. The drawback is that this procedure requires the programs to deal with balanced panels.

A balanced panel is a panel in which all cross-sections have a complete set of observations: There are no holes in the panel. Since the programs are written to handle balanced data, the panels that are used for the panel regressions in this thesis require that all firms included in a panel must have observations for all the years used in that particular panel. This greatly reduces the number of cross-sections available for analysis and therefore introduces survival bias. This also affects the number of years used to set up a panel, as well as how outliers are treated. See section L.4 on page 213 for further details on this topic.

There are several panel regression models available for assessing the regression parameters. To avoid ex ante affecting the results by choosing an estimation model (e.g., Cubbin & Geroski 1987; Dechow et al. 1999; Fairfield et al. 1996; Finger 1994; Jacobsen 1988; Jacobson & Aaker 1985; Mueller 1977; Mueller 1990; Ou & Penman 1989b; Penman & Zhang 2002; Waring 1996), I consider three alternative estimation models. These models are the pooled regression model, the random effect model, and the fixed-effect model. The estimation models are tested against each other using a specification test, which allows the most appropriate panel regression model to be used to estimate the regression parameters.

The pooled regression model assumes no unmodeled heterogeneity between the firms (the cross sections). That is, all differences between units are accounted for by the difference in the regressors. It also implies that all firms obey the same equation, i.e. have identical behavior. Despite its widespread use in accounting research, the model is highly restrictive and is thus not considered a priori to be a likely estimation model for the data in this thesis.

The random effects model assumes that any unmodeled heterogeneity is uncorrelated with the regressors and that it is a random panel element. That is, the unmodeled heterogeneity randomly varies across the firms. According to Green (2003, p. 293), the random-effect model is appropriate if, e.g., the sample of firms is drawn from a much larger population. However, the sample of firms covers the bulk of the complete Swedish manufacturing industry and hence the random-effect model is not expected a priori to fit the data very well.

The third method assumes that the unmodeled heterogeneity is correlated with the regressors (Greene 2003). If Homo comperiens is a valid description, unmodeled heterogeneity should be present in the data; given the alternative hypothesis, it is expected that the heterogeneity will be correlated with the regressors. The fixed model is therefore likely the panel regression model that best meets the theoretical propositions and hence is most appropriate in this thesis.

If the random- or fixed-effects method is applied, no substantive conclusions can be drawn from the intercepts in [EQ 6-10] and [EQ 6-11]. This is because they contain firm-specific errors. The random- and fixed-effects models have the benefit that they provide an unbiased estimation of β (see, e.g., Green (2003, p. 283-374) for a discussion on panel regression methods).

The appropriateness of the pooled regression model, the random-effects model, and the fixed-effects model depends on the presence and the type of unmodeled heterogeneity. Three specification tests are used in the thesis to choose the most appropriate regression model. Those tests and their results are discussed in Appendix M (p. 233).

L.3 Alternative covariance structures for the fixed effect method

Alternative covariance structures are needed when the basic assumptions on the disturbance process are violated. Using different covariance structures affect how regression parameters, standard errors,

and fit statistics are calculated. This means that the assumptions of covariance structure can have a significant effect on the hypotheses tests carried out.

Subsection L.2 assumes that the fixed-effects model a priori best fits the theory of Homo comperiens. Therefore, to reduce complexity only alternative covariance structures are considered for the fixed-effects model.

The thesis uses a lagged dependent variable as explanatory variable (as in [EQ 6-10] and [EQ 6-11]). According to Beck & Katz (2004), this implies that the disturbances are, by design, serially correlated. First-order serially correlated disturbances are considered in the alternative covariance structures.

Because Homo comperiens suggests that firms are heterogeneous, there is no particular reason why disturbances should be homoscedastic. A more likely situation is the presence of panel heteroscedasticity, i.e. each firm's disturbances have an own, constant, variance. I consider covariance structures affected by panel heteroscedasticity.

The serially correlated disturbance structure exists by design and thus any applied covariance structure must deal with serial correlated disturbances. Panel heteroscedasticity, however, is not part of the structure by design so the test in subsection M.6 (p. 238) is needed to ascertain whether it is present in the panels.

The data set consists of almost the whole population of incorporated firms within the Swedish manufacturing industry. This means that I expect that the behavior of one firm has an effect on other firms in the panels, and the panels are therefore likely to be affected by contemporaneous correlated disturbances.

According to Beck & Katz (1995), OLS regression with spherical disturbances is an inefficient method. They propose that the standard errors from the OLS estimation procedure should be corrected for both panel heteroscedasticity and contemporaneous correlated disturbances. Their method for correcting the standard errors is known as the panel corrected standard errors (PCSE). The covariance structure proposed by Beck and Katz is applied on the fixed effect model whenever the panel heteroscedasticity test indicates that the assumptions of spherical disturbances are violated.

The point of reference when considering covariance structures is the structure having spherical disturbances. Spherical disturbances are covered in this section to provide an overview of the applied method. First-order serial correlation is added to the spherical disturbances in subsection L.3.4. Subsection L.3.3 covers the PCSE covariance structure and it is adjusted to allow for first-order serially correlated disturbances in subsection L.3.5.

Only two of four covariance structures are implemented because serially correlated disturbances exist by construction and the implemented covariance structures are the Spherical AR(1) covariance structure and the PCSE AR(1) covariance structure. Because the assumptions of spheri-

cal disturbances presume homoscedasticity and because homoscedasticity is not expected to be present in the data set, a priori, the PCSE AR(1) covariance structure is expected to dominate the others.

This section begins with a brief subsection that contains this section's notational structure. This is followed by a subsection with the spherical disturbances. After the spherical disturbances, the section having panel corrected standard errors (PCSE) is presented. These two covariance structures come first since they are base cases.

After the base cases, follows the spherical disturbances that are affected by the serial correlation but that is transformed using the Prais-Winsten method. The Prais-Winsten method is also applied to the PCSE covariance structure.

L.3.1 Notational structure

Before the different covariance structures are presented, a common notational system is presented. Let $\mathbf{D} = \mathbf{i}_n \otimes \mathbf{I}_T$ be the selector matrix, i.e. a vector of dummies for the firms (Baltagi 2003). Let $\mathbf{y}$ be the $n \cdot T \times 1$ matrix with the independent observations for the firms. Let $\mathbf{X}$ be the $n \cdot T \times K$ matrix of regressors. Finally, let $\mathbf{Z} = [\mathbf{D}, \mathbf{X}]$ be the full regressor matrix.

By using these matrices, the fixed-effect model of the present thesis is:

$$\mathbf{y} = \mathbf{Z} \cdot \boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad \text{[EQ L-2]}$$

This fixed-effect model is similar to Green's (2003, p. 287) equation 13-2, and since $\mathbf{Z}$ consists of both the dummies and the regressors, it follows that $\boldsymbol{\beta}$ includes the firm-specific constants. Another assumption maintained throughout the analysis is that the disturbances are expected to be zero:

$$\mathrm{E}[\boldsymbol{\varepsilon}] = 0 \qquad \text{[EQ L-3]}$$

The general description of the covariance structure is the $n \times n$ matrix:

$$\mathrm{E}\left[\boldsymbol{\varepsilon}^\top \boldsymbol{\varepsilon}\right] = \mathbf{V} = \sigma^2 \cdot \Omega = \begin{pmatrix} \sigma_{11}\Omega_{11} & \cdots & \sigma_{1n}\Omega_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1}\Omega_{n1} & \cdots & \sigma_{nn}\Omega_{nn} \end{pmatrix} \qquad \text{[EQ L-4]}$$

The covariance matrix is focused by attributing it with different properties as the analysis proceeds.

L.3.2 Spherical disturbances

Typical regression analysis assumes as a point of reference the covariance structure with spherical disturbances. This is also done in this thesis. A spherical disturbance structure exists when the disturbances are white noise disturbances (Greene 2003):

$$\mathrm{E}[\boldsymbol{\varepsilon}] = 0$$

$$\mathrm{E}\left[\varepsilon^\top \varepsilon\right] = \mathbf{V} = \sigma^2 \mathbf{I}_n = \begin{pmatrix} \sigma^2 \cdot 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma^2 \cdot 1 \end{pmatrix} \qquad \text{[EQ L-5]}$$

where $\mathbf{I}_n$ is the $n \times n$ identity matrix. The variance in this thesis is assumed constant across firms.

It follows that when the disturbances are spherical; the fixed-effect model estimated with OLS is the best linear unbiased estimator (BLUE). In this thesis, the regression parameters are estimated as below when the disturbances are spherical:

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{Z}^\top \cdot \mathbf{Z}\right)^{-1} \cdot \mathbf{Z}^\top \cdot \mathbf{y} \qquad \text{[EQ L-6]}$$

The hat-sign identifies estimated matrices.

In the thesis, the fixed-effects panel-regressions model's fit statistics is the sum of squared disturbances, the total sum of squares, the coefficient of determination, the mean squared error, the root mean squared error, and the degrees of freedom (Green 2003):

$$SSE = \varepsilon^\top \varepsilon \text{ [EQ L-7]}$$

$$SST = \mathbf{y}^\top \cdot \mathbf{M}^0 \cdot \mathbf{y} \qquad \text{[EQ L-8]}$$

$$SSR = SST - SSE \qquad \text{[EQ L-9]}$$

$$R^2 = 1 - SSE \cdot SST^{-1} \qquad \text{[EQ L-10]}$$

$$MSE = SSE \cdot dfe^{-1} \qquad \text{[EQ L-11]}$$

$$RMSE = \sqrt{MSE} \qquad \text{[EQ L-12]}$$

$$dfe = n \cdot T - n - K \qquad \text{[EQ L-13]}$$

where $\mathbf{M}^0 = \mathbf{I}_{n \cdot T} - (n \cdot T)^{-1} \cdot \mathbf{i}_{n \cdot T} \cdot \mathbf{i}_{n \cdot T}^\top$, $\mathbf{i}_{n \cdot T}$ is $n \cdot T \times 1$ matrix of 1's, and $\mathbf{I}_{n \cdot T}$ is an $n \cdot T \times T \cdot n$ identity matrix.

The asymptotic covariance matrix for spherical disturbances in this thesis is calculated as (note that MSE is a scalar):

$$\mathrm{var}\left(\hat{\beta}\right) = \left(\mathbf{Z}^\top \mathbf{Z}\right)^{-1} \cdot MSE \text{ where } \omega_{i,j} \in \mathrm{var}\left(\hat{\beta}\right) \qquad \text{[EQ L-14]}$$

The thesis collects the standard errors from the asymptotic covariance matrix's vector diagonal, where the standard errors are the square root of the vector diagonal. Thus, the standard error for the first regression parameter (the constant for the first firm) is:

$$stde_1 = \sqrt{\omega_{1,1}}$$

Next, is the covariance structure when panel heteroscedasticity and contemporaneous correlated disturbances are present.

225

L.3.3 Panel-corrected standard errors (PCSE)

When panel heteroscedasticity and contemporaneous correlated disturbances are present, the spherical $\mathbf{V}$ matrix, [EQ L-5], is incorrect. This cascades into incorrect standard errors. Since $[\mathbf{Z}, \mathbf{y}]$ is not affected by these problems, $\hat{\beta}$ remains unaffected and hence it can still be estimated using [EQ L-6].

The standard errors must be corrected for heteroscedasticity and contemporaneous correlation. This is done in the present thesis with the PCSE estimated $\hat{\mathbf{V}}$ matrix (Beck & Katz 1995):

$$\hat{\mathbf{V}} = \begin{pmatrix} \hat{\Sigma} & 0 & 0 & \cdots & 0 \\ 0 & \hat{\Sigma} & \mathbf{0} & \cdots & \vdots \\ 0 & 0 & \hat{\Sigma} & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \hat{\Sigma} \end{pmatrix} = \hat{\Sigma} \otimes \mathrm{I}_n \qquad \text{[EQ L-15]}$$

where the estimated contemporaneous covariance matrix is:

$$\hat{\Sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \cdots & \sigma_{nn} \end{pmatrix} = \frac{\varepsilon^{\top} \cdot \varepsilon}{T} \qquad \text{[EQ L-16]}$$

The elements in the contemporaneous covariance matrix are:

$$\sigma_{ij} = T^{-1} \cdot \sum_{t=1}^{T} \varepsilon_{it} \cdot \varepsilon_{jt} \qquad \text{[EQ L-17]}$$

Note that if no contemporaneous correlation, $\rho_{ij} = 0$, is assumed, the estimated covariance matrix could be adjusted by calculating the diagonal elements as $\sigma_{ii} = T^{-1} \cdot \sum_{t=1}^{T} \varepsilon_{it} \cdot \varepsilon_{it}$, and setting the off-diagonal elements to zero, since $\sigma_{ij} = \sigma_{ii} \cdot \sigma_{jj} \cdot \rho_{ij} \Rightarrow \sigma_{ij} = 0$ when $\rho_{ij} = 0$. However, contemporaneous correlation is likely to be present in the data set and thus the off-diagonal elements are allowed to be non-zero.

The thesis applies the PCSE estimated $\hat{\mathbf{V}}$ matrix to adjust the estimated covariance matrix (Beck & Katz 1995):

$$\mathrm{var}\left(\hat{\beta}\right) = \left(\mathbf{Z}^{\top} \cdot \mathbf{Z}\right)^{-1} \cdot \mathbf{Z}^{\top} \cdot \hat{\mathbf{V}} \cdot \mathbf{Z} \cdot \left(\mathbf{Z}^{\top} \cdot \mathbf{Z}\right)^{-1} \qquad \text{[EQ L-18]}$$

The standard errors from [EQ L-18] are found as the square root of the vector diagonal. This also follows the procedure for spherical disturbances.

Since PCSE only adjusts the standard errors of the estimated regression parameters, the fit statistics is calculated using [EQ L-7] to [EQ L-13].

L.3.4 Spherical disturbances with first-order serial correlation and the Prais-Winsten transformation

The estimation of the regression parameters using [EQ L-6] is no longer BLUE when the spherical disturbances are serially correlated (Greene 2003). There are several methods available to cope with

serial correlation; in this thesis, the Prais-Winsten transformation of $[\mathbf{Z}, \mathbf{y}]$ is applied. The Prais-Winsten method is the preferred method by both Green (2003) and Baltagi (2003).

When serial correlation affects the disturbances, it follows that $\mathbf{V}$, [EQ L-4], is affected and its off-diagonal block are (Greene 2003, p. 326):

$$\sigma_{ij} \cdot \Omega_{ij} = \frac{\sigma_u^2}{1 - \rho^2} \cdot \begin{pmatrix} 1 & \rho^1 & \rho^2 & \cdots & \rho^{T-1} \\ \rho^1 & 1 & \vdots & \vdots & \vdots \\ \rho^2 & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \cdots & \cdots & \cdots & 1 \end{pmatrix} \qquad \text{[EQ L-19]}$$

The serial correlation, ρ, is assumed constant. More advanced serial correlation structures can be concocted (e.g., the serial correlation can be assumed firm-specific), however, this is not practical given the limitation of the panel: T is too small for a good estimation of a firm-specific serial correlation.

The panel is transformed using the Prais-Winsten transformation to remove the first-order serial correlation. There are several methods to estimate the first-order serial correlation coefficient. See, e.g., Greene (2003) who discusses one method; however, Baltagi (2003) reports that Greene's method performs poorly when T is small. Baltagi's method is used in this thesis since its panels' T is small. Baltagi's (2003) proposed method to estimate the first-order serial correlation coefficient is:

$$\hat{\rho} = \left(\tilde{Q}_1 - \tilde{Q}_2 \right) \cdot \left(\tilde{Q}_0 - \tilde{Q}_1 \right)^{-1} \qquad \text{[EQ L-20]}$$

where $\tilde{Q}_s = \left[\sum_{i=1}^{n} \sum_{t=s+1}^{T} \varepsilon_{it} \cdot \varepsilon_{it-s} \right] \cdot \left[n \cdot (T - s) \right]^{-1}$. The disturbances originate from the pooled regression model.

Let the Prais-Winsten transformation matrix be (Baltagi 2003):

$$\mathbf{C} = \begin{pmatrix} \sqrt{1 - \hat{\rho}^2} & 0 & \cdot & \cdot & 0 \\ -\hat{\rho} & 1 & 0 & \cdot & \cdot \\ 0 & -\hat{\rho} & 1 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & -\hat{\rho} & 1 \end{pmatrix} \qquad \text{[EQ L-21]}$$

Let the block-diagonal augmented Prais-Winsten transformation matrix be (Baltagi 2003):

$$\mathbf{P}_* = \mathbf{I}_n \otimes \mathbf{C}$$

By premultiplying the $[\mathbf{X}, \mathbf{y}]$ observations with the block-diagonal augmented Prais-Winsten transformation matrix, the first-order serial correlation is removed from the observations (Baltagi 2003):

$$\mathbf{X}_* = \mathbf{C} \cdot \mathbf{X}, \text{ and} \qquad \text{[EQ L-22]}$$
$$\mathbf{y}_* = \mathbf{C} \cdot \mathbf{y}$$

$$\mathbf{Z}_* = [\mathbf{D}, \mathbf{X}_*]$$

The transformed observations are used to estimate the unbiased regression parameters:

$$\hat{\beta}_* = \left(\mathbf{Z}_*^\top \cdot \mathbf{Z}_*\right)^{-1} \cdot \mathbf{Z}_*^\top \cdot \mathbf{y}_* \qquad \text{[EQ L-23]}$$

In addition, the fit statistics uses the transformed data to provide unbiased results. The fit statistics is calculated:

$$SSE = \varepsilon_*^\top \cdot \varepsilon_* \qquad \text{[EQ L-24]}$$

$$SST = \mathbf{y}_*^\top \cdot \mathbf{M}^0 \cdot \mathbf{y}_* \qquad \text{[EQ L-25]}$$

$$SSR = SST - SSE \qquad \text{[EQ L-26]}$$

$$R^2 = 1 - SSE \cdot SST^{-1} \qquad \text{[EQ L-27]}$$

$$MSE = \sigma_\varepsilon^2 \cdot \left(1 - \hat{\rho}^2\right)^{-1} \qquad \text{[EQ L-28]}$$

$$RMSE = \sqrt{MSE} \qquad \text{[EQ L-29]}$$

where $\sigma_\varepsilon^2 = SSE \cdot \left[n \cdot (T-1)\right]^{-1}$, and $\mathbf{M}^0 = \mathbf{I}_{n \cdot T} - (n \cdot T)^{-1} \cdot \mathbf{i}_{n \cdot T} \cdot \mathbf{i}_{n \cdot T}^\top$. The disturbance variance σ_ε^2 is estimated based on Baltagi (2003) and not on Green (2003) since $\hat{\rho}$ is estimated using Baltagi's method and not using Greene's method. The difference lies in that Baltagi uses $T-1$, where Green uses T.

Assuming AR(1) disturbances are an improvement to the spherical covariance structure for spherical errors. An even more likely covariance structure is the PCSE with AR(1) disturbances, which is considered next.

L.3.5 PCSE with first-order serial correlation and the Prais-Winsten transformation

The PCSE covariance structure with first-order serial correlated disturbances is affected in a similar manner as the covariance structure with spherical disturbances and first-order serial correlation (subsection L.3.4). This means that the PCSE estimated $\mathbf{V}$ matrix's off-diagonal blocks can be described using [EQ L-19].

Since the serial correlation affects $\mathbf{V}$ (the same no matter the two base scenarios), the Prais-Winsten transformation can also be applied to the PCSE case. Recall from [EQ L-16] that: $\hat{\Sigma} = \varepsilon^\top \cdot \varepsilon \cdot T^{-1}$. The contemporaneous covariance matrix in this case is:

$$\hat{\Sigma}_* = \varepsilon_*^\top \cdot \varepsilon_* \cdot \left[T \cdot \left(1 - \hat{\rho}^2\right)\right]^{-1} \qquad \text{[EQ L-30]}$$

The $\hat{\Omega}$ in this thesis is:

$$\hat{\Omega} = \begin{pmatrix} 1 & \rho^1 & \rho^2 & \cdots & \rho^{T-1} \\ \rho^1 & 1 & \vdots & \vdots & \vdots \\ \rho^2 & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \cdots & \cdots & \cdots & 1 \end{pmatrix}$$ [EQ L-31]

The $\hat{\mathbf{V}}$ matrix is:

$$\hat{\mathbf{V}}_* = \hat{\Sigma}_* \otimes \hat{\Omega}$$ [EQ L-32]

The unbiased regression parameters are estimated using [EQ L-23] but using the Prais-Winsten corrected $\hat{\mathbf{V}}$ allows an unbiased adjusted estimated covariance matrix to be expressed as:

$$\text{var}\left(\hat{\beta}_*\right) = \left(\mathbf{Z}_*^\top \cdot \mathbf{Z}_*\right)^{-1} \cdot \mathbf{Z}_*^\top \cdot \hat{\mathbf{V}}_* \cdot \mathbf{Z}_* \cdot \left(\mathbf{Z}_*^\top \cdot \mathbf{Z}_*\right)^{-1}$$ [EQ L-33]

Subsection L.3.3 to subsection L.3.5 considers several more advanced models than the straightforward fixed-effect model, which occurs with spherical disturbances in subsection L.3.2. By allowing panel heteroscedasticity and contemporaneous correlation, more robust covariance matrices can be designed. However, it is not certain that those more advanced structures are needed.

The specification test in M.6 (p. 238) ascertains whether panel heteroscedasticity is present in the disturbances. If the test finds that panel heteroscedasticity is present, then the covariance structure in subsection L.3.5 is applied. If not, the covariance structure in subsection L.3.4 is implemented.

L.4 Formation of panels, the identification, and the treatment of outliers in the fit periods

This section presents how the panels are created that are used in the panel regressions. All firms in a panel must have observations for all the panels' years since the panels must be balanced. The panel length is therefore an important choice; the choice of panel length is the section's first topic. The second topic considers the presence of outliers.

It should be noted that the panel length considered in this section is the fit period's length, i.e. the period on which the regressions are estimated. The requirement for balanced panel demands that a full set of observations exists for the complete fit period and also for the year that precedes the fit period since the regression equations use a lagged dependent variable (see [EQ 6-10] and [EQ 6-11]). Therefore, a panel with a four-year period have five years with uninterrupted observations.

L.4.1 The choice of fit period length

One big fit period with 16 years of observations could be used to test the hypotheses, but since the panel must be balanced, it requires all firms in the fit period to have 16 years of uninterrupted observations. Only very few firms have such a long period of uninterrupted observations available,

which therefore shrinks the number of permissible firms substantially. Earlier research (e.g., Albrecht et al. 1977; Holthausen & Larcker 1992; Ou & Penman 1989a, b; Lev 1983) also indicates that there may be time instabilities that, with a 16-year fit period, adversely affect the reliability of the regressions estimates.

The choice between long time series versus a large number of firms is a trade-off needed to be considered as the panels' lengths are decided. Ideally, a panel's length is long enough to allow any dynamics of learning to have impact, but not long enough to allow serious time instabilities to affect the reliability and validity of the thesis. As panel length increases, the number of firms decreases and a trade-off between them must determine that there are enough firm-years to be able to achieve statistical significant results.

The descriptive data in Table L-11shows that it appears as if risk-adjusted RRNOA diminish rather rapidly. Initially, there are a total of 5,731 time-series reporting significantly positive relative risk-adjusted RRNOA but there are only 314 time-series consistently reporting significantly positive relative risk-adjusted RRNOA for six years in a row.

Given that the descriptive data indicate a quick dynamic process, the panels are set up having a length of four years, which requires five years of uninterrupted observations. Reducing the panel length from 16 to 4 years increases the number of permissible firms considerably since a panel then consists of all firms having five years of uninterrupted observations. Reducing panel length from 16 to 4 years also reduces the potential effect of time instabilities and thus there is an increase in the reliability and validity of the tests.

Risk-adjusted RRNOA						
Number of observations per prediction						
Sign	T	T+1	T+2	T+3	T+4	T+5
POS	5,731	2,586	1,381	809	500	314
NEG	6,218	2,696	1,251	623	330	165

Table L-11: The dynamics of relative risk-adjusted RRNOA measured using number of observations and the relative risk-adjusted RRNOA.

Nickell (1981) notes that the bias in the regression parameter estimates from a dynamic fixed-effect model does not disappear as $n \rightarrow \infty$ because the asymptotic process depends on large T and not on large n. There are methods to deal with the bias. One way is to remove the bias by first taking differences and then applying an instrumental variable estimation method (Green 1993). The cost for such a method in this thesis is very large since it requires three years of data to perform a regression for the first year. Green (2003) notes that the Nickell bias is approximately $1 \cdot T^{-1}$, which means that the bias can be approximately 25 percent with four years of observations. This implies the need to be careful when drawing far-reaching conclusions from cases where β is close to one or when it is close to zero.

Not only regression analysis is performed to asses the theory of Homo comperiens. The results are also validated using an out-of sample method with fixed-size rolling window forecasts (Chapter 7, p. 117). This validation method requires regressions to be fitted on fit periods. Setting the panel length to four years allows these panels to be the validations' fit periods: the estimated regressions can then work as inputs into the forecasts. Consequently, the choice of panel length is also subjected to the need to validation.

The observations span from 1978 to 1994[32]. This means that, since a panel has four years, there are 13 panels (one panel per period as depicted in Figure L-1 on which panel regressions are fitted.
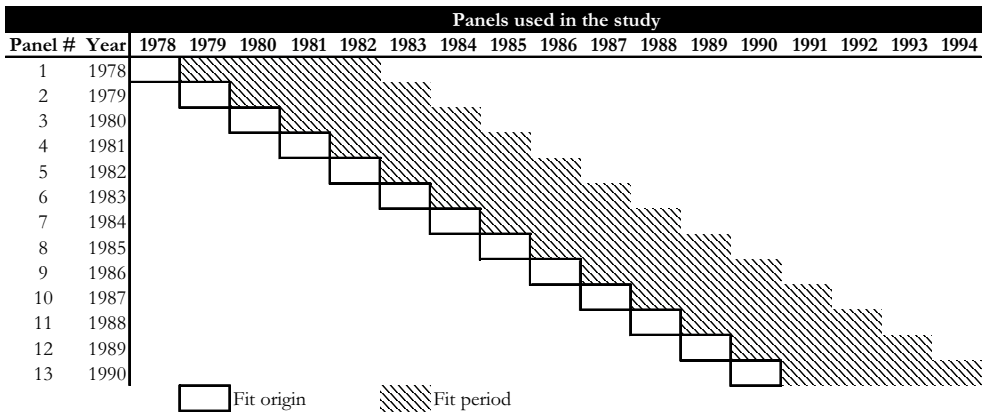
Figure L-1: The panels with fit periods used to estimate the panel regressions.

Having 13 panels also facilitates detection of significant time inconsistencies. Next, the presence of outliers in the panels is considered.

L.4.2 The presence of outliers in the explanatory variable and in the independent variable

It is clear from subsection 6.2.4 that outliers affect the distributions of the risk-adjusted RROE and risk-adjusted RRNOA. If the outliers remain in the panels, it renders the regression estimates useless since the variances are enormous. This means that the estimated regression parameter will be biased and the corresponding standard error will be so large that it invalidates any statistical tests. The influence of outliers must be reduced before any regression analysis.

Common methods to reduce the influence of outliers are Winsorizing and Trimming of the distributions before an estimation of location is performed. Two common methods for assessing robust scale are MAD and the interquartile range. Subsection I.2 discusses another more efficient method for finding robust estimates of location and scale.

---

[32] The years 1995—1996 are not part of the test since the change in the annual reporting structure from 1995 adversely affects comparability.

Since outlier identification is not concerned with robust confidence intervals, it can focus on separately identifying efficient robust location estimates and efficient robust scale estimates. The method used in subsection I.2 is thus not necessary.

Mosteller & Tukey (1977) report that the biweight estimate with $c = 9$ is highly efficient in large samples and thus that estimate of both the location and the scale are used to identify outliers.

An observation is classified as an outlier if it is farther away from the biweight location estimate than $\pm$ 3 biweight standard deviations. This implies that the six standard deviations include 99.99% of all observations in a Gaussian distribution. That is, only the most extreme of the extreme observations are classified as outliers.

The estimation of robust location and scale is performed per year. Hence, an outlier is identified based on the observation's distance to its year-specific robust location estimate, and the distance is compared with the permissible interval implied by the year-specific robust scale and the six sigma rule.

All firms identified as having at least one outlier are deleted from the panel since the panels require five years of consecutive data without outliers.

L.5 Summary

This is a technical appendix devoted to discuss the panel regression tests. The appendix considers three panel regression models. These are the pooled regression model, the random-effect model, and the fixed-effect model.

Several alternative covariance structures are also considered but the most likely candidate to the type of data, as well as to the type of econometric model chosen, is the PCSE with AR(1) errors. This covariance structure is considered in subsection L.3.5.

The formation of panels for the hypotheses tests are, by necessity, restricted to require complete sets of observations because of restrictions in the statistical program used.

A set of 12 panels is used to asses the hypotheses. A robust method using biweight estimates of location and scale with c=9 is applied to eliminate outliers in the panels.

APPENDIX M—PANEL REGRESSION SPECIFICATION TESTS

M.1 Introduction

This appendix presents four specification tests and their results. The first three tests are used to select an appropriate panel regression model while the fourth method is used to asses whether panel heteroscedasticity is present in the data, which then requires an adaptation of the covariance structure used in the model.

The appendix is structured such that it presents each specification test together with the accompanying results before moving to the next test.

M.2 Specification test plan

The appropriateness of the pooled regression model, the random-effect model, and the fixed-effect regression model depends on the presence unmodeled heterogeneity and, if it is present, if it is correlated or not with the regressors. To choose the most appropriate of the regression models is therefore a matter of step-wise elimination of them.

The first step is the Breusch-Pagan LM specification test for the presence of random effects. It test which of the pooled regression and random-effect models that is preferable given the behavior of the disturbances. The second step follows a similar strategy, where the pooled regression model is poised against the fixed-effect model.

The final step is necessary only when the pooled regression model is rejected in the previous steps. The third test tests for the presence of correlation between the random firm effects and the regressors; it poises the random-effect model against the fixed-effect model.

These tests are discussed in this appendix in the same order as above.

M.3 The Breusch-Pagan LM test for presence of random effects

M.3.1 The Breusch-Pagan LM specification test

The Breusch-Pagan Lagrange multiplier test uses the disturbances from the pooled regression model. With the pooled regression disturbances, the specification test tests the null hypothesis that the variance for the firm random effect is zero. If the null hypothesis is rejected, there is unmodeled heterogeneity in the data to such an extent that the random-effect model should replace the pooled regression model.

The Lagrange multiplier test statistic that the test uses is defined as (Greene 2003):

$$LM = \frac{n \cdot T}{2 \cdot (T-1)} \cdot \left(\frac{\sum_{i=1}^{n}\left[\sum_{t=1}^{T}\varepsilon_{it}\right]}{\sum_{i=1}^{n}\cdot\sum_{t=1}^{T}\varepsilon_{it}^2} - 1 \right)^2 \qquad \text{[EQ M-1]}$$

The decision rule is (Green 2003):

$$\text{Reject H}_0 \text{ if } LM > X_{1,\alpha}^2$$

Where n is the number of firms in the panel, T is the number of years in the panel, $X_{1,\alpha}^2$ is the cut-off point for a chi-square distribution having one degrees of freedom and with significance level α. In this test a five percent significance level is applied

M.3.2 Results from the Breusch-Pagan LM specification tests

The Breusch-Pagan LM test investigates whether there is any remaining and randomly distributed unmodeled heterogeneity. If the null hypothesis is rejected, the test has detected such randomly distributed unmodeled heterogeneity. In such a case, the random-effect model is preferable for parameter estimation rather than the pooled regression model. If the null hypothesis is not rejected, the pooled regression model is the preferable model.

Table M-12 shows the results from the Breusch-Pagan LM test. In the table, LM is the test statistic, CHIC is the chi-square cut-off point at the five percent significance level having one degree of freedom.

Table M-12 shows that using the risk-adjusted residual return on equity (risk-adjusted RROE), the null hypothesis of no unmodeled heterogeneity is rejected for panels with fit origins 1979, 1984, 1989, and 1990.

Variable	Field	Fit origin												
		1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
RROE	LM	1.38	8.23	2.16	0.32	1.00	2.48	9.88	0.52	3.69	1.75	0.26	10.26	5.41
	CHIC	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84
	PVALUE	24.0%	0.4%	14%	57%	32%	11.5%	0.2%	47%	5.5%	19%	61%	0.1%	2.0%
RRNOA	LM	0.02	4.42	2.89	1.37	0.38	0.09	18.79	9.16	4.21	0.58	0.20	8.34	8.49
	CHIC	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84
	PVALUE	89%	3.6%	9%	24%	54%	77%	0.0%	0.2%	4.0%	45%	66%	0.4%	0.4%

Table M-12: The outcome of the Breusch-Pagan LM tests for presence of random effects using the variables risk-adjusted RROE and risk-adjusted RRNOA.

This means that, according to the specification tests, the pooled regression model is preferable for all panels except those with fit origins 1979, 1984, 1989, and 1990. For the panels having fit origins 1979, 1984, 1989, and 1990, the random-effect model is the preferred choice according to the table.

Table M-12 also shows the results using the risk-adjusted residual return on net operating assets (risk-adjusted RRNOA) variable too; its null hypotheses also fail to be rejected for panels ex-

cept those with fit origins 1979, 1984, 1989, and 1990. In addition to these panels, it also fails to reject the null hypothesis for the panels having 1985 and 1986 as fit origins.

Thus, it appears as though the appropriate model for RROE is on balance not the random-effect model but for RRNOA it appears as though it is almost even between choosing or not choosing the random-effect model.

The Breusch-Pagan LM test only compares the pooled regression model against the random-effect model. It is also necessary to compare the pooled regression model with the fixed-effect model. Such a specification tests is discussed in the next subsection.

M.4 The F-test for the presence of firm effects

M.4.1 The F-test specification test

The F-test for firm effects is a specification test in which the null hypothesis presumes that the constant terms are equal across firms. That is, the pooled regression model is applicable to the data. If the null hypothesis of equal constant terms is rejected, it implies that so much unmodeled heterogeneity exists in the data that the pooled regression model is inferior to the fixed-effect model. The F-statistic for testing firm effects is defined as (Greene 2003):

$$F = \frac{\left(R^2_{Fixed} - R^2_{Pooled}\right) \cdot (n-1)^{-1}}{\left(1 - R^2_{Fixed}\right) \cdot (n \cdot T - n - K)^{-1}}$$

[EQ M-2]

The decision rule is (Green 2003):

Reject H$_0$ if $F > F_{n-1, n \cdot T - n - K, \alpha}$

where $F_{n-1, n \cdot T - n - K, \alpha}$ is the cut-off point for a F-distribution with significance level α having $n-1$ numerator degrees of freedom and $n \cdot T - n - K$ denominator degrees of freedom. In this test, a five percent significance level is applied. R^2_{Fixed} is the coefficient of determination for the fixed-effect regression model and R^2_{Pooled} is the coefficient of determination for the pooled regression.

If neither the Breusch-Pagan LM test for presence of random effects nor the F-test for the presence of firm effects rejects its null hypothesis, the pooled regression is the preferable model to use for estimating the regression model. If the Breusch-Pagan LM test for presence of random effects rejects its null hypothesis and the F-test for the presence of firm effects does not, the random-effect model should be applied for estimating the regression model. If the F-test for the presence of firm effects rejects its null hypothesis and the Breusch-Pagan LM test for presence of random effects does not, the fixed-effect model should be applied for estimating the regression model.

If both the Breusch-Pagan LM test for presence of random effects and the F-test for the presence of firm effects reject their respective null hypothesis, the pooled regression is not prefera-

ble. A third specification test is therefore needed to discriminate between the random-effect model and the fixed-effect model. This test is the Hausman test, which is presented in subsection M.5.

M.4.2 Results from the F-tests for the presence of firm effects

The null hypothesis in the F-test assumes that no unmodeled heterogeneity remains in the data and hence that the intercepts are equal across firms. If the null hypothesis is not rejected, the pooled regression model is the preferable choice. In this specification test the pooled regression model is confronting the fixed-effect model, which means that if the null hypothesis is rejected, the fixed-effect model is the preferable regression model.

The results from the test using the risk-adjusted RROE and risk-adjusted RRNOA are presented in Table M-13. The table's FVALUE is the test statistic and FCRI is the F-distribution cut-off point at the five percent significance level. The cut-off point has NDF numerator degrees of freedom and DDF denominator degrees of freedom.

Variable	Field	Fit origin												
		1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
RROE	NDF	299	354	635	642	656	669	641	662	616	608	582	574	618
	DDF	899	1,064	1,907	1,928	1,970	2,009	1,925	1,988	1,850	1,826	1,748	1,724	1,856
	FVALUE	1.92	1.84	1.61	1.61	1.35	1.19	1.52	1.42	1.58	1.41	1.40	1.76	1.89
	FCRI	1.16	1.15	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.12	1.12	1.11
	PVALUE	0.00%	0.00%	0.00%	0.00%	0.00%	0.22%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
RRNOA	NDF	289	353	658	639	643	634	614	609	554	578	559	573	626
	DDF	869	1,061	1,976	1,919	1,931	1,904	1,844	1,829	1,664	1,736	1,679	1,721	1,880
	FVALUE	1.70	1.85	1.70	1.80	1.44	1.37	1.71	1.92	1.88	1.39	1.40	1.80	2.13
	FCRI	1.17	1.15	1.11	1.11	1.11	1.11	1.11	1.11	1.12	1.12	1.12	1.12	1.11
	PVALUE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table M-13: The results of the F-tests for presence of firm effects measured using both risk-adjusted RROE and risk-adjusted RRNOA.

The table shows that the null hypotheses are rejected in favor of the alternative hypotheses for all panels and for both variables. The results are so strong that the null hypotheses are rejected even at the one percent significance level. No significant difference is discernable between the two variables and thus the results appear robust to the choice of measurement variable.

Using the risk-adjusted RROE, the results in Table M-12 in conjunction with Table M-13 imply that the fixed-effect model should be used rather than any of the pooled regression model and the random-effect model for all panels except those with fit origins 1979, 1984, 1989, and 1990,. No Hausman test is needed in order to reach that conclusion. When using the risk-adjusted RROE and panels with fit origins from 1979, 1984, 1989, and 1990, it is necessary to apply Hausman specification tests to discriminate between the fixed-effect model and the random-effect model.

As Table M-13 shows, the F-tests for the presence of firm effects reject all the null hypotheses at the 0.0 percent significance level when they are tested using the risk-adjusted RRNOA. When considering Table M-12 too, the fixed-effect model should be employed using the risk-adjusted RRNOA for the panels not starting in 1979, 1984, 1985, 1986, 1989, and 1990. The results

are strong and consistent enough that no Hausman test is necessary. The results of the Hausman specification tests nevertheless are presented in subsection M.5.2. The Hausman specification tests should corroborate the findings above concerning risk-adjusted RRNOA and the findings for risk-adjusted RROE.

M.5 The Hausman test for correlation between random firm effects and the regressors

M.5.1 The Hausman specification test

The Hausman test is a chi-square test that positions the results from the random effect against the fixed effect. The test's null hypothesis presumes that there is no correlation between random firm effects and the regressors. If there is no correlation, the random-effect regression model is preferable (Greene 2003). The fixed-effect regression model is preferable if the null hypothesis is rejected.

To test the hypotheses the estimated covariance matrix is needed and it is defined as (Green 2003):

$$\hat{\Psi} = Var\left[\hat{\beta}_{Fixed}\right] - Var\left[\hat{\beta}_{Random}\right] \qquad \text{[EQ M-3]}$$

Using the estimated covariance matrix, the Wald criterion is defined as (Green 2003):

$$W = \left[\hat{\beta}_{Fixed} - \hat{\beta}_{Random}\right] \cdot \hat{\Psi}^{-1} \cdot \left[\hat{\beta}_{Fixed} - \hat{\beta}_{Random}\right] \qquad \text{[EQ M-4]}$$

The decision rule is (Green 2003):

$$\text{Reject H}_0 \text{ if } W > \text{X}^2_{K-1,\alpha}$$

Where $\text{X}^2_{K-1,\alpha}$ is the cut-off point for a chi-square distribution having $K-1$ degrees of freedom and with significance level α. A five percent significance level is applied in this specification test.

The Hausman test also provides an implicit test of Homo comperiens. Homo comperiens postulates that there should be some firm-specific effect and hence that there should be heterogeneity in the data. Furthermore, it postulates that firm-specific effects are non-random. If the Hausman test rejects the null hypothesis of random effect in favor of fixed effects, the theory of Homo comperiens is supported.

This section focuses on finding the most appropriate panel regression model, given the potential existence of heterogeneity. Another important issue concerns the potential existence of heteroscedasticity in the models, which is the topic of subsection M.6.

M.5.2 Results from the Hausman tests for correlation between random firm effects and the regressors

The Hausman test's null hypothesis assumes that there is no correlation between random firm effects and the regressors, suggesting that the random-effect model is the preferable estimation model if the null hypothesis is not rejected.

Table M-14 reports the results from the Hausman tests on all panels using both the risk-adjusted RROE and the risk-adjusted RRNOA. W is the Hausman test statistic, CHIC is the chi-square cut-off point.

Variable	Field	Fit origin												
		1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
RROE	W	283	252	484	537	487	422	360	448	463	347	405	419	525
	CHIC	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84
	PVALUE	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**
RRNOA	W	257	278	519	601	506	442	357	519	486	309	365	440	602
	CHIC	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	3.84	
	PVALUE	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**	**0.00%**

Table M-14: The results from the Hausman test for correlation between random firm effects and the regressors when estimated using both the risk-adjusted RROE and the risk-adjusted RRNOA.

The results rejects that the random-effect model should be employed for all panels using either the risk-adjusted RROE or the risk-adjusted RRNOA. Therefore, it implies that the fixed-effect model dominates over the random-effect model. This means that the fixed-effect model is implemented onto all panels as the regression parameters are estimated.

The next subsection presents the results from the tests of panel heteroscedasticity. These specification tests are conducted using the fixed-effect model since it is the preferred estimation model based on the analysis above.

M.6 The likelihood test for the presence of panel heteroscedasticity in the fixed-effect model

M.6.1 The likelihood specification test

The likelihood test for panel heteroscedasticity tests the null hypothesis of homoscedasticity against the alternative hypothesis of panel heteroscedasticity. It tests this by investigating whether the variance of the disturbances is constant across the firms. The test statistic is (Greene 2003):

$$-2\left(\ln L_0 - \ln L_1\right) = n \cdot \ln\left(\frac{\varepsilon^\top \cdot \varepsilon}{n}\right) - \sum_{i=1}^{n}\left[T \cdot \ln\left(\frac{\varepsilon_i^\top \cdot \varepsilon_i}{T}\right)\right] \qquad \text{[EQ M-5]}$$

As previously, n is the number of firms in the panel and T is the panel length. Since section L.3 limits itself to consider alternative specifications for a fixed-effect model, ε is the disturbance matrix from a fixed-effect regression with spherical disturbances and ε_i is the disturbance matrix from an individual regression on firm i while assuming spherical disturbances.

The decision rule is (Green 2003):

$$\text{Reject } H_0 \text{ if } -2\left(\ln L_0 - \ln L_1\right) > X_{n-1,\alpha}^2$$

where $X_{n-1,\alpha}^2$ is the cut-off point for a chi-square distribution having $n-1$ degrees of freedom and with significance level α. A five percent significance level is used in this test.

M.6.2 Results from the tests for panel heteroscedasticity in the fixed-effect model

The fixed-effect model dominates both the pooled regression model and the random-effect model in the reported specification test, which means that the fixed-effect estimates are used in the hypotheses tests. Running an OLS fixed-effect regression model requires spherical disturbances or else the standard errors will be biased (see subsection L.3.3, p. 226), which affect the reliability of the hypotheses tests. The alternative is to allow for panel heteroscedasticity by using a more complex covariance structure.

Table M-15 reports the results from the specification tests. In the table, LM is the test statistic, CHIC is the cut-off point at the five percent significance level from a chi-square distribution that has DF degrees of freedom.

Variable	Field	Fit origin												
		1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
RROE	DF	299	354	635	642	656	669	641	662	616	608	582	574	618
	-2(LN0-LN1)	4,154	4,866	8,597	8,747	9,082	9,483	9,821	9,896	9,330	7,948	7,257	6,963	7,358
	CHIC	340	399	695	702	717	730	701	723	675	666	639	631	677
	PVALUE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
RRNOA	DF	289	353	658	639	643	634	614	609	554	578	559	573	626
	-2(LN0-LN1)	5,477	6,585	12,080	11,800	11,958	11,864	12,065	11,694	10,639	10,054	9,195	9,343	10,068
	CHIC	330	398	719	699	703	694	673	668	610	635	615	630	685
	PVALUE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table M-15: Test for the presence of panel heteroscedasticity using both the risk-adjusted RROE and the risk-adjusted RRNOA.

The test statistics are extremely large, which leads to the rejection of the null hypothesis of no panel heteroscedasticity in favor of its alternative of panel heteroscedasticity. This implies that a covariance structure is needed that allows for panel heteroscedasticity in order to achieve unbiased standard errors from the panel regressions.

A covariance structure that allows for panel-corrected standard errors (PCSE) (and first-order correlated disturbances) is applied to the panels with risk-adjusted RROE and to the panels with risk-adjusted RRNOA. See subsection L.3.5 (p. 228) for a covariance structure having PCSE and AR(1) disturbances.

M.7 Conclusions from the specification tests

The appendix rejects the pooled regression model as applicable to the accounting data used in this thesis. The appendix also rejects the random-effect model as applicable to the accounting data. In the end, the model that perseveres through all specification tests is the fixed-effect model, which consequently is the applied model in this research.

The panel heteroscedasticity specification tests clearly reject the null hypothesis of no panel heteroscedasticity, which implies that the covariance structure that yields PCSE is implemented as the regression model.

L.3 concludes that, by design, the estimated regression models are affected by serial correlation. Together with this appendix finding, it means that the preferred regression model on all panels and for both variables is a fixed-effect model having PCSEs after they are adjusted for first-order serial correlation. This is equivalent to the covariance model discussed in subsection L.3.5.

It should be noted that using the fixed-effect model with dynamic econometric models, as in this research, introduces the Nickell bias, which may effect the regression parameter estimation. Thus, caution is called for when interpreting the results from the hypotheses tests if they are weakly significant or if they are weakly insignificant.

APPENDIX N—DOES RANDOM WALK IN THE RISK-ADJUSTED RESIDUAL ACCOUNTING RATES-OF-RETURNS?

N.1 The hypotheses to be tested

A random-walk process of some variable implies that successive changes of the variable are independent (Fama 1965a). Using probabilities, independence implies that the probability distribution for the change of the variable at a given period is independent of the sequence of the variable's changes from the periods preceding the current time period (Fama 1965a). That is:

$$x_{t+1} = x_t + \varepsilon_{t+1} = x_{t-1} + \varepsilon_t + \varepsilon_{t+1} = \cdots = x_0 + \sum_{i=1}^{t} \varepsilon_i$$

Where ε_t is a white noise disturbance, which means that it has:

$$E_t\left[\varepsilon_{t+1}\right] = 0$$

$$\sigma_\varepsilon^2 = \sigma^2$$

$$corr\left(\varepsilon_{t+1}, \varepsilon_t\right) = 0$$

Recall the following proposition:

> **Proposition 4-7:** In a market that meets the conjectures of the theory of Homo comperiens (Proposition 2-4, Proposition 3-2) and with unbiased accounting the limit values of risk-adjusted subjective expected RROE and RRNOA are zero. That is: $\lim_{t \to \infty}\left(E_{K0}^*\left[_{t-1}RROE_t\right]\right) = 0$, and $\lim_{t \to \infty}\left(E_{K0}^*\left[_{t-1}RRNOA_t\right]\right) = 0$.

The process in Proposition 4-7 is a direct contradiction to the definition of a random-walk process since it implies variable dependency. That is, in Proposition 4-7 it is possible to predict future variables based on the sequence of the variable's changes from the preceding periods.

The variables in Proposition 4-7 are operationalized as [EQ 6-8] and [EQ 6-9] (p. 98).

This appendix assesses the following two hypotheses (one per variable):

H_{1A}: The theory of Homo comperiens argues that the risk-adjusted subjective expected residual return on equity (RROE) is not randomly walking. Using [EQ 6-8] this can be expressed as:

The serial correlation in $_tRROE_{it+1}^* = {}_{t-1}RROE_{it}^* + \varepsilon_{t+1}$ is not zero, that is:

$$corr\left(\varepsilon_{t+1}, \varepsilon_t\right) \neq 0$$

H_{0A}: The risk-adjusted subjective expected RROE is randomly walking. This can be expressed as:

$$corr\left(\varepsilon_{t+1}, \varepsilon_t\right) = 0.$$

H₁ʙ: The theory of Homo comperiens argues that the risk-adjusted subjective expected residual return on net operating assets (RRNOA) is not randomly walking. Using [EQ 6-9] this can be expressed as:

The serial correlation in $_tRRNOA^*_{it+1} = {}_{t-1}RRNOA^*_{it} + \varepsilon_{t+1}$ is not zero, that is:

$$corr\left(\varepsilon_{it+1}, \varepsilon_{it}\right) \neq 0$$

H₀ₐ: The risk-adjusted subjective expected RRNOA is randomly walking. This can be expressed as:

$$corr\left(\varepsilon_{it+1}, \varepsilon_{it}\right) = 0.$$

Indeed, an even harsher test could be devised since the proposition implies $corr\left(\varepsilon_{t+1}, \varepsilon_t\right) < 0$. This could be tested against the null hypothesis of $corr\left(\varepsilon_{t+1}, \varepsilon_t\right) \geqslant 0$. Such a test is not formally carried out since it is thought not to contribute much to the test already reported in section 6.4 (p. 108).

Nevertheless, the tests reported in this appendix do not have the same strength as those reported in section 6.4. A rejection of the null hypotheses in this appendix does not exclude $corr\left(\varepsilon_{t+1}, \varepsilon_t\right) > 0$, but such a rejection does, together with the findings reported in section 6.4, provide, I argue, strong support for Proposition 4-7.

The appendix is organized such that it first presents the test method followed by the results from the tests.

N.2 The goodness-of-fit test
The goodness-of-fit test focuses on the independence-dependence difference in the stated hypotheses.

The test is performed by classifying all observations in a period into equal-sized bins. If the distribution is completely random, as the null hypotheses assume, then the probability for observing an observation in a certain bin is equal to observing it in any other bin, i.e. the bin-probability is constant.

Let $k \in K$ be a bin and K the total number of bins. The unconditional probability for an observation to be assigned to a bin $k \in K$ is $\pi = 1 \cdot K^{-1}$. Let O_k be the number of observations assigned to a bin and let E be the expected number of observations in a bin, which is $E = n \cdot \pi$, where n is the number of observations for a period. By comparing the difference between the numbers of observations per bin to the expected number of observations per bin, a chi-square test statistic can be measured as (Newbold 1995):

$$\text{X}^2 = \sum\nolimits_{k=1}^{K} \frac{\left(O_k - E\right)^2}{E} \qquad \text{[EQ N-1]}$$

According to Newbold (1995), the decision rule is:

$$\text{Reject H}_0 \text{ if } X^2 > X^2_{K-1,\alpha}$$

where $X^2_{K-1,\alpha}$ is the cut-off point for a chi-square distribution having $K-1$ degrees of freedom and with significance level α.

The goodness-of-fit test above tests the distributional characteristics of a period and needs to be extended in order to focus on the independence-dependence since the difference is exacerbated as a longer perspective is used. The test above is therefore extended to cover more than one period.

N.3 A multi-period goodness-of-fit-test

If the null hypotheses are correct, the observations are serially independent. The independence assumption allows not only the use of unconditional probabilities but also the use of conditional probabilities. Conditional probabilities make it possible to extend the test into a multi-period setting.

Let the conditional probability for observing the variable from a firm in the same bin for two consecutive periods be $\pi_2 = \left(1 \cdot K^{-1}\right)^2$. Analogously, the expected number of variables that are observed in the same bin for two consecutive periods is $E_2 = n_2 \cdot \pi_2$, where n_2 is the number of complete time series of variables for two consecutive periods.

With the conditional probability perspective, it becomes possible to count the number of observations of variables (stemming from the same firm) that are assigned to the same bin for two consecutive periods and to measure E_2. These variables are then used to measure the chi-square test statistic according to [EQ N-1] and to evaluate it against the decision rule in section N.2.

The multi-period goodness-of-fit tests the ability to discover non-randomness increases as the number of periods is added.

Newbold (1995) notes that the goodness-of-fit test works well for samples in which the expected number of observations per bin is greater than five. Having $E>5$ severely limits the possibility to extend the test into the future: The required sample size increases dramatically since the conditional probability decreases fast. For example, assume there are five bins, which implies an unconditional probability of 20 percent (this requires n to be 25). Assume now that you wish the test to assess the independency in five periods. The conditional probability that the variable from a firm remains in the same bin is then only $0.2^5=0.032$ percent and n must therefore be at least 15,625.

The multi-period goodness-of-fit test is designed so that it chooses the maximum number of bins that meets the criteria of having $E>5$. The exact number of feasible bins is a function of the minimum number of expected observations, the conditional probability for the forecast length, and the number of firms. It is calculated as shown below:

$$bins = \frac{1}{\pi_1} = \frac{1}{\left(E_t \cdot n_t^{-1}\right)^{t^{-1}}} \qquad \text{[EQ N-2]}$$

For $t = 4$ the number of bins must be greater than or equal:

$$E_4 > 5 \Rightarrow bins > \frac{1}{\pi_1} = \frac{1}{\left(5 \cdot n_4^{-1}\right)^{4^{-1}}}$$

N.4 Test method

The first test uses a two-year conditional probability and thus uses only a firm time series having two complete years of observations. The second test uses a three-year conditional probability and thus uses only a firm time series having three complete years of observations. This continues up to the fourth test, which uses a firm time series having five complete years of observations.

The formation of the set of time series for the first test is done using a rolling-window method that starts in year 1978 and collects all time series having observations in 1978 and 1979. The window then rolls forward to 1979, and all time series having observations in 1979 and in 1980 are collected and added to those previously found. The process so continues to the horizon year 1993, which collects all time series having observations in 1993 and in 1994 and adds them to all other time series.

The time series are apportioned into the initial period's, $t = 1$, equal-sized bins based on the ranking of the variable (see below). From this, the observations are counted that remain in the same bin for the following years. The observations are compared with the expectations and a chi-square statistic is calculated on the differences following [EQ N-1].

Since the time series are apportioned into initial equal-sized bins, it follows that the product of the number of bins and the observations per bin must equal the total available time series. If this is not true are time series deleted so that the condition is fulfilled. The time series are deleted from the center of the distribution.

Because the time series in a set originates from different years, there is a chance that their levels will differ, which can affect the reliability of the test while it presumes that they all come from the same distribution. To make the risk-adjusted residual accounting rates-of-returns comparable between different years, this test normalizes them per year. This is a method used elsewhere (Mueller (1977).

The variables used in the tests, [EQ 6-8] and [EQ 6-9], are normalized against their industry-year's profitability ratio. The industry year's profitability ratio is measured according to I.2 (p. 211). The relative ratios are defined as:

$$_{t-1}\text{rel-}RROE_{it}^* = {}_{t-1}RROE_{it}^* \cdot {}_{t-1}ROE_{It}^{-1} \qquad \text{[EQ N-3]}$$

$$_{t-1}\text{rel-}RRNOA_{it}^* = {}_{t-1}RRNOA_{it}^* \cdot {}_{t-1}RNOA_{It}^{-1} \qquad \text{[EQ N-4]}$$

N.5 Results from the multi-period goodness-of-fit tests

The goodness-of-fit test ascertains the assumption in Proposition 4-7 that the risk-adjusted residual accounting rates-of-returns are dependent on their sequence of changes against the null hypotheses of random walks in the variables. The tests are performed using conditional probabilities to capture the dynamics in the process.

The section is organized where descriptive outputs from the test are discussed followed by the formal outcome of the hypotheses tests.

N.5.1 Descriptive outputs from the tests

The goodness-of-fit tests are preformed using the relative risk-adjusted RROE and the relative risk-adjusted RRNOA for two-year[33], three-year, four-year, and five-year conditional probabilities. All ratios are relative ratios based on the definitions [EQ N-3] and [EQ N-4].

The number of bins is chosen so that maximum permissible number of bins is applied while keeping the expected number of observations per bin greater than five. The two-year conditional probability tests apply 56 bins and the unconditional probabilities are $\pi_1 = 56^{-1} = 1.79\%$, while the two-year conditional probabilities are $\pi_2 = \left(56^{-1}\right)^2 = 0.0319\%$. For the three-year tests, 12 bins are used. Six bins are used for the four-year tests and four bins are used for the five-year tests.

Probablility table			
t=2	t=3	t=4	t=5
1.79%	0.59%	0.46%	0.10%

Table N-16: The conditional probabilities

Since the test method requires that the firm's time series are evenly apportioned into the first-year bins, it follows that some time series are deleted to ascertain equal-sized bins. The table below provides information of how many time series that are deleted. The time series are deleted from the center of the distributions. While this is an arbitrary choice, it is not likely to have a material affect on the results since the number of deleted time series is small in relation to the total time series available for analysis. See Table N-18 for information on total number of available time series.

Number of deleted cpl firm time-series				
Variable (relative)	t=2	t=3	t=4	t=5
RROE	N/A	6	2	2
RRNOA	8	6	1	0

Table N-17: Number of deleted complete firm times-series.

---

[33] No goodness-of-fit test is performed using the risk-adjusted RROE with a two-year conditional probability since the ranking procedure in SAS gives equal ranking to identical RROE. This poses a problem since they are assigned to bins on their rankings.

The amount of work deemed to solve this problem does not stand in proportion to the gain from performing this test when considering that three two-year conditional probability goodness-of-fit tests are performed and that three other goodness-of-fit tests are performed using the risk-adjusted RROE. Fifteen goodness-of-fit tests are performed compared with potentially 16 tests.

At most eight complete time series are deleted. Deleting nine time series has a negligible effect since the statistical test uses many observations after deletion of the nine time series. The number of complete time series that I use after the nine time series are deleted is presented in Table N-18.

Number of cpl firm time-series available for GoF-tests				
Variable (relative)	t=2	t=3	t=4	t=5
RROE	N/A	13,208	10,938	8,972
RRNOA	15,272	12,506	10,224	8,312

Table N-18: Number of used complete firm time series that is available for the goodness-of-fit tests. These time series are the time series available after the trimming explicated in Table N-17.

Below are three tables (Table N-19—Table N-21) that show the number of observations per bin and variable for $t = 5$, $t = 4$, and $t = 3$. No such information is provided for $t = 2$ since that implies information on 56 bins. It is considered to cause too much clutter in the table.

Table N-19 shows that of 8,972 time series with risk-adjusted RROE, only 218 manage to be ranked into the same bin for five consecutive years. Still, that is much more than what was expected since only 35 time series were expected to be consistently ranked into the same bin for all five years. Similar results appear for the other three variables.

No of obs and expectations per bin for t=5				
	rel-RROE		rel-RRNOA	
Bin	Obs	Exp	Obs	Exp
1	64	8.76	130	8.12
2	40	8.76	21	8.12
3	48	8.76	51	8.12
4	66	8.76	76	8.12
Σ	218	35.0	278	32.5

Table N-19: Number of empirical observations (Obs) and the expected number of observations (Exp) per bin for t=5.

Another result from Table N-19 is that bin #1 and bin #4 have more persistent time series than the other bins. From this casual analysis, it appears as if the variables are non-independent.

It also appears as if risk-adjusted residual accounting rates-of-returns measured before financial activities (risk-adjusted RRNOA) are more persistent than when measured after financial activities (risk-adjusted RROE).

No of obs and expectations per bin for t=4				
	rel-RROE		rel-RRNOA	
Bin	Obs	Exp	Obs	Exp
1	59	8.44	113	7.89
2	34	8.44	20	7.89
3	27	8.44	11	7.89
4	27	8.44	34	7.89
5	40	8.44	37	7.89
6	75	8.44	86	7.89
Σ	262	50.6	301	47.3

Table N-20: Number of empirical observations and the expected number of observations per bin for t=4.

For $t = 4$, six bins are used, which is an improvement when compared with $t = 5$. It is possible to use more bins in the shorter period since it has more of complete firm time series. The results interpretable from Table N-20 are comparable with those from Table N-19, and indicate that the variables are dependent.

Much more observations remain in their bins than expected if they were truly random and if they are independently distributed. Again, the start and end (bin #1 and #6) are those bins that have the most persistent time series.

No of obs and expectations per bin for t=3				
	rel-RROE		rel-RRNOA	
Bin	Obs	Exp	Obs	Exp
1	36	6.01	67	5.69
2	17	6.01	26	5.69
3	15	6.01	18	5.69
4	19	6.01	9	5.69
5	14	6.01	12	5.69
6	22	6.01	7	5.69
7	13	6.01	13	5.69
8	10	6.01	13	5.69
9	23	6.01	21	5.69
10	29	6.01	20	5.69
11	15	6.01	12	5.69
12	22	6.01	20	5.69
13	59	6.01	49	5.69
Σ	294	78.2	287	74.0

Table N-21: Number of empirical observations and the expected number of observations per bin for t=3.

Table N-21 reports the results from $t = 3$. These results are consistent with those reported in Table N-19 and Table N-20. Table N-21 shows that the observed persistence is much greater than what should be the case with independent variables.

For $t > 3$, it appears as though risk-adjusted RRNOA is less independent than risk-adjusted RROE. This effect is almost nullified at $t = 3$ since the aggregate difference between the two variables is small. An ocular investigation thus indicates that even though some firms manage to report relatively stable risk-adjusted RRNOA, this does not carry through to risk-adjusted RROE. Thus, it appears that adding the financial effects remove apparent stability from the firm's operations.

N.5.2 Results from the hypotheses tests

Subsection N.5.1 shows that it appears as though the variables are serially dependent, which supports Proposition 4-7. A causal ocular analysis therefore rejects the null hypotheses of random walk in favor of the alternative hypotheses of non-random walk.

This subsection provides the results from the formal hypotheses tests performed on $t = 2$ to $t = 5$. The outcomes are presented in descending order since the tests with a high t are more valid than the tests with a low t.

Summary statistics, t=5		
Field	rel-RROE	rel-RRNOA
DFE	3	3
CHIC	20.9	30.2
PVALUE	0.01%	0.00%

Table N-22: Summary statistics for the goodness-of-fit tests where t=5.

Table N-22 shows the results from the goodness-of-fit tests, where $t = 5$. DFE is the degrees of freedom, CHIC is the chi-square statistics as defined in [EQ N-1], and the PVALUE is the p-values.

The causal ocular analysis from the previous subsection is supported by the statistical test having $t = 5$. The test rejects the null hypotheses of independent variables in favor of the alternative hypotheses of dependent variables. The result meets the cut-off point of five percent and the results are even significant at the 0.0 percent level.

Summary statistics, t=4		
Field	rel-RROE	rel-RRNOA
DFE	5	5
CHIC	25.0	32.2
PVALUE	0.01%	0.00%

Table N-23: Summary statistics for the goodness-of-fit tests where t=4.

The table above shows the goodness-of-fit tests with $t = 4$. Also at $t = 4$ the hypotheses tests reject the null hypotheses of independency in favor of the alternative hypotheses of dependency. These results are significant at the 0.0 percent level.

Summary statistics, t=3		
Field	rel-RROE	rel-RRNOA
DFE	12	12
CHIC	35.9	37.4
PVALUE	0.03%	0.02%

Table N-24: Summary statistics for the goodness-of-fit tests where t=3.

At $t = 3$, (see Table N-24) the goodness-of-fit tests are also rejects the null hypotheses of independency in favor of the alternative hypotheses of dependency. These results are significant at the 0.0 percent level.

Summary statistics, t=2		
Field	rel-RROE	rel-RRNOA
DFE	N/A	55
CHIC	N/A	39.5
PVALUE	N/A	93.07%

Table N-25: Summary statistics for the goodness-of-fit tests where t=2.

The hypotheses tests that investigate the behavior of the variables between two consecutive periods, i.e. $t = 2$, fail to reject the null hypotheses of independence in favor of the alternative hypotheses of dependence.

It appears as if the process between two consecutive periods is a random-walk process. When a longer perspective is applied, the random-walk process gives way for a non-random process. That is, from a short-run perspective it appears as if the assumption of random walk cannot be rejected, but as the perspective is prolonged, the variables become serially dependent.

The result in this section is similar to findings on tests of market efficiency. Kothari (2001) reports that in short-window event studies there are hardly no abnormal returns (thus, random walk works), whereas the long-window event-studies show the possibility to earn abnormal returns, which implies that the process is not randomly walking.

N.6 Conclusions from the multi-period goodness-of-fit tests

Both the descriptive data and the goodness-of-fit tests that investigate the alternative hypotheses based on Proposition 4-7 reject the null hypotheses of independent distributed variables in all multi-period tests, except for $t = 2$, which favors the alternative hypotheses.

The results can be interpreted in many ways. The number of bins is 56 for $t = 2$. Having extremely many bins makes it almost impossible to retain the same bin assignment over time since an extremely small change in the variable mandates a change of the assigned bin. I believe that this may drive the results for $t = 2$, especially considering the consistent and strong rejections of the null hypotheses for $t > 2$, where the number of bins is no more than 13. Indeed, when researchers (e.g., Mueller 1977; Penman 1991) use a bin structure to analyze the behavior of accounting rates-of-returns, they do not use more than 20 bins.

However, the results of $t > 2$ are more affected by survival bias than for $t = 2$. This may also drive the results in the direction of dependency. On the other hand, the number of bins is not drastically different between $t = 2$ and $t = 3$, which makes me reluctant to assign too much importance to a survival bias effect.

I therefore find that the most likely difference between results from the hypotheses tests for $t = 2$ compared with the hypotheses tests for $t > 2$ is due to an excessive use of bins.

The general conclusion is therefore that the goodness-of-fit tests find consistent and strong support for Proposition 4-7 since the null hypotheses of independence is rejected in favor of the

alternative hypothesis of dependence. That is, randomness does not walk in residual accounting rates-of-returns.

ACKNOWLEDGEMENTS

Abarbanell, J., Bushee, B., 1997, "Fundamental analysis, future earnings, and stock prices, *Journal of Accounting Research*, vol. 35, no. 1, p. 1-24.

Abarbanell, J., Bushee, B., 1998, "Abnormal returns to a fundamental analysis strategy," *Accounting Review*, vol. 73, p. 19-45.

Ahlburg, D., 1992, "A commentary on error measures," *International Journal of Forecasting*, vol. 8, p. 99-100.

Albrecht, W, Lookabill, L., McKeown, J., 1977, The Time-Series Properties of Annual Earnings, *Journal of Accounting Research*, vol. 15, no. 2, p. 227-244.

Armstrong, J., Collopy, F., 1992, "Error measures for generalizing about forecasting methods: Empirical comparisons," *International Journal of Forecasting*, vol. 8, p. 69-80.

Baginski, S., Lorek, K., Willinger, G., Branson, B., 1999, "The relationship between economic characteristics and alternative annual earnings persistence measures, *Accounting Review*, vol. 74, no. 1, p. 105-120.

Ball, R., Watts, R., 1972, "Some time series properties of accounting income," *Journal of Finance*, vol. 27, p. 663-682.

Baltagi, B., 2003, *Econometric analysis of panel data*, 2nd edn, Wiley, Chichester.

Barberis N., Shleifer, A., Vishny, R., 1998, A model of subject sentiment, *Journal of financial economics*, vol. 49, p. 307-343.

Baumol, W., 1961, *Economic theory and operations analysis*, Prentice-Hall, London.

Beaver, W., 1968, "The information content of annual earnings announcements," *Journal of Accounting Research* (Supplement), vol. 6, p. 67-92.

Beaver, W., 1970, "The time series behavior of earnings," *Journal of Accounting Research*, vol. 8, p. 62-99.

Beaver, W., 1989, *Financial reporting: an accounting revolution*, Prentice-Hall, New Jersey.

Beaver, W., 2002, "Perspectives on recent capital market research," *Accounting Review*, vol. 77, no. 2, p. 453-474.

Beaver, W., Kettler, P., Scholes, M., 1970, "The association between market determined and accounting determined risk measures," *The Accounting Review*, vol. 45, no. 4, p. 654-682.

Beaver, W., Manegold, J., 1975, "The association between market-determined and accounting-determined measures of systematic risk: some further evidence," *Journal of Financial and Quantitative Analysis*, vol. 10, no. 2, p. 231-284.

Beck, N., Katz, J., 1995, "What to do (and to do) with time-series cross-section data," *American political science review*, vol. 89, no. 3, p. 634-647.

Beck, N., Katz, J., 2004, "Time-Series–Cross-Section Issues: Dynamics, 2004," Working Paper, July 26.

Bernard, V., 1989, "Capital market research in accounting during the 1980's: a critical review." In: Frecka, T., (eds.) *The state of accounting research as we enter the 1990s*. University if Illinois, Urbana-Champaign, p. 72-120.

Bernard, V., Thomas, J., 1990, "Evidence that stock prices do not fully reflect the implications of current earnings for future earnings," *Journal of Accounting and Economics*, vol. 13, p. 305-340.

Brooks, L., Buckmaster, D., 1976, "Further evidence of the time series properties of accounting income," *Journal of Finance*, vol. 31, no. 5, p. 1359-1373.

Brown, P., Ball, R., 1967, "Some preliminary findings on the association between the Earnings of a Firm, its industry, and the market," *Journal of Accounting Research*, vol. 5, p., 55-77.

Brozen, Y., 1970, "The antitrust task force deconcentration recommendation," *Journal of Law and Economics*, vol. 13, no. 2, p. 279-292.

Buzzell, R., Gale, T., 1987, *The PIMS principles : linking strategy to performance*, Free Press, New York.

Callen, J., Morel, M., 2001, "Linear Accounting Valuation When Abnormal Earnings Are AR(2)," *Review of Quantitative Finance and Accounting*, vol. 16, p. 191-203.

Collins, D., Kothari, S., 1989, "An analysis of intertemporal and cross-sectional determinants of earnings response coefficients," *Journal of Accounting and Economics*, vol. 11, no. 2-3, p. 143-181.

Congleton, R., 2001, "In defense of ignorance: On the significance of a neglected form of incomplete information," *Eastern Economic Journal*, vol. 27, no. 4, p. 391-407.

Conlisk, J., 1996, "Why Bounded Rationality?," *Journal of Economic Literature*, vol. 34, no. 2, p. 669-700.

Copeland, T., Koller, T., Murrin, J., 2000, *Valuation : measuring and managing the value of companies*, Wiley, New York..

Cubbin, J., Geroski, P., 1987, "The convergence of profits in the long run: Inter-firm and inter-industry comparisons," *The Journal of Industrial Economics*, vol. 35, no. 4, p. 427-442.

Damodaran, A., 2000, *Corporate Finance: theory and practice*, 2nd edn, Wiley, New York.

Daniel, K., Hirshleifer, D., & Subramanyam, A., 1998, "Subject psychology and security market under- and overreactions," *Journal of Finance*, vol. 53, p. 1839–1885.

DeBond, W., Thaler, R., 1985, "Does the stock market overreact?" *Journal of Finance*, vol. 40, p. 793–805.

Debreu, A., 1959, *Theory of value: an axiomatic analysis of economic equilibrium*, Wiley, New York.

Dechow, P., Hutton, A., Sloan, R., 1999, "An empirical assessment of the residual income valuation model," *Journal of accounting and economics*, vol. 26, p. 1-34.

Dekel, E., Lipman, B., Rustichini, A., 1998, "Standard state-space models preclude unawareness," *Econometrica*, vol. 66, no. 1, p. 159-173.

Demski, J., 1972, *Information analysis*, Addison-Wesley, Reading.

Dielman, T., 1989, *Pooled cross-sectional and time series data analysis*, Dekker, New York.

Dothan, M., 1990, *Prices in financial markets*, Oxford University Press, New York.

Duffie, D., 1992, *Dynamic assets pricing theory*, Princeton University Press, Princeton.

Easton, P., Zmijewski, M., 1989, "Cross-sectional variation in the stock market response to accounting earnings announcements," *Journal of Accounting and Economics*, vol. 11, no. 2-3, p. 117-141.

Eichberger, J., Harper, I., 1997, *Financial economics*, Oxford University Press, New York.

Eriksson, J., 2003, "Treatment and editing of tax data for Swedish structural business," Working paper, *United Nations Statistical Commission and Economic Commission for Europe*, Conference of European statisticians, WP.12.

Fairfield, P, Sweeney, R., Yohn, T., 1996, "Accounting classification and the predictive content of earnings," *Accounting Review*, vol. 71, no. 3, p. 337-355.

Fama, E., 1965a, "The Behavior of Stock Market Prices," *Journal of Business*, vol. 38, no. 1, p. 34-105.

Fama, E., 1965b, "Random Walks in Stock Market Prices," *Financial Analysts Journal*, vol. 21, no. 5, p. 55-59.

Fama, E., 1970, "Efficient Capital Markets: A Review of the theory and Empirical Work," *Journal of Finance*, p. 383-417.

Fama, E., 1991, "Efficient Capital Markets: II," *Journal of Finance*, vol. 46, no. 5, p. 1575-1617.

Fama, E., French, K., 2000, "Forecasting profitability and earnings," *Journal of Business*, vol. 73, no. 2, p. 161-175.

Fama, E., Miller, M., 1972, *Theory of finance*, Holt, Rinehart and Winston, New York.

FARs Samlingsvolym, 1994, FARs förlagsservice, Stockholm.

Feltham, G., Ohlson, J., 1995, "Valuation and clean surplus accounting for operating and

financial activities," *Contemporary Accounting Research*, vol.11, p. 689-731.

Feltham, G., Ohlson, J., 1996, "Uncertainty resolution and theory of depreciation measurement," *Journal of Accounting Research*, vol.34, no. 2, p. 209-234.

Feltham, G., Ohlson, J., 1999, "Residual earnings valuation with risk and stochastic interest rates," *The Accounting Review*, vol. 74, p. 165-183.

Fieldsend, S., Longford, N., McLeay, S., 1987, "Industry effects and the proportionality conjecture in ratio analysis: A variance component analysis," *Journal of Business Finance & Accounting*, vol. 14, no. 4, p. 497-517.

Fildes, R., 1992, "The evaluation of extrapolative forecasting methods," *International Journal of Forecasting*, vol. 8, p. 81-98.

Finger, C., 1994, "The ability of earnings to predict future earnings and cash flow, *Journal of accounting research*, vol. 32, no. 2, p. 210-223.

Finansanalytikernas rekommendationer, 1994, Sveriges finansanalytikers service, Stockholm.

Fishburn, P., 1979, *Utility theory for choice making*, Huntington, New York.

Fisher, I., 1930, *Theory of interest*, Macmillan, New York.

Freeman, R., Ohlson, J., Penman, S., 1982, "Book rate-of-return and prediction of earnings changes: An empirical investigation," *Journal of Accounting Research*, vol. 20, no. 2., part II, p. 639-653.

Friedman, M., 1953, The methodology of positive economics, In Friedman, M., 1953, *Essays in positive economics*, University of Chicago Press, Chicago.

Geroski, P., Jacquemin, A., 1988, "The persistence of profits: A European comparison," *Economic Journal*, vol. 98, no. 391, p. 375-389.

Giner, B., Iñiguez, R., 2006, "An empirical assessment of the Feltham-Ohlson models considering the sign of abnormal earnings," *Accounting and Business Research*, vol. 36, no. 3, p. 169-190.

Gonedes, N., 1975, "A note on accounting-based and market-based estimates of systematic risk," *Journal of Financial and Quantitative Analysis*, vol. 10, no. 2, p. 355-365.

Goodall, C., 1983, "M-estimators of location: An outline of the theory," In Hoaglin, Mosteller, Tukey, (eds.) 1983, *Understanding Robust and Exploratory Data Analysis*, Wiley, New York, p. 339-403.

Graflund, A., 2001, "Some Time Serial Properties of the Swedish Real Estate Stock Market, 1939-1998," *Department of Economics*, Lund university.

Gravelle, H., Rees, R., 1998, *Microeconomics*, 2nd edn, Longman, London.

Greene, W., 1993, *Econometric analysis*, 3rd edn, Pearson education, Upper Saddle River.

Greene, W., 2003, *Econometric analysis*, 5th edn, Pearson education, Upper Saddle River.

Gregory, A., Saleh, W., Tucker, J. 2005, "A UK test of an inflation-adjusted Ohlson model," *Journal of Business Finance & Accounting*, vol. 32., no. 3-4, p. 487-534.

Gross, A., 1976, "Confidence interval robustness with long-tailed symmetric distributions," *Journal of American Statistical Association*, vol. 71, no. 354, p. 409-416.

Grossman, S., 1981, "An introduction to the theory of rational expectations under asymmetric information," *The Review of Economic Studies*, vol. 48, no. 4, p. 541-559.

Gupta, M., Huefner, R., 1972, "A cluster analysis study of financial ratios and industry characteristics," *Journal of Accounting Research*, vol. 10, no. 1, p. 77-95.

Hayek, F., 1936, "Economics and knowledge," In Hayek, F., 1949, *Individualism and economic order*, Routledge and Kegan Paul Ltd., London.

Hayek, F., 1945, "The use of knowledge in society," In Hayek, F., 1949, *Individualism and economic order*, Routledge and Kegan Paul Ltd., London.

Hamberg, M., 2000, *Risk, uncertainty, and profitability. An accounting-based study of industrial firms' financial performance*, Doctoral thesis no. 80, Uppsala University, Department of Business Studies, Uppsala.

Hirshleifer, J., 1970, *Investment, interest and capital*, Prentice-Hall, Englewood Cliffs.

Holthausen, R., Larcker, D., 1992, "The prediction of stock returns using financial statement information," *Journal of Accounting and Economics*, vol. 15, no. 2/3, p. 3373-411.

Hong and Stein, 1999, "A unified theory of underreaction, momentum trading, and overreaction in asset markets," *Journal of Finance*, vol. 54, no. 6, p. 2143-2184.

Huang, C. F., Litzenberger, R., 1988, *Foundations for financial economics*, Prentice Hall, Englewood Cliffs.

Hyndman, R., Koehler, A., 2005, "Another look at measures of forecast accuracy," Working Paper, May 20.

Iglewicz, B., 1983, "Robust scale estimators and confidence intervals for location," In Hoaglin, Mosteller, Tukey, (eds.), 1983, *Understanding Robust and Exploratory Data Analysis*, Wiley, New York, p. 404-431.

Jacobsen, R., 1988, "The persistence of abnormal returns," *Strategic Management Journal*, vol. 9, no. 5, 415-430.

Jacobsen, J., Voetmann. T., 1999, "Post-acquisition performance in the short and long run," *Department of finance*, Copenhagen Business School.

Jacobson, R., Aaker, D., 1985, "Is market share all that it's cracked up to be?," *Journal of Marketing*, vol. 49, no. 4, p. 11-22.

Kahneman, D., 2003, "Maps of bounded rationality: Psychology for behavioral economics," *American Economic Review*, vol. 93, no. 5, p. 1449-1475.

Kahneman, D., Tversky, A., 1979, "Prospect theory: An analysis of decision under risk," *Econometrica*, vol. 47, no. 2, p. 262-292.

Kirzner, I., 1973, *Competition and entrepreneurship*, University of Chicago, Chicago.

Kirzner, I. 1992, *The meaning of market process: essays in the development of modern Austrian economics*, Routledge, London.

Kothari, S., 2001, "Capital market research in accounting," *Journal of Accounting and Economics*, vol. 31, p. 105-231.

Kreps, D., 1988, *Notes on theory of choice*, Westview, Boulder.

Kreps, D., 1990, *Game Theory and Economic Modelling*, Oxford University Press, Oxford.

Landsman and Maydew, 2002, "Has the Information Content of Quarterly Earnings Announcements Declined in the Past Three Decades?, *Journal of Accounting Research*, vol. 40, no. 3, p. 797-808.

Lee, C., 1985, "Stochastic properties of cross sectional financial data," *Journal of Accounting Research*, vol. 23, no. 1, p. 213-227.

Lee, C., 2001, "Market efficiency and accounting research: a discussion of 'capital market research in accounting' by S.P. Kothari, 2001," *Journal of Accounting and Economics*, vol. 31, p. 233-253.

Lev, B., 1974, "On the association between operating leverage and risk," *Journal of Financial and Quantitative Analysis*, vol. 9, no. 4, p. 627-641.

Lev, B., 1983, "Some economic determinants of time-series properties of earnings," *Journal of Accounting and Economics*, vol. 5, p. 31-48.

Lev, B., Ohlson, J., 1982, "Market based empirical research in accounting: a review, interpretations, and extensions," *Journal of Accounting Research (Supplement)*, vol. 20, p. 249-322.

Lev, B., Sougiannis, T., 1996, "The capitalization, amortization, and value-relevance of R&D," *Journal of Accounting and Economics*, vol. 21, p. 107-138.

Lev, B., Sunder, S., 1979, "Methodological issues of financial ratios," *Journal of Accounting and Economics*, vol. 1, no. 3, p. 187-210.

Levin, J., 1998, *Essays in company valuation*, Doctoral thesis, Economic Research Institute, Stockholm School of Economics, Stockholm.

Lipman, B., 1995, "Information processing and bounded rationality: A Survey," *The Canadian Journal of Economics / Revue canadienne d'Economique,* vol. 28, no. 1, p. 42-67.

Liu and Thomas, 2000, "Stock returns and accounting earnings," *Journal of Accounting Research*, vol. 38, no. 1, p. 71-101.

Lo, K., Lys, T., 2000, "The Ohlson model: Contribution to valuation theory, limitations, and empirical applications," *Journal of Accounting, Auditing, and Finance*, vol. 15, no. 3, p. 337-367.

Lookabill, L. 1976, "Some additional evidence on the time series properties of accounting earnings," *Accounting Review*, vol. 51, no. 4, p. 724-738.

Lundholm, R., 1995, "A tutorial on the Ohlson and Feltham/Ohlson models: Answers to some frequently asked questions," *Contemporary accounting research*, vol. 11, no. 2, p. 749-761.

Lundholm, R., O'Keefe, T., 2001, "Reconciling value estimates from the discounted cash flow model and the residual income model," *Contemporary Accounting Research*, vol. 18, no. 2, p. 311-335.

Mas-Colell, A., Whinston, M., Green, J., 1995, *Microeconomic theory*, Oxford University Press, New York.

Malkiel, B., 2003, "The efficient market hypothesis and its critics," *Journal of Economic Perspectives*, vol. 17, no. 1, p. 59-82.

McCrae, M., Nilsson, H., 2001, "The explanatory and predictive power of different specifications of the Ohlson (1995) valuation models," *European Accounting Review*, vol. 10, no. 2, p. 315-341.

McDonald, B., Morris, M., 1984, "The statistical validity of the ratio method in financial analysis: An empirical examination," *Journal of Business Finance & Accounting*, vol. 11, no. 1, p. 89-97.

McLeay, S., Fieldsend, S., 1987, "Sector and size effects in ratio analysis: An indirect test of ratio proportionality," *Accounting and Business Research*, vol. 17, no. 66, p. 133-140.

Meyers, J, 1999, "Implementing residual income valuation with linear information dynamics," *Accounting Review*, vol. 74, no. 1, p. 1-28.

Mises, von L., 1949, *Human action: a treatise on economics*, London.

Modica, S., Rustichini, A., 1994, "Awareness and partitional information structures," *Theory and Decision*, vol. 37, no. 1, p. 107-124.

Modica, S., Rustichini, A., 1999, "Unawareness and partitional information structures," *Games and Economic Behavior*, vol. 27, p. 265-298.

Modigliani, F., Miller, M., 1958, "The cost of capital, corporation finance, and theory of investment," *American Economic Review*, vol. 48, no. 3, p. 261-297.

Mosteller, F., Tukey, J., 1977, *Data analysis and regression: a second course in statistics*, Addison-Wesley, Reading.

Mueller, D., 1977, "The persistence of profits above the norm," *Economica*, vol. 44, no. 176, p. 369-380.

Mueller, D., 1990, in Mueller, D., 1990, *The dynamics of company profits : an international comparison*, Cambridge University Press, Cambridge.

Muth, J., 1961, "Rational expectations and the theory of price movements," *Econometrica*, vol. 29, no. 3, p. 315-335.

Nagel, E., 1963, "Assumptions in economic theory," *American Economic Review*, vol. 53, no. 2, p. 211-219.

Nerlove, M., 1958, "Adaptive expectations and cobweb phenomena," *Quarterly Journal of Economics*, vol. 72, no. 2, p. 227-240.

Nerlove, M., 1972, "Lags in economic behavior," Econometrica, vol. 40, no. 2, p. 221-251.

Newbold, P., 1995, *Statistics for business and economics*, Prentice Hall, Englewood Cliffs.

Nickell, S., 1981, "Biases in dynamic model with fixed effect," *Econometrica*, vol. 49, no. 6, p. 1417-1426.

Nissim, D., Penman, S., 2001, "Ratio analysis and equity valuation: From research to practice," *Review of accounting studies*, vol. 6, p. 109-154.

Ohlson, J., 1987, *Theory of financial markets and information*, North-Holland, New York.

Ohlson, J., 1995, "Earnings, book values, and dividends in equity valuation," *Contemporary Accounting Research*, vol. 11, p. 661-687.

Ohlson, J., 2003, "Positive (Zero) NPV projects and the behavior of residual earn-

ings," *Journal of Business Finance & Accounting*, vol. 30, no. 1, p. 7-15.

Ou, J., Penman, S., 1989a, "Financial statement analysis and the prediction of stock returns," *Journal of Accounting and Economics*, vol. 11, p. 295-329.

Ou, J., Penman, S., 1989b, "Accounting measurement, price-earnings ratio, and the information content of security prices," *Journal of Accounting Research (Supplement)*, vol. 27, p. 111-144.

Pareto, V., 1906, *Manual of political market*.

Penman, S., 1991, An evaluation of accounting rate-of-return, *Journal of Accounting, Auditing & Finance*, vol. 6, no. 2, p. 233-255.

Penman, S., 2004, *Financial statement and security valuation*, 2nd edn, McGraw-Hill, New York.

Penman, S. , Zhang, X.-J., 2002, "Accounting conservatism, the quality of earnings, and stock returns, *Accounting Review*, vol. 77, no. 2, p. 237-264.

Persky, J., 1995, Retrospectives: The ethnology of the perfect rational choice. *Journal of Economic Perspectives*, vol. 9, no. 2, p. 221-231.

Piotroski, J., 2000, "Value investing: The use of historical financial statement information to separate winners from losers," *Journal of Accounting Research*, vol. 38, Supplement: studies on accounting information and the economics of the firm, p. 1-41.

Pliska, S., 1997, *Introduction to mathematical finance: discrete time models*, Blackwell, Malden.

Popper, K., 1959, *The logic of scientific discovery*, Hutchinson, London.

Preinreich, G. 1938, "Annual survey of economic theory: Theory of depreciation," *Econometrica*, vol. 6, p. 219-231.

Preinreich, G., 1936, "The Law of Goodwill," *The Accounting Review*, vol. 11, p. 317-329.

Preinreich, G., 1937a, "Goodwill in accountancy," *Journal of Accountancy*, vol. 93, p. 28-51.

Preinreich, G., 1937b, "Valuation and amortization," *The Accounting Review*, vol. 12, p. 209-226.

Rosenberger, J., Gasko, M., 1983, "Comparing location estimators: Trimmed means, medians, and trimean." In Hoaglin, Mosteller,

Tukey, (eds.), 1983, *Understanding Robust and Exploratory Data Analysis*, Wiley, New York, p. 297-338.

Rubinstein, A., 1998, *Modeling bounded rationality*, MIT Press, Cambridge.

Rubinstein, M., 1976, "The valuation of uncertain income streams and the pricing of options," *Bell Journal of Economics*, vol. 7, p. 407-425.

Rumsfeld, D., 2002, "Transcript: Defense Department Briefing, February 12, 2002" (Electronic). Available: <http://www.globalsecurity.org/military/library/news/2002/02/mil-020212-usia01.htm> (May 5, 2007).

Runsten, M. 1998, *The association between accounting information and stock prices: model development and empirical tests based on Swedish data*, Doctoral thesis, Economic Research Institute, Stockholm School of Economics, Stockholm.

Samuelson, L., 2004, "Modeling knowledge in economic analysis," *Journal of Economic Literature*, vol. 42, no. 2, p. 367-403.

Savage, L., [1954] 1972, *The foundations of statistics*, John Wiley, New York.

Simon, H., 1955, "A behavioral model of rational choice," *Quarterly Journal of Economics*, vol. 69, no. 1, p. 99-118.

Simon, H., 1956, "Rational choice and the structure of the environment," *Psychological Review*, vol. 63, no. 2, p. 129-138.

Simon, H., 1986, "Rationality in psychology and economics," *Journal of Business*, vol. 59, no. 4, p. S209-S224.

Simon, H., 1990, "Invariants of human behavior," *Annual Review of Psychology*, vol. 41, p. 1-19.

Simon, H., 1993, "Altruism and economics," *American Economic Review*, vol. 83, no. 2, p. 156-161.

Shefrin, H., 2002, *Beyond greed and fear: Understanding behavioral finance and the psychology of investing*, Oxford University Press, New York.

Shleifer, A., 2000, *Inefficient markets: an introduction to behavioral finance*, Oxford University Press, Oxford.

Shiller, R., 2003, "From efficient markets theory to behavioral finance," *Journal of Economic Perspectives*, vol. 17, no. 1, p. 83-104.

Statistics Sweden, 2006, [Online], available at http://www.foretagsregistret.scb.se/sni/.

Stewart, B., 1991, *The quest for value: the EVA® management guide*, Harper Business, New York.

Stickney, P., Brown, P., 1999, *Financial reporting and statement analysis – a strategic perspective*, Dryden, Orlando.

Stigler, G., "The economics of information," *Journal of Political Economy*, vol. 69, no. 3, p. 213-225.

Sydsaeter, K., Strom, A., Berck, P., 1999, *Economists' mathematical manual*, Springer, New York

Tashman, L., 2000, Out-of-sample tests of forecasting accuracy: an analysis and review, *International Journal of Forecasting*, vol. 16, no. 4, p. 437–450.

Tversky, A., Kahneman, D., 1981, "The framing of decisions and the psychology of choice," *Science*, vol. 211, p. 453-458.

Walras, L., [1874] 1954, *Elements of pure economics or theory of social wealth / Translated by Jaffé, W.,* Irwin, Homewood.

Waring, G., 1996, "Industry differences in the persistence of firm-specific returns," *American Economic Review*, vol. 86, no. 5, p. 1253-1265.

Watt, R., Leftwich, R., 1977, "The time series of annual accounting earnings," *Journal of Accounting Research*, vol. 15, no. 2, p. 253-271.

Watts, R., Zimmerman, J., 1986, *Positive accounting theory*, Prentice-Hall, Englewood Cliffs.

DOCTORAL THESES

1 Nellbeck, Lennart, 1967, *Trävaruexportens distributionsvägar och förbrukning* och *Trävaruexportens distributionsled en modell.* Stockholm: Scandinavian University Books.

2 Ramström, Dick, 1967, *The Efficiency of Control Strategies.* Stockholm: Almqvist & Wiksell.

3 Landström, Lars, 1970, *Statligt kontra privat företagande. En jämförande organisationsteoretisk studie av det statliga företagets beteende.* Uppsala: Företagsekonomiska institutionen.

4 Skår, John, 1971, *Produksjon og produktivitet i detaljhandelen. En studie i teori, problem og metoder.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 3.

5 Wadell, Birgitta, 1971, *Daghemsbarns frånvaro ett kommunalt planeringsproblem.* Uppsala: Företagsekonomiska institutionen.

6 von der Esch, Björn, 1972, *Skatt, inflation, eget kapital: en ekonometrisk studie av lantbruksföretag.* Uppsala: Företagsekonomiska institutionen.

7-8 Hörnell, Erik & Vahlne, Jan-Erik, 1972, *The Deciding Factors in the Choice of a Subsidiary as the Channel for Exports.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 6.

9 Mattsson, Anders, 1972, *The Effects of Trade Barriers on the Export Firm.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 5.

10 Tornberg, Georg, 1972, *Internationell marknadskommunikation.* Stockholm: Prisma.

11 Wiedersheim-Paul, Finn, 1972, *Uncertainty and Economic Distance. Studies in International Business.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 7.

12 Söderbaum, Peter, 1973, *Positionsanalys vid beslutsfattande och planering.* Stockholm: Läromedelsförlagen.

13 Håkansson, Håkan, 1975, *Studies in Industrial Purchasing with Special Reference to Determinants of Communication Patterns.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 9.

14 Okoso-Amaa, Kweku, 1975, *Rice Marketing in Ghana. Uppsala:* Nordiska Afrikainstitutet.

15 Olson, Hans Christer, 1975, S*tudies in Export Promotion: Attempts to Evaluate ExportStimulation Measures for the Swedish Textile and Clothing Industries.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 10.

16 Wootz, Björn, 1975, *Studies in Industrial Purchasing with Special Reference to Variations in External Communication.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 8.

17   Åkerblom, Mats, 1975, *Företag i inflation*. Uppsala: Research Report nr 7 (mimeo).

18   Johansson, Sven-Erik, 1976, *Fåmansbolag*. Uppsala: Research Report nr 1976:1. (mimeo).

19   Samuelsson, Hans-Fredrik, 1977, *Utländska direkt investeringar i Sverige*. (mimeo).

20   Lundberg, Lars, 1982, *Från Lag till Arbetsmiljö*. Malmö: Liberförlag.

21   Hallén, Lars, 1982, *International Industrial Purchasing. Channels, Interaction, and Governance Structures*. Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 13.

22   Jansson, Hans, 1982, *Interfirm Linkages in a Developing Economy. The Case of Swedish Firms in India*. Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 14.

23   Axelsson, Björn, 1982, *Wikmanshyttans uppgång och fall*. Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 15.

24   Sharma, Deo D., 1983, *Swedish Firms and Management Contracts*. Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 16.

25   Sandberg, Thomas, 1982, *Work Organizations and Autonomous Groups*. Lund: Liberförlag.

26   Ghauri, Pervez, 1983, *Negotiating International Package Deals*. Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr. 17.

27   Joachimsson, Robert, 1984, *Utlandsägda dotterbolag i Sverige. En analys av koncerninterna transaktionsmönster och finansiella samband*. Stockholm: Liberförlag.

28   Kallinikos, Jannis, 1984, *Control and Influence Relationships in Multinational Corporations: The Subsidiary's Viewpoint. Application of the Resource Dependence Perspective for Studying Power Relationships in Multinational Corporations*. Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 19.

29   Hadjikhani, Amjad, 1985, *Organization of Manpower Training in International Package deal Projects Temporary Organizations for Transfer of Technology*. Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 21.

30   Klint, Mats B., 1985, *Mot en konjunkturanpassad kundstrategi*. Uppsala: Företagsekonomiska institutionen.

31   Larsson, Anders, 1985, *Structure and Change Power in the Transnational Enterprise*. Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 23.

32   Lorendahl, Bengt, 1986, *Regionutvecklings- och lokaliseringsprocesser. Beslut och handling i kommunal näringspolitik och industriell lokalisering*. Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 24.

33   Borg, Malcolm, 1987, *International Transfers of Managers in Multinational Corporations. Transfer Patterns and Organizational Control.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 27.

34   Thunman, Carl G., 1987, *Technology Licensing to Distant Markets Interaction Between Swedish and Indian Firms.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 28.

35   Hyder, Syed Akmal, 1988, *The Development of International Joint Venture Relationships. A Longitudinal Study of Exchange of Resources, Control and Conflicts.* Uppsala: Department of Business Studies.

36   Gunnarsson, Elving, 1988, *Från Hansa till Handelshögskola. Svensk ekonom-undervisning fram till 1909.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 29.

37   Wallerstedt, Eva, 1988, *Oskar Sillén. Professor och praktiker. Några drag i företagsekonomiämnets tidiga utveckling vid Handelshögskolan i Stockholm.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 30.

38   Smith, Dag, 1989, *Structure and Interpretation of Income Models.* Uppsala: Department of Business Studies

39   Laage-Hellman, Jens, 1989, *Technological Development in Industrial Networks.* Comprehensive Summaries of Uppsala Dissertations from the Faculty of Social Sciences nr 6.

40   Waluszewski, Alexandra, 1989, *Framväxten av en ny mekanisk massateknik.* Uppsala: Acta Universitatis Upsaliensis, Studia Oeconomiae Negotiorum nr 31.

41   Seyed-Mohamed, Nazeem, 1990, *Structural Modelling of Business Relation-ships.* Comprehensive Summaries of Uppsala Dissertations from the Faculty of Social Sciences nr 21.

42   Snehota, Ivan, 1990, *Notes on a Theory of Business Enterprise.* Uppsala: Department of Business Studies.

43   Hultbom, Christina, 1991, *Intern handel. Köpar/säljarrelationer inom stora företag.* Uppsala: Företagsekonomiska institutionen.

44   Lindvall, Jan, 1991, *Svenska industriföretags internationella företagsförvärv, inriktning och utfall.* Uppsala: Företagsekonomiska institutionen.

45   Levinson, Klas, 1991, *Medbestämmande i strategiska beslutsprocesser. Facklig medverkan och inflytande i koncerner.* Uppsala: Företagsekonomiska institutionen.

46   Lee, Joong-Woo, 1991, *Swedish Firms Entering the Korean Market Position Development in Distant Industrial Networks.* Uppsala: Department of Business Studies

47   Molin, Roger, 1991, *Organisationen inom facket. Organisationsutvecklingen inom de till Landsorganisationen anslutna förbunden.* Stockholm: Carlssons Bokförlag.

48   Henders, Barbara, 1992, *Marketing Newsprint in the UK Analyzing Positions in Industrial Networks.* Uppsala: Department of Business Studies.

49   Lampou, Konstantin, 1992, *Vårt företag. En empirisk undersökning av några organisatoriska självuppfattningar och identiteter.* Uppsala: Företagsekonomiska institutionen.

50   Jungerhem, Sven, 1992, *Banker i fusion.* Uppsala: Företagsekonomiska institutionen.

51   Didner, Henrik, 1993, *Utländskt ägande av svenska aktier.* Uppsala: Företags-ekonomiska institutionen.

52   Abraha, Desalegn, 1994, *Establishment Processes in an Underdeveloped Country. The Case of Swedish Firms in Kenya.* Uppsala: Department of Business Studies.

53   Holm, Ulf, 1994, *Internationalization of the Second Degree.* Uppsala: Department of Business Studies.

54   Eriksson, Kent, 1994, *The Inter-relatedness of Environment Technology and Structure A Study of Differentiation and Integration in Banking.* Uppsala: Department of Business Studies.

55   Marquardt, Rolf, 1994, *Banketableringar i främmande länder.* Uppsala: Företagsekonomiska institutionen.

56   Awuah, Gabriel B., *1994, The Presence of Multinational Companies (MNCs) in Ghana A Study of the Impact of the Interaction between an MNC and Three Indigenous Companies.* Uppsala: Department of Business Studies.

57   Hasselbladh, Hans, 1995, *Lokala byråkratiseringsprocesser, institutioner, tolkning och handling.* Uppsala: Företagsekonomiska institutionen.

58   Eriksson, Carin B., 1995, *Föreställningar och värderingar i en organisation under förändring - en studie av identitetsuppfattningar inom konsument-kooperationen.* Uppsala: Företagsekonomiska institutionen.

59   Jonsson, Tor, 1995, *Value Creation in Mergers and Acquisitions: A Study of Swedish Domestic and Foreign Takeovers.* Uppsala: Department of Business Studies.

60   Furusten, Staffan, 1995, *The Managerial Discourse - A Study of the Creation and Diffusion of Popular Management Knowledge.* Uppsala: Department of Business Studies.

61   Pahlberg, Cecilia, 1996, *Subsidiary - Headquarters Relationships in International Business Networks.* Uppsala: Department of Business Studies.

62   Sjöberg, Ulf, 1996, *The Process of Product Quality - Change Influences and Sources - A Case from the Paper and Paper-Related Industries.* Uppsala: Department of Business Studies.

63 Lind, Johnny, 1996, *Ekonomistyrning och verksamhet i utveckling - Ekonomiska rapporters utformning och användning när verksamheten flödesorienteras*. Uppsala: Företagsekonomiska institutionen.

64 Havila, Virpi, 1996, *International Business-Relationships Triads - A Study of the Changing Role of the Intermediating Actor*. Uppsala: Department of Business Studies.

65 Blankenburg Holm, Desirée, 1996, *Business Network Connections and International Business Relationships*. Uppsala: Department of Business Studies.

66 Andersson, Ulf, 1997, *Subsidiary Network Embeddedness. Integration, Control and Influence in the Multinational Corporation*. Uppsala: Department of Business Studies.

67 Sanner, Leif, 1997, *Trust Between Entrepreneurs and External Actors. Sensemaking in Organising New Business Ventures*. Uppsala: Department of Business Studies.

68 Thilenius, Peter, 1997, *Subsidiary Network Context in International Firms*. Uppsala: Department of Business Studies.

69 Tunisini, Annalisa, 1997, *The Dissolution of Channels and Hierarchies - An Inquiry into the Changing Customer Relationships and Organization of the Computer Corporations*. Uppsala: Department of Business Studies.

70 Majkgård, Anders, 1998, *Experiential Knowledge in the Internationalization Process of Service Firms*. Uppsala: Department of Business Studies.

71 Hjalmarsson, Dan, 1998, *Programteori för statlig företagsservice*. Uppsala: Företagsekonomiska institutionen.

72 Avotie, Leena, 1998, *Chefer ur ett genuskulturellt perspektiv*. Uppsala: Företagsekonomiska institutionen.

73 Arnesson, Leif, 1998, *Chefsrörlighet*. Uppsala: Företagsekonomiska institutionen.

74 Dahlqvist, Jonas, 1998, *Knowledge Use in Business Exchange – Acting and Thinking Business Actors*. Uppsala: Department of Business Studies.

75 Jonsson, Eskil, 1998, *Narrow Management. The Quest for Unity in Diversity*. Uppsala: Department of Business Studies.

76 Greve, Jan, 1999, *Ekonomisystem och affärsstrategier*. Uppsala: Företagsekonomiska institutionen.

77 Roxenhall, Tommy, 1999, *Affärskontraktets användning*. Uppsala: Företagsekonomiska institutionen.

78 Blomgren, Maria, 1999, *Pengarna eller livet?* Uppsala: Företagsekonomiska institutionen.

79 Bäckström, Henrik, 1999, *Den krattade manegen: Svensk arbetsorganisatorisk utveckling under tre decennier*. Uppsala: Företagsekonomiska institutionen

80   Hamberg, Mattias, 2000, *Risk, Uncertainty & Profitability. An Accounting-Based Study of Industrial Firms' Financial Performance*. Uppsala: Department of Business Studies.

81   Sandberg, Eva, 2000, *Organiseringens dynamik – Strukturskapande processer i ett telematikföretag.* Uppsala: Företagsekonomiska institutionen.

82   Nordin, Dan, 2000, *Två studier av styrning i kunskapsintensiva organisationer*. Uppsala: Företagsekonomiska institutionen.

83   Wedin, Torkel, 2001, *Networks and Demand. The Use of Electricity in an Industrial Process.* Uppsala: Department of Business Studies.

84   Lagerström, Katarina, 2001, *Transnational Projects within Multinational Corporations*. Uppsala: Department of Business Studies.

85   Markgren, Bertil, 2001, *Är närhet en geografisk fråga? Företags affärs-verksamhet och geografi – En studie av beroenden mellan företag och lokaliseringens betydelse*. Uppsala: Företagsekonomiska institutionen.

86   Carlsson, Leif, 2001, *Framväxten av en intern redovisning i Sverige – 1900-1945*. Uppsala: Företagsekonomiska institutionen.

87   Silver, Lars, 2001*, Credit Risk Assessment in Different Contexts – The Influence of Local Networks for Bank Financing of SMEs.* Uppsala: Department of Business Studies.

88   Choi, Soon-Gwon, 2001, *Knowledge Translation in the Internationalization of Firms*. Uppsala: Department of Business Studies.

89   Johanson, Martin, 2001*, Searching the Known, Discovering the Unknown. The Russian Transition from Plan to Market as Network Change Processes*. Uppsala: Department of Business Studies.

90   Hohenthal, Jukka, 2001, *The Emergence of International Business Relationships. Experience and performance in the internationalization process of SMEs*. Uppsala: Department of Business Studies.

91   Gidhagen, Mikael, 2002, *Critical Business Episodes. The Criticality of Damage Adjustment Processes in Insurance Relationships*. Uppsala: Department of Business Studies.

92   Löfmarck Vaghult, Anna, 2002, *The Quest for Stability. A Network Approach to Business Relationship Endurance in Professional Services*. Uppsala: Department of Business Studies.

93   Grünberg, Jaan, 2002, *Problematic Departures. CEO Exits in Large Swedish Publicly Traded Corporations*. Uppsala: Department of Business Studies.

94   Gerdin, Jonas, 2002, *Ekonomisystems utformning inom produktionsavdelningar: En tvärsnittsstudie*. Uppsala: Företagsekonomiska institutionen.

95   Berggren, Björn, 2002, *Vigilant Associates – Financiers Contribution to the Growth of SMEs*. Uppsala: Department of Business Studies.

96   Elbe, Jörgen, 2002, *Utveckling av turistdestinationer genom samarbete*. Uppsala: Företagsekonomiska institutionen.

97   Andersson, Maria, 2003, *Creating and Sharing Subsidiary Knowledge within Multinational Corporations*. Uppsala: Department of Business Studies.

98   Waks, Caroline, 2003, *Arbetsorganisering och professionella gränsdragningar. Sjukgymnasters samarbete och arbetets mångfald.* Uppsala: Företagsekonomiska institutionen.

99   Bengtson, Anna, 2003, *Framing Technological Development in a Concrete Context – the Use of Wood in the Swedish Construction Industry*. Uppsala: Department of Business Studies.

100   Bäcklund, Jonas, 2003, *Arguing for Relevance – Global and Local Knowledge Claims in Management Consulting*. Uppsala: Department of Business Studies.

101   Levay, Charlotta, 2003, *Medicinsk specialisering och läkares ledarskap*: *En longitudinell studie i professionell kollegialitet och konkurrens*. Uppsala: Företagsekonomiska institutionen.

102   Lindholm, Cecilia, 2003, *Ansvarighet och redovisning i nätverk. En longitudinell studie om synliggörande och osynliggörande i offentlig verksamhet*. Uppsala: Företagsekonomiska institutionen.

103   Svensson, Birgitta, 2003, *Redovisningsinformation för bedömning av små och medelstora företags kreditvärdighet*. Uppsala: Företagsekonomiska institutionen.

104   Lindstrand, Angelika, 2003, *The Usefulness of Network Experiential Knowledge in the Internationalization Process*. Uppsala: Department of Business Studies.

105   Baraldi, Enrico, 2003, *When Information Technology Faces Resource Interaction. Using IT Tools to Handle Products at IKEA and Edsbyn*. Uppsala: Department of Business Studies.

106   Prenkert, Frans, 2004, *On Business Exchange Activity. Activity Systems and Business Networks*. Uppsala: Department of Business Studies.

107   Abrahamsson, Gun & Helin, Sven, 2004, *Problemlösningsarbete på låg organisatorisk nivå. Två studier om implementering respektive konkretisering av idéer om kundorderstyrd tillverkning.* Uppsala: Företagsekonomiska institutionen.

108   Wedlin, Linda, 2004, *Playing the Ranking Game: Field formation and boundary-work in European management education*. Uppsala: Department of Business Studies.

109   Hedmo, Tina, 2004, Rule-making in the Transnational Space. The Development of European Accreditation of Management Education. Uppsala: Department of Business Studies.

110   Holmström, Christine, 2004, *In search of MNC competitive advantage: The role of foreign subsidiaries as creators and disseminators of knowledge.* Uppsala: Department of Business Studies.

111 Ciabuschi, Francesco, 2004, *On the Innovative MNC. Leveraging Innovations and the Role of IT Systems.* Uppsala: Department of Business Studies.

112 Ståhl, Benjamin, 2004, *Innovation and Evolution in the Multinational Enterprise.* Uppsala: Department of Business Studies.

113 Latifi, Mohammad, 2004, *Multinational Companies and Host Partnership in Rural Development. A Network Perspective on the Lamco Case.* Uppsala: Department of Business Studies.

114 Lindbergh, Jessica, 2005, *Overcoming Cultural Ignorance: Institutional Knowledge Development in the Internationalizing Firm.* Uppsala: Department of Business Studies.

115 Spencer, Robert, 2005, *Strategic Management of Customer Relationships. A Network Perspective on Key Account Management*, Uppsala: Department of Business Studies.

116 Neu, Elizabeth, 2006, *Lönesättning i praktiken. En studie av chefers handlingsutrymme,* Uppsala: Företagsekonomiska institutionen.

117 Gebert Persson, Sabine, 2006, *Crash-Landing in a Turbulent Transition Market. A Legitimating Activity?* Uppsala: Department of Business Studies.

118 Persson, Magnus, 2006, *Unpacking the Flow - Knowledge Transfer in MNCs.* Uppsala: Department of Business Studies.

119 Frimanson, Lars, 2006, *Management Accounting and Business Relationships from a Supplier Perspective.* Uppsala: Department of Business Studies.

120 Ström, Niklas, 2006, *Essays on Information Disclosure. Content, Consequence and Relevance*, Uppsala: Department of Business Studies.

121 Grafström, Maria, 2006, *The Development of Swedish Business Journalism. Historical Roots of an Organisational Field*, Uppsala: Department of Business Studies.

122 Flöstrand, Per, 2006, *Valuation Relevance. The Use of Information and Choice of Method in Equity Valuation*, Uppsala: Department of Business Studies.

123 Windell, Karolina, 2006, *Corporate Social Responsibility under Construction. Ideas, Translations, and Institutional Change*, Uppsala: Department of Business Studies.

124 Wictorin, Bo, 2007, *Är kluster lönsamma? En undersökning av platsens betydelse för företags produktivitet*, Uppsala: Företagsekonomiska institutionen.

125 Johed, Gustav, 2007, *Accounting, Stock Markets and Everyday Life*, Uppsala: Department of Business Studies.

126 Maaninen-Olsson, Eva, 2007, *Projekt i tid och rum – Kunskapsintegrering mellan projektet och dess historiska och organisatoriska kontext*, Uppsala: Företagsekonomiska institutionen.

127 Scherdin, Mikael, 2007, *The Invisible Foot. Survival of new art ideas in the swedish art arena – An autoethnographic study of nontvtvstation*, Uppsala: Department of Business Studies.