MECHANICAL BEHAVIOR OF CELLULAR STRUCTURES: A FINITE ELEMENT STUDY

A THESIS PRESENTED

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ABSTRACT

Cellular solids, such as foams are widely used in engineering applications. In these applications, it is important to know their mechanical properties and the variation of these properties with the presence of defects. Several models have been proposed to obtain the mechanical properties of cellular materials. However, some of these models are based on idealized unit cell structures, and are not suitable for finding the mechanical properties of cellular materials with defects. Also, a gradual increase in the cell size distribution, can impart many properties such as mechanical shock resistance and thermal insulation. Furthermore, functional gradation is one of the characteristic features of living tissue. Bio-inspired functionally graded materials open new approaches for manufacturing implants for bone replacement.

The objective of this work is to understand the effect of missing walls and filled cells on mechanical and creep behavior of both the regular hexagonal and non-periodic Voronoi structures using finite element analysis. Furthermore, mechanical properties of functionally graded cellular structures as a function of density gradient have not been previously addressed. In this study, the finite element method is used to investigate the compressive uniaxial and biaxial behavior of functionally graded Voronoi structures. The effect of missing cell walls on its overall mechanical (elastic, plastic, and creep) properties is also investigated.

The results showed that the missing walls have a significant effect on overall elastic properties of the cellular structure. For both regular hexagonal and Voronoi materials, the yield strength of the structure decreases by more than 60% by introducing

10% missing walls. In contrast, the results indicated that filled cells have much less effect on the mechanical properties of both regular hexagonal and Voronoi materials.

The finite element analysis also showed that the overall effective elastic modulus and yield strength of structures increased by increasing the density gradient. However, the overall elastic modulus of functionally graded structures was more sensitive to density gradient than the overall yield strength. The study also showed that the functionally graded structures with different density gradient had similar sensitivity to random missing cell walls. Creep analysis suggested that the structures with higher density gradient had lower steady-state creep rate compared to that of structures with lower density gradient.

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CHAPTER 1

INTRODUCTION

1. Introduction to Cellular Solids

A cellular solid is made up of an interconnected network of solid struts or plates which form the edges and faces of cells. The simplest form of cellular structure is a two-dimensional array of polygons which pack to fill a plane area like the hexagonal cells of the bee (Fig. 1); and for this reason we call such two-dimensional cellular materials *honeycombs*. More commonly, the cells are polyhedral which pack in three dimensions to form cellular materials which are called *foams*.

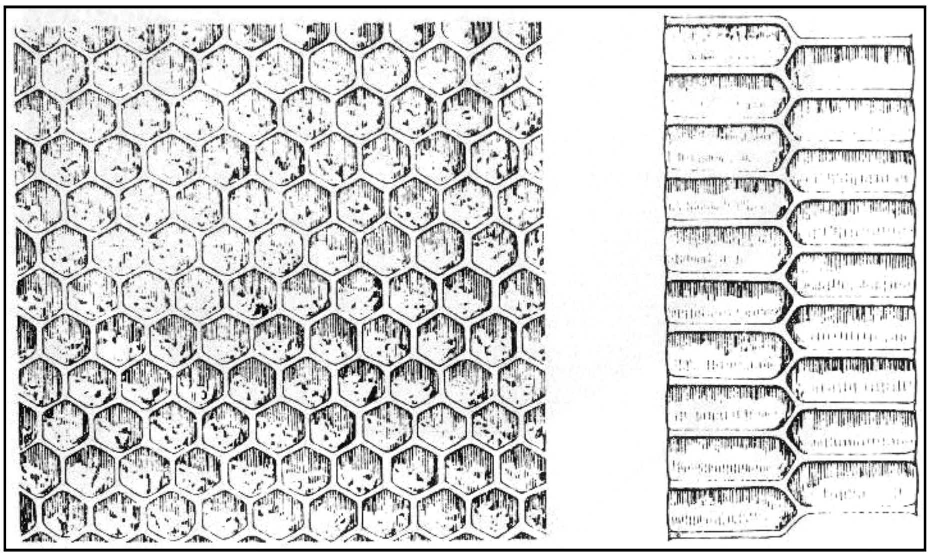


Figure 1 – The honeycomb of a bee

One of the most important feature of a cellular solid is its *relative density*, ρ^*/ρ_s ; where ρ^* is the density of the cellular material and ρ_s is the density of the solid from which the cell walls are made.

Polymeric foams used for cushioning, packaging and insulation have relative densities which are usually between 0.05 and 0.20; cork is about 0.14; and most softwoods are between 0.15 and 0.40. As the relative density increases, the cell walls thicken and the pore space shrinks; above about 0.30 there is a transition from a cellular structure to one which is better thought of as solid containing isolated pores. Here we just considered the true cellular solids with relative densities of less than 0.30.

Cellular structures extend the range of properties available to the engineer. Cellular solids have physical, mechanical and thermal properties which are measured by the same methods as those used for fully dense solids. The low densities permit the design of light, stiff components such as sandwich panels and large portable structures, and flotation devices. The low thermal conductivity allows cheap, reliable thermal insulation. The low stiffness makes foams ideal for a wide range of cushioning applications; for example elastomeric foams are the standard materials for seating. The low strengths and large compressive strains make foams attractive for energy-absorbing applications.

2. Applications of Cellular Structures

Four major areas of application of cellular materials can be considered as: thermal insulation, packaging, structural use, and buoyancy. The largest application of polymeric and glass foams are related to thermal insulation. Modern buildings, transport systems (refrigerated trucks and railway cars), and even ships (partially those designed to carry liquid natural gas) all take advantage of the low thermal conductivity of expanded plastic foams. A particular advantage of foams for ultra-low-temperature research is reducing the amount of refrigerant needed to cool the insulation itself because of their low density. The same is true, at higher temperatures, in the design of kilns and furnaces; the lower the thermal mass, the greater the efficiency. The thermal mass of a foam is proportional to its relative density.

Another major use of cellular solids is in packaging. An effective package must absorb the energy of impacts without subjecting the contents to damaging stresses. Foams are particularly well suited for this. The strength of a foam can be adjusted over a wide range by controlling its relative density. Furthermore, foams can undergo large compressive strains at almost constant stress, so that large amount of energy can be absorbed without generating high stresses. Using foams also has another advantage as packaging materials. The low density means the package is light, reducing handling and shipping costs.

The application of natural cellular materials such as wood can be traced to the beginning of civilization. Wood is still the world's most widely used structural material. Man-made foams and honeycombs are used in applications in which they perform a truly structural function. The most obvious example is their use in sandwich panels which are used in wide range of applications where weight is critical such as: space vehicles, skis, racing yachts, automobile industry and portable buildings. Sandwich panels are also found in nature: the skull is made up of two layers of dense, compact bone separated by a lightweight core of spongy, cancellous bone.

Models which predict their compressive failure behavior have broad applicability to both engineered and natural cellular materials. In designing lightweight sandwich structures, or in critical packaging applications, the engineer requires precise information about the way in which the foam will behave under different stress states.

Unit cell models have proven to be useful theoretical tools for understanding some of the key aspects of the mechanical behavior of cellular solids, such as effective elastic stiffness and the dependence of failure properties on relative density and on the failure mode of individual cells [1].

In contrast to an idealized unit cell, most cellular materials have inherent imperfections and inhomogeneities in their microstructure, such as different cell wall thickness, missing cell walls, filled cells and non-periodic arrangement of cell walls. These microstructural variations are most obvious in natural cellular materials and ones that are manufactured using random physical processes (e.g. foams), but also occur in manufactured honeycombs. Thus, unit cell models do not accurately represent the mechanical behavior of most real cellular materials and there exists a need for models that can be used to quantify the influence of microstructural variability on the mechanical properties of cellular materials.

Most honeycombs have hexagonal cells, although honeycombs with square or triangular cells can also be made (Fig. 2). In three dimensions more variety of cell shapes is possible, some of which are shown in Fig. 3. Most foams are not regular packing of identical units but contain cells of different sizes and shapes with different numbers of faces and edges. In general, cellular materials are anisotropic, due to the anisotropy of the cell wall.

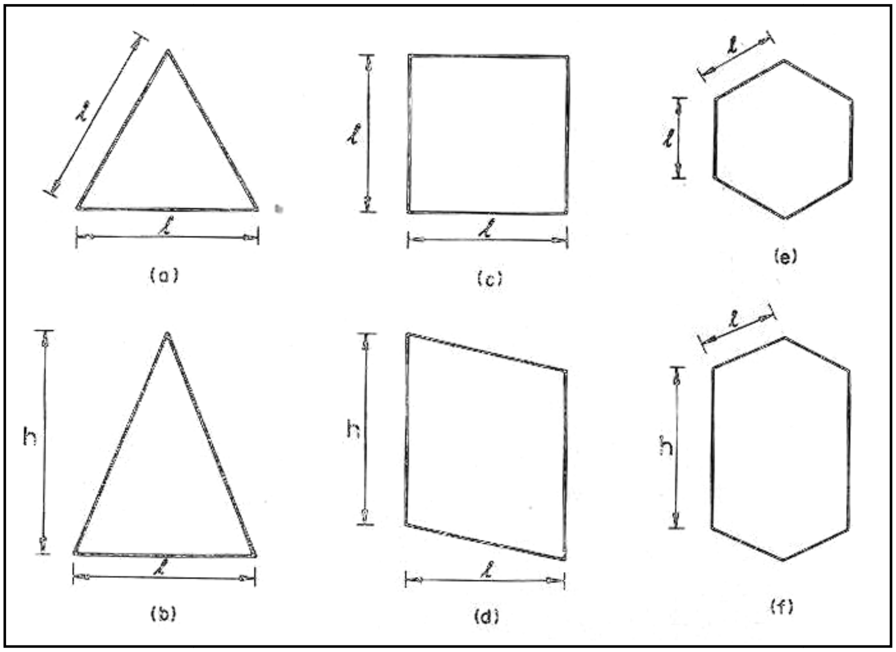
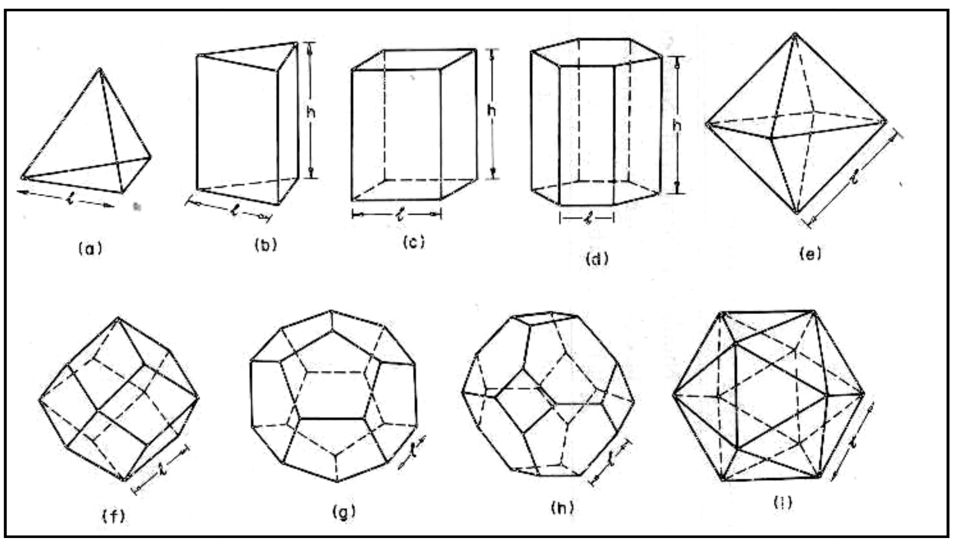


Figure 2 – Polygons found in two-dimensional cellular materials: (a) equilateral triangle, (b) isosceles triangle, (c) square, (d) parallelogram, (e) regular hexagon, (f) irregular hexagon. Note that any triangle, quadrilateral or hexagon with a centre of symmetry will fill the plane.;



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3. Literature Review

Different models have been developed to predict the mechanical behavior of cellular materials in order to find and analyze the failure mechanism by which cell walls deform under load. Mechanical properties of cellular materials have been studied extensively, and can be found in several comprehensive surveys [1,2].

Early models for the uniaxial elastic behavior, only assumed axial loading for the cell walls [3-5]. This simplification causes an inconsistency with the experimental data: that the elastic properties vary linearly with the density. Later studies found that bending deformation of cell walls has more important contribution to the mechanical properties of the structures [6-14].

In uniaxial compression, cells fail by elastic buckling, plastic yielding or brittle crushing [3,8,9,11,12,15-24]. For honeycombs, with their regular periodic geometry, an exact analysis is possible and its results have been confirmed by experiments on elastic and elastic-plastic honeycombs with a wide range of geometries [11,25]. This analysis can be extent to the uniaxial collapse of foams [1,12].

Anisotropy of cellular materials has also been investigated in previous works. The mechanical properties of simple honeycomb material made up of hexagonal cells can be related to the cell geometry, the volume fraction of solids, and the cell wall properties through an analytical approach [10,11]. The level of anisotropy in the mechanical properties can easily be found base on the cell geometry. Harrigan and Mann [26] have shown that the shape of the cells can be characterized by an anisotropy tensor. Investigations on the anisotropy in young’s modulus have assumed that it is related to the uniaxial deformation of the cell walls [27-30]; this neglects the important contribution of cell wall bending. Huber and Gibson [31] analyzed the effect of shape anisotropy on material properties in a systematic manner and considered cell wall bending. They derived relationships for the elastic moduli, the elastic-plastic and brittle collapse stresses for both axisymmetric and orthotropic materials, and compared them with measurements of cell shape and mechanical properties.

It is now well established that the in-plane hydrostatic strength of a perfect hexagonal honeycomb is governed by cell wall stretching and is proportional to the relative density ρ, while its deviatory strength is set by cell wall bending and scales with ρ [1]. Thus, the yield surface is elongated along the hydrostatic axis in biaxial stress space. Using simple beam theory, Klintworth and Stronge [25] proposed failure envelopes for regular honeycombs with respect to various elastic and plastic cell crushing modes; these are used together with the associated flow rule of plasticity to describe the in-plane indentation of a honeycomb by a plane punch. Gibson et al. [32] studied the biaxial yield surface of 2-dimensional honeycombs and the triaxial yield surface of 3-dimensional open-celled foams.

These studies suggest that the plastic yield surface of regular honeycombs may be truncated by elastic buckling in compression and by brittle fracture in tension. Several attempts have been made to account for the effects of morphological defects on the elastic and plastic properties of cellular solids. Warren and Kraynik [13] and Kraynik et al. [33] found that the presence of Plateau borders (non-uniform wall thickness) has only a small effect on the elastic response of honeycombs. Simone and Gibson [34] considered the effect of Plateau borders on the mechanical properties of hexagonal honeycombs and of idealized cellular foams with closed tetrakaidecahedral cells. Their finite element results suggest that the distribution of material in the cell walls has little effect upon the Young's modulus and has only a moderate influence upon the uniaxial yield strength. They argued that the maximum bending moments appear at the joints of the honeycomb and so the presence of Plateau borders has only a small effect on the Young's modulus. Grenestedt and Tanaka [35] used the finite element method to study the influence of non-uniform cell wall thickness on the shear and bulk modulus of a flat-faced Kelvin structure, consisting of 14-sided closed cells in a BCC arrangement. It was found that both moduli are rather insensitive to thickness variations, at a fixed overall relative density.

The effects of cell face curvature and wiggles on the mechanical properties of regular honeycombs and of tetrakaidecahedral closed cell foams were studied by Simone and Gibson [36]. They found that wavy imperfections can reduce significantly the Young's modulus and the uniaxial yield strength of the foam. Grenestedt (1998) has shown that wavy imperfections give a bigger reduction in the Voigt upper bound bulk modulus for open cell foams than for closed cell foams. Grenestedt [37] has also studied the effect of wavy imperfections on the yield behavior of open cell foams. In agreement with Gibson and Ashby [1] he argued that the hydrostatic strength of the perfect foam is governed by cell wall stretching and scales with $\boldsymbol{\rho}$. Silva et al. [38] used finite element method to model a 2D random Voronoi distribution of cells and found that `the relations between microstructural and elastic properties for non-periodic honeycombs are, on average, not different from those for periodic honeycombs'. Silva and Gibson [39] investigated the influence of random cellular microstructures and missing cell walls on the Young's modulus and uniaxial yield strength of 2D Voronoi honeycombs. They found that the uniaxial compressive yield strength of a Voronoi honeycomb is about 30% less than that of a perfect honeycomb at the same relative density level, and those defects, introduced by removing some of the cell walls at random locations, lead to a sharp decrease in the uniaxial stiffness and strength of both Voronoi and perfect honeycombs. Through a combination of analytical and finite element techniques Triantafyllidis and Schraad [40] found that the yield surface of a perfect hexagonal honeycomb provides an upper bound for the yield surfaces of honeycombs with microstructural imperfections.

Chen et al. [41] investigated the influence of different types of morphological imperfection (waviness, non-uniform cell wall thickness, cell-size variations, fractured cell walls, cell-wall misalignment and missing cells) on the yielding of 2D cellular solids. They also performed a finite element study to find the effects of holes on elastic modulus and yield strength of regular honeycombs under biaxial loading. Microstructures with missing or fractured cell walls have been investigated for finite regular hexagonal [42] and Voronoi cell honeycombs [39].

The picture for the mechanical behavior of three-dimensional cellular solids under multiaxial loads is more confusing. Shaw and Sata [43] measured the combination of stresses required to cause failure of a polystyrene foam which yields plastically under several loading conditions. Their results showed that the failure is governed by the maximum principal stress. The elastic buckling collapse of isotropic foams under multiaxial stress was modeled by Zhang [44] by analyzing the elastic buckling of four struts in space which meet in at equal angel of 108˚ (assuming pentagonal dodecahedral cell). Assuming a single buckling mode, he found that the failure envelope in stress space is nearly box-like in the compressive octant.

In addition to the mechanical behavior of cellular materials under static loading, there are number of investigations on the mechanical behavior of cellular materials under dynamic loadings [45,46]. Xue and Hutchinson [47] introduced a continuum model for high rate deformation of square honeycomb cores which can be used to simulate core behavior in large structural calculations.

4. Objectives

The present investigation builds upon prior works and considers the effects of randomly missed cell walls through the structure and also randomly filled cell on the mechanical behavior of the structure. We have considered both periodic (regular hexagonal honeycombs) and non-periodic (Voronoi honeycombs) two dimensional microstructures. In contrast to previous works, we have chosen individual missing cell walls throughout the structure as opposed to missing cell clusters. In addition, we also considered the effect of a new defect, filled cells, on the mechanical behavior of cellular materials. For plastic behavior, effect of using materials with different rate of hardening in plastic region was also investigated. In chapter 4, we developed a class of cellular solids which are gradually graded in one direction. The graded materials exhibit a linear cell size variation in the direction of the gradient.

Mechanical Behavior under uniaxial and biaxial loading of three different cellular materials and the effect of different defects on their mechanical properties were investigated using finite element analysis.

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CHAPTER 2

METHODOLOGY

1. The Mechanics of Honeycombs

Polymer, metal and ceramic honeycombs are used in variety of applications. Polymer and metal honeycombs are used as the cores in sandwich panels; metal honeycombs are used as energy-absorbing component in several applications. If such materials are used as load-bearing structures, an understanding of their mechanical behavior is important. Furthermore, studying honeycombs helps in understanding the mechanics of much more complex three-dimensional foams.

A honeycomb is loaded in-plane when the applied stresses act in the $X_1 - X_2$ plane as shown in Fig. 1.

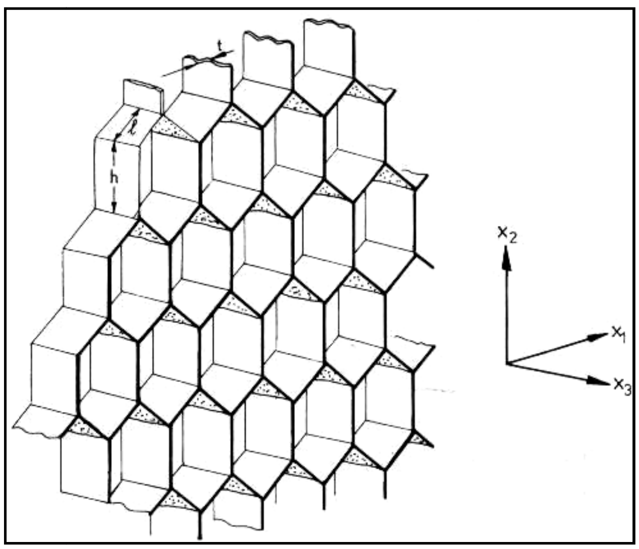


Figure 1 – A honeycomb with hexagonal cells. The in-plane properties are those relating to loads applied in the $X_1 - X_2$ plane. Responses to loads applied to the faces normal to X_3 are referred to as the out-of-plane properties.

When a honeycomb is compressed, the cell walls start to bend and cause a linear elastic deformation. Beyond a critical strain the cells collapse by elastic buckling, plastic yielding, creep or brittle fracture, depending on the nature of the cell wall material. Whenever the opposing cell walls contact each other, cell collapse ends. As the cells close up the structure densifies and its stiffness increases rapidly. The in-plane stiffness and strength are the lowest because the cell walls respond to external loads by bending, and subsequent buckling, yielding, or fracturing. The out-of-plane (as shown in Fig. 1) stiffness and strength are much larger since they require axial deformation of the cell walls. In this chapter, the in-plane strength of hexagonal honeycombs is studied.

2. The in-plane Properties of Honeycombs: Uniaxial Loading

A unit cell of a hexagonal honeycomb is shown in Fig. 2a. If the hexagon is regular (the sides are equal and the angles are all 120°) and the cell walls are all of the same thickness, then the in-plane properties are isotropic. A regular structure has two independent elastic moduli (a Young's modulus E and a shear modulus G) and a single value of the yield stress, σ. However, for irregular hexagon or when the thickness is not same for the walls, the properties are anisotropic and the structure has four elastic constants (E_1, E_2, G_{12} and ϑ_{12}which is Poisson's ratio) and two values for the yield stress (σ_1 and σ_2).

A general hexagonal is shown in Fig. 2a with an arbitrary cell wall angle. The isotropic properties can easily be obtained from the result for the case of general hexagon. Simple geometry analysis gives the relative density ρ^*/ρ_s , as below:

$$\frac{\rho^*}{\rho_s} = \frac{t/l\,(h/l+2)}{2\cos\theta(h/l+\sin\theta)} \tag{1}$$

which reduces to:

$$\frac{\rho^*}{\rho_s} = \frac{2}{\sqrt{3}}\frac{t}{l} \tag{2}$$

when the cells are regular ($h = l;\ \theta = 30°$). For the theoretical approach, it is also assumed that the deformations are sufficiently small that changes in geometry can be neglected.

Many commercial honeycombs are made by expanding strip-glued sheets. Then each cell has four walls of thickness t and two (those of height h in Fig. 2a) which are doubled and have thickness of $2t$. The doubling of this pair of cell walls does not change the values of the in-plane Young's moduli calculated below.

2.1. Linear Elastic Deformation

When a honeycomb, loaded in the X_1 or X_2 direction, deforms in a linear-elastic way, the cell walls bend [1-5]. The Young's moduli in each direction are calculated by the method shown in Fig. 2.

A stress σ_1 parallel to X_1 causes one set of cell walls – those of length l - to bend; one cell wall is shown in the figure. Equilibrium requires that the component of force C parallel to X_2 be zero. The moment M tending to bend the cell wall which is treated as a beam of length, l, thickness, t, depth, b, and Young's modulus, E_s, is:

$$M = \frac{Pl\sin\theta}{2} \tag{3}$$

where

$$P = \sigma_1(h + l\sin\theta)b \tag{4}$$

From standard beam theory, the wall deflects by:

$$\delta = \frac{Pl^3 \sin\theta}{12E_s I} \tag{5}$$

Where I is the second moment of inertia of the cell wall ($= bt^3/12$ for a wall of uniform thickness t). Of this deflection, a component $\delta \sin\theta$ is parallel to X_1 axis and the strain would be:

$$\epsilon_1 = \frac{\delta \sin\theta}{l\cos\theta} = \frac{\sigma_1(h + l\sin\theta)bl^2 \sin^2\theta}{12E_s I \cos\theta} \tag{6}$$

The Young's modulus parallel to X_1 is just $E_1^* = \sigma_1/\epsilon_1$, giving:

$$\frac{E_1^*}{E_s} = \left(\frac{t}{l}\right)^3 \frac{\cos\theta}{(h/l + \sin\theta)\sin^2\theta} \tag{7}$$

Loading in the X_2 direction is shown in Fig. 1c; the forces acting on the cell wall of length, l, and depth, b, are shown in the bottom part. By equilibrium $F = 0$ and $W = \sigma_2 lb\cos\theta$, giving:

$$M = \frac{Wl\cos\theta}{2} \tag{8}$$

The wall deflects by:

$$\delta = \frac{Wl^3 \cos\theta}{12E_s I} \tag{9}$$

Of this, a component $\delta\cos\theta$ is parallel to the X_2 axis, giving a strain:

$$\epsilon_1 = \frac{\delta \cos\theta}{h + l\sin\theta} = \frac{\sigma_2 b l^4 \cos^3\theta}{12 E_s I (h + l\sin\theta)} \tag{10}$$

from which the Young's modulus parallel to X_2 is simply $E_2^* = \sigma_2/\epsilon_2$ giving:

$$\frac{E_2^*}{E_s} = \left(\frac{t}{l}\right)^3 \frac{(h/l + \sin\theta)}{\cos^3\theta} \tag{11}$$

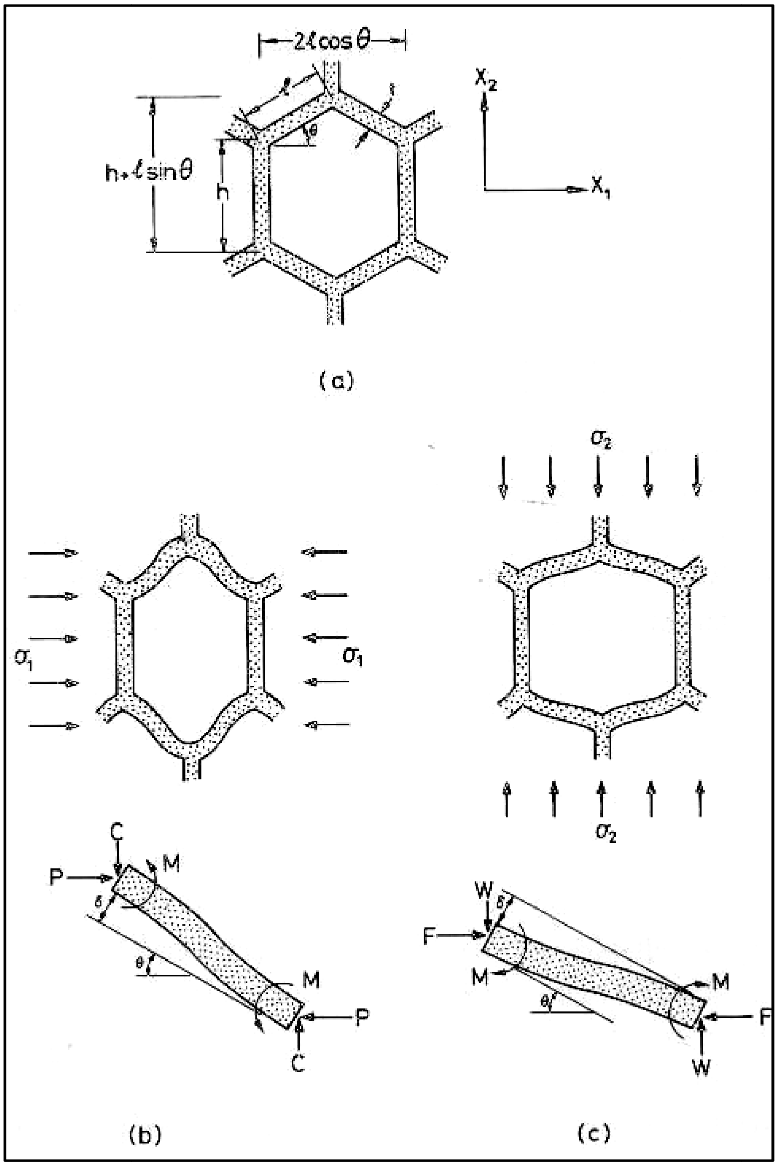


Figure 2 - Cell deformation by cell wall bending, giving linear-elastic extension or compression of the honeycomb: (a) the undeformed honeycomb; (b) and (c) the bending caused by loads in the X_1 and X_2 directions. [1]

For regular hexagonal with walls of uniform thickness, both Young's moduli, E_1^* and E_2^*, reduce to the same value:

$$\frac{E_1^*}{E_s} = \frac{E_2^*}{E_s} = 2.3\left(\frac{t}{l}\right)^3 \tag{12}$$

Which indicates that honeycombs made up of regular hexagonal are isotropic.

2.2. Elastic Buckling

The cell walls most nearly parallel to the loading direction behave like an end-loaded column; such a column buckles when the load exceeds the Euler buckling load:

$$P_{cr} = \frac{n^2 \pi^2 E_s I}{h^2} \tag{13}$$

The factor, n, describes the rotational stiffness of the node where three cell walls meet. Fig. 3 shows the buckling mode observed when honeycombs are compressed in the X_2 direction. The load per column (column *EB,* for example) is related to the remote stress, as before, by:

$$P = 2\sigma_2 lb \cos\theta \tag{14}$$

Elastic collapse occurs when $P = P_{cr}$, giving the elastic collapse stress as:

$$\frac{(\sigma_{el}^*)_2}{E_s} = \frac{n^2 \pi^2}{24} \frac{t^3}{lh^2} \frac{1}{\cos\theta} \tag{15}$$

The factor, n, depends on the degree of constraint to rotation at the node B caused by the walls AB and BC; if rotation is freely allowed, $n = 0.5$; if no rotation is possible, $n = 2$. The constraint on the vertical wall caused by the walls to which it is connected lies between these limits; then n is greater than 0.5 and less than 2 and for regular hexagonal honeycomb, $n = 0.69$. (Values of n as a function of h/l is derived and given in [1]). Therefore, the collapse stress for the regular hexagons is:

$$\frac{(\sigma_{el}^*)_2}{E_s} = 0.22\left(\frac{t}{l}\right)^2 \tag{16}$$

2.3. Plastic Collapse

Metals, and many polymers, are elastic-perfectly plastic solids. Honeycombs made of them collapse plastically when the bending moment in the cell walls reaches the fully plastic moment. This will give a stress-strain curve with a plateau both in compression and in tension at the plastic collapse stress σ_{pl}^*.

Consider loading in the X_1 direction in Fig. 4. An upper bound on the plastic collapse stress is given by equating the work done by the force:

$$P = \sigma_1(h + l\sin\theta)b \tag{17}$$

During a plastic rotation $\emptyset$ of the four plastic hinges A, B, C, and D to the plastic work done at the hinges giving:

$$4M_p\emptyset \geq 2\sigma_1 b(h + l\sin\theta)\emptyset l\sin\theta \tag{18}$$

where M_p is the fully plastic moment of the cell wall in bending:

$$M_p = \frac{1}{4}\sigma_{ys} b t^2 \tag{19}$$

and where σ_{ys} is the yield stress of the cell wall material. It follows that:

$$\frac{\left(\sigma_{pl}^*\right)_1}{\sigma_{ys}} = \left(\frac{t}{l}\right)^2 \frac{1}{2(h/l + \sin\theta)\sin\theta} \tag{20}$$

a lower bound is given by equating the maximum moment in the beam to M_p.

this maximum moment is:

$$(M_{max})_1 = \frac{1}{2}\sigma_1 (h/l + \sin\theta) b l \sin\theta \tag{21}$$

from which:

$$\frac{\left(\sigma_{pl}^*\right)_1}{\sigma_{ys}} = \left(\frac{t}{l}\right)^2 \frac{1}{2(h/l + \sin\theta)\sin\theta} \tag{22}$$

The lower and upper bounds are identical, and thus define the exact solution to the problem. For regular uniform hexagons it reduces to:

$$\frac{\sigma_{pl}^*}{\sigma_{ys}} = \frac{2}{3}\left(\frac{t}{l}\right)^2 \tag{23}$$

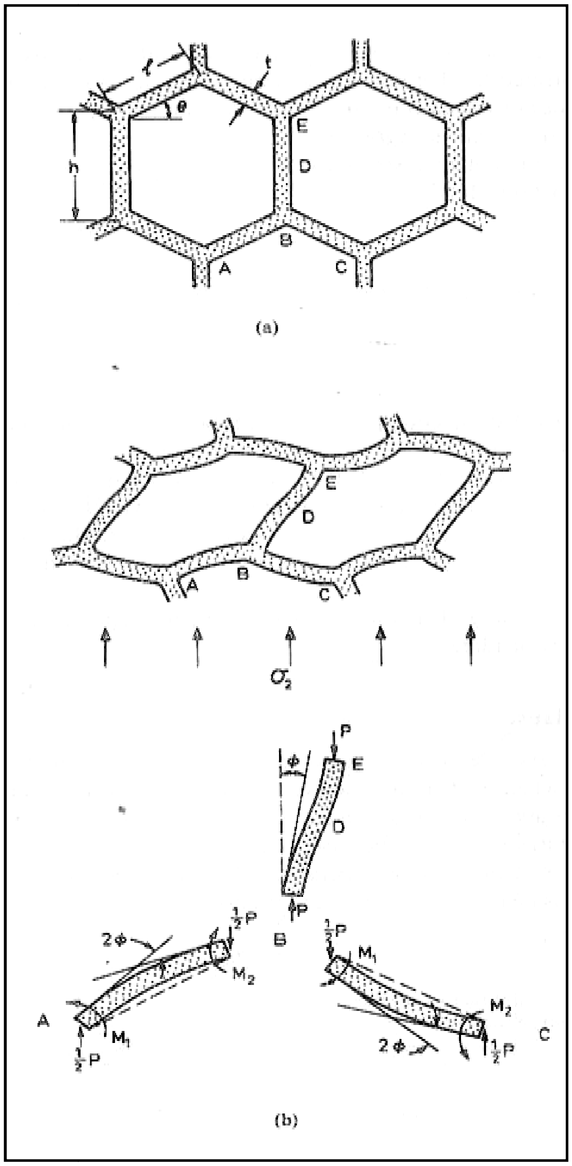


Figure 3 – Cell deformation by elastic buckling: (a) the undeformed honeycomb; (b) the buckling mode in uniaxial loading, and associated forces, moments, displacement and rotations [1]

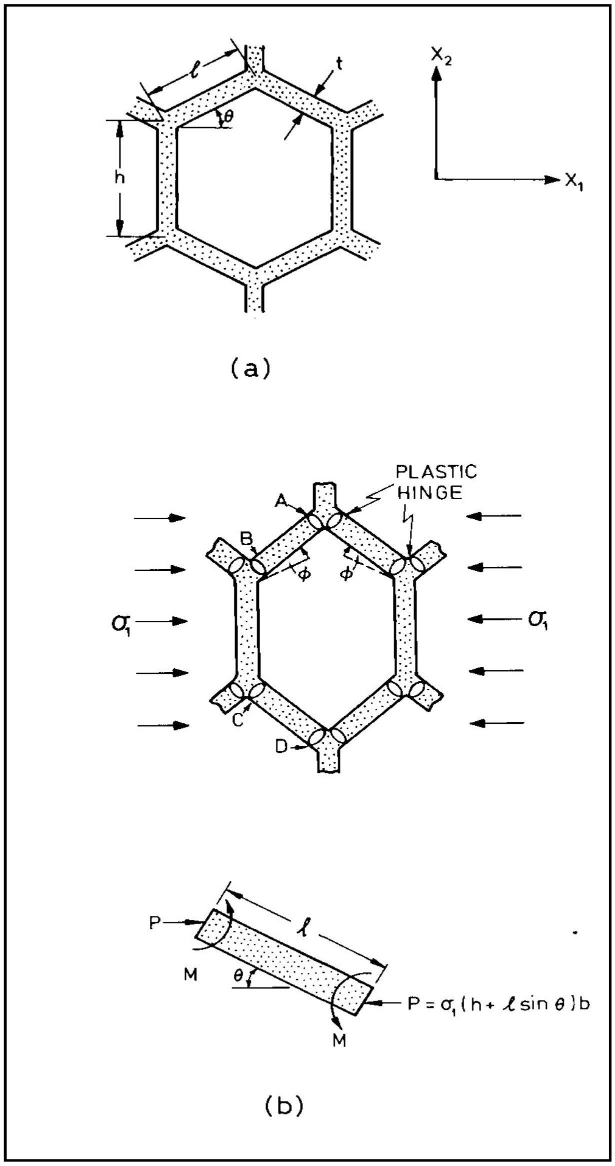


Figure 4 – Cell deformation by plastic collapse: (a) the undeformed honeycomb, (b) the rotation, forces and moments for loading in the X_1 direction [1]

3. Finite Element Model

Models of two-dimensional cellular solids with both periodic and non-periodic microstructures were generated. For periodic structures, we used regular hexagonal honeycombs which are widely used as fundamental geometry for modeling cellular solids. The cell walls have length of ***l,*** thickness ***t,*** and the unit depth. Regular hexagonal cell geometry and a representative diagram of regular hexagonal honeycomb structures are shown in Fig. 5 and Fig. 6, respectively.

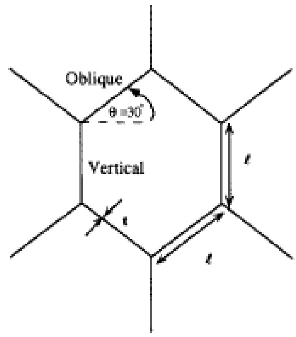


Figure 5 - Schematic drawing of single regular hexagonal cell

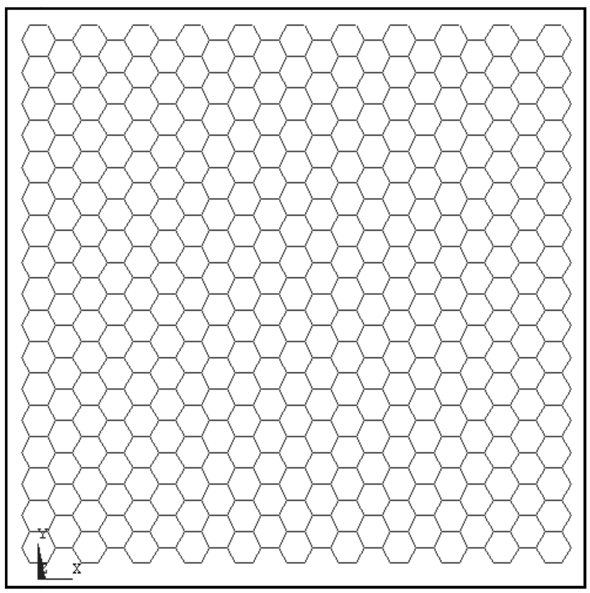


Figure 6 - Regular hexagonal honeycomb structure

For non-periodic structures, Voronoi Honeycomb, we used random nodes as the nuclei of honeycomb cells. The cell walls were then generated using Matlab® (The MathWorks, Inc., Natick, MA). Fig. 7 is output from the Matlab code using unconstrained random generation. Fig. 4 is a Voronoi structure generated while enforcing a minimum cell size. In order to produce cells of approximately uniform size, as shown in Fig. 8, the nucleus points were spaced no closer than a pre-set minimum distance .

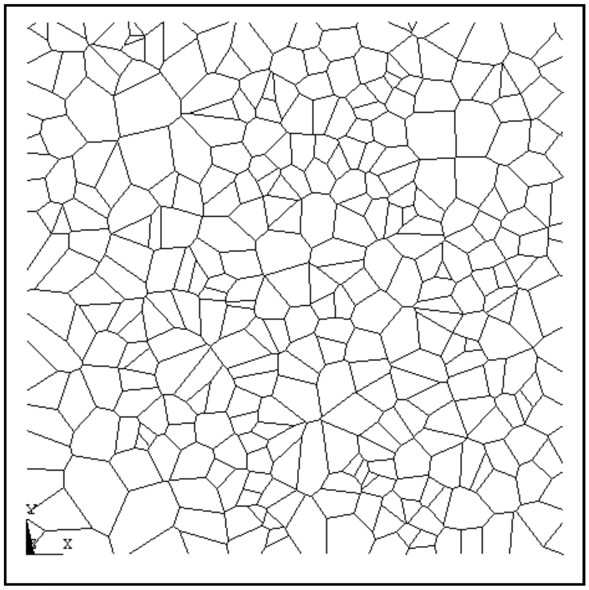


Figure 7 - Voronoi honeycomb structure

The compressive stress-strain behavior of the honeycombs structures is determined using finite element analysis (ANSYS®, ANSYS Inc., Canonsburg, PA). Cell walls of the honeycombs are modeled as Beam23 elements which are capable to model both elastic and plastic behavior and also consider the bending, axial and shear deformation.

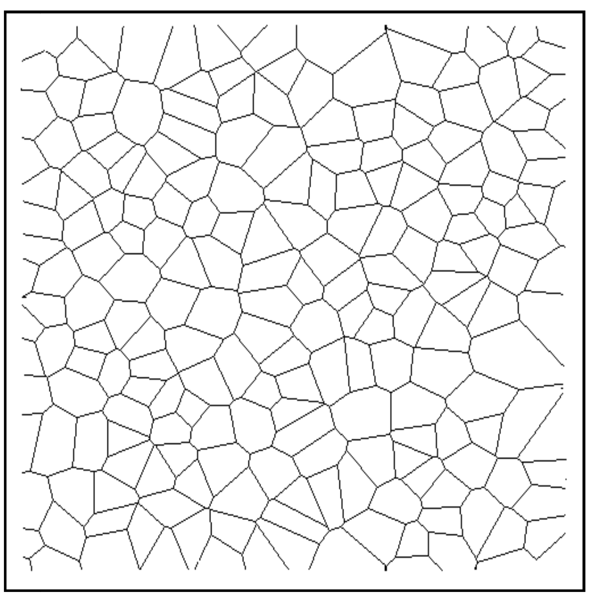


Figure 8 - Voronoi honeycomb with a pre-set minimum distance between the nuclei of cells

We assumed the cell wall material to be linearly elastic for the moduli and elastic buckling calculations and linear elastic-perfectly plastic for the plastic collapse calculations. The cell wall Young's modulus, yield strength and Poisson's ratio were taken as $E = 70\ GPa$, $\sigma_y = 130\ MPa$ and $\vartheta\ = 0.3$, respectively**.**

3.1. Effective Elastic Modulus

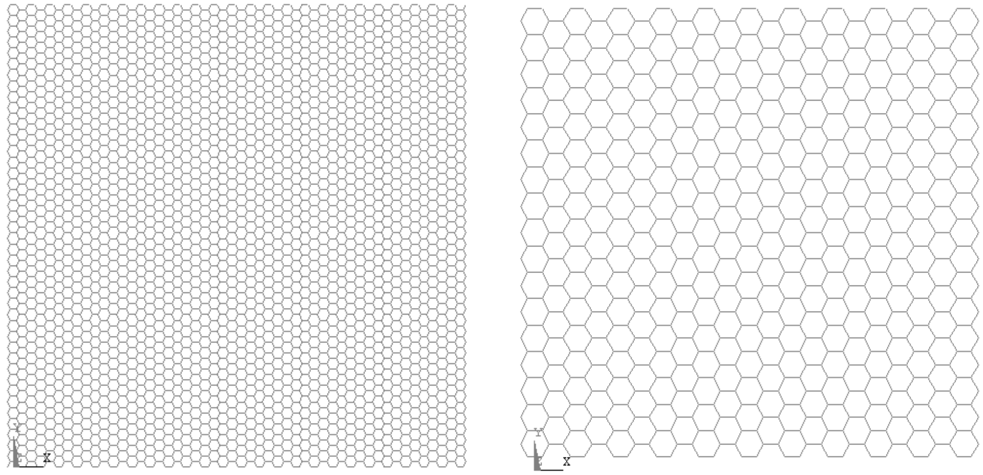
Finite element analysis was performed to calculate the elastic moduli in both X and Y direction. Effective elasticity of the honeycomb structure in each direction was calculated as:

$$E_x = \frac{F_x . L_x}{L_y . u_x} \tag{24}$$

$$E_y = \frac{F_y . L_y}{L_x . u_y} \tag{25}$$

Where F is the force applied and *u* is the resultant displacement, L_x and L_y are the structure dimensions in X and Y direction, respectively.

To calculate the modulus in the X direction, E_x , a horizontal force was applied to the right edge of the model. All cells on the right edge were coupled to have the same horizontal displacement. The left edge was supported by rollers at each cell and both side edges were allowed to translate in the vertical direction. For elastic moduli in Y direction, E_y, boundary conditions were similar but in vertical direction. The calculations were repeated for different ratio of t/l while varying *t* and *l* independently. The effect of varying t while holding l constant is shown in Fig. 9a. Fig. 9b illustrates the effect of varying l while holding t constant.



(a) (b)

Figure 9 - Finite element model for different ratio of t/l (t/l = 1/4) which is achieved by: (a) changing the length of cell walls and keeping the thickness constant. (b) changing the thickness, the length of cell walls is constant.

3.2. Elastic Buckling

The critical uniaxial buckling stress in the X direction was computed from the buckling eigenvalue evaluation analysis in ANSYS® (ANSYS Inc., Canonsburg, PA). The cell walls were assumed to be linearly elastic and isotropic. The calculation was repeated for different ratios of t/l (different relative density).

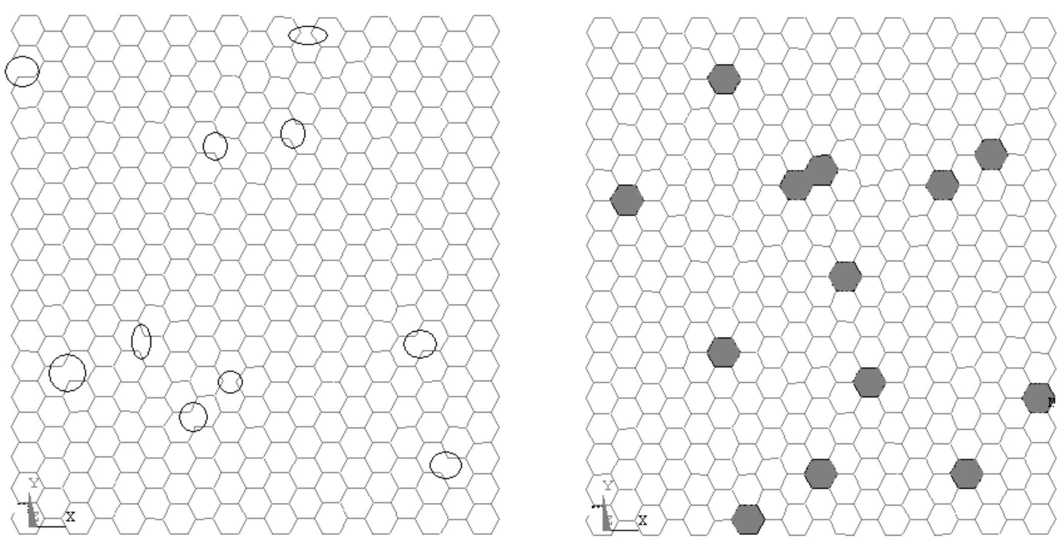
3.3. Plastic Behavior

To evaluate the post-yield behavior, we assumed a linear elastic-perfectly plastic constitutive relation for the cell wall, and applied incremental horizontal displacement on the right edge using non-linear, large deformation option in ANSYS® (ANSYS Inc., Canonsburg, PA).

In this study, we first considered the intact honeycomb structure and perform the finite element analysis to calculate the effective elasticity, elastic buckling and elastic-plastic behavior of the structure. We then repeated the analysis while introducing defects.

The defects which we considered in this study were randomly missed cell walls and randomly filled cells, as shown in Fig. 10. Random cells were selected using Matlab® (The MathWorks, Inc., Natick, MA). In case of filled cells, we used the same material properties as those used for the cell walls; element type *plane42* was used to model the filled area.

As a part of this investigation, we evaluated the effective elastic moduli, buckling load and mode shapes, and elastic–plastic behavior of regular and Voronoi materials with various defects. The results are presented first for regular honeycomb structures followed by the results for Voronoi structures.



(a) (b)

Figure 10 - Finite element model of: (a) missing cell wall defect (3% missed wall) [location of missed walls are circled] and (b) filled cell defect (5% filled)

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CHAPTER 3

RESULTS AND DISCUSSIONS

1. Summary

Cellular solids such as foams are widely used in engineering applications. In these applications, it is important to know their mechanical properties and the variation of these properties with the presence of defects. Several models have been proposed to obtain the mechanical properties of cellular materials. However, these models are usually based on idealized unit cell structures, and are not suitable for finding the mechanical properties of cellular materials with defects. The objective of this work is to understand the effect of missing walls and filled cells on elastic-plastic behavior of both regular hexagonal and non-periodic Voronoi structures using finite element analysis.

The results show that the missing walls have a significant effect on overall elastic properties of the cellular structure. For both regular hexagonal and Voronoi materials, the yield strength of the cellular structure decreases by more than 60% by introducing 10% missing walls. In contrast, the results indicate that filled cells have much less effect on the mechanical properties of both regular hexagonal and Voronoi materials.

2. Introduction

Models of two-dimensional cellular solids with both periodic and non-periodic microstructures were generated. For periodic structures, we used regular hexagonal honeycombs which are widely used as fundamental geometry for modeling cellular solids. The cell walls have length of ***l,*** thickness ***t,*** and the unit depth.

For non-periodic structures, Voronoi Honeycomb, we used random nodes as the nuclei of honeycomb cells. The cell walls were then generated using Matlab® (The MathWorks, Inc., Natick, MA).

The compressive stress-strain behavior of the honeycombs structures is determined using finite element analysis (ANSYS®, ANSYS Inc., Canonsburg, PA). Cell walls of the honeycombs are modeled as Beam23 elements which are capable to model both elastic and plastic behavior and also consider the bending, axial and shear deformation.

We assumed the cell wall material to be linearly elastic for the moduli and elastic buckling calculations and linear elastic-perfectly plastic for the plastic collapse calculations. The cell wall Young's modulus, yield strength and Poisson's ratio were taken as $E = 70\ GPa$, $\sigma_y = 130\ MPa$ and $\vartheta\ = 0.3$, respectively.

Finite element analysis was performed to calculate the elastic moduli in both X and Y direction. To calculate the modulus in the X direction, E_x , a horizontal force was applied to the right edge of the model. All cells on the right edge were coupled to have the same horizontal displacement. The left edge was supported by rollers at each cell and both side edges were allowed to translate in the vertical direction. For elastic moduli in Y direction, E_y, boundary conditions were similar but in vertical direction. The calculations were repeated for different ratio of t/l while varying t and l independently.

The critical uniaxial buckling stress in the X direction was computed from the buckling eigenvalue evaluation analysis in ANSYS® (ANSYS Inc., Canonsburg, PA). The cell walls were assumed to be linearly elastic and isotropic. The calculation was repeated for different ratios of t/l (different relative density).

To evaluate the post-yield behavior, we assumed a linear elastic-perfectly plastic constitutive relation for the cell wall, and applied incremental horizontal displacement on the right edge using non-linear, large deformation option in ANSYS® (ANSYS Inc., Canonsburg, PA).

In this study, we first considered the intact honeycomb structure and perform the finite element analysis to calculate the effective elasticity, elastic buckling and elastic-plastic behavior of the structure. We then repeated the analysis while introducing defects.

3. Results and Discussion

Before exploring the influence of defects on the elastic stiffness and yield strength of elastic-perfectly plastic honeycombs, the mechanical properties of intact honeycombs structure were evaluated based on the unit cell model in order to ensure reliability of the finite element method. As discussed in chapter two, the deformation of a perfect hexagonal honeycomb is governed by cell-wall stretching under hydrostatic loading and by cell-wall bending under deviatory loading and the macroscopic Young's modulus, E^*, elastic buckling strength, σ_{el}^*, and uniaxial yield strength, σ_{pl}^*, are given by:

$$\frac{E_1^*}{E_s} = \frac{E_2^*}{E_s} = 2.3\left(\frac{t}{l}\right)^3 \tag{1}$$

$$\frac{(\sigma_{el}^*)_2}{E_s} = 0.22\left(\frac{t}{l}\right)^2 \tag{2}$$

$$\frac{\sigma_{pl}^*}{\sigma_{ys}} = \frac{2}{3}\left(\frac{t}{l}\right)^2 \tag{3}$$

where E_s and σ_{ys} are Young's modulus and yield strength of the cell-wall material, respectively.

In this study, the relative density of the structure was defined as below:

$$\rho = \frac{t \sum_n l_i}{L_x \times L_y} \tag{4}$$

where l_i is the length of cell walls and t is the thickness of cell walls. L_x and L_y are the structure dimensions in X and Y directions, respectively.

3.1. Elastic Properties of Regular Hexagonal Honeycombs

The normalized effective elastic moduli in the X and Y direction, while varying the relative density are compared with analytical solutions, based on unit cell model. For relative density less than 0.1, our results are in agreement with previous works and analytical solution, however for cellular structure with relative density greater than 0.15, the finite element analysis showed less stiffness for the structure in X and Y direction compared with the analytical solution. This difference was more pronounced for higher relative densities.

For small relative densities $\rho \leq 0.10$, finite element model was 7% less stiff in X direction and near 1% in Y direction while for $0.10 \leq \rho \leq 0.30$ the comparison showed 20% and 15% difference in x and y directions, respectively. The results are shown in Fig.1 and Fig. 2. Although the analytical solution predicts the same effective Young's modulus in both X and Y directions, our results showed they are slightly different (6% less stiffness in x-direction) as shown in Fig. 3.

In order to change the relative density in a honeycomb structure, we can either change the length of the cell edge or change the thickness of the cell walls. Our analysis showed less than 3% difference in resultant elasticity for the structures with same relative density but different length and/or thickness. The results indicated that for the elastic behavior the effect of relative density could be investigated by only changing the thickness of the cell walls. The results are plotted in Fig. 4 and Fig. 5.

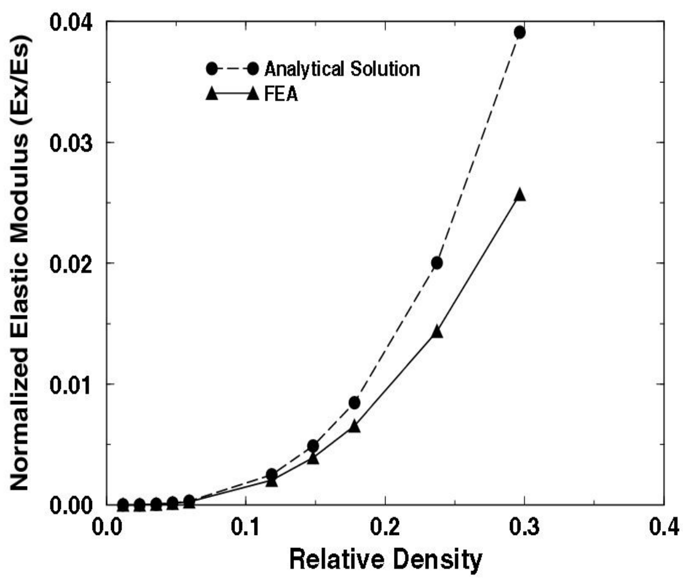


Figure 1 - Normalized elastic modulus vs. relative density (X direction)

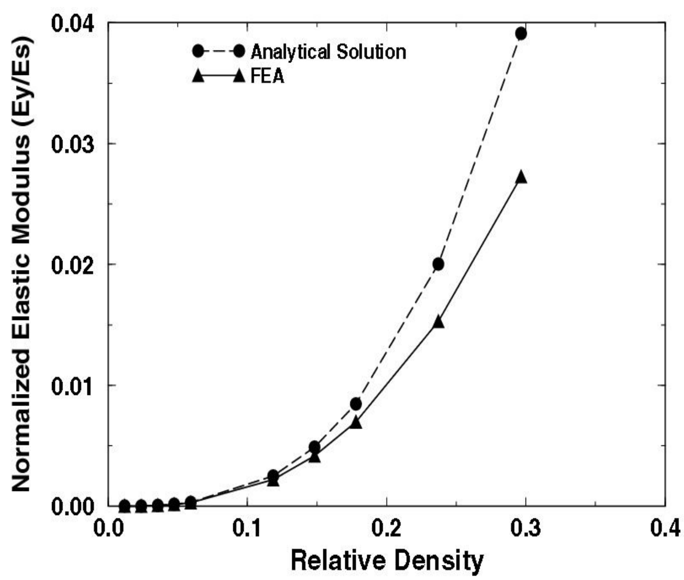


Figure 2 - Normalized elastic modulus vs. relative density (Y direction)

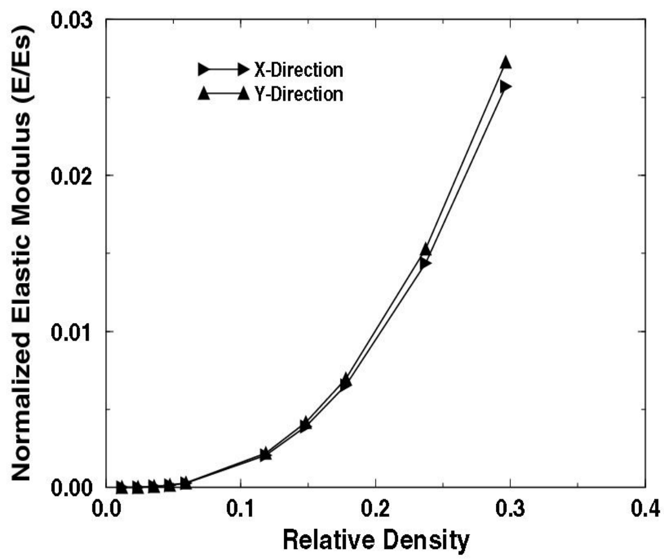


Figure 3 - Elastic modulus in the *X* and *Y* directions as a function of honeycomb density

3.1.1. Uniaxial Elastic Buckling

Finite element analysis for elastic buckling strength of honeycomb structure showed an average of 8% lower buckling mode compared with the analytical solution. This is shown in Fig. 6. The lowest buckling mode shape is shown in Fig. 7 and it is in agreement with those presented by Gibson et al. 1977.

3.1.2. Effect of Missing Cell Walls

In order to investigate the effect of missing cell walls on the elastic behavior of honeycomb structures, six different honeycomb structures were randomly produced with the same percentage of missing cell walls. MATLAB software was used to generate the random numbers used to select walls for deletion. Fig. 8 shows the effect of random missing cell walls on effective elastic stiffness of the regular honeycomb structure. On Average, the elastic modulus of the structure decreased by more than 45% for hexagonal structure with 7% missing cell walls.

The effect of missing cell clusters on the overall stiffness of the structure was also investigated. Examples of missing cell clusters can be seen in Fig. 9. The result demonstrated that the defect location has an apparently negligible effect on the elastic behavior of cellular structure (Fig. 10).

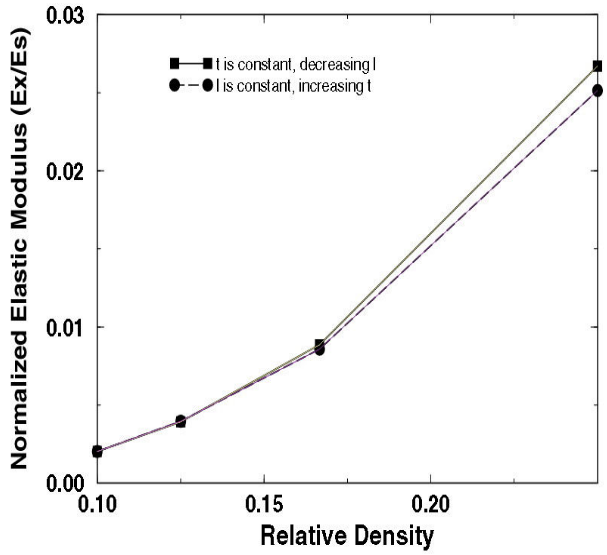


Figure 4 - The effect of cell wall thickness and cell wall length on the elastic modulus of the honeycomb structure in X direction

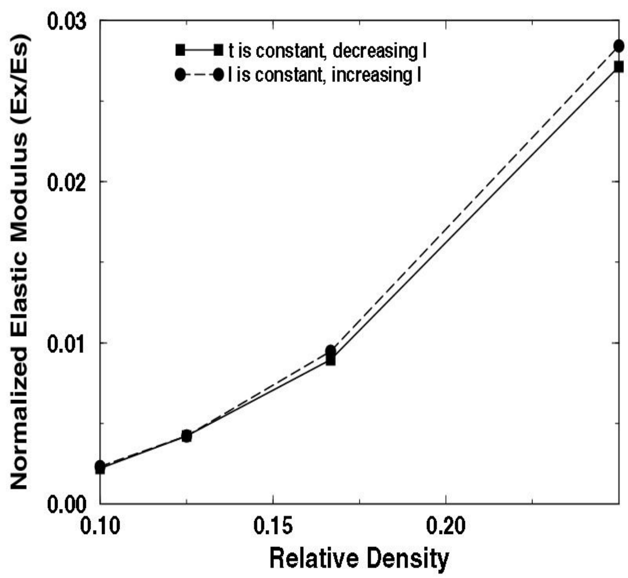


Figure 5 - The effect of cell wall thickness and cell wall length on the elastic modulus of the honeycomb structure in Y direction

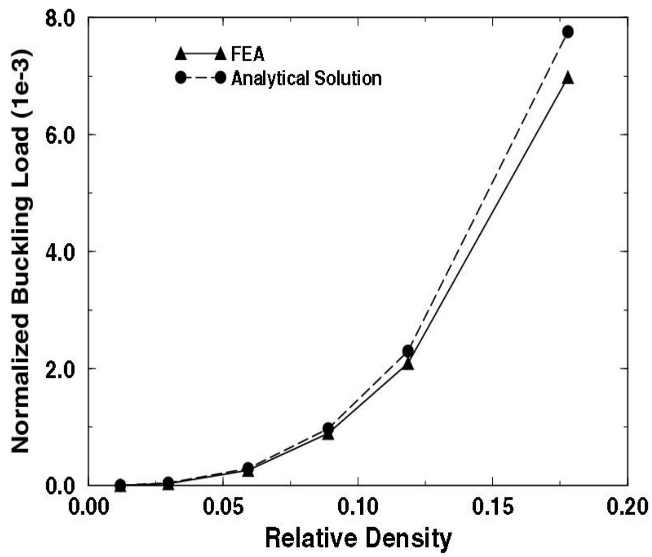


Figure 6 - Elastic buckling stress in X direction

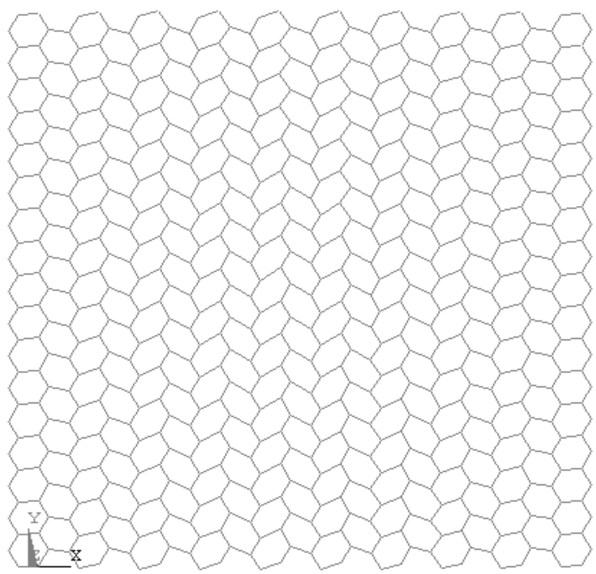


Figure 7 - First buckling mode of a regular honeycomb structure in x-direction

3.1.3. Effect of Random Filled cells

The effect of randomly filled cells on the mechanical behavior of honeycomb structures was also investigated. Fig. 11 shows that the elastic modulus of the structure increases with increasing filled cell volume fraction. The results indicate that an average of 11% increase in effective elastic modulus with 5% filled honeycomb cells. The results also show that location of filled cell have apparently negligible effect on the elastic modulus of the honeycomb structure.

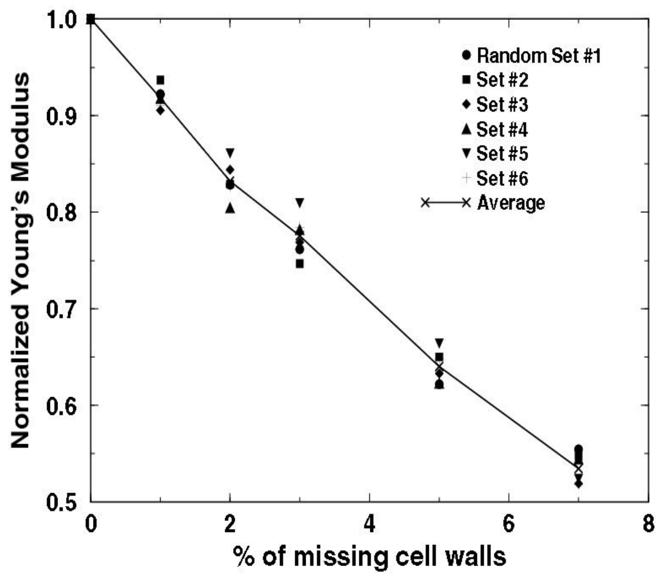
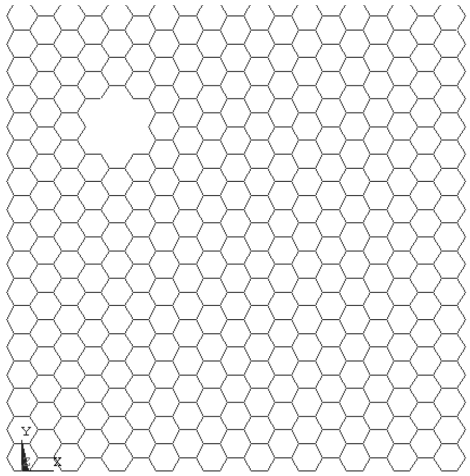


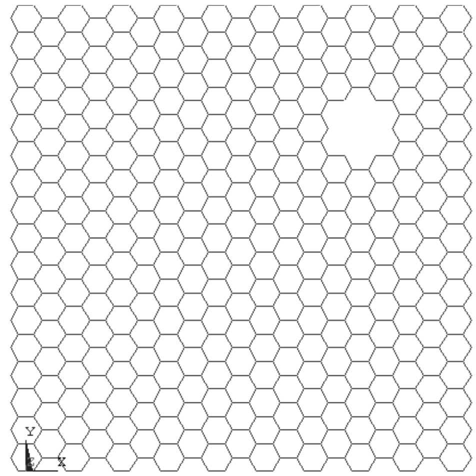
Figure 8 - Effect of missing walls on elastic modulus of regular honeycomb structure

3.2. Plastic Behavior of Regular Hexagonal Honeycombs

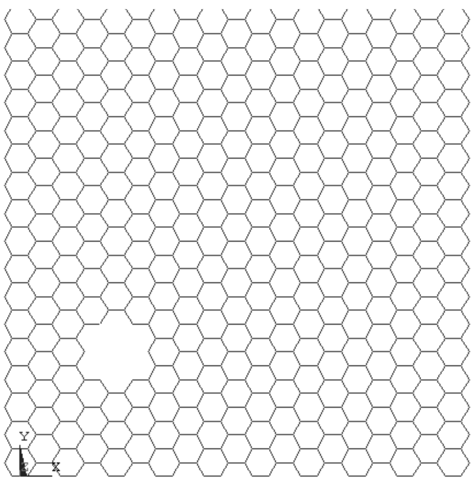
For the plastic analysis, a linear elastic-perfectly plastic constitutive relation was assumed for the honeycomb cell walls. Using the non-linear, large deformation option in ANSYS® an incremental horizontal displacement on the right edge was applied and reaction forces on the left side were used to define the stress-strain relation of the structure.



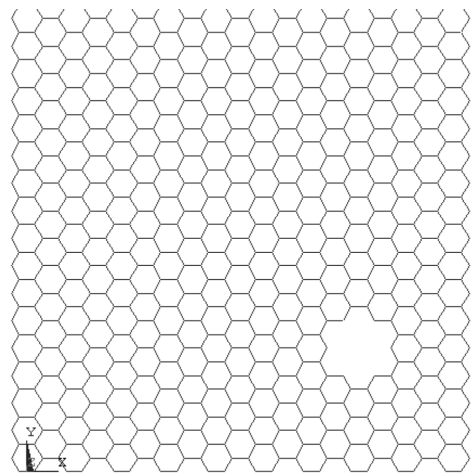
location (1)



location (2)



location (3)



location (4)

Figure 9 - Finite element models for varying missing cell cluster location

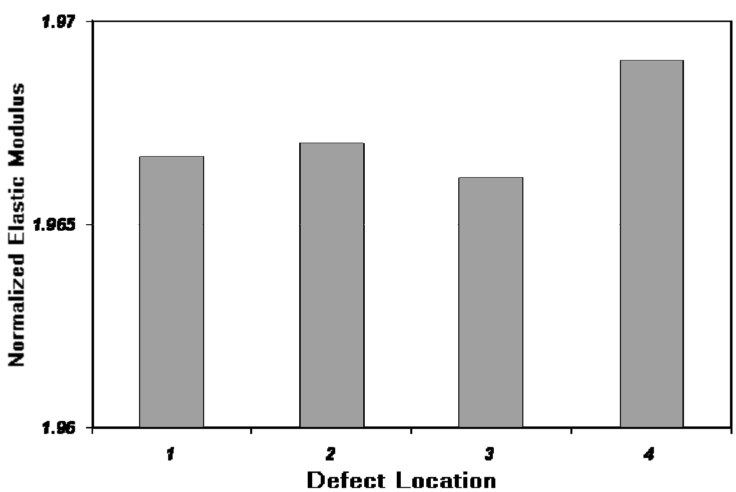


Figure 10 - Normalized elastic modulus of regular honeycomb structure with missing cells at different location

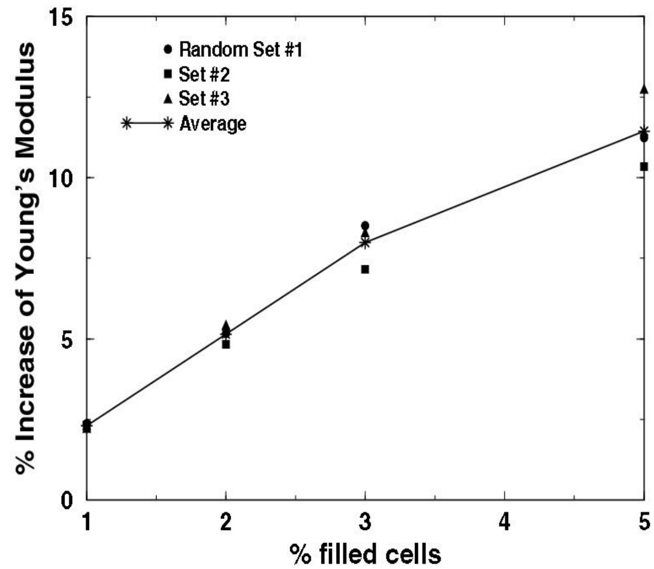


Figure 11 - The effect of filled cell on the elastic modulus of the honeycomb structure.

Fig.12 shows a typical stress-strain curve for a regular hexagonal honeycomb. The stress-strain curve exhibits a linear region, for small loading followed by a bilinear region which indicates initial localized yielding. This is followed by the global yielding of the cellular structure.

The plastic analysis was performed for different relative densities of honeycomb structure by changing the thickness of the cell walls. The yields stress was defined based on 0.2% offset plastic strain. Our results indicate that the yields stress of the material is in agreement with the proposed closed form solution. The finite element analysis was on average 10% below the yield stress calculated via theory. Fig. 13 shows the effects of honeycomb density on the yield stress of the structure.

3.2.1. Effect of Missing Cell Walls

Finite element analysis was performed to understand the effect of missing walls on yield strength of hexagonal structure. Our results for randomly selected missing walls are shown in Fig.14. The yield strength of the structure decreased by more than 60% compared with intact structure for 10% missing walls. This result is consistent for different random cell walls.

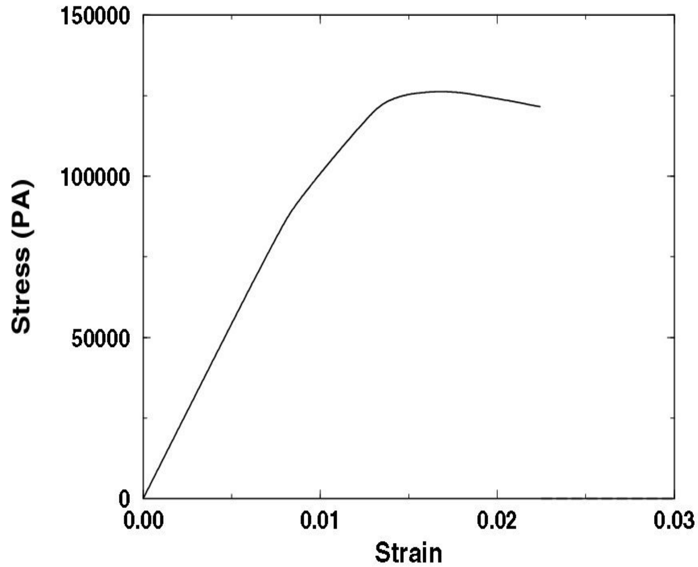


Figure 12 - Stress-Strain curve of a honeycomb structure with the relative density of ρ=0.1

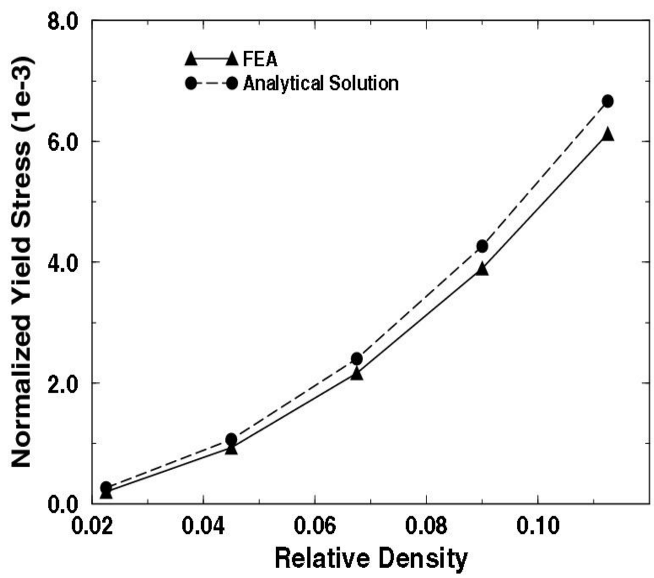


Figure 13 - Normalized yield stress vs. relative density for regular hexagonal honeycomb

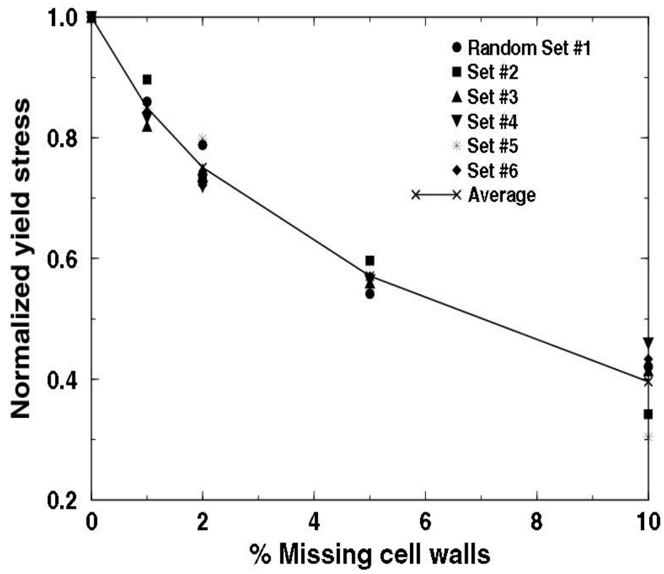


Figure 14 - Effect of missing walls on the yield stress of the honeycomb structure

3.2.2. Effect of Filled Cells

Finite element analysis demonstrated that filling up to 20 percent of the cells does not have a significant effect on the yield strength of the structure. Figure 15 shows the stress strain curve for regular hexagonal honeycombs with different percentage of filled cells. Based on 0.2% offset plastic strain, the yield strength of the structures was found to increase by less than 5% for hexagonal honey comb structure with 20% filled cells compared to the intact structure.

3.3. Elastic Properties of Voronoi Honeycomb Structures

Six different Voronoi network was produced using MATLAB®. Finite element analysis showed that Voronoi structures are less stiff compared with hexagonal honeycomb with the same relative density (Fig. 16).

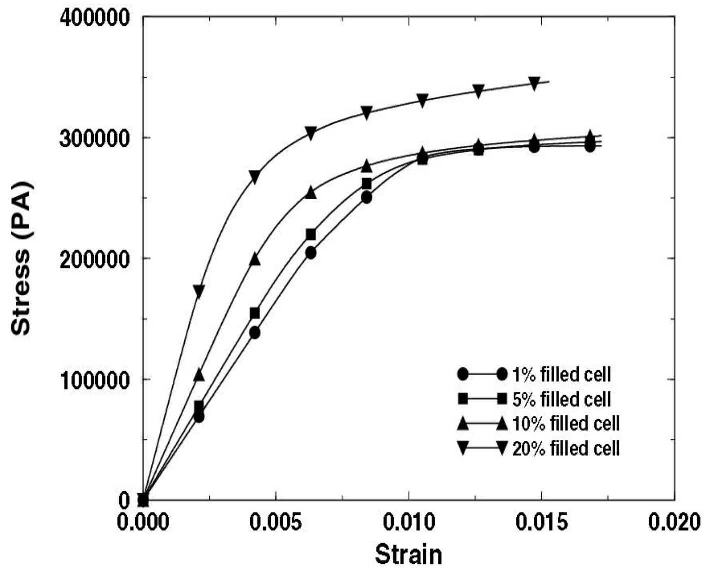


Figure 15 - Effect of filled cells on the yield strength of the honeycomb structure

3.3.1. Effect of Missing Cell Walls on Elastic Properties

The effect of missing walls on elastic properties of the Voronoi structure was investigated by randomly removing cell walls from the structure using MATLAB software. . Six different set of randomly selected cell walls were analyzed to understand the effects of missing cell walls on the mechanical properties of the Voronoi model.

Each randomly generated set had the same volume fraction of the removed cell wall. Fig 17 shows the elastic modulus of the Voronoi structures decreases drastically with the increasing cell wall removal. The results further indicate that the elastic modulus is much more sensitive to the cell wall removal in the Voronoi structure than the hexagonal structure with the same relative removed cell wall density.

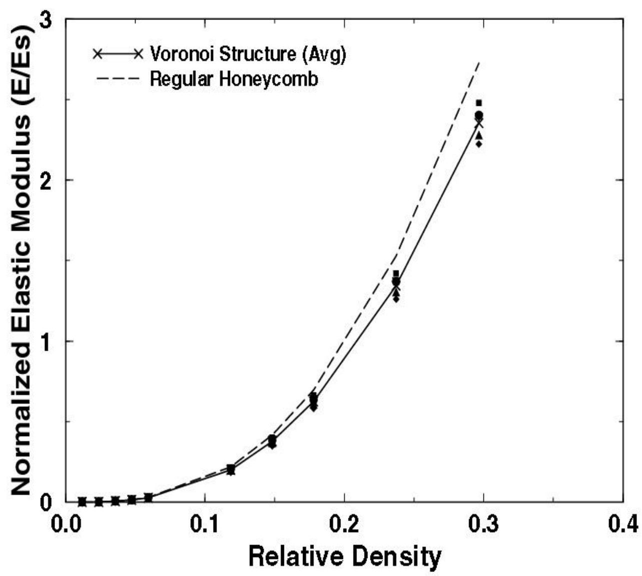


Figure 16 - Elastic modulus of Voronoi and regular honeycomb structures, showing the regular structure is stiffer than the Voronoi structure

3.3.2. Effect of Filled Cells on Elastic Properties

To investigate the effect of randomly filled cells on the elastic behavior of Voronoi structures, a Voronoi structure consisting of 150 cells was created. We considered four different random sets of cells for each case. Figure 18 shows the effect of filled cell on the elastic modulus of Voronoi structure.

The results indicate that elastic modulus is sensitive to the filled cell location. The results further indicate that Voronoi structures are more sensitive to filled cells compared to the hexagonal structure. For example, for 5% filled cells, the elastic modulus of regular honeycomb and Voronoi structure increased about 11% and 23%, respectively.

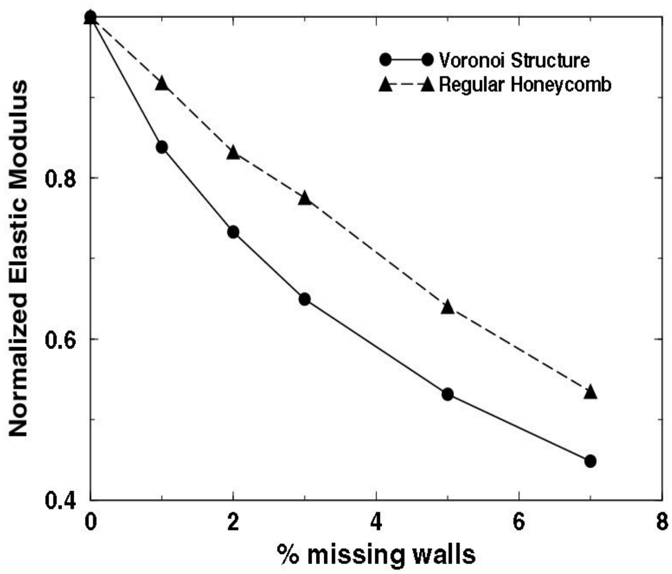


Figure 17 - Effect of missing wall on elastic modulus of Voronoi structure

3.4. Plastic Behavior of Voronoi Structures

For the plastic analysis, a linear elastic-perfectly plastic constitutive relation was assumed for honeycomb cell walls. An incremental horizontal displacement on the top edge using non-linear, large deformation option in ANSYS® was applied and reaction forces on the other side were used to define the stress-strain relation of the structure.

Fig.19 shows the equivalent yield stress, based on 0.2% offset plastic strain method, for different Voronoi relative densities. Our results indicate that the yield strength of the Voronoi honeycombs is less than regular hexagonal honeycomb structure with the same relative density.

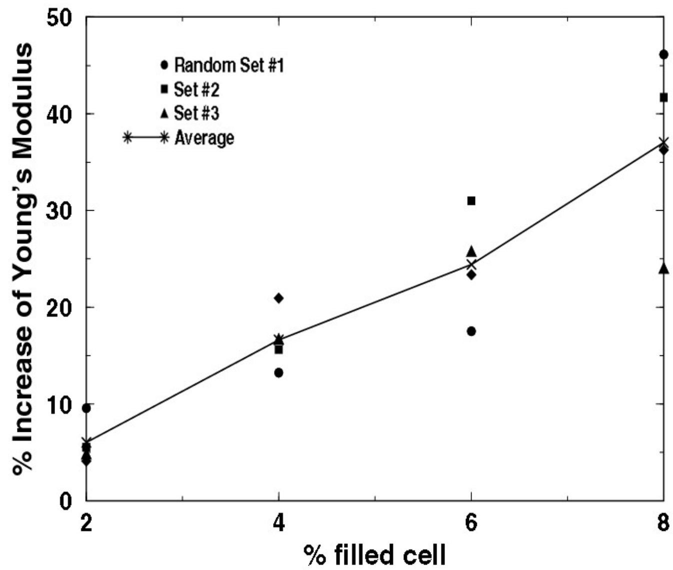


Figure 18 - Effect of filled cells on elastic behavior of Voronoi structure

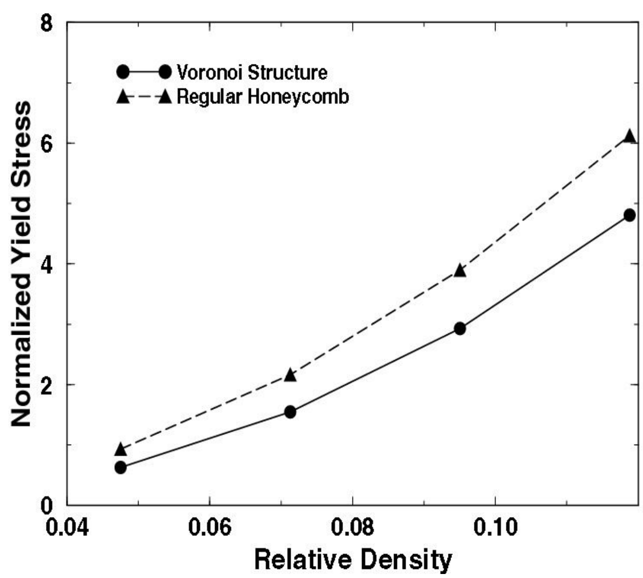


Figure 19 - Normalized yield stress vs. relative density for Voronoi structure and regular hexagonal structure

3.4.1. Effect of Missing Cell Walls on Plastic Behavior

Six different sets of randomly selected cell walls were used to investigate the effect of missing cell walls on the yield stress of the Voronoi structure. The results indicate that yield strength of the Voronoi structure decreases with increasing cell wall removal. However, in contrast to the filled cell, the results indicate that both Voronoi structure and hexagonal structure exhibit a similar sensitivity to the cell wall removal. Fig.20 shows the normalized yield strength of Voronoi structure versus the percentage of missing cell walls. Fig.21 compares the decrease of yield strength for both Voronoi and regular hexagonal structure due to missing walls. The results also suggest the yield stress in the Voronoi structure is sensitive to the missing cell walls location in a random set, Fig. 20.

3.4.2. Effect of Filled Cells on Plastic Behavior

Similar to regular honeycomb structures, filled cells did not have a significant effect on the overall plastic behavior of the structure. Our investigation showed less than a 4% variation in the yield stress when comparing defect-free and 8% filled-cell structures.

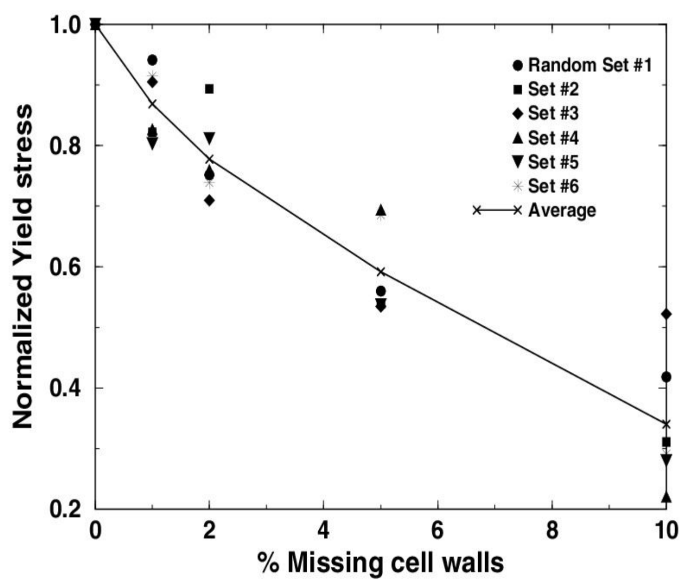


Figure 20 - Effect of missing walls on yield stress for Voronoi structures

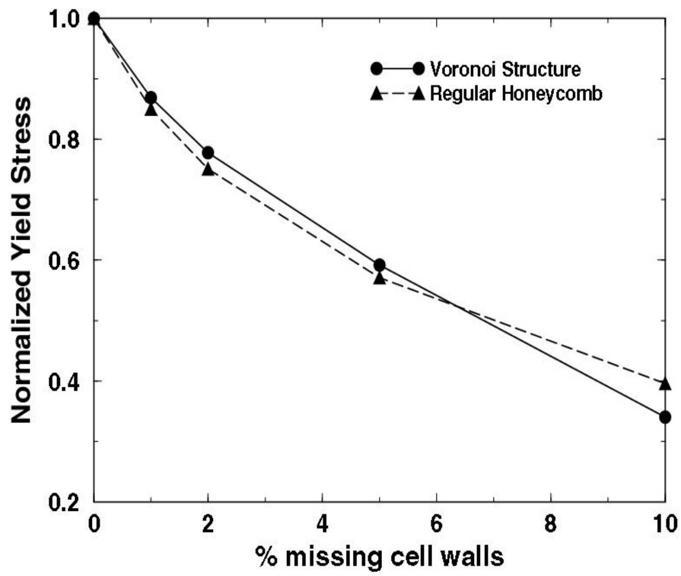


Figure 21 - Compare missing walls effect on yield stress point for Voronoi and regular hexagonal

3.5. Effect of Hardening on Plastic Behavior of Structures

For plastic behavior, we also considered bilinear constitute equation for the cell walls material. Here, we considered bilinear material properties with different hardening rate (tangent modulus) in the plastic regime. Fig.22 shows a schematic diagram of the stress-strain curve used in our investigation in order to understand the effect of strain hardening on the elastic-plastic behavior of both regular and Voronoi structures.

The effect of strain hardening on the yield stress of the cellular materials is presented in figure 23. Here, the results are presented as the percentage of increase of yield stress compared with the yield stress of elastic-perfectly plastic material. The change in the material yield stress with strain hardening is much more pronounced for Voronoi structures. For example, the yield stress of the regular honeycomb and Voronoi structure increased by 5% and 22%, respectively for material with 10% tangent modulus compared to elastic-perfect plastic material.

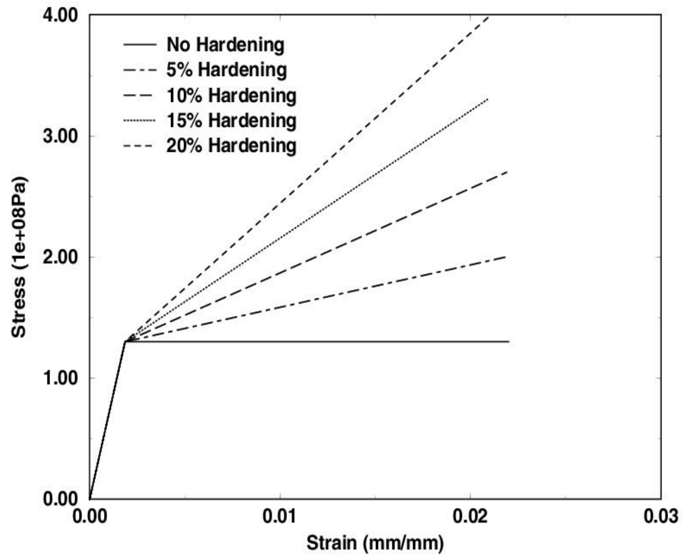


Figure 22 - Schematic stress-strain curve for materials with different hardening rate

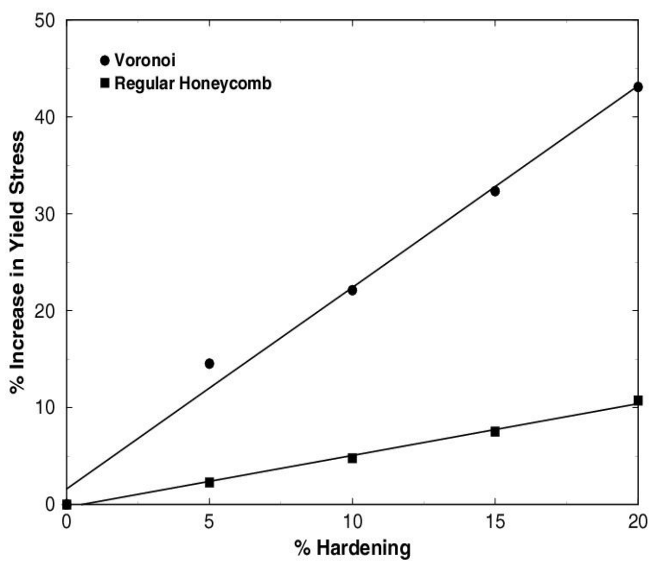


Figure 23 – Effect of hardening on overall yield strength of regular and Voronoi structures

4. Conclusion

The effects of various defects on the elastic and plastic behavior of both regular hexagonal honeycombs and 2D Voronoi structures were investigated using finite element method.

The finite element analysis showed that on average the non periodic structures have inferior mechanical properties compared to that of the regular hexagonal structures with the same relative density. The yield stress of Voronoi structures was, on average, 27% lower than that of hexagonal structure with the same relative densities.

Defects, introduced by removing cell walls at random locations, caused a sharp decrease in the effective mechanical properties of both Voronoi and periodic hexagonal honeycombs. However, our results indicated that elastic properties of Voronoi Structures are more sensitive to missing walls when compared to those of regular honeycombs. The yield strength of Voronoi and regular honeycombs exhibited the same sensitivity to cell wall removal.

In the case of filled cells, regular honeycomb structures showed less sensitivity to the defect compared to Voronoi structures. The overall elastic modulus of the structure increased by 11% when 5% of cells were filled in regular hexagonal honeycombs while for Voronoi structure it had more significant effect (approximately 22% increase).

The results also show that filled cell did not have a significant effect on yield strength of the regular and Voronoi structures.

CHAPTER 4

FUNCTIONALLY GRADED CELLULAR STRUCTURES

1. Abstract

Functionally graded cellular structures such as bio-inspired functionally graded materials for manufacturing implants or bone replacement, are a class of materials with low densities and novel physical, mechanical, thermal, electrical and acoustic properties. A gradual increase in cell size distribution, can impart many improved properties which may not be achieved by having a uniform cellular structure.

The material properties of functionally graded cellular structures as a function of density gradient have not been previously addressed within the literature. In this study, the finite element method is used to investigate the compressive uniaxial and biaxial behavior of functionally graded Voronoi structures. Furthermore, the effect of missing cell walls on its overall mechanical (elastic, plastic, and creep) properties is investigated.

The finite element analysis showed that the overall effective elastic modulus and yield strength of structures increased by increasing the density gradient. However, the overall elastic modulus of functionally graded structures was more sensitive to density gradient than the overall yield strength. The study also showed that the functionally graded structures with different density gradient had similar sensitivity to random missing cell walls. Creep analysis suggested that the structures with higher density gradient had lower steady-state creep rate compared to that of structures with lower density gradient.

2. Introduction

Functionally graded materials (FGM) consist of a gradual change in the volume fraction or mechanical properties of constituents in a direction. Application of these materials tends to reduce stresses resulting from material property mismatch, increases the bonding strength, improves the surface properties and provides protection against adverse thermal and chemical environment. Functionally graded materials are ideal for applications involving severe thermal gradients, ranging from thermal structures in advanced aircraft and aerospace engines to computer circuit boards [1-10].

Cellular structures are a class of materials with low densities and novel physical, mechanical, thermal, electrical and acoustic properties. A gradual increase in the cell size distribution, can impart many properties such as mechanical shock resistance and thermal insulation. The understanding of mechanical properties of functionally graded cellular structures as a function of cell size gradient is important in the proper application and utilization of these materials. Furthermore, the presence of defects may affect the material properties of graded cellular structures.

Several efforts have been made to investigate the mechanical behavior and the effects of imperfections on the mechanical properties of cellular materials; most of which are based on the finite element method (FEM) [11-23]. Chen et al. [16] investigated the influence of different types of morphological imperfections (waviness, non-uniform cell wall thickness, cell-size variations, fractured cell walls, cell-wall misalignment and missing cells) on the yielding of 2D cellular solids. They also performed a finite element study to determine the effects of holes on elastic modulus and yield strength of regular honeycombs under biaxial loading. Wang and McDowell [22] and Silva and Gibson [23] investigated the effects of missing or fractured cell walls on mechanical properties of regular hexagonal and Voronoi cellular materials.

The mechanical properties of cellular structures in which the density is functionally graded in a specific direction have not been addressed. In this study, the finite element method is used to investigate the compressive uniaxial and biaxial behavior of functionally graded Voronoi structure. Furthermore, the effect of missing cell walls on the overall mechanical (elastic, plastic, and creep) properties of these structures is investigated.

3. Methodology

A two-dimensional square Voronoi structure with the length L, was considered. The structure was divided into four equal length segments ($\Delta y = L/4$), Fig. 1. The local density gradient for segments j and $j+1$, ($for\ j = 1,2,3$) was defined as below:

$$\nabla \rho = \frac{\partial \rho}{\partial y} = \frac{\rho_{j+1} - \rho_j}{\Delta y} = \frac{\frac{[\sum l_i t_i]_{j+1}}{L.\Delta y} - \frac{[\sum l_i t_i]_j}{L.\Delta y}}{\Delta y} \qquad (1)$$

where l_i and t_i are the length and the thickness of each cell wall located in each segment, respectively. The density gradient was kept constant through the structure. Fig.1 shows a functionally graded Voronoi structure with relative density gradient of 0.53% in y-direction.

Models of two dimensional non-periodic Voronoi structure with different relative density gradient in y-direction were developed using Matlab® 6.1 (The MathWorks, Inc., Natick, MA) and the density gradient, as described, was kept constant while developing Voronoi structures.

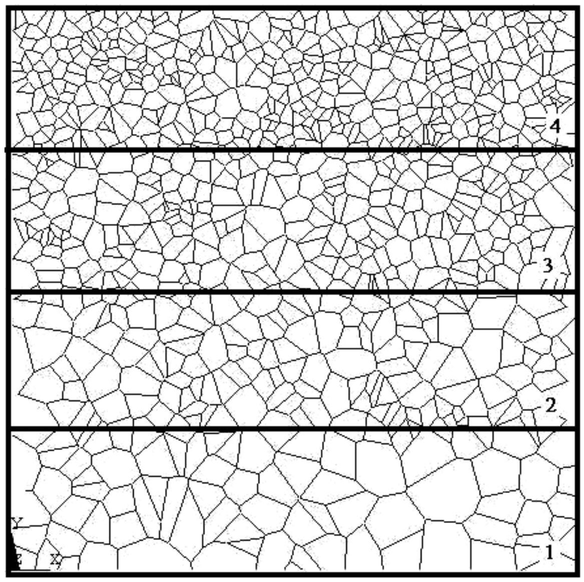


Figure 1 - Schematic FEM model for Voronoi structure with density gradient

In this study, four different density gradients were studied. Two dimensional Voronoi structure with relative gradient equals to 0.02% and relative density of 0.1 was first developed, as shown in Fig. 2; then the relative gradient was changed by adding cell walls to the structure in accordance with the density gradient, as shown in Fig. 1; the thickness of cell walls was kept constant. The relative density of Voronoi structures is defined as:

$$\rho^* = \frac{\sum t_i . l_i}{L_1 . L_2} \quad (2)$$

where l_i and t_i are the length and the thickness of each cell walls and L_1 and L_2 are the structure dimensions in X and Y direction, respectively. This density (0.1) constituted the base line density in developing graded materials with higher densities.

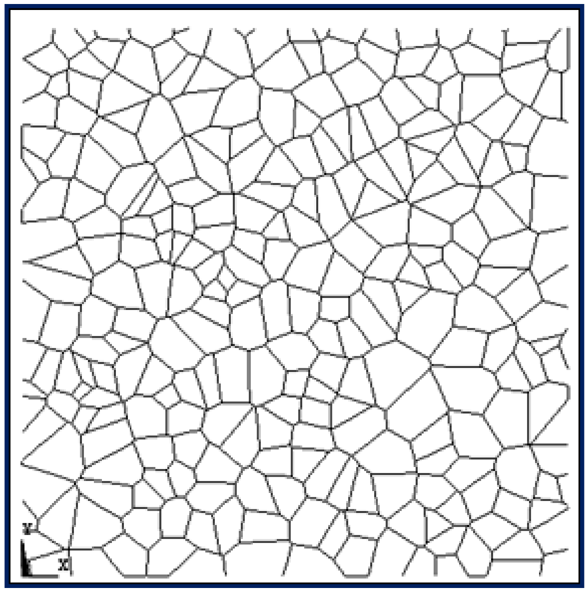


Figure 2 – Finite element model of a Voronoi honeycomb structure with density gradient of zero and relative density of 0.1

The compressive stress-strain curve of each structure was evaluated using the nonlinear solver of finite element package ABAQUS 6.6 (SIMULIA, Warwick, RI). Both uniaxial and biaxial loadings were considered in evaluating the stress-strain curve. Furthermore, the effect of defects on the mechanical behavior of functionally graded cellular materials was also investigated.

Based on the cell wall length, each cell wall in the structures was modeled using two to six BEAM22 elements which is capable of capturing the axial, bending and shear deformations. The material property of aluminum was used for this study. The cell wall material was assumed to be linearly elastic for the moduli calculations and linear elastic-perfectly plastic for the plastic collapse calculations. The cell wall Young's modulus, yield strength and Poisson's ratio were taken as $E = 70\,GPa$, $\sigma_y = 130\,MPa$ and $\vartheta\ = 0.3$, respectively [1].

For the honeycomb structure with thickness of unity, the effective elastic modulus in Y was calculated as:

$$E = \frac{F.L_2}{L_1.u} \tag{3}$$

where F is the applied force and u is the resultant displacement, L_1 and L_2 are the structure dimensions in X and Y direction, respectively. To calculate the modulus, a vertical (compressive) force was applied to the top surface of the model. All cells on the top edge were coupled to have the same vertical displacement. The bottom edge was supported by rollers at each cell and both side edges were allowed to translate in the horizontal direction. To evaluate the post-yield behavior, a linear elastic-perfectly plastic constitutive relation for the cell walls was assumed. An incremental vertical displacement on the top edge was applied and reactions at the base supports were recorded. The Yield strength was calculated using 0.2% strain offset method for each structure.

In this study, the finite element analysis was first performed for intact graded cellular structures to calculate the effective elastic modulus and yield strength of the structures. Then the analysis was repeated while introducing defects.

4. Results and Discussion

The elastic modulus of functionally graded cellular structures as a function of its density gradient is shown in Fig. 3. As expected, the overall effective elastic modulus increased by increasing the density gradient. The results indicate that for the range of the density gradient considered, the elastic modulus increases linearly with the density gradient, Fig. 3. Figure 4 shows the normalized yield strength for Voronoi structures with different density gradient. The elastic modulus and yield strength are presented as a normalized value respect to the Voronoi structure with no gradient (The uniform Voronoi structure constitutes the bottom layer of the functionally graded material, which had a density of 0.1).

It is observed that for the structures with higher density gradient, the yield strength of the graded structure did not change significantly compared to the lower gradient structures; this can be explained by considering that the yield strength is controlled by the weakest section of the structure, Fig. 1. For example, the yield strength of a functionally graded Voronoi structure with a gradient of 0.38% increased 58% compared with uniform Voronoi structure while this increase was about 62% for structure with the gradient of 0.53%. These results also showed that the overall elastic modulus of functionally graded structures was more sensitive to density gradient than the overall yield strength.

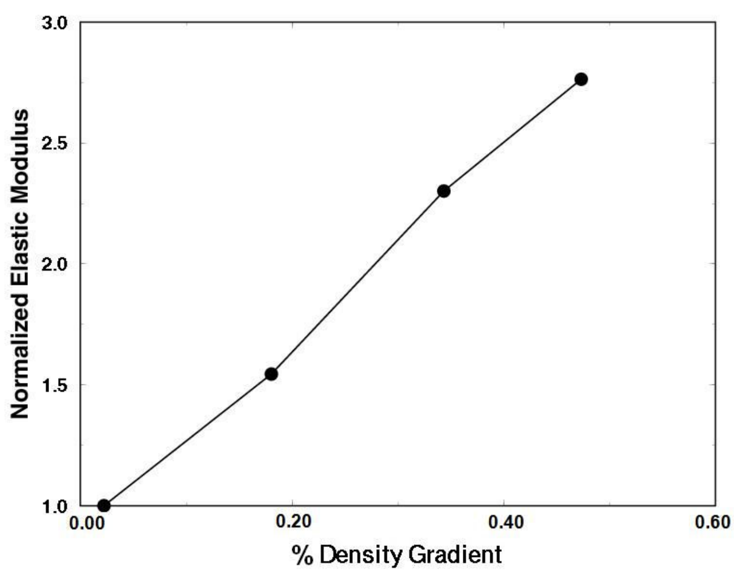


Figure 3 – Normalized elastic modulus of a functionally graded cellular structure as a function of the density gradient

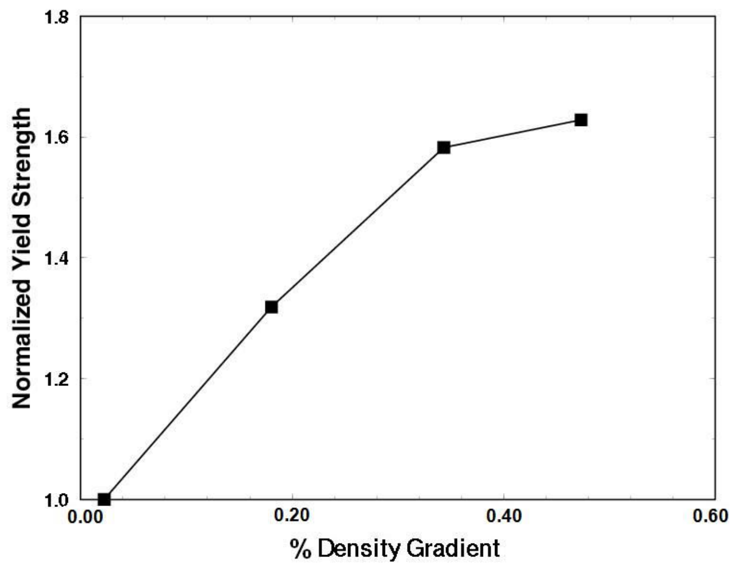


Figure 4 - Normalized yield strength of a functionally graded cellular structure as a function of the density gradient

4.1. Effect of Missing Cell Walls on Elastic and Plastic Behavior

Three different random sets were developed using Matlab® to investigate the effect of randomly missing cell walls on the overall mechanical behavior of functionally graded cellular structures.

Fig. 5 shows the reduction of the overall elastic modulus for various functionally graded structures as a function of missing walls percentage. The result suggested that the functionally graded structures with different density gradient had similar sensitivity to random missing cell walls. For example, 5% random missing cell walls decreased the overall elastic modulus of functionally graded Voronoi structures by 45%, 41% and 45% for structures with density gradient of 0.02%, 0.20% and 0.38%, respectively.

Same random sets were used to investigate the plastic behavior of defected structures. Fig. 6 shows the effect of missing walls on yield strength of structures. The results indicated that the effective yield strength also exhibit similar sensitivity to the missing walls for different density gradient. The results furthermore indicated that the yield strength is slightly more sensitive to the missing walls compared to the overall elastic modulus.

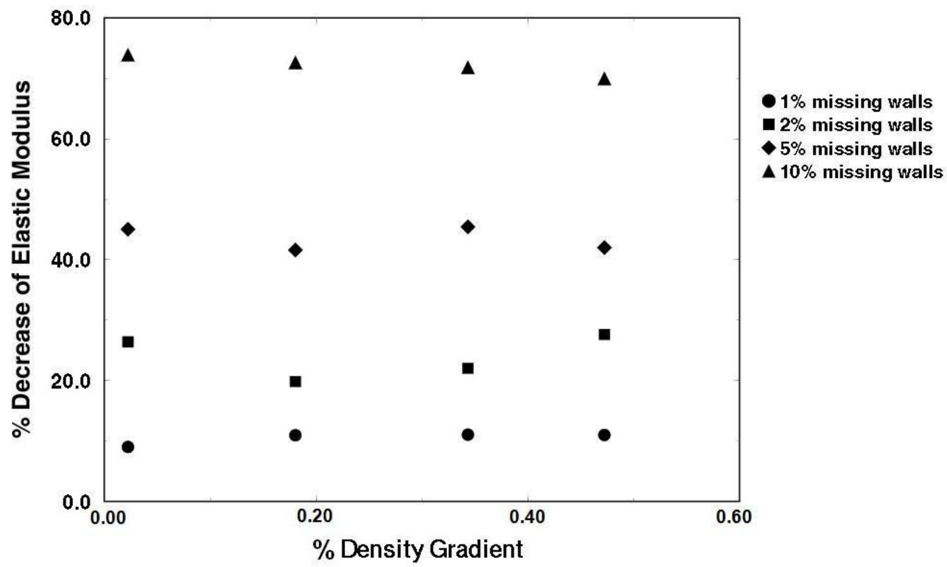


Figure 5 - Effect of missing walls on elastic behavior of functionally gradient structures

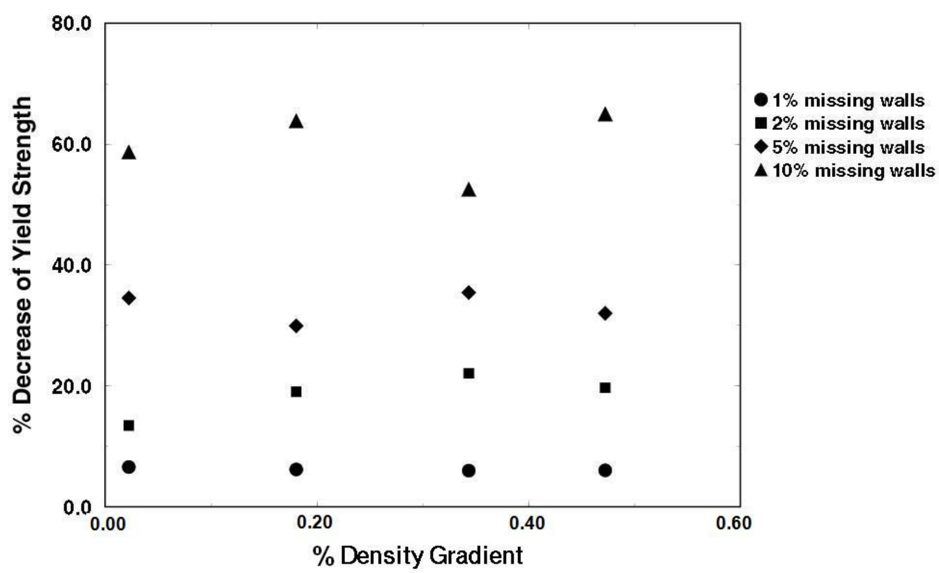


Figure 6 - Effect of missing walls on plastic behavior of functionally graded structures

4.2. Effect of Hardening on Plastic Behavior

In addition to elastic-perfectly plastic behavior, the cell walls were also considered as elastic-bilinear hardening in the plastic regime. Here, materials with different hardening rate (tangent modulus) in plastic regime were considered [11]. The results is shown in a form of percentage of increase in the effective yield strength of structures for different gradient and different hardening rate (Fig. 7)

The results suggested that higher the density gradient of structure, the higher the percentage of increase in yield strength for a constant rate of hardening in plastic region. Furthermore, the effect of hardening rate on the effective yield strength of structures was more pronounced for higher hardening rates. [For example, for structure with gradient of 0.53% and 20% strain hardening rate, the yield strength increased about 80% compared to an elastic-perfectly plastic structure with same relative density gradient.]

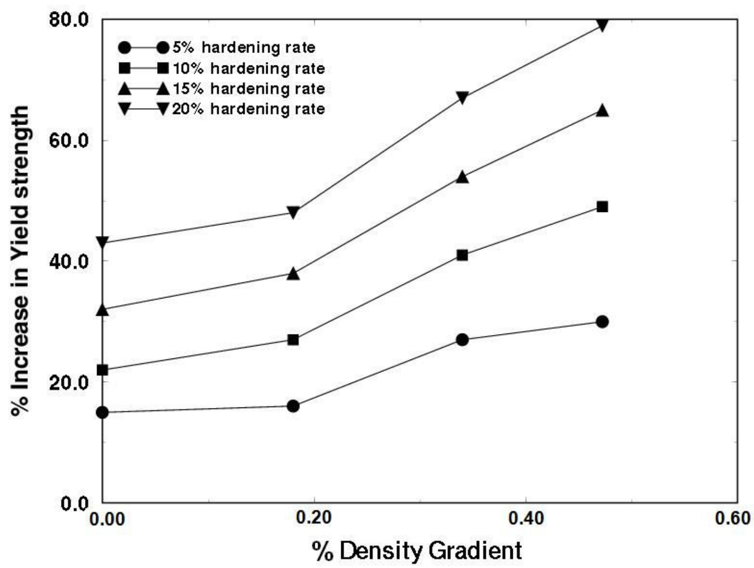


Figure 7 - Effect of strain hardening on normalized yield stress of gradually graded structures

4.3. Creep Behavior of Functionally Graded Cellular Structures

Before finding the effect of defects on creep response of the graded structures, the steady state creep rate of cellular structures (regular honeycomb and Voronoi structures) were fist investigated.

To simulate creep, each node on the top surface of structures was subjected to a constant load over time and the strain was calculated from the overall displacement of structure. The steady-state creep rate was then obtained from the plot of deflection versus time. The material of the cell walls was assumed to obey power-law creep and creep property of Aluminum was used for this investigation [15]. The creep rate was calculated for a range of stresses and temperatures.

Fig.8 compares the creep rate for regular hexagonal honeycomb and Voronoi structure for different relative densities. The results indicated that the creep rate was a strong function of structures relative density and the creep rate in Voronoi structure was higher than that of regular honeycomb structure with the same density. The difference was more pronounced for higher relative densities. Creep analysis was performed at the temperature of 250°C and the constant stress of $\sigma = 0.17\ MPa$.

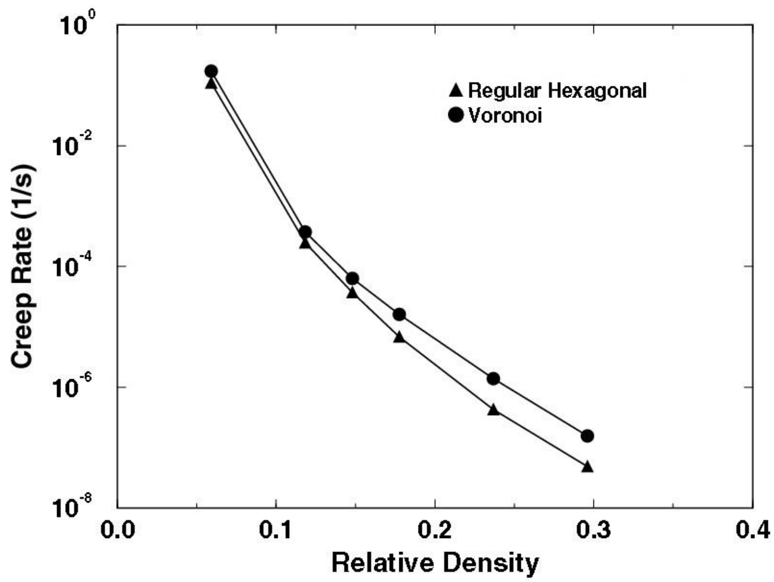


Figure 8 - Effect of relative density on steady-state creep rate of cellular structures

The creep rate was obtained for different temperature and applied stress. Fig. 9 shows creep rate versus 1/T (the absolute temperature). The results showed that the cellular structures obey the power-law creep. Constant stress of $\sigma = 0.17\ MPa$ was applied for this analysis.

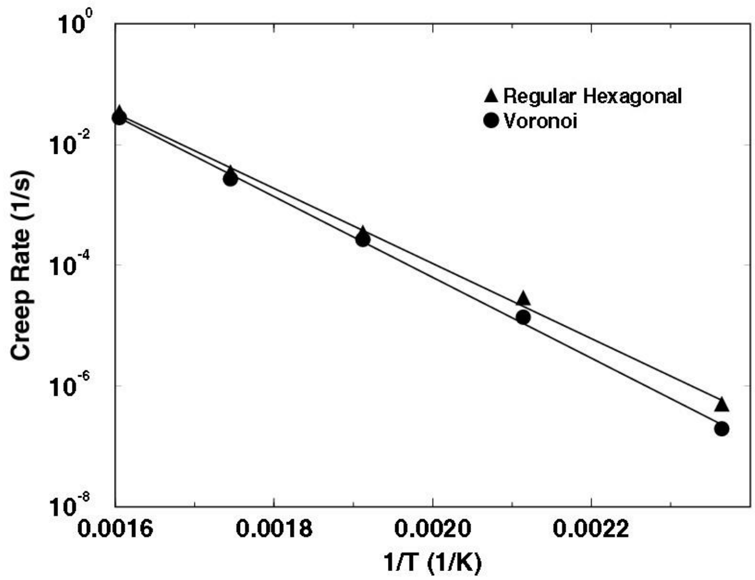


Figure 9 - Effect of temperature on steady-state creep rate of cellular structures

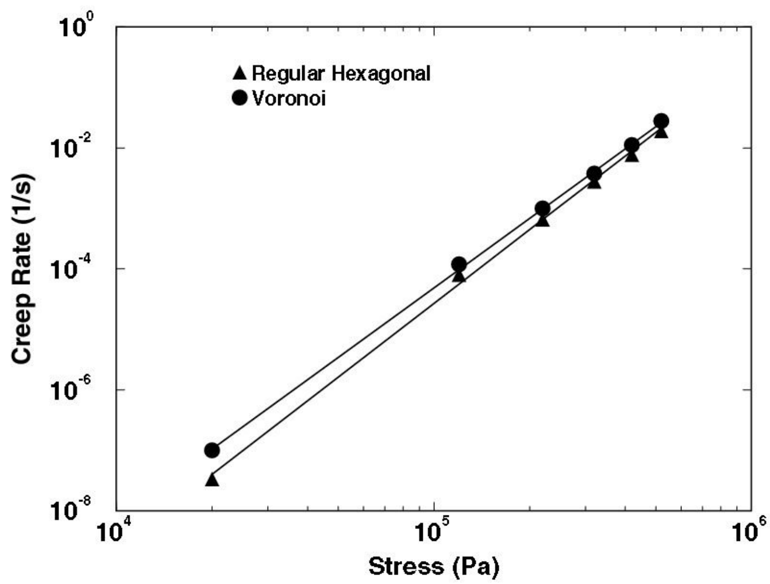


Figure 10 - Effect of different applied stress on the steady-state creep rate of cellular structures

The effect of different applied stressed on the creep response of the structure at a constant temperature (250°C) was also investigated. Fig. 10 shows the creep rate also follows a power-law relation with the applied stress.

The effect of missing walls on the creep rate of uniform hexagonal and Voronoi structure was investigated. The results showed a dramatic effect on the creep rate for various percentage of missing walls. For instance, removing only 5 percent of cell walls caused the creep rate of the regular hexagonal and Voronoi structure to increase by 520% and 480%, respectively. The average of creep rate for three different random sets for each percentage of missing walls and the temperature of 250 °C and the applied stress of $\sigma = 0.17\ MPa$ were taken as the representative of its behavior and is reported here.

The finite element analysis for the effect of different density gradient on the overall steady-state creep behavior of Voronoi structures is shown in Fig. 11. The results suggested that the structures with higher density gradient had lower steady-state creep rate compared to that of structures with lower density gradient. This could be explained by the overall increase in the density of the structures. Similar to regular hexagonal and Voronoi structures, the presence of defects (missing cell walls) caused a significant increase in the steady-state creep behavior of functionally graded structures.

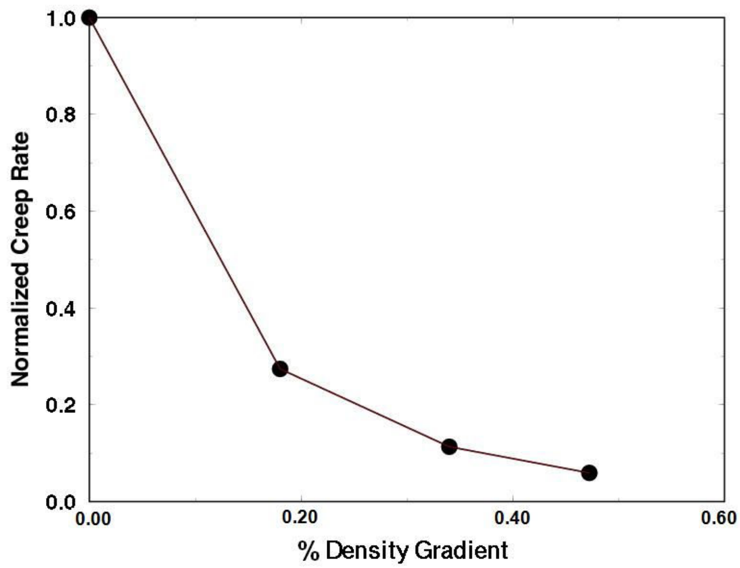


Figure 11 - Steady-state creep behavior for functionally graded structures

4.4. Biaxial Loading

Biaxial behavior of regular hexagonal honeycomb and Voronoi structure (with no gradient) were first investigated. In this study, only compressive-compressive behavior of structures was studied. Fig. 12 shows the yield surface for both regular and Voronoi structure with the same relative density (ρ=0.13). Ajdari *et al.* 2007 [11] showed that Voronoi structures are less stiff compare to regular hexagonal structures with the same relative density, and for biaxial behavior the results is in agreement with previous works.

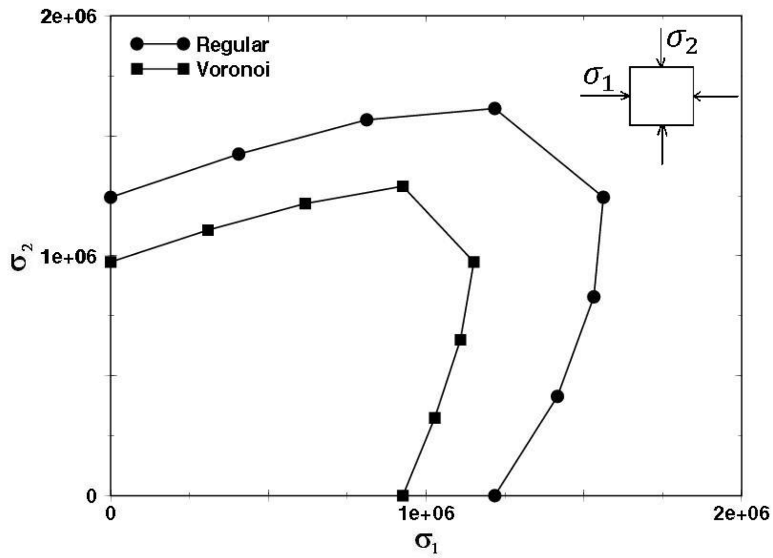


Figure 12 - Comparison of yield surface of regular hexagonal and Voronoi structure with same relative density

The result of finite element study on the effect of density gradient on the resultant yield surface is shown in Fig. 13. The results indicate that for structures with higher density gradient, the compressive-compressive yield surface expands in the x-direction, the direction in which the structure is not graded. This behavior is more pronounced for structures with higher density gradient.

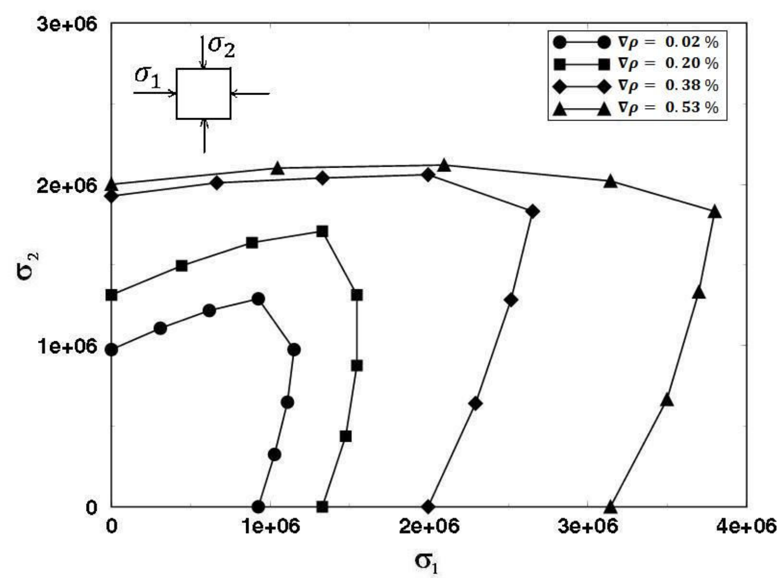


Figure 13 - Effect of density gradient on yield surface of functionally graded structures

The effect of missing walls on the overall compressive-compressive yield surface for both regular hexagonal honeycomb and Voronoi structure was investigated. The results are shown in Fig. 14 and 15. The results are the average result for three different random sets for regular hexagonal honeycombs and Voronoi structures, respectively. Missing cell walls shrinks the compressive-compressive yield surface of both regular and Voronoi structures. Both structures, showed similar sensitivity to the missing walls.

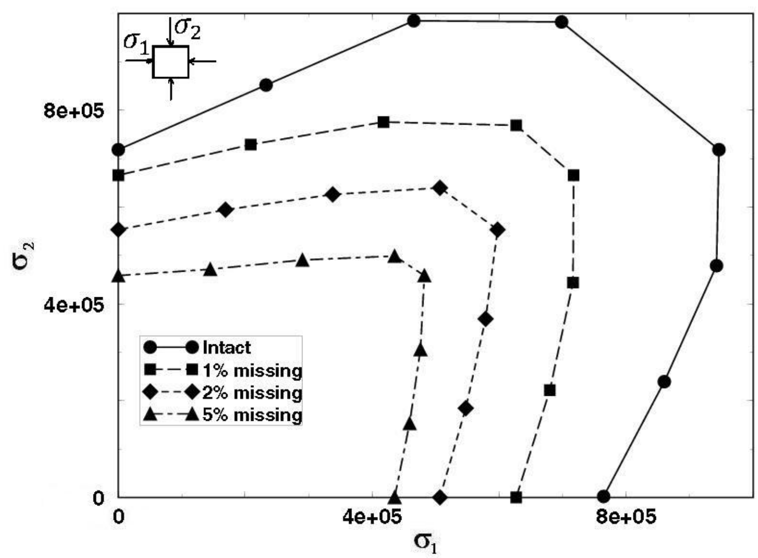


Figure 14 - Effect of missing wall on yield surface for regular hexagonal honeycombs

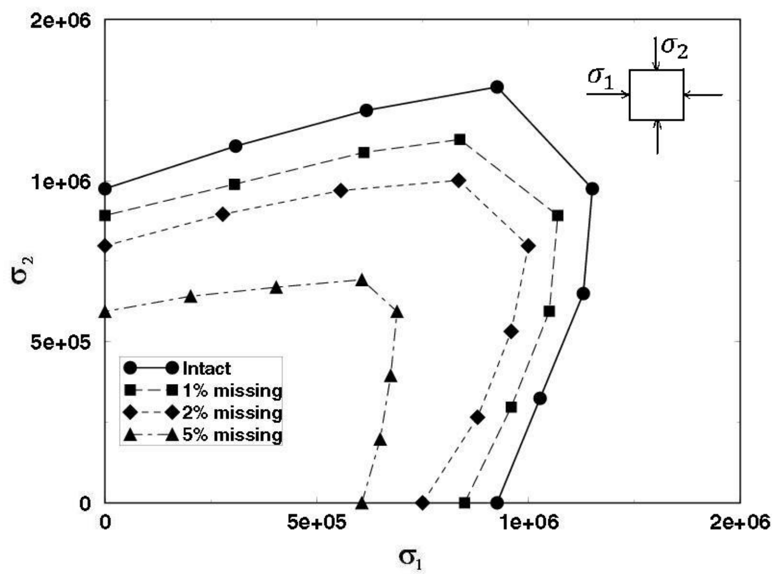


Figure 15 - Effect of missing walls on yield surface for Voronoi structures

5. Conclusion

Models of two dimensional graded Voronoi structures were generated using Matlab. Finite element method was used to study the behavior of structures. Both elastic and plastic behavior of structures and the effect of missing cell walls were considered in this study.

The overall effective elastic modulus and yield strength of structures increased by increasing the density gradient however the overall elastic modulus of functionally graded structures was more sensitive to density gradient than the overall yield strength. The study also showed that the functionally graded structures with different density gradient had similar sensitivity to random missing cell walls.

The yield strength of functionally graded cellular structures with cell walls made of material exhibiting elastic-bilinear hardening behavior showed that the overall yield strength increased compared to elastic-perfectly plastic material. This increase was much more pronounced for material with higher strain hardening and higher density gradient.

Creep analysis showed that the structures with higher density gradient had lower steady-state creep rate compared to that of structures with lower density gradient. The finite element analysis indicated that the compressive-compressive yield surface of structures with higher density gradient expanded in the direction of which the structure is not graded. The yield stress is a strong function of applied stress in the gradient direction and independent off applied stress in the not graded direction.

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