A FRAMEWORK FOR SIMULATION-BASED MULTI-ATTRIBUTE OPTIMUM DESIGN WITH IMPROVED CONJOINT ANALYSIS

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Alex Ruderman

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A FRAMEWORK FOR SIMULATION-BASED MULTI-ATTRIBUTE OPTIMUM DESIGN WITH IMPROVED CONJOINT ANALYSIS

Approved by:

Dr. Seung-Kyum Choi, Advisor
School of Mechanical Engineering
Georgia Institute of Technology

Dr. Chris Paredis
School of Mechanical Engineering
Georgia Institute of Technology

Dr. Janet Allen
School of Mechanical Engineering
Georgia Institute of Technology

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SUMMARY

Decision making is necessary to provide a synthesis scheme to design activities and identify the most preferred design alternative. There exist several methods that address modeling designer preferences in a graphical manner to aid the decision making process. For instance, the Conjoint Analysis has been proven effective for various multi-attribute design problems by utilizing a ranking- or rating-based approach along with the graphical representation of the designer preference. However, the ranking or rating of design alternatives can be inconsistent from different users and it is often difficult to get customer responses in a timely fashion. The high number of alternative comparisons required for complex engineering problems can be exhausting for the decision maker. In addition, many design objectives can have interdependencies that can increase complexity and uncertainty throughout the decision making process. The uncertainties apparent in the attainment of subjective data as well as with system models can reduce the reliability of decision analysis results. To address these issues, the use of a new technique, the Improved Conjoint Analysis, is proposed to enable the modeling of designer preferences and trade-offs under the consideration of uncertainty. Specifically, a simulation-based ranking scheme is implemented and incorporated into the traditional process of the Conjoint Analysis. The proposed ranking scheme can reduce user fatigue and provide a better schematic decision support process. In addition, the incorporation of uncertainty in the design process provides the capability of producing robust or reliable products. The efficacy and applicability of the proposed framework are demonstrated

with the design of a cantilever beam, a power-generating shock absorber, and a mesostructured hydrogen storage tank.

CHAPTER 1: INTRODUCTION

1.1. Motivation

As exponential growth of technology and engineering capabilities continues to accelerate, our understanding and handling of these advances will get out of control. Literally, the progress will be too rapid that it will rupture engineers' capability to follow it. For instance, these leaps forward bring greater complexity to designs and a larger number of decisions that need to be made to develop better products. This appreciation has brought the focus of design to the need for new methods to facilitate and simplify the design process with addressing Decision Maker's (DM) preferences.

The realization that making decisions is an intricate part of engineering design has stimulated a great deal of research in areas of decision analysis and multi-attribute decision making [1-3]. The main goal of decision-making processes is the improvement of decision quality and the creation of the most profitable product [4]. Techniques to model design decisions can be used to incorporate designer preferences in the development of a product to assist in accomplishing this goal. The modeling of DM preferences and trade-off analysis is typically done through surveys and other methods of gaining subjective data from the DM that further accompanies objective data to determine an optimal design based on the attained subjective data. It is often stated DM can be indecisive when comparing many different alternatives in the multi-attribute decision making process. For example, the traditional decision making techniques [1, 5-9] are usually highly iterative and require exhaustive questioning for the DM. In addition, the most popular methods in the research area, such as Conjoint Analysis (CA) [5, 7, 10, 11], Quality Function Deployment [12, 13] and Survey Design [14], that reflect a customer driven survey technique often require a large amount of alternative comparisons and many respondents in order to be accurate. Moreover the lottery elicitation in von-Neumann Morgenstern utility theory and Multi-Attribute Utility Theory, require questions to determine indifference points which are less systematic than a more simple ranking survey [15-17]. These qualities cause some of the uncertainty in the decision making process when it comes to modeling multiple DMs' preferences.

Increased complexity in a system adds an increased amount of uncertainty with design variables, boundary conditions, system modeling and simulation, etc. Thus, advanced methods of uncertainty analysis have had enormous attentions for last two decades to accurately quantify and propagate uncertainties in engineering design problems. One of the major uncertainty analysis approaches is a probabilistic approach which is based on the assumption of known probability density function (PDF) information. The benefit of probabilistic analysis is the ability to produce comprehensive analysis results, not a single result from a mean design point. These probabilistic analysis techniques have successfully been used for many types of design analysis such as bridge failure assessment, multi-criteria decision analysis, reliability of steel connections, etc. [18-21]. In addition, it has been shown that the incorporation of uncertainty-based analysis or reliability-based design can aid in the reduction of risks in designs by accounting for various uncertainties in the design process [22].

1.2. Research Questions and Hypothesis

The above details describe the basis for the following research question for this thesis.

1) *How can customer survey driven decision analysis methods be integrated with Reliability-based Design methods to reduce uncertainty?*

It is hypothesized that utilizing objective or measurable data from a simulation model to represent the decision-maker's preferences will allow for subjective data to be modeled without the need for customer surveys or iterative questioning. To elaborate, if a preference can be given that can be assumed to be accurate for the majority of decision-makers preferences and is based on a measureable value then the subjective data can be represented with objective data. If this is accomplished, then it would allow for preferences to be used to compare alternatives through the simulation of a physical model.

This prediction leads to a secondary hypothesis for this research question. If a majority preference is made based on a reliability constraint, such as risk properties or probability of failure which is measurable through the use of Reliability-based Analysis methods, then the subjective data can be modeled based on the most reliable alternative.

These hypotheses were arrived at from the difficulties and inaccuracies apparent in methods anchored in customer survey type subjective data. It is stated the decision-maker can be indecisive sometimes when comparing alternatives. This causes some of the uncertainty in these methods when it comes to modeling multiple decision-makers' preferences. However, in many engineering problems the solution is determined by finding the most reliability solution. In other words, the designer has a preference for the system that is more reliable, which is true for the majority of customers. It would be a rare case that the decision-maker would want a solution to a problem to not be reliable. With the use of reliability-based analysis methods, the reliability of a specific alternative can be calculated to give a relative preference value for comparison with another alternative.

In this thesis, one of the objectives of the current research is to implement approaches to reduce or appropriately handle complexity and uncertainty of the system during the design process. Thus, it is critical to create a more systematic approach for reducing the complexity of the design process by correctly modeling subjective data or DM's preferences in multi-attribute decision analysis. It is also critical to implement an approach to address uncertainty of the system parameters and customers' survey driven data. These issues will be addressed in the current research by exploring a novel framework to appropriately handle complexity and uncertainty in the design process by utilizing decision support design methods and probabilistic approaches. The focus is on the integration of probabilistic analysis for the systematic ranking of alternatives in modeling DM's preferences and trade-offs with modification of the CA. We anticipate that with appropriate modifications in the traditional framework of the CA, it is possible to utilize objective or measurable data from a simulation model to represent DM's preferences. It will allow for subjective data to be modeled without the need for customer surveys or iterative questioning which often causes user fatigue issues.

1.3. Outline of Thesis

The objective of the current study is the implementation of a novel framework to utilize decision-analysis and uncertainty-based design methods. The focus is on the integration of probabilistic analysis for the systematic ranking of alternatives in modeling designer preferences and trade-offs within the framework of CA which is one of the most useful methods for multi-objective problems. In Chapter 2, a description of previously developed decision analysis methods is given. The background research leads into a description of the focus of the improved method followed by the research gaps. Chapter 3 will present a detailed description of the Multi-Attribute Optimization via Conjoint Analysis leading into the development of the proposed framework for the Improved Conjoint Analysis (iCA). The proposed framework integrates the CA and an improved simulation-based ranking scheme to improve the current method and account for uncertainties accompanying traditional customer survey methods. In Chapter 4, the proposed framework is then applied to the design of a cantilever beam, a Power-Generating Shock Absorber (PGSA) and a mesostructured hydrogen storage tank. The first two examples will show the efficacy of the proposed method for a simple example and a practical engineering design. The hydrogen storage tank example gives a description of a second practical engineering problem for the design of an improved pressure vessel utilizing mesostructures for fuel-cell applications. This example shows the flexibility of the simulation-based ranking of alternatives by evaluating a structural problem. Chapter 5 gives a discussion of the developed decision analysis design framework utilizing the simulation-based ranking technique as well as proposed future work.

CHAPTER 2: BACKGROUND OF PREVIOUS RESEARCH

2.1. Decision Analysis Methodology

For the past 300 years, decision science has been a focus for engineering research in the hopes to aid in the decision making process for the design of a product. Decision analysis methods have been developed to assist in decision analysis and design selection. Decisions in engineering design can be on any number of topics from system dimensions, to fluid flow velocity, to geographical locations. The number of different decisions that need to be made in order to solve a design problem is large and any range of different designers may have vastly different preferences towards these decisions. Although the decisions that need to be made are great for every design solution, the methodology used for analyzing them is similar for all problems. Hazelrigg states that the three main elements of decision making are to identify options, determine expectation on each option (usually probabilistic), and an expression of value. With the value of each option determined, a decision is made such that the decision with the expectation of the highest values is most preferred [1].

In a similar statement, Keeney describes a decision analysis paradigm as an introduction to his development of Multi-Attribute Utility Theory (MAUT). He describes the five steps of decision analysis as preanalysis, structural analysis, uncertainty analysis, utility or value analysis, and optimization analysis [16].

Preanalysis: This step is be defined as the formulation of the problem. The need for decision analysis arises when either a single designer or design group is unable to make a decision on a particular action necessary to solve a problem. The statement of the decision(s) that needs to be made as well as the feasible action alternatives is made so further decision analysis can be performed. This step sets up the initial framework of the methodology. Full knowledge and understanding of what needs to be decided on is important for the development of the solution.

Structural Analysis: In the second step, the DM(s) lay out the structure of the problem qualitatively. The task is done by determining details such as a timeline of what decisions need to be made now and which ones need to be made in the future. The DM also needs to determine how information is gathered and what information has an effect on decisions in the future. The purpose of structural analysis of the decisions is an organization of the problem(s) at hand. The effects of specific decisions that are made as well as how information is gathered to aid in finding a feasible solution are structured through the creation of a decision tree. The decision tree can be used to map out necessary decisions that the DM can control (decision nodes) and those that are not under the control of the DM (chance nodes) in a tree style format as shown at the left side of Figure 1and Figure 2. The square node represents a decision node whereas the two circular nodes represent chance nodes.

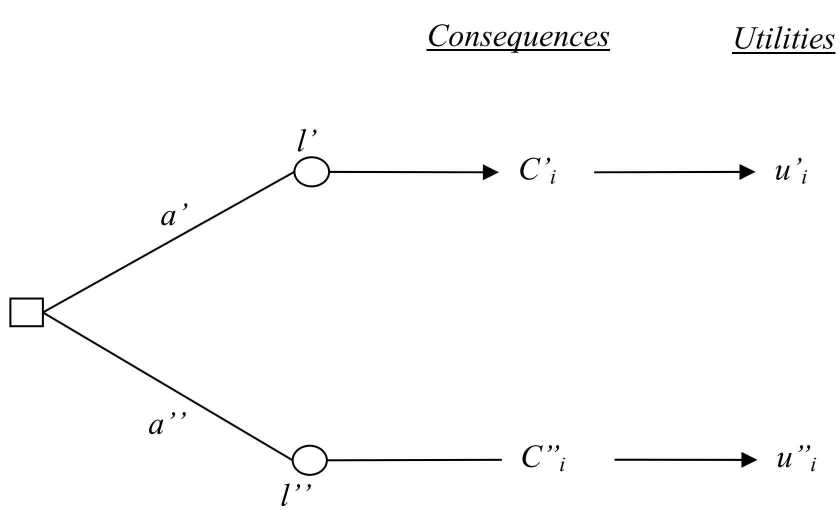


Figure 1: Decision Tree for Choice Problem Under Certainty

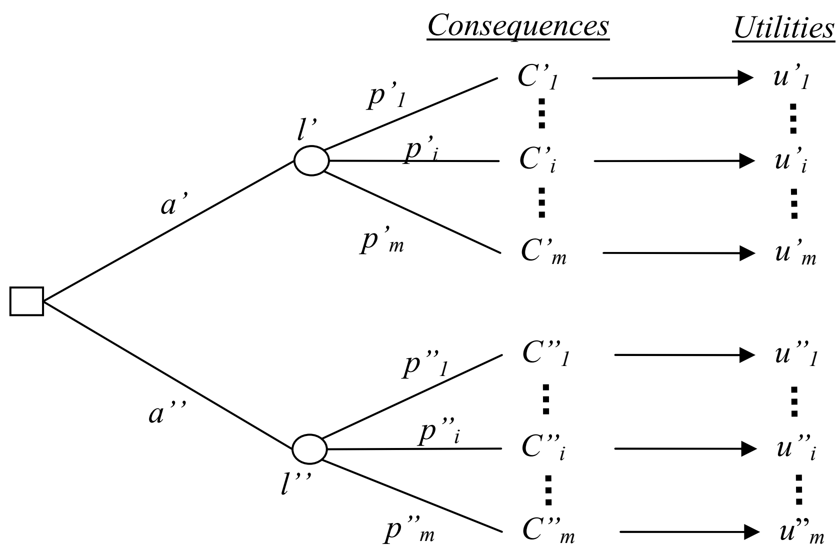


Figure 2: Decision Tree with Corresponding Utility for a Choice Problem Under Uncertainty [16]

Uncertainty Analysis: Once the structure of the decision analysis problem is formed, the expression of the preferences for each alternative is needed. In order to express these preferences it must be known the type of problem that is being analyzed. There are two types of problems that involve decision making: 1) decision making under certainty and 2) decision making under uncertainty.

Decision making under certainty (Figure 1), also known as value theory [8, 15, 23], constitutes problems in which the outcome of a given alternative is known exactly. In other words, the DM preferences elicited for a specific alternative are given such that the outcome of choosing that alternative is known without uncertainty.

On the other hand, decision making under uncertainty (Figure 2), also known as utility theory [8, 15, 23], represent problems in which the outcome of choosing a specific alternative is unknown and can have multiple consequences. For problems such as these, there are certain probabilities for each path coming from chance nodes because the DM does not have control over the outcome. The probabilities for each path are strictly determined from the DM(s). The values are assigned from a number of different sources such as past experience, expert advices from knowledgeable sources, evaluation of stochastic models and the subjective input of the DM(s).

Utility or Value Analysis: For decision making under certainty, the value for a given alternative is directly specified by the DM preferences. Since there is no uncertainty associated with the outcome of choosing a given alternative, the elicitation of the DM preferences can be carried out easily with a comparison of possible alternatives. A value function can be created for this type of problem to represent the DM value for all possible alternatives. A common method for forming the value functions associated with given attributes of a problem is through rankings of a discrete number of alternatives to gauge the DM preferences for the chosen attributes.

In the case of decision making under uncertainty, many different consequences may be possible for a given alternative. Once the uncertainty analysis is evaluated for this case, the consequences corresponding to paths through the decision tree are given utility values in terms of cardinal utility values [23]. A typical method for eliciting these utility values is through the use of lotteries. The lotteries are used to represent the presence of uncertainty in the outcome of choosing a given alternative. The use of a lottery is meant to determine the probabilities associated with different consequences based on the DM preferences. The use of this measurement allows for the modeling of preferences towards specific outcomes from a path representing an ordinal ranking of different consequences. For example, Figure 2 shows the decision that needs to be made between choice *a'* and *a"*. Each choice has a corresponding lottery which leads to a set of consequences, *C*. Utility values need to be given to each consequence such that the DMs preferences match the following.

$$(a'\, is\ preferred\ over\ a'') \Leftrightarrow \left(\sum_{i=1}^{m} p'_i\, u'_i > \sum_{j=1}^{n} p''_j\, u''_j \right) \tag{1}$$

The assignment of utilities as stated above allows for the suitable criterion for the DM's optimal choice to be modeled by the maximization of *expected* utility. A benefit for the use of cardinal utility theory for this step is that it gives consequences a value with a fixed size allowing for comparisons of preferences across persons.

Optimization Analysis: The final step in decision analysis is to determine the optimal path of action to follow in order to solve the problem. This is done by determining the strategy that maximizes the expected utility as mentioned in the previous step.

The above paradigm formed the introduction to Keeney's description of the MAUT decision analysis framework. Other decision analysis techniques have been developed in previous research based on the similar principles as the previous methodology such as Quality Function Deployment, weighted sum of product attributes, and physical programming [6, 24, 25]. For the multi-attribute decision design case, marketing tools, namely Conjoint Analysis, have also been incorporated into these methods. A few of the main types of methods are described below as a background of information for this thesis. Each method described below has its own advantages and disadvantages which will lead to the realization of a research gap to motivate further advancement in the field of decision science.

2.2. Weighted Sum of Product Attributes

The weighted sum of product attributes type of decision analysis or decision selection method is based on the creation of a utility function. The hypothesis in this type of method is that the suggested optimal design decision is the one that maximizes this function. In this approach, a measure is determined for each of the most important attributes of the problem and is understood to be proportional to the utility of each attribute. The weighted sum of each of the attribute measures forms the utility function to be maximized. The determination of the weight values for each attribute is a focus for many methods with this general framework. The chosen weights can typically be chosen arbitrarily related to relative importance; however random choice adds more uncertainty to the problem as opposed to physical calculation of weight values. Other methods utilize comparison of different solution alternatives to gain a model of designer preferences in order to calculate the weight values under the presence of uncertainty. Research on methods of weighted sum of product attributes under uncertainty has been done in a large extent by John von Neumann and Oskar Morgenstern in the form of utility theory [23] as well as Ralph Keeney in the form of Multi-Attribute Utility theory [8, 15, 16].

2.3. Multi-Attribute Utility Theory

The framework for MAUT is a method for determining and analyzing the decision process for a generic decision problem [16]. The basis for this method is that there is a problem in which the designer must make a decision between alternatives to choose the optimal one. The application of such a method can be performed for any type of problem; however, the process may be too complex for simple problems with a low amount of alternatives. The method has been noted for accurately representing the preferences and trade-offs of a DM for problems with multiple objectives based on decision making under uncertainty. In most problems, especially in engineering, decisions made for design attributes can be difficult and one decision can affect the outcome of another. The use of a multi-attribute utility function utilizing weight factors to quantify designer preferences and trade-offs can help reduce the complexity involved in interdependent design decisions.

The design objectives, design variables and attributes to quantify the measurement of each design objective are determined. With this information the utility elicitation can be conducted to generate an individual utility function for each attribute. The utility elicitation, as described by the von-Neumann Morgenstern utility theory [23], is performed by asking a series of questions relating to the DM participation in a lottery between two difference scenarios to determine certainty equivalents. These certainty equivalents are used with interpolation or approximation methods to form a continuous utility plot for each attribute. The concept of the utility plot is for comparison between each attribute which requires that the utility of each attribute be normalized between zero and one.

Trade-off analysis is performed to aggregate the individual utility functions into a weighted multi-attribute utility function. The use of the weight values will represent the preferences towards both the individual attributes as well as the interactive preferences. The trade-off analysis method involves lottery questions similar to that for von-Newmann Morgenstern utility theory and other preference independency questions.

The form of a multi-attribute utility function depends on the independence conditions of mutual utility independence and mutual preferential independence, which are described in more detail in Chapter 3. The general form for the multi-attribute utility function satisfying the condition that each attribute is mutually utility independent is as follows.

$$\begin{aligned} u(x) = \sum_{i=1}^{n} k_i u_i(x_i) + K \sum_{i-1, j>i}^{n} k_i k_j u_i(x_i) u_j(x_j) + K^2 \sum_{i=1, j>i, l>j}^{n} k_i k_j k_l u_i(x_i) u_j(x_j) u_l(x_l) + \\ \ldots + K^{n-1} k_1 k_2 \ldots k_n u_1(x_1) u_2(x_2) \ldots u_n(x_n) \end{aligned} \tag{2}$$

where x_i is the value of the i^{th} attribute, $u_i(x_i)$ is the normalized individual utility function, k_i is the preferential weight factor, and K is a scaling factor which is the solution to

$$1 + K = \prod_{i=1}^{n} \left(1 + Kk_i\right) \tag{3}$$

If the value of K=0 then Equation (1) is simplified to an additive assumption representing both preferential and utility independence as follows.

$$u(x) = \sum_{i=1}^{n} k_i u_i\left(x_i\right) \tag{4}$$

To calculate the weight factors, k_i, comparisons or lotteries are made to determine sets of alternatives that have equivalent total utility values. These indifference combinations can be found by asking elicitation questions similar to that of the von-Newmann Morgenstern utility method.

The derived multi-attribute utility function can be used as the objective function for an optimization problem. The objective is to maximize this function (Equation (2)). The set of design variables that maximizes the overall utility is the design suggestion that is most preferable to the DM.

2.4. Physical Programming

As stated by Dr. Achille Messac [6, 26], Physical Programming (PP) is a method for expressing preferences directly as opposed to running through continuous iterations involved in modification of weight values in hopes to find a convergence. There are two main parts of the use of Physical Programming to create a preference function for each design objective:

qualitative and quantitative. The qualitative part is the decision of preference class to determine the form of the preference function plot. Also in the qualitative part is the definition of regions or ranges similar to "fuzzy" variables to represent the designer's preferences in the form of degrees of desirability: *unacceptable, highly undesirable, undesirable, tolerable, desirable, and highly desirable*. The quantitative portion relates to the numerical ranges of the different degrees of desirability and the creation of the aggregate function representing the combination of the different preference functions. The process for PP is shown in Figure 3 as compared to a traditional weight-based approach.

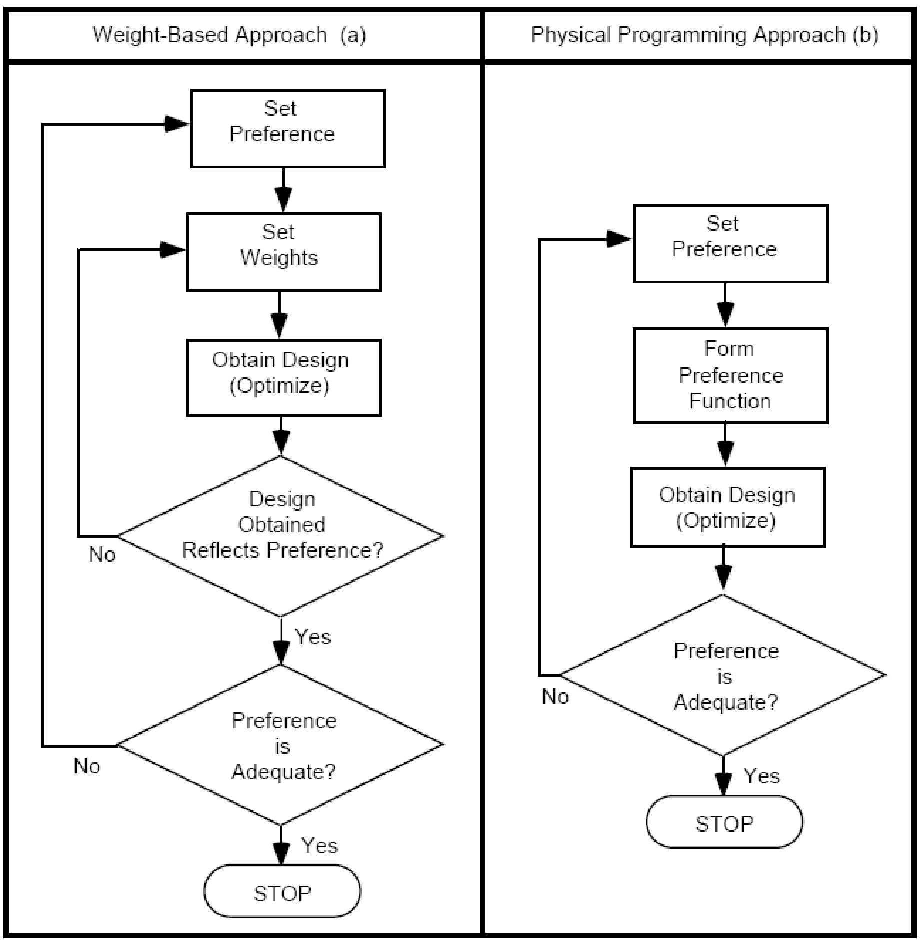


Figure 3: Physical Programming Design Process

As can be seen in Figure 3,one of the benefits of PP as mentioned by Messac [6] is the elimination of the inner loop that is apparent in the weight-based approach. This reduces the amount of computation time involved in the process.

In order to generate the preference functions for each objective using Physical Programming, the class type is defined (i.e. Class 1-Smaller-Is-Better, Class 2-Larger-Is-Better, or Class 3-Center-Is-Better) to determine the general shape of the graph. The degrees of desirability (as described earlier in this section) are used to denote the designer's preferences and are determined either from the problem statement or the DM's previous knowledge. The method of determining a desirable design alternative is based on reduction of one or all criterion across one or more of the degree(s) of desirability as stated in what is called the One vs. Others criteria rule (OVO rule) [6].

2.5. Customer Survey

A popular method for gaining the customer's preferences in order to assist in the design process is the use of customer driven surveys. The surveys are given to either a selected group of customers or a random assortment of customers in the hopes to gain ideas on the preferences of the people who will directly use the product. The main purpose of the survey is to present the customer with information regarding the product and ask for the customer's preference in regards to different design alternatives. For example, 100 customers could be surveyed on their preferences towards color of a product and five different colors are given to each customer in hopes to gain the most preferred color. The presentation and judgment criteria can change (i.e. list all possible colors and have customer rank each in order, or simply ask what is your favorite color), but the outcome will still be given as the color preferred by the most amount of customers should be the selected color.

2.6. Quality Function Deployment

Quality Function Deployment (QFD) [12, 13, 27] is meant to assist in creating engineering characteristics for a product or service based on the voice of the customer. This method is developed due to the realization of the impracticality of employing customer surveys based on the full range of possible alternatives for a given set of attributes. Instead, QFD focuses on the customer's preferences based on each attribute. The preferences on the individual attribute is then aggregated and used to determine the preferred product. For example, assume that preferences on three products, A, B, and C, are required and three customers are surveyed. Each product has two instantiations, A_1, and A_2 for attribute A, B_1 and B_2 for attribute B, and C_1 and C_2 for attribute C. The preferences are given such that a customer labels each instantiation as "Great" if he/she prefers that instantiation, "OK" if it is almost as good, or "Hate" if the customer will not buy the product. Table 1 shows the preferences given by each of the three customers.

Table 1: QFD Customer Preferences

	Attribute					
	A		*B*		*C*	
Customer	A_1	A_2	B_1	B_2	C_1	C_2
I	Hate	Great	Great	OK	Great	OK
II	Great	OK	Hate	Great	Great	OK
III	Great	OK	Great	OK	Hate	Great
Group Preference	A_1		B_1		C_1	

According to the QFD procedure the suggested product combines the instantiations that are most preferred to all three customers according to the group preference, i.e. $A_1B_1C_1$. The reason for this is because these three instantiations has the most preference over all three customers. However as can be seen from this implementation, the combination $A_2B_2C_2$ is liked by all customers even though the overall preferences designated by a label of "Great" is not better than the other combination according to the QFD procedure. This is a known flaw to the well noted method [27].

2.7. Evaluation of System Reliability

2.7.1. Probability of Failure

Reliability is the probability that a system will perform its function over a specified amount of time and under specified service conditions. Primarily, reliability-based design consists of minimizing an objective function while satisfying reliability constraints. The reliability constraints are based on the failure probability corresponding to each failure mode or a single failure mode decreasing the system failure. The estimation of failure probability is usually performed by reliability analysis. In the case of structural optimization the structure is under the influence of loads and boundary conditions and the response also depends on the stiffness and mass properties. The responses that are critical for the reliability of the structure such as critical location stresses, resonant frequencies, displacements, etc. are considered satisfactory when the design requirements imposed on the structural behavior are well within the degree of certainty. Each of these requirements is called *limit-state*. The probability of violation of the limit state is a metric for quantifying the reliability of the structure under consideration. Once the limit state has been violated the structure is believed to have undergone failure for the sake of calculations. By determining the number of times the structure failed out of the number of evaluations the probability of failure can be determined. Once the probability has been determined the next step will be to choose design alternatives that improve structural reliability and minimize the risk of failure.

Generally the limit state indicates the margin of safety between the resistance and the load of structures. The limit-state function, *g*(.), and probability of failure, P_f, can be defined as

$$g(X) = R(X) - S(X) \tag{5}$$

$$P_f = P[g(.) < 0] \tag{6}$$

where R is the resistance and S is the loading of the system. Both *R*(.) and *S*(.) are functions of random variables *X*. Here *g*(.) = 0 represents the failure surface *g*(.) < 0 and *g*(.) > 0 represent the failure region and safe region respectively.

The mean of the limit state *g*(.) can be expressed as in Equation (7), where μ_R and μ_S represent the means of *R* and *S* respectively.

$$\mu_g = \mu_R - \mu_S \tag{7}$$

The standard deviation of *g*(.) is

$$\sigma_g = \sqrt{\sigma_R^2 + \sigma_S^2 - 2\rho_{RS}\sigma_R\sigma_S} \tag{8}$$

where, ρ_{RS} is the correlation coefficient between *R* and *S*, and σ_R and σ_S are the standard deviations of *R* and *S*, respectively. The safety index reliability index is then defined as

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2 - 2\rho_{RS}\sigma_R\sigma_S}} \tag{9}$$

The safety index indicates the distance of the mean margin of safety from *g(.)=0*. The idea behind the safety index is that the design is more reliable if μ_g is farther away from the limit state surface.

For a special case, if the resistance R and the loading S are assumed to be normally distributed and uncorrelated, then the probability density function of the limit-state function can be represented as

$$f_g(g) = \frac{1}{\sigma_g \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{g - \mu_g}{\sigma_g} \right)^2 \right] \tag{10}$$

The probability of failure can then be represented as

$$P_f = \int_{\infty}^{0} f_g(g)dg \tag{11}$$

Other strategies have also been used in the past for probabilistic analysis for designing reliable structures. Sampling methods and such as Monte Carlo Simulation and Latin-Hypercube Sampling are some of the most commonly used methods for conducting reliability analysis.

2.7.1.1. Monte Carlo Simulation

Monte Carlo methods were originally practiced under more generic names such as statistical sampling, and the name is a reference to the famous casino in Monaco. The methods use of randomness and iterative procedure is similar to a casino’s activities. In Monte Carlo Sampling (MCS) [28] the inverse transform method is used to generate random variables with specified probability distributions. This method can be applied to variables for which the cumulative distribution function has been obtained from direct observation, or where an analytic expression for the inverse cumulative function, $F^{-1}(.)$, exists [29].

Let $F_X(x_i)$ be the Cumulative Distribution Function (CDF) of random variable xi. Since the value of CDF can only lie between 0 and 1, $F(.)$ has a value between 0 and 1. If u is the uniformly distributed random variable that is generated using MCS then the inverse transfer method is used to equate u to $F_X(x_i)$ as follows:

$$F_X(x_i) = u \tag{12}$$

or

$$x_i = F_X^{-1}(u) \tag{13}$$

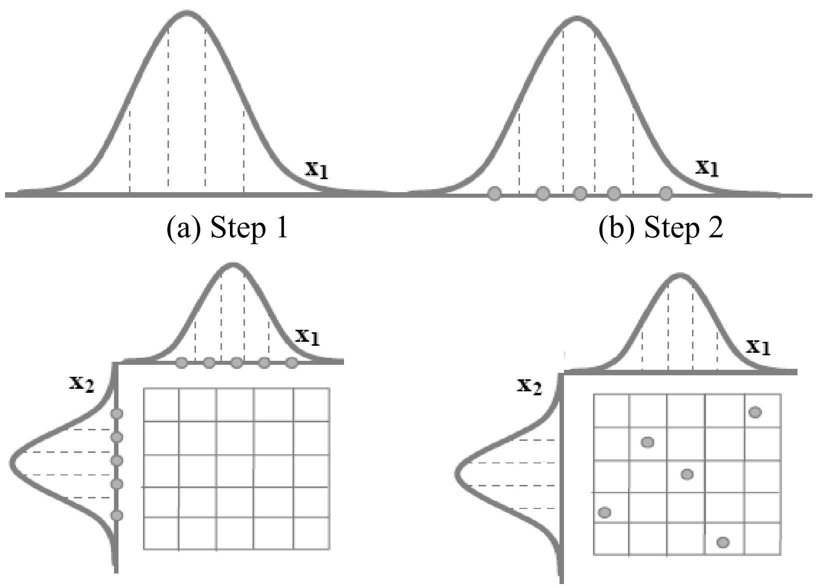
This method can be applied to variables for which a cumulative distribution function has been obtained from experiments or where an expression for the inverse cumulative function exists. The process starts with the random number generator producing random numbers between 0 and 1 based on randomly selected seed values. The corresponding CDF value of the uniform distribution and target distribution can easily be obtained using the random numbers that were generated. The final step is to obtain the random number for the target PDF using Equation (12).

Monte Carlo sampling can be very computationally expensive since they are random in nature. In order to make MCS less computationally expensive sometimes variance reduction techniques are integrated. Latin Hypercube Sampling is an excellent variance reduction technique that reduces the computational requirement for the simulation as well as increasing the accuracy with the same number of runs.

2.7.1.2. Latin Hypercube Sampling

In order to reduce the computational cost of the reliability assessment, a variance reduction sampling method, namely Latin Hypercube Sampling (LHS) [30], is introduced. LHS, also known as the stratified sampling technique, represents a multivariate sampling method that guarantees non-overlapping designs. In LHS, the distribution for each random variable can be subdivided into n equal probability intervals or bins. Each bin has one analysis point. There are n analysis points, randomly mixed, so each of the n bins has 1/n of the distribution probability. Figure 4 shows the basic steps for the general LHS method, which are:

1. Divide the distribution for each variable into n non-overlapping intervals on the basis of equal probability.
2. Select one value at random from each interval with respect to its probability density.
3. Repeat steps (1) and (2) until you have selected values for all random variables, such as $x_1, x_2, \ldots, x_k$.
4. Associate the n values obtained for each x_i with the n values obtained for the other $x_{j} \neq i$ at random



(a) Step 1 (b) Step 2

(c) Step 3 (d) Step 4

Figure 4: Basic Steps of LHS for Two Variables and Five Realizations

The regularity of probability intervals on the probability distribution function ensures that each of the input variables has all portions of its range represented, resulting in relatively small variance in the response. At the same time, the analysis is much less computationally expensive to generate. The LHS method also provides flexible sample sizes while ensuring stratified sampling; i.e., each of the input variables is sampled at *n* levels.

2.7.1.3. Probability of failure calculation

The sampling methods can be used to calculate the probability of failure where the limit state function involves complex functions, and direct evaluation of the limit state is not possible. The following steps are taken to calculate the probability of failure P_f:

1. Generate a sampling set of random variables according to the corresponding probability density functions.
2. Set the mathematical model of the limit-state, which can determine failures for the drawing samples of the random variables.
3. The simulation is executed and for each run the limit state is evaluated.
4. If the limit-state function $g(.)$ is violated, the structure or the structural element has "failed".
5. The trial is repeated many times to guarantee convergence of the statistical results.
6. If N trials are conducted, the probability of failure is given approximately by

$$P_f = \frac{N_f}{N} \tag{14}$$

where N_f is the number of trials for which the limit state function is violated out of the N experiments conducted.

2.7.2. Multi-Scale Design via Inductive Design Exploration Method

Aside from sampling methods such as those described in the previous section, other methods exist for the evaluation of system reliability. In particular, the Inductive Design Exploration Method focuses on the consideration of uncertainty for the purpose of improving the reliability of systems involving multi-scale design.

The objective of the IDEM is to find design ranges which pass design process in a top-down manner maintaining "design accuracy" as much as possible. Figure 5 shows the overall procedure of the IDEM.

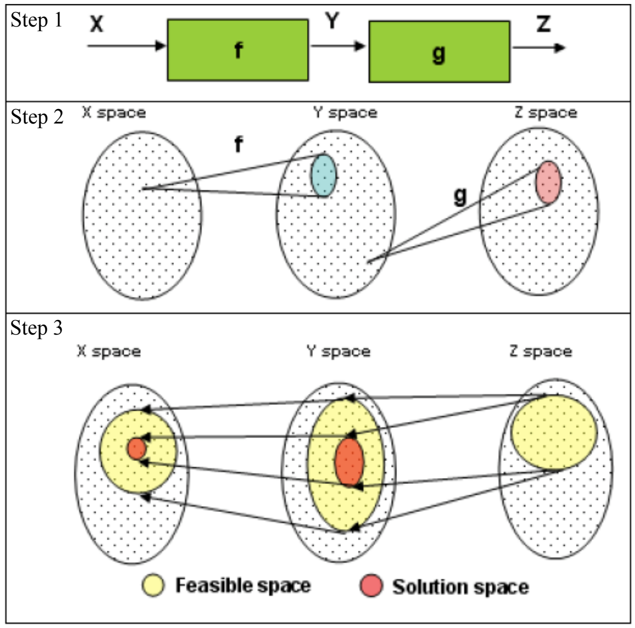


Figure 5: Steps of IDEM Procedure [31]

In Figure 5, X and Y are the input design space and the intermediate design space, respectively. Z is the final response (output design space) in this model chain. In the IDEM process, the design requirements representing the feasible response space, Z, are used to determine the feasible Y space. The feasible input design space, X, is then calculated in the same fashion by utilizing the function $Y = f(X)$ as the new design requirements. The procedure of the IDEM includes the following steps. In step 1, the designer defines the rough design space from which the discrete points in each of the spaces are generated. The purpose of step 1 is primarily for setting up the problem as well as initially refining the exploration space. This step can be difficult and is highly dependent on the user's previous knowledge and experience with the specific application. For this reason this step required a good amount of familiarity with the problem. This Model information flow, such as the scale-specific and shared design variables, is also established in this step. In step 2, the generated discrete points are evaluated based on the functions $f(.)$ and $g(.)$, and corresponding input and output data points are stored in a database. Feasible regions in Y and X spaces are sequentially identified with a given final performance range in Z space. By trading off the values in each space within the allowable margin for variation, the feasible regions in each space can be identified. In step 3, the maximum, minimum, and mean responses are obtained through the mapping function. In this step, the best feasible design solution space can be obtained by comparing the Error Margin Index (EMI) [31]. The EMI is metric indicating the degree of reliability of a decision that satisfies systems constraints or bounds. The EMI can be used in search algorithms to find ranged sets of design specifications that meet a range of system requirements. This metric can be calculated as follows

$$\begin{cases} EMI_{all} = -1, \; for\ non-feasible\ region & (15) \\ EMI_i = Min\left(\dfrac{|mean_i - b_{j,i}|}{|mean_i - b^i_{j,i}|}\right), \; for\ feasible\ region & (16) \end{cases}$$

where i is the number of the direction, j is the number of discrete points on the constraint boundaries, $mean_i$ is the i^{th} component of the mean vector of an output range, B_j is a discrete point vector on constraints boundaries, $b_{j,i}$ is the i^{th} component of B_j, B_{ij} is the projected vector of B_j on the nearest boundary of output range along i^{th} direction, and $b_{ij,i}$ the i^{th} component B_{ij} as shown in Figure 6.

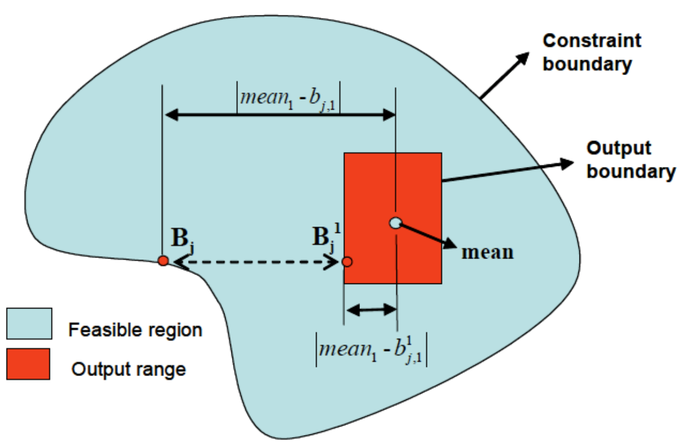


Figure 6: EMI Calculation in a Direction [31]

The EMI is introduced as a metric for checking if the projection of each discrete point forms an input space to an output space that is within a given satisfying output range. To calculate EMI it is required to determine whether the mean of an output range is in the feasible range or not. When the mean of an output range is not in the feasible range, the EMI is -1. An EMI value of -1 represents the input point is unsatisfactory for the constraint bound in the output space. If the mean is inside the feasible range, then it is required to calculate the EMI in each output direction.

EMI has an individual value in each output direction which is defined as EMI_i for the value in the i^{th} direction. The EMI_i value is the minimum of all the EMIs evaluated at the discrete points on a constraint boundary (B) as shown in Figure 6. The projected points on a boundary of an output range in i direction (B^i) and the mean of the output range (*mean*) are also required for calculating the EMI. As the EMI increases the output range moves farther than the constraint boundary and is therefore more likely satisfactory.

In Figure 6, the output space is two-dimensional and the feasible range is depicted by a contour, which is the constraint boundary. The output range, which is a rectangular region, can be a hypercube in a multidimensional case. The mean of the rectangular region is also shown in the Figure 6. Points on the constraint boundary (for example B_j) are projected on the output range boundary. The projection cannot pass through the output range so it does not produce any projection in the other side of the output range. For example the projected point corresponding to B_j will be B_j^1. The EMI values can be calculated for each point projected from the constraint boundary. In multi-dimensional cases the EMI can also be calculated likewise. The EMI can be used to calculate the discrete boundary points in the input space. More details about this process can be found in [31].

Although Choi et al. [31] shows the possibility of achieving robust designs using the IDEM, there are still unresolved important issues. For instance, the IDEM does not have clear steps to combine realistic stochastic models at different scales in order to obtain a manageable model of the entire multi-scale system. Furthermore, the current procedure of the IDEM utilizes the exhaustive search method which induces unnecessary high computational costs and discretization errors of the design space [9].

2.8. Research Gaps

The factor that links all methods mentioned above is subjective data. Each method has been developed as an alternate method for gaining subjective data, or observational data, from the customer, producer, specialist, or other DM. The subjective data is used to model the preferences of the DM(s) involved in choosing the final solution in hopes to link preferences to objective data, or quantitative data, for the system model. As described in the previous sections, subjective data can be obtained by many ways such as direct customer surveys of hypothetical design alternatives, iterative questioning in regards to individual attributes, and lottery elicitation in regards to uncertain outcomes.

There are a few difficulties with these methods. One complexity is due to the highly iterative nature of attaining an accurate representation of the preferences of the majority of DMs. For instance with MAUT, the lottery elicitations are performed on each DM. When there is only one DM, the lottery questions are used to ask the DM to choose between winning a specific amount and a 50/50 lottery of winning either a highly preferred value or an unfavorable value. The elicitations are performed continuously until an indifference point is determined. In addition, these indifference points are used to form the utility plots for each attribute and multiple indifference points are required to form an accurate plot. Therefore many elicitations are required for complex problems which would be very exhaustive for the DM. In addition, more DMs increase the level of user fatigue.

For a second example of the iterative nature of attaining subjective data, customer survey based methods involve a large amount of user fatigue. A customer survey could be as simple as asking which product is preferred between two hypothetical alternatives. For problems with many alternatives, the number of pair-wise comparisons necessary for the customer to choose from increase exponentially. The larger the number of questions asked to the customer increases the stress on the customer. As the amount of fatigue increases on the customer the level of uncertainty in the results of the survey will increase as well.

Another difficulty with the methods of gaining subjective data is the ability to accurately model the preferences for all possible DMs. The preferences for each attribute of a design will vary drastically from one person to another. For instance, in weighted-sum of product attribute methods, like the ones mentioned above, the weighted values for each attribute will more than likely vary from customer to customer. The common method for solving this problem is a running average of all of the weight values for each DM involved in the problem formulation. Although this will give a result that is precise for the different possible preferences for many DMs, the reliability of the final result is low when the range of DM preferences is great.

The recognized research gaps dealing with the amount of user fatigue and uncertainty involved in the modeling of DM preferences is the motivation for the current research given in this thesis. The focus for the proposed framework is on filling these gaps as they pertain to customer survey driven decision analysis techniques. In customer survey-based methods, the research gaps can be specified as (a) the amount of fatigue put on the customer when answering surveys with a high number of attributes and alternatives and (b) the amount of uncertainty associated with the number of customers involved in the preference modeling process. These gaps give need for an improved method that will fill these gaps in research and improve the reliability when attaining subjective data.

2.9. Summary

This chapter gave an overview of a general methodology for decision analysis in engineering problems as well as gave a description of a few common methods developed for making decisions based on modeling preferences. The outlined gaps provide the motivation for the research presented in this thesis. In an effort to fill these gaps, an improved method is presented in the following chapter. As previously mentioned the proposed framework is focused on filling the gaps in relation to customer survey-based methods and in particular improving the existing method of CA. Chapter 3 gives a detailed description of this base method as well as the devised improvements devised to answer the research questions. The proposed framework is then applied to three design examples to demonstrate the efficacy of the improved method in Chapter 4.

CHAPTER 3: PROPOSED METHOD

To fill the research gaps of this thesis an integrated method is proposed to form a framework for multi-attribute decision analysis utilizing reliability-based analysis to objectively rank alternatives. The base framework in which this thesis is weighted in is Multi-Attribute Optimization based on CA. In order to answer the stated research questions, a new simulation-based ranking method was devised to improve the ranking operation and reduce the subjectivity and user fatigue present in traditional methods. In the following sub-sections a detailed description of the CA is given to properly understand the in which this thesis is anchored in. An explanation of the developed simulation-based ranking is then provided along with the description of the proposed framework of the iCA.

3.1. Multi-Attribute Optimization Based on Conjoint Analysis

Conjoint Analysis is a method used in the marketing field for determining a quantitative value for a DM’s preferences during the evaluation of a multi-attribute problem. CA is beneficial as a decision-making process since the tool is a systematic method for creating and ranking a set of many design configurations based on design attributes to model designer preferences. Since the ranking of design alternatives is used in the elicitation of DM preferences, the CA is a method for decision making under certainty as it pertains to value theory as described in Chapter 2.1. This process creates a discrete number of configurations to choose from and rank and utilizes multiple regression analysis, which allows for an easy, systematic method for modeling preferences. A design selection method based on a rank ordering of all design alternatives is beneficial because it tells which is ‘‘best,’’ and gives insights as to the ordering of preference of the other alternatives [27]. As an alternative, fractional factorials can be used instead of full factorials to create configurations helping to simplify even complex design problems with a large number of design objectives. CA is a technique that breaks down attributes to derive the part-worth associated with each level of a product based on the overall preferences of choice alternatives by a group of respondents [11]. These respondents can be customer or producers. The respondent can also be the DM or DMs in the case of a design team conducting the Conjoint Analysis. This type of analysis helps evaluate the contribution of all attributes towards the determination of a final solution by estimating part-worths from respondent(s) preferences on different design concepts.

According to Orme [21] CA can be divided into three types: (i) Conjoint Value Analysis [32], (ii) Adaptive Conjoint Analysis [33], and (iii) Choice-based Conjoint Analysis [34].

Conjoint Value Analysis (CVA), also known as the traditional full profile CA, is a simple evaluation of CA and can be implemented with pencil and paper or faster with computers. CVA can be used for preference problems with up to 6 attributes as stated by Green and Srinivasan [10]. The main downfall to the use of this method is that the complexity and fatigue on the DM increases greatly as the number of attributes increase causing the possibility of error.

Adaptive Conjoint Analysis is an improved method developed to handle more complex problems with more attributes than with CVA. This method utilizes a hybrid approach combining state evaluations of attributes and levels with pair-wise comparisons. This feature allows for a reduction in the number of comparisons the DM must make at one time. The interviewing process for the implementation of ACA adapts to the respondent's answers as the survey progresses. What is meant by this is that the previous questions are used to determine the next question to ask making this method more difficult to implement.

Choice-Based Conjoint Analysis (CBC) is a more complex implementation of Conjoint Analysis that imitates products in the competing market. Respondents choose which product they would purchase from a set of possible alternatives rather than using rating or ranking scales. In addition, the respondent may also choose to not purchase any of the products as in the real world.

Conjoint analysis has been integrated with multi-attribute optimization as a method applying this marketing tool to engineering applications [5]. More specifically, Conjoint Value Analysis is used due to the simplicity to implement and the application for engineering systems with less than six attributes. The method is used to create part-worth plots based on the CA results and optimizes the addition of each individual part-worth value to determine the best design suggestion. The flow chart shown in Figure 7 represents this framework. The following sections give a details description of each step.

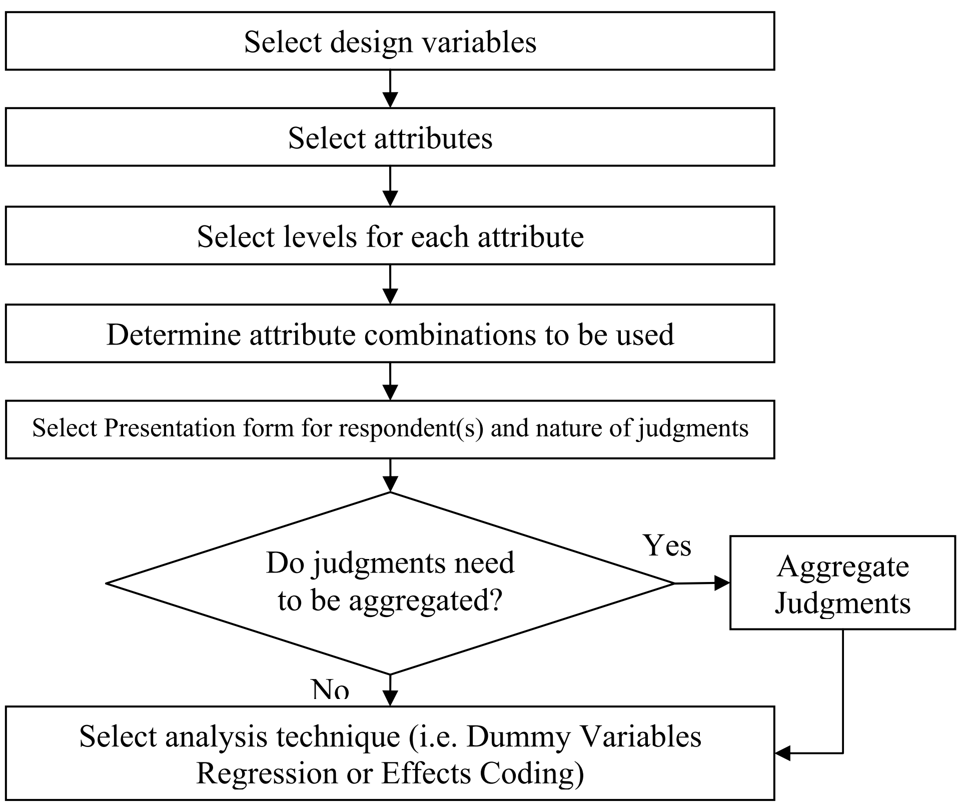


Figure 7: Flow Chart for Conjoint Analysis[5]

3.1.1. Determine Attributes and Levels

The framework in Figure 7 is a weighted sum of product attributes method as described in Section 2.4. The first step is to determine the most important design attributes for the specific problem. This can be done by any means from a simple design team discussion to an in-depth analysis of the problem involving customers, designers, etc. to see which design objectives are most important. These objectives must be a function of the necessary design variables. The attribute of each design objective is a measurable quantity that can be used to represent the value of each objective. For example, the attribute for cost of a product would be number of dollars spent for each unit.

Once the attributes are determined levels must be created for each. Choosing levels can be difficult for engineering applications as most of them involve continuous attribute values rather than known discrete values. The decision can be simplified if there are specific bounds on an attribute based on the specific design problem, previous expertise, or existing knowledge of the system. For example if it is known that the stress acting on a beam must be between 10 and 100 kPa then these would represent the upper and lower levels to be used in the analysis. The number of levels to use once a starting level is chosen requires some knowledge about the system to be analyzed. This is where expert knowledge may be required to solve the problem accurately. The number of levels chosen has a direct relation to the correct modeling of designer preference. The simple method is to divide the levels evenly across the chosen bounds.

3.1.2. Determine Attribute Combinations

Once the levels are chosen, they are used to create a number of design alternatives. The number of combinations has a direct impact on the complexity of the evaluation process and on the accuracy of the part-worth calculation. Factorial evaluations [35] can be used to determine the number of combinations. A full factorial design will give the most accurate evaluation as it uses every combination of each level possible. As a downfall for a design with 4 attributes divided into 5 levels each will give a total of 625 combinations. For problems such as this where the number of combinations in a full factorial are too large, a fractional factorial can used to lower the number of alternatives. A fractional factorial design will take an adequate fraction of combinations from the full factorial design with as little effect on the overall represented results as possible. For example a 1/5 fractional factorial, 5^{4-1}, results in a total of 125 alternatives which is still a large number but significantly less than the original design. Previous knowledge of factorial designs is required to be able to determine an appropriate fractional factorial design to use in order to retain the proper depiction of possible alternatives. The number of combinations necessary for a good conjoint design can be based on the number of parameters. The number of parameters can be determined by Equation (17).

$$\#\,of\ \ parameters\,to\,be\,estimated = (\#\,levels) - (\#\,attributes) + 1 \tag{17}$$

Traditionally there are more observations (combinations) than parameters (usually 1.5 to 3 times) to be estimated. These designs usually lead to more stable estimates of respondent utilities [7].

3.1.3. Select Presentation Form and Nature of Judgment

After the combinations are made, the method of presentation of alternatives and the nature of the judgment is chosen. The most basic methods of presentation are verbal, paragraph, and pictorial description. Then, the presentation form for judging is selected (e.g. ranking or rating) to measure which alternatives are more favorable. This is where the DM's preferences are incorporated in the design. Due to the applicability of CA to gaining input from multiple DMs, this portion of the method could be done for one or many rankings or ratings. Depending on the number of DM's the results may need to be aggregated to get the preferences models representing the entire decision population. In cases such as this a running average is a simple and accurate method for aggregation especially when mass customer surveys are involved.

3.1.4. Calculate Part-Worths via Dummy-Variable Regression

The part-worth values for each level of each attribute represent the relationship between the objective attribute values and the corresponding DM's preferences. Regression techniques are common for the determination of these values and provide high accuracy. Dummy-Variable Regression technique [5], which is employed in this method, uses a binary matrix representation of each attribute combination to determine the part-worth values. An example problem is solved to properly explain Dummy –Variable Regression process.

The following example is taken from Orme [7]. Consider a problem with three attributes: Brand, Color, and Price. Brand is divided into three levels designated A, B, and C. Color is divided into levels for Red and Blue. Price is considered to have three levels $50, $100, and $150. The set of combinations is created using a full factorial design resulting in 18 total possible combinations.

$$3\,Brands \times 2\,Colors \times 3\,Prices = 18 \tag{18}$$

The chosen presentation form and nature of judgment is a comparison of all 18 combinations based on a rating scale from 1 to 10 (with 10 being best) for simplicity. To further simplify the example, it is assumed there is only one DM for this analysis. Table 2 shows the total set of combinations in the full factorial design.

Table 2: Full Factorial Design for Dummy-Variable Example

Combination	***Brand***	***Color***	***Price***
1	A	Red	$50
2	A	Red	$100
3	A	Red	$150
4	A	Blue	$50
5	A	Blue	$100
6	A	Blue	$150
7	B	Red	$50
8	B	Red	$100
9	B	Red	$150
10	B	Blue	$50
11	B	Blue	$100
12	B	Blue	$150
13	C	Red	$50
14	C	Red	$100
15	C	Red	$150
16	C	Blue	$50
17	C	Blue	$100
18	C	Blue	$150

The next step is to gain the respondent's preferences on a rating scale for each of the above combinations. This would be done through customer surveys, computer programs, or elicitations from designers. As mentioned in the introduction to this chapter, the respondent can be a customer or user of the product or even the designer or DM conducting the CA. In either case the subjective data elicited are based on the attributes and preferences on the respondent. Therefore the suggested final design will represent the preferred design as pertains to the respondent(s) giving the rating/ranking data.

The ratings for this example are presented in Table 3. With the respondent's preferences given, coding of the combinations and rating must be performed. In dummy-variable coding a binary representation is used to form the regression problem. For the presence of an attribute level in a combination, a '1' is used and a '0' symbolizes the absence of an attribute level. The ending result is a n x m table, where n is the total number of attribute levels and m is the number of combinations, containing only ones and zeros in the left section and the far right column depicting the rating of the respondent as shown in Table 4,

Table 3: Respondent Rating of Attribute Combinations

Combination	*Brand*	*Color*	*Price*	*Rating*
1	A	Red	$50	5
2	A	Red	$100	5
3	A	Red	$150	0
4	A	Blue	$50	8
5	A	Blue	$100	5
6	A	Blue	$150	2
7	B	Red	$50	7
8	B	Red	$100	5
9	B	Red	$150	3
10	B	Blue	$50	9
11	B	Blue	$100	6
12	B	Blue	$150	5
13	C	Red	$50	10
14	C	Red	$100	7
15	C	Red	$150	5
16	C	Blue	$50	9
17	C	Blue	$100	7
18	C	Blue	$150	6

Table 4: Dummy-Variable Binary Representation

Combination	*Brand*			*Color*		*Price*			*Rating*
	A	*B*	*C*	*Red*	*Blue*	*$50*	*$100*	*$150*	
1	1	0	0	1	0	1	0	0	5
2	1	0	0	1	0	0	1	0	5
3	1	0	0	1	0	0	0	1	0
4	1	0	0	0	1	1	0	0	8
5	1	0	0	0	1	0	1	0	5
6	1	0	0	0	1	0	0	1	2
7	0	1	0	1	0	1	0	0	7
8	0	1	0	1	0	0	1	0	5
9	0	1	0	1	0	0	0	1	3
10	0	1	0	0	1	1	0	0	9
11	0	1	0	0	1	0	1	0	6
12	0	1	0	0	1	0	0	1	5
13	0	0	1	1	0	1	0	0	10
14	0	0	1	1	0	0	1	0	7
15	0	0	1	1	0	0	0	1	5
16	0	0	1	0	1	1	0	0	9
17	0	0	1	0	1	0	1	0	7
18	0	0	1	0	1	0	0	1	6

The above data has a linear dependency which represents a complication in the analysis. Multiple regression is used to determine the part-worth values from the above data. In this analysis no independent variable can be perfectly predictable from the value of any other independent variable or combination of variables [7]. The linear dependency is resolved by omitting one column of data from each attribute. The omission of one of the levels implicitly denotes an attribute level as a reference (i.e. part-worth of zero) for the other levels. The specific level does is not important and does not affect the outcome of the regression.

The rating/ranking data is fit to a regression model of the form,

$$y = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_n x_n + e \tag{19}$$

where y is the rating/ranking value, b_0 is an intercept term, $b_1, b_2, \ldots, b_n$ are the part worth utilities of the $x_1, x_2, \ldots x_n$ attribute levels, and e is an error term. There are different criteria for the use of a rating scheme for preference elicitation or ranking. In the case of rating, Equation (17) may be used directly where y is the given rating from the DM. In the case where ranking is used to measure the customer's preference a logit recode of the given ranking value is required. The reason is because Ordinary Least Squares regression methods are not appropriate for conjoint data consisting of rank orders [36, 37]. This is due to the different between the representation of a rating and a ranking. In a rating the data is scaled so that real differences in combinations are communicated by the arithmetic differences in their value [32]. In other words, the difference between a rating of a 1 and 2 is the same as the different between a rating of 9 and 10. In rankings, the same assumption cannot be true. For instance, a combination with a ranking of 4 is necessarily twice as preferred as the combination ranked 2.

To perform the logit coding [38], a probability value, p, of each ranking is,

$$p = \frac{\bar{y} - \min + 1}{\max - \min + 2} \tag{20}$$

where $\bar{y}$ is the ranking value given by the respondent and *min* and *max* are the minimum and maximum ranking value used. The *p* value is then used to calculate the logit coded ranking value, y_L,

$$y_L = \ln\left(\frac{p}{1-p}\right) \tag{21}$$

This recode is performed for each ranking value and used to evaluate Equation (19) for the regression problem. The logit coding is a transformation of the ranking values into a scaled value in which it is appropriate to use an Ordinary Least Squares regression method such as multiple regression.

When the Dummy-Variable regression is conducted, there is a possibility to get very different part-worth utilities depending on the value of the intercept term (i.e. zero or non-zero). This can be a critical issue since the intercept term may represent a reference point for the each attribute level. This is the main reason of considering Effects Coding [39, 40] as an alternative to Dummy Variable Regression for determining the part-worth utilities due to the possibility of the statistically significant intercept term b_0 as shown in Equation (19).

For Effects Coding, the reference level for each attribute is assigned a value of '-1' for all combinations as opposed to removing the level completely as in Dummy-Variable Regression. The binary matrix is formed in the same manner by representing the presence of an attribute level in a combination with a '1' and the absence of a level with a '0'. The ranking/rating data from the DM is represented in the far right column of the binary matrix. The presence of the '-1' in Effects Coding helps to define the reference level as the negative sum of the estimated coefficients (i.e. the part-worth values of the other levels). In other words, the reference point is internalized in the *b* variables in Equation (19) as opposed to being carried over on the intercept term.

The solution to the multiple regression analysis minimizes the sum of squares of the errors over all observations. A regression equation is typically solved for each respondent. Thus it is required to evaluate a minimum of one combination per parameter for an accurate estimate of the part-worth utilities [32]. However, if only the minimum is done then there is no room to account for respondent error so traditionally more combinations are assessed to provide a better approximation.

In this example, the attributes are described through discrete values making the division of attributes into levels simple. However, for many engineering problems attributes are have continuous values. In cases such as this a continuous plot is required to represent all possible part-worth utilities. With the use of dummy-variable regression discrete values are determined and further analysis is required to approximate the values in between.

3.1.5. Formation of Part-Worth Plots

The part-worth values determined from the dummy coding represent the relationship between the objective data from the design objectives and the subjective data gained from the designer preferences. With the use of the attribute levels and the chosen analysis technique, part-worth utilities are only calculated for discrete attribute values. For optimization problems, all possible alternatives need to be considered giving the requirement of an approximation for the data points between the discrete level part-worth values of each attribute to form a continuous function. In the Multi-attribute Optimization with CA process, a piece-wise linear interpolation and extrapolation is suggested to create a continuous part-worth plot for each attribute separately. It is stated that this method is accurate enough because in engineering applications the many preferences are monotonic, meaning always has a positive or negative or zero slope which is correctly captured with this approximation [5]. In order to visualize this step, Figure 8 is shown as an example of a typical part-worth plot for a given attribute.

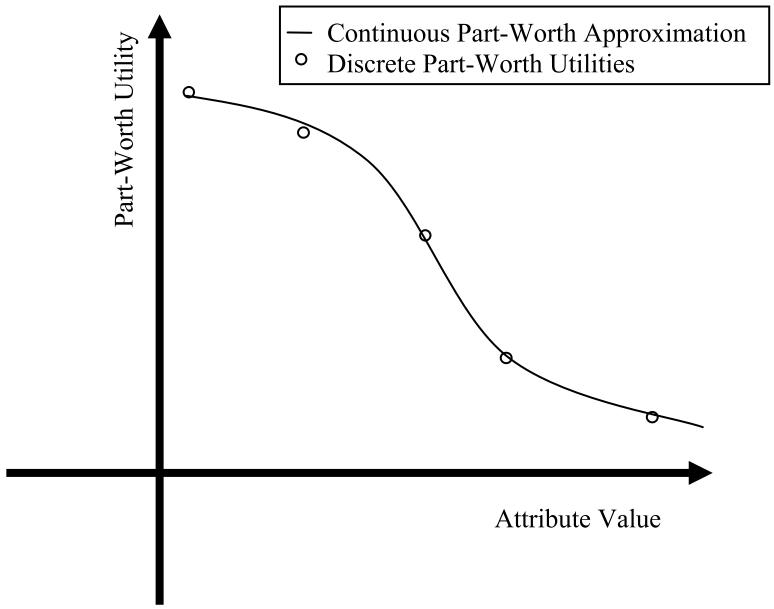


Figure 8: Example Part-Worth Plot

3.1.6. Optimization

Calculating the optimal design suggestion is done through the use of an optimization algorithm. Each attribute in the design problem has a continuous part-worth plot, p_1, p_2,…,p_k where k is the number of attributes, modeling the DM's preferences. These part-worth plots are a function of the attribute value which can be determined from a set of design variables, x. In order to find the total part worth of a specific set of design variables, each attribute plot is combined in an additive model forming the corresponding optimization problem as shown below.

$$\begin{aligned} &Minimize: && -\{p_1(f_1(x)) + p_2(f_2(x)) + ... + p_k(f_k(x))\} \\ &Subject\ to: && x_L \le x \le x_U \end{aligned} \tag{22}$$

where $f_k(x)$ is the calculated attribute value as a function of the set of design variables x, and x_L and x_U are the corresponding lower and upper bounds on each design variable. According to Equation (22), the set of design variables that gives the highest overall part-worth value is the optimal suggested design. The use of an additive model in Equation (22) is based on assumption of mutual preferential independence as stated by Keeney and Raiffa [16]. Mutual preferential independence an ordinal utility assumption meaning that it does not capture the strength of preferences but only ranking. As defined by Keeney, attribute X is preferentially independent of attribute Y if preferences for a value of X do not depend on the value of Y.

In addition to the assumption of mutual preferential independence, some assumptions are made in order to be able to utilize the CA. The first and most important assumption is that the DM is capable of eliciting his/her preferences. Any indecision from the DM increases uncertainty in the modeling of preferences. Another assumption is that the respondent of the survey has a complete understanding of the problem. Before the respondent(s) is asked to elicit his/her preferences, they must understand what type of information is required from them and what that information is used for. Without a full understanding of problem and decisions needing to be made, inaccuracies may occur when analyzing the subjective data.

3.2. A Discussion of Preference Aggregation based on Donald Saari

The assumptions for the CA are necessary to guarantee accurate representation of respondent preferences and the determination of the most preferred design. As mentioned in the previous section, aggregation of multiple respondent preferences are used to determine the preferred design by all DM affected by the decision to be made. The validity of aggregated preferences has received a significant amount of attention as it pertains to the ability to suggest the best outcome.

Donald Saari, Distinguished Professor of Mathematics and Economics, has given extensive evidence to flaws in the use of preference aggregation in voting theory [41]. According to Saari's work, caution should be used when evaluating methods of determining a preferred design from multiple voters whether it is by plurality vote, pair-wise comparisons, runoff voting, or any other method of aggregation. Saari states that there exist many 'voting paradoxes' that may alter the perception of the 'optimal' decision. To better understand some of these paradoxes, an example is given based on an electoral fable described in Ref. [41].

The chairman for a mythical academic department is in charge of deciding the preferred drink (milk, beer, or wine) to serve at the fall banquet. In order to save money, only one beverage is to be served. The preferences on drink choice from all 15 members of the department are:

A. Six preferred milk to wine to beer (i.e. milk > wine > beer)
B. Five specified beer > wine > milk.
C. Four specified wine > beer >milk.

Plurality vote is utilized such that each person votes for her/his favorite drink and based on this rule the department's choice for preferred drink is obviously *milk* > *beer* > *wine*. The decision is clear for the chairman to serve milk at the banquet since six people preferred milk whereas only five preferred beer and four preferred wine.

A closer look at the results of the vote proves that the interpretation is completely false since nine of the department members prefer beer and wine to milk. If the votes are tallied in a point-wise fashion such that the most preferred drink receives two points, the middle choice receives one point, and no points are given to the least preferred, the 'true ranking' is shown to be *wine* >

beer > *milk* with 19 points for wine, 14 points for beer, and 12 points for milk. Based on this comparison, it is obvious misinterpretation is highly possible with the use of aggregated preferences. Saari demonstrates additional examples and descriptions exemplifying the discovered flaws in aggregated preferences [41].

In addition to the use of aggregated preferences from multiple respondents in CA, rankings are typically given based on the use of pair-wise comparisons of alternatives. However, Saari has demonstrated the existence of a critical flaw in the use of pair-wise comparisons. For voting theory, the critical assumption is that the respondent is rational. Conversely, Saari states that there is an inability to distinguish between transitive (rational) and intransitive (irrational) preferences. Take the example that Susie states that she prefers strawberries to apple pie, and apple pie to raspberries. Based on the rule of transitivity, if a person prefers $c_1 > c_2$ and $c_2 > c_3$ we can expect that the same person prefers $c_1 > c_3$. However, is it irrational to for Susie to state that she prefers raspberries to strawberries? Many reasonable scenarios can entertain the possibilities of such intransitive voting. For cases with many pair-wise comparisons to be made, it becomes increasingly possible for a rational person to elicit irrational preferences. Saari elaborates on this critical flaw as well as other discovered problems with the use of pair-wise comparisons in Ref. [41].

One of the main objectives of the current research is to consider the uncertainties in the use of customer surveys and the known flaws with the use of aggregated preferences and pair-wise voting by improving the current procedures of the CA. The following section describes the details of the proposed framework, which utilizes the simulation-based ranking. It will show that the simulation-based ranking method can allow for preferences to be modeled accurately when system models are available. In addition, the subjectivity and the time required to receive a significant amount of responses to customer surveys can be reduced. The paradoxes demonstrated by Donald Saari's work are avoided with the used of objective data to perform the ranking process as opposed to the use of aggregation methods from multiple respondents or pair-wise comparisons that can have inconsistencies to rational voting. Although some of the paradoxes may be surpassed with the use of the simulation-based ranking, other paradoxes that were not demonstrated above may be possible. The discovery of these paradoxes is not in the scope of this thesis and may have significant impact on the accuracy of the proposed framework. The descriptions of Dr. Saari's works are given as a reference for possible limitations in voting processes and the iCA.

3.3. Proposed Framework

To utilize the benefit from the rank ordering process, and to account for accompanied uncertainties in the modeling of designer preferences and the use of customer surveys by means of probabilistic assessment, an improvement of the CA is proposed. The main advantage of the proposed framework is the use of a simulation-based ranking scheme to replace the traditional customer survey for the modeling of DM's preferences via the use of stochastic simulation. The simulation-based ranking scheme is performed through known uncertainty data allowing for objective data to accurately be used to represent subjective data. The implemented framework is depicted in Figure 9.

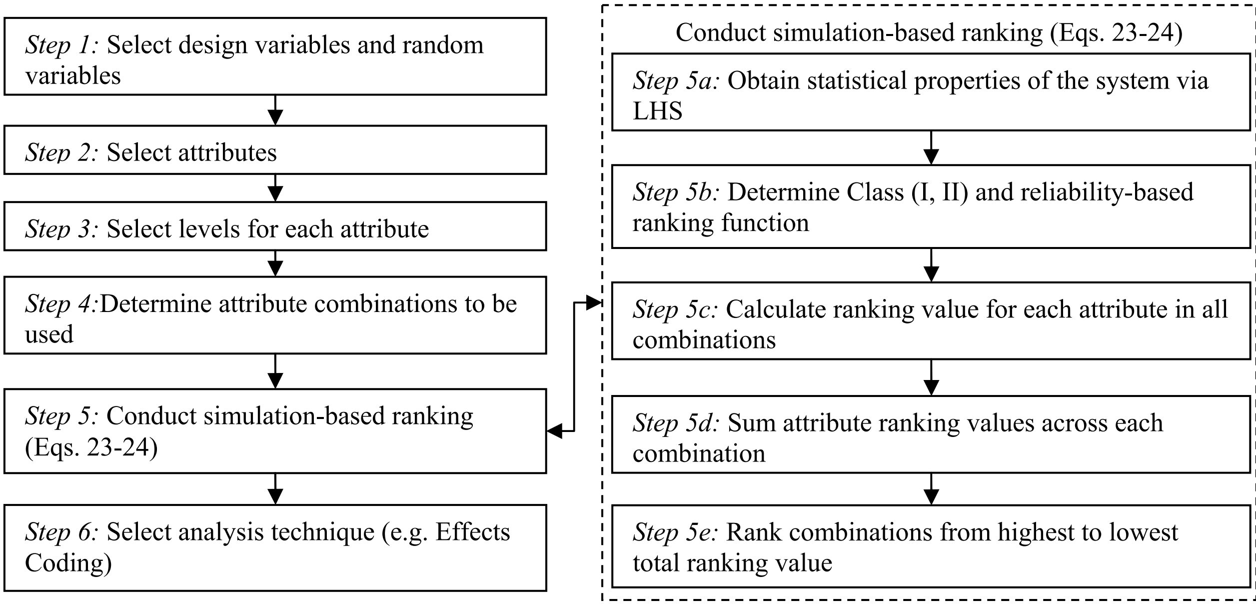


Figure 9: Proposed Framework for Improved Conjoint Analysis

3.3.1. Select Design Variables and Random variables

As shown in Figure 9, step 1 of the iCA is the same as the traditional CA with the determination of design objectives and design variables. However, the random variables that contribute to significant model uncertainty are also determined to be considered in the ranking of alternatives. Uncertain variables can be used in traditional Conjoint Analysis in the final design selection method; however, the consideration of model uncertainties is not directly used to rank alternatives based on preferences. The use of random variables in the iCA is the initial benefit for the common case where the DM would like a more reliable solution to a design problem. Once the random variables are determined a corresponding PDF is chosen for each (i.e. uniform, triangular, Gaussian, etc.) to accurately represent the uncertainty based on assumptions, previous knowledge, or experimental data.

3.3.2. Selection of Attributes and Attribute Levels.

Step 2 and step 3 in iCA is to determine the attributes and attribute levels for each design objective. The difference in this step between iCA and CA is the selection of the bounds for the attribute values. In the CA the maximum and minimum can be any value chosen by the DM. The design space can be refined with the use of a Design of Experiments (DOE) performed on a system model but this is not required. However, for the iCA method the ranking scheme is based on multiple simulations of a system model which will be discussed in the Chapter 3.2.4. Due to the necessity for simulation of a system model, the attribute levels chosen are required to be at feasible values. Therefore, a second improvement of the iCA is the refinement of the explored design space to only consider the preferences of the DM for feasible attribute values which reduces any unnecessary computation cost. Once the bounds are determine from the system model (i.e. DOE), the attributes are divided into discrete attribute levels as in the CA to be used to form alternatives for the ranking/rating process.

3.3.3. Determine Attribute Combinations

The discrete attribute levels are used to form hypothetical design alternatives in step 4. A factorial design (full or fractional) is used to determine these attribute level combinations for the ranking/rating process. The use of a fractional factorial design for more complex engineering design problem will reduce the computation cost and fatigue apparent with a full factorial design. The number of combinations required to solve an accurate iCA problem can be determined from Equation (17) as is used in the CA. The combinations chosen for the iCA must be determined based on what is feasible based on the system model just as was the case for the attribute bounds.

The fact needs to be considered when choosing the appropriate design alternatives to be ranked/rated.

3.3.4. Simulation-Based Ranking

The main modifications of the proposed iCA is to incorporate the use of a simulation-based ranking for the determination of part-worth utilities as an improvement to the use of timely and often uncertain customer surveys. The simulation-based ranking does not utilize presented surveys or questionnaires for respondents to answer. Alternatively, in step 5 the ranking is performed by a single DM based on the determination of a ranking value for each attribute combination calculated from measureable data attained from stochastic simulation of a system model. The steps for the simulation-based ranking are described in the following sections.

3.3.4.1. Obtain Statistical Properties of the System via Sampling Method

The first step of the simulation-based ranking scheme (step 5a) is to obtain the statistical properties of the system labeled as the performance (attribute value for a combination) and the variability (variance modeling the uncertainty in the attribute value of a combination based on the influence from the random variables). These statistical properties are determined by using sampling methods such as Latin-Hypercube Sampling [42] as described in Chapter 2.7.1. The sampling method is evaluated with a chosen set of design variables and the stochastic sampling for each of the random variables. The set of design variables is chosen such that the resulting mean value, μ_i, (performance value) for each attribute matches the attribute level in a given combination. In this case, the standard deviation, σ_i, represents the variability of that specific attribute due to the influence of the random variables. These two parameters are the basis for the calculation of the ranking value for a given alternative. It is important to note that the obtained stochastic responses of the system based on the given random inputs can facilitate the ranking process in the CA to reduce the user fatigue and uncertainty between multiple respondents due to the use of a system model in place of surveys or questionnaires.

3.3.4.2. Determine Class (I, II) and Reliability-based Ranking Function

Step 5b is to choose the class of the ranking function for each attribute. The class of the function determines the correct ranking function to use. This class is based on what value of the attribute is more preferred. There can be two classes; the DM can have a preference for a smaller attribute value (Class 1) or the DM can prefer a larger attribute value (Class 2),

$$Class\ 1(Smaller - Is - Better) \rightarrow R_i(w, COV) = \exp(-w_i * COV_i) \tag{23}$$

$$Class\ 2(Larger - Is - Better) \rightarrow R_i(w, COV) = \exp(\frac{-COV_i}{w_i}) \tag{24}$$

where R_i is the ranking value for a given i^{th} attribute in an alternative, COV represents the coefficient of the variance; $COV_i = \sigma_i / \mu_i$; σ_i and μ_i denote the standard deviation and the mean values of the attribute level distribution due to the effects of the random variables respectively, and w_i is the a weight factor determined by normalizing the attribute level between 0 and 1 to emphasize the performance value of the i^{th} attribute.

The reason for considering two different Classes is that the DM's preferences can be modeled with the assumption that the lowest uncertainty or variability is preferable to all DM due to manufacturing costs and reliability of the products. The hypothesis is that the higher the variability due to the random variables, the lower the reliability or robustness of the product. Related to a ranking scheme, the lower the reliability related to a specific alternative the lower the rank. The ranking function is modeled in the same manner such that a higher COV of an attribute will result in a lower reliability. In addition to the variance, the performance or value of the attribute is important as well, which is the reason for considering the weighting factor. The *COV* has been applied previously to model the sensitivity to uncertain or noise variables in Taguchi's approach for robust design. In Taguchi's approach a robust design is achieved by minimizing performance deviation from target values while simultaneously brining mean performance to target. These goals are accomplished through the measure of the signal-to-noise ratio which is the inverse of the *COV*. More information on the use of the signal-to-noise ratio and Taguchi approach for robust design can be found in Refs. [43-46].

3.3.4.3. Calculate the Ranking Value for Each Combination and Rank

Based on the simulation of the system model, each attribute in a combination will have a corresponding performance weight, μ, and variance, *COV*, for a given combination. In step 5c and 5d, the R_i value is calculated for each attribute in a specific alternative and then summed to give the total ranking value. As can be seen from Equations (23) and (24) the total ranking value for each alternative takes into account both the attribute level (i.e. the performance weight) and the specific variance due to uncertainties. The benefit of utilizing the ranking function to rank alternatives is the use of objective data to measure preference. In engineering design problems where the DM knows that he/she prefers a more reliable system, the proposed framework can be used without the need for multiple respondents and surveys. Instead, model simulations are used to measure the reliability of each combination and rank each accordingly.

3.3.5. Select Analysis Technique and Conduct Optimization

As shown in Figure 9, once all of the alternatives are ranked, step 6 is performed similar to the traditional CA. As mentioned in Chapter 3.1.4, Effects Coding is an alternative to Dummy-Variable regression to determine the part-worth utilities due to the possibility of the statistically significant intercept term b_0 as shown in Equation (17). For this reason Effects coding is chosen as the preferred method of regression for determining the discrete part-worth utilities for the proposed method. This implementation allows for the reference levels to be considered in the regression and allows them to be uncorrelated with the intercept value. The calculated part-worth utilities at each attribute level can be used to create a continuous part-worth plot as described in Chapter 3.1.5. At this point, the optimization problem shown in Equation (19) can be used to determine the most preferred solution. Since this portion of the iCA is the same as in the traditional CA method the corresponding assumptions of mutual preferential independence must hold true. These assumptions are required in order to utilize an additive objective function as stated in Chapter 3.1.5.

In the proposed framework (Figure 9), the benefits of discreetly ranking or rating to determine part-worths (CA) are combined with the advantages a simulation-based ranking scheme. Although using a multi-attribute ranking method like the traditional CA for ranking alternatives is less mentally taxing on the DM [27], it has been often criticized that there still exists a relatively large user fatigue when many alternatives are compared. Thus, the proposed simulation-based ranking method based on the performance and variance is a beneficial addition to the CA. The use of the simulation-based ranking method lowers the user fatigue by assuming a consistent general preference on less variability of the product performance for all DM. With this observation, the method used provides an algebraic ranking scheme that can easily be used to rank even a large number of alternatives. Therefore, this framework provides a method for modeling designer preferences that is less subjective and less mentally exhausting to the designer(s). In this way the proposed method also reduces the uncertainty associated with differing preferences from multiple respondents by allowing for the subjective data to be modeled with measureable data from simulations of a system model. The use of the simulation-based ranking does not require multiple respondent elicitations and does not necessitate the need for inaccurate augmentation. However, this does not eliminate the possibility of voting and preference modeling paradoxes as demonstrated by Donald Saari. As mentioned in Section 3.2, the discovery of these paradoxes for the proposed framework was not in the scope of the current research.

In addition, the use of the simulation-based ranking is beneficial for incorporating the effects of aleatory uncertainties associated with system models into the ranking/rating process. The improved ranking scheme is based on simulations of the system model to gain statistical data used to rank/rate each alternative based on reliability. The impact of model uncertainties can have a significant effect on the reliability of a system and, hence, the DM preferences. For problems in which the reliability of the system is important, the proposed framework is advantageous by considering the effects of uncertainties on the preferences of the DM.

Additionally, two classes of ranking functions are given to ensure the accuracy of the modeled designer preferences. The use of multiple classes allows for the shape of the part-worth plot as well as the value to be able to match the preferences of the DM no matter whether it is a customer or designer. For example, some attributes, such as cost, may have different preferences based on who the DM (respondent conducting the iCA) is. If the DM is a consumer, the preference for cost is to be as small as possible. However, in many engineering problems the designer is the DM and may not want to consider the preferences of the customer in the suggested decision. In this case, the DM may prefer to have a larger cost representing a larger profit for him/her. In either case, the iCA can provide an accurate model of subjective data by allowing the DM to choose the class of the ranking function.

In order to demonstrate the efficacy of the proposed framework and elaborate on the details of each step, the following chapter demonstrates the application towards three engineering design problems.

CHAPTER 4: APPLICATION OF PROPOSED FRAMEWORK

Validation must now be performed to show that the proposed framework indeed answers the posed research questions. To perform this validation and demonstrate the efficacy and applicability of the proposed method, a cantilever beam, a Power-Generating Shock Absorber (PGSA) [47], and mesostructured hydrogen storage tank design problems are considered. The cantilever beam application is used to demonstrate the efficacy of the iCA as compared to the existing CA method. The PGSA example is meaningful to show the utilization of the proposed framework on a more practical engineering problem. The purpose of the mesostructured hydrogen storage tank example is to demonstrate the advantages of the simulation-based ranking scheme through the flexibility of the ranking function. In addition the example shows the application of the proposed framework towards the development of a beneficial variable design concept. By validating the ability to both gain more reliable or robust design solutions and reduce the time and subjectivity involved in attaining a design decision, I hypothesis that the development of the iCA will fill the proposed research gaps.

4.1. Cantilever Beam

4.1.1. Problem Background

Consider the cantilever beam depicted in Figure 10. The beam is fixed at one end with a load, *P*, acting downward at the other. The beam is made of aluminum alloy with density, ρ, of 2786 kg/m^3. It is assumed to design the beam to minimize the mass, *m*, and the maximum displacement, *d*. The significant design variables are chosen to be the width, *b*, and the height, *h*, of the beam with upper and lower bounds chosen by the designer to be 1.5 m and 0.1 m. There are three chosen random variables to account for possible uncertainties in the design which are assumed to be normally distributed: the Young's Modulus, E~N(71.7, 18) GPa, the applied load, P~N(4,1) kN, and the beam length, L~N(5,0.25) m.

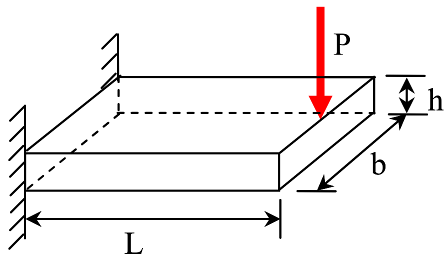


Figure 10: Cantilever Beam

The density equation, $m=\rho V$, is considered to characterize the mass of the beam as a function of the design variables. In addition the equation for maximum displacement of a cantilever beam, $\delta=PL^3/EI$, is used to relate the design variables to the displacement attribute where I is the area moment of inertia ($I=bh^3/12$).

4.1.2. Multi-Attribute Decision Analysis via CA and iCA

It is chosen by the designer to set a constraint on the mass and displacement. For this case, the mass should be less than 1000 kg and the displacement must be less than L/30 m or 0.1667 m. Based on the experience of the designer the lower bound of the m and δ is chosen to be 500 m and 0.02 m. Four levels will be used to evaluate the CA and iCA methods. Since there are only two attributes and a total number of 16 possible combinations a full factorial design is used to generate each alternative. Table 5 and Table 6 summarize the levels selected and the corresponding alternatives chosen to be ranked using CA and iCA.

Table 5 Cantilever Beam Attribute Levels

Mass (kg)	Displacement (m)
500	0.02
700	0.05
900	0.10
1000	0.15

Table 6: Combinations for CA and iCA Beam Design

Alternative #	Mass (kg)	Deflection (m)
1	500	0.02
2	500	0.05
3	500	0.1
4	500	0.15
5	700	0.02
6	700	0.05
7	700	0.1
8	700	0.15
9	900	0.02
10	900	0.05
11	900	0.1
12	900	0.15
13	1000	0.02
14	1000	0.05
15	1000	0.1
16	1000	0.15

Based on the CA, the combinations displayed in Table 6 are ranked by the DM from 1 to 16 based on his/her preferences where 16 represents the most preferred combination. The ranking values can either be done from direct ranking of each alternative or through the use of pair-wise comparisons via CA software as developed by Sawtooth Software [32-34]. A direct ranking is used in this case with one respondent to determine the ranking of each combination. Table 7 displays the rankings for each alternative achieved from the use of the traditional CA.

For the iCA the rankings are determined from the use of the ranking functions (Eqs. (23) and (24)) to calculate a total ranking value for each combination based on the statistical properties of the system. The simulation-based ranking method as described in Chapter 3.3.4 is used. The first step for the evaluation of the simulation-based ranking is to utilize sampling methods to determine these statistical properties for each combination of attribute levels. To begin a DOE is run on the system with a full-factorial design to determine the sets of design variables that will achieve a value for each of the attributes that matches that of each combination.

Table 7: Combination Ranking via CA from Single Respondent

Alternative #	Mass (kg)	Deflection (m)	Ranking Result (CA)
1	500	0.02	16
2	500	0.05	15
3	500	0.1	14
4	500	0.15	11
5	700	0.02	13
6	700	0.05	12
7	700	0.1	10
8	700	0.15	6
9	900	0.02	9
10	900	0.05	8
11	900	0.1	5
12	900	0.15	3
13	1000	0.02	7
14	1000	0.05	4
15	1000	0.1	2
16	1000	0.15	1

Once the DOE is performed, a LHS with 1000 samples is used to determine the performance and variability for each attribute level in each combination. The LHS based on a given set of design variables (i.e. width and height of the beam) and the assumed uncertainty of the random variables (i.e. Young's Modulus, applied load, and length of the beam) will give a corresponding distribution of values for each attribute (i.e. mass and displacement). The mean value of the mass and displacement must match the specific combination. The exact attribute level may be difficult to match perfectly so the mean value may be slightly larger or smaller than the attribute level. However, the difference should be small in comparison to the attribute level and assumed to be negligible.

Once the statistical properties are determined the class of each attribute is chosen. Given that both of these attribute are to be minimized, Equation (23) (i.e. Class 1 – Smaller-Is-Better) is used for both attributes to determine the ranking value. In order to better visualize the step of calculating the ranking value for each combination, the statistical properties, the ranking value for each combination and the rank for all alternatives is given in Table 8.

Table 8: Ranking Value Calculation for iCA

Alt. #	Mass (kg)	Performance Weight	COV_m	Defl. (m)	Performance Weight	COV_δ	Ranking Value	Ranking Result (iCA)
1	500	0.507	0.04999	0.02	0.124	0.4976	1.915	16
2	500	0.484	0.05020	0.05	0.275	0.4687	1.855	12
3	500	0.479	0.04998	0.1	0.577	0.4649	1.741	8
4	500	0.521	0.04997	0.15	0.866	0.4372	1.659	4
5	700	0.679	0.04994	0.02	0.130	0.5046	1.903	15
6	700	0.689	0.04995	0.05	0.324	0.4459	1.832	11
7	700	0.706	0.05009	0.1	0.644	0.5189	1.681	5
8	700	0.702	0.05001	0.15	0.885	0.5413	1.585	2
9	900	0.889	0.05009	0.02	0.130	0.5046	1.893	13
10	900	0.905	0.04997	0.05	0.303	0.4373	1.832	10
11	900	0.893	0.05011	0.1	0.687	0.4507	1.690	6
12	900	0.879	0.05003	0.15	1.000	0.4685	1.583	1
13	1000	0.962	0.05010	0.02	0.121	0.4975	1.895	14
14	1000	1.000	0.05005	0.05	0.271	0.6681	1.786	9
15	1000	0.988	0.04999	0.1	0.618	0.4510	1.708	7
16	1000	0.986	0.04998	0.15	0.891	0.5017	1.591	3

Once the combinations are ranked using both methods, Effects Coding is used to calculate the part-worth utilities associated with each attribute level and create the part-worth utility plots. Since ranking is used for this problem for the evaluation of both CA and iCA, a logit recode of the rankings is required as described in Chapter 3.1. To elaborate on the description, the logit coding values for the CA and iCA are given in Table 9.

The logit coding values are used to determine the part-worth utilities for each attribute via Effects Coding. The next step is to create the binary matrix to form the regression model for the calculation of the part-worth utilities. As described in Chapter 3.1, the binary matrix is formed by representing the presence of an attribute level with a '1' and the absence of an attribute level with a '0'. The binary matrix for the design of the cantilever beam is shown in Table with the logit coded ranking values for both CA and iCA

Table 9: Logit Coding of CA and iCA Ranking

Alternative #	Conjoint Analysis		Improved CA	
	Original Ranking	Logit Coding	Original Ranking	Logit Coding
1	16	2.77	16	2.77
2	15	2.01	12	0.88
3	14	1.54	8	-0.12
4	11	0.61	4	-1.18
5	13	1.18	15	2.01
6	12	0.88	11	0.61
7	10	0.36	5	-0.88
8	6	-0.61	2	-2.01
9	9	0.12	13	1.18
10	8	-0.12	10	0.36
11	5	-0.88	6	-0.61
12	3	-1.54	1	-2.77
13	7	-0.36	14	1.54
14	4	-1.18	9	0.12
15	2	-2.01	7	-0.36
16	1	-2.77	3	-1.54

Table 10: Binary Matrix for Effects Coding Regression

Alternative #	Mass				Displacement				CA	iCA
	500	*700*	*900*	*1000*	*0.02*	*0.05*	*0.1*	*0.15*		
1	1	0	0	0	1	0	0	0	2.77	2.77
2	1	0	0	0	0	1	0	0	2.01	0.88
3	1	0	0	0	0	0	1	0	1.54	-0.12
4	1	0	0	0	0	0	0	1	0.61	-1.18
5	0	1	0	0	1	0	0	0	1.18	2.01
6	0	1	0	0	0	1	0	0	0.88	0.61
7	0	1	0	0	0	0	1	0	0.36	-0.88
8	0	1	0	0	0	0	0	1	-0.61	-2.01
9	0	0	1	0	1	0	0	0	0.12	1.18
10	0	0	1	0	0	1	0	0	-0.12	0.36
11	0	0	1	0	0	0	1	0	-0.88	-0.61
12	0	0	1	0	0	0	0	1	-1.54	-2.77
13	0	0	0	1	1	0	0	0	-0.36	1.54
14	0	0	0	1	0	1	0	0	-1.18	0.12
15	0	0	0	1	0	0	1	0	-2.01	-0.36
16	0	0	0	1	0	0	0	1	-2.77	-1.54

In order to account for the linear dependency, a modified binary matrix is formed by choosing a reference level for which the part-worth utilities are based on. For Effects Coding, the reference level is represented in the matrix with a value of '-1'. The reference values for each attribute are chosen to be a beam mass of 500 kg and a displacement of 0.02 m. The modified Effects Coding matrix is shown in

Table 11: Modified Binary Matrix for Effects Coding Regression

Alternative #	**Mass**				**Displacement**				CA	iCA
	500	*700*	*900*	*1000*	*0.02*	*0.05*	*0.1*	*0.15*		
1	-1	0	0	0	-1	0	0	0	2.77	2.77
2	-1	0	0	0	-1	1	0	0	2.01	0.88
3	-1	0	0	0	-1	0	1	0	1.54	-0.12
4	-1	0	0	0	-1	0	0	1	0.61	-1.18
5	-1	1	0	0	-1	0	0	0	1.18	2.01
6	-1	1	0	0	-1	1	0	0	0.88	0.61
7	-1	1	0	0	-1	0	1	0	0.36	-0.88
8	-1	1	0	0	-1	0	0	1	-0.61	-2.01
9	-1	0	1	0	-1	0	0	0	0.12	1.18
10	-1	0	1	0	-1	1	0	0	-0.12	0.36
11	-1	0	1	0	-1	0	1	0	-0.88	-0.61
12	-1	0	1	0	-1	0	0	1	-1.54	-2.77
13	-1	0	0	1	-1	0	0	0	-0.36	1.54
14	-1	0	0	1	-1	1	0	0	-1.18	0.12
15	-1	0	0	1	-1	0	1	0	-2.01	-0.36
16	-1	0	0	1	-1	0	0	1	-2.77	-1.54

Excel regression add-on tool is used to follow out the multiple regression based on the regression model (Equation 19) described in Chapter 3.1. In order to calculate the part-worth utilities for each attribute level (b_{500}, b_{700}, b_{900}, b_{1000}, $b_{0.02}$, $b_{0.05}$, $b_{0.1}$, $b_{0.15}$), the regression model is solved such that the binary matrix represents the independent variables and the logit recoded rankings represent the dependent variables. The resulting part-worth utilities along with the regression statistics are shown

Table 12: Regression Statistics for CA

Regression Statistics

Multiple R	0.973855
R Square	0.948394
Adjusted R Square	0.91399
Standard Error	0.290661
Observations	16

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	6	13.97358	2.32893	27.56651	2.64E-05
Residual	9	0.760356	0.084484		
Total	15	14.73393			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.379988	0.192254	1.976483	0.079508
X Variable 1 (500)	0	0	65535	#NUM!
X Variable 2 (700)	-0.5405	0.205529	-2.62982	0.027367
X Variable 3 (900)	-1.26003	0.205529	-6.13069	0.000173
X Variable 4 (1000)	-2.10626	0.205529	-10.248	2.92E-06
X Variable 5 (0.02)	0	0	65535	#NUM!
X Variable 6 (0.05)	-0.27571	0.205529	-1.34144	0.212641
X Variable 7 (0.10)	-0.71801	0.205529	-3.4935	0.006793
X Variable 8 (0.15)	-1.31822	0.205529	-6.41382	0.000123

Table 13: Regression Statistics for iCA

Regression Statistics

Multiple R	0.972373
R Square	0.94551
Adjusted R Square	0.686961
Standard Error	0.298673
Observations	16

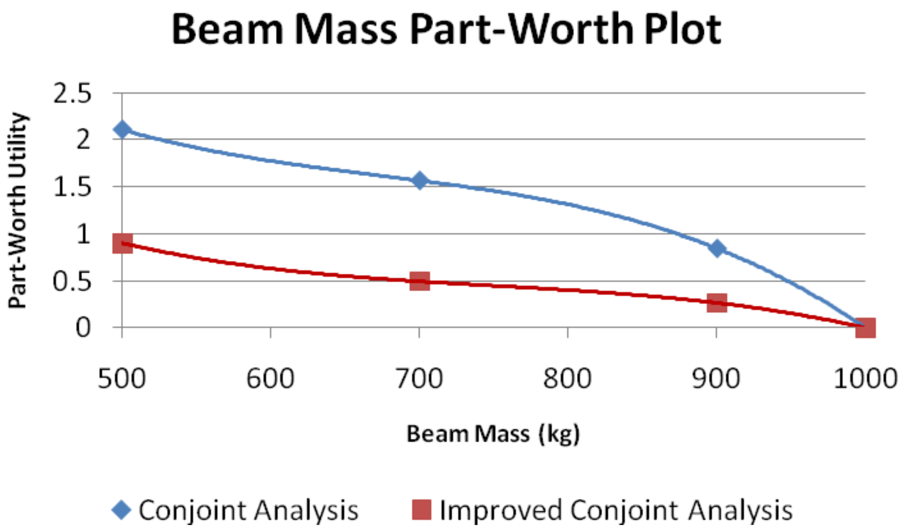
ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	8	13.93108	1.741386	26.02805	0.000154
Residual	9	0.80285	0.089206		
Total	17	14.73393			

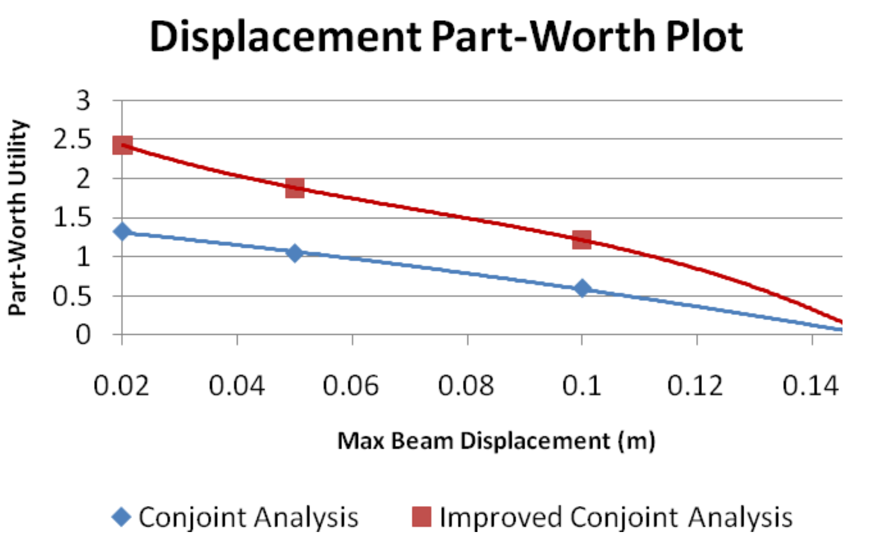
	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.204655	0.197554	1.035948	0.327258
X Variable 1 (500)	0	0	65535	#NUM!
X Variable 2 (700)	-0.40615	0.211194	-1.92311	0.086624
X Variable 3 (900)	-0.63377	0.211194	-3.0009	0.014935
X Variable 4 (1000)	-0.9828	0.211194	-1.41235	0.191477
X Variable 5 (0.02)	0	0	65535	#NUM!
X Variable 6 (0.05)	-0.54339	0.211194	-2.57293	0.030043
X Variable 7 (0.10)	-1.2111	0.211194	-5.73454	0.000282
X Variable 8 (0.15)	-2.42473	0.211194	-11.481	1.12E-06

The intercept values represent the corresponding part-worth utilities of each of the attribute levels used in the analysis. The ANOVA regression statistics verify that the part-worth utilities have a good fit to the regression model formed by the chosen combinations and the corresponding ranking functions for both the CA and the iCA. The part-worths for the reference levels are equal to zero and the values for the other intercepts correspond to the preferences in reference to these levels. In this evaluation the part-worth utilities are scaled to be positive by adding the minimum utility from a specific attribute to all other levels for that attribute.

Once the discrete part-worth utilities are scaled, a trend line is fit to the data for each attribute in order to form the continuous plots used to determine the optimal suggested design. The calculated part-worth utility plots for both mass and displacement using the CA and the iCA methods are shown in Figure 11. As can be seen, the methods of CA and iCA takes into account the trade-offs between the attributes and adds a preference weighting to each attribute based on the ranking given. The preference weight value for each attribute is given as the highest part-worth value of the specific attribute. Therefore, it can be deduced from this comparison that the existing CA method based on the customer survey ranking scheme results in a higher weight on the mass of the beam. However, the iCA method based on the simulation-based ranking scheme results in a higher weight being placed on the displacement of the beam. Based on the use of the simulation-based ranking scheme which focuses on the uncertainty of the attribute, it is concluded that the displacement has a larger effect on the total part-worth of a design alternative than the mass.



(a)



(b)

Figure 11: Part-Worth Utility Plots from CA and iCA

For the design of the cantilever beam, the following optimization formulation can be considered,

$$\begin{aligned} &\textit{Minimize}: && -\{p_m(f_m(b,h)) + p_\delta(f_\delta(b,h))\} \\ &\textit{Subject to}: && 0.1 \le b \le 1.5 \\ & && 0.1 \le h \le 1.5 \end{aligned} \tag{25}$$

In Equation (25) $f_m(.)$ and $f_\delta(.)$ are the functions relating the design variables to the mass and displacement attributes, and $p_m(.)$ and $p_\delta(.)$ are the part-worth utility plots (Figure 11) showing the relationship between the mass and displacement values (objective data) and the designer's preferences (subjective data).

Sequential-quadratic programming is used to solve the optimization problem (Equation (25)) for both methods. The optimal set of design variables utilizing the CA part-worth plots is a width, *b*, of 0.1 m and a height, *h*, of 0.2736 m. The suggested design based on the part-worth plots determined from the iCA method is a width of 0.1 m and a height of 0.4136 m. Table 14 shows the corresponding mean attribute values and *COV* for both sets of results.

Table 14 Comparison Results for Beam Design

	Conjoint Analysis	**Improved Conjoint Analysis**	**% Different**
Final Mass (kg)	378.6	572.4	+51.2
COV_m	0.0501	0.05	-0.2
Final Displacement (m)	0.035	0.0153	-56.3
COV_δ	0.668	0.42	-37.1

As can be seen from Table 14, using the proposed method the optimal solution has a suggested 33.9 % increase in beam mass; however there is an accompanied 56.3 % decrease in beam displacement. In addition, to the difference in mean value of the attribute values there is a 34.5 % decrease in *COV* when comparing the two methods. Therefore due to the consideration of the uncertainty in the system in the ranking scheme the final solution is more reliable in terms of a lower variation in attribute values. Consequently, this example shows that the proposed framework can be useful to produce a more reliable product while reducing the user fatigue with the consideration of uncertainties in the system. In addition to the reduction of variability, the proposed solution is valid from a design perspective. As can be seen from the results for the use of both methods to minimize the weight, the lower bound for the width is suggested for the optimal design. This suggestion is convincing because the applied load acts parallel to the height of the beam. Therefore more support is required in this direction to allow for a safer design. In addition, the suggested design when utilizing the iCA has a larger height which is consistent with the statement that a more reliable beam will have a larger height.

It should be noted that the use of decision analysis methods is not meant to give what is traditionally considered to be the "*optimum*" design in optimization analysis. Instead, decision analysis methods such as CA and iCA give a suggested design to enhance further decision making in the design problem based on the preferences given for the problem. In the cantilever beam case it is noted that for problems where the DM would prefer a more reliable system, it is validated that the iCA method is the preferred method to use. A similar evaluation is to be displayed in the following sections for the Power-Generating Shock Absorber and Mesostructured Hydrogen Storage Tank design problems.

4.2. Power-Generating Shock Absorber

In the following section, the design of a Power-Generated Shock Absorber (PGSA) is considered to show the applicability of the proposed method for practical engineering problems. The problem formulation and the application of the Multi-Attribute Optimization with iCA method will be described.

4.2.1. Problem Background

The system consists of a spring to count any vertical motion of the car and the PGSA to soften the recoil of the spring. The PGSA uses electromagnetic induction to dampen the amount of recoil instead of a traditional fluid-based damper which uses the compression of a fluid. The PGSA converts kinetic energy from suspension travel into electrical energy that supplements the electrical system of the car. To simplify the decision process the focus will be on the three important design attributes determined from sensitivity analysis. A depiction of the system is shown in Figure 12.

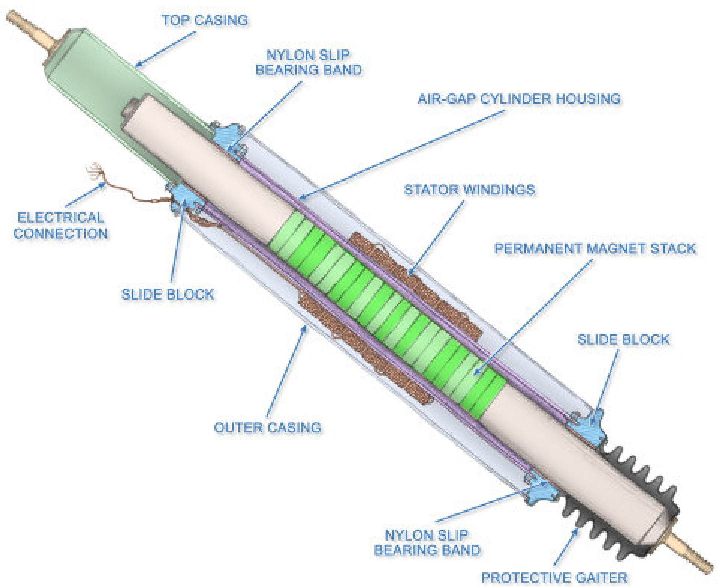


Figure 12: Power-Generating Shock Absorber [47]

It is important to design the PGSA to maximize comfort and generated energy and minimize the cost. The attributes chosen to represent these objectives is to minimize the vertical acceleration, maximize the energy generated by the linear motor, and minimize the cost of manufacturing 10,000 units. The chosen design variables that have a significant impact on these attributes were the strength of the magnet in Tesla and the length (in meters) of the copper wire used to make the stator coil. In addition to these controllable design variables, uncontrollable (random) variables of car mass in kilograms and variations in spring constant due to possible manufacturing defects were considered for uncertainty. The determined bounds for each design variable are shown in Table 15.

Table 15 Design Variable Bounds

Length of Wire (m)	Magnet Strength (T)	Car Mass (kg)	Spring Constant (N/m)
min=20	min=0.00001	μ_m=1630.9	μ_k=87654
max=150	max=0.75	σ_m=76	σ_k=1751

The random variables are assumed to be normal distributions and corresponding values of the mean and standard deviations are also given in Table 15. The randomness for car mass was based on the changing number of passengers in the vehicle. A 2% variability has been considered to the common spring as a result of manufacturing defects.

To represent the relationship between each of the design variables and the attributes, an algebraic model was formed using Dymola [48]. The model, shown in Figure 13, uses predefined relations to represent the spring force, mass, gravity, and electrical components. A linear motor model was created to incorporate the contribution of the magnetic strength and wire length on the vertical acceleration and the amount of generated energy. A detailed description of the PGSA and analytical equations are available in Refs. [47].

The amount of generated energy is calculated by integrating the power generated from the PGSA which is determined from Joule's Law. As can be seen from Figure 13b, a model of the entire car was made with four PGSA suspension systems attached to four tire models, a mass to represent the weight of the car, and a constant load acting on the mass to represent gravity. The total generated energy is determined to be a combination of the energy produced by all four PGSA's. The vertical acceleration is calculated by using a predefined accelerometer model in Dymola. The following equation is considered to estimate the cost of the PGSA

$$C_{total} = 10000\,(C_{magnet} + C_{wire}) \tag{26}$$

where C_{magnet} and C_{wire} represent the cost of the magnet and stator coil wire. It is important to note that this is meant as a relative cost measurement and does not represent the actual cost of the product.

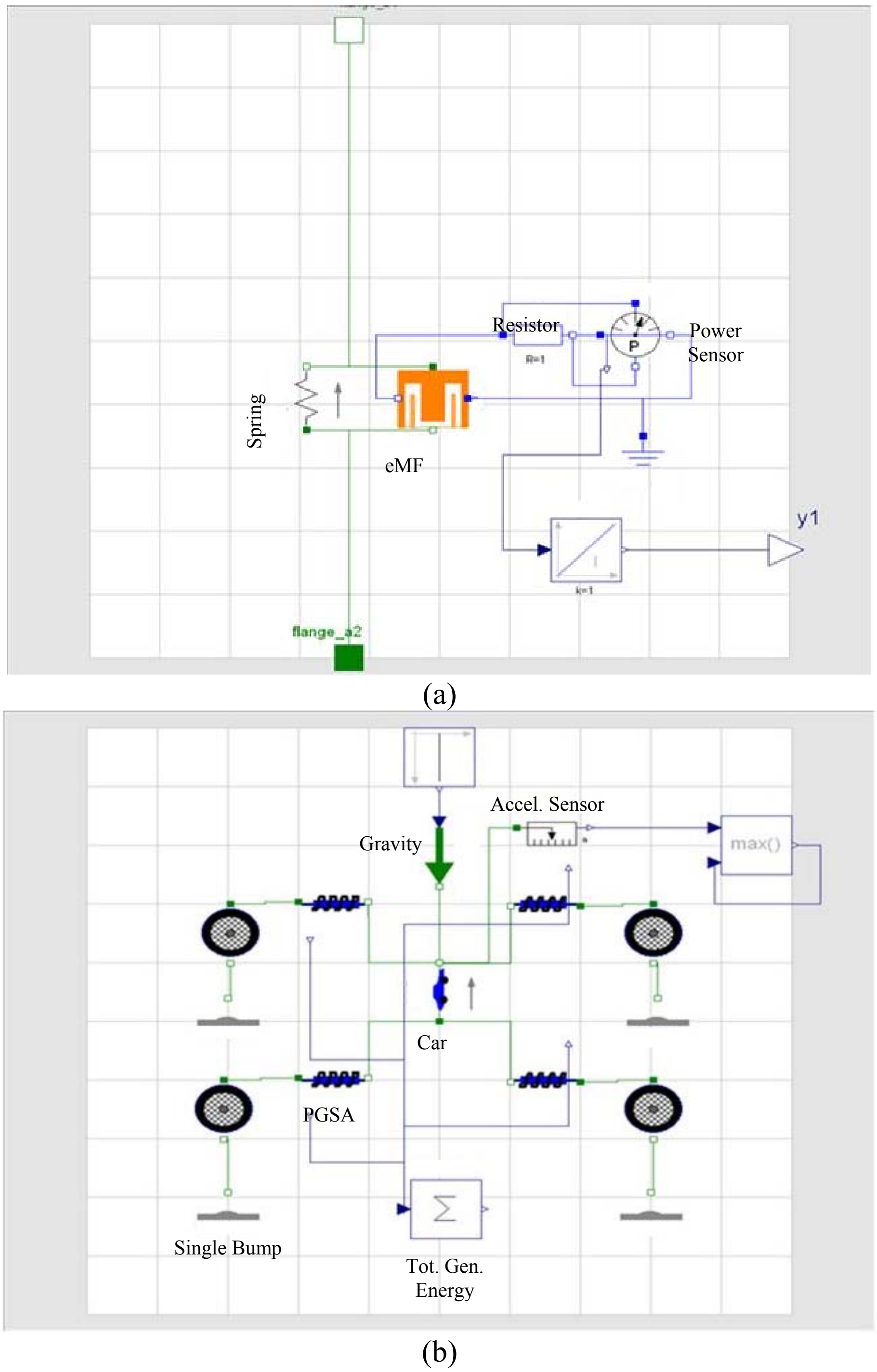


Figure 13: (a) Dymola Model for Suspension System with PGSA (b) Dymola Model of Full Car

Two case studies were conducted similar to the cantilever beam example. The first is to evaluate the design of the PGSA utilizing the existing CA method based on a customer survey. The second case will be evaluated with the ICA based on the simulation-based ranking scheme.

4.2.2. Case 1: Multi-Attribute Optimization with Traditional Conjoint Analysis

For the CA case, the chosen attribute bounds and levels are summarized in Table 16. Using a full-factorial design with three attributes and five levels gives 125 different combinations. To simplify this analysis, a fractional factorial design was used to create 25 different combinations. The fractional factorial design and ranks representing the DM's preferences are shown in Table 17. The combinations are ranked independently by 6 respondents during this case.

The part-worth utilities of each attribute level is first calculated for each respondent. The final part-worth utilities are determined by averaging the individual values for each level determined for each respondent. As can be seen from the multiple DM rankings of the combinations (Table 17), there is a large difference in preference rankings from each respondent. Although there are similarities between user preferences, it can be observed that there is a large amount of variance from user to user making it difficult to give an accurate ranking that is acceptable by all respondents. For this reason the current research is suggested using the simulation-based ranking which takes into account the preferences on the specific performance value of each attribute as well as the variance. To show the benefits of using the simulation-based ranking based on model simulation rather than a customer survey from many DM, the following section will depict the detailed steps of the iCA for the same problem.

Table 16 PGSA Attribute Levels

Vertical Acceleration (m/s^2)	Generated Energy (J)	Cost of PGSA ($)
0	355	225070
1	655	265000
2	950	300000
3	1250	345000
4	1550	381924

Table 17 DM preference Ranks for Fractional Factorial Design

Vertical Acceleration (m/s^2)	Power Generated (J)	Cost of PGSA ($)	Respondents					
			1	*2*	*3*	*4*	*5*	*6*
0	355	225070	17	12	18	21	5	18
0	655	265000	20	13	19	22	9	19
0	950	300000	22	21	20	23	13	21
0	1250	345000	21	22	21	24	16	22
0	1550	381924	15	15	22	25	17	20
1	355	265000	13	8	13	16	4	14
1	655	300000	16	11	14	17	8	16
1	950	345000	12	14	15	18	11	15
1	1250	381924	11	20	17	19	14	17
1	1550	225070	25	25	25	20	25	25
2	355	300000	8	5	16	11	3	8
2	655	345000	9	4	12	12	7	9
2	950	381924	4	6	11	13	10	10
2	1250	225070	24	24	23	14	23	23
2	1550	265000	23	23	24	15	24	24
3	355	345000	2	2	7	6	2	2
3	655	381924	3	3	6	7	6	6
3	950	225070	18	17	10	8	19	11
3	1250	265000	19	19	9	9	21	13
3	1550	300000	14	18	8	10	22	12
4	355	381924	1	1	1	1	1	1
4	655	225070	10	7	5	2	12	3
4	950	265000	7	16	4	3	15	4
4	1250	300000	6	9	3	4	18	5
4	1550	345000	5	10	2	5	20	7

4.2.3. Case 2: Simulation-based Multi-Attribute Optimization with Improved Conjoint Analysis

As explained in the previous section, levels of each attribute are created and combinations (alternatives) are created and ranked in order to create the part-worth plots. For the iCA, an alternative set of levels are used to determined design combinations due to the fact that the ranking approach is based on model simulations. For this reason only the attribute values that are attainable based on the model are considered for the levels. This altered set of attribute levels is beneficial as the range of values considered for the preference modeling decreases as much as possible and only considers values that are actually possible based on the model. In order to determine the new levels a design space exploration is performed using a full factorial design. After conducting the model simulations the new attribute levels are obtained as in Table 18. A fractional factorial design is used to create attribute combinations and depicted in Table 19.

Table 18: Improved Conjoint Analysis Attribute Levels

Vertical Acceleration (m/s^2)	Generated Energy (J)	Cost of PGSA ($)
0	400	231862
0.5	550	258352.5
1.5	700	284793
2.5	850	311133.5
3.5	1000	337524

Table 19: Improved Conjoint Analysis Alternatives

Vertical Accel. (m/s^2)	Generated Energy (J)	Cost ($)	Vertical Accel. (m/s^2)	Generated Energy (J)	Cost ($)
0.0	1000	311133.5	3.0	700	231862
0.5	1000	284793	0.0	550	337524
1.0	1000	258352.5	0.5	550	311133.5
1.5	1000	284793	1.0	550	311133.5
0.0	850	311133.5	1.5	550	284793
0.5	850	284793	2.0	550	258352.5
1.0	850	258352.5	2.5	550	258352.5
1.5	850	231862	3.0	550	231862
2.0	850	231862	3.5	550	231862
0.0	700	337524	1.0	400	337524
0.5	700	284793	1.5	400	311133.5
1.0	700	284793	2.0	400	311133.5
1.5	700	258352.5	2.5	400	284793
2.0	700	258352.5	3.0	400	258352.5
2.5	700	231862	3.5	400	231862

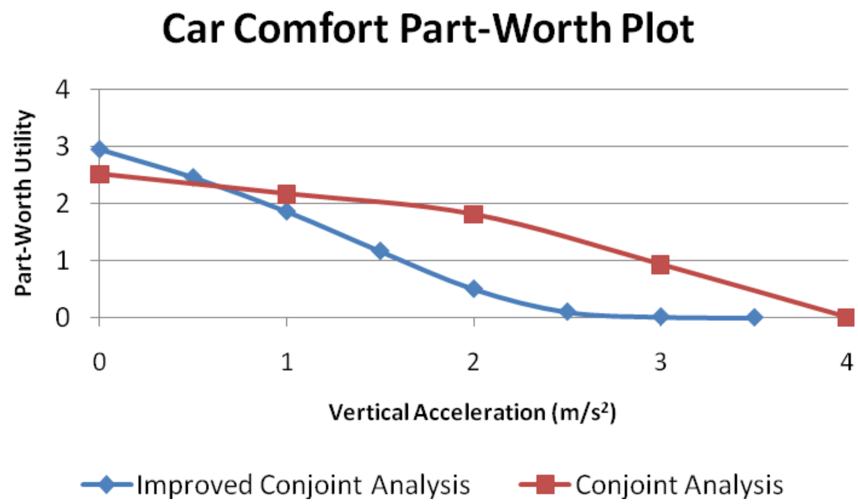
To implement the simulation-based ranking of these alternatives, ModelCenter is used to run simulations of the PGSA model and determine the effects of the randomness on the attributes.

For the stochastic simulation step, Latin Hypercube sampling with 1000 iteration is ran to gain statistical information for each attribute based entirely on the contribution from the given random variables in the same manner as the cantilever beam problem. The obtained stochastic simulation results are then used to determine the ranking value using Equation (23) and (24). In this example, the Class 1 equation is used for vertical acceleration and cost and the Class 2 equation is used for generated energy. To aid in determining which combination of design variables will achieve the particular mean value distribution needed, a full factorial design of experiments with 100 levels was applied. As can be seen from the formulation the value of cost has no dependence on the uncertain variables. In order to account for this independence for the ranking value calculation, the same COV is used for all combinations making the ranking value strictly a function of the performance.

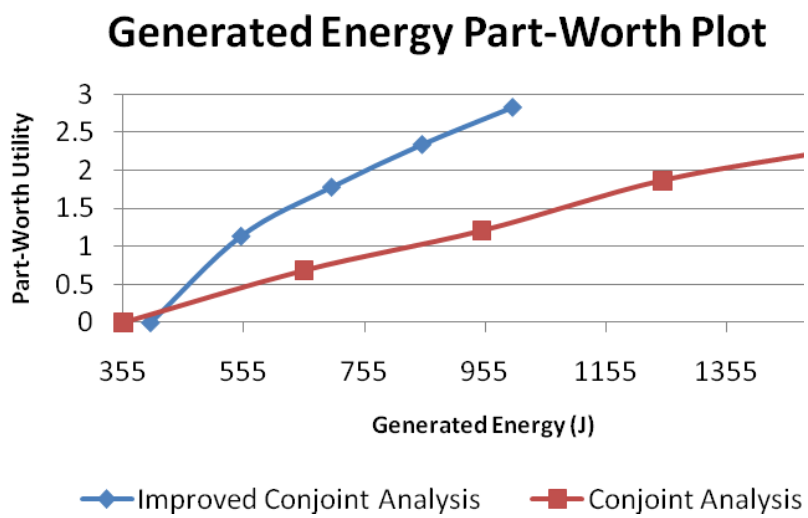
The results of both the CA and iCA are given in the following section. A comparison of the resulting part-worth plots and final design suggestions is given and useful observations described.

4.2.4. Comparison between CA and iCA results

The part-worth plots are compared for both methods. Figure 14 shows the part-worth plot for all three attributes: car comfort, generated energy, and cost. The plots were used to conduct the analysis to determine the optimal design suggestion for both methods. During the optimization process, Latin Hypercube sampling method was conducted to gain statistical properties of the response for a specific set of design variables based on the values shown in Table 15. A full-factorial design exploration with 100 levels was first performed in order to find a starting value for the optimizer.



(a)



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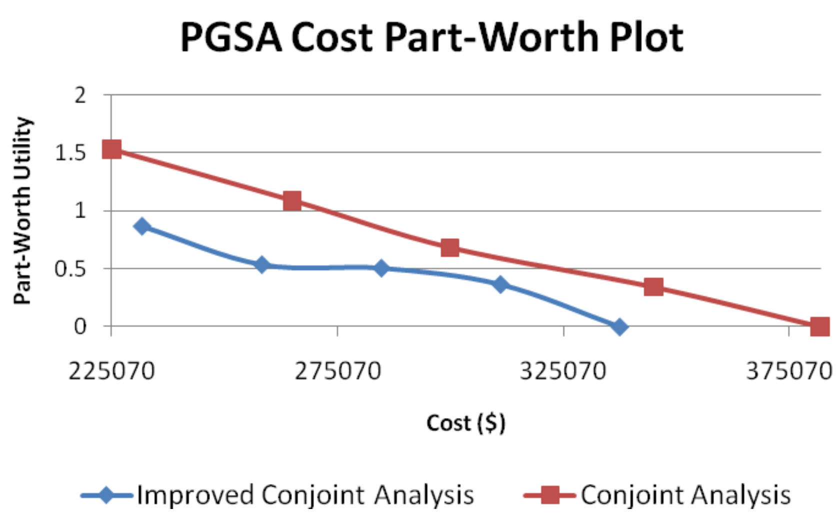


(c)

Figure 14: Comparison of Part-Worth Plots for (a) Car Comfort, (b) Generated Energy, and (c) Total Cost

The quasi-Newton optimization algorithm is used to determine the maximum within the design space. The set of design variables for the maximum part-worth utility represent the optimal design suggestion for the PGSA. The optimization results for both cases are shown in Table 20.

Table 20 Optimization Results for Design of PGSA

	Conjoint Analysis	Improved Conjoint Analysis	% Difference
Wire Length (m)	74.348	37.1	-50.1
Magnetic Strength (T)	0.75	0.75	N/A
Vertical Acceleration (m/s^2)	0.517	1.47	+184
COV_{VA}	1.15	0.813	-29.6
Generated Energy (J)	706.968	726.997	+2.83
COV_{GE}	0.0255	0.0220	-13.86
Cost ($)	321943	303319	-5.78

As represented from the results, the *COV* for the attribute values of the final solution is less when applying the iCA method representing a more robust solution. Although there is an increase in the vertical acceleration of the suggested design, there is an increase in final generated energy and a decrease in cost when comparing the CA and iCA results. In addition, the COV value is decreased by approximately 30% for the vertical acceleration and approximately 14% for the generated energy. This verifies the inclusion of the simulation-based ranking scheme is useful for determining a solution that is more reliable due to uncertainties. In addition to the validity of the iCA, the resulting design must make sense to the designer. As can be seen the wire length for both designs seems long. However, considering an example of a stator coil that is 1 inch in diameter, 75 meters of wire is equal to approximately 500 turns of wire. Some stator coils can be in excess of 10,000 turns. Therefore, the values for wire length on both designs may actually be small for similar stator coils. However, the design is meant to consist of one stator coil on each wheel corresponding to four in total. In addition, the magnetic strength for the suggested design using both methods is at the upper bound of 0.75 Tesla which is very high. As a point of reference, most MRI machines are 1.5 Tesla. Therefore, with the increased magnetic strength and number of PGSA used on a given automobile, the suggested design for both methods is valid.

Another important aspect to note through this design example is the situation where multiple respondents are needed which is required for customer survey methods. As was seen from the survey of the 6 respondents, there was a large amount of variation in the preferences. When conducting a survey with 300-600 respondents as is traditionally done, the uncertainty apparent in the preferences will increase greatly. In addition, an increase amount of time would be required to attain a significant number of customer responses. The use of the group preference assumption described in Chapter 3 in regards to the ranking function reduces the variance in preferences from person to person. For different problems this function can be modified to better represent the respondent's preferences based on previous experience with the problem. In addition, the ability to rank based on stochastic simulations reflects comprehensive stochastic information of the system and lowers the user fatigue that accompanies traditional comparison surveys with a large number of alternatives.

4.3. Mesostructured Hydrogen Storage Tank

In nature, systems with behavior encompassing interaction on a collection of different scales are a common occurrence. Thus, accounting for these multi-scale aspects in analysis and design has long been a spotlight in science and engineering. Design of these multi-scale systems do not fit within the classical framework due to the significant dependence on behaviors that are non-distinctively coupled through the multiple spatial and temporal scales. In such situations, consistent and physically realistic mathematical descriptions of the coupling behavior of the various scales are necessary to obtain robust and predictive computational simulations. Over the last decade, the need to account for this behavior in complex physical systems has risen drastically. As such, the physical and mathematical complications that occur in multi-scale systems represent one of the major obstacles to future development in numerous fields of science and engineering. Accordingly, scientists and engineers are seeking to simulate, analyze and even control the design of more complex systems. To successfully build future engineering systems in extremely competitive environments, an integrated approach of advanced multi-scale modeling techniques must be developed.

Although there has been a tremendous increase in computing power, measurement, and characterization tools over the past twenty five years, further progress in many fields of science and engineering is still impeded by the physical and mathematical complications that are inherent in multi-scale systems. One of the major tasks of the multi-scale research is to develop computational design tools which can facilitate the development of prototype multi-scale problems and challenging applications with sufficient consistency, stability, convergence, and accuracy. Thus, there is an urgent need to address the following key issues: 1) understanding how to represent the relationship of information gained from models at various scales to ensure the coupling effects can be controlled, 2) ensuring that errors occurring from the transfer of solutions and representations for component models are sufficiently controlled, and 3) quantifying the effects of uncertainty that propagates from one scale to another through the determination of parameters which accurately represent each component model at the various scales of the system. With the advent of new technologies, the exponential pace of engineering capabilities became too fast for product developments. The educational period needed to effectively learn about the performance and perception of these new technologies has not been reduced at equal pace. These leaps forward bring greater complexity to designs and a larger number of decisions that need to be made. Specifically, increased complexity in a system adds an increased amount of uncertainty. Successful product innovation cannot be achieved without adequate tools to analyze and manage these complexities and uncertainties. It is unlikely that a current design process will be able to capture all of these issues raised. Therefore, there is an urgent need for delivering new methodologies to determine and manage the rapidly increasing complexity and uncertainty of most engineered systems.

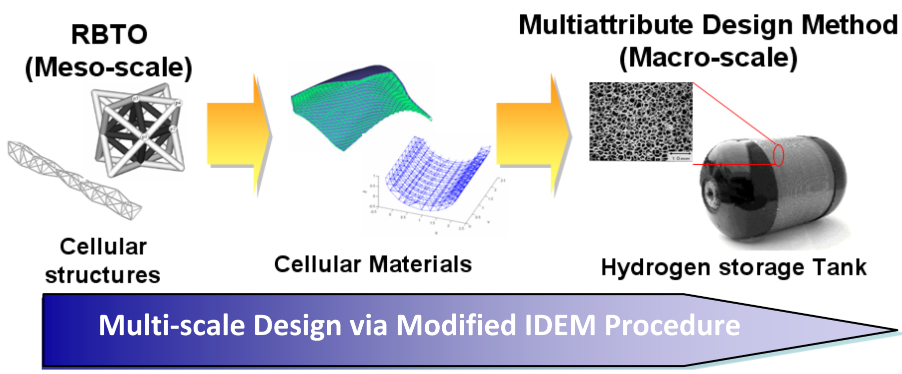


Figure 15: Multi-scale Design for Hydrogen Storage Tank

The purpose of the current research is to develop a new framework which integrates meso- and macro-scale systems design process. The feasibility of the proposed method will be demonstrated by designing and modeling cellular material structures so that an improved automotive component, a hydrogen storage tank for a fuel cell vehicle, can be designed. To develop robust cellular structures, the current research has two main directions: 1) propagating and quantifying the deformation and failure behaviors of loaded cellular structures and their variability, and 2) designing robust cellular structures that perform well under specific loading, displacement, and shape conditions. The Reliability-based Topology Optimization (RBTO) [49] will be introduced to design the cellular material structure. The proposed algorithms include topology-based risk mitigation that provides feedback on the design process and improves the reliability of the evolution of the cellular material structure. Thus, a reliability-based material design technique will be developed to mitigate the risk of structural failure via enhancements of conventional topology optimization techniques. As shown in Figure 15, the RBTO is used for the determination of optimal topologies for cellular materials at the meso-scale. The simulation-based multi-attribute design method [50] supports decision making on macro-scale design parameters. The Inductive Design Exploration Method (IDEM) [31] is integrated for the benefit of concurrent design on multiple scales providing an approach for integration of the other two methods. The focus is on accounting for the uncertainties of system parameters through the integration of simulation-based multi-attribute design at the macro-scale, RBTO at the meso-scale, and IDEM for concurrent multi-scale design procedure.

In the following sections, a description of the related research work will be given followed by the development of the proposed framework for multi-scale design. The proposed framework is then applied to the design of a robust hydrogen storage tank for fuel cell applications which utilizes cellular materials to provide a strong, light-weight system. In order to model the improved properties, if any, of the cellular structured hydrogen storage tank a comparison will be made between the overall strength, weight, and hydrogen capacity of the designed cellular structure tank and a typical solid wall tank.

The proposed method is meant to aid in the variant design of an improved compressed-gas hydrogen storage tank. The improved storage tank features a mesostructure wall as opposed to a solid wall. Mesostructured materials are materials that have a characteristic cell length in the range of 0.1 to 10 mm such as small truss structures, honeycombs, and foams [49]. The advantage of a mesostructured wall would be to drastically decrease the overall weight of the tank by only placing material within the walls of the tank where it is needed most.

4.3.1. Robust Multi-Scale Design Framework

The three methods (RBTO, iCA and IDEM as described in Chapter 2.7) form the proposed multi-scale framework. As shown in Figure 16 for the multi-scale design framework, the simulation-based ranking approach for iCA provides an effective method for performing design on the macro-scale, while RBTO is incorporated to the design of meso-scale systems. To propagate the impact of the design variables between different scales, the IDEM is induced to reduce these uncertainties by utilizing concurrent design of the other two methods. The order of operations from the flow chart in Figure 16 is based on IDEM which is a top-down design approach. Based on a range of feasible solutions in phase 1, a range of model inputs on both scales are used to determine a set of feasible designs using phase 2 and 3. From these feasible designs, the range of feasible design variables can be determined.

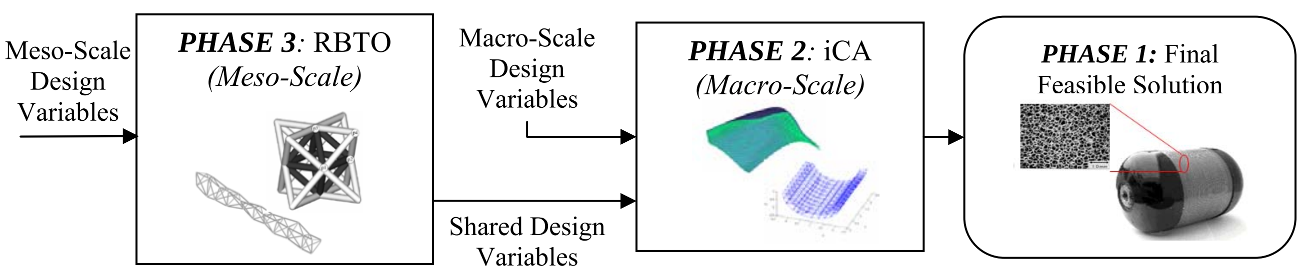


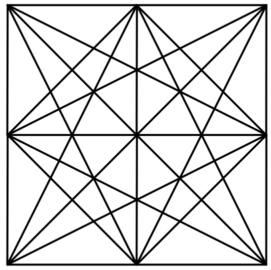
Figure 16: Proposed Multi-Scale Framework

There will be three different types of design variables defined in the framework: meso-scale design variables, macro-scale design variables and those that are shared by both meso- and macro-scale methods. Once these design variable ranges are determined, phase 1 of the multi-scale design framework begins with the definition of the feasible solution space as defined in the

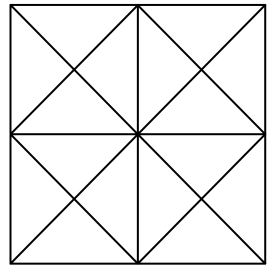
IDEM procedure. This design space is to be used to determine the feasible design variables used in the evaluation of both the iCA and RBTO methods.

Once the initial range of these variables is determined, phase 2 consists of the evaluation of the iCA method in order to determine the macro-scale objective function. This objective function is used to determine the preferences associated with the macro-scale attribute values for a given set of design variables. As stated in Chapter 4, the attributes for the design problem are determined on the macro-scale which are affected by the chosen design variables (macro-scale and shared). Once the attributes are determined, the attribute levels are determined and design alternatives created. The simulation-based ranking (Chapter 3.3.4) is used to determine the discrete part-worth values and form the part-worth utility plots for each of the macro-scale attributes. The next step is conducting the optimization process. The additive objective function (Equation 22) is used to model the maximum part-worth value which assists to determine the feasibility of the macro-scale designs. As described in Chapter 2.7, the EMI is utilized to denote the feasibility and determine the design space to be used for the next phase of the design framework.

In Phase 3, the RBTO approach is used for the optimization of cellular materials for the stiffest structure [51]. The topological model used for the topology optimization is in the form of either a ground truss or a unit cell structure as shown in Figure 17. Typically, the ground truss structure is defined as the topology in which truss elements connect each node to all other nodes. The unit cell structure is defined as a unit consisting of truss elements connected between only the nearest nodes.



(a) Ground Truss



(b) Unit Cell

Figure 17: Topological Model for RBTO Framework

From the topological models shown in Figure 17, the interest is in using RBTO to find the safe stiffest structure with the consideration of uncertainties. The statistical nature of constraints and design problems are defined in the objective function and probabilistic constraints. Thus, the formation of RBTO is similar to that of deterministic optimization,

Min/ Max: $f(b)$

Subject to:
$$P_j[g_j(b,\underline{x}) < 0] \le P_{Rj} \tag{27}$$

$$\sum_{i=1}^{N} A_i L_i - V^* \le 0 \tag{28}$$

$$A_l \le A \le A_u \tag{29}$$

$$Ku = F \tag{30}$$

where $f(.)$ represents the objective function, $g_j(.)$ represents the limit-state function [22, 52], b is the vector of deterministic design variables, and $\underline{x}$ is the random vector, which can be random design variables or random parameters of the system.

In Equation (27), $P_j[.]$ denotes the probability of the event and the probability of failure, P_f, can be defined as $P_j[g_j(.) < 0]$. P_{Rj} is the specified probability of failure level. A_i is the cross-sectional area of the elements and L_i is the length of that particular element. V^* denotes the volume of material that can be used in the final design. A_l and A_u are the upper and lower bounds on the cross-sectional area of the elements, respectively. K is the global stiffness matrix, u is the global nodal displacement vector and F is the nodal load vector.

The RBTO method is evaluated with the specific applied forces, volume fraction, unit cell dimensions, and cross-sectional area boundary conditions based on the reduced design space determined from the macro-scale evaluation. Some of these values may be linked to outputs or inputs from the decision support method and some may be specific to the meso-scale design. The use of IDEM and the EMI assists to determine the feasible design space on the meso-scale based on the previously determined results on other scales. Once the meso-scale parameters are defined based on the results from Phase 1 and 2, the RBTO can be conducted to determine the optimal material structures utilizing Equations (27)-(30). More information on the RBTO procedure can be found in Ref. [49].

In the above framework, IDEM is used to moderate the identification of a feasible design in the macro-scale which is utilized to map the feasible design space in the meso-scale. Through an exhaustive search of all discrete combinations of the macro- and meso-scale design variables, the EMI is used to determine the feasibility of the results gained from each discrete combination of design variables. Through the use of the IDEM exhaustive search approach propagation uncertainties are reduced by mapping the design space based on the combined effects from the macro- and meso-scales as opposed to each separately. Through the above multi-scale design framework a robust multi-scale design can be achieved.

In this thesis, the application of the iCA for the design of the tank is the main focus. Therefore, the following sections will give a description of the problem background, application, and results for the iCA applied to the design of the hydrogen storage tank on a macro-scale. The formulation of the multi-scale framework and the details for the meso-scale evaluation can be found in Refs [49, 53].

4.3.2. Problem Background

The two common methods for using hydrogen as an energy source is as a fuel cell to produce electricity which is intern used to power an electric motor or as a hydrogen powered combustion engine similar to the traditional gasoline engine. For both methods, there exist technical difficulties in the use of hydrogen for commercial-level products. For instance, hydrogen has about three times greater energy content by weight than gasoline, but around four times less energy content by volume. For this reason, it is a difficult task to store hydrogen within the size and weight constraints for vehicular applications. One of the most technically difficult tasks impeding widespread use of hydrogen as an energy source is developing safe, reliable, compact, and cost-effective methods for storing hydrogen. This is a challenging task due to the significant amount of space required to store enough quantities of hydrogen. For light-duty vehicular applications the available compressed hydrogen tanks are larger and heavier than necessary. A higher amount of hydrogen is able to be stored in liquefied hydrogen tanks as compared to compressed hydrogen storage; however energy is required to liquefy hydrogen and the required tank insulation has large impact on the weight and allowable volume of hydrogen stored [54, 55]. This paper demonstrates the proposed multi-scale framework that includes reliability as part of the objective for designing a novel high-pressure hydrogen tank with cellular materials.

High pressure storage tanks available in the market are commonly made of steel, but typically do not have a large enough capacity for fuel cell applications [56]. To design a tank for an increased capacity, one has to increase the pressure which results in an increase in mass and corresponding increase in cost of the storage tank. Light materials, high in yield strength and non-reactive with hydrogen have been used as materials for the storage tank as a method for weight reduction.

In order to design the hydrogen storage tank to meet the specification mentioned above, specific objectives and constraints must be defined. The objectives chosen for the design of the tank are to minimize the volume of the gas, V_{gas}, and the tank material volume, V_{tank}, and to maximize the mass of hydrogen, m_{H2}. The constraints chosen for these objectives are based on the goals for hydrogen storage for fuel cell applications. The main targets for fuel cell technology for the years 2010 and 2015 are summarized in Table 21. For this thesis the goals for 2010 tank design are chosen as the basis for the constraints.

Table 21: Targets for Hydrogen Storage for 2010 and 2015 [57]

	Targets for 2010	Targets for 2015
Gravimetric Density (wt%)	6	9
Volumetric Density (kg/m^3)	45	81
System Mass (kg)	83	55.6
System Volume (m^3)	0.111	0.062
Min Operating Temp. (°C)	-30	-30
Max Operating Temp. (°C)	85	85

Most hydrogen storage tanks are cylindrical in shape with spherical ends as shown in Figure 18. The represented variables shown are the height, h, tank wall thickness, t, inner radius, r_{inner}, and outer radius, r_{outer}, which designate the main geometric design variables.

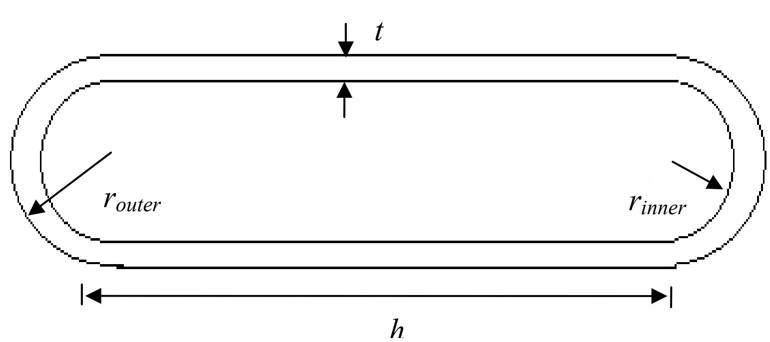


Figure 18: Cross Section of Hydrogen Storage Tank

The volume of the gas is equal to the inner volume of the storage tank, shown in Figure 18, and is calculated as,

$$V_{gas} = \pi r_{inner}^2 h + \frac{4}{3}\pi r_{inner}^3 \tag{31}$$

The volume of the tank material can be calculated using a similar equation with the addition of the outer radius term. The equation for the tank material volume, V_{tank}, is obtained as,

$$V_{tank} = \pi(r_{outer}^2 - r_{inner}^2)h + \frac{4}{3}\pi(r_{outer}^3 - r_{inner}^3) \tag{32}$$

In addition to the volume calculations, a model equation is needed to calculate the mass of hydrogen for a given set of design variables. Since hydrogen is the lightest element, it needs to be compressed at high pressures to be able to store it. Increasing pressures cause gases, including hydrogen, to lose their compressibility. For situations such as this, the equation of state is given by

$$PV_{sgas}=zRT' \tag{33}$$

where P is the pressure, V_{sgas} the specific volume of the gas, z the compressibility factor, R is the universal gas constant (8.314 m^3 Pa K^{-1} mol^{-1}) and T is the temperature.

There are different methods for estimating the impact of increased pressure on the compressibility of gases. The Benedict-Webb-Rubin equation [58] has shown to be an accurate predictor of hydrogen state at high pressures which incorporates available compressibility. From the Benedict-Webb-Ruben equation the compressibility factor can be expressed as follows,

$$z = 1 + (B_0 - \frac{A_0}{RT} - \frac{C_0}{T^3})\rho + (b - \frac{a}{RT})\rho^2 + \frac{a\alpha}{RT}\rho^5 + \frac{c\rho^2}{RT^3}(1 + \gamma\rho^2)\exp(-\gamma\rho^2) \qquad (34)$$

where a, A_0, b, B_0, c, C_0, α, and γ are Benedict-Webb-Rubin constants defined in [58]. This equation combined with Equation (33) shows the relationship between the volumetric density and the pressure inside the tank. However, the equation is a 6th order polynomial making evaluation of the density of hydrogen difficult. A more simple equation [59] to evaluate the compressibility accurately at high pressures is given by,

$$z = 0.99704 + 6.4149 \times 10^{-9} P \qquad (35)$$

Substituting this equation and the definition of specific volume into Equation (33) produces,

$$m_{H_2} = \frac{0.002 P V_{gas}}{RT(0.99704 + 6.4149 \times 10^{-9} P)} \qquad (36)$$

The constant in the beginning of the equation represents the conversion from *mol* of hydrogen to *kg* of hydrogen based on the units of the universal gas constant, *R*. Equation (36) is used in this thesis to determine the mass of hydrogen in the tank for a specific set of design variables. Equation (36) relates the calculated volume of the tank, temperature, and pressure to the mass of hydrogen. To utilize this equation the pressure and temperature must be determined. For this evaluation the temperature is going to be taken as an uncertain variable that is normally distributed. The temperature is assumed to be a Gaussian distribution with the mean of 293.15 K with a standard deviation of 20 K based on the target specifications given in Table 21.

In order to calculate the pressure inside the tank, a relationship with the design variables is required. The maximum stress acting on the tank wall gives an appropriate link to be used for a simulation-based model. The minimum pressure acting on the wall is determined using the equation for maximum stress on a thin-wall pressure vessel,

$$P = \frac{t\ S_y}{r_{inner}} \tag{37}$$

where P is the pressure, and S_y is the yield strength of the tank material.

To create the design space for the problem the inner tank radius is varied between 10 cm- 30 cm, and the total height, h_{total} is less than 1.35 m [60]. Taking into account future targets for storage capacity of hydrogen fuel cell tanks, safety concerns and other works on hydrogen storage pressure tanks [55, 61], a pressure range from 10 to 100 MPa is considered for the analysis of the improved design. Hydrogen storage tanks in production can store hydrogen at pressures as high as 70 MPa [62]. These values are utilized to expand calculation beyond what is currently commercially available.

Based on the above information, the optimization problem for the macro-scale design of the hydrogen storage tank is given in Equations (38)-(43).

Minimize: $V_{tank}(r_{inner}, t, h)$, $V_{gas}(r_{inner}, h)$

Maximize: $m_{gas}(\rho, r_{inner})$

Subject to: $0.25 \leq r_{inner} \leq 0.55\ m$ (38)

$0.50 \leq h \leq 1.35\ m$ (39)

$0.01 \leq t \leq 0.250\ m$ (40)

$10 \leq P \leq 100\ MPa$ (41)

$\rho_{grav} \geq 6\ wt\%$ (42)

$\rho \geq 45\ kg/m^3$ (43)

where h is the height of cylindrical portion of the tank, ρ_{grav} is the gravimetric density, and ρ is the volumetric density.

The volumetric and gravimetric densities are used as the main constraints for designing the storage tank due to the necessary targets that need to be achieved by the year 2010 as given in Table 21. The volumetric density (ρ) is defined as the mass amount of hydrogen to the cylinder volume. The gravimetric density (ρ_{grav}) is the ratio of hydrogen mass to the tank mass and expressed as hydrogen mass percentage.

The following section presents the formulation of the iCA for the design of the hydrogen storage tank. In order to validate the results the optimization problem for the design, a traditional optimization problem under uncertainty (Equations (38) to (43)) will be performed using a sequential-quadratic programming (SQP). The objective function for this problem will be a simple addition of each of the attribute values. The results of both the iCA and SQP will be compared to validate the efficacy of the proposed framework and the flexibility of the ranking function.

4.3.3. Improved Conjoint Analysis

For the design of the hydrogen storage tank, the same evaluation of the iCA is performed as with the other two examples. The three attributes are given bounds and divided into attribute levels. The bounds are determined from a design space exploration using a full-factorial design for the constraints mentioned for the design variables. Based on experience with the use of the traditional CA and the formulation of the iCA, each attribute is divided into 5 levels. Similar to the case of the Power-Generating Shock Absorber, the number of combinations possible for a full factorial design of these levels will be much too great to evaluate. Therefore a fractional factorial design is used. As mentioned in Chapter 3.1.2, the number of combinations should be between 1.5 and 3 times larger than the number of parameters for a good design. For this problem there are 12 parameters to estimate. Therefore, there needs to be between 28 and 36 total combinations. From experience with conjoint designs, it is chosen to use 31 combinations to be ranked. From the design space exploration used before to determine the attribute bounds, the 30 combinations are chosen. The attribute levels and combinations are given in Table 22 and Table 23

Table 22: Attribute Levels for Storage Tank Design

Volume of Gas (m^3)	**Volume of Tank Material (m^3)**	**Mass of H_2 (kg)**
0.1	0.05	2
0.25	0.2125	4
0.4	0.375	6
0.55	0.5375	8
0.7	0.7	10

Table 23: Attribute Level Combinations

Volume of Gas (m^3)	**Vol. of Tank Material (m^3)**	**Mass of H_2 (kg)**	**Volume of Gas (m^3)**	**Vol. of Tank Material (m^3)**	**Mass of H_2 (kg)**
0.1	0.05	2	0.4	0.2125	6
0.1	0.2125	4	0.4	0.2125	8
0.1	0.375	6	0.4	0.375	10
0.1	0.5375	8	0.4	0.5375	10
0.1	0.5375	6	0.4	0.7	10
0.1	0.7	6	0.55	0.05	2
0.1	0.7	8	0.55	0.05	4
0.1	0.7	10	0.55	0.2125	8
0.25	0.05	4	0.55	0.2125	10
0.25	0.2125	6	0.55	0.375	10
0.25	0.375	8	0.7	0.05	2
0.25	0.5375	8	0.7	0.05	4
0.25	0.375	10	0.7	0.2125	10
0.25	0.7	8	0.7	0.2125	8
0.4	0.05	2	0.7	0.375	10
0.4	0.05	4			

The purpose of this example is to demonstrate the flexibility of the ranking function (Equations (23) and (24)). The proposed ranking function given in Chapter 3.3 is not required. The DM is able to choose a different ranking function based on the specific problem to model designer preferences better.

The design of the hydrogen storage tank is constrained problem based on specific targets given for necessary deadlines in hydrogen storage technology. For problems under uncertainty such as this, probability of failure calculation is a commonly used method of reliability analysis. This quantity can be solved for in any number of ways. A popular method is by means of evaluating a limit-state function. For structural applications, the condition beyond which a structure or part of a structure is unable to perform as required is the limit-state [52]. A system is unreliable if the failure probability of the limit-state exceeds the required value. Therefore, the limit-state function is the difference between the resistance load(s) from the structure and the load(s) acting on the structure.

In the hydrogen storage tank problem the limit state is chosen to be the value of the volumetric and gravimetric densities. Therefore, a failure in the design is defined when the volumetric density *OR* the gravimetric density of a given design is be less than the target values given in Table 21. The choice of these two values as the limit-states is due to the relation to the design attributes. The volumetric density can demonstrate the preference relationship between the mass of hydrogen and the volume of hydrogen in the tank. The gravimetric density can show the preference relationship between the mass of hydrogen and the mass of the tank material, which is a function of the volume of the tank material and the density. The material chosen for the tank material is steel alloy which has a density of 7860 kg/m^3.

The ranking function for the storage tank design is based on the probability of failure calculation as described above. In this case, Equations (23) and (24) can be written in a more general form with the same definitions for Class 1 and Class 2. In either case the assumption given in Chapter 3.3 is constant where a lower variability due to uncertain variables is more preferred.

$$\begin{cases} Class\ 1(Smaller-Is-Better): R_i(w,C) = \exp(-w_i * C_i) & (44) \\ Class\ 2(Larger-Is-Better): R_i(w,C) = \exp(\dfrac{-C_i}{w_i}) & (45) \end{cases}$$

where w_i is a performance weighting value for attribute i determined by a normalized value of the attribute level, and C_i is represents the variability of the system due to the uncertain variables and can be given as either the coefficient of variance (*COV*) or the probability of failure (P_{fi}) based on attribute i [22]. For the design of the hydrogen storage tank, the variability is represented as the P_f as mentioned previously. In this case, the attribute used to determine the performance weight is not the design attributes but rather the volumetric and gravimetric densities. The performance weight in the ranking functions is used to show the preference towards the mean value of the volumetric density or gravimetric density and how much higher or lower it is from the target. The ranking value is calculated for both density values based on the uncertainty in temperature as described in Chapter 4.3.1 and then added together to get the total ranking value of a specific combinations of design attributes.

To validate the results gained from the above analysis, the iCA solution will be compared to an evaluation of the same problem using the SQP algorithm in Matlab. In this method the objective function is a weighted sum of the value of each attribute calculated from a set of design variables as given in Equation (46).

$$z = w_1 V_{H_2} + w_2 V_{tank} - w_3 m_{H_2} \tag{46}$$

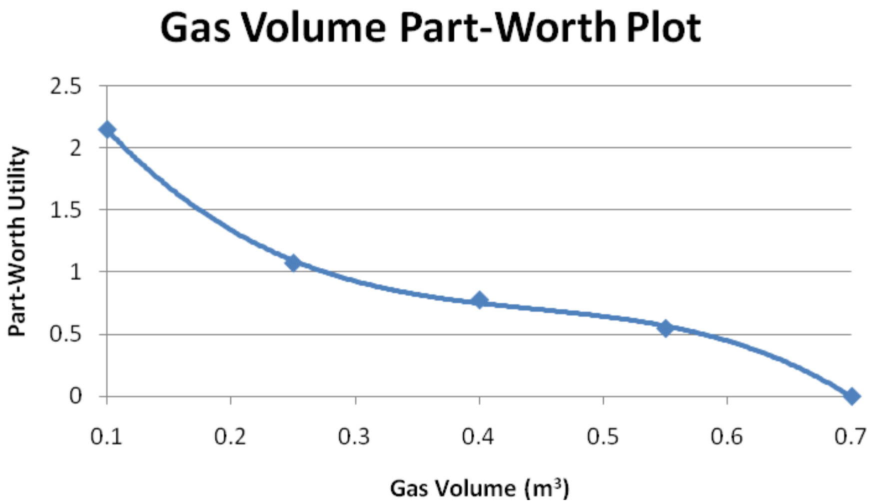
where w_1, w_2, and w_3 correspond to a chosen weight each attribute chosen by the designer, and z is the objective function value to be maximized. The weight value for this evaluation is chosen to be 0.33 for all three weight values. The evaluation is carried out in the same fashion as described for the optimization problem given by Equations (38) to (43) accounting for the same temperature uncertainty. The optimum design for the evaluation of this problem is the set of design variables that minimizes the objective function shown in Equation (46).

The results of both the iCA and Sequential Quadratic Programming optimization with weighted-sum of attributes method are given in the following section. A comparison of the resulting design suggestions is given and useful observations described.

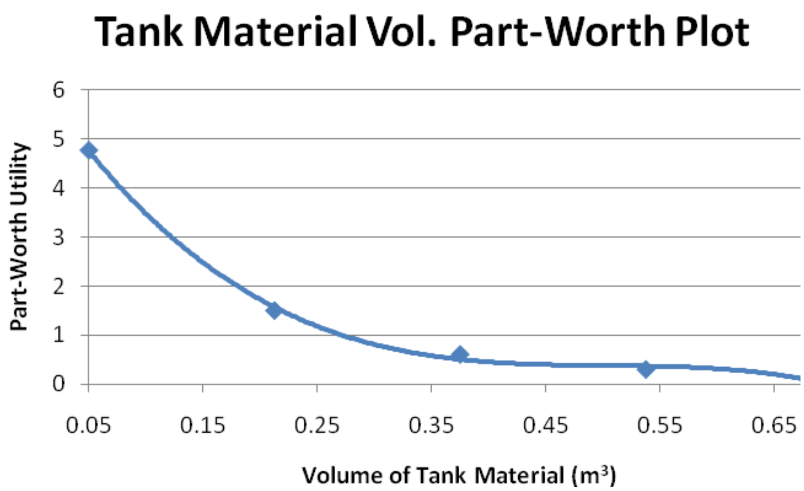
4.3.4. Results and Discussion

Based on the varied ranking functions given in Equations (44) and (45), the part-worth plots are displayed in Figure 19.

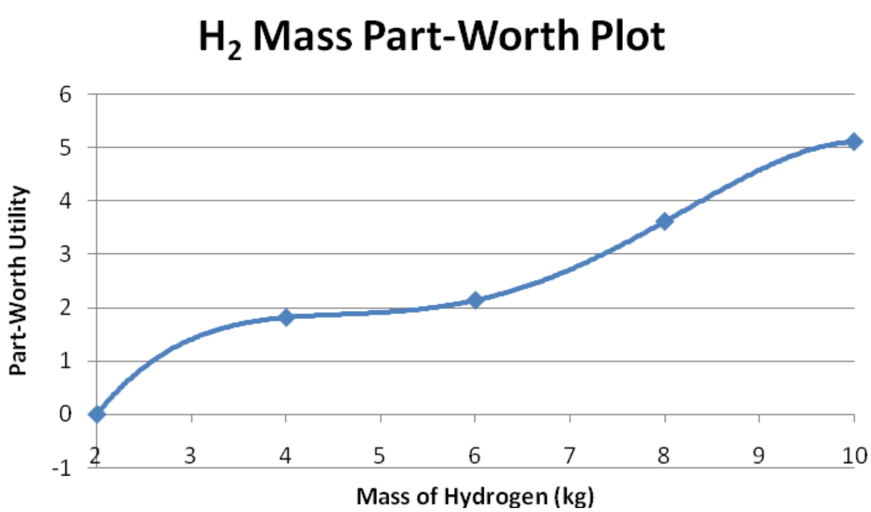
As can be seen from these plots, the use of the new ranking function still achieves the shape that is expected for the preferences of each of the attributes. It is accurate to say that a lower volume of gas is preferred because this will decrease the overall size of the tank. It is also precise to model the preferences on the volume of the tank material such that a lower volume is preferred because this will allow for a lighter tank. Finally, the modeled preferences make sense for the mass of hydrogen since it is necessary for the targets of hydrogen storage in the future to be able to store large amounts of hydrogen. The determined shape of the part-worth plots validates the ability to model the designer's preferences accurately using a flexible ranking function.



(a) Volume of gas



(b)Volume of material in tank wall



(c) Mass of Hydrogen

Figure 19: Part-Worth Plots for Hydrogen Storage Tank Attributes

In order to validate the results of the iCA, the given solution for both methods described in Chapter 4.3.2 are shown in Table 24.

Table 24: Results for Evaluation of Macro-Scale Design for Hydrogen Storage Tank

	Sequential Quadratic Programming	**Improved Conjoint Analysis**	**% Difference**
Inner Radius (m)	0.5	0.3095	-0.38
Wall Thickness (m)	0.0543	0.0683	-0.26
Height (m)	1.35	1.344	0
Volume of Gas (m^3)	0.7985	0.3424	-0.57
Vol. of Tank Material (m^3)	0.2527	0.2088	-0.17
Mass of H_2 (kg)	12.97	10	-0.23
Gravimetric Density (wt%)	0.6529	0.6093	-0.07
$P_{f,grav}$	0.1134	0.4350	+2.84
Volumetric Density (kg/m^3)	16.24	29.21	+0.8
$P_{f,vol}$	0.997	0.6780	-0.320

The interpretation of the results for both evaluation methods gives a validation of the ability of the iCA to achieve accurate results for a practical engineering problem. As can be seen, the suggested design variables for the iCA are relatively close to the solution using the traditional optimization method. Although the wall thickness is 26% larger, due to the magnitude, these two values are very similar. The determined inner radius is more of a significant difference which is the main contribution to the improvements based on the use of the iCA which will be explained later. In addition, the storage tank design for both methods gives a valid set of design variables. The upper limit of the height of the tank is based on an estimated width of a standard trunk on a mid-sized car. Many of the current tank designs for automotive applications require a tank of such size to allow for a significant travel distance (~300 miles). Therefore, the suggested height makes sense to be as large as possible given the constraints. In addition, the pressure relating to the suggested design for the iCA results is 25.4 MPa. This value is well in the range of what is currently available today as described in the previous section. The corresponding mass of hydrogen for this suggested design is also in the acceptable range (>5 kg). Although this design is in the acceptable range for these parameters, the main concern is the weight of the tank which is meant to be overcome with the use of the mesostructured tank wall described in Section 4.3.1. Based on the details shown in Table 24, the design is valid based on current designs and constraints for the above problem, however, further analysis is required to validate this design for the use of the mesostructures. In order to go into more details on the validation of the iCA method, the values of the design attributes and the volumetric and gravimetric densities are compared.

From the results of the attribute values based on the suggested design, there is a significant difference (57% decrease) in the gas volume between the weighted-sum and iCA methods. In addition there is a decrease in tank material volume when comparing results of the two methods. Both of these observations imply that the iCA results in a more preferred solution. There is an opposite effect when the mass of hydrogen stored is examined. For comparison of these observations, the value of the volumetric density and gravimetric density is scrutinized. The gravimetric density is acceptable for both the design solution for the SQP and iCA methods. However, the results of the iCA suggest a design that has an 80% increased volumetric density when compared to that of the SQP. This is highly advantageous for the concept of the mesostructured hydrogen storage tank design. Although both design suggestions result in an undesirable volumetric density when compared to the targets of 2010, the application of mesostructures on the wall of this design will drastically decrease the weight of the tank. Assuming the design of the mesostructure tank is sound, the suggested design for the iCA method will be more likely to be an acceptable design. This statement can be further validated with the comparison of the P_f for each of the chosen ranking attributes. As shown in Table 24, the P_f for both gravimetric and volumetric density is less than 1 for the suggested design using SQP and iCA and that the P_f is much lower for the SQP results. However, the P_f for the volumetric density using the SQP is very close to 1 representing complete failure. The P_f for both density values using the iCA method represents a more reliable design for both attributes rather than just one. Based on these observations the details given for the above problem validate the ability of the iCA method to achieve accurate results as compared to another common multi-attribute optimization method. The results show that, as long as the chosen ranking function is able to accurately model the designer's preferences towards the chosen attributes, the ranking function is flexible and based on the DM's experience and knowledge of the system at hand.

4.4. Discussion

The Improved Conjoint Analysis approach integrates the proposed simulation-based ranking with the traditional CA. The benefits as described in Chapter 3 of the proposed method is to incorporate model uncertainties in the ranking process as well as simplifying the attainment of DM preferences and reduce subjective related uncertainties through the elimination of the need for surveys and multiple respondent aggregation. The previous sections demonstrated the applicability of the proposed framework through three practical engineering design problems and the flexibility of the simulation-based ranking approach to fit the modeling of different types of preferences.

The results from the comparisons of the iCA method with both the traditional CA and existing optimization solutions demonstrate a validation of the posed hypotheses to the research questions given in Chapter 1. The hypotheses pose that the use of objective data in the form of reliability-based analysis calculations will aid the modeling of DM preferences to lead to a more reliable suggested solution. As was demonstrated by the cantilever beam and PGSA design examples, the use of the iCA suggested a design that corresponds to a lower variability due to the uncertain variables as described by the use of the *COV* when compared to that of the CA. The lower *COV* relates to the reliability measurement of the final design solution and a lower *COV* will represent a more reliable system. Therefore, since each attribute for the iCA suggested design has a lower *COV*, this design is said to satisfy the stated hypothesis and validate the benefits of the proposed framework.

For the design of the mesostructured hydrogen storage tank, the use of the probability of failure is utilized to model the DM preferences. The SQP optimization is used as the solution that minimizes the P_f representing the most reliable design solution without the consideration of DM preferences and trade-offs. As was shown with the design suggestion from the use of the iCA, the suggested design had system properties and a P_f similar to that of the SQP. These comparative results validate the ability of the proposed framework to suggest a design that is more reliable.

In addition to the benefits represented by the results of using the proposed framework, the validity of the intermediate steps is demonstrated through the above applications. As shown from the part-worth plots generated from the use of the iCA, the ability of the proposed framework to be used to form mathematical representations of preferences is validated based on the acceptable shape of the part-worth utility plots and the ability to suggest an acceptable design solution. The resulting design for all three design problems is valid based on the knowledge of existing system solutions and the suggested design based on the use of an existing valid decision design framework, namely Conjoint Analysis.

CHAPTER 5: DISCUSSION AND FUTURE WORK

This chapter gives an overview of the information described in this thesis and proposed future work for the current research. The summary includes an explanation of the answers to the posed research questions. The future work provides a critical review of the proposed framework showing the required improvement. In addition, the future proposed applications of the iCA are given.

5.1. Summary of Thesis

The ability to model DM preference information is beneficial to engineering problems by aiding in the decision making process for multi-attribute designs under uncertainty. This thesis is comprised of four chapters dedicated to presenting an improved framework to gain this subjective data in a process that is hypothesized to improve the traditional customer survey-based scheme. This framework incorporates a new simulation-based ranking scheme to be applied to the ranking/rating of hypothetical design alternatives as a means of improving the traditional CA. Chapter 1 provides an overview of the area of multi-attribute decision design and probabilistic analysis as an introduction to the current research. This chapter gives the necessity for an improved decision analysis method which requires less fatigue on the DM and reduces the uncertainty accompanied by highly complex engineering problems and large numbers of respondents. In order to understand the proposed framework better, background information on the common methodology for multi-attribute decision analysis is given in Chapter 2. A brief outline of a few of the popular multi-attribute methods is provided along with the recognized gaps in current research that provide the need for an improved framework. The area of focus in decision analysis for this thesis is the improvement of traditional customer surveys for the purpose of ranking or rating alternatives. CA has been proven to accurately model DM preferences for problems involving ranking/rating and has been chosen as a basis for the proposed framework. Chapter 3 provides an overview of the CA and leads into the description of the iCA and the development of the simulation-based ranking scheme. Chapter 4 gives details on the application of the iCA towards the design of a cantilever beam, a Power-Generating Shock Absorber, and a mesostructured hydrogen storage tank. The applications described are given to demonstrate the efficacy of the proposed method and validate the hypotheses for answering the research question posed in Chapter 1.

The research question posed based on the determine research gaps in the current multi-attribute decision analysis methods is

1) *How can customer survey driven decision analysis methods be integrated with Reliability-based Design methods to reduce uncertainty?*

Two hypotheses were given to the method for answering this question.

Hypothesis 1 → Utilize objective (measurable) data to rank alternatives based on a general preference for a more reliable system

Hypothesis 2 → Reliability-based Analysis methods can be used to accurately determine reliability of a system under uncertainty to provide the objective data for the ranking/rating of alternatives.

The design problems given in Chapter 4 represent the validation of these hypotheses to answer the given research questions. Based on the results of the cantilever beam and PGSA problems, the iCA method based on the simulation-based ranking scheme suggested a more reliable design in terms of the robustness due to uncertain variables when compared to the traditional CA for a simple and complex design. In addition as demonstrated with the PGSA example, the iCA method does not require aggregation of preferences from multiple respondents. As a result time is saved and uncertainty based on differing preferences and increased user fatigue apparent in the traditional approaches is reduced.

The third design example is used to validate the flexibility of the ranking function for the simulation-based ranking scheme. The ranking function is decided by the DM and should accurately represent the global preference assumption for the majority of respondents. As shown in the design of the hydrogen storage tank, the ranking function is chosen to be based on the probability of failure of a given alternative based on a limit state. The limit state is chosen to be the volumetric and gravimetric densities which are quantities that are a function of the design attributes. The results show that when using an alternative ranking function based on the structural reliability analysis, the use of the iCA suggests a design that is close to that suggested by a traditional multi-objective optimization evaluation under uncertainty.

The comparative results from all three design problems are a step towards the full validation of the proposed framework. The stated hypotheses are validated based on the ability of the proposed framework to account for model uncertainties as well as suggest a design based on the preferences of a more reliable system. The method is also validated to remove the requirement for aggregated preferences and voting paradoxes as described by Donald Saari's work that are apparent in the traditional CA. However, further improvements of the proposed framework as well as additional applications will aid to fill known gaps in the current research. Also further validation may be required to fully determine if any other voting paradoxes are present in the use of the proposed framework. Some of the limitations of the iCA and required future work are elaborated on in the following section.

5.2. Future Work

5.2.1. Limitations and Future Improvements

The proposed framework is shown to be an appropriate method for modeling DM preferences for multi-attribute design problem under uncertainty based on a global preference assumption. The method utilizes a simulation-based ranking scheme which is based on approximated probability statistics for the system. Although the framework has been validated for the given examples there are some limitations for the use of this framework and a couple characteristics that were beyond the scope of this thesis. These items are presented below as possible future work for improvement of the given framework.

The current framework has certain limitations that should be taken into consideration for future applications. The iCA is meant as an improvement to the CA for problems in which the DM has a preference for a more reliable system in the sense that less variability is more reliable. This assumption may not be the case for some applications. In the case where more variability is preferred a modified ranking function may be derived to match the DM preferences. In addition, the use of the simulation-based ranking is beneficial in applications where it may be more costly to have hundreds of respondents answer a survey. The use of the proposed framework is meant for applications in which one or two DM can utilize computational models to accurately represent subjective data with measurable data. For more simple problems with very little uncertainty information, the traditional CA may be more appropriate.

The proposed framework is also based on certain assumption on the availability of important system information which may limit its applicability. For the proposed framework it is assumed that there is known PDF information for the random variables which have a significant effect on the outcome of the design. In addition it is assumed that an accurate system model is available to demonstrate these dependencies and the relationship between the design variables and attributes. The iCA method should not be utilized if this information is not available. In addition, the assumption of mutual preferential independence is required for the use of the additive objective function. Although this assumption holds true for many engineering applications, a validation of this assumption should be made prior to evaluation of the proposed framework.

Other limitations in the iCA pertain to the consideration of uncertainties in the elicitation of preferences. As stated in Chapter 3, the improvement of the simulation-based ranking scheme is the consideration of uncertainties in system parameters in order to bridge a cap between the CA, which is based on decision making under certainty, and the benefits of decision making under uncertainty. Typically, decision problems are decomposed in a fashion that allows us to elicit preferences in a fashion that is independent of the actual uncertainty for a particular problem. This has the advantage that when you learn more about the problem and hence reduce the uncertainty in some of the variables, you can still use the same preference characterizations you elicited previously. In the proposed framework uncertainty is incorporated into the elicitation process. This may make it easier to express preferences under uncertainty. However, a limitation to this benefit is the requirement for the DM to re-elicit his/her preferences whenever the uncertainty changes.

Aside from the limitations stated above, future work may be performed to improve the applicability of the proposed framework. The first area of future work is in regards to the incorporation of uncertain variables in the simulation-based ranking scheme. The current simulation-based ranking scheme is demonstrated by ranking/rating alternatives based on the dependence of the design variables on Type I or aleatory uncertainty. Type II and III or epistemic uncertainty was not explored in the current research. Epistemic uncertainty is that which deals with the uncertainty associated with a lack of knowledge about the system or problem at hand. All types of uncertainty have an impact on the outcome of the design. In the case of epistemic uncertainty, if the designer has a lack of knowledge of the system or even the methodology being used the uncertainty will propagate to the DM or respondent which supplies his/her preferences. The inclusion of this type of uncertainty would be beneficial to the reliability of the proposed framework. More information on all three types of uncertainty and methods of accounting for them can be found in Refs. [22, 63-66].

The second topic for future work is on the improvement of the combinations which are ranked/rated using the simulation-based ranking scheme. The current method ranks each combination based on the total ranking value calculated from the ranking functions described in Chapter 3. For the implementation, the ranking function is based on the variability of each of the design attributes of a given combination due to the uncertain variables. This variability is calculated from model simulations for a given set of the design variables. In the current framework, a one-to-one mapping between the set of design variables and the combination of attributes is assumed for the determination of the ranking value. To clarify, it is assumed that there is only one set of design variables that will achieve one given combination of attributes. This assumption may not always be true. Multiple sets of design variables could achieve the same attribute combination and have a completely different dependence on the uncertainty variables. The current methodology simply looks at each set of design variables from a factorial design and the DM chooses the one with the design variable combination with the lowest variability. However, this method may not be accurate. Future work is required to determine a proper method of taking into account the fact that multiple design variable combinations can attain the same combination of attribute values.

5.2.2. Future Applications

To accompany future research steps presented above, there are two applications proposed for the future regarding the framework presented in this thesis. The two future applications are the development of the multi-scale design method proposed for the design of the mesostructure hydrogen storage tank (Chapter 4.3) and for the design of grid portals for global computing and information sharing.

The design of the mesostructured hydrogen storage tank was presented in Chapter 4 of this thesis. The design began with the development of a multi-scale design method for complex engineering systems. As mentioned previously, the proposed multi-scale method incorporates the iCA for macro-scale design and Reliability-based Topology Optimization for meso-scale design. In multi-scale designs uncertainties in one scale will propagate throughout all scales. The use of Inductive Design Exploration Method is given to account for the propagated uncertainties and determine a robust design on all scales. An outline of the initial proposed method is given in Ref. [53]. The future work required for this method is the validation of the multi-scale method. The initial results have been gained for the mesostructured hydrogen storage tank; however, validation of these results through structural comparisons with current storage tanks must be performed to show the benefits, if any. In addition other applications for multi-scale design must be performed and validated to properly demonstrate the efficacy of this method.

In addition, further applications of iCA can be improved with the use of Grid Portals and grid computing. Grid computing is a technique for solving a single problem through the application of several computers at one time. This computing method is typically useful for scientific or technical problems which require high levels of data transfer and computer processing speed (www.globus.org). A grid portal is a web-based application server enhanced with necessary software to communicate to Grid services and resources. This allows for a single access point to Grid resources for customized views of software and hardware for specific problems domains that the user already has authorized access to. In other words, a grid portal is a central web-based access point to multiple Grids used for grid computing. More information on grid portals can be found in Refs. [67-70]. These benefits of shared Grid resources such as computation speed and model and uncertainty information are a huge leap forward in technology. With the use of Grid Portals, the possibilities of model simulation and the accuracy of uncertainty quantification would be beneficial to the use of the simulation-based ranking scheme. Faster computing speed and better probabilistic information will increase the accuracy and advantages already shown from the development of the iCA towards the modeling of designer preferences for the design of complex engineering systems.

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