

**COMBINING MATHEMATICAL PROGRAMMING AND SYSML
FOR COMPONENT SIZING AS APPLIED TO HYDRAULIC
SYSTEMS**

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SYSTEMS**

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SUMMARY

In this research, the focus is on improving a designer's capability to determine near-optimal sizes of components for a given system architecture. Component sizing is a hard problem to solve because of the presence of competing objectives, requirements from multiple disciplines, and the need for finding a solution quickly for the architecture being considered. In current approaches, designers rely on heuristics and iterate over the multiple objectives and requirements until a satisfactory solution is found. To improve on this state of practice, this research introduces advances in the following two areas: a.) Formulating a component sizing problem in a manner that is convenient to designers and b.) Solving the component sizing problem in an efficient manner so that all of the imposed requirements are satisfied *simultaneously* and the solution obtained is mathematically optimal.

In particular, an acausal, algebraic, equation-based, declarative modeling approach is taken to solve component sizing problems efficiently. This is because global optimization algorithms exist for algebraic models and the computation time is considerably less as compared to the optimization of dynamic simulations. In this thesis, the mathematical programming language known as GAMS (General Algebraic Modeling System) and its associated global optimization solvers are used to solve component sizing problems efficiently.

Mathematical programming languages such as GAMS are not convenient for formulating component sizing problems and therefore the Systems Modeling Language developed by the Object Management Group (OMG SysML™) is used to formally capture and organize models related to component sizing into libraries that can be reused

to compose new models quickly by connecting them together. Model-transformations are then used to generate low-level mathematical programming models in GAMS that can be solved using commercial off-the-shelf solvers such as BARON (Branch and Reduce Optimization Navigator) to determine the component sizes that satisfy the requirements and objectives imposed on the system. This framework is illustrated by applying it to an example application for sizing a hydraulic log splitter.

CHAPTER 1

INTRODUCTION

This research focuses on improving a designer's capability to determine component sizes, such as during the architecture exploration phase in the design process. This can lead to more efficient ways of exploring large design spaces and ultimately allow a designer to consider more alternatives. The need to consider more alternatives is increasing because the design of modern systems is becoming increasingly complex, not only due to the associated core technology of the system, but also due to the large number of often competing requirements that the system must *simultaneously* satisfy. These requirements come from a multitude of stakeholders involved in different engineering domains [38]. This makes the process of determining component sizes harder and therefore a different approach is necessary. In order to determine a different approach it is necessary to explore the problem of component sizing in more detail, starting with understanding the importance of component sizing in design. t

1.1 Component Sizing and Architecture Exploration in Design

The process of design can be considered as problem solving involving a repeated sequence of two steps: Synthesis and Analysis. Synthesis involves the process of generating a complete specification of a system. This includes the architecture (also known as topology) as well as the sizes for the components of the system. With a complete specification available, the analysis process involves determining the extent to which the system satisfies the requirements. For instance, a dynamic simulation or

traditional machine design for a system is a type of analysis. Therefore in this context of design, component sizing is a part of the *synthesis* process in which appropriate sizes for a particular architecture are determined to enable its subsequent analysis. This is a subtle difference, mainly because the result of component sizing is a set of specifications while in analysis the result is a set of performance metrics. Therefore, during the architecture exploration phase, component sizing is an important type of analysis because it is possible to reject or not even consider a near-optimal solution due to improper component sizing methods. This is mainly due to the fact that component sizing problems are hard to solve and formulating them is also time-consuming.

1.1.1 Component Sizing is Hard to Solve and Formulate

Component sizing problems are hard to solve because of a variety of different factors, some of which are as follows. The large number of requirements imposed on the system result in multiple competing objectives, each of which must be measured, predicted or modeled by some means. In addition these competing objectives can come from multiple types of analyses, such as cost, mass, performance, or reliability, all of which need to be handled *simultaneously*. Moreover, the requirements themselves are often formulated as inequalities, such as “*The force shall be greater than $x N$* ” or “*The cost shall be less than y dollars*”. In such cases, it becomes non-trivial to find good components that satisfy all the requirements *simultaneously* and is near-optimal, i.e. it is difficult to find a better solution than the one obtained.

In addition to being hard to solve, the formulation of component sizing problems is a time consuming effort. Due to the presence of numerous inequality relations it is

often necessary to change the problem formulation based on the assumptions that have been made. For instance, a designer may use a different method to size a system given an engine specification versus sizing a system given a cylinder specification. Moreover, it is often difficult to formulate a representation that can take into account all of the aspects of the problem (multiple analyses, requirements in terms of inequalities, competing objectives).

Therefore, the goal of this research is to provide a tool that can help designers not only find “good” component sizes quickly but also help in formulating the problem during the design phase. In order to do this, it would be helpful to gain a perspective on how designers solve such problems currently.

1.1.2 Current Approaches to Component Sizing

In spite the difficulties described above, practicing designers encounter these problems often and tackle them successfully. However, this does not mean that their methods are ideal. Designers make use of the limited resources available and make tradeoffs when necessary. For instance, they may use predefined “best” practices, heuristics or spreadsheets that have been developed previously or make certain assumptions to limit the number of available choices. A designer goes through multiple iterations, mainly based on trial and error, and the solution obtained is largely dependent on the experience of the designer [35]. Such compromises are made because the process of design is ultimately one of *value*, in which a method or tool is used only if it provides value to the designer. Therefore, the question is: *How can a designer do better than the current practices described above?*

1.1.3 Constrained Optimization as a Better Way for Component Sizing

The central idea in this research is to formulate the component sizing problem in terms of a *constrained optimization* problem instead of using heuristics and assumptions related to what is known and unknown prior to solving the problem.

As described in the previous sections, component sizing is hard because of factors such as the presence of inequalities, multiple objectives, and different analyses. As a result there is no predetermined single sequence of steps that can be used to solve the equations and arrive at a solution. Consequently, the problem becomes one of optimization in which a single or multi-variable objective needs to be optimized, such as “*Find the component sizes that minimizes the total cost*”.

In particular, the class of optimization involved for component sizing is Mixed-Integer Nonlinear Constrained Global Optimization, also known as MINLP (Mixed-Integer Nonlinear Programming) problems. Component sizing falls under the nonlinear class of optimization because the models involved commonly have nonlinear relations (e.g., $F = \frac{\pi}{4}d^2p$ where F , d , p are variables). In addition, component sizing problems usually consist of a mix of continuous and discrete variables. Discrete variables arise due to the nature of the design space for the components. When making decisions at the system level, detailed component behavior models are often not available or are computationally too intensive. For instance, a system-level variable such as mass of a cylinder is dependent on the cylinder’s detailed geometry, which is unknown or too complex to model during the system-level design phase. Alternatively, system-level attribute information can be obtained from manufacturers’ catalogs, which are usually discrete in nature. This has the advantage of describing system-level information without

the need for complex low-level parametric relations but at the same time makes it harder to solve as compared to using purely continuous variables [41].

Even without the discrete nature of component sizing, global nonlinear optimization problems are hard to solve [46]. Since the term “global optimization” is used throughout this thesis, an important clarification is required. The term global optimization is used purely in the context of optimizing the mathematical representation of the problem being considered and not with the entire design process. Traditional approaches for solving global optimization problems involved the use of imperative techniques based on sampling such as gradient-based, stochastic and evolutionary algorithms. However, these approaches have certain limitations when applied to the class of component sizing problems. Sampling based algorithms treat the optimization problem as a black box and therefore it is difficult for the algorithm to guarantee global optimality. Since the design space is sampled, there is always the possibility that a better solution may exist in an unsampled region. As a result, such algorithms are inefficient when dealing with situations requiring global optimization. Gradient based methods are also not applicable when dealing with discrete variables and MINLP problems. Moreover, these techniques are imperative in nature, i.e. equations consist of a left hand side representing unknown variable and right hand side representing known variables. As a result, the equations would change depending on what is assumed to be known and unknown. This makes it hard to formulate the component sizing problem, since multiple models would be needed depending on the objective being optimized.

Thus, a different approach to constrained optimization is required for component sizing.

1.1.4 Acausal Algebraic Equation Based Declarative Modeling for Constrained Optimization in Component Sizing

As discussed in the previous section, traditional optimization approaches are not ideal when dealing with MINLP problems, such as component sizing problems. Therefore, in this thesis, the use of equation-based declarative modeling is proposed for component sizing problems.

One of the benefits associated with using a declarative programming approach is the ability to describe an equation without any consideration to the order of execution of its elements. This frees a designer to create representations that are more reusable than in traditional methods. In addition, unlike traditional approaches, declarative based models are not black boxes for a solver because they provide additional problem-specific information that can be used during optimization. For instance, declarative modeling languages support operations such as symbolic manipulation, which is used to rearrange and determine the order of execution of equations at run-time. As a result, in addition to using sampling points similar to traditional approaches, declarative based solvers can make use of additional knowledge about a model. This additional knowledge can be in the form of intervals that represent the feasible bounds of a variable. Solvers can perform operations on intervals using interval arithmetic to logically determine optimal solutions. This has led to the development of algorithms such as branch-and-bound, which are better suited for global optimization as compared to traditional sampling based techniques. Moreover, these algorithms can ensure global optimality under certain assumptions, which is not possible with traditional sampling-based approaches.

Therefore, in this research, the use of declarative equation based modeling for component sizing is proposed. So the next question then is: *What kind of declarative modeling language should be used?*

Different declarative modeling languages exist depending on the type of models involved in a problem, such as dynamic or algebraic models. As the computing resources available increases, there is a trend to go towards more complex models that describe a system. To this end dynamic models, which are based on differential equations, are able to model complex time-dependent phenomena better than algebraic models, which are time-independent. However, in the case of component sizing, the use of dynamic models may prove infeasible due to certain limitations which are discussed below.

Dynamic modeling languages such as Modelica [26] are commonly used to simulate the dynamic behavior of a system *given* the complete specification of the system at an initial time. This is also known as initial-value problems. However, in component sizing, the specifications of the system are unknown and are to be determined based on the requirements imposed on the system. Therefore, in the case of dynamic models, it becomes a boundary-value problem in which the sizes (considered as variables with derivative equal to zero) are to be determined given boundary conditions in the form of requirements. This can be very time consuming due to the large number of simulations required and ensuring global optimality becomes very difficult.

Therefore, in this research, declarative algebraic models are used to represent component behavior instead of dynamic models. Since algebraic models are not time-dependent and do not contain derivative terms, component sizing can be formulated as solving a number of algebraic equations *simultaneously*, which is considerably faster than

for a similar dynamic model formulation. Moreover, the limitation of algebraic models can be overcome by performing a dynamic simulation to verify the performance once sizing has been performed with algebraic models [35].

To summarize the line of thought presented in this section, component sizing is an important part of architecture exploration. However, for a particular architecture, it is non-trivial to find “good” sizes for components that both satisfies all the requirements and is near-optimal. It is also time consuming to formulate the problem when trying to explore different scenarios for the same architecture (e.g. minimize cost, minimize mass, maximize force, etc.).

The goal of this research, therefore, is to provide designers with a capability to represent and solve component sizing problems for a given architecture more efficiently. Integrating such a method within architecture exploration would increase the *value* associated with exploring more system architectures early in the design phase, thereby increasing the likelihood of designing better systems that satisfy all of the requirements.

1.2 Research Questions and Hypotheses

The ideas presented in the previous section lead to the following research questions and hypotheses:

RQ: Is it possible for designers to represent and solve component sizing problems more efficiently?

The above question can be divided into two parts: a.) Solving component sizing problems efficiently and b.) Formulating the problem in a manner that is both convenient

to designers and can be solved using the method proposed. The answers to these two questions forms the basis for the hypotheses defended in this thesis.

H1: Through the use of mathematical programming and constraint satisfaction techniques, designers can solve component sizing problems involving algebraic models more efficiently.

Based on the discussion in the previous section, the idea is to use declarative algebraic equations to solve component sizing problems efficiently. Mathematical programming is a type of algebraic declarative language that can be used to solve mixed integer nonlinear optimization problems such as those encountered in component sizing. In addition, by using the global optimizers available in mathematical programming languages it is possible to determine sizes with a possibility of optimality.

Along with solving component sizing problems more efficiently, designers care equally, if not more, about the ease with which problems can be formulated and represented. This becomes more important as the complexity of problems increases and it is no longer feasible to manually create models that can be executed. Mathematical programming is good for solving complex algebraic models. However, it lacks the semantics necessary to represent engineering design problems in an easy-to-use manner. Therefore, a method for representing component sizing problems in a more convenient manner is developed using the Systems Modeling Language (SysML™) [33] developed by the Object Management Group (OMG). Thus, in order to increase the *value* associated with using Mathematical Programming for solving component sizing problems, the following hypothesis is also studied in this thesis:

H2: It is possible to extend traditional mathematical programming using SysML and model transformations to provide designers with improved capabilities for representing and formulating component sizing problems.

Since component sizing can be applied to many types of problems the scope of this research is limited to one application domain, which is the hydraulic systems domain. The motivation for using hydraulic systems as an application domain is provided in the next section.

1.3 Hydraulic Systems as an Example Application Domain

The term *Component Sizing Problem* is very broad in scope and can be applied to many different domains and disciplines. Therefore, in order to take the first steps towards addressing the research question proposed in the previous section, it is necessary to identify the domain over which component sizing problems will be considered.

In this thesis, the domain under consideration is the Fluid Power or Hydraulics domain. From the perspective of design automation and systems engineering, fluid power systems have an interesting characteristic in that they are circuit-like. This is because fluid power systems can easily be decomposed into a number of *modular* components that connect together to form complex systems. This modularity also ensures the presence of a consistent interface between different components, such as fluid ports. As a result, the systems can be specified in terms of independent component models that can be connected together, just as in the actual systems. These independent component models refer to two types of models: behavior models as well as selection models, such as supplier catalog information. Moreover, hydraulic systems consist of components

belonging to different domains, such as motors, engines, and cylinders. This is an important characteristic that helps broaden the scope of component sizing being considered in this thesis. The hydraulic system used in this thesis for an example application is based on a practical application of a log splitter, which is discussed in Chapter 4.

1.4 Thesis Organization

The remainder of this thesis is organized as follows:

In the next chapter, related work is reviewed and the problem background is provided. This includes related literature on solving of component sizing problems as well as literature on representing constraint satisfaction problems (CSPs). Based on this related work, the use of CSP-based formulation for component sizing is discussed. Thereafter, an introduction to the mathematical programming language GAMS and general modeling language SysML is provided.

In Chapter 3 the framework for component sizing is described, in which mathematical programming is extended using SysML. The framework is based on the use of domain specific languages, metamodels and model transformations to automatically generate executable GAMS code from SysML models.

This framework is then applied to an example application for a hydraulic log splitter in Chapter 4. This chapter details the process of representing the problem in SysML and its subsequent solution using GAMS. The results obtained for different scenarios are then presented.

Finally, in Chapter 5 the research questions and hypotheses are reviewed along with a discussion about the research contributions, limitations and future work.

CHAPTER 2

RELATED WORK & PROBLEM BACKGROUND

This chapter provides a review of the underlying principles along with a discussion on the related work that is applicable for this research. A basic premise of this research is the use of Mathematical Programming for solving component sizing problems and use of SysML for its representation. Therefore, related work in solving component sizing problems as well as for representing Constraint Satisfaction Problems (CSPs) is discussed. Thereafter, the motivation for describing component sizing problems in terms of CSP and based on mathematical programming is discussed. Finally, a brief introductory background regarding the use of GAMS (General Algebraic Modeling System) and SysML (Systems Modeling Language) is provided to familiarize the reader with concepts that will be used in the framework presented in this thesis.

2.1 Related Work on Solving Component Sizing Problems

The following is a review of other literature related to the solving of problems related to component sizing of systems. The focus of this research is to develop an automated tool for component sizing; therefore, two main approaches are reviewed: knowledge-based engineering (KBE) efforts and efforts based on Constraint Programming (or CSP).

In addition to the research in automated sizing of systems, a more conventional approach known as the *Component Sizing Procedure* is also commonly used in industry, in which pre-defined procedures are used to guide the designer in selecting a particular

component. For instance, companies like Sauer-Danfoss [43] and Eaton [12] publish manuals that provide procedures for selecting a particular component based on assumptions made regarding loading, performance, life requirements, etc. A disadvantage of such procedures is that they limit the designer's ability to experiment with different alternatives by forcing the designer to assume certain starting values for variables and check the feasibility of the system. For instance, a designer may be required to start with assumptions on the engine output and then sequentially size the remaining components of the system. Another disadvantage with such procedures is that they are company dependent i.e., a manual from Eaton uses components by Eaton only and therefore mixing components from multiple manufacturers can be difficult to implement in the form of a procedure.

2.1.1 Knowledge-Based Engineering Efforts

The idea of automating design tasks and capturing knowledge through computers gained momentum through the use of Knowledge-Based Engineering (KBE) in the 1980s with the advent of artificial intelligence and expert systems [7, 11]. These efforts were characterized by two main features:

- a. Use of detailed design knowledge, and
- b. Heuristics for sizing.

For instance, this initial effort was strongly focused on the generation of geometry during the detailed design phase, which resulted in a variety of commercial software based around CAD tools, such as Knowledge Fusion (part of NX by Siemens PLM) or KnowledgeWare (part of Catia by Dassault Systèmes). These tools were typically add-

ons to existing mechanical CAD tools and most often required low-level design knowledge that involved using relationships based on physical principles (e.g., modeling mass based on complex relations between material properties and detailed geometry). Concurrently, in the hydraulics domain, a few efforts toward KBE have been reported in the literature [9, 10, 18, 22, 42, 48]. In particular, da Silva developed an expert system for configuring hydraulic components based on a high-level characterization of loading conditions [9]. This expert system is entirely rule-based and does not involve any analysis models. Its heuristics can identify a reasonable configuration among the known hydraulic circuit configurations, but does not attempt any component-level or system-level optimization.

The framework presented in this research differs from the above mentioned approaches in two distinct areas, namely:

- a. The use of tradeoff models [24] instead of low-level models that rely on physical principles
- b. The use of analysis models instead of heuristics

Low-level models are used to establish relations between the sizing attributes of components, such as maximum power output, cost or mass. However, such low-level models are not usually available during system-level decision making. As an alternative, tradeoff models that consist of discrete observational data from existing components (supplier catalogs) are used. By definition, a tradeoff model is an “abstract representation of a system in terms of a predictive relationship between its top-level attributes” [24]. Therefore, discrete component data is utilized to establish system-level relations between component attributes.

In addition to tradeoff models, analysis models are used in place of heuristics. In the context of this research, the analysis models consist of algebraic equations that relate to a model's performance as well as the physical laws that it must obey at component interfaces where energy flow takes place. For instance, in the electrical domain this refers to the two Kirchhoff Laws, in which the potentials between two connections are equal and the currents flowing in and out of each connector sum-to-zero. These principles of equality and sum-to-zero are found in almost all domains in which some kind of energy flow takes place between components.

Consequently, it is possible to define self-contained analysis models that can be connected together to form larger systems. The approach for automating this connection behavior is similar to approaches used in Modelica [16, 26] (a modeling language for dynamic simulations of energy-based systems). In addition to these characteristics, the framework proposed in this research relies on principles used in solving CSPs and so the next section reviews related literature that utilizes CSP-based approaches.

2.1.2 Constraint-Satisfaction Problem (CSP) Based Approaches

The analysis models described in the previous section consist of constraints or equations over variables, which must be satisfied simultaneously in order for the selection of components to be valid. As discussed in Section 2.3, the resulting system model consisting of a number of components can be treated as a Constraint Satisfaction Problem (CSP). CSPs have been commonly used in many different areas, such as artificial intelligence, operations research, engineering design, and computer science since the 1960's [37]. Moreover, algorithms to solve such problems have also been in

development and have become increasingly powerful at solving problems belonging to a wide variety of domains [14]. Based on the type of variables, constraints or domains encountered, CSPs can be classified as: discrete (integer and boolean), continuous (real), linear, nonlinear, finite and infinite bounded. In the field of engineering and engineering design the most common type of CSP encountered is the mixed-integer nonlinear type, consisting of a combination of integer, real and boolean variables along with both linear and nonlinear constraints. In the literature, the use of CSP for engineering design has been reported by Chenouard *et al.* [6], O'Sullivan [31], Wielinga [49] and others. Depending on the type of CSP, different solvers are available. Table 1 includes a comparison of some commonly used CSP tools. Continuous constraint support is a must for engineering problems due to the presence of continuous variables and non-linear constraints.

It is clear from the related literature that CSP techniques are a powerful tool for solving problems and component sizing problems clearly fit within the framework of CSP-based modeling. However, the current implementations for CSP are limited in their ability to effectively represent the engineering problem to be solved. Therefore in the next section, literature related to the representation of CSPs is reviewed.

Table 1 Comparison of different types of CSP solvers

Solver	Continuous Constraint Support	Math based Syntax	OS	Modeling Language	License	Development Status
BARON [40]	Y	Y	Windows / Unix	GAMS	Commercial	Current (by GAMS & BARON)
Choco ¹	Y	N	Independent	Java	Open Source	Current
Elisa ² (Gaol ³ , Mathlib)	Y	N	Linux / GCC	C++	Open Source	2005 (Gaol: 2008)
GlobSol ⁴	Y	N	Windows	FORTRAN	Boost License	2003
Ilog ⁵	Y	Y	Windows	Multiple	Commercial	Current (by IBM)
RealPaver [20]	Y	N	Linux / GCC	C++	Open Source	2004
JaCoP ⁶	N	N	Independent	Java	Open Source	Current
Koalog ⁷	N	N	Independent	Java	Commercial	Current

1 <http://www.emn.fr/z-info/choco-solver/index.html>

2 <http://sourceforge.net/projects/elisa/>

3 <http://sourceforge.net/projects/gaol/>

4 <http://interval.louisiana.edu/GlobSol/>

5 <http://www-01.ibm.com/software/websphere/products/optimization/>

6 <http://jacop.osolpro.com/>

7 <http://www.koalog.com>

2.2 Related Work on Representing CSPs

A common feature for representing CSPs and declarative programming involves the separation between *defining* the problem to be solved and specifying *how* to solve it. Towards this, there has been a recent trend in the CSP tools described in the previous section to separate the process of defining and solving problem, such as in Zinc [28] for discrete CSPs or in GAMS [19] for continuous optimization problems (and CSPs). However, there is close to limited or no support for object-oriented representation of CSPs, even if the tool itself is encoded in an object-oriented language such as Java or

C++. This is a major limitation when dealing with engineering systems due to their hierarchical nature, in which systems can be decomposed into multiple levels of subsystems and modular units.

As an example of research in this direction, Chenouard *et al.* have developed a custom implementation (s-COMMA GUI) that allows users to graphically define constraint models [5]. However, some of its features limits its use in engineering design, such as limited support for defining continuous CSPs and limited support for continuous CSP solvers (only RealPaver is currently supported), as well as a custom user interface in which only constraint models can be defined. Another example is the development of ASCEND [36], which is an object-oriented mathematical modeling system used mainly for chemical process modeling. A common limitation of these tools is the difficulty involved in integrating the custom representations within other tools that are used in the design process.

Based on the related work, it is clear that component sizing problems fit within the framework of CSPs. The use of CSP formulation approach that is based on mathematical programming is discussed in the next section.

2.3 Component Sizing as a Constraint Satisfaction Problem (CSP)

The process of component sizing for a particular architecture can be viewed as a two-part process. First, constraints are specified to limit a designer's selection and then different alternatives that satisfy *all* of these constraints are explored, with the best alternative being the solution [21]. There are different types of constraints associated with

the sizing of a system, such as behavioral constraints (fundamental physical laws) and selection constraints (catalog information) that are not controllable by the designer, as well as requirements and objectives that reflect a designer's preferences and goals. When taken together, component sizing becomes a constrained optimization problem, which can be solved in various ways. The approach taken in this thesis is to model component sizing in terms of a Constraint Satisfaction Problem (CSP), which is based on Mathematical Programming principles. The motivation for using Mathematical Programming is discussed in the next section.

Motivation for Mathematical Programming:

There are two main approaches to defining and solving constrained optimization problems: a declarative and imperative approach. Imperative programming is based on explicitly specifying the *sequence of statements* necessary to model a problem. For instance, a designer may define a model to calculate the force produced by a cylinder. The model would take certain inputs such as pressure and return an output force. This same model cannot be used to determine the pressure required to generate some known force. From a designer's perspective, the same model for a cylinder should be able to calculate force given pressure as well as pressure given force. Thus, imperative programming limits the expressivity of a designer, because multiple models are needed depending on what is known and unknown. This influences the way designers solve optimization problems. For instance when using *fmincon* [25], a non-linear optimizer in MATLAB, a designer is required to specify non-linear constraints and objectives in terms of functions with predetermined causality, i.e. a left- and right-hand side with inputs and

outputs respectively (Figure 1). Therefore, in order to use the imperative programming approach designers must make some assumptions regarding what is known and unknown at the time of execution, a valid assumption in certain cases.

$$\begin{aligned}
 & \min_x f(x) \text{ subject to} \\
 & \quad c(x) \leq 0 \\
 & \quad ceq(x) = 0 \\
 & \quad A \cdot x \leq b \\
 & \quad Aeq \cdot x = beq \\
 & \quad lb \leq x \leq ub \\
 & \text{where the RHS is constant}
 \end{aligned} \tag{1}$$

Figure 1 Imperative programming based approach for nonlinear optimization. The optimizer referred to above is *fmincon*, a nonlinear optimizer in MATLAB [25]

During component sizing, however, this is a difficult assumption to make since the different components of a system are coupled while at the same time are independent. For instance, the behavior of a pump, engine and cylinder can be modeled independently but their selection is coupled at the system-level. The force requirement on cylinder influences pressure requirements in the circuit, thereby influencing torque requirements on the pump which ultimately affects the output of the engine. Thus a different approach is required when it is not possible to identify what is known and unknown, and *declarative programming* is an approach that can handle such situations.

Declarative programming, in contrast to imperative programming, involves specifying properties of a valid solution for a problem instead of specifying *how* to solve it. From the perspective of component sizing designers can specify constraints on variables without any mention of inputs or outputs. The *acausal* nature of constraints allows the designer to experiment with different objectives without changing the models.

For instance, consider a constraint for a cylinder in which force is related to bore diameter and pressure (see Table 2). In the imperative approach, the constraint would be

formulated differently depending on what the designer assumes as input and output. Declarative equations, on the other hand, specify a relation and impose causality only at the time of solving. Consequently, it is possible to use the same equation for different problem formulations.

Table 2 Imperative versus Declarative Implementation of a constraint

Constraint formulated by designer	Implementation of Constraint in a Solver			
	Imperative Approach (MATLAB, C, Java)		Declarative Approach (Mathematical Programming)	
$F = p \cdot \frac{\pi}{4} \cdot d^2$	Output: F	$F = p \cdot \frac{\pi}{4} \cdot d^2$	Output: F, p or d	$F = p \cdot \frac{\pi}{4} \cdot d^2$
	Output: p	$p = F \cdot \frac{4}{\pi} \cdot \frac{1}{d^2}$		
	Output: d	$d = \sqrt{F \cdot \frac{4}{\pi} \cdot \frac{1}{p}}$		

Therefore in this research, a mathematical programming approach, which is based on the declarative programming, is used to formulate and solve component sizing problems. In mathematical programming, by modeling variables and constraints over these variables, a variable is optimized as opposed to an objective function (see Figure 2). This means that there is no restriction on the way constraints are formulated. As a result, issues of causality are taken care of during runtime by the solver.

$$\min v_j \text{ given } f_1, f_2, f_3, \dots \text{ and variables } v_1, v_2, v_3, \dots, v_n$$

$$\text{where } c_1, c_2, c_3, \dots \text{ are constraints (linear/nonlinear) of the form} \quad (2)$$

$$f(v) \leq g(v), f(v) = g(v), \text{ or } f(v) \geq g(v)$$

Figure 2 Optimization form for mathematical programming. Constraints represent the behavior of a model and not how to solve it. Symbolic manipulation is performed at runtime to determine order of execution of equations.

In particular the mathematical programming language GAMS (General Algebraic Modeling System) is used in this research and a brief discussion of the relevant features of GAMS is discussed in the next section.

2.3.1 Using Solvers in GAMS for Solving Component Sizing Problems

A benefit of using CSPs to solve problems is that a designer can specify a problem without needing to specify *how* to solve it. This relates to the concept of separation between modeling and solving of a problem. Such concepts have been popular in mathematics and operations research, in which the same model can be solved by a number of different solvers. In engineering, however, the trend has been to define specialized solving techniques that are tailored for a particular problem [17].

Traditional approaches included the use of sampling-based techniques such as stochastic, gradient-based or evolutionary algorithms to find solutions. These approaches are imperative in nature, i.e. the model may change depending on the assumptions imposed regarding the knowns and unknowns in the problem [6]. In addition, engineering problems typically consist of a combination of continuous and discrete variables as well as linear and non-linear constraints. One such example is the combination of continuous variables such as force and integer variables such as number of gear teeth or a variable used to select a gear out of a set of potential gears from a supplier catalog. Such problems are commonly known as MINLP (Mixed-Integer Non Linear Programming) problems and are a special type of CSPs that use specialized algorithms based on interval arithmetic and branch-and-bound frameworks [2], such as those included within the algebraic modeling environment known as GAMS [19].

In this research, the solver used is BARON (Branch and Reduce Optimization Navigator), which is available within the GAMS (General Algebraic Modeling System) language. BARON is a global optimization solver that can be used to solve purely continuous nonlinear programs (NLP), purely integer, and mixed-integer nonlinear programs (MINLP) [40]. According to a comparison carried out by Neumaier *et al.*, BARON is the fastest and most robust global optimization solver among available global solvers [29].

GAMS (General Algebraic Modeling System) is a high-level modeling language for mathematical programming and optimization. According to [19], GAMS is intended for “*complex, large scale modeling applications, and allows [a designer] to build large maintainable models that can be adapted quickly to new situations.*” To this end, GAMS consists of a modeling language and a number of integrated solvers which can be changed according to the type of problem (LP, NLP, MINLP, etc.). GAMS models consist of purely algebraic statements, which is compatible with this research’s use of algebraic constraints for modeling component sizing problems.

Although GAMS is suitable for representing constraints declaratively, it is limited in its ability to describe engineering systems. This is discussed in the next section.

2.3.2 Limitations of GAMS for Representing Component Sizing Problems

GAMS is a text-based language with semantics based on the characteristics of optimization problems typically found in Operations Research. One of the main features of GAMS is the separation between the characteristics of a problem and the data that it uses. For instance, in a transportation problem the model is defined independently of the

size of the supply and demand. This is similarly found in component sizing problems, in which the same model can be used irrespective of the number of potential catalog components being considered. To facilitate such modeling, the common components of a GAMS model are described in Table 3. Through these components, it is clear that GAMS is well-suited for problems found in mathematical programming, such as operations research, in which problems can be described without the need for subsystems and individual components.

Table 3 The basic components of a GAMS model [3]

<p>Inputs:</p> <ul style="list-style-type: none"> • Sets: Container for elements. Represents “collections” • Data (Scalars, Parameters, Tables): Used to store constant data in one, two or multiple dimensions. • Variables: Same as traditional variables. Its value changes during the process of solving • Assignment of bounds and/or initial values (optional) • Equations: Used to define the <u>symbolic</u> algebraic relationships • Model and Solve statements: Model is used to collect equations into a group; Solve solves the set of equations included in the Model for the objective to be optimized and using the specified solver and • Display statement (optional) 	<p>Outputs:</p> <ul style="list-style-type: none"> • Echo Print • Reference Maps • Equation Listings • Status Reports • Results
--	--

However, GAMS is not well suited for describing engineering design problems due to a number of reasons, one of which is the hierarchical nature of engineered systems. Engineered systems are commonly composed of multiple levels of subsystems that ultimately consist of individual component models. In addition, there is a large amount of model reuse in engineering systems and corresponding design problems, such as reusing the same component (e.g. Cylinder) multiple times in the same circuit as well in other problems. These characteristics of design problems imply the need for an object-oriented perspective along with additional language constructs, which GAMS does not

support. In Figure 3, an example of manual creation of a GAMS model for engineering problems is shown. In particular, note the manual duplication and unique variable naming required when using the same cylinder component again.

In order to support the modeling and formulation of component sizing problems using mathematical programming, the use of the Systems Modeling Language (SysML) [33] is proposed. SysML is a general modeling language developed by the Object Management Group (OMG). Therefore, prior to discussing the proposed framework in Chapter 3, a brief introduction to SysML is provided in the next section.

```
* To use the same cylinder model twice, a copy with unique names must be created.
* There is no concept of objects, or model hierarchies, or reuse of the same model.

* Cylinder Model 1
set cylinderCatalog1 / SAE-64508, SAE-64008, HMW-5008, PMC-5608 /;
parameter boreDiameterData1 / SAE-64508 0.1143, SAE-64008 0.1016, HMW-5008 0.127, PMC-5608 0.1016 /;
variable cylinder_f1, cylinder_bore1, cylinder_rod1, cylinder_portA_p1, cylinder_portB_p1;
equation cylinder_f_eq1;
cylinder_f_eq1.. cylinder_f1 =e= Pi*0.25*( (sqr(cylinder_bore1)*cylinder_portA_p1)- cylinder_portB_p1*
(sqr(cylinder_bore1)-sqr(cylinder_rod1)) );

* Cylinder Model 2
set cylinderCatalog1 / SAE-64508, SAE-64008, HMW-5008, PMC-5608 /;
parameter boreDiameterData1 / SAE-64508 0.1143, SAE-64008 0.1016, HMW-5008 0.127, PMC-5608 0.1016 /;
variable cylinder_f1, cylinder_bore1, cylinder_rod1, cylinder_portA_p1, cylinder_portB_p1;
equation cylinder_f_eq1;
cylinder_f_eq1.. cylinder_f1 =e= Pi*0.25*( (sqr(cylinder_bore1)*cylinder_portA_p1)- cylinder_portB_p1*
(sqr(cylinder_bore1)-sqr(cylinder_rod1)) );

model m /cylinder_f_eq1, cylinder_f_eq2/;
solve m using minlp maximizing cylinder_f1;
display cylinder_f1.l, cylinder_f2.l;
```

Figure 3 Example of manual input of GAMS model for an engineering problem. There is duplication of variables and equations, making it difficult to reuse a model. Also, variable naming must be unique

2.4 Introduction to SysML

In order to familiarize the reader with the terminology used in this thesis, some general background is provided regarding the features of SysML. SysML is an extension

of the Unified Modeling Language (UML) [34], both of which have been standardized by the OMG. UML is widely used in software engineering and has been extended to support the modeling of systems of all types through SysML. The following are some of the common SysML entities used throughout this thesis. These descriptions are based on the book by Friedenthal *et al.* [15] and the SysML specification [33].

SysML Blocks:

A block is the primary modeling unit in SysML. The analogous of a block in software engineering is a class. A block can be used to represent various parts of a system, such as a process, function, model, behavior or the system itself. Blocks can be combined together to form subsystems and systems that collectively describe the problem being modeled. In addition, blocks can contain other entities like properties and ports to describe the problem in more detail. Thus, blocks provide a modular way for a designer to represent the system in a decomposable manner.

SysML Properties:

SysML properties are an extension of UML properties and can be classified as value properties and part properties. Value properties are commonly used to specify variables while part properties are used to define local usages for a block within another block. This is similar to the concept of class *definition* versus class *usage* in object-oriented programming. This translates well for component sizing problems, in which systems can be decomposed using part properties and variables can be modeled using value properties.

SysML Flow Ports:

In order to enable model reuse in SysML, ports are used to clearly define the interfaces through which information can be exchanged [33]. By connecting together the ports of different blocks, it is possible to model the *flow* between various parts of the system. Depending on the system being modeled, the concept of what *flows* can be different such as energy flow between components in energy-based system models.

Stereotypes & Profiles:

In order to customize SysML for a specific domain such as GAMS or fluid power, UML (and SysML) provide a construct known as a *stereotype* that can be used to extend existing SysML constructs like blocks and properties. A stereotype is more precise than the existing SysML entities. Stereotypes are organized within *profiles*, which represent a collection of customizations for a specific domain or application.

2.5 Summary

Through a review of literature related to solving of component sizing problems and their representation, it is clear that the CSP approach is a powerful solving technique that can be used in a variety of problems. In particular, component sizing problems clearly fit within the framework of CSP-based solving. However, limitations in the expressivity of the current modeling capabilities of CSP tools such as GAMS limits it from being used to solve problems in engineering design such as component sizing.

Consequently, the framework proposed in this thesis involves using a general modeling language such as SysML to extend the current modeling formalisms of GAMS in order to support a more efficient representation of component sizing problems. This framework is described in the next chapter.

CHAPTER 3

EXTENDING MATHEMATICAL PROGRAMMING USING SysML

In this chapter, the framework for representing and solving component sizing problems more efficiently using SysML and GAMS is presented. In this framework, SysML is used to extend current mathematical programming formalisms of GAMS in order to provide a designer with improved capabilities for modeling the component sizing problem to be solved. The actual solving of the problem is still done using the integrated solvers included in GAMS. In this research, the solver BARON is used to solve the problem after it has been modeled in SysML.

This process is based on using the principles of Model Driven Software Development (MDSD) [45], which is commonly used in software engineering. This process involves the specification of metamodels, domain specific languages (DSLs) and automated model transformations. In particular, the process of representing the analysis knowledge related to component sizing in a form that is convenient to designers within SysML is presented. Along with capturing this analysis knowledge, the process of transforming such a representation into a form that can be solved by external solvers in GAMS is also discussed. In order to use the advantages of both languages (SysML and GAMS), a combination of DSLs and model transformations are used to create a consistent representation in both languages. The approach presented in this thesis involves the following steps:

1. Formal Capture of GAMS Domain Using Metamodels.
2. Representing GAMS Compliant Models in SysML using Profiles
3. Model Transformations to Support Hierarchical Object-Oriented Modeling

Each step is discussed in the following sections.

3.1 Formal Capture of GAMS Domain Using Metamodels

In order to provide designers with improved capabilities for representing component sizing problems, SysML is used as a formal object-oriented modeling language, which can then be passed to GAMS and subsequently solved. This requires a different approach than when done in a single tool, e.g. entirely in GAMS. In a single tool, this process would be done through internal data-models (*language compilers*) that are customized for the particular software tool, such as the source code for GAMS. In order to integrate multiple tools a common metamodel is used, which describes the concepts that can appear in a valid model as well as represents the links between these concepts, such as inheritance and composition. To support such model and metamodel driven systems, the OMG established the Meta Object Facility (MOF) standard, which provides a framework for “defining, manipulating, and integrating meta-data and data in a platform independent manner” [23, 32]. A metamodel represents the *abstract syntax* for a domain, since the relations are defined using classes and associations that are independent of any particular encoding.

To extend the functionality supported by GAMS, the approach taken is to convert the implicit metamodel for GAMS (i.e., the data structures used internally – refer Table 3) into a formal and explicit metamodel compliant with the MOF standard. In addition, existing constructs are extended through additional features such as object-oriented modeling that are added to the metamodel.

The GAMS metamodel is shown in Figure 4, in which the constructs in GAMS are represented as classes (`GamsSolve`, `Model`, `GamsVariable`, etc.). According to mathematical programming formalism of GAMS, the model that is passed to a solver consists only of equations and there is no concept of ownership. Moreover, a GAMS file consists of a number of variables, parameters, sets, equations, a model statement and a solve statement, and some display statements, all of which are modeled at the same level. As a result, a GAMS model is flat i.e., there is no concept of an object, ownership, or visibility (`public`, `private`). This lack of expressivity severely limits a designer's ability to describe systems in terms of modular components. To overcome this, existing GAMS constructs are mapped to different objects (similar to `class` in object-oriented programming) in the metamodel shown in Figure 4. In order to introduce the concept of ownership and hierarchical modeling, *associations* are defined between the objects such as `A_owner_ownedModels` and `A_model_variables`. This enables a designer to define and limit the scope of GAMS constructs used within models. These associations can be described as follows.

The `GamsSolve` corresponds to the `solve` statement in GAMS and it represents the top-most level in the resultant model hierarchy. Just as a `solve` statement specifies the model to solve, a `GamsSolve` object has an association `A_gamsSolve_model` which specifies that a `GamsSolve` object owns a `Model`. This is as far as the similarity between GAMS and the metamodel goes. Unlike GAMS, the metamodel allows a `Model` to own the following: other models, variables, sets, parameters and equations.

In this way, it is possible for a designer knowing GAMS syntax to define a modular class-based system with object-oriented constructs.

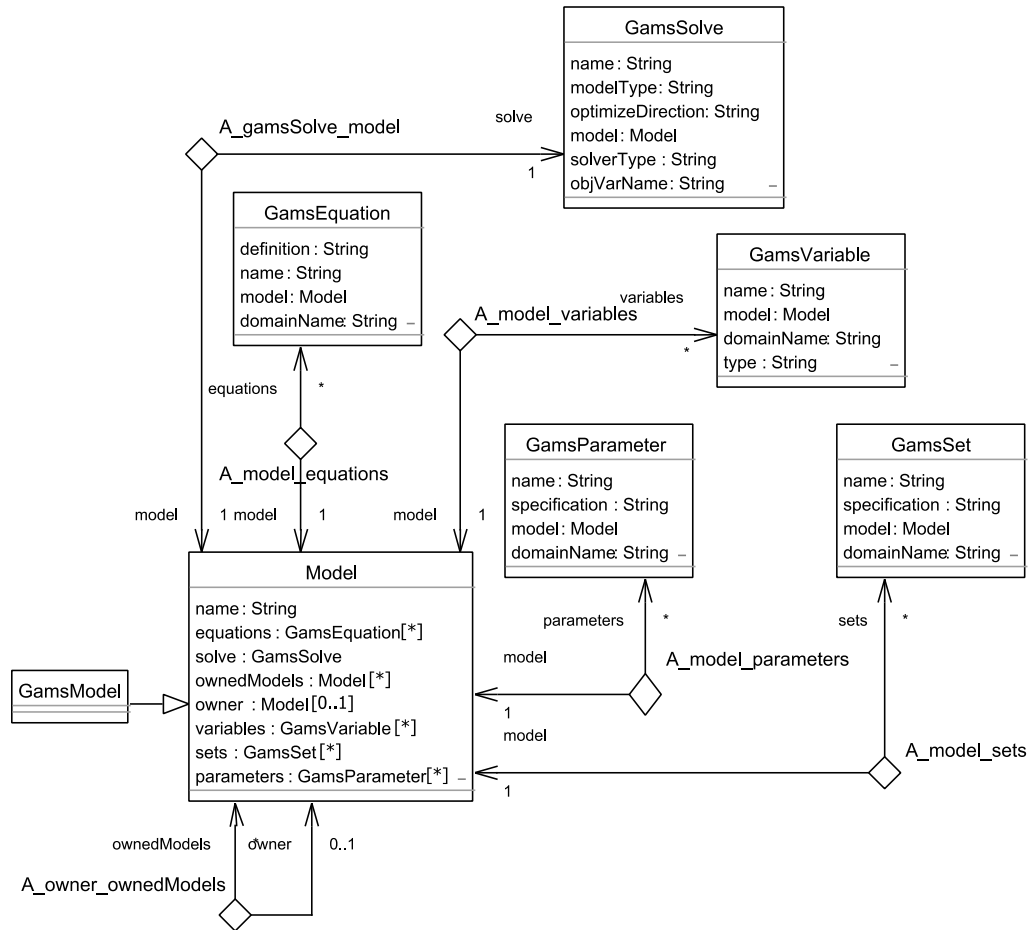


Figure 4 GAMS Metamodel Definition. Semantics Of GAMS are Represented as Objects in the Metamodel

An interesting feature of the metamodel described above is that it is not specific for component sizing problems. The abstract syntax described in this metamodel can be used to represent any GAMS model and can therefore be used for other applications as well that consist of decomposable systems.

In order to provide additional capabilities specifically for component sizing, new language constructs are added to the *concrete syntax*, in this case SysML. This involves customizing SysML through the use of profiles, and then using model transformations to

automatically generate an executable representation in GAMS. The customization of SysML is discussed in the next section.

3.2 Representing Component Sizing Problems in SysML

Since SysML is a general purpose modeling language, it lacks the detailed, formal semantics needed for representing a problem in a domain-specific way [4]. For instance, there is no SysML concept that can represent GAMS-specific semantics like `variable` or `parameter`. They could all be modeled by using the same SysML construct, such as `Property`, and using the name to represent a variable or parameter (e.g. `variable_x` as the name of a `Property` in SysML). This would lead to ambiguity at the time of converting from SysML to GAMS. In addition, the lack of precise problem-specific semantics can make it cumbersome for domain experts to create models in SysML due to the large amount of repetitive tasks involved. This can limit the acceptance of general SysML for specific domains and problems. Therefore, in order for SysML to be used for modeling a particular type of problem, the necessary semantics associated with the problem must be included within SysML through customizations.

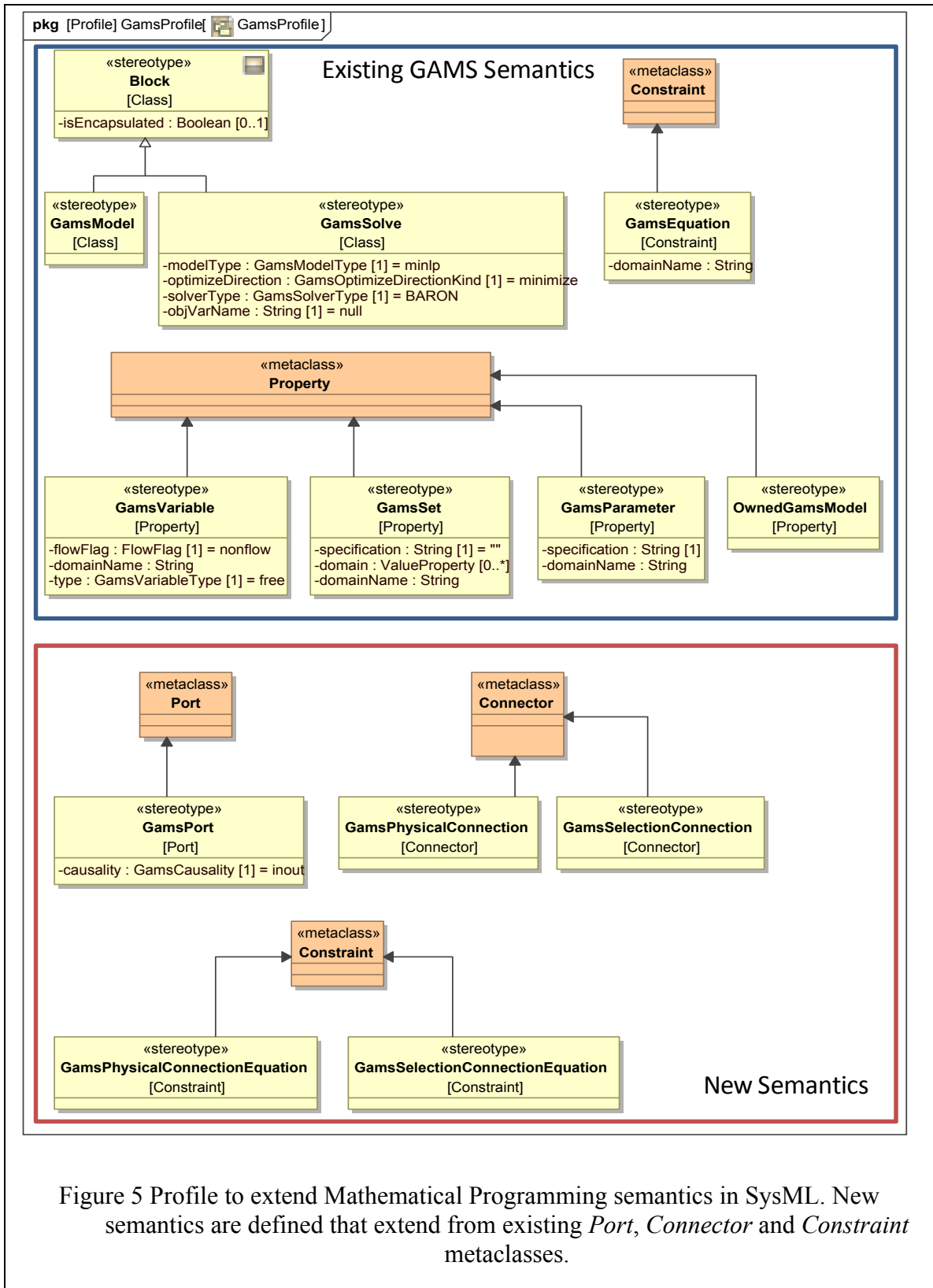
Therefore, in this section, the process of customizing SysML for component sizing is discussed. There are two parts to this: capturing existing GAMS semantics in SysML and defining new semantics that are relevant from a component sizing perspective.

3.2.1 Capturing GAMS Semantics in SysML Using Profiles

SysML (UML) provides several mechanisms for customization, such as extending the UML metamodel, creating new profiles that extend existing SysML/UML constructs or defining a completely new language. Profiles are preferred since they do not modify the underlying UML metamodel, thereby retaining existing tool support [47]. A portion of a profile created for representing component sizing problems based on GAMS semantics is shown in Figure 5.

The profile is constructed as per the MOF metamodel (Figure 4). For instance `variable`, `parameter` and `set` each have their own stereotype defined but all extend the SysML `Property` class. The GAMS construct for equation is defined by a stereotype `GamsEquation` that extends the SysML `Constraint` class.

Since both the `model` and `solve` constructs extend the SysML `Block`, all of the characteristic of a `Block` are available to objects stereotyped as `GamsModel` and `GamsSolve`. For instance, SysML supports hierarchical modeling through composition associations and this is automatically available when using `GamsModel` stereotype to create models.



3.2.2 New Constructs in SysML to Support Representation of Component Sizing Problems

Existing SysML constructs are extended using stereotypes to make it easier for a designer to define component sizing problems. These new features include support for hierarchical modeling, embedding the physics related to energy-based systems that are typically encountered during engineering design, as well as support for explicitly defining dependencies between component models and their associated selection models (supplier catalogs).

Hierarchical modeling is established through existing composition associations in SysML and this allows the designer to logically decompose a system into its individual components. It also enables a designer to store models and reuse them in multiple contexts in the same problem or across different problems. An important effect of modularization is the possibility of using *connections* to connect the components in different ways, which is similar to the process engineers use when assembling together components in the real world.

The concept of connecting components exists at multiple levels during the design process. Moreover, the connections can have different meanings depending on the context in which they are used. For instance, when connections are used to create schematics they refer to the graphical representation. For this and other situations in which connections can be used, the existing SysML constructs of `Port` and `Connector` are customized depending on the context of use. Based on the different contexts, it is possible to encode knowledge by customizing SysML in order to make it more convenient for designers to formulate component sizing problems. This reduces the

amount of repetitive and error-prone manual modeling that a designer would otherwise have to do. In particular, three types of connections are considered in this research:

1. Connections used to describe a system architecture
2. Connections used for energy-based analysis models
3. Connections used to establish relations between multiple analysis models and corresponding catalog models

In this research, a system architecture is assumed to exist and component sizing is performed on the given architecture. Therefore, the capability to model a system architecture by connecting components together is provided by customizing existing SysML constructs of `Port` and `Connector`. The additional knowledge that is encoded in `Port` and `Connector` can be used to automatically generate equations that a designer would otherwise have to manually define.

A common feature in models for energy-based systems is the existence of standard interfaces through which energy is transferred between components. This process of energy transfer can be captured in terms of equations that can be used to generate system-level models from component-level models and their connections. This logic is based on the law of energy conservation and can generally be formulated through two equations:

- Sum-to-zero equation for *flow* variables (e.g. force, flow, torque)
- Equality equation for *potential* variables (e.g. pressure, velocity, angular speed).

This is a generic logic that applies to multiple domains including fluid power, mechanics (translational / rotational), thermal, etc. Therefore, to aid the designer in

creating system-level analysis models from component models, `Port` and `Connector` are customized to encode this logic for energy transfer (refer to `<<GamsPhysicalConnection>>` stereotype in the GAMS profile in Figure 5). This allows for automatic generation of equations, which would otherwise be done manually by the designer. As a result, it is easier for designers to create new architectures and analyze them.

The final type of connection that is customized refers to the process of relating a component to its use across multiple analyses. As discussed previously, component sizing involves satisfying a number of requirements *simultaneously*. These requirements come from multiple analyses such as cost analysis, mass analysis, and hydraulic performance analysis. Different component models exist for each analysis and therefore it is necessary to ensure that ultimately the same component is referred to across all of the analyses. This is done by defining a `SystemSizingModel`, in which all of the components to be sized are included, and then explicitly defining connections between each component in the `SystemSizingModel` and its usage in each analysis model. The `Connector` class is extended through the `<<GamsSelectionConnection>>` stereotype. The corresponding logic to be encoded involves the creation of equality constraints between each variable in the component model in `SystemSizingModel` and its corresponding usage in an analysis.

Thus, by defining *current* GAMS semantics along with *new* constructs related to component sizing problems in SysML, a designer is provided with additional capabilities to represent component sizing problems. Moreover, the combination of profiles and

metamodels provides the framework in which model transformations can be used to define the logic described above in order to automatically generate models that can be solved in GAMS. This is discussed in the next section.

3.3 Model Transformations to Support Hierarchical Object-Oriented Modeling

As discussed in the previous sections, a combination of profiles and metamodels are used to provide new capabilities to designers for representing component sizing problems in SysML and solving them by solvers in GAMS. In order to encode the logic behind these new constructs (such as <<GamsPhysicalConnections>>) as well as generate GAMS-compliant executable code, model transformations are used to automatically perform these tasks.

Model transformations are used to convert the SysML model into an intermediate object-oriented GAMS model, which is then transformed into a flat executable model that can be solved within GAMS. Since the domain MOF metamodel and SysML profile can be described in terms of graphs [1], model transformations can be defined in which the domain semantics (metamodel and profile objects) represent the nodes, and associations between objects represent the edges. As is shown in Figure 6, the transformations are defined in a declarative fashion at the meta-model level and are then compiled into an executable form that operate at the user-model level.

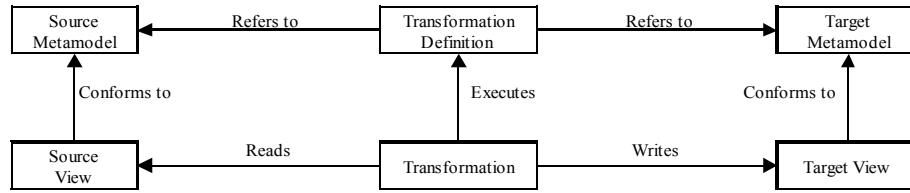
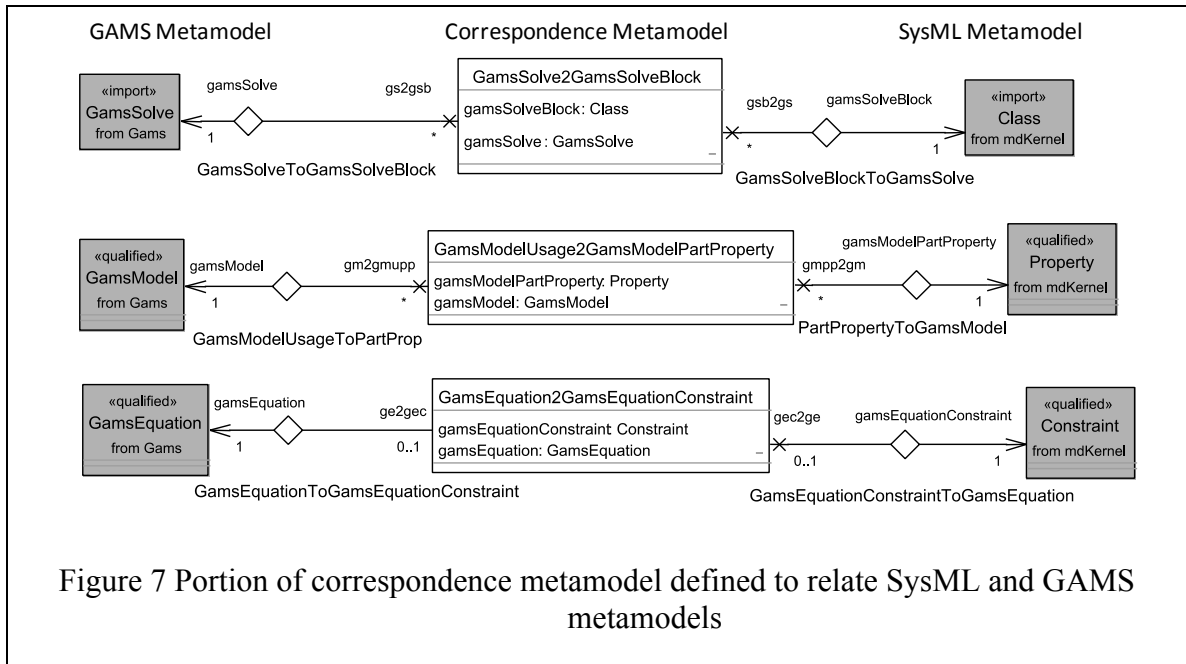


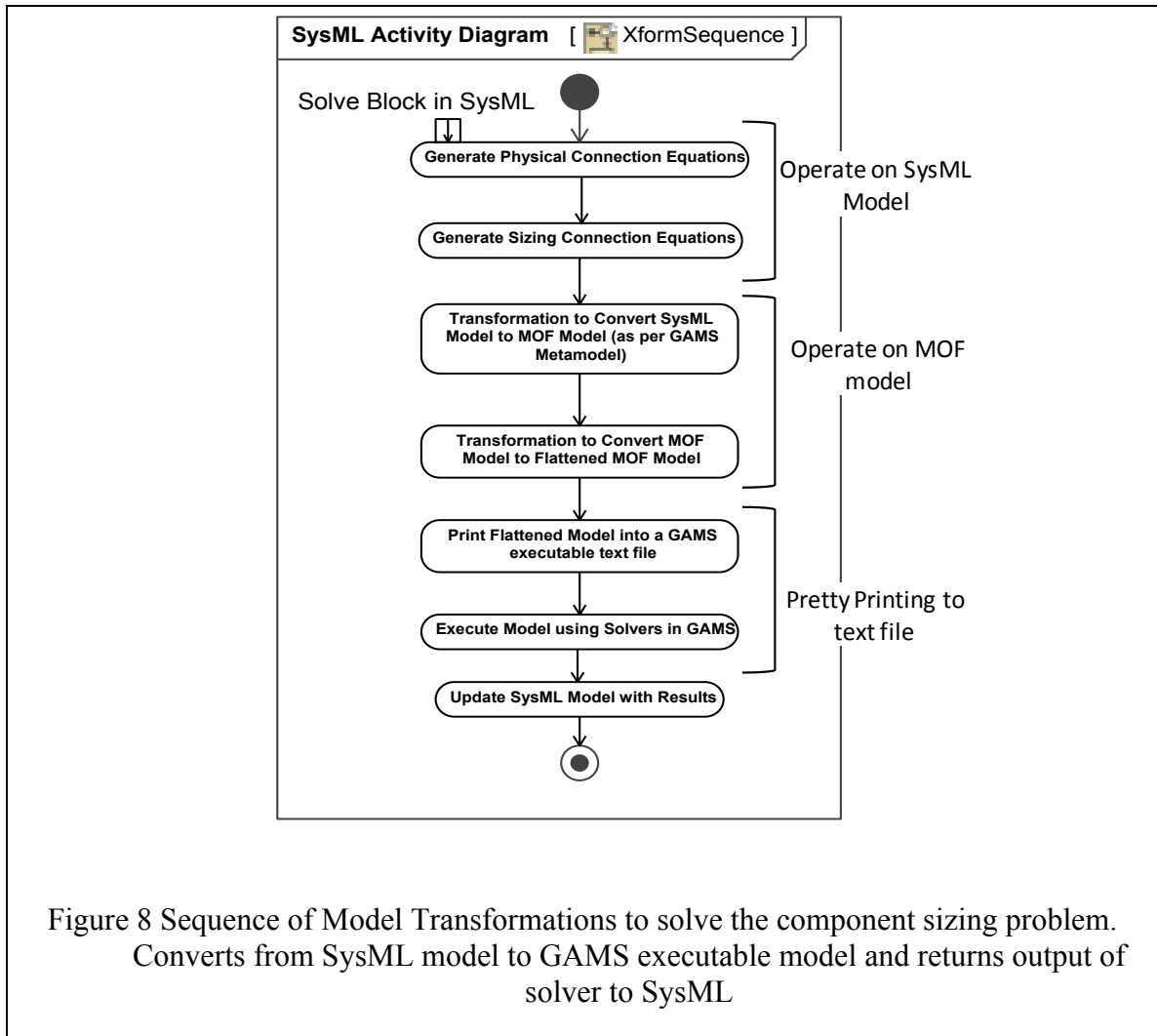
Figure 6 Process Of Model Transformation from Source to Target Model (Czarnecki *et al.* [8])

A correspondence metamodel is used to maintain relations between the elements of the input SysML model and the resulting GAMS model [23, 44]. This is necessary when retrieving information from the solver's output to update the SysML model. An example of a correspondence link is shown in the correspondence metamodel in Figure 7. It involves the use of object `gmu2gmpp` of type `GamsModelUsage2GamsModelPartProperty` to link a `topLevelGamsModelPartProp` object of type `Property` in SysML to a `topLevelGamsModel` object of type `GamsModel` in the GAMS metamodel (refer to the GAMS metamodel in Figure 4).



Model transformations are then defined using the correspondence metamodel to relate elements of the source and target views with one another. The transformations are written in a declarative and graphical manner through the use of story diagrams [13]. The model transformations used in this research are defined in MOFLON [27], which automatically generates Java Metadata Interface (JMI) code that implements the transformations in Java. This JMI code is then combined with a JMI-compliant SysML tool, such as Magic Draw [30], in the form of a plugin that can be executed from within SysML.

The sequence of model transformations executed to solve a component sizing problem described in SysML using solvers in GAMS is shown in a SysML activity diagram (Figure 8). Each action represents a transformation that is performed.



The transformation that converts an input SysML model of the problem into an intermediate representation based on the MOF metamodel is shown in Figure 9. One such transformation is shown in, in which the input is a SysML block (stereotyped with `GamsSolve`) that contains solver information and the model to be solved. The output of this model transformation is the creation of an equivalent model based on the GAMS metamodel that was defined previously. The mechanism for transformation involves matching a pattern (left-hand side) and applying a replacement pattern (right-hand side) if successful. The pattern to be matched is defined in black and the replacement pattern is shown in green (with `<<create>>` tag visible).

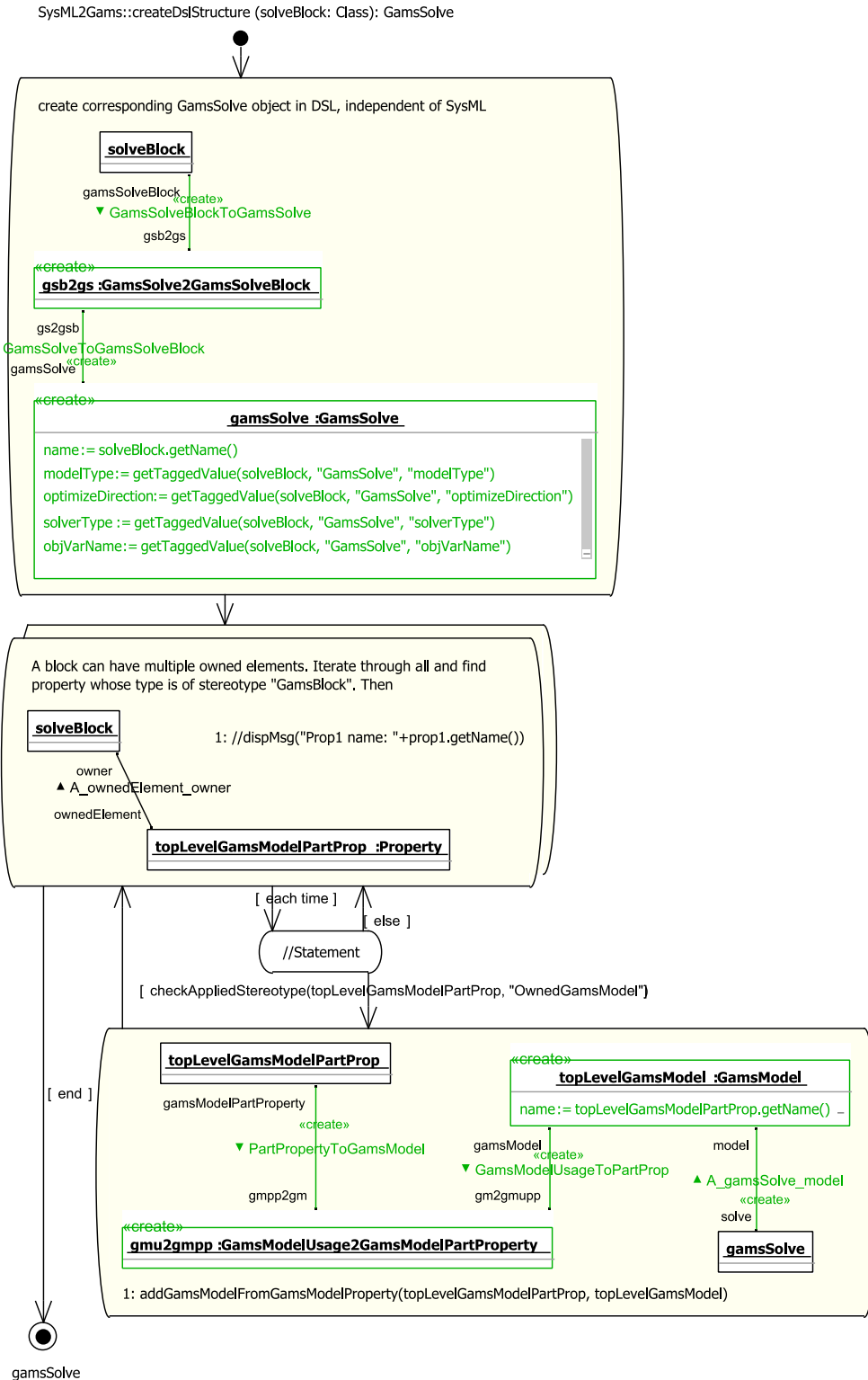


Figure 9 Model Transformation to convert SysML model to MOF model (as per GAMS Metamodel)

3.4 Summary

In this chapter, a framework using Model-Driven Software Development concepts is presented for representing component sizing problems in SysML and their solving in GAMS. This framework provides improved capabilities to designers for more efficiently formulating component sizing problems in a hierarchical, model-based manner, thereby addressing the limitations of current tools in terms of expressivity. Moreover, the use of SysML enables this framework to be integrated within larger frameworks of design, such as Model-Based Systems Engineering. In the next chapter, an example application is presented which explores the use of this framework for representing and solving component sizing problems more efficiently.

CHAPTER 4

EXAMPLE APPLICATION: COMPONENT SIZING FOR A HYDRAULIC LOG SPLITTER

In this chapter, an example application involving the sizing of a hydraulic log splitter is presented in order to apply the framework presented in this research. First, the motivation for the use of fluid power as an example domain is provided. Thereafter, the modeling of the problem in SysML using the proposed framework is discussed. Finally, results obtained from solving the problem under different scenarios are presented.

4.1 Problem Description and Motivation for Fluid Power

To validate the framework presented in the previous chapter for automated component sizing, it is applied to an example belonging to the fluid power domain. The use of fluid power as an example domain for component sizing was discussed in Section 1.3. To summarize, some of the desirable characteristics of the fluid power domain include: systems are circuit-like in that they can be modularized and connected together, well defined interfaces exist between components, and there is large amount of catalog data available for different components which makes component sizing problems combinatorially hard to solve. .

The hydraulic system considered in this example application is that of a horizontal acting hydraulic log splitter (Figure 10). A log splitter is a system used to divide roughly cylindrical pieces of wood into two or more pieces, generally longitudinally along the grain of the wood. An operator loads a piece of wood (of varying length) and actuates a

control to drive a wedge along the grain of the wood. Log splitters are usually portable and so the critical requirements include the ram force available to split the log, total cycle time involved, total mass, and total cost of the machine. These attributes represent competing objectives, out of which the designer must make tradeoffs to find a specification that satisfies all of the requirements simultaneously.

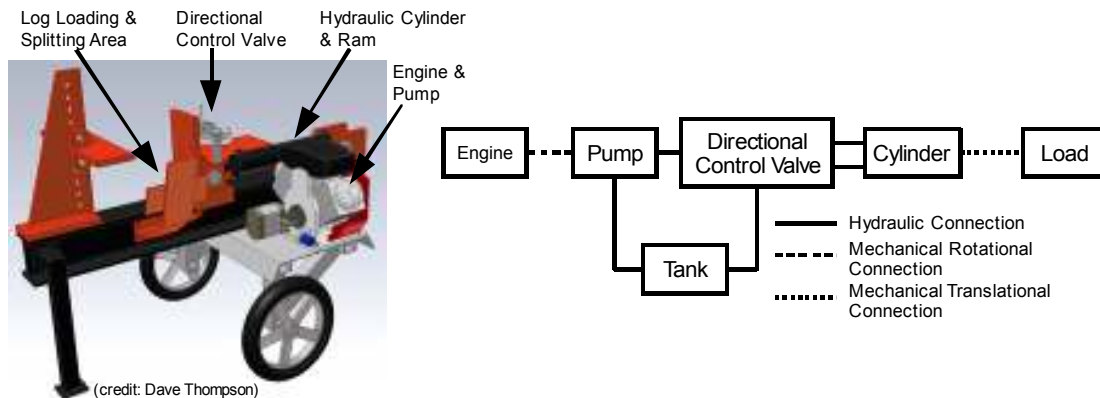


Figure 10 An assembly and block diagram for a horizontal acting hydraulic log splitter

The scope of component sizing is limited to the hydraulic subsystem; the mechanical structure is not considered. In Figure 10, a block diagram of the log splitter architecture considered in this example application is shown. There are different system architectures that can be used, for instance open center circuit with constant pump displacement versus closed center circuit with variable pump displacement. In this example, the open-center circuit is used.

The components that are considered include: a gas engine, hydraulic fixed displacement pump, directional control valve, double acting cylinder, load and tank. Since the system is horizontal, only a horizontal load requirement is considered. As discussed in Chapter 1, this example is restricted to a single architecture for which component sizing is to be performed.

The log splitter problem is a valid example for the proposed framework because it possesses characteristics that belong to larger and more complex models. This includes the presence of multiple types of interfaces (hydraulic, translational, rotational), competing objectives (minimize cost versus maximize force), as well as multiple types of analyses (cost, mass, fluid power performance). In this example, four components – engine, pump, cylinder and valve – are considered for sizing, and their possible sizes are taken from component catalog information that has been obtained from industrial component manufacturers.

In the following sections, the modeling of the log splitter in SysML and its subsequent solving in GAMS is discussed.

4.2 Modeling the Log Splitter Problem in SysML

As described in the previous section, the log splitter possesses different aspects that are commonly found in component sizing problems. From a designer's perspective, the modeling steps that would be involved are as follows.

The first step involves the specification of requirements and their relation to a particular component or variable. Since a single architecture is considered, the descriptive modeling step is not considered in which the architecture to be sized is modeled. This is discussed more in the Future Work section in the next chapter. Thereafter, the modeling of hydraulic performance is considered in which energy-based modeling principles are used. As part of the hydraulic performance analysis, the designer needs to consider multiple use-phases based on the problem requirements. After modeling the hydraulic performance with multiple use phases, other types of analyses are modeled. Finally, in

order to ensure that all of the analyses and use phases are considered simultaneously the designer needs to ensure that a common sizing for each component is used throughout the entire model and is associated with one catalog model.

In order to evaluate the usefulness of the proposed framework, each aspect is discussed separately in the same order that the designer would approach the modeling task without a framework.

4.2.1 Requirements Modeling in SysML

One of the features of SysML is the ability to model requirements and assign dependencies between requirements and model elements. In the case of component sizing, requirements modeling is the first step for guiding the designer in defining the composition of the system in terms of relevant analyses as well as the mathematical constraints associated with the requirements. In Figure 11, a SysML requirements model is shown in which requirements are decomposed hierarchically until they can be described mathematically. For instance, the forward phase for the hydraulic analysis consists of two requirements: a.) The force produced by the cylinder should be greater than a specified limit and b.) The maximum pressure in the circuit should be less than the specified max pressure. In this way, requirements modeling helps to derive mathematical constraints that are then included in the different SysML analysis models, which is described in the next section.

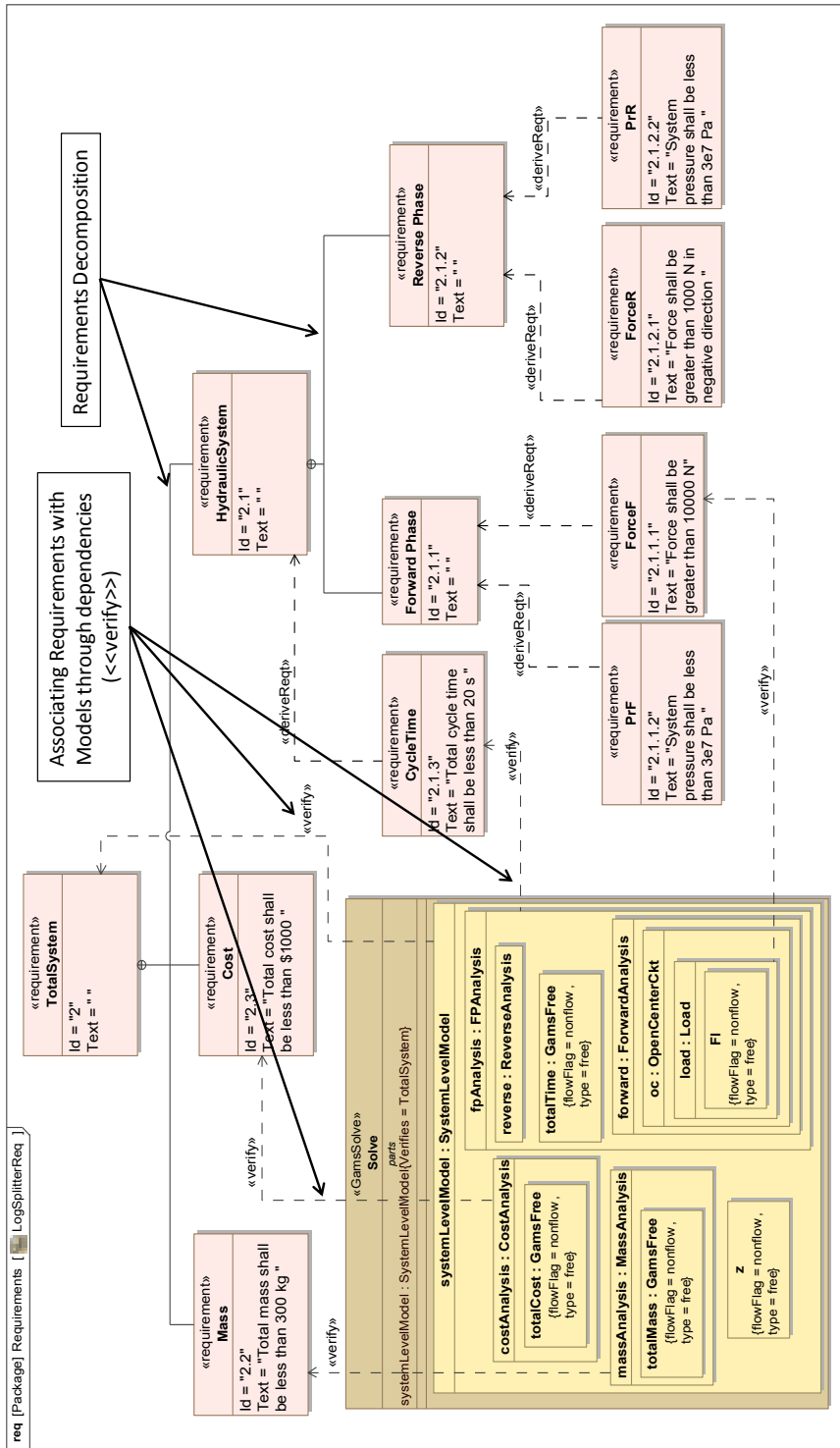


Figure 11 SysML model for requirements and associating them with corresponding component models through dependencies (<<verify>>). Requirements modeling helps to decompose the problem into different analyses and use phases.

Before discussing the details of the SysML models used to formulate the sizing problem, the overall system model is shown in Figure 12. The details of these models (such as constraints) have been hidden to allow the reader to understand the overall model hierarchy that is common to general component sizing problems.

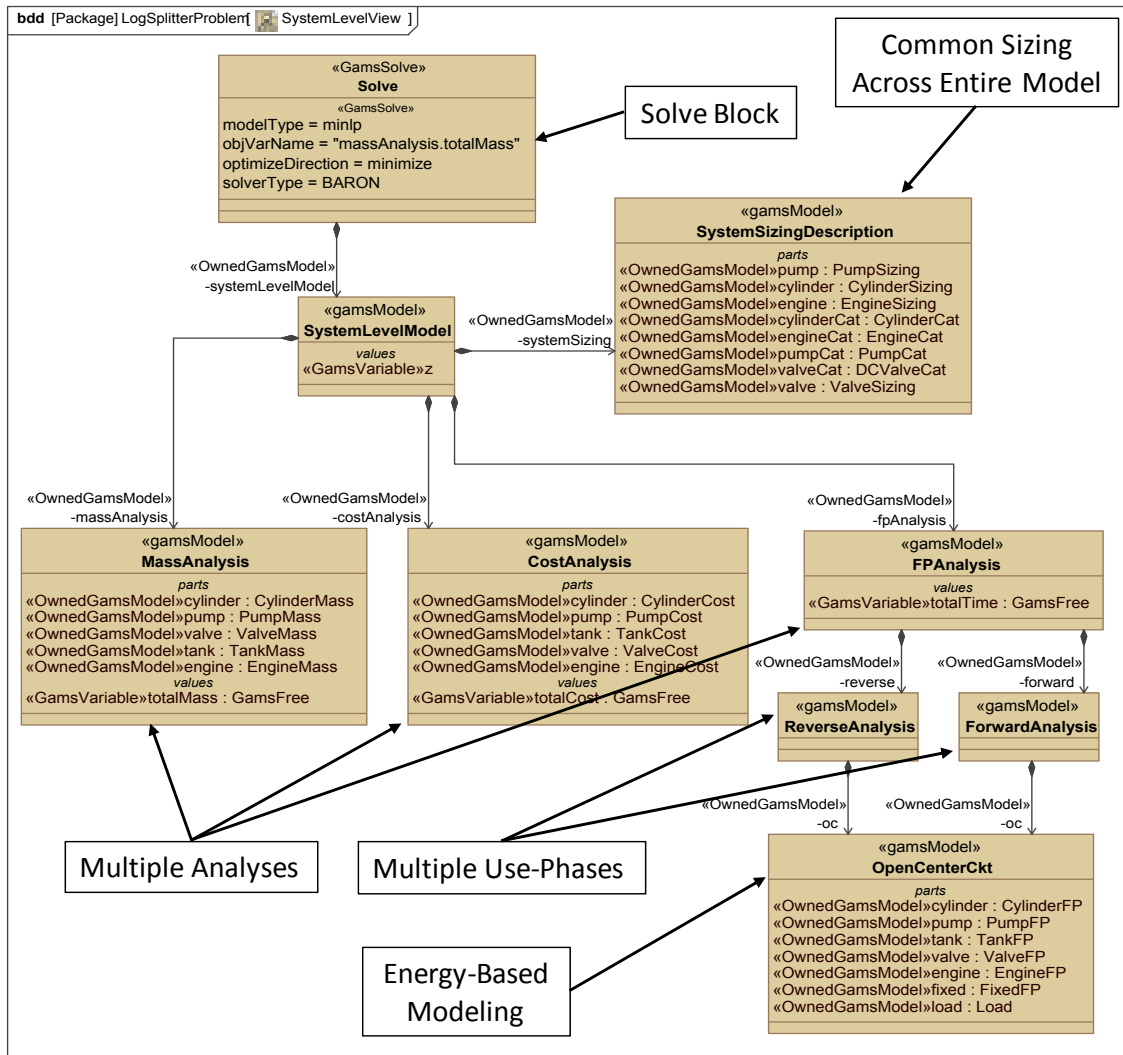


Figure 12 System Level View highlighting the features found in component sizing problems. This type of model hierarchy would be common for such problems in general.

4.2.2 Energy-Based Modeling & Multiple Use-Phases for Fluid Power Systems

In order to analyze the hydraulic performance and ensure that the requirements are satisfied, algebraic equations are used to model the behavior of the individual hydraulic components as well as the combined system behavior. Since algebraic equations are used instead of dynamic simulations, steady state behavior is assumed for the system. Assuming a single steady state operation for a problem is not feasible and therefore multiple use-phases are considered, each representing a particular steady state phase. In the case of the log splitter, two use-phases are considered: the forward motion of the wedge and the reverse motion of the wedge. These two phases can be assumed to occur at constant velocity and therefore the steady state equations can be used. In Figure 13, a portion of the SysML model is shown in which the same hydraulic circuit is used for two phases. For more complex problems, it is possible to discretize the system into a number of time-steps, each of which can be assumed to operate at steady state. This is discussed further in the section on Future Work in the next chapter.

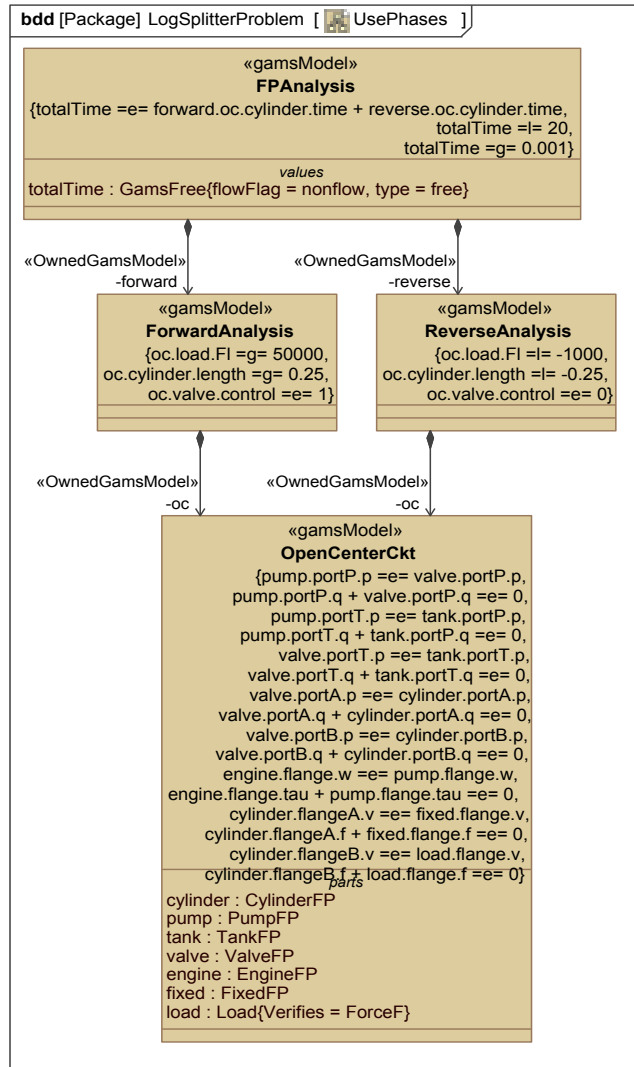


Figure 13 Modeling multiple use-phases for a problem. In this case, there are two use-phases, a *ForwardAnalysis* and *ReverseAnalysis*. The use-phases are for the same hydraulic circuit, as represented by the common *OpenCenterCkt* Block.

Energy-based modeling principles are used to define analysis models for a particular use-phase. This is used when connecting individual component models together to form a system level model of the fluid power circuit. In Figure 14, an Internal Block Diagram (IBD) for the hydraulic circuit is shown. Through the use of transformations to automatically generate equations based on the connections between components, it is possible to generate complete analysis models by combining individual

components and connecting them together. Energy-based principles are used when connecting components as well as at the time of defining individual component behavior. Through the use of proper sign conventions and standardized port-based interfaces, it is possible to define components that can potentially be reused in other problems as well. Component models and the sign conventions used are described in Appendix A.

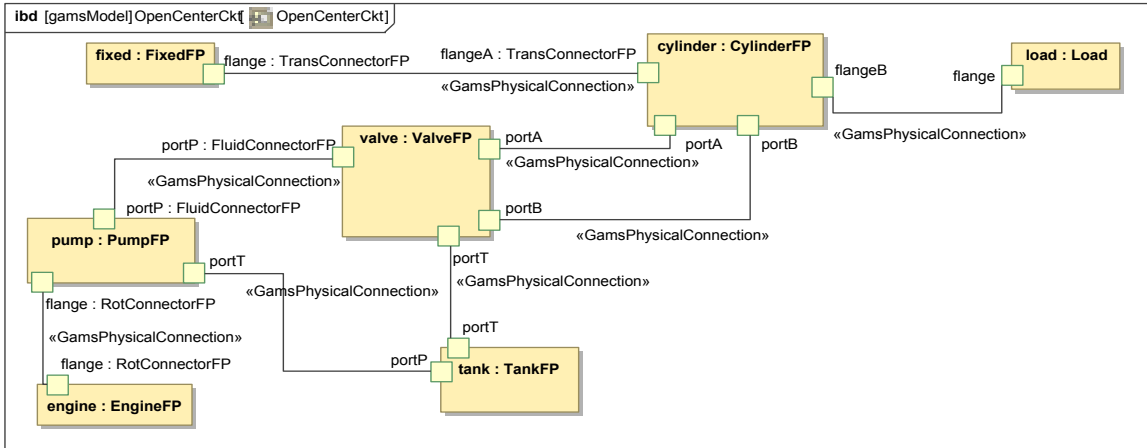


Figure 14 A Internal Block Diagram for the open center circuit used in the problem. The connections between ports are stereotyped with `<<GamsPhysicalConnection>>` and automatically generate the connection equations, based on conservation of energy.

Along with the energy-based analysis such as hydraulic performance, there are also other types of analyses that are needed to determine if requirements are satisfied. The modeling of multiple analyses and organizing them in a model hierarchy is discussed in the next section.

4.2.3 Multiple Analyses & Hierarchical Modeling

In addition to energy-based analyses such as fluid power performance, other analyses are also needed depending on the requirements specified by the designer. For

instance, the requirement on total mass and total cost of the system cannot be included within the analysis described in the previous section, because defining cost in multiple use-phases does not make sense from a modeling perspective and would also result in duplication of the same constraint, resulting in model ambiguity. Different analyses can be modeled in the same way as energy-based analyses and can be arranged in a model hierarchy under a single top-level model. By grouping the analyses under a common model, it is ensured that the resultant executable model contains all of the different analyses and, if applicable, multiple use-phases (see cost, mass and fluid power analysis all acting simultaneously in Figure 12).

In this way, hierarchical modeling in SysML can be used to model the component sizing problem in a logical and modular fashion using individual component models that can be reused in the same problem or in different problems. It is important to note that each analysis and use-phase uses different (or duplicate) component models to ensure that there is no overlap in variable usage. However, since all of the analyses and use-phases are supposed to involve the same components, a mechanism is needed in SysML to define the relations between a component sizing model and its multiple usages across analyses and use-phases. This is discussed in the next section.

4.2.4 Common Sizing Description for the Entire Model

When a problem is divided into different analyses and use-phases that exist independently of each other, it is important to remember that sizing is determined by considering *all* of the analyses simultaneously. However, since each analysis and use-phase model use different usages for the *same* component model, it is necessary to

establish a relation between all of the usages throughout the model. For instance, the forward and reverse use phases and the mass and cost analyses each use a different cylinder model that has all of the sizing variables. From a systems perspective, the selection of the Cylinder depends on all the analyses. Therefore, to ensure that the same sizing variable is referred to throughout the different analyses and use-phases, a separate system sizing model is defined that contains the sizing variables used throughout the system (`SystemSizingDescription` – see Figure 12).

There are two main reasons for using a system sizing model:

- a. Specifying a component model and its associated catalog model *once* for the entire problem, irrespective of the number of use-phases or analyses (see the component catalog library in Figure 15)
- b. Ensuring that all analyses and use-phases refer to the appropriate component model by explicitly connecting the component model with its usages in each analysis and use-phase. (see Figure 17)

In order to specify the sizes a component can assume, a two-step process is used, in which first a catalog model is populated with data and then equations are defined to ensure that the sizing variables only take values from the catalog model. By storing supplier data in a problem-independent model library, it is possible to populate a component catalog model, which is problem-specific, with possible component values by establishing dependencies between the sizing variables used in the problem with the corresponding parameters (constants) found in the model library. Model transformations are used to automatically populate the component model with information found from the model library (see Figure 15).

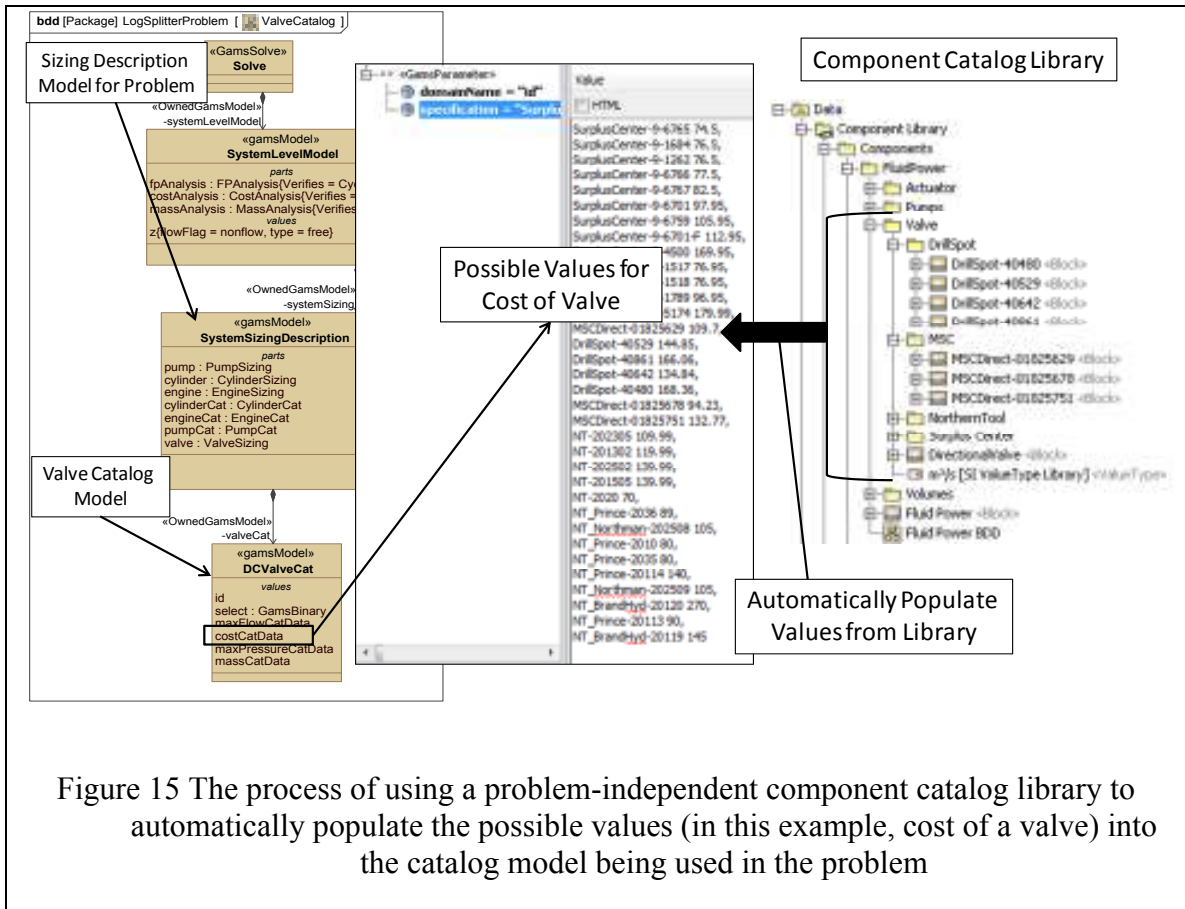


Figure 15 The process of using a problem-independent component catalog library to automatically populate the possible values (in this example, cost of a valve) into the catalog model being used in the problem

After defining the catalog model, constraints are defined to ensure that sizing variables for a component assume values only from the set of possible values contained in the catalog model. For instance, it is not meaningful for a cylinder to have sizing variables like: boreDiameter = 0.3m, stroke = 1m, mass = 1kg, cost = \$10 because it is physically impossible. Therefore, a Boolean variable is used to determine which component from the catalog has actually been selected. An example of the collection of equations used for a cylinder is provided in Figure 16. In this way, it is possible to ensure that all of the sizing variables take values from a particular catalog entry.

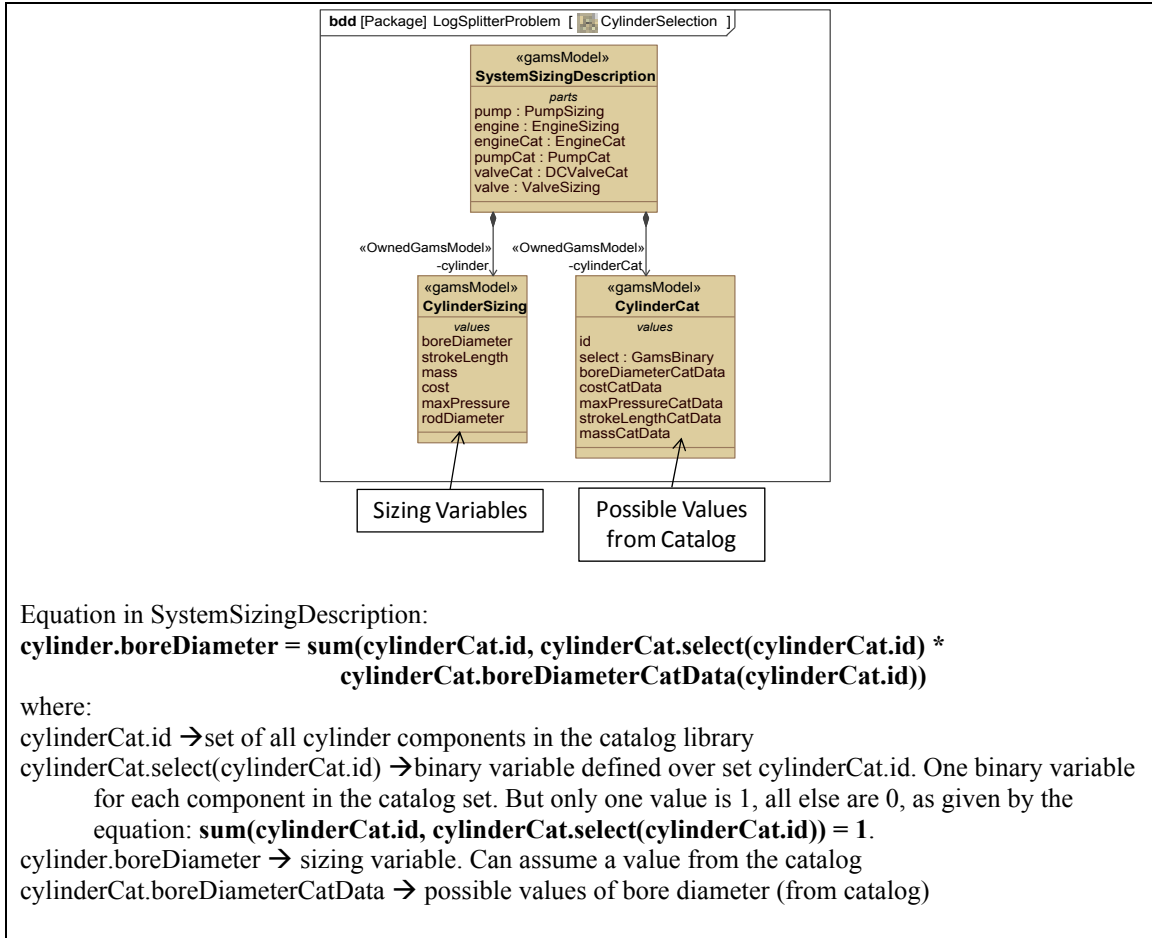


Figure 16 Equations used to associate a component’s sizing variables (boreDiameter) with the corresponding catalog values from supplier (boreDiameterCatData)

The equations above involve the use of a binary variable (0 or 1) that is defined over the set of all possible components. From the summation equation ($\text{sum}(\text{cylinderCat.id}, \text{cylinderCat.select}(\text{cylinderCat.id})) = 1$), only one of the binary values in the entire set can be 1 and this represents the component that is selected by the solver.

After defining the relation between component and catalog connections are defined (stereotyped with `<<GamsSelectionConnection>>`) between the component models in the system sizing model with the component models used in the analyses (see Figure 17). Based on the model transformation logic defined in the framework, constraints are automatically generated for all of the variables included at

each end of the connection, since both ends of the connection refer to the same sizing model. In Figure 17, an example of such a constraint is shown in the SystemLevelModel Block in which the bore diameter in the forward phase is equated to the bore diameter in the system sizing model.

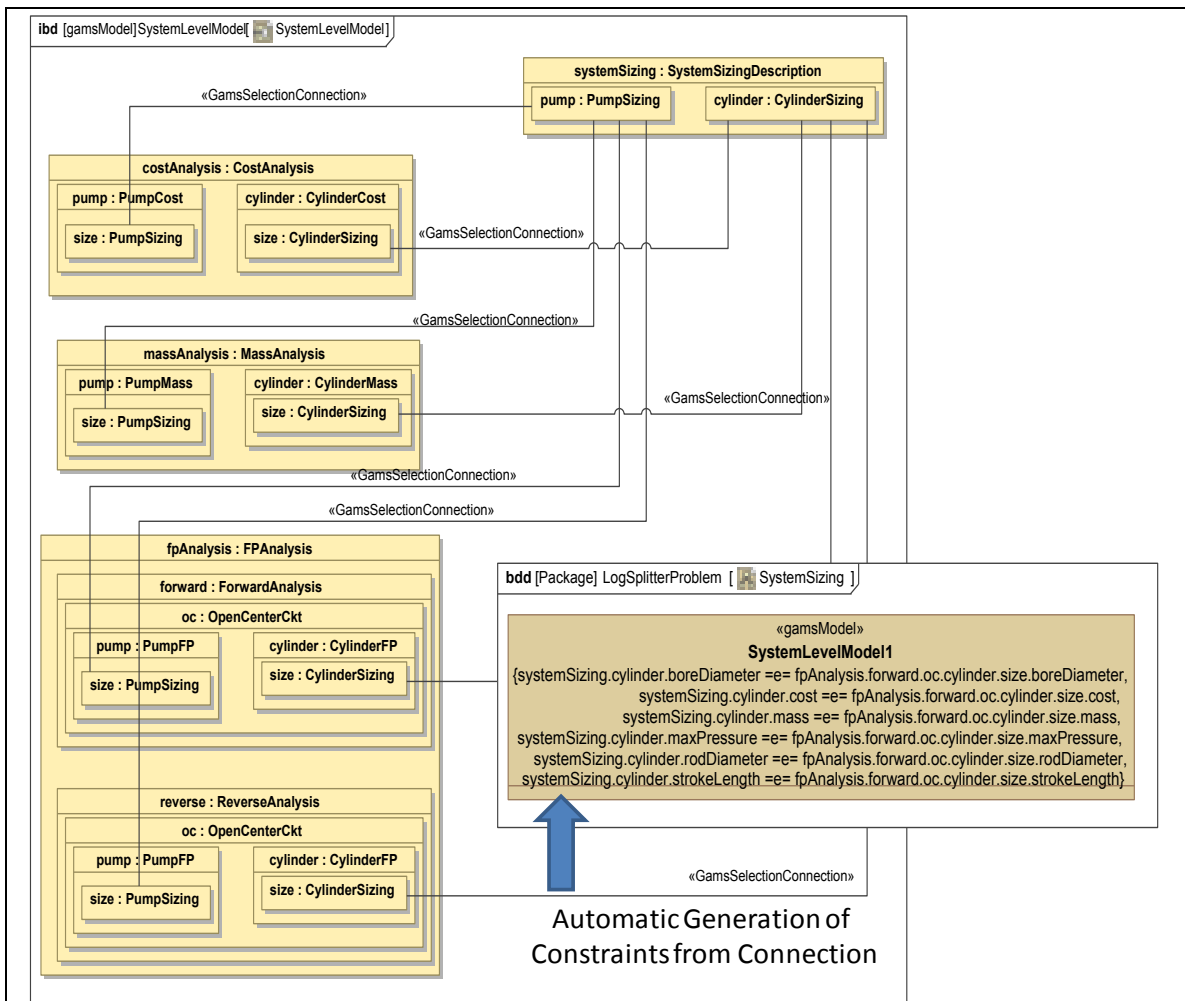


Figure 17 Through the use of a customized connection (`<<GamsSelectionConnection>>`), it is possible to ensure that common sizing description is used across the entire model. Equations are automatically generated to equate the variable used in an analysis or use-phase with the corresponding variable in the sizing description model.

4.2.5 Defining the Solve Block & Solving in GAMS

Once all the analysis models have been defined along with a system sizing model, it is possible to define a `Solve` Block of stereotype `<<GamsSolve>>` that specifies solver related properties such as solver name, model type, objective variable and so on. After the top-level `SystemLevelModel` Block of stereotype `<<GamsModel>>` is connected with the `Solve` block, the next step is to execute the model transformations in the form of a plugin (refer to Section 3.3) in order to solve the problem externally in GAMS.

In this way, a complete component sizing model for the log splitter is constructed in SysML. Then, through model transformations, executable GAMS code is generated that can be solved using the solver specified in the SysML model. The results obtained from this model are discussed in the next section.

4.3 Results for Different Scenarios

In this section, results obtained from solving the log splitter component sizing problem for different scenarios are presented. As discussed in the Introduction, the motivation for this research is the investigation of new frameworks that can represent and solve component sizing problems more efficiently. Therefore, in addition to describing the actual solutions obtained for various scenarios, a discussion is provided regarding the use of SysML versus other tools such as MATLAB or GAMS. In the current scope of this research, investigation into the global optimality of solutions is not considered; such validation is beyond the current scope of this research and is left for future work.

Based on available component catalog data, sizing was performed on four components: a gas engine with 45 possible options, a fixed displacement pump with 64 possible options, a double acting cylinder with 158 possible options, and an open center directional control valve with 34 possible options.

Depending on the constraints defined by the requirements as well as the variable to be “optimized”, the component selections obtained are different. The global MINLP solver BARON [39] in GAMS is used to solve the log splitter problem. Figure 18 lists a summary of the model and solution statistics for one scenario as provided by BARON. The model statistics remain the same for all scenarios, since they are only a variation of the problem. These statistics are useful in evaluating the complexity of the problem from a solver’s perspective. In this case, there are 68 nonlinear coefficient entries, 256 equations and 298 discrete variables (representing the catalog components). The code length of 503 provides some indication of the complexity of the nonlinearity of the model and represents the amount of code that GAMS passes onto the non-linear solver (For more information, refer to GAMS User Guide, Ch. 10 [3]).

An important aspect to note is that the component models used in this example do not consider losses such as leakage or friction, i.e. they model ideal physical behavior. This assumption has been made because these models represent the first time that GAMS-compliant declarative models have been used. Since losses do not alter the fundamental behavior of the component, including them will only serve to make the model more complex, but will not invalidate the use of GAMS for solving of this class of problems.

```

General Algebraic Modeling System
Model Statistics SOLVE m1 Using MINLP From line 2827

MODEL STATISTICS

BLOCKS OF EQUATIONS    256  SINGLE EQUATIONS    256
BLOCKS OF VARIABLES    209  SINGLE VARIABLES    503
NON ZERO ELEMENTS      2,342  NON LINEAR N-Z      68
DERIVATIVE POOL        11  CONSTANT POOL        19
CODE LENGTH            503  DISCRETE VARIABLES   298

GENERATION TIME = 0.094 SECONDS 4 Mb
EXECUTION TIME = 0.109 SECONDS 4 Mb

Solution Report SOLVE m1 Using MINLP From line 2827

      S O L V E   S U M M A R Y

MODEL m1      OBJECTIVE v135
TYPE MINLP    DIRECTION MINIMIZE
SOLVER BARON  FROM LINE 2827

**** SOLVER STATUS 1 Normal Completion
**** MODEL STATUS 8 Integer Solution
**** OBJECTIVE VALUE 657.4000

RESOURCE USAGE, LIMIT 2.450 1000.000
ITERATION COUNT, LIMIT 0 2000000000
EVALUATION ERRORS 0 0

```

Figure 18 Model and solver statistics for the scenario of minimizing total cost. The model statistics are same for all scenarios since they are for the same problem. The solution is provided by the solver BARON.

Five scenarios are considered: maximizing the cylinder force during the forward phase, minimizing total cost, minimizing total mass, minimizing total time, and minimizing a variable that is defined by a multi-objective function. The requirements imposed on the system, in terms of constraints, are shown in Equation 3. These requirements act in addition to the constraints already imposed by individual component behavior models (see Appendix A for details of component models).

$$\begin{aligned}
 \text{Force produced in forward phase: } F_{L_{forward}} &\geq 50,000 \text{ N} \\
 \text{Total time taken by system: } t_{total} &\leq 20 \text{ s} \\
 \text{Total Cost of Components: } C_{total} &\leq \$1,000 \\
 \text{Total Mass of Components: } m_{total} &\leq 150 \text{ kg}
 \end{aligned}
 \tag{3}$$

In order to understand the results obtained for the scenarios, it is necessary to identify the coupling between the different components of the log splitter. The log splitter's function is to split wood by using force generated by a cylinder. The force produced by a cylinder directly depends on the bore diameter and pressure of the fluid inside it. Therefore, to produce more force, either the bore diameter or pressure can increase or both. Higher pressure at the cylinder means that the pump needs a greater input torque, which places a demand on the engine for higher torque. An increase in the bore diameter results in a decrease in flow rate in the cylinder, which increases the time taken to split the wood. Moreover, the maximum flow that can be handled by the system is limited by the valve that is used. Thus, with inequalities in the constraints it is difficult to determine manually what the best solution is to a given scenario.

Before discussing the individual scenarios, provides an overall comparison of the results obtained. It appears that the results are appropriate, since the objective to be optimized in each scenario is better (smaller or larger, depending on optimization direction) than its corresponding value in the other scenarios. In addition to the overall results, the actual component sizes for each scenario are also described below, along with a discussion to understand the logic behind the values obtained.

Table 4 Comparison of results for different scenarios. Component Sizing is represented in terms of the selection from the corresponding component catalogs.

Scenario	Component Sizing (Selection Id from Catalog)				Selected Variable Values					CPU Execution Time (s)
	Cylinder Id	Pump Id	Engine Id	Valve Id	Forward Force (N)	Total Mass (kg)	Total Cost (\$)	Total Time (s)	z	
Maximize Force (N)	HMW-5032	SKP1NN_012	DP340E	NT-2020	139,833	94.9	993.5	20	0.4027	2.82
Minimize Total Time (s)	HMW-3010	SKP1NN_012	DP390E	NT_Prince-2036	50,000	51.87	843.97	4.896	0.2529	3.54
Minimize Total Cost (\$)	HMW-4010	SKP1NN_012	DP240	NT-2020	53,698	51.3	657.4	9.69	0.26116	2.45
Minimize Total Mass (kg)	PMC-5414	SNP2NN_4_0	DP160V	MSCDirect-01825629	52,013	32.25	708.6	9.15	0.2528	78.13
Minimize Multiobjective z	HMW-5010	SKP1NN_012	DP390	NT-2020	147,437	71.53	866.3	13.79	-0.3968	5.65

$$z = 0.25*((totalMass/300) + (totalTime/20) + (totalCost/1000) - (forwardForce/50000))$$

Maximize Force Produced by Cylinder in Forward Phase:

In this scenario, the problem is to find the sizes for the components so that the maximum force can be produced to split the log, which is the force produced in the forward phase. The component sizes obtained are presented in Table 5.

Table 5 Component Sizes to Produce Maximum Log Splitting Force (N)

Cylinder: HMW-5032	Bore Diameter (m)	0.13	Engine: DP340E	Max Torque (N-m)	23.4
	Stroke Length (m)	0.81		Speed at Max Torque (rpm)	2500
	Max Operating Pressure (Pa)	17200000		Max Power (W)	8200
	Cost (\$)	293.5		Speed at Max Power (rpm)	3600
	Mass (kg)	56.1		Cost (\$)	399.99
			Mass (kg)	32.65	
Pump: SKP1NN_012	Displacement (m ³ /rev)	1.1995E-05	Valve: NT-2020	Max Flow (m ³ /s)	0.0016
	Max Operating Pressure (Pa)	11997000		Max Operating Pressure (Pa)	1.38E+07
	Max Operating Speed (rpm)	2000		Cost (\$)	70
	Cost (\$)	230		Mass (kg)	4.53
	Mass (kg)	1.65			

Since this scenario only cares about maximizing force, it is logical to assume that the other requirements (Equation 3) would remain at the bounds. The cylinder selected has a large bore diameter, resulting in greater force but increasing the time taken to complete one cycle. This is reflected in the total time lying at the boundary of the constraint, i.e. 20s. Similarly, the total cost (\$993) is very close to the boundary of the constraint on total cost (\$1000). The large bore diameter results in smaller flow for the same pressure and in this case it is only 0.0004 m³/s. Therefore, the consideration for valve selection lies mainly on the cost in order to maintain a total cost below the requirement. Since the objective is to maximize force, the possibility is to increase bore diameter and increase the pressure. The increase in bore diameter has been taken into account by selecting a large size cylinder. Higher system pressure is possible if the input torque to pump is higher, which leads to engines with higher possible torques. The logic underlying the selection of engine and pump is shown in Figure 19.

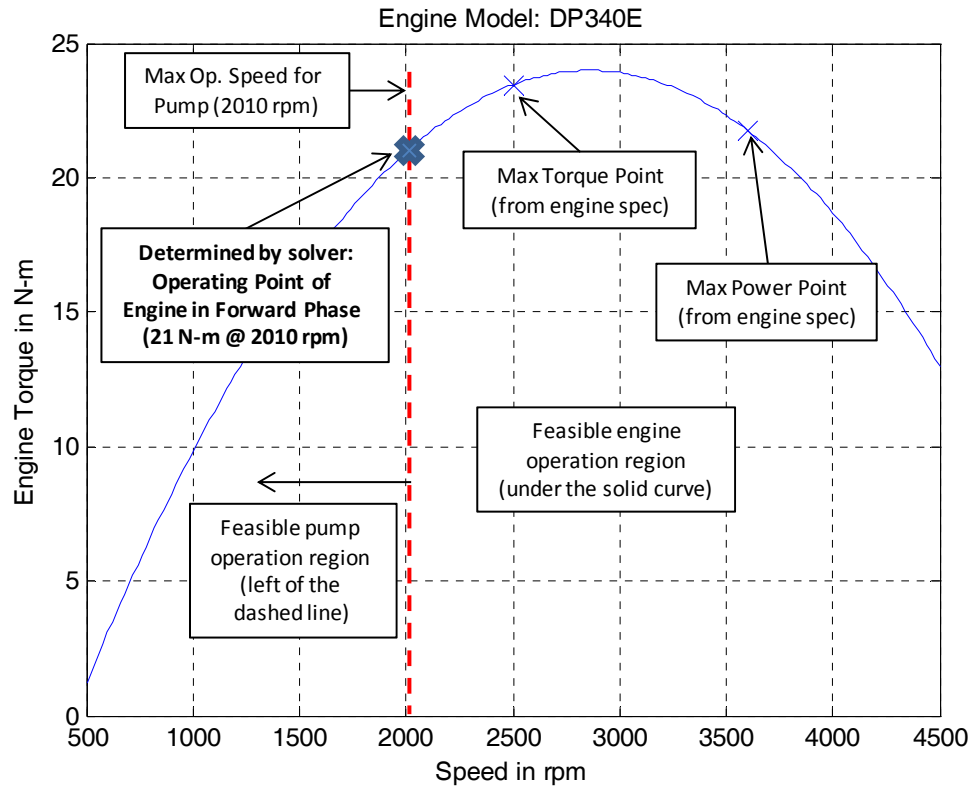


Figure 19 Engine operating point for forward phase of operation, as determined by solver. The operating point is below the speed at max torque (as provided by engine specification), which is counterintuitive to a designer

In Figure 19 the torque-speed curve for the engine selected by the solver is shown, in which the region below the curve represents the feasible operating region of the engine. A clarification regarding the position of the max torque point is required: the curve represents a quadratic curve fitted through the two operating points specified by the vendor (see Appendix for more details). Also shown is the maximum operating speed for the selected pump (red dashed line). Therefore, based on the intersection of these two feasible regions (engine and pump), the solver determined the operating point to be 21 Nm @ 2010 rpm.

This result is counter-intuitive to what a designer would normally expect. A designer may assume that to achieve maximum torque input to pump, the maximum torque point for the engine should be considered first. Thereafter, a pump with suitable maximum operating speed would be selected. In that case, the designer would have outright rejected the pump selected by the solver for being too slow. Therefore, the question is: *Why did the solver select this pump?*

A closer look at the selected pump reveals the answer. In Table 6, the selected pump is compared with some of the other pumps available in the catalog. Since the total time is 20 seconds, the flow rate is low and consequently the pump displacement needed is also small. The two pumps with smaller displacements (SKP1NN_78 and SKP1NN_010) cannot generate enough flow in the system to have the total time under 20s. Pumps with larger displacements can be chosen, but their costs are much higher (around \$400 compared to \$230 for the selected pump). This comes with the disadvantage of a much lower maximum operating speed of 2010 rpm.

Table 6 Comparison of selected pump (SKP1NN_012) with other pumps

Pump Id	Displacement (m ³ /rev)	Maximum operating Pressure (Pa)	Maximum operating RPM (rpm)	Mass (kg)	Cost (\$)
SKP1NN_78	7.58721E-06	2.00E+07	3000	1.39	225.45
SKP1NN_010	9.94695E-06	1.50E+07	2000	1.55	227.99
SKP1NN_012	1.19953E-05	1.20E+07	2010	1.65	230.01
SNP3NN_022	2.21225E-05	2.50E+07	3000	6.80	410.94
SNP3NN_026	2.62193E-05	2.50E+07	3000	6.80	415.67
SNP3NN_033	3.31019E-05	2.50E+07	3000	7.17	426.41

Thus, the solver selects the pump SKP1NN_012 and chooses the engine operating points to be at 2010 rpm and 21 Nm, at the bounds of both the engine and pump feasible

operating regions. This is interesting because such a solution would likely have been overlooked by a designer performing sizing manually.

For the remaining scenarios, the component sizes are presented in the following tables. For the case of minimizing the total time ($t_{forward} + t_{reverse}$), the same logic applies. The main characteristic is that a cylinder with a much smaller bore is selected (0.08m versus 0.13m) and in this case the force produced lies at the boundary of the requirement. Refer Table 7.

Table 7 Component Sizes for Fastest Log Splitter Operation (Total Time in seconds)

Cylinder: HMW-3010	Bore Diameter (m)	0.08	Engine: DP390E	Max Torque (N-m)	26.4
	Stroke Length (m)	0.25		Speed at Max Torque (rpm)	2500
	Max Operating Pressure (Pa)	2.07E+07		Max Power (W)	9694
	Cost (\$)	74.97		Speed at Max Power (rpm)	3600
	Mass (kg)	12.3		Cost (\$)	449.99
			Mass (kg)	32.9	
Pump: SKP1NN_012	Displacement (m3/rev)	1.1995E-05	Valve: NT_Prince- 2036	Max Flow (m3/s)	0.0016
	Max Operating Pressure (Pa)	11997000		Max Operating Pressure (Pa)	1.38E+07
	Max Operating Speed (rpm)	2000		Cost (\$)	89
	Cost (\$)	230		Mass (kg)	4.98
	Mass (kg)	1.65			

In Table 8, the component sizes for minimizing the total cost of the system are presented. In this case, the force lies at the bounds and the cheapest components are selected.

Table 8 Component Sizes for the Cheapest Log Splitter (\$)

Cylinder: HMW-4010	Bore Diameter (m)	0.1	Engine: DP240	Max Torque (N-m)	16.6
	Stroke Length (m)	0.25		Speed at Max Torque (rpm)	2500
	Max Operating Pressure (Pa)	2.06E+07		Max Power (W)	5966
	Cost (\$)	107		Speed at Max Power (rpm)	3600
	Mass (kg)	20.2		Cost (\$)	249.99
			Mass (kg)	24.94	
Pump: SKP1NN_012	Displacement (m3/rev)	1.1995E-05	Valve: NT-2020	Max Flow (m3/s)	0.0016
	Max Operating Pressure (Pa)	11997000		Max Operating Pressure (Pa)	1.38E+07
	Max Operating Speed (rpm)	2000		Cost (\$)	70
	Cost (\$)	230		Mass (kg)	4.53
	Mass (kg)	1.65			

In Table 9, the component sizes for minimizing the total mass of the system are presented. In order to decrease the mass, smaller components are selected in general.

Table 9 Component Sizes for Least Mass (kg)

Cylinder: PMC-5414	Bore Diameter (m)	0.06	Engine: DP160V	Max Torque (N-m)	7.9
	Stroke Length (m)	0.36		Speed at Max Torque (rpm)	2700
	Max Operating Pressure (Pa)	1.72E+07		Max Power (W)	4101
	Cost (\$)	99.93		Speed at Max Power (rpm)	3600
	Mass (kg)	9.98		Cost (\$)	279.99
				Mass (kg)	15.42
Pump: SNP2NN_4_0	Displacement (m3/rev)	3.93E-06	Valve: MSCDirect- 01825629	Max Flow (m3/s)	0.0016
	Max Operating Pressure (Pa)	2.50E+07		Max Operating Pressure (Pa)	1.89E+07
	Max Operating Speed (rpm)	4000		Cost (\$)	109.7
	Cost (\$)	218		Mass (kg)	4.53
	Mass (kg)	2.3133			

Finally, a multi-objective function is constructed by using a weighted normalized sum of the four individual objectives considered above. The variable z is defined as

$$z = 0.25 \cdot \left(\frac{m_{total}}{300} + \frac{t_{total}}{20} + \frac{C_{total}}{2000} - \frac{F_{L_{forward}}}{50000} \right)$$

The mass, time and cost are added because they are minimized while the force is subtracted because it is maximized. In Table 10, the component sizes for minimizing the multi-objective function are provided. Since $F_{L_{forward}}$ is normalized by its lower bound, the solver tries to maximize the force produced in order to lower the value of z by the largest amount. This is reflected in the selection of components that generates a large amount of force.

Table 10 Component Sizes that Minimize Multi-objective function

Cylinder: HMW-5010	Bore Diameter (m)	0.13	Engine: DP390	Max Torque (N-m)	26.4
	Stroke Length (m)	0.25		Speed at Max Torque (rpm)	2500
	Max Operating Pressure (Pa)	1.72E+07		Max Power (W)	9694
	Cost (\$)	166.3		Speed at Max Power (rpm)	2500
	Mass (kg)	34.5		Cost (\$)	399.99
Pump: SKP1NN_012	Displacement (m3/rev)	1.1995E-05	Valve: NT-2020	Max Flow (m3/s)	0.0016
	Max Operating Pressure (Pa)	11997000		Max Operating Pressure (Pa)	1.38E+07
	Max Operating Speed (rpm)	2000		Cost (\$)	70
	Cost (\$)	230		Mass (kg)	4.53
	Mass (kg)	1.65			

4.4 Summary

In this chapter, an example application problem was used to demonstrate the framework presented in this thesis for representing and solving component sizing problems. Through the steps described in this chapter, it is possible to formulate component sizing problems in SysML which can then be executed and solved within GAMS using the specified solver (BARON, in this case). The SysML representation is a structured and object-oriented model composed of individual component models that can be connected together in different ways. By encoding the logic common to component sizing problems, such as generation of equations for energy-based connections, a designer can formulate new architectures quickly by connecting components in different ways.

After formulating the SysML model, different scenarios were run simply by modifying the objective to be optimized in the block stereotyped with <<GamsSolve>>. From the execution time of the solver to the type of solutions obtained, it is clear that MINLP solvers such as BARON are efficient at solving component sizing problems. The solutions presented in this example application are *near-optimal* (as stated in output file

by BARON) but global optimality *can* be ensured by BARON through different methods, such as specifying upper and lower bounds on variables. In a comparison of global optimization solvers, BARON is considered to be the fastest and most robust [29]. Therefore, these initial results indicate that BARON is well suited for solving component sizing problems encountered in engineering design.

Global optimality cannot be ensured by conventional sampling-based solvers and in the worst case, an exhaustive search would involve searching $45 \cdot 64 \cdot 158 \cdot 34 = 15,471,360$ alternatives. This number would increase as more components are considered and larger component catalog sizing data are used.

To conclude, this example application shows that mathematical programming and GAMS is well-suited for solving component sizing problems while SysML is well-suited for representing them.

CHAPTER 5

DISCUSSION AND CLOSURE

5.1 Review and Evaluation

The high-level motivation for this research involves the automated exploration of system architectures, with the ultimate goal being to automatically synthesize and select the “best” architecture for a problem given a set of requirements. For this to happen there are a number of different steps that must be integrated together; this research is a first step towards addressing this by considering the case of analyzing one system architecture.

In particular, this research aims to provide designers with improved capabilities for both representing and solving of component sizing problems. This leads to the following research question and hypotheses:

RQ: Is it possible for designers to represent and solve component sizing problems more efficiently?

H1: Through the use of mathematical programming and constraint satisfaction techniques, designers can solve component sizing problems involving algebraic models more efficiently.

H2: It is possible to extend traditional mathematical programming using SysML and model transformations to provide designers with improved capabilities for representing and formulating component sizing problems.

This research does not claim to validate the hypotheses completely; such validation is beyond the current scope. By applying the proposed framework to an

example application involving a hydraulic log splitter, this research is intended to provide a foundation and basis for future research that can help in validating the hypotheses more rigorously.

Evidence to support Hypothesis 1:

The results obtained in the example show that mathematical programming solvers such as BARON (within GAMS) can find solutions quickly in approximately 2-5 seconds depending on the particular scenario. When the same log splitter problem was modeled in MATLAB using an exhaustive search, the computation time involved was approximately 30 minutes to consider around 16 million combinations. However, some issues encountered when using MATLAB include:

- The causal nature of MATLAB requires that some variables be assumed as known and reorder equations accordingly.
- Defining inequality constraints in MATLAB is often not possible and very cumbersome if possible. If a variable in an inequality is assumed as known, then an assumption is that value occurs at the boundary. This is different from mathematical programming, in which values can lie between bounds as well.

The above issues highlight the difficulties in formulating *and* solving a component sizing problem in conventional imperative programming languages such as MATLAB. As a result, it is clear that mathematical programming *can* solve component sizing problems in an efficient manner as compared to similar approaches in MATLAB or other tools.

Moreover, solvers like BARON fall under the category of global optimization solvers [29], which is important when dealing with non-linear design spaces that are found in component sizing problems.

Evidence to support Hypothesis 2:

The second hypothesis in this research refers to the use of SysML and model transformations to make it easier to formulate component sizing problems in terms of mathematical programming. Although “easier” is subjective, there are certain characteristics of component sizing problems that can be used to illustrate the usefulness of SysML for component sizing; in particular, scalability and reduction in the time required to model the system.

As discussed in the example (Section 4.2), component sizing problems consist of different models such as multiple analyses and use-phases, energy-based analysis models, catalog models, etc. This results in a large number of SysML models, as evident from Section 4.2. Since the example presented in this thesis is not very complex, it can be argued that the problem could have been formulated manually and directly in GAMS instead of in SysML in a similar time frame. However, as larger and more complex problems are considered, it quickly becomes cumbersome and error prone to formulate the problem manually. Therefore, as problems become more complex, a formal representation becomes more important to a designer. The framework using SysML and model transformations presented in this thesis supports this requirement for a formal representation by reducing the time required to model component sizing problems. The time required to model a problem stems from two aspects: creating models and then composing them together to form system-level models.

Through the use of energy-based modeling, ports, and a common sign convention it is possible to define independent and self-contained models that captures its steady-state behavior. Moreover, additional constructs related to inheritance (similar to those found in Modelica) would enable the creation of a standard library that can be used for a wide variety of problems. Although there is some effort required to initially create models, this time is offset by the savings obtained when models are reused, such as across multiple use-phases or multiple components in a system. In addition, with a library of components available, system models can quickly be composed by connecting them together through their ports.

Arguably the largest benefit of using this framework for representing component sizing problems is the model-based graphical nature of SysML, which is similar to the way designers construct schematics and other models. Thus, without these model transformations or SysML, a designer would have to manually define the entire problem directly in terms of an executable model consisting of equations and variables. This is non-intuitive for designers and would increase the occurrence of errors that are unrelated to the problem being solved.

To summarize, the primary research contributions in this thesis are:

1. This thesis presents an initial framework for representing *and* solving component sizing problems using SysML and GAMS.
2. Demonstrating the use of mathematical programming (GAMS and its solvers) for solving of engineering design problems.

3. Initial implementation of a language in SysML that can be used for object-oriented algebraic modeling of energy-based systems. This is achieved through the use of domain specific languages and model transformations.

5.2 Limitations and Future Work

This thesis provides only a first step towards defining a framework for representing and solving component sizing problems. Therefore, many of the limitations of the current work serve as a basis for future work in addition to the existing open research questions for this work.

The framework presented in this thesis was applied to a single example application involving a hydraulic log splitter, a non-trivial problem to solve but not as complex compared to other real-world systems. In addition, the hydraulic component behavior models used in this work do not take into account phenomena such as losses that would be found in practical components (refer Appendix A). Making the component models more complex by including losses, increasing the variety of components modeled, and applying the framework to more complex problems are all future work that can address these limitations.

As more complex problems are considered, there is a greater need for a standard model library of algebraic component models (similar to the Modelica standard library [16, 26]). Since more complex problems would involve many different use-phases, a standard way of defining components also needs to be investigated. Such approaches may lead to a new modeling language similar to Modelica, but for the mathematical programming of engineering design problems. In addition to better tools for modeling,

such a language could provide better support for debugging. For instance, it is not always easy to determine a bug in the model from the debugging support available in existing tools like GAMS. This can be improved by using the existing debugging output of GAMS and analyzing it to provide additional information that is specifically related to engineering problems instead of general mathematical problems.

In addition to bigger problems and better debugging, another aspect of future work involves improving the support for a larger number and more complex steady-states. In this thesis independent use-phases are considered; however, it is not always the case. There are instances in which use-phases may be related to each other i.e., the initial and final values obtained in a use-phase tie into the previous and next use-phase. For instance a system can be discretized into a number of time steps, in which each time step represents a steady state, and optimization can be performed over all of the time steps. This can result in a more standardized way of defining models as well as the ability to solve more complex models.

Another area that can be investigated is the use of continuous catalog models (see Malak *et al.* [24]) instead of discrete catalog models that are used in this research. This refers to the use of continuous models to capture the high-level dependencies between component attributes instead of catalog models that capture the dependencies discretely. Continuous models are useful in cases where a system may benefit from custom designed components instead of COTS components from a supplier. Some questions related to this include: “*What is involved in defining acausal continuous catalog models?*”, and “*How does solver performance change for continuous catalog models?*”

Although the use of mathematical programming is proposed in this research, it is not clear how far this capability can extend. *What is the limit after which mathematical programming is no longer feasible as a solution technique?*

In conclusion, the “Model Based Mathematical Programming” framework presented in this thesis provides designers with the ability to quickly define systems using model libraries and explore solutions for different requirements and objective functions.

APPENDIX A

COMPONENT MODELS & ASSOCIATED CATALOG DATA

This appendix gives an overview of the different components used in the example application as well as the mathematical models for each of these components. As discussed in the thesis, the component models are all algebraic in nature, in which an assumption of steady-state conditions is applied. The components considered in this thesis include a double-acting cylinder, fixed displacement pump, 4/3 directional control valve, and a gas engine.

An important aspect to note is that the component models in this thesis do not consider losses such as leakage or friction, i.e. they model ideal physical behavior. This assumption has been made because these models represent the first time that GAMS-compliant declarative models have been used. Since losses do not alter the fundamental behavior of the component, including them will only serve to make the model more complex, but will not invalidate the use of GAMS for solving of this class of problems.

Double Acting Cylinder:

In Figure 20, a SysML model is shown that captures the hydraulic behavior of a double-acting cylinder in steady-state. As discussed previously, losses are not considered and the model captures the steady-state behavior for a cylinder. The model consists of sizing variables (`boreDiameter`, `strokeLength`, `mass`, `cost`, `maxPressure`, `rodDiameter`) that are contained within a separate `CylinderSizing` block as well as state variables (similar to time-dependent) like `time`, `force`, `vel`, and `length`. It also contains four ports, two translational and two fluid power ports. This is similar to the

interfaces found in actual cylinders. These ports represent the interfaces through which components can be connected together. The ports also have variables, depending on the type of port. For instance, the fluid power port has variables pressure and flow, while translational port has variables for force and velocity.

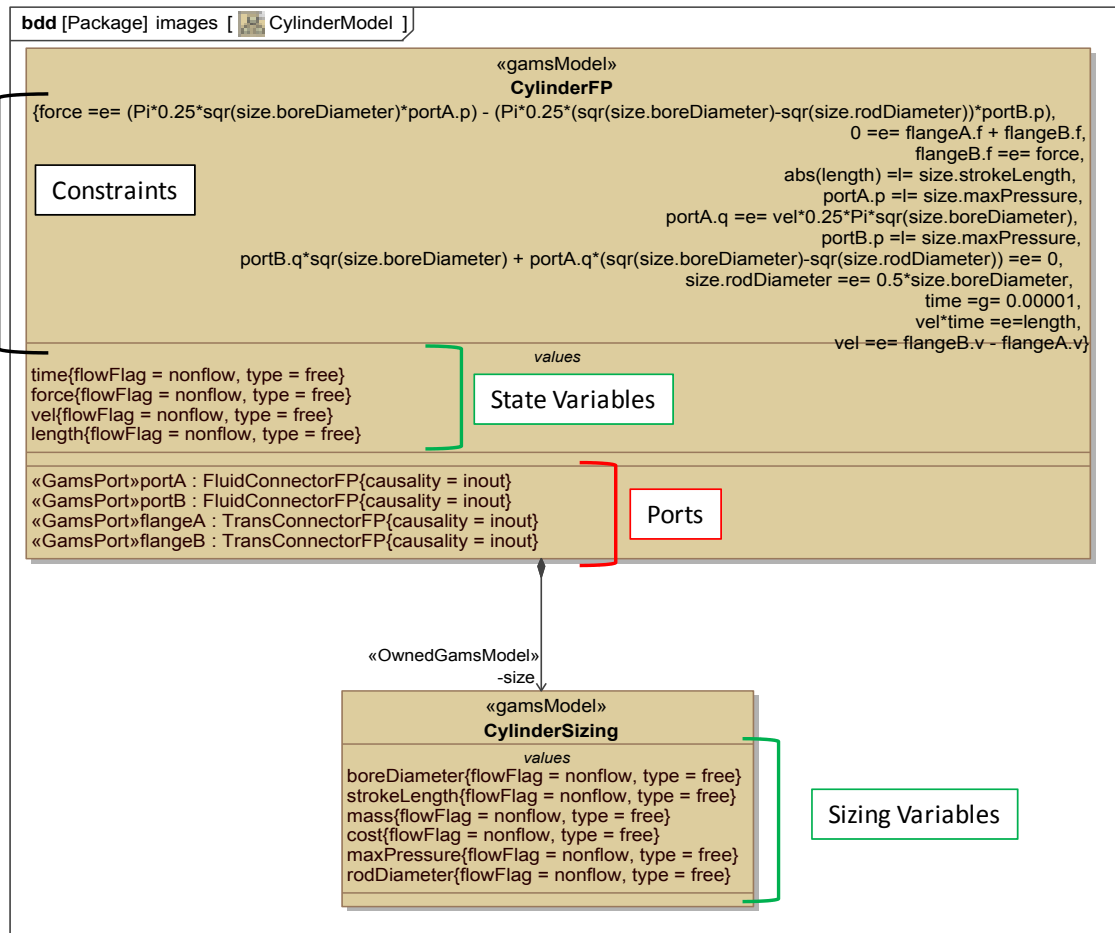


Figure 20 GAMS-compliant SysML model to capture the idealized hydraulic behavior for a double acting cylinder. The equations used to model the cylinder are displayed in the Constraints area in the CylinderFP Block

Based on these variables, declarative constraints are established to model cylinder behavior. For instance, consider the equation for force generated:

$$force = (Pi \cdot 0.25 \cdot sqr(boreDiameter) \cdot portA.p) - (Pi \cdot 0.25 \cdot (sqr(boreDiameter) - sqr(rodDiameter)) \cdot portB.p); \quad (4)$$

This is a declarative equation and therefore it does not matter what variable is known or unknown; the necessary symbolic manipulation is done at the time of solving by the solver.

Fixed Displacement Pump:

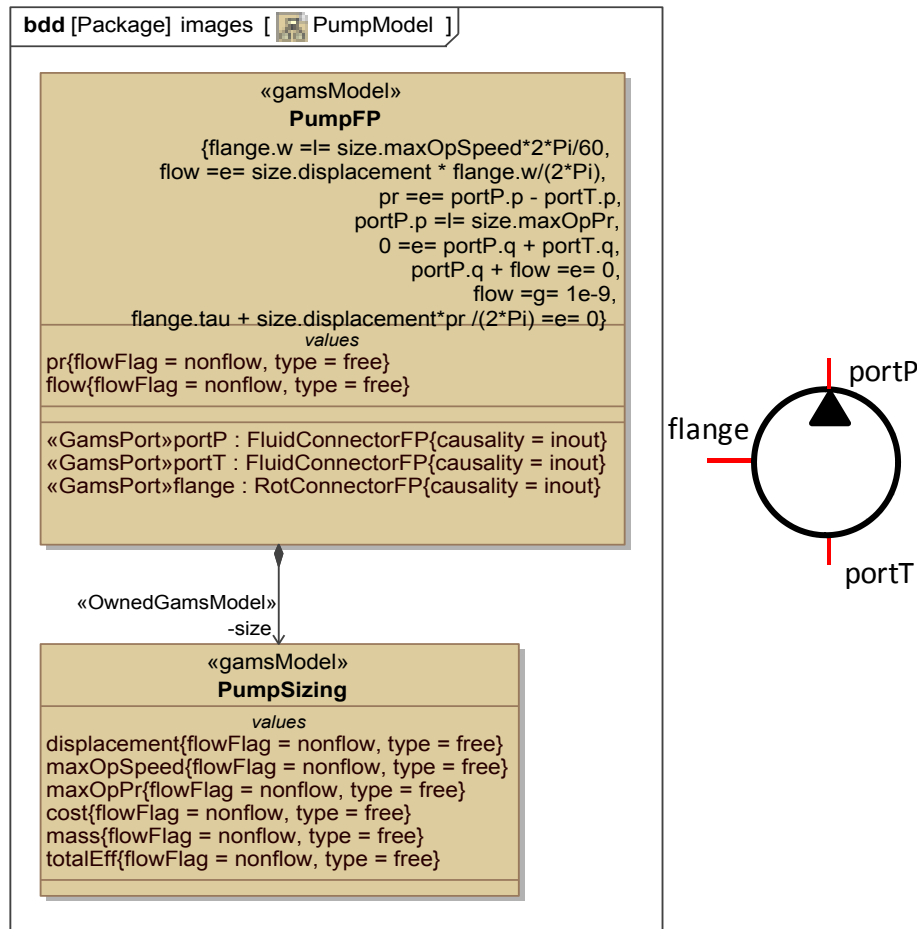


Figure 21 GAMS-compliant SysML model to capture the idealized hydraulic behavior for a fixed displacement pump.

Similar to the cylinder, a model for a fixed displacement pump is shown in Figure 21. The pump has three ports: one rotational flange and two fluid power ports. The sizing variables, state variables, ports and constraints are shown, similar to the cylinder model.

4/3 Directional Control Valve:

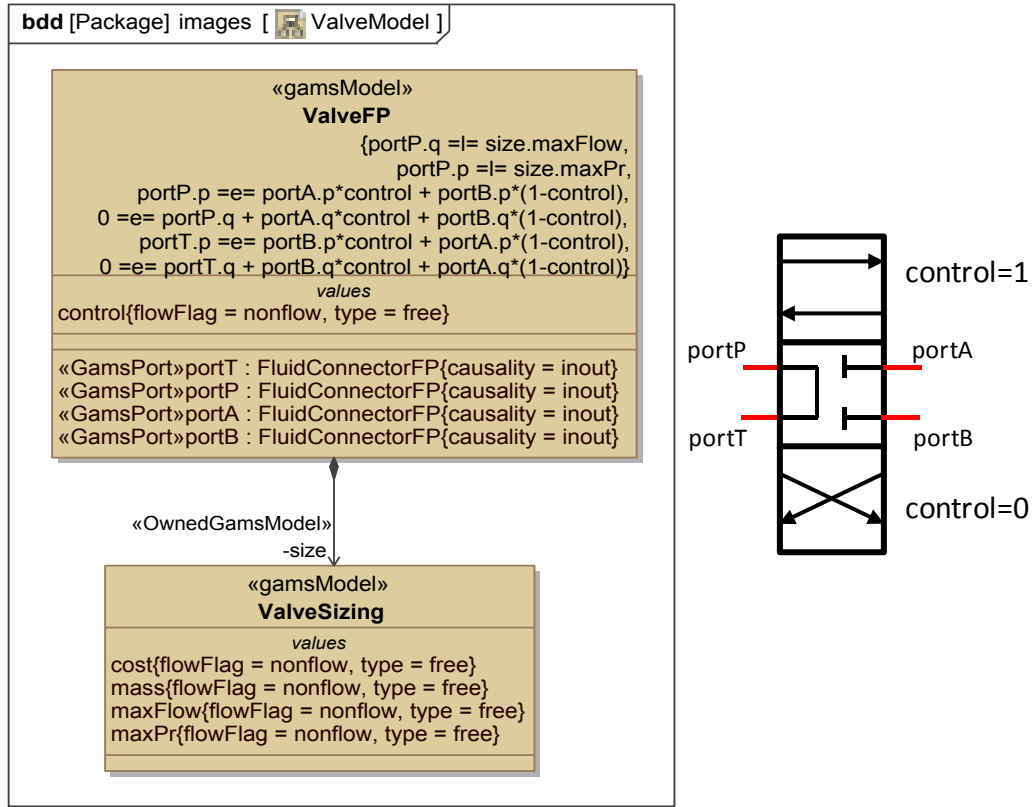


Figure 22 GAMS-compliant SysML model to capture the idealized hydraulic behavior for a 4-way 3-position open center directional control valve.

An open center type valve is chosen for the log splitter problem, and is shown in Figure 22. It has four ports: portP connecting to pump, portT connecting to tank, and ports A and B connecting to actuator. A *control* variable is used to determine the position of the valve; two positions are considered, corresponding to forward and reverse phases (*control* = 1 and 0 respectively). In this way, the internal connections are established depending on the control value.

It is assumed that the valve changes position instantaneously, which is different from the gradual opening that normally occurs. Moreover, pressure losses in the input

and output side of the valve is not considered. These losses can be modeled by modifying the equations relating the pump pressure to the actuator side pressure. In order to model gradual opening of the valve, it may be possible to divide the two use-phases considered in this problem into multiple use-phases to model the change in position of valve. This is left to future work.

Gas Engine:

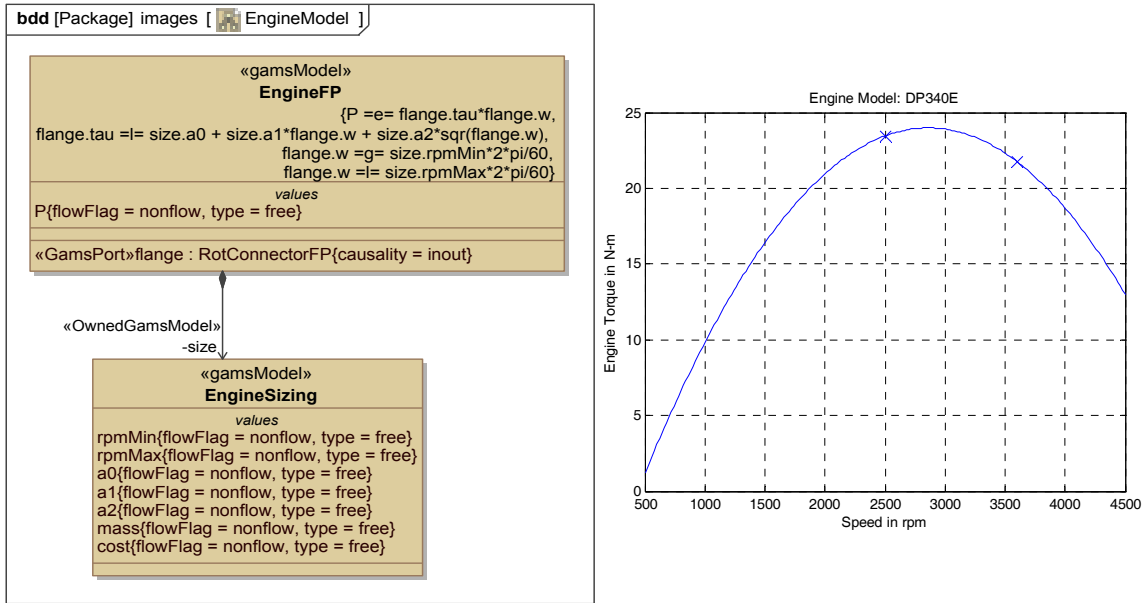


Figure 23 GAMS-compliant SysML model to capture the idealized behavior for an IC gas engine.

Similar to the previous components, the engine consists of variables, ports, and constraints. Based on the supplier information (two engine points – max torque and max power), a quadratic curve is fitted through these two points. The curve is of the form:

$$\begin{aligned}
 \tau &= a_0 + a_1 w + a_2 w^2 \\
 P &= \tau w \\
 \frac{d\tau}{dw} &= a_1 + 2a_2 w \\
 \frac{dP}{dw} &= a_0 + 2a_1 w + 3a_2 w^2
 \end{aligned} \tag{5}$$

Four equations are possible: max torque, max power, derivatives for torque and power at max conditions. Based on these equations, quadratic interpolation is performed for each engine in the supplier catalog. The model of the engine is represented by an inequality in which the torque must lie under the curve.

$$\begin{bmatrix} 1 & w_{\tau_{max}} & w_{\tau_{max}}^2 \\ 0 & 1 & 2w_{\tau_{max}} \\ 1 & w_{P_{max}} & w_{P_{max}}^2 \\ 1 & 2w_{P_{max}} & 3w_{P_{max}}^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \tau_{max} \\ 0 \\ \frac{P_{max}}{w_{max}} \\ 0 \end{bmatrix} \quad (6)$$

The use of a curve within the model highlights the ability of the solver to handle searching both across alternative engines as well as below the curve for each engine. Again, losses have not been considered; but they can be added to these models without affecting other components.

Supplier Catalog Models for Components:

For the component models described above, the actual supplier catalog information for each is presented below. This data was collected by Richard Malak and Lina Tucker from manufacturers and vendors. The highlighted cells represent the components selected in the different scenarios for the example problem discussed in Chapter 4.

Table 11 Cylinder Catalog Data

Cylinder Id	Stroke Length (m)	Bore Diameter (m)	Maximum Operating Pressure (Pa)	Mass (kg)	Cost (kg)
SAE-64508	0.20	0.11	2.068E+07	24.49	217.88
SAE-64008	0.20	0.10	2.068E+07	19.05	192.79
HMW-5008	0.20	0.13	1.724E+07	32.60	158.75
PMC-5608	0.20	0.10	1.724E+07	15.88	149.87
PMC-5508	0.20	0.09	1.724E+07	11.79	118.97
PMC-8308	0.20	0.08	1.724E+07	9.98	103.35
HMW-4008	0.20	0.10	2.068E+07	18.50	96.53
PMC-5408	0.20	0.06	1.724E+07	7.71	92.72
HMW-3508	0.20	0.09	2.068E+07	14.80	81.63
HMW-3008	0.20	0.08	2.068E+07	11.20	69.31
HMW-2508	0.20	0.06	2.068E+07	9.00	61.96
HMW-1508	0.20	0.04	2.068E+07	5.20	61.39
HMW-2008	0.20	0.05	2.068E+07	6.10	57.00
HMW-5010	0.25	0.13	1.724E+07	34.50	166.30
HMW-4010	0.25	0.10	2.068E+07	20.20	107.40
HMW-3510	0.25	0.09	2.068E+07	16.20	85.16
HMW-3010	0.25	0.08	2.068E+07	12.30	74.97
HMW-2510	0.25	0.06	2.068E+07	9.90	66.36
HMW-1510	0.25	0.04	2.068E+07	5.80	65.21
HMW-2010	0.25	0.05	2.068E+07	6.90	60.71
SAE-64512	0.30	0.11	2.068E+07	28.12	227.49
SAE-64012	0.30	0.10	2.068E+07	21.77	199.77
HMW-5012	0.30	0.13	1.724E+07	36.51	187.68
PMC-5612	0.30	0.10	1.724E+07	18.60	153.38
PMC-5512	0.30	0.09	1.724E+07	13.15	127.51
HMW-4012	0.30	0.10	2.068E+07	21.90	114.20
PMC-8312	0.30	0.08	1.724E+07	11.79	111.99
PMC-5412	0.30	0.06	1.724E+07	9.07	97.43
HMW-3512	0.30	0.09	2.068E+07	17.60	89.98
HMW-3012	0.30	0.08	2.068E+07	13.40	81.63
HMW-2512	0.30	0.06	2.068E+07	10.90	68.63
HMW-2012	0.30	0.05	2.068E+07	7.70	63.52
HMW-5014	0.36	0.13	1.724E+07	38.41	203.61
PMC-5614	0.36	0.10	1.724E+07	20.41	157.63
PMC-5514	0.36	0.09	1.724E+07	14.51	131.81
HMW-4014	0.36	0.10	2.068E+07	23.51	123.79
PMC-8314	0.36	0.08	1.724E+07	13.15	116.98
PMC-5414	0.36	0.06	1.724E+07	9.98	99.93
HMW-3514	0.36	0.09	2.068E+07	18.90	94.06
HMW-3014	0.36	0.08	2.068E+07	14.50	85.16
HMW-2514	0.36	0.06	2.068E+07	11.90	73.71
HMW-2014	0.36	0.05	2.068E+07	8.50	67.05
SAE-64516	0.41	0.11	2.068E+07	32.21	239.45
HMW-5016	0.41	0.13	1.724E+07	40.41	216.07
SAE-64016	0.41	0.10	2.068E+07	24.95	207.11
PMC-5616	0.41	0.10	1.724E+07	21.77	161.89
PMC-5516	0.41	0.09	1.724E+07	15.42	136.10
HMW-4016	0.41	0.10	2.068E+07	25.21	128.44
PMC-8316	0.41	0.08	1.724E+07	14.06	121.88
HMW-3516	0.41	0.09	2.068E+07	20.30	104.47
PMC-5416	0.41	0.06	1.724E+07	10.43	102.52
HMW-3016	0.41	0.08	2.068E+07	15.60	88.86
HMW-2516	0.41	0.06	2.068E+07	12.90	76.43
HMW-2016	0.41	0.05	2.068E+07	9.30	72.15
HMW-5018	0.46	0.13	1.724E+07	42.41	228.73
HMW-4018	0.46	0.10	2.068E+07	26.80	133.52
HMW-3518	0.46	0.09	2.068E+07	21.60	110.79
HMW-3018	0.46	0.08	2.068E+07	16.70	92.25
HMW-2518	0.46	0.06	2.068E+07	13.90	80.05
HMW-2018	0.46	0.05	2.068E+07	10.10	76.23
SAE-64520	0.51	0.11	2.068E+07	36.29	250.64
HMW-5020	0.51	0.13	1.724E+07	44.31	239.91
SAE-64020	0.51	0.10	2.068E+07	28.12	214.92
PMC-5620	0.51	0.10	1.724E+07	25.40	170.29
PMC-5520	0.51	0.09	1.724E+07	17.24	144.70
HMW-4020	0.51	0.10	2.068E+07	28.50	139.43
PMC-8320	0.51	0.08	1.724E+07	15.88	130.47
HMW-3520	0.51	0.09	2.068E+07	23.01	116.74
PMC-5420	0.51	0.06	1.724E+07	12.25	107.23
HMW-3020	0.51	0.08	2.068E+07	17.80	97.45
HMW-2520	0.51	0.06	2.068E+07	14.90	86.48
HMW-2020	0.51	0.05	2.068E+07	10.90	79.25
SAE-64524	0.61	0.11	2.068E+07	40.37	263.15
HMW-5024	0.61	0.13	1.724E+07	48.21	248.55
SAE-64024	0.61	0.10	2.068E+07	31.30	221.90
9-6890	0.61	0.10	2.068E+07	32.21	199.95
PMC-5624	0.61	0.10	1.724E+07	28.12	177.32
PMC-5524	0.61	0.09	1.724E+07	19.96	148.16
HMW-4024	0.61	0.10	2.068E+07	31.81	146.66

Cylinder Id	Stroke Length (m)	Bore Diameter (m)	Maximum Operating Pressure (Pa)	Mass (kg)	Cost (kg)
PMC-8324	0.6096	0.0762	1.724E+07	18.60	139.11
HMW-3524	0.6096	0.0889	2.068E+07	25.71	125.88
PMC-5424	0.6096	0.0635	1.724E+07	13.61	111.80
HMW-3024	0.6096	0.0762	2.068E+07	20.00	109.05
HMW-2524	0.6096	0.0635	2.068E+07	16.90	92.96
HMW-2024	0.6096	0.0508	2.068E+07	12.50	86.48
HMW-5028	0.7112	0.127	1.724E+07	52.21	260.03
HMW-4028	0.7112	0.1016	2.068E+07	35.11	167.88
HMW-3528	0.7112	0.0889	2.068E+07	28.40	146.78
HMW-3028	0.7112	0.0762	2.068E+07	22.20	121.09
HMW-2528	0.7112	0.0635	2.068E+07	18.80	102.64
HMW-2028	0.7112	0.0508	2.068E+07	14.10	89.94
SAE-64530	0.762	0.1143	2.068E+07	47.17	279.51
HMW-5030	0.762	0.127	1.724E+07	54.11	270.73
SAE-64030	0.762	0.1016	2.068E+07	35.83	233.17
PMC-5630	0.762	0.1016	1.724E+07	32.66	190.62
HMW-4030	0.762	0.1016	2.068E+07	36.80	180.06
PMC-5530	0.762	0.0889	1.724E+07	21.77	160.73
HMW-3530	0.762	0.0889	2.068E+07	29.81	160.69
PMC-8330	0.762	0.0762	1.724E+07	20.87	151.81
HMW-3030	0.762	0.0762	2.068E+07	23.31	126.13
PMC-5430	0.762	0.0635	1.724E+07	15.88	118.78
HMW-2530	0.762	0.0635	2.068E+07	19.80	114.96
HMW-2030	0.762	0.0508	2.068E+07	14.90	108.94
HMW-5032	0.8128	0.127	1.724E+07	56.11	293.57
SAE-64532	0.8128	0.1143	2.068E+07	48.08	284.73
SAE-64032	0.8128	0.1016	2.068E+07	37.65	237.93
PMC-5632	0.8128	0.1016	1.724E+07	33.57	198.80
HMW-4032	0.8128	0.1016	2.068E+07	38.41	189.53
HMW-3532	0.8128	0.0889	2.068E+07	31.21	165.80
PMC-5532	0.8128	0.0889	1.724E+07	23.59	164.84
PMC-8332	0.8128	0.0762	1.724E+07	21.77	155.93
HMW-3032	0.8128	0.0762	2.068E+07	24.30	133.04
HMW-2532	0.8128	0.0635	2.068E+07	20.80	123.75
PMC-5432	0.8128	0.0635	1.724E+07	18.60	122.80
HMW-2032	0.8128	0.0508	2.068E+07	15.80	116.35
HMW-5036	0.9144	0.127	1.724E+07	60.01	309.55
SAE-64536	0.9144	0.1143	2.068E+07	52.16	295.03
SAE-64036	0.9144	0.1016	2.068E+07	40.82	252.80
PMC-5636	0.9144	0.1016	1.724E+07	36.29	205.08
HMW-4036	0.9144	0.1016	2.068E+07	41.71	202.31
HMW-3536	0.9144	0.0889	2.068E+07	33.91	180.43
PMC-5536	0.9144	0.0889	1.724E+07	25.40	173.16
PMC-8336	0.9144	0.0762	1.724E+07	23.59	164.42
HMW-3036	0.9144	0.0762	2.068E+07	26.50	142.86
HMW-2536	0.9144	0.0635	2.068E+07	22.80	128.00
PMC-5436	0.9144	0.0635	1.724E+07	19.96	127.93
HMW-2036	0.9144	0.0508	2.068E+07	17.40	121.32
HMW-5040	1.016	0.127	1.724E+07	63.91	345.05
SAE-64540	1.016	0.1143	2.068E+07	56.25	311.76
SAE-64040	1.016	0.1016	2.068E+07	43.54	255.21
PMC-5640	1.016	0.1016	1.724E+07	38.56	219.54
HMW-4040	1.016	0.1016	2.068E+07	45.01	216.57
HMW-3540	1.016	0.0889	2.068E+07	36.60	189.37
PMC-5540	1.016	0.0889	1.724E+07	27.22	181.43
HMW-3040	1.016	0.0762	2.068E+07	28.70	170.51
PMC-8340	1.016	0.0762	1.724E+07	25.40	168.68
HMW-2540	1.016	0.0635	2.068E+07	24.80	147.24
HMW-2040	1.016	0.0508	2.068E+07	19.01	134.64
SAE-64542	1.0668	0.1143	2.068E+07	58.06	336.75
SAE-64042	1.0668	0.1016	2.068E+07	45.36	262.42
PMC-5642	1.0668	0.1016	1.724E+07	41.73	223.38
PMC-5542	1.0668	0.0889	1.724E+07	29.03	185.59
PMC-8342	1.0668	0.0762	1.724E+07	26.76	172.84
PMC-5442	1.0668	0.0635	1.724E+07	21.32	147.70
HMW-5048	1.2192	0.127	1.724E+07	71.71	381.43
SAE-64548	1.2192	0.1143	2.068E+07	66.68	339.57
SAE-64048	1.2192	0.1016	2.068E+07	52.16	284.22
HMW-4048	1.2192	0.1016	2.068E+07	51.71	253.33
PMC-5648	1.2192	0.1016	1.724E+07	45.36	234.19
PMC-5548	1.2192	0.0889	1.724E+07	31.75	224.39
HMW-3548	1.2192	0.0889	2.068E+07	42.01	214.85
HMW-3048	1.2192	0.0762	2.068E+07	33.11	189.40
PMC-8348	1.2192	0.0762	1.724E+07	29.48	188.82
HMW-2548	1.2192	0.0635	2.068E+07	28.70	185.94
HMW-2048	1.2192	0.0508	2.068E+07	22.20	161.20
SAE-64560	1.524	0.1143	2.068E+07	80.29	404.30
SAE-64060	1.524	0.1016	2.068E+07	62.60	284.04
PMC-5660	1.524	0.1016	1.724E+07	54.43	273.41

Table 12 Pump Catalog Data

Pump Id	Displacement (m3/rev)	Max operating Pr (Pa)	Max operating RPM (rpm)	Weight (kg)	Cost (\$)
SNP2NN_4_0	3.9329E-06	2.499E+07	4000	2.31	218.96
SNP2NN_6_0	6.06321E-06	2.499E+07	4000	2.40	222.66
SNP2NN_8_0	8.3574E-06	2.499E+07	4000	2.49	228.18
SNP2NN_011	1.08155E-05	2.499E+07	4000	2.63	232.49
SNP2NN_014	1.44206E-05	2.499E+07	3500	2.86	254.02
SNP2NN_017	1.67148E-05	2.499E+07	3000	2.95	258.31
SNP2NN_019	1.91729E-05	2.099E+07	3000	3.04	264.47
SNP2NN_022	2.2778E-05	1.800E+07	3000	3.18	266.95
SNP2NN_025	2.52361E-05	1.600E+07	3000	3.31	269.40
SKP2NN_8_0	8.3574E-06	2.499E+07	4000	2.49	228.18
SKP2NN_011	1.08155E-05	2.499E+07	4000	2.63	232.49
SKP2NN_014	1.44206E-05	2.499E+07	3500	2.86	254.02
SKP2NN_017	1.67148E-05	2.499E+07	3000	2.95	258.31
SKP2NN_019	1.91729E-05	2.399E+07	3000	3.04	264.47
SKP2NN_022	2.2778E-05	2.099E+07	3000	3.18	266.95
SKP2NN_025	2.52361E-05	1.900E+07	3000	3.31	269.40
DE1L-07	7.04644E-06	2.758E+07	3400	7.17	348.86
DE1L-10	9.5045E-06	2.758E+07	3400	7.30	350.58
DE1L-13	1.2618E-05	2.758E+07	3400	7.48	352.31
DE1L-14	1.42567E-05	2.758E+07	3400	7.57	353.16
DE1L-17	1.70425E-05	2.758E+07	3400	7.76	354.03
DE1L-19	1.9009E-05	2.758E+07	3400	7.89	354.89
DE1L-21	2.04838E-05	2.758E+07	3400	7.94	361.78
DE1L-23	2.24503E-05	2.758E+07	3400	8.07	363.51
DE1L-25	2.53999E-05	2.758E+07	3400	8.26	366.10
DE1L-29	2.90051E-05	2.758E+07	3200	8.44	367.82
DE1L-32	3.17909E-05	2.758E+07	3000	8.62	383.34
DE1L-38	3.8018E-05	2.275E+07	2750	8.98	386.79
DE1L-41	4.09677E-05	2.068E+07	2500	9.16	391.96
CPB-020	3.2938E-05	2.482E+07	3200	8.75	837.80
CPB-023	3.6707E-05	2.482E+07	3200	8.89	843.95
CPB-026	4.16231E-05	2.482E+07	3200	9.07	850.21
CPB-030	4.78502E-05	2.482E+07	3200	9.30	858.56
CPB-032	5.14554E-05	2.482E+07	3200	9.48	866.67
CPB-035	5.5716E-05	2.482E+07	3200	9.66	871.34
CPB-040	6.35818E-05	2.482E+07	3200	10.07	883.77
CPB-045	7.16115E-05	2.482E+07	3000	10.48	897.61
CPB-050	7.94773E-05	2.275E+07	2750	10.89	907.37
CPB-055	8.78347E-05	2.068E+07	2500	11.29	921.06
CPB-060	9.57005E-05	1.862E+07	2500	11.70	934.97
SKP1NN_12	1.17987E-06	2.499E+07	4000	1.03	213.05
SKP1NN_17	1.57316E-06	2.499E+07	4000	1.05	213.85
SKP1NN_22	2.09754E-06	2.499E+07	4000	1.09	216.18
SKP1NN_26	2.62193E-06	2.499E+07	4000	1.11	218.49
SKP1NN_32	3.14632E-06	2.499E+07	4000	1.14	220.04
SKP1NN_38	3.65432E-06	2.499E+07	4000	1.18	220.81
SKP1NN_43	4.19509E-06	2.499E+07	3000	1.20	222.35
SKP1NN_60	5.88296E-06	2.299E+07	3000	1.30	223.89
SKP1NN_78	7.58721E-06	1.999E+07	3000	1.39	225.45
SKP1NN_010	9.94695E-06	1.500E+07	2000	1.55	227.99
SKP1NN_012	1.19953E-05	1.200E+07	2000	1.65	230.01
SNP3NN_022	2.21225E-05	2.499E+07	3000	6.80	410.94
SNP3NN_026	2.62193E-05	2.499E+07	3000	6.80	415.67
SNP3NN_033	3.31019E-05	2.499E+07	3000	7.17	426.41
SNP3NN_038	3.8018E-05	2.499E+07	3000	7.30	429.54
SNP3NN_044	4.40812E-05	2.499E+07	3000	7.48	433.76
SNP3NN_048	4.80141E-05	2.310E+07	3000	7.62	436.99
SNP3NN_055	5.50605E-05	2.310E+07	2500	7.85	443.51
SNP3NN_063	6.34179E-05	4.168E+07	2500	8.12	449.80
SNP3NN_075	7.43973E-05	1.820E+07	2500	8.48	456.87
SNP3NN_090	8.81624E-05	1.500E+07	2500	8.89	468.79

Table 13 Valve Catalog Data

Valve Id	Max Operating flow (m3/s)	Max Operating Pressure (Pa)	Weight (kg)	Cost (\$)
SurplusCenter-9-6765	0.001261804	20684271.87	5.44	74.50
SurplusCenter-9-1684	0.001577255	15513203.9	4.54	76.50
SurplusCenter-9-1262	0.001577255	13789514.58	4.99	76.50
SurplusCenter-9-6766	0.001261804	20684271.87	5.44	77.50
SurplusCenter-9-6767	0.001261804	20684271.87	5.44	82.50
SurplusCenter-9-6701	0.001577255	20684271.87	7.26	97.95
SurplusCenter-9-6759	0.001577255	20684271.87	7.26	105.95
SurplusCenter-9-6701-F	0.001577255	20684271.87	7.26	112.95
SurplusCenter-9-4500	0.000315451	17236893.23	2.27	169.95
SurplusCenter-9-1517	0.001577255	18960582.55	3.63	76.95
SurplusCenter-9-1518	0.001387984	15513203.9	4.54	76.95
SurplusCenter-9-1789	0.001892706	13789514.58	6.35	96.95
SurplusCenter-9-5174	0.001135624	20684271.87	3.18	179.99
MSCDirect-01825629	0.001577255	18960582.55	4.54	109.70
DrillSpot-40529	0.001009443	20684271.87	4.54	144.85
DrillSpot-40861	0.001892706	20684271.87	6.80	166.06
DrillSpot-40642	0.001261804	20684271.87	4.54	134.84
DrillSpot-40480	0.001892706	20684271.87	6.80	168.36
MSCDirect-01825678	0.001892706	20684271.87	4.54	94.23
MSCDirect-01825751	0.001261804	24131650.52	5.44	132.77
NT-202305	0.000675065	31026407.81	1.81	109.99
NT-201302	0.001059915	31026407.81	1.81	119.99
NT-202502	0.001665581	31026407.81	5.90	139.99
NT-201505	0.001249186	31026407.81	5.90	139.99
NT-2020	0.001577255	13789514.58	4.54	70.00
NT_Prince-2036	0.001577255	13789514.58	4.99	89.00
NT_Northman-202508	0.001577255	20684271.87	4.08	105.00
NT_Prince-2010	0.001261804	20684271.87	4.99	80.00
NT_Prince-2035	0.001577255	20684271.87	4.99	80.00
NT_Prince-20114	0.001892706	20684271.87	4.54	140.00
NT_Northman-202509	0.001577255	20684271.87	4.08	105.00
NT_BrandHyd-20120	0.002839059	20684271.87	7.71	270.00
NT_Prince-20113	0.001261804	20684271.87	4.54	90.00
NT_BrandHyd-20119	0.001135624	20684271.87	3.63	145.00

Table 14 Engine Catalog Data

Engine Id	Max Power (W)	RPM at max Power (rpm)	Max torque (N-m)	RPM at max torque (rpm)	Weight (kg)	Cost (\$)
CS4T-901502	2983	3600	7.46	2600	19.05	312.00
CS6T	4474	3600	10.98	2400	18.14	356.04
CS8_5T	6338	3600	16.54	2400	30.39	608.00
CS12T	8948	3600	24.95	2400	41.28	736.00
CH18S	13423	3600	41.35	2600	40.82	1350.83
CH20S	14914	3600	44.06	2600	40.82	1506.34
CH22S	16405	3600	44.74	2600	40.82	1397.60
CH15T	11185	3600	32.27	2400	40.05	937.12
CS10T	7457	3600	19.93	2400	31.98	688.17
CV15T	11185	3600	32.27	2400	39.46	801.21
CV18S	13423	3600	41.35	2600	40.82	1607.78
CV20S	14914	3600	44.06	2600	40.82	1562.58
CV22S	16405	3600	44.74	2600	40.82	1539.33
CV25S	18642	3600	54.23	2200	42.64	1906.75
DP160	4101	3600	10.85	2500	15.00	229.99
DP200	4847	3600	13.02	2500	16.00	249.99
DP240	5966	3600	16.68	2500	24.95	249.99
DP270	6711	3600	18.98	2500	24.95	299.99
DP340	8203	3600	18.98	2500	24.95	349.99
DP390	9694	3600	26.44	2500	30.84	399.99
DP120V	2983	3600	5.97	2700	12.47	199.99
DP160V	4101	3600	8.00	2700	15.42	279.99
DP225	5593	3600	16.00	2500	18.14	249.99
DP200E	4847	4000	12.88	2500	16.74	349.99
DP240E	5966	3600	16.68	2500	26.31	349.99
DP270E	6711	4000	18.98	2500	29.48	399.99
DP340E	8203	3600	23.46	2500	32.66	399.99
DP390E	9694	3600	26.44	2500	32.93	449.99
DP420E	11931	3600	28.47	2500	36.29	499.99
DP225E	5593	3600	16.00	2500	19.96	349.99
NT-Honda-6059	4101	3900	10.71	2900	17.24	364.99
NT-Honda-60242	15287	3600	45.96	2500	43.09	1299.99
NT-Honda-60694	3878	3600	11.25	2500	13.79	259.99
NT-Honda-6066	8203	3600	25.08	2500	30.98	674.99
NT-Honda-60863	5294	3600	15.32	2500	24.99	699.99
NT-Honda-60968	5966	3600	17.76	2500	24.99	589.99
NT-Honda-6032	2610	3600	7.32	2500	13.02	349.99
NT-Honda-605921	3579	3600	10.30	2500	13.02	374.99
NT-Honda-6067	4101	3600	12.34	2500	16.01	399.99
Honda-GC160	3728	3600	10.30	2500	13.52	280.00
Honda-GX160	4101	3600	10.85	2500	15.42	310.00
Honda-GX270	6711	3600	18.98	2500	25.40	590.00
Honda-GX120	2983	3600	6.78	2500	13.15	370.00
Honda-GX200	4474	3600	13.29	2500	20.87	400.00
Honda-GX240	5966	3600	16.27	2500	25.40	579.00

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