نموذج رقم (1)

إقـــــرار

أنا الموقع أدناه مقدم الرسالة التي تحمل العنوان:

صياغة هاملتون الأمامية وتكامل المسار لفعل بوليكوف داي برين ثابت المقياس

Light-Front Hamiltonian and Paht Integral Formulations of the Conformally Gauge-Fixed Polyakov D1 Brane Action

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INSTANT FORM THEORY AND LIGHT-FRONT THEORY HAMILTONIAN FORMULATION OF THE CONFORMALLY GAUGE-FIXED POLYAKOV D1 BRANE ACTION

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نتيجة الحكم على أطروحة ماجستير

بناءً على موافقة شئون البحث العلمي والدراسات العليا بالجامعة الإسلامية بغزة على تشكيل لجنة الحكم على أطروحة الباحثة/ فدوى علي سليمان المصري لنيل درجة الماجستير في كلية العلوم قسم الفيزياء وموضوعها:

صياغة هاملتون الأمامية وتكامل المسار لفعل بوليكوف داي برين ثابت المقياس

Light-Front Hamiltonian and Path Integral formulations of the Conformally Gauge-Fixed Polyakov D1 Brane Action

وبعد المناقشة التي تمت اليوم الثلاثاء 03 ذو القعدة 1436هـ، الموافق2015/08/18م الساعة العاشرة صباحاً، اجتمعت لجنة الحكم على الأطروحة والمكونة من:

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وبعد المداولة أوصت اللجنة بمنح الباحثة درجة الماجستير في كلية العلوم/ قسم الفيزياء.

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ABSTRACT

We have studied the instant form quantization and light front quantization of conformally gauge fixed polyakov D1 brane action with and without scalar dilation field and showed these theories when consider in the presence of background gauge fields such as the 2- form gauge field $B_{\alpha\beta}(\sigma,\tau)$ or in the presence of $U(1)$ gauge field $A_{\alpha}(\sigma,\tau)$ and constant scalar axion field $C(\sigma,\tau)$.

The instant form quantization is studied in the equal world sheet time framework on hyperplanes defined by world $\sigma^{o} = \tau =$ constant and light front quantization in equal light cone world sheet time framework in hyperplane defined by $\sigma^{+} = (\tau + \sigma)$ = constant.

The Hamilton formulation is given by two approaches: The first is Dirac approach while the second is Güler approach. The equal equations of motion are obtained as total differential equations in many variables.

These equations of motion are in exact agreement with those equations that had been obtained using Dirac's method.

ملـــــخص

صياغة هاملتون لمجال بيليوكوف

تـم دراسـة تكمـيم نظـام المجـال البليوكـوفي ذو الفعـل ببعـد D1 بوجـود نمـوذجين للمجـال $B_{\alpha\beta}(\sigma,\tau)$ مع وجود U(1) والمجال $A_{\alpha}(\sigma,\tau)$ مع وجود $C(\sigma,\tau)$

كمـا تـم دراسـة النمـوذج اللحظـي لنظـام المجـال البليوكـوفي فـي المسـتوى الزمنـي الثابـت علـى المستويات العليا باعتبار (ثابت= $\sigma^{o}=\tau$).

ودراسـة المسـتويات المخروطيـة فـي المسـتويات العليـا باعتبـار (ثابـت= $\sigma^{+}=(\tau+\sigma)$) باسـتخدام الصياغة الهاملتونية باستخدام طريقتي ديراك وجولر لكتابة معادلات الحركة لهذه الانظمة وقد تبين أن الطريقتين تعطيا نفس النتائج.

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CHAPTER ONE

INTRODUCTION

1.1 Introduction

In 1933, Dirac made the observation that action plays a central role in classical mechanics [1]. He considered the Lagrangian formulation of classical mechanics to be more fundamental than the Hamiltonian formulation, but that it seemed to have no important role in quantum mechanics as it was known at that time. He speculated how this situation might be rectified, and he arrived at conclusion that the propagator in quantum mechanics "corresponds to" exp $iS/\hbar$ where S is the classical action evaluated along the classical path.

In 1948, Feynman developed Dirac's suggestion, and succeeded in deriving a third formulation of quantum mechanics, based on the fact that the propagator can be written as a sum over all possible paths (not just the classical one) between the initial and final points. Each path contributes exp $iS/\hbar$ to the propagator [2].

The Hamiltonian formulation of constrained system in classical mechanics was initiated by Dirac in 1933 [1,2], who set up a formalism for treating singular systems and the constraints involved. He showed that, in the presence of constraints, the number of degrees of freedom of the dynamical system was reduced. His approach are subsequently extended to continuous systems [3].

Following Dirac, there is another approach for quantizing constrained systems of classical singular theories, which was initiated by Feynman Kernel [4,5], who first set up a formalism of the path integral formalism when only first-class constraints in the canonical gauge are present [6,7]. The generalization of the methods to theories with second-class constraints is given by Senjanovic [8]. Fradkin and Vilkovisky [9,10] rederived both results in a broader context, where they improved Faddeev's procedure mainly to include covariant constraints; also they extended this procedure to the Grossman variables. When the dynamical system possesses some second-class constraints there exists another method given by Batalin and Fradkin [11]: the BFV-BRST operator quantization method. More, Gitman and Tyutin [12] discussed the canonical quantization of singular theories as well as the Hamiltonian formalism of gauge theories in an arbitrary gauge. An alternative approach was developed by Bukenhout, Sprague and Faddeev [13,14] without following Dirac step by step. In this formalism there is no need to distinguish between first and second –class or primary and secondary constraints, where the primary constraint is a set of relations connecting between the momenta and the coordinates. The general formalism is then applied to several problems, quantization of the massive Yang-Mills field theory, Light-Cone quantization of the self interacting scalar field, and quantization of a local field theory of magnetic monopolies, etc.

In 1992, Güler developed a formalism based on Hamilton Jacobi formulation of constrained system [15,16] which has been developed to investigate the constrained systems. Several constrained systems were investigated by using the Hamilton-Jacobi approach [17-36]. The Cathodovy equivalent Lagrangian method is used to obtain the set of Hamilton-Jacobi Partial Differential Equations (HJPDE). In this approach, the distinction between the first and second-class constraints is not necessary. The equations of motion are written as total differential equations in many variables, which require the investigation of the integrability conditions. In other words, the integrability conditions may lead to new constraints. Moreover, it is shown that gauge fixing, which is an essential procedure to study singular system by Dirac's method, is not necessary if the Hamilton-Jacobi approach is used.

Following Hamilton-Jacobi approach, there is another approach for quantizing constrained systems of classical singular theories by path integral quantization [37].

In the following two sections a brief review of the last two formulations will be given.

1.2 Dirac's Method

The standard quantization methods can't be applied directly to the singular Lagrangian theories. However, the basic idea of the classical treatment and the quantization of such systems were presented a long time by Dirac [1,2], and is now widely used in investigating the theoretical models in a contemporary elementary particle physics and applied in high energy physics, especially in the gauge theories[12].

The presence of constraints in such theories makes one careful on applying Dirac's method, especially when first-class constraints arise. This is because the first-class constraints are generators of gauge transformation which lead to the gauge freedom [36].

Let us consider a system which is described by the Lagrangian

$$L \equiv L(q_i, \dot{q}_i; \tau), \qquad i = 1, \ldots, n. \tag{1.1}$$

such that the rank of the Hessian matrix is $(n\text{-}r)$, $r < n$.

$$A_{ij} = \frac{\partial^2 L}{\partial \dot{q}_i \partial \dot{q}_j}, \qquad i, j = 1, \ldots, n, \tag{1.2}$$

Since the rank of the Hessian matrix is $(n - r)$, the momenta components will be functionally dependent. The first $(n - r)$ equations of the momenta

$$p_i = \frac{\partial L}{\partial \dot{q}_i}, \tag{1.3}$$

can be solved for the $(n - r)$ components of $\dot{q}_i$ in terms of q_i as well as the first $(n - r)$ components of p_i and the last r components of $\dot{q}_i$.

In other words

$$\dot{q}_a = \omega_a(q_i, p_a, \dot{q}_\mu), \tag{1.4}$$

$$a = 1, \ldots, n - r \qquad \mu = 1, \ldots, r, \qquad i = 1, \ldots, n$$

The singular system characterized by the fact that all velocities $\dot{q}_i$ are not uniquely determined in terms of the coordinates and momenta only. In other words, not all momenta are independent, and there must exist a certain set of relations among them, of the form:

$$\chi_m(p_i, q_i) = 0, \qquad m = 1, 2, \ldots r \tag{1.5}$$

The q's and the p's are the dynamical variables of the Hamiltonian theory. They are connected by the relations eq.(1.5) which are called primary constraints of the Hamiltonian formalism.

Now the usual Hamiltonian H_0 for any dynamical system is defined as

$$H_0(p_i, q_i) = p_i \dot{q}_i - L\,, \tag{1.6}$$

(Here the Einstein summation rule is used which is a convention when repeated indices are implicitly summed over).

H_0 will not be uniquely determined, since we may add to it any linear combinations of the primary constraints χ'_μ's which are zero, so that the total Hamiltonian is [2,37]

$$H_T = H_0 + \lambda_\mu \chi'_\mu, \tag{1.7}$$

Where $\lambda_\mu(q,p)$ being some unknown coefficients, they are simply Lagrange's undetermined multipliers. Making use of the Poisson brackets, one can write the total time derivative of any function $g(q,p)$ as

$$\dot{g} \equiv \frac{dg}{d\tau} \approx \{g, H_T\} = \{g, H_0\} + \lambda_\mu \{g, \chi'_\mu\}, \tag{1.8}$$

Where Dirac's symbol (≈) for weak equality has been used in the sense that one can't consider $\chi'_\mu = 0$ identically before working out the Poisson brackets. Thus the equations of motion can be written as

$$\dot{q}_i \approx \{q_i, H_T\} = \{q_i, H_0\} + \lambda_\mu \left\{q_i, \chi_\mu{}'\right\} \tag{1.9}$$

$$\dot{p}_i \approx \{p_i, H_T\} = \{p_i, H_0\} + \lambda_\mu \left\{p_i, \chi_\mu{}'\right\} \tag{1.10}$$

Subject to the so-called consistency conditions. This means that the total time derivative of the primary constraints should be zero;

$$\dot{\chi}'_\mu \equiv \frac{d\,\chi'_\mu}{d\,\tau} \approx \left\{\chi'_\mu, H_T\right\}$$
$$= \left\{\chi'_\mu, H_0\right\} + \lambda_\nu \left\{\chi'_\mu, \chi'_\nu\right\} \approx 0, \qquad \mu,\nu = 1, \ldots r. \qquad (1.11)$$

These equations may be reduced to 0 = 0, where it is identically satisfied as a result of primary constraints, else they will be lead to new conditions which are called secondary constraints. Repeating this procedure as many times as needed, one arrives at a final set of constraints or/and specifies some of λ_μ. Such constraints are classified into two types, a) First-class constraints which have vanishing Poission brackets with all other constraints. b) Second-class constraints which have non-vanishing Poisson brackets. The second-class constraints could be used to eliminate conjugate pairs of the *p's* and *q's* form the theory by expressing them as functions of the remaining *p's* and *q's*.

The Poisson bracket of some arbitrary functions $A(q_i, p_i), B(q_i, p_i)$ of the canonical variables q_i, p_i is defined by

$$[A, B]_p = \frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i} - \frac{\partial B}{\partial q_i}\frac{\partial A}{\partial p_i} \qquad (1.12)$$

where the repeated indices are summed. One can check that it has the following properties

$$[A, B]_p = -[B, A], \qquad (1.13a)$$
$$[A, BC]_p = [A, B]_p C + B[A, C]_p, \qquad (1.13b)$$
$$[A, [B, C]_p]_p + [B, [C, A]_p]_p + [C, [A, B]_p]_p = 0, \qquad (1.13c)$$

Usually, we quantize this classical system by defining the commutators of two function of the canonical variables by

$$[A, B] = i\hbar\,[A, B]_p, \qquad (1.14)$$

However, if the canonical variables satisfy some constraints $\chi_n(q_i, p_i) = 0$, using eq.(1.14) to quantize the classical system would lead into some inconsistencies. For example, the constraint equations imply $\left[A, \chi_n\right] = 0$ for all χ_n and any function

$A(q_i, p_i)$ because we want to require the constraints being also satisfied quantum mechanically, but $[A, \chi_n]_p$ is in general nonzero even if the constraint equations are imposed[1].

The constraints can be divided into two classes. A constraint is called first class if its Poisson bracket with all the other constraints vanishes when we impose the constraints

$$[\chi_a, \chi_n]_p = f_{a,n}^{\ m} \chi_m, \tag{1.15}$$

where a runs through all the first class constraints, and m , n runs through all the constraints, and $f_{a,n}^{\ m}$ can be arbitrary functions of canonical variables.

The remaining constraints are called second class. We will introduce a way of quantization when the system has no first class constraints[1,38]. In such case, the Poisson brackets of the constraints can be summarized into

$$[\chi_m, \chi_n]_p = C_{mn}, \tag{1.16}$$

where C_{mn} is a nonsingular matrix, $\det C \neq 0$.

Dirac suggested a way of quantizing this system by defining the Dirac brackets

$$[A, B]_D = [A, B]_p - [A, \chi_m](C^{-1})^{mn} [\chi_n, B]_p, \tag{1.17}$$

The Dirac bracket has the same properties (1.13) as the Poisson bracket. It further satisfies

$$[\chi_n, A]_D = 0, \tag{1.18}$$

for all constraint χ and function *A*. Now, it is straightforward to define the commutator

$$[A, B] = i\hbar [A, B]_D, \tag{1.19}$$

1.3 Hamilton-Jacobi method (Güler Method)

Now we would like to discuss the constrained systems by Hamilton-Jacobi treatment [3,4], and demonstrate the fact that the gauge-fixing problem is solved naturally.

This method is a completely different method to investigate singular systems. The system that is described by the Lagrangian $L(q_i, \dot{q}_i, t)$ or L(φ, $\partial\varphi$) in field theory), $i = 1, \ldots, n$, is singular if the Hess matrix eq.(1.2) has a rank $(n - p)$, $p < n$. in this case we have P momenta are dependent of each other. In this case, the generalized momenta P_i corresponding to the generalized coordinates q_i are defined as

$$P_a = \frac{\partial L}{\partial \dot{q}_a}, \qquad a = 1, \ldots, n - p, \tag{1.20}$$

$$P_\mu = \frac{\partial L}{\partial \dot{q}_\mu}, \qquad \mu = n - p + 1, \ldots, n. \tag{1.21}$$

Since the rank of the Hess matrix is ($n - p$), one may solve (1.20) for $\dot{q}_a$ as

$$\dot{q}_a = \dot{q}_a(q_i, \dot{q}_\mu, P_b) \equiv \omega_a. \tag{1.22}$$

Substituting (1.22) into (1.21), we obtain relations in q_i, P_a, $\dot{q}_\nu$ and t in the form

$$P_\mu = \left.\frac{\partial L}{\partial \dot{q}_\mu}\right|_{\dot{q}_a = \omega_a} = -H_\mu(q_i, \dot{q}_\nu, \dot{q}_a = \omega_a, P_a, t), \qquad \nu = n - p + 1, \ldots, n. \tag{1.23}$$

The canonical Hamiltonian H_0 is defined as

$$H_0 = -L(q_i, \dot{q}_\mu, \dot{q}_a = \omega_a, t) + P_a \omega_a + \dot{q}_\mu P_{\mu|P_\nu = -H_\nu}. \tag{1.24}$$

The set of Hamilton-Jacobi partial differential equations (HJPDE) is expressed as

$$H'_\alpha(q_\beta; q_a; P_a = \frac{\partial S}{\partial q_a}; P_\mu = \frac{\partial S}{\partial q_\mu}) = 0, \qquad \alpha, \beta = 0, 1, \ldots, p, \tag{1.25}$$

where

$$H_0' = P_0 + H_0, \tag{1.26}$$
$$H_\mu' = P_\mu + H_\mu. \tag{1.27}$$

With $q_0 = t$ and S being the action. The equations of motion are obtained as total differential equations in many variables such as,

$$dq_a = \frac{\partial H_\alpha'}{\partial P_a} dt_\alpha, \tag{1.28}$$

$$dP_r = -\frac{\partial H_\alpha'}{\partial q_r} dt_\alpha, \qquad r = 0,1, \ldots, n. \tag{1.29}$$

$$dZ = (-H_\alpha + P_a \frac{\partial H_\alpha'}{\partial P_a}) dt_\alpha. \tag{1.30}$$

where Z = S(t_α,q_a). These equations are integrable if and only if

$$dH_0' = 0, \tag{1.31}$$

and

$$dH_\mu' = 0, \qquad \mu = n - p + 1, \ldots, n. \tag{1.32}$$

If the conditions (1.31) and (1.32) are not satisfied identically, we consider them as new constraints and we examine their variations. Thus repeating this procedure, one may obtain a set of constraints such that all the variations vanish, taking into account if the system is completely (where the set of equations of motion and the action function is integrable) or partially (where the set of equations of motion is only integrable).

1.4 The Bosonic String

In string theory, D-branes are a class of extended objects upon which open strings can end with Dirichlet boundary conditions, after which they are named. D-branes were discovered by Dai, Leigh and Polchinski, and independently by Hořava in 1989. In 1995, Polchinski identified D-branes with black p-brane solutions of supergravity, a discovery that triggered the Second Superstring Revolution and led to both holographic and M-theory dualities [39, 40].

D-branes are typically classified by their spatial dimension, which is indicated by a number written after the D. A D0-brane is a single point, a D1-brane is a line (sometimes called a "D-string"), a D2-brane is a plane, and a D25-brane fills the highest-dimensional space considered in bosonic string theory. There are also instantonic D(-1)-branes, which are localized in both space and time [39, 40].

We can easily generalize this construction to a string, which is a one-dimensional object described by a two-dimensional "worldsheet" that the string sweeps out as it moves in time with coordinates

$$\left(\xi^0,\xi^1\right)=\left(\tau,\sigma\right), \qquad \tau,\sigma\in R^n\subset\mathbb{R}^2 \tag{1.33}$$

Where R^n domain within a sset $\mathbb{R}^2$ of real number, and $0\le\sigma\le\pi$ is the spatial coordinate along the string, while $\tau\in\mathbb{R}$ describes its propagation in time. The string's evolution in time is described by functions $X^{\mu}(\tau,\sigma), \mu=0,1,...,d-1$ giving the shape of its worldsheet in the target spacetime (Fig. 1.1). The "induced metric" h_{ab} on the string worldsheet corresponding to tits embedding into space-time is given by the "pullback" of the flat Minkowski metric $\eta_{\mu\nu}$ to the surface,

$$h_{ab}=\eta_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu}, \tag{1.34}$$

where

$$\partial_a\equiv\frac{\partial}{\partial\xi^a}\ ,\ \ a=0,1. \tag{1.35}$$

If we move from ξ^{α} to $\xi^{\alpha}+d\xi^{\alpha}$ on the worldsheet, the corresponding change induced in the flat Mankowski spacetime through the string moves is $X^{\mu}\rightarrow X^{\mu}+\partial_a X^{\mu}d\xi^{\alpha}$.

Consequently, the (distance)^2 between two points ξ^{α} and $\xi^{\alpha}+d\xi^{\alpha}$ on the worldsheet is given by $d\xi^{\alpha}\partial_a X^{\mu}d\xi^{\beta}\partial_b X^{\nu}$.

An elementary calculation shows that the invariant, infinitesimal area element on the worldsheet is given by

$$\mathrm{dA} = \sqrt{-\det(h_{ab})}\, d^2\xi, \tag{1.36}$$

where the determinate is taken over the indices $a,b = 0,1$ of the 2×2 symmetric nondegenerate matrix (h_{ab})

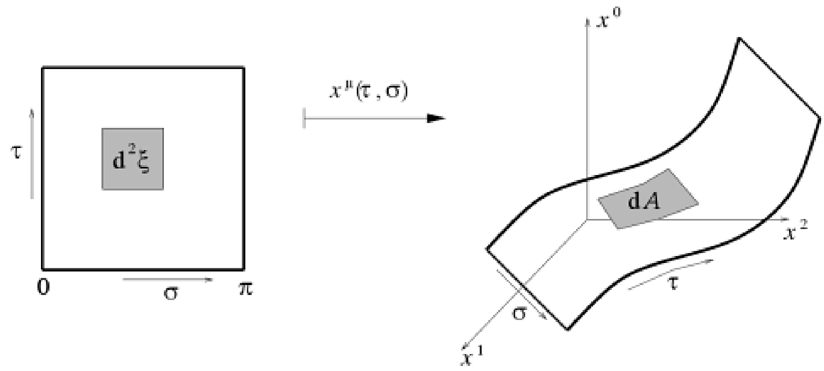

Fig. 1.1 The embedding $(\tau,\sigma)\mapsto x^{\mu}(\tau,\sigma)$ of a string trajectory into *d*-dimensional spacetime. As *τ* increases the string sweeps out its two-dimensional worldsheet in the target space, with *σ* giving the position along the string.

In analogy to the point particle case, we can now write down an action whose variational law minimizes the total area of the string worldsheet in spacetime

$$S[X] = -T\int \mathrm{dA} = -T\int \mathrm{d}^2\xi\sqrt{-\det(\partial_a X^{\mu}\partial_b X^{\nu})}. \tag{1.37}$$

The quantity T has dimensions of mass per unit length and is the tension of the string. It is related to the "intrinsic length" of the ℓ_s string by

$$T = \frac{1}{2\pi\alpha'}, \qquad \alpha' = \ell_s^2. \tag{1.38}$$

The parameter α' is called the "universal Regge slope", because the string vibrational modes all lie on linear parallel Regge trajectories with slope α'. the action eq.(1.37) defines a 1+1-dmenstional field theory on the string worldsheet with bosonic fields $X^{\mu}(\tau,\sigma)$.

Evaluating the determinate explicitly in (1.37) leads to the form

$$S[X] = -T\int \mathrm{d}\tau \mathrm{d}\sigma \sqrt{\dot{X}X'^2 - (\dot{X}\cdot X')^2} \tag{1.39}$$

where

$$\dot{X}^{\mu} = \frac{\partial X^{\mu}}{\partial \tau}, \quad X'^{\mu} = \frac{\partial X^{\mu}}{\partial \sigma}. \tag{1.40}$$

This is the form that the original string action appeared in and is known as the "Nambu-Goto action" [41]. However, the square root structure of this action is somewhat ackward to work with. It can, however, be eliminated by the fundamental observation that the Nambu-Goto action is classically equivalent to another action which does not have the square root

$$\begin{aligned} S[X,\gamma] &= -\frac{T}{2}\int d^2\xi \sqrt{-\gamma}\gamma^{ab} h_{ab} \\ &= -\frac{T}{2}\int d^2\xi \sqrt{-\gamma}\gamma^{ab}\partial_a X^{\mu}\partial_b X^{\nu}. \end{aligned} \tag{1.41}$$

Here the auxiliary rank two symmetric tensor field has a natural interpretation as a metric on the string worldsheet, and we have defined

$$\gamma = \det(\gamma_{ab}), \quad \gamma^{ab} = (\gamma^{-1})^{ab}. \tag{1.42}$$

The action (1.41) is called the "Polyakov action" [41].

1.4.1 Worldsheet Symmetries

The conditions T_{ab}=0 are often refered to as "Virasoro constraints" [42] and they are equivalent to two local "gauge symmetries" of the Polyakov action, namely the "reparametrization invariance"

$$(\tau,\sigma) \mapsto \left(\tau(\tau',\sigma'),\, \sigma(\tau',\sigma')\right), \tag{1.43}$$

and the "Weyl invariance" (or "conformal invariance")

$$\gamma_{ab} \mapsto e^{2\rho(\tau,\sigma)}\gamma_{ab}, \tag{1.44}$$

where $\rho(\tau,\sigma)$ is an arbitrary function on the worldsheet. These two local symmetries of $S[x,\gamma]$ allow us to select a gauge in which the three functions residing in the symmetric 2×2 matrix (γ_{ab}) are expressed in terms of just a single function. A particularly convenient choice is the "conformal gauge"

$$\left(\gamma_{ab}\right)= e^{\phi(\tau,\sigma)}\left(\eta_{ab}\right)=e^{\phi(\tau,\sigma)}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}. \tag{1.45}$$

In this gauge, the metric is γ_{ab} said to be "conformally flat", because it agrees with the Minkowski metrix η_{ab} of a flat worldsheet, but only up to the scaling function e^{ϕ}. Then, at the classical level, the conformal factor e^{ϕ} drops out of everything and we are left with the simple gauge-fixed action $S[x,e^{\phi}\eta]$, that is the Polyakov action in the conformal gauge, and the constraints T_{ab}=0.

$$\begin{aligned}
&S[X,e^{\phi}\eta]=T\int \mathrm{d}\tau \mathrm{d}\sigma(\dot{X}^{2}-X'^{2}),\\
&\quad T_{01}=T_{10}=\dot{X}\cdot X'=0,\\
&\quad T_{00}=T_{11}=\frac{1}{2}(\dot{X}^{2}+X'^{2})=0.
\end{aligned} \tag{1.46}$$

Note that apart from the constraints, eq.(1.46) defines a free (noninteracting) field theory.

1.5 String Equations of Motion

The equations of motion for the bosonic string can be derived by applying the variational principle to the 1+1-dimensional filed theory eq.(1.48). varying the Polyakov action in the conformal gauge with respect to the X^{μ} gives

$$\delta S[X,e^{\phi}\eta]=T\int \mathrm{d}\tau \mathrm{d}\sigma(\eta^{ab}\partial_a\partial_b X_{\mu})\delta X^{\mu}-\int \mathrm{d}\tau X'_{\mu}\,\delta X^{\mu}\Big|_{\sigma=0}^{\sigma=\pi}. \tag{1.47}$$

The first term in eq.(1.47) yields the usual bulk equations of motion which here correspond to the two-dimensional wave equation

$$\left(\frac{\partial^2}{\partial\sigma^2}-\frac{\partial^2}{\partial\tau^2}\right)X^{\mu}(\tau,\sigma)=0. \tag{1.48}$$

The second term comes from the integration by parts required to arrive at the bulk differential equation, which involves a total derivative over the spatial interval $0\le\sigma\le\pi$ of string. In order that the total variation of the action be zero, these boundary terms must vanish as well. The manner in which we choose them to vanish depends crucially on whether we are dealing with closed or open strings. The solutions of classical equations of motion then correspond to solutions of the wave equation (1.48) with the appropriate boundary conditions.

Closed Strings: Here we tie the two ends of the string at $\sigma = 0$ and $\sigma = \pi$ together by imposing periodic boundary conditions on the string embedding fields (Fig. 1.2):

$$\begin{aligned} X^{\mu}(\tau,0) &= X^{\mu}(\tau,\pi), \\ X'^{\mu}(\tau,0) &= X'^{\mu}(\tau,\pi). \end{aligned} \tag{1.49}$$

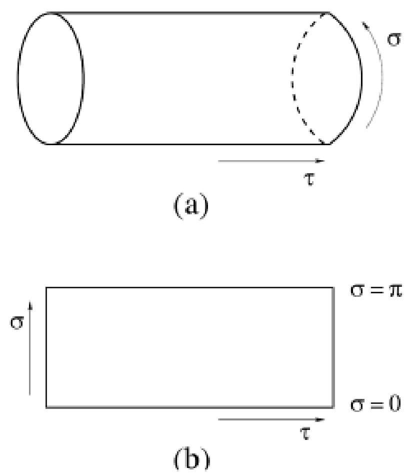

Fig. 1.2 The worldsheet of (a) a closed string is an infinite cylinder $\mathbb{R}\times S^1$, and of (b) an open string is an infinite strip $\mathbb{R}\times I^1$.

Open Strings: Here there are two canonical choices of boundary conditions. Neumann boundary conditions are defined by

$$X'^{\mu}(\tau,\sigma)\Big|_{\sigma=0,\pi} = 0. \tag{1.50}$$

In this case the ends of the string can sit anywhere in spacetime. Dirichelt boundary conditions, on the other hand, are defined by

$$\dot{X}^{\mu}(\tau,\sigma)\Big|_{\sigma=0,\pi} = 0. \tag{1.51}$$

Integrating the condition (1.51) over specifies a spacetime location on which the string ends, and so Dirichlet boundary conditions are equivalent to fixing the endpoints of the string

$$\delta X^{\mu}(\tau,\sigma)\big|_{\sigma=0,\pi} = 0. \tag{1.52}$$

We will see later on that this spacetime point corresponds to a physical object called a "D-brane". For the time being, however, we shall focus our attention on Neumann boundary conditions for open strings [42].

1.5.1 Mode Expansions

To solve the equations of motion, we write the two-dimensional wave equation (1.48) in terms of worldsheet light-cone coordinates

$$\partial_{+}\partial_{-}X^{\mu} = 0\ , \tag{1.53}$$

where

$$\partial_{\pm} = \frac{\partial}{\partial \xi^{\pm}} \quad , \quad \xi^{\pm} = \tau \pm \sigma. \tag{1.54}$$

The general solution of (1.53) is then the sum of an analytic function of ξ^{+} alone, which we will call the "left-moving" solution, and an analytic function of ξ^{-} alone, which we call the "right-moving" solution, $X^{\mu}(\tau,\sigma) = X_{L}^{\mu}(\xi^{+}) + X_{R}^{\mu}(\xi^{-})$ [42].

The precise form of the solutions now depends on the type of boundary conditions.

Closes Strings: The periodic boundary conditions (1.49) accordingly restrict the Taylor series expansions of the analytic functions which solve (1.53), and we arrive at the solution [42]

$$\begin{aligned} X^{\mu}(\tau,\sigma) &= X_{L}^{\mu}(\xi^{+}) + X_{R}^{\mu}(\xi^{-}), \\ X_{L}^{\mu}(\xi^{+}) &= \frac{1}{2}X_{0}^{\mu} + \alpha' p_{0}^{\mu}\xi^{+} + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_{n}^{\mu}}{n}e^{-2in\zeta^{+}}, \\ x_{R}^{\mu}(\xi^{-}) &= \frac{1}{2}x_{0}^{\mu} + \alpha' p_{0}^{\mu}\xi^{-} + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_{n}^{\mu}}{n}e^{-2in\zeta^{-}}. \end{aligned} \tag{1.55}$$

We have appropriately normalized the terms in these Fourier-type series expansions, which we will refer to as "mode expansions", according to physical dimension. Reality of the string embedding function X^{μ} requires the integration constants X_0^{μ} and P_0^{μ} to be real, and

$$\left(\tilde{\alpha}_n^{\mu}\right)^* = \tilde{\alpha}_{-n}^{\mu} \quad , \qquad \left(\alpha_n^{\mu}\right)^* = \alpha_{-n}^{\mu} \ . \tag{1.56}$$

By integrating X^{μ} and $\dot{X}^{\mu}$ over $\sigma \in [0, \pi]$ we see that X_0^{μ} and P_0^{μ} represent the center of mass position and momentum of string, respectively.

The $\tilde{\alpha}_n^{\mu}$ and α_n^{μ} represent the oscillatory modes of string. The mode expansions (1.55) correspond to those of left and right moving travelling waves circulating aroung the string in opposite directions.

Open Strings: For open strings, the spatial worldsheet coordinate lives on a finite interval rather than a circle. The open string mode expansion may be obtained from that of the closed string through the "doubling trick", which indentifies $\sigma \sim -\sigma$ on the circle $\mathbf{S}^1$ and thereby maps it onto the finite interval [0,π] (Fig. 1.3). The open string solution to the equations of motion may thereby be obtained from (1.55) by imposing the extra condition $X^{\mu}(\tau, \sigma) = X^{\mu}(\tau, -\sigma)$. This is of course still compatible with the wave equation (1.48) and it immediately implies the Neumann boundary conditions (1.50). We therefore find

$$X^{\mu}(\tau,\sigma) = X_0^{\mu} + 2\alpha' P_0^{\mu}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^{\mu}}{n}e^{-in\tau}\cos(n\sigma). \tag{1.57}$$

The open string mode expansion has a standing wave for its solution, representing the left and right moving sectors reflected into one another by the Neumann boundary condition (1.50) [42,43].

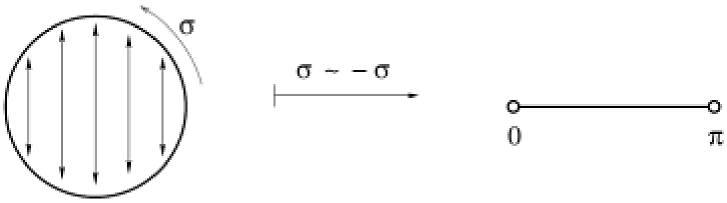

Fig. 1.3 The doubling trick identifies opposite points on the circle and maps it onto a finite interval.

1.5.2 Mass-Shell Constraints

The final ingredients to go into the classical solution are the physical constraints $T_{ab}= 0$. In the light-cone coordinates (1.54), the components T_{+-} and T_{-+} are identically zero, while in the case of the closed string the remaining components are given by

$$T_{++}(\xi^{+}) = \frac{1}{2}\left(\partial_{+}X_{L}\right)^{2} = \sum_{n=-\infty}^{\infty} \tilde{L}_{n} e^{2in\xi^{+}} = 0,$$
$$T_{--}(\xi^{-}) = \frac{1}{2}\left(\partial_{-}X_{R}\right)^{2} = \sum_{n=-\infty}^{\infty} \tilde{L}_{n} e^{2in\xi^{-}} = 0, \qquad (1.58)$$

where we have defined

$$\tilde{L}_{n} = \frac{1}{2}\sum_{m=-\infty}^{\infty} \tilde{\alpha}_{n-m}\cdot\tilde{\alpha}_{m} \ , \ L_{n} = \frac{1}{2}\sum_{m=-\infty}^{\infty} \alpha_{n-m}\cdot\alpha_{m} \qquad (1.59)$$

with

$$\tilde{\alpha}_{0}^{\mu} = \alpha_{0}^{\mu} = \sqrt{\frac{\alpha'}{2}}P_{0}^{\mu}. \qquad (1.60)$$

For open strings, we have only the constraint involving untitled quantities, and the definition of the zero modes (1.60) changes to $\alpha_{0}^{\mu} = \sqrt{2\alpha'}P_{0}^{\mu}$.

This gives an infinite number of constraints corresponding to an infinite number of conserved currents of the 1+1-dimensitonal field theory. They are associated with the residual, local, infinite-dimensional "conformal symmetry" of the theory which preserves the conformal gauge condition (1.45)

$$\zeta^{+} \mapsto \zeta'^{+} = f(\zeta^{+}),$$
$$\zeta^{-} \mapsto \zeta'^{-} = g(\zeta^{-}), \tag{1.61}$$

where f and g are arbitrary analytic functions. Only the conformal factor ϕ in (1.45) is affected by such coordinate transformations, and so the entire classical theory is invariant under them. They are known as "conformal transformations" and they rescale the induced worldsheet metric while leaving preserved all angles in two-dimensions. This "conformal invariance" of the worldsheet field theory makes it a "conformal field theory" [44,45], and represents one of the most powerful results and techniques of perturbative string theory.

The thesis is arranged as follows: In chapter two the instant of theory will be discussed. In chapter three the Light Front Quantization will be presented. The fourth chapter is devoted to conclusion.

CHAPTER TWO

INSTANT FORM THEORY

2.1 Recapitulation of Instant Form Theory

We first recapitulate very briefly the instant form theory. The Polyakov D1 brane action in a *d*-dimensional curved background $h_{\alpha\beta}$ (with d = 10 for the fermionic and d = 26 for bosonic D1 brane) is defined by [1,46, 47]

$$L = -\frac{T}{2}\sqrt{-h}\,h^{\alpha\beta}G_{\alpha\beta}\ ,\ h = \ \det\ \left(h_{\alpha\beta}\right) \tag{2.1}$$

Where

$$G_{\alpha\beta} = \partial_\alpha X^{\ \mu}\partial_\beta X^{\ \nu}\eta_{\mu\nu}\,, \tag{2.2}$$

$$\eta_{\mu\nu} = diag\,(-1,+1,\ldots,+1), \tag{2.3}$$

$$\mu,\nu = 0,1,\ \ldots\ ,(d-1), \qquad \alpha\beta = 0,1$$

Here $\sigma^\alpha = (\tau,\sigma)$ are the two parameters describing the worldsheet. The over dots and primes would denote the derivatives with respect to τ and σ. $G_{\alpha\beta}$ is the induced metric on the worldsheet and $X^{\ \mu}(\tau,\sigma)$ are the maps of the worldsheet into the d – dimensional Minkowski space and describe the strings evolution in space-time [46-47]. $h_{\alpha\beta}$ are the auxiliary fields (which turn out to be proportional to the metric tensor $\eta_{\alpha\beta}$ of the two-dimensional surface swept out by the string). One can think of $\tilde{S}$ as the action describing d massless scalar fields x_μ in two dimensions moving on a curved background $h_{\alpha\beta}$ [53].

Also because the metric components $h_{\alpha\beta}$ are varied in the above equation, the 2-dimensional gravitational field $h_{\alpha\beta}$ is treated not as a given background field, but rather as an adjustable quantity coupled to the scalar fields [46-47].

The action $\tilde{S}$ possesses the well-known three local gauge symmetries given by the two-dimensional worldsheet reparametrization invariance and the Weyl invariance [46-47]:

$$
\begin{aligned}
&X^{\mu} \rightarrow \tilde{X}^{\mu} = [X^{\mu} + \delta X^{\mu}], \\
&\delta X^{\mu} = [\zeta^{\alpha}(\partial_{\alpha} X^{\mu})], \\
&h^{\alpha\beta} \rightarrow \tilde{h}^{\alpha\beta} = [h^{\alpha\beta} + \delta h^{\alpha\beta}], \\
&\delta h^{\alpha\beta} = [\zeta^{\gamma}\partial_{\gamma} h^{\alpha\beta} - \partial_{\gamma}\zeta^{\alpha} h^{\gamma\beta} - \partial_{\gamma}\zeta^{\beta} h^{\alpha\gamma}], \\
&h_{\alpha\beta} \rightarrow [\Omega h_{\alpha\beta}]\ ;\ \Omega(\tau,\sigma) = e^{(2\omega(\tau,\sigma))}.
\end{aligned}
\tag{2.4}
$$

The worldsheet reparametrization invariance (WSRI) is defined by the first four equations in (2.4) involving the two gauge parameters ζ^{α} and the Weyl invariance is defined by the last equation and is specified by the gauge parameter Ω (or equivalently by ω).

Here $\zeta^{\alpha}(\tau,\sigma)$ is a gauge parameter corresponding to the (WSRI) and $\Omega(\tau,\sigma) = e^{(2\omega(\tau,\sigma))}$ is a gauge parameter corresponding to the Weyl symmetry.

Also the above theory being a gauge-invariant theory (possessing the local gauge symmetries including two worldsheet reparametrization invariance and one Weyl invariance symmetries), could be studied under appropriate gauge-fixing the way one likes.

2.2 Conformal Gauge Theory (Gauge Invariant Theory)

Possessing the local gauge symmetries including two worldsheet reparametrization invariance and one Weyl invariance symmetries, could be studied under appropriate gauge-fixing the way one likes.

However, one could also use the above three local gauge symmetries of the theory to choose $h_{\alpha\beta}$ to be of a particular form [46-47].

$$h_{\alpha\beta} = \eta_{\alpha\beta} \equiv \begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix}. \tag{2.5}$$

This is called Conformal Gauge. In the Conformal Gauge we have

$$\sqrt{-h} = \sqrt{-\det h_{\alpha\beta}} = +1. \tag{2.6}$$

Now the action $\bar{S}$ in Conformal Gauge becomes

$$S_1 = \int L_1 d^2\sigma, \tag{2.7}$$

$$L_1 = -\frac{T}{2}\sqrt{-h}\,h^{\alpha\beta}G_{\alpha\beta}, \tag{2.8}$$

where T is the string tension. Substituting of Eq. (2.4),(2.6) into (2.8) we obtain

$$\begin{aligned}
\tilde{S} &= \int -\frac{T}{2}\sqrt{-h}\,h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}\, d^2\sigma \\
&= \int -\frac{T}{2}\sqrt{-h}\,h^{\alpha\beta}\left[\partial_\alpha\left[X^\mu + \delta X^\mu\right]\partial_\beta\left[X^\nu + \delta X^\nu\right]\eta_{\mu\nu}\right] d^2\sigma \\
&= \int -\frac{T}{2}\sqrt{-h}\left[h^{\alpha\beta} + \delta h^{\alpha\beta}\right]\left[\partial_\alpha X^\mu + \partial_\alpha\left(\xi^\alpha\left(\partial_\alpha X^\mu\right)\right)_\alpha\right]\left[\partial_\beta X^\nu + \partial_\beta\left(\xi^\beta\left(\partial_\beta X^\nu\right)\right)\right] d^2\sigma
\end{aligned} \tag{2.9}$$

$$\tilde{S} = \int -\frac{T}{2}\sqrt{-h}\left[h^{\alpha\beta} + \xi^\gamma\partial_\gamma h^{\alpha\beta} - \partial_\gamma \xi^\alpha h^{\gamma\beta}\right]\cdot\left[\partial_\beta X^\nu + \partial_\beta\left(\xi^\beta\left(\partial_\beta X^\nu\right)\right)\right]\cdot\eta_{\mu\nu} d^2\sigma, \tag{2.10}$$

$$\begin{aligned}
\tilde{S} = \int -\frac{T}{2}\sqrt{-h}\Big[& h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu + h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta\left(\xi^\beta\left(\partial_\beta X^\nu\right)\right) + h^{\alpha\beta}\partial_\alpha\left(\xi^\alpha\left(\partial_\alpha X^\mu\right)\right)\partial_\beta X^\gamma \\
& + h^{\alpha\beta}\partial_\alpha\left(\xi^\alpha\left(\partial_\alpha X^\mu\right)\right)\partial_\beta\left(\xi^\beta\left(\partial_\beta X^\nu\right)\right) + \zeta^\gamma\left(\partial_\gamma h^{\alpha\beta}\right)\partial_\alpha X^\mu \partial_\beta X^\nu \\
& + \zeta^\gamma\left(\partial_\gamma h^{\alpha\beta}\right)\partial_\alpha X^\mu \partial_\beta \xi^\beta\left(\partial_\beta X^\nu\right) + \zeta^\gamma\left(\partial_\gamma h^{\alpha\beta}\right)\partial_\alpha \xi^\alpha \partial\left({}_\alpha X^\mu\right)\partial_\beta X^\nu \\
& + \zeta^\gamma\left(\partial_\gamma h^{\alpha\beta}\right)\partial_\alpha \xi^\alpha\left(\partial_\alpha X^\mu\right)\partial_\beta \xi^\beta\left(\partial_\beta X^\nu\right) - \partial_\gamma \xi^\alpha h^{\gamma\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \\
& - \partial_\gamma \xi^\alpha h^{\gamma\beta}\partial_\alpha X^\mu \partial_\beta\left(\xi^\beta\left(\partial_\beta X^\nu\right)\right) - \partial_\gamma \xi^\gamma h^{\gamma\beta}\partial_\alpha \xi^\alpha \partial_\alpha X^\mu \partial_\beta X^\nu \\
& - \partial_\gamma \xi^\alpha h^{\gamma\beta}\partial_\alpha \xi^\alpha\left(\partial_\alpha X^\mu\right)\partial_\beta \xi^\beta\left(\partial_\beta X^\nu\right) - \partial_\gamma \xi^\beta h^{\alpha\gamma}\partial_\alpha X^\mu \partial_\beta X^\nu \\
& - \partial_\gamma \xi^\beta h^{\alpha\gamma}\partial_\alpha X^\mu \partial_\beta \xi^\beta\left(\partial_\beta X^\nu\right) - \partial_\gamma \xi^\beta h^{\alpha\gamma}\partial_\alpha \xi^\alpha\left(\partial_\alpha X^\mu\right)\partial_\beta X^\nu \\
& - \partial_\gamma \xi^\beta h^{\alpha\gamma}\partial_\alpha \xi^\alpha\left(\partial_\alpha X^\mu\right)\partial_\beta \xi^\beta \partial_\beta X^\nu\Big]\cdot\eta_{\mu\nu} d^2\sigma
\end{aligned} \tag{2.11}$$

$$\tilde{S} = \int -\frac{T}{2}\sqrt{-h}\,h^{\alpha\beta}\partial_\alpha X^{\mu}\partial_\beta X^{\mu} d^2\sigma, \qquad \mu=\nu$$

$$= \int -\frac{T}{2}\sqrt{-h}\left[h^{00}\partial_0 X^{\mu}\partial_0 X^{\mu} + h^{11}\partial_1 X^{\mu}\partial_1 X^{\mu}\right],$$

$$= \int -\frac{T}{2}\left[-\partial_0 X^{\mu}\partial_0 X^{\mu} + \partial_1 X^{\mu}\partial_1 X^{\mu}\right] d^2\sigma. \qquad (2.12)$$

Substituting from eq (2.5) and eq. (2.6) into eq. (2.12) with, $\mu=\nu$,$\eta_{\mu\nu}=1$, we obtain

$$\tilde{S}_1 = \int -\frac{T}{2}\left[\left(\frac{\partial X^{\mu}}{\partial\sigma}\right)^2 - \left(\frac{\partial X^{\mu}}{\partial\tau}\right)^2\right] d^2\sigma, \qquad (2.13)$$

$$\tilde{S}_1 = \int -\frac{T}{2}\left[\left(X'\right)^2 - \left(\dot{X}\right)^2\right] d^2\sigma. \qquad (2.14)$$

where: $\frac{\partial X^{\mu}}{\partial\tau} = \dot{X}^{\mu}$, $\frac{\partial X^{\mu}}{\partial\sigma} = X'^{\mu}$, $\sqrt{-h} = +1$

In the following sections we will denote the derivation as

$$\partial_\tau = \frac{\partial}{\partial\tau}, \quad \partial_\sigma = \frac{\partial}{\partial\sigma}$$

This is the Conformal Gauge Fixed Polyakov one dimension D1 Brane action.

2.2.1 The Canonical Momenta Conjugate

According to the definition (1.3), the canonical momenta conjugate to the canonical variables X^{μ} are

$$P^{\mu} = \frac{\partial L_1}{\partial(\partial_\tau X^{\mu})} = T\,\partial_\tau X_{\mu}, \qquad (2.15)$$

Solving eq.(2.15) for the velocities, one gets

$$\partial_\tau X^{\mu} = \frac{1}{T}P^{\mu}. \qquad (2.16)$$

This theory is easily seen to be an unconstrained system in the sense of Dirac [1]. It may be important to remark here that an unconstrained system like this represents a gauge noninvariant theory and is some what a kind to a gauge fixed gauge-invariant theory which makes it a gauge-noninvariant system[48].

The Canonical Hamiltonian Density Corresponding to L_1 definition in eq.(1.7)

$$H^C = \sum_{i=1}^{n} p_i \dot{q}_i - L, \tag{2.17}$$

From eq.(2.16) and eq.(2.17) we obtain

$$\begin{aligned}
H_1^C &= P^{\mu}(\partial_\tau X_\mu) - \left[-\frac{T}{2}\left[(X')^2 - (\dot{X})^2 \right] \right], \\
&= P^{\mu}\partial_\tau X_\mu + \frac{T}{2}(X')^2 - \frac{T}{2}(\partial_\tau X^{\mu}\partial_\tau X^{\mu})^2, \\
&= \frac{1}{T}P^{\mu}P_{\mu} + \frac{T}{2}(X')^2 - \frac{T}{2}(\partial_\tau X^{\mu}\partial_\tau X^{\mu})^2, \\
&= \frac{1}{T}P^{\mu}P_{\mu} + \frac{T}{2}(X')^2 - \frac{T}{2}P^{\mu}P_{\mu}, \\
&= \left[\frac{1}{2T}P^{\mu}P_{\mu} + \frac{T}{2}(X')^2 \right].
\end{aligned} \tag{2.18}$$

The quantization of the system is trivial which are described by

$$\left[X^{\mu}(\sigma,\tau), P_\nu(\sigma',\tau) \right] = iS_\nu^{\mu}\delta(\sigma - \sigma'). \tag{2.19}$$

where $\delta(\sigma-\sigma')$ is the one-dimensional Dirac distribution function[1,2].

2.3 Dirac Method of The Conformal Gauge Theory in Presence of Scalar Dilation Field

In presence of scalar dilation field $\phi = \phi(\sigma,\tau)$ in d-dimension flat background reads as [34]

$$S_2 = \int L_2 d\sigma^2, \tag{2.20}$$

where:

$$L_2 = e^{-\phi} L_1 = \left(-\frac{T}{2} e^{-\phi} \sqrt{-h} h^{\alpha\beta} G_{\alpha\beta} \right), \tag{2.21}$$

Substituting from eq.(2.1), we obtain

$$\begin{aligned} L_2 &= -\frac{T}{2} e^{-\phi} \partial_\alpha X^{\mu} \partial_\beta X^{\nu}, \\ &= -\frac{T}{2} e^{-\phi} \left((X'^2) - (\dot{X}^2) \right). \end{aligned} \tag{2.22}$$

The canonical momenta conjugated to ϕ and X^{μ} are respectively

$$\Pi = \frac{\partial L_2}{\partial(\partial_\tau \phi)} = 0, \tag{2.23}$$

$$P^{\mu} = \frac{\partial L_2}{\partial(\partial_\tau X^{\mu})} = T e^{-\phi} \partial_\tau X^{\mu}, \tag{2.24}$$

Solving for the velocities $\dot{X}^{\mu}$, we get

$$\dot{X}^{\mu} \equiv \partial_\tau X^{\mu} = \frac{1}{T} e^{\phi} P^{\mu}. \tag{2.25}$$

The canonical Hamiltonian density corresponding to L_2 is

$$H^C = P^{\mu} \partial_\tau X^{\mu} - \left[-\frac{T}{2} e^{-\phi} \left((X'^2) - (\dot{X}^2) \right) \right]. \tag{2.26}$$

Substituting from eq. (2.24) we obtain

$$\begin{aligned} H &= \frac{1}{T} e^{\phi} P^{\mu} P_{\mu} + \frac{T}{2} e^{-\phi} (X')^2 - \frac{T}{2} e^{\phi} \dot{X}^2, \\ &= \frac{1}{T} e^{\phi} P^{\mu} P_{\mu} + \frac{T}{2} e^{-\phi} (X')^2 - \frac{T}{2} e^{-\phi} \left(\frac{1}{T^2} e^{2\phi} P^{\mu} P_{\mu}\right), \\ &= \frac{1}{2T} e^{\phi} P^{\mu} P_{\mu} - \frac{T}{2} e^{-\phi} (X')^2. \end{aligned} \tag{2.27}$$

2.3.1 The Primary and Secondary Constraints

The Conformal gauge theory in Presence of scalar dilation field is easily seen to possess two constraints [48]

$$\rho_1 = \Pi \simeq 0, \tag{2.28}$$

$$\begin{aligned}\rho_2 &= [\rho_1, H]_{\Pi,\phi} = \frac{\partial\rho_1}{\partial\Pi}\frac{\partial H}{\partial\phi} - \frac{\partial\rho_1}{\partial\phi}\frac{\partial H}{\partial\Pi}, \\ &= \left[\frac{1}{2T}e^{\phi}P^{\mu}P_{\mu} - \frac{1}{2}Te^{-\phi}\left(X'\right)^2\right] \simeq 0.\end{aligned} \tag{2.29}$$

where ρ_1 is a primary constraint and ρ_2 is a secondary Gauss law constraint, P^{μ} and Π here are the momenta conjugate canonically respectively to X^{μ} and ϕ .

The matrix of the Poisson brackets of the constraints ρ_1 and ρ_2 is seen to be nonsingular implying that the set of these constraints is second-class and that the theory is gauge-noninvariant (which does not respect the usual string gauge symmetries worldsheet and Weyl invariance).

The Hamiltonian formulations of this theory have been studied in reference[48]. It may be worth mentioning here that the Instant Form theory in the absence of a scalar dilation field, is not a constrained system in the sence of Dirac (implying that theory is equivalent to a gauge-fixed gauge-invariant theory) whereas the theory in the presence of a scalar dilation field represents a constrained system in the sence of Dirac possessing a set of two second-class constraints where one constraint is primary and the other one is the secondary Gauss law constraint [49].

2.4 Hamilton Jacobi Formulation of Instant form theory in presence of scalar dilation field ϕ

Now we propose the Conformed Gauge theory in the presence of scalar dilation field action

$$S_2 = \int -\frac{T}{2}e^{-\phi}\left((X')^2 - (\dot{X}^2)\right)d\sigma^2, \tag{2.30}$$

with Lagrangian

$$L = -\frac{T}{2} e^{-\phi} \left((X')^2 - (\dot{X}^{2}) \right). \tag{2.31}$$

The canonical momenta defined in (1.20) and (1.21) take the forms

$$\Pi = \frac{\partial L_2}{\partial_+(\partial\phi)} = 0 \equiv -H_\phi, \tag{2.32}$$

and

$$p^\mu = \frac{\partial L_2}{\partial\left(\partial_\tau X^\mu\right)} = T e^{-\phi} \partial_\tau X^\mu = T e^{-\phi} \dot{X}^\mu, \tag{2.33}$$

According to eq.(1.22) the velocity $\dot{X}$ can be written from eq.(2.33) as

$$\dot{X}^\mu \equiv \left(\partial_\tau X^\mu\right) = \frac{1}{T} e^{\phi} p^\mu, \tag{2.34}$$

The used Hamiltonian H_o is

$$H_o = P^\mu (\partial_\tau X^\mu) - L_2, \tag{2.35}$$

The Hamilton-Jacobi Partial derivative equation are

$$H_o' = P_o \quad + \quad H_o \tag{2.36}$$

$$H_o' = P_o + \frac{T}{2} e^{\phi} P^\mu P_\mu + \frac{T}{2} e^{+\phi} (X')^2 = 0, \tag{2.37}$$

$$H_\phi' = \Pi = 0. \tag{2.38}$$

The total differential equation for the characteristic read as

$$\begin{aligned} dX^\mu &= \frac{\partial H_o'}{\partial P^\mu} d\tau + \frac{\partial H_\phi'}{\partial P^\mu} d\phi, \\ &= \frac{1}{T} e^{\phi} P^\mu d\tau. \end{aligned} \tag{2.39}$$

$$\mathrm{d}P^\mu = \frac{dH_o'}{\partial(\partial X^\mu)} d\tau + \frac{dH_\phi'}{\partial(\partial X^\mu)} d\phi, \tag{2.40}$$

$$\mathrm{d}P^\mu = 0, \tag{2.41}$$

$$d\Pi = \frac{\partial H_o'}{\partial \phi} d\tau + \frac{\partial H_\phi'}{\partial \phi} d\phi, \tag{2.42}$$

$$= \left[\frac{1}{2T} e^{\phi} P^{\mu} P_{\mu} + \frac{T}{2} e^{-\phi} \left(X' \right)^2 \right] d\tau, \tag{2.43}$$

2.5 Instant Form Quantization in presence of A2- Form Gauge Field $B_{\alpha\beta}$

We now consider this conformed Gauge Fixed Polyakove D1-Brane action in the presence of a constant background antisymmetric 2-Form gauge field $B_{\alpha\beta}$ [50].

This theory is defined by the action

$$S_B = \int L_B d^2\sigma, \tag{2.44}$$

$$L_B = \left[L^C + L^B \right], \tag{2.45}$$

where $$L^C = \left[\lambda L^N \right] = \left[-\frac{T}{2} \right] \left[\lambda \partial^B X^{\mu} \partial_B X_{\mu} \right], \tag{2.46}$$

and

$$L^B = \left[-\frac{T}{2} \right] \left[\Lambda \varepsilon^{\alpha\beta} B_{\alpha\beta} \right], \tag{2.47}$$

where $$\lambda = \sqrt{1 + \Lambda^2}, \tag{2.48}$$

Λ constant,

$$\varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{2.49}$$

$$B_{\alpha\beta} = \partial_\alpha X^{\mu} \partial_\beta X^{\nu} B_{\mu\nu}, \tag{2.50}$$

$$B_{\alpha\beta} = \begin{pmatrix} 0 & B \\ -B & 0 \end{pmatrix}, \tag{2.51}$$

$$B = B_{01} = -B_{10} \tag{2.52}$$

$$\alpha, \beta = 0,1, \qquad \mu, \nu = 0,1,i, \qquad i = 2,3, \cdots, 25$$

Here the 2-form gauge field $B_{\alpha\beta}$ is a scalar field in the target-space whereas it is constant anti-symmetric tensor field in worldsheet space.

The action reads in Instant Form Quantization

$$S_3 = \int L_3 d\tau d\sigma, \tag{2.53}$$

where

$$L_3 = \left[-\frac{\lambda T}{2}\left[X'^2 - \dot{X}^2 \right] - \Lambda TB \right]. \tag{2.54}$$

The canonical momenta are

$$P^\mu = \frac{\partial L_3}{\partial\left(\partial_\tau X_\mu\right)} = \left[\lambda T \dot{X}^\mu \right], \tag{2.55}$$

$$\Pi_B = \frac{\partial L_3}{\partial\left(\partial_\tau B\right)} = 0. \tag{2.56}$$

Here P^μ and Π_B are the canonical momenta conjugate respectively to X_μ and B.

The Light Form Quantization is thus seen to posses one primary constraint

$$\Phi_L = \Pi_B \simeq 0, \tag{2.57}$$

The canonical Hamiltonian density of this theory is

$$H_3^C = \left[P^\mu (\partial_\tau X_\mu) + \Pi_B (\partial_\tau B) - L \right], \tag{2.58}$$

$$= \left[\left(P^\mu \frac{1}{\lambda T} P_\mu \right) + \left[\left[\frac{\lambda T}{2} \right] \left[X'^2 - \dot{X}^2 \right] \right] + \Lambda TB \right], \tag{2.59}$$

$$= \left[\frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2} \left(X' \right)^2 + \Lambda TB \right]. \tag{2.60}$$

The total Hamiltonian density of the theory could be written as

$$H_3^T = \left[\frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2} \left(X' \right)^2 + \Lambda TB + u\Pi_B \right]. \tag{2.61}$$

Also, the momenta canonically conjugate to u is denoted by P^{μ}, and the Hamilton's equations obtained from of total Hamiltonian

$$H_3^T = \int H_3^C d\sigma. \tag{2.62}$$

The Poission bracket of constraint of the theory with itself is seen to be zero implying that the constraints is first-class and that theory is Gauge Instant.

It is indeed seen to posses three local gauge symmetries given by the 2-dimentional worldsheet reparametrization invariance and the Weyl invariance defined by eq(2.4)

$$B_{\alpha\beta} \to \tilde{B}_{\alpha\beta} = \left[B_{\alpha\beta} + \delta B_{\alpha\beta}\right], \tag{2.63}$$

$$\delta B_{\alpha\beta} = \left[\xi^{\alpha}\partial_{\alpha} B_{\alpha\beta}\right]. \tag{2.64}$$

It is important to recollect here that 2-form gauge filed $B_{\alpha\beta}$ in a scalar filed in the target-space whereas it is constant anti-symmetric tensor filed in the world sheet space and consequently we have $\delta B_{\alpha\beta} = 0$ [50].

The matrix of the Poisson brackets of these constraints are seen to be nonsingular implying that the corresponding set of constraints is second-class.

The Dirac quantization procedure in the Hamiltonian formulation, the nonvanishing equal wolrdsheet time (EWST) Commutation relations of the theory under the above gauge are obtained as

$$\left[X^{\mu}(\sigma,\tau), P^{\nu}(\sigma',\tau)\right] = i\,\delta^{\mu\nu}\delta(\sigma-\sigma'). \tag{2.65}$$

2.6 Instant-Form Quantization in Presence of a Scalar Axion Field *C* and An *U*(1) Gauge Field A^{μ}

In this section, we study the Instant-Form Quantization of conformal gauge form Polyakov D1 brane action in the presence of a $U(1)$ gauge field $A_{\alpha}\left(\equiv A_{\alpha}(\tau,\sigma)\right)$ and a constant scalar axion field $C\left(\equiv C\left(\tau,\sigma\right)\right)$ [49,50] by using the equal wolrdsheet time (EWST) framework, on the hyperplanes defined by the wolrdsheet time $\sigma^0 \equiv \tau =$ constant. The Instant-Form action reads

$$S_4 = \int L_4 d\tau d\sigma, \tag{2.66}$$

$$L_4 = L^C + L^A, \tag{2.67}$$

where $$L^A = \left(-\frac{T}{2}\right)\left(\Lambda C\,\varepsilon^{\alpha\beta}F_{\alpha\beta}\right), \tag{2.68}$$

and

$$F_{\alpha\beta} = \left(\partial_\alpha A_\beta - \partial_\beta A_\alpha\right), \tag{2.69}$$

$$f = F_{01} = -F_{10}, \quad \alpha,\beta = 0,1 \quad , \ \mu = 0,1,i \quad , \ i = 2,3,\cdots,25, \tag{2.70}$$

$$L_4 = \left[-\frac{\lambda T}{2}\left(\left(X'\right)^2 - \left(\dot{X}\right)^2\right) + \Lambda TCf\right], \tag{2.72}$$

Substituting from eq(2.45), (2.48) and (2.68) from eq (2.67)

$$L_4 = L^C + L^A, \tag{2.73}$$

$$= \left[-\frac{\lambda T}{2}\left(\left(X'\right)^2 - \left(\dot{X}\right)^2\right) - \frac{T}{2}\left(\Lambda C\,\varepsilon^{\alpha\beta}F_{\alpha\beta}\right)\right], \tag{2.74}$$

$$\begin{aligned}
L_4 &= -\frac{\lambda T}{2}\left(\left(X'\right)^2 - \left(\dot{X}\right)^2\right) - \frac{T}{2}\Lambda C\left[\varepsilon^{0\beta}F_{0\beta} + \varepsilon^{1\beta}F_{1\beta}\right], \\
&= -\frac{\lambda T}{2}\left(\left(X'\right)^2 - \left(\dot{X}\right)^2\right) - \frac{T}{2}\Lambda C\left[\varepsilon^{00}F_{00} + \varepsilon^{10}F_{10} + \varepsilon^{01}F_{01} + \varepsilon^{11}F_{11}\right], \\
&= -\frac{\lambda T}{2}\left(\left(X'\right)^2 - \left(\dot{X}\right)^2\right) - \frac{T}{2}\Lambda C\left[F_{10} - F_{01}\right] = -T\,\Lambda C F_{10} = -T\,\Lambda C f\ , \\
&= -\frac{\lambda T}{2}\left(\left(X'\right)^2 - \left(\dot{X}\right)^2\right) - T\,\Lambda C\left[\partial_1 A_\beta - \partial_\beta A_1\right] \\
&= -\frac{\lambda T}{2}\left(\left(X'\right)^2 - \left(\dot{X}\right)^2\right) - T\,\Lambda C\left[\partial_1 A_0 - \partial_0 A_1\right].
\end{aligned} \tag{2.75}$$

Overdotes and primes denote derivatives with respect to τ and σ respectively. The canonical momenta obtained are

$$P^{\mu}=\frac{\partial L_4}{\partial\left(\partial_\tau X_{\mu}\right)}=\left[\lambda T\dot{X}^{\mu}\right], \tag{2.76}$$

$$\partial_\tau X_{\mu}=\dot{X}^{\mu}=\frac{1}{\lambda T}P^{\mu}, \tag{2.77}$$

$$\Pi_c=\frac{\partial L_4}{\partial\left(\partial_\tau C\right)}=0, \tag{2.78}$$

$$\Pi^0=\frac{\partial L_4}{\partial\left(\partial_\tau A_0\right)}=0, \tag{2.79}$$

and ,

$$E\left(\equiv\Pi^1\right)=\frac{\partial L_4}{\partial\left(\partial_\tau A_1\right)}=\Lambda TC\ , \tag{2.80}$$

where $P^{\mu},\Pi^0,E\left(\equiv\Pi^1\right)$ and Π_c are the canonical momenta conjugate respectively to X_{μ},A_0,A_1 and C.

The theory is thus seen to possess three primary constraints

$$\Psi_1=\Pi^0\approx 0, \tag{2.81}$$

$$\Psi_2=\left(E-\Lambda TC\right)\approx 0, \tag{2.82}$$

$$\Psi_3=\Pi_c\approx 0. \tag{2.83}$$

Canonical Hamiltonian density corresponding to above Lagrangian density is

$$H_4^c=\left[P^{\mu}(\partial_\tau X_{\mu})+\Pi^0(\partial_\tau A_0)+E(\partial_\tau A_1)+\Pi_c(\partial_\tau C)-L_5\right], \tag{2.84}$$

Substituting from eq(2.72) , (2.76) and (2.80) into eq(2.84), we obtain

$$H_4^c=\frac{1}{\lambda T}P^{\mu}P_{\mu}+\Lambda TC\,\partial_\tau A_1+\frac{\lambda T}{2}\left(X'\right)^2-\frac{\lambda T}{2}\frac{1}{\lambda T}P^{\mu}\frac{1}{\lambda T}P_{\mu}+\Lambda TC\,\partial_1 A_0-\Lambda TC\,\partial_0 A_1, \tag{2.85}$$

$$=\frac{1}{2\lambda T}P^{\mu}P_{\mu}+\frac{\lambda T}{2}\left(X'\right)^2+\Lambda TCA_0'. \tag{2.86}$$

where $A_0' = \partial_\sigma A_0,$

$$\dot{A}_1 = \partial_\tau A_1 = \partial_0 A_1. \tag{2.87}$$

After incorporating the primary constraints of the theory in the canonical Hamiltonian density with the help of Lagrange multiplier fields $u(\tau,\sigma)$,$v(\tau,\sigma)$ and $w(\tau,\sigma)$ (treated as dynamical) the total Hamiltonian density of the theory becomes

$$H_4^T = \left[H_4^c + u\Psi_1 + v\Psi_2 + w\,\Psi_3\right], \tag{2.88}$$

Substituting from eq(2.85) and eqs.(2.81 – 2.83), we obtain

$$H_4^T = \left[\frac{1}{2\lambda T}P^\mu P_\mu + \frac{\lambda T}{2}\left(X'\right)^2 + \Lambda TCA_0' + \Pi^0 u + \left(E - \Lambda TC\right)v + \Pi_c w\right]. \tag{2.89}$$

Momenta canonically conjugate to u, v and w are denoted respectively by p_u, p_v and p_w. Hamiltons equations obtained from the Hamiltonian $H_4^T = \int H_4^C d\sigma$, for the closed string with periodic conditions are

$$\partial_\tau X^\mu = \frac{\partial H_4^T}{\partial P_\mu} = \left[\frac{1}{\lambda T}\right]P^\mu, \tag{2.90}$$

$$-\partial_\tau P^\mu = \frac{\partial H_4^T}{\partial\left(\partial X_\mu\right)} = 0, \tag{2.91}$$

$$\partial_\tau C = \frac{\partial H_4^T}{\partial \Pi_C} = w\,, \tag{2.92a}$$

$$-\partial_\tau \Pi_C = \frac{\partial H_4^T}{\partial C} = \Lambda T\,\left(A_0' - v\right) = \Lambda T\,\left(A_0' - \partial_\tau A_1\right), \tag{2.92b}$$

$$\partial_\tau A_0 = \frac{\partial H_4^T}{\partial \Pi^0} = u\,, \tag{2.92a}$$

$$-\partial_\tau \Pi^0 = \frac{\partial H_4^T}{\partial A_0} = 0\,, \tag{2.92b}$$

$$\partial_\tau A_1 = \frac{\partial H_4^T}{\partial E} = v\,, \tag{2.93a}$$

$$-\partial_\tau E = \frac{\partial H_4^T}{\partial A_1} = 0, \tag{2.93b}$$

$$\partial_\tau u = \frac{\partial H_4^T}{\partial p_u} = 0, \tag{2.94a}$$

$$-\partial_\tau p_u = \frac{\partial H_4^T}{\partial u} = \Pi^0 = 0, \tag{2.94b}$$

$$\partial_\tau v = \frac{\partial H_4^T}{\partial p_v} = 0, \tag{2.95a}$$

$$-\partial_\tau p_v = \frac{\partial H_4^T}{\partial v} = \left(E - \Lambda TC\right), \tag{2.95b}$$

$$\partial_\tau w = \frac{\partial H_4^T}{\partial p_w} = 0, \tag{2.96a}$$

$$-\partial_\tau p_w = \frac{\partial H_4^T}{\partial w} = \Pi_C = 0, \tag{2.96b}$$

These are the equations of motion of the theory that preserve the constraints of the theory in the course of time. Demanding that the primary constraints of the theory be preserved in the course of time one does not get any further constraints.

The theory is thus seen to posses only three constraints Ψ_1, Ψ_2 and Ψ_3.

Matrix of the Poission brackets of these constraints is seen to be singular implying that the constraints form a set of first-class constraints and that the theory gauge instant and posses three local gauge symmetries given by the 2-dimentianl worldsheet reparametrization invariance and the Weyl invariance defined by (2.4) and

$$C \rightarrow \tilde{C}_\alpha = \left[C + \delta C\right], \tag{2.97a}$$

$$\delta C = \left[\xi^\alpha \partial_\alpha C\right], \tag{2.97b}$$

The first order Lagrangian density of the theory is

$$\begin{aligned} L_4^{IO} = \Big[& P^\mu\left(\partial_\tau X_\mu\right) + \Pi^0\left(\partial_\tau A_0\right) + E\left(\partial_\tau A_1\right) + \Pi_c\left(\partial_\tau c\right) \\ & + p_u\left(\partial_\tau u\right) + p_v\left(\partial_\tau v\right) + p_w\left(\partial_\tau w\right) - H_5^T\Big], \end{aligned} \tag{2.98}$$

$$= \left[\frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2}\left(X'\right)^2 + \Lambda TCf\right]. \tag{2.99}$$

The theory could be quantized under appropriate gauge-fixing. To study the Hamiltonian of this theory under gauge-fixing, we could choose the gauge

$$\xi = A_0 \approx 0, \tag{2.100}$$

Corresponding to this choice of gauge the total set of constraints of the theory under which the quantization of the theory could be studied becomes

$$\xi_1 = \Psi_1 = \Pi^0 \approx 0, \qquad (2.101a)$$
$$\xi_2 = \Psi_2 = \left(E - \Lambda TC\right) \approx 0, \qquad (2.101b)$$
$$\xi_3 = \Psi_3 = \Pi_c \approx 0. \qquad (2.101c)$$
$$\xi_4 = \xi = A_0 \approx 0. \qquad (2.101d)$$

We now calculate the matrix $M_{\alpha\beta}\left(=\left\{\Psi_\alpha,\Psi_\beta\right\}_{PB}\right)$of the Poisson brackets of the constraints Ψ_i . The nonvanishing elements of the matrix are obtained as

$$\Lambda TM_{14} = -\Lambda TM_{41} = M_{23} = -M_{32} = \left[-\Lambda T\right]\delta\left(\sigma-\sigma'\right), \tag{2.102}$$

The matrix $M_{\alpha\beta}$ is seen to be nonsingular implying that the corresponding set of constraints is a set of second-class constraints. The determinant of the matrix $M_{\alpha\beta}$ is given by

$$\left[\left\|\det\left(M_{\alpha\beta}\right)\right\|\right]^{1/2} = \Lambda T\,\delta^2\left(\sigma-\sigma'\right), \tag{2.103}$$

and the nonvanishing elements of the inverse of the matrix $M_{\alpha\beta}$ (i.e., the elements of matrix$\left(M^{-1}\right)_{\alpha\beta}$) are obtained as

$$\left(M^{-1}\right)_{14} = -\left(M^{-1}\right)_{41} = \Lambda T\,\left(M^{-1}\right)_{23}, \tag{2.104}$$
$$= \Lambda T\,\left(M^{-1}\right)_{23} = \delta\left(\sigma-\sigma'\right). \tag{2.105}$$

$$\int M\,(\sigma,\sigma')M^{-1}(\sigma'',\sigma')d\,\sigma'' = \mathbf{1}_{4\times4}\delta(\sigma-\sigma'), \tag{2.106}$$

Following the Dirac quantization procedure in the Hamiltonian formulation, the nonvanishing EWST CR's of the theory under the gauge $\xi = A_0 \approx 0$ (with the arguments being suppressed) are obtained as

$$\left[X^{\mu}(\sigma,\tau), P_{\nu}(\sigma',\tau)\right] = i\,\eta^{\mu\nu}\delta(\sigma-\sigma'). \tag{2.107}$$

2.7 Hamilton-Jacobi Treatment of Instant-Form Quantization in Presence of a scalar Axion Field *C* and An *U*(1) Gauge Field A^{μ}

The action of Instant-Form of Conformal Gauge formulation of Polyakove D1 Brane action in presence of an *U*(1) Gauge field and constant scalar axion field is given by

$$\begin{aligned} S_4 &= \int L_4 d\tau d\sigma, \\ &= \int \left[-\frac{\lambda T}{2}\left((X')^2 - (\dot{X})^2\right) + \Lambda T C f \right] d\tau d\sigma, \end{aligned} \tag{2.108}$$

The Lagrangian is

$$L_4 = -\left[\frac{\lambda T}{2}\right]\left((X')^2 - (\dot{X})^2\right) + \Lambda T C f , \tag{2.109}$$

The canonical momenta are given as

$$P^{\mu} = \frac{\partial L_4}{\partial(\partial_\tau X_{\mu})} = \frac{\partial L_4}{\partial \dot{X}^{\mu}} = \lambda T \dot{X}^{\mu}, \tag{2.110}$$

$$\Pi_C = \frac{\partial L_4}{\partial(\partial_\tau C)} = 0 = -H_C, \tag{2.111}$$

$$\Pi^0 = \frac{\partial L_4}{\partial(\partial_\tau A_0)} = \frac{\partial L_4}{\partial \dot{A}_0} = 0 = -H_{A_0}, \tag{2.112}$$

and

$$E = \Pi^1 = \frac{\partial L_4}{\partial(\partial_\tau A_1)} = \frac{\partial L_4}{\partial \dot{A}_1} = 0 = \Lambda TC = -H_{A_1}. \qquad (2.113)$$

Now the velocities $\dot{X}^\mu$ can be expressed in terms of the momenta P^μ as

$$\dot{X}^\mu = \frac{1}{\lambda T} P^\mu, \qquad (2.114)$$

The canonical Hamiltonian H is obtained as

$$H_0 = \frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2}\left(X'\right)^2 + \Lambda TCA_0', \qquad (2.115)$$

The set of Hamilton-Jacobi deferential equations are

$$H' = P + H_0$$

$$H' = P + \frac{1}{2\lambda T} P^\mu P_\mu + \frac{\lambda T}{2}\left(X'\right)^2 + \Lambda TCA_0' = 0 \qquad (2.116)$$

$$H_C' = \Pi_C = 0, \qquad (2.117)$$

$$H_{A_0}' = \Pi^0 = 0, \qquad (2.118)$$

and

$$H_{A_1}' = E - \Lambda TC = 0. \qquad (2.119)$$

The total differential equation for characteristics read as

$$dX^\mu = \frac{\partial H'}{\partial P^\mu} d\tau + \frac{\partial H_C'}{\partial P^\mu} dC + \frac{\partial H_{A_0}'}{\partial P^\mu} dA_0 + \frac{\partial H_{A_1}}{\partial P^\mu} dA_1, \qquad (2.120)$$

$$dX^\mu = \frac{1}{\lambda T} P^\mu d\tau, \qquad (2.121)$$

$$dP^\mu = \frac{\partial H'}{\partial X^\mu} d\tau + \frac{\partial H_C'}{\partial X^\mu} dC + \frac{\partial H_{A_0}'}{\partial X^\mu} dA_0 + \frac{\partial H_{A_1}}{\partial X^\mu} dA_1, \qquad (2.122)$$

$$= 0, \qquad (2.123)$$

$$d\Pi_C = \frac{\partial H'}{\partial C} d\tau + \frac{\partial H'_C}{\partial C} dC + \frac{\partial H'_{A_0}}{\partial C} dA_0 + \frac{\partial H'_{A_1}}{\partial C} dA_1, \tag{2.124}$$

$$= \Lambda T A'_0 d\tau - \Lambda T dA_1, \tag{2.125}$$

$$dC = \frac{\partial H'}{\partial \Pi_C} d\tau + \frac{\partial H'_C}{\partial \Pi_C} d\tau + \frac{\partial H'_{A_0}}{\partial \Pi_C} d\tau + \frac{\partial H'_{A_1}}{\partial \Pi_C} d\tau = d\tau \tag{2.126}$$

$$d\Pi^0 = \frac{\partial H'}{\partial A_0} d\tau + \frac{\partial H'_C}{\partial A_0} dC + \frac{\partial H'_{A_0}}{\partial A_0} dA_0 + \frac{\partial H'_{A_1}}{\partial A_0} dA_1, \tag{2.127}$$

$$= 0, \tag{2.128}$$

$$dA_o = \frac{\partial H'}{\partial \Pi_o} d\tau + \frac{\partial H'_C}{\partial \Pi_o} d\tau + \frac{\partial H'_{A_0}}{\partial \Pi_o} d\tau + \frac{\partial H'_{A_1}}{\partial \Pi_o} d\tau = d\tau \tag{2.129}$$

$$dE = d\Pi' + \frac{dH'}{dA_1} d\tau + \frac{dH'_C}{dA_1} dC + \frac{dH'_{A_0}}{dA_1} dA_0 + \frac{dH'_{A_1}}{dA_1} dA_1, \tag{2.130}$$

$$= 0. \tag{2.131}$$

CHAPTER THREE

LIGHT-FRONT QUANTIZATION

3.1 Dirac Approach of Conformal Gauge in Light Front Quantization

In Light Front Quantization we use three local gauge symmetries of the theory to choose $h_{\alpha\beta}$ to be of particular form as [53]

$$h_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} 0 & -1/2 \\ -1/2 & 0 \end{pmatrix}, \tag{3.1}$$

$$h^{\alpha\beta} = \eta^{\alpha\beta} = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix}, \tag{3.2}$$

and $$\sqrt{-h} = \sqrt{-\det(h_{\alpha\beta})} = (+1/2). \tag{3.3}$$

This is the so-called conformal gauge in the Light Front Quantization of the theory. Also, in the Light Front Quantization we use the Light-Cone variables defined by [47-48]:

$$\sigma^{+} = (\tau \pm \sigma) \ \ and \ \ X^{\pm} = \frac{(X^{o} \pm X^{1})}{\sqrt{2}}. \tag{3.4}$$

where $X^{o} = \tau d$, d is constant.

$X^{1} = \frac{L}{2}\sin(\tau)\cos(\sigma)$, with L is the length of the string.

In the instant form quantization of field theories one studies the theory on the hyper surfaces defined by instant form time: $\tau = X^{o}$ = constant.

On the other hand, in the Light Front Quantization of field theories, one studies the theories on the hyper surfaces of light front defined by light cone time

$\tau = X^{+} = \frac{X^{o} + X^{1}}{\sqrt{2}} =$ constant.

The action in the Conformal Gauge in Light Front Quantization reads [49]

$$S_5 = \int L_5 d\sigma^+ d\sigma^- . \tag{3.5}$$

with Lagrangian

$$L_5 = -\frac{T}{2}\eta^{\alpha\beta}\cdot\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}. \tag{3.6}$$

$$\mu,\nu = 0,1,i\,,$$

$$\alpha,\beta = \tau,+,-$$

Substituting from eqs. (3.1) , (3.2) and (3.3), (3.6) becomes

$$\begin{aligned}
L_5 &= -\frac{T}{2}\left[\eta^{+\beta}\partial_+ X^\mu \partial_\beta X^\nu \eta_{\mu\nu} + \eta^{-\beta}\partial_- X^\mu \partial_\beta X^\nu \eta_{\mu\nu}\right],\\
&= -\frac{T}{2}\left[\eta^{++}\partial_+ X^\mu \partial_+ X^\nu \eta_{\mu\nu} + \eta^{-+}\partial_- X^\mu \partial_+ X^\nu \eta_{\mu\nu} + \eta^{+-}\partial_+ X^\mu \partial_- X^\nu \eta_{\mu\nu} + \eta^{--}\partial_+ X^\mu \partial_- X^\nu \eta_{\mu\nu}\right],\\
&= -\frac{T}{2}\left[(-2)\partial_- X^\mu \partial_+ X^\nu \eta_{\mu\nu} + (-2)\partial_+ X^\mu \partial_- X^\nu \eta_{\mu\nu}\right],\\
&= -\frac{T}{2}(-2)\left[\partial_- X^\mu \partial_+ X^\nu \eta_{\mu\nu} + \partial_+ X^\mu \partial_- X^\nu \eta_{\mu\nu}\right],\\
&= T\begin{bmatrix}\partial_- X^+ \partial_+ X^\nu \eta_{+\nu} + \partial_+ X^+ \partial_- X^\nu \eta_{+\nu} + \partial_- X^- \partial_+ X^\nu \eta_{-\nu} + \partial_+ X^- \partial_- X^\nu \eta_{-\nu}\\ +\partial_- X^i \partial_+ X^\nu \eta_{i\nu} + \partial_+ X^i \partial_- X^\nu \eta_{i\nu}\end{bmatrix},\\
&= T\begin{bmatrix}\partial_- X^+ \partial_+ X^+ \eta_{++} + \partial_+ X^+ \partial_- X^+ \eta_{++} + \partial_- X^- \partial_+ X^+ \eta_{-+} + \partial_+ X^- \partial_- X^+ \eta_{-+}\\ +\partial_- X^i \partial_+ X^+ \eta_{i+} + \partial_+ X^i \partial_- X^+ \eta_{i+} + \partial_- X^+ \partial_+ X^- \eta_{+-} + \partial_+ X^+ \partial_- X^- \eta_{+-}\\ +\partial_- X^- \partial_+ X^- \eta_{--} + \partial_+ X^- \partial_- X^- \eta_{--} + \partial_- X^i \partial_+ X^- \eta_{i-} + \partial_+ X^i \partial_- X^- \eta_{i-}\\ +\partial_- X^+ \partial_+ X^i \eta_{+i} + \partial_+ X^+ \partial_- X^i \eta_{+i} + \partial_- X^- \partial_+ X^i \eta_{-i} + \partial_+ X^- \partial_- X^i \eta_{-i}\\ +\partial_- X^i \partial_+ X^i \eta_{ii} + \partial_+ X^i \partial_- X^i \eta_{ii}\end{bmatrix},
\end{aligned} \tag{3.7}$$

$$= -\frac{T}{2}\left[\partial_+ X^+ \partial_- X^- + \partial_+ X^- \partial_- X^+ + \partial_+ X^i \partial_- X^i\right]. \tag{3.8}$$

We now study the Light Front Quantization of the above Polyakov (D1) brane action as the Hamiltonian System using Dirac's Approach.

The canonical momenta P^+, P^- and P^i (i=2,3, …, 25) conjugated to X^-, X^+ *and* X^i respectively, can be obtained from eq.(3.7) are

$$P^{+} = \frac{\partial L_5}{\partial(\partial_{+}X^{-})} = -\frac{T}{2}\partial\_X^{+}, \tag{3.9a}$$

$$P^{-} = \frac{\partial L_5}{\partial(\partial_{+}X^{+})} = -\frac{T}{2}\partial\_X^{-}, \tag{3.9b}$$

$$P^{i} = \frac{\partial L_5}{\partial(\partial_{+}X^{i})} = -\frac{T}{2}\partial\_X^{i}, \tag{3.9c}$$

The equation (3.8), however, imply that the theory possesses twenty six primary constraints

$$\chi_1 = \left[P^{+} + \frac{T}{2}\partial\_X^{+}\right] \simeq 0, \tag{3.10a}$$

$$\chi_2 = \left[P^{-} + \frac{T}{2}\partial\_X^{-}\right] \simeq 0, \tag{3.10b}$$

$$\chi_i = \left[P^{i} + \frac{T}{2}\partial\_X^{i}\right] \simeq 0, \qquad i = 2,3,\ldots 25 \tag{3.10c}$$

The Canonical Hamiltonian Density Corresponding to L_5 is

$$\begin{aligned} H_5^C &= \sum_{i=1}^{n}\left(P_i\dot{q}_i - L_5\right) \\ &= P^{+}(\partial_{+}X^{-}) + P^{-}(\partial_{+}X^{+}) + P^{i}(\partial_{+}X^{i}) - L_5 \simeq 0 \\ &= P^{+}(\partial_{+}X^{-}) + P^{-}(\partial_{+}X^{+}) + P^{i}(\partial_{+}X^{i}) \\ &\qquad -\frac{T}{2}\Big[(\partial_{+}X^{+})(\partial\_X^{-}) + (\partial_{+}X^{-})(\partial\_X^{+}) + (\partial_{+}X^{i})(\partial\_X^{i})\Big] \simeq 0, \end{aligned} \tag{3.11}$$

The total Hamiltonian can be constructed from the canonical Hamiltonian density H_3^C with the help of Lagrange multipliers u , v, w_i .

$$H_5^T = u\left(P^{+} + \frac{T}{2}\partial\_X^{+}\right) + v\left(P^{-} + \frac{T}{2}\partial\_X^{-}\right) + w_i\left(P_i + \frac{T}{2}\partial\_X^{i}\right) \tag{3.12}$$

3.1.1 The Closed Bosnic Strings with Periodic Boundary Conditions

We now treat u, ν *and* w_i as dynamical parameters, the total Hamiltonian is obtained from

$$H_5^T = \int H_5^C d\sigma^- . \tag{3.13}$$

The equations of motion are obtained as

$$\dot{X} = \left[X \ , H_5^T \right], \tag{3.14a}$$

$$+\partial_+ X^- = \frac{\partial H_5^T}{\partial P^+} = u, \tag{3.14b}$$

$$+\partial_+ X^+ = \frac{\partial H_5^T}{\partial P^-} = \nu, \tag{3.14c}$$

$$+\partial_+ X^i = \frac{\partial H_5^T}{\partial P^i} = w_i, \tag{3.14d}$$

$$+\partial_+ u = \frac{\partial H_5^T}{\partial P_u} = 0, \tag{3.14e}$$

$$+\partial_+ \nu = \frac{\partial H_5^T}{\partial P_\nu} = 0, \tag{3.14f}$$

$$+\partial_+ w_i = \frac{\partial H_5^T}{\partial P_{w_i}} = 0. \tag{3.14g}$$

and

$$\dot{P} = \left[P \ , H_5^T \right], \tag{3.15a}$$

$$-\partial_+ P^+ = \frac{\partial H_5^T}{\partial_- X^-} = -\frac{T}{2}\partial_- \nu, \tag{3.15b}$$

$$-\partial_+ P^- = \frac{\partial H_5^T}{\partial_- X^+} = -\frac{T}{2}\partial_- u, \tag{3.15c}$$

$$-\partial_+ P_i = \frac{\partial H_5^T}{\partial_- X^i} = -\frac{T}{2}\partial_- w_i, \tag{3.15d}$$

$$-\partial_+ P_u = \frac{\partial H_5^T}{\partial_- u} = \left[P^+ + \frac{T}{2}\partial_- X^+ \right], \tag{3.15e}$$

$$-\partial_+ P_\nu = \frac{\partial H_5^T}{\partial_- \nu} = \left[P^- + \frac{T}{2}\partial_- X^- \right], \tag{3.15f}$$

$$-\partial_+ P_{w_i} = \frac{\partial H_5^T}{\partial_- w_i} = \left[P^i + \frac{T}{2}\partial_- X^i \right], \tag{3.15g}$$

These are the equations of motion of the theory that preserve the constraints of the theory in the course of time. Demanding that the primary constraints χ_1, χ_2 *and* χ_i , $i = (2,3,...,25)$ be preserved in the course of time one does not get any secondary constraints. The theory is thus seen to possess only twenty six constraints χ_1, χ_2 *and* χ_i , $i = (2,3,...,25)$.

3.1.2 Lagrangian Density of The Theory

The Lagrangian Density of light front L_5 is

$$L_5 = \left[P^+ \left(\partial_+ X^-\right) + P^- \left(\partial_+ X^+\right) + P_i \left(\partial_+ X^i\right) + P_u \left(\partial_+ u\right) + P_\nu \left(\partial_+ \nu\right) + P_{w_i} \left(\partial_+ w_i\right) - H_5^T \right] \tag{3.16}$$

Substituting from eq.(3.11), we obtain

$$L_5 = \left(\frac{T}{2}\right)\left[u \left(\partial_- X^+\right) + \nu \left(\partial_- X^-\right) + w_i \left(\partial_- X^i\right)\right]. \tag{3.17}$$

The matrix of Poisson brackets of constrains χ_j , are constructed as

$$M_{\alpha\beta}(\sigma,\sigma') = \left\{\chi_\alpha(\sigma), \chi_\beta(\sigma')\right\}_{PB} . \tag{3.18}$$

The nonvanishing elements of this matrix are obtained as

$$M_{12} = M_{21} = M_{ii} = T\,\partial_- \delta(\sigma - \sigma'). \tag{3.19}$$

The matrix $M_{\alpha\beta}$ is seen to be nonsingular with determinant

$$\left[\left\|\det(M_{\alpha\beta})\right\|\right]^{1/2} = T\,\partial_-(\delta(\sigma - \sigma')), \tag{3.20}$$

and the nonvanishing elements of inverse of the matrix $\left(M_{\alpha\beta}\right) = \left(M^{-1}\right)_{\alpha\beta}$ are obtained as

$$\left(M^{-1}\right)_{12} = \left(M^{-1}\right)_{21} = \left(M^{-1}\right)_{ii} = \frac{1}{2T}\varepsilon\left(\sigma - \sigma'\right), \tag{3.21}$$

with

$$\int M(\sigma,\sigma'')M^{-1}(\sigma'',\sigma')d\sigma''=\delta(\sigma-\sigma'), \tag{3.22}$$

The step function $\varepsilon(\sigma-\sigma')$ is defined as

$$\varepsilon(\sigma-\sigma')=\begin{cases}+1 & for \quad (\sigma-\sigma')>0\\ -1 & for \quad (\sigma-\sigma')<0\end{cases} \tag{3.23}$$

3.1.3 Commutators of The Light Front Quantization Theory

$$\left[X^{+}(\sigma^{-},\sigma^{+}),P^{-}(\sigma'^{-},\sigma^{+})\right]=\frac{i}{2}\delta(\sigma^{-}-\sigma'^{-}), \tag{3.24a}$$

$$\left[X^{-}(\sigma^{-},\sigma^{+}),P^{+}(\sigma'^{-},\sigma^{+})\right]=\frac{i}{2}\delta(\sigma^{-}-\sigma'^{-}), \tag{3.24b}$$

$$\left[X^{i}(\sigma^{-},\sigma^{+}),P_{i}(\sigma'^{-},\sigma^{+})\right]=\frac{i}{2}\delta(\sigma^{-}-\sigma'^{-}), \tag{3.24c}$$

$$\left[X^{-}(\sigma^{-},\sigma^{+}),X^{+}(\sigma'^{-},\sigma^{+})\right]=\frac{i}{2T}\varepsilon(\sigma^{-}-\sigma'^{-}), \tag{3.24d}$$

$$\left[X^{i}(\sigma^{-},\sigma^{+}),X^{i}(\sigma'^{-},\sigma^{+})\right]=\frac{i}{2T}\varepsilon(\sigma^{-}-\sigma'^{-}), \tag{3.24e}$$

$$\left[P^{+}(\sigma^{-},\sigma^{+}),P^{-}(\sigma'^{-},\sigma^{+})\right]=-\frac{iT}{4}\partial_{-}\delta(\sigma^{-}-\sigma'^{-}), \tag{3.24f}$$

$$\left[P_{i}(\sigma^{-},\sigma^{+}),P_{i}(\sigma'^{-},\sigma^{+})\right]=-\frac{iT}{4}\partial_{-}\delta(\sigma^{-}-\sigma'^{-}), \tag{3.24g}$$

3.2 Hamilton-Jacobi formulation of conformal Gauge Light-Front Quantization (Light Front Quantization)

In this section we use Hamilton-Jacobi method to obtain the equations of motion for light front problem without scalar dilation filed.

We consider the action of the light front Quantization of Polyakov D_1 brane in Conformal Gauge given in eq.(3.5)

$$S_5 = \int -\left[\frac{T}{2}\right]\left[\left(\partial_+ X^+\right)\left(\partial_- X^-\right)+\left(\partial_+ X^-\right)\left(\partial_- X^+\right)+\left(\partial_+ X^i\right)\left(\partial_- X^i\right)\right] d\sigma^+ d\sigma^-, \qquad (3.25)$$

with the Lagrangian density:

$$L_5 = -\left[\frac{T}{2}\right]\left[\left(\partial_+ X^+\right)\left(\partial_- X^-\right)+\left(\partial_+ X^-\right)\left(\partial_- X^+\right)+\left(\partial_+ X^i\right)\left(\partial_- X^i\right)\right], \qquad (3.26)$$

The canonical momenta which is defined in eqs.(1.20) and (1.21) take the forms

$$P^+ = \frac{\partial L_5}{\partial(\partial_+ X^-)} = -\frac{T}{2}\left(\partial_- X^+\right) = -H^+, \qquad (3.27a)$$

$$P^- = \frac{\partial L_5}{\partial(\partial_+ X^+)} = -\frac{T}{2}\left(\partial_- X^-\right) = -H^-, \qquad (3.27b)$$

$$P^i = \frac{\partial L_5}{\partial(\partial_+ X^i)} = -\frac{T}{2}\left(\partial_- X^i\right) = -H^i, \qquad (3.27c)$$

We obtain Hamiltonian-Jacobi partial differential equation as

$$\left(H^+\right)' = P^+ + H^+ = P^+ + \frac{T}{2}\partial_- X^+ = 0, \qquad (3.28a)$$

$$\left(H^-\right)' = P^- + H^- = P^- + \frac{T}{2}\partial_- X^- = 0, \qquad (3.28b)$$

$$\left(H^i\right)' = P^i + H^i = P^i + \frac{T}{2}\partial_- X^i = 0, \qquad (3.28c)$$

The Hamiltonian density H_0 defined in eq. (1.24) is now

$$\begin{aligned} H_0 &= P^+(\partial_+ X^-) + P^-(\partial_+ X^+) + P^i(\partial_+ X^i) \\ &\quad + \frac{T}{2}\left[\partial_+ X^+\left(\frac{-2}{T}\right)P^- + \partial_+ X^-\left(\frac{-2}{T}\right)P^+ + \partial_+ X^i\left(\frac{-2}{T}\right)P^i\right], \end{aligned} \qquad (3.29)$$

Explicitly, H_o becomes

$$H_0 = \left[P^+\left(\partial_+ X^-\right)+P^-\left(\partial_+ X^+\right)+P^i\left(\partial_+ X^i\right)-P^-\left(\partial_+ X^+\right)-P^+\left(\partial_+ X^-\right)-P^i\left(\partial_+ X^i\right)\right] \simeq 0, \qquad (3.30)$$

The canonical Hamiltonian may be written as:

$$H_0 = \int d\sigma^+ d\sigma^- H_o, \qquad (3.31)$$

$$H_0 = \int d\sigma^+ d\sigma^- \left[\left(\partial_+ X^+\right)\left(\partial_- X^-\right) + \left(\partial_+ X^-\right)\left(\partial_- X^+\right) + \left(\partial_+ X^i\right)\left(\partial_- X^i\right)\right], \tag{3.32}$$

$$H_0' = P_0 + H_0, \tag{3.33}$$

$$\begin{aligned} H_0' = P^+ + P^- + P^i &+ \left[P^+\left(\partial_+ X^-\right) + P^-\left(\partial_+ X^+\right) + P^i\left(\partial_+ X^i\right)\right] \\ &- \left[P^-\left(\partial_+ X^+\right) + P^+\left(\partial_+ X^-\right) + P^i\left(\partial_+ X^i\right)\right], \end{aligned} \tag{3.34}$$

According to (1.32) and (1.30), the set of Hamilton-Jacobi Partial Diferential Equation (3.32) leads to the following total differential equation

$$\left(d_+ X^-\right) = \frac{\partial H^{+\prime}}{\partial P^+} d\tau^+ + \frac{\partial H^{-\prime}}{\partial P^+} d\tau^- + \frac{\partial H^{i\prime}}{\partial P^+} d\tau^i = 1d\tau^{+1} = 1dt, \tag{3.35a}$$

$$\left(d_+ X^+\right) = \frac{\partial H^{+\prime}}{\partial P^-} d\tau^+ + \frac{\partial H^{-\prime}}{\partial P^-} d\tau^- + \frac{\partial H^{i\prime}}{\partial P^-} d\tau^i = 1d\tau^{-1} = 1dt, \tag{3.35b}$$

$$\left(d_+ X^i\right) = \frac{\partial H^{+\prime}}{\partial P^i} d\tau^+ + \frac{\partial H^{-\prime}}{\partial P^i} d\tau^- + \frac{\partial H^{i\prime}}{\partial P^i} d\tau^i = 1d\tau^i = 1dt, \tag{3.35c}$$

$$\left(dP^+\right) = \frac{\partial H^{+\prime}}{\partial X^-} dX^+ + \frac{\partial H^{-\prime}}{\partial X^-} dX^+ + \frac{\partial H^{i\prime}}{\partial X^-} dX^i = 0, \tag{3.36a}$$

$$\left(dP^-\right) = \frac{\partial H^{+\prime}}{\partial X^+} dX^- + \frac{\partial H^{-\prime}}{\partial X^+} dX^+ + \frac{\partial H^{i\prime}}{\partial X^+} dX^i = 0, \tag{3.36b}$$

$$\left(dP^i\right) = \frac{\partial H^{+\prime}}{\partial X^i} dX^- + \frac{\partial H^{-\prime}}{\partial X^i} dX^+ + \frac{\partial H^{i\prime}}{\partial X^i} dX^i = 0, \tag{3.36c}$$

3.3 Dirac approach for Light-Front Quantization in presence of scalar dilation field

We now study Light-Front Quantization of this theory in the presence of scalar dilation field Q defined in Light-Front coordinates by the action [52,53].

$$S_6 = \int L_6 d\sigma^+ d\sigma^-,$$

with Lagrangian density

$$L_6 = e^{-\phi} L_5 = \left[-\frac{T}{2} e^{-\phi} \right] \left[\partial^{\beta} X^{M} \partial_{\beta} X_{\mu} \right],$$
$$= -\frac{T}{2} e^{-\phi} \left[\partial_+ X^+ \partial\_ X^- + \partial_+ X^- \partial\_ X^+ + \partial_+ X^i \partial\_ X^i \right], \tag{3.37}$$
$$\mu, \nu = +, -, 2, \ldots (d-1) \quad , \quad i = 2, 3, \ldots (d-1)$$

where (as before) d = 10 for the fermionic string and d=26 for the bosonic string. In the following we would study the Light-Front Quantization of the above action S_6 (which describes the Polyakov D_1 -brane action in the Light-Front coordinates).

The canonical momenta Π, P^+, P^- and P_i conjugate respectively to ϕ, X^-, X^+ and X^i obtained from [46,51] are

$$\Pi = \frac{\partial L_6}{\partial(\partial_+ \phi)} = 0, \tag{3.38}$$

$$P^+ = \frac{\partial L_6}{\partial(\partial_+ X^-)} = -\frac{T}{2} e^{-\phi} \left(\partial\_ X^+ \right), \tag{3.39a}$$
$$P^- = \frac{\partial L_6}{\partial(\partial_+ X^+)} = -\frac{T}{2} e^{-\phi} \left(\partial\_ X^- \right), \tag{3.39b}$$
$$P_i = \frac{\partial L_6}{\partial(\partial_+ X^i)} = -\frac{T}{2} e^{-\phi} \left(\partial\_ X^i \right), \tag{3.39c}$$

The equations (3.37) imply that the theory possesses four primary constraints:

$$\Omega_1 = \Pi = 0, \tag{3.40a}$$
$$\Omega_2 = \left(P^+ + \frac{T}{2} e^{-\phi} \left(\partial\_ X^+ \right) \right) = 0, \tag{3.40b}$$
$$\Omega_3 = \left(P^- + \frac{T}{2} e^{-\phi} \left(\partial\_ X^- \right) \right) = 0, \tag{3.40c}$$
$$\Omega_4 = \left(P_i + \frac{T}{2} e^{-\phi} \left(\partial\_ X^i \right) \right) = 0, \tag{3.40d}$$

The Canonical Hamiltonian density corresponding to L_6 is

$$H_6^C = \sum_{i=1}^{n} \left(P\dot{q} - L_6 \right), \tag{3.41}$$

Explicitly,

$$H_6^C = \left[\Pi(\partial_+\phi) + P^+(\partial_+ X^-) + P^-(\partial_+ X^+) + P_i(\partial_+ X^i) - L_6\right] \simeq 0,$$
$$= \Pi(\partial_+\phi) + P^+(\partial_+ X^-) + P^-(\partial_+ X^+) + P_i(\partial_+ X^i)$$
$$-\frac{T}{2}e^{-\phi}\left[(\partial_+ X^+)(\partial\_X^-) + (\partial_+ X^-)(\partial\_X^+) + (\partial_+ X^i)(\partial\_X^i)\right] = 0. \quad (3.42)$$

3.3.1 The Total Hamiltonian

After including the primary constraints Ω_i in the canonical Hamiltonian density H_6^C with help of Lagrangian multiplier fields μ, ν,w and z the total Hamiltonian density H_6^T could be written as

$$H_6^T = u\,\Omega_1 + \nu\,\Omega_2 + w\,\Omega_3 + z\,\Omega_4 + H_6^C, \quad (3.43)$$

$$H_6^T = \left[u\,\Pi + \nu\left(P^+ + \frac{T}{2}e^{-\phi}\partial\_X^+\right) + w\left(P^- + \frac{T}{2}e^{-\phi}\partial\_X^-\right) + z_i\left(P_i + \frac{T}{2}e^{-\phi}\partial\_X^i\right)\right] + H_6^C, \quad (3.44)$$

3.3.2 The Equations of Motion

We now treat the Lagrange multiplier fields u , ν,w and z_i as dynamical variables.

The Hamilton equations of motion can be obtained from the total Hamiltonian

$$H_6^T = \int H_6^T d\sigma^-, \quad (3.45)$$

The closed bosonic strings with periodic boundary conditions are now defined as

$$+\partial_+ X^- = \frac{\partial H_6^T}{\partial P^+} = \nu, \quad (3.46)$$

$$-\partial_+ P^+ = \frac{\partial H_6^T}{\partial X^-} = \left(-\frac{T}{2}\right)\frac{\partial}{\partial X^-}\left[\frac{\partial w e^{-\phi}\phi}{\partial X^-}\right]$$
$$= -\frac{T}{2}\left[e^{-\phi}\partial\_w - w e^{-\phi}\partial\_\varphi\right] = -\frac{T}{2}e\phi\left[\partial\_w - w\,\partial\_\phi\right], \quad (3.47)$$

$$+\partial_+ X^+ = \frac{\partial H_6^T}{\partial P^+} = w, \quad (3.48)$$

$$-\partial_+ P^- = \frac{\partial H_6^T}{\partial X^+}$$
$$= -\frac{T}{2}\left[e^{-\phi}\partial_- \nu - \nu e^{-\phi}\partial_- \phi\right] = -\frac{T}{2}e^{-\phi}\left[\partial_- \nu - \nu\partial_- \phi\right], \tag{3.49}$$

$$-\partial_+ X^i = \frac{\partial H_6^T}{\partial P^i} = z\,, \tag{3.50}$$

$$-\partial_+ P_i = \frac{\partial H_6^T}{\partial X^i}$$
$$= \left[-\frac{T}{2}e^{-\phi}\right]\left[\partial_- z - z\,\partial_- \phi\right], \tag{3.51}$$

$$+\partial_+ \phi = \frac{\partial H_6^T}{\partial \Pi} = u\,, \tag{3.52}$$

$$-\partial_+ \Pi = \frac{\partial H_6^T}{\partial \phi} = -\frac{T}{2}e^{-\phi}\left[\nu\,\partial_- X^+ + w\,\partial_- X^- + z\,\partial_- X^i\right],$$
$$= -\frac{T}{2}e^{-\phi}\left[\partial_+ X^-\,\partial_- X^+ + \partial_- X^-\partial_+ X^- + \partial_- X^i\partial_+ X^i\right], \tag{3.53}$$

$$+\partial_+ u = \frac{\partial H_6^T}{\partial P_u} = 0, \tag{3.54}$$

$$-\partial_+ P_u = \frac{\partial H_6^T}{\partial u} = \Pi, \tag{3.55}$$

$$+\partial_+ \nu = \frac{\partial H_6^T}{\partial P_\nu} = 0, \tag{3.56}$$

$$-\partial_+ P_\nu = \frac{\partial H_6^T}{\partial \nu} = \left[P^+ + \frac{T}{2}e^{-\phi}\partial_- X^+\right], \tag{3.57}$$

$$+\partial_+ w = \frac{\partial H_6^T}{\partial P_w} = 0, \tag{3.58}$$

$$-\partial_+ P_w = \frac{\partial H_6^T}{\partial w} = \left[P^- + \frac{T}{2}e^{-\phi}\partial_- X^-\right], \tag{3.59}$$

$$+\partial_+ z_i = \frac{\partial H_6^T}{\partial P_{z_i}} = 0, \tag{3.60}$$

$$-\partial_+ P_{z_i} = \frac{\partial H_6^T}{\partial z_i} = \left[P_i + \frac{T}{2} e^{-\phi} \partial_- X^i \right], \tag{3.61}$$

The Lagrangian density of the theory is

$$L_6 = \Big[\Pi(\partial_+\phi) + P^+(\partial_+ X^-) + P^-(\partial_+ X^+) + P_i(\partial_+ X^i) + P_u(\partial_+ u) + P_\nu(\partial_+\nu) + P_w(\partial_+ w)$$
$$+ P_{z_i}(\partial_+ z_i) - H_6^T \Big] \tag{3.62}$$

Substituting from eq.(3.44) we obtain

$$L_6 = \left[\Pi(\partial_+\phi) + P^+(\partial_+ X^-) + P^-(\partial_+ X^+) + P_i(\partial_+ X^i) - u\pi - \nu \left[P^+ + \frac{T}{2} e^{-\phi}(\partial_- X^+) \right] \right.$$
$$\left. - w \left[P^- + \frac{T}{2} e^{-\phi}(\partial_- X^-) \right] - z_i P \left[P_i + \frac{T}{2} e^{-\phi}(\partial_- X^i) \right] \right], \tag{3.63}$$

and from equations of motion eqs.(3.43-58), we get

$$L_6 = u\pi + P^+\nu + P^- w + P_i z_i - u\pi - \nu P^+ - w P^- - z_i P_i$$
$$- \left[\frac{T}{2} e^{-\phi} \right] \left[\nu(\partial_- X^+) + w(\partial_- X^-) + z_i(\partial_- X^i) \right], \tag{3.64}$$

$$L_6 = \left[-\frac{T}{2} e^{-\phi} \right] \left[\nu(\partial_- X^+) + w(\partial_- X^-) + z_i(\partial_- X^i) \right] \tag{3.65}$$

3.3.3 The matrix of the Poisson brackets of the constraints

The matrix of Poisson brackets of constraints Ω_i in eq. (3.40a – 3.40d) namely

$$R_{\alpha\beta}(\sigma,\sigma') := \left\{ \Omega_\alpha(\sigma), \Omega_\beta(\sigma') \right\}_{PB} \tag{3.66}$$

is then calculated, and result is

$$R_{\alpha\beta}=\begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14}\\ R_{21} & R_{22} & R_{23} & R_{24}\\ R_{31} & R_{32} & R_{33} & R_{34}\\ R_{41} & R_{42} & R_{43} & R_{44}\end{pmatrix} \tag{3.67}$$

The nonvanishing elements of matrix $R_{\alpha\beta}(\sigma,\sigma')$ are obtained as

$$R_{12}=-R_{21}=\frac{T}{2}e^{-\phi}\left(\partial_{-}X^{+}\right)\delta(\sigma-\sigma'), \tag{3.68a}$$

$$R_{13}=-R_{31}=\frac{T}{2}e^{-\phi}\left(\partial_{-}X^{-}\right)\delta(\sigma-\sigma'), \tag{3.68b}$$

$$R_{14}=-R_{41}=\frac{T}{2}e^{-\phi}\left(\partial_{-}X^{i}\right)\delta(\sigma-\sigma'), \tag{3.68c}$$

$$R_{23}=R_{32}=R_{44}=Te^{-\phi}\partial_{-}\delta(\sigma-\sigma'). \tag{3.68d}$$

Here $\delta(\sigma-\sigma')$ is the Dirac distribution function.

The matrix $R_{\alpha\beta}$ is seen to be nonsingular with the determinant given by

$$\left[\left\|\det(R_{\alpha\beta})\right\|\right]^{1/2}=\frac{1}{2}R\cdot T^{2}e^{-2\phi}\left[\partial_{-}\delta(\sigma-\sigma')\right]. \tag{3.69}$$

$$R^{2}=\left[2\left(\partial_{-}X^{+}\right)\left(\partial_{-}X^{-}\right)+\left(\partial_{-}X^{i}\right)^{2}\right]. \tag{3.70}$$

The nonvanishing elements of inverse of matrix $R_{\alpha\beta}$

$$(R^{-1})_{\alpha\beta}=\begin{pmatrix} R^{-1}_{11} & R^{-1}_{12} & R^{-1}_{13} & R^{-1}_{14}\\ R^{-1}_{21} & R^{-1}_{22} & R^{-1}_{23} & R^{-1}_{24}\\ R^{-1}_{31} & R^{-1}_{32} & R^{-1}_{33} & R^{-1}_{34}\\ R^{-1}_{41} & R^{-1}_{42} & R^{-1}_{43} & R^{-1}_{44}\end{pmatrix}, \tag{3.71}$$

are

$$(R^{-1})_{11}=\frac{4}{Te^{-\phi}\left[2\left(\partial_{-}X^{+}\right)\left(\partial_{-}X^{-}\right)+\left(\partial_{-}X^{i}\right)^{2}\right]}\partial_{-}\delta(\sigma-\sigma'), \tag{3.72a}$$

$$(R^{-1})_{12}=-(R^{-1})_{21}=\frac{-2}{Te^{-\phi}\left[2\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)+\left(\partial\_X^{i}\right)^{2}\right]}\partial\_X^{-}\delta(\sigma-\sigma'), \qquad (3.72b)$$

$$(R^{-1})_{13}=-(R^{-1})_{31}=\frac{-2}{Te^{-\phi}\left[2\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)+\left(\partial\_X^{i}\right)^{2}\right]}\partial\_X^{+}\delta(\sigma-\sigma'), \qquad (3.72c)$$

$$(R^{-1})_{14}=-(R^{-1})_{41}=\frac{-2}{Te^{-\phi}\left[2\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)+\left(\partial\_X^{i}\right)^{2}\right]}\partial\_X^{i}\delta(\sigma-\sigma'), \qquad (3.72d)$$

$$(R^{-1})_{22}=\frac{-1}{Te^{-\phi}\left[2\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)+\left(\partial\_X^{i}\right)^{2}\right]}\left(\partial\_X^{-}\right)^{2}\varepsilon(\sigma-\sigma'), \qquad (3.72e)$$

$$(R^{-1})_{23}=+(R^{-1})_{32}=\frac{\left[\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)+\left(\partial\_X^{i}\right)^{2}\right]}{Te^{-\phi}\left[2\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)+\left(\partial\_X^{i}\right)^{2}\right]}\varepsilon(\sigma-\sigma'), \qquad (3.72f)$$

$$(R^{-1})_{24}=+(R^{-1})_{42}=\frac{-\left(\partial\_X^{i}\right)\left(\partial\_X^{-}\right)}{2Te^{-\phi}\left[2\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)+\left(\partial\_X^{i}\right)^{2}\right]}\varepsilon(\sigma-\sigma'), \qquad (3.72g)$$

$$(R^{-1})_{33}=+\frac{-\left(\partial\_X^{+}\right)^{2}}{2Te^{-\phi}\left[2\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)+\left(\partial\_X^{i}\right)^{2}\right]}\varepsilon(\sigma-\sigma'), \qquad (3.72h)$$

$$(R^{-1})_{34}=+(R^{-1})_{43}=\frac{\left(\partial\_X^{i}\right)\left(\partial\_X^{+}\right)}{2Te^{-\phi}\left[2\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)+\left(\partial\_X^{i}\right)^{2}\right]}\varepsilon(\sigma-\sigma'), \qquad (3.72i)$$

$$(R^{-1})_{44}=+\frac{\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)}{Te^{-\phi}\left[2\left(\partial\_X^{+}\right)\left(\partial\_X^{-}\right)+\left(\partial\_X^{i}\right)^{2}\right]}\varepsilon(\sigma-\sigma'), \qquad (3.72j)$$

and

$$\int R(\sigma,\sigma'')R^{-1}(\sigma'',\sigma')=|_{4\times4}\delta(\sigma-\sigma'), \qquad (3.73)$$

3.3.4 The Dirac brackets of the theory

The nonvanishing Dirac brackets of the theory described by the Polyakov D_1 bran action S_4 in the presence of scalar dilation field ϕ are formally obtained as [1,2]:

$$\left\{\phi, X^M\right\}_D = \left[\frac{-2}{tR^2}\right]\partial_- X^M \delta(\sigma-\sigma'), \tag{3.74}$$

$$\left\{X^+, P_i\right\}_D = \left\{X^i, P^+\right\}_D = \left[\frac{1}{2R^2}\right](D_1)\delta(\sigma-\sigma'), \tag{3.75a}$$

$$\left\{X^-, P_i\right\}_D = \left\{X^i, P^-\right\}_D = \left[\frac{1}{2R^2}\right](D_2)\delta(\sigma-\sigma'), \tag{3.75b}$$

$$\left\{X^i, P\right\}_{iD} = \left[1-\left(\frac{1}{R^2}\right)(D_3)\right]\delta(\sigma-\sigma'), \tag{3.75c}$$

$$\left\{X^+, P^+\right\}_D = \left[\frac{1}{2R^2}(D_4)\right]\delta(\sigma-\sigma'), \tag{3.75d}$$

$$\left\{X^-, P^-\right\}_D = \left[\frac{1}{2R^2}(D_5)\right]\delta(\sigma-\sigma'), \tag{3.75e}$$

$$\left\{X^+, P^-\right\}_D = \left\{X^-, P^+\right\}_D = \left[1-\left(\frac{1}{2R^2}\right)(D_7)\right]\delta(\sigma-\sigma'), \tag{3.75f}$$

$$\left\{\phi, \Pi\right\}_D = \left[1-\left(\frac{1}{R^2}\right)(D_7)\right]\delta(\sigma-\sigma'), \tag{3.76}$$

$$\left\{X^+, X^i\right\}_D = \left[-\frac{1}{(tR^2)}\right](D_1)\varepsilon(\sigma-\sigma'), \tag{3.77a}$$

$$\left\{X^-, X^i\right\}_D = \left[-\frac{1}{(2tR^2)}\right](D_2)\varepsilon(\sigma-\sigma'), \tag{3.77b}$$

$$\left\{X^i, X^i\right\}_D = \left[\frac{2}{(tR^2)}\right](D_3)\varepsilon(\sigma-\sigma'), \tag{3.77c}$$

$$\left\{X^{+}, X^{+}\right\}_D = \left[-\frac{1}{(2tR^2)}\right](D_4)\varepsilon(\sigma-\sigma'), \qquad (3.77d)$$

$$\left\{X^{-}, X^{-}\right\}_D = \left[-\frac{1}{(2tR^2)}\right](D_5)\varepsilon(\sigma-\sigma'), \qquad (3.77e)$$

$$\left\{X^{-}, X^{+}\right\}_D = \left[-\frac{1}{(2tR^2)}\right](D_7)\varepsilon(\sigma-\sigma'), \qquad (3.77\text{f})$$

$$\left\{\phi, \phi\right\}_D = \left[\left(\frac{4}{tR^2}\right)\right]\partial_{\_}\delta(\sigma-\sigma'). \qquad (3.78)$$

$$\left\{\phi, P^{M}\right\}_D = \left[\frac{1}{R^2}\right]\partial_{\_}X^{M}\delta_{\_}(\sigma-\sigma'), \qquad (3.79)$$

$$\left\{P^{+}, P_i\right\}_{iD} = \left[\frac{t}{(4R^2)}(D_1)\right]\delta_{\_}(\sigma-\sigma'), \qquad (3.80a)$$

$$\left\{P^{-}, P_i\right\}_{iD} = \left[\frac{t}{(4R^2)}(D_2)\right]\delta_{\_}(\sigma-\sigma'), \qquad (3.80b)$$

$$\left\{P_i, P\right\}_{iD} = \left[\frac{-t}{(4R^2)}(D_3)\right]\delta_{\_}(\sigma-\sigma'), \qquad (3.80c)$$

$$\left\{P^{+}, P^{+}\right\}_D = \left[\frac{t}{(4R^2)}(D_4)\right]\delta_{\_}(\sigma-\sigma'), \qquad (3.80d)$$

$$\left\{P^{-}, P^{-}\right\}_D = \left[\frac{t}{(4R^2)}(D_5)\right]\delta_{\_}(\sigma-\sigma'), \qquad (3.80e)$$

Where

$$D_1 = \left(\partial_{\_}X^{i}\right)\left(\partial_{\_}X^{+}\right), \qquad (3.81a)$$
$$D_2 = \left(\partial_{\_}X^{i}\right)\left(\partial_{\_}X^{-}\right), \qquad (3.81b)$$
$$D_3 = \left(\partial_{\_}X^{+}\right)\left(\partial_{\_}X^{-}\right), \qquad (3.81c)$$
$$D_4 = \left(\partial_{\_}X^{+}\right)^2, \qquad (3.81d)$$

$$D_5 = \left(\partial_- X^-\right)^2, \tag{3.81e}$$

$$D_6 = \left(\partial_- X^i\right)^2, \tag{3.81f}$$

$$D_7 = \left(D_3 + D_6\right)^2, \tag{3.81g}$$

and $\quad t = Te^{-\phi}.$ (3.82)

3.4 Hamilton Jacobi Formulation of (Light Front Quantization) in presence of scalar dilation field ϕ:

We studied the following Light Front Quantization in present of scalar dilation field by the action

$$S_6 = \int -\frac{T}{2} e^{-\phi}\left[\partial_+ X^+ \partial_- X^- + \partial_+ X^- \partial_- X^+ + \partial_+ X^i \partial_- X^i\right] d\sigma^+ d\sigma^-, \tag{3.83}$$

With the Lagrangian density

$$L_6 = -\frac{T}{2} e^{-\phi}\left[\partial_+ X^+ \partial_- X^- + \partial_+ X^- \partial_- X^+ + \partial_+ X^i \partial_- X^i\right], \tag{3.84}$$

The canonical momenta defined in (1.20) and (1.21) take the form

$$\Pi = \frac{\partial L_6}{\partial(\partial_+ \phi)} = H_\phi, \tag{3.85a}$$

$$P^+ = \frac{\partial L_6}{\partial(\partial_+ X^-)} = -\frac{T}{2} e^{-\phi}\left(\partial_- X^+\right) = -H^+, \tag{3.85b}$$

$$P^- = \frac{\partial L_6}{\partial(\partial_+ X^+)} = -\frac{T}{2} e^{-\phi}\left(\partial_- X^-\right) = -H^-, \tag{3.85c}$$

$$P_i = \frac{\partial L_6}{\partial(\partial_+ X^i)} = -\frac{T}{2} e^{-\phi}\left(\partial_- X^i\right) = -H^i, \tag{3.85d}$$

The Hamiltonian density H_o is

$$H_o = \left[\Pi(\partial_+ \phi) + P^+\left(\partial_+ X^-\right) + P^-\left(\partial_+ X^+\right) + P^i\left(\partial X^i\right) - L_6\right] \simeq 0 \tag{3.86}$$

$$=\Pi(\partial_+\phi)+P^+\left(\partial_+X^-\right)+P^-\left(\partial_+X^+\right)+P^i\left(\partial_+X^i\right)$$
$$+\frac{T}{2}e^{-\phi}\left[\left(\partial_+X^+\right)\left(\partial_-X^-\right)+\left(\partial_+X^-\right)\left(\partial_-X^+\right)+\left(\partial_+X^i\right)\left(\partial_-X^i\right)\right], \quad (3.87)$$

$$=\Pi(\partial_+\phi)+P^+\left(\partial_+X^-\right)+P^-\left(\partial_+X^+\right)+P^i\left(\partial_+X^i\right)-P^-\left(\partial_+X^+\right)-P^+\left(\partial_+X^-\right)-P^i\left(\partial_+X^i\right), \quad (3.88)$$

The set of Hamilton- Jacobi Partial Differential Equation eqs.(1.26) and (1.27) are

$$H^{+\prime}=P^+ +\frac{T}{2}e^{-\phi}\left(\partial_-X^+\right), \quad (3.89a)$$
$$H^{-\prime}=P^- +\frac{T}{2}e^{-\phi}\left(\partial_-X^-\right), \quad (3.89b)$$
$$H^{i\prime}=P^i +\frac{T}{2}e^{-\phi}\left(\partial_-X^i\right), \quad (3.89c)$$

and

$$H'_\phi=\Pi_\phi=0, \quad (3.90)$$

Using (1.28) and(1.29) ,the set of Hamilton- Jacobi Partial Differential Equation (3.88) leads to the following total differential equation:

$$dP^+=\frac{\partial H^{+\prime}}{\partial X^-}dX^- +\frac{\partial H^{-\prime}}{\partial X^-}dX^+ +\frac{\partial H^{i\prime}}{\partial X^-}dX^i +\frac{\partial H'_\phi}{\partial X^-}d\phi=0, \quad (3.91a)$$
$$dP^-=\frac{\partial H^{+\prime}}{\partial X^+}dX^- +\frac{\partial H^{-\prime}}{\partial X^+}dX^+ +\frac{\partial H^{i\prime}}{\partial X^+}dX^i +\frac{\partial H'_\phi}{\partial X^+}d\phi=0, \quad (3.91b)$$
$$dP^i=\frac{\partial H^{+\prime}}{\partial X^i}dX^- +\frac{\partial H^{-\prime}}{\partial X^i}dX^+ +\frac{\partial H^{i\prime}}{\partial X^i}dX^i +\frac{\partial H'_\phi}{\partial X^i}d\phi=0, \quad (3.91c)$$

$$d\Pi=\frac{\partial H^{+\prime}}{\partial\phi}dX^- +\frac{\partial H^{-\prime}}{\partial\phi}dX^+ +\frac{\partial H^{i\prime}}{\partial\phi}dX^i +\frac{\partial H_\varphi}{\partial\phi}d\phi,$$
$$=-\frac{T}{2}e^{-\varphi}\left(\partial_-X^+\right)dX^- -\frac{T}{2}e^{-\phi}\left(\partial_-X^-\right)dX^+ -\frac{T}{2}e^{-\phi}\left(\partial_-X^i\right)dX^i,$$
$$=-\frac{T}{2}e^{-\phi}\left[\left(\partial_-X^+\right)dX^- +\left(\partial_-X^-\right)dX^+ +\left(\partial_-X^i\right)dX^i\right]. \quad (3.92)$$

$$dX^{-} = \frac{\partial H^{+\prime}}{\partial P^{+}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial P^{+}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial P^{+}} d\tau^{i} + \frac{\partial H_{\phi}^{\prime}}{\partial P^{+}} d\tau^{\phi} = 1d\tau, \qquad (3.93a)$$

$$dX^{+} = \frac{\partial H^{+\prime}}{\partial P^{-}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial P^{-}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial P^{-}} d\tau^{i} + \frac{\partial H_{\phi}^{\prime}}{\partial P^{-}} d\tau^{\phi} = 1d\tau, \qquad (3.93a)$$

$$dX^{i} = \frac{\partial H^{+\prime}}{\partial P^{i}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial P^{i}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial P^{i}} d\tau^{i} + \frac{\partial H_{\phi}^{\prime}}{\partial P^{i}} d\tau^{\phi} = 1d\tau, \qquad (3.93c)$$

3.5 Dirac Method of the Light front Quantization in presence a 2-Form Gauge field $B_{\alpha\beta}$

The action of the light front Quantization of the theory reads

$$S_7 = \int L_7 d\sigma^{+} d\sigma^{-}, \qquad (3.94)$$

$$L_7 = \left[-\frac{\lambda T}{2}\right]\left[\left(\partial_{+}X^{+}\right)\left(\partial_{-}X^{-}\right)+\left(\partial_{+}X^{-}\right)\left(\partial_{-}X^{+}\right)+\left(\partial_{+}X^{i}\right)\left(\partial_{-}X^{i}\right)-\Lambda TB\right], \qquad (2.95)$$

where $B = B_{+-} = -B_{-+}$ $\quad ,\alpha\beta = +,-$

$\mu = +,-,i \quad , \; i = 2,3,\ldots,25$

the canonical momenta Π_B, P^{+}, P^{-} and P_i conjugate respectively to B, X^{-}, X^{+} and X_i are

$$\Pi_B = \frac{\partial L_7}{\partial(\partial_{+}B)} = 0, \qquad (3.96a)$$

$$P^{+} = \frac{\partial L_7}{\partial(\partial_{+}X^{-})} = -\frac{\lambda T}{2}\left(\partial_{-}X^{+}\right), \qquad (3.96b)$$

$$P^{-} = \frac{\partial L_7}{\partial(\partial_{+}X^{+})} = -\frac{\lambda T}{2}\left(\partial_{-}X^{-}\right), \qquad (3.96c)$$

$$P_i = \frac{\partial L_7}{\partial(\partial_{+}X^{i})} = -\frac{\lambda T}{2}\left(\partial_{-}X^{i}\right), \qquad i = 2,3,\cdots,25 \qquad (3.96d)$$

The equations (3.95) imply that the theory possesses the following 27 primary constraints

$$\Psi_1 = \Pi_B \approx 0, \tag{3.97a}$$

$$\Psi_2 = \left[P^+ + \frac{\lambda T}{2}\left(\partial_- X^+\right)\right] \approx 0, \tag{3.97b}$$

$$\Psi_3 = \left[P^- + \frac{\lambda T}{2}\left(\partial_- X^-\right)\right] \approx 0, \tag{3.97c}$$

$$\Psi_i = \left[P_i + \frac{\lambda T}{2}\left(\partial_- X^i\right)\right] \approx 0, \tag{3.97d}$$

The canonical Hamiltonian density corresponding to L is

$$H_7^C = \left[\Pi_B\left(\partial_+ B\right) + P^+\left(\partial_+ X^-\right) + P^-\left(\partial_+ X^+\right) + P^i\left(\partial_+ X^i\right) - L_7 \right], \tag{3.98}$$

The total Hamiltonian density of (LFQ) with help of Lagrangian multiplier field $S(\sigma^+,\sigma^-)$, $u(\sigma^+,\sigma^-)$, $v(\sigma^+,\sigma^-)$ and $w_i(\sigma^+,\sigma^-)$ which treat as dynamical, could be written as

$$H_7^T = \left[H_7^C + S\Psi_1 + u\Psi_2 + v\Psi_3 + w_i\Psi_i \right], \tag{3.99}$$

$$\begin{aligned} H_7^T = \Bigg[& \Lambda T B + S\Pi_B + u\left[P^+ + \frac{\lambda T}{2}\left(\partial_- X^+\right)\right] + v\left[P^- + \frac{\lambda T}{2}\left(\partial_- X^-\right)\right] \\ & + w_i\left[P_i + \frac{\lambda T}{2}\left(\partial_- X^i\right)\right] \Bigg] \end{aligned} \tag{3.101}$$

The equation of motions

$$\partial_+ X^- = \frac{\partial H_7^T}{\partial P^+} = u \quad , \tag{3.102a}$$

$$\partial_+ X^+ = \frac{\partial H_7^T}{\partial P^-} = v \ , \tag{3.102b}$$

$$\partial_+ X^i = \frac{\partial H_7^T}{\partial P^i} = w_i \ , \tag{3.102c}$$

$$\partial_+ u = \frac{\partial H_7^T}{\partial P_u} = 0 \ , \tag{3.102d}$$

$$\partial_+ \nu = \frac{\partial H_7^T}{\partial P_\nu} = 0 \ , \tag{3.102e}$$

$$\partial_+ w_i = \frac{\partial H_7^T}{\partial P_{w_i}} = 0 \ , \tag{3.102f}$$

$$\partial_+ B_i = \frac{\partial H_7^T}{\partial \Pi_B} = S \ , \tag{3.102d}$$

and

$$\partial_+ P^+ = \frac{\partial H_7^T}{\partial X^-} = \frac{\lambda T}{2} \partial_- \nu \ , \tag{3.103a}$$

$$\partial_+ P^+ = \frac{\partial H_7^T}{\partial X^+} = \frac{\lambda T}{2} \partial_- u \ , \tag{3.103b}$$

$$\partial_+ P^i = \frac{\partial H_7^T}{\partial X^i} = \frac{\lambda T}{2} \partial_- w \ , \tag{3.103c}$$

$$\partial_+ P_u = \frac{\partial H_7^T}{\partial u} = \left(P^+ + \frac{\lambda T}{2} \left(\partial_- X^+ \right) \right) \ , \tag{3.103e}$$

$$\partial_+ P_\nu = \frac{\partial H_7^T}{\partial \nu} = \left(P^- + \frac{\lambda T}{2} \left(\partial_- X^- \right) \right) \ , \tag{3.103f}$$

$$\partial_+ P_w = \frac{\partial H_7^T}{\partial w} = \left(P_i + \frac{\lambda T}{2} \left(\partial_- X^i \right) \right) \ , \tag{3.103g}$$

$$\partial_\tau \Pi_B = \frac{\partial H_7^T}{\partial B} = \Lambda^T \ , \tag{3.103h}$$

and

$$\left[X^+, P^- \right] = \left[X^-, P^+ \right] = \left[X^i, P_i \right] = \frac{i}{2} \delta\left(\sigma - \sigma' \right), \tag{3.104a}$$

$$\left[X^-, X^+ \right] = \left[X^i, P^i \right] = \frac{1}{2\lambda T} \varepsilon\left(\sigma - \sigma' \right), \tag{3.104b}$$

$$\left[P^+, P^- \right] = \left[P_i, P_i \right] = \frac{-iT\lambda}{4} \partial_- \delta\left(\sigma - \sigma' \right). \tag{3.104c}$$

3.6 Hamilton-Jacobi of Light Front Quantization of Conformal Fixed Polyakov D1-Brane action in presence of 2-Form Gauge Field $B_{\alpha\beta}$

The action of Light Front Quantization of the theory is

$$S_7 = \int L_7 d\sigma^+ d\sigma^- ,$$

The Lagrangian is

$$L_7 = \left[-\frac{\lambda T}{2}\right]\left[\left(\partial_+ X^+\right)\left(\partial_- X^-\right) + \left(\partial_+ X^-\right)\left(\partial_- X^+\right) + \left(\partial_+ X^i\right)\left(\partial_- X^i\right) - \Lambda T B\right],$$

The canonical Momenta defined in (1.20) and (1.21) read as

$$P^+ = \frac{\partial L_7}{\partial(\partial_+ X^-)} = -\frac{\lambda T}{2}\left(\partial_- X^+\right) = -H^+, \tag{3.105a}$$

$$P^- = \frac{\partial L_7}{\partial(\partial_+ X^+)} = -\frac{\lambda T}{2}\left(\partial_- X^-\right) = -H^-, \tag{3.105b}$$

$$P_i = \frac{\partial L_7}{\partial(\partial_+ X^i)} = -\frac{\lambda T}{2}\left(\partial_- X^i\right) = -H^i, \tag{3.105c}$$

$$\Pi_B = \frac{\partial L_7}{\partial B} = 0 = -H_B, \tag{3.105d}$$

The canonical Hamiltonian is obtain as

$$\begin{aligned} H_7^L &= \left[\Pi_B\left(\partial_+ B\right) + P^+\left(\partial_- X^+\right) + P^-\left(\partial_+ X^+\right) + P^i\left(\partial_+ X^+\right) - L_7\right], \\ &= \Pi_B\left(\partial_+ B\right) - \frac{\lambda T}{2}\left(\Lambda T B\right) + P^+\left(\partial_- X^+\right) + P^-\left(\partial_+ X^+\right) + P^i\left(\partial_+ X^+\right) \\ &\quad + \frac{\lambda T}{2}\left[\partial_+ X^+ \partial_- X^- + \partial_+ X^- \partial_- X^+ + \partial_+ X^i \partial_+ X^i\right], \end{aligned} \tag{3.106}$$

$$H_7^L = \Pi_B\left(\partial_+ B\right) - \frac{\lambda \Lambda T^2 B}{2}, \tag{3.107}$$

The set of Hamilton- Jacobi Partial Differential Equations are

$$H^{+\prime} = \left(P^{+} + \frac{\lambda T}{2} \left(\partial_{-} X^{+} \right) \right) = 0, \tag{3.108a}$$

$$H^{-\prime} = \left(P^{-} + \frac{\lambda T}{2} \left(\partial_{-} X^{-} \right) \right) = 0, \tag{3.108b}$$

$$H^{i\prime} = \left(P^{i} + \frac{\lambda T}{2} \left(\partial_{-} X^{i} \right) \right) = 0, \tag{3.108c}$$

and,

$$H_B{}' = \Pi_B = 0, \tag{3.108d}$$

The differential equations for characteristics read as

$$dP^{+} = \frac{\partial H^{+\prime}}{\partial X^{-}} dX^{-} + \frac{\partial H^{-\prime}}{\partial X^{-}} dX^{+} + \frac{\partial H^{i\prime}}{\partial X^{-}} dX^{i} + \frac{\partial H_B'}{\partial X^{-}} dB = 0, \tag{3.109a}$$

$$dP^{-} = \frac{\partial H^{+\prime}}{\partial X^{+}} dX^{-} + \frac{\partial H^{-\prime}}{\partial X^{+}} dX^{+} + \frac{\partial H^{i\prime}}{\partial X^{+}} dX^{i} + \frac{\partial H_B'}{\partial X^{+}} dB = 0, \tag{3.109b}$$

$$dP^{i} = \frac{\partial H^{+\prime}}{\partial X^{i}} dX^{-} + \frac{\partial H^{-\prime}}{\partial X^{i}} dX^{+} + \frac{\partial H^{i\prime}}{\partial X^{i}} dX^{i} + \frac{\partial H_B'}{\partial X^{i}} dB = 0, \tag{3.109c}$$

$$d\Pi_B = \frac{\partial H^{+\prime}}{\partial B} dX^{-} + \frac{\partial H^{-\prime}}{\partial B} dX^{+} + \frac{\partial H^{i\prime}}{\partial B} dX^{i} + \frac{\partial H_B'}{\partial B} dB = 0, \tag{3.109d}$$

and

$$dX^{+} = \frac{\partial H^{+\prime}}{\partial P^{-}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial P^{-}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial P^{-}} d\tau^{i} + \frac{\partial H_B'}{\partial P^{-}} d\tau = d\tau^{-}, \tag{3.109e}$$

$$dX^{-} = \frac{\partial H^{+\prime}}{\partial P^{+}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial P^{+}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial P^{+}} d\tau^{i} + \frac{\partial H_B'}{\partial P^{+}} d\tau = d\tau^{+}, \tag{3.109f}$$

$$dX^{i} = \frac{\partial H^{+\prime}}{\partial P^{i}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial P^{i}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial P^{i}} d\tau^{i} + \frac{\partial H_B'}{\partial P^{i}} d\tau = d\tau^{i}, \tag{3.109g}$$

$$dB = \frac{\partial H^{+\prime}}{\partial \Pi_B} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial \Pi_B} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial \Pi_B} d\tau^{+} + \frac{\partial H_B'}{\partial \Pi_B} d\tau = d\tau, \tag{3.109h}$$

3.7 Dirac Method of Light-Front Quantization of conformal Gauge Fixed Polyakov D1-Brane action in presence of a scalar Axion Field C and U(1) Gauge Field A^{μ}

In Light-Front Quantization, the action of the theory reads

$$S_8 = \int L_8 d\sigma^+ d\sigma^-, \tag{3.110}$$

$$L_8 = \left[-\frac{\lambda T}{2}\right]\left[\left(\partial_+ X^+\right)\left(\partial_- X^-\right) + \left(\partial_+ X^-\right)\left(\partial_- X^+\right) + \left(\partial_+ X^i\right)\left(\partial_- X^i\right) + \Lambda TCf\ \right], \tag{3.111}$$

The canonical momenta $P^+, P^-, P_i, \Pi_c, \Pi^+$ and Π^- conjugate respectively to X^-, X^+, X_i, C, A^- and A^+ are obtained as

$$\Pi^+ = \frac{\partial L_8}{\partial(\partial_+ A^-)} = 0, \tag{3.112a}$$

$$\Pi^- = \frac{\partial L_8}{\partial(\partial_+ A^+)} = \Lambda TC, \tag{3.112b}$$

$$\Pi_c = \frac{\partial L_8}{\partial(\partial_+ C)} = 0, \tag{3.112c}$$

$$P^+ = \frac{\partial L_8}{\partial(\partial_+ X^-)} = -\frac{\lambda T}{2}\left(\partial_- X^+\right), \tag{3.112d}$$

$$P^- = \frac{\partial L_8}{\partial(\partial_+ X^+)} = -\frac{\lambda T}{2}\left(\partial_- X^-\right), \tag{3.112e}$$

$$P_i = \frac{\partial L_8}{\partial(\partial_+ X^i)} = -\frac{\lambda T}{2}\left(\partial_- X^i\right), \tag{3.112f}$$

$$i = 2, 3, \cdots, 25$$

The above equations however, imply that the theory possesses 29 primary constraints

$$\chi_1 = \Pi^+ \approx 0, \tag{3.113a}$$

$$\chi_2 = \left(\Pi^- - \Lambda TC\right) \approx 0, \tag{3.113b}$$

$$\chi_3 = \Pi_c \approx 0, \tag{3.113c}$$

$$\chi_4 = \left[P^+ + \frac{\lambda T}{2} \left(\partial_- X^+ \right) \right] \approx 0, \qquad (3.113e)$$

$$\chi_5 = \left[P^- + \frac{\lambda T}{2} \left(\partial_- X^- \right) \right] \approx 0, \qquad (3.113f)$$

$$\chi_i = \left[P_i + \frac{\lambda T}{2} \left(\partial_- X^i \right) \right] \approx 0, \qquad (3.113g)$$

$$i = 2, 3, \cdots, 25$$

Canonical Hamiltonian density of this theory is

$$h_8^C = \left[P^+ \left(\partial_+ X^- \right) + P^- \left(\partial_+ X^+ \right) + P_i \left(\partial_+ X^i \right) + \Pi^+ \left(\partial_+ A^- \right) + \Pi^- \left(\partial_+ A^+ \right) \right. \qquad (3.114)$$
$$\left. + \Pi_C \left(\partial_+ C \right) - L_8 \right]$$

$$= \left[\Lambda T C \left(\partial_- A^- \right) \right], \qquad (3.115)$$

After including the above 29 primary constraints in the canonical Hamiltonian density H_8^C with the help of Lagrange multiplier fields $\nu_1(\sigma^+, \sigma^-)$, $\nu_2(\sigma^+, \sigma^-)$, $\nu_3(\sigma^+, \sigma^-)$, $\nu_4(\sigma^+, \sigma^-)$, $\nu_5(\sigma^+, \sigma^-)$ and $\nu_i(\sigma^+, \sigma^-)$ (which we treat as dynamical), the total Hamiltonian density of the theory could be written as

$$H_8^T = \left[H_8^c + s_1 \chi_1 + s_2 \chi_2 + s_3 \chi_3 + u \chi_4 + v \chi_5 + w_i \chi_i \right], \qquad (3.116)$$

$$H_8^T = \left[\Lambda T C \left(\partial_- A^- \right) + u \left[P^+ + \left(\frac{\lambda T}{2} \right) \left(\partial_- X^+ \right) \right] + v \left[P^- + \left(\frac{\lambda T}{2} \right) \left(\partial_- X^- \right) \right] \right.$$
$$\left. + w_i \left[P_i + \left(\frac{\lambda T}{2} \right) \left(\partial_- X^i \right) \right] + s_1 \Pi^+ + s_2 \left(\Pi^- - \Lambda T C \right) + s_3 \Pi_c \right] \qquad (3.117)$$

The Hamiltons equations of motion of the theory that preserve the constraints of theory in the course of time obtained from the total Hamiltonian

$$H_8^T = \int h_8^T d\sigma^-, \qquad (3.118)$$

e.g., for the closed strings with periodic BC's are obtained as

$$\partial_+ X^- = \frac{\partial H_8^T}{\partial P^+} = u \qquad ,$$
$$-\partial_+ P^+ = \frac{\partial H_8^T}{\partial X^-} = \left[-\frac{\lambda T}{2}\right]\left(\partial_- v\right), \qquad (3.119)$$

$$\partial_+ X^+ = \frac{\partial H_8^T}{\partial P^-} = v \qquad ,$$
$$-\partial_+ P^- = \frac{\partial H_8^T}{\partial X^+} = \left[-\frac{\lambda T}{2}\right]\left(\partial_- u\right), \qquad (3.120)$$

$$\partial_+ X^i = \frac{\partial H_8^T}{\partial P_i} = w_i \qquad ,$$
$$-\partial_+ P_i = \frac{\partial H_8^T}{\partial X^i} = \left[-\frac{\lambda T}{2}\right]\left(\partial_- w_i\right), \qquad (3.121)$$

$$\partial_+ u = \frac{\partial H_8^T}{\partial p_u} = 0 \qquad ,$$
$$-\partial_+ p_u = \frac{\partial H_8^T}{\partial u} = \left(P^+ + \frac{\lambda T}{2}\partial_- X^+\right), \qquad (3.122)$$

$$\partial_+ v = \frac{\partial H_8^T}{\partial p_v} = 0 \qquad ,$$
$$-\partial_+ p_v = \frac{\partial H_8^T}{\partial v} = \left(P^- + \frac{\lambda T}{2}\partial_- X^-\right), \qquad (3.123)$$

$$\partial_+ w_i = \frac{\partial H_8^T}{\partial p_{w_i}} = 0 \qquad ,$$
$$-\partial_+ p_{w_i} = \frac{\partial H_8^T}{\partial w_i} = \left(P_i + \frac{\lambda T}{2}\partial_- X^i\right), \qquad (3.124)$$

$$\partial_+ C = \frac{\partial H_8^T}{\partial \Pi_C} = s_3 \qquad ,$$
$$-\partial_+ \Pi_C = \frac{\partial H_8^T}{\partial C} = \Lambda T\left(\partial_- A^- - s_2\right),$$
$$= \Lambda T\left(\partial_- A^- - \partial_+ A^+\right) = \Lambda T dA^+. \qquad (3.125)$$

where $dA^+ = \left(\partial_- A^- - \partial_+ A^+\right)$.

$$\partial_+ A^+ = \frac{\partial H_8^T}{\partial \Pi^-} = s_2 \qquad ,$$
$$-\partial_+ \Pi^- = \frac{\partial H_8^T}{\partial A^+} = 0, \tag{3.126}$$

$$\partial_+ A^- = \frac{\partial H_8^T}{\partial \Pi^+} = s_1 \qquad ,$$
$$-\partial_+ \Pi^+ = \frac{\partial H_8^T}{\partial A^-} = 0, \tag{3.127}$$

$$\partial_+ s_1 = \frac{\partial H_8^T}{\partial p_{s_1}} = s_2 \qquad ,$$
$$-\partial_+ p_{s_1} = \frac{\partial H_8^T}{\partial s_1} = \Pi^+, \tag{3.128}$$

$$\partial_+ s_2 = \frac{\partial H_8^T}{\partial p_{s_2}} = s_2 \qquad ,$$
$$-\partial_+ p_{s_2} = \frac{\partial H_8^T}{\partial s_2} = \left(\Pi^- - \Lambda T C\right), \tag{3.129}$$

$$\partial_+ s_3 = \frac{\partial H_8^T}{\partial p_{s_3}} = 0 \qquad ,$$
$$-\partial_+ p_{s_3} = \frac{\partial H_8^T}{\partial s_3} = \Pi_C\,, \tag{3.130}$$

Demanding that the primary constraints of the theory be preserved in the course of time one does not get any secondary constraints. The theory is thus seen to possess only 29 constraints: $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ and χ_i. Further the matrix of the Poisson brackets of these 29 constraints among themselves is easily seen to be singular, implying that the set of these 29 constraints is first-class. This in turn implies that the theory is a gauge invariant (GI) (and consequently gauge anomalous). The theory is indeed seen to possess three local gauge symmetries given by the 2D WorldSheet reparametrization invariant (WSRI) and the Weyl invariance (WI). The theory could now be quantized under appropriate gauge-fixing.

The first-order Lagrangian density of the theory is

$$L_8^{IO} = \Big[\Pi_c \left(\partial_+ C\right) + P^+ \left(\partial_+ X^-\right) + P^- \left(\partial_+ X^+\right) + P_i \left(\partial_+ X^i\right) + \Pi^+ \left(\partial_+ A^-\right) + \Pi^- \left(\partial_+ A^+\right)$$
$$+ p_{s_1} (\partial_+ s_1) + p_{s_2} (\partial_+ s_2) + p_{s_3} (\partial_+ s_3) + p_u (\partial_+ u) + p_v (\partial_+ v) + p_{w_i} (\partial_+ w_i) - H_8^T \Big], \quad (3.131)$$

$$L_8^{IO} = \left[\Lambda T C \left(s_2 - \partial_- A^-\right) + \frac{\lambda T}{2} \left[u \left(\partial_- X^+\right) + v \left(\partial_- X^-\right) + w_i \left(\partial_- X^i\right) \right] \right], \quad (3.132)$$

To study the Hamiltonian and path integral formulations of the theory under gauge-fixing, we could as example, choose the gauge:

$$\theta = A^- \approx 0, \quad (3.133)$$

Corresponding to this gauge choice, the total set of constraints of the theory under which the quantization of the theory could be studied becomes

$$\eta_1 = \chi_1 = \Pi^+ \approx 0, \quad (3.134a)$$
$$\eta_2 = \chi_2 = \left(\Pi^- - \Lambda T C\right) \approx 0, \quad (3.134b)$$
$$\eta_3 = \chi_3 = \Pi_c \approx 0, \quad (3.134c)$$
$$\eta_4 = \theta = A^- \approx 0, \quad (3.134d)$$

$$\eta_5 = \chi_4 = \left[P^+ + \frac{\lambda T}{2} \left(\partial_- X^+\right) \right] \approx 0, \quad (3.134e)$$
$$\eta_6 = \chi_5 = \left[P^- + \frac{\lambda T}{2} \left(\partial_- X^-\right) \right] \approx 0, \quad (3.134f)$$
$$\eta_i = \chi_i = \left[P_i + \frac{\lambda T}{2} \left(\partial_- X^i\right) \right] \approx 0, i = 2, 3, \cdots, 25 \quad (3.134g)$$

We now calculate the matrix

$$R_{\alpha\beta}\left(\sigma, \sigma'\right) = \left(\left\{ \eta_\alpha \left(\sigma\right), \eta_\beta \left(\sigma'\right) \right\}_{PB} \right), \quad (3.135)$$

of the Poisson brackets of these above 30 constraints. The nonvanishing elements of the matrix $R_{\alpha\beta}$ are obtained as

$$R_{56} = +R_{65} = R_{ii} = \left(\lambda T\right) \partial_- \delta\left(\sigma - \sigma'\right), i = 2, 3, \cdots, 25 \quad (3.136)$$

$$\Lambda T R_{14} = -\Lambda T R_{41} = R_{23} = -R_{32} = \left(-\Lambda T\right)\delta\left(\sigma - \sigma'\right) \tag{3.137}$$

The matrix $R_{\alpha\beta}$ is seen to be nonsingular implying that the corresponding set of these 30 constraints is second-class. The determinant of the matrix $R_{\alpha\beta}$ is given by

$$\left[\left\|\det\left(R_{\alpha\beta}\right)\right\|\right]^{1/2} = \left[\left(\Lambda T\,\delta^2\left(\sigma - \sigma'\right)\right)\left(\Lambda T\,\partial_-\delta\left(\sigma - \sigma'\right)\right)^{13}\right] \tag{3.138}$$

Nonvanishing elements of the inverse of this matrix $R_{\alpha\beta}$ (i.e. the elements of the matrix $\left(R^{-1}\right)_{\alpha\beta}$) are

$$\left(R^{-1}\right)_{56} = \left(R^{-1}\right)_{65} = \left(R^{-1}\right)_{ii} = \frac{1}{2\lambda T}\varepsilon\left(\sigma - \sigma'\right), i = 2, 3, \cdots, 25 \tag{3.139}$$

$$\left(R^{-1}\right)_{14} = -\left(R^{-1}\right)_{41} = \left(R^{-1}\right)_{ii} = \Lambda T\,\left(R^{-1}\right)_{23} = -\Lambda T\,\left(R^{-1}\right)_{32} = \delta\left(\sigma - \sigma'\right), \tag{3.140}$$

with

$$\int R\left(\sigma, \sigma''\right) R^{-1}\left(\sigma'', \sigma'\right) d\sigma'' = \mathbf{1}_{30\times 30}\delta\left(\sigma - \sigma'\right). \tag{3.141}$$

$$\left[X^+, P^-\right] = \left[X^-, P^+\right] = \left[X^i, P_i\right] = \left[i/2\right]\delta\left(\sigma - \sigma'\right), \tag{3.142a}$$

$$\left[X^-, X^+\right] = \left[X^i, X^i\right] = \left[i/\left(2\lambda T\right)\right]\varepsilon\left(\sigma - \sigma'\right), \tag{3.142b}$$

$$\left[P^+, P^-\right] = \left[P_i, P_i\right] = \left[-iT\,\lambda/4\right]\partial_-\delta\left(\sigma - \sigma'\right), \tag{3.142c}$$

$$\left[A^+, \Pi^-\right] = \Lambda T\,\left[A^+, C\right] = \left(i\right)\delta\left(\sigma - \sigma'\right). \tag{3.142d}$$

3.8 Hamilton-Jacobi Method of Light Front Quantization of Conformal Gauge Fixed Polyakov D1-Brane action in Presence of a Scalar Axion Field *C* and An *U*(1) Gauge Field A^μ

The action of Light Front Quantization

$$S_8 = \int L_8 d\sigma^+ d\sigma^-, \tag{3.143}$$

The Lagrangian is

$$L_8 = \left[-\frac{\lambda T}{2}\right]\left[\left(\partial_+ X^+\right)\left(\partial_- X^-\right) + \left(\partial_+ X^-\right)\left(\partial_- X^+\right) + \left(\partial_+ X^i\right)\left(\partial_- X^i\right) - \Lambda T C f\right], \tag{3.144}$$

The canonical momenta defined in (1. 20) and (1. 21) as

$$\Pi^+ = \frac{\partial L_8}{\partial(\partial_+ A^-)} = 0 = -H_{\Pi^+}, \tag{3.145a}$$

$$\Pi^- = \frac{\partial L_8}{\partial(\partial_+ A^+)} = \Lambda T C = -H_{\Pi^-}, \tag{3.145b}$$

$$\Pi_c = \frac{\partial L_8}{\partial(\partial_+ C)} = 0 = -H_C, \tag{3.145c}$$

$$P^+ = \frac{\partial L_8}{\partial(\partial_+ X^-)} = -\frac{\lambda T}{2}\left(\partial_- X^+\right) = -H^+, \tag{3.145d}$$

$$P^- = \frac{\partial L_8}{\partial(\partial_+ X^+)} = -\frac{\lambda T}{2}\left(\partial_- X^-\right) = -H^-, \tag{3.145e}$$

and

$$P_i = \frac{\partial L_8}{\partial(\partial_+ X^i)} = -\frac{\lambda T}{2}\left(\partial_- X^i\right) = -H^i, \tag{3.145f}$$

The canonical Hamilton density is obtained as

$$H_8^C = \Big[P^+\left(\partial_+ X^-\right) + P^-\left(\partial_+ X^+\right) + P_i\left(\partial_+ X^i\right) + \Pi^+\left(\partial_+ A^-\right) + \Pi^-\left(\partial_+ A^+\right) + \Pi_C\left(\partial_+ C\right) - L_8\Big] \tag{3.146}$$

$$= \left[\Lambda T C\left(\partial_- A^-\right)\right], \tag{3.147}$$

The set of HJPDE's are

$$H'_{\Pi^+} = \Pi^+ = 0, \tag{3.148a}$$

$$H'_{\Pi^-} = \Pi^- - \Lambda T C = 0, \tag{3.148b}$$

$$H'_C = \Pi_C = 0, \tag{3.148c}$$

$$H^{+\prime} = P^{+} + \frac{\lambda T}{2}\left(\partial_{-} X^{+}\right) = 0, \quad (3.149a)$$

$$H^{-\prime} = P^{-} + \frac{T}{2}\left(\partial_{-} X^{-}\right) = 0, \quad (3.149b)$$

and

$$H^{i\prime} = P^{i} + \frac{T}{2}\left(\partial_{-} X^{i}\right) = 0. \quad (3.149c)$$

The total differential equations for characteristics are

$$d\Pi^{+} = \frac{\partial H'_{\Pi^{+}}}{\partial A^{-}} dA^{-} + \frac{\partial H'_{\Pi^{-}}}{\partial A^{-}} dA^{+} + \frac{\partial H'_{C}}{\partial A^{-}} dC + \frac{\partial H'^{+}}{\partial A^{-}} dX^{-} + \frac{\partial H'^{-}}{\partial A^{-}} dX^{+} + \frac{\partial H'^{i}}{\partial A^{-}} dX^{i} = 0, \quad (3.150a)$$

$$d\Pi^{-} = \frac{\partial H'_{\Pi^{+}}}{\partial A^{+}} dA^{-} + \frac{\partial H'_{\Pi^{-}}}{\partial A^{+}} dA^{+} + \frac{\partial H'_{C}}{\partial A^{+}} dC + \frac{\partial H'^{+}}{\partial A^{+}} dX^{-} + \frac{\partial H'^{-}}{\partial A^{+}} dX^{+} + \frac{\partial H'^{i}}{\partial A^{+}} dX^{i} = 0, \quad (3.150b)$$

$$d\Pi_{C} = \frac{\partial H'_{\Pi^{+}}}{\partial C} dA^{-} + \frac{\partial H'_{\Pi^{-}}}{\partial C} dA^{+} + \frac{\partial H'_{C}}{\partial C} dC + \frac{\partial H'^{+}}{\partial C} dX^{-} + \frac{\partial H'^{-}}{\partial C} dX^{+} + \frac{\partial H'^{i}}{\partial C} dX^{i} = -\Lambda T dA^{+}, \quad (3.150c)$$

$$dP^{+} = \frac{\partial H'_{\Pi^{+}}}{\partial X^{-}} dA^{-} + \frac{\partial H'_{\Pi^{-}}}{\partial X^{-}} dA^{+} + \frac{\partial H'_{C}}{\partial X^{-}} dC + \frac{\partial H'^{+}}{\partial X^{-}} dX^{-} + \frac{\partial H'^{-}}{\partial X^{-}} dX^{+} + \frac{\partial H'^{i}}{\partial X^{-}} dX^{i} = 0, \quad (3.150d)$$

Similarly

$$dP^{-} = \frac{\partial H'_{\Pi^{+}}}{\partial X^{+}} dA^{-} + \frac{\partial H'_{\Pi^{-}}}{\partial X^{+}} dA^{+} + \frac{\partial H'_{C}}{\partial X^{+}} dC + \frac{\partial H'^{+}}{\partial X^{+}} dX^{-} + \frac{\partial H'^{-}}{\partial X^{+}} dX^{+} + \frac{\partial H'^{i}}{\partial X^{+}} dX^{i} = 0, \quad (3.150e)$$

$$dP^{i} = \frac{\partial H'_{\Pi^{+}}}{\partial X^{i}} dA^{-} + \frac{\partial H'_{\Pi^{-}}}{\partial X^{i}} dA^{+} + \frac{\partial H'_{C}}{\partial X^{i}} dC + \frac{\partial H'^{+}}{\partial X^{i}} dX^{-} + \frac{\partial H'^{-}}{\partial X^{i}} dX^{+} + \frac{\partial H'^{i}}{\partial X^{i}} dX^{i} = 0, \quad (3.150f)$$

and

$$dX^{-} = \frac{\partial H^{+\prime}}{\partial P^{+}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial P^{+}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial P^{+}} d\tau^{i} + \frac{\partial H^{+\prime}_{\Pi^{-}}}{\partial P^{+}} d\tau^{A-} + \frac{\partial H^{+\prime}_{\Pi^{+}}}{\partial P^{+}} d\tau^{A+} + \frac{\partial H^{+\prime}_{\Pi^{C}}}{\partial P^{+}} d\tau^{C} = d\tau^{+}, \quad (3.151a)$$

$$dX^{+} = \frac{\partial H^{+\prime}}{\partial P^{-}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial P^{-}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial P^{-}} d\tau^{i} + \frac{\partial H^{+\ \prime}_{\Pi^{-}}}{\partial P^{-}} d\tau^{A-} + \frac{\partial H^{+\ \prime}_{\Pi^{+}}}{\partial P^{-}} d\tau^{A+} + \frac{\partial H^{+\ \prime}_{\Pi^{C}}}{\partial P^{-}} d\tau^{C} = d\tau^{-},$$

(3.151b)

$$dX^{i} = \frac{\partial H^{+\prime}}{\partial P^{i}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial P^{i}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial P^{i}} d\tau^{i} + \frac{\partial H^{+\ \prime}_{\Pi^{-}}}{\partial P^{i}} d\tau^{A-} + \frac{\partial H^{+\ \prime}_{\Pi^{+}}}{\partial P^{i}} d\tau^{A+} + \frac{\partial H^{+\ \prime}_{\Pi^{C}}}{\partial P^{i}} d\tau^{C} = d\tau^{i},$$

(3.151c)

Similarly

$$dA^{-} = \frac{\partial H^{+\prime}}{\partial \Pi^{+}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial \Pi^{+}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial \Pi^{+}} d\tau^{i} + \frac{\partial H^{+\ \prime}_{\Pi^{-}}}{\partial \Pi^{+}} d\tau^{A-} + \frac{\partial H^{+\ \prime}_{\Pi^{+}}}{\partial \Pi^{+}} d\tau^{A+} + \frac{\partial H^{+\ \prime}_{\Pi^{C}}}{\partial \Pi^{+}} d\tau^{C} = d\tau^{+},$$

(3.152a)

$$dA^{+} = \frac{\partial H^{+\prime}}{\partial \Pi^{-}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial \Pi^{-}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial \Pi^{-}} d\tau^{i} + \frac{\partial H^{+\ \prime}_{\Pi^{-}}}{\partial \Pi^{-}} d\tau^{A-} + \frac{\partial H^{+\ \prime}_{\Pi^{+}}}{\partial \Pi^{-}} d\tau^{A+} + \frac{\partial H^{+\ \prime}_{\Pi^{C}}}{\partial \Pi^{-}} d\tau^{C} = d\tau^{-},$$

(3.152b)

$$dC = \frac{\partial H^{+\prime}}{\partial \Pi_{C}} d\tau^{+} + \frac{\partial H^{-\prime}}{\partial \Pi_{C}} d\tau^{-} + \frac{\partial H^{i\prime}}{\partial \Pi_{C}} d\tau^{i} + \frac{\partial H^{+\ \prime}_{\Pi^{-}}}{\partial \Pi_{C}} d\tau^{A-} + \frac{\partial H^{+\ \prime}_{\Pi^{+}}}{\partial \Pi_{C}} d\tau^{A+} + \frac{\partial H^{+\ \prime}_{\Pi^{C}}}{\partial \Pi_{C}} d\tau^{C} = d\tau^{A}.$$

(3.152c)

CHAPTER 4

CONCLUSION

This work aimed to study of the constrained systems in instant form theory and light front theory using both Dirac approach and Hamilton-Jacobi approach.

The two methods, represent the Hamiltonian treatment of the constrained systems. Dirac's approach hinges on introducing primary constraints, then constructing the total Hamiltonian by adding the primary constraints. All other constraints are obtained from these conditions. The equations of motion are obtained using Poisson brackets, are in ordinary differential equations forms. The gauge fixing conditions, which are not an easy task in this approach, are necessary in order to determine the unknown Lagrange multipliers.

The Hamilton-Jacobi formulation of singular systems arrived to important result in physics, that is we first exhibit the fact that a singular system can be treated as a system with many independent variables.

In other words, the equations of motion are not ordinary differential equations but total differential ones in many variables. In general mathematically speaking, it is not possible to solve the equations of motion of singular systems unless they satisfy the integrability conditions. If these conditions are not identically satisfied, it will be considered as new constraints. This process will continue until we obtain a complete system. The gauge fixing conditions are not necessary in the Hamilton-Jacobi formulation since one does not need to introduce Lagrange multipliers.

The previous two methods have been applied classically in chapter two and chapter three.

Instant form theory and light front theory of this thesis are discussed in the frame work of two method Dirac's and the Hamilton-Jacobi.

The methods, represent the Hamiltonian treatment of instant form theory and light front theory of conformally Gauge light front Polyakov-D1 Brane action in a d-dimensional curved back ground with d=10 for fermionic and d=26 for bosonic D1 brane.

This theory is an unconstrained system in the sense of Dirac and presence of a scalar dilation field represents a canstrained system in the sense of Dirac possessing a set of two second class constraints where one constraint is primary and other one is the secondary Gauss Law constraints.

Light front theory is discussed in chapter three, this theory is easily seen to possess twenty six primary constraints and does not get any secondary constraints, and we study LFQ of this theory in presence of scalar dilation field ϕ which describes the polyakov D1-brane action LF coordinates and possessing a set of 27 primary constraints when consider in the presence of scalar dilation field.

Also in this thesis, Instant form theory and light front theory conformally Gauge light front Polyakov-D1 Brane action in the presence of a constant scalar axion field and a discussion of different models of Instant form theory and light front quantization which treated as singular system to vistigated by Hamilton-Jacobi Method (or Güler approach).

The final results of the two Methods are found the same, and the Hamilton-Jacobi Method simpler than Dirac's Method.

REFERENCES

[1] Dirac P. A. M., Generalized Hamiltonian Dynamics, Can. J. Math. Phys. **2**, 129 (1950).

[2] Dirac P. A. M., Lectures on Quantum Mechanics, Yashiva University, New York, (1964).

[3] Sundermeyer K., Lecture notes in physics, Spring-Verlag, Berlin, 1982.

[4] Feynman R. P., Rev. Mod. Phys., **20**, 367 (1948).

[5] Feynman R. P., Quantum Mechanics and Path Integrals (McGraw-Hill, New York, N.Y.) (1965).

[6] Faddeev L. D., Theor. Math. Phys. **1**, 1 (1970).

[7] Faddeev L. D. and POPOV V. M., Phys. Lett. **B24**, 29 (1967).

[8] Senjanovic P., Ann. Phys. (N. Y.), **100**, 227 (1976).

[9] Fradkin E. S. and Vilkosisky G. A., Phys. Rev. **D8** 4241 (1973).

[10] Fradkin E. S. and Vilkovisky G. A., Phys. Lett. **B55** 4241 (1975).

[11] Batalin I. A. and Fradkin E. S., Nuct. Phys. **B270** 514 (1968).

[12] Gitman D. M. and Tyutin I. V., Quantization of Fields with Constraints (Spring-Verlag, Berlin, Heidelberg) (1990).

[13] Buekenhout F. and Sprague A., Polar Spaces having some line of cardinality two, J. Comb. Th. (A), **33**, 223, (1982).

[14] Faddeev L. D. and Jackiw R., Phys. Rev. Lett. 60, 1962, (1988).

[15] Güler, Y., Nuovo Cimento **B107**, 1389 (1992).

[16] Güler, Y., Nuovo Cimento **B107**, 1143 (1992).

[17] Farahat N. I. and Güler, Y., Nuovo Cimento **B111**, 513 (199).

[18] W. I. Eshraim, and N.I. Farahat, “Hamilton-Jacobi Approach to the Rlativistic Local Free Field with Linear velocity of Dimension D” Hadronic Journal, **29,** (2006), 553.

[19] O.Shihada, M. Sadallah, , S.I.Muslih and N.I.Farahat, “Hamilton-Jacobi quantization of regular systems with higher-order Lagrangian”, the first international conferece on Mathematical Sciences, Al-Azhar University-Gaza, 15-17 May (2006).

[20] N. I. Farahat, and H. A. Elegla, “Hamilton-Jacobi Formulation of Siegle Superparticle”, ”, The second international Conference for Science and development, Faculty of science, Islamic University of Gaza, 6-7 March 2007, and Tur. J. Phys, **30,** (2006), 473.

[21] W. I. Eshraim, and N.I. Farahat, “Quantization of the scalar field coupled minimally to the vector potential” Electronic journal of theotetical Physics, **14**, (2007), 61.
[22] W. I. Eshraim, and N.I. Farahat, “Hamilton-Jacobi formulation of the scalar ϕ coupled to two flavours Fermionic through Yukawa couplings ”, The second international Conference for Science and development, Faculty of science, Islamic University of Gaza, 6-7 March 2007, and The Islamic University Journal, **15**, (2007).
[23] N. I. Farahat, and H. A. Elegla,” Path integral quantization of superstring”, Modern physics Letters A, 25, No.2,(2010), 135.
[24] N. I. Farahat, and H. A. Elegla, “Path integral quantization of spinning superparticle”, Turkish J. of Phys, **32**, (2008), 1.
[25] W. I. Eshraim, and N.I. Farahat, “ Hamilton-Jacobi treatment of Lagrangian with fermionic and scalar field”, Romanian J. of Phys, **53**, (2008), 437.
[26] W. I. Eshraim, and N.I. Farahat, ““ Hamilton-Jacobi formulation of a non-abelian Yang-Mills theories”, Electronic J. of Theoretical Phys, **17**, (2008), 65.
[27] N. I. Farahat, and H. A. Elegla, “Path Integral Quantization of Brink-Schwarz Superparticle ” Electronic J of Theor. Phys, **19**, (2008), 1.
[28] W. I. Eshraim, and N.I. Farahat, "Path Integral Quantization of the Electromagnetic Field Coupled to A Spinor**",** *Electronic Journal of Theoretical Physics***, 22,** (2009) 189.
[29] N. I. Farahat, and H. A. Elegla, “” Hamilton-Jacobi formulation of of Green-Schwarz Superstring”, Int J. Theor. Phys, **49**, (2010), 384.
[30] N. I. Farahat, and H. A. Elegla, “Path Integral Quantization of Superparticle coupled to external superpotential” Submitted for publication …
[31] O.Shihada, N.I.Farahat, and S.I.Muslih,”The Canonical Path integral Quantization of Regular continuous systems with Higher Order Lagrangians” Submitted for publication in Hadronic Journal.
[32] N. I. Farahat, and H. A. Elegla, “Path Integral Quantization of Superparticle with 1/4 Supersymetry Breaking", J of App. Math. And Phys., **1**, (2013)105.
[33] Muslih S. I. and Güler, Y., Nuovo Cimento **B112**, 97 (1997).
[34] Muslih S. I., Nuovo Cimento **B115**, I (2000).
[35] Muslih S. I., Path intehral Formulation of Constrained Systems, to appear in Hedonic Journal **23** (2000).
[36] Carathoedary C., Calculus of Variation and Partial Differential Equation of the First Order. London: Holden-Day, INC. (1967).
[37] Faddeev L.D. and POPOV V. M., Phys. Lett. **B24**, 29 (1967).
[38]Weinberg, Steven, *The Quantum Theory of Fields,* Volume 1. Cambridge University Press, 1995
[39] Bachas, C. P. "Lectures on D-branes" (1998). arXiv:hep-th/9806199.

[40] Giveon, A. and Kutasov, D. "Brane dynamics and gauge theory", *Rev. Mod. Phys.* **71**, 983 (1999). arXiv:hep-th/9802067

[41] Nambu, Y., *Strings, monopoles, and gauge fields.* Physical Review D, 1974. **10**(12): p. 4262.

[42] *Szabo, R.J., An Introduction to String theory and D-brane Dynamics. 2004: World Scientific.*

[43] Polyakov, A.M., *Quantum geometry of bosonic strings.* Physics Letters B, 1981. **103**(3): p. 207-210.

[44] Sakita, B. and M. Virasoro, *Dynamical model of dual amplitudes.* Physical Review Letters, 1970. **24**(20): p. 1146.

[45] Belavin, A.A., A.M. Polyakov, and A.B. Zamolodchikov, Infinite conformal symmetry in two-dimensional quantum field theory. Nuclear Physics B, 1984. 241(2): p. 333-380.

[46] D. Luest and s. Theisen "Lectures in String Theory". Lecture Notes in Physics, Springer Verlag, Berlin, Vol. 346, 1989.

[47] D. S. Kulshreshtha, "String gauge Systemrie in Light front Polyakov D1 Bran action". Invited talk at the international conference Light-fone 2010: Relativistic Hadronic and particle physics (LC 20110), Valencia. June 2010, Published in Pos LC 2010: 006, 2010 SISSA, Trieste, Italy.

[48] Usha Kulshreshtha and D. S. Kulshreshtha, "Hamiltonian and path Integral Quantization of conformally Gauge Fixed Polyatov D1 Brane action in the presence of scalar Dilation Field". Intenatioanl Journal of theoretical physics, Vol. 48, No. 4, 2009, PP 937 – 944.

[49] Usha Kulshreshtha and D. S. Kulshreshtha, "String Gauge Symmetries in the Confromally Gauge-Fixed Polyakov D1-Brane Action in the Presence of a Scalar Dilation Field". Journal of Modern physics, Vol. 3, No. 1, 2012, PP. 110-115. doi:10.4236/jmp.2012.31015.

[50] D. S. Kulshreshtha, " Light -Front Quantization of Conformally Gauge-Fixed Polyakov D1-Brane Action in the Presence of a Scalar Axion Field an U(1) Gague Field," Few Body Systems, Vol. 52, 2012, pp. 463-467. doi:10.1007/s00601-011-0279-9.

[51] S. J. Brodsky, H. C. Pauli and S. S.Pinsky, "Qunatum Chromodynamics and Other Field Theories on the Light-Cone," Physics Reports, Vol. 301, No. 4-6, 1998, pp. 229-486. doi:10.1016/S0370-1573(97)00089-6.

[52] U. Kulshreshtha, "Hamiltonian and BRST Formulations of the Nelsen-Olesen Model," International Journal of Therortical physics, Vol. 41, No.2, 2002, pp. 273-291. doi:10.1023/A:1014058806710.
[53] Usha Kulshreshtha and D. S. Kulshreshtha, "Light-Front Hamiltonian and path Integral formulation of conformally Gauge Fixed Polyatov D1 Brane action". Journal of Modern Physics, Vol. 2, No. 5, 2011, PP 335 – 340.