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# Graphical Game Theory with Mobility 

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A Thesis presented for the degree of Doctor of Philosophy

Algorithms and Complexity in Durham<br>School of Engineering and Computing Sciences<br>University of Durham<br>England<br>January 2015

# Mobile Graphical Game Theory 

## Adam Symonds

Submitted for the degree of Doctor of Philosophy<br>January 2015


#### Abstract

This study aimed to resolve disparities between the human behaviour predicted by game theoretic models and the behaviours observed in the real world. The existing model of graphical games was analysed and expanded to create a new model in which agents can move themselves around the graph over time. By adopting different configurations of variables, this model can simulate a very wide range of different scenarios. The concept of meta-games was applied to expand this range yet further and introduce more real-world applications. The interactions between different elements of the configuration were investigated to develop an understanding of the model's emergent properties. The study found that this new model is more accurate and more widely applicable than all other pre-existing candidate models. This suggests that human irrationality can generally be accounted for with a better understanding of the environment within which interaction is occurring.


## Declaration

The work in this thesis is based on research carried out within the Algorithms and Complexity in Durham research group, at the School of Engineering and Computing Sciences, Durham, England. No part of this thesis has been submitted elsewhere for any other degree or qualification and it is all my own work unless referenced to the contrary in the text.

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## Chapter 1

## Introduction

### 1.1 Context

For over 50 years, classical game-theoretic tools have been used to attempt to identify the 'best' approach to playing games and, consequently, how players in a game should act. However, the field as a whole is notoriously poor at predicting the behaviour of human players in all but the most artificial and sterile environments. For example, a standard analysis of the Iterated Prisoners' Dilemma, eliminating strictly dominated strategies and applying backwards induction, would lead a 'rational' player to defect or play aggressively at each step, regardless of previous actions, or any communication or even agreements between the players involved, as this is the only stable solution (or 'Nash equilibrium') for this game.

In practice, however, many human players cooperate regardless, and pursue a wide variety of other effective strategies. In short, the existing tools and techniques in this field are demonstrably insufficient to analyse the ways in which humans approach and play games in real-world situations [1].

Because of this, many bemoan human play as 'irrational', something which cannot be predicted or accounted for on an individual level. However, experimental results
have shown that humans can outperform 'rational' agents by considering the impact of their actions beyond the immediate scope of the game. For example, in the Prisoners' Dilemma, players who ignore the 'rationality' advocated in the previous example and instead cooperate can earn larger payoffs than those who defect [2].

This effect, and others similar to it, can help human players gain utility and resources over repeated games which strictly 'rational' (or artificial) players disregard as unattainable. There are, at present, no wholly satisfying or widely accepted explanations to this central problem of why humans don't play 'rationally', and how they outperform carefully designed autonomous agents while doing so.

Furthermore, many people believe in some sort of overpowering external source of justice or fairness, whether societal or supernatural, which (irrespective of the accuracy of such beliefs) further skew the strategies and payoffs available in the minds of those agents. For instance, experiments conducted with the 'Ultimatum Game', in which players can make and accept fair or unfair offers, consistently show that humans tend to perceive their choices in the game as being judged in relation to their social standing.

Because of this, such players generally act more cooperatively than classic game theoretic indicators (such as the Nash equilibrium) would predict, and expect indirect reciprocity through external mechanisms such as reputation- even in situations where such mechanisms have been specifically excluded from the experimental setup [3]. Because of this, efforts to isolate the external influences which cause humans to deviate from the classical models seem misguided. I argue that, instead of specifically dismissing them, we must formulate a new model which takes these factors into account.

In addition to this systemic failure to forecast the outcome of games, the question of when to play games relative to a natural setting so as to achieve meaningful results has not yet been well addressed. Games are frequently considered to be played 'in
vitro', without reference to or situation in any sort of environment. Many studies, from rigorous game theoretic competitions of strategy and tactics (such as those run by Axelrod [4] and, more recently, those marking the 20th anniversary of his original experiments [5]) through to broader psychological evaluations of human behaviour, use highly unrealistic structural assumptions about the manner in which players interact.

For instance, it is typically specified that each player will interact with each other player a fixed number of times, consecutively, and will never again interact with that player- presumably, not even outside of the experimental setup, even though the experimenters cannot guarantee this. Worse still, it is often assumed that the players learn nothing from the experience, neither while it is ongoing nor on reflection before the start of the next interaction.

However, in the real world, humans are almost always learning and incorporating information into their world-view in an effort to make good decisions. In short, connections between players in experimental conditions are universally arbitrary or random, whereas real-world networks are almost always formed with an element of purposefulness and active effort on behalf of those involved [6].

However, the dynamics of any pre-existing group of preferential agents (such as human beings) can be arbitrarily more complex than any experimental setup. It should be obvious to any human being that we are not equally influenced by a large number of others, but rather are strongly influenced by a relatively small number of people (such as friends, colleagues, superiors, celebrities, technical experts, and so on), each on a relatively narrow range of topics. From psychology, according to the results of Milgram's 'small world' experiments, relationships in human society can typically be modelled as a large number of connections to those close to you, and less-common links to those further away- not as an amorphous clique [7].

There is also an inherent element of randomness regarding which players have rela-
tionships with which others, due to the vagaries of human contact, interaction, and socialisation- none of which are generally well represented in behavioural experiments or simulations. This further compounds the problem of accurately representing the effects and interactions between players.

### 1.2 Motivation

Clearly, it would be in our interests to overcome these persistent limitations to game theory, and try to develop a robust, generic framework which can approximate the essential components of real-world interaction without becoming overly complex. Such a system would likely enable human behaviour to be more finely approximated than existing experimental setups (though probably not predicted flawlessly). This enhanced fidelity could be used to evaluate many potential games and strategies for such games, perhaps producing the next baseline model for assessing the strengths of competing player types and strategies- much like the Prisoners' Dilemma and then the Iterated Prisoners' Dilemma were previously and are at present [4] [5].

Though my main motivation is academic, other possible applications include:

- Type selection in a variety of scenarios (for example, what opening sequence to utilise in a chess tournament, or which of several tournaments to attend given a preference for a specific opening sequence)
- Assisting mechanism design in the creation of any system in which participation is desired, but not globally mandatory (for example, limiting collusion in an auction)
- Contributing to ongoing attempts to effectively predict the various strategies of agents in diverse and dynamic populations (for example, in complex competitive multi-agent systems, and human communities)

Additionally, if we do not clearly understand the environment, larger circumstances, and finer details of any interactive system (game or otherwise) in which strategies are deployed, we cannot understand which qualities in the environment are related or unrelated to the performance of those strategies. The development of a more clearly structured framework in this area may therefore lead to new ideas as to which strategies are considered 'strongest' for a given game played in a given situation.

### 1.3 Statement of the Problem

Given the above context, it should be clear that current game-theoretic methods, models, and techniques are poorly suited to ongoing attempts to develop a deeper understanding of the nuanced interactions and emergent properties of these complex systems; whether between artificially-designed agents in fields such as artificial intelligence and multi-agent systems, or with real-world agents in fields such as psychology and economics. Somewhat ironically, even artificial agents are deployed into a world far more complex and intricate than the experimental setups used to evaluate their approach. Conversely, the challenge of any simulation is to recreate the same or at least similar results from a much coarser input and rules of interactionotherwise, the task of 'simulating' the problem becomes akin to outright solving the problem itself.

More specifically, although much game-theoretic research is focused on finding the 'best' strategy or approach for a given situation, there is disagreement over what the 'best' strategy is, even for games as well-studied as the Prisoners' Dilemma. Though some strategies, such as Tit-For-Tat, or the related Tit-For-Two-Tats, are typically touted as being optimal or near-optimal [4], the situation is not so clear cut.

For example, 'Grim' (also known as 'Grudge') is extremely strong in experimental setups where players can somehow prove they are using this strategy- in such situ-
ations, the opponent can do no better than by also playing Grim [8]. 'Master and slave' strategies, consisting of pre-determined coalitions of players in different roles, can trump all of these when the environment has certain properties and the setup permits them [5].

Finally, in any experimental situation in which the general disposition of the population can be determined or estimated in advance, a novel strategy can be designed to exploit this information. This occurs with some complex real-world games, and is known as the 'meta-game', indicating that type-selection can, in itself, be considered a strategic choice within the context of a larger 'game' [9].

The limitations of these different tools and approaches often produce confusion and disagreement, particularly between different scholarly fields as they approach these common problems in from different directions. In short, because of a lack of recognition regarding the fundamental circumstances in which games are played, the contrasting and comparison of more specific concepts within the field presents a surprisingly difficult obstacle which has, thus far, not been adequately addressed.

### 1.4 Aims, Objectives, and Focus

Taking the above into consideration, I aimed to develop a model which can:

- Tie together the strengths of different game-theoretic approaches to different scenarios into a single generic system.
- Provide explanations for why certain strategies do or do not work effectively in different scenarios.
- Holistically and accurately represent the nature of interactions between entities in different environments.
- Eliminate some of the systemic inaccuracies of game theory as a field and by
doing so provide an increase in fidelity over existing models.
- Advance our understanding of how people approach and play games outside of a laboratory setting.
- Be applied to an extremely large range of situations.
- Be more intuitively comprehensible than any previous system, allowing results to be conveyed more clearly.

This is my proposed 'Graphical Gaming with Mobility' model which I will explain in detail over the course of this thesis. The central question I seek to answer through my research is "When players interact across a naturally mobile population with different possible parameters and configurations, how does this affect the strategies used for interaction and the dynamic distribution of the population?". Addressing this question will enable me to understand the intricacies of my proposed model, and identify its capabilities and any areas for possible improvement and/or future research. As such, I will refer to it, and the goals for my model above, throughout my results in Chapter 6.

Although this research is rather broad in order to best tackle the diverse elements of the underlying challenges, there are nevertheless limitations in its scope. I have not attempted to 'solve' the Prisoners' Dilemma or indeed any existent game, nor design some new strategy for it which outplays others in specific situations, as this has been researched thoroughly [10] and even incremental progress on this topic is extremely difficult outside of rather narrow research. I have not aimed to provide a definitive guide or approach for any given type of agent as to how best approach a given problem to ensure the greatest payoff, or to map out the ideal movement strategy within my model for a given player type, for similar reasons. Finally, I have avoided detailed statistical analysis of the outcomes of individual parameterisations in favour of broader analysis of more general patterns and trends. I believe this approach will be applicable over a wider variety of games, graph structures, timing
modes, and so on, and more effectively demonstrate the ability of the model I've designed to simulate a very wide range of possible scenarios.

In addition to these objectives, there are several additional hypotheses regarding the functionality of my system which I formed after studying the existing literature. These are introduced throughout Chapter 3 as appropriate.

### 1.5 Contribution

The idea of adding an underlying structure to multiple interacting games in an attempt to rectify some of these problems has been previously explored, and is known as 'graphical games'. In a graphical game, players interact only with a subset of all players in accordance with the layout of a graph. As such, graphical games are excellent for simulating situations where each player's overall outcome is dependent on a few strong influences, rather than being equally subject to the whims of every other player in the environment- which would seem to be a more accurate description of human interaction than being affected equally by all humans in any given group [11]. Though such influences can be modelled by more basic, pre-existing systems, the difficulties involved make doing so impractical for anything beyond simple demonstrations.

However, this definition of graphical games is still incomplete when compared to actual scenarios. I realised that, in the real world, any person who feels they are not deriving sufficient utility from their interactions with other people can reasonably expend some effort to find new, more compatible people to interact with in future. This idea neatly extends into spheres which commonly feature real-world applications of game theory, such as economics and business- a worker unhappy with their colleagues may tolerate the discomfort up to a point, or a business which faces material delays may incur considerable costs to arrange a replacement supplier.

Additionally, any person who feels let-down by their friends may go to great lengths to meet new people with a view to forming new relationships. In each case, the existence of this capability is self-evident.

I think that these situations, and the deeper facet of humans having the capability to choose who they interact with subject to certain restrictions, must be understood and modelled if we hope to improve our knowledge in this field. If we attempt to model the above situations without doing so, the result we derive will be observably inaccurate and sub-optimal- typically that the agents will continue to interact non-cooperatively. More fundamentally, even in the most basic and constrained experimental conditions, the humans taking part have all decided to participate for some reason, and this 'response bias' must be considered and accounted for in experimental design [12].

As such, for my main contribution, I have expanded the graphical gaming model to cover situations in which agents in the population have some capacity to take action to situate themselves in a neighbourhood of their choosing- in essence, they can move around the graph. I've studied various ways this functionality can be implemented, so that it can be used to simulate and assist with understanding given instances of these redesigned 'Graphical Games with Mobility'.

I have evaluated the performance of different strategies under various parameterisations of this model, with a view to comprehensively understanding this environment and how each of its elements impacts all the others. This has primarily been achieved through the use of simulations, with some human experimentation to confirm and double-check findings from the simulations as well as test the applicability of my model to real-world behaviour. Of the work done previously with graphical games, most of it has focused on complexity results and computational bounds [13], leaving questions of the broader applications of this model unaddressed until now.

I have also investigated how various strategies for movement, based on which types
are perceived to be favourable or unfavourable opponents for a given agent, can positively or negatively impact the utility gained. I believe that this model displays many properties of interest, and is a strong candidate for a model of generic interaction, mechanism and type evaluation, having many benefits and no significant drawbacks over those previously available.

## Chapter 2

## Background

In this chapter, we will first introduce and then proceed to explain in detail some of the core game theoretic concepts used throughout this research. After this, we will extend those basic concepts using my central contribution of mobility.

### 2.1 Fundamental Game Theory

The fundamentals of game theory are widely known, but included here for completeness. A 'game' consists of a set of players (or 'agents'). Each player has a set of 'strategies'- options they can choose which correspond to the actions they will take in the game. Each combination of strategies has an associated 'payoff' for each player. Without loss of generality, this can be reduced to two players simultaneously selecting from two strategies. Players aim to maximise their individual utility, which is typically (but not always) equivalent to the magnitude of the payoff received.

Games can be 'iterated' (or 'repeated'), which means the same agents play the same game more than once and can use their memory of previous games to help choose their action in the current game. Common concepts include 'social utility', which is the combined utility of all players. Although some outcomes favour particular
players, others may produce a higher total amount of utility- these outcomes are 'socially optimal' and often of particular interest to mechanism designers aiming to induce or incentivise particular behaviours. There is also the concept of 'Nash equilibria', which are combinations of strategies from which no players have an incentive to unilaterally deviate (as doing so would decrease their payoff, assuming no other agents simultaneously deviate).

There are several traditional methods of representing the information contained within games. A 'tree'- a directed graph without any cycles- is used in the 'extensive form'. In this form, at each level of the tree, each branch splits in accordance with one player's possible actions at that point. If two or more players would act simultaneously, they are simply given consecutive levels in the tree (with the ordering of these levels being irrelevant). Each leaf is labelled with the payoffs received by each player at that point, and it is trivial to prove that all possible outcomes are covered (as each possible action from each possible point of play is covered).

There is also the 'normal form', a matrix with each dimension corresponding to one of the players, and one row in that dimension corresponding to each action available to that player. Normal form games can be nested, but tend to be used to represent simple, classical games, or one step of a much larger game. A demonstration that all possible outcomes are covered is again trivial, as every possible intersection of actions is covered. While the extensive form is useful for larger and/or more complex scenarios, the normal form allows the results of each players' actions to be looked up more easily.

A game is said to be one of 'perfect' or 'complete' information if (aside from the types and/or decision making processes of the players) all the elements of the game and its state are known to all players at all times. If this is not the case, the game is said to contain 'imperfect', 'hidden', or 'incomplete information'. For example, chess is a game with complete information (each player can see where every piece
is at all times), whereas poker is a game with incomplete information (namely, the face-down cards only viewable to their holder).

Most classic academic games, including all the games used in this research, use perfect information. Typically players choose their actions simultaneously, such that neither can simply use the knowledge of the action their opponent is playing to respond optimally [8].

More formally:

Definition 1 A game consists of $n$ players, each with a finite set of pure strategies or actions available to them, along with a specification of the payoffs to each player. We use $a_{i}$ to denote the action chosen by player $i$. For simplicity we will assume a binary action space, so $a_{i} \in\{0,1\}$. The payoffs to player $i$ are given by a table or matrix $M_{i}$, indexed by the joint action $\vec{a} \in\{0,1\}^{n}$. The value $M_{i}(\vec{a})$, which we assume without loss of generality to lie in the interval [0, 1], is the payoff to player $i$ resulting from the joint action $\vec{a}$. Multiplayer games described in this way are referred to as 'normal form' games. [11]

A 'strategy' is simply any method for determining which action to take in a given situation. Note that these ideas can be expanded to include games with more than two actions without loss of generality. Though fundamental, the concept of 'strategy' used here only operates on an academic level, especially the idea of mixed strategies in which one determines one's chosen action purely probabilistically. In practice, the 'strategies' developed in my research take into account higher-level considerations (for instance, Tit-For-Tat chooses the same action its opponent chose previously), and human players will likely have even more detailed strategies.

Definition 2 The actions ' 0 ' and ' 1 ' are the 'pure strategies' of each player, while a 'mixed strategy' for player $i$ is given by the probability $p_{i} \in[0,1]$ that the player will play 0. For any joint mixed strategy, given by a product distribution $\vec{p}$, we define
the expected payoff to player $i$ as $M_{i}(\vec{p})=E_{\vec{a} \sim \vec{p}}\left[M_{i}(\vec{a})\right]$, where $\vec{a}^{\sim} \vec{p}$ indicates that each $a_{j}$ is 0 with probability $p_{j}$ and 1 with probability 1- $p_{j}$ independently. [11]

Games with dominant pure strategies- that is, those which can be 'won' by always playing the same action- are of little academic interest. However, I make occasional use of agents playing pure strategies as one of several types in my populations, in order to study the reaction of more advanced types to them. It's worth noting that, in the Prisoners' Dilemma, the cooperative strategy is strictly dominated, meaning one will always gain greater utility from defecting than cooperating, regardless of the action selected by the opponent. However, the situation becomes practically rather more complex (though not any more mathematically complex) once the game becomes iterated, as the possibility of building a reputation with the opponent becomes a factor which should be considered- even though the actual matrix itself remains unchanged.

Definition 3 We use $\vec{p}\left[i: p_{i}^{\prime}\right]$ to denote the vector (product distribution) which is the same as $\vec{p}$ except in the ith component, where the value has been changed to $p_{i}^{\prime}$. A Nash equilibrium (NE) for the game is a mixed strategy $\vec{p}$ such that for any player $i$, and for any value $p_{i}^{\prime} \in[0,1], M_{i}(\vec{p}) \geq M_{i}\left(\vec{p}\left[i: p_{i}^{\prime}\right]\right)$. (We say that $p_{i}$ is a best response to the rest of $\vec{p}$.) In other words, no player can improve their expected payoff by deviating unilaterally from an NE. The classic theorem of Nash (1951) states that for any game, there exists an NE in the space of joint mixed strategies. [11]

Although issues pertaining to Nash equilibria and their related concepts are not a focus of my research, it is helpful to be aware of how they are derived for a general discussion of game theory.

### 2.1.1 The Prisoners' Dilemma

The Prisoners' Dilemma is probably the most researched game in academia [4] [5] [10] [14] [15] and arguably the most interesting non-iterated, simultaneous, binary 2-player game. Despite its apparent simplicity, it is applicable to a large number of real-world situations in fields such as economics, biology, psychology, and politics [16] [17] [18] [19]. It occasionally goes by other names such as 'Hawks and Doves'. In the game, each of two players is given a simple choice- cooperate, or defect. If both choose to cooperate, they both receive a good payoff. However, if one attempts to cooperate while the other defects, the defector gets an even higher payoff, while the cooperator receives next to nothing. Finally, if they both try to defect, they both receive a poor payoff.

More formally:

Definition 4 Let ' $A$ ' be the payoff received by a player who cooperates when their opponent defects, ' $B$ ' be the payoff received by a player who defects when their opponent defects, ' $C$ ' be the payoff received by a player who cooperates when their opponent cooperates, and ' $D$ ' be the payoff received by a player who defects when their opponent cooperates. $A<B<C<D$. In the iterated case, we also require that $2 C>A+D$ so that a higher payoff is received from two agents cooperating rather than alternating between betraying each other (though this would require even greater coordination than mere cooperation).

The game is of particular interest as its Nash equilibrium is to defect- meaning that you always earn a greater payoff by defecting, regardless of what action your opponent takes. However, the 'socially optimal' outcome- the one which generates the highest total utility amongst all players- is mutual cooperation. This inherent tension between doing what is best for you versus what is best for a larger group, or, in the iterated case, of short-term versus long-term advantage, makes the game
extremely interesting.
The 'Prisoners' Dilemma' was chosen as the main game to use throughout this thesis, as it has been thoroughly researched and documented in the literature, allowing areas where my results differed from the norm to be identified with ease while minimising the chance that some novel mechanism or unexplored quirk of the game could cloud my data. This means that, for each individual 2-player game within the much larger graphical game, the form of the Prisoners' Dilemma illustrated below was used. Other simple, established academic games, such as the 'Ultimatum Game' [20] and simple variants of 'Rock, Paper, Scissors' were used occasionally with human subjects later on, as covered in more detail in Chapters 5 and 6 .

| X | Cooperate | Defect |
| :---: | :---: | :---: |
| Cooperate | 3,3 | 0,5 |
| Defect | 5,0 | 1,1 |

Figure 2.1: Payoff matrix for the Prisoners' Dilemma.

| X | Rock | Paper | Scissors |
| :---: | :---: | :---: | :---: |
| Rock | 0,0 | 0,1 | 1,0 |
| Paper | 1,0 | 0,0 | 0,1 |
| Scissors | 0,1 | 1,0 | 0,0 |

Figure 2.2: Payoff matrix for Rock, Paper, Scissors.

| X | Fair | Unfair | Very Unfair |
| :---: | :---: | :---: | :---: |
| Accept | 5,5 | 7,3 | 9,1 |
| Reject | 0,0 | 0,0 | 0,0 |

Figure 2.3: Payoff matrix for the Ultimatum Game.

I devised this simplified, discrete formation of the Ultimatum Game such that it could work with the discrete behaviours pre-existing in my agents. I disregarded the
possibility of splits in favour of the receiving agent, and of the first agent attempting to take the entire pot, as unrealistic.

### 2.2 Fundamental Graph Theory

A graph consists of a set of 'vertices' and 'edges'. Each vertex, or 'node', can be connected to any number of other vertices by edges. Although multiple edges between the same two nodes, or an edge connecting a node to itself, are technically possible, these possibilities are usually discounted, as they have been in the graphs I used in this research. Edges can be 'directed', signifying a one-way, rather than the typical 'undirected' two-way, relationship between the nodes. In this case, the connection is also one way- for instance, $A$ may be connected (by a directed edge) to $B$, but $B$ would not then be connected to $A$. If a graph contains at least one directed edge the graph is also 'directed', otherwise it too is 'undirected'. Finally, edges can be 'weighted', meaning there is some value assigned to them. The usage of this value varies with the usage of the graph, but is typically used to imbue certain connections with additional importance and/or cost relative to others.

### 2.3 Graphical Games

Having introduced the fundamentals of game theory, we can now progress to definitions from the graphical games model originally proposed by Kearns [11].

To formulate a graphical game, we begin with an undirected graph. Any particular graph with some desired structure or properties can be used. There are also players equal to the number of vertices in the graph. Each player is then associated with a single vertex- if we consider each vertex to be a position, each player is 'at' their associated vertex. Next, each vertex (and/or player, depending on your point of
view) is associated with a game, as defined above. These players play the games of the players they're connected to with that player, and all others in that player's neighbourhood.

Each player's total payoffs are summated from all the games they play, and, as such, entirely determined by their interactions with the connected agents, especially those in their neighbourhood who they are certain to play against multiple times (at minimum twice, from their game and the agent's own). Graphical games are a kind of 'sparse game', in which most of the payoffs for interactions between agents are zero and can thus be ignored (in this case, those between disconnected agents).

Kearns defines these more formally:

Definition 5 A graph $G$ consists in its most basic form of a vertex set $V(G)$ and an edge set $E(G)$ which maps connections between the vertices. Edges may be 'directional', in which case they are considered to link one vertex to another, but not vice versa. Edges may be 'weighted', which assigns a value to them (typically an integer) which is then used for further computation. [11]

Although graphical games on directed graphs are possible and model one-way influences, Kearns seemingly ignores this possibility. It is, however, possible to produce a similar effect by modifying the payoffs of the individual games themselves, as I discuss when describing the properties of the model in Chapter 4.

Definition 6 In a 'graphical game', each player $i$ is represented by a vertex in an undirected graph $G$. We use $N(i) \subseteq 1, \ldots, n$ to denote the neighbourhood of player $i$ in $G$ - that is, those vertices $j$ such that the edge $(i, j) \in E(G)$. By convention $N(i)$ includes $i$ itself... [11]

A 'joint action' is the combined result of each action chosen by each player. Any situation where an agent would play a game against itself is of little interest, and can practically be ignored without loss of generality. Alternatively, it can be assumed
that all agents are smart enough to coordinate with themselves and always resolve such situations optimally, which permits the results of these games to be subtracted and discounted from results for clarity.

Definition 7 A graphical game is a pair $(G, \mathcal{M})$, where $G$ is an undirected graph over the vertices $1, \ldots, n \in N$, and $\mathcal{M}$ is a set of $n$ local payoff matrices. For any joint action $\vec{a}$, the local game matrix $M_{i} \in \mathcal{M}$ specifies the payoff $M_{i}\left(\vec{a}^{i}\right)$ for player $i$, which depends only on the actions taken by the players in $N(i)$. [11]

In my research, I will be looking at situations where all local matrices in $\mathcal{M}$ are identical in order to avoid obscuring the results with extra variables.

### 2.4 Proposed Graphical Games with Mobility Model

As graphical games with mobility form an extension of the basic graphical game model, which is itself an extension of basic game theory, many of the definitions and basic elements are unchanged from those frequently used elsewhere- for example, the description of a game in matrix form, and how each player's actions are compared to determine an outcome and the associated payoff for each player. If weighted edges are used, they represent the potential value of the interaction occurring across that edge (so the payoffs from a game weighted as ' 2 ' will be twice as great as those from a typical game weighted at ' 1 ').

As the starting point for my research, I have developed an alternative formulation of the original graphical games model. My new system is somewhat different to the original phrasing, even before the new elements unique to graphical games with mobility are added. By expanding the detail in some areas and restricting it in others, and thinking about these basic concepts in a novel way, we arrive at a different way of looking at the different levels of interaction occurring within the graphical game.

For example, Kearns stresses that, for any given agent $i$, a matrix is used such that only the agents in its neighbourhood $N(i)$ affect $i$ 's payoff. However, this system has a property which is particularly undesirable for my model, in that meaningful information can be transferred between non-adjacent vertices (and thus players) at distance 2 through the medium of a game with a shared neighbour. For example, if one player can observe the action chosen by another player, even though it was in a game with a third player that did not directly impact it.

This can be inferred from the definitions above, and occurs even if the payoffs between these agents are deliberately fixed at 0 , and even if the potential for further 'action at a distance' (perhaps due to the same disconnected players playing further games with each other or more agents even further away) is ignored. Agents are still able to view each other's approach to the shared game and use this information to inform their strategy in later games. For example, a Tit-For-Tat player could copy the defection they've already seen in the previous example, rather than beginning each new game with cooperation. This has even greater effects in a system with mobility, as it could also potentially inform each agent's movement, as I'll discuss later in Chapters 4 and 5.

Although each matrix is an $n$-player game played 'at a vertex', by performing the above step and formally restricting potential non-zero payoffs to adjacent agents, the matrix can be redesigned in such a way that it becomes equivalent to $n$ simultaneous 2-player games between the player at the vertex and each adjacent player without any undesired functional changes. This is achieved simply by making a new $M$ for each agent in $N(i)$, which has a strategy for each combination of choices and calculating the payoffs deterministically based on the interactions in the underlying scenario. This can then be repeated for each player (ignoring duplications). Finally, the rules of the system can be changed such that agents cannot observe the results of games they do not directly take part in. The final result is that games can be thought of as occurring 'at an edge' in those situations where it makes a more natural
example, which is typically how I will discuss them throughout this thesis.
These changes, particularly the restriction to 2-player games, cause a vast reduction in the number of interlinked subcomponents, and a corresponding reduction in complexity, which could otherwise have caused my results to be obfuscated by a myriad of small factors. Additionally, I believe that this interpretation is more naturally intuitive to humans, both when analysing graphical games as a researcher, and when interacting within graphical games as a player, as it is more similar to both the general understanding of games as played by humans in the real world, and similarly, common methods of interaction between individuals.

Definition 8 A graphical game can also be considered as a pair $(G, \mathcal{M}) . G$ is an undirected, unweighted graph over the vertices $1, \ldots, n$. $m$ is a symmetric game matrix. For simplicity and without loss of generality, we assume all $M_{i j} \in \mathcal{M}$ are identical to $m$. For each agent $j$ in $N(i)$ (which does not include $i$ ), and any joint action $\vec{a}$ between $i$ and $j$, the payoff for player $i$ is $m_{i j}\left(\vec{a}^{i j}\right)$.

Though each individual edge of the graph can have its own game with any number of strategies and any coherently-formed payoff matrix, for this research I am using a single game with a single payoff matrix uniformly across the whole graph for clarity and simplicity. Likewise, although it seems quite trivial to expand the basic definition to include directed graphs, and by doing so open up the possibility of using asymmetric games like the Ultimatum Game, I will restrict myself to undirected graphs for this research.

Finally, we can go on to define mobility:

Definition 9 Each player has exactly one associated vertex. We say player $i$ is 'at' $v_{i}$ if $i$ is associated with a given vertex $v_{i} . i$ is 'adjacent' to $j$ iff there is an edge between $v_{i}$ and $v_{j} . t$ is an integer which increments with each iteration of $G$ until $t_{\text {max }}$ is reached. In each iteration, each $i \in n$ plays each game $M_{i k}$ with $k$, for
each player in $N(i)$. After each player has determined their $a_{i}$, the joint action $\vec{a}$ is revealed and payoffs are allocated to all players according to $m_{i k}\left(\vec{a}^{i} k\right)$ for each m. Next, each player specifies an ordered list $L_{i}$ of players in $N(i)$ plus $i$. (This is equivalent to the list $L_{v} i$ of vertices associated with players in $N(i)$ plus $v_{i}$. The complete set of lists $L$ is used as input to a restricted variant of the Stable Marriage Problem [21], and a maximal assignment $R$ is sought. Then $R$ is applied to $G, t$ is incremented, and the next iteration begins.

The choice of $L_{i}$ can be considered as either a decision occurring after all games have been played in a round, which is more naturally intuitive, or as strategic choice occurring within the bounds of a single game, which helps clarify that its specification is an expression of a player's type. The mobility algorithm will be detailed extensively in Chapter 5.

### 2.5 Classic Strategies

I selected a wide range of simple, pre-existing strategies to populate my simulations and provide a starting point for the development of more interesting types. Though much of the history and usage of these strategies is within the Prisoners' Dilemma, these strategies can be adapted in relatively obvious ways for other games- for instance, 'EverDove' in the Ultimatum game would always make fair offers and accept any offer. The strategies are as follows:

- EverDove - Always plays cooperatively. Also known as 'AllC'.
- EverHawk - Always plays non-cooperatively. Also known as 'AllD'.
- Grim - Always plays cooperatively until the opponent defects, and defects each step thereafter. Also known as 'Grudge'.
- Tit-For-Tat - Initially plays against each new opponent cooperatively. There-
after, repeats the last action they played against it back to them in each round, using memory of older games for the first play of each new game. Also known as 'TFT'.
- Tit-For-Two-Tats - Plays the same way as 'Tit-For-Tat', except that it only switches to defecting after two consecutive defections from its opponent. Also known as 'TFTT'.
- Pavlov - Initially plays against each new opponent cooperatively. Thereafter, repeats the action it played previously, unless the outcome of the previously chosen action was undesirable (that is, mutual non-cooperation, or the opponent betraying it while it tried to cooperate), in which case it instead selects the other action. Also known as 'Win-Stay-Lose-Shift'.
- Tester - Plays the same way as 'Tit-For-Tat', except that it sometimes defects with small probability rather than responding to whatever its opponent did last round. If it does defect, it will continue to do so until its opponent also defects.
- Random - Randomly chooses between cooperating and defecting each round for each game.

As part of integrating the ideas represented by these strategies into my model, I devised an appropriate movement schema for each of them, in addition to combining and developing them into new types more suited to the environment I was developing. This will be discussed in Chapter 4 when we look at the player types used in more detail.

## Chapter 3

## Literature Review

The literature I have analysed prior to and over the course of this research is drawn from a broad base of game theory, psychology, distributed computing, graph and network theory, evolutionary biology, anthropology, as well as more generalised mathematics and statistical modelling. In this section I review a cross-section of this work in detail, which provides background and motivation to many of the decisions made within this thesis. I have grouped related papers into rough categories for ease of reading, though much of the work here is difficult to place in a single discipline. For instance, Maynard Smith has noted that "'paradoxically, it has turned out that game theory is more readily applied to biology than to the field of economic behaviour for which it was originally designed"' [22].

### 3.1 Game Theory

Game theory can also be described as 'interactive decision theory', or 'behavioural theory'. It broadly looks at interactions between agents, and the decision-making processes they invoke to determine how best to interact with one another.

## Graphical Games (Kearns 2007) [11]

This is the main paper outlining the pre-existing graphical games model (without mobility). Graphical games are a subset of all possible games, and all games can be viewed graphically, in the worst case, as a complete graph. This is important, as it proves that any and all results applicable to games in general will also be applicable to graphical games- and, by extension, mobile graphical games, due to the possibilities for conversion between the two I outlined earlier.

The advantages of graphical games are described as threefold:

- The relatively compact and sparse structure of graphical games is derived from several restrictions on an otherwise large and unwieldy game. By exploiting the nature of these restrictions, more efficient computation of certain problems can be achieved.
- They are a potentially fruitful area for further research, due in part to similarities to models of probabilistic inference such as Bayesian and Markov networks.
- They "provide a powerful framework in which to examine the relationships between the network structure and strategic outcomes". This is particularly noteworthy, as it describes almost exactly the tool I need to address my central research question.

Potential applications of 'correlated equilibria' are discussed, in which a trusted party picks an outcome in a game and tells each player what action to perform to ensure it occurs. However, this is only effective, and indeed desirable, if no player has an incentive to deviate from the action suggested for them assuming that no other players will deviate from their own suggested actions- in other words, if the chosen outcome is a Nash equilibrium.

For example in the game of Chicken [23], both players would prefer not to swerve, but face a relatively catastrophic utility hit if neither does. By adding a single
random bit observable to both players, we can define a very basic protocol which suggests that one player swerves while the other continues straight on depending on the value of the bit. If each player assumes the other will follow the course of action suggested for them, they can perform no better than by performing their own recommended action. A similar, every-day example of such a scenario is a traffic light (the trusted party) telling one car (the first player) at a crossroads to go and another car (the second player) to wait (and in doing so, recommending an action for each of them to take).

I considered using a form of this to give agents some limited, more easily-digestible information about the state of the population beyond their locality. However, this would disadvantage the more aggressive, predatory strategies which most other players aim to avoid, as well as complicating the investigation into the fundamental dynamics of mutable populations.

This paper also contains detailed description and analysis of several known algorithms which efficiently compute the Nash equilibria of graphical games, particularly on highly restricted graphs such as trees. However, as there has been no work done into examining how the addition of mobile agents alters the nature of a graphical game, all of these assume that the players' positions on the graph and strategic types remain fixed, and so are of limited use to my own research.

## Games on Grids (Nowak \& Sigmund 2000) [24]

This work is something of a survey, covering many different types of spatial gamesthose in which the locality of one or more players is in some way a considerationand attempts to draw comparisons between them. Some of the elements I seek to integrate into my model have been analysed in relation to known systems- for instance, the effects of noise on some different player types, and how the complexity and effectiveness of other types changes with varying amounts of memory.

Although there are a few known results for some specific scenarios, the aggregation of these has been hampered by their distribution across several fields. For instance, much discourse has been previously produced regarding whether 'player types' are most akin to 'species', 'individuals', or 'genetic lines', as well as what exactly the propagation of a type to a new node represents- but I believe that all of these are different ways of phrasing the fundamental question of 'what is the best way to determine the type of the player at a given node in a given scenario?'. I believe to have sidestepped this dispute quite elegantly- my model should be able to successfully run simulations from all such areas, regardless of the nature of or driving force behind the mobility.

It is important to note that, due to the complex interplay between different parameters, there cannot really be considered to be a single 'definitive' version of any matrix-form game such as the Prisoners' Dilemma. Altering the precise values of the payoffs even slightly can produce a very different equilibrium, while keeping the fundamental nature of the game unchanged. Graphs are provided showing how qualitative and entirely unintuitive state-changes can emerge in the behaviour of the overall system, arising from very minor quantitative differences in initial configuration. As the effect of varying these payoffs has already been explored, I will keep them fixed in order to focus on the other parameters I've identified in my research.

## Beyond Nash Equilibrium: Solution Concepts for the 21st Century (Halpern 2008) [25]

Halpern identifies three major faults with the concept and general application of the Nash equilibrium:

- It cannot account for 'faulty' players (those choosing, for whatever reason, strategies which would classically be termed 'irrational') or coalitions of colluding players.
- It does not take into account implicit costs of computation.
- It assumes all agents have perfect knowledge of the game.

However, it seems self-evident that most real world situations (which I aim to model) include one or more of these aspects. Though all mathematical models must include some amount of simplification by necessity, these are quite significant structural issues, which I believe are, at the very least, improved upon in my model:

- Real people tend to have complex utility functions, including factors such as disproportionate altruism towards their 'friends' and possibly spite towards their 'enemies'. They're also likely to collude with these other players for mutual benefit where possible, even in situations where the penalties for doing so are relatively high.
- Real people simply do not weigh up every possible action, reaction and interaction which could have an impact on the outcome of a scenario. The 'cost of computation' for humans is akin to the amount of 'effort' that the person expends trying to decide which action is best.
- Real scenarios are almost never fully explained to or understood by all participants in them. No one can ever really have 'perfect' knowledge of anything but the most simple real-world situation to the same extent that this term is employed in academia. Furthermore, even these simple situations can be misinterpreted or misunderstood, even if such information is theoretically available.

Halpern also suggests that the dissonance between predicted and observed results in game theory is due to these three factors. For example, in fixed-length iterated Prisoners' Dilemma, calculating the Nash equilibrium requires performing backwards induction, a task which could be reasonably expected to be challenging for inexperienced human players. Because of this, players may 'incorrectly' play other
strategies such as Tit-For-Tat, or cooperating in all but the final turn.
This is interesting because it provides a testable hypothesis for the classically observed 'irrational' behaviour of players in such games. If this is the case, then by providing the players with more processing power in the form of more time and less pressure to consider their actions, there should be a notable decrease in deviation from the predicted strategies for these games. I have tested this hypothesis in the course of my research, and will discuss these findings alongside my results in Chapter 6.

## Coordinating Team Players Within a Noisy Iterated Prisoner's Dilemma Tournament (Rogers et al. 2007) [5]

To commemorate the 20th anniversary of Axelrod's original competitions using the iterated Prisoners' Dilemma, several new competitions were organised. As the only change from Axelrod's original ruleset, each researcher was invited to submit multiple entries, rather than being limited to one as was the previous norm. Rogers at al. proved their intuition that a team of colluding agents could outperform others not just by unconditionally cooperating, but by adopting 'master' and 'slave' roles to ensure some of the players on the team achieved extremely high scores at the expense of the others.

As there was no explicit communication permitted between agents- a restriction which I feel is absurd in experiments which are, at their core, about cooperation and communication- the team had to use a pre-defined sequence of plays to identify themselves to each other.

Finally, they show that this system of collusion is robust in a large number of theoretical experimental structures, so long as multiple entries from each group are permitted, as the 'slave' players can actually outperform a fair number of other
strategies overall by blindly cooperating with each other.
There is a question of how interesting this result is, as it is both well-known and makes strong intuitive sense that colluding agents can outperform others in a wide variety of situations [26] [27]. One could, however, make the argument that in the real-world, agents often can and do make a strategic decision to collude, even in circumstances where it risks severe penalties (such as criminal prosecution), due to a considered and calculated attempt to achieve the large potential gains which collusion can make available. As such, it would seem that any attempt to comprehensively understand real-world behaviour should also be able to understand and account for the possibility of external collusion.

Though the arguments presented in this paper are persuasive, the form of collusion studied here presupposes the existence of absolute trust between two or more players. Though this is not completely beyond the realms of possibility in the real-world, it is clearly so in the phrasing of most of the real-world scenarios we'd like to model, such as business, economics, and competitive gaming. Even outside of these scenarios, the question of absolute trust is problematic. As such, I decided to discount this possibility in my research.

Allowing agents to naturally express their preferences through their type, strategy selection, and free communication should provide the best system for modelling existent behaviour, as this provides the closest representation of how people actually interact in such situations. In reference to the idea of meta-games (which I explain further in Chapter 5), collusion is simply cooperation which occurs at one 'level higher' than usual, which, like any form of interaction between agents, can be succinctly represented by graphical games with mobility system I have designed- for instance, by designing and running an additional simulation to model the discussion and negotiation occurring between agents before the beginning of the scenario in the previous model.

### 3.2 Graph Theory

The structural core of the system I've designed is a classic 'nodes-and-edges' graph. These graphs have been studied for many years, and there are a large number of known facts and techniques. Here I will expand on some which are directly relevant to my contribution.

## Bargaining Solutions in a Social Network (Chakraborty \& Kearns) [28]

This paper uses a variant of the Ultimatum game to study the impact of negotiation across a graphical game. Players act 'myopically'- based only on information available from the neighbourhood of the players they're immediately connected to. Edges have weights corresponding to the amount of resource available for those players to divide, but the special case where all edges have unit weight is frequently considered. Although they assume that all players partake of the same global utility function, the effect of different global utility functions and models of rationality on negotiations is discussed. I have used a simple linear global utility function, in which the utility of each player directly corresponds to the magnitude of their summated payoff in the final round (that is, players are unaffected by traits such as altruism or spite) for my simulated agents. This means that each player is only concerned with their own payoff- they do not 'feel' better or worse depending on other factors such as the payoffs of other players.

A general solution to the 'network bargaining problem' of finding a stable distribution of a certain resource across the graph is sought. A distribution is considered 'stable' if each edge satisfies the given model of rationality. Additionally, Chakraborty \& Kearns are interested in how the topology of the network shapes the final distribution.

It seems reasonable that a player's strategy may change depending on whether they have a dearth or over-abundance of opponents to engage with. A player with only one link may choose to foster a cooperative arrangement with their opponent as they have only a single potential supplier. Conversely, a player with many such options could play more aggressively and attempt to extort a disproportionate amount of resources from its neighbours, threatening to 'go elsewhere' if its demands aren't met. Indeed, this paper finds that the relative 'bargaining power' of a node can be described by a function relating the size of its neighbourhood to those of its neighbours' neighbourhoods.

The paper goes on to show that at least one stable equilibrium exists for all such problems in both the 'Nash bargaining solution' and 'proportional bargaining solution' models. Unfortunately, the problem rapidly becomes intractable if every player tries to integrate signals from beyond their locality into their strategy. Because of this and other considerations, I opted to use myopic players in my simulations, who can only observe events in their neighbourhoods and as such do not have access to current information outside of that limited area. However, my results were still in broad agreement that high bargaining power is correlated with utility gain, as I will discuss in Chapter 6.

## Power Exchange in Networks: A Power-Dependence Formulation (Cook \& Yamagishi 2002) [29]

This paper further explores the relationship between the number and magnitude of potential payoffs and 'structural power'- the bargaining strength of a player. It starts by drawing parallels to classic network connectivity questions- the removal of one node may adversely impact the flow through the graph to a greater or lesser extent dependent on a number of factors. Cook \& Yamagishi demonstrate a correlation between the nodes whose removal causes a relatively high impact on the maximal
flow, and the nodes with high structural power.
They describe situations in which each player may only form one deal, referring to them as 'negatively connected'. Structural power for a given agent in a given deal is determined by the increase in utility over the 'alternative' (typically zero) for the opponent, and the best offer the agent could get by negotiating elsewhere. The Nash bargaining solution is approximated as each agent, over time, tends to negotiate its most profitable deal to the point where both agents receive amounts directly proportional to their structural power.

This is referred to as the 'equi-dependency principle', and it is concluded that the equilibrium eventually reached will be in accordance with (and can indeed be calculated by) this principle. It's worth noting that the links not used in negotiation are nonetheless critical in determining structural power, and thus the eventual equilibrium- these are referred to as 'latent relations'. This means the entire structure of the graph must be considered at each step- no simplification is possible.

The case where multiple deals can be struck in each time step- a 'positively connected' scenario- is also considered. Surprisingly, the determination of structural power in this instance is not much more complex, simply comparing the weakest deal in the set of potential deals to the deal in consideration, as opposed to the strongest remaining deal in the neighbourhood. This occurs as it is the least profitable deal which will inevitably be dropped in favour of the formation of a stronger one if a fixed number of deals must be made.

This approach appears to provide a simple solution for playing games across a population in a positively connected manner. It yields satisfying holistic results while accounting for network topology and relative bargaining power, and does so iteratively and with reference to other players. It should also be noted that 'structural power' is very similar to the notion of 'bargaining power' presented in 'Bargaining Solutions in a Social Network' [28].

Finally, Cook \& Yamagishi note that "While the original motivation of defining graphical games was computational, we believe that an important property of the model is that it enables an investigation of many natural structural properties of games." This idea provided my motivation for many of the changes I made to Kearns' original formulation of graphical games, in order to emphasise and maximise those natural structural properties of the graphical gaming model which enable this investigation.

## The Local and Global Price of Anarchy in Graphical Games (Ben-Zwi \& Ronen) [30]

The 'price of anarchy' (or 'PoA') is the ratio between the 'worst' possible Nash equilibrium (that is, the one which provides the lowest total payoffs across all agents) and the optimum social welfare (that is, the course of action which provides the highest total payoffs across all agents) for any given game. Conversely, the 'price of stability' is the measure between the best possible Nash equilibrium and the optimum social welfare.

This research presents these as natural measurements of games and, indirectly, as natural measurements of the strategies with which these games are played (via Nash equilibria), arising whenever myopic agents act selfishly. In a standard graphical game (without mobility), the 'local price of anarchy' is a measure of the effect in a specific locality of a graphical game. By comparing this to the global price of anarchy, inferences can be made about both the game and its topology:

Definition 10 Let $G$ be a graphical game. The local price of anarchy of a set of players $S_{i}$ is at least $\alpha$, if for every set of actions of its neighbours, the PoA of the induced sub-game is at least $\alpha$. Let $S=S_{1}, S_{2}, \ldots, S_{l}$ be a cover of $V[G]$. We say that the local PoA of $G$ with respect to $S\left(L P o A_{S}(G)\right)$ is at least $\alpha$, if the local price
of anarchy of every subset $S_{i} \in S$ is at least $\alpha$.

The naming used is somewhat unintuitive- a game with a high price of anarchy (approaching the maximum of ' 1 ') naturally produces utility near the optimal level, and so does not lose as much when players act myopically and selfishly. Similarities can clearly be drawn between this and correlated equilibria- the total utility which can be gained from a game can increase dramatically if players are able to coordinate even on a rudimentary level.

Although this metric was mainly designed to help in evaluating certain computational properties of graphs, they are unfortunately not the ones my research focuses on. However, a strength of this metric is that it can be assessed at any scale across a graphical game- this is the 'local' price of anarchy. I hypothesise a correlation should exist between a low local price of anarchy and the proportion of agents attempting to move in that locality, once mobility becomes a consideration.

Generally speaking, an agent in such a situation could expect their situation to improve elsewhere in the graph, whether due to a different structure, and/or a different distribution of player types. In the worst-case scenario when everyone is acting selfishly- which seems to be the fundamental assumption for most of the situations I will be attempting to model (and a reasonable starting assumption for cautious agents, such as humans, to make) they would still stand to gain. I have tested this hypothesis and outline these findings along with my other results in Chapter 6 later.

### 3.3 Psychology

As contributing towards attempts to model human behaviour in real-world situations is one of my main goals, a brief review of human behaviour as it relates to the features of the model I am designing should be helpful.

## Altruism, Spite, and Competition in Bargaining Games (Montero 2005) [31]

There are two opposing attributes which players can display; 'altruism', in which their utility increases with the increasing utility of other players, and 'spite', in which their utility increases with the decreasing utility of those players. The effect of differing levels of altruism and spite on players in a range of different scenarios forms the focus of this paper.

Montero shows that spiteful or aggressive players fare better in bilateral negotiations in which a deal must be reached, so long as their opponents are able to anticipate their nature. In these situations the opponent, who is hoping for a deal with greater utility than the 'status quo', is willing to make concessions to the spiteful player in order to ensure that any deal is successfully reached. Conversely, altruistic players find it easier to make deals more quickly, as they're more comfortable accepting less. However, the analysis of which type fares better is much more complicated in multiplayer games, in which players have some control over which opponents they form deals with. The main dilemma is that, while altruistic players may be able to find many more players willing to bargain with them, spiteful players are able to extort much better payoffs from the fewer games they do play.

This paper focuses on such situations. In the first game studied, from three initial players, any two must form a coalition which then splits an amount of money, while the excluded third player receives nothing. Interestingly, both spite and altruism cause a decrease in the average money gained by an otherwise selfish or neutral player in this situation. The most spiteful player, who would demand the best deal from negotiation, is excluded by the other two. Then the less altruistic of the remaining players gets a larger share of the money from the coalition.

In the next game analysed, each of three players has an equal random chance of
being the 'proposer' in each discrete time round. The proposer makes an offer of a definitive split of some money to another player. If the second player accepts, the two share the money and the third player receives nothing. If the offer is rejected, no one receives anything and a new round begins. The conclusion reached is that, if all players are perfectly patient, altruism and spite have no bearing on the outcome of such games.

However, in more realistic environments where players must reach an agreement expediently or face decreased payoffs (modelling the common real-world occurrences of time pressure and/or resource depletion), the altruistic players, who are happier to accept any given deal so that some players will receive utility, fare somewhat better by (counter-intuitively) acting 'impatiently' and accepting early deals that others would reject. They once again find themselves preferentially invited to coalitions at the expense of the spiteful players, who would be happy to wait much longer for a deal to be made. But if a player is too altruistic, the deals offered to them, and gladly accepted by them, will be so unbalanced in favour of the proposer that it is once again the more selfish players who fare best.

Some other results are briefly mentioned, highlighting the fact that altruistic agents may, in the right circumstances, increase the efficiency of a population and even outperform spiteful players when both are paired with other altruistic agents. Although other players can moderate their demands to find partners in multilateral negotiations, altruistic agents will always be more popular opponents. In other words, a known spiteful player can reduce their levels of spite, but will still be a less popular opponent than a known altruist.

This research is also of interest as I will have a large number of agents who have a limited capacity to control their opponents by moving around a graph. My players will generally display cooperative or aggressive types, which roughly correspond to the idea of altruistic and spiteful agents respectfully- more so if the given situation can
be modelled on different levels. This closely mirrors the cooperative-vs-competitive core of situations like the Prisoners' Dilemma, though operating one lever higher in the pros and cons of selecting various types- I will expand upon these 'meta-games' in more detail in Chapter 5.

In my preliminary experiments, I was concerned that the initial distribution of player types across the population was causing dramatic variance in the final reckoning of which strategies performed best. However, I am now confident that this is due to the properties of the game played across the population. With something like the Ultimatum Game, aggressive strategies could be less effective at repeatedly exploiting the more altruistic ones, resulting in a more balanced dynamic distribution. Ultimately, different games can be used as different models of interaction, causing the system as a whole to display very different behaviours and properties.

## Behavioural Experiments on a Network Formation Game (Kearns, Judd \& Vorobeychik 2012) [6]

Three multi-stage experiments with 36 participants each were designed and carried out to assess the performance of humans in collaboratively building a network to solve a simple coordination problem. The players were tasked with ensuring they all picked identically from one of two colours, but could only see the colours of those in their neighbourhood, and could not communicate beyond publicly changing their colour preference. Each player could incur some cost to create a new link to another specified player, which would only be subtracted from their final payoff if it was positive. Some players had preferences, receiving a greater payoff if one colour or the other was uniformly chosen- but all players received no payoff if the task was not completed successfully.

The critical result presented is that the networks created by the human players involved were poorly suited to the task at hand, and, in some cases, actively unsuited
to it. This is despite numerous attempts to guide them into creating better solutions, such as seeding the population with a pre-existing 'good' network structure or the structure from a previous experiment. The participants seemed generally unable to use their greater capacity for foresight and planning to outperform comparatively simple heuristic agents, and were in fact hindered by a variety of factors (including the selfishness of players who attempted to earn a greater payoff by defying the will of the group as a whole, and those who attempted to conserve their resources and maximise their potential payoff by purchasing as few edges as possible).

The results from these experiments strengthen my certainty that the outcomes of my simulations using relatively simple, myopic agents (who cannot meaningfully interpret events beyond their locality) should be generally representative of humans operating in the same sorts of situations, as human players seem broadly unable to leverage their more advanced cognitive faculties to assist with this category of problems.

### 3.4 Alternative Models

Before settling on using the graphical gaming model as the basis for my research, I investigated a number of other candidate systems. However, using graphical games as the basis for my model had a number of advantages over other candidate systems. Critically, the restricted nature of interaction in a graphical game enabled the natural and intuitive addition of mobility while retaining straightforward comparison to both the original model and deeper game theory. Additionally, graphical games have more versatility than the other models, and are able to model a wider variety of existent scenarios, without having to resort to some of the stranger examples used with other models to justify their particular adaptations and idiosyncrasies.

## An Introduction to Population Protocols (Aspnes \& Ruppert 2007) [32]

A 'population protocol' is a model designed to simulate extremely basic agents with a fixed number of states interacting unpredictably to compute some predicate on an input initially distributed across the population. When two agents interact, they both read each others states and update their own state in accordance with a global transition function, aiming for all agents to eventually converge and stabilise on the correct output. Such protocols are 'uniform' in that they operate identically regardless of the number of agents in the population, and 'homogeneous' in that all agents execute the same program and do not possess unique identifiers.

Definition $11 \bullet Q$, a finite set of possible states for an agent,

- $\Sigma$, a finite input alphabet,
- $\iota$, an input map from $\Sigma$ to $Q$,
- $\iota(\sigma)$, the initial state of the agent whose input is $\iota$,
- $\omega$, an output map from $Q$ to the output range $Y$, where $\omega(q)$ represents the output value of an agent in state $q$, and
- $\delta \subseteq Q^{4}$, a transition relation that describes how pairs of agents can interact.

To refer to the example presented in the paper, imagine a flock of birds, with each creature having a tiny, cheap sensor with a wireless transmitter attached to it. Each sensor can assess the bird's health, and communicate with other sensors within a meter, but has only linear memory. We wish to determine whether a certain number of birds, or some percentage of the population, are ill. It is shown that this, along with any other predicate expressible in Presburger arithmetic ('semilinear' predicates) comprise everything stably computable by a general population protocol.
'Fairness' is important in this model- interaction scheduling is typically considered to be adversarial, and without some restrictions a subset of agents could be made to interact infinitely to the exclusion of all others. Clearly, this is not a good model and could not be expected to produce reasonable results. Various conditions for enforcing fairness, and their effect on the speed of computation, are assessed. Generally, it is assumed that each possible transition occurs arbitrarily often.

Aspnes \& Ruppert also define an 'interaction graph', which connects agents at the vertices to other agents with whom interaction is possible (but not ensured). Restricted interaction graphs are known to increase the computational power of the model, in some cases even enabling it to simulate a Turing machine and compute any predicate or function in LINSPACE- much like a graphical game. It is noted that agents can be considered to be moving around the graph by exchanging their states in this model, and examples of this being used to solve problems such as graph colouring are provided.

It is easy to see how this could have formed a base for my research- my artificial agents will be extremely simplistic, and although they will move around a graph limiting their interactions, they cannot completely control with whom they interact at each step. However, there are also a large number of flaws which render population protocols unsuitable for further investigation. My agents are not homogeneous, as each has a discrete type which is effectively initialised before play begins. They are also unable to determine the disposition of their opponents clearly even after interaction is completed.

## Even Small Birds are Unique: Population Protocols with Identifiers (Guerraoui \& Ruppert 2007) [33]

This paper considers 'community protocols', which are an extension of the population protocol model. There is some criticism of the initial population protocol
model, arguing that in even the most basic possible scenarios scenarios, agents are able to recognise each other unless specifically prevented from doing so. Such a capability provides a number of demonstrable benefits to each agent's computational power, and therefore the power of the whole system.

This would also appear to be both intuitively correct, and backed up by mown results showing that agents who recognise and respond to their opponents outperform those which do not. As such, the main alteration is that each agent has a unique identifier in addition to the prior model. Guerraoui \& Ruppert go to great lengths to design a scenario such that the ability of agents to recognise others does not, in and of itself, grant additional computational power- for instance, identifiers cannot be altered or operated on except to test for equality.

Despite the simplicity of this change, community protocols are notably more powerful than population protocols, able to compute any symmetric function in $\operatorname{NSPACE}(n \log (n))$.

As a generalisation of population protocols, the formal definition is very similar:

Definition 12 Let $U$ be an infinite set that contains a special symbol $\perp$ and all possible identifiers. An algorithm where agents get inputs from a finite set $\Sigma$ and produce outputs from a finite set $Y$ is specified by:

- a finite set, B, of possible basic states,
- a non-negative integer d representing the number of identifiers that can be remembered by an agent,
- an input map $\iota: \Sigma \rightarrow B$,
- an output map $\omega: B \rightarrow Y$, and
- a transition relation $\delta \subseteq Q^{4}$, where $Q=B \times U^{d}$.

As in the previous paper, there is much discussion of the various types of failures which can be encountered by distributed systems and the robustness of these different models to them. This interesting problem lies outside the scope of my more theoretical research.

It seems self-evident that, in any vaguely 'realistic' example, players should be able to recognise their opponents, even if only on the most basic level of whether or not they've met a given player before and what the interactions with them were. This enables more complex interactions to develop- for example, an intelligent, typically cooperative player in a game with a player who is known to have played aggressively in every prior game would do well to recognise that they should change their approach. In fact, I would argue that identity and recognition are essential aspects of any iterated system, as agents will find it extremely difficult to gauge how to react to each opponent if they're unable to recall the relevant memories.

## Population Protocols that Correspond to Symmetric Games (Bournex, Chalopin, Cohen \& Koegler 2009) [34]

Population protocols appears very similar to the structure of a simple game. In a population protocol, agents interact in pairs according to some rules, and update their states automatically according to the state each was in previously. In a game, two players choose a strategy and receive payoffs based on the strategies chosen. These similarities are investigated in this paper, which is an update of 'Playing with Population Protocols' from 2008 by the same authors [35].
'Pavlov' is a strategy for the iterated Prisoners' Dilemma in which the player begins by cooperating and henceforth makes the same play as in the previous game, unless they received a 'bad' payoff (from mutual defection or from cooperating while their
opponent defected), in which case the other strategy is used. Because of this fundamental behaviour, Pavlovian strategies are also known as 'Win-Stay, Lose-Shift' strategies. Pavlov is known to quickly stabilise into a wholly cooperative population in any restricted graphical game, regardless of topology. Importantly, Pavlovian behaviours are 'Markovian', meaning their actions at each stage depends wholly on the chain of previous actions from themselves and their adversary, which can be implemented using the simple identification and memory systems available to my agents.

A simple construction is given to show that symmetric games can be associated to Pavlovian population protocols without loss of generality. Pavlovian solutions to classic problems such as logical AND/OR, leader election, and majority detection are described. Finally, a proof by construction is used to show that any semilinear predicate can be computed by a symmetric population protocol- in other words, the requirement of symmetry between the players is not a restriction.

The simplistic, deterministic nature of Pavlovian protocols is attractive, as is the ease of which they can converge and stabilise. I had initially planned to have roughly equal numbers of different types of agents in my populations to form a 'default', 'balanced' population- however, my experimentation indicated that such a population can cause unexpected and unhelpful results. For instance, a very simple player which always defects and never attempts to move can attain perfect performance if irrevocably paired with an equally simple agent which always cooperates and also never attempts to move. As such, I decided to generally seed my populations with a higher percentage of Pavlovian agents. When paired with various movement preferences, should help smooth out some of the extremes and produce more meaningful results.

## Stochastic Games (Shapley 1953) [36]

A 'stochastic game' is a two player simultaneous sequential game with a set of payoff matrices. Each outcome of each matrix is probabilistically mapped to a subset of the other matrices, including a non-zero chance of ending the game at each step (which can alternatively be thought of as an additional mapping to a special matrix). When an outcome is reached, the game transitions to another matrix in accordance with the associated probability. There is a defined initial matrix, and whenever a matrix is transitioned to, that matrix is used to define the payoffs in the next step of the sequential game. Payoffs accumulate throughout the course of a stochastic game.

Definition 13 Assume a finite number, $N$, of positions, and finite numbers $m_{k}$ and $n_{k}$ of choices at each position; nevertheless, the game may not be bounded in length. If, when at position $k$, the players choose their ith and jth alternatives, respectively, then with probability $s_{i} j^{k}>0$ the game stops, while with probability $p_{i} j^{k} l$ the game moves to position l. Define $s=\min (k, i, j) s_{i}^{k} j$

By specifying a starting position we obtain a particular game $\Gamma^{k}$. The term 'stochastic game' will refer to the collection $\Gamma^{k}=\Gamma^{k} \mid k=1,2, \ldots . N$.

Since $s$ is positive, the game ends with probability 1 after a finite number of steps, because, for any number $t$, the probability that it has not stopped after $t$ steps is not more than $(1-s) t$. Payments accumulate throughout the course of play: the first player takes $a_{i} j^{k}$ from the second whenever the pair $i, j$ is chosen at position $k$.

This general framework contains several special cases which have been effectively analysed elsewhere, such as games with temporally discounted payoffs and simple matrix games with durations dependent on the strategies used. Conversely, the idea can be expanded without loss of generality to an n-player game with a randomised initial state. As such, a graphical game with a mutable structure is fundamentally equivalent to a (very large) stochastic game, with each possible configuration of the
graph corresponding to a different state and a different matrix.
This would, however, require some redefinition of the transition probabilities so as to ensure, for example, that the simulation would run for a minimum length. As graphical games provide a far more succinct representation with far fewer random elements outside of the control of any agent, I decided to instead use them as the basis for my research.

## Urn Automata (Angluin, Aspnes, Diamadi \& Fischer 2003) [37]

Urn Automata are a precursor to population protocols, which retain many of the properties of more classic systems but are clearly motivated by the search for solutions to the same problems inherent in models of cheap, distributed computation. The model consists of an 'urn' containing tokens drawn from a finite selection of colours, a finite-state controller, and an input tape. Tokens drawn from the urn are done so at random, reflecting the structureless, amorphous storage of a group of anonymous distributed agents. As such, the model is inherently probabilistic.

Definition 14 An urn automaton is a tuple ( $Q, q_{0}, \Sigma, T, \Delta$ ), where:

1. $Q$ is the finite set of states for the controller;
2. $q_{0} \in Q$ is the initial state;
3. $\Sigma \supseteq \$_{L}, \$_{R}$ is the finite input alphabet;
4. $T$ is the finite token alphabet, that is, the set of available token colours;
5. $\Delta$ is the finite transition relation, which is a subset of $Q \times \Sigma \times T * \times Q \times T *$ $\times(L,-, R, A C C E P T, R E J E C T)$.
$A$ state of an urn automaton is a 3-tuple ( $q, x, i$ ), where $q$ is the state of the finitestate controller, $x$ is a multiset of tokens from the token set $T$, and $i$ is the position of the input tape head, with the leftmost position assigned index 0. In an initial
state, $q=q_{0}$ and $i=1$; and the initial contents of the urn are taken to be part of the input.

A transition $(q, x, i) \rightarrow\left(q^{\prime}, x^{\prime}, i^{\prime}\right)$ can occur if there exists a transition $\left(q, \sigma, t, q^{\prime}, t^{\prime}, O P\right)$ such that: 1. The symbol in cell $i$ of the input tape is $\sigma$;
2. The tokens in $t$ are a submultiset of $x$;
3. The new urn state $x^{\prime}$ equals $x-t+t^{\prime}$, where addition and subtraction of sequences of tokens from and to multisets of tokens are interpreted in the obvious way; and 4. Depending on the type of $O P$ :
(a) If op is $L$, then $i^{\prime}=i-1$;
(b) If op is $R$, then $i^{\prime}=i+1$;
(c) If op is - , ACCEPT, or REJECT, then $i^{\prime}=i$.

The model considered in the paper uses deterministic transitioning and uniform sampling. It is also 'conservative', in that exactly one token is added to the urn for each one removed, and has 'constant width', in that it always reads $k$ tokens sequentially before transitioning. Depending on the structure of the input, an urn automata with $n$ tokens can be at least as powerful as a probabilistic Turing machine with $\log (n)$ tape cells. Some of the classic problems of population protocols such as 'leader election', in which a unique agent must be selected by the algorithm during execution, appear here in a similar context- namely, ensuring there exists exactly one token with a given colour.

The approach described in this paper is markedly different from most others. The idea of using a single, centralised memory to emulate an arbitrary number of distributed agents initially seems counter-intuitive, but could conceivably form the basis of a model which addresses my goals. I considered the idea of using an urn to model the general 'atmosphere' of a population, particularly how frequently cooperation or competition was occurring, and using that to inform the future choices of my agents.

For example, some players could add 'green' tokens to an urn when they successfully cooperate, and 'red' tokens when they are betrayed. Tokens could then be sampled to determine whether or not an agent could reasonably expect to be betrayed in a given round, and moves determined accordingly. This would be more akin to a system of background chatter and overall perception of the environment informing the agent's actions rather than any specific pieces of information.

However, this model assumes there is no structure whatsoever in the underlying distribution, and doesn't have a way to associate known information or samples to any particular entity. This is a level of anonymity even stronger than that of population protocols, which has itself been challenged as unrealistically restrictive [33]. Because of this, it seems unlikely that this model can be adapted to overcome these limitations without a fundamental overhaul.

### 3.5 Cellular Automata

Cellular automata have many important qualities in common with graphical games. The grids they take place on are similar or even equivalent to the model of restricted connections used across a graphical game, and the simple propagation or replacement of types across nodes is superficially similar to agents negotiating to swap positions as in my system. Most interestingly, cellular automata systems often display fascinating and strongly emergent behaviour, and are renowned for creating extremely complex, strongly stochastic behaviour based on the initial state of just a few simple variables [38]. Again, this description could also be applied to the configuration and resultant behaviour of graphical games with mobility.

## Spatial Games and the Maintenance of Cooperation (Bonhoeffer, May \& Nowak 1994) [39]

A 'spatial game' takes place across an n-regular graph. At each step, each agent plays a game against every other agent in their neighbourhood. After this, each agent assumes the type of whichever agent in its neighbourhood, itself included, performed the best in the previous round. Alternatively, a less deterministic method can be used, in which each agent assumes each type with probability correlated to how well that type performed in its neighbourhood.

The issue of 'discrete vs continuous' time (or, more commonly and correctly 'synchronous vs asynchronous' time) is discussed, and a similar conclusion to that of the population protocols model is reached. I will explain these systems in more detail in Chapter 4. Both having all agents play against all others agents in each time step, and the introduction of truly continuous time, add complexity to different parts of the model. Having all agents interact fully with their neighbourhoods in each step is, in addition, unrealistic for modelling most biological and social situations.

However, if one agent, chosen at random, interacts with its neighbours and updates accordingly, a slight element of randomness is introduced, though simplifying the overall model. Both systems appear to produce broadly similar behaviour (at least in this scenario), but as there are some variations, I will include both in my model and compare their effects on different simulations in my results.

In the simplest case studied, with two agent types interacting deterministically on a 4-regular grid with boundaries, the environment is very similar to classic cellular automata such as 'Conway's Game of Life'. The paper progresses to applying these ideas to a more natural graphical game. On a 200 by 200 grid, $5 \%$ of the cells were initially seeded with agents, and then an 'interaction radius' $r$ was used to limit the neighbourhood of each agent to those within distance $r$.

However, the patterns which emerged here were far more static, and tended towards non-cooperative behaviour as $r$ increased and the diameter of the graph consequently decreased. This would correlate with known results from the $n$-player Prisoners' Dilemma (also known as 'The Tragedy of the Commons' to contrast it with the standard 2-player version), which has an extremely strong attraction towards noncooperative behaviour as $n$ increases [40].

The overall conclusion presented is that, for a surprisingly wide range of environments, cooperative and non-cooperative behaviours can rationally co-exist without one or the other eventually gaining the upper hand and forcing the other to become extinct, as might be expected. This is an excellent example of emergence, as it uses fixed-type agents playing a very simple game, yet produces extremely chaotic results, but ultimately stable- a description which can also be applied to my own model.

## Evolutionary Dynamics on Graphs (Liberman, Hauert \& Nowak 2004) [41]

Evolutionary dynamics is a concept from biology which bears some resemblance to concepts of strategies and dominance from game theory. It is concerned with questions about the relationship between the quality of naturally arising mutations, and the probability of that mutation propagating through and eventually conquering a population, known as 'invading' that population. This is described as the mutant strategy 'fixing' itself within that population. Similarly, an 'evolutionarily stable strategy' (or 'ESS') is one which, for a given environment, cannot be invaded.

Initially the vertices of a graph are each given some colour. In each time step, a vertex is selected with probability correlated to the 'fitness' of its current colour to copy its colour over another selected vertex. Fitness is a scalar value which can be considered to be analogous to the strength of its type in a given environment- for
example, we might describe Tit-For-Tat in the classic iterated Prisoners' Dilemma as having high fitness. This second vertex is chosen with probability correlated to the weight of the edge connecting it to the first vertex. The 'Moran process' is a special case which uses a complete graph with unit-weight edges.
'Drift' is the systemic characteristic of different mutations moving haphazardly throughout a population, while 'selection' is the competing characteristic of mutations persisting or vanishing based on their relative fitness. Interestingly, the paper finds that different graph structures can have an arbitrarily large effect on the relative power of drift and selection. For instance, a simple directed path graph confers virtually no benefit to advantageous mutations, ensuring they can never fix themselves completely, while a star or super-star graph all but ensures beneficial mutations, no matter how slight, fix themselves in the population.

It is noted that, instead of using an arbitrary 'fitness' criterion, the agents can play a matrix-form game to determine if the reproducer successfully overwrites another vertex. This alteration enables vertex colour to behave somewhat more like a player type, implying a preference and strategy selection schema. However, as each vertex with a given colour plays identically and the final result is binary (propagation or lack thereof) this seems to be little more than an abstracted fitness criterion, using the payoff matrix from game theory as a sort of 'look-up table' of results.

However, these results demonstrate that carefully tuned network topology can have a pronounced influence on which strategies will prove successful in a given population. Unfortunately, this model fails to consider the situation of different players being relatively strong or weak against each other in different situations, to say nothing of the payoffs, preferences or positioning of individual agents as instantiations of a type. However, their conclusion that 'The vast array of cases [of game, graph, and orientation] constitutes a rich field for future study' reinforces the notion that my model is exploring an academically interesting avenue.

## Chapter 4

## Features of the Proposed Model

The model I've designed consists of a number of novel additions and alterations to the basic graphical games model in order to create a new system, which is better suited to simulating real-world scenarios. In this chapter I will outline the alterations I've made, the form they take, why they should be considered as improvements to the pre-existing graphical game model, and their potential effects.

### 4.1 Constant Features

After extensive experimentation and prototyping, a number of potential parameters, features, and characteristics were chosen to remain fixed throughout the thesis as follows:

1. Identity. If agents cannot be identified then the relative potency of hawk strategies increases, as they are likely to have a window of time with which to exploit their new opponents whenever they change position before they are identified as non-cooperative, even by players who've already interacted with them. As such, each agent is recognisable only by a unique identifier, which contains no information in and of itself. This enables agents to recognise others
they'd played games with previously, and those they're playing against for the first time.
2. Memory. If agents can be identified, the possibility of agents having the ability to store their past encounters and choosing their strategies dependent on previous interactions becomes open. Indeed, the use of memory would seem to be a requirement for a complex iterated system to display interesting behaviour. The question then is how much should agents be able to remember? Once again, there is a huge amount of information in the model which could be considered tactically relevant- a player's total payoff to date, the number of times a player has moved, even a complete history of all players strategies in each game they've played.

This problem rapidly approaches intractability if even a small amount of this information is evaluated by every player for every decision in every game. As a compromise, in the basic model, each player will be able to flawlessly recall their own play history with each other player. The agents can then use this information to help decide their strategies and to make type inferences about their opponents- if they've played aggressively for their past 100 interactions with you, it would seem unlikely they're going to cooperate this time round. Though human players have access to additional information, it appears they do not make significant use of it, as I show in my results later.
3. Perception. My agents are 'myopic', in that they can see how many neighbours they have and who they are, but not beyond their neighbourhood. They do not observe the games others play or the results of those games. If each agent were able to observe the payoffs and movements of all other agents in the graph, and attempt to factor even some of these into their own strategies, attempts to determine the causes of effects in the model would quickly bloom into intractability. However, within each game, both players can perfectly
perceive which actions were played (unless noise is applied, as described in the next section) and the overall outcome.

As I discuss alongside my human experimental setup later, the data I gathered showed that even humans tend not to consider occurrences beyond distance 2 (that is, further away than the other players their opponents are interacting with), and even then only infrequently and when their capacity to control their position is strong. This gives me confidence that I can approximate human behaviour and provide a robust framework, even with myopic agents.
4. Undirectedness. There doesn't appear to be any particular advantage to the power of the model to include directed links and one-way relationships as a separate feature. If desired, this functionality can be recreated by modifying individual payoff matrices (such that, in a given situation, only one player influences the actual payoffs from that situation).
5. Intelligence. My artificial agents are simple implementations of player 'types'that is, strategies and approaches decided in advance. Agents do not attempt to 'map out' the graph or predict what sorts of players are likely to be occupying nodes beyond their vision.
6. Topological Immutability. The possibility for the graph to somehow change its structure during play was also considered. Although the number of vertices could never be lower than the number of players, additional vertices could also be added, leaving some vertices without players at each step. Extra edges could be added or removed at various points. These mutations could be properties of the graphical game itself, occurring randomly or when certain conditions (such as time passing, or players achieving a certain score) were met, but they could also be the result of strategies chosen by the players themselves. Once again, the limited intelligence possessed by my simulated agents meant that this addition did not meaningfully affect the results in my preliminary
experimentation, and even human players appear unable to make consistent use of this additional tool to improve their own situation in any significant manner [6].
7. Utility Functions. Specific players and/or types could have customised utility functions, in which their final utility is no longer equivalent to the total payoffs received. Common traits which can modify this include spite and altruism (as detailed in Chapter 3), though more complex modifications are possible, such as those based on distribution of types across the graph in a graphical game with mobility. However, non-standard utility functions can be modelled effectively through the use of player types. For instance, consider two similar types which, when both confronted with the same unclear situation, decide to cooperate and defect respectively. From the point of view of an external observer, who cannot see the internal workings to each agent's decision, this behaviour is identical to two players of the same player type, but with somewhat differing utility functions.
8. Fairness. In asymmetric games- those in which the players fulfil different roles and have different actions available to them- the roles are assigned randomly. To decide which pair of connected agents would be selected to play the next game when asynchronous time was used, a random selection was made, with a small additional weighting taking in favour of agents who have been waiting longer since their last game. In any other instance where a player needed to be determined for something (most notably movement negotiations), this was determined randomly, unless otherwise specified.
9. Fundamental Graph Structure. I have ruled out the use of loops (which connect a vertex to itself) and multiple edges (which connect a vertex to another vertex more than once), and addressed the consequent situation in which a player would interact with themselves, back in Chapter 2. Multiple edges can be
emulated satisfactorily using weighting.
10. Number of Players Per Game. Though many multi-player games are both academically interesting and relatively unexplored, I decided to restrict my examples to two player games, again to retain as much of the restriction on interaction created by the graph structure as possible and the resultant clarity. Fundamentally, however, it is already possible to model situations of this type using the model I have created, by giving part of a larger, multi-player payoff matrix to each 2-player game (as mentioned in Chapter 2). In addition, as I will discuss with the notion of 'meta-games' in Chapter 5, it is possible to model more complex forms of interaction- such as multi-player games- by instead considering the real-world situation leading to this interaction, and modelling it at a different stage or level.
11. Mobility. As the main addition to the graphical games model, agents will have some ability to control their position in the graph. Generally speaking, as time progresses, each agent's position in the graph will change, bringing them into contact with new opponents and new strategies. As one of the main additions of this research, many forms of mobility were investigated in the preliminary stage (explained in more detail in Chapter 6), listed here from lowest to highest levels of control by the agents:

- Agents move entirely at random at every step. There is no possible differentiation in movement strategies between agents.
- Agents can indicate that they wish to move- if they do, they then move at random as above. Otherwise, they remain stationary. Movement strategies are reduced to a binary decision at each step.
- Agents can arrange to swap position with another agent. If they don't, they remain stationary.
- Agents can move to any adjacent vertex, displacing any other agent there. In the event of multiple agents wishing to move to the same location, a tie-breaking algorithm would have been used to get each agent as close as possible to their desired location. Ideas from physical space could have been incorporated, with displaced agents being 'pushed away' from the node the displacing agent previously occupied.
- Agents can directly specify any vertex to move to (including non-adjacent ones), displacing any other agent there. Conflicts would have been resolved as above.

Ultimately, after lengthy preliminary experimentation, I decided to use 'agents arranging to swap positions with adjacent agents' in my model. There are many reasons which make this the only satisfactory candidate form of mobility for my model:

- It is sufficiently complex to model the two quintessential aspects of questions about voluntary change- firstly, 'should I act to change my present situation?', and secondly 'if I do so, then what manner of change should I affect?'. These questions are simply not adequately captured in lesscontrolled forms of movement.
- It does not require especially sophisticated and/or novel player types to make use of it, compared to the stronger forms of mobility which requires all players to understand and analyse not just their own situation, but their probable situation in other potential scenarios in order to be effective. Additionally, types would likely require at least some awareness of the graph's structure to make informed decisions.
- It enables the structure of players' positions in the graph to be meaningfully developed gradually over time despite the movement of agents inside it. If agents can relocate to any position on a round-by-round ba-
sis, there becomes little relationship between the distributions of players from one round to the next, and it becomes much more difficult to see how to model or represent different scenarios which we are interested in.
- It can always be computed locally. Each agent can ensure they move according to the rules of the game, without central oversight, and in a way which prevents exploitation. Agents can also easily enforce this restriction on each other regardless of their myopia.
- It reduces what could otherwise be a very nuanced decision-making process, depending heavily on the configuration of the rest of the system, down to a binary decision (in the first instance) plus selecting one of a limited number of players or positions (in the second instance). This retains the essence of the decision-making process and allows a huge variety of types to be expressed, while ensuring the problems which must be solved within the model remain tractable. This is an extremely attractive quality, as I aim for my system to successfully model a very large number of possible scenarios without ever becoming overwhelmingly complex.
- By using a relatively simple and intuitive mobility mechanism, I lower the chance of introducing idiosyncrasies into my system which will need to be specifically accounted for in future work. As entities inhabiting physical space, it seems both natural and fundamental that, in order to relocate to somewhere further away, we must first pass through the intervening space. This contrasts sharply with some of the fixed parameters found in other models (such as agents being unable to be uniquely identified in population protocols.
- As both agents must consent to any swap, it allows for an additional layer of decision-making which can be used to represent more complex negotiation. This means that, in larger or more complex scenarios, any
and all available methods of bargaining (such as offering resources or making (potentially enforceable) promises about future actions) could come into play very naturally as modifiers for influencing this simple decision-making.

In order for the swapping to occur, each agent produces an ordered list of its preferred locations, including its current location. The problem of using these lists to actually move the agents in this manner is effectively a restricted stable roommates problem [42], which is discussed in more depth in the next chapter.

### 4.2 Configurable Features

Conversely, as a generic model which can be used to simulate a large range of situations, there are a correspondingly large number of parameters and settings which can be configured to change the way in which the simulations will be initialised, executed, and iterated. These features enable a wide variety of scenarios to be modelled accurately:

1. Graph Structure The shape of the graph on which individual games are played can have a dramatic impact on the simulation and its eventual outcome. A variety of different graphs were tested- most notably paths, cycles, stars, regular grids, loosely connected cliques, and complete graphs. Generally, the important properties are the average degree of each vertex- that is, whether or not the graph is 'well-connected'- and the diameter of the graph. Higher connectivity makes it more difficult for cooperative types to form tight-knit groups which exclude non-cooperative types. Higher diameter increases the length of time before most players have played against each other at least a few times, and so increases the window which aggressive strategies have to exploit cooperative ones. Any size and structure of graph can be used.
2. Timing System. Time can either be 'synchronous' or 'asynchronous'. With synchronous time, every game is played across each edge in the graph simultaneously. Players cannot use information gained from one of their games in each timestep in any others, or try to move until all the games from that timestep are complete. Conversely, in asynchronous time, in each timestep an edge of the graph is randomly selected and its corresponding game played. After this, the agents involved update their memories and have the opportunity to try and move. Note that some papers abuse the notation somewhat, referring to asynchronous time as 'continuous' when both timing systems are actually discrete. Synchronous time is also referred to elsewhere as 'discrete' or 'parallel' time, while asynchronous time is also referred to as 'continuous' or 'sequential' time.

Asynchronous systems may behave somewhat more 'realistically' in certain scenarios- such as those concerned with more organic and/or fluid situations. However, this comes with the (usually undesirable) possibility of some agents playing many games in the same 'time' it takes another agent to play just one [39]. The addition of fairness conditions- such as those used for population protocols- can successfully mitigate this.

More importantly for my model, synchronous time decreases the effective control of agents over their position for any given mode of movement by increasing the chance that their information an neighbourhood they're travelling towhich they used to inform their original decision- will be out of date and more-or-less irrelevant by the time they actually arrive. As all agents move simultaneously in synchronous time, the vertex they select to attempt to reach (by whatever method) may have had its neighbourhood dramatically changed by the time they get there. As such, I hypothesise that the use of synchronous time in a simulation will behave analogously to increasing the diffusion (as outlined below), or affecting other changes which reduce agents' effective control
over their movements.
3. Games and Payoffs. The games played across the graph can be any theoretical or existent game. Though each edge of the graph could represent a different game, my model will use the same game across the whole graph to produce more consistent results. Although the Prisoners' Dilemma was used for the majority of experimentation as the basic results for it are well understood, any single-stage, two-player game, including novel games, can be modelled asis using this system. A single-stage game is one where the entire game occurs (or, alternatively, can be modelled as occurring) in a single moment.

For example, the 'Ultimatum Game' and some simple variants of 'Rock, Paper, Scissors' were used during research, as shown in Chapter 2. In addition to dramatically changing the nature of the game, more subtle adjustments can be made to the payoffs for specific outcomes (for example, to make the 'sucker's payoff' for betrayal more or less painful) to investigate their effects on the overall system. Even extremely complicated games with multiple stages can be approximated for the purposes of modelling by considering which types are naturally strong against which others, as explained more in Chapter 5 under 'meta-games'.
4. Player Types. A 'player type' (or 'type' for brevity) designates the probability that a given strategy will be chosen in each set of circumstances. It is more general than a strategy on a payoff matrix, as a player's type also incorporates decisions related to the game outside of the game itself. Notably in my model, this includes when and how to move on the graph itself. The basic model will assume non-mutable types; that is, players which do not change their overall strategic approach midway through the simulation.

This is analogous to human players selecting a 'meta-strategy' to follow for the entirety of the graphical game before it begins, or designing a finite-state
or Turing machine to play the game on their behalf, and should help keep the effect of the graph structure's restriction on interaction clearly visible. Such a machine or algorithm has access to all the same information (as input) as that player would have at each decision-making stage. Each game has an arbitrarily large number of effective player types- any possible construct of these forms can be used. One of the central aims of this model is to determine the effectiveness of different types for their respective games in different populations of players, graph structures, and simulation environments.
5. Noise. This is a probability that an agent's action will be incorrectly reported to their opponent (both directly and in their memory of that game), although payoffs are still silently allocated based on their actual actions. Again, this stands to advantage hawk strategies, as some of their exploitation will be reported as cooperation and potentially engender a more open response from their opponent. Conversely, a dove whose cooperation is reported as defection may face undeserved retribution from their opponent. Typically a low level of noise (around $5 \%$ ) will be used, as this opens up more varied behaviour by emphasising the strengths and weaknesses of several of my core types.
6. Initial Distribution. This is the initial position of individual players, and thus the distribution of types across the graph. I used three general distribution types: 'clustered', in which agents were preferentially placed adjacent to other agents which shared a type, 'spaced', in which agents were preferentially placed adjacent to other agents with a different type, and a wholly random distribution. In simulations involving a large number of different types, I placed each type into a broad category- cooperative, aggressive, or neutral- and preferred agents be placed next to others of the same category as a secondary criterion.
7. Population. Different numbers and proportions of each player type were used in different simulations, which led to different densities of strategy within the
graph and population. Any number of players, each with any type, can be used, ranging from a population composed entirely of a single type through to each agent having their own unique type.
8. Duration. The model can be iterated for any number of rounds. It's worth noting that one round of synchronous time will consist of every game being played, whereas only a single game is played during a round of asynchronous time. To correct for this, I'll treat one round of synchronous play as being equivalent to a number of rounds of asynchronous play equal to the number of edges in the graph when comparing the two directly.

The use of fixed-duration simulations can cause some issues, such as inviting classical backwards-induction to conceptually reduce the game to a noniterated format, or, more practically, increasing the probability that agents will temporally discount their payoffs, and so change behaviour and become more aggressive as the end of the simulation approaches. Though this was occasionally observed with my human experiments, my simulated agents simply weren't complex enough to attempt this behaviour, and a simple correction is to not inform the players how many rounds the simulation will run for.

Note that, unless I was specifically investigating a property related to long duration, I did not run each simulation for an arbitrarily high number of rounds until it converged into a more-or-less stable distribution. Fundamentally, not all simulations converge, such as those with high diffusion or a large proportion of especially mobile types (as a trivial example, consider any simulation with $100 \%$ diffusion or composed entirely of 'Random’ agents).

Each simulation goes through a variety of qualitatively different behaviours as time passes, and agents learn more about their opponents and expend effort to situate themselves relative to 'good' opponents. It is the progression of the different agents in these different situations which is of most interest to me,
as this informs the understanding of both the model and the scenarios it is modelling, rather than the form of the eventual stable distribution, which is more related to computational questions.
9. Diffusion 8 Anti-Diffusion. Similar to noise, diffusion is a probability for an agent to involuntarily move at random, instead of according to its preferences, representing general chaos and uncertainty in a scenario. Diffusion could also be modelled as a temporally increasing payoff offered to agents who move away from their current position, incentivising movement in otherwise disadvantageous situations. As I'll discuss in Chapter 6, diffusion typically disproportionately advantages hawk strategies, as doves tend to fare better when they can stay adjacent to other doves for long periods of time.

As the converse of diffusion, the mobility of agents may be impaired. This can be modelled by having some percentage of movement attempts fail, much like diffusion causes some percentage of attempts to stay stationary to fail. After my experiments with humans, I added a few more specific implementations of this feature to model risk-aversiveness in the population, namely the ability to begin with a high level of anti-diffusion in the population which decreases over time, and another which makes specific agents less willing to move depending on their rough expectation about the amount of unknowns in their would-be new environment, determined by the number of edges at the new vertex which aren't shared with the agent's current vertex.

Both forms of diffusion can be considered as a form of 'meta-noise', creating uncertainty in opponent preference just as noise creates uncertainty in strategic preference. In practice, I kept both diffusion and anti-diffusion coupled by using a third parameter which completely randomised a given agent's movement preferences with some probability. This could result in an agent which wished to stay still moving, or one which wished to move staying still.

### 4.3 General Impact of Mobility

Before I can begin to outline how I added my idea of 'mobility' to the graphical gaming model, we should first prove that it could have any impact on graphical games to which it is added. In order for this to be true, there must be enough variety in the types of players in the whole game that a player could realistically alter their prospects by moving 'well' or 'poorly', and by doing so come into contact with different types. More generally, viable mixed strategies should exist for the games to be played which are, in some way, strong against some other strategies, and weak against others.

The simplest demonstration of this would be three strategies existing in a cycle, such that ' A ' is strong against ' B ', ' B ' is strong against ' C ', and ' C ' is strong against ' $A$ '. A demonstration that such a scenario can and does exist in the real-world is trivial in the game 'Rock, Paper, Scissors'. At least three such strategies exist for the iterated Prisoners' Dilemma, which forms the basis for almost all my research.

First, we must define what we mean when we say one strategy can successfully 'invade' another. This means that the second strategy is outperformed by the first strategy, when a smaller number of players using the first strategy is introduced into a homogeneous population using the second strategy, for a given comprehensive method of determining individual competition. This criterion of invasion is the default which I will use for determining whether one strategy is strong against another, given the huge number of possible configurations and sub-scenarios which can arise from using this model, unless otherwise specified.

A population of EverHawks can be successfully invaded by a group of Tit-For-Tat players, as they can successfully cooperate with each other, but remain hostile to the hawks [4]. Then, a population of Tit-For-Tat can be invaded by Pavlov, so long as there exists a small amount of noise in communication. Tit-For-Tat is unable to recover from miscommunication, resulting in runs of alternating backstabbing
against itself, whereas Pavlov rapidly recovers from such mistakes, and all agents otherwise freely cooperate [43] [44]. Finally, Pavlov can be invaded by EverHawks, as its alternating cooperations and defections in a hopeful attempt to find a better equilibrium are repeatedly exploited by the simpler strategy [45].

So, The Prisoners' Dilemma- a game used the world over for its simplicity, in which there are only two discrete options available to each player at each step- contains this cycle of three strategies. This demonstrates that any existent (and much more complex) game is almost certain to contain this same situation, or something comparable to it. If a game contains any scenario which reduces to or even resembles the iterated Prisoners' Dilemma at any point, this example will also apply to that game. As such, we can conclude that any addition which gives players an option to 'move towards' and increase their chance of interacting with players they are strong against, while 'moving away from' and avoiding those they are vulnerable to, will have a noteworthy impact on the execution of the game.

A great many systems display emergent behaviour which would appear to be much more complex than their simple rules could generate alone, particularly when interaction graphs are involved. Cellular automata are excellent examples of this, as they can generate a bewildering array of complex shapes and high-level interactions from just a few very simple rules of binary interaction on a regular grid. The classic example of such a simulation is 'Conway's Game of Life' [46], which despite being discovered over 40 years ago is still not fully understood [38].

Another illustration is the phenomenon of real-world traffic congestion, which is fundamentally a network flow optimisation problem, but has so many interacting component-problems that it has a vast array of relatively complex mathematics at its core [47]. These emergent phenomena provide additional evidence that even the simple scenarios will contain some pattern which reduces to this basic cycle of strategies, such that the addition of mobility is certain to have a significant impact
on the resolution of an otherwise standard graphical game.
Of course, mobility will have a reduced impact in any scenario where each agent can alter its own type or other inherent preferences- for example, an intelligent agent which could determine its opponent is resolutely non-cooperative could change its overall approach to incorporate this new information. However, such an agent would still, in all likelihood, expend effort to find a more cooperative opponent if possible, as doing so would likely improve its overall situation.

Although many agents, particularly humans, do alter their types during play in this manner, they also tend to have a general type preference in how they approach the game and conduct themselves within it, while remaining within the framework of maximising utility. One need only glance at any complex, real-world game to see that its experts, while each playing to win, each have their own unique strengths and weaknesses which lead to preferred strategies and types of opposition.

### 4.4 Types of Mobility

Various types of mobility were designed and tested by simulation in the preliminary investigation stage before the final implementation of mobility in my model was settled upon.

The first type of mobility considered was the most freeform, allowing any agent to attempt to move to any position on the entire graph. In the likely event of conflict for a position, agents would have been moved to vertices with minimal distance from their desired position, or brokered swaps like the current model. However, the situations this system created were very abstract and not particularly useful from a game theoretic or behavioural modelling point of view, as each agent was essentially asked to calculate the optimal position for its type in the current graph (regardless of its unique experiences or immediate environment). To prevent all agents coming
to the same conclusion, knowledge of the system needed to be somehow restricted to each individual, yet, simultaneously, they needed to understand and act on all the information available in the remainder of the simulation in order to make 'intelligent' decisions. This form of mobility ultimately proved unworkable and was discarded.

Another possibility was to disassociate the paths along which games are played from those along which movement occurs. An early prototype involved agents negotiating to form pairs which only played with each other. This could be considered as a special case of the current model, effectively a 'forest' or group of 'disconnected pairs'. In practice, each node would be connected to every other by value 0 links, (ensuring that games are played with all other players, but do not affect each agent's payoffs) except for the partner, which would be connected as normal.

Definition 15 Let there be a pair $(G, \mathcal{M}) . G$ is an undirected, weighted graph over the vertices $1, \ldots, n$, where $n / 2=0 . m$ is a symmetric game matrix. For simplicity and without loss of generality, we assume all $M_{i j} \in \mathcal{M}$ are identical to $m$. For each agent $j$ in $N(i)$ (which does not include $i$ ), and any joint action $\vec{a}$ between $i$ and $j$, the payoff for player $i$ is $m_{i j}\left(\vec{a}^{i j}\right)$. Each vertex is connected to each other vertex by a 0-weight edge, except for $i$ where $i / 2=0$, which is connected to $i+1$ by a unit-weight edge.

Again, similar problems arose- as time went on, each agent gained awareness of the optimal partner for their type, abstracting the question at the heart of the simulation far above how a particular agent will act in a particular situation. The lack of structural detail in both these systems also made it difficult to model specific real-world scenarios with fidelity.

Finally, a prototype was created in which agents could negotiate a movement to any adjacent node, so long as its current occupier guaranteed it would be vacated during the movement step, and that they had not made the same offer to another agent. This enabled long cycles of movements to occur, even if some of the agents
involved in the movement were not adjacent to each other. For instance, in a ring graph, each player could move to the adjacent 'clockwise' or 'anti-clockwise' position simultaneously, if all players agreed to do so.

However, this system had a number of downsides. Global oversight and moderation would be needed to extend the algorithm used to real-world situations- and such oversight is often non-existent and/or prohibitively costly to achieve. Additionally, due to the limitations of the intelligence of the agents, the results observed from this method were almost identical to those observed using the final method of direct, adjacent swaps only. By combining these investigations with the higher-level considerations discussed earlier, I settled on the system of mobility I ultimately implemented for the remainder of this research, which I describe fully in Chapter 5.

### 4.5 Player Types

Based on the well-known existing strategies and approaches commonly used in the Prisoners' Dilemma which I discussed earlier in Section 2.5, I devised the following player types which were used in the bulk of my experimentation:

- EverDove - Always plays cooperatively. Prefers to stay where it is, orders remaining nodes randomly.
- EverHawk - Always plays non-cooperatively. Prefers to stay where it is, orders remaining nodes randomly.
- DoveShift - Always plays cooperatively. Prefers to move to any other node at random, placing itself last.
- HawkShift - Always plays non-cooperatively. Prefers to move to any other node at random, placing itself last.
- DoveTilHawk - Always plays cooperatively. Orders other nodes randomly, then
inserts its current node into the rankings depending on how many opponents defected against it last round. For example, if 1 out of 3 opponents defected, it would rank its current position approximately one third of the way up from the bottom of the ranking. It will always prefer to move if all opponents defect.
- HawkTilHawk - As with DoveTilHawk, but always plays non-cooperatively.
- Tit-For-Tat - Initially plays against each new opponent cooperatively. Thereafter, repeats the last action they made against it back to them in each round, using memory of older games for the first play of each new game. Prefers to swap with players it recently cooperated with. This allows it to respond to its opponents and encourage cooperation by competing and gaining retribution against players who compete against it.
- Tit-For-Two-Tats - Like Tit-For-Tat, except that it doesn't begin to play noncooperatively against its opponents until they defect twice in a row.
- Pavlov - Initially plays against each new opponent cooperatively. Thereafter, repeats the action it played previously, unless the outcome of the previously chosen action was undesirable (that is, mutual non-cooperation, or the opponent betraying it while it tried to cooperate), in which case it instead selects the other action. More formally, it switches actions if it failed to get at least $50 \%$ of the theoretically available utility from that action on the previous round. Pavlov has the interesting property that any population consisting solely of Pavlov players, regardless of size, distribution, connectivity, and starting moves, stabilises with permanent universal cooperation within two rounds in the absence of noise. Moves using the same approach as DoveTilHawk.
- Random - Randomly chooses between cooperating and defecting each round for each game. Moves randomly.
- Hunter - Always defects. Keeps a list of agents who it has historically scored
well against. If it is adjacent to such an agent, it does not move- otherwise it moves, ideally towards the last known position of an agent it scored well against.

Additional, more basic player types were used in early experiments, such as 'Grim', which never moves and plays cooperatively, until an opponent betrays it, at which point it will never again cooperate with that player. Many of these were incorporated into more advanced player types- for example, DoveTilHawk, which cooperates and generally stays close to a given player until they start to defect, can be seen as an amalgamation of Grim and Tit-For-Tat.

It is possible to use this concept to model behaviours observed in human players which have traditionally been considered to be 'irrational', such as cooperation in the non-iterated Prisoners' Dilemma. This is achieved by by altering the relationship between the payoffs awarded in games and the utility ultimately received by the agent. For example, an altruistic type might receive a fraction of the utility awarded to its opponents, whereas a spiteful type could receive a flat bonus when an opponent is successfully exploited. For mobility, a particularly mobile type might receive utility whenever it completes a swap, while a notably static type could be rewarded for remaining motionless.

Though I investigated this possibility in my preliminary experimentation, it is not clear that this feature in-and-of-itself grants any additional power or reach to the model as a whole. More importantly, through my research I have aimed to understood the behaviour of human players and show how, in the correct environment, their beneficial quirks can be explained as beneficial, rather than simply hamstring artificial players with an uninformed interpretation of these quirks themselves.

### 4.6 Example Configurations

For illustrative purposes, I've included a few representative configurations and settings of the model's features below. The reasons behind these configurations illustrating these particular properties are detailed throughout this thesis. Any reader interested in investigating this model themselves could perhaps begin with one such configuration and, much like I did, alter the settings one-by-one until a more comprehensive intuition of the behaviours of this complex model is achieved:

|  | Effects of Move- <br> ment Enhanced | Effects of Move- <br> ment Diminished | Approximation of <br> Cellular Automata | Approximation of <br> Human Behaviour |
| :--- | :--- | :--- | :--- | :--- |
| Graph <br> Structure | Ring | Randomly <br> constructed 6-regular <br> graph | 4-regular grid | Ring, with occa- <br> sional connections <br> to otherwise distant <br> nodes. |
| Timing <br> System | Asynchronous | Synchronous | Either | Asynchronous |
|  <br> Payoffs | Classic Prisoner's <br> Dilemma. | Any game with el- <br> ements of random <br> chance. | A biologically inspired <br> academic game, such as <br> Chicken | Any real-world game |
| Player <br> Types | One cooperative, <br> one competitive, <br> and one neutral. | Types which do not <br> play or move intel- <br> ligently. | A single, hybrid type, eg <br> Tit-For-Tat which <br> occasionally competes | Types which play <br> and move intelli- <br> gently. |
| Noise | $0 \%$ | $50 \%$ | $0 \%$ | $5 \%$ |
| Diffusion | $0 \%$ | $50 \%$ | $0 \%$ | $5 \%$ |
| Anti- <br> diffusion | $0 \%$ | $50 \%$ | $0 \%$ | $20 \%$ |
| Initial <br> Distribu- <br> tion | Random | Any | Any | 100 |
| Duration <br> (rounds) | 100 | 50 | Any size, 100\% the <br> chosen type. | $\sim 50, \sim 70 \%$ coop- <br> erative types. |
| Population | $\sim 20$, |  |  |  |
| $\sim 33 \%$ each type. | Any. |  |  |  |

Figure 4.1: Sample Configurations

### 4.7 Reduction

The fundamental mathematics underpinning a graphical game with mobility can be modelled using a number of existing methods. Most fundamentally, they could comprise an $n$-dimensional nested payoff matrix(where ' $n$ ' is the number of players),
in which each combination of strategies not only allocates payoffs, but also leads to a new payoff matrix with updated values to reflect the repositioning undertaken by the agents. However this repositioning does, in and of itself, introduce an additional multiplier to the branching factor of the game, even before the players' chosen strategies are considered.

Because of this huge branching factor, such a formation rapidly approaches such a high level of computational complexity as to be intractable. Speaking practically, this system is so overwhelmingly unwieldy as to be near-impossible to work with. The depth of the nested payoff matrix is equal to the number of rounds in the game, as after each game we need to transition to a new matrix representing the current position of each player, their score, and their interaction history with each other player. The branching factor for this has a trivial lower bound at the number of games being played multiplied by the number of unique outcomes for each game (which is itself obtained by multiplying together the number of actions available to each player).

For simpler agents which cannot meaningfully interpret their entire interaction history to choose a course of action, a better comparison may be a finite-stateautomaton, with each state encoding information about the position of each agent and the 'disposition' of each agent towards each other agent- such as whether or not they're cooperating, competing, trying to swap with or away from, or any number of other behaviours. However, this would still contain a number of states equal to the factorial of the number of players, multiplied by the number of dispositions for each agent, multiplied by each other agent they could posses each disposition towards. The transition function also becomes rather inelegant when compared to the more natural concept of agents moving between nodes.

Alternatively, the graphical games could be represented by, say, a rather complex state transition diagram, with arrangements of the players on the vertices of the
graph corresponding to states, the transitions to states dependent on the strategies chosen by players, and the outputs relating to payoffs [48]. However, this would once again be so large and clumsy as to be near-impossible to work with effectively. There are many other existing systems which can theoretically model graphical games with the addition of mobility, but, as outlined in Chapter 3, none can do so succinctly in a way which aids comprehension. So, although the additions and alterations I have devised do not, in and of themselves, increase the computational power of the model, it does allow many more such situations to be understood intuitively and the effects of elements within them on the overall outcome quantified in such a way as to be of practical relevance. Additionally, this reducibility to graphical games without mobility means that results for more general games will also apply to graphical games with mobility.

## Chapter 5

## Methodology

In this chapter I describe the methods by which results were obtained, both for artificial and real-world agents. I will also explain precisely the system by which these agents can utilise their new-found mobility and move around the graphical structure in a fair and consistent manner.

### 5.1 Terminology

As some terms are used in different ways in different areas of the literature, and could otherwise have potentially ambiguous meanings when used to refer to various aspects of the system I've designed, it will be helpful to give some definitions specific to this thesis before we proceed further:

A 'player type' or just 'type' is a meta-strategy for deciding which option to play in each game, and how to rank nodes in the movement phase. This strategy may refer to the agent's memory and past experience, but cannot be changed once the game is underway- though more complex strategies which switch between discrete modes, giving the appearance of a shift in strategy, would be possible. Though this idea appears very occasionally in the literature [9], I have developed and refined it
here as an important component of my model.
A 'player' or 'agent' is a specific instantiation of a type, situated within a graphical game. The two terms will be used interchangeably here.
'Nature' is a term occasionally used to refer to events occurring outside of the agency or control of the players. For instance, which card is drawn from a shuffled deck, or the initial position of a player when they is determined randomly.

A 'round' is the period beginning with game/s being played, and ending after agents have updated their position on the graph. Note that while only a single game will be played each round in asynchronous setups, a game will be played along each edge of the graph when synchronous time is used. As such, when comparing the two directly, I will treat one round of synchronous play as being equivalent to a number of rounds of asynchronous play equal to the number of edges in the graph, unless explicitly stated otherwise.
'Playing dove', 'cooperating', and 'working together' are all synonyms for playing the cooperative 'C' option in the Prisoners' Dilemma, or, more generally, for choosing an action which attempts to cooperate with your opponent/s. Similarly, 'playing hawk', 'betraying', 'backstabbing', 'defecting', and 'exploiting' are all synonyms for playing the non-cooperative 'D' option in the Prisoners' Dilemma or, more generally, for choosing an action which attempts to take advantage of your opponent/s for your own gain.
'Payoffs', 'rewards', 'penalties' and 'utilities' are all treated as equivalent unless otherwise specified.

To avoid confusion between the applications of pre-existing strategy and the new addition of mobility, I will mostly avoid using 'move' to refer to the selection of a specific 'action' or 'play' at a given point in a game, in favour of these latter terms. Likewise, 'strategy' refers to the algorithm used by each agent or type to select an
action and move in given circumstances, not the action itself.
Even setting aside the question of how types are selected, there are still two levels of games taking place in the system I have designed- the entire graphical game taken as a whole including the application of mobility, and the selection of an option in each individual interaction. Generally I will use 'graphical game' to refer to the former and 'individual game' or simply 'game' to refer to the latter.

### 5.2 The Meta-Game

The 'meta-game' is a broad term used to refer to the notion of selecting a type before an individual game begins, given some expectation of the type one's opponent $/ \mathrm{s}$ will adopt. As the name indicates, it is extremely similar to the classic notion of choosing an action in a game based on the expectation of the action/s chosen by one's opponent/s, but operating 'one level above' it, hence the 'meta' prefix. Though an important and well-understood concept among players of some real-world games [49], the concept has not been well-explored academically.

As it is relevant to both my research and understanding game theory as a whole, I will attempt to codify it here as part of my contribution. As the notion of metagames should be applicable to all multi-player games with type selection which have multiple steps of interaction (thus creating an iterated component), it should also be applicable to a wide variety of real-world games. As such, when codifying this notion, it makes sense to speak as generally and generically as possible about what constitutes a 'game'.

All real-world games (that is, those more complex than the abstractions typically studied in academic game theory) from ancient boardgames to the most complex modern video games consist of multiple interacting elements, for example:

- Pieces, cards, units, and so on, which hold and represent different pieces of
information.
- A board, map, or playing area to indicate the position and relation of those elements.
- Hidden information, only viewable by a subset of players (possibly none).
- Rules, which exist to set limitations on what choices can be made by each player as a combination of these elements (that is, players cannot freely change the game state).
- The individual skills and styles of the players, such as being able to choose a 'good' action quickly, or thoroughly iterate through many possibilities to determine a near-optimal action.

Not all games will have all of these, but even at this point, without making any reference to any specific game, we can come up with some generic advice to players in games.

Where possible, players should acquire hidden information, so that they are not surprised and can select actions and strategies in accordance with this information. Players should attempt to reduce limitations on their choices (such as by obtaining resources, conserving existing resources, or moving pieces to positions from which they have more possible moves) so that they have a wider range of possible choices in future. Finally, each player should select a play-style (and, as my research has shown, even a game) which matches their natural strengths- a quick player should play differently (and play different games from a methodical player), even if both wish to accomplish the same goals.

Conversely, preventing your opponent/s from accessing hidden information (or confusing their analysis by bluffing), limiting their options, and trying to force them into a position where they are unable to deploy their strengths, should all generally disadvantage them (and thus advantage you). Already, we can see that a great
number of features of games can be abstracted away, while still leaving the core of competitive interaction intact.

Using chess as a more specific example, it is almost always good to have:

- More pieces than your opponent (a resource advantage)
- More powerful pieces than your opponent (which can attack many squares at once, giving you more options in future turns)
- Your pieces in more advantageous positions on the board (such as in the centre or towards your opponent's side, giving you more options and limiting those of your opponent)
- Your pieces supporting each other (such that they are difficult for your opponent to attack, ensuring your resources are maintained)

Most crucially, different players have different styles which can emphasise one or more of these elements over another in ways which may initially seem counter-intuitive, but which play to the innate strengths of their user. The number of possible ways to play any game can be enormously large, and some strategies may well be 'stronger' (by a given metric) than others. However, which strategy is the strongest and gains the most utility for a given player employing it is dependent upon that player's innate skills and preferences- in other words, their type.

For example, in Chess, the 'fried liver attack' is a potent opening sequence in which a player attacks so aggressively that they lose a large number of powerful pieces, all in an effort to simply move the opponent's relevant pieces out of position for a rapid checkmate [50]. In other words, for a player with a particularly aggressive and/or exploratory type who chooses to play this particular strategy, one or two elements (in this instance, positioning and turn advantage, or 'tempo', over the opponent) are emphasised over all other
considerations (such as defending one's pieces and controlling key areas of the board). However, a different player may reverse these preferences entirely, opting for a much slower, more defensively-orientated game.

Even the very first decision of the game- whether the opening move on a board, which resource to develop, or the selection of which units or resources are available to use, is akin to type selection. This is because the decision immediately starts that player down a certain strategic path. From that point on it will, generally speaking, be easier to continue to expend effort to progress in that direction rather than radically changing approach. Doing so will likely, at best, involve starting from scratch on the new approach and giving up much of the progress already made with the old approach, but may require spending time to undo, or even outright sacrifice, this earlier progress.

In some games you choose which resources you have the ability or potential to access for the whole game before it begins. In such games, significantly changing strategy may not be possible at all. Such real-world games display strong type selection, and as such tend to have nuanced and varied meta-games based on intuitions about your opponent's type.

Other examples of meta-game choices would be choosing certain players for one's team based on the known strong players on the opposing team in a game such as football, or selecting a character or team which an opponent has previously shown themselves to be weak against in a video game. Generally, I avoid considering explicit questions of which type should be selected in specific situations, as they can be modelled by choices made within the system I have designed by using mobility.

In summary, the following types of decision can all be modelled using the framework I have designed:

- The individual actions and reactions taken by players (and nature) in any
game. (This is the classic understanding of a 'game').
- The 'style' and 'preference' of each player. This can be expressed simply using a type, or viewed as a modification to players' utility functions such that they gain more utility (or, perhaps more accurately, expend less effort) from selecting and executing a preferred action correctly than a non-preferred action correctly.
- The decision of which opponent/s to play against (which, as with any of the previous choices, may be restricted given the greater context of events). This can be expressed simply by utilising the available movement options within a standard graphical game with mobility.
- Which 'game' to play. The payoff matrices used could be nested or multiplied-through, with an initial or primary selection determining which matrix is transitioned to. These subsequent matrices can themselves be designed to simulate any game as normal.
- Whether or not to play a 'game' at all. As above, but with the addition of a null matrix indicating zero interaction occurring.

It can be difficult to see where the line between type selection and game selection falls, given that a player's natural type will inform which games they want to play! This aspect could be considered a 'meta-meta-game', or more simply, a recognition of the many levels on which decisions are made and potential for competitive interaction exists. Critically, the ability of my system to model these considerations at any level is a key strength, which is outside the capabilities of any previous model.

### 5.3 Technical Detail

To perform the bulk of the simulations, a program was written in Java. Despite some preliminary concern that Java would be too slow for the extensive computation required for this research, experimentation showed that a comprehensive simulation would complete in a timely manner for even the largest, longest, and/or most unwieldy setups. For instance, a 200 round game on a randomly constructed dense graph with 100 players (which is much larger and more complex than the average simulation to be performed) completed in under 30 seconds using synchronous time and 15 seconds using asynchronous time- this latter increase in speed being due to movement negotiations completing much faster, as far fewer players are involved at each step, even though the number of actual rounds is much higher.

Randomisation was required for practical implementation details such as some initial distributions of players over the graph, the actions taken by certain types in certain situations, selecting a pair of linked agents in each round of asynchronous time, and to allow the mobility algorithm to perform fairly in situations with multiple allocations which are both feasible and correct. This was ensured by the use of Java's native 'Random' class.

Object-orientation allows the program's structure to mimic the natural structure of the problem, making use of Graph, Game, Player, and Node objects. This made the integrity of the program easy to verify. The program receives initial input on its various parameters from formatted text files, simulates every element of the graphical game internally (such as each game, each player, and their choices), and then outputs the results to another text file.

Though features such as running time and computational complexity, are not the focus of my research, it is straightforward to determine a loose upper-bound of $O\left(n^{2}\right)$ for a single round, where $n$ is the number of players. This occurs in
the event that each agent will have to play against each other agent in each time step (when the complete graph is used), which proved low enough to be satisfactory for my research. Though I strongly recommended a computerised solution for anything beyond the most basic simulation, this model could be executed by hand if desired.

### 5.4 Experimental Setup

One of the central difficulties of this project was viewing the sheer volume of inter-linked data in a way which made human comprehension possible. Even a relatively simple graphical game with few players and few rounds contains a huge amount of information- the position of each agent, the actions they decide to play in each game against each adjacent agent, their movement preferences (and whether or not they were randomised), and where they actually move to, all multiplied by the number of players and again by the number of rounds. Derived values, such as the average score across various player types or for players occupying certain positions on the graph, are crucial for more detailed analysis and must also be calculated and included.

As such, I investigated and ultimately discarded a range of approaches, particularly those which display the information in real-time as the game occurs, as being ineffective and/or too distracting from the focus of the research to implement. I eventually settled on a carefully structured and detailed plain text output for individual games (as shown below), and created a spreadsheet to compile results from multiple games into a table, from which general patterns could be discerned both for individual simulations and this structure as a whole.

After studying this spreadsheet, I could 'zoom in' (see below) and examine the
comprehensive log of whichever simulations were of particular interest in full to gain a more complete view of what exactly was causing any given behaviour, as well as using this information to create graphs and tables from calculated or derived data.

Unfortunately, the completeness of the data included in the final output had to be reduced to maintain readability- information such as the outcome of every game played at every timestep is simply too large and dense to be meaningfully comprehended, let alone for patterns within it to be discerned. However, this information can be obtained during the execution of the program, or output from the program by use of a certain mode, to confirm its functional completeness. The information listed in my text output includes, but is not limited to, the main details of the initial parameterisation (at the top), the position of each agent at each round (in the middle), and the overall performance of each agent and its associated type (at the bottom). An examples of this is included below:

ROUNDS: 20
TIME: S
SHAKE\%: 10.0

| $1$ | LAYERS== <br> (HAWK N SWITCH) |
| :---: | :---: |
| 2 | (HAWK N SWITCH) |
| 3 | (HAWK N SWITCH) |
| 4 | (HAWK N SWITCH) |
| 5 | (DOVE-TIL-HAWK) |
| 6 | (DOVE-TIL-HAWK) |
| 7 | (DOVE-TIL-HAWK) |
| 8 | (DOVE-TIL-HAWK) |
| 9 | (RANDOM) |
| 10 | (RANDOM) |
| 11 | (RANDOM) |
| 12 | (RANDOM) |

==ADJACENCY MATRIX==
$\mathrm{X}, 1,0,0,1,0,0,0,0,0,0,1$, $1, x, 1,0,0,0,0,0,1,0,0,0$, $0,1, x, 1,0,0,0,0,0,0,0,0$, $0,0,1, x, 1,0,1,0,0,0,0,0$, $1,0,0,1, x, 1,0,0,0,0,0,0$, $0,0,0,0,1, x, 1,0,0,0,0,0$, $0,0,0,1,0,1, x, 1,0,1,0,0$, $0,0,0,0,0,0,1, x, 1,0,0,0$, $0,1,0,0,0,0,0,1, x, 1,0,0$, $0,0,0,0,0,0,1,0,1, x, 1,0$, $0,0,0,0,0,0,0,0,0,1, x, 1$,
==PLAYER POSITIONS==


| $==$ ABSOLUTE PLAYER | SCORES== |  |
| ---: | ---: | ---: |
| 1 (HAWK N SWITCH) | -126 |  |
| 4 (HAWK N SWITCH) | -156 |  |
| 2 (HAWK N SWITCH) | -162 |  |
| 12 (RANDOM) | -191 |  |
| 3 (HAWK N SWITCH) | -192 |  |
| 9 (RANDOM) | -211 |  |
| 6 (DOVE-TIL-HAWK) | -222 |  |
| 7 (DOVE-TIL-HAWKK) | -246 |  |
| 10 | (RANDOM) | -263 |
| 11 | (RANDOM) | -286 |
| 8 (DOVE-TIL-HAWK) | -288 |  |
| 5 (DOVE-TIL-HAWK) | -307 |  |

==ABSOLUTE TYPE SCORES==
HAWK N SWITCH: -159
RANDOM: - 238

DOVE-TIL-HAWK: -266

|  | ELATIVE PLAYER |  |
| :---: | :---: | :---: |
| 1 | (HAWK N SWITCH): | 95 |
| 4 | (HAWK N SWITCH): | 65 |
| 2 | (HAWK N SWITCH): | 59 |
| 12 | (RANDOM) : | 30 |
| 3 | (HAWK N SWITCH): | 29 |
| 9 | (RANDOM) : | 10 |
| 6 | (DOVE-TIL-HAWK) : | -1 |
| 7 | (DOVE-TIL-HAWK) : | -25 |
| 10 | (RANDOM) : | -42 |
| 11 | (RANDOM) : | -65 |
| 8 | (DOVE-TIL-HAWK): | -67 |
| 5 | (DOVE-TIL-HAWK) : | -86 |



Figure 5.1: Example output from my simulation program, placed into two columns for brevity.

The number of rounds, timing system used (either 'S' ynchronous or 'A'synchronous)
and probability of a given preference list being randomised are clearly shown at the top. This is followed by an adjacency matrix, which shows the structure
of the graph. A ' 1 ' indicates a link is present between two nodes, while a ' 0 ' indicates no link. As all links are bi-directional, the table produced is always symmetrical along the lines of ' X 's in the main diagonal, which show that nodes cannot be directly connected to themselves. Next, the identity of each player at each node after each round is shown, with the number in brackets at the end of the line showing the number of successful swaps which occurred in that round.

In this depiction, a player's number is the same as that of the node they occupied at the beginning of the simulation, but serves no additional purpose beyond readability and, importantly, cannot be used to verify an opponent's type. Finally, the scores at the end of the simulation are displayed, both individually and grouped by player type, and absolute (those payoffs actually obtained during the game) and relative scores (for ease of reading and comparison). Recall that, in the Prisoners' Dilemma, all payoffs are negative, and a less-negative score is desirable.

A series of simulations with no mobility at all were run to act as a control against which the effects of different forms of mobility could be measured (and to confirm that, as graphical games without mobility can be considered a special case of mobile graphical games, there were no surprising behaviours to report). The full range of graph structures, player types and initial distributions detailed earlier were used.

Even before these simulations began, it became clear that, with the types I'm using (which don't use the memory or experience of playing against one opponent to inform their play against each other) and mobility removed, there is no method by which information can propagate through the graph. As such, a series of isolated iterated games are created, each with length equal to the length of the simulation- much like a more traditional experimental setup.

In these simulations, the different player types performed as classically expectedindividual EverHawk players achieved the highest scores, but strategies such as Tit-For-Tat and Pavlov performed better on average. Without mobility, the only variance in the performance of both individual players and overall types arose from:

- Their initial distribution (which determined the subset of all the possible iterated games which was actually played- for instance, an EverHawk adjacent only to EverDoves will never receive anything other than a perfect iterated game.)
- The relative sizes of each population (affecting both the types of players involved in games, and the relation between individual and average scores for a type- if there is just a single EverHawk player, the 'average' score of this strategy can be very high!)
- The shape and structure of the graph itself. For instance, in a star or super-star graph, the type of the player at the central node has a large impact on which games are played, though this is an extreme case. In other graphs where the degree of each node is closer to the average (ie $k$-regular graphs), this effect is less pronounced.

As the system of graphical games with mobility is so complex, it took many months of preliminary experimentation to gain a comprehensive understanding of the many interacting elements, and thus a working model of what changes were likely to produce what patterns of effects. These preliminary experiments were performed for an almost exhaustive variety of parameterisations interacting in almost every combination, in an attempt to isolate which of the possible variables were having meaningful effects on the execution and outcome of each simulation, and which were having minimal or overwhelming effects. This process informed the decision-making which led to the mutability
(or immutability) of the possible variables as listed earlier.
A second wave of more formal simulations was designed to quantitatively measure the interactions between each possible pair of variables at a variety of levels and in more detail. The different combinations of parameters, and the subsequent result on the dominant various strategies, are demonstrated later as the focus on my results.

### 5.5 Mobility Algorithm

At first glance, the problem of pairing up entities in the desired manner based on their preferences appears to have already been solved. The 'Stable Roommates Problem' is a slightly different form of the more widely-known 'Stable Marriage Problem', and involves a scenario in which an amorphous group of people must be paired off such that no pairing is 'unstable', meaning that, for each pair, each person prefers their partner to the other viable options available [42].

There are, however, a few important differences to the problem as it appears in my model, compared to the stable roommates problem:

- A complete matching does not need to be found. This is actually impossible to find in certain graphs, with a trivial example being those with an odd number of players (as each pairing requires 2 players).
- Not all pairing options are valid among the set of all agents. The absence and presence of edges restricts possible pairings such that a complete matching, or even a majority matching, is typically impossible even before preferences are considered (such as in a star graph). Agents can swap positions only with another connected agent.
- Even if a specific pairing is possible and desired by both players, if one of them would rather remain in their current position than form the pairing, the pairing will not be formed. Some types may prefer to be unmatched in certain situations, even if a 'good' matching (one which is preferable to certain other options) is possible for them.
- The matching used does not need to be stable if no stable matching is possible. In such a case, a random matching which contains a minimal number of blocking pairs (hypothetical pairings which would be preferred over actual pairing/s) will be sought. This enables cycles of preferences (in which $A$ prefers $B, B$ prefers $C$, and $C$ prefers $A$ ) to be broken fairly

A solution to these difficulties must be found in order to devise a fair system for agents to express their preferences for moving around the graph as such changing the overall distribution of player types. This is, in and of itself, a non-trivial problem. To overcome it, I created a modification of Irving's algorithm [51], as described in the following section.

My algorithm has the important advantage that computation by this method is achieved locally by the agents, without central direction or oversight beyond simple timekeeping and the generation of randomness- which, in the real world, would both occur naturally. For example, randomness was used to decide which agents send their preferences first, whereas in the real world some agents would naturally send such information earlier or later than others. This feature allows my algorithm to be deployed in a much wider range of distributed environments than algorithms which require higher levels of control over, or guarantees of, behaviours from potentially uncooperative agents.

Each agent begins by ranking the position of their neighbours and their current position in an ordered list, indicating their preference for graph position by the end of the movement phase. Agents may prefer to keep their current position
for whatever reason, and cannot be forced out of position by other players unless affected by diffusion. Recall that diffusion is a chance, defined at the beginning of the simulation, that each given list will be randomly shuffled, causing agents to move haphazardly- this is calculated and applied to the selected agents' lists at this point.

Then, one at a time in a random order, each agent sends a message to the agent occupying its preferred position. If an agent receives such a message from the agent occupying the position it most wants to move to, the process is complete for these agents, as they have reached an agreement whereby they can swap positions and end up exactly where they want to be. In the more likely event that an agent receives an offer from one or more lower ranked choices, they will provisionally accept the best such offer they receive. Both agents will note that their best option thus far is to swap positions with each other. If all the proposals are ranked lower than the agent's own position, or no proposals are received, the agent notes that it's best off staying in its current position. Each agent which made a proposal and has it rejected will default to their 'best offer' of staying where they are. In each subsequent round the process repeats.

There is a simple, but convincing argument that my algorithm functions as required, similar to one often used for the Stable Marriage Problem, making use of contradiction. Assume that $A$ and $B$ are agents who would prefer to swap places to whatever current offer they hold, but, somehow, have not agreed to swap. $A$ must have proposed to $B$ at some point, because each agent will keep proposing until it reaches its own position in the rankings- so if it has not done so, $A$ ranks its own position higher than that of $B . A$ cannot be paired with a position lower than its own in its rankings, as it would instead have defaulted to its own position. So, $B$ received a proposal from $A$, and in that case would have accepted it in lieu of the currently held proposal.

### 5.5.1 Pseudocode

In simplified pseudocode, this algorithm can be written as follows:

```
for each agent A do
    make a new list 'prefs';
    for each agent A', at distance 1 or less from A do
        add A' to prefs, ranked according to how much A wants to swap with A' ;
        end
end
if random < threshold then
    shuffle(prefs);
end
return prefs;
```

Figure 5.2: Pseudocode algorithm for the creation of ranked preference lists

```
integer attempt = 0;
agent asknext = prefs.attempt;
while partner = null do
    response = trypair(asknext);
    if response = 'hardyes' then
        partner = asknext;
        sorted = true;
    end
    if response = ' no' then
        donothing;
    end
    if response = 'softyes' then
        if prefs.attempt < getPos(offerfrom, prefs) then
            offerfrom = prefs.listpos;
        end
    end
    attempt++;
    if attempt >= prefs.size then
        partner = offerfrom;
    end
end
```

Figure 5.3: Pseudocode algorithm for the querying of other players for pair formation for mobility
agent bestoffer;
integer bestrank;
while nextQuery() != null do
agent offerfrom $=$ nextQuery ()$;$
integer fromrank $=\operatorname{getPos}($ offerfrom, prefs);
if fromrank $=0$ then sendReponse(offerfrom, 'hardyes');
partner $=$ offerfrom;
sorted $=$ true;
end
if sorted $=$ false AND fromrank $<$ bestrank then
sendReponse(bestoffer, 'no');
sendReponse(offerfrom, 'softyes');
bestoffer $=$ offerfrom;
bestrank $=$ fromrank;
end
else
sendReponse(offerfrom, "no"');
end
end

Figure 5.4: Pseudocode algorithm for responding to queries for pair formation for mobility

### 5.5.2 Example Execution

The method by which we arrive at these ranked lists doesn't matter- perhaps some of them were produced by relatively advanced player types, while others were simply ordered randomly, or shuffled by the parameters of the graphical
game. As this shuffling can cause an agent to incorrectly express its preferences and so, in common parlance, make a decision it would not be 'happy' with, I will use the term 'expressed preferences' to sidestep this issue. Remember that each agent's list only includes adjacent agents and themselves, not the whole set of agents in the graphical game. (For completeness, unconnected agents can be considered to be ranked below the ranking agent in the final list if desired, as this does not functionally change anything.)

Recall that we are not seeking an 'optimal' matching in which every agent's expressed preferences are met as closely as possible, or a 'stable' matching in which no pairings would prefer to be paired differently. Neither such matching may exist (as in the classic case where $A$ prefers $B, B$ prefers $C$, and $C$ prefers A).

Even if an optimal and stable matching does exist for a given configuration, as each agent is selfishly pursuing their own goals, and acting only on locally available information and their own incomplete memory, it is often impossible to practically coordinate such a matching. As such, we instead aim for a 'good' matching, in which no agent decides to pair with another, or stay in their current position, if they could instead have reasonably expected to pair with a third agent who ranks higher on their expressed preferences (or would prefer to stay in their current position).

In each step, each agent sends a query to the next agent on its list, which is its most preferred partner out of the remaining unqueried options. This can be itself, which is a special case- the response is always 'hard accept'. In other cases, the queried agent responds depending on the responses it itself has received from the agents it has queried. If the agent sending the query is the highest ranked agent in the preference list, excluding those who have 'reject'ed previous queries, a 'hard accept' response is sent.

On sending or receiving a 'hard accept', the agents confirm each other as partners who will swap positions at the end of the phase. They stop querying other agents, and respond 'reject' to any and all queries sent to them.

Otherwise, a 'soft accept' response is sent. This indicates the offer is currently the best one that the agent has received, but it may receive a better offer later on as it continues to query other agents. The sending agent also sends a 'reject' to any other soft accepts it was previously holding.

Similarly, if an agent receives a 'soft accept', it first notes it. Then, if it has more than one soft accept stored, it sends 'reject' to all but the most-preferred offer- this prevents the possibility of an agent having a soft accept from another agent who has hard accepted another offer. By this method the ' $A>B>C>$ $A$ ' case can be overcome, as one pairing will be formed 'randomly', mimicking the real-world situation in which one person will eventually settle for secondbest rather than unrealistically waiting indefinitely for their preferred option and risking being excluded entirely.

If a 'reject' is received, the agent eliminates the sender from its internal preference list, as there is now no possible circumstance under which the queried agent will pair up with this one.

At the end of the phase, all remaining soft accepts are converted into hard accepts before movement occurs. This will always result in paired agents and never in cycles, due to the way signals are sent, processed, and then accepted or rejected. An agent who has accepted its own query will not move, whereas all other agents will swap position simultaneously with their chosen partners.

By this point, all agents have a hard accept, as they themselves must appear at some point in their own preference rankings, and this entry will inevitably be queried and accepted if no better option exists.

This algorithm works for both synchronous and asynchronous time, with asynchronous time simply being a special case in which only two agents send queries and each ultimately swap with an adjacent agent (including perhaps each other).

We can show this makes 'good' matchings as previously defined by considering the following. Assume an agent gets finally paired with another, when they could instead have paired with a third agent who ranked higher on their expressed preferences. They must have queried that agent at some point (as agents will continue to send queries until matched with the best possible partner, or all agents have been queried). If that agent had responded 'hard accept', they would now be paired together. If it had responded 'reject', there is no possibility of them pairing with that agent. Finally, if they had responded 'soft accept', either the pairing would have eventually solidified, meaning no better offer was found by either party, or it was rejected, meaning that one or more agents found a better offer, and is thus unwilling to keep the originally proposed pairing.


Figure 5.5: Example Graph

| A | D | C | B | A |
| :--- | :--- | :--- | :--- | :--- |
| B | C | A | B |  |
| C | A | E | C | B |
| D | D | A |  |  |
| E | C | E |  |  |

Figure 5.6: Example Preference Rankings

1. Each agent queries their ideal partner, and responds to any queries they receive. $D$ queries themselves, and finds they are receiving a query from their first preference (namely, themselves). $D$ hard accepts $D . D$ then subsequently rejects $A$, as it has already committed to not moving. $A$ soft accepts $C$ 's query, as they'd prefer that pairing to staying in their current position. $C$ soft accepts $E$, and rejects $B$ in line with their preferences.
2. $D$ sends no queries, as it has already firmly decided not to swap. $A$ queries $C$, who is now their number 1 preference. $C$ hard confirms, as $A$ is also their number 1 preference. $E$ is then rejected by $C$, but immediately hard accepts its own offer. $B$ queries $A$, and is again rejected.
3. Only $B$ is left without a hard accepted offer. $B$ queries itself, its third preference, and hard accepts.

All agents have hard accepted an offer, so the algorithm is complete. A swaps with $C$, and $B, D$, and $E$ stay where they are. Note that this occurs despite $B$ and $E$ strongly desiring to move, due to the structure of the graph and the decisions made by the other players.

In both the coded implementation and the real world, some of these offers may arrive in a slightly less convenient order. This can mean that an agent soft accepts an offer, then rejects it as they hard confirm another in the same step (for example, if $C$ queries $E$ before receiving $A$ 's query in step 2). However,
this doesn't alter the final result.

### 5.6 Human Experimentation

One of the central aims of this research was to devise a new model to more accurately model existent human behaviour using game theoretic concepts. As such, I also designed some small-scale human experiments, mainly to verify that the general influences and traits I was observing in my simulations were also applicable to interactions between humans in the same circumstances. Rather than attempt to artificially limit human players to the level of my agents (who are myopic, and only communicate simplistically using 'costly' signals within the scope of the graphical game itself), I instead designed an experimental setup much like my simulations in which the effect of these additional traits could be observed and quantified for human subjects.

16 experiments were conducted over the course of 6 sessions, as detailed below:

The participants were physically seated in a simple arrangement corresponding to the shape of the graph to be used. I restricted these experiments to simple graph structures in order to present this information clearly and avoid creating confusion for the participants. The initial position for each player was determined randomly for each experiment. Each round consisted of each game being played simultaneously (unless asynchronous time was used, in which case only a single, random game was played in each round), negotiating and then completing movements.

The default round duration was 2 minutes long, as preliminary investigation showed that about $80 \%$ of players are able to comfortably make all decisions within this time. The length of each round was always declared in advance.

| Players | Rounds | Time <br> $($ mins $)$ | Diffusion <br> $(\%)$ | Graph <br> Used | Game <br> Used | Timing | Other |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 5 | 2 | 0 | Cycle | PD | S |  |
|  | 5 | 2 | 0 | Cycle | Ult | S |  |
|  | 5 | 2 | 0 | Path | PD | S |  |
|  | 5 | 2 | 20 | Cycle | PD | S |  |
|  | 5 | 2 | 0 | Cycle | PD | A |  |
| 8 | 6 | 5 | 5 | Cycle | PD | S |  |
|  | 6 | 1 | 0 | Cycle | PD | S |  |
|  | 6 | $1 / 2(\mathrm{a})$ | 0 | Cycle | PD | S |  |
| 12 | 30 | 2 | 10 | Cycle | PD | S | (c) |
|  | 10 | 2 | 0 | Star | PD | S |  |
| 10 | 30 | 1 | 0 | Cycle | Ult | A |  |
| 5 | 15 | 2 | 5 | Cycle | PD | S | $(c)$ |
|  | 15 | 2 | 0 | Cycle | SSP | S |  |
| 4 | 4 | $15(\mathrm{~b})$ | 5 | Cycle | PD | S |  |
|  | 5 | 2 | 0 | Clique | PD | S |  |

(a): $50 \%$ of players were given 2 minutes, the rest were given 1 minute.
(b): All players were ready to progress after 10 minutes at most.
(c): Players were told when the simulation would end.

Each graphical game was simulated this way for between 5 and 30 rounds. The number of rounds in each simulation was not conveyed to the participants in advance unless noted (in order to mitigate backward-inductive strategies), aside from a range of possible experimental durations for practicality. Though the majority of experiments used the Prisoners' Dilemma, the other games and corresponding payoff matrices from chapter 2 were also investigated.

I wanted to minimise the effects of altruism, spite, and other 'irrationality' specifically outside the scope of my investigations for my human experiments. As such, I endeavoured to recruit subjects who were sociable, competitive, and used to playing games in general. 29 of my 43 subjects were found from advertising through gaming-related societies within Durham University, and others were recruited from the larger student body. No participants were reused between experimental groups, and no subjects left any session before it reached its natural conclusion.

The experimental setup was carefully detailed to all participants. The steps of the game, being the same as those in the simulation, were first explained to all subjects; each player selects an action for each player they're linked to, outcomes are produced deterministically based on the actions of each player, potential movement is discussed, pairs are determined, movement between pairs occurs, and then the process begins anew. Reminders of this sequence were made available to the players. Outcomes were selected using simple marked cards, and revealed simultaneously.

Unlike my simulated scenarios, players could see the actions made by each opponent in each game at each stage. This change by itself did not appear to cause significant divergence from the expected results, as detailed further in Chapter 6. I kept track of the position of each player at each point, including swaps when they occurred, and oversaw proceedings to ensure the rules were followed at all times. Before the experiment began, players were given the opportunity to ask questions about the setup and rules, to the point where they each expressed confidence in understanding the situation. I gave no advice or suggestions on what strategies to use and, though the players could freely communicate, they were generally (and unsurprisingly) unwilling to share their own thoughts and analysis with the group.

Players used pen and paper to record their actions against each opponent and corresponding outcomes, their attempts to move as well as successful moves and the players they swapped with, and finally their more general thoughts on movement and strategy. Due to the complexity of even these tightly controlled tests, I asked players to write down and record their thought processes throughout the experiment however came most naturally to them (so long as they could be interpreted later), rather than using a standardised form.

In particular, I asked each player to summarise what they thought their general
strategy would be before the game, any insights which arose during play, especially if those insights caused them to re-evaluate the game and their approach to it, and what strategy they actually ended up using by the game's conclusion. Each player produced, on average, about 350 words of meaningful notes for each experiment. The output produced per round was much higher for experiments with fewer rounds. Delays were added between rounds in order for players to accurately record their thoughts, but no actions or communications were permitted during this time.

Payoffs were allocated using the matrices presented in chapter 2. One unit of utility was represented by one small sweet of the player's choice from a mixed selection. Before this round of experimentation, I performed some preliminary testing to establish the validity of my setup, in which I trialled three methods of rewarding or penalising players based on the outcomes of their games. Firstly, all results were negative or neutral, with bad outcomes subtracting from a pool of sweets they began with containing sweets equal to the maximum number they could theoretically lose. Next, all results were neutral or positive, with players gaining sweets.

Finally, I tried a mostly positive system, but where players forfeited sweets to their opponent if they cooperated while their opponent defected. Out of all of these, the second format seemed to produce the most balanced results, as players feared losing a sweet disproportionately greater than they valued gaining one, even if the ultimate utility was obviously the same. This seems consistent with earlier work done on risk aversion [52].

There is not a broad consensus at present on the general effects of communication on the outcome of games, especially 'cheap talk' or 'small talk'- free communication between players which carries no particular weight or mechanical meaning- outside of coordination games [53]. However, some of the more
focused research in this area provides compelling evidence that there is, at the very least, some sort of effect on human players [15]. I decided to allow all players to communicate at all times, as I felt this was the method most generally representative of naturally occurring scenarios- at the very least, you could argue that (outside of a laboratory setting), some form of communication is a requirement for any sort of interaction.

However, I did ensure that it is not possible for communication to progress beyond small talk, as there is no explicit method of costly signalling (sending messages which incur some inconvenience or negative utility to the sender, thus adding some weight to their content) [54] or central pact enforcement (which could ensure players abide by contracts or agreements they make to, say, cooperate with each other or move in a certain manner) beyond the vague and indirect effects of social reputation. In particular, players were not allowed to offer or trade their sweets to other players (which were, in practice, given out at the end of each complete game rather than in-between rounds).

## Chapter 6

## Results

In this chapter I will describe and illustrate my main findings regarding the operation and functionality of the model I have developed. Most of my findings are in the form of behaviours observed over a range or grouping of the many configurations possible in this model, and display small variations in continuous values such as frequency, timing, probability and so forth across that range. Additionally, due to the emergent nature of many of the model's properties, slight variations in parameterisation can dramatically alter the nature of the overall simulation, making precision and extrapolation problematic [24].

As such, many of the observations presented here are in the form of qualitative descriptions, which allow for this natural variance in the configuration and settings of the simulated environment in which each given effect was observed, while still describing the effect comprehensively, or as graphs which can illustrate a more detailed variation within a narrower area. Additionally, a selection of the data analysed has been included in the form of an appendix (Chapter 9). Though this is not intended to be read by the naked eye, I will refer to experiments performed within these data occasionally to enhance my descriptions.

### 6.1 Preliminary Investigations

As I've explained previously in Chapter 3, graph-based systems with many different settings and continuous variables often display highly chaotic emergent behaviours, which can completely alter the running of the entire simulation as threshold values are approached and crossed, even by relatively small amounts [38]. As such, in order to even begin to understand the complex interactions between different aspects of my model, I first had to develop a rough working knowledge of the system which would then enable me to determine the sorts of simulations and experiments from which I could then develop a deeper, more rigorous understanding.

This was accomplished through many months of preliminary investigations, consisting of repeatedly running simulations with a variety of settings, carefully sifting through the data produced for interesting or novel behaviour, modifying the settings slightly, and noting the change/s (if any) on the behaviour. The behaviours I was searching for at this stage were mainly extreme clumps of data or outliers which seemed to be outside of the expected range of their data. For instance, when investigating star graphs this way, I noticed that the entire result set was extremely variable, which, on later, more detailed investigation, led to the results detailed under 'Variation in Results' later in this chapter.

My early understanding was also guided by intuitions regarding which settings were most likely to quickly lead to meaningful insights and comprehension. It seemed apparent from the very beginning of this model that it would likely display extremely complex, emergent behaviour, with the overall behaviour of the graphical game changing dramatically depending on the initial settings, due to its similarity to other models which display such behaviour [46]. The prediction that some of the underlying behaviours well-known from preexisting, simpler systems (such as basic strategy for the Prisoners' Dilemma)
would generally carry over into my model was ultimately proven accurate, as shown throughout my results. This allowed areas of interest to be identified and check more rapidly than mere blind experimentation or brute force would have permitted if an unstudied game had been used instead.

To confirm the intuition that my system with the mobility wholly removed is identical to the pre-existing and relatively well-studied model of graphical games, I ran simulations without any mobility whatsoever. This process was detailed more fully in my experimental setup. I was eager to finalise the form of mobility which would be used (recall I listed the candidates in Chapter 4, and discussed some further implications in Chapter 5) as soon as possible, as I quickly realised that this was the most critical variable in the entire system, and would have a substantial impact on practically all other results gathered from my model. This form of 'pre-preliminary' experimentation led me to use 'adjacent agents agreeing to swap positions' as the basic implementation of mobility in my research, as explained in detail throughout these earlier chapters.

As an additional part of these investigations, I conducted smaller experiments with human volunteers to help inform the configuration of the simulations, and identify pertinent features to include or exclude from the new model- such as memory and myopia, as detailed previously within the features of the model. Such observations also guided me towards configurations and qualities which would assist artificial simulations in producing human-like behaviour- such as the relatively high proportion of intelligent cooperative strategies. Obviously, it is extremely difficult (and some would say impossible) for any game theoretic simulation to accurately consider the myriad possible quirks of human interaction. However, it seems reasonable that, if we can understand these behaviours qualitatively in terms of their interaction with this model, we can interpret the results from our artificial simulations more intelligently, account
for some of the human behaviours, and thus increase its predictive power.
This process, though both lengthy and empirical, was relatively casual compared to the extensive cataloguing and cross-comparison which took place in the main bulk of my experimentation as described below. Any findings which this process uncovered were investigated more thoroughly in the main stage of my experimentation. Although this process was vital to the progression of my research- enabling me to obtain the information I needed to devise a fundamentally sound model before I began more rigorous testing and analysis on itI will henceforth disregard it in favour of discussion of the more detailed, later experiments which it informed, as detailed in the bulk of my results below.

### 6.2 Data Collection

The data from which I drew my results were obtained by three main methods. Firstly, individual simulations were run and analysed in detail, yielding insights into the small-scale movements and patterns that can lead to certain outcomes from different configurations.

Secondly, those same simulations were repeated for between 10 and 1000 iterations with the same settings, in order to eliminate this small-scale variance and allowed broader conclusions on the less-apparent behaviour of these systems to be drawn. These two methods are strongly linked, not only as they were used on the same data sets, but also as they allow connections to be discerned between the behaviours of individual graphical games and more general properties of the model when used in concert.

Although the configurations used in these investigations ranged across the entire available spectrum, all of the following observations are drawn from 'average', 'stable' parameters- relatively balanced populations, consisting of
roughly equal numbers of cooperative and competitive strategies, with very low noise and diffusion- unless otherwise specified. The vast majority of the quantitative results discussed below are derived from these two methods. Finally, experiments were run with human subjects to confirm the applicability of these results to real-world situations, and provide additional insight into the sorts of thought processes and approaches more intelligent agents could take towards this category of games. These experiments do not, and were not intended to, provide robust, quantitative data independent of the results from the automated simulations. However, they do provide some compelling evidence that human behaviour is broadly similar to the behaviour of even my simple autonomous agents, so long as the simulation is configured to allow this behaviour, as mentioned previously in Chapter 4 and further on in these results.

### 6.3 Observations from Simulations

To obtain these results, each setting was investigated as fully as was reasonably possible, informed by earlier indications from my preliminary investigations. For instance, probability-based features such as diffusion and noise were tested at $5 \%$ intervals from $0 \%$ to $100 \%$ for each combination of other settings, with additional testing between $1 \%$ and $10 \%$ as the preliminary investigations showed the effect within this range to be particularly variable. Durations between 1 and 1000000 rounds were used in the preliminary investigations, which enabled me to say with confidence that no significant chances to the behaviour of the system occur after 500 rounds for any reasonably sized graph structure ( $<30$ players), and so exclude this from consideration in my more rigorous analysis.

## Volatility \& Variation in Results

|  | Mostly Hawks (75\%) | Split (50\%/50\%) | Mostly Doves (75\%) |
| :--- | :--- | :--- | :--- |
| Line | Low variance. <br> Convergence towards a <br> clump of doves takes <br> $100+$ rounds, speeded <br> up by diffusion. <br> Hawks win by $\sim 8$ points <br> per round. | With a spaced initial distribution, <br> hawks often get stuck at the 1-degree <br> ends, making it almost impossible for <br> the doves to form a clump due to <br> pairing issues. <br> Low diffusion removes this quirk. <br> Hawks win by $\sim 7$ points per round. | Low variance as hawks <br> end up between <br> clumps of doves <br> regardless of other <br> parameters. <br> Hawks win by $\sim 4$ <br> points per round. |
| Ring | Converges $\sim 30 \%$ faster <br> than the same <br> population on a line <br> graph, due to the <br> additional connection <br> across the furthest <br> points of graph, with all <br> of the doves forming a <br> clump. <br> Hawks win by $\sim 1$ point <br> per round, but with an <br> addition of an extra <br> dove this becomes a <br> dove victory of $\sim 1$ point <br> per round. | With clusted intial distribution and <br> $0 \%$ diffusion, there was no movement <br> other than swaps between hawks. <br> If the number of doves was odd, <br> eventually a single dove would <br> seperate from one end of the clump <br> and become stuck between two <br> hawks. <br> Doves win by $\sim 2$ points per round <br> with 0\% diffusion, otherwise scores <br> are extremely dependent on which <br> forced movements occur when, with <br> those occuring at the edge of the <br> dove clump being worst for the <br> doves. | line graph. |

Figure 6.1: A non-exhaustive table of some qualified observations from slight variations to just two parameters, namely graph structure and population. 'Dove-TilHawk' was used as the dove strategy and 'Hawk-Til-Hawk' as the hawk strategy. 12 players and synchronous time were used unless specified. The Milgram ring used here consisting of one additional connection between opposite sides of a ring graph. Scores indicated are the average of all scores from players of that type.

The figure above demonstrates how stochastic and volatile my results can be, with multiple emergent behaviours appearing and disappearing based on
relatively minor changes in configuration. This supports my decision when designing the model to actively reduce the number of complex variables, as detailed in Chapters 2 and 4.

Pavlov performed very well across the board, infrequently outperforming the other, newer player types, across a wide range of configurations, but especially when a graph structure with low connectivity was used. Although a Pavlovian population is able to successfully cooperate internally more times than not (even with relatively high levels of noise in the environment), victories (in which this type scored higher than each other in a given simulation) and good scores (those which earned at least $50 \%$ of the available payoffs) were also observed when the amount of Pavlov players in the population was relatively low (in some instances as low as 10\%). For example, experiments 232 and 233 in Chapter 9 display this property.

The Pavlov strategy I used took a similar approach to its moves as to its actions, changing position if it was generally scoring poorly (due to competition from the majority of its opponents), but remaining stationary if it was doing well (due to cooperation). This was one of the more advanced movement strategies used (based as it was on analysing the actions chosen by opposing players), and, combined with a relatively intelligent and adaptive action strategy, resulted in a strong type overall which performed above the average score set by other types in just over $50 \%$ of simulations tested.

In environments with little movement (due to lack of diffusion and/or relatively static player types), this movement schema acted as a sort of 'ratchet', locking Pavlov players into favourable neighbourhoods while shuffling them out of unfavourable ones. Though a lack of focus and specialisation for any particular environment meant it wasn't often the best-performing strategy, its generic strengths combined with the ability to stabilise quickly and recover from noise
(which can cripple Tit-For-Tat based agents) enabled it to occasionally edge out the other types.

The overall variation in results for multiple instances of simulations with the same configuration was reduced when non-responsive types, such as EverDove, EverHawk, and Random, were swapped out for those which immediately take the actions of their opponents into account, such as Tit-For-Tat and especially Pavlov (with its innate capacity to self-stabilise). The presence of these types creates the risk of two immobile agents becoming irrevocably paired, creating extreme fluctuations in the overall results of a simulation dependent on whether or not, and how many, of these pairs are created. The case where a permanently cooperative agent is permanently paired with a permanently competitive agent is particularly extreme, resulting in the competitive agent scoring perfectly from that one interaction and thus extremely highly overall, whereas the converse is true for the would-be cooperator.

Taking $0 \%$ diffusion as the baseline, adding a low level of diffusion $(<5 \%)$ reduced the variation in results caused by agents which did not move voluntarily, as it prevented these clumps from persisting indefinitely. This can be seen in experiments However, high levels of diffusion ( $>50 \%$ ) resulted in variation only slightly lower than $0 \%$ diffusion, regardless of agent types, especially when combined with short durations ( $<20$ rounds). This is because diffusion acts to randomise movement, impeding the decision-making processes of the agents and instead causing them to approximate a wholly random movement strategy, with all the variation between otherwise identical simulations which that entails. Additionally shorter duration prevent the agents from spreading out and spending roughly equal time interacting with each opponent

In a related manner, variation in results from running the same simulation multiple times increased sharply when exploitative types, such as Tester and

Hunter, were able to interact with types which could not effectively combat them, especially EverDove. A population of $50 \%$ EverDove and $50 \%$ Tester displayed variation on average scores for each player type between simulations of just under $400 \%$. Depending on the time taken for each exploitative agent to identify and position themselves next to a non-responsive agent or agents, these two types can act as 'super-aggressors', actively seeking-out and exploiting the helpless EverDoves even more effectively than a lucky EverHawk, given a small amount of diffusion.

Shorter durations also increased variation across a wide variety of configurations, especially those which would ultimately stabilise, as the environment had less time to converge towards whatever pattern it would take if given an arbitrary length of time. However, with all that said, the overall variation between average type scores in multiple instantiations of identical simulations was generally low.

## Impact of Graph Structure

The first result I noticed is that hawkish types, and traditionally strong strategies such as Tit-For-Tat, fare better as the connectivity of the graph increases, and thoroughly dominate in all instances where the complete graph is used. This serves to confirm an expected result for the model- in these instances, the simulation effectively defaults to the specific case which has been studied for decades, in which each player plays every other (as discussed in Chapters 2 and 4).

With a complete graph, all players play each other simultaneously and mobility has no effect- as, no matter where you move to, you'll always be playing the complete set of other agents, and the possibility of adapting your strategy through experience is the same as in prior experiments. Additionally, many
of the other variables, such as initial distribution, population, and even the system of movement itself are effectively ignored or at least relegated, further reducing the impact of the novel features of my model.

Conversely, dove strategies fared relatively well in those situations where the graph was only loosely connected and the average degree of each vertex was relatively low. That is, instances where the graph had low average cut- the number of edges which need to be removed to cleanly partition the graph, relative to the number of vertices in the graph- such as a ring graph. This enabled clusters of cooperative players to form without being infiltrated or broken up by hawks.

Depending on the distribution, and the outcomes of random chance determining things such as agent movement, one or two extremely competitive players could manage to situate themselves in amongst a cluster of extremely cooperative players, and by doing so attain extremely high scores (though the average score of all cooperative players was still typically much higher than those of the non-cooperative players). More intelligent cooperative types which could identify and respond to such threats, such as Tit-For-Tat, were not as severely affected by the presence of a small number of non-cooperative players inside of their group. This is illustrated in figures 6.5 and 6.6 later in the chapter.

The overall outcome of star graphs is extremely dependent on which type occupies the central position for longest- with little or no diffusion, the results from these graphs are some of the most extreme. Exactly half of the games played in a star graph involve the centrally positioned agent. The general disposition of the central agent had a significant impact on the social utility produced by the simulation, with a central dove resulting in as much as a $300 \%$ increase over a central hawk. Such structures also have extremely low mobility, as an agent in the central position who chooses not to move also
prevents all other movement in the graph.
I observed a relatively high number of outcomes in which Pavlov had the highest type score in low duration simulations- roughly $30 \%$ across the range of all tested configurations. Though my agents are not advanced enough to understand and apply the concept of bargaining power [28], Pavlov is perhaps best-suited to the general concept of altering your stance based on your interactions with others to produce the best result, and so performs especially well in this situation.

Loosely-connected cliques (groups in which each vertex is connected to each other vertex in the group) are an excellent demonstration of the multi-layered properties of this model. Within each clique, their behaviour is almost identical to a smaller graphical game simulating just that group on a complete graph. However, if the clique has just 1 or 2 connections to other cliques, its functionality in the graph can be approximated by a single node. Depending on the properties of the clique itself, this then can be further refined and abstracted- for example, a highly mobile clique which is $50 \%$ cooperative and $50 \%$ competitive can be loosely modelled as a single agent with a new type which randomly chooses between these two strategies.

## Impact of Initial Distribution

Some of the most successful parameters for cooperative strategies were short durations with clustered initial type distributions, in which similar types were closer to each other. This reduced the time needed for cooperative types to find cooperative opponents, as they already began close to each other in the graph, whereas the competitive players spent much time competing amongst themselves, and the simulation ended before the clusters could be broken up by player action and/or diffusion.

As the distribution became more even, the effectiveness of aggressive strategies sharply increased for a similar reason- they began the graphical game interspersed between more passive players they could exploit. In this case, a short running time favoured the hawks, allowing them to exploit all of their opponents for almost all rounds of the game before it ended, and leaving the doves unable to cluster together for mutual gain within this narrow window. In addition, these shorter durations consistently produce more extreme scores, as there are fewer data points to average out.

Similarly, anything which reduced the effective control of the agents over their positions, such as increased diffusion, tended to even out the distribution of types (much like 'diffusion' of chemicals in the real world) and so favoured hawk strategies. However, over time, the impact of the initial distribution decreases the longer the simulation runs for, as the starting position of each player and/or type becomes less important the longer each player has to move around. These results are illustrated in the graphs below:


Figure 6.2: Relative performance of various types in short-length games with a variety of configurations. In each instance the simulation was repeated 10 times and the scores averaged.


Figure 6.3: Relative performance of various types in medium-length games with a variety of configurations. In each instance the simulation was repeated 10 times and the scores averaged.

## Type Performance

As expected, strategies which were unable to respond to their changing environment fared extraordinarily badly. Strategies like Tester and Hunter successfully exploited Everdove, and to a lesser degree Random, across a very wide range of configurations, failing only in the presence of extremely high levels of noise and diffusion ( $>40 \%$ ). On the other hand, strategies which were able to respond to changes in their opposition (caused by the motion of others) and take action to face more favourable opponents in future (utilising their own motion effectively) universally outperformed those which did not- even if the responsive strategy was otherwise generally immobile.

Critically, Tit-For-Tat fared relatively poorly, despite being one of the strongest strategies historically (arguably the strongest strategy) in the Prisoners' Dilemma [4]. Although it was able to outperform basic strategies with historical roots outside of the graphical game with mobility model, they were outperformed by one or more of my newer strategies across a wide variety of parameterisations.

More than a simple deficiency in utilising movement optimally, this appears to be due to the nature of mobility negating the traditional advantages of this strategy. The idea behind traditional Tit-For-Tat is to repeat the last action played by the opponent in each step, cooperating with cooperative players while punishing aggressive ones. However, most of the hawkish player types in my simulation repeatedly defected against all other players in each round, while trying to move away from other hawks using the same or similar approaches.

This is in some ways a similar concept to Tit-For-Tat- namely, of cooperating with cooperative players and not with uncooperative players. However, by making use of their newfound mobility, these new strategies could work to completely avoid unfavourable opponents, allowing them to ruthlessly exploit naïve types while minimising contact with aggressive ones- making it superior to traditional Tit-For-Tat in all respects within my model.

The relative inferiority of Tit-For-Tat to Hawk-Til-Hawk is shown below. For these simulations, the population was composed of one-third each of Hawk-TilHawk, Tit-For-Tat, and Random, across a variety of configurations as shown. A 'Milgram ring' is simply a ring graph with a small number of additional connections between non-adjacent agents [7].


Figure 6.4: Performance of Hawk-Til-Hawk and Tit-For-Tat

## Impact of Timing Systems

Although previous work found little effective difference between synchronous and asynchronous timing, I predicted back in Chapter 4 that this would have a greater impact in my model. My results have demonstrated that this is the case. The use of asynchronous time notably improved the performance of the more intelligent, and more cooperative strategies, especially at low connectivity and in more diverse populations. Conversely, in synchronous time, all agents negotiate and move simultaneously, often degrading and occasionally wholly invalidating the original reasoning for movement in a manner analogous to increased diffusion in the environment.

For example, consider a 4 -cycle containing players $A, B, C$, and $D . A$ is aggressive, while the other players are cooperative. $B$ is not doing well against
$A$, and so arranges to swap with $C$, reasonably expecting to move out of range of $A$ and come into contact with $D$. However, simultaneously, $A$ and $D$ also agree to swap. After the movement is complete, $B$ is no better off, despite having correctly selected and negotiated a good move, because of the simultaneous choices of other agents outside their control. However, with asynchronous time, this scenario is much less likely to occur, as only two connected players out of the set of all players interact and attempt movement in each round.

This is a positive result for my model, as this asynchronicity more closely resembles the somewhat haphazard nature of real-world interaction. Unfortunately, ensuring that all players play a roughly equal number of games with a roughly equal delay in the system I have devised is non-trivial, due to the mobile nature of the players and linked nature of the games. This is something of a concern, as in a real-world situation using truly continuous time, all players would have roughly the same amount of useful time at their disposal in most situations, and so complete a roughly similar number of interactions regardless of additional factors. The addition of a basic fairness criterion to the asynchronous timing system, which increases the probability of agents who have played fewer games being selected, helped prevent extreme results and appears to produce clearer and more accurate results across all configurations because of this.

Even in the worst case scenario using asynchronous time, where an agent must wait an extremely long time between playing games, their situation is not much worse than it would be with synchronous time, as their knowledge of their potential opponents will, in both situations, be curtailed by other movements which may have occurred in the meantime. However, in the event of a given player playing two games in close succession, their knowledge of their immediate environment, their opponents, and consequently historical
moves or actions informing their present decision, is much improved, allowing their movements to be much more directed and thus more useful. On average, though, an agent would expect the same number of third-party movements to occur between its games in asynchronous time as occur simultaneously during synchronous time.

As a result, in asynchronous time the knowledge agents have of their neighbourhoods remains effectively accurate for longer, making decisions on when and where to move more informed. For example, assume a player plays a game during asynchronous time, cooperates effectively, and decides not to attempt to swap. They are more likely to stay in their current position, with their neighbourhood intact, until their next game is played.

Conversely, in synchronous time, even if they cooperate with each of their neighbours effectively, as each has the possibility of moving in the movement step before any further games are played, the quality of the neighbourhood is more likely to deteriorate before the agent gets a chance to respond. Similarly, suppose an agent was about to move into a position surrounded by cooperative players. In asynchronous time, the move would be completed and another game would be played. But with synchronous time, there's a reasonable chance that every single one of those desirable agents will have similarly relocated in the same movement step.

As with other features of the model, it should be possible to design player types which identify the configuration of the simulation they are placed in, by observing the alterations in themselves and their neighbours over time if required. Such types would be able to anticipate and respond to at least some of the many possible situations which could arise, and so outperform agents who did not. This would essentially be a capacity for 'meta-meta-awareness' of their environment, analogously to how agents with sound movement types
are 'meta-aware' and agents with sound action types are 'aware'. I will discuss this possibility further in Chapter 8 .

## Risks of Mobility

As agents are myopic and cannot perceive events or graph structure beyond their local neighbourhood, moving to a new position will always be something of a risk. In certain situations, particularly those within environments which generate large amounts of movement (for instance by using synchronous time, highly connected graphs, a high proportion of mobile types, and/or large amounts of diffusion), it may be less risky to remain stationary due to the expectation that some of your neighbours will change after movement. However, in all other scenarios, agents performed better when they made moves determined with any reasonable heuristic (i.e. not randomly), than they did when they remained stationary.

For example, if you swap positions with another player, you are guaranteed to still be adjacent to them at the end of the movement phase (as each agent can move, at most, once per round). As such, movement strategies which preferentially swapped with cooperative players tended to outperform those which, when they did decide to swap, did so randomly, and also those strategies which preferred to remain stationary. This behaviour was frequently observed with human players, who almost never swapped with players they hadn't first cooperated with.

As I predicted in Chapter 3, agents generally performed better when moving from a locality with a low price of anarchy to one with a high price of anarchy [30]. Although not a perfect comparison, as in my model the same payoff matrix is used uniformly across the whole graphical game, the same idea can be applied to individual games between agents based on their types. For
instance, if one of the two agents will always choose to play a particular action, a reduced payoff matrix with a new price of anarchy can be formed containing only those outcomes which could be relevant, allowing a new price of anarchy to be calculated.

Stationary dove strategies which ignore the mobility available in the graph are, as seen in classical studies of this scenario, repeatedly exploited by hawks. On occasion, they find themselves in the worst possible situation of being completely surrounded by player who refuse to move and constantly betrayed by those same players. Stationary hawks fare better than stationary doves, as their strategy is inherently stronger- a 'mindless' dove player will still find themselves exploited by neutral strategies such as Tester and Hawk-Til-Hawk, whereas a 'mindless' hawk has no such vulnerability- but still score much lower than other types which make good use of mobility such as Hawk-Til-Hawk. This is seen, for example, in Experiments 177 and 178.

## Impact of Duration

Increasing the number of rounds in the simulation can have a number of different effects, highly dependent on the other parameters. If there is a high proportion of more intelligent agents and/or low diffusion and/or a more restricted graph, the relative score of cooperative players increases sub-linearly with the duration of the simulation. This is because locating other cooperative agents and demonstrating cooperative intent takes time, but yields benefits in the long run. In cases with very high diffusion and/or simplistic types unable to take advantage of the increased knowledge they gain of their opponents over time, extending the length of the simulation simply gives aggressive strategies more opportunities to exploit their opponents- the cooperative players are unable to coordinate effectively, regardless of how many chances they are given
to do so.

Longer simulations with high diffusion and more restricted graphs roughly approximate shorter simulations with less restricted graphs. This occurs as, in both cases, each player will face each opponent a roughly equal, small number of times. Asymmetric timing also amplifies this similarity. In every simulation, increasing the number of rounds reduces the impact of the initial distribution on the final outcome, as it becomes just one of many situations from which the distribution of players will develop.

## Population Distribution Over Time



Figure 6.5: Variations in the number of completed swaps over time in a ring graph with $0 \%$ noise and $0 \%$ diffusion, using synchronous time, consisting of 12 'Dove-TilHawk' and 12 'Hawk-Til-Hawk' players placed in alternating initial position

In the preceding figure, note that the average, but also the maximum and minimum number of swaps goes down over time. This is caused by the cooperative players gradually building a larger and larger cluster, which the aggressive agents could not re-enter once they found themselves wholly outside of it. Eventually, all the cooperative players are fixed in position, leaving the aggressors endlessly swapping as they fruitlessly attempt to search out a more favourable neighbourhood. This type of convergence- into a subset of possible configurations which repeat or even loop indefinitely- is typical of graphical games with mobility. Total stability with no movement occurring is only achieved in simulations with an extremely high proportion of cooperative or specifically immobile agents- typically at most one non-cooperative agent.

## Impact of Initial Distribution

The effect of the initial distribution on the final results is most noticeable on heavily restricted graphs, simulations running for a relatively low number of rounds, and/or simulations with little to no diffusion, as each of these factors reduces the time available for each agent to explore the graph and reach a favourable position. As time goes on and the agents move around, the impact of the initial distribution diminishes and approaches insignificance as each agent's own decision making enables it to approach the position/s on the graph where it wants to be. Experiments indicate that for $k$-regular graphs with $k>2$, with 20 or fewer players, this parameter has no observable effect beyond 200 rounds.

After this time, most agents have found a position they're happy with, or are at least unable to find a better one (perhaps being trapped amidst other, content players), and mobility dips as the distribution of the population stabilises. The arrangement they form is, at worst, a shallow local maximum of the overall,
combined preferences of the agents.
Due to the inherently competitive nature of the fundamental interactions between agents in this system, there is no distribution which is truly or completely stable for an arbitrary length of time, other than those only involving simple deterministic agents. Rarely, however, a distribution will be reached which is optimal for a given type, or, more rarely, socially optimal (given each agent's type), in which each agent of a given type scores perfectly for the duration of a simulation against all other agents.

Often, even in such a 'stable' situation, movement does still continue to occur at the local level, with two agents swapping back and forth endlessly as they each try to improve their outcomes. Frequently, the randomised querying of agents from the mobility algorithm or diffusion can knock the total system out of this state, even after some time has passed, and results in more agents becoming mobile and once again searching for a better position as circumstances change. However, the arrangement caused when the graph has stabilised once again often closely resembles, or is even isomorphic to, the previous arrangement. These behaviours are illustrated in the following figures:


Figure 6.6: Mobility arising from a clustered distribution in a ring graph over time. Red indicates Hawk-Til-Hawk players, purple indicates Pavlov players, and blue indicates Dove-Til-Hawk players. Note how, with the absence of noise and diffusion, the initial clump of cooperative players cannot be penetrated by hawks or Pavlov players attempting to escape them- though both can mitigate their utility loss by positioning themselves at the edge of the group.


Figure 6.7: Mobility arising from a spaced distribution in a ring graph over time. Note how, with the absence of noise and diffusion, cooperative strategies are eventually able to form a solid block which the hawks cannot penetrate.

## Impact of Diffusion

Diffusion generally has a lesser impact on the outcome of the graphical game than predicted in Chapter 4. This seems to be because the control any single agent has over its position is relatively low, even with zero diffusion in the configuration. First, our agent forms an ordered list of preferences based on its own myopic perception, which is unlikely to contain all the information
required to make an optimal decision. Then, the agent negotiates swaps based on this.

However, as all players are seeking more-or-less the same thing in this simulationto be surrounded by cooperative players, whether to reciprocate that cooperation or to exploit it- not many are likely to get their first pick and must make do with various less-optimal arrangements. With all this taken into consideration, the total payoff is not significantly impacted if a given agent occasionally has its preferences randomised.

The largest impact from diffusion comes on heavily-restricted graphs containing a diverse population of competitive and cooperative types. In such a situation, even a low level of diffusion can make it impossible for cooperative types to eternally form blocks which the competitive players cannot infiltrate, and so correspondingly can cause the scores of cooperative types to be significantly lower (as in experiments 53 and 64).

## Impact of Type Balance

Hawks performed better in dove-heavy populations than in balanced ones. While somewhat counter-intuitive, it makes sense that the score of a single hawk invading a population of doves will be higher than the average dove score, as a single hawk in a population of doves will never be betrayed, and even has a small initial advantage in a population of smarter types such as Tit-For-Tat or Pavlov, as it will always successfully exploit each player of these types once. In a large and mobile population, which ensures the invader will get a chance to interact with and betray each other player, and a relatively short number of rounds, which emphasises the effects of these earlier games, it is even possible for the invading hawk to outperform these other strategies.

Doves, naturally, also fare well in dove-heavy populations, as this enables them to cooperate successfully. More advanced types which cooperate first and keep cooperating for as long as the opponent does (such as Tit-For-Tat and Pavlov) are equally equitable to these doves. As two interacting doves prefer the socially optimal outcome of mutual cooperation, it should be no surprise that most types generally scored higher in dove-rich environments, as more of the possible utility is gained overall by players in such an instance. This can be seen in the contrast between experiments 865 and 875 (once the lone EverDove is accounted for).

### 6.4 Observations from Humans

The main purposes of my detailed human experimentation process was to observe how humans interacted within the model I created, compare and contrast their behaviours with those of my simulated players, and attempt to understand these behaviours within the context of previously known results. There were also opportunities to test some conjectures made by myself (such as 'players will not make significant use of information regarding events beyond their locality', which I hypothesised in Chapter 3) and others (such as 'human irrationality is partially caused by lack of computational ability' [25]), which I will discuss alongside the relevant results.

## Communication

Interestingly, the vast majority of players responded truthfully when other players asked them about the actions of third parties and the general game state. I believe the reasons for this are two-fold. Firstly, the amount to be gained from deception was relatively low. For one round, that player is likely
to receive the best possible outcome, but any future rounds are jeopardised by the betrayal. Conversely, information (and so the potential for deception) is greatest earlier in the game before players have a chance to collect their own information. Secondly, even this would require high levels of mobility to manoeuvre into a position where the deception could be useful. Thirdly, with no restrictions on communication, it was relatively easy for another player to contradict any liars, and, as doing so gained them standing in the eyes of the other players, all were eager to do so.

Conversation and negotiations with other players regarding possible movement during the experiments were generally fluid, with players waiting for others they wished to talk to complete their current negotiation, or joining them in conversation with a third party if time was limited. Players who were at more connected nodes often had several other players vying for their attention at once. However, few players meaningfully or repeatedly interrogated other players beyond their neighbourhood, as I'll discuss more later in this section.

## Mobility

Many players are initially reluctant to make use of their mobility, even in extreme cases such as when the most recent round of games produced the lowest possible total payoff for them. The reasons players gave for this behaviour were diverse, but there was generally an impression that they could improve their performance by responding to the known opponent, even in cases where they seemed to be at a disadvantage. This made them unwilling to take the risk of moving to a new neighbourhood and having to 'expose' themselves to an unknown opponent until more information could be gathered.

The overall lack of awareness displayed by players throughout the experimentation process of the symmetry of these situations- namely, that the known
opponent would also be responding to the information they gained from the previous round, and that a new opponent would, just like them, be making a decision without any information about their type or preferences- was striking. This could be a genuine blind spot in the logical faculties of almost all human subjects, or simply an expression of the innate risk-aversiveness often found in such situations.

As expected from my preliminary experimentation, the utility of the players who remained stationary in these scenarios was markedly less than those who utilised the movement options available to them. Unlike the often decried 'irrational' human behaviour which causes streaks of high-scoring cooperation rather than the 'optimal' competition, I believe this is symptomatic of a more fundamental kind of 'irrationality' which does actually impede effective playnamely, risk aversion. Despite steps being taken to normalise the nature of the payoffs in order to create as neutral an experimental setup as possible, this behaviour was still frequently observed across all experiments with human subjects [52].

However, as time continued, more rounds were played, and humans gained familiarity with both the games and the experimental setup, the prevalence of this phenomenon decreased. This could also be partially due to players gaining external or 'meta' knowledge about the types and actions typically employed by particular opponents between games, and as such becoming more confident in their movements towards players with whom they cooperated well with in previous games and away from those they didn't work well with. Though such explicit reasoning was conspicuously absent from all but a few of my players' notes and responses when asked, it may have had an impact on a less-concious level.

This phenomenon also decreased as the diffusion across the graph increased,
which forced the players to move out of their comfort zone- though about $35 \%$ of players then attempted to move back to their previous locations after being moved unwillingly.

## Attentiveness and Memory

Almost all players quickly took to recording the actions played by their opponents in case they encountered them again in future- players who did not were betrayed often, earned the ire of even the most cooperative players (who expected reciprocal cooperation), and generally performed very poorly. Conversely, a small number of players $(<5 \%)$ went far beyond this, extensively cataloguing every play made by each player in each round. However, these players did not outperform the more typical players by a statistically meaningful amount, and in some experiments actually fared worse. I observed other players generally treating such players with distrust; paying 'too much' attention to events outside your neighbourhood implies that you are not interested in building alliances with your current neighbours, increasing the chance they will decide to 'pre-emptively' betray you instead of playing cooperatively. It would appear that this effect neatly counteracts the value of the information gained.

Similarly, players generally paid far more attention to scores and movement within their neighbourhood than they did outside of it, with only the most diligent looking beyond that, and only then in scenarios where they had some confidence they could move effectively towards better locales if they found any. On investigation, a majority of players perceived events outside their locality in the graph as being less-relevant then those happening close-by, which confirms my hypothesis.

## Backwards Induction

In instances where players did cooperate for long periods, a significant minority were observed switching to backstabbing or even mutual defection when informed that the next round would be the final round (at which point the nature of the game arguably changes from repeated to non-repeated). On closer inspection of the notes made by each player, the majority commented to the effect that they thought this was the best action at this point as their opponent would not have a chance to retaliate. Even those who would rather have cooperated often defected to 'guard' against this possibility of defection from their opponent/s.

One or two even commented that they believed this was the right action to make according to 'game theory'- perhaps an ironic example of human players playing sub-optimally (in the case of mutual defection where each receives a lower payoff than if they had continued their successful cooperation) because of, rather than in spite of, classical beliefs about rationality.

Additionally, not a single player in a single game played EverHawk (defecting at each opportunity for the duration of a simulation), and very few engaged in long sequences of repeated defections, especially against all opponents simultaneously. This would seem to provide evidence both that backwards induction is not a reliable method for determining human behaviour, and that humans can and do perform some basic game theoretic analysis of their situation (or, at the very least, that these players were sufficiently knowledgable about game theory).

## Time Pressure

There was not a statistically significant difference ( $<10 \%$ in average scores achieved) observed in the performance of players who were given the standard ( 2 minutes), generous ( 5 minutes), and extremely generous ( 15 minutes) amounts of time to determine their actions and potential movements. This was observed to be the case at all stages of the experiments, including those lasting 30 rounds, and was unaffected by other variables such as graph structure and the payoff matrix used. This evidence would seem to disprove the hypothesis presented by Halpern that humans often miss out on 'rational' play because they are computationally limited and unable to calculate the correct course of action [25], as giving them more time to decide their actions would, at least, mitigate this.

Players given 15 minutes to determine each action were extremely confident in both the plays and agreements they made, often re-examining them in full and making little to no changes to their plans, and expressing a desire to progress to the next stage of the experiment. Conversely, a large minority of players were quite vocal about getting as much time as they could to negotiate and discuss with other players where such an opportunity was offered, even though the effects of additional time on their performance was negligible. These players expressed the belief that their play would indeed be improved by making full use of the time available to determine their actions.

That said, the performance of players given a limited amount of time to select their actions and movements (one minute or less) suffered noticeably when they faced off against players with more generous time constraints. This was implemented by having the players with less time available confirm their decisions and cease discussion prematurely, while the unrestricted players continued. Even when they only needed to select one action from a possible set of two,
and a move from a possible set of three (two adjacent opponents, or remaining stationary), the overall score of these time-constrained players was up to $50 \%$ lower than the average attained by less-pressured players.

There are explanations for this other than a direct impact on 'computational power' though. Players without time-constraints expressed a level of uncertainty interacting with rushed players, and thought they were more likely to defect than others. The time pressure also made it difficult for them to build a rapport with other players and so engender cooperation. Simulations in which all players were time-constrained were notably less cooperative than averagein other simulations around $70 \%$ of all actions chosen were cooperative, but this dropped to $40 \%$ in time-constrained experiments.

## Bargaining Power

The concept of high bargaining power (as explained in Chapter 3), whether relative or absolute, and a correlated effect on utility and payoffs was also investigated. No statistically significant relationship between the two was found for my artificial agents. This is likely because they were unable to perceive their bargaining power in any meaningful way, as they were both myopic and unable to map the structure of the graph as they moved around, and therefore also unable to adapt their strategies to exploit such advantages when they occurred.

However, a very strong correlation between the two metrics was observed with human players. Almost all human players quickly saw the advantages of being in a position with high bargaining power, and leveraged their position to coerce cooperation from opponents with limited other options, refusing to move from such a position under any circumstances. This resulted in them scoring extremely highly across all configurations except those with non-trivial diffu-
sion, which made holding a position with high bargaining power indefinitely untenable.

## Action Selection Strategies

Unlike the fixed types used by my artificial agents, human players were not limited to expressing a single type, and could use their natural intelligence to determine which action to play and move to make at each step. As expected, the human players were generally very flexible, with many of them shifting strategy over the course of the game. Many of them approached the games with a strategy in mind at the beginning, but almost all of them ended up modifying their plans as they learned more about their relative position in the graph and the types of the opponents they were facing.

However for each game played other than the very first they were much more stable in their overall approach. This phenomenon is likely attributed to the players taking some time to learn the intricacies of the game and find good strategies.

Rather than each human player being wholly unique, they can be broadly simulated by a reasonably-sized group of advanced, but fixed types, such as (but not limited to) Dove-Til-Hawk and Hunter. Generally, successful (those which performed above average in their experiment, and/or earned at least half of the total utility available to them) players cooperated early and often, built alliances, and manoeuvred themselves into a good position to take advantage of those alliances.

## Advanced Cooperation and Negotiation

Some players attempted to garner a reputation as being cooperative, both by playing cooperatively and asking their opponents to validate their claims of being a cooperative player in discussion. This strategy was not especially strong, as the strategies they declared in advance were easily exploited by less scrupulous players, especially in shorter simulations where the cooperative players as a whole had less time to set up.

Similarly, the results of overt attempts to get a group of cooperative players to form a clique varied wildly in their success. On some occasions, such a group was formed without incident and scored extremely highly. In others, it was infiltrated by non-cooperative players or prevented from forming by other players for various reasons (including a subset spitefully attempting to prevent others from scoring higher than them).

Even when such a group did form, its circumstances highlighted that maintaining a high level of trust was often difficult, especially in the face of external pressures. In one instance, three players had worked to position themselves at three adjacent nodes in a ring graph, on the understanding that they would constantly exchange the favourable central position. On attaining this position, one player refused to swap with the next player in their plan, leaving the others with little recourse that would not also degrade their own payoffs (there was no diffusion in this experiment) and ultimately allowing the betraying player to outperform all others in that particular experiment.

Although there was no 'resource system' enabling players to perform advanced bargaining, some players were observed offering to allow themselves to be exploited (that is, they would cooperate knowing their opponent would defect) in return for a favourable positional swap. Though uncommon in and of itself, most of these deals, when accepted, were upheld after the swap occurred, and
almost always reverted to strong mutual cooperation after the terms of the deal had been met by both sides.

I theorise that each player's commitment to deals such as these (accepting a potentially less favourable position, and outright allowing another player to exploit you) functions as a costly signal, demonstrating the commitment of each player to their partnership [55]. This struck me as an excellent demonstration that the capacity of human players to go above and beyond any given experimental setup in an effort to communicate and find others with whom they can interact favourably should not be underestimated!

## Chapter 7

## Conclusion

### 7.1 Analysis of Objectives

In order to more accurately assess whether my research has met its stated goals, I will expand upon each objective I identified in the first chapter and discuss to whether, and to what extent, it can be said to have been fulfilled.

Tie together the strengths of different game-theoretic approaches to different scenarios into a single generic system.

As covered in chapter 3, there are many different existent approaches to emulating and studying scenarios in which agents interact, which, on closer inspection, can be incorporated under the umbrella of game theory. For example, approaches from biology tend to be excellent at representing structure, but downplay the importance of individual interaction, while computationallyorientated models such as population protocols focus entirely on transmitting information through interaction, but have no real structure to speak of. My system has both structure and an emphasis on individual interaction, alongside many other desirable properties found in other systems, as detailed in chapter
4. As such, it clearly combines the strengths of multiple existing systems. In addition, my model is field-agnostic, and capable of simulating computational, biological, and other scenarios with equivalent ease.

Provide explanations for why certain strategies do or do not work effectively in different scenarios.

Many specific examples of strategies performing significantly above or below expectations have been discussed throughout chapter 6 , along with justified reasoning which attempts to explain why this may have been observed. A particularly clear example of this was those players who gathered a lot of information from beyond their locality in my human experiments. Although, on paper, having as much information as possible on hand would trivially seem to be a dominant strategy, by viewing it through the lens of reciprocal social interaction, we can understand and explain the observation that it had no significant impact.

Holistically and accurately represent the nature of interactions between entities in different environments.

This has been discussed previously in chapter 4 . The relatively simple swapping system succinctly summarises the vast spectrum of possible negotiations into a single variable, as (outside of randomised diffusion) it requires both involved agents to communicate and consent. As any interaction between any number of entities can be reduced to this relatively simple outcome, it follows that my system can accommodate even the most complex negotiations, using multiple stages and resources. I will discuss this further in chapter 8 .

Eliminate some of the systemic inaccuracies of game theory as a field and by doing so provide an increase in fidelity over existing models.

This was easily my most ambitious goal- even a small increase in the success rate of predictive models including humans could have long-reaching implications. Nonetheless, I believe this has been achieved, albeit for a relatively narrow range of scenarios. At the absolute minimum, I have studied and discussed a small range of quirky human behaviours which were not visible prior to the development of my model- and this new knowledge of a hitherto unstudied phenomenon should allow it to be taken into account going forward, and directly reduce the impact of the unknown on the accuracy and fidelity of our models.

Advance our understanding of how people approach and play games outside of a laboratory setting.

Although my human experimentation was limited, my less formal methods of data collection and observation allowed the perceptions and reasoning behind the behaviours demonstrated by participants in my experiments to be more clearly understood. Though it could be argued that a more formal methodology would have gathered more defined and quantifiable results, I believe that this misses one of the fundamental points of my thesis. As I have argued consistently, human beings simply do not think about their interactions in such rigid terms- to improve our understanding further we must accept this and find new ways to accommodate these phenomena, rather than trying to force human beings to adjust to our existing models and systems.

## Be applied to an extremely large range of situations.

This is closely related to the first point- in order for my model to be applicable to an 'extremely large range' of situations and scenarios, it should incorporate the features and strengths of multiple existing models which allows each of them to model a narrower range of such scenarios independently. I believe I have effectively demonstrated these qualities.

Be more intuitively comprehensible than any previous system, allowing results to be conveyed more clearly.

Although the bulk of the data included in the appendices is extremely unclear, this includes the minutae of the system which can largely be ignored when analysing behaviour at a higher level. The higher level information and derived arguments as presented in chapter 6 should be considered to be more illustrative. Fundamentally, I believe that, in comparison to every other system analysed in chapter 3, it was clear that the system that I have designed is immediately more accessible on several levels and, indeed, more intuitive and comprehensible.

### 7.2 Final Summary

The addition of mobility has a significant impact on the established model of graphical games, creating new tools for the investigation of these scenarios as well as opening up many potential avenues of further investigation. In addition, the incorporation of mobility is guaranteed to have a significant impact on basically every restricted-interaction multi-player game (as demonstrated in Chapters 2, 4 and 5).

By using a widely researched and well-understood baseline model of interaction (i.e. the Prisoners' Dilemma) to underpin my research, the results gathered from both artificial and human players can be compared to those available from other sources. These have shown that the model I have designed is a natural extension of pre-existing models (most clearly, the 'graphical game' model), adding features which genuinely and significantly improve both the range and the fidelity of scenarios it simulates.

My investigations of my new model have revealed many complex inter-relationships between the different qualities which all such scenarios inherently possess, regardless of their apparent complexity. Though there have been no wholly counter-intuitive behaviours observed, many parameters affect each other in complex ways, causing otherwise identical simulations to behave extremely differently with only very small changes (as discussed throughout Chapter 6). Although challenging, a holistic understanding of these strongly emergent effects has been formed and presented throughout this thesis to assist others in their use of this model and its further refinement. Various hypotheses and conjectures from both myself and others were raised (Chapters 3, 4, and 5), tested (Chapters 5 and 6), and discussed (Chapters 6 and 8).

All of the methods and results obtained in the course of this research are reproducible from the information contained within this thesis. The features of the model can be combined with known graph and game theory to create the basic graphical games model. The mobility algorithm which I've detailed and provided psuedocode for can then be added to this to create a full implementation of my graphical games model with mobility, recreating the same system I used throughout this research. After this, one of the basic configurations I've provided in Chapter 4 can be used as a starting point for verifying my results and conducting experiments of one's own using this system.

All in all, the evidence suggests that my model of graphical games with mobility, as I've presented in this research, is at least as good as existing game theoretic models of interaction, type evaluation, and mechanism design, according to the metrics below. In addition, this new model has a number of powerful features and improvements over other systems:

- It can simulate any scenario which can be simulated by pre-existing models, in addition to a wider range of scenarios pertaining to the movement of individual agents among a restricted population.
- It can also be reduced to an established and effective pre-existing model without loss of generality, meaning almost all previously discovered results for graphical games (for example, computational complexity limits) can be applied to it in a straightforward manner. A reduction to a nested payoff matrix or finite state automata has been provided previously.
- It enables most scenarios to be modelled in a more natural and intuitive manner than with pre-existing models, without concern for minutiae such as the exact method of propagation of types through the graph, and avoids under-valuing larger aspects such as the relevance of these amorphous types in the first instance.
- It provides enhanced fidelity at all levels of interaction by making use of the concept of meta-games. Different aspects of graphical games with mobility- such as the act of moving itself- can be modelled with additional simulations using different payoff matrices to study these alternate forms of interaction between agents in more detail.
- It successfully produces, and provides explanations for, behaviours generally resembling those observed in the real-world to a greater extent than pre-existing models.

Once configured correctly for the specific scenario to be studied, the behaviour of human players using this system appears generally analogous to the behaviour of artificial agents, even those using strategically and/or computationally simple types. This observation has been reinforced across a wide range of simulations, each iterated hundreds of times, and from cross-comparison with results from human testing. However, without a much larger study applying this model to multiple complex, real world scenarios, using games far more complex than the Prisoners' Dilemma, it would be incautious to claim this with certainty.

That said, most compellingly, my results would appear to support a testable hypothesis for future research- namely, that human 'irrationality' is pre-dominantly caused by a systemic failure in experiment and mechanism design to take into account human nature as social animals with a variety of different experiences which exist in the real-world. By taking steps to broadly replicate such features and properties, and taking advantage of the many emergent properties that are created because of this, my research shows we can dramatically curtail the divergence between predicted and observed behaviour in human subjects.

## Chapter 8

## Further Research

A number of interesting avenues have been opened up for subsequent research due to this work. As I outlined earlier, there are a large number of natural continuations of this model which will be of interest to anyone attempting to research the graphical gaming model in more detail. For starters, due to the inherently complex nature of the systems being modelled and the emergent behaviour they display, any of the parameters not explicitly investigated could have unexpected interactions with any other parameters (explicitly investigated or otherwise)- though some were discarded as preliminary testing indicated this was unlikely.

The clearest area of expansion would be to design more intelligent player types (perhaps using heuristics, and/or any learning system such as reinforcement learning or neural nets), and to simultaneously remove some of the restrictions on the model which would prevent these players from bringing their new qualities fully to bear. The myopia, which prevents agents from viewing events beyond their neighbourhood, is probably the most limiting restriction- even if the players only use any newfound perception to occasionally observe other games nearby, this would still give them potentially critical information about
some opponents they would otherwise be facing as complete unknowns for the first time.

The limit between what information human and artificial players have access to- even if it is almost always discarded without use by the more perceptive humans- strikes me as the largest remaining limitation to more closely synchronising the behaviours of synthetic and organic agents within these environments.

Related to the idea of enhancing the computational ability and information available to the agents, it would be interesting to see how a particular type, or several closely related types, could make use of the Urn Automata model [37] (or a similar system) in comparison to other types within my model- although the Urn Automata model itself does not seem to be particularly useful or of further academic interest. My intuition is that, with the correct configuration, a variant of the 'Tester' strategy, which defects with some probability based on the amount of defections occurring across the graphical game, could be quite an effective type, as mentioned in Chapter 3.

However, this would raise larger questions of communication, privacy, and type-recognition within the system. Assuming more human-like agents who can communicate freely regardless of distance, what mechanism would these 'Urn-Testers' use to inform the others of their findings and so update the shared memory? If such a mechanism to verify one's type to other agents of the same type can be created, it would certainly be of interest to other agents. For example, hawks could immediately move away upon contact, neutral strategies could freely cooperate, and doves could preferentially swap with each other to exclude non-doves. Much like the question of balancing for predefined collusion, I doubt these behaviours are of rigorous academic interestbut if addressed correctly, the addition more structured and/or specific types
by this method could be an asset to the overall model.
Similarly, with several of the more advanced player types I designed, the question of coalitions, particularly with regard to their formation and efficacy, arose. Though explicitly pre-defined coalitions through the use of player types is probably not of deep academic interest, the question of emergently formed coalitions between players in competition is certainly intriguing. My intuition is that this situation could be well-modelled at present using a meta-game, in which cooperation between two agents indicates a deeper trust and coalition in the lower game.

As a corollary, more advanced player types could specifically be designed to take more strategic advantage of the mobile graphical games model, and their ability to perform under a variety of situations assessed. For instance, the distribution of types over time could be tracked, and this information extrapolated for use by agents within a given graphical game. Learning agents could themselves gain an understanding of how the different settings affect the execution of the simulation, and learn to read those settings, either directly (from the graphical game itself) or indirectly (for instance, by keeping track of how often its preferences have been ignored to calculate the chance of diffusion), and modify their behaviour accordingly to gain strategic advantage.

Experiments could be conducted using a combination of human and artificial agents. Configuring the parameters of such a simulation would be challenging, due to the differing innate abilities and affinities of these different types of players. For instance, either myopia would have to be removed for the artificial agents, necessitating the creation of more complex types, or added for the human agents, necessitating a more complex experimental setup. However, this may help bridge the behavioural gap between these player types, and consequently lead to new insights into both the model and general behaviours
of these entities.
There are many possible applications to wider models of graphical computation, or even just multi-agent computation. For instance, if the agents had some limited knowledge of the graphical structure, such as the ability to determine the shortest route to a specified node, they could move towards or away from it depending on the results of their games. This could provide a novel system for examining or understanding classical, or perhaps novel, computational problems.

Topological mutability, as discussed previously in 'Constant Features of the Model', would certainly be an interesting addition to this model. I do not think that it would increase the clarity of the model or enable it to model novel scenarios, as it's much harder to see what exactly such an addition would represent (especially compared to other potential features). However, there may be applications in other areas such as network theory and general computational complexity.

My intuition is that there exists a subclass of problems to which it could provide an elegant solution, but outside of this such an addition would do little more than create confusion and work to reduce the graph to a highlyconnected, amorphous cluster as discussed in Chapter 6. This could be mitigated by the addition of more intelligent agents which were able to deploy specific alterations in a focused and meaningful manner.

There are many natural games and situations which contain more than two players. Though these can be modelled with the system I've designed, there may be ways to further clarify such systems so that they are as easy to understand and use as possible. For example, hyperedges could be added to connect multiple players through the same element (which would still be representing a game or single mode of interaction). Likewise, rulesets which further dis-
tinguish or interweave connections between the edges along which games are played and movement options may be of interest, as they could allow more complex structures to be simulated (for example, incurring a penalty to payoff when one wishes to move).

The ideas of varying connection and structural power from Cook and Yamagishi [29] could be expanded and re-applied to graphical games with mobility. More simply, agents could be allocated some resource/s which they must choose to distribute between all the games they play in each (synchronous) round, such as 'money' in an Ultimatum Game, or a limited number of uses of each strategy. This would bring the model back towards the ' $n$ players at nodes' form originally envisaged by Kearns [11], but could probably be structured in such a way as to retain most of the advantages of more clearly defined binary links between agents.

In my human experiments, cooperative players frequently expressed a desire to 'push away' non-cooperative players, while those players often wanted to 'jump' into the middle of a dense pack of cooperative players. A model with more advanced movement rules, including but not limited to players moving each other, players moving along a structure not identical to the graph along which games are played, players having some mechanism to move to a specific position, or players having some capacity to alter the structure of the graph, could be of deep tactical interest if time was spent developing artificial players which could take full advantage of it, or with further experiments conducted with human players.

As with other aspects of the decision making which is made by the players, this could either be incorporated into the payoff matrix of an existing game, by multiplying it through by the number of new options available, or considered as a separate choice occurring simultaneously or immediately afterwards.

Again, more complex strategies would be needed to take advantage of this new capability in an intelligent (that is, non-random) manner.

Though the reluctance of some humans to make use of mobility for various reasons can be generally understood as aversion to risk, further investigation may produce more detailed hypotheses more closely related to this model. A simple study focused on this behaviour should be able to draw out and identify more accurately why it was occurring, and under what circumstances it can be suppressed or exacerbated.

Mobility is clearly the most interesting feature in the model (as without it, few of the other features would have a meaningful impact), and now that the basic form has been researched and documented, it should open the way for different types of mobility to be studied, such that the type used can be chosen to help model a given scenario. For instance, the concepts I have developed could be joined with those from spatial games [39], resulting in a continuous 2-dimensional space with agents interacting within a fixed radius. Weighting could be used to increase the value of games played with opponents closer to a given agent, additional costs could be invoked for operating in a 'dense' environment, and so forth.

In final summary, the full power of this model should be realised when applied to a situation with a highly complex graphical structure, highly complex game, and highly complex player types- such as those found in many real-world, modern games. If these situations could be accurately translated into this model, this model should be able to identify useful real-world information, which could, in turn, be used to identify successful strategies within those situations.

## Chapter 9

## Appendix

This chapter contains a selection of data extracted from my program and used to support the results and conclusions I have drawn.

### 9.1 Populations Used

| Pop <br> Label | Type 1 | $\begin{array}{r} \text { \# of } \\ \text { Type } \\ 1 \\ \hline \end{array}$ | Type 2 | \# of <br> Type | Type 3 | $\begin{array}{r} \text { \# of } \\ \text { Type } \\ 3 \\ \hline \end{array}$ | Type 4 | \# of <br> Type <br> 4 | Type 5 | $\begin{aligned} & \text { \# of } \\ & \text { Type } \\ & 5 \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | HAWK- <br> TIL- <br> HAWK | 4 | DOVE- <br> TIL- <br> HAWK | 2 | $\begin{aligned} & \text { RAN- } \\ & \text { DOM } \end{aligned}$ | 1 | $\begin{aligned} & \text { TIT- } \\ & \text { FOR- } \\ & \text { TAT } \end{aligned}$ | 2 | $\begin{aligned} & \text { PAV- } \\ & \text { LOV } \end{aligned}$ | 1 |
| B | HAWK- <br> TIL- <br> HAWK | 3 | $\begin{aligned} & \text { DOVE- } \\ & \text { TIL- } \\ & \text { HAWK } \end{aligned}$ | 3 | $\begin{aligned} & \text { RAN- } \\ & \text { DOM } \\ & \hline \end{aligned}$ | 1 | $\begin{aligned} & \text { TIT- } \\ & \text { FOR- } \\ & \text { TAT } \end{aligned}$ | 2 | $\begin{aligned} & \text { PAV- } \\ & \text { LOV } \end{aligned}$ | 1 |
| C | HAWK- <br> TIL- <br> HAWK | 2 | $\begin{aligned} & \text { DOVE- } \\ & \text { TIL- } \\ & \text { HAWK } \end{aligned}$ | 4 | $\begin{aligned} & \text { RAN- } \\ & \text { DOM } \\ & \hline \end{aligned}$ | 1 | $\begin{aligned} & \text { TTT- } \\ & \text { FOR- } \\ & \text { TAT } \end{aligned}$ | 2 | $\begin{aligned} & \text { PAV- } \\ & \text { LOV } \end{aligned}$ | 1 |
| D | EVER- <br> HAWK | 2 | $\begin{aligned} & \text { EVER- } \\ & \text { DOVE } \end{aligned}$ | 2 | RAN- <br> DOM | 3 | $\begin{aligned} & \text { TIT- } \\ & \text { FOR- } \\ & \text { TAT } \end{aligned}$ | 1 | $\begin{aligned} & \text { PAV- } \\ & \text { LOV } \end{aligned}$ | 2 |
| E | EVER- HAWK | 2 | $\begin{aligned} & \text { EVER- } \\ & \text { DOVE } \end{aligned}$ | 2 | $\begin{aligned} & \text { DOVE- } \\ & \mathrm{N}- \\ & \text { SWITCH } \end{aligned}$ | 2 | HAWK- <br> TIL- <br> HAWK | 2 | $\begin{aligned} & \text { DOVE- } \\ & \text { TIL- } \\ & \text { HAWK } \end{aligned}$ | 2 |
| F | EVERDOVE | 2 | HAWK- <br> N- <br> SWITCH | 1 | DOVE- <br> N- <br> SWITCH | 1 | $\begin{aligned} & \text { TTT- } \\ & \text { FOR- } \\ & \text { TAT } \end{aligned}$ | 2 | $\begin{aligned} & \text { PAV- } \\ & \text { LOV } \end{aligned}$ | 4 |

Table 9.1: The expanded population information used in the experiments detailed below.

### 9.2 Configurations

| Exp. ID | Graph | Timing | Duration (Rounds) | Diffusion (\%) | Pop. Set | Distribution |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Ring | Synchronous | 10 | 0.0 | A | Clustered |
| 2 | Ring | Synchronous | 10 | 0.0 | A | Distributed |
| 3 | Ring | Synchronous | 10 | 0.0 | B | Clustered |
| 4 | Ring | Synchronous | 10 | 0.0 | B | Distributed |
| 5 | Ring | Synchronous | 10 | 0.0 | C | Clustered |
|  | Ring | Synchronous | 10 | 0.0 | C | Distributed |
| 7 | Ring | Synchronous | 10 | 0.0 | D | Clustered |
|  | Ring | Synchronous | 10 | 0.0 | D | Distributed |
| 9 | Ring | Synchronous | 10 | 0.0 | E | Clustered |
| 10 | Ring | Synchronous | 10 | 0.0 | E | Distributed |
| 11 | Ring | Synchronous | 10 | 0.0 | F | Clustered |
| 12 | Ring | Synchronous | 10 | 0.0 | F | Distributed |
| 13 | Ring | Synchronous | 10 | 5.0 | A | Clustered |
| 14 | Ring | Synchronous | 10 | 5.0 | A | Distributed |
| 15 | Ring | Synchronous | 10 | 5.0 | B | Clustered |
| 16 | Ring | Synchronous | 10 | 5.0 | ${ }^{\text {B }}$ | Distributed |
| 17 | Ring | Synchronous | 10 | 5.0 | C | Clustered |
| 18 | Ring | Synchronous | 10 | 5.0 | C | Distributed |
| 19 | Ring | Synchronous | 10 | 5.0 | D | Clustered |
| 20 | Ring | Synchronous | 10 | 5.0 | D | Distributed |
| 21 | Ring | Synchronous | 10 | 5.0 | E | Clustered |
| 22 | Ring | Synchronous | 10 | 5.0 | E | Distributed |
| 23 | Ring | Synchronous | 10 | 5.0 | F | Clustered |
| 24 | Ring | Synchronous | 10 | 5.0 | F | Distributed |
| 25 | Ring | Synchronous | 10 | 20.0 | A | Clustered |
| 26 | Ring | Synchronous | 10 | 20.0 | A | Distributed |
| 27 | Ring | Synchronous | 10 | 20.0 | B | Clustered |
| 28 | Ring | Synchronous | 10 | 20.0 | B | Distributed |
| 29 | Ring | Synchronous | 10 | 20.0 | C | Clustered |
| 30 | Ring | Synchronous | 10 | 20.0 | C | Distributed |
| 31 | Ring | Synchronous | 10 | 20.0 | D | Clustered |
| 32 | Ring | Synchronous | 10 | 20.0 | D | Distributed |
| 33 | Ring | Synchronous | 10 | 20.0 | E | Clustered |
| 34 | Ring | Synchronous | 10 | 20.0 | E | Distributed |
| 35 | Ring | Synchronous | 10 | 20.0 | F | Clustered |
| 36 | Ring | Synchronous | 10 | 20.0 | F | Distributed |
| 37 | Ring | Synchronous | 10 | 50.0 | A | Clustered |
| 38 | Ring | Synchronous | 10 | 50.0 | A | Distributed |
| 39 | Ring | Synchronous | 10 | 50.0 | B | Clustered |
| 40 | Ring | Synchronous | 10 | 50.0 | B | Distributed |
| 41 | Ring | Synchronous | 10 | 50.0 | C | Clustered |
| 42 | Ring | Synchronous | 10 | 50.0 | C | Distributed |
| 43 | Ring | Synchronous | 10 | 50.0 | D | Clustered |
| 44 | Ring | Synchronous | 10 | 50.0 | D | Distributed |
| 45 | Ring | Synchronous | 10 | 50.0 | E | Clustered |
| 46 47 | ${ }_{\text {Ring }}$ | ${ }_{\text {Synchronous }}$ | 10 10 | 50.0 50.0 | $\stackrel{\text { E }}{\text { F }}$ | ${ }^{\text {Distributed }}$ Clustered |
| 48 | Ring | Synchronous | 10 | 50.0 | F | Distributed |
| 49 | Ring | Synchronous | 25 | 0.0 | A | Clustered |
| 50 | Ring | Synchronous | 25 | 0.0 | A | Distributed |
| 51 | Ring | Synchronous | 25 | 0.0 | B | Clustered |
| 52 | Ring | Synchronous | 25 | 0.0 | B | Distributed |
| 53 | Ring | Synchronous | 25 | 0.0 | C | Clustered |
| 54 | Ring | Synchronous | 25 | 0.0 | C | Distributed |
| 55 | Ring | Synchronous | 25 | 0.0 | D | Clustered |
| 56 | Ring | Synchronous | 25 | 0.0 | D | Distributed |
| 57 58 58 | Ring | $\stackrel{\text { Synchronous }}{\text { Synchronous }}$ | $\stackrel{25}{25}$ | 0.0 0.0 | $\underset{\mathrm{E}}{\mathrm{E}}$ | Clustered |
| 59 | Ring | Synchronous | 25 | 0.0 | F | Clustered |
| 60 | Ring | Synchronous | 25 | 0.0 | F | Distributed |
| 61 | Ring | Synchronous | 25 | 5.0 | A | Clustered |
| 62 | Ring | Synchronous | 25 | 5.0 | A | Distributed |
| 63 | Ring | Synchronous | 25 | 5.0 | B | Clustered |
| 64 | Ring | Synchronous | 25 | 5.0 | B | Distributed |
| 65 | Ring | Synchronous | 25 | 5.0 | C | Clustered |
| 66 | Ring | Synchronous | 25 | 5.0 | C | Distributed |
| 67 | Ring | Synchronous | 25 | 5.0 | D | Clustered |
| 68 | Ring | Synchronous Synchronous | 25 25 | 5.0 5.0 | D | $\xrightarrow{\text { Distributed }}$ |
| 70 | Ring | Synchronous | 25 | 5.0 | E | Distributed |
| 71 | Ring | Synchronous | 25 | 5.0 | $\underset{\text { F }}{\text { F }}$ | Clustered |
| 72 | Ring | Synchronous | 25 | 5.0 | F | Distributed |
| 74 | Ring | Synchronous | 25 | 20.0 | A | Distributed |
| 75 | Ring | Synchronous | 25 | 20.0 | B | Clustered |
| 76 | Ring | Synchronous | 25 | 20.0 | B | Distributed |
| 77 | Ring | Synchronous | 25 | 20.0 | ${ }^{\text {C }}$ | Clustered |
| 78 | Ring | Synchronous | 25 | 20.0 | C | Distributed |
| 79 | Ring | Synchronous | 25 | 20.0 | D | Clustered |
| 80 81 | Ring | Synchronous | 25 25 | 20.0 | D | Distributed |
| 82 | Ring | Synchronous | 25 | 20.0 | E | Distributed |
| 83 | Ring | Synchronous | 25 | 20.0 | F | Clustered |
| 84 | Ring | Synchronous | 25 | 20.0 | F | Distributed |
| 85 | Ring | Synchronous | 25 | 50.0 | A | Clustered |
| 86 | Ring | Synchronous | 25 | 50.0 | A | Distributed |
| 87 88 | Ring | Synchronous | 25 | 50.0 | B | Clustered |
| 88 | Ring Ring | $\xrightarrow[\text { Synchronous }]{\text { Synchronous }}$ | 25 25 | 50.0 50.0 | $\stackrel{\text { B }}{\text { C }}$ | $\xrightarrow{\text { Distributed }}$ |
| 90 | Ring | Synchronous | 25 | 50.0 | C | Distributed |
| 91 | Ring | Synchronous | 25 | 50.0 | D | Clustered |
| 92 | Ring | Synchronous | 25 | 50.0 | D | Distributed |
| 93 | Ring Ring | $\xrightarrow[\text { Synchronous }]{\text { Synchronous }}$ | 25 25 | 50.0 50.0 | $\underset{\mathrm{E}}{\mathrm{E}}$ | Clustered Distributed |
| 95 | Ring | Synchronous | 25 | 50.0 | F | Clustered |
| 96 | Ring | Synchronous | 25 | 50.0 | F | Distributed |
| 97 98 | Ring | $\stackrel{\text { Synchronous }}{\text { Synchronous }}$ | 50 50 | 0.0 0.0 | A | Clustered Distributed |
| 99 | Ring | Synchronous | 50 | 0.0 | B | Clustered |
| 100 | Ring | Synchronous | 50 | 0.0 | B | Distributed |
| 101 | Ring | Synchronous | 50 | 0.0 | C | Clustered |
| 102 | Ring | Synchronous | 50 | 0.0 | C | Distributed |
| 103 | Ring Ring | $\xrightarrow[\text { Synchronous }]{\text { Synchronous }}$ | 50 50 | 0.0 0.0 | D | Clustered |
| 105 | Ring | Synchronous | 50 | 0.0 | E | Clustered |
| 106 | Ring | Synchronous | 50 | 0.0 | E | Distributed |
| 107 | Ring | Synchronous | 50 | 0.0 | F | Clustered |



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| 0.0 | A | Clustered |
| :---: | :---: | :---: |
| 0.0 | A | Distributed |
| 0.0 | B | Clustered |
| 0.0 | B | Distributed |
| 0.0 | C | Clustered |
| 0.0 | C | Distributed |
| 0.0 | D | Clustered |
| 0.0 | D | Distributed |
| 0.0 | E | Clustered |
| 0.0 | E | Distributed |
| 0.0 | F | Clustered |
| 0.0 | F | Distributed |
| 5.0 | A | Clustered |
| 5.0 | A | Distributed |
| 5.0 | B | Clustered |
| 5.0 | B | Distributed |
| 5.0 | C | Clustered |
| 5.0 | C | Distributed |
| 5.0 | D | Clustered |
| 5.0 | D | Distributed |
| 5.0 | E | Clustered |
| 5.0 | E | Distributed |
| 5.0 | F | Clustered |
| 5.0 | F | Distributed |
| 20.0 | A | Clustered |
| 20.0 | A | Distributed |
| 20.0 | B | Clustered |
| 20.0 | B | Distributed |
| 20.0 | C | Clustered |
| 20.0 | C | Distributed |
| 20.0 | D | Clustered |
| 20.0 | D | Distributed |
| 20.0 | E | Clustered |
| 20.0 | E | Distributed |
| 20.0 | F | Clustered |
| 20.0 | F | Distributed |
| 50.0 | A | Clustered |
| 50.0 | A | Distributed |
| 50.0 | B | Clustered |
| 50.0 | B | Distributed |
| 50.0 | C | Clustered |
| 50.0 | C | Distributed |
| 50.0 | D | Clustered |
| 50.0 | D | Distributed |
| 50.0 | E | Clustered |
| 50.0 | E | Distributed |
| 50.0 | F | Clustered |
| 50.0 | F | Distributed |
| 0.0 | A | Clustered |
| 0.0 | A | Distributed |
| 0.0 | B | Clustered |
| 0.0 | B | Distributed |
| 0.0 | C | Clustered |
| 0.0 | C | Distributed |
| 0.0 | D | Clustered |
| 0.0 | D | Distributed |
| 0.0 | E | Clustered |
| 0.0 | E | Distributed |
| 0.0 | F | Clustered |
| 0.0 | F | Distributed |
| 5.0 | A | Clustered |
| 5.0 | A | Distributed |
| 5.0 | B | Clustered |
| 5.0 | B | Distributed |
| 5.0 | C | Clustered |
| 5.0 | C | Distributed |
| 5.0 | D | Clustered |
| 5.0 | D | Distributed |
| 5.0 | E | Clustered |
| 5.0 | E | Distributed |
| 5.0 | F | Clustered |
| 5.0 | F | Distributed |
| 20.0 | A | Clustered |
| 20.0 | A | Distributed |
| 20.0 | B | Clustered |
| 20.0 | B | Distributed |
| 20.0 | C | Clustered |
| 20.0 | C | Distributed |
| 20.0 | D | Clustered |
| 20.0 | D | Distributed |
| 20.0 | E | Clustered |
| 20.0 | E | Distributed |
| 20.0 | F | Clustered |
| 20.0 | F | Distributed |
| 50.0 | A | Clustered |
| 50.0 | A | Distributed |
| 50.0 | B | Clustered |
| 50.0 | B | Distributed |
| 50.0 | C | Clustered |
| 50.0 | C | Distributed |
| 50.0 | D | Clustered |
| 50.0 | D | Distributed |
| 50.0 | E | Clustered |
| 50.0 | E | Distributed |
| 50.0 | F | Clustered |
| 50.0 | F | Distributed |
| 0.0 | A | Clustered |
| 0.0 | A | Distributed |
| 0.0 | B | Clustered |
| 0.0 | B | Distributed |
| 0.0 | C | Clustered |
| 0.0 | C | Distributed |
| 0.0 | D | Clustered |
| 0.0 | D | Distributed |
| 0.0 | E | Clustered |
| 0.0 | E | Distributed |
| 0.0 | F | Clustered |
| 0.0 | F | Distributed |
| 5.0 | A | Clustered |
| 5.0 | A | Distributed |
| 5.0 | B | Clustered |
| 5.0 5.0 | $\stackrel{\mathrm{C}}{\mathrm{C}}$ | ${ }_{\text {Distributed }}^{\text {Clustered }}$ |








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| 0.0 | D | Clustered |
| :---: | :---: | :---: |
| 0.0 | D | Distributed |
| 0.0 | E | Clustered |
| 0.0 | E | Distributed |
| 0.0 | F | Clustered |
| 0.0 | F | Distributed |
| 5.0 | A | Clustered |
| 5.0 | A | Distributed |
| 5.0 | B | Clustered |
| 5.0 | B | Distributed |
| 5.0 | C | Clustered |
| 5.0 | C | Distributed |
| 5.0 | D | Clustered |
| 5.0 | D | Distributed |
| 5.0 | E | Clustered |
| 5.0 | E | Distributed |
| 5.0 | F | Clustered |
| 5.0 | F | Distributed |
| 20.0 | A | Clustered |
| 20.0 | A | Distributed |
| 20.0 | B | Clustered |
| 20.0 | B | Distributed |
| 20.0 | C | Clustered |
| 20.0 | C | Distributed |
| 20.0 | D | Clustered |
| 20.0 | D | Distributed |
| 20.0 | E | Clustered |
| 20.0 | E | Distributed |
| 20.0 | F | Clustered |
| 20.0 | F | Distributed |
| 50.0 | A | Clustered |
| 50.0 | A | Distributed |
| 50.0 | B | Clustered |
| 50.0 | B | Distributed |
| 50.0 | C | Clustered |
| 50.0 | C | Distributed |
| 50.0 | D | Clustered |
| 50.0 | D | Distributed |
| 50.0 | E | Clustered |
| 50.0 | E | Distributed |
| 50.0 | F | Clustered |
| 50.0 | F | Distributed |
| 0.0 | A | Clustered |
| 0.0 | A | Distributed |
| 0.0 | B | Clustered |
| 0.0 | B | Distributed |
| 0.0 | C | Clustered |
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| 0.0 | D | Clustered |
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| 0.0 | E | Clustered |
| 0.0 | E | Distributed |
| 0.0 | F | Clustered |
| 0.0 | F | Distributed |
| 5.0 | A | Clustered |
| 5.0 | A | Distributed |
| 5.0 | B | Clustered |
| 5.0 | B | Distributed |
| 5.0 | C | Clustered |
| 5.0 | C | Distributed |
| 5.0 | D | Clustered |
| 5.0 | D | Distributed |
| 5.0 | E | Clustered |
| 5.0 | E | Distributed |
| 5.0 | F | Clustered |
| 5.0 | F | Distributed |
| 20.0 | A | Clustered |
| 20.0 | A | Distributed |
| 20.0 | B | Clustered |
| 20.0 | B | Distributed |
| 20.0 | C | Clustered |
| 20.0 | C | Distributed |
| 20.0 | D | Clustered |
| 20.0 | D | Distributed |
| 20.0 | E | Clustered |
| 20.0 | E | Distributed |
| 20.0 | F | Clustered |
| 20.0 | F | Distributed |
| 50.0 | A | Clustered |
| 50.0 | A | Distributed |
| 50.0 | B | Clustered |
| 50.0 | B | Distributed |
| 50.0 | C | Clustered |
| 50.0 | C | Distributed |
| 50.0 | D | Clustered |
| 50.0 | D | Distributed |
| 50.0 | E | Clustered |
| 50.0 | E | Distributed |
| 50.0 | F | Clustered |
| 50.0 | F | Distributed |
| 0.0 | A | Clustered |
| 0.0 | A | Distributed |
| 0.0 | B | Clustered |
| 0.0 | B | Distributed |
| 0.0 | ${ }^{\text {C }}$ | Clustered |
| 0.0 | C | Distributed |
| 0.0 | D | Clustered |
| 0.0 | D | Distributed |
| 0.0 | E | Clustered |
| 0.0 | E | Distributed |
| 0.0 | F | Clustered |
| 0.0 | F | Distributed |
| 5.0 | A | Clustered |
| 5.0 | A | Distributed |
| 5.0 | B | Clustered |
| 5.0 | B | Distributed |
| 5.0 | C | Clustered |
| 5.0 | C | Distributed |
| 5.0 | D | Clustered |
| 5.0 | D | Distributed |
| 5.0 | E | Clustered |
| 5.0 | E | Distributed |
| 5.0 | F | Clustered |


| 1464 | Milgram Ring | Asynchronous | 50 | 5.0 | F | Distributed |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1465 | Milgram Ring | Asynchronous | 50 | 20.0 | A | Clustered |
| 1466 | Milgram Ring | Asynchronous | 50 | 20.0 | A | Distributed |
| 1467 | Milgram Ring | Asynchronous | 50 | 20.0 | B | Clustered |
| 1468 | Milgram Ring | Asynchronous | 50 | 20.0 | B | Distributed |
| 1469 | Milgram Ring | Asynchronous | 50 | 20.0 | C | Clustered |
| 1470 | Milgram Ring | Asynchronous | 50 | 20.0 | C | Distributed |
| 1471 | Milgram Ring | Asynchronous | 50 | 20.0 | D | Clustered |
| 1472 | Milgram Ring | Asynchronous | 50 | 20.0 | D | Distributed |
| 1473 | Milgram Ring | Asynchronous | 50 | 20.0 | E | Clustered |
| 1474 | Milgram Ring | Asynchronous | 50 | 20.0 | E | Distributed |
| 1475 | Milgram Ring | Asynchronous | 50 | 20.0 | F | Clustered |
| 1476 | Milgram Ring | Asynchronous | 50 | 20.0 | F | Distributed |
| 1477 | Milgram Ring | Asynchronous | 50 | 50.0 | A | Clustered |
| 1478 | Milgram Ring | Asynchronous | 50 | 50.0 | A | Distributed |
| 1479 | Milgram Ring | Asynchronous | 50 | 50.0 | B | Clustered |
| 1480 | Milgram Ring | Asynchronous | 50 | 50.0 | B | Distributed |
| 1481 | Milgram Ring | Asynchronous | 50 | 50.0 | C | Clustered |
| 1482 | Milgram Ring | Asynchronous | 50 | 50.0 | C | Distributed |
| 1483 | Milgram Ring | Asynchronous | 50 | 50.0 | D | Clustered |
| 1484 | Milgram Ring | Asynchronous | 50 | 50.0 | D | Distributed |
| 1485 | Milgram Ring | Asynchronous | 50 | 50.0 | E | Clustered |
| 1486 | Milgram Ring | Asynchronous | 50 | 50.0 | E | Distributed |
| 1487 | Milgram Ring | Asynchronous | 50 | 50.0 | F | Clustered |
| 1488 | Milgram Ring | Asynchronous | 50 | 50.0 | F | Distributed |
| 1489 | Milgram Ring | Asynchronous | 100 | 0.0 | A | Clustered |
| 1490 | Milgram Ring | Asynchronous | 100 | 0.0 | A | Distributed |
| 1491 | Milgram Ring | Asynchronous | 100 | 0.0 | B | Clustered |
| 1492 | Milgram Ring | Asynchronous | 100 | 0.0 | B | Distributed |
| 1493 | Milgram Ring | Asynchronous | 100 | 0.0 | C | Clustered |
| 1494 | Milgram Ring | Asynchronous | 100 | 0.0 | C | Distributed |
| 1495 | Milgram Ring | Asynchronous | 100 | 0.0 | D | Clustered |
| 1496 | Milgram Ring | Asynchronous | 100 | 0.0 | D | Distributed |
| 1497 | Milgram Ring | Asynchronous | 100 | 0.0 | E | Clustered |
| 1498 | Milgram Ring | Asynchronous | 100 | 0.0 | E | Distributed |
| 1499 | Milgram Ring | Asynchronous | 100 | 0.0 | F | Clustered |
| 1500 | Milgram Ring | Asynchronous | 100 | 0.0 | F | Distributed |
| 1501 | Milgram Ring | Asynchronous | 100 | 5.0 | A | Clustered |
| 1502 | Milgram Ring | Asynchronous | 100 | 5.0 | A | Distributed |
| 1503 | Milgram Ring | Asynchronous | 100 | 5.0 | B | Clustered |
| 1504 | Milgram Ring | Asynchronous | 100 | 5.0 | B | Distributed |
| 1505 | Milgram Ring | Asynchronous | 100 | 5.0 | C | Clustered |
| 1506 | Milgram Ring | Asynchronous | 100 | 5.0 | C | Distributed |
| 1507 | Milgram Ring | Asynchronous | 100 | 5.0 | D | Clustered |
| 1508 | Milgram Ring | Asynchronous | 100 | 5.0 | D | Distributed |
| 1509 | Milgram Ring | Asynchronous | 100 | 5.0 | E | Clustered |
| 1510 | Milgram Ring | Asynchronous | 100 | 5.0 | E | Distributed |
| 1511 | Milgram Ring | Asynchronous | 100 | 5.0 | F | Clustered |
| 1512 | Milgram Ring | Asynchronous | 100 | 5.0 | F | Distributed |
| 1513 | Milgram Ring | Asynchronous | 100 | 20.0 | A | Clustered |
| 1514 | Milgram Ring | Asynchronous | 100 | 20.0 | A | Distributed |
| 1515 | Milgram Ring | Asynchronous | 100 | 20.0 | B | Clustered |
| 1516 | Milgram Ring | Asynchronous | 100 | 20.0 | ${ }^{\text {B }}$ | Distributed |
| 1517 | Milgram Ring | Asynchronous | 100 | 20.0 | C | Clustered |
| 1518 | Milgram Ring | Asynchronous | 100 | 20.0 | C | Distributed |
| 1519 | Milgram Ring | Asynchronous | 100 | 20.0 | D | Clustered |
| 1520 | Milgram Ring | Asynchronous | 100 | 20.0 | D | Distributed |
| 1521 | Milgram Ring | Asynchronous | 100 | 20.0 | E | Clustered |
| 1522 | Milgram Ring | Asynchronous | 100 | 20.0 | E | Distributed |
| 1523 | Milgram Ring | Asynchronous | 100 | 20.0 | F | Clustered |
| 1524 | Milgram Ring | Asynchronous | 100 | 20.0 | F | Distributed |
| 1525 | Milgram Ring | Asynchronous | 100 | 50.0 | A | Clustered |
| 1526 | Milgram Ring | Asynchronous | 100 | 50.0 | A | Distributed |
| 1527 1528 | Milgram Ring Milgram Ring | Asynchronous Asynchronous | 100 100 | 50.0 50.0 | B ${ }_{\text {B }}$ | Clustered Distributed |
| 1529 | Milgram Ring | Asynchronous | 100 | 50.0 | C | Clustered |
| 1530 | Milgram Ring | Asynchronous | 100 | 50.0 | C | Distributed |
| 1531 | Milgram Ring | Asynchronous | 100 | 50.0 | D | Clustered |
| 1532 | Milgram Ring | Asynchronous | 100 | 50.0 | D | Distributed |
| 1533 | Milgram Ring | Asynchronous | 100 | 50.0 | E | Clustered |
| 1534 | Milgram Ring | Asynchronous | 100 | 50.0 | E | Distributed |
| 1535 | Milgram Ring | Asynchronous | 100 | 50.0 | F | Clustered |
| 1536 | Milgram Ring | Asynchronous | 100 | 50.0 | F | Distributed |

tently throughout these data. The Prisoners' Dilemma payoff matrix from Chapter 2 was used in each instance.

### 9.3 Scores \& Deviations

| Exp. 1D | Type A |  | Type B |  | Type C |  | Type D |  | Type E |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | SD | Mean | SD | Mean | SD | Mean | SD | Mean | SD |
| 1 | -3.82 | 0.38 | -5.84 | 0.92 | -5.36 | 1.13 | -5.36 | 0.45 | -3.25 | 1.15 |
| 2 | -3.13 | 0.21 | -7.03 | 0.67 | -4.99 | 1.10 | -5.72 | 0.55 | -4.28 | 0.81 |
| 3 | -3.01 | 0.37 | -4.72 | 0.71 | -5.50 | 1.13 | -5.00 | 0.68 | -3.09 | 0.50 |
| 4 | -2.24 | 0.34 | -6.02 | 0.50 | -3.89 | 0.42 | -4.97 | 0.37 | -3.38 | 0.92 |
| 5 | -2.08 | 0.43 | -4.19 | 0.62 | -4.73 | 0.93 | -4.26 | 0.53 | -2.24 | 0.86 |
| 6 | -1.75 | 0.52 | -4.46 | 0.52 | -3.53 | 0.81 | -3.88 | 0.49 | -3.08 | 1.00 |
| 7 | -3.80 | 0.10 | -3.97 | 0.31 | -4.24 | 0.35 | -3.91 | 0.44 | -4.23 | 0.52 |
| 8 | -1.44 | 0.22 | -7.52 | 0.84 | -4.76 | 0.30 | -4.60 | 0.88 | -3.74 | 0.29 |
| 9 | -3.28 | 0.80 | -3.45 | 0.70 | -4.58 | 1.46 | -2.50 | 0.74 | -4.28 | 1.14 |
| 10 | -0.78 | 0.32 | -6.92 | 0.67 | -4.85 | 0.80 | -1.65 | 0.51 | -5.59 | 0.39 |
| 11 | -4.42 | 0.78 | -2.01 | 0.83 | -3.52 | 1.09 | -3.77 | 0.90 | -2.55 | 0.84 |
| 12 | -6.96 | 0.93 | -2.58 | 0.27 | -7.66 | 2.02 | -4.60 | 0.67 | -2.44 | 0.46 |
| 13 | -3.83 | 0.20 | -5.88 | 0.91 | -6.31 | 0.88 | -4.69 | 0.74 | -3.13 | 1.16 |
| 14 | -3.13 | 0.28 | -7.16 | 0.89 | -4.29 | 0.98 | -6.00 | 0.41 | -4.45 | 0.37 |
| 15 | -2.97 | 0.51 | -5.03 | 0.93 | -4.35 | 1.52 | -4.75 | 0.74 | -3.01 | 1.23 |
| 16 | -2.28 | 0.51 | -6.01 | 0.57 | -3.57 | 1.00 | -4.59 | 0.39 | -3.78 | 0.65 |
| 17 | -2.11 | 0.36 | -4.00 | 0.34 | -4.13 | 0.70 | -4.54 | 0.62 | -2.99 | 0.42 |
| 18 | -1.41 | 0.41 | -4.72 | 0.62 | -3.61 | 1.39 | -3.78 | 0.37 | -2.76 | 1.19 |
| 19 | -3.76 | 0.42 | -4.01 | 1.16 | -3.94 | 0.34 | -3.68 | 0.80 | -4.29 | 0.64 |















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|  | 0.5 |






































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[^5]| 1515 | -2.39 | 0.14 | -5.48 | 0.14 | -3.71 | 0.29 | -5.25 | 0.22 | -2.77 | 0.25 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1516 | -2.37 | 0.16 | -5.40 | 0.26 | -3.85 | 0.22 | -5.33 | 0.27 | -2.70 | 0.33 |
| 1517 | -1.79 | 0.14 | -4.51 | 0.09 | -2.99 | 0.29 | -4.32 | 0.27 | -2.02 | 0.28 |
| 1518 | -1.68 | 0.13 | -4.49 | 0.17 | -3.06 | 0.22 | -4.45 | 0.20 | -1.80 | 0.25 |
| 1519 | -2.94 | 0.18 | -6.42 | 0.25 | -4.58 | 0.04 | -6.28 | 0.26 | -3.21 | 0.18 |
| 1520 | -2.84 | 0.15 | -6.38 | 0.20 | -4.64 | 0.13 | -6.21 | 0.31 | -3.21 | 0.19 |
| 1521 | -1.87 | 0.17 | -5.11 | 0.17 | -4.95 | 0.17 | -1.97 | 0.16 | -4.90 | 0.15 |
| 1522 | -1.94 | 0.14 | -5.13 | 0.24 | -4.92 | 0.09 | -1.89 | 0.20 | -4.75 | 0.17 |
| 1523 | -5.93 | 0.29 | -2.54 | 0.20 | -5.90 | 0.36 | -5.70 | 0.24 | -2.57 | 0.06 |
| 1524 | -5.95 | 0.22 | -2.44 | 0.16 | -5.96 | 0.36 | -5.77 | 0.21 | -2.56 | 0.12 |
| 1525 | -3.06 | 0.17 | -6.51 | 0.18 | -4.72 | 0.16 | -6.15 | 0.28 | -3.27 | 0.18 |
| 1526 | -3.08 | 0.12 | -6.47 | 0.20 | -4.61 | 0.34 | -6.21 | 0.15 | -3.38 | 0.24 |
| 1527 | -2.41 | 0.13 | -5.49 | 0.16 | -3.72 | 0.16 | -5.26 | 0.26 | -2.67 | 0.16 |
| 1528 | -2.29 | 0.11 | -5.59 | 0.21 | -3.88 | 0.24 | -5.29 | 0.16 | -2.64 | 0.25 |
| 1529 | -1.72 | 0.13 | -4.51 | 0.19 | -3.04 | 0.22 | -4.34 | 0.23 | -1.85 | 0.17 |
| 1530 | -1.76 | 0.16 | -4.43 | 0.14 | -2.89 | 0.22 | -4.24 | 0.17 | -1.96 | 0.19 |
| 1531 | -3.02 | 0.14 | -6.46 | 0.18 | -4.63 | 0.10 | -6.28 | 0.28 | -3.15 | 0.14 |
| 1532 | -2.88 | 0.15 | -6.41 | 0.19 | -4.69 | 0.20 | -6.39 | 0.20 | -3.11 | 0.17 |
| 1533 | -1.96 | 0.21 | -5.03 | 0.22 | -5.07 | 0.31 | -1.94 | 0.20 | -4.99 | 0.22 |
| 1534 | -1.99 | 0.07 | -5.02 | 0.23 | -5.03 | 0.23 | -2.03 | 0.12 | -4.98 | 0.12 |
| 1535 | -5.96 | 0.20 | -2.57 | 0.16 | -5.98 | 0.23 | -5.76 | 0.20 | -2.63 | 0.14 |
| 1536 | -5.91 | 0.25 | -2.52 | 0.16 | -6.05 | 0.33 | -5.78 | 0.30 | -2.69 | 0.08 |

given previously. Accurate to 2 decimal places. Each simulation was run 10 times and the
results averaged. Recall that less negative scores are better- a player scores 0 for defection vs cooperation, -1 for cooperation vs cooperation, -6 for defection vs defection, and -10 for cooperation vs defection.

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