2008-01-01

Fleet Sizing and Scheduling Model of Container Carriers between Two Ports

Alaa Mustapha Elyamak
University of Miami, elyamak.alaa@gmail.com

UNIVERSITY OF MIAMI

FLEET SIZING AND SCHEDULING MODEL OF CONTAINER CARRIERS BETWEEN TWO PORTS

By

Alaa Mustapha Elyamak

A THESIS

Submitted to the faculty
of the University of Miami
in partial fulfillment of the requirements for
the degree of Master of Science

Coral Gables, Florida

December 2008

UNIVERSITY OF MIAMI

A thesis submitted in partial fulfillment of
the requirements for the degree of
Master of Science

FLEET SIZING AND SCHEDULING MODEL OF CONTAINER CARRIERS
BETWEEN TWO PORTS

Alaa Mustapha Elyamak

Approved:

____
Murat Erkoc, Ph.D.
Assistant Professor of
Industrial Engineering

____
Terri A. Scandura, Ph.D.
Dean of the Graduate School

____
Shihab S. Asfour, Ph.D.
Professor of Industrial Engineering

____
Mohamed Abdel-Mottaleb, Ph.D.
Professor of Electrical and
Computer Engineering

MUSTAPHA ELYAMAK, ALAA (M.S., Industrial Engineering)
Fleet Sizing and Scheduling Model (December 2008)
of Container Carriers Between Two Ports

Abstract of a thesis at the University of Miami.

Thesis supervised by Assistant Professor Murat Erkoc.
No. of pages in text. (77)

Globalization and containerization have changed the shipping industry and carriers are challenged to reshape their operational planning in order to maintain their market share. The objective of this paper is to formulate a model to determine the optimal fleet size and sailing frequency that minimizes total shipping and inventory (wait) costs for a container shipping company. The proposed model assumes an arrival process that follows a Poisson rate. We first consider unlimited ship capacity and propose a solution to determine the required fleet size and the optimal sailing frequency. We then extend the work to consider limited ship capacity. Furthermore, we introduce a cost component associated with outsourcing shipments due to insufficient capacity. The outsourced shipment is utilized when the number of containers at a port exceeds the available capacity. In the general case, a closed form solution could not be derived. Therefore, a simulation study is undertaken to analyze optimal fleet sizing, scheduling, and outsourcing policies under varying paramaters. Our study investigates the trade-off between building capacity and outsourcing in the context of cargo shipment. The model proves to be a reliable tool to determine optimal delay time at ports and optimal fleet size.

Acknowledgments

This thesis is the result of my work at the University of Miami, where I have been encouraged and supported by many people. I now have the opportunity to express my gratitude to each one of them.

First of all, I would like to thank my advisor Dr. Murat Erkoc, with whom I worked as a teaching assistant. Without him this work would not have been possible. I appreciate his guidance throughout my stay at the University of Miami, and the opportunity he gave me to enrich my knowledge.

I also would like to thank Dr. Shihab Asfour for all his support and patience. He is my mentor in both my academic and personal life.

I would like to dedicate everything I have gained to my parents Mustapha Elyamak and Nada Elyamak. Their unconditional support and encouragement are my source of confidence. This thesis is dedicated to them and to my brothers, Bilal and Hani.

Table of Contents

List of Figures

List of Tables

CHAPTER 1

1.1 INTRODUCTION AND MOTIVATION

Container shipping is the major mode of international trade used today. The world fleet size is experiencing a continuous growth, and the world's seaborne shipping is also growing with the continuous growth in the world population, the change in the standard of living, the increased globalization pushing international companies to collaborate and merge, greater product specialization, and the limitation of local resources (Christiansen 2003).

Containers were first introduced in 1956 by entrepreneur Malcom Mclean who had no maritime experience whatsoever. Mclean was a trucker in southeastern North Carolina who thought of building something to transport large amount of goods more efficiently, faster, and with much less cost. Before the introduction of containers, goods were loaded and unloaded onto ships item by item by longshoremen. Mclean's invention brought a revolution in the shipping industry that changed the world from an old to a new one. He sold his idea to people who had the same vision and invested huge amount of money to build specialized ports and machinery. Today, the world without containers would be a very different place, and the world's fleet is expanding steadily, with the capacity of pure containerships rising almost 10 percent per year from 2001 through 2005. Also ships themselves are getting bigger and bigger creating more economies of scale, making it still cheaper and easier to move products around the world.

Cargo transported on board of vessels between ports is comprised of two types:

a. Bulk shipping of large cargo quantities like oil, ore, grain, etc. This type of cargo is transferred using built to purpose specialized vessels called bulk carriers.

b. Containerized shipping in which small cargo are packed and shipped using steel containers. Containers may not fill the top of the ship on regular services. Containers have dimensions (in feet) of $20*8*8.5$ or $20*8*9.5$, called 20-ft containers, or the $40*8*8.5$ or $40*8*9.5$ called 40-ft containers. A container vessel capacity is measured in TEU (twenty-foot equivalent unit). A 40 ft container is counted as 2 TEUs.

It's also important to list the three basic modes of operation of ships :

1 – Industrial Shipping: where the owner of the cargo also own a ship or a fleet of ships. The objective of industrial shippers is always to send their shipments at minimal cost.

2 – Tramp Shipping. In this mode a tramp shipping company follows available cargoes like a taxi (Chritiansen 2003). A tramp shipping operators usually try to maximize their profits from available cargoes.

3 – Liner Shipping. Liners operate exactly like a bus or train. They have a published itinerary. Container shipping usually falls in the realm of liner shipping.

We need to mention that a ship operating in one of the modes described above can be easily transferred to another mode. In this thesis, however, we only consider cargo transported in containers; We consider tranporting containers in both Tramp and Liner shipping modes.

With the eliminitation of trade barriers, World Trade is continuously increasing, creating an increasing need for container shipping. However, the ocean shipping industry is becoming a monopoly with the introduction of huge capacity ships. The world's leading container carriers currently control more than 70% of the world's total container shipping.

Table 1.1 lists the top 20 container carriers in 2007 and their transport capacities.

Table 1 : Top 20 ocean carriers in 2007.

AP Moller-Maersk	1,600,012 TEU
Mediterranean Shipping Co	937,145
CMA CGM	597,677
Evergreen	539,801
Hapag-Lloyd	448,840
Cosco	385,368
China Shipping Container Lines	339,545
Hanjin Shipping	328,307
APL	323,319
NYK	313,049
Mitsui OSK Lines	284,848
OOCL	268,502
CSVA	249,885
k Line	241,772
Yangming	223,192
Hamburg Sud	217,018
Zim Integrated Shipping Services	213,795
Hyundai Merchant Marine	153,850
Pacific International Lines	141,391
Wan Hai Lines	117,767
TOTAL	**7,925,083**
Top 20 Share	**72.70%**

Source: Containerisation International

This means competition is fierce for small carriers who strive to gain market share. Considering the high capital investment in owning a ship which is typically $20,000/TEU (Agarwal 2007), and the fact that some assets are owned and others are leased, in order to survive this intense competition, carriers are required to make effective planning at both strategic and operational levels. The increased operational complexities and the various variable and operational costs must be well managed and controlled while maintaining high customer satisfaction. With such a very capital-intensive industry, for most shipments freights, revenues only account for a very small portion of the shipment's total value.

Table 1.1.2 shows the wide variability in cost bases.

Table 2 shows all costs in container shipping calculated as percentage of gross revenue.

Gross Revenue	100%
Cargo Costs	50 – 60%
Vessel and Voyage Costs	20 – 25%
Equipment and Repositioning Costs	15 – 20%
Gross Profit	(5) – 15%
Business and Administrative Expenses	10 – 20%
Operating Results	(25) – 5%

Source: ROI Container Cargo Alliance

With profit margins becoming razor thin, voyage management has great impact on total profitability. The main challenge an ocean transportation company is facing today is how to increase revenue by optimizing the ship planning[1].

All related literature deals with maximizing profit by improving routes and schedules and picking the best mix of containers. Mathematical programming models in the literature will be presented in related section where linear programming based and mixed integer programming based optimization models are the most commonly used tools in the maritime transport industry.

To the best of our knowledge, some of the work in the literature concentrates on costs in maritime shipping, but no work deals with minimizing shipping cost by finding the optimal shipping cycle time and the fleet size while container arrivals are stochastic. We believe that managing the frequency of service is very essential for ocean carriers to maintain a market share. Shipping lines need to operate reliable and frequent services at minimum cost. Most of carriers try to have one outbound shipment from a port on a service route every week. Maintaining this type of service will require to determine an optimal fleet size. However, costs associated with containers wait time and operations have a great impact on determining the optimal frequency of service and fleet size.

In this thesis, we consider two ports, and containers arrive according to a Poisson rate. We assume that all containers have the same size and ship capacity is measured in terms of the total number of containers it can carry. We also assume that all ships in the fleet are of the same size, and have same operational cost. In the first part of this thesis, we look at the uncapacitated case, that is, total ship capacity is not a constraint and all arriving containers can be carried. We develop a cost function expressed in two decision variables: the total voyage time T, and the fleet size Q. We show that the cost function is a convex function of T, indicating that there is a unique value of T that minimizes the total expected cost. We look at the ships inter-departure time $\frac{T}{Q}$ because the optimal T is a function of the fleet size. We prove that if the optimal T is greater than the actual voyage time, the carrier should maintain a service with one ship and manages to keep the total voyage time at optimal T to achieve minimum cost. On the other hand, the cost function is also a convex function of the fleet size Q, and the minimum fleet size is derived as a function of the voyage time. We proved that when the minimum actual voyage time is greater than optimal T, the carrier should maintain a service with the minimum travel time and determine the optimal fleet size from the derived value of Q. An interesting observation in this case is that the inbalance between the arrival rates at the two ports does not affect the total cost.

In the other case, we look at a more general situation. We consider that when the total number of containers available for loading is more than the ship capacity, the ship carries the maximum number of containers, in this case the total capacity k, and then the remaining containers are outsourced and shipped by a different carrier. In this case the carrier is incurred an additional cost for each outsourced container. We develop another cost function to include this cost. A closed-form solution for determining the optimal voyage time and the fleet size can not be found, thus, we develop a discrete-event simulation model to analyze this situation. We prove that the model built is a reliable and sufficient tool to make sound managerial decisions. An interesting observation is that the cost function behaves like a convex function which is a sufficient condition for optimality. We show that the inbalance of containers does not affect the total cost as long as the total number of containers is much less than the total capacity of the ship. However, we observe that when there is inbalance and the total number of containers is close or greater than the available capacity, our optimal decision changes and the total cost varies. With many variables involved, the simulation model built is a great tool to test different scenarios for making sound decisions.

The next section discusses and summarizes research on ship routing, scheduling, and related problems. In Chapter Two, we look at the two different cases we discussed before. In Chapter Three, we analyze the simulation model built for the uncapacitated case. Chapter Four concludes the thesis.

1.2 LITERATURE REVIEW

As stated in the previous section, most of the work in the literature deals with ship routing and scheduling. In 1983, Ronen published the first survey in ship routing and scheduling and related problems. Ten years later, Ronen (1993) presented a second review on ship scheduling and issues in maritime transport. Since 1993, the literature has shown a positive and significant increase in the number of papers on ship scheduling and related problems. Christiansen, Fagerholt, and Ronen (2004) surveyed the most recently published research on the subject. In general, fleet scheduling work has been developed for industrial carriers and bulk carriers. In liner and tramp shipping modes, heuristics, rather than analytic optimization models are predominant. The uninitiated reader may benefit from browsing the former three reviews to get acquainted with the background of this thesis. Some of the materials presented in the reviews will be discussed in this section along with papers that are directly related to our work.

At this point it is worthwhile to clarify some terms used throughout this section. *Shipping* refers to moving cargoes by ships. A *schedule* is defined as visiting a sequence of nodes, including the arrival time at each node.

All problems related to the maritime transportation industry can be solved by optimization method like linear programming. We start with the work of Christiansen, Fagerholt, and Ronen (2004). They developed a mathematical formulation for an industrial ship routing and scheduling problem. The objective of this model is to minimize the total cost of ships in the fleet while ensuring that all cargoes are lifted. Although the developed model minimizes total tranportation cost, it assumes that the cargo to be shipped is deterministic, that is, its quantity is known during the planning horizon.

Another minimization problem is considered by Brown (1987) where an oil company is shipping crude oil from the Middle East to Europe and North America. The cargoes can be transported by either a ship in the fleet or outsourced. This work was extended by several other papers.

As mentioned before, tramp shipping operators follow cargoes like a taxi. They have contracted cargoes that must be shipped, and may have optional cargo in the market to carry if space on ship is available. Christiansen, Fagerholt, and Ronen (2003) also developed a mathematical model to maximize the profit of a tramp shipping operator. They included in the model the revenues and costs of carrying the cargoes on different routes by different ships. This model also differs from our work in the sense that we are looking to minimize the cost of the tramp shipper, and also we assume that cargo arrival is stochastic.

Zdenka Zenzerovie and Svjetlana Beslie (2003) presented a linear fractional model addressing shipping containers on a selected route. The objective of the model is to choose the best mix of containers of various masses available on a loading port in order to achieve maximum transport profitability per one voyage. The profitability of the transport process is defined as the ratio between profit yielded from transport of various types of cargo and time spent for realization of such transport process. The developed model can be used in operative planning when making shipping decisions, however it does not include the cost of containers stay at the port had the company commited to carry these containers.

When studies on maritime transportation are conducted, the terms port, and container terminal are used. A container terminal is the place where a ship arrives and empties its inbound containers and load export containers at a port. Although the main focus in this work is not on terminal operations, it's very important to mention that terminal efficiency plays a big role in the ocean transportation industry and in fact a lot of research today is investigating how to integrate ocean shipping into a multimodal[2] door to door supply chain. Work in this area was done by Katta Murty, Jiyin Liu, Yat Wan, and Richard Linn (2003), where a decision support system for daily operations at a container terminal is presented . The decisions made include the resources needed for handling the workload at the terminal, the waiting time of customer trucks, the congestion on the road inside the terminal while minimizing the berthing time of vessels. The problems are modeled and solved using mixed linear programming models. The study is addressed to terminals in Hong Kong which are considered the most competitive in the global transportation logistics industry.

Ship transportation problems can be analysed using network optimization approach. The most common applied method is the minimum cost flow problem. The problem treats a network as sets of nodes and arcs. For example, in the maritime shipping industry, nodes are the locations of ports, and arcs are transportation movements between ports. A heuristic[3]algorithm is developed by Sveuciliste Dubrovniku and Odjel Racunarstva (2004) to find optimal transportation plan and loading sequence to minimize shipping and loading expenses, transshipment costs, and cost of ship's stay in port, while fullfilling all cargo demands. In this model, the ship with defined cargo capacity is shipping from first port to last. The objective is to find a loading and transshipment strategy that minimizes the total cost incured over the whole voyage route consisting of multiple ports. The problem is modeled and solved with network optimization problem regarded as a minimum cost flow problem. While the model is a minimization problem, it assumes that the number of various container types is known at all ports. Also, the model does not include the cost associated with container stay at the port. The costs included in the model are the transshipment costs between ports, and the loading and discharging cost including the cost of container ship stay at the different ports.

Network design in ocean transportation is considered by Richa Agarwal (2007) but from different perspective. Here the authors consider collaboration between carriers where a number of shipping companies bring their fleets into a pool and operate them together. The goal of the study is to determine allocation of capacity on a transportation network among the carriers in the alliance. Game theory concepts and mathematical programming techniques were used to model and solve the problem. Although this work is completely outside the scope of our focus, it presents an important issue in the industry because of the fact that economies of scale lead to surplus space onboard of the vessels that shipping lines are eager to fill.

James S.K Ang, Chengxuan Cao, and Heng-Qing Ye (2006) proposed two heuristics algorithm that can solve large scale problems in a multi-period sea cargo mix problem in the international maritime shipping industry. The problem is formulated as a multi-dimensional multiple knapsack[4] (MDMKP) problem. The goal in this study is to find the optimal profit generated by all accepted cargo, and determine which cargo to accept during the time horizon and which to shut out from the time horizon given that demands, available capacities, and the information of price and cost for each shipment are all given.

Inventory-routing problems have rarely been analysed in marine context. Mabel Chou, Mark Goh, Miao Song, and Chung-Piaw Teo (2004) considered a study on a large shipping company where its client owns three mines in Australia and the ores from the mines are consumed as raw material by some factories in Asia. The company has opened several supply warehouses in selected ports and regularly transports the ores from Sydney to these demand ports by a fleet of ships. The aim of the study is determine the most cost effective distribution strategy to move the ores. The problem is analysed under two conditions. First a direct service model where a direct port-to-port service exists for any port pair, and all ships in the fleet visit all ports taking the same route. In the second condition, the problem is treated as a hub-and-spoke[5] model, where cargoes are first delivered to a hub and then distributed to different destinations by feeder[6] systems. Costs functions are developed to determine optimal decisions.

A simulation model for empty containers allocation is described by K.K Lai, Kokin Lam, and W.K.Chan (1995). The study is aimed at a shipping company in Hong Kong that needs to find a policy to allocate empty containers that are transported from the Middle East to ports in the Far East. This study differs form our work in that this company will face lost sales due to unavailability of empty containers when demanded by shippers; However, we assume that containers are always available at any port of call. In addition, this study does not include the portion of the actual transportation, it's aimed at a container company that acctually leases, stores, picks up and drops off containers from/to customers, but does not involve in the shipping process. The company faces already set vessel schedules. The simulation model helped deciding the necessary number of containers required to meet future expected demand.

A similar work was developed by Qiang Gao (1994). Here the author modeled and solved a linear programming model to minimize costs of acquisition and movement of containers. This study treats costs not related to voyage. The author aims to minimize costs related to purchase and lease of containers, the storage of empty containers, the repair, maintenance, and insurance of containers, and the reallocation of empty containers.

Shih-Chan, and Gwo-Hshiung Tzeng (2003) developed a dynamic programming (DP)[7] model to improve ship scheduling and clarifies costs factors. In this model, a liner has a specific service route and fleet size and tries to find the best routing strategy that minimizes total fleet operating cost. The model captures very important details like time to berth[8], time to departure, speeds between two ports, and quay crane[9] dispatching. The model was implemented on a Trans-Atlantic service of a Taiwanese carrier where 5 full-container vessels with similar speed are deployed on this route. Our study differs from this work in certain aspects. We consider that containers arrive according to a probability distribution, however the described model assume containers numbers are known at different ports. In addition, we consider cost of holding containers an important factor in our study whereas the paper described does not include this cost. One final difference is that we are optimizing the fleet size for a minimum cost, the authors in this paper has a certain fleet size and optimize the deployment of the ships.

E. Sambracos, J.A Paravantis, C.D. Tarantilis, and C.T. Kiranoudis (2004) attempt to optimize a marine freight transport problem. This study addresses the Aegan Sea in Greece in an effort to reeingineer coastal marine transport. The problem is dealt with under two dimensions. At first, the authors introduce a linear programming formulation for the determination of vessel traffic under known supply and demand constraints where total fuel costs and port dues are minimized. A network of 13 ports and 25 sea links were taken into consideration. The second approach determines a more realistic fleet size by formulating the problem as one of vehicle routing problem nature. This work disregards time scheduling which is an important decision in the work we are presenting. Another difference is that fleet size is determined based on known number containers whereas our work seeks to find the optimal fleet size that minimizes the total cost while container arrive based on a probability distribution. The authors of this paper only consider cost of fuel and port fees, disregarding containers stay at ports.

Maritime hub-and-spoke container network are also considered by Chaug-Ing Hsu, and Yu-Ping Hsieh (2006). This study is focusing on developing a two objective model to decide best routes for ships, ship size, and sailing frequency while minimizing shipping costs and inventory costs. The authors consider shipping costs and inventory costs as two seperate objectives. They used the Pareto[10] optimality concept to conduct the study. Different from our work which focuses on making single cost function to minimize shipping cost, this study minimized shipping cost and inventory costs seperately. Desicions in this study include the desicion to either route containers through a hub or directly to the destination. Also, in this study, the fleet size is not considered, the authors assume that they have a fixed fleet size with different ship types and try to optimize deployment of the different ships on different routes.

To summarize, because our work considers issues as time scheduling and fleet size, it can be classified according to Ronen's useful classification of ship routing and scheduling problems (1983).

CHAPTER 2

OPTIMIZATION MODEL

2.1 MODEL BASICS

In this section we present the methodology used to address the issues stated before. We consider a network of two nodes, that is one port of origin and one port of destination, and goods are transported in containers of the same size. If only ports A and B were considered, let T denotes the total shipping time per round voyage. It follows that T can be written as:

$$T = t_A + t_B + t_{AB} + t_{BA}$$

The ship first spends t_A unit of time , i.e., day. t_A includes the time to arrival at the assigned berth[11], working time for unloading and loading containers, and the departure from port A. The ship than sails from port A and travels on sea for t_{AB} unit of time until she arrives at port B. The time spent in port B includes similar components as port A. Finally, the ship leaves port B to reach port A again in t_{BA} unit of time. We assume that the arrival process of containers at a loading port is a Poisson process with parameter λ. In this study, we consider that containers arrive at both ports, A and B; Thus, λ_A and λ_B denote containers arrivals at port A and B respectively. The arrival rates at ports A and B are not necessarily equal. In fact arrival rates are an interesting parameters and their roles will be investigated via post-optimization sensitivity analysis. Furthermore, the expected number of events occuring within the cycle time T is λT, and the number of container arrival within T has a Poisson distribution with mean λT. If we denote the number of ships in the fleet Q, it follows that container arrival is a random variable with a probability mass function:

$$\frac{e^{-\lambda T/Q}(\lambda T/Q)^n}{n!} \qquad \text{for } n = 0, 1, 2, 3.... \qquad \textbf{(2.1)}$$

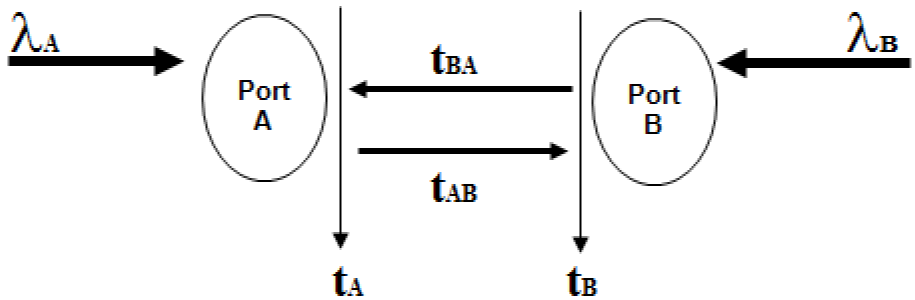
Where n is the number of containers.

Figure 1 shows the fundamental of our network.

In this study, we assume that all deployed ships are of same size, and have same operating and capital costs. Capital and operating costs correspond to the total expenses paid for using the ship per unit of time. This cost includes cost of owning the ship, crew wages and meals, ship maintenance and repair, insurance, materials, and supplies and so on. Capital and operating costs will be denoted W and U respectively throughout the model.

As mentioned before, this study constructs an analytical model to decide the best shipping time and optimal fleet size that minimizes total expected annual cost. In this study, we assume that all costs are borne by the carrier. We consider inventory costs related to the container shipping process are crucial factor affecting the revenues of carriers. Inventory costs are related to containers waiting to be shipped in a loading port. The cost component, including the holding cost at ports, and delay cost will be denoted by C.

Figure 1
Example of two ports and container arrivals



In the mathematical model the following notation will be used.

G : Total cost per one voyage

H : Total time per one year

t_x : Time spent at port x, $x = A, B$.

t_{xy} : Voyage time between port x and port y, $x = A, B$, $y = A, B$.

λ_x : Container arrival rate at port x, $x = A, B$.

K : Ship's transport capacity

W : Ship operating cost per round trip

Q : Number of ships

C_x : Cost of container stay at port x per unit of time, $x = A, B$.

U : Unit capital cost for each ship per unit of time (tax, insurance...)

2.2 PROBLEM FORMULATION

In this section we present the methodology used to address the issues stated before. As explained, the goal of this study is to minimize the average yearly cost of containers carriers by determining the optimal voyage time T, or frequency of service and the optimal fleet size Q. In this study we assume that the time in the sea depends on the speed of the ship and the distance between ports A and B. However, optimizing ship speed is outside the scope of this work. Since T is also a function of t_A and $t_{B,}$ we are interested in the total sum of the times spent at ports A and B. In other words, we believe that the total sum of the times in T might be managed and controlled. Thus, T and Q are our decision variables, and T_{min} includes the minimum time a ship can stay at ports A and B. For simplicity, we first focus our analysis on the case where ship transport capacity is not considered. As described in Chapter 1, a container ship's capacity is measured in twenty-foot equivalent unit. In our study, the capacity of the ship is determined by the maximum number of containers the ship can carry. We first assume that all containers can be loaded on board. Next we extend the model to the case where ship capacity is limited, and we develop a cost function based on expected container arrivals. The impact and roles of different parameters will be investigated via post-optimization sensitivity analysis.

2.2.1 UNCAPACITATED SHIPMENT SCHEDULING

We first write an expected cost function where all containers are picked regardless of the maximum number of containers a ship can carry. We can consider that containers can be carried by any ship in the fleet. If we denote by G the expected cost per one voyage, and H is the total time in one year, the expected annual cost is $\frac{H}{T}$ G.

It follows that our annual expected cost $\widehat{G}$ can be written as

$$\frac{HQ}{T}\left[\left(\frac{C_{\mathrm{A}}\lambda_{\mathrm{A}}+C_{\mathrm{B}}\lambda_{\mathrm{B}}}{2}\right)\frac{T^2}{Q^2}+W+\left(C_{\mathrm{A}}\lambda_{\mathrm{A}}t_{\mathrm{AB}}+C_{\mathrm{B}}\lambda_{\mathrm{B}}t_{\mathrm{BA}}\right)\frac{T}{Q}\right)\Big]+HUQ \quad \textbf{(2.2)}$$

Equation 2.2 can be further simplified as follows:

$$H\left(\left(\frac{C_{\mathrm{A}}\lambda_{\mathrm{A}}+C_{\mathrm{B}}\lambda_{\mathrm{B}}}{2}\right)\frac{T}{Q}+\frac{WQ}{T}+C_{\mathrm{A}}\lambda_{\mathrm{A}}t_{\mathrm{AB}}+C_{\mathrm{B}}\lambda_{\mathrm{B}}t_{\mathrm{BA}}\right)+HUQ \qquad \textbf{(2.3)}$$

$\left(\frac{C_{\mathrm{A}}\lambda_{\mathrm{A}}+C_{\mathrm{B}}\lambda_{\mathrm{B}}}{2}\right)\frac{T}{Q}$ captures the cost of containers staying at ports A and B until picked based on the expected number of containers arrivals during T. The term WQ represents the total operating costs of the whole fleet. $C_{\mathrm{A}}\lambda_{\mathrm{A}}t_{\mathrm{AB}}+C_{\mathrm{B}}\lambda_{\mathrm{B}}t_{\mathrm{BA}}$ represent the cost of the delay of containers while being transported. In this study, the cost parameters C_{A} and C_{B} involve waiting time cost and shipping time cost at ports A and B. The waiting time cost is incurred from waiting in the loading port, and the shipping time cost is related to time while containers are shipped on a ship. The last term HUQ captures the total capital costs of all ship in the fleet per one year. Now we can present the following observation:

Lemma 2.1 *Assuming* $T_{min} \leq \sqrt{\frac{2W}{C_A\lambda_A+C_B\lambda_B}}$, *the optimal policy that minimizes the total expected cost is to employ the minimum number of ships that can achieve the optimal inter-departure time.*

Proof First we need to prove that $\widehat{G}$ is a convex function of the total voyage time T. Since our objective is to minimize the total expected cost, and T is a decision variable, we need to find the optimal T that minimizes $\widehat{G}$. It follows that

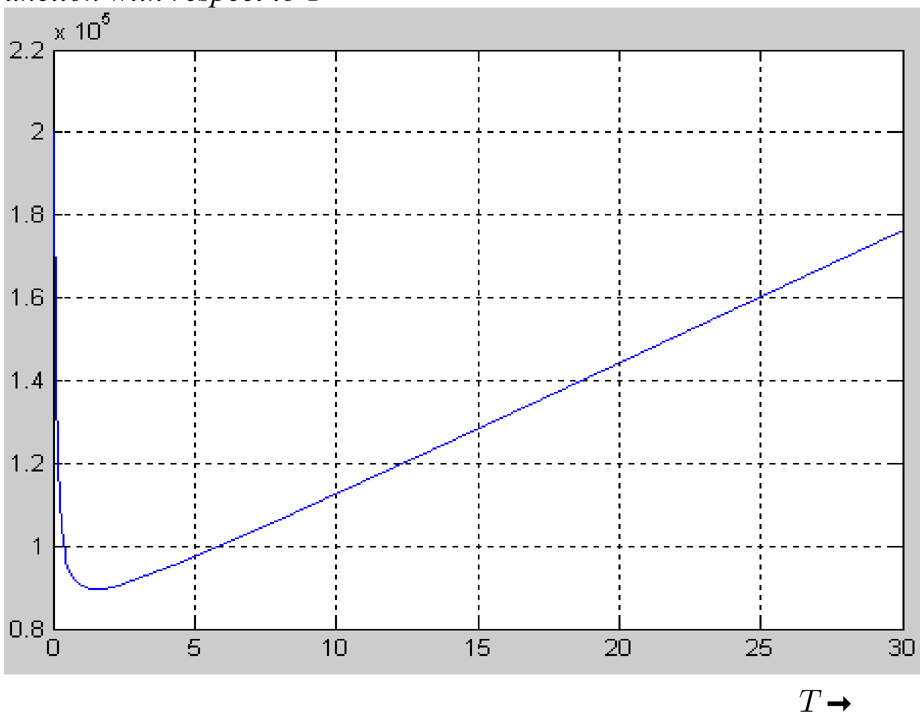
$$\frac{\partial \widehat{G}}{\partial T} = H\left(\frac{1}{2Q}\left(C_{\mathrm{A}}\lambda_{\mathrm{A}}+C_{\mathrm{B}}\lambda_{\mathrm{B}}\right)-\frac{WQ}{T^2}\right)$$

and

$\frac{\partial^2 \widehat{G}}{\partial^2 T} = \frac{2HWQ}{T^3} > 0$ for $T > 0$.

Since $\frac{\partial^2 \widehat{G}}{\partial^2 T} > 0$, it follows that $\widehat{G}$ is a convex function of T. We can conclude that there is unique value of T that minimizes the total expected cost. A typical cost function behaves as illustrated in Figure 2.

Figure 2
The expected annual cost function with respect to T



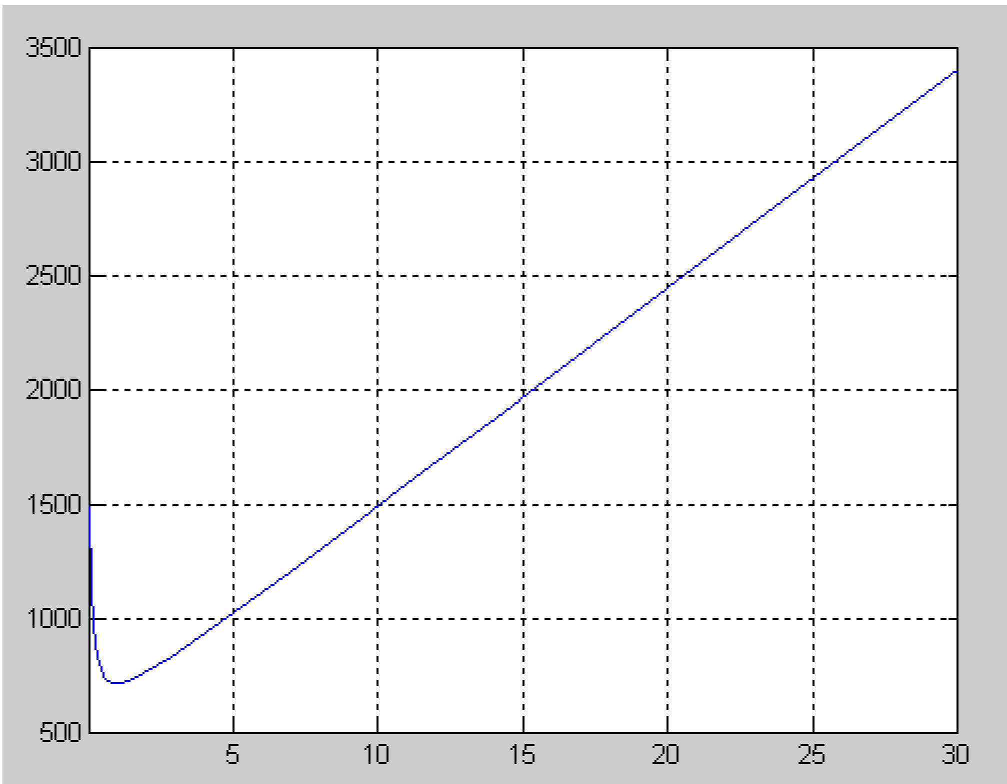
The optimal value of $\widehat{G}$ occurs where $\frac{\partial \widehat{G}}{\partial T} = 0$, which gives

$$T^* = \left(\sqrt{\frac{2W}{C_A \lambda_A + C_B \lambda_B}} \right) Q \qquad \textbf{(2.4)}$$

Since T^* is a function of the fleet size Q, we look at ships inter-departure time $\frac{T}{Q}$. It's straightforward to see from equation (2.3) that $\frac{T}{Q}$ needs to be kept at minimum. This implies that T should be very small and Q should be large. However, the last term of equation (2.3) indicates that a large Q will increase the total expected cost function, thus, Q should be small as well. We note that for $T = T^*$, $\widehat{G}$ is strictly decreasing in Q as depicted in figure 2-1-3.

This analysis implies that when $T^* \geq t_A + t_B + t_{AB} + t_{BA}$, the optimal inter-departure time can be achieved with only one ship.☐

Figure 3
The expected annual cost function with respect to Q When $T = T^*$



Q→

The foregoing lemma implies that if a carrier can complete a voyage with $\sqrt{\frac{2W}{C_{\mathrm{A}}\lambda_{\mathrm{A}}+C_{\mathrm{B}}\lambda_{\mathrm{B}}}}$ unit of time, it's optimal to deploy only one ship on the route. In other word, ships inter-departure time should be managed to be maintained at $\sqrt{\frac{2W}{C_{\mathrm{A}}\lambda_{\mathrm{A}}+C_{\mathrm{B}}\lambda_{\mathrm{B}}}}$ unit of time to achieve minimum cost.

Having obtained an optimal solution for our voyage time to minimize total expected cost, we test the sensitivity of the formulation to changing values of the containers arrival rates, ships operating costs, and the costs associated with containers wait time at ports A and B.

Let's suppose that a container shipping carrier has maintained a service with one ship between two ports A and B. The ship's operation costs amount 16386 USD, which is based on a real-life example of a feeder ship owned by "Losinjska Plovidba", operating on route Rijeka-Gioia Tauro. Let's suppose that containers of the same type and size arrive to ports A and B according to a poisson process with rates 20 and 40 containers per day respectively. Let's also suppose that both ports A and B charge the same cost 10 USD per day for each container waiting. We note that λ_{A} and λ_{B} can be written as:

$\lambda_{\mathrm{A}} = \lambda + x$

$\lambda_{B} = \lambda - x$

Where $\lambda = \frac{\lambda_{\mathrm{A}}+\lambda_{B}}{2}$ and x is just the difference between λ and λ_{A} or λ_{B}. For our example, the value of λ can be calculated to be 30. It follows that (2.4) can be written as :

$$T^{*} = \left(\sqrt{\frac{W}{C\lambda}}\right)Q$$

By replacing the values from our example, we obtain an optimal voyage time of 7 days. In other words, the carrier should complete its voyage with only one ship in 7 days to achieve minimum cost. Now let's study the impact of the various parameters on the optimal voyage time. We note that

$$\frac{\partial \mathrm{T}}{\partial \mathrm{W}} = \left(\frac{1}{2\sqrt{CW\lambda}}\right)Q > 0,$$

and

$$\frac{\partial \mathrm{T}}{\partial C} = -\left(\frac{W}{\sqrt{C\lambda}}\right)\frac{Q}{2C} < 0,$$

and

$$\frac{\partial \mathrm{T}}{\partial \lambda} = -\left(\frac{W}{\sqrt{C\lambda}}\right)\frac{Q}{2\lambda} < 0.$$

We notice that as the ship operating cost increases while keeping the other parameters constant, the optimal voyage time increases. This can be explained by the cruising speed of the ship. If the ship consumes a lot of fuel, it's better to cruise at a lower speed to keep the cost low, hence a longer voyage time. If we look at the inter-departure time, increasing the operating cost will increase the inter-departure time, and this is straightforward, because if the operating cost is high, the carrier should wait longer between ships departure to keep the cost low.

Parallel to this analysis, if we change the holding cost C and keep the other parameters, we notice that optimal voyage time decreases. This is rational because if ports apply higher charges while containers are waiting, and similarly if the delay cost while containers are transported is high, it's better to make faster trips to save on costs. Same analysis is true if we look at ships inter-departure time. It's better to have a high frequency of service to save on cost.

Another obervation can be noted when containers arrival rates increase. We also notice a decrease in the voyage time. This is also expected because as containers arrive at higher rates, the carrier is charged a higher costs due to waiting cost. Thus, it's better to maintain a higher frequency of service.

One last interesting observation is that we can see from equation (2.3), that if the cost of container delay at ports A and B is the same, the inbalance of containers does not affect the total cost.

We use The What-If Analysis tools available with Excel to see the observations stated above, and the results are available graphically in Figures 4 through 6 and their data is available in Tables 3 through 5.

Table 3 : Data of Figure 4.

W	T^*
16386	7.39
17386	7.61
18386	8.03
19386	8.63
20386	9.37
21386	10.22
22386	11.16
23386	12.16
24386	13.21
25386	14.30

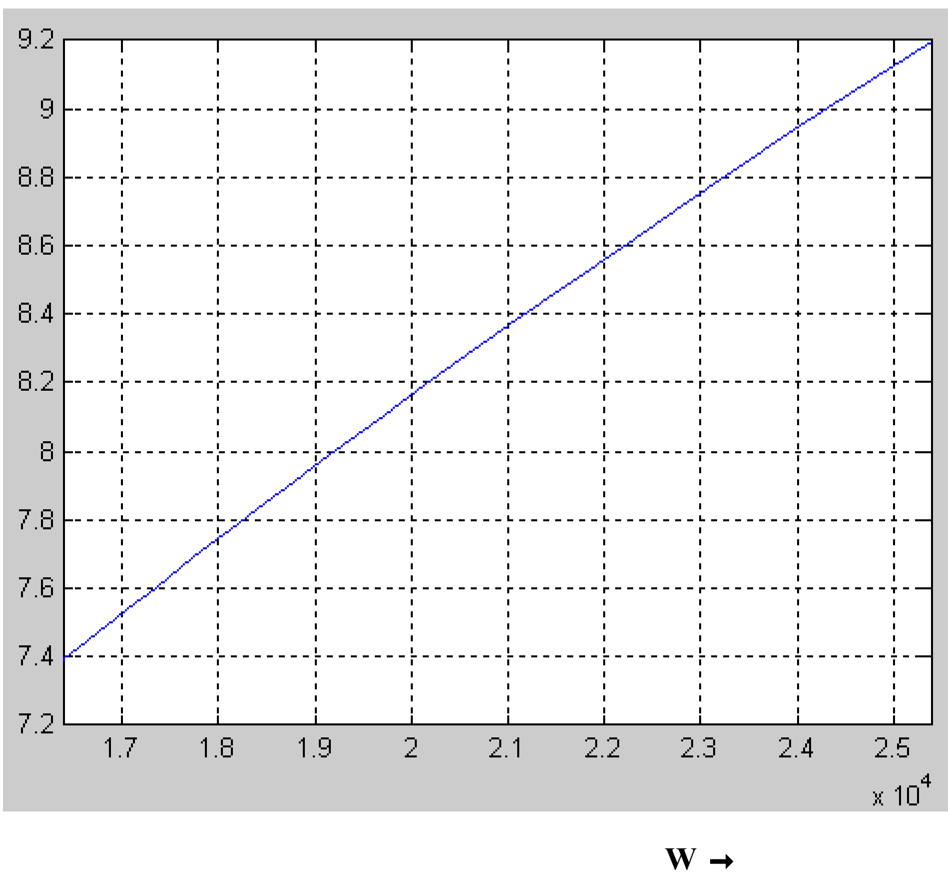


Figure 4
The optimal voyage time T^*
when the operating cost increases

Table 4 : Data of Figure 5.

C	T^*
10	7.39
15	6.03
20	4.67
25	3.69
30	3.01
35	2.53
40	2.17
45	1.90
50	1.69
55	1.52

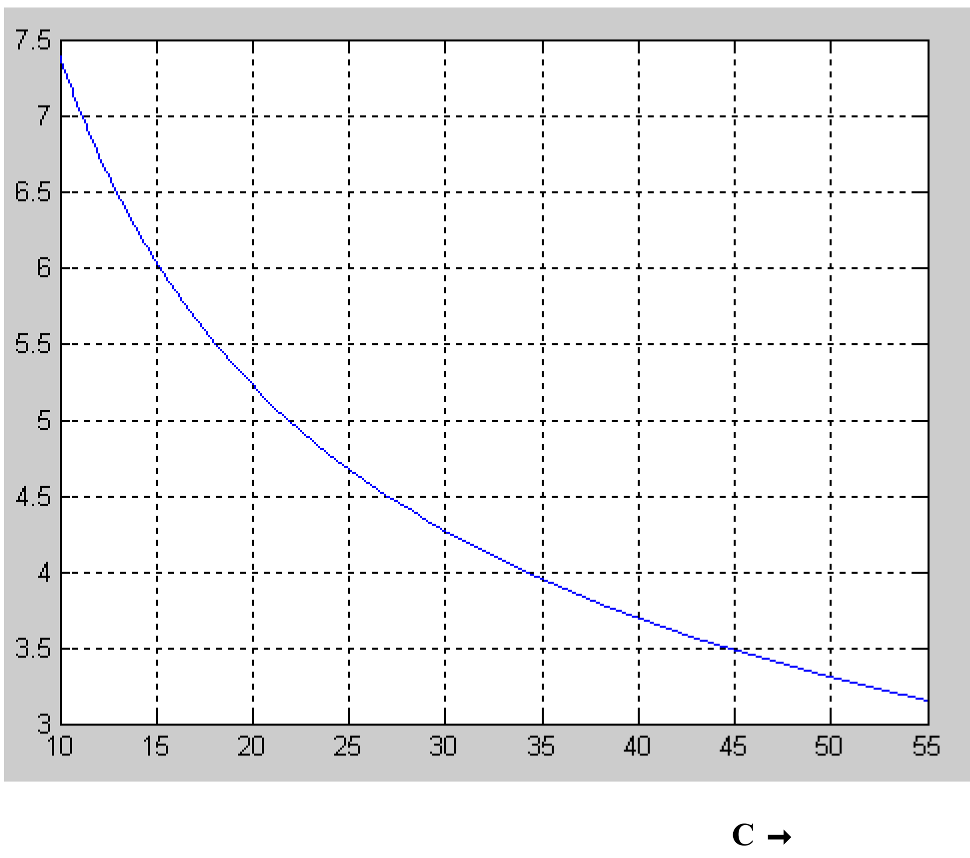


Figure 5
The optimal voyage time T^*
when the holding cost increases

Table 5 : Data of Figure 6.

λ	T^*
30	7.39
35	6.84
40	6.03
45	5.22
50	4.52
55	3.95
60	3.48
65	3.10
70	2.79
75	2.53

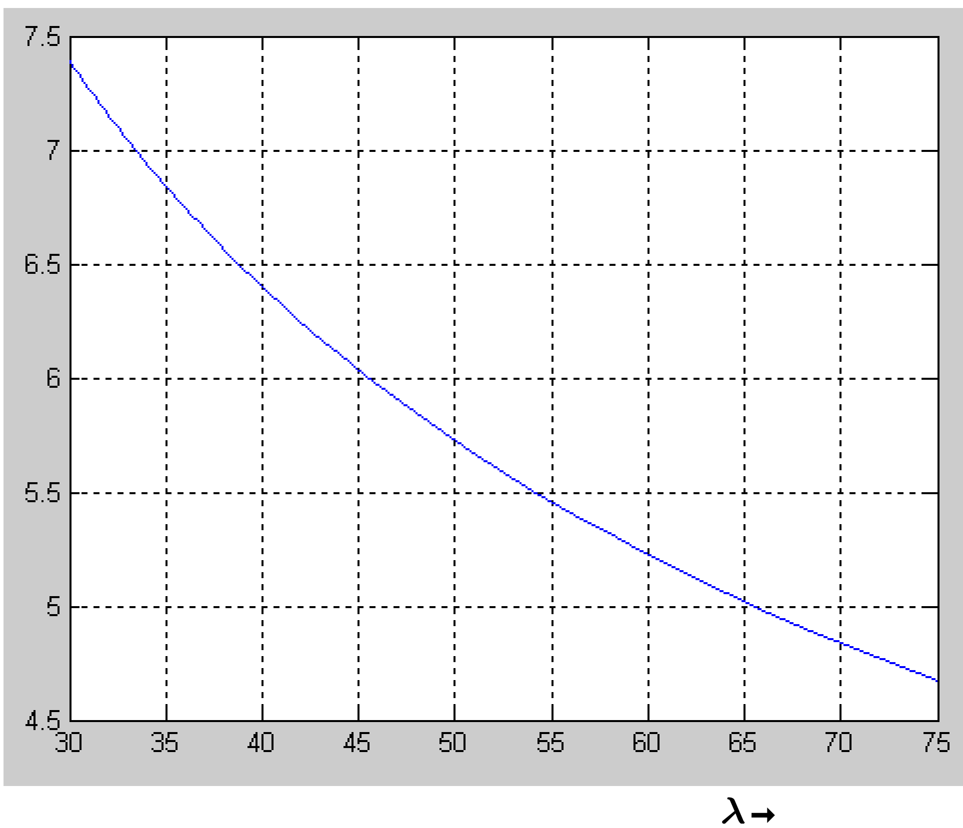


Figure 6
The optimal voyage time T^*
when containers arrival rates increase

In real life, carriers may not be able to complete a voyage with the optimal time if only one ship is deployed. This is due to distances between ports, and maximum speed a ship can reach. In this case management should consider adding more ships to the fleet. Since owning a ship is a very high capital investment as discussed in chapter 1, the number of ships in the fleet and the frequency of service should be carefully managed. We now observe the following :

<u>Lemma 2.2</u> *There is a unique value of Q that minimizes the total expected cost, or equivalently* $Q^* = argmin\ \widehat{G}$.

<u>Proof</u> This is an unconstrained optimization problem. We prove that $\widehat{G}$ is a convex function of the fleet size Q. It follows that:

$$\frac{\partial \widehat{G}}{\partial Q} = H\ [-\ \frac{(C_A\lambda_A + C_B\lambda_B)\,T}{2Q^2} + \frac{W}{T}] + HU$$

and

$$\frac{\partial^2 \widehat{G}}{\partial^2 Q} = \frac{H(C_A\lambda_A + C_B\lambda_B)T}{Q^3} > 0 \text{ for } Q > 0.$$

The minimum Q is the solution of $\frac{\partial \widehat{G}}{\partial Q} = 0$. Hence,

$$Q^* = \sqrt{\frac{C_A * \lambda_A + C_B * \lambda_B}{2(W + U * T)}} * T \ \square \qquad (2.5)$$

The optimal fleet size is a function of the actual travel time. As stated before, the optimal voyage time with one ship can not be realistic, in this case, management should determine the optimal fleet size and the optimal ships inter-departure time that minimizes total cost. We are faced with two options:

1- Carrier should determine the number of ships in the fleet that can achieve the optimal frequency of service derived from equation (2.4), that is $\sqrt{\frac{2W}{C_A\lambda_A+C_B\lambda_B}}$.

2- Carrier should determine the number of ships in the fleet from equation (2.5) based on the minimum actual voyage time.

Now, we observe the following:

Lemma 2.3 *A carrier should determine the optimal fleet size and sailing frequency according to the following policy:*

(a) $Q = ROUND\left(\sqrt{\frac{C_A*\lambda_A+C_B*\lambda_B}{2(W+U*T)}}*T\right)$ when

$$T_{min} > \sqrt{\frac{2W}{C_A\lambda_A+C_B\lambda_B}} \text{ and } T^* = T_{min}$$

(b) $Q = 1$ and $T^* = \sqrt{\frac{2W}{C_A\lambda_A+C_B\lambda_B}}$ when $T_{min} \leq \sqrt{\frac{2W}{C_A\lambda_A+C_B\lambda_B}}$

Proof **(a)** Suppose the carrier can not complete a voyage with the optimal voyage time when only one ship is deployed, that is, $T_{min} > \sqrt{\frac{2W}{C_A\lambda_A+C_B\lambda_B}}$. It follows that the optimal inter-departure time can be maintained if we increase the number of ships Q

from 1. In other words, the ratio $\frac{T}{Q}$ can be adjusted for a given T for it to be equal to $\sqrt{\frac{2W}{C_A\lambda_A+C_B\lambda_B}}$.

In this case the carrier needs to consider adding ships to its fleet. However, as proven from equation (2.3), increasing the fleet size will result in higher costs. So carriers are better off choosing the number of ships in the fleet that minimizes total cost. In this case fleet size needs to be calculated according to equation (2.5)☐

(b) This is straighforward because $\sqrt{\frac{2W}{C_A\lambda_A+C_B\lambda_B}}$ can be achieved with only one. Since one is the minimum number of ships a carrier can have, the carrier can adjust its voyage time to reach $\sqrt{\frac{2W}{C_A\lambda_A+C_B\lambda_B}}$ in order to guarantee minimum cost. This can be realized if the ship speed can be reduced so optimal time is achieved.☐

Let's consider again our first example. The parameters of this illustration are:

$\lambda_A = 20$ $\quad C_A = 10$ $\quad W = 16386$ $\quad U = 2000$

$\lambda_B = 40$ $\quad C_B = 10$ $\quad H = 240$ $\quad T = 30$ or Changing

From the data above we can calculate the optimal voyage time if one ship is available from equation (2.4):

$$T^* = 7.4$$

It follows that the carrier should deploy only one ship for a total voyage time of 7.4 unit of time. However, the actual voyage time is 30, so in order for the carrier to maintain the optimal inter-departure time, more ships should be deployed. In this example,we can choose Q $= 4$. In this case, the carrier should deploy 4 ships with inter-departure time of 7.4 unit of time.

We can calculate the total annual cost from equation (2.3) to be:

$\widehat{G} = 2,984,957\ USD$

Parallel to this calculation, we calculate the optimal fleet size from equation (2.5) when $T = 30$:

$Q^* = 2$

It follows that the carrier can deploy one ship every 15 unit of time with a total of 2 ships. The total annual cost can be calculated from equation (2.3) to be:

$\widehat{G} = 2,290,238\ USD$

As expected, this cost is lower that the one calculated before.

Now let us consider other examples in which T changes. The data used and results are available in Table 2.4.

Table 6 : Costs when $\frac{T}{Q} = \sqrt{\frac{2*W}{C_A*\lambda_A+C_B*\lambda_B}}$, and when $Q^* = \sqrt{\frac{C_A*\lambda_A+C_B*\lambda_B}{2(W+U*T)}}*T$

T	Q	Expected Cost	Q*	Expected Cost
5	1	1,626,528	1	1,626,528
15	2	2,024,352	1	1,822,176
25	3	2,511,916	2	2,174,461
30	4	2,984,957	2	2,290,238
35	5	3,465,805	2	2,444,722
40	5	3,949,896	2	2,596,632

From the example above, we can conclude that one ship is not always optimal, that is, in cases where the minimum voyage time can not be achieved with the optimal voyage time, multiple ships should be deployed.

2.2.2 CAPACITATED SHIPMENT, SCHEDULING, FLEET SIZING, AND OUTSOURCING ANALYSIS

In this model, we consider the case where ships have transport capacities that can not be exceeded. In some cases, the fleet can not guarantee to service all containers. In such a case, the over capacity containers can be outsourced and serviced by spot charters[12]. We introduce a variable P that represents the loss of opportunity cost or the penalty cost because containers can not be loaded.

Let $E(w(\frac{T}{Q}))$ be the expected waiting time for the containers. It's true to assume that one container waits approximately up to one voyage until loaded from port if only one ship is available.It follows that according to the memoryless property, if the interarrival time of containers is exponentially distributed, all containers have the same expected wait time. Considering multiple ships in the fleet, the wait time is uniformely distributed within the interval $[0\ \frac{T}{Q}]$, and the expected wait time is $\frac{T}{2Q}$.

Hence,

$$E(w(\tfrac{T}{Q}) \,|\, N(\tfrac{T}{Q}) = n \leq K) = E\left[\sum_{i=1}^{n} (\tfrac{T}{Q} - U_i) \right] = \sum_{i=1}^{n} E(\tfrac{T}{Q} - U_i)$$

$$= \sum_{i=1}^{n} \frac{T}{2Q} = \frac{nT}{2Q}$$

$$E(w(\tfrac{T}{Q})) = \sum_{n=0}^{K} E\left[w(\tfrac{T}{Q}) \,|\, N(\tfrac{T}{Q}) = n \right] \frac{e^{-\lambda \frac{T}{Q}} (\frac{\lambda T}{Q})^n}{n!} +$$

$$\sum_{n=K+1}^{\infty} E\left[w_k | N(T) = n\right] \frac{e^{-\lambda\frac{T}{Q}}(\frac{\lambda T}{Q})^n}{n!} \ =$$

$$\sum_{n=0}^{K}\left[\left(\frac{nT}{2Q} \ + \ nt_{AB}\right) \frac{e^{-\frac{\lambda T}{Q}}(\frac{\lambda T}{Q})^n}{n!}\right] +$$

$$\sum_{n=K+1}^{\infty}\left[\left(\frac{KT}{2Q} \ + Kt_{AB}\right)\frac{e^{-\frac{\lambda T}{Q}}\frac{\lambda T}{Q}^n}{n!}\right.$$

Since in this section we consider that containers over ship capacity are outsourced, the associated cost must be included in our cost function. The expected number of outsourced containers is given by:

$$E(L) \ = \sum_{n=K+1}^{\infty} n \ \frac{e^{-\lambda t}\lambda t^n}{n!}$$

The expected annual cost can be written as

$$G = \frac{HQ}{T}\left[E_A(w(t))C_A \ + \ P_A E_A(L) \ + \ E_B(w(t))C_B \ + \ P_B E_B(L) + W\right] + HUQ$$

$$G = \frac{HQ}{T} \ \left[\left[C_A\left(\sum_{n=1}^{K}(\frac{nT}{2Q})\frac{e^{-\lambda_A\frac{T}{Q}}(\frac{\lambda_A T}{Q})^n}{n!} + \sum_{n=K+1}^{\infty}\left((\frac{KT}{2Q})\frac{e^{-\lambda_A\frac{T}{Q}}(\frac{\lambda_A T}{Q})^n}{n!}\right)\right) +\right.\right.$$

$$C_A t_{AB}\left(\ \sum_{n=1}^{K} \ \frac{ne^{-\lambda_A\frac{T}{Q}}(\frac{\lambda_A T}{Q})^n}{n!} + \sum_{n=K+1}^{\infty} K \ \frac{e^{-\lambda_A\frac{T}{Q}}(\frac{\lambda_A T}{Q})^n}{n!}\right) +$$

$$\left. P_A\sum_{n=K+1}^{\infty} \frac{(n-K)\,e^{-\lambda_A\frac{T}{Q}}(\frac{\lambda_A T}{Q})^n}{n!}\right] + \left[C_B\left(\sum_{n=1}^{K}(\frac{nT}{2Q})\frac{e^{-\lambda_B\frac{T}{Q}}(\frac{\lambda_B T}{Q})^n}{n!} +\right.\right.$$

$$\sum_{n=K+1}^{\infty}\left(\left(\frac{KT}{2Q}\right)\frac{e^{-\lambda_B\frac{T}{Q}}\left(\frac{\lambda_B T}{Q}\right)^n}{n!}\right)\right) + C_B t_{BA}\left(\sum_{n=1}^{K}\frac{ne^{-\lambda_B\frac{T}{Q}}\left(\frac{\lambda_B T}{Q}\right)^n}{n!} +\right.$$

$$\left.\sum_{n=K+1}^{\infty} K\frac{e^{-\lambda_B\frac{T}{Q}}\left(\frac{\lambda_B T}{Q}\right)^n}{n!}\right) + P_B\sum_{n=K+1}^{\infty}\frac{(n-K)e^{-\lambda_B\frac{T}{Q}}\left(\frac{\lambda_B T}{Q}\right)^n}{n!}\Bigg]$$

$$\left. + W\right] + HUQ \qquad \textbf{(2.6)}$$

The first term $E_A(w(t))C_A$, captures the cost of the containers waiting to be picked at port A. The second term $P_A E_A(L)$ represents the penalty incurred by containers outsourced to different carriers because of limited ship capacity. The third and fourth terms capture similar costs at port B. W represents the ship operating cost. The final term HUQ represents total capital costs of all ship in the fleet per one year.

Equation (2.6) is an unconstrained optimization problem. Unfortunately, a closed form solution for equation (2.6) cannot be derived. We decided to develop a discrete-event simulation to model the network in Figure 2.1.1 but when excess containers are outsourced and additional costs are incurred for the carrier.

CHAPTER 3

THE SIMULATION STUDY

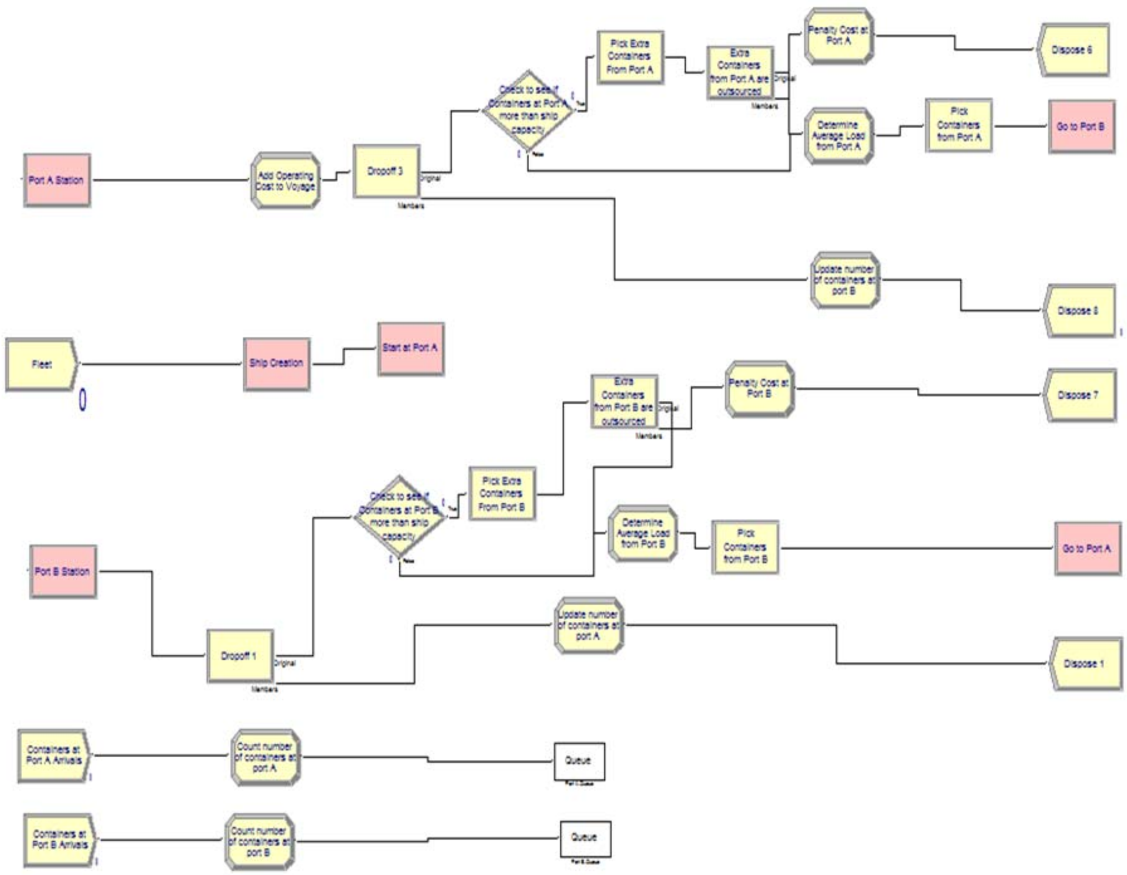
In this section, we present a simulation model carried out with Arena software. The model built basically mimics the behavior of the cost function (2.6) under different control approaches.

The literature does not include extensive references to scientific research studying simulation of containers transportation in ports, perhaps because of the complexity of such systems. We found the simulation tool very important for managers to make sound decisions in this risky capital-intensive industry.

The model that is built run as follows: containers of one type arrive at two different ports with inter-arrival time being exponentially distributed. Containers wait in two separate queues until a ship with available capacity arrives to load the containers. Before loading the containers, the model checks whether the ship has enough capacity to carry the containers. If capacity is not available, extra containers exit the system, and a penalty cost is added with each extra container. The ship then leaves full to the next port and disposes of its load upon arrival. Unloaded containers exit the system. The ship then picks the containers from the second port, and the whole cycle repeats.

If more than one ship are available in the fleet, ships are deployed according to the following: $\frac{T}{Q}$. We consider that the operating cost of the ship is added every time it completes a voyage and leaves for another voyage. Figure 7 depicts our network modules.

Figure 7
System modules.



Statistics for average percentage of capacity of ship used, the average penalty cost, the average wait cost, the average operating cost, and the total average cost were all collected on a daily basis.

We did 36 simulations to test different scenarios when our fleet contains 1, 2, 3 and 4 ships and when the operating cost of the ship changes from $1,600 to $100,000. Waiting time at ports from one hour to twenty four hours were simulated.

The objective is to determine a policy when containers arrival rates vary, but their summation always adds up to a specific number. In other words, we test how the balance between container arrivals in both ports affects our decision.

3.1 SIMULATION RESULTS

The simulation of the model presented allows to determine optimal wait time at ports and fleet size under varying parameters. The variables considered to assess our network are the following:

- Fleet Size. This variable controls the number of ships deployed . We simulated runs with 1, 2, 3, and 4 ships
- Port Containers Current Number. This variable is used to collect statistics on the daily average wait cost of containers from the moment they arrive to the system until shipped and leave the system.
- Penalty Cost. When the number of containers exceeds the ship capacity, they are outsourced and sent outside the system to be served by a spot charter. This variables allows to add the penalty cost for outsourcing containers.
- operating cost. This variable is added to the total cost everytime a ship departs on a new voyage. We assume that all ships deployed have the same operating cost.
- Arrival rate. Variable describing the arrival pattern to the two different ports. Our study

assumes an arrival rate according to a Poisson process.

- Ship Capacity. This variable determines the maximum number of containers that the ship can carry. We assume that all ships deployed have the same capacity.
- Average load. This variable allows us to look at the average capacity of the ship used expressed as a percentage, or in other word, the utilization of the ship.

These variables provide a logistic picture of our network's performance. Notice that the capital cost is not included in the model. Decisions involving capital cost will be discussed in later section.

To ensure that the model is an actual representation to the system we are considering, we used few numerical values and compared the output of the simulation results to the results obtained from the cost function. The results were close, giving us a confidence in the model accuracy.

The parameters of this model are:

$\lambda_A = 10$ or Changing $\quad P_A = 350 \quad C_A = 50 \quad W = 1600$ or Changing

$\lambda_{\mathrm{B}} = 10$ or Changing $\quad P_B = 350 \quad C_B = 50 \quad T = 1.5$ or Changing

The simulation is run for 365 days with 8 replications to assure accurate statistical results over all scenarios. For all simulation results under diffrent scenarios please see Appendix

An important observation from the simulation analysis is that the total cost function behaves like a convex function which is a sufficient confition for optimality. In other words, carriers can decide on this model to determine the optimal delay time at ports. The parameters of the examples we considered are as follows:

$(\lambda_A,\ \lambda_{\mathrm{B}}) = (10, 10)$

$(\lambda_A, \ \lambda_B) = (8, 12)$

$(\lambda_A, \ \lambda_B) = (1, 19)$

The results are available between Figures 8, 9, and 10 respectively.

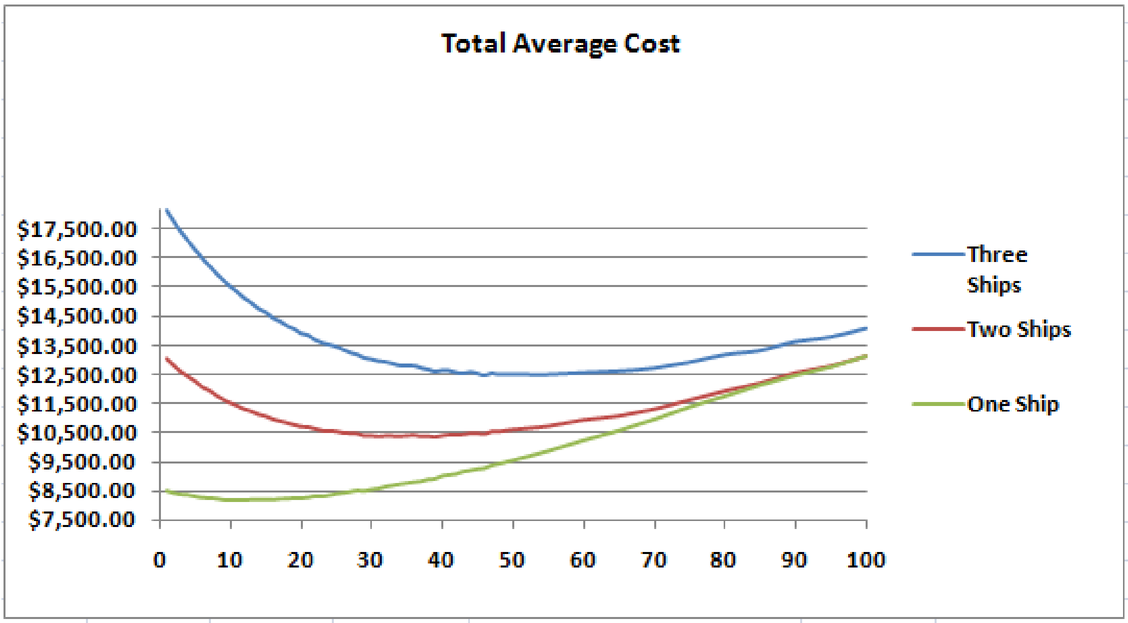


Figure 8 $T \rightarrow$

The total average cost when

$\lambda_a = 10, \ \lambda_b = 10, W = \$1600$

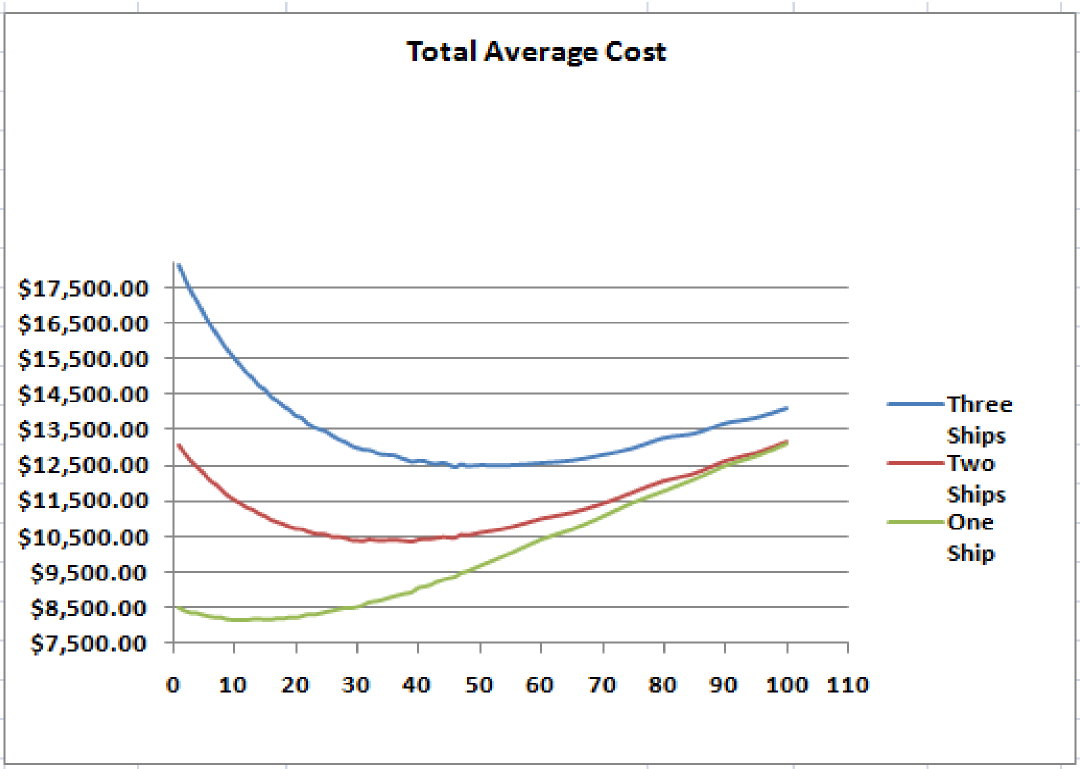


Figure 9 $T \rightarrow$

The total average cost when
$\lambda_a = 8,\ \lambda_b = 12, W = \$1600$

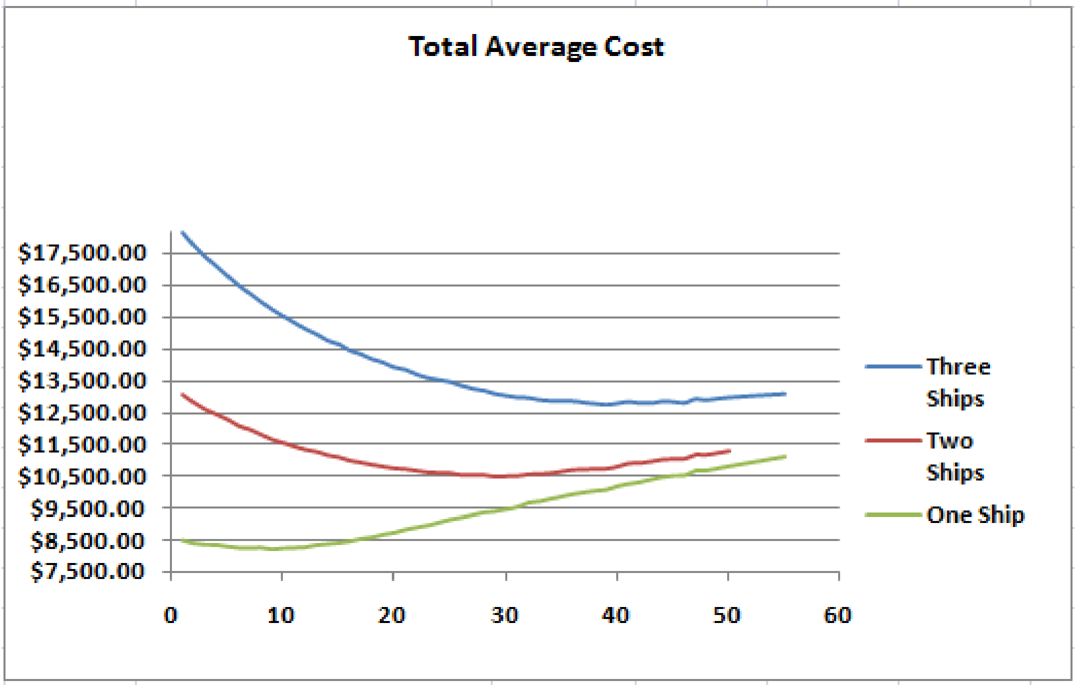


Figure 10 $T \rightarrow$

The total average cost when
$\lambda_a = 1,\ \lambda_b = 19, W = \$1600$

We observe that when the sum of the arrival rates at both ports is much less than the capacity of the ship, only one ship is needed regardless of the inbalance of the arrival rates. However, this should be the case in our example because of the short travel time in the sea which is 1.5 days. If the travel time increases, we will have more containers buil-up at the ports, which definitely suggests adding a ship to the fleet if all other parameters remanin the same.

We also observe that the fleet sizing decision is affected by inbalance of the containers arrival rates when arrival rates increase. The results of this analysis is available in Tables 7 to 15.

Table 7:

Optimal voyage time when $\lambda_A = 10$, $\lambda_B = 10$, *and* $K = 80$

Fleet Size	Optimal Wait Time(Hours)	Total Cost
1	47	$8,138.74
2	75	$10,371.42
3	82	$12,477.09

Table 8:

Optimal voyage time when $\lambda_A = 8$, $\lambda_B = 12$, *and* $K = 80$

Fleet Size	Optimal Wait Time(Hours)	Total Cost
1	47	$8,134.69
2	75	$10,366.61
3	82	$12,470.69

Table 9:

Optimal voyage time when $\lambda_A = 1$ *and* $\lambda_B = 19$, *and* $K = 80$

Fleet Size	Optimal Wait Time(Hours)	Total Cost
1	81	$8,200.92
2	101	$10,471.13
3	111	$12,753.98

Table 10:

Optimal voyage time when $\lambda_A = 30$ and $\lambda_B = 30$

Fleet Size	Optimal Wait Time(Hours)	Total Cost
1	37	$16,809.43
2	42	$17,296.90
3	50	$20,484.41
4	56	$23,579.48

Table 11:

Optimal voyage time when $\lambda_A = 5$ and $\lambda_B = 55$

Fleet Size	Optimal Wait Time(Hours)	Total Cost
1	37	$22,434.32
2	37	$18,826.27
3	42	$21,567.86
4	45	$25,090.73

Table 12:

Optimal voyage time when $\lambda_A = 70$ and $\lambda_B = 70$

Fleet Size	Optimal Wait Time(Hours)	Total Cost
1	37	$50,823.03
2	37	$36,773.43
3	37	$30,968.65

Table 13:

Optimal voyage time when $\lambda_A = 10$ and $\lambda_B = 130$

Fleet Size	Optimal Wait Time(Hours)	Total Cost
1	37	$55,080.69
2	37	$48,207.14
3	37	$44,692.26
4	37	$42,173.90
5	37	$41,633.19
6	37	$45,081.86

Table 14:

Optimal voyage time when $\lambda_A = 70$, $\lambda_B = 70$, and $W = 100,000$

Fleet Size	Optimal Wait Time(Hours)	Total Cost
1	46	$77,424.06
2	67	$84,133.19
3	86	$94,098.77

Table 15:

Optimal voyage time when $\lambda_A = 10$, $\lambda_B = 130$, and $W = 100,000$

Fleet Size	Optimal Wait Time(Hours)	Total Cost
1	53	$79,890.90
2	81	$87,610.31
3	101	$96,249.89
4	111	$105,146.98

CHAPTER 4

CONCLUSIONS

With the increase of international trade, container shipping became a very important component in the supply chain. Shipping companies face today challenges on the operational level due to the increased world fleet and the monopolization of the container shipping industry. Shipping companies must have a competitive strategy in order to stay in operation. Managing the fleet size, sailing frequency, and the wait time at ports can result into significant improvemens in financial results. In this work, we formulated a model to minimize the cost in the container shipping industry while finding the optimal fleet size, the optimal sailing frequency, and the optimal wait time at ports. We first derived a cost function for the case where ship capacity is not considered. The derived function was found to be convex of the fleet size and the voyage time. Thus, we were able to determine the optimal values of these two parameters that minimize our total annual cost. In real life however, voyage time can be managed to a certain extent, and thus can be higher than the optimal time. We developed a policy to determine the optimal fleet size under two scenarios. First, when the optimal ships inter-departure time can not be reached using one ship, or in other words the actual voyage time is longer than the optimal time when one ship is used. On the other hand we consider the case where the actual voyage time is less than the optimal time needed to minimize cost. In the latter case we found that carrier should maintain the service with one ship and manage to keep the voyage time at optimal T. The model is then extended to consider ships' limited capacity, and introduced a new cost related to outsourcing containers. We derived a new cost function to capture the more realistic situation. A closed-form solution for the derived function could not be derived, and we followed a new approach to analyze the function. We decided to develop a simulation model to study the behavior of the function.

The simulation model, developed with Arena software proved to be an important and reliable tool to make sound managerial decisions. The statistical analysis from the simulation results lead to the assumption that the cost function is actually convex and hence, unique values of the voyage time and fleet size exist that minimize the total cost. Sensistivity analysis were conducted to see how our decisions are affected by changing parameters values.

This thesis can stimulate and support further research in the container shipping industry. This work can be extended to include:

- Considering multiple ports of loading and unloading.
- Considering ships with different sizes and operational costs.
- Considering different container sizes with different arrival rates of each type of containers.
- Analysing the case where outsourcing containers is not permitted, that is the carrier is commited to carry all containers. In this case containers may have to wait for more than one voyage until picked. A high frequency of service and more ships may be needed in this case to save on cost of containers waiting.
- Studying the effect on total cost if a carrier deploys its ships when a certain number of containers are available. While it might be more efficient if ships leave at full capacity, carriers may have additional savings if ships depart when a predetermined number of containers are available. Decisions in this case may include the optimal number of containers to carry.

References

1. A. Fabri, & P. Recht. (2006). On dynamic pickup and delivery vehicle routing with several time windows and waiting times. Transportation Research, 40 (4), 335.

2. Anthony J. Hayter. (2002). Probability and statistics for engineers and scientists (2nd Edition ed.). United States of America: Duxbury Thomson Learning.

3. Chaug-Ing Hsu, & Yu-Ping Hsieh. (2007). Routing, ship size, and sailing frequency decision-making for a maritime hub-and-spoke container network. Mathematical and Computer Modelling, 45 (7)

4. Chengxuan Cao, James S.K.Ang, & Hengqing Ye. (2005). Cargo mix and shipment planning problem with uncertainty. 2005 International Conference on Services Systems and Services Management, , 1 305-310.

5. M. Christiansen, K. Fagerholt, & D. Ronen. (2004). Ship routing and scheduling: Status and perspective. Transportation Science, 38 (1), 1.

6. E. Sambracos, J.A Paravantis, C.D. Tarantilis, & C.T. Kiranoudis. (2004). Dispatching of small containers via coastal freight liners: The case of the aegean sea. European Journal of Operational Research, 152 (2), 365.

7. Gao Qiang. (1994). An operational approach for container control in liner shipping. Logistics and Transportation Review, 30 (3), 267.

8. James S.K Ang, Chengxuan Cao, & Heng-Qing Ye. (2006). Model and algorithms for multi-period sea cargo mix problem. European Journal of Operational Research, 180 (3), 1381.

9. John Fossey. (2007, Global and seamless. Containerisation International, 40(6) 7.

10. K Fagerholt, & M Christiansen. (2000). A combined ship scheduling and allocation problem. Journal of the Operation Research Society, 51(7), 834.

11. K.K Lai, Kokin Lam, and K. Chan. (1995). Shipping container logistics and allocation. Journal of the Operation Research Society, 46(6), 687-697.

12. Katta G. Murty, , J. L., , Y. W., & , R. L. (2003). A decision support system for operations in a container terminal. Science Direct,

13. Mabel Chou, Mark Goh, Miao Song, & Chung-Piaw Teo. (2004). Value of transhipment hub in sea freight inventory-routing problems.

14. Marc Levinson. (2006). The box. Princeton, New Jersey 08540: Princeton University Press.

15. Profit optimisation for container carriers(2002). ROI Container Cargo Alliance.

16. Richa Agarwal, & Ozlem Ergun. (2007). Network design and allocation mechanisms for carrier alliances in liner shipping. Georgia Institute of Technology). Operations Research,

17. Shih-Chan Ting, & Gwo-Hshiung Tzeng. (2003). Ship scheduling and cost analysis for route planning in liner shipping. Maritime Economics and Logistics,5 (4), 378.

18. Steven Nahmas. (2001). Production and operations analysis (4th Edition ed.). Singapore: McGraw-Hill Higher Education.

19. Sveuciliste u Dubrovniku, & Odjel Racunarstva. (2004). Application of the minimum cost flow problem in container shipping. 46th International Symposium Electronics in Marine, Zadar, Croatia. 466.

20. Teodor Gabrie Crainic, Michel Gendreau, & Pierre Dejax. (1993). Dynamic and stochastic models for the allocation of empty containers. Operations Research, 41(1), 102.

21. Theo E. Notteboom. (2004). Container shipping and ports: An overview. Review of Network Economic, 3(2)

22. Xiaojing Li. (2006). Network design and fleet allocation model for vessel operation. Massachusetts Institute of Technology).

23. Zdenka Zenzerovic, & Svjetlana Beslic. (2003). Optimization of cargo transport with a view to cost efficient operation of container ship. 25th Int. Conf. Information Technology Interfaces, Cavtat, Croatia. 531-536.

Appendix

Table 16: Data of Figure 11
Q = 1, W = $1600, K=80

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	39.792	39.583	0	0	$1,544.80	$1,546.69	$5,379.45	$8,470.94
2	2	41.458	40.625	0	0	$1,597.24	$1,593.72	$5,198.63	$8,389.59
3	3	42.292	41.875	0	0	$1,637.62	$1,644.34	$5,063.01	$8,344.98
4	4	47.5	40.208	0	0	$1,678.52	$1,677.20	$4,972.60	$8,328.32
5	5	43.75	42.708	0	0	$1,723.28	$1,719.20	$4,836.99	$8,279.47
6	6	45.833	44.375	0	0	$1,763.16	$1,764.09	$4,701.37	$8,228.62
7	7	48.333	44.375	0	0	$1,803.93	$1,804.18	$4,610.96	$8,219.07
8	8	48.333	43.958	0	0	$1,843.11	$1,839.14	$4,520.55	$8,202.80
9	9	51.458	44.375	0	0	$1,894.78	$1,878.39	$4,384.93	$8,158.10
10	10	45.833	47.708	0	0	$1,923.86	$1,928.75	$4,294.52	$8,147.14
11	11	47.5	47.708	0	0	$1,967.84	$1,966.79	$4,204.11	$8,138.74
12	12	54.583	48.333	0	0	$2,019.01	$2,014.11	$4,113.70	$8,146.83
13	13	54.167	48.333	0	0	$2,051.77	$2,046.81	$4,068.49	$8,167.07
14	14	54.167	49.792	0	0	$2,095.77	$2,093.10	$3,978.08	$8,166.95
15	15	55	52.083	0	0	$2,135.20	$2,138.17	$3,887.67	$8,161.04
16	16	52.292	51.875	0	0	$2,181.66	$2,178.88	$3,797.26	$8,157.81
17	17	56.042	54.167	0	0	$2,216.71	$2,219.88	$3,752.06	$8,188.64
18	18	52.708	55.417	0	0	$2,270.73	$2,264.04	$3,661.64	$8,196.42
19	19	58.958	56.875	0	0	$2,305.56	$2,297.34	$3,616.44	$8,219.34
20	20	60.833	55.625	0	0	$2,338.84	$2,344.49	$3,526.03	$8,209.36
21	21	60.625	57.917	0	0	$2,377.94	$2,382.87	$3,480.82	$8,241.63
22	22	60.833	59.375	0	0	$2,426.65	$2,423.77	$3,435.62	$8,286.04
23	23	64.375	59.583	0	0	$2,458.83	$2,470.90	$3,345.21	$8,274.93
24	24	57.5	60.833	0	0	$2,510.24	$2,505.73	$3,300.00	$8,315.97
25	25	64.583	59.792	0	0	$2,554.36	$2,555.77	$3,254.80	$8,364.92
26	26	65.417	64.167	0	0	$2,592.49	$2,597.79	$3,209.59	$8,399.87
27	27	65.417	63.958	0	0	$2,635.97	$2,637.05	$3,164.38	$8,437.40
28	28	67.917	66.458	0	0	$2,683.11	$2,679.38	$3,119.18	$8,481.67
29	29	69.583	66.042	0	0	$2,718.64	$2,718.77	$3,028.77	$8,466.18
30	30	71.667	67.083	0	0	$2,766.54	$2,764.97	$2,983.56	$8,515.07
31	31	71.25	68.333	0.479	0	$2,808.79	$2,798.33	$2,938.36	$8,545.96
32	32	71.458	69.792	0	0.959	$2,837.30	$2,837.37	$2,938.36	$8,613.99
33	33	71.042	71.042	0	0.479	$2,886.81	$2,873.50	$2,893.15	$8,653.94
34	34	74.583	76.25	1.598	0.799	$2,927.03	$2,920.81	$2,847.95	$8,698.18
36	36	69.792	70	0.32	3.196	$3,010.27	$2,997.17	$2,757.53	$8,768.49
37	37	73.333	75.208	0.16	3.995	$3,039.73	$3,041.93	$2,712.33	$8,798.15
38	38	79.167	77.083	3.196	5.434	$3,082.68	$3,099.21	$2,667.12	$8,857.65
39	39	78.125	79.375	3.356	5.114	$3,138.26	$3,115.95	$2,621.92	$8,884.60
40	40	78.542	83.958	2.237	4.635	$3,180.99	$3,175.15	$2,621.92	$8,984.93
41	41	77.292	79.583	2.078	9.749	$3,221.96	$3,205.30	$2,576.71	$9,015.80
42	42	82.5	80.208	7.511	10.228	$3,254.16	$3,251.98	$2,531.51	$9,055.39
43	43	82.083	84.583	7.032	15.502	$3,297.58	$3,285.49	$2,531.51	$9,137.12
44	44	78.125	78.958	11.826	23.014	$3,332.74	$3,321.97	$2,486.30	$9,175.85
45	45	82.708	83.75	12.306	20.137	$3,382.70	$3,368.50	$2,441.10	$9,224.73
46	46	84.375	86.042	12.306	21.416	$3,419.51	$3,401.82	$2,395.89	$9,250.94
47	47	84.375	82.917	19.658	34.68	$3,451.78	$3,449.20	$2,395.89	$9,351.21
48	48	86.875	86.25	20.776	35.479	$3,506.25	$3,484.68	$2,350.69	$9,397.88
50	50	85.625	88.125	34.521	48.584	$3,580.41	$3,551.70	$2,305.48	$9,520.70
55	55	92.917	92.708	84.224	104.041	$3,751.36	$3,735.48	$2,169.86	$9,844.97
60	60	92.708	93.333	160.457	175.639	$3,905.73	$3,891.74	$2,079.45	$10,213.01
65	65	95.625	98.958	269.932	276.005	$4,028.14	$4,034.29	$1,943.84	$10,552.20
70	70	97.917	100	393.311	393.47	$4,152.32	$4,145.01	$1,853.43	$10,937.54
75	75	100	100	517.489	508.219	$4,268.90	$4,263.19	$1,808.22	$11,366.01
80	80	98.958	100	638.95	631.279	$4,382.88	$4,371.52	$1,717.81	$11,742.45
85	85	100	100	766.804	736.758	$4,488.01	$4,504.50	$1,627.40	$12,123.47
90	90	100	100	857.9	849.909	$4,588.77	$4,584.79	$1,582.19	$12,463.56
95	95	100	100	963.699	927.1	$4,681.31	$4,689.05	$1,491.78	$12,752.94
100	100	100	100	1065.342	1023.47	$4,784.18	$4,805.27	$1,446.58	$13,124.84

Table 17: Data of Figure 11
Q = 2, W = $1600, K =80

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	21.458	16.875	0	0	$1,165.90	$1,166.69	$10,713.70	$13,046.29
2	2	17.5	21.667	0	0	$1,199.03	$1,203.54	$10,397.26	$12,799.83
3	3	23.958	16.25	0	0	$1,234.55	$1,232.81	$10,126.03	$12,593.39
4	4	24.583	16.667	0	0	$1,263.26	$1,266.04	$9,900.00	$12,429.30
5	5	21.458	22.5	0	0	$1,298.99	$1,300.47	$9,673.97	$12,273.43
6	6	25.625	16.042	0	0	$1,334.99	$1,334.36	$9,402.74	$12,072.09
7	7	20	24.167	0	0	$1,367.95	$1,365.91	$9,221.92	$11,955.78
8	8	28.75	17.917	0	0	$1,399.65	$1,397.50	$8,995.89	$11,793.04
9	9	28.542	17.917	0	0	$1,439.31	$1,434.81	$8,769.86	$11,643.98
10	10	21.042	16.042	0	0	$1,468.93	$1,470.08	$8,589.04	$11,528.05
11	11	21.042	16.875	0	0	$1,510.96	$1,503.06	$8,408.22	$11,422.23
12	12	31.667	16.875	0	0	$1,542.99	$1,541.79	$8,227.40	$11,312.18
13	13	34.583	16.667	0	0	$1,574.16	$1,575.55	$8,091.78	$11,241.49
14	14	20.833	19.792	0	0	$1,612.49	$1,614.56	$7,910.96	$11,138.01
15	15	17.5	34.167	0	0	$1,651.56	$1,653.90	$7,775.34	$11,080.80
16	16	21.042	16.875	0	0	$1,689.08	$1,689.97	$7,594.52	$10,973.57
17	17	21.458	19.792	0	0	$1,725.98	$1,726.59	$7,458.90	$10,911.47
18	18	22.917	16.042	0	0	$1,768.80	$1,759.37	$7,323.29	$10,851.47
19	19	20.208	19.375	0	0	$1,801.52	$1,795.45	$7,187.67	$10,784.64
20	20	41.25	16.667	0	0	$1,836.10	$1,835.15	$7,052.06	$10,723.30
21	21	17.5	40.417	0	0	$1,874.26	$1,870.01	$6,961.64	$10,705.92
22	22	42.083	19.583	0	0	$1,911.11	$1,913.49	$6,826.03	$10,650.63
23	23	44.792	16.667	0	0	$1,943.52	$1,951.69	$6,690.41	$10,585.62
24	24	22.917	16.875	0	0	$1,989.71	$1,985.84	$6,600.00	$10,575.55
25	25	17.917	42.917	0	0	$2,027.00	$2,028.26	$6,509.59	$10,564.85
26	26	19.792	17.708	0	0	$2,069.79	$2,066.10	$6,373.97	$10,509.86
27	27	20.833	19.792	0	0	$2,108.99	$2,105.17	$6,283.56	$10,497.72
28	28	50	19.583	0	0	$2,145.91	$2,141.31	$6,193.15	$10,480.37
29	29	51.042	20.208	0	0	$2,172.36	$2,176.79	$6,057.53	$10,406.68
30	30	53.125	20	0	0	$2,222.34	$2,221.50	$5,967.12	$10,410.96
31	31	53.333	20.208	0	0	$2,256.89	$2,253.89	$5,876.71	$10,387.50
32	32	54.583	19.167	0	0	$2,289.66	$2,289.81	$5,831.51	$10,410.98
33	33	20.833	19.167	0	0	$2,334.51	$2,328.54	$5,741.10	$10,404.15
34	34	19.792	16.667	0	0	$2,373.16	$2,370.63	$5,650.69	$10,394.47
36	36	22.917	53.125	0	0	$2,451.71	$2,459.06	$5,515.07	$10,425.83
37	37	19.583	17.083	0	0	$2,483.31	$2,481.96	$5,424.66	$10,389.93
38	38	60.417	20	0	0	$2,531.04	$2,531.91	$5,334.25	$10,397.20
39	39	20.833	19.167	0	0	$2,567.10	$2,560.49	$5,243.84	$10,371.42
40	40	17.5	16.25	0	0	$2,606.73	$2,609.36	$5,198.63	$10,414.72
41	41	19.583	60	0	0	$2,642.46	$2,648.78	$5,153.43	$10,444.66
42	42	65.625	20	0	0	$2,684.67	$2,690.17	$5,063.01	$10,437.85
43	43	20.208	16.25	0	0	$2,716.11	$2,722.66	$5,017.81	$10,456.58
44	44	22.917	60.208	0	0	$2,766.27	$2,758.04	$4,972.60	$10,496.91
45	45	18.958	17.917	0	0	$2,801.20	$2,809.86	$4,882.19	$10,493.25
46	46	21.458	16.667	0	0	$2,842.93	$2,835.84	$4,791.78	$10,470.55
47	47	21.042	64.375	0	0.32	$2,881.49	$2,880.42	$4,791.78	$10,554.01
48	48	72.5	20	0	0	$2,934.99	$2,921.66	$4,701.37	$10,558.01
50	50	19.167	17.292	0.479	0.16	$3,010.02	$2,997.09	$4,610.96	$10,618.71
55	55	17.292	20.208	1.918	3.676	$3,193.40	$3,196.51	$4,339.73	$10,735.24
60	60	22.917	77.708	4.795	8.151	$3,390.83	$3,385.40	$4,158.90	$10,948.08
65	65	20	16.667	19.178	24.452	$3,574.60	$3,577.75	$3,887.67	$11,083.64
70	70	21.458	16.042	51.142	50.502	$3,761.72	$3,741.17	$3,706.85	$11,311.38
75	75	18.333	19.583	103.402	106.758	$3,934.57	$3,910.17	$3,571.23	$11,626.13
80	80	18.75	16.875	175.479	180.114	$4,085.74	$4,059.49	$3,435.62	$11,936.44
85	85	98.958	16.042	277.443	259.543	$4,200.51	$4,207.79	$3,254.80	$12,200.08
90	90	16.875	16.042	387.557	380.205	$4,320.73	$4,301.80	$3,164.38	$12,554.67
95	95	21.458	20.417	499.749	473.699	$4,410.84	$4,413.87	$2,983.56	$12,781.72
100	100	100	16.875	607.785	576.621	$4,510.93	$4,532.55	$2,893.15	$13,121.04

Table 18: Data of Figure 11
Q = 3, W = $1600, K = 80

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	14.375	8.75	0	0	$1,037.13	$1,038.66	$16,047.95	$18,123.74
2	2	13.125	10.833	0	0	$1,067.55	$1,067.92	$15,595.89	$17,731.37
3	3	16.875	10.208	0	0	$1,097.92	$1,098.07	$15,189.04	$17,385.03
4	4	18.542	11.667	0	0	$1,126.66	$1,127.86	$14,827.40	$17,081.91
5	5	15.208	15.833	0	0	$1,157.16	$1,158.12	$14,465.75	$16,781.03
6	6	20.833	8.542	0	0	$1,190.21	$1,189.47	$14,104.11	$16,483.78
7	7	13.125	17.917	0	0	$1,219.33	$1,221.02	$13,787.67	$16,228.02
8	8	12.5	10.208	0	0	$1,254.52	$1,250.99	$13,471.23	$15,976.74
9	9	22.5	12.5	0	0	$1,283.76	$1,286.04	$13,154.80	$15,724.59
10	10	10.625	8.542	0	0	$1,318.06	$1,321.44	$12,883.56	$15,523.06
11	11	10.625	8.75	0	0	$1,353.31	$1,355.97	$12,612.33	$15,321.61
12	12	25.625	12.917	0	0	$1,385.04	$1,388.45	$12,341.10	$15,114.59
13	13	26.875	10.208	0	0	$1,416.95	$1,420.84	$12,115.07	$14,952.86
14	14	15	12.708	0	0	$1,455.71	$1,455.73	$11,843.84	$14,755.28
15	15	15.625	26.25	0	0	$1,486.75	$1,489.17	$11,663.01	$14,638.94
16	16	14.167	13.125	0	0	$1,521.30	$1,523.69	$11,391.78	$14,436.78
17	17	15.208	13.542	0	0	$1,559.80	$1,561.16	$11,210.96	$14,331.92
18	18	12.708	8.542	0	0	$1,597.71	$1,592.35	$10,984.93	$14,174.99
19	19	13.333	12.083	0	0	$1,632.09	$1,633.55	$10,804.11	$14,069.75
20	20	33.542	11.667	0	0	$1,666.26	$1,671.48	$10,578.08	$13,915.82
21	21	13.125	34.167	0	0	$1,707.44	$1,701.98	$10,442.47	$13,851.89
22	22	15	12.5	0	0	$1,738.27	$1,737.33	$10,216.44	$13,692.03
23	23	37.083	10.208	0	0	$1,775.04	$1,779.81	$10,035.62	$13,590.47
24	24	12.708	10.417	0	0	$1,812.38	$1,814.39	$9,900.00	$13,526.77
25	25	12.083	38.333	0	0	$1,849.18	$1,850.10	$9,764.38	$13,463.67
26	26	14.375	13.333	0	0	$1,888.72	$1,884.79	$9,583.56	$13,357.07
27	27	15.625	13.958	0	0	$1,928.18	$1,924.69	$9,402.74	$13,255.61
28	28	14.375	13.333	0	0	$1,958.02	$1,961.86	$9,267.12	$13,187.00
29	29	13.125	13.75	0	0	$1,991.79	$1,999.11	$9,086.30	$13,077.19
30	30	12.5	13.333	0	0	$2,041.92	$2,041.99	$8,950.69	$13,034.59
31	31	13.125	12.292	0	0	$2,073.84	$2,078.51	$8,815.07	$12,967.42
32	32	14.375	11.875	0	0	$2,104.20	$2,110.57	$8,724.66	$12,939.43
33	33	15.625	14.583	0	0	$2,144.59	$2,147.77	$8,589.04	$12,881.40
34	34	14.375	12.708	0	0	$2,183.83	$2,186.08	$8,453.43	$12,823.33
36	36	12.708	12.917	0	0	$2,267.85	$2,265.78	$8,272.60	$12,806.23
37	37	15.208	12.917	0	0	$2,302.74	$2,296.46	$8,136.99	$12,736.19
38	38	54.792	11.458	0	0	$2,337.98	$2,344.33	$8,001.37	$12,683.68
39	39	15	14.583	0	0	$2,372.53	$2,374.71	$7,865.75	$12,612.99
40	40	13.125	12.083	0	0	$2,420.30	$2,414.28	$7,820.55	$12,655.13
41	41	15.625	10.625	0	0	$2,450.59	$2,458.79	$7,730.14	$12,639.52
42	42	58.75	11.458	0	0	$2,488.99	$2,493.98	$7,594.52	$12,577.48
43	43	13.333	12.5	0	0	$2,519.22	$2,521.71	$7,504.11	$12,545.03
44	44	12.708	56.042	0	0	$2,574.52	$2,563.21	$7,458.90	$12,596.63
45	45	12.083	10.625	0	0	$2,608.46	$2,609.81	$7,323.29	$12,541.55
46	46	15.208	12.708	0	0	$2,644.81	$2,644.61	$7,187.67	$12,477.09
47	47	14.167	58.125	0	0	$2,683.52	$2,684.32	$7,187.67	$12,555.51
48	48	65.625	11.667	0	0	$2,734.45	$2,728.79	$7,052.06	$12,515.30
50	50	11.667	12.917	0	0	$2,802.03	$2,799.18	$6,916.44	$12,517.65
55	55	11.667	13.75	0.16	0	$2,991.02	$3,008.15	$6,509.59	$12,508.92
60	60	12.708	71.667	0.16	1.758	$3,191.71	$3,185.03	$6,193.15	$12,571.80
65	65	13.125	12.292	4.795	6.393	$3,390.50	$3,389.27	$5,831.51	$12,622.46
70	70	15.208	11.667	12.945	15.822	$3,578.23	$3,561.21	$5,560.27	$12,728.48
75	75	12.083	11.042	43.63	43.63	$3,763.54	$3,740.70	$5,334.25	$12,925.74
80	80	12.917	10.417	74.635	93.014	$3,941.08	$3,921.78	$5,153.43	$13,183.93
85	85	95.208	12.708	148.151	145.913	$4,073.87	$4,077.72	$4,882.19	$13,327.84
90	90	13.542	8.542	235.571	242.443	$4,214.33	$4,189.81	$4,746.58	$13,628.73
95	95	10.417	14.375	349.041	335.137	$4,314.14	$4,315.85	$4,475.34	$13,789.51
100	100	14.375	10.625	457.237	431.667	$4,416.34	$4,435.84	$4,339.73	$14,080.81

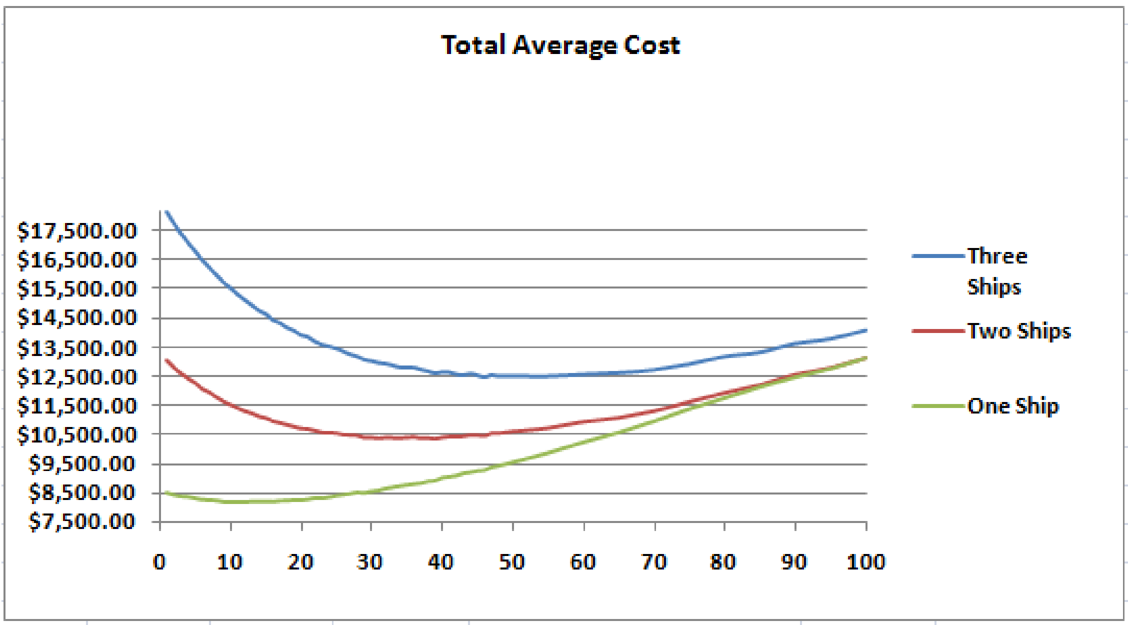


Figure 11 $T \rightarrow$

The total average cost when
$\lambda_a = 10,\ \lambda_b = 10, W = \$1600$

Table 19: Data of Figure 12
Q = 1, W = $1600, K = 80

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	31.458	46.458	0	0	$1,233.71	$1,865.20	$5,379.45	$8,478.37
2	2	31.042	48.333	0	0	$1,278.14	$1,912.28	$5,198.63	$8,389.04
3	3	32.708	49.792	0	0	$1,306.06	$1,968.93	$5,063.01	$8,337.99
4	4	34.375	50	0	0	$1,340.70	$2,014.54	$4,972.60	$8,327.85
5	5	35.833	51.458	0	0	$1,375.00	$2,063.99	$4,836.99	$8,275.97
6	6	36.042	52.708	0	0	$1,410.81	$2,124.39	$4,701.37	$8,236.57
7	7	38.333	56.042	0	0	$1,434.03	$2,164.69	$4,610.96	$8,209.68
8	8	37.292	55.625	0	0	$1,471.55	$2,214.53	$4,520.55	$8,206.63
9	9	37.708	55.208	0	0	$1,509.73	$2,259.90	$4,384.93	$8,154.57
10	10	36.667	57.292	0	0	$1,533.85	$2,313.83	$4,294.52	$8,142.20
11	11	38.125	58.125	0	0	$1,567.76	$2,362.83	$4,204.11	$8,134.69
12	12	40	59.375	0	0	$1,611.79	$2,411.28	$4,113.70	$8,136.77
13	13	41.875	60.625	0	0	$1,635.99	$2,459.25	$4,068.49	$8,163.73
14	14	44.583	58.542	0	0	$1,676.81	$2,517.59	$3,978.08	$8,172.48
15	15	41.042	65.208	0	0	$1,701.43	$2,565.16	$3,887.67	$8,154.26
16	16	42.917	63.125	0	0.799	$1,737.93	$2,612.12	$3,797.26	$8,148.11
17	17	45.208	61.875	0	0	$1,767.19	$2,668.08	$3,752.06	$8,187.32
18	18	45.625	66.042	0	0	$1,802.86	$2,714.92	$3,661.64	$8,179.43
19	19	48.333	66.25	0	0	$1,840.63	$2,758.59	$3,616.44	$8,215.66
20	20	46.875	67.5	0	1.119	$1,868.37	$2,810.16	$3,526.03	$8,205.68
21	21	47.917	71.667	0	0	$1,901.91	$2,860.88	$3,480.82	$8,243.61
22	22	48.75	68.75	0	0.959	$1,942.61	$2,914.48	$3,435.62	$8,293.66
23	23	49.583	72.917	0	2.237	$1,973.29	$2,966.21	$3,345.21	$8,286.94
24	24	49.167	73.125	0	3.676	$2,006.89	$3,010.70	$3,300.00	$8,321.27
25	25	51.875	76.875	0	6.233	$2,034.87	$3,068.92	$3,254.80	$8,364.82
26	26	52.917	73.542	0	4.635	$2,071.52	$3,110.15	$3,209.59	$8,395.90
27	27	53.333	75.208	0	9.11	$2,102.38	$3,165.02	$3,164.38	$8,440.89
28	28	53.958	76.875	0	9.589	$2,136.95	$3,207.69	$3,119.18	$8,473.41
29	29	54.792	77.292	0	11.986	$2,167.90	$3,256.37	$3,028.77	$8,465.03
30	30	56.042	79.375	0	12.785	$2,199.23	$3,306.59	$2,983.56	$8,502.17
31	31	55.208	79.792	0	21.735	$2,247.41	$3,354.43	$2,938.36	$8,561.93
32	32	57.292	81.667	0	17.1	$2,273.53	$3,414.39	$2,938.36	$8,643.37
33	33	58.542	82.917	0	31.164	$2,307.13	$3,440.91	$2,893.15	$8,672.35
34	34	61.25	85.833	0	36.438	$2,333.02	$3,489.60	$2,847.95	$8,707.01
36	36	58.958	87.292	0	55.457	$2,412.77	$3,585.05	$2,757.53	$8,810.81
37	37	62.5	88.958	0	66.804	$2,435.13	$3,638.02	$2,712.33	$8,852.28
38	38	61.042	90.625	0	93.493	$2,458.72	$3,673.43	$2,667.12	$8,892.76
39	39	64.792	92.083	0	107.078	$2,504.59	$3,692.30	$2,621.92	$8,925.89
40	40	62.708	95	0	118.425	$2,549.91	$3,761.78	$2,621.92	$9,052.04
41	41	65.833	91.667	0	141.438	$2,569.69	$3,790.04	$2,576.71	$9,077.87
42	42	65.625	93.958	0	156.781	$2,598.86	$3,841.46	$2,531.51	$9,128.60
43	43	68.333	96.25	0	171.963	$2,634.98	$3,878.60	$2,531.51	$9,217.05
44	44	66.458	96.042	0	214.954	$2,669.51	$3,906.12	$2,486.30	$9,276.89
45	45	69.167	96.875	0	230.297	$2,702.70	$3,945.67	$2,441.10	$9,319.76
46	46	69.792	98.75	0	258.904	$2,733.84	$3,975.03	$2,395.89	$9,363.67
47	47	70.417	97.083	0	289.909	$2,766.89	$4,018.04	$2,395.89	$9,470.73
48	48	69.375	97.708	0	322.192	$2,808.41	$4,037.40	$2,350.69	$9,518.69
50	50	71.875	99.167	0	385.959	$2,866.63	$4,106.10	$2,305.48	$9,664.17
55	55	78.333	100	2.557	559.68	$3,031.77	$4,260.37	$2,169.86	$10,024.24
60	60	81.25	100	3.676	744.429	$3,197.13	$4,390.10	$2,079.45	$10,414.79
65	65	83.125	100	10.388	897.694	$3,345.49	$4,500.61	$1,943.84	$10,698.02
70	70	86.458	100	26.21	1050	$3,501.26	$4,626.03	$1,853.43	$11,056.92
75	75	91.042	100	51.941	1183.447	$3,660.23	$4,756.76	$1,808.22	$11,460.59
80	80	96.458	100	88.059	1329.521	$3,792.19	$4,855.95	$1,717.81	$11,783.53
85	85	94.167	100	146.073	1429.726	$3,912.42	$5,000.77	$1,627.40	$12,116.39
90	90	99.792	100	213.196	1551.986	$4,024.14	$5,131.71	$1,582.19	$12,503.23
95	95	98.958	100	289.11	1619.429	$4,128.49	$5,235.80	$1,491.78	$12,764.61
100	100	100	100	369.018	1720.274	$4,214.46	$5,371.50	$1,446.58	$13,121.82

Table 20: Data of Figure 12
Q = 2, W = $1600, K = 80

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost
1	1	14.792	23.333	$0.00	$0.00	$929.21	$1,403.40	$10,713.70
2	2	16.875	23.75	$0.00	$0.00	$960.82	$1,443.32	$10,397.26
3	3	16.042	23.125	$0.00	$0.00	$988.07	$1,479.48	$10,126.03
4	4	18.125	22.292	$0.00	$0.00	$1,007.96	$1,520.63	$9,900.00
5	5	13.958	28.75	$0.00	$0.00	$1,035.86	$1,563.47	$9,673.97
6	6	18.542	23.333	$0.00	$0.00	$1,064.39	$1,601.19	$9,402.74
7	7	14.792	31.25	$0.00	$0.00	$1,089.85	$1,642.46	$9,221.92
8	8	22.5	19.792	$0.00	$0.00	$1,119.06	$1,681.87	$8,995.89
9	9	21.458	23.125	$0.00	$0.00	$1,146.31	$1,723.50	$8,769.86
10	10	17.708	23.333	$0.00	$0.00	$1,173.35	$1,771.00	$8,589.04
11	11	17.708	23.333	$0.00	$0.00	$1,205.52	$1,808.23	$8,408.22
12	12	23.75	22.708	$0.00	$0.00	$1,232.65	$1,853.18	$8,227.40
13	13	27.083	19.792	$0.00	$0.00	$1,259.05	$1,898.46	$8,091.78
14	14	13.542	24.375	$0.00	$0.00	$1,290.72	$1,942.79	$7,910.96
15	15	16.458	40.833	$0.00	$0.00	$1,313.29	$1,983.82	$7,775.34
16	16	14.583	22.917	$0.00	$0.00	$1,351.19	$2,027.46	$7,594.52
17	17	13.958	25.208	$0.00	$0.00	$1,373.32	$2,073.57	$7,458.90
18	18	16.25	23.333	$0.00	$0.00	$1,410.50	$2,119.88	$7,323.29
19	19	13.958	25.625	$0.00	$0.00	$1,436.21	$2,155.26	$7,187.67
20	20	32.083	22.292	$0.00	$0.00	$1,468.07	$2,206.77	$7,052.06
21	21	16.875	50.833	$0.00	$0.00	$1,502.26	$2,248.34	$6,961.64
22	22	34.167	24.792	$0.00	$0.00	$1,522.39	$2,296.69	$6,826.03
23	23	35.625	19.792	$0.00	$0.00	$1,554.47	$2,346.46	$6,690.41
24	24	16.25	22.708	$0.00	$0.00	$1,586.18	$2,387.80	$6,600.00
25	25	17.292	55.833	$0.00	$0.00	$1,616.89	$2,432.35	$6,509.59
26	26	13.958	24.167	$0.00	$0.00	$1,644.30	$2,476.86	$6,373.97
27	27	13.75	26.25	$0.00	$0.00	$1,682.93	$2,523.54	$6,283.56
28	28	39.375	25	$0.00	$0.00	$1,707.96	$2,568.17	$6,193.15
29	29	39.167	24.583	$0.00	$0.00	$1,736.72	$2,612.61	$6,057.53
30	30	40.417	23.75	$0.00	$0.00	$1,765.79	$2,664.31	$5,967.12
31	31	40.625	24.792	$0.00	$0.00	$1,802.19	$2,703.80	$5,876.71
32	32	43.125	25.625	$0.00	$0.00	$1,831.13	$2,759.21	$5,831.51
33	33	13.75	24.375	$0.00	$0.80	$1,861.53	$2,802.79	$5,741.10
34	34	13.958	21.667	$0.00	$0.00	$1,891.67	$2,851.52	$5,650.69
36	36	16.25	67.292	$0.00	$0.00	$1,953.53	$2,949.19	$5,515.07
37	37	13.958	22.917	$0.00	$0.00	$1,986.95	$2,987.99	$5,424.66
38	38	46.458	21.25	$0.00	$1.44	$2,010.88	$3,035.29	$5,334.25
39	39	13.542	24.375	$0.00	$0.80	$2,048.54	$3,073.44	$5,243.84
40	40	16.875	20.417	$0.00	$1.76	$2,088.23	$3,130.78	$5,198.63
41	41	14.792	72.917	$0.00	$1.44	$2,102.94	$3,182.26	$5,153.43
42	42	50	21.25	$0.00	$1.44	$2,148.83	$3,227.74	$5,063.01
43	43	13.958	22.083	$0.00	$2.88	$2,169.83	$3,273.41	$5,017.81
44	44	16.25	77.083	$0.00	$4.32	$2,205.33	$3,308.22	$4,972.60
45	45	14.792	19.792	$0.00	$3.84	$2,231.57	$3,360.06	$4,882.19
46	46	13.958	21.667	$0.00	$8.47	$2,268.60	$3,402.16	$4,791.78
47	47	14.583	80.625	$0.00	$11.99	$2,301.65	$3,452.50	$4,791.78
48	48	55.208	21.667	$0.00	$10.23	$2,333.29	$3,493.58	$4,701.37
50	50	16.667	22.5	$0.00	$14.54	$2,399.29	$3,585.78	$4,610.96
55	55	15.208	24.583	$0.00	$53.86	$2,551.00	$3,820.89	$4,339.73
60	60	16.25	95.208	$0.00	$117.95	$2,714.70	$4,006.99	$4,158.90
65	65	14.792	21.042	$0.00	$234.29	$2,865.24	$4,176.90	$3,887.67
70	70	13.958	20.417	$0.00	$367.42	$3,029.04	$4,315.60	$3,706.85
75	75	16.667	20.833	$2.40	$517.17	$3,196.18	$4,450.36	$3,571.23
80	80	14.583	22.708	$4.00	$702.40	$3,353.15	$4,554.90	$3,435.62
85	85	83.75	20.625	$12.63	$814.59	$3,491.49	$4,689.34	$3,254.80
90	90	14.167	23.333	$29.73	$945.64	$3,657.11	$4,806.88	$3,164.38
95	95	17.708	22.708	$54.66	$1,077.17	$3,791.00	$4,923.56	$2,983.56
100	100	93.75	20	$97.33	$1,182.17	$3,916.96	$5,051.20	$2,893.15

Table 21: Data of Figure 12
Q = 3, W = $1600, K = 80

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	10.625	15.417	0	0	$829.39	$1,249.24	$16,047.95	$18,126.58
2	2	8.958	15.417	0	0	$852.74	$1,283.47	$15,595.89	$17,732.10
3	3	11.875	15.417	0	0	$876.05	$1,317.89	$15,189.04	$17,382.97
4	4	12.917	16.458	0	0	$900.83	$1,356.08	$14,827.40	$17,084.30
5	5	9.167	22.083	0	0	$924.95	$1,391.85	$14,465.75	$16,782.55
6	6	13.542	15.417	0	0	$948.72	$1,429.07	$14,104.11	$16,481.89
7	7	10.208	25	0	0	$973.92	$1,470.74	$13,787.67	$16,232.34
8	8	9.583	15.833	0	0	$1,000.72	$1,503.70	$13,471.23	$15,975.65
9	9	16.25	16.25	0	0	$1,025.09	$1,548.63	$13,154.80	$15,728.51
10	10	11.25	15.417	0	0	$1,054.94	$1,586.72	$12,883.56	$15,525.22
11	11	11.25	15.417	0	0	$1,080.81	$1,627.15	$12,612.33	$15,320.29
12	12	18.542	14.792	0	0	$1,108.34	$1,669.00	$12,341.10	$15,118.44
13	13	21.458	15.417	0	0	$1,133.38	$1,709.73	$12,115.07	$14,958.18
14	14	9.375	12.917	0	0	$1,162.49	$1,749.73	$11,843.84	$14,756.06
15	15	9.375	31.458	0	0	$1,188.80	$1,791.05	$11,663.01	$14,642.87
16	16	12.292	16.25	0	0	$1,216.90	$1,834.39	$11,391.78	$14,443.07
17	17	9.167	14.375	0	0	$1,241.45	$1,873.13	$11,210.96	$14,325.54
18	18	11.875	15.417	0	0	$1,278.35	$1,919.16	$10,984.93	$14,182.44
19	19	8.75	13.333	0	0	$1,303.60	$1,963.69	$10,804.11	$14,071.39
20	20	26.458	16.458	0	0	$1,332.04	$2,006.01	$10,578.08	$13,916.13
21	21	8.958	40.833	0	0	$1,364.66	$2,046.71	$10,442.47	$13,853.83
22	22	9.375	11.875	0	0	$1,386.36	$2,087.00	$10,216.44	$13,689.80
23	23	29.792	15.417	0	0	$1,418.05	$2,136.92	$10,035.62	$13,590.59
24	24	11.875	15.625	0	0	$1,446.64	$2,177.06	$9,900.00	$13,523.70
25	25	8.958	48.125	0	0	$1,473.69	$2,222.88	$9,764.38	$13,460.95
26	26	9.167	19.167	0	0	$1,505.69	$2,263.04	$9,583.56	$13,352.29
27	27	9.375	14.792	0	0	$1,537.65	$2,308.09	$9,402.74	$13,248.47
28	28	9.167	13.125	0	0	$1,563.04	$2,354.34	$9,267.12	$13,184.50
29	29	10.208	12.708	0	0	$1,590.22	$2,403.04	$9,086.30	$13,079.57
30	30	9.583	11.875	0	0	$1,621.59	$2,443.94	$8,950.69	$13,016.21
31	31	10.208	11.667	0	0	$1,653.98	$2,495.07	$8,815.07	$12,964.12
32	32	9.167	13.542	0	0	$1,684.89	$2,539.90	$8,724.66	$12,949.44
33	33	9.375	17.917	0	0	$1,718.66	$2,578.84	$8,589.04	$12,886.54
34	34	9.167	16.875	0	0	$1,742.45	$2,630.30	$8,453.43	$12,826.18
36	36	11.875	14.792	0	0	$1,808.50	$2,717.13	$8,272.60	$12,798.23
37	37	9.583	14.792	0	0	$1,839.13	$2,759.33	$8,136.99	$12,735.44
38	38	42.5	14.583	0	0	$1,865.59	$2,807.90	$8,001.37	$12,674.86
39	39	9.375	17.917	0	0	$1,897.82	$2,855.21	$7,865.75	$12,618.79
40	40	8.958	14.792	0	0	$1,932.44	$2,898.67	$7,820.55	$12,651.66
41	41	11.458	15.625	0	0	$1,954.93	$2,954.74	$7,730.14	$12,639.80
42	42	44.792	14.583	0	0	$1,985.98	$2,994.81	$7,594.52	$12,575.31
43	43	8.75	17.083	0	0.32	$2,017.18	$3,038.09	$7,504.11	$12,559.69
44	44	11.875	70.625	0	0	$2,056.27	$3,077.49	$7,458.90	$12,592.66
45	45	10.417	16.042	0	0	$2,079.26	$3,134.90	$7,323.29	$12,537.45
46	46	9.167	16.875	0	1.918	$2,110.33	$3,170.78	$7,187.67	$12,470.69
47	47	12.292	73.333	0	2.078	$2,139.41	$3,223.45	$7,187.67	$12,552.60
48	48	50	13.542	0	0.799	$2,175.25	$3,271.75	$7,052.06	$12,499.85
50	50	8.958	15.625	0	4.795	$2,242.38	$3,360.69	$6,916.44	$12,524.29
55	55	9.167	12.708	0	14.703	$2,392.89	$3,607.00	$6,509.59	$12,524.18
60	60	11.875	89.167	0	31.644	$2,556.04	$3,806.76	$6,193.15	$12,587.59
65	65	10.208	14.792	0	97.968	$2,710.77	$4,013.89	$5,831.51	$12,654.13
70	70	9.167	14.583	0	195.297	$2,868.23	$4,186.16	$5,560.27	$12,809.96
75	75	8.958	13.75	0.799	312.123	$3,030.99	$4,328.79	$5,334.25	$13,006.95
80	80	10.208	15.625	0.799	496.233	$3,185.28	$4,451.10	$5,153.43	$13,286.84
85	85	80.208	14.792	2.717	609.224	$3,336.30	$4,581.81	$4,882.19	$13,412.24
90	90	10.625	15.417	10.548	748.105	$3,496.44	$4,700.41	$4,746.58	$13,702.08
95	95	12.292	14.167	25.411	890.822	$3,644.76	$4,816.95	$4,475.34	$13,853.28
100	100	9.167	13.333	51.142	997.1	$3,789.65	$4,941.08	$4,339.73	$14,118.69

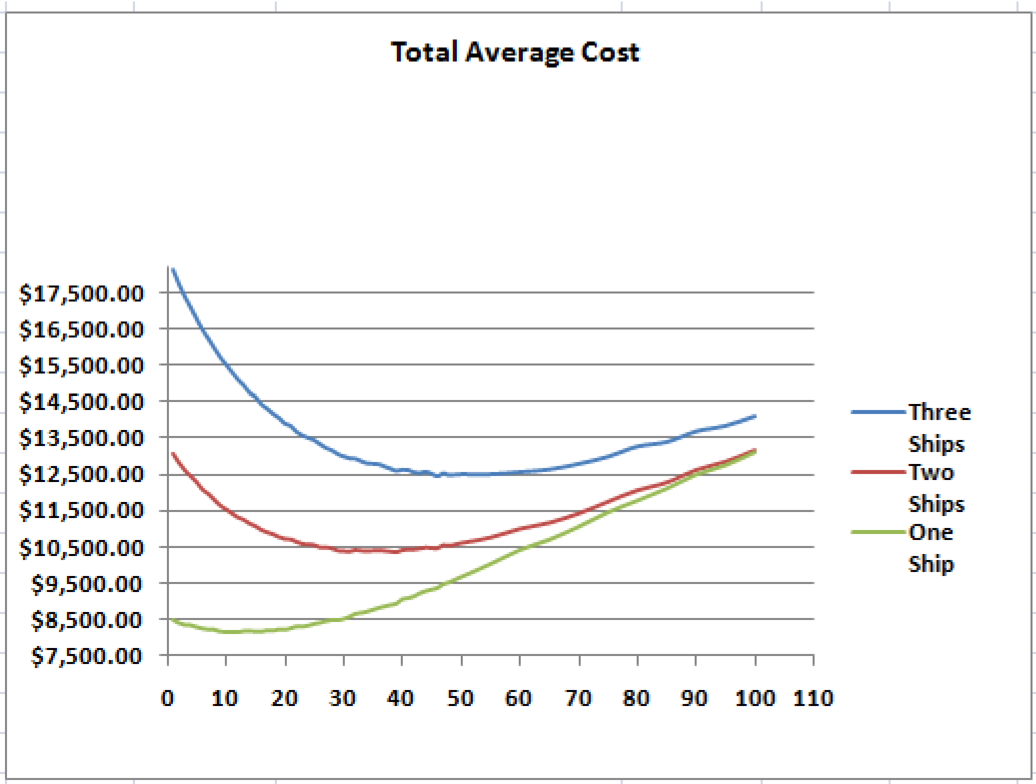


Figure 12

$T \rightarrow$

The total average cost when
$\lambda_a = 8,\ \lambda_b = 12, W = \$1600$

Table 22: Data of Figure 13
Q = 1, W = $1600, K = 80

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	4.792	75.833	0	$2.40	$159.40	$2,945.65	$5,379.45	$8,486.90
2	2	3.75	77.917	0	$5.59	$162.68	$3,019.02	$5,198.63	$8,385.92
3	3	5.417	78.333	0	$4.00	$169.14	$3,110.83	$5,063.01	$8,346.97
4	4	4.167	82.083	0	$11.83	$173.28	$3,179.96	$4,972.60	$8,337.66
5	5	4.375	82.708	0	$13.90	$178.09	$3,256.94	$4,836.99	$8,285.92
6	6	5.833	85.208	0	$20.78	$181.66	$3,342.44	$4,701.37	$8,246.24
7	7	4.583	87.5	0	$32.12	$183.65	$3,408.41	$4,610.96	$8,235.14
8	8	4.792	87.5	0	$54.34	$189.45	$3,485.86	$4,520.55	$8,250.20
9	9	5.417	90.625	0	$69.52	$194.20	$3,552.26	$4,384.93	$8,200.92
10	10	4.167	91.042	0	$101.80	$198.24	$3,637.57	$4,294.52	$8,232.13
11	11	4.583	93.125	0	$137.28	$201.85	$3,701.33	$4,204.11	$8,244.58
12	12	6.25	94.792	0	$179.16	$206.12	$3,764.54	$4,113.70	$8,263.52
13	13	5.417	94.375	0	$224.70	$209.61	$3,825.93	$4,068.49	$8,328.73
14	14	5	93.333	0	$286.39	$215.01	$3,888.09	$3,978.08	$8,367.57
15	15	5	96.875	0	$344.57	$221.08	$3,943.45	$3,887.67	$8,396.76
16	16	4.792	98.542	0	$431.19	$223.15	$3,995.64	$3,797.26	$8,447.24
17	17	5.833	95.208	0	$493.68	$230.12	$4,055.93	$3,752.06	$8,531.78
18	18	5.625	99.167	0	$579.34	$232.32	$4,105.35	$3,661.64	$8,578.65
19	19	6.042	97.5	0	$661.80	$238.56	$4,146.58	$3,616.44	$8,663.38
20	20	7.292	99.583	0	$755.30	$240.92	$4,200.02	$3,526.03	$8,722.27
21	21	5.625	100	0	$845.27	$244.98	$4,247.58	$3,480.82	$8,818.65
22	22	6.458	98.542	0	$925.50	$251.55	$4,277.13	$3,435.62	$8,889.80
23	23	6.875	100	0	$1,017.08	$255.94	$4,333.28	$3,345.21	$8,951.49
24	24	6.042	100	0	$1,114.57	$258.36	$4,368.90	$3,300.00	$9,041.83
25	25	6.25	100	0	$1,200.87	$261.65	$4,418.35	$3,254.80	$9,135.66
26	26	6.042	100	0	$1,271.83	$264.05	$4,450.24	$3,209.59	$9,195.70
27	27	6.875	100	0	$1,353.65	$271.28	$4,489.25	$3,164.38	$9,278.56
28	28	6.667	100	0	$1,437.88	$276.02	$4,526.29	$3,119.18	$9,359.36
29	29	6.875	100	0	$1,515.55	$283.15	$4,564.37	$3,028.77	$9,391.83
30	30	6.875	100	0	$1,593.22	$279.73	$4,600.16	$2,983.56	$9,456.68
31	31	6.875	100	0	$1,663.70	$285.38	$4,640.79	$2,938.36	$9,528.23
32	32	7.292	100	0	$1,734.98	$298.66	$4,690.78	$2,938.36	$9,662.77
33	33	7.083	100	0	$1,802.58	$296.12	$4,714.08	$2,893.15	$9,705.93
34	34	7.292	100	0	$1,868.11	$301.48	$4,768.88	$2,847.95	$9,786.41
36	36	7.5	100	0	$2,021.85	$310.92	$4,843.60	$2,757.53	$9,933.90
37	37	7.708	100	0	$2,080.50	$314.53	$4,881.27	$2,712.33	$9,988.63
38	38	8.75	100	0	$2,132.76	$319.13	$4,921.68	$2,667.12	$10,040.70
39	39	7.917	100	0	$2,182.47	$321.84	$4,946.17	$2,621.92	$10,072.40
40	40	7.917	100	0	$2,232.97	$326.04	$5,008.72	$2,621.92	$10,189.65
41	41	7.5	100	0	$2,318.31	$328.51	$5,034.29	$2,576.71	$10,257.83
42	42	9.375	100	0	$2,362.10	$331.86	$5,085.50	$2,531.51	$10,310.96
43	43	8.542	100	0	$2,402.06	$338.97	$5,124.79	$2,531.51	$10,397.32
44	44	7.917	100	0	$2,485.00	$344.20	$5,157.81	$2,486.30	$10,473.32
45	45	7.708	100	0	$2,521.44	$349.69	$5,203.68	$2,441.10	$10,515.91
46	46	8.542	100	0	$2,553.40	$351.11	$5,228.20	$2,395.89	$10,528.60
47	47	8.125	100	0	$2,633.31	$358.37	$5,286.96	$2,395.89	$10,674.53
48	48	10	100	0	$2,660.32	$362.81	$5,312.99	$2,350.69	$10,686.80
50	50	8.75	100	0	$2,765.00	$367.30	$5,379.57	$2,305.48	$10,817.34
55	55	9.792	100	0	$2,960.30	$389.23	$5,598.91	$2,169.86	$11,118.30

Table 23: Data of Figure 13
Q = 2, W = $1600, K=80

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	2.5	34.375	$0.00	$0.00	$119.91	$2,213.10	$10,713.70	$13,046.71
2	2	2.292	40.417	$0.00	$0.00	$124.21	$2,279.49	$10,397.26	$12,800.96
3	3	3.125	35.417	$0.00	$0.00	$127.51	$2,337.88	$10,126.03	$12,591.42
4	4	2.083	33.75	$0.00	$0.00	$131.27	$2,404.07	$9,900.00	$12,435.34
5	5	2.292	44.167	$0.00	$0.00	$134.20	$2,467.84	$9,673.97	$12,276.01
6	6	3.333	35.208	$0.00	$0.00	$137.43	$2,531.38	$9,402.74	$12,071.55
7	7	1.458	48.958	$0.00	$0.00	$140.27	$2,591.06	$9,221.92	$11,953.25
8	8	2.917	34.583	$0.00	$0.00	$145.10	$2,659.25	$8,995.89	$11,800.24
9	9	3.333	35	$0.00	$0.00	$146.90	$2,724.20	$8,769.86	$11,640.97
10	10	2.708	35.208	$0.00	$0.00	$150.81	$2,797.49	$8,589.04	$11,537.34
11	11	2.708	34.375	$0.00	$0.00	$156.10	$2,857.64	$8,408.22	$11,421.96
12	12	4.167	34.583	$0.00	$0.00	$158.92	$2,930.68	$8,227.40	$11,317.00
13	13	3.542	34.792	$0.00	$0.00	$162.43	$2,997.74	$8,091.78	$11,251.95
14	14	2.917	39.375	$0.00	$0.00	$166.98	$3,065.32	$7,910.96	$11,143.26
15	15	2.5	66.042	$0.00	$0.00	$170.97	$3,135.76	$7,775.34	$11,082.08
16	16	2.083	35.417	$0.00	$0.00	$173.77	$3,207.10	$7,594.52	$10,975.40
17	17	2.292	38.542	$0.00	$0.32	$177.39	$3,276.56	$7,458.90	$10,913.18
18	18	2.083	35.208	$0.00	$0.64	$181.17	$3,340.01	$7,323.29	$10,845.10
19	19	1.875	39.167	$0.00	$0.64	$185.85	$3,410.76	$7,187.67	$10,784.92
20	20	5.417	33.75	$0.00	$3.04	$189.35	$3,484.72	$7,052.06	$10,729.16
21	21	2.292	81.458	$0.00	$3.20	$194.08	$3,547.45	$6,961.64	$10,706.36
22	22	4.375	39.792	$0.00	$6.87	$196.28	$3,625.86	$6,826.03	$10,655.04
23	23	5.208	34.792	$0.00	$13.90	$201.44	$3,699.43	$6,690.41	$10,605.18
24	24	2.083	35.208	$0.00	$16.62	$204.42	$3,767.10	$6,600.00	$10,588.14
25	25	2.292	87.292	$0.00	$29.73	$209.46	$3,829.95	$6,509.59	$10,578.72
26	26	2.083	36.458	$0.00	$34.04	$212.02	$3,901.74	$6,373.97	$10,521.77
27	27	2.292	37.917	$0.00	$54.18	$216.42	$3,963.87	$6,283.56	$10,518.03
28	28	5	38.958	$0.00	$71.28	$222.08	$4,034.22	$6,193.15	$10,520.73
29	29	4.792	39.375	$0.00	$93.33	$225.21	$4,095.05	$6,057.53	$10,471.13
30	30	4.792	39.167	$0.00	$135.21	$227.35	$4,153.59	$5,967.12	$10,483.27
31	31	5.208	40.417	$0.00	$177.08	$232.93	$4,202.77	$5,876.71	$10,489.49
32	32	5.208	39.375	$0.00	$205.53	$236.88	$4,269.70	$5,831.51	$10,543.61
33	33	2.292	38.125	$0.00	$265.46	$238.27	$4,307.08	$5,741.10	$10,551.91
34	34	2.083	33.75	$0.00	$307.49	$247.00	$4,374.27	$5,650.69	$10,579.45
36	36	2.083	99.167	$0.00	$452.28	$253.69	$4,461.94	$5,515.07	$10,682.98
37	37	1.667	33.958	$0.00	$507.74	$255.63	$4,501.63	$5,424.66	$10,689.66
38	38	6.667	36.458	$0.00	$580.94	$258.86	$4,543.08	$5,334.25	$10,717.12
39	39	2.917	38.125	$0.00	$639.11	$266.40	$4,569.67	$5,243.84	$10,719.02
40	40	2.292	32.917	$0.00	$697.12	$267.70	$4,625.16	$5,198.63	$10,788.62
41	41	1.875	100	$0.00	$802.76	$271.71	$4,658.25	$5,153.43	$10,886.15
42	42	7.292	36.458	$0.00	$856.46	$274.95	$4,703.04	$5,063.01	$10,897.47
43	43	1.875	33.333	$0.00	$910.48	$282.85	$4,737.20	$5,017.81	$10,948.33
44	44	2.083	100	$0.00	$987.67	$284.16	$4,762.94	$4,972.60	$11,007.38
45	45	1.875	34.583	$0.00	$1,053.04	$286.72	$4,808.68	$4,882.19	$11,030.63
46	46	2.292	33.75	$0.00	$1,118.72	$291.65	$4,832.56	$4,791.78	$11,034.71
47	47	2.083	100	$0.00	$1,189.04	$295.67	$4,882.78	$4,791.78	$11,159.27
48	48	7.917	38.542	$0.00	$1,246.10	$301.96	$4,908.92	$4,701.37	$11,158.34
50	50	1.042	33.958	$0.00	$1,387.06	$308.62	$4,968.95	$4,610.96	$11,275.58

Table 24: Data of Figure 13
Q = 3, W = $1600, K = 80

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.875	22.708	$0.00	$0.00	$106.78	$1,972.37	$16,047.95	$18,127.09
2	2	0.833	24.375	$0.00	$0.00	$110.45	$2,027.04	$15,595.89	$17,733.39
3	3	2.5	23.333	$0.00	$0.00	$113.25	$2,085.01	$15,189.04	$17,387.30
4	4	1.25	24.375	$0.00	$0.00	$116.47	$2,140.67	$14,827.40	$17,084.54
5	5	1.25	33.75	$0.00	$0.00	$119.69	$2,199.73	$14,465.75	$16,785.18
6	6	2.917	23.125	$0.00	$0.00	$123.34	$2,259.46	$14,104.11	$16,486.91
7	7	2.292	37.083	$0.00	$0.00	$125.89	$2,318.96	$13,787.67	$16,232.52
8	8	2.5	22.292	$0.00	$0.00	$128.97	$2,379.19	$13,471.23	$15,979.39
9	9	2.5	24.792	$0.00	$0.00	$131.84	$2,446.16	$13,154.80	$15,732.79
10	10	1.458	23.125	$0.00	$0.00	$134.13	$2,506.12	$12,883.56	$15,523.81
11	11	1.458	22.708	$0.00	$0.00	$139.34	$2,568.55	$12,612.33	$15,320.21
12	12	3.333	23.542	$0.00	$0.00	$142.95	$2,636.50	$12,341.10	$15,120.55
13	13	2.5	23.333	$0.00	$0.00	$145.78	$2,698.85	$12,115.07	$14,959.70
14	14	1.458	24.167	$0.00	$0.00	$149.72	$2,765.17	$11,843.84	$14,758.72
15	15	1.042	53.125	$0.00	$0.00	$153.39	$2,826.96	$11,663.01	$14,643.37
16	16	1.458	25.208	$0.00	$0.00	$157.79	$2,895.90	$11,391.78	$14,445.47
17	17	1.25	25.625	$0.00	$0.00	$159.48	$2,959.62	$11,210.96	$14,330.05
18	18	1.875	23.125	$0.00	$0.00	$163.65	$3,028.58	$10,984.93	$14,177.16
19	19	1.042	23.542	$0.00	$0.00	$168.98	$3,099.12	$10,804.11	$14,072.20
20	20	4.375	24.375	$0.00	$0.00	$172.24	$3,167.15	$10,578.08	$13,917.47
21	21	0.833	66.042	$0.00	$0.00	$176.43	$3,227.19	$10,442.47	$13,846.09
22	22	1.458	23.75	$0.00	$0.00	$178.67	$3,300.00	$10,216.44	$13,695.11
23	23	4.583	23.333	$0.00	$0.00	$182.25	$3,373.32	$10,035.62	$13,591.19
24	24	1.875	24.167	$0.00	$0.00	$187.69	$3,439.40	$9,900.00	$13,527.09
25	25	0.833	76.042	$0.00	$1.44	$193.25	$3,506.30	$9,764.38	$13,465.37
26	26	1.667	27.708	$0.00	$1.92	$194.79	$3,572.10	$9,583.56	$13,352.37
27	27	1.042	25	$0.00	$5.91	$200.11	$3,645.64	$9,402.74	$13,254.40
28	28	1.667	25.417	$0.00	$5.59	$203.09	$3,720.35	$9,267.12	$13,196.15
29	29	2.292	23.75	$0.00	$5.91	$206.95	$3,795.60	$9,086.30	$13,094.76
30	30	2.5	23.333	$0.00	$15.02	$209.95	$3,859.57	$8,950.69	$13,035.23
31	31	2.292	23.958	$0.00	$24.45	$215.41	$3,928.17	$8,815.07	$12,983.10
32	32	1.667	24.375	$0.00	$33.88	$215.52	$3,994.94	$8,724.66	$12,968.99
33	33	1.042	26.667	$0.00	$55.62	$217.92	$4,045.31	$8,589.04	$12,907.88
34	34	1.667	26.042	$0.00	$68.08	$226.87	$4,125.69	$8,453.43	$12,874.06
36	36	1.875	23.542	$0.00	$116.67	$233.77	$4,238.97	$8,272.60	$12,862.01
37	37	1.667	24.583	$0.00	$153.11	$235.37	$4,296.43	$8,136.99	$12,821.89
38	38	6.458	22.083	$0.00	$188.11	$239.33	$4,355.84	$8,001.37	$12,784.64
39	39	1.458	26.667	$0.00	$242.76	$244.76	$4,400.71	$7,865.75	$12,753.98
40	40	0.833	22.083	$0.00	$278.72	$247.69	$4,449.08	$7,820.55	$12,796.04
41	41	1.667	25	$0.00	$359.91	$252.51	$4,503.59	$7,730.14	$12,846.15
42	42	7.083	22.083	$0.00	$401.14	$254.90	$4,550.19	$7,594.52	$12,800.75
43	43	1.042	26.875	$0.00	$446.53	$260.79	$4,588.17	$7,504.11	$12,799.59
44	44	1.875	100	$0.00	$521.96	$264.34	$4,613.99	$7,458.90	$12,859.19
45	45	0.833	22.292	$0.00	$583.97	$266.25	$4,669.33	$7,323.29	$12,842.84
46	46	1.25	26.042	$0.00	$661.01	$271.69	$4,693.87	$7,187.67	$12,814.23
47	47	1.458	100	$0.00	$722.85	$276.63	$4,743.91	$7,187.67	$12,931.07
48	48	7.292	22.708	$0.00	$797.17	$283.03	$4,770.82	$7,052.06	$12,903.07
50	50	1.25	24.583	$0.00	$941.01	$289.54	$4,834.02	$6,916.44	$12,980.99
55	55	1.458	23.75	$0.00	$1,236.51	$310.17	$5,034.40	$6,509.59	$13,090.67

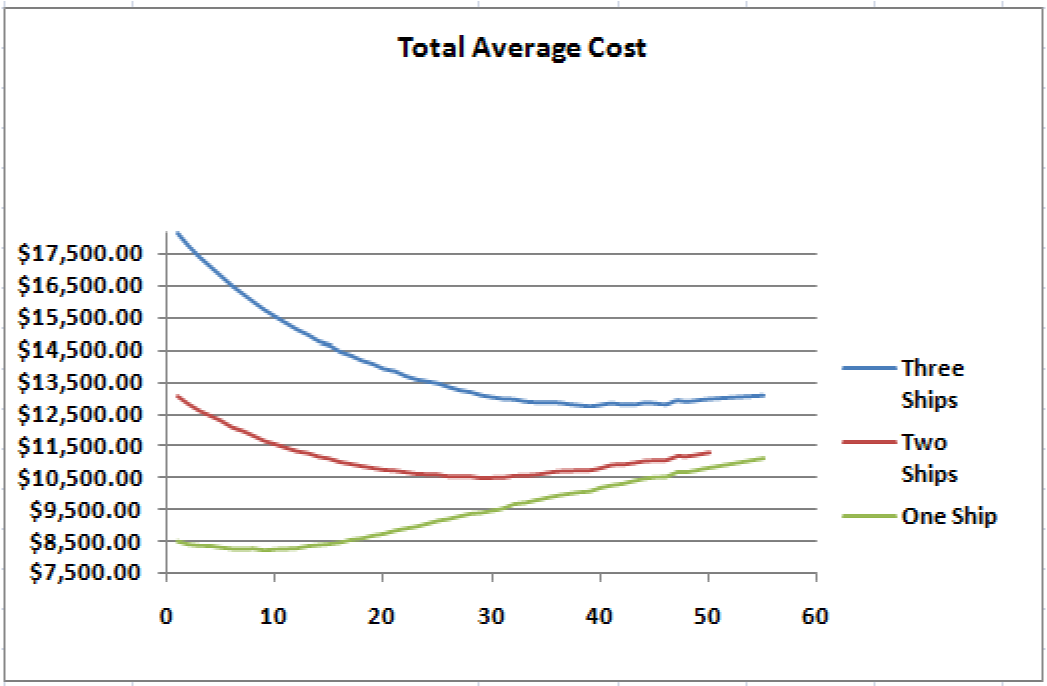


Figure 13 $T \rightarrow$

The total average cost when
$\lambda_a = 1,\ \lambda_b = 19, W = \$1600$

Table 25 : Data of Figure 14
Q = 1, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	97.083	99.792	$1,459.93	$1,413.11	$4,278.41	$4,278.54	$5,379.45	$16,809.43
2	2	1.5	80	100	100	$1,670.73	$1,631.10	$4,344.84	$4,339.83	$5,198.63	$17,185.13
3	3	1.5	80	98.958	100	$1,893.68	$1,836.78	$4,419.21	$4,404.91	$5,063.01	$17,617.59
4	4	1.5	80	97.917	100	$2,099.04	$2,039.75	$4,468.84	$4,465.50	$4,972.60	$18,045.73
5	5	1.5	80	100	100	$2,288.27	$2,249.11	$4,538.14	$4,525.10	$4,836.99	$18,437.60
6	6	1.5	80	100	100	$2,493.47	$2,428.58	$4,597.11	$4,597.08	$4,701.37	$18,817.62
7	7	1.5	80	100	100	$2,662.72	$2,623.08	$4,658.95	$4,656.68	$4,610.96	$19,212.38
8	8	1.5	80	100	100	$2,845.71	$2,778.11	$4,735.04	$4,711.99	$4,520.55	$19,591.39
9	9	1.5	80	100	100	$3,019.43	$2,950.55	$4,792.95	$4,783.28	$4,384.93	$19,931.14
10	10	1.5	80	100	100	$3,156.71	$3,115.64	$4,856.29	$4,839.88	$4,294.52	$20,263.03
11	11	1.5	80	100	100	$3,310.14	$3,270.66	$4,899.48	$4,913.15	$4,204.11	$20,597.54
12	12	1.5	80	100	100	$3,488.81	$3,413.06	$4,969.59	$4,969.45	$4,113.70	$20,954.61
13	13	1.5	80	100	100	$3,624.82	$3,550.82	$5,035.89	$5,019.99	$4,068.49	$21,300.01
14	14	1.5	80	100	100	$3,750.75	$3,675.64	$5,090.70	$5,090.05	$3,978.08	$21,585.23
15	15	1.5	80	100	100	$3,871.10	$3,839.29	$5,151.44	$5,146.05	$3,887.67	$21,895.55
16	16	1.5	80	100	100	$3,982.17	$3,945.41	$5,220.84	$5,208.35	$3,797.26	$22,154.02
17	17	1.5	80	100	100	$4,125.05	$4,043.70	$5,295.12	$5,267.62	$3,752.06	$22,483.53
18	18	1.5	80	100	100	$4,220.94	$4,188.65	$5,336.74	$5,321.83	$3,661.64	$22,729.80
19	19	1.5	80	100	100	$4,353.90	$4,269.04	$5,409.46	$5,391.34	$3,616.44	$23,040.19
20	20	1.5	80	100	100	$4,486.07	$4,402.17	$5,464.00	$5,459.49	$3,526.03	$23,337.75
21	21	1.5	80	100	100	$4,555.27	$4,529.70	$5,521.66	$5,530.56	$3,480.82	$23,618.02
22	22	1.5	80	100	100	$4,671.30	$4,584.84	$5,576.70	$5,572.31	$3,435.62	$23,840.78
23	23	1.5	80	100	100	$4,786.37	$4,701.51	$5,658.35	$5,654.98	$3,345.21	$24,146.41
24	24	1.5	80	100	100	$4,834.64	$4,811.14	$5,718.80	$5,717.07	$3,300.00	$24,381.65

Table 26: Data of Figure 14
Q = 2, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	58.333	56.458	$0.00	$5.75	$3,456.42	$3,448.56	$10,713.70	$17,624.43
2	2	1.5	80	51.25	65	$0.00	$6.55	$3,550.98	$3,539.74	$10,397.26	$17,494.53
3	3	1.5	80	66.875	55.625	$0.00	$7.35	$3,652.15	$3,643.40	$10,126.03	$17,428.92
4	4	1.5	80	65	57.5	$0.00	$7.83	$3,745.47	$3,734.77	$9,900.00	$17,388.07
5	5	1.5	80	56.875	74.375	$0.96	$9.91	$3,844.08	$3,832.86	$9,673.97	$17,361.78
6	6	1.5	80	75.208	57.708	$0.96	$13.58	$3,946.50	$3,933.12	$9,402.74	$17,296.90
7	7	1.5	80	55.417	79.583	$4.16	$16.94	$4,044.28	$4,031.47	$9,221.92	$17,318.75
8	8	1.5	80	76.875	60.208	$8.15	$21.26	$4,152.32	$4,133.97	$8,995.89	$17,311.59
9	9	1.5	80	80.208	60	$20.14	$35.96	$4,251.81	$4,238.17	$8,769.86	$17,315.93
10	10	1.5	80	53.125	57.708	$37.24	$55.62	$4,350.21	$4,336.81	$8,589.04	$17,368.91
11	11	1.5	80	53.125	56.458	$71.60	$85.98	$4,434.73	$4,434.01	$8,408.22	$17,434.54
12	12	1.5	80	91.25	60.417	$120.82	$138.40	$4,534.02	$4,523.96	$8,227.40	$17,544.60
13	13	1.5	80	90.625	56.667	$200.09	$221.03	$4,616.20	$4,598.54	$8,091.78	$17,727.64
14	14	1.5	80	58.542	54.792	$289.11	$304.29	$4,691.79	$4,687.27	$7,910.96	$17,883.43
15	15	1.5	80	52.5	96.667	$400.02	$413.13	$4,762.86	$4,758.82	$7,775.34	$18,110.17
16	16	1.5	80	52.292	59.792	$528.84	$540.50	$4,829.65	$4,819.64	$7,594.52	$18,313.15
17	17	1.5	80	56.875	60.417	$631.92	$654.77	$4,907.32	$4,883.20	$7,458.90	$18,536.11
18	18	1.5	80	53.542	57.708	$852.31	$845.27	$4,952.02	$4,937.68	$7,323.29	$18,910.57
19	19	1.5	80	55.625	59.792	$946.44	$971.53	$5,012.63	$5,003.01	$7,187.67	$19,121.28
20	20	1.5	80	98.125	57.5	$1,141.42	$1,146.21	$5,059.65	$5,058.13	$7,052.06	$19,457.47
21	21	1.5	80	51.25	100	$1,292.76	$1,266.71	$5,108.95	$5,114.45	$6,961.64	$19,744.51
22	22	1.5	80	100	59.375	$1,485.18	$1,411.03	$5,156.13	$5,152.05	$6,826.03	$20,030.42
23	23	1.5	80	98.75	56.667	$1,605.37	$1,559.98	$5,218.96	$5,223.10	$6,690.41	$20,297.81
24	24	1.5	80	53.542	54.375	$1,727.31	$1,674.89	$5,273.94	$5,268.08	$6,600.00	$20,544.21

Table 27: Data of Figure 14
Q = 3, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	40.625	39.167	$0.00	$5.75	$3,073.06	$3,071.06	$16,047.95	$22,197.82
2	2	1.5	80	36.667	36.875	$0.00	$6.55	$3,159.56	$3,155.69	$15,595.89	$21,917.69
3	3	1.5	80	49.167	38.542	$0.00	$7.35	$3,245.83	$3,239.40	$15,189.04	$21,681.62
4	4	1.5	80	48.125	34.583	$0.00	$7.83	$3,336.06	$3,331.57	$14,827.40	$21,502.86
5	5	1.5	80	36.458	51.875	$0.00	$9.27	$3,426.27	$3,422.26	$14,465.75	$21,323.55
6	6	1.5	80	58.542	38.75	$0.00	$10.07	$3,523.33	$3,514.72	$14,104.11	$21,152.22
7	7	1.5	80	37.292	61.875	$0.00	$11.03	$3,616.72	$3,608.77	$13,787.67	$21,024.19
8	8	1.5	80	38.542	39.375	$0.00	$11.83	$3,718.49	$3,702.64	$13,471.23	$20,904.18
9	9	1.5	80	63.333	36.458	$0.00	$12.79	$3,806.59	$3,802.30	$13,154.80	$20,776.48
10	10	1.5	80	34.792	38.75	$0.00	$13.11	$3,905.92	$3,904.04	$12,883.56	$20,706.63
11	11	1.5	80	34.792	39.167	$0.00	$13.90	$4,006.30	$4,000.33	$12,612.33	$20,632.86
12	12	1.5	80	75.208	36.25	$0.00	$16.46	$4,102.99	$4,097.76	$12,341.10	$20,558.31
13	13	1.5	80	76.667	38.542	$6.71	$19.34	$4,203.09	$4,193.77	$12,115.07	$20,537.97
14	14	1.5	80	38.542	37.917	$10.55	$29.25	$4,302.62	$4,298.17	$11,843.84	$20,484.41
15	15	1.5	80	36.042	86.875	$21.26	$36.92	$4,405.99	$4,392.45	$11,663.01	$20,519.62
16	16	1.5	80	34.792	36.458	$41.07	$61.69	$4,498.49	$4,497.03	$11,391.78	$20,490.06
17	17	1.5	80	36.458	35.417	$54.82	$88.22	$4,601.22	$4,586.95	$11,210.96	$20,542.16
18	18	1.5	80	34.583	38.75	$115.07	$148.47	$4,687.05	$4,671.67	$10,984.93	$20,607.18
19	19	1.5	80	35.833	35.208	$179.32	$187.47	$4,767.58	$4,763.85	$10,804.11	$20,702.32
20	20	1.5	80	94.167	34.583	$274.89	$278.08	$4,847.63	$4,844.23	$10,578.08	$20,822.90
21	21	1.5	80	36.667	96.667	$368.86	$362.79	$4,925.04	$4,906.59	$10,442.47	$21,005.74
22	22	1.5	80	38.542	36.667	$513.33	$500.71	$4,977.67	$4,973.27	$10,216.44	$21,181.41
23	23	1.5	80	94.792	38.542	$607.15	$586.05	$5,045.61	$5,048.52	$10,035.62	$21,322.95
24	24	1.5	80	34.583	37.083	$735.32	$690.57	$5,102.18	$5,099.57	$9,900.00	$21,527.65

Table 28: Data of Figure 14
Q = 4, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	30	27.5	$0.00	$5.75	$2,882.53	$2,878.68	$21,382.19	$27,149.16
2	2	1.5	80	27.708	27.292	$0.00	$6.55	$2,961.76	$2,961.33	$20,794.52	$26,724.16
3	3	1.5	80	38.542	27.708	$0.00	$7.35	$3,048.89	$3,044.73	$20,252.06	$26,353.03
4	4	1.5	80	39.792	27.708	$0.00	$7.83	$3,134.99	$3,130.69	$19,754.80	$26,028.30
5	5	1.5	80	28.125	40.833	$0.00	$9.27	$3,222.22	$3,215.20	$19,302.74	$25,749.42
6	6	1.5	80	49.167	28.125	$0.00	$10.07	$3,310.36	$3,307.23	$18,805.48	$25,433.14
7	7	1.5	80	29.583	50	$0.00	$11.03	$3,399.02	$3,392.26	$18,398.63	$25,200.94
8	8	1.5	80	26.667	25.833	$0.00	$11.83	$3,492.19	$3,486.48	$17,946.58	$24,937.07
9	9	1.5	80	55	26.667	$0.00	$12.79	$3,583.97	$3,579.61	$17,539.73	$24,716.09
10	10	1.5	80	24.167	28.125	$0.00	$13.11	$3,681.44	$3,677.35	$17,178.08	$24,549.98
11	11	1.5	80	24.167	27.5	$0.00	$13.90	$3,776.33	$3,770.82	$16,816.44	$24,377.49
12	12	1.5	80	66.875	27.083	$0.00	$14.54	$3,870.19	$3,866.82	$16,454.80	$24,206.34
13	13	1.5	80	66.667	28.333	$0.64	$16.30	$3,970.01	$3,959.49	$16,138.36	$24,084.79
14	14	1.5	80	30.833	33.75	$0.16	$18.38	$4,071.58	$4,067.65	$15,821.92	$23,979.68
15	15	1.5	80	29.167	76.042	$4.32	$19.82	$4,168.88	$4,158.90	$15,505.48	$23,857.39
16	16	1.5	80	26.458	27.292	$6.39	$22.53	$4,265.10	$4,264.92	$15,189.04	$23,747.99
17	17	1.5	80	28.125	26.458	$5.27	$29.89	$4,369.67	$4,364.94	$14,917.81	$23,687.58
18	18	1.5	80	25.208	28.125	$18.06	$49.22	$4,473.60	$4,459.58	$14,646.58	$23,647.04
19	19	1.5	80	29.167	25.625	$39.80	$57.37	$4,564.19	$4,559.19	$14,375.34	$23,595.89
20	20	1.5	80	88.958	27.708	$66.48	$91.26	$4,660.88	$4,656.75	$14,104.11	$23,579.48
21	21	1.5	80	27.708	92.083	$105.00	$133.93	$4,754.18	$4,732.79	$13,878.08	$23,603.98
22	22	1.5	80	27.708	28.333	$195.62	$201.05	$4,829.97	$4,818.98	$13,606.85	$23,652.46
23	23	1.5	80	92.083	28.333	$245.96	$241.32	$4,914.19	$4,909.84	$13,380.82	$23,692.13
24	24	1.5	80	25.208	27.083	$345.53	$315.16	$4,985.04	$4,979.87	$13,200.00	$23,825.60

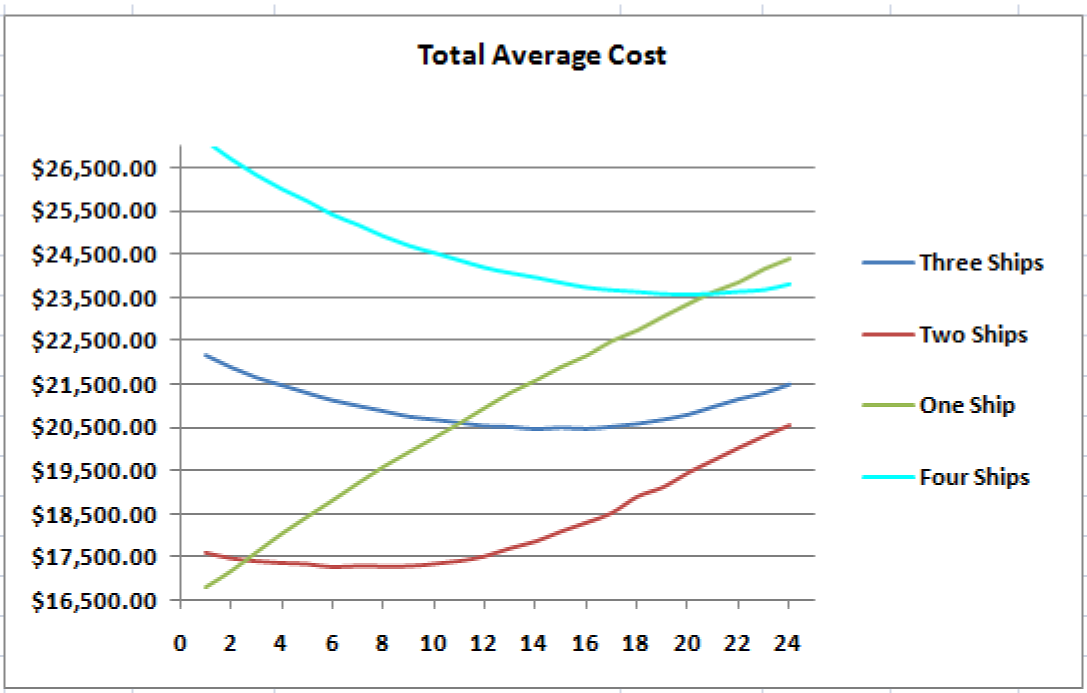


Figure 14 $T \rightarrow$

The total average cost when
$\lambda_a = 30,\ \lambda_b = 30, W = \$1600$

Table 29: Data of Figure 15
Q = 1, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	21.875	100	$0.00	$10,078.88	$756.29	$6,219.70	$5,379.45	$22,434.32
2	2	1.5	80	21.667	100	$0.00	$10,325.32	$774.89	$6,325.28	$5,198.63	$22,624.12
3	3	1.5	80	22.917	100	$0.00	$10,544.11	$796.40	$6,436.13	$5,063.01	$22,839.66
4	4	1.5	80	24.167	100	$0.00	$10,740.05	$814.70	$6,549.53	$4,972.60	$23,076.87
5	5	1.5	80	24.167	100	$0.00	$11,005.66	$833.08	$6,662.15	$4,836.99	$23,337.88
6	6	1.5	80	25	100	$0.00	$11,151.10	$857.70	$6,788.74	$4,701.37	$23,498.91
7	7	1.5	80	24.375	100	$0.00	$11,380.59	$873.77	$6,896.92	$4,610.96	$23,762.24
8	8	1.5	80	28.958	100	$0.00	$11,481.28	$899.66	$6,999.82	$4,520.55	$23,901.31
9	9	1.5	80	26.042	100	$0.00	$11,668.74	$918.69	$7,121.23	$4,384.93	$24,093.60
10	10	1.5	80	23.958	100	$0.00	$11,841.51	$940.29	$7,221.82	$4,294.52	$24,298.13
11	11	1.5	80	24.375	100	$0.00	$11,996.69	$963.99	$7,367.95	$4,204.11	$24,532.74
12	12	1.5	80	27.708	100	$0.00	$12,134.61	$980.22	$7,465.02	$4,113.70	$24,693.55
13	13	1.5	80	30.833	100	$0.00	$12,258.63	$998.75	$7,561.40	$4,068.49	$24,887.28
14	14	1.5	80	28.75	100	$0.00	$12,362.83	$1,024.31	$7,689.15	$3,978.08	$25,054.37
15	15	1.5	80	26.875	100	$0.00	$12,592.01	$1,035.97	$7,804.76	$3,887.67	$25,320.41

Table 30: Data of Figure 15
Q = 2, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	10.625	96.458	$0.00	$1,538.56	$569.58	$6,004.43	$10,713.70	$18,826.27
2	2	1.5	80	10.833	94.375	$0.00	$1,919.57	$583.72	$6,078.13	$10,397.26	$18,978.68
3	3	1.5	80	11.667	95.833	$0.00	$2,346.60	$600.99	$6,158.78	$10,126.03	$19,232.39
4	4	1.5	80	14.583	93.542	$0.00	$2,748.22	$614.67	$6,216.96	$9,900.00	$19,479.84
5	5	1.5	80	11.042	99.792	$0.00	$3,156.71	$629.91	$6,292.98	$9,673.97	$19,753.57
6	6	1.5	80	14.167	96.875	$0.00	$3,497.60	$648.09	$6,366.60	$9,402.74	$19,915.04
7	7	1.5	80	11.667	100	$0.00	$3,864.54	$664.53	$6,440.48	$9,221.92	$20,191.48
8	8	1.5	80	17.917	91.667	$0.00	$4,201.76	$682.92	$6,512.39	$8,995.89	$20,392.95
9	9	1.5	80	16.458	95.833	$0.00	$4,552.56	$699.05	$6,589.52	$8,769.86	$20,610.99
10	10	1.5	80	10.833	96.875	$0.00	$4,880.50	$717.74	$6,660.45	$8,589.04	$20,847.74
11	11	1.5	80	10.833	96.458	$0.00	$5,157.79	$732.06	$6,756.79	$8,408.22	$21,054.85
12	12	1.5	80	18.125	95.625	$0.00	$5,436.03	$751.16	$6,837.92	$8,227.40	$21,252.50
13	13	1.5	80	19.792	92.083	$0.00	$5,710.75	$769.43	$6,908.15	$8,091.78	$21,480.12
14	14	1.5	80	10.208	95.208	$0.00	$5,990.43	$784.72	$6,998.97	$7,910.96	$21,685.08
15	15	1.5	80	12.5	100	$0.00	$6,280.82	$803.16	$7,086.87	$7,775.34	$21,946.20

Table 31: Data of Figure 15
Q = 3, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	6.458	68.125	$0.00	$57.53	$506.59	$5,633.40	$16,047.95	$22,245.46
2	2	1.5	80	9.375	69.375	$0.00	$68.08	$520.24	$5,789.72	$15,595.89	$21,973.93
3	3	1.5	80	7.5	69.583	$0.00	$103.40	$534.47	$5,938.93	$15,189.04	$21,765.84
4	4	1.5	80	10.625	66.667	$0.00	$179.00	$548.65	$6,084.58	$14,827.40	$21,639.62
5	5	1.5	80	8.75	93.75	$0.00	$342.97	$562.54	$6,213.81	$14,465.75	$21,585.06
6	6	1.5	80	10.208	67.5	$0.00	$560.96	$579.22	$6,323.57	$14,104.11	$21,567.86
7	7	1.5	80	5.208	97.292	$0.00	$867.01	$594.75	$6,418.04	$13,787.67	$21,667.47
8	8	1.5	80	4.792	67.917	$0.00	$1,211.74	$612.36	$6,493.67	$13,471.23	$21,789.00
9	9	1.5	80	12.5	68.75	$0.00	$1,572.76	$625.54	$6,575.92	$13,154.80	$21,929.03
10	10	1.5	80	7.917	67.5	$0.00	$2,008.90	$641.02	$6,630.29	$12,883.56	$22,163.77
11	11	1.5	80	7.917	68.125	$0.00	$2,252.79	$658.67	$6,730.20	$12,612.33	$22,253.99
12	12	1.5	80	14.167	69.792	$0.00	$2,635.87	$675.15	$6,788.38	$12,341.10	$22,440.49
13	13	1.5	80	16.667	69.583	$0.00	$3,005.05	$690.89	$6,847.31	$12,115.07	$22,658.31
14	14	1.5	80	7.708	63.75	$0.00	$3,299.91	$708.61	$6,930.55	$11,843.84	$22,782.91
15	15	1.5	80	7.708	100	$0.00	$3,626.10	$725.00	$7,006.05	$11,663.01	$23,020.16

Table 32: Data of Figure 15
Q = 4, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	3.75	52.292	$0.00	$55.94	$474.26	$5,283.71	$21,382.19	$27,196.10
2	2	1.5	80	6.25	50.208	$0.00	$58.65	$487.48	$5,431.50	$20,794.52	$26,772.15
3	3	1.5	80	4.792	51.667	$0.00	$60.57	$501.28	$5,591.53	$20,252.06	$26,405.43
4	4	1.5	80	8.125	51.875	$0.00	$65.21	$514.51	$5,742.61	$19,754.80	$26,077.13
5	5	1.5	80	3.333	79.375	$0.00	$77.19	$530.25	$5,899.32	$19,302.74	$25,809.50
6	6	1.5	80	7.5	51.042	$0.00	$114.59	$544.70	$6,052.67	$18,805.48	$25,517.44
7	7	1.5	80	5	90.208	$0.00	$185.71	$558.79	$6,197.74	$18,398.63	$25,340.87
8	8	1.5	80	3.125	52.083	$0.00	$329.22	$576.24	$6,330.43	$17,946.58	$25,182.47
9	9	1.5	80	10	51.667	$0.00	$516.85	$587.68	$6,446.48	$17,539.73	$25,090.73
10	10	1.5	80	6.25	51.042	$0.00	$847.03	$603.27	$6,531.42	$17,178.08	$25,159.81
11	11	1.5	80	6.25	52.292	$0.00	$1,079.57	$621.05	$6,634.46	$16,816.44	$25,151.51
12	12	1.5	80	11.667	54.167	$0.00	$1,469.36	$635.42	$6,698.52	$16,454.80	$25,258.10
13	13	1.5	80	14.583	53.542	$0.00	$1,835.18	$652.50	$6,756.45	$16,138.36	$25,382.48
14	14	1.5	80	3.958	51.667	$0.00	$2,183.11	$668.41	$6,834.15	$15,821.92	$25,507.58
15	15	1.5	80	5	100	$0.00	$2,501.62	$685.93	$6,906.71	$15,505.48	$25,599.73

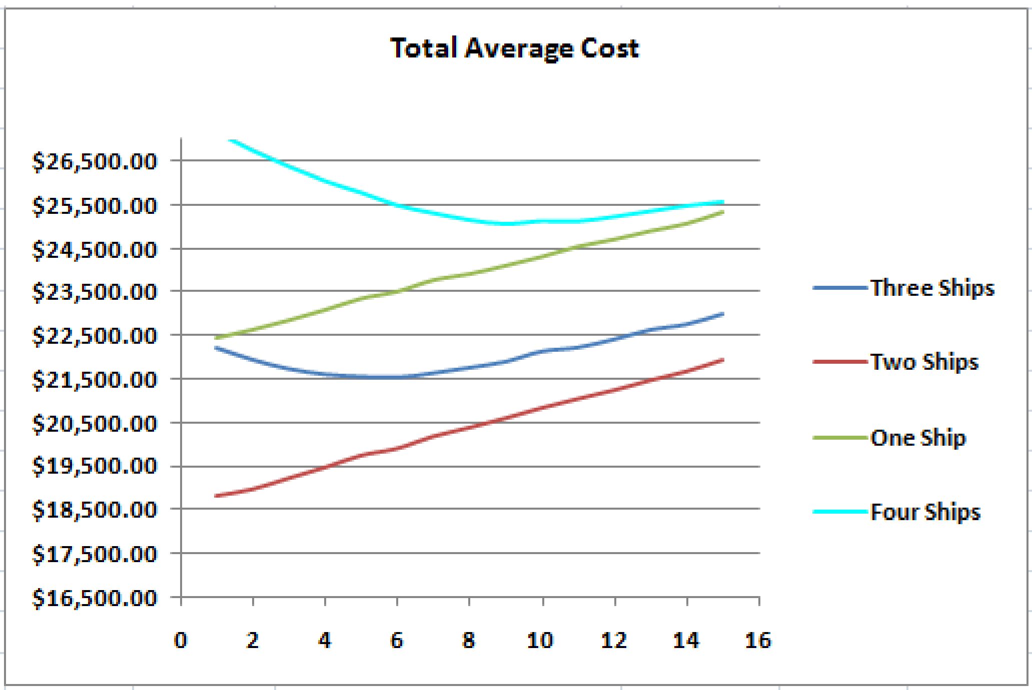


Figure 15 $T \rightarrow$

The total average cost when
$\lambda_a = 5,\ \lambda_b = 55, W = \$1600$

Table 33: Data of Figure 16
Q = 1, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	100	100	$15,374.11	$15,319.29	$7,390.91	$7,359.27	$5,379.45	$50,823.03
2	2	1.5	80	100	100	$15,497.01	$15,570.05	$7,525.65	$7,501.67	$5,198.63	$51,293.01
3	3	1.5	80	100	100	$15,843.49	$15,784.36	$7,680.26	$7,651.37	$5,063.01	$52,022.50
4	4	1.5	80	100	100	$16,031.92	$15,968.31	$7,812.44	$7,797.71	$4,972.60	$52,582.98
5	5	1.5	80	100	100	$16,190.94	$16,268.61	$7,955.06	$7,938.95	$4,836.99	$53,190.54
6	6	1.5	80	100	100	$16,457.19	$16,389.75	$8,081.07	$8,099.84	$4,701.37	$53,729.22
7	7	1.5	80	100	100	$16,557.56	$16,642.10	$8,242.17	$8,235.58	$4,610.96	$54,288.37
8	8	1.5	80	100	100	$16,777.63	$16,703.47	$8,401.58	$8,363.35	$4,520.55	$54,766.57
9	9	1.5	80	100	100	$16,979.80	$16,903.08	$8,550.05	$8,525.70	$4,384.93	$55,343.56
10	10	1.5	80	100	100	$16,988.43	$17,080.16	$8,662.97	$8,665.11	$4,294.52	$55,691.19
11	11	1.5	80	100	100	$17,141.85	$17,237.10	$8,828.05	$8,803.46	$4,204.11	$56,214.58
12	12	1.5	80	100	100	$17,454.45	$17,370.71	$8,968.20	$8,947.48	$4,113.70	$56,854.54
13	13	1.5	80	100	100	$17,571.92	$17,482.42	$9,117.17	$9,114.29	$4,068.49	$57,354.30
14	14	1.5	80	100	100	$17,668.61	$17,577.19	$9,257.87	$9,247.92	$3,978.08	$57,729.66
15	15	1.5	80	100	100	$17,738.77	$17,844.41	$9,404.52	$9,384.44	$3,887.67	$58,259.81
16	16	1.5	80	100	100	$17,783.84	$17,897.31	$9,544.08	$9,529.78	$3,797.26	$58,552.26
17	17	1.5	80	100	100	$18,032.03	$17,929.43	$9,705.39	$9,677.91	$3,752.06	$59,096.81
18	18	1.5	80	100	100	$18,040.02	$18,154.13	$9,849.95	$9,818.46	$3,661.64	$59,524.21
19	19	1.5	80	100	100	$18,257.22	$18,147.10	$9,985.01	$9,987.36	$3,616.44	$59,993.12
20	20	1.5	80	100	100	$18,451.71	$18,346.39	$10,125.89	$10,130.78	$3,526.03	$60,580.81
21	21	1.5	80	100	100	$18,412.08	$18,534.82	$10,271.16	$10,262.36	$3,480.82	$60,961.23
22	22	1.5	80	100	100	$18,589.16	$18,475.21	$10,436.62	$10,404.49	$3,435.62	$61,341.08
23	23	1.5	80	100	100	$18,749.45	$18,633.11	$10,567.59	$10,544.40	$3,345.21	$61,839.76
24	24	1.5	80	100	100	$18,653.72	$18,784.45	$10,712.00	$10,682.83	$3,300.00	$62,133.01

Table 34: Data of Figure 16
Q = 2, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	100	100	$6,324.77	$6,371.28	$6,684.98	$6,678.70	$10,713.70	$36,773.43
2	2	1.5	80	100	100	$6,776.90	$6,827.88	$6,764.13	$6,757.04	$10,397.26	$37,523.20
3	3	1.5	80	100	100	$7,252.35	$7,293.43	$6,847.83	$6,843.40	$10,126.03	$38,363.03
4	4	1.5	80	100	100	$7,671.87	$7,706.87	$6,929.66	$6,929.67	$9,900.00	$39,138.07
5	5	1.5	80	100	100	$8,078.29	$8,140.30	$7,018.99	$7,015.61	$9,673.97	$39,927.16
6	6	1.5	80	100	100	$8,482.15	$8,514.27	$7,109.22	$7,106.72	$9,402.74	$40,615.09
7	7	1.5	80	100	100	$8,829.59	$8,894.80	$7,198.80	$7,193.74	$9,221.92	$41,338.85
8	8	1.5	80	100	100	$9,183.90	$9,209.32	$7,301.52	$7,287.20	$8,995.89	$41,977.83
9	9	1.5	80	100	100	$9,539.82	$9,564.11	$7,391.26	$7,391.13	$8,769.86	$42,656.18
10	10	1.5	80	100	100	$9,805.43	$9,894.93	$7,491.96	$7,489.74	$8,589.04	$43,271.10
11	11	1.5	80	100	100	$10,110.05	$10,207.53	$7,593.40	$7,590.30	$8,408.22	$43,909.50
12	12	1.5	80	100	100	$10,474.91	$10,491.37	$7,692.38	$7,679.83	$8,227.40	$44,565.89
13	13	1.5	80	100	100	$10,747.88	$10,756.19	$7,801.44	$7,805.45	$8,091.78	$45,202.73
14	14	1.5	80	99.583	100	$11,011.42	$10,998.31	$7,896.39	$7,901.08	$7,910.96	$45,718.15
15	15	1.5	80	100	100	$11,240.43	$11,324.02	$8,014.30	$8,004.20	$7,775.34	$46,358.29
16	16	1.5	80	100	100	$11,442.76	$11,553.20	$8,133.15	$8,123.54	$7,594.52	$46,847.17
17	17	1.5	80	100	100	$11,759.04	$11,735.39	$8,237.44	$8,235.30	$7,458.90	$47,426.07
18	18	1.5	80	100	100	$11,927.33	$12,043.36	$8,340.45	$8,341.79	$7,323.29	$47,976.21
19	19	1.5	80	100	100	$12,215.80	$12,184.00	$8,462.13	$8,463.07	$7,187.67	$48,512.67
20	20	1.5	80	100	100	$12,470.87	$12,465.43	$8,570.89	$8,569.73	$7,052.06	$49,128.97
21	21	1.5	80	100	100	$12,607.67	$12,705.32	$8,700.08	$8,689.41	$6,961.64	$49,664.13
22	22	1.5	80	100	100	$12,836.85	$12,819.11	$8,797.19	$8,815.18	$6,826.03	$50,094.35
23	23	1.5	80	100	100	$13,074.82	$13,057.56	$8,921.25	$8,925.01	$6,690.41	$50,669.05
24	24	1.5	80	100	100	$13,153.61	$13,286.26	$9,048.23	$9,026.93	$6,600.00	$51,115.03

Table 35: Data of Figure 16
Q = 3, W = $1600

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	88.542	85.417	$307.81	$407.37	$7,110.32	$7,095.20	$16,047.95	$30,968.65
2	2	1.5	80	79.375	84.375	$599.80	$724.13	$7,241.47	$7,225.52	$15,595.89	$31,386.80
3	3	1.5	80	96.458	85.417	$988.95	$1,131.03	$7,350.51	$7,337.77	$15,189.04	$31,997.30
4	4	1.5	80	98.125	87.708	$1,517.31	$1,653.31	$7,431.34	$7,418.81	$14,827.40	$32,848.16
5	5	1.5	80	78.958	100	$2,066.28	$2,188.06	$7,506.58	$7,489.64	$14,465.75	$33,716.31
6	6	1.5	80	99.583	86.042	$2,647.06	$2,649.29	$7,568.76	$7,582.44	$14,104.11	$34,551.66
7	7	1.5	80	81.25	100	$3,099.02	$3,184.52	$7,652.79	$7,651.12	$13,787.67	$35,375.11
8	8	1.5	80	81.25	88.75	$3,552.90	$3,695.78	$7,739.48	$7,720.35	$13,471.23	$36,179.74
9	9	1.5	80	100	87.5	$4,015.25	$4,102.67	$7,821.91	$7,808.93	$13,154.80	$36,903.55
10	10	1.5	80	88.125	86.042	$4,489.11	$4,554.80	$7,889.75	$7,893.81	$12,883.56	$37,711.03
11	11	1.5	80	88.125	85.417	$4,867.56	$4,995.73	$7,994.40	$7,973.58	$12,612.33	$38,443.59
12	12	1.5	80	100	86.667	$5,343.49	$5,362.67	$8,069.41	$8,067.57	$12,341.10	$39,184.24
13	13	1.5	80	100	85.417	$5,703.72	$5,734.41	$8,164.31	$8,177.19	$12,115.07	$39,894.69
14	14	1.5	80	80.625	86.875	$6,074.34	$6,114.77	$8,257.88	$8,253.99	$11,843.84	$40,544.81
15	15	1.5	80	80.417	100	$6,430.25	$6,535.09	$8,355.52	$8,345.03	$11,663.01	$41,328.90
16	16	1.5	80	82.917	87.5	$6,752.92	$6,883.65	$8,445.71	$8,445.20	$11,391.78	$41,919.27
17	17	1.5	80	78.958	81.875	$7,056.26	$7,155.98	$8,563.60	$8,544.64	$11,210.96	$42,531.43
18	18	1.5	80	88.125	86.042	$7,340.89	$7,511.42	$8,659.09	$8,655.04	$10,984.93	$43,151.37
19	19	1.5	80	76.458	87.292	$7,733.88	$7,728.29	$8,760.54	$8,764.26	$10,804.11	$43,791.08
20	20	1.5	80	100	87.708	$8,107.37	$8,054.32	$8,859.05	$8,863.18	$10,578.08	$44,462.00
21	21	1.5	80	79.375	100	$8,272.47	$8,375.39	$8,974.57	$8,960.79	$10,442.47	$45,025.68
22	22	1.5	80	80.625	82.083	$8,537.44	$8,601.37	$9,082.29	$9,071.64	$10,216.44	$45,509.19
23	23	1.5	80	100	85.417	$8,867.47	$8,881.05	$9,181.79	$9,176.31	$10,035.62	$46,142.23
24	24	1.5	80	88.125	86.875	$9,033.36	$9,181.83	$9,296.26	$9,274.19	$9,900.00	$46,685.64

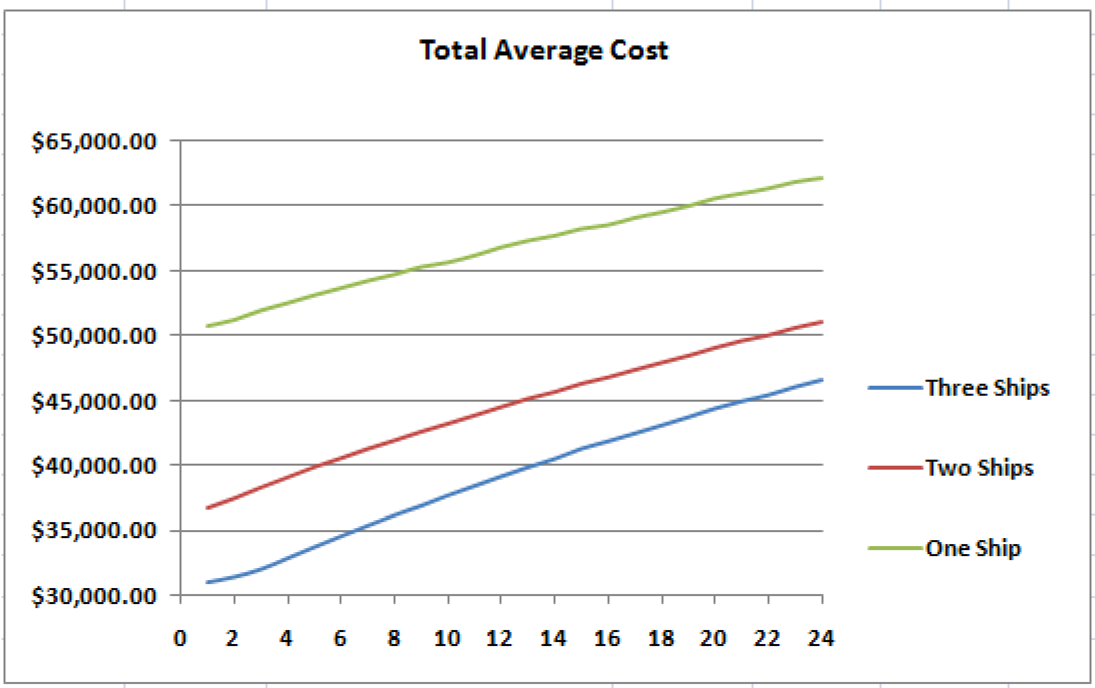


Figure 16 $T \rightarrow$

The total average cost when
$\lambda_a = 70,\ \lambda_b = 70, W = \$1600$

Table 36: Data of Figure 17
Q = 1, W = $100,000

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	100	100	$15,374.11	$15,319.29	$7,390.91	$7,359.27	$32,602.74	$78,046.32
2	2	1.5	80	100	100	$15,497.01	$15,570.05	$7,525.65	$7,501.67	$31,506.85	$77,601.23
3	3	1.5	80	100	100	$15,843.49	$15,784.36	$7,680.26	$7,651.37	$30,684.93	$77,644.42
4	4	1.5	80	100	100	$16,031.92	$15,968.31	$7,812.44	$7,797.71	$30,136.99	$77,747.36
5	5	1.5	80	100	100	$16,190.94	$16,268.61	$7,955.06	$7,938.95	$29,315.07	$77,668.62
6	6	1.5	80	100	100	$16,457.19	$16,389.75	$8,081.07	$8,099.84	$28,493.15	$77,521.00
7	7	1.5	80	100	100	$16,557.56	$16,642.10	$8,242.17	$8,235.58	$27,945.21	$77,622.61
8	8	1.5	80	100	100	$16,777.63	$16,703.47	$8,401.58	$8,363.35	$27,397.26	$77,643.28
9	9	1.5	80	100	100	$16,979.80	$16,903.08	$8,550.05	$8,525.70	$26,575.34	$77,533.97
10	10	1.5	80	100	100	$16,988.43	$17,080.16	$8,662.97	$8,665.11	$26,027.40	$77,424.06
11	11	1.5	80	100	100	$17,141.85	$17,237.10	$8,828.05	$8,803.46	$25,479.45	$77,489.92
12	12	1.5	80	100	100	$17,454.45	$17,370.71	$8,968.20	$8,947.48	$24,931.51	$77,672.35
13	13	1.5	80	100	100	$17,571.92	$17,482.42	$9,117.17	$9,114.29	$24,657.53	$77,943.34
14	14	1.5	80	100	100	$17,668.61	$17,577.19	$9,257.87	$9,247.92	$24,109.59	$77,861.17
15	15	1.5	80	100	100	$17,738.77	$17,844.41	$9,404.52	$9,384.44	$23,561.64	$77,933.78
16	16	1.5	80	100	100	$17,783.84	$17,897.31	$9,544.08	$9,529.78	$23,013.70	$77,768.70
17	17	1.5	80	100	100	$18,032.03	$17,929.43	$9,705.39	$9,677.91	$22,739.73	$78,084.48
18	18	1.5	80	100	100	$18,040.02	$18,154.13	$9,849.95	$9,818.46	$22,191.78	$78,054.35
19	19	1.5	80	100	100	$18,257.22	$18,147.10	$9,985.01	$9,987.36	$21,917.81	$78,294.49
20	20	1.5	80	100	100	$18,451.71	$18,346.39	$10,125.89	$10,130.78	$21,369.86	$78,424.65
21	21	1.5	80	100	100	$18,412.08	$18,534.82	$10,271.16	$10,262.36	$21,095.89	$78,576.30
22	22	1.5	80	100	100	$18,589.16	$18,475.21	$10,436.62	$10,404.49	$20,821.92	$78,727.38
23	23	1.5	80	100	100	$18,749.45	$18,633.11	$10,567.59	$10,544.40	$20,273.97	$78,768.52
24	24	1.5	80	100	100	$18,653.72	$18,784.45	$10,712.00	$10,682.83	$20,000.00	$78,833.01
25	25	1.5	80	100	100	$18,790.05	$18,925.57	$10,830.53	$10,836.10	$19,726.03	$79,108.28
26	26	1.5	80	100	100	$18,917.42	$18,784.93	$10,994.47	$11,006.75	$19,452.06	$79,155.63
27	27	1.5	80	100	100	$19,033.45	$18,901.12	$11,116.98	$11,121.56	$19,178.08	$79,351.19
28	28	1.5	80	100	100	$19,133.97	$19,007.08	$11,262.71	$11,262.11	$18,904.11	$79,569.97
29	29	1.5	80	100	100	$19,228.11	$19,098.65	$11,416.87	$11,413.08	$18,356.16	$79,512.88
30	30	1.5	80	100	100	$19,309.13	$19,176.64	$11,607.67	$11,533.37	$18,082.19	$79,709.00
31	31	1.5	80	100	100	$19,381.53	$19,245.37	$11,721.15	$11,684.49	$17,808.22	$79,840.75
32	32	1.5	80	100	100	$19,440.82	$19,300.98	$11,860.04	$11,823.46	$17,808.22	$80,233.52
40	40	1.5	80	100	100	$19,869.61	$19,704.36	$12,994.38	$13,031.02	$15,890.41	$81,489.78
41	41	1.5	80	100	100	$19,831.58	$20,014.73	$13,147.90	$13,148.65	$15,616.44	$81,759.29
50	50	1.5	80	100	100	$20,222.49	$20,433.29	$14,461.09	$14,405.27	$13,972.60	$83,494.74

Table 37: Data of Figure 17
Q = 2, W = $100,000

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	100	100	$6,324.77	$6,371.28	$6,684.98	$6,678.70	$64,931.51	$90,991.24
2	2	1.5	80	100	100	$6,776.90	$6,827.88	$6,764.13	$6,757.04	$63,013.70	$90,139.64
3	3	1.5	80	100	100	$7,252.35	$7,293.43	$6,847.83	$6,843.40	$61,369.86	$89,606.87
4	4	1.5	80	100	100	$7,671.87	$7,706.87	$6,929.66	$6,929.67	$60,000.00	$89,238.07
5	5	1.5	80	100	100	$8,078.29	$8,140.30	$7,018.99	$7,015.61	$58,630.14	$88,883.32
6	6	1.5	80	100	100	$8,482.15	$8,514.27	$7,109.22	$7,106.72	$56,986.30	$88,198.65
7	7	1.5	80	100	100	$8,829.59	$8,894.80	$7,198.80	$7,193.74	$55,890.41	$88,007.34
8	8	1.5	80	100	100	$9,183.90	$9,209.32	$7,301.52	$7,287.20	$54,520.55	$87,502.48
9	9	1.5	80	100	100	$9,539.82	$9,564.11	$7,391.26	$7,391.13	$53,150.69	$87,037.00
10	10	1.5	80	100	100	$9,805.43	$9,894.93	$7,491.96	$7,489.74	$52,054.80	$86,736.86
11	11	1.5	80	100	100	$10,110.05	$10,207.53	$7,593.40	$7,590.30	$50,958.90	$86,460.18
12	12	1.5	80	100	100	$10,474.91	$10,491.37	$7,692.38	$7,679.83	$49,863.01	$86,201.50
13	13	1.5	80	100	100	$10,747.88	$10,756.19	$7,801.44	$7,805.45	$49,041.10	$86,152.05
14	14	1.5	80	99.583	100	$11,011.42	$10,998.31	$7,896.39	$7,901.08	$47,945.21	$85,752.39
15	15	1.5	80	100	100	$11,240.43	$11,324.02	$8,014.30	$8,004.20	$47,123.29	$85,706.24
16	16	1.5	80	100	100	$11,442.76	$11,553.20	$8,133.15	$8,123.54	$46,027.40	$85,280.05
17	17	1.5	80	100	100	$11,759.04	$11,735.39	$8,237.44	$8,235.30	$45,205.48	$85,172.65
18	18	1.5	80	100	100	$11,927.33	$12,043.36	$8,340.45	$8,341.79	$44,383.56	$85,036.49
19	19	1.5	80	100	100	$12,215.80	$12,184.00	$8,462.13	$8,463.07	$43,561.64	$84,886.64
20	20	1.5	80	100	100	$12,470.87	$12,465.43	$8,570.89	$8,569.73	$42,739.73	$84,816.64
21	21	1.5	80	100	100	$12,607.67	$12,705.32	$8,700.08	$8,689.41	$42,191.78	$84,894.26
22	22	1.5	80	100	100	$12,836.85	$12,819.11	$8,797.19	$8,815.18	$41,369.86	$84,638.19
23	23	1.5	80	100	100	$13,074.82	$13,057.56	$8,921.25	$8,925.01	$40,547.95	$84,526.58
24	24	1.5	80	100	100	$13,153.61	$13,286.26	$9,048.23	$9,026.93	$40,000.00	$84,515.03
25	25	1.5	80	100	100	$13,368.72	$13,479.00	$9,140.29	$9,148.01	$39,452.06	$84,588.07
26	26	1.5	80	100	100	$13,571.05	$13,521.51	$9,286.87	$9,277.36	$38,630.14	$84,286.92
27	27	1.5	80	100	100	$13,759.32	$13,709.29	$9,418.81	$9,377.99	$38,082.19	$84,347.60
28	28	1.5	80	100	100	$13,918.17	$13,887.01	$9,510.78	$9,519.49	$37,534.25	$84,369.70
29	29	1.5	80	100	100	$14,090.46	$14,054.02	$9,658.96	$9,647.21	$36,712.33	$84,162.97
30	30	1.5	80	100	100	$14,246.44	$14,210.96	$9,786.90	$9,753.29	$36,164.38	$84,161.98
31	31	1.5	80	100	100	$14,395.71	$14,356.87	$9,877.08	$9,887.09	$35,616.44	$84,133.19
32	32	1.5	80	100	100	$14,531.87	$14,492.88	$10,003.48	$10,006.26	$35,342.47	$84,376.96
40	40	1.5	80	100	100	$15,521.30	$15,430.53	$11,014.15	$11,039.43	$31,506.85	$84,512.26
41	41	1.5	80	100	100	$15,564.77	$15,718.84	$11,180.67	$11,147.00	$31,232.88	$84,844.15
50	50	1.5	80	100	100	$16,418.20	$16,620.85	$12,338.82	$12,292.04	$27,945.21	$85,615.10

Table 38: Data of Figure 17
Q = 3, W = $100,000

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	88.542	85.417	$307.81	$407.37	$7,110.32	$7,095.20	$97,260.27	$112,180.98
2	2	1.5	80	79.375	84.375	$599.80	$724.13	$7,241.47	$7,225.52	$94,520.55	$110,311.46
3	3	1.5	80	96.458	85.417	$988.95	$1,131.03	$7,350.51	$7,337.77	$92,054.80	$108,863.06
4	4	1.5	80	98.125	87.708	$1,517.31	$1,653.31	$7,431.34	$7,418.81	$89,863.01	$107,883.78
5	5	1.5	80	78.958	100	$2,066.28	$2,188.06	$7,506.58	$7,489.64	$87,671.23	$106,921.79
6	6	1.5	80	99.583	86.042	$2,647.06	$2,649.29	$7,568.76	$7,582.44	$85,479.45	$105,927.00
7	7	1.5	80	81.25	100	$3,099.02	$3,184.52	$7,652.79	$7,651.12	$83,561.64	$105,149.09
8	8	1.5	80	81.25	88.75	$3,552.90	$3,695.78	$7,739.48	$7,720.35	$81,643.84	$104,352.35
9	9	1.5	80	100	87.5	$4,015.25	$4,102.67	$7,821.91	$7,808.93	$79,726.03	$103,474.79
10	10	1.5	80	88.125	86.042	$4,489.11	$4,554.80	$7,889.75	$7,893.81	$78,082.19	$102,909.66
11	11	1.5	80	88.125	85.417	$4,867.56	$4,995.73	$7,994.40	$7,973.58	$76,438.36	$102,269.62
12	12	1.5	80	100	86.667	$5,343.49	$5,362.67	$8,069.41	$8,067.57	$74,794.52	$101,637.67
13	13	1.5	80	100	85.417	$5,703.72	$5,734.41	$8,164.31	$8,177.19	$73,424.66	$101,204.28
14	14	1.5	80	80.625	86.875	$6,074.34	$6,114.77	$8,257.88	$8,253.99	$71,780.82	$100,481.79
15	15	1.5	80	80.417	100	$6,430.25	$6,535.09	$8,355.52	$8,345.03	$70,684.93	$100,350.82
16	16	1.5	80	82.917	87.5	$6,752.92	$6,883.65	$8,445.71	$8,445.20	$69,041.10	$99,568.58
17	17	1.5	80	78.958	81.875	$7,056.26	$7,155.98	$8,563.60	$8,544.64	$67,945.21	$99,265.68
18	18	1.5	80	88.125	86.042	$7,340.89	$7,511.42	$8,659.09	$8,655.04	$66,575.34	$98,741.78
19	19	1.5	80	76.458	87.292	$7,733.88	$7,728.29	$8,760.54	$8,764.26	$65,479.45	$98,466.42
20	20	1.5	80	100	87.708	$8,107.37	$8,054.32	$8,859.05	$8,863.18	$64,109.59	$97,993.51
21	21	1.5	80	79.375	100	$8,272.47	$8,375.39	$8,974.57	$8,960.79	$63,287.67	$97,870.88
22	22	1.5	80	80.625	82.083	$8,537.44	$8,601.37	$9,082.29	$9,071.64	$61,917.81	$97,210.56
23	23	1.5	80	100	85.417	$8,867.47	$8,881.05	$9,181.79	$9,176.31	$60,821.92	$96,928.53
24	24	1.5	80	88.125	86.875	$9,033.36	$9,181.83	$9,296.26	$9,274.19	$60,000.00	$96,785.64
25	25	1.5	80	81.25	100	$9,357.47	$9,443.77	$9,388.40	$9,388.41	$59,178.08	$96,756.13
26	26	1.5	80	83.958	83.333	$9,609.98	$9,562.67	$9,512.95	$9,525.60	$58,082.19	$96,293.39
27	27	1.5	80	80.417	83.333	$9,867.76	$9,809.91	$9,614.79	$9,613.55	$56,986.30	$95,892.32
28	28	1.5	80	83.958	81.25	$10,106.53	$10,055.71	$9,728.64	$9,736.06	$56,164.38	$95,791.32
29	29	1.5	80	81.25	85	$10,310.14	$10,291.92	$9,843.30	$9,849.56	$55,068.49	$95,363.41
30	30	1.5	80	81.25	85.417	$10,496.64	$10,520.94	$9,993.40	$9,955.10	$54,246.58	$95,212.66
31	31	1.5	80	81.25	80.417	$10,707.76	$10,729.98	$10,081.45	$10,075.14	$53,424.66	$95,018.99
32	32	1.5	80	83.958	87.708	$10,919.20	$10,932.79	$10,196.20	$10,195.53	$52,876.71	$95,120.43
40	40	1.5	80	79.375	92.5	$12,304.02	$12,229.54	$11,153.27	$11,189.35	$47,397.26	$94,273.43
41	41	1.5	80	82.708	86.458	$12,409.98	$12,528.56	$11,295.90	$11,282.64	$46,849.32	$94,366.40
50	50	1.5	80	89.792	87.292	$13,567.06	$13,787.28	$12,430.29	$12,396.34	$41,917.81	$94,098.77

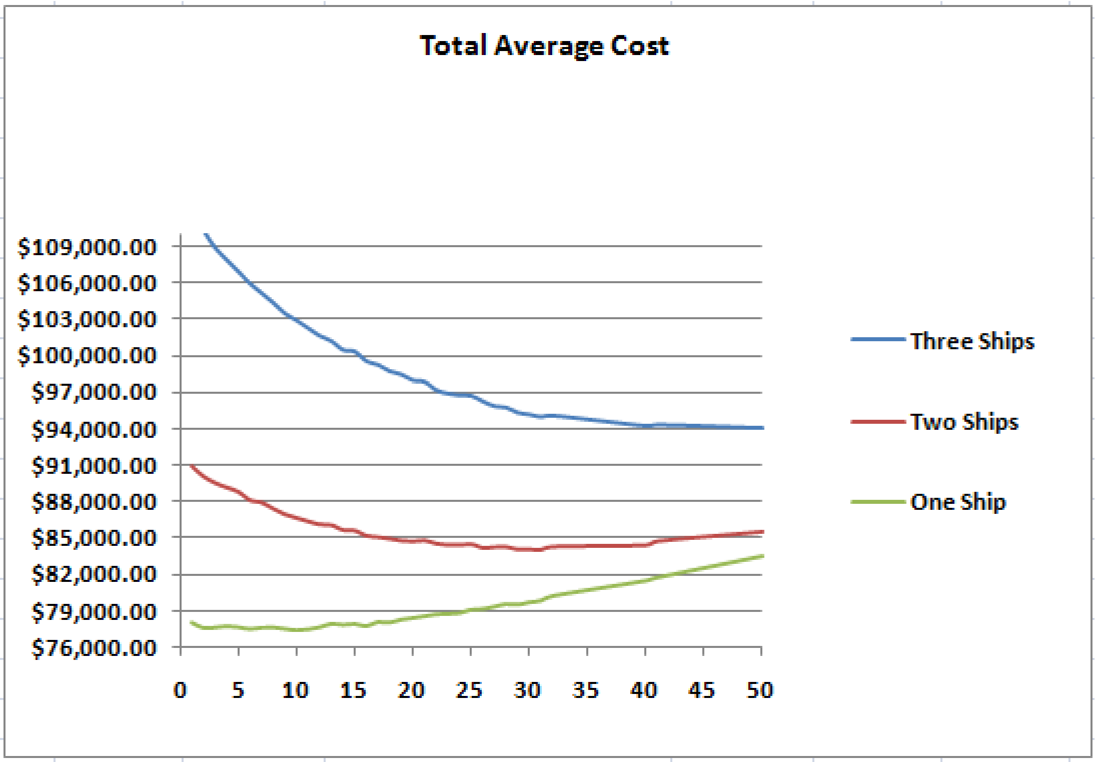


Figure 17 $T \rightarrow$

The total average cost when
$\lambda_a = 70,\ \lambda_b = 70, W = \$100{,}000$

Table 39: Data of Figure 18
Q = 1, W = $100,000

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	35.417	100	$0.00	$36,202.79	$1,544.23	$11,954.22	$32,602.74	$82,303.98
2	2	1.5	80	40	100	$0.00	$36,467.76	$1,589.16	$12,238.86	$31,506.85	$81,802.63
3	3	1.5	80	37.083	100	$0.00	$36,673.13	$1,626.83	$12,501.32	$30,684.93	$81,486.21
4	4	1.5	80	37.708	100	$0.00	$36,815.21	$1,666.08	$12,778.99	$30,136.99	$81,397.27
5	5	1.5	80	41.25	100	$0.00	$37,236.32	$1,712.37	$13,061.13	$29,315.07	$81,324.89
6	6	1.5	80	40.625	100	$0.00	$37,271.16	$1,754.54	$13,346.49	$28,493.15	$80,865.34
7	7	1.5	80	44.792	100	$0.00	$37,600.23	$1,793.66	$13,592.89	$27,945.21	$80,931.98
8	8	1.5	80	42.083	100	$0.00	$37,519.52	$1,833.52	$13,840.63	$27,397.26	$80,590.93
9	9	1.5	80	43.542	100	$0.00	$37,764.52	$1,877.10	$14,111.95	$26,575.34	$80,328.91
10	10	1.5	80	45.417	100	$0.00	$37,961.58	$1,918.01	$14,387.80	$26,027.40	$80,294.78
11	11	1.5	80	46.458	100	$0.00	$38,120.59	$1,957.76	$14,642.55	$25,479.45	$80,200.36
12	12	1.5	80	46.25	100	$0.00	$38,239.02	$1,995.76	$14,909.66	$24,931.51	$80,075.95
13	13	1.5	80	47.292	100	$0.00	$38,310.94	$2,042.35	$15,198.83	$24,657.53	$80,209.65
14	14	1.5	80	50.833	100	$0.00	$38,352.65	$2,087.27	$15,474.50	$24,109.59	$80,024.01
15	15	1.5	80	52.5	100	$0.00	$38,786.07	$2,129.01	$15,747.11	$23,561.64	$80,223.83
16	16	1.5	80	48.333	100	$0.00	$38,755.55	$2,168.29	$16,004.51	$23,013.70	$79,942.05
17	17	1.5	80	53.542	100	$0.00	$38,677.56	$2,209.68	$16,263.94	$22,739.73	$79,890.90
18	18	1.5	80	52.083	100	$0.00	$39,035.55	$2,257.37	$16,526.69	$22,191.78	$80,011.39
19	19	1.5	80	56.042	100	$0.00	$38,884.68	$2,288.36	$16,825.86	$21,917.81	$79,916.70
20	20	1.5	80	54.792	100	$0.00	$39,193.29	$2,327.52	$17,108.60	$21,369.86	$79,999.27
21	21	1.5	80	56.458	100	$0.00	$39,476.48	$2,372.57	$17,370.18	$21,095.89	$80,315.13
22	22	1.5	80	58.333	100	$0.00	$39,233.72	$2,419.40	$17,607.36	$20,821.92	$80,082.41
23	23	1.5	80	59.792	100	$0.00	$39,461.62	$2,458.97	$17,867.59	$20,273.97	$80,062.15
24	24	1.5	80	58.125	100	$0.00	$39,679.29	$2,500.56	$18,141.48	$20,000.00	$80,321.33
25	25	1.5	80	58.958	100	$0.00	$39,870.27	$2,544.07	$18,420.56	$19,726.03	$80,560.94
35	35	1.5	80	68.75	100	$0.64	$40,688.38	$2,958.03	$21,141.61	$16,986.30	$81,774.96
45	45	1.5	80	78.958	100	$7.83	$40,968.06	$3,372.72	$23,755.03	$14,794.52	$82,898.16
55	55	1.5	80	88.542	100	$84.70	$41,321.26	$3,731.84	$26,389.36	$13,150.69	$84,677.85
65	65	1.5	80	94.583	100	$264.02	$41,380.71	$4,015.02	$29,044.62	$11,780.82	$86,485.18
75	75	1.5	80	99.167	100	$503.58	$41,495.78	$4,269.05	$31,731.48	$10,958.90	$88,958.80
85	85	1.5	80	100	100	$743.31	$41,952.69	$4,468.40	$34,328.74	$9,863.01	$91,356.16

Table 40: Data of Figure 18
Q = 2, W = $100,000

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	18.75	100	$0.00	$27,331.48	$1,161.97	$8,999.98	$64,931.51	$102,424.94
2	2	1.5	80	16.458	100	$0.00	$27,722.56	$1,194.53	$9,150.92	$63,013.70	$101,081.71
3	3	1.5	80	20.417	100	$0.00	$28,260.98	$1,224.26	$9,302.07	$61,369.86	$100,157.17
4	4	1.5	80	20	100	$0.00	$28,635.11	$1,258.12	$9,459.49	$60,000.00	$99,352.73
5	5	1.5	80	17.083	100	$0.00	$29,104.82	$1,290.79	$9,624.37	$58,630.14	$98,650.11
6	6	1.5	80	23.333	100	$0.00	$29,473.52	$1,325.84	$9,784.06	$56,986.30	$97,569.72
7	7	1.5	80	16.458	100	$0.00	$29,852.28	$1,359.01	$9,956.70	$55,890.41	$97,058.41
8	8	1.5	80	23.333	100	$0.00	$30,108.63	$1,394.58	$10,133.47	$54,520.55	$96,157.23
9	9	1.5	80	25.833	100	$0.00	$30,501.14	$1,428.25	$10,308.80	$53,150.69	$95,388.88
10	10	1.5	80	18.333	100	$0.00	$30,854.34	$1,463.63	$10,488.97	$52,054.80	$94,861.73
11	11	1.5	80	18.333	100	$0.00	$31,167.10	$1,499.50	$10,695.64	$50,958.90	$94,321.15
12	12	1.5	80	28.542	100	$0.00	$31,437.67	$1,532.98	$10,867.79	$49,863.01	$93,701.46
13	13	1.5	80	28.542	100	$0.00	$31,667.97	$1,570.55	$11,089.31	$49,041.10	$93,368.92
14	14	1.5	80	17.917	100	$0.00	$31,860.39	$1,607.84	$11,277.11	$47,945.21	$92,690.54
15	15	1.5	80	16.25	100	$0.00	$32,265.53	$1,646.00	$11,471.88	$47,123.29	$92,506.69
16	16	1.5	80	19.583	100	$0.00	$32,490.55	$1,680.93	$11,683.58	$46,027.40	$91,882.45
17	17	1.5	80	17.083	100	$0.00	$32,568.86	$1,717.90	$11,886.76	$45,205.48	$91,379.00
18	18	1.5	80	17.708	100	$0.00	$33,002.28	$1,755.18	$12,102.23	$44,383.56	$91,243.25
19	19	1.5	80	16.667	100	$0.00	$33,008.36	$1,783.65	$12,309.67	$43,561.64	$90,663.33
20	20	1.5	80	36.042	100	$0.00	$33,391.28	$1,828.98	$12,526.05	$42,739.73	$90,486.03
21	21	1.5	80	16.458	100	$0.00	$33,646.35	$1,866.67	$12,728.21	$42,191.78	$90,433.01
22	22	1.5	80	39.375	100	$0.00	$33,658.33	$1,901.29	$12,956.33	$41,369.86	$89,885.81
23	23	1.5	80	40.417	100	$0.00	$33,969.34	$1,935.81	$13,161.23	$40,547.95	$89,614.33
24	24	1.5	80	17.708	100	$0.00	$34,259.09	$1,974.28	$13,377.70	$40,000.00	$89,611.07
25	25	1.5	80	16.25	100	$0.00	$34,423.70	$2,019.17	$13,612.43	$39,452.06	$89,507.36
35	35	1.5	80	16.25	100	$0.00	$36,008.93	$2,409.04	$15,930.56	$33,972.60	$88,321.13
45	45	1.5	80	19.167	100	$0.00	$37,009.38	$2,783.52	$18,228.36	$29,589.04	$87,610.31
55	55	1.5	80	16.042	100	$2.24	$37,817.74	$3,187.62	$20,664.36	$26,301.37	$87,973.33
65	65	1.5	80	16.458	100	$15.66	$38,268.90	$3,563.76	$23,132.32	$23,561.64	$88,542.30
75	75	1.5	80	16.667	100	$86.94	$38,686.19	$3,924.37	$25,725.74	$21,643.84	$90,067.08
85	85	1.5	80	96.042	100	$268.17	$39,375.80	$4,178.61	$28,156.83	$19,726.03	$91,705.44

Table 41: Data of Figure 18
Q = 3, W = $100,000

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	13.125	100	$0.00	$18,293.49	$1,029.99	$9,320.84	$97,260.27	$125,904.59
2	2	1.5	80	10.625	100	$0.00	$19,020.18	$1,061.42	$9,431.12	$94,520.55	$124,033.26
3	3	1.5	80	14.792	100	$0.00	$19,684.70	$1,091.09	$9,551.25	$92,054.80	$122,381.83
4	4	1.5	80	14.792	100	$0.00	$20,336.12	$1,118.73	$9,668.93	$89,863.01	$120,986.79
5	5	1.5	80	10.833	100	$0.00	$20,973.31	$1,151.15	$9,793.14	$87,671.23	$119,588.83
6	6	1.5	80	17.917	100	$0.00	$21,510.62	$1,182.22	$9,929.48	$85,479.45	$118,101.76
7	7	1.5	80	11.667	100	$0.00	$22,104.34	$1,212.57	$10,068.10	$83,561.64	$116,946.65
8	8	1.5	80	12.708	100	$0.00	$22,574.84	$1,247.78	$10,223.45	$81,643.84	$115,689.90
9	9	1.5	80	20.625	100	$0.00	$23,121.26	$1,278.17	$10,370.78	$79,726.03	$114,496.24
10	10	1.5	80	11.042	100	$0.00	$23,581.85	$1,315.00	$10,520.48	$78,082.19	$113,499.52
11	11	1.5	80	11.042	100	$0.00	$24,046.92	$1,344.40	$10,685.98	$76,438.36	$112,515.65
12	12	1.5	80	23.333	100	$0.00	$24,516.62	$1,379.07	$10,838.00	$74,794.52	$111,528.21
13	13	1.5	80	23.333	100	$0.00	$24,901.14	$1,413.97	$11,027.64	$73,424.66	$110,767.41
14	14	1.5	80	11.25	100	$0.00	$25,248.58	$1,448.83	$11,200.91	$71,780.82	$109,679.14
15	15	1.5	80	11.25	100	$0.00	$25,744.98	$1,485.61	$11,380.18	$70,684.93	$109,295.71
16	16	1.5	80	12.083	100	$0.00	$26,109.52	$1,518.55	$11,561.65	$69,041.10	$108,230.82
17	17	1.5	80	10.833	100	$0.00	$26,342.53	$1,554.35	$11,743.91	$67,945.21	$107,586.00
18	18	1.5	80	11.042	100	$0.00	$26,803.77	$1,585.50	$11,945.08	$66,575.34	$106,909.69
19	19	1.5	80	10.625	100	$0.00	$27,012.81	$1,621.33	$12,123.54	$65,479.45	$106,237.12
20	20	1.5	80	30.833	100	$0.00	$27,470.37	$1,660.61	$12,316.29	$64,109.59	$105,556.85
21	21	1.5	80	10.625	100	$0.00	$27,816.21	$1,695.16	$12,505.06	$63,287.67	$105,304.10
22	22	1.5	80	11.25	100	$0.00	$27,968.36	$1,726.49	$12,710.92	$61,917.81	$104,323.57
23	23	1.5	80	33.958	100	$0.00	$28,353.20	$1,762.99	$12,905.67	$60,821.92	$103,843.77
24	24	1.5	80	11.042	100	$0.00	$28,676.03	$1,800.66	$13,098.41	$60,000.00	$103,575.10
25	25	1.5	80	11.042	100	$0.00	$28,977.12	$1,841.42	$13,319.64	$59,178.08	$103,316.26
35	35	1.5	80	11.042	100	$0.00	$31,329.48	$2,216.30	$15,503.97	$50,684.93	$99,734.67
45	45	1.5	80	12.708	100	$0.00	$32,926.21	$2,592.73	$17,721.94	$44,383.56	$97,624.44
55	55	1.5	80	12.917	100	$0.00	$34,196.76	$2,990.34	$20,074.22	$39,452.06	$96,713.38
65	65	1.5	80	11.667	100	$5.27	$35,035.96	$3,369.07	$22,497.12	$35,342.47	$96,249.89
75	75	1.5	80	10.833	100	$29.57	$35,758.81	$3,746.12	$25,028.24	$32,328.77	$96,891.50
85	85	1.5	80	93.125	100	$148.15	$36,683.04	$4,053.45	$27,426.20	$29,589.04	$97,899.88

Table 42: Data of Figure 18
Q = 4, W = $100,000

Loading Time at Port A (hr)	Loading Time at Port B (hr)	Voyage Time (Days)	Ship Capacity	Average Load from Port A	Average Load from Port B	Average Penalty Cost at port A	Average Penalty Cost at port B	Average Wait Cost from Port A	Average Wait Cost from Port B	Average Operating Cost	Total Average Cost
1	1	1.5	80	11.042	99.167	$0.00	$9,350.27	$968.35	$10,473.08	$129,589.04	$150,380.75
2	2	1.5	80	8.75	99.583	$0.00	$10,282.97	$995.77	$10,559.94	$126,027.40	$147,866.08
3	3	1.5	80	12.708	99.167	$0.00	$11,192.65	$1,023.31	$10,653.13	$122,739.73	$145,608.81
4	4	1.5	80	11.875	98.75	$0.00	$12,042.56	$1,052.25	$10,762.14	$119,726.03	$143,582.97
5	5	1.5	80	8.333	100	$0.00	$12,877.60	$1,081.67	$10,861.28	$116,986.30	$141,806.85
6	6	1.5	80	14.792	100	$0.00	$13,633.38	$1,111.63	$10,978.51	$113,972.60	$139,696.12
7	7	1.5	80	7.917	100	$0.00	$14,389.32	$1,140.68	$11,114.12	$111,506.85	$138,150.96
8	8	1.5	80	10	100	$0.00	$15,050.64	$1,173.80	$11,243.87	$108,767.12	$136,235.43
9	9	1.5	80	17.708	100	$0.00	$15,736.10	$1,203.39	$11,375.52	$106,301.37	$134,616.38
10	10	1.5	80	8.333	100	$0.00	$16,379.84	$1,234.27	$11,514.05	$104,109.59	$133,237.74
11	11	1.5	80	8.333	99.167	$0.00	$17,006.80	$1,266.48	$11,661.19	$101,917.81	$131,852.28
12	12	1.5	80	20.417	100	$0.00	$17,591.10	$1,301.36	$11,810.02	$99,726.03	$130,428.50
13	13	1.5	80	20.208	100	$0.00	$18,150.46	$1,333.57	$11,979.41	$97,808.22	$129,271.65
14	14	1.5	80	9.167	100	$0.00	$18,633.27	$1,368.47	$12,135.82	$95,890.41	$128,027.97
15	15	1.5	80	8.125	100	$0.00	$19,260.87	$1,400.57	$12,306.76	$93,972.60	$126,940.80
16	16	1.5	80	8.333	99.583	$0.00	$19,713.31	$1,434.74	$12,476.77	$92,054.80	$125,679.61
17	17	1.5	80	8.333	100	$0.00	$20,122.92	$1,469.33	$12,656.35	$90,410.96	$124,659.56
18	18	1.5	80	8.125	100	$0.00	$20,680.05	$1,502.40	$12,833.23	$88,767.12	$123,782.80
19	19	1.5	80	7.083	100	$0.00	$21,011.19	$1,539.56	$13,012.38	$87,123.29	$122,686.42
20	20	1.5	80	27.708	98.75	$0.00	$21,539.54	$1,575.05	$13,194.66	$85,479.45	$121,788.71
21	21	1.5	80	8.75	100	$0.00	$22,015.00	$1,610.05	$13,374.78	$84,109.59	$121,109.42
22	22	1.5	80	8.75	100	$0.00	$22,275.34	$1,641.95	$13,584.97	$82,465.75	$119,968.02
23	23	1.5	80	30.625	100	$0.00	$22,737.69	$1,677.02	$13,752.34	$81,095.89	$119,262.95
24	24	1.5	80	8.125	98.542	$0.00	$23,161.53	$1,715.12	$13,955.60	$80,000.00	$118,832.24
25	25	1.5	80	8.542	100	$0.00	$23,550.53	$1,753.75	$14,168.24	$78,630.14	$118,102.65
35	35	1.5	80	8.542	100	$0.00	$26,676.23	$2,121.67	$16,249.92	$67,671.23	$112,719.06
45	45	1.5	80	9.792	100	$0.00	$28,830.25	$2,488.49	$18,446.02	$59,178.08	$108,942.84
55	55	1.5	80	8.958	100	$0.00	$30,564.91	$2,889.40	$20,746.92	$52,602.74	$106,803.97
65	65	1.5	80	7.917	100	$2.72	$31,792.79	$3,270.28	$23,146.15	$47,123.29	$105,335.22
75	75	1.5	80	7.708	100	$15.98	$32,809.70	$3,653.24	$25,654.36	$43,013.70	$105,146.98
85	85	1.5	80	10	100	$102.12	$33,970.78	$3,979.98	$28,006.31	$39,452.06	$105,511.25

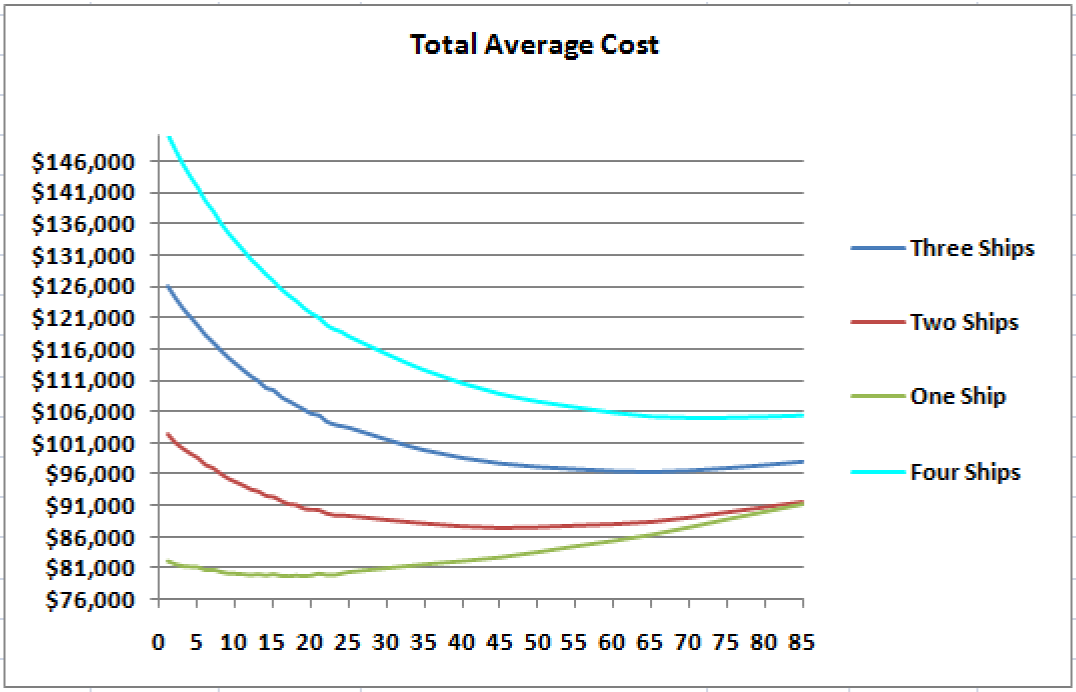


Figure 18 $T \rightarrow$

The total average cost when
$\lambda_a = 10,\ \lambda_b = 130, W = \$100,000$