2010-05-11

Optimal Capacity Investment, and Pricing Across International Markets Under Exchange Rate Uncertainty and Duopoly Competition

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OPTIMAL CAPACITY INVESTMENT AND PRICING ACROSS INTERNATIONAL MARKETS UNDER EXCHANGE RATE UNCERTAINTY AND DUOPOLY COMPETITION

By

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A DISSERTATION

Submitted to the Faculty
of the University of Miami
in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

Coral Gables, Florida

May 2010

UNIVERSITY OF MIAMI

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AHMED, ANAS A. (Ph.D., Industrial Engineering)
Optimal Capacity Investment and Pricing Across International Markets Under Exchange Rate Uncertainty and Duopoly Competition. (May 2010)

Abstract of a dissertation at the University of Miami.

Dissertation supervised by Assistant Professor Murat Erkoc
No. of pages in text. (94)

In this dissertation we investigate joint optimal capacity investment, pricing and production decisions for a multinational manufacturer who faces exchange rate uncertainties. We consider a manufacturer that sells its product in both domestic and foreign markets over a multiperiod season. Because of long-lead times, the capacity investment must be committed before the selling season begins. The exchange rate between the two countries fluctuates across period and the demand in both markets is price dependent. In the first part, the model considers three scenarios: (1) *early commitment to price and quantity with central sourcing*, (2) *postponement of prices and quantities with central sourcing*, and (3) *local sourcing*. We derive the optimal capacity and the optimal prices for each scenario, and investigate the impact of the exchange rate parameters and the length of the selling season on optimal capacity investment, production allocation, and pricing decisions. We observe that while the price and production decisions in the domestic market are independent of the exchange rate under early commitment and local sourcing scenarios, the exchange rate between two countries directly impacts these decisions under the postponement setting. We identify thresholds and gain insights on investment costs, market potentials, exchange rate drifts, and selling season length for the choice of entering a foreign market under all scenarios.

In the second part of this dissertation, we consider a duopoly competition in the foreign country. We consider a single period setting and we model the exchange rate as a random variable. We assume two scenarios: (1) *Exogenous Model*, where the price of the foreign manufacturer is set a priori, and (2) *Endogenous Model*, where the prices are set simultaneously based on a Nash Game outcome. In the *Exogenous Model*, we study the impact of exchange rate and foreign manufacturer's price on optimal capacity and prices. In the *Endogenous Model*, we investigate the impact of competition and exchange rate on optimal capacities and optimal prices. We show how competition can impact the decision of the home manufacturer to enter the foreign market.

To the most important people of my life

Ahmed Mohammed Mahmoud

Nafisah Alshibani

Nora Alshibani

Acknowledgments

My thanks to Allah Almighty for giving me the wisdom, knowledge, strength, and patience to finish this research.

I would like to send my deepest thanks to my advisor, Dr. Murat Erkoc for his patience, encouragement, and never giving up on me even in difficult times.

My deepest thanks also goes to Dr. Shihab S. Asfour, Professor and Associate Dean of Engineering, for his advice during my time at the University of Miami. He is a role model and an inspiration to me.

I would like to extend my thanks and gratitude to my other committee members, Dr. Haresh Gurnani and Dr. Dingxi Qiu, for their suggestions and help.

My ultimate thanks goes to my parents, Ahmed Mohammed Mahmoud and Nafisah Alshibani, for their unconditional love and support. They have been with me every step of the way since my first graduation. I thank them for their prayers and I wish I could repay them for what they have done for me.

Last but not least, I could not have gone through this experience without the help and support of my wife, Nora Alshibani. She has supported me throughout my undergraduate, Master's, and Ph.D. studies. I will forever be grateful to her.

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Chapter 1

Introduction and Literature Review

1.1 Introduction

Many manufacturers sell their products and services in international markets so as to increase their market share and profitability. Exchange rate is one of the key factors for a multinational manufacturer determining its profitability. Exchange rate fluctuations from 1% a day to 20 % a year is common and this changes could affect capacity and cost, which will ultimately affect the overall profit significantly (Dornier 1998). During the past 20 years, firms are expanding their product and services to foreign market, (Lowe et al. 2002). For a global firm to continue operating internationally and stay in business, it must be flexible and proactive in responding to uncertain shocks in exchange rate movements and adjust its operations based on unexpected movements in exchange rate. Therefore, multinational firms incorporate multiple risk-management techniques to handle exchange rate uncertainty. Unfavorable conditions in exchange rates may result in financial losses for the manufacturers' international operations or even end their operations in the market altogether. For example, in late 70's and early 80's a British airline company called Lakers Airlines was facing increasing demand for its low cost tickets (Jet Blue is an example of current companies that uses their technique). To expand its capacity, the company made investments in US dollars. When the value of dollar increased against pounds significantly, the company collapsed in 1982 because of the imbalance between its revenues in pounds and expenses in dollars (Anderson 1997). Similarly, according to Dominguez (1998), between the time from 1985 to 1997 the Yen was appreciating against the Dollar reaching its highest value in 1995. Therefore the Japanese companies revised their operations significantly by shifting their production abroad to low cost countries including the US (Smithson 1998). Pollack (1993) mentioned that the sole reason behind that move was that the Japanese companies were paying their employees in a strong Yen and making revenue in weak Dollars. It is imperative for a multinational company not to ignore the impact of exchange rate in its investment, production, and pricing decisions. Specifically, when a manufacturing firm makes a decision to enter a foreign market, based on the prospect of a favorable market potential and exchange rate, it must coordinate its capacity investments and supply chain operations across borders properly in order to realize higher profits. To achieve this goal, the firm has to allocate its investments wisely across markets and fix the prices accordingly.

In this dissertation, we consider a single manufacturer that produces its product in home country and sells it in both domestic and foreign countries over a planning horizon that consists of multiple periods. The goal of the manufacturer is to maximize the total net present value (NPV) of its profits (before tax) over the planing horizon in two markets; domestic and international. Quite a few companies prefer manufacturing their products at home and sell them both domestically and internationally. Sargento Cheese Company, Inc., for example, is a food company that has a factory in Plymouth, Wisconsin. The company is famous for producing quality cheese products which are sold in the local and international markets.

In our setting, the manufacturer must first decide on its capacity investment in the home country before the selling season. Then, based on demand and exchange rate conditions, the company is to decide how to allocate capacity across markets and how to price them. The selling season is composed of multiple periods across which the exchange rate fluctuates. The fluctuations in exchange rate is stochastic and modeled by the Geometric Brownian Motion. The demands in both markets, on the other hand, are assumed to be downward slopping with selling prices. We study the effects of exchange rate parameters, specifically, drift and volatility on a multinational manufacturer's capacity investment and allocation decisions under three different scenarios. In the first scenario we consider *early commitment with central sourcing*, where the manufacturer builds capacity only in the domestic market. Furthermore it must determine and commit to its pricing and production quantities for the selling season in both markets before the selling season starts. The selling price, hence demand in each market, stays constant throughout the selling season regardless of how the exchange rate fluctuates. Such situations occur when the manufacturer makes long term contracts with its distributors or buyers. Typically, those contracts stipulate price and quantity that the manufacturer must provide for its buyers over a specified term. This type of contracts are common especially for food and beverage producers. The second model (*Postponement of Prices and Quantities*) also assumes that the overall capacity will be built in the domestic market before the selling season, however it considers *postponement with central sourcing* where the price and the allocation of the production can be adjusted in each period for each market once the exchange rate for that period is observed.

As an alternative to exporting its products, we also consider the case where the manufacturer has the option of investing in capacity in the foreign country. This case investigates *local sourcing* where the manufacturer builds capacity in both markets and satisfies the demand in each market locally. In this case, since both the manufacturing cost and the revenue from sales are based on the same currency the early and delayed commitments are effectively undistinguishable. Traditional manufacturers of the 20th Century used to carry out their main production operations in their home country and export their goods to other nations. However, with the recent wave of globalization many large scale manufacturers open and operate manufacturing facilities at international locations. Moving some of the manufacturing to other countries provide the firms with chances to improve their service and delivery efficiency in local markets, product availability, savings in logistics, and tax benefits. On the other hand, it exposes them to financial risks due to exchange rate fluctuations and additional overhead costs. In the car industry, luxury car manufactures prefer to produce their cars locally instead of exporting them to the foreign market, because they can react more efficiently to consumer satisfactions and exchange rate fluctuations (Taipei Time 2003). As an example, Lexus is one of the successful car manufacturers that produces luxury vehicles. Although it manufactures its certain models in Japan and exports them to foreign markets, it has also invested in capacity to manufacture the RX model in Canada. The investment for overseas production could be explained by the huge demand for the RX in the United States (The Age 2006).

By analyzing the aforementioned models we investigate the conditions under which the manufacturer should invest for capacity for local production in the foreign market, export to the foreign market, or not to enter the foreign market at all. We also study the impact of the length of the planing horizon on the manufacturer's optimal investment, allocation, and pricing policies under all scenarios.

In this dissertation, we will try to address the following questions:

In Part I:

1. What is the impact of exchange rate parameters on capacity and prices?
2. When is it profitable to enter the foreign market?
4. When is it profitable to invest capacity in the foreign?

In part II

5. How competition is going to affect capacity allocation?
6. How is competition going to affect the firm's home sales?
7. which exchange rate value is going to help the firm competing?

1.2 Dissertation Outline

This dissertation is divided into two parts, Part I (Monopoly) and Part II (Competition). In Part I, the manufacturer relies on the exchange rate to enter the foreign market where in Part II the manufacturer relies on exchange rate and competition factors to enter the foreign markets.

Part I includes: the preceding section, Chapter 2, Chapter 3, Chapter 4, and Chapter 5. In the preceding section, we present the literature review. In Chapter 2, we present our basic settings, assumptions, and nomenclature. In Chapter 3, we present the Early Commitment to Price and Quantity Model where the manufacturer commits to prices and quantities at the beginning of the planning horizon. In Chapter 4, we present the Postponement of Prices and Quantities Model where the manufacturer flexes its prices and quantities from period to period. In Chapter 5, we present the Capacity Investment in the Foreign Market Model (LS) where we show when it is profitable for the manufacturer to build capacity at the foreign country. For each model we will discuss the allocation scenarios under capacity investment at home and in both countries.

Part II includes chapter 6 where we assume that there is a competition at the foreign market only and the manufacturer postpones prices and quantities until the realization of exchange rate. In this chapter, we present our basic settings, assumptions, and nomenclature, Exogenous Model, Endogenous Model, and we drive the optimal capacity for each model.

In Chapter 7, we will have a summary of key results and future work.

1.3 Literature Review

To manage the risk caused by the exchange rates a global manufacturer can employ two types of strategies: financial hedging and operational hedging. In financial hedging the multinational firm buys option contracts such as the one that grants the right to buy or sell currency at a specified exchange rate during a specified period of time. The multinational firm can opt out of the contract if the future exchange rate is not favorable. According to O'Brien (1996), currency option is used regularly by firms to protect themselves against exchange rate volatility. Another example that is being used frequently is forward exchange contract where an agreement is made between two parties to exchange a specified amount of one currency for another currency at a specified foreign exchange rate on a future date. The financial hedging has its disadvantages that could hit the firm with financial losses: the firm could lose profit because it cannot catch the upside of the exchange rate volatility and it could be risky for a firm to enter foreign market, which leads to financial losses if the firm is not hitting the targeting sales and the exchange rate is weak, and/or any cancellation of the contract will result in a financial loss for the multinational firm (Huchzermeier and Cohen 1996).

Our work focuses on operational hedging; where we propose operational hedging mechanism for the firm to implement to hedge against exchange rate fluctuations. Ding et al. (2007) define operational hedging strategies as "real compound options that are exercised in response to the demand, price, and exchange-rate contingencies faced by the firm in a global supply chain context." Such operational techniques (real options) can be found in the literature and include postponing prices (Kazaz et al. 2005), switching production between countries (Kogut and Kutalika 1994, Dasu and Li 1997, Kouvelis et al. 2001), and reserving capacity (Cohen and Huchzermeier 1999).

Huchzermeier and Cohen (1996) consider a supply chain consisting of supplying countries, production facilities, and markets. Similar to our model, capacity decision is made before realizing the exchange rate. They developed model to select the optimal supply chain. For example, one optimal supply chain option is to supply the material from home country and manufacture the product at all factories and each factory sells it to its market. They implement a multinomial approach to approximate exchange rate. Our model differs from this work in the sense that we assume a multiperiod model with price dependent demand. Kouvelis et al. (2001) consider three types of strategies: exporting with capacity being only at home, partnership with a firm in the foreign market, and investing capacity in the foreign market. They study policies that shift between those strategies based on exchange rate volatility. They observe that increasing volatility would force the firm to implement the exporting strategy. In their work, they assume that the firm always enters the foreign market, whereas in our work the manufacturer has the option of not entering the market in a setting with price dependent demand.

Kazaz et al. (2005) developed a two-staged stochastic program. The goal of their model is to determine optimal capacity investment and allocation policies that can be employed as a hedge against uncertain fluctuations in exchange rate. This program chooses the optimal allocation policy based on the realized exchange rate. In their model, capacity is built and products are manufactured before the realization of the exchange rate. Once the exchange rate is observed the firm decides how it allocates its quantities to the markets. Similar to our setting, they consider a multiple period model.

In another paper, Ding et al. (2007) present a multinational firm that invests capacity in either one of the two markets or in both markets. They assume that the firm commits to capacity before the realization of the exchange rate and demand in both markets. In their paper, they consider both financial hedging and operational hedging in order for the firm to protect itself against exchange rate uncertainty. The multinational firm employs financial hedging at the time of capacity building by purchasing financial option contracts. This strategy is based on the schemes presented by previous researchers such as Mello et al. (1995) and Chowdhry and Howe (1999). The other method is operational hedging where the multinational firm postpones its allocation of capacity to both markets until the demand and exchange rate are observed. In this sense their model is similar to Kazaz et. al. (2005) where both papers utilize a two-stage stochastic program. In the first stage, both of them invest capacity before the realization of the exchange rate and in the second stage they allocate capacity after the realization of the exchange rate. As mentioned above, the main difference is that Ding et al. (2007) incorporate financial hedging techniques into the first stage whereas Kazaz et al. (2005) also considers a multiperiod setting.

There are two key differences that distinguish our model from Kazaz *et. al.* and Ding et al. (2007): how the exchange rate fluctuations are modeled and the price dependent demand function. In our model we assume that the exchange rate follows a geometric Brownian motion whereas Kazaz et. al. (2005) assume a general probability distribution function and Ding et al. (2007) assume that exchange rate follows a lognormal distribution.

In a relevant work, Aytekin and Birge (2004) compare both financial and operational hedging strategies. Their conclusions favor financial hedging when exchange rate has a small volatility and operational hedging when exchange rate has a strong volatility. They consider three scenarios where production is done at home, production is done at home and foreign country where home production supply the shortage at the foreign market, and production is done in both countries and each production country can serve either market.

Chapter 2

Basic Settings, Assumptions, and Nomenclature

2.1 Basic Settings and Assumptions

In this study we investigate for optimal capacity investment, pricing, and production allocation policies for a global firm that manufacturers its products at "home" country and sells them to domestic and foreign markets. As an alternative to exporting its products, the manufacturer has the option of investing in capacity at the foreign country. Traditional manufacturers of the 20th Century used to carry out their main production operations in their home country and export their goods to other nations. However, with the recent wave of globalization many large scale manufacturers open and operate manufacturing facilities at international locations. Moving some of the manufacturing to other countries provide the firms with chances to improve their service and delivery efficiency in local markets, product availability, savings in logistics, and tax benefits. On the other hand, it exposes them to financial risks due to exchange rate fluctuations and additional overhead costs. As such, there are still quite a few companies that prefer manufacturing their products only at home and export them to other countries. Sargento Cheese Company, Inc., is an example of a food company that has a factory in Plymouth, Wisconsin. The company is famous for producing quality cheese products and their products are sold to local and international markets. In the car industry, luxury car manufactures prefer to produce their cars locally and export them to the foreign market, because they can react more efficiently to consumer satisfactions and exchange rate fluctuations (Taipei Time, 2003). The main reason for a luxury car manufacturer to spend a lot of money to invest in capacity aboard is to increase their sales (Taipei Time 2003). Lexus is one of the successful car manufacturers that produces luxury vehicles. According to Table 2.1, Lexus manufactures all of there models in Japan and export them to foreign market except for the RX model which is produced in Japan and Canada. This could be explained by the huge demand of the RX in the United States (Morley 2006).

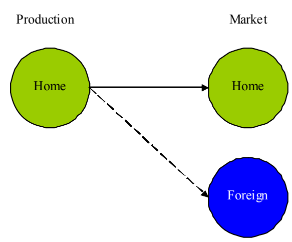
Table 2.1 Lexus Production Location

Model	**Production Location**
LS	Tahara, Japan
LS Hybrid	Tahara, Japan
GS	Tahara, Japan
GS Hybrid	Tahara, Japan
ES	Kyushu, Japan
IS	Kyushu and Tahara, Japan
IS F	Tahara, Japan
SC	Kanto Jidosha, Japan
LX	Araco, Japan
GX	Tahara, Japan
RX	Kyushu, Japan and Cambridge, Ontario
RX Hybrid	Kyushu, Japan

We consider a firm that manufactures and sells a single type product to two markets: the home country and a foreign market. The firm must decide on its production capacity, selling prices at both markets, and the production allocations throughout the selling season of multiple periods. In the basic model, the capacity investment is irreversible and must be made in the home country.

The choice of capacity is made ahead of the selling season, before the exchange rates between the currencies of the home and the foreign countries are observed. This is often times the case for many industries with long capacity lead times such as the pharmaceutical and high-tech industries (Anupindi and Jiang 2008). Also, many multinational car manufacturers invest in capacity before the realization of the exchange rates and postpone their prices and allocations to local and foreign market until after the realization of the exchange rates (for more examples see Mieghem and Dada 1999).

If the manufacturer chooses to build all the capacity at home, both the domestic and the foreign market demands are satisfied from the home country (Figure 2.1.a). We referred to this model as the *Central Sourcing Model*. The manufacturer also has the option of building capacity in the foreign market to satisfy the demand therein. In this case, both markets are served locally and independently (Figure 2.1.b). We call this model as the *Local Sourcing Model*.



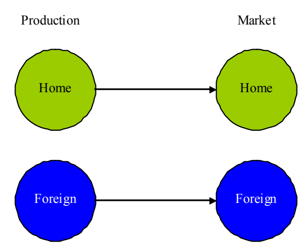


Figure 2.1.a Central Sourcing Model **Figure 2.1.b** Local Sourcing Mode

The domestic and the foreign markets differ in their demand base and disparity in currencies. In each market demand is assumed to be linear decreasing in price which is expressed in local currency. The demand base is steady. While the demand is assumed to be a deterministic function of price, the exchange rate is stochastic and fluctuating across periods. It is assumed that the manufacturer does not carry any inventory across periods. The product can be produced and shipped to both markets within the same selling period.

We study the manufacturer's decision problem under two scenarios. In the first scenario (*Early Commitment*), the manufacturer has to commit to its capacity, price, and production before the selling season as illustrated in Figure 2.2. Once the season starts no decision change can be made. This is a common practice especially for agricultural products where the suppliers and buyers lock prices and quantities before the harvesting season based on their prospect in future market demand and supply.

Figure 2.2 Sequence of operational decisions under early commitment

In the second scenario (*postponement of prices and quantities*), the manufacturer has more flexibility in that she can postpone the price and quantity decisions until after the exchange rates are realized (Figure 2.3).

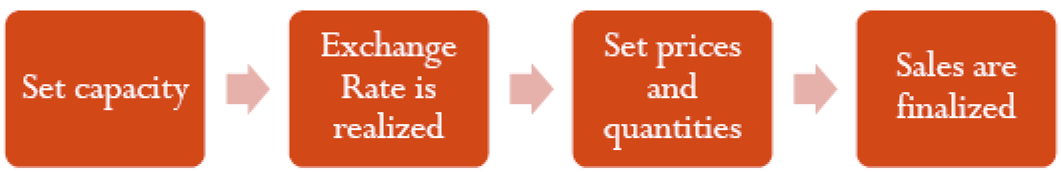


Figure 2.3 Sequence of operational decisions under price and quantity postponement

2.2 Nomenclator

<u>Indices</u>

i: market index where $i = H$ for the home market, and $i = F$ for the foreign market

j: investigated scenario index where scenarios $j = 1,2,$ and 3 are as explained above

t: period index (t=1..T)

<u>Parameters</u>

I_o: The initial foreign exchange rate expressed in home-currency per unit foreign-currency.

$I(t)$: exchange rate between the currency of home country and foreign country

c_H: unit manufacturing cost at home country expressed in home currency

c_F: unit manufacturing cost at foreign country expressed in foreign currency

r unit shipment cost to foreign market expressed in home currency

u_H: unit capacity cost at home in home currency

u_{FH}: portion of the marginal capacity cost for the foreign country paid in home currency

u_{FF}: portion of the marginal capacity cost for the foreign country paid in local currency

u_F: unit capacity cost in the foreign country where $u_F = u_{FH} + I_o u_{FF}$

Decision Variables

K^z_{Hj}: manufacturing capacity built at home country expressed in number of units under

scenario j for model z ($z = s$ for the SSM and l for LSM)

K^z_{Fj}: manufacturing capacity built at the foreign country expressed in number of units

under scenario j for model z.

p^z_{ijt}: unit selling price for the product in market i, scenario j, in period t in local currency and model z

Q^z_{ijt}: quantity allocated to market i under scenario j in period t and model z

To model the price dependent demand we employ the linear additive demand function. Specifically, at any given period t ($t = 1..T$)

$$D_{ijt} = \alpha_i - \beta_i p_{ijt} \tag{2.1}$$

where D_{ijt} denotes the demand in market i while α_i and β_i represent the demand potential and price coefficient in market i respectively. Similar demand function is common in the literature and some include Mills (1959), Zabel (1972), Thowsen (1975), Petruzzi and Dada (1999), Chen and Simchi-Levi (2004), Chen et al. (2006), and Gurnani et al. (2007).

In our setting, the exchange rate fluctuations across periods are modeled by a Weiner Process, $B(t)$, where $B(t) = \epsilon\sqrt{t}$. Here, ϵ is the random error term that follows the Standard Normal distribution. As such, it is assumed that the exchange rate follows a Geometric Brownian motion:

$$dI(t) = \mu I(t)dt + \sigma I(t)dB(t) \tag{2.2}$$

where $I(t)$ is driven by the Ito process. The parameters μ and σ are the mean and the standard deviation of the Normal exchange rate drift. The solution to equation (2.2) is given by Davis (2001) as $I(t) = I_0 e^{((\mu - \frac{1}{2}\sigma^2)t + \sigma B(t))}$. Assuming $B(t) = \epsilon\sqrt{t}$ and replacing in (2.2), we get

$$I(t) = I_0 e^{\left((\mu - \frac{1}{2}\sigma^2)t + \sigma\epsilon\sqrt{t}\right)} \tag{2.3}$$

Hence, the expected value of $I(t)$ at time t is $I_0 e^{\mu t}$.

Chapter 3

Early Commitment to Price and Quantity

3.1 Background

In this section, we consider the case where the manufacturer must commit to her capacity, domestic and foreign market prices, and quantity allocation before the beginning of the selling season. Once fixed, prices and sales quantities stay unchanged across all periods. Clearly, since the demand is deterministic, the manufacturer's revenue from the domestic sales stays constant in all periods. On the other hand, since the exchange rate is random she may loose money in some periods due to low realized exchange rates. We note that in the foreign market the manufacturer collects its revenues in local currencies, whereas, both the production and transportation costs are paid in home currency. Therefore, by early commitment, the manufacturer takes the risk of incurring negative profits in some periods from her sales in the foreign country. As such, she accepts to make a commitment for the foreign market only if the overall expected profits increase by doing so.

3.2 The Model

All products are manufactured at home and shipped to both markets. In this case, the prices and quantities, and hence demand are unchanged across periods. Consequently, the manufacturer's objective is to maximize the following profit function:

$$\underset{K_H^e, p_H^e, p_F^e, Q_H^e, Q_F^e \geq 0}{\text{Maximize}} \; \Pi^e = -u_H * K_H^e + E\left(\sum_{t=1}^{T} e^{-\rho t}((p_H^e - c_H)Q_H^e + (p_F^e I(t) - c_H - r)Q_F^e\right) \tag{3.1}$$

$$s.t. \;\; Q_i^e \leq \alpha_i - \beta_i p_i^e \;\; \forall i = H, F \tag{3.2}$$

$$Q_H^e + Q_F^e \leq K_H^e \tag{3.3}$$

In (3.1), $E(\,\cdot\,)$ returns the expected value and the exponential term is used to capture the net present value of the profit using the constant discount rate ρ. The first constraint set ensures that sales in any market do not exceed the production allocated to that market. The constraint in (3.3) is the capacity constraint for the production. We note that since the sales quantities have constant values for all periods and production and capacity decisions are given simultaneously, there will be no incentive for the manufacturer to deviate from the market prices that clear the market and also it will not be economically justified to build capacity beyond the total production levels. Consequently, it is straightforward to see that both constraints (3.2) and (3.3) must be binding. Hence, assuming Ito process for the exchange rates we can rewrite the manufacturer's model as follows:

$$\underset{p_H^e, p_F^s \geq 0}{\text{Maximize}} \; \Pi^e = -u_H(\alpha_H + \alpha_F - \beta_H p_H^e - \beta_F p_F^e) + \sum_{t=1}^{T} e^{-\rho t}(p_H^e - c_H)(\alpha_H - \beta_H p_H^e)$$
$$+ \left(\int_{-\infty}^{\infty} \sum_{t=1}^{T} e^{-\rho t} (p_F^e I_0 e^{\left((\mu - \frac{1}{2}\sigma^2)t + \sigma\epsilon\sqrt{t}\right)} - c_H - r)(\alpha_F - \beta_F p_F^e)\phi(\epsilon) d\epsilon \right) \tag{3.4}$$

which can be further reduced to

$$\underset{p_H^e, p_F^s \geq 0}{\text{Maximize}} \; \Pi^e = -u_H(\alpha_H + \alpha_F - \beta_H p_H^e - \beta_F p_F^e)$$
$$+ L_o(p_H^e - c_H)(\alpha_H - \beta_H p_H^e) + (L_1 p_F^e I_0 - (c_H + r)L_o)(\alpha_F - \beta_F p_F^e) \tag{3.5}$$

where $\phi(\epsilon)$ is the standard Normal density function,

$$L_o = \frac{e^{-\rho}(1 - e^{-\rho T})}{(1 - e^{-\rho})} \text{ and } L_1 = \frac{e^{(\mu - \rho)}(e^{(\mu - \rho)T} - 1)}{(e^{(\mu - \rho)} - 1)}. \tag{3.6}$$

A quick analysis of (3.5) will reveal that the expected profit function is jointly concave in p_H^e and p_F^e. Hence using the first order optimality conditions we get

$$p_H^{e*} = \frac{\alpha_H}{2\beta_H} + \frac{c_H L_o + u_H}{2L_o} \tag{3.7}$$

$$p_F^{e*} = \frac{\alpha_F}{2\beta_F} + \frac{(c_H + r)L_o + u_H}{2I_o L_1} \tag{3.8}$$

Thus,

$$Q_H^{e*} = \frac{\alpha_H}{2} - \frac{\beta_H(c_H L_o + u_H)}{2L_o} \tag{3.9}$$

$$Q_F^{e*} = \frac{\alpha_F}{2} - \frac{\beta_F((c_H + r)L_o + u_H)}{2I_o L_1} \tag{3.10}$$

$$K_H^{e*} = Q_H^{e*} + Q_{\mathrm{F}}^{e*} \tag{3.11}$$

From the optimality conditions we can make the following observation:

Lemma 3.1: *The optimal price for the foreign market decreases in the drift on the exchange rate (i.e., μ). Moreover, both optimal market prices are decreasing in T.*

Proof. First we need to write the first derivative of the foreign market price given in (3.8):

$$\frac{\partial p_F^{e*}}{\partial \mu} = -\frac{(c_H + r)}{2I_o} \cdot \frac{Te^{(\mu-\rho)T}(e^{(\mu-\rho)} - 1) - (e^{(\mu-\rho)T} - 1)}{e^{(\mu-\rho)}(e^{(\mu-\rho)T} - 1)^2} \tag{3.12}$$

The derivative of the numerator in the right hand side with respect to T is

$$e^{(\mu-\rho)T}(e^{(\mu-\rho)} - (\mu - \rho) - 1 + (\mu - \rho)Te^{(\mu-\rho)}) \tag{3.13}$$

Observe that $e^z - z \geq 1$ for any value of z implying that the foregoing function is strictly positive. Hence, from (3.12), we conclude that p_F^{e*} is strictly decreasing in μ.

To investigate the impact of T on prices we first write down the first derivative of p_H^{e*} as follows:

$$\frac{\partial p_H^{e*}}{\partial T} = -\frac{u_H \rho e^{-\rho T}(1 - e^{-\rho})}{2(1 - e^{-\rho T})} \tag{3.14}$$

Clearly, the foregoing function is negative implying that the optimal home market price is monotonically decreasing in T. To analyze the foreign market price we compute the first derivative of p_F^{e*} with respect to T:

$$\frac{\partial p_F^{e*}}{\partial T} = \frac{(c_H + r)}{2I_o}(e^{(\mu-\rho)} - 1)\big(\rho(1 - e^{(\mu-\rho)T}) - (\mu - \rho)e^{(\mu-\rho)T}(1 - e^{-\rho T})\big) - \frac{u_H(\mu - \rho)e^{(\mu-\rho)(T+1)}\rho e^{-\rho T}(1 - e^{-\rho})}{2I_o(e^{(\mu-\rho)} - 1)} \tag{3.15}$$

It is easy to see that the last term in the function above is always positive. A closer look reveals that the first term is always negative. Consequently, we can conclude that the foreign market price is decreasing in T as well. □

We notice from (3.8) and (3.10) that exchange rate volatility does not affect optimal foreign price and optimal foreign quantity because the firm commits to price and quantity at the beginning of the planning horizon and it would not change, so exchange rate shocks does not affect them.

We note that higher drift implies a prospect for more favorable exchange rates for the firm. Since higher exchange rates imply increased profit margins for the firm in the foreign market. Increased profit margins allow the firm to drop the prices in this market in return for increased demand eventually leading to higher profits overall. As expected, higher exchange rate makes the foreign market more attractive for the firm. Under this scenario, the home country price is clearly independent of the exchange rate drift. Return on investment on capacity is higher also when the selling season gets longer. This allows the firm drop the product's prices in both markets. As the number of periods increases prices converge to the following levels:

$$\underset{T\to\infty}{p_H^{e*}} = \frac{\alpha_H + \beta_H(c_H + u_H(e^\rho - 1))}{2\beta_H} \tag{3.16}$$

$$\underset{\mu>\rho,T\to\infty}{p_F^{e*}} = \frac{\alpha_F}{2\beta_F} \tag{3.17}$$

$$\underset{\mu<\rho,T\to\infty}{p_F^{e*}} = \frac{\alpha_F}{2\beta_F} + \frac{(c_H + r + u_H(e^\rho - 1))(e^\rho - e^\mu)}{2e^\mu(e^\rho - 1)I_0} \tag{3.18}$$

Equations (3.17) and (3.18) hint us that the foreign market price is in general more sensitive to the selling season length when $\mu > \rho$. In this case since the drift in exchange rate outstrips the discount rate there will be additional return on investment due to exchange rates providing more room for further price cuts when T increases. Equations (3.16) through (3.18) imply the following:

$$\underset{T\to\infty}{Q_H^{e*}} = \frac{\alpha_H - \beta_H(c_H + u_H(e^{\rho}-1))}{2} \tag{3.19}$$

$$\underset{\mu>\rho,T\to\infty}{Q_F^{e*}} = \frac{\alpha_F}{2} \tag{3.20}$$

$$\underset{\mu<\rho,T\to\infty}{Q_F^{e*}} = \frac{\alpha_F}{2} - \frac{\beta_F(c_H + r + u_H(e^{\rho}-1))(e^{\rho}-e^{\mu})}{2e^{\mu}(e^{\rho}-1)I_0} \tag{3.21}$$

and thus,

$$\underset{\mu>\rho,T\to\infty}{K^{e*}} = \frac{\alpha_H + \alpha_F - \beta_H(c_H + u_H(e^{\rho}-1))}{2} \tag{3.22}$$

$$\underset{\mu<\rho,T\to\infty}{Ke^{*}} = \frac{\alpha_1 + \alpha_2 - \beta_H(c_H + u_H(e^{\rho}-1))}{2} - \frac{\beta_F(c_H + r + u_H(e^{\rho}-1))(e^{\rho}-e^{\mu})}{2e^{\mu}(e^{\rho}-1)I_0} \tag{3.23}$$

To keep the analysis interesting we assume that $\alpha_H > \beta_H(c_H + u_H)$ so that it is always profitable for the manufacturer to manufacture and sell its products in the home market. On the other hand willingness of the manufacturer committing to the price and quantity in the foreign country depends on its prospect on the exchange rate fluctuations. A risk neutral manufacturer participates in the foreign market under this scenario only if the "expected" profit from sales in the foreign market over the selling season is positive. The following lemma lays out the condition under which this criterion is satisfied:

Lemma 3.2: *The manufacturer accepts early commitment for the foreign market if and only if*

$$\alpha_F > \frac{\beta_F((c_H + r)L_o + u_H)}{L_1 I_o} = \alpha_F^s. \tag{3.24}$$

Otherwise the manufacturer does not enter the foreign market at all, figure 3.1.

Proof. It is straightforward to observe from (3.5) that selling to the foreign market is profitable for the manufacturer if and only if

$$(L_1 p_{F1}^s I_0 - (c_H + r)L_o - u_H)(\alpha_F - \beta_F p_{F1}^s) > 0. \tag{3.25}$$

By plugging the optimal price given in (3.8) in the left hand side we get the inequality in (3.24). □

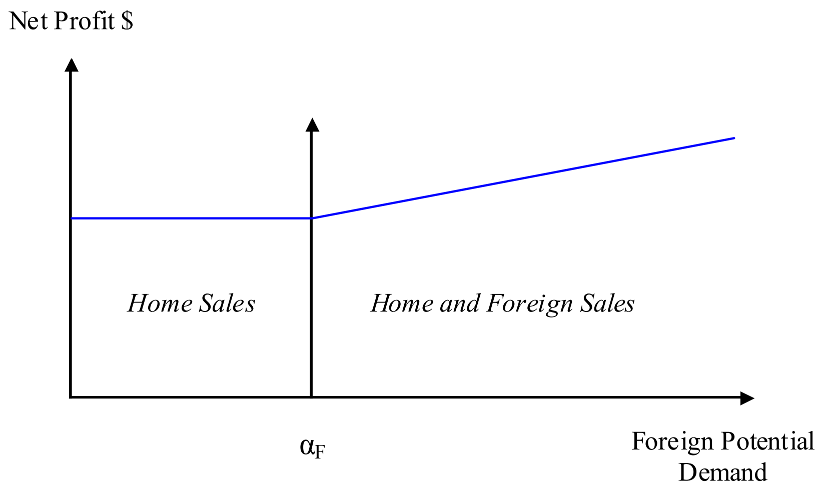


Figure 3.1 Early Commitment to Price and Quantity Capacity Allocation

3.3 Summary

The result above indicates that entering the foreign market with central sourcing is preferable for the firm if the demand potential in that market is above a certain threshold. We note that the threshold decreases in the average exchange rate drift (μ) and the initial exchange rate (I_o). As such, everything else is fixed, the foreign market is appealing to the firm if and only if the exchange rate drift is above a certain level. This is intuitive since high values for these parameters increase the ROI. On the other hand, increased costs and higher sensitivity to price in the market makes it more difficult for the firm to participate in the foreign market.

From the first order derivative we can observe that the right hand side in (3.24) is decreasing in the number of periods, T, when $\mu > \rho$, that is, when the mean drift on exchange rate is higher than the discount factor. In this case, a longer horizon makes the foreign market more appealing for the firm. It is easy to see from (3.6) that in this case as $T \rightarrow \infty$, the market potential threshold approaches to zero. In contrast, when the drift is slow or negative (i.e., $\mu < \rho$), the manufacturer will be more reluctant to enter the foreign market as the length of the selling season increases.

Chapter 4

Postponement of Prices and Quantities

4.1 Background

In this section, we consider the case where the manufacturer is able to adjust its prices and quantities sold in each market at the beginning of each period after the exchange rate for that period is observed. This flexibility will give the manufacturer the opportunity to allocate more capacity to the foreign market by decreasing its foreign price when exchange rate is rising, and less capacity to the foreign market by increasing its foreign price when exchange rate is falling. The manufacturer still has to decide on capacity before the beginning of the selling season.

Similar to the previous case all products are manufactured at home and sold in both markets. To facilitate our post-investment analysis we first suppose that capacity is ample (i.e., unlimited). We employ this assumption at the beginning to find the maximum capacity that manufacturer would utilize to satisfy demand at home and in the foreign market. We also attempt to compute the exchange rate levels at which the manufacturer exports its product to the foreign market. This implies that the home price and hence the sales volume in the domestic market stay constant across all periods, that is, $p^n_{Ht} = p^n_H$ and $Q^n_{Ht} = Q^n_H$ for all t. Under this assumption, one of the two scenarios could occur: (1) serve only home market and (2) serve both home and foreign markets. As such, the manufacturer faces two decisions: when is it optimal to serve both markets and when is it optimal to serve only the home market?

4.2 The Model

The manufacturer's profit at any given period can be written as:

$$\max_{p_H^n, p_{Ft}^n, Q_H^n, Q_{Ft}^n \geq 0} \Pi_t^{n^+} = (p_H^n - c_H)(\alpha_H - \beta_H p_H^n) + (I(t)p_{Ft}^n - c_H - r)(\alpha_F - \beta_F p_{F3}^n)$$

$$+ (I(t)p_{Ft}^n - c_H - r)(\alpha_F - \beta_F p_{Ft}^n) \quad (4.1)$$

$$s.t.\ \ Q_H^n = \alpha_H - \beta_H p_H^n \quad (4.2)$$

$$Q_{Ft}^n = \alpha_H - \beta_H p_{Ft}^n \quad (4.3)$$

A quick analysis of (4.1) will reveal that the expected profit function is jointly concave in p_H^n and p_{Ft}^n. Hence using the first order optimality conditions we get

$$p_H^{n+} = \frac{\alpha_H + c\beta_H}{2\beta_H} \quad (4.4)$$

$$p_{Ft}^{n+} = \frac{I(t)\alpha_F + \beta_F(c_H + r)}{2\beta_F I(t)} \quad (4.5)$$

Thus,

$$Q_H^{n+} = \frac{\alpha_H - c_H\beta_H}{2} \quad (4.6)$$

$$Q_{Ft}^{n+} = \frac{I(t)\alpha_F - \beta_F(c_H + r)}{2I(t)} \quad (4.7)$$

Optimal home quantity shows that the manufacturer will always sell to home market, since $\alpha_H - c_H\beta_H$ is assumed to be a non-negative value. Also, it shows that home sales are independent of exchange rate and demand potential at the foreign market because we assume ample capacity. The main question is: when is it profitable to export to foreign market?

Lemma 4.1: *The manufacturer will export to the foreign market if*

$$I(t) > \frac{\beta_F(c_H + r)}{\alpha_F} = I_z. \quad (4.8)$$

Otherwise, the manufacturer will sell only to home market.

Proof. In order for the manufacturer to export to the foreign market, Q_{Ft}^{n+} must be greater than 0, which implies from (4.7) that $I(t)\alpha_F - \beta_F(c_H + r) > 0$. This inequality is equivalent to (4.8). □

Lemma 4.1 indicates that the manufacturer exports to the foreign market if the foreign potential demand is high, the cost of production and transportation are sufficiently low, and the price sensitivity is low. This strategy translates into operational hedging; once the capacity is committed, at any period t, if the desired exchange rate is not met, i.e., $I(t) < I_z$, then the manufacturer does not export to the foreign market. Say, if in the next period the desired exchange rate is met, then the manufacturer will export to the foreign market. Suppose that the desired exchange rate is not met at a given period. Then the manufacturer profit for that period can be written as:

$$\Pi_{t,I(t)<I_z}^{n^+} = \frac{(\alpha_H - c_H\beta_H)^2}{4\beta_H} \tag{4.9}$$

Substituting optimal home price (4.4) and optimal home quantity (4.6) in (4.1), we get (4.9). We notice that the profit is constant and independent of exchange rate since no sales occur in the foreign market.

Suppose at any given period, the desired exchange rate is met, $I(t) > I_{z,}$. The manufacturer exports to the foreign market and the profit increases due to the portion of sales coming from the foreign market, Figure 4.1.

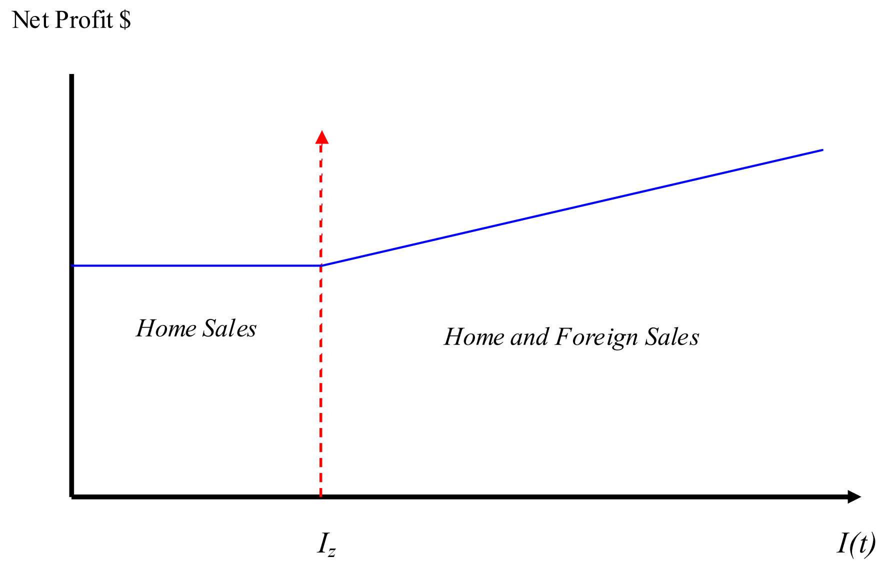


Figure 4.1 Capacity Allocation for Postponement of Prices and Quantities (Unlimited Capacity)

In this case, substituting equations (4.4-4.7) in (4.1), we get:

$$\Pi^{\mathrm{n}^+}_{\mathrm{t},I(t)>I_z} = \frac{(\alpha_H - c_H\beta_H)^2}{4\beta_H} + \frac{(I(t)\alpha_F - \beta_F(c_H + r))^2}{4\beta_F I(t)} \tag{4.10}$$

Since this model is a multiperiod model, the net profit over all periods can be written as

$$\Pi^{\mathrm{n}} = \sum_{t=1}^{T} e^{-\rho t}\left(\frac{(\alpha_H - c_H\beta_H)^2}{4\beta_H} + \int_{A(I_z)}^{\infty} \frac{\left(I_0 e^{\left(\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma\epsilon\sqrt{t}\right)}\alpha_F - \beta_F(c_H+r)\right)^2}{4\beta_F I_0 e^{\left(\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma\epsilon\sqrt{t}\right)}}\phi(\epsilon)d\epsilon\right) \tag{4.11}$$

The total capacity used in period t is the sum of the quantities sold to home and foreign markets in the same period:

$$K^{n^+}_{Ht} = \frac{\alpha_H + \alpha_F - c_H\beta_H}{2} - \frac{\beta_F(c_H + r)}{2I(t)} \tag{4.12}$$

As expected, total used capacity increases in the foreign exchange rate, while decreasing in production and transportation costs. Since the price of the product decreases in the exchange rate in the foreign market, the sales volume will increase. The sales in the domestic market will not be affected in this case by the exchange rate fluctuations.

Now, we relax the unlimited capacity assumption. This implies that the manufacturer can not always fully satisfy demand in both markets at the same period. Unlike the unlimited capacity case, now the manufacturer must decide on its capacity allocation. The condition that the manufacturer will export to the foreign market if $I(t) > I_z$ is still valid as long as the manufacturer can satisfy the home demand.

When capacity is limited, the manufacturer must decide on which market to serve first. We have a full operational hedging mechanism in that the manufacturer can react to exchange rate fluctuations by changing the selling prices and quantities in both markets.

At any given period t the manufacturer faces the following optimization problem:

$$\text{Maximize}_{Q_{Ht}^n, Q_{Ft}^n, p_{Ht}^n, p_{Ft}^n \geq 0} \Pi_t^n = (p_{Ht}^n - c_H)(\alpha_H - \beta_H p_{Ht}^n) + (I(t)p_{Ft}^n - c_H - r)(\alpha_F - \beta_F p_{Ft}^n) \tag{4.13}$$

$$s.t. \;\; Q_{it}^n \leq \alpha_i - \beta_i p_{it}^n \;\; \forall i = H, F \tag{4.14}$$

$$Q_{Ht}^n + Q_{Ft}^n \leq K_H^n \tag{4.15}$$

As in the previous case we can easily observe that the inequalities in (4.14) must be binding. Using this observation, we can employ Lagrangian relaxation and rewrite the model as follows:

$$\text{Maximize}_{p_{Ht}^n, p_{Ft}^n \geq 0} \Pi_t^n = (p_{Ht}^n - c_H)(\alpha_H - \beta_H p_{Ht}^n) + (I(t)p_{Ft}^n - c_H - r)(\alpha_F - \beta_F p_{Ft}^n)$$
$$+ \lambda(K_H^n - \alpha_H + \beta_H p_{Ht}^n - \alpha_F + \beta_F p_{Ft}^n) \tag{4.16}$$

It can be shown that the function in (4.16) is jointly concave in p_{Ht}^n and p_{Ft}^n. Hence using the first order optimality conditions we get

$$p_{Ht}^{n^*} = \frac{\alpha_H + \beta_H(c_H + \lambda(t))}{2\beta_H} \tag{4.17}$$

$$p_{Ft}^{n^*} = \frac{I(t)\alpha_F + \beta_F(c_H + r + \lambda(t))}{2I(t)\beta_F} \tag{4.18}$$

$$\text{where } \lambda(t) = \frac{I(t)(\alpha_H + \alpha_F - 2K_H^n - \beta_H c_H) - \beta_F r - \beta_F c_H}{I(t)\beta_H + \beta_F} \tag{4.19}$$

Thus,

$$Q_{Ht}^{n^*} = \frac{\alpha_H\beta_F - \beta_H(I(t)(\alpha_F - 2K_H^n) - \beta_F r)}{2(I(t)\beta_H + \beta_F)} \tag{4.20}$$

$$Q_{Ft}^{n^*} = \frac{I(t)\alpha_F\beta_H - \beta_F(\alpha_H - 2K_H^n - \beta_H r)}{2(I(t)\beta_H + \beta_F)} \tag{4.21}$$

Proposition 4.1: *At a given period, the manufacturer's capacity is fully utilized (and hence limited) if and only if*

$$K_H^n < \frac{I(t)(\alpha_H + \alpha_F - \beta_H c_H) - \beta_F r - \beta_F c_H}{2I(t)} \tag{4.22}$$

Proof. It is straightforward to observe from (4.19) that $\lambda(t)$ must be > 0 in order for the capacity to be limited (i.e., the capacity constraint is binding). This implies that (4.22) must hold. ☐

We note that the right hand side in (4.22) equals $K_{Ht}^{n^+}$ given in (40). The inequality in (4.12) implies that while the capacity utilization in a period increases with the observed exchange rate and market potentials, it decreases in price sensitivities, the production cost, and the transportation cost. This result implies that the unconstrained optimal solution given in (4.4-4.7) is obtained when

$$I(t) \le \frac{\beta_F(r + c_H)}{\alpha_H + \alpha_F - 2K_H^n - \beta_H c_H} = I_t. \tag{4.23}$$

The above lemma shows that the manufacturer can implement the unconstrained solution and hence does not need to split her capacity across markets if

$I(t) < I_t$. In contrast, when the inequality in (4.23) does not hold the capacity is limited. As such, the manufacturer faces the problem of allocating the capacity between the two markets. Depending on the market potentials and the realized exchange rates the manufacturer may also choose to sell only in one of the markets. The next proposition indicates that if the exchange rate is significantly high, the manufacturer in fact may opt out of the domestic market.

Proposition 4.2. *The manufacturer chooses to sell in the domestic market in a given period if and only if*

$$I(t) < \frac{\beta_F(\alpha_H + \beta_H r)}{\beta_H(\alpha_F - 2K_H^n)} = I_3 \tag{4.24}$$

otherwise, the manufacturer sells only in the foreign market. On the other hand, she will export her product to the foreign market at any period if and only if

$$I(t) > \frac{\beta_F(\alpha_H - 2K_H^n + \beta_H r)}{\beta_H \alpha_F} = I_4 \tag{4.25}$$

Hence, the manufacturer splits the capacity between the domestic and foreign markets only when $I_4 < I(t) < I_3$.

Proof. The inequality in (4.24) directly follows from the fact that the manufacturer will sell to the domestic market if $Q_{Ht}^n > 0$ implying that the numerator in (4.20) must be non-negative. This is satisfied when (4.24) holds. Similarly, $Q_{Ft}^n > 0$ implies that (4.25) holds. Next we show that $I_4 < I_3$. First observe that $I_4 < I_3$ implies that

$$\alpha_F(\alpha_H + \beta_H r) > (\alpha_F - 2K_H^n)(\alpha_H - 2K_H^n - \beta_H r),$$

which can be rewritten as

$$\alpha_F \beta_H r > -K_{H3}^n(\alpha_F + \alpha_H - 2K_H^n - \beta_H r).$$

Since $\alpha_F + \alpha_H - 2K_{H3}^s - \beta_H r > 0$, this inequality must hold. □

We notice from (4.24) that the manufacturer will always sell to the domestic market if $K_H^n \geq \frac{\alpha_F}{2}$. Clearly, when the capacity is ample it is always profitable to sell in the domestic market. However when capacity is scarce, as the foreign exchange rate (or equivalently the potential demand in the foreign market) increases, it becomes less appealing to allocate capacity to the domestic market. Similarly, we observe from (4.25) that the manufacturer exports to the foreign market at any given time if $K_H^n \geq \frac{\alpha_H+\beta_H r}{2}$. Therefore, low transportation cost and home potential demand makes the foreign market appealing to the manufacturer. Proposition 4.2 implies that the optimal policy for the manufacturer at any given period is a "cherry-picking" policy when the capacity is scarce. Namely, when the exchange rate is significantly high the manufacturer sells only in the foreign market whereas when it is too low she will serve exclusively the domestic market. For exchange rates not too high or low, the capacity is split between the markets. The optimal splitting policy is investigated next.

4.3 Optimal Capacity Allocation

Based on Proposition 4.2 we conclude that in any period the optimal pricing and allocation policy falls in one of the following five scenarios:

HU: The capacity is ample (unconstrained) and the firm sells only in the domestic market.

SU: The capacity is ample (unconstrained) and the firm sells in both markets.

HC: The capacity is limited and the firm allocates all of its capacity to the domestic market.

FC: The capacity is limited and the firm allocates all of its capacity to the foreign market.

SC: The capacity is limited and the firm splits all of its capacity between the two markets.

First we note that when capacity is ample, i.e., $K_H^n \geq K_{Ht}^{n^+}$, there is always incentive for the firm to sell in the domestic market. From Lemma 4.1, the firm sells in the foreign market in any period if the exchange rate is above the threshold given in (4.8). In case of scarce capacity any one of the three scenarios (home only, foreign only or split capacity) may occur depending on the exchange rate as shown in Proposition 4.2.

Suppose that capacity is too scarce (high-cost case), that is, lower than Q_H^{s+} given in (4.6). This situation is more likely when the cost of capacity is too high that the firm cannot fully satisfy the demand in the domestic market alone. In this case, the capacity is fully utilized. As depicted in Figure 4.2, the firm allocates all of its capacity to the domestic market (HC) if the exchange rate is below I_4. In this case the low exchange rate does not justify exporting to the foreign market. The firm chooses to split the capacity between two markets (SC) if the exchange rate falls between I_3 and I_4. The optimal allocation is then given by (4.20) and (4.21). If the exchange rate is too high (above I_3) the revenue from sales in the foreign market justifies the full allocation of the capacity to this market (FC).

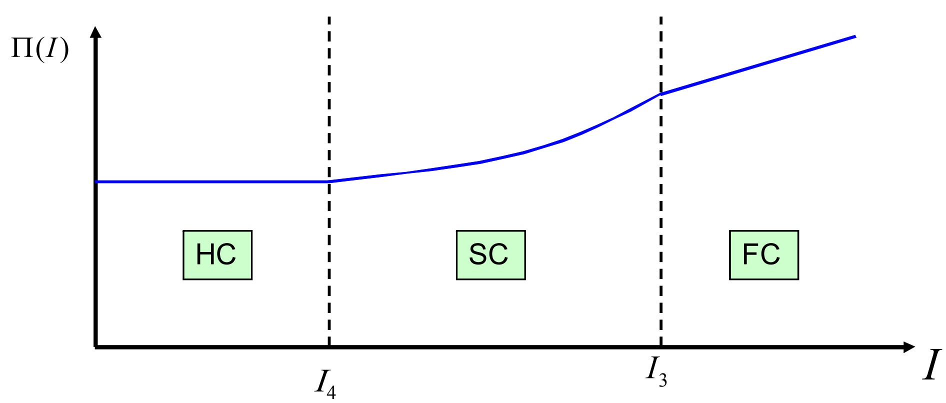


Figure 4.2 Scarce Capacity Allocation

Under this scenarios *exante* expected profit function before the start of the selling season can be written as follows:

$$\underset{I(t)>I_z, K_H^{n-}<Q_H^{n+}}{\text{Maximize}} \quad \Pi^{n^-} = -u_H K_H^{n-}$$
$$+\sum_{t=1}^{T} e^{-\rho t}\left(\int_{-\infty}^{A(I_4)} \Pi_{HC}\phi(\epsilon)d\epsilon + \int_{A(I_4)}^{A(I_3)} \Pi_{SC}\phi(\epsilon)d\epsilon + \int_{A(I_3)}^{\infty} \Pi_{FC}\phi(\epsilon)d\epsilon\right) \qquad (4.26)$$

where from (2.3)

$$A(I) = \epsilon(t) = \frac{ln(I/I_0) - (\mu - \frac{1}{2}\sigma^2)t}{\sigma t}$$

$$\Pi_{HC} = \frac{\alpha_H - K_H^n - \beta_H c_H}{\beta_H} K_H^n,$$

$$\Pi_{SC} = (p_{Ht}^{n^*} - c_H)Q_{H3t}^{n^*} + (I(t)p_{Ft}^{n^*} - c_H - r)Q_{Ft}^{n^*} \text{and}$$

$$\Pi_{FC} = \frac{I(t)*(\alpha_H - K_H^n) - \beta_F(c_H + r)}{\beta_F} K_H^n$$

To compute the optimal capacity we look at the first order optimality condition:

$$\frac{d\Pi^{n^-}}{dK_H^{n-}} = -u_H + \sum_{t=1}^{T} e^{-\rho t}\left(\int_{-\infty}^{A(I_4)} \frac{\alpha_H - 2K_H^{n-} - \beta_H c_H}{\beta_H}\phi(\epsilon)d\epsilon + \int_{A(I_4)}^{A(I_3)} \lambda(t)\phi(\epsilon)d\epsilon\right) + \sum_{t=1}^{T} e^{-\rho t}\left(\int_{A(I_3)}^{\infty} \frac{I(t)(\alpha_F - 2K_H^{n-}) - \beta_F(c_H + r)}{\beta_F}\phi(\epsilon)d\epsilon\right) = 0 \qquad (4.27)$$

where λ(t) is the Lagrangian multiplier defined in (4.19). It is straightforward to see from the function above that the second derivative with respect to K_H^{n-} is strictly negative for any $K_H^{n-} \geq 0$ implying concavity. Hence solution to (4.27) gives the unique optimal value for the capacity to be built at home in advance of the selling season. Unfortunately, there is no close form solution. However, the optimal capacity can be calculated easily with a simple line search.

Recall that the condition given in (4.27) applies when the capacity scarce. This is expected to occur when capacity is very expensive. Suppose that capacity is bigger than home demand and less than the total unconstrained demand. Then, the allocation decision follows Figure 4.3:

(1) if the spot exchange rate is below I_z, then the manufacturer satisfies all the demand in the domestic market and stays out of the foreign market (Lemma 4.1).

(2) if the spot exchange rate falls between I_z and I_t, then the manufacturer satisfies all demand in both markets (Proposition 4.1).

(3) if the spot exchange rate falls between I_t and I_3, then the manufacturer splits its limited capacity between the two markets (Proposition 4.2).

(4) if the spot exchange rate is higher than I_3, then the manufacturer allocates all the capacity to the foreign market (Proposition 4.2).

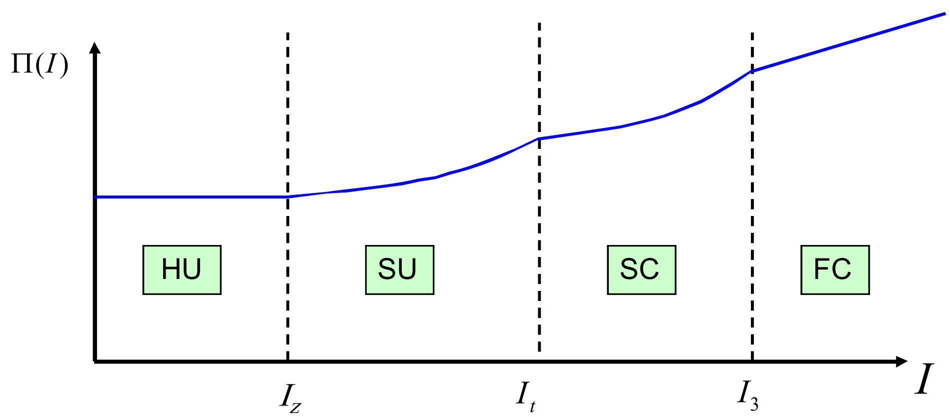


Figure 4.3. Allocation of medium capacity

Consequently, the manufacturer's net profit in this case can be written as:

$$\underset{I(t)>I_z, Q_F^{n+}+Q_H^{n+}>K_H^n>Q_H^{n+}}{\text{Maximize}} \Pi^n = -u_H K_H^n + \sum_{t=1}^{T} e^{-\rho t}\left(\int_{-\infty}^{A(I_z)} \Pi_{HU}\phi(\epsilon)d\epsilon\right) +$$

$$\sum_{t=1}^{T} e^{-\rho t}\left(\int_{A(I_z)}^{A(I_t)} \Pi_{SU}\phi(\epsilon)d\epsilon + \int_{A(I_t)}^{A(I_3)} \Pi_{SC}\phi(\epsilon)d\epsilon + \int_{A(I_3)}^{\infty} \Pi_{FC}\phi(\epsilon)d\epsilon\right) \quad (4.28)$$

In order to find the optimal capacity, once again we look at the first order optimality condition based on (56):

$$\frac{d\Pi^n}{dK_H^n} = -u_H + \sum_{t=1}^{T} e^{-\rho t}\left(\int_{A(I_t)}^{A(I_3)} \frac{I(t)(\alpha_H + \alpha_F - 2K_{H3}^s)}{I(t)\beta_H + \beta_F}\phi(\epsilon)d\epsilon\right) +$$

$$\sum_{t=1}^{T} e^{-\rho t}\left(\int_{A(I_3)}^{\infty} \frac{I(t)(\alpha_F - 2K_{H3}^s) - \beta_F(c_H + r)}{\beta_F}\phi(\epsilon)d\epsilon\right) = 0 \quad (4.29)$$

Although we cannot find closed form solutions both for (5.27) and (5.29) we can derive the following conclusions from concavity

Proposition 4.3. *There exits a capacity cost threshold u_H^* such that if $u_H \geq u_H^*$ the optimal capacity is obtained from (5.27). Otherwise, solution to (5.29) gives the optimal capacity for the firm.*

Proof. First we observe that when the cost of capacity is too high optimal, $K_H^n = Q_H^{n^+}$ in (4.28), implying that the optimal capacity investment strategy is determined by (4.27) since the medium cost case is but a special case of the other. Similarly when the cost is too low $K_H^{n-} = Q_H^{n^+}$ in (4.26) implying that the optimal capacity investment strategy is determined by (4.29) since the high-cost case is a special case of the medium-cost case. In between we can observe that the first derivative given in the left hand side of (4.29) is greater than that of (4.27). This implies that in this region $K_H^{n-} < K_H^n$ at optimality. From the Envelop Theorem, the derivative of the difference between the optimal expected profits with respect to the cost of capacity ($i.e.$, u_H) is $K_H^{n-} - K_H^n$, which is negative. As such, the difference between profits in the medium-cost case and the high-cost case is strictly increasing in this region. Consequently we can conclude that there exists a capacity cost value that falls in this region beyond which the optimal capacity is computed by (4.27) and below which by (4.29). □

4.4 Summary

Figure 4.4 shows all allocation scenarios with regards to the capacity cost and the exchange rate. When capacity is inexpensive and the foreign currency is weak (*i.e.*, $I > I_z > I_t$), the firm stays in the home market and fully satisfies the demand. For $I_z > I > I_t$, both market demands are filled. In both cases the capacity is underutilized. When the capacity is expensive and foreign currency is weak, the firm sells only to the home market and fully utilizes the capacity. As exchange rate increases, the firm splits its capacity between the two markets. When the foreign currency is too strong, the firm dedicates all of its capacity to the foreign market and leaves the home market completely. When capacity cost is expensive, depending on the exchange rate, the firm allocates all of its capacity to the home market ($I < I_4$) or to the foreign market ($I > I_3$), or split it between both markets ($I_3 > I > I_4$). As the capacity cost increases the allocation decision becomes "cherry picking" problem where the scarce capacity is dedicated to one of the markets. Since capacity will be too small, splitting capacity does not make economical sense.

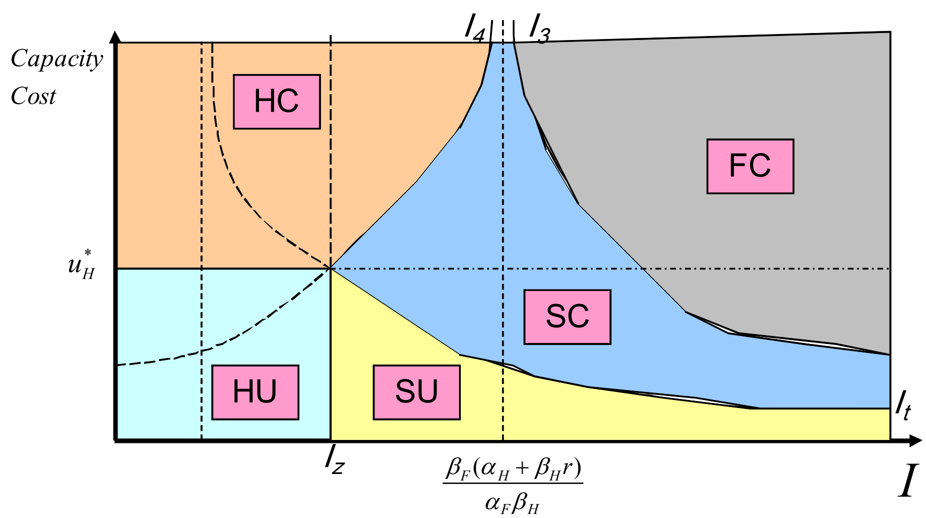


Figure 4.4 Limited Capacity Allocations

Our numerical analysis indicates that the optimal capacity increases in number of periods regardless of the drift on the exchange rate. Clearly as the number of periods (*i.e.*, the selling season) increases the return on investment in capacity increases. As such, the firm can invest for more capacity. As expected, it is also observed that the optimal capacity increases in exchange rate drift. A higher exchange rate drift diminishes the risk in capacity investment for the firm. As shown in Figure 4.5, the increase in optimal capacity investment is stronger when the drift is large. Figure 4.6 shows the impact of the consumer's price sensitivity on optimal capacity. We notice that optimal capacity is higher when both home and foreign consumer price sensitivity are low, and it is lower when they are high.

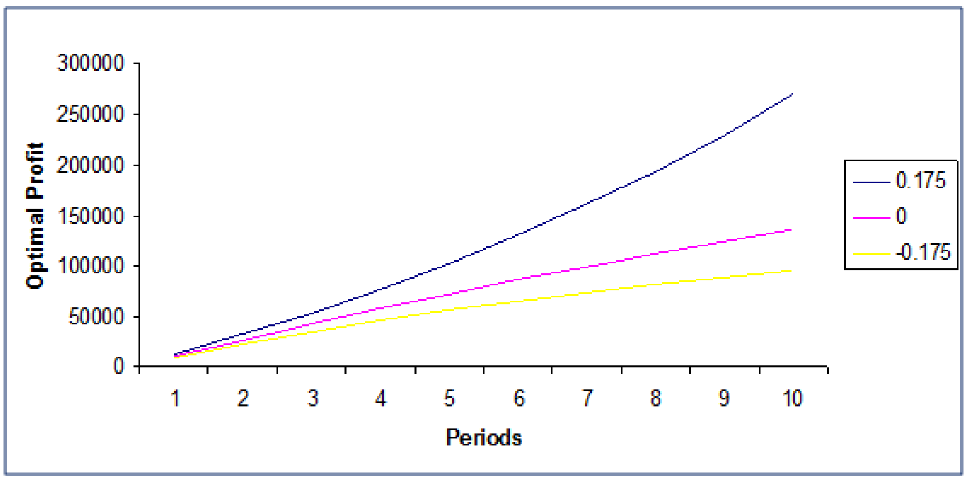


Figure 4.5 Optimal Profit Vs. Drift

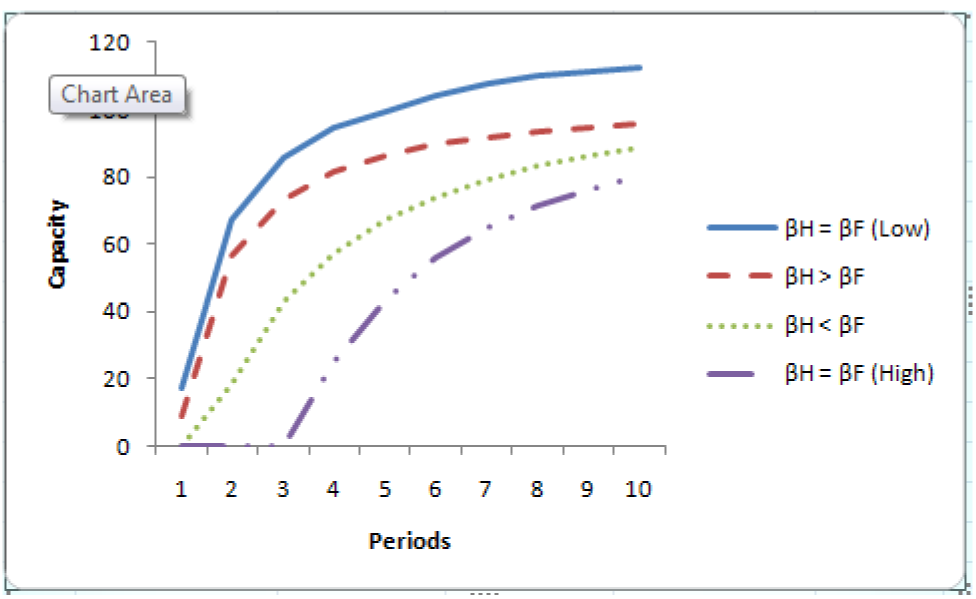


Figure 4.6 Effect of Consumer Price Sensitivity on Optimal Capacity

Chapter 5

Capacity Investment in the Foreign Market *(LS)*

5.1 Overview

We now consider the case where the manufacturer invests for capacity in both markets to serve them locally. We refer to this case as the *Local Sourcing Model* (LS). The unit cost for capacity built in the foreign market is $u_{FH} + I_o u_{FF}$. In this case, the capacity built in a market is exclusively allocated to that market. The first part of the cost is incurred at home that may include transfer of technology and equipment and recruitment of qualified workforce at home. The latter part represents the necessary expenditures made in the local market that may include facility and local workforce expansion. The production cost in the foreign market is c_F and expressed in the foreign currency. We first note that since both markets are served locally early commitment and the postponement models discussed earlier are identical in this case. That is, once the capacities are determined the price and quantity decisions will not change throughout the planning period since these decision are made locally.

5.2 The Model

The firm's decision model is as follows:

$$\begin{aligned}\underset{p^l_{H1},p^l_{F1}\geq 0}{\text{Maximize}} \quad \Pi^l =& -u_H(\alpha_H - \beta_H p^l_H) - (u_{FH} + I_o u_{FF})(\alpha_F - \beta_F p^l_F) \\ &+ \sum_{t=1}^{T} e^{-\rho t}\left(p^l_H - c_H\right)(\alpha_H - \beta_H p^l_H) \\ &+ \left(\int_{-\infty}^{\infty}\sum_{t=1}^{T} e^{-\rho t} I_0 e^{\left((\mu - \frac{1}{2}\sigma^2)t + \sigma\epsilon\sqrt{t}\right)}(p^l_F - c_F)(\alpha_F - \beta_F p^l_F)\phi(\epsilon)d\epsilon\right) \quad (5.1)\end{aligned}$$

which reduces to

$$\underset{p_{H1}^{l},p_{F1}^{l}\geq 0}{\text{Maximize}}\ \Pi^{l} = -u_H(\alpha_H - \beta_H p_H^l) - (u_{FH} + I_o u_{FF})(\alpha_F - \beta_F p_F^l)$$

$$+ L_o\big(p_H^l - c_H\big)(\alpha_H - \beta_H p_H^l) + L_1 I_o(p_F^l - c_F)(\alpha_F - \beta_F p_F^l) \qquad (5.2)$$

A straightforward comparison between profit functions in (3.5) and (5.2) reveals that $p_H^{l*} = p_H^{e*}$ and $K_H^{l*} = Q_H^{l*} = Q_H^{e*}$. We compute the optimal foreign market price and capacity from the first order optimality conditions as follows:

$$p_F^{l*} = \frac{\alpha_F + \beta_F c_F}{2\beta_F} + \frac{u_{FH} + I_o u_{FF}}{2 I_o L_1} \qquad (5.3)$$

Thus,

$$K_F^{l*} = Q_F^{l*} = \frac{\alpha_F - \beta_F c_F}{2} - \frac{\beta_F(u_{FH} + I_o u_{FF})}{2 I_o L_1} \qquad (5.4)$$

Hence similar to the previous case we observe the following:

Lemma 5.1: *The optimal price for the foreign market decreases in the drift on the exchange rate (i.e., μ) in the LS model. Moreover, both optimal market prices are decreasing in T.*

The proof is similar to Lemma 3.1 and from the first derivatives. For infinite horizon the prices and sales converge to what follows:

$$\underset{\mu>\rho,T\to\infty}{p_F^{l*}} = \frac{\alpha_F + \beta_F c_F}{2\beta_F} \qquad (5.5)$$

$$\underset{\mu<\rho,T\to\infty}{p_F^{l*}} = \frac{\alpha_F + \beta_F c_F}{2\beta_F} + \frac{(u_{FH} + I_o u_{FF})(e^{\rho} - e^{\mu})}{2 I_o e^{\mu}} \qquad (5.6)$$

$$\underset{\mu>\rho,T\to\infty}{Q_F^{s*}} = \frac{\alpha_F - \beta_F c_F}{2} \qquad (6.7)$$

$$\underset{\mu<\rho,T\to\infty}{Q_F^{*}} = \frac{\alpha_F\text{-}\beta_F c_F}{2} - \frac{\beta_F(u_{FH} + I_o u_{FF})(e^{\rho} - e^{\mu})}{2 I_o e^{\mu}} \qquad (5.8)$$

We observe from (3.17) and (5.5) that when $\mu > \rho$ long-run foreign market price is always higher in this case compared to early commitment with central sourcing. To see the intuition first observe that the production cost is incurred in the foreign country in the latter case. When the currency of the foreign country has a high average drift, the effective variable cost for the manufacturer increases significantly in terms of home currency, driving the manufacturer towards setting a higher price in the foreign market. Eventually, when $T \to \infty$ and the foreign currency is stronger than the home currency, the manufacturer can sell the product cheaper in the foreign market by manufacturing it at home and exporting. Thus, the sales volume will be higher under the export option. This is interestingly the case regardless of the capacity and labor costs. Whereas, for $\mu < \rho$, the comparison depends on the capacity and the labor costs incurred in the home and foreign markets.

Similar to the previous case, participation of the manufacturer in the foreign market depends on the demand base, price sensitivity, and the average exchange drift:

Lemma 5.2: *The manufacturer has incentive to build capacity in the foreign market if and only if*

$$\alpha_F > \beta_F c_F + \frac{\beta_F(u_{FH} + I_o u_{FF})}{L_1 I_o} = \alpha_F^l. \tag{5.9}$$

The proof is similar to that of Lemma 2. Likewise, the average exchange rate drift, price sensitivity, and cost factors have impact on the manufacturer's decision about entering the foreign market. Inequality given in (5.9) specifies the conditions for the manufacturer's market entry. Observe that the right hand side of the equation is decreasing in T for any values of μ implying that investing for capacity is more appealing as the product life cycle gets longer regardless of the drift. At this point, the interesting question is "how will the manufacturer allocate its capacity investment should it enter the foreign market?" In what follows we investigate the answer to this question.

5.3 Comparison between Capacity Expansion Strategies

As a risk neutral decision maker, the manufacturer will choose between exporting its products and production based on the expected profits over the planning horizon. First consider the early commitment case. Since all decisions are to be made before the beginning of the selling season, the comparison of the profits under two cases is easy to get from the cost margins.

Proposition 5.1. *Producing at home and exporting to the foreign market generates higher expected profits for the manufacturer if and only if*

$$\boldsymbol{G} = L_1 I_o c_F - L_o(c_H + r) - (u_H - u_{FH} - I_o u_{FF}) > 0 \qquad (5.10)$$

The proof is straightforward from the comparison of the margins in the profit functions given by (3.5) and (5.2). This observation shows that the manufacturer's choice is a direct result of the net present value of the effective marginal cost of goods sold. Higher capacity and/or production costs in one market creates incentives for the manufacturer to divert the production to the other market.

We note that the exchange rate indeed has an impact on this decision. Specifically, since L_1 is increasing in μ, we can conclude when the exchange rate is expected to get stronger against the home currency over time, producing at home and exporting becomes more appealing for the manufacturer. The intuition is that with sufficiently strong foreign currency the variable production cost becomes substantial for the manufacturer and it becomes difficult to economically justify production in the foreign country. The impact of the length of the selling season is reflected in L_1 and L_o and depends on the tradeoff between the future direction of the exchange rate drift and the investment costs. To investigate the role of the season length (T) we first make the following observation:

Lemma 5.3 *The difference in profits,* $\boldsymbol{G}$, *given in (5.10) is unimodular in T in* $(-\infty, \infty)$. *Moreover, it has at most one maximizer if* $\mu < 0$ *and at most one minimizer if* $\mu > 0$.

Proof. To make the proof first we take the first derivative of $\boldsymbol{G}$ with respect to T:

$$G' = L_1' I_o c_F - L_o'(c_H + r).$$

Note that at a stationary point $L_1' I_o c_F = L_o'(c_H + r)$ must hold. Second we compute the second order derivative as follows:

$$G' = (\mu - \rho) L_1' I_o c_F + \rho L_o'(c_H + r)$$

From the above observation it is straightforward to observe that at a stationary point the second derivative is $\mu L_1' I_o c_F$. Since $L_1' > 0$, for $\mu < 0$ $(\mu > 0)$ all stationary points must be maximizers (minimizers) implying unimodularity. □

This result is quite useful to capture the interplay between the optimal investment strategy and the planning horizon. The other factor that determines the long-run investment strategy is the gap between the capacity investments that must be finalized before the beginning of the selling season. The difference in investment amounts is captured by $u_H - u_{FH} - I_o u_{FF}$. Let $\boldsymbol{H}$ denote this difference. As such a positive value for $\boldsymbol{H}$ indicates that it is more costly for the firm to invest for capacity at home. Reverse is true for any negative value. Based on these observations we can summarize the optimal investment strategies as a function of the length of the planning horizon as shown in Table 5.1.

Table 5.1 Central v.s. local capacity investment preferences as a function of exchange rate drift and investment costs

Drift Condition	Cost Condition	Scenarios	Initial Exchange Rate Condition ($I_o C_F$)
$\mu < 0$	$H > 0$	Always LS	Small
		LS → SS	Large
		LS → SS → LS	Medium
	$H < 0$	Always SS	Large
		SS → LS	Small to Medium
$0 < \mu < \rho$	$H > 0$	LS → SS	Small
		Always LS	Large
	$H < 0$	Always SS	Large
		SS → LS → SS	Medium
		SS → LS	Small
$\mu > \rho$	$H > 0$	LS → SS	small
	$H < 0$	Always SS	Large

When the drift on the exchange rate is negative the firm expects a weaker currency for the foreign market. In general, weak foreign currency makes local sourcing strategy (LS) appealing due to decreasing production costs. When the initial exchange rate is sufficiently small local investment is a preferred strategy regardless of the planning horizon. Interestingly for higher initial exchange rates, depending on the magnitude of the drift, central sourcing strategy (CS) becomes preferable as the planning horizon increases. Under this case, the production cost in the foreign market is high. Nevertheless, for short planning horizon LS may still be preferable due to high cost of investment at home. However as the planning period lengthens the high cost of production in the foreign market weighs in and as such, investment at home becomes more profitable for the firm. For longer horizon the decreasing value of the foreign currency may lead back to LS policy. Even when capacity in the foreign market costs more, while the firm is better off with central investment for relatively short horizon, it prefers the local investment strategy as the horizon increases unless the initial exchange rate is high.

When the drift on the exchange rate is positive but below the discount rate, central capacity investment becomes more appealing for the firm due to increasing production cost in the foreign market. This is especially the case when the capacity investment is cheaper in the home country. However, when cost of capacity is high at home, local capacity investment is preferable for short planning horizon. When the drift is above the discount rate, central investment is more profitable in general except when the home capacity is expensive. In the latter case, the central investment still becomes appealing as the planning horizon grows.

It is straightforward to observe from (5.9) that building capacity in the foreign market (LS) becomes more appealing if the initial exchange rate is low and the drift is negative On the other hand, staying with home capacity (CS) is more profitable, when the initial exchange rate is high and the drift is positive, Figure 5.1. The impact of u_{FF} is similar to the impact of initial exchange rate and it is illustrated in Figure 5.2.

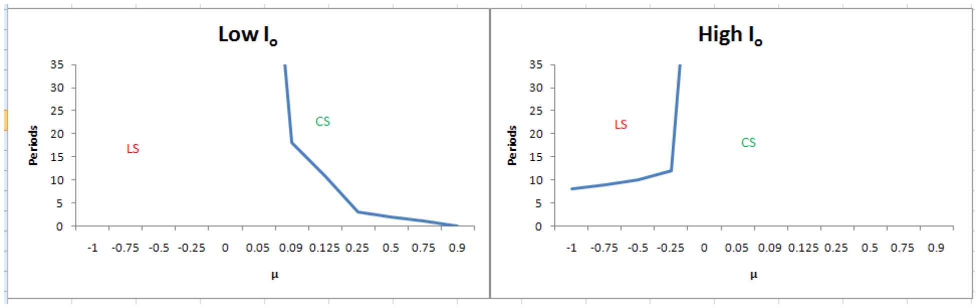


Figure 5.1 Impact of Initial Exchange Rate on Capacity Investment Strategy

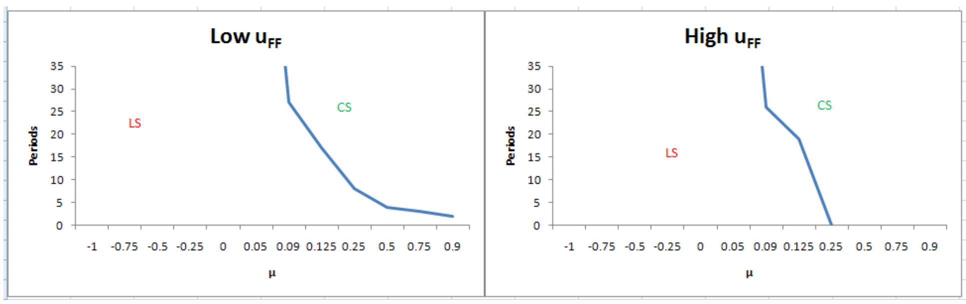


Figure 5.2 Impact of Foreign Capacity Cost on Capacity Investment Strategy

Clearly, the postponement strategy discussed in Chapter 4 yields higher profits for the firm since the price and quantity decisions are *ex-post*. As such the centralized capacity investment generates relatively more profits for the firm under this scenario. Otherwise, the general structure of the strategy shift between capacity location structures is similar to the comparison discussed above. The main difference is that the central sourcing remains to be a preferable strategy for longer time horizon. The advantage of this strategy is more significant under higher exchange rate volatility (variance) as the expected profit for the firms increases in σ under the postponement case as shown in Figure 5.3.

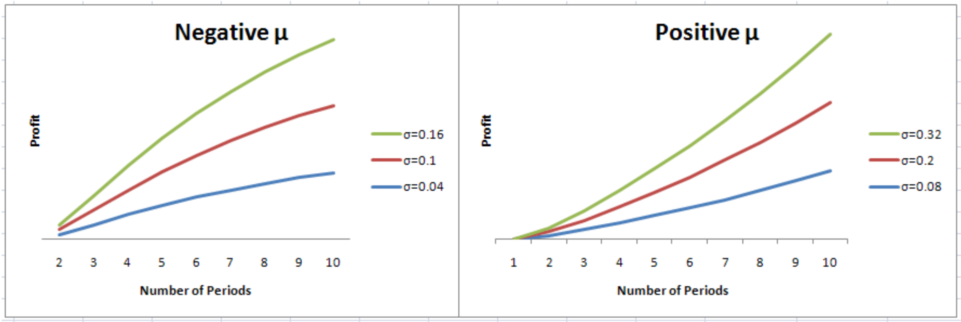


Figure 5.3 Impact of the standard deviation in exchange rate drift on profits

Part II Duopoly

Chapter 6

Competition

6.1 Introductory Remarks

In Part I, we discussed the importance of exchange rate fluctuation in determining whether the firm is better off entering the foreign market or not and how the firm utilizes its capacity between the two markets. In Part II, we give a comprehensive analysis of a duopoly competition between two manufacturers who sell substitutable product under exchange rate uncertainty and market competition.

In this part, we consider an international firm that competes with a foreign manufacturer in a foreign market while also selling its product in her home country as a monopolist. We study two scenarios under a single period setting. In the first scenario, we assume that the price of the product sold by the foreign manufacture in the foreign market is given and known (*Exogenous Model*). In the second scenario, the price of the foreign manufacturer is modeled as a decision variable (*Endogenous Model*). Under each scenario, there are two cases concerning capacity constraint for the international firm: (1) the capacity constraint is inactive or non binding, meaning that capacity is ample, and (2) the capacity constraint is active or binding, meaning that capacity is scarce.

In the *Exogenous Model*, the international firm will build its capacity and then observed the spot exchange rate and the foreign manufacturer's price. Then, the firm decides on its capacity allocation for both home and foreign markets. In the *Endogenous Model*, the capacity decision depends on exchange rate and competition parameters; however, both prices are affected by each other. The price decisions for the foreign market are decided simultaneously by both firms. As such, we assume a Nash Game setting. In the first capacity scenario, the capacity is assumed to be ample enough for any possible production amount. In this scenario, once the exchange rate between countries is observed both firms move simultaneously and set their prices for all markets that they serve. Specifically, the international firm determines the market prices for both home and foreign markets. At the same time, the foreign manufacturer sets its price in its own market. Under the second capacity scenario, the international firm has to decide how much capacity to build in the home country before the exchange rate is observed. Once capacity is built (and the exchange rate is observed) the prices are determined simultaneously as in the former scenario. It is assumed that the foreign manufacturer in the foreign country has always ample capacity.

Competition has a major impact on pricing. Bitran and Caldentey (2003) claim that selecting a certain pricing policy depends mainly on the price competition among players. Chan et al. (2004) claim that product price and service are the major factors that firms compete on and that should be taken into account by firms when setting the pricing policies. The work done in the area of competition is ample, however most of the reported studies are trying to drive the optimal prices and quantities under demand uncertainty for different demand functions. Some examples include: Bernstein and Federgruen (2004), Mieghem and Dada (1999), and Anupindi and Jiang (2008).

Bernstein and Federgruen (2004) address the problem of two echelon system (single supplier and competing retailers) under demand uncertainty. They derive the Nash equilibrium in terms of price and base-stock level for each retailer. Also, they show the impact of different parameters such as cost, and distribution parameters on optimal price and optimal base-stock level of a retailer. Mieghem and Dada (1999) show how price postponement strategy can generate more profit to the firm under competition and demand uncertainty. Anupindi and Jiang (2008) show that flexible firms (price and quantity postponement) can make more profit than inflexible firms (price postponement) under demand uncertainty (multiplicative and additive demand). However, Bernstein and Federgruen (2004), Mieghem and Dada (1999), and Anupindi and Jiang (2008) did not consider the impact of exchange rate uncertainty on capacity investment and price.

In this chapter, we are presenting a new model where we integrate competition between two manufacturers under exchange rate uncertainty. We will drive the optimal prices and quantities for both manufactures and show the impact of exchange rate under price competition.

6.2 Basic Settings, Assumptions, and Nomenclature

We consider a multinational firm that produces a single product at home and sells it to domestic and foreign markets. At the foreign market, we consider a foreign manufacturer that manufacturers the same product and sells it to the foreign market only. Therefore, the former firm is competing with the foreign manufacturer on market share at the foreign market only. We consider a monopolistic setting at domestic market, and a duopoly setting at the foreign market as illustrated in Figure 6.1.

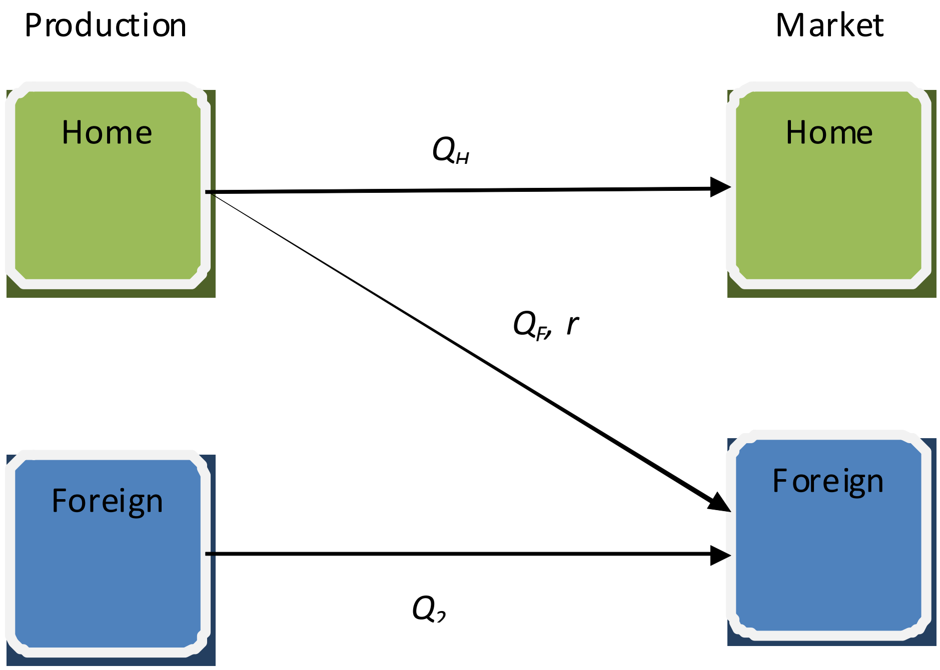


Figure 6.1 The Proposed Model

We consider a single period where the international firm must decide on its initial capacity investment before the selling season, and selling price at each market, and the production allocation across the selling season after the realization of the exchange rate. In this model, the capacity investment is irreversible and must be made in the home country only. The domestic and the foreign markets differ in their demand base and disparity in currencies. In each market, demand is assumed to be linearly decreasing in price which is expressed in local currency. The demand base is steady. In this part, we assume a deterministic demand function of prices and can be "interpreted as expected demand in many applications" (Gallego and Hu 2006). We model the exchange rate as a random variable. It is assumed that the firm does not carry any inventory. The product can be produced and shipped to both markets at the beginning of the selling season. We use the following nomenclator in our analysis:

<u>Parameters</u>

i: manufacturer index where i = l for the international firm, and i = g for the foreign manufacturer

j: market index where $j = h$ for the home market, and $j = f$ for the foreign market

k: scenario index where $k = 1$ for *Exogenous Model*, and $k = 2$ for *Endogenous Model*

n: capacity index where $n = +$ for ample or unlimited capacity, and $n = -$ for limited capacity

I_o: The initial foreign exchange rate expressed in home-currency per unit foreign-currency.

I: observed exchange rate between the currency of home country and foreign country

c_i: unit manufacturing cost at country i expressed in country i currency

r: unit shipment cost to foreign market expressed in home currency

u_l: unit capacity cost at home in home currency

<u>Decision Variables</u>

K_{lk}: firm's capacity under scenario k

p_{ijk}^n: unit selling price set by manufacturer i for market j

Q_{ijk}^n: quantity sold by manufacturer i market j

We assume that demand function is differentiable and satisfies the following conditions:

$$\frac{\partial D_{lf}}{\partial P_{lf}}, \frac{\partial D_g}{\partial P_g} \leq 0 \qquad (C1)$$

$$\frac{\partial D_{lf}}{\partial P_g}, \frac{\partial D_g}{\partial P_{lf}} \geq 0 \qquad (C2)$$

$(C1)$ and $(C2)$ are common conditions in the oligopoly pricing literature (see Bernstein and Federgruen (2005) and Gallego and Hu (2006)). $(C1)$ shows that each manufacturer's demand decrease in its price. $(C2)$ indicates that each manufacturer's demand increase in the other manufacturer price.

We are considering the additive demand function mentioned in Varian (1992), which satisfies $(C1)$ and $(C2)$.

$$D_{ij} = \alpha_{ij} - \beta_{ij} P_{ijk} + \theta P_{\text{-}ijk} \tag{6.1}$$

Where D_{ij} denotes the demand for manufacturer i in market j while α_{ij} and β_{ij} represent the demand potential and price coefficient for manufacturer i in market j respectively. θ is the "cross-price effect" which is symmetric for "well-behaved consumer demand function". θ measures the degree of competition between the two manufacturers (Vives (1999) and Varian (1992)). In this study we are focusing on noncomplementary product (substitutable product). Positive θ (e.g., an increase in the foreign manufacturer price increases the demand of the firm) means the products of both manufacturer are substitutable and if θ is a negative value that means that the two products are complements (Baumann et al. 1998). Therefore, equation (6.1) can be formulated as follows:

$$D_{lh} = \alpha_{lh} - \beta_{lh} P_{lh}$$

$$D_{lf} = \alpha_{lf} - \beta_{lf} P_{lf} + \theta P_g$$

$$D_g = \alpha_g - \beta_g P_g + \theta P_{lf}$$

To be consistent with the first part of the study, the exchange rate is modeled by a Weiner Process, $B(t)$, where $B(t) = \epsilon\sqrt{t}$. Here, ϵ is the random error term that follows the Standard Normal distribution. As such, it is assumed that the exchange rate follows a Geometric Brownian motion:

$$dI(t) = \mu I(t)dt + \sigma I(t)dB(t) \tag{6.2}$$

where $I(t)$ is driven by the Ito process. The parameters μ and σ are the mean and the standard deviation of the Normal exchange rate drift. The solution to equation (6.2) is given by Davis (2001) as $I(t) = I_0 e^{((\mu-\frac{1}{2}\sigma^2)t+\sigma B(t))}$. Assuming $B(t) = \epsilon\sqrt{t}$ and replacing in (6.2), we get

$$I(t) = I_0 e^{\left((\mu-\frac{1}{2}\sigma^2)t+\sigma\epsilon\sqrt{t}\right)} \tag{6.3}$$

6.3 The Exogenous Model

In this section, we assume that the firm knows the price of the foreign manufacturer before it set its local and foreign prices. In order to facilitate our post-investment analysis we first suppose that capacity is ample (i.e., unlimited). We employ this assumption at the beginning to find the maximum capacity that firm would utilize to satisfy demand at home and in the foreign market. We also attempt to compute the exchange rate levels at which the firm exports its product to the foreign market. This implies that the home price and hence the sales volume in the domestic market are not affected by exchange rate fluctuations or the price of the foreign manufacturer. Under this assumption, one of the two scenarios could occur: (1) serve only home market and (2) serve both home and foreign markets. Later, we assume that capacity is limited, where exchange rate and price of the foreign manufacturer can affect the local price and sales at both markets.

6.3.1 The Exogenous Model with Ample Capacity

In this section, we assume that the firm has ample capacity where capacity cost is negligible and the firm can produce up to demand in both markets. Consequently, the firm's profit can be written as:

$$\underset{p_{lh1},p_{lf1},Q_{lh1},Q_{lf1}\geq 0}{\text{Max}} \Pi_{l1}^{+} = (P_{lh1} - C_l)(\alpha_{lh} - \beta_{lh}P_{lh1}) +$$

$$(IP_{lf1} - C_l - r)(\alpha_{lf} - \beta_{lf}P_{lf1} + \theta P_g) \tag{6.4}$$

$$s.t.\ \ Q_{lh1} = \alpha_{lh} - \beta_{lh}P_{lh1} \tag{6.5}$$

$$Q_{lf1} = \alpha_{lf} - \beta_{lf}P_{lf1} + \theta P_g \tag{6.6}$$

A quick analysis of (6.4) will reveal that the expected profit function is jointly concave in P_{lh1} and P_{lf1}. Hence using the first order optimality conditions we get the optimal prices and quantities:

$$P_{lh1}^{*^{+}} = \frac{\alpha_{lh} + \beta_{lh}C_l}{2\beta_{lh}} \tag{6.7}$$

$$P_{lf1}^{*^{+}} = \frac{I(\alpha_{lf} + \theta P_g) + \beta_{lf}(C_l + r)}{2\beta_{lf}I} \tag{6.8}$$

Thus,

$$Q_{lh1}^{*^{+}} = \frac{\alpha_{lh} - \beta_{lh}C_l}{2} \tag{6.9}$$

$$Q_{lf1}^{*^{+}} = \frac{I(\alpha_{lf}{+}\theta P_g) - \beta_{lf}(C_l + r)}{2I} \tag{6.10}$$

A quick analysis to equation (6.8) shows that firm's foreign price decreases in exchange rate, however as exchange rate gets large, $P_{lf1}^{*^{+}}$ converges to $(\alpha_{lf} + \theta P_g)/2\beta_{lf}$ and $Q_{lf1}^{*^{+}}$ converges to $(\alpha_{lf} + \theta P_g)/2$. Therefore, the firm drops its foreign price further if the exchange rate is high to export more quantities to the foreign market and to capture the entire foreign market. Also, equation (6.10) shows that the exported quantities to the foreign market increases in foreign manufacturer's price. The effect of high P_g on $Q_{lf1}^{*^{+}}$ is stronger if θ is close to β_{lf} (see EQ. 6.6). However, low θ will offset the contribution of high P_g.

Equation (6.9) reveals that the firm will always sell to domestic market since $\alpha_{lh} > C_l\beta_{lh}$. This is expected since the capacity is ample, there is no competition at the domestic market (home market share belongs to the firm), and exchange rate does not affect the quantity sold to domestic market. Therefore, the firm makes profit even if exchange rate is not favorable. P_g or θ does not affect the local price and quantity because of ample capacity.

Lemma 6.1: *The firm will export to the foreign market if*

$$I > \frac{\beta_{lf}(C_l + r)}{\alpha_{lf} + \theta P_g} = I_{z1} \tag{6.11}$$

Otherwise the firm will sell only to home market.

Proof. In order for the manufacturer to export to the foreign market, Q_{lf1}^{*+} must be > 0. The denominator of equation (6.10) is clearly > 0, however the numerator of (6.10) must be > 0 : $I(\alpha_{lf} + \theta P_g) - \beta_{lf}(C_l + r) > 0$. From the last inequality, we can get the exchange rate threshold, I_{z1} given in (6.11) . □

The exchange rate threshold, I_{z1}, shows that it would be difficult for the firm to export its product to the foreign market if the foreign manufacturer's price is low or if the "cross-price" is low. Therefore, if the firm is fortunate to export to the foreign market ($I > I_{z1}$) while the foreign manufacturer's price is low, the firm would drop its price according to (6.8) to compete with foreign manufacturer. I_{z1} shows that if the foreign customers are very sensitive to price, the firm might not export its product to the foreign market. Also, demand potential plays an important role in shipping to foreign market, the higher the foreign potential demand, the more likely the firm would export to foreign market.

Suppose the exchange rate is $> I_{z1}$, meaning the firm is exporting its product to foreign market, then the equation (6.4) can be written as:

$$\Pi_{l1}^{+} = \frac{(\alpha_{lh} - \beta_{lh} C_l)^2}{4\beta_{lh}} + \frac{(I(\alpha_{lf} + \theta P_g) - \beta_{lf}(C_l + r))^2}{4I\beta_{lf}} \tag{6.12}$$

The first term of (6.12) is the profit of the quantities sold at the domestic market where the second term is the profit of the quantities exported to foreign market. The profit clearly decreases in consumer price sensitivity, cost per unit, and transportation cost and increases in potential demands, exchange rate, "cross-price", and foreign manufacturer's price.

The total capacity used by the firm is the sum of the quantities sold to home and foreign market:

$$K_{l1}^{+} = \frac{\alpha_{lh} + \alpha_{lf} - \beta_{lh} C_l + \theta P_g}{2} - \frac{\beta_{lf}(C_l + r)}{2I(t)} \tag{6.13}$$

As expected, total used capacity increases in the foreign exchange rate and the price of the foreign manufacturer, while decreasing in production and transportation costs. Since the price of the product decreases in the exchange rate in the foreign market, the sales volume will increase. The sales in the domestic market will not be affected in this case by the exchange rate fluctuations or the price of the foreign manufacturer.

6.3.2 The Exogenous Model with Limited Capacity

Now, we relax the unlimited capacity assumption. This implies that the firm can not always fully satisfy demand in both markets at the same period. Unlike the unlimited capacity case, now the firm must decide on its capacity allocation. The condition that the firm will export to the foreign market, Lemma 6.1, is still valid as long as the firm can satisfy the home demand.

Two factors will impact capacity allocations: available capacity and exchange rate. In this model, we are going to assume two capacity levels: expensive capacity (available capacity is low) and medium capacity (capacity cost is not expensive nor cheap). Under each capacity level, the firm must decide on capacity allocations. The process is as follow: depending on the capacity cost (expensive or medium), the firm determines the capacity level then the manufacturer looks at the spot exchange rate and decide on allocation of capacity. Later, we will discuss all allocation scenarios.

6.3.2.1 The Model

The firm faces the following optimization problem:

$$\underset{P_{lh1},P_{lf1}\geq 0}{\text{Maximize}}\ \Pi_{l1}^{-} = (P_{lh1} - C_l)(\alpha_{lh} - \beta_{lh}P_{lh1}) + (IP_{lf1} - C_l - r)(\alpha_{lf} - \beta_{lf}P_{lf1} + \theta P_g) \quad (6.14)$$

$$s.t.\ \ Q_{lh1} \leq \alpha_{lh} - \beta_{lh}P_{lh1} \quad (6.15)$$

$$Q_{lf1} \leq \alpha_{lf} - \beta_{lf}P_{lf1} + \theta P_g \quad (6.16)$$

$$Q_{lh1} + Q_{lf1} \leq K_{l1} \quad (6.17)$$

we can easily observe that the inequalities in (6.15) and (6.16) must be binding. Using this observation, we can employ Lagrangian relaxation and rewrite the model as follows:

$$\underset{\lambda_{l1},K_{l1},P_{lh1},P_{lf1}\geq 0}{\text{Maximize}}\ \Pi_{l1}^{-} = (P_{lh1} - C_l)(\alpha_{lh} - \beta_{lh}P_{lh1}) + (IP_{lf1} - C_l - r)(\alpha_{lf} - \beta_{lf}P_{lf1} + \theta P_g) +$$

$$\lambda_{l1}(K_{l1} - \alpha_{lh} + \beta_{lh}P_{lh1} - \alpha_{lf} + \beta_{lf}P_{lf1} - \theta P_g) \quad (6.18)$$

Equation (6.18) shows that the expected profit function is jointly concave in P_{lh1} and P_{lf1}. Hence, using the first order optimality conditions we get

$$P_{lh1}^{*-} = \frac{\alpha_{lh} + \beta_{lh}(C_l + \lambda_{l1})}{2\beta_{lh}}$$

$$P_{lf1}^{*-} = \frac{I(\alpha_{lf} + \theta P_g) + \beta_{lf}(C_l + r + \lambda_{l1})}{2I\beta_{lf}}$$

where

$$\lambda_{l1} = \frac{I(\alpha_{lh} + \alpha_{lf} + \theta P_g - 2K_{l1} - \beta_{lh}C_l) - \beta_{lf}(C_l + r)}{I\beta_{lh} + \beta_{lf}} \tag{6.19}$$

Substituting (6.19) in P^{*-}_{lh1} and P^{*-}_{lf1}, we get the optimal prices for the firm in terms of P_g.

$$P^{*-}_{lh1} = \frac{\beta_{lh}I(2\alpha_{lh} + \alpha_{lf} - 2K_{l1} + \theta P_g) + \beta_{lf}(\alpha_{lh} - \beta_{lh}r)}{2\beta_{lh}(I\beta_{lh} + \beta_{lf})} \tag{6.20}$$

$$P^{*-}_{lf1} = \frac{\beta_{lh}I(\alpha_{lf} + \theta P_g) + \beta_{lf}(2\alpha_{lf} + \alpha_{lh} - 2K_{l1} + 2\theta P_g + \beta_{lh}r)}{2\beta_{lf}(I\beta_{lh} + \beta_{lf})} \tag{6.21}$$

Thus, the optimal quantities in terms of P_g are

$$Q^{*-}_{lh1} = \frac{\beta_{lh}I(2K_{l1} - \alpha_{lf} - \theta P_g) + \beta_{lf}(\alpha_{lh} + \beta_{lh}r)}{2(I\beta_{lh} + \beta_{lf})} \tag{6.22}$$

$$Q^{*-}_{lf1} = \frac{\beta_{lh}I(\alpha_{lf} + \theta P_g) - \beta_{lf}(\alpha_{lh} - 2K_{l1} + \beta_{lh}r)}{2(I\beta_{lh} + \beta_{lf})} \tag{6.23}$$

Due to the limited capacity, we can witness the effect of P_g and θ on domestic price and quantity (P^{*-}_{lh1} and Q^{*-}_{lh1}). As P_g or θ increase, the firm increases its domestic price to sell less quantities to the domestic market because it is more attractive to allocate more capacity to the foreign market. At the foreign market, the firm increases its foreign price and allocate more capacity under high P_g and θ is close to β_{lf}.

Proposition 6.1: *the firm's capacity is fully utilized (and hence limited) if and only if*

$$K_{l1} < \frac{I(\alpha_{lh} + \alpha_{lf} - \beta_{lh}C_l + \theta P_g) - \beta_{lf}(C_l + r)}{2I} \tag{6.24}$$

Proof. It is straightforward to observe from (6.19) that λ_{l1} must be > 0 in order for the capacity to be limited (i.e., the capacity constraint is binding). This implies that (6.24) must hold. □

We note that the right hand side in (6.24) equals K^{+}_{l1} given in (6.13). The inequality in (6.24) implies that while the capacity utilization in a period increases with the observed exchange rate and market potentials, it decreases in price sensitivities, the production cost, and the transportation cost. This result implies that the unconstrained optimal solution given in (6.20-6.23) is obtained when

$$I(t) \leq \frac{\beta_{lf}(C_l + r)}{(\alpha_{lh} + \alpha_{lf} - 2K_{l1} - \beta_{lh} C_l + \theta P_g)} = I_{t1} \tag{6.25}$$

Eq. (6.25) shows that the firm can implement the unconstrained solution and hence does not need to split her capacity across markets if $I(t) < I_{t1}$. In contrast, when the inequality in (6.25) does not hold the capacity is limited. As such, the firm faces the problem of allocating the capacity between the two markets. Depending on the market potentials, the realized exchange rates, and the price of the foreign manufacturer the firm may also choose to sell only in one of the markets. The next proposition indicates that if the exchange rate is significantly high, the firm in fact may opt out of the domestic market.

Proposition 6.2. *The firm chooses to sell in the domestic market at the beginning of the period if and only if*

$$I < \frac{\beta_{lf}(\alpha_{lh} + \beta_{lh} r)}{\beta_{lh}(\alpha_{lf} + \theta P_g - 2K_{l1})} = I_{h1} \tag{6.26}$$

otherwise, the firm sells only in the foreign market. On the other hand, she will export her product to the foreign market at any period if and only if

$$I > \frac{\beta_{lf}(\alpha_{lh} - 2K_{l1} + \beta_{lh} r)}{\beta_{lh}(\alpha_{lf} + \theta P_g)} = I_{f1} \tag{6.27}$$

Hence, the firm splits the capacity between the domestic and foreign markets only when $I_{f1} < I(t) < I_{h1}$.

Proof. The inequality in (6.26) directly follows from the fact that the manufacturer will sell to the domestic market if $Q_{lh1}^{*-} > 0$ implying that the numerator in (6.22) must be non-negative. This is satisfied when (6.26) holds. Similarly, $Q_{lf1}^{*-} > 0$ implies that (6.27) holds. Next we show that $I_{f1} < I_{h1}$. First observe that $I_{f1} < I_{h1}$ implies that

$$\frac{\beta_{lf}(\alpha_{lh} + \beta_{lh}r)}{\beta_{lh}(\alpha_{lf}+\theta P_g - 2K_{l1})} > \frac{\beta_{lf}(\alpha_{lh} - 2K_{l1} + \beta_{lh}r)}{\beta_{lh}(\alpha_{lf}+\theta P_g)},$$

which can be rewritten as

$$\frac{(\alpha_{lh} + \beta_{lh}r)}{(\alpha_{lfF}+\theta P_g - 2K_{l1})} > \frac{(\alpha_{lh} - 2K_{l1} + \beta_{lh}r)}{(\alpha_{lf}+\theta P_g)}.$$

It is clear that from the last argument that $(\alpha_{lh} + \beta_{lh}r) > (\alpha_{lh} - 2K_{l1} + \beta_{lh}r)$ and $(\alpha_{lf} + \theta P_g - 2K_{l1}) < (\alpha_{lf} + \theta P_g)$, this inequality must hold. □

We notice from (6.26) that the firm will always sell to the domestic market if $K_{l1} \geq \frac{\alpha_{lf}+\theta P_g}{2} = K_{h1}$. Clearly, when the capacity is ample it is always profitable to sell in the domestic market. However when capacity is scarce, as the foreign exchange rate (or equivalently the potential demand in the foreign market and the price of the foreign manufacturer) increases, it becomes less appealing to allocate capacity to the domestic market. Similarly, we observe from (6.27) that the firm exports to the foreign market at any given time if $K_{l1} \geq \frac{\alpha_{lh}+\beta_{lh}r}{2}$ as long as inequality in (6.11) holds. Therefore, low transportation cost and home potential demand make the foreign market appealing to the firm. Proposition 6.2 implies that the optimal policy for the firm at any given period is a "cherry-picking" policy when the capacity is scarce. Namely, when the exchange rate is significantly high the firm sells only in the foreign market whereas when it is too low she will serve exclusively the domestic market. For exchange rates not too high or low, the capacity is split between the markets. The optimal splitting policy is investigated in the next section. Now, we will show how exchange rate thresholds are related.

Lemma 6.2: *For any given K_{l1}, $I_{h1} > I_{t1}$*

Proof.

$$\frac{\beta_{lf}(\alpha_{lh}+\beta_{lh}r)}{\beta_{lh}(\alpha_{lf}+\theta P_g-2K_{l1})} > \frac{\beta_{lf}(C_l+r)}{(\alpha_{lh}+\alpha_{lf}-2K_{l1}-\beta_{lh}C_l+\theta P_g)}$$

$$\frac{(\alpha_{lh}+\beta_{lh}r)}{\beta_{lh}(\alpha_{lf}+\theta P_g-2K_{l1})} > \frac{(C_l+r)}{(\alpha_{lh}+\alpha_{lf}-2K_{l1}-\beta_{lh}C_l+\theta P_{g2})}$$

It is clear that $(\alpha_{lh}+\beta_{lh}r) > (C_l+r)$, and for any given K_{l1}, $(\alpha_{lh}+\alpha_{lf}-2K_{l1}-\beta_{lh}C_l+\theta P_{g2}) > \beta_{lh}(\alpha_{lf}+\theta P_g-2K_{l1})$. □

Lemma 6.3: *For any given capacity level,* $I_{h1} > I_{z1}$

Proof.

$$\frac{\beta_{lf}(\alpha_{lh}+\beta_{lh}r)}{\beta_{lh}(\alpha_{lf}+\theta P_g-2K_{l1})} > \frac{\beta_{lf}(C_l+r)}{\alpha_{lf}+\theta P_g}$$

It is clear that $(\alpha_{lh}+\beta_{lh}r) > (C_l+r)$, and for any given K_{l1}, $(\alpha_{lf}+\theta P_g) > \beta_{lh}(\alpha_{lf}+\theta P_g-2K_{l1})$. □

Lemma 6.4: $I_{z1} > I_{f1}$ *for* $K_{l1} > Q^{*+}_{lh1}$ *otherwise* $I_{f1} > I_{z1}$

Proof.

$$\frac{\beta_{lf}(C_l+r)}{\alpha_{lf}+\theta P_g} > \frac{\beta_{lf}(\alpha_{lh}-2K_{l1}+\beta_{lh}r)}{\beta_{lh}(\alpha_{lf}+\theta P_g)}$$

Simplifying the last argument we get: $0 > \alpha_{lh}-2K_{l1}+\beta_{lh}r$. The last inequality indicates that: at $K_{l1} = Q^{*+}_{lh1}$, we find that $I_{z1} = I_{f1}$, and at $K_{l1} > Q^{*+}_{lh1}$, we get $I_{z1} > I_{f1}$, and at $K_{l1} < Q^{*+}_{lh1}$, we get $I_{f1} > I_{z1}$. □

Lemma 6.5: $I_{t1} > I_{z1}$ *if and only if* $K_{l1} > Q^{*+}_{lh1}$

Proof. if capacity level falls below Q^{*+}_{lh1}, then I_{t1} is irrelevant because capacity is scarce for both markets. We are interested in the situation where $K_{l1} > Q^{*+}_{lh1}$:

$$\frac{\beta_{lf}(C_l+r)}{(\alpha_{lh}+\alpha_{lf}-2K_{l1}-\beta_{lh}C_l+\theta P_g)} > \frac{\beta_{lf}(C_l+r)}{\alpha_{lf}+\theta P_g}$$

Simplifying the last inequality we get: $0 > \alpha_{lh}+\alpha_{lf}-2K_{l1}-\beta_{lh}C_l$. Therefore, for $K_{l1} > Q^{*+}_{lh1}$, we get $I_{t1} > I_{z1}$. □

Lemma 6.2 through Lemma 6.5 show the relationship between exchange rate thresholds with respect to capacity and those Lemmas are important in order to construct Figure 6.2.

6.3.2.2 The Exogenous Model Optimal Capacity

Based on Proposition 6.2 we conclude that at the beginning of the planning horizon the optimal pricing and allocation policy falls in one of the following five scenarios:

HU: The capacity is ample (unconstrained) and the firm sells only in the domestic market.

SU: The capacity is ample (unconstrained) and the firm sells in both markets.

HC: The capacity is limited and the firm allocates all of its capacity to the domestic market.

FC: The capacity is limited and the firm allocates all of its capacity to the foreign market.

SC: The capacity is limited and the firm splits all of its capacity between the two markets.

First we note that when capacity is ample, i.e., $K_{h1} \geq K_{l1}^{+}$, there is always incentive for the firm to sell in the domestic market. From Lemma 6.1, the firm sells in the foreign market in the beginning of the planning horizon if the exchange rate is above the threshold given in (6.11). In case of scarce capacity any one of the three scenarios (home only, foreign only or split capacity) may occur depending on the exchange rate as shown in Proposition 6.2.

Suppose that capacity is too scarce (high-cost case), that is, lower than $Q_{lh1}^{*^{+}}$ given in (6.9). This situation is more likely when the cost of capacity is too high that the firm cannot fully satisfy the demand in the domestic market alone. In this case, the capacity is fully utilized. As depicted in Figure 6.2, the firm allocates all of its capacity to the domestic market (HC) if the exchange rate is below I_{f1}. In this case the low exchange rate does not justify exporting to the foreign market. The firm chooses to split the capacity between two markets (SC) if the exchange rate falls between I_{h1} and I_{f1}. The optimal allocation is then given by (6.22) and (6.23). If the exchange rate is too high (above I_{h1}) the revenue from sales in the foreign market justifies the full allocation of the capacity to this market (FC).

Under this scenarios *exante* expected profit function before the start of the selling season can be written as follows:

$$\underset{K_{l1}<Q^{*+}_{lh1}}{\text{Maximize}}\ \Pi^{-}_{l1} = -u_l K_{l1} + \int_0^{I_{f1}} \Pi_{HC} A(I)dI$$

$$+\int_{I_{f1}}^{I_{h1}} \Pi_{SC} A(I)dI + \int_{I_{h1}}^{\infty} \Pi_{FC} A(I)dI \qquad (6.28)$$

Where:

$$\Pi_{HC} = \frac{\alpha_{lh} - K_{l1} - \beta_{lh} C_l}{\beta_{lh}} * K_{l1},$$

$$\Pi_{SC} = (P^{*-}_{lh1} - C_l)Q^{*-}_{lh1} + (IP^{*-}_{lf1} - C_l - r)Q^{*-}_{lf1}, \text{ and}$$

$$\Pi_{FC} = \frac{I(\alpha_{lh} - K_{l1} + \theta P_g) - \beta_{lf}(C_l + r)}{\beta_{lf}} * K_{l1}$$

To find the optimal capacity, we need to take the derivative of (6.28) with respect to K_{l1}

$$\frac{d\Pi_{l1}^{-}}{dK_{l1}} = -u_l + \int_0^{\frac{I_{f1}}{I_o}} \frac{\alpha_{lh} - 2K_{l1} - \beta_{lh}C_l}{\beta_{lh}} f(\epsilon)d\epsilon + \int_{\frac{I_{f1}}{I_o}}^{\frac{I_{h1}}{I_o}} \lambda_{l1} f(\epsilon)d\epsilon + \int_{\frac{I_{h1}}{I_o}}^{\infty} \frac{I_o\epsilon_i(\alpha_{lf} - 2K_{l1} + \theta P_g) - \beta_{lf}(C_l + r)}{\beta_{lf}} f(\epsilon)d\epsilon = 0 \qquad (6.29)$$

where λ_{l1} is the Lagrangian multiplier defined in (6.19). It is straightforward to see from the function above that the second derivative with respect to K_{l1} is strictly negative for any $K_{l1} \geq 0$ implying concavity. Hence solution to (6.29) gives the unique optimal value for the capacity to be built at home in advance of the selling season. Unfortunately, there is no close form solution. However, the optimal capacity can be calculated easily with a simple line search.

Recall that the condition given in (6.29) applies when the capacity scarce. This is expected to occur when capacity is very expensive. Suppose that capacity is bigger than home demand and less than K_{h1}. Then, the allocation decision follows Figure 6.2:

(1) if the spot exchange rate is below I_{z1}, then the firm satisfies all the demand in the domestic market and stays out of the foreign market (Lemma 6.1).

(2) if the spot exchange rate falls between I_{z1} and I_{t1}, then the firm satisfies all demand in both markets (Proposition 6.1).

(3) if the spot exchange rate falls between I_{t1} and I_{h1}, then the firm splits its limited capacity between the two markets (Proposition 6.2).

(4) if the spot exchange rate is higher than I_{h1}, then the firm allocates all the capacity to the foreign market (Proposition 6.2).

Consequently, the firm's net profit in this case can be written as:

$$\underset{K_{h1}>K_{l1}>Q^{*+}_{lh1}}{\text{Maximize}} \Pi^{-}_{l1} = -u_l K_{l1} + \int_{0}^{\frac{I_{z1}}{I_o}} \Pi_{HU} f(\epsilon) d\epsilon +$$

$$\int_{\frac{I_{z1}}{I_o}}^{\frac{I_{t1}}{I_o}} \Pi_{SU} f(\epsilon) d\epsilon + \int_{\frac{I_{t1}}{I_o}}^{\frac{I_{h1}}{I_o}} \Pi_{SC} f(\epsilon) d\epsilon + \int_{\frac{I_{h1}}{I_o}}^{\infty} \Pi_{FC} f(\epsilon) d\epsilon \qquad (6.30)$$

We want to find the optimal capacity, we need to take the derivative of equation (6.30) with respect to K_{l1} and we find:

$$\frac{d\Pi^{-}_{l1}}{dK_{l1}} = -u_l + \int_{\frac{It1}{I_o}}^{\frac{I_{h1}}{I_o}} \lambda_{l1} f(\epsilon) d\epsilon +$$

$$\int_{\frac{I_{h1}}{I_o}}^{\infty} \frac{I_o \epsilon_i (\alpha_{lf} - 2K_{l1} + \theta P_g) - \beta_{lf}(C_l + r)}{\beta_{lf}} f(\epsilon) d\epsilon = 0 \qquad (6.31)$$

Now, remember that in (6.31) the capacity is somehow not scares, however the firm above certain exchange rate threshold, I_{h1}, will dedicate its capacity to the foreign market. Lets assume that capacity falls between K_{h1} and K^{+}_{l1} where the firm will always satisfy demand at home market. Then, the allocation decision follows Figure 6.2:

(1) if the spot exchange rate is below I_{z1}, then the firm satisfies all the demand in the domestic market and stays out of the foreign market (Lemma 6.1).

(2) if the spot exchange rate falls between I_{z1} and I_{t1}, then the firm satisfies all demand in both markets (Proposition 6.1).

(3) if the spot exchange rate falls between I_{t1} and I_{h1}, then the firm splits its limited capacity between the two markets (Proposition 6.2).

Consequently, the firm's net profit in this case can be written as:

$$\underset{K_{t1}>K_{l1}>K_{h1}}{\text{Maximize}} \Pi_{l1}^{-} = -u_l K_{l1} + \int_0^{\frac{I_{z1}}{I_o}} \Pi_{HU} f(\epsilon) d\epsilon +$$

$$\int_{\frac{I_{z1}}{I_o}}^{\frac{I_{t1}}{I_o}} \Pi_{SU} f(\epsilon) d\epsilon + \int_{\frac{I_{t1}}{I_o}}^{\infty} \Pi_{SC} f(\epsilon) d\epsilon \tag{6.32}$$

We want to find the optimal capacity, we need to take the derivative of equation (6.32) with respect to K_{l1} and we find:

$$\frac{d\Pi_{l1}^{-}}{dK_{l1}} = -u_l + \int_{\frac{It1}{I_o}}^{\infty} \lambda_{l1} f(\epsilon) d\epsilon \tag{6.33}$$

Although we cannot find closed form solutions for (6.29), (6.31) and (6.33) we can derive the following conclusions from concavity

Proposition 6.3. *There exit two capacity cost thresholds* u_l^{t1} *and* u_l^{t2} *such that* $u_l^{t1} > u_l^{t2}$.

if $u_l > u_l^{t1}$*, then* K_{l1}^* *will be obtained from (6.29), if* $u_l^{t2} > u_l > u_l^{t1}$*, then* K_{l1}^* *will be obtained from (6.31), and if* $u_l^{t2} > u_l$*, then* K_{l1}^* *will be obtained from (6.33).*

The above proposition shows that when capacity cost is expensive, then the optimal capacity falls between 0 and $Q_{lh1}^{*^{*^{+}}}$. However, as capacity cost get cheaper, then optimal capacity falls between $Q_{lh1}^{*^{*^{+}}}$ and K_{h1} if we assume that $K_{h1} > Q_{lh1}^{*^{*^{+}}}$. Otherwise, the optimal capacity falls between $Q_{lh1}^{*^{*^{+}}}$ and K_{t1}.

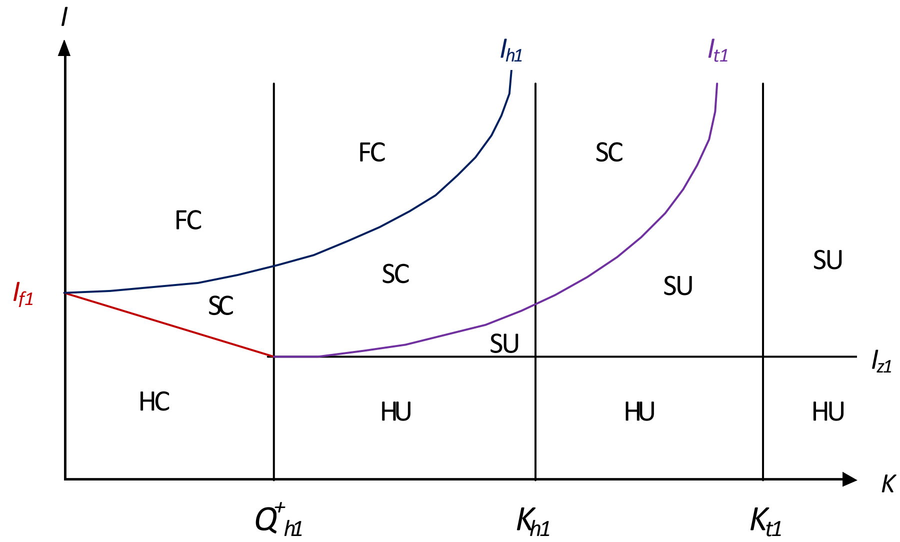


Figure 6.2 Capacity Allocation (Exogenous Model)

6.3.2.3 Discussion

Figure 6.2 shows all allocation scenarios with regards the exchange rate and the available capacity. When capacity is inexpensive and the foreign currency is weak, the firm stays in the home market and fully satisfies the demand. For $I_{z1} > I > I_{t1}$, both market demands are filled. In both cases the capacity is underutilized. When the capacity is expensive and foreign currency is weak, the firm sells only to the home market and fully utilizes the capacity. As exchange rate increases, the firm splits its capacity between the two markets. When the foreign currency is too strong, the firm dedicates all of its capacity to the foreign market and leaves the home market completely. When capacity cost is expensive (equivalently less available capacity), depending on the exchange rate, the firm allocates all of its capacity to the home market ($I < I_{f1}$) or to the foreign market ($I > I_{h1}$), or split it between both markets ($I_{h1} > I > I_{f1}$). As the capacity cost increases the allocation decision becomes "cherry picking" problem where the scarce capacity is dedicated to one of the markets. Since capacity will be too small, splitting capacity does not make economical sense.

Also, we notice from Figure 6.3, Table 6.1 and Figure 4.4 that optimal capacity increases in P_g and θ which makes it more attractive for the firm to export to the foreign market when the price of the foreign manufacturer is high. Also, optimal capacity decreases in σ. Table 6.2 shows the effect of P_g on exchange rate thresholds. I_{z1} and I_{f1} decreases in P_g, which as we stated earlier makes it compelling to export to foreign market.

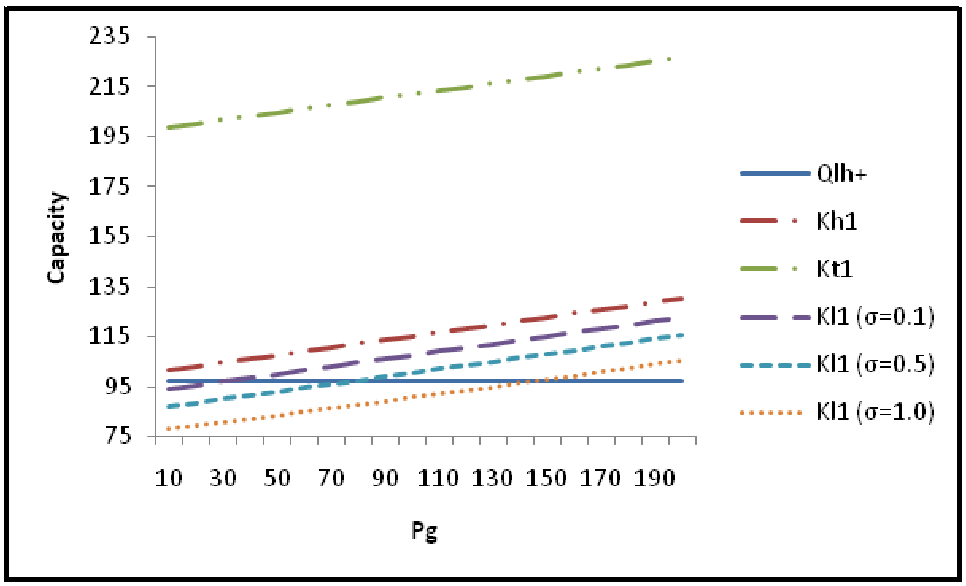


Figure 6.3 Effect of Sigma on Optimal Capacity

Table 6.1 Effect of P_g and σ on Optimal Capacity (K_{l1})

P_g	Q^{*+}_{lh1}	K_{h1}	K_{t1}	$K_{l1}(\sigma=0.1)$	$K_{l1}(\sigma=0.5)$	$K_{l1}(\sigma=1)$
10	97	101.5	198.5	94.22	87.21	78.43
20	97	103	200	95.72	88.66	79.75
30	97	104.5	201.5	97.22	90.11	81.09
40	97	106	203	98.72	91.57	82.43
50	97	107.5	204.5	100.22	93.04	83.794
60	97	109	206	101.72	94.51	85.16
70	97	110.5	207.5	103.22	95.99	86.55
80	97	112	209	104.72	97.48	87.94
90	97	113.5	210.5	106.22	98.98	89.35
100	97	115	212	107.72	100.48	90.77
110	97	116.5	213.5	109.22	101.98	92.20
120	97	118	215	110.72	103.48	93.64
130	97	119.5	216.5	112.22	104.98	95.10
140	97	121	218	113.72	106.48	96.56
150	97	122.5	219.5	115.22	107.98	98.05
160	97	124	221	116.72	109.48	99.55

Table 6.2 Effect of P_g and σ on Exchange Rate Thresholds

P_g	I_{z1}	I_{t1}			I_{f1}			I_{h1}		
		0.1	0.5	1.0	0.1	0.5	1.0	0.1	0.5	1.0
10	0.03				0.06	0.13	0.22	13.88	7.07	4.37
20	0.03				0.05	0.11	0.20	13.88	7.04	4.34
30	0.03	7.89				0.10	0.19	13.88	7.02	4.31
40	0.03	8.01				0.08	0.17	13.88	7.00	4.28
50	0.03	8.14				0.07	0.16	13.88	6.98	4.26
60	0.03	8.26				0.05	0.14	13.88	6.97	4.23
70	0.03	8.39				0.04	0.13	13.88	6.96	4.21
80	0.03	8.53	7.91				0.11	13.88	6.96	4.19
90	0.03	8.66	8.03				0.10	13.88	6.96	4.18
100	0.03	8.81	8.16				0.08	13.88	6.96	4.16
110	0.03	8.96	8.28				0.07	13.88	6.96	4.15
120	0.03	9.11	8.42				0.06	13.88	6.96	4.14
130	0.03	9.27	8.55				0.04	13.88	6.96	4.13
140	0.03	9.43	8.69				0.03	13.88	6.96	4.13
150	0.03	9.60	8.83	7.96				13.88	6.96	4.13
160	0.03	9.78	8.98	8.08				13.88	6.96	4.13
170	0.03	9.96	9.14	8.20				13.88	6.96	4.13
180	0.03	10.15	9.30	8.33				13.88	6.96	4.13

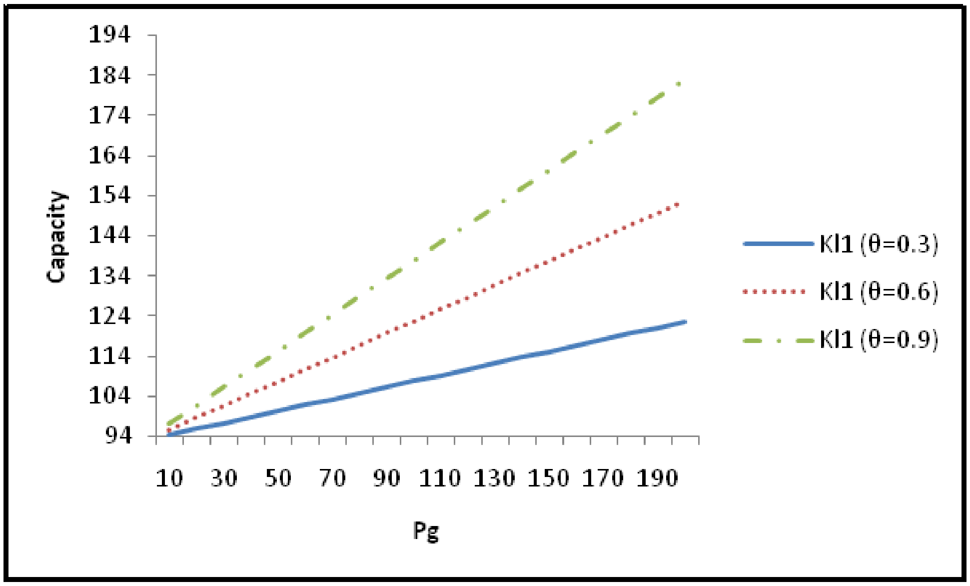


Figure 6.4 Effect of Cross-Price (θ) on Optimal Capacity

6.4 The Endogenous Model

In this section, we are assuming that both firms make their prices simultaneously at the foreign market. We are going to make two assumptions: both firms has ample capacities and the firm has limited capacity and the foreign manufacturer has ample capacity. We will drive the Nash Game outcome (optimal prices and quantities) for each firm and show the impact of exchange rate and competition parameters on the firm's capacity allocations, hence the foreign manufacturer does not face a capacity issue.

6.4.1 The Endogenous Model with Ample Capacity

The assumption of ample capacity help us finding the exchange rate threshold which will allow the firm to export to the foreign market. Also, it will give an idea how much the firm can sell to the local market. Therefore, the firm's profit was given in (6.4) and the home optimal price and quantity are given in (6.7) and (6.9) respectively. Also, the firm's foreign price and quantity are given in (6.8) and (6.10), respectively. However, both optimal foreign price and quantity are function of the foreign manufacturer's price, P_g, therefore we need to drive the optimal foreign manufacturer price and substitute it in (6.8) and (6.10) to obtain optimal prices, optimal quantities and the exchange rate threshold (I_{z2}) where the firm exports its product to foreign market. The profit function of the foreign manufacturer can be written as:

$$\underset{p_g, Q_g \geq 0}{\text{Maximize}} \Pi_g^+ = (P_g - C_g)(\alpha_g - \beta_g P_g + \theta P_{lf2}) \tag{6.34}$$

$$s.t.\ Q_g = \alpha_g - \beta_g P_g + \theta P_{lf2} \tag{6.35}$$

A quick analysis of (6.34) will reveal that the expected profit function is jointly concave in P_g. Hence using the first order optimality conditions we get

$$p_g^* = \frac{\alpha_g + C_g\beta_g + \theta P_{lf2}}{2\beta g} \tag{6.36}$$

Thus, substituting (6.36) in (6.8), we get

$$P_{lf2}^{*^+} = \frac{I(2\alpha_{lf}\beta_g + \theta(\alpha_g + C_g\beta_g)) + 2\beta_{lf}\beta_g(C_l + r)}{I(4\beta_{lf}\beta_g - \theta^2)} \tag{6.37}$$

Substituting (6.37) in (6.36), we get

$$P_g^{*^+} = \frac{I(\theta\alpha_{lf} + 2\beta_{lf}(\alpha_g + C_g\beta_g)) + \theta\beta_{lf}(C_l + r)}{I(4\beta_{lf}\beta_g - \theta^2)} \tag{6.38}$$

We notice that both optimal prices decreases in exchange rate, however $P_{lf2}^{*^+}$ decreases in exchange rate faster rate than $P_g^{*^+}$ if demand base and consumer price sensitivity are the same ($\alpha_{lf} = \alpha_g$ and $\beta_{lf} = \beta_g$) since $2\beta_g > \theta$. As exchange rate increases, the firm would decrease its price, $P_{lf2}^{*^+}$, and as consequence reaction by the foreign manufacturer, $P_g^{*^+}$ would decreases too. Therefore, the local manufacturer acts as a leader and the foreign manufacturer acts as a follower.

Lemma 6.6: *The firm will drop its price below the foreign manufacturer's price,* $P_{lf2}^{*^+} < P_g^{*^+}$, *if and only if*

$$I > \frac{\beta_{lf}(C_l + r)(2\beta_g - \theta)}{(2\beta_f - \theta)(\alpha_g + C_g\beta_g) - (2\beta_g - \theta)\alpha_{lf}} = I_p$$

where $\alpha_{lf} \geq (2\beta_f - \theta)(\alpha_g + C_g\beta_g)/(2\beta_g - \theta)$, *and if* $\alpha_g = \alpha_{lf}$, *and* $\beta_g = \beta_{lf}$, *then* $P_{lf2}^{*^+} < P_g^{*^+}$ *occurs if and only if*

$$I > \frac{C_l + r}{C_g} = \overline{I_p}$$

otherwise, $P_{lf2}^{*^+} > P_g^{*^+}$, *Figure 6.5.*

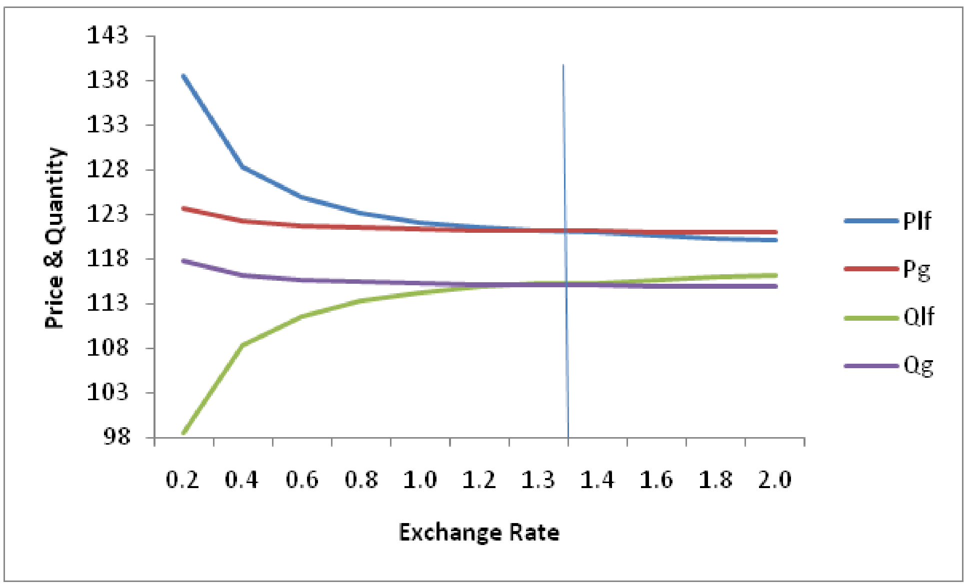


Figure 6.5 Effect of Exchange Rate on Optimal Prices and Quantities

The goal of each manufacturer is to drop its price below the competitor manufacturer's price to increase sales; The above lemma shows the price competition between the two manufacturers in terms of exchange rate where the firm can drop its price below the foreign manufacturer's price faster as β_{lf}, C_l, r, and α_{lf} decreases or as α_g, β_g, and C_g increases. We notice that θ does not affect the price competition if potential demands and prices sensitivity are the same.

Now, we will drive the optimal quantities for the firm and the foreign manufacturer. Substituting (6.37) and (6.38) in (6.10), we get

$$Q_{lf2}^{*^+} = \frac{I\beta_{lf}(2\alpha_{lf}\beta_g + \theta(\alpha_g + C_g\beta_g)) - \beta_{lf}(C_l + r)(2\beta_{lf}\beta_g - \theta^2)}{I(4\beta_{lf}\beta_g - \theta^2)} \quad (6.39)$$

Substituting (6.37) and (6.38) in (5.35), we get

$$Q_g^{*^+} = \frac{I\beta_g(\beta_{lf}(2\alpha_g - 2C_g\beta_g) + \theta(\alpha_{lf} + \theta C_g)) + \theta\beta_{lf}\beta_g(C_l + r)}{I(4\beta_{lf}\beta_g - \theta^2)} \quad (6.40)$$

***Lemma 6.7**: Suppose $\beta_{lf} = \beta_g = \beta$, $\alpha_{lf} = \alpha_g = \alpha$, then*

$$P_{lf2}^{*^+} = \frac{I((2\beta + \theta)\alpha + \theta\beta C_g) + 2\beta^2(C_l + r)}{I(4\beta^2 - \theta^2)}$$

$$P_g^{*^+} = \frac{I((2\beta + \theta)\alpha + 2\beta^2 C_g) + \theta\beta(C_l + r)}{I(4\beta^2 - \theta^2)}$$

$$Q_{lf2}^{*^+} = \frac{\beta(I((2\beta + \theta)\alpha + \theta\beta C_g) - (2\beta^2 - \theta^2)(C_l + r))}{I(4\beta^2 - \theta^2)}$$

$$Q_g^{*^+} = \frac{\beta(I((2\beta + \theta)\alpha - C_g(2\beta^2 - \theta^2)) + \theta\beta(C_l + r))}{I(4\beta^2 - \theta^2)}$$

and if $I = 1$, and $C_l + r = C_g = C$, then

$$P_{lf2}^{*^+} = P_g^{*} = \frac{(\theta + 2\beta)(\alpha + \beta C)}{4\beta^2 - \theta^2}$$

$$Q_{lf2}^{*^+} = Q_g^{*^+} = \frac{\beta(\alpha - C(\beta - \theta))}{2\beta - \theta}$$

The above lemma shows that if everything is symmetric, where consumers have the same reaction to price changes for each manufacturer and both currencies have the same value, then equilibrium prices and sales are identical.

Lemma 6.8: *The firm will export to foreign market if*

$$I > \frac{(C_l + r)(2\beta_{lf}\beta_g - \theta^2)}{2\alpha_{lf}\beta_g + \theta(\alpha_g + C_g\beta_g)} = I_{z2} \tag{6.41}$$

Otherwise, the firm will sell only to domestic market.

Proof. The firm will export to the foreign market only if $Q_{lf2}^{*+} > 0$. From (6.39) we know that $4\beta_{lf}\beta_g - \theta^2 > 0$, since $\beta_{ij} > \theta$. So, we are left with numerator which must be > 0 which is given in (6.41). □

I_{z2} shows that it is more attractive to export to foreign market as θ, β_g, C_g, α_{lf}, and α_g increase. On the other hand, it becomes less attractive to export to foreign market as β_{lf}, C_l, and r, Table 6.3.a, 6.3.b, and 6.3.c. The interesting outcome is that if the potential demand of the foreign manufacturer increase, the firm has more incentive to enter the foreign market.

Table 6.3.a Impact of θ

θ	I_{z2}
0.1	0.056
0.2	0.053
0.3	0.050
0.4	0.047
0.5	0.043
0.6	0.040
0.7	0.036
0.8	0.033

Table 6.3.b Impact of β_g

β_g	I_{z2}
0.4	0.083
0.5	0.073
0.6	0.066
0.7	0.061
0.8	0.056
0.9	0.053
1.0	0.050
1.1	0.047

Table 6.3.c Impact of α_g

α_g	I_{z2}
160	0.0517
170	0.0514
180	0.0510
190	0.0507
200	0.0504
210	0.0500
220	0.0497
230	0.0494

We are going to assume that the exchange rate is $> I_{z2}$ where the firm can export to foreign market and compete with the foreign manufacturer, then the firm's profit can be written as:

$$\Pi_{l2}^{+} = \frac{\left(\alpha_{lh} - C_l\beta_{lh}\right)^2}{4\beta_{lh}} +$$

$$\frac{\beta_{lf}\left(I(2\alpha_{lf}\beta_g + \theta(\alpha_g + C_g\beta_g)) - (C_l + r)(2\beta_{lf}\beta_g - \theta^2)\right)^2}{I\left(4\beta_{lf}\beta_g - \theta^2\right)^2} \tag{6.42}$$

We notice from (6.42) that the profit generated from home market is independent of the price of the foreign market and the exchange rate since capacity is unlimited. The second term is the profit generated from the sales at the foreign market and this profit is zero if the exchange rate falls below I_{z2}.

The foreign manufacturer's profit can be written as:

$$\Pi_g^{+} = \frac{\beta_g\left(I(\beta_{lf}(\alpha_g - C_g\beta_g) + \theta(\alpha_{lf} + C_g\theta)) + \theta\beta_{lf}(C_l + r)\right)^2}{\left(I(4\beta_{lf}\beta g - \theta^2)\right)^2} \tag{6.43}$$

Lemma 6.9: *The firm will generate more profit than the foreign manufacture at the foreign market if*

$$I > \frac{1}{P_{lf2}^{*+}}\left(\frac{\left(P_g^* - C_g\right)\left(\alpha_g - \beta_g P_g^* + \theta P_{lf2}^{*+}\right)}{\alpha_{lf} - \beta_{lf}P_{lf2}^{*+} + \theta P_g^*} + C_l + r\right) = I_{eq} \tag{6.44}$$

otherwise, if $I_{z2} < I < I_{eq}$, *then the foreign manufacturer will generate more profit.*

Proof. Solving equations (6.43) and (6.44) in terms of exchange rate (*I*).

The above lemma shows how the exchange rate can affect the profit of each manufacturer; if the foreign currency is strong, then the firm is more profitable than the foreign manufacturer because looking at (6.43) we notice that an increase in the exchange rate is disadvantageous for the foreign manufacturer.

Lemma 6.10: *The foreign manufacturer can not make sales if*

$$I > \frac{\theta\beta_{lf}(C_l + r)}{2\beta_{lf}(C_g - \alpha_g) - \theta(\alpha_{lf} + \theta C_g)} = I_v \tag{6.45}$$

where

$$C_g > \frac{2\beta_{lf}\alpha_g + \theta\alpha_{lf}}{2\beta_{lf} - \theta^2} = C_v \tag{6.46}$$

if $C_g < C_v$, then the firm can not force the foreign manufacturer out of the game.

Proof. From (5.7) where $Q_g^{*^+} \leq 0$.

The above lemma shows that if the exchange rate is above a certain thresholds, I_v, then the foreign manufacturer will have zero sales and the firm will be the only player in the game. However, in order for this to occur the cost of production, C_g, must be really high as we notice from (6.46).

Lemma 6.11: *There is no competition if*

$$C_g > \frac{(\beta_{lf}\alpha_g + \theta\alpha_{lf})(4\beta_{lf}\alpha_g - \theta^2)}{4\beta_{lf}^2\beta_{g-}\theta^2(2\beta_{lf} + 3\beta_{lf}\beta_g - \theta^2)} = C_{high} \tag{6.47}$$

Proof. Solving for C_g when $I_{z2} > I_v$

Lemma 6.12: *There exist c_m, such that $I_z > Ieq$.*

$$C_g < \frac{(C_l + r)(2\beta_{lf}\beta_g - \theta^2) - 2\alpha_{lf}\beta_g(I_{eq} - 1) - \theta\alpha_g}{\beta_g I_{eq}}$$

By solving the above equation we can obtain C_m.

Lemma 6.13: $C_{high} > C_m > C_v$

Lemma 6.14: *The interactions between the firm and the foreign manufacturer in terms of the cost of production and exchange rate are as follows:*

1. *if $C_g > C_{high}$ no competition at all, where*
   - *if $I < I_v$ foreign manufacturer's sales only*
   - *if $I_v < I < I_z$ No sales*
   - *if $I_z < I$ firm's sales only*
2. *if $c_m < c_g < c_{high}$, then*
   - *if $I < I_z$ foreign manufacturer's sales only*

- *if $I_z < I < I_v$ advantage for the firm*
- *if $I_v < I$ firm's sales only*

3. *if $C_v < C_g < C_m$, then*
   - *if $I < I_z$ foreign manufacturer's sales only*
   - *if $I_z < I < I_{eq}$ advantage for the foreign manufacturer*
   - *if $I_{eq} < I < I_v$ advantage for the firm*
   - *if $I_v < I$ firm's sales only*

4. *if $C_g < C_v$, then more and more competition*
   - *if $I < I_z$ foreign manufacturer's sales only*
   - *if $I_z < I < I_{eq}$ advantage for the foreign manufacturer*
   - *if $I_{eq} < I < I_v$ advantage for the firm*

Table 6.4 Effect of Exchange Rate on Optimal Prices, Quantities, and Profits

I	P_{lf2}^{*+}	P_g^{*+}	Q_{lf2}^{*+}	Q_g^{*+}	Π_{lf2}^{+}	Π_g^{+}
$I_{z2} = 0.033$	241.78	139.26	0	133.274	0	17760
0.2	138.56	123.78	98.56	117.78	1943	4646
$I_{eq} = 0.382$	128.81	122.32	107.88	116.32	4449	4449
0.8	123.22	121.48	113.22	115.48	10255	4338
$I_p = 1.333$	121.176	121.176	115.17	115.17	17687	4298
2	120.15	121.02	116.15	115.02	26983	4278

The above table illustrates lemmas 6.9, 6.10, and 6.14 where it shows the impact of the exchange rate thresholds on optimal prices, optimal quantities, and profits. We notice at $I \leq I_{z2}$, the firm does not export to the foreign market and the foreign manufacturer generates the maximum profit because it is the only player in the market. When exchange rate increases (above I_{z2}), the firm starts exporting to the foreign market. Then, the foreign manufacturer starts decreasing its price due to the competition, however, the foreign manufacturer still generates more profit than the firm as long as $I < I_v$. At $I = I_v$, both manufacturers generates the same profits, however, prices are different because $I_v \neq I_p$. At $I = I_p$, both optimal prices and quantities are equal, however the firm generates more profit due to a high value of exchange rate. Clearly, as exchange rate increases the foreign manufacturer's profit converges while the firm's profit increases linearly, Figure 6.6.

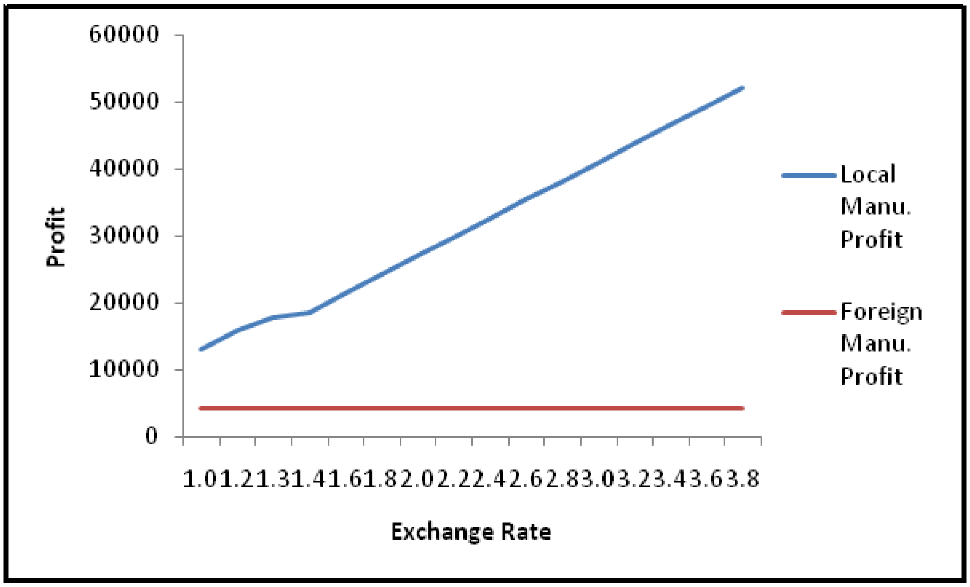


Figure 6.6 Effect of Exchange Rate on Profits

6.4.2 The Endogenous Model with Limited Capacity

Now, we assume that the firm's capacity is limited. However, we are going to assume that the foreign manufacturer has unlimited capacity. This will give an advantage to the foreign manufacturer. However, we would like to see how the firm is going to react to this leverage. Unlike the ample capacity case, the firm must decide on its capacity allocation to maximize the net profit. We expect the firm to miss some sales opportunities due to capacity limitation. The assumption is still valid the firm will export to the foreign market if $I > I_{z2}$ and if the capacity level is above the home demand (Q^{*+}_{lh1}).

The firm has to look at the available capacity and the spot exchange rate. In this model we are going to assume two capacity levels: expensive capacity and medium capacity. Under each capacity level, the firm must decide on capacity allocation. The process is as follow: depending on the capacity cost, that manufacturer determines the capacity level, then the firm looks at the spot exchange rate and decide on allocation of capacity. Later, we will discuss all allocation scenarios.

6.4.2.1 The Model

In this section, we will drive the optimal prices and quantities for the firm and the foreign manufacturer. By substituting (6.38) in (.20) and (6.21) we get:

$$P_{lh2}^{*^-} = \frac{I\beta_{lf}\beta_g(4\alpha_{lh} + 2\alpha_{lf} - 4K_{l2} + \theta C_g)}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)} + \frac{I\theta(\theta(\alpha_{lh} + K_{l2}) + \beta_{lf}\alpha_g)}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)} + \frac{\beta_{lf}(\alpha_{lh} - \beta_{lh}r)(2\beta_{lf}\beta_g - \theta^2)}{\beta_{lh}(I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2))} \tag{6.48}$$

$$P_{lf2}^{*^-} = \frac{I\beta_{lh}(2\beta_g\alpha_{lf} + \theta(\alpha_g + C_g\beta_g))}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)} + \frac{2\beta_{lf}(\beta_g(\alpha_{lh} + 2\alpha_{lf} - 2K_{l2} + \theta C_g + \beta_{lh}r) + \theta\alpha_g)}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)} \tag{6.49}$$

Substituting $P_{lf2}^{*^-}$ in $P_g^{*^+}$ we get

$$P_g^{*^-} = \frac{I\beta_{lh}(2\beta_{lf}(\alpha_g + \beta_g C_g) + \theta\alpha_{lf})}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)} + \frac{\beta_{lf}((2\beta_{lf}(\alpha_g + \beta_g C_g) + \theta\alpha_{lf}) + \theta(\alpha_{lh} + 2\alpha_{lf} - 2K_{l2} + \beta_{lh}r))}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)} \tag{6.50}$$

Thus,

$$Q_{lh2}^{*^-} = \frac{I\beta_{lh}((\beta_{lf}\beta_g)(4K_{l2} - 2\alpha_{lf} - \theta C_g) - \theta(\beta_{lf}\alpha_g + \theta K_{l2}))}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)}$$

$$+ \frac{\beta_{lf}(\alpha_{lh} + \beta_{lh}r)(2\beta_{lf}\beta_2 - \theta^2)}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)} \tag{6.51}$$

$$Q_{lf2}^{*^-} = \frac{I\beta_{lf}\beta_{lh}(\beta_g(2\alpha_{lf} + \theta C_g) + \theta\alpha_g)}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)}$$

$$- \frac{\beta_{lf}(\alpha_{lh} - 2K_{l2} + \beta_{lh}r)(2\beta_{lf}\beta_2 - \theta^2)}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)} \tag{6.52}$$

$$Q_g^* = \frac{\beta_g(I\beta_{lh}\theta(\alpha_{lf} + \theta C_g) + 2\beta_{lf}(\alpha_g - \beta_g C_g)(I\beta_{lh}\beta_{lf} + 1))}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)}$$

$$+ \frac{\beta_g\beta_{lf}\theta(\alpha_{lh} + 2\alpha_{lf} - 2K_{l2} + 2\theta C_g + \beta_{lh}r)}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)} \tag{6.53}$$

Substituting P_g^{*+} in (6.19), we get

$$\lambda_{l2} = \frac{I(2\beta_{lf}\beta_g(2\alpha_{lh} + 2\alpha_{lf} - 4K_{l2} + \theta C_g - 2\beta_{lh}C_l) + 2\theta\beta_{lf}\alpha_g)}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)}$$

$$+ \frac{I\theta^2(-\alpha_{lh} + 2K_{l2} + \beta_{lh}C_l) - 2\beta_{lf}(C_l + r)(2\beta_{lf}\beta_g - \theta^2)}{I\beta_{lh}(4\beta_{lf}\beta_g - \theta^2) + \beta_{lf}(4\beta_{lf}\beta_g - 2\theta^2)} \tag{6.54}$$

Hence, if we assume that demand base and consumer price sensitivity are the same for both manufacturers at the foreign market, then the optimal prices and quantities can be written as:

$$P_{lh2}^{*^-} = \frac{I\beta^2(4\alpha_{lh} + 4\alpha - 8K_{l2} + 2\theta C_g - 4C_l\beta_{lh})}{(\beta + I\beta_{lh})(4\beta^2 - \theta^2)}$$

$$- \frac{I(\theta^2(\alpha_{lh} - K_{l2}) - \theta\beta\alpha)}{(\beta + I\beta_{lh})(4\beta^2 - \theta^2)}$$

$$+\frac{\beta(4\beta^2(\alpha_{lh}-\beta_{lh}C_l)-\theta^2(\alpha_{lh}-\beta_{lh}(C_l+r)))}{2\beta_{lh2}(\beta+I\beta_{lh})(4\beta^2-\theta^2)}$$

$$P_{lf2}^{*^-}=\frac{I(\beta_{lh}(2\beta\alpha+\theta\alpha+\theta\beta C_g)+2\beta^2\alpha_{lh})}{4\beta^2(\beta+I\beta_{lh})}$$

$$+\frac{2\beta^2(2\alpha-2K_{l2}+\theta C_g+\beta_{lh}r)}{4\beta^2(\beta+I\beta_{lh})}$$

$$-\frac{\theta^2(\alpha_{lh}-2K_{l2}-C_l\beta_{lh})+4\theta\beta\alpha}{2(\beta+I\beta_{lh})(4\beta^2-\theta^2)}$$

$$\frac{\theta^2(C_l+r)}{I(\beta+I(t)\beta_{lh})(4\beta^2-\theta^2)}$$

$$P_g^{*^-}=\frac{I\beta_{lh}(2\beta(\alpha+\beta C_g)+\theta\alpha)}{I\beta_{lh}(4\beta^2-\theta^2)+\beta(4\beta^2-2\theta^2)}+$$

$$\frac{\beta_{lf}((2\beta(\alpha+\beta C_g)+\theta\alpha_{lf})+\theta(\alpha_{lh}+2\alpha-2K_{l2}+\beta_{lh}r))}{I\beta_{lh}(4\beta^2-\theta^2)+\beta(4\beta^2-2\theta^2)}$$

Thus,

$$Q_{lh2}^{*^-}=\frac{I\beta_{lh}((\beta^2)(4K_{l2}-2\alpha-\theta C_g)-\theta(\beta\alpha+\theta K_{l2}))}{(\beta+I\beta_{lh})(4\beta^2-\theta^2)}$$

$$+\frac{\beta(4\beta^2(\alpha_{lh}+\beta_{lh}r)-\theta^2\beta(\alpha_{lh}+\beta_{lh}(C_l+2r)))}{2(\beta+I\beta_{lh})(4\beta^2-\theta^2)}$$

$$Q_{lf2}^{*^-}=\frac{I\beta\beta_{lh}(2\alpha(2\beta+\theta)+\theta\beta C_g)}{(\beta+I\beta_{lh})(4\beta^2-\theta^2)}$$

$$\frac{\beta(4\beta^2(2K_{l2}-\alpha_{lh}-\beta_{lh}r)+\theta^2(\alpha_{lh}-2K_{l2}+\beta_{lh}(C_l+r)))}{2(\beta+I\beta_{lh})(4\beta^2-\theta^2)}$$

$$Q_g^* = \frac{\beta(I\beta_{lh}\theta(\alpha+\theta C_g)+2\beta(\alpha-\beta C_g)(I\beta_{lh}\beta+1))}{I\beta_{lh}(4\beta^2-\theta^2)+\beta_{lf}(4\beta^2-2\theta^2)}$$

$$+\frac{\beta^2\theta(\alpha_{lh}+2\alpha-2K_{l2}+2\theta C_g+\beta_{lh}r)}{I\beta_{lh}(4\beta^2-\theta^2)+\beta(4\beta^2-2\theta^2)}$$

Proposition 6.4*: The firm's capacity is fully utilized if and only if*

$$I \geq \frac{2\beta_{lf}(C_l+r)\left(2\beta_{lf}\beta_g-\theta^2\right)}{2\beta_{lf}\beta_g(2\alpha_{lh}+2\alpha_{lf}-4K_{l2}+\theta C_g-2\beta_{lh}C_l)+\theta^2(-\alpha_{lh}+2K_{l2}+C_l\beta_{lh})+2\theta\beta_{lf}\alpha_g} = I_{t2} \tag{6.55}$$

or

$$K_{l2} < \frac{2\beta_{lf}\beta_g(2\alpha_{lh}+2\alpha_{lf}+\theta C_g-2C_l\beta_{lh})+\theta^2(-\alpha_{lh}+C_l\beta_{lh})+2\theta\alpha_g\beta_{lf}}{2\left(4\beta_{lf}\beta_g-\theta^2\right)} = K_{t2} \tag{6.56}$$

Otherwise, the firm will sell constrained optimal.

Proof. We drove I_{t2} via $\lambda_{l2} \geq 0$. We drove K_{t2} via $I_{t2} > 0$. □

The above lemma shows that the firm can implement the unconstrained solution and hence does not need to split her capacity across markets if $I(t) < I_{t2}$. In contrast, when the inequality in (6.55) does not hold the capacity is limited. As such, the firm faces the problem of allocating the capacity between the two markets. Depending on the market potentials and the realized exchange rates the firm may also choose to sell only in one of the markets. The next proposition indicates that if the exchange rate is significantly high, the firm in fact may opt out of the domestic market.

Proposition 6.5*. The firm chooses to sell in the domestic market if and only if*

$$I < \frac{\beta_{lf}(\alpha_{lh}+\beta_{lh}r)(2\beta_{lf}\beta_2-\theta^2)}{\beta_{lh}((\beta_{lf}\beta_g)(2\alpha_{lf}-4K_{l2}+\theta C_g)+\theta(\beta_{lf}\alpha_g+\theta K_{l2}))} = I_{h2} \tag{6.57}$$

otherwise, the firm sells only in the foreign market. On the other hand, she will export her product to the foreign market if and only if

$$I > \frac{(\alpha_{lh} - 2K_{l2} + \beta_{lh}r)(2\beta_{lf}\beta_2 - \theta^2)}{\beta_{lh}(\beta_g(2\alpha_{lf} + \theta C_g) + \theta\alpha_g)} = I_{f2} \tag{6.58}$$

Hence, the firm splits the capacity between the domestic and foreign markets only when $I_{f2} < I(t) < I_{h2}$.

Proof. The inequality in (6.57) directly follows from the fact that the manufacturer will sell to the domestic market if $Q^{*-}_{lh2} > 0$ implying that the numerator in (6.51) must be non-negative. This is satisfied when (6.57) holds. Similarly, $Q^{*-}_{lf2} > 0$ implies that (6.52) holds. Next we show that $I_{f2} < I_{h2}$. First observe that $I_{f2} < I_{h2}$ implies that

$$\frac{\beta_{lf}(\alpha_{lh} + \beta_{lh}r)}{((\beta_{lf}\beta_g)(2\alpha_{lf} - 4K_{l2} + \theta C_g) + \theta(\beta_{lf}\alpha_g + \theta K_{l2}))} > \frac{(\alpha_{lh} - 2K_{l2} + \beta_{lh}r)}{(\beta_g(2\alpha_{lf} + \theta C_g) + \theta\alpha_g)},$$

which can be rewritten as

$$2\beta_{lf}\beta_g(2\alpha_{lh} + 2\alpha_{lf} + \theta C_g) + \theta(2\beta_{lf}\alpha_g + \alpha_{lh} + \beta_{lh}r) > 2K_{l2}(4\beta_{lf}\beta_g + \theta).$$

It is clear that the above inequality holds. □

We notice from (6.57) that the manufacturer will always sell to the domestic market if $K_{l2} \geq \frac{\beta_{lf}(\theta(\alpha_g + \beta_g C_g) + 2\beta_g\alpha_{lf})}{4\beta_{lf}\beta_g - \theta^2} = K_{lh2}$. Clearly, when the capacity is ample it is always profitable to sell in the domestic market. However when capacity is scarce, as the foreign exchange rate increases, it becomes less appealing to allocate capacity to the domestic market. Similarly, we observe from (6.58) that the manufacturer exports to the foreign market at any given time if $K_{l2} \geq \frac{\alpha_{lh} + \beta_{lh}r}{2}$. Therefore, low transportation cost and home potential demand makes the foreign market appealing to the firm. We also notice that how completion can increase the appeal to enter the foreign market especially when cost of production (C_g) potential demand, alpha (α_g) for the foreign manufacturer and cross price effect (θ) increase. Proposition 6.5 implies that the optimal policy for the manufacturer at any given period is a "cherry-picking" policy when the capacity is scarce. Namely, when the exchange rate is significantly high the manufacturer sells only in the foreign market whereas when it is too low she will serve exclusively the domestic market. For exchange rates not too high or low, the capacity is split between the markets. The optimal splitting policy is investigated next. Now, we will show how exchange rate thresholds are related.

Lemma 6.15: *For any given* K_{l2}, $I_{h2} > I_{t2}$

Proof. The proof is similar to the proof of Lemma 6.2 □

Lemma 6.16: *For any given capacity level,* $I_{h2} > I_{zf2}$

Proof. The proof is similar to the proof of Lemma 6.3 □

Lemma 6.17: $I_{z2} > I_{f2}$ *for* $K_{l2} > Q^{*+}_{lh1}$ *otherwise* $I_{f2} > I_{z2}$

Proof. The proof is similar to the proof of Lemma 6.4 □

Lemma 6.18: $I_{t2} > I_{z2}$ *if and only if* $K_{l2} > Q^{*+}_{lh1}$

Proof. The proof is similar to the proof of Lemma 6.5 □

6.4.2.2 The Endogenous Model Optimal Capacity

Based on Proposition 6.5 we conclude that at the beginning of the planning horizon the optimal pricing and allocation policy falls in one of the following five scenarios:

HU: The capacity is ample (unconstrained) and the firm sells only in the domestic market.

SU: The capacity is ample (unconstrained) and the firm sells in both markets.

HC: The capacity is limited and the firm allocates all of its capacity to the domestic market.

FC: The capacity is limited and the firm allocates all of its capacity to the foreign market.

SC: The capacity is limited and the firm splits all of its capacity between the two markets.

First we note that when capacity is ample, i.e., $K_{l2} \geq K_{t2}$, there is always incentive for the firm to sell in the domestic market. From Lemma 6.8, the firm sells in the foreign market in the beginning of the planning horizon if the exchange rate is above the threshold given in (6.41). In case of scarce capacity any one of the three scenarios (home only, foreign only or split capacity) may occur depending on the exchange rate as shown in Proposition 6.5.

Suppose that capacity is too scarce (high-cost case), that is, lower than Q_{lh1}^{*+} given in (6.9). This situation is more likely when the cost of capacity is too high that the firm cannot fully satisfy the demand in the domestic market alone. In this case, the capacity is fully utilized. As depicted in Figure 6.7, the firm allocates all of its capacity to the domestic market (HC) if the exchange rate is below I_{f2}. In this case the low exchange rate does not justify exporting to the foreign market. The firm chooses to split the capacity between two markets (SC) if the exchange rate falls between I_{h2} and I_{f2}. The optimal allocation is then given by (6.51) and (6.52). If the exchange rate is too high (above I_{h2}) the revenue from sales in the foreign market justifies the full allocation of the capacity to this market (FC).

Under this scenarios *exante* expected profit function before the start of the selling season can be written as follows:

$$\underset{I(t)>I_{zf}, K_{l2}<Q_{lh1}^{*+}}{\text{Maximize}} \Pi_{l2}^{-} = -u_l K_{l2} + \int_0^{I_{f2}} \Pi_{HC} A(I) dI$$

$$+ \int_{I_{f_2}}^{I_{h2}} \Pi_{SC} A(I) dI + \int_{I_{h2}}^{\infty} \Pi_{FC} A(I) dI \qquad (6.59)$$

Where:

$$\Pi_{HC} = \frac{\alpha_{lh} - K_{l2} - \beta_{lh} C_l}{\beta_{lh}} * K_{l2},$$

$$\Pi_{SC} = (P^{*-}_{lh2} - C_l)Q^{*-}_{lf2} + (I(t)P^{*-}_{lf2} - C_l - r)Q^{*-}_{lf2}\text{, and}$$

$$\Pi_{FC} = \frac{I(\alpha_{lh} - K_{l2} + \theta P^*_g) - \beta_g(C_l + r)}{\beta_{lf}} * K_{l2}$$

To find the optimal capacity, we need to take the derivative of (6.59) with respect to K_{l2}

$$\frac{d\Pi^-_{l2}}{dK_{l2}} = -u_l + \int_0^{\frac{I_{f2}}{I_o}} \frac{\alpha_{lh} - K_{l2} - \beta_{lh}C_l}{\beta_{lh}} f(\epsilon)d\epsilon + \int_{\frac{I_f}{I_o}}^{\frac{I_{h2}}{I_o}} \lambda_{l2} f(\epsilon)d\epsilon + \int_{\frac{I_{h2}}{I_o}}^{\infty} \frac{I(\alpha_{lh} - K_{l2} + \theta P^*_g) - \beta_g(C_l + r)}{\beta_{lf}} f(\epsilon)d\epsilon = 0 \qquad (6.60)$$

where λ_{l2} is the Lagrangian multiplier defined in (6.54). It is straightforward to see from the function above that the second derivative with respect to K_{l2} is strictly negative for any $K_{l2} \geq 0$ implying concavity. Hence solution to (6.60) gives the unique optimal value for the capacity to be built at home in advance of the selling season. Unfortunately, there is no close form solution. However, the optimal capacity can be calculated easily with a simple line search.

Recall that the condition given in (6.60) applies when the capacity scarce. This is expected to occur when capacity is very expensive. Suppose that capacity is bigger than home demand and K_{lh2}. Then, the allocation decision follows Figure 6.7:

(1) if the spot exchange rate is below I_{z2}, then the manufacturer satisfies all the demand in the domestic market and stays out of the foreign market (Lemma 6.8).

(2) if the spot exchange rate falls between I_{z2} and I_{t2}, then the manufacturer satisfies all demand in both markets (Proposition 6.4).

(3) if the spot exchange rate falls between I_{t2} and I_{h2}, then the manufacturer splits its limited capacity between the two markets (Proposition 6.5).

(4) if the spot exchange rate is higher than I_{h2}, then the manufacturer allocates all the capacity to the foreign market (Proposition 6.5).

Consequently, the manufacturer's net profit in this case can be written as:

$$\underset{I>I_z, Q_{lf2}^{*+}+Q_{lh2}^{*+}>K_{l2}>Q_{lh1}^{*+}}{\text{Maximize}} \Pi_{l2}^{-} = -u_l K_{l2} + \int_0^{\frac{I_{z2}}{I_o}} \Pi_{HU} f(\epsilon) d\epsilon +$$

$$\int_{\frac{I_{z2}}{I_o}}^{\frac{I_{t2}}{I_o}} \Pi_{SU} f(\epsilon) d\epsilon + \int_{\frac{I_{t2}}{I_o}}^{\frac{I_h}{I_o}} \Pi_{SC} f(\epsilon) d\epsilon + \int_{\frac{II_h}{I_o}}^{\infty} \Pi_{FC} f(\epsilon) d\epsilon \qquad (6.61)$$

We want to find the optimal capacity, we need to take the derivative of equation (6.51) with respect to K_{l2} and we find:

$$\frac{d\Pi_{l2}^{-}}{dK_{l2}} = -u_l + \int_{\frac{It1}{I_o}}^{\frac{I_{h2}}{I_o}} \lambda_{l2} f(\epsilon) d\epsilon +$$

$$\int_{\frac{I_{h2}}{I_o}}^{\infty} \frac{I_o \epsilon_i \left(\alpha_{lf} - 2K_{l2} + \theta P_g^*\right) - \beta_{lf}(C_l + r)}{\beta_{lf}} f(\epsilon) d\epsilon = 0 \qquad (6.62)$$

Now, remember that in (6.62) the capacity is somehow not scares, however the firm above certain exchange rate threshold, I_{h2}, will dedicate its capacity to the foreign market.

Lets assume that capacity falls between K_{h2} and K_{t2} where the firm will always satisfy demand at home market. Then, the allocation decision follows Figure 6.7:

(1) if the spot exchange rate is below I_{z2}, then the manufacturer satisfies all the demand in the domestic market and stays out of the foreign market (Lemma 6.8).

(2) if the spot exchange rate falls between I_{z2} and I_{t2}, then the manufacturer satisfies all demand in both markets (Proposition 6.4).

(3) if the spot exchange rate falls between I_{t2} and I_{h2}, then the manufacturer splits its limited capacity between the two markets (Proposition 6.5).

Consequently, the manufacturer's net profit in this case can be written as:

$$\underset{K_{t2}>K_{l2}>K_{h2}}{\text{Maximize}}\ \Pi_{\overline{l2}} = -u_l K_{l2} + \int_0^{\frac{I_{z2}}{I_o}} \Pi_{HU} f(\epsilon) d\epsilon +$$

$$\int_{\frac{I_{z2}}{I_o}}^{\frac{I_{t2}}{I_o}} \Pi_{SU} f(\epsilon) d\epsilon + \int_{\frac{I_{t2}}{I_o}}^{\infty} \Pi_{SC} f(\epsilon) d\epsilon \tag{6.63}$$

We want to find the optimal capacity, we need to take the derivative of equation (6.63) with respect to K_{l2} and we find:

$$\frac{d\Pi_{\overline{l2}}}{dK_{l2}} = -u_l + \int_{\frac{It2}{I_o}}^{\infty} \lambda_{l2} f(\epsilon) d\epsilon \tag{6.64}$$

Although we cannot find closed form solutions for (6.60), (6.62) and (6.64) we can derive the following conclusions from concavity

Proposition 6.6 *There exit two capacity cost thresholds* u_l^{t1} *and* u_l^{t2} *such that* $u_l^{t1} > u_l^{t2}$.

if $u_l > u_l^{t1}$, *then* K_{l2}^* *will be obtained from (6.60), if* $u_l^{t2} > u_l > u_l^{t1}$, *then* K_{l2}^* *will be obtained from (6.62), and if* $u_l^{t2} > u_l$, *then* K_{l2}^* *will be obtained from (6.64)*

The above proposition shows that when capacity cost is expensive, then the optimal capacity falls between 0 and $Q_{lh1}^{*^{*+}}$. However, as capacity cost get cheaper, then optimal capacity falls between $Q_{lh1}^{*^{*+}}$ and K_{h1} if we assume that $K_{h2} > Q_{lh1}^{*^{*+}}$. Otherwise, the optimal capacity falls between $Q_{lh1}^{*^{*+}}$ and K_{t2}.

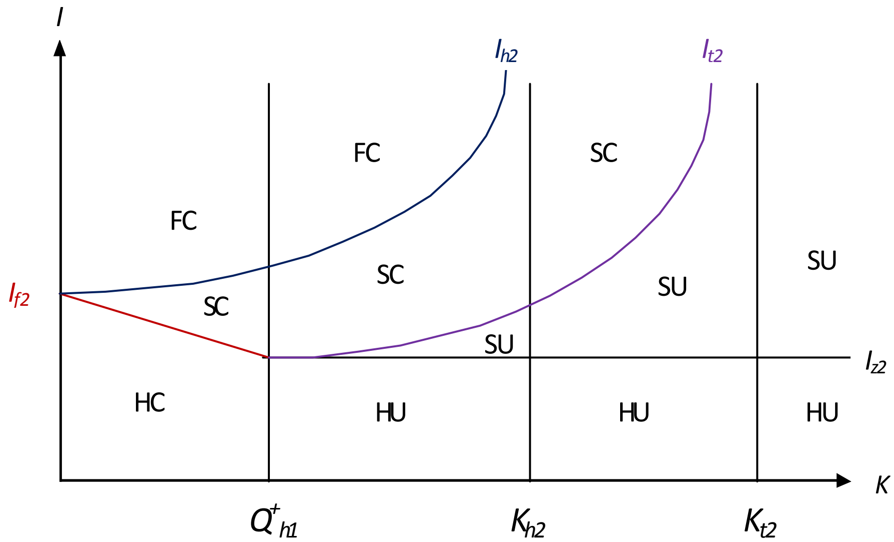


Figure 6.7 Optimal Capacity Allocation

6.4.2.3 Discussion

As we mentioned in the previous section that equations (6.60), (6.62) and (6.64) can not be solved analytically; therefore we performed a numerical analysis to get some insight on the effect of certain parameters on the optimal capacity where u=100, α_{lh}=200, α_{lf}=200, α_g=200, β_{lh}=1, β_{lf}=1, β_g=1, σ=0.1, 0.2, and 0.3, C_l=6, C_g=6, I_o=1, r=2, and θ=0.3

We notice the optimal capacity decreases in σ and β_g (Table 6.5 (a, b, and c) and Figure 6.8) and it increases in α_g, θ, and μ (Table 6.5 (a, b, and c) and Figure 6.8).

Table 6.5.a Impact of β_g

β_g	K_{l2}
0.4	142.12
0.5	131.87
0.6	125.17
0.7	120.45
0.8	116.94
0.9	114.23
1.0	112.08
1.1	110.32

Table 6.5.b Impact of α_g

α_g	K_{l2}
160	109.01
170	109.77
180	110.54
190	111.31
200	112.08
210	112.84
220	113.61
230	114.38

Table 6.5.c Impact of θ

θ	K_{l2}
0.2	54.81
0.3	62.53
0.4	71.52
0.5	82.01
0.6	94.28
0.7	108.70
0.8	125.78
0.9	146.22

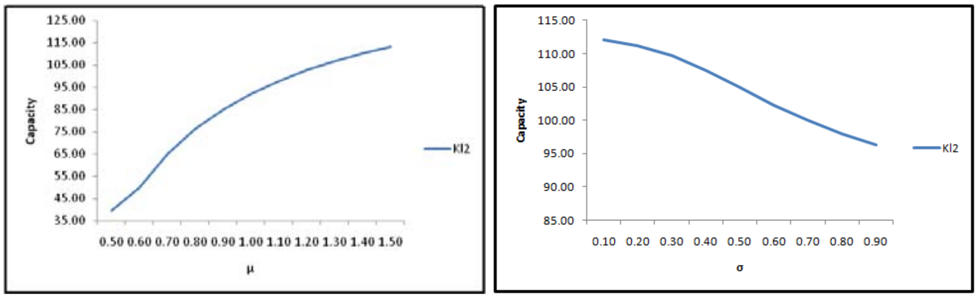


Figure 6.8 Effect of μ and σ on Optimal Capacity (K_{l2})

Chapter 7

Conclusions and Future Work

7.1 Summary of Key Results

Exchange rate fluctuations and competition impact the manufacturers' decision to enter a foreign market for exporting a product or even building capacity for manufacturing. In the Monopoly setting, the former scenario almost all the costs of overhead, production and transportation are incurred in home currency while the realized revenue is based on the foreign currency. In the latter case, most of the associated costs are realized in units of the foreign currency. As such, the behavior of the exchange rate between the home and foreign countries play a substantial role in the manufacturer's strategical and operational decisions. Another factor that significantly affects the firm's strategy is the length of the selling season and the commitment requirements of the markets. Often times the capacity investment must be finalized at the beginning of the season. When the manufacturer has the flexibility of postponing its pricing and production decisions throughout the selling season, the fluctuations in exchange rate can be hedged by adjusting the price and quantity of the product every period. However, this is not possible if these decisions must be locked at the beginning and can't be changed throughout the selling season. In this case, an alternative way of hedging may be to invest for capacity in the foreign market to fill the demand locally. This paper investigates the interplay among important factors such as exchange rate, cost of capacity and production, demand, and selling season length on the manufacturing's decisions regarding production capacity investment, pricing, and production policies in international markets.

The study first considers the *early commitment model with central sourcing*, where the manufacturer builds all the production capacity in the home country and allocate the production between the domestic and foreign markets. In this case all decisions including capacity, price, and production amount are decided and fixed at the beginning of the selling season based on the prospect of the behavior of the exchange rate over multiple periods. Our results show that entering the foreign market with central sourcing is preferable for the firm if the exchange rate drift is above a certain level. This is intuitive since high values for these parameters increase the ROI. On the other hand, increased costs and higher sensitivity to price in the market makes it more difficult for the firm to participate in the foreign market. We also observe that with positive exchange rate drift a longer selling season is more appealing for the manufacturer for entering the foreign market. On the other hand, when the drift is negative, the manufacturer will be more reluctant to enter the foreign market as the length of the selling season increases. In this setting, the home market prices (production quantities) are independent of the exchange rate fluctuations in the foreign market but decrease (increase) in selling season length.

Under *postponed commitment model with central sourcing*, the manufacturer has the flexibility of postponing the price and production decisions until the foreign exchange rate is observed in each period. However it still has to make its capacity investment decision before the beginning of the selling season. As such, the manufacturer faces the problem of allocating its capacity across markets in each period. Consequently, not only the prices and quantities fluctuate in the foreign market but also home market prices and quantities change every period as well. We observe that optimal capacity increases in drift on the exchange rate, demand potential, and number of periods, and decreases in price sensitivity, transportation cost, discount factor, exchange rate volatility, and capacity cost. Our study derives the conditions on the exchange rate movement for varying optimal allocation policies.

The manufacture is more likely to split capacity across domestic and foreign markets when: drift on the exchange rate is positive, planing horizon is long, capacity cost is less expensive, price sensitivity in the domestic market is high, domestic market potential is low, and production cost is low. This setting is substantially better for the manufacturer when the exchange rate movements are highly volatile.

Our analysis also investigates the *local sourcing* policy where the manufacturer opens plants in both markets. We first observe that under this policy all variable costs are realized in local currencies. As such postponement does not offer any advantage. The home market prices and quantities are identical to the early commitment case with central sourcing and similarly are independent of the exchange rate movements. We identify the thresholds on investment costs, market potentials, exchange rate drifts, and selling season length for choice of this option for the manufacturer.

In the Competition setting, we showed the impact of competition on optimal prices, quantities, and capacity under two scenarios: *Exogenous Model* and *Endogenous Model*. Under the Exogenous model, we showed the impact of the foreign manufacturer price on local sales and the firm's decision to enter the foreign market. As the foreign manufacturer's price increases, it becomes more appealing for the firm to enter the foreign manufacturer, sales at home country decreases, and the optimal capacity increases. Also, as exchange rate volatility increases, optimal capacity decreases.

Under the *Endogenous Model,* where capacity is ample, it becomes attractive to export to the foreign market when "cross-price", cost per unit for the foreign manufacturer, potential demand at the foreign country, and consumer sensitivity for the foreign price increases. We presented exchange rate thresholds when the firm generate more money than the foreign manufacturer, and when the prices at the foreign country are identical. Also, we showed that high cost per unit for the foreign manufacturer makes the firm the only player at the foreign market. When capacity is ample, sales at the home market stays constant. However, when capacity is limited, the firm must allocate its capacity properly between the two markets to attain optimal profit. We drove exchange rate thresholds where: the firm split capacity between the two markets, sell only to home market, and sell only to foreign market. We showed that an increase in the drift of exchange rate, potential demand, and "cross-price" increase optimal capacity. On the other hand, an increase in consumer price sensitivity, and exchange rate volatility decreases optimal capacity.

7.2 Future Work

The future work is to make demand uncertain. Also, using different demand functions such as Cobb-Douglas function (see Gallego and Hu (2006)), the CES function (see Varian (1992)), or the Transcendental Logarithmic (translog) function (see Christensen et al. (1973)). In addition to exchange rate, we also aim to integrate the impact of after-tax profit calculations and tariffs into our analysis.

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