University of Miami
Scholarly Repository

Open Access Dissertations | Electronic Theses and Dissertations

2017-05-17

A Game Theoretical Approach for Load-Shifting and Energy Storage in the Smart Grid

Eeyad Al-Ahmadi
University of Miami, e.alahmadi@umiami.edu

Follow this and additional works at: https://scholarlyrepository.miami.edu/oa_dissertations

Recommended Citation
Al-Ahmadi, Eeyad, "A Game Theoretical Approach for Load-Shifting and Energy Storage in the Smart Grid" (2017). *Open Access Dissertations*. 1828.
https://scholarlyrepository.miami.edu/oa_dissertations/1828

This Embargoed is brought to you for free and open access by the Electronic Theses and Dissertations at Scholarly Repository. It has been accepted for inclusion in Open Access Dissertations by an authorized administrator of Scholarly Repository. For more information, please contact repository.library@miami.edu.

UNIVERSITY OF MIAMI

A GAME THEORETICAL APPROACH FOR LOAD-SHIFTING AND ENERGY STORAGE IN THE SMART GRID

By

Eeyad Mohammed Al-ahmadi

A DISSERTATION

Submitted to the Faculty
of the University of Miami
in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

Coral Gables, Florida

May 2017

©2017
Eeyad Mohammed Al-ahmadi
All Rights Reserved

UNIVERSITY OF MIAMI

A dissertation submitted in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

A GAME THEORETICAL APPROACH FOR LOAD-SHIFTING AND ENERGY STORAGE IN THE SMART GRID

Eeyad Mohammed Al-ahmadi

Approved:

Murat Erkoc, Ph.D. Associate Professor of Industrial Engineering	Shihab Asfour, Ph.D. Professor of Industrial Engineering
Nurcin Celik, Ph.D. Associate Professor of Industrial Engineering	Guillermo Prado, Ph.D. Dean of the Graduate School

Moataz Eltoukhy, Ph.D.
Assistant Professor of Kinesiology and Sport Sciences

AL-AHMADI, EEYAD MOHAMMED (Ph.D., Industrial Engineering)
A Game Theoretical Approach for Load-Shifting and Energy Storage in the Smart Grid (May 2017)

Abstract of a dissertation at the University of Miami.

Dissertation supervised by Professor Murat Erkoc.
No. of pages in text. (138)

This study presents an electricity market composed of a single energy provider and multiple customers to evaluate the effects of energy storage and load-shifting as part of a smart grid demand response through the use of Stackelberg game models. In these Stackelberg game models, the energy provider is the leader and the customers are the followers. The leader moves first and offers price discounts across different time slots to motivate customers to shift their consumption away from peak consumption periods. The followers respond by deciding whether or not to shift their consumption from their nominal demand, and how much of their load they should shift. In this model, the aim of the energy provider is to maximize its profits, while the consumers aim to minimize their total costs related both to the energy consumption and inconvenience of deviating from the nominal demand. Within this setting, a procedure is proposed to obtain equilibrium outcomes. We begin by evaluating the effects that different types of customers, a market with different degrees of diversity, and a market of different sizes have on the equilibrium discounts and the peak-to-average ratios (PAR). We then continue into a second model in which we incorporate individual inconvenience costs for each customer and each period and evaluate the effects that having a homogeneous market or a heterogeneous market has on the equilibrium discount and PAR. Finally, we introduce a customer side energy storage and evaluate the effects that this system has on the

equilibrium discount and PAR in both cases, when it is controlled by the energy provider or when it is controlled by the customers. Our results show that price discounts provide significant leverage to the energy generator and that the use of energy storage is very effective in the reduction of the peak-to-average ratios. The use of both of these tactics provides effective ways to improve profits. Furthermore, when the energy provider controls the energy storage, it deploys them more effectively and achieves its maximum profits and the lowest PAR. When the customers control the energy storage, the equilibrium discounts are higher, but the PAR is also higher. Lastly, our results show that the use of load-shifting always reduces the customers' total costs, but this reduction is diminished by the implementation of energy storage.

Dedication

This dissertation is dedicated to my parents, my wife, my sisters and brother, and my kids.

Acknowledgments

I am grateful and thankful to God, the most kind, for all opportunities and successes. I praise God for giving me the power and capability to reach another step in my life. “Praise be to Allah.”

I would also like to thank my parents. My lovely mother, Badriyyah Alahmadi: You taught me how to live my life with love. I remember how happy you were when I told you that I decided to do my Ph.D. Mom, it will be soon. My inspirational father, Dr. Mohammed Alahmadi: You taught me how to live my life with wisdom. Father and Mother, I always feel your continuous prayers; I do not know how to express all my feelings and thanks for the support and encouragement you have given to me.

Dr. Murat Erkoc, thank you for your guidance during my Ph.D. journey. I have been extremely lucky to have a supervisor who cared so much about my research and who responded to my questions so promptly. Thank you for the patient guidance, encouragement, and advice you have provided throughout. I could not have imagined having a better advisor and mentor for my Ph.D.

In the same vein, I would like to extend great thanks to Dr. Shihab Asfour. Thank you for giving me the opportunity to work and participate in the Industrial Assessments Center at The University of Miami. Your guidance and knowledge has been a major factor in my ability to excel in my research. I appreciate the fact that you always made yourself available. More importantly, thank you for warmly welcoming me into your office any time I needed help.

I would like to thank my thesis committee members, Dr. Nurcin Celik and Dr. Moataz Eltoukhy, for all of their guidance, encouragement, insightful comments, and hard questions throughout this process.

Thanks to my lovely wife, Noha Almarwni: Thank you for always standing by my side during this journey. Thank you for being patient and for taking care of everything while I was busy. You gave me the support and the environment that I needed. Thank you for taking care of our kids while I was studying. Your support has given me the time to study without having to worry.

Thanks to my sister, Hadeel, and my brother, Hammam, for supporting me and for taking care of our parents and covering my responsibilities during my absence. My younger sister, Tasneem: You were standing beside me whenever I needed help. Thank you for your support and help. Sisters and brother, your thoughtfulness is greatly appreciated and will never be forgotten. Thanks to my kids, Mohammed and Jana: I am sorry for not spending enough time with you; thank you for your patience with me.

Thanks to my best friend, Abdulkader Hanbazazah, for being by my side, and for supporting me every step of the way. I can never thank you enough for being such a wonderful and caring friend. Thank you so much for being with me in my joys and sorrows. I thank you for treating our friendship as a responsibility, and for being the mirror which hides my weaknesses and makes my strengths more visible.

Fahad Algheshyan, Anas Ahmad, Juan Saenz, Talal Alyamani, and Sercan Demir: You were always there with a word of encouragement or listening ear. I am very grateful to you for giving me your friendship and for providing me with lifts and practical help. Thank you all for being in my life.

TABLE OF CONTENTS

List of Figures viii
List of Tables x
Chapter 1 Introduction 1
1.1. Overview 1
1.2. Literature Review 3
1.3. Modeling Overview 11
Chapter 2 A Game Theoretic Approach For Load-Shifting In The Smart Grid 13
2.1. Overview 13
2.2. System Model and Game Formulation 13
2.2.1. Energy Provider's Model 14
2.2.2. Customer's Model 15
2.3. Equilibrium Analysis 17
2.4. Numerical Analysis 24
2.5. Conclusions 30
Chapter 3 Price Discounts and Consumer Load-Shifting Behavior in the Smart 32
3.1. Overview 32
3.2. System Model and Game Formulation 33
3.2.1. Energy Provider's Model 34
3.2.2. Customer's Model 34
3.3. Equilibrium Analysis 36
3.4. Numerical Analysis 43
3.5. Conclusions 53
Chapter 4 A Game Theoretic Approach for Load-Shifting and Energy Storage 56
4.1. Overview 56
4.2. Energy Storage Setting 57
4.3. Customer Controlled Energy Storage 58
4.3.1. Game Formulation 58
4.3.2. Model 59
4.3.3. Equilibrium Analysis 60
4.3.4. Numerical Analysis 66
4.4. Provider Controlled Energy Storage 76
4.4.1. Model 76
4.4.2. Equilibrium Analysis 77

4.4.3. Numerical Analysis 85
4.5. Comparison and coordination 92
4.6. Conclusions 100
Chapter 5 Conclusions and Future Work 102
5.1. Conclusions 102
5.2. Future Work 104
References 106
APPENDIX A 110
APPENDIX B 115
APPENDIX C 121
APPENDIX D 133

List of Figures

Figure 1: Aggregate demand across the time horizon. 25
Figure 2: Consumer inconvenience and energy provider's average price discount. 26
Figure 3: Consumer inconvenience and PAR values..... 27
Figure 4: Consumer inconvenience and energy consumption. 27
Figure 5: Consumer diversity, equilibrium discounts, and PAR. 28
Figure 6: Number of consumers and equilibrium discount averages. 29
Figure 7: Number of consumers and PAR. 30
Figure 8: Aggregate demand across the time horizon, customer behavior..... 44
Figure 9: Consumer inconvenience levels across the clusters of periods under the DM scenario, customer behavior..... 46
Figure 10: Consumer inconvenience levels and electricity provider's average price discount, customer behavior 47
Figure 11: Consumer inconvenience and PAR values, customer behavior 48
Figure 12: Consumer inconvenience and profit, customer behavior 49
Figure 13: Consumer inconvenience and load-shift, customer behavior..... 50
Figure 14: Load shift during peak periods (Cluster 2)..... 50
Figure 15:Average consumption costs..... 52
Figure 16: Aggregate demand across the time horizon; storage controlled by customers. 67
Figure 17: Consumer inconvenience and energy provider's average price discount; storage controlled by customers..... 68
Figure 18: Consumer inconvenience and PAR values; storage controlled by customers. 69
Figure 19: Consumer diversity, equilibrium discounts, and PAR; storage controlled by customers. 71
Figure 20: Number of consumers and equilibrium discount averages; storage controlled by customers 72
Figure 21: Number of consumers and PAR; storage controlled by customers..... 73
Figure 22:Storage capacity and equilibrium discount averages; storage controlled by customers. 75
Figure 23: Storage capacity and equilibrium PAR; storage controlled by customers. 76
Figure 24: Aggregate demand across the time horizon; storage controlled by provider. . 86
Figure 25: Consumer inconvenience and energy provider's average price discount; storage controlled by provider. 87
Figure 26: Consumer inconvenience and PAR values; storage controlled by provider. .. 87
Figure 27: Consumer diversity, equilibrium discounts, and PAR; storage controlled by provider. 89
Figure 28:Number of consumers and equilibrium discount averages; storage controlled by provider. 90
Figure 29: Number of consumers and PAR; storage controlled by provider. 91
Figure 30:Storage capacity and equilibrium discount averages; storage controlled by provider. 92
Figure 31:Storage capacity and equilibrium PAR; storage controlled by provider..... 92
Figure 32:Comparison consumer inconvenience and average price discount. 95
Figure 33:Comparison consumer inconvenience and PAR. 95

Figure 34:Comparison consumer inconvenience and energy provider's profit..... 98
Figure 35:Customer inconvenience multiplier and PAR, coordination..... 99
Figure 36: Customer inconvenience multiplier and energy provider' profit, coordination. 99
Figure 37: Customer inconvenience multiplier and average discount, coordination..... 100
Figure 38: Customer inconvenience multiplier and customers total cost, coordination. 100

List of Tables

Table 1: procedure to find equilibrium solution 23
Table 2: Aggregate demand (kwh) 24
Table 3: Procedure to find equilibrium solution, customer behavior 43
Table 4: Aggregate demand (kwh), customer behavior 44
Table 5: Inconvenience patterns under the DM scenario, customer behavior 46
Table 6: Procedure to find equilibrium solution, storage is controlled by customer 64
Table 7: Aggregate demand (kwh), storage is controlled by customers 67
Table 8: Procedure to find equilibrium solution, Storage is controlled by provider 81
Table 9: Aggregate demand (kwh), Storage is controlled by provider 86
Table 10: "Consumptions" table from the data file when number of customers is two – no energy storage 113
Table 11: "Customers" table from the data file when number of customers is two – no energy storage 113
Table 12:"Timeslots" table from the data file when number of customers is two– no energy storage 114
Table 13: "Consumptions" table from the data file for number of customers is two – inconvenience varies across time periods 118
Table 14: "Customers" table from the data file for two customers – inconvenience varies across time periods 119
Table 15: "cost" table from the data file for two customers – inconvenience varies across time periods 119
Table 16: "Timeslots" table from the data file– inconvenience varies across time periods 120
Table 17: "Consumptions" table from the data file for number of customers is two – Storage is controlled by customers 131
Table 18: "Customers" table from the data file for two customers – Storage is controlled by customers 131
Table 19: "Timeslots" table from the data file– Storage is controlled by customers 132
Table 20: "Consumptions" table from the data file when number of customers is two – Storage is controlled by provider 137
Table 21: "Customers" table from the data file when number of customers is two – Storage is controlled by provider 137
Table 22: "Timeslots" table from the data file when number of customers is two– Storage is controlled by provider 138

Chapter 1 Introduction

1.1. Overview

Over the past four decades, there has been a steady growth in electricity consumption (U.S. Energy Information Administration 2015) that is expected to continue driven by economic growth, population, and the introduction of new technologies, especially with the increase in the adoption of electric vehicles (Trigg, Telleen, Boyd, & Cuenot, 2013). Moreover, on-peak demand is increasing to a higher rate over the recent years, which causes a growing peak-to-average ratios (PAR). Therefore, this increase leads to problems for electricity suppliers in order to ensure that their generation capacity meets the demand at the on-peak levels. Moreover, the high changes in load levels increase the stress that is applied to the electrical infrastructure that may cause brownouts or blackouts. Furthermore, the operation with a high PAR is more costly for the suppliers, since to meet high peak demand they are often forced to use fuel that is more costly and has high carbon emissions (Sims, Rogner, & Gregory, 2003; Soliman & Leon–Garcia, 2014). When the electricity demand is low, the electricity supplier just operates the base load plant such as nuclear sources. As the electricity demand increases to an intermediate load, the utility runs the intermediate load plant, which uses fossil fuel and has a high carbon emission compared to other sources. The peaking plant is operated when the demand reaches its peak (U.S. Energy Information Administration, 2014). Moreover, the operating expense for the fossil fuel is 3.7¢/KWH, which is more expensive than the nuclear sources, which have operating expenses around 2.5¢/KWH (U.S. Energy Information Administration, 2015).

Smart grids, with two-way communication between the supplier and consumer have provided tools for the efficient and reliable energy balance and safeguard of the electrical system (Sáenz, Celik, Xi, Son, & Asfour, 2013). The smart grid allows sides, providers and customers, to be aware of the levels of supply and demand at any given time and to respond accordingly. The deployment of smart grids has encouraged energy providers and researchers to develop and employ demand-side management (DSM) in order to improve the service reliability and reduce costs.

DSM has become a remarkable tool to control both the increase in electricity demand as well as the fluctuation between different periods. Two popular DSM approaches include energy efficiency and demand response (DR). The approach focused on energy efficiency provides for lifelong effects in the electricity demand by using devices that consume less energy without modifying the consumers' actions. The DR approach focuses on altering the customers' short-term actions by encouraging them to reduce or shift the demand during the peak time (Siano, 2014).

The effective deployment of DR will benefit all of the stakeholders in the smart grid. Consumers benefit from reduced electricity bills, while electricity suppliers benefit from better grid efficiency, reliability, and lower production cost. DR has been defined by The US Department of Energy (DOE, 2006, p. v) as "a tariff or program established to motivate changes in electric use by end-use customers in response to changes in the price of electricity over time, or to give incentive payments designed to induce lower electricity use at times of high market prices or when grid reliability is jeopardized."

Electricity consumption varies throughout the day and reaches a consumption peak during the afternoon and early evening hours. These hours are considered on-peak

while the rest of the day is considered off-peak (DOE, 2006). The use of dynamic pricing where an extra tariff is charged during on-peak consumption hours or an incentive provided during off-peak hours is one DR approach that encourages customers to reduce or shift their consumption from on-peak to off-peak periods.

Furthermore, energy storage provides a great alternative as an energy management method. When it is employed by energy producers, it uses off-peak generation to meet on-peak demand, so when it is used by customers it effectively lowers on-peak consumption. Furthermore, the use of storage enhances the network's reliability by effectively decreasing the PAR. There are multiple different energy storage technologies that provide different benefits that are highlighted by the characteristics of the energy demand, such that choosing the right technology is specific to each situation where the efficiency, capacity, and response time must all be taken into account.

Various mathematical techniques have been used to analyze DR in order to provide effective strategies for the reduction of PAR. One technique for this analysis involves the use of game theory, and the modelling of the network as a Stackelberg game. Using this approach, the optimal dynamic pricing strategy is found through a sequential multi-stage game. In this game the leader moves first and makes his decision; then in the second stage, the follower responds with their decision to the leader's decision.

1.2. Literature Review

Recently, game theory approaches have become significant tools for studying the interaction between the different objectives of the smart grid stakeholders. A game theory models have been applied to DR in order to investigate equilibrium strategies that aim to reduce PAR, minimize consumption costs, and maximize the electricity provider's profit.

A review of the application of game theoretical approaches and models for the smart grid is provided (Saad et al., 2012), categorizing the applications into three areas: microgrid systems, demand side management, and communication within the smart grid.

DR management between the electricity provider and consumers has been classified by R. Deng, Yang, Chow, and Chen (2015). In their review, they categorized the mathematical models used in demand response into three clusters: peak clipping, valley filling, and load-shifting.

Some works in the first two clusters that apply a game theory include Bu, Yu, and Liu (2011), Chai, Chen, Yang, and Zhang (2014), Chen, Li, Low, and Doyle (2010), Chen, Yang, and Guan (2012), Y. Chen et al. (2012), Maharjan, Zhu, Zhang, Gjessing, and Basar (2013), and Mohsenian–Rad, Wong, Jatskevich, Schober, and Leon-Garcia (2010). In these papers, typically, the game had two levels composed of energy providers and customers. The energy provider determined the electricity source and/or how much energy to buy in order to maximize profit. The customers decided the amount of energy they needed to consume in order to maximize their utility functions. Some of these papers considered a single provider (Bu et al., 2011; J. Chen et al., 2012; Y. Chen et al., 2012), while Maharjan et al. (2013) and Chai et al. (2014) extended the analysis to multiple utility companies that competed with each other in a noncooperative game. In this noncooperative game, a competition price was set in the first stage by each utility, while the customers' responses regarding how much to buy and from which provider was set in the second stage. Some of these papers formulated their models under either a single period setting or independently for each time period. While Chen et al. (2010) considered energy pricing and customer demand choices over multiple depending periods.

Some of these papers considered the customer preference. Bu et al. (2011) provided a multistage Stackelberg game. In this model the total cost function was formulated as a linear function where the price was a function of the total electricity supply; furthermore, this model did not consider the effects that on-peak and off-peak demand has on prices and actual consumption. Another approach that used Stackelberg games to evaluate DR techniques is the one presented by J. Chen et al. (2012). The model evaluated the effect of on-peak and off-peak consumption. The model was limited by an assumption that all of the customers may easily shift their demand at any given moment as a response to the incentives provided by the electricity provider. Maharjan et al. (2013) proposed a model where Stackelberg games were used to find a point of equilibrium in a multi-provider and multi-customer setting. Limitations to this approach arise from the different loads to which the electrical grid may be subjected during on-peak and off-peak consumption time slots, which were not considered.

Another approach using Stackelberg games is presented in Nekouei, Alpcan, and Chattophadhyay (2015) where load curtailment was the goal. Here, the game employed multiple utilities and a demand response aggregator (DRA). In this case the DRA was the leader and provided demand reduction bids; the utilities, as followers, competed to maximize their profits based on the reduced demand. Once these decisions were made, the interaction between the DRA and the individual customers was modeled as a mechanism design problem where the aggregate inconvenience was to be minimized while ensuring that the targeted load curtailment was achieved.

In contrast, in this study, we studied demand response from the perspective of load shifting rather than load shedding. Load shifting has been modeled by the use of explicit

appliance schedules as in Chen, Kishore, and Snyder (2011), Meng and Zeng (2013), and Mohsenian-Rad et al. (2010) and as part of a satisfaction/dissatisfaction function for the demand shifts as in Jiang and Low (2011), Logenthiran, Srinivasan, and Vanessa (2014), and Yang, Tang, and Nehorai (2013). Mohsenian–Rad et al. (2010) proposed a model to solve the interaction among consumers in an energy system, and the model proposed helps to reduce the PAR. However, the proposed model did not consider the consumer type or the provider as decision maker. One further approach employed Stackelberg games in Meng and Zeng, (2013) to evaluate DR techniques. The model uses the reduction of the PAR as a strategy to achieve its objectives and characterizes the demand as shiftable, non-shiftable and curtailable loads. Chen et al. (2011) proposed a model of a Stackelberg game in order to find the equilibrium for the energy provider and the energy consumers. The proposed model aimed to maximize the retailer's profits while minimizing the customers' costs by determining the optimal start time of each of the customers' appliances. This model used an inconvenience cost for the customers who had to shift the start time of their appliances from their original desired start to that proposed by the model. This model has demonstrated to achieve the optimal schedule for a system with a single customer with a single appliance. Meng and Zeng (2013) and Mohsenian–Rad et al. (2010) focused on the cost of usage by the customers, while C. Chen et al. (2011) focused on the inconvenience caused by shifting the schedules. This study combined consumption cost and inconvenience cost. In papers not based on appliance scheduling, in order to match electricity supply and demand, Jiang and Low (2011) present a joint optimization model between a single utility and multiple customers; Yang et al. (2013) used a game theoretical approach in a model with a single utility and a single customer.. The model considered the

gap between the nominal demand and actual consumption of the customer as a cost of inconvenience. However, they recognized the difference between total actual consumption and the total nominal demand.

Logenthiran et al. (2014) formulate optimization model to study the customer behavior toward load shifting , where the customer aims to minimizes his total cost of energy consumption and load-shifting inconvenience across a finite time horizon. However, the energy provider is not a strategic decision maker.

Recently, the deployment of energy storage has become a significant area of research with literature focusing on the different technologies available for energy storage and their application within energy networks. Vazquez, Lukic, Galvan, Franquelo, and Carrasco (2010) provided a review of energy storage and considered two kinds of applications for its deployment: transport and utility applications. Transport applications included road and rail transport, while utility applications included the increase in renewable energy penetration, load leveling, and energy arbitrage, among others. Ibrahim, Ilinca, and Perron (2008) provided a review of different energy storage technologies with their characteristics while also comparing the different storage techniques and their application along with renewable energy generation technologies. Dunn, Kamath, and Tarascon (2011) categorized the energy storage technologies in terms of the mechanism used to store the energy into mechanical, electrical, chemical, and electrochemical storage systems. Hittinger, Whitacre, and Apt (2012) proposed an engineering–economic model to evaluate energy storage technologies and select the appropriate method for different applications in term of cost effectiveness.

Multiple sources present models for the application of energy storage as part of DR. Kumar Nunna and Doolla (2013) presented an agent-based model for microgrids with DR and distributed energy storage. The proposed model aimed to minimize electricity costs and reduce on-peak demand by distributed energy, distributed storage, and demand response. Huang, Walrand, and Ramchandran (2012) proposed a model that minimized the customers' electricity cost while minimizing a disutility function that measures the customers' comfort. Further, this model considered that customers have a renewable energy source and may decide the energy consumption for each slot time separately.

Wang, Gu, Li, Member, and Bale (2013) provided a model to find the optimal balance between energy supply and demand by the deployment of energy storage technologies that have shared ownership and control. The model stated that sharing ownership of the energy storage is an effective way to minimize the electricity price and to reduce the congestion in the energy grid. Zheng, Meinrenken, and Lackner (2014) presented an agent-based model and concluded that energy storage has more economic viability than DR tariffs for typical households in the US. They also proposed an optimal capacity for the storage.

Some literature focuses on the evaluation of storage levels and the charging/discharging during each time slot in order to minimize the electricity cost while integrating renewable energy resources. Qin, Chow, Yang, and Rajagopal (2014) formulated a stochastic control problem to operate energy storage under uncertain net demand and with a marginal electricity cost. Wu, Tazvinga, and Xia (2015) studied a demand-side management problem with a model that aimed to find the optimal operation management by scheduling the storage to maximize the utilization of the energy that

comes from solar resources. This model reduces the amount of energy that the consumers draw from the grid as well as their electricity cost. Another storage scheduling problem that utilizes renewable energy resources was proposed by Zhou, Pan, and Cai (2014) where Lyapunov optimization techniques were employed. Xu and Tong (2015) studied an operation problem that aims to find the optimal energy storage levels with storage resources controlled by consumers.

Nakayama, Zhao, Bic, Dillencourt, and Brouwer (2013) proposed a model that aimed to minimize the cost of energy storage and the power generation by finding the optimal real-time power flow. Yu and van der Schaar (2014) proposed a model that aimed to deploy energy storage in order to maximize energy consumption with minimum electricity cost. Deng, Liu, Jin, and Wu (2013) employed efficiency Lyapunov optimization techniques for a power supply system that employed a datacenter that scheduled for the multiple sources of energy.

There are many game theory models that focus on energy storage where the energy provider is not a strategic decision maker. Atzeni, Ordonez, Scutari, Palomar, and Fonollosa (2013) elaborated a non-cooperative game model among multiple users based on consumers' abilities to produce energy, store energy, and buy energy from the grid. The strategies of each user depended on their energy production and/or storage profiles in order to minimize the payoff cost function while meeting their energy demand. In order to avoid a high demand when consumers loaded their storage at the same time, Vytelingum, Voice, Ramchurn, Rogers, and Jennings (2010) presented an agent-based simulation model that employed a game theory framework to adopt efficient storage behaviors.

Nguyen, Song, and Han (2012) proposed a non-cooperative game to find the equilibrium of the of consumers' interaction in order to minimize their electricity bills. Here, the user's strategic decisions were based on the optimization of the scheduling of energy consumption and storage. Furthermore, in order to minimize the electricity consumption cost, Forouzandehmehr, Esmalifalak, Mohsenian–Rad, and Han (2015) created a stochastic differential game to study the interaction between the electricity market and consumers. The consumers aimed to find the optimal storage level for each time slot as well as the optimal air conditioner usage based on the electricity price and weather conditions. They considered the comfort of the consumers. Moreover, Wang, Saad, Han, Poor, and Basar (2014) presented a system in which an energy storage unit was presented as a seller for a microgrid in which different elements acted as buyers. In this system a game was used to find the equilibrium amount of energy to be traded, and they presented a heuristic that relied on an auction mechanism to find selling prices.

All these approaches studied and investigated different aspects of improving the electricity market and reducing the total electricity consumption and PAR. However, this study focused on the load-shifting aspect, and as such we do not consider demand shedding. Rather than providing specific schedules or solutions to the load-shifting problem, our goal was to investigate the interactions between cost, price, and load-shifting behaviors; and derive insights for the energy market. We formulated a Stackelberg game such that the energy provider was a strategic player with multiple customers of varying types. This model considers the inconvenience cost that is incurred when there is as difference between the nominal intended demand and actual consumption of a customer; while ensuring that the total energy consumed during the daily planning horizon is equal

to the total nominal demand of said day. We evaluated the effect of consumer types, the diversity and the size of the market, and energy generation cost structure on the equilibrium decisions. Additionally, we deployed an energy storage alongside the load shifting to evaluate its impact on the electricity market, both when it is controlled by the provider and when controlled by the customers. We evaluated the influence of the storage capacity and storage efficiency on the equilibrium decisions.

1.3. Modeling Overview

This study introduce a Stackelberg game-based formulation to study the load-shifting and energy storage problem within the aspect of smart grid demand response for an electricity market composed of a single energy provider and multiple customers. In the model presented in Chapter 2, the provider acted as the channel leader and offered price discounts to affect the behavior of the customers who responded by shifting their consumption from on-peak to off-peak hours according to their nominal demand and their inconvenience cost incurred by changing their intended behavior. The energy provider aimed to maximize profit, while the customers aimed to minimize their total consumption cost. We evaluated the effects that customers' individual inconvenience costs had on the overall equilibrium in terms of the energy provider's profit, customers' cost, and the effectiveness in the PAR. In this chapter, we investigated the model when the customer had the same attitude toward load-shifting throughout the day. In Chapter 3, we investigated the model when customers' individual behavior was not necessarily the same throughout the day, so that they may have had different attitudes towards load-shifting at different times, which and may affect the overall equilibrium of the market.

In Chapter 4, we incorporated an energy storage into the model from Chapter 2 to evaluate the effects of deploying energy storage and load-shifting, as energy management technologies, to the overall equilibrium. In the first part of this chapter, we analyzed and investigated the model when the energy storage was controlled by the customers, while in the second half we analyzed and investigated the model when the energy storage was controlled by the energy provider.

We obtain the equilibrium conditions and present numerical studies to evaluate the different models, and derive managerial insights about the effect of factors, (consumer types, market diversity, market size, and storage capacity) on the interactions between the energy provider and its customers.

Chapter 2 A Game Theoretic Approach For Load-Shifting In The Smart Grid

2.1. Overview

This chapter describes an electricity market composed of a single energy provider and multiple consumers in terms of load-shifting. We formulate a Stackelberg game in which the energy provider, acting as leader, moves first and offers price discounts across a finite time horizon to encourage customers to shift their energy consumption from on-peak periods. The customers, acting as followers, react by determining their effective demand and how much they are willing to shift across periods. In this model, the energy provider aims to maximize profit, while the aim of the customers is to minimize their total costs related both to the energy consumption and inconvenience of deviating from the nominal demand. We proposed a procedure to obtain equilibrium results on the interactions between the energy provider and its customers. We investigated the impact of various factors, including consumer types, market diversity, and market size. Our results show that price discounts may provide significant influence to reduce the PAR while improving the energy provider profit and customers' total cost. Moreover, the results show that the equilibrium outcomes are not depending only on the size of the demand (market depth) but also on the number of customers (market breadth).

2.2. System Model and Game Formulation

We formulated a Stackelberg game between a single energy provider, as the leader, and N customers, as followers, who have nominal energy consumption demands across a final time horizon with length T. The energy provider aimed maximize its profit by deciding a percentage discount, γ_t, over a predetermined base price, P, for time slot t ($t = 1 \ldots T$). The customers aimed to minimize their cost by deciding how much energy

to consume during each time slot t and how much consumption to shift from time slots with low or no discount to time slots with a higher discount based on the tradeoff between price discounts and cost of load-shifting inconvenience. We let y_{it} denote the nominal demand of customers *i* for electricity in kWh at time slot *t*, and x_{it} represented actual consumption in kWh consumed by consumer *i* during time slot *t*. Clearly, the load shift for customer *i* for time slot *t* is defined as the difference between x_{it} and y_{it}.

2.2.1. Energy Provider's Model

In this model, we considered a case in which the long-term electricity base price had already been set and short-term incentives (by means of the percentage discount γ_t) were used to control fluctuation in the short-term demand. The establishment of this base price, which is usually set as a result of market conditions and long-term aggregate demand curves, falls outside the scope of this study. The use of discount schemes on nominal prices to manage demand is a common and widespread revenue management method in the travel and hospitality industry (Choi & Mattila, 2014; Pachon, Erkoc, & Iakovou, 2007). It dampens demand variability in supply chain management (Chopra & Meindl, 2016). The electricity provider, as the leader of the Stackleberg game, selected the discount rates for all of the periods of the planning horizon in order to maximize their profit. As such, the effective price for each period t was then $\mathrm{p_t} = (1 - \gamma_t)\mathrm{P}$, where $0 < \gamma_t \leq 1$.

The energy generation cost for the energy provider was convex, increasing with the generation amount because the utility provider is using a less expensive energy source to produce electricity, as the demand increase it start to use expensive sources to meet the demand (Soliman & Leon–Garcia, 2014). As such, the electricity generation cost was

convex, increasing with the generation amount. Without loss of generality, the model used an electricity generation cost at time t with the following form:

$$C_t(X_t) = aX_t^2 + bX_t + c \tag{1}$$

Here, X_t was the total electricity generated during period t and was equal to the total consumption of that period such that $X_t = \sum_{\forall N} x_{it}$. The increasing return in cost was a commonly used approximation for the energy provider (Meng & Zeng, 2013; Mohsenian–Rad et al., 2010). With this in place, the profit maximization for the energy provider is written as follows:

$$\text{Maximize } \Pi_s = \sum_{t=1}^{T}((1-\gamma_t)PX_t - C(X_t)) \tag{2}$$

$$s.t. \qquad 0 \leq \gamma_t \leq 1 \qquad \forall t \in \mathbb{T} \tag{3}$$

The objective function includes the electricity provider's net profit based on the given aggregate consumption across the planning horizon. The constraint in (3) ensures that the percentage discount used by the electricity provider is within 0 and 100 percent. The total consumption for each period was determined by the customers as a response to the discounts provided by the electricity provider and was optimized using the consumers' model. We defined the provider's feasible strategies space as $\Omega_s = \{\gamma_t | \gamma_t \in R,\ t \in \mathbb{T}, 0 \leq \gamma_t \leq 1\}$.

2.2.2. Customer's Model

The goal of the consumers, who acted as followers in the Stackelberg game, was to minimize their total cost, both from energy consumption and inconvenience. In order to do this, they determined the optimal levels of energy to consume during each time slot by shifting their energy usage across the different slots while ensuring that the total energy consumed during the daily planning horizon was equal to the total intended demand for

said day. Based on the different discounts offered by the energy provider during some of the time slots, the customers' cost of energy consumption depended not only on the consumption amount but also on the time slots in which this consumption occurs. With this behavior energy customers acted similarly to buyers in a supply chain context who have to determine the amounts to procure using forward contracts or delayed procurements as a response to price fluctuations. In order to model the customers' disposition towards shifting their consumption across time slots, the inconvenience cost was modeled with a quadratic function that increases as the difference between the actual consumption x_{it} and nominal demand y_{it} of a time slot deviate (in a fashion similar to that of Avci, Erkoc, Rahmani, & Asfour, 2013; Jiang & Low, 2011; Yang et al., 2013). It is important to highlight that the obtained results may be easily generalized to any convex inconvenience cost function. The individual tolerance to load-shifting of customer i was modeled through the use of an inconvenience parameter α_i, which is included with the quadratic inconvenience cost.

With these considerations in place, the optimization model for customer i is as follows:

$$Min\ \Pi_i = \sum_{t=1}^{T}\left((1-\gamma_t)Px_{it} - a_i(x_{it}-y_{it})^2\right) \tag{4}$$

$$s.t. \qquad \sum_{t=1}^{T} x_{it} = \sum_{t=1}^{T} y_{it} \tag{5}$$

$$x_{it} \geq m_{it} \qquad \forall t \in \mathbb{T} \tag{6}$$

Here, the objective function encompasses the total cost for the customers, which was driven by the actual consumption during each time slot x_{it}, and the deviation from the actual consumption and the nominal demand $x_{it} - y_{it}$. The sensitivity of a customer to load-shifting depends on α_i such that customers with low values of α_i were flexible and

achieved lower total costs by shifting demand, while customers with high values of α_i were inflexible and did not achieve lower total costs by shifting demand. The first constraint of this customers' optimization model ensures that the total energy consumed across all of the time slots is equal to the total energy demand for the time period, while the second constraint ensures that the consumption of each time slot is at least the minimum required consumption m_t, which may not be shifted.

2.3. Equilibrium Analysis

Backward induction is an appropriate solution concept for calculating the Stackelberg equilibrium. In this approach, the later decision's best response is mapped to the actions of the earlier decision. Therefore, we analyzed the consumers' best responses for a known price discount scheme. We used Lagrangian Relaxation in order to analyze the consumer's model. We let λ be the Lagrange multiplier for constraint (5) and μ_t for constraint set given in (6). Thus, the consumer's objective is equivalent to minimizing the following Lagrange function for x_{it}, λ, and μ_t:

$$Max\ LR = \sum_{t=1}^{T}(1-\gamma_t)P\,x_{i,t} + \sum_{t=1}^{T}\alpha_i\left(x_{i,t}-y_{i,t}\right)^2 - \lambda\left(\sum_{t=1}^{T}y_t - \sum_{t=1}^{T}x_{i,t}\right) - \sum_{t=1}^{12}\mu_t(x_{i,t}-m_{it}) \tag{7}$$

The Kuhn-Tucker conditions yielded the following set of equations and inequaltities:

$$\frac{\partial LR}{\partial x_t} = (1-\gamma_t)P + 2\alpha_i\left(x_{i,t}-y_{i,t}\right) + \lambda - \mu_t = 0 \qquad \forall t \tag{8}$$

$$\frac{\partial LR}{\partial \lambda} = \sum_{t=1}^{12} y_{i,t} - \sum_{t=1}^{12} x_{i,t} = 0 \tag{9}$$

$$x_{i,t} \geq m_{it} \qquad \forall t \tag{10}$$

$$\mu_t(\, x_{i,t} - \, m_{it}) = 0 \qquad \forall t \tag{11}$$

$$\mu_t \geq 0 \qquad \forall t \tag{12}$$

Proposition 1: At optimality, the consumer's electricity consumption x_{it}^* is

$$x_{i,t}^* = \begin{cases} y_{i,t} + \dfrac{(\gamma_t - \bar{\gamma})P}{2\alpha}, & y_{i,t} - \, m_{it} > \dfrac{(\,\bar{\gamma} - \, \gamma_t)P}{2\alpha} \\ m_{it}, & y_{i,t} - \, m_{it} \leq \dfrac{(\,\bar{\gamma} - \, \gamma_t)\, P}{2\alpha} \end{cases} \tag{13}$$

Proof: The solution to the model using equation (7) has two candidate cases: the first solution when the constraints from equation (10) are strictly holding, $x_{i,t} > m_{it}$, and the second when the constraints from equation (10) are binding, $x_{i,t} = m_{it}$.

In the first case, we assumed that the customer consumption x_{it} is strictly holding during each time slot. In this case, it can clearly be seen from the complementary slackness conditions of equation (11) that $\mu_t = 0$ because $x_{i,t} - \, m_{it} > 0$. From equation (8)

$$x_{i,t}^* = \, y_{i,t} - \frac{\lambda - (1 - \gamma_t)P}{2\alpha} \tag{14}$$

Substituting equation (14) into equation (9) leads to equation (15) where we determined the optimal Lagrange multipliers λ:

$$\lambda = (\,\bar{\gamma} - 1)P \tag{15}$$

By substituting equation (15) into equation (14), we derived the optimal consumption $x_{i,t}^*$ for each customer i during each timeslot t, to be as shown in equation (16):

$$x_{i,t}^* = \, y_{i,t} + \frac{(\gamma_t - \bar{\gamma})\, P}{2\alpha} \tag{16}$$

By substituting equation (16) into $x_{i,t} - \, m_{it} > 0$, we had

$$y_{i,t} - m_{it} > \frac{(\bar{\gamma} - \gamma_t)P}{2\alpha} \tag{17}$$

In the second case, $x_{i,t} = m_{it}$ when the constraint in equation (10) is binding, and in order to satisfied the complementary slackness conditions in equation (11) $\mu_t \geq 0$, from equation (8):

$$\mu_t = (1 - \gamma_t)P + 2\alpha_i\left(m_{i,t} - y_{i,t}\right) + \lambda \tag{18}$$

By substituting equation (15) into equation (18),

$$\mu_t = (1 - \gamma_t)P + 2\alpha_i\left(m_{i,t} - y_{i,t}\right) + (\bar{\gamma} - 1)P \tag{19}$$

By using equation (19), we can rewrite into equation (20) as

$$y_{i,t} - m_{it} \leq \frac{(\bar{\gamma} - \gamma_t)\,P}{2\alpha} \tag{20}$$

■

Here, $\bar{\gamma}$ is the average discount price offered throughout the planning horizon, such that $\bar{\gamma} = \frac{1}{T}\sum_{t=1}^{T}\gamma_t$, and x_{it}^* is composed of two parts: the nominal demand y_{it} and the deviation. This deviation is positive in the time slots in which the offered discount is higher than average and negative when the opposite is true.

If the resultant consumption x_{it}^* from equation (16) are strictly larger than the minimum consumption, the constraints given by equation (6) are redundant and the solution to the consumer model is given by the values found in equations (15) and (16). This situation is more likely to arise in cases when the customers are insensitive to the prices and the values of α_i are high.

Some of the constraints in equation (6) may be binding. This is more likely when the consumer is more of a price taker and less sensitive to demand deviation. To analyze this case, let $\mathbf{T_1}$ be the set of time periods with corresponding constraints in (6) that are

strictly holding and $\mathbf{T_2}$ be the set of those that are binding. We also defined $\hat{T}$ as the cardinality of set $\mathbf{T_1}$. For a given discount scheme, these sets can be easily obtained by solving (4–6) with an over-the-counter nonlinear optimization tool. We can then use (7) for this case to generate optimal results from Karush–Kuhn–Tucker conditions. For the following results, let $\bar{\gamma}_1$ be the mean discount rate for time slots in $\mathbf{T_1}$and $\hat{x}_{it}^*$ be the optimal energy usage values for this case.

Proposition 2: Given $\mathbb{T}_{i1}$ and $\mathbb{T}_{i2}$, at optimality we get

$$\hat{x}_{it}^* = \begin{cases} y_{it} + \frac{1}{\hat{T}} \sum_{\bar{t}=1}^{\bar{T}} (y_{it} - m_{it}) + \frac{(\gamma_t - \bar{\gamma})P}{2\alpha} & , t \in \mathbb{T}_1 \\ m_{it} & , t \in \mathbb{T}_2 \end{cases} \tag{21}$$

$$\lambda = (\bar{\gamma}_{\hat{t}} - 1)P - \frac{2\alpha}{\hat{T}} \sum_{\bar{t}=1}^{\bar{T}} (y_{\bar{t}} - m_{it}) \tag{22}$$

$$\mu_{i,t}^* = \begin{cases} 0 & , t \in \mathbb{T}_1 \\ (\bar{\gamma}_1 - \gamma_{\hat{t}})P - 2\alpha_i (y_{it} - m_{it} + \frac{1}{\hat{T}} \sum_{t \in \mathbb{T}_2} (y_{it} - m_{it})) & , t \in \mathbb{T}_2 \end{cases} \tag{23}$$

Proof: Given $\mathbb{T}_{i1}$ and $\mathbb{T}_{i2}$, the Lagrangean relaxation becomes as follows:

$$\begin{aligned} LR = {} & \sum_{t=1}^{T} (1 - \gamma_t) P \left(\sum_{i \in \mathbb{T}_1} x_{\widehat{i,t}} + \sum_{i \in \mathbb{T}_2} m_{\overline{it}} \right) + \sum_{t \in \mathbb{T}_1}^{\hat{T}} \alpha_i \left(x_{\widehat{i,t}} - y_{\widehat{i,t}} \right)^2 \\ & + \sum_{t \in \mathbb{T}_2}^{\bar{T}} \alpha_i \left(m - y_{\overline{i,t}} \right)^2 + \lambda \left(\sum_{\hat{t}}^{\hat{T}} x_{\widehat{i,t}} + \sum_{\bar{t}}^{\bar{T}} m_{\overline{it}} - \sum_{t=1}^{12} y_t \right) \\ & - \sum_{t=1}^{12} \mu_t \left(x_{i,t} - m_{it} \right) \end{aligned} \tag{24}$$

At optimality, the following KKT conditions must hold

$$\frac{\partial LR}{\partial x_{\widehat{it}}} = (1 - \gamma_{\hat{t}})P + 2\alpha_i (x_{\widehat{it}} - y_{\widehat{it}}) + \lambda - \mu_{\hat{t}} = 0 \qquad \forall \hat{t} \tag{25}$$

$$\frac{\partial LR}{\partial \lambda} = \sum_{t=1}^{12} y_{i,t} - \left(\sum_{\hat{t}}^{\hat{T}} x_{\widehat{i,t}} + \sum_{\bar{t}}^{\bar{T}} m_{\overline{it}} \right) = 0 \tag{26}$$

$$x_{i,t} \geq m_{it} \qquad \forall t \tag{27}$$

$$\mu_t \left(x_{i,t} - m_{it} \right) = 0 \qquad \forall t \tag{28}$$

$$\mu_t \geq 0 \qquad \forall t \tag{29}$$

We can rewrite equation (25) as follows:

$$(1-\gamma_{\hat{t}})P + 2\alpha_i(x_{\hat{\imath}t} - y_{\hat{\imath}t}) + \lambda = \mu_{\hat{t}} \tag{30}$$

By substituting equation (30) in equation (28),

$$(x_{i,t} - m_{it})\big((1-\gamma_{\hat{t}})P + 2\alpha_i(x_{\hat{\imath}t} - y_{\hat{\imath}t}) + \lambda\big) = 0 \tag{31}$$

When the constraint in equation (27) is strictly , than $x_{i,t} > m_{it}$ and $\mu_{\hat{t}} = 0$ than

$$(1-\gamma_{\hat{t}})P + 2\alpha_i(x_{\hat{\imath}t} - y_{\hat{\imath}t}) + \lambda = 0 \tag{32}$$

From equation (32), we can define actual consumption, $x^*_{\hat{\imath},t}$, as follows:

$$x^*_{\hat{\imath},t} = y_{\hat{\imath},t} - \frac{\lambda-(1-\gamma_{\hat{t}})P}{2\,\alpha} \tag{33}$$

Substituting equation (33) into equation (26) leads to equation (36), where we determined the optimal Lagrange multipliers λ, which may be developed as shown in equation (34) and (35):

$$\sum_{t=1}^{12} y_{i,t} - \sum_{\hat{t}=1}^{\hat{T}} \left(y_{\hat{\imath},t} - \frac{\lambda-\gamma_{\hat{t}}P}{2\,\alpha}\right) - \sum_{\bar{t}}^{\bar{T}} m_{\bar{\imath}t} = 0 \tag{34}$$

$$\sum_{\hat{t}}^{\hat{T}} \frac{\lambda}{2\,\alpha} = -\sum_{t=1}^{12} y_{it} + \sum_{\hat{t}}^{\hat{T}} y_{\hat{\imath}t} - \sum_{\hat{t}}^{\hat{T}} \frac{(1-\gamma_{\hat{t}})P}{2\,\alpha} + \sum_{\bar{t}}^{\bar{T}} m_{\bar{\imath}t} \tag{35}$$

$$\lambda = (\bar{\gamma}_1 - 1)P - \frac{2\alpha}{\hat{T}} \sum_{t\in\mathbb{T}_2} (y_{\bar{\imath}t} - m_{\bar{\imath}t}) \tag{36}$$

By substituting equation (36) into equation (33), we derived the optimal consumption $x^*_{\hat{\imath},t}$ for each customer i during each timeslot t, to be as shown in equation (37), when $t \in \mathbb{T}_1$:

$$x^*_{\hat{\imath},t} = y_{\hat{\imath},t} + \frac{1}{\hat{T}} \sum_{t\in\mathbb{T}_2} (y_{\bar{\imath}t} - m_{\bar{\imath}t}) + \frac{(\gamma_{\hat{t}} - \bar{\gamma}_1)P}{2\,\alpha} \tag{37}$$

In the second case, when $t \in \mathbb{T}_2$, the constraint in equation (27) is binding. Then $x^*_{\hat{\imath},t} = m_{it}$ and $\mu_{\hat{t}}$ will find it by substituting equation (36) in equation (30):

$$\mu_{\hat{t}} = (\bar{\gamma}_1 - \gamma_{\hat{t}})P - 2\alpha_i(y_{it} - m_{it} + \frac{1}{\hat{T}} \sum_{t \in \mathbb{T}_2} (y_{it} - m_{it})) \tag{38}$$

■

Here, $\overline{\gamma_1}$ is the average discount rate for the time slots in $\mathbb{T}_1$, and $\hat{x}_{it}^*$ is the optimal energy consumption.

The energy provider determines the optimal discount price by inferring the customers' optimal responses, such that the results from equations (15), (16), (21), (22), and (23) may be included into the provider's optimization model. Computing the equilibrium optimal discounts may take more than one iteration given that the closed-form solution from the customers' stage depends on the constraints from equation (6). We proposed a procedure in order to derive the equilibrium solutions. First, we ignored constraint (6) and included the consumer best response obtained in (16) into the energy provider's model. The total energy consumption values (i.e., X_t) in the energy provider's profit function in (2) are replaced by

$$X_t = \sum_{i=1}^{N} (y_{it} + (\gamma_t - \bar{\gamma}) \frac{P}{2\alpha_i}) \tag{39}$$

Definition 1: The point $(\gamma_t^*, x_{it}^*(\gamma_t^*))$, which satisfied the constraints in (2–3) and (4–6), is an equilibrium result of the Stackelberg game $G = \{\mathbb{N}, \mathbb{T}, \Pi_s, \Pi_i\}$ if and only if

$$\Pi_s(\gamma_t^*, x_{it}^*(\gamma_t^*)) \geq \Pi_s(\gamma_t, x_{it}(\gamma_t)), \forall\, t \in \mathbb{T} \; and \; i \in \mathbb{N} \tag{40}$$

We can solve energy provider optimization model by using a nonlinear optimization tool. The equilibrium results will be obtained *only if* the optimal price discounts lead to a state where constraints in (6) are satisfied for all consumers. Otherwise, we need to carry out additional iterations as outlined in Table 1.

Table 1: procedure to find equilibrium solution

1: include the consumer's best response from equation (39) into the energy provider's model in equation (2–3).
2: solve energy provider's model
3: if $x_{i,t}^{*} \geq m_{it}\ \forall\, i \in \mathbb{N}, t \in \mathbb{T}$, then STOP.
4: else for each consumer for which constraint (6) is violated in any time *t*
5: identify and update $\mathbb{T}_1$ and $\mathbb{T}_2$
6: use (21) instead of (39) for these consumers in the energy provider's model
7: solve the utility firm's model again including these updates.
8: go to line 3.

Clearly, when the nominal demand is uniform across the planning horizon, the energy provider does not benefit from load-shifting. In such cases, the energy provider will not offer a price discount. The price discounts benefit the provider when there is a fluctuation on the nominal demand, which reduce the generation costs. Therefore, the price discounts will only be used to shift demand from on-peak periods to relatively off-peak periods. Consequently, we can make the following conclusion:

COROLLARY 1. *At equilibrium, the price discount will be zero for at least one period.*

The price discount will be applied if and only if there is a fluctuation in the nominal demand over the time horizon. Clearly, when there is a peak demand period, the supplier's model will determine no discount to encourage the consumers to shift load from the peak period to off-peak periods.■

2.4. Numerical Analysis

In this section we performed a numerical study to evaluate the model under different scenarios. We investigated the effect of the consumer types in terms of their willingness to shift load on the energy provider's profits, price discounts, and PARs. We are not only concerned in the customers' willingness levels but also in the diversity in the consumer type. Moreover, we investigated the impact of the number of customers (market breadth) on the equilibrium outcomes.

In all of the scenarios, we divided the time horizon into twelve time slots. For the energy generation cost function, we set the parameter a equal to 0.0035 $/KWh^2 and b and c equal to zero. We assumed m_{it} was zero for all consumers and periods; the base price, P, was set to 0.25 $/kWh. Customers had different total nominal demands; the numbers for the nominal demands were gathered from a randomly selected residential electricity bill. Table 2 shows the aggregate nominal demand that we used for each period, and Figure 1 depicts the demand pattern across the planning horizon.

Table 2: Aggregate demand (kwh)

Period (t)	1	2	3	4	5	6	7	8	9	10	11	12
Aggregate Demand	10.19	15.72	22.28	29.23	33.31	39.14	73.43	61.27	58.1	37.2	17.53	15.01

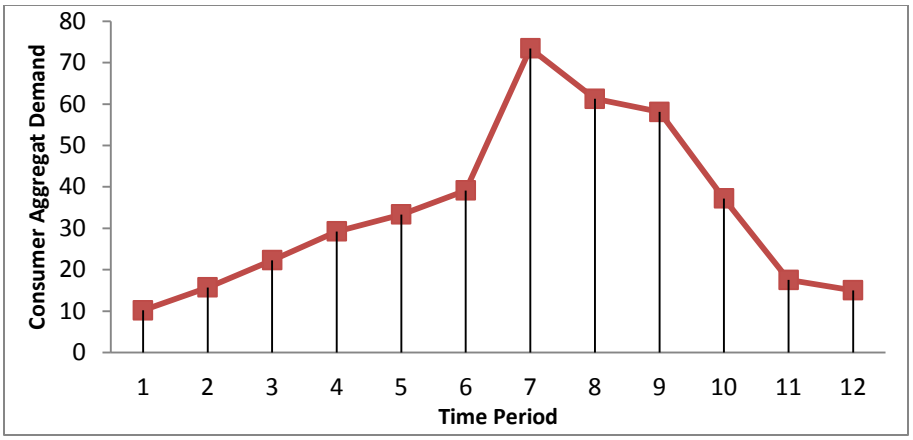

Figure 1: Aggregate demand across the time horizon.

The first scenario focused on the consumer type and considered seven consumers with varying types. The base values for the inconvenience factors across seven customers (α_i) were 0.02, 0.021, 0.00025, 0.00875, 0.0003, 0.0004, and 0.007 with an average of 0.0082. We scaled these values with a multiplier: z in [1, 15]. Clearly, higher z values result in lower sensitivity to price discounts for the consumers. Figure 2 and 3 show the effect of consumers' sensitivity to load-shifting on the equilibrium average discounts (γ) and PAR values.

We observed three segments in the graph, shown in Figure 2. In the first segment when $z = 0.5,1,2$, the customers were price takers, and some of the constraints in (6) were binding for some customers. In this segment, the energy provider had no incentive to provide large discounts since the customers achieved a high load-shifting with low discount. In the second segment (z is from 3 to 6) as the consumers became more resistant to load shifts and the set of constraints (6) are strictly holding, the price discounts increased since the provider still can influence the customers' behavior toward load-shifting. After a certain threshold, the price discount began to decrease because resistance became sufficiently strong so the energy provider did not benefit from

applying high discounts. The PAR on Figure 3 increased as the customers' inconvenience increased.

The consumption at equilibrium for different consumer markets is shown in Figure 4. Here we can see the nominal energy demand and the effects of high inconvenience averages (HS) and moderate inconvenience averages (MS). When there are moderate inconveniences, the equilibrium is reached with no discounts during only four time slots, while when there are high inconveniences, the equilibrium is reached with no discounts in 10 of the 12 time slots.

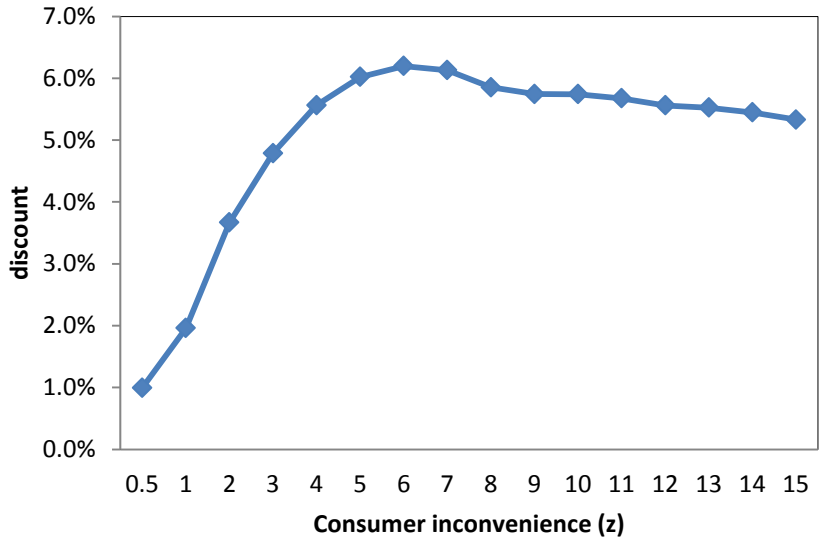

Figure 2: Consumer inconvenience and energy provider's average price discount.

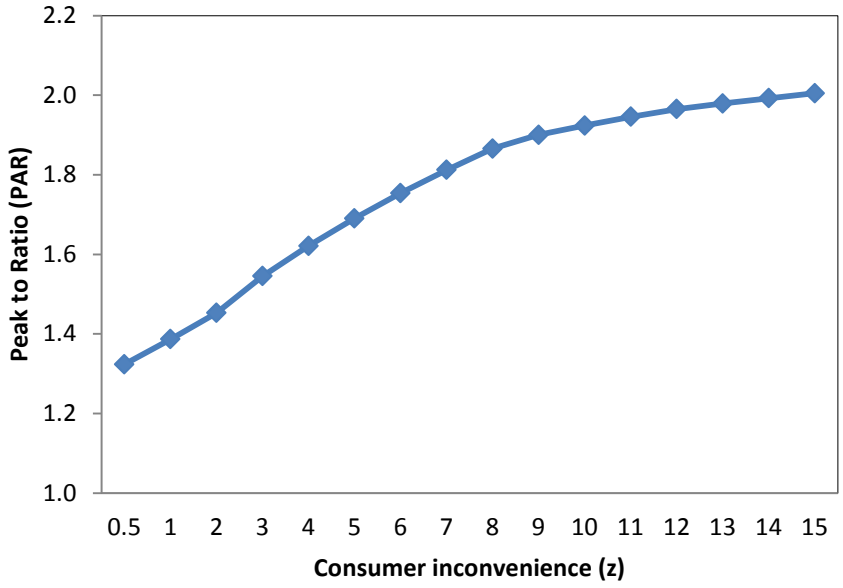

Figure 3: Consumer inconvenience and PAR values.

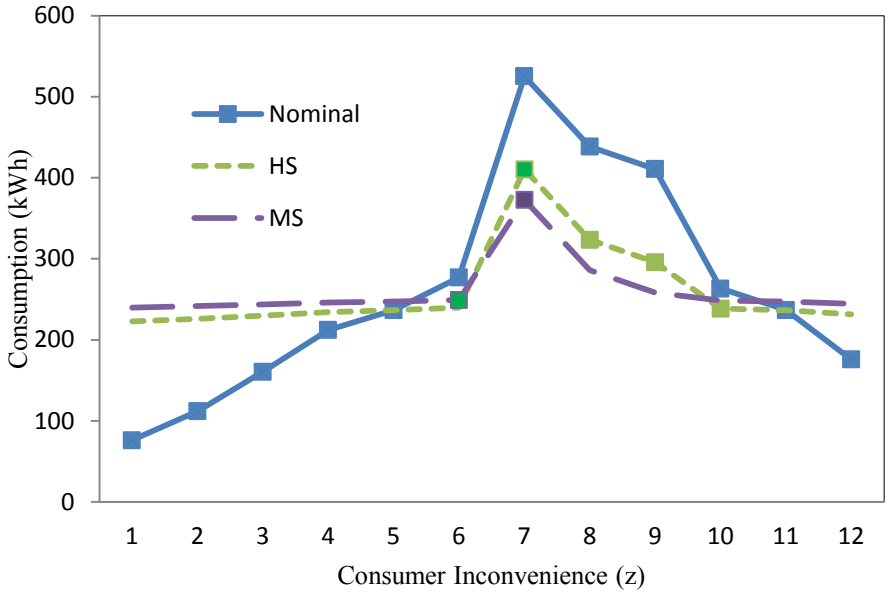

Figure 4: Consumer inconvenience and energy consumption.

Next, we studied the variety of the customers in terms of their behavior towards load-shifting. We kept the average of the inconvenience factors at 0.0082. Coefficient of

variation (COV) for α_i ranged from 0 to 1.05 Figure 5 shows the effect of diversity on the equilibrium average discounts and PAR values.

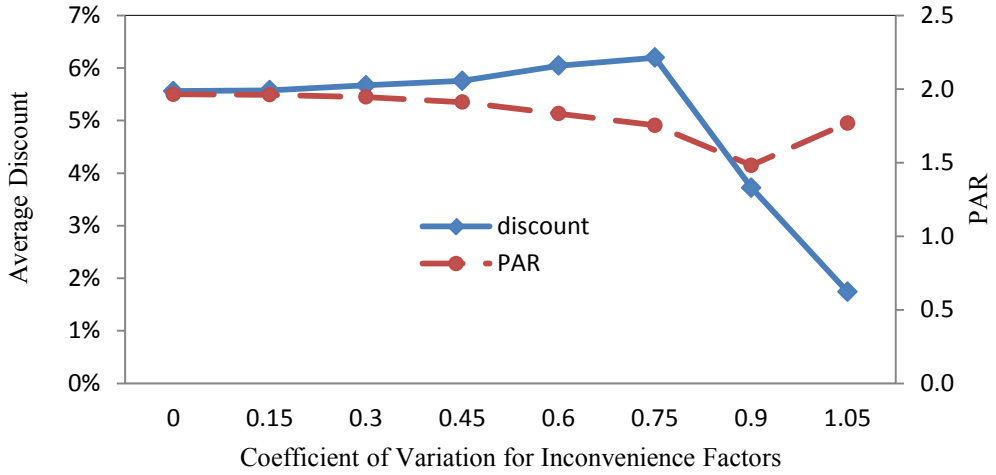

Figure 5: Consumer diversity, equilibrium discounts, and PAR.

As shown in Figure 5, the average price discount first increased as the COV increased until a certain threshold, after which the discount began to decrease. We believe this is because when the customers were more similar to one another, their behavior was relatively less sensitive to price discount. When the diversity increased, the energy provider was incentivized to make load-shifting more attractive as more and more customers would react. However, when diversity increased, the customers' types were too distant from each other, and some were price takers while others would not shift their loads regardless of the discount. Thus, a lower discount was required to shift the loads of the subgroup of customers who were willing to do any load shits. The decrease in the discount was significant as some of the constraints in equation (6) become engage.

In the third scenario, we evaluated the effects that having multiple customers has on the Stackelberg equilibrium. Here, we used a number of customers ranging from 1 to 7 while keeping the total nominal demand of the system equal to the aggregate demand in

Table 2. Furthermore, we assumed that all of the customers had the same inconvenience $\alpha = 0.001$ and used the inconvenience multiplier z. Figure 6 and 7 show the effects of having a different number of customers at different inconvenience levels.

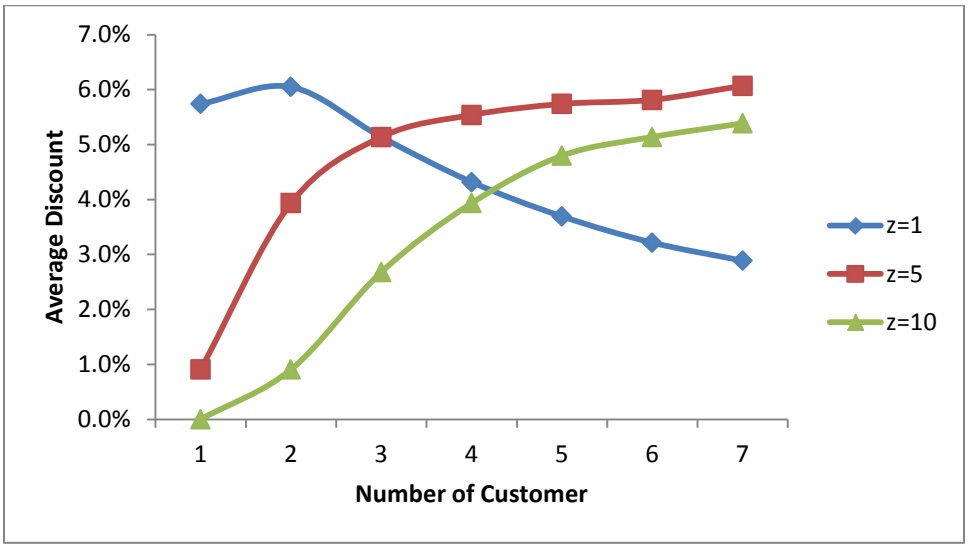

Figure 6: Number of consumers and equilibrium discount averages.

Clearly, as the number of consumers increased, the load shift for each consumer was less costly. This is mainly due to the fact that the inconvenience costs are convex, increasing with the deviation between consumption and nominal demand.

When the consumers were less resistant to load shifting ($z = 1$), the graph had two distinct fragments. In the first fragment, the market has up to two customers, and price discounts were very effective ways to incentivize load-shifting; thus, high discounts (6%) were offered. Past this point, the market had more customers the effects of load shifting may be achieved with lower discounts, which decreased as the number of customers increased.

When $z = 5$ customers were not as sensitive to price discounts, and shifting large loads led to high inconvenience. As such, when the market had fewer customers (1 or 2),

they were quite price insensitive. As the market grew in number (3 to 7), the individual amount of nominal demand that must be shifted by each customer was smaller and thus became more price sensitive, and the equilibrium discount increased to high equilibrium and discounts of 6.1% were reached.

When $z = 10$ customers were price insensitive. In this case the equilibrium discount slowly increased as the number of customers increased, but even when the market has seven customers, the equilibrium discount only reached 5.4%.

When investigating the effect on the PAR, we saw that for every level of inconvenience, the increase in the number of customers led to a decrease in the PAR.

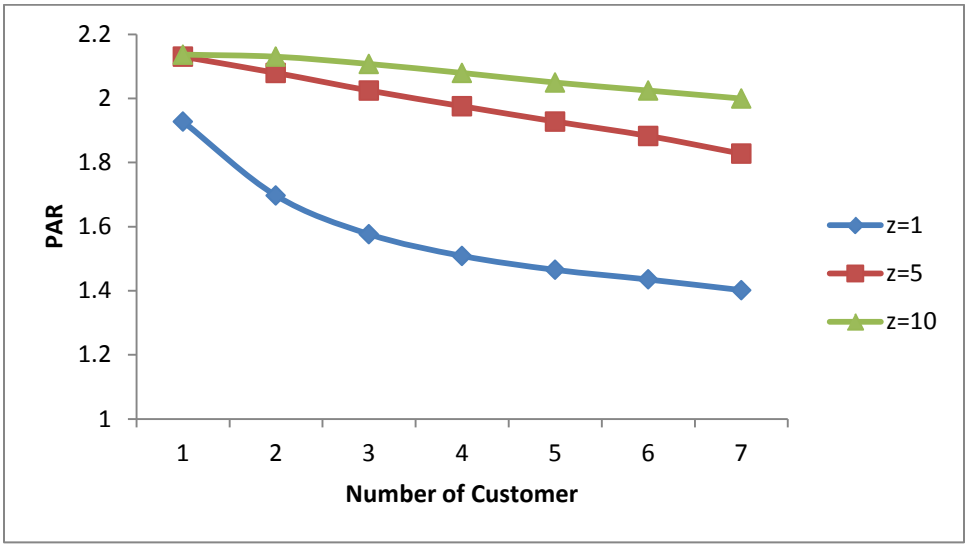

Figure 7: Number of consumers and PAR.

2.5. Conclusions

In this chapter, we introduced and studied a game-theoretic model for the load-shifting problem in the smart grid in order to derive managerial insights that can help energy providers offer a price discount for consumers to shift their on-peak energy consumption to the off-peak in order to reduce the demand fluctuations. Our results show

that price discounts may provide significant influence for achieving lower PARs while improving the energy provider's profits. Furthermore, when the customers were moderate and/or the customer population was moderately diverse in terms of their inconvenience factors, the provider offered higher discounts. Discounts had a low impact in reducing PAR when the customers were mostly alike in their inconvenience factors. Moreover, the number of customers (i.e., market breadth) affected the equilibrium discounts and energy provider profits.

Chapter 3 Price Discounts and Consumer Load-Shifting Behavior in the Smart

3.1. Overview

Customers' individual behavior is not necessarily the same throughout the day, so they may have different attitudes towards load-shifting at different times, which may affect the overall equilibrium of the market. A customer who has a high average inconvenience may have a few periods where he or she is eager to shift demand since these specific periods are not personally significant (such as when to run a washing machine) and thus provide flexibility, while a customer with an average low inconvenience may be very resistant to load-shifting during specific periods (such as when to cook dinner and use the stove).

This chapter studies the impact of consumers' individual attitudes towards load-shifting in electricity consumption in an electricity market that includes a single energy provider and multiple consumers. Based on the model presented in Chapter 2, a Stackelberg game model was formulated in which the provider used price discounts over a finite number of periods in order to induce incentives for consumers to shift their peak period loads to off-peak periods. Consumers reacted to the proposed discounts by determining how much they were willing to shift across periods based on the trade-off between their consumption costs and inconvenience costs. We investigated the equilibrium outcomes for the proposed model and derived analytical results for this type of market where not only the response behaviors of independent consumers are diverse but also an individual consumer's valuation of electricity consumption varies across periods. Using both analytical and numerical analyses, we obtained insights regarding the impact

of the diversity of the electricity market and the varying attitudes of the consumers on the energy provider's proposed discounts and profits, shifts in electricity consumption across the planning period, and PARs. Our results demonstrate that consumer sensitivities to price discounts significantly impact price discounts and load-shifts, which are not necessarily monotonic. We also observed that a diverse market led to lower peak-to-average values and provider payoffs compared to a homogenous market unless the latter one is composed of consumers with relatively lower inconvenience costs during the peak periods.

3.2. System Model and Game Formulation

In order to evaluate the effects of having various different prices on the customers' behavior in a smart grid, we provided a decision-making framework based on a Stackelberg game. The game was formulated across finite time horizon with length of T periods between a single energy provider and N customers. The set of customers $\mathbb{N}$ is defined as $\mathbb{N} \triangleq \{1, 2, \dots, \mathrm{N}\}$ and the planning horizon $\mathbb{T}$ is defined as $\mathbb{T} \triangleq \{1, 2, \dots, \mathrm{T}\}$. Our proposed framework included two decision stages for the Stackelberg game. The first level was where the energy provider, acting as the leader, took strategic actions to maximize their profit by determining the optimal percentage discount γ_t (from a predetermined nominal price P) for each period within the planning horizon. In the second decision stage, customers, acting as the followers, reacted by shifting their consumption from their nominal demand across the planning horizon based on the tradeoff between the provider's price discounts and consumers' cost of load-shifting inconvenience. In this setting, consumer i has an original electricity demand in kWh for period t, denoted by y_{it}.

Given the electricity provider's price discounts, the consumer's decision was the actual consumption for each period *t*, as represented by x_{it}. As such, the difference between y_{it} and x_{it} measured the load-shift for customer *i* in period *t*. Since shifting incurred an inconvenience cost for the customer, the customer was better off with shifting his or her demand only if the price discounts could counterbalance this cost. It is important to highlight that customers fulfilled their total original demand throughout the planning horizon, that is, $\sum_{t\in T} y_{it} = \sum_{t\in T} x_{it}$. We captured the customers' inconvenience type via α_{it}, which denoted the cost coefficient for customer *i* for period *t*. The coefficient not only varied across customer but also across periods for a given consumer.

3.2.1. Energy Provider's Model

In this model, the setting of energy provider was similar to the one presented in section 2.2.1. In this case, we used the model that present on equation (2)–(3) as follows:

$$\text{Maximize } \Pi_s = \sum_{t=1}^{T}((1-\gamma_t)PX_t - C(X_t)) \tag{41}$$

$$s.t. \qquad 0 \le \gamma_t \le 1 \qquad \forall t \in \mathbb{T} \tag{42}$$

3.2.2. Customer's Model

Customers, as the followers in the Stackelberg game, attempted to minimize their total consumption cost by determining how they modified their consumption from their nominal demand based on their inconvenience costs and the discounts offered by the electricity provider. At the end, the total energy consumption for each customer must equal to his or her total demand across the whole time horizon. The following equation enforced this constraint:

$$\sum_{t=1}^{T} x_{it} = \sum_{t=1}^{T} y_{it} \tag{43}$$

The balance is analogous to the use of forward and delayed procurements as a response to price fluctuations in a supply chain setting.

Not all electrical consumption in a given period can be shifted, such as electricity consumed by refrigerators or security alarms. Therefore, the customers' model included a minimum required consumption for each customer for period m_{it}, which was ensured by the following constraint:

$$x_{it} \geq m_{it} \qquad \forall t \in \mathbb{T} \qquad (44)$$

The adverse effect on the customer of shifting consumption from one period to another was captured by the inconvenience cost function $\varphi(x_{it})$, which was modeled using a quadratic function that convex increases as the difference between the actual consumption x_{it} and original demand y_{it} increases:

$$\varphi(x_{it}) = \alpha_{it}(x_{it} - y_{it})^2 \qquad \forall t \in \mathbb{T} \qquad (45)$$

It is important to highlight that any convex inconvenience cost function may be easily applied to our methodology without loss of generality in the results.

We modeled heterogeneous customers in terms of their attitudes towards load-shifting. As such, each customer i had a positive inconvenience coefficient α_{it}, which drove each customer's sensitivity to electricity price discounts. Customers were more willing to change their consumption in periods with a lower α_{it}, as opposed to periods with a higher α_{it}. Periods that have a low α_{it} were price sensitive, and customers were more likely to change their demand in order to achieve lower total costs. Those periods may include time periods where customers were not constrained with other tasks in their daily lives or did not necessarily need to use particular equipment to the full extent (e.g., air conditioning units during the hours their homes are not occupied). However, there

were periods where customers were less willing to divert their routines (e.g., laundry, dishes, TV). In those periods, the customers employed higher α_{it} values, and as such, they were rather insensitive to electricity price discounts. Moreover, not all consumers were expected to have the same sensitivity to price and convenience in general. Therefore, the inconvenience cost parameters also varied across customers.

Each customer's objective function attempted to minimize the total cost composed of consumption costs and load-shifting inconvenience costs. Consequently, the overall model can be written as follows:

$$Min\ \Pi_i = \sum_{t=1}^{T}\left((1-\gamma_t)Px_{it} - \alpha_{it}(x_{it}-y_{it})^2\right) \tag{46}$$

$$s.t. \qquad \sum_{t=1}^{T} x_{it} = \sum_{t=1}^{T} y_{it} \tag{47}$$

$$x_{it} \geq m_{it} \qquad \forall t \in \mathbb{T} \tag{48}$$

We defined the customers' feasible strategies space as $\Omega_i = \{x_{it} | x_{it} \in R,\ t \in \mathbb{T}, x_{it} \geq m_{it}, \sum_{t=1}^{T} x_{it} = \sum_{t=1}^{T} y_{it}\}$. Hence, interaction between the electricity provider and the customers in the proposed Stackelberg game was summarized by $G = [\,\Omega_s, \Pi_s, \mathbb{N}, \{\Pi_i\}, \{\Omega_i\}]$.

3.3. Equilibrium Analysis

We obtained the equilibrium price discount and load-shifting strategies using backwards induction. As such, we began our analysis with the consumer's problem for a given array of price discounts over the planning horizon. Once this was complete, inferring from the consumers' best responses, the leader, that is the electricity provider, decided on the price discounts. To solve the customers' model, we used Lagrangian Relaxation. Here, we let λ_i be the Lagrange multiplier for constraint (47) and μ_{it} as the multiplier for the constraint set given in (48) in consumer *i*'s model. Thus, the

consumer's objective was equivalent to finding $x_{it}, \lambda_i,$ and μ_{it} that maximized the following Lagrangian function:

$$Max\ LR = \sum_{t=1}^{T}(1-\gamma_t)P\,x_{i,t} + \sum_{t=1}^{T}\alpha_{it}\left(x_{i,t}-y_{i,t}\right)^2 - \lambda\left(\sum_{t=1}^{T}y_t - \sum_{t=1}^{T}x_{i,t}\right) - \sum_{t=1}^{12}\mu_t(x_{i,t}-m_{it}) \tag{49}$$

The Kuhn–Tucker conditions yielded the following set of equations and inequaltities:

$$\frac{\partial LR}{\partial x_t} = (1-\gamma_t)P + 2\alpha_{it}\left(x_{i,t}-y_{i,t}\right) + \lambda - \mu_t = 0 \qquad \forall t \tag{50}$$

$$\frac{\partial LR}{\partial \lambda} = \sum_{t=1}^{12} y_{i,t} - \sum_{t=1}^{12} x_{i,t} = 0 \tag{51}$$

$$x_{i,t} \geq m_{it} \qquad \forall t \tag{52}$$

$$\mu_t(\,x_{i,t} - m_{it}) = 0 \qquad \forall t \tag{53}$$

$$\mu_t \geq 0 \qquad \forall t \tag{54}$$

These conditions led to the following observation:

Proposition 3: At optimality, the consumer's electricity consumption x_{it}^* was

$$x_{i,t}^* = \begin{cases} y_{it} + (\gamma_t - \Lambda_{it})\frac{P}{2\alpha_{it}}, & y_{it} - m_{it} > (\Lambda_{it} - \gamma_t)\frac{P}{2\alpha_{it}} \\ m_{it}, & o/w \end{cases} \tag{55}$$

$$\text{where } \Lambda_{it} = \frac{\sum_t^T\left(\frac{\gamma_t}{2\alpha_{it}}\right)}{\left(\sum_t^T\frac{1}{2\alpha_{it}}\right)}$$

proof: There were two cases for the solution to the system given in (49)–(54). In the first case, the constraint in (52) strictly held in period *t* (i.e., $x_{it}^* > m_{it}$), whereas in the second case it was $x_{it}^* = m_{it}$. Under the first case, we assumed that consumption x_{it}^* exceeded the minimum m_{it} for every period, and thus, $\mu_{it} = 0$ as a result of the complimentary

slackness condition given in (53). Then, using (50), we computed the optimal consumption x_{it}^*, as a function of λ_i:

$$x_{i,t}^* = y_{i,t} - \frac{\lambda-(1-\gamma_t)P}{2\alpha} \tag{56}$$

We substituted the result of (56) into (51):

$$\lambda_i = -\frac{\sum_t^T((1-\gamma_t)P/2\alpha_{it})}{\sum_t^T 1/2\alpha_{it}} \tag{57}$$

Once we had the optimal Lagrange multipliers, we could substitute λ from equation (57) into equation (56), leading to the optimal consumption $x_{i,t}^*$ at equation (60), which may be developed as shown in Eqs. (58) and (59):

$$x_{i,t}^* = y_{i,t} - \frac{(1-\gamma_t)P}{2\alpha_{it}} + \frac{\sum_t^T((1-\gamma_t)P/2\alpha_{it})}{\left(\sum_t^T \frac{1}{2\alpha_{it}}\right) 2\alpha_{it}} \tag{58}$$

$$x_{i,t}^* = y_{i,t} + \frac{P}{2\alpha_{it}}(-1+\gamma_t + \frac{\sum_t^T\left(\frac{1}{2\alpha_{it}}\right)}{\sum_t^T\left(\frac{1}{2\alpha_{it}}\right)} - \frac{\sum_t^T\left(\frac{\gamma_t}{2\alpha_{it}}\right)}{\left(\sum_t^T \frac{1}{2\alpha_{it}}\right)}) \tag{59}$$

$$x_{i,t}^* = y_{i,t} + (\gamma_t - \frac{\sum_t^T\left(\frac{\gamma_t}{2\alpha_{it}}\right)}{\left(\sum_t^T \frac{1}{2\alpha_{it}}\right)})\frac{P}{2\alpha_{it}} \tag{60}$$

Substituting (60) in $x_{it}^* - m_{it} > 0$, yielded

$$y_{it} - m_{it} > (\frac{\sum_t^T\left(\frac{\gamma_t}{2\alpha_{it}}\right)}{\left(\sum_t^T \frac{1}{2\alpha_{it}}\right)} - \gamma_t)\frac{P}{2\alpha_{it}} \tag{61}$$

The inequality in (61) is the condition given in (52). Clearly, when this condition does not hold, $x_{it}^* = m_{it}$, and $\mu_{it} > 0$ must hold due to complimentary slackness.

■

It is clear from this result that each individual consumer's consumption decision was going to be driven by Λ_{it}. Specifically, if the discount offered by the electricity provider in period t, γ_{it} was greater than Λ_{it}, customer i's consumption for that period would be above his or her nominal intended consumption; the converse is true if the discount was below this threshold. It is straightforward to notice that the magnitude of deviation between the nominal and actual consumption was directly proportional to the peak electricity price and inversely proportional to the inconvenience cost coefficient.

If the actual consumption was strictly larger than the minimum consumption for all periods, then the solutions to the consumer model were given by (57) and (60) as the constraint set given by (52) became redundant. This was most likely the case when the values of α_{it} were sufficiently high, implying that the consumers' response level to price discounts was limited. On the other hand, it was expected that one or more constraints defined by (52) would be binding for the price-sensitive consumers. In order to analyze and capture the latter case in the solution process further, for each consumer, we introduced two subsets of periods, namely, $\mathbb{T}_{i1}$ and $\mathbb{T}_{i2}$, where $\mathbb{T}_{i1}$ was composed of periods for which the constraint (52) strictly held and $\mathbb{T}_{i2}$ composed of periods for which the constraint (52) was binding. Moreover, we let $\hat{T}_i$ denote the cardinality of subset $\mathbb{T}_{i1}$. Subsets $\mathbb{T}_{i1}$ and $\mathbb{T}_{i2}$ can be easily identified by solving the problem given by equations (46)–(48), using any over-the-counter nonlinear optimization tool. Once this was done, we rewrote the optimal results based on $\mathbb{T}_{i1}$ and $\mathbb{T}_{i2}$.

Proposition 2: Given $\mathbb{T}_{i1}$ and $\mathbb{T}_{i2}$, at optimality yielded

$$\hat{x}_{it}^{*} = \begin{cases} y_{it} + \dfrac{P}{2\,\alpha_{it}}\left(\gamma_t - \widehat{\Lambda}_i\right) + \dfrac{\sum_{t\in\mathbb{T}_2}(y_{it} - m_{it})}{\sum_{t\in\mathbb{T}_1}(1/2\,\alpha_{it})2\,\alpha_{it}} & , t \in \mathbb{T}_1 \\ m_{it} & , t \in \mathbb{T}_2 \end{cases} \tag{62}$$

$$\lambda_i^{*} = -\frac{1}{\sum_{t\in\mathbb{T}_1}(1/2\,\alpha_{it})}\left(\sum_{t\in\mathbb{T}_1}((1-\gamma_t)P/2\,\alpha_{it}) + \sum_{t\in\mathbb{T}_2}(y_{it} - m_{it})\right) \tag{63}$$

$$\mu_{i,t}^{*} = \begin{cases} 0 & , t \in \mathbb{T}_1 \\ \left(\widehat{\Lambda}_i - \gamma_t\right)P + 2\alpha_{it}(m_{it} - y_{it}) - \dfrac{\sum_{t\in\mathbb{T}_2}(y_{it} - m_{it})}{\sum_{t\in\mathbb{T}_1}(1/2\,\alpha_{it})} & , t \in \mathbb{T}_2 \end{cases} \tag{64}$$

where $\widehat{\Lambda}_i = \dfrac{\sum_{t\in\mathbb{T}_1}\left(\frac{\gamma_t}{2\alpha_{it}}\right)}{\left(\sum_{t\in\mathbb{T}_1}\frac{1}{2\alpha_{it}}\right)}$

proof: Given $\mathbb{T}_{i1}$ and $\mathbb{T}_{i2}$, the Lagrangean relaxation became the following:

$$\begin{aligned} LR = {} & \sum_{t=1}^{T}(1-\gamma_t)P\left(\sum_{i\in\mathbb{T}_1} x_{\widehat{i,t}} + \sum_{i\,\epsilon\,\mathbb{T}_2} m_{\overline{it}}\right) + \sum_{t\in\mathbb{T}_1}^{\hat{T}} \alpha_{it}\left(x_{\widehat{i,t}} - y_{\widehat{i,t}}\right)^2 \\ & + \sum_{t\,\epsilon\,\mathbb{T}_2}^{\bar{T}} \alpha_{it}\left(m - y_{\overline{i,t}}\right)^2 + \lambda\left(\sum_{\hat{t}}^{\hat{T}} x_{\widehat{i,t}} + \sum_{\bar{t}}^{\bar{T}} m_{\overline{it}} - \sum_{t=1}^{12} y_t\right) \\ & - \sum_{t=1}^{12} \mu_t\,(x_{i,t} - m_{it}) \end{aligned} \tag{65}$$

At optimality, the following KKT conditions must hold:

$$\frac{\partial LR}{\partial x_{\widehat{it}}} = (1-\gamma_{\hat{t}})P + 2\alpha_{it}(x_{\widehat{it}} - y_{\widehat{it}}) + \lambda - \mu_{\hat{t}} = 0 \qquad \forall \hat{t} \tag{66}$$

$$\frac{\partial LR}{\partial \lambda} = \sum_{t=1}^{12} y_{i,t} - \left(\sum_{\hat{t}}^{\hat{T}} x_{\widehat{i,t}} + \sum_{\bar{t}}^{\bar{T}} m_{\overline{it}}\right) = 0 \tag{67}$$

$$x_{i,t} \geq m_{it} \qquad \forall t \tag{68}$$

$$\mu_t\,(x_{i,t} - m_{it}) = 0 \qquad \forall t \tag{69}$$

$$\mu_t \geq 0 \qquad \forall t \tag{70}$$

We can rewrite equation (66) as follows:

$$(1-\gamma_{\hat{t}})P + 2\alpha_{it}(x_{\widehat{it}} - y_{\widehat{it}}) + \lambda = \mu_{\hat{t}} \tag{71}$$

By substituting equation (71) in equation (67),

$$((x_{i,t} - m_{it})\big((1-\gamma_{\hat{t}})P + 2\alpha_{it}(x_{\hat{i}t} - y_{\hat{i}t}) + \lambda\big) = 0 \tag{72}$$

When the constraint in equation (68) was strictly holding than $x_{i,t} > m_{it}$ and $\mu_{\hat{t}} = 0$, then

$$(1-\gamma_{\hat{t}})P + 2\alpha_{it}(x_{\hat{i}t} - y_{\hat{i}t}) + \lambda = 0 \tag{73}$$

From equation (73), we defined actual consumption, $x^*_{\hat{i},\hat{t}}$, as follows:

$$x^*_{\hat{i},\hat{t}} = y_{\hat{i},\hat{t}} - \frac{\lambda - (1-\gamma_{\hat{t}})P}{2\,\alpha_{it}} \tag{74}$$

Substituting equation (74) into equation (67) led to equation (77), where we determined the optimal Lagrange multipliers λ, which may be developed as shown in equation (75) and (76):

$$\sum_{t=1}^{12} y_{i,t} - \sum_{\hat{t}=1}^{\hat{T}} \left(y_{\hat{i},\hat{t}} - \frac{\lambda - (1-\gamma_{\hat{t}})P}{2\,\alpha_{it}}\right) - \sum_{\bar{t}}^{\bar{T}} m_{\bar{i}\bar{t}} = 0 \tag{75}$$

$$\sum_{\hat{t}}^{\hat{T}} \frac{\lambda}{2\,\alpha_{it}} = -\sum_{t=1}^{12} y_{it} + \sum_{\hat{t}}^{\hat{T}} y_{\hat{i}\hat{t}} - \sum_{\hat{t}}^{\hat{T}} \frac{(1-\gamma_{\hat{t}})P}{2\,\alpha_{it}} + \sum_{\bar{t}}^{\bar{T}} m_{\bar{i}\bar{t}} \tag{76}$$

$$\lambda = -\frac{1}{\sum_{\hat{t}}^{\hat{T}} (1/2\,\alpha_{it})} \left(\sum_{\hat{t}}^{\hat{T}} ((1-\gamma_{\hat{t}})P/2\,\alpha_{it}) + \sum_{t \in \mathbb{T}_2} (y_{\bar{i}t} - m_{\bar{i}t})\right) \tag{77}$$

By substituting equation (77) into equation (74), we derived the optimal consumption $x^*_{\hat{i},\hat{t}}$ at equation (78) for each customer i during each timeslot t when $t \in \mathbb{T}_1$:

$$x^*_{\hat{i},\hat{t}} = y_{\hat{i},\hat{t}} + \frac{P}{2\,\alpha_{it}} \left(\gamma_{\hat{t}} - \frac{\sum_{\hat{t}}^{\hat{T}} \left(\frac{\gamma_{\hat{t}}}{2\alpha_{\hat{i}\hat{t}}}\right)}{\left(\sum_{\hat{t}}^{\hat{T}} \frac{1}{2\alpha_{\hat{i}\hat{t}}}\right)}\right) + \frac{\sum_{t \in \mathbb{T}_2} (y_{\bar{i}t} - m_{\bar{i}t})}{\sum_{\hat{t}}^{\hat{T}} (1/2\,\alpha_{it}) 2\,\alpha_{it}} \tag{78}$$

In the second case, when $t \in \mathbb{T}_2$, the constraint in equation (68) is binding, than $x^*_{\bar{i}\bar{t}} = m_{it}$ and $\mu_{\hat{t}}$ will find it by substituting equation (77) in equation (71):

$$\mu_{it}^{*} = \left(\frac{\sum_{\hat{t}}^{\hat{T}} \left(\frac{\gamma_{\hat{t}}}{2\alpha_{i\hat{t}}} \right)}{\left(\sum_{\hat{t}}^{\hat{T}} \frac{1}{2\alpha_{i\hat{t}}} \right)} - \gamma_t \right) P + 2\alpha_{it}(m_{it} - y_{it}) - \frac{\sum_{t \in \mathbb{T}_2} (y_{\bar{t}t} - m_{\bar{t}t})}{\sum_{\hat{t}}^{\hat{T}} (1/2\, \alpha_{it})} \tag{78}$$

■

In this case, $\widehat{\Lambda}_i$ is the threshold for customer i in the periods that apply to subset $\mathbb{T}_1$. The electricity provider optimized its strategic price discount decision by inferring the customers' optimal responses. As such its decision model incorporated the results from equations (57), (60), and (62)–(64). Computing the equilibrium price discounts may require multiple iterations as the constraints from equation (52) drove the closed-form solution of the consumers' model.

In order to find the equilibrium solutions, we followed this proposed procedure. We began by inserting the best responses obtained in equation (60) into the electricity provider's model and ignoring the constraints from equation (52), as in the previous stage. As such, the electricity provider's profit function replaced the total energy consumption X_t with the following:

$$X_t = \sum_i^N (y_{i,t} + (\gamma_t - \Lambda_{it}) \frac{P}{2\alpha_{it}}) \tag{79}$$

Definition 2: The Stackleberg game $G = [\, \Omega_s, \Pi_s, \mathbb{N}, \{\Pi_i\}, \{\Omega_i\}]$ has an equilibrium result at the point $(\gamma_t^*\, , x_{it}^*(\gamma_t^*))$ if and only if

$$\Pi_s\big(\gamma_t^*\, , x_{it}^*(\gamma_t^*)\big) \geq \Pi_s(\gamma_t\, , x_{it}(\gamma_t)\,)\, , \forall\, t \in \mathbb{T}\, and\, i \in \mathbb{N} \tag{80}$$

We solved the electricity provider's model by using AMPL (but it may be solved using any other nonlinear optimization tool). However, only when the constraints from equation (52) are met for all the customers, could we say that the solution with this characterization of electricity consumption mapping gave us the equilibrium results. If this was not the case, we needed to perform iterations, as outlined in Table 3.

It is not hard to see that the electricity provider did not benefit from load-shifting if the nominal demand was uniform across the planning horizon. As such, the electricity provider was better off without providing any discounts. Price discounts were helpful to the electricity provider when they reduced demand variations from period to period, as these variations elevated electric generation costs. Thus, price discounts would only be used to shift demand from periods with high demand to periods with lower consumption.

Table 3: Procedure to find equilibrium solution, customer behavior

1: include the consumers' best response from equation (79) into the energy provider's model in equation (41–42).
2: solve energy provider's model
3: if $x_{i,t}^{*} \geq m_{it}\ \forall\, i \in \mathbb{N}, t \in \mathbb{T}$ then STOP.
4: else for each consumer for which constraint (52) is violated in any time *t*
5: identify and update $\mathbb{T}_1$ and $\mathbb{T}_2$
6: use (62) instead of (79) for these consumers in the energy provider's model
7: solve the utility firm's model again including these updates.
8: go to line 3.

3.4. Numerical Analysis

We created numerical scenarios to evaluate the effects of consumer types (in terms of their different inconvenience cost profiles) on the equilibrium outcome of the price discounts, electricity supplier's profit, PAR values, load-shift, and the consumers' total cost. We considered a time horizon of 24 hours, which were aggregated into 12 2-hour

periods. These periods were arranged into three clusters. The first cluster started at midnight and ended at 8:00 a.m. that included low demand or off-peak hours. The second cluster started at 8:00 a.m. and ended at 4:00 p.m., and all of the periods in this cluster were considered high demand or peak by the electricity supplier. The last cluster started at 4:00 p.m. and ended at midnight, in which the first two periods were considered high demand by the electricity supplier while the last two periods faced relatively lower demand. Table 4 shows the aggregate nominal demand that we used for each period, and Figure 8 depicted the demand pattern across the planning horizon.

Table 4: Aggregate demand (kwh), customer behavior

Cluster	1				2				3			
Period (t)	1	2	3	4	5	6	7	8	9	10	11	12
Aggregate Demand	30	47	67	88	100	117	220	184	174	111	52	45

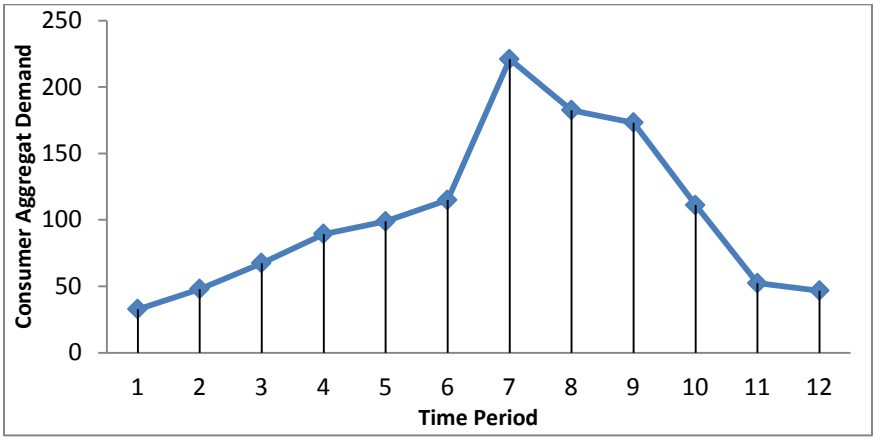

Figure 8: Aggregate demand across the time horizon, customer behavior

In this study, we included 21 independent consumers whose nominal demand had been randomly generated based on a typical residential electricity bill. Each of these consumers had three levels of inconvenience (characterized as high, medium, and low),

which expressed their relative willingness to deviate from their nominal demand in any given period. Inconvenience factors varied across consumers and time periods.

In the electricity supplier's energy production cost function $C(X_t)$, parameter a was set to 0.0009 \$/KWH^2, and parameters b and c were assumed to be zero without loss of generality. We also normalized minimal consumption m_{it} to 0 without loss of generality. Finally, the base electricity price P was set to 20¢/KWH.

We considered four scenarios to study the effect of consumer attitudes towards load-shifting so as to analyze their impact on the overall equilibrium profit of the electricity provider, the total cost for the consumers, and the PAR. Under scenarios one to three, consumers in the market were homogenous in that they exhibit similar behavior as response to price discounts. In the first scenario, every consumer had a high level (H) of inconvenience during the periods in the first cluster, a medium level (M) of inconvenience during the periods in the second cluster, and a low level (L) of inconvenience during the periods of the third cluster. As such, we referred to this scenario as the homogenous market HM–HML scenario. In the second scenario, every consumer had a low level of inconvenience during the periods in the first cluster, a high level of inconvenience during the periods in the second cluster, and a medium level of inconvenience during the periods of the third cluster. We referred to this scenario as the homogenous market HM–LHM scenario. In the third scenario, every consumer had a medium level of inconvenience during the periods in the first cluster, a low level of inconvenience during the periods in the second cluster, and a high level of inconvenience during the periods of the third cluster. We referred to this scenario as the homogenous market HM–MLH scenario.

In the fourth scenario, we considered a market with diverse consumers where the consumers were split into three groups, and all groups had the same average of the inconvenience factor, which was 0.001377. The first group's behavior was similar to the HM–HML scenario, while the second group's behavior was similar to the HM–MLH scenario. The third group's behavior was similar to the HM–LHM scenario. These inconvenience patterns are illustrated in Table 5. We referred to this situation as the diverse market (DM) scenario. In homogenous market and DM scenarios, the average for the inconvenience factor (α_{it}) in the low level was 0.000416. In the medium level, it was 0.00152, and in the high level 0.003316. The consumer inconvenience levels across the clusters of periods under the DM scenario are depicted in Figure 9.

Table 5: Inconvenience patterns under the DM scenario

	Cluster 1	Cluster 2	Cluster 2
Group 1	H	M	L
Group 2	M	L	H
Group 3	L	H	M

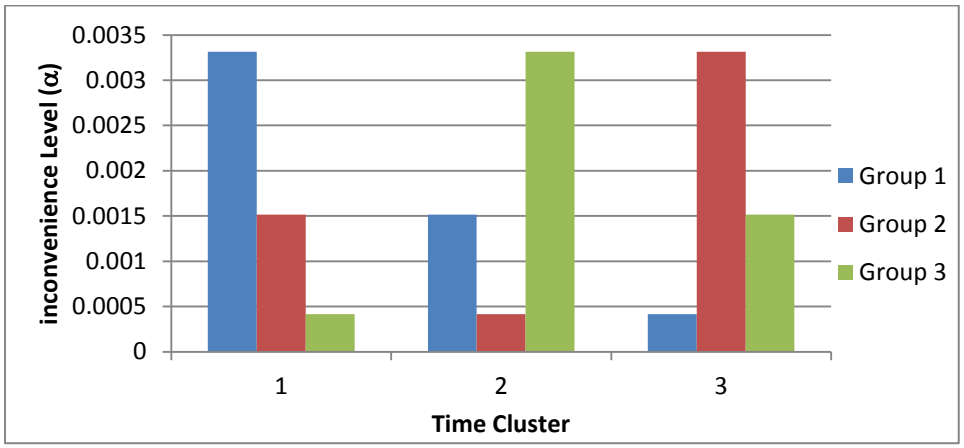

Figure 9: Consumer inconvenience levels across the clusters of periods under the DM scenario, customer behavior

In all scenarios, we used multiplier z to scale the inconvenience factors, such that at higher values of z, the consumers had lower sensitivity to price discounts and hence became more resistant to demand shifts. Specifically, we varied the inconvenience cost coefficient (i.e., inconvenience level) by multiplying them by z, whose value varied between 0.5 and 15. Figures 10 and 11 depict the impact of consumer attitudes on the overall equilibrium average discounts (γ) and PAR. When z was low, consumers were more responsive to price discounts in general. In this case, the provider needed to offer higher price discounts to the –-LHM consumers to shift their demand away from peak periods since these consumers had the highest inconvenience costs during the peak hours. As z increased, in general, the responsiveness of the consumers to discounts became so low that it was no more economical for the provider to offer deeper discounts. Therefore, discounts took a downward trend as observed in Figure 10. Consistent to this observation, from Figure 10, we observed that the highest discounts were offered to the HM–MLH consumers under high values of z.

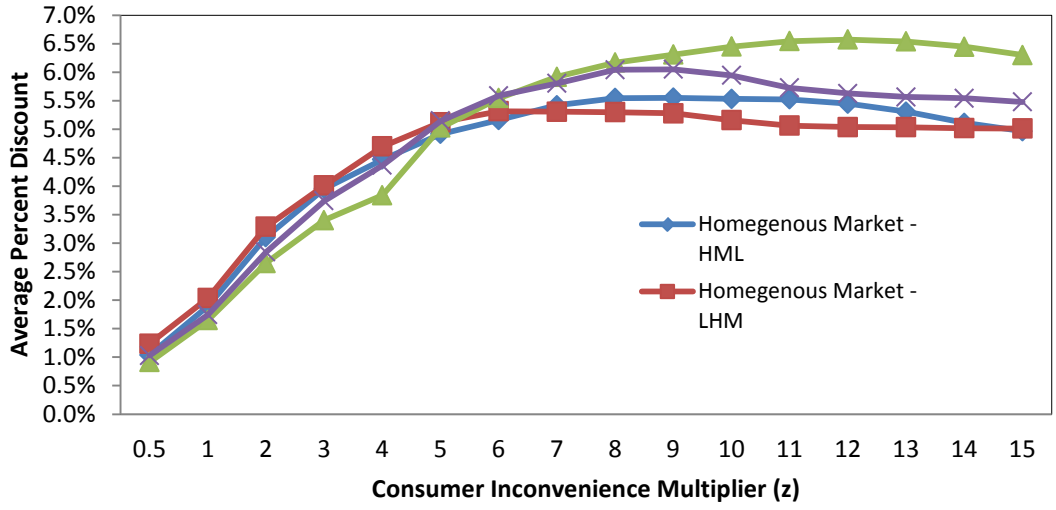

Figure 10: Consumer inconvenience levels and electricity provider's average price discount, customer behavior

Overall, as illustrated in Figure 11, the PAR values increase with inconvenience levels. This was expected since the load shift diminished as consumers became less sensitive to price discounts. The PAR levels were higher in the HM–LHM market since the consumers had the highest inconvenience levels during the peak periods in this case. On the other hand, PAR values were strictly lower for the HM–MLH where consumers were the most responsive to price discounts during the peak periods. Typically, PAR and the provider's profits were inversely related. As depicted in Figure 12, the provider profits were relatively higher with the HM–MLH scenario where the PAR values were the lowest. The opposite was true under the HM–LHM scenario.

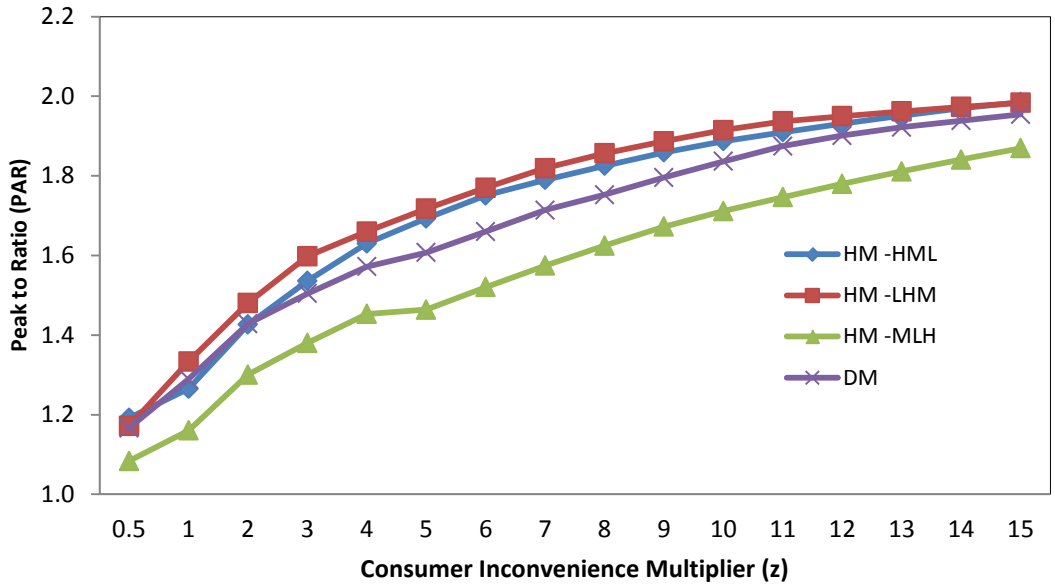

Figure 11: Consumer inconvenience and PAR values, customer behavior

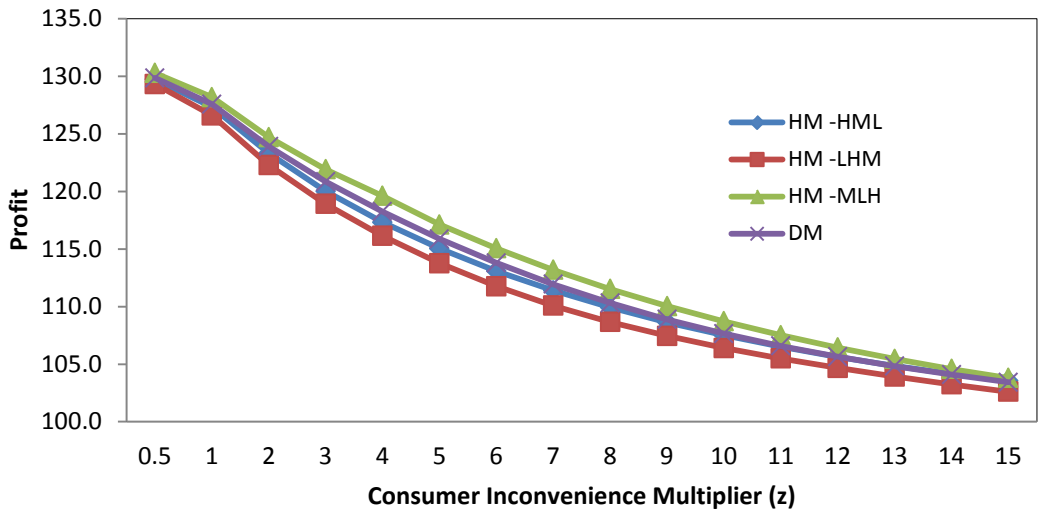

Figure 12: Consumer inconvenience and profit, customer behavior

Interestingly, we observed that although the lowest PAR was attributed to HM–MLH, the amount of load-shift was higher in the DM as depicted in Figure 13. First, this indicates that the relation between PAR and load shift was not necessarily perfect. More demand shift did not always result in lower PAR. In the diverse market, the consumers took advantage of the price discounts by shifting their load in varying directions depending on the attitude group to which they belonged. Whereas, in a homogenous market all consumers reacted to price discounts similarly. As such, price discounts did not result in mixed reactions in terms of how the consumers shifted their consumption. Consequently, under the HM–MLH scenario, where all consumers were more willing to move away from the peak period usage, the electricity provider could dampen the load during these periods more than in any other scenario. In fact, we observe in Figure 13 that when the inconvenience costs were low (i.e., z was sufficiently low), total load-shift amounts are the least under the HM–MLH scenario even though this scenario always led to the lowest PAR values. This was an indication that the main factor that influenced the PAR values was the consumers' reaction to price discounts during the peak periods. A

closer look at the consumer responses during the peak period is given in Figure 14, where we observe that the peak period consumption changes were always higher under HM–MLH, where the average inconvenience cost was the lowest during those periods (Cluster 2).

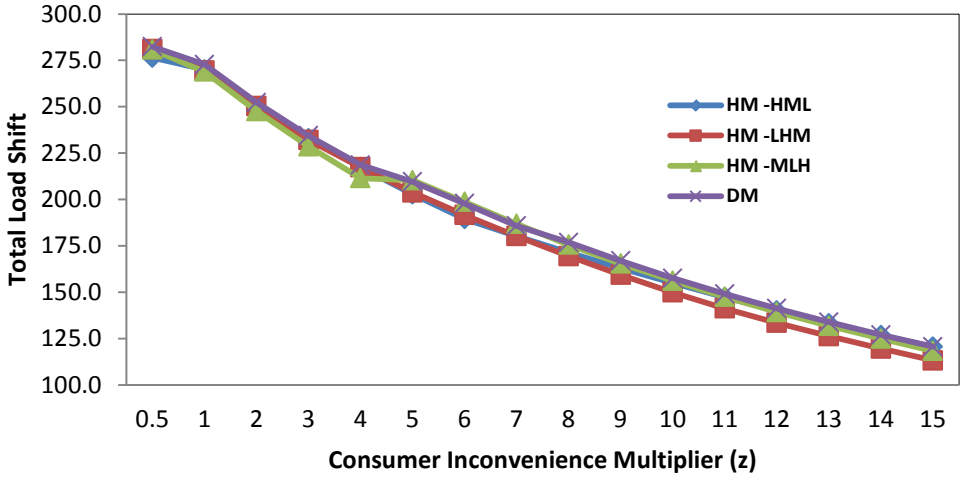

Figure 13: Consumer inconvenience and load-shift, customer behavior

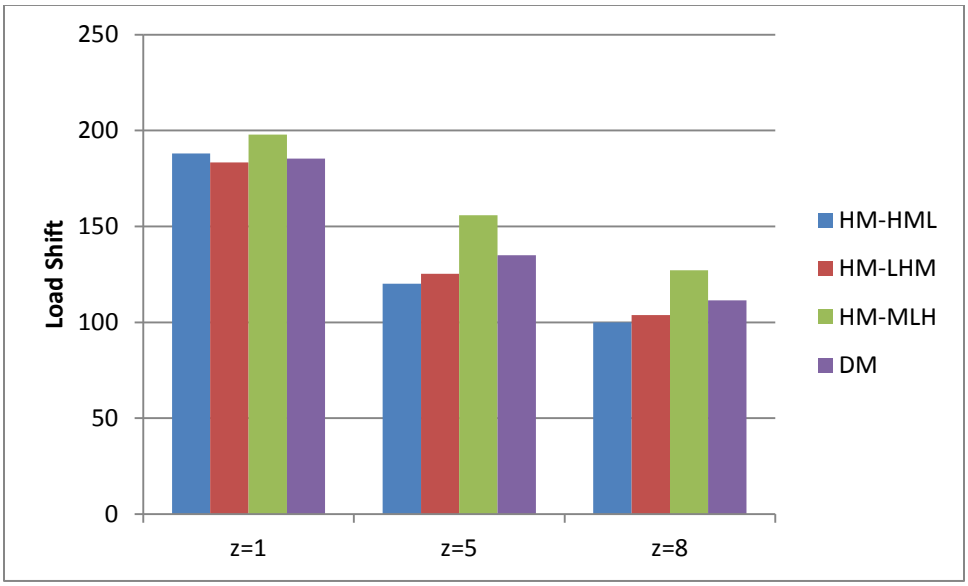

Figure 14: Load shift during peak periods (Cluster 2).

As expected, price discounts not only alleviated PAR values and helped the provider realize better profits, they could also reduce the consumers' total cost for use of electricity. To analyze the trade-off between the consumer inconvenience levels and the consumers' electricity bill amounts further, we compared the equilibrium outcomes between the homogenous and the diverse markets. We plotted the average electricity usage costs, which were captured by the first term in the consumer objective given in (46), for the same seven consumers in both markets in Figure 15. For example, in Figure 15(a) we computed and plotted the average usage costs for Group 1 (HML) in the HM–HML and DM scenarios. In HM–HML, all consumers were alike in terms of their attitude types towards load-shifting whereas the DM was composed of three groups of consumers given in Table 5. By this comparison we aimed to capture the impact of market diversity on equilibrium consumer costs. We repeated the same process for Groups 2 and 3 in plots given by Figures 15(b) and 15(c) respectively.

We first noted that in all cases as the inconvenience levels increased, the provider's extended discounts led to lower energy bills up to a certain point. Consistent with our observation in Figure 10 p.47, when the inconvenience levels become too high, the electricity provider gives up on furthering its price discounts, resulting with an increase in energy bills. Consequently, we can conclude that the consumers enjoy lowest bills when their inconvenience levels are neither too low nor too high.

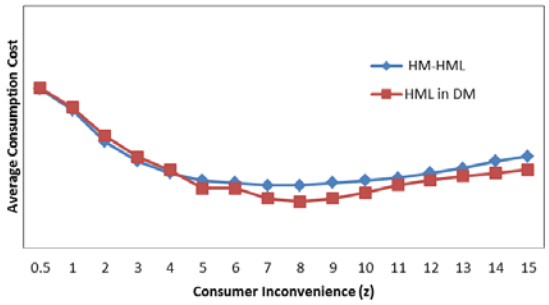

Figure 15(a): Average consumption costs for Group 1

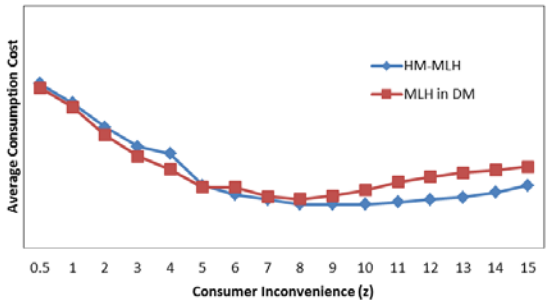

Figure 15(b): Average consumption costs for Group 2

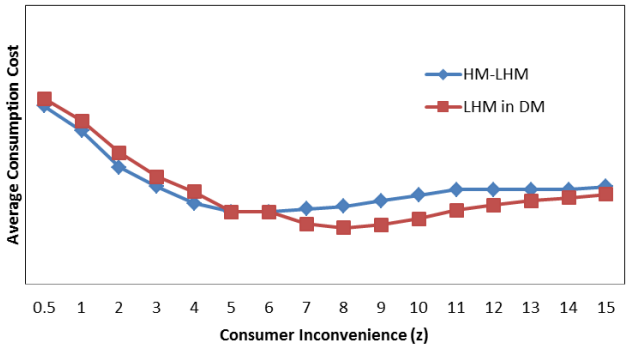

Figure 15(c): Average consumption costs for Group 3

Figure 15:Average consumption costs.

In the case of Group 1, as illustrated in Figure 15(a), no apparent gap was observed for the consumer energy bills across scenarios for relatively low levels of inconvenience. However, under relatively higher inconvenience levels, the consumers of this type were better off in the DM setting. We noted that for consumers in this group, the lowest inconvenience levels were experienced during Cluster 3 where the demand was neither too low nor too high. Since the DM setting included other consumers with inconvenience levels either low during the on-peak periods or during the off-peak periods, the electricity provider's price discounts were more aggressive compared to the HM–HML market as they generated higher returns in profits when z was high.

Similar to Group 1, Group 3 (LHM) consumers experienced larger bills in the homogenous market when z was high because under high inconvenience costs the provider focused on the Group 2 consumers, thus offering higher discounts in the DM setting compared to the HM–LHM setting (see Figure 15c). However, when the overall inconvenience scale (i.e., z was small), the opposite occurs. In this case, the electricity provider took a more balanced approach in its discounts under the DM, leading to better outcomes for Group 3.

Consumers of Group 2 (MLH) experienced a different effect than others as illustrated in Figure 15(b). When the inconvenience scale was low, these consumers did not require high discounts in the HM–MLH since the off-peak resistance was moderate and the on-peak resistance was low. As such, they were better off under the DM setting. On the other hand, as observed in Figure 10 p.47, the provider was willing to offer higher price discounts when z was high to these consumers in the HM–MLH since the return on discounts were higher. That is, more consumption would be shifted from the on-peak periods with relatively lower inconvenience levels.

3.5. Conclusions

In this paper we proposed a model based on a Stackelberg game to analyze the load-shifting problem in the smart grid, where an electricity provider offered price discounts across the planning horizon in order to incentivize consumers to rearrange their consumption habits. The provider was motivated to offer price discounts due to the high costs it incurred during the peak periods. The consumers reacted with their decisions on consumption shifts. In order to capture the variations in consumers' response behaviors and their resistance to load-shifting, we modeled consumer types based on a mapping of

inconvenience that they suffered by altering their consumption amounts from their nominal demand in a period. Specifically, we considered the case where not only do the consumers have different types, but their inconvenience levels also varied across the planning horizon.

In order to investigate the impact of consumer inconvenience and derive managerial insights, we carried out an extensive numerical analysis. We showed that when consumers were price takers, that is, their inconvenience costs were low, it was relatively easier for the electricity provider to dampen the PAR and enjoy higher payoffs with smaller discounts. As the inconvenience levels increase, the provider was compelled to offer deeper discounts up to a point. When the inconvenience levels became too high, the consumer resistance to load-shifting was too strong to justify price discounts. As such, after a point, the provider, in fact, began to cut back on the discounts.

In our analysis, we employed a variety of consumer groups who differed in their reactions to price discounts at on-peak and off-peak periods. We observed that when the overall scale for the inconvenience levels was low, the provider adopted more aggressive discounts for a market with consumers who showed relatively higher resistance during the on-peak periods and lower discounts for consumers who had lower inconvenience costs during the off-peak periods. Consistent with our earlier observation, the opposite occurred when the inconvenience scale was very high. In addition, our analysis revealed that the structure of the market in terms of consumer diversity and the consumer types jointly impacted the equilibrium discounts and PAR levels. In general, a diverse market led to lower PAR values and higher provider payoffs compared to a homogenous market unless

the latter one was composed of consumers with relatively lower inconvenience costs during the peak periods.

Chapter 4 A Game Theoretic Approach for Load-Shifting and Energy Storage

4.1. Overview

This chapter presents an electricity market composed of a single energy provider and multiple customers to evaluate the effects of energy storage and load-shifting as part of a smart grid demand response. The system was modeled using a Stackelberg game in which the energy provider was the leader and the customers were the followers. The leader moved first and offered price discounts across different time slots to motivate customers to shift their consumption away from peak consumption periods. The followers responded by deciding whether or not to shift their consumption from their nominal demand and how much of their load to shift. Under this scenario, the goal of the energy provider was to maximize their profits, while the goal of the customers was to minimize their total cost, both from energy consumption and from the inconvenience generated by load-shifting and deviating from their originally intended nominal demand. In this particular model, we evaluated and compared the effects that the control of customer-side energy storage had on the equilibrium. When the energy storage were controlled by the energy generator, they reduced energy generation fluctuations by changing the energy generation amounts to different time slots without significantly changing the customers' nominal demand. When these energy storage were controlled by the consumers, they were able shift their effective demand to periods with high discounts while keeping the nominal demand relatively unchanged. Using this model, we proposed a procedure to obtain the equilibrium discount and PAR. Insight was

gained by evaluating the effects that having different customer types, a different number of customers, and different customer attitudes towards load-shifting have on the system. Our results show that the use of energy storage was very effective in the reduction of PARs, regardless of the customers' attitude toward load-shifting. Price discounts continued to provide significant leverage to the energy generator, and the use of both of these mechanisms provided successful ways to improve profits. Furthermore, when the energy provider controlled the energy generation systems, it deployed them more effectively and achieved its maximum profits and the lowest PAR. When the customers controlled the energy storage , the equilibrium discounts were higher, but the PAR was also higher. Lastly, our results showed that the use of load-shifting always reduced the customers' total costs, but this reduction was diminished by the implementation of energy storage.

4.2. Energy Storage Setting

As part of our model for the storage profile, we defined storage decision variables s_{it} *and* u_{it} as the amount of charging and discharging, for customer i during timeslot t, respectively. Further, we let w_{it} denote the storage level at customer i during time slot t as

$$w_{it} = \rho\, w_{i,t-1} + (1 - \tau)\, s_{it} - (1 + \beta) u_{it} \tag{81}$$

Where ρ, τ *and* β presented the storage, charging, and discharging efficiencies respectively (In a fashion similar to that of (Atzeni et al., 2013) and (Vytelingum et al., 2010)). These efficiencies determined the energy lost by the use of the energy storage.

In order to control the storage profiles and to avoid excesses in the charging and discharging of the storage systems, we determined the storage capacity as W^{max} and a

minimum stored energy requirement as $zero$, so that w_{it} was bound by these conditions as

$$0 \leq \mathrm{w_{it}} \leq \mathrm{W^{max}} \tag{82}$$

4.3. Customer Controlled Energy Storage

4.3.1. Game Formulation

In order to evaluate the effects of the introduction of energy storage, when these systems were controlled by the consumers in a smartgrid we formulated a Stackelberg game based on the model provided in Chapter 3. We incorporated energy storage on the customer side in order to investigate the effect of the energy storage on the equilibriums outcome. In our proposed game there were two levels. The provider, as the leader of the game, made the first move in order to maximize its profit by deciding on a percentage discount, γ_t, with a schedule based on the predetermined base price P. While the customers, acting as followers, responded by making two strategic decisions in order to minimize their total cost. First, they chose the amount of the consumption they were willing to shift from low or no discount time slots to time slots with a higher discount. The second decision was to determine an amount of energy to store or deploy from storage for each time slot by charging the system when the discounts were high and discharging the system during points of low or no discounts. The customer tradeoff between the electricity cost and inconvenience costs, which was incurred by the deviation in their consumption from their original intended demand, or the storage cost that incurred by storage leak. To model the smartgrid, we let $\mathbb{N}$ be the set of customers, defined as $\mathbb{N} \triangleq \{1, 2, \dots, N\}$ and $\mathbb{T}$ be the set of time slots defined as $\mathbb{T} \triangleq \{1, 2, \dots, T\}$.

4.3.2. Model

The goal of the consumers, who acted as followers in the Stackelberg game, was to minimize their total cost, both from their energy bill and inconvenience. In order to do this, they determined the optimal levels of energy to buy and consume during each time slot by shifting their energy usage across the different slots and/or by determining the amount of energy to charge or discharge from the storage. Based on the different discounts offered by the energy provider during some of the time slots and the effect of storage efficiency, the customers' energy bill depended not only on the consumption amount but also on the time slots in which this consumption occurred. We used the same inconvenience cost function modeled in section 2.2.2, which is a quadratic function that increases as the difference between the actual consumption x_{it} and nominal demand (y) of a time slot deviates. Since the amount that customer i bought from the electricity provider was different from the actual consumption during each time t. Thus, d_{it} denoted the amount of energy that customer i bought from the electricity provider during time slot t. Based on this we defined $d_{it} = x_{it} - u_{it} + s_{it}$ where x_{it} was the actual consumption profile for customer i during time slot t.

By including the condition in equations (81) and (82) to the customer model presented in equations (4)–(6), we rewrote an optimization model for the customer who aimed to minimize his total cost as

$$Min\ \Pi_i = \sum_{t=1}^{T}\left((1-\gamma_t)P\ d_{it} + \alpha_i(x_{it} - y_{it})^2\right) \tag{83}$$

s.t

$$\sum_{t=1}^{T} x_{it} = \sum_{t=1}^{T} y_{it} \tag{84}$$

$$x_{it} \geq m_{it} \qquad \forall t \in \mathbb{T} \tag{85}$$

$$w_{it} = \rho\, w_{i,t-1} + (1-\tau)\, s_{it} - (1+\beta) u_{it} \quad \forall t \in \mathbb{T} \tag{86}$$

$$0 \le \mathrm{w_{it}} \le \mathrm{W^{max}} \quad \forall t \in \mathbb{T} \tag{87}$$

$$s_{it}, u_{it} \ge 0 \quad \forall t \in \mathbb{T} \tag{88}$$

The strategy technique for the electricity provider was identical to that presented in section 2.2.1, which offeried a higher price discount when the nominal demand was low in order to encourage customers to change their consumption behavior by shifting their consumption from the peak time to the off-peak time and/or charging the storage during the off-peak time for use during the on-peak time. Since the amount that customer i bought from the electricity provider was different from the actual consumption during each time t, we rewrote the optimization model for the electricity provider presented in Eqs. (2)–(3) as

$$Max\, \Pi_s = \sum_{t=1}^{T} \Big((1-\gamma_t) P\,(d_{it}) - C\big(a(\textstyle\sum_{i=1}^{N}(d_{it}))^2 + b \textstyle\sum_{i=1}^{N}(d_{it}) + c\big)\Big) \tag{89}$$

$$s.t. \qquad 0 \le \gamma_t \le 1 \qquad \forall t \in \mathbb{T} \tag{90}$$

4.3.3. Equilibrium Analysis

In terms of finding the equilibrium result of the Stackelberg game, we began to analyze the customer model using backwards induction. To simplify the customer model, we substituted the storage balance constraint, equation (86), in terms of the variable decision s_{it} into the objective function, equation (83). We redefined the customer model as follows:

$$Min\, \Pi_i = \sum_{t=1}^{T} \Big((1-\gamma_t)\, P\, \Big(x_{it} - u_{it} + \frac{w_{it} - \rho\, w_{i,t-1} + (1+\beta) u_{it}}{(1-\tau)}\Big) + \alpha_i (x_{it} - y_{it})^2 \Big) \tag{91}$$

s.t

$$\sum_{t=1}^{T} x_{it} = \sum_{t=1}^{T} y_{it} \tag{92}$$

$$x_{it} \geq m_{it} \qquad \forall t \in \mathbb{T} \tag{93}$$

$$\frac{w_{it} - \rho\, w_{i,t-1} + (1+\beta) u_{it}}{(1-\tau)} \geq 0 \qquad \forall t \in \mathbb{T} \tag{94}$$

$$w_{it} \leq W^{max} \qquad \forall t \in \mathbb{T} \tag{95}$$

$$w_{it}, u_{it} \geq 0 \qquad \forall t \in \mathbb{T} \tag{96}$$

We used Lagrangian Relaxation and assigned λ_i, μ_{1it}, μ_{2it}, and μ_{3it} as Lagrangian multipliers to constraints in equations (92)–(95) respectively. The following Lagrange function is equivalent to the customer objective:

$$\begin{aligned} \text{Min } LR = \Pi_i - \lambda_i\left(\sum_{t=1}^{T} y_{it} - \sum_{t=1}^{T} x_{it}\right) - \sum_{t=1}^{T} \mu_{1t}(x_{ti} - m_{it}) \\ -\mu_{2it}\left(\frac{w_{it} - \rho\, w_{i,t-1} + (1+\beta) u_{it}}{(1-\tau)}\right) + \mu_{3it}(w_{it} - W^{max}) \end{aligned} \tag{97}$$

The Kuhn-Tucker conditions yielded the following set of equations and inequaltities:

$$\frac{\partial LR_i}{\partial x_{it}} = (1-\gamma_t)P + 2\alpha_i(x_{it} - y_{it}) + \lambda_i - \mu_{1it} = 0 \qquad \forall t \tag{98}$$

$$\frac{\partial LR_i}{\partial \lambda_i} = \sum_{t=1}^{12} y_{it} - \sum_{t=1}^{12} x_{it} = 0 \tag{99}$$

$$x_{it} \geq m_{it} \qquad \forall t \tag{100}$$

$$\mu_{1it}(x_{it} - m_{it}) = 0 \qquad \forall t \tag{101}$$

$$\frac{\partial LR_i}{\partial u_{it}} = (1-\gamma_t)P\left(\frac{(1+\beta)}{(1-\tau)} - 1\right) - \frac{(1+\beta)}{(1-\tau)}\mu_{2it} \geq 0 \qquad \forall t \tag{102}$$

$$u_{it}\left(\frac{\partial LR}{\partial u_t}\right) = 0 \qquad \forall t \tag{103}$$

$$\begin{aligned} \frac{\partial LR_i}{\partial w_{it}} = (1-\gamma_t)P\left(\frac{1}{(1-\tau)}\right) - (1-\gamma_{t+1})P\left(\frac{\rho}{(1-\tau)}\right) + \mu_{3it} \\ -\frac{1}{(1-\tau)}\mu_{2it} + \frac{\rho}{(1-\tau)}\mu_{2it+1} \geq 0 \qquad \forall t \end{aligned} \tag{104}$$

$$w_{it}\left(\frac{\partial LR}{\partial u_t}\right) = 0 \qquad \forall t \qquad (105)$$

$$w_{it} \leq W^{max} \qquad \forall t \qquad (106)$$

$$\mu_{3it}(w_{it} - W^{max}) = 0 \qquad \forall t \qquad (107)$$

$$\frac{w_{it} - \rho\, w_{i,t-1} + (1+\beta)u_{it}}{(1-\tau)} \geq 0 \qquad \forall t \qquad (108)$$

$$\mu_{2it}\left(\frac{w_{it} - \rho\, w_{i,t-1} + (1+\beta)u_{it}}{(1-\tau)}\right) = 0 \qquad \forall t \qquad (109)$$

$$w_{it}, u_{it}, \mu_{1it}, \mu_{2it} \text{ and } \mu_{3it} \geq 0 \qquad \forall t \qquad (110)$$

Since the customers' actual consumption x_{it} and Lagrange multipliers λ_i have the same conditions, equations (98)–(100), as that in section 2.3. Thus, we used the result obtained in that section. In the case where the equations in set (100) is strictly holding, then $\mu_{1it} = 0$ for all t. Thus, we used the result obtained in equations (15) and (16) in order to find x_{it} and λ_i where

$$x^*_{i,t} = y_{i,t} + \frac{(\gamma_t - \bar{\gamma})\,P}{2\alpha} \geq 0 \qquad \forall t \qquad (111)$$

$$\lambda = (\bar{\gamma} - 1)P \qquad (112)$$

It is clear from equation (111) that the customer made his or her decision for the actual consumption $x^*_{i,t}$ depending on the price discount and the inconvenience factor. Moreover, he or she will decide whenever to get the actual consumption directly from the grid, $x^*_{i,t} - u_{it}$, and/or to get it from the storage, u_{it}.

Since the inconvenience cost function was quadratic, increasing with the deviation between the nominal demand and actual consumption, the energy storage helped reduce the cost without significantly changing the nominal demand. This situation occurred most often when the customer had a high inconvenience factor α_i, where the load-shifting increased his or her payoff significantly. However, due to the high energy

waste, the efficiency of the energy storage had a direct impact on the customer's decision. When the storage efficiency was low, getting the energy from storage may be more expensive compared to taking the demand directly from the grid.

When customers were more sensitive to the price changes, some of the constraints from equation (100) may not have been satisfied. In order to optimize this case, the time slots were separated into two subsets, $\mathbb{T}_1$ and $\mathbb{T}_2$, and $\hat{\mathrm{T}}$ was defined as the cardinality of subset $\mathbb{T}_1$. Here, the time slots in which the constraint from equation (100) simply held were placed in subset $\mathbb{T}_1$ and those in which the constraint was binding were placed in set $\mathbb{T}_2$. In this case, we used the result obtained in equations (21), (22), and (23) in order to find x_{it} ,λ_i and μ_{1it}, where

$$\hat{x}_{it}^{*} = \begin{cases} y_{it} + \frac{1}{\hat{T}} \sum_{\bar{t}=1}^{\bar{T}} (y_{it} - m_{it}) + \frac{(\gamma_t - \bar{\gamma})P}{2\alpha} & , t \in \mathbb{T}_1 \\ m_{it} & , t \in \mathbb{T}_2 \end{cases} \tag{113}$$

$$\lambda = (\bar{\gamma}_{\hat{t}} - 1)P - \frac{2\alpha}{\hat{T}} \sum_{\bar{t}=1}^{\bar{T}} (y_{\bar{t}} - m_{it}) \tag{114}$$

$$\mu_{i,t}^{*} = \begin{cases} 0 & , t \in \mathbb{T}_1 \\ (\bar{\gamma}_1 - \gamma_{\hat{t}})P - 2\alpha_i (y_{it} - m_{it} + \frac{1}{\hat{T}} \sum_{t \in \mathbb{T}_2} (y_{it} - m_{it})) & , t \in \mathbb{T}_2 \end{cases} \tag{115}$$

The electricity provider needed to infer the customer strategies in order to find the optimal discount. Thus, equations (111)–(115) and the KKT conditions in equations (102)–(110) must be included into the electricity provider's decision model. Since some constraints in equation (100) may not hold, the procedure proposed in Table 1 (p.23) can be used to derive the equilibrium solution. By adding equations (102)–(110) to the procedure, Table 6 presents the outline of the algorithm.

Definition 3: The point $(\gamma_t^*, x_{it}^*(\gamma_t^*), u_{it}^*, s_{it}^*, w_{it}^*)$, which satisfied the constraints in (89)–90) and (83)–(88), was an equilibrium result of the Stackelberg game $G = \{\mathbb{N}, \mathbb{T}, \Pi_s, \Pi_i\}$ if and only if

$$\Pi_s(\gamma_t^*, x_{it}^*(\gamma_t^*), u_{it}^*, s_{it}^*, w_{it}^*) \geq \Pi_s(\gamma_t, x_{it}(\gamma_t), u_{it}, s_{it}, w_{it}), \forall\, t \in \mathbb{T}\ and \in \mathbb{N} \quad (116)$$

Table 6: Procedure to find equilibrium solution, storage is controlled by customer

1: include equation (111) and (102)–(110) into the electricity provider's model in equation (89–90).
2: solve energy provider's model
3: if $x_{i,t}^* \geq m_{it}\ \forall\, i \in \mathbb{N}, t \in \mathbb{T}$ then STOP.
4: else for each consumer for which constraint (100) is violated in any time *t*
5: identify and update $\mathbb{T}_1$ and $\mathbb{T}_2$
6: use (113) instead of (111) for these consumers in the energy provider's model
7: solve the utility firm's model again including these updates.
8: go to line 3

The leader strategic on the price discount technique depended on the consumers' inconvenience coefficient α, the distribution of the nominal demand across the planning horizon, and storage system efficiency parameters. The price discount technique helped reduce the demand fluctuations if the customers were sensitive to price discounts and were willing to shift. Otherwise, if the customers had high resistance, the utility did not receive any added benefits from this strategy. Moreover, the energy provider may have

still benefitted from offering a high discount during off-peak times in order to encourage customers to charge the storage during off-peak times and use the stored energy during on-peak times. However, the electricity provider more conservative on offering the discount, especially when the customers were not willing to shift and the storage have high efficiency and large capacity. This meant that if the energy provider offerred a high discount, the customers' response may have moved the on-peak time to another time slot. Thus, the energy provider, as leader of the game, offered discount prices that controlled the customer's selfish reaction.

Proposition 5: The customer cannot charge and discharge the storage system during the same timeslot t:

$$u_{it}s_{it} = 0 \qquad \forall\, i \in \mathbb{N}, t \in \mathbb{T} \tag{117}$$

Proof: Using Lagrange Relaxation for the customer model in equations (83)–(88), where λ_i, μ_{1it}, μ_{3it}, and δ_{it} as Lagrangian multipliers to constraints in equations (92)–(95) respectively

$$\begin{aligned} \text{Min } LR = \Pi_i - \lambda_i\left(\textstyle\sum_{t=1}^{T} y_{it} - \sum_{t=1}^{T} x_{it}\right) - \textstyle\sum_{t=1}^{T} \mu_{1t}(x_{ti} - m_{it}) \\ -\delta_{it}\left(w_{it} - \rho\, w_{i,t-1} - (1-\tau)s_{it}\ (1+\beta)u_{it}\right) \\ + \mu_{3it}(w_{it} - W^{max}) \end{aligned} \tag{118}$$

The Kuhn–Tucker conditions that are related to s_{it} and u_{it} are as follows:

$$\frac{\partial LR}{\partial s_{it}} = (1-\gamma_t)\,P + \delta_t\,(1-\tau) \ \geq 0 \qquad \forall t \in \mathbb{T} \tag{119}$$

$$\frac{\partial LR}{\partial u_{it}} = -(1-\gamma_t)\,P - \delta_t(1+\beta) \ \geq 0 \qquad \forall t \in \mathbb{T} \tag{120}$$

$$s_{it}\left(\frac{\partial LR}{\partial u_t}\right) = 0 \qquad \forall t \in \mathbb{T} \tag{121}$$

$$u_{it}\left(\frac{\partial LR}{\partial u_t}\right) = 0 \qquad \forall t \in \mathbb{T} \qquad (122)$$

To prove the statement, we must satisfy the following:

(I). $\frac{\partial LR}{\partial s_{it}} = 0$ and $\frac{\partial LR}{\partial u_{it}} \neq 0$, or

(II). $\frac{\partial LR}{\partial s_{it}} \neq 0$ and $\frac{\partial LR}{\partial u_{it}} = 0$

Assuming that $s_{it} > 0$, so to satisfy the slackness condition on equation (121) $\partial LR/\partial s_{it} = 0$. It follows from equation (119) and (121) that $\partial LR/\partial s_{it} \neq \partial LR/\partial u_{it}$. Therefore, to satisfy the slackness condition from equation (122), $u_{it} = 0$. ■

4.3.4. Numerical Analysis

In this section, we evaluate the effects that the availability of energy storage for the customers have on the Stackelberg equilibrium, the total costs for both the energy supplier and the individual customers, the PAR, and the average discount offered by the energy supplier. In order to do this, we expanded on the case study presented in Section 2.4, to which we added an energy storage with a capacity of 5 KWh and an efficiency $\rho = 0.99$, $\tau = 0.01$ and $\beta = 0.01$ for every customer.

Just as in the previous case study, the planning horizon was divided into twelve slots; the energy production cost function $C(d_t)$ had parameters $a = 0.0035\ \$/KWh^2$, $b = 0\ \$/KWh$, and $c = 0\ \$$. We assumed the minimum required consumption $m_t = 0$ for every customer and period. The base price was set to $P = 0.25\ \$/kWh$, and we generated the nominal demand from residential electricity bills gathered randomly, as seen in Table 7 and Figure 16.

Table 7: Aggregate demand (kwh), storage is controlled by customers

Period (t)	1	2	3	4	5	6	7	8	9	10	11	12
Aggregate Demand	10.19	15.72	22.28	29.23	33.31	39.14	73.43	61.27	58.1	37.2	17.53	15.01

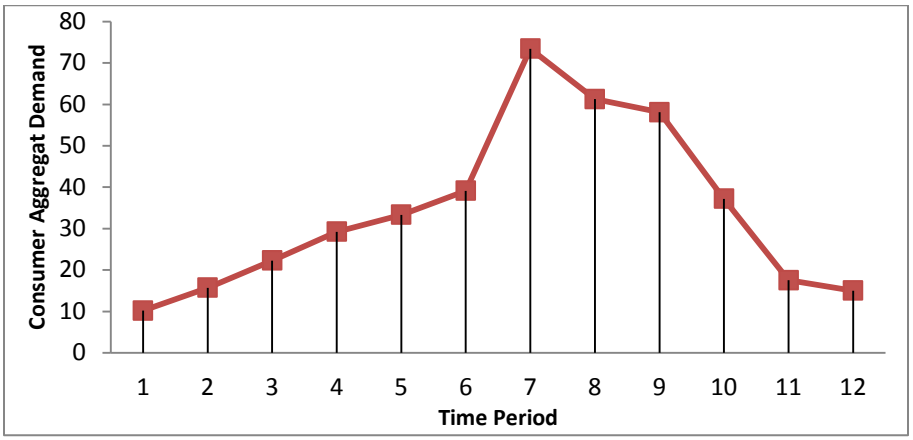

Figure 16: Aggregate demand across the time horizon; storage controlled by customers.

In this setting, we evaluated the effects of having different customer types (in terms of their nominal demands and willingness to shift load), the effects of having markets with different sizes (market breadth), and the effects that the capacity of the energy storage had on the equilibrium outcomes.

In the first scenario, we focused on the effects of having consumers of different types. We used four customers and an inconvenience multiplier z, which ranged from 0.5 to 15 and scaled the inconvenience factor α_i. The individual inconvenience factors were 0.02, 0.021, 0.00025, 0.00875, 0.0003, 0.0004, and 0.007, respectively. Clearly, as the value of z increased, the customers' willingness to shift their nominal demand decreased. Figures 17 and 18 show the effect of consumers' sensitivity to load-shifting on the equilibrium average discounts (γ) and the PAR values.

Figure 17 reveals that when the customers were price takers ($z \in [\frac{1}{2}, 2]$), the equilibrium discount increased sharply with increases in the inconvenience multiplier, as the incentives provided by higher discounts were very effective at incentivizing customer load-shifting.

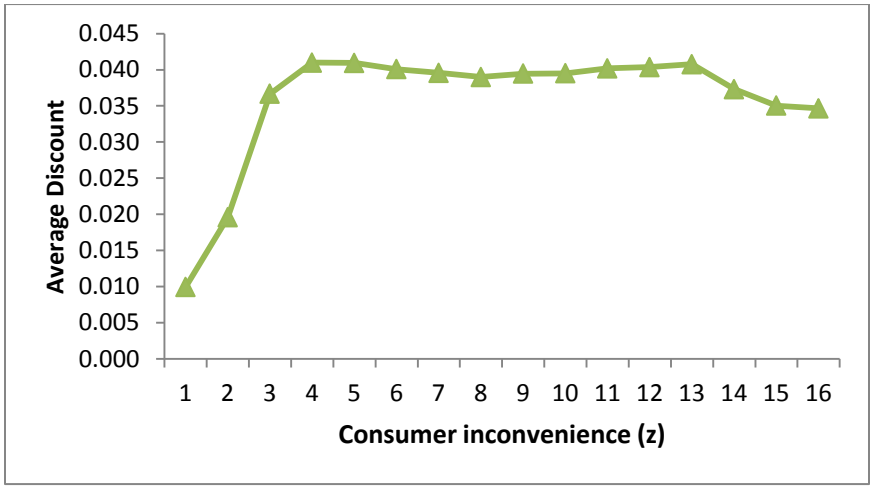

Figure 17: Consumer inconvenience and energy provider's average price discount; storage controlled by customers

Past this point ($z \in [3,12]$), the effects of customer discounts were not effective, and thus the equilibrium discount plateaued at 4%. Once the point of inflexibility on behalf of customers was reached ($z \in [13,15]$), the discounts were very ineffective and higher profits were achieved by lowering discounts.

Figure 18 reflects the effects of these three customer behaviors on the PAR. When the customers were price takers ($z \in [\frac{1}{2}, 2]$), it was more effective for customers to shift their loads and take all of their demand directly from the grid; thus, there was a small increase in the PAR as the inconvenience multiplier increased. Customers began to benefit from their ability to store energy past this point ($z \in [3,12]$), and as such when

$z = 4$ there was a drop in the PAR to a minimum of 1.272. In this fragment of the graph, the PAR increased as the inconvenience multiplier increased as less loads were shifted. In the final fragment, once the point of inflexibility on behalf of customers was reached ($z \in [13,15]$), the PAR shifted from 1.825 to 2.025 as the discounts decreased and the equilibrium storage also decreased.

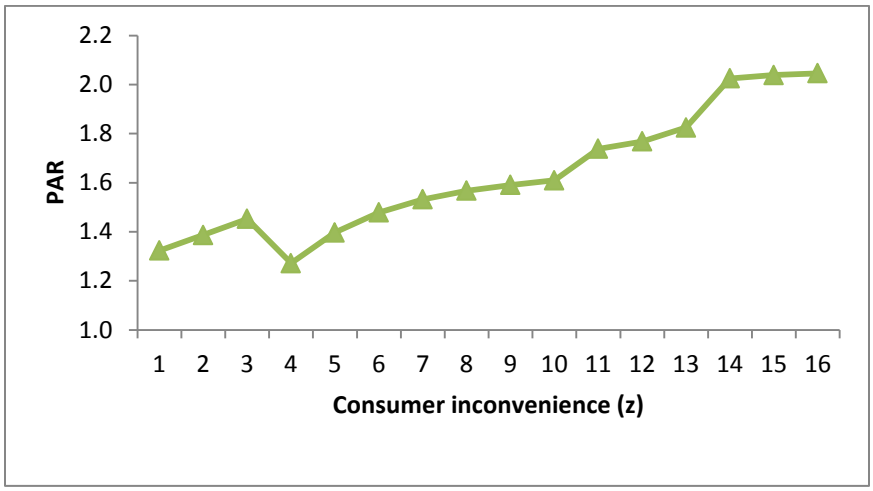

Figure 18: Consumer inconvenience and PAR values; storage controlled by customers.

The behavior reflected in Figures 17 and 18 can be attributed to the effects of the constraints from equation (85). In the first three cases ($z \in [\frac{1}{2}, 2]$), some of these constraints were binding as consumers were very sensitive to price discounts and there was no incentive for the electricity provider to offer high discounts. As such, the optimal way for customers to meet their needs was through the use of extensive load-shifting and buying all of their electricity directly from the grid.

Past this point, ($z \in [3,12]$), some of the constraints in equation (85) become strictly holding, but the discounts still continued to affect load-shifting. Customers began to take advantage of the energy storage and there was a drop in the PAR from 1.452 when $z = 3$ to 1.272 when $z = 4$. During this segment the price discounts reached their

maximum effectiveness and maximal storage was reached with discounts of 4%. In this segment, the PAR increased as the inconvenience multiplier increased, since less loads were being shifted.

In the final segment, ($z \in [13,15]$), the customers were very insensitive to price discounts and as such the electricity provider did better to lower discounts and increase revenues as its ability to influence the PAR diminished. This shift in the PAR (from 1.825 to 2.025) led to higher costs for the electricity supplier and lower profits.

In the second scenario, we evaluated the effects of the customers' attitude toward load-shifting (market diversity). In order to do this, we introduced change to the individual inconvenience factors α_i while keeping the average inconvenience at 0.0125 (as in the prior case). We set the coefficient of variation ($COV \in [0,1.05]$) for a_i, so that a higher COV constituted higher diversity in the consumer market. Figure 19 shows the effects of different COV levels on the average discounts, the PAR, and the energy storage levels.

There are three distinct segments to the graph shown in Figure 19. In the first segment ($COV \in [0,0.15]$), there is very low market diversity and all of the customers had the same behavior in terms of load-shifting and energy storage. In this segment the high discount leads to the use of energy storage on behalf of the customers but the low market diversity leads to low load shifting.

In the second segment ($COV \in [0.3,0.9]$), as market diversity increased, there were more and more customers willing to shift more of their loads, and as such the equilibrium discount was lower. This low equilibrium discount did not promote the use of

energy storage on behalf of the customers; thus, there was the shift in PAR from 1.644 to 1.812.

In the final segment ($COV > 0.9$), there is great diversity in the market such that some customers were very price sensitive and some customers were very price insensitive. At this point the electricity supplier had a very large incentive to encourage load-shifting through the use of high discounts. Furthermore, these high discounts also encouraged the use of energy storage. As such, at this point there was a shift in the equilibrium discount from 3.7% to 4% while the PAR continued to decrease. Out of all of the scenarios we studied, this was the one where the electricity provider achieved the highest profits.

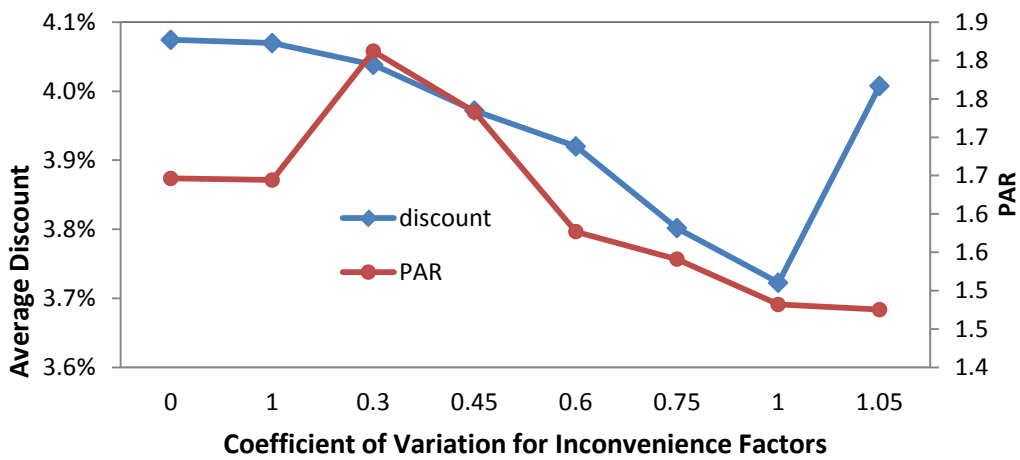

Figure 19: Consumer diversity, equilibrium discounts, and PAR;

storage controlled by customers.

In our third scenario, we evaluated the effects that having multiple customers had on the Stackelberg equilibrium. Here, we used a number of customers ranging from 1 to 7, while keeping the total nominal demand and total storage capacity of the system equal. Furthermore, we assumed that all of the customers had the same inconvenience $\alpha =$

0.001 and used the inconvenience multiplier z. Figures 20 and 21 show the effects of having a different number of customers at different inconvenience levels.

When $z = 1$ and customers were price sensitive, the graph has two distinct fragments. In the first fragment, the market had up to 4 customers, and price discounts were very effective ways to incentivize load-shifting. Thus, high discounts (4%) are offered.

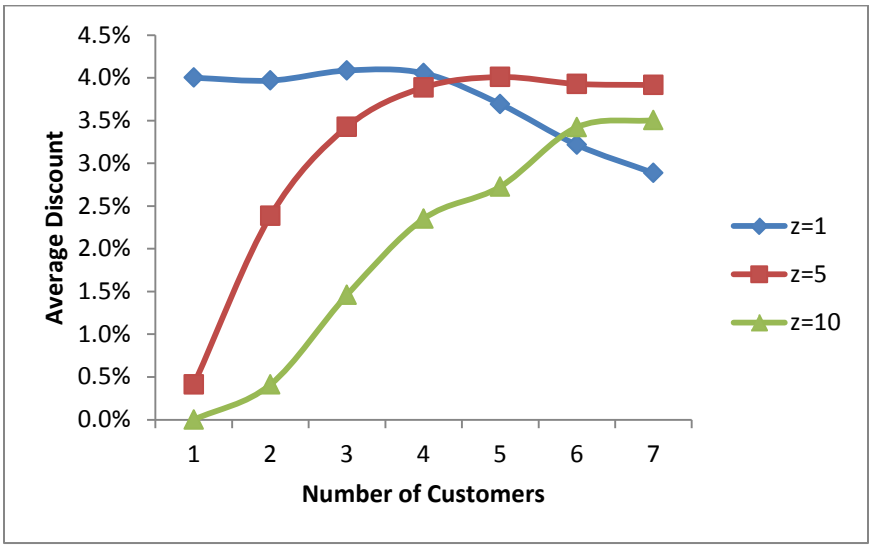

Figure 20: Number of consumers and equilibrium discount averages;

storage controlled by customers

Past this point, the market has more customers the effects of load-shifting may be achieved with lower discounts, which decreased as the number of customers increased.

When $z = 5$ customers were not as sensitive to price discounts, and shifting large loads led to high inconvenience. As such, when the market had fewer customers (1 or 2), they were quite price insensitive. As the market grew in number (3, 4, or 5), the individual amount of nominal demand that had to be shifted by each customer was smaller; thus, they became more price sensitive and the equilibrium discount increased.

After this, customers were fairly price sensitive, and high equilibrium discounts of 4% were reached.

When $z = 10$ customers were price insensitive. In this case the equilibrium discount slowly increased as the number of customers increased, but even when the market had 7 customers, the equilibrium discount only reached 3.5%.

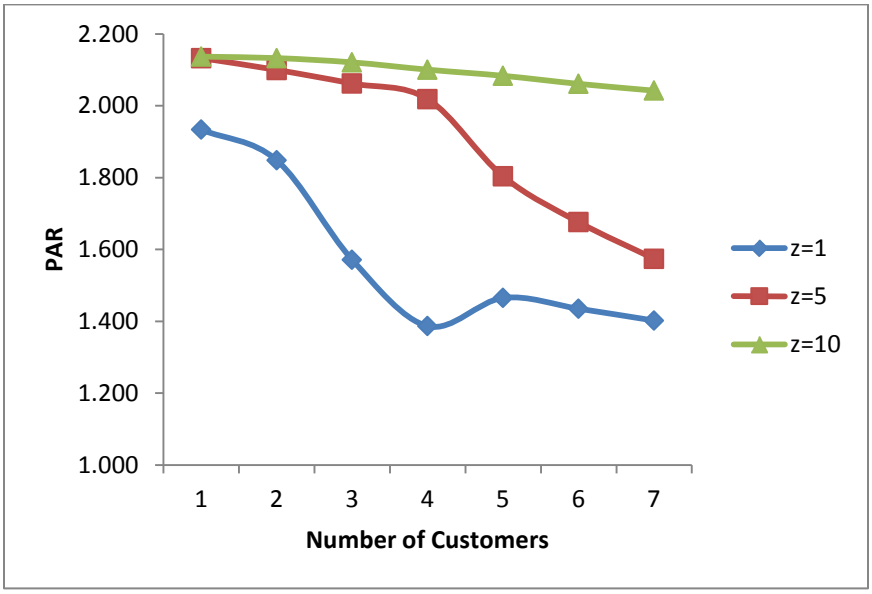

Figure 21: Number of consumers and PAR; storage controlled by customers.

When evaluating the PAR, we can see that for every level of inconvenience the increase in the number of customers led to a decrease in the PAR. However, it is important to point out the role that load-shifting and energy storage had on these effects. When there were high discounts (i.e., discounts greater than 3.9%), energy storage became part of the optimal strategy for the customers, and as such the PAR was lowered. This explained the increase in the PAR when the customers were price sensitive and the market grew from 4 to 5 customers, since at this point the size of the market led to an optimal discount that does not incentivize the use of energy storage.

In the final scenario, we evaluated the effects that having energy storage of different capacities had on the Stackelberg equilibrium. We used a market with 7 independent customers with an a_i that averaged 0.0082 and an aggregate total nominal demand equal to the demand shown in Table 7 p. 67. Furthermore, we used the inconvenience multiplier z at 3 different levels ($z = 1, 5, 13$) and had the storage capacity range from 0 to 12KW.

In the first case in which customers were price takers (z=1), the storage capacity was irrelevant to the Stackelberg equilibrium. Customers were very eager to shift their nominal demand with low discounts, and as such the discount offered by the electricity supplier never reached the threshold needed for energy storage to become a feasible alternative.

In the second case, when customers could still be influenced by the energy discounts provided by the electricity supplier ($z = 5$), the storage capacity became a binding constraint whenever it was below 4KW. In this segment the Stackelberg equilibrium led to relatively high discounts (i.e., greater than 4%) in which case customers would store the maximum electricity possible. Past this point, the customers had enough storage capacity to achieve the maximum possible shift in their nominal demand through energy storage, and as such the discount provided by the electricity supplier plateaued at 4.00%.

In the case where customers were very reluctant to shift their nominal demand ($z = 13$), the Stackelberg equilibrium behaved similarly to the case where the electricity provider still had some influence on the customers ($z = 5$). However, in this case the equilibrium discount was lower and stabilized at 3.73%, while the PAR achieved a

minimum of 1.87 when there was a relatively small energy storage capacity (1KW). This response from the PAR may be explained by the lower discounts when the storage capacity is of 1KW. The equilibrium discount achieved actually led to some load-shifting as well as maximum storage; thus, the PAR was lowered. However, past this point, since the nominal load shifts were minimized and the maximum benefit from the use of energy storage had been achieved, there were not enough incentives for the electricity supplier to provide high discounts. As such the PAR stabilized at 2.02.

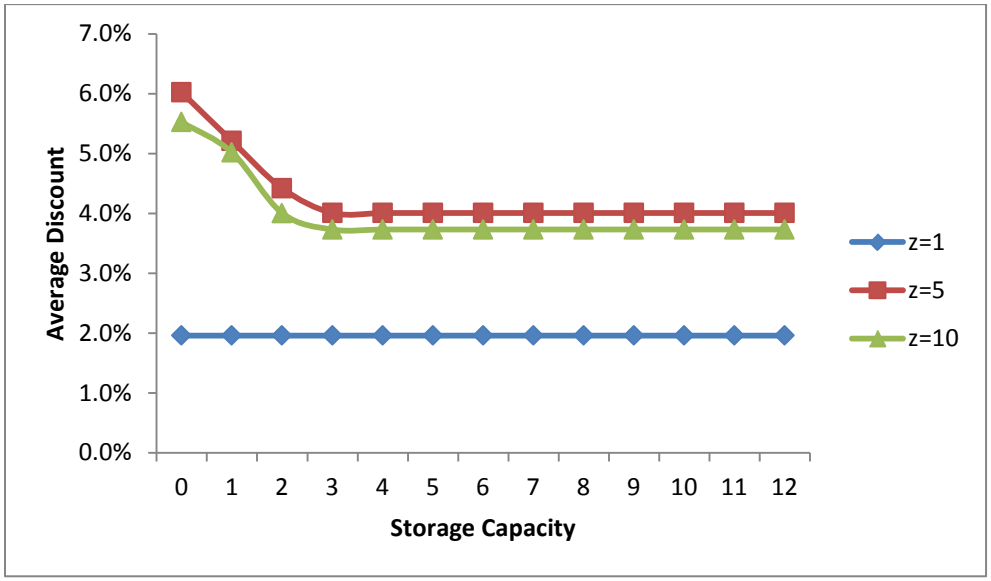

Figure 22:Storage capacity and equilibrium discount averages;

storage controlled by customers.

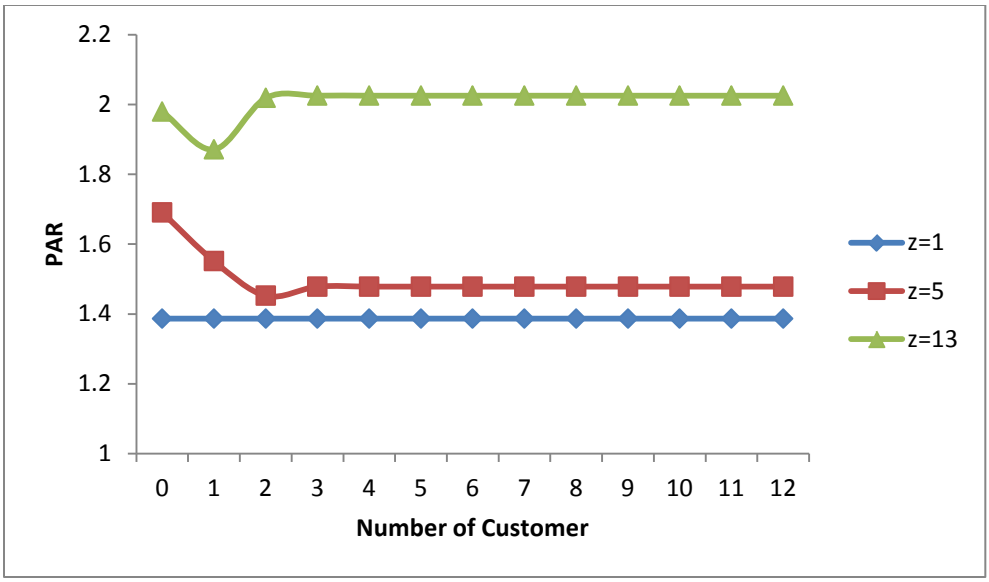

Figure 23: Storage capacity and equilibrium PAR; storage controlled by customers.

4.4. Provider Controlled Energy Storage

4.4.1. Model

Our proposed game had three levels. In the first level, the provider first decided the price discount to offer the customers in order to encourage them to shift loads from time slots with high demand to off-peak times. In the second level, the customers first decided their consumption for each time slot based on the announced discount and on the inconvenience generated by shifting demand. In the final level, the supplier made energy storage decisions, which depended on how consumers responded to the price discount. To model the smartgrid, we let $\mathbb{N}$ be the set of customers, defined as $\mathbb{N} \triangleq \{1, 2, \dots, N\}$ and $\mathbb{T}$ be the set of time slots defined as $\mathbb{T} \triangleq \{1, 2, \dots, T\}$.

As a strategy for short-term demand management, the energy provider employed two techniques: energy storage and price discount. Energy storage was employed to shift the generation amount throughout the different time slots, while percentage discounts were used to shift the consumers' nominal consumption. To model the first decision

taken by the energy provider, we defined γ_t as the percentage discount applied to the base price during time slot t.

Further, l_{it} denoted the amount of energy produced by the energy provider for each customer i during time slot t. Based on this we can see that $l_{it} = x_{it} - u_{it} + s_{it}$, and we can define the total energy produced in each time slot t as L_t, where $L_t = \sum_{i=1}^{N} l_{it}$.

By including the condition in equations (81) and (82) in the energy provider model present in equations (89)–(90), we can reach an optimization model for the energy provider that maximizes profit as

$$Max\ \Pi_s = \sum_{t=1}^{T}\left((1-\gamma_t)P\ (X_{it}) - C(a(\sum_{i=1}^{N}(L_{it}))^2 + b\sum_{i=1}^{N}(L_{it}) + c)\right) \quad (123)$$

s.t

$$0 \le \gamma_t \le 1 \qquad \forall t \in \mathbb{T} \quad (124)$$

$$w_{it} = \rho\, w_{i,t-1} + (1-\tau)\, s_{it} - (1+\beta)u_{it} \qquad \forall t \in \mathbb{T} \quad (125)$$

$$0 \le \mathrm{w_{it}} \le \mathrm{W^{max}} \qquad \forall t \in \mathbb{T} \quad (126)$$

$$s_{it}, u_{it} \ge 0 \qquad \forall t \in \mathbb{T} \quad (127)$$

As the strategy decision that make by the customer is the same as it present in section 2.2.2. Where optimization model for customer i is as follows:

$$Min\ \Pi_i = \sum_{t=1}^{T}\left((1-\gamma_t)Px_{it} - a_i(x_{it} - y_{it})^2\right) \quad (128)$$

$$s.t. \qquad \sum_{t=1}^{T} x_{it} = \sum_{t=1}^{T} y_{it} \quad (129)$$

$$x_{it} \ge m_{it} \qquad \forall t \in \mathbb{T} \quad (130)$$

4.4.2. Equilibrium Analysis

In order to solve the formulated game, we used the Stackelberg equilibrium. This equilibrium was calculated using backwards induction, an approach in which the later decision's best response is mapped to the actions of the prior decision. In this way the

optimal storage decision was included in the customer's optimization problem, and the customers' optimal decisions were included into the energy supplier's price discount optimization problem. Then the equilibrium point was calculated. If we analyze the provider's payoff, which is in equation (123), we can find the gross profit earned by using the electricity storage system, so that in the final stage of the game, the electricity storage system optimization problem was as follows:

$$Max\, \Pi_{storage} = \sum_{t=1}^{T}\left((1-\gamma_t)P\ \sum_{i=1}^{N} u_{it} - \sum_{i=1}^{N} s_{it}\frac{C(L_t)}{L_t}\right) \tag{131}$$

s.t

$$w_{it} = \rho\, w_{i,t-1} + (1-\tau)\, s_{it} - (1+\beta)u_{it} \quad \forall\, i \in \mathbb{N}, t \in \mathbb{T} \tag{132}$$

$$0 \le w_{it} \le W^{max} \qquad \forall\, i\in\mathbb{N}, t\in\mathbb{T} \tag{133}$$

$$s_{it}, u_{it} \ge 0 \qquad \forall\, i\in\mathbb{N}, t\in\mathbb{T} \tag{134}$$

Since the storage decision taken by the energy provider did not affect the solution of the customer optimization model, we did not need to input the best response of the electricity storage decisions into the consumers' model. Because of this, we combined the electricity storage system optimization problem with the price discount model, as is present in equation (123) into equation (127). As such, we first analyzed the customers' optimal response to a given energy discount scheme. We used Lagrangian Relaxation to analyze the consumers' model, so that λ became the Lagrange multiplier for the constraint in equation (129), and μ_t for the constraints given by equation (130). Therefore, minimizing the consumers' optimization model was equivalent to minimizing the following Lagrange function for x_{it}, λ, *and* μ_t:

$$Max\ LR = \sum_{t=1}^{T}(1-\gamma_t)P\,x_{i,t} + \sum_{t=1}^{T}\alpha_i\left(x_{i,t}-y_{i,t}\right)^2 - \lambda\left(\sum_{t=1}^{T}y_t - \sum_{t=1}^{T}x_{i,t}\right) - \sum_{t=1}^{12}\mu_t(x_{i,t}-m_{it}) \tag{135}$$

In the first case, we assumed that the customer consumption x_{it} was strictly holding during each time slot, in this case $\mu_t = 0$, and we used the result obtained in equations (15) and(16) in order to find x_{it} and λ_i where

$$x_{i,t}^{*} = y_{i,t} + \frac{(\gamma_t - \bar{\gamma})\,P}{2\alpha} \geq 0 \qquad \forall t \tag{136}$$

$$\lambda = (\bar{\gamma} - 1)P \tag{137}$$

In the second case, the constraint in equation (130) was binding, so $x_{i,t} = m_{it}$ and $\mu_t \geq 0$. In order to optimize this case, we used the same procedure that presented in section 2.3 where time slots were separated into two subsets, $\mathbb{T}_1$ and $\mathbb{T}_2$. Here, the time slots in which the constraint from equation (130) simply held were placed in subset $\mathbb{T}_1$, and those in which the constraint was were are placed in set $\mathbb{T}_2$. In this case, we used the result obtained in Proposition 2 in order to find x_{it} ,λ_i and μ_{1it}:

$$\hat{x}_{it}^{*} = \begin{cases} y_{it} + \dfrac{1}{\hat{T}}\displaystyle\sum_{\bar{t}=1}^{\bar{T}}(y_{it}-m_{it}) + \dfrac{(\gamma_t - \bar{\gamma})P}{2\alpha} & ,t \in \mathbb{T}_1 \\ m_{it} & ,t \in \mathbb{T}_2 \end{cases} \tag{138}$$

$$\lambda = (\bar{\gamma}_{\hat{t}} - 1)P - \frac{2\alpha}{\hat{T}}\sum_{\bar{t}=1}^{\bar{T}}(y_{\bar{t}} - m_{it}) \tag{139}$$

$$\mu_{i,t}^{*} = \begin{cases} 0 & ,t \in \mathbb{T}_1 \\ (\bar{\gamma}_1 - \gamma_{\hat{t}})P - 2\alpha_i(y_{it} - m_{it} + \frac{1}{\hat{T}}\sum_{t\in\mathbb{T}_2}(y_{it}-m_{it})) & ,t \in \mathbb{T}_2 \end{cases} \tag{140}$$

The equilibrium for the system was reached when the constraints given by equation (130) were satisfied for all customers i in each timeslot t. We found an initial answer to the

consumers' best response by relaxing these constraints in equation (130) and then inputting equation (136) into the supplier's model, for which nonlinear optimization tools could be used to determine the optimal solution. Once the solution for the supplier model was found, if the constraints from equation (130) were satisfied $\forall\, i \in \mathbb{N}\ and\ t \in \mathbb{T}$, then the equilibrium point is reached. Otherwise, the use the proposed algorithm that been used. Table 1 p. 23. gives additional iterations in order to find the equilibrium solution. Table 8 presents the outline of the algorithm.

Definition 4: The point $(\gamma_t^*, x_{it}^*(\gamma_t^*), u_{it}^*, s_{it}^*, w_{it}^*)$, which satisfied the constraints in (123–127) and (128–130), is an equilibrium result of the Stackelberg game $G = \{\mathbb{N}, \mathbb{T}, \Pi_s, \Pi_i\}$ if and only if

$$\Pi_s(\gamma_t^*, x_{it}^*(\gamma_t^*), u_{it}^*, s_{it}^*, w_{it}^*) \geq \Pi_s(\gamma_t, x_{it}(\gamma_t), u_{it}, s_{it}, w_{it}), \forall\, t \in \mathbb{T}\ and\ i \in \mathbb{N} \tag{141}$$

The leader stratgy on the price discount technique depeneds on the consumers' inconvenience coefficient α, the distribution of the nominal demand across the planning horizon and storage system efficiency parameters. The price discount technique helped reduce the demand fluctuations, if the customers were sensitive to price discounts and willing to shift. Otherwise, if the consumers had high resistance, the utility did not receive any added benefits from this strategy.

Moreover, the utility has another possible technique to reduce demand fluctuations by using an energy storage. The energy storage helped reduce the PAR, which means that the fluctuation in the generation amount was reduced. The energy storage was charged, by amount of s_{it} , when the nominal demand was low, and discharged by amount of u_{it}

when the nominal demand was high. Moreover, the storage system decision depended on the consumer willingness to perform load-shifting.

Table 8: Procedure to find equilibrium solution, Storage is controlled by provider

1: include the consumers' best response from equation (136) into the energy provider's model in equation (123–127).
2: solve energy provider's model
3: if $x_{i,t}^{*} \geq m_{it}\ \forall\, i \in \mathbb{N}, t \in \mathbb{T}$ then STOP.
4: else for each consumer for which constraint (130) is violated in any time *t*
5: identify and update $\mathbb{T}_1$ and $\mathbb{T}_2$
6: use (138) instead of (136) for these consumers in the energy provider's model
7: Solve the utility firm's model again including these updates.
8: Go to line 3

Proposition 6: The energy provider could not charge and discharge the storage system during the same timeslot t.

$$u_{it} s_{it} = 0 \qquad \forall\, i \in \mathbb{N}, t \in \mathbb{T} \tag{142}$$

Proof: We assumed that the customers, as followers, were not too sensitive to the price discount, and on every time slot t equation (130) was strictly holding. From equation (136), we take the derivative of x_{it} with respect to γ_t:

$$\frac{\partial x_{it}}{\partial \gamma_t} = \frac{P}{2\,\alpha}\left(1 - \frac{1}{T}\right) = \Delta \tag{143}$$

Using Lagrange multipliers for the supplier model,

$$Max\ LR = \sum_{t=1}^{T}((1-\gamma_t)P\ x_{it} \qquad - (a(\sum_{i=1}^{N}(x_{it} - u_{it} + s_{it}))^2$$
$$+b\sum_{i=1}^{N}(x_{it} - u_{it} + s_{it}) + c\ )$$
$$+\ \delta_{it}\ (w_{it} - \rho\ w_{i,t-1} - (1-\tau)s_{it} + (1+\beta)u_{it})$$
$$+\ \sigma\ (W^{max} - w_{it}) + \mu_{ut}u_{it} + \mu_{\gamma t}\gamma_t + \mu_{st}s_{it} + \mu_{wt}w_{it}\ ) \tag{144}$$

The Kuhn–Tucker conditions related to s_{it} and u_{it} are as follows:

$$\frac{\partial LR}{\partial s_{it}} = -\ 2\ a\ \left(y_{i,t} + \frac{(\gamma_t - \overline{\gamma})P}{2\alpha} - u_{it} + s_{it}\right) - b - \delta\ (1-\tau) \ \leq 0 \tag{145}$$

$$\frac{\partial LR}{\partial u_{it}} = 2\ a\ \left(y_{i,t} + \frac{(\gamma_t - \overline{\gamma})P}{2\alpha} - u_{it} + s_{it}\right) + b + \delta(1+\beta) \ \leq 0 \tag{146}$$

$$s_{it}\left(\frac{\partial LR}{\partial u_t}\right) = 0 \tag{147}$$

$$u_{it}\left(\frac{\partial LR}{\partial u_t}\right) = 0 \tag{148}$$

To prove the statement, we must satisfy the following:

$\frac{\partial LR}{\partial s_{it}} = 0$ and $\frac{\partial LR}{\partial u_{it}} \neq 0$, or

$\frac{\partial LR}{\partial s_{it}} \neq 0$ and $\frac{\partial LR}{\partial u_{it}} = 0$

Assume that $s_{it} > 0$, so to satisfy the slackness condition on equation (147) $\partial LR/\partial\, s_{it} = 0$. It follows from equation (145) and (146) that $\partial LR/\partial\, s_{it} \neq \partial LR/\partial\, u_{it}$. Therefore, to satisfy the slackness condition from equation (148), $u_{it} = 0$. ■

Proposition 7: The energy storage usage will increase when the customers are more reluctant to load-shifting.

As we define the Lagrange function in equation (144) , When $s_{it} > 0$ the $\frac{\partial LR}{\partial s_{it}} = 0$, equation (145). So the function optimal charging decision is:

$$s_{it} = -\ y_{i,t} - \frac{(\gamma_t - \overline{\gamma})P}{2\alpha} - \frac{b}{2\ a} - \frac{\delta}{2\ a}\ (1-\tau) \qquad \forall\ i \in \mathbb{N}, t \in \mathbb{T} \tag{149}$$

Here, v represented the amount of change from the nominal demand, which is $\frac{(\gamma_t - \overline{\gamma})P}{2\alpha}$. The charging decision and amount of change were negatively correlated because $\partial s_{it}/\partial v < 0$. Moreover, $u_{it} > 0$ when the demand reached the peak and the $\frac{\partial LR}{\partial u_{it}} = 0$, so the function optimal discharging decision u_{it} was

$$u_{it} = y_{i,t} + \frac{(\gamma_t - \overline{\gamma})P}{2\alpha} + \frac{b}{2\,a} + \frac{\delta}{2\,a}(1 + \beta) \qquad \forall\, i \in \mathbb{N}, t \in \mathbb{T} \qquad (150)$$

In this case, the price discount was less than the average discount, so the amount of change was negative. So when the amount of change decreased, the discharging amount increased, and the opposite was true. Since the $\partial s_{it}/\partial v < 0$ and $\partial u_{it}/\partial v < 0$, the energy storage profile increased when the consumers' willingness to shift decreased. ■

The possibility of using energy storage was greatly affected by their efficiency. If there was a significant difference between the amounts of energy used to charge the system and the energy actually stored, as well as the energy available at the time for discharge and the energy effectively output by the storage system, the system had low possibilities of implementation due to the high energy waste. Furthermore, the overall efficiency of the storage system was also affected by the storage efficiency, ρ, which relates to the energy lost to the environment as it was kept in storage for multiple periods. As the storage became more efficient, the PAR was high, or the customers were more reluctant to load-shifting. The storage capacity limitation became a significant factor in improving the supplier payoff.

Proposition 8: The storage capacity, W^{max} became a significant factor that impacted the supplier payoff if and only if

$$\frac{(1+\beta)}{\rho}\left(y_{i,t} + \frac{(\gamma_t - \overline{\gamma})P}{2\alpha} + \frac{b}{2a}\right) > W^{max} \qquad \forall\, i \in \mathbb{N}, t \in \mathbb{T} \qquad (151)$$

Proof: Let time t', where $t' \in \mathbb{T}$ denoted a peak demand timeslot where the discharging amount $u_{it'}$ need was equal to the total storage capacity. Because of that, the constraint in equation (126) on $t'-1$ was binding, $w_{it'-1} = W^{max}$. Taking into account Proposition 6, the supplier could not charge the storage in time t' so that $s_{it'} = 0$, $w_{it'} = 0$. equation (126) became the following:

$$u^*_{it'} = \rho\, W^{max}/(\,1+\,\beta) \qquad \forall\, i \in \mathbb{N}, t \in \mathbb{T} \qquad (152)$$

After that, we relaxed the binding constraints in equation (126) $\forall t \in \mathbb{T}$ and resolved the $u_{it'}$, assuming that $u_{it'}$ would consume all the available energy in the storage. Moreover, you substituted the storage balance constraint, equation (125), in terms of the variable decision s_{it} into the objective function equation (123). We redefined the model as follows:

$$\begin{aligned} \text{Max } \Pi = & \textstyle\sum_{t=1}^{T}((1-\gamma_t)P\, x_{it} \\ & - (a\left(\textstyle\sum_{i=1}^{N}\left(x_{it} - u_{it} + \frac{w_{it} - \rho\, w_{i,t-1} + +(\,1+\beta)u_{it}}{(1-\tau)}\right)\right)^2 \\ & + b\textstyle\sum_{i=1}^{N}\left(x_{it} - u_{it} + \frac{w_{it} - \rho\, w_{i,t-1} + +(\,1+\beta)u_{it}}{(1-\tau)}\right) + c\,)) \end{aligned} \qquad (153)$$

s.t.

$$0 \le \gamma_t \le 1 \qquad \forall\, i \in \mathbb{N}, t \in \mathbb{T} \qquad (154)$$

$$s_{it}, u_{it} \ge 0 \qquad i \in \mathbb{N}, t \in \mathbb{T} \qquad (155)$$

Since we assumed that $u_{it'}$ would consume all the available energy in the storage, by using Lagrange Relaxation, than Kuhn–Tucker condition $\frac{\partial LR}{\partial u_{it'}} = 0$, which presented as

$$\frac{\partial LR}{\partial u_{it'}} = -\,2\,a\left(\frac{1+\beta}{1-\tau} - 1\right)\left(y_{i,t} + \frac{(\gamma_t - \bar{\gamma})P}{2\alpha} - u_{it}\right) - b\left(\frac{1+\beta}{1-\tau} - 1\right) = 0 \qquad (156)$$

We rewrote the equation (156) as $u^R_{it'}$ function:

$$u_{it'}^{R} = y_{i,t} + \frac{(\gamma_t - \bar{\gamma})P}{2\alpha} + \frac{b}{2a} \qquad \forall\, i \in \mathbb{N}, t \in \mathbb{T} \quad (157)$$

When $u_{it'}^{R}$ was greater than $u_{it'}^{*}$, the optimal storage capacity was larger than the maximum installed capacity, and thus the capacity needed to increase. ■

Since the cost function was convex, increasing with the generation amount, the utility was better off as the fluctuation in generation was reduced. The deployment of an energy storage would help reduce this fluctuation without a significant change to the nominal demand. Consequently, the supplier may reach similar reductions in generation fluctuation without the need to apply the same price discount as it would if this energy storage were not deployed. However, the efficiency of the energy storage had a direct impact on the price discount decision, and the price discount increased when the efficiency of the energy storage was low. The utility may decide to use one or both of the techniques for PAR reduction. However, the price discount and energy storage were unprofitable to the utility firm if the nominal demand was uniform across the planning horizon.

4.4.3. Numerical Analysis

In this section, we performed a numerical analysis by using a case study and applying our model to different scenarios. We examined the utility firm's profits, price discounts, energy storage decisions, and PARs under the influence of different consumer types in terms of their nominal demand and willingness to shift load. In this study, we investigated the consumers' willingness levels and the diversity in the consumer portfolio with this respect. Furthermore, we considered how the number of customers (market breadth) influenced the equilibrium outcomes.

We divided the time horizon into twelve slots for all of the scenarios. For the energy production cost function $C(L_t)$, we set the parameter a to 0.0035 $\$/KWh^2$, and we assumed that parameters b and c were zero. Moreover, we assumed that the energy storage efficiencies ρ, τ, and β were 0.99, 0.01 and 0.01, respectively. For every consumer and period, we assumed that the minimum required consumption m_t was zero. The base price, P, was set to 0.25 $\$/kWh$. We gathered the nominal demand randomly from residential electricity bills, which show in Table 9 and Figure 24.

Table 9: Aggregate demand (kwh), Storage is controlled by provider

Period (t)	1	2	3	4	5	6	7	8	9	10	11	12
Aggregate Demand	10.19	15.72	22.28	29.23	33.31	39.14	73.43	61.27	58.1	37.2	17.53	15.01

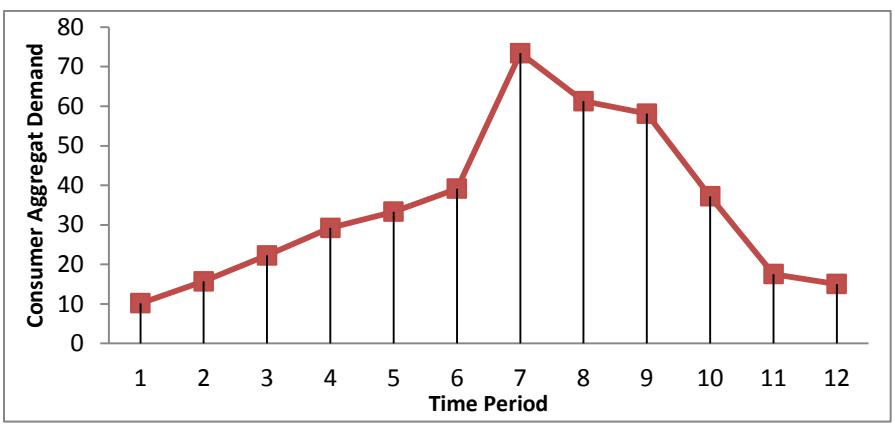

Figure 24: Aggregate demand across the time horizon; storage controlled by provider.

In our first scenario, we focused on the effects of different consumer types given by their willingness to shift load. We used 4 customers and a multiplier *z*, ranging from .0 to 15, to scale the inconvenience factor α_i. The individual (α_i) factors were 0.02, 0.021, 0.00025, 0.00875, 0.0003, 0.0004, and 0.007 respectively. Clearly, as the value of z increased the customers' willingness to shift load decreased. Figures 25 and 26 show the

effect of consumers' sensitivity to load-shifting on the equilibrium average discounts (γ) and the PAR values.

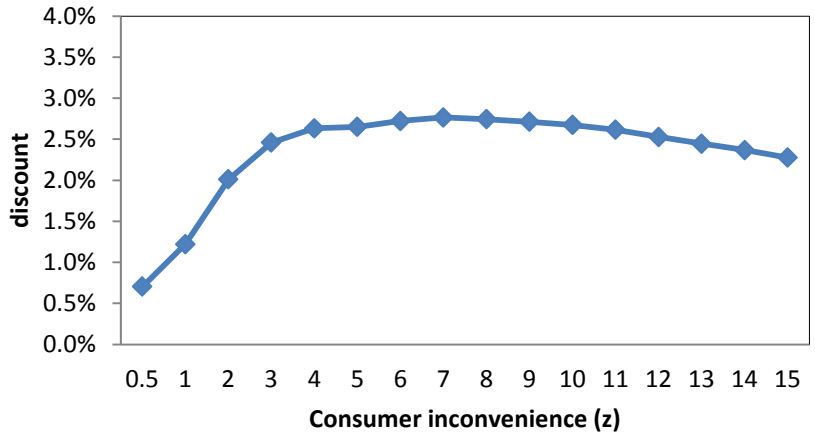

Figure 25: Consumer inconvenience and energy provider's average price discount; storage controlled by provider.

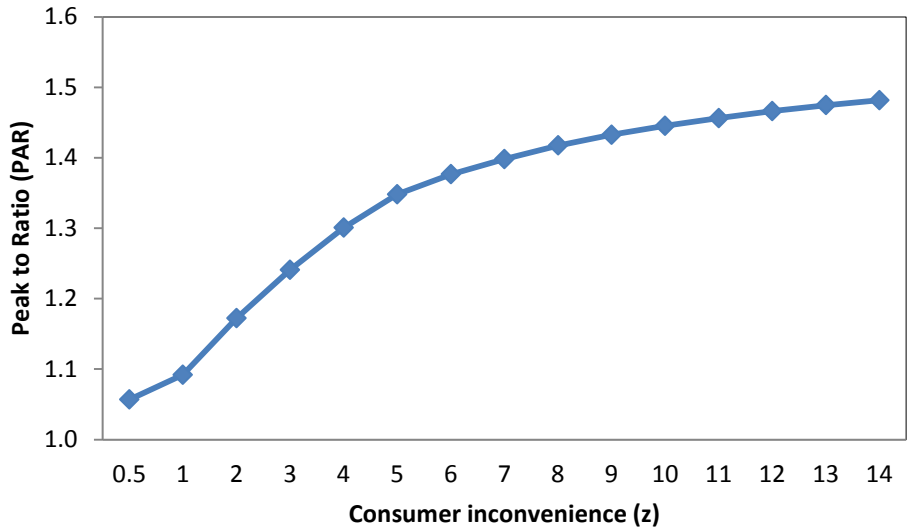

Figure 26: Consumer inconvenience and PAR values; storage controlled by provider.

We can see that in the first three cases (z in [0.5,2]) some of the constraints from equation (130) were binding. The utility firm had no incentive to apply a high discount or a high use of the energy storage since consumers were too sensitive to price discount. Consequently, the small price discount and the low use of the energy storage had an effective way to reduce the fluctuation in the generation amount. Past these points the

constraints from equation (130) stopped being binding, and the discounts continued to increase since they still influenced customer behavior and a point of diminishing returns (z=7) had not been reached. Furthermore, as the inconvenience multiplier increased, the sensitivity to price decreased and the energy storage became more cost efficient.

After this, discounts persistently decreased since their effect on the customers was weaker every time and the use of energy storage was more effective in the reduction of the PAR. Furthermore, there were very small amounts of load-shifting and the effects on overall PAR reduction decrease and the supplier became more dependent on the storage, so that the PAR was 1.4 when z = 7 and increased to 1.5 when z = 15.

In order to evaluate the effects of the customers' attitude toward load-shifting, we changed the individual inconvenience factors α_i while keeping the average inconvenience at 0.0082 as in the prior case) We set the coefficient of variation (COV) for α_i between 0 to 1.05, so that a higher COV constituted higher diversity in the consumer market. Figure 27 showed the effects of different COV levels on the average discounts and the PAR.

The average equilibrium price discount increased as the COV increased up to a certain point, after which the discount started to decrease. We believe this is because when the customers' were more alike, their behavior was relatively less sensitive to price incentives. As the diversity increased, the utility fir was incentivized to make load-shifting more attractive as more and more customers reacted. However, a level in diversity may be reached when customers are too distant from each other and some are very sensitive to low discounts and will not shift their load regardless of the discount. Thus, a lower discount would be required to shift the loads of the subset of customers

who are actually willing to do any load shits. The drop in the discount was significant as some of the constraints in equation (130) become binding.

Something that we found interesting is that the PAR consistently reduced as the COV increased, which, in turn, drives lower needs for energy storage, and there was a point in which increases in the COV led to decreases in all the discount levels and PAR. This can be seen in Figures 27, and we believe this was because of the constant increase in customers who were highly sensitive to price incentives, which led to increasing in the load-shifting amount.

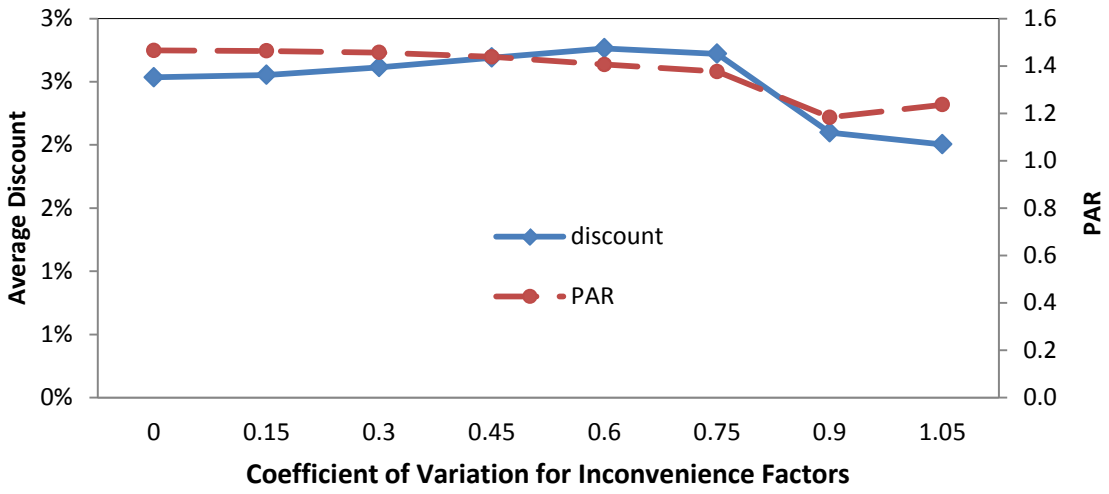

Figure 27: Consumer diversity, equilibrium discounts, and PAR; storage controlled by provider.

In the third scenario, we evaluated the effects of different market sizes on the different discounts, based on the number of customers. In order to single out only the impact of having a different number of customers, we considered customers of identical types. We generated different instances by varying the number of customers between 1 and 7 with inconvenience factors at 0.001 and the inconvenience multipliers at 1, 5, and 10. In order to keep the market volume constant, we kept the total nominal demand and total storage capacity of the system equal and distributed it equally among the different

customers. The effects of the number of customers on the equilibrium result can be seen in Figures 28 and 29.

We can see in Figure 28 that the number of customers had a significant effect on the equilibrium discount, such that when customers were less resistant to load-shifting (when z=1), the discounts decreased as the number of customers increased. When customers are more resistant to load-shifting (z = 1, 10), the opposite was true; the discounts increased as the number of customers increased. We believe this effect is a result of the fact that the inconvenience costs were convex, increasing with the deviation between nominal demand and consumption, as well as the fact that as the number of customers increased it was less costly for the utility firm to shift each of the individual customer's demand.

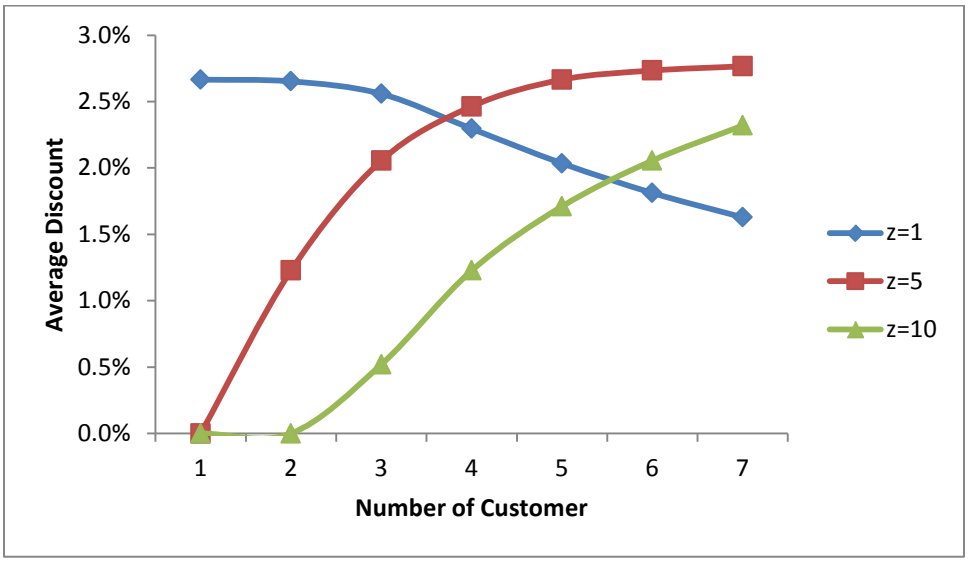

Figure 28:Number of consumers and equilibrium discount averages; storage controlled by provider.

In terms of PAR, we can see that an increase in the number of customers always led to a reduction, as individual customers became less significant and their individual load-shifting became less costly as their share of the market decreased.

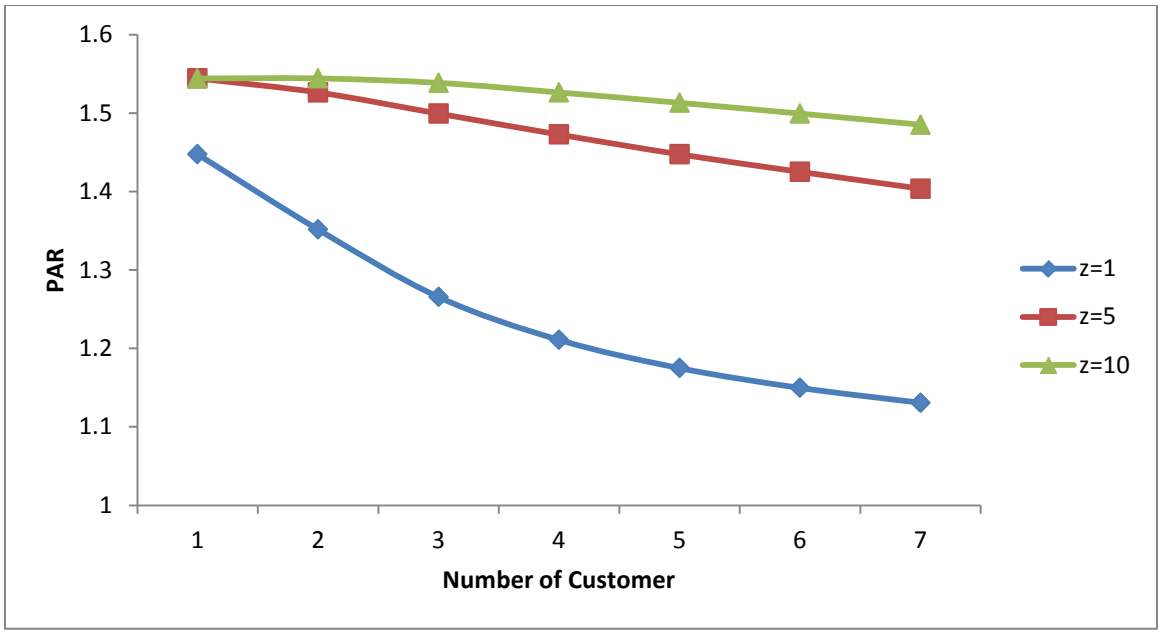

Figure 29: Number of consumers and PAR; storage controlled by provider.

Finally, we evaluated the effects that having different storage capacity had on the Stackelberg equilibrium. We had 7 independent customers with an a_i that averaged 0.0082, deployed an energy storage with a capacity that ranged from 0 to 12 KW, and evaluated the results using three different levels of the inconvenience multiplier z ($z = 1, 5, 13$).

Figure 30 has two distinct segments: a first segment in which the average discount decreased as the storage capacity increased and a second segment in which the average discount stabilized. In the first segment, the energy provider took advantage of the increasing energy storage capacity to decrease the average discount and maximize their profit. This behavior from the equilibrium discount may be explained by the structure of the demand peaks, as the energy supplier may take advantage of the off-peak periods before the peak of demand to charge the storage system. After the peak has occurred the storage is no longer available, and the supplier's discounts incentivized load-shifting to

these latter periods. In terms of the PAR, there was a decrease as the storage capacity increased in the first segment; while in the second segment, the PAR became stable as the storage system reached the maximum capacity, as shown in Figure 31.

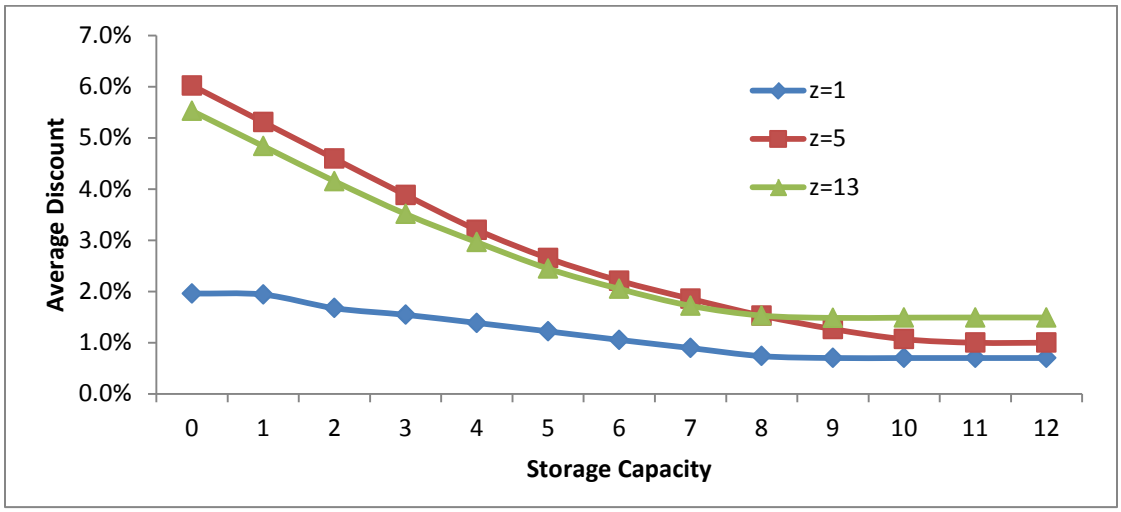

Figure 30:Storage capacity and equilibrium discount averages;

storage controlled by provider.

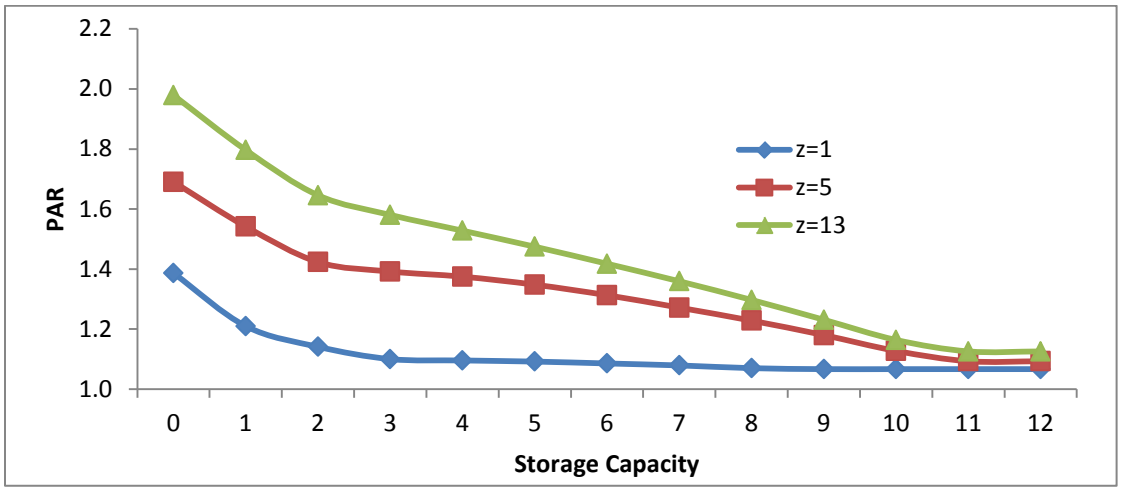

Figure 31:Storage capacity and equilibrium PAR; storage controlled by provider.

4.5. Comparison and coordination

We formulated a Stackelberg game to evaluate the impact of the three proposed models: load-shifting with no storage, load-shifting with customer-controlled storage, and

load-shifting with supplier-controlled storage. In the first model, equilibrium was achieved with an optimal discount in which the electricity provider maximized its profit after taking into account the customers' reaction to a proposed discount. In the second model, we built on the first model by adding the availability of electricity storage for each customer who then decided how much to shift his or her nominal consumption and how much energy to store as a reaction to the discounts proposed by the electricity supplier. In the third model, the energy storage available at each customer was placed under the control of the electricity provider who then decided in which periods it wanted to store and unload the energy storage. We evaluated these three models in terms of the optimal payoff for each of the players and the effects on the PAR.

Conjecture 1: At the equilibrium point, the PAR will be lower when the energy storage is controlled by the electricity provider.

Since the electricity provider was the leader of the game, their strategic decisions controlled the game path. The electricity provider aimed to maximize its profit, and when it controlled the energy storage, it had more degrees of freedom in order to maximize this profit. In order to minimize the electricity production cost, the electricity provider was better off when the fluctuations in electricity generation were reduced since the cost function was convex, increasing with respect to the generation amount. Moreover, since the game was non-cooperative, the players were selfish in their decisions, which led the provider to be more conservative in its strategies. This means that if the electricity supplier offered a high discount, the customers' reactions may move the on-peak time entirely to another time slot (a slot where the high incentive provided by the discount led both to load-shifting and energy storage) ■.

We have observed that in some cases, when the customers are in control of the energy storage, alternative optimal solutions arise for the customer model. In these cases, there are more than one set of decision variables (for both energy consumption and storage) that lead to the same minimum cost. These cases pose a non-trivial challenge to the energy provider as they can have significantly different levels of PAR, and thus lead to significantly different profits. We believe that these cases arise when the discount offered by the energy provider makes the customers indifferent between using the electricity from the grid for consumption or for charging of the energy storage, during the off-peak periods. However, since they are indifferent to which of the multiple solutions to employ, we assume that the customers will react in the same way that the equilibrium results display. Therefore, this possibility is another disadvantage for the energy supplier, when the customers control the energy storage.

When the customer controlled the energy storage, we observed that the customer may respond with alternative optimal solution. The alternative optimal solutions mean that the customer may have two or more solutions equal in the total cost, but they are different on the decision variables. We believed that the alternative solution occurred because the supplier offered a discount that made indifferent on the peak time for the customer to get his or her actual consumption directly from the grid or charging the storage on the off-peak time then using it during on-peak time. Since the alternative solutions were indifferent for the customer, we assumed that the customer reacted same as it obtained on the equilibrium results. However, in reality, if the customer reacted with an alternative solution, the PAR may change, which affected the electricity generation cost. That consider as disadvantage of make the storage controlled by the customer.

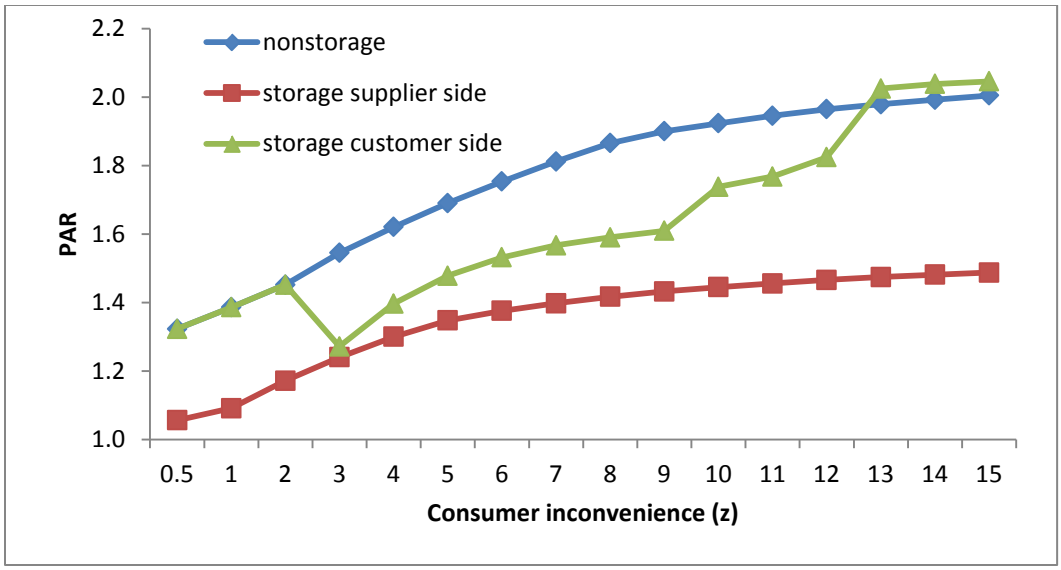

Figure 32:Comparison consumer inconvenience and average price discount.

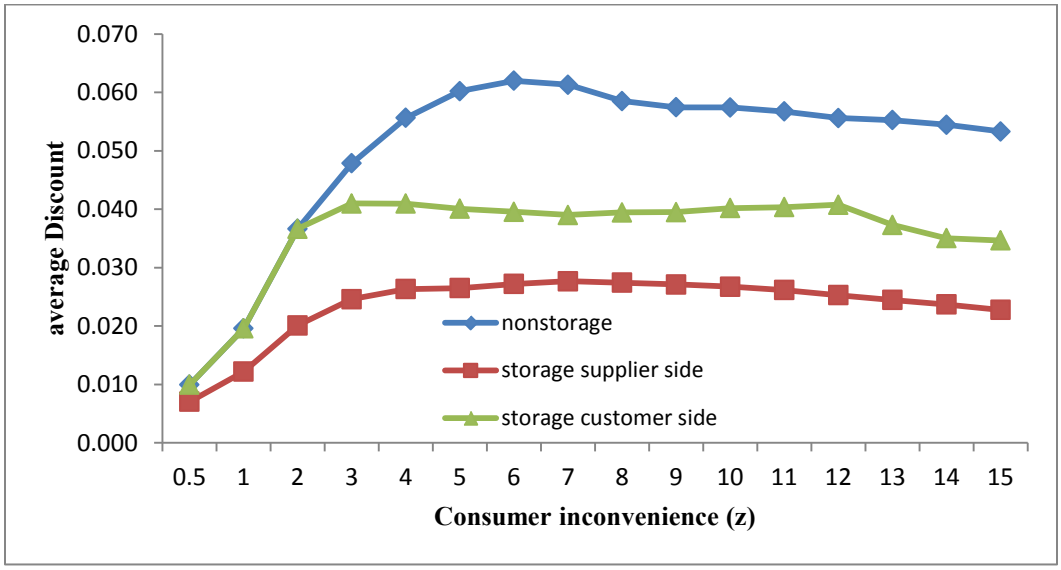

Figure 33:Comparison consumer inconvenience and PAR.

Figure 32 reflects the effects of these three models the PAR. There are three distinct segments on the graph. The first segment is when $z \in [\frac{1}{2}, 2]$ where the customers were price takers and the minimum consumption constraints were binding during some periods. In this segment, there was no difference on the PAR in the cases of no storage and storage controlled by the customer, since in both cases there was no energy storage.

While in the third case, storage controlled by the supplier, the PAR was lower since the energy storage was being employed.

Past this point ($z \in [3,12]$), the minimum consumption constraints started to become strictly holding Eqs. (6), (85), and (130). As such, the PAR of the first two cases started to become different since the customers started to store energy. However, these amounts of energy storage were lower than those achieved in the third case, and the PAR was higher. After the point ($z > 12$), the customers become highly resistant to price incentives and, as such, the optimal price discount dropped. This drop in the price discount led to no energy being stored by the customers, less shifting than in the case with no energy storage available, and a higher PAR.

Looking from the customers' point of view, we can see that average price discount decreased as energy storage was added to the system, as shown in Figure 33. This is because as storage was introduced into the system, the utility firm had a tool that decreased the generation fluctuations without having to incentivize load shifts via discounts as heavily.

Conjecture 2. In equilibrium, if the utility firm does not employ an energy storage, it will provide higher percentage discounts than if it employs an energy storage.

We define the Kuhn–Tucker condition, which is the derivative of the Lagrange function on equation (144) with respect to γ_t, as follows:

$$\frac{\partial LR}{\partial \gamma_t} = -P\ (y_{i,t} + \frac{(\gamma_t - \bar{\gamma})P}{2\alpha}) + P\,(1-\gamma_t)\Delta - 2\,a\,\Delta\,(y_{i,t} + \frac{(\gamma_t - \bar{\gamma})P}{2\alpha} - u_{it} + s_{it}) - b\Delta \leq 0 \tag{158}$$

If we solve the supplier model without storage, as in Erkoc, Al-Ahmadi, Celik, and Saad, (2015), we will find

$$\frac{\partial LR}{\partial \gamma_t} = -P\ \left(y_{i,t} + \frac{(\gamma_t - \bar{\gamma})P}{2\alpha}\right) + P\,(1-\gamma_t)\Delta$$
$$-\,2\,a\,\Delta\left(y_{i,t} + \frac{(\gamma_t - \bar{\gamma})P}{2\alpha}\right) - b\Delta \leq 0 \tag{159}$$

When we compare the difference between equation (158) and equation (159), we can find the impact of the charging and discharging decisions in the price discount. As the supplier decided to charge the storage, $s_{it} > 0$, the price discount decreased because they were negatively correlated, as shown in equation (158). On the other hand, when the energy storage was discharged, $\mathrm{u_{it}} > 0$, the price discount increased. However, if the discharging decision occurred during the peak time, the price discount was bound by the non-negativity constraint, and most of the time it remained binding even after adding the storage energy to the system. Therefore, the discharging decision did not have a significant impact on the price discount, which changed from zero to zero. ■

Since the utility firm's cost function was convex, increasing with the generation amount, it perceived the highest benefits as the generation fluctuation was minimized. The implementation of energy storage into the system allowed the utility firm to reduce generation fluctuation without a significant change to the nominal demand; consequently, the need for price discounts diminished. The effect of this can be seen in Figure 33. Furthermore, our analyses show that as the utility firm deployed the energy storage in the system, the PAR ,and discounts decreased compared to the case with no storage.

Finally, the energy provider's profits increased as energy storage was introduced to the system. This is because energy storage allowed the utility firm to participate in Energy Arbitrage, which is "earning a profit by charging ESS with cheap electricity when the demand is low and selling the stored energy at a higher price when the demand is

high" (Vazquez et al., 2010). This can be seen in Figure 35 where the scenarios with storage and no storage were compared.

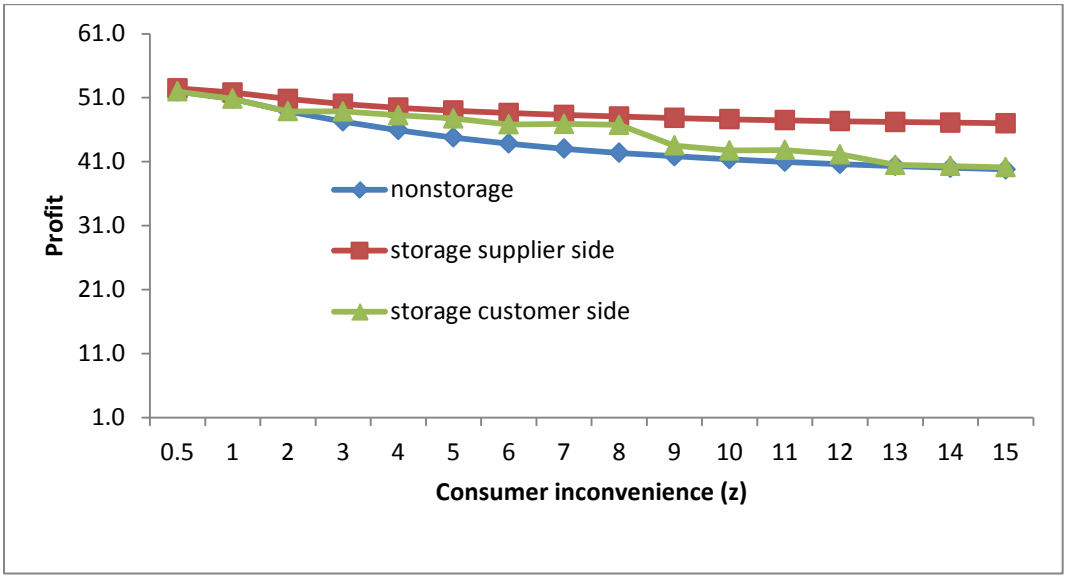

Figure 34:Comparison consumer inconvenience and energy provider's profit.

When there is a central authority that wants the lowest energy prices and PAR, regulating the market, we propose the following procedure to reach the equilibrium:

1. Find the optimal price discount and the actual consumptions for customers using the model with no storage from the Algorithm in Table 1.
2. Set the price discount and the customer actual consumption from step 1 as a fixed and substitute it into the model from equations (123)–(127) in order to find the storage decision variables.

The results from this procedure are shown in Figures 36, 37, 38 and 39, where we can see that the lowest PAR is achieved when the set of the minimum consumption constraints ($x_{it} \geq m_{it}$) are not binding and the energy provider's profits decrease around 2.9%. While the price discount and the customers total cost is that of the model with no storage.

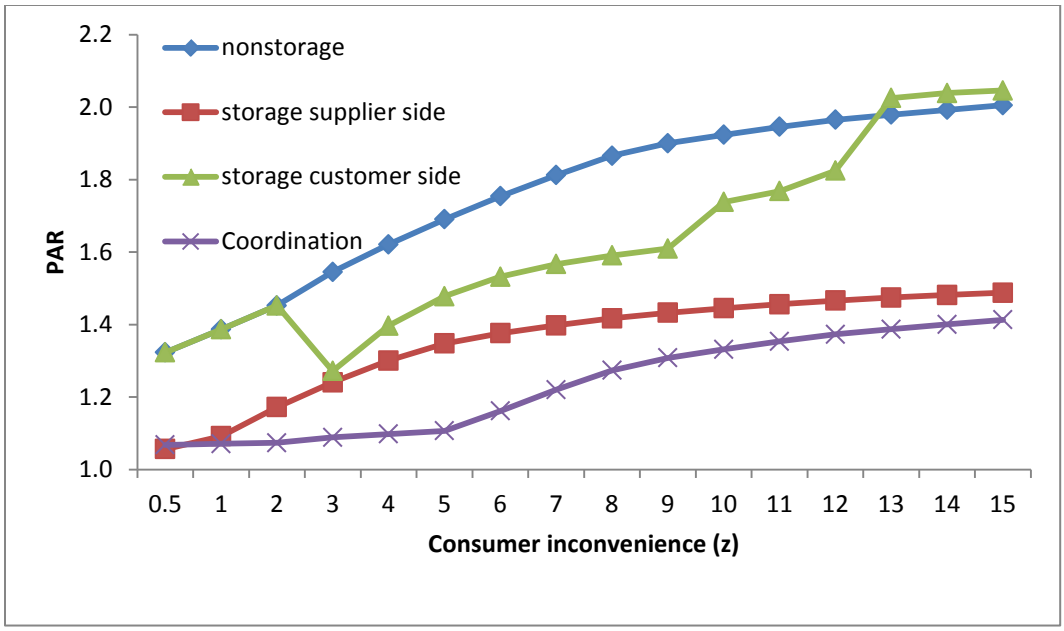

Figure 35:Customer inconvenience multiplier and PAR, coordination.

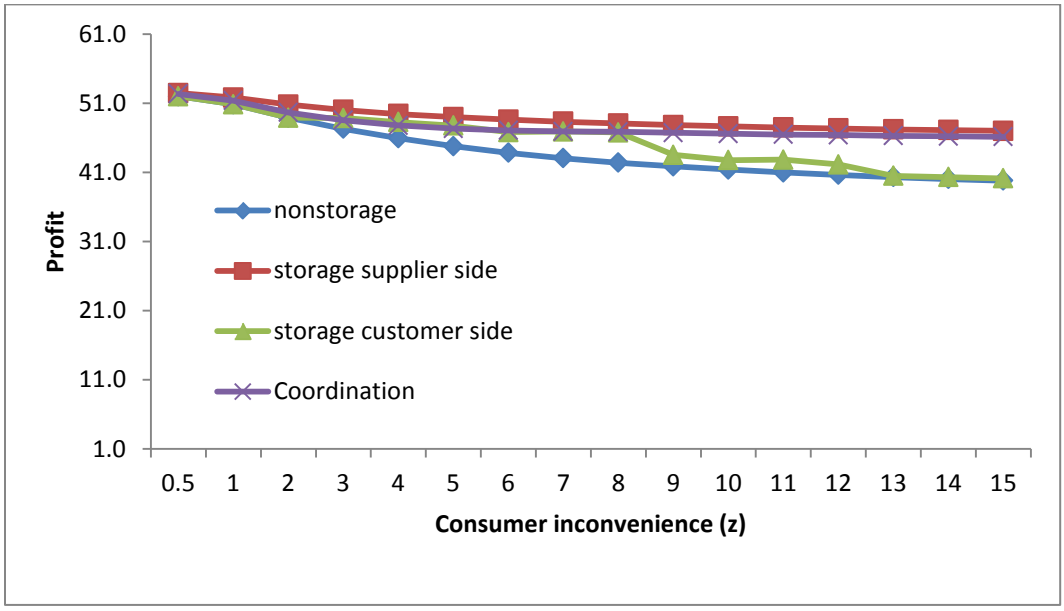

Figure 36: Customer inconvenience multiplier and energy provider' profit, coordination.

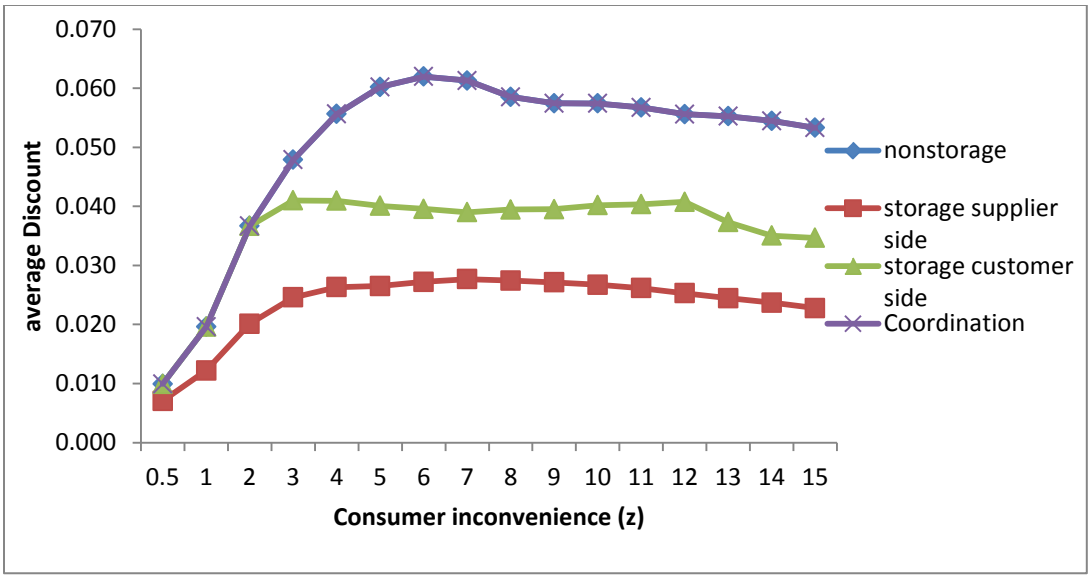

Figure 37: Customer inconvenience multiplier and average discount, coordination

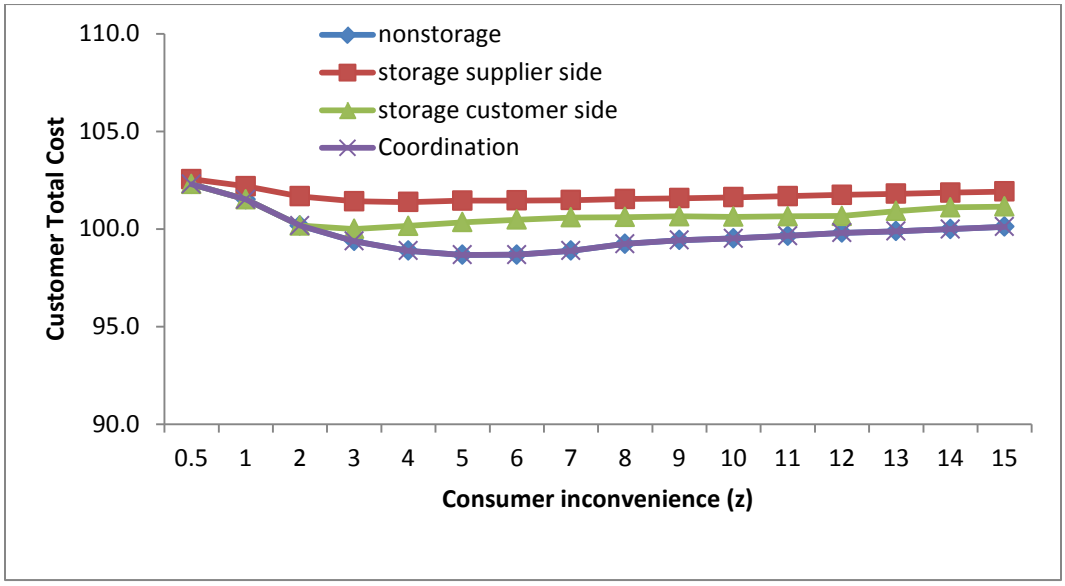

Figure 38: Customer inconvenience multiplier and customers total cost, coordination

4.6. Conclusions

In this chapter, we have evaluated the use of energy storage as part of a strategy for demand response within an electricity market with one energy provider and multiple customers. Our results show that the deployment of energy storage controlled by the

energy provider along with the use of price discounts led to more effective PAR reductions than when the customers control the energy storage. As such, the deployment of energy storage led to lower discounts and higher profits for the energy provider. Furthermore, when customers were more sensitive to price changes, there was less reliability on the energy storage at equilibrium, but as customers became less sensitive to price, the reliance on energy storage increased.

Chapter 5 Conclusions and Future Work

5.1. Conclusions

We have used Stackelberg game models to study an electricity market composed of a single energy provider and multiple customers and evaluated the effects of energy storage and load-shifting as part of a smart grid demand response. In these Stackelberg game models, the energy provider was the leader and the customers were the followers as they intended to maximize their profits and minimize their costs, respectively. The customer optimal cost included the consumption cost and inconvenience cost. A technique to reach the equilibrium discount prices has been detailed. We built three separate models to evaluate the effects that having different types of customers, a market with different degrees of diversity and a market of different sizes, a homogeneous or heterogeneous market, and energy storage have on the equilibrium discount and PAR.

In the first model, we studied the characteristics of load-shifting with no energy storage. The game had two levels, in the first level the provider makes his strategic decisions by determining the price discount; and then, in the second level, the customers respond to the announced discounts by shifting some of their consumption from On-peak time slots to Off-peak. Moreover, we investigated the model when customers had the same attitude towards load-shifting and when the customers had different attitudes and the customers' inconvenience level varied across the planning horizon.

In the second model, we studied the aspect of load-shifting with energy storage. We introduced an energy storage to the customer's model, in which the customers are in control of the energy storage. Again, we formulate the game with two levels, the provider moves first by deciding a price discount in each time slot on order to encourage

customers to shift their consumption from the On-peak time slots to Off-peak, and/or charge the storage system in the Off-peak time slots and deploy it on the On-peak time slots. In the second level, customers react to the announced discounts by making two strategic decisions, the actual consumption and the storage profiles for each time slot.

In third model, we investigated and studied the effects on the equilibrium, profits and PAR, when the energy storage are controlled by the energy provider. In this game there are three levels. In the first level, the provider decides the price discounts; in the second level the customers, respond to the announced discount by shifting their nominal demand; and in the third level, the provider makes his storage decisions depending on the customer's response to the price discounts.

Our results showed that higher discounts were needed when customer inconvenience levels were moderate and/or the consumer population was moderately diverse in terms of their customer types. Further, discounts were not as effective in reducing PAR when the customer population was small or customer types werte alike. Moreover, when consumers were price takers, it was relatively easier for the electricity provider to dampen the PAR and enjoy higher payoffs with smaller discounts. However, when the inconvenience levels became too high, the consumer resistance to load-shifting was too strong to justify price discounts. As such, after a point, the provider, in fact, began to cut back on the discounts. Finally, the results showed that the use of energy storage was very effective in the reduction of the PARs, and the uses of both of these tactics (i.e., price discounts and energy storage) provided effective ways to improve profits. Moreover, in terms of which controlling the storage, our result showed the PAR is lower when the provider controlled the storage, while the price discount was higher

when the storage was controlled by the customers. Furthermore, results showed that the use of load-shifting always reduced the customers' total costs, but this reduction was diminished by the implementation of energy storage.

In terms of the equilibrium results of the three models, the PAR is lowest when the energy storage is controlled by the energy provider; while the average discount price is highest when there is no energy storage. Because of this we propose the use of coordination in order to maximize discounts and minimize the PAR simultaneously. For this the model with no storage is run to find the optimal discount and actual consumption during each period. These levels are then input to the model with provider controlled storage to find the storage profile decisions. Our results show that this method lead to the lowest PAR, while the price discount is that of the model with no storage. This proposal leads to a slightly lower profit for the energy provider, compared to the model where he controls the energy storage and uses it as part of the discount optimization, however this difference is very small, and we believe that in the long run the lower PAR achieved from the coordination model may lead to greater benefits, especially if future possible regulations are taken into account.

5.2. Future Work

In our future work, a possible research direction involves combining renewable energy sources to the customers' model, studying and evaluating of the effects of the uncertainty of renewable energy on the discount price and the PAR. Furthermore, each customer may become an interactive part of the microgrid where he or she can produce some electricity needs and also buy and sell to the electricity market depending on their nominal demand and inconvenience cost. This model can combine two games: a

Stackelberg game between the provider and customers and a Nash equilibrium game for each of the customers.

Another approach considered as future work is the use of price discrimination where the level of consumption pricing and the time of consumption pricing are combined. In each time slot, the energy provider offers a two-discount price. The two discounts work by use of a defined threshold. The first discount is offered to all customers, while the second is offered to the customers who are more willing to shift demand beyond the threshold. Moreover, defining the inconvenience factor is another avenue for future study, by investigating and analyzing the factors that impact the customer inconvenience.

Other avenues for future research include the use of a simulated game, in which agent based simulation is employed to evaluate the behavior of both the energy provider and the customers, and their real-time response to the provided discounts. Furthermore, the proposed framework may be evaluated with an empirical study by employing a selected group of customers to whom discounts are offered during different periods of the day. Their behavior can be evaluated and compared with their historical usage and to other control users, who are not offered discounts.

References

(DOE), U. S. D. of E. (2006). Benefits of demand response in electricity markets and recommendations for achieving them. *U.S. Department of Energy*, (February), 122.

Atzeni, I., Ordonez, L. G., Scutari, G., Palomar, D. P., & Fonollosa, J. R. (2013). Demand-side management via distributed energy generation and storage optimization. *IEEE Transactions on Smart Grid*, *4*(2), 866–876.

Avci, M., Erkoc, M., Rahmani, A., & Asfour, S. (2013). Model predictive HVAC load control in buildings using real-time electricity pricing. *Energy and Buildings*, *60*, 199–209.

Bu, S., Yu, F. R., & Liu, P. X. (2011). A game-theoretical decision-making scheme for electricity retailers in the smart grid with demand-side management. *2011 IEEE International Conference on Smart Grid Communications, SmartGridComm 2011*, 387–391.

Chai, B., Chen, J., Yang, Z., & Zhang, Y. (2014). Demand response management with multiple utility companies: A two-level game approach. *IEEE Transactions on Smart Grid*, *5*(2), 722–731.

Chen, C., Kishore, S., & Snyder, L. V. (2011). 2011—An innovative RTP-based residential power scheduling scheme for smart grids, 5956–5959.

Chen, J., Yang, B., & Guan, X. (2012). Optimal demand response scheduling with Stackelberg game approach under load uncertainty for smart grid. *2012 IEEE 3rd International Conference on Smart Grid Communications, SmartGridComm 2012*, 546–551.

Chen, L. C. L., Li, N. L. N., Low, S. H., & Doyle, J. C. (2010). Two market models for demand response in power networks. *Smart Grid Communications (SmartGridComm), 2010 First IEEE International Conference on*, 397–402.

Chen, Y., Lin, W. S., Han, F., Yang, Y. H., Safar, Z., & Liu, K. J. R. (2012). A cheat-proof game theoretic demand response scheme for smart grids. *IEEE International Conference on Communications*, 3362–3366.

Choi, C., & Mattila, A. S. (2014). The effects of promotion framing on consumers' price perceptions: The moderating role of a personal sense of power. *Journal of Service Management*, *25*(1), 149–160. https://doi.org/10.1108/JOSM-11-2012-0234

Chopra, S., & Meindl, P. (2016). *Supply chain management: Strategy, planning, and operation* (6th ed.). Boston: Pearson.

Deng, R., Yang, Z., Chow, M.–Y., & Chen, J. (2015). A survey on demand response in smart grids: Mathematical models and approaches. *IEEE Transactions on Industrial Informatics*, *11*(3), 1–1.

Deng, W., Liu, F., Jin, H., & Wu, C. (2013). SmartDPSS: Cost-minimizing multi-source power supply for datacenters with arbitrary demand. *Proceedings - International Conference on Distributed Computing Systems*, 420–429.

Dunn, B., Kamath, H., & Tarascon, J. M. (2011). "Electrical energy storage for the grid: A battery of choices." *Science*, *334*(6058), 928–935.

Erkoc, M., Al-Ahmadi, E., Celik, N., & Saad, W. (2015). A game theoretic approach for load-shifting in the smart grid. *2015 IEEE International Conference on Smart Grid Communications (SmartGridComm)*, 187–192.

Forouzandehmehr, N., Esmalifalak, M., Mohsenian–Rad, H., & Han, Z. (2015). Autonomous Demand Response Using Stochastic Differential Games. *IEEE Transactions on Smart Grid*, *6*(1), 291–300.

Hittinger, E., Whitacre, J. F., & Apt, J. (2012). What properties of grid energy storage are most valuable? *Journal of Power Sources*, *206*, 436–449.

Huang, L., Walrand, J., & Ramchandran, K. (2012). Optimal demand response with energy storage management. *2012 IEEE Third International Conference on Smart Grid Communications (SmartGridComm)*, 61–66.

Ibrahim, H., Ilinca, A., & Perron, J. (2008). Energy storage systems—Characteristics and comparisons. *Renewable and Sustainable Energy Reviews*, *12*(5), 1221–1250.

Jiang, L., & Low, S. (2011). Real-time demand response with uncertain renewable energy in smart grid. *2011 49th Annual Allerton Conference on Communication, Control, and Computing, Allerton 2011*, 1334–1341.

Kumar Nunna, H. S. V. S., & Doolla, S. (2013). Energy management in microgrids using demand response and distributed storage: A multiagent approach. *IEEE Transactions on Power Delivery*, *28*(2), 939–947.

Logenthiran, T., Srinivasan, D., & Vanessa, K. W. M. (2014). Demand side management of smart grid: Load shifting and incentives. *Journal of Renewable and Sustainable Energy*, *6*(3).

Maharjan, S., Zhu, Q., Zhang, Y., Gjessing, S., & Basar, T. (2013). Dependable demand response management in the smart grid: A Stackelberg game approach. *Smart Grid, IEEE Transactions on*, *4*(1), 120–132.

Men, F. L., & Zeng, X. J. (2013). A Stackelberg game-theoretic approach to optimal real-time pricing for the smart grid. *Soft Computing*, *17*(12), 2365–2380.

Mohsenia–Rad, A. H., Wong, V. W. S., Jatskevich, J., Schober, R., & Leon–Garcia, A. (2010). Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid. *IEEE Transactions on Smart Grid*, *1*(3), 320–331.

Nakayama, K., Zhao, C., Bic, L. F., Dillencourt, M. B., & Brouwer, J. (2013). Distributed real-time power flow control with renewable integration. *2013 IEEE International Conference on Smart Grid Communications, SmartGridComm 2013*, 516–521.

Nekouei, E., Alpcan, T., & Chattopadhyay, D. (2015). Game-theoretic frameworks for demand response in electricity markets. *IEEE Transactions on Smart Grid*, *6*(2), 748–758.

Nguyen, H., Song, J., & Han, Z. (2012). Demand side management to reduce peak-to-average ratio using game theory in smart grid. ... *Workshops (INFOCOM WKSHPS ...*, 91–96.

Pachon, J., Erkoc, M., & Iakovou, E. (2007). Contract optimization with front-end fare discounts for airline corporate deals. *Transportation Research Part E: Logistics and Transportation Review*, *43*(4), 425–441.

Qin, J., Chow, Y., Yang, J., & Rajagopal, R. (2014). Online modified greedy algorithm for storage control under uncertainty *IEEE Transactions on Power Systems, 31*(3), 11.

Saad, W., Han, Z., Poor, H. V., & Basar, T. (2012). Game-theoretic methods for the smart grid: An overview of microgrid systems, demand-side management, and smart grid communications. *IEEE Signal Processing Magazine*, *29*(5), 86–105.

Sáen, J. P., Celik, N., Xi, H., Son, Y. J., & Asfour, S. (2013). Two-stage economic and environmental load dispatching framework using particle filtering. *International Journal of Electrical Power and Energy Systems*, *48*(1), 93–110.

Siano, P. (2014). Demand response and smart grids—A survey. *Renewable and Sustainable Energy Reviews*, *30*, 461–478.

Sims, R. E. H., Rogner, H. H., & Gregory, K. (2003). Carbon emission and mitigation cost comparisons between fossil fuel, nuclear and renewable energy resources for electricity generation. *Energy Policy*, *31*(13), 1315–1326.

Soliman, H. M., & Leon–Garcia, A. (2014). Game-theoretic demand-side management with storage devices for the future smart grid. *IEEE Transactions on Smart Grid*, *5*(3), 1475–1485.

Trigg, T., Telleen, P., Boyd, R., & Cuenot, F. (2013). Global EV Outlook: Understanding the Electric Vehicle Landscape to 2020. *Iea*, (April), 1–41. Retrieved from https://www.iea.org/publications/freepublications/publication/GlobalEVOutlook_2013.pdf

U.S. Energy Information Administration. (2014). Levelized Costs of New Electricity Generating Technologies, 12.

U.S. Energy Information Administration. (2015). Table 8.4. Average power plant operating expenses for major U.S. investor-owned electric utilities, 2005 through 2015 (mills per kilowatthour). Retrieved from https://www.eia.gov/electricity/annual/html/epa_08_04.html

Vazquez, S., Lukic, S. M., Galvan, E., Franquelo, L. G., & Carrasco, J. M. (2010). Energy storage systems for transport and grid applications. *IEEE Transactions on Industrial Electronics*, *57*(12), 3881–3895.

Vytelingum, P., Voice, T. D., Ramchurn, S. D., Rogers, A., & Jennings, N. R. (2010). Agent-based micro-storage management for the smart grid. *Aamas*, 39–46.

Wang, Y., Saad, W., Han, Z., Poor, H. V, & Basar, T. (2014). A game-theoretic approach to energy trading in the smart grid. *IEEE Transactions on Smart Grid, 5*(3), 1439–1450.

Wang, Z., Gu, C., Li, F., Member, S., & Bale, P. (2013). Active demand response using shared energy storage for husehold energy management. *IEEE Transactions on Smart Grid, 4*(4), 1888–1897.

Wu, Z., Tazvinga, H., & Xia, X. (2015). Demand side management of photovoltaic-battery hybrid system. *Applied Energy*, *148*, 294–304.

Xu, Y., & Tong, L. (2015). On the operation and value of storage in consumer demand response. *53rd IEEE Conference on Decision and Control*, Los Angeles, CA, 205–210.

Yang, P., Tang, G., & Nehorai, A. (2013). A game-theoretic approach for optimal time-of-use electricity pricing. *IEEE Transactions on Power Systems*, *28*(2), 884–892.

Yu, Z., & van der Schaar, M. (2014). Structure-aware atochastic atorage management in smart grids. *IEEE Journal of Selected Topics in Signal Processing, 8*(6), 1098–1110.

Zheng, M., Meinrenken, C. J., & Lackner, K. S. (2014). Agent-based model for electricity consumption and storage to evaluate economic viability of tariff arbitrage for residential sector demand response. *Applied Energy*, *126*, 297–306.

Zhou, K., Pan, J., & Cai, L. (2014). Optimal combined heat and power system scheduling in smart grid. *Proceedings—IEEE INFOCOM*, 2831–2839.

APPENDIX A

Model's file (Load shifting with no energy storage) – Using Ampl

```
param T:= 12;            # timeslot
param N := 7;   # Number of Customers
param p:= 0.25;   # base price
param alpha {1..N};              # inconvenience factor
param y {1..T,1..N};    # The original amount of electricity demanded
param a {1..T};         # parameter of the generation cost function
param b {1..T};         # parameter of the generation cost function
param c {1..T};         # parameter of the generation cost function
param m {1..T,1..N};      # Minimum consumption that cannot be shifted to another
timeslot
var discount {t in 1..T} >= 0, <= 1;        # price discounts
var x {1..T,1..N};                    # actual consumption
maximize Total_Profit:
sum {t in 1..T} ((1 - discount[t]) * p * sum {n in 1..N} x[t,n]
                - a[t] * (sum {n in 1..N} x[t,n])^2 - b[t] * (sum {n in 1..N} x[t,n]) - c[t]);
subject to demand {t in 1..T,n in 1..N}:
 x[t,n] = y[t,n] + ((discount[t] - ((sum {k in 1..T} discount[k])/T)) * p/(2* alpha[n]));
```

Run's file (Load shifting with no energy storage) – Using Ampl

```
model Load_shifting_with_no_storage.mod;
param cost1 {1..N};  # consumption cost
param cost2 {1..N};  # Inconvenience cost
```

```
param shift {1..N};
## Reading the data from Excel file###
table pdiscount "ODBC" "load_shifting_with_no_storage_data.xlsx" "Timeslots":
   [T], a IN, b IN, c IN,
  discount OUT;
table consumers "ODBC" "load_shifting_with_no_storage_data.xlsx" "Customers":
   [N], alpha IN, cost1 OUT, cost2 OUT, shift OUT;
table amount "ODBC" "load_shifting_with_no_storage_data.xlsx" "Consumptions":
  [T,N], y IN,
  x OUT;
read table pdiscount;
read table consumers;
read table amount;
solve;
param h {i in 1..T,j in 1..N};   #Loop parameter
param q;                         #Loop parameter
param v {1..T};                  #Loop parameter
param k {1..N};  # T hat
param tbar {1..N};
subject to demand1 {t in 1..T,n in 1..N: h[t,n] > 0}:
x[t,n] = y[t,n] + tbar[n]/ k[n] + (discount[t] - ((sum {i in 1..T} if h[i,n] > 0  then
discount[i] else 0)/ k[n])) * p/2/alpha[n] ;
subject to demand2 {t in 1..T,n in 1..N: h[t,n] <= 0}:
```

```
x[t,n] = 0;
problem subopt: discount, x, Total_Profit, demand2, demand1;
repeat optimal_loop {
let {i in 1..T,j in 1..N} h[i,j]:= x[i,j];
for {i in 1..T} { let v[i]:= 0;
};
for {i in 1..T} {for {j in 1..N} {if h[i,j] >= 0 then {let v[i]:= v[i] + 1;   # step2
};};};
let q:= sum{i in 1..T} v[i];
if q = N * T then break optimal_loop;    # step2
for {i in 1..N} { let k[i]:= 0;
};
for {i in 1..N} {for {j in 1..T} {if h[j,i] > 0 then {let k[i]:= k[i] + 1;
};};};
let {j in 1..N} tbar[j]:= sum {i in 1..T} if h[i,j] <= 0 then y[i,j] else 0;
solve subopt; } ;
let {j in 1..N} cost1[j]:= sum {i in 1..T} x[i,j] * (1- discount[i]) * p;
let {j in 1..N} cost2[j]:= sum {i in 1..T} (x[i,j] - y[i,j])^2 * alpha[j] ;
let {j in 1..N} shift[j]:= sum {i in 1..T} if  (y[i,j]- x[i,j]) > 0 then (y[i,j]- x[i,j]) else 0  ;
write table pdiscount;
write table amount;
write table consumers;
```

Table 10: “Consumptions” table from the data file when number of customers is two – no energy storage

T	N	y	x
1	1	1.71	3.50
1	2	1.53	3.36
2	1	2.38	3.53
2	2	2.03	3.21
3	1	3.34	3.74
3	2	3.01	3.42
4	1	4.16	3.76
4	2	4.32	3.91
5	1	4.51	3.66
5	2	4.77	3.89
6	1	5.67	4.82
6	2	5.59	4.71
7	1	10.66	9.81
7	2	10.54	9.66
8	1	8.58	7.73
8	2	8.96	8.08
9	1	8.10	7.25
9	2	8.43	7.55
10	1	5.17	4.32
10	2	5.27	4.39
11	1	2.36	3.31
11	2	2.56	3.53
12	1	2.40	3.63
12	2	2.10	3.37

Table 11: “Customers” table from the data file when number of customers is two – no energy storage

N	alpha	cost1	cost2	shift
1	0.008139	14.169280258	0.094695461	5.520829090
2	0.00794	14.211110408	0.097061139	5.658750236

Table 12:“Timeslots” table from the data file when number of customers is two– no energy storage

T	a	b	c	discount
1	0.0035	0	0	0.179822152
2	0.0035	0	0	0.130824399
3	0.0035	0	0	0.072700475
4	0.0035	0	0	0.011121027
5	0.0035	0	0	0.000000000
6	0.0035	0	0	0.000000000
7	0.0035	0	0	0.000000000
8	0.0035	0	0	0.000000000
9	0.0035	0	0	0.000000000
10	0.0035	0	0	0.000000000
11	0.0035	0	0	0.114787160
12	0.0035	0	0	0.137115250

APPENDIX B

Model's file (Load shifting with no energy storage- inconvenience varies across time periods) – Using Ampl:

```
param T:= 12;            # timeslot
param N := 21;   # Number of Customers
param p:= 0.25;   # base price
param alpha {1..T,1..N};   # inconvenience factor
param y {1..T,1..N};    # The original amount of electricity demanded
param a {1..T};         # parameter of the generation cost function
param b {1..T};         # parameter of the generation cost function
param c {1..T};         # parameter of the generation cost function
param m {1..T,1..N};      # Minimum consumption that cannot be shifted to another timeslot
var discount {t in 1..T} >= 0, <= 1;      # price discounts
var x {1..T,1..N};                        # actual consumption
maximize Total_Profit:
sum {t in 1..T} ((1 - discount[t]) * p * sum {n in 1..N} x[t,n] - a[t] * (sum {n in 1..N} x[t,n])^2 - b[t] * (sum {n in 1..N} x[t,n]) - c[t]);
subject to demand {t in 1..T,n in 1..N}:
 x[t,n] = y[t,n] - (1-discount[t])* p/(2* alpha[t,n]) +  (sum {k in 1..T} (1-discount[k])* p /(2* alpha[k,n]))/ (sum {s in 1..T} 1/(2* alpha[s,n]))/ (2* alpha[t,n]) ;
```

Run's file (Load shifting with no energy storage- inconvenience varies across time periods) – Using Ampl:

```
model model_different_alpha.mod;
param cost1 {1..N};  # cosumption cost
param cost2 {1..N};  # Inconvenience cost
param shift {1..N};
table pdiscount "ODBC" "different_alpha_data.xlsx" "Timeslots":
   [T], a IN, b IN, c IN,discount OUT;
table consumers "ODBC" "different_alpha_data.xlsx" "Customers":
   [T,N], alpha IN ;
table consumers1 "ODBC" "different_alpha_data.xlsx" "cost":
   [N], cost1 OUT, cost2 OUT, shift OUT;
table amount "ODBC" "different_alpha_data.xlsx" "Consumptions":
  [T,N], y IN,x OUT;
read table pdiscount;
read table consumers;
read table amount;
solve;
param h {i in 1..T,j in 1..N};   #Loop parameter
param q;                         #Loop parameter
param v {1..T};                  #Loop parameter
param k {1..N};  # T hat
param tbar {1..N};
```

```
subject to demand1 {t in 1..T,n in 1..N: h[t,n] > 0}:
 x[t,n] = y[t,n]  - (1-discount[t])* p/(2* alpha[t,n])
     + (1/(sum {s in 1..T} if h[s,n] > 0  then  1/(2* alpha[s,n]) else 0) / (2* alpha[t,n]))*
(tbar[n]+ (sum {i in 1..T} if h[i,n] > 0 then (1-discount[i])* p /(2* alpha[i,n]) else 0)) ;
subject to demand2 {t in 1..T,n in 1..N: h[t,n] <= 0}:
x[t,n] = 0;
problem subopt: discount, x, Total_Profit, demand2, demand1;
repeat optimal_loop {
let {i in 1..T,j in 1..N} h[i,j]:= x[i,j];
for {i in 1..T} { let v[i]:= 0;};
for {i in 1..T} {for {j in 1..N} {if h[i,j] >= 0 then {let v[i]:= v[i] + 1; };};};  # step2
let q:= sum{i in 1..T} v[i];
if q = N * T then break optimal_loop;    # step2
for {i in 1..N} { let k[i]:= 0;};
for {i in 1..N} {for {j in 1..T} {if h[j,i] > 0 then {let k[i]:= k[i] + 1; };};};};
let {j in 1..N} tbar[j]:= sum {i in 1..T} if h[i,j] <= 0 then y[i,j] else 0;
solve subopt; } ;
let {j in 1..N} cost1[j]:= sum {i in 1..T} x[i,j] * (1- discount[i]) * p;
let {j in 1..N} cost2[j]:= sum {i in 1..T} (x[i,j] - y[i,j])^2 * alpha[i,j] ;
let {j in 1..N} shift[j]:= sum {i in 1..T} if  (y[i,j]- x[i,j]) > 0 then (y[i,j]- x[i,j]) else 0  ;
write table pdiscount;
write table amount;
write table consumers1;
```

Table 13: “Consumptions” table from the data file for number of customers is two – inconvenience varies across time periods

T	N	y	x
1	1	1.71	3.90
1	2	1.53	3.18
2	1	2.38	3.97
2	2	2.03	3.23
3	1	3.34	4.23
3	2	3.01	3.68
4	1	4.16	4.31
4	2	4.32	4.42
5	1	4.51	4.05
5	2	4.77	4.43
6	1	5.67	4.40
6	2	5.59	4.67
7	1	10.66	8.57
7	2	10.54	9.04
8	1	8.58	6.49
8	2	8.96	7.46
9	1	8.10	5.15
9	2	8.43	6.54
10	1	5.17	3.81
10	2	5.27	4.39
11	1	2.36	4.82
11	2	2.56	4.11
12	1	2.40	5.35
12	2	2.10	3.96

Table 14: “Customers” table from the data file for two customers – inconvenience varies across time periods

T	N	alpha
1	1	0.003567
1	2	0.004687
2	1	0.003567
2	2	0.004687
3	1	0.003567
3	2	0.004687
4	1	0.003567
4	2	0.004687
5	1	0.002767
5	2	0.003887
6	1	0.002767
6	2	0.003887
7	1	0.002767
7	2	0.003887
8	1	0.002767
8	2	0.003887
9	1	0.001967
9	2	0.003087
10	1	0.001967
10	2	0.003087
11	1	0.001967
11	2	0.003087
12	1	0.001967
12	2	0.003087

Table 15: “cost” table from the data file for two customers – inconvenience varies across time periods

N	cost1	cost2	shift
1	31.83771	0.107972	10.224287
2	31.95995	0.074560	7.037451

Table 16: “Timeslots” table from the data file– inconvenience varies across time periods

T	a	b	c	discount
1	0.0009	0	0	0.049411216
2	0.0009	0	0	0.041739349
3	0.0009	0	0	0.032638545
4	0.0009	0	0	0.022996687
5	0.0009	0	0	0.016397388
6	0.0009	0	0	0.008309326
7	0.0009	0	0	0.000000000
8	0.0009	0	0	0.000000000
9	0.0009	0	0	0.000000000
10	0.0009	0	0	0.011357074
11	0.0009	0	0	0.038645614
12	0.0009	0	0	0.042141654

APPENDIX C

Model's file (Load shifting with customers controlled storage) – Using Ampl:

```
param T:= 12;            # timeslot
param N := 7;            # Number of Customers
param p:= 0.25;   # base price
param alpha {1..N};              # inconvenience factor
param y {1..T,1..N};   # The original amount of electricity demanded
param a {1..T};         # parameter of the generation cost function
param b {1..T};         # parameter of the generation cost function
param c {1..T};         # parameter of the generation cost function
param m {1..T,1..N};      # Minimum consumption that cannot be shifted to another
timeslot
param ro:= 0.99;         # storage efficiency
param tau:= 0.01;        # charging efficiency
param beta:= 0.01;      # discharging efficiency
param cap:= 12;                  # storage capacity
var discount {t in 1..T} >= 0, <= 1;       # price discounts
var x {1..T,1..N};                    # actual consumption
var xcharge {1..T,1..N} >=0;     #amount of charging
var xstored {1..T,1..N} >=0, <=cap;  # storage level
var mu2 {1..T,1..N} >= 0;   #lagrangian multiplier for discharged non-negativity
constraints
var mu3 {1..T,1..N} >= 0;   #lagrangian multiplier for the storage capacity constraints
```

```
maximize Total_Profit:
(1 - discount[1]) * p * ( sum {n in 1..N} (x[1,n] + xcharge[1,n] - (- xstored[1,n] + (1- tau)
* xcharge[1,n])/(1+ beta)))
                                - a[1] * (sum {n in 1..N} (x[1,n] + xcharge[1,n] - (- xstored[1,n] +
(1- tau) * xcharge[1,n])/(1+ beta)))^2
                                - b[1] * ( sum {n in 1..N} (x[1,n] + xcharge[1,n] - (- xstored[1,n]
+ (1- tau) * xcharge[1,n])/(1+ beta))) - c[1]
+ sum {t in 2..T} ((1 - discount[t]) * p * ( sum {n in 1..N} (x[t,n] + xcharge[t,n] - (-
xstored[t,n]+ ro * xstored[t-1,n] + (1 - tau) * xcharge[t,n])/(1+ beta)))
                                - a[t] * (sum {n in 1..N} (x[t,n] + xcharge[t,n] - (-xstored[t,n]+ ro
* xstored[t-1,n] + (1 - tau) * xcharge[t,n])/(1+ beta)))^2
                                - b[t] * ( sum {n in 1..N} (x[t,n] + xcharge[t,n] - (-xstored[t,n]+
ro * xstored[t-1,n] + (1 - tau) * xcharge[t,n])/(1+ beta)))- c[t]);
subject to demand {t in 1..T,n in 1..N}:
x[t,n] = y[t,n] + ((discount[t] - ((sum {k in 1..T} discount[k])/T)) * p/(2* alpha[n])) ;
subject to charge {t in 1..T,n in 1..N}:
 (1 - discount[t]) * p * (1 - (1- tau)/(1+ beta)) - mu3[t,n] * (1- tau)/(1+ beta) >= 0;
subject to charge_s {t in 1..T,n in 1..N}:
 xcharge[t,n] * ((1 - discount[t]) * p * (1 - (1- tau)/(1+ beta)) - mu3[t,n] * (1- tau)/(1+
beta)) = 0;
#t=12
subject to stored12 {n in 1..N}:
(1 - discount[12]) * p/(1 + beta) + (1/(1+ beta)) * mu3[12,n]
```

```
            + mu2[12,n] >= 0;
subject to s512 {n in 1..N}:
((1 - discount[12]) * p/(1 + beta) + (1/(1+ beta)) * mu3[12,n]
            + mu2[12,n]) * xstored[12,n]= 0;
#t=1
subject to stored1 {n in 1..N}:
(1 - discount[1]) * p/(1 + beta) - (1 - discount[2]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[1,n] - (ro/(1 + beta)) * mu3[2,n]
            + mu2[1,n] >= 0;
subject to s51 {n in 1..N}:
((1 - discount[1]) * p/(1 + beta) - (1 - discount[2]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[1,n] - (ro/(1 + beta)) * mu3[2,n]
            + mu2[1,n]) * xstored[1,n]= 0;
subject to stored222 {n in 1..N}:
(1 - discount[2]) * p/(1 + beta) - (1 - discount[3]) * p * ro/ (1 + beta)
         + (1/(1+ beta)) * mu3[2,n] - (ro/(1 + beta)) * mu3[3,n]
            + mu2[2,n] >= 0;
subject to s52 {n in 1..N}:
((1 - discount[2]) * p/(1 + beta) - (1 - discount[3]) * p * ro/ (1 + beta)
         + (1/(1+ beta)) * mu3[2,n] - (ro/(1 + beta)) * mu3[3,n]
            + mu2[2,n]) * xstored[2,n]= 0;
#t=3
```

```
subject to stored3 {n in 1..N}:
(1 - discount[3]) * p/(1 + beta) - (1 - discount[4]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[3,n] - (ro/(1 + beta)) * mu3[4,n]
            + mu2[3,n] >= 0;
subject to s53 {n in 1..N}:
((1 - discount[3]) * p/(1 + beta) - (1 - discount[4]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[3,n] - (ro/(1 + beta)) * mu3[4,n]
            + mu2[3,n]) * xstored[3,n]= 0;
#t=4
subject to stored4 {n in 1..N}:
(1 - discount[4]) * p/(1 + beta) - (1 - discount[5]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[4,n] - (ro/(1 + beta)) * mu3[5,n]
            + mu2[4,n] >= 0;
subject to s54 {n in 1..N}:
((1 - discount[4]) * p/(1 + beta) - (1 - discount[5]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[4,n] - (ro/(1 + beta)) * mu3[5,n]
            + mu2[4,n]) * xstored[4,n]= 0;
#t=5
subject to stored5 {n in 1..N}:
(1 - discount[5]) * p/(1 + beta) - (1 - discount[6]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[5,n] - (ro/(1 + beta)) * mu3[6,n]
            + mu2[5,n] >= 0;
```

```
subject to s55 {n in 1..N}:
((1 - discount[5]) * p/(1 + beta) - (1 - discount[6]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[5,n] - (ro/(1 + beta)) * mu3[6,n]
            + mu2[5,n]) * xstored[5,n]= 0;
#t=6
subject to stored6 {n in 1..N}:
(1 - discount[6]) * p/(1 + beta) - (1 - discount[7]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[6,n] - (ro/(1 + beta)) * mu3[7,n]
            + mu2[6,n] >= 0;
subject to s56 {n in 1..N}:
((1 - discount[6]) * p/(1 + beta) - (1 - discount[7]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[6,n] - (ro/(1 + beta)) * mu3[7,n]
            + mu2[6,n]) * xstored[6,n]= 0;
#t=7
subject to stored7 {n in 1..N}:
(1 - discount[7]) * p/(1 + beta) - (1 - discount[8]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[7,n] - (ro/(1 + beta)) * mu3[8,n]
            + mu2[7,n] >= 0;
subject to s57 {n in 1..N}:
((1 - discount[7]) * p/(1 + beta) - (1 - discount[8]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[7,n] - (ro/(1 + beta)) * mu3[8,n]
            + mu2[7,n]) * xstored[7,n]= 0;
#t=8
```

```
subject to stored8 {n in 1..N}:
(1 - discount[8]) * p/(1 + beta) - (1 - discount[9]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[8,n] - (ro/(1 + beta)) * mu3[9,n]
            + mu2[8,n] >= 0;
subject to s58 {n in 1..N}:
((1 - discount[8]) * p/(1 + beta) - (1 - discount[9]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[8,n] - (ro/(1 + beta)) * mu3[9,n]
            + mu2[8,n]) * xstored[8,n]= 0;
#t=9
subject to stored9 {n in 1..N}:
(1 - discount[9]) * p/(1 + beta) - (1 - discount[10]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[9,n] - (ro/(1 + beta)) * mu3[10,n]
            + mu2[9,n] >= 0;
subject to s59 {n in 1..N}:
((1 - discount[9]) * p/(1 + beta) - (1 - discount[10]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[9,n] - (ro/(1 + beta)) * mu3[10,n]
            + mu2[9,n]) * xstored[9,n]= 0;
#t=10
subject to stored10 {n in 1..N}:
(1 - discount[10]) * p/(1 + beta) - (1 - discount[11]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[10,n] - (ro/(1 + beta)) * mu3[11,n]
            + mu2[10,n] >= 0;
subject to s510 {n in 1..N}:
```

```
((1 - discount[10]) * p/(1 + beta) - (1 - discount[11]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[10,n] - (ro/(1 + beta)) * mu3[11,n]
            + mu2[10,n]) * xstored[10,n]= 0;
#t=11
subject to stored11 {n in 1..N}:
(1 - discount[11]) * p/(1 + beta) - (1 - discount[12]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[11,n] - (ro/(1 + beta)) * mu3[12,n]
            + mu2[11,n] >= 0;
subject to s511 {n in 1..N}:
((1 - discount[11]) * p/(1 + beta) - (1 - discount[12]) * p * ro/ (1 + beta)
        + (1/(1+ beta)) * mu3[11,n] - (ro/(1 + beta)) * mu3[12,n]
            + mu2[11,n]) * xstored[11,n]= 0;
subject to s2 {t in 1..T,n in 1..N}:
mu2[t,n]*(cap- xstored[t,n]) = 0;
subject to maxcap {t in 1..T,n in 1..N}:
  xstored[t,n] <= cap;
subject to nonnegtive_discharged1 {n in 1..N}:
(- xstored[1,n] + (1- tau) * xcharge[1,n])/(1+ beta) >= 0;
subject to nonnegtive_discharged1_S {n in 1..N}:
mu3[1,n] * ((- xstored[1,n] + (1- tau) * xcharge[1,n])/(1+ beta)) = 0;
subject to nonnegtive_discharged {t in 2..T,n in 1..N}:
(-xstored[t,n]+ ro * xstored[t-1,n] + (1 - tau) * xcharge[t,n])/(1+ beta) >= 0;
subject to nonnegtive_discharged_S {t in 2..T,n in 1..N}:
```

```
mu3[t,n] * ((-xstored[t,n]+ ro * xstored[t-1,n] + (1 - tau) * xcharge[t,n])/(1+ beta)) = 0;
```

Run's file (Load shifting with customers controlled storage) – Using Ampl:

```
model Storage_customer_controlled_mod.mod;
option omit_zero_rows 1;
option display_1col 0;
option display_eps .0000000001;
param cost1 {1..N};  # consumption cost
param cost2 {1..N};  # Inconvenience cost
param shift {1..N};
param storage {1..N};
table pdiscount "ODBC" "Storage_customer_controlled_data.xlsx" "Timeslots":
   [T], a IN, b IN, c IN, discount OUT;
table consumers "ODBC" "Storage_customer_controlled_data.xlsx" "Customers":
   [N], alpha IN, cost1 OUT, cost2 OUT, shift OUT, storage OUT;
table amount "ODBC" "Storage_customer_controlled_data.xlsx" "Consumptions":
  [T,N], y IN, x OUT,  xcharge OUT,  xstored OUT , mu3 OUT, mu2 OUT   ;
read table pdiscount;
read table consumers;
read table amount;
solve;
param h {i in 1..T,j in 1..N};   #Loop parameter
param q;                         #Loop parameter
param v {1..T};                  #Loop parameter
```

```
param k {1..N};  # T hat
param tbar {1..N};
subject to demand1 {t in 1..T,n in 1..N: h[t,n] > 0}:
 x[t,n] = y[t,n] + tbar[n]/ k[n] + (discount[t] - ((sum {i in 1..T} if h[i,n] > 0  then
discount[i] else 0)/ k[n])) * p/2/alpha[n] ;
subject to demand2 {t in 1..T,n in 1..N: h[t,n] <= 0}:
x[t,n] = 0;
problem subopt: discount, x, Total_Profit, demand2, demand1, xcharge, xstored, mu2 ,
mu3,
                charge, charge_s, stored12, s512, stored1, s51, stored222, s52, stored3, s53,
                  stored4, s54,  stored5, s55, stored6, s56, stored7, s57,
                     stored8, s58, stored9, s59, stored10, s510, stored11, s511,
                        s2, maxcap, nonnegtive_discharged1, nonnegtive_discharged1_S,
nonnegtive_discharged, nonnegtive_discharged_S ;
repeat optimal_loop {
let {i in 1..T,j in 1..N} h[i,j]:= x[i,j];
for {i in 1..T} { let v[i]:= 0;};
for {i in 1..T} {for {j in 1..N} {if h[i,j] >= 0 then {let v[i]:= v[i] + 1;   # step2
};};};
let q:= sum{i in 1..T} v[i];
if q = N * T then break optimal_loop;    # step2
for {i in 1..N} { let k[i]:= 0;};
for {i in 1..N} {for {j in 1..T} {if h[j,i] > 0 then {let k[i]:= k[i] + 1;
```

```
};};};
let {j in 1..N} tbar[j]:= sum {i in 1..T} if h[i,j] <= 0 then y[i,j] else 0;
solve subopt; } ;
let {n in 1..N} cost1[n]:= (1 - discount[1]) * p * ( x[1,n] + xcharge[1,n] - (- xstored[1,n] +
(1- tau) * xcharge[1,n])/(1+ beta)) + sum {t in 2..T} ((1 - discount[t]) * p * (x[t,n] +
xcharge[t,n]
- (-xstored[t,n]+ ro * xstored[t-1,n] + (1 - tau) * xcharge[t,n])/(1+ beta)));
let {j in 1..N} cost2[j]:= sum {i in 1..T} (x[i,j] - y[i,j])^2 * alpha[j] ;
let {j in 1..N} shift[j]:= sum {i in 1..T} if  (y[i,j]- x[i,j]) > 0 then (y[i,j]- x[i,j]) else 0  ;
let {j in 1..N} storage[j]:= sum {i in 1..T} xstored[i,j]  ;
let {t in 2..T, n in 1..N} xdischarge[t,n] := (- xstored[t,n] + (1 - tau) * xcharge[t,n] + ro *
xstored[t-1,n])/(1+beta);
let {n in 1..N} xdischarge[1,n] := (- xstored[1,n] + (1 - tau) * xcharge[1,n] )/(1+beta);
write table pdiscount;
write table amount;
write table consumers;
```

Table 17: “Consumptions” table from the data file for number of customers is two – Storage is controlled by customers

T	N	y	x	xcharge	xstored	mu2	mu3
1	1	1.71	10.01	4.44946	4.40496	0	0.00460
1	2	1.53	1.77	0.00000	0.00000	0	0.00460
2	1	2.38	9.15	0.00000	4.36091	0	0.00465
2	2	2.03	2.23	3.08187	3.05105	0	0.00465
3	1	3.34	8.56	0.00	4.32	0	0.00
3	2	3.01	3.17	0.801509	3.814031	0	0.004697
4	1	4.16	7.81	0	4.274132	0	0.004745
4	2	4.32	4.43	1.22E-08	3.77589	0	0.004745
5	1	4.51	6.58	0.276209	4.504837	0	0.004793
5	2	4.77	4.84	0.921573	4.650488	0	0.004793
6	1	5.67	6.14	0.40	4.86	0	0.00
6	2	5.59	5.61	0.01	4.61	0	0.00
7	1	10.66	6.26	0.00	0.00	0	0.00
7	2	10.54	10.42	0.00	0.00	0	0.00
8	1	8.58	2.52	0.00	0.00	0	0.00
8	2	8.96	8.79	0.00	0.00	0	0.00
9	1	8.10	2.02	0.00	0.00	0	0.00
9	2	8.43	8.26	0.00	0.00	0	0.00
10	1	5.17	0.00	0.00	0.00	0	0.00
10	2	5.27	5.09	0.00	0.00	0	0.00
11	1	2.36	0.00	0.00	0.00	0	0.00
11	2	2.56	2.45	0.00	0.00	0	0.00
12	1	2.40	0.00	0.00	0.00	0	0.00
12	2	2.10	2.04	0.00	0.00	0	0.00

Table 18: “Customers” table from the data file for two customers – Storage is controlled by customers

N	alpha	cost1	cost2	shift	storage
1	0.00075	13.920752	0.218258	26.480295	26.719231
2	0.02625	14.390253	0.006785	0.800339	19.900891

Table 19: “Timeslots” table from the data file– Storage is controlled by customers

T	a	b	c	discount
1	0.0035	0	0	0.088415300
2	0.0035	0	0	0.079207374
3	0.0035	0	0	0.069906438
4	0.0035	0	0	0.060511554
5	0.0035	0	0	0.051021771
6	0.0035	0	0	0.041436133
7	0.0035	0	0	0.012193138
8	0.0035	0	0	0.002215290
9	0.0035	0	0	0.002098202
10	0.0035	0	0	0.000000000
11	0.0035	0	0	0.013646749
12	0.0035	0	0	0.024202800

APPENDIX D

Model's file (Load shifting with provider controlled storage) – Using Ampl:

```
param T:= 12;            # timeslot
param N := 7;            # Number of Customers
param p:= 0.25;   # base price
param alpha {1..N};              # inconvenience factor
param y {1..T,1..N};   # The original amount of electricity demanded
param a {1..T};         # parameter of the generation cost function
param b {1..T};         # parameter of the generation cost function
param c {1..T};         # parameter of the generation cost function
param m {1..T,1..N};      # Minimum consumption that cannot be shifted to another
timeslot
param ro:= 0.99;         # storage efficiency
param tau:= 0.01;        # charging efficiency
param beta:= 0.01;      # discharging efficiency
param cap:= 12;                  # storage capacity
var discount {t in 1..T} >= 0, <= 1;       # price discounts
var x {1..T,1..N};                    # actual consumption
var xcharge {1..T,1..N} >=0;     #amount of charging
var xdischarge {1..T,1..N} >=0;  #amount of discharging
var xstored {1..T,1..N} >=0, <=cap;  # storage level
maximize Total_Profit:
sum {t in 1..T} ((1 - discount[t]) * p * sum {n in 1..N} x[t,n]
```

```
            - a[t] * (sum {n in 1..N} (x[t,n]-xdischarge[t,n]+xcharge[t,n]))^2
              - b[t] * (sum {n in 1..N} (x[t,n]-xdischarge[t,n]+xcharge[t,n])) - c[t]);
subject to demand {t in 1..T,n in 1..N}:
x[t,n] = y[t,n] + ((discount[t] - ((sum {k in 1..T} discount[k])/T)) * p/(2* alpha[n]));
## Storage balance constirants##
subject to xini_CB {i in 1..N}:
(1-tau)*xcharge[1,i]-(1+beta)*xdischarge[1,i]-xstored[1,i]=0;
subject to xC_Balance {t in 2..T, i in 1..N}:
ro*xstored[t-1,i]+(1-tau)*xcharge[t,i]-(1+beta)*xdischarge[t,i]-xstored[t,i]=0;
```

Run’s file (Load shifting with provider controlled storage) – Using Ampl

```
model Load_shifting_w_provider_controlled_storage.mod;
param cost1 {1..N};  # cosumption cost
param cost2 {1..N};  # Inconvenience cost
param shift {1..N};
param storage {1..N};
table pdiscount "ODBC" "Load_shifting_w_provider_controlled_storage.xlsx"
"Timeslots":
   [T], a IN, b IN, c IN,
  discount OUT;
table consumers "ODBC" "Load_shifting_w_provider_controlled_storage.xlsx"
"Customers":
   [N], alpha IN, cost1 OUT, cost2 OUT, shift OUT, storage OUT;
```

```
table amount "ODBC" "Load_shifting_w_provider_controlled_storage.xlsx"
"Consumptions":
  [T,N], y IN,
  x OUT, xcharge OUT, xdischarge OUT, xstored OUT  ;
read table pdiscount;
read table consumers;
read table amount;
option omit_zero_rows 1;
option display_1col 0;
option display_eps .000001;
solve;
param h {i in 1..T,j in 1..N};   #Loop parameter
param q;                         #Loop parameter
param v {1..T};                  #Loop parameter
param k {1..N};  # T hat
param tbar {1..N};
subject to demand1 {t in 1..T,n in 1..N: h[t,n] > 0}:
x[t,n] = y[t,n] + tbar[n]/ k[n] + (discount[t] - ((sum {i in 1..T} if h[i,n] > 0  then
discount[i] else 0)/ k[n])) * p/2/alpha[n] ;
subject to demand2 {t in 1..T,n in 1..N: h[t,n] <= 0}:
x[t,n] = 0;
problem subopt: discount, x, Total_Profit, demand2, demand1, xini_CB, xC_Balance,
xcharge, xdischarge, xstored  ;
```

```
repeat optimal_loop {
let {i in 1..T,j in 1..N} h[i,j]:= x[i,j];
for {i in 1..T} { let v[i]:= 0;};
for {i in 1..T} {for {j in 1..N} {if h[i,j] >= 0 then {let v[i]:= v[i] + 1;   # step2
};};};
let q:= sum{i in 1..T} v[i];
if q = N * T then break optimal_loop;    # step2
for {i in 1..N} { let k[i]:= 0;};
for {i in 1..N} {for {j in 1..T} {if h[j,i] > 0 then {let k[i]:= k[i] + 1;
};};};
let {j in 1..N} tbar[j]:= sum {i in 1..T} if h[i,j] <= 0 then y[i,j] else 0;
solve subopt;
} ;
let {j in 1..N} cost1[j]:= sum {i in 1..T} x[i,j] * (1- discount[i]) * p;
let {j in 1..N} cost2[j]:= sum {i in 1..T} (x[i,j] - y[i,j])^2 * alpha[j] ;
let {j in 1..N} shift[j]:= sum {i in 1..T} if  (y[i,j]- x[i,j]) > 0 then (y[i,j]- x[i,j]) else 0  ;
let {j in 1..N} storage[j]:= sum {i in 1..T} xstored[i,j]  ;
write table pdiscount;
write table amount;
write table consumers;
```

Table 20: “Consumptions” table from the data file when number of customers is two – Storage is controlled by provider

T	N	y	x	xcharge	xdischarge	xstored
1	1	1.71	6.95	5.05	0.00	5.00
1	2	1.53	1.68	2.87	0.00	2.84
2	1	2.38	5.79	0.05	0.00	5.00
2	2	2.03	2.13	2.21	0.00	5.00
3	1	3.34	4.71	0.05	0.00	5.00
3	2	3.01	3.05	0.05	0.00	5.00
4	1	4.16	3.41	0.05	0.00	5.00
4	2	4.32	4.30	0.05	0.00	5.00
5	1	4.51	2.20	0.05	0.00	5.00
5	2	4.77	4.70	0.05	0.00	5.00
6	1	5.67	1.57	0.05	0.00	5.00
6	2	5.59	5.47	0.05	0.00	5.00
7	1	10.66	6.56	0.00	4.90	0.00
7	2	10.54	10.42	0.00	4.90	0.00
8	1	8.58	4.48	0.00	0.00	0.00
8	2	8.96	8.84	0.00	0.00	0.00
9	1	8.10	4.00	0.00	0.00	0.00
9	2	8.43	8.31	0.00	0.00	0.00
10	1	5.17	3.23	0.00	0.00	0.00
10	2	5.27	5.21	0.00	0.00	0.00
11	1	2.36	7.59	0.00	0.00	0.00
11	2	2.56	2.71	0.00	0.00	0.00
12	1	2.40	8.54	0.00	0.00	0.00
12	2	2.10	2.28	0.00	0.00	0.00

Table 21: “Customers” table from the data file when number of customers is two – Storage is controlled by provider

N	alpha	cost1	cost2	shift	storage
1	0.00075	14.28786	0.13716	21.39331	30.00000
2	0.02625	14.58101	0.00392	0.61124	27.84238

Table 22: “Timeslots” table from the data file when number of customers is two– Storage is controlled by provider

T	a	b	c	discount
1	0.0035	0	0	0.056020394
2	0.0035	0	0	0.045059047
3	0.0035	0	0	0.032839592
4	0.0035	0	0	0.020115395
5	0.0035	0	0	0.010725773
6	0.0035	0	0	0.000000000
7	0.0035	0	0	0.000000000
8	0.0035	0	0	0.000000000
9	0.0035	0	0	0.000000000
10	0.0035	0	0	0.012948395
11	0.0035	0	0	0.055947938
12	0.0035	0	0	0.061456777