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Decomposition Methods for In-Transit Freight Consolidation Problems

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UNIVERSITY OF MIAMI

DECOMPOSITION METHODS FOR IN-TRANSIT FREIGHT CONSOLIDATION PROBLEMS

By

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In-transit merging operations involve efficient scheduling of the delivery of shipments from a list of origin points to one or more destinations. It focuses on optimizing the way that individual loads are aggregated so that total transportation and handling costs are minimized, while ensuring that each of the delivery-time requirements are met. The success of in-transit merging operations is critical for competitive advantage for third party logistics (3PL) companies who compete in transportation and handling costs. An effective consolidation strategy helps carriers offer better prices for their customers without hampering their profits. One important factor in transshipment planning is whether the shipment orders can be broken into pieces that are then scheduled separately in terms of both routing and timing. In such cases, the shipments are referred to as "divisible" shipments and typically allow for more consolidation opportunities. In other cases, the shipments are "nondivisible," where the carrier is required to transport a shipment order as one parcel throughout the network.

In this study, we consider both cases, and propose models and efficient solution methods for the in-transit freight consolidation problems, which are typically quite difficult to solve optimally, due to computational complexity and size. Motivated by our collaborations with a major global 3PL company, we tackle three versions of the general problem that differ from each other in cost structure, in addition to the shipments' divisibility. In all cases, we consider a three-echelon network that involves suppliers, consolidation points (referred to as terminals or gateways), and the customer. For each version of the problem we develop a mixed-integer programming (MIP) formulation that involves transshipments of multiple products using multiple transportation modes over a planning horizon. The first version tackles the problem with divisible shipments, where we propose a redesigned Benders decomposition approach that significantly speeds up the computational performance. In the second part, we modify our model for nondivisible shipments. With the understanding that Benders decomposition does not provide the same effect for the nondivisible case, we develop a novel decomposition based on LP relaxations and valid cuts. In the third part, we introduce cost breaks for shipment amounts, which result in piecewise linear objective function. We show that our decomposition method for this case always leads to optimality. We demonstrate the computational competence of our solution methods using real-life case studies. We also conduct sensitivity analysis to investigate the impact of problem parameters on computational performance.

Keywords: in-transit consolidation, integer programming, 3PL, transshipment with time windows, divisible shipment, nondivisible shipment, decomposition, piecewise cost function

Dedication

This thesis is dedicated to my parents, my wife, my brothers and sister, and my kids for their endless love, support, and encouragement.

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I praise God the Almighty, merciful and passionate, for providing me this opportunity and granting me the capability to proceed successfully. I cannot express enough thanks to God, without whom nothing is possible.

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There is a wisdom behind everything! In every hardship, Joseph Kennedy said, “When life gets tough, the tough get going.” The journey of this dissertation was tough, and I have my own challenges by being married to a student and raising three kids while doing my Ph.D. I was not alone in that; by my side, there were caring hearts and encouraging souls. I feel so blessed to have known each one of them. I share some great memories with them and I wish to give my warmest thanks to the following:

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Chapter 1: Introduction

One of the biggest challenges in operations management is to minimize the logistics cost while meeting adequate customer satisfaction levels (Christopher, 2016). 3PL logistics providers strive to meet customer deadlines while reducing total logistics cost, including costs such as inventory holding, transportation, and insurance (Rushton, Croucher, & Baker, 2014). Efficient models and solution methodologies in terms of providing high-quality solutions and computation times can significantly contribute to the profits and market share of the logistics providers. Motivated by our collaboration with a major global 3PL logistics provider, our goal in this study is to develop and propose such models and solution methods that can be employed to reduce operational costs while maintaining excellent levels of customer satisfaction. Specifically, we investigate cost efficiency by means of effective in-transit consolidation in transshipments between supply and demand locations.

In-transit consolidation concerns transportation and delivery of goods where goods from different sources are sent to a distribution center, where they are consolidated into containers and shipped to the customer. The general network structure is depicted in Figure 1. In a typical network, shipments from individual suppliers, who are oftentimes spread across locations, are first transported to consolidation points, often referred to as "terminals" or "gateways." From there, they are forwarded to the customer's location. Our approach focuses on leveraging the use of full container load (FCL) shipments at consolidation points as they incur less-per-unit costs compared to the less-than-container load (LCL) shipment options. The FCL enables economies of scale, and is useful when sufficient volumes of shipments can be aggregated. The FCL usage can be maximized with careful planning that involves delaying downstream shipments at consolidation points without violating the prescribed delivery due dates.

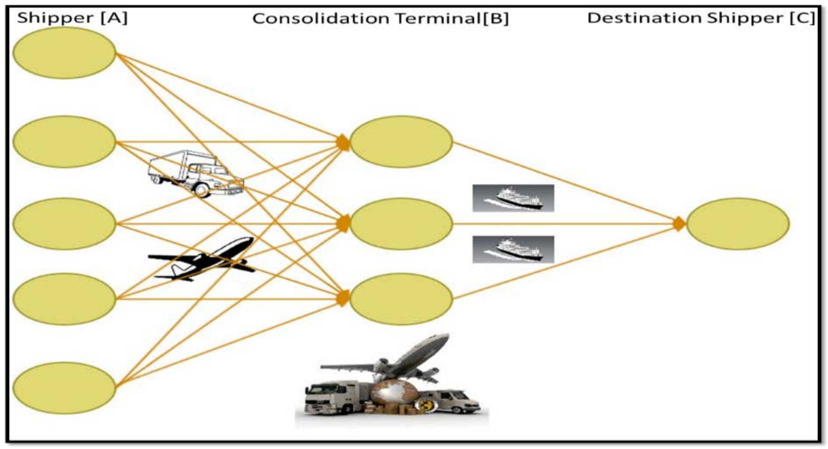

Figure 1. Merge-in-transit network with transportation modes

We propose novel mixed-integer linear programming (MIP) models and solution methods for three groups of in-transit merging optimization problems that we have observed in practice. The problems differ from each other mainly in two aspects: the divisibility of shipments, and shipment cost structures. “Divisibility” refers to whether the shipments from a supplier picked up on a prescribed data can be broken into pieces that are then scheduled separately in terms of consolidation-point allocations and delay times therein. In the case this is feasible, the shipments are known to be “divisible,” which allows for more consolidation opportunities. In other cases, the shipments are “nondivisible,” where the carrier is required to transport a shipment order as one parcel throughout the network.

In the literature, the shipment costs are typically assumed to be strictly linear in shipment amounts. Although such cases exist, the more common practice involves cost breaks. The freight shipment typically involves minimum charges up to a threshold quantity. From that point on, quantity discounting applies. Typically, there are multiple break points for shipment quantities where the unit shipment cost decreases after each break point. This leads to piecewise linear cost functions.

In all cases, we consider a three-echelon network that involves suppliers, consolidation points, and the customer, as depicted in Figure 1, and attempts to minimize total transshipment costs across multiple products and multiple transportation modes over a finite planning horizon.

The first version of the in-transit freight consolidation model considers divisible shipments with strictly linear costs. The model has integrality requirements for the number of FCL shipments across all periods, which renders the solution time-consuming for realistic size problems. In fact, for real-life problems that we have encountered, exact solution solvers such as CPLEX were not able to solve the problem at all. To tackle the challenge on computational times, we introduce a redesigned Benders decomposition approach that significantly speeds up the computational performance for this case in Chapter 2.

In Chapter 3, we modify our model to capture the case with nondivisible shipments, where every shipment collected from a supplier in a period has to be transported as a single parcel. To model this case we need to employ binary variables. Unfortunately, the inclusion of new binary variables impedes the execution of the Benders decomposition proposed for the divisible case. With this understanding, we develop and propose a novel decomposition method based on LP relaxations and valid cuts in this chapter. Although optimality is not guaranteed, our numerical analysis reveals that the method can generate upper bounds that are quite tight.

In Chapter 4, we go back to the divisible case, but this time with a more realistic and common setting where the shipment costs are piecewise linear in quantities due to cost breaks. In this case, a set of binary variables is introduced to capture the piecewise structure of the objective function, which, similar to the nondivisible case, renders the Benders decomposition useless. For this case, we slightly modify our decomposition approach developed in Chapter 4 and apply it to the problem in question. In this case, we are able to show that our decomposition method always leads to optimality. We demonstrate the computational competence of our solution methods using real-life case studies for all scenarios. We also conduct sensitivity analysis to investigate the impact of problem parameters on computational performance.

In-transit freight consolidation has been used by transportation and logistics companies for multiple years, achieving significant reduction in the cost of transporting goods. Furthermore, there are multiple sources of literature devoted to the study of the in-transit merging problem; however, to our knowledge, none of the reported work captures all aspects of the problem that we consider in this study together. To the best of our knowledge our work is a novel approach, and the in-transit consolidation problem has not been approached highlighting all aspects together in a model before. In what follows, we relate our work to the existing literature.

1.2 Literature Review

The literature on in-transit freight consolidation problem is vast and includes various nuances that change the problem complexity as well as the research focus. A thorough survey of this literature is presented by Guastaroba, Speranza, and Vigo (2016), who focuses on the use of intermediate facilities in freight transportation planning and their application on three different settings: vehicle routing problems; transshipment problems; and service network design problems.

Another review is presented by Aguezzoul (2014), who focuses on the selection of 3PL providers and identify 11 key criteria in their analysis. They conclude that cost is the prominent selection criteria, and that it is followed by the factors of relationship, services, and quality. This review covers articles published within 1994–2013, and finds that the 3PL selection is empirical in nature and varies based on region, industry, and the type of services that are outsourced.

A good introduction to the efforts to address the freight consolidation problem is presented in the works of Popken (1994), Cole and Parthasarathy (1998), and Croxton, Gendron, and Magnanti (2003). These works all include approaches that highlight the benefits provided by the consolidation of cargo for its delivery. Popken involves consolidation, and the way in which it impacts the logistics costs. The author proposes a composite algorithm for optima and heuristic search to provide local improvement, which shows significant saving opportunities compared to single-attribute techniques. A model based on binary decisions for the construction of a logistics network is presented by Cole and Parthasarathy; the model focuses on consolidating so that deliveries may be done using FCL freight. Furthermore, the authors propose the integration of GIS and MS Excel for the development of an application capable of providing solutions for small-freight consolidation problems. Integer programming formulations and solution methods to address some of the operational issues that arise in freight consolidation distribution systems have been developed by Croxton et al. (2003). The proposed models account for features including the integration of inventory and transportation decisions, the dynamic and multimodal components of the application, and the nonconvex piecewise linear structure of the cost functions. A practical application of these methods is presented using a case study based on the computer industry.

Croxton et al. (2003) developed one of the more comprehensive mathematical models for the in-transit freight consolidation problem. Their MIP formulation addresses some of the operational issues arising in merge-in-transit distribution systems. The model formulation accounts for various complex yet necessary features of an in-transit freight consolidation problem, and includes the integration of inventory and transportation decisions, the dynamic and multimodal components of the application, and the nonconvex piecewise linear structure of the cost functions. In the aforementioned papers, it is established that the in-transit freight consolidation problem is NP-complete; and hence, researchers have been focusing on developing heuristic approaches to scale-up, as well as speed up, the problem.

Further, the use of a 3PL provider is introduced into the freight consolidation problem by the works by Tyan, Wang, & Du (2003) and Song, Hsu, & Cheung (2008), including the effects of having pickup and delivery schedules. A model to compute the total cost of freight consolidation where weekly shipment forecasts from manufacturers are used by the 3PL provider for daily morning and afternoon pickups is developed by Tyan et al. (2003. The proposed model was able to lower the logistics cost by 20%. A setting where shipments are coordinated between multiple suppliers and customers through the use of a consolidation center is presented by Song et al. (2008). This model incorporates the pickup and delivery times, transportation options, and inventory holding costs. They show that the proposed solutions are on average within 3.24% of the global optimal.

The use of multi-shipping units is introduced in the approaches from Lim, Miao, Rodrigues, & Xu (2005) and Jin and Muriel (2009), and the effect of having multiple sources for the cargo that is to be consolidated and delivered is studied. Polynomial-time algorithms for transshipment through cross docks while considering time windows, warehouse capacity, and transportation schedules are developed by Lim et al. (2005). The model's objectives include meeting the demand on time, and to minimize the delays to shipments at the cross docks. Jin and Muriel propose Lagrangian decomposition methods for single-warehouse, multi-retailer systems with FTL shipments, to establish some structural properties of optimal solutions while satisfying demand. The authors propose three algorithms: one for the single-stage dynamic lot sizing problem; one for a single-retailer-single-warehouse system; and a third to find the shortest routes from a warehouse to multiple retailers.

Other variants of the freight consolidation problem are studied by Moccia, Cordeau, Laporte, Ropke, & Valentini (2011), Miao, Fu, and Yang (2012), and Musa, Arnaout, & Jung (2010), among others. These variants highlight the importance of the use of time windows, the use of inventory at the transshipment gateways, networks with one source and one destination (one-to-one), and networks with multiple sources and multiple destinations (many-to-many). Shipment consolidation options in multimodal network with time windows, timetables, and flexible-time transportation are considered by Moccia et al. (2011). Here, they propose an algorithm based on decomposition for feasible path bounds. Furthermore, the authors develop column generation algorithms to find feasible solutions within the previously described bounds.

A single-shipment-single-delivery variant of the problem that includes hard time windows and preferred service time intervals is studied by Miao et al. (2012). The study focuses on the penalties incurred when the constraints are violated. The authors use tabu search and genetic algorithms to solve their integer linear-programming formulation. Musa et al. (2010) proposed an ant colony optimization algorithm to minimize the total transportation cost of a variant of the problem with one source and one destination.

A case study of a 3PL operating a consolidation warehouse in China of products that must be distributed across the United States is presented by Qin (2013). The problem is modeled as an integer programming model, and is solved with a proposed memetic algorithm. The effectiveness of the model is argued by its actual implementation by the 3PL in the study.

A method where items are aggregated heuristically, and the aggregated problem is solved with MIP methods, is developed by Melo (2015). The proposed methods are shown to be effective for problems with small items, and to reach the best solutions in 88.9% of the cases with large items. Chen (2015) proposes an analytical model that focuses on long-run profit, to enhance managerial insight regarding the consolidation of perishable goods. Further, the authors provide a scenario where a supplier has multiple retailers who are sensitive to price, delivery-time, and the product quality.

The potential of receiver-led consolidation programs is evaluated by Holguín-Veras (2015). The authors provide a case study for the city of New York. Here they focus on the reduction in vehicle-miles-traveled and the impacts that the reduction in the number of deliveries have both in the city’s traffic and on the carrier’s delivery costs.

Bookbinder (2014) evaluated the air cargo consolidation problem in which unit load devices are charged at under-pivot rate, and over-pivot rate. This scheme is optimized using a local branching heuristic with relaxation-induced neighborhood search. It is shown to achieve solutions within 3.4% of the optimal for problems with up to 400 shipments and 80 containers.

A model for the design of a hazardous material (HAZMAT) transportation network is presented by Mohammadi (2016). This model is a MILP model that includes an integration of chance-constrained programing with a possibilistic programming that is aimed at minimizing the risk of an incident. A short-term ship fleet-planning problem, taking into account container transshipment and uncertain shipment demand, is assessed by Meng (2012). The authors formulate the problem as a two-stage stochastic integer programming model and provide a solution algorithm that integrates dual decomposition with Lagrangian relaxation.

All of these approaches address several different important aspects of the freight consolidation problem; however, we have been unable to find a model in the literature where the use of divisible and nondivisible shipments and specific time windows has been leveraged to improve the computational efficiency of the scheduling algorithm, in order to reach the optimal solution. Therefore, to the best of our knowledge, we propose novel models to address the freight consolidation problem that includes a multiple supplier-single customer setting, with a constrained delivery time window and nondivisible shipments.

1.3 Modeling Overview

In order to study the in-transit freight consolidation problem, we start with a model formulation in which the cargo is divisible into multiple shipments, and employ a redesigned version of the Benders decomposition technique to solve the problem. Using this model, we are able to reach the optimal solution with reasonable computation times. The specifics of this model and the methodology employed are detailed in Chapter 2.

We expanded our study to include the in-transit consolidation problems in which the cargo is nondivisible in nature, and must be shipped and delivered together. We develop a solution technique that employs decomposition and valid cuts. Although we have no proof optimality for this solution procedure, we employ numerical analysis to demonstrate that the method can generate remarkably tight upper bounds. Chapter 3 presents the details of this specific setting and our solution technique as well as a numerical analysis of structure on a real-life based case study.

Finally, we evaluate a scenario in which there are different breaks in the transportation cost function. Chapter 4 discusses the details of the in-transit merging problem under this setting and presents an extensive computational study evaluating the effects of changing the input data, such as perturbing the time windows and the container breakeven points. The proposed models we have developed are generic enough so that we can apply them to most of the in-transit merging optimization scenarios, after little or no modification.

Overall, all of our proposed models share some very similar constraints. These shared constraints include the following:

- Demand satisfaction: This group of constraints ensures that all of the freight expected by a customer is delivered within a prescribed time window. All of our proposed problem formulations employ these time windows as such. In all of the models these constraints ensure that all of the freight is delivered within the time window;

- Container capacity: These constraints ensure that the total weight of all of the freight shipped using a particular container doesn't exceed the container's capacity;
- Mass balance at consolidation terminals: This group of constraints ensures that there is a balance in flow of products at each of the consolidation terminals. These constraints guarantee that all of the packages entering a consolidation terminal are accounted for and that they all eventually leave that same consolidation terminal; and
- Customer balance: This constraint ensures that all of the shipment amounts a customer receives are equal to that customer's demand, and that these are all eventually delivered. These constraints further ensure that there is no inventory of shipped goods at the consolidation terminals at the end of the considered planning horizon.

Chapter 2: A Benders Decomposition Approach for In-Transit Consolidation with Divisible Freight

2.1 Overview

The growth in online shopping and third party logistics (3PL) has seen a revival of interest in finding optimal solutions to the large-scale in-transit freight consolidation problem (Xiaomin, 2017). The 3PL providers try and consolidate the shipments going to a customer, and use economies of scale and reduction in package count to provide important cost savings. The consolidation problem requires determining what products to consolidate into one shipment at an intermediate gateway or terminal versus what to ship individually to a customer such that the shipment costs are minimized while the delivery time windows are honored. This study and the solutions herein are motivated by our involvement with a major 3PL provider. We study the problem of in-transit container consolidation of products being shipped from n shippers to a single business customer via m consolidation points (called gateways henceforth) within a predefined time window.

Typically a business or corporate customer that employs 3PL has standing orders from multiple suppliers for multiple products across a planning horizon, where each product has a prespecified shipment date and delivery time window. The 3PL providers pick up the products from the suppliers on given shipment dates and deliver to the customer within the delivery time windows. All products are first shipped to intermediate gateways before being forwarded to the customer. A 3PL company usually has more than one gateway that provides flexibility pertaining to shipment costs and consolidation options. The routing decisions therefore need to be made for two legs: from suppliers to gateways, and from gateways to the final customer. The first leg decision involves assigning the shipment to a particular gateway and selection of the transportation mode. The consolidation-related decisions are made at the gateway. In the second leg, the carrier ships the products either as consolidated shipments or as is, so as to minimize shipment costs without violating the constraints set by the delivery time windows. When a shipment is not consolidated into a container, it is forwarded to the customer as individual shipment, which is typically more expensive.

We formulate the in-transit freight consolidation problem described above as a mixed-integer programming (MIP) problem, and our key contribution is the development of a Benders decomposition-based solution approach that provides a significant scale-up in the performance of the solver. The decomposition replaces a large number of integer "freight-consolidation" variables by a small number of continuous variables that reduce the size of the problem in terms of both the number of variables and constraint without impacting the optimality. Using our approach, we can solve to optimality a large-scale case with more than 10,046 million variables and 231 million constraints that would be otherwise unsolvable using CPLEX on a 64 GB RAM server.

The literature on in-transit freight consolidation problem is vast and includes various nuances that change the problem complexity as well as researchers focus. A thorough survey of this literature is presented by Guastaroba et al. (2016) who focused on the use of intermediate facilities in freight transportation planning and their application on three different settings: vehicle routing problems; transshipment problems; and service network design problems. Croxton et al. (2001) developed one of the more comprehensive mathematical models for the in-transit freight consolidation problem. Their MIP formulation addresses some of the operational issues arising in merge-in-transit distribution systems. The model formulation accounts for various complex yet necessary features of an in-transit freight consolidation problem, and includes the integration of inventory and transportation decisions, the dynamic and multimodal components of the application, and the nonconvex piecewise linear structure of the cost functions. These two papers together give a good insight into the general setting of the problem as well as the specifics about the modeling and operationalization that the reader may look up for details.

Both papers establish that the in-transit freight consolidation problem is NP complete, and researchers have been focusing on developing heuristic approaches to scale-up as well as speedup the problem. Researchers have relied on (a) dual-based solution methods (Song et al., 2008), (b) column generation algorithms (Dondo & Mendez 2014; Moccia et al., 2011), (c) cutting-plane procedures and branch-and-bound heuristics (Croxton et al., 2003), (d) heuristic search (Golias, Saharidis, Boile, & Theofanis, 2012; Popken, 1994), (e) simulations (Qian & Xu, 2012), and (f) decomposition based heuristics (Jin & Muriel, 2009) to achieve the dual objective of scale-up and speedup without compromising the quality of the solution. To the best of our knowledge, none of the papers in the domain have looked at Benders decomposition-based approach to solve large scale in-transit freight consolidation problem. That apart, Fischetti, Ljubić, & Sinnl (2016) recently proposed a redesigned Benders decomposition for solving large scale MIP that uses a projected decision space for a "thinned out" version of the classic decision problem and show that the method enables significant scale-up and speedup without impacting the optimality of the solution. The decomposition takes advantage of the new hardware and software technologies such as multi-core processors.

Our model builds on the model proposed in Croxton et al. (2003) along with the redesigned Benders decomposition approach proposed in Fischetti et al. (2016). We include two linear cost structures that correspond to shipment from the shipper to the consolidation point, and from the consolidation point to the customer, respectively, and a time constraint on each shipment in addition to the constraints accounted for by Croxton et al. (2003). Our setting is relevant to 3PL providers who need to solve the large-scale in-transit freight consolidation problem on a frequent basis.

The remaining of the chapter is organized into five sections. We present a brief literature review on the freight consolidation problem in Section 2. There are several useful research outputs reported in the literature on both the in-transit freight consolidation problem as well as Benders decomposition, and we point the reader to the relevant reviews. The proposed MIP model is presented in Section 3 and the Benders decomposition-based reformulation of our model is presented in Section 4. Finally, a detailed case study elucidating the efficacy of the decomposition approach for solving large-scale in-transit freight consolidation problems is presented in Section 5 with the conclusions and potential extensions discussed in Section 6.

2.2 Model Formulation

In this section, we introduce the MIP model for the studied problem. The model tackles the case of in-transit consolidation of products being shipped from n geographically-spread shippers to a single final destination through m gateways. Under a multiperiod setting, products must be picked from the suppliers and routed to the destination within a given time window. Our model assumes that the freight from a supplier to the customer is divisible into different loads, which can be transshipped via different gateways; and the 3PL provider may choose this option for consolidation opportunities. The customer has predetermined pickup dates and due date windows for each product from suppliers that the 3PL provider is aware of, and delivery deadlines are imposed as hard constraints.

The problem involves two stages. In the first stage, products are shipped from suppliers (shippers) located in different locations to one of several gateways, such as ports. There are alternatives for the mode of transportation (usually land or air). For each transportation mode, the cost of shipment is linear-increasing in the amount of shipment. The transportation cost typically depends on the distance between the supplier and the gateway.

At gateways, the products are forwarded to the customers either as less-than-container-load (LCL) shipments or full-container-load (FCL) shipments. For the LCL option, the shipment cost is linear-increasing in the shipment weight. If sufficient volume of products can be consolidated into a container without violating the delivery time windows of the products, a more economic option of FCL shipment can be exercised. The goal of the models is to identify the optimal shipment routes and schedules over a planning horizon that minimizes the total transportation costs.

As mentioned earlier, it is assumed that the shipments from the suppliers can be broken into pieces and routed to separate gateways on their way to the end delivery point, i.e., the customer. The carrier may choose the option of dividing the products picked up from a supplier into subsets, if doing so provides opportunities for FCL consolidation at the gateways. A shipment can be stalled at a gateway before it is moved to the second stage so that it can be coupled with other shipments and consolidated into a container. However, as mentioned earlier, products cannot be delayed beyond a certain point in time, which results in late delivery. If they cannot be consolidated into a container in a timely fashion, they must be forwarded as LCL shipments so as to make their respective delivery deadlines. Keeping products at the gateway incurs holding costs for the carrier, which is typically low in comparison to savings obtained from consolidation. FCL consolidation necessitates the introduction of integers variables that represent the number of containers used at each gateway in each period.

The model attempts to minimize the total cost over a set of multiple periods, D (typically days in this context). It incorporates a set of shippers, S, a set of products, P, and a set of gateways, H. We denote $c1l_{s,h}$ and $c1a_{s,h}$ as the unit cost of shipment from supplier s to consolidation gateway h by land and by air, respectively. Likewise, $c2_h$ is the cost of sending one pound from gateway h to the final customer; $c3_h$, is the cost of sending one container from gateway h to the customer; and ci_h is the inventory cost per pound realized by keeping the shipment at consolidation gateway h for one time period. A time period in this context is typically a day. As such, in the rest of the chapter we employ "day" as our time unit.

Decision variables for the model are as follows: $X_{p,s,h,d}$ is the weight in pounds of product p sent from shipper s to gateway h on day d by land; $Y_{p,s,h,d}$is the weight in pounds of the items of product p sent from shipper s to gateway h on day d by air; $Z_{p,h,d}$is the weight in pounds of the items of product p sent from gateway h to the final customer on day d as a LCL shipment; $U_{p,h,d}$ is the weight in pounds of the items of product p sent from gateway h to the final customer on day d as a FCL shipment; $T_{h,d}$ is the number of containers shipped from gateway h on day d; $N_{p,d}$ is the total inventory in pounds of product p delivered to the final customer on day d; and $I_{p,h,d}$is the inventory of product p in pounds at consolidation gateway h on day d.

As for other parameters, $d_{p,s,h}$ is the weight of the items in pounds that must be picked up from shipper s on day d; k is the maximum capacity in pounds per container; $t1l_{s,h}$ is the number of days that a shipment takes by land from shipper s to gateway h; $t1a_{s,h}$ is the number of days that a package takes by air from shipper s to gateway h; $t2_{s,h}$ is the number of days it takes a package to ship from gateway h to the final customer; t_w is the length of the time window;, and finally *D* is the total number of periods (days) in the planning horizon.

The MIP model is given below:

$$\min \sum_{\forall p,s,h,d} \left(c1l_{s,h} X_{p,s,h,d} + c1a_{s,h} Y_{p,s,h,d} + c2_h Z_{p,h,d} + c3_h T_{h,d} + ci_h I_{p,h,d} \right) \qquad (1)$$

St:

$$\sum_{\forall h} X_{p,s,h,d} + \sum_{\forall h} Y_{p,s,h,d} = d_{p,s,d} \qquad \forall p,s,d \qquad (2)$$

$$\sum_{p} U_{p,h,d} \leq kT_{h,d} \qquad \forall p,h,d \qquad (3)$$

$$U_{p,h,d} + Z_{p,h,d} + I_{p,h,d+1} = \sum_{\forall s} X_{p,s,h,d-t1l_{sh}} + \sum_{\forall s} Y_{p,s,h,d-t1a_{s,h}} + I_{p,h,d} \qquad \forall p,h,d \qquad (4)$$

$$\sum_{\forall h} U_{p,h,d-t2_{s,h}} + \sum_{\forall h} Z_{p,h,d-t2_{s,h}} + N_{p,d} = \sum_{\forall s} d_{p,s,d-t_w} + N_{p,d+1} \qquad \forall p,d\colon d \geq t_w \qquad (5)$$

$$\sum_{\forall h: d \geq t2_{s,h}} U_{p,h,d-t2_{s,h}} + \sum_{\forall h: d \geq t2_{s,h}} Z_{p,h,d-t2_{s,h}} + N_{p,d} = N_{p,d+1} \qquad \forall p,d\colon d < t_w \qquad (6)$$

$$T_{h,d} \in \mathbb{N} \qquad \forall h,d \qquad (7)$$

$$X_{p,s,h,d}, Y_{p,s,h,d}, Z_{p,h,d}, U_{p,h,d}, I_{p,h,d}, N_{p,d} \geq 0 \qquad \forall p,s,h,d \qquad (8)$$

The objective of the model is shown in Equation (Eq.) (1), where we want to minimize the total shipping cost, which is composed of the fixed and variable costs of land freight and air freight from shippers to gateways, the cost of freight from gateways to the final customer broken into LCL and FCL shipments, and the cost of inventory held at the gateways. Equation (2) ensures that scheduled pickups are carried out and shipped to gateways from a given supplier on a given day. Equation (3) ensures that the amount of products shipped from gateway to the customer via containers does not exceed the capacity of the containers. Equation (4) enforces that inbound shipments, shipments on-hold, and outbound shipments are balanced at the gateways for a product type on a given day. Equations (5) and (6) keep track of the flow balance at the customer site and ensure that the products are delivered to the customer by their due dates. Equations (7) and (8) are the integrality and nonnegativity constraints, respectively.

The incorporation of the limits on the time windows enforces the feasibility constraints, guaranteeing that the maximum time span that a shipment may take from pick-up at any given supplier location to delivery at the final customer does not exceed certain time duration, t_w. As such, typically the consolidation of all products at gateways may not be possible across the time horizon. Moreover, the inclusion of the holding costs at the gateways may deter the storage of shipments until full truckload containers are completely loaded for shipment. The optimal solution is typically a mix of individual LCL and FCL shipments. If the time windows are sufficiently large, air option is usually not utilized, except for consolidation purposes at the gateways since they are usually much more expensive.

2.3 Benders Decomposition

Benders decomposition is a method that is usually used for large mixed binary and integer optimization problems, where the problem is divided into smaller subproblems, which enable the global solution of the problem to be achieved. Fischetti et al. (2016) recently proposed a redesigned Benders decomposition for solving large-scale MIP that uses a projected decision space for a “thinned out” version of the classic decision problem, and shows that the method enables significant scale-up and speedup without impacting the optimality of the solution. The decomposition takes advantage of the new hardware and software technologies, such as multi-core processors.

The model presented in Eqs. (1)–(8) grows in size as p, s, h, d increase and most of the real-life 3PL in-transit consolidation problems cannot be solved to optimality on account of constraints on computational resources. We therefore tailor the Benders decomposition method with the “thin-out” approach presented by Fischetti et al. (2016) to solve large-scale in-transit freight consolidation problems. Our methodology involves two major parts.

In the first part, we obtain the linear programming (LP) relaxation of the original model by relaxing the integrality constraints given in Eq. (7). The solution of the LP relaxation model provides us with a lower bound for the problem, which we can use as a starting point for the overall implementation. Next, we use the principles of Benders decomposition to find an upper bound solution for our problem. We adapt its solution philosophy to our problem under the following considerations:

a. Let us organize our objective function in the following two parts: $FO = f(X) + g(Z) + h(T)$, where $f(X)$ is the cost for the first leg of the transshipments, that is cost of shipping products from suppliers to gateways. Here, X is the array of all decision variables of the first leg. Let $g(Z)$ involve the costs related to the LCL shipments from gateways to the customer and holding inventory at the gateways. Finally, $h(T)$ is the part of the objective function that captures the FCL shipment costs from gateways to the customer.

b. Under Benders' strategy, when we fix *T*, our integer variable, the problem left to solve is of LP class. Under this view, we can rewrite our problem:

$$Min\; q(T) + h(T) \tag{9}$$

St:

$$T \in \mathbb{N} \tag{10}$$

Here, $q(T)$ is the solution to the following problem:

$$Min\; f(X) + g(Z) \tag{11}$$

St:

$$\sum_{p} U_{p,h,d} \le k\bar{T}_{h,d} \qquad \forall p, h, d \tag{12}$$

$$\sum_{\forall h} X_{p,s,h,d} + \sum_{\forall h} Y_{p,s,h,d} = d_{p,s,d} \qquad \forall p, s, d \tag{13}$$

$$U_{p,h,d} + Z_{p,h,d} + I_{p,h,d+1} = \sum_{\forall s} X_{p,s,h,d-t1l_{sh}} + \sum_{\forall s} Y_{p,s,h,d-t1a_{s,h}} + I_{p,h,d} \qquad \forall p, h, d \tag{14}$$

$$\textstyle\sum_{\forall h} U_{p,h,d-t2_{s,h}} + \sum_{\forall h} Z_{p,h,d-t2_{s,h}} + N_{p,d} = \sum_{\forall s} d_{p,s,d-t_w} + N_{p,d+1} \qquad \forall p, d: d \ge t_w \tag{15}$$

$$\sum_{\forall h: d \ge t2_{s,h}} U_{p,h,d-t2_{s,h}} + \sum_{\forall h: d \ge t2_{s,h}} Z_{p,h,d-t2_{s,h}} + N_{p,d} = N_{p,d+1} \qquad \forall p, d: d < t_w \tag{16}$$

$$X_{p,s,h,d}, Y_{p,s,h,d}, Z_{p,h,d}, U_{p,h,d}, I_{p,h,d}, N_{p,d} \ge 0 \qquad \forall p, s, h, d \tag{17}$$

In the above model, $\bar{T}$ is a given integer value rather than a decision variable. Note that if the above model is unbounded for some $\bar{T} \in \mathbb{N}$, then the mathematical model given in Eqs. (9)–(10) is also unbounded, which in turn implies unboundedness of the original problem. If the model defined by Eqs. (11)–(17) is bounded, than we can obtain $q(T)$ by solving its dual. Furthermore, assuming feasibility of the region of the dual, we can enumerate all extreme points $(\alpha_p^1, \dots, \alpha_p^I)$ and extreme rays $(\alpha_r^1, \dots, \alpha_r^J)$. Notice that by solving our problem for $q(T)$, we can also access its dual α_p^i, α_r^j variables. This implies that the mathematical model defined by Eqs. (11)–(17) can be viewed as a subproblem. Let q represent the optimal objective function value of this subproblem. Consequently, our master problem becomes:

$$Min\ q + h(T) \tag{18}$$

St:

$$\left(\alpha_p^i\right)'(b - BT) \leq q \qquad \forall i = 1, \dots, I \tag{19}$$

$$\left(\alpha_r^j\right)'(b - BT) \leq 0 \qquad \forall i = 1, \dots, J \tag{20}$$

$$T \in \mathbb{N} \tag{21}$$

Here Eq. (18) represents Benders' optimality cut, and Eq. (19) represents Benders' feasibility cut, where *B* is a parameter matrix whose elements come from the coefficients of all constraints that involve the integer variable *T*; and *b* is a vector whose elements are the parameters from the coefficients of constraints in Eqs. (13)–(16).

Given that there exists an exponential number of extreme points and extreme rays of the dual of $q(T)$, generating all constraints of the type of Eqs. (19) and (20) is not practical. Instead, we solve our Benders decomposition starting with a subset of these constraints and solving a relaxed master problem, which yields a candidate solution. We iterate solving the subproblem and the master problem until the bounds meet, i.e., q converges to a value.

2.4 Case Study

In this section, we introduce a case study that illustrates the implementation our suggested solution methodology discussed in the previous section. The case is based on a real-life problem that a major 3PL provider in the United States faces frequently. In the case study considered here, the customer is a manufacturer of generic drugs based out of

Puerto Rico, and it provided the 3PL company with the supply data—i.e., product details, quantity, shipping date, shipping location, and delivery time window for one calendar year.

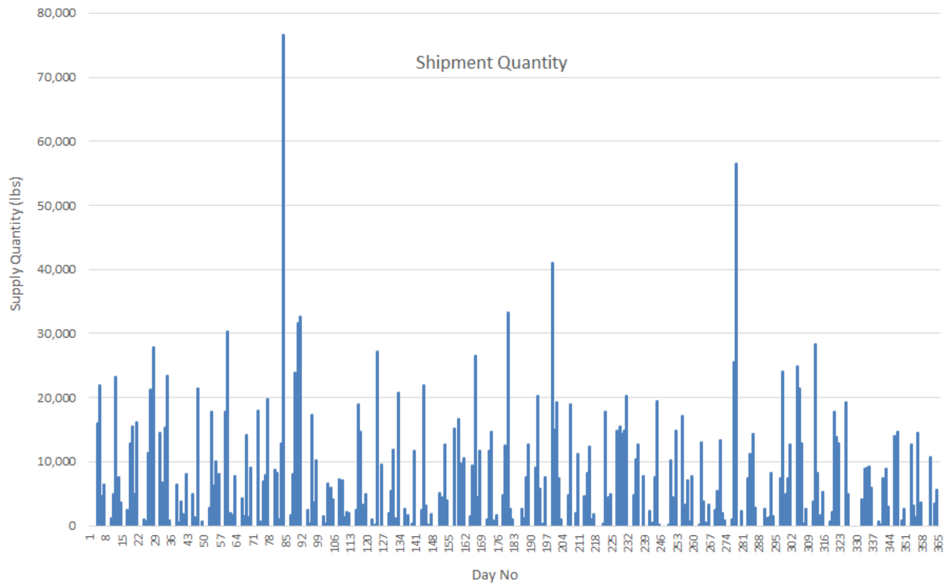

*Figure 2.*Demand for shipment across the 365-day time horizon

There exists a total of 722 products originating from 104 supply locations spread over 25 states in the United States. The descriptive statistics of these products and a summary of the shipping locations are presented, respectively, in Table 1 and Table 2. The expected delivery pattern, i.e., the quantity to be picked up from the supply location on a specific day is presented in Figure 2. Products are aggregated based on their weight rather than volume, since the latter one is relatively insignificant.

Table 1. *Descriptive Statistics of Supply Data*

	Min	1st Qu.	Median	Mean	3rd Qu.	Max.	Obs.
Products (lbs.)	15	417.2	1414	3028	4520	40000	722
Daily shipment quantity (lbs.)	0	0	2054	5975	9051	76537	365

Table 2. *Number of Scheduled Pickups across States and 365-Day Time Horizon*

Origin State	**Total**	**Origin State**	**Total**	**Origin State**	**Total**
AL	1	MA	27	PA	63
AZ	19	MD	3	SD	9
CA	98	MN	3	TN	1
DL	5	MO	10	TX	22
FL	8	NC	24	UT	5
GA	48	NJ	37	VA	1
IL	91	NM	5	WI	59
IN	72	NY	33	Grand Total	722
KY	37	OH	41		

The customer requires pickup dates from the suppliers and strict constraints on tardiness; all the products need to reach the manufacturer's site set in Puerto Rico within a nine-day window. As pointed out above, the 3PL company's objective is to determine the lowest cost at which it could satisfy the customer's shipment requirements. The 3PL company operates three gateways on the East Coast of the United States, which are located at Port of Elizabeth, NJ; Port of Jacksonville, FL; and Port of Miami, FL. In the first leg, products are collected from the supplier locations summarized in Table 2 and shipped to one of these gateway locations using trucking. At gateways, the shipments are consolidated into LCL and FCL shipments and forwarded to San Juan, PR, via ocean freight. The local delivery of the products in San Juan is omitted from the problem since the impact of this stage to the overall problem is negligible.

Table 3. *LCL and FCL Costs*

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	D4	E1	E2
Shipment Time (in days)														
Time	2	2	3	3	3	4	4	4	5	5	5	5	6	6
Variable Cost Per Pound (in cents)														
Cost per Pound	29	33	33	37	41	37	41	44	37	41	44	46	44	46

The unit transportation costs in the first stage depend on the location of the supplier and where the product will be shipped. The costing at the 3PL company is done based on zones. Locations in United States are allocated to these zones for costing purposes. The zones are shown in Figure 3. The zone matrix is given in Table 4. Zone matrix is used to identify the transportation times and unit transportation costs to gateways.

We note that Zones 9 and 12 are used as destination zones since the gateways are located in these zones. The zoning structure and cost values are slightly modified to protect 3PL company's private information. The company's goal is to gain cost advantage by consolidating multiple products into containers at gateway locations before shipping them to Puerto Rico. The 3PL provider's problem requires making decisions on what products to ship through what consolidation center so that the transportation cost from supplier to the manufacturing unit is minimized, while all the due date constraints are met. Clearly, this problem can be modeled using the MIP introduced in Section 3, where there are 722 items, 104 supply points, 3 gateways, and 365 time periods.

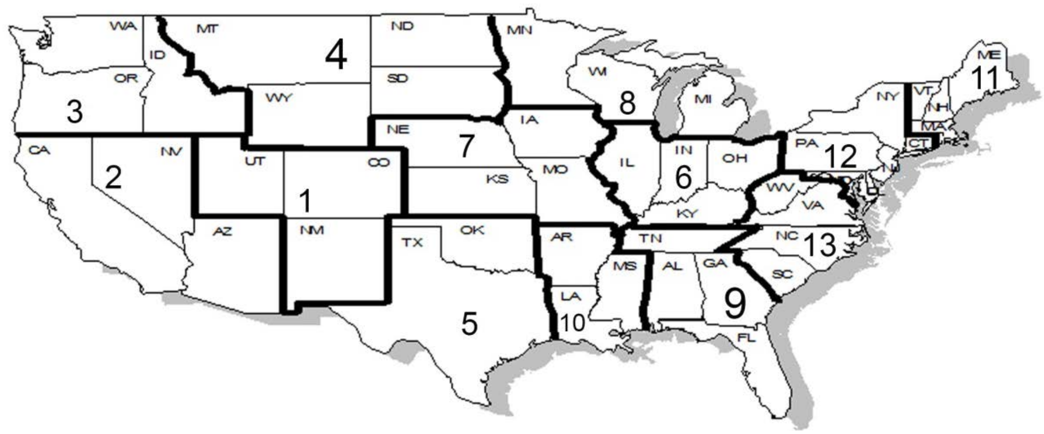

Figure 3. Zoning within the United States and zone matrix.

FROM	TO ZONE											
ZONE	1	2	3	4	5	6	7	8	9	10	11	12	13
1	A1	B2	C1	C1	B3	C2	B2	C2	D3	C3	D3	D3	D4
2	B2	A1	B1	D2	B3	C2	C2	C2	D4	D4	D4	D4	D4
3	C1	B1	A1	C1	D2	D2	D2	D2	D4	D4	D4	D4	D4
4	C1	D3	C1	B2	E1	D1	D3	C2	E2	E2	E2	E2	E2
5	B3	B3	D2	E1	A1	B2	B2	B3	B3	B2	C3	C3	C3
6	C2	C2	D2	D1	B2	A1	B1	B1	B2	B3	B3	B3	B3
7	B2	C2	D2	E	B2	B1	A1	B1	C2	B3	C3	C3	C2
8	C2	C2	D2	C2	B3	B1	B1	A1	B3	C3	B3	B3	B3
9	D3	D4	D4	E2	B3	B2	C2	B3	A1	B1	B3	B2	B1
10	C3	D4	D4	E2	B2	B3	B3	C3	B1	A1	C3	C3	B2
11	D3	D4	D4	E2	C3	B3	C3	B3	B3	C3	A1	A2	B2
12	D3	D4	D4	E2	C3	B3	C3	B3	B2	C3	A2	A1	B2
13	D4	D4	D4	E2	C3	B3	C2	B3	B1	B2	B2	B2	A1

Figure 4. Zone Matrix

We note that typically there is also a fixed cost for pickup at the supplier site in the first stage. However, since this cost is fixed and identical across all locations ($80 per pickup in the case study) and it applies before the shipments are split to gateways, they do not affect the optimality of the solution obtained by the proposed MIP model. The resultant MIP model has more than 27.5 million variables and 9.2 million constraints. Attempts were made to solve the MIP using CPLEX on an Intel (R) Xeon (R) CPU Es-268 WO @ 3.10 GHz (dual processor) with 64 GB RAM machine; however, all attempts at solving the MIP failed on account of lack of sufficient computational resources. Subsequently, the solution methodology presented in Section 4 was applied and results are discussed next.

2.5 Results

In order to solve the in-transit merging optimization problem using the Benders decomposition approach, we divided the problem into integer and linear parts. The linear part consists of the delivery of the packages sent from the shippers to the gateways, while the integer parts consisted of the merger of products at the consolidation stations and their shipment using FCL containers, as well as individual shipments (using LCL containers). The total cost is then the result of adding the individual values of the three cost components: the cost of freight from suppliers to the gateways (the linear part of the model); the cost of freight from the gateways to the clients using FCL containers; and the cost of freight from the gateways to the clients using LCL containers. The fixed costs of the pickups at the first stage are added to the solution of the model so as to find the overall annual cost. The results obtained for this case are summarized in Table 5.

Solving the linear relaxation of the model yielded the following results: The linear part of the model had a total cost of $657,399.67; the FCL part of the model resulted in a total cost of $163,469.83; and the LCL part had a cost of $0. It is straightforward to see that the relaxed problem allocates all shipments of the second stage to FCL containers, since the unit cost is lower, and fractional numbers for containers are allowed due to LP relaxation. This resulted in a total cost of $876,629.50.

Problem	No. of Containers	Shipping Costs			
		f(x)		g(t,z)	Total
		fix cost	variable cost		
LP Relaxation	0	$55,760.00	$657,399.67	$163,469.83	$876,629.50
Benders decomposition (delivery exactly on the 9th day)	6	$55,760.00	$657,400.00	$235,000.82	$948,160.82
Benders decomposition (delivery within 9 days)	13	$55,760.00	$660,207.00	$207,534.15	$923,501.15

Figure 5. LP Relaxation and Benders Decomposition Results

When implementing Benders decomposition to solve the in-transit merging problem, we first consider the scenario, where the customer expects a delivery exactly nine days after a pickup. Occasionally, early delivery is regarded as inconvenience by the client since he or she schedules the pickup dates based on just-in-time production, and avoid carrying input inventory. We capture this case by simply removing the variable $N_{p,d}$ from the proposed MIP model. In this case, we obtain an optimal objective value of $948,160.82. Overall, only six FCL containers were possible under this scenario. At the end, we observed that about 75% of the costs were incurred in the first stage in this case.

When we introduce the inventory option at the client's site, that is, early delivery is allowed, more consolidation alternatives become feasible. As expected, this leads to improvement in the optimal solution. In this case, applying the Benders decomposition approach we get a total cost of $923,501.15 at optimality. This indicates a reduction of about $25,000 in total costs. The reduction is due to the introduced flexibility of early delivery option resulting in seven more FCL (13 in total) consolidations.

One further advantage from the schedule provided by our model is that it ensures that all of the deliveries are carried out within the time window. This is a significant improvement for the company who delivered about 20% of the shipments outside the delivery time window. Their shipment time performance is depicted Figure 6.

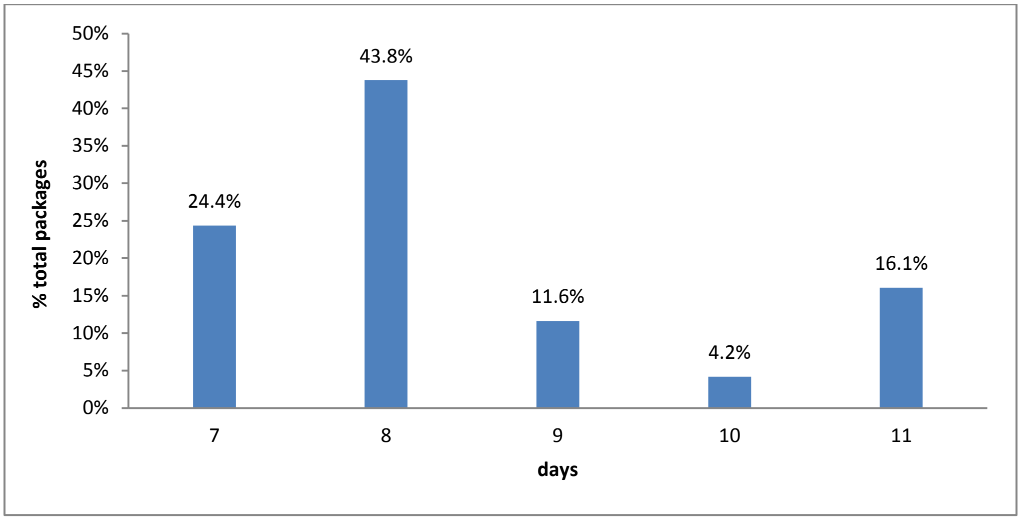

Figure 6. 3PL actual data—distribution of delivery days for all products.

On the other hand, the optimization model (with early delivery permitted) produced a solution with delivery performance depicted in Figure 7. The solution suggests a more uniform distribution in terms of delivery times. Approximately 25% of the shipments are consolidated into FCL shipments at gateways in the suggested solution. We believe that providing customer satisfaction by guaranteeing timely deliveries is paramount in the freight industry; and the implementation of our proposed models ensure that highest quality service can be provided by the 3PL company.

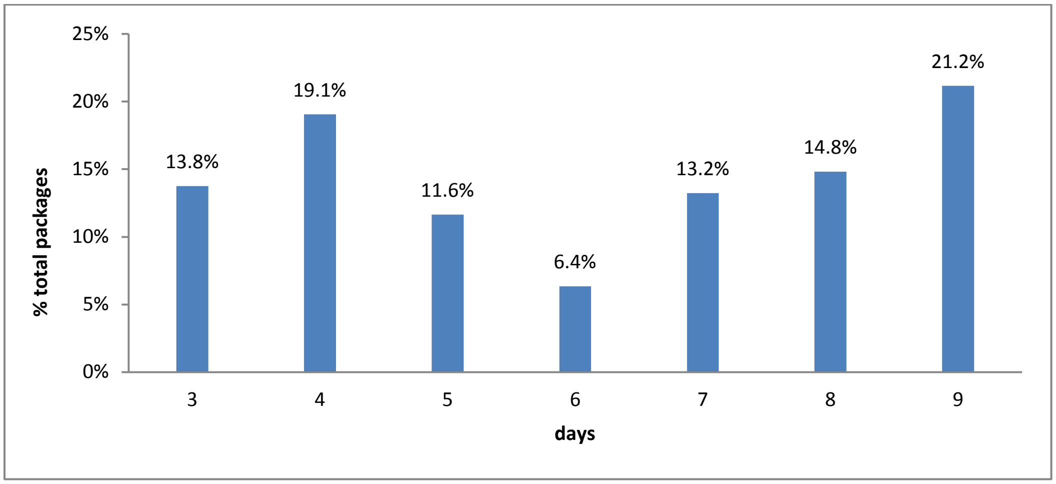

Figure 7. Distribution of delivery days for all products

2.6 Conclusions

We developed a MIP model that considers the case of in-transit consolidation of products being shipped from multiple shippers to a single business customer via multiple gateways that serve as consolidation points. The shipments have prespecified pickup dates with delivery time windows across a multiperiod time horizon. The problem is composed of two legs. In the first leg, products are shipped from suppliers to gateway locations, where shipment cost is a linear function of the package weight and distance between the supply point and the selected gateway. The shipments are forwarded from gateways to the customer's site either using LCL or FCL. The latter one is the cheaper option with lower unit costs; however, it is possible only if a sufficient amount of shipments from the first stage can be consolidated at a given gateway. The delivery time windows impose constraints on consolidation opportunities since products must be delivered before their respective deadlines.

Due to complexity of the problem, the proposed model cannot be used to solve realistic size instances in its monolithic form. To facilitate practical use of the model, we propose a decomposition approach adapted from the Benders decomposition method, where the large numbers of integer freight-consolidation variables are replaced by a small number of continuous, so as to reduce the size of the problem without impacting the optimality. Using a case study adopted from real-life application, we showed that Benders decomposition provides a significant scale-up in the performance of the solver, and we can solve a large-scale case with more than 27.5 million variables and 9.2 million constraints to optimality that was otherwise unsolvable using CPLEX on a 64 GB RAM server. Thus, the proposed redesigned Benders decomposition-based approach solves large-scale in-transit freight consolidation problems optimally and efficiently.

Our solution has several practical benefits as well. The implementation of such a method will not only reduce the total costs for the 3PL providers, but will also enable them to solve larger problems. In future work, we plan to extend the scope of our model to multiple customers that potentially facilitates more consolidation options at gateways.

Chapter 3: A Novel Decomposition Approach for In-Transit Consolidation with Non-Divisible Freight

3.1 Overview

In this chapter, we consider the time-constrained freight consolidation problem with nondivisible shipments (TCCP-ND) for a single customer that acquires from multiple suppliers from different regions. Orders are set for a specified planning horizon with predetermined delivery windows, and transportation is outsourced to a 3PL service provider. The 3PL has to pick up the shipments from the different supplier on predetermined dates and deliver them to the customer within a specific time window. Given the nature of the shipments, no shipment may be divided into multiple parcels, and all of the shipment must be transported together from the supplier to the customer. The 3PL picks up the shipments and takes them to intermediary facilities where they may be consolidated before being sent to the customer either using FCL or LCL transport.

Given the scale of the 3PL provider, we are assuming that the shipment pickups from the suppliers and transport to consolidation terminals is charged at a rate based on the specific consolidation terminal used, while the charge for the transport of shipments from the consolidation terminals to the customer depends on the type of transport used. There is a rate for LCL transport and there is another (lower) rate for FCL transport. It is important to highlight that while the LCL transport is charged based on the weight of the shipments sent, the FCL transport is charged based on the total number of containers used. We assume that there is sufficient space at the consolidation terminals for the storage of shipments, so that a shipment may be held in inventory for consolidation so long as its actual delivery does not exceed its delivery window.

We propose a binary mixed-integer linear programming (MILP) model for the TCCP-ND, taking into account the existence of having different shipments from different suppliers along the planning horizon, which each of these shipments has their own time window, that these shipments are nondivisible into different parcels, and that FCL consolidation is desired at consolidation terminals. The details of our proposed model are shown as follows, while the detailed notation is provided in Appendix B.

The rest of the chapter is organized as follows. Section 3.2 describes our proposed formulation for the in-transit merging problem with nondivisible shipments, as well as the details of our three-phase formulation. Section 3.3 describes the solution algorithm, which explain the three steps for our solution; section 3.5 and 3.6 describes the case study and results obtained to demonstrate the proposed approach. Section 3.7 describes our sensitivity analysis; and finally, Section 4.8 summarizes the conclusions that may be drawn from this study, as well as the future avenues of research that stem from the presented work.

This model uses the formulation of Section 2.1. Given the data information, no air shipment is available. In addition, we set all initial inventory and early delivery variables equal to zero at day 0.

3.2 NonDivisible Binary Mixed-Integer Linear Programming (NDBMILP) Model

Our proposed model is presented below as follows: The objective function is presented in Eq. (23), and the cost link constraints are in Eqs. (24)–(26). Flow balance constraints are shown in Eqs. (27)–(31), time window and demand constraints are presented in Eqs. (32)–(34), and nonnegativity constraints are Eqs. (35)–(36).

3.2.1 Objective Function

$$Minimize \sum_{\forall p,s,h,d} C_{p,h,d} + \sum_{\forall p,h,d} L_{p,h,d} + \sum_{\forall h,d} \bar{c}_h T_{h,d} \tag{23}$$

Equation (23) presents the minimization of the total cost associated with the inbound transportation of shipments to the consolidation terminals; the cost associated with the outbound transportation of shipments from the consolidation terminals to the final customer using LCL shipments; and the costs associated with the outbound transportation of shipments from the consolidation terminals to the final customer using LCL shipments.

3.2.2 Cost Link Constraints

$$C_{p,h,d} = c_{p,h} B_{p,h,d} X_{p,h,d} \qquad \forall p, h, d \tag{24}$$

$$L_{p,h,d} = l_h Z_{p,h,d} \qquad \forall p, h, d \tag{25}$$

$$C_{p,h,d} = \bar{c_h} T_{h,d} \qquad \forall p, h, d \tag{26}$$

Equation (24) calculates the inbound transportation cost based on the rate for each terminal for each product ($c_{p,h}$); the binary variable that decides which terminal is used by each shipment ($B_{p,h,d}$); and the size of each shipment ($X_{p,h,d}$). Equation (25) calculates the outbound transportation cost based on the rate for LCL transportation (l_h); and the total amount of shipments that use this mode of transportation ($Z_{p,h,d}$). Equation (26) calculates the outbound transportation cost based on the rate for FCL transportation ($\bar{c_h}$); and the total number of containers used in this mode of transportation ($T_{h,d}$).

3.2.3 Flow-Balance Constraints

$$X_{p,h,d} = d_p B_{p,h,d} \qquad \forall p,h,d \qquad (27)$$

$$\sum_h B_{p,h,d} = 1 \qquad \forall p,h,d \qquad (28)$$

Equations (27) and (28) ensure that the shipments sent to each consolidation terminal ($X_{p,h,d}$) are equal to the total number of shipments scheduled (d_p), while the variable $B_{p,h,d}$ ensures that each shipment only goes to one single consolidation terminal, and ensures that the nondivisibility of shipments is not violated.

$$Z_{p,h,d} = d_p V_{p,h,d} \qquad \forall p,h,d \qquad (29)$$

$$U_{p,h,d} = d_p Q_{p,h,d} \qquad \forall p,h,d \qquad (30)$$

$$\sum_p U_{p,h,d} \leq kT_{h,d} \qquad \forall h,d \qquad (31)$$

Equation (29) determines which shipments will be sent from the consolidation terminals to the customer using LCL transportation ($V_{p,h,d}$), and ensures mass balance. Equation (30) is similar, but is related to the shipments that use FCL transportation, which are decided by the variable $Q_{p,h,d}$. Equation (31) calculates the total number of containers required for transportation ($T_{h,d}$) based on container capacity (k).

3.2.4 Time Window and Demand Constraints

$$\sum_{h,d \in [R_p + t1_{p,h}, R_p + tw - t2_h]} V_{p,h,d} + \sum_{h,d \in [R_p + t1_{p,h}, R_p + tw - t2_h]} Q_{p,h,d} = 1 \qquad \forall p \qquad (32)$$

$$\sum_{d \in [R_p + t1_{p,h}, R_p + tw - t2_h]} Z_{p,h,d} + \sum_{d \in [R_p + t1_{p,h}, R_p + tw - t2_h]} U_{p,h,d} = X_{p,h,d} \qquad \forall p,h \qquad (33)$$

$$\sum_{h,d \in [R_p + t1_{p,h}, R_p + tw - t2_h]} Z_{p,h,d} + \sum_{h,d \in [R_p + t1_{p,h}, R_p + tw - t2_h]} U_{p,h,d} = d_p \qquad \forall p \qquad (34)$$

Equation (32) ensures that every shipment must be sent either by LCL or FCL transportation so that its time window is met. Every shipment only has an opportunity to select the mode of transportation within its feasible delivery week, which at the consolidation terminals is within the predetermined shipment date (R_p) to the final feasible delivery date ($R_p + tw$), shortened by the inbound transit time ($t1_{p,h}$) and the outbound transit time ($t2_h$). Equation (33) is similar to Eq. (10), but it calculates the amounts shipped and ensures mass balance at the terminals. Equation (34) ensures that the demand for every product is met at the customer.

3.2.5 Variable Types

$$T_{h,d} \in \mathbb{N} \qquad \forall h, d \qquad (35)$$

$$X_{p,h,d}, U_{p,h,d}, Z_{p,h,d} \geq 0 \qquad \forall p, h, d \qquad (36)$$

$$B_{p,h,d}, V_{p,h,d}, Q_{p,h,d} \quad binary \qquad (37)$$

Equation (35) defines the number of containers ($T_{h,d}$); Eq. (36) ensures nonnegativity for the amounts transported to the consolidation terminals ($X_{p,h,d}$), and from the consolidation terminals to the customer using LCL ($Z_{p,h,d}$) and FCL ($U_{p,h,d}$) transportation. The NDBMILP model is an NP-complete problem, given the binary nature of the decision variables.

3.3 Solution Methodology

In order to find a solution to the proposed model, we propose a three-phase strategy. The first stage includes an LP relaxation of the container constraint; the second stage uses decomposition in order to generate subproblems that may be independently solved; and the third stage uses a valid cut approach to solve the individual subproblems generated in the second phase.

3.3.1 Container Relaxation

In this phase, the binary nature of Eq. (35) is relaxed, so that a partial number of containers may be transported each day from each terminal. However, in order to evade a trivial solution in which no loads are consolidated, an incentive (n) is added to the objective function where inventory $(I_{p,h,d})$ held at each consolidation terminal is promoted. As such, the objective function is updated as shown below in Eq. (38).

$$Minimize \sum_{\forall p,s,h,d} C_{p,h,d} + \sum_{\forall p,h,d} L_{p,h,d} + \sum_{\forall h,d} \bar{c}_h T_{h,d} - n \sum_{\forall p,h,d} I_{p,h,d} \tag{38}$$

The solution to this problem provides as with a lower bound where the maximum feasible consolidation is achieved and gives us an appropriate initial solution for the second phase. Every single shipment of the container relaxation problem is done using a real positive number of containers—as those positive values provide initial candidates for consolidation. Furthermore, the solution to the container relaxation problem can be used to eliminate degeneracy.

3.3.2 Decomposition

The second phase is used to generate subproblems that can be solved independently using mixed-integer linear programming by decomposing the complete problem. In order to accomplish this, we look for any gaps within the planning horizon, as well as for demand cycles within the problem's configuration. The decomposition begins by looking for blocks within the data that have the size of the delivery time window and where there are less than two shipments predetermined by the suppliers. Any such block that is identified is then added to an individual subproblem, since any shipments within that subproblem may not be consolidated with shipments from any other date and can be easily solved. Once these subproblems are removed, the rest of the data will also be split into several different blocks, which will also be independently solved. Once these initial subproblems are generated, the breakeven consolidation amount (F_h) is found as shown in Eq. (39).

$$F_h = {}^{\frac{\bar{c}_h}{l_h}}\!\Big/_{k} \qquad \forall h, d \quad (39)$$

Conjecture 1. For any number of consecutive days of length $\mathrm{tw} + \tau$ where the total amount of shipments is less than $\mathrm{F_h}$, the first τ days may be separated into an individual subproblem since there will be no feasible consolidation.

3.3.3 Valid Cuts

The third phase is used to perform valid cuts from the subproblems in order to reduce the problem's complexity and improve the computational efficiency. The cuts, leveraging the information gained from the first two stages, zeros out consolidation variables where there cannot be any consolidation. In any subproblem that is longer than two delivery time windows, and for which in the first stage there is no consolidation of containers, the container variables ($T_{h,d}$) may be zeroed out.

Conjecture 2. Independently-cut problems may be solved individually within their constrained solution space and aggregated into a global solution, without loss of optimality.

3.4 Solution Algorithm

The overall solution algorithm, integrating the three mentioned phases, is presented in Table 4.

Table 4. Solution Algorithm

1. Solve the container relaxation problem, where maximal consolidation is incentivized

$$\max \quad z = \sum_{\forall p,h,d} C_{p,h,d} + \sum_{\forall p,h,d} L_{p,h,d} + \sum_{\forall d,h} \bar{c}_h T_{h,d} - n \sum_{\forall p,h,d} I_{p,h,d}$$

   where n is a small positive real number.

2. Calculate the cost breakeven point for each terminal h

$$F_h = \frac{\bar{c}_h}{l_h} \Big/ k$$

3. Let $i = 1$ on the first day of the problem
4. If $i \geq 365$, all the days in a year have been explored, go to Step 11.
5. Let $a = 0$
6. Let $m = \sum_{\forall p} \sum_{d=i+a}^{i+a+tw} U_{p,h,d}$
7. If $m < F_h$, let $a = a + 1$ and go to Step 5
8. Else create a subproblem starting on day i and ending on day $i + a$
9. Let $i = i + a + 1$
10. Go to Step 4
11. Add any block of consecutive days that are not already in a subproblem into independent subproblems
12. Add the cut $T_{h,d} = 0$ for any subproblem identified in Step 8
13. Solve each subproblem as a MIBLP problem

3.5 Case Study

We consider the case of Freight Logistics, a major 3PL logistic solutions provider in the United States. Freight Logistics is working on an annual bid to provide 3PL services for a manufacturer of generic drugs based out of Puerto Rico. The manufacturer has provided Freight Logistics with the anticipated shipping data, i.e., product details, quantity, scheduled date, origin location, and due date. There are a total of 722 products originating from 104 locations spread over 25 states. The descriptive statistics of these products and a summary of the origin locations are presented in Tables 1 and 2, respectively. The expected delivery pattern, i.e., the quantity to be delivered on a specific day, is presented in Figure 1. All products are in a form such that there is a near 1:1 relationship between volume and weight.

Other than providing these data, the manufacturer laid down two constraints:

- Every product's scheduled shipments cannot be split into parts.
- Assuming all the products are ready to be shipped at the scheduled time, they need to reach the destination in Puerto Rico by their respective due dates.

Freight Logistics provider's internal objective is to determine the lowest cost at which it may satisfy the manufacturer's shipment requirements. The Freight Logistics provider has three consolidation terminals: one in Newark, NJ; a second in Miami, FL; and a third in Jacksonville, FL. Its goal is to gain a cost advantage over its competitors by consolidating multiple product shipments into containers before shipping them to Puerto Rico.

Freight Logistics provider's decision problem requires determining which products to ship through which consolidation terminal so that the transportation cost from supplier to the destination is minimized, while all the due date constraints are met. The problem has an approximate total of 6.692 million variables and 5.856 million constraints (as a result of having 722 products, 104 suppliers, 3 consolidation terminals, and 365 days). As discussed in Section 3.2, our solution relies on three steps: container relaxation; decomposition; and cut generation.

3.5.1 Container Relaxation

Given the rate for FCL container shipments, as well as the LCL rate per 100 pounds, it can be seen that there exists a threshold (in lbs.) up to which the shipping cost increases a function of weight, and beyond, which the shipping cost per container is fixed. In this phase we remove the integrality constraint for the containers so that partial containers may be shipped at the FCL rate, and we add an incentive for inventory held at the consolidation terminals such that the maximum possible consolidation is encouraged. The container relaxation of the model yielded the use of 45.38 containers, at a cost of $823,127, with a computation time of 265 seconds. This solution provides an upper bound on the number of containers that can be used without violating the delivery time constraint.

3.5.2 Decomposition

Using the procedure, we were able to decompose the 365-day planning horizon into 13 independent subproblems (shown in Table 4), and where the largest subproblem has 98 days. The time to solve this subproblem was 10,391 seconds, which is then the maximum taken to solve the problem, as the other 13 subproblems can be run independently in parallel as shown in given the rate for FCL container shipments, as well as the LCL rate per 100 pounds. It can be seen that there exists a threshold (in lbs.) up to which the shipping cost increases a function of weight, and beyondm which the shipping cost per container is fixed. These levels are presented as FCL breakeven threshold in Table *5*.

Table 5. *Summary of Scenarios*

SP	start	end	# of days
SP1	1	98	97
SP2	99	109	10
SP3	110	140	30
SP4	141	151	10
SP5	152	220	65
SP6	221	245	24
SP7	246	251	5
SP8	252	264	12
SP9	265	269	4
SP10	270	328	58
SP11	329	339	10
SP12	340	360	20
SP13	361	365	4

3.5.3 Valid Cuts

We are performing our valid cuts from the subproblems to reduce the problem's complexity and improve the computational efficiency. Before we start to present the solution, we are trying to evaluate the effectiveness of our proposed NDBMILP model. When comparing, the computational time taken to reach the optimal solution to that of using CPLEX on AMPL. CPLEX was able to solve up to 60-day planning horizon problems, but after 60 days, CPLEX was unable to reach a solution after more than 138 hours. It is important to highlight that in an instance of 60 days with 134 products, the

NDBMILP model reached the optimal solution in 717.60 seconds—9.20% of the 7800.00 seconds taken by AMPL, as shown in *Table 6* and Figure 8.

Table 6. *Computational Comparison*

# days	# Product	CPLEX time (Sec)	NDBMILP time (Sec)
10	20	1.25	0.906
20	44	7.343	2.671
30	69	33.078	18.844
40	93	445.25	242.36
50	110	2486.08	10672
60	134	7800	717.609
70	154	496800	96497.3
80	172	496800	201.375
90	195	496800	194.359
95	205	496800	11267.7
98	215	496800	10391.1

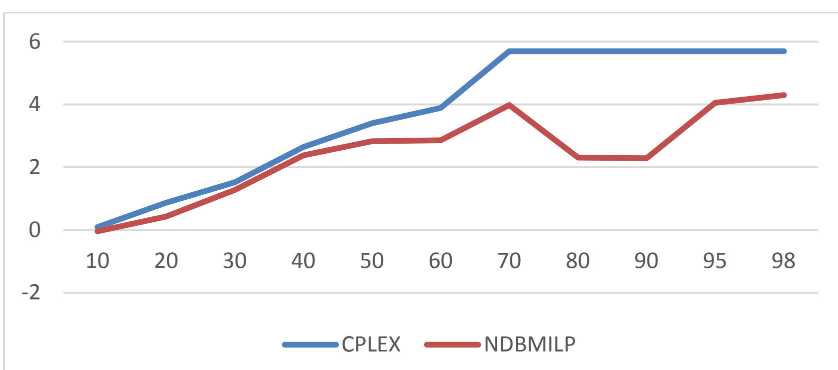

Figure 8. Computational comparison.

3.5.4 Case Study Results

Using the procedure from the NDBMILP model presented in Section 3.1, and the results from the container relaxation from Section 3.2.1, we were able to reduce the computational for the different subproblems. Specifically, the number of containers to consolidate in subproblems 2, 4, 7, 9, 11, and 13 has been determined as 0 *a priori*, since there is no consolidation in the days immediately before these, nor in the days of these subproblems in the container relaxation. Furthermore, we had a maximum number of containers to be consolidated in every other subproblem, which leads to the consolidation of 37 FCL shipments across the complete planning horizon. The complete details of each subproblem, as well as the container relaxation and the overall solution are shown in Table 7.

The thirteen subproblems into which the problem is decomposed range in length from 98 days in the case, so subproblem 1, to five days in subproblems 9 and 13. Subproblem 1 was the hardest to solve and took almost 2 hours and 54 minutes to reach a solution, while the total computational time used to solve the decomposed problems was of almost 3 hours and 9 minutes. It is important to highlight the effectiveness of the decomposition and cutting procedure, as 10 of the 13 subproblems generated reached a solution within 5 seconds, while the other two not previously mentioned took almost 14 minutes, and almost 3 minutes to solve, respectively.

Our proposed NDBMILP model reaches an aggregate solution of $861,193.33 with 37 consolidated containers. This represents a total increase in cost of 4.62%, and a reduction in the container consolidation of 18.49% from the relaxed lower cost bound and upper consolidation bound.

Table 7. *Performance of the NDBMILP Model*

Solution Method		start	end	C (\$)	L (\$)	$\bar{c}$ (\$)	Total Cost (\$)	FCL	Elapsed time
Container Relaxation		Total		659.752.0	-	163,375.00	823,127.00	45.38	265.15
Total		1	365	673,130.01	61,789.82	126,273.50	861,193.33	37	11,330.04
ND-BMILP	SP1	1	98	220,228.00	22,141.30	40,656.00	283,025.30	12	10,391.08
	SP2	99	109	10,071.10	3,778.19	-	13,849.29	0	0.75
	SP3	110	140	46,227.90	2,360.91	10,164.00	58,752.81	3	4.89
	SP4	141	151	9,327.66	4,131.21	-	13,458.87	0	0.88
	SP5	152	217	133,970.00	6,882.10	27,104.00	167,956.10	8	128.59
	SP6	218	245	56,051.10	100.00	13,552.00	69,703.10	4	2.53
	SP7	246	251	3,540.70	1,804.12	-	5,344.82	0	0.53
	SP8	252	264	22,631.80	2,966.68	3,388.00	28,986.48	1	0.89
	SP9	265	269	2,044.40	725.73	-	2,770.13	0	0.42
	SP10	270	328	124,349.00	9,395.66	24,633.50	158,378.16	7	797.31
	SP11	329	339	10,261.30	4,523.12	-	14,784.42	0	0.75
	SP12	340	360	28,431.40	716.41	6,776.00	35,923.81	2	0.72
	SP13	361	365	5,995.65	2,264.39	-	8,260.04	0	0.70

It is important to highlight that we cannot evaluate the efficiency of our proposed NDBMILP model since when we used the CPLEX solver in AMPL we ran out of computational resources.

3.6 Sensitivity Analysis

3.6.1 Generation of Test Instances

As we mentioned in Section 2.3, this is a study of a real-life application where we ship from the United States from four different locations to one customer in Portico, through two consolidation points. The planning horizon is 180 and 360 days, and three different cost breaks: *C1*, *C2*, and *C3*; where *C1* is the most expensive, and *C3* is the shipsets. We have three different types of demand structure: *d1*, *d2*, and *d3*; and finally four instances: *1*, *2*, *3*, *4*, once the item arrives to the terminal. All the test instances make used of three demand types. The demand structures included a scenario where there was a week with a peak demand, while the rest of the week had a similar lower level, and the shipments were all scheduled during weekdays only; the second scenario was similar to the first one, but the peak was repeated throughout; finally, the third scenario didn't have such a schedule, and the daily demand was random—as shown in Figure 9.

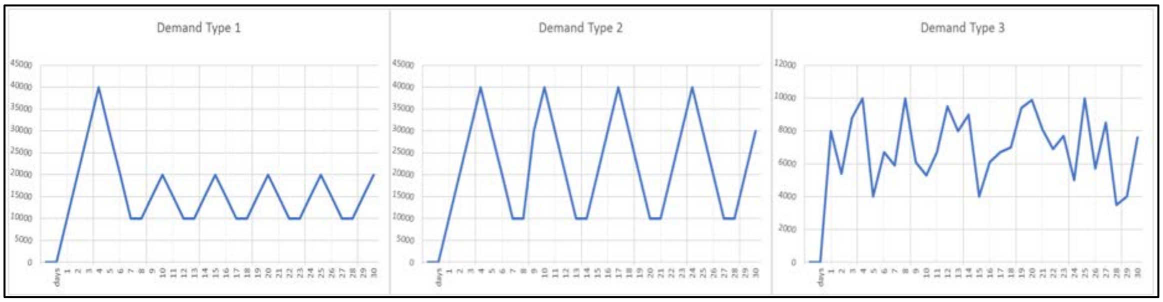

Figure 9. Demand type.

We evaluated the statistical significance and effects of the proposed to the total cost. The investigated factors are shown in *Table 8*. We used all created statistically-significant scenarios to test the effectiveness of our proposed NDBMILP.

Table 8. *Summary of Scenarios*

Factor	Levels
Planning Horizon	2
Consolidation Terminals	2
Cost Rates	3
Demand Structure	3
Replications	4

The greatest source of variation came from the demand structure, such that having a random peak in the demand was a significant cost driver, and having only one peak in demand during the planning horizon led to the highest cost increase. The number of consolidation terminals was also an important driver of the cost, where having three consolidation terminals was more cost effective than having only two. The effects of the proposed factors using of the total cost are shown in Figure 10, Figure 11 and Figure 12.

Factor Information

Factor	Type	Levels	Values
COST	Fixed	3	c1, c2, c3
TERMINAL	Fixed	2	g2, g3
D STRUCTURE	Fixed	3	d1, d2, d3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
COST RATE	2	45436661569	22718330784	14.70	0.000
TERMINAL	1	2.14100E+11	2.14100E+11	138.51	0.000
D STRUCTURE	2	3.66041E+12	1.83020E+12	1184.07	0.000
Error	138	2.13305E+11	1545688220		
Lack-of-Fit	12	2.10234E+11	17519483364	718.77	0.000
Pure Error	126	3071174012	24374397		
Total	143	4.13325E+12			

Regression Equation

Total Cost= 482000 + 6497 COST RATE_c1 + 17767 COST RATE_c2 - 24264 COST RATE_c3
+ 38559 TERMINAL_g2 - 38559 TERMINAL_g3 + 127587 D STRUCTURE_d1
+ 97205 D STRUCTURE_d2 - 224792 D STRUCTURE_d3

Figure 10. General linear model for total cost.

Further, we found that there is an interaction effect between the number of consolidation terminals and the type of demand, such that there is a difference in the total cost when there are random demand peaks (demand type 3) and the number of available terminals for consolidation, while having three terminals available for consolidation significantly decrease the total cost; while with one peak in demand (demand type 1) and two terminals, the higher the total cost will go. As the number of consolidation terminal increases, the higher the possibility of FCL will occur.

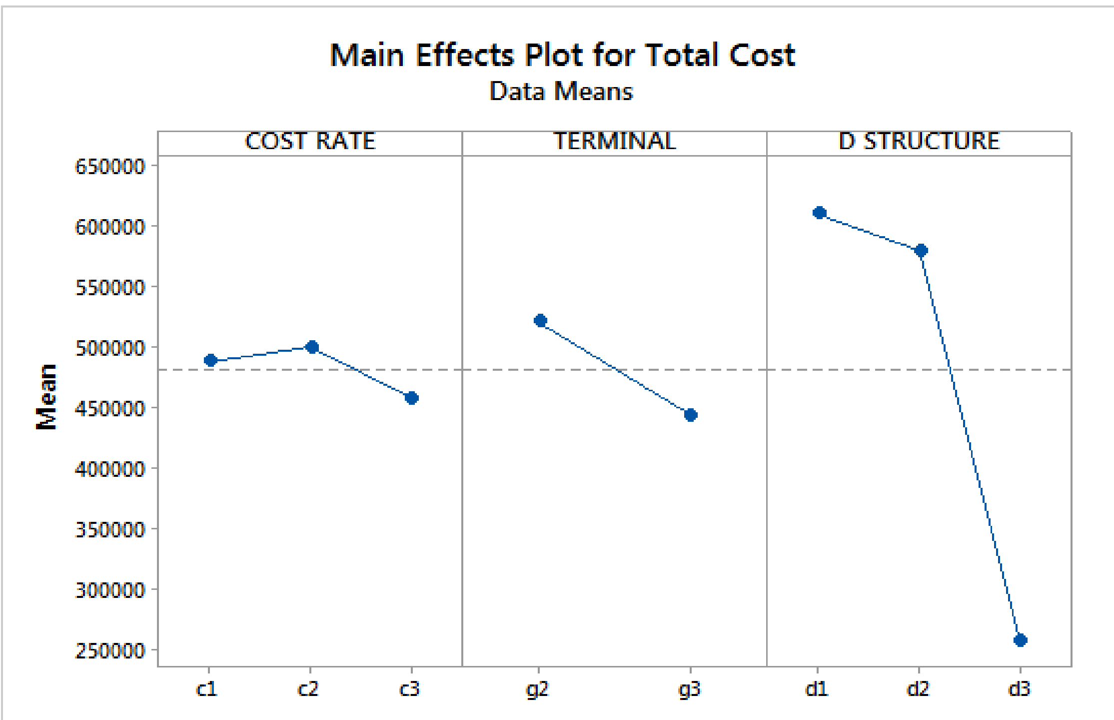

Figure 11. Main effects plot for total cost.

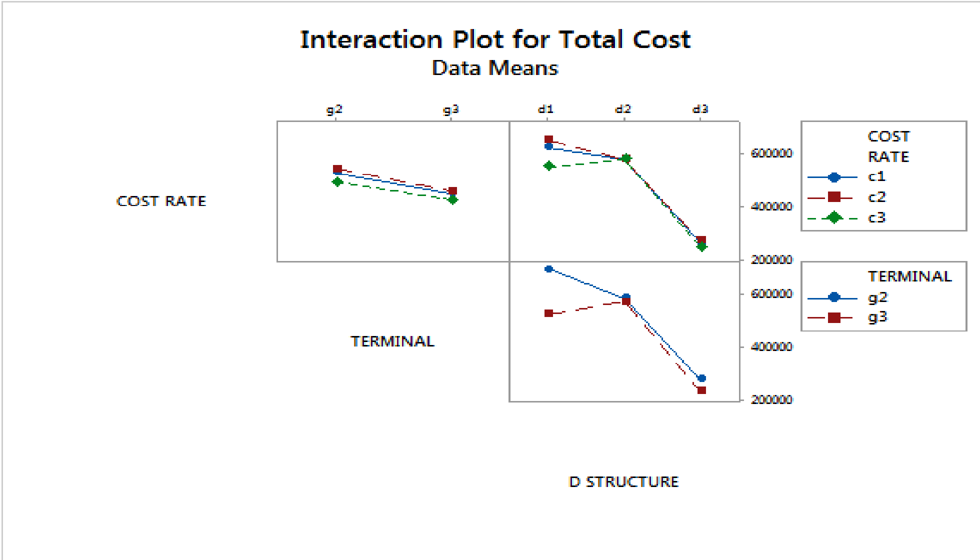

Figure 12. Joint effect on demand and terminal.

Factor Information

Factor	Type	Levels	Values
COST RATE	Fixed	3	c1, c2, c3
TERMINAL	Fixed	2	g2, g3
D STRUCTURE	Fixed	3	d1, d2, d3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
COST RATE	2	509618	254809	1.00	0.370
TERMINAL	1	53705451	53705451	210.85	0.000
D STRUCTURE	2	5944354	2972177	11.67	0.000
Error	138	35149810	254709		
Lack-of-Fit	12	17867311	1488943	10.86	0.000
Pure Error	126	17282499	137163		
Total	143	95309233			

Regression Equation

Time = 2155.1 - 83.0 COST RATE_c1 + 29.4 COST RATE_c2+ 53.6 COST RATE_c3 + 610.7 TERMINAL_g2- 610.7 TERMINAL_g3- 274.3 D STRUCTURE_d1 + 62.9 D STRUCTURE_d2+ 211.3 D STRUCTURE_d3

Figure 13. General linear model for time.

When assessing the advantages of decomposition, we used the computational time taken as a response and evaluated the previously mentioned factors. We used this as a metric since this is the cap to the computational performance when the decomposed subproblems are solved in parallel. We found that demand structures and the number of terminals also statistically significant factors, while the cost rates are not statistically significant. The model's computational time is reduced when there are three consolidation terminals, and the demand has only one peak (demand type 1), as shown in Figure 14.

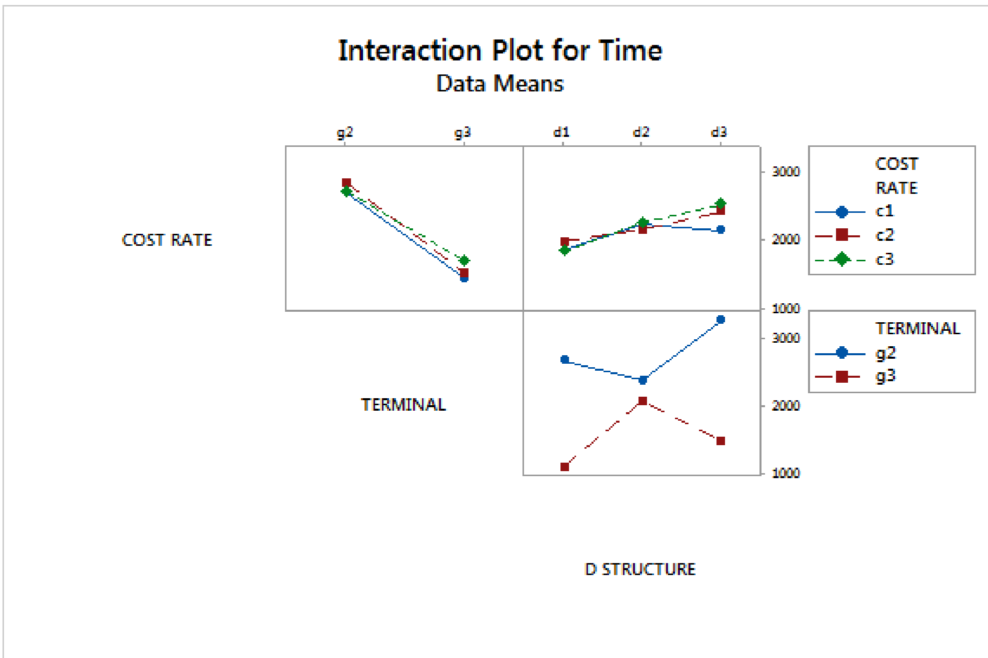

Figure 14. Interaction effect of total time.

Further, we found that there is an interaction effect between the number of consolidation terminals and the type of demand, such that there is very little difference in the model's performance when there are multiple demand peaks (demand type 2) and the number of available terminals for consolidation. Having three terminals available for consolidation significantly increases the speed with which the optimal is reached when there is only one peak in demand (demand type 1), or the demand is random (demand type 3). These effects are shown below in Figure 15.

Factor Information

Factor	Type	Levels	Values
COST RATE	Fixed	3	c1, c2, c3
TERMINAL	Fixed	2	g2, g3
D STRUCTURE	Fixed	3	d1, d2, d3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
COST RATE	2	511426	255713	1.00	0.371
TERMINAL	1	54185686	54185686	211.83	0.000
D STRUCTURE	2	5795269	2897635	11.33	0.000
Error	138	35300676	255802		
Lack-of-Fit	12	18025949	1502162	10.96	0.000
Pure Error	126	17274727	137101		
Total	143	9579305			

Regression Equation

Total Diff = 1458.2 + 83.1 COST RATE_c1 - 29.5 COST RATE_c2 - 53.6 COST RATE_c3
- 613.4 TERMINAL_g2 + 613.4 TERMINAL_g3 + 271.6 D STRUCTURE_d1
- 64.9 D STRUCTURE_d2 - 206.7 D STRUCTURE_d3

Figure 15. NDBMILP model vs CPLEX total different

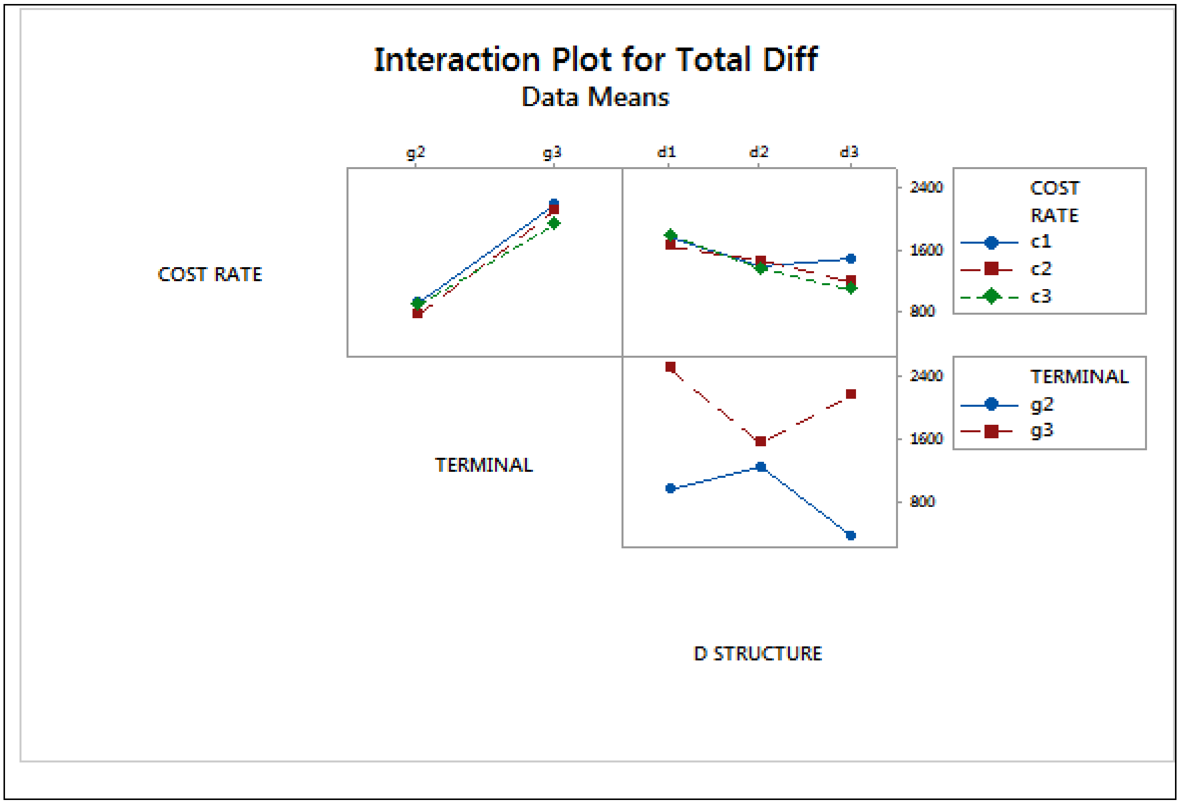

Figure 16. Interaction plot for total different

Additionally, when evaluating this expediency with our proposed NDBMILP model, we are using the difference between the times taken for CPLEX to get to a feasible solution, to the total computational time taken to solve every subproblem of the same instance. We fort CPLEX to a one-hour time limit, as well as the NDBMILP model. In both, the time horizon is 180 and 360 days, and we have 6 problems and 12 subproblems, respectively. Then, we randomly distribute the one hour among the subproblems. Comparing between the elapsed time for NDBMILP and CPLEX, we are faster than CPLEAX. In the 180-planning horizon, NDBMILP's average time is 2,107 seconds, while it was 3,609 seconds for CPLEX, which makes NDBMILP 42% faster than CPLEX. Alike, the case with 360 planning horizons, the average of NDBMILP was 2,202 seconds, and the average of CPLEX was 3,616 seconds, which make NDBMILP 39% faster than CPLEX.

The number of terminals and the demand structure are significant factors to expediency. In this case, having three terminals greatly improves our model's performance, while having only one peak in demand (demand type 1) hinders it, as shown below in Figure 10. Again, it is important to point out that the average difference between our proposed model and the CPLEX solution in this case is beyond 24 minutes; and on average, our NDBMILP model reaches the optimal two-and-a-half times faster as shown in Figure 17.

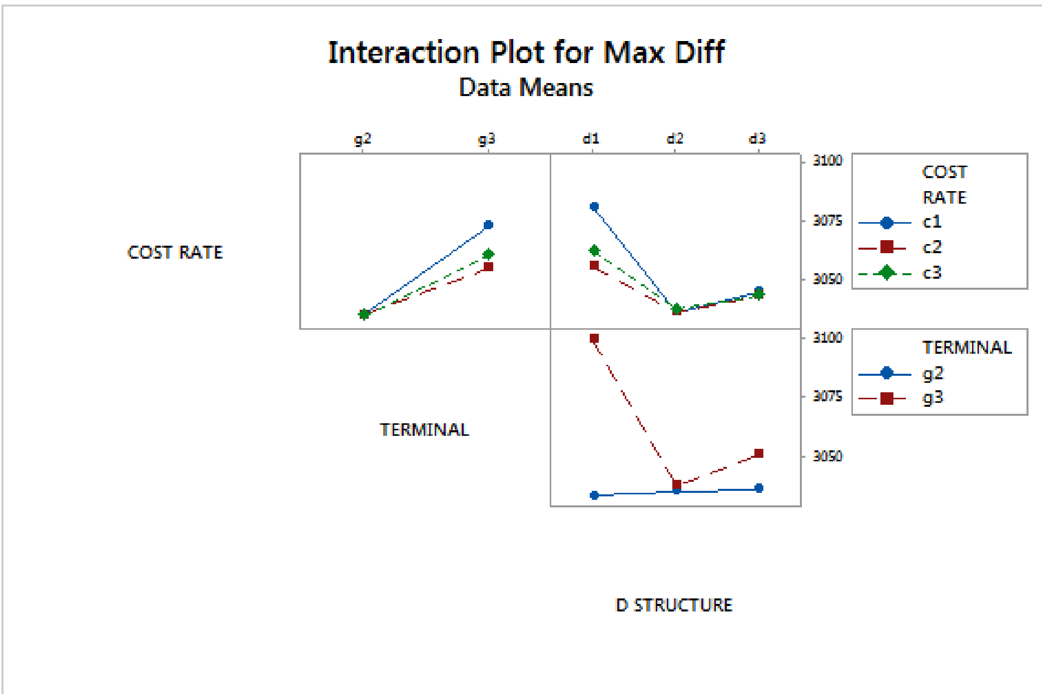

Figure 17. Main effects for subproblem computational time.

3.7 Conclusions

We have developed a novel binary mixed-integer linear programming strategic model for the optimization of freight consolidation with nondivisible shipments. Our focus on nondivisible shipments provides increased insight in a situation that isn't too uncommon, but that to the best of our knowledge had been overlooked by the literature.

Our model relies on three phases to reach the optimal solution. The linear relaxation phase is used to generate initial solutions for a linear decomposition that leads to effective cuts. By developing these subproblems, we tackle an NP problem and reach the optimal solution. Our approach provides a comprehensive solution that optimizes the shipments for the complete network, and includes a two-echelon solution that may be used at a strategic level.

The implementation of our three-phase method was successful at reaching an optimal solution for the few instances in which the mixed-integer formulation employing CPLEX on AMPL was able to reach the optimal solution. The computational efficiency of our proposed approach was able to reach solutions in multiple cases where just using AMPL was impossible as the program never reached a solution within five days before running out of computational resources.

We have found that freight consolidation provides significant advantages for 3PL providers for cost reduction, while meeting customer demands and time windows completely. The use of freight consolidation offers the 3PL provides the opportunity to consolidate cargo and use 37 FCL shipments reaching 81.54% of the lower bound from the linear relaxation.

Our proposed model is very efficient in reaching the optimal solution, and its computational performance improves in settings with more consolidation terminals. It is encouraging to see that the decomposition technique leads to similar computational times regardless of the planning horizon length used, as our model may be used for long-term contracts without hindering the efficiency.

Further avenues of research include, but are not limited to, the consolidation of cargo with different shipping costs for individual parcels; the consolidation of shipments for different customers with similar destinations; consolidation integrating two or more networks; and the inclusion of different inventory holding costs at the consolidation terminals.

Chapter 4: Freight Consolidation with Divisible Shipments and Piecewise Transportation Costs

4.1 Overview

This chapter focuses on a heuristic solution to the problem in which three stages are proposed: in stage one, a container load relaxation is developed; stage two deploys a decomposition; and stage three uses a cutting method to reach the optimal solution. We consider a network with a single firm supplied by multiple products from multiple physically scattered shippers via a 3PL provider. The shipments are charged either a minimum shipping cost or a cost that is based on a piecewise linear increasing cost function that depends on the weight of the shipment. We validate and illustrate our modeling and solution approach using a real-life-based case study obtained from a major 3PL company. The case study involves annual planning of scheduling and routing of more than 700 shipments from hundreds of locations on a large-scale time-space network. We demonstrate the effectiveness of the proposed algorithm and present a sensitivity analysis using several variations of the case problem obtained by varying container costs and delivery time windows. We observe that the proposed method's computational time efficiency increases with higher FCL costs and smaller delivery time windows mainly because the problem can be decomposed into a larger number of smaller subproblems.

The rest of the chapter is organized as follows. Section 4.1 presents the proposed formulation of the freight consolidation problem, while Section 4.2 discusses our proposed three-phase solution methodology. Section 4.4 presents the case study that demonstrates the proposed approach. Finally, discusses our sensitivity analysis.

4.2 Mathematical Model

The specific version of the time constrained freight consolidation problem with divisible shipments (TCFCP-DS) that we consider involves a single manufacturer that procures from multiple suppliers dispersed over different regions. The firm orders multiple items from these suppliers over a certain time horizon with scheduled shipment dates and delivery time windows. The transportation of the goods is outsourced to a 3PL service provider that picks up the shipments from the suppliers on predetermined shipment dates and delivers them to the firm within a prescribed time interval. The transshipment network is composed of two echelons. The picked shipments are first moved to intermediary facilities. Any particular shipment at any origin can be divided into subshipments that can be transported to different terminals. In the next echelon, the items are forwarded to the customer (manufacturer) using LCL and/or FCL shipments.

The shipment cost is piecewise linear with the shipment amount and determined by the cost breaks. For each segment in the first echelon there is a minimum fixed charge if the shipment amount on any route is below a certain threshold. Beyond the threshold, the shipment cost increases linearly as a function of the shipment weight until the next threshold. The cost breaks provide quantity discounts and the specific cost values may vary across origin and terminal pairs. The set of origins in the first echelon includes all locations where products are to be received by the carrier at specific times. At these points, overnight storage for the carrier is not available and as such, any good picked by the shipper must be shipped on the same day.

The shipment cost in the second echelon depends on whether the items are shipped as individual packages or in full containers. There is a terminal dependent variable cost for the first option. In the FCL option, the cost is calculated on a per container basis. There is sufficient storage capacity at the terminals so that goods that arrive to a terminal on a certain day do not necessarily have to leave the terminal on that same day and be sent to the final customer. The goods can be held as inventories at the terminal points for consolidation into full containers. However, since late delivery is not allowed, the maximum time that a shipment can be kept at any given terminal is constrained by its delivery deadline.

We propose a binary mixed-integer linear programming (MLP) model for the TCFCP-DS, which incorporates piecewise linear functions for the shipping costs, delivery time windows, shipment of multiple products over a planning horizon, and FCL consolidation options at the terminals. In what follows, we discuss the details of the proposed model. The complete notation is given in Appendix C.

4.2.1 Objective Function

The goal of our proposed MLP model is to minimize the total cost of shipping cargo from multiple suppliers to a single customer via a set of terminals. We employ the following objective function:

$$Minimize \sum_{\forall p,s,h,d} C_{p,s,h,d} + \sum_{\forall p,h,d} L_{p,h,d} + \sum_{\forall h,d} \bar{c}_h T_{h,d} \tag{42}$$

Equation (42) is composed of the total cost associated with the use of inbound shipments to the terminals using ground ($C_{p,s,h,d}$); the cost associated with the individual ($L_{p,h,d}$); and consolidated ($\bar{c}_h T_{h,d}$) outbound shipments from the terminals to the final customer. These costs are explained in depth in the following subsections.

4.2.2 Cost Breaks

The cost structure has the following characteristics: There is a minimum charge for any shipment under a certain minimum weight threshold. Once that minimum weight is reached, the cost increases as a linear function of the weight of the shipped goods until the next threshold. The thresholds are represented by cost breaks that are used to enable quantity discounts. Figure 11 illustrates a typical shipment cost function. The cost breaks and the ultimate shipment cost function depend on the segments. The piecewise functions necessitate the use of binary variables in the mathematical model as discussed next.

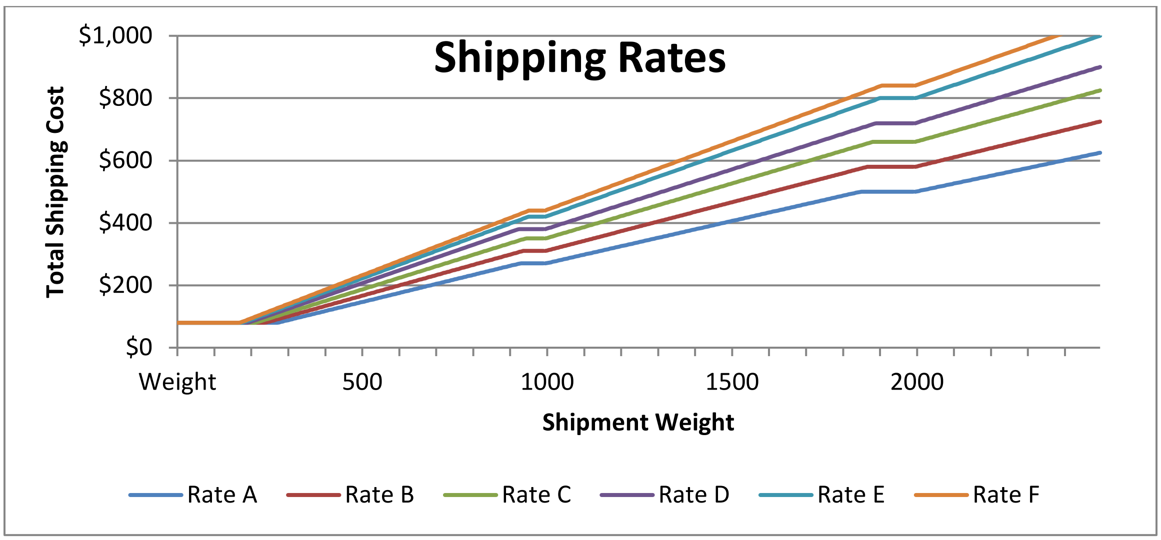

Figure 18. Cost breaks for shipping costs

As mentioned earlier, the piecewise cost structures for shipments are modeled via binary variables that represent cost breaks. The binary values are mapped into the cost variables given in the above objective function. The following constraints are employed to control the cost variables: $C_{p,s,h,d}$:

$$\sum_{\forall b} \delta_{p,s,h,d,b} = 1 \qquad \forall p,s,h,d \qquad (43)$$

$$\delta_{p,s,h,d,1} \leq y_{p,s,h,d,1} \qquad \forall p,s,h,d \qquad (44)$$

$$\delta_{p,s,h,d,i} \leq y_{p,s,h,d,i} + y_{p,s,h,d,i-1} \qquad \forall p,s,h,d,i > 1 \qquad (45)$$

$$\sum_{\forall b} y_{p,s,h,d,b} = 1 \qquad \forall p,s,h,d \qquad (46)$$

In Eqs. (43)–(46), δ is a variable that represents the proportion of the cargo that is being sent at cost rate interval b from the shippers to the consolidation terminals, while y is a binary variable that indicates if the total cargo shipped reaches cost rate interval b. This ensures that the correct shipping costs are used between the shippers and the consolidation terminals. Likewise, the following constraints are used to generate LCL shipment costs from the terminals to the end customer ($L_{p,h,d}$):

$$\sum_{\forall b} \eta_{p,h,d,b} = 1 \qquad \forall p,h,d \qquad (47)$$

$$\eta_{p,h,d,1} \leq z_{p,h,d,b} \qquad \forall p,h,d \qquad (48)$$

$$\eta_{p,h,d,i} \leq z_{p,h,d,i} + z_{p,h,d,i-1} \qquad \forall p,h,d,i > 1 \qquad (49)$$

$$\sum_{\forall b} z_{p,h,d,b} = 1 \qquad \forall p,h,d \qquad (50)$$

The following constraints ensure that the amounts of cargo sent can be directly obtained from the interval where the piecewise convex function is active. In this set of equations parameters, q and r represent the threshold amounts that correspond to the cost breaks at the first and second echelons, respectively:

$$X_{p,s,h,d} = \sum_{\forall b} \delta_{p,s,h,d,b} q_{s,h,b} \qquad \forall p,s,h,d \qquad (51)$$

$$Z_{p,h,d} = \sum_{\forall b} \eta_{p,h,d,b} r_{h,b} \qquad \forall p,h,d \qquad (52)$$

In (51) and (52), the shipment amounts are represented by convex of the threshold amounts. Now, total costs can be derived by means of the following constraints:

$$C_{p,s,h,d} = \sum_{\forall b} c_{p,s,h,d,b} X_{p,s,h,d,b} \qquad \forall p,s,h,d \quad (53)$$

$$L_{p,h,d} = \sum_{\forall b} l_{h,d,b} Z_{p,h,d} \qquad \forall p,h,d \quad (54)$$

4.2.3 Delivery Time Window

The time window determines the maximum time that may elapse between pickup of products from the shippers and their final delivery to the end customer. Under the current setting, there is no penalty for being early, so products can be delivered any time within the time window. On the other hand, tardiness is not allowed. The restrictions imposed by the time window render the transportation options and transportation times between the shippers and the terminals critical. A feasible transshipment schedule identifies the transportation schedule, shipper-terminal pairs, and the time spent in the terminals that result with final deliveries within prescribed time windows. Consequently, the optimization model must incorporate the time windows into the flow-balance constraints as follows:

$$\sum_{\forall h,d \in [i,i+t]} X_{p,s,h,d} \geq D_{p,s,d} \qquad \forall p,s,d \quad (55)$$

$$\sum_{\forall p} U_{p,h,d} \leq k\ T_{h,d} \qquad \forall h,d \quad (56)$$

$$\sum U_{p,h,d} + Z_{p,h,d} + I_{p,h,d+1} = \sum_{\forall s,m} X_{p,s,h,d} + I_{p,h,d} \qquad \forall p,h,d \quad (57)$$

$$\sum_{\forall h} U_{p,h,d} + \sum_{\forall h} Z_{p,h,d-t} + N_{p,d} = \sum_{\forall s} D_{p,s,d-t} + N_{p,d+1} \qquad \forall p,d: t \leq d \leq \Omega + t \quad (58)$$

The constraint in (55) ensures that the demanded amount for a product from a given shipper at a given period is picked and shipped to the terminals. Inequality given in (56) ensures that the consolidated inbound shipments consolidated into containers do not exceed the total capacity of the containers. The constraint (56) establishes the flow balance between the inbound and outbound shipments at a terminal on a given period. Finally, constraint (57) guarantees that the deliveries from the terminals to the end customer occur within the given time window.

Depending on the context, the flow amounts and container capacities can be measured in weight and/or volume units. The real-life application that has motivated this study focuses on weights as the volume is not a limiting factor. However, the volume based constraints and variables can be easily incorporated to the model in similar fashion.

4.2.4 Nonnegativity and Integrality

Finally, the following constraints are employed to enforce nonnegativity bounds and integrality constraints:

$$y_{p,s,h,d}, z_{p,h,d,b} \in \{0,1\} \qquad \forall p, s, h, d, b \tag{59}$$

$$T_{s,h,d} \geq 0, \text{ integer} \qquad \forall s, h, d \tag{60}$$

$$C_{p,s,h,d}, L_{p,h,d,b}, I_{p,h,d}, \delta_{p,s,h,d,b,}, \eta_{p,h,d,b}, X_{p,s,h,d}, Z_{p,h,d}, U_{p,h,d}, N_{p,d} \geq 0 \qquad \forall p, s, h, d, b \tag{61}$$

Clearly, the above model includes binary variables employed to capture the piecewise nature of the shipment costs and integer variables that represent the number of containers used at terminals. Thus, the objective function and the constraints given in (1)–(20) result with a mixed-integer programming model. As will be discussed later, obtaining exact solutions to realistic size problems through conventional commercial optimizers in reasonable times is quite difficult, if not impossible, for this problem. Therefore, an efficient methodology is developed and introduced in the following section.

4.3 Solution Methodology

To assess the complexity of TCFCP-DS, we first consider the special case where the delivery time windows are sufficiently large so that all shipments can be held at the terminals for consolidation for as long as necessary. We observe that this problem can be reduced to a facility location problem with discrete capacity increment options represented by the container sizes when the LCL shipment costs are sufficiently high. This problem is shown to be NP-hard by Jacobsen (1990). As such, we can conclude that TCFCP-DS is also NP-hard. Consistent with this observation, our preliminary experiments with real-life problems reveal that the computational time requirements geometrically increase with the number of terminal locations and the planning period. The planning horizon for real-life problems in the 3PL context typically extends to a whole year. For most of the real-life problem instances, our attempts to solve the TCFCP-DS using CPLEX 12.0 on Intel (R) Xeon (R) CPU Es-268 WO @ 3.10 GHz (dual processor) with 64 GB RAM machines have failed to produce any solutions. The problem has an approximate total of 10,046 million variables and 231 million constraints (as a result of having 722 products, 104 suppliers, 3 consolidation terminals, and 365 days, 4 cost breaks, 5 zons). As discussed in Section 4.2, our solution relies on three steps: container relaxation; decomposition; and cut generation.

Consequently, to obtain high-quality solutions for the realistic size TCFCP-DS within practical time frames, we propose a holistic heuristic method (CLR-D-C) implemented in three phases. The proposed phases include: the container load relaxation phase; the decomposition phase; and the valid cuts phase.

The first phase is implemented so as to obtain a lower bound for the given problem instance. The relaxed solution is used to initialize the problem's variables. The second stage is designed to divide the problem into subproblems that can be solved independently using mixed-integer linear programming. The decomposition over time segments allows us to reduce the computational times significantly. To achieve decomposition, we take advantage of potential time gaps and cycles in the problem's structure. In the final stage, the concept of valid cuts that constrain the number of containers to be used is employed to further reduce the computational times. The cuts help us limit the solution space without eliminating the sets of potential optimal solutions to the subproblems generated in the second phase. The following section explains these three phases in detail.

4.3.1 Phase 1: Container Load Relaxation (CLR)

The first phase addresses the TCFCP-DS by employing a linear programming relaxation, and provides a lower bound, which is used to initialize the decomposition phase that follows. In this stage, the integrality conditions of the container variables are relaxed and the following objective function is used:

$$Min\ \varpi = \sum_{\forall p,s,d,h} c_{p,s,d,h} X_{p,s,h,d} + \sum_{\forall p,d,h} l_{p,d,h} Z_{p,h,d} + \sum_{\forall d,h} \bar{c}_h T_{h,d} - n \sum_{\forall p,d,h} I_{p,h,d}\ \forall s,h,d \tag{62}$$

The above objective function introduces an incentive to ship each order on its latest possible date from the terminals h to the customer where n is a small positive real number. The use of n when properly scaled creates an incentive in the model that encourages the retention of inventory and leads the relaxed model into larger consolidation of shipments. In the case that this term is not included into the model because of the relaxed nature of the integrality constraint for the FCL shipments, the model may consolidate each individual shipment into a fraction of an FCL shipment and not reach a practical solution, where multiple shipments are consolidated together.

Clearly, the optimal solution to this relaxed version of the problem provides a lower bound. The binary variables that are utilized to capture the piecewise nature of the shipment costs are not included in this relaxation. The solution to the CLR model, with objective function (62) and constraints (42)–(61), provides an adequate initial solution for the second stage. Since the variable $T_{s,h,d}$ has been relaxed, every single shipment of the CLR problem is made using this relaxed container variable. In addition, the solution to the relaxed problem is given by real positive container values, and indicates initial candidates for consolidation. This solution helps us eliminate degeneracy and potential multiple solutions in the problem.

4.3.2 Phase 2: Decomposition

The second phase is designed to decompose the problem into subproblems that can be solved independently using mixed-integer linear programming. To achieve this, we take advantage of potential time gaps and demand cycles in the problem's structure.

We calculate the breakeven cost point (F_h) (typically in weight units) for each terminal h by:

$$F_h = \frac{\bar{c}_h}{l_h} \Big/ k \tag{63}$$

where $\bar{c}_h$ is the cost of shipping one container from terminal h; l_h is the cost of shipping one pound as parcel from terminal h to the end customer; and k is the container capacity.

Notice that the breakeven cost point expresses the percentage that a container needs to be filled such that its shipping cost is equal to the cost of shipping the items individually (represented by the variable $Z_{p,h,d}$).

Starting from day one, we construct a block of size equal to the time window of the problem. For every single day in this block, and for every terminal we compare the breakeven cost points to the percentage of containers that can be shipped on a specific day. We calculate this percentage by dividing the lagged sum of the obtained relaxed solutions $U_{p,h,d}$ from Step 1 to the container capacity. For example, for the block from day 1 to day tw, the lagged sum of day 1 is $\sum_p U_{p,h,d}$, for day 2 $\sum_{d=1}^{2} \sum_p U_{p,h,d}$, until day tw $\sum_{d=1}^{tw} \sum_p U_{p,h,d}$. Notice that for any given day, if the obtained percentage value is less than the breakeven point, no container is shipped on that day. In general, if for all the days in the block and all the terminals every calculated percentage value is less than their corresponding breakeven-cost points, the problem of the starting day (day 1 in this case) can be viewed as independent; thus the day is removed from the original problem and placed on a bucket of independent suproblems. However, if this condition does not hold, the exploration needs to continue. The block is marked as the initial period of a potentially smaller subproblem (smaller in comparison to the original problem); and a new block is constructed starting from day $tw + 1$ to $2tw + 1$. The comparison process is repeated for this new block, such that if day $tw + 1$ can be viewed as an independent problem, it is removed from the original problem. Otherwise, the block $tw + 1$ to $2tw + 1$ is added to the block previous block and the exploration continues.

Theorem 1. For any period of length $\tau > 2t$, where the total cargo demanded is not enough to surpass the breakeven cost of a full container load shipment, the cargo from the first $\tau - t$ days will not be consolidated and may be solved independently from the rest of the problem and thus a subproblem is generated with these days; and the global optimal solution is not affected.

Proof of Optimality

Let O^* be the optimal solution of problem $\mathcal{S}$ with optimal cost $C^* = X^* + U^* + Z^*$. Moreover, let $\mathcal{S}_1, \mathcal{S}_2$ be subproblems of $\mathcal{S}$ with optimal solution of problem $\mathcal{S}$ with optimal cost $C_i^* = S_i^* + U_i^* + Z_i^*$ for $i = 1{,}2$. If $\mathcal{S}_1, \mathcal{S}_2$ are subproblems obtained using our method, then $C^* = C_1^* + C_2^*$. In order to complete the proof we need to analyze two conditions:

Condition 1: The inequality $C_1^* + C_2^* < C^*$ never holds. Let us assume that $C_1^* + C_2^* < C^*$. In this case, there would exist a $\hat{C}^* = C_1^* + C_2^*$, which is an optimal cost for problem $\mathcal{S}$. Nonetheless, this is not possible since C^* is the optimal cost by definition (contradiction). Then, condition (1) holds.

Condition 2: the inequality $C_1^* + C_2^* > C^*$ never holds. Since the cost structure is composed of three elements, we need to analyze the three potential sources of variation.

$X_1^* + X_2^* > X^*$ with $U_1^* + U_2^* = U^*$ and $Z_1^* + Z_2^* = Z^*$

For the same length of the subproblems $\mathcal{S}_1, \mathcal{S}_2$, we can rewrite the optimal cost from the shippers as $X_1^* + X_2^* > X^* = \hat{X}_1^* + \hat{X}_2^*$. Furthermore, let us assume that the cost difference is only generated because $X_1^* > \hat{X}_1^*$. This condition implies that there exist at least o_{ijdh} a shipment of product i from shipper j on day d that went into terminal h such that:

$U_1^* + U_2^* > U^*$ with $X_1^* + X_2^* = X^*$ and $Z_1^* + Z_2^* = Z^*$

However, we can easily see that packages at day t_1 and t_2 from subproblems $\mathcal{S}_1$ and $\mathcal{S}_2$ cannot be merged together, given that package at t_2 was shipped by container. Also, packages at day t_1 and t_2 from subproblems $\mathcal{S}_1$ and $\mathcal{S}_2$ cannot be merged together, given that package at t_2 is shipped by pound.

$Z_1^* + Z_2^* > Z^*$ with $U_1^* + U_2^* = U^*$ and $X_1^* + X_2^* = X^*$

The above cannot be true due to contradiction by costs and definitions, which follows from the first argument.

4.3.3 Phase 3: Valid Cuts

The final stage uses a cutting method to solve each of the individual subproblems that were generated in the second stage. Here, given the information from the two previous stages, a large subset of variables may be equaled to zero *a priori*, and are thus cut from the solution space. These are instances in which we know that there may not be any consolidation and for these days the container variables are zeroed out.

Further, we can include restrictions where the number of containers that may be consolidated within each time window is bounded above by the optimal solution from the linear programming relaxation. This procedure speeds up the process of finding the optimal solution by introducing additional constraints to the problem that limit the solution space, without eliminating the sets of potential optimal solutions. Once each subproblem has been optimized, the solutions are aggregated and a final solution for the time constrained freight consolidation problem with multiproduct divisible packages problem is achieved.

The final stage uses a cutting method to solve each of the individual subproblems that were generated in the second stage. We include a cutting restriction where the number of containers that may be consolidated within each time window is bounded below by the optimal solution from the linear programing relaxation, and an upper bound cut on the number of containers that can be shipped by day by terminal is also used. This procedure speeds up the process of finding the optimal solution by introducing additional constraints to the problem that limit the solution space without eliminating the sets of potential optimal.

Moreover, given the information from the two previous stages a large subset of variables may be equaled to zero *a priori*. Once each subproblem has been optimized, the solutions are aggregated and a final solution for the time constrained freight consolidation problem with multiproduct divisible packages problem is achieved.

4.4 Freight Consolidation Solution Algorithm

The overall algorithm is presented in *Table 9*.

Table 9. *CLR-D-C Algorithm*

Steps
1. Solve a modified LP relax problem of the original problem introducing a penalization to ship each order on its latest possible date from the terminals h to the customer $$z = \sum_{\forall p,s,d,h} c_{p,s,d,h} + \sum_{\forall p,d,h} l_{p,d,h} + \sum_{\forall d,h} \bar{c}_h T_{h,d} - n \sum_{\forall p,d,h} I_{p,h,d}$$ where n is a small positive real number.
2. Calculate the cost breakeven point for each terminal h $$F_h = \frac{\bar{c}_h}{l_h} \Big/ k$$
3. Initialize $s^* = 1$ starting at the first day of the problem
4. Let $s^{**} = s^* + tw$
5. Let $s^{***} = s^*$
6. If $s^{***} = 365$, all the days in a year have been explored, go to Step 12
7. Make the subproblem $S = \{\text{start} = s^{***}, end = \text{start} + tw\}$
8. Using the solution from Step 1, calculate the ratio between the total amount to be shipped from terminal h for the subproblem S and the container capacity

$$\Upsilon_{h,d=s^{***}} = \frac{\sum_{\forall p} \sum_{d=s^*}^{s^*+tw} U_{p,h,d}}{cap}$$

9. If $\Upsilon_{h,s^{***}} \geq F_h$, let $s^{***} = end + 1$, $s^{**} = s^{***} + tw$ and go to Step 6
10. If $\Upsilon_{h,s^{***}} < F_h$ and $s^{***} \neq s^{**}$, set $s^{***} = s^{***} + 1$ and go to Step 6
11. If $\Upsilon_{h,s^{***}} < F_h$ and $s^{**} = s^{***}$, an independent subproblem i has been found. Separate $S_i = \{s^*, end\}$ from the complete problem. Let $s^* = end + 1$ and go to Step 4
12. Solve each subproblem as a MIBLP problem introducing the cut $T_{h,d} \leq round(1 + \Upsilon_{h,s^{***}})$

FROM ZONE	TO ZONE 1	2	3	4	5	6	7	8	9	10	11	12	13
1	A1	B2	C1	C1	B3	C2	B2	C2	D3	C3	D3	D3	D4
2	B2	A1	B1	D2	B3	C2	C2	C2	D4	D4	D4	D4	D4
3	C1	B1	A1	C1	D2	D2	D2	D2	D4	D4	D4	D4	D4
4	C1	D3	C1	B2	E1	D1	D3	C2	E2	E2	E2	E2	E2
5	B3	B3	D2	E1	A1	B2	B2	B3	B3	B2	C3	C3	C3
6	C2	C2	D2	D1	B2	A1	B1	B1	B2	B3	B3	B3	B3
7	B2	C2	D2	E	B2	B1	A1	B1	C2	B3	C3	C3	C2
8	C2	C2	D2	C2	B3	B1	B1	A1	B3	C3	B3	B3	B3
9	D3	D4	D4	E2	B3	B2	C2	B3	A1	B1	B3	B2	B1
10	C3	D4	D4	E2	B2	B3	B3	C3	B1	A1	C3	C3	B2
11	D3	D4	D4	E2	C3	B3	C3	B3	B3	C3	A1	A2	B2
12	D3	D4	D4	E2	C3	B3	C3	B3	B2	C3	A2	A1	B2
13	D4	D4	D4	E2	C3	B3	C2	B3	B1	B2	B2	B2	A1

Figure 19. Zone matrix.

4.5 Case Study

As a case study, we consider again the 3PL service provider. The logistics provider is working on an annual bid for 3PL services from a pharmaceutical manufacturer located in Puerto Rico. The contract contains the expected conditions for 3PL services, including product details (722 products), scheduled shipments, locations (104 locations in 25 states), and due dates. The characteristics of the products are such that a linear relationship between volume and weight may be assumed. The logistics provider aims to minimize the cost at which it may satisfy the customer's service requirements for the 3PL service.

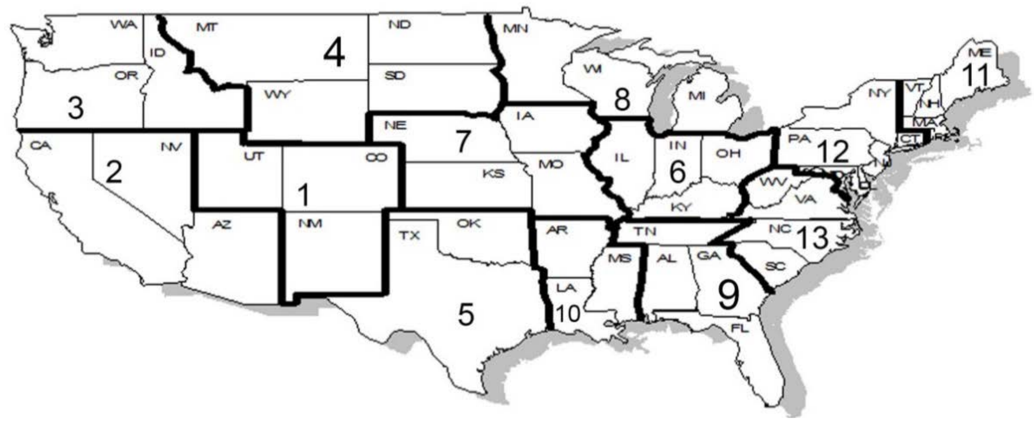

Figure 20. Zoning within the USA

Table 10. *Shipment Time and Cost Based on Zoning and Zone Matrix*

	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	D4	E1	E2
Shipment Time (In Days)														
Time	2	2	3	3	3	4	4	4	5	5	5	5	6	6
Fixed Cost Per Shipment (In Dollars)														
Cost	80	80	80	80	80	80	80	80	80	80	80	80	80	80
Variable Cost Per Shipment (In Cents)														
0-99 lbs.	29	33	33	37	41	37	41	44	37	41	44	46	44	46
100-999 lbs.	27	31	31	35	38	35	38	42	35	38	42	44	42	44
> 1000 lbs.	25	29	29	33	36	33	36	40	33	36	40	42	40	42

The logistics provider has three consolidation centers: Elizabeth, NJ; Miami, FL; and Jacksonville, FL. It utilizes these locations to consolidate multiple inbound shipments into containers before shipping them to Puerto Rico. Table 12 shows the shipping costs and transit times for the overall network. We note that there is a minimum cost of $80 for each shipment with three cost rates across three quantity regions.

In the logistics provider's optimization problem we have $n = 722$, $k = 3$, $m = 1$, and $t = 365$. That way, it may find what products to ship through what consolidation center so that the transportation cost is minimized, while meeting all of the due date constraints.

The resultant MIP has more than 27.5 million variables and 9.2 million constraints. All attempts at solving the MIP failed on account of lack of sufficient computational resources, using CPLEX on an Intel (R) Xeon (R) CPU Es-268 WO @ 3.10 GHz (dual processor) with 64 GB RAM machine. Our proposed methodology from Section 3 (price linearization, decomposition, and cut generation) was applied with the results presented below.

4.5.1 Data

All real-life applications are bounded by the availability of accurate data. For our case study, we are using data from a healthcare company that uses a 3PL services supplier to transport healthcare supplies and products to their final destination. This dataset consists of 704 products, 104 local 3PL shipper facilities, 3 consolidation terminals, and 365 days. Furthermore, data is available for the ground and maritime shipping rates both in terms of pounds (LCL) and full containers (FCL), detailed daily customer demand, and container capacity. Finally, the specified time window for this case study is nine days, such that every shipment must arrive within its own time-window.

While the provided data had a long list of advantages, it also included limitations, especially regarding the thoroughness of the shipment volumes. Because of these restrictions, during the development of our framework, we were unable to include constraints based on shipment volume and had to only use the weight of the shipments as a parameter for the shipping rates. The data available for our case study does not consistently include the volume of the shipments, such that 54% of the data regarding the volume of the shipments is blank. Therefore, to avoid incurring in unjustifiable assumptions that may be unrealistic, we have decided not to include a volume restriction at this stage. We are aware that volume restrictions are important to consider and may significantly affect the optimal shipping schedule; however, our model will still provide a sound solution for the 3PL freight consolidation optimization. Descriptive statistics for the volume and weight characteristics of each shipment are shown in Table 13, while Figure 21 shows the shipments from their sources through the consolidation terminals (in green) and from the consolidation terminals to their final destination (in black).

Table 11. *Descriptive Statistics for Weight and Volume Data—3PL Problem*

Variable	Min	1st Qu.	Median	Mean	3rd Qu.	Max.	N. Obs	Missing
Volume (ft^3)	0	44.81	161	323.4	500.3	3943	722	395
Weight (lb)	15	417.2	1414	3028	4520	40000	722	0

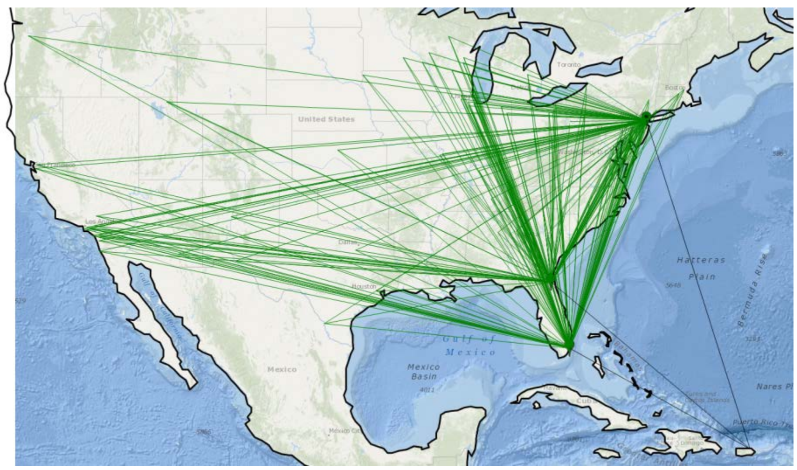

Figure 21. Sources of cargo

The cargo that is being handled at the three consolidation terminals originates from 26 different states, with five major sources that represent more than 50% of the total cargo, as shown in Table 12 and Figure 22 .

Table 12. *Total Shipper Origins and States*

Origin State	Total	Origin State	Total	Origin State	Total
AL	1	MA	27	PA	63
AZ	19	MD	3	SD	9
CA	98	MN	3	TN	1
DL	5	MO	10	TX	22
FL	8	NC	24	UT	5
GA	48	NJ	37	VA	1
IL	91	NM	5	WI	59
IN	72	NY	33	Grand Total	722
KY	37	OH	41		

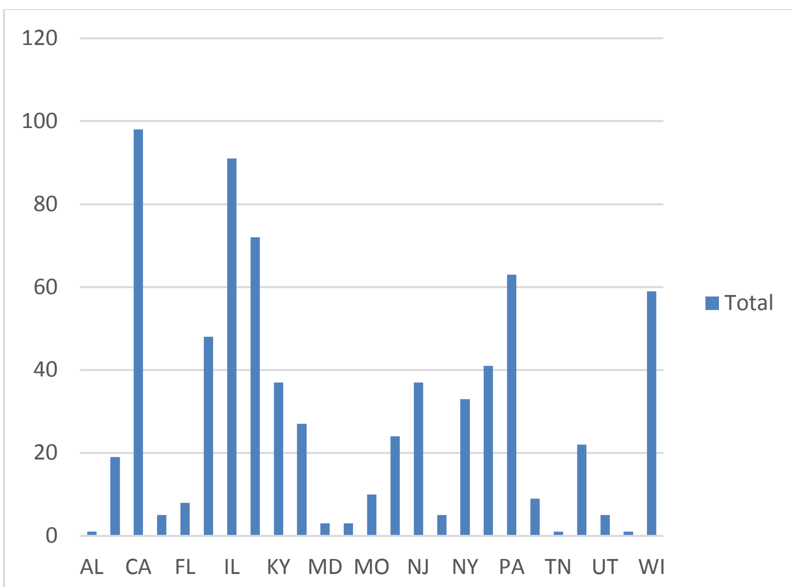

Figure 22 . Total shipper origins and states.

4.5.2 Price Linearization

Given the cost of a full container and the rate per 100 pounds, it can be seen that there exists a threshold (in lbs.) up to which the shipping cost in a container increase linearly as a function of weight, and beyond, which the shipping cost per container is fixed. These levels are presented as threshold percents in Table 4. In the first step, we 1), convert this piecewise linear rate into a linear rate; and 2), determine the solution for the JIT delivery problem.

The price linearization of the model yielded the use of 45.38 containers, at cost of $841,342, and with a computation time of 265 seconds. This solution provides an upper bound on the number of containers that can be used without violating the delivery time window constraint.

4.5.3 Decomposition

We have 14 subproblems, where the largest subproblem includes 68 days. The maximum of time taken to solve the entire problem is determined by the time to solve each subproblem individually, as the other subproblems can be solved in parallel. The total solving time was of 161 seconds, with a maximum subproblem solving time of 46 seconds. Table 13 shows the start and end day of the 14 subproblems.

Table 13. *Decomposition of the Monolithic Problem into Subproblems*

Subproble m	Start day	End day	# day
SP1	1	39	38
SP2	40	98	58
SP3	99	109	10
SP4	110	138	28
SP5	139	145	6
SP6	146	151	5
SP7	152	220	68
SP8	221	246	25
SP9	247	269	22
SP10	270	290	20
SP11	291	328	37
SP12	329	339	10
SP13	340	348	8
SP14	349	365	16

4.5.4 Cut Generation

The use of cut generation relied on the solutions from the prize linearization and decomposition in order to determine that the number of containers to be consolidated in subproblems 3, 5, 6, 12, and 14, was 0—as all of the shipments were done individually and could be done immediately as soon as they reached the consolidation centers. The effectiveness of these cuts can be seen in the computational time of these subproblems, which was never past one second. The details of the solution are shown in Table 14.

The total cost comes to \$890,183, which is composed of \$692,117 from sending the products to the consolidation centers; \$80,122 from individual product shipments from the consolidation centers to Puerto Rico; and \$117,944 from sending 34 consolidated full container loads from the consolidation centers to Puerto Rico. It is important to highlight that 74.92% of all the products were held for consolidation at the centers. The individual shipping of 25.08% of the products was at an effective rate of 103% more than that of the consolidated 74.92% of products.

Table 14. *Cost Breakdown Solution of the Case Study*

Subproblem	Start day	End day	Days	C (\$)	L (\$)	$\bar{C}$(\$)	Total Cost(\$)	CPU Time	Number of Containers
SP1	1	39	38	83,862	10,110	13,552	107,523	46	4
SP2	40	98	58	142,649	7,584	32,327	182,560	40	9
SP3	99	109	10	10,433	3,778	0	14,211	1	0
SP4	110	138	28	43,954	1,816	10,164	55,934	1	3
SP5	139	145	6	11,225	4,549	0	15,774	1	0
SP6	146	151	5	2,208	874	0	3,082	0	0
SP7	152	220	68	136,987	10,829	27,104	174,920	35	8
SP8	221	246	25	56,644	8,676	6,776	72,096	3	2
SP9	247	269	22	29,079	5,535	3,388	38,001	1	1
SP10	270	290	20	49,586	6,423	6,776	62,785	2	2
SP11	291	328	37	79,537	7,835	14,470	101,841	29	4
SP12	329	339	10	10,725	4,384	0	15,109	0	0
SP13	340	348	8	15,075	1,098	3,388	19,561	0	1
SP14	349	365	16	20,153	6,630	0	26,784	1	0
Total				692,117	\$80,122	117,944	890,183	161	34

4.6 Computational Study

A computational study was conducted by keeping the demand pattern the same but by changing the following:

- The delivery time window (7, 8, 9, 10, and 11 days).
- The shipping costs (5%, +/- 10%, +15% and +20% of the container breakeven cost) using a time window of nine days.
- The shipping costs (to +/- 5%, +/- 10%, +15% and +20% of the actual cost) using a time window of eight days as the 3PL.

These manipulations change the pressure, as well as the opportunity to consolidate. For example, when the delivery time window increases, there is a greater opportunity to consolidate and we can expect the objective function values to decrease. Similarly, when the shipping cost decreased, there is a greater value achievable from consolidation.

4.6.1 Time Window Sensitivity

In these scenarios, the time window was changed from the original eight days used in all of the other scenarios, to be between 6 days and 11 days, while the breakeven cost remains at the same levels. In the first of these scenarios, our proposed MLP model generated 30 subproblems, and the effective consolidation conditions were met 17 times at the consolidation terminal from where 28 containers were shipped.

In this scenario, the total cost increased by 7% to $938,851, and the total number of containers used for shipping decreased from 39 to 28. This scenario yields total increase of $62,241; this is driven by increased shipments to the consolidation terminals, and of individual shipments from the terminal. These increases are not offset by the decrease in the cost of consolidated shipments of $39,102. The details for each subproblem of these scenarios are shown in Table 27 in Appendix C.

In the second scenario, a time window of seven days was implemented. In this case the total cost increased by 2% to $925,303 and the total number of containers used for shipping decreased from 31 to 29. This scenario yields a total increase of $43,371, which is driven by increased shipments to the consolidation terminals, and of individual shipments from the terminal. The details for each subproblem of these scenarios are shown in Table 37 in Appendix C.

In the scenario with the nine-day time window, the total cost only changed by $23, with an increase in FCL container shipments of $12,634 that was offset by the decrease in individual shipments. In this case, 13 scenarios were generated, and consolidation occurred in nine of them. The details for each subproblem of these scenarios are shown in Table 38 in Appendix C.

In the fourth scenario, with a time window of 10 days, the total cost decreased by 1% to $869,528, and the total number of containers used for shipping increased from 39 to 40. This scenario yields total cost decrease of $7,059, which is driven by decreased shipments to the consolidation terminals, and of individual shipments from the terminal; these decreases account for a reduction in cost of $28,332, and are not offset by the increase in the cost of consolidated shipments of $16,940. The details for each subproblem of these scenarios are shown in Table 39 in Appendix C.

In the final scenario, a time window of 11 days was used, and the total cost decreased by 2% to $862,804, with a total of 41 consolidated FCL shipments. This scenario yields total cost decrease of $29,134.55; this is driven by $36,828.05 of savings in the shipments to the consolidation terminals, and of individual shipments from the terminal. These are not offset by the increase in the cost of consolidated shipments of $13,783. The details for each subproblem of these scenarios are shown in Table 38 in Appendix C, while the summary is shown in Table 40.

Table 15. *Scenario Result of Changing Time Window*

Scenario	Elapse time	Total container	# subproblem	Objective function
6 days	27	28	30	$938,851
7 days	70	29	13	$925,303
8 days	161	34	14	$890,183
9 days	20,687	39	13	$876,609
10 days	119,165	40	6	$869,528
11 days	112,931	41	5	$862,804

Furthermore, we found that reducing the time window to 6 days is the lowest we can guarantee for on-time delivery; and this also leads us to generate more subproblems, and find the optimal solution with more ease.

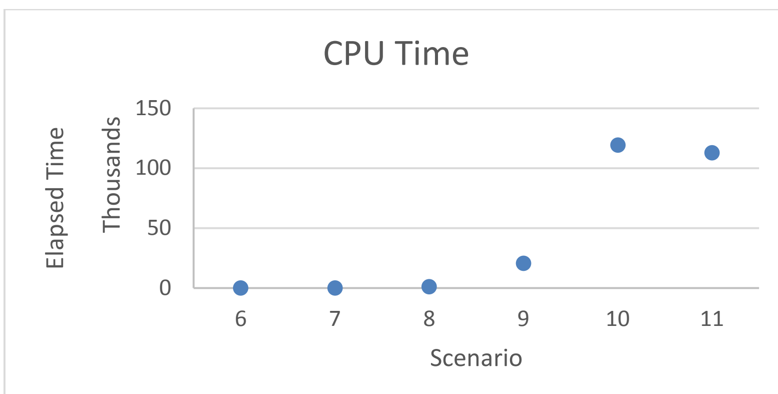

Figure 23. Time window vs. CPU time.

However, with time window larger than 11 days, the amount of cargo sent within each potential subproblem could always be consolidated, and our MLP could not provide any cuts while guaranteeing that the optimal solution would not be cut from the search space. That has a direct impact of enormous reducing the elapsed time, and fewer chances to FCL. This is shown in Figure 23 through Figure 25.

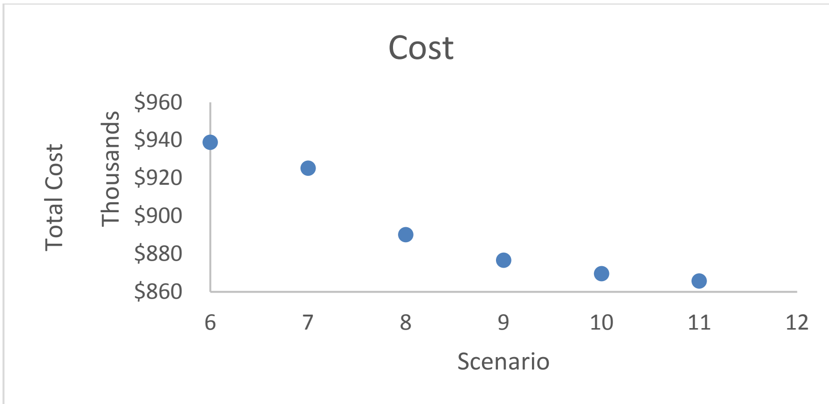

Figure 24. Time window vs. total cost.

As we reduce the time window that also impacts the total cost since there are fewer chances to ship FCL and thus the cost of individual shipments between the consolidation terminals and the final destination increased.

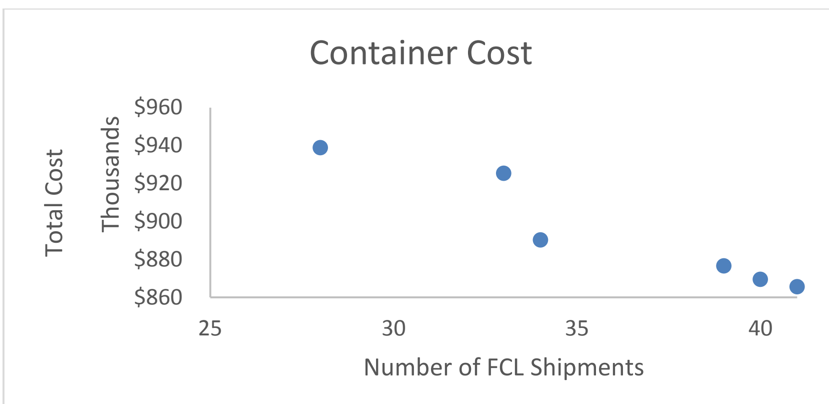

Figure 25. Total cost to container chart

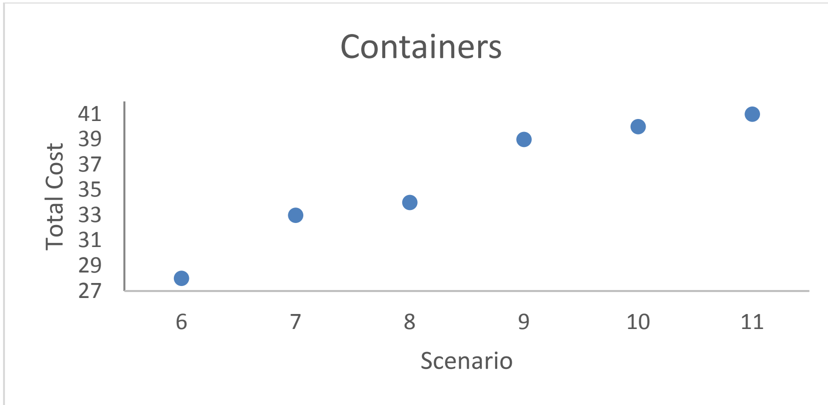

Figure 26. Number of containers to scenarios.

4.6.2 Container Breakeven Cost

Here, we investigate the impact of container breakeven cost on the computational performance. In our analysis, we consider the case where the time window is 9 days. As shown in Table 16, in the first scenario (-10%), our proposed BMILP model generated only eight subproblems, the total cost decreased by 2.1%, and the total number of containers used for shipping increased from 39 to 46. This may be attributed to reduce the individual shipment.

In the second scenario, the breakeven cost for the containers was decreased by 5% of its original point at the consolidation terminals. Our proposed BMILP model also generated eight subproblems, like in the scenario with the 10% decrease, although these subproblems did not include the same number of days in every case. The total cost decreased by 1.2%, and the total number of containers used for shipping increased from 39 to 40. This scenario yields total savings of $10,198.93, 63% of which may be attributed to reduced consolidated shipment costs.

Table 16. *Result of Cost-Breakdown Solution with Nine-Day Time Window*

Scenario	Time Window	Elapse time	Total container	# subproblem	Objective function	CLR Objective	GAP
(-10%)	9	167,507	46	8	$858,371.67	$816,306	4%
(-5%)	9	710,43	40	8	$866,410.54	$828,555	5%
3PL	9	207,64	39	13	$876,609.47	$840,648	4%
(+5%)	9	3,763	32	12	$891,516.21	$853,665	4%
(+10%)	9	3,69	30	13	$896,962.85	$840,621	7%

In the third scenario, the breakeven cost for the containers was increased by 5% of its original point at the consolidation terminals. In this scenario, our proposed BMILP model generated 12 subproblems, where 32 containers were shipped. Here, the total cost increased by 1.7% to $891,516.21, and the total number of containers used for shipping decreased from 39 to 32. This scenario yields a total cost increase; this is driven by an increase of individual shipments which isn't fully offset by the decrease in the cost of consolidated shipments.

In the fourth scenario, the breakeven cost for the containers was increased by 10% of its original point at consolidation terminals. In this scenario our proposed BMILP model generated 13 subproblems and the total number of containers used for shipping decreased from 39 to 30. This scenario yields total increase of $20,353.38, which is driven by an increase of individual shipments of $55,947.08, which isn't offset by the decrease in the cost of consolidated shipments of $13,387.50.

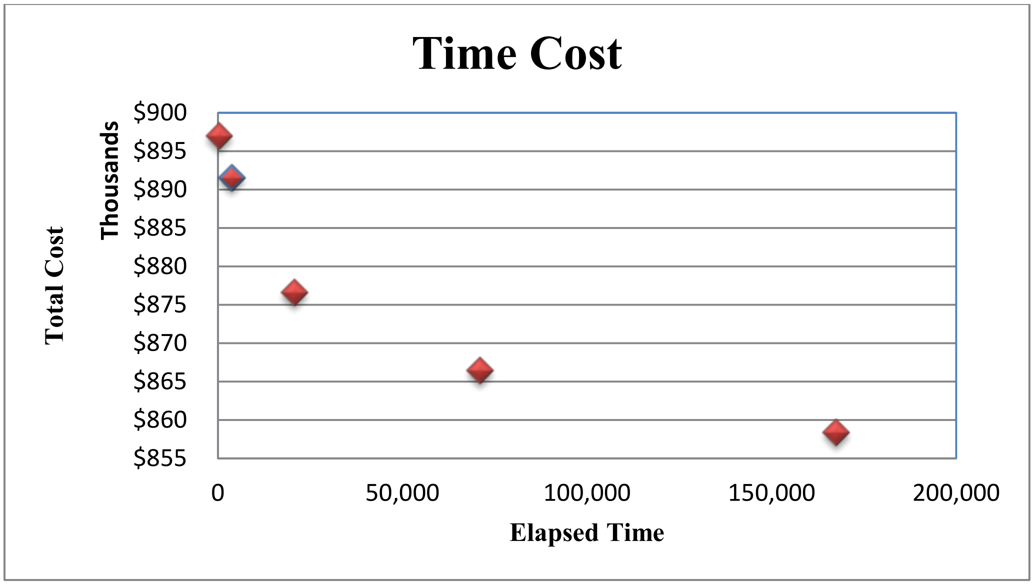

Figure 27. Time to cost chart

Figure 27 shows the relation between total cost and total elapsed time. In these scenarios, we find that reducing the total cost resulted in more time elapsed. This may be attributed to reduced individual shipment costs while shipping more FCL. Shipping more FCL will lead the total cost to reduction, which can be attributed directly to decrease the individual shipments from the consolidation point to the final customer.

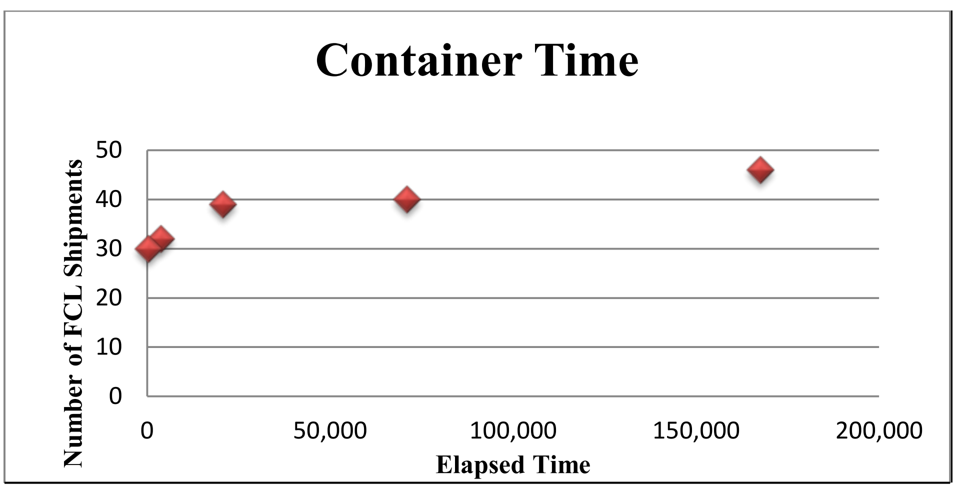

Figure 28. Container to elapsed time chart

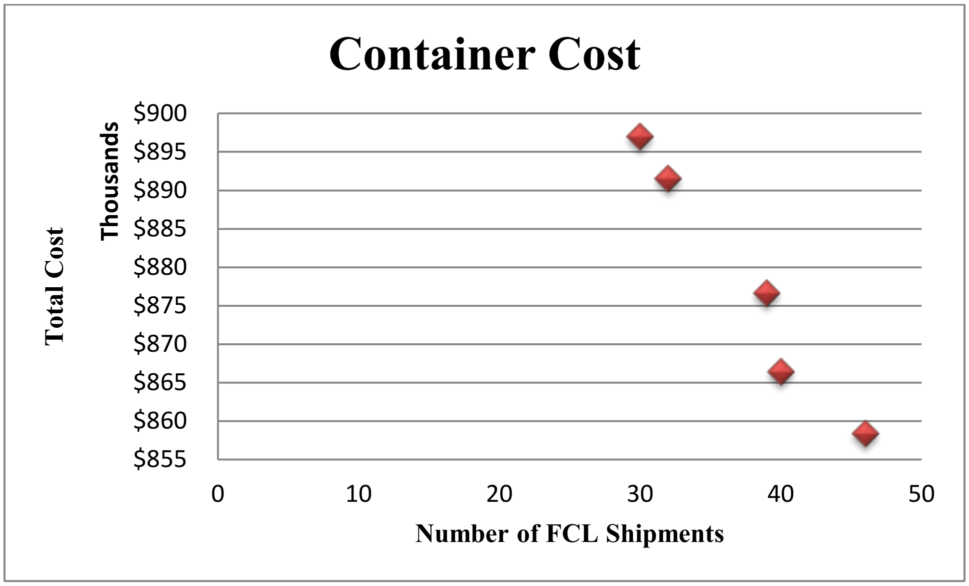

Figure 29. Total cost to container chart

We have found that freight consolidation provides significant advantages for 3PL providers for cost reduction, while meeting customer demands and time windows completely. The use of freight consolidation offers the 3PL provides the opportunity to consolidate cargo and use 39 FCL shipments reaching 82.65% of the lower bound from the linear relaxation. Furthermore, we have discovered that reducing the delivery time windows from 9 days to 6 days leads to a total cost increase of 7.1%, which is distributed into an increase of 3.74% for shipments to the consolidation terminals, and 19.7% in the shipments from the consolidation terminals to the final destination; furthermore, under this scenario, the use of FCL shipments for consolidation drops by 25%.

When evaluating the impact of the FCL shipping rates on the total transportation costs, we have found that reducing the breakeven cost for consolidation by 10% leads to a total cost decrease of 2.08%, which can be attributed almost entirely to a cost decrease of

32.8% in the individual shipments from the consolidation stations to the final customer; with this cost change, the use of FCL shipments for consolidation increases by 17.95%.

When increasing the breakeven cost for consolidation by 10%, there is a total cost increase of 2.32%, which may be attributed to a 80.28% cost increase in the individual shipments from the consolidation stations to the final customer, that is not offset in the cost decrease of 10.71% of consolidated shipments; in this case, the use of FCL shipments for consolidation drops by 23.08%.

We now consider the case where the time window is eight days so as to study the joint effect of time windows and container cost breaks. When the time window is reduced, the optimal number of containers to consolidate can change, and it provides us with different scenarios to evaluate the efficacy of the decomposition algorithm, and the cutting procedure.

In the first scenario (-10%), our proposed MLP model generated only 8 subproblems, the total cost decreased by 2.1%, and the total number of containers used for shipping increased from 39 to 46. This may be attributed to reduce the individual shipment. The details for each subproblem of this scenario are shown in Table 30.

In the second scenario, the breakeven cost for the containers was decreased by 5% of its original point at the consolidation terminals. Our proposed MLP model also generated six subproblems. The total cost decreased by 2%, and the total number of containers used for shipping increased from 31 to 37. This scenario yields total savings of $16,625, of which 29% may be attributed to reduced consolidated shipment costs. The details for each subproblem of this scenario are shown in Table 31

Table 17. *Scenario Result of Cost-Breakdown Solution for Eight-Day Time Window*

Scenario	CPU time w/cut	Total containers	# subproblems	Objective function with cut
(-10%)	3,207	40	8	$870,384
(-5%)	186	36	13	$881,008
3PL	161	34	14	$890,183
(+5%)	105	24	15	$906,729
(+10%)	76	19	18	$912,377
(+15%)	46	15	21	$917,267
(+20%)	32	11	22	$920,823

The first scenario (-10%): Our proposed MLP model generated only eight subproblems; the total cost decreased by 2%, and the total number of containers used for shipping increased from 34 to 40. This may be attributed to reduce the individual shipment 26%. The details for each subproblem of this scenario are shown in Table 30 in Appendix C.

In the second scenario, the breakeven cost for the containers was decreased by 5% of its original point at the consolidation terminals. Our proposed MLP model also generated 13 subproblems. The total cost *decreased* by 1%, and the total number of containers used for shipping increased from 34 to 36. The details for each subproblem of this scenario are shown in Table 31 in Appendix C.

In the third scenario, the breakeven cost for the containers was increased by 5% from its original point at the consolidation terminals. In this scenario our proposed MLP model generated 15 subproblems, where 24 containers were shipped. Here the total cost increased by 2% to $906,729, and the total number of containers used for shipping decreased from 34 to 24. This is driven by an increase of individual shipments of 57%, which isn't fully offset by the decrease in the cost of consolidated shipments. The details for each subproblem of this scenario are shown in Table 32 in Appendix C.

In the fourth scenario, the breakeven cost for the containers was increased by 10% of its original point at consolidation terminals. In this scenario our proposed MLP model generated 18 subproblems and the total number of containers used for shipping decreased from 34 to 19. This scenario yields total increase of 2%; this is driven by an increase of individual shipments of 85%, which isn't offset by the decrease in the cost of consolidated shipments of the decrease of 31%. The details for each subproblem of this scenario are shown in Table 33 in Appendix C.

In the fifth scenario, the breakeven cost for the containers was increased by 15% of its original point at consolidation terminals. In this scenario our proposed MLP model generated 21 subproblems and the total number of containers used for shipping decreased from 34 to 15. This scenario yields total increase of $27,083; this is driven by an increase of individual shipments of 111%, which isn't offset by the decrease in the cost of consolidated shipments of 43%. The details for each subproblem of this scenario are shown in Table 34 in Appendix C.

Finally, in the sixth scenario, the breakeven cost for the containers was increased by 20% of its original point at consolidation terminals. In this scenario, our proposed MLP model generated 22 subproblems and the total number of containers used for shipping decreased from 34 to 11. This scenario yields total increase of $30,640. The details for each subproblem of this scenario are shown in Table 35 in Appendix C.

Figure 6 shows the total cost for the different scenarios. As the breakeven cost increases, the total cost increases. Further, Figure 7 shows how as the breakeven cost increases, the number of consolidated shipments decreases. Additionally, the increase in the breakeven cost leads to more days that may be cut from the problem since they will not have any consolidated shipments, and thus the elapsed time for the model to complete the optimization decreases, as shown in Figure 29.throw Figure 32 shows how the total cost decreases as the number of FCL containers increases.

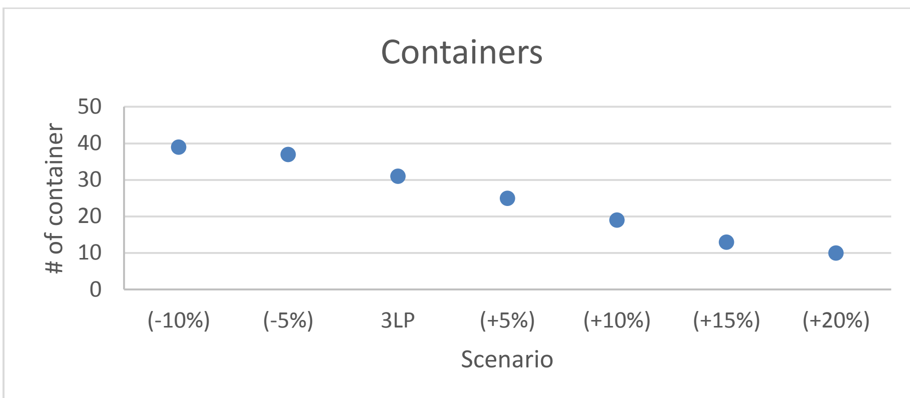

Figure 30. Total shipment cost

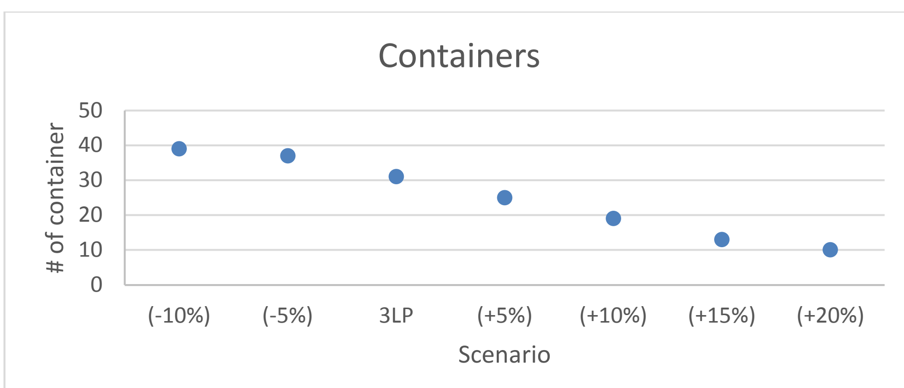

Figure 31. Consolidated shipment

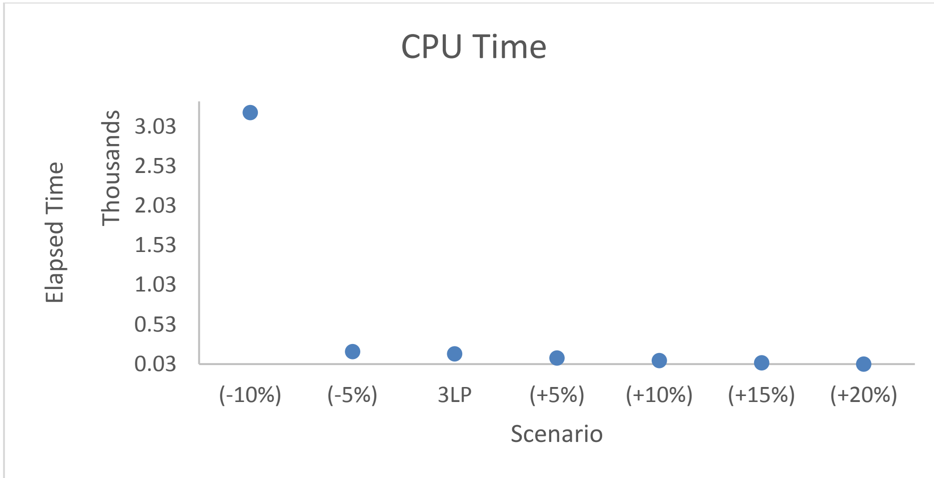

Figure 32. Elapsed time

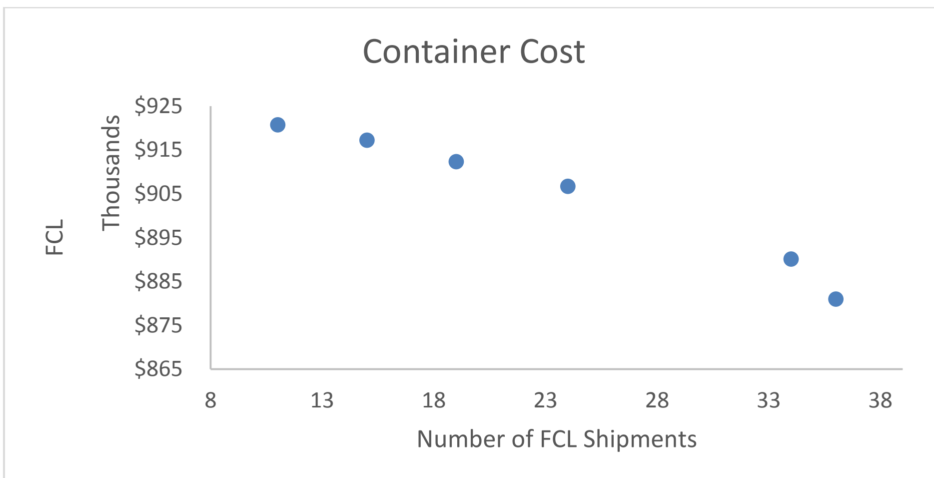

Figure 33. Cost vs. FCL shipments.

4.6.3 Decomposition

The test scenarios discussed above generated 25 cases, which we solved both with and without decomposition. These test cases, along with the computational time required to solve with and without decomposition, are presented in Figure 33. We observe that:

- All 25 cases yielded a very similar solution, indicating that the optimality is not impacted by the problem decomposition.

- The average speedup achieved due to the decomposition was 134.2 times. The speedup displayed a strong relationship with the problem size; the speedup increased quadratically with the size of the problem.

The largest problem that we could solve using the standard formulation (without decomposition) was a 218-day problem.

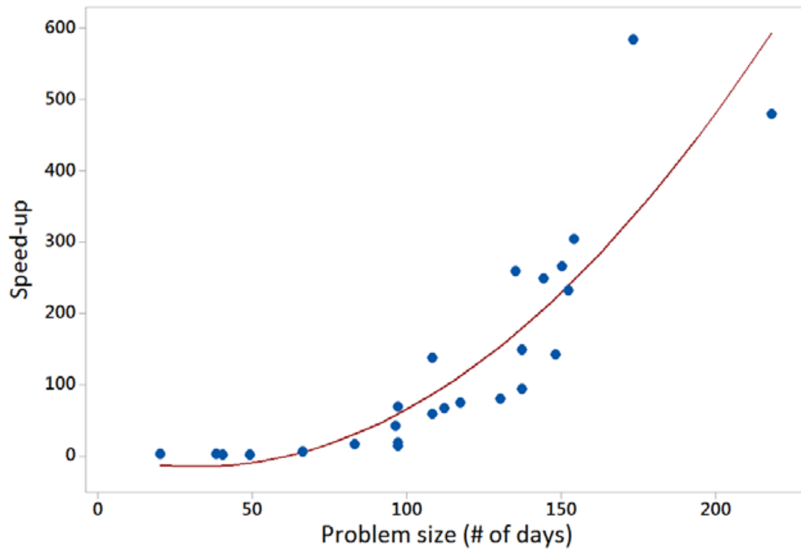

Figure 34. Computational speedup achieved using the proposed time decomposition approach.

Table 18. *Performance of the Decomposition Algorithm*

Number of Subproblems	Days	T.W	With Decomposition		Without Decomposition		Speedup
			Obj.	Time	Obj.	Time	
SP1+SP2	20	7	89,045	3	89,045	6	2.33
SP1+SP2+SP3	40	7	143,079	13	143,089	28	2.16
SP1+SP2+SP3+SP4	49	7	158,494	14	158,474	31	2.22
SP1+SP2+SP3+SP4+SP5	66	7	203,673	16	203,659	89	5.64
SP1+SP2+SP3+SP4+SP5+SP6	83	7	273,388	18	273,395	281	15.95
SP1+SP2+SP3+SP4+SP5+SP6+SP7	96	7	320,401	20	320,422	829	42.26
SP1+SP2+SP3+SP4+SP5+SP6+SP7+SP8	112	7	341,013	21	341,065	1,389	67.19
SP1+SP2+SP3+SP4+SP5+SP6+SP7+SP8+SP9	135	7	393,682	23	393,685	5,913	259.67
SP1+SP2+SP3+SP4+SP5+SP6+SP7+SP8+SP9+ SP10	154	7	422,872	25	422,877	7,505	304.22
SP1+SP2+SP3+SP4+SP5+SP6+SP7+SP8+SP9+SP10 +SP11	173	7	477,101	26	477,119	15,403	584.53
SP1+SP2	97	8	290,084	86	290,087	5,978	69.71
SP1+SP2+SP3	108	8	304,295	86	304,269	11,880	137.69
SP1+SP2+SP3+SP4	137	8	360,229	87	360,197	12,980	148.45
SP1+SP2+SP3+SP4+SP5	144	8	376,003	88	375,975	21,904	248.74
SP1+SP2+SP3+SP4+SP5+SP6	150	8	379,085	88	379,036	23,515	265.75
SP1+SP2	38	9	110,357	10	110,358	30	3.01
SP1+SP2+SP3	97	9	297,148	52	297,169	983	18.89
SP1+SP2+SP3+SP4	117	9	330,638	54	330,619	3,994	74.54
SP1+SP2+SP3+SP4+SP5	130	9	357,621	54	357,614	4,329	79.88
SP1+SP2+SP3+SP4+SP5+SP6	152	9	395,496	57	395,462	13,169	232.69
SP1+SP2	97	10	286,586	104	286,585	1,496	14.43
SP1+SP2+SP3	108	10	300,797	104	300,781	6,105	58.55
SP1+SP2+SP3+SP4	137	10	355,936	106	355,908	9,911	93.91
SP1+SP2+SP3+SP4+SP5	148	10	372,444	106	372,451	15,156	142.77
SP1+SP2+SP3+SP4+SP5+SP6	218	10	547,307	148	547,282	70,925	479.42

4.6.4 Cut Generation

The test scenarios discussed above generated 185 subproblems that we solved both with and without cuts. These test cases, along with the computational time required to solve with and without decomposition, are presented in Table 19. We observe that:

- All 185 subproblems yielded the same solution (gap in the objective function values = 0) indicating that the optimality is not impacted by the introduction of the cuts.
- The average speedup achieved due to the cuts was 1.55 with a variance of 0.36. The speedup achieved was thus statistically significant (with a hypothesis test of equality of mean speedup to 1 yielding a t-stat of 11.45) and cut down the mean time to solve a subproblem from about 32 seconds to 24 seconds.

Table 19. *Performance of the Cutting Procedure*

Scenario	T.W	# of SP's	Total	CPU Time with cut	Time without cut	Mia	NY	Total
Case Study	8	14	890,183	161.01	330.44	31	3	34
Cost -10%	8	8	870,384	3,207.11	3,825.72	36	4	40
Cost -5%	8	13	881,008	186.43	502.83	33	3	36
Cost +5%	8	15	906,729	105.17	138.12	22	2	24
Cost +10%	8	18	912,377	76.22	98.47	17	2	19
Cost +15%	8	21	917,267	45.81	74.03	14	1	15
Cost +20%	8	22	920,823	31.66	38.45	10	1	11
T.W. 6 DAYS	6	30	938,851	27.92	42.61	27	1	28
T.W. 7 DAYS	7	20	928,527	70.25	109.09	31	2	33
T. W. 9 DAYS	9	13	876,610	20,686.87		36	3	39
T. W. 10 DAYS	10	6	869,678	202,127.43	Cannot be solved without	36	4	40
T.W. 11 DAYS	11	5	865,676	205,977.16	algorithm	73	8	81

4.7 Conclusions

We have developed a novel binary mixed-integer linear programming strategic model for the optimization of freight consolidation, and the evaluation of different rates for FCL shipments and different delivery time windows. Our proposed model relies on the linear approximation to the model to generate initial solutions for a linear decomposition that leads to effective cuts and the generation of subproblems. By using these subproblems, we can tackle an NP problem and reach an optimal solution. Our approach provides a comprehensive solution that optimizes the shipments for the complete network and includes a multi-echelon solution that may be used at a strategic level and is not limited to the individual optimization of each consolidation terminal.

We have found that freight consolidation provides significant advantages for 3PL providers for cost reduction, while meeting customer demands and time windows. The use of freight consolidation offers the 3PL provides the opportunity to consolidate cargo and use 39 FCL shipments reaching 82.65% of the lower bound from the linear relaxation. Further, we have discovered that reducing the delivery time windows from 8 days to 6 days leads to a total cost increase of 7.1%, which is distributed into an increase of 3.74% for shipments to the consolidation terminals, and 19.7% in the shipments from the consolidation terminals to the final destination; furthermore, the use of FCL shipments for consolidation drops by 28%. Moreover, increasing the delivery time windows from 9 days to 11 days leads to a total cost drop of 0.98%, with a use of FCL shipments for consolidation that increases by 5%.

When evaluating the impact of the FCL shipping rates on the total transportation costs, we have found that reducing the breakeven cost for consolidation by 10% leads to a total cost decrease of 2.08% that can be attributed almost entirely to a cost decrease of 32.8% in the individual shipments from the consolidation stations to the final customer. With this cost change, the use of FCL shipments for consolidation increases by 17.95%. When increasing the breakeven cost for consolidation by 10%, there is a total cost increase of 2.32%, which may be attributed to a 80.28% cost increase in the individual shipments from the consolidation stations to the final customer that is not offset in the cost decrease of 10.71% of consolidated shipments; in this case, the use of FCL shipments for consolidation drops by 23.08%.

Given that the cost is concentrated in the shipment between the sources of the cargo and the consolidation stations, our proposed decomposition method may be limited when solving a problem with a different cost composition. The costs of shipping to the consolidation terminals range from 76–80% of the total cost, while the costs of consolidated cargo from the consolidation terminal to the final customer only range from 10–16% of the total cost. This gives us two avenues for further research.

Chapter 5: Conclusions and Future Work

5.1 Conclusions

The effective use of in-transit freight consolidation is a powerful tool for 3PL service providers to lower operational costs, while at the same time improving service quality and customer satisfaction. In order to study the in-transit freight consolidation problem, we have developed models for three distinct scenarios. We started with a formulation in which the cargo was divisible, and used the Bender decomposition technique to reach the optimal solution. Based on the results from our model, we can conclude that 3PL service providers should consider the implementation of an in-transit merging consolidation schedule. The implementation of such a method will not only reduce the total costs, but it will improve the quality of service provided to their customers, leading to higher customer satisfaction and preventing customer losses due to substandard quality. Implementing bender decomposition with the nondivisible scenarios result in heuristic approach which led us to adapt the decomposition approach in Chapter 3.

We expanded our study to include in-transit merging problems in which the cargo is nondivisible, and developed a solution technique that employs decomposition and valid cuts to reach the optimal solution. By developing these subproblems, we tackle an NP problem and reach the optimal solution. Our approach provides a comprehensive solution that optimizes the shipments for the complete network and includes a two-echelon solution that may be used at a strategic level. With the numerical analysis, we found that the demand type and the number of consolidated terminal effect the total price and the elapse time to solve the problems. This result motivates us to include different cost group and the minimum cost, which led us to the next model with incorporating the piecewise function in Chapter 4.

Finally, we evaluated a scenario in which there are different breaks in the transportation cost function, and presented an extensive computational study evaluating the effects of manipulating the time windows and the container breakeven point. Further, we evaluated our model and implemented several scenarios by manipulating the date by changing the time window and the cost breaks. We came up with a conclusion that by increasing the number of days in the time window, more consolidation options will be available; and hence, the total price will decrease. The same outcome with decreasing the cost break will result in more FCL and reducing the cost break; on the contrary, increasing the cost break will result in more LCL, which will lead to increase to the total price.

We have shown the effectiveness of using Bender decomposition and efficient cutting techniques to reduce computational times to reach optimal consolidation schedules up to nine times compared to CPLEX. As such, our developed models may be used to provide timely insight into the effects of changing specific parameters of the problem, such as time windows or container breakeven costs, and help managers develop effective strategies to manage their shipment schedules and contract bids.

5.2 Future Work

There are multiple avenues for further research for the in-transit consolidation problem, some of these include: the evaluation of multiple customers; multiple destinations; multiple types of cargo; and stochastic demand, among many others.

The inclusion of multiple customers brings about multiple potential scenarios, some where the customers have all similar destinations and consolidation can be done with the products of multiple customers; some where the customers have different destinations but have similar origins, and consolidation may be done up to an intermediate consolidation terminal, after which all of the products to consolidate have to be from the same customer. Furthermore, the inclusion of multiple customers allows the 3PL to use different prices and discount policies in order to incentivize more customers to use their services and allow for more efficient consolidation. This scenario leads to the use of game theoretical approaches in which the price schemes and discounts are used to entice customers, and reach operational efficiencies.

The inclusion of multiple destinations leads to scenarios in which a hub-and-spoke model may be evaluated, where consolidation between hubs may be compared to consolidation or direct shipments between the spokes.

The inclusion of stochastic demands, time windows, freight times, and other of the model's parameters will certainly increase the problem's complexity, and may be addressed in future research.

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APPENDIX A

A BENDER DECOMPOSITION APPROACH FOR IN-TRANSIT CONSOLIDATION WITH DIVISIBLE FREIGHT (M1)

Table 20. *Indices*

Set	Description
i	Index for warehouses $i \in I$
j	Index for vendors $j \in J$
k	Index foe shipping modes $k \in K$
m	Index for consolidation terminals $m \in M$
t	Index for times $t \in T$

Table 21. *Model M1 Notation*

Group	Notation	Description
Set	D	Set of time periods
	S	Set of shippers
	H	Set of gateways
	P	Set of products
Parameters	$c1l_{s,h}$	Cost of sending 1 lbs from supplier s to gateway h by land
	$c1a_{s,h}$	Cost of sending 1 lbs from shipper s to gateway h by air
	$c2_h$	Cost of sending 1 lbs from gateway h to the final customer
	$c3_h$	Cost of sending 1 container from gateway h to the final customer
	ci_h	Inventory cost per lbs at gateway h per period
	$d_{p,s,h}$	Weight of product p in lbs sent from shipper s on day d
	k	Maximum capacity in lbs per container
	$t1l_{s,h}$	Number of days that a package takes by land to arrive from shipper s to gateway h
	$t1a_{s,h}$	Number of days that a package takes by air to arrive from shipper s to gateway h
	$t2_{s,h}$	Number of days it takes a package to arrive from gateway h to the final customer
	t_w	Length of the time window
Decision Variables	$X_{p,s,h,d}$	Weight in lbs of product p sent from shipper s to gateway h on day d by land
	$Y_{p,s,h,d}$	Weight in lbs of product p sent from shipper s to gateway h on day d by air
	$Z_{p,h,d}$	Weight in lbs of product p sent from gateway h to the final customer on day d
	$I_{p,h,d}$	Weight of the of product p in lbs at gateway h on day d.
	$U_{h,d}$	Weight in lbs sent from gateway h to the customer at day d using a container
	$N_{p,d}$	Weight of the inventory (items delivered early) in lbs at the final customer on day d

APPENDIX B

A DECOMPOSITION APPROACH FOR IN-TRANSIT CONSOLIDATION WITH NONDIVISIBLE FREIGHT (M2)

Table 22. *Indices*

Set	Description
i	Index for warehouses $i \in I$
j	Index for vendors $j \in J$
k	Index foe shipping modes $k \in K$
m	Index for consolidation terminals $m \in M$
t	Index for times $t \in T$

Table 23. *Model M2 Notation*

Group	Notation	Description
Set	D	Set of time periods
	S	Set of shippers
	H	Set of gateways
	P	Set of products
Parameters	$c1l_{s,h}$	Cost of sending 1 lbs from shipper s to consolidation terminal h by land
	$c1a_{s,h}$	Cost of sending 1 lbs from shipper s to consolidation terminal h by air
	$c2_h$	Cost of sending 1 lbs from 3PL terminal h to the final customer
	$c1l_{fix}$	Fixed cost of sending items from shipper s to consolidation terminal h by land
	$c1a_{fix}$	Fixed cost of sending items from shipper s to consolidation terminal h by air
	$c3_h$	Cost of sending 1 container from consolidation terminal h to the final customer
	ci_h	Inventory cost per kg at consolidation terminal h
	k	Maximum capacity in lbs per container.
	$t1l_{s,h}$	Number of days that a package takes by land to arrive from shipper s to consolidation terminal h
	$t1a_{s,h}$	Number of days that a package takes by air to arrive from shipper s to consolidation terminal h
	$t2_{s,h}$	Number of days it takes a package to arrive from consolidation terminal h to the final customer
	N_d	Weight of the inventory (items delivered early) in lbs at the final customer on day$d$
Decision Variables	$X_{p,s,h,d}$	Weight in lbs of the items of product p sent from shipper s to consolidation terminal h on day d by land
	$Y_{p,s,h,d}$	Weight in lbs of the items of product p sent from shipper s to consolidation terminal h on day d by air
	$V1_{p,s,h,d}$	Binary that indicates if items of product p are being sent from shipper s to consolidation terminal h on day d by land
	$V2_{p,s,h,d}$	Binary that indicates if items of product p are being sent from shipper s to consolidation terminal h on day d by air
	$Z_{p,h,d}$	Weight in lbs of the items of product p sent from consolidation terminal h to the final customer on day d
	$I_{p,h,d}$	Weight of the inventory of product p in lbs at consolidation terminal h on day d.
	$d_{s,h}$	Weight of the items in lbs sent from shipper s on day d
	$U_{h,d}$	Weight in lbs sent from consolidation terminal h to the customer at day d using a container

Table 24. *Result for M2 180 Planning Horizon*

Scanarios							LP	AMPL-1 hr limit			OUR METHOD FULL MODEL TOTAL		
#	PLANNING HORIZON	COST STRUCTURE (1,2)	GATEWAY (2,3)	TIME WINDOW (5)	DEMAND STRUCTURE (1,2,3)	Replication (1,2,3,4)	Obj	Obj	Time	Gap	Obj	Time	Gap-1hr
1	180d	c1	g2	tw5	d1	i1	$ 352,993.56	$ 355,221.79	3610.938	1%	$ 355,027.04	2288.405	0.1%
2	180d	c1	g2	tw5	d1	i2	$ 347,223.51	$ 349,948.98	3608.594	1%	$ 349,891.39	2657.703	0.0%
3	180d	c1	g2	tw5	d1	i3	$ 359,253.87	$ 361,952.15	3611.344	1%	$ 361,709.30	2311.452	0.1%
4	180d	c1	g2	tw5	d1	i4	$ 368,866.77	$ 371,871.08	3610.235	1%	$ 371,812.25	2567.156	0.0%
5	180d	c1	g2	tw5	d2	i1	$ 292,238.95	$ 294,629.59	3610.703	1%	$ 294,590.08	2467.999	0.0%
6	180d	c1	g2	tw5	d2	i2	$ 289,260.35	$ 291,338.70	3610.672	1%	$ 291,019.16	2588.546	0.1%
7	180d	c1	g2	tw5	d2	i3	$ 293,249.25	$ 295,019.68	3608.391	1%	$ 294,866.27	2599.263	0.1%
8	180d	c1	g2	tw5	d2	i4	$ 289,487.60	$ 291,667.70	3610.687	1%	$ 291,647.74	2078.016	0.0%
9	180d	c1	g2	tw5	d3	i1	$ 138,564.22	$ 140,804.87	3609.234	2%	$ 140,672.43	2892.264	0.1%
10	180d	c1	g2	tw5	d3	i2	$ 138,163.19	$ 140,392.55	3610.578	2%	$ 140,290.70	3246.561	0.1%
11	180d	c1	g2	tw5	d3	i3	$ 140,532.41	$ 142,765.49	3609.375	2%	$ 142,459.23	3371.218	0.2%
12	180d	c1	g2	tw5	d3	i4	$ 143,340.63	$ 146,347.70	3609.531	2%	$ 146,129.07	3395.798	0.1%
13	180d	c1	g3	tw5	d1	i1	$ 266,726.02	$ 270,937.41	3610.093	2%	$ 270,937.41	32.893	0.0%
14	180d	c1	g3	tw5	d1	i2	$ 264,500.44	$ 268,869.19	3609.953	2%	$ 268,869.19	784.625	0.0%
15	180d	c1	g3	tw5	d1	i3	$ 269,823.55	$ 274,238.08	3609.828	2%	$ 274,238.08	1527.03	0.0%
16	180d	c1	g3	tw5	d1	i4	$ 278,140.90	$ 283,024.65	3610.125	2%	$ 283,024.65	1548.329	0.0%
17	180d	c1	g3	tw5	d2	i1	$ 283,958.33	$ 286,701.96	3609.875	1%	$ 286,964.46	1556.925	-0.1%
18	180d	c1	g3	tw5	d2	i2	$ 280,783.87	$ 283,259.52	3609.282	1%	$ 283,201.14	2818.907	0.0%
19	180d	c1	g3	tw5	d2	i3	$ 284,953.22	$ 286,681.10	3610.578	1%	$ 286,418.60	1929.549	0.1%
20	180d	c1	g3	tw5	d2	i4	$ 281,150.90	$ 283,976.64	3609.203	1%	$ 283,976.64	1553.108	0.0%
21	180d	c1	g3	tw5	d3	i1	$ 113,988.90	$ 117,686.54	3610.937	3%	$ 117,629.16	792.391	0.0%
22	180d	c1	g3	tw5	d3	i2	$ 114,282.37	$ 118,164.14	3609.797	3%	$ 118,060.93	963.952	0.1%
23	180d	c1	g3	tw5	d3	i3	$ 115,022.79	$ 118,856.21	3613.031	3%	$ 118,749.27	1534.111	0.1%
24	180d	c1	g3	tw5	d3	i4	$ 117,635.16	$ 122,205.65	3610.625	4%	$ 122,121.74	878.889	0.1%
25	180d	c2	g2	tw5	d1	i1	$ 367,420.54	$ 372,918.74	3610.156	1%	$ 372,318.74	2967.282	0.2%
26	180d	c2	g2	tw5	d1	i2	$ 374,375.98	$ 378,746.94	3610.078	1%	$ 378,746.94	2793.922	0.0%
27	180d	c2	g2	tw5	d1	i3	$ 364,586.26	$ 369,475.65	3609.828	1%	$ 369,175.65	2936.296	0.1%
28	180d	c2	g2	tw5	d1	i4	$ 358,327.73	$ 363,546.24	3609.234	1%	$ 363,546.24	2856.641	0.0%
29	180d	c2	g2	tw5	d2	i1	$ 287,061.45	$ 290,116.50	3609.656	1%	$ 289,516.50	1932.764	0.2%
30	180d	c2	g2	tw5	d2	i2	$ 293,851.35	$ 296,239.50	3609.703	1%	$ 295,939.50	2588.58	0.1%
31	180d	c2	g2	tw5	d2	i3	$ 287,595.75	$ 289,327.50	3609.781	1%	$ 289,327.50	2603.845	0.0%
32	180d	c2	g2	tw5	d2	i4	$ 289,005.60	$ 291,612.00	3609.297	1%	$ 291,012.00	1563.547	0.2%
33	180d	c2	g2	tw5	d3	i1	$ 142,082.56	$ 145,723.26	3609.438	2%	$ 145,536.90	3274.731	0.1%
34	180d	c2	g2	tw5	d3	i2	$ 145,094.66	$ 149,276.43	3609.094	3%	$ 148,899.71	3638.203	0.3%
35	180d	c2	g2	tw5	d3	i3	$ 141,929.20	$ 145,156.18	3609.453	2%	$ 144,628.92	3646.202	0.4%
36	180d	c2	g2	tw5	d3	i4	$ 140,597.07	$ 144,453.87	3611.64	3%	$ 144,474.10	3272.892	0.0%
37	180d	c2	g3	tw5	d1	i1	$ 273,291.55	$ 281,665.02	3609.312	3%	$ 281,665.02	778.705	0.0%
38	180d	c2	g3	tw5	d1	i2	$ 273,646.67	$ 281,890.34	3609.641	3%	$ 281,890.34	549.985	0.0%
39	180d	c2	g3	tw5	d1	i3	$ 272,687.84	$ 280,498.67	3610.594	3%	$ 280,498.67	910.233	0.0%
40	180d	c2	g3	tw5	d1	i4	$ 269,988.25	$ 277,883.29	3611.094	3%	$ 277,883.29	1022.83	0.0%
41	180d	c2	g3	tw5	d2	i1	$ 278,780.83	$ 281,604.00	3609.39	1%	$ 281,604.00	1704.922	0.0%
42	180d	c2	g3	tw5	d2	i2	$ 285,374.87	$ 288,252.00	3609.75	1%	$ 287,952.00	2449.595	0.1%
43	180d	c2	g3	tw5	d2	i3	$ 279,299.72	$ 281,077.50	3610.484	1%	$ 281,077.50	1928.874	0.0%
44	180d	c2	g3	tw5	d2	i4	$ 280,668.90	$ 283,737.00	3608.156	1%	$ 283,662.00	2079.125	0.0%
45	180d	c2	g3	tw5	d3	i1	$ 115,508.73	$ 120,831.26	3611.609	4%	$ 120,831.26	1527.564	0.0%
46	180d	c2	g3	tw5	d3	i2	$ 116,558.67	$ 122,843.46	3611.125	5%	$ 122,729.58	1651.375	0.1%
47	180d	c2	g3	tw5	d3	i3	$ 115,881.02	$ 121,465.03	3610.812	5%	$ 121,465.03	1142.173	0.0%
48	180d	c2	g3	tw5	d3	i4	$ 115,644.57	$ 121,602.28	3611.547	5%	$ 121,320.62	1530.717	0.2%
49	180d	c3	g2	tw5	d1	i1	$ 326,393.79	$ 330,632.01	3608.781	1%	$ 330,495.62	2912.671	0.0%
50	180d	c3	g2	tw5	d1	i2	$ 300,378.86	$ 304,975.46	3608.922	2%	$ 304,505.19	3011.185	0.2%
51	180d	c3	g2	tw5	d1	i3	$ 306,470.58	$ 310,682.98	3609.203	1%	$ 310,228.92	2606.997	0.1%
52	180d	c3	g2	tw5	d1	i4	$ 311,136.72	$ 315,255.73	3603.172	1%	$ 315,231.88	1483.249	0.0%
53	180d	c3	g2	tw5	d2	i1	$ 287,458.10	$ 290,213.15	3609.125	1%	$ 290,213.15	1928.064	0.0%
54	180d	c3	g2	tw5	d2	i2	$ 291,322.69	$ 293,710.84	3608.75	1%	$ 293,410.84	2589.125	0.1%
55	180d	c3	g2	tw5	d2	i3	$ 289,201.48	$ 291,233.23	3610.937	1%	$ 290,933.23	2637.156	0.1%
56	180d	c3	g2	tw5	d2	i4	$ 289,391.20	$ 291,697.60	3610	1%	$ 291,697.60	1557.735	0.0%
57	180d	c3	g2	tw5	d3	i1	$ 132,534.17	$ 135,552.05	3610.062	2%	$ 134,870.90	3244.064	0.5%
58	180d	c3	g2	tw5	d3	i2	$ 127,429.70	$ 130,772.30	3610	3%	$ 129,950.04	3639.564	0.6%
59	180d	c3	g2	tw5	d3	i3	$ 128,260.94	$ 131,127.89	3609.891	2%	$ 130,614.25	3645.139	0.4%
60	180d	c3	g2	tw5	d3	i4	$ 129,897.52	$ 132,325.07	3609.688	2%	$ 132,341.87	3269.826	0.0%
61	180d	c3	g3	tw5	d1	i1	$ 239,680.57	$ 246,762.29	3609.188	3%	$ 246,630.93	1154.847	0.1%
62	180d	c3	g3	tw5	d1	i2	$ 227,215.81	$ 234,457.35	3608.844	3%	$ 234,259.71	1533.378	0.1%
63	180d	c3	g3	tw5	d1	i3	$ 229,643.05	$ 236,126.07	3609.078	3%	$ 236,126.07	769.237	0.0%
64	180d	c3	g3	tw5	d1	i4	$ 232,556.42	$ 239,194.06	3608.344	3%	$ 239,180.77	321.642	0.0%
65	180d	c3	g3	tw5	d2	i1	$ 279,177.48	$ 282,113.15	3609.734	1%	$ 282,000.65	1941.298	0.0%
66	180d	c3	g3	tw5	d2	i2	$ 282,846.21	$ 285,723.34	3610.094	1%	$ 285,348.34	2911.156	0.1%
67	180d	c3	g3	tw5	d2	i3	$ 280,905.45	$ 283,058.23	3609.891	1%	$ 282,945.73	1933.486	0.0%
68	180d	c3	g3	tw5	d2	i4	$ 281,054.50	$ 284,122.60	3609.188	1%	$ 284,347.60	2324.123	-0.1%
69	180d	c3	g3	tw5	d3	i1	$ 107,626.62	$ 111,389.59	3610.781	3%	$ 111,169.36	1558.344	0.2%
70	180d	c3	g3	tw5	d3	i2	$ 106,597.91	$ 110,554.39	3615.125	4%	$ 110,483.99	1633.47	0.1%
71	180d	c3	g3	tw5	d3	i3	$ 106,091.74	$ 110,047.58	3610.593	4%	$ 110,031.53	2390.734	0.0%
72	180d	c3	g3	tw5	d3	i4	$ 107,257.77	$ 111,296.68	3611.5	4%	$ 111,171.78	1010.328	0.1%
Min							$ 106,091.74	$ 110,047.58	3603.172	1%	$ 110,031.53	32.893	-0.1%
Max							$ 374,375.98	$ 378,746.94	3615.125	5%	$ 378,746.94	3646.202	0.6%
median							$ 278,460.87	$ 281,777.68	3609.8515	2%	$ 281,777.68	2078.5705	0.1%
mean							$ 237,415.69	$ 241,243.44	3609.935722	2%	$ 241,086.08	2107.540431	0.1%

Table 25. *Result for M2 for 360 Planning Horizons*

Scenarios							LP	AMPL-1 hr limit			OUR METHOD FULL MODEL TOTAL		GAP
#	PLANNING HORIZON	COST STRUCTURE (1,2)	GATEWAY (2,3)	TIME WINDOW (5)	DEMAND STRUCTURE (1,2,3)	Replication (1,2,3,4)	Obj	Obj	Time	Gap	Obj	Time	Gap-1hr
1	360	c1	g2	tw5	d1	i1	$707,699.49	$713,327.72	3610.75	1%	$ 713,077.29	3092.232	0.4%
2	360	c1	g2	tw5	d1	i2	$716,635.41	$722,496.73	3610.922	1%	$ 721,808.92	2459.345	0.3%
3	360	c1	g2	tw5	d1	i3	$694,802.84	$700,012.90	3611.797	1%	$ 699,732.71	2011.185	0.3%
4	360	c1	g2	tw5	d1	i4	$703,574.50	$708,944.74	3610.39	1%	$ 708,675.67	2762.062	0.4%
5	360	c1	g2	tw5	d2	i1	$587,946.45	$592,085.58	3610.203	1%	$ 592,318.44	3034.705	0.5%
6	360	c1	g2	tw5	d2	i2	$579,572.45	$583,052.52	3611.781	1%	$ 583,359.36	2619.516	0.4%
7	360	c1	g2	tw5	d2	i3	$582,883.60	$586,801.20	3612.765	1%	$ 587,231.88	1987.157	0.3%
8	360	c1	g2	tw5	d2	i4	$585,433.65	$589,914.74	3611.735	1%	$ 589,502.36	2660.128	0.5%
9	360	c1	g2	tw5	d3	i1	$278,940.52	$284,844.29	3610.938	2%	$ 284,025.96	2616.395	0.9%
10	360	c1	g2	tw5	d3	i2	$282,877.22	$288,706.97	3612.782	2%	$ 287,671.43	2805.065	1.0%
11	360	c1	g2	tw5	d3	i3	$276,554.70	$281,727.90	3611.203	2%	$ 281,260.69	3177.106	1.1%
12	360	c1	g2	tw5	d3	i4	$277,917.62	$283,244.26	3616.766	2%	$ 282,155.07	3343.533	1.2%
13	360	c1	g3	tw5	d1	i1	$527,395.75	$536,366.29	3611.906	2%	$ 536,629.13	1534.687	0.3%
14	360	c1	g3	tw5	d1	i2	$538,054.97	$547,217.85	3612.547	2%	$ 547,415.49	1559.125	0.3%
15	360	c1	g3	tw5	d1	i3	$525,798.25	$533,998.50	3611.875	2%	$ 534,055.51	1111.064	0.2%
16	360	c1	g3	tw5	d1	i4	$526,973.10	$535,883.84	3613.703	2%	$ 535,967.16	1181.954	0.2%
17	360	c1	g3	tw5	d2	i1	$571,008.96	$575,677.50	3612.812	1%	$ 575,597.20	2003.062	0.3%
18	360	c1	g3	tw5	d2	i2	$562,847.02	$567,702.40	3612.875	1%	$ 567,702.40	1668.593	0.3%
19	360	c1	g3	tw5	d2	i3	$566,068.34	$572,307.66	3612.516	1%	$ 571,736.66	2155.906	0.4%
20	360	c1	g3	tw5	d2	i4	$568,832.94	$574,259.66	3613.188	1%	$ 574,475.32	2103.122	0.4%
21	360	c1	g3	tw5	d3	i1	$228,317.07	$237,614.02	3688.672	4%	$ 237,273.88	539.692	0.2%
22	360	c1	g3	tw5	d3	i2	$232,171.07	$241,222.65	3643.734	4%	$ 240,897.36	1549.17	0.6%
23	360	c1	g3	tw5	d3	i3	$227,985.14	$236,260.77	3614.282	4%	$ 236,203.48	1552.923	0.7%
24	360	c1	g3	tw5	d3	i4	$226,795.24	$235,242.44	3614.078	4%	$ 235,149.04	1549.89	0.7%
25	360	c2	g2	tw5	d1	i1	$734,059.84	$744,815.05	3612.734	1%	$ 744,215.05	3024.343	0.4%
26	360	c2	g2	tw5	d1	i2	$746,701.00	$757,353.67	3610.516	1%	$ 756,753.67	3171.095	0.4%
27	360	c2	g2	tw5	d1	i3	$744,299.31	$752,159.53	3610.36	1%	$ 752,197.03	2569.764	0.3%
28	360	c2	g2	tw5	d1	i4	$716,974.92	$727,431.53	3612.313	1%	$ 727,431.53	2983.905	0.4%
29	360	c2	g2	tw5	d2	i1	$587,166.45	$592,066.50	3611.391	1%	$ 592,066.50	2998.282	0.5%
30	360	c2	g2	tw5	d2	i2	$579,814.95	$583,711.50	3611.406	1%	$ 584,311.50	2360.423	0.4%
31	360	c2	g2	tw5	d2	i3	$582,929.10	$587,907.00	3612.016	1%	$ 587,607.00	1969.831	0.3%
32	360	c2	g2	tw5	d2	i4	$575,491.15	$579,785.50	3613.235	1%	$ 580,085.50	2323.312	0.4%
33	360	c2	g2	tw5	d3	i1	$285,286.32	$294,384.34	3611.844	3%	$ 293,077.62	3319.469	1.1%
34	360	c2	g2	tw5	d3	i2	$289,967.75	$296,747.18	3612.437	2%	$ 295,867.25	3124.234	1.1%
35	360	c2	g2	tw5	d3	i3	$288,506.83	$295,470.90	3612.125	2%	$ 294,205.35	3192.873	1.1%
36	360	c2	g2	tw5	d3	i4	$281,511.20	$288,771.35	3611.469	3%	$ 287,510.59	3474.078	1.2%
37	360	c2	g3	tw5	d1	i1	$548,651.14	$564,451.66	3612.735	3%	$ 564,451.66	1192.423	0.2%
38	360	c2	g3	tw5	d1	i2	$549,070.99	$564,492.39	3612.219	3%	$ 564,492.39	1186.784	0.2%
39	360	c2	g3	tw5	d1	i3	$547,462.97	$561,336.77	3613.234	2%	$ 561,674.27	1732.127	0.3%
40	360	c2	g3	tw5	d1	i4	$536,976.35	$553,939.13	3611.985	3%	$ 553,939.13	844.127	0.2%
41	360	c2	g3	tw5	d2	i1	$570,228.96	$575,266.50	3613.188	1%	$ 575,529.00	1973.333	0.3%
42	360	c2	g3	tw5	d2	i2	$563,089.52	$568,036.50	3613.453	1%	$ 567,999.00	1717.533	0.3%
43	360	c2	g3	tw5	d2	i3	$566,113.84	$572,307.00	3612.375	1%	$ 572,569.50	2153.313	0.4%
44	360	c2	g3	tw5	d2	i4	$558,890.44	$564,448.00	3614.531	1%	$ 564,110.50	2113.672	0.4%
45	360	c2	g3	tw5	d3	i1	$233,068.31	$246,335.88	3637.125	5%	$ 245,679.18	1569.565	0.6%
46	360	c2	g3	tw5	d3	i2	$233,797.35	$244,924.02	3614.985	5%	$ 244,761.84	1214.797	0.5%
47	360	c2	g3	tw5	d3	i3	$232,769.73	$243,272.45	3680.094	4%	$ 242,814.80	1600.642	0.7%
48	360	c2	g3	tw5	d3	i4	$230,301.20	$241,259.84	3615.485	5%	$ 241,035.37	1692.858	0.7%
49	360	c3	g2	tw5	d1	i1	$619,330.34	$628,084.74	3610.875	1%	$ 627,458.41	2622.06	0.4%
50	360	c3	g2	tw5	d1	i2	$612,709.97	$621,208.44	3609.672	1%	$ 621,304.02	2565.158	0.4%
51	360	c3	g2	tw5	d1	i3	$628,533.90	$637,301.02	3611.844	1%	$ 636,664.25	2700.578	0.4%
52	360	c3	g2	tw5	d1	i4	$614,833.45	$623,269.83	3611.719	1%	$ 622,746.44	2617.704	0.4%
53	360	c3	g2	tw5	d2	i1	$581,353.32	$586,253.37	3610.469	1%	$ 586,253.37	3160.96	0.5%
54	360	c3	g2	tw5	d2	i2	$584,173.37	$588,969.92	3609.891	1%	$ 588,369.92	2319.513	0.4%
55	360	c3	g2	tw5	d2	i3	$583,754.73	$588,732.63	3611.218	1%	$ 588,432.63	1984.517	0.3%
56	360	c3	g2	tw5	d2	i4	$581,337.25	$585,631.60	3610.641	1%	$ 585,931.60	2332.674	0.4%
57	360	c3	g2	tw5	d3	i1	$258,480.86	$264,534.09	3611.61	2%	$ 263,631.89	3378.106	1.3%
58	360	c3	g2	tw5	d3	i2	$258,937.46	$265,521.40	3613.031	2%	$ 264,005.88	2721.902	1.0%
59	360	c3	g2	tw5	d3	i3	$262,278.67	$268,181.33	3610.906	2%	$ 266,973.76	3481.466	1.3%
60	360	c3	g2	tw5	d3	i4	$257,578.64	$262,986.31	3612.141	2%	$ 262,189.46	3108.715	1.2%
61	360	c3	g3	tw5	d1	i1	$457,169.43	$470,304.49	3613.421	3%	$ 470,146.70	1138.952	0.2%
62	360	c3	g3	tw5	d1	i2	$462,312.84	$476,088.62	3613.578	3%	$ 475,465.28	1279.968	0.3%
63	360	c3	g3	tw5	d1	i3	$469,829.54	$481,911.69	3613	3%	$ 482,140.67	1044.564	0.2%
64	360	c3	g3	tw5	d1	i4	$457,398.64	$471,536.66	3612.984	3%	$ 471,621.73	1568.171	0.3%
65	360	c3	g3	tw5	d2	i1	$564,415.83	$569,453.37	3619.656	1%	$ 569,715.87	2092.251	0.4%
66	360	c3	g3	tw5	d2	i2	$567,447.94	$572,394.92	3626.063	1%	$ 572,619.92	2198.576	0.4%
67	360	c3	g3	tw5	d2	i3	$566,939.47	$572,607.63	3612.312	1%	$ 573,132.63	2163.89	0.4%
68	360	c3	g3	tw5	d2	i4	$564,736.54	$570,031.60	3613.547	1%	$ 570,256.60	2105.594	0.4%
69	360	c3	g3	tw5	d3	i1	$212,016.14	$220,532.12	3614.344	4%	$ 220,069.54	1698.734	0.8%
70	360	c3	g3	tw5	d3	i2	$215,511.01	$224,364.46	3667.562	4%	$ 223,489.29	1785.796	0.8%
71	360	c3	g3	tw5	d3	i3	$216,170.45	$224,541.32	3653.188	4%	$ 223,650.69	2355.344	1.0%
72	360	c3	g3	tw5	d3	i4	$212,037.61	$220,024.36	3613.922	4%	$ 219,386.08	1585.596	0.7%
Min							$212,016.14	$220,024.36	3609.67	0.60%	$219,386.08	539.69	0.15%
Max							$746,701.00	$757,353.67	3688.67	5.39%	$756,753.67	3481.47	1.30%
median							$553,980.72	$564,472.02	3612.48	1.49%	$564,472.02	2154.61	0.41%
mean							$474,576.43	$482,278.52	3616.74	1.93%	$481,999.50	2202.66	0.53%

APPENDIX C

FREIGHT CONSOLIDATION WITH DIVISIBLE SHIPMENTS, DELIVERTY TIME WINDOWS, AND PIECEWISE TRANSPORTATION COSTS (M3)

Table 26. *Model M3 Notation*

Group	Notation	Description
SET	$\boldsymbol{P}$	Set of products, indexed by p
	$\boldsymbol{S}$	Set of shippers, indexed by s
	$\boldsymbol{H}$	Set of consolidation terminals, indexed by h
	$\boldsymbol{D}$	Set of days, indexed by d
	$\boldsymbol{B}$	Set of cost rate intervals defined by the cost breaks, indexed by b
Parameters	$\bar{\boldsymbol{c}}_{\boldsymbol{h}}$	Cost of sending a container from consolidation terminal h to the final customer
	$\boldsymbol{c}_{\boldsymbol{p,s,h,d,b}}$	Cost rate b of cargo of product p shipped from shipper s to consolidation terminal h on day d
	$\boldsymbol{C}_{\boldsymbol{p,s,h,d}}$	Total cost of cargo of product p shipped from shipper s to consolidation terminal h on day d
	$\boldsymbol{L}_{\boldsymbol{p,h,d}}$	Total cost of cargo of product p shipped individually from consolidation terminal h to the final customer on day d
	$\boldsymbol{F}_{\boldsymbol{h}}$	Breakeven cost point of consolidation terminal h for container shipments
	$\boldsymbol{R}_{\boldsymbol{h,d}}$	Ratio between the cargo shipped from consolidation terminal h to the final customer on day d and the breakeven cost F_h
	$\boldsymbol{q}_{\boldsymbol{s,h,b}}$	Lower threshold for cost break rate b of product p shipped from shipper s to consolidation terminal h
	$\boldsymbol{r}_{\boldsymbol{h,b}}$	Lower threshold for cost break rate b of products shipped from consolidation terminal h to the final custome
	$\boldsymbol{D}_{\boldsymbol{p,s,d}}$	Total pounds of cargo of product p from shipper s demanded by the final customer in the time window that ends on day d
	$\boldsymbol{t}$	Length of the delivery time window
	$\boldsymbol{k}$	Maximum capacity of a single container
Decision variables	$\boldsymbol{T}_{\boldsymbol{h,d}}$	Number of containers shipped from consolidation terminal h to the final customer on day d
	$\boldsymbol{I}_{\boldsymbol{p,h,d}}$	Amount of inventory of product p at consolidation terminal h at the end of day d
	$\boldsymbol{y}_{\boldsymbol{p,s,h,d,b}}$	Binary variable that indicates if the shipment of product p from shipper s to consolidation terminal h on day d occurs at cost rate b
	$\boldsymbol{\delta}_{\boldsymbol{p,s,h,d,b}}$	Variable that indicates the proportion of the shipments of product p from shipper s to consolidation terminal h on day d that corresponds to cost rate b
	$\boldsymbol{z}_{\boldsymbol{p,h,d,b}}$	Binary variable that indicates if the shipments of product p from the consolidation terminal h on day d occurs at cost rate b
	$\boldsymbol{\eta}_{\boldsymbol{p,h,d,b}}$	Variable that indicates the proportion of the shipments of product p from consolidation terminal h on day d that corresponds to cost rate b
	$\boldsymbol{X}_{\boldsymbol{p,s,h,d}}$	Total pounds of cargo of product p shipped from shipper s to consolidation terminal h on day d
	$\boldsymbol{Z}_{\boldsymbol{p,h,d}}$	Total pounds of cargo of product p shipped individually by weight from consolidation terminal h to the final customer on day d
	$\boldsymbol{U}_{\boldsymbol{p,h,d}}$	Total pounds of cargo of product p sent from consolidation terminal h to the customer at day d using a container
	$\boldsymbol{N}_{\boldsymbol{p,d}}$	Total pounds of cargo of product p in inventory at the customer with an early arrival at the end of day d

Table 27. *Six-Day Time Window Solution*

Subproblem	Start day	End day	problem size	Total	CPU Time with cut	time without cut	cost break Miami	cost break NJ	scenario
SP1	1	16	15	35,991	0.782	2	3,388	4,305.45	Time Window 6 days
SP2	17	22	5	20,827	0.5	0			
SP3	23	32	9	40,626	1.25	2			
SP4	33	51	18	36,397	0.766	0			
SP5	52	64	12	43,227	1.156	2			
SP6	65	71	6	43,227	1.156	2			
SP7	72	95	23	100,032	4.796	4			
SP8	96	114	18	28,596	0.75	0			
SP9	115	121	6	18,852	0.593	1			
SP10	122	128	6	15,896	0.359	1			
SP11	129	134	5	17,055	0.407	1			
SP12	135	156	21	30,335	1.343	0			
SP13	157	162	5	21,802	0.328	1			
SP14	163	167	4	17,593	0.328	1			
SP15	168	175	7	17,192	0.453	0			
SP16	176	184	8	21,246	1.094	1			
SP17	185	188	3	4,654	0.343	0			
SP18	189	205	16	57,221	0.562	3			
SP19	206	211	5	15,868	0.343	0			
SP20	212	223	11	23,112	0.719	0			
SP21	224	238	14	50,499	0.968	1			
SP22	239	259	20	39,815	0.562	0			
SP23	260	273	13	21,554	0.438	0			
SP24	274	288	14	51,652	0.844	3			
SP25	289	300	11	24,056	0.641	1			
SP26	301	316	15	49,757	2.063	2			
SP27	317	337	20	45,058	1.484	0			
SP28	338	351	13	21,246	0.859	0			
SP29	352	360	8	16,975	0.344	0			
SP30	361	364	3	8,489	0.5	0			
Total				**938,851**	**26.731**	**28**			

Table 28. *Scenario Result of Changing Eight-Day Time Window*

Time Window	CPU time	Total containers	# subproblem	Objective function	Container relaxation	Gap
6	28	28	30	938,851	841,409	11.58%
7	70	33	20	928,527	841,387	10.36%
8	161	34	14	890,183	841,342	5.81%
9	20,686	39	13	876,610	840,648	4.28%
10	202,127	40	6	869,678	841,379	3.36%
11	205,977	41	5	865,676	839,269	3.15%

Table 29. *Scenario Result of Changing Eight-Day Time Window Cost Breakdown*

Scenario	CPU time	Total containers	# subproblems	Objective function	CLR Objective	Gap
(-10%)	3,198	40	8	870,384	816,777	6.56%
(-5%)	186	36	13	881,008	829,079	6.26%
3PL	161	34	14	890,183	841,342	5.81%
(+5%)	105	24	15	906,729	864,921	4.83%
(+10%)	76	19	18	912,377	877,173	4.01%
(+15%)	46	15	21	917,267	889,448	3.13%
(+20%)	32	11	22	920,823	901,745	2.12%

Table 30. *Eight-Day Time Window Cost Decreased—10%*

Subproblem	**Start day**	**End day**	**problem size**	**Total**	**CPU Time with cut**	**time without cut**	**Total container**	**T. W**	**cost break Miami**	**cost break NJ**	**scenario**
SP1	1	98	97	283,092	3042.5	3,609.22	13	8	2,858.00	3,727.00	**TW 8 COST -%10**
SP2	99	109	10	14,211	0.6	1.18	0	8			
SP3	110	149	39	70,586	2.2	2.89	4	8			
SP4	150	219	69	172,743	49.6	87.54	8	8			
SP5	220	251	31	75,882	1.8	2.38	4	8			
SP6	252	269	17	31,991	0.9	1.5	1	8			
SP7	270	338	68	175,863	108.0	118.95	8	8			
SP8	339	365	26	46,016	1.5	2.06	2	8			
Total				870,384	3,207.11	3,825.72	870,384	3207.1			

Table 31. *Eight-Day Time Window Cost Decreased—5%*

Subproblem	Start day	End day	Problem size	Total	CPU Time with cut	Time without cut	Total container	Cost break Miami	Cost break NJ	Scenario
SP1	1	39	38	106,460	43.7	57.64	4	3,123.00	4,016.10	**T.W 8 Cost -%5**
SP2	40	98	58	180,126	59.99	222.44	9			
SP3	99	109	10	14,211	0.57	0.95	0			
SP4	110	138	28	55,139	1.27	1.82	3			
SP5	139	149	10	16,507	0.63	1.2	1			
SP6	150	219	69	174,864	41.78	66.95	8			
SP7	220	255	35	90,073	2.95	4.9	3			
SP8	256	269	13	19,433	0.84	1.49	0			
SP9	270	290	20	62,249	1.33	2.32	3			
SP10	291	328	37	100,757	31.5	139.49	4			
SP11	329	339	10	15,109	0.56	1.01	0			
SP12	340	348	8	19,296	0.42	0.87	1			
SP13	349	365	16	26,784	0.89	1.75	0			

Table 32. *Eight-Day Time Window Cost Increased—5%*

Subproblem	Start day	End day	problem size	Total	CPU Time with cut	time without cut	Total container	cost break Miami	cost break NJ	scenario
SP1	1	20	19	51,931	2.47	2.94	1			
SP2	21	39	18	58,425	7.39	9.46	2			
SP3	40	98	58	186,792	42.16	53.37	7			
SP4	99	118	19	33,490	1.56	2.13	0			
SP5	119	131	12	26,983	0.61	0.92	1			
SP6	132	153	21	37,875	2.41	3.25	0			
SP7	154	214	60	164,274	27.3	38.68	6			
SP8	215	219	4	6,522	0.55	0.56	0	3,918.00	4,883.50	**T.W 8 Cost+%5**
SP9	220	245	25	73,157	1.89	2.67	2			
SP10	246	269	23	38,182	1.25	1.98	0			
SP11	270	285	15	61,723	1.13	2.12	2			
SP12	286	296	10	7,705	0.69	0.97	0			
SP13	297	327	30	98,140	13.8	15.69	3			
SP14	328	349	21	35,042	1.22	1.96	0			
SP15	350	365	15	26,488	0.77	1.42	0			
Total				906,729	105.17	138.12	24			

Table 33. *Eight-Day Time Window Cost Increased—10%*

Subproblem	Start day	End day	problem size	Total	CPU Time with cut	time without cut	Total container	cost break Miami	cost break NJ	scenario
SP1	1	20	19	51,931	2.44	2.76	1	4,183.00	5,172.46	**T.W 8 COST +%10**
SP2	21	39	18	58,955	5.95	7.65	2			
SP3	40	53	13	26,322	1	1.45	0			
SP4	54	68	14	44,416	1.44	2.22	1			
SP5	69	102	33	118,949	9.24	11.34	4			
SP6	103	122	19	35,446	3.56	5.65	0			
SP7	123	131	8	24,121	0.46	0.86	0			
SP8	132	153	21	37,875	2.25	2.98	0			
SP9	154	208	54	153,973	21.48	29.59	4			
SP10	209	219	10	18,083	0.72	1.13	0			
SP11	220	238	18	61,132	1.74	2.23	2			
SP12	239	250	11	16,337	0.45	0.68	0			
SP13	251	269	18	34,399	1.17	1.94	0			
SP14	270	285	15	62,250	1.33	1.56	2			
SP15	286	296	10	7,705	0.7	1.23	0			
SP16	297	328	31	98,955	20.05	22.78	3			
SP17	329	349	20	35,042	1.36	1.45	0			
SP18	350	365	15	26,487	0.89	0.97	0			
	Total			912,377	76.22	98.47	19			

Table 34. *Eight-Day Time Window Cost Increased—15%*

Subproblem	Start day	End day	problem size	Total	CPU Time with cut	time without cut	Total container	cost break Miami	cost break NJ	scenario
SP1	1	20	19	52,164	1.69	1.98	0	4,448.00	5,461.46	**T.W 8 COST +%15**
SP2	21	35	14	56,731	2.24	3.48	1			
SP3	36	53	17	29,096	1.17	2.45	0			
SP4	54	68	14	44,681	1.5	2.94	1			
SP5	69	102	33	120,025	5	10.82	4			
SP6	103	122	19	35,449	2.86	4.87	0			
SP7	123	131	8	24,121	0.41	0.98	0			
SP8	132	153	21	37,875	1.58	3.78	0			
SP9	154	161	7	24,324	0.39	0.97	0			

SP10	162	208	46	130,703	16.31	17.97	4			
SP11	209	220	11	18,083	0.73	1.21	0			
SP12	221	238	17	61,662	1.5	2.33	2			
SP13	239	250	11	16,337	0.48	0.87	0			
SP14	251	269	18	34,399	1.33	1.97	0			
SP15	270	285	15	62,690	1.7	2.45	1			
SP16	286	296	10	7,705	0.69	1.24	0			
SP17	297	305	8	42,358	2.25	3.58	1			
SP18	306	319	13	28,303	0.92	1.67	1			
SP19	320	328	8	29,033	0.84	2.94	0			
SP20	329	349	20	35,042	1.31	2.97	0			
SP21	350	365	15	26,487	0.91	2.56	0			
	Total			917,267	45.81	74.03	15			

Table 35. *Eight-Day Time Window Cost Increased—20%*

Subproblem	**Start day**	**End day**	**problem size**	**Total**	**CPU Time with cut**	**time without cut**	**Total container**	**cost break Miami**	**cost break NJ**	**scenario**
SP1	1	20	19	52,164	0.95	0.83	0			
SP2	21	35	14	56,996	2.09	2.83	1			
SP3	36	53	17	29,096	1.17	1.43	0			
SP4	54	69	15	45,586	2.59	3.02	1			
SP5	70	102	32	120,382	5.34	6.95	3			
SP6	103	122	19	35,446	1.97	2.33	0	4,713.00	5,750.60	**T. W 8 COST +%20**
SP7	123	131	8	24,121	0.41	0.53	0			
SP8	132	153	21	37,875	1.66	2.56	0			
SP9	154	161	7	24,324	0.38	0.69	0			
SP10	162	186	24	58,392	2.11	3.55	2			
SP11	187	208	21	73,209	1.58	1.98	1			

SP12	209	220	11	18,083	0.7	1.02	0
SP13	221	238	17	62,192	1.53	2.42	2
SP14	239	250	11	16,337	0.48	0.55	0
SP15	251	270	19	36,604	1.27	1.56	0
SP16	271	285	14	60,732	1.53	2.09	0
SP17	286	296	10	7,705	0.69	1.11	0
SP18	297	305	8	42,449	1.14	0.16	0
SP19	306	319	13	28,568	1.03	1.53	1
SP20	320	328	8	29,033	0.73	0.26	0
SP21	329	349	20	35,042	1.47	0.16	0
SP22	350	365	15	26,487	0.83	0.91	0
	Total			920,823	31.66	38.45	11

Table 36. *Six-Day Time Window*

Subproblem	**Start day**	**End day**	**problem size**	**Total**	**CPU Time with cut**	**time without cut**	**Total container**	**Miami**	**cost break NJ**	**scenario**
SP1	1	16	15	35,991	0.78	0.96	2			
SP2	17	22	5	20,827	0.5	0.12	0			
SP3	23	32	9	40,626	1.25	2.23	2			
SP4	33	51	18	36,397	0.77	1.34	0			
SP5	52	64	12	43,227	1.16	1.93	2			
SP6	65	71	6	43,227	1.16	2.14	2			
SP7	72	95	23	100,032	4.8	7.54	4			
SP8	96	114	18	28,596	0.75	1.26	0			
SP9	115	121	6	18,852	0.59	0.96	1	3,388.00	4,305.46	**T. W 6 DAYS**
SP10	122	128	6	15,896	0.54	0.89	1			
SP11	129	134	5	17,055	0.5	0.21	1			
SP12	135	156	21	30,335	1.34	2.34	0			
SP13	157	162	5	21,802	0.5	0.24	1			
SP14	163	167	4	17,593	0.5	0.41	1			
SP15	168	175	7	17,192	0.5	0.3	0			
SP16	176	184	8	21,246	1.09	2.43	1			
SP17	185	188	3	4,654	0.5	0.45	0			
SP18	189	205	16	57,221	0.56	0.69	3			

SP19	206	211	5	15,868	0.5	0.45	0
SP20	212	223	11	23,112	0.72	1.32	0
SP21	224	238	14	50,499	0.97	1.79	1
SP22	239	259	20	39,815	0.56	1.02	0
SP23	260	273	13	21,554	0.5	0.23	0
SP24	274	288	14	51,652	0.84	1.46	3
SP25	289	300	11	24,056	0.64	1.18	1
SP26	301	316	15	49,757	2.06	3.75	2
SP27	317	337	20	45,058	1.48	2.92	0
SP28	338	351	13	21,246	0.86	1.54	0
SP29	352	360	8	16,975	0.5	0.41	0
SP30	361	364	3	8,489	0.5	0.1	0
	Total			938,851	27.92	42.61	28

Table 37. *Seven-Day Time Window*

Subproblem	**Start day**	**End day**	**problem size**	**Total**	**CPU Time with cut**	**time without cut**	**Total container**	**cost break Miami**	**cost break NJ**	**scenario**
SP1	1	7	6	52,547.90	0.83	1.75	1	3,388.00	4,305.46	**T. W 7 DAYS**
SP2	8	21	13	36,496.96	1.7	3.03	2			
SP3	22	41	19	54,034.32	10.4	15.55	2			
SP4	42	50	8	15,414.81	0.81	1.56	0			
SP5	51	67	16	45,179.38	2	2.15	2			
SP6	68	84	16	69,746.98	1.9	2.72	5			
SP7	85	97	12	47,014.35	1.98	2.8	2			
SP8	98	113	15	20,655.95	1.05	2.85	0			
SP9	114	136	22	52,604.79	2.1	3.23	3			
SP10	137	155	18	29,180.46	1.9	2.56	0			
SP11	156	174	18	54,245.06	1.68	2.45	2			
SP12	175	187	12	23,650.45	0.86	1.56	1			
SP13	188	221	33	88,007.72	17.8	25.32	4			

SP14	222	236	14	57,298.33	2.41	3.76	2			
SP15	237	272	35	64,114.30	2.12	4.44	0			
SP16	273	286	13	52,192.66	1.7	2.49	3			
SP17	287	299	12	22,995.45	0.88	1.76	1			
SP18	300	319	19	53,119.45	8.73	14.67	2			
SP19	320	325	5	26,240.50	0.5	0.97	1			
SP20	326	365	39	63,787.60	8.9	13.47	0			
	Total			928,527	70.25	109.09	33			

Table 38. *Nine-Day Time Window*

Subproblem	**Start day**	**End day**	**problem size**	**Total**	**CPU Time with cut**	**time without cut**	**Total container**	**cost break Miami**	**cost break NJ**	**scenario**
SP1	1	98	97	285,853	19,825.43	Cannot be solve without algorithm	14	3,388.00	4,305.46	**TIME WINDOW 9 DAYS**
SP2	99	109	10	14,211	0.75		0			
SP3	110	140	30	60,219	4.89		3			
SP4	141	151	10	13,806	0.88		0			
SP5	152	217	65	171,349	51.06		8			
SP6	221	245	24	70,995	2.53		4			
SP7	246	251	5	5,481	0.53		0			
SP8	252	264	12	29,580	0.89		1			
SP9	265	269	4	2,869	0.42		0			
SP10	270	328	58	161,929	797.31		7			
SP11	329	339	10	15,109	0.75		0			
SP12	340	360	20	36,718	0.72		2			
SP13	361	365	4	8,489	0.7		0			
	Total			876,610	20,686.87		39			

Table 39. *Ten-Day Time Window*

Subproblem	Start day	End day	problem size	Total	CPU Time with cut	time without cut	Total container	cost break Miami	cost break NJ	scenario
SP1	1	98	97	283,137	86,567.32	Cannot be solve without algorithm	14	3,388.00	4,305.46	**T.W 10 DAYS**
SP2	99	106	7	7,769	0.67		0			
SP3	107	148	41	77,410	61.72		3			
SP4	149	252	103	254,089	86,564.42		12			
SP5	253	336	83	201,864	28,929.80		9			
SP6	337	365	28	45,409	3.5		2			
Total				869,678	202,127.43		40			

Table 40. *Eleven-Day Time Window*

Subproblem	Start day	End day	problem size	Total	CPU Time with cut	time without cut	Total container	cost break Miami	cost break NJ	scenario
SP1	1	107	106	289,792	86,564.21	Cannot be solve without algorithm	15	3,388.00	4,305.46	**T.W 11 DAYS**
SP2	108	149	41	77,586	25.69		3			
SP3	150	253	103	252,116	86,476.80		11			
SP4	254	337	83	202,173	32,908.09		10			
SP5	338	365	27	44,009	2.38		2			
Total				865,676	205,977.16		81			