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On the Computation of Heterogeneous Agent Models and Its Applications

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ON THE COMPUTATION OF HETEROGENEOUS AGENT MODELS AND ITS APPLICATIONS

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This thesis has two parts, each with a different subject. Part 1 studies the macroeconomic implications of alternative health care reforms. Part 2 studies the computation and simulation of dynamic competitive equilibria in models with heterogeneous agents and market frictions.

In 2007, 44.5 million non-elderly in the U.S did not have health insurance coverage. Empirical studies suggest that there are serious negative consequences associated with uninsurance. Consequently, there is wide agreement that reforming the current health care system is desirable and several proposals have been discussed among economists and in the political arena. However, little attention has been paid to quantify the macroeconomic consequences of reforming the health insurance system in the U.S. The objective of this section is to develop a theoretical framework to evaluate a broad set of health care reform plans. I build a model that is capable of reproducing a set of key facts of health expenditure and insurance demand patterns, as well as key macroeconomic conditions of the U.S. during the last decade. Then, I use this model to derive the macroeconomic implications of alternative reforms and alternative ways of funding these reforms.

The second part of this thesis studies the computation and simulation of dynamic competitive equilibria in models with heterogeneous agents and market frictions. This type of models have been of considerable interest in macroeconomics and finance to analyze the effects of various macroeconomic policies, the evolution of wealth and income distribution, and the variability of asset prices. However, there is no reliable algorithm available to compute their equilibria. We develop a theoretical framework for the computation and

simulation of dynamic competitive markets economies with heterogeneous agents and market frictions. We apply these methods to some macroeconomic models and find important improvements over traditional methods.

To my parents for their unconditional love,

to Jia for her supportive love,

to Lucas for his pure smile.

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Chapter 1

Health insurance markets and health care reform in the U.S.

One of the major social policy issues facing the United States in the first decade of the 21^{st} century is the large number of Americans lacking health insurance. There is wide agreement that reforming the current health care system is desirable and several proposals have been discussed among economists and in the political arena. Reform of the health insurance system could potentially affect macroeconomic variables by distorting the labor market through changes in tax rates, creating deadweight loss, and ultimately changing the number of uninsured and the aggregate health expenditure. In recent years, a number of analyses have been devoted to understanding the health insurance system in the United States as well as to exploring the impacts of health care reforms on the macro-economy. The purpose of this chapter is to review the implications of this literature.

In this chapter I proceed as follows. I begin by reviewing the important facts on the insurance market in the U.S. I then focus on existing explanations for why the U.S. has such a large number of uninsured individuals, and why this may be undesirable. I then describe the major existing health care reform proposals as well as the current state of the literature in estimating the impacts of alternative health care reforms on macroeconomic variables, such as labor supply, GDP, health expenditure, number of uninsured and welfare.

1.1 The key features of the health insurance market in the United States.

The health insurance market in the U.S. is a mixture of private and public health insurance. More than half (67%) of non-elderly Americans, who are under age 65, have private insurance. While Medicare covers virtually all those who are 65 years or older, the non-elderly who do not have access or cannot afford private insurance go without health coverage unless they qualify for public insurance through the Medicaid program, State Children's Health Insurance Program, or other state-subsidized programs. The gap in the private and public health insurance systems left 44.5 million non-elderly in the U.S without health insurance coverage in 2007. The distribution of insurance coverage in the U.S. is shown in table 1.1.[1]

Table 1.1: Health Insurance Coverage of the Non-Elderly Population, 2007

	People (Millions)	Percentage of Population
Total Population	261.4	100.0%
Private	175.1	67.0%
Employment-based	159.5	61%
Individually purchased	15.7	6%
Public	41.8	16%
Medicare	6.5	2.5%
Medicaid	34.9	11.2%
TRICARE/CHAMPVA	7.1	2.3%
Uninsured	44.5	17%

1.1.1 Private health insurance

Private insurance is the most important source of health insurance in the United States. Among those with insurance, 80 percent are covered with private insurance. The vast majority of this group purchase insurance through their employer. Only 10 percent of those

[1] Data source: Kaiser foundation

with private insurance purchase insurance individually through the non-group insurance market.

Most, but not all, employers offer group health insurance to their employees and to their employees' families. About half of those insured through employer-sponsored health plans are covered by their own employer (52%) and half are covered as an employee's dependent (48%). Employer-sponsored health insurance is voluntary. Employers are not legally required to offer a health benefit, and employees can choose not to take the offer. Large firms and workers with higher wages are more likely to offer coverage. Employers also charge employees some share of the costs of insurance, averaging 16 percent of insurance cost for individuals and 27 percent for families. Even when businesses offer health benefits, some employees are ineligible because they are part-time workers or they cannot afford the required share of the premium. On average 80 percent of employees who are offered health benefits enroll in the health insurance plan.

There are two reasons why employer-sponsored insurance is predominant in the private insurance market. The first is risk pooling. The insurer minimizes the cost by creating large insurance pools with a predictable distribution of medical risks. When the pools are large and are constructed for reasons independent of health risk, it is easier to estimate the distribution.

The second reason is that the government subsidizes the purchasing of employer-sponsored insurance coverage. When a health plan is sponsored by an employer, typically both the employer and the employee who wishes to sign up for coverage contribute to paying the premium of the health plan. The employer portion of the premium is not included as taxable income, reducing employee tax liability. This lowers the amount workers owe to federal and state governments for income taxes, and for the payroll taxes paid to help support Medicare and Social Security. And while many employees pay for their share of the premium with after-tax income, federal law also allows employees to contribute to premiums with pre-tax income, in which case taxes would also not be paid on the employee share.

1.1.2 Public health insurance

There are two major public health insurance programs. The first is Medicare, which provides health insurance coverage for all people over age 65 and disabled people under age 65. This program is funded by a payroll tax.

The other major public health insurance program in the U.S. is Medicaid, which provides health care for the poor. Medicaid covers about 14 percent of the non-elderly population. Medicaid primarily covers four main categories of non-elderly low-income individuals (typically below 200 percent of the federal poverty level): children, their parents, pregnant women, and individuals with disabilities. Individuals who do not fall into one of these groups may be ineligible for Medicaid regardless of their income. Although Medicaid covers 45 percent of those below the poverty level, the categorical requirement leaves 35 percent of low-income individuals without insurance coverage.

The government also provides health insurance for those currently or formerly in the military and their dependents. Tri-Care provides civilian health benefits for military personnel, military retirees, and their dependents, including some members of the Reserve Component. The Civilian Health and Medical Program (CHAMPVA) is a health benefit program from the Department of Veteran's Affairs that is awarded to spouses, widows, widowers and children of veterans who have been rated disabled due to a service connected disability while living or at the time of death. Together, these two programs cover about 7 million Americans.

1.1.3 The uninsured

The health insurance system in the U.S. leaves 45 million Americans without any insurance coverage. According to CPS data, over 80 percent of the uninsured come from working families. About two-third of the uninsured are individuals and families with income below the poverty level ($21,203 for a family of four in 2007) or between one and two times the federal poverty level. These individuals are less likely to be offered employer-sponsored coverage or to be able to afford to purchase their own coverage. Uninsured adults are more likely than the insured to be under age 35, unmarried, and single parents, likely because of the role of lower health risk.

1.2 The problem of uninsured

Almost all existing health care reform proposals share the same target, which is to reduce the number of uninsured. To study the impacts of these reforms, we must first understand why individuals are uninsured and why we should care about the uninsured.

1.2.1 Why are people uninsured

A simple model with concave utility predicts that individuals would purchase health insurance at a cost of actuarially fair premiums so that they are fully insured for medical risks. Why then do 18 percent of non-elderly lack health insurance coverage?

Individuals may be unwilling to purchase insurance if it is not available at an actuarially fair price. There are at least two reasons why the price may not be actuarially fair. Private insurance in the U.S. has administrative costs averaging about 12 percent of premiums paid. Adverse selection in the insurance market also raises the cost of insurance by screening potential applicants to identify the most costly cases and through the standard lemons pricing effect [Akerlof (1970)][2]. Such deviations from actuarial fairness can cause individuals with lower levels of risk aversion to forgo insurance.

Another reason why individuals may not be willing to purchase insurance is that they are implicitly insured by hospital uncompensated care, if their medical risks are primarily catastrophic. According to current law, a hospital that receives Medicare money from the government has to treat any individuals who show up in the emergency room, regardless of their ability to pay. Uninsured patients have the liability but they can avoid such costs in the

[2]There is clear evidence of adverse selection within health insurance markets [see Cutler and Zeckhauser (2000), Cutler and Reber (1998)]

limit through personal bankruptcy. When the uninsured are unable to pay for the health care they receive, that uncompensated care is paid for through a patchwork of federal, state and private funds amounting to approximately $57 billion in 2008 according to Kaiser (2008). The possibility of receiving free care for emergent health conditions provides a valuable option to primarily healthy individuals. Rask and Rask (2000) and Herring (2005) suggest that individuals are more likely to be uninsured when more free care is available. It is important to have a model to capture the above facts and to study how health care reform affects individual's demand for health insurance.

1.2.2 Why should we care about the uninsured

It has been taken as given by the public that the large number of uninsured is a major social policy issue. However, what are the economic arguments suggesting we should reform the current system to cover these uninsured?

The common argument for reducing insurance coverage is that there are externalities associated with the uninsured. For example, uninsured people are less likely to receive vaccinations and care for communicable diseases and thus they impose physical externalities on the population. There are also significant financial externalities imposed by the uninsured on the insured through uncompensated care, whose costs were estimated to be about $30 billion in 2005 [Gruber and Rodriguez(2007)].

Uninsured adults are far more likely to postpone accessing health care or to forgo it altogether and are less able to afford prescription drugs or follow through with recommended treatments. A 2003 report by the Institute of Medicine states that the uninsured have a more rapid decrease in general health and a higher risk of dying prematurely than the insured. According to their estimation, the cost of diminished health and shorter life span due to lack of insurance was between $65 and $130 billion in 2003.

Another potential inefficiency associated with uninsurance is distortions in the labor market caused by employer-sponsored insurance coverage. The employee may be unwilling to move to a more suitable job for fear of losing health insurance. This phenomena is referred to as job lock [Gruber (2000)]. Madrian (1994) finds that job lock decreases job mobility by as much as 25 percent. However, the welfare cost of such inefficiency is rather small. Gruber and Madrian (2004) estimate the welfare cost from the reduced job mobility is on the order of 0.1-0.2 percent of GDP.

The major motivation for covering the uninsured comes from the concern that being uninsured is bad for the individual's health. The Institute of Medicine recently reviewed the major studies in the literature considering the health problems associated with uninsurance. It found that uninsured individuals use only half as much medical care as the insured, and have a mortality risk that is 25 percent higher, with over 18000 people dying each year because of lack of insurance.

The final reason why we should care about the uninsured is redistribution. The uninsured are a disproportionately low income group. Therefore they may be a group to whom we want to redistribute health care resources. Providing health insurance to poor children can fix the failure of intra-household utility maximization by offsetting the failures of their parents to sufficiently provide for their care. More important, many believe that to cover the uninsured ensures the basic rights of the uninsured poor.

The literature suggests that it would be interesting to have a framework to study the under-use of medical service and worse health problems associated with uninsurance and how changes in health care policy may affect individual's medical usage decisions and health status. In order to do that it is important to explicitly model health investment as well as the health insurance decision.

1.3 Alternative reform proposals to the health care system

The negative effects associated with the large number of uninsured have encouraged policymakers to consider substantial changes to the U.S. health care system. In recent years alternative proposals have been brought forth in an effort to cover the uninsured. They can be grouped into three broad categories. The first group consists of incremental health insurance expansion plans, such as tax credits for individual purchase of non-group insurance, expansion of the existing State Children's Health program (SCHIP) to cover all uninsured children, or expanding Medicare to cover people between the ages of fifty-five and sixty-five. A second class of reforms consist of combinations of subsidy expansion and an individual mandate. A version of this reform has been enacted in Massachusetts and proposed in California and Pennsylvania. Under this plan all individuals are required to carry a minimum level of health insurance and the government expands the public health insurance to cover more low income families. The third type of reform is a single-payer plan, which offers publicly financed health insurance to all citizens. This approach is based on the expansion of the traditional Medicare program to the whole population or on the Canadian health care system.

There are many empirical studies that explore the impacts of health care reforms on individual's behavior such as crowd-out by public insurance, medical usage and health status. The expansion of public insurance coverage, such as Medicaid expansion, can shift some individuals from existing private insurance coverage to public coverage. This is because the Medicaid insurance package is much more generous than the typical private insurance plan and it does not cost anything. It might be attractive to some individuals to leave private insurance for public insurance when the government expands the eligibility of public insurance programs. This crowding-out phenomenon of expanded public insurance is reviewed in Gruber and Simon (2009). Culter and Gruber (1996) suggest that private insurance coverage can decline by half as much as the government public insurance enrollment. In addition, Lo Sasso and Buchmueller (2004) show that about 50 percent of enrolles in SCHIP previously had private insurance.

Once a health care reform that improves insurance coverage (such as a Single-payer system) is instituted, the newly insured will consume more medical services and have better health status through better health care access and lower prices (under the current system, the uninsured are charged more for comparable services than the insured). Such a reform may decrease the health disparity and increase aggregate health status and labor productivity, which encourages economic activity. Cheng and Chiang (1997) found that after the introduction of universal health insurance in Taiwan, the newly insured consumed more than twice the amount of outpatient physician visits and hospital admissions than before universal health insurance was implemented. Hanratty (1996) studied the impact of the introduction of national health insurance in Canada and found that it was associated with a 4 percent decline in the infant mortality rate and an 8.9 percent decrease in the incidence of low birth weight among single mothers. While the study by Lurie et al. (1984) indicated that health deteriorated significantly after the state of California removed the eligibility for public insurance for a large group of individuals. Currie and Gruber (1996a, 1996b) found that the expansion of public insurance across the U.S. states in the 1980s and 1990s led to an 8.5 percent reduction in infant mortality and a 5 percent reduction in child mortality. Decker and Remler (2004) suggest that the availability of universal health insurance reduces the health disparity.

Some recent studies compare the effects of health insurance reform proposals. Gruber (2008) uses a micro-simulation model to estimate the impact of alternative health policies targeted at insurance coverage. He finds that alternative reforms to provide health insurance can have different effects on aggregate health expenditure, insurance coverage and private and public sector health care cost. Meara et al (2008) use CPS data to examine the effects of three different health insurance reforms on insurance coverage, health expenditure, wages and employment. They suggest that Medicaid expansion will increase insurance coverage as well as employment. They also find that tax credits have negligible effects on labor supply and require substantial public funding.

The literature reviewed above provides valuable insights about how health care reform can affect individuals, as well as a comprehensive view of the pros and cons associated with alternative health policies. However, health care reform may have important general equilibrium effects on macroeconomic variables that have been unexplored by the empirical literature.

Reforming the health system will affect the household's demand for health insurance. This in turn alters the pool of agents insured, which affects insurance premiums. Similarly, different insurance decisions result in changing health status and worker productivity, which in turn affect wages and hours worked. A change in labor income tax may be required to fund the reform, which will in turn affect individual's labor supply decisions. A reform will also affect agents' saving behavior (and thus the aggregate capital stock and factor prices) because health insurance influences precautionary saving motives. Under reforms that increase insurance coverage, agents have a decreased exposure to health shocks, which decrease the demand for precautionary saving. Better health implies longer life expectancy and thus higher saving incentive. These complicated tradeoffs can only be fully captured in a general equilibrium framework.

The classic works of Bewley (1986), Imrohoroglu (1992), Huggett (1993) and Aiyagari (1994) have set up a framework to study uninsurable labor productivity risk. Many recent papers introduce exogenous health expenditure shocks into Bewley-type models to add realism. For example, Palumbo (1999) and De Nardi, French and Jones (2006) incorporate heterogeneity in medical expenses in order to understand the pattern of saving among the elderly. Jeske and Kitao (2008) study the welfare costs of a tax policy change associated with health insurance. A few papers endogenize health expenditures as investments in health following the seminal work of Grossman (1972). Suen (2006) endogenizes households' medical expenditure decision to explain the rapid growth in health expenditure. Jung and Tran (2008) use an OLG model built upon Jeske and Kitao (2008) to analyze the effect of the Health Saving Accounts on the health expenditure and individual's insurance decision.

The labor supply decision is absent from most of the existing macro-literature regarding health, and consequently labor income tax revenues are obtained distortion free. However, having endogenous labor supply is critically important for welfare analysis. The cost of reforms in terms of labor supply distortions is weighed against the benefits of reforms, such as higher productivity and a larger risk pool (reduced adverse selection).

To that end, I build up a dynamic general equilibrium overlapping generations model with idiosyncratic health shocks and endogenous labor supply in the following chapter. This model enables us to compare the welfare effect of policy experiments, changes in the aggregate health expenditure as well as labor supply. Moreover, the model can take into account important general equilibrium effects of reforms, including the distortions associated with a change in taxes, as well as the interaction between the medical usage demand and labor supply that affects factor prices. My study is also related to the literature on taxation and labor supply (Prescott (2004), Rogerson (2007)). In my model, the government adjusts tax rates to fund the reforms, which creates distortions in labor supply. The main contribution of my work is to develop a tool to quantify the effects of alternative health care reforms. I use the Medical Expenditure Panel Survey to calibrate the model and succeed in closely matching the current pattern of health expenditure and insurance demand as observed in the data. Numerical simulations indicate that reforming the health insurance system has a quantitatively relevant impact on the number of uninsured, hours worked, and welfare.

Chapter 2

Macroeconomic consequences of alternative reforms to the health insurance system in the U.S.

This chapter examines the macroeconomic and welfare implications of alternative reforms to the U.S. health insurance system. In particular, I study the effect of the expansion of Medicare to the entire population, the expansion of Medicaid, an individual mandate, the removal of the tax break to purchase group insurance and providing a refundable tax credit for insurance purchases. This chapter is organized as follows: section 1 introduces the model; section 2 explains the calibration of the model; section 3 details some reform proposals and presents the numerical results both from the benchmark and from policy experiments; the last section concludes.

2.1 Benchmark Model

2.1.1 Demographics

This economy has overlapping generations of agents who live a maximum of three periods as *young*, *middle-aged*, and *old*. Let $g \in \{1,2,3\}$ denote the age. In the first period, the measure of newly born agents is normalized to 1. Individuals alive in period t survive to the next period with a certain probability. For old people this probability is always 0. For young and middle-aged people, the survival probability is given by $\rho(h_g)$, which depends on the health status h_g at the end of age g as described below. The population of young individuals grows at a constant rate n, implying that the population of young in period t is $(1+n)^t$. I denote the relative size of age g to the population as μ_g, which is determined in the equilibrium.

2.1.2 Agent types

All individuals enter the economy with the same level of health $\bar{h}_0$, an idiosyncratic endowment e_0, and idiosyncratic health types i_h. Health type determines the probability of drawing a certain health shock $\varepsilon_t \in \Omega_\varepsilon = \{\varepsilon^1, ..., \varepsilon^{N_\varepsilon}\}$. The probability distribution of the shock is assumed to be age-type-dependent. Specifically, the probability of drawing $\varepsilon \in \Omega_\varepsilon$ by type i_h agent at age g is denoted by $p_{g,i_h}(\varepsilon)$, with $\Sigma_{\varepsilon \in \Omega_\varepsilon} p_{g,i_h}(\varepsilon) = 1$ for all (g, i_h). A typical history of shocks up to time t is denoted by $\sigma_t \equiv \{\varepsilon_0, ..., \varepsilon_t\}$, with $\sigma_{t+1} = \{\sigma_t, \varepsilon_{t+1}\}$. Agents are endowed with a fixed amount of time per period that can be allocated to leisure or labor. Agents participate in the labor market during the first two periods and receive a wage income $\tilde{w}e^{\zeta h}l$. Here ζ measures the effect of health on labor productivity.[1] Health is an important form of human capital. It can enhance workers' productivity by increasing their physical capacities, such as strength and endurance, as well as their mental capacities. I postulate a positive relationship between health and productivity.

During their work stage agents receive income in the form of wages and profit Π_t from the firm. They can also save a_g units of the consumption good using a storage technology with gross rate of return $R_{t+1} = 1 + r$. Retired agents have income through previous saving and profit, and consume all of their income at their last period of life.

The type of an agent is a triple (g, i_h, x), where $g \in \{1,2,3\}$ is age; $i_h \in \{healthy, unhealthy\}$ is health risk type; and $x \in R_+$ is their disposable resources at the beginning of each period defined as follows:

[1]See Bloom and Canning (2005). They model the human capital of the worker by $v = e^{\phi_s s + \phi_h h}$, where s represents years of schooling and h represents health. Here we normalize the effect of schooling.

$$x = \begin{cases} e_0, & if\ g=1 \\ (1+r)a_1, & if\ g=2 \\ (1+r)a_2 & if\ g=3 \end{cases}$$

2.1.3 Preferences

Preferences over stochastic sequences of consumption, leisure and health are given by

$$U = E_t \sum_{g=1}^{3} \beta^{g-1} \Pi \rho(h_{g-1}) \cdot u(c_g, L_g, h_g) \tag{2.1}$$

where β denotes the discount factor, ρ survival probability, c consumption, L leisure and h health status. E_t denotes the conditional expectation with the information available when the agent is born.

2.1.4 The evolution of health

I use the idea of health capital introduced by Grossman (1972a). The price of medical care p_m is exogenously given so that each unit of consumption good can be transformed into $\frac{1}{p_m}$ units of medical care. In my model medical care m can be used to produce new units of health. Each agent chooses an optimal amount of health expenditure m to build up health capital h. The accumulation process of health is given by:

$$h' = (1-\delta_h)h + \frac{\varepsilon}{\exp\left[A_m m^{\zeta}\right]}. \tag{2.2}$$

where A_m measures the medical technology. I assume that technological progress in the production of medical service A_m is exogenously given.

In Jeske and Kitao (2007) the health expenditure is an exogenous random shock. Each period in time individuals must pay the full amount for necessary health care after the shock, independent of their income level and current health stock. I, instead, endogenize medical expenditures. Hence, agents may choose the optimal amount of health care usage to build up health stock. For agents who have the same levels of health and face the same health shocks, richer agents will spend more on health care to build up better health stock[2]. Richer individuals have higher levels of consumption and lower marginal utility from consumption goods, therefore they will substitute some health for consumption goods.

Conditional on being alive at the current age with end of period health stock h, a given agent will survive to the next period with probability $\rho(h)$. Death is certain when health falls below zero ($\rho(h) = 0$ if $h \leq 0$). I assume that $\rho'(h) > 0$. Deceased agents leave their savings a as an accidental bequest that is collected by the government as revenues.

2.1.5 Medical expenses and health insurance

Young agents can have one out of three possible insurance states labeled as $in = \{1,2,3\}$. Private health insurance is $in = 1$, $in = 2$ denotes that the agent has Medicaid, and $in =$ 3 indicates the agent has no insurance. The out of pocket health expenditure will be $(1-\tilde{q}(p_m m, 1))p_m m$ if the agent chooses to buy insurance and $(1-\tilde{q}(p_m m, 2))p_m m$ when he/she is covered by the government program. It will cost the entire expenditure $p_m m$ ($\tilde{q}(p_m m, 3) = 0$) if the agent does not have insurance. Here $\tilde{q}(p_m m, in)$ is function that represents the coinsurance rate and varies with the health insurance state in as we discuss in the following subsection. Agents take it as exogenously given and it is calibrated from the data. Retired agents are insured under Medicare.

[2]Wobus, Diana Z. and Gary Olin (2005) found that the average health expenditures per person with expense decrease as you have higher income level in 2002. However, the low income has lower health insurance coverage rate and worse health status. For people age under 65, the un-insurance rate among person in families with income less than 200% of poverty line is 24.5%, while the number is only 8.7% among person in middle and high income families. The price of medical services is much higher for uninsured due to the cost shifting (see Anderson (2007)), which implies the prices of medical care paid by low income families are higher. There are 52.4% people from low income families who report their health status are very good or excellent, compared to 69.1% for middle and high income person. Taking these factors into account, it is plausible that rich agent consumes more medical service than the poor agent given the same level of health shock.

2.1.5.1 Private health insurance

To simplify the analysis, the only available private health insurance I considered is the Employer-Sponsored Health Insurance *(EHI)*. Even when an employer offers health insurance, not all workers get coverge. Some choose not to enroll, perhaps because they are young or very healthy and feel that health insurance is not a pressing need, and others' incomes are so low that they cannot afford insurance. These tradeoffs will be present in the benchmark simulation.

Once an agent chooses to purchase EHI a constant premium π_E must be paid to the insurance company, and a fraction $q_E(p_m m)$ of the total medical expenditure will be paid by the health insurance company. The premium is not dependent on prior health history or any individual states. This accounts for the practice that group health insurance does not price-discriminate the insured by such individual characteristics.

2.1.5.2 Public health insurance

The government supplies two type of health insurances, Medicaid and Medicare, to the individuals.

Medicaid Medicaid is a joint federal-state program that provides health insurance coverage to low-income children, parents, seniors and people with disabilities. The main criterion for Medicaid eligibility is limited income and financial resources. I assume that young and middle-aged individuals are eligible to receive Medicaid if their disposable resources at the beginning of the period is lower than the poverty line Y_{ma}. There is also an exogenous probability χ of getting a Medicaid offer. This captures the fact that Medicaid is only eligible for child and adults with children. The program will cover the fraction $q_{ma}(p_m m)$ of the total medical expenditure. Medicaid is a part of government spending.

Medicare I assume that all retirees are enrolled in the Medicare program. Each retiree pays a fixed premium π_{mr} for Medicare and the program will cover the fraction $q_{mr}(p_m m)$ of the total medical expenditures. Medicare is funded by the Medicare tax τ_{mr} that is proportional to the worker's labor income.

2.1.6 The representative agent's problem

A representative agent of generation $g = \{1,2\}$ enters each period with characteristics $s_g = (i_h, x, h_{g-1}, i_{ma})$, where i_h is the risk type of the agent, x is the disposable resources, h_{g-1} is the health status at the beginning of the period, and i_{ma} is the indicator function that signals the availability of the Medicaid benefit in the current period. Since all old agents are enrolled in the Medicare program and leave the labor market, their characteristics simply are $s_3 = (i_h, x, h_2)$. The distribution of households over their state space is given by $f_g(s_g, \sigma_t)$, which is endogenously determined in the equilibrium and evolves over time.

Agents observe s_g at the beginning of the period. They take prices as given and make the insurance decision $in_g(s_g)$ and choose a set of state-contingent decision rules, which can be denoted by $\{c_g(s_g, \varepsilon_g), a_g(s_g, \varepsilon_g), m_g(s_g, \varepsilon_g), L_g(s_g, \varepsilon_g)\}$, to solve the following problem.

$$\max E_t \left\{ \sum_{g=1}^{3} \beta^{g-1} \Pi \rho(h_{g-1}) \cdot u\left[c_g(s_g, \varepsilon_g), L_g(s_g, \varepsilon_g), h_g(s_g, \varepsilon_g)\right] \mid \sigma_t \right\} \tag{2.3}$$

subject to the budget constraint and a no-borrowing constraint

$$\begin{aligned} &(1+\tau_c)c_1(s_1, \varepsilon_1) + [1 - \tilde{q}(p_m m_1, in)] \cdot p_m m_1(s_1, \varepsilon_1) + \tilde{\pi}(in) + a_1(s_1, \varepsilon_1) \\ &\leq e_0 + \Pi_t + (1 - 0.5\tau_{mr}) \left[\tilde{w}_t e^{\zeta h_1} l_1(s_1, \varepsilon_1) - 1_{\{in=1\}} \tilde{\pi}(in) \right] - T(y_1) \end{aligned} \tag{2.4}$$

$$a_1(s_1, \varepsilon_1) \geq 0 \tag{2.5}$$

when young;

$$\begin{aligned}&(1+\tau_c)c_2(s_2,\varepsilon_2)+[1-\tilde{q}(p_m m_2,in)]\cdot p_m m_2(s_2,\varepsilon_2)+\tilde{\pi}(in)+a_2(s_2,\varepsilon_2)\\ &\leq R_{t+1}a_1(s_2,\varepsilon_2)+\Pi_{t+1}+(1-0.5\tau_{mr})\left[\tilde{w}_{t+1}e^{\zeta h_2}l_2(s_2,\varepsilon_2)-1_{\{in=1\}}\tilde{\pi}(in)\right]-T(y_2)\end{aligned} \tag{2.6}$$

$$a_2(s_2,\varepsilon_2)\geq 0 \tag{2.7}$$

when middle-aged; and

$$\begin{aligned}&(1+\tau_c)c_3(s_3,\varepsilon_3)+[1-q_{mr}(p_m m_3)]\cdot p_m m_3(s_3,\varepsilon_3)+\pi_{mr}\\ \leq\ & R_{t+2}a_2(s_3,\varepsilon_3)+\Pi_{t+2}-T(y_3)\end{aligned} \tag{2.8}$$

when old, where

$$h_g = (1-\delta_h)h_{g-1}+\frac{\varepsilon_g}{\exp[A_m m_g^{\zeta}(s_g,\varepsilon_g))]} \tag{2.9}$$

$$\tilde{w}_t = (1-0.5\tau_{mr})w_t \tag{2.10}$$

$$\Pi_t = \frac{(1-\alpha)Y_t}{\sum_{g=\{1,2,3\}}\mu_g\int f_g ds_g} \tag{2.11}$$

$$\tilde{\pi}(in) = \begin{cases}\pi_E, & \text{if } in=1\\ \pi_{ma}, & \text{if } in=2\\ 0 & \text{if } in=3\end{cases} \tag{2.12}$$

$$\tilde{q}(p_m m_1,in) = \begin{cases}q_E(p_m m_1), & \text{if } in=1\\ q_{ma}(p_m m_1), & \text{if } in=2\\ 0 & \text{if } in=3\end{cases} \tag{2.13}$$

$$y_g = \begin{cases}\tilde{w}_t e^{\zeta h_1}l_1(s_1,\varepsilon_1)+\Pi(\sigma_t)-1_{\{in=1\}}\tilde{\pi}(in), & \text{if } g=1\\ ra_1(s_1,\varepsilon_1)+\tilde{w}_{t+1}e^{\zeta h_2}l_2(s_2,\varepsilon_2)+\Pi(\sigma_{t+1})-1_{\{in=1\}}\tilde{\pi}(in), & \text{if } g=2\\ ra_2(s_2,\varepsilon_2)+\Pi(\sigma_{t+2}) & \text{if } g=3\end{cases} \tag{2.14}$$

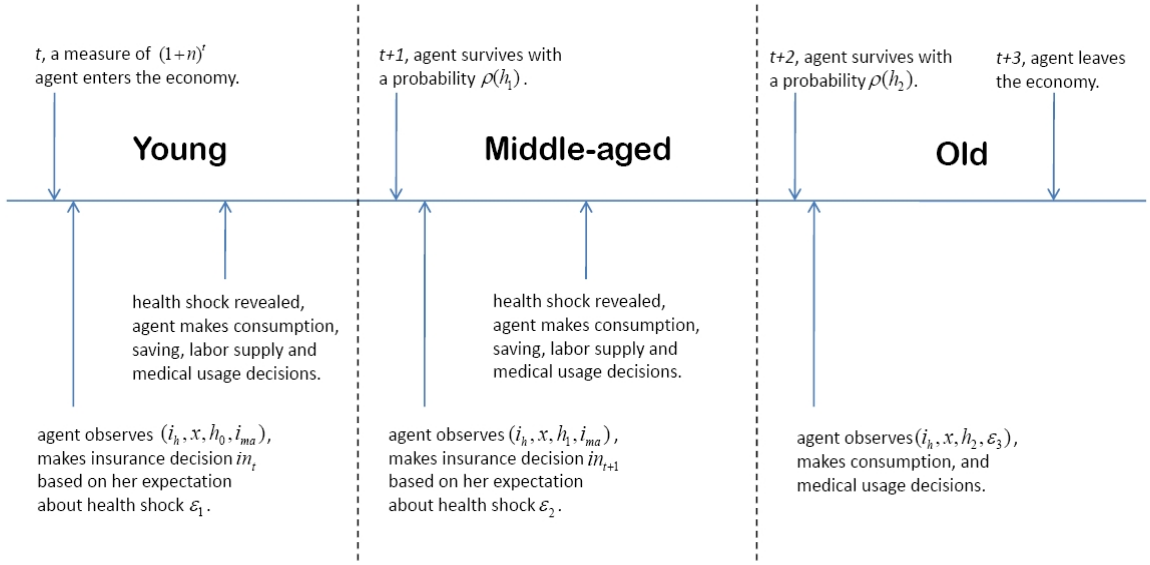

Figure 2.1: Timeline for the generation born in period t

The timeline for the generation who was born in period t is shown in Figure 2.1.

Each agent born at t is endowed with e_0. They save some storage goods $\{a_g(\sigma_{t+g-1}, s_g)\}_{g=1,2}$ to attain desirable amounts of consumption. Equation (2.10) presents the individual's after-Medicare-tax adjusted wage rate. Agents survive to the next period with probability $\rho(h_g)$. The firm needs to share the Medicare tax τ_{mr} with the agent. Hence, in equilibrium a fraction $0.5\tau_{mr}$ of tax is subtracted from the wage. Profit Π_t will be uniformly distributed to the household as payment as displayed in equation (2.11). Equations (2.12) and (2.13) explain the insurance premium paid by the individual and the co-payment rate, which vary with his health insurance state. Income taxes are imposed on the labor income paid to a worker plus accrued interest on savings and profit from the firm. Equation (2.14) represents the income tax base, which depends on the agent's age. $T(\cdot)$ is a progressive income tax function.

2.1.7 Aggregate production function

The consumption goods are produced by a neoclassical production function. The aggregate production function takes a nested Cobb-Douglas specification in the following form.

$$Y_t = A_t E_t^{\alpha} \tag{2.15}$$

$$E_t = \sum_{g=\{1,2\}} \mu_g(t) \int \left[e^{\xi h_g} l_g(s_g, \varepsilon_g) \right] f_g ds_g \tag{2.16}$$

where A_t is a total factor productivity, and E_t is an aggregate efficiency labor input, which depends on individual worker's health status. The firm's profit maximization problem is

$$\max_{\{E_t\}} A_t E_t^{\alpha} - w_t E_t. \tag{2.17}$$

Profits Π_t are distributed back to households in a lump-sum payment.

2.1.8 The government

I impose a government balanced budget constraint period by period. The government has three different types of outlays: general public consumption, Medicaid and Medicare expenses. The government collects revenues from various sources: income taxation according to a progressive tax function $T(\cdot)$, consumption taxation at rate τ_c, Medicare taxation at rate τ_{mr}, Medicare premium π_{mr}, Medicaid premium π_{ma}, and accidental bequests B collected from deceased agents.

$$\begin{aligned}
G_t + \sum_{g=\{1,2\}} \mu_g(t) \int [q_{ma}(p_m m_g) p_m m_g - \pi_{ma}] \cdot 1_{\{in=3\}} f_g ds_g + \\
\mu_3(t) \int [q_{mr}(p_m m_3) p_m m_3 - \pi_{mr}] f_3 ds_3 \\
= R_t B_t + \sum_{g=\{1,2\}} \mu_g(t) \int \tau_{mr} \left[\tilde{w}_t e^{\xi h_g} l_g - 0.5 \cdot 1_{\{in=1\}} \pi_E \right] f_g ds_g + \\
\sum_{g=\{1,2,3\}} \mu_g(t) \int [\tau_c c_g + T(y_g)] f_g ds_g
\end{aligned} \tag{2.18}$$

where y_g is the taxable income for age g agent.

2.1.9 Health insurance company

The health insurance company is competitive. Hence, in equilibrium the premium π_E is charged such that expected expenditures on the insured are precisely covered.

$$\pi_E = \frac{\sum_{g=\{1,2\}} \mu_g(t) \int \left[q_E(p_m m_g) p_m m_g \cdot 1_{\{in=1\}} \right] f_g ds_g}{\sum_{g=\{1,2\}} \mu_g(t) \int 1_{\{in=1\}} f_g ds_g} \tag{2.19}$$

Notice the coverage ratio functions $q_E(\cdot)$ are taken as exogenously given.

2.1.10 Stationary competitive equilibrium

Let $i_h \in I^2 = \{healthy, unhealthy\}, x \in \mathbb{R}_+, h_g \in \mathbb{R}_+, i_{ma} \in I^2 = \{0,1\}, \varepsilon_g \in \mathbb{R}_-$. The state space for age $g = \{1,2\}$ year old agents is $S_g = I^2 \times \mathbb{R}_+ \times \mathbb{R}_+ \times I^2 \times \mathbb{R}_-$, and the state space for the old is $S_3 = I^2 \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_-$.

Definition 2.1.1 *A* ***stationary competitive equilibrium*** *is i) fiscal variables* $\{G, \tau_c, T(\cdot), \tau_{mr}\}$*; ii) a sequence of prices for medical services* p_m*; iii) health insurance choices* $\{in(s_g)\}_{g=1,2}$*, a set of state-contingent decision rules* $\{c_g(s_g,\varepsilon_g), a_g(s_g,\varepsilon_g), m_g(s_g,\varepsilon_g), L_g(s_g,\varepsilon_g)\}_{g=1,2,3}$ *for the agents; iv) a state-contingent sequence of labor demand E; v) insurance premium* π_E*; vi) distributions of agents* $f_g(s_g)$ *over the state space S such that*

1. $\{in(s_g), c_g(s_g,\varepsilon_g), a_g(s_g,\varepsilon_g), m_g(s_g,\varepsilon_g), L_g(s_g,\varepsilon_g)\}_{g=1,2,3}$ solve the consumers problem (2.3) taking prices and taxes as given;

2. given the distribution f_g^* of households, the insurance companies choose π_E such that the budget constraint of insurance companies (2.19) holds;

3. the government sets τ_{mr}, and $T(\cdot)$ such that (2.18) holds;

4. given price w, the labor market clears

$$E = \sum_{g=\{1,2\}} \mu_g \int e^{\zeta h_g} l_g(s_g,\varepsilon_g) f_g ds_g \tag{2.20}$$

5. the accidental bequests matches the remaining assets.

$$B = \sum_{g=\{1,2\}} \mu_g \int a_g(s_g,\varepsilon_g)\cdot(1-\rho(h_g,\varepsilon_g)) f_g ds_g \tag{2.21}$$

6. the aggregate resource constraint holds

$$\begin{aligned} G + \sum_{g=\{1,2,3\}} \mu_g \int \left[c_g(s_g,\varepsilon_g) + p_m m_g(s_g,\varepsilon_g)\right] f_g ds_g + \sum_{g=\{1,2\}} \mu_g \int a_g(s_g,\varepsilon_g) f_g ds_g \\ = \mu_1 \int e_0 f_1 ds_1 + \sum_{g=\{1,2\}} \mu_g \int R_t \cdot a_g(s_g,\varepsilon_g) f_g ds_g + Y + B \end{aligned} \tag{2.22}$$

7. there is a consistency between beliefs and the actual prices.

8. the relative size of age g to the population μ_g is recursively determined by

$$\mu_g = \frac{\int \rho(h_{g-1},,\varepsilon_{g-1}) f_{g-1} ds_{g-1}}{1+n} \mu_{g-1} \tag{2.23}$$

9. the law of motion for the distribution of agents over the state space S satisfies

$$f_g^{t+1} = \int \rho(h_{g-1},\varepsilon_{g-1}) f_{g-1}^t ds_{g-1} \tag{2.24}$$

2.2 Calibration

In this section I outline the calibration of the model. Table B.4 summarizes the values and describes the parameters.

Most parameters can be independently estimated. However, there are 16 parameters that cannot be determined independent of each other as I discuss below. These include parameters of preference $(\gamma_{3,g}, \eta)$, the health production function (A_m, ζ), the survival probability function (a_ρ, b_ρ), the magnitude of the negative health shock $(\varepsilon^1, \varepsilon^2)$, the probability distribution of the shock p_{g,i_h} and the price of medical service p_m. Hence, I use a minimization procedure to determine these parameter values. More specifically, I pick parameter values such that the distance between key moments in the stationary distribution of the benchmark model and the real-world statistics listed in Table C.1 are minimized. Formally, let ψ denotes the vector of parameters, and Γ be the vector of selected real-world moments. Given ψ, a prediction $\hat{\Gamma}(\psi)$ on Γ can be computed in the stationary distribution of the benchmark. The minimization procedure can be defined as the following problem:

$$\min_{\psi} \left\| \hat{\Gamma}(\psi) - \Gamma \right\| \tag{2.25}$$

2.2.1 Data sources

The data used for estimating the process of health insurance decision and health production come from the Household Component of the Medical Expenditure Panel Survey (MEPS), which is based on a series of national surveys conducted by the U.S. Agency for Health Care Research and Quality (AHRQ). The MEPS consists of eight two-year panels from 1996/1997 up to 2003/2004 and includes data on demographics, income and most importantly health status and insurance.

2.2.2 Demographics

One period is defined as 20 years. Agents enter the economy at the age of 25 ($g = 1$) and survive up to the maximum age of 85 ($g = 3$). In line with Suen (2006), I assume that the survival probability function $\rho(\cdot)$ takes the form of the cumulative Weibull distribution function:

$$\rho(h) = 1 - \exp(-a_\rho h^{b_\rho}) \tag{2.26}$$

with $a_\rho > 0$ and $b_\rho > 0$. The endogenous survival probability rules out the case that agents survive to the next period with negative health stock.

I consider a yearly population growth of 1.25%. Together with the survival probability $\rho(h)$, the ratio of retired people to active population (the dependency ratio) is equal to 18.6% (19.2% according to the 2000 Population Census for the United States). The initial level of health at age 1, $\bar{h}_0$, is assumed to be constant over time and is normalized to 100.

2.2.3 Preferences and technology

Agents have period utility over consumption, leisure and health:

$$u(c_g, L_g, h_g) = \log c_g + \gamma_{2,g} \log L_g + \gamma_{3,g} \frac{h_g^{1-\eta}}{1-\eta} \tag{2.27}$$

The parameter $\gamma_{2,g}$ is age-dependent and I choose parameter values such that the average fraction of the time endowment allocated to market work is 0.33, which implies $\gamma_{2,1} = 1.3$, and $\gamma_{2,1} = 0.85$. Notice old agents retire from the labor market and they spend all time on leisure. For simplicity I set $\gamma_{2,3} = \gamma_{2,1}$. $\gamma_{3,g}$, which is age-dependent as is $\gamma_{2,g}$, measures the importance of health and η denotes the coefficient of relative risk aversion of health.

The annual subjective discount factor is taken to be 0.97, so $\beta = (0.97)^{20} = 0.5936$. The average annual interest rate in the U.S is 4%, so $r = (1 + 0.04)^{20} - 1 = 1.19$.

2.2.4 Production of health and health shocks

The health measure h used in this paper is the Physical Component Summary scores formed from the answers to the Short-Form 12 questions. For people aged between 25 and 85, the lowest health level is 4.56 and the highest level is 72.17 in the MEPS data.[3] This paper assumes that human beings can live up to 85 years without any accident or illness. We choose δ_h such that $72.17 \times (1-\tilde{\delta}_h)^{60} = 4.56$, where $\tilde{\delta}_h$ refers to annual health depreciation rate. I also assume that the depreciation rate increases with age. Therefore I choose depreciation rate of $\{0.4, 0.4, 0.5\}$.

The transition of agent's health is described by equation (2.2). Agents can offset the negative effect of a health shock by purchasing medical care. The productivity of medical care is captured by A_m, and the price of medical care is p_m. Both are exogenously given.

Brown (2006) found that uninsured people in California pay 65% more for common prescription drugs than the federal government does for the same medications. Anderson (2007) found that the uninsured patients pay up to 2.5 times for hospital service than health insurers. I assume that uninsured consumers pay a 60% higher price for medical services than the insured, so that $p_m^u = 1.6 \times p_m^i$. This is similar to Jung and Tran (2008). I assume that the relative price of medical service p_m is the weighted average price paid by the insured and the uninsured, i.e. $p_m = (1-\theta)p_m^i + \theta p_m^u$, where θ is the fraction of uninsured in the population. According to Kaiser (2007), the value of θ was 18% in 2006. Therefore, I pick $p_m^i = 0.9145 p_m$, and $p_m^u = 1.4605 p_m$.

I differentiate agents into two groups, which are high-risk and low-risk, by using the estimation procedure of Bundorf, M. Kate et al (2005).[4] The health shocks take two possible values $\{\varepsilon^1, \varepsilon^2\}$. For the same age cohort high-risk people are different from low-risk people in terms of the probabilities $p_{g,i_h}(\varepsilon)$ of getting the same shock ε. The health shocks $\varepsilon \in \Omega_\varepsilon = \{\varepsilon^1, \varepsilon^2\}$ and the probability distribution of the shock $p_{g,i_h}(\varepsilon)$ are chosen so that

[3]As for how to calculate these summary scores, please refer to Ware et al, How to Score the SF-12(r) Physical and Mental Health Summary Scales, QualityMetric,Inc., Lincoln, RI.

[4]Please refer to the technical appendix of Bundorf, M. Kate et al (2005) for the detailed procedure.

the health insurance take-up rate (percentage of workers buying private insurance per age-type group) and the share of health expenditure in GDP is approximated.

2.2.5 Health insurance

Private health insurance The coverage rate increases in the health expenditures incurred by the patients. Similar to Jeske and Kitao (2007) I assume that the coverage ratio is a function of total health expenditure $p_m m$ and takes the following form.

$$q_E(p_m m) = \beta_0^E + \beta_1^E \log(p_m m) + \beta_2^E \left[\log(p_m m)\right]^2 \tag{2.28}$$

I estimate the set of parameters $\{\beta_0^E, \beta_1^E, \beta_2^E\}$ using the MEPS data. I rank the health expenditure and use 5 bins for health expnediture data. I specify the bins of uniform size. Therefore the first bin contains individuals whose health expenditure is between zero and 20-quantile. The 20% spending the most on health care belongs to the fifth bin. I plug in the health expenditure data to attain the average coverage ratio for each bin.

The coverage ratios of Medicaid and Medicare are estimated by the same procedure. I report the parameter values and coverage ratios for each expenditure grid in table B.2 and B.3. In table B.3, the standard errors in brackets and all coefficient estimates are significant at the 1% level. The insurance premium π_E is determined in the equilibrium to ensure zero profits for the insurance company.

Medicaid I use Medicaid as a proxy of public health insurance for the non-elderly population, which includes S-CHIP. I use the MEPS data to calculate the acceptance rate of Medicaid $\chi = 0.6$. The beneficiaries of Medicaid typically do not pay anything for enrolling in the program. I pick $\pi_{ma} = 0$ in the simulation.

Medicaid is funded by general government revenue. The income level characteristic of Medicaid is typically 100% to 133% of the federal poverty line (FPL) and SCHIP is

200%.[5] I set $Y_{ma} = \$12,000$ or about 34% of annual per capita GDP in the benchmark.

Medicare I assume that every old agent is enrolled in Medicare. Medicare taxes are levied on all labor income and split between employer and employee contributions. The Medicare premium was \$799.20 annually in 2004 or about 2.11% of annual GDP. The Medicare tax rate τ_{mr} is determined within the model so that the government budget is balanced.

2.2.6 Firms

I choose a standard labor share in production of $\alpha = 0.66$ from NIPA. Total factor productivity is normalized to $A = 8$ such that the average labor income equals 10 in the benchmark. In line with Bloom and Canning (2005), I assume that individual worker's health status affects the efficiency of labor input by a factor of $e^{\xi h}$. Therefore, labor income is given by $we^{\xi h}l$, where w is the average wage rate. I estimate the parameter ξ that fits the following equation using the MEPS data.

$$\log(LaborIncome) = \xi h + \log(AverageWage \times WorkingHours) + \varepsilon \tag{2.29}$$

where h is the Physical Component Summary scores that measure the individual's health status ranging from 0 to 100. I normalize the average labor income observed in the data to be 10.0 and I calculate $\xi = 0.1393$ in the benchmark.

2.2.7 Government

The value for G is exogenously given and is fixed across all policy experiments. I calibrate it to 27.5% to match the share of government consumption, social security and gross investment excluding transfers, at federal, state and local levels (The Economic Report of the

[5] Source: Genevieve M. Kenney, Jennifer M. Haley, Alexandra Tebay. Children's Insurance Coverage and Service Use Improve. Urban Institute. July 31, 2003. http://www.urban.org/publications/310816.html

President, 2004). This number is bigger than the standard value of 18% because I do not model a Social Security program and Social Insurance as in Jeske and Kitao (2007). The consumption tax rate is 5.67% as in Mendoza, Razin, and Tesar (1994).

The income tax function follows the functional form studied by Gouveia and Strauss (1994), which is given as

$$T(y) = b_0 \left(y - (y^{-b_1} + b_2)^{-1/b_1} \right) + \tau_y y \tag{2.30}$$

Parameter b_0 is the limit of marginal taxes in the progressive part as income goes to infinity, b_1 denotes the curvature of marginal taxes and b_2 is a scaling parameter. I use the parameters estimated by Gouveia and Struss (1994), which are $\{b_0, b_1, b_2\} = \{0.258, 0.768, 0.716\}$. When they calibrate the tax function, the income has been normalized to the range of $[0, 1]$. In my model, I divide taxable income of every agent by the maximum income observed in the simulated economy to get the normalized income. Then I use this normalized income directly in (2.30) to get the tax rate. The parameter τ_y in the proportional term of the income tax equals 10% in the benchmark.

2.3 Numerical results

All potential reforms start from the same initial steady state calibrated to the current U.S. economy and end in a different final steady state with an alternative health insurance system. Therefore, I first compare moments of associated invariant distributions. Then I discuss the quantitative aspects of the transitions and welfare analysis associated with each of the reforms considered.

2.3.1 Benchmark model

Table C.1 reports the main features of the benchmark simulation. Under the baseline parameterizations the model is able to match the main features of the current economy in the U.S. The fraction of insured agents among all young and middle-aged agents is 84.8%, which is slightly higher than 82% in the data. Among non-elderly, 12.3% are covered by the Medicaid program (12.9% in the data). The model overstates total health expenditure as a ratio of GDP, which is about 15.8% according to Department of Health & Human Services (2006). The model reports 16.6%. The model matches working hours fairly well, which is 30.6% of total non-sleeping time (33.3% in the data). The gross saving rate is 25.8% (21% in the data).

Next, I examine the model's predictions on the life-cycle patterns of medical spending and consumption. Panel 1 of Figure 2.2 displays medical spending over various age groups. According to MEPS, the average health expenditure is roughly constant from ages 25 to 64 and almost triples afterwards. The benchmark model is able to replicate the increasing pattern. However, the magnitude of the health expenditure is bigger than in the data, especially for non-elderly agents. In the steady state, a representative agent age between 25 to 44 spends $5,697 or about 14.5% of per capita GDP (7.48% in the data). Agents between ages 45 to 64 years old on average spend $6,783, or about 12.7% of per capita GDP (11.02% in the data). Agents over 65 spend $13,283, or about 29.8% of per capita GDP (32.59% in the data).

Panel 2 of Figure 2.2 shows the consumption over various age groups. Fernandez-Villaverde and Krueger (2002) estimated the life-cycle consumption profiles using data from the Consumer Expenditure Survey. They found that non-durable consumption peaked at age 52 and was about 29% higher than at age 25. The current model is able to generate similar hump-shaped patterns. However the peak level is only about 13.7% higher than that in ages between 25 and 44.

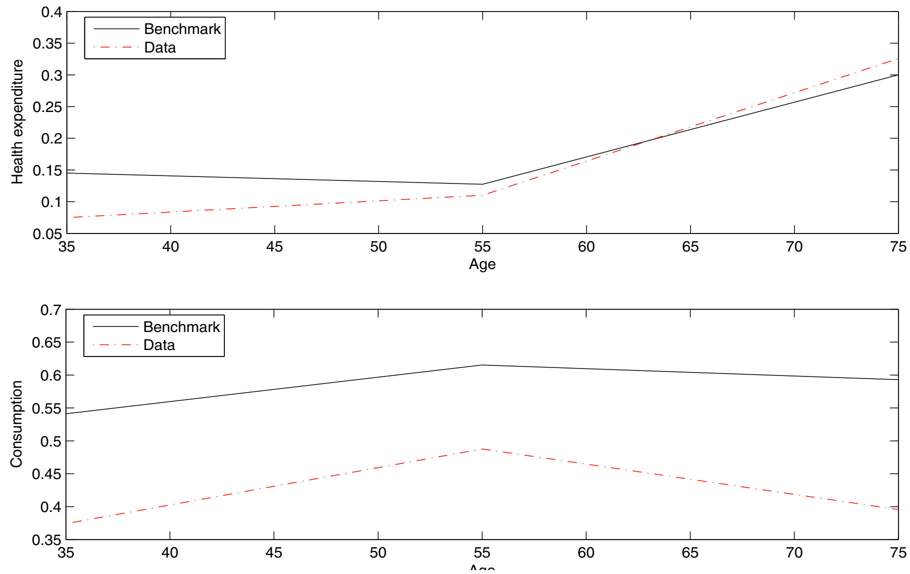

Figure 2.2: Health expenditure and consumption life profiles

2.3.2 Policy experiments

I now conduct experiments to determine the effect of reforming the health insurance system. I am interested in changes in health expenditure as a ratio of GDP, the change in taxes that balances the government budget, aggregate labor supply, aggregate health status, savings rate and output. I treat changes in government revenue as follows: expenditures G, consumption tax rate τ_c, the progressive part of income tax function $T(\cdot)$ and the proportional income tax rate τ_y remain unchanged from the benchmark. I adjust the medicare tax τ_{mr} to balance the government's budget.

In each experiment I first compute a steady state outcome under the stationary equilibrium and then the transition dynamics. In line with Conesa and Krueger (1999), I measure the welfare effect of a reform by computing the consumption equivalent variation (CEV). I quantify the welfare change of a given policy reform for an individual of type (i_h, x, i_{ma}) by asking by how much (in percent) this individual's consumption has to be increased in all future periods and contingencies (keeping health expenditure, leisure and health insurance status constant) in the old steady state so that his expected life-time utility equals that under a specific policy reform. I denote it with $CEV(i_h, x, i_{ma})$. For example, a $CEV(i_h, x, i_{ma})$ of -10% implies that if the given policy reform is put into place, then an individual of type (i_h, x, i_{ma}) will experience an decrease in welfare due to the reform equivalent to sacrifice 10% of his consumption in the initial steady state with leisure, health insurance and health expenditure constant at the initial steady-state choices.

Alternative sources of revenue to fund these reforms are also considered. I first consider supporting the reform by adjusting the income tax. I also conduct companion experiments where the government funds the reform through a payroll tax and through a lump-sum transfer separately.

2.3.2.1 Policy experiment A: expansion of Medicare to the entire population

In this experiment the private health insurance and the Medicaid program are abolished. Non-elderly will be covered by a uniform health insurance program, which is sponsored by the government, with premium π_{mr} and coverage rate $q_E(\cdot)$. Specifically, non-elderly pay for a premium that equals 2.11% of the per capita GDP. A fraction $q_E(p_m m)$ of their health expenditure will be paid by the government. Compared to the benchmark, 72.5% of non-elderly who purchase private insurance pay an actuarially fair premium π_E, which is about 10.9% of the per capita GDP.

I assume that the price for medical service equals the average price for medical service in the equilibrium of the benchmark, which means $p_m^{\exp} = p_m^{ben} = (1-\theta)p_m^i + \theta p_m^u$. The medical technology A_m is constant and exogenously given. I can also consider a case in which the technology slows down (or speeds up) as a result of the reform.

Experiment results are summarized in Table 2.1. The top section displays some statistics on aggregate variables: the fraction of insured non-elderly, the Medicare tax rate, the average effective income tax rate, average working hours, average effective working hours, and the health expenditure as a ratio of GDP. The lower section displays the welfare effects of each reform. *% w/ CEV > 0* indicates the fraction of young agents in the benchmark that would experience a welfare gain (positive *CEV*) if the alternative reform is taken place.

Expansion of Medicare to the entire population achieves a universal coverage as shown in the fraction of insured non-elderly. The aggregate health expenditure as a ratio of GDP increases by 0.3%. This is because those newly insured non-elderly will consume more medical service and incur higher amounts of health expenditure as the reform provides them with cheaper health insurance. The program needs to cover 15.2% of the non-elderly who would be uninsured in the benchmark and to pay for part of the expenditure of the insured, who pay a premium of π_{mr} after the reform, which is about 20% of the premium they paid in the benchmark. Therefore, the government raises the proportional income tax rate by 4.5%. As a consequence, average working hours decreases by 4.8% to 28.7. The average health stock of the non-elderly increases from 46 to 47, which implies a long life expectancy and a higher saving incentive. A decreased exposure to the health shocks lowers the precautionary saving demand, but this effect is dominated by the previous one and the aggregate saving rate slightly increases by 0.8%.

Although the proportional income tax rate τ_y is higher than in the benchmark, the cheaper health insurance program from the government is enough to compensate this cost for most agents. As shown in *% w/ CEV > 0,* 72.6% of young agents would experience a welfare gain from this reform, and the average welfare effect is in the order of 2.6% in terms of consumption in every state. However, low income agents, especially those with Medicaid offers, will suffer because the new insurance program from such a reform is less generous than Medicaid. On average, low-income individuals would experience a welfare loss equivalent to 4.27% of consumption. While agents who have income above the poverty line have a welfare gain of 5.96% of consumption.

A-1: Medicare expansion.

A-2: Medicare expansion with Medicaid.

I also consider a experiment A-2 to test whether an expansion of Medicare can improve all individuals' well-being. In this experiment, the government offers low-income agents

Table 2.1: Policy Experiment A

	Bench.	A-1	A-2
Insured non-elderly (in %)	84.8	100	100
Medicare tax (in %)	2.5	2.5	2.5
Ave. income tax (in %)	24.6	29.4	30.39
Ave. Working hrs.	30.6	28.7	28.5
Ave. Effective Working hrs.	61.06	57.3	56.97
Health exp. (in % of GDP)	16.6	16.91	17.7
π_E (in % of per capita GDP)	10.1	2.11	2.11
Output	100	97.96	98.06
Aggregate saving rate (in %)	25.8	26.6	26.9
Average consumption	100	97.1	95.6
Average health stock	46.6	46.88	46.84
CEV from transition			
all young (in %)	–	2.6	2.8
young w/ $e_0 > Y_{ma}$ (in %)	–	5.96	4.85
young w/ $e_0 \leq Y_{ma}$ (in %)	–	-4.27	-1.39
% w/ CEV > 0 (young)	–	72.6	76.7

with Medicaid and keeps the rest the same as in experiment A-1. Specifically, non-elderly whose incomes are below the poverty line will be covered by Medicaid. Agents whose income are above the poverty line need to pay a premium equal to 2.11% of the per capita GDP. A fraction $q_E(p_m m)$ of their health expenditure will be paid by the government. Apparently, the tax rate needs a bigger increase. This can be explained by the fact that this reform is more generous to low income individuals and they will spend more in health. However, the benefit from such a guaranteed Medicaid coverage cannot offset the loss due to a higher tax rate, which is used to supply generous Medicaid program to low income agents. As shown in CEV from transition, young agents with $e_0 \leq Y_{ma}$ still experience a welfare loss, but at a much smaller magnitude of 1.39%. The welfare gain of higher income young agents decreases to 4.85% from 5.96% in experiment A-1. On average, young agents have a welfare gain in the order of 2.8% in terms of consumption in every state. From this experiment, it seems possible to make expansion of Medicare a welfare improving program for everybody by appropriately funding the reform.

2.3.2.2 Policy experiment B: expansion of public health insurance

Policy experiment B involves expansion of the public health insurance, including Medicaid/S-CHIP (Jonathan Gruber, 2001). Approaches that follow this model generally build on existing public programs by raising income limits to include many more needy people and do away with all tests of eligibility except income. In experiment B-1, I increase the Medicaid offer rate to $\chi = 1$. Specifically, agents who meet the maximum income requirement will be covered by Medicaid with probability 1, compared to a probability of 0.6 in the benchmark. While in experiment B-2, I leave the Medicaid offer rate χ unchange and increase the maximum income requirement to 300% of the poverty line. I report experiment results in Table 2.2.

When the government extends Medicaid to include all agents who meet the maximum income requirement, the spending in Medicaid as a ratio of GDP increases from 1.65% to 2.46%. As these newly insured people consume more medical services than in the benchmark, aggregate health expenditure increases as well. The proportional income tax rate has been raised by 1.5% to match this spending. As a consequence, average working hours decrease by 1.3%. Medicaid expansion alone cannot achieve "universal health care". This reform will leave 10.5% of the non-elderly without insurance coverage. These agents choose not to purchase private insurance because they are relatively healthy and expect to have a smaller health shock.

When the government increases the maximum income requirement in experiment B-2, some previously insured agents will choose to apply for Medicaid and at the risk of being uninsured. Consequently, the insured as a fraction of non-elderly decreases to 81.2%. However the aggregate health expenditure increases to 17.4% of GDP. This is because Medicaid is more generous than private insurane in terms of the coverage ratio. The expansion of Medicaid by raising the income standard requires a bigger increase in income tax rate as it covers another 18% of non-elderly. The average working hours decreses by 3%.

B-1: Public health insurance expansion 1.

Table 2.2: Policy Experiment B

	Bench.	B-1	B-2
Insured non-elderly (in %)	84.8	89.5	81.2
Medicare tax (in %)	2.5	2.5	2.5
Ave. income tax (in %)	24.6	25.9	30.6
Ave. Working hrs.	30.6	30.2	29.7
Ave. Effective Working hrs.	61.1	60.4	59.3
Health exp. (in % of GDP)	16.6	17.02	17.4
π_E (in % of per capita GDP)	10.8	10.8	9.7
Output	100	99.9	98.7
Aggregate saving rate (in %)	25.8	25.9	27.2
Average consumption	100	99.2	97.1
Average health stock	46.6	46.7	46.79
CEV from transition			
all young (in %)	–	-0.28	-2.4
young w/ $e_0 > Y_{ma}$ (in %)	–	-1.07	-1.64
young w/ $e_0 \leq Y_{ma}$ (in %)	–	1.34	-2.73
% w/ CEV > 0 (young)	–	10.96	0

B-2: Public health insurance expansion 2.

Now let's look at the welfare effect. Public insurance expansion as in experiment B-1, which includes all agents who meet the maximum income requirement, is beneficial to low-income agents. They experience a welfare gain in the order of 1.34% in terms of consumption in all states in B-1. They benefit from these reforms with a guaranteed public insurance coverage and in exchange pay a higher income tax. Given the small size of the program, the benefit is enough to compensate for the loss due to a tax increase. This type of reform is welfare decreasing for high income agents who do not qualify the maximum income requirement. This is because their health benefits are intact and they need to pay for a higher tax to support the expanded Medicaid program. They will suffer a loss equivalent to more than 1% in terms of consumption in all states.

While to increase the maximum income requirement makes everybody worse off. Agents whose income is below the existing maximum income requirement have the same public insurance coverage as in the benchmark. However they are are required to pay for a higher tax rate. As a consequence, they experience a welfare loss of the order of 2.73% in terms of consumption in all states. High income agents benefit from the reform with a cheaper insurance or a chance of being covered by Medicaid depending on their income level. While the cost of higher income tax cannot be offset by this benefit. Consequently, they experience a welfare loss of the order of 1.64% in terms of consumption, which is in a smaller magnitude compared with low income agents who do not benefit from this reform.

2.3.2.3 Policy experiment C: individual mandate

Table 2.3: Policy Experiment C

	Bench.	C
Insured non-elderly (in %)	84.8	100
Medicare tax (in %)	2.5	2.5
Ave. income tax (in %)	24.6	25.2
Ave. Working hrs.	30.6	30.66
Ave. Effective Working hrs.	61.1	61.3
Health exp. (in % of GDP)	16.6	17.04
π_E (in % of per capita GDP)	10.8	9.5
Output	100	100.2
Aggregate saving rate (in %)	25.8	26.7
Average consumption	100	99.1
Average health stock	46.6	47.1
CEV from transition		
all young (in %)	–	-0.63
young w/ $e_0 > Y_{ma}$ (in %)	–	-0.29
young w/ $e_0 \leq Y_{ma}$ (in %)	–	-1.32
% w/ CEV > 0 (young)	–	0

In this experiment about 15% of non-elderly are forced to purchase private insurance, who are relatively healthier. Their entry into the insurance market makes the risk pool better and the insurance premium lower. Consequently, the price of private insurance decreases by 12%. The aggregate health expenditure as a ratio of GDP increases to 17.04% as everybody has insurance coverage. The aggregate health status becomes better and the average working hours increases by 0.2% even though the reform requires an higher income tax rate. In terms of welfare, an individual mandate makes everybody worse off. Such a reform imposes a higher income tax rate, whose cost cannot be offset by a cheaper insurance for high income agents. Among low income agents, only a small fraction holds private insurance. Consequently they benefit less from the cheaper insurance and they experience a welfare loss at the magnitude of 1.32% in terms of consumption in all states, compared with a loss at the order of 0.29% for high income agents.

2.3.2.4 Policy experiment D: abolishing tax deductibility of private insurance premiums and providing a tax credit

Compared with the above experiments, policy experiment D-1 is a market-based reform rather than a government program. Under this experiment, the deductibility of the insurance premium for income tax is abolished. Taxes are now collected on the entire portion of the premium and the taxable income is given as

$$y_g = \begin{cases} \tilde{w}_t e^{\zeta h_1} l_1(s_1,\varepsilon_1) + \Pi(\sigma_t), & \text{if } g = 1 \\ r a_1(s_1,\varepsilon_1) + \tilde{w}_t e^{\zeta h_2} l_2(s_2,\varepsilon_2) + \Pi(\sigma_t), & \text{if } g = 2 \\ r a_2(s_2,\varepsilon_3) + \Pi(\sigma_t) & \text{if } g = 3 \end{cases} \tag{2.31}$$

At the same time, the government will provide agents with a refundable tax credit in experiment D-2. This tax credit is only given to agents who purchase private insurance.

Experiment results are summarized in Table 2.4. Removing the tax subsidy in D-1 leads to a partial collapse of the private insurance market as found by Jeske and Kitao (2007). The fraction of non-elderly who purchase private insurance falls from 72.5% to 37.5%.[6] More than 1/3 of the non-elderly opt out of the private insurance market and choose to be self-insured. Those are the agents in a better health condition who face a lower

[6]This experiment is similar to experiment A in Jeske and Kitao (2007). The magnitude of the decrease here is bigger than in their paper. This result can be explained by the fact that I model the health expenditure as endogenous decision. The demand for medical services by healthy individuals is more elastic to price change than unhealthy individuals as found by Bajari, Hong and Khwaja (2006). A model with exogenous health expenditure as in Jeske and Kitao (2007) cannot capture this effect and the change in the number of insured will be smaller.

probability of suffering a bad health shock. The exit of these agents out of the insurance market deteriorates the risk pool and the price of the private insurance jumps by 15%. The aggregate health expenditure as a ratio of GDP falls 1.2% as those self-insured spend less on health. The income tax rate falls as the income base increases with the removal of the tax deductability for premium. As a consequence, average working hours slightly increase by 0.5%.

A tax credit creates incentives for individuals to purchase private insurance as in experiment D-2. The fraction of insured non-elderly jumps to 94.2% as the tax credit goes to agents who purchase private insurance. Consequently, the price of private insurance falls to 9.68% of per capita GDP and the health expenditure rises to 16.89% of GDP.

Table 2.4: Policy Experiment D

	Bench.	D-1	D-2
Insured non-elderly (in %)	84.8	49.1	94.2
Medicare tax (in %)	2.5	2.5	2.5
Ave. income tax (in %)	24.6	23.1	27.5
Ave. Working hrs.	30.6	30.76	29.8
Ave. Effective Working hrs.	61.1	61.11	59.43
Health exp. (in % of GDP)	16.6	15.38	16.89
π_E (in % of per capita GDP)	10.8	12.14	9.68
Output	100	100.01	99.15
Aggregate saving rate (in %)	25.8	25.76	26.17
Average consumption	100	102.4	98.3
Average health stock	46.6	46.35	46.78
CEV from transition			
all young (in %)	–	1.76	−0.22
young w/ $e_0 > Y_{ma}$ (in %)	–	1.58	0.81
young w/ $e_0 \leq Y_{ma}$ (in %)	–	2.14	−2.3
% w/ CEV > 0 (young)	–	73.97	67.1

C-1: Abolish private insurance deductibility from income tax base.

C-2: Abolish private insurance deductibility from income tax base and provide credit for individuals who purchase private insurance.

In terms of welfare, the removal of the subsidy for purchasing private health insurance is welfare improving, as 74% of the young would experience a welfare gain. For most individuals, a lower income tax rate is enough to compensate for the welfare loss due to the lower insurance coverage and increased exposure to health shocks. On average a young individual will benefit from this reform in the order of 1.76% in terms of consumption in all states. In D-2, A tax credit to private insurance buyers would encourage health insurance market participation. While the proportional tax rate τ_y is higher than in the benchmark due to the tax credit, it cannot be offset by the benefit from the higher insurance coverage. On average, a young agent would experience a welfare loss equivalent to 0.22% in terms of consumption.

2.3.3 Alternative approaches of funding the reforms

2.3.3.1 Income tax v.s. payroll tax

In order to understand how the macroeconomic effects of these proposals change in response to how the government finances the reform, I also consider funding the reform by changing the payroll tax τ_{mr}. Now, government expenditure G, consumption tax rate τ_c and the progressive part of income tax function $T(\cdot)$, as well as the proportional tax rate τ_y remain unchanged from the benchmark. I adjust the payroll tax rate τ_{mr} to balance the government's budget.

As shown in average working hours in table 2.5, to adjust the payroll tax creates bigger distortions compared with income taxes.[7] Notice I change some policy targets in order to make the experiment meaningful. In experiment A, the Medicare premium doubles from 2.11% of GDP to 4.22%. Otherwise the payroll tax rate will skyrocket and partially crash the labor market as some agents will leave the market. To finance the reform with payroll tax requires the Medicare tax to increases from 2.5% to 7.87%. As a consequence, average

[7]There is no capital in my model. The profit Π is distributed back to the agent as a payment, which is inelastic supply to the individual. The interest rate is exogenous and the demand for saving is inelastic as well. Furthermore, the tax base of income tax is broader than labor tax. These facts explain why tax labor income creates more distortion than to tax income.

working hours decrease by 5.6%. The welfare of an average agent decreases compared to funding the reform through the income tax change.

Given the relatively small size of the Medicaid program, public insurance expansion (experiment B-1, B-2) requires a gradual increase in the Medicare tax. Average working hours decrease by 4.6% in B-1 and 6.5% in B-2 (1.3% and 0.9% when the reforms are funded through payroll taxes). Again, welfare decreases compared to the experiments when the government funds the reform through income tax.

Similar to experiment A, the tax credit has been decreased to $500 in D-2. When reform D-1 is funded through the labor tax, a larger tax rate drop leads to a 6.7% rise in average working hours and the young agent experiences a welfare gain of more than double. Even though the tax credit in D-2 is much smaller than in the experiment when the reform is funded through income tax, we still can observe a decrease in working hours of 3.3%.

Table 2.5: Policy Experiments - payroll tax

	Bench.	A-1	A-2	B-1	B-2	C	D-1	D-2
Insured non-elderly (in %)	84.8	100	100	89.5	84.6	100	49.1	74.7
Medicare tax (in %)	2.5	7.87	11.2	7.81	2.7	5.56	-7.77	9.3
Ave. income tax (in %)	24.6	24.7	24.5	24.1	24.6	24.2	26.2	24.6
Ave. Working hrs.	30.6	28.87	27.8	29.18	30.5	30.06	32.67	28.9
Ave. Effective Working hrs.	61.1	57.73	55.8	58.4	60.93	60.15	64.86	57.5
Health exp. (in % of GDP)	16.6	16.84	17.73	17.06	16.59	17.06	15.33	16.32
π_E (in % of GDP)	10.8	4.22	4.22	10.05	10.1	9.66	12.17	10.5
Average consumption	100	97.2	95.1	97.4	99.9	98	105.9	95.8
Average health stock	46.6	46.88	46.83	46.69	46.82	47.4	46.39	46.4
CEV from transition								
all young (in %)	–	1.81	2.52	-1.57	1.09	-1.32	3.84	-3.5
% young w/ $e_0 > Y_{ma}$	–	5.45	4.37	-1.92	3.09	0.76	2.89	-2.87
% young w/ $e_0 \leq Y_{ma}$	–	-5.63	1.26	-0.85	0.11	-2.46	5.78	-4.8
% w/ CEV > 0 (young)	–	72.6	76.7	9.59	15.07	0	75.3	0

A-1: Medicare expansion.

A-2: Medicare expansion with Medicaid.

B-1: Public health insurance expansion 1.

B-2: Public health insurance expansion 2.

C: Individual mandate.

D-1: Abolish private insurance deductibility from income tax base.

D-2: Abolish private insurance deductibility from income tax base and provide credit for individuals who purchase private insurance.

2.3.3.2 Changing tax rates vs. Lump-sum transfer

The analysis so far indicates that the change in taxes may play a dominant role in how the reform affects the macroeconomy. In order to isolate the effect of tax changes, I also conducted companion exercises in which the government funds the reform through a lump sum transfer. In the companion experiments, the tax rates are kept intact as in the benchmark. The government returns a lump sum transfer to each individual. The transfer is determined so that the government's budget is balanced.

The results in Table 2.6 confirm the above conjecture. The greatest labor supply effect is observed in experiment D-1 with a 2.3% decrease in average working hours, compared to an average 4% change when the reforms are funded through the income tax.

A-1: Medicare expansion.

A-2: Medicare expansion with Medicaid.

B-1: Public health insurance expansion 1.

B-2: Public health insurance expansion 2.

C: Individual mandate.

D-1: Abolish private insurance deductibility from income tax base.

D-2: Abolish private insurance deductibility from income tax base and provide credit for individuals who purchase private insurance.

Health insurance reforms that can decrease the number of uninsured (as in A-1, A-2, B-1, C, and D-2) will improve the aggregate health status even though the effect might be small. As the insured consume more medical service, the aggregate health spending rises

Table 2.6: Policy Experiments - Lump sum transfer

	Bench.	A-1	A-2	B-1	B-2	C	D-1	D-2
Insured non-elderly (in %)	84.8	100	100	89.5	80.44	100	49.1	94.2
Medicare tax (in %)	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
Ave. income tax (in %)	24.6	25.4	25.4	24.7	25.0	24.6	25.2	25.3
Ave. Working hrs.	30.6	30.56	30.6	30.66	31.08	30.81	29.89	30.7
Ave. Effective Working hrs.	61.06	60.97	61.2	61.26	61.94	61.53	59.41	61.2
Health exp. (in % of GDP)	16.6	16.67	17.5	16.98	17.3	16.81	15.46	16.8
π_E (in % of GDP)	10.1	2.11	2.11	10.1	9.7	9.66	11.18	9.64
Average consumption	100	99.3	98.6	99.8	98.9	99.6	101.2	99.6
Average health stock	46.6	46.82	46.85	46.7	46.8	46.96	46.3	46.78
CEV from transition								
all young (in %)	–	2.73	3.0	0.25	−2.51	−0.38	1.75	−0.14
% young w/ $e_0 > Y_{ma}$	–	5.97	4.94	−1.1	−1.44	−0.1	1.59	0.81
% young w/ $e_0 \leq Y_{ma}$	–	−3.89	−0.87	1.49	−3.04	−0.95	2.07	−2.1
% w/ CEV > 0 (young)	–	72.6	76.7	10.96	0	5.48	73.97	67.12

as well. Better health encourages labor supply as labor productivity increases with health stock. As shown in experiment C, average working hours increase by 0.7% as average health stock increases by 0.5%. Among the reforms I considered, only experiment B-2 and D-1 fail to decrease the number of the uninsured. Aggregate health expenditure decreases as fewer people have insurance coverage in experiment D-1. The average health stock falls as well. In experiment C-1, poorer health status discourages labor supply and the average working hours decreases by 2.3%, which is substantial.

In terms of welfare, the implication is almost identical to when the government finances the reforms with the income tax, but with a slightly different magnitude.

2.4 Conclusion

I build a micro-founded dynamic general equilibrium model to study the impact of alternative health care reform proposals on the aggregate labor supply, health expenditure, saving, welfare, and on the fraction of adults with no health insurance. As opposed to some papers in the literature, I consider a model with a labor-leisure choice. This is important because a health care reform affects the demand for medical usage, which in turn affects the individual's health status and labor productivity. A reform may create distortions on the labor supply by requiring additional tax revenues to fund such reform. The magnitude of the distortion depends on the details of the reform as well as how to fund the reforms.

As policymakers evaluate alternative approaches to reforming the health insurance system in the U.S., they should consider several tradeoffs: the reduction in the number of uninsured, alternative distortions of the labor market, deadweight loss and the cost of raising public funds to cover government programs. These complicated tradeoffs can only be fully captured in a general equilibrium framework, similar to the one employed in my analysis. My results suggest that Medicare expansion and an individual mandate are good candidates for achieving universal health care, while a removal of the tax subsidy to purchase private insurance would result in a significant reduction in the insurance market. For all proposals studied, the aggregate health expenditure rises as the number of insured increases. Funding the reform through payroll taxes does not seem promising because such a policy can heavily distort the labor market, especially in the case of the expansion of Medicare and providing tax credit to the insured.

Regarding quantitative implications of the reforms, I find that the impact on the aggregate labor supply may vary between -9.1% and 6.8%, depending on the details of the reforms and how they are funded. In some reforms, such as the expansion of Medicare to the entire population and the expansion of Medicaid, cheaper insurance means a better health risk pool, lower premiums and better health, which in turn increases labor productivity and working hours. However, some reforms require higher taxes which result in lower working hours, for example the expansion of Medicare and an individual mandate. Quantitatively, I find that the expansion of Medicaid funded with income taxes results in the smallest change in hours worked because the government only needs to collect tax revenue to include about 5% of the non-elderly into the public insurance program. Similarly, the change with the strongest impact on hours worked is the removal of the tax break to purchase the group insurance funded through the labor tax. This is because a larger fraction of non-elderly (72.5%) pay a tax for the insurance premium, which is income tax free in the benchmark. Consequently, a lower labor tax rate is needed to balance the government budget.

In terms of welfare implications, an increase in insurance coverage does not always improve welfare. Both Medicare expansion and individual mandate can achieve universal insurance coverage. Medicare expansion improves the aggregate welfare by offering cheaper insurance. In contrast, an individual mandate may deteriorate welfare even though the risk pooling becomes more inclusive and the premiums go down as agents are forced to purchase insurance. This is because the government needs to increase other taxes so that the newly insured can enjoy the subsidy for purchasing insurance. The removal of the tax subsidy to purchase private insurance makes agent better off by lowering the tax rate, which is enough to compensate the loss due to lower insurance coverage.

Since I focus on the effect of reforming the health insurance system, I chose not to alter the health production sector along the transition. However, as the demand for medical service changes after the reform is instituted, the supply side may be affected as well. An interesting extension of the current paper would be to ask how medical technology and the price of medical services are determined and how they will be affected by health insurance reforms.

Part II

Computation and simulation of nonoptimal economies

Chapter 3

Numerical simulation of nonoptimal dynamic equilibrium models [1]

3.1 Introduction

In this paper we present a recursive method for the computation of sequential competitive equilibria for dynamic economic models in which the welfare theorems may fail to hold because of the presence of incomplete agents' participation, taxes, externalities, incomplete financial markets, and other financial frictions. These models have become central to analyze the effects of various macroeconomic policies, the evolution of wealth and income distribution, and the variability of asset prices. However, computation of their equilibrium solutions may be a formidable task. Indeed, dynamic programming arguments may fail to apply, and a continuous Markov equilibrium may not exist. Therefore, existing numerical techniques cannot be readily extended to non-optimal economies.

We shall address the following issues for the computation and simulation of dynamic equilibrium solutions: *(i) Existence: Lack of Markov equilibria.* Even though the model may have a recursive structure, a Markovian equilibrium may not exist – or no Markov equilibrium may be continuous – over a natural space of state variables. We prove existence of a Markov equilibrium over an expanded state space. *(ii) Computation: Non-convergence*

[1] joint with M. Santos: University of Miami. J. Miao: Boston University. A. Peralta-Alva: Research Division, Federal Reserve Bank of Saint Louis.

of the algorithm. Backward iteration over a candidate equilibrium function may not reach a Markovian equilibrium solution. Contraction arguments underlying dynamic programming methods usually break down for non-optimal economies. We prove convergence of our algorithm to a fixed-point solution that can generate all sequential competitive equilibria. *(iii) Approximation: Accuracy properties of the computed solution.* Approximation errors may cumulate over time. Consequently, as we refine the approximation we need to ensure that discretized versions of the algorithm approach an exact solution. Again, contraction arguments cannot be invoked to guarantee good approximation properties of the algorithm. We establish convergence of the computed solution to the set of competitive equilibria. *(iv) Simulation: Convergence of the moments from sample paths.* Standard laws of large numbers require certain regularity conditions – such as continuity of the law of motion – that would be rather imposing for the equilibria of these economies. We present a discretized method in which the moments from sample paths approach the set of moments of the invariant distributions of the model.

In dynamic competitive-markets economies with frictions the existence of Markovian equilibria has been well established under certain monotonicity properties on the equilibrium dynamics [e.g., see Bizer and Judd (1989), Coleman (1991), and Datta, Mirman and Reffett (2002)]. But existence of Markov equilibria remains largely unexplored in many other models in which these monotonicity conditions may not be satisfied. Regular examples of non-existence of Markovian equilibria have been found in one-sector growth models with taxes and externalities [Santos (2002)], in exchange economies with incomplete financial markets [Krebs (2004) and Kubler and Schmedders (2002)], and in overlapping generations (OLG) economies [Kubler and Polemarchakis (2004)]. For the canonical one-sector growth model with taxes and externalities, monotonicity conditions follow from fairly mild restrictions on the primitives, but monotone dynamics are much harder to obtain in multi-sector models with heterogeneous agents and incomplete financial markets. Duffie *et al*. (1994) dispense with such monotonicity requirements by expanding the state space with endogenous variables such as asset prices and individual consumptions. By a suitable randomization of the equilibrium correspondence [Blume (1982)] they then prove the existence of an ergodic invariant distribution for a wide class of discrete-time infinite-horizon models with exogenous short-sale constraints on asset holdings. Building on these methods, Kubler and Schmedders (2003) prove the existence of a Markovian equilibrium for a class of financial economies with collateral requirements.

We extend this existence result to various types of economies. Our state space includes agents' shadow values of investment. This choice of the state space seems suitable for computation. The set of all Markov equilibria can be characterized as the fixed-point solution of a convergent iterative procedure. (A key factor of convergence is that our operator is acting over candidate equilibrium sets on a compact domain.) Then, we develop a computable version of the theoretical algorithm. This numerical algorithm is shown to approximate the original fixed-point solution. Moreover, the moments derived from simulated paths of the computed solution converge to a set of moments of the invariant distributions of the model. We apply our methods to two growth economies, a stochastic OLG economy with money, and an asset pricing model with incomplete financial markets and heterogeneous agents. We illustrate the applicability of our algorithm by comparing our numerical solution with those generated from some other standard methods. These other methods may display low accuracy properties, fail to converge to the equlibrium solutions, or capture only one of the possible existing equilibria.

The computation of competitive equilibria for non-optimal economies has been of considerable interest in macroeconomics and finance [e.g., Castaneda, Diaz-Gimenez and Rios-Rull (2003), Krusell and Smith (1998), Heaton and Lucas (1996), Marcet and Singleton (1999), and Rios-Rull (1999)], but most of this literature does not deal with the problem of existence of a Markovian equilibrium. Kubler and Schmedders (2003) refine the analysis of Duffie *et al*. (1994) and develop a reliable computational algorithm over an expanded state space. But in the implementation of this algorithm they iterate over continuous equilibrium functions, and such iteration process does not guarantee convergence to a fixed-point solution. Also, their state space includes additional variables which seem hard to compute, and so their algorithm may not be computationally efficient.

The idea of enlarging the state space with the shadow values of investment was first suggested by Kydland and Prescott (1980) in their seminal study of time inconsistency. Abreu, Pierce and Stacchetti (APS, 1990) use a similar approach for the computation of sequential perfect equilibria in which they expand the state space with continuation utility values. The analyses of Kydland and Prescott and APS have been extended in several directions involving strategic decisions [e.g., Atkenson (1991), Chang (1998), Judd, Yeltekin and Conklin (2003), Marcet and Marimon (1998) and Phelan and Stacchetti (2001)], but none of these papers are concerned with the computation of sequential equilibria for competitive-market economies with heterogeneous agents. To the best of our knowledge, the only related paper is Miao (2006) who sets forth a recursive solution method for the model of Krusell and Smith (1998). However, as in the original APS approach Miao's state space includes expected continuation utilities over the set of sequential competitive equilibria, and this choice of the state space does not seem operative for the computation of equilibrium solutions in the present framework.

Finally, for nonoptimal economies convergence properties of numerical algorithms and convergence of the simulated moments remain largely unexplored. As already discussed, Duffie *et al*. (1994) show existence of an ergodic distribution (which validates a law of large numbers for these economies). This result is not practical for computational purposes as it is usually hard to locate the ergodic set. In the absence of continuity of the equilibrium law of motion, other ways to validate laws of large numbers for these economies would be to resort to monotonicty assumptions on the equilibrium dynamics [Santos (2008)] or to artificial expansions of the noise space [Blume (1979)]. These latter approaches seem less attractive for these economies.

We proceed as follows. In Section 2 we present our general framework and lay out our theoretical algorithm. Section 3 studies the numerical implementation of our algorithm and its convergence properties. Sections 4-6 explore the existence and computation of recursive equilibria for various families of models. We conclude in Section 7.

3.2 General Theory

In this section, we first set out a general analytic framework that encompasses various competitive equilibrium models. We then present our numerical approach and main results on existence and global convergence to the Markovian equilibrium correspondence.

3.2.1 The Analytical Framework

Time is discrete, $t = 0, 1, 2, \cdots$. The state of the economy includes a state vector of endogenous variables x and vector of exogenous shocks z. Vector x belongs to a compact domain X and contains all predetermined variables, such as agents' holdings of physical capital, human capital, and financial assets. The exogenous state vector follows a Markov chain $(z_t)_{t\geq 0}$ over a finite set Z. This Markovian process is described by positive transition probabilities $\pi(z'|z)$ for all $z, z' \in Z$. The initial state, $z_0 \in Z$, is known to all agents in the economy. Then $z^t = (z_1, z_2, ..., z_t) \in Z^t$ is a history of shocks, often called a date-event or node. Let y denote the vector of all other endogenous variables. These variables could be equilibrium prices or choice variables such as consumption and investment.

In all our models the dynamics of the state vector x is conformed by a system of nonlinear equations:

$$\varphi(x_{t+1}, x_t, y_t, z_t) = 0. \tag{3.1}$$

Function φ incorporates technological constraints as well as individual budget constraints. For some models, such as those analyzed in Section 4, function φ is known and we can explicitly solve for x_{t+1} as a function of (x_t, y_t, z_t). In other applications such as in various models with adjustment costs, vector x_{t+1} may not admit an analytical representation.

Let m denote a vector of shadow values of the marginal investment return for all assets and all agents. This vector lies in a compact space M, and it will be a function of existing variables such as prices, rates of interest, and marginal utilities and productivities:

$$m_t = h(x_t, y_t, z_t). \tag{3.2}$$

Let us assume that a sequential competitive equilibrium exists and can be represented by a sequence $(x_t(z^t), y_t(z^t))_{t=0}^{\infty}$ satisfying (3.1), (3.2) and the additional sytem of equations

$$\Phi(x_t, y_t, z_t, E_t[m_{t+1}]) = 0, \tag{3.3}$$

where $E[m]$ is the expectations operator. Function Φ may describe individual optimality conditions (such as Euler equations), market-clearing conditions, various types of budget restrictions, and resource constraints. We assume that equations (3.1)-(3.3) fully characterize any sequential competitive equilibrium, and that φ, h, and Φ are continuous functions.

3.2.2 Recursive Equilibrium Theory

In order to compute the set of equilibria for the model economy we define the equilibrium correspondence $V^*(x,z)$ containing all the equilibrium vectors m for any given state (x,z). From this correspondence V^* we can generate recursively the set of sequential competitive equilibria as V^* is the fixed point of an operator $B : V \longmapsto B(V)$ that links state variables to future equilibrium states. Operator B embodies all equilibrium conditions such as agents' optimization and market-clearing conditions from any initial node z to all immediate successor states z_+. This operator is analogous to the expectations correspondence defined in Duffie *et al*. (1994), albeit it is defined over a smaller set of endogenous variables.

More precisely, let $B(V)(x,z)$ be the set of all values $m = h(x,y,z)$ satisfying the following temporary equilibrium conditions: For given x, z there exist y and $m_+(z_+) \in V(x_+, z_+)$ with $z_+ \in Z$ such that

$$\Phi(x,y,z,\sum_{z_+\in Z}\pi\left(z_+|z\right)m_+\left(z_+\right))=0,$$

and

$$\varphi\left(x_+,x,y,z\right)=0. \tag{3.4}$$

Note that operator B is well defined as a sequential competitive equilibrium is assumed to exist. Also, B is monotone in the sense that if $V\subset V'$ then $B(V)\subset B(V')$.[2] Moreover, if V has a closed graph then $B(V)$ also has a closed graph since the above functions φ,h,Φ are all assumed to be continuous. Indeed, in all our models below operator B satisfies the following

Assumption 3.2.1 *Operator B preserves compactnes in the sense that if V is compact valued, then $B(V)$ is also compact valued.*

Using this assumption we can show existence of a fixed-point solution V^* which is globally convergent for every initial guess $V_0\supset V^*$. Convergence, should be understood as pointwise convergence[3] in the Hausdorff metric [e.g., see Hildenbrand (1974)]. If V^* is a continuous correspondence then convergence will be uniform over all points (x,z).

Theorem 3.2.1 *(convergence) Let V_0 be a compact-valued correspondence such that $V_0\supset V^*$. Let $V_n=B\left(V_{n-1}\right),n\geq 1$. Then, $V_n\to V^*$ as $n\to\infty$. Moreover, V^* is the largest fixed point of the operator B, i.e., if $V=B(V)$, then $V\subset V^*$*

Theorem 3.2.1 provides the theoretical foundations of our algorithm. The iterative process starts as follows: For all (x,z), pick a sufficiently large compact set $V_0\left(x,z\right)\supset V^*\left(x,z\right)$. Then apply operator B to V_0 and iterate until a desirable level of convergence is attained. This is possible since $\lim_{n\to\infty}V_n$ equals the equilibrium correspondence V^*. An important

[2]For correspondences V,V' we say that $V\subset V'$ if $V\left(x,z\right)\subset V'\left(x,z\right)$ for all (x,z).

[3]Later, we will establish uniform convergence of the simulated moments even though the equilibrium correspondence V^* is only upper semicontinuous.

advantage of our approach is that if there are multiple equilibria, we can find all of them. Finally, under assumption 2.1 by the measurable selection theorem [Hildenbrand (1974)] it follows that from operator B we can select a measurable policy function $y = g^{y}(x,z,m)$, and a transition function $m_{+}(z_{+}) = g^{m}(x,z,m;z_{+})$, for all $z_{+} \in Z$. These functions give a Markovian characterization of a dynamic equilibrium in the enlarged state space.

Note that the equilibrium shadow value correspondence V^{*} may not be single-valued; hence, there could be multiple equilibrium selections in which none of them is continuous. Moroever, there may not be an equilibrium function $y = g(x,z)$, and hence a simple recursive equilibrium may not exist.[4] Kubler and Schmedders (2002) construct an example economy with multiple equilibria. They show that the model does not admit a recursive solution $g(x,z)$:

$$\Phi(x, g(x,z), z, \sum_{z_{+}\in Z} \pi(z_{+}|z)\, h(f(x, g(x,z), z), g(f(x, g(x,z)), z_{+})) = 0. \tag{3.5}$$

where $x_{+} = f(x,y,z)$. Kubler and Schmedders (2003) propose a computation procedure to recover such Markov equilibria numerically by a related expansion of the state space. But their computational algorithm relies on the assumption that the policy correspondence is a continuous function. Their algorithm may fail if there are multiple equilibria or if the policy function is not continuous. Our approach overcomes this problem as we illustrate by the various examples in the coming sections.

3.3 Numerical Implementation

Numerical implementation of our theoretical results requires the construction of a computable algorithm that approximates the fixed point of operator B. In this section we develop and study properties of such an algorithm.

[4] Of course, if the competitive equilibrium is always unique then there is a continuous function $y = g(x,z)$.

We first partition the state space into a finite set of simplices $\{X^j\}$ with non-empty interior and maximum diameter h. Over this partition we define a family of step correspondences which take constant values over each X^j. To obtain a computer representation of a step correspondence we resort to an outer approximation in which each set-value is defined by N elements. Using these two simplifications we get a discretized version of operator B, which we denote by $B^{h,N}$. By a suitable selection of an initial condition V_0, the sequence $\{V_{n+1}^{h,N}\}$ defined recursively as $V_{n+1}^{h,N} = B^{h,N} V_n^{h,N}$ converges to a limit point $V^{*,h,N}$ containing the equilibrium correspondence V^*. Moreover, the sequence of fixed points $\{V^{*,h,N}\}$ approaches the equilibrium correspondence V^* as the accuracy of the discretizations goes to the limit. It should be understood that convergence is uniform in economies where the equilibrium corespondence is continuous. At a later stage, we address the issue of convergence of the moments obtained from simulations of our numerical approximations. This problem has been hardly addressed in the literature, and again it has to cope with the fact that the equilibrium correspondence may not be continuous.

3.3.1 The Numerical Algorithm

Let $\{X^j\}$ be a finite family of simplices with non-empty interior such that $\cup_j X^j = X$ and $int(X^j) \cap int(X^i)$ is empty for every pair X^i, X^j. Define the *mesh size* h of this discretization as

$$h = \max_j diam\left\{X^j\right\}.$$

Consider a correspondence $V : X \times Z \to 2^M$ that takes values in space M. Then, its step approximation V^h over the partion $\{X^j\}$ takes constant set-values $V^h(x,z)$ on each simplex X^j and is conformed by the union of sets $V(x,z)$ for $x \in X^j$ for given z. That is, for each z

$$V^h(x,z) = \cup_{x \in X^j} V(x,z). \tag{3.6}$$

Accordingly, we can define operator B^h that takes a correspondence V into the step correspondence $[B(V)]^h$. By similar arguments as above, we can prove that B^h has a fixed point solution V^{*h}. Moreover, we shall soon clarify the sense in which the correspondence V^{*h} constitutes an approximation to V^*.

As already mentioned, to obtain a computer representation of the step correspondence we also perform a discretization on the image space. We say that $\mathscr{C}^N(V(x,z)) \supseteq V(x,z)$ is an N-element outer approximation of $V(x,z)$ if $\mathscr{C}^N(V(x,z))$ can be generated by N elements. In what follows we assume that this approximation satisfies a strong uniform convergence property.[5] Namely, for any $\varepsilon > 0$ there is $0 < N^* < \infty$ such that $d[\mathscr{C}^N(V(x,z)), V(x,z)] \leq \varepsilon$ for all $N > N^*$, and all $V(x,z)$. For instance, this later property can be satisfied if the outer approximation is generated by convex combinations of N points as M is a compact set.

We are now ready to put forward a computable version of operator B. That is, we can define a new operator $B^{h,N}$ that sends a correspondence V to the step correspondence $[B(V)]^h$ and then each set-value is adjusted with the N-element outer approximation so as to get $\mathscr{C}^N\left([B(V)]^h\right)$. Sections 4 to 6 illustrate examples of this type of operators, and their application in different dynamic models. Let us first show that our discretized operator has good convergence properties: The fixed point of this operator $V^{*,h,N}$ contains the equilibrium correspondence V^* and it approaches V^* as we refine the discretizations. The proof of this result extends the convergence arguments of Beer (1980) to a dynamic setting.

Theorem 3.3.1 *Suppose that $V_0 \supseteq V^*$ is an upper-semicontinuous correspondence. Consider the recursive sequence defined by $V_{n+1}^{h,N} = B^{h,N} V_n^{h,N}$ for given h and N and with initial condition V_0. Then: (i) $V_n^{h,N} \supseteq V^*$ for all n; (ii) $V_n^{h,N} \to V^{*,h,N}$ uniformly as $n \to \infty$; and (iii) $V^{*,h,N} \to V^*$ as $h \to 0$ and $N \to \infty$.*

The output of our numerical algorithm is summarized by the equilibrium correspondence $V_n^{h,N}$ from operator $B^{h,N}$. By Theorem 3.3.1, we have that $graph[\mathscr{C}^N\left([B(V_n^{h,N})]^h\right)]$ can be made arbitrarily close to $graph[B(V^*)]$ for appropriate choices of n, h, and N. As

[5]Again, convergence should be understood in the Hausdorff metric d (see *opt. cit.*).

$graph[\mathscr{C}^N\left([B(V_n^{h,N})]^h\right)]$ is compact, by the measurable selection theorem [Hildenbrand (1974)] we can choose an approximate equilibrium selection $y = g_n^{y,h,N}(x,z,m)$, and a transition function $m_+(z_+) = g_n^{m,h,N}(x,z,m;z_+)$. From these approximate equilibrium functions we can generate simulated paths $(x_t(z^t), y_t(z^t))_{t=0}^{\infty}$.

3.3.2 Convergence of the Simulated Moments

To assess model's predictions, analysts usually calculate moments of the simulated paths $(x_t(z^t), y_t(z^t))_{t=0}^{\infty}$ from a numerical approximation. The idea is that the simulated moments should approach steady-state moments of the true model. Under continuity of the policy function, Santos and Peralta-Alva (2005) establish various convergence properties of the simulated moments. They also provide examples of non-existence of stochastic steady-state solutions for non-continuous functions, and lack of convergence of emprirical distributions to some invariant distribution of the model. Hence, it is not clear how economies with distortions should be simulated, since for these economies the continuity of the policy function does not usually follow from standard economic assumptions.

We now outline a reliable simulation procedure that circumvents the lack of continuity of the equilibrium law of motion. We append two further steps to the standard model simulation. First, we discretize the image space of the approximate equilibrium selection so that this function can take on a finite number of points. Then, the simulated moments are generated by a finite Markov chain that has an invariant distribution, and every empirical distribution from the simulated paths converges almost surely to some ergodic invariant distribution of the Markov chain. Second, following Blume (1982) and Duffie *et al*. (1994) we randomize over continuation values of operator B. We construct a new operator B^{cv} that is a convex-valued correspondence in the space of probability measures. It follows then that there is an invariant distribution $\mu^* \in B^{cv}(\mu^*)$. Moreover, as we refine the approximations the simulated moments from our numerical approximations are shown to converge to the moments of some invariant distribution μ^*.

(i) *Discretization of the equilibrium law of motion*: In order to make the analysis more transparent, let $S = X \times M$. Let $\chi_n^{h,N} : S \times Z \to S \times Z$ be a selection from $graph[\mathscr{C}^N\left([B(V_n^{h,N})]^h\right)]$. Note that function $\chi_n^{h,N}$ is simply defined from the above functions $y = g_n^{y,h,N}(x,z,m)$, and $m_+(z_+) = g_n^{m,h,N}(x,z,m;z_+)$ and the law of motion for state variable x as given by equation (1). Then, $\chi_n^{h,N}$ gives rise to a time-homogeneous Markov process $(s,z) \to s_+(z_+)$ for $s = (x,m)$ and all $z_+ \in Z$. Now, let A_γ be a set with a finite number of points in S such that $d(A_\gamma, S) < \gamma$ so that each point in S is within a γ-ball of some point in A. Let $\chi_n^{h,N,A_\gamma}(s,z) = \arg min_{s_+ \in A_\gamma} d(s_+, \chi_n^{h,N}(s,z))$. If there are various solution points s_+ we just pick arbitrarily one solution s_+. Hence, the new discretized function χ_n^{h,N,A_γ} takes values in the finite set $A_\gamma \times Z$, and gives rise to a Markov chain that has an invariant distribution ν_n^{*,h,N,A_γ}. Further, the moments of a simulated path $(s_t, z^t)_{t=0}^{\infty}$ converge almost surely to those of some ergodic invariant distribution ν_n^{*,h,N,A_γ} [e.g., see Stokey, Lucas and Prescott (1989), Ch. 11].

(ii) *Randomization over continuation equilibrium sequences*: We can view operator $B : V^* \to V^*$ as a correspondence in the space of probability measures μ on $S \times Z$. That is, $\nu \in B(\mu)$ if there is a selection χ of B such that $\nu = \chi \cdot \mu$, where $\chi \cdot \mu$ denotes the action of function χ on probability measure μ [e.g., see Stokey, Lucas and Prescott (1989)]. Following Blume (1982) and Duffie *et al*. (1994) we convexify the image of B. Thus, if ν and ν' are two probability measures that belong to the range of B we assume that every convex combination $\lambda\nu + (1-\lambda)\nu'$ also belongs to the range of B. We let B^{cv} denote the convexification[6] of operator B over the space of probability measures μ on $S \times Z$. The new operator B^{cv} is a convex-valued, upper semicontinuous correspondence. Since $S \times Z$ is assumed to be compact, the set of probability measures μ on $S \times Z$ is also compact in the weak topology of measures. Therefore, operator B^{cv} has a fixed point solution; i.e., there exists an invariant probability, $\mu^* \in B^{cv}(\mu^*)$.

[6]Duffie et *al*. (1994) argue that such convexification amounts to a weak form of sunspot equilibria since the randomization proceeds over equilibrium distributions rather than over an external parameter or extraneous sunspot variable.

(iii) $Convergence\ of\ the\ simulated\ moments\ to\ population\ moments\ of\ the\ model$: For given function χ_n^{h,N,A_γ} and a randomly selected sequence $(z^t)_{t=0}^{\infty}$, we generate an approximate equilibrium path $(s_t)_{t=0}^{\infty}$. Let $f : S \times Z \to R_+$ be a function of interest. Then, $\frac{1}{T}\sum_{t=0}^{T} f(s_t, z_t)$ represents a simulated moment or some other statistic. Since χ_n^{h,N,A_γ} defines a Markov chain, it follows that $(s_t, z_t)_{t=0}^{\infty}$ must enter an ergodic set in finite time. Therefore, $\frac{1}{T}\sum_{t=0}^{T} f(s_t, z_t)$ must converge almost surely to $\int f(s,z) d\nu_n^{*,h,N,A_\gamma}$ as $T \to \infty$for some ergodic invariant distribution ν_n^{*,h,N,A_γ}. We now link convergence of invariant distributions $\nu_n^{*,h,N,A_\gamma}$of numerical approximations to some invariant distribution of the original model μ^*so that the simulated statistics converge almost surely to those of some invariant distribution μ^*.

Theorem 3.3.2 *Let* $\left(\nu_n^{*,h,N,A_\gamma}\right)$ *be a sequence of invariant distributions corresponding to functions* $\left(\chi_n^{h,N,A_\gamma}\right)$. *Then, every limit point of* $\left(\nu_n^{*,h,N,A_\gamma}\right)$ *converges weakly to some invariant distribution* $\mu^* \in B^{cv}(\mu^*)$.

To summarize our work in this section, convergence of the simulated moments involves discrete approximations over the following margins:

1. $Discretization\ of\ the\ domain$: h mesh size of the family of simplices $\{X^j\}$.
2. $Discretization\ of\ the\ image$: N number of elements in every outer approximation.
3. $Finite\ iterations$: n number of iterations over operator $B^{h,N}$.
4. $Finite\ Markov\ chain$: γ maximum distance of every point in S to some point in set A_γ.
5. $Finite\ simulations$: T lenght of a simulated path $(s_t, z_t)_{t\geq 0}$.
6. $Convexification\ of\ equilibrium\ distributions$: B^{cv} regularized operator in the space of distributions with a convex image.

Thus, for every $\varepsilon > 0$ we can make the aforementioned parameters sufficiently close to their respective limits so that for a given path $(s_t, z_t)_{t=0}^{\infty}$ generated under function χ_n^{h,N,A_γ}, there are invariant distributions μ^*, μ'^* of B^{cv} such that $\int f(s,z)d\mu^* + \varepsilon \leq \frac{1}{T}\sum_{t=0}^{T} f(s_t, z_t) \leq \int f(s,z)d\mu'^* - \varepsilon$ almost surely. Therefore, for a sufficiently fine approximation the moments from simulated paths are close to the set of moments of the invariant distributions of the model. Of course, if B^{cv} has a unique invariant distribution μ^* then $\mu'^* = \mu^*$ and the above expression reads as $\int f(s,z)d\mu^* + \varepsilon \leq \frac{1}{T}\sum_{t=0}^{T} f(s_t, z_t) \leq \int f(s,z)d\mu^* - \varepsilon$.

3.4 Non-Optimal Growth Models

In this section we present a standard stochastic growth model with taxes, heterogeneous agents, and incomplete markets. This framework comprises several macroeconomic models that are often simulated by numerical methods. We illustrate the applicability of our algorithm with two simple specifications of the model, and contrast its performance against standard numerical methods. In the first application, we study a representative-agent deterministic economy with capital income taxes. We show that a continuous Markov equilibrium may not exist; moreover, standard computation methods would usually fail to converge or yield inaccurate solutions. In the second application, we consider a stochastic economy with heterogeneous agents. For our simple parameterization, the competitive equilibrium is unique, and hence there is a continuous Markovian solution. We compare the solution of our accurate algorithm against a simplified algorithm that uses an approximate aggregation strategy. We show that this latter algorithm yields a rather poor approximation of the equilibrium correspondence and simulated statistics are strongly biased. Therefore, the first numerical experiment alerts us of the dangers of using continuous approximations when the true solution may not be continuous, and the second numerical experiment alerts us of the dangers of using heuristic simplifications as they may introduce large errors in the equilibrium law of motion.

3.4.1 Economic Environment

The economy is populated by a finite number of agents, I, who live forever. The vector of shocks z affects the overall productivity level, as well as individual income and preferences. Capital is the only asset in this economy, and hence financial markets are technically incomplete.

Each agent i has preferences represented by the intertemporal objective

$$E\left[\sum_{t=0}^{\infty}\left(\beta^{i}\right)^{t}u^{i}\left(c_{t}^{i},z_{t}\right)\right], \tag{3.7}$$

where $\beta \in (0,1)$ is the discount factor, and c_t is consumption of the aggregate good at a given node z^t. Function $u(\cdot,z_t)$ is increasing, strictly concave, and twice continuously differentiable.

Stochastic consumption plans $\left(c_t^i\right)_{t\geq 0}$ are financed from after-tax capital returns, wages, profits, and commodity endowments. These values are expressed in terms of the single good, which is taken as the numeraire commodity of the system at each date-event. For a given rental rate r_t and wage w_t household i offers $k_t^i \geq 0$ units of capital to the production sector, and supplies inelastically $l_t^i(z_t) \geq 0$ units of labor. For simplicity, we abstract from leisure considerations.

Each household i is subject to the following sequence of budget constraints

$$\begin{aligned} k_{t+1}^{i}\left(z^{t}\right)+c_{t}^{i}\left(z^{t}\right) &= (1-\delta)k_{t}^{i}\left(z^{t-1}\right)+(1-\tau_{k}(K))r_{t}\left(z^{t}\right)k_{t}^{i}\left(z^{t-1}\right)+ \\ &\quad +w_{t}\left(z^{t}\right)l_{t}^{i}\left(z^{t}\right)+e_{t}^{i}\left(z^{t}\right)+T_{t}^{i}\left(z^{t}\right)+\pi^{i}(z^{t}) \\ k_{t+1}^{i}\left(z^{t}\right) &\geq 0,\ \text{for all state histories } z^{t}=(z_{0},\ldots,z_{t}),\ \text{and } k_{0}^{i}\ \text{given.} \end{aligned} \tag{3.8}$$

Capital income is taxed according to function τ_k, which depends on the aggregate capital stock, K_t. This tax function is assumed to be positive, continuous, and bounded away from 1. Tax revenues are rebated back to consumers as lump-sum transfers T_t^i. π_t^i denotes profits.

The production sector is made up of a continuum of identical units that have access to a constant returns to scale technology in individual factors. Hence, without loss of generality we shall focus on the problem of a representative firm. After observing the current shock vector z the firm hires K units of capital and L units of labor. The total quantity produced of the single aggregate good is given by a production function $A_t F(K_t, L_t)$, where A_t is the firm's total factor productivity and $F(K_t, L_t)$ is the direct contribution of the firm's inputs to the production of the aggregate good. Hence, at each date event z^t, the representative firm seeks to maximize one-period profits by an optimal choice of factors (K, L),

$$\pi_t = max A_t F(K_t, L_t) - r_t K_t - w_t L_t. \tag{3.9}$$

We shall maintain the following standard conditions on function F:

Assumption 3.4.1 *$F : \mathbf{R}_+ \times \mathbf{R}_+ \to \mathbf{R}_+$ is increasing, concave, continuous and linearly homogeneous. This function is continuously differentiable at each interior point (K, L); moreover,* $\lim_{K\to\infty} D_1 F(K, L) = 0$ *for each given* $L > 0$.

3.4.2 Sequential and Recursive Competitive Equilibrium

The present model contemplates several deviations from a frictionless world and so a competitive equilibrium cannot usually be recast as the solution to an optimal planning program. The model includes individual uninsurable shocks to preferences and labor, capital income taxes, and an aggregate shock to production. Households can hold capital to transfer wealth, but they may be unable to smooth out consumption since there is only one single asset and capital holdings must be non-negative.

Definition 3.4.1 *A sequential competitive equilibrium (SCE) is a tax function $\tau_k(K)$, and a collection of vectors $\left(\left\{c_t^i(z^t), k_{t+1}^i(z^t)\right\}_i, K_t(z^t), L_t(z^t), w_t(z^t), r_t(z^t)\right)_{t\geq 0}$ that satisfy*

(i) Constrained utility maximization: For each household i, the sequence $\left(c_t^i, k_{t+1}^i\right)_{t\geq 0}$ maximizes the objective (3.7) subject to the sequence of budget constraints (3.8).

(ii) Profit maximization: For each z^t, vector $(K_t(z^t), L_t(z^t))$ maximizes profits (3.9).

(iii) Market clearing: For each z^t and its predecessor node z^{t-1},

$$K_t\left(z^t\right) + \sum_{i=1}^{I} c_t^i\left(z^t\right) = A_t F\left(K_t\left(z^t\right), L_t\left(z^t\right)\right) + (1-\delta)K_t\left(z^t\right) + \sum_{i=1}^{I} e_t^i\left(z^t\right),$$

$$\sum_{i=1}^{I} k_t^i\left(z^{t-1}\right) = K_t\left(z^t\right) \text{ and } \sum_{i=1}^{I} l^i\left(z_t\right) = L_t\left(z^t\right).$$

Note that the equilibrium quantities $(K_t(z^t), L_t(z^t))_{t\geq 0}$ may be inferred from households' aggregate supply of these factors. Hence, we may refer to a SCE as simply a sequence of vectors $\left(\left\{c_t^i(z^t), k_{t+1}^i(z^t)\right\}_i, r_t(z^t), w_t(z^t)\right)_{t\geq 0}$. There does not seem to be a general proof of existence of competitive equilibria for infinite-horizon economies with distortions. We are aware of a related contribution by Jones and Manuelli (1999), but their analysis is not directly applicable to models with incomplete markets or externalities. Hence, in the Appendix we outline a proof of the following result.

Proposition 3.4.2 *A SCE exists.*

For computational purposes we need to bound the equilibrium values of the key variables of the model. In the Appendix below we show that there are positive constants K^{max} and K^{min} such that for every equilibrium sequence of physical capital vectors $\left(k_{t+1}^i(z^t)\right)\right)_{t\geq 0}$ if $K^{max} \geq \sum_{i=1}^{I} k_0^i(z^0) \geq K^{min}$ then $K^{max} \geq \sum_{i=1}^{I} k_{t+1}^i(z^t) \geq K^{min}$ for all z^t. Moreover, $K^{min} > 0$ if $\lim_{K\to 0} D_1 F(K,L) = \infty$ for some positive L. Hence, in what follows the domain of aggregate capital will be restricted to the interval $[K^{min}, K^{max}]$, and it should be understood that $K^{min} = 0$ only if $\lim_{K\to 0} D_1 F(K,L)$ is bounded for all given $L > 0$. This implies that every equilibrium sequence of factor prices $(r_t(z^t), w_t(z^t))_{t\geq 0}$ is bounded.

We also need to bound the equilibrium shadow values of investment. To accomplish this task, we define an auxiliary value function of an individual sequential optimization problem. For a given sequence of factor prices and aggregate capital $(\mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0}))$ $= (r_t(z^t), w_t(z^t), K_t(z^t))_{t\geq 0}$, let

$$J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0})) = \max E \sum_{t=0}^{\infty} \beta^t u^i(c_t(z^t), z_t)$$

$$s.t.$$

$$k_{t+1}^i\left(z^t\right) + c_t^i\left(z^t\right) = (1-\delta)\, k_t^i\left(z^{t-1}\right) +$$

$$(1-\tau_k(K_t\left(z^t\right)))r_t\left(z^t\right) k_t^i\left(z^{t-1}\right) +$$

$$+w_t\left(z^t\right) l_t^i + e_t^i + T_t^i\left(z^t\right) + \pi_t^i,$$

$$k_{t+1}^i\left(z^t\right) \geq 0, k_0^i \text{ given.}$$

For every bounded sequence $(\mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0})) = (r_t(z^t), w_t(z^t), K_t\left(z^t\right))_{t\geq 0}$, the value function $J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0}))$ is well defined, and continuous. Moreover, mapping
$J^i(\cdot, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0}))$ is increasing, concave, and differentiable with respect to the initial condition k_0^i. Further, the partial derivative $D_1 J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0}))$ varies continuously with $(k_0^i, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0}))$ [cf. Rincon-Zapatero and Santos (2009)]. The next result readily follows from these regularity properties of the value function.

Proposition 3.4.3 *For all SCE* $\left(\left\{c_t^i(z^t), k_{t+1}^i(z^t)\right\}_i, r_t(z^t), w_t(z^t)\right)_{t\geq 0}$ *with* $K^{max} \geq \sum_{i=1}^I k_0^i(z^0) \geq K^{min}$*, there is a constant* γ *such that* $0 \leq D_1 J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0})) \leq \gamma$ *for all* z^t.

Observe that these bounds apply to the following types of utility functions: (i) Both function $u(\cdot, z)$ and its derivative are bounded, (ii) function $u(\cdot, z)$ is bounded, and its derivative function is unbounded, and (iii) both function $u(\cdot, z)$ and its derivative are unbounded. Phelan and Stacchetti (2001) deal with case (i) and Krebs (2004) and Kubler and Schmedders (2003) consider utility functions of type (iii). We provide a uniform method of proof that covers all the three cases, as well as production functions with bounded and unbounded derivatives. As a matter of fact, Proposition 3.4.3 fills an important gap in the literature, since no general results are available on upper and lower bounds for factor prices and marginal utilities for production economies with heterogeneous consumers and market frictions.

For any initial distribution of capital k_0 and a given shock z_0, we define the Markov equilibrium correspondence $V^*: \mathbf{K} \times \mathbf{Z} \to \mathbf{R}_+^I$ as

$$V^*(k_0, z_0) = \left\{ \begin{array}{c} \left(\cdots, D_1 J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0})), \cdots\right) : \\ \left(\left\{c_t^i(z^t), k_{t+1}^i(z^t)\right\}_i, r_t, w_t\right)_{t \geq 0} \text{ is a SCE} \end{array} \right\}, \tag{3.10}$$

where $\mathbf{K} = \{k : K^{max} \geq \sum_{i=1}^I k^i \geq K^{min}\}$. Hence, the set $V^*(k_0, z_0)$ contains all current equilibrium shadow values of investment $m_0 = \left(\cdots, m_0^i, \cdots\right)$, for every household i.

Corollary 3.4.4 *Correspondence V^* is nonempty, compact-valued, and upper semicontinuous.*

This corollary is a straightforward consequence of Propositions 3.4.2 and 3.4.3. Note that by the envelope theorem we must have $D_1 J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0})) \geq (1 - \delta + (1 - \tau_k) r_0(z_0)) D_1 u^i(c_0^i, z_0)$, with equality when $c_i^0 > 0$. Moreover, Proposition 3.4.3 implies $0 \leq D_1 J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}), \mathbf{K}(\mathbf{z_0})) \leq \gamma$, and so $c_0^i = 0$ is only possible if the derivative of the utility function u^i is bounded at $c_0^i = 0$.

The second key element of our analysis is operator B which is defined as follows. For any given correspondence $V : K \times Z \to R_+^I$ let $B(V)(k, z)$ be the set of values $m = (\cdots, m^i, \cdots)$, with $0 \leq m^i \leq \gamma$ for all i, such that there is some vector $(c, k_+, r, w, m_+, \lambda, \zeta) \in R_+^I \times R_+^I \times R_+ \times R_+ \times \left(R_+^I\right)^N \times R_+^I \times R_+^I$, with $m_+(z_+) \in V(k_+, z_+)$ for all $z_+ \in Z$ that satisfies all individual and aggregate *temporary equilibrium conditions*.

3.4.3 Numerical example 1: A model with capital income taxes

Let us first consider a deterministic version of the above model with a representative agent and capital income taxes. To further simplify our analysis, assume that capital is the only production factor with full depreciation $\delta = 1$, and the utility function is logarithmic. The production function and discount rate are given by

$$f(k) = k^{1/3}, \beta = 0.95. \tag{3.11}$$

Assume that there is a piecewise linear, tax schedule given by

$$\tau(K) = \begin{cases} 0.10 & \text{if } K \leq 0.160002 \\ 0.05 - 10(K - 0.165002) & \text{if } 0.160002 \leq K \leq 0.170002 \\ 0 & \text{if } K \geq 0.170002. \end{cases} \tag{3.12}$$

Then, a continuous Markov equilibrium fails to exist [cf. Santos (2002, Prop. 3.4)]. However, it follows from the foregoing analysis that a recursive equilibrium in an adequately expanded state space does exist.

Implementation of our algorithm

Following the notation of our general theoretical framework, we can write:

$$\varphi(k_{t+1}, c_t) = f(k_t) - c_t - k_{t+1}, \text{ and} \tag{3.13}$$

$$m_t = h(k_t, c_t) = \frac{r_t\,(1 - \tau_k(K_t))}{c_t} = \frac{\frac{1}{3}k_t^{-2/3}(1 - \tau(k_t))}{c_t}. \tag{3.14}$$

Similarly, aggregate feasibility and the intertemporal optimality conditions for the household can be summarized by the Euler equation

$$\Phi(k_t, c_t, m_{t+1}) = \frac{1}{c_t} - \beta m_{t+1}. \tag{3.15}$$

The, let $B(V)(k_t)$ be the set of values m_t such that there is (c_t, k_{t+1}) and $m_{t+1} \in V(k_{t+1})$ satisfying the temporary equilibrium conditions (3.13- 3.15).

For the numerical implementation of our algorithm we exploit the low dimensionality of the state space and compactness of the equilibrium correspondence. Specifically, notice that for each given k_t the shadow values of investment, $m(k_t)$, lie in a compact interval $[\underline{m}(k_t), \overline{m}(k_t)]$. Hence, our numerical algorithm starts by approximating the upper and lower bound functions $\underline{m}(k_t)$ and $\overline{m}(k_t)$ using step functions. Notice, however, that these functions may be discontinuous. Hence, the standard strategy of approximating these functions only at the vertex points of the triangulation may not work. In our case it is necessary to obtain bounds for all values within each of the simplices. Some technical details are relegated to Appendix B. Here, we just illustrate some properties of our numerical approximation.

Figure 3.1 presents our initial guess (left panel), $V_0^{h,N}$, and the correspondence defined by the area (right panel) between the upper and lower approximated functions $\underline{m}(k_t)$ and $\overline{m}(k_t)$. A useful feature of this example is that the backward shooting algorithm can be used to obtain highly accurate solutions. (Of course, for stochastic versions the shooting method no longer works.) The dots in the Figure below represent an approximate solution derived via backward shooting.

Since the limiting correspondence is not single valued near the middle steady state, our method is signaling the possibility of a multiple valued equilibrium correspondence. The resulting policy correspondence is illustrated in Figure 3.2 below together with the solution obtained via the shooting method.

In this specific example both our method and the shooting algorithm yield highly accurate solutions. However, we remark that shooting cannot be used for stochastic models whereas our algorithm will be use below in several examples with uncertainty.

Comparing with other computational algorithms

A standard practice in quantitative analysis is to assume that a continuous policy function exists. Hence, let

$$k_1 = g(k, \xi),$$

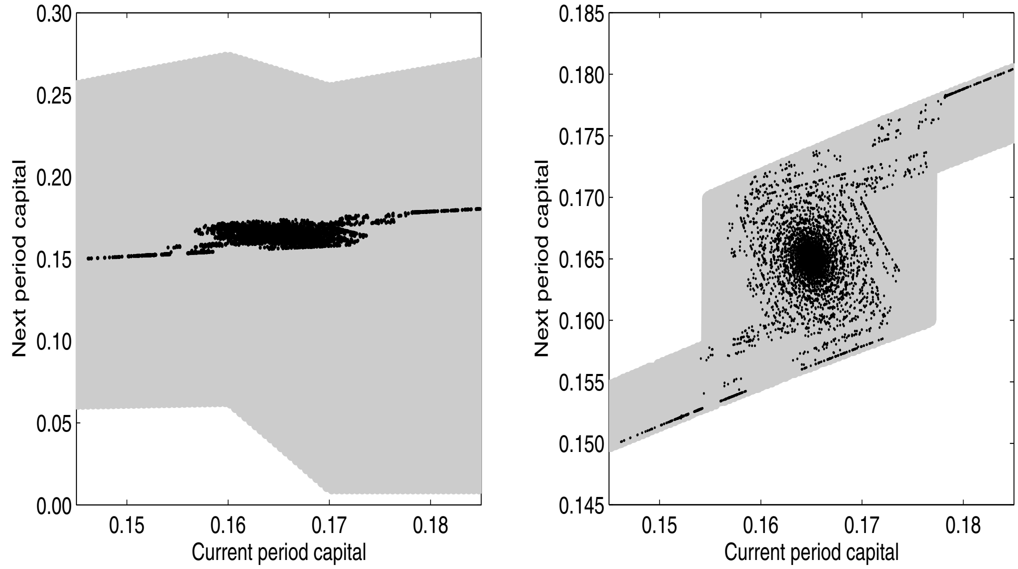

Figure 3.1: Initial (left grey area) and limiting correspondence (right grey area) vs solution obtained via the backward shooting method (black dots)

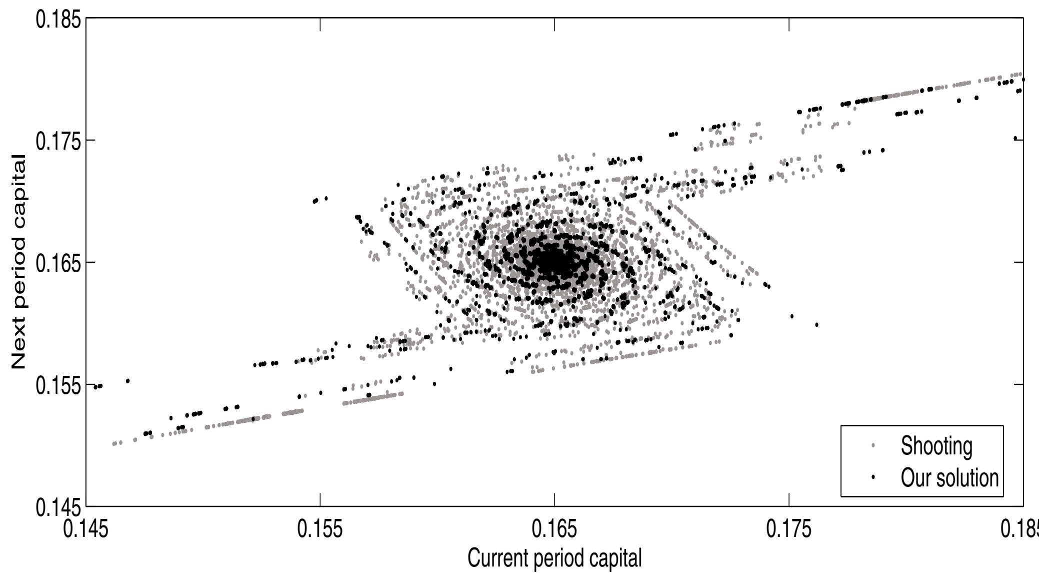

Figure 3.2: Equilibrium correspondence computed with our method vs the backwards shooting method

where g is a continuous approximation defined by a finite vector of parameters ξ. We obtain an estimate for ξ by forming an Euler equation system over as many points as the dimensionality of the parameter space

$$u'(k^i, g(k^i, \xi)) = \beta u'(g(k^i, \xi), g(g(k^i, \xi), \xi)) \cdot \left[f'(g(k^i, \xi))(1 - \tau(g(k^i, \xi))) + (1 - \delta) \right].$$

The choice of the grid points, k^i, for the Euler equation may be dependent on the functional approximations for the policy function (e.g. Chebyshev polynomials could be evaluated at the Chebyshev nodes). Here, we assume that $g(k, \xi)$ belongs to the class of piecewise linear functions. First, we should note that this approximation failed to converge in several instances. In particular, we found that vector ξ could oscillate with no discernible pattern across different iterations. As expected, the area of the domain where the lack of convergence occurred was close to the middle steady state. Figure 3.3 below displays some representative functions from different iterations of the algorithm. Second, in some other cases the distance between candidate policy functions was relatively small, but this does not mean that these policies are close to the true solution. Of course, for points near the middle steady state solution a continuous policy function will arbitrarily redirect the convergence of initial conditions to one of the remaining two competitive steady-states solutions.

In summary, the equilibrium correspondence of this model cannot be represented by a continuous law of motion. Traditional computational methods based on iterations of continuous functions may either fail to converge or yield inaccurate solutions that highly distort the dynamics of competitive equilibria.

3.4.4 Numerical example 2: A model with two agents and no taxes

We now consider a specification of the model with two agents who face idiosyncratic and aggregate uncertainty. There are no taxes. Both agents have the same utility function,

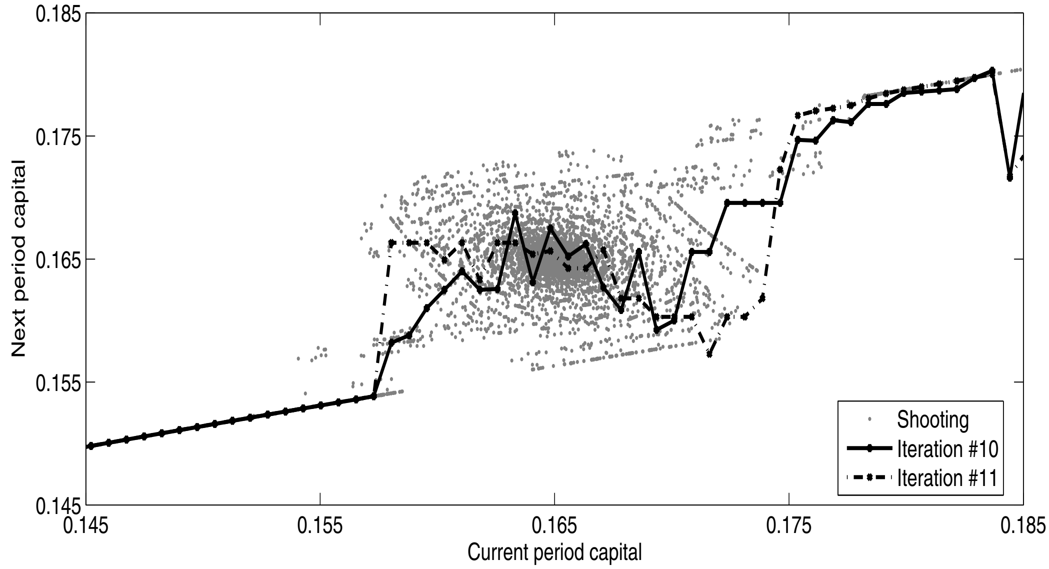

Figure 3.3: Different iterations of a standard solution method vs Shooting solution

$u^i(c^i) = \frac{(c^i)^{1-\sigma}}{1-\sigma}$, with discount factor, $\beta^1 = \beta^2 = 0.95$. The capital share is $\alpha = 0.34$ and depreciation rate $\delta = 0.06$. Total factor productivity is a random variable with two possible values: $A_g = 1.0807$ and $A_b = 1.0593$. Each agent has a random endowment of labor, l^i, which can take two possible values, $l_b = 0$ and $l_g = 1$. These idiosyncratic shocks do not affect the aggregate labor supply; that is, $l_t^1 + l_t^2 = 1$ at all date-events. Productivity and labor endowments are assumed to be jointly driven by a Markov process with transition matrix

$$\Pi = \begin{pmatrix} 0.5 & 0.4 & 0.06 & 0.04 \\ 0.6 & 0.3 & 0.06 & 0.04 \\ 0.45 & 0.35 & 0.15 & 0.05 \\ 0.5 & 0.3 & 0.15 & 0.05 \end{pmatrix}.$$

Here, entry $\Pi_{p,q}$ is the probability of moving to state p from the current state q.[7]

Implementation of our algorithm

Mapping this model into the notation of our general theoretical framework is simple.

[7]Notice there are four possible states $(A_g, l_g, l_b), (A_g, l_b, l_g), (A_b, l_g, l_b)$, and (A_b, l_b, l_g).

The vector of endogenous predetermined variables is given by the capital holdings of each agent, $x_t = (k_t^1, k_t^2)$, while the vector of current endogenous variables contains the consumption and investment choices of each agent, $y_t = (c_t^1, c_t^2, i_t^1, i_t^2)$. Equilibrium interest rates and wages can be explicitly written in terms of the aggregate capital K_t and unit labor supply. Hence, capital and the shadow multipliers for investment are determined by the following equations

$$\begin{aligned} \varphi(x_{t+1}, x_t, y_t, z_t) &= (i_t^1 + (1-\delta)k_t^1 - k_{t+1}^1, i_t^2 + (1-\delta)k_t^2 - k_{t+1}^2), \quad (3.16) \\ \left(m_t^1, m_t^2\right) &= h(x_t, y_t, z_t) = ((r_t + 1 - \delta)\left(c_t^1\right)^{-\sigma}, (r_t + 1 - \delta)\left(c_t^2\right)^{-\sigma}). \quad (3.17) \end{aligned}$$

with $r_t = \theta A_t K_t^{\theta-1}, w_t = (1-\theta) A_t K_t^{\theta}$. Finally, intertemporal optimality and all individual and aggregate constraints are collected in a function Φ defined as $\Phi(x_t, y_t, z_t, E_t[m_{t+1}]) =$

$$\begin{gathered} ((r_t + 1 - \delta) k_t^i + w_t l_t^i - c_t^i - i_t^i, \quad (3.18) \\ \left(c_t^j\right)^{-\sigma} - \beta E_t m_{t+1}^j(z_{t+1}) + \lambda_t^j, \text{ for } j = 1, 2, \\ \sum_i (c_t^i + i_t^i) - A_t K_t^{\theta}, \end{gathered}$$

where λ_t^i is the multiplier associated to the constraint $k_{t+1}^i \geq 0$.

Our algorithm operates as follows. Let V be any given correspondence, then $BV(x,z)$ is the set of all values $\left(m_t^1, m_t^2\right)$ for which one can find values $c_t^1, c_t^2, i_t^1, i_t^2, k_{t+1}^1, k_{t+1}^2$, and $\left(m_{t+1}^1, m_{t+1}^2\right) \in V(k_{t+1}^1, k_{t+1}^2, z_{t+1})$ at all successors z_{t+1} that satisfy (3.16-3.18). Appendix B explains further details of the operation of this algorithm that considers multiple agents.

Comparing with other computational algorithms

A commonly employed method to solve this type of models is the "approximate aggregation" procedure pioneered by Krusell and Smith (1998). A key insight of this method is that in equilibrium aggregate variables may be well approximated as functions of simple statistics. In particular, the stochastic process driving aggregate capital is assumed to be characterized by a finite vector of moments. Individual decisions are computed on the basis of such expectations for aggregate variables. And a fixed point is reached if the simulated moments from the individual decision rules match those of the law of motion for aggregate capital.

In our baseline model, the algorithm is applied in the following way. Start with a guess on a parameterized functional form for the first moment of aggregate capital. Then, use value function to compute the problem of the representative household

$$\begin{aligned} v(k^i;K,z) &= \max\{U(c)+\beta E[v(k'^i;K',z')|z,\varepsilon]\} \qquad (3.19)\\ s.t.\ \ c+k^{i\prime} &= r(K,z)k^i + w(K,z)\varepsilon^i + (1-\delta)k^i \\ k^{i\prime} &\geq B \\ \log K_{t+1} &= a(z)\log K_t + b(z) \qquad (3.20) \end{aligned}$$

The algorithm estimates coefficients $(a(z_g),b(z_g),a(z_b),b(z_b))$ and individual policy functions in the following fashion: (i) Start with initial parameter estimates; (ii) Solve the dynamic programming problem of each agent (3.19); (iii) Construct aggregate capital time series by aggregating the resulting individual time series simulations; (iv) Perform a regression over the stationary region to obtain new estimates for such coefficients. This process stops when there is no variation in the coefficient estimates and the R^2 and standard error of the aforementioned regression are sufficiently good.

An obvious advantage of “Approximate Aggregation" is computational cost. Indeed, the algorithm can accommodate an arbitrary number of agents and idiosyncratic shocks. Surprisingly, relatively little is known about the accuracy properties of the solutions and of the simulated moments for this type of algorithms. In Table 3.1 below, we compare some quantitative properties of the “Approximate Aggregation" method described above to those of our algorithm. EE_i refers to the Euler equation residuals, and $Mean(k_i)$ is the average of simulated capital values for each agent $i=1,2$.

Table 3.1: Euler equation residuals and simulated moments of alternative solution methods

Method	$Mean(\lvert EE_1\rvert)$	$Mean(\lvert EE_2\rvert)$	$Mean(k_1)$	$Mean(k_2)$
Approx. Aggregation	1.57×10^{-2}	2.71×10^{-2}	2.8196	4.5210
Our Algorithm	5.14×10^{-4}	7.58×10^{-4}	3.0898	3.8623

Even though in this case the model always has a unique competitive equilibrium which may be generated by a continuous policy function, we can see that our method yields higher accuracy of approximation as measured by Euler equation residuals. Further, our non-linear equilibrium approximation results in substantially different simulated statistics for individual wealth from those of the approximate aggregation method. Indeed, approximation errors for these simple moments are of the order of 10 percent.

3.5 A Stochastic OLG Economy

Overlapping generations (OLG) models have become central in the analysis of several macro issues such as the funding of social security, the optimal profile of savings and investment over the life cycle, the effects of various fiscal and monetary policies, and the evolution of future interest rates and asset prices under current demographic trends.[8]

As already stressed, there are no known convergent procedures for the computation of sequential competitive equilibria in OLG models even for frictionless economies with complete financial markets. We now illustrate that our approach delivers a reliable, computable algorithm for the solution of competitive equilibria in a general class of OLG models.

[8]For instance, see Champ and Freeman (2002), Conesa and Krueger (1999), Geanakoplos, Magill and Quinzii (2003), Gourinchas and Parker (2002), Imrohoroglu, Imrohoroglu, and Joines (1995), Storelesletten, Telmer and Yaron (2004), and Ventura (1999).

3.5.1 Economic Environment

The economy is conformed by a sequence of overlapping generations that live for two periods. The primitive characteristics of the economy are defined by a stationary Markov chain. At every time period $t = 0, 1, 2, \cdots$ a new generation is born. Each generation is made up of I agents, who are present in the economy for two periods. More specifically, for a household of type i born at time t preferences are defined over consumption bundles of the goods available at times t and $t+1$, and the agent can only trade goods and assets in these two periods. The economy starts with an initial generation who is only present in the initial period $t = 0$. This generation is endowed with the aggregate supply of assets θ_0. At each node z^t, there exist spot markets for the consumption good and J securities. These securities are specified by the current vector of prices, $q_t(z^t) = (\cdots, q_t^j(z^t), \cdots)$, and the vectors of future dividends $d_r(z^r) = (\cdots, d_r^j(z^r), \cdots)$ promised to deliver at future information sets $z^r|z^t$ for $r > t$. We assume that the vector of security prices $q_t(z^t)$ is non-negative – a condition implied by free disposal of securities – and the vector of dividends $d_t(z^t)$ is positive in all components and depends only on the current realization of the vector of shocks z_t; hence, $(d_t(z_t))_{t \geq 0}$ is a time invariant Markov chain.

For simplicity, we assume that every utility function U^i is separable over consumption of different dates. For an agent i born in period t, let $c_{y,t}^i(z^t)$ denote the consumption of the aggregate good in period t over the history of shocks z^t, and let $c_{o,t+1}^i\left(z^{t+1}|z^t\right)$ denote the consumption in period $t+1$ for every successor node $z^{t+1}|z^t$ of z^t. Then the intertemporal objective U^i is defined as

$$U^i\left(c_y^i, c_0^i; z^t, z^{t+1}\right) = u^i\left(c_{y,t}^i\left(z^t\right), z_t\right) + \beta \sum_{z^{t+1} \in \mathbf{Z}} v^i\left(c_{o,t+1}^i\left(z^{t+1}\right), z_{t+1}\right) \pi\left(z^{t+1}|z^t\right) \quad (3.21)$$

The one-period utilities u^i and v^i satisfy the following conditions:

Assumption 3.5.1 *For each $z \in \mathbf{Z}$ the one-period utility functions $v^i(\cdot, z), u^i(\cdot, z) : \mathbf{R}_+ \rightarrow$*

$\mathbf{R} \cup \{-\infty\}$ *are increasing, strictly concave, and continuous. These functions are also continuously differentiable at every interior point* $c > 0$.

Each agent i born at $t = 1, 2, \cdots$ is endowed with a vector of goods $e_t^i = (e_{y,t}^i, e_{o,t+1}^i)$ and trades an asset portfolio θ^i to attain desirable amounts of consumption. The endowment process $(e_t^i(z^t)) = (e_{y,t}^i(z^t), e_{o,t+1}^i(z^{t+1}|z^t))$ follows a time invariant Markov chain; hence $e_{y,t}^i(z^t) = e_y^i(z_t)$, and $e_{o,t+1}^i(z^{t+1}|z^t) = e_o^i(z_{t+1})$ for every agent i and every t. Given prices $(q_t(z^t))_{t\geq 0}$, a consumption-savings plan $(c_{y,t}^i(z^t), c_{o,t+1}^i(z^{t+1}), \theta_t^i(z^t))$ must obey the following two-period budget constraints:

$$\theta_{t+1}^i\left(z^t\right) \cdot q_t\left(z^t\right) + c_{y,t}^i\left(z^t\right) \leq e_{y,t}^i\left(z_t\right), \text{ for } \theta_{t+1}^i\left(z^t\right) \geq 0, \tag{3.22}$$

$$c_{o,t+1}^i\left(z^{t+1}\right) \leq \theta_{t+1}^i\left(z^t\right) \cdot \left(q_{t+1}\left(z^{t+1}\right) + d_t\left(z_{t+1}\right)\right) + e_{o,t+1}^i\left(z_{t+1}\right), \text{ all } z^{t+1}|z^t. \tag{3.23}$$

For an initial stock of securities θ_0^i each agent i at time $t = 0$ seeks to maximize the total quantity of consumption $c_{o,0}^i(z_0)$ for given endowments of the aggregate good e_o^i and the vector of securities θ_0^i. More precisely,

$$c_{o,0}^i\left(z_0\right) = \theta_0^i \cdot \left(q_0\left(z_0\right) + d_0\left(z_0\right)\right) + e_o^i\left(z_0\right). \tag{3.24}$$

3.5.2 Sequential and Recursive Competitive Equilibrium

In this economy the aggregate commodity endowment is bounded by a portfolio-trading plan [Santos and Woodford (1997)], and hence asset pricing bubbles cannot exist in a SCE.

Definition 3.5.1 *A SCE is a collection of vectors* $\{\left(c_{y,t}^i\left(z^t\right), c_{o,t+1}^i\left(z^{t+1}|z^t\right), \theta_{t+1}^i\left(z^t\right)\right)_{i=1}^I,$ $q_t\left(z^t\right)\}_{t\geq 0}$ *such that*

(i) Utility maximization: For every household i and all t, vector $(c_{y,t}^i\left(z^t\right), c_{o,t+1}^i\left(z^{t+1}|z^t\right),$ $\theta_{t+1}^i\left(z^t\right))$ *maximizes the objective (3.21) subject to (3.22)-(3.23). For every household i of the starting generation,* $c_{o,0}^i\left(z_0\right)$ *satisfies (3.24).*

(ii) Market clearing: For each z^t,

$$\begin{aligned}\sum_{i=1}^{I}\left(c_{y,t}^{i}\left(z^{t}\right)+c_{o,t}^{i}\left(z^{t}\right)\right) &= \sum_{j=1}^{J}d_{t}^{j}\left(z_{t}\right)+\sum_{i=1}^{I}\left(e_{yt}^{i}\left(z_{t}\right)+e_{ot}^{i}\left(z_{t}\right)\right)\\ \sum_{i=1}^{I}\theta_{t+1}^{ji}\left(z^{t}\right) &= 1, j=1,\cdots,J.\end{aligned}$$

Note that to circumvent technical issues concerning existence of a SCE, we still maintain the short-sale constraint $\theta_t \geq 0$ for all t. Then, the existence of a SCE can be established by standard methods [e.g., Balasko and Shell (1980), and Schmachtenberg (1988)]. Moreover, by similar arguments used by these authors it is easy to show that every sequence of equilibrium asset prices $\left(q_t\left(z^t\right)\right)_{t\geq 0}$ is bounded.

Then, we define the Markov equilibrium correspondence $V^{*}:\theta\times\mathbf{Z}\rightarrow\mathbf{R}_{++}^{JI}$ as

$$V^{*}\left(\theta_{0},z_{0}\right)=\left\{\left(\ldots\left(q_{0}^{j}\left(z_{0}\right)+d_{0}^{j}\left(z_{0}\right)\right)D_{1}v^{i}\left(c_{0}^{i}\left(z_{0}\right),z_{0}\right)\ldots\right):\left(c_{y},c_{o},\theta,q\right)\text{ is a SCE}\right\}.$$

From the above results on existence of SCE for OLG economies we obtain

Proposition 3.5.2 *Correspondence V^* is nonempty, compact-valued, and upper semicontinuous.*

3.5.3 Numerical Example: A monetary model

We consider a simplified version of the OLG model with money of Benhabib and Day (1982) and Grandmont (1985). This simple model is useful because it can be solved with arbitrary accuracy. Hence, it is possible to compare the true solution of the model with other numerical approximations. Extensions to a stochastic environment are easy to handle by our algorithm, but may become problematic when using other algorithms.

Each individual receives an endowment e_1 of the perishable good when young and e_2 when old. There is a single asset, money, that pays zero dividends at each given period.

The initial old agent is endowed with the existing money supply M. Let P_t be the price level at time t. An agent born in period t chooses consumption c_{1t} when young, c_{2t+1} when old, and money holdings M_t to solve

$$\max u(c_{1t}) + \beta v(c_{2t+1})$$

subject to

$$\begin{aligned} c_{1t} + \frac{M_t}{P_t} &= e_1, \\ c_{2t+1} &= e_2 + \frac{M_t}{P_{t+1}}. \end{aligned}$$

A sequential competitive equilibrium for this economy is a sequence of prices $(P_t)_{t\geq 0}$, and sequences of consumption and money holdings $\{c_{1t}, c_{2t+1}, M_t\}_{t\geq 0}$ such that individual solves the budget-constrained utility maximization problem and markets clear:

$$c_{1t} + c_{2t} = e_1 + e_2, \text{ and } M_t = M \text{ for all } t.$$

A sequential competitive equilibrium can be characterized by the following first-order condition:

$$\frac{1}{P_t} u'\left(e_1 - \frac{M}{P_t}\right) = \frac{1}{P_{t+1}} \beta v'\left(e_2 + \frac{M}{P_{t+1}}\right).$$

Let $b_t = M/P_t$ be real money balances at t. Then,

$$b_t u'(e_1 - b_t) = b_{t+1} \beta v'(e_2 + b_{t+1}).$$

Hence, all competitive equilibria can be generated by an offer curve in the (b_t, b_{t+1}) space.[9] A simple recursive equilibrium would be described by a function g such that $b_{t+1} = g(b_t)$.

In the remainder of this section, we restrict our attention to the following parameteriza-

[9] We can also use the (c_{1t}, c_{2t+1}) space as in Cass, Okuno, and Zilcha (1979).

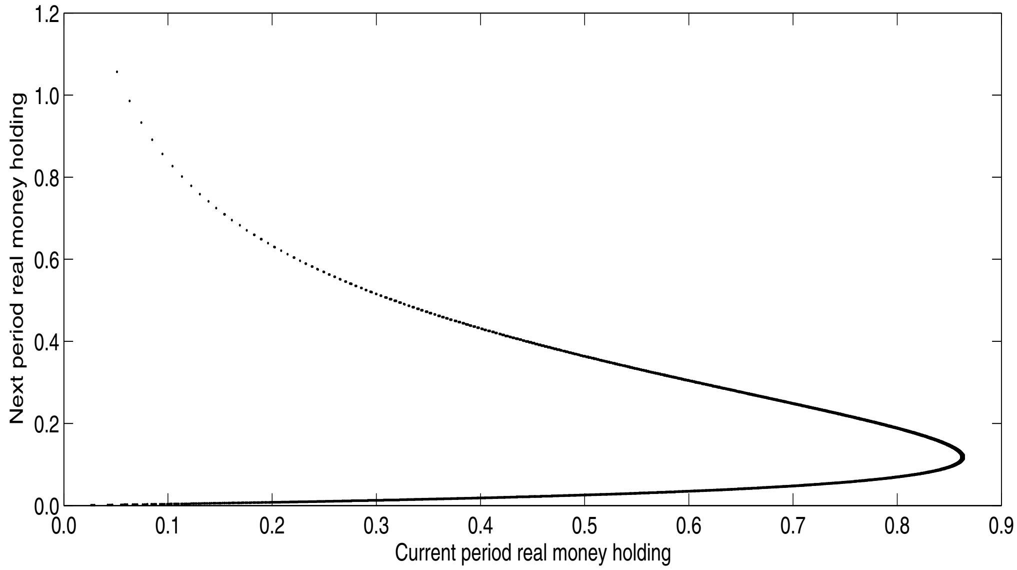

Figure 3.4: "Exact" offer curve for the OLG model

tions:

$$u(c) = c^{0.45},\ v(c) = -\frac{1}{7}c^{-7},\ \beta = 0.8,$$

$M = 1$, $e_1 = 2$, and $e_2 = 2^{6/7} - 2^{1/7}$. For this simple example, the offer curve is backward bending. Hence, the equilibrium correspondence is multi-valued, and standard methods – based on the computation of a continuous equilibrium function $b_{t+1} = g(b_t)$ – may portray a partial view of the equilibrium dynamics.

The solution is illustrated in Figure 3.4.

Implementation of our algorithm.

Note that the implementation our numerical algorithm of Section 3 is fairly straightforward. In fact, since the shadow multipliers of investment lie on a compact subset of R, we can follow the same computational steps as in the growth model of the previous section. Then, upper and lower bound functions are selected to compute the fixed point that can generate all competitive equilibria. The results from this algorithm are reported in Figure 3.5 where the dark grey area represents the initial correspondence, the light grey area represents the fixed point of algorithm $B^{h,N}$, and the dotted line is the equilibrium

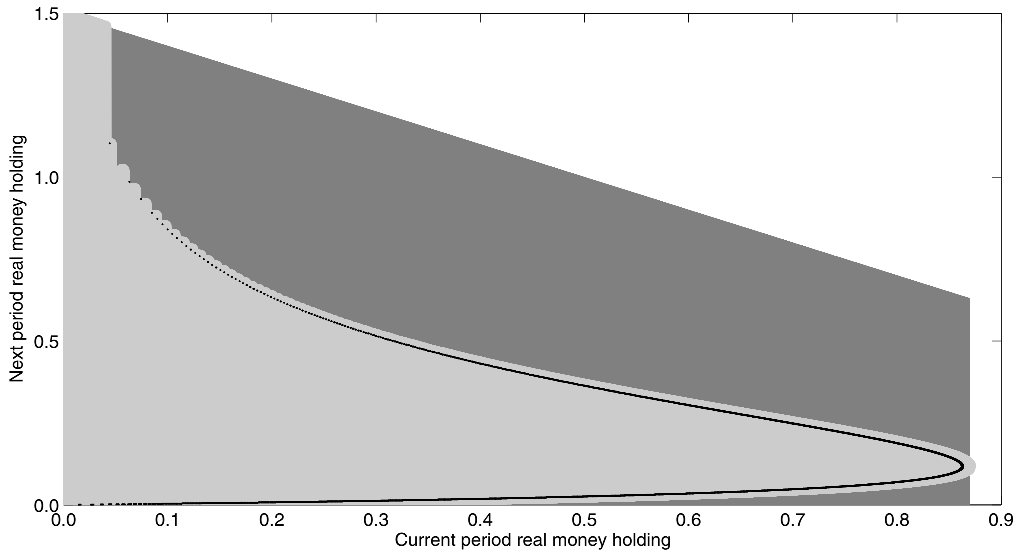

Figure 3.5: Initial guess, limiting correspondence, and approximated equilibrium policy correspondence from operator $B^{h,N}$.

correspondence constructed using the equilibrium selection algorithm of Section 3.

For this example, we find that the policy correspondence and time series from our method generate an Euler equation residual of order 10^{-6}, so that the solution obtained with our algorithm is indistinguishable from the "exact" solution.

Comparing with other computational algorithms

A common practice in OLG models is to start the search with an equilibrium guess function of the form $b' = g(b)$. In several numerical experiments we obtained that either the upper part or the lower part of the offer curve. Which one one will obtain depends on the initial guess. This strong dependence on initial conditions is a rather undesirable feature of this method. In particular, note that for initial conditions where the method yields the lower part of the actual equilibrium correspondence all competitive equilibria converge to autarchy. Indeed, zero real monetary holdings are the unique absorbing steady state associated with the lower part of the equilibrium correspondence. Hence, even in the deterministic version of the model, we need a global approximation of the equilibrium

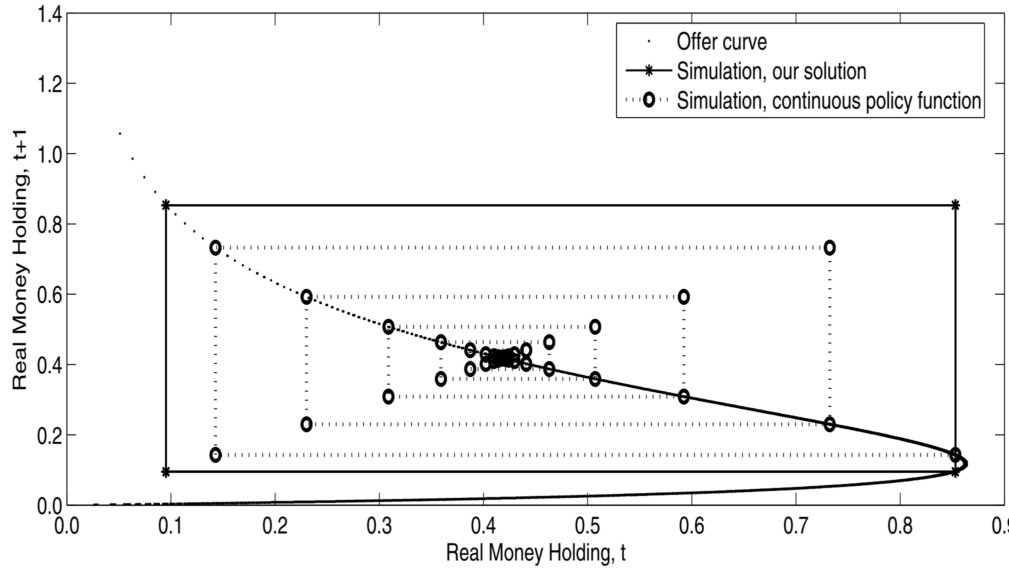

Figure 3.6: Time-series behavior of different numerical solutions

correspondence to analyze the various predictions of the model. As shown in Figure 3.6, in the approximate equilibrium correspondence there is cyclical equilibrium in which real money holdings oscillate between 0.85296237892 and 0.09517670718. It is also known that the model has a three-period cycle. But if we just iterate over the upper part of the offer curve we find that money holdings converge monotonically to $\frac{\bar{M}}{p} = 0.418142579084$, as illustrated by the dashed line of Figure 3.6. Indeed, the upper part of the equilibrium correspondence is monotonic, and can at most have cycles of period two, whereas the model generates lots of equilibrium cycles of various periodicities.

In conclusion, for OLG economies standard computational methods based on iteration of continuous functions may miss some important properties of the equilibrium dynamics.

3.6 Asset Pricing Models with Incomplete Markets

There is an important family of macroeconomic models that incorporate financial frictions in the form of sequentially incomplete markets, borrowing constraints, transactions costs, cash-in-advance constraints, and margin and collateral requirements. These models are commonly used to assess the effects of monetary policies, and the variability of macroeconomic aggregates such as asset prices, consumption, interest rates, and inflation.[10] Fairly general conditions rule out the existence of financial bubbles in these economies, and hence equilibrium asset prices are determined by the expected value of future dividends [Santos and Woodford (1997)]. However, in the presence of financial frictions the equivalence of competitive equilibria and optimal allocations breaks down, and standard computational methods are of limited application. The purpose of this section is to illustrate the applicability of our proposed algorithm in a model with collateral requirements taken from Kubler and Schmmeders (2003). Our choice of the state space simplifies the computations, and becomes instrumental to solve the model by a reliable iteration procedure.

3.6.1 Economic Environment

The economy is populated by a finite number of agents. At each date, agents can trade quantities of the unique aggregate good as well as a fixed set of assets that span the horizon of the economy. There are various financial frictions: Incomplete markets, collateral requirements, and short-sale constraints.

Each agent i maximizes the intertemporal objective

$$E\left[\sum_{t=0}^{\infty}\left(\beta^{i}\right)^{t}u^{i}\left(c_{t}^{i}\right)\right], \tag{3.25}$$

subject to a sequence of budget constraints. We assume that $\beta^i \in (0,1)$, and u^i is strictly increasing, strictly concave and continuously differentiable with derivative $(u'^i(0) = \infty$. At each node z^t, there exist spot markets for the consumption good and J securities. These securities are specified by the current vector of prices, $q_t(z^t) = (\cdots, q_t^j(z^t), \cdots)$, and the

[10]For instance, see Campbell (1999), Heaton and Lucas (1996), Huggett (1993), Krebs and Wilson (2004), Mankiw (1986), and Telmer (1993). For some monetary models see Bewley (1980), Lucas (1982), and Santos (2006).

vectors of dividends $d(z^r) = (\cdots, d^j(z^r), \cdots)$ promised to deliver at future information sets $z^r|z^t$ for $r > t$. The vector of security prices $q_t(z^t)$ is non-negative, and that the vector of dividends $d_t(z_t)$ is positive and depends only on the current realization of the vector of shocks z_t. Also, at each node z^t the agent receives $e^i(z_t) > 0$ units of the consumption good.

There is also a market for one-period bonds available at all times. A bond is a promise to one unit of the consumption good at all successor nodes $z^{t+1}|z^t$. Bonds are are in zero net supply, and are specified by the price vector $p_t(z^t)$. Agents can default on bond payments, and hence they required to hold at least $k^j \geq 0$ units worth of each security j as collateral. In case of default, the buyer of the bond will garnish the collateral wealth.

For a given a price process $(q_t(z^t), p_t(z^t))_{t\geq 0}$, each agent i chooses desired quantities of consumption, real securities and bond holdings $(c_t^i(z^t), \theta_{t+1}^i(z^t), \phi_{t+1}^i(z^t))_{t\geq 0}$ subject to the following sequence of budget constraints

$$c_t^i\left(z^t\right) - \phi_t^i(z^{t-1}) \min\left\{1, \sum_j k^j \frac{q_t^j(z^t)}{q_{t-1}^j(z^{t-1})}\right\} + \theta_{t+1}^i\left(z^t\right) \cdot q_t\left(z^t\right) = \tag{3.26}$$

$$e^i(z_t) + \theta_t^i\left(z^{t-1}\right) \cdot \left(q_t\left(z^t\right) + d\left(z_t\right)\right) - \phi_{t+1}^i(z^t) p_t(z^t),$$

$$-k^j \phi_{t+1}^i(z^t) \leq q_t^j(z^t) \theta_{t+1}^{ij}(z^t), \text{ for } j = 1..J, \tag{3.27}$$

$$0 \leq \theta_{t+1}^i\left(z^t\right), \text{ all } z^t, \ \theta_0^i \text{ given.} \tag{3.28}$$

Note that (3.28) imposes non-negative holdings of real securities, and (3.27) is meant to limit the amount of bond debt to a fraction of collateral wealth. The minimum in expression (3.26) above reflects that it is optimal to default on previous bond short-sales whenever the promised payment is larger than the cost of loosing the collateral.

3.6.2 Sequential and Recursive Competitive Equilibrium

Definition 3.6.1 *A sequential competitive equilibrium (SCE) for this economy is a collection of vectors* $(c_t(z^t), \theta_{t+1}(z^t), \phi_{t+1}(z^t), p_t(z^t), q_t(z^t))_{t\geq 0}$ *such that (i) for each agent i the* *plan* $(c_t^i(z^t), \theta_{t+1}^i(z^t), \phi_{t+1}^i(z^t))_{t\geq 0}$ *maximizes the objective (3.25) subject to (3.26)-(3.28), and (ii) markets clear:*

$$\sum_i^I c_t^i\left(z^t\right) = \sum_j^J d^j(z_t) + \sum_i^I e_t^i\left(z^t\right), \tag{3.29}$$

$$\sum_i^I \theta_{t+1}^{ji}\left(z^t\right) = 1, \textit{ for } j = 1, \cdots, J, \tag{3.30}$$

$$\sum_i^I \phi_{t+1}^i\left(z^t\right) = 0, \textit{ at all } z^t. \tag{3.31}$$

For the recursive specification of equilibria the state space includes the space of exogenous shocks Z, the space of possible values for share prices, Q, the distribution of shares $\Theta = \left\{\theta \in R_+^{JI} : \sum_{i=1}^I \theta^{ji} = 1 \text{ for all } j\right\}$ and bond holdings $\Delta = \left\{\phi \in R_+^I : \sum_{i=1}^I \phi^i = 0\right\}$. The equilibrium shadow value correspondence $V^* : Q \times \Theta \times \Delta \times Z \to R_+^{JI}$ is then defined as

$$V^*(q_-, \theta_0, \phi_0, z_0) = \left\{ \begin{array}{c} \left(\ldots, \left(q_0^j(z_0) + d^j(z_0)\right) U_1^i\left(c_0^i(z_0)\right), \ldots\right) : \\ (c_t, \theta_{t+1}, \phi_{t+1}, q_t, p_t, \lambda_t, \gamma_t)_{t\geq 0} \text{ is a SCE} \end{array} \right\}.$$

Observe that, for every $(q_-, \theta_0, \phi_0, z_0)$, the set $V^*(q_-, \theta_0, \phi_0, z_0)$ contains all equilibrium JI-vectors $m_0 = (\cdots, m_0^{ji}, \cdots)$ of shadow values of investing in each asset j for every agent i. It follows that operator $B : V \longmapsto B(V)$ is defined as: For each $(q_-, \theta, \phi, z) \in Q \times \Theta \times \Delta \times Z$, the set $B(V)(q_-, \theta, \phi, z)$ contains all values $m = (\cdots, m_+^{ji}, \cdots)$ such that there is some vector $(c, \theta_+, \phi_+, q, q_+, p, \lambda, \gamma)$ satisfying all the equilibrium conditions with $m_+ = \left(\ldots, m_+^{ji}(z_+), \ldots\right) \in V(q, \theta_+, \phi_+, z_+)$ for each $z_+ \in Z$.

Under similar regularity conditions Kubler and Schmedders (2003) show existence and compactness of the equilibrium set. Building on the previous literature we can then derive the following result.

Proposition 3.6.2 *Correspondence V^* is nonempty, compact-valued, and upper semicontinuous.*

We now illustrate an application of our algorithm for a model with two agents and two assets.

3.6.3 Numerical Example

There are two infinitely lived agents $i = 1,2$, and a real security that generates a sequence of random dividends. Following Kubler and Schmedders (2003) we choose the auxiliary variable,

$$\omega = \frac{\theta q + \phi \min\left\{1, k\frac{q_+}{q}\right\}}{q},$$

Then, the set of predetermined variables is reduced to $y = \left(\omega, d_t, \left\{e_t^h\right\}_{h=1}^2\right)$. Further, the budget constraints also simplify to

$$c_t^1 = e_t^1 + \omega_t q_t + \theta_t \left(d_t - q_t\right) - \phi_t p_t \tag{3.32}$$

$$c_t^2 = e_t^2 + \left(1 - \omega_t\right) q_t + \left(1 - \theta_t\right)\left(d_t - q_t\right) + \phi_t p_t \tag{3.33}$$

$$0 \leq \theta_t \leq 1.$$

With this simplification it is no longer necessary to keep track of last period or next period prices. This change of variable is actually not needed for our methods but it will speed up computations.

Implementation of our algorithm

Under the above change of variable, it becomes easier to consider the related shadow value

$$\hat{m}_t^i \equiv (q_t) u'^i(c_t^i). \tag{3.34}$$

From the above definition, and the individual constraints (3.32-3.33) we can solve for θ_t and q_t as functions of $\hat{m}_t^1, \hat{m}_t^2, y_t, p_t, \phi_t$. Hence, given a correspondence V, we have that $\left(\hat{m}_t^1, \hat{m}_t^2\right)$ will belong to BV if we can find bond holdings ϕ_t and prices $\left(p_t, q_{t+1}\right)$, a wealth level, ω_{t+1}, and continuation values for the shadow investment values, $\left(\hat{m}_{y_{t+1}}^1, \hat{m}_{y_{t+1}}^2\right) \in V\left(y_{t+1}\right)$ for all successor nodes, which satisfy the individual budget constraints as well as the intertemporal optimality conditions

$$(d_t - q_t) U_1^i \left(c_t^i\right) + \beta E_t m^i_{y_{t+1}} + q_t \lambda^i_{c,t} + \lambda^i_{ss} = 0 \tag{3.35}$$

$$-p_t U_1^i(c_t^i) + \beta E_t \left[\frac{k}{q_t} m^i_{y_{t+1}} | \Omega_A\right] + \beta E_t \left[\frac{m^i_{y_{t+1}}}{q_{t+1}} | \Omega_B\right] + k\lambda^i_{c,t} = 0 \tag{3.36}$$

where $\Omega_A = \left\{(b_t, q_t, q_{t+1}) : \min\left\{b_t, k\frac{q_{t+1}}{q_t}\right\} = k\frac{q_{t+1}}{q_t}\right\}$, $\Omega_B = \left\{(b_t, q_t, q_{t+1}) : \min\left\{b_t, k\frac{q_{t+1}}{q_t}\right\} = b_t\right\}$, and

$$\omega_{t+1} = \frac{\theta_t q_{t+1} + \phi_t \min\left\{b_t, k\frac{q_{t+1}}{q_t}\right\}}{q_{t+1}}. \tag{3.37}$$

Comparing with other computational algorithms

Kubler and Schmedders (2003) enlarge the state space with all exogenous and endogenous variables, and wealth. Recursive equilibrium is constructed from a correspondence that maps the enlarged state space into the set of all endogenous variables. As we have seen in our previous examples, the computational cost of approximating a set operator grows exponentially in the dimension of the domain and range of the operator. Hence, in the end these authors proceed with a computational algorithm that iterates over *functions* from the enlarged state space into the set of all endogenous variables. Unfortunately, iteration over functions does not guarantee of convergence to the equilibrium correspondence, and can only identify one one equilibrium at a time. In contrast, our proposed algorithm constructs recursive equilibria from an operator that maps the enlarged state space into the space of shadow multipliers of investment. This is a lower dimensional object that makes the algorithm more amenable to computation.

To illustrate the performance of our algorithm, assume both agents have identical utilities $u = \frac{c^{1-\sigma}}{1-\sigma}$, with a common coefficient of risk aversion of $\sigma = 2$ and $\beta_1 = \beta_2 = 0.95$. There are four possible values for the aggregate endowment, $\overline{e} \in (9.9, 10.5, 9.9, 10.5)$, with

Table 3.2: Simulated moments from alternative solution methods

	mean c_1 $(\sigma(c_1))$	mean c_2 $(\sigma(c_2))$	$mean(\lvert EE_1\rvert)$	$mean(\lvert EE_2\rvert)$
Continuous Markov equilibrium	4.96 (0.78)	5.26 (0.78)	5.05×10^{-6}	3.27×10^{-8}
Our algorithm	4.96 (0.78)	5.26 (0.78)	2.41×10^{-5}	9.01×10^{-6}

dividends $d = 0.3\cdot\bar{e}$, and individual endowments

$$\begin{aligned} e^1 &\in (1.386, 2.205, 5.544, 5.145), \\ e^2 &= 0.7\,\bar{e} - e^1. \end{aligned}$$

Also, the transition matrix driving individual shocks

$$\Pi(z'|z) = \begin{bmatrix} 0.4 & 0.4 & 0.1 & 0.1 \\ 0.4 & 0.4 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.4 & 0.4 \\ 0.1 & 0.1 & 0.4 & 0.4 \end{bmatrix}.$$

We simulate the economy using the decision rules obtained from our method, as well as from the algorithm based on continuous value function iteration. The resulting simulated statistics are summarized in Table 3.2 below. As before, EE_i denotes the Euler equation residual over the computed solution and c_i is consumption for $i = 1,2$.

Note that for this benchmark calibration both methods deliver identical simulated moments. As a straightforward consequence of Theorem 3.3.1, we obtain that convergence to a continuous function can only occur when the equilibrium is always unique. Hence, we have shown uniqueness of equilibria for the present model. Of course, iterations over a continuous functions may fail to converge, and for models in which there is no continuous equilibrium selection this procedure may lead to poor approximations of the equilibrium dynamics.

3.7 Concluding Remarks

This paper provides a theoretical framework for the computation and simulation of dynamic competitive-markets economies in which the welfare theorems may fail to hold because of market frictions or the existence of an infinite number of generations. We have applied these methods to some macroeconomic models with heterogeneous agents, taxes, sequentially incomplete markets, borrowing limits, short-sales, and collateral requirements.

For optimal economies, sequential competitive equilibria are generated by a continuous policy function which is the fixed-point solution of a contractive operator. Continuity of the policy function allows for various methods of approximation and functional interpolation, and it is essential to validate laws of large numbers for the simulated paths. Moreover, differentiability and contractive properties are useful for the derivation of error bounds that can guide the computation process. But for OLG models and economies with distortions several papers [e.g. Krebs (2004), Kubler and Polemarchakis (2004), Kubler and Schmedders (2002), and Santos (2002)] have shown that a continuous Markov equilibrium may not exist. We establish a general result on the existence of a Markovian equilibrium solution in a suitably expanded space of state variables. We construct a numerical algorithm that has desirable approximation properties and guarantees convergence of the moments computed from simulated paths.

There are three main features of our algorithm that should be of interest for quantitative work in this area. First, the existence of a Markovian competitive equilibrium is obtained in an enlarged space of state variables. Our choice of the marginal utility values of assets returns is dictated by computational considerations. This is a minimal addition to the state space to restore existence of a Markovian equilibrium and with the property that the extra added variables enter linearly into the Euler equation. Second, the algorithm iterates in a space of candidate equilibrium sets – rather than in a space of functions. Iteration over candidate equilibrium sets guarantees convergence to the fixed-point solution even if Markov equilibria are not continuous. Moreover, we also establish some desirable approximation properties of the computed solutions. And third, the algorithm provides a reliable method for model simulation. We resort to a further discretization of the equilibrium law of motion so that it becomes a Markov chain. It should be stressed that the usual simulation over a continuum of values cannot be justified on theoretical grounds: The simulated moments may fail to converge to the set of moments of the invariant distributions of the model. Other ways to restore laws of large numbers for the simulated paths of these economies would be by imposing monotonicity assumptions on the equilibrium dynamics [Santos (2008)] or by expanding artificially the noise process [e.g., Blume (1979)]. These latter approaches seem to be of more limited economic interest.

Of course, our methods have to face some computational challenges. Iteration over sets is computationally much more costly than iteration over functions. Therefore, the expansion of the state space along with iteration over sets should certainly be manifested into an additional computational burden. Besides, our general convergence results lack error bounds. This lack of accuracy should be expected since our models cannot be restated as optimization programs, and miss some common concavity, differentiability and contractive properties. In terms of numerical implementation the innovative techniques for error estimation proposed by Judd, Yeltekin, and Conklin (2003) seem to be of limited application for our economies. These authors use outer and inner approximations over convex sets. It is not clear to us that an outer approximation over convex sets will converge to the convex hull of the equilibrium correspondence. Moreover, inner convex approximations may be hard to find. Still, these techniques may work well in some applications.

There are several directions in which our analysis can be extended. For example, in the preceding sections we considered exogenous short-sales constraints and exogenous borrowing limits. We could incorporate borrowing constraints that depend on future income [e.g., Miao and Santos (2005)]. These general borrowing schemes arise in financial models and in the modeling of the public sector so as to allow for various types of fiscal policy rules. In most quantitative studies of recursive equilibrium with fiscal policy, the government must balance the budget in each state of the world. This is a rather strong assumption. Another extension is to the area of policy games. As our algorithm includes all the shadow values of investment, it can deal with heterogeneity and market frictions. For example, we can generalize the model of Phelan and Stacchetti (2001) to include heterogeneous agents and various types of financial frictions.

Appendix A

Computation algorithm to stationary equilibrium

Given the parameter values as shown in the text, I compute the stationary equilibrium as follows:

Step 1. Discretize the state space $S = (i_h, x, h, i_{ma}, \varepsilon)$.

Step 2. Start with an arbitrary pair of the steady state values of aggregate labor supply E, tax rate τ_{mr}, bequest B, and EHI premium π_E. Define $\Theta = \{E, \tau_{mr}, B, \pi_E\}$. Compute the value w.

Step 3. Agents solve their optimization problem.

Step 4. Simulate the economy:

4.1. Set $t = 0$, there are N_{ppl} agents live in the economy, who are randomly assigned the values of $(i_h, x, h_{g-1}, i_{ma}, \varepsilon_g)$ if young or middle-aged, and $(i_h, x, h_{g-1}, \varepsilon_g)$ if retired.

4.2. Given shocks agents choose whether to insure, how much to save, and how much to spend;

4.3. New period starts, $t = t + 1$, $g = g + 1$, the government collects the assets left behind by the accidentally deceased.

4.4. A sequence of time series is generated by repeating step 4.2 & 4.3;

4.5. Store the distribution of $\left\{(i_h, x, h_g, i_{ma}, \varepsilon_g, in_g)\right\}_{g=1}^{3}$ with $\{\Psi_g\}_{g=1}^{3}$;

4.6. Stop the process if the economy enters the stationary distribution.

Step 5. Compute the insurance premium π_E^{new}, aggregate labor supply E^{new}, bequest

B^{new}, and tax rate τ_{mr}^{new} based on the distribution $\{\Psi_g\}_{g=1}^{3}$ according to equations (2.19), (2.16), (2.21), and (2.18). Denote $\Theta' = \{E^{new}, \tau_{mr}^{new}, B^{new}, \pi_E^{new}\}$.

Step 6. Find the fixed point of Θ by iteration. If $\|\Theta' - \Theta\| > \delta$, set $\Theta = \frac{(\Theta+\Theta')}{2}$ and return to step 3. Otherwise set $\Theta^* = \Theta'$ and define

$$c_g = G_{c_g}(in, i_h, x, h_{g-1}, i_{ma}, \varepsilon_g; \Theta^*) \tag{A.1}$$

$$l_g = G_{l_g}(in, i_h, x, h_{g-1}, i_{ma}, \varepsilon_g; \Theta^*) \tag{A.2}$$

$$m_g = G_{m_g}(in, i_h, x, h_{g-1}, i_{ma}, \varepsilon_g; \Theta^*) \tag{A.3}$$

$$a_g = G_{a_g}(in, i_h, x, h_{g-1}, i_{ma}, \varepsilon_g; \Theta^*) \tag{A.4}$$

$$in_g = G_{in}(i_h, x, h_{g-1}, i_{ma}, \varepsilon_g; \Theta^*) \tag{A.5}$$

Appendix B

Calibration

Table B.1: Health shocks by age group

Age	Shock 1	Shock 2
25-44	-0.5	-10.0
45-64	-2.5	-10.0
65-85	-10.0	-20.0

Table B.2: Coverage ratio for each expenditure grids

bin	1	2	3	4	5
$q_E(p_m m)$	0.55487	0.61017	0.65671	0.70503	0.78060
$q_{ma}(p_m m)$	0.76524	0.81319	0.85763	0.88673	0.94784
$q_{mr}(p_m m)$	0.49942	0.57952	0.63345	0.69578	0.77799

Table B.3: Parameter values in the coverage ratio functions

	q_E	q_{ma}	q_{mr}
β_0	0.63632(0.00144)	0.83671(0.00353)	0.51344(0.00416)
β_1	0.05444(0.00079)	0.02315(0.00165)	0.03223(0.00266)
β_2	0.00546(0.00371)	0.00349(0.00067)	0.01477(0.00094)
R^2	0.0863	0.0475	0.1634

Table B.4: Parameters of the model

Parameter	Description	Values
n	population growth rate	1.25%
$\{a_\rho, b_\rho\}$	parameters in survival probability	$\{0.35895, 1.0\}$
β	discount factor	0.97
$\gamma_{2,g}$	preference on leisure	$\{1.3, 0.85, 1.3\}$
$\gamma_{3,g}$	preference on health	$\{0.05, 0.5, 2.5\}$
η	relative risk aversion over health	1.35
$\{A_m, \vartheta\}$	health production	$\{1.96, 0.52\}$
ξ	parameter in health on labor	0.1393
ε_g	health shock	see table B.1
δ_h	health depreciation	see text
p_m	price for medical service	see text
A	total factor productivity	8.0
α	labor share	0.66
r	interest rate	4%
$\{b_0, b_1, b_2\}$	income tax parameters (progressive part)	$\{0.258, 0.768, 0.716\}$
τ_y	income tax parameter (proportional part)	10%
τ_c	consumption tax	5.67%
τ_{mr}	medicare tax	2.5%
G	government expenditure	27.5% of GDP
$q_{ma}(\cdot)$	Medicaid coverage rate	see text
π_{ma}	Medicaid premium	see text
$q_{mr}(\cdot)$	Medicare coverage rate	see text
π_{mr}	Medicare premium	see text
$q_E(\cdot)$	private insurance coverage rate	see text
π_E	private insurance premium	see text

Appendix C

Numerical results

Table C.1: Data vs. model

Parameters	Data	Benchmark
All insured (in % of non-elderly)	82	84.8
w/ Private insurance (in % of non-elderly)	71.1	72.5
w/ Medicaid (in % of non-elderly)	12.9	12.3
Health Expenditures (in % of GDP)	15.8	16.6
Labor supply (in % of total time)	33.3	30.6
Ratio of retired to active population (in %)	19.2	18
Marginal income tax at 10% quantile	15	20
Marginal income tax at 50% quantile	26	25.4
Marginal income tax at 99% quantile	35	27
Medicare tax (in %)	2.9	2.5
Ave. insurance premium (in % of per capita GDP)	10.9	10.1
Size of Medicaid & Medicare (in % of GDP)	4.6	4.8
Consumption and health expenditure profiles		see figure 2.2
Gross saving rate (in %)	21	25.8

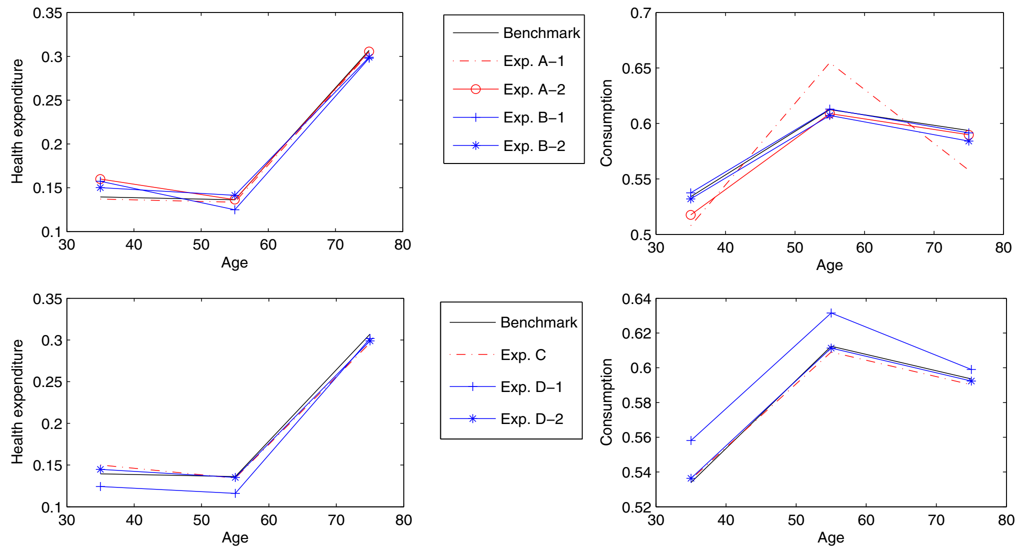

Figure C.1: Life cycle profiles of health expenditure and consumption

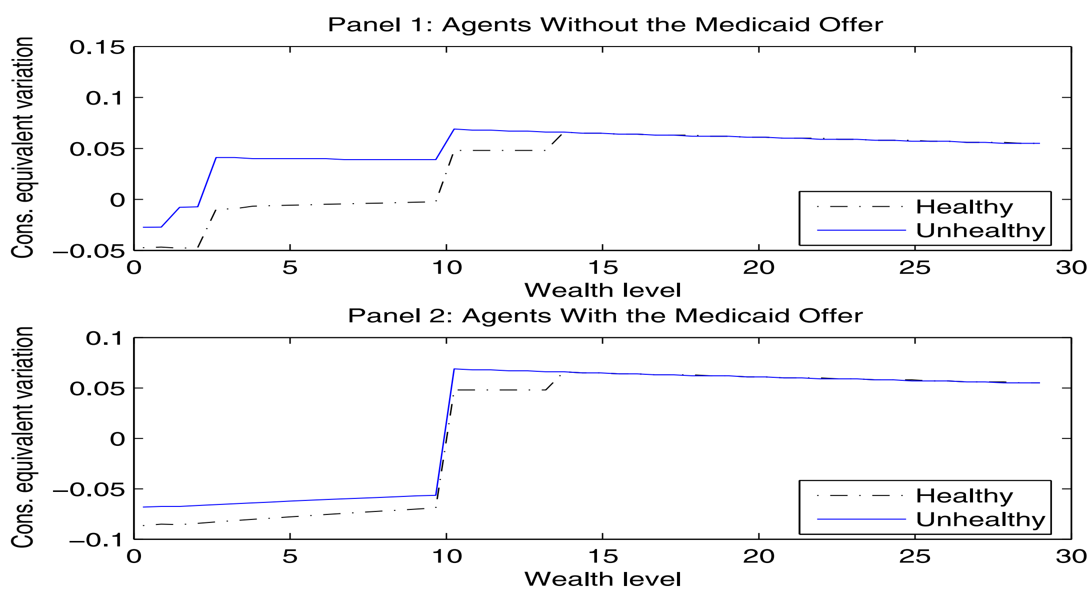

Figure C.2: Welfare effects of reform A-1

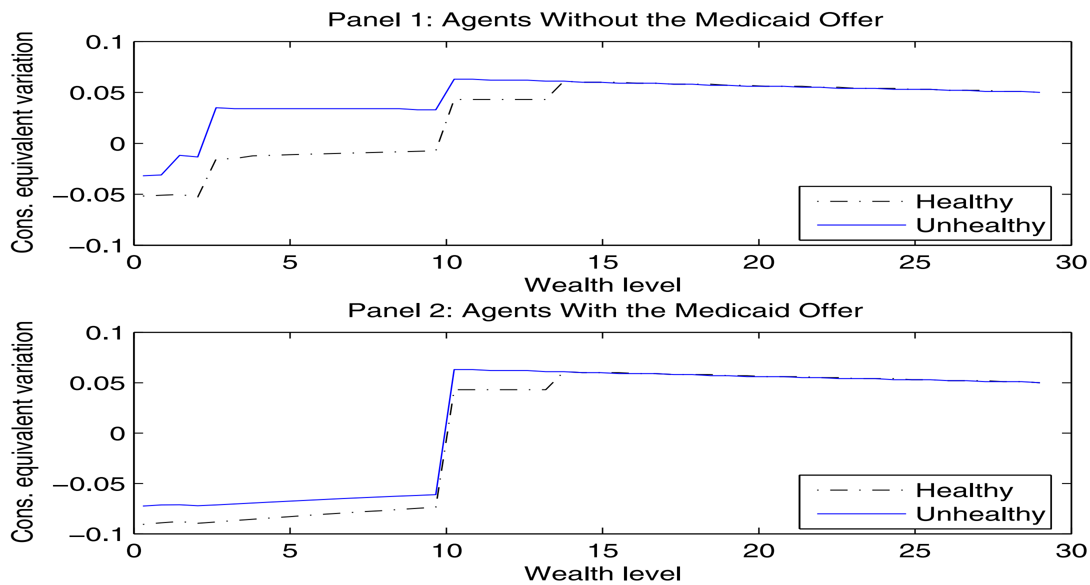

Figure C.3: Welfare effects of reform A-2

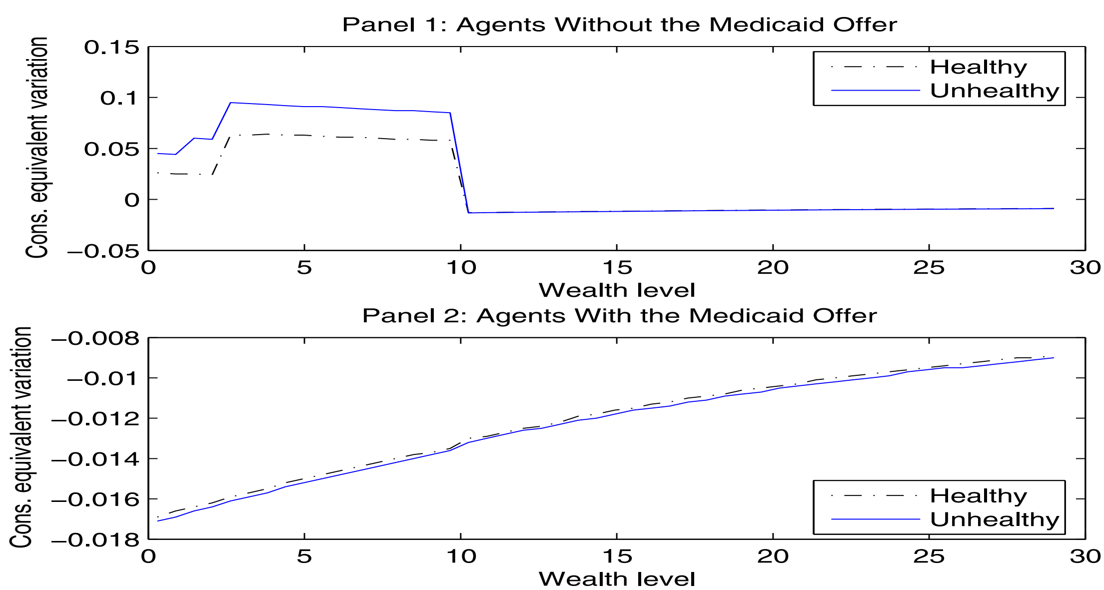

Figure C.4: Welfare effects of reform B-1

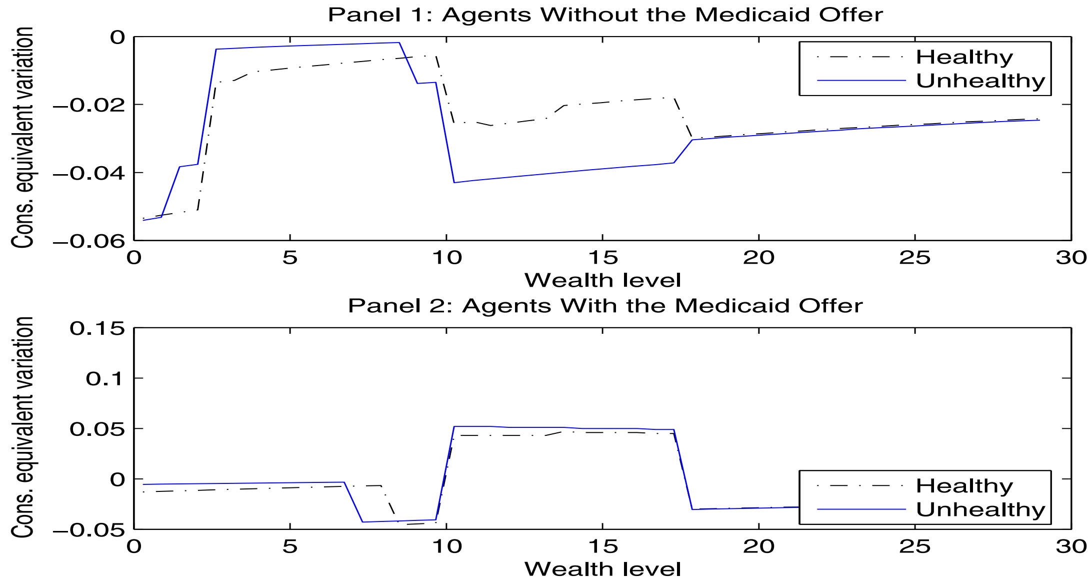

Figure C.5: Welfare effects of reform B-2

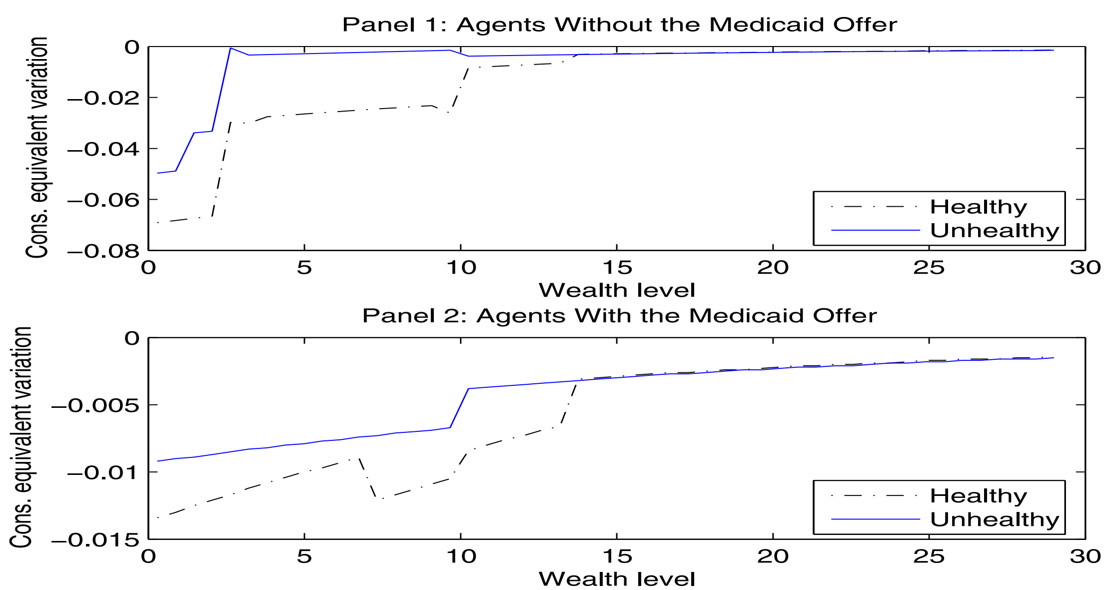

Figure C.6: Welfare effects of reform C

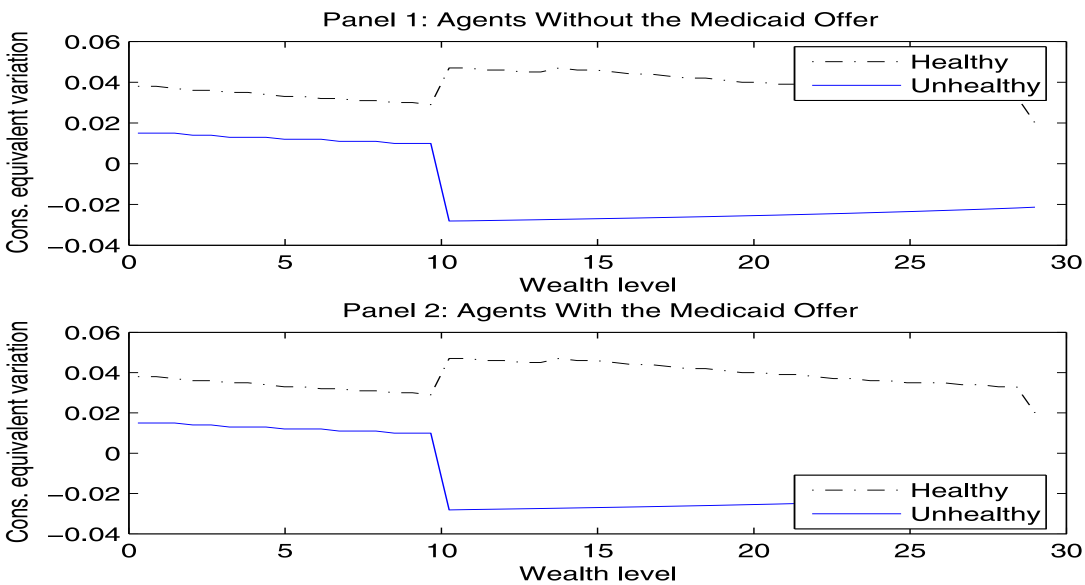

Figure C.7: Welfare effects of reform D-1

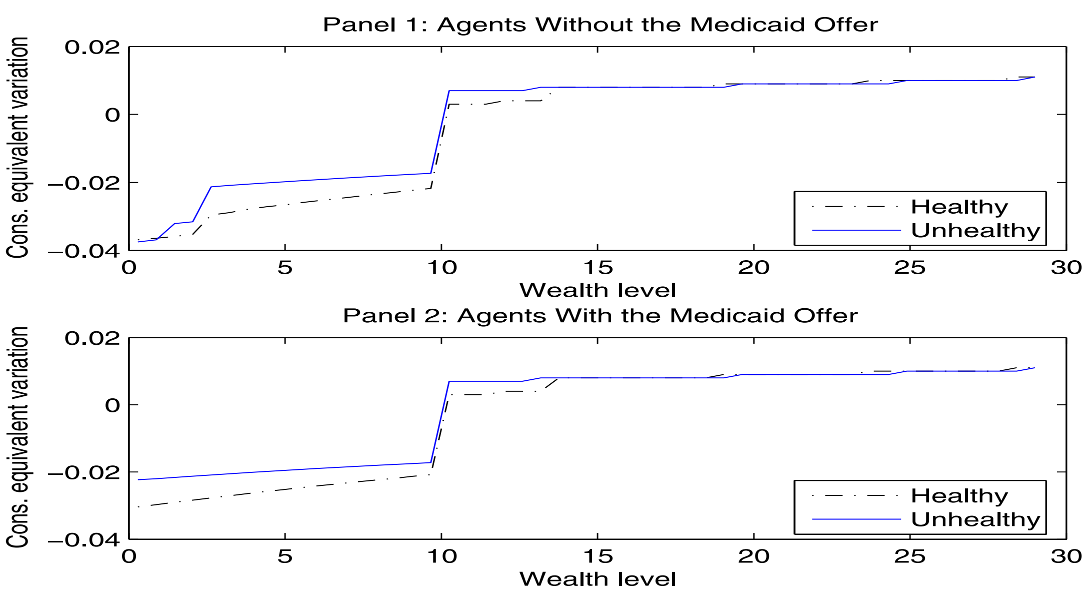

Figure C.8: Welfare effects of reform D-2

Appendix D

Proofs

In this Appendix we prove all results formally stated in Section 3 and Proposition 4. All remaining results follow from similar arguments.

Proof of Proposition 1 (Sketch): As in the original work of Bewley (1972), the existence of a SCE can be established by approximating the infinite-horizon economy by a sequence of finite economies. This is the strategy followed by Jones and Manuelli (1999), but their proof is incomplete and does not apply to sequential competitive economies. As is usual in this approximation argument the hardest part of the proof is to provide upper bounds for equilibrium allocations and prices over all the finite-horizon economies. We nevertheless skip this part since these bounds follow from the proof of Proposition 2 below.

Hence, following Jones and Manuelli (1999), we consider the following steps for the proof of a SCE: (i) Existence of an equilibrium for a finite horizon economy. This result is covered by the general proofs of existence of competitive equilibria for economies with taxes and externalities of Arrow and Hahn (1971), Mantel (1975), and Shafer and Sonneschein (1976). (ii) Uniform bounds for equilibrium allocations and prices of finite-horizon economies. As already pointed out, these bounds can be established by the method of proof of Proposition 2. (iii) Existence of SEC as a limit point of finite equilibria. The preceding steps (i) and (ii) guarantee that there is a collection of vectors

$\left(c_t(z^t), k_{t+1}(z^t), K_t(z^t), L_t(z^t), \overline{K}_t(z^t), w_t(z^t), r_t(z^t)\right)$ that can be obtained as a limit of equilibria of finite economies. It is obvious that for such limiting solution the market clearing conditions must be satisfied at each z^t, and that one period-profits must be maximized. Moreover, for each agent i the limiting allocation $\left(c_t^i(z^t), k_{t+1}^i(z^t)\right)$ must satisfy the sequence of budget constraints (2), and it is optimal since the discounted objective (1) is continuous in the product topology over the set of feasible consumption plans $\left(c_t^i(z^t)\right)_{t\geq 0}$ which are preferred to the endowment allocation $\left(e_t^i(z_t)\right)_{t\geq 0}$. This is because feasible consumption plans $\left(c_t^i(z^t)\right)_{t\geq 0}$ are bounded above (see below) and the endowment process $\left(e_t^i(z_t)\right)_{t\geq 0}$ is bounded below by a positive quantity.

Proof of Proposition 2: We first show that there are positive constants K^{max} and K^{min} such that for every equilibrium sequence of physical capital vectors $\left(k_{t+1}^i(z^t))\right)_{t\geq 0}$ if $K^{max} \geq \sum_{i=1}^I k_0^i(z^0) \geq K^{min}$ then $K^{max} \geq \sum_{i=1}^I k_t^i(z^{t+1}) \geq K^{min}$ for all z^t. The existence of K^{max} follows directly from Assumptions 2 and 3. In particular, A is bounded by Assumption 2, and by Assumption 3 the marginal productivity of capital converges to zero as K goes to ∞ for every fixed $L > 0$. Also, it obvious that $K^{min} \geq 0$. We now claim that if $\lim_{K\to 0} D_1F(K,L) = \infty$ for some given positive L, then $K^{min} > 0$. For if not, there is a sequence of equilibrium capitals $\left(k_{t+1}^i(z^t)\right)_{t\geq 0}$ such that $\sum_{i=1}^I k_t^i(z^{t+1})$ is arbitrarily close to 0 for some z^{t+1}. Under the system of budget constraints (2), it follows that there is an arbitrarily small number $\varepsilon \geq 0$ such that $c_t^i(z^t) \geq e_t^i(z^t) - \varepsilon$ for every i. Therefore, modulo an arbitrarily small number the derivative $D_1u(c_t^i(z^t), z_t)$ is bounded by $D_1u(e_t^i(z^t), z_t)$, and $D_1F(K_t, L_t)$ is arbitrarily large. These latter two conditions together are not compatible with utility maximization, since the existence of K^{max} implies that future consumption $c_t^i(z^r|z^t)$ for $r > t$ is uniformly bounded. Consequently, if $\lim_{K\to 0} D_1F(K,L) = \infty$ for some positive L, then $K^{min} > 0$.

Since L takes on a finite number of positive values, our bounds K^{max} and K^{min} imply that there are constants r^{max} and w^{max} such that for every equilibrium sequence of factor prices $\left(r_t^i(z^t), w_t^i(z^t)\right)_{t\geq 0}$ we have $0 \leq r_t(z^t) \leq r^{max}$ and $0 \leq w_t(z^t) \leq w^{max}$ for all z^t. Hence, the value function $J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}))$ is well defined, and as already pointed out the derivative $D_1 J^i(\cdot, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}))$ is continuous in $(k_0^i, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}))$. Moreover, by a simple notational change it follows from (2) that function J^i can be rewritten as $J^i(a_0^i(z_0), z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}))$, where $a_0^i = e_0^i(z_0) + r_0 k_0^i$. Then we can conclude that $0 \leq D_1 J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0})) \leq \gamma$, since $e_0^i(z_0)$ is bounded below by a positive number, and as shown above all feasible vectors $(k_0^i, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}))$ lie in a compact set.

Proof of Theorem 1: For the proof of Theorem 1, we shall invoke the following version of Bellman's equation.

$$J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0})) = \max u^i(c_0^i(z_0), z_0) + \beta E[J^i(k_1^i(z^1), z^1, \mathbf{r_1}(\mathbf{z^1}), \mathbf{w_1}(\mathbf{z^1}))]$$

$$\text{s. t.} \qquad k_1^i(z_0) + c_0^i(z_0) = r_0(z_0)\, k_0^i(z_0) + w_0(z_0)\, l_0^i(z_0) + e_0^i(z_0),$$
$$k_1^i(z^t) \geq 0,\ k_0^i \ \text{ given.}$$

We now divide the proof into three parts:

(i) $V^* \subset B(V^*)$: This part essentially follows from (BE). Since $m_0^i = D_1 J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}))$ is the derivative of the value function, and (5)-(6) in the definition of operator B provide necessary conditions for utility maximization. Conditions (7)-(10) are also satisfied in every SCE.

(ii) $B(V^*) \subset V^*$: This is the sufficiency part; again, the most difficult step of the proof follows from (BE). More specifically, since value function $J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}))$ is concave in k_0^i and $m_0^i = D_1 J^i(k_0^i, z_0, \mathbf{r_0}(\mathbf{z_0}), \mathbf{w_0}(\mathbf{z_0}))$, conditions (5)-(7) imply that (BE) holds. But by the well-known arguments of dynamic programming the Bellman equation (BE) implies that the original sequential optimization problem (SOP) attains a global solution. Hence, constrained utility maximization in the definition of SCE is satisfied. Conditions (8)-(9) imply profit maximization, and condition (10) implies market clearing.

(iii) *V^* is the largest fixed point of B.* First note that by the same arguments as in the proof of Proposition 2 we can show that every fixed-point solution $\hat{W} = B(\hat{W})$ is a compact correspondence. Now, as in Section 3 we may consider a measurable selection $f(k,z,m)$ for every (k,z,m) in the graph of $\hat{W}$. Let $(k_{t+1}(z^t),\ z^{t+1},\ m_{t+1}(z^{t+1}),\ c_t(z^t),\ r_t(z^t),\ w_t(z^t),\ \lambda_t(z^t),\ \delta_t(z^t)) = f(k_t(z^{t-1}), z_t, m_t(z^t))$ for each $z^{t+1}|z^t$. Then we claim that $(c_t(z^t), k_{t+1}(z^t), r_t(z^t), w_t(z^t))_{t\geq 0}$ is a SCE. Indeed, operator B is compact, and hence the sequence of factor prices $(r_t(z^t), w_t(z^t))_{t\geq 0}$ is bounded. Moreover, individual consumptions and capital holdings $\left(c_t^i(z^t), k_{t+1}^i(z^t)\right)_{t\geq 0}$ are bounded, and the sequence of shadow values of investment $\left(m_{t+1}(z^{t+1})\right)_{t\geq 0}$ is bounded. Along these sequences, the Euler equations and the budget constraints are satisfied for every agent i, and so the individual (SOP) attains a global maximum [e.g., see Rincon-Zapatero and Santos (2009)]. Also, Conditions (8)-(9) imply profit maximization, and condition (10) implies market clearing. It follows that every selection f generates a SCE. Therefore, $\hat{W} = V^*$.

Proof of Theorem 2: Let $\hat{W} = \bigcap_n W_n$. Hence, $\hat{W} = \{(k,z,m) : m \in B^n(W)(k,z) \text{ for every } n\}$. Consequently, $B(\hat{W}) = \{(k,z,m) :$ There are some continuation values (k_+, z_+, m_+) such that $m_+ \in B^n(W)(k_+, z_+)$ for every $n \geq 1$ and all $z_+ \in \mathbf{Z}\}$.

It follows that $B(\hat{W}) \subset \{(k,z,m) : m \in B^{n+1}(W)(k,z) \text{ for every } n \geq 1\}$. Therefore, $B(\hat{W}) \subset \hat{W}$ as the sequence $(W_n)_{n\geq 0}$ is decreasingly monotone. We next show that $\hat{W} \subset B(\hat{W})$.

Consider any (k,z,m) that belongs to the graph of $\hat{W}$. Hence, $m \in B^n(W)(k,z)$ for every n. Let Ψ_n be the set of continuation values (k_+, z_+, m_+) of (k,z,m) such that $m_+ \in B^{n-1}(W)(k_+, z_+)$. This set is non-empty and compact, and so $\Psi = \bigcap_n \Psi_n$ is not empty. Consequently, for every (k,z,m) that belongs to the graph of $\hat{W}$, there exists a non-empty set of continuation values (k_+, z_+, m_+) that belong to the the graph of $\hat{W}$. This proves that (k,z,m) belongs to the graph of $B(\hat{W})$, and so $\hat{W} \subset B(\hat{W})$.

We thus obtain that $\hat{W} = B(\hat{W})$. Finally, by the monotonicity of B the assumed condition $V^* \subset W$ implies $V^* \subset B^n(W)$ for every n. Hence, $V^* \subset \hat{W}$. Moreover, $\hat{W} \subset V^*$ since V^* is the largest fixed point of B. We thus obtain $V^* = \hat{W}$.

Proof of Theorem 2.1: Let $V_0 \supset V_0^*$, let $V_n = B(V_{n-1}), n \geq 1$.

Consider, $V_N^U = \cup_{n=N}^{\infty} V_n$. Then $V_{N+1}^U = B(V_N^U)$ and $V_{N+1}^U \subset V_N^U$ for all $N \geq 1$. It follows that the sequence $\{V_N^U\}$ must converge to a non-empty set V^{*U}.

Moreover, $V^{*U} = B(V^{*U})$, since $V^{*U} = \cap_{N=n} V_N^U$, for all $n \geq 1$. It is easy to see that $V^{*U} = V_t^*$, Indeed, by the monotonicity of operator B we get that V^{*U} is a fixed point that contains V^*, and $V^{*U} \subset V^*$ since every fixed point conforms an equilibrium – given that the transversality conditions are trivially satisfied in this model.

To complete the proof of the theorem, just note that $V^* \subset V_n^* \subset V_n^U$ for all $n \geq 1$. Since we have already established that $V_n^U \to V^*$, we get that $V_n \to V^*$.

Proof of Theorem 3: This follows trivially from part (iii) of the proof of Theorem 1.

Proof of Theorem 3.1: (i) Obvious. Operator $B^{h,N}$ is monotone, $V_0 \supseteq V^*$ and $B^{h,N}(V^*) \supset V^*$.

(ii) Proof follows similar arguments as in proof of Theoreom 2.1. Actually, it is possible that $V_n^{h,N} \subset V^{*,h,N}$.

(iii) Note that operator $B^{h,N}$ varies continuously with h and N. Hence, the fixed point of $B^{h,N}$ is an uppersemicontinuous correspondence on parameter values h and N. Since $V^* \subset V^{*,h,N}$, it follows that $V^{*,h,N} \to V^*$ as $h \to 0$ and $N \to \infty$.

Proof of Theorem 3.2: The proof follows directly from Blume (1982), Theorem 2.1. The sequence of operators $\{B^{h,N,A_\gamma}\}$ converges to B. Moreover, the convexfication of operator B^{cv} has a fixed point $\mu^* \in B^{cv}(\mu^*)$.

Proof of Proposition 4: This result is proved along the lines of Levine and Zame (1996) and Magill and Quinzii (1994). For every agent i the objective in (1) satisfies Assumption (A.2) of Santos and Woodford (1997). Hence, for every optimal consumption-portfolio plan $\{c_t^i(z^t), \theta_{t+1}^i(z^t)\}_{t\geq 0}$ the sequence of portfolio values $\{\theta_{t+1}^i(z^t)q_t(z^t)\}_{t\geq 0}$ is bounded above, and a uniform bound can be found that applies to all equilibrium sequences of asset prices $\{q_t(z^t)\}_{t\geq 0}$. Since there is a finite number I of agents and one unit of the asset, equilibrium condition (18) implies the existence of the lower bound $-M$.

The second part of the proposition is proved by contradiction. Note that the dividend process $(d_t(z_t))_{t\geq 0}$ is bounded below by a positive number. Also, by the argument above all equilibrium sequences of asset prices $\{q_t(z^t)\}_{t\geq 0}$ are bounded above. By a similar argument, it is easy to see that $\{q_t(z^t)\}_{t\geq 0}$ is bounded below by a positive number. Hence, it follows from (19) that any initial small debt $\theta_0 < 0$ that is rolled over at every period, it will grow to $-\infty$. Indeed, at every z^t the debt θ_{t+1} must be incremented to pay for the dividend $d(z^t)$, and these negative increments can be bounded uniformly. Since every sequence of asset prices $\{q_t(z^t)\}_{t\geq 0}$ is bounded below, the value $\{q_t(z^t)\theta_{t+1}(z^t)\}_{t\geq 0}$ must also converge to $-\infty$. Now, by the definition of $\underline{\pi}_{e^i}(z^t)$, if (22) is violated at some date-event z^t, it means that the debt cannot be repaid in finite time, and hence it must grow without bound along some history of date-events $\{z^r|z^t\}_{t\geq 0}$. The proposition is thus established.

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